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Empirical Essays
in Macroeconomics and Finance

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Ph.D. in Economics

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Economics consists of theories that are not borne out by the data and observed empirical regularities for which there is no theory.

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I declare that all chapters of the thesis represent my own work except where clearly referenced to others.
Introduction and Motivation

This thesis explores the relationship between macroeconomics and finance from an empirical perspective.

Macroeconomics analyses the economy as a whole focusing on the dynamics of aggregate quantities. It involves the analysis of different aspects of the economy; in this study we mainly consider the evolution of some key variables such as output, unemployment and inflation. A key issue in macroeconomic theory concerns the distinction between the short and the long run. Price stickiness characterizes the Keynesian world in the short run; while, all prices are assumed to be fully flexible in the long run, as stressed by the neo-classical doctrine. The temporal distinction is also important from a policy point of view. For instance, monetary policy actions display deferred effects on the economy; such a delay might procrastinate even several quarters depending on the phase of the business cycle.

Modern finance develops methods and techniques with the aim of determining the fair price of any tradable asset. Although challenging, asset pricing theory is fascinating since it implies the existence of a metric to evaluate uncertainty. Fair pricing hinges on the possibility of identifying an objective, rather than subjective, measure of risk. Risk is also a time factor. According to common wisdom, the longer the investment horizon, the greater the uncertainty, the higher the premium required by investors. The term structure of interest rates, which is a core subject throughout the thesis, is capable of capturing the aforementioned chain; bond pricing can be viewed as a specific case of asset pricing. For instance, in Chapter 2 we claim that medium term maturities may reflect uncertainty more effectively than long maturities. In particular, long term bonds are risky when held over short horizons thus justifying a compensation for bearing risk required by investors.

In this work the link between finance and macroeconomics is represented by the term structure of interest rates, since its shape is acknowledged to reflect agents’ expectations about the future stance of monetary policy. While the short end of the term structure is directly influenced by the monetary authority decision of setting a target value for the policy rate, the dynamics of long term yields depends upon a great variety of factors. In particular, the propagation of the impulse to the short end of the term structure along the entire spectrum of maturities reflects agents expectations about future monetary policy interventions. The term structure of interest rates thus represents a crucial step of the monetary policy transmission, that is the mechanism through which monetary policy affects aggregate spending, i.e. the real economy. To summarize, the term structure of
interest rates offers an effective opportunity of synthesizing the above mentioned macroeconomic distinction between the short and the long run.

The importance of analyzing the link between macroeconomics and finance has been emphasized in recent contributions. Cochrane (2006) points out that there exists a close connection between finance, uncertainty, and the real economy. Recent research, in fact, has achieved a common view about the role played by the stochastic discount factor, which is used to price any asset in the economy. Individuals tackle the trade-off between saving and consuming by comparing the marginal utilities of present versus future consumption. Therefore, according to this story the decision of postponing consumption is equivalent to the choice of buying an asset today. The dynamics of the stochastic discount factor evolves consistently with the idea that people prefer holding an asset that performs greatly when they feel poor and consume little rather than when they feel rich and can afford luxurious good. Risk aversion thus reflects uncertainty and represents the link between financial theory and aggregate spending decisions.

Applied econometrics has become more and more sophisticated in recent years giving the opportunity of deepening the analysis of the macro-finance environment. The importance of instrumental variables estimation and the diffusion of non linear methods have contributed to enhance the quality of empirical research. In this work we make extensive use of non linear models which we believe effective in emphasizing expectations. For instance, Kalman filtering is an appealing device to describe how rational agents adjust expectations in a Bayesian fashion as soon as new information becomes available.

Hence, the thesis reserves a key role to expectations. In several parts of the thesis, in fact, we stigmatize the traditional dogma of rational expectations. The assumption of perfect and symmetric information, which is a side of rationality, seems extreme in social sciences; the idea of bounded rationality (Simon, 1957) suits definitely better. Linear models fail describing agents’ expectations-based approach; while empirical methods employed in this work handle with expectations in various ways. As a matter of fact, expectations are often conditioned to the evolution of leading economic indicators. Alternatively, we emphasize the dynamic process of revisiting expectations when new information becomes available. We presume that an extensive use of non linear frameworks improves the analysis of information flows.

The scrupulous attention dedicated to expectations in this work, and, more generally, in recent research, derives from the devastating effect of the Lucas (1976) critique. Lucas argued that parameter estimates of macroeconometric models are unstable because of rational expectations; in particular, parameters are unlikely to remain invariant since they depend on agents’ expectations about policy actions. Two sources of instability can be identified. On the one hand, the socio-
economic environment is variable over time; on the other hand, policymakers can change their preferences. Expectations have become such a relevant issue in macroeconomic theory that it is now common practice for the monetary authority to monitor private sector expectations.

This thesis contributes to the advancements of applied macro-finance research in different directions. First, we provide an extensive analysis of the term structure of interest rates. From a financial point of view we are interested to check the strength of the expectations hypothesis, which has always been doubted in empirical studies. From a macroeconomic perspective, instead, we aim at highlighting the relationships between the yield curve, monetary policy and real activity. Second, we provide a deep analysis of the term structure of interest rates. We analyze not only the overall dynamics of the yield curve, but we also focus on its underlying factors. In particular, we assess whether the latent factors of the term structure have a reasonable macroeconomic counterpart. Third, we test the predictive ability of the yield spread in different contexts. The slope of the yield curve is believed to carry information regarding the future evolution of both interest rates and macroeconomic variables, since it reflects agents’ expectations about the incoming monetary policy stance. Fourth, we exploit the informative content of the slope of the yield curve to determine under which circumstances the Lucas (1973) hypothesis dominates the Friedman (1977) hypothesis. Finally, we assess the predictive content of risk premia and their volatility.

Examining the term structure of interest rates, or the yield curve, is equivalent to dealing with bond pricing since bond prices are just the expected value of the discount factor (Cochrane, 2006). In absence of uncertainty the discount factor merely matches the yield to maturity. Bond pricing is a recurrent theme in the macro-finance literature for several reasons (Piazzesi, 2003). Monetary policy is the first reason since monetary policy actions can only affect the short end of the yield curve, while aggregate spending decisions are more closely related to long term yields. The monetary transmission mechanism thus is accomplished through yield curve’s movements. Fiscal policy is the second reason as long as the Treasury has to manage the maturity structure of public debt. Forecasting is a third issue since long term yields are risk-adjusted expectations of short yields; hence, the yield spread, and, more generally, the term structure, contains valuable information about the future evolution of the economy. Finally, understanding the dynamics of the term structure of interest rates is crucial for pricing derivative securities; moreover, financial engineering exploits the dynamics of interest rates for hedging risk.

Recent research has analyzed bond pricing in macroeconomic settings; in particular, a strand of the macro-finance literature has focused on the interpretation of three unobservable component underlying the term structure of interest rates, namely level, slope, and curvature (Litterman and Scheinkman, 1991). Three factors, in fact, are enough to explain almost all movements of the yield
The yield curve over time. The level is usually associated to the medium term inflation rate targeted by the monetary authority. The slope reflects the dynamics of the policy rate set by the monetary authority in response to changing conditions of the economy. So far more controversial has been the interpretation of curvature. In Chapter 2 we show evidence that curvature may reflect the cyclical fluctuations of the economy. In particular, data evidence suggests that a negative shock to curvature seems either to anticipate or to accompany a slowdown in economic activity. To show results are robust we also develop and estimate a structural macroeconometric model for curvature and industrial production.

The informative content of the term structure of interest rates has been traditionally examined to test the expectations hypothesis. In Chapter 3 we exploit the potentiality of non linear models to tackle typical weaknesses affecting the empirical test of the expectations hypothesis such as time variation in term premia, risk aversion, monetary policy uncertainty. In line with Campbell (1995) and Thornton (2005), data evidence suggests that the variance of term premia versus rationally expected yields’ changes and the variance of long to short term yields respectively matter in the corroboration of the expectations hypothesis. Moreover, we find evidence that the informative content of the yield spread about future economic activity is contingent to the magnitude of term premia; in particular, the slope of the term structure tends to anticipate fast economic growth when actual term premia are low. Finally, results highlight that the ability of the yield spread to predict future inflation depends on uncertainty regarding the incoming monetary policy stance.

Time variation in term premia is a crucial issue in chapter 4. Several contributions have emphasized the role of both the yield spread and the term premium to predict future economic activity. Hamilton and Kim (2002) have also investigated whether interest rate volatility is useful to predict future GDP growth. We continue along the path traced by previous research since we explore the possibility that the conditional volatility of term premia forecast errors contains valuable information to infer the future dynamics of industrial production. We find evidence that term premia conditional variance helps to predict future real activity. We speculate that the variability of term premia might capture a pessimistic sentiment akin to financial distress.

Finally, we wish to examine whether empirical evidence contrarian to the Lucas’ view, that there is an inverse relationship between the variance of nominal shocks and the magnitude of output response to nominal shocks, might be revisited in light of the information content of the term structure of interest rates. In particular we find evidence supporting the Lucas (1973) conjecture when the yield curve is either flat or downward sloping. On the other hand, data evidence tends to corroborate the alternative Friedman (1977) hypothesis, that the variability of inflation reduces the level of output, when the term structure has a positive slope.
The structure of the thesis is the following. Chapter 1 contains an extensive review of the literature about the analysis of the term structure of interest rates. Chapter 2 focuses on the macroeconomic interpretation of the unobservable components of the term structure. We revisit recent evidence and propose an appealing and original interpretation of curvature. Chapter 3 focuses on the expectations hypothesis and the predictive ability of the yield spread. Data evidence suggest that the ability of the term structure of being informative about future yields’ movements depends on the level of term premia. In Chapter 4 we provide evidence suggesting that the conditional volatility of term premia contains valuable information to infer the future evolution of economic activity. Chapter 5 is an empirical assessment of the Lucas hypothesis that there is an inverse correlation between the variance of nominal shocks and the magnitude of output response to nominal shocks.
The aim of this chapter is to offer a mere, but extensive, review of the analysis of the term structure of interest rates, its relation with monetary policy and, more generally, with macroeconomics. Several empirical studies are presented in order to highlight how the examination of the yield curve has evolved over time. A lot of technical works and many scientific contributions about the term structure have appeared in journals and on the web, therefore the selection has been challenging. Academic contributions on the yield curve have mostly focused on the empirical analysis of the expectations hypothesis. After a brief description of financial theories underlying the term structure of interest rates, we discuss the empirical work by Campbell and Shiller (The Review of Economic Studies, 1991). This is the first study that provides with a complete inspection of the entire spectrum of maturities; the main conclusion seems paradoxical “the slope of the term structure almost always gives a forecast in the wrong direction for the short term change in the yields on longer bonds, but gives a forecast in the right direction for long term changes in short term rates”. The survey of the literature continues investigating how important the role of term premia is in the empirical analysis of the expectations hypothesis. Finally, we examine the relation between the yield curve, monetary regimes, and macroeconomic variables.
1.1 Introduction

The term structure of interest rates is believed to carry useful information regarding the future state of the economy. The shape of the yield curve, in fact, reflects agents’ expectations about the incoming stance of monetary policy. Policy interventions mostly affect the short end of the yield curve, while movements in long term yields are determined by a complex combination of factors. Hence, examining the term structure (henceforth TS) primarily means to find a link between short and long rates. The expectations hypothesis (hereafter EH) is an appealing attempt to address this issue. The theory suggests that the yield spread should anticipate future yields movements. Unfortunately, EH has almost always been rejected in empirical studies; in particular, data do not seem to support the idea that forward rates can be implicitly derived by projecting spot yields into the future. Data evidence in support of EH is so weak that is not unreasonable to wonder whether “there really is any attempt to forecast” (Macaulay, 1938).

In early studies authors focused on the empirical examination of EH at short horizons; then they moved on exploring more distant maturities. Former contributions have been methodological as well as empirical; economists and econometricians have figured out appropriate techniques for testing EH. For instance, Mishkin (1988) has emphasized the importance of using appropriate covariance matrices to perform consistent inference in presence of overlapping residuals.

The lack of empirical support for EH has inspired plenty of studies. Time-varying term premia, as well as model misspecification have been proposed as possible explanations of the aforementioned “anomaly”. EH contrarian evidence might also depend on change in risk perception, issues related to relative assets supply, measurement errors, irrational rather than rational agents’ behaviour, overreaction of long rates to expected changes in short rates. Non linearity in the EH equation has also been advocated to justify the empirical rejection of the expectations theory.

Abandoned the mere financial perspective, researchers have turned to the interesting field of forecasting macroeconomic variables by exploiting the information content of the yield curve (Estrella and Mishkin, 1997; Estrella and Hardouvelis, 1991). On the same wavelength empirical research has evolved by considering the possibility that term premia, rather than the yield spread, are informative about macro variables.

Modelling the interaction between macroeconomic and financial variables has become more and more sophisticated in recent years. Bonds are priced by means of affine models, and the term structure dynamics is typically investigated either in small macro-finance models or in non linear frameworks. Financial economists have finally developed a parallel strand of literature regarding the investigation of term premia’s macroeconomic determinants.
The rest of the chapter is organized as follows. Theories of the term structure are presented in Section 1.2. Section 1.3 discusses some empirical works. In Section 1.4 we present evidence regarding the relation between TS and monetary regimes. Section 1.5 is dedicated to time-varying term premia. Section 1.6 offers a brief summary about the ability of both the spread and the term premium to predict macroeconomic variables. Finally, in Section 1.7 the most recent macro-finance literature is discussed.

1.2 The Term Structure of Interest Rates and the Expectations Hypothesis

Four basic theories have been proposed to explain yields dynamics over time: the expectations hypothesis, preference for liquidity, market segmentation, and, finally, preferred habitat. The pure expectations hypothesis\(^1\) (PEH) moves from a number of simplifying assumptions: risk neutral agents with rational expectations, liquid markets, absence of transaction costs, perfect and symmetric information. PEH states that forward (implicit) rates are the best forecast of future interest rates; or, equivalently, that forward rates equal the market expectations of future spot rates.

Few equivalent statements define EH (Cochrane, 2006). a) the \(n\)-period yield can be expressed as the average of expected future one-period yields; b) forward rates equal expected future spot rates; c) expected holding period returns are equal on bond of all maturities. The long term \(n\)-period rate \(y_t^*\) is a weighted average of short term \(m\)-period rates \(y_{r,mt}\) \((c\) is a constant term premium):

\[
y_t^* = \left( \frac{m}{n} \right) \sum_{j=0}^{n-m} E_t \left( y_{r,m_i} \right) + c \tag{1.1}
\]

Forward rates are, up to a constant, unbiased predictor of future realized spot rates implying that if investors observe an upward sloping yield curve they would expect short rates to rise in the future. In this vein, EH represents essentially a no-arbitrage condition. No profitable opportunities exist in the bond market since the long/short spread precisely anticipates the evolution over time of short rates. A simple example can clarify this concept. Any investor can implement two alternative trading strategies that cover the same horizon length. The first is a maturity strategy: the investor buys a zero-coupon (pure discount) bond and holds it till its expiration date; the rate of return of this riskless action equals the bond’s yield to maturity. The second strategy is a roll-over: the investor buys a bond and sells it before maturity; the money obtained, uncertain in the amount, are immediately invested in a new bond with the same maturity date of the bond just sold. In perfect

\(^1\) Following Lutz (1940) we distinguish PEH from EH. The pure version of EH states that the expected excess return is simply zero; EH, instead, assumes that it is constant over time.
financial markets both strategies must guarantee the same rate of return ruling out any arbitrage. In the latter case, when a bond is sold before maturity, the holding period return may differ from its yield to maturity, i.e. the investor take a risk. In the former case, instead, the investor knows with certainty that the bond price at maturity will equal its face value; the holding period return exactly matches the yield to maturity. If during the life of a zero-coupon security an unexpected price change takes place, the investors would expect some form of compensation, for instance a price change in the opposite direction in the future (before the bond expires). Variations in nominal returns on a given bond are thus negatively correlated, and hence forecastable. The basic reasoning behind EH is that two equivalent investment options should offer the same expected return; if not, investors may well arbitrage away any difference. The relation tying the yield to maturity and the instantaneous forward rate at that specific maturity is analogous to that linking the average and marginal costs. Whenever the marginal cost curve (spot forward rates) is below the average cost curve (yield to maturity), average cost decreases. Conversely, if forward rates are high relative to the yield to maturity, the yield to maturity is expected to raise. This is because the yield to maturity represents the cost of borrowing, while forward rates measure the cost of extending the loan a period further. It can be shown that the \( n \)-period ahead instantaneous forward rate equals the yield on a \( n \)-period ZCB plus \( n \) times the slope of the yield curve to maturity.

Different theories explain how interest rates evolve over time. The preference for liquidity (Hicks\(^2\)) posits that risk averse investors prefer short to long term investments; agents thus require an incentive, a positive liquidity premium, to invest on long horizons. Market segmentation (Culbertson\(^2\)) implies that the bond market is split into different sections each characterized by trading activity put forward by investors with different preferences about the horizon’s length. Assets are not perfect substitutes and TS is determined by equilibria achieved independently at each maturity. Preferred habitat (Modigliani and Sutch\(^2\)) is a particular case of market segmentation. Both borrowers and lenders compare long and short rates before deciding which strategy they should implement. Moving into a particular maturity set depends only on expected profitability. The more homogeneous the expectations, the closer the model to PEH. With heterogeneous expectations, instead, there is a significant mismatch between demand and supply of financial assets; hence, a liquidity premium is necessary since investors claim a compensation for adapting preferences.

Research regarding the term structure has evolved in continuous time, where TS models represent an advanced area of application of financial theory. We briefly present affine models since later we estimate a discrete-time affine model to derive the latent components of the yield curve (Chapter 2).

Cox, Ingersoll, and Ross (1985) derive a continuous time TS model in a general equilibrium framework, in which consumers are assumed to maximise the Von Neumann-Morgenstern utility of consumption. Bond yields are expressed in terms of the stochastic discount factor, or pricing kernel:

$$\frac{d\Lambda}{\Lambda} = -y dt - \sigma'_\lambda(\bullet) dw$$

(1.2)

The subjective discount factor, an individual time preference rate, becomes stochastic when modelled through the diffusion process (1.2); $y$ represents the short term interest rate, $dt$ is the differenced time process; $dw$ is the Brownian motion term which drives the stochastic volatility of the overall process. The stochastic discount factor represents consumers’ intertemporal marginal rate of substitution and it is used for asset pricing, i.e. to discount security forward expected prices. The subjective discount factor $\delta$ is an individual time preference rate that becomes stochastic when modelled through the diffusion process (1.2). The stochastic discount factor $z_{t+1}$ thus can be seen as a function of the ratio between the marginal utility $u'(\bullet)$ of future and current consumption $c$:

$$z_{t+1} = \delta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \right]$$

(1.3)

Essentially the pricing kernel is a density function that associates a stochastic weight to alternative states of the world; hence, it is a useful tool to describe agents preferences through time and across states. In particular, in many financial applications the stochastic discount factor has become a convenient expedient to determine, or predict, future asset prices under uncertainty. In the above expression. The short rate process is driven by a drift term $\mu_y(\bullet)$ and a diffusion process $\sigma'_y(\bullet) dw$:

$$dy = \mu_y(\bullet) dt + \sigma'_y(\bullet) dw$$

(1.4)

The rate starts out as a state variable for the drift of the discount factor in (1.2); the expected value of the pricing kernel becomes the continuous series of the short rate process $E(d\Lambda/\Lambda) = -y dt$ since the Wiener process is a zero-mean normally distributed process (the Levy theorem guarantees that any Wiener process, adapted to a certain filtration -a dynamic information set-, is a Brownian motion). In absence of the diffusion term, the above equations would simply be ordinary differential equations. The stochastic differential equation (1.4) that drives the short rate process $y_t$ needs appropriate specification for being of practical use. The basic model is by Merton (1971, 1974) and Ho and Lee (1986). Vasicek (1977) has suggested the following mean-reverting Markov specification for the short rate ($dy$):

$$\frac{d\Lambda}{\Lambda} = -y dt - \sigma_\Lambda dw \quad \text{with} \quad dy = \rho(\bar{y} - y) dt + \sigma_y dz$$

(1.5)
Where $\overline{y}$ is the long-run equilibrium level for the short rate ($\rho$ is a coefficient that captures the effect of mean reversion). The main drawback is that (1.5) allows for negative interest rates, which are economically a nonsense; in addition, the price of risk is not permitted to change sign thus contradicting factual evidence. Moreover, homoscedastic single-factor models cannot generate inverted hump-shape forward curves.

Semi-affine models have become increasingly popular in macro-finance applications since they allow for time-varying sign-changing term premia (Duarte, 2002). A model is said to be affine when the variable can be expressed in terms of a constant and a linear term. In the term structure literature the price of a bond is expressed as a linear function of (a constant and) a state vector through coefficients that are, in turn, functions of the bond’s maturity. The affine class is typically associated to term structure models that feature the absence of arbitrage condition (Piazzesi, 2003).

A popular specification is by Cox, Ingersoll, and Ross (1985):

$$
\frac{d\Lambda}{\Lambda} = -y dt - \sigma \sqrt{\overline{y}} d\zeta \quad \text{with} \quad dy = \rho (\overline{y} - y) dt + \sigma \sqrt{y} dz
$$

The square root constrains the short rate to non-negative values. The restriction $\sigma_y \leq 2 \rho \overline{y}$, also known as the Feller condition, is necessary to prevent the spot rate from degenerating to zero. As for the homoscedastic case, also the square root process cannot generate inverted hump-shape curves. Multiple-factor TS models have been introduced since two or three independent factors guarantees the yield curve to assume any possible shape.

Both (1.5) and (1.6) hinge on the existence of a risk-neutral probability measure $Q$ which is the Radon-Nikodym transformation of the actual probability $P$. Bond pricing is determined after specifying a structure for the discount factor process (1.2). It holds:

$$
P_t^r = E_t \left( \frac{d\Lambda_{t+T}}{\Lambda_t} \right)
$$

The stochastic discount factor is usually assumed to be log-Normal. Solutions of these models are obtained either by applying expectations or solving analytically the differential equations (Duffie and Kan, 1996). A typical solution represents the (log) bond price as:

$$
-p_t^r = A(T) + B(T) \overline{y}
$$

which is recursively solved forward in the coefficients $A(T)$ and $B(T)$. The price of a bond is inversely related to the yield (here we abuse of notation since $\overline{y}$ denotes any state variable). A more general version of equation (1.8), i.e. the empirical version of (1.7), shows that the price of an asset at time $t$ is simply the stochastically discounted value of the price a period ahead:
Vasicek has proposed a completely affine model\(^1\). A drawback of affine and semi-affine models is that the price of risk is a strictly increasing function of instantaneous volatility \(\sigma_t\), alias the compensation for bearing risk (expected excess returns) is proportional to interest rates’ volatility, i.e. to the risk itself. Although semi-affine models are a parsimonious generalization of completely affine models they perform poorly; in particular, the magnitude of term premia turns out to be systematically smaller than that observed in the data.

Duffee (2002) introduces the class of essentially affine models to break the vicious connection between the price of risk and state variables’ volatility. The diffusion is affine under both the historical probability measure \(P\) and the risk-neutral probability \(Q\). The main feature of essentially affine models is that the price of risk becomes independent of the volatility matrix. Duffee provides evidence that essentially affine models forecast future yields better than affine do.

1.3 The Expectations Hypothesis: Some Empirical Evidence

The empirical review starts with the contribution by Campbell and Shiller (1991) since they offer an extensive analysis of the entire TS maturity spectrum. They employ a linearized version of EH testing whether the yield spread anticipates future changes in interest rates. For each couple of maturities \((n, m)\), with \(n > m\), they estimate ordinary least square regressions of the theoretical, or perfect foresight, spread onto the actual spread. To correct for overlapping residuals they adopt Newey-West consistent covariance matrix.

The first important result is that the yield spread fails to predict movements of long term rates; more precisely, the spread returns forecasts in the wrong direction (\(\beta < 0\)) for the short term change of long rates. The contrarian test (Thornton, 2005) for predicting future changes of long yields is:

\[
\left(\frac{n-m}{m}\right) \left(y^n_{t+m} - y^n_t\right) = \alpha + \beta \left(y^n_t - y^m_t\right) + \varepsilon_{t+m}
\]  

Campbell and Shiller use \(y^n_{t+m}\) as an approximation of \(y^n_{t+m}\) when estimating the above regression for many couples of maturities \((n, m)\).

\(^1\) In completely affine models all processes are affine: the diffusion under the actual probability measure \(P\); the diffusion under the risk-neutral probability measure \(Q\); the log prices of bonds; the square of the discount factor, and the quadratic form linking the state variables’ covariance matrix and the discount factor through the square root of the state variables. In the semi-affine class of TS models the Girsanov transformation does not allow both \(P\) and \(Q\) to be affine; thus the model is affine only under the synthetic risk-neutral probability measure \(Q\). In essentially affine models the square of the price of risk vector is not necessarily affine in the state variables. Essentially affine models nest completely the affine class of TS models.
The second result concerns the prediction of short rate over the life of long term bonds. Results are partially supportive of EH, the estimated slope coefficient $\beta$ turns out to be positive and close to one for many pairs of maturities. The conventional test (Thornton, 2005) regression is:

$$\left(\frac{m}{n}\right)\sum_{t=0}^{n-m} y^m_t - y^m_t = \alpha + \beta \left(y^m_t - y^m_t\right) + \epsilon_{t+n-m}$$  \hspace{1cm} (1.11)

Equation (1.11) can be consistently estimated under the assumption of rational expectations, since disturbances are not correlated with the effective yield spread. The analysis is performed with U.S. end-of-month data from 1952 to 1987; they conclude “we thus see an apparent paradox: the slope of the term structure almost always gives a forecast in the wrong direction for the short term change in the yields on longer bonds, but gives a forecast in the right direction for long term changes in short term rates”. They suggest that long rates tend to overreact to rationally expected changes in short rates. Excessively high spreads cause long rates to fall when short rates rise; short term changes in long rates are thus negatively correlated with long term changes in short rates. The negative correlation is amplified by the magnitude of the weighting coefficient that depends on maturities’ temporal distance. Shiller, Campbell, and Schoenholtz (1983) point out that linear specifications of EH inappropriately assume constant bonds’ duration. In particular, in line with Ando and Kennickell (1983), high interest rates reduce long term bonds’ duration affecting EH tests. A better fit for EH is achieved once weighting coefficients are corrected for duration.

