Essays on Monetary and Fiscal Policy

Raffaele Rossi

Submitted in fulfilment of the requirements for the

Degree of Doctor of Philosophy

Department of Economics

Faculty of Law, Business and Social Sciences

University of Glasgow

2009

Abstract

This thesis is composed by four chapters on New Keynesian macroeconomics.

Chapter 1 develops a small New Keynesian model augmented with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC henceforth) as in Galí et al. (2004, 2007). This chapter focuses on the consequences for the design of monetary and fiscal rules, of the bifurcation on the demand side of the economy generated by the presence of ROTC, in the absence of Ricardian equivalence. When fiscal policy follows a balanced budget rule, the share of ROTC determines whether an active and/or a passive monetary policy in the sense of Leeper (1991) guarantees determinacy. When a short run public debt asset is introduced, the amount of ROTC determines whether equilibrium determinacy requires a mix of active (passive) monetary policy and a passive (active) fiscal policy or a mix where both policies are active or passive.

Chapter 2 studies the equilibrium determinacy of a New Keynesian model augmented with trend inflation, public debt and distortionary taxation. Both the level of long run inflation as well as the stock of steady state public debt have to be explicitly taken into consideration for the characterisation of the equilibrium dynamics between monetary and fiscal policy.

Chapter 3 considers the implications of external habits for optimal monetary policy in an otherwise standard New Keynesian model, when those habits exist at the level of individual goods as in Ravn et al. (2006). External habits generate an additional distortion in the economy, which implies that the flex-price equilibrium will no longer be efficient and that policy faces interesting new trade-offs and potential stabilisation biases. The endogenous mark-up behaviour, which emerges with deep habits, significantly affects the optimal policy response to shocks and the stabilising properties of standard simple rules.

Chapter 4 analyses both optimal monetary and fiscal policy in a New Keynesian model augmented with deep habits and valuable government spending. We find that, in

line with the general consensus in the macro literature, fiscal policy adds very little to optimal monetary policy as a stabilisation device.

Contents

List of Figures

Acknowledgements

I would like to express my gratitude to my supervisors, Professor Campbell Leith and Doctor Ioana Moldovan for their guidance, advice, wisdom and superb supervision. Moreover, I would like to thank my mentors at the European Central Bank, Massimo Rostagno and Leopold von Thadden for their support and the participants to the 4th European Macroeconomic workshop, the department seminar series at the University of Glasgow, University of Milan Bicocca, for their comments on the chapters of this thesis.

Special thanks to Dr. Lucy Reynolds, without whom most of this would have been impossible.

Declaration

The material contained in this thesis has not been previously submitted for a degree in this or any other university.

Preface

This thesis is composed of four chapters on New Keynesian macroeconomics. We use this introduction to describe the common features of these chapters, the methodological techniques adopted and for a review of the literature.

The purpose of this thesis is twofold. First, it aims to study the equilibrium determinacy of two New Keynesian (NK henceforth) models in which Ricardian equivalence of fiscal policy does not hold. Second, it analyses the optimal policy problem in a basic NK model where households and government are affected by consumption habits.

The NK models integrate Keynesian elements such as imperfect competition and nominal rigidities, into a dynamic general equilibrium framework that until the early '90s was largely associated with the Real Business Cycle (RBC henceforth) school.

In contrast to the traditional Keynesian models, i.e. the textbook *IS-LM* framework, the dynamic general equilibrium approach implies that the equilibrium conditions for aggregate variables are derived from the optimal behaviour of economic agents, i.e. all agents face well-defined decision problems and behave optimally, and are consistent with the simultaneous clearing of all markets.

In its basic formulation, a NK model ignores the endogenous variations in the capital stock[1] and features one nominal rigidity modelled as a constraint on the firms' ability to optimally reset their prices.[2] Despite the existence of several other popular methods of modelling this feature[3] we adopt the Calvo (1983) price setting mechanism throughout

[1]Throughtout this thesis we follow McCallum and Nelson (1999) and Cogley and Nason (1995). They argue that the response of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by the NK models and that, at least for the US data, there is little evidence of correlation between capital stock and output at business cycle frequencies.

[2]Note that in its basic formulation the NK model postulates that nominal wages are allowed to fluctuate freely.

[3]See for example the staggered-overlapping contracts as in Taylor (1980) or the quadratic adjustment cost as in Rotemberg (1982).

this thesis. This implies that in each period only an exogenous fraction of firms can optimally reset their prices, while the rest have to keep their prices unchanged. This constraint throws light on particular features of the nature of inflation dynamics. Firms re-setting their prices today recognise that the prices they choose are likely to stay in place for more than one period, and are unresponsive to developments within the period. Therefore firms find it optimal, when making their current pricing decisions, to take into account their expectations regarding future cost and demand conditions. This implies that changes in the aggregate price level are a consequence of current pricing decisions, and therefore it follows that inflation has an important forward looking component. This property appears clearly reflected in the so called New Keynesian Phillips curve (NKPC henceforth). Furthermore, this nominal rigidity introduces a source of monetary non neutralities, which creates an explicit role for monetary policy: changes in the nominal interest rate have real effects on the economy.

These characteristics yield a NK framework which has strong and sound theoretical foundations, yet a simple and straightforward analytical tractability and it is useful for exploring a number of policy issues. For this reason this approach has gained increasing fame in both theoretical and empirical macroeconomics over the last decade as a benchmark specification for policy analysis.

In this thesis we extend the basic NK model in several directions in order to analyse different macro-policy issues.

First we study the problem of equilibrium determinacy. In so doing, we postulate the behaviour of economic policy by assuming that the policy makers commit to simple rules. This allows us to explicitly derive the conditions under which these rules ensure the equilibrium to be determinate in the sense of Blanchard and Khan (1983). Following Blanchard and Khan (1983), we write the dynamic model in matrix form as

$$AE_t x_{t+1} = Bx_t \tag{0.1}$$

where x_t of size $n \times 1$ is a vector representing the model's endogenous and exogenous variables. A and B are square matrix of size $n \times n$. Let us define $J = A^{-1}B$, m as the number of non-predetermined variables in x, $n - m$ the number of predetermined variables in x and q the number of eigenvalues of J that are greater than one in absolute value, i.e. explosive eigenvalues. If $q = m$, the system is determinate (determinacy). In other words the solution to (0.1) is unique and converges to the steady state for any given initial state of the economy. If $q < m$ there are an infinite number of solutions to (0.1), the system is therefore indeterminate (indeterminacy). Ultimately, if $q > m$ there is no solution to (0.1) and the system is unstable (instability).

Second we study optimal policy problems. For this part we follow the *utility-based welfare analysis* of Woodford (2003). This technique allows one to analyse the welfare consequences of alternative policies, and can thus be used as the basis for the design of an optimal (or, at least, desirable) policy.

Part 1, chapters 1 and 2: determinacy analysis and the interactions between monetary and fiscal policy.

The analysis of the properties of macro-policy rules has been one of the central themes of the recent literature on monetary and fiscal policy (Leeper, 1991, Taylor, 1993, Galí et al., 1999, 2004, Leith and Wren-Lewis, 2000 Schmitt-Grohe and Uribe, 1997) . This field of research has shown that simple rules seem to explain relatively well the observed policy choices as well as their role in different macroeconomic episodes. While this point of view is widely shared, most of the literature makes convenient assumptions, i.e. a fiscal policy which implies Ricardian equivalence, that allows monetary and fiscal policy rules to be studied separately. However, these assumptions are often questionable, and therefore it has been argued that the resulting conclusions of this approach could be misleading.

The main criticism of this approach is that it ignores the impact of monetary policy on the government's finances and in turn ignores the consequences that different types of fiscal policy may have on the conduct of monetary policy. In fact, there are several ways in which monetary policy can affect the government's budget constraint and in turn the conduct of fiscal policy and *vice versa.* Typical examples are the seigniorage problem, the relationship between debt service costs and inflation stabilisation, the size of the tax base and the need for fiscal transfers when prices are sticky. Leith and Wren-Lewis (2000), Linnemann (2006), Davig and Leeper (2006) and Schmitt-Grohe' and Uribe (2007) are some of the recent authors that point out how the assumptions regarding the interactions between monetary and fiscal policy are of crucial importance in understanding macro-policy rules.

In particular, a common point of all these works is that, when, for any reason, Ricardian equivalence does not hold, fiscal policy cannot be recursively separated by the rest of the model and the equilibrium dynamics are determined by a genuine interaction between monetary and fiscal policy, see inter alia Leith and von Thadden (2008).

The traditional benchmark results of this field of research are the following: a) an active monetary policy, i.e. a monetary policy which reacts to inflation raising the real interest rate, delivers a unique rational expectation equilibrium if and only if fiscal policy adopts a *passive* tax policy role, i.e. it raises tax revenues when public debt rises. However, if fiscal policy does not adopt a tax policy which implies public debt stabilisation-active fiscal policy- a fiscal policy that responds to increases in public debt cutting the tax revenues- monetary policy has to abandon the Taylor principle, embracing a passive role. A passive/passive policy mix delivers indeterminacy while an active/active policy mix implies instability, i.e. no solution. This result can be found in Leeper (1991) in a simple maximising model with money in the utility function and lump-sum taxes. Leith and Wren-Lewis (2000) and Linnemann (2006) have similar results in a NK model. b)the first type of regime (active monetary/passive fiscal) is more likely to deliver low inflation and a sustainable path for public debt. c) periods of passive monetary policy can substantially alter the propagation mechanism of the shocks to the fundamentals, Lubik and Schorfeide (2004).

In the first two chapters of this thesis we extend these benchmark results in two different NK models where Ricardian equivalence does not hold.

In Chapter 1 we study the consequences for the equilibrium dynamics of the interactions between monetary and fiscal policy rules in a basic NK model with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC) as in Galí et al. (2004, 2007) and Bilbiie (2008). These consumers, who are not allowed to participate in financial markets, i.e. they cannot hold public debt in order to smooth consumption over time, but consume their available labour income in each period, stand next to standard forward looking agents. From this, and independently of the tax instrument adopted, lump-sum taxes or proportional labour income taxation, the presence of ROTC implies a clear departure from Ricardian equivalence: both types of consumer pay the burden of public debt but only the optimisers benefit from it. Hence public debt becomes net wealth and therefore a relevant state variable which has to be taken into account for the equilibrium dynamics of the system. In particular the aim of this chapter is to study the consequences for the design of monetary and fiscal policy rules of the bifurcation on the demand side of the economy, see for example Bilbiie et al. (2004) and Bilbiie (2008), generated by the presence of ROTC.

In Chapter 2 we study the interactions between monetary and fiscal policy rules in a NK model augmented with trend inflation, as in, for example, Ascari and Ropele (2007, 2009), a steady state level of public debt and a fiscal policy which levies a proportional labour income tax. As in the previous chapter, due to the distortive nature of fiscal policy, Ricardian equivalence does not hold and the equilibrium dynamics are determined by genuine interactions between monetary and fiscal policy. The aim of this paper is to explicitly analyse the role of trend inflation on the setting of monetary and fiscal policy rules.

Part 2, chapters 3 and 4: optimal monetary and fiscal policy.

In recent years the NK framework has been largely used for normative policy analysis, i.e. optimal policy. Within this set of models, computing optimal policy means a specific use of the policy instruments in order to maximise a well defined objective function, given frictions in the economic environment and the behaviour of the economic agents. To this extent, a recurrent assumption in the optimal policy literature is the one of the *benevolent policy maker,* see for example Ramsey (1927) and Lucas and Stokey (1983).[4] This implies that the policy maker uses the utility, i.e. the welfare, of the households as the objective function in the maximisation process. This approach to optimal policy is generally defined as *utility-based welfare analysis,* Galí (2001).

The fact that NK models are based on the optimal behaviour of the economic agents and are consistent with the simultaneous clearing of all markets is of fundamental importance for this approach to optimal policy. Indeed, the utility-based welfare analysis in such models is conceptually straightforward because the preferences of private agents, which are connected in the structural relations that determine the effects of alternative policies, provide a natural welfare criterion. Furthermore, in the context of sticky-price models with monopolistic competition, the utility-based approach to welfare analysis not only allows the evaluation of different policies (mostly in terms of optimal policy), but also helps in quantifying the welfare costs of the various forms of real or nominal rigidities.

There are several approaches to computing optimal policy in a NK model. Yun (2005) constructs optimal monetary policy as a Ramsey problem. He maximises the utility functions subject to the structural equations in non-linear form. Schmitt-Grohe' and Uribe (2004, 2005, 2007*a*, 2007*b*) study optimal policy as a second order approximation to the exact Ramsey problem, i.e. they approximate to the second order around the non-stochastic steady state both the utility function and the structural equations of the model and then they compute the maximisation problem. Woodford (2001) analyses

[4]Although Ramsey (1927) and Lucas and Stokey (1983) do not consider a NK economy, their works are pioneering in the utility based optimal policy literature.

optimal policy maximising the second order approximation to the utility function of the representative consumer subject to the log-linear approximations of the structural equation around the non-stochastic steady state.

The first two techniques can be applied to a broad range of models without relying on ad-hoc assumptions regarding the steady state. However they can be used only for the solution under commitment (time inconsistent policy). Furthermore, they often lack a straightforward analytical solution.

On the other hand, the validity of the technique proposed by Woodford (2001), often referred as the *linear quadratic approach*, relies on particular assumptions, i.e. small steady state distortions, small shocks, no capital accumulation, but can be used both for commitment solutions as well as for the solution under discretion (time consistent policy). Furthermore it is often possible with the linear quadratic apparatus to find an analytical solution to the optimal policy problem.

In Chapters 3 and 4 we analyse optimal monetary policy using the linear quadratic approach as in Woodford (2001), in a NK model augmented with habit formation.

Traditionally, the basic NK model has been augmented with habit formation in order to capture the hump-shaped output response and the persistency in inflation and consumption, to changes in monetary policy one typically finds in the data.

The habits effects can either be internal (see for example, Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), Leith and Malley (2005)) or external (see, for example, Smets and Wouters (2007)) the latter reflecting a *catching up with the Joneses* effect, whereby households fail to internalise the externality their own consumption causes on the utility of other households.

Both forms of habits behaviour can help the New Keynesian monetary policy model capture the persistence found in the data (see, for example Kozicki and Tinsley (2002)), although the policy implications are likely to be different. More recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of habits behaviour, which they label as 'deep'. Deep habits occur at the level of individual goods rather than at the level of an aggregate consumption basket ('superficial' habits). While this distinction does not affect the dynamic description of aggregate consumption behaviour relative to the case of superficial habits, it does render the individual firms' pricing decisions intertemporal and, in the flexible price economy considered by Ravn, Schmitt-Grohe, and Uribe (2006), can produce a counter-cyclical mark-up which significantly affects the responses of key aggregates to shocks.

In Chapter 3 we extend the benchmark sticky-price NK economy to include deep external habits in consumption. This implies that there is an externality associated with fluctuations in consumption which implies that the flexible price equilibrium will not usually be efficient, thereby creating an additional trade-off for policy makers, which may give rise to further stabilisation biases if policy is constrained to be time consistent. We also consider the implications of habit formation effects for the nature of simple policy rules. The ability of policy to influence the time profile of endogenously determined mark-ups can significantly affect the monetary policy stance and how it differs across discretion and commitment and across different exogenous shocks.

In Chapter 4 we extend the policy analysis conducted in Chapter 3 with an endogenous fiscal policy. This manifests in the model under the form of endogenous government spending that, entering in the utility function of the representative consumer, is valuable from a Social Planner point of view. Furthermore, we assume that as private consumption, public spending is also affected by external deep habits formation. This setting allows us to characterise both the optimal fiscal and monetary policy under private and public deep habit formation.

CHAPTER 1

Designing monetary and fiscal policy rules in a New Keynesian model with rule-of-thumb consumers

This chapter develops a small New Keynesian model augmented with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC henceforth) as in Gali'et al. (2004; 2007). The paper focuses on the consequences for the design of monetary and fiscal rules, of the bifurcation generated by the presence of ROTC on the demand side of the economy, in the absence of Ricardian equivalence. We find that, when fiscal policy follows a balanced budge rule, the amount of ROTC determines whether an active and/or a passive monetary policy in the sense of Leeper (1991) guarantees determinacy. When short run public debt assets are introduced, the amount of ROTC determines whether equilibrium determinacy requires a mix of active (passive) monetary policy and a passive (active) fiscal policy or a mix where policies are both active or passive. This set of equilibria has the potential to explain the empirical evidence on the U.S. postwar data on monetary and fiscal policy interactions.

1.1. Introduction

The analysis[1] of the properties of macro-policy rules has been one of the central themes of the recent literature on monetary and fiscal policy. This field of research has shown that simple rules seem to explain relatively well the observed policy choices as well as their role in different macroeconomic episodes. While this point of view is widely shared, most of the literature makes convenient assumptions, i.e. a fiscal policy which implies *Ricardian* equivalence, that allows monetary and fiscal policy rules to be studied

[1]I am grateful for useful comments to Florin Bilbiie, Campbell Leith, and Ioana Moldovan, as well as all the participants at the 4th European Macroeconomic Workshop and at seminars at Glasgow and Milan-Bicocca Universities.

separately. However, these assumptions are often questionable, and therefore it has been argued that the resulting conclusions of this approach could be misleading. Leith and Wren-Lewis (2000), Linnemann (2006), Davig and Leeper (2006) and Schmitt-Grohe' and Uribe (2007) are some of the recent works that point out how the assumptions regarding the interactions between monetary and fiscal policy are of crucial importance in understanding macro-policy rules. In particular, a common point of all these works is that, when, for any reason, Ricardian equivalence does not hold, fiscal policy cannot be recursively separated from the rest of the model and the equilibrium dynamics are determined by the interactions between monetary and fiscal policy.

In this paper we augment a standard New Keynesian (NK) model with a steady state level of public debt and a share of rule-of-thumb consumers (ROTC) as in Galí et al. (2004, 2007). These consumers, who are not allowed to participate in financial markets, i.e. they cannot hold public debt in order to smooth consumption over time, but consume their available labour income in each period, stand next to standard forward looking agents (OPTC). From this, and independently of the tax instrument adopted, lump-sum taxes or proportional labour income taxation, the presence of ROTC implies a clear departure from Ricardian equivalence: both types of consumers pay the burden of public debt but only the optimisers benefit from it. Hence public debt becomes net wealth, therefore a relevant state variable which has to be taken into account for the equilibrium dynamics of the system.

While the behavior that we assume for rule-of-thumb consumers is admittedly simplistic (and justified only on tractability grounds), we believe that their presence captures an important aspect of actual economies which is missing in conventional models. Empirical support of non-Ricardian behavior among a substantial fraction of households in the U.S. and other industrialized countries can be found in Campbell and Mankiw (1989). It is also consistent, at least prima facie, with the findings of a myriad of papers rejecting the permanent income hypothesis on the basis of aggregate data.

Moreover, as stressed in the literature (Galí et al.; 2004, Di Bartolomeo and Rossi; 2007, Colciago; 2008, Bilbiie, 2008), the introduction of a set of ROTC can drastically change the determinacy conditions of an otherwise standard NK model. On this subject the main contribution can be found in Bilbiie (2008). The author shows that in a NK model with no capital accumulation, a Walrasian labour market and no fiscal policy, the presence of a share of ROTC may generate a bifurcation in the conduct of monetary policy. In particular, with a small share of ROTC, the traditional results on equilibrium determinacy hold: necessary and sufficient condition for determinacy is to have, using Leeper's (1991) definition, an active monetary policy, whereby nominal interest rate is adjusted such that the real rate increases in response to positive inflation. However, when the share of ROTC is above a specified threshold, determinacy requires a passive monetary policy, whereby nominal interest rate is adjusted such that the real rate decreases in response to positive inflation.

The basic intuition for this result is that when the monetary authority increases the interest rate, the system experiences downward pressure on wages, that are, by assumption, fully flexible.[2] This, combined with a sticky price environment, implies an increase in profits which are held only by the optimiser consumers (OPTC henceforth). With a high share of ROTC, the increase in OPTC wealth caused by the increase in profits may generate an increase in total demand, putting, via the Phillips curve, upward pressure on prices. A monetary authority wishing to stabilise the price level may therefore need to cut the real interest rate in the face of an inflationary shock.

The main contribution of this paper is to study the bifurcation effect generated by the presence of ROTC on the interactions between monetary and (a non Ricardian) fiscal policy.

To this end we conduct several exercises. We start by studying the equilibrium dynamics of the interactions between monetary and fiscal policy. We assume that monetary

[2]Colciago (2008) shows that in a NK model with ROTC and sticky wages the Taylor principle could be restored through an ad hoc monetary policy rule.

policy adopts a contemporaneous interest rate rule which is a function only of the inflation rate, i.e. a Taylor rule as in Clarida et al. (2000), and fiscal policy adjusts the labour income tax rate in every period in order to generate enough revenues to pay a level of public spending and service the long run level of public debt, without releasing short run public debt assets.

This type of fiscal rule, commonly known as *balanced budget rule,* has been studied in detail by Schmitt-Grohe and Uribe (1997) in a Real Business Cycle model with capital accumulation, and by Linnemann (2006) in a NK model with a contemporaneous monetary rule and no capital accumulation. While both works stress the destabilising role of such a fiscal rule, given the NK elements of our model, we use Linnemann's (2005) results as a benchmark for ours.

He finds that with a balanced budget rule, an active monetary policy rule that reacts "*too strongly*" to inflation leads easily to the possibility of self fulfilling expectations, i.e. indeterminacy. In other words, in Linnemann's (2006) model, monetary policy has an upper limit in its active strength, and this upper limit is tighter the higher the long run level of public debt.

This result is a direct consequence of the distortive nature of fiscal policy and its interaction with monetary policy: if monetary policy increases the real interest rate in order to contrast higher inflation expectations, via a traditional reduction of current output through the demand channel, the burden of the service of public debt increases, therefore forcing fiscal policy to increase taxation in order to collect extra revenues. This increase in taxation feeds back on the endogenous variables of the model, inflation and output, via the supply side of the economy, the Phillips curve, generating a positive wedge between tax rate and current inflation which could make the initial expectations of higher inflation self fulfilling, generating endogenous sunspots fluctuations. In our paper we show that even with a small share of ROTC, the upper bound on monetary policy gets looser, in turn helping to reestablish the validity of the Taylor principle. This is because a small proportion of ROTC strengthens the validity of the Taylor principle or, in other words, it increases the sensitivity of aggregate demand to interest rate movements. Hence monetary policy can reduce output to the desired level to contrast inflation with lower movements in interest rates, therefore generating a weaker fiscal response, avoiding sunspot fluctuations.

Furthermore, we find that, when the share of ROTC is above a specified threshold similar to the one found by Bilbiie (2008), both a strongly passive or a strongly active monetary policy can lead to equilibrium determinacy. As described above, a passive monetary policy, through its effect on aggregate profits and financial portfolio, can reduce aggregate demand and, *ceteris paribus*, decreases the cost of servicing the public debt, avoiding the perverse effect of an increase in the tax rate on current inflation. On the other hand, a strong active monetary policy can expand aggregate demand. While higher output can have a destabilising effect on inflation stabilisation, it increases, *ceteris paribus,* government revenues, potentially implying a decrease in the tax rate and this, via the Phillips curve, can act as stabilisation device, leading to determinacy.

Next we assume a more general fiscal policy rule in which the fiscal authority is allowed to release short run public debt assets in order to balance its budget. This type of fiscal policy, jointly with a traditional interest rate type of monetary rule, allows us to analyse the equilibrium dynamics of our model under the active/passive logic of Leeper (1991).

The traditional benchmark results of this field of research are the following: a) an active monetary policy delivers a unique rational expectation equilibrium if and only if fiscal policy adopts a passive tax policy role, i.e. it raises tax revenues when public debt rises. However, if fiscal policy does not adopt a tax policy which implies public debt stabilisation- active fiscal policy- monetary policy has to abandon the Taylor principle, embracing a passive role. A passive/passive policy mix delivers indeterminacy while an active/active policy mix implies instability, i.e. no solution. b)the first type of regime (active monetary/passive fiscal) is more likely to deliver low inflation and a sustainable path for public debt.c) periods of passive monetary policy can substantially alter the propagation mechanism of the shocks to the fundamentals, Lubik and Schorfeide (2004).

However, as pointed by Favero and Monacelli (2005) and by Davig and Leeper (2006), the active/passive policy logic is not able to capture the macro-evidence of the US post-war data on monetary and fiscal policy regimes. Indeed the empirical investigations in these papers show long periods of policy regime mixes, i.e. both policies active or both passive, which are incompatible with the traditional results of the literature on monetary and fiscal policy interactions. While Favero and Monacelli (2005) remain completely agnostic on a possible theoretical explanation of their findings, Davig and Leeper (2006) explain the unconventional policy mixes resulting from the data with the introduction of macro-policy switches. They show that a standard New Keynesian model, where in each period macro policies have a probability of switching from active to passive and this probability is taken into account by the agents, is able to deliver a unique rational expectation equilibrium for any policy combination.

The results we present in this paper could be considered as complementary to the ones of Davig and Leeper (2006). In particular we find that when the share of ROTC is below the threshold previously described, determinacy requires either an active monetary policy jointly with a passive fiscal one or *viceversa*. When instead the share of ROTC is above the threshold, determinacy requires for monetary and fiscal policy to be both either active or passive.

Intuitively, this result is driven by the consequences of a share of ROTC on the demand side of the economy. Suppose, for example, that our system is affected by a large share of ROTC so that we are above the threshold previously described. When fiscal policy adopts a debt stabilisation policy, i.e. passive fiscal policy, monetary authority is free to stabilise inflation. As shown by Bilbiie (2008) and Leith and von Thadden (2008) this is ensured by a passive monetary policy. If instead fiscal policy follows an active role, monetary policy has to abandon the inflation stabilisation policy, adopting an active role.

The remainder of the paper proceeds as follows: section 1.2 derives the model, section 1.3 outlines the results, section 1.4 conducts some robustness analysis with a more general specification of the monetary policy rules and different fiscal arrangements, and section 1.5 concludes.

1.2. The model

The economy consists of two types of households, a continuum of firms producing differentiated goods in a monopolistic competitive-sticky price environment, a perfectly competitive labour market, a central bank in charge of monetary policy and a government in charge of fiscal policy.

The totality of households is normalised to unity. Of this, a fraction $(1-\lambda)$, with $\lambda \leq 1$, behaves in a traditional forward-looking, optimising way. Hence they maximise their (infinite) lifetime utility, hold profits coming from the monopolistic nature of the goods market, and participate in perfect and complete financial markets. We define the remaining λ households as rule-of-thumb consumers (ROTC) as in Galí *et al.*(2004, 2007). They care only for their current disposable income and they hold no financial assets nor any profit shares. For these consumers all their wealth is represented by their after tax wages and therefore they cannot smooth consumption over time. Variables with the suffix o and r indicate OPTC and ROTC respectively. A variable without time index identifies its steady state value.

1.2.1. Optimisers

The (lifetime) OPTC utility function has a standard form and it simply includes consumption and labour

$$U_t^o = E_0 \sum_{t=0}^{+\infty} \beta^t u^o \left(C_t^o, N_t^o\right) \tag{1.1}$$

where $\beta \in (0,1)$ is the discount factor, E_t is the rational expectations operator, $u^o(\cdot,\cdot)$ represents instantaneous utility. We assume, in line with most of the literature, that

$\frac{du^o}{dC_t^o} > 0$ and $\frac{du^o}{dN_t^o} < 0$. The shape of u^o is[3]

$$u^o\left(C_t^o, N_t^o\right) = \log C_t^o - \theta \frac{\left(N_t^o\right)^{1+\eta}}{1+\eta} \tag{1.2}$$

where C_t^o is the level of consumption of the OPTC, N_t^o is the OPTC labour supply. The parameter θ, with $\theta \in (0, \infty)$ indicates how leisure is valued relative to consumption. The parameter $\eta > 0$ is the inverse of the Frisch elasticity of labour supply and represents the risk aversion to variations in leisure.

The nominal OPTC flow budget constraint is

$$\int_0^1 P_t(j)\, C_t^o(j)\, dj + R_t^{-1} \frac{B_{t+1}}{1-\lambda} + \frac{E_t\left(Q_{t,t+1} V_{t+1}\right)}{1-\lambda} = \begin{bmatrix} W_t N_t^o \left(1-\tau_t\right) + \frac{D_t}{1-\lambda} + \\ + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t S^o \end{bmatrix} \tag{1.3}$$

where $P_t(j)$ is the price level of the variety of good j, W_t is the nominal wage, D_t are the nominal profits coming from the monopolistic competitive structure of the goods market, B_{t+1} is the nominal payoff of the one period risk-less bond purchased at time t, R_t is the gross nominal return on bonds purchased in period t, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoff and V_t is nominal payoff of a state-contingent asset portfolio.[4] The government is assumed to pay a level of public spending, G_t and the service of debt, levying a proportional labour income tax, τ_t. S^o is a steady state transfer such that at steady state the two types of agents have the same level of consumption and supply the same amount of labour.

OPTC must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. This, combined with the CES Dixit-Stiglitz aggregator, results in a

[3]We assume this shape of the utility function in order to make our results comparable with the existing literature on ROTC, i.e. Bilbiie et al.(2004), Gali' et al.(2007), Bilbiie(2008), Leith and Von Thadden(2008).

[4]Note that given the definition the OPTC, $V_{t+1}^o = \frac{V_{t+1}}{1-\lambda}$. The same holds for bonds and profits.

demand function for any single good that is downward sloping in the current price of the specific j good.

$$C_t^o(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t^o$$

where the price index is found by

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$$

at the optimum we have

$$\int_0^1 P_t(j)\, C_t^o(j)\, dj = P_t C_t^o \tag{1.4}$$

where the parameter ε represents the elasticity of substitution among goods and it is a measure of the market power held by each firm.

The budget constraint can be therefore rewritten as

$$P_t C_t^o + R_t^{-1}\frac{B_{t+1}}{1-\lambda} + \frac{E_t(Q_{t,t+1}V_{t+1})}{1-\lambda} = W_t N_t^o(1-\tau_t) + \frac{D_t}{1-\lambda} + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t S^o \tag{1.5}$$

Next the OPTC have to decide their labour supply and their intertemporal consumption allocation. This problem involves maximising the utility (1.1) subject to the budget constraint (1.5). The first order condition for the intertemporal consumption allocation is

$$\beta\left(\frac{C_t^o}{C_{t+1}^o}\right)\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\left[\left(\frac{C_t^o}{C_{t+1}^o}\right)\left(\frac{P_t}{P_{t+1}}\right)\right] = 1 \tag{1.6}$$

Where $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is implied by the non arbitrage condition. This expression is the familiar *Euler equation* for consumption. It describes the desire to smooth consumption over time once the opportunity cost implied by the real interest rate has been taken into account. The first order condition with respect to labour states that the *marginal rate of substitution* between labour and consumption must be equal to the after tax real wage

$$\theta \left(N_t^o\right)^\eta C_t^o = \frac{W_t}{P_t}\left(1-\tau_t\right) \tag{1.7}$$

From the last expression one can see that taxation distorts the leisure-consumption choice. Any change in the tax rate has a direct effect on real wage and therefore on the marginal rate of substitution between consumption and labour.

1.2.2. Rule of Thumb Consumers

The ROTC utility function is represented by a single period expression. In particular, following Galí *et al.*(2004, 2007), it is assumed that the shape of the *instantaneous* utility is the same for the two types of consumer. Therefore

$$U_t^r = \log C_t^r - \theta \frac{\left(N_t^r\right)^{1+\eta}}{1+\eta} \tag{1.8}$$

As stressed above, the ROTC do not participate in financial markets and do not hold any profits. Their budget constraint can be expressed as follows

$$\int_0^1 P_t\left(j\right) C_t^r\left(j\right) dj = W_t N_t^r \left(1-\tau_t\right) - P_t S^r \tag{1.9}$$

Where $C_t^r\left(j\right)$ and N_t^r are the level of consumption of each j product and the labour supply of the ROTC. Furthermore, it is assumed that similarly to the behaviour of the OPTC, the ROTC exploit any relative price differences in creating their consumption basket. Hence, at the optimum

$$P_t C_t^r = \int_0^1 P_t\left(j\right) C_t^r\left(j\right) dj \tag{1.10}$$

On the consumption side the ROTC are forced to consume all their income in each period, therefore consumption can easily be inferred by combining (1.9) with (1.10). The first order condition for the optimal supply of labour implies

$$\theta \left(N_t^r\right)^\eta C_t^r = \frac{W_t}{P_t}\left(1-\tau_t\right) \tag{1.11}$$

The last two expressions state the ROTC *"hand to mouth"* attitude towards consumption. This means that they consume in every period all their resources which, as previously stated, are equal to their after tax labour income. The optimal supply of labour takes the same analytical form as that of the OPTC.

1.2.3. Firms

In this economy, firms are assumed to possess an identical production technology. This production function is linear in labour and can be written as

$$Y_t\left(j\right) = N_t\left(j\right) \tag{1.12}$$

Furthermore, it is worth noting that each firm faces the following demand function

$$Y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\varepsilon} Y_t \tag{1.13}$$

where

$$Y_t = \left[\int_0^1 Y_t\left(j\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.14}$$

Following the NK literature it is assumed that prices are sticky. We model this feature of the economy following Calvo (1983). In each period there is a (randomly selected) set of firms, $(1-\alpha)$ with $\alpha < 1$, who reset their price optimally, while the remaining α keep their prices fixed. When a firm is allowed to reset its prices, it takes into account the expected future stream of profits discounted for the probability of not resetting its prices.

In particular the maximisation problem of a price setter can be written in real terms as

$$\max_{P_t^*(j)} E_t \sum_{i=0}^{+\infty} \alpha^i q_{t,t+i} \left(\left(\frac{P_t^*(j)}{P_{t+i}} \right) Y_{t+i}(j) - mc_{t+i} Y_{t+i}(j) \right) \tag{1.15}$$

Where $q_{t,t+1} = \beta \left(\frac{C_t^o}{C_{t+1}^o} \right)$ is the real stochastic discount factor and $mc_t = W_t / P_t$ represents the real marginal costs. The first order condition with respect to $P_t^*(j)$ is

$$\frac{P_t^*(j)}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o} \right) \left(mc_{t+i} (P_{t+i})^{\varepsilon} Y_{t+i} \right)}{E_t \sum_{i=0}^{+\infty} \alpha^i \beta^i \left(\frac{C_t^o}{C_{t+i}^o} \right) (P_{t+i})^{\varepsilon} P_{t+i}^{-1} Y_{t+i}} \tag{1.16}$$

while the price level follows

$$P_t^{(1-\varepsilon)} = \left[(1-\alpha) P_t^{*(1-\varepsilon)} + \alpha P_{t-1}^{(1-\varepsilon)} \right] \tag{1.17}$$

1.2.4. Aggregation rules and market clearing condition

The aggregate expressions for consumption and labour are simply the weighted average of the single consumer type variables. Therefore aggregate consumption follows

$$C_t = \lambda C_t^r + (1-\lambda) C_t^o \tag{1.18}$$

and aggregate labour

$$N_t = \lambda N_t^r + (1-\lambda) N_t^o \tag{1.19}$$

In the absence of capital accumulation, everything produced must be consumed in the same period. Furthermore each product j can be purchased by the private sector or by the government

$$Y_t(j) = C_t(j) + G_t(j) \tag{1.20}$$

In aggregate, given the price dispersion implied by Calvo price setting

$$Y_t s_t = N_t \tag{1.21}$$

where $s_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$. Given our assumption of zero steady state inflation, fluctuations of s_t around the steady state are of second-order importance[5], and therefore can be ignored in the present analysis which employs a linearised framework. In equilibrium total demand is equal to total supply and therefore

$$Y_t = C_t + G_t \tag{1.22}$$

1.2.5. The Government

The government uses labour income tax revenues, $N_t \tau_t W_t$ to finance a stream of public spending, $P_t G_t$[6], and the service of public debt. Therefore the government budget constraint can be expressed as

$$R_t^{-1} B_{t+1} = B_t - \tau_t W_t N_t + P_t G_t \tag{1.23}$$

where $P_t G_t - \tau_t N_t W_t$ is the primary deficit. The government budget constraint can be expressed in real terms as

$$R_t^{-1} b_{t+1} = \frac{b_t}{\pi_t} - \tau_t w_t N_t + G_t \tag{1.24}$$

where $b_{t+1} = \frac{B_{t+1}}{P_t}$, $w_t = \frac{W_t}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$.

1.2.6. Monetary Policy

Monetary policy sets the nominal interest rate, R_t, in every period. Following the literature on monetary policy, for example Clarida *et al.* (2000), we approximate monetary

[5]A detailed discussion of this can be found in Woodford (2003).

[6]As the private sector, the government exploits any price differences in the market to form its consumption basket G_t. This jointly with a CES aggregator gives the following downward sloping demand function for each single public spending good. $G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} G_t$

policy by a simple Taylor rule of the type

$$R_t = R(\pi_t)^{\phi_\pi} \tag{1.25}$$

Where $R = \frac{1}{\beta}$ is the steady state interest rate.[7] The single policy parameter ϕ_π in (1.25) is the Taylor coefficient, as discussed in the literature on interest rate rules inspired by Taylor (1993). Accordingly, following Leeper (1991), monetary policy is called *active* (or *passive)* if the nominal interest rate, R_t, rises more (or less) than one-for-one with the current inflation rate, i.e. if $\phi_\pi > 1$ ($\phi_\pi < 1$).

1.2.7. Fiscal Policy

Regarding fiscal policy, we assume a government revenue rule of the type

$$\tau_t = \delta_0 + \delta_1 \frac{\tau}{b}(b_t - b) + \delta_2 \frac{\tau}{Y}(Y_t - Y) \tag{1.26}$$

where $\delta_0 = \frac{(1-\beta)b+G}{wN}$ and δ_1 and δ_2 are policy parameters identifying the relative weight given to debt stabilisation and output stabilisation. This fiscal rule has the characteristic of being steady state neutral (at steady state the fiscal rule collapses to $\tau = \frac{(1-\beta)b}{wN} + \frac{G}{wN}$ which is equal to $\tau = \delta_0$).

Unlike monetary policy, there is no widely accepted specification for fiscal policy. The rule we assume is similar to the one considered in Linnemann (2006), Davig and Leeper (2006, 2007) and Schmitt-Grohe and Uribe (2007). This type of rule has two main advantages. The first is that it allows the study of the interactions between monetary and fiscal policy under the logic of Leeper's(1991).[8] Second is that these rules are receiving

[7]A variable without time index refers to its steady state value.

[8]Following the definition of Leeper (1991), we call the fiscal rule (1.26) *passive* if $\delta_1 > \left(\frac{1}{\beta} - 1\right)$, i.e positive fiscal response to increase in public debt from its steady state value, while it is *active* in the opposite case of $\delta_1 < \left(\frac{1}{\beta} - 1\right)$.

particular attention from an empirical point of view, given their ability to capture many stylised fiscal facts of US postwar data.[9]

Several special cases of fiscal policy will be specified and discussed in detail below. One prominent example is a fiscal policy which follows a balanced budget rule, i.e. no short run public debt fluctuations, in the fashion of Schmitt-Grohe and Uribe (1997) and Linnemann (2006). In this case fiscal policy has to collect enough revenues to repay the cost of public debt and a level of government spending. Its specification derives directly from (1.24) in which one has to impose that $b_t = b\ \forall t$. It can be described as

$$\tau_t w_t N_t = G_t + b\left(\frac{1}{\pi_t} - \frac{1}{R_t}\right) \tag{1.27}$$

1.2.8. Equilibrium

The non linear structural equations of the model are log-linearised around the non stochastic steady state.[10] Furthermore, we present the model in terms of aggregate variables. These equations are: the New Keynesian Phillips curve (NKPC)[11]

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\left(\frac{1}{\gamma_c} + \eta\right)\widehat{Y}_t - \frac{(1-\gamma_c)}{\gamma_c}\widehat{G}_t + \frac{\tau}{1-\tau}\widehat{\tau}_t\right) \tag{1.28}$$

where $\gamma_c = \frac{C}{Y}$, the dynamic IS curve augmented for the presence of ROTC

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \Theta\left(\frac{1-\gamma_c}{\gamma_c}\right)\left(\widehat{G}_{t+1} - \widehat{G}_t\right) - \Theta\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{1.29}$$

We define Θ as the elasticity of the demand side of the economy to changes in real interest rate. This parameter is defined as $\Theta = \left(\frac{1}{\gamma_c} - \eta\frac{\lambda}{1-\lambda}\right)^{-1}$.

[9]See inter alia Perotti (2007).

[10]Algebrical details are provided in the appendix of this chapter. We impose, through a transfer defined in the appendix, that the two agents have the same level of consumption and supply the same level of labour at steady state. Hence the heterogeneity between the two consumers is only along the business cycle.

[11]Note that $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$. This notation is innocuous since we assume no trend inflation.

The market clearing condition,

$$\widehat{Y}_t = \gamma_c \widehat{C}_t + (1-\gamma_c)\,\widehat{G}_t \tag{1.30}$$

the monetary policy rule,

$$\widehat{R}_t = \phi_\pi \pi_t \tag{1.31}$$

and the fiscal policy, described by the government budget constraint and the tax rule when public debt is allowed to fluctuate along the business cycle as

(1.32)

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left\{\widehat{b}_t - \pi_t - \frac{\tau w}{\gamma_b}\left[\left(\frac{1}{\gamma_c}+\eta+1\right)\widehat{Y}_t + \frac{1}{1-\tau}\widehat{\tau}_t\right] + \left(\frac{1-\gamma_c}{\gamma_b} + \frac{\tau w\,(1-\gamma_c)}{\gamma_b\gamma_c}\right)\left(\widehat{G}_t\right)\right\}$$

$$\widehat{\tau}_t = \delta_1 \widehat{b}_t + \delta_2 \widehat{Y}_t \tag{1.33}$$

or simply by the (log-linearised) government budget constraint where $\widehat{b}_t = 0\ \forall t$ in the case of balanced budget fiscal policy

(1.34)

$$\widehat{R}_t + \frac{1}{\beta}\left\{\left(\frac{1-\gamma_c}{\gamma_b} + \frac{\tau w\,(1-\gamma_c)}{\gamma_b\gamma_c}\right)\left(\widehat{G}_t\right) - \pi_t - \frac{\tau w}{\gamma_b}\left[\left(\frac{1}{\gamma_c}+\eta+1\right)\widehat{Y}_t + \frac{1}{1-\tau}\widehat{\tau}_t\right]\right\} = 0$$

A few points are worth stressing. Firstly, this model displays a clear departure from the so called *Ricardian* equivalence of fiscal policy. Both types of consumer pay the burden of public debt, but only the optimisers benefit from it, holding public debt assets. Therefore public debt is net wealth and, independently of how it is financed, it implies a wealth transfer from the ROTC to the OPTC. Moreover, fiscal policy levies a proportional labour income tax, which distorts the marginal rate of substitution between consumption and leisure. This feeds back directly into the NKPC via the labour supply, i.e. a higher tax rate induces OPTC to substitute leisure from the future to the present, lowering labour supply, increasing the firms' real marginal cost, and thus generating a positive wedge between the tax rate and inflation. These properties of the model, together with the non neutral effects of monetary policy due to sticky prices, imply that: a) the government budget constraint cannot be separated from the rest of the model, i.e. government debt turns into a relevant state variable which needs to be accounted in the analysis of local equilibrium dynamics, b) that equilibrium dynamics are driven by a genuine interaction of monetary and fiscal policy.

Secondly, the presence of ROTC dramatically affects the dynamic IS equation (1.29), i.e. the demand side of the economy, via Θ, the elasticity of the aggregate demand to changes in real interest rate. This parameter is linked in a non-linear way to λ, the share of ROTC, and to η, the inverse of the Frisch elasticity of labour. Both the size and the sign of Θ can potentially alter the transmission mechanism and local determinacy properties of the model. The intuition for this result is as follows. Assume the monetary authority suddenly increases the real interest rate. This increase shifts downward the consumption of the optimisers, through the usual intertemporal Euler equation channel. This, *ceteris paribus*, generates a reduction in labour demand and therefore in nominal wages. The reduction in wages lowers firms marginal costs. Consequently prices fall, via the NKPC. Due to the *Walrasian* structure of the labour market and to the Calvo price mechanism, nominal wages decrease more than prices, implying as a result lower real wages.

Furthermore, the form of the utility function, i.e. log-consumption, together with the assumption of no capital accumulation and the shape of the tax structure, causes the ROTC to supply labour inelastically[12] and therefore to pass through their consumption any change in real wage. This is not all. The asymmetric decrease in wages and prices, i.e real wages decrease more than real prices, generates an increase in profits. Note that the OPTC hold all the financial activities present in the system, i.e. profits share and public debt bonds. In particular they hold $(1-\lambda)^{-1}$ of total firms share. If, for

[12]Although this assumption simplifies the algebra and the economic mechanism behind our results, it does not drive them. This is shown when other types of fiscal arrangment are introduced.

example, profits increase by one unit, dividend income of asset holders (OPTC) increases by $\frac{1}{1-\lambda} > 1$ units. The same thing is true for public debt bonds: a unit of increase in the real return of public debt generates a $\frac{1}{1-\lambda} > 1$ increase in the optimisers' wealth.[13] These financial effects work in the opposite direction relative to the traditional intertemporal Euler equation: while the latter imply a contractionary effect of higher real interest rate, the opposite is true for the former.

As argued by Bilbiie et al. (2004) and Bilbiie (2008), the sign of Θ determines which of these two channels prevails. Of course, the sign of Θ depends on the share of ROTC, i.e. the higher λ, the higher the financial channel of interest rate, and on the elasticity of labour supply (of the OPTC), i.e. the higher η, the higher the sensibility of real wage to interest rate movements.[14] A necessary condition for $\Theta > 0$ is

$$\lambda < \frac{1}{(1 + \eta\gamma_c)} \tag{1.35}$$

Figure 1.1 sketches the sign of Θ for a given value of γ_c in the $(\lambda - \eta)$ space. As one can see, Θ remains positive for combinations of high values of the Frisch elasticity of labour supply, i.e. low η, and high shares of ROTC, i.e high λ, or *vice versa*. The reason is now understood: when the share of ROTC is low (or the total labour supply is inelastic), the intertemporal Euler equation transmission channel prevails on the financial one: an increase in the real interest rate decreases the economic activity. Furthermore inside the parameter values where Θ is positive an increase in the share of ROTC increases the sensitivity of aggregate demand to interest rate movements, i.e. lower real wages imply lower consumption for the ROTC and the traditional intertemporal effect prevails on the financial one for the optimisers. This ceases to be true when $\Theta < 0$: an increase in the real rate could potentially expand aggregate demand.[15]

---

[13]Note that these effects of interest rate movements on financial portfolio would be irrelevant if $\lambda = 0$, i.e. no ROTC.

[14]High sensitivity of real wage to interest rate movements enhances the financial effects described.

[15]Bilbiie (2008) refers to this as the "*inverted aggregate demand logic*". We use the same terminology in section 1.5.

It is quite intuitive that these effects have dramatic consequences on the equilibrium dynamics: as discussed in Bilbiie (2008), a monetary economy with a share of ROTC that displays a negative Θ requires, for the RE equilibrium to be unique, the monetary policy to abandon the Taylor principle and adopt a passive monetary rule.

Here we explore the consequences of the sign of Θ on the RE equilibrium determinacy in a model where, due to the presence of a distortive fiscal policy, equilibrium dynamics are driven by a genuine interaction of monetary and fiscal policy.

1.2.9. Determinacy

Given the focus of the paper on the equilibrium dynamics of the model we assume that non fundamental shocks hit the economy.[16] We further assume that government spending is always at its steady state level, i.e. $G_t = G \ \ \forall t$.

We combine (1.28)-(1.33) to obtain a system of difference equations describing the equilibrium dynamics of our economy. After some algebraic substitutions we can reduce the system to one involving three variables

$$AE_t\left\{x_{t+1}\right\} = B\left\{x_t\right\} \tag{1.36}$$

where $x_t \equiv \left(\ \widehat{y}_t,\ \pi_t,\ \widehat{b}_t\right)'$ and $A = \begin{bmatrix} 1 & \Theta & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$B = \begin{bmatrix} 1 & \Theta\phi_\pi & 0 \\ -\kappa\left(\frac{1}{\gamma_c} + \eta + 1 + \frac{\tau}{1-\tau}\delta_2\right) & 1 & -\kappa\left(\frac{\tau}{1-\tau}\delta_1\right) \\ -\frac{1}{\beta}\frac{\tau w}{\gamma_b}\left(\frac{1}{\gamma_c} + \eta + 1 + \frac{1}{1-\tau}\delta_2\right) & \phi_\pi - \frac{1}{\beta} & \frac{1}{\beta}\left(1 - \frac{\tau w}{\gamma_b(1-\tau)}\delta_1\right) \end{bmatrix}$$

In order to study the determinacy of the system we need to analyse the eigenvalues of $J = A^{-1}B$. Given that the x vector displays two non-predetermined variables (inflation and output) and one predetermined (public debt), determinacy requires the J matrix to

[16] The absence of shocks does not affect the determinacy analysis as the eigenvalue associated with any shock is assumed (if stationarity is imposed) to be inside the unit circle.

have two eigenvalues outside the unit circle and one inside the unit circle. Alternatively if more than one eigenvalue of J lie inside the unit circle, the system is locally undetermined: from any initial value of the stock of public debt there exists a continuum of equilibrium paths converging to the steady state, and the possibility of sunspots fluctuations arises. If instead there are no eigenvalues inside the unit circle, there is no solution to (1.36) that converges to the steady state.[17]

1.2.10. Calibration

The model is calibrated to a quarterly frequency.[18] We assume the elasticity of substitution among goods, ε, is equal to 6. This implies a steady state markup of 20%, which is in line with most of the macro literature. The discount factor β has been fixed at 0.99. As a consequence, the real annual interest rate is 4%. θ, the parameter of relative disutility of labour to consumption, has been chosen to obtain an average steady state labour supply of 1/3. The steady state ratio between private consumption and total output, γ_c, is 0.75. This value implies a steady state ratio of government spending over output of 25%, which is in line with the level of public consumption in most of the industrialised countries, see Galí et al.(2007). As in most of the NK literature, we assume that prices remain unchanged on average for one year. Therefore α, the parameter ruling the degree of price stickiness, is fixed at 0.75. When not differently specified, these parameters are kept at their baseline values throughout the determinacy exercise. Next we turn to the parameters for which some sensitivity analysis is conducted, by examining a range of values in addition to their baseline settings. Given the aim of the paper, the model has been solved with several pairs of λ, the share of ROTC and η, the inverse Frisch elasticity

[17] Unless the initial level of the public debt stock is at its steady state value, in which $x_t = 0$ for all t is the only non explosive solution.

[18] We insert this paragraph on calibration before presenting the analytical results. This is because in the section where we present the analytical results, we use simple numerical examples based on the calibration presented here, in order to generate the economic intuitions behind our results.

of labour supply, depending whether we want to study a situation where Θ is positive or negative.[19]

In the case of a balanced budget fiscal policy rule, the determinacy has been studied for different values of γ_b, the steady state level of public debt to GDP ratio, while in the case of general fiscal rules, we fix $\gamma_b = 2.4$, a value which implies an annual steady state ratio of public debt to output equal to 60% and a steady state level of taxation of $32,8\%$ of total output. The determinacy, and consequently the calibration exercise, has been studied with different values of δ_2, the fiscal policy parameter of the output gap. A value of $\delta_2 = 0$ implies a policy rule very similar to the one studied by Leeper (1991), and describes a situation in which the tax rates do not respond to output fluctuations. We furthermore define a countercyclical (procyclical) fiscal policy in terms of output if $\delta_2 > 0$ ($\delta_2 < 0$). Similarly, in order to describe the active-passive policy mix, the determinacy conditions is analysed for a broad range of policy parameters[20], ϕ_π and δ_1.

1.3. Results

1.3.1. Balanced Budget Rule

As a first step in analysing the interaction between monetary and fiscal policy with a share of ROTC, we study the equilibrium dynamics of the model in the case where the government has to balance its budget in every period without accessing to short run public debt assets. Such a fiscal policy implies that the tax rate is fixed in every period to satisfy[21]

$$\frac{1}{1-\tau}\widehat{\tau}_t = \frac{\gamma_b}{\tau w}\left(\beta \widehat{R}_t - \pi_t\right) - \left(\frac{1}{\gamma_c} + \eta + 1\right)\widehat{Y}_t \tag{1.37}$$

Thus it is assumed there is a historical inherited stock of real public debt, on which interest has to be paid by the government, but this stock never changes because the tax

[19] In particular we allow η to vary in a range between 0.25 and 4 and λ between 0.05 and 0.5. These values are consistent with most empirical literature.

[20] In particular we allow $\phi_\pi \in (-2, 6)$ and $\delta_1 \in (-1, 2)$.

[21] We continue to assume that $G_t = G \; \forall t$.

rate is adjusted appropriately. With a balanced rule of this type the dynamic system can be written as

$$E_t\left\{x_{t+1}\right\} = J^{br}\left\{x_t\right\} \tag{1.38}$$

where $x_t = \{\widehat{Y}_t, \pi_t\}$, $J^{br} = \begin{bmatrix} 1+\frac{\Lambda_1\Theta}{\beta} & \Theta\phi_\pi - \frac{\Theta(1+(1-\phi_\pi\beta)\Psi)}{\beta} \\ -\frac{\Lambda_1}{\beta} & \frac{(1+(1-\phi_\pi\beta)\Psi)}{\beta} \end{bmatrix}$, with $k = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$, $\Psi = \frac{k\gamma_b\varepsilon}{(\varepsilon-1)}$ and $\Lambda_1 = k\left(\frac{1-\tau}{\gamma_c} + (1-\tau)\eta - \tau\right)$.

We assume that $\tau < \left(1+\frac{1}{\frac{1}{\gamma_c}+\eta}\right)^{-1}$. This implies that $\Lambda_1 > 0$.

The restriction on τ greatly simplifies the algebra and it is mild in empirical terms. Consider for example a standard parametrisation where $\gamma_c = 0.75$ and $\eta = 1$. The assumption on τ implies that the tax rate has to be smaller than 60%. With $\gamma_c = 0.75$ and $\eta = 4$, the restriction implies that τ has to be smaller than 84%.

Given that both variables are non-predetermined, determinacy requires both eigenvalues of J^{br} lying, in absolute values, outside the unit circle. As previously stated the sign of the elasticity of demand to the real interest rate, Θ, changes markedly the dynamic properties of the model. Let us first assume $\Theta > 0$. In this case[22], necessary and sufficient conditions for determinacy require

$$\text{If } \Lambda_1 > 2\frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi > 1 \tag{1.39}$$

$$\text{else if } \Lambda_1 < \frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \min\left\{\xi_1, \xi_2\right\} \tag{1.40}$$

$$\text{else if } \frac{\Psi\beta}{\Theta} < \Lambda_1 < 2\frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \xi_2 \tag{1.41}$$

With $\xi_1 = \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}$ and $\xi_2 = \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Theta\Lambda_1}$. (1.39) represents the case with no or very low level of steady state public debt, or high values of Θ. As in any standard New Keynesian sticky price model, the only condition for equilibrium determinacy is to have an active monetary policy, i.e. $\phi_\pi > 1$. Two main reasons drive this result. First, with $\Theta > 0$, the effect of an interest rate change on the economy follows the standard "Taylor

[22] Formal proof of this determinacy results is provided in the appendix of this chapter.

principle" logic: a higher interest rate generates a contraction in aggregate demand and, through the NKPC, downward pressure on inflation. For any given level of $\phi_\pi > 1$, this contraction of aggregate demand is positively correlated with Θ. Therefore the higher Θ the easier it is for monetary policy to keep inflation under control. Second, for values of the steady state ratio of public debt to output, γ_b, close to zero, the feedback of monetary policy on the government budget constraint is very limited. The tax rate moves only to balance changes in output and this movement does not imply any major feedback on the endogenous variables of the model.

This stops being partly true when (1.40) or (1.41) are verified. As in the previous case monetary policy has to adopt an active role, but this is now constrained by some upper bounds which are functions of the structural parameters of the model. They depend, among other things, on the long run level of debt, the share of ROTC, the Frisch elasticity of labour supply and the degree of price stickiness. Note that when $\frac{\Psi\beta}{\Theta} < \Lambda_1 < 2\frac{\Psi\beta}{\Theta}$, ξ_1 is not binding, meanwhile, when $\Lambda_1 < \frac{\Psi\beta}{\Theta}$, ξ_1 is more likely to bind than ξ_2 for standard parameter values. For example, suppose that $\alpha = 0.75$, $\beta = 0.99$, $\gamma_b = 2.4$, $\eta = 1$, $\lambda = 0.3$, $\varepsilon = 6$. This set of parameters implies $\Lambda_1 = 0.16$, $\Psi = 0.24$, $\Theta = 1.10$, $\xi_1 = 2.01$ and $\xi_2 = 12.34$. With instead $\gamma_b = 3$ (all other parameters constant), $\xi_1 = 1.67$ and $\xi_2 = 9.48$.

These upper bounds are directly generated by the distortive nature of fiscal policy. let us instance assume agents suddenly expect higher inflation. Monetary policy adopting an active role increases the real interest rate so as to decrease current aggregate demand and thus stabilise inflation via the NKPC. The magnitude of the effect of an interest rate increase on aggregate output via the IS equation depends in a non-trivial way on Θ, i.e the higher Θ, the more sensitive the aggregate demand on monetary policy. On the other hand a higher interest rate feeds back on the government budget constraint, generating an increase in the cost of the service of public debt and therefore an upward pressure on the tax rate. Note that the higher the level of steady state public debt, the higher the tax rate increase for each increase in interest rate. Furthermore, the contractionary effect of monetary policy on output implies a decrease in the government revenues tax base, which causes a further increase in the tax rate. Moreover each increase in the tax rate feeds back positively, via the NKPC, on current inflation. This positive wedge can, for high levels of public debt or, *ceteris paribus*, for high responses of monetary policy to inflation, neutralise the initial attempt of monetary policy to stabilise inflation via a reduction of output, making the initial expectations of higher inflation self-fulfilling.

Figure 1.2 displays determinacy analysis in the $(\phi_\pi - \gamma_b)$ space for different parameter combinations of the share of ROTC, λ, and the inverse Frisch elasticity of labour supply, η. With low levels of public debt, the only condition for determinacy is to have $\phi_\pi > 1$, i.e. the Taylor principle. Furthermore, the constraints on the monetary policy parameter are less likely to bind the higher is the share of ROTC or the lower is the Frisch elasticity on labour supply (high η). The reason is now well understood. Within the parameter values where $\Theta > 0$, a high share of ROTC, or a more elastic aggregate labour supply, increases the effect of an interest rate changes on aggregate demand, preventing the fiscal policy feedback on the supply side of the economy to generate self-fulfilling expectations. When, for example, $\eta = 3$ and $\lambda = 0.3$ the only condition to obtain determinacy is $\phi_\pi > 1$.

We now turn to study of the determinacy properties of the model when $\Theta < 0$. Necessary and sufficient conditions for determinacy require

$$\phi_\pi < \min\{1, \xi_1\} \;\cup\; \phi_\pi > \max\{1, \xi_2\} \tag{1.42}$$

In this case there are two determinacy spaces. In the first one, monetary policy has to adopt a passive role, i.e. $\phi_\pi < 1$. This conduct may have an upper limit represented by ξ_1. In the other determinacy space monetary policy needs to adopt an active role, with a potential downward limit represented by ξ_2. As before, both ξ_1 and ξ_2 depend crucially on the structural parameters of the model. In particular, ξ_1 is increasing in λ, η and γ_b, while ξ_2 is increasing in λ and η, and decreasing[23] in γ_b. We start by explaining the first determinacy space, i.e. $\phi_\pi < 1$. Let us assume agents suddenly expect higher inflation. The monetary authority, adopting a passive role, cuts the real interest rate. This cut contracts aggregate demand through a decrease in the financial activities held by the optimiser consumers. A decrease in the real rate and in output have opposite effects on the government budget constraint: a lower real interest rate, cutting the cost of the service of public debt, implies a decrease in the tax rate, while a decrease in output, lowering the government revenue base, generates an opposite effect. As stressed above an increase in the tax rate could have, through the supply side of the economy, a destabilising effects on inflation. Therefore if the changes of the real interest rate and output generates an increase in the tax rate, the initial expectations could be self-fulfilling. This situation is more likely to happen with low values of γ_b, i.e. lower monetary feed back on the tax rate, low values of λ and η, i.e. higher sensitivity of aggregate demand[24] to interest rate movements.

The second area of determinacy requires monetary policy to be active with a lower bound represented by $\xi_2 = \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Theta\Lambda_1}$. As before, let us assume agents suddenly expect higher inflation. The monetary authority would increase the real rate which, given its effect on financial assets, expands aggregate demand. This expansion feeds back on the government budget constraint generating an increase in the tax base and therefore a reduction of the tax rate. A reduction of the tax rate can stabilise inflation through the NKPC. However, a higher interest rate feeds back on fiscal policy causing an increase in tax rate and this, *ceteris paribus*, puts upward pressure on prices. It is therefore important for determinacy that the effect of output on fiscal policy overcompensates the monetary one. When this happens the decrease in the tax rate stabilises current inflation contrasting the initial expectations of higher inflation. As stressed before, the higher λ and η, the lower the sensitivity of aggregate demand to monetary policy, and therefore

[23]For example with $\varphi = 3$, $\lambda = 0.35$ and $\gamma_b = 2$, $\xi_1 = 0.19$ and $\xi_2 = 2.96$, while with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 2$, $\xi_1 = 0.64$ and $\xi_2 = 8.38$. Finally with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 3$, $\xi_1 = 0.77$ and $\xi_2 = 6.25$.

[24]Note that with $\lambda = 0.35$ and $\varphi = 3$, $\Theta = -3.54$, while with $\lambda = 0.5$ and $\varphi = 4$, $\Theta = -0.37$.

more likely that, for each increase in the real interest rate, tax rate increases, raising the possibility of sunspot fluctuations.

Figure 1.3 displays determinacy in the $(\phi_\pi - \gamma_b)$ space for different values of λ and η. As stressed above, increasing these two parameters, (when Θ is negative) lowers the sensitivity of aggregate demand to interest rate movements. This increases the possibility of determinacy when monetary policy adopts a passive role, while the opposite is true when monetary policy is active.

This simple exercise helps us to motivate and explain the importance of inserting fiscal policy when analysing the effects on the equilibrium determinacy of a share of ROTC. First of all, when $\Theta > 0$, a balanced budget rule delivers determinacy for parameter values which are consistent with the empirical evidence. This result is a clear departure from Linnemann (2006). Linnemann finds that in a standard NK model with a balanced budget fiscal rule, an active monetary policy could, through its feedback on fiscal policy and the feedback of fiscal policy on aggregate supply, easily lead to indeterminacy even for low positive values of long run public debt. The differences of our results stem from an increased sensitivity of aggregate demand to monetary policy due to the presence of ROTC.

Similarly, when $\Theta < 0$, the presence of balanced budget fiscal rule, through its feedback on the endogenous variables of the model, helps to reestablish the Taylor principle within realistic monetary policy responses to inflation fluctuations. This result extends the ones found by Bilbiie (2008) in the absence of fiscal policy, where a contemporaneous active monetary policy rule could deliver determinacy only for implausibly high levels of the monetary parameter ϕ_π.

1.3.2. Endogenous Debt

Here we study a more general version of monetary/fiscal policy mix. Short run public debt fluctuations are allowed and fiscal policy can be represented by the government budget constraint (1.32) and by the tax rate rule (1.33). Note that, differently from the previous case of a balanced budget rule, here the tax rate is a policy instrument which can be discretionally set according to δ_1 and δ_2. Given the lack of a straightforward and intuitive analytical result for the determinacy analysis of this exercise, we rely on numerical solutions.

Figure 1.4 displays the determinacy analysis in the $(\phi_\pi - \delta_1)$ space for different values of the fiscal parameter on output, δ_2, when Θ is positive, i.e. $\eta = 1$ and $\lambda = 0.3$.

As previously described, when $\Theta > 0$ the monetary policy effects on the system follow the common wisdom. Hence the presence of ROTC does not alter Leeper's (1991) logic. In other words equilibrium determinacy is guaranteed by an active (passive) monetary policy, $\phi_\pi > 1$, $(\phi_\pi < 1)$ and a passive (active) fiscal policy, $\delta_1 > \left(\frac{1}{\beta} - 1\right)$, $\left(\delta_1 < \left(\frac{1}{\beta} - 1\right)\right)$.[25] When both policies are passive, the system displays an infinite number of solutions and the possibility of endogenous sunspot fluctuations arises. When both policies are active there is no solution to (1.36). Furthermore it is interesting to note that the fiscal parameter on output is not relevant for equilibrium determinacy.

The intuition for these results goes as follows. Let us assume agents suddenly expect higher inflation. Monetary policy, following an active role, raises the real interest rate. Higher interest rate increases the cost of the service of public debt. A fiscal policy which follows a passive role increases government revenues raising the tax rate.[26] The combined effect of a higher interest rate and a higher tax rate, reduces disposable real wages, potentially lowering consumption of the ROTC and that of the OPTC. On the other hand, both lower real wages, through an increase in profit share, and higher return on public debt, generate an increase in the financial portfolio of the optimisers. For low values of λ, these positive financial effects of monetary policy on the optimisers' wealth are overcompensated by the traditional Euler equation channel: an increase in real interest

[25] Note that this definition of fiscal policy is a simplifying approximation, given that it refers to an environment where the Ricardian equivalence holds. For a detailed discussion see Leith and Wren-Lewis(2000). For our benchmark parameterisation when $\Theta > 0$, a stable public debt dynamics requires $\delta_1 > 0.011$, which is very close to $\frac{1}{\beta} - 1$.

[26] The distortive nature of fiscal policy implies a Laffer curve in the government revenues. However the peak of the Laffer curve happens for steady state tax rate values which are far above to the ones assumed in this analysis. For a detailed discussion see Schmitt-Grohe and Uribe (1997).

rate reduces aggregate demand. Via the NKPC, this reduction in aggregate demand puts downward pressure on current inflation[27]: initial expectations of higher inflation are not self-fulfilled and the combination of monetary and fiscal policy stabilises both the price level and the public debt.

Consider instead a passive/passive policy mix. The monetary authority would respond to the initial expectations of higher inflation by cutting the real interest rate. This would feed back on public debt generating a tax rate cut. The combined effect of lower interest rate and lower taxation would increase consumption of both type of consumer, expanding aggregate demand and in turn putting upward pressure on prices. The initial expectations of higher inflation are self-fulfilled, so generating local indeterminacy.

The active/active policy mix generates a perverse path for inflation and public debt which in turn leads the system to be unstable.

Figure 1.5 displays the determinacy analysis in the $(\phi_\pi - \delta_1)$ space for different values of the fiscal parameter on output, δ_2, when Θ is negative, i.e. $\eta = 3$ and $\lambda = 0.5$. A necessary condition for determinacy is that monetary and fiscal policy are both either active or passive. As previously described, the reason for these results is that when $\Theta < 0$, the financial effects of an interest rate change overturn the traditional transmission mechanism of monetary policy on aggregate demand. As a consequence, an active monetary policy $(\phi_\pi > 1)$, through an increase in the return of the optimiser consumers' financial activities has the potential to expand aggregate demand. The intuition for these results goes as follows. Let us assume that both monetary and fiscal policy adopt an active rule $(\phi_\pi > 1,\ \delta_1 < \frac{1}{\beta} - 1)$. Let us further assume that agents suddenly expect higher public debt. Fiscal policy reacts to this, cutting the tax rate (active fiscal policy) and therefore generating an explosive path for public debt. Moreover the tax rate cut feeds back on aggregate demand, generating, *ceteris paribus,* an increase in output and a downward pressure on inflation via the NKPC. Monetary policy through an active rule expands

[27]Note that higher taxation *per se* puts upward pressure on prices via (1.28). However this effect is overcompensated by the decrease in aggregate demand.

further output, i.e. $\Theta < 0$, causing, via the NKPC an explosive path on inflation, which in turn deflates the cost of public debt, implying a stable RE equilibrium.

Let us assume now that both policies are passive ($\phi_\pi < 1$, $\delta_1 > \frac{1}{\beta} - 1$) and that agents suddenly expect higher inflation. Monetary policy contrasts these expectations by cutting the real interest rate (passive rule). A lower interest rate lowers current output, putting downward pressure on prices via the NKPC, and therefore stabilising inflation. At the same time, monetary policy has two opposite effects on fiscal policy. On one hand, a lower interest rate implies, via a reduction in the optimiser consumers' wealth, a reduction in aggregate demand and therefore of the tax base, while on the other hand, a lower interest rate implies a lower cost of the service of public debt. These effects imply an important role for the equilibrium determinacy, of δ_2, the fiscal policy parameter on output. In particular the determinacy region increases when δ_2 decreases. This is due to the fact that if fiscal policy reacts to a decrease in output with a further cut in the tax rate ($\delta_2 > 0$), it could potentially fail to generate enough revenues to balance its budget, causing an explosive path for public debt and therefore generating indeterminacy. This destabilising situation can be partially avoided with a procyclical, in terms of output, fiscal rule, i.e. $\delta_2 < 0$. Note that within this monetary/fiscal policy mix determinacy requires a stronger fiscal policy, i.e. high δ_1, the closer ϕ_π is to unity, i.e. constant real interest rate.

With $\Theta < 0$, a policy mix of active monetary ($\phi_\pi > 1$) passive fiscal ($\delta_1 > \frac{1}{\beta} - 1$) policy generates indeterminacy. Monetary policy responds to higher inflation expectations, increasing the real interest rate. This generates an increase in the cost of the service of public debt and therefore an increase in the tax rate. However as previously described, the initial increase in the interest rate would expand aggregate demand, putting further pressure on current inflation and making the initial expectation self-fulfilling. Similarly, a policy mix of passive monetary policy ($\phi_\pi < 1$) and active fiscal policy ($\delta_1 < \frac{1}{\beta} - 1$) generates the stabilisation of inflation and the destabilisation of public debt, generating indeterminacy for positive values ($0 < \phi_\pi < 1$) of ϕ_π and instability for negative values of ϕ_π.

1.4. Robustness

1.4.1. General Monetary Policy Rules

We extend the determinacy analysis for a more general class of monetary policy rules of the type

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right) \text{ } with \text{ } i = -1, 0, 1 \tag{1.43}$$

where ρ identifies the nominal interest rate smoothing parameter, while ϕ_Y is the output parameter on monetary policy. In particular, when $i = -1$, (1.43) reduces to a backward looking rule. When $i = 0$ it corresponds to a contemporaneous rule, and when $i = 1$ it becomes a forward looking rule. Figure 1.6 reports the determinacy analysis in the ($\phi_\pi - \delta_1$) space when Θ, the parameter capturing the elasticity of aggregate demand to interest rate, is positive, with λ and η calibrated as in the previous section, and $\delta_2 = 0$. Scrolling down the figure changes i (contemporaneous, backward-looking and forward-looking), while scrolling the figure from left to right changes the parameter values on ρ and ϕ_Y. Obviously, with i, ϕ_Y and ρ equal to zero, (1.43) collapses[28] to (1.31). For what concerns the first two top rows, i.e. contemporaneous rule and backward-looking rule, the adoption of more general monetary rules does not change the equilibrium dynamics of the model. In other words, the presence of a response in output ($\phi_Y > 0$) or the persistence of the interest rate ($\rho > 0$) does not alter, or does so only marginally, the logic of Leeper (1991). As in the previous section, in order to have a unique RE equilibrium when $\Theta > 0$, monetary policy has to be active (passive) and fiscal policy has to be passive (active).

With a forward-looking monetary policy rule (last row), this stops being true.

[28]For the sake of clarification, note that the case represented in the top left quadrant in figure (1.6) is the same as the one at the left bottom in figure (1.4).

Forward-looking monetary rules where originally proposed by Bernanke and Woodford (1997), and estimated by Clarida et al. (1998, 2000). As noted by Bernanke and Woodford (1997) and Bullard and Mitra (2002), this type of rule can change markedly the equilibrium conditions of a standard monetary sticky price model respect its contemporaneous counterpart. In particular, when monetary policy is active, equilibrium determinacy imposes an upper limit on ϕ_π and δ_1, which in turn depends on ρ and ϕ_Y. In other words, when in (1.43) $i = 1$, there is an upper bound to the size of the response to expected inflation that must be satisfied. If that upper bound is overcome, the equilibrium becomes indeterminate. Galí et al. (2004) find a similar result in a monetary model with capital accumulation and ROTC.

Here, while there are no changes in the case of active fiscal policy/passive monetary policy, in the case of active monetary/passive fiscal policy, the upper limit on ϕ_π depends, other than on ϕ_Y and ρ, on δ_1, the response of fiscal policy to public debt fluctuations. The upper limit on the monetary policy response to inflation expectations is present only for high responses of the tax rate to public debt. When fiscal policy reacts too strongly to public debt fluctuations, the implied increase in the tax rate feeds back on the supply side of the system via the NKPC, generating a destabilising effect on the attempt of monetary policy to contain inflation expectations and hence, causing indeterminacy. With an increase of interest rate inertia, via ρ or an increase in monetary response to output, via ϕ_Y, the upper limit on ϕ_π disappears. These results are consistent with the ones presented in Galí et al. (2004).

Figure 1.7 reports the same exercise when $\Theta < 0$ ($\eta = 3$ and $\lambda = 0.5$). As in the previous section, in this case the logic of Leeper (1991) is reversed: necessary conditions for a unique RE equilibrium are that monetary and fiscal policy are both either active or passive. these results changes only marginally for the timing of the monetary rule and for the parameters of monetary policy inertia and output stabilisation.

Secondly, with a forward-looking monetary rule, the upper limit on ϕ_π and δ_1, which characterised the case where $\Theta > 0$, disappears. When fiscal policy adopts a destabilising public debt policy, monetary policy has to inflate the system through an active policy. the potentially explosive path of both inflation and public debt overshoot the importance of the fiscal feedback on the supply side of the economy, leading to determinacy.

1.4.2. Different fiscal arrangements: the case of lump sum taxation.

One might rightly wonder if the results thus far presented depend on the particular specification of fiscal policy or instead are robust to a different specification of fiscal policy, i.e. lump sum taxation. This represents a natural extension of the analysis under several points of view. First of all, lump sum taxation maintains the distortive nature of fiscal policy: both types of consumers pay the burden of public debt but only the OPTC hold public debt assets. Therefore the government budget constraint cannot be separated from the rest of the model, and public debt remains an important state variable which has to be taken into account in the dynamics of the model. Secondly, despite the shape of the utility function (log-consumption), with lump sum taxation, ROTC do not supply labour inelastically.

The model with lump sum taxation is very similar to the one presented in the literature, as in Bilbiie et al. (2004), Galí et al. (2008). We therefore relegate to the appendix the standard derivation of the model, while here we only present the log-linearised equilibrium equations. When not differently specified we maintain the same notation and the same calibration as in the case of labour income taxation. As before, we assume public spending to be always at its steady state value. The equilibrium can be described by this set of equations. The NKPC

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\left(\frac{1}{\gamma_c}+\eta\right)\widehat{Y}_t\right) \tag{1.44}$$

the government budget constraint

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{b}_t - \pi_t - \frac{\tau^{ls}}{b}\widehat{\tau}_t^{ls}\right) \tag{1.45}$$

where τ^{ls} identifies the steady state value of lump sum taxes, the monetary policy

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right) \; with \; i = -1, 0, 1 \tag{1.46}$$

the fiscal policy rule[29]

$$\widehat{\tau}_t^{ls} = \delta_1 \widehat{b}_t + \delta_2 \widehat{Y}_t \tag{1.47}$$

and aggregate demand

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \Gamma_c^{-1}\Theta^{ls}(1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) + \Gamma_\tau \Gamma_c^{-1}\Theta^{ls}\left(E_t\widehat{\tau}_{t+1}^{ls} - \widehat{\tau}_t^{ls}\right) \tag{1.48}$$

where $\Gamma_c = \left(1 - \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}\right)$, $\Gamma_n = \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}$, $\Theta^{ls} = (1/\gamma_c - \Gamma_n/\Gamma_c)^{-1}$ and $\Gamma_\tau = \frac{\lambda\tau^{ls}\eta}{Y(1+\eta\mu\gamma_c)}$.

A few things are worth noticing. The distortive nature of fiscal policy is represented by the feedback of taxation on the endogenous variables of the model via (1.48), the demand side of the economy. The sign and size of this effect depend crucially on the share of ROTC[30] and the elasticity of labour supply. Similarly, the sign of the elasticity of aggregate demand to interest rate movements depends on Θ^{ls} while its size depends on Γ_c. despite being a less straightforward analytical expression, the economic intuition for the sign of Θ^{ls} is the same to Θ, introduced in the labour income taxation environment. A necessary condition for $\Theta^{ls} > 0$ is

$$\lambda < \frac{1+\gamma_c\mu\eta}{1+\gamma_c+\eta+\gamma_c\eta} \tag{1.49}$$

Figure 1.8 sketches in the $(\lambda - \eta)$ space the sign of Θ^{ls}. As in the case with labour income taxation, Θ^{ls} remains positive for high values of the Frisch elasticity of labour supply, i.e. low η, and the higher, the higher the share of ROTC, i.e. high λ. We repeat the exercise conducted in 3.1 assuming that fiscal policy balances its budget in every period without accessing short term public debt asset but has to repay the interest on the long

[29]From now on we ignore the term on output in the fiscal rule, i.e. $\delta_2 = 0$.
[30]Note that in the limiting case of no ROTC, $\Gamma_n = \Gamma_\tau = 0$ and $\Gamma_c = 1$.

term public debt. The behavior of fiscal policy can be represented in log-linear form as

$$\widehat{\tau}_t^{ls} = \frac{\gamma_b Y}{\tau^{ls}} \left(\widehat{R}_t - \frac{1}{\beta} \widehat{\pi}_t \right) \tag{1.50}$$

While we assume that monetary policy implements (1.46) with $i = \rho = \phi_Y = 0$, i.e. contemporaneous rule. The dynamic system can be written as

$$E_t \{x_{t+1}\} = J^{ls} \{x_t\}$$

where, as before, $x_t = \left\{ \widehat{Y}_t, \pi_t \right\}$ and $J^{ls} = \begin{bmatrix} 1 + \frac{\Gamma_c^{-1} \Theta^{ls} \chi (\beta - \Upsilon + \beta \Upsilon \phi_\pi)}{\beta^2} & \frac{\Gamma_c^{-1} \Theta^{ls} (\beta + (\beta - 1) \Upsilon)(\phi_\pi \beta - 1)}{\beta^2} \\ -\frac{\chi}{\beta} & \frac{1}{\beta} \end{bmatrix}$ with $\Upsilon = \frac{\lambda \eta \gamma_b}{(1 + \eta \mu \gamma_c)} > 0$ and $\chi_1 = k \left(\frac{1}{\gamma_c} + \eta \right) > 0$. As in the case with labour income taxation, determinacy requires J^{ls} to have both eigenvalues outside the unit circle. When $\Theta^{ls} > 0$, necessary and sufficient condition for determinacy is

$$\phi_\pi > 1 \tag{1.51}$$

The upper bound on active monetary policy that was present in the case of labour income taxation, disappear in the case of lump sum taxation. In other words, a monetary policy which follows the Taylor principle, i.e. $\phi_\pi > 1$, always delivers determinacy when only lump sum taxes are available and fiscal policy follows (1.50). This is due to the lack of direct feedback of a tax change on the supply side of the economy together with the ordinary effect of an interest rate change on the demand side of the economy. A graphical inspection of determinacy can be found in figure 1.9.

When $\Theta^{ls} < 0$, necessary and sufficient conditions for determinacy require

$$\phi_\pi < \min(1, \xi_3) \cup \phi_\pi > \max(1, \xi_4) \tag{1.52}$$

where $\xi_3 = \frac{\beta^2 - \beta + \Upsilon \Gamma_c^{-1} \chi_1 \Theta^{ls}}{\beta \Gamma_c^{-1} \chi_1 \Theta^{ls} (1 + \Upsilon)}$ and $\xi_4 = \frac{-2\beta - 2\beta^2 - \beta \Gamma_c^{-1} \chi_1 \Theta^{ls} + 2\Gamma_c^{-1} \Upsilon \chi_1 \Theta^{ls}}{\beta \Gamma_c^{-1} \chi_1 \Theta^{ls} (1 + 2\Upsilon)}$. As in the case with labour income taxation, when $\Theta^{ls} < 0$, there are two determinacy areas, one in which monetary policy follows an active rule and one in which monetary policy follows a passive one. Both ξ_3 and ξ_4 are functions of the structural parameters of the model. In particular while ξ_3 is increasing in λ, η and γ_b, while ξ_4 is increasing in λ and η, and decreasing[31] in γ_b. Despite a less intuitive expression, ξ_3 and ξ_4 have the same interpretation ξ_1 and ξ_2, respectively. Figure 1.10 displays the determinacy results in the case of Θ^{ls}. The economic intuition for these results is very similar to the analogous case with labour income taxation. We therefore refer to paragraph 1.3.1 for a detailed discussion.

Finally, we repeat the exercise conducted in (1.3.2) for the case of lump sum taxes. Fiscal policy is allowed to release short run public debt assets and it balances its budget following[32] (1.47). The analysis can therefore be conducted under the active/passive logic of Leeper(1991).

Figures 1.11 and 1.12 report the determinacy analysis with lump sum taxation with positive ($\eta = 1$ and $\lambda = 0.3$) and negative ($\eta = 3$ and $\lambda = 0.5$) Θ^{ls}. In both cases there are no noticeable differences with the labour income taxation scenario and the policy mix which guarantees determinacy is mainly driven by the sign of Θ^{ls}.

While, as detailed in Bilbiie et al. (2004), different fiscal arrangements imply important consequences for the transmission mechanism of macro policies, they do not cause important changes for the equilibrium dynamics.

1.5. Concluding Remarks

The introduction of ROTC has dramatic consequences for the equilibrium dynamics of a standard NK model. While most of the literature focuses only on the monetary policy aspect of these consequences, for example Galí et al.(2004) and Bilbiie (2008), we concentrate on the effects of a share of ROTC on fiscal policy and its interaction with monetary policy. In doing so, we analyse a broad range of monetary and fiscal policy rules. To this end this paper contributes to enrich the theoretical literature on

[31]For example with $\varphi = 3$, $\lambda = 0.35$ and $\gamma_b = 2$, $\xi_3 = 0.421$ and $\xi_4 = 1.78$, while with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 2$, $\xi_3 = 0.63$ and $\xi_4 = 6.81$. Finally with $\varphi = 4$, $\lambda = 0.5$ and $\gamma_b = 3$, $\xi_3 = 0.72$ and $\xi_4 = 7.19$.
[32]For semplicity we fix $\delta_2 = 0$ through this exercise.

macro-policy rules and has the potential to explain the U.S. postwar empirical evidence on monetary and fiscal policy regimes.

We summarise our results as follow.

1) When the share of ROTC and the elasticity of labour supply guarantee that the elasticity of demand follows the common wisdom, i.e. negative relation between interest rate and aggregate demand, monetary policy adopts a contemporaneous interest rate rule and fiscal policy balances its budget constraint without releasing short run public debt, an active monetary policy rule is necessary but not sufficient condition for determinacy. In other words, a monetary policy which respects the Taylor principle and reacts 'too strongly' against inflation might lead to indeterminacy for high levels of long run public debt. This upper bound on monetary policy gets tighter the higher the level of public debt and tends to disappear with an increase of the share of ROTC and a more elastic labour supply.

2) When the combination of ROTC and the elasticity of labour supply inverts the elasticity of aggregate demand to the interest rate and monetary and fiscal policy rules follow from 1), equilibrium determinacy can be guaranteed by both an active and a passive monetary policy rule. While a passive monetary rule leads to a unique RE equilibrium with an upper bound which is increasing in the share of ROTC, labour supply elasticity and long run public debt, an active monetary rule leads to determinacy with a lower bound which in turn is increasing in the share of ROTC and the elasticity of labour supply and decreasing in the long run level of public debt.

3) When fiscal policy is allowed to realise short run public debt and it follows a tax revenue rule as in (1.33), equilibrium determinacy requires, following the definition of Leeper (1991), to have an active (passive) monetary rule together with a passive (active) fiscal rule when the aggregate demand responds negatively to increases in real interest rate or both monetary and fiscal policy simultaneously active or passive when the aggregate demand logic is inverted.

4) Results 1, 2 and 3 survive to different specifications of monetary rules (contemporaneous, forward-looking, backward-looking) and different specifications of fiscal arrangements (labour income tax, lump sum tax).

We interpret our results in several directions. Results 1 and 2 represent a clear extension on fiscal balanced budget rule. While for many reasons this type of fiscal policy might not be a wise policy choice, i.e. it increases business cycle fluctuations and generates significant welfare losses (Barro, 1979; Lucas and Stokey, 1983;) and it may lead to indeterminacy, thus inducing belief-driven aggregate instability and endogenous sunspot fluctuations (Schmitt-Grohe and Uribe, 1997; Linnemann, 2005), its analysis represents a recurrent theme of debate in many countries. The present work does not deal with business cycle fluctuations nor with welfare analysis, but only with the determinacy properties of a balanced budget fiscal policy. To this respect, we find that, within reasonable parameter values, an active monetary policy together with a moderate level of ROTC, i.e usual aggregate demand logic, guarantees determinacy with a balanced fiscal policy rule. This result can be compared to that of Linnemann (2006). The author finds that in a similar model, although with no ROTC, an active monetary policy with a balanced budget fiscal policy can easily lead to indeterminacy. The difference in our results are crucially driven by the presence of ROTC.

On the other hand, result 2 can be seen as an extension of Bilbiie (2008). He shows that when the share of ROTC, or *ceteris paribus,* the elasticity of labour supply, imply an inverted aggregate demand logic, monetary policy has to be passive in order to guarantee a unique RE equilibrium. In this paper we argue that when the aggregate demand logic is inverted and fiscal policy follows a balanced budget rule, an active monetary policy can realistically lead to determinacy.

Favero and Monacelli (2005) and Davig and Leeper (2006) find that in the US postwar macro policy regimes, alongside with periods in which monetary and fiscal policy respect the active/ passive logic of Leeper (1991), there are periods in which monetary and fiscal policy are both active or passive. This evidence cannot generally be explained with a traditional Real Business Cycle or NK model in which macro policies are not allowed to switch from active to passive and *viceversa*. We show in result 3, that within a reasonable parameters region, our model[33] has the potential to explain this empirical evidence.

[33]A similar result in a continuous time NK model with ROTC and lump sum taxation is obtained by Leith and Von Thadden (2007).

1.6. Figures

Figure 1.1. Sign of Θ. Black spots, $\Theta > 0$, white area $\Theta < 0$.

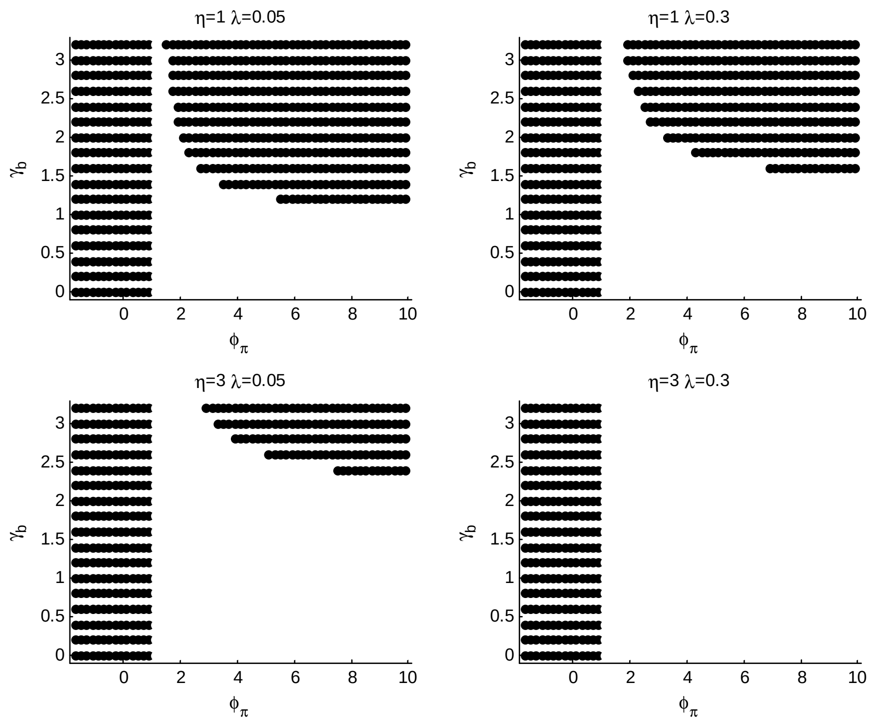

Figure 1.2. Determinacy analysis with a balanced budget fiscal policy, positive Θ. White area, determinacy. Black area, indeterminacy.

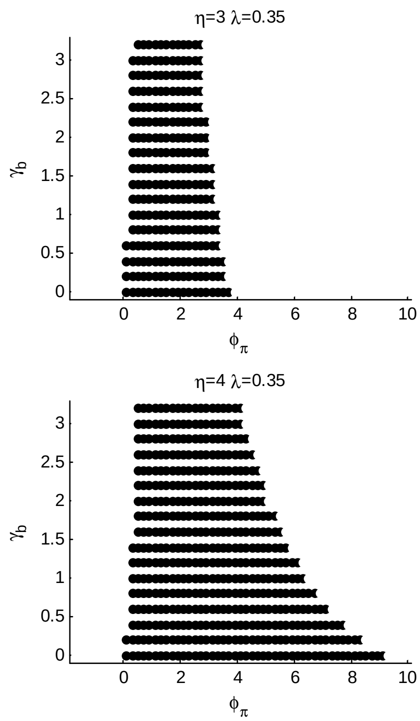

Figure 1.3. Determinacy analysis with a balanced budget fiscal policy, negative Θ. White area, determinacy. Black area, indeterminacy.

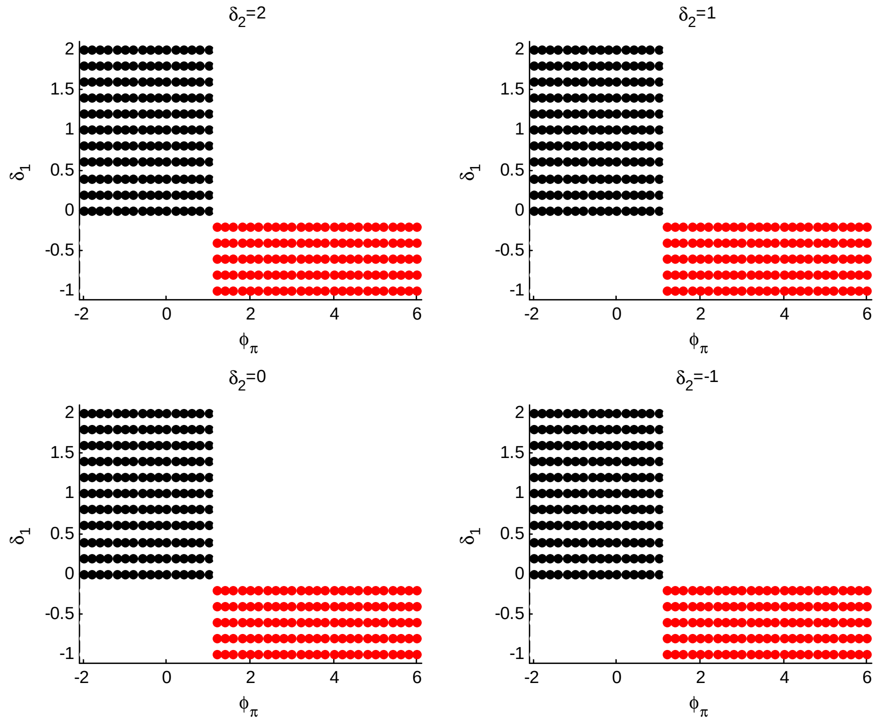

Figure 1.4. Determinacy area with contemporaneous monetary rule and a fiscal rule of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t + \delta_2 \widehat{Y}_t$ and positive Θ ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.

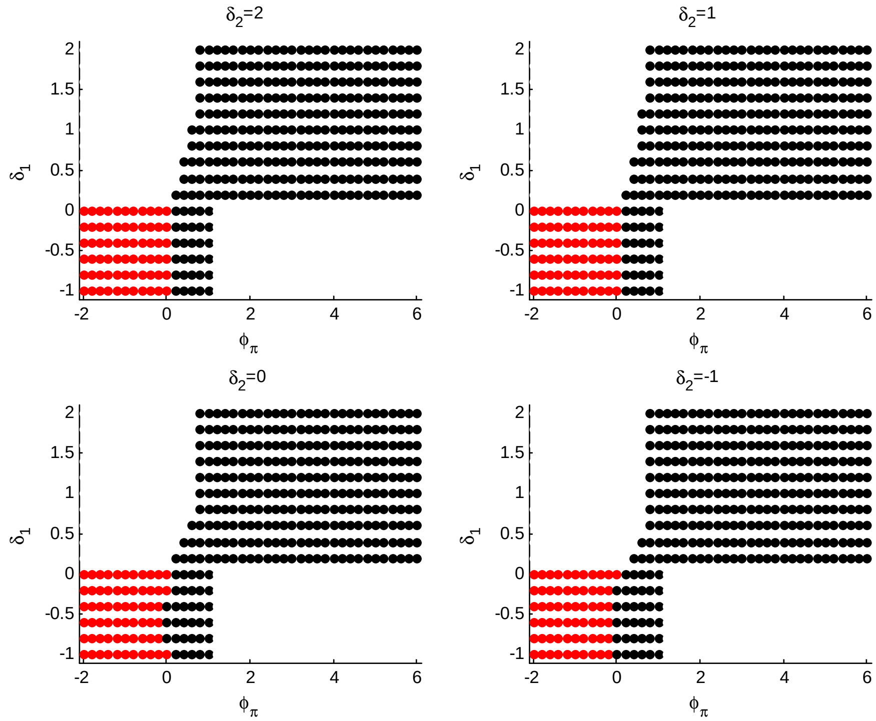

Figure 1.5. Determinacy area with contemporaneous monetary rule and a fiscal rule of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t + \delta_2 \widehat{Y}_t$ and positive Θ ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.