Thornton (2005) gives a simple, but effective, mathematical rationalization of EH empirical failure arguing that EH estimates are inherently biased when EH does not hold. If expectations are not rational the true data generating process is not necessarily coherent with EH. In fact, if both long and short rates follow a first-order autoregressive process, as they usually do, EH coefficients’ estimates are biased. In particular, the expected value of coefficient estimates is positive for the short term rate prediction (1.11), and negative in equation (1.10) that forecasts long rates. This is because the actual short rate appears symmetrically on both sides of (1.11); whereas the long rate appears with opposite sign on different sides of (1.10). These results can be generalized under the alternative hypothesis using asymptotic theory. The probability limit for $\beta$ in (1.10) is affected by a relative variance factor$^4$

$$p \lim_{N \to \infty} \beta = 1 + \beta' \frac{n}{m} \left[ \frac{1 - \rho \delta^{1/2}}{1 - 2 \delta^{1/2} + \delta} \right]$$  \hspace{1cm} (1.12)

$^4$ The relative variance factor depends both upon the ratio of the variances $\delta$ and the correlation coefficient between long and short term interest rates $\rho$.
The relative variance factor (in square brackets) can assume both positive and negative values, so that the slope coefficient $\beta$ is not necessarily one. The relative variance factor is strictly negative when $\rho < \delta^{-1/2}$. For the conventional test (1.11) the limit in probability is:

$$ p \lim_{N \to \infty} \beta = \beta + \beta' \left[ \frac{\rho \delta^{1/2} - \delta}{1 - 2 \delta^{1/2} + \delta} \right] $$

(1.13)

Mishkin (1988) provides a refinement of Fama’s (1984) work on the predicting power of the yield spread achieving similar conclusions. Forwards rates seem reliable predictors of future changes in spot rates with forecasting power increasing after 1982. In September 1982 the Fed abandoned the non borrowed reserve operating procedure (stabilizing monetary aggregates) in favour of the interest rate targeting regime. Mishkin’s main contribution is methodological, since he discusses econometric issues for testing EH. Ordinary least square applied to equations (1.10) and (1.11) generates serially correlated errors, because of the overlapping nature of variables. Autocorrelated disturbances affect standard errors invalidating inference procedures. Various corrections have been proposed: Hansen and Hodrick (1980), White (1980), and Newey and West (1985). Newey and West, for instance, suggest multiplying off-diagonal elements of the covariance matrix by some weights in order to scale down residuals’ autocovariances.

Mankiw and Miron (1986, henceforth MM) analyse EH throughout an extensive sample starting in 1890. MM focus on the short end of the maturity spectrum and identify specific monetary regimes. They find evidence in favour of EH only before the creation of the Fed. After ruling out the possibility of both seasonal effect and financial turmoil (occurred in 1980, 1893, and 1907), they conclude that EH holds before 1915 because of easy predictability of short rates due to their mean reverting pattern. In particular, when the short rate is above its mean by 100 basis points, it follows an adjustment of 57 bps within the following period. After the foundation of the Fed the mean reverting behaviour has progressively faded away due to the policy of stabilizing, or even pegging, interest rates. Data evidence after 1915 suggests that the short rate is a martingale. In case of monetary policy inertia (interest rates smoothing), the Fed keeps the expected change of the short

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5 Before October 1979 the monetary regime was an interest rate targeting as well.

6 Data before 1910 were collected by the National Monetary Commission, which was the precursor of the Federal Reserve System founded in 1915. In this sample data are the interest rates banks charged for loans of fixed maturities.

7 The first sub-sample ends just before the foundation of the Fed in 1915. The second regime terminates at the introduction of the New Deal banking reforms, sample which also approximates both the end of the classic Gold Standard and the beginning of the interest rate pegging (early 1930s). The third regimes goes from 1934 to 1951, which ends with the Accord, i.e. the agreement between the Treasury and the Fed of abandoning the interest rate pegging. The fourth regime lasts till 1958, when the market for 3- and 6-month Treasury Bills actively begins. The entire sample finishes in October 1979 when the Fed changed the conduct of monetary policy. As mentioned earlier, Volcker became president of the Fed in 1979 and the interest rate targeting policy was abandoned in favour of the non borrowed reserve operating procedure.

8 The regression for the 1980-1915 sub-sample returns a particularly high goodness of fit, much higher than in other sub-samples. In addition, the null hypothesis that all coefficients are jointly null is rejected at 1% significance level.
rate equal to zero; hence the spread would always match a random term premium without containing any predictive power. MM believe that the existence of a time-varying risk premium can explain deviations from EH after 1915. They also mention few other potential causes that may account for the empirical failure of EH, such as change in risk perception, change in relative asset supplies, measurement errors, and near rational, rather than rational, expectations.

A reconsideration of MM’s work is provided by Kool and Thornton (2004) who show that, once financial panics in 1907-1908 are properly taken into account, the evidence in favour of EH is not different before and after the creation of the Federal Reserve; so that EH is rejected also before 1915. Kool and Thornton argue that the eventual support for EH before 1915 is merely due to the presence of extreme observations in the data sample.

Rudebusch (1995) think that MM’s thesis is plausible but it has not been formalized. Focusing on cases in which the long maturity is exactly twice the short term maturity ($n = 2m$) concluding “The ability of TS to predict changes in short rates is quite good for forecast horizons that are no longer than about a month. As the forecast horizon increases, the predictive power disappears, and $\beta$s are insignificantly different from zero from three months to one year. However, at horizons longer than one or two years, there is some evidence that predictive power appears to improve. This U-shaped pattern of the predictive ability of the yield curve traces out some of the results this paper will try to explain”.

Rudebusch splits the entire sample into two sub-samples reflecting different Fed’s regimes (Sept. 1974 - Sept. 1979; Mar. 1984 - Sept. 1992). To test the random walk hypothesis proposed by MM, he builds a probability model of the deviation of the interest rate from its targeted level $\delta_t = \overline{fr}_t - fr_{t-1}$. The daily effective fed funds rate is expressed as the sum of the targeted level plus the delta deviation $^\text{10}$.

$$E_{t-1}(\overline{fr}_t) = \overline{fr}_{t-1} + P_t^* E(\eta) - P_t^* E(\eta) \quad (1.14)$$

If MM theory holds, the probability of positive and negative adjustments should be the same at each date $P_t^* = P_t^* \forall t$. Interest rate smoothing occurs when the fed funds target randomly oscillates around its past value, being zero, and thus unpredictable, the mean of deviations. Conversely, if changes in the target for federal funds are followed by changes in the same direction there is evidence of gradualism in the conduct of monetary policy. The estimated hazard functions do not display duration dependent behaviour when the horizon is longer than five weeks. Deviations from the target rate are large but not persistent; they tend to disappear within the following trading day. In line with MM, Rudebusch concludes “after four weeks the random walk nature of the federal funds

---

$^9$ Hardouvelis (1994) argues that time-varying term premia do not matter in explaining persistent negative estimates of the slope coefficient $\beta$. He, instead, supports the idea of the overreaction hypothesis originally proposed by Campbell and Shiller (1991).

$^{10}$ The mean absolute value of these deviations is 17 basis points.
rate target asserts itself*. Moreover, consistently with Fama and Bliss (1987), Rudebusch comments “Spreads involving long term bonds -specifically, for maturities longer than one or two years- appear to have some predictive content for movements in future interest rates. This probably reflects the fact that markets expected the Fed to restrain inflation and that the Fed has indeed been successful in obtaining such restraints, at least at business cycle frequencies. Coupled with a stationary real rate, the Fed’s expected and actual containment of inflation during the sample has probably generated the significant coefficients associated with long-maturity nominal rate spread”.

Fama and Bliss (1987) concentrate on the information content of forward rates concluding that forward rates have substantial predictive power which is increasing with the forecast horizon. They argue this is due to the mean reverting property of the 1-year interest rate. The 1-year forward rate is also informative about the 1-year bonds’ expected returns. The inverse relation between yields $y$ and $(\log)$ prices $p$ of an $n$-period zero coupon bond with face value $l$ is:

$$y^n_t = -\frac{1}{n} \ln P^n_t$$

(1.15)

The one-period spot rate is $y^1_t$. It is common to look at bond prices in logarithm for the suitable interpretation. If the price of a ZCB is 0.95, i.e. 95 cents per dollar face value, the log-price is $-0.051$, indicating that the bonds sells at 5 percent discount. The holding period return on an $n$-year bond which is hold for 1 year is:

$$hpr^n_{t+1} = p^{n-1}_{t+1} - p^n_t$$

(1.16)

The forward rate obtained comparing two different points in time of the maturity spectrum is

$$f^{n,n+1}_t = p^n_t - p^{n+1}_t$$

(1.17)

Equivalently, the forward rate can be expressed either as difference of bond prices or as difference between yields. Expression (1.18) gives a measure of the current evaluation (time $t$ prediction) of the interest rate on a 1-year ZCB with maturity $n+1$ future. The price of a discount bond can be seen as a sequence of expected discounted payoffs; it is immediate to use forward rates:

$$p^n_t = (p^n_t - p^{n-1}_t) + (p^{n-1}_t - p^{n-2}_t) + \ldots + (p^2_t - p^1_t) + p^1_t = $$

$$= -f^{n-1,n}_t - f^{n-2,n-1}_t - \ldots - f^{1,2}_t - y^1_t$$

(1.18)

Prices thus contain rational forecasts of equilibrium expected returns. The forward-spot spread is:

$$f^{n,n+1}_t - y^1_t = [E_t y^1_{t+1} - y^1_t] + [E_t hpr^n_{t+1} - y^1_t]$$

(1.19)

Fama and Bliss estimate\(^{11}\) the following regression:

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\(^{11}\)Cook and Hahn (1989) use similar explanatory variables to analyse the term structure and term premia.
A positive and significant $\beta$ indicates that the spread carries valuable information for predicting future changes in the $l$-year spot rate ($n$-$l$) years ahead. Equation (1.20) is complementary to:

$$hpr_{t+1}^{n-l} - y_t^l = -\alpha + (1 - \beta)\left[f_t^{n-l} - y_t^l\right] + u_{t+1}$$

The following equation, instead, shows that the forward-spot spread (LHS) is composed by the time $t$ expected premium of the $l$-year return on an $n$-year bond over the $l$-year spot rate (the first RHS term) and the expected change from ($t+1$) to $t$ in the yield on ($n$-$l$)-year bond (second RHS term):

$$f_t^{n,l} - y_t^l = [E_t hpr_{t+1}^n - y_t^l] + [E_t y_{t+1}^{n-1} - y_t^{n-1}]$$

In regression (1.23) the dynamics of the term premium is a function of the forward-spot spread:

$$hpr_{t+1}^{n-l} - y_t^l = \alpha + \beta\left[f_t^{n-l} - y_t^l\right] + u_{t+1}$$

A positive and statistically significant $\beta$ is interpreted as evidence of a time-varying term premium. Fama and Bliss interpret their results in line with Shiller (1979): when bond prices are rationally determined, the volatility of long rates affects the variability of both term premia and expected returns. Fama and Bliss’ major finding is “the 1-year forward rate calculated from the prices of 4- and 5-year bonds explains 48 percent of the variance of the change in the 1-year interest rate 4 years ahead. This forecast power is largely due to a slow mean reverting tendency in interest rates which is more apparent over long horizons”. This conclusion is also coherent with MM’s idea.

Campbell (1995) finds that exceptionally high spreads do not help infer future movements in long rates. EH implies unpredictable excess returns on long over short bonds. As Thornton (2005) would argue later, Campbell suggests: “the long rate appears both in the regressor with positive sign (as part of the yield spread) and in the dependent variable with a negative sign (as part of the change in long yield), measurement error would tend to produce negative estimates of the coefficient”.

An instrument correlated with the spread could help (Campbell and Shiller 1991); however, negative estimates of EH coefficients seem robust to different instruments. Campbell’s explanation of the paradox hinges on the presence of a measurement error, which may derive from changing rational expectations. Measurement errors affect positively explanatory variables, while negatively the dependent variable. Campbell focuses on the unusual response of long yields in 1994 after a tight policy implemented by the Fed. Long term rates have increased more than short rates, widening the spread substantially above the post war average. The exceptionally steep yield curve has induced the Treasury to shortening the average maturity of Federal debt in order to save on interest costs.
The Fed increased the fed funds rate\textsuperscript{12} by 1.25 percent in four months starting in January 1994. Two possible explanations for the overreaction of long rates. Investors could have found the Fed behaviour surprisingly severe since Greenspan had previously established a reputation for gradualism. The atypical conduct might have been misinterpreted by investors as they judged the Fed having private information about future inflation, so they anticipated future contractionary actions. Unfortunately, this explanation is not entirely supported by factual evidence since the Fed conduct was transparent. The Fed clearly aimed at choking any inflationary pressures off. Alternatively, investors merely required elevated excess returns on long term maturities to compensate for high risk reflected in soaring volatility of long rates. At the end of May forward rates dropped achieving the January level. Campbell: \textit{“it seems more likely that bond yields were affected by a temporary change in the required excess bonds returns. This story is consistent with the evidence against EH”}. Another attempt to rationalize the systematic rejection of EH is offered by Froot (1989)\textsuperscript{13}. He shows that the perverse effect affecting short term changes in long rates is actually due to a violation of the rationality principle rather than to time variation of risk premia. Froot emphasizes that deviations from EH consist of two components, an expectations error and a term premium. He finds that the perception of risk becomes increasingly important for pricing bonds with long duration. Term premia extracted from survey data are positively correlated with both inflation and the level of nominal rates. The market surveys are determined by market expectations on the forward rate plus a zero-mean expectational error $E(\varepsilon_{t+j}\mid R_{t+j}) = 0$ uncorrelated with expectations:

$$s_{t+j} = \varepsilon_{t+j} + \varepsilon_{t,j}$$  

(1.24)

$e_{t+j} = E(v_{t+j}) - y_{t+j}$ represents the market expectations about the yield on a \((n-j)\)-period bond issued at time \(t+j\) minus the yield to maturity at time \(t\) on a \(j\)-period bond. Forward premia are also proportional to the yield spread, as pointed out by Campbell, Shiller and Schoenholtz (1983). The term premium $\theta_{t+j}^{(n,j)}$ on a \(n\)-period bond held from period \(t\) to period \((t+j)\), is defined as the difference between the forward premium and the expected future change in the interest rate:

$$\theta_{t+j}^{(n,j)} = fP_{t+j}^{(n,j)} - e_{t+j}^{(n,j)}$$  

(1.25)

Consequently, EH can be tested by estimating the following equation:

\textsuperscript{12} The federal funds rate is the interest rate at which depository institutions lend balances at the Federal Reserve to other depository institutions overnight. The effective federal funds rate is a weighted average interest rate reflecting federal funds transactions by a group of brokers who report daily to the Fed of New York.

\textsuperscript{13} The empirical analysis is based on market survey data from Goldsmith-Nagan Bond and Money Market Letter; this dataset reveals investors’ beliefs about interest rates on securities with different maturities. Survey data gives the crucial opportunity of decomposing the spread’s biased predictions into an expectational error measure and a term premium. Estimations are carried out for both the conventional and the contrarian tests using quarterly data from 1969 to 1986. The conventional test returns positive $\beta$ estimation but significantly lower than unity; the contrarian test returns negative $\beta$ coefficients.
\[ y_{i+j}^{(n-j)} - y_i^j = \alpha + \beta \cdot f_{p_i^{(j,n)}} + u_{i+j} \]  

(1.26)

EH implies a null premium as forward rates should equal future spot rates; so that EH is respected when both \( \beta = 1 \) and \( \alpha = 0 \). The slope coefficient comes out to be:

\[ \beta = \frac{\text{cov}(e_i^{(j,n)}, f_{p_i^{(j,n)}}) + \text{cov}(u_{i+j}, f_{p_i^{(j,n)}})}{\text{var}(f_{p_i^{(j,n)}})} \]  

(1.27)

\( \beta \) can be expressed also as the summation of a premium and an expectations error \( \beta = 1 + \beta_p + \beta_e \)

where \( \beta_p = -\frac{\text{cov}(e_i^{(j,n)}, f_{p_i^{(j,n)}})}{\text{var}(f_{p_i^{(j,n)}})} \) and \( \beta_e = -\frac{\text{cov}(u_{i+j}, f_{p_i^{(j,n)}})}{\text{var}(f_{p_i^{(j,n)}})} \)  

(1.28)

\( \beta_p \) is null when also the variance of the premium is zero; this is the case of EH. \( \beta_e \) is zero in absence of systematic expectational error. Froot extracts from data the following components:

\[ \theta_i^{(j,n)} = f_{p_i^{(j,n)}} + s_{(n-j)}^{(i)} + \nu_i \]  

and \[ u_{i+j} = (i^{(n-j)} - i^i) - s_{(n-j)}^{(i)} + \nu_{i,j} \]  

(1.29)

Kuttner (2001) investigates how monetary policy surprises affect interest rates. Using data from the fed funds future market, he discriminates between anticipated and unanticipated policy actions. Results confirm a strong reaction to unanticipated ones. The rate response of a security is

\[ \Delta r_i = \alpha + \beta_e \Delta \tilde{r}^e + \beta_u \Delta \tilde{r}^u + \nu_i \]  

(1.30)

\( \Delta \tilde{r}^e \) is the expected component of monetary policy actions while \( \Delta \tilde{r}^u \) represents the unanticipated one. The total change of the target rate gives the nominal size of monetary policy intervention \( \Delta \tilde{r} = \Delta \tilde{r}^e + \Delta \tilde{r}^u \). \( r_i \) is the interest rate associated to 3-, 6-, and 12-month bills, the 2-, 5-, 10-year notes and the 30-year bond. Results indicate important responses of securities yields to the surprise component: \( \beta_u \) is much larger than \( \beta_e \). The Wald test rejects the null \( \beta_e = \beta_u \) for all securities. In addition, the intensity of the response gets smaller as the maturity increases. Consistently with EH “the smaller response of long rates; the less than one-for-one response of bill rates; and, the significant coefficient on expected target rate changes in the monthly bill rate regression”. As in Cook and Hahn (1989), the fed funds’ mean reversion implies softer reaction for long bonds. Moreover, they point out that surprises have more to do with timing of actions rather than size.

\[ ^{14} \text{An important strand of literature analyses the reaction of interest rates to monetary policy announcements. Roley and Sellon find that the relationship between policy announcements and the behaviour of long term rates is variable over the business cycle. They conclude that there is a strong but variable connection between monetary policy actions and long rates; moreover, “to a considerable degree, long term rates appear to anticipate policy changes, moving well in advance of policy actions. Monetary policy actions are likely to be most effective in changing long term rates when these actions are seen as persistent”. Roley and Sellon (1998) have also documented the eventual reaction to policy non-announcements “our empirical results suggest a small, but statistically significant, reversal of rates when an expected federal funds target rate change does not occur”.} \]
Kozicki and Sellon (2005) analyse a particular phenomenon, later labelled “the conundrum” by chairman Greenspan, occurred after policy tightening in 2004. It happened that long rates declined substantially after severe monetary policy thus contradicting common wisdom and pre-existing evidence “while long term sometimes decline as short term rates increase, this phenomenon typically occurs towards the end of the policy tightening, rather than at the beginning of the cycle. Thus, what is particularly unusual about the current episode is that the decline in long term rates occurred at the beginning of a tightening cycle rather than at the end”. At the beginning of a policy tightening short rates raise to remove previous accommodation; this jump is intended to curb excess demand and contain inflationary pressures. On the other hand, long rates reflect agents’ expectations about structural or institutional changes in the economy. They attribute the unusual long rates dynamics, and the subsequent flattening of the yield curve, to a decrease in the inflation term premium required by investors (interpretable as a sign of both reputation and credibility).

Favero claims that EH rejection might be due to a misleading way of processing information. He undertakes a reversal engineering approach to obtain the forward path of short rates, which is used to generate EH-consistent long rates. He employs a reduced-form VAR defined on the following variables: inflation, a measure of the output gap, and the IMF world commodity price index. The most endogenous variable, the policy rate, is placed at the end of the VAR sequence. Such a specification implies a Taylor rule reaction function for the short rate. Rolling estimates and stochastic dynamic simulations generate forecasts for the policy rate $i^F_t$ used to construct the EH-consistent long rate $y^*_t$: 

$$y^*_{t,T} = \frac{1}{T-t} \sum_{j=1}^{T} i^F_{t-j}$$

(1.31)

Results are interpreted as evidence in favour of EH since there is a clear tendency of the generated series to co-move with actual data. Actual data, in fact, lie within the 95% confidence interval bands. Similarly, Carriero, Kaminska, and Favero (2004) argue that inappropriate approach to model expectations underlie EH empirical failure. Using a macro model augmented with financial factors they simulate EH-consistent long rates and construct a series of the yield spread which is used to test EH. The comparison between simulated and actual data allow them not to reject the null EH.

1.4 The Expectations Hypothesis and Monetary Regimes

In this Section we explore whether the corroboration of EH is related to monetary regimes. Hardouvelis (1988) provides evidence that the predictive power of the spread is not necessarily reduced by the intention of the monetary authority to stabilize the short rate, as instead Mankiw and
Miron (1986) suggested. Three samples are considered: the interest rate targeting between January 1972 and October 1979; the non borrowed reserve operating procedures between 1979 and October 1982; and, again, the partial interest rate targeting till November 1985. Despite forward rates inherently incorporate a time-varying premium, data evidence suggests forward rates generating respectable predictions of future change in interest rates. Hardouvelis finds evidence regarding the predictive power of the spread also after October 1982, during the partial interest rate targeting regime. Moreover, between October 1979 and October 1982 the predictive power of forward rates lasts only six weeks into the future, while after 1982 it endures till nine weeks ahead. He concludes “Results show no necessary connection between interest rate predictability and the degree to which the Fed adheres to interest rate targeting. Changes in the TS’s predictive power across the different monetary regimes are not necessarily due to changes in predictability since they can also originate from change in parameters that characterize the evolution of expected risk premia”.

Recently a variety of critical contributions on the predictive power of the yield spread have been proposed by Thornton. In 2004 he offers empirical evidence in contrast with Simon (1990) and Roberds, Runkle and Whiteman (1996).

Simon (1990) offers empirical evidence supporting the predictive power of the spread during the non borrowed reserve operating procedure (October 1979 - October 1982). He focuses on the short end of TS considering the spread between the 3-month Treasury bill and the federal funds rate finding that the spread is informative about future interest rates when the short rate is highly volatile, which occurs when the Fed targets monetary aggregates (M1).

Roberds, Runkle and Whiteman (1996) analyse the predictive power of the spread using interest rates data for settlement Wednesdays. With daily observations from 1974 and 1991, they find evidence in favour EH focusing on maturities shorter than 6 months. They document that during the non borrowed reserves operating procedure regime, when interest rates were highly volatile, the spread carried substantial predictive power; whereas, after October 1982, when interest rates’ volatility dropped, the slope coefficient $\beta$ was approximately zero. The corroboration of EH thus seems connected to the volatility of the short rate. They conclude “under the non borrowed reserves targeting regime the short end of TS was substantially more informative about movements in future interest rates”.

In this note we report the measures of interest rates’ volatility in the sub-samples to give a flavour of how different the periods were from a financial point of view. From October 1979 to October 1982 volatility, on a weekly basis, has reached its peak around 140 basis points; before 1979 it was just 31 basis points, and after 1982 it decreased from 140 to 65 basis points.

Initially the target of the Federal Reserve was the monetary aggregate M1, then it became non-borrowed reserves.

These are the days when banks are required to meet Federal Reserve-imposed reserve requirements.

Before October 1979 the regime was an explicit interest rate targeting. Between 1980 and October 1982 the Fed aimed at stabilizing monetary aggregates. After October 1982 the operating procedure passed from non-borrowed to borrowed reserves, but several economists agree on describing that passage as a return, at least partially, to an interest rate targeting. A change in the reserve accounting occurred in 1984. Before 1984 the system was a lagged reserve accounting, that is required reserves were calculated over a week-long computation period and had to be maintained over the two following weeks. After 1984 the accounting procedure was known as the contemporaneous reserve accounting, required reserves were computed over a two-week period, and had to be maintained for a two-day lag.
short rates”; moreover “Results are consistent with the idea advanced by Mankiw and Miron that the information content of TS is strongly linked to the volatility of short term interest rates”. Finally they emphasize “Results indicates that after 1984 equation (*) fits best for a volatile sub-sample of our dataset; that is, on settlement Wednesdays when the short rate is an overnight rate... the market believes the Fed is committed to returning to the pre-settlement overnight funds rate after settlement Wednesdays, no matter how much rates move on settlement. Thus yield spreads on settlement Wednesdays are good predictors of future movements in short rates”.

Thornton (2004) claims that results by both Hardouvelis and Roberds, Runkle and Whiteman are somehow awkward. His basic idea is that it seems difficult to accept that the spread predicts changes in short rates when the target of the monetary authority is expressed in terms of monetary aggregates. The first criticism hits the considerable predictive power ascribed to the spread during the non borrowed operating procedure sample. Conventional wisdom suggests easier prediction of short rates when the Fed is actually targeting the funds rate. Two different reasons support this view. On the one hand, the monetary authority acts to offset the effects of shocks to supply and demand in the funds market, which makes the fed funds deviate from its targeted level; hence the funds rate may well display a mean reverting forecastable pattern; on the other hand, rational market participants may well anticipate the Fed’s target for the fed funds rate. In addition, if the monetary authority makes transparency an important institutional objective, investors can also infer future movements of the policy rate by analysing the macroeconomic scenario. The second criticism concerns the settlement Wednesday issue. Even though Roberds et al. (1996) clearly motivate their appealing theory, Thornton sustains the argument tautological as long as he argues that spikes in the funds rate, which typically occur on settlements Wednesdays, are merely idiosyncratic, in the sense that they are not generally reflected by other short term T-bills rates. Thornton thus attributes the anomalous findings to the presence of extreme observations since OLS estimates are particularly sensitive to outliers.