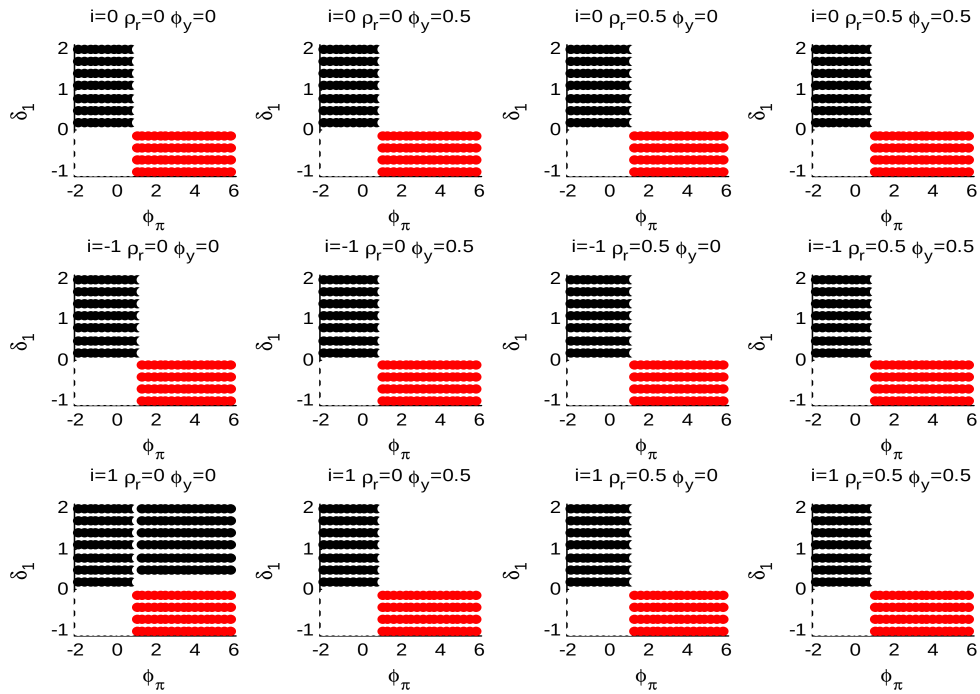

Figure 1.6. Determinacy area with monetary rule of the type $\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$ and a fiscal rule of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t - \widehat{Y}_t$ and positive Θ ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.

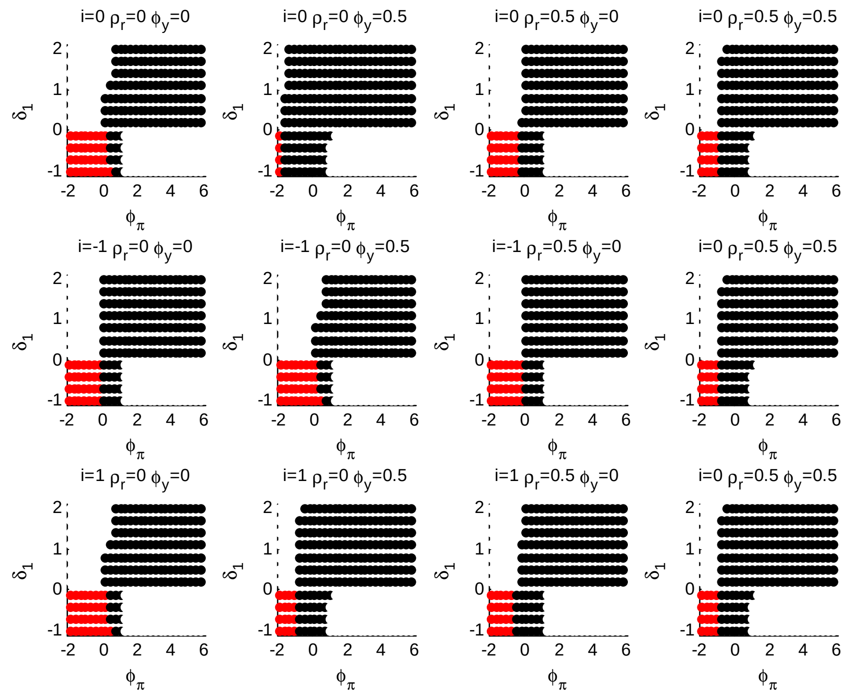

Figure 1.7. Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$ and a fiscal rule of the type $\widehat{\tau}_t = \delta_1\widehat{b}_t - \widehat{Y}_t$ and negative Θ ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.

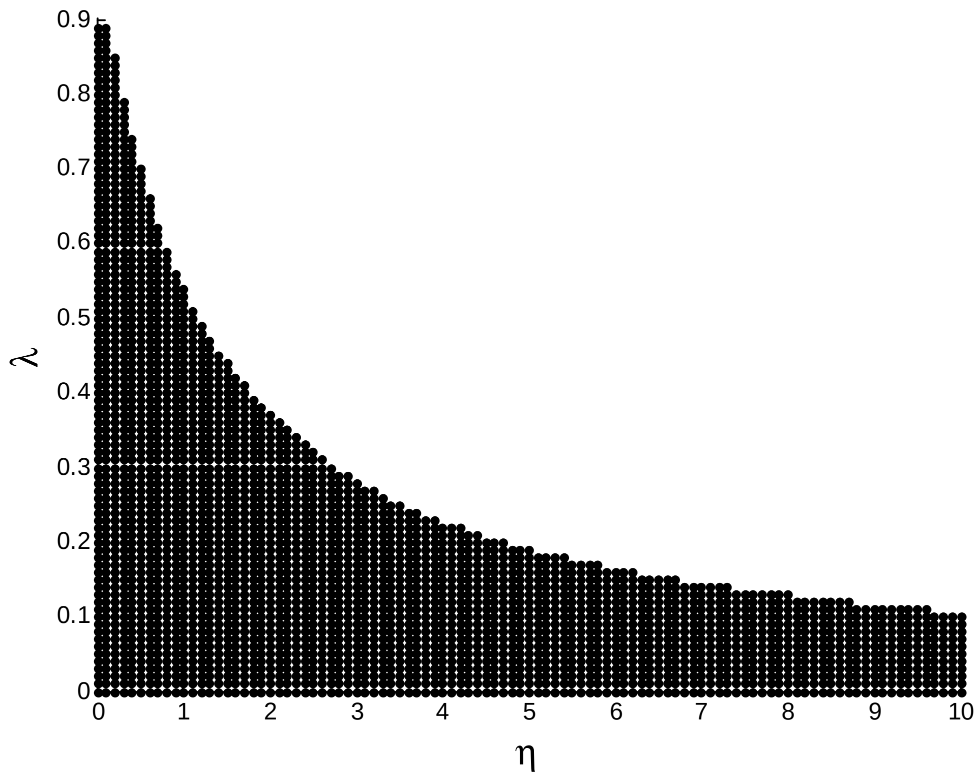

Figure 1.8. Sign of Θ^{ls}. Black spots, $\Theta^{ls} > 0$, white area $\Theta^{ls} < 0$.

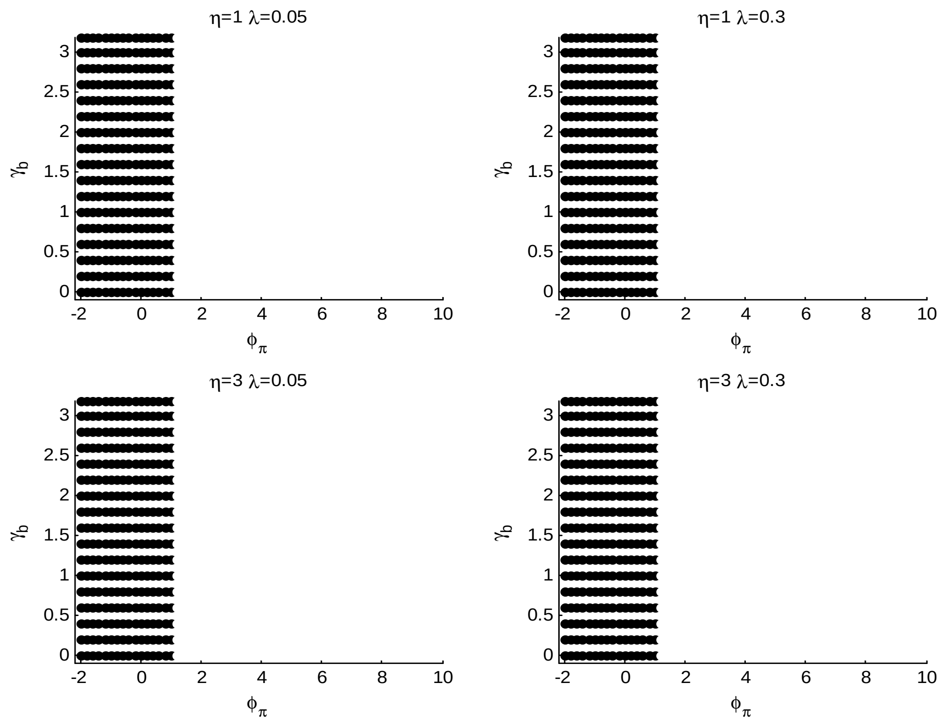

Figure 1.9. Determinacy analysis with a balanced budget fiscal policy, positive Θ^{ls}. White area, determinacy. Black area, indeterminacy.

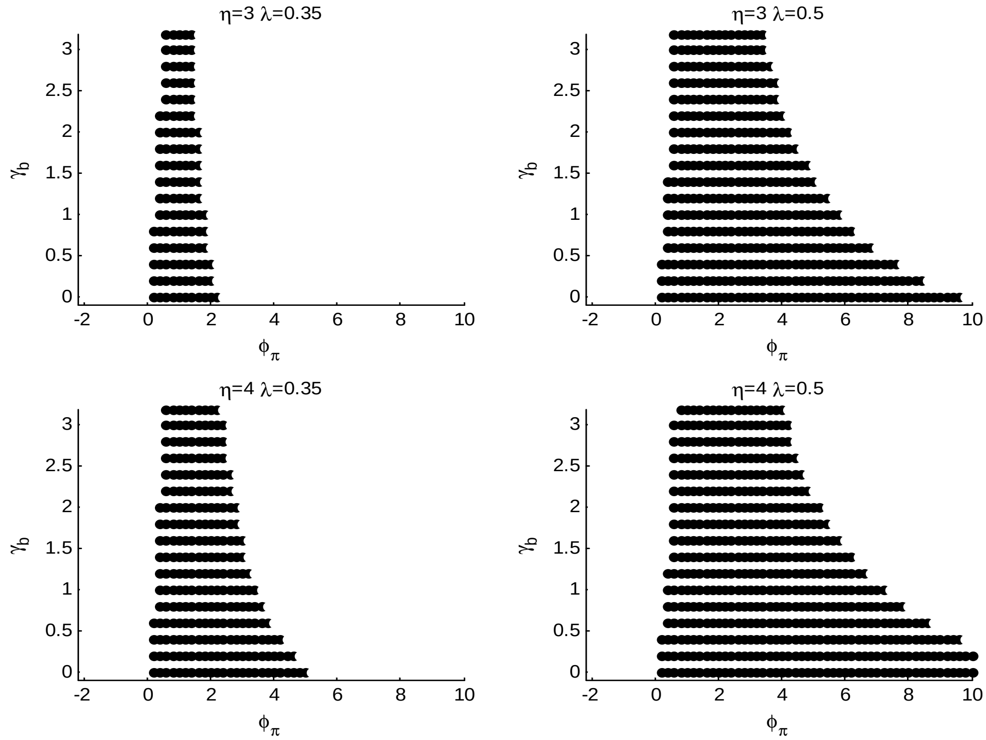

Figure 1.10. Determinacy analysis with a balanced budget fiscal policy, negative Θ^{ls}. White area, determinacy. Black area, indeterminacy.

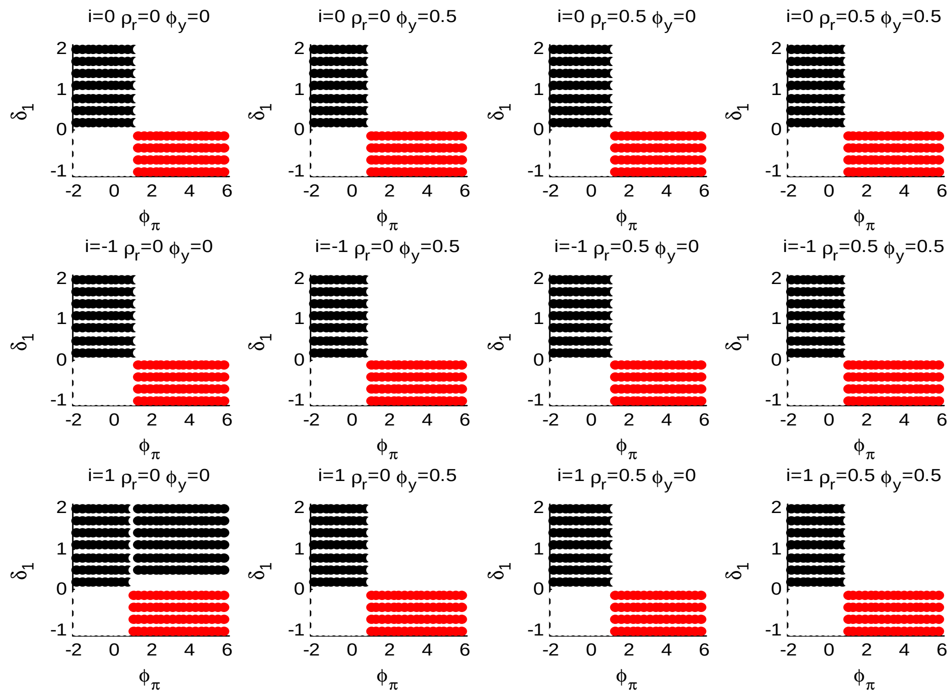

Figure 1.11. Determinacy area with monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$, a fiscal rule of the type $\widehat{\tau}_t^{ls} = \delta_1\widehat{b}_t$ and positive Θ^{ls} ($\lambda = 0.3$ and $\varphi = 1$). White area, determinacy, grey area instability, black area indeterminacy.

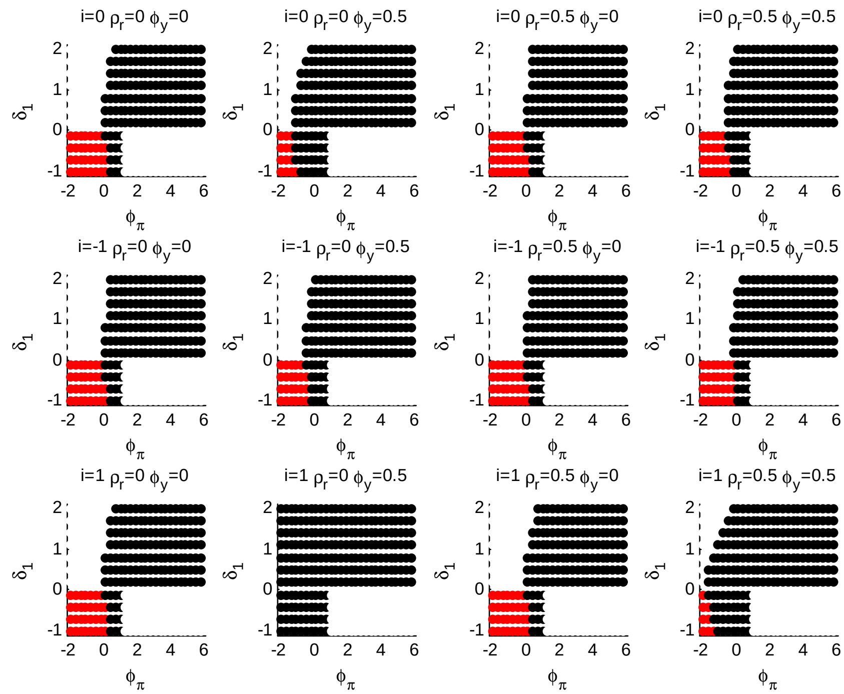

Figure 1.12. Determinacy area with monetary rule of the type $\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)\left(\phi_\pi E\pi_{t+i} + \phi_Y E\widehat{Y}_{t+i}\right)$ with $i = -1, 0, 1$, a fiscal rule of the type $\widehat{\tau}_t^{ls} = \delta_1 \widehat{b}_t$ and negative Θ^{ls} ($\lambda = 0.5$ and $\varphi = 3$). White area, determinacy, grey area instability, black area indeterminacy.

1.A. Appendix

1.A.1. Steady state with labour income taxation

This section describes the steady state of the model with labour income taxation. A few points are worth stressing. First of all, we impose, through a transfer, that the two agents have the same level of consumption and supply the same level of labour at steady state. Hence the heterogeneity between the two consumers is only along the business cycle. Price are normalised to unity and we fix $\frac{G}{Y} = 1 - \gamma_c$. The OPTC budget constraint is

$$C^o = WN^o (1 - \tau) + \frac{D}{1 - \lambda} + \left(1 - R^{-1}\right) \frac{B}{1 - \lambda} + S^o \tag{1.53}$$

Where S^o is the OPTC transfer. The steady state ROTC budget constraint is

$$C^r = (WN^r)(1 - \tau) + S^r \tag{1.54}$$

where S^r is the ROTC transfer. Furthermore we need to impose

$$(1 - \lambda) S^o + \lambda S^r = 0 \tag{1.55}$$

From the steady state Euler equation it is possible to find the steady state interest rate

$$\frac{1}{\beta} = R$$

While the steady state profits follow

$$D = (1 - W) Y \tag{1.56}$$

Homogeneity requires

$$C^o = WN^o (1 - \tau) + \frac{D}{1 - \lambda} + (1 - \beta) \frac{B}{1 - \lambda} + S^o = C^r = WN^r (1 - \tau) + S^r \tag{1.57}$$

Therefore

$$S^o = -\frac{\lambda}{1-\lambda}\left(D + (1-\beta)\,B\right) \tag{1.58}$$

and

$$S^r = -\frac{(1-\lambda)}{\lambda} S^o \tag{1.59}$$

From the firm's marginal cost

$$w = \frac{\varepsilon - 1}{\varepsilon} = \frac{1}{\mu}$$

The steady state government budget constraint can be written as

$$\tau w N = (1-\beta)\,b + G \tag{1.60}$$

Given that $\frac{C}{Y} = \gamma_c$,$\frac{G}{Y} = (1-\gamma_c)$, $Y = N$ and that $\frac{b}{Y} = \gamma_b$ we can rewrite the last equation as

$$\tau = \frac{(1-\beta)\,\gamma_b + (1-\gamma_c)}{w} \tag{1.61}$$

The steady state optimal labour supply it yields

$$\theta\gamma_c\,(N)^{\eta+1} = w\,(1-\tau) \tag{1.62}$$

After rearranging, the latter yields the steady state level of labour supply

$$Y = N = \left(\frac{W\,(1-\tau)}{\theta\gamma_c}\right)^{\frac{1}{\eta+1}} \tag{1.63}$$

Consequently

$$\begin{aligned} G &= (1-\gamma_c)\,Y \\ C &= \gamma_c Y \end{aligned}$$

These equations give us to have a full description of the steady state variables.

1.A.2. Log linearisation with labour income taxation

This section presents a log-linearised version of the model with labour income taxation around the non stochastic steady state. Henceforth, all the upper hat variables identify the variable percentage deviation from its steady state value (i.e.$\widehat{X}_t = \log\left(\frac{X_t}{X}\right)$). While $\pi_t = \log P_t - \log P_{t-1}$ identifies the inflation rate.

The log linearisation of the OPTC Euler equation and optimal supply of labour are

$$\widehat{C}_t^o = E_t\widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{1.64}$$

$$\widehat{C}_t^o + \eta\widehat{N}_t^o = \widehat{w}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{1.65}$$

where $\widehat{w}_t = \widehat{W_t} - \widehat{P}_t$. The ROTC consumption and labour supply follow

$$\widehat{C}_t^r = \widehat{w}_t + \eta\widehat{N}_t^r - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{1.66}$$

$$\eta\widehat{N}_t^r + \widehat{C}_t^r = \widehat{w}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{1.67}$$

Log linearising (1.16) and (1.17) around a zero inflation steady state yields to the traditional New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t\pi_{t+1} + \kappa(\widehat{mc}_t) \tag{1.68}$$

Where $\kappa = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}$. The log linearisation of the aggregation rules for consumption and labour yield

$$\widehat{C}_t = \lambda\widehat{C}_t^r + (1-\lambda)\,\widehat{C}_t^o \tag{1.69}$$

$$\widehat{N}_t = \lambda \widehat{N}_t^r + (1-\lambda)\,\widehat{N}_t^o \tag{1.70}$$

while the market clearing condition follows

$$\widehat{Y}_t = \gamma_c \widehat{C}_t + (1-\gamma_c)\,\widehat{G}_t \tag{1.71}$$

where $\gamma_c = \frac{C}{Y}$. Furthermore from the production function

$$\widehat{Y}_t = \widehat{N}_t \tag{1.72}$$

The log linearisation of the monetary and fiscal rule yields

$$\widehat{R}_t = \phi_\pi \pi_t \tag{1.73}$$

$$\widehat{\tau}_t = \delta_1 \widehat{b}_t + \delta_2 \widehat{Y}_t$$

.Finally, a log linearisation of the government budget constraint can be written as

(1.74)

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left\{\widehat{b}_t - \pi_t - \frac{\tau w}{\gamma_b}\left[\left(\frac{1}{\gamma_c} + \eta + 1\right)\widehat{Y}_t + \frac{1}{1-\tau}\widehat{\tau}_t\right] + \left(\frac{1-\gamma_c}{\gamma_b} + \frac{\tau w\,(1-\gamma_c)}{\gamma_b \gamma_c}\right)\left(\widehat{G}_t\right)\right\}$$

1.A.3. Equilibrium with labour income taxation

This section presents the equilibrium of the model. Further analysis is simplified by rewriting the model as a function of aggregate variables only. First, combining (1.66) with (1.67), we obtain

$$\widehat{N}_t^r = 0 \tag{1.75}$$

and

$$\widehat{C}_t^r = \widehat{w}_t - \frac{\tau}{1-\tau}\widehat{\tau}_t \tag{1.76}$$

From the last two expressions one can see that the introduction of distortive taxation is completely internalised in the ROTC consumption, while their labour supply remains constant at the steady state level.[34] Therefore changes in the tax rate over the business cycle do not have any effect on the ROTC labour supply.

Combining the last expression with the optimal labour supply of the OPTC yields

$$\widehat{C}_t^o + \eta \widehat{N}_t^o = \widehat{C}_t^r \tag{1.77}$$

Furthermore, combining (1.70) with (1.75) it is possible to rewrite the total supply of labour as

$$\widehat{N}_t = (1-\lambda)\,\widehat{N}_t^o \tag{1.78}$$

Therefore aggregate labour fluctuations are just a function of changes in OPTC labour supply. Moreover, plugging these results into the equation for total consumption yields

$$\widehat{C}_t = \lambda\left[\widehat{C}_t^o + \frac{\eta}{1-\lambda}\widehat{N}_t\right] + (1-\lambda)\,\widehat{C}_t^o$$

Simplifying gives

$$\widehat{C}_t = \widehat{C}_t^o + \eta\frac{\lambda}{1-\lambda}\widehat{N}_t \tag{1.79}$$

From the latter we can rewrite the Euler equation in terms of aggregate consumption as

$$\widehat{C}_t = E_t\left(\widehat{C}_{t+1}\right) - \left(\widehat{R}_t - E_t\pi_{t+1}\right) - \eta\frac{\lambda}{1-\lambda}E_t\Delta\widehat{N}_{t+1} \tag{1.80}$$

Substituting in the latter the market clearing condition and the production function one can obtain the dynamic IS equation presented in the main text. On the supply side, using the market clearing condition and the definition of real marginal cost, we can express the New Keynesian Phillips Curve (NKPC) in terms of aggregate variables as

[34]For the ROTC the substitution effect on the labour supply is equal to the income effect.

follows

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(\left(\frac{1}{\gamma_c} + \eta \right) \widehat{Y}_t - \frac{(1-\gamma_c)}{\gamma_c} \widehat{G}_t + \frac{\tau}{1-\tau} \widehat{\tau}_t \right) \tag{1.81}$$

1.A.4. Model with lump-sum taxes

This model shares with its labour income taxation counterpart the shape of the utility function, the production sector, the aggregation and the monetary policy rules. The optimiser budget constraint is

(1.82)

$$P_t C_t^o + R_t^{-1} \frac{B_{t+1}}{1-\lambda} + \frac{E_t \left(Q_{t,t+1} V_{t+1}\right)}{1-\lambda} = \left(W_t N_t^o + \frac{D_t}{1-\lambda} \right) + \frac{B_t}{1-\lambda} + \frac{V_t}{1-\lambda} - P_t \tau_t^{ls} - P_t S^o$$

Where τ_t^{ls} identifies the level (common to the two types of consumer) of lump sum taxes.

The optimisers first order conditions are

$$\beta R_t E_t \left[\left(\frac{C_t^o}{C_{t+1}^o} \right) \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{1.83}$$

Where as before $R_t = \frac{1}{E_t(Q_{t,t+1})}$ is implied by the non arbitrage condition. This expression is the familiar *Euler equation* for consumption. The first order condition with respect to labour states that the *marginal rate of substitution* between labour and consumption must be equal to the real wage

$$\theta \left(N_t^o \right)^\eta C_t^o = \frac{W_t}{P_t} \tag{1.84}$$

The budget constraint for the ROTC is

$$P_t C_t^r = W_t N_t^r - \tau_t^{ls} - S^o \tag{1.85}$$

The ROTC first order condition is

$$\theta \left(N_t^r \right)^\eta C_t^r = \frac{W_t}{P_t} \tag{1.86}$$

while the optimum level of consumption is directly derived from (1.85).

The government budget constraint is

$$R_t^{-1} b_{t+1} = \frac{b_t}{\pi_t} - \tau_t^{ls} + G_t \tag{1.87}$$

Steady state with lump sum taxation

This section sketches the steady state for the model with lump-sum taxation.

$$C^o = WN^o + \frac{D}{1-\lambda} + \left(1 - R^{-1}\right) \frac{B}{1-\lambda} - \tau^{ls} + S^o \tag{1.88}$$

Where S^o is the OPTC transfer. The steady state ROTC budget constraint is

$$C^r = (WN^r) - \tau^{ls} + S^r \tag{1.89}$$

where S^r is the ROTC transfer. Furthermore we need to impose

$$(1-\lambda) S^o + \lambda S^r = 0 \tag{1.90}$$

From the steady state Euler equation it is possible to find the steady state interest rate

$$\frac{1}{\beta} = R$$

While the steady state profits follow

$$D = (1 - W) Y \tag{1.91}$$

Homogeneity requires

$$C^o = \left(WN^o + \frac{D}{1-\lambda}\right) + (1-\beta) \frac{B}{1-\lambda} + S^o = C^r = WN^r + S^r \tag{1.92}$$

Therefore

$$S^{o}=-\frac{\lambda}{1-\lambda}\left(D+\left(1-\beta\right)B\right) \tag{1.93}$$

and

$$S^{r}=-\frac{\left(1-\lambda\right)}{\lambda}S^{o} \tag{1.94}$$

The steady state government budget constraint can be written as

$$\tau^{ls}=\left(1-\beta\right)B+G \tag{1.95}$$

Given that $\frac{C}{Y}=\gamma_c$,$\frac{G}{Y}=(1-\gamma_c)$ and that $\frac{B}{Y}=\gamma_b$ we can rewrite the last equation as

$$\frac{\tau^{ls}}{Y}=\left(1-\beta\right)\gamma_b+\left(1-\gamma_c\right) \tag{1.96}$$

Combining the fact that at steady state $Y=N$ with the steady state optimal labour supply it yields

$$\theta\gamma_c\left(N\right)^{\eta+1}=W \tag{1.97}$$

After rearranging, the latter yields the steady state level of labour supply

$$Y=N=\left(\frac{W}{\theta\gamma_c}\right)^{\frac{1}{\eta+1}} \tag{1.98}$$

Consequently

$$G = \left(1-\gamma_c\right)Y \tag{1.99}$$

$$C = \gamma_c Y \tag{1.100}$$

1.A.5. Log-linearisation and equilibrium with lump sum taxation

This paragraph derives the log-linearisation of the demand side of the economy with lump sum taxes. Log linearisation of the first order conditions for both types of consumers yields

$$\widehat{C}_t^o = E_t\widehat{C}_{t+1}^o - \left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{1.101}$$

$$\widehat{C}_t^o + \eta\widehat{N}_t^o = \widehat{w}_t \tag{1.102}$$

$$\widehat{C}_t^r = \left(\frac{1}{\mu\gamma_c}\right)\left(\widehat{N}_t^r + \widehat{w}_t\right) - \frac{\tau^{ls}}{C}\left(\widehat{\tau}_t^{ls}\right) \tag{1.103}$$

where $W = \frac{1}{\mu}$ and $\frac{N}{C} = \frac{1}{\gamma_c}$.

$$\widehat{C}_t^r + \eta\widehat{N}_t^r = \widehat{w}_t \tag{1.104}$$

From the aggregation rules

$$\widehat{C}_t = \lambda\widehat{C}_t^r + (1-\lambda)\,\widehat{C}_t^o \tag{1.105}$$

$$\widehat{N}_t = \lambda\widehat{N}_t^r + (1-\lambda)\,\widehat{N}_t^o \tag{1.106}$$

The market clearing conditions are

$$\widehat{Y}_t = \gamma_c\widehat{C}_t + (1-\gamma_c)\,\widehat{G}_t \tag{1.107}$$

$$\widehat{Y}_t = \widehat{N}_t \tag{1.108}$$

Therefore the total labour supply follows

$$\widehat{C}_t + \eta\widehat{N}_t = \widehat{w}_t \tag{1.109}$$

Plugging the aggregation rules and the labour supply into (1.103) it yields

$$\widehat{C}_t^r = \left(\frac{(1+\eta)\,\eta}{1+\eta\mu\gamma_c}\right)\widehat{N}_t + \frac{1+\eta}{1+\eta\mu\gamma_c}\widehat{C}_t - \frac{\mu\tau^{ls}\eta}{Y\,(1+\eta\mu\gamma_c)}\left(\widehat{\tau}_t^{ls}\right) \tag{1.110}$$

Substituting the aggregation rule into the optimisers' Euler equation one obtains

$$\widehat{C}_t - \lambda\widehat{C}_t^r = E_t\widehat{C}_{t+1} - E_t\widehat{C}_{t+1}^r - (1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{1.111}$$

Using (1.110) we can write the aggregate demand as

$$\Gamma_c\widehat{C}_t = \Gamma_c E_t\widehat{C}_{t+1} - \Gamma_n E_t\Delta\widehat{N}_{t+1} + \Gamma_T E_t\Delta\widehat{\tau}_{t+1}^{ls} - (1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{1.112}$$

where $\Gamma_c = \left(1 - \frac{\lambda(1+\eta)}{1+\eta\mu\gamma_c}\right)$, $\Gamma_n = \frac{\lambda(1+\eta)\eta}{1+\eta\mu\gamma_c}$, $\Gamma_T = \frac{\lambda\tau^{ls}\eta}{Y(1+\eta\mu\gamma_c)}$ and $\Delta\widehat{X}_t = \widehat{X}_t - \widehat{X}_{t-1}$. Finally, using the market clearing condition and imposing $\widehat{G}_t = 0$ at all time it yields

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \Gamma_c^{-1}\Theta^{ls}\,(1-\lambda)\left(\widehat{R}_t - E_t\pi_{t+1}\right) + \Gamma_\tau\Gamma_c^{-1}\Theta^{ls}\left(E_t\widehat{\tau}_{t+1}^{ls} - \widehat{\tau}_t^{ls}\right) \tag{1.113}$$

as in the main text.

The log-linearisation of the government budget constraint is

$$\widehat{b}_{t+1} = \widehat{R}_t + \frac{1}{\beta}\left(\widehat{b}_t - \pi_t - \frac{\tau^{ls}}{b}\widehat{\tau}_t^{ls}\right) \tag{1.114}$$

1.A.6. Analytical determinacy analysis: the case of a balanced budget rule

1.A.6.1. Case with labour income taxation. After few algebraical substitutions we can write the model with a balanced budget rule as

$$E_t x_{t+1} = J^{br} x_t \tag{1.115}$$

where x is defined in the main text and

$$J^{br} = \begin{bmatrix} 1 + \frac{\Lambda_1\Theta}{\beta} & \Theta\phi_\pi - \frac{\Theta(1+(1-\phi_\pi\beta)\Psi)}{\beta} \\ -\frac{\Lambda_1}{\beta} & \frac{(1+(1-\phi_\pi\beta)\Psi)}{\beta} \end{bmatrix}$$

Where $\Lambda_1 = k\left(\frac{1-\tau}{\gamma_c} + (1-\tau)\eta - \tau\right)$, $\Psi = \frac{k\gamma_b}{w} \geq 0$. We assume that $\tau < \left(1 + \frac{\gamma_c}{1+\gamma_c\eta}\right)^{-1}$. As explained in the main text, the latter assumption implies that $\Lambda_1 > 0$. Given that the x vector contains two jump variables, determinacy requires that both eigenvalues of J^{br} lye outside the unit circle. Determinant and trace of J^{br} are respectively $Det(J) = \frac{1+\Lambda_1\Theta\phi_\pi+\Psi-\phi_\pi\beta\Psi}{\beta}$ and $Tr(J) = \frac{1+\beta+\Lambda_1\Theta+\Psi-\phi_\pi\beta\Psi}{\beta}$. We start from the case where $\Theta > 0$. Following Woodford (2003, appendix C), every determinate equilibrium satisfies either criterion I with

$$\text{(I.a):} \qquad Det(J) > 1 \Longleftrightarrow \frac{1+\Lambda_1\Theta\phi_\pi+\Psi-\phi_\pi\beta\Psi}{\beta} > 1$$

$$\text{(I.b):} \qquad Det(J) - Tr(J) > -1 \Longleftrightarrow \Lambda_1\Theta(\phi_\pi - 1) > 0$$

$$\text{(I.c):} \qquad Det(J) + Tr(J) > -1 \Longleftrightarrow \frac{2+\beta+\Lambda_1\Theta+\Lambda_1\Theta\phi_\pi+2\Psi-2\beta\Psi\phi_\pi}{\beta} > -1$$

or criterion II as

$$\text{(II.a):} \qquad Det(J) - Tr(J) < -1 \Longleftrightarrow \Lambda_1\Theta(\phi_\pi - 1) < 0$$

$$\text{(II.b):} \qquad Det(J) + Tr(J) < -1 \Longleftrightarrow \frac{2+\beta+\Lambda_1\Theta+\Lambda_1\Theta\phi_\pi+2\Psi-2\beta\Psi\phi_\pi}{\beta} < -1$$

We want to express the determinacy conditions in terms of the monetary policy parameter ϕ_π. (I.a) implies that

$$\begin{aligned} \text{if } \Lambda_1 &< \frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi < \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1} \\ \text{elseif } \Lambda_1 &> \frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi > \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1} \end{aligned}$$

while (I.b) implies

$$\phi_\pi > 1$$

and (I.c)

$$\begin{aligned}\text{if } \Lambda_1 &< 2\frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi < \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\\ \text{elseif } \Lambda_1 &> 2\frac{\Psi\beta}{\Theta} \Longrightarrow \phi_\pi > \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\end{aligned}$$

Putting things together criterion I implies

$$\begin{aligned}\text{if } \Lambda_1 &> 2\frac{\Psi\beta}{\Theta} \Longrightarrow \phi > 1\\ \text{elseif } \frac{\Psi\beta}{\Theta} &< \Lambda_1 < 2\frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\\ \text{elseif } \Lambda_1 &< \frac{\Psi\beta}{\Theta} \Longrightarrow 1 < \phi_\pi < \min\left\{\frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}, \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\right\}\end{aligned}$$

Criterion II can be ruled out due to sign restrictions.[35]■

Let now turn to the case when $\Theta < 0$. (I.b) implies $\phi_\pi < 1$, while (I.a) is verified when

$$\phi_\pi < \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1} \tag{1.116}$$

and (I.c)

$$\phi_\pi < \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} \cap \Lambda_1 \in \left(0, -2\frac{(1+\beta+\Psi)}{\Theta}\right) \tag{1.117}$$

Therefore when $\Theta < 0$ criterion I implies[36]

$$\phi_\pi < \min\left\{1, \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}\right\} \tag{1.118}$$

When $\Theta < 0$, criterion II cannot be ruled out, therefore (II.a) implies

$$\phi_\pi > 1 \tag{1.119}$$

[35]In fact (II.a) implies $\phi_\pi < 1$, while (II.b) requires $\phi_\pi > \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} \cap \Lambda_1 < 2\frac{\Psi\beta}{\Theta}$. Note that if $\Lambda_1 < 2\frac{\Psi\beta}{\Theta}$, $\frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}$ is greater than one.

[36]The condition $\Lambda_1 \in \left(0, -2\frac{(1+\beta+\Psi)}{\Theta}\right)$ is always verified within standard parametrisations. This also imples that $\frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} > 1$. Hence (I.c) is not binding for standard parametrisation.

and (II.b)

$$\phi_\pi > \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta} \tag{1.120}$$

This yields, for criterion II, to

$$\phi_\pi > \max\left\{1, \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\right\} \tag{1.121}$$

Therefore, putting things together, when $\Theta < 0$ there are two determinacy spaces

$$\phi_\pi < \min\left\{1, \frac{1+\Psi-\beta}{\beta\Psi-\Theta\Lambda_1}\right\} \;\cup\; \phi_\pi > \max\left\{1, \frac{2+2\beta+2\Psi+\Lambda_1\Theta}{2\beta\Psi-\Lambda_1\Theta}\right\} \quad \blacksquare \tag{1.122}$$

1.A.6.2. Case with lump sum taxation. The relevant matrix is

$$J^{ls} = \begin{bmatrix} 1+\frac{\Gamma_c^{-1}\Theta^{ls}\chi(\beta-\Upsilon+\beta\Upsilon\phi_\pi)}{\beta^2} & \frac{\Gamma_c^{-1}\Theta^{ls}(\beta+(\beta-1)\Upsilon)(\phi_\pi\beta-1)}{\beta^2} \\ -\frac{\chi}{\beta} & \frac{1}{\beta} \end{bmatrix}$$

Note that $\Upsilon = \frac{\lambda\eta\gamma_b}{(1+\eta\mu\gamma_c)} \geq 0$ and $\chi = k\left(\frac{1}{\gamma_c}+\eta\right) > 0$. Given that the x vector contains two jump variables, determinacy requires that both eigenvalues of J^{ls} lye outside the unit circle. Determinant and trace of J^{ls} are respectively $Det\left(J^{ls}\right) = \frac{1}{\beta} + \frac{\Gamma_c^{-1}\chi\Theta^{ls}(\beta\phi_\pi(1+\Upsilon)-\Upsilon)}{\beta^2}$ and $Tr\left(J^{ls}\right) = 1+\frac{\Gamma_c^{-1}\Theta^{ls}\chi(\beta-\Upsilon+\beta\Upsilon\phi_\pi)}{\beta^2}+\frac{1}{\beta}$. In the same fashion adopted in the case of labour income taxation, we follow Woodford (2003, appendix C). Every determinate equilibrium satisfies either criterion I

$$\text{(I.a)}: \quad Det\left(J^{ls}\right) > 1$$

$$\text{(I.b)}: \quad Det\left(J^{ls}\right) - Tr\left(J^{ls}\right) > -1$$

$$\text{(I.c)}: \quad Det\left(J^{ls}\right) + Tr\left(J^{ls}\right) > -1$$

or criterion II

$$\text{(II.a):} \qquad Det\left(J^{ls}\right) - Tr\left(J^{ls}\right) < -1$$

$$\text{(II.b):} \qquad Det\left(J^{ls}\right) + Tr\left(J^{ls}\right) < -1$$

Let start from criterion I when $\Theta^{ls} > 0$. It is easy to show that (I.b) is verified if and only if $\phi_\pi > 1$. Furthermore, if (I.b) holds, (I.a) and (I.c) are verified as well.

We can rule out criterion II due to sign restrictions.[37] ■

Now we turn to study the determinacy conditions when $\Theta^{ls} < 0$. Let start with criterion I. (I.a) implies

$$\phi_\pi < \xi_3 \tag{1.123}$$

With $\xi_3 = \frac{\beta^2 - \beta + \Upsilon\Gamma_c^{-1}\chi\Theta^{ls}}{\beta\Gamma_c^{-1}\chi\Theta^{ls}(1+\Upsilon)}$. (I.b) implies

$$\phi_\pi < 1 \tag{1.124}$$

while (I.c) implies

$$\phi_\pi < \xi_4 \tag{1.125}$$

where $\xi_4 = \frac{-2\beta - 2\beta^2 - \beta\Gamma_c^{-1}\chi_1\Theta^{ls} + 2\Gamma_c^{-1}\Upsilon\chi_1\Theta^{ls}}{\beta\Gamma_c^{-1}\chi_1\Theta^{ls}(1+2\Upsilon)}$.

Let now analyse criterion II. (II.a) implies

$$\phi_\pi > 1 \tag{1.126}$$

while (II.b) implies

$$\phi_\pi > \xi_4$$

[37]As in the analogous case with labour income taxation, (II.a) requires $\phi_\pi < 1$ while (II.b) requires $\phi_\pi > 1$.

Summing up the results: when $\Theta^{ls} < 0$ necessary and sufficient conditions for determinacy require

$$\phi_{\pi} < \min(1, \xi_3, \xi_4) \cup \phi_{\pi} > \max(1, \xi_4) \quad \blacksquare \tag{1.127}$$

CHAPTER 2

Indeterminacy with trend inflation and fiscal policy rules

This chapter studies the equilibrium determinacy of a New Keynesian model augmented with trend inflation (see for example Ascari and Ropele (2007), Schmitt-Grohe and Uribe (2007), Ascari and Rossi (2009)), public debt and distorionary taxation. Interest rate policies influence the government budget constraint and distortionary taxation feeds back on the model endogenous variables. When the fiscal authority follows a balanced budget rule without access to short run public debt and the system experiences a long run level of inflation, the model easily displays indeterminacy regardless of the conduct of monetary policy. When short run deficits are allowed, the equilibrium determinacy requires a stronger monetary response to inflation, the stronger is the fiscal response to short run fluctuations in public debt and the higher is the level of trend inflation. In this case we find also that, ceteris paribus, a higher level of steady state public debt increases the possibility of determinacy.

2.1. Introduction

Assuming[1] zero long run inflation in the standard New Keynesian model, as in Woodford (2003), may lead to policy analyses which are empirically unrealistic and theoretically misleading. Empirically unrealistic because in the developed economies the long run rate of inflation is moderately different from zero, theoretically misleading because the zero long run inflation assumption and the Calvo price setting mechanism, relegates to a second order importance many relations between variables, such as the correlation between output and price dispersion. These have to be taken into account studying the more general case of positive long run inflation. The economic implications of long run

[1]This chapter was thought and written during my visiting period at the Monetary Policy Strategy division at the European Central Bank. I would like to thank Massimo Rostagno, Jean-Pier Vidal and Leopold Von Thadden as well as all the participants at the ECB division seminar for useful comments.

inflation can be divided into three fields: implications for the transmission mechanism of shocks to fundamentals (Ascari and Ropele, 2007; Amano et al. 2007), implications for the conduct of optimal policy (Schmitt-Grohe and Uribe, 2007) and implications for equilibrium determinacy (Ascari and Ropele, 2009) . This paper explores the last of these points.

There is a large literature that studies the equilibrium determinacy conditions in the New Keynesian (NK henceforth) model.[2] The basic results of this literature could be described as follows. In a simple monetary NK model with no capital accumulation, equilibrium determinacy is guaranteed by a monetary policy that reacts to an increase in inflation by raising the real interest rate. This result is commonly known as the *Taylor Principle* (*TP* henceforth), as discussed in the literature on interest rate rules inspired by Taylor (1993). Let us consider that agents suddenly expect higher inflation. Monetary policy reacting to these expectations by raising the interest rate contracts current demand and therefore current output. Given the positive correlation between current output and inflation implied by the *New Keynesian Phillips Curve*, this contraction in output reduces current inflation, invalidating the initial expectations of inflation.

In a NK model with distortive fiscal policy and public debt equilibrium determinacy is guaranteed by a particular mix of monetary and fiscal policy. In particular, when fiscal policy reacts to increases in the stock of public debt by raising the tax rate (*passive* fiscal policy), monetary policy can control inflation through the adoption of a policy that respects the TP (*active* monetary policy). On the other hand, if fiscal policy does not control the stock of public debt (*active* fiscal policy), in order to have a unique rational expectations equilibrium, monetary policy must deflate the cost of public debt by cutting the real rate in response to inflation (*passive* monetary policy). This result is due to Leeper (1991) and extended by Leith and Wren-Lewis (2000) and is commonly known as the *Active-Passive policy mix*.

[2] See inter alia Woodford (2003) and Leeper (1991).

In a monetary NK model with trend inflation and the Calvo price mechanism, equilibrium determinacy requires monetary policy to be more aggressive against inflation the higher the level of long run inflation.[3] As shown by Ascari and Ropele (2007, 2009), with trend inflation, inflation expectations increase their importance in the NKPC on the determination of current inflation relative to the output component. Furthermore, an increase in inflation generates price dispersion that in this context is a relevant variable of first order importance (Schmitt-Grohe and Uribe, 2007). This endogenous variable, which in the traditional NK model with zero trend inflation is relegated to second order importance, affects output negatively and inflation positively. Therefore pinning down expectations of future inflation requires monetary policy to move output through the traditional demand channel more than in the case with no long run inflation (Ascari and Ropele, 2009; Ascari and Rossi, 2009a, b).

This paper studies the equilibrium requirements of monetary and fiscal policy in a NK model with a distortive labour income tax, a steady state level of public debt and a positive level of steady state inflation. In order to do so we approximate the conduct of monetary and fiscal policies by simple rules. In particular we assume that monetary policy fixes the nominal interest rate as a function of current inflation rate while fiscal policy collects proportional labour income taxes.

We run two determinacy exercises. In the first one we assume that fiscal policy balances the government budget constraint in each period without accessing to public debt. In this case the only aim is collecting taxes to repay the service of the steady state level of public debt, and finance a time-independent level of public spending. This type of fiscal rule was first introduced by Schmitt-Grohe and Uribe(1997) in a simple *Real Business Cycle* model with capital accumulation and then studied in a traditional *NK* model with no capital accumulation by Linnemann (2006). In the former the combination of a balanced budget fiscal rule and capital accumulation could easily lead to indeterminacy

[3]Benhabbib and Eusebi (2005) and Ascari and Rossi(2009a, b) analyse an economy with steady state trend inflation and Rotemberg price setting.

for parameter values consistent with the empirical evidence, in the latter indeterminacy is instead generated by the combination of a balanced budget rule and the presence of a steady state level of public debt, regardless of whether or not monetary policy[4] is active.

When we analyse a balanced budget rule in our model we find that even in the case of zero steady state public debt and no capital accumulation, a balanced budget rule and a moderately positive level of trend inflation lead to situations of sunspots fluctuation regardless of the conduct of monetary policy. This result is driven by the combination of fiscal feedback on the supply side of the economy, i.e. positive correlation between tax rate and current inflation, and by the monetary policy feedback on the government budget constraint, i.e. with trend inflation monetary policy has to react more strongly against inflation in order to pin down inflation expectations and this increases the costs of serving public debt. This result can be considered as an extension along the line of Linnemann (2006): adopting a balanced budget rule in the contest of a NK model may not be a wise idea given that, regardless of the conduct of monetary policy, it can easily lead to indeterminacy.

In our second exercise we allow fiscal policy to access public debt. We therefore approximate the conduct of fiscal policy in the fashion of Leeper (1991), Leith and von Thadden (2008). This allows us to analyse the policy mix from an *active-passive* perspective. We find that the introduction of trend inflation does not affect the determinacy properties of passive monetary policy/ active fiscal policy while it does change the equilibrium analysis for the combination of active monetary/passive fiscal policy. When describing the latter policy mix, we find that the greater is the fiscal policy reaction to fluctuations of short run public debt, the more aggressive monetary policy has to be against inflation in order to guarantee determinacy. Increasing the level of trend inflation reduces the determinacy area to combinations of strong fiscal reactions to public debt and strong monetary policy reactions to inflation. These results are driven by the joint

[4]SGU(1997) show that in a RBC with no capital accumulation, a balanced budget rule does not lead to indeterminacy. For this reason is crucial for indeterminacy the presence of long run debt in the NK model analysed in Linnemann(2005).

effects of fiscal policy feedback on the supply side of the economy and the difficulties pinning down inflation expectations with positive trend inflation.

The paper is organized as follows. Section 2.1 outlines the model, section 2.2 describes the determinacy results, section 2.3 concludes.

2.2. Model

The economy is represented here by a New Keynesian model (NK) with a government which can balance its budget by levying a proportional labour income tax, and has to deal with a steady state level of public debt. Furthermore, the system is affected by a (low) level of trend inflation. Hence, the economy is populated by four type of agents: a continuum of identical consumers who decide how much to consume and work, a continuum of monopolistically competitive firms that decide how much to produce and at which price to sell, a monetary authority that fixes the nominal interest rate in every period and a fiscal authority that, using different specifications, balance its budget in every period.

2.2.1. Households

There is a continuum of households normalized to one. Each of these has a lifetime utility function defined as

$$U_0 = E_0 \sum_{t=0}^{+\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta} \right) \tag{2.1}$$

where E_t is the rational expectation (RE) operator, $\beta \in (0,1)$ is the discount factor, C_t represents the consumption bundle, N_t is the labour supplied in a Walrasian labour market, $\sigma \geq 1$ is the $CRRA$ parameter and $\eta \geq 0$ is the inverse-Frisch parameter. The consumer problem consists in maximising (4.1) subject to a (nominal) budget constraint

$$\int_0^1 P_{it} C_{it} di + E_t Q_{t,t+1} D_{t+1} = \Gamma_t + W_t N_t \left(1 - \tau_t\right) + D_t \tag{2.2}$$

where P_{it} is the price of variety i, D_{t+1} is the nominal payoff of a state contingent asset, $E_t Q_{t,t+1}$ is the stochastic discount factor, Γ_t is the representative household's share of profits in the imperfectly competitive firms, W_t are wages and τ_t is the labour income tax rate. Households must first decide how to allocate a given amount of expenditure across the i products. In doing so they exploit any price difference present in the economy so as to minimise a total expenditure function subject to the *CES* Dixit-Stiglitz (1977) aggregator of the type

$$C_t = \left(\int_0^1 C_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3}$$

It is easy to show that the minimisation problem results in a demand for each single i product which is decreasing in P_{it} and increasing in total consumption as

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} C_t \tag{2.4}$$

The parameter of substitutability among goods, ε, identifies the degree of monopolistic competition present in the system(i.e. $\varepsilon \to +\infty$ implies perfect competition). Furthermore, from the same minimization problem we can infer the general price level P_t as

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \tag{2.5}$$

Hence we can write $\int_0^1 P_{it} C_{it} di = P_t C_t$. Maximizing (2.1) subject to (2.2) results in the household's first order conditions

$$\beta R_t \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) = 1 \tag{2.6}$$

$$\chi C_t^{\sigma} N_t^{\eta} = \frac{W_t}{P_t} (1 - \tau_t) \tag{2.7}$$

where R_t is the one period risk free nominal rate and it is derived from the non arbitrage condition

$$R_t = \frac{1}{E_t Q_{t,t+1}}$$

Expression (2.7) identifies the household labour decision: the marginal rate of substitution between consumption and leisure must be equal to the after tax real wage. Fiscal policy is distortive because the tax rate changes the consumption-leisure decision of the representative household. Taking the conditional expectation of (2.6) on both side and rearranging it yields

$$\beta E_t R_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \left(\frac{P_t}{P_{t+1}}\right) = 1 \tag{2.8}$$

This is the traditional *Euler* equation, which indicates the consumer propensity of smoothing consumption across time, having taken into account the opportunity cost represented by the real interest rate.

For future reference it is useful to define the *real* stochastic discount factor as

$$q_{t,t+z} = \beta^{z} \left(\frac{C_t}{C_{t+z}}\right)^{\sigma} \tag{2.9}$$

2.2.2. Government

The government allocates its total consumption G_t, exploiting optimally, as the households, any price difference present in the economy. It can be shown that the government demand for each (public) consumption good is given by

$$G_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} G_t \tag{2.10}$$

In order to finance its expenditure the government levies a proportional labour income tax, τ_t and releases one (beginning of) period state-contingent risk-less bonds.[5] The

[5]The assumption of complete markets is neutral to the aim of the paper.

government budget constraint can be written as

$$W_t N_t \tau_t + R_t^{-1} B_{t+1} = P_t G_t + B_t \tag{2.11}$$

let us define real public debt as $b_t = \frac{B_t}{P_{t-1}}$ and the real wage as $w_t = \frac{W_t}{P_t}$. The government budget constraint in real term becomes

$$w_t N_t \tau_t + R_t^{-1} b_{t+1} = G_t + \frac{b_t}{\pi_t} \tag{2.12}$$

Furthermore, for the sake of simplicity we assume that government spending never deviates from its steady state level, i.e. $G_t = G \; \forall t$.

2.2.3. Monetary Policy

We define the behavior of monetary policy by (standard) simple rules. In particular monetary policy sets the nominal interest rate as

$$R_t = \phi_s \left(\pi_t\right)^{\phi_1} \tag{2.13}$$

with $\phi_s = \left(\frac{R}{\pi^{\phi_1}}\right)$. The single policy parameter ϕ_1 in (2.13) is the Taylor-coefficient, as discussed in the literature on interest rate rules inspired by Taylor (1993). Accordingly, monetary policy is called '*active*' ('*passive*') if the real interest rate rises (falls) in the current inflation rate, i.e. if $\phi_1 > 1$ ($\phi_1 < 1$). The literature is quite familiar with this type of rule, e.g. Clarida (2000), Benhabib and Eusebi (2005), Ascari and Ropele (2007), Leith and von Thadden (2008) because it allows a relative simple policy analysis in the fashion of Leeper (1991).

2.2.4. Firms

We assume there is a continuum of monopolistic competitive firms indexed by $i \in [0,1]$. Each of these firms uses a linear technology in labour of the type

$$Y_{it} = N_{it} \tag{2.14}$$

We introduce sticky prices as in Calvo (1983). In each period each firm has a fixed probability $1-\alpha$ with $\alpha \in (0,1)$, to optimally reset its price. This implies that when it can reset its prices it takes into account the expected future discounted sum of profits for the periods in which it cannot do so. Hence the pricing problem becomes dynamic in nature. The optimal behavior of a re-setter can be formalized as follows

$$\max_{P_{it}^*} E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(\frac{P_{it}^*}{P_{t+z}} Y_{it+z} - mc_{it+z} Y_{it+z} \right)$$

subject to the demand function obtained from the aggregation of private and public consumption as $Y_{it+z} = \left(\frac{P_{it}^*}{P_{t+z}}\right)^{-\varepsilon} Y_{t+z}$. The variable mc_{it+z} identifies the real marginal cost function of the i firm and, given that capital accumulation is not considered here, is simply equal to the *Walrasian* real wage, i.e. $\frac{W_t}{P_t}$. As previously described we do not restrict the analysis to the special case of no long run inflation. On the contrary, we consider scenarios in which steady state inflation is greater than zero. It is easy to show that the first order conditions with respect to P_{it}^* is

$$P_{it}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{C_t}{C_{t+z}}\right)^{\sigma} \left(P_{t+z}^{\varepsilon} Y_{t+z} mc_{t+z}\right)}{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{C_t}{C_{t+z}}\right)^{\sigma} \left(P_{t+z}^{\varepsilon-1} Y_{t+z}\right)} \tag{2.15}$$

Given the presence of long run inflation, i.e. $\pi > 1$, it is useful to re-express (2.15) including the cumulative gross rate of inflation

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{C_t}{C_{t+z}}\right)^{\sigma} \left(\Pi_{t+1,t+z}^{\varepsilon} Y_{t+z} mc_{t+z}\right)}{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{C_t}{C_{t+z}}\right)^{\sigma} \left(\Pi_{t+1,t+z}^{\varepsilon-1} Y_{t+z}\right)} \tag{2.16}$$

where we define

$$\Pi_{t,t+z-1} = \left(\frac{P_{t+z-1}}{P_{t-1}}\right) \quad for\ z \geq 1 \text{ or}$$
$$= 1\ for\ z = 0$$

2.2.5. Market Clearing

The market clearing conditions are

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} (G + C_t) \text{ and } N_{it} = Y_{it} = C_{it} + G_i$$

Therefore we can write

(2.17)
$$N_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t$$

Integrating the last expression over the i products it yields

(2.18)
$$N_t = \int_0^1 N_{it} di = \int_0^1 \left(\frac{P_{it}}{P}\right)^{-\varepsilon} di Y_t = s_t Y_t$$

The variable $s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$ measures the relative price dispersion across firms. Its law of motion can be written as

(2.19)
$$s_t = (1-\alpha)\left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} + \alpha \pi_t^{\varepsilon} s_{t-1}$$

In the case with no trend inflation, i.e. $\pi = 1$, the variable s_t is not relevant up to a second order. In the case where $\pi > 1$ the relative price dispersion starts to matter up to a first order, i.e. in a log-linearized world.

The general price level is a weighted average between the re-setters and those that do not reset their price as

(2.20)
$$P_t^{1-\varepsilon} = \left[(1-\alpha)\left(P_t^*\right)^{1-\varepsilon} + \alpha P_{t-1}^{1-\varepsilon}\right]$$

where we drop the i subscript due that all the re-setters will choose the same price. Dividing on both side for $P_t^{1-\varepsilon}$ yields

$$1 = (1-\alpha)\left(p_t^*\right)^{\varepsilon-1} + \alpha\pi_t^{\varepsilon-1} \tag{2.21}$$

where π_t represents the inflation rate and p_t^* is the relative optimal price.

2.2.6. Fiscal Policy and Determinacy

We run two exercises on local determinacy analysis. The first one consists of analysing the equilibrium dynamics of the model presented above and with a fiscal policy which balances its budget without accessing to short run public debt, i.e. $b_t = B\ \forall t$. This type of fiscal policy is similar to the one presented by Schmitt-Grohe and Uribe (1997) and Linnemann (2006). It implies that the tax rate is adjusted in order to guarantee in each period that

$$w_t N_t \tau_t = G + b\left(\frac{1}{\pi_t} - \frac{1}{R_t}\right) \tag{2.22}$$

In the second exercise we allow taxes and short run debt to vary along the business cycle. In this more general case, fiscal authority changes the tax rate in each period following a rule of the type

$$\tau_t = \theta_s + \delta_1\frac{\tau}{b}\left(b_t - b\right) + \delta_2\frac{\tau}{Y}\left(Y_t - Y\right) \tag{2.23}$$

$$with\ \theta_s = \frac{G}{wN} + \left(\frac{1}{\pi} - \frac{1}{R}\right)\frac{b}{wN} \tag{2.24}$$

in order to balance in each period the government budget constraint as defined in (2.12). Following the logic of Leeper (1991), we call the fiscal rule (2.23) '*passive*'[6] if $\delta_1 > \frac{1}{\pi} - \frac{1}{\beta}$, while[7] it is '*active*' in the opposite and non-stabilising case of $\delta_1 < \frac{1}{\pi} - \frac{1}{R}$.

The first step in studying local determinacy consists of log-linearising the structural equations of the model around the non-stochastic steady state. A hatted variable represents the variable deviation from its steady state value, i.e. $\hat{K}_t = \log\left(\frac{K_t}{K}\right)$.

These equations are: the NKPC augmented with trend inflation and distortive labour income tax

$$(2.25)\quad \widehat{\pi}_t = \beta\eta_1 E_t\widehat{\pi}_{t+1} + k\left(\sigma\widehat{C}_t + \eta\widehat{Y}_t + \eta\widehat{s}_t + \left(\frac{\tau}{1-\tau}\right)\widehat{\tau}_t\right) + \eta_2\left(\sigma\widehat{C}_t - \widehat{Y}_t + E_t\widehat{\gamma}_{t+1}\right)$$

with $k = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$, $\eta_1 = \left(\pi + (\pi-1)\left(1-\alpha\pi^{\varepsilon-1}\right)(\varepsilon-1)\right)$ and $\eta_2 = \left(1-\alpha\pi^{\varepsilon-1}\right)\beta(\pi-1)$. $\hat{\gamma}_t$ is an auxiliary variable with no particular economic intuition.[8] Its log-linearised law of motion can be written as

$$(2.26)\quad \widehat{\gamma}_t = \left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t - \sigma\widehat{C}_t\right) + \alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1}\right)$$

The price dispersion

$$(2.27)\quad \widehat{s}_t = \omega_1\widehat{\pi}_t + \alpha\pi^{\varepsilon}\widehat{s}_{t-1}$$

with $\omega_1 = \varepsilon\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}(\pi-1)$. The Dynamic Euler equations

$$(2.28)\quad \widehat{C}_t = E_t\widehat{C}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

[6]This definition refers to a Ricardian environment. When the Ricardian equivalence does not hold, the critical value on δ_1 to identify a passive fiscal policy is greater than $\left(\frac{R}{\pi} - 1\right)$, see for a detailed discussion Leith and Wren-Lewis (2000). However from numerical results we show that this critical value is very close to $\left(\frac{R}{\pi} - 1\right)$.
[7]For example, assume that $\beta = 0.99$ and $\pi = 1.005$. In order to have a passive fiscal policy δ_1 must be greater than 0.015.
[8]For a detailed discussion about the auxiliary variable $\hat{\gamma}_t$, see Ascari and Ropele (2007).

The market clearing condition

$$\hat{Y}_t = \lambda \hat{C}_t \tag{2.29}$$

with λ representing the steady state ration of private consumption over total output as $\lambda = \frac{C}{Y}$. The monetary policy rule

$$\hat{R}_t = \phi_1 \hat{\pi}_t \tag{2.30}$$

the government budget constraint

$$\frac{1}{1-\tau}\widehat{\tau}_t + \left(1 + \frac{\sigma}{\lambda} + \eta\right)\widehat{Y}_t + (1+\eta)\,\widehat{s}_t = \frac{\gamma_b}{\pi w s \tau}\left(\widehat{b}_t - \widehat{\pi}_t\right) - \frac{\gamma_b}{R w s \tau}\left(\widehat{b}_{t+1} - \widehat{R}_t\right) \tag{2.31}$$

and the tax rate rule

$$\hat{\tau}_t = \delta_1 \hat{b}_t - \hat{Y}_t \tag{2.32}$$

Note that in the case of balanced budget rule, fiscal policy is defined only by the government budget constraint where $\hat{b}_t = 0 \ \forall t$.

The second step in the study of local determinacy consists of writing the model in space form, finding its impact matrix and calculating its eigenvalues.[9] Following Blanchard and Khan (1981) a linear model of difference equations has a unique rational expectations equilibrium if and only if the impact matrix displays a number of eigenvalues outside the unit circle equal to the numbers of *non-predetermined* variables. From this, if the impact matrix has a number of eigenvalues which exceed the unit circle inferior to the number of *non-predetermined* variables the system is *indeterminate* (infinite number of solutions). In the remaining case the system is *unstable* (no solution).

[9] For a detailed discussion see paragraph 1.2.10

2.2.7. Calibration

The model is calibrated to a quarterly frequency. Its structural parameters are: α, β, σ, ε, η, λ, γ_b, ϕ_1, δ_1 and δ_2. The parameter of Calvo price setting, α, is fixed to 0.75. This in turn implies that on average firms keep their price fixed for a year. This value is consistent with a large amount of empirical evidence such as Clarida, Galí and Gertler (2000). The discount factor has been calibrated so that the steady state real interest rate is 2% a year. This implies that $\beta = 0.99$. The CRRA parameter of consumption, σ, is fixed to 2. This value has been largely used in the literature. Following SGU (2007) we calibrate η, the inverse of Frisch elasticity of labour to 1 and $\varepsilon = 6$. The latter implies that the steady state mark-up is around 20%. We fix the steady state ratio of $\frac{C}{Y} = \lambda = 0.75$, which is in line with government consumption of most *OECD* countries. When not otherwise stated, we fix the steady state debt to output ratio, γ_b, to 2.4. This implies a annual debt to output equal to 60%. Given the aim of this work we study the determinacy for a wide range of ϕ_1 and δ_1, while for the sake of simplicity we fix $\delta_2 = -1$.

2.3. Results

2.3.1. Constant debt and variable tax rate

Let us first analyse the equilibrium conditions with a balanced budget rule similar to the type introduced by SGU (1997). In our setting, this implies that the tax rate is adjusted in every period in order to collect enough revenues to finance public spending and the service of a steady state level of public debt. The log-linearised version of this rule is

$$\frac{1}{1-\tau}\widehat{\tau}_t = \frac{\gamma_b \beta}{ws\tau}\widehat{R}_t - \frac{\gamma_b}{ws\tau\pi}\widehat{\pi}_t - \left(1 + \frac{\sigma}{\lambda} + \eta\right)\widehat{Y}_t - (1+\eta)\,\widehat{s}_t \tag{2.33}$$

As previously described, during this exercise the fiscal authority cannot access short run public debt to satisfy the government budget constraint, and public spending is always at its steady state level, i.e. $b_t = b$, $G_t = G$ $\forall t$. We rely on numerical results for this exercise. These are displayed in figure 2.1. Consider the case of no trend inflation. A necessary condition for a unique rational equilibrium is to have an active monetary rule, i.e. $\phi_1 > 1$. The logic behind this condition stems from the well known demand channel, i.e. Woodford (2003). However, as one can see, adopting an active monetary policy rule is not sufficient for equilibrium determinacy. Indeed both the level of debt to output ratio and the strength of monetary policy play an important role in the equilibrium determinacy. With low or no long run public debt, an active monetary policy rule is enough to guarantee a unique equilibrium. This ceases to be true when the ratio of annual public debt to GDP is around 50%. In this case determinacy requires an active but "*not too aggressive*" monetary policy.

The intuition for this result goes as follow. Let us assume agents suddenly expect higher inflation. The monetary authority following the active role raises the real interest rate. For each increase in the real rate, the fiscal authority, *ceteris paribus,* through the government budget constraint, increases the tax rate by $\frac{\gamma_b \beta(1-\tau)}{ws\tau}$. This increase has a *supply side* effect, through the NKPC, of $(1-\alpha)\frac{1-\alpha\beta}{\alpha}\left(\frac{\tau}{1-\tau}\right)$ which directly feeds back on current inflation. In other words, an increase in tax rate could potentially lead to an increase in current inflation neutralising the attempt of monetary policy to pin down expectations of future inflation. Therefore, due to the effects of changes in interest rate on the government budget constraint, a "*strong*" active monetary policy in response to inflationary expectations might make these self-fulfilling. These self-fulfilling effects depend on the long run debt to output ratio both directly (the higher γ_b, the higher is the monetary feed back on fiscal policy), and indirectly (the higher τ, the higher is the fiscal policy feed back on inflation). For these reasons equilibrium determinacy shows an upper bound in the level of debt to output.

An increase in trend inflation increases the possibility of endogenous sunspots fluctuations. When inflation is at 2% per year, the presence of even a mild level of steady state public debt, or *ceteris paribus*, a strongly active monetary policy, leads to indeterminacy. With trend inflation of 4% per year, a unique RE equilibrium requires both the absence of long run public debt and a very aggressive monetary policy, i.e. $\phi_1 > 6$. With higher levels of trend inflation, there is no combination of γ_b and monetary policy parameter (for the parameters ranges we consider in the present analysis) that guarantees determinacy. As before, consider agents suddenly have expectations of higher inflation. Let us further assume that an active monetary policy tries to reduce these expectations, increasing the real interest rate. The monetary policy feedback on the government budget constraint and the consequent fiscal feed back via the *NKPC* may cause an increase current inflation which in turn could lead to self-fulfilling prophecies for expected inflation. As shown by Ascari and Ropele (2009) with trend inflation, inflation expectations have a stronger impact on current inflation and therefore they are more difficult to pin down. This is captured by η_1, the parameter in the NKPC identifying the importance of inflation expectations on the determination of current inflation, that with positive trend inflation, becomes greater than 1. They find that this feature implies for equilibrium determinacy a stronger monetary policy reaction to inflation.[10]

Moreover, in the case of $\pi > 1$, price dispersion, $\hat{s}_t$, becomes a relevant variable for the equilibrium determinacy and, given the assumption of Calvo price setting, is positively related to inflation and positively affects inflation. The fiscal feedback with its supply side effect increases inflation by $k\left(\frac{\tau}{1-\tau}\right)$. This generates an increase in $\hat{s}_t$ of ω_1, the parameter which puts further pressure on prices. Hence, when trend inflation is positive it is easier for the monetary policy response to expected inflation to generate endogenous sunspots fluctuations. As one can see from figure 2.1, these effects increase with the increase of steady state inflation. Moreover higher price dispersion means lower output. A lower output will feed back on fiscal policy generating an increase in the tax rate, which in turn causes a potential increase in current inflation, generating higher price dispersion. This is why increasing trend inflation may lead to sunspots fluctuations even in the case of zero debt. This result is clearly an extension of the one obtained by Ascari and Ropele(2009). In their analysis they consider a NK model with trend inflation but

[10] In particular Ascari and Ropele (2009) show that when steady state inflation is greater than zero, the Taylor coefficient on the monetary rule must be much greater than one in order to ensure the determinacy of the system.

without fiscal policy. Their main result is that the higher is trend inflation the stronger monetary policy must be against inflation in order to guarantee a unique equilibrium. With a fiscal policy that relies on distortive taxation and follows a balanced rule similar to the one introduced by SGU (1997), and a level of trend inflation greater than 4%, the system displays indeterminacy even in the case of zero steady state public debt and regardless of the monetary policy parameter ϕ_1.

2.3.2. Endogenous debt and tax rate

The second exercise stems from analysing the equilibrium conditions assuming a fiscal policy that, in contrast with the last section, has the possibility to balance the government budget constraint changing along the business cycle both the short run debt and the level of labour income tax. The system is closed assuming a fiscal rule as in (2.23). For sake of simplicity and in order to make this analysis comparable to most of the literature[11], we consider a fiscal policy in which $\delta_2 = -1$. This rule implies a procyclical response of the tax rate to output. Log linearisation of this fiscal policy is

$$\widehat{\tau}_t = \delta_1 \widehat{b}_t - \widehat{Y}_t$$

Results of this exercise are reported in figure 2.2. Scrolling figure 2.2 from left to right, increases the level of trend inflation, while from up to bottom increases the steady state level of public debt. In the case with no-trend inflation the usual Leeper (1991) result holds: equilibrium determinacy requires a policy mix characterised by an active monetary policy and a passive fiscal policy or *vice versa.* As stressed before, with positive trend inflation, expectations of future inflation are harder to stabilise, i.e. η_1, the parameter in the NKPC identifying the importance of inflation expectations on the determination of current inflation, becomes greater than 1. Furthermore the fiscal policy feedback on current inflation raises the price dispersion variable, s_t, which, *ceteris paribus,* puts further upward pressure on current inflation. Moreover with trend inflation each increase in

[11]Inter alia Linnemann (2006), Leeper (1991).

inflation expectations reduces total output. At the same time, the government's access to short run public debt reduces in intensity the fiscal feedback of monetary policy with respect to the case of balanced budget rule analysed in the first exercise. An active interest rate rule in response to inflation expectations reduces the economic activity through the traditional demand channel but at the same time it implies, *via* the government budget constraint, an increase in the service of debt. A moderate positive response of fiscal policy guarantees a unique RE equilibrium. If instead the reaction of fiscal policy is too strong, i.e. raises the tax rate "*too much*" in response to the fiscal feedback of monetary policy, the system could display indeterminacy.

The intuition behind this result is similar to the case described above. A higher tax rate, via its supply side effects, puts upward pressure on current inflation, which in turn may make the expectations of inflation self-fulfilling and therefore generate endogenous sunspots. In order to avoid indeterminacy, monetary policy has to react more strongly to inflation expectations the higher is the fiscal policy parameter, δ_1. All these effects imply that for a given level of δ_1, monetary policy has to be more aggressive, the higher the level of trend inflation, to avoid sunspots fluctuations.

Ceteris paribus, increasing the level of steady state public debt reduces the fiscal reaction to increases in short run public debt, spreading the determinacy area, for any given level of trend inflation, for the combination of active monetary policy and passive fiscal policy.

2.4. Conclusions

This paper analyses the determinacy properties of a New Keynesian model with trend inflation, long run public debt and a distortive fiscal policy. We assumed, following the mainstream NK literature that monetary policy is concerned with inflation stabilisation and fiscal policy with public debt stabilisation. The message of the paper is simple: a steady state level of inflation and a distortive fiscal policy augment the difficulty for economic policies to reach determinacy. In particular, in the case of a balanced budget rule, where fiscal policy is not allowed to access short run public debt, determinacy is impossible even for a moderate level of inflation and/or low levels of long run public debt. When instead fiscal policy has the ability to access short run public debt, determinacy requires that, for any levels of fiscal policy reaction to public debt fluctuations, monetary policy reacts more aggressively to inflation fluctuations the higher the level of steady state inflation.

2.5. Figures

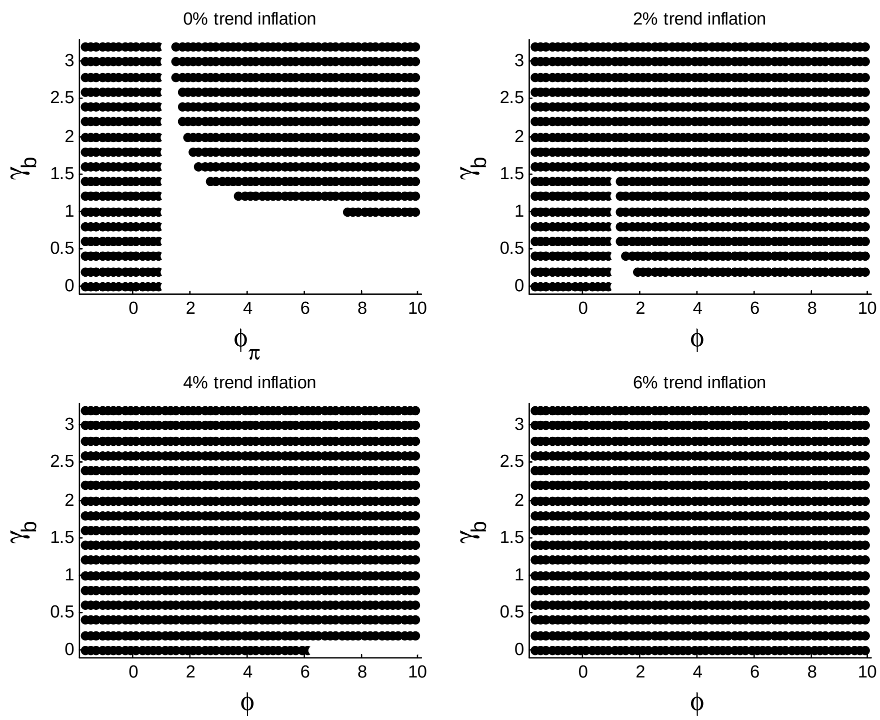

Figure 2.1. Determinacy with a balanced budget fiscal policy. Determinacy, white area, indeterminacy, black area.

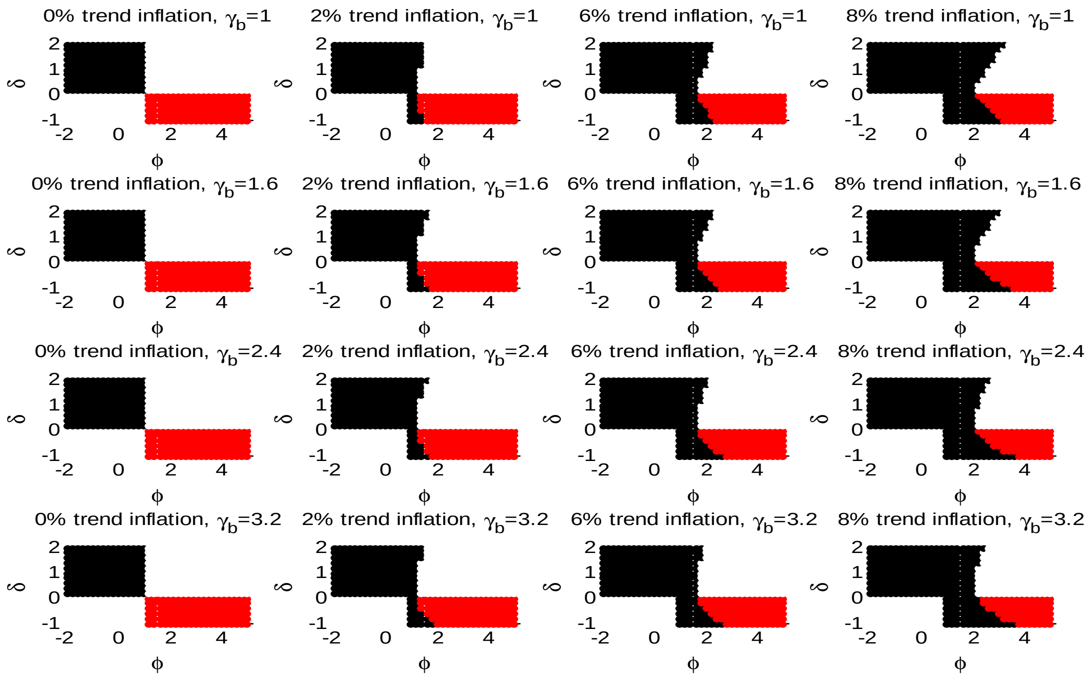

Figure 2.2. Determinacy analysis with fiscal rules of the type $\widehat{\tau}_t = \delta_1 \widehat{b}_t - \widehat{Y}_t$. Determinacy, white area, indeterminacy, black area, instability, red area.

2.A. Appendix

2.A.1. Log linear equilibrium

The log-linearised version of the model is derived from the first order conditions of the households the NKPC, the government budget constraint, a definition of monetary and fiscal policy. As

$$\widehat{\gamma}_t = \left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t - \sigma\widehat{C}_t\right) + \alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1}\right) \tag{2.34}$$

$$\widehat{\pi}_t = \beta\eta_1 E_t\widehat{\pi}_{t+1} + k\left(\sigma\widehat{C}_t + \eta\widehat{Y}_t + \eta\widehat{s}_t + \left(\frac{\tau}{1-\tau}\right)\widehat{\tau}_t\right) + \eta_2\left(\sigma\widehat{C}_t - \widehat{Y}_t + E_t\widehat{\gamma}_{t+1}\right) \tag{2.35}$$

$$\widehat{s}_t = \omega_1\widehat{\pi}_t + \alpha\pi^{\varepsilon}\widehat{s}_{t-1} \tag{2.36}$$

$$\widehat{C}_t = E_t\widehat{C}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{2.37}$$

$$\widehat{Y}_t = \lambda\widehat{C}_t \tag{2.38}$$

$$\widehat{R}_t = \phi_1\widehat{\pi}_t \tag{2.39}$$

$$\frac{1}{1-\tau}\widehat{\tau}_t + \left(1+\frac{\sigma}{\lambda}+\eta\right)\widehat{Y}_t + (1+\eta)\,\widehat{s}_t = \frac{\gamma_b}{\pi w s\tau}\left(\widehat{b}_t - \widehat{\pi}_t\right) - \frac{\gamma_b}{R w s\tau}\left(\widehat{b}_{t+1} - \widehat{R}_t\right) \tag{2.40}$$

$$\widehat{\tau}_t = \delta_1\widehat{b}_t - \widehat{Y}_t \tag{2.41}$$

Where the first three equations represent the NKPC augmented with trend inflation, the fourth one is the dynamic IS, the fifth is the market clearing condition together with the assumption that G is constant, the sixth one is the government budget constraint, the seventh is the definition of the Taylor type monetary policy rule and the eighth is the fiscal policy revenue rule. Note that (2.40) corresponds to the case where the tax rate is free to move along the business cycle. In the case of a balanced budget rule as in Schmitt-Grohe and Uribe(1997) the log linearised version of the fiscal policy takes the form of

$$\frac{1}{1-\tau}\widehat{\tau}_t = \frac{\gamma_b \beta}{ws\tau}\widehat{R}_t - \frac{\gamma_b}{ws\tau\pi}\widehat{\pi}_t - \left(1+\frac{\sigma}{\lambda}+\eta\right)\widehat{Y}_t - (1+\eta)\,\widehat{s}_t \tag{2.42}$$

2.A.2. Derivation of the NKPC

we start from the expression

$$\frac{P_{it}^{*}}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{z=0}^{+\infty}(\alpha\beta)^z\left(\frac{1}{C_{t+z}}\right)^{\sigma}\left(\Pi_{t+1,t+z}^{\varepsilon}Y_{t+z}mc_{t+z}\right)}{E_t\sum_{z=0}^{+\infty}(\alpha\beta)^z\left(\frac{1}{C_{t+z}}\right)^{\sigma}\left(\Pi_{t+1,t+z}^{\varepsilon-1}Y_{t+z}\right)} \tag{2.43}$$

where given the production function $mc_{t+z} = \frac{W_{t+z}}{P_{t+z}} = w_{t+z}$.

2.A.2.1. Quasi-differentiate the optimal relative price. Let first rewrite (2.15) as

$$\frac{P_{it}^{*}}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{\zeta_t}{\gamma_t} \tag{2.44}$$

where

$$\zeta_t = E_t\sum_{z=0}^{+\infty}(\alpha\beta)^z\left(\frac{1}{C_{t+z}}\right)^{\sigma}\left(\Pi_{t+1,t+z}^{\varepsilon}Y_{t+z}mc_{t+z}\right) \tag{2.45}$$

$$\gamma_t = E_t\sum_{z=0}^{+\infty}(\alpha\beta)^z\left(\frac{1}{C_{t+z}}\right)^{\sigma}\left(\Pi_{t+1,t+z}^{\varepsilon-1}Y_{t+z}\right) \tag{2.46}$$

we need to find a *recursive* formulation for ζ_t and γ_t.

$$\begin{aligned}\zeta_t &= mc_tY_tC_t^{-\sigma} + \alpha\beta E_t\left[\Pi_{t+1,t+1}^{\varepsilon}Y_{t+1}mc_{t+1}C_{t+1}^{-\sigma}\right] + \\ &\quad +(\alpha\beta)^2E_t\left[\Pi_{t+1,t+2}^{\varepsilon}Y_{t+2}mc_{t+2}C_{t+2}^{-\sigma}\right] + ...\end{aligned}$$

this can be rewritten as

$$\zeta_t = mc_t Y_t C_t^{-\sigma} + \alpha\beta E_t \left\{ \begin{array}{c} \Pi_{t+1,t+1}^{\varepsilon} \times \\ \times \left[Y_{t+1} mc_{t+1} C_{t+1}^{-\sigma} + \alpha\beta \Pi_{t+1,t+2}^{\varepsilon} Y_{t+2} mc_{t+2} C_{t+2}^{-\sigma} + ... \right] \end{array} \right\}$$

noting that the second line of the last expression is simply equal to $E_t \zeta_{t+1}$ we can rewrite the last expression as

$$\zeta_t = mc_t Y_t C_t^{-\sigma} + \alpha\beta E_t \left(\pi_{t+1}^{\varepsilon} \zeta_{t+1} \right) \tag{2.47}$$

where $\Pi_{t+1,t+1} = E_t \pi_{t+1}$. With the same fashion

$$\gamma_t = Y_t C_t^{-\sigma} + \alpha\beta E_t \left(\pi_{t+1}^{\varepsilon-1} \gamma_{t+1} \right) \tag{2.48}$$

2.A.2.2. Steady state. We evaluate the last expressions at steady state (i.e. a variable without the time index corresponds to its steady state value)

$$\frac{P_i^*}{P} = \frac{\varepsilon}{\varepsilon - 1} \frac{\zeta}{\gamma} \tag{2.49}$$

$$\zeta = \frac{mcYC^{-\sigma}}{1 - \alpha\beta\pi^{\varepsilon}} \tag{2.50}$$

$$\gamma = \frac{YC^{-\sigma}}{1 - \alpha\beta\pi^{\varepsilon-1}} \tag{2.51}$$

$$1 = (1-\alpha) \left(p_i^* \right)^{1-\varepsilon} + \alpha\pi^{\varepsilon-1} \tag{2.52}$$

Furthermore using the definition of steady state mc and the household first order condition

$$mc = w = \frac{Y^{\eta+\sigma} s^{\eta} \lambda^{\sigma}}{1 - \tau} \tag{2.53}$$

where we use the fact that at steady state $\frac{C}{Y}=\lambda$. Therefore we can re-write (2.50) and (2.51)as

$$\zeta = \frac{Y^{1+\eta}s^{\eta}}{(1-\alpha\beta\pi^{\varepsilon})(1-\tau)} \tag{2.54}$$

$$\gamma = \frac{Y^{1-\sigma}\lambda^{-\sigma}}{1-\alpha\beta\pi^{\varepsilon-1}} \tag{2.55}$$

the steady state optimal re-setter is

$$\frac{P_i^*}{P} = p_i^* = \frac{\varepsilon}{\varepsilon-1}\frac{\sum_{z=0}^{+\infty}(\alpha\beta\pi^{\varepsilon})^z mc}{\sum_{z=0}^{+\infty}(\alpha\beta\pi^{\varepsilon-1})^z} = \frac{\varepsilon}{\varepsilon-1}\frac{1-\alpha\beta\pi^{\varepsilon-1}}{1-\alpha\beta\pi^{\varepsilon}}mc \tag{2.56}$$

Note that in order for the last expression to hold $\alpha\beta\pi^{\varepsilon}<1$.

2.A.2.3. Log-linearisation. Taking the log linearisation (step-by-step) of (2.47). Let consider it at steady state

$$1 = \frac{mcY}{C^{\sigma}\zeta} + \frac{\alpha\beta\pi^{\varepsilon}\zeta}{\zeta}$$

then we take a first order approximation. Hatted variables represents its log deviation from steady state i.e. $\widehat{z}_t = \log\left(\frac{z_t}{z}\right)$.

$$1 \approxeq \frac{mcY}{C^{\sigma}\zeta}\left(1+\widehat{mc}_t+\widehat{Y}_t-\sigma\widehat{C}_t-\widehat{\zeta}_t\right)+\alpha\beta\pi^{\varepsilon}\left(1+\varepsilon E_t\widehat{\pi}_{t+1}+E_t\widehat{\zeta}_{t+1}-\widehat{\zeta}_t\right)$$

Rearranging and using the definition of ζ it yields

$$\widehat{\zeta}_t = (1-\alpha\beta\pi^{\varepsilon})\left(\widehat{mc}_t+\widehat{Y}_t-\sigma\widehat{C}_t\right)+\alpha\beta\pi^{\varepsilon}\left(\varepsilon E_t\widehat{\pi}_{t+1}+E_t\widehat{\zeta}_{t+1}\right) \tag{2.57}$$

Log-linearisation of (2.48)

$$\widehat{\gamma}_t = \left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t-\sigma\widehat{C}_t\right)+\alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1}+E_t\widehat{\gamma}_{t+1}\right) \tag{2.58}$$

Log-linearisation of (2.47)

$$\widehat{p}_{it}^* = \frac{\alpha\pi^{\varepsilon-1}}{(1-\alpha)(p_i^*)^{1-\varepsilon}}\widehat{\pi}_t \tag{2.59}$$

Plugging in the last expression $(p_i^*)^{1-\varepsilon} = \left(1-\alpha\pi^{\varepsilon-1}\right)\frac{1}{1-\alpha}$ yields

$$\widehat{p}^*_{it} = \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t \tag{2.60}$$

Furthermore from (2.44) it is easy to see that

$$\widehat{p}^*_{it} = \widehat{\zeta}_t - \widehat{\gamma}_t \tag{2.61}$$

combining the last two equations yields

$$\widehat{\zeta}_t = \widehat{\gamma}_t + \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t \tag{2.62}$$

Then we substitute the latter into (2.57)

$$\begin{aligned}\widehat{\gamma}_t &= \left(1-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{mc}_t + \widehat{Y}_t - \sigma\widehat{C}_t\right) - \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t + \\ &\quad +\alpha\beta\pi^{\varepsilon}\left(\varepsilon E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1} + \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}E_t\widehat{\pi}_{t+1}\right)\end{aligned} \tag{2.63}$$

Then let plug in the last expression in (2.58)

$$\begin{aligned}\left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t - \sigma\widehat{C}_t\right) &= -\alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1}\right) + \\ &\quad + \left(1-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{mc}_t + \widehat{Y}_t - \sigma\widehat{C}_t\right) - \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t + \qquad (2.64) \\ &\quad +\alpha\beta\pi^{\varepsilon}\left(\varepsilon E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1} + \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}E_t\widehat{\pi}_{t+1}\right) \qquad (2.65)\end{aligned}$$

Expressing in term of current inflation

$$\begin{aligned}-\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t &= \left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t - \sigma\widehat{C}_t\right) + \alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1}\right) - \\ &\quad -\alpha\beta\pi^{\varepsilon}\left(\varepsilon E_t\widehat{\pi}_{t+1} + E_t\widehat{\gamma}_{t+1} + \frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}E_t\widehat{\pi}_{t+1}\right) + \qquad (2.66) \\ &\quad - \left(1-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{mc}_t + \widehat{Y}_t - \sigma\widehat{C}_t\right) \qquad (2.67)\end{aligned}$$

Rearranging

$$
\begin{aligned}
\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t &= \left(1-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{mc}_t+\widehat{Y}_t-\sigma\widehat{C}_t\right)-\left(1-\alpha\beta\pi^{\varepsilon-1}\right)\left(\widehat{Y}_t-\sigma\widehat{C}_t\right)+ \\
&+\alpha\beta\pi^{\varepsilon}\left(\varepsilon E_t\widehat{\pi}_{t+1}+E_t\widehat{\gamma}_{t+1}+\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}E_t\widehat{\pi}_{t+1}\right)+ \qquad (2.68)\\
&-\alpha\beta\pi^{\varepsilon-1}\left((\varepsilon-1)E_t\widehat{\pi}_{t+1}+E_t\widehat{\gamma}_{t+1}\right) \qquad (2.69)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t &= \left(1-\alpha\beta\pi^{\varepsilon}\right)\widehat{mc}_t+\left(\alpha\beta\pi^{\varepsilon-1}-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{Y}_t-\sigma\widehat{C}_t\right)+ \\
&+\left(\alpha\beta\pi^{\varepsilon}\left(\varepsilon+\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\right)-\alpha\beta\pi^{\varepsilon-1}\left(\varepsilon-1\right)\right)E_t\widehat{\pi}_{t+1}+ \qquad (2.70)\\
&+\left(\alpha\beta\pi^{\varepsilon}-\alpha\beta\pi^{\varepsilon-1}\right)E_t\widehat{\gamma}_{t+1} \qquad (2.71)
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\pi}_t &= \frac{\left(1-\alpha\beta\pi^{\varepsilon}\right)\left(1-\alpha\pi^{\varepsilon-1}\right)}{\alpha\pi^{\varepsilon-1}}\widehat{mc}_t+\frac{1-\alpha\pi^{\varepsilon-1}}{\alpha\pi^{\varepsilon-1}}\left(\alpha\beta\pi^{\varepsilon-1}-\alpha\beta\pi^{\varepsilon}\right)\left(\widehat{Y}_t-\sigma\widehat{C}_t-E_t\widehat{\gamma}_{t+1}\right)+ \\
(2.72)&+\frac{1-\alpha\pi^{\varepsilon-1}}{\alpha\pi^{\varepsilon-1}}\left(\alpha\beta\pi^{\varepsilon}\left(\varepsilon+\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\right)-\alpha\beta\pi^{\varepsilon-1}\left(\varepsilon-1\right)\right)E_t\widehat{\pi}_{t+1}
\end{aligned}
$$

simplifying

$$\widehat{\pi}_t=\beta\eta_1E_t\widehat{\pi}_{t+1}+k\widehat{mc}_t+\eta_2\left(\sigma\widehat{C}_t-\widehat{Y}_t+E_t\widehat{\gamma}_{t+1}\right) \qquad (2.73)$$

where $\eta_1=\left(\pi+(\pi-1)\left(1-\alpha\pi^{\varepsilon-1}\right)(\varepsilon-1)\right)$, $\eta_2=\left(1-\alpha\pi^{\varepsilon-1}\right)\beta\left(\pi-1\right)$, $k=\frac{\left(1-\alpha\beta\pi^{\varepsilon}\right)\left(1-\alpha\pi^{\varepsilon-1}\right)}{\alpha\pi^{\varepsilon-1}}$. To double check this expression: fix $\pi=1$ (no steady state inflation); (2.73) collapses to the standard NKPC with no trend inflation

$$\pi_t=\beta E_t\pi_{t+1}+\frac{(1-\beta\alpha)(1-\alpha)}{\alpha}\widehat{mc}_t \qquad (2.74)$$

Furthermore, with no government spending and an utility function in log consumption $C_t=Y_t$ and therefore (2.73) reduces to equation (13) in the Ascari and Ropele(2007b)

paper in the case of no indexation. Log linearisation of

$$s_t = \int_0^1 \frac{P_{it}}{P_t} di \tag{2.75}$$

First of all we need to write this expression in a recursive form. Applying the same technique used for γ and ζ and remembering the basic mechanism behind the Calvo price setting, we can write

$$s_t = (1-\alpha)\left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} + \alpha\pi_t^{\varepsilon} s_{t-1} \tag{2.76}$$

At steady state the latter collapses to

$$\begin{aligned} s &= (1-\alpha)\left(p_i^*\right)^{-\varepsilon} + \alpha\pi^{\varepsilon} s \\ s &= \frac{1-\alpha}{1-\alpha\pi^{\varepsilon}}\left(p_i^*\right)^{-\varepsilon} \end{aligned} \tag{2.77}$$

log linearising (2.76)

$$\widehat{s}_t = -\varepsilon\left(1-\alpha\pi^{\varepsilon}\right)\widehat{p}_{it}^* + \alpha\pi^{\varepsilon}\left(\varepsilon\widehat{\pi}_t + \widehat{s}_{t-1}\right) \tag{2.78}$$

plugging in the latter the definition of $\widehat{p}_{it}^*$

$$\widehat{s}_t = -\varepsilon\left(1-\alpha\pi^{\varepsilon}\right)\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\widehat{\pi}_t + \alpha\pi^{\varepsilon}\left(\varepsilon\widehat{\pi}_t + \widehat{s}_{t-1}\right) \tag{2.79}$$

rearranging

$$\widehat{s}_t = \omega_1\widehat{\pi}_t + \alpha\pi^{\varepsilon}\widehat{s}_{t-1} \tag{2.80}$$

where $\omega_1 = \varepsilon\frac{\alpha\pi^{\varepsilon-1}}{1-\alpha\pi^{\varepsilon-1}}\left(\pi-1\right)$.