1.5 The Yield Curve and Time-Varying Risk Premia


In this Section we report the model by McCallum (1994, 2005). He allows for the possibility of time variation in risk premia and monetary policy inertia assuming that the short rate responds to the yield spread. McCallum develops the analysis in a simple two-period case, and, then,
generalizes the model considering \( n \) periods. EH states that the 2-period long rate is an average of expected future spot (one-period) rates plus a constant term premium \( \xi_t \)

\[
y_t^2 = 0.5 \left[ y_t + E_t(y_{t+1}) \right] + \xi_t
\]

(1.32)

The expectations error is defined as follows

\[
e_{t+1} = y_{t+1} - E_t(y_{t+1})
\]

(1.33)

Substituting the latter in the former yields:

\[
0.5 \left[ y_{t+1} - y_t \right] = \left( y_t^2 - y_t \right) - \xi_t + 0.5 \epsilon_{t+1}
\]

(1.34)

Assuming a constant premium \((\xi_t = \bar{\xi})\), and imposing orthogonality of \( e_{t+1} \) with both \( y_t^2 \) and \( y_t \), it is possible to derive the traditional Campbell-Shiller equation (1.11). The stochastic process generating \( \xi_t \) is assumed to be covariance stationary but not necessarily a white noise; more specifically, for better tractability, McCallum imposes \( \xi_t \) to be a first-order autoregressive AR(1) process, thus time-varying, with white noise disturbances (stationarity implies \( \rho < 1 \))

\[
\xi_t = \rho \xi_{t-1} + u_t
\]

(1.35)

The policy rule for the short term rate allows for interest rate smoothing

\[
y_t = \sigma y_{t-1} + \lambda \left( y_t^2 - y_t \right) + \xi_t
\]

(1.36)

\( \sigma \) is close to unity and \( 0 < \lambda < 2 \). \( \sigma \) captures the smoothing behaviour of monetary policy, whilst \( \lambda \) denotes a tendency to tighten whenever the spread is too high, i.e. when monetary policy is loose. The fundamental, or bubble-free, solution is obtained by combining (1.32) and (1.36)

\[
(1 - \lambda) y_t = \sigma y_{t-1} + \lambda \left[ 0.5 \left( y_t - E_t(y_{t+1}) \right) + \xi_t \right] + \xi_t
\]

(1.37)

Substituting and taking expectations yields

\[
E_t(y_{t+1}) = \phi_0 + \phi_1 \left[ \phi_0 + \phi_1 y_{t-1} + \phi_2 \xi_t + \phi_3 \xi_{t-1} \right] + \phi_2 \rho \xi_t
\]

(1.38)

A solution for the short rate is assumed to be of the form

\[
y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 \xi_t + \phi_3 \xi_{t-1}
\]

(1.39)

The short rate depends on both a time-varying risk premium and innovations in the monetary policy rule. For (1.39) to be a solution we need to impose some parameter restrictions. When \( \sigma = 1 \), which is the interest smoothing case, the first difference of the short rate equals the yield spread. Parametrically the solutions are: \( \phi_0 = 0 \), \( \phi_1 = \phi_3 = 1 \), and \( \phi_2 = \lambda/(1 - 0.5 \lambda \rho) \). In a mere AR(1) model for the short rate, \( \sigma = 1 \) implies a random walk process. It follows that
\[ E_t(y_{t+1}) - y_t = \phi_2 \rho \xi_t \]  

(1.40)

The term premium thus drives the expected theoretical spread. The yield spread is expressed as:

\[ y_t^2 - y_t = 0.5 \left[ E_t(y_{t+1}) - y_t \right] + \xi_t = \frac{1}{1 - 0.5 \rho \lambda} \xi_t \]  

(1.41)

In this equation \( \xi_t \) is not orthogonal to \( E_t(y_{t+1}) - y_t \). The spread in (1.41) depends on the evolution of the term premium over time. Substituting and reordering, yields

\[ 0.5 (y_t - y_{t-1}) = 0.5 \lambda \rho (y_{t-1}^2 - y_{t-1}) + \frac{0.5 \lambda}{1 - 0.5 \rho \lambda} u_t + 0.5 \xi_t \]  

(1.42)

This represents a more general version of the Campbell-Shiller equation (1.11); \( \beta \) is nothing more than a consistent estimator of \( 0.5 \lambda \rho \) which is clearly lower than one (since \( \sigma = 1 \)). Therefore, adding a simple monetary policy rule to the framework for testing EH leads to the conclusion that the estimation of \( \beta \) is trivially lower than one; as well as Thornton (2005) pointed out. When \( \lambda = 0 \), i.e. monetary policy does not respond to the spread, it holds the MM random walk hypothesis for the short rate. Finally, with \( \sigma < 1 \) we need to include a non-zero intercept in the policy rule to permit a stationary equilibrium, being \( E(\xi_t) = 0 \). Hence, McCallum has shown that a stochastic time-varying risk premium can affect the empirical corroboration of EH.

### 1.6 The Term Structure of Interest Rates, Term Premia, and Macroeconomic Variables

In the first part of this Section we discuss some evidence regarding the ability of the yield spread to anticipate future movements in macroeconomic variables; while, in the second part we focus on the predictive ability of the term premium.

In 1977 Shiller and Siegel provide an empirical assessment of the so-called Gibson Paradox. Using British data, Gibson (1923) found a strong positive correlation between the (log) series of a price index and long term interest rates. Shiller and Siegel using spectral techniques confirm this relation, but highlight that there is a short cycle correlation as far as short rates are concerned.

Mishkin (1990a, 1990b) investigates whether TS helps forecast future inflation. Data suggest that interest rates on bonds with maturities less than or equal to six months do not provide any relevant information about the future path of inflation; however, the short end of TS contains information about the real TS of interest rates. On the other hand, for maturities between nine and twelve months TS is informative about future inflation. Mishkin starts the analysis with a very simple equation where the \( m \)-period inflation is simply regressed onto the \( m \)-period interest rate:
\[ \pi^n_t = \alpha_n + \beta_n y^n_t + u^n_t \] (1.43)

The model unfortunately fails to forecast inflation. The rejection of the null hypothesis of constancy of the real interest rate \( (\beta_n = 1) \) is interpreted as a sign that nominal rates are informative about real interest rates. Before October 1979, \( \hat{\beta}_n \) is greater than one; so that negative values for the quantity \( 1 - \beta_n \) indicate negative correlation between nominal and real rates. Mishkin derives an equation to analyse how changes in inflation over time are explained by changes in interest rates:

\[ \pi^n_t - \pi^m_t = \alpha_{m,n} + \beta_{m,n} (y^n_t - y^m_t) + u^{m,n}_t \] (1.44)

Estimates performed with U.S. T-bills data document that during the non borrowed reserves operating procedure the ability of nominal rates to anticipate future inflation has decreased. Mishkin concludes: “evidence in this paper suggests that some caution should be exercised in using the TS of interest rates as a guide for assessing inflationary pressures in the economy”. The null hypothesis \( 1 - \beta_{m,n} = 0 \) is rejected only for short maturities, thus the short end of the nominal TS carries valuable information for inferring the dynamics of its real counterpart. The null hypothesis, however, is not rejected for longer maturities supporting the view that the nominal spread helps forecast inflation.

Frankel and Lown (1994) propose a refinement of Mishkin’s works. They argue that an appropriate indicator of expected inflation should exploit information along the entire spectrum of the yield curve rather than using merely a spread between two points. They suggest looking at the real stance of monetary policy and claim that TS can be thought an inverse indicator of the real interest rate. Basically the difference between long and short rates approximate the real rate since long rates reflect inflation expectation better than short rates. Accommodative policy increases the steepness of the yield curve and lowers the real rate. Expansionary policy actions drive short rates down, while inflation, due to the stimulus to aggregate demand, is expected to rise and long rates along with it. Frankel and Lown estimate the steepness of a non linear transformation of TS. The real interest rates is time-variant following a first-order continuous-time stochastic differential equation:

\[ dy_t = -\delta \left( y_t - r_0^\pi - rr \right) + \sigma dw_t \] (1.45)

\( y_t \) is the instantaneous short rate; \( r_0^\pi \) is the exogenous long run expected inflation; \( rr \) is the constant steady state value of the real rate (not directly observable). Parameter \( \delta \) indicates the speed of adjustment of the real rate towards the equilibrium. \( dw_t \) is a standard brownian motion with normally distributed independent increments \( dw_t \sim N(0, \sqrt{t}) \). The Fisher equation \( rr + r_0^\pi \) represents the value of the nominal interest rate. The economic rationale follows. Consider the monetary authority is going to expand money supply raising consequently expected inflation “if short rates were to rise instantly,
real money demand would fall, which is not possible when the price level and thus the real money supply are fixed. But over time, prices rise in response to excess goods demand. Thus, over time the real money supply is free to fall” short rates then rise to reflect worries about expected inflation.

According to EH an equation for the interest rate on a bond with maturity $\tau$ issued in $t = 0$ the short term interest rate is the average of instantaneous interest rates between time 0 and $\tau$

$$y^*_0 = \frac{1}{\tau} \int_0^\tau E_o y_t \, dt + k_t$$

(1.46)

$k_t$ is a possible liquidity premium; integrating (1.46) and using appropriate weights, yields:

$$y^*_0 = k_t + y_o + (\pi^*_0 - rr - y_o)(1 - \omega_t)$$

(1.47)

The unknown $(\pi^*_0 - rr - y_o)$ represents the correct measure of the steepness of TS. At any point in time, the $\tau$ -term interest rate is expressed as a weighted average of theoretical long nominal rates $(\pi^*_0 + rr)$. The longer the maturity the higher the weight given to long term rates and the lower the weight given to short rates. Frankel and Lown obtain positive and significant estimates of the slope both before and after September 1979. Coefficients are greater than one; they thus claim their results are superior in forecasting inflation compared Mishkin’s results.

Caporale and Pittis (1998) exploit the uncovered interest rates parity to assess whether yield spreads are useful to forecast inflation differentials among countries. Working with European data they find evidence suggesting that the predictive power has increased when the exchange rate became an objective of European countries and realignments occurred less frequently.

Estrella and Hardouvelis (1991) investigate whether the TS slope is informative about future activity. Using U.S. average quarterly data they find evidence that the interest rate spread between the 10-year T-bond and the 3-month T-bill predicts real GNP change

$$X_{t,k} = (400/k)[\ln(X_{t+k}/X_t)]]$$

The regression is estimated for different horizons, from 3 to 20 quarters ahead. The slope coefficient is significantly positive; the intercept is positive as well, which means that a negative slope not necessarily implies a negative future GNP growth. $W_t$ is a vector of other information variables. They find that “the forecasting accuracy in predicting cumulative changes in highest 5 to 7 quarters ahead is very impressive, especially, because, ..., the lagged value of real GNP growth has very little predictive power”. The spread can explain more than one-third of the variation in future output changes.
Estrella and Mishkin (1997) perform a similar analysis for both U.S. and some European countries. They examine whether the spread is sensitive to rates at different maturities and the effect of the spread on output and inflation. They adopt a VAR model to capture the double-sided relation

\[
(y_{t}^{40q} - y_{t}^{1q}) = \alpha + \sum_{i=0}^{6} \beta_i CB_{t-i} + \sum_{i=0}^{6} \gamma_i y_{t-i}^{1q} + \sum_{i=0}^{6} \delta_i y_{t-i}^{40q} + \varepsilon_t
\]  

(1.49)

\(CB\) is the central bank rate; \(y_{t}^{1q}\) is the 3-month T-bill rate, and \(y_{t}^{40q}\) is the 10-year T-bond rate. VAR models are usually specified with exogenous variables at the beginning and endogenous (policy) variables at the end; the ordering reflects how variables influence one another. They defend the choice of placing the policy rate in the first position “we are essentially examining the relationship between two endogenous variables. In setting the central bank rate, the monetary authority is influenced by past economic indicators as well as by expectations of future economic variables….., it would not be appropriate to think of the central bank rate as a purely exogenous variable”. Results differ across countries; a percentage point rise in the central bank rate generates a decline in the spread, ranging from 20 basis points in the case of Italy to almost 90 in France. The effect of the spread on real output is significant for horizons of 4 to 8 quarters ahead. In France, Germany, and U.S. the effect holds also at shorter horizons. Moreover, a probit regression reveals the ability of the spread to anticipate recessions; however the sensitivity of the probabilities to change in the spread are highly non linear. Finally, Estrella and Mishkin argue that TS could be an intermediate target for monetary policy pointing out that the spread is undoubtedly an indicator of the monetary policy stance.

Despite empirical evidence largely supports the predictive power of the spread, no theoretical frameworks have been proposed to analyse the relationship between the spread and the economy. Estrella (2004) develops a rational expectations model to explore the relation between the yield curve and macroeconomic variables. The model matches an important feature of existing empirical evidence which regards the greater accuracy of the spread to anticipate business cycle fluctuations rather than future inflation. The simple macro model is composed by three basic equations, an IS (AD) curve, a Phillips (AS) curve, and a monetary policy rule that allows for interest rate smoothing. Results are summarized by the following equations; (1.50) highlights the dependence of future output gap upon the spread, the difference between the interest rate and inflation, the inflation deviation from the targeted steady state level, and the current level of the output gap.

\[
E_i(gap_{t+1}) = \frac{\alpha}{g} \left( y_{t}^{2} - y_{t} \right) + f_1(y_{t} - \pi^*) + f_2(\pi^* - \pi_t) + f_3(gap_t)
\]  

(1.50)

\[
E_i(\Delta\pi_{t+2}) = \frac{\alpha}{g} \left( y_{t}^{2} - r_{t} \right) + a f_1(y_{t} - \pi^*) + a f_2(\pi^* - \pi_t) + a f_3(gap_t)
\]  

(1.51)
Equation (1.51) shows how agents form inflation expectations. In both cases the predictive power of the spread is inversely related to the policy parameter $g$, which reflects the reaction of the central bank to the deviations of output around its potential level. The more incline the monetary authority to stabilize output, the weaker the predictive ability of the TS slope. Parameter $a$ captures the effect that current output exerts on actual inflation. The variability of the ratio between $a$ and $g$ over time accounts for the instability of the spread to predict future inflation; it explains also the relative inaccuracy of the spread for forecasting future inflation rather than output.

Now we turn our attention to the recent strand of literature examining whether term premia contain predictive power to infer future level of output.

Fama and French (1989) find evidence suggesting that term premia are inversely related to the business cycle. Cochrane (2005) reinforces this view. A good security, in fact, is expected to offer high returns, i.e. premia, in bad states, when investors feel poor. This basic idea underlies modern asset pricing through the stochastic discount factor, which reflects the intertemporal marginal utility of consumption. Earlier Hicks (1946) has emphasized the financial role of the term premium, since it is linked to the capital risk resulting from interest rates’ volatility. Investors require a compensation to bear risk associated to highly volatile rates. A negative correlation between term premia and the economic cycle is also documented by Cook and Hahn (1989).

Hamilton and Kim (2002) provide a decomposition of the spread using ex-post observed short term rates data instead of ex-ante expected rates. The spread is split into two components: the expected future changes in short rates and a premium. They show that both components help predict real GDP growth; the estimated effect of both components on future output growth is significantly positive. Cyclical movements of interest rates’ volatility are not explicitly related to GDP; however, volatility seems an important empirical determinant of both the spread and the term premium. Ang, Piazzesi and Wei (2006) confirm that the distinction of the aforementioned components is important to obtain a clear understanding of the forecasting model. The spread reflects the expected stance of monetary policy; while premia are related to agents’ risk aversion. Although the risk factor is not statistically significant, evidence highlights that the ability of the model to forecast future output improves substantially when the spread is split into the aforementioned components.

Kim and Wright (2005) employ a standard arbitrage-free dynamic latent factor TS model to derive risk premia. They ascribe the above mentioned conundrum to a fall in term premia. Wright (2006) finds that risk premia help predict recessions over a six-quarter horizon, but not from two to four quarters ahead. Consistently with previous research, he remarks that lower premia raise the probability of a recession in the future.
Favero, Kaminska, and Soderstrom (2005) analyse the role of the yield spread to predict variations in output ($\Delta x$) since it has been argued that the spread forecasting ability has decreased in recent times (Dotsey, 1998). The reduced predictive power can be attributed either to the changing behaviour of the Fed, more concerned about price stability, or to the effect of term premia. As Hamilton and Kim (2002), Favero et al. (2005) thus isolate the term premium. The 20-quarter rate can be expressed as the average of expected future 3-month rates $y_{\tau+1}$ plus a time-varying premium

$$y_{\tau} = \frac{1}{20} \sum_{j=0}^{19} E_t\left(y_{\tau+j} | I_t \right) + tp_t$$

(1.52)

$tp$ can be seen as the sum of a risk term and a liquidity premium. After simulating a macro-finance VAR model forward to obtain forecasts of the short rate $y^*_{\tau+1}$, they generate the rationally expected ($ER$) component of the yield spread:

$$\hat{ER}_t = \sum_{j=1}^{19} \frac{20-j}{j} E_t\left(\Delta y^*_{\tau+j} | \Omega_t \right)$$

(1.53)

The spread is thus the summation of $ER$ and $tp$. Discrepancies between effective and expected spread exist since then information set $I_t$ used by agents is not as rich as the continuously updated information set $\Omega_t$ available to the econometricians. The correlation between $ER$ and $tp$ is negative. The predictive power of both components of the spread results from the following regression:

$$\Delta_4 x_{t+4} = \gamma_0 + \gamma_1 ER_t + \gamma_2 TP_t + \gamma_3 y^*_t + \gamma_4 \pi_t + u_t$$

(1.54)

Recursive estimations show that the term premium is highly significant in explaining future output ($x$) fluctuations, as well as inflation ($\pi$) and the short rate. Inflation and the expectations-based rate series share important co-movements. The positive correlation between $ER$ and inflation implies that the expectational term is negatively correlated with the real interest rate “hence, a predictive model for GDP growth based on the spread, the nominal short term rate, and inflation may deliver “wrong” or non significant sign on the real short rate”. Favero et al. (2005) relate term premia behaviour to time-varying investors’ risk aversion “when risk aversion is high, and hence the term premium is large, monetary policy has less power in determining output fluctuations than when risk aversion, and then term premium, is low”.

Feroli (2004) explores the interaction between monetary policy and TS. He suggests that, assuming both interest rate smoothing and counter cyclical monetary policy, the spread is informative about future output. Smoothing rates implies that the first difference of the short term policy rate becomes an instrument of monetary policy. Feroli considers four measures of monetary policy: the federal funds rate, the first difference of the fed funds, residuals from an identified VAR (policy...
innovations), and a narrative measure of monetary policy stance (Boschen and Mills, 1995). It turns out that the significance of the spread is robust to different specification of equation (1.54). The spread Granger determines output. Monte Carlo simulations highlights that after 1979 the predictive ability of the spread has diminished due to a change in the policy preferences. Feroli says that the ability of the spread to predict economic growth crucially depends on expectations about future policy concluding that the ability to predict is somewhat different from the ability to cause\(^{19}\).

Empirical research has largely explored the possibility that premia could anticipate future economic activity; however, Rudebusch, Sack, and Swanson (2007) observe that no theoretical formalization has been proposed\(^{20}\). Despite data evidence, they show that there is not a structural relationship between term premia and future GDP growth. We thus conclude this Section by presenting a theoretical model of TS and the economy\(^{21}\). Only optimizing agents live in the economy. Firms maximize expected discounted profits given the technology \(A_t\) (assumed to be an AR(1) process):

\[
E_t \sum_{j=0}^{\infty} \xi^j z_{t+j} \left[ p_t(f) q_{t+j}(f) - w_{t+j} l_{t+j}(f) \right]
\]

(1.55)

\(z_{t+j}\) is the stochastic discount factor between \(t\) and \(t+j\); \(p\) and \(q\) denote respectively the price and the quantity of final output (\(f\) is the firm index); \(w\) represents the wage rate, while \(l\) the amount of labour. The optimal price is set as a mark-up on marginal costs. The production function is a typical Cobb-Douglas with constant returns to scale:

\[
Q_t = A_t K_t^\alpha L_t^{1-\alpha} = \left[ \int_0^1 q_t(f)^{Y_{t+\theta}} df \right]^{1+\theta}
\]

(1.56)

Intermediate goods are produced by monopolistic competitive firms and are purchased by a perfectly competitive final good sector that produces final goods with constant elasticity of substitution (\(\theta\)). The typical downward sloping demand curve and the constant elasticity of substitution price aggregator on the continuum firms space \(f \in [0, 1]\) are respectively

\[
y_t(f) = \left[ \frac{p_t(f)}{P_t} \right]^{-\frac{1}{\theta}} Q_t \quad \text{and} \quad P_t = \left[ \int_0^1 p_t(f)^{-\frac{1}{\theta}} df \right]^{-\theta}
\]

(1.57)

Consumers maximize their lifetime utility:

\[
E_t \sum_{i=0}^{\infty} \delta^i \left( \frac{(c_t - bh_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{L_t^{1+\chi}}{1+\chi} \right)
\]

(1.58)

\(^{19}\) Granger definition of causality: a time series \(Y_t\) is said to Granger cause \(X_t\) if predictions of \(X_t\) based on past values of both \(Y_t\) and \(X_t\) are more accurate than predictions obtained using past values of \(X_t\) only.

\(^{20}\) Estrella (2004), briefly discussed above, develops a rational expectations model to examine the relationship between the slope of the yield curve and macro variables. He concludes that the relationship is not structural; however, in line with Feroli (2004), he finds that TS dynamics influences output and inflation through monetary policy actions.

\(^{21}\) Bekaert, Cho, and Moreno (2005) derive a model combining a typical DSGE macro model with no-arbitrage TS framework.
\( \delta \) is the subjective time-preference discount factor, \( c \) denotes household’s consumption, and \( h_t = C_{t-1} \) represents a shock to consumption habits (\( b \) and \( \chi \) are parameter). The pricing kernel is

\[
z_{t,t+1} \equiv \delta^r \left( \frac{\left( \frac{c_t - b C_{t-1}}{c_t} \right)^\gamma}{\frac{P_t}{P_{t+1}}} \right)
\]  

(1.59)

Optimizing households’ behaviour leads to both an intratemporal condition

\[
w_t = \frac{\beta L_t^G}{(c_t - b C_{t+1})^\gamma}
\]  

(1.60)

and to the classical intertemporal Euler condition with consumption habit:

\[
(c_t - b C_{t+1})^\gamma = \delta \exp(y_t) \frac{E_t \left( \frac{c_{t+1} - b C_{t+1}}{c_t} \right)^\gamma \left( \frac{P_t}{P_{t+1}} \right)}
\]  

(1.61)

\( y_t \) is the continuously compounded interest rate. Private consumption \( C_t \) and government spending \( G_t \) which is first-order autoregressive with idiosyncratic shock) form the aggregate demand of a closed economy. The central bank responds to both output and inflation deviations from the respective natural and targeted level; the monetary policy rule allows for interest rate smoothing. Finally, the financial sector is described by the usual asset pricing condition:

\[
p_t = E_t \left\{ z_{t+1} P_{t+1} \right\}
\]  

(1.62)

The price of any asset equals its the stochastic discounted future values. In the case of a perpetuity, the \((\log)\) nominal consol price satisfies:

\[
p_t^* = 1 + E_t \left\{ z_{t+1} P_{t+1}^* \right\}
\]  

(1.63)

The risk neutral \((Q\) probability measure) \(\log\) price of the consol can be expressed as follows:

\[
p_t^* \bigg|_Q = 1 + \exp(-y_t) E_t \left\{ p_{t+1}^* \bigg|_Q \right\}
\]  

(1.64)

Thus, the implied term premium is:

\[
\log \left( \frac{p_t^*}{p_t^* - 1} \right) - \log \left( \frac{p_t^* \bigg|_Q}{p_t^* \bigg|_Q - 1} \right)
\]  

(1.65)

Considering two finite maturities \( n \) and \( m \) \((n > m)\), the term premium satisfies the following:

\[
p_t^{\pi,m} = y_t - \left( \frac{m}{n} \right) \sum_{i=0}^{n-m} E_t \left( y_{t+i}^{n_m} \right) = \ldots = -\frac{1}{n} \log \left( \prod_{i=1}^{n} z_{t+i} \right) + \left( \frac{1}{n} \right) E_t \left( \sum_{i=1}^{n-m} \log E_{t+i-n} z_{t+i} \right)
\]  

(1.66)

Rudebusch et al. (2007) find that the first-order approximation \((\log\)-linearization) of the model around the steady state makes the term premium disappear; the second-order approximation...
produces a constant premium. Only the third-order approximation allows a time-varying premium. Results do not highlight any structural relationship between term premia and macro variables. However, impulse response analysis generates interesting results. A positive monetary policy shock, a rise in the fed funds rate, increases the premium and lowers output; the response of the premium vanishes away after few quarters. Following a positive technology shock, the premium drops and output jumps upward. Therefore an increase in the premium seems associated to future weak economic growth. A positive innovation to government spending increases the term premium as well as output; in addition, the upward shift of the premium is quite persistent since the government budget worsen. Consistently with the model outcome, but contrarian to previous results, they find evidence suggesting that faster economic growth is related to a decline in the term premium.

1.7 Macroeconomic Models, Latent Factors, and the Term Structure

In this Section I briefly present the strand of literature regarding the macroeconometric analysis of dynamic TS models. The contributions discussed in this part challenge to merge two different fields: macroeconomics and finance. Financial economists have found a suitable way to model bonds pricing by means of arbitrage-free affine TS models. The relative simple tractability of affine versus non-affine (Black, 1976) models has contributed to the success of modelling TS within affine frameworks. Affine models imply that the price of a zero-coupon bond \( P_t \) with maturity \( T \) is exponentially affine (constant plus linear term) in the real function \( A_t(T) \) and in the vector function \( B_t(T) \):

\[
P_t^T = \exp\left\{ A_t(T) + B_t(T)^\prime \ X_t \right\} = \exp\left\{ -Y_t(T) \cdot T \right\}
\]

(1.67)

\( X_t \) is the vector of state variables. The inverse relationship between bonds’ prices and yields implies:

\[
y_t^T = A_t(T) + B_t(T)^\prime \ x_t
\]

(1.68)

where small-case letters denote log transformation. Affine models are appealing since they summarize the dynamics of bond pricing by means of few state variables since available evidence suggests that almost all movements of the yield curve can be captured by the joint effect of only three factors. The name of the yield curve factors comes from Litterman and Scheinkman (1991). A shock to the level affects uniformly yields of all maturities generating a parallel shift of the entire yield curve. A positive shock to the slope increases the steepness of the yield curve, since short

---

22 Hamilton and Kim (2002), as well as Favero, Kaminska, and Soderstrom (2005), find that lower term premia predicts slower output growth. Also Wright (2006) argues that “low term premia raises the odd of a recession”. 
rates raise less than long rates. Finally, a curvature shock affects intensely the middle part of the maturity spectrum accentuating the hump-shape form of the yield curve.

Ang and Piazzesi (2003) pioneered the approach including macroeconomic variables in dynamic arbitrage-free TS models. They examine the influence of macro variables on the yield curve. Bond yields are determined by the three unobservable variables (level, slope, and curvature), a measure of inflation, and an indicator of real economic activity. Results show that incorporating inflation and real activity into asset-pricing models improves forecasting movements of yield curve both in- and out-of-sample. However, such effects are sometimes limited. They find evidence that approximately 85% of bond yields variation is attributed to the impact of macro factors. Macro factors, obtained by principal components explain movements of short and medium term yields (up to maturity of one year); whereas, movements of long yields are related to unobservable factors. Therefore, they conclude that macroeconomic variables cannot shift the level of the yield curve.

Wu (2001) investigates the relationship between the Federal Reserve’s monetary policy surprises and the slope factor finding a strong correlation. Policy surprises are identified in different ways to make the analysis more robust; although they account for 80% to 90% of the movements of slope factor, such influences usually dissipate in one to two months. The level factor is apparently independent from monetary policy, Wu thus concludes that, after 1982, the Fed has affected the yield curve primarily through changing its slope.

Ang, Bekaert and Wei (2008) decompose nominal interest rates into movements in real rates and expected inflation. Since real rates and expected inflation are not directly observable, they build a model to infer these factors from their impact on other variables in the economy. They find that short term real rates tend to be highly volatile, while long rates are smooth and persistent. The flat shape of the real TS allows them to conclude that the positive slope typically present in the nominal TS is due to an inflation risk premium which is increasing in maturity. Moreover, real rates mainly follow pro-cyclical dynamics whereas nominal rates show a counter-cyclical pattern. They also find that changes in expected inflation and in inflation risk premia explain about 80% of the variations of nominal rates.

Rudebusch and Wu (2004) develop a no-arbitrage macro-finance model to examine the joint movement of TS and macro variables. They consider only two latent factors as they argue two factors are enough to capture about 99% of yields’ dynamics. Results indicate that the TS level is associated with the central bank inflation target; while, the slope factor closely reproduces the dynamics of the policy rate implied by a Taylor-type reaction function. Rudebusch and Wu (2005) propose a refinement of their previous work to analyse the shift of TS occurred during the 1980s. They argue that the volatility of term premia has declined in the last decades together with the price
of risk reflected in the level factor. Risk premia are thus closely linked to the expected inflation rate targeted by the monetary authority; hence, the severe conduct of monetary policy started by Volcker and, to some extent, continued by Greenspan, has influenced the TS level. They conclude that modified risk premia reflected in the level factor can definitely account for TS shifts over time.

Hordahl, Tristani, and Vestin (2006) estimate a joint macroeconomic and finance model for the German yield curve. The model is developed on the crucial assumption of no-arbitrage, and includes three basic equations: an equation for the output gap, an equation for inflation (the new Keynesian Phillips curve), and a monetary policy Taylor rule for the short rate. The macroeconometric model appears to fit German data quite well and reproduces salient features related to the German TS. Data evidence, in fact, indicates a regular decline in the targeted inflation rate from 4 to 1% in the last three decades; in addition, an increase in the inflation target would lead to a substantial increase in the risk premium. They document by means of the impulse response analysis that inflationary shocks affect yields at medium maturities, accentuating TS curvature. Risk premia significantly respond to output gap shocks particularly after the German reunification. Investors seem to prefer long term bonds during recessions since risk premia are related to the expected state of the economy. Finally, they adopt a reverse-engineering technique to generate a measure of risk premia compatible with the solution of the EH puzzle (given a specific set of parameter values which maximizes the log-likelihood function derived in their model). The model-implied population coefficients replicate quite closely the pattern of the sample coefficients; in line with Hardouvelis (1994) and Bekaert and Hodrick (2001), they thus conclude that the expectations puzzle is less severe using German data than for U.S.

Dewachter, Lyrio and Maes (2004) employ a continuous-time affine TS model to analyse yields and the economy allowing for time-varying risk premia. Inflation and the output gap are observable, while latent factors represent respectively the real interest rate, the central tendencies of inflation, and the central tendencies of the real rate. Evidence suggests that the output gap and inflation do not adequately fit the dynamic properties of TS at long maturities; whereas, the variability of the long end of the yield curve is mainly driven by the central tendency of inflation. Medium term yields, from six months to two years, appear to be responsive to the central tendency of the real rate. They also find that both macro and latent factors affect bond excess holding returns.

Diebold and Li (2003), and Diebold, Piazzesi and Rudebusch (2005) employ the Nelson-Siegel method to examine bonds pricing within a dynamics latent factor approach. Nelson and Siegel (1987) have proposed a parsimonious model for the yield curve capable of reproducing a great variety of shapes, and, more importantly, of fitting U.S. data quite well. In continuous-time the yield to maturity \( i(n) \) on a \( n \)-period bill can be expressed as the average of future forward rates:
\[ i(n) = \frac{1}{n} \int_{0}^{n} y(\bullet) \, dx \quad (1.69) \]

The following interpolant is essentially a Laguerre function, i.e. a polynomial function characterized by an exponential decay term \( \lambda \):

\[ y^{(\infty)} = \beta_0 + \beta_1 \left( \frac{1 - \exp\left\{-\lambda n\right\}}{\lambda n} \right) + \beta_2 \left( \frac{1 - \exp\left\{-\lambda n\right\}}{\lambda n} - \exp\left\{-\lambda n\right\} \right) \quad (1.70) \]

Parameters \( \beta_0 \), \( \beta_1 \), and \( \beta_2 \) embody the long, short, and medium properties of the yield curve respectively; hence, they can be interpreted as level, slope, and curvature. The loading of \( \beta_0 \) does not decay to zero as the maturity increases, i.e. the multiplying coefficient one is constant. The \( \beta_1 \) loading starts at one and decays monotonically towards zero. Finally, the loading of \( \beta_2 \) starts at zero, increases and then goes back to zero, as a curvature factor is expected to behave. Nelson and Siegel find that the proposed model can account for almost 96% of the variations in bill yields between 1981 and 1983. Nelson and Siegel (1988) focusing on the long run properties of the model show that, under some regularity conditions, the flattening of the yield curve at long maturities is approximately proportional to the reciprocal of time to maturity.

Before concluding it is interesting to mention some works reflecting the most recent evolution of macro-TS research. Recently there has been an attempt to include latent factor dynamics into New Keynesian macro dynamic stochastic general equilibrium (DSGE) models. Bekaert, Cho, and Moreno (2005) combine the structural New Keynesian macroeconomics with no-arbitrage TS theory. They find that changes in the inflation target dominate the dynamics of the level factor. In addition, they relate both slope and curvature to monetary policy shocks. Impulse responses of TS factors to macroeconomic shocks confirm the above interpretation.

In a similar vein Wachter (2006) develops a macro-finance no-arbitrage TS model. The innovative feature introduced by Wachter is external habit on consumption. In such a framework bond prices depend on expected inflation and past consumption through habit. She finds evidence that bond term premia are increasing with maturity (Ang, Bekaert, and Wei 2008; Boudoukh, Richardson, Smith, and Whitelaw, 1999). Finally, model simulations generate a pattern for time-varying term premia that accounts for the expectations puzzle; since the model implies that investors’ marginal utility becomes highly volatile during periods of slowdown in economic activity. This line of research has been followed by Garcia and Luger (2005) who have developed an equilibrium macro-finance model with a reference level of consumption, i.e. a benchmark denoting the predictable component of consumption.
Chapter 2

A Macroeconometric Analysis of the Unobservable Components of the Yield Curve: Evidence from U.S. and Canada

Abstract

This work extends the strand of literature that examines the relation between the term structure of interest rates and macroeconomic variables. We focus on three unobservable components to describe the evolution of the term structure over time. The level reflects the medium term inflation rate targeted by the monetary authority. The slope factor is related to the nominal stance of monetary policy; in particular, the slope tracks the annual change of the effective federal funds rate. The second unobservable component thus captures the adjusting preferences of the monetary authority over the business cycle. Finally, we address the challenging issue of attributing a macroeconomic interpretation to the curvature factor. We find evidence suggesting that curvature reflects the fluctuations of the business cycle. In particular, curvature is positively correlated with industrial production growth and inversely related to unemployment. Interestingly, this result holds in spite of whether curvature is extracted from the nominal or the real term structure. A negative shock to curvature seems either to anticipate or to accompany a slowdown in economic activity. The curvature effect thus complements the transition from an upward sloping yield curve to a flat one.
2.1 Introduction

Examining the relation between yields of different maturities is crucial for both macroeconomists and financial economists. From a macro perspective, the short rate is a policy instrument controlled by the monetary authority; from a financial perspective, short rate movements help predict long rates, since long term yields are the expected average of risk-adjusted future short yields.

Traders and financial analysts often attribute term structure movements to monetary policy actions; expectations on policy announcements exert a significant impact on the dynamics of the term structure. In addition, macroeconomists believe that the shape of the yield curve gives information about the future path of macroeconomic variables; moreover, the yield curve itself tends to respond to macro news.

Only recently macro variables have been included in TS models leaving space to explore the way further. In this chapter, in fact, we aim to establish a clear connection between factors underlying term structure’s movements and macroeconomic variables. In particular, we focus on the interpretation of curvature, or the butterfly factor, which has been mostly ignored before; secondly, we propose an innovative interpretation of the slope factor, which seems related to the evolution of the nominal stance of monetary policy rather than to the level of the policy rate.

We consider the bond market of the U.S. economy between January 1984 and June 2007. Our analysis thus covers both the final part of the Volcker’s mandate at the Federal Reserve System and the Greenspan era; in particular, we focus on a sample characterized by both price stability and relative homogeneity of the monetary regime (explicit interest rate targeting). We also analyse the Canadian term structure of interest rates between January 1986 and June 2006. Data evidence suggests that almost all TS movements can be summarized by few underlying factors. We thus estimate two alternative TS factor models in order to extract three latent components, namely level, slope and curvature. The terminology refers to the effect that a shock to these unobservable factors exerts on the shape of the yield curve (Litterman and Scheinkman, 1991).

An important strand of literature has recently focused on the macroeconomic interpretation of these factors. The existing empirical literature associates the TS level to some measures of inflation. Rudebusch and Wu (2004), as well as Bekaaert, Cho and Moreno (2005), suggest the level factor reflects the inflation rate targeted by the monetary authority, alias the medium-long run equilibrium inflation rate. Dewachter, Lyrio, and Maes (2006) emphasize the level is an indicator of the central tendency of inflation. There is also general consensus about the interpretation of the slope, which is believed to be a monetary policy factor. Rudebusch and Wu (2004) provide evidence that the slope tracks a fitted Taylor-type monetary policy rule; Bekaaert, Cho, and Moreno (2005) relate the slope to monetary policy shocks. A negative slope shock increases short rates by more than long rates reducing the yield spread, which is what generally occurs after a policy tightening. The economic
interpretation of curvature is more controversial. It has been argued it is either related to monetary policy shocks (Cho, Bekaert, and Moreno, 2005), or to the real stance of monetary policy (Dewachter, Lyrio, and Maes, 2006), or, eventually, to the expected future path of interest rates (Giese, 2008). Dewachter and Lyrio (2002) suggest that curvature represents a clear independent monetary policy factor; in particular, curvature reflects movements of the real interest rates that are orthogonal to any other macroeconomic variables. Finally, Hordahl, Tristani, and Vestin (2006) emphasize the effect of both inflation and output shocks on medium term maturities of the yield curve.

The empirical work contained in this chapter finds its inspiration in the following diagrams (Figure 2.1). Grey shaded areas highlight NBER recessions; while yellow shadings call attention to the economic slowdown following Volcker’s disinflation.

![Figure 2.1](image)

**Figure 2.1**

In the left panel of Figure 2.1, we plot the level factor together with both the CPI and the core-CPI inflation measures. As expected, the series display important co-movements. The negative trend of the level factor is not regularly reproduced by the price series though. The constant decline of the level might be due both to the augmented credibility of the U.S. monetary regime and to the consolidation of the U.S. monetary authority’s reputation. The central panel of Figure 2.1 reveals that the slope factor is closely associated to the annual change of the federal funds rate, which we consider an important indicator of how the nominal stance of monetary policy evolves over time. Finally, curvature is plotted in the right panel of Figure 2.1. We suggest curvature being related to the real economy. A visual inspection of the right diagram of Figure 2.1 is striking in this sense. There is a clear drop of curvature during periods of slowdown in real economic activity. In particular, curvature seems to be contemporaneously linked to the industrial production growth, while it follows with some delay the change in the labour market conditions.
The empirical analysis of curvature is developed along few parallel lines. Firstly, we compare two different models in order to discern the economic nature of the third unobservable component. In one model curvature is a function of some monetary variables; while, the second model links curvature to variables that reflect the conjunctural evolution of the real economy. Empirical evidence supports the view that curvature is more closely related to the cyclical fluctuations of real activity. In addition, we estimate a typical forward-looking aggregated demand (IS) curve for both curvature and the industrial production gap; results suggest that curvature approximates quite well the aforementioned gap.

Harvey (1988) and Chapman (1997) highlight a significant relationship between the real term structure of interest rates and consumption growth. We thus extract a measure of curvature from the real TS finding that it is significantly related to the business cycle. In particular, although curvature from the nominal TS seems unrelated to consumption, evidence suggests that the real counterpart of the third unobservable component is inversely correlated with consumption.

Finally, we present and estimate a joint macroeconometric model for curvature and real activity which confirms the aforementioned economic interpretation of curvature.

The rest of the chapter is organized as follows. Section 2.2 presents a brief review of the literature. In Section 2.3 we present the Nelson-Siegel latent factor model. The core of the empirical analysis is contained in Sections 2.4, 2.5, and 2.6, where we provide evidence to support the macroeconomic interpretation of curvature. In particular, in Section 2.5 we estimate a cyclical model for curvature; while in Section 2.6 we develop and estimate a joint macroeconometric model for curvature and real activity. Empirical evidence in Section 2.7 concerns the macroeconomic interpretation of both the level and the slope of the yield curve. Sections 2.8 presents evidence for Canada. Finally, Section 2.9 concludes. Data are presented in Appendix A2.I. Estimations of TS models are in Appendix A2.II.

2.2 Literature Review

Two main classes of TS models have been considered in the literature. The classical affine TS class is based on the no-arbitrage assumption; and the Nelson-Siegel class where yields are assumed to be a function of the latent factors through Laguerre functions of their maturities.

The success of the affine class might be due to the relatively simple tractability compared to the computational difficulties implied by the non-affine class (Black, 1976). In addition, arbitrage opportunities are quickly traded away in bond markets, since bonds are exchanged in well organized efficient markets. Affine models are appealing since they explain bonds’ price dynamics by means of few state variables, i.e., the latent factors. In particular, almost all movements of TS are due to the effect of a small number of components. Dai and Singleton (2000) show that 99% of the variations in the yield
curve can be attributed only to three factors. Rudebusch and Wu (2004) argue that level and slope account for over than 99% of the variation in the yield curve dynamics.

Ang and Piazzesi (2003) estimate a macro-finance model of TS and the economy investigating the influence of both inflation and a real factor on the yield curve in an arbitrage-free asset pricing framework. Bond yields are determined both by the three unobservable factors (level, slope, and curvature) and by two common factors that reflect respectively a measure of inflation and an indicator of real economic activity. Incorporating macro factors into TS models improves the ability to forecast yields’ movements both in- and out-of-sample. However, such effects are sometimes limited. Ang and Piazzesi argue that approximately 85% of bond yields variation is attributable to the impact of macro factors; macroeconomic variables explain movements at short and medium term maturities (up to one year); movements of long term bond yields, on the other hand, are due to the effect of financial factors.

A joint macro-finance TS model for the Euro area is worked out by Hordahl, Tristani, and Vestin (2006). The model includes three equations: an equation for the output gap, i.e. the AD-IS relation; an equation for inflation, which is an empirical specification of the new Keynesian Phillips curve; and, finally, a monetary policy rule for the short term rate. The macroeconometric model appears to fit German data quite well. Impulse response analysis shows that inflationary shocks affect yields mostly at medium maturities, increasing the curvature of the yield curve; while, monetary policy shocks affect the short end of the yield curve. Risk premia tend to respond to output gap real shocks.

Rudebusch and Wu (2004) develop an affine macro-finance model of TS and the economy to examine the joint evolution of the yield curve and macro variables. They find that the level is closely associated with the central bank’s long run inflation target; while the slope reflects the central bank’s reaction function, i.e. the short term interest rate set in response to the cyclical evolution of inflation and real activity.

Ang, Bekaert and Wei (2008) offer a decomposition of the nominal TS into the combined effect of real interest rates and expected inflation. Since neither real interest rates nor expected inflation are observable, they infer these factors from their impact on other variables in the economy. Short term real interest rates are volatile, whereas long term rates are smooth and persistent. Moreover, data evidence suggests that the positive slope typically present in the nominal TS is caused by an inflation risk premium which is increasing in maturity; the real term structure is thus flat since it is not affected by inflation. Real rates tend to follow pro-cyclical dynamics whereas nominal rates show counter-cyclical movements. Variations in expected inflation and in inflation risk premia explain about 80% of the variation in nominal interest rates and that these variables are also the main determinants of nominal interest rate spreads at long horizons.

Dewachter, et al. (2004) employ a continuous-time affine TS model to investigate the dynamic relation between yields and macroeconomic variables. The adopted framework also allow for time-varying risk
premia. The model includes two observable macroeconomic variables, i.e. inflation and the output gap, and three unobservable components. They propose the latent factors to represent the real interest rate, the central tendency of inflation, and the central tendency of the real interest rate respectively. They achieve conclusions in line with Ang and Piazzesi (2003). Macroeconomics variables are not capable of explaining movements at the long end of the yield curve. The variability of the long end of the yield curve is instead related to the central tendency of inflation. Medium term interest rates, from six months to two years, appear to be responsive to the real interest rate central tendency. They recognize that both observable and unobservable components influence risk premia and bond excess returns.

Diebold and Li (2003), Diebold, Piazzesi and Rudebusch (2005), Diebold, Rudebusch, and Aruoba (2006) have employed the Nelson-Siegel interpolant to examine bond pricing. Nelson and Siegel (1987) have proposed a parsimonious model for the yield curve, which is capable of reproducing a great variety of shapes that the yield curve can assume over time. The model fits U.S. data quite well. The interpolant, which is essentially a polynomial function, is flexible enough to fit a wide range of shapes displayed by the yield curve. The loadings of these exponential functions embody the long, short, and medium properties of the yield curve respectively; so that these parameters can be interpreted as level, slope, and curvature respectively (see Section 2.3). NS find that the model can account for almost 96% of the variations in yields during the period between 1981 and 1983. They also show that this model implies that, under some regularity conditions, the flattening of the yield curve at long maturities is approximately proportional to the reciprocal of the time to maturity.

The most recent strand of literature has mixed TS model with macroeconomic theory, including latent factor dynamics into New Keynesian macroeconomic general equilibrium framework. Bekaert, Cho, and Moreno (2005) have developed a model in which they combine the structural New Keynesian macroeconomics and theno-arbitrage term structure theory. This line of research has been followed by Wachter (2006) and Garcia and Luger (2005) who have considered a consumption-based equilibrium macro-finance model.

### 2.3 The Nelson-Siegel Factor Model

Nelson and Siegel (1987, 1988) propose a factor model for TS which has become increasingly popular to the extent that also central banks have adopted it. Diebold and Li (2006) employ the NS approach without imposing the arbitrage-free condition (Hull and White, 1990; Ang and Piazzesi, 2003) and excluding the equilibrium approach (Dai and Singleton, 2000; Piazzesi and Schneider, 2006). The yield on a bond with maturity $n$ ($y^{(n)}$) is set to be a polynomial function of the maturity:
\[ y^{(s)} = L_t + S_t \left( \frac{1 - \exp \left\{ -\frac{\lambda}{n} t \right\}}{\frac{\lambda}{n}} \right) + C_t \left( \frac{1 - \exp \left\{ -\frac{\lambda}{n} t \right\}}{\frac{\lambda}{n}} - \exp \left\{ -\frac{\lambda}{n} t \right\} \right) \] (2.1)

Parameter \( \lambda \) governs the exponential decay\(^{23} \); as Diebold, Rudebusch, and Aruoba (2006), we set \( \lambda \) equal to 0.077. It follows a plot of the loadings:

![Figure 2.2](image)

L: loading of the level factor.  
S: loading of the slope factor.  
C: loading of the curvature factor.

The first loading, i.e. the unity coefficient multiplying \( L_t \), is a constant and it is interpreted as the level factor; the loading of \( S_t \) is interpreted as the slope; it is an exponential function that starts at one and decays monotonically toward zero. The loading of \( C_t \) starts at zero, increases with maturity \( n \) and then declines, it finally approaches to zero. The shape of the loadings resembles the effect exerted on TS by a shock that hits the specific latent factors (Litterman and Scheinkman, 1991).

The latent factors follow a first-order VAR process. Differently from standard assumptions in canonical affine TS model (Dai and Singleton, 2000), we do not restrict the transition matrix (\( \Phi_{SS} \)) to be lower triangular. So that the actual value of each factor depends on the first lag of all the other factors. The transition matrix is (3 x 3) and contains nine parameters, while the mean state vector is (3 x 1). The latent components are stacked in the state vector: \( F_t = [L_t, S_t, C_t] \); the transition, or state, equation of the state-space representation is:

\[ F_t = \mu_F + \Phi_{SS} \cdot F_{t-1} + \omega_{t,F} \] (2.2)

The disturbance \( \omega_{t,F} \) is i.i.d. Normal with zero mean and diagonal covariance matrix (\( \Omega \)). The initial state vector \( F_0 \) is orthogonal to the disturbances \( \omega_{t,F} \) of the transition equation. The observation equation of the state-space system is:

\[ y_t = \Phi F_t + \nu_t \] (2.3)

\(^{23}\) Nelson and Siegel suggest fixing it equal to 0.06, since this is the value that maximizes the third loading, i.e. the exponential function which multiplies the curvature factor. We have estimated the model also with this value of the parameter; results are similar.
The disturbance term is \( i.i.d. \) Normally distributed with zero mean and variance \( (\sigma^2 \cdot I) \). We assume a white noise transition disturbances \( \nu_t \), orthogonal to the initial state vector \( F_0 \). The measurement equation links yields to the unobservable components:

\[
\begin{bmatrix}
y^{(1)}_t \\
y^{(3)}_t \\
\vdots \\
y^{(120)}_t
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{1 - \exp(-\lambda)}{\lambda} & \frac{1 - \exp(-\lambda)}{\lambda} - \exp(-\lambda) \\
1 & \frac{1 - \exp(-3\lambda)}{3\lambda} & \frac{1 - \exp(-3\lambda)}{3\lambda} - \exp(-3\lambda) \\
\vdots & \vdots & \vdots \\
1 & \frac{1 - \exp(-120\lambda)}{120\lambda} & \frac{1 - \exp(-120\lambda)}{120\lambda} - \exp(-120\lambda)
\end{bmatrix}
\begin{bmatrix}
L_t \\
S_t \\
C_t
\end{bmatrix}
+ \begin{bmatrix}
\nu_{t,1} \\
\nu_{t,3} \\
\vdots \\
\nu_{t,120}
\end{bmatrix}
\] (2.4)

We consider nine yields with maturities 1-, 3-, 6-, 12-, 24-, 36-, 48-, 60-, and 120-month; it seems enough to achieve a dense representation of the maturity spectrum domain. Nominal TS estimations\(^{24}\) suggest high persistency of all latent factors and weak cross-factor dynamics. The first autoregressive coefficients of the latent factors are 0.98, 0.92, and 0.90 for \( L_t \), \( S_t \), and \( C_t \) respectively. Estimations for the real TS are different, since the most persistent factor becomes curvature. The first autoregressive coefficients of the latent factors extracted from the real TS are 0.94, 0.91, and 0.95 for \( L_t \), \( S_t \), and \( C_t \) respectively.