2.A.2.4. Remaining log linear equations. Log linearisation of (2.7) and (2.8) yield

$$\sigma\widehat{C}_t + \eta\widehat{N}_t = \widehat{w}_t - \left(\frac{\tau}{1-\tau}\right)\widehat{\tau}_t \tag{2.81}$$

$$\widehat{C}_t = E_t\widehat{C}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{2.82}$$

Log-linearisation of the market clearing conditions lead to

$$\widehat{Y}_t = \lambda\widehat{C}_t + (1-\lambda)\,\widehat{G}_t \tag{2.83}$$

and

$$\widehat{N}_t = \widehat{Y}_t + \widehat{s}_t \tag{2.84}$$

The government budget constraint at steady state is

$$G = \tau wsY + \beta b - \frac{b}{\pi} \tag{2.85}$$

where is the real term value of debt defined as $b = \frac{B}{P}$while steady state interest rate follows

$$R = \frac{\pi}{\beta} \tag{2.86}$$

Therefore log linearisation of (2.11) yields

$$\frac{1}{1-\tau}\widehat{\tau}_t + \left(1 + \frac{\sigma}{\lambda} + \eta\right)\widehat{Y}_t + (1+\eta)\,\widehat{s}_t = \frac{\gamma_b}{\pi ws\tau}\left(\widehat{b}_t - \widehat{\pi}_t\right) - \frac{\gamma_b}{Rws\tau}\left(\widehat{b}_{t+1} - \widehat{R}_t\right) \tag{2.87}$$

Real marginal cost are given by

$$mc_t = \frac{W_t}{P_t} = w_t \tag{2.88}$$

log linearising and using (2.81) it yields to

$$\widehat{mc}_t = \sigma\widehat{C}_t + \eta\widehat{Y}_t + \eta\widehat{s}_t + \left(\frac{\tau}{1-\tau}\right)\widehat{\tau}_t \tag{2.89}$$

Plugging this into the NKPC it yields

$$
\begin{aligned}
\widehat{\pi}_t &= \beta\eta_1 E_t\widehat{\pi}_{t+1} + k\left(\sigma\widehat{C}_t + \eta\widehat{Y}_t + \eta\widehat{s}_t + \left(\frac{\tau}{1-\tau}\right)\widehat{\tau}_t\right) + \\
&\quad +\eta_2\left(\sigma\widehat{C}_t - \widehat{Y}_t + E_t\widehat{\gamma}_{t+1}\right) \qquad (2.90)
\end{aligned}
$$

2.A.3. Steady State

This section describes the steady state of the model. The steady state equilibrium condition are

$$
\begin{aligned}
\text{Utility function:} &\quad \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\eta}}{1+\eta} \\
\text{Consumer budget constraint} &: \quad C + B = wN(1-\tau) + \Gamma + R^{-1}B \\
\text{Euler equation:} &\quad R = \frac{\pi}{\beta} \\
\text{Labour supply:} &\quad C^{\sigma}N^{\eta} = w(1-\tau) \\
\text{Government budget constraint} &: \quad \tau = \frac{(1-\lambda) + \left(\frac{1}{\pi} - \frac{1}{R}\right)\gamma_b}{ws} \\
\text{Market clearing conditions} &: \quad N = sY \\
Y &= C + G \\
\text{Price level} &: \quad 1 = (1-\alpha)\left(p_i^*\right)^{1-\varepsilon} + \alpha\pi^{\varepsilon-1} \\
\text{Re-setter price} &: \quad p_i^* = \frac{\varepsilon}{\varepsilon-1}\frac{1-\alpha\beta\pi^{\varepsilon-1}}{1-\alpha\beta\pi^{\varepsilon}}mc = \frac{\varepsilon}{\varepsilon-1}\frac{\zeta}{\gamma} \\
\text{Real marginal cost:} &\quad mc = w \\
\text{Price dispersion} &: \quad s = \frac{1-\alpha}{1-\alpha\pi^{\varepsilon}}\left(p_i^*\right)^{-\varepsilon} \\
\zeta &= \frac{Y^{1+\eta}s^{\eta}}{(1-\alpha\beta\pi^{\varepsilon})(1-\tau)} \\
\gamma &= \frac{YC^{-\sigma}}{1-\alpha\beta\pi^{\varepsilon-1}}
\end{aligned}
$$

In order to have a first analytical expression for the steady state variables we operate few substitutions. First of all, let rewrite

$$p_i^* = \left(\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\pi^{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}} \tag{2.91}$$

With the latter we can find an expression function of the sole parameters of s and w. The we can rewrite the government budget constraint as

$$Ysw\tau = G + \left(\frac{1}{R} - \frac{1}{\pi}\right) b \tag{2.92}$$

Dividing on both side for Y and imposing $\frac{G}{Y} = 1 - \lambda$ and $\frac{b}{Y} = \gamma_b$, the latter becomes

$$\tau = \frac{(1-\lambda)}{ws} + \left(\frac{1}{\pi} - \frac{1}{R}\right) \frac{\gamma_b}{ws} \tag{2.93}$$

Therefore

$$\tau = \frac{(1-\lambda)}{ws} + \left(\frac{1}{\pi} - \frac{1}{R}\right) \frac{\gamma_b}{ws} \tag{2.94}$$

Using the household labour supply we can infer the steady state values of Y as

$$\begin{aligned}
(\lambda Y)^{\sigma} Y^{\eta} s^{\eta} &= w(1-\tau) \\
(Y)^{\sigma+\eta} &= \frac{w(1-\tau)}{\lambda^{\sigma} s^{\eta}} \\
Y &= \left(\frac{w(1-\tau)}{\lambda^{\sigma} s^{\eta}}\right)^{\frac{1}{\sigma+\eta}} \qquad (2.95)
\end{aligned}$$

2.A.4. Matrix Representations

2.A.4.1. Balanced budget rule. In order to check for determinacy we write the model in matrix form as $Ax_{t+1} = Bx_t$.

$$A = \begin{pmatrix} \alpha\beta\pi^{\varepsilon-1} & (\varepsilon-1)\,\alpha\beta\left(\pi^{\varepsilon-1}\right) & 0 & 0 \\ \eta_2 & \eta_1 & 0 & k\eta \\ 0 & \left(\frac{1}{\sigma}\right) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad x_{t+1} = \begin{pmatrix} E_t\gamma_{t+1} \\ E_t\pi_{t+1} \\ E_tC_{t+1} \\ s_t \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & -\left(1-\alpha\beta\pi^{\varepsilon-1}\right)(\lambda-\sigma) & 0 \\ 0 & \eta_4 & -\eta_3 & 0 \\ 0 & \frac{1}{\sigma}\phi_1 & 1 & 0 \\ 0 & \omega_1 & 0 & \alpha\pi^{\varepsilon} \end{pmatrix}$$

$$H = A^{-1} \times B$$

Determinacy requires three eigenvalues of the H matrix outside the unit circle and one inside.

$$\eta_3 = \left(k\left(\sigma+\eta\lambda\right)+\eta_2\left(\sigma-\lambda\right)-k\lambda\tau\left(1+\tfrac{\sigma}{\lambda}+\eta\right)\right),\ \eta_4 = \left(1+k\gamma_b\left(\tfrac{1}{\pi}-\beta\phi_1\right)\right).$$

2.A.4.2. Endogenous tax and short run debt. In order to check for determinacy we write the model in matrix form as $Ax_{t+1} = Bx_t$. Note that for sake of simplicity we set $\widehat{G}_t = 0$. And $\delta_2 = 0$. Following this $\widehat{Y}_t = \lambda \widehat{C}_t$.

$$A = \begin{pmatrix} \alpha\beta\pi^{\varepsilon-1} & (\varepsilon-1)\,\alpha\beta\pi^{\varepsilon-1} & 0 & 0 & 0 \\ \eta_2 & \beta\eta_1 & 0 & 0 & k\eta \\ 0 & \frac{1}{\sigma} & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\gamma_b\beta}{ws\tau} & (1+\eta) \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad x_{t+1} = \begin{pmatrix} E_t\gamma_{t+1} \\ E_t\pi_{t+1} \\ E_tC_{t+1} \\ b_{t+1} \\ s_t \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & -\left(1-\alpha\beta\pi^{\varepsilon-1}\right)(\lambda-\sigma) & 0 & 0 \\ 0 & 1 & -k\left(\sigma+\eta\lambda-\lambda\frac{\tau}{1-\tau}\right)-\eta_2(\sigma-\lambda) & -k\delta\frac{\tau}{1-\tau} & 0 \\ 0 & \frac{1}{\sigma}\phi & 1 & 0 & 0 \\ 0 & -\frac{\gamma_b}{ws\tau}\left(\beta\phi_1-\frac{1}{\pi}\right) & \left(1+\frac{\sigma}{\lambda}+\eta-\frac{1}{1-\tau}\right)\lambda & -\left(\frac{1}{1-\tau}\delta_1-\frac{\gamma_b}{\pi ws\tau}\right) & 0 \\ 0 & \omega_1 & 0 & 0 & \alpha\pi^{\varepsilon} \end{pmatrix}$$

In particular the system has a unique rational expectation solution iff H has 3 eigenvalues outside the unit circle and 2 inside the unit circle. Where H is defined as

$$H = A^{-1}B$$

In the above matrix representation the first line is (2.34), the second line (2.35), the third line(2.37), the fourth line is(2.40) and the last line represents(2.36).

CHAPTER 3

Optimal Monetary Policy in a New Keynesian Model with Deep Habits Formation.

While consumption habits have been utilised as a means of generating a humpshaped output response to monetary policy shocks in sticky-price New Keynesian economies, there is relatively little analysis of the impact of habits (particularly, external habits) on optimal policy. In this paper we consider the implications of deep external habits ('deep' habits: see Ravn, Schmitt-Grohe, and Uribe (2006)) for optimal monetary policy. External habits generate an additional distortion in the economy, which implies that the flex-price equilibrium will no longer be efficient and that policy faces interesting new trade-offs and potential stabilisation biases. Furthermore, the endogenous mark-up behaviour, which emerges with deep habits, can also significantly affect the optimal policy response to shocks, as well as dramatically affecting the stabilising properties of standard simple rules.

3.1. Introduction

Within[1] the benchmark New Keynesian analysis of monetary policy (see, for example, Woodford (2003)), monetary policy typically influences the economy through the impact of interest rates on a representative household's intertemporal consumption decision. It has often been felt that the purely forward-looking consumption dynamics that such basic intertemporal consumption decisions imply, are unable to capture the hump-shaped output response to changes in monetary policy one typically finds in the data. As a means of accounting for such patterns, some authors have augmented the benchmark model with various forms of habits effects in consumption. The habits effects can either

[1]This chapter is part of the paper "Optimal monetary policy in a new Keynesian model with consumption habits" (with Campbell Leith and Ioana Moldovan) ECB working paper series, No. 1076.

be internal (see for example, Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), Leith and Malley (2005)) or external (see, for example, Smets and Wouters (2007)) the latter reflecting a catching up with the Joneses effect whereby households fail to internalise the externality their own consumption causes on the utility of other households.

Both forms of habits behaviour can help the New Keynesian monetary policy model capture the persistence found in the data (see, for example Kozicki and Tinsley (2002)), although the policy implications are likely to be different. More recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of habits behaviour, which they label as 'deep'. Deep habits occur at the level of individual goods rather than at the level of an aggregate consumption basket ('superficial' habits). While this distinction does not affect the dynamic description of aggregate consumption behaviour relative to the case of superficial habits, it does render the individual firms' pricing decisions intertemporal and, in the flexible price economy considered by Ravn, Schmitt-Grohe, and Uribe (2006), can produce a counter-cyclical mark-up which significantly affects the responses of key aggregates to shocks. While the focus of the papers listed above is on the dynamic response of economies which feature some form of habits, they do not consider the implications for optimal policy of such an extension. In contrast, Amato and Laubach (2004) consider optimal monetary policy in a sticky-price New Keynesian economy which has been augmented to include internal (but superficial) habits. Since the form of habits is internal (households care about their consumption relative to their own past consumption, rather than the consumption of other households), there is no additional externality associated with consumption habits themselves, and, given an efficient steady-state, the flexible price equilibrium in the neighbourhood of that steady-state remains efficient. Accordingly, as in the benchmark New Keynesian model, there is no trade-off between output gap and inflation stabilisation in the face of technology shocks and interesting policy trade-offs require the introduction of additional inefficiencies (such as mark-up shocks or a desire for interest rate smoothing).

In this paper we extend the benchmark sticky-price New Keynesian economy to include deep external habits in consumption. This implies that there is an externality associated with fluctuations in consumption which implies that the flexible price equilibrium will not usually be efficient, thereby creating an additional trade-off for policy makers, which may give rise to further stabilisation biases if policy is constrained to be time consistent. We also consider the implications for optimal policy. The ability of policy to influence the time profile of endogenously determined mark-ups can significantly affect the monetary policy stance and how it differs across discretion and commitment and across different exogenous shocks.

In addition to examining optimal policy, we also consider how the introduction of habits affects the conduct of policy through simple rules. We find that the introduction of deep habits can induce problems of indeterminacy, as the tightening of monetary policy can induce inflation through variations in mark-up behaviour, such that an interest rate rule which satisfies the Taylor principle (where nominal interest rates rise more than one for one with increases in inflation above target) may not be sufficient to ensure determinacy of the local equilibrium.

The plan of the paper is as follows: in the next section we outline our model with deep superficial habits. In section 3.3 we consider the determinacy properties of a simple Taylor rule. In section 3.4 we consider optimal policy under both commitment and discretion when the economy is hit by a technological and a government spending shock, where the policy-maker's objective function is derived from a second order approximation to households' utility. Section 3.5 concludes.

3.2. The Model

The economy is comprised of households, two monopolistically competitive production sectors, a monetary authority and a government. There is a continuum of final goods that enter the households' and the government's consumption baskets, each final good being produced as an aggregate of a continuum of intermediate goods. The households and the government form external consumption habits at the level of each final good in their basket. Ravn et al. (2006) label this type of habits as '*deep*'.

3.2.1. Households

The economy is populated by a continuum of perfectly rational, infinitely-lived households uniformly distributed on the unit interval and indexed by κ. Each of them has preferences over a set of differentiated types of products (i.e. wine, cheese etc.), C_{it}^{κ}. Types of product are indexed by i. Moreover, each of these types of goods is composed by a continuum of specific " *brand* " products, C_{jit}^{κ}, indexed by j. Furthermore the households derive disutility from labour effort, N_t^{κ} which is supplied in a perfectly competitive labour market, and have access to perfect and complete financial markets. Following Ravn et al. (2006), it is assumed that preferences show external habit formation at the level of each type of products i rather than, as in Abel (1990), at a final composite good level. For this reason our assumption on habit formation is commonly defined as "*Deep habits*". In particular, households derive utility from X_t^{κ} such that

$$X_t^{\kappa} = \left(\int_0^1 \left(C_{it}^{\kappa} - \theta C_{it-1} \right)^{1-\frac{1}{\vartheta}} di \right)^{1/(1-1/\vartheta)} \quad \forall \kappa \tag{3.1}$$

where $C_{it-1} \equiv \int_0^1 C_{it-1}^{k} d\kappa$ denotes the cross sectional average level of consumption variety i consumed at $t-1$ which is taken as exogenous by the households. The parameter θ measures the degree of external habit formation. The parameter ϑ represents the elasticity of substitution of habit-adjusted types of consumption goods i and C_{it}^{κ} is a consumption basket of a single type of consumption good (i.e. j are single brands of cheese while $i's$ identify the totality of cheese consumed by the households) formed as

$$C_{it}^{\kappa} = \left(\int_0^1 \left(C_{jit}^{\kappa} \right)^{1-1/\varepsilon} dj \right)^{1/(1-1/\varepsilon)} \tag{3.2}$$

where the parameter ε identifies the elasticity of substitution among the j products. In forming the last consumption basket the consumers exploit any price differences present in the system. Doing so they minimise the total expenditure for each product j, $\int_0^1 P_{jit} C^{\kappa}_{jit} dj$, subject to (3.2). The optimal demand for good j is therefore defined as

$$C^{\kappa}_{jit} = \left(\frac{P_{jit}}{P^m_{it}} \right)^{-\varepsilon} C^{\kappa}_{it} \tag{3.3}$$

where $P^m_{it} \equiv \left[\int_0^1 P^{1-\varepsilon}_{jit} dj \right]^{1/(1-\varepsilon)}$. At the optimum we have $P^m_{it} C^{\kappa}_{it} = \int_0^1 P_{jit} C^{\kappa}_{jit} dj$. Furthermore for any given level of X^{κ}_t, purchases of each variety i in period t must again solve the minimisation problem of $\int_0^1 P_{it} C^{\kappa}_{it} di$, with $P^m_{it} < P_{it}$,[2] subject to the consumption bundle defined in (3.1). The optimal level of C^{κ}_{it} is given by

$$C^{\kappa}_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\vartheta} X^{\kappa}_t + \theta C_{it-1} \tag{3.4}$$

where $P_t \equiv \left[\int_0^1 P^{1-\vartheta}_{it} di \right]^{1/(1-\vartheta)}$. The demand function for each variety i defined in (3.4) is decreasing in its relative price $\frac{P_{it}}{P_t}$ and increasing in past aggregate consumption of the variety in question. At the optimum we have $P_t X^{\kappa}_t = \int_0^1 P_{it} \left(C^{\kappa}_{it} - \theta C_{it-1} \right) di$.

The utility function is defined as

$$U^{\kappa}_t = E_0 \sum_{t=0}^{+\infty} \beta^t u \left(X^{\kappa}_t, N^{\kappa}_t \right) \tag{3.5}$$

where $\frac{du}{dX} > 0$, $\frac{du}{dN} < 0$ and $\beta \in (0,1)$, denoting the discount factor. Let us assume that $u \left(X^{\kappa}_t ; N^{\kappa}_t \right) = \frac{(X^{\kappa}_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(N^{\kappa}_t)^{1+\eta}}{1+\eta}$ where the parameter $\sigma > 0$ represents the inverse of the intertemporal elasticity of habit-adjusted consumption and $\eta > 0$ corresponds to the inverse of the Frisch elasticity of labour supply and represents the risk aversion to variations in leisure. The intertemporal nominal budget constraint follows

$$P_t X^{\kappa}_t + P_t \epsilon_t + E_t Q_{t,t+1} D^{\kappa}_{t+1} = D^{\kappa}_t + W_t N_t + \Phi_t + P_t T_t \tag{3.6}$$

[2] The reason why $P^m_{it} < P_{it}$ is discussed in detail below.

where $\epsilon_t = \theta \int_0^1 \left(\frac{P_{it}}{P_t}\right) C_{it-1} di.$[3] The variable Φ_t denotes the profits coming from monopolistic competitive firms, W_t is the nominal wage and $E_t Q_{t,t+1}$ is the one period nominal stochastic discount factor. $T_t = T \; \forall t$, is a steady state lump sum tax which is used to subsidise producer firms. The household problem consists of choosing $\left\{X_t^{\kappa}; N_t^{\kappa}; D_{t+1}^{\kappa}\right\}_{t=0}^{+\infty}$, taking as given the processes for W_t, ϵ_t, Φ_t, P_t and the initial asset holding D_0^{κ} as to maximise (3.5) subject to (3.6). The first order conditions are

$$-\frac{u_N\left(X_t^{\kappa}; N_t^{\kappa}\right)}{u_X\left(X_t^{\kappa}; N_t^{\kappa}\right)} = \frac{W_t}{P_t} \tag{3.7}$$

$$1 = \frac{u_X\left(X_{t+1}^{\kappa}; N_{t+1}^{\kappa}\right)}{u_X\left(X_t^{\kappa}; N_t^{\kappa}\right)} \frac{P_t}{P_{t+1}} \frac{\beta}{Q_{t,t+1}} \tag{3.8}$$

Taking expectations from the last expression

$$\beta R_t E_t \left(\frac{u_X\left(X_{t+1}^{\kappa}; N_{t+1}^{\kappa}\right)}{u_X\left(X_t^{\kappa}; N_t^{\kappa}\right)} \frac{P_t}{P_{t+1}} \right) = 1 \tag{3.9}$$

where $R_t = \frac{1}{E_t(Q_{t,t+1})}$, implied by the non arbitrage condition, represents the nominal return on a riskless one period bond paying off a unit of currency in $t+1$. Condition (3.7) simply states that real wage is equal to the marginal rate of substitution between consumption and leisure, while condition (3.9) is the traditional Euler equation. It states that households tend to smooth habit-adjusted consumption across periods taking into account the opportunity cost represented by the real interest rate, such that the marginal rate of substitution is the same across periods.

3.2.2. Firms

The production sector is assumed to be formed of two groups. One group, that we call for simplicity "*production group*", is formed of a continuum of firms indexed by j, each

[3]This term is necessary in order to have just present consumption in the budget constraint. It can be obtained substituting for X_t its optimum $\int_0^1 P_{it} \left(C_{it} - \theta C_{it-1}\right) di$.

of whom produces in a monopolistically competitive environment a single variety of j products.

In each period each j firm sells all its products to the second group, formed again by a continuum of firms indexed by i, that we call "*final group*", which aggregates the j products creating the i ones, and sells them in a monopolistically competitive environment to the households. Both types of firm are assumed to be price setters and to take as exogenous all the actions of other firms of the same group.

3.2.2.1. Production Group. This group is assumed to have a linear labour intensive production function of the type $Y_{jit} = A_t N_{jit}$ where A_t identifies the common technology, Y_{jit} the total production of variety j and N_{jit} the total labour input required to produce Y_{jit}. Each firm of this group has two constraints. The first is given by the demand of each good $Y_{jit} = \left(\frac{P_{jit}}{P^m_{it}}\right)^{-\varepsilon} Y_{it}$ where $Y_{jit} = C_{jit}$, $\varepsilon > 1$ and P^m_{it} is a measure of the general producer price level. The firm's cost minimisation problem implies that

$$MC^m_t = (W_t/A_t)(1 - \varkappa)$$

where MC^m_t identifies the nominal marginal cost[4] for a firm j at time t and $\varkappa$ represents a steady state subsidy financed by consumers with a lump sum tax which will be discussed in detail later. In real terms

$$mc^m_t = \frac{MC^m_t}{P_t} = \frac{W_t/P_t}{A_t}(1 - \varkappa) \tag{3.10}$$

The firm's j real profits follow $\frac{\Pi_{jit}}{P_t} = \left(\frac{P_{jit}}{P_t} - mc^m_t\right) Y_{jit}$, and the profits in the production sector as a whole follow

$$\int_0^1 \int_0^1 \frac{\Pi_{jit}}{P_t} dj di = \frac{\Pi^m_t}{P_t} \tag{3.11}$$

[4]Given the assumption on the labour market that marginal costs are common across the production group, we dropped the index j.

When all the firms can adjust their prices in each period, they set their prices according to

$$\frac{P_{it}^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) mc_t^m = \mu^m mc_t^m$$

where μ^m represents the production sector mark up due to the monopolistic competitive environment.

Furthermore we assume that each production firm in order to change optimally its prices has to participate in the "*Calvo lottery*". This is the second constraint faced by the production sector firms. If it is extracted (with probability $1 - \alpha$) it can optimally reset its prices, otherwise (with probability α) it keeps its prices unchanged. When a firm can change its prices it takes into account the expected discounted value of current and future profits. The problem can be formalised as follows

$$\max_{P_{jit}^*} E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(\left(\frac{P_{jit}^*}{P_{t+z}} \right) Y_{jit+z} - mc_{t+i}^m Y_{jit+z} \right) \tag{3.12}$$

$$z.t. \quad Y_{jit+z} = \left(\frac{P_{jit}}{P_{it+z}^m} \right)^{-\varepsilon} Y_{it+z} \tag{3.13}$$

Where $q_{t,t+z}$ is the real discount factor defined as

$$q_{t,t+z} = \beta^z \frac{u_x\left(X_{t+z}; N_{t+z}\right)}{u_x\left(X_t; N_t\right)} = \beta^z \left(\frac{X_t}{X_{t+z}} \right)^{\sigma} \tag{3.14}$$

or alternatively

$$q_{t,t+z} = Q_{t,t+z} \frac{P_{t+z}}{P_t}$$

given that all the j companies that re-optimise operate the same choice, the first order condition with respect to P_{it}^{m*} can be expressed as follows

$$\frac{P_{it}^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(mc_{t+z}^m \left(P_{it+z}^m\right)^{\varepsilon} Y_{it+z} \right)}{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(P_{it+z}^m\right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t}\right)^{-1} Y_{it+z}} \tag{3.15}$$

while the aggregate price level for the production group follows

$$P_{it}^{m(1-\varepsilon)} = \left[(1-\alpha) P_{it}^{m*(1-\varepsilon)} + \alpha P_{it-1}^{m(1-\varepsilon)} \right] \tag{3.16}$$

3.2.2.2. Final product group. The final product group uses the j products as an input in order to produce the i products according to the technology

$$Y_{it} = F(Y_{jit}) = \left[\int_0^1 (Y_{jit})^{1-1/\varepsilon} dj \right]^{1/(1-1/\varepsilon)} \tag{3.17}$$

Firms are price setters. In exchange they must stand ready to satisfy demand at the announced prices, formally firm i must satisfy $\left[\int_0^1 (Y_{jit})^{1-1/\varepsilon} dj \right]^{1/(1-1/\varepsilon)} \geq C_{it}$. Given (3.17) firm's i nominal profits in period t are

$$\Pi_{it} = P_{it} Y_{it} - \int_0^1 P_{jit} Y_{jit} dj \tag{3.18}$$

On average each i firm pays P_{it}^m to produce an additional unit[5] of Y_{it} and charges, for the same product, P_{it} to the households. The marginal cost for each firm i is therefore $MC_{it} = P_{it}^m$, or in real terms $mc_{it} = \frac{P_{it}^m}{P_t}$, while the (real) profit function can be expressed as

$$\frac{\Pi_{it}}{P_t} = \left(\frac{P_{it}}{P_t} - mc_{it} \right) Y_{it} = \left(\frac{P_{it} - P_{it}^m}{P_t} \right) Y_{it} \tag{3.19}$$

The mark up of the generic firm i is defined as $\mu_{it} = \frac{P_{it}}{MC_{it}}$ and the average mark up charged in the economy

$$\mu_t = \frac{P_t}{MC_t} = \frac{P_t}{P_t^m} \tag{3.20}$$

[5]This can be found formally from the cost minimization problem of the firm

$$\min_{y_{jit}} \int_0^1 P_{jit} Y_{jit} + \zeta_t \left(Y_{it} - \left[\int_0^1 (Y_{jit})^{1-1/\varepsilon} dj \right]^{1/(1-1/\varepsilon)} \right)$$

where ζ_{jit} the Lagrangian multiplier, identifying the marginal costs, is equal to P_{it}^m.

while the aggregate demand for each i product can be expressed as

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\vartheta} (X_t) + \theta Y_{it-1} \tag{3.21}$$

where $X_t = \int_0^1 X_t^{\kappa} d\kappa$ is a measure of aggregate demand. This demand function generates a procyclical behaviour of its price elasticity. Indeed, when for any reason there is an upward shift in the aggregate demand X_t, the importance in (3.21) of the price elastic term $\left(\frac{P_{it}}{P_t}\right)^{-\vartheta}$ increases hence reducing the relative importance of θY_{it-1}, which, given its habit origin, is by definition inelastic. Hence as pointed out by Ravn et al. (2006), this generates a co-movement between aggregate demand and price elasticity of demand. Given the negative relation between markup and price elasticity, this feature of the model implies countercyclical mark ups at the final group level.

The firm's problem consists of choosing processes P_{it} and Y_{it} given the processes $\{P_{it}^m, P_t, Q_{t,t+z}, X_t\}$ so as to maximise the present discounted value of real profits

$$E_t \sum_{z=0}^{+\infty} q_{t,t+z} \frac{\Pi_{it+z}}{P_{t+z}} \tag{3.22}$$

subject to the demand constraint in (3.21). The Lagrangian can be written as

$$\Lambda = E_o \sum_{t=0}^{+\infty} q_{0,t} \left\{ \left(\frac{P_{it} - P_{it}^m}{P_t}\right) Y_{it} + \omega_{it} \left[\left(\frac{P_{it}}{P_t}\right)^{-\vartheta} (X_t) + \theta Y_{it-1} - Y_{it} \right] \right\}$$

where ω_{it} is the Lagrangian multiplier related to (3.21). The first order conditions are

$$\frac{d\Lambda}{dY_{it}} = 0 \Rightarrow \omega_{it} = \left(\frac{P_{it} - P_{it}^m}{P_t}\right) + \theta E_t q_{t,t+1} \omega_{it+1} \tag{3.23}$$

$$\frac{d\Lambda}{dP_{it}} = 0 \Rightarrow Y_{it} = \vartheta \omega_{it} \left(\frac{P_{it}}{P_t}\right)^{-(\vartheta+1)} X_t \tag{3.24}$$

With the market clearing conditions $Y_{jit} = C_{jit}$ and $Y_{it} = C_{it}$.

The variable ω_{it}, representing the Lagrangian multiplier to the final group firm problem, can be interpreted as the shadow value of profits given by the sale of an extra unit of good i at time t. Indeed, (3.23) has two components: the first one, represented by $\left(\frac{P_{it}-P_{it}^m}{P_t}\right)$, identifies the contemporaneous increase in marginal profit derived by an extra unit sold in time t. The second derives directly from the deep habits assumption. In fact, given the shape of habits, for each unit sold at any time t, the firms will sell θ units of the same good at the time $t+1$. This intertemporal effect on marginal profits is here represented by $\theta E_t q_{t,t+1}\omega_{it+1}$. On the other hand, (3.24) states that each i firm chooses its optimal price P_{it} where the marginal benefit of a unit increase in prices, identified by Y_{it}, is equal to its marginal cost (in terms of reduced demand) represented by $\omega_{it}\left(\frac{P_{it}}{P_t}\right)^{-(\vartheta+1)} X_t$.

3.2.3. Equilibrium

The equilibrium is represented by (3.1), (3.7), (3.9), (3.10), (3.15), (3.16), (3.20), (3.23) and (3.24). In order to have a complete description of the equilibrium we need to add to this set of conditions the expression for the total profits present in the economy

$$\Phi_t = \Pi_t + \Pi_t^m \tag{3.25}$$

and

$$N_t = \frac{Y_t}{A_t}\int_0^1\int_0^1\left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} dj di \tag{3.26}$$

The first of these two equations simply states that the totality of state contingent assets held by the households are the sum of the profits coming from the monopolistic environment of the production sector and from the monopolistic environment of the final sector. The second represents the market clearing condition of total labour demand. It includes a term of price dispersion $\int_0^1\int_0^1\left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} didj$ which is not relevant up to the second order.

Here we present in detail the equilibrium conditions as log deviations from the non stochastic efficient steady state.[6] Henceforth $\widehat{K}_t \equiv \log\left(\frac{K_t}{K}\right)$, where K is the steady state

[6] For a detailed description of log-linearization and the steady state see the appendix of this chapter .

level of a variable, represents the log deviation of a variable from its non stochastic steady state. The log linear equilibrium can be defined as

$$\widehat{X}_t = \frac{\widehat{C}_t - \theta \widehat{C}_{t-1}}{1-\theta} \tag{3.27}$$

$$\eta \widehat{N}_t + \sigma \widehat{X}_t = \widehat{W}_t - \widehat{P}_t \tag{3.28}$$

$$\widehat{X}_t = E_t \widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t \pi_{t+1}\right) \tag{3.29}$$

$$\frac{1}{\omega\mu}\left(\widehat{\mu}_t\right) = \widehat{\omega}_t - \theta\beta\left(E_t \pi_{t+1} - \widehat{R}_t + E_t \widehat{\omega}_{t+1}\right) \tag{3.30}$$

$$\widehat{\omega}_t = \widehat{Y}_t - \widehat{X}_t \tag{3.31}$$

$$\widehat{Y}_t = \widehat{C}_t \tag{3.32}$$

$$\widehat{Y}_t = \widehat{A}_t + \widehat{N}_t \tag{3.33}$$

$$\widehat{A}_t = \rho_a \widehat{A}_{t-1} + \varepsilon_t^a \text{ with } \varepsilon^a \sim N(0,1) \tag{3.34}$$

$$\pi_t^m = \beta E_t \pi_{t+1}^m + k\left(\sigma \widehat{X}_t + \eta \widehat{N}_t - \widehat{A}_t + \widehat{\mu}_t\right) \tag{3.35}$$

$$\pi_t = \pi_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{3.36}$$

$$\pi_t = \hat{P}_t - \hat{P}_{t-1} \tag{3.37}$$

$$\pi_t^m = \hat{P}_t^m - \hat{P}_{t-1}^m \tag{3.38}$$

This model shares with Ravn et al. (2006) the equations (3.27)-(**??**). The only difference is represented by (3.35), the New Keynesian Phillips Curve (NKPC henceforth) introduced by the presence of sticky prices a la Calvo (1983) in the production sector. In common with the traditional "*superficial*" external habits models (i.e. Abel (1990)) it shares the optimal labour supply (3.28) and the dynamic IS curve (3.29). Within this class of models the macroeconomic propagation of shocks generates (through the demand channel) a high persistence of aggregate variables. The main intuition for this result lies in the shape of (3.29). Given the definition of X_t as a quasi difference between current and past consumption, it is indeed easy to see that in the dynamic IS curve current consumption is a function of a combination of both future and past consumption.

Amato and Laubach (2004) show that indeed a superficial habits model augmented with sticky prices generates a higher persistence not only to the real variable but also in the inflation rate. The presence of $\widehat{X}_t$ in the NKPC causes in fact a longer impact of any output fluctuation on actual and expected inflation.

As stressed above, the introduction of deep habits creates other dynamic effects in this model. First of all the pricing problem of final group firms becomes dynamic. As a result of (3.30) and (3.31), we can guess the implied dynamic behaviour of markup and marginal profits. An increase in current demand generates, *ceteris paribus,* an increase in the price elasticity of demand, causing a negative relation through (3.31), between output and marginal profit[7]. This intratemporal effect is the price elasticity effect of deep habits on mark ups. Furthermore, it is clear from (3.30) that current mark up depends negatively on future values of profits, $E_t\widehat{\omega}_{t+1}$. The intuition behind this result is that a higher future value of profits generates an incentive to increase the future market share, and given the presence of deep habits this can be obtained by lowering the price today. On the other hand, current mark up depends positively on real interest rate. The reason is that with a higher interest rate firms discount more future profits, having therefore less incentive to increase the current market share.

The introduction of sticky prices creates a further complication to the setting. This feature generates in fact two more interactions in the model. On one hand from equation (3.35) current producer inflation depends positively on contemporaneous movement of the final group mark up. Indeed, the countercyclical behavior of $\widehat{\mu}_t$ seems to act as automatic stabiliser for the producer inflation rate. The intuition is the following. When for any reason there is an increase in current demand, producer inflation increases through the NKPC. The same increase in current demand generates a countercyclical movement in the final group mark up which puts downward pressure on producer prices. On the other

[7]To better see this it is enough to substitute in (2.89) the definition of X_t and the market clearing condition so that

$$\widehat{\omega}_t = -\frac{\theta}{1-\theta}\left(\widehat{Y}_t - \widehat{Y}_{t-1}\right)$$

hand the presence of staggered prices gives a role for monetary policy, as moving the interest rate affects final group mark ups via (3.30). These two effects will play a crucial role in the transmission mechanism of the model and in the setting of economic policies.

3.3. Determinacy and the Taylor Principle

This section describes the determinacy analysis of the model. In order to check the equilibrium properties of the model we close the system assuming a simple monetary rule of the type

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + \phi_\pi \pi_t^m + \phi_y \widehat{Y}_t \tag{3.39}$$

This formulation of monetary policy implies that movements in the nominal interest rate are directly linked to producer inflation, and changes in output and past nominal interest rate. The choice of target is justified by the fact that sticky prices (and therefore price dispersion) are present at the production group level, therefore we believe it is sensible for monetary policy to respond to producer inflation. As in Schmitt-Grohe and Uribe (2007) and Leith et al. (2009) the monetary rule also includes the response of the nominal interest rate, to output and to the past interest rate.

3.3.1. Calibration

The model is calibrated to a quarterly frequency. The model's structural parameters are β, σ, η, ε, ϑ, ϕ_π, ϕ_y, α, ρ and θ. The risk aversion parameter σ is set to 2 while η equals 0.25.[8] Following the literature, we impose $\varepsilon = \vartheta = 11$. This values imply a total (production plus final sector) steady state mark up over the real marginal costs (for $\theta = 0$) of 20%, which is in line with the empirical evidence. The discount factor, β, is fixed to 0.99. This value implies an annual steady state interest rate of 4%, which is in line with the average interest rate of the last 20 years of most OECD countries. In

[8] η is the inverse of the Frisch elasticity of labour supply. While micro estimates of this elasticity are rather small, they tend not to fit well in macro models. Here, differently than chapters 1 and 2, we follow the macroeconomic literature and choose a larger value of 4.0.

order to give persistence to the model we fix ρ_a, the parameter ruling the autoregressive process of technology, equal to 0.9. The steady state value of the final group mark up depends upon θ. In fact $\mu = \left(\frac{1}{(1-\theta)\vartheta}(\theta\beta - 1) + 1\right)^{-1}$. In particular the steady state mark up is increasing in θ (i.e. the higher θ the more inelastic the demand function). For the same reason ω, with $\omega = \frac{1}{\vartheta(1-\theta)}$, the steady state shadow value of profit is increasing in θ. Determinacy analysis is conducted for a wide range of the deep habits parameter, and the monetary policy rule ρ_r, ϕ_π and ϕ_y.

3.3.2. Determinacy Results

Figure 3.1 displays the determinacy analysis[9] with a monetary rule as in (3.39). Each sub-plot details the combinations of ϕ_π and ϕ_y which ensure determinacy (white area), indeterminacy (black area) and instability (red area). Moving from left to right across subplots increases the degree of interest rate inertia in the rule, ρ_r, while moving down the page increases the extent of habits formation, θ. Consider the first sub-plot in the top left hand corner with $\rho_r = 0$ and $\theta = 0$, which re-states the stability properties of the original Taylor rule. Here, the importance of the Taylor principle is revealed as $\phi_\pi > 1$. As we move across the page from left to right we increase the extent of interest rate inertia in the rule. In this case, as Woodford (2003) shows, the Taylor principle needs to be rewritten in terms of the long-run interest rate response to excess inflation, $\frac{\phi_\pi}{1-\rho_r} > 1$. As a result, the determinacy region in the positive quadrant spreads further into the adjacent quadrants (where $\phi_\pi < 1$) since a given level of instantaneous policy response to inflation ϕ_π has a far greater long-run effect. It is also interesting to note that a second region of determinacy exists where the interest rate rule fails to satisfy the Taylor principle, such that $\phi_\pi < 1$, and the response to the output is strongly negative. This region is not often discussed in the literature, but is mentioned in Rotemberg and Woodford (1999) and in Leith et al. (2009). Typically, when monetary policy fails to satisfy the Taylor principle, inflation can be driven by self-fulfilling expectations which are validated by monetary

[9] A technical analysis of determinacy can be found in the appendix of this chapter.

policy. However, when the output response is sufficiently negative there is an additional destabilising element in the policy, which overturns the excessive stability generated by a passive monetary policy, implying a unique saddle-path where any deviation from that saddlepath will imply an explosive path for inflation.

If the extent of habits formation is relatively low, the determinacy properties of the model are similar to those observed in the case of no habit formation. However, when the degree of habits formation exceeds $\theta > 0.77$, then there are some significant differences. The usual determinacy region tends to disappear and the system becomes indeterminate. This indeterminacy is linked to the additional dynamics displayed in the final goods sectors, where the markup, due to the deep habits formation, is time-varying. Suppose economic agents expect an increase in inflation. Given an active interest rate rule, $\phi_\pi > 1$, this will give rise to a tightening of monetary policy. Typically, such a policy would lead to a contraction in aggregate demand, invalidating the inflation expectations. However, in the presence of deep habits, the higher real interest rates will encourage final goods firms to raise current mark-ups as they discount the lost future sales such price increases would imply more heavily. If the size of habits effects is sufficiently large, then this increase in mark-ups can validate the initial increase in inflationary expectations, leading to self-fulfilling inflationary episodes and indeterminacy.

Furthermore, the excessive stability implied by endogenous markup behaviour implies that the only determinate rule in the presence of a large deep habits effect is where the rule is passive, $\frac{\phi_\pi}{1-\phi_R} < 1$, and the policy response to the output is sufficiently strongly negative.

Finally, when we combine a moderate the deep habits effect (θ around 0.4) with interest rate inertia, it becomes possible to induce instability in our economy when the rule is passive, $\frac{\phi_\pi}{1-\phi_R} < 1$ and the interest rate response to the output gap is negative, ϕ_Y < 0. The relatively slow evolution of consumption under habits combined with interest rate inertia and a perverse policy response to output gaps and inflation serves to induce a cumulative instability in the model.

3.4. Optimal Policy

First we consider the Social Planner problem, and then we compare this with the non-stochastic steady state in order to derive the optimal subsidy, $\varkappa$, which, financed with the lamp-sum tax T, can ensure that the steady state variables are at their socially optimal level.[10] Next we derive the policy maker loss function as a second order approximation of the utility function of the representative consumer which assesses the extent to which endogenous variables differ from the efficient equilibrium due to the nominal inertia and the overconsumption generated by external habit formation. Finally, we minimise this loss function subject to the log-linearised structural equations of the model in order to determine the optimal behaviour of interest rate.

3.4.1. The Social Planner Problem and the Optimal Subsidy

The social planner problem can be defined as the maximisation of the utility function of the representative consumer subject to the market clearing condition, the production function and the definition of habits. Once the maximisation takes place we compare the social planner's outcome in steady state with the outcome resulting from the non stochastic steady state that emerges from the decentralised equilibrium. Imposing equality between these two, one can obtain the optimal subsidy as[11]

$$(1-\varkappa) = \frac{\varepsilon - 1}{\varepsilon}\left(1 - \frac{1}{(1-\theta)\,\vartheta}(1-\theta\beta)\right)\frac{1}{1-\beta\theta} \tag{3.40}$$

When in place, this subsidy guarantees the steady state to be socially optimal. It is decreasing in ε and ϑ, (i.e. the lower the monopolistic competition the lower the steady state inefficiency). Furthermore it is greatly affected by θ, the habit parameter. Figure 3.2 sketches the value of the subsidy as a function of the degree of habit presents in the system. As one can see the subsidy is positive for low values of θ and it turns negative

[10]This procedure allows us to obtain an accurate expression for welfare involving only second-order terms.

[11]Details of the social planner problem can be found in the appendix of this chapter.

for high values of θ. The intuition for this is as follows. The system is affected by two distortions: the market power of firms and the externality of consumption. While the former generates a situation of under production, i.e. the natural level of output is below the efficient one, the latter induces a situation of over production as households fail to internalise the impact of their consumption decisions on others. For low θ, the distortion generated by the monopolistic power in the goods market is greater than the distortion generated by the externality in consumption, while the opposite is true for high values of θ. This means that for low values of external habits formation, in order to reach the Social Planner's equilibrium is necessary to subside intermediate firms' marginal costs, while when habits effects are large the social planner's equilibrium is implemented through a tax.

3.4.2. Policy Maker Loss Function

Appendix 3.A presents the step-by-step derivation of the second order approximation of the representative household's utility function around the efficient non stochastic steady state.

$$L = -\left((1-\theta)^{-\sigma}(C)\right)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{+\infty}\beta^t\Lambda_t + t.i.p + o(3) \tag{3.41}$$

where Λ_t, representing the instantaneous loss function is

$$\Lambda_t = (1-\theta\beta)\,\eta\left(\widehat{Y}_t - \frac{(1+\eta)}{\eta}\widehat{A}_t\right)^2 + \sigma(1-\theta)\,\widehat{X}_t^2 + (1-\theta\beta)\,\frac{\varepsilon}{\kappa}\,(\pi_t^m)^2$$

This loss function contains quadratic terms in inflation, which reflects the cost of price dispersion, output and habit-adjusted consumption which can be interpreted as the cost associated with deviation from the steady state of the real side of the economy. This formulation is particularly appealing as the weight associated with each component of the loss function derives in a microfounded way from the deep parameters of the model. A few aspects are worth stressing. First of all, the presence of the optimal steady state subsidy is a key assumption for the derivation of a quadratic expression suitable for policy analysis. Secondly, due to the presence of a dynamic real rigidity along the business cycle (i.e. deep habits), and despite the steady state subsidy, the flexible price equilibrium implied by the model is not efficient outside the non-stochastic steady state. For this reason we decide to keep the loss function and therefore the model in log deviation from steady state, rather than expressing the policy problem in gap variable from the flexible price equilibrium. Indeed, in order to find the efficient level of the associated flexible price equilibrium one needs to log linearise the first order conditions of the social planner and then subtract this measure from the log deviation from steady state level of the variable.