### 2.4 Curvature and Business Cycle Fluctuations

A better understanding of TS dynamics can be achieved only by exploring the macroeconomic underpinning of the yield curve. We start by giving a macroeconomic interpretation to the curvature factor which is so far an unresolved issue. The first part of this Section gives an insight about the inspiration of the work that follows. Initially we focus both on correlations between curvature and real variables and on the visual inspection of the dynamics of the variables of interest. The intuition is more formally developed in the second part of this Section, as well as in the following Sections of this Chapter, where we perform a rigorous empirical analysis to support the intuition.

A curvature shock influences medium term yields giving a hump-shaped form to the yield curve\(^{25}\). Hence, a negative shock generates an inverted hump-shaped yield curve, which is sometimes observed in the data. As shown in diagram 2.3, curvature seems related to the business cycle.

\(^{24}\) Estimations reported in Appendix A2.II.

\(^{25}\) One-factor TS models cannot generate the inverted hump-shape. Two-factor TS models allow for greater flexibility and can account for an inverted hump of the yield curve; however, the effect on curvature depends on the combined effect of the first two factors. Three-factor TS models provide a suitable framework to analyse the economic function of curvature.
recessions (grey shaded areas) are accompanied by a sharp curvature drop. Curvature dynamics is sensitive to the rise of the unemployment and to the reduction of industrial production. The yield curve seems hit by a significant curvature shock each quarter preceding NBER recessions. However, we wish to point out that this is not in contrast with previous evidence indicating the slope as a good predictor of future economic activity (Stock and Watson, 1989; Estrella and Hardouvelis, 1991).

Our focus on curvature provides a more general analysis of TS since also intermediate, rather than only extreme, maturities, are examined. Any shock affecting the short end of the yield curve, typically a monetary policy shock, generates only a moderate and delayed reaction of long yields, which are smooth and persistent; so that, any shock affecting the short end of TS can be considered a shock to the slope. Our interest in medium maturities arises from the idea that yields at the medium end represent an important link between the extremely dynamic short end and the smooth TS long end. In particular, in this chapter we argue that the propagation of shocks from the short to the long end reflects the evolution of economic conditions over the business cycle.

The empirical macro-finance literature has proposed different measures of curvature (see legend of Table 2.1). Correlations are positive, but not so high as expected; although not reported, the plot of different curvature series share important co-movements. In the following part of this Section we focus on curvature obtained by Kalman filtering the NS factor model of TS, since it fits quite well all the theoretical measures proposed in the literature.

A preliminary examination of the correlations between curvature and some economic indicators suggests a considerable connection between the financial factor and the real economy, as shown in

\[ \text{Table 2.1} \]

Different measures of curvature. PC: third principal components. A-LT (kf): curvature extracted by Kalman filtering a standard Affine TS model with lower triangular transition matrix. NS (kf): curvature from Kalman filtering the NS model (Section 2.3). AP: theoretical measure of curvature (Ang and Piazzesi, 2003) computed as \( y(\text{1m}) + y(\text{60m}) - 2y(\text{12m}) \), where \( m \) indicates the maturity in months. BCM: Bekker, Cho, and Moreno (2005) propose \( y(\text{3m}) + y(\text{60m}) - 2y(\text{12m}) \). NS-DL: Nelson and Siegel (1987), Diebold and Li (2006), and Diebold, Rudebusch, and Aruoba (2006), compute curvature as \( 2(y(\text{24m}) - y(\text{120m}) - y(\text{3m})) \).

49
Table 2.2. Curvature seems related both to the unemployment rate and its variation over time. Moreover, curvature is correlated to the rate of growth of industrial production computed over different horizons, from a quarter ($m = 3$) to three years ($m = 36$). Finally, curvature is also linked to the real personal consumption expenditure and tracks different measures of the output gap.

<table>
<thead>
<tr>
<th>Correlations: Curvature and Real Variables</th>
<th>NS (kf)</th>
<th>NS (kf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>un grw</td>
<td>-0.511</td>
<td>0.321</td>
</tr>
<tr>
<td>un grw (-12)</td>
<td>-0.639</td>
<td>0.373</td>
</tr>
<tr>
<td>un</td>
<td>-0.507</td>
<td>0.470</td>
</tr>
<tr>
<td>un (-3)</td>
<td>-0.432</td>
<td>0.607</td>
</tr>
<tr>
<td>un (-6)</td>
<td>-0.345</td>
<td>0.679</td>
</tr>
<tr>
<td>r-cons grw</td>
<td>0.335</td>
<td>0.666</td>
</tr>
<tr>
<td>r-cons grw (-12)</td>
<td>0.458</td>
<td>0.440</td>
</tr>
</tbody>
</table>

U.S. data, NS (kf): curvature factor extracted by Kalman filtering the NS model (Section 2.3). un grw: annual growth of the unemployment rate. un: unemployment rate (civilian, seasonally adjusted, FRED). r-cons grw: annual rate of growth of the seasonally adjusted real personal consumption expenditures (FRED). IP grw: annual rate of growth of the seasonally adjusted industrial production index (FRED). IP gap BK: industrial production gap obtained with the Baxter King band pass filter. IP gap HP: industrial production gap obtained with the Hodrick-Prescott filter. Negative numbers in parenthesis indicate lags (months); while positive numbers in parenthesis indicate leads (months).

Figure 2.4 plots curvature together with some cyclical economic indicators: the unemployment rate of growth (left panel); the industrial production gaps (right panel).

Figure 2.5 plots curvature and with real consumption (left panel) and the 12-month lag series of real personal consumption expenditures (right panel). Curvature co-moves quite closely with real variables. In particular, as mentioned above, a sharp reduction of the curvature factor occurs immediately before economic slowdown.

Evidence seems to support the conjecture that curvature is informative beyond the slope about business cycle fluctuations. We speculate that negative shocks to curvature seem either to anticipate or
to accompany a decline in economic activity. Moreover, available empirical evidence is consistent with the idea that the curvature effect complements the transition from an upward sloping yield curve to a flat one.

It has been argued that the curvature factor is either related to monetary policy shocks (Cho, Bekaert, and Moreno, 2005), or to the real stance of monetary policy (Dewachter, Lyrio, and Maes, 2006), or again to the expected future path of interest rates (Giese, 2008). In the following analysis we show that curvature is more closely related to the condition of the real economy rather than to monetary variables. The right panel of Figure 2.6 plots curvature and the annual rate of change of the federal funds rate; the series show a close fit. The right panel plots curvature and CPI inflation; in this case the relation seems to be weaker.

![Figure 2.6](image)

To assess whether curvature is closely related to real variables rather than to monetary variables we estimate two different equations. The monetary model (M) is:

\[
cur_t = \delta_0 + \delta_1 \Delta ffr_{t-12} + \delta_2 \Delta M1_{t-12} + \delta_3 \pi_t + \epsilon_{t,M}
\]  

(2.5)

Where \( \Delta ffr_{t-12} \) is the annual change in the federal funds rate; \( \Delta M1_{t-12} \) is the annual rate of growth of the money stock M1; \( \pi_t \) represents CPI inflation, i.e. the annual change of the seasonally adjusted consumer price index. The real-variable model (R) relates curvature to some business cycle indicators:

\[
cur_t = \rho_0 + \rho_1 \Delta IP_{t-12} + \rho_2 \Delta rc_{t-12} + \rho_3 \Delta un_{t-12} + \rho_4 gap_t + \epsilon_{t,R}
\]  

(2.6)

\( \Delta IP_{t-12} \) is the annual rate of growth of the seasonally adjusted industrial production; \( \Delta rc_{t-12} \) represents the annual change in the real personal consumption expenditures; \( \Delta un_{t-12} \) is the annual variation in unemployment; and, \( gap_t \) is either the Hodrick-Prescott or the Baxter-King de-trended series.

\[\text{After estimating a model for curvature using the annual change in the fed funds rate and several lags (including the intercept and 6 lags of the fed funds change) we found a poor goodness of fit, anyway lower than that obtained through the real-variable model (R). Forecasting curvature with such a model returns worse “predictive accuracy” indicators than those obtained after forecasting curvature with model (R). Predictive accuracy tests employed in this check are as follows: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Theil Inequality Coefficient (Theil IC). RMSE is the square root of the sum of squared differences between predicted and actual series. MAE is the sum of the absolute value of the differences between the predicted and the actual series. High values of both the RMSE and the MAE indicate poor fit. The Theil IC is the ratio between the RMSE and the sum of the square root of (sum of) the squared predicted values and the square root of (sum of) the actual values. The Theil coefficient is thus normalized between 0 and 1, the lower the coefficient the more accurate the prediction.}\]
of (log) IP. In order to show that our results are robust, the above equations have been estimated both by ordinary least squares and by instrumental variables methods. In addition, OLS estimations have been performed allowing different structures of the variance-covariance matrix.

---

### Table 2.3

<table>
<thead>
<tr>
<th>Model (R) – Equation (2.6)</th>
<th>Model (M) – Equation (2.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 )</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>( \delta_0 )</td>
</tr>
<tr>
<td>OLS</td>
<td>-1.6088</td>
</tr>
<tr>
<td>WH</td>
<td>(0.236)</td>
</tr>
<tr>
<td>[6.7]</td>
<td>[2.1]</td>
</tr>
<tr>
<td>HH (12)</td>
<td>(0.345)</td>
</tr>
<tr>
<td>[4.6]</td>
<td>[0.9]</td>
</tr>
<tr>
<td>NW (18)</td>
<td>(0.352)</td>
</tr>
<tr>
<td>[4.6]</td>
<td>[0.9]</td>
</tr>
<tr>
<td>s-HH</td>
<td>(0.827)</td>
</tr>
<tr>
<td>[1.9]</td>
<td>[0.6]</td>
</tr>
<tr>
<td>( R^2_{adj} )</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Standard error in parenthesis; t-statistics in square brackets.

---

Among real variables only real consumption turns out to be unrelated to curvature; however, consumption growth is significantly related to curvature of real TS, as shown later. Estimated coefficients suggest that increasing unemployment tends to lower medium term yields, generating the inverted hump-shape form of the yield curve. On the contrary, a positive growth of the IP index tends to increase yields at medium maturities. Results are consistent with evidence that a reduction in the IP growth, occurring when economic conditions worsen, is reflected by an inverted hump-shaped yield curve. Hence, this is in line with the idea that the shape of the yield curve changes over time.
the business cycle. Again we remark that the curvature effect is not incompatible with the fact that also the TS slope varies across the business cycle.

A flat yield curve is usually interpreted as a sign of imminent recession as long as high short rates, relative to long rates, should reflect severe monetary policy (Bernanke and Blinder, 1992). While an upward sloping TS reflects accommodative policies, and thus incorporates expectations of a thriving economy. Hence, suppose that the economy is growing fast; strong aggregate demand is likely to generate inflationary pressures. Suppose further that the monetary authority raises interest rates to preserve inflation stability; two effects follow. On the one hand, the yield spread shrinks, since short yields are likely to increase more than long yields; on the other hand, aggregate demand weakens following the reduction of private investments. This process needs time to take place, it seems thus reasonable to expect that the policy tightening affects medium term rates more intensely than long rates. The propagation along the entire spectrum of maturities generates a temporary spike in the medium end of the yield curve. Therefore, both the dynamics of the yield curve and the evolution of macroeconomic conditions occur at the same time. Expectations may either accelerate or anticipate the process. The contrary happens before a recession. Expectations of accommodative policy exert a negative pressure on TS medium maturities reducing curvature. In this chapter we do not intend to establish any causality relation between curvature and real economy, we simply suggest that curvature reflects the cyclical behaviour of the economy. In short, the curvature effect seems to accompany the transition of the yield curve from a positively sloped one, prevailing during booms, to a flat one, which is believed to anticipate recessions.

Estimations of the monetary equation (2.5) suggest curvature being significantly related to the annual change of both the fed funds and M1. Hence, evidence does not entirely reject the hypothesis that curvature is related to monetary policy shocks, as put forward by Bekaert, Cho, and Moreno (2005); however, we provide evidence that the link between curvature and the real economy is stronger than the link between curvature and monetary variables. In line with Rudebusch and Wu (2004, 2005), we find that the slope is more closely related to a Taylor-type policy equation.

---

30 The monetary equation (2.5) for the slope returns significant coefficients robust to different standard errors (White, HH, NW, s-HH). The goodness of fit is 55%. Monetary variables predict the slope more accurately than curvature.

31 We jointly estimated two Taylor-type reaction functions by maximum likelihood. One based on the fed funds rate; the other based on the slope factor which, following Rudebusch and Wu (2004), is considered as a proxy for the policy rate. Both dependent variables are assumed to respond to inflation and the HP IP gap. We have also included the first lag of the dependent variable among regressors allowing for monetary policy inertia. The Wald test cannot reject the null hypothesis of equal respective coefficients in the two equations. This evidence suggests that the response of the slope to macro variables is similar to that of the effective fed funds rate. Similar results are obtained after GMM estimating the forward-looking Taylor policy functions where the dependent variables are assumed to react to expected rather than to actual inflation.
Our results are robust to different computation of the IP gap. Table 2.4 shows, in fact, estimations of equation (2.6) where the IP gap is obtained by applying the Baxter-King frequency filter.

We forecast curvature using both models (R) and (M). In Figure 2.7 forecasts are plotted together with (twice) the standard errors bands. Both models return accurate forecasts since the actual value of curvature never breaks the forecast standard errors. However, forecast errors from the real model (left panel) tend to oscillate closer to the zero line indicating better fit.

<table>
<thead>
<tr>
<th>Model (R) – Equation (2.6)</th>
<th>( \hat{\rho}_0 )</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
<th>( \hat{\rho}_3 )</th>
<th>( \hat{\rho}_4 )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>-2.2535</td>
<td>0.2018</td>
<td>-0.1732</td>
<td>-0.6823</td>
<td>0.4018</td>
<td>98.44</td>
</tr>
<tr>
<td>WH</td>
<td>(0.254)</td>
<td>(0.032)</td>
<td>(0.079)</td>
<td>(0.070)</td>
<td>(0.061)</td>
<td>96.45</td>
</tr>
<tr>
<td>NW (S)</td>
<td>(0.396)</td>
<td>(0.059)</td>
<td>(0.076)</td>
<td>(0.072)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-5.7]</td>
<td>[3.4]</td>
<td>[-2.2]</td>
<td>[-9.7]</td>
<td>[6.5]</td>
<td></td>
</tr>
<tr>
<td>( R_{adj}^2 )</td>
<td>0.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model (M) – Equation (2.6)</th>
<th>( \hat{\rho}_0 )</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
<th>( \hat{\rho}_3 )</th>
<th>( \hat{\rho}_4 )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>-3.311</td>
<td>0.1859</td>
<td>-0.1476</td>
<td>-0.7774</td>
<td>0.3547</td>
<td>74.43</td>
</tr>
<tr>
<td></td>
<td>(0.525)</td>
<td>(0.065)</td>
<td>(0.088)</td>
<td>(0.009)</td>
<td>(0.120)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-4.4]</td>
<td>[2.8]</td>
<td>[-0.8]</td>
<td>[-5.6]</td>
<td>[2.9]</td>
<td></td>
</tr>
<tr>
<td>( R_{adj}^2 )</td>
<td>0.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4

A battery of tests are performed to assess the predictive accuracy of both the monetary and the real-variable equations. Statistical results, reported in Table 2.5, suggest that real rather than monetary variables generate more accurate predictions of curvature. RMSE and MAE measure how closely the forecasted series track actual data; low values of these statistics indicate a good fit. The Theil
inequality coefficient can assume a range of values between \textit{zero} and \textit{one}; the lower the coefficient greater the predictive accuracy.

<table>
<thead>
<tr>
<th>Predictive Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>model</td>
</tr>
<tr>
<td>M</td>
</tr>
<tr>
<td>R</td>
</tr>
</tbody>
</table>


Results are in line with evidence presented so far. The Diebold-Mariano test rejects the null that both models have equal predictive accuracy. The Morgan-Granger-Newbold test confirms the significant difference in the forecasting accuracy of model (R) over model (M). Both the Sign and the Wilcoxon’s Signed-Rank tests reject the null hypothesis of \textit{zero}-median loss differential.

As a further robustness check we examine whether curvature from the real TS\textsuperscript{32} is related to the economy. Removing the effect of inflation from TS should not affect the curvature factor, since inflation is mainly reflected in long term yields. Ang, Bekaert, and Wei (2008) point out that the real TS does not show any clear upward trend, so that inflation risk premia are incorporated in long yields. If we believe this, we also think the medium part of the yield curve should be unaffected after ruling out the effect of inflation. Therefore, curvature extracted from the real TS should track curvature obtained from the nominal TS. The correlation between real TS and nominal TS curvature is above 0.95. However, the series of real TS curvature is smoothed compared to its nominal counterpart.

After ruling out inflation, we re-estimate both the monetary and the real equations for the real TS curvature. Curvature is still significantly explained by real variables (Table 2.6). Curvature from the real TS is significantly related to consumption growth, whereas curvature from the nominal TS is not. This result is consistent with previous findings that the real TS of interest rate is informative about the future path of consumption (Harvey, 1988; Chapman, 1997).

\textsuperscript{32} Real rates can be computed as the difference between nominal yields and expected inflation (the Fisher effect) since, for small values, the ratio can be approximated by the log-difference. Assuming rationality and perfect information expected inflation matches its realization one period ahead. As expected, and consistently with common sense, the level factor obtained from the nominal TS dominates in magnitude the level extracted from the real TS which is almost flat.
We thus use both (M) and (R) models to predict real TS curvature. Left panel of Figure 2.8 shows the forecast of curvature obtained with the real-variable model (R), while the right diagram plots the forecast with the monetary model (M)\textsuperscript{33}. The forecast series are plotted with twice the standard errors bands. For model (R), the actual value of real TS curvature stays within the standard errors bands of the forecasted series; whereas, the actual series of the curvature breaks the standard errors lines of the forecast generated by the monetary (M) model. Forecast errors associated to the monetary model appear to be serially correlated. In particular, during the weakening of U.S. economy in early 1980s it seems that the monetary model (M) systematically under-predicts the curvature of the real term structure. Early 1980s were characterized by a strong disinflation process pursued by Paul Volcker, the chairman of the Federal Reserve System. The severe monetary policy conduct carried out in that era is summarized by an effective epithet: “leaning against the wind”. Data thus suggest that removing the effect of inflation in an era of prominent disinflation makes monetary variables less informative about the term structure evolution, and, in particular, about financial factors underlying the term structure.

\textsuperscript{33} The scale for the forecast error series is different in the two diagrams.
As a further robustness check, we perform some predictive accuracy tests. Statistics in Table 2.7 confirm that real variables have a significantly larger predictive power.

<table>
<thead>
<tr>
<th>Predictive Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>M</td>
</tr>
<tr>
<td>R</td>
</tr>
</tbody>
</table>


As a final inspection, we wish to verify whether the curvature factor is related to the aggregate demand (AD) curve, which is usually assumed to describe the state of the economy.

\[
\begin{align*}
\text{gap}_t &= \psi_{0, g} \quad + \psi_{1, g} E_t \left( \text{gap}_{t+1} \right) + \left( 1 - \psi_{1, g} \right) \left[ \psi_{2, g} \quad \text{gap}_{t-1} \right] + \psi_{3, g} \left[ ffr_t - E_t \left( \pi_{t+1} \right) \right] + \epsilon_{t, g} \\
\text{cur}_t &= \psi_{0, c} \quad + \psi_{1, c} E_t \left( \text{cur}_{t+1} \right) + \left( 1 - \psi_{1, c} \right) \left[ \psi_{2, c} \quad \text{cur}_{t-1} \right] + \psi_{3, c} \left[ ffr_t - E_t \left( \pi_{t+1} \right) \right] + \epsilon_{t, c}
\end{align*}
\]

Figure 2.8

(2.7)

A traditional AD (IS) curve implies that the output gap\(^{34}\) is a function of its forward-looking component, its lagged realizations, and the expected real interest rate. We jointly estimate equations (2.7); we thus compare the actual AD curve with its latent component counterpart. \(ffr_t\) is the effective fed funds rate; \(\pi_t\) is CPI inflation; \(E_t\) denotes the expectations operator. The forward-looking real component in the AD equation captures both consumption smoothing behaviour, which is an empirical regularity, and the expectations reflecting the sentiment about the future state of the economy. The system is GMM estimated thus matching a twofold objective. We need instruments

\(^{34}\) As shown in the Data Appendix, both the Hodrick-Prescott and the Baxter-King filtered series of (log) seasonally adjusted IP return accurate measures of the cycle. The correlations of these cyclical indicators with the rate of growth of IP are very high; in addition, both the HP and the BK detrended series perfectly match the NBER recessions. Here we use the HP filtered series of log IP.
because expected future (unobserved) variables appear in both equations; instruments are also required to back generated regressors in the second equation. In both cases variables may be eventually measured with errors, so that the GMM allows obtaining robust estimates\(^{35}\). In addition, GMM estimation handles with heteroscedasticity and serial correlation of unknown forms.

<table>
<thead>
<tr>
<th>IP gap</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_{0,g})</td>
<td>(\psi_{0,c})</td>
</tr>
<tr>
<td>(\psi_{1,g})</td>
<td>(\psi_{1,c})</td>
</tr>
<tr>
<td>(\psi_{2,g})</td>
<td>(\psi_{2,c})</td>
</tr>
<tr>
<td>(\psi_{3,g})</td>
<td>(\psi_{3,c})</td>
</tr>
</tbody>
</table>

GMM

\[ -0.0494 \quad 0.8931 \quad 0.8292 \quad 0.0246 \]

\[ -0.0608 \quad 0.7366 \quad 0.9711 \quad 0.0197 \]

\[ 0.035 \quad 0.089 \quad 0.339 \quad 0.013 \]

\[ [-1.4] \quad [10] \quad [2.4] \quad [1.9] \]

\[ 0.183 \quad 0.292 \quad 0.209 \quad 0.042 \]

\[ [-0.3] \quad [2.5] \quad [4.6] \quad [0.5] \]

\[ R^2_{\text{adj}} \quad 0.88 \]

\[ R^2_{\text{adj}} \quad 0.95 \]

Standard errors in parenthesis; \(t\)-statistics in square brackets.

Table 2.8

If estimated coefficients of the first equation are similar to the respective coefficients of the second equation, we may presume that curvature can proxy the IP gap, i.e. curvature reflects the cyclical fluctuations of real activity. Estimations are reported in the Table 2.8. The magnitude of coefficients is comparable. The higher goodness of fit of the curvature equation might be due to the higher persistence of the financial factor.

Residuals obtained from both equations are serially uncorrelated and sufficiently homoscedastic. Figure 2.10 plots the autocorrelation functions of residuals (36 lags included). Both correlograms suggest absence of serial correlation.

---

\(^{35}\) GMM estimation does not require distributional assumptions, and, therefore, provides a useful alternative to other estimation methods, which, nevertheless, are nested by the generalized method of moments. GMM is a large sample estimator; each equation is estimated on a sample of 270 observations. Instruments: the annual rate of growth of industrial production and its first lag, both of which are highly correlated with both the industrial production gap and the curvature factor; the realized real interest rate, computed as the difference between the first lag of the federal funds rate and actual inflation.
The Wald test permits to check whether the estimated parameters in the IP gap equation are equal to the respective counterparts in the curvature equation. We perform the test to check for both individual and joint coefficient equality. In both cases restrictions cannot be rejected thus reinforcing our finding that similarities between the equations are statistically significant.

Finally, we substitute the dependent variable of the curvature equation with the IP gap; we thus impose the IP gap to be a function of curvature as shown by this equation:

\[
gap_t = \psi_{0,c} + \psi_{1,c} E_t(\text{cur}_t^{c+1}) + (1 - \psi_{1,c}) \cdot \psi_{2,c} \text{cur}_{t-1} + \psi_{3,c} \left[ \delta r_t - E_t(\pi_{t+1}) \right] + \epsilon_{t,gc} \quad (2.8)
\]

The forecast obtained from the above equation tracks quite well the actual series of the industrial production gap. The plot of both series are displayed in the Figure 2.11. Despite the forecast is volatile and erratic, it seems to capture quite well the core dynamics of the IP gap.

### 2.5 A Cyclical Model for Curvature

In this Section we provide some more evidence relating the curvature factor to the cyclical component extracted from a structural time series model for industrial production; while, in the next Section, we develop and simulate joint macro-econometric model of curvature and industrial production. In both case we find evidence supporting our main conjecture, that is curvature reflects business cycle fluctuations.
The basic structural time series model for industrial production has been introduced by Harvey (1989). The industrial production is assumed to have a stochastic trend and a cyclical component; the former represents the long-term movement in the time series, while the latter determines the entity of economic fluctuation, i.e. the dynamics of the cycle36. Two random walk processes underlie the stochastic trend ($\mu_t$):

$$\mu_t = \mu_{t-1} + \beta_{\infty} + \eta_t$$  \hspace{1cm} (2.9)

$$\beta_t = \beta_{t-1} + \xi_t$$  \hspace{1cm} (2.10)

$\eta_t$ and $\xi_t$ are white noise mutually uncorrelated disturbances with zero means and standard deviations $\sigma_{\eta}$ and $\sigma_{\xi}$ respectively. Both the upward and downward movements of the trend are driven by the $\eta_t$ component; while the steepness of the trend depends on $\xi_t$. Whenever the variance of the disturbances collapses to zero the stochastic trend turns into a deterministic one; on the other hand, the larger the variances the greater the stochastic movements of the trend.