3.4.3. Optimal Policy Results

3.4.3.1. Technology Shock. We start the analysis of the consequences of deep habit formation for the setting of optimal commitment policy by computing, for different values of θ, the optimal response plan in the face of an unexpected 1% technology shock described in (**??**). Details of the reduced form, matrix representation and numerical approach for the policy problem are reported in the appendix of this chapter. We define the optimal response plan in the case where $Y_t = C_t$, as a particular stochastic-response process of the quadruple $\{\pi_t^m, \widehat{Y}_t, \widehat{R}_t, \widehat{\mu}_t\}$ which minimises (3.41) subject to the structural equations of the model (3.27)-(3.33) and (3.35) for all $t \geq 0$. The impulse response functions (IRF henceforth) are reported in figure 3.3. For $\theta = 0$ (the solid line), the policy problem collapses to a standard New Keynesian case. At the time of the shock the monetary authority lowers the nominal interest rate, so achieving the complete stabilisation of inflation and output gap. Output increases while the final group markup does not move from its steady state level.

This result is well established in the macroeconomic literature (Walsh, 2003) and it takes various names such as *the Divine Coincidence* (Galí and Blanchard, 2007). It states that in a simple NK model with no capital accumulation, monetary policy is able, through movements in the nominal interest rate to fully stabilise the economy (i.e.

to replicate the efficient flexible price equilibrium) in the event of a technology shock. Therefore the policy maker does not face any trade off between stabilising the output-gap and inflation. As θ increases this ceases to be true. The divine coincidence disappears causing a stabilisation problem. Indeed, with the presence of habit formation monetary policy faces an endogenous trade off: in the face of a technology shock it is not possible to fully replicate the efficient flexible price equilibrium.

The main intuition behind this result is that, while with $\theta = 0$ the monetary authority has to stabilise the inefficiency (and only the inefficiency) created by price dispersion, for $\theta > 0$, on the other hand, given the presence of habit formation both in the loss function and in the structural equations of the model, monetary policy has to correct, with just one instrument, two model distortions: price dispersion generated by staggered prices a la Calvo and the externality of consumption caused by habit formation. At the time of the shock output increases while the monetary authority decreases the nominal interest rate. As a consequence, final firms have a double incentive to lower their markups. On one side, the increase in output generates, through the presence of deep habits a strong incentive for the firms to lower their prices as to increase their sale base and future profits. On the other side, a similar effect is generated by a lower interest rate. As stressed in the previous section, a decrease in the mark up generates a downward pressure on producer prices, which adds up to a fall in nominal marginal cost induced by the technology shock by itself. As a result producer inflation decreases. As shown in figure 3.3, during the optimal plan the effect of a technology shock on inflation is increasing in θ. This is not surprising. Augmenting the importance of deep habit formation increases the incentive of the final firms to decrease their markups putting increasingly downward pressure on prices.

Figure 3.3 displays the response of output for different values of θ. For $\theta = 0$ the pattern of output is downward sloping, following the pattern of the technology shock. The reason for this is that with no habit formation the combination of movements in the nominal rate and of the technology shock, consistent with the forward-looking rational expectations of the agents, generates the greatest impact on output in the first period. With $\theta > 0$ this stops being true. First of all, on impact, the response of output is decreasing in θ. This is due to the fact that increasing θ it increases the importance of lagged output in the habit-adjusted Euler equation. For the same reason, positive values of θ generate hump-shaped response of output to shocks, see for example Christiano et al. (2005). Furthermore as θ increases, the stabilisation policy trade off gets worse, implying a widening output gap. It is also interesting to note that once the degree of habits passes a certain level, real interest rates actually rise initially, as policy makers seek to dampen the initial rise in consumption which imposes an undesirably externality on households.

Figure 3.4 shows the IRF under commitment and discretionary policy. The difference between the two types of policy is relatively small. This is because the variable patterns are mainly driven by the persistence implied by habit formation, rather than on the type of policy adopted. The main difference is represented by the price level stabilisation which is achieved only under commitment. The welfare loss in terms of steady state consumption is 1.92% higher under discretion than under commitment with the baseline calibration, i.e. $\theta = 0.75$[12].

3.4.3.2. Government Spending Shock. Figure 3.5 reports the IRF to a 1% government spending shock under optimal commitment. The model is augmented with a (inefficient) government spending[13]. The government spending, being excluded from the representative household utility function has a completely exogenous behaviour along the business cycle and takes the form of

[12] We measure the welfare cost of a particular policy as the fraction of permanent consumption that must be given up in order to equal welfare in the stochastic economy to that of the efficient steady state as

$$E\sum_{t=0}^{+\infty}\beta^{t}u\left(X_{t},N_{t}\right)=\left(1-\beta\right)^{-1}u\left(\left(1-\theta\right)\left(1-\varpi\right)C,N\right)$$

Given the utility function adopted the exspression for ϖ in percentage terms is

$$\varpi=\left[1-\frac{\left(\left(1-\sigma\right)\Upsilon\right)^{\frac{1}{1-\sigma}}}{\left(1-\theta\right)C}\right]\times 100$$

where $\Upsilon\equiv\left(1-\beta\right)\varsigma+\chi\frac{N^{1+\eta}}{1+\eta}$ and $\varsigma\equiv E\sum_{t=0}^{+\infty}u\left(X_{t},N_{t}\right)$ represents the unconditional expectation of lifetime utility in the stochastic equilibrium.

[13] Details of the log-linearized version of the model are discussed in appendix C

$$\widehat{G}_t = \rho\widehat{G}_{t-1} + \varepsilon_t^g \text{ with } \varepsilon_t^g \tilde{} iid\left(0; \sigma_g^2\right)$$

At the steady state the market clearing condition is now $Y = C + G$ with $\frac{C}{Y} = \psi$. A few points are worth stressing. Firstly, given the exogeneity of government spending, the shape of the loss function remains unchanged. Therefore the policy evaluation is carried out minimising (3.41) subject to the structural equations of the model augmented with government spending. For the same reason, in this case the social planner problem is trivial: given that the public spending has no value for the representative household, the social planner will always choose an allocation where $\psi = 1$. Hence, the output gap (i.e. the difference between the actual level of output and its efficient level counterpart) has not been carried out for this exercise. Moreover public spending is financed with a balanced tax rule of the type

$$T_t = G_t \quad \forall t$$

Where T_t is a lump sum tax paid by the households.

At the time of the shock, output increases through the market clearing condition. At the same time, for the same reasons explained in the previous section, final sector firms cut their markup. Of course the fall in markup is increasing in θ: the higher is the deep habit parameter in the model, the higher is the incentive for final firms to cut their markup so as to increase their sale base in period of *"high demand"*. The effect on producer price inflation is somehow not straighforward. In fact, if on one side producers prices have an upward pressure given by the expansion of total demand, on the other the countercyclical movement of the final group mark up puts downward pressure on the producers' real marginal cost and therefore on producers inflation. The numerical simulations show that the second effect overtakes the first one, leading to a decrease in producer price inflation. It is interesting to note how this effect is present even for low values of θ.

While the nominal interest rate increases in face of a government shock, representing the desire of the monetary authority to decrease the ineffecient high level of output, the real interest rate falls. This is due to the combined effect of a decrease in both the producer and the total price level generated by the dynamic behaviour of the final group mark up.

As in the previous exercise, and for the same reason, private consumption shows a hump-shaped pattern in response to a government spending shock. Furthermore, as one can see from figure 3.5 the qualitative behaviour of private consumption to a government spending shock is strongly dependent on the magnitude of θ. In fact, for low values of θ, private consumption responds negatively to public spending. For high values of θ, on the other hand, public spending crowds in on impact, private consumption. In order to understand this result we need to clarify a few points. First of all the so called *Ricardian Equivalence* holds in equilibrium: given the presence of lump-sum taxes, the timing of how public spending is financed is not important. In the first period, consumers internalise the dynamic effects of a change in the government spending (and therefore a change in taxation). When the deep habit parameter is low we fall into the traditional real business cycle result: for each increase in government spending, and independently on how this is financed, the after tax labour income of the consumer is reduced. Through the marginal rate of substitution between leisure and consumption, they transfer this reduction offering more work (which is indeed needed given the increase in total demand) and consuming less. For high values of θ, public spending crowds in private consumption. The main intuition for this result lays in the fact that deep habit formation causes a decrease in the general price level and therefore, ceteris paribus an increase, on impact in the real wages which is stronger than the decrease in the after tax income induced by an increase in G_t. From the simulation it appears that this increase is big enough to generate on impact the crowding-in of public spending on private consumption.

3.5. Conclusion and Future Research

This paper derives a simple and tractable New Keynesian model augmented with deep habit formation. Monetary policy is analysed both with a simple rule a la Taylor and in a welfare maximising environment (i.e. optimal policy).

With respect to a simple rule we find that the deep habit formation feature of the model creates a mechanism of transmission of monetary policy which leads easily to a situation of indeterminacy. Indeed we prove numerically that this indeterminacy is completely independent of the type of monetary rule assumed and instead it depends crucially on the degree of deep habit formation present in the system.

Regarding optimal policy, we derive a reasonably straightforward policy loss function which depends in a microfounded way on the structural parameters of the model and that displays both forward looking variables, such as output and producer inflation, and a backward looking one, represented by the habit-adjusted variable of consumption, X_t. Furthermore, we find that the introduction of external habit formation introduces a stabilisation trade off for the policy maker: in face of a technology shock the monetary policy is unable (in contrast with a traditional NK model), to stabilise both inflation and the output-gap when faced with technology shocks. As a result, at the time of the (technology) shock inflation decreases while output gap increases. The implications for optimal policy are that, as in Ravn et al.(2006), markups display a countercyclical behaviour while consumption, at least on impact, reacts positively to a government spending shock. Moreover the presence of sticky prices, and therefore a real effects of monetary policy on the real variables, creates an hump-shape in the IRF of consumption and markups which better replicates the stylized facts of the business cycle than its flexible price, real business cycle counterpart. The next step in the analysis of deep habits is to develop a model in which government spending is chosen endogenously as an active instrument of economic policy. Given the positive correlation between private and public consumption, this feature may result in interesting outcomes concerning optimal fiscal policy and its interaction with monetary policy.

3.6. Figures

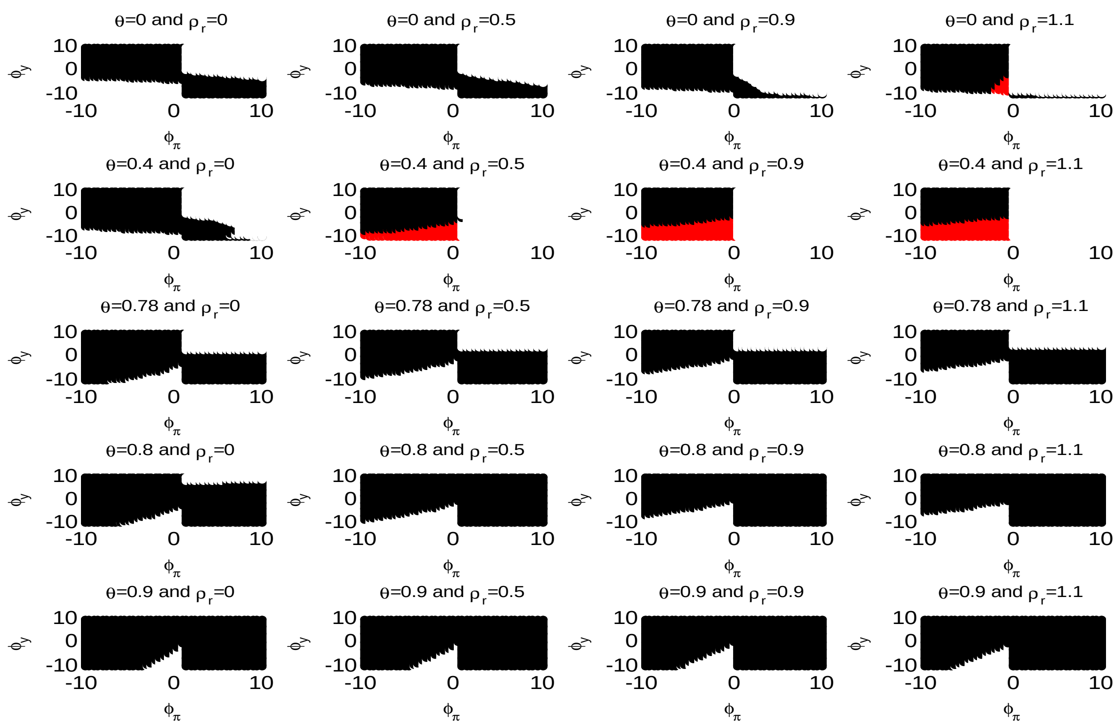

Figure 3.1. Determinacy of the model with a monetary rule of the type $\widehat{R}_t = \rho\widehat{R}_{t-1} + \phi_\pi \pi_t^m + \phi_y \widehat{Y}_t$. Determinacy (white area), indeterminacy (black area), instability (red area).

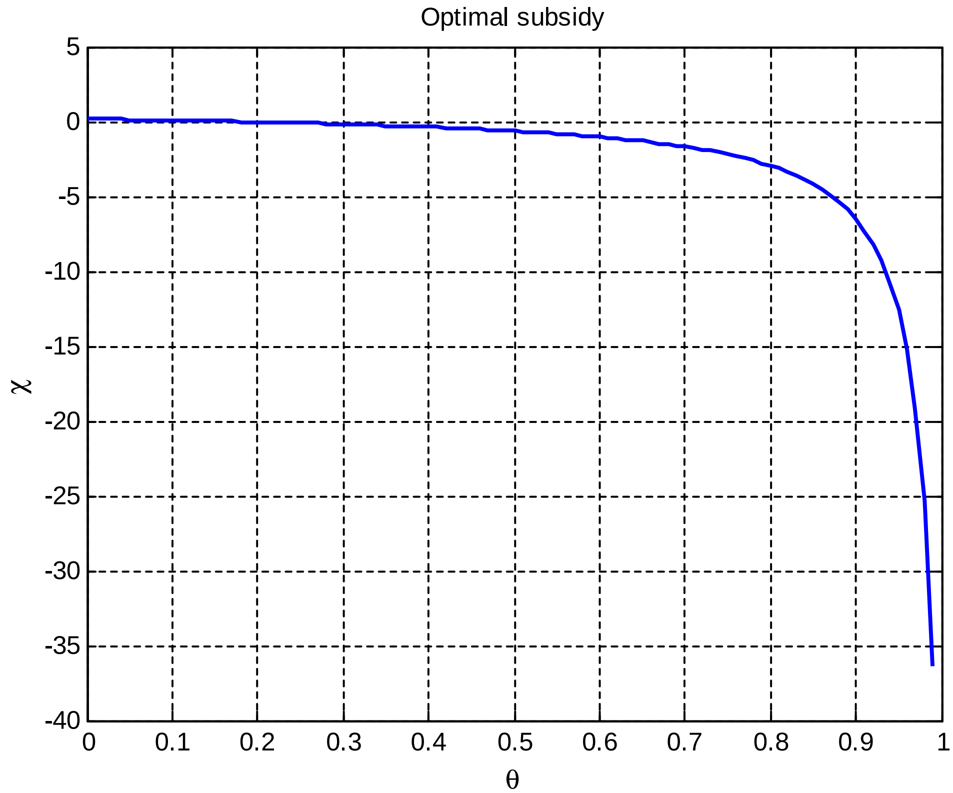

Figure 3.2. Optimal subsidy as function of the habit parameter θ. β, ε, η at their baseline values.

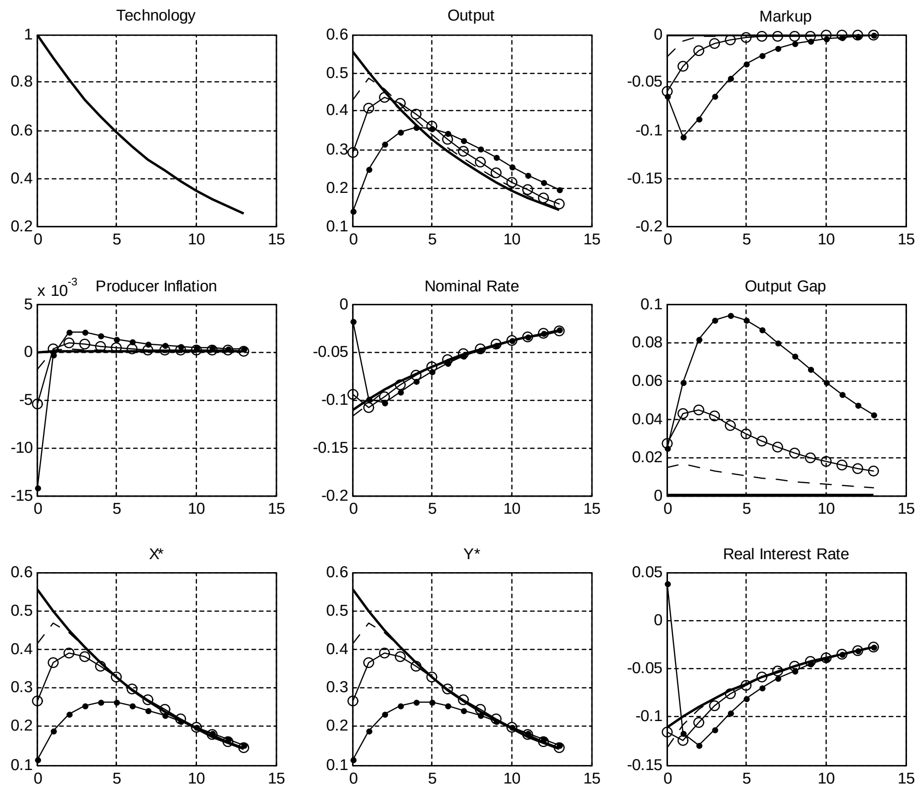

Figure 3.3. IRF to a 1% technology shock under optimal commitment. Solid line $\theta = 0$, dashed $\theta = 0.25$, circles $\theta = 0.55$, dots $\theta = 0.75$.

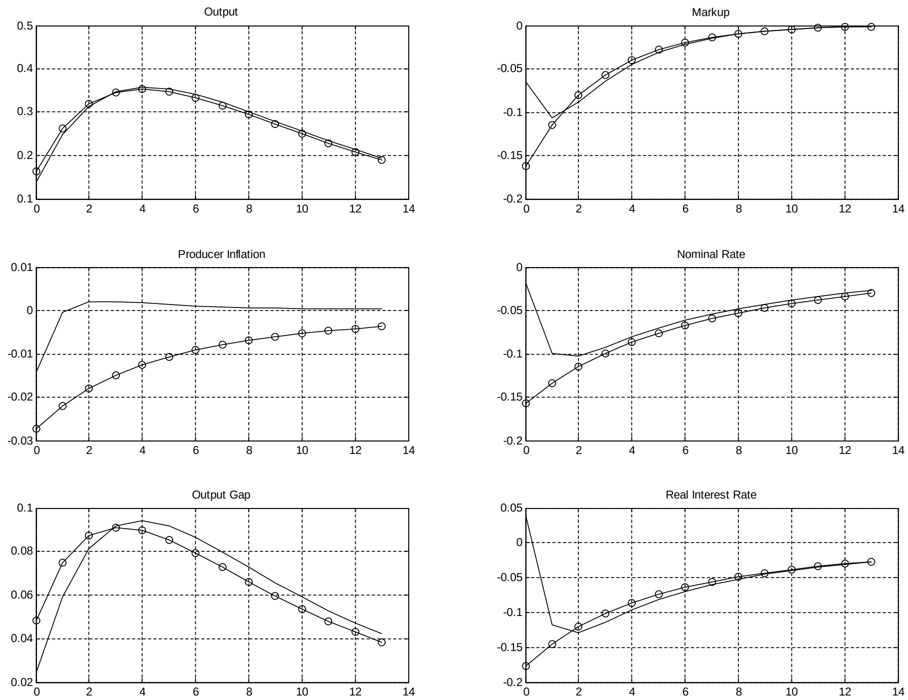

Figure 3.4. IRF to a 1% technology shock under commitment (solid line) and discretion (circles). Baseline calibration, $\theta = 0.75$.

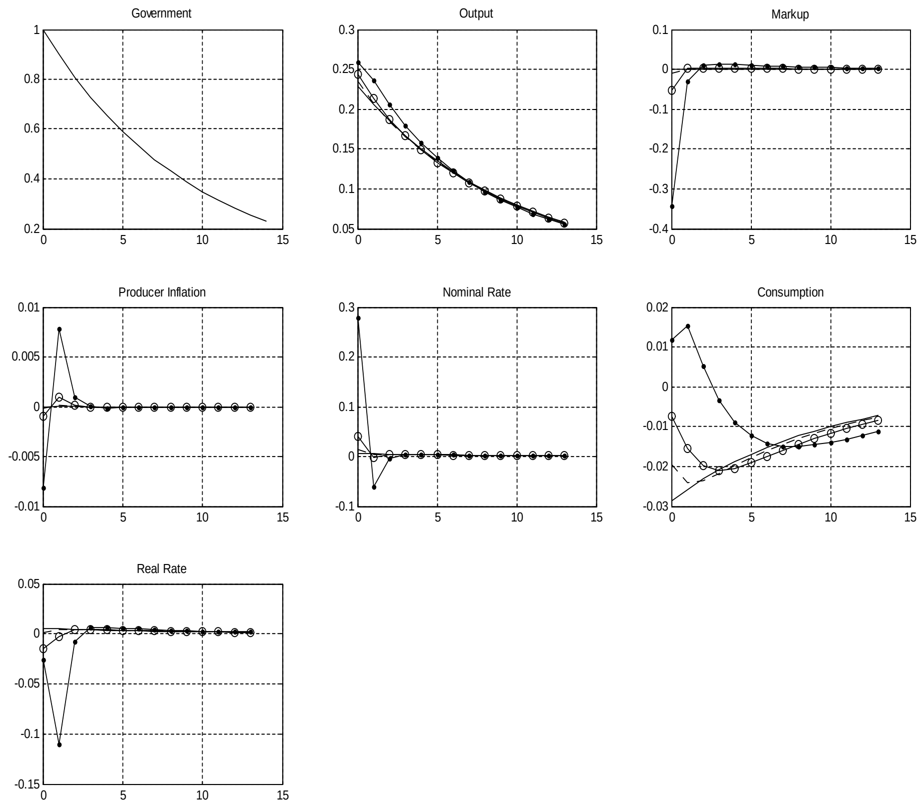

Figure 3.5. IRF to a 1% government spending shock under commitment. Solid line $\theta = 0$, dashed $\theta = 0.25$, circles $\theta = 0.55$, dots $\theta = 0.75$.

3.A. Appendix

3.A.1. Equilibrium conditions

The non-linear equilibrium conditions can be identified by these 11 equations. Since all consumers are identical, we can drop the κ index. We focus on symmetric equilibria. We can therefore drop the ι and the j indices.$(P_{it} = P_t; P_{it}^m = P_t^m)$

$$X_t = C_t - \theta C_{t-1} \tag{3.42}$$

$$u_x(X_t; N_t) = \beta R_t E_t u_x(X_{t+1}; N_{t+1}) \frac{P_t}{P_{t+1}} \tag{3.43}$$

$$\frac{W_t}{P_t} = -\frac{u_N(X_t; N_t)}{u_X(X_t; N_t)} \tag{3.44}$$

$$\Phi_t = \Pi_t + \Pi_t^m \tag{3.45}$$

$$Y_t = C_t = \vartheta\omega_t (C_t - \theta C_{t-1}) \tag{3.46}$$

$$\omega_t = \theta\beta E_t \left(\frac{X_t}{X_{t+1}}\right)^{\sigma} \omega_{t+1} + 1 - \frac{P_t^m}{P_t} = \theta\beta E_t \left(\frac{X_t}{X_{t+1}}\right)^{\sigma} \omega_{t+1} + 1 - \frac{1}{\mu_t} \tag{3.47}$$

$$N_t = \frac{Y_t}{A_t} \int_0^1 \int_0^1 \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} dj di \tag{3.48}$$

$$\mu_t^m = A_t \left(\frac{P_t^m}{W_t(1-\varkappa)}\right) \tag{3.49}$$

$$\mu_t = \frac{P_t}{P_t^m} = \frac{P_t}{\mu_t^m W_t(1-\varkappa)} A_t \tag{3.50}$$

$$\frac{P_t^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(mc_{t+z}^m \left(P_{t+z}^m\right)^{\varepsilon} Y_{t+z}\right)}{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(P_{t+z}^m\right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t}\right)^{-1} Y_{t+z}} \tag{3.51}$$

$$P_t^{m(1-\varepsilon)} = \left[(1-\alpha) P_t^{m*(1-\varepsilon)} + \alpha P_{t-1}^{m(1-\varepsilon)}\right] \tag{3.52}$$

Where (3.46) comes from the symmetry properties of the equilibrium given by (3.24).

3.A.1.1. Price elasticity and the intertemporal effects of deep habits. Iterating equation (3.47) forward and assuming the transversality condition $\lim_{j\to+\infty} \theta^j E_t q_{t,t+j}\omega_{t+j} = 0$ we can write that

$$\omega_t = E_t \sum_{j=0}^{+\infty} \theta^j q_{t,t+j} \left(\frac{\mu_{t+j}-1}{\mu_{t+j}}\right) = E_t \sum_{j=0}^{+\infty} \theta^j q_{t,t+j} \left(1 - \frac{P_{t+j}^m}{P_{t+j}}\right) \tag{3.53}$$

and using (3.46) and $Y_t = \frac{1}{\frac{1}{Y_t}}$

$$\omega_t = \frac{1}{\vartheta\left(1-\theta\frac{Y_{t-1}}{Y_t}\right)} \tag{3.54}$$

The denominator of the last expression is the short-run price elasticity for each variety of good in equilibrium where $\vartheta > \vartheta\left(1-\theta\frac{Y_{t-1}}{Y_t}\right)$. Furthermore, *ceteris paribus*, each increase in current demand Y_t relative to habitual demand θY_{t-1}, increases the short-run demand elasticity. Substituting (3.54) into (3.47) we obtain the dynamic evolution of the final firm markup

$$\mu_t = H\left(E_tY_{t+1}, Y_t, Y_{t-1}, R_t\right) = \left(1 - \frac{1}{\vartheta\left(1-\theta\frac{Y_{t-1}}{Y_t}\right)} + \theta E_t\left(\frac{X_t}{X_{t+1}}\right)^{\sigma}\frac{1}{\vartheta\left(1-\theta\frac{Y_t}{Y_{t+1}}\right)}\right)^{-1} \tag{3.55}$$

3.A.2. Steady state

This section reports the analytical derivation of the non stochastic steady state (steady state henceforth). The steady state equilibrium conditions are:

$$X = C - \theta C \tag{3.56}$$

$$u_x\left(X;N\right) = \beta R u_x\left(X;N\right) \tag{3.57}$$

$$\frac{W}{P} = -\frac{u_N\left(X;N\right)}{u_X\left(X;N\right)} \tag{3.58}$$

$$Y = C = \vartheta\omega\left(C-\theta C\right) \tag{3.59}$$

$$\Phi = \Pi + \Pi^m \tag{3.60}$$

$$\omega = \theta\frac{1}{R}\omega + 1 - \frac{1}{\mu} \tag{3.61}$$

$$Y = AN = C \tag{3.62}$$

$$\mu^m = A\left(\frac{P^m}{W\left(1-\varkappa\right)}\right) \tag{3.63}$$

$$\mu = \frac{P}{P^m} = \frac{P}{\mu^m W\left(1-\varkappa\right)}A \tag{3.64}$$

At steady state $A=1$. From the Euler equation (3.57) we can obtain the long run interest rate $R=\beta^{-1}$. The elasticity of substitution among intermediate goods is ε, therefore, imposing $P^m=1-\varkappa$, nominal wages are equal to $W=\frac{\varepsilon-1}{\varepsilon}$ and nominal producer sector markup is $\mu^m=\frac{\varepsilon}{\varepsilon-1}$ (3.63). From (3.59) $\omega=\frac{1}{(1-\theta)\vartheta}$. Plugging this in (3.61) we can obtain the steady state final group mark up

$$\frac{1}{(1-\theta)\,\vartheta}=\theta\beta\frac{1}{(1-\theta)\,\vartheta}+1-\frac{1}{\mu} \tag{3.65}$$

Solving for μ yields

$$\mu=\left(\frac{1}{(1-\theta)\,\vartheta}\,(\theta\beta-1)+1\right)^{-1} \tag{3.66}$$

From (3.64) it easy to show that $P=\mu\,(1-\varkappa)$. Assuming a standard CRRA utility function of the type

$$U=\frac{X^{1-\sigma}}{1-\sigma}-\gamma\frac{N^{1+\eta}}{1+\eta} \tag{3.67}$$

equation (3.58) implies

$$\frac{W}{P}=\gamma X^{\sigma}N^{\eta} \tag{3.68}$$

Using (3.56) and (3.62)

$$\frac{W}{P}=\gamma\,((1-\theta)\,C)^{\sigma}\,N^{\eta} \tag{3.69}$$

Substituting in the latter for the real wage it yields

$$\frac{\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{(1-\theta)\vartheta}\,(\theta\beta-1)+1\right)}{1-\varkappa}=\gamma\,(1-\theta)^{\sigma}\,N^{\sigma+\eta} \tag{3.70}$$

Solving for N

$$N = Y = C = \left(\frac{\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{(1-\theta)\vartheta}(\theta\beta-1)+1\right)}{\gamma(1-\theta)^{\sigma}} \frac{1}{1-\varkappa} \right)^{\frac{1}{\sigma+\eta}} \tag{3.71}$$

3.A.3. Social Planner

In order to find the optimal subsidy that achieves efficiency at steady state, we solve the social planner problem. This problem consists of maximising the representative consumer's utility function subject to economic constraints, once taken into account the symmetry conditions. The problem can be formalised as follow

$$Max_{\{X_t;C_t;N_t\}} E_t \sum_{t=0}^{+\infty} \beta^t \left(\frac{X_t^{1-\sigma}}{1-\sigma} - \gamma \frac{N_t^{1+\eta}}{1+\eta} \right)$$

$$\begin{aligned} s.t.\ Y_t &= C_t \\ Y_t &= A_t N_t \\ X_t &= C_t - \theta C_{t-1} \end{aligned}$$

The first order conditions are

$$\begin{aligned} X_t^{-\sigma} &= -\delta_t \\ \frac{\gamma N_t^{\eta}}{A_t} &= -\lambda_t \\ \lambda_t - \delta_t + \theta\beta E_t \delta_{t+1} &= 0 \end{aligned}$$

where λ_t and δ_t are the Lagrangian multipliers for the two constraints. Combining the three solutions yields

$$X_t^{-\sigma} = \theta\beta E_t X_{t+1}^{-\sigma} + \gamma \frac{N_t^{\eta}}{A_t} \tag{3.72}$$

At steady state

$$\begin{aligned} X^{-\sigma} &= \theta\beta X^{-\sigma} + \gamma N^{\eta} \\ (1-\theta\beta)\left((1-\theta)C\right)^{-\sigma} &= \gamma N^{\eta} \end{aligned}$$

Using (3.69) and (3.70)

$$\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{(1-\theta)\vartheta}(\theta\beta-1)+1\right) = (1-\varkappa)\gamma\left((1-\theta)C\right)^{\sigma} N^{\eta}$$

so the optimal subsidies is

$$(1-\varkappa) = \frac{\varepsilon-1}{\varepsilon}\left(1-\frac{1}{(1-\theta)\vartheta}(1-\theta\beta)\right)\frac{1}{1-\beta\theta} \tag{3.73}$$

The optimal subsidy offsets the steady state distortions caused by the monopolistic competition at the production level as well as at the final level and the distortion caused by habit formation. If (3.73) is in place the steady state levels of the variables is efficient and the first best is reached.

3.A.4. Log-linearisation

Log linearisation of (3.56) and (3.57) (where hatted variables identify a variable log deviation from its steady state value i.e. $\widehat{K}_t = \log\frac{K_t}{K}$ and $\pi_t = \widehat{P}_t - \widehat{P}_{t-1}$)

$$\begin{aligned} \widehat{X}_t &= \frac{\widehat{C}_t - \theta\widehat{C}_{t-1}}{1-\theta} \\ \widehat{X}_t &= E_t\widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\pi_{t+1}\right) \end{aligned}$$

The log linearisation of the optimal labour supply follows

$$\sigma\widehat{X}_t + \eta\widehat{N}_t = \widehat{W}_t - \widehat{P}_t$$

and the log linearisation of (3.46) is

$$\widehat{Y}_t = \widehat{\omega}_t + \frac{\widehat{C}_t - \theta \widehat{C}_{t-1}}{1-\theta}$$

The log linearisation of the production function follows

$$\widehat{Y}_t = \widehat{A}_t + \widehat{N}_t$$

and the market clearing condition

$$\widehat{Y}_t = \widehat{C}_t$$

3.A.5. The NKPC

We show above that the optimal price setter resets its price following

$$\frac{P_t^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(\frac{MC_{t+z}^m}{P_{t+z}} \left(P_{t+z}^m\right)^{\varepsilon} Y_{t+z}\right)}{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(P_{t+z}^m\right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t}\right)^{-1} Y_{t+z}} \tag{3.74}$$

Using the definition of the stochastic discount factor

$$q_{t,t+z} = \beta^z \frac{u_x\left(X_{t+z}; N_{t+z}\right)}{u_x\left(X_t; N_t\right)} = \beta^z \left(\frac{X_t}{X_{t+z}}\right)^{\sigma}$$

Therefore (3.74) can be rewritten as

$$\frac{P_t^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{X_t}{X_{t+z}}\right)^{\sigma} \left(\frac{MC_{t+z}^m}{P_{t+z}} \left(P_{t+z}^m\right)^{\varepsilon} Y_{t+z}\right)}{E_t \sum_{z=0}^{+\infty} (\alpha\beta)^z \left(\frac{X_t}{X_{t+z}}\right)^{\sigma} \left(P_{t+z}^m\right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t}\right)^{-1} Y_{t+z}} \tag{3.75}$$

Collecting all the terms which are not dependent on s and then log linearising the expression yields

$$\widehat{P}_t^{m*} = (1-\alpha\beta) \sum_{z=0}^{+\infty} (\alpha\beta)^z \, \widehat{MC}_{t+z}^m$$

Quasi-differentiating the last expression

$$\frac{1}{1-\alpha\beta}\widehat{P}_t^{m*} = \frac{\alpha\beta}{1-\alpha\beta}E_t\widehat{P}_{t+1}^{m*} + \widehat{MC}_t^m \tag{3.76}$$

Log linearisation of (3.16) and (3.50) yields

$$\widehat{P}_t^m = \left[(1-\alpha)\widehat{P}_t^{m*} + \alpha\widehat{P}_{t-1}^m\right] \tag{3.77}$$

and

$$\widehat{\mu}_t = \widehat{P}_t - \widehat{P}_t^m$$

Combining the last two expressions with (3.76) yields

$$\frac{1}{1-\alpha\beta}\left(\frac{1}{1-\alpha}\left(\widehat{P}_t - \widehat{\mu}_t\right) - \frac{\alpha}{1-\alpha}\left(\widehat{P}_{t-1} - \widehat{\mu}_{t-1}\right)\right) = \frac{\alpha\beta}{1-\alpha\beta}E_t\begin{pmatrix} \frac{1}{1-\alpha}\left(\widehat{P}_{t+1} - \widehat{\mu}_{t+1}\right) + \\ -\frac{\alpha}{1-\alpha}\left(\widehat{P}_t - \widehat{\mu}_t\right)\end{pmatrix} + \widehat{MC}_t^m \tag{3.78}$$

This can be solved (subtracting on both side $\frac{\widehat{P}_t}{\alpha}$) as

$$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\widehat{MC}_t^m - \widehat{P}_t\right) - \beta E_t\widehat{\mu}_{t+1} + \frac{1+\alpha^2\beta}{\alpha}\widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{3.79}$$

While the log linearisation of the production sector marginal cost yields

$$\widehat{MC}_t^m = \widehat{W}_t - \widehat{A}_t \tag{3.80}$$

Plugging in (3.79) (3.80) and (3.77) yields

$$\pi_t = \beta E_t\pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\sigma\widehat{X}_t + \eta\widehat{N}_t - \widehat{A}_t\right) - \beta E_t\widehat{\mu}_{t+1} + \frac{1+\alpha^2\beta}{\alpha}\widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{3.81}$$

In terms of producer prices the latter can be expressed as

$$\pi_t^m = \beta E_t\pi_{t+1}^m + k\left(\sigma\widehat{X}_t + \eta\widehat{N}_t - \widehat{A}_t + \widehat{\mu}_t\right)$$

Derivation equation(3.30)

$$\omega_t = \theta\beta E_t \left(\frac{X_t}{X_{t+1}}\right)^{\sigma} \omega_{t+1} + 1 - \frac{1}{\mu_t}$$

A first order approximation yields

$$\omega\left(1+\widehat{\omega}_t\right) = \theta\beta\omega\left(1+\sigma\widehat{X}_t - \sigma E_t\widehat{X}_{t+1} + E_t\widehat{\omega}_{t+1}\right) + 1 - \frac{1}{\mu}\left(1-\widehat{\mu}_t\right)$$

At steady state the latter collapses to

$$1 = \frac{\theta\beta\omega X^{\sigma}}{\omega X^{\sigma}} + \frac{1}{\omega} - \frac{\mu^{-1}}{\omega} \tag{3.82}$$

Collecting terms and constants and using (3.30)

$$\widehat{\omega}_t = \theta\beta\left(\sigma\widehat{X}_t - \sigma E_t\widehat{X}_{t+1} + E_t\widehat{\omega}_{t+1}\right) + \frac{1}{\omega\mu}\left(\widehat{\mu}_t\right) \tag{3.83}$$

3.A.6. Determinacy

This section gives technical details of the determinacy exercise. Substituting (3.32) in (3.27), (3.31) in (3.30), (3.33) in (3.35), (3.39) in (3.29) and (3.30), and (3.27) into (3.29), (3.30), and (3.35), we can rewrite the monetary model as

$$\widehat{Y}_t = \frac{E_t\widehat{Y}_{t+1}}{1+\theta} + \frac{\theta}{1+\theta}\widehat{Y}_{t-1} - \left(\frac{1-\theta}{1+\theta}\right)\frac{1}{\sigma}\left(\widehat{R}_t - E_t\pi_{t+1}\right) \tag{3.84}$$

$$\pi_t^m = \beta E_t\pi_{t+1}^m + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\left(\frac{\sigma}{1-\theta}+\eta\right)\widehat{Y}_t - \sigma\frac{\theta}{1-\theta}\widehat{Y}_{t-1} + \widehat{\mu}_t\right) \tag{3.85}$$

$$-\left(\frac{\theta+\theta^2\beta}{1-\theta}\right)\widehat{Y}_t + \frac{\theta^2\beta}{1-\theta}E_t\widehat{Y}_{t+1} + \frac{\theta}{1-\theta}\widehat{Y}_{t-1} = \theta\beta\left(E_t\pi_{t+1} - \widehat{R}_t\right) + \frac{1}{\mu\omega}\widehat{\mu}_t \tag{3.86}$$

$$\pi_t = \pi_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{3.87}$$

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + \phi_\pi \pi_t^m + \phi_y \widehat{Y}_t \tag{3.88}$$

We represent the model in matrix form as

$$A_0 X_{t+1} = A_1 X_t \tag{3.89}$$

with

$$A_0 = \begin{pmatrix} \frac{\theta^2\beta}{1-\theta} & 0 & -\frac{1}{\mu\omega} & -\theta\beta & 0 & \theta\beta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{1+\theta} & 0 & 0 & \frac{1}{\sigma}\left(\frac{1-\theta}{1+\theta}\right) & 0 & -\frac{1}{\sigma}\left(\frac{1-\theta}{1+\theta}\right) \\ 0 & k\left(\frac{\sigma}{1-\theta}+\eta\right) & k & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} \left(\frac{\theta^2\beta+\theta}{1-\theta}\right) & -\frac{\theta}{1-\theta} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 1 & -\frac{\theta}{1+\theta} & 0 & 0 & 0 & 0 \\ 0 & k\sigma\left(\frac{\theta}{1-\theta}\right) & 0 & 0 & 1 & 0 \\ \phi_y & 0 & 0 & 0 & \phi_\pi & \rho_r \end{pmatrix}$$

$$X_t = \begin{pmatrix} Y_t \\ Y_{t-1} \\ \mu_{t-1} \\ \pi_t \\ \pi_t^m \\ R_{t-1} \end{pmatrix}$$

The determinacy follows from the analysis of $H = A_0^{-1}A_1$. The system is determined when H has three eigenvalues outside the unit circle and three inside.

3.A.7. Welfare Function

We take the second order Taylor expansion to the utility function

$$U_t = E_0 \sum_{t=0}^{+\infty} \beta^t \left(\frac{X_t^{1-\sigma}}{1-\sigma} - \gamma \frac{N_t^{1+\eta}}{1+\eta} \right)$$

The first argument can be approximated as

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \frac{X^{1-\sigma}}{1-\sigma} + X^{1-\sigma} \left(\widehat{X}_t + \frac{1}{2}(1-\sigma)\widehat{X}_t^2 \right) + o(3) \tag{3.90}$$

while the second argument

$$\frac{N_t^{1+\eta}}{1+\eta} = \frac{N^{1+\eta}}{1+\eta} + N^{1+\eta} \left(\widehat{N}_t + \frac{1}{2}(1+\eta)\widehat{N}_t^2 \right) + o(3) \tag{3.91}$$

Now we need to relate the labour input to output and a measure of price dispersion. Using (3.23) and noting that there is no price dispersion across the i sectors we can write

$$N_t = \frac{Y_t}{A_t} s_t^m \tag{3.92}$$

where $s_t^m = \int_0^1 \left(\frac{P_{jit}}{P_t^m}\right)^{-\varepsilon} dj$. The latter expression can be written, following Woodford (2003), as

$$\widehat{N}_t = \widehat{Y}_t - \widehat{A}_t + var_j \frac{\varepsilon}{2} (p_{jit}) \tag{3.93}$$

therefore

$$\frac{N_t^{1+\eta}}{1+\eta} = N^{1+\eta} \left(\widehat{Y}_t + \frac{1}{2} (1+\eta) \left(\widehat{Y}_t - \widehat{A}_t \right)^2 + var_j \frac{\varepsilon}{2} (p_{jit}) \right) + t.i.p. + o(3)$$

where $t.i.p.$ includes all the terms which are independent of policy at time t. Using $X = (1-\theta) C$ and the second order approximation to the definition of X_t

$$\widehat{X}_t = \frac{\widehat{C}_t - \theta \widehat{C}_{t-1}}{1-\theta} - \frac{1}{2} \widehat{X}_t^2 + \frac{1}{2} \frac{1}{1-\theta} \widehat{C}_t^2 - \frac{1}{2} \frac{\theta}{1-\theta} \widehat{C}_{t-1}^2 + t.i.p. + o(3) \tag{3.94}$$

and putting all together, we can write the single period utility as

$$\begin{aligned} \Lambda_t &= (1-\theta)^{-\sigma} C^{1-\sigma} \left(\widehat{C}_t - \theta \widehat{C}_{t-1} - \frac{1}{2} (1-\theta) \widehat{X}_t^2 + \frac{1}{2} \widehat{C}_t^2 - \frac{1}{2} \theta \widehat{C}_{t-1}^2 + \frac{(1-\sigma)(1-\theta)}{2} \widehat{X}_t^2 \right) + \\ &\quad -\gamma N^{1+\eta} \left(\widehat{Y}_t + \frac{1}{2} (1+\eta) \left(\widehat{Y}_t - \widehat{A}_t \right)^2 + \frac{\varepsilon}{2} var_j (p_{jit}) \right) + t.i.p + o(3) \end{aligned} \tag{3.95}$$

Collecting terms, using the infinite sum property of the loss function and the efficient level of C and N implied by the steady state subsidy, $(1-\theta\beta) \left((1-\theta)^{-\sigma} (C) \right)^{1-\sigma} = \gamma N^{\eta+1}$, we can write the loss function as

$$L = \Omega E_0 \sum_{t=0}^{+\infty} \beta^t \left\{ \begin{array}{c} (1-\theta\beta) \left(\widehat{C}_t - \widehat{Y}_t \right) - \theta \widehat{C}_{t-1} - \frac{1}{2} \theta \widehat{C}_{t-1}^2 + \frac{1}{2} (1-\theta\beta) \widehat{C}_t^2 + \\ -\sigma \frac{(1-\theta)}{2} \widehat{X}_t^2 - (1-\theta\beta) \left(\frac{1}{2} (1+\eta) \left(\widehat{Y}_t - \widehat{A}_t \right)^2 + \frac{\varepsilon}{2} var_j (p_{jit}) \right) \end{array} \right\} + t.i.p + o(3)$$

where $\Omega = \left((1-\theta)^{-\sigma} (C) \right)^{1-\sigma}$. Using the second order approximation to the market clearing condition

$$\widehat{C}_t - \widehat{Y}_t = \frac{1}{2} \widehat{Y}_t^2 - \frac{1}{2} \widehat{C}_t^2 + o(3) \tag{3.96}$$

we can write the loss function as

$$L=-\Omega\frac{1}{2}E_0\sum_{t=0}^{+\infty}\beta^t\left\{\begin{array}{c}-\left(1-\theta\beta\right)\widehat{Y}_t^2+\sigma\left(1-\theta\right)\widehat{X}_t^2+\\ +\left(\left(1-\theta\beta\right)\varepsilon var_j\left(p_{jit}\right)\right)+\left(1-\theta\beta\right)\left(\left(1+\eta\right)\left(\widehat{Y}_t-\widehat{A}_t\right)^2\right)\end{array}\right\}+t.i.p+o(3)$$

With this representation we assume that $-\Omega\theta\widehat{C}_{t-1}$, $-\Omega\theta\widehat{C}_{t-1}^2$ are $t.i.p.$ and we use the fact that

$$\lim_{i\to\infty}\beta^{t+i}\left(1-\theta\beta\right)\widehat{Y}_{t+i}=\lim_{i\to\infty}\beta^{t+i}\widehat{C}_{t+i}=\lim_{i\to\infty}\beta^{t+i}\widehat{C}_{t+i}^2=0$$

Furthermore collecting terms, we exploit the identity

(3.97)
$$\widehat{Y}_t^2+\frac{(1+\eta)}{\eta}\widehat{A}_t^2-2\frac{(1+\eta)}{\eta}\widehat{Y}_t\widehat{A}_t=\left(\widehat{Y}_t-\frac{(1+\eta)}{\eta}\widehat{A}_t\right)^2+\left(\frac{(1+\eta)}{\eta}-\left(\frac{(1+\eta)}{\eta}\right)^2\right)\widehat{A}_t^2$$

Moreover, noting that $-\Omega\left(\frac{1-\theta\beta}{\eta}\right)\left(\frac{(1+\eta)}{\eta}-\left(\frac{(1+\eta)}{\eta}\right)^2\right)\widehat{A}_t^2$ are included in the $t.i.p.$ term and using, following Woodford (2003),

$$\sum_{t=0}^{+\infty}\beta^t var\left(p_{jit}\right)=\frac{1}{k}\sum_{t=0}^{+\infty}\beta^t\left(\pi_t^m\right)^2+t.i.p+o(3) \tag{3.98}$$

where $k=\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}$, we can write the linear quadratic expression for the policy maker loss function as

$$L=-\Omega\frac{1}{2}E_0\sum_{t=0}^{+\infty}\beta^t\left\{\begin{array}{c}\left(1-\theta\beta\right)\eta\left(\widehat{Y}_t-\frac{(1+\eta)}{\eta}\widehat{A}_t\right)^2+\\ +\sigma\left(1-\theta\right)\widehat{X}_t^2+\left(1-\theta\beta\right)\frac{\varepsilon}{\kappa}\left(\pi_t^m\right)^2\end{array}\right\}+t.i.p+o(3) \tag{3.99}$$

3.A.8. Efficient flexible price equilibrium and gap variables

Given the dynamic real inefficiency along the business cycle represented by the presence of deep habits, the efficient equilibrium (in log-linear form) is carried out as the log-linearisation of the social planner first order conditions. A hatted star variable, i.e. $\widehat{Y}_t^*$, represents the social planner log deviation of a variable from its steady state value (i.e.$\widehat{Y}_t^*=\log\left(\frac{Y_t^*}{Y^*}\right)$). The first step to obtain the gap variable is to log linearise the social planner first order conditions

$$X_t^{*-\sigma} = \theta\beta E_t X_{t+1}^{*-\sigma} + \gamma\frac{N_t^{*\eta}}{A_t} \tag{3.100}$$

$$X_t^* = C_t^* - \theta C_{t-1}^* \tag{3.101}$$

$$Y_t^* = A_t N_t^* \tag{3.102}$$

$$Y_t^* = C_t^* \tag{3.103}$$

Log-linearisation of (3.100) (step-by-step)

$$\begin{aligned}
1 &= \frac{\theta\beta E_t X_{t+1}^{*-\sigma}}{X_t^{*-\sigma}} + \gamma\frac{N_t^{*\eta}}{A_t X_t^{*-\sigma}} \\
1 &\cong \theta\beta\left(1 + \sigma\widehat{X}_t^* - \sigma\widehat{X}_{t+1}^*\right) + \gamma\frac{N^\eta}{X^{-\sigma}}\left(1 + \sigma\widehat{X}_t^* + \eta\widehat{N}_t^* - A_t\right) \\
0 &\cong \theta\beta\left(\sigma\widehat{X}_t^* - \sigma\widehat{X}_{t+1}^*\right) + \gamma\frac{N^\eta}{X^{-\sigma}}\left(\sigma\widehat{X}_t^* + \eta\widehat{N}_t^* - A_t\right) \\
0 &\cong \theta\beta\left(\sigma\widehat{X}_t^* - \sigma\widehat{X}_{t+1}^*\right) + \gamma\left(1-\theta\right)^\sigma Y^{\sigma+\eta}\left(\eta\widehat{Y}_t^* + \sigma\widehat{X}_t^* - \left(1+\eta\right)A_t\right)
\end{aligned}$$

We can therefore write the log linearisation of (3.100) as

$$\Gamma_1\widehat{X}_t^* = \sigma\theta\beta E_t\widehat{X}_{t+1}^* + \Gamma\left(\left(1+\eta\right)A_t - \eta\widehat{Y}_t^*\right) \tag{3.104}$$

where $\Gamma = \gamma\left(1-\theta\right)^\sigma Y^{\sigma+\eta}$ and $\Gamma_1 = \sigma(\theta\beta + \Gamma)$. Log-linearisation of (3.101)[14] is

$$\widehat{X}_t^* = \frac{\widehat{Y}_t^* - \theta\widehat{Y}_{t-1}^*}{1-\theta} \tag{3.105}$$

The gap variables are therefore defined as $\tilde{K}_t = \hat{K}_t - \hat{K}_t^*$.