The cycle ($\psi$) is technically constructed by means of both the sine and the cosine wave functions. The length of a cycle is called the period, which represents the time taken to go through its complete range of values ($2\pi f_H$); while the frequency ($f_H$) measures how often the cycle is repeated in the unit of time37. The cycle is then characterized by few other parameters, the amplitude ($A$) and the phase shift ($\theta$). The cyclical component is thus expressed as follows:

$$\psi_t = A \cos(\lambda t - \theta)$$  \hspace{1cm} (2.11)

A complete formulation for representing the cycle combines both the sine and the cosine waves:

$$\psi_t = a \cos(\lambda t) + b \sin(\lambda t)$$  \hspace{1cm} (2.12)

The time series of industrial production cycle thus can be seen as the summation of the above cyclical component plus a white noise error term with zero mean. A stochastic pattern for the cycle requires parameters $a$ and $b$ to evolve over time; so that, in order to preserve time series continuity we make use of the following recursion:

$$\begin{bmatrix} \psi_t \\ \psi_{t-1} \end{bmatrix} = \begin{bmatrix} \cos \lambda & \sin \lambda \\ -\sin \lambda & \cos \lambda \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_{t-1} \end{bmatrix}$$  \hspace{1cm} (2.13)

with initial states $\psi_0 = a$ and $\psi_{-1} = b$; and, where $\epsilon_t$ and $\epsilon_{t-1}$ are white noise disturbances. The model is identified if either we assume that two disturbances have the same variance or if they are

36 The cyclical component extracted by Kalman filtering the structural trend-cycle model is a reliable indicator of the economic conjuncture. It is highly correlated with the growth rate of industrial production and with both the Hodrick-Prescott and Baxter-King filtered series of industrial production. The cyclical component is also negatively correlated with unemployment and the unemployment annual change. The cyclical pattern of all these variables appear to be significantly related.

37 Both the sine and the cosine functions have period $2\pi$ and frequency $1/2\pi$. The function $\cos(\omega t)$ has period $2\pi/\omega$ with frequency $\lambda = \omega/2\pi$. 

60
uncorrelated. Finally, we introduce a dumping factor ($\rho$) affecting the amplitude of the cycle to allow for more flexibility; the following system summarizes the entire structural model for IP:

\[
\begin{bmatrix}
\mu_t \\
\beta_t \\
\psi_t
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \rho \cos \lambda & \rho \sin \lambda \\
0 & 0 & -\rho \sin \lambda & \rho \cos \lambda
\end{bmatrix} \begin{bmatrix}
\mu_{t-1} \\
\beta_{t-1} \\
\psi_{t-1}
\end{bmatrix} + \begin{bmatrix}
\nu_t \\
\epsilon_t \\
\zeta_t
\end{bmatrix}
\]

Equation (2.14) is the state or transition equation of the state-space representation. The transition matrix on RHS describes the evolution of the unobservable components, so that it captures the stochastic behaviour of both the trend and the cyclical components.

We have not considered the seasonal component since we deal with the seasonally adjusted series of industrial production. The IP series is thus decomposed into a trend and a cycle (plus a residual component). The model is estimated for both the level of the seasonally adjusted IP series and for its log transformation. The estimation of the amplitude is 0.9357 (p-value: 0); it is a stable solution ($|\rho| < 1$) that denotes a cycle with decreasing amplitude (i.e. convergence).

Figure 2.12 plots the cyclical component together with different indicators of the business cycle; they appear significantly related. There seems to be a positive and significant relationship with the output gap computed with both the Hodrick-Prescott and the Baxter-King frequency filters; while, there exists an evident inverse relation with the annual change in the unemployment rate.

![Figure 2.12](image)

In the core analysis that follows we consider curvature obtained with the Nelson-Siegel method. The forecast of the cyclical component is obtained by Kalman filtering the structural model for IP. The following diagram shows that the cyclical component of IP (continuous blue line) is highly correlated with the curvature factor one-year ahead (semi-dotted black line). The correlation coefficient

---

38 The state space system is composed by a measurement equation and a transition equation. The observation, or measurement, equation relates the actual series of IP to its unobservable components (trend and cycle). The state space system is estimated by means of Kalman filtering, an iterative procedure based on the maximum likelihood estimation method.

39 The estimation of the of the variance of the disturbances are the following. Measurement equation: (0.1588)$^2$ (p-value: 0); trend component: (0.0564)$^2$ (p-value: 0.0040); and cyclical component: (0.3358)$^2$ (p-value: 0). The estimate of the amplitude of the cycle extracted from the log series of the IP is 0.9461; the variance of disturbances are the following. Measurement equation: (0.0016)$^2$ (p-value: 0); trend component: (0.006)$^2$ (p-value: 0.0030); and cyclical component (0.004)$^2$ (p-value: 0). In both cases convergence is achieved after quite a few iterations.
between these two series is about 0.71. Moreover, it is interesting to notice that curvature slightly breaks the forecast standard error (red) bands only during periods of economic downturn.

Strikingly the curvature factor lies within the standard error bands in the entire sample, suggesting how significant the relation between the economic cycle and curvature is.

The right panel of Figure 2.14 plots both the actual series of the seasonally adjusted IP and its predicted value obtained through Kalman filtering the cyclical model. The predicted series seems to track quite well the actual one. The forecast error series is serially uncorrelated and does not show any evident heteroscedastic pattern. The forecast error series is stationary, as suggested by both the autocorrelation function and the unit root tests. Stationarity of the forecast error series is supported by both the ADF (augmented Dickey-Fuller) and the PP (Phillips-Perron) tests that reject the null hypothesis of unit root. In addition the KPSS (Kwiatkoswki-Phillips-Schmidt-Shin) test confirms stationarity. The correlogram of the forecast error series is in the right panel:

In what follows we repeat the same analysis for the log series of industrial production. We expect to achieve similar results in the sense that (appropriately scaled) the curvature factor should resemble the cyclical pattern shown by the cyclical component of the IP log series. The following diagram shows the forecast of the cyclical component obtained through Kalman filtering the log IP series (continuous
blue line) with its standard errors, and the (rescaled) curvature factor obtained by applying the Nelson-Siegel procedure. The plots confirm a close relation between the aforementioned series.\(^{40}\)

![Figure 2.15](image)

The Kalman filter prediction of log IP co-moves very closely with the actual series (Figure 2.16). The zero-mean prediction errors are serially uncorrelated and apparently homoscedastic. The correlogram of the forecast error is reported in the right panel.

![Figure 2.16](image)

As argued in previous Sections, in order to show that also the curvature from the real TS is informative about the business cycle we repeat the experiment above with the series of real TS curvature. The correlation coefficient between the cyclical component extracted by Kalman filtering the seasonally adjusted IP series and real TS curvature is almost 0.70. The left panel of the diagram below (Figure 2.17) highlights how similar the path of both series are. The series of curvature from the real TS is smoothed, so it tracks quite well the IP series without breaking the standard error bands. Finally, we verify how robust the relation between the (re-scaled) real TS curvature and the cyclical component of seasonally adjusted log IP is. As before, we expect to obtain similar results. The right diagram of Figure 2.17 shows the pattern over time of both series with the standard errors of the cyclical component.\(^{41}\)

\(^{40}\) In this case it would not make sense to judge whether the curvature series breaks or not the standard error bands of the cyclical component, since the curvature is scale-adjusted in order to match the log scale of the IP series.

\(^{41}\) The correlation coefficient between real TS curvature and the Kalman filtered (log) business cycle indicator is 0.67.
2.6 A Joint Macroeconometric Model for Curvature and Industrial Production

In this Section we propose an experiment in order to provide more evidence about the economic relationship between the curvature factor and business cycle. We develop and estimate a joint structural macroeconometric model for both curvature and IP. The measurement equations of the model are:

\[
\begin{bmatrix}
\log \left( \frac{\text{IP}_t}{\text{cur}_t} \right)
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
\mu_{t-1} \\
\beta_{t-1} \\
\psi_{t-1} \\
\psi_{t-1}^\psi
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{t-1} \\
\varepsilon_{t-1}^\psi
\end{bmatrix}
\]

(2.15)

where \(\text{cur}_t\) is the simulated trended curvature series. In the model above we assume that both the trends follow first-order integrated stochastic process, and the cyclical components are a combination of sine and cosine waves. The model is represented by the following system:

\[
\begin{bmatrix}
\mu_{t-1} \\
\beta_{t-1} \\
\psi_{t-1} \\
\psi_{t-1}^\psi
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho \cos \lambda & \rho \sin \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\rho \sin \lambda & \rho \cos \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho \cos \lambda & \rho \sin \lambda & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho \sin \lambda & \rho \cos \lambda
\end{bmatrix} \begin{bmatrix}
\mu_{t-1} \\
\beta_{t-1} \\
\psi_{t-1} \\
\psi_{t-1}^\psi
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{t-1} \\
\varepsilon_{t-1}^\psi
\end{bmatrix}
\]

(2.16)

\[\text{As stressed before the curvature component is a stationary cyclical series. In order to build a joint model with industrial production we need to add a stochastic trend to the curvature factor. We estimate the trend and the intercept of the IP series; then we run a stochastic simulation (with 1000, 5000, and 10000, repetitions achieving similar results) in order to get the trended series for curvature. The OLS regression of (log) industrial production onto the constant and the trend returns an estimate of 3.41 and 0.0026 respectively; both coefficients are statistically significant with null p-values. The OLS estimations have been confirmed by both the White and the Newey-West corrections.}\]
The model has been estimated with data from January 1987 to June 2007. The estimated amplitude of the cycle is 0.9311 for IP (\( \rho \)) and 0.7741 for the simulated curvature factor (\( \varphi_2 \)). These results are coherent with a decreasing amplitude of the cycle over time (stability). The covariance between the cycles has been imposed to be approximately zero. Estimations from January 1984 return very similar results regarding the fluctuations of the cyclical component. The amplitude coefficients are 0.9411 (\( \rho \)) and 0.5864 (\( \varphi_2 \)); both estimates are again lower than one denoting stable solutions.

The left panel Figure 2.18 shows the evolution over time of both the predicted states of the cyclical components (right scale) and the actual cyclical indicators (left scale); the co-movements with the IP growth and the IP gap (HP filtered log IP series) are important.

As far as the trend component is concerned, Figure 2.19 plots the estimated deterministic trend of log IP and the trend series obtained after Kalman filtering the joint model. The predicted series displays a slightly larger variance than the estimated one (left panel); both series fluctuate regularly around the deterministic trend though. We now show how the decomposition of the seasonally adjusted series of log IP into a cyclical component and a trend reliable is. We thus consider the error term of the measurement equation for (log) IP, i.e. \( \xi_e \) in equation (2.15).
The residual series is covariance stationary. The augmented Dickey-Fuller test decisively rejects the null hypothesis of unit root, as well as the Phillips-Perron test. On the other hand, the Kwiatkoswki-Phillips-Schmidt-Shin test does not reject the null of stationarity. In the right panel of Figure 2.19 we plot the correlogram of the series; the autocorrelations confirm that the noise series is stationary. In addition, the Jarque and Bera statistics suggests the normal distribution of the error term. Before concluding, we repeat the same experiment using the curvature series obtained from the real TS. The simulated trended curvature series is obtained by as described above. The model has been estimated from January 1987 to June 2007. The estimated dumping factor that affects the amplitude of the cycle is 0.9365 for IP and 0.6926 for simulated curvature. The right panel of Figure 2.18 shows both the predicted and estimated states of the cycle, together with the annual IP growth rate and the output gap (constructed by removing the HP filtered log IP from the actual series). There is an evident relationship between the series extracted by Kalman filtering and the real economic indicators.

2.7 The Level and the Slope: Empirical Analysis and Macroeconomic Interpretation

The empirical analysis contained in this Section focuses on the interpretation of the first two latent factors of the term structure of interest rates, namely level and slope. The TS level typically reflects the inflation rate of an economy, since a constant real interest rate requires that high inflation is offset by high nominal yields. More precisely, the TS level depends on agent’s expectations about future inflation; the level factor, which is actually unobservable, can thus be interpreted as the inflation rate targeted by the central bank as perceived by private investors, i.e. the expected medium-long run equilibrium rate of inflation. Apart from short run deviations, in fact, on average the central bank will achieve its objectives in terms of inflation.

The slope factor, instead, is associated to the nominal stance of monetary policy; Rudebusch and Wu (2004) show that the slope tracks fitted values from the monetary authority’s reaction function a la Taylor. According to the Taylor rule, the central bank has a twofold objective: minimizing the fluctuations of output around its long run natural level and stabilizing inflation around a socially acceptable low level.

43 We have also run an auxiliary OLS estimation of the log industrial production onto the trend and the cycle. The residual series obtained from this regression turns out to be homoscedastic and not serially correlated. The statistical properties of both the error series from the measurement equation and the residuals from this auxiliary regression are almost identical. Moreover, since the regressors employed in the aforementioned auxiliary regression are generated series (KF estimated trend and cycle), we have also employed the instrumental variables method, using as instruments the lagged values of IP. The IV estimated coefficients are actually the same, and the pattern of the residuals almost identical. Trivially, as largely expected, the goodness of fit of the auxiliary regression is practically 1.
In this chapter we provide evidence that the slope is informative about the change in the nominal stance of monetary policy, as captured by the annual variation of the effective federal funds rate.

These diagrams show the level factor and some inflation measures constructed using the CPI, the core-CPI, and the PPI indexes. The left panel highlights that the core-CPI inflation follows with some delay the CPI inflation, being the former smoother than the latter.

At a first sight the series seem to share important co-movements. However, there is an evident decline of the level factor over time which is not reproduced by the inflation measures\textsuperscript{44}; it might be due to the structural stability of monetary conditions achieved in the 1990s during the Greenspan’s era. The initial drop of the level factor is certainly determined by the successful strong disinflation strategy pursued by president Volcker. Afterwards, the Fed has built a significant reputation for gradualism, while it has been promptly responsive to choke off any inflationary pressure.

Sharp drops in the price level start during recessions and continues onward; consistently with conventional economic theory weak aggregate demand pushes prices down\textsuperscript{45}. The general downward direction of both CPI and core-CPI inflation in the 1990s, instead, is due to structural reasons rather than to conjunctural matters. For instance, the role played by the technological progress was determinant to improve efficiency. The boost in productivity the U.S. economy experienced in that fortunate decade coupled with relatively low energy prices might explain the gradual lowering of inflation. Many other factors might have contributed, such as a relatively accommodative monetary policy conduct, the credibility of the monetary regime, the reputation built to tackle the dynamic inconsistency issue, a balanced combination of fiscal and monetary policies, a high degree of trade liberalization and the consequent increased international competition, etc...

\textsuperscript{44} Only the core-CPI inflation exhibits the downward trend; the level factor, in fact, tracks more closely the core-CPI inflation rather than mere CPI-inflation. In addition, the level factor is higher than CPI-inflation when the actual CPI-inflation rate is particularly low; whereas the level is lower than CPI-inflation when this is high. This feature reinforces our conjecture that the cyclical behaviour of the level can be considered an appropriate measure of the inflation rate targeted by the central bank.

\textsuperscript{45} At the beginning of the 1990s the reduction in price level occurred after the recession since tensions created by the Gulf War have kept the price of raw materials, particularly the price of oil, unusually high.
The correlation matrix confirms our previous intuition. In particular, the level factor co-moves closely with the core-CPI inflation, which can be regarded as the medium-long run equilibrium inflation level, since it rules out the more volatile price components (food and energy).

<table>
<thead>
<tr>
<th>Correlations</th>
<th>lev NS (kf)</th>
<th>lev A-LT (kf)</th>
<th>CPI infl.</th>
<th>core-CPI infl.</th>
<th>PPI infl.</th>
</tr>
</thead>
<tbody>
<tr>
<td>lev NS (kf)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lev A-LT (kf)</td>
<td>0.975</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI infl.</td>
<td>0.532</td>
<td>0.495</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>core-CPI infl.</td>
<td>0.843</td>
<td>0.791</td>
<td>0.678</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>PPI infl.</td>
<td>-0.103</td>
<td>-0.102</td>
<td>0.663</td>
<td>-0.039</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.9

All major monetary institutions agree that their primary task is to deliver price stability; central bankers thus operate in order to maintain the inflation rate on a desired path. The level of TS reflects agent’s perception about the evolution over time of the aforementioned path. It is also plausible to presume that the inflation rate targeted by the monetary authority is somewhat changeable over time, since it necessarily depends upon the conditions of the economy. Short run deviations from the targeted level are thus inevitable, but on average, in the medium-long run, the central bank meets the inflation target.

The adaptive learning measure of inflation implies that past intended values of inflation are corrected for the observed actual realizations:

\[ L_t = (1 - \rho_L) \pi_t + \rho_L L_{t-1} + \varepsilon_{L,t} \]  

(2.17)

Coefficient \( \rho_L \) measures the weight given to observed discrepancies between actual \( (L_t) \) and perceived lagged inflation \( (L_{t-1}) \); roughly speaking it can be regarded as a smoothing parameter according to which inflation expectations adjust over time. As actual inflation \( (\pi_t) \) changes, the level factor gets linearly updated by news about current and future inflation. We recall that the level factor represents the medium term inflation target as perceived by private investors. The OLS estimation\(^{46}\) of equation (2.17) returns \( \hat{\rho}_L = 0.973 \) for the sample January 1984 – June 2006.

In the New-Keynesian literature, the forward-looking Phillips curve implies that current inflation depends on expectations about its future values:

\(^{46}\) The coefficient is statistically significant (p-value: 0). Residuals are homoscedastic and serially uncorrelated; the Jarque and Bera test suggests residuals are normally distributed. The Wald test rejects the null hypothesis of unity coefficient. The adjusted goodness of fit is 0.983 and the standard error of regression is 0.24. As a further robustness check we have performed the GMM estimation of equation (2.17) including the first lag of regressors as instrumental variables; results are statistically significant with an estimated AR(1) coefficient 0.971. In addition, equation (2.17) has been estimated replacing CPI inflation with core-CPI inflation; OLS estimation returns an AR(1) coefficient of 0.965. In both cases the CUSUM test indicates stability of the coefficients’ estimates over time. If we restrict the analysis to the Greenspan period (June 1987 – December 2005) results are similar to those obtained by Rudebusch and Wu (2004). Using CPI inflation the estimated AR(1) coefficient is 0.976, while using the core-CPI inflation the AR(1) coefficient becomes 0.962. For both specifications the standard error of regression decreases to 0.21 in the Greenspan era.
\[ \pi_t = \rho_{\pi} L_t + (1 - \rho_{\pi}) \left[ \mu_{x1} \pi_{t-1} + \mu_{x2} \pi_{t-2} \right] + \mu_{g} \text{gap}_t + \varepsilon_{\pi,t} \]  

(2.18)

Inflation expectations are captured by the latent factor \((L_t)\); in a sense this forward-looking component reflects price stickyness which drives firms’ pricing behaviour.

### Table 2.10

<table>
<thead>
<tr>
<th>( \rho_{\pi} )</th>
<th>( \mu_{x1} )</th>
<th>( \mu_{x2} )</th>
<th>( \mu_{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0320</td>
<td>1.1915</td>
<td>-0.2363</td>
<td>0.0365</td>
</tr>
<tr>
<td>(0.0014)</td>
<td>(0.0000)</td>
<td>(0.0006)</td>
<td>(0.0236)</td>
</tr>
<tr>
<td>[3.23]</td>
<td>[17.7]</td>
<td>[-3.47]</td>
<td>[2.28]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho_{\pi} )</th>
<th>( (1 - \rho_{\pi}) \mu_{x1} )</th>
<th>( (1 - \rho_{\pi}) \mu_{x2} )</th>
<th>( \mu_{g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0320</td>
<td>0.1534</td>
<td>-0.2287</td>
<td>0.0365</td>
</tr>
<tr>
<td>(0.0014)</td>
<td>(0.0000)</td>
<td>(0.0006)</td>
<td>(0.0236)</td>
</tr>
<tr>
<td>[3.23]</td>
<td>[17.5]</td>
<td>[-3.49]</td>
<td>[2.28]</td>
</tr>
</tbody>
</table>

p-values in parenthesis; t-statistics in square brackets

Equation (2.18) is a traditional aggregate supply (AS) curve which can be obtained by solving macro dynamic stochastic general equilibrium models with nominal rigidities. In particular, equation (2.18) implies that prices are set as a mark-up on marginal costs\(^{47}\). The key parameter \( \rho_{\pi} \) captures the importance of forward- relative to backward-looking pricing behaviour. Inflation inertia enters equation (2.18) through coefficients \( \mu_{x1} \) and \( \mu_{x2} \). Finally, actual inflation depends upon the IP gap which reflects aggregate demand spending. Estimations for the Greenspan sample reported in Table 2.10 are similar to those obtained by Rudebusch and Wu (2004).

Now we turn to investigate the slope factor. The diagrams show how closely the slope tracks both the effective federal funds rate (left diagram) and its annual percentage variation (right diagram).

\(^{47}\) In a perfectly competitive labour market marginal costs and the output gap are linearly related. In DSGE models it is commonly assumed that firms set price according to the so-called Calvo pricing rule, i.e. each period only a fraction of firms are allowed to change prices.
During recession the slope follows the reduction of the fed funds, which is a clear sign of policy loosening. In Table 2.11 we report the correlations between the slope factor and the effective fed funds.

<table>
<thead>
<tr>
<th>Correlations</th>
<th>slo NS (kf)</th>
<th>slo A-LT (kf)</th>
<th>ffr</th>
<th>ffr grw</th>
</tr>
</thead>
<tbody>
<tr>
<td>slo NS (kf)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slo A-LT (kf)</td>
<td>0.777</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ffr</td>
<td>0.531</td>
<td>0.745</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>ffr grw</td>
<td>0.555</td>
<td>0.535</td>
<td>0.247</td>
<td>1</td>
</tr>
</tbody>
</table>

| slo NS (kf): slope factor extracted by Kalman filtering the NS term structure model (Section 2.3). slo A-LT (kf): slope factor extracted by Kalman filtering a standard affine TS model with lower triangular transition matrix. ffr: effective federal funds rate. ffr grw: annual rate of growth of the effective federal funds rate. |

Table 2.11

Correlations are positive and particularly high. We need to consider whether the slope factor is an appropriate indicator of either the effective fed funds rate or of its change over time. Therefore, the following distributed-lag models are considered:

\[ S_t = \alpha_0 + \alpha_1 ffr_t + \alpha_2 ffr_{t-1} + u_{S,t} \]  
\[ S_t = \beta_0 + \beta_1 A ffr_t + \beta_2 A ffr_{t-1} + u_{S,t} \]

(2.19)  
(2.20)

Both models are estimated and used to generate predictions. Results support the second specification. Simple statistics to compare predictive accuracy are reported in Table 2.12. Also a visual inspection of Figure 2.22 suggests that the annual change in the fed funds (right panel) gives a better prediction of the slope factor than the effective federal funds itself (left panel).

**Table 2.12**

<table>
<thead>
<tr>
<th>Federal Funds Rate – Level</th>
<th>Equation (2.19)</th>
<th>Federal Funds Rate - Annual Change</th>
<th>Equation (2.20)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_1 )</td>
<td>( \alpha_2 )</td>
<td>( \beta_1 )</td>
</tr>
<tr>
<td></td>
<td>0.8334</td>
<td>-0.4900</td>
<td>-0.4112</td>
</tr>
<tr>
<td></td>
<td>(0.0111)</td>
<td>(0.1316)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td></td>
<td>[2.55]</td>
<td>[-1.51]</td>
<td>[-3.86]</td>
</tr>
<tr>
<td></td>
<td>s.e. 1.314</td>
<td>s.e. 1.314</td>
<td>s.e. 1.220</td>
</tr>
<tr>
<td>p-values in parenthesis; t-statistics in square brackets; s.e. standard error of regression.</td>
<td></td>
<td>p-values in parenthesis; t-statistics in square brackets; s.e. standard error of regression.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RMSE</th>
<th>MAE</th>
<th>Theil IC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3070</td>
<td>1.1112</td>
<td>0.2470</td>
</tr>
<tr>
<td>RMSE</td>
<td>MAE</td>
<td>Theil IC</td>
</tr>
<tr>
<td>1.2133</td>
<td>1.0530</td>
<td>0.2272</td>
</tr>
</tbody>
</table>

RMSE: Root Mean Squared Error. MAE: Mean Absolute Error. Theil IC: Theil inequality coefficient.

48 The choice of including only the first lag is supported by both the Akaike and Schwarz criteria. These statistics drop when we add the first lag and remain stable if we add subsequent lags. In addition, the adjusted goodness of fit increases substantially when adding the first lag, while only marginally if we include more lags.
It follows the core analysis which allow us to establish the macroeconomic link between the slope factor and the nominal stance of monetary policy. We thus perform some estimates of simple Taylor rule-type equations for both the slope and the annual change in the effective fed funds. Equation (2.21) represents the monetary rule in terms of the slope factor:

\[ S_t = \alpha + \varphi_\pi (\pi_t - \hat{L}_t) + \varphi_s x_t + \varepsilon_{S,t} \tag{2.21} \]

Coefficient \( \varphi_\pi \) represents the weight given to deviations of inflation from its targeted level; while its complement captures the sensitivity to the cyclical fluctuations of the economy. Specification (2.21) may well be considered a forward-looking policy rule, since the level factor represents expected inflation. GMM estimations suggest that both inflation and real variables are important determinant of the slope. The estimated inflation deviation coefficient is \( \hat{\varphi}_\pi = 0.644 \) (p-value: 0); while the industrial production gap coefficient is \( \hat{\varphi}_s = 0.336 \) (p-value: 0.002). The adjusted goodness of fit of the equation is 0.22.

A similar equation has been GMM estimated for the effective fed funds rate. The change of the fed funds, rather than its mere level, gives a measure of how the monetary policy stance varies over time:

\[ \Delta \text{ffr}_t = \alpha + \varphi_\pi (\pi_t - \hat{L}_t) + \varphi_s x_t + \varepsilon_t \tag{2.22} \]

Figure 2.23 shows that the fitted series of the variations in the fed funds go hand in hand with the slope factor. Slope is more persistent and follows with some delay the movements of the predicted series.

---

49 The measure of the output gap is either the Hodrick-Prescott or the Baxter-King IP gap. As an alternative cyclical measure we have also employed the annual change in the unemployment rate. We report the estimates of the Taylor type monetary rules with the HP industrial production gap, since results using the BK gap are similar.

50 Since expected inflation, i.e. the level factor, and the output gap are generated regressors we need to back the explanatory variables using their first lag as instruments.

51 When the cyclical variable is the annual change in unemployment the estimated coefficient of inflation is 0.724 (p-value: 0); while the unemployment coefficient is -0.052 (p-value: 0). The adjusted goodness of fit increases to 0.28.

52 The first lag of the regressors have been chosen as instruments, since they are highly correlated with the explanatory variables but not with the disturbances. The estimated inflation deviation coefficient is 0.166 (p-value: 0.001); while the industrial production gap coefficient is 0.147 (p-value: 0). The adjusted goodness of fit is approximately 0.40. When the cyclical variable is the rate of change of unemployment in equation (2.22), the inflation deviation coefficient becomes 0.205 (p-value: 0); while the unemployment coefficient is -0.025 (p-value: 0). The adjusted goodness of fit of the monetary policy equation with unemployment rises to 0.73. The intercept is significant only in equation (2.21) (approximately -1.9).
During recessions the slope factor decreases dramatically as well as the predicted series of the federal funds rate changes, reflecting agents’ expectations about the incoming accommodative stance of monetary policy. The slope factor, instead, rises substantially both during the recovery in early 1990s after the Gulf War (and the World recession\textsuperscript{53}) and during the recovery after the recession that took place at the beginning of the new millennium (2001-2002).

Figure 2.23

A visual inspection of Figure 2.24 suggests that the slope factor dynamics is influenced by both monetary and real variables. The left diagram plots the slope factor against inflation, while the right one against the M1 growth rate. During recessions, or immediately after, weak aggregate demands causes a drop of the inflation rate. The monetary authority reacts to stimulate the economy by expanding money supply, and the slope factor decreases, since it follows the reduction in the fed funds.

Figure 2.24

Figure 2.25 plots the slope factor with real variables. In the left panel we plot the IP gap (HP-filtered) and the IP rate of growth; while in the right panel it is shown the unemployment series\textsuperscript{54}.

\textsuperscript{53} Economists agree the Gulf War was not the determinant of the world recession; the war simply made the recession long-lasting and more severe.

\textsuperscript{54} We recall that the annual rate of change of unemployment has been considered as explanatory variable in the above equations.
The sharp decrease of the slope factor during recessions is symptomatic of how concern the monetary authority is about the weakening of the economy. This vigorous response also works as an important signal for banks and lenders that promptly expand credit and help the economy to recover. The estimations, in fact, seem to support the idea that the U.S. monetary policy is extremely sensitive to the cyclical fluctuations of the economy, and, in particular, to the changing conditions in the labour market. These slope factors simply reflect agents’ expectations which are incorporated in the TS of interest rates. We now investigate whether predictions generated by equations (2.21) and (2.22) are influenced by some key macroeconomic variables. In the left panel of Figure 2.26 we plot the difference between the actual and the predicted slope factor obtained from model (2.21) against the annual standard deviation of inflation. It is evident the cyclical behaviour of the forecast errors; slope predictions tend to be less accurate when inflation is highly volatile. Mapping the discrepancies between actual and predicted slope using the business cycle seems to be more controversial; apparently large deviations tend to occur during periods of economic slowdown.

In the right diagram inflation volatility is plotted against the deviations between the slope factor and the fitted values of the federal funds rate changes as of implied by equation (2.22). Again when inflation is particularly volatile deviations appear to be large in absolute values. In this case also cyclical fluctuations matter in explaining the deviations between the slope and the federal funds dynamics.
In order to detect whether there exists a significant relation between prediction errors and the business cycle we run the following regressions:

\[ S_t - \hat{S}_t = c + \text{dummy}(t = 1| \text{recession}) + \nu_t \]  
(2.23)

\[ S_t - \Delta\hat{r}_t = c + \text{dummy}(t = 1| \text{recession}) + \nu_t \]  
(2.23')

The dummy variable assumes value 1 in correspondence of periods of economic slowdown, as highlighted by shaded areas in Figure 2.26. The dummies turn out to be statistically significant in both equations.

Finally, we allow for the possibility that residual from the monetary rules (2.20) and (2.21) are eventually correlated. Rudebusch and Wu (2004) argue that discrepancies between the actual slope series and the fitted one, occurred mainly in 1992 and 1993, can be attributed to a misspecification of the model rather than to the so-called “credit crunch”. We thus need to estimate the following specification:

\[
\begin{align*}
S_t &= \alpha + \varphi_{\pi} (\pi_t - L_t) + \varphi_{x,} x_t + \varepsilon_{S,t} \\
\varepsilon_{S,t} &= \rho \varepsilon_{S,t-1} + u_t \\
\end{align*}
\]  
(2.24)

Estimations confirm our previous results. The current output gap becomes not significant though. Parameter \(\varphi_{x,}\) is still significant if we replace the current value of output gap with its first lag \((x_{t-1})\).

In this Section we have provided evidence offering a macroeconomic interpretation of the first two unobservable components of the term structure of interest rates.\(^{55}\)

### 2.8 The Canadian Term Structure: Latent Factors and Macroeconomic Variables

Section 8 provides evidence that the main conclusions of our analysis fit also Canadian data.\(^{56}\) The Nelson-Siegel model presented in Section 2.3 has been estimated with Canadian yield data from January 1986 to June 2006. The maturity spectrum consists of the 3-, 6-, 12-, 24-, 36-, 48-, 60-, 72-, and 120-month yields. Estimations results of the system composed by equations (2.2) and (2.3) are reported in Appendix A2.II. As expected the level factor is highly persistent. Figures 2.27 and 2.28 plot the latent factors together with their macroeconomic counterparts (shaded areas represents NBER recessions). The level and the slope are reported in the following Figure:

\(^{55}\) We have also estimated both equations (2.21) and (2.22) imposing the restriction \(\varphi_{\pi} = 1 - \varphi_{x,}\). Estimation results remain significant. Finally, we also have estimated equation (2.21) allowing for monetary policy inertia. The interest rate smoothing coefficient is statistically significant (0.95). It does not shadow the relevance of the inflation and the output gap though.

\(^{56}\) Data are described in Appendix A2.I. For simplicity in this Section we treat the Canadian economy as if it was a closed one. So that we do not consider neither the dynamics of the exchange rate, nor any international commodity price index, nor the eventual effect of the U.S. federal funds rate.
The correlation between the level and the CPI inflation is almost 0.50. The correlation between the slope factor and the overnight rate is about 0.8. Although the slope tracks more closely the annual change in the overnight rate, the correlation is just above 0.55. The visual evidence in favour of the interpretation of curvature as cyclical variable is surprising. Co-movements between curvature and the HP filtered IP gap are certainly important, as outlined by the plots in Figure 2.28. There is also a significant inverse relation between curvature and unemployment.

The GMM estimation of the following specification for the New Keynesian Phillips (aggregate supply) curve returns significant coefficient for both the backward- and the forward-looking inflation components.

\[
\pi_t = \rho_{\pi L} L_t + (1 - \rho_{\pi L}) \left[ \mu_{\pi} \pi_{t-1} \right] + \mu_{\pi} \text{gap}_t + \epsilon_{\pi,t}
\]  

The slope of the term structure should reflect the nominal stance of monetary policy as captured by a Taylor rule:

---

57 The estimate of the forward-looking coefficient is 0.021 (p-value: 0.059). The coefficient of the backward-looking component is 0.954 (p-value: 0).

The output gap coefficient is 0.05. The second and the third lags of inflation have been used as instruments as well as the first lag of the industrial production HP gap.
\[ S_t = \alpha + \varphi_x (\pi_t - L_t) + \varphi_s x_t + \varepsilon_{S,t} \]  

The GMM estimation\(^{58}\) of (2.25) suggests that the slope is sensitive to the inflation deviation from the target \((\pi_t - L_t)\). The estimated coefficient is 0.21 and statistically significant \((p\text{-value: } 0.05)\).

According to our interpretation, whenever the inflation rate is one percentage point above the targeted level, the Canadian monetary authority would raise the policy rate by an amount capable of reducing the slope of the term structure by 21 basis points. We can obtain a rule of thumb policy measure by performing the following exercise. If the overnight rate is placed as dependent variable of equation (2.26) the estimate of the inflation deviation coefficient turns out to be 0.48. We can only speculate on our results, by saying that, roughly speaking, the monetary authority should raise the policy rate by 48 basis points in order to flatten the yield curve by 21 bps.

We now turn the economic interpretation of curvature. We start estimating a single equation relating curvature to some real variables: the IP gap and the annual change in the unemployment rate.

\[
cur_t = \rho_0 + \rho_1 \text{gap}_t + \rho_2 \Delta \text{un}_{t-12} + \varepsilon_{t,C} \tag{2.27}
\]

Results indicate that curvature can be explained by the actual values of the aforementioned variables; both OLS and GMM\(^{59}\) estimates are statistically significant. The goodness of fit is not particularly high though. The \(F\) test rejects the null hypothesis that coefficients are jointly zero.

<table>
<thead>
<tr>
<th>Curvature – Canada – Equation (2.27)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\rho}_0 )</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>coef</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>WH</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>NW, 4</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
</tbody>
</table>

- Standard errors in parenthesis; t-statistics in square brackets

\(^{58}\) The explanatory variables have been instrumented by their first lag.

\(^{59}\) Both the White and the Newey-West corrections have been employed to show results’ robustness. In addition, as a further robustness check, also the GMM method has been used, since IV estimation is acknowledged to handle with both serial correlation and heteroscedasticity of unknown form. The first lags of the regressors have been chosen as instruments, since they are highly correlated with the explanatory variables.
Again we need to point out that curvature is positively linked to the output gap and inversely correlated with the rate of growth of unemployment. As it happens with U.S. data, also Canadian evidence supports the view that curvature can be considered a countercyclical indicator of the economic conjuncture.

As previously done in Section 2.5, we estimate the state-space model for the industrial production series in order to derive the cyclical component. The Kalman filtered prediction of the state variable is thus compared with the curvature factor$^{60}$. Figure 2.29 contains the plots of the series

The curvature factor lies within the standard error bands of the economic cycle predicted series. The correlation coefficient between curvature and the cyclical component of industrial production is about 0.32. An analogous exercise has been repeated with the log series of industrial production achieving comparable results$^{61}$.

![Figure 2.29](image)

Finally we use Canadian data to estimate the following system of aggregate demand (IS) curves:

$$
\begin{align*}
\text{gap}_t &= \psi_{0,g} + \psi_{1,g} E_t \left( \text{gap}_{t+1} \right) + \left( 1 - \psi_{1,g} \right) \left[ \left( 1 - \psi_{2,g} \right) \text{gap}_{t-1} + \psi_{3,g} \left[ r_{t-1} - \pi_t \right] + \epsilon_{t,g} \right] \\
\text{cur}_t &= \psi_{0,c} + \psi_{1,c} E_t \left( \text{cur}_{t+1} \right) + \left( 1 - \psi_{1,c} \right) \left[ \left( 1 - \psi_{2,c} \right) \text{cur}_{t-1} + \psi_{3,c} \left[ r_{t-1} - \pi_t \right] + \epsilon_{t,c} \right]
\end{align*}
$$

(2.28)

Where the industrial production gap is a function of both its future value (the forward-looking term) and its past value (the backward-looking component); the gap is also a function of the realized real interest rate, i.e. the (log) difference between the past nominal rate and actual inflation$^{62}$. The second equation of the system represents an IS curve expressed in terms of the curvature factor.

GMM and OLS estimation results are reported in the following Table$^{63}$. The specification in terms of the curvature factor (right panel) presents more stability than the IP gap specification. The

$^{60}$ The estimated amplitude of the cycle is 0.9656 (p-value: 0); value which guarantees convergence. It follows the estimates of the variance of the disturbances. For the measurement equation there estimated variance is (0.3513)$^2$ with 0 p-value. The trend component: (0.0312)$^2$ (p-value: 0.0146); and, finally, the cyclical component: (0.5024)$^2$ (p-value: 0).

$^{61}$ It goes without saying that the scale of the predicted cyclical series has to be adjusted in order to match curvature.

$^{62}$ The nominal rate is either the overnight rate or the yield with maturity 120 months, since aggregate demand spending depends on the long end of the yield curve. We try also the ex-ante expected future value of the real interest rate, but coefficients remain not significant in both equations.

$^{63}$ To perform the GMM estimation, the explanatory variables have been backed by the contemporaneous value of the output gap, its second lag, and the lag of the real interest rate in the form employed in the equation.
magnitude of the coefficient associated to the forward-looking component is absolutely comparable to that of the multiplier of the backward-looking term.

<table>
<thead>
<tr>
<th>IP gap</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi_{0,g})</td>
<td>(\psi_{0,c})</td>
</tr>
<tr>
<td>OLS</td>
<td></td>
</tr>
<tr>
<td>-0.0092</td>
<td>0.0377</td>
</tr>
<tr>
<td>(0.077)</td>
<td>(0.0102)</td>
</tr>
<tr>
<td>[0.1]</td>
<td>[14]</td>
</tr>
<tr>
<td>GMM</td>
<td></td>
</tr>
<tr>
<td>-0.0226</td>
<td>0.0228</td>
</tr>
<tr>
<td>(0.140)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>[0.1]</td>
<td>[2.5]</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.55</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 2.14

The Wald test cannot reject the null hypothesis of equality between the correspondent coefficients. In particular individual coefficient tests suggest that \(\psi_{1,c}\) is not different from \(\psi_{1,g}\) (the statistics of the test is \(\chi^2 = 0.15\); p-value: 0.70). Moreover, the null hypothesis of equality between \(\psi_{2,c}\) and \(\psi_{2,g}\) cannot be rejected. (\(\chi^2 = 1.21\)). More important, the equality of the weights \((1-\psi_{1,g})\psi_{2,g} = (1-\psi_{1,c})\psi_{2,c}\) cannot be rejected by the Wald test (the \(\chi^2\) statistics is about 0.16 with an associated p-value of 0.69).

### 2.9 A Structural VAR Approach

In this Section we estimate a structural vector autoregressive model including the latent components of the term structure of interest rates. We mean to derive the impulse responses in order to assess the interrelation between the economy to the yield curve factors. A general form for VAR is

\[
A \, X_t = C(L) \, X_{t-1} + B \, u_t
\]

(2.29)

Matrix \(A\) describes the contemporaneous relations among the endogenous variables; \(C(L)\) is a finite-order matrix of polynomial lags; vector \(X_t = [Y_t, F_t, P_t]^T\) contains the most exogenous variables at first, latent factors thereafter, and, finally, policy variables. Macro variables are stacked in vector \(Y_t = [x_t, \pi_t]^T\), where \(x_t\) is the (log) total capacity utilization and \(\pi_t\) is the CPI inflation.
Theoretical latent factors are stacked in vector \( F_t = [C_t, S_t, L_t] \). Consistently with the view expressed in previous Section, our policy variable, \( P_t \), is simply the annual change of the federal funds rate. The vector of structural innovations is \( u_t = [u_t^r, u_t^e, u_t^p] \). Structural disturbances are assumed to be orthogonal, i.e. the covariance matrix is the identity matrix \( E(u_t'u_t') = I \).

Pre-multiplying both sides of (2.29) by the inverse of matrix \( A \) we obtain the reduced form:

\[
A^{-1} A_t X_t = A^{-1} C(L) X_{t-1} + A^{-1} B u_t, \tag{2.30}
\]
\[
X_t = D(L) X_{t-1} + e_t. \tag{2.31}
\]

This is the reduced form of the Structural VAR, \( e_t = [e_t^r, e_t^e, e_t^p] \) is the reduced form residuals with covariance matrix \( \Sigma = E(e_t'e_t') \). There are many possible structural models (2.29) that can be represented by a generic reduced form model (2.31). Identification restrictions are imposed since the number of parameters to be estimated in the reduced form is smaller than that of the structural form. Structural VAR disturbances are linear combinations of the reduced form residuals:

\[
A e_t = B u_t, \tag{2.32}
\]
\[
A \Sigma A' = B I B' \tag{2.33}
\]

Identification requires \( k(k+1)/2 \) restrictions both on matrix \( A \) and \( B \). The ordering of the \( k \) variables has been described above. The particular form of the upper triangular part of matrix \( A \) depends upon the relation which links the latent factors, so that it is consistent with the transition matrix of the latent factor state space system.

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
\alpha_{21} & 1 & 0 & 0 & 0 & 0 \\
\alpha_{31} & \alpha_{32} & 1 & \alpha_{34} & \alpha_{35} & 0 \\
\alpha_{41} & \alpha_{42} & 0 & 1 & \alpha_{45} & 0 \\
\alpha_{51} & \alpha_{52} & 0 & 0 & 1 & 0 \\
\alpha_{61} & \alpha_{62} & \alpha_{63} & \alpha_{64} & \alpha_{65} & 1
\end{bmatrix}, \quad B = \begin{bmatrix}
\beta_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & \beta_{66}
\end{bmatrix} \tag{2.34}
\]

Structural identification requires also that macroeconomic variables do not simultaneously respond to policy shocks, whilst the relation is permitted in the opposite direction. Traditionally VAR analysis has been exploited to assess monetary policy shocks using data with quarterly frequency (Christiano

---

64 The level is simply the long term yield with maturity 120-month. The slope is the difference between the 60- and the 3-month yield. Following Nelson and Siegel (1987), and Diebold and Li (2006), we compute curvature as the difference between twice the 24-month yield and the summation of the 3- and the 120-month yields \((2*y_{24m} - y_{120m} - y_{3m})\).
et al., 1996); despite the extensive and popular use of VAR models to gauge monetary policy, the reliability of VAR methodology has been questioned and turns out to be controversial\textsuperscript{65}.

Estimations are performed with U.S. data starting from January 1985. We compute the impulse response functions and we plot them together with the associated standard errors. Each response to a shock is measured as a percentage deviation from the steady state level.

The first panel on the left shows that a one standard deviation innovation that hits the total capacity utilization generates a significant response by the curvature factor. After a soft spike the effect remains persistent; it vanishes away after 24 months.

![Figure 2.30](image)

More interesting seems to be the effect of the effective federal funds rate shock onto the slope factor (central panel). A negative response of slope follows a positive shock that affects the annual change of the effective federal funds. Tightening monetary policy, i.e. an increase of the fed funds, raises short rates more than long term rates flattening the yield curve. Or the other way around, when monetary policy becomes accommodative, the slope increases since short term rates lower relative to long rates. We recall that monetary policy actions exert a direct influence only on the short end of the term structure, which is linked to the dynamics of the effective federal funds rate.

A shock to inflation tends to raise the level of TS as shown in the right panel of Figure 2.30. After a significant initial response, the effect of the inflationary shock fades away in six months.

Canadian impulse responses draw a similar pattern as shown in Figure 2.31. The left panel shows that the output gap is sensitive to a shock that hits curvature. Consistently with the view expressed in previous sections, curvature either predicts or accompanies the flattening of the yield curve, thus anticipating a slowdown in economic activity i.e. the gap enlarges. A positive shock to the annual change in the overnight rate generates a reduction of the slope as shown in the central panel. An increase of the overnight rate flattens the yield curve since it reflects tightening monetary policy.

\textsuperscript{65} Granger tests have been performed in order to select the ordering of the variables; as shown in the Appendix we use only stationary variables. I have selected a 3\textsuperscript{rd} order VAR, after comparing the Akaike and Schwarz criteria with both the 6\textsuperscript{th} and the 12\textsuperscript{th} order VAR. Therefore, with monthly data we assume that only the most recent quarter matters.
2.10 Concluding Remarks

Both macroeconomists and financial economists have always paid scrupulous attention to the bidirectional relation that links macroeconomics and finance. The yield curve certainly represents an appealing bridge to explore the aforementioned relation. In this vein, term structure models provide an effective framework to summarize in few factors all the information contained in the yield curve, which is regarded to be a leading economic indicator. So far, the empirical literature has expressed a certain consensus about the macroeconomic interpretation of only two components underlying the term structure, namely the level and the slope. The former is associated to the rate of inflation targeted by the monetary authority, while the latter is considered an indicator of the monetary policy stance. This study offers a refinement of traditional factor-models of the term structure since we focus also on the third latent factor.

Working with U.S. and Canadian data, we provide significant evidence that curvature reflects the cyclical behaviour of the economy, as represented by the dynamics of unemployment and industrial production. We find evidence, in fact, that a negative shock to curvature seems either to anticipate or to accompany a slowdown in economic activity. The curvature effect thus appears to complement the transition from an upward sloping yield curve, prevailing during expansions, to a flat yield curve, that is thought to anticipate recessions. Interestingly, our main results hold despite the curvature factor is extracted from the real or the nominal term structure of interest rates. In particular, U.S. data suggest that curvature from the real term structure of interest rates is related to the consumption growth.

Furthermore, on the basis of the empirical analysis developed in this chapter, we propose that the slope factor is related to the annual variation of the effective federal funds rate rather than to its level. We thus believe that the yearly change in the fed funds reflects the evolution of the nominal stance of monetary policy over time as well as the adjusting preferences of the monetary authority.
Appendix A2.I - Data

Financial variables. All data employed in this chapter have monthly frequency. U.S. yields data between January 1984 and December 1998 are from both the McCulloch-Kown database (3-month, 6-month, and 10-year) and from the Fama-Bliss dataset (1-, 2-, 3-, 4-, 5-year). After January 1999 all yields data are from Datastream (U.S. ZCB yields). In this work we consider the sample between January 1984 and June 2007. The effective federal funds rate is from the Federal Reserve Economics Database (FRED). Roughly speaking, the cyclical behaviour of nominal yields reveals that the monetary authority has lowered interest rates to help recovering from unfavourable economic conditions. Table 2.15 reports some descriptive statistics about yields associated to bonds with different maturities. The mean is increasing with maturity; this may be due to a positive liquidity (or inflation risk) premium. The standard deviation tends to be larger at short maturities, while it is substantially lower at long maturities. Data confirm that long term yields are more persistent than short term yields.

According to the Jarque and Bera test, short term yields tend to be normally distributed around the mean; however, this feature no longer holds for longer term yields. Both the augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) suggest the series are stationary over the sample 1984-2007. The ADF test leads to the rejection of the null hypothesis of unit root; consistently, the null hypothesis of stationarity cannot be rejected by the KPSS test. A visual inspection of the correlograms shows that the autocorrelations decay faster than linearly; the partial autocorrelation function suggests the first-order autoregressive structure of yields. Thus, we have estimated an AR(1) regression for each yield obtaining a coefficient of approximately 0.98. However, in line with results from the ADF test, the Wald test leads to the rejection of the null hypothesis of coefficient equal to one in the first-order autoregressive models for each

![Table 2.15](image)

Sample: jan84-jun07 (282 obs). Normality and ADF tests: p-values in parenthesis. KPSS test statistics reported. Exogenous included:*Intercept;**Intercept and trend.

66 McCulloch data are available from the Gregory R. Duffee web page; while the Fama-Bliss yields data are from Cochrane and Piazzesi (AER, 2005).

67 The autocorrelation of yields increases with maturity if we consider the entire sample 1964 - 2007. However, as noted by Piazzesi (2003), there is evidence that the persistence of short term rates has increased over time. In our sample (1984-2007) the magnitude of the estimated AR(1) coefficients decreases with the maturity of yields.

68 To match the monthly frequency of data, the selected number of lags in the auxiliary regression is always 12. The automatic lag selection based on various criteria (Akaike, Schwarz, Hannan-Quinn) leads to similar results of the stationarity test. The KPSS critical values are 0.739 (1%), 0.463 (5%), and 0.347 (10%) if only the intercept is included in the model. The KPSS test critical values if both the intercept and the trend are included are 0.216, 0.146, and 0.119 at 1%, 5%, and 10% significance levels respectively. We are unable to reject the null of stationarity when the empirical KPSS statistics (reported in the Table) is below the critical values.
yield. According to both the ADF and the KPSS test, the annual change of the federal funds rate is stationary. In the sample from January 1984 to June 2006 only the level of federal funds rate might be considered non-stationary; however, we point out that the federal funds rate in the level is never used in the following analysis.

**Monetary variables.** The inflation series is the annual change in the seasonally adjusted Consumer Price Index for all urban consumers (all items) available from the FRE-Database (source: U.S. Department of Labor, Bureau of Labor Statistics). Monetary aggregate M1 is the seasonally adjusted M1 money stock from the FRED (source: Board of Governors of the Federal Reserve System). In our analysis we consider the annual rate of growth. Both the ADF and the KPSS test confirm the series are stationary. The auxiliary regression contains 11 lags. The KPSS test 1%, 5%, and 10% critical values are 0.74, 0.46, and 0.35 respectively. The $p$-values associated to the ADF test is 0.019 and 0.068 for inflation and M1 growth respectively; thus rejecting the null of unit root. The KPSS test cannot reject the null of stationarity.

**Indicators of the real economy.** The monthly seasonally adjusted series of industrial production (IP) is from the FRE-Database. Different measures of the output gap have been generated: a) the annual (log) rate of growth of industrial production (blue line, right scale); b) the Hodrick-Prescott filter of (log) IP (red line, left scale); c) the Baxter-King cyclical component of log IP (green line, left scale); d) the Christiano-Fitzgerald cyclical component of log IP (brown line, left scale). All cyclical indicators are highly correlated. As shown in the diagram below, the Baxter and King band-pass filter is particularly effective, that is the actual frequency series matches closely the optimal frequency function.

The seasonally adjusted civilian unemployment rate series is from the FRED (source: U.S. Department of Labor, Bureau of Labor Statistics). The seasonally adjusted real personal consumption expenditures is from FRED (source: U.S. Department of Commerce, Bureau of Economic Analysis). In both cases we consider the annual growth rate. According to both the ADF and the KPSS tests (either 11 or 12 lags considered) all the real variables series can be considered stationary in the sample between January 1984 and June 2006.

---

69 The ADF test performed with 11 lags in the auxiliary regression and no exogenous variables returns a probability value of 0.003. The KPSS test (exogenous: intercept) cannot reject the null of stationarity.