[14]in the expression we implicitly use the social planner market clearing condition.

3.A.9. Analytical representation of the policy problem

The policy maker seeks to minimise

$$\min\left((1-\theta)^{-\sigma}(C)\right)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{+\infty}\beta^t\Lambda_t$$

subject to

$$\widehat{X}_t = \frac{\widehat{Y}_t-\theta\widehat{Y}_{t-1}}{1-\theta}$$

$$\widehat{X}_t = \frac{1}{1-\theta}E_t\widehat{Y}_{t+1}-\frac{\theta}{1-\theta}\widehat{Y}_t-\frac{1}{\sigma}\left(R_t-E_t\pi^m_{t+1}-E_t\widehat{\mu}_{t+1}-\widehat{\mu}_t\right) \tag{3.106}$$

$$\frac{1}{\mu\omega}\widehat{\mu}_t = \delta_1\widehat{Y}_t-\delta_2E_t\widehat{Y}_{t+1}-(1+\theta\beta\sigma)\widehat{X}_t \tag{3.107}$$

$$\pi^m_t = \beta E_t\pi^m_t+k\left(\sigma\widehat{X}_t+\eta\widehat{Y}_t-(1+\eta)A_t+\widehat{\mu}_t\right) \tag{3.108}$$

$$A_t = \rho_aA_{t-1}+\varepsilon_t \text{ with } \varepsilon\sim N(0,1) \tag{3.109}$$

With $\delta_1=1-\frac{\theta^2\beta(1+\sigma)}{1-\theta}$, $\delta_2=\theta\beta-\frac{\theta\beta(1+\sigma)}{1-\theta}$. Where (3.106), which represents the demand side of the economy, is obtained combining the definition of habits, the definition of producer price inflation and the market clearing condition with the dynamic habit-adjusted IS curve. The evolution of the mark up, (3.107), is derived plugging into (3.30) the expression for the shadow value of final group profit (3.31) , the market clearing condition and the definition of habit. Finally, (3.108) is the combination of the NKPC with the production function and (3.109) is the technological progress.

Given the complexity of the minimisation we provide a numerical solution to the policy problem.

3.A.10. Matrix Representation of the Optimal Policy

The model is augmented to include the log-linearised solution of the Social Planner's problem. In matrix form can be written as

$$Ax_{t+1}=Bx_t+Cu_t+\varepsilon_{t+1} \tag{3.110}$$

where x_t is a $n \times 1$ vector of non-predetermined and predetermined variables, u_t is a $k \times 1$ vector of policy instruments and ε_{t+1} is a $n \times 1$ vector of innovations with covariance matrix Σ. A, B and C are matrices defined as

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1-\theta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \delta_1 & 0 & -\left(1+\sigma\theta\beta\right) & -\delta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{1-\theta} & -\frac{1}{\sigma} & -\frac{1}{\sigma} & 0 \\ 0 & 0 & 0 & \sigma k & 0 & 0 & \beta & 0 \\ 0 & 0 & -\Gamma & 0 & 0 & 0 & 0 & \sigma\theta\beta \end{pmatrix}$$

$$B = \begin{pmatrix} \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{\theta}{1-\theta} & 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{\theta}{1-\theta} & 0 & 0 & \frac{1}{1-\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\mu\omega} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\theta}{1-\theta} & -\frac{1}{\sigma} & 0 & 0 \\ k\left(1+\eta\right) & 0 & 0 & 0 & -k\eta & -k & 1 & 0 \\ \left(1+\eta\right)\Gamma & 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{\sigma} \\ 0 \\ 0 \end{pmatrix}$$

and the vectors of the model's variables and instruments are defined as

$$x_t = \begin{pmatrix} A_t \\ \widehat{Y}_{t-1} \\ \widehat{Y}^*_{t-1} \\ \widehat{X}_{t-1} \\ \widehat{Y}_t \\ \widehat{\mu}_t \\ \pi^m_t \\ \widehat{X}^*_t \end{pmatrix}, \quad u_t = \widehat{R}_t,\ \varepsilon_{t+1} = \varepsilon^a_{t+1} \text{ and } \Sigma = 1$$

In particular, the first row represents the technological process, the second row the identity

$$\widehat{Y}_t = \widehat{Y}_t$$

the fourth row identifies the definition of habits, the third and the last row identify (3.104) and (3.105) which represent the log-linearised equation of the Social planner's solution. The fifth row represents the evolution of the markup, the sixth row is the dynamic IS equation and the seventh row the NKPC augmented with the final group markup. Following Soderlind (1999) we represent the loss function (3.41) as

$$L = E_0 \sum_{t=0}^{+\infty} \beta^t \left(x_t^{'} Q x_t + x_t^{'} R u_t + u_t^{'} U u_t \right) \tag{3.111}$$

Given that in (3.41) there are no instruments terms, both R and U are matrix of zeros and Q is defined consistently as

$$Q = \frac{1}{2}\Omega \begin{pmatrix} 0 & 0 & 0 & 0 & -2\left(1+\eta\right)\left(1-\theta\beta\right) & 0 & 0 & 0 \\ 0 & \sigma\theta^2/\left(1-\theta\right) & 0 & 0 & -2\frac{\theta\sigma}{1-\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(1-\theta\beta\right)\eta + \frac{\sigma}{1-\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \left(1-\theta\beta\right)\frac{\varepsilon}{k} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The policy problem consists in maximising (3.110) subject to (3.111).

3.A.11. Model with Exogenous Government Spending

With respect to the model presented above the market clearing condition (log-linearised version) is

$$\widehat{Y}_t = \psi\widehat{C}_t + \left(1-\psi\right)\widehat{G}_t \tag{3.112}$$

Consistently the structural equations of the model are

$$\widehat{X}_t = \frac{1}{1-\theta}E_t\widehat{C}_{t+1} - \frac{\theta}{1-\theta}\widehat{C}_t - \frac{1}{\sigma}\left(R_t - E_t\pi^m_{t+1} - E_t\widehat{\mu}_{t+1} - \widehat{\mu}_t\right) \tag{3.113}$$

$$\pi^m_t = \beta E_t\pi^m_{t+1} + k\left(\sigma\widehat{X}_t + \eta\widehat{Y}_t - \left(1+\eta\right)\widehat{A}_t + \widehat{\mu}_t\right) \tag{3.114}$$

$$-\left(\frac{\theta+\theta^2\beta}{1-\theta}\right)\widehat{Y}_t + \frac{\theta}{1-\theta}\widehat{Y}_{t-1} = \left[\begin{array}{c} \theta\beta\sigma\widehat{X}_t - \left(\frac{\sigma\theta\beta+\theta^2\beta\psi}{1-\theta}\right)E_t\widehat{C}_{t+1} + \\ -\left(1-\psi\right)\frac{\theta^2\beta}{1-\theta}\widehat{G}_{t+1} + \frac{1}{\mu\omega}\widehat{\mu}_t + \frac{\sigma\beta\theta^2}{1-\theta}\widehat{C}_t \end{array}\right] \tag{3.115}$$

CHAPTER 4

Optimal Monetary and Fiscal Policy in a New Keynesian Model with Deep Habit Formation

Recent work on optimal policy in sticky price models suggests that demand management through fiscal policy adds little to optimal monetary policy. We explore this consensus assignment in an economy subject to 'deep' habits at the level of individual goods where the counter-cyclicality of mark-ups this implies can result in government spending crowding-in private consumption in the short run. We explore the robustness of this mechanism to the existence of price discrimination in the supply of goods to the public and private sectors. We then describe optimal monetary and fiscal policy in our New Keynesian economy subject to the additional externality of deep habits. Consistently with the mainstream literature (e.i. Gali' and Monacelli (2005), Eser et al. (2009)) we find that government spending adds little in the optimal stabilisation process. The stabilisation burden is entirely left to monetary policy.

4.1. Introduction

We address the issue of how monetary and fiscal policy should be set optimally as stabilisation management tools along the business cycle. We do so in a New Keynesian model, i.e. optimising agents, monopolistic competition and Calvo prices, augmented with a level of valuable government spending and external deep habit formation in private and public consumption in the sense of Ravn et al. (2006). The external habit is formed at the level of a single good rather than on the aggregate level of consumption as in, for example, Abel (1990). Monetary policy sets the nominal interest rate in every period while fiscal policy manages the level of public spending, balancing its budget constraint in every period with a non-distortive lump-sum taxation. The model so developed presents a nominal rigidity implied by the Calvo price mechanism, and two real rigidities generated by the externality in private and public consumption that external deep habits imply. This causes an endogenous policy stabilisation trade off[1] between inflation, the consumption, output and government spending gaps. Furthermore, as shown by Ravn et al. (2006, 2007), deep habit formation implies a further dynamic property in the firms' price setting behaviour, generating, *ceteris paribus,* an extra transmission channel for economic policies, see for example Leith et al. (2009), and potentially a positive correlation between private and public consumption.

The aim of this paper is to analyse how the policy trade-off generated by habit formation changes the optimal conduct of monetary and fiscal policy with respect to its basic New Keynesian model counterpart, see for example Eser et al. (2009). These authors find that in a basic New Keynesian model augmented with a level of valuable government spending, optimal policy involves a mute response of government spending gap to shocks, i.e. government spending is always at its Social Planner- efficient level. In other words, the policy maker does not use fiscal policy as a stabilisation device, leaving the whole "*stabilisation burden*" in the hands of monetary policy.

The intuition for this result goes as follows: changing the government spending gap is clearly costly in terms of welfare, because it moves government spending from its optimal level. At the same time fiscal policy is inherently inefficient in adjusting inflation compared to monetary policy. In fact while monetary policy acts both to reduce demand, by reducing consumption, but also to raise supply, as workers reduce their leisure in line with consumption, government spending acts only on the demand side. Therefore moving government spending from its efficient level worsens the welfare of the representative household and it is less effective than monetary policy in stabilising price dispersion.

[1]As shown in Amato and Laubach (2004), the policy trade off is not present when habits are of the internal type.

In this paper we aim to determine whether the introduction of the dynamic inefficiencies generated by the presence of external habits in consumption and government spending leads to a use of government spending gap as a stabilisation device.

This exercise can be seen as a natural extension of the monetary policy analysis in a NK model augmented with deep habits presented in Leith et al. (2009). The authors find that the presence of a policy trade off generated by the introduction of external deep habits, results in monetary policy (under full commitment) aiming to fully stabilise the price level as its principal policy objective, leaving the output gap (and therefore consumption gap) to rise above its efficient level. Here we study whether and how the policy maker uses fiscal policy in order to reduce this over consumption.

The main finding of the paper is that, as in the basic New Keynesian model analysed in Eser et at. (2009), the government spending gap plays a very small role in the stabilisation of the economy following a shock. The presence and the use of endogenous government spending is negligible and the policy analysis that emerges is almost isomorphic to the one presented in Leith et al. (2009). Both qualitatively and quantitatively the differences present in the impulse response functions analysis of this model and in that of Leith et al.(2009) are very small and negligible. As in Eser et al. (2009) fiscal policy does not improve the policy trade off and is dynamically inefficient as a stabilisation device when compared to its monetary counterpart.

The remainder of the paper is as follows: section 4.2 presents the model, section 4.3 discusses optimal full commitment policy and finally section 4.4 concludes.

4.2. The Model

The economy is comprised of households, two monopolistically competitive production sectors, a monetary authority and a government. There is a continuum of final goods that enter the households' and the government's consumption baskets, each final good being produced as an aggregate of a continuum of intermediate goods. The households and the government form external consumption habits at the level of each final good in their basket. Ravn et al. (2006) label this type of habits as *'deep'*.

4.2.1. The Households

The economy is populated by a continuum of perfectly rational, infinitely-lived households uniformly distributed on the unit interval and indexed by k. Each of these has preferences over a set of differentiated types of products (i.e. wine, cheese etc.), C_{it}^{k}. Types of product are indexed by i. Moreover, each of these types of goods is formed by a continuum of specific *"brand"* products, C_{jit}^{k}; indexed by j. Moreover households derive disutility from labour effort, N_{t}^{k}, which is supplied in a perfectly competitive labour market, and derive utility from a composite level of habit-adjusted public spending X_{t}^{g}, and have access to perfect and complete financial markets. The introduction of government spending in the utility function is a commonly used shortcut to give value to public consumption, see for example Galí and Monacelli (2005) and Leith and Wren-Lewis (2008). Following Ravn et al. (2006), it is assumed that preferences show external habit formation at the level of each type of product i, rather than, as in Abel (1990), at a final composite good level. For this reason our assumption on habit formation is commonly defined as *"deep habits"*. This can be formulated as

$$X_{t}^{c,\kappa}=\left(\int_{0}^{1}\left(C_{it}^{\kappa}-\theta C_{it-1}\right)^{\frac{\eta-1}{\eta}}di\right)^{\frac{\eta}{\eta-1}} \tag{4.1}$$

where $X_{t}^{c,\kappa}$ represents the habit-adjusted consumption basket of the consumer κ, C_{it}^{κ} identifies the amount of consumption of each good i, C_{it-1} is the cross sectional average of aggregate consumption of the generic good i in period $t-1$, θ represents the deep habits parameter in consumption habit and η is the elasticity of substitution among i goods and is a measure of monopolistic power. The cost minimisation problem implies that the representative consumer minimises, exploiting any price differences present in the system, the total expenditure as

$$\min_{\{C_{it}^{\kappa}\}} \int_0^1 P_{it} C_{it}^{\kappa} di \tag{4.2}$$

subject to (4.1). In (4.2), P_{it} identifies the price of good i. From the minimisation problem, one can infer the demand for each good i, as $C_{it}^{\kappa} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} X_t^{\kappa} + \theta C_{it-1}$ and the aggregate consumer price level as $P_t = \left(\int_0^1 (P_{it})^{(1-\eta)} di\right)^{\frac{1}{1-\eta}}$.

The representative consumer's stream of utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(X_t^{\kappa})^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^{\kappa})^{1+\varphi}}{1+\varphi} + \chi^G \frac{(X_t^{g})^{1-\sigma}}{1-\sigma} \right) \tag{4.3}$$

where β is the discount factor, E is the rational expectation operator, N_t^{κ} is the amount of labour supplied in the Walrasian labour market by the consumer κ, X_t^g is the habit-adjusted public spending consumption, σ is the CRRA parameter, φ is the Frisch inverse parameter on the disutility of labour, and χ and χ^G are the relative weights consumers put on labour and public consumption. The utility maximisation problem consists in maximising (4.3) subject to the nominal budget constraint

$$P_t X_t^{\kappa} + P_t \vartheta_t + E_t Q_{t,t+1} D_{t+1}^{\kappa} = W_t N_t^{\kappa} + D_t^{\kappa} + \Phi_t^{\kappa} - P_t T_t^{\kappa} \tag{4.4}$$

where $\vartheta_t = \theta \int_0^1 \left(\frac{P_{it}}{P_t}\right) C_{it-1} di$. $Q_{t,t+1}$ is the stochastic discount factor, D_{t+1}^{κ} is the beginning of the period portfolio of state contingent assets, W_t is the nominal wage, Φ_t are the profits from the ownership of firms and T_t^{κ} is a lump sum taxation. The standards first order conditions are the habit-adjusted Euler equation

$$1 = \beta R_t E_t \left[\left(\frac{X_t^{\kappa}}{X_{t+1}^{\kappa}} \right)^{\sigma} \frac{P_t}{P_{t+1}} \right] \tag{4.5}$$

and the habit-adjusted consumption-leisure decision

$$\chi \left(N_t^{\kappa}\right)^{\varphi} \left(X_t^{\kappa}\right)^{\sigma} = \frac{W_t}{P_t} \tag{4.6}$$

where $R_t = \frac{1}{E_t(Q_{t,t+1})}$, implied by the non arbitrage condition, represents the nominal return on a riskless one period bond paying off a unit of currency in $t+1$. Condition (4.6) simply states that real wage is equal to the marginal rate of substitution between consumption and leisure, while condition (4.5) is the habit-adjusted Euler equation. It states that households tend to smooth habit-adjusted consumption across periods taking into account the opportunity cost represented by the real interest rate, such that the marginal rate of substitution is the same across periods.

4.2.2. The Government

While it is natural to think of households failing to internalise the impact of their consumption decisions on the utility of others, it is less obvious that government spending decisions are subject to a similar externality if spending is on global public goods. However, if public goods are local then the externality in government consumption can occur at a local level. Controversies over 'post-code lotteries' in health care and other local services (Cummins, Francis, and Coffey (2007)) and comparisons of regional per capita government spending levels (MacKay (2001)) indicate that households care about their local government spending levels relative to those in other constituencies. We therefore allow for these effects in public consumption, but will assess how optimal policy varies as we alter the extent of such externalities. It is important to note that, although the national government is aware of the externality in the households' perception of public goods provision, in allocating public spending across goods, they are bound by the experience of that spending within each household.

In other words, this is not a model of pork-barrel politics where local politicians over-provide local services which are financed from universal taxation[2], but simply one in which public goods are local in nature and households care about the provision of individual public goods in their constituency relative to other constituencies.

[2]For a model of pork-barrel politics with vote-trading and alternative voting mechanisms, see Chari and Cole (1995).

Assuming, for simplicity, that each household defines an area associated with a local public good, the government decides for each household on the provision of individual public goods so as to maximize the aggregate $X_t^{g,\gamma}$ that enters household κ's utility function, given the allocated level of aggregate spending, G_{it-1}, from the previous period, the problem can be formalised as follows

$$\max_{\{G_{it}^{\gamma}\}} X_t^{g,\gamma} = \left(\int_0^1 \left(G_{it}^{\gamma} - \theta G_{it-1} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{4.7}$$

$$s.t \int_0^1 P_{it} G_{it}^{\gamma} di \leq P_t G_t^{\gamma} \tag{4.8}$$

where θ represents the government's constituency habits parameter and within this chapter we maintain the assumption that consumers and government have the same degree of habits.

In the same fashion as for the consumer problem, it is relatively straightforward to infer the demand of each i good

$$G_{it}^{\gamma} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t^{g,\gamma} + \theta G_{it-1} \tag{4.9}$$

Furthermore the each government department balances its budget as

$$P_t X_t^{g,\gamma} + P_t \vartheta_t^g = P_t T_t^{\gamma} \tag{4.10}$$

where $\vartheta^g = \theta \int_0^1 \left(\frac{P_{it}}{P_t} \right) G_{it-1} di$.

4.2.3. Firms

The production sector is assumed to be formed by two groups. One group, that we call for simplicity "*production group*", is formed by a continuum of firms indexed by j, each of whom produces in a monopolistic competitive environment a single variety of j products.

In each period each j firm sells all its products to the second group, formed again by a continuum of firms indexed by i, that we call "*final group*", which aggregates the j

products creating the i ones, and sells them in a monopolistic competitive environment to the households. Both types of firm are assumed to be price setters and to take as exogenous all the actions carried out by other firms of the same group.

4.2.3.1. Production Group. This group is assumed to have a linear labour intensive production function of the type $Y_{jit} = A_t N_{jit}$ where A_t identifies the common technology, Y_{jit} the total products of variety j, and N_{jit} the total labour input required to produce Y_{jit}. Each firm of this group has two constraints. The first is given by the demand of each good $Y_{jit} = \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} Y_{it}$, where $Y_{it} = C_{it} + G_{it}$, $\varepsilon > 1$ and P_{it}^m is a measure of the general producer price level. The firm's cost minimisation problem implies that

$$MC_t^m = (W_t/A_t)(1 - \varkappa)$$

where MC_t^m identifies the nominal marginal cost[3] for a firm j at time t and $\varkappa$ represents a steady state subsidy financed by the consumers which will be discussed in detail later. In real terms

$$mc_t^m = \frac{MC_t^m}{P_t} = \frac{W_t/P_t}{A_t}(1 - \varkappa) \tag{4.11}$$

The firm $j's$ real profits are given by $\frac{\Phi_{jit}}{P_t} = \left(\frac{P_{jit}}{P_t} - mc_t^m\right) Y_{jit}$, and the profits in the production sector as a whole follow

$$\int_0^1 \int_0^1 \frac{\Phi_{jit}}{P_t} dj di = \Phi_t^m \tag{4.12}$$

When all the firms can adjust their prices in each period, they set their prices according to

$$\frac{P_{it}^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) mc_t^m = \mu^m mc_t^m$$

where μ^m represents the production sector mark up due to the monopolistic competitive environment.

[3]Given the assumption on the labour market that marginal costs are common across the production group, we dropped the index j.

However, we assume that in order to change optimally its prices each production firm has to participate to the "*Calvo lottery*". If it is chosen (with probability $1-\alpha$), it can optimally reset its prices, otherwise (with probability α) it keeps its prices unchanged. When a firm can change its prices it takes into account the expected discounted value of current and future profits. The problem can be formalised as follows

$$\max_{P^*_{jit}} E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(\left(\frac{P^*_{jit}}{P_{t+z}} \right) Y_{jit+z} - mc^m_{t+i} Y_{jit+z} \right) \tag{4.13}$$

$$z.t. \quad Y_{jit+z} = \left(\frac{P_{jit}}{P^m_{it+z}} \right)^{-\varepsilon} Y_{it+z} \tag{4.14}$$

where $q_{t,t+z}$ is the real discount factor defined as

$$q_{t,t+z} = \beta^z \frac{u_x \left(X_{t+z}; N_{t+z}\right)}{u_x \left(X_t; N_t\right)} = \beta^z \left(\frac{X_t}{X_{t+z}} \right)^{\sigma} \tag{4.15}$$

or alternatively

$$q_{t,t+z} = Q_{t,t+z} \frac{P_{t+z}}{P_t}$$

Given that all the j companies that re-optimise operate the same choice, the first order condition with respect to P^{m*}_{it} can be expressed as follows

$$\frac{P^{m*}_{it}}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(mc^m_{t+z} \left(P^m_{it+z} \right)^{\varepsilon} Y_{it+z} \right)}{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(P^m_{it+z} \right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t} \right)^{-1} Y_{it+z}} \tag{4.16}$$

while the aggregate price level for the production group follows

$$P^{m(1-\varepsilon)}_{it} = \left[(1-\alpha) P^{m*(1-\varepsilon)}_{it} + \alpha P^{m(1-\varepsilon)}_{it-1} \right] \tag{4.17}$$

4.2.3.2. Final product group. The final product group uses the j products as an input in order to produce i products according to the technology

$$Y_{it} = F\left(Y_{jit}\right) = \left[\int_0^1 \left(Y_{jit}\right)^{1-1/\varepsilon} dj\right]^{1/(1-1/\varepsilon)} \tag{4.18}$$

Firms are price setters. In exchange they must stand ready to satisfy demand at the announced prices. Formally firm i must satisfy $\left[\int_0^1 \left(Y_{jit}\right)^{1-1/\varepsilon} dj\right]^{1/(1-1/\varepsilon)} \geq Y_{it}$. Given (4.18) the nominal profits of each firm i in period t are

$$\Phi_{it} = P_{it}Y_{it} - \int_0^1 P_{jit}Y_{jit}dj = \left(P_{it} - P_{it}^m\right)Y_{it} \tag{4.19}$$

On average each i firm pays P_{it}^m to produce an additional unit[4] of Y_{it} and charges, for the same product, P_{it} to the households. The marginal cost for each firm i is therefore $MC_{it} = P_{it}^m$, or in real terms $mc_{it} = \frac{P_{it}^m}{P_t}$, while the (real) profit function can be expressed as

$$\frac{\Phi_{it}}{P_t} = \left(\frac{P_{it}}{P_t} - mc_{it}\right)Y_{it} = \left(\frac{P_{it} - P_{it}^m}{P_t}\right)Y_{it} \tag{4.20}$$

The mark up of the generic firm i is defined as $\mu_{it} = \frac{P_{it}}{MC_{it}}$, and the average mark up charged in the economy

$$\mu_t = \frac{P_t}{MC_t} = \frac{P_t}{P_t^m} \tag{4.21}$$

while the aggregate demand for each i product can be expressed as

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta}\left(X_t + X_t^g\right) + \theta Y_{it-1} \tag{4.22}$$

where $X_t = \int_0^1 X_t^\kappa d\kappa$ and $X_t^g = \int_0^1 X_t^g dg$ are measures of aggregate demand. This demand function generates a procyclical behaviour of its price elasticity. Indeed, when

[4]This can be found formally from the cost minimization problem of the firm

$$\min_{y_{jit}} \int_0^1 P_{jit}Y_{jit}dj + \zeta_t\left(Y_{it} - \left[\int_0^1 \left(Y_{jit}\right)^{1-1/\varepsilon} dj\right]^{1/(1-1/\varepsilon)}\right)$$

where ζ_{jit}, the Lagrangian multiplier, identifying the marginal costs, is equal to P_{it}^m.

for any reason there is an upward shift in the aggregate demand X_t or *ceteris paribus,* X_t^g, the importance in (4.22) of the price elastic term $\left(\frac{P_{it}}{P_t}\right)^{-\eta}$ increases, hence reducing the relative importance of θY_{it-1}, which, given its habit origin, is by definition inelastic. Hence as pointed out by Ravn et al (2006), this generates a co-movement between aggregate demand and price elasticity of demand. Given the negative relationship between markup and price elasticity, this feature of the model implies countercyclical mark ups at the final group level.

The firm's problem consists of choosing processes P_{it} and Y_{it} given the processes $\{P_{it}^m, P_t, Q_{t,t+z}, X_t, X_t^g\}$ so as to maximise the present discounted value of real profits

$$E_t \sum_{z=0}^{+\infty} q_{t,t+z} \frac{\Phi_{it+z}}{P_{t+z}} \tag{4.23}$$

subject to (4.22). The Lagrangian can be written as

$$\Lambda = E_o \sum_{t=0}^{+\infty} q_{0,t} \left\{ \left(\frac{P_{it} - P_{it}^m}{P_t} \right) Y_{it} + \omega_{it} \left[\left(\frac{P_{it}}{P_t} \right)^{-\eta} (X_t + X_t^g) + \theta Y_{it-1} - Y_{it} \right] \right\}$$

where ω_{it} is the Lagrangian multiplier related to (4.22). The first order conditions are

$$\frac{d\Lambda}{dY_{it}} = 0 \Rightarrow \omega_{it} = \left(\frac{P_{it} - P_{it}^m}{P_t} \right) + \theta E_t q_{t,t+1} \omega_{it+1} \tag{4.24}$$

$$\frac{d\Lambda}{dP_{it}} = 0 \Rightarrow Y_{it} = \eta \omega_{it} \left(\frac{P_{it}}{P_t} \right)^{-(\eta+1)} (X_t + X_t^g) \tag{4.25}$$

with the market clearing conditions $Y_{it} = C_{it} + G_{it}$.

The variable ω_{it}, representing the Lagrangian multiplier to the final group firm problem, can be interpreted as the shadow value of profits given by the sale of an extra unit of good i at time t. Indeed, (4.24) has two components: the first one, represented by $\left(\frac{P_{it}-P_{it}^m}{P_t}\right)$, identifies the contemporaneous increase in marginal profit derived by an extra unit sold in time t. The second derives directly from the deep habit assumption. In fact, given the shape of habits, for each unit sold today the firms will sell θ units of the same good in the next period. This intertemporal effect on marginal profits is here represented by $\theta E_t q_{t,t+1}\omega_{it+1}$. On the other hand, (4.25) states that each i firm chooses its optimal price P_{it} where the marginal benefit of a unit increase in prices, identified by Y_{it}, is equal to its marginal cost (in terms of reduction in demand) represented by $\omega_{it}\left(\frac{P_{it}}{P_t}\right)^{-(\eta+1)}(X_t + X_t^g)$.

4.2.4. Aggregation

This section describes the model in terms of aggregate variables. At the intermediate level the market clearing condition implies

$$\left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon}(Y_{it}) = A_t N_{jit} \qquad \forall j, \forall i$$

Aggregating over $j's$ yields

$$s_{it}\left(Y_{it}\right) = A_t N_{it} \tag{4.26}$$

where $s_{it} = \int_0^1 \left(\frac{P_{jit}}{P_{it}^m}\right)^{-\varepsilon} dj$ and it represents the price dispersion in the intermediate producer sector. Given the symmetry in the final sector we can drop the i index

$$s_t Y_t = A_t N_t \tag{4.27}$$

where $s_t = \int_0^1 \left(\frac{P_{jt}^m}{P_t^m}\right)^{-\varepsilon} di$. While aggregate real profits are

$$\phi_t = Y_t - (1-\varkappa)\frac{W_t}{P_t}N_t \tag{4.28}$$

and on the demand side the market clearing condition implies

$$Y_t = C_t + G_t \tag{4.29}$$

Given the presence of two production sectors and therefore of two different price levels, P_t, i.e. consumer price index (CPI) and P_t^m, i.e. producer price index (PPI), the system has two different inflation rates as well, π_t and π_t^m. These are related as

$$\pi_t = \frac{\mu_t}{\mu_{t-1}} \pi_t^m \tag{4.30}$$

4.2.5. Log-linear system

We focus on a symmetrical equilibrium. This is represented by (4.1), (4.5), (4.6), (4.7), (2.54), (4.17), (4.21), (4.24), (4.25), (4.27), (4.29) and (4.30) from which, given the homogeneity across households (they all supply the same amount of labour and consume the same basket of goods), production firms (when extracted from the Calvo lottery choose the same price) and final firms (all the i firms choose the same price and supply the same quantity of goods), it is possible to eliminate from the above condition the superscript and the subscripts i and j. We log-linearise the equilibrium conditions around the non-stochastic-zero inflation steady state. We define a hatted variable as the variable log-deviation from its steady state value, i.e. $\widehat{K}_t = \log\left(\frac{K_t}{K}\right)$. This gives us the following system of equations

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{C}_t - \theta \widehat{C}_{t-1}\right) \tag{4.31}$$

$$\widehat{X}_t^g = \frac{1}{1-\theta}\left(\widehat{G}_t - \theta \widehat{G}_{t-1}\right) \tag{4.32}$$

$$\sigma \widehat{X}_t + \varphi \widehat{N}_t = \widehat{w}_t \tag{4.33}$$

$$\widehat{X}_t = E_t \widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1}\right) \tag{4.34}$$

$$\widehat{Y}_t = \widehat{\omega}_t + \left(\psi \widehat{X}_t + (1-\psi)\,\widehat{X}_t^g\right) \tag{4.35}$$

$$\widehat{\omega}_t = \frac{1}{\mu\omega}\widehat{\mu}_t + \theta\beta E_t \widehat{\omega}_{t+1} + \theta\beta\left(\widehat{X}_t - E_t \widehat{X}_{t+1}\right) \tag{4.36}$$

$$\widehat{Y}_t = \widehat{A}_t + \widehat{N}_t \tag{4.37}$$

$$\widehat{Y}_t = \psi \widehat{C}_t + (1 - \psi)\,\widehat{G}_t \tag{4.38}$$

$$\widehat{P}_t^m = \alpha \widehat{P}_{t-1}^m + (1 - \alpha)\,\widehat{P}_t^{m*} \tag{4.39}$$

$$\pi_t^m = \widehat{P}_t^m - \widehat{P}_{t-1}^m \tag{4.40}$$

$$\widehat{\mu}_t = \widehat{P}_t - \widehat{P}_t^m$$

$$\pi_t = \pi_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{4.41}$$

$$\pi_t^m = \beta E_t \pi_{t+1}^m + \frac{(1 - \alpha\beta)\,(1 - \alpha)}{\alpha}\left(\sigma \widehat{X}_t + \varphi \widehat{Y}_t - (1 + \varphi)\,\widehat{A}_t + \widehat{\mu}_t\right) \tag{4.42}$$

This framework shares the basic building block with the NK model augmented with external deep habits presented in Leith et al. (2009). The only difference is represented by the presence of public spending that enters in the utility function of the representative consumer. This short cut is used to give intrinsic value from a social planner point of view to government spending, see, for example, Galí and Monacelli (2008) and Leith and Wren-Lewis (2008). As a consequence, government spending becomes an endogenous policy instrument which can be used as a stabilisation device.

4.3. Optimal Policy

We compute optimal policy following the technique proposed by Woodford (2003). First we consider the Social Planner problem, and then we compare this with the non-stochastic steady state in order to derive the optimal subsidy which can ensure that the steady state variables are at their socially optimal level. Next we derive the policy maker loss function as a second order approximation of the utility function of the representative consumer which assesses the extent to which endogenous variables differ from the efficient equilibrium due to the nominal inertia and the private and public overconsumption generated by external habit formation. Finally, we minimise this loss function subject to the log-linearised structural equations of the model.

4.3.1. The Social Planner's Problem

The social planner is not constrained by the price mechanism and simply maximises the representative household's utility, (4.3), subject to the definition of both private and public habits formation (4.1) and (4.7), to the production function, (4.27), and resource constraints, (4.29). This yields the following first order conditions[5],

$$\frac{\chi\left(N_t^*\right)^{\varphi}}{\left(X_t^*\right)^{-\sigma}} = A_t\left[1-\beta\theta E_t\left(\frac{X_{t+1}^*}{X_t^*}\right)^{-\sigma}\right]$$

$$\left(X_t^*\right)^{-\sigma}-\theta\beta E_t\left(X_{t+1}^*\right)^{-\sigma} = \chi^g\left(X_t^{g*}\right)^{-\sigma}-\chi^g\theta\beta E_t\left(X_{t+1}^{g*}\right)^{-\sigma}$$

where we introduce the '*' superscript to identify the efficient level of that variable. Not surprisingly, given the dynamic nature of habit persistence the Social Planner's problem has a dynamic nature. Calculating the Social Planner's steady state, we can derive the optimal subsidy as

$$\varkappa = 1-\frac{1}{1-\theta\beta}\left[\left(1-\frac{1-\theta\beta}{(1-\theta)\,\eta}\right)\frac{\varepsilon-1}{\varepsilon}\right]$$

[5]A detailed derivation of the Social planner problem can be found in Appendix A.

while the optimal government spending rule is such that it implements

$$\left(\frac{\psi^*}{1-\psi^*}\right)^{-\sigma} = \chi^g$$

The previous Social Planner's first order conditions can be log-linearised around the socially optimal steady state so as to obtain

$$\widehat{Y}_t^* = \widehat{C}_t^* = \widehat{G}_t^* \tag{4.43}$$

It is therefore optimal to have equal fluctuations in the various components of the aggregate output. From this we can derive the Social Planner's Euler equation in terms of output as

$$\widehat{Y}_t^* = \theta\beta\varsigma E_t \widehat{Y}_{t+1}^* + \theta\varsigma\widehat{Y}_{t-1}^* + \left(\frac{1+\varphi}{\zeta}\right)\varsigma\widehat{A}_t \tag{4.44}$$

where $\zeta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$ and $\varsigma = \frac{1}{1+\theta^2\beta+\frac{\varphi}{\zeta}}$. Henceforth we define a gap variable as the difference between the log-deviation level of a variable and its correspondent social planner log-linearised level, i.e. $\widehat{K}_t^{gap} = \widehat{K}_t - \widehat{K}_t^*$.

4.3.2. Policy Maker Loss Function

The policy maker loss function can be written as[6]

$$L = \frac{1}{2}\Omega\psi E_0 \sum_{t=0}^{+\infty} \beta^t \Lambda_t + t.i.p + o(3) \tag{4.45}$$

where Λ_t, representing the instantaneous loss function, is

$$\Lambda_t = \left\{ \begin{array}{c} (1-\theta\beta)\,\varphi\left(\widehat{Y}_t - \frac{(1+\varphi)}{\varphi}\widehat{A}_t\right)^2 + \psi\sigma\,(1-\theta)\,\widehat{X}_t^2 + \\ +(1-\theta\beta)\,\frac{\varepsilon}{\kappa}\,(\pi_t^m)^2 + (1-\psi)\sigma\,(1-\theta)\,(\widehat{X}_t^g)^2 \end{array} \right\}$$

and $\Omega = \left((1-\theta)^{-\sigma}(C)\right)^{1-\sigma}$.

[6] Appendix A presents the step-by-step derivation of the second order approximation of the representative household's utility function around the efficient non stochastic steady state.

This loss function contains quadratic terms of producer price inflation, which reflects the cost of price dispersion, output and habit-adjusted private and public consumption which can be interpreted as the cost associated with deviation from the steady state of the real side of the economy. This formulation is particularly appealing as the weight associated with each component of the loss function derives in a microfounded way from the deep parameters of the model.

A few aspects are worth stressing. First of all, the presence of the optimal steady state subsidy is a key assumption for the derivation of a quadratic expression suitable for policy analysis. Secondly, while this welfare measure has the same basic components as a benchmark New Keynesian model augmented with government spending (without externalities due to consumption habits), see for example Leith and Wren-Lewis (2008), this welfare measure looks different, in that it does not contain a single real "variable gap", defined as the difference between a variable and its flex-price level. However, the current set-up is conceptually similar. The variables gap terms in the standard analysis captures the extent to which a variable deviates from its efficient level (typically because of nominal inertia, rather than any other distortion). In a model with external private and public habits, there are additional externalities which means that the flexible price equilibrium is unlikely to be efficient, such that it is not possible to rewrite variables in gap form due to the presence of a dynamic real rigidity along the business cycle (i.e. Deep Habits), and despite the steady state subsidy, the flexible price equilibrium implied by the model is not efficient. For this reason we decide to keep the loss function and therefore the model in log deviation from steady state, rather than expressing the policy problem in gap variable from the flexible price equilibrium.

4.3.3. Optimal Commitment and Calibration

If the central authority can credibly commit to following its policy plans, it then chooses, through an appropriate pattern of nominal interest rate and government spending, the policy that maximises households' welfare subject to the private sector's optimal behaviour, as summarised in equations (4.31) - (4.42), and given the exogenous process for technology. Appendix A gives analytical details of the optimal commitment policy. Here we present the numerical results.

The model is calibrated to a quarterly frequency. We fix the discount rate β to 0.99. This value implies an annual real interest rate of 4% which is in line with most of the macroeconomic literature. The relative weight on labour χ and that on government χ^g in the utility function are assumed to be 3 and 0.75 respectively. The risk aversion parameter σ is set at 2.0, while φ equals 0.25. We set these parameters' value following the estimation and calibration results of Galí et al. (2007) and Leith and Malley (2005). Consistent with the empirical evidence, the level of price inertia parameter, α, is set at 0.75. This value implies that on average prices remain fixed on average for one year. The degree of market power is 1.21, split approximately equally between the two monopolistically competitive sectors of our economy. The steady state value of the markup in the final goods sector is given as $\mu = \left(1 - \frac{1-\theta\beta}{\eta-\eta\theta}\right)^{-1}$, and depends on both the elasticity of substitution between final goods η and the degree of habit formation θ. However, the impact of θ on the markup η is minimal and we therefore set $\eta = \varepsilon = 11$. For the habit formation parameter θ, we use a benchmark value of 0.65, which falls within the range of estimates identified in the literature, see for example Smets and Wouters (2008). However, we allow θ to vary in the $[0, 1)$ interval as we conduct sensitivity analyses of our results. The steady state ratio between private consumption and total output, ψ, is fixed to 0.75, a value most used in the literature, see for example Galí et al (2007). Technology shocks are assumed persistent with persistence parameter $\rho = 0.9$.

In face of such a shock, the policy maker cannot simultaneously stabilise producer price inflation, the output gap and government spending gap. The reason for this is that the central authority has two policy instruments- the nominal interest rate and government spending- while the system displays three rigidities: a nominal rigidity, i.e. the price dispersion, generated by Calvo price setting at the production level, and two real rigidities, i.e. consumption externality both at private and public level, generated by external (deep) habits both at the consumer and government level. Instead, while nominal inertia points to a relaxation of monetary policy in the face of a positive technology shock to boost aggregate demand, private and public consumption externalities suggest that the higher aggregate demand this entails need not be desirable.

Figure 4.1 reports the impulse response functions to a 1% technology shock under optimal monetary and fiscal full commitment policy. At the time of the shock monetary policy cuts the nominal interest rate in order to boost aggregate demand and therefore stabilise price dispersion. As a consequence, private households substitute their current consumption and leisure from future to the present. Therefore, aggregate output increases and hours worked decrease. These two effects put pressure on the demand side of the labour market, generating an increase in real wages. Furthermore, the presence of deep habits causes final firms to respond to this increase in total demand cutting their prices in order to expand their sales base and induce consumption habits in their product, generating a further increase in real wages and total demand. Because the policy is expansionary, we can implicitly say that the inefficiency due to price stickiness is dominating over the real rigidities caused by the consumption externality. As the degree of importance of habits increases, inflation stabilisation remains the primary goal and the policy maker suffers a widening (positive) output and consumption gap due to both private and public consumption externality. However, once the degree of habits passes a certain level ($\theta = 0.75$), real interest rates actually rise initially, as policy makers seek to dampen the initial rise in consumption which imposes an undesirable externality on households and government as they fail to internalise the impact of their consumption decisions on others.

Fiscal policy reacts to the positive technology shock increasing government spending, however keeping it very close to the social planner level, i.e. the government spending gap is negligible. Government spending gap is not used as a stabilisation instrument despite the presence of a policy trade off. In other words, when monetary policy is unconstrained, government spending gap does not respond to shocks.

This result was first noted by Eser et al. (2009). These authors analyse a simple cashless NK model augmented with a level of (valuable) government spending. They show that in a small open economy in the fashion of Gali and Monacelli (2005), the optimal policy response to shocks, independently of whether these shocks are efficient or not, implements a zero government spending gap, therefore leaving the whole "*stabilisation burden*" to monetary policy.

The intuition for this result goes as follows: changing the government spending gap is clearly costly because it moves government spending away from the optimal provision of public goods. At the same time such a policy does not improve the stabilisation trade-off. The reason for this lies in the fact that fiscal policy is inherently inefficient compared to monetary policy in adjusting inflation. In fact while monetary policy acts both to reduce demand, by reducing consumption, but also to raise supply, as workers reduce their leisure in line with consumption, government spending acts only on the demand side.

This result holds in our model and is almost insensitive to changes in the deep habit parameter, i.e. θ. Independently from private and public consumption externality generated by external habits, monetary policy remains the most efficient stabilisation instrument influencing both the demand side of the economy through the Euler equation and the supply side of the economy both at the production level, via a change in the supply of labour and at the final level, via the intertemporal effect of a change in the interest rate on the final group pricing decisions.

4.4. Conclusions

This paper derives a small New Keynesian model augmented with deep habits formation in private and public consumption in the sense of Ravn et al. (2006) and a level of valuable government spending.