70 To fit the monthly frequency of data the lowest and highest cycle periods have been chosen 18 and 96 respectively; in the Hodrick-Prescott filter the smoothing parameter has been set equal to 14400. Grey shaded areas indicate NBER recessions; while the yellow shaded area indicates the weakening of the economy in mid 1980s.
**Table 2.16**

<table>
<thead>
<tr>
<th>Stationarity</th>
<th>IP grw</th>
<th>HP gap</th>
<th>real C grw</th>
<th>un grw</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>(0.021)</td>
<td>(0.000)</td>
<td>(0.056)*</td>
<td>(0.016)</td>
</tr>
<tr>
<td>KPSS</td>
<td>0.146*</td>
<td>0.106*</td>
<td>0.179*</td>
<td>0.193*</td>
</tr>
</tbody>
</table>

Sample: jan84-jun06. *Intercept.

Candian data. Yields data are from the Bank of Canada. Descriptive statistics reveal that the mean increases with maturity, while the standard deviation follows the opposite pattern. Both the ADF and the KPSS\(^{71}\) test support stationarity of all yields. The Jarque and Bera test rejects the null of normality.

**Table 2.17**

<table>
<thead>
<tr>
<th>Yields</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity</td>
</tr>
<tr>
<td>stdev</td>
</tr>
<tr>
<td>skew</td>
</tr>
<tr>
<td>kurt</td>
</tr>
<tr>
<td>norm</td>
</tr>
<tr>
<td>ADF</td>
</tr>
<tr>
<td>KPSS</td>
</tr>
</tbody>
</table>

Sample January 1988 - June 2006. ADF tests are performed with 12 lags in the auxiliary regression, thus matching the monthly frequency of data (p-values reported). Intercept and trend not included. KPSS test statistics (12 lags plus trend and intercept).

Stationarity of the annual change in the overnight rate is supported by both the ADF and the KPSS tests. The CPI price index from which the inflation rate is computed comes from the Bank of Canada. The ADF test can rejects the null of unit root only at 10% significance level. While, the KPSS test cannot reject the null of stationarity at 5% when the intercept is included in the model.

![Fixed length symmetric (Baxter-King) filter](image1)

Real variables data are obtained from Datastream. The IP gap series are stationary. The ADF test rejects the null of unit root for the IP rate of growth of with p-value 0.04. Results for the Hodrick-Prescott filtered gap and the unemployment rate are similar. Both KPSS and the Phillips-Perron test confirm stationarity. In these

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\(^{71}\) The KPSS test critical values are 0.216, 0.146, and 0.119 at 1%, 5%, and 10% significance levels respectively.
diagrams we have highlighted the NBER recession; however, Canada suffered the recession of 1990-1991 with few months delay with respect to U.S.

Appendix A2.II - Estimations

### Affine Term Structure Model

<table>
<thead>
<tr>
<th>U.S. Nominal TS</th>
<th>$L_{t-1}$</th>
<th>$S_{t-1}$</th>
<th>$C_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_t$</td>
<td>0</td>
<td>0.9876</td>
<td></td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_t$</td>
<td>0</td>
<td>0.0062</td>
<td>0.9766</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.095)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td>$C_t$</td>
<td>0</td>
<td>0.0036</td>
<td>-0.0142</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.325)</td>
<td>(0.133)</td>
<td>(0.019)</td>
</tr>
</tbody>
</table>

| diag(Σ)        | 5.7e-0.7  | 2.2e-0.7  | 1.9e-0.7  |
| $p$-val        | (0.000)   | (0.000)   | (0.000)   |

<table>
<thead>
<tr>
<th>const</th>
<th>$L_t$</th>
<th>$S_t$</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_L$</td>
<td>0.0813</td>
<td>0.0210</td>
<td>0.0131</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.091)</td>
<td>(0.002)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>$\Lambda_S$</td>
<td>-0.0509</td>
<td>0.0075</td>
<td>0.0581</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.151)</td>
<td>(0.084)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>$\Lambda_C$</td>
<td>-0.0036</td>
<td>-0.0090</td>
<td>0.0159</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.232)</td>
<td>(0.220)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Table 2.18

### Nelson-Siegel Model

<table>
<thead>
<tr>
<th>U.S. Nominal TS</th>
<th>$\mu_F$</th>
<th>$L_{t-1}$</th>
<th>$S_{t-1}$</th>
<th>$C_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_t$</td>
<td>0.1087</td>
<td>0.9853</td>
<td>-0.0062</td>
<td>-0.0091</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.997)</td>
<td>(0.999)</td>
</tr>
<tr>
<td>$S_t$</td>
<td>0.0137</td>
<td>0.0113</td>
<td>0.9239</td>
<td>0.0493</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.158)</td>
<td>(0.205)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$C_t$</td>
<td>0.0047</td>
<td>-0.0028</td>
<td>0.0724</td>
<td>0.9043</td>
</tr>
<tr>
<td>$p$-val</td>
<td>(0.364)</td>
<td>(0.583)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

| diag(Ω)        | 0.0576  | 0.0739    | 0.5848    |
| $p$-val        | (0.000) | (0.000)   | (0.000)   |

Table 2.19

As expected, the intercept is significant only for the level.
**Nelson-Siegel Model**

<table>
<thead>
<tr>
<th></th>
<th>( \mu_t )</th>
<th>( L_{t-1} )</th>
<th>( S_{t-1} )</th>
<th>( C_{t-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_t )</td>
<td>0.1287</td>
<td>0.9855</td>
<td>0.0006</td>
<td>-0.0382</td>
</tr>
<tr>
<td>( p\text{-val} )</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.480)</td>
<td>(0.990)</td>
</tr>
<tr>
<td>( S_t )</td>
<td>0.0143</td>
<td>0.0227</td>
<td>0.9514</td>
<td>0.0262</td>
</tr>
<tr>
<td>( p\text{-val} )</td>
<td>(0.030)</td>
<td>(0.015)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>( C_t )</td>
<td>0.1152</td>
<td>0.0058</td>
<td>0.0618</td>
<td>0.8694</td>
</tr>
<tr>
<td>( p\text{-val} )</td>
<td>(0.000)</td>
<td>(0.341)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

\( \text{diag}(\Omega) \) | 0.0709 | 0.2433 | 0.0685 |
| \( p\text{-val} \) | (0.000) | (0.000) | (0.000) |

*p-values in parenthesis.*

Table 2.20
Chapter 3

A Non Linear Approach for the Term Structure of Interest Rates, Term Premia, and Monetary Policy Expectations

Abstract

According to the expectations theory rational agents can exploit the informative content of the term structure to predict future changes in both interest rates and inflation. However, although appealing, the expectations hypothesis has found weak empirical support. Risk aversion, time-varying term premia, as well as monetary policy uncertainty may represent significant sources of non linearity in the analysis of the expectations theory. In this chapter we thus propose to investigate the predictive power of the yield spread within threshold models. We find evidence that the informative content of the term structure crucially depends either on expectations or on uncertainty about the future stance of monetary policy. In addition, we document that the ability of the yield spread to predict future output growth is inversely proportional to the level of term premia. In line with previous research our approach highlights that an upward sloping yield curve predicts faster output growth. More importantly, we provide evidence suggesting that low levels of term premia seem to be of great stimulus to economic activity; we interpret this effect as a sign of self-fulfilling expectations.
“I suggested (but certainly did not prove) that the expectations theory fails because long rates are far more sensitive to short rates than rational pricing models predict. This hypothesis may or may not be correct. My main purpose in calling attention to the term structure puzzle here is not to resolve it, but rather to urge central bank research departments to give it high priority. It may be the piece of monetary transmission mechanism about which we are most in the dark.”

Alan S. Blinder, “Monetary Policy Today,” 2006 (Chapter 3).
3.1 Introduction

The term structure of interest rates is regarded to be a fundamental indicator of the state of the economy. The slope of the yield curve is, in fact, a useful predictor of both future inflation and economic growth. In addition, according to the expectations hypothesis, also movements in interest rates can be anticipated by observing current values of the yield spread. In particular, the expectations theory asserts that long rates are a maturity-weighted average of spot rates. Unfortunately, although appealing, EH has been almost invariably rejected in empirical studies. The expectations theory of the term structure is of primary importance for both macroeconomists and financial economists. From a macroeconomic perspective, the level of long rates exerts influence on aggregate-spending decisions, while short rates represent the opportunity cost of holding money. From a financial perspective, the pricing of derivative securities depends on the evolution of interest rates over time. Additionally, understanding the relationship between short and long term rates is relevant for policymakers since it has important implications for the conduct of monetary policy and for the management of public debt maturity.

The predictive power of the spread has been usually investigated in linear models. However, in this chapter we argue that linear models may be inaccurate to capture the effective ability of the spread to anticipate the future path of both macroeconomic and financial variables. For instance, risk aversion as well as uncertainty regarding the future stance of monetary policy may introduce significant non linear effects in the analysis of EH. We thus propose to analyse the predictive ability of the yield spread in a multiple regime framework which, on one side, allows for time variation in term premia, and, on the other side, reflects more effectively agents’ expectations about the future stance of monetary policy.

One of the main advantages of analysing EH within threshold models is that this technique accounts for the criticism by Thornton (2004), and Kool and Thornton (2004) who attribute “anomalous” empirical findings in favour of EH to the presence of extreme observations. Hence, clustering observations into sub-regimes, as we do, diminishes substantially the effect of outlier data. Previous results in the literature suggest that the TS slope is helpful to forecast long term changes in short rates (Fama and Bliss, 1987; Campbell and Shiller, 1991); in addition, empirical research has highlighted that the spread, at medium-long horizons, is informative about future inflation changes (Mishkin, 1990a, 1990b; Estrella and Mishkin 1997) and output growth (Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002; Ang, Piazzesi, and Wei, 2006; Wright, 2006). However, detection of non linearity can potentially improve the forecasting ability of the spread, since linear models can be viewed as constrained non linear models. We find evidence, in fact, that threshold effects are

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relevant in the empirical analysis of EH. Working with U.S. data we examine the informative content of the U.S. term structure from 1964 to 2002.

Our analysis is developed along three different lines. First we assess the ability of the spread to anticipate future movements in short rates within a threshold model for term premia. Second, we examine whether the yield spread is informative about future inflation once agents’ expectations about the incoming stance of monetary policy are appropriately adjusted. Third and finally, we investigate the extent to which a regime-switching model for term premia influences the prediction of future economic growth.

The interest in non linear models in general, and in threshold models in particular, is motivated by the fact that EH has been traditionally rejected in linear settings. Firstly, our approach accounts for the evidence attributing the empirical failure of EH in single equation models to the presence of time-varying term premia (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tzavalis and Wickens, 1997). Secondly, inspired by Favero, Kaminska, and Soderstrom (2005) we condition EH analysis to a more appropriate informative set by allowing either for non linearity or for non neutrality of monetary policy (Feroli, 2004; Ravenna and Seppala, 2006).

Our work hinges on the crucial assumption that the difference between long and short term rates reflects expectations regarding the incoming monetary policy stance, as suggested by Laurent (1988, 1989), Bernanke and Blinder (1992), Bernanke (1990), Bernanke and Mihov (1998). The rationale works as follows. The monetary authority can easily influence short rates through policy interventions; however, long rates are market-driven and depends on a complex variety of factors. Moreover, given high persistency of long rates, which can be regarded as a proxy for the equilibrium level of short rates, in a Wicksellian sense, the yield spread provides with as a measure of relative policy tightness.

Our findings suggest that the yield spread is extremely informative about future movements in both interest rates and macroeconomic variables when the informative content of TS is conditioned to non linearity in either term premia or monetary policy expectations.

On the one hand, our results can be compared with previous studies on the asymmetric effects of monetary policy (Morgan, 1993; Rhee and Rich, 1995; Karras and Stokes, 1999; Feroli, 2004). It is, in fact, acknowledged that loose monetary policy exerts a weaker impact on output growth than severe policy (the traditional Keynesian asymmetry). On the other hand, our findings highlight that term premia are inversely related to the business cycle (Cochrane and Piazzesi, 2005; Ludvigson and Ng, 2006; Backus and Wright, 2007; Mele, 2007). The countercyclical pattern of term premia is coherent with both agents’ consumption smoothing behaviour and basic financial theory, which recommends to buy and hold a security offering high payoff in bad times rather than in good times.
The rest of the chapter is organized as follows. In Section 3.2 we briefly discuss a selected survey of the literature. Section 3.3 presents details of the threshold methodology. In Section 3.4 we discuss EH and present empirical evidence about the prediction of interest rates in non-linear models. Section 3.5 deals with the non-linear relationship linking EH, monetary policy expectations, and inflation prediction. In Section 3.6 we examine the ability of the spread to anticipate future growth in economic activity within a threshold model for term premia. Section 3.7 concludes. Data are presented in Appendix A3.1.

### 3.2 Literature Review

An extensive review on the empirical examination of the EH is presented in Chapter 1; hence, we only summarize some results succinctly in this Section.

The expectations theory has found weak support in empirical studies. Economists have offered several attempts to rationalize this tricky puzzle; Shiller, Campbell, and Schoenholtz (1983) point out that linear specifications for testing EH are inappropriate so long as bonds duration is not constant over time. Campbell and Shiller (1991) find “an apparent paradox: the slope of TS almost always gives a forecast in the wrong direction for the short term change in the yields on longer bonds, but gives a forecast in the right direction for long term changes in short term rates”.

The empirical failure of EH might be due to the over-reaction of long rates to the expected change in short rates (Campbell and Shiller, 1991; Hardouvelis, 1994) In addition, Campbell (1995) and Hardouvelis (1994) attribute the weird sign of long term rates forecasts to large measurement errors.

EH equations imply a constant term premium over time, thus a time-varying term premium correlated with the spread may well account for the failure of the theory (Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tsavalis and Wickens, 1997). Along this line also McCallum (1994).

Mankiw and Miron (1986) find that the TS slope is informative about future movements in short rates only before the creation of the Fed suggesting that the high predictability is merely due to a mean reverting process followed by the short term policy rate. After 1915, instead, the interest rate smoothing policy has affected the predictability of short rates thus diminishing the predictive power of the spread. Kool and Thornton (2004) find that the apparent support for EH before the creation of the Fed is due to the presence of extreme observations (financial panics in 1907–1908).

EH has been examined in different monetary regimes. Hardouvelis (1988) shows that the spread carries substantial predictive power between October 1979 and October 1992. In the same vein, Simon (1990) finds that the TS slope significantly anticipates future changes in interest rates during the non-borrowed reserve operating procedure. Roberds, Runkle, and Whiteman (1996) provide
evidence in favour of EH using daily data for settlement Wednesdays. Thornton (2005) believes these results are contrary to common wisdom. He rationalizes as follows: “these results are anomalous in that they suggest that the funds rate is more predictable (1) during periods when the Fed is targeting monetary aggregates than when it is explicitly targeting the federal funds rate and (2) on days when there are large idiosyncratic shocks to the federal funds rate”.

Mishkin (1990) examines whether the yield spread is useful to predict future inflation. For maturities between nine and twelve months the TS slope carries information about future inflation but it is not informative about the real TS. Evidence suggests that an inverted yield curve reflects expectations of falling inflation. Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997) investigate whether the spread is a significant predictor of future output growth. Although results differ across countries, they are generally supportive of the predictive power of the spread. A recent strand of research has examined whether the term premium, rather than the yield spread, is informative about future GDP growth (Hamilton and Kim, 2002; Favero, Kaminska, and Soderstrom, 2005; Ang, Piazzesi, and Wei, 2006).

As outlined in this Section, almost all evidence regarding the predictive power of the spread has been worked out in linear models. Although useful linear models might fail to fit data when economic variables display non linear features. In particular, there is substantial evidence about the non linear conduct of monetary policy. Ruge-Murcia (2003) documents an important asymmetric behaviour of the monetary authority towards inflation depending on whether actual inflation is above or below the targeted rate. Analogously Dolado et al. (2005) find that combining a non linear Phillips with a typical quadratic loss function leads to a non linear optimal policy rule. Data evidence suggest that the asymmetric conduct of monetary policy is stronger in the Euro area than in U.S. Cukierman and Muscatelli (2002) show that in G7 countries the preferences of the monetary authority vary over the business cycle and depend on the credibility of the monetary regime.

There is considerable evidence also linking the steepness of the term structure to the expected stance of monetary policy (Bernanke and Blinder, 1992). Hence, the informative content of the TS slope should reflect non linearity in the monetary policy conduct thus affecting the corroboration of EH. In this chapter we suggest that exploiting the informative content of TS in linear models might constitute a violation of the rationality principle leading to the rejection of EH. We thus propose a non linear regime-dependent model relating the predictive power of the spread to the asymmetric behaviour of monetary policy and to agents’ risk aversion.
3.3 Examining Non Linearity in Threshold Models

The non linear nature of economic data is documented in many empirical works; financial series, in particular, tend to display non linear features. In this Section we thus review a bit of theory about threshold models widely adopted throughout the Chapter. A natural approach for examining non linear relationships seems to define some states of the world, or regimes, allowing for the possibility of different performance or asymmetric behaviour of economic variables.

Threshold models offer the possibility of analysing the same issue into different sub-regimes since they provide with a cluster of available information, as summarized by the dynamics of a pre-determined threshold variable. Threshold models can be regarded as a deterministic version of Markov switching models\(^{72}\), in which the transition between regimes occurs whenever the threshold variable crosses a specific value known *ex-ante*. The empirical framework supplied by threshold modelling allows exploring the same phenomenon in different regimes, or, basically, from different perspectives.

One of the goals of this chapter is to understand why EH is weakly supported by data evidence. EH suggests that the TS slope should be informative to the extent of anticipating future movements of interest rates and inflation. We believe that using linear models to analyse EH provides only with a partial and incomplete view on the issue. For instance, the assumption of rationality which lies beneath EH might be excessive in some circumstances; it thus seems reasonable to assume that rationality might be contingent to the state of the economy. Financial distress as well as imperfect information might affect agents risk aversion at an aggregate level thus bounding rationality (Simon, 1957, 1991).

Suppose \( x_i \) \((n x 1)\) is the variable of interest, i.e. the dependent variable (where \( n \) is the overall number of observations); suppose further that a theoretical model suggests \( x_i \) being related to some variables \((z_a)\) stacked in matrix \( Z \) \((n x k)\). A typical linear model for examining the relationship between \( Z \) and \( x \) would be:

\[
x_i = Z_i \theta + \varepsilon_i
\]

\( \theta \) is the \((k x 1)\) vector of parameters that captures the static relationship between \( Z \) and \( x \). However, if the relationship linking \( Z \) and \( x \) is not constant over time, it would be recommendable to split the entire sample into two (or more) sub-samples, each representing a specific regime. In a similar context parameters \( \theta \) would become time-varying or regime-dependent.

\(^{72}\) In Markovian models the probability of switching regimes has to be estimated since the transition across regimes is not known in advance. The unobservable switch is governed by a first-order stochastic process; hence, the probability of regime shifting depends only on the regime one period before. Markov switching models can thus be considered a stochastic version of deterministic structural change models.
and \( Z_{t_2} \) are appropriate time-dependent partitions of matrix \( Z_t \). Both \( Z_{t_1} \) and \( Z_{t_2} \) contain all the explanatory variables present in \( Z_t \); they differ from each other because of the temporal distribution of observations. In particular matrix \( Z_{t_1} \) \((n_1 \times k)\) contains realizations of \( Z_t \) which corresponds chronologically to the observations of the threshold variable below a precise estimated value. While \( Z_{t_2} \) \((n_2 \times k)\) includes realizations of \( Z_t \) associated to the values of the threshold variable above the estimated threshold.

\[
x_t = Z_{t_1}	heta_1 + Z_{t_2}	heta_2 + \varepsilon_t
\]

The difference between the values of estimated parameters in different regimes is the \textit{delta} threshold effect \((\delta_\theta = \hat{\theta}_2 - \hat{\theta}_1)\).

The central limit theorem guarantees that the distribution of a sufficiently high number of \textit{i.i.d.} random variables, with finite mean and variance, is approximately Normal. Furthermore, the consistency property of \textit{least squares} estimators states that for very large sample size the estimated coefficients approach the true population value\(^73\), thus suggesting a unique and stable relationship between the dependent and the explanatory variables. Ideally, if it was possible to observe the true data generating process of an infinitely large number of observations, we would figure out the exact linear relationship existing among variables without the need of estimating such relationship. In particular, the threshold effect should be null \((\delta_\theta \to 0)\) as \( n \) approaches to infinity \((n \to \infty)\). Since finite samples estimations are computed on a partial and incomplete informative set, there is space to analyse data with sophisticated techniques that allow detecting for eventual non linearity.

The key difference between threshold and structural change models is also the main reason behind our choice. In a standard two-regime structural change model the entire sample is split at one point in time, hence regimes are defined temporally. In structural change models a continuous sequence of chronologically ordered data characterizes both regimes. This work, instead, focuses on the possibility that regimes can switch back and forth depending on the estimated value of a pre-determined threshold variable; hence, time continuity is not a matter here. Structural change and thresholds models are conceptually similar but accomplish different purposes. Threshold modelling is a flexible tool to capture switches in regime that occur frequently over time; the evolution of a key variable, in fact, determines a regular stream of new information which allows agents to re-examine expectations.

Although analogous structural change and threshold models are technically different. Structural change models usually imply a time trend which affects the distribution of the threshold variable;

\(^73\) The probability limit that the absolute difference between the estimated coefficient and the true population parameter is smaller than any arbitrarily tiny positive number tends to one.
while, the threshold variable should be covariance stationary in threshold models. Equation (3.2) can be re-written using the indicator function $I(\bullet)$

$$x_i = Z_i, \theta_i [1 - I(thr_i > \gamma)] + Z_i, \theta_i [I(thr_i > \gamma)] + \epsilon_i$$

(3.3)

where $thr_i$ denotes the selected threshold variable and $\gamma$ is the estimated threshold value. Dummy variables are useful econometric tools to represent the indicator function:

$$d_{1i}(\gamma) = I(thr_i \leq \gamma) \quad \text{and} \quad d_{2i}(\gamma) = I(thr_i > \gamma)$$

(3.4)

Equation (3.3) becomes

$$x_i = d_{1i}(\gamma) Z_i, \theta_1 + d_{2i}(\gamma) Z_i, \theta_2 + \epsilon_i$$

(3.5)

Partitioning the regressors as explained above yields

$$x_i = Z_i(\gamma) \theta_1 + Z_2(\gamma) \theta_2 + \epsilon_i$$

(3.6)

The explanatory variables are functions of the estimated threshold. In a more compact form:

$$X = Z(\gamma) \phi(\gamma) + \epsilon$$

(3.7)

Equation (3.7) can thus be estimated by ordinary least square; the unbiased estimator is merely

$$\hat{\phi}(\gamma) = [Z^T(\gamma) Z(\gamma)]^{-1} Z^T(\gamma) X$$

(3.8)

After fitting the model and computing residuals we consider the sum of squared residuals

$$S(\gamma) = \hat{\epsilon}^T(\gamma) \hat{\epsilon}(\gamma)$$

(3.9)

Minimizing the above quantity is necessary to obtain the threshold estimate

$$\hat{\gamma} = \arg\min[S(\gamma)] \quad | \gamma \in \Gamma_n$$

(3.10)

Finally we derive the coefficient estimates $\hat{\phi}(\gamma) = \hat{\phi}(\hat{\gamma})$. From a computational perspective, the minimization process is simply a grid search over the range $\Gamma_n = \Gamma \cap \{\gamma_1, \gamma_2, ..., \gamma_{n-1}\}$. Once the threshold value is known, two separate equations can be estimated; leading to the same outcome of a regression with deterministic dummy variables. A clear representation of the two-regime threshold model is the following:

$$\begin{cases} 
  x_{1i} = Z_1, \theta_1 + \epsilon_{1i}, & thr \leq \hat{\gamma} \\
  x_{2i} = Z_2, \theta_2 + \epsilon_{2i}, & thr > \hat{\gamma}
\end{cases}$$

(3.11)
The optimization can be successfully computed if some regularities conditions are satisfied. In particular, threshold estimations require processes \( \{z_{it}, thr_t, \epsilon_i\} \) to be strictly stationary and ergodic, thus excluding time-trended and integrated variables. Residuals should also be zero-mean. The fourth moment of both explanatory variables (\(z_{it}\)) and noise (\(\epsilon_i\)) has to be finite. In addition, the quantity \(E_{z_{it}}\) should be finite for all regressors (\(\forall i\)). Finally, some full-rank conditions are imposed on the covariance matrixes both to guarantee non-degenerated asymptotic distributions and to rule out multi-collinearity.

### 3.4 Regime-Dependent Term Premia and the Expectations Hypothesis

According to the expectations hypothesis any long term rate can be expressed as a maturity-weighted average of short rates, thus the slope of the term structure should be informative about future movements in interest rates.

Unfortunately, single-equation models fail to corroborate EH when deviations between the expected theoretical spread and the ex-post observed spread are large. Such large deviations are usually coupled with soaring volatility in the term premium. As Campbell (1995) argues, in fact, the conventional test of EH does not generate theory-consistent coefficient estimates when the variance of rationally expected changes in short rates is small relative to the variance of the term premium.

In addition, there is lot of evidence that time-varying term premia can be considered responsible of the empirical failure of EH (Fama, 1986; Cook and Hahn, 1989; McCallum, 1994, 2005; Campbell, 1995; Lee, 1995; Tzavalis and Wickens, 1997; Hejazi and Li, 2001). As shown later, term premia provide with a measure of the unexpected effect of monetary policy actions onto the TS of interest rates, therefore they captures agents’ perception of uncertainty.

We believe that a non linear model for term premia, rather than a simple linear equation, reflects more accurately how investors process available information.

In this Section we thus estimate a threshold model for EH that allows for a regime-dependent behaviour of term premia. Evidence suggests that the yield spread is informative about future movements in short rates once the risk-averse attitude of economic agents is properly taken into account.

Threshold models offer the possibility of clustering available information allowing heterogeneous behaviour across regimes