We compute optimal commitment monetary and fiscal policy using a linear quadratic technique. In the presence of a nominal rigidity due to sticky prices and two real rigidities due to externality in consumption and government spending, the policy maker faces a stabilisation trade off even in the face of a technology shock. Furthermore we find that despite the policy trade off, deviations of government spending from its efficient level are negligible. In other words, government spending is not used as a policy stabilisation device while all the "*stabilisation burden*" is left to monetary policy. As in the monetary economy counterpart of this model, see for example Leith et al (2009), monetary policy's principal objective is price stabilisation. As a result the system experiences both positive output and consumption gap. Moreover, due to the presence of deep habits formation, optimal policy implies a countercyclical behaviour of firms' markup together with a hump-shaped behaviour of all the real variables.

4.5. Figures

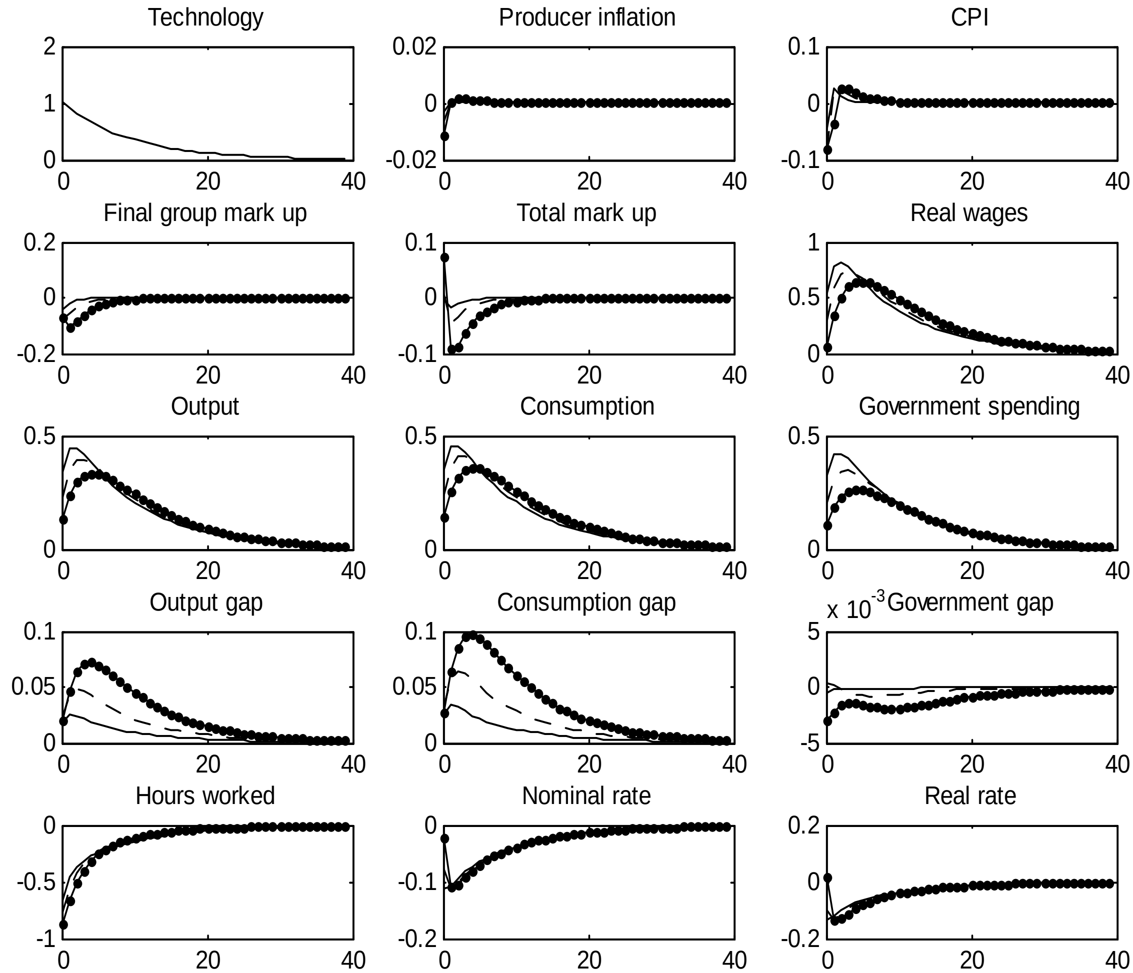

Figure 4.1. IRF's to a 1% technology shock. Optimal commitment policy. Solid line $\theta = 0.4$, dashed line $\theta = 0.65$ (baseline value), line dots $\theta = 0.75$.

4.A. Appendix

4.A.1. Equilibrium

Here we list the equilibrium condition described as system of non-linear equations

$$X_t^c = C_t - \theta C_{t-1} \tag{4.46}$$

$$X_t^g = G_t - \theta G_{t-1} \tag{4.47}$$

$$\chi (N_t)^{\varphi} (X_t^c)^{\sigma} = \frac{W_t}{P_t} = w_t \tag{4.48}$$

$$1 = \beta R_t E_t \left[\left(\frac{X_t}{X_{t+1}} \right)^{\sigma} (\pi_{t+1})^{-1} \right] \tag{4.49}$$

$$\omega_t = \left(1 - \frac{1}{\mu_t}\right) + \theta E_t q_{t,t+1} \omega_{t+1} \tag{4.50}$$

$$Y_t = \eta \omega_t \left(\frac{P_{it}}{P_t} \right)^{-(\eta+1)} (X_t + X_t^g) \tag{4.51}$$

$$G_t + \varkappa w_t N_t = T_t \tag{4.52}$$

$$s_t = \int_0^1 \left(\frac{P_{it}^m}{P_t^m} \right)^{-\varepsilon} di = (1-\alpha) \left(\frac{P_t^{m*}}{P_t^m} \right)^{-\varepsilon} + \alpha \left(\pi_t^m\right)^{-\varepsilon} s_{t-1} \tag{4.53}$$

$$s_t Y_t = A_t N_t \tag{4.54}$$

$$Y_t = C_t + G_t \tag{4.55}$$

$$(P_t^m)^{1-\varepsilon} = \alpha \left(P_{t-1}^m\right)^{1-\varepsilon} + (1-\alpha)\left(P_t^{*m}\right)^{1-\varepsilon} \tag{4.56}$$

$$\frac{P_t^{m*}}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(mc_{t+z}^m \left(P_{it+z}^m\right)^{\varepsilon} Y_{it+z}\right)}{E_t \sum_{z=0}^{+\infty} \alpha^z q_{t,t+z} \left(P_{it+z}^m\right)^{\varepsilon} \left(\frac{P_{t+z}}{P_t}\right)^{-1} Y_{it+z}} \tag{4.57}$$

$$mc_t = \frac{w_t}{A_t} \tag{4.58}$$

$$\phi_t = Y_t - (1-\varkappa) w_t N_t \tag{4.59}$$

$$\pi_t^m = \pi_t \frac{\mu_{t-1}}{\mu_t} \tag{4.60}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \xi_t \text{ with } \xi \ i.i.d. \left(0, \sigma_\xi^2\right) \tag{4.61}$$

$$E_t q_{t,t+1} = \beta E_t \left(\frac{X_t}{X_{t+1}}\right)^{\sigma} \tag{4.62}$$

$$\mu_t = \frac{P_t}{P_t^m} \tag{4.63}$$

$$\pi_t^m = \frac{P_t^m}{P_{t-1}^m} \tag{4.64}$$

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{4.65}$$

4.A.2. Steady State

This paragraph describes the non-stochastic steady state. We make the assumption that there is no trend inflation. Therefore $s = 1$.

$$X = (1-\theta)\, C \tag{4.66}$$

$$X^g = (1-\theta)\, G \tag{4.67}$$

$$\chi\,(N)^{\varphi}\,(X)^{\sigma} = \frac{W}{P} = w \tag{4.68}$$

$$1 = \beta R \tag{4.69}$$

$$\omega = \left(1 - \frac{1}{\mu}\right) + \theta\beta\omega \tag{4.70}$$

$$Y = \eta\omega\,(X + X^g) \tag{4.71}$$

$$G + \varkappa W N = T \tag{4.72}$$

$$Y = N \tag{4.73}$$

$$Y = C + G \tag{4.74}$$

$$P^m = \frac{\varepsilon}{\varepsilon - 1} MC \tag{4.75}$$

$$mc = \frac{w}{A} \tag{4.76}$$

$$\phi = Y - (1-\varkappa)\, wN \tag{4.77}$$

$$\mu = \frac{P}{P^m} \tag{4.78}$$

$$p^* = \frac{P^*}{P} \tag{4.79}$$

Therefore

$$R = \frac{1}{\beta} \text{ Interest Rate} \tag{4.80}$$

$$\omega = (\eta\,(1-\theta))^{-1} \quad \text{Shadow price of private consumption} \tag{4.81}$$

$$\mu = (1 - (1-\theta\beta)\,\omega)^{-1} \text{ goods mark up} \tag{4.82}$$

$$mc = \frac{1}{\mu}\left(\frac{\varepsilon-1}{\varepsilon}\right) \text{ Marginal costs producers} \tag{4.83}$$

$$w = \frac{1}{\mu}\left(\frac{\varepsilon-1}{\varepsilon}\right) \text{ Wages} \tag{4.84}$$

$$\frac{P^*}{P} = \frac{1}{\mu} \text{ Producer optimal relative price} \tag{4.85}$$

We fix the steady state ratios

$$\frac{C}{N} = \frac{C}{Y} = \psi \tag{4.86}$$

The labour supply becomes

$$\begin{aligned}\chi\,(N)^{\varphi}\,((1-\theta)\,\psi N)^{\sigma} &= w \\ \chi\,(1-\theta)^{\sigma}\,\psi^{\sigma} N^{\varphi+\sigma} &= \frac{1}{\mu}\left(\frac{\varepsilon-1}{\varepsilon}\right) \qquad (4.87)\end{aligned}$$

4.A.3. Social Planner

The Social planner maximises

$$\max_{\{X_t^*, X_t^{g*}, N_t^*, C_t^*, G_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(X_t^*)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t^*)^{1+\varphi}}{1+\varphi} + \chi^g \frac{(X_t^{g*})^{1-\sigma}}{1-\sigma}\right)$$

subject to

$$\begin{aligned}A_t N_t^* &= C_t^* + G_t^* \\ X_t^* &= C_t^* - \theta C_{t-1}^* \\ X_t^{*g} &= G_t^* - \theta G_{t-1}^*\end{aligned}$$

Calling L^{sp} the associated Lagrangian to this maximisation problem and $z_{1,t}$, $z_{2,t}$ and $z_{3,t}$ the Lagrangian multipliers of each constraint, the first order conditions are

$$\begin{aligned}\frac{\delta L^{sp}}{\delta X_t^{c*}} &= 0 \rightarrow (X_t^*)^{-\sigma} = -z_{2,t} \\ \frac{\delta L^{sp}}{\delta X_t^{g*}} &= 0 \rightarrow \chi^g\,(X_t^{g*})^{-\sigma} = -z_{3,t} \\ \frac{\delta L^{sp}}{\delta N_t^*} &= 0 \rightarrow \frac{\chi\,(N_t^*)^{\varphi}}{A_t} = z_{1,t} \\ \frac{\delta L^{sp}}{\delta C_t^*} &= 0 \rightarrow -z_{1,t} + z_{2,t} - \beta\theta E_t z_{2,t+1} = 0 \\ \frac{\delta L^{zp}}{\delta G_t^*} &= 0 \rightarrow -z_{1,t} + z_{3,t} - \beta\theta E_t z_{3,t+1} = 0\end{aligned}$$

After some manipulations

$$\frac{\chi\,(N_t^*)^{\varphi}}{(X_t^{c*})^{-\sigma}} = A_t\left[1 - \beta\theta E_t\left(\frac{X_{t+1}^*}{X_t^*}\right)^{-\sigma}\right] \qquad (4.88)$$

$$(4.89) \qquad (X_t^*)^{-\sigma} - \theta\beta E_t \left(X_{t+1}^*\right)^{-\sigma} = \chi^g (X_t)^{-\sigma} - \chi^g \theta\beta E_t \left(X_{t+1}^*\right)^{-\sigma}$$

At steady state the two above conditions collapse to

$$(4.90) \qquad \chi (N^*)^{\varphi} (X^*)^{\sigma} = (1-\theta\beta)$$

and

$$(4.91) \qquad (1-\theta\beta)(1-\theta)^{-\sigma} (C^*)^{-\sigma} = \chi^g (1-\theta\beta)(1-\theta)^{-\sigma} (G^*)^{-\sigma}$$

The latter implies

$$(4.92) \qquad \left(\frac{C^*}{G^*}\right)^{-\sigma} = \chi^g$$

Now defining the Social Planner steady state ratio as

$$(4.93) \qquad \frac{C^*}{N^*} = \psi^*$$

we can write the expression for N^* as

$$(4.94) \qquad (N^*)^{\sigma+\varphi} = \frac{(1-\theta\beta)}{\chi (1-\theta)^{\sigma} \psi^{*\sigma}}$$

and

$$(4.95) \qquad \left(\frac{\psi^*}{1-\psi^*}\right)^{-\sigma} = \chi^g$$

Therefore the optimal subsidy follows

$$(4.96) \qquad \varkappa = 1 - \frac{1}{1-\theta\beta}\left[\left(1 - \frac{1-\theta\beta}{(1-\theta)\eta}\right)\frac{\varepsilon-1}{\varepsilon}\right]$$

Next we log-linearise the Social Planner's equation around the steady state. This methodology allows us to derive the welfare relevant gap variables.

$$\widehat{X}_t^* = \theta\beta E_t \widehat{X}_{t+1}^* + \frac{1-\theta\beta}{\sigma}\left(\widehat{A}_t - \varphi \widehat{N}_t^*\right) \tag{4.97}$$

$$\widehat{X}_t^* - \theta\beta E_t \widehat{X}_{t+1}^* = \widehat{X}_t^{g,*} - \theta\beta E_t \widehat{X}_{t+1}^{g,*} \tag{4.98}$$

$$\widehat{Y}_t^* = \widehat{A}_t + \widehat{N}_t^* \tag{4.99}$$

$$\widehat{Y}_t^* = \left(\frac{C^*}{Y^*}\right)\widehat{C}_t^* + \left(1 - \frac{C^*}{Y^*}\right)\widehat{G}_t^* \tag{4.100}$$

$$\widehat{X}_t^* = \frac{1}{1-\theta}\left(\widehat{C}_t^* - \theta \widehat{C}_{t-1}^*\right) \tag{4.101}$$

$$\widehat{X}_t^{g,*} = \frac{1}{1-\theta}\left(\widehat{G}_t^* - \theta \widehat{G}_{t-1}^*\right) \tag{4.102}$$

Using the aggregate constraint and the definitions of habit-adjusted private and public consumption, the Euler equation can be re-written as

$$\left(1 + \theta^2\beta + \psi^* \frac{\varphi}{\zeta}\right)\widehat{C}_t^* = \theta\beta E_t \widehat{C}_{t+1}^* + \theta \widehat{C}_{t-1}^* - (1-\psi^*)\frac{\varphi}{\delta}\widehat{G}_t^* + \left(\frac{1+\varphi}{\zeta}\right)\widehat{A}_t \tag{4.103}$$

While the equation on consumption becomes

$$\left(1+\theta^2\beta\right)\left(\widehat{C}_t^* - \widehat{G}_t^*\right) = \theta\beta E_t\left(\widehat{C}_{t+1}^* - \widehat{G}_{t+1}^*\right) + \theta\left(\widehat{C}_{t-1}^* - \widehat{G}_{t-1}^*\right) \tag{4.104}$$

The solution of the latter takes the form $\left(\widehat{C}_t^* - \widehat{G}_t^*\right) = a\left(\widehat{C}_{t-1}^* - \widehat{G}_{t-1}^*\right)$. In order to have a stationary solution, the coefficient a has to be less that one in modulus. In order to check this, it is easy to show one should solve the quadratic expression $\theta\beta a^2 - \left(1+\theta^2\beta\right)a + \theta = 0$.

The only solution less than one is $a = \theta$. Therefore the stationary solution for the private public consumption balance is $\left(\widehat{C}_t^* - \widehat{G}_t^*\right) = \theta\left(\widehat{C}_{t-1}^* - \widehat{G}_{t-1}^*\right)$. Assuming that $\widehat{C}_{-1}^* = \widehat{G}_{-1}^* = 0$, it is optimal to have equal fluctuation of the two components of aggregate output

$$\widehat{C}_t^* = \widehat{G}_t^*$$

Given this allocation, we can write the Euler equation in terms of output as

$$\widehat{Y}_t^* = \theta\beta\varsigma E_t\widehat{Y}_{t+1}^* + \theta\varsigma\widehat{Y}_{t-1}^* + \left(\frac{1+\varphi}{\zeta}\right)\varsigma\widehat{A}_t \tag{4.105}$$

where $\zeta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$ and $\varsigma = \frac{1}{1+\theta^2\beta+\frac{\varphi}{\zeta}}$.

4.A.4. Loss Function

Here we derive the second order approximation to the utility function

$$U = \sum_{t=0}^{+\infty}\beta^t\left(\frac{X_t^{1-\sigma}}{1-\sigma} - \gamma\frac{N_t^{1+\varphi}}{1+\varphi} + \chi\frac{X_t^{g1-\sigma}}{1-\sigma}\right) \tag{4.106}$$

The first term can be approximated as

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \frac{X^{1-\sigma}}{1-\sigma} + X^{1-\sigma}\left(\widehat{X}_t + \frac{1}{2}(1-\sigma)\widehat{X}_t^2\right) + o(3)$$

the second as

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\left(\widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2\right) + o(3)$$

and the third one as

$$\frac{(X_t^g)^{1-\sigma}}{1-\sigma} = \frac{(X^g)^{1-\sigma}}{1-\sigma} + (X^g)^{1-\sigma}\left(\widehat{X}_t^g + \frac{1}{2}(1-\sigma)(\widehat{X}_t^g)^2\right) + o(3) \tag{4.107}$$

Furthermore, we can write that

$$N_t = \frac{Y_t}{A_t}\Delta_t \tag{4.108}$$

where $\Delta_t = \int_0^1 \left(\frac{P_{jit}}{P_t^m}\right)^{-\varepsilon} dj$. The latter expression can be written, following Woodford (2003) as

$$\widehat{N}_t = \widehat{Y}_t - \widehat{A}_t + var_j \frac{\varepsilon}{2}\left(p_{jit}\right) \tag{4.109}$$

the labour term can be rewritten as

$$\frac{N_t^{1+\varphi}}{1+\varphi} = N^{1+\varphi}\left(\widehat{Y}_t + \frac{1}{2}\left(1+\varphi\right)\left(\widehat{Y}_t - \widehat{A}_t\right)^2 + var_j \frac{\varepsilon}{2}\left(p_{jit}\right)\right) + t.i.p. + o\left(3\right)$$

Using that $X^g = (1-\theta)\,G$, and the second order approximation to the definition of X_t^g

$$\widehat{X}_t^g = \frac{\widehat{G}_t - \theta \widehat{G}_{t-1}}{1-\theta} - \frac{1}{2}(\widehat{X}_t^g)^2 + \frac{1}{2}\frac{1}{1-\theta}\widehat{G}_t^2 - \frac{1}{2}\frac{\theta}{1-\theta}\widehat{G}_{t-1}^2 + t.i.p. + o\left(3\right) \tag{4.110}$$

$\frac{1}{\psi}C = N$ and $G = \frac{1-\psi}{\psi}C$, the single period loss function can be therefore written as

$$\Lambda_t^g = \Omega \left[\begin{array}{c} \left(\widehat{C}_t - \theta \widehat{C}_{t-1} + \frac{1}{2}\widehat{C}_t^2 - \frac{1}{2}\theta \widehat{C}_{t-1}^2 + \frac{-\sigma(1-\theta)}{2}\widehat{X}_t^2\right) + \\ -\frac{1}{\psi}\left(1-\theta\beta\right)\left(\widehat{Y}_t + \frac{1}{2}\left(1+\varphi\right)\left(\widehat{Y}_t - \widehat{A}_t\right)^2 + \frac{\varepsilon}{2}var_j\left(p_{jit}\right)\right) + \\ +\frac{1-\psi}{\psi}\left(\widehat{G}_t - \theta\widehat{G}_{t-1} + \frac{1}{2}\widehat{G}_t^2 - \frac{1}{2}\theta\widehat{G}_{t-1}^2 + \frac{-\sigma(1-\theta)}{2}(\widehat{X}_t^g)^2\right) \end{array}\right] + t.i.p. + o(3) \tag{4.111}$$

Where $\Omega = \left((1-\theta)^{-\sigma}\left(C\right)^{1-\sigma}\right.$. We use the properties of the infinite sum to collect terms. Furthermore we exploit the fact that at SS the efficient level the variables are

$$\begin{aligned} \left(1-\theta\beta\right)\left(1-\theta\right)^{-\sigma} C^{1-\sigma} &= \gamma N^{1+\varphi}\psi \\ \chi\left(1-\theta\beta\right)\left(1-\theta\right)^{-\sigma} G^{1-\sigma} &= \gamma N^{1+\varphi}\left(1-\psi\right) \end{aligned}$$

$$L=\Omega E_0\sum_{t=0}^{+\infty}\beta^t\left\{\begin{array}{c}(1-\theta\beta)\left(\widehat{C}_t-\frac{1}{\psi}\widehat{Y}_t+\frac{1-\psi}{\psi}\widehat{G}_t\right)+\frac{1}{2}\widehat{C}_t^2-\frac{1}{2}\theta\widehat{C}_{t-1}^2+\\ -(1-\theta\beta)\frac{1}{\psi}\left(\frac{1}{2}(1+\varphi)\left(\widehat{Y}_t-\widehat{A}_t\right)^2+\frac{\varepsilon}{2}var_j\left(p_{jit}\right)\right)+\\ +\frac{-\sigma(1-\theta)}{2}\widehat{X}_t^2+\frac{1-\psi}{\psi}\left(\frac{-\sigma(1-\theta)}{2}(\widehat{X}_t^g)^2+\frac{1}{2}\widehat{G}_t^2-\frac{1}{2}\theta\widehat{G}_{t-1}^2\right)\end{array}\right\}+t.i.p+o(3)$$

Using the second order approximation to the market clearing condition

$$\widehat{C}_t-\frac{1}{\psi}\widehat{Y}_t+\frac{1-\psi}{\psi}\widehat{G}_t=\frac{1}{2}\frac{1}{\psi}\widehat{Y}_t^2-\frac{1}{2}\widehat{C}_t^2-\frac{1}{2}\frac{1-\psi}{\psi}\widehat{G}_t^2+o(3) \tag{4.112}$$

$$L=\Omega E_0\sum_{t=0}^{+\infty}\beta^t\left\{\begin{array}{c}(1-\theta\beta)\left(\frac{1}{2}\frac{1}{\psi}\widehat{Y}_t^2-\frac{1}{2}\widehat{C}_t^2-\frac{1}{2}\frac{1-\psi}{\psi}\widehat{G}_t^2\right)+\\ -(1-\theta\beta)\frac{1}{\psi}\left(\frac{1}{2}(1+\varphi)\left(\widehat{Y}_t-\widehat{A}_t\right)^2+\frac{\varepsilon}{2}var_j\left(p_{jit}\right)\right)+\\ +\frac{1-\psi}{\psi}\left(\frac{-\sigma(1-\theta)}{2}(\widehat{X}_t^g)^2\right)+\frac{-\sigma(1-\theta)}{2}\widehat{X}_t^2+\end{array}\right\}+t.i.p+o(3)$$

Now rearranging, moving into $t.i.p.$ $-\Omega\left(\frac{1-\theta\beta}{\varphi\psi}\right)\left(\frac{(1+\varphi)}{\varphi}-\left(\frac{(1+\varphi)}{\varphi}\right)^2\right)\widehat{A}_t^2$, using that $\sum_{t=0}^{+\infty}\beta^t var\left(p_{jit}\right)=\frac{1}{k}\sum_{t=0}^{+\infty}\beta^t\left(\pi_t^m\right)^2+t.i.p+o(3)$ and assuming that

$$\lim_{i\to\infty}\beta^{t+i}\frac{(1-\theta\beta)}{\psi}\widehat{Y}_{t+i}=\lim_{i\to\infty}\beta^{t+i}\widehat{C}_{t+i}=\lim_{i=\infty}\beta^{t+i}\frac{1-\psi}{\psi}\widehat{G}_{t+i}=\lim_{i\to\infty}\beta^{t+i}\widehat{C}_{t+i}^2=\lim_{i\to\infty}\beta^{t+i}\frac{1-\psi}{\psi}\widehat{G}_{t+i}^2=$$

we can write the linear quadratic second order loss function as

$$L=-\frac{1}{2}\Omega\psi E_0\sum_{t=0}^{+\infty}\beta^t\left[\begin{array}{c}(1-\theta\beta)\varphi\left(\widehat{Y}_t-\frac{(1+\varphi)}{\varphi}\widehat{A}_t\right)^2+\psi\sigma(1-\theta)\widehat{X}_t^2+\\ +(1-\theta\beta)\frac{\varepsilon}{\kappa}\left(\pi_t^m\right)^2+(1-\psi)\sigma(1-\theta)(\widehat{X}_t^g)^2\end{array}\right]+t.i.p+o(3) \tag{4.113}$$

4.A.5. Optimal Policy

The central authority seeks to minimise

$$L=-\frac{1}{2}\Omega\psi E_0\sum_{t=0}^{+\infty}\beta^t\left[\begin{array}{c}(1-\theta\beta)\varphi\left(\widehat{Y}_t-\frac{(1+\varphi)}{\varphi}\widehat{A}_t\right)^2+\psi\sigma(1-\theta)\widehat{X}_t^2+\\ +(1-\theta\beta)\frac{\varepsilon}{\kappa}\left(\pi_t^m\right)^2+(1-\psi)\sigma(1-\theta)(\widehat{X}_t^g)^2\end{array}\right]+t.i.p+o(3)$$

subject to the structural log-linearised equations that characterised the decentralised equilibrium. These are

$$\widehat{X}_t = E_t\widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

$$\pi_t^m = \beta E_t\pi_{t+1}^m + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left(\sigma\widehat{X}_t + \varphi\widehat{Y}_t - (1+\varphi)\widehat{A}_t + \widehat{\mu}_t\right)$$

$$\widehat{\omega}_t = \frac{1}{\mu\omega}\widehat{\mu}_t + \theta\beta E_t\widehat{\omega}_{t+1} + \theta\beta\left(\widehat{X}_t - E_t\widehat{X}_{t+1}\right)$$

$$\widehat{Y}_t = \widehat{\omega}_t + \left(\psi\widehat{X}_t + (1-\psi)\widehat{X}_t^g\right) \tag{4.114}$$

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{C}_t - \theta\widehat{C}_{t-1}\right) \tag{4.115}$$

$$\widehat{X}_t^g = \frac{1}{1-\theta}\left(\widehat{G}_t - \theta\widehat{G}_{t-1}\right) \tag{4.116}$$

$$\widehat{Y}_t = \psi\widehat{C}_t + (1-\psi)\widehat{G}_t \tag{4.117}$$

$$\pi_t = \pi_t^m + \widehat{\mu}_t - \widehat{\mu}_{t-1} \tag{4.118}$$

We utilise (4.114)-(4.118) to substitute for the Lagrangian multiplier ω_t, CPI inflation, and habit-adjusted private and public consumption in the Euler equation, NKPC, the evolution of markup and in the loss function. After few analytical passages we obtain

$$\left(\frac{1+\theta}{1-\theta}\right)\widehat{C}_t = \frac{1}{1-\theta}E_t\widehat{C}_{t+1} + \frac{\theta}{1-\theta}\widehat{C}_{t-1} + \frac{1}{\sigma}\left(E_t\pi_{t+1}^m + E_t\widehat{\mu}_{t+1} - \widehat{R}_t - \widehat{\mu}_t\right) \tag{4.119}$$

$$\pi_t^m = \beta E_t\pi_{t+1}^m - \kappa_1\widehat{C}_t - \kappa_2\widehat{C}_{t-1} - \kappa_3\widehat{G}_t - \kappa_4\widehat{A}_t + \kappa\widehat{\mu}_t \tag{4.120}$$

$$\widehat{\mu}_t = \gamma_1\beta E_t\widehat{C}_{t+1} - \gamma_2\widehat{C}_t - \gamma_3\widehat{C}_{t-1} + \gamma_4\beta E_t\widehat{G}_{t+1} - \gamma_5\widehat{G}_t - \gamma_6\widehat{G}_t \tag{4.121}$$

where $\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$, $\kappa_1 = -\kappa\left(\frac{\sigma}{1-\theta} + \varphi\psi\right)$, $\kappa_2 = \kappa\frac{\sigma\theta}{1-\theta}$, $\kappa_3 = -\kappa\varphi(1-\psi)$, $\kappa_4 = \kappa(1+\varphi)$, $\gamma = \mu\omega\frac{\theta}{1-\theta}$, $\gamma_1 = \gamma(\psi\theta+\sigma)$, $\gamma_2 = \gamma[\psi(1+\theta\beta)+\beta\sigma(1+\theta)]$, $\gamma_3 = -\gamma(\psi+\theta\beta\sigma)$, $\gamma_4 = \gamma(1-\psi)\theta$, $\gamma_5 = \gamma(1-\psi)(1+\theta\beta)$, $\gamma_6 = -\gamma(1-\psi)$.

Applying the same substitutions in the period loss function, it yields

$$L_t = -\frac{1}{2}\Omega\psi\left\{\begin{array}{c} \psi(\delta+\varphi\psi)\widehat{C}_t^2 - 2\theta\delta\psi\widehat{C}_t\widehat{C}_{t-1} + \\ +2\varphi\psi(1-\psi)\widehat{C}_t\widehat{G}_t - 2(1+\varphi)\psi\widehat{C}_t\widehat{A}_t + \theta^2\delta\psi\widehat{C}_{t-1}^2 + \\ +(1-\psi)[\delta+\varphi(1-\psi)]\widehat{G}_t^2 + \\ -2\theta\delta(1-\psi)\widehat{G}_t\widehat{G}_{t-1} - 2(1+\varphi)(1-\psi)\widehat{G}_t\widehat{A}_t + \\ \theta^2\delta(1-\psi)\widehat{G}_{t-1}^2 + \frac{\varepsilon}{\kappa}(\widehat{\pi}_t^m)^2 \end{array}\right\}$$

Where $\delta = \frac{\sigma}{(1-\theta\beta)(1-\theta)}$. Given that the interest rate appears only in one equation, the dynamic IS curve is not binding. The maximisation problem can be therefore represented as

$$\Lambda = E_t\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c} L_t - \xi_t\left(\pi_t^m - \beta E_t\pi_{t+1}^m - \kappa_1\widehat{C}_t + \kappa_2\widehat{C}_{t-1} - \kappa_3\widehat{G}_t + \kappa_4\widehat{A}_t - \kappa\widehat{\mu}_t\right) + \\ -\varpi_t\left(\widehat{\mu}_t - \gamma_1\beta E_t\widehat{C}_{t+1} + \gamma_2\widehat{C}_t + \gamma_3\widehat{C}_{t-1} - \gamma_4\beta E_t\widehat{G}_{t+1} + \gamma_5\widehat{G}_t + \gamma_6\widehat{G}_t\right) \end{array}\right\}$$

The optimal commitment problem consists in choosing a path for $\left\{\widehat{\mu}_t\ \widehat{C}_t\ \widehat{G}_t\ \widehat{\pi}_t^m\right\}_{t=0}^{\infty}$, given the exogenous process $\widehat{A}_t$; once this path is obtained, one can find through the dynamic IS the path for the nominal interest rate $\widehat{R}_t$.

From the first order condition on $\widehat{\mu}_t$ we can find the static relationship between the two Lagrangian multipliers

$$\varpi_t = \kappa\xi_t$$

and for inflation

$$\widehat{\pi}_t^m = -\frac{\kappa}{\varepsilon\Omega}(\xi_t - \xi_{t-1})$$

The first order conditions for private and public spending are

$$\left[\begin{array}{c} -\Omega\delta\psi\upsilon_c\widehat{C}_t+\Omega\delta\theta\psi\widehat{C}_{t-1}-\Omega\varphi\psi\left(1-\psi\right)\widehat{G}_t+ \\ +\Omega\left(1+\varphi\right)\psi\widehat{A}_t-\kappa_1\xi_t-\gamma_2\varpi_t \\ +\gamma_1\varpi_{t-1}+\beta E_t\left(\Omega\delta\theta\psi\widehat{C}_{t+1}-\kappa_2\varpi_{t+1}-\gamma_3\xi_{t+1}\right) \end{array}\right] = 0$$

$$\left[\begin{array}{c} -\Omega\delta(1-\psi)\upsilon_g\widehat{G}_t+\Omega\delta\theta\left(1-\psi\right)-\Omega\delta\theta\left(1-\psi\right)\widehat{G}_{t-1}+ \\ -\Omega\varphi\psi\left(1-\psi\right)\widehat{C}_t+\Omega\left(1+\varphi\right)\left(1-\psi\right)\widehat{A}_t \\ -\kappa_3\xi_t-\gamma_5\varpi_t+\gamma_4\varpi_{t-1} \end{array}\right] = 0$$

Where $\upsilon_c = 1+\theta^2\beta+\frac{\varphi}{\delta}\psi$ and $\upsilon_g = 1+\theta^2\beta+\frac{\varphi}{\delta}(1-\psi)$. We can now use the relation between the two Lagrangian multiplier to eliminate ϖ_t,

$$(4.122)\quad \upsilon_c\widehat{C}_t+\frac{\varphi}{\delta}\left(1-\psi\right)\widehat{G}_t+\left(\frac{\kappa_1+\kappa\gamma_2}{\delta\Omega\psi}\right)\xi_t = \left\{\begin{array}{c} \theta\beta E_t\widehat{C}_{t+1}-\left(\frac{\kappa_2+\kappa\gamma_3}{\delta\Omega\psi}\right)\beta E_t\xi_{t+1}+ \\ +\theta\widehat{C}_{t-1}+\left(\frac{\kappa\gamma_1}{\delta\Omega\psi}\right)\xi_{t-1}+\left(\frac{1+\varphi}{\delta}\right)\widehat{A}_t \end{array}\right\}$$

$$(4.123)\quad \upsilon_g\widehat{G}_t+\frac{\varphi}{\delta}\psi\widehat{C}_t+\left(\frac{\kappa_3+\kappa\gamma_5}{\delta\Omega\left(1-\psi\right)}\right)\xi_t = \left\{\begin{array}{c} \theta\beta E_t\widehat{G}_{t+1}-\left(\frac{\kappa\gamma_6}{\delta\Omega(1-\psi)}\right)\beta E_t\xi_{t+1} \\ \theta\widehat{G}_{t-1}+\left(\frac{\kappa\gamma_4}{\delta\Omega(1-\psi)}\right)\xi_{t-1}+\left(\frac{1+\varphi}{\delta}\right)\widehat{A}_t \end{array}\right\}$$

Finally we substitute for the markup $\widehat{\mu}_t$ and for inflation in terms of Lagrangian multipliers to write the NKPC as

(4.124)

$$\frac{\kappa}{\varepsilon\Omega}\left(1+\beta\right)\xi_t-\left(\kappa_1+\kappa\gamma_2\right)\widehat{C}_t-\left(\kappa_3+\kappa\gamma_5\right)\widehat{G}_t = \left\{\begin{array}{c} \frac{\kappa}{\varepsilon\Omega}\beta E_t\xi_{t+1}-\kappa\gamma_1\beta E_t\widehat{C}_{t+1}-\kappa\gamma_4\beta E_t\widehat{G}_{t+1}+ \\ +\frac{\kappa}{\varepsilon\Omega}\xi_{t-1}+\left(\kappa_2+\kappa\gamma_3\right)\widehat{C}_{t-1}+ \\ +\kappa\gamma_6\widehat{G}_{t-1}+\kappa_4\widehat{A}_t \end{array}\right\}$$

The expressions (4.122)-(4.124) together with the exogenous process for the technology shock form a system of equations which can be solved for $\left\{\widehat{C}_t,\widehat{G}_t,\xi_t,\widehat{A}_t\right\}_{t=0}^{\infty}$.

References

[1] Abel, A. B. (1990): "Asset Prices under Habit Formation and Catching Up with the Joneses," American Economic Review, 80, 2 (May 1990), 38-42.

[2] Alesina, A. and Perotti, R. (1996). "Fiscal discipline and the budget process" American Economic Review, Papers and Proceedings, Vol. 86, pp. 401–7

[3] Amato, J. D., and Thomas Laubach (2003). "Rule-of-Thumb Behavior and Monetary Policy" European Economic Review, Vol. 47, pp. 791-831.

[4] Amato, J. D., and T. Laubach (2004): "Implications of Habit Formation for Optimal Monetary Policy," Journal of Monetary Economics, 51, 305—325.

[5] Ascari, Guido (2004) "Staggered prices and trend inflation: some nuisances" Review of Economic Dynamics.

[6] Ascari, G and T. Ropele (2009) "Trend Inflation, Taylor Principle and Indeterminacy" Journal of Money Credit and Banking, *forthcoming*.

[7] Ascari, G and L. Rossi (2009*a*) " Real Wage Rigidities and Disinflation Dynamics: Calvo vs. Rotemberg Pricing" University of Pavia WP series.

[8] Ascari, G and L. Rossi (2009*b*) "Trend Inflation, Non-linearities and Firms Price-Setting: Rotemberg vs. Calvo" University of Pavia WP series.

[9] Azariadis, C. (1993). "Intertemporal Economics" MIT press.

[10] Barro, R. (1979). "On the determination of public debt", Journal of Political Economy, Vol. 87, pp. 940–71.

[11] Benhabib, J and S. Eusebi(2005) "The Design of Monetary and Fiscal Policy: A Global Perspective" Journal of Economic Theory,Volume 123.

[12] Bernanke B., and M. Woodford (2001) ""Inflation Forecasts and Monetary Policy" Journal of Money, Credit and Banking, Vol. 29. pp. 653-685.

[13] Bilbiie F. O.(2008). "Limited Asset Markets Participation, Monetary Policy and (Inverted) Keynesian Logic" Journal of Economic Theory, Volume 140, pp. 162-196 .

[14] Bilbiie F. O. and R. Straub (2004). "Fiscal Policy, Business Cycles and Labor-Market Fluctuations" Mimeo.

[15] Blanchard, O. and Khan C.M. (1980). "The Solution of Linear Difference Models Under Rational Expectations" Econometrica, Vol. 48, pp. 1305-1311.

[16] Bullard, J. and K. Mitra (2002). "Learning about monetary policy rules" Journal of Monetary Economics, Vol. 49(6), pp. 1105-1129,

[17] Clarida, Richard, Jordi Galí and Mark Gertler (1999). "The Science of Monetary Policy: A New Keynesian Perspective" Journal of Economic Literature, Vol. 37, pp. 1661-1707.

[18] Clarida, Richard, Jordi Galí, and Mark Gertler (2000). "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory" Quarterly Journal of Economics, Vol. 115, pp. 147-180.

[19] Cogley, Timothy & Nason, James M, (1995):"Output Dynamics in Real-Business-Cycle Models," American Economic Review, vol. 85(3), pages 492-511, June.

[20] Colciago A. (2008). "Rule of Thumb consumers Meets Sticky Wages" University of Milan-Bicocca working paper.

[21] Canzoneri M. B., Robert E. Cumby and Behzad T. Diba (2000)"Is the Price Level Determined by the Needs of Fiscal Solvency?" American Economic Review, Vol. 91(5), pp. 1221-1238.

[22] Chari, V. V., and H. L. Cole (1995): "A Contribution to the Theory of Pork Barrel Spending," Federal Reserve Bank of Minneapolis, Staff Report No. 156.

[23] Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," Journal of Political Economy, 113(1), 1—45.

[24] Clarida, R., Galí, J., and Gertler, M.(1999). " The science of monetary policy: a new Keynesian perspective" Journal of Economic Literature, Vol. 37, pp. 1661-1707.

[25] Clarida, R., Galí, J., and Gertler, M.(2000)." Monetary policy rules and macroeconomic stability: evidence and some theory, Quarterly Journal of Economics, 115,pp. 147-80.

[26] Cummins, J., R. Francis, and R. Coffey (2007): "Local Solutions or Postcode Lotteries," Office of Public Management, May.

[27] Davig T. and E. Leeper (2006)"Fluctuating Macro Policies and the Fiscal Theory," NBER Macroeconomics Annual 2006, Vol. 21, pp. 247-316

[28] Davig T. and E. Leeper (2007) "Generalizing the Taylor principle": American Economic Review, Vol. 97(3), pp. 607-635.

[29] Di Bartolomeo, G. and L. Rossi (2007):"Effectiveness of monetary policy and limited asset market participation: Neoclassical versus Keynesian effects," International Journal of Economic Theory, Vol. 3, pp. 213-218.

[30] Eser F., C. Leith and Simon Wren-Lewis, (2009) "When is monetary policy all we need?," Working Papers 2009_18, Department of Economics, University of Glasgow.

[31] Favero C. and T. Monacelli (2005). "Fiscal Policy Rules and Regime (In)Stability: Evidence from the U.S." Mimeo IGIER Bocconi University.

[32] Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," American Economic Review, 90(June), 367—390.

[33] Galí, J., Lopez-Salido and Valles (2004). " Rule-of-Thumb Consumers and the Design of Interest Rate Rules" Journal of Money, Credit, and Banking, Vol. 36, pp. 739-763.

[34] Galí, J., Lopez-Salido and Valles (2007). "Understanding the Effect of Public Spending on Consumption" Journal of European Economic Association, Vol. 5, pp. 227-270.

[35] Galí J. and O. Blanchard (2007): " Real Wage Rigidities and the New Keynesian Model" Journal of Money, Credit, and Banking, supplement to vol. 39, n^{o}1, 2007, 35-66.

[36] Galí, J. and T. Monacelli, (2005) "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," Review of Economic Studies, Blackwell Publishing, vol. 72(3), pages 707-734, 07.

[37] Galí , J. and T. Monacelli (2008). Optimal monetary and fiscal policy in a currency union, Journal of International Economics 76: 116-132.

[38] Greenwood, J. and Huffman, G. W. (1991). Tax analysis in a real business cycle model , Journal of Monetary Economics, Vol. 27, pp. 167–90.

[39] King, R. G., Plosser, C. I. and Rebelo, S. (1988). "Production, growth and business cycles I: the basic neoclassical growth model" , Journal of Monetary Economics, vol. 21, pp. 195–232.

[40] Leeper E. (1991) "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies" Journal of Monetary Economics, Vol. 27, pp. 129-147.

[41] Leith, C., and J. Malley (2005): "Estimated General Equilibrium Models for the Analysis of Monetary Policy in the US and Europe," European Economic Review, 49(8), 2137—2159.

[42] Leith, C., I. Moldovan and R. Rossi (2008): "Optimal Monetary Policy in a New Keynesian Model with Habits in Consumption" , ECB working paper series No.1976.

[43] Leith, C. and S. Wren-Lewis(2000): "Interactions between monetary and fiscal policy rules" Economic Journal, Vol 110, pp.93-108.

[44] Leith, C. and L. von Thadden (2007): "Monetary and Fiscal Interactions in a New Keynesian Model with Capital Accumulation and non-Ricardian Consumers" Journal of Economic Theory, vol. 140(1), pages 279-313.

[45] Linnerman L. (2006). "Interest Rate Policy, Debt, and the Indeterminacy with Distortive Taxation" Journal of Economic Dynamics & Control, Vol. 30,pp. 487–510.

[46] Ljungqvist, L., and H. Uhlig (2000): "Tax Policy and Aggregate Demand Management under Catching up with the Joneses," American Economic Review, 90(3), 356—366.

[47] Lubik T. A. and F. Schorfheide, (2004). "Testing for Indeterminacy: An Application to U.S. Monetary Policy" American Economic Review, Vol. 94, pp. 190-217.

[48] Lucas R. B. and N. Stokey (1983): " Optimal fiscal and monetary policy in an economy without capital," Journal of Monetary Economics, vol. 12(1), pages 55-93.

[49] MacKay, R. R. (2001): "Regional Taxing and Spending: The Search for Balance," Regional Studies: The Journal of the Regional Studies Association, 35(6), 563—575.

[50] McCallum, B.T. and N., Edward, (1999): "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis," Journal of Money, Credit and Banking,vol. 31(3), pages 296-316, August.

[51] Perotti R. (2007). "In Search of the Transmission Mechanism of Fiscal Policy" NBER Working Papers 13143.

[52] Ramsey F.P. (1927), "A Contribution to the Theory of Taxation," Economic Journal, Vol. 37, No 145, pp. 47–61

[53] Ravn, M., S. Schmitt-Grohe, and M. Uribe (2006): "Deep Habits," Review of Economic Studies, 73, 195—218.

[54] Ravn, M., S. Schmitt-Grohe, and M. Uribe (2007): "Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate," NBER Working Papers 13328, National Bureau of Economic Research.

[55] Rotemberg, Julio J, (1982): "Sticky Prices in the United States," Journal of Political Economy, vol. 90(6), pages 1187-1211, December.

[56] Schmitt-Grohe' S. and M. Uribe (1997). "Balanced budget rule, distortionary taxes and aggregate instability" Journal of Political Economy, Vol. 105, pp. 976-1000.

[57] Schmitt-Grohé, Stephanie & Uribe, Martín, (2004)"Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the US Business Cycle," CEPR Discussion Papers 4654, C.E.P.R. Discussion Papers.

[58] Stephanie Schmitt-Grohe & Martin Uribe, (2005) "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model,"NBER Working Papers 11854, National Bureau of Economic Research, Inc.

[59] Schmitt-Grohe' S. and M. Uribe (2007*a*)." Optimal simple and implementable monetary and fiscal rules" Journal of Monetary Economics, Vol. 54, pp. 1702–1725.

[60] Smets, F., and R. Wouters (2003): “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,” Journal of the European Economic Association, 1(5), 1123—1175.

[61] Smets, F., and R. Wouters (2007): “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” The American Economic Review, 97(3), 586—606.

[62] Soderlind, P. (1999): “Solution and Estimation of Rational Expectations Macro Models with Optimal Policy,” European Economic Review, 43, 813—823.

[63] Taylor, J. (1993). "Discretion versus policy rules in practice", Carnegie Rochester Conference Series on Public Policy, 39, 195-214.

[64] Taylor, J. (1980): "Aggregate Dynamics and Staggered Contracts" Journal of Political Economy, 88(1),1-24.

[65] Woodford M. (1998) " Public Debt and the Price Level" Mimeo.

[66] Woodford M. (2001)." Fiscal Requirements for Price Stability" Journal of Money Credit and Banking, Vol. 33, pp. 669-728.

[67] Woodford M. (2003). "Interest and Prices" Princeton University Press.

[68] Yun, T. (1996) "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles,” Journal of Monetary Economics, vol. 37, pp. 345-370.

[69] Yun, T. (2005): ”Optimal Monetary Policy with Relative Price Distortions,” American Economic Review, vol. 95, pp. 89-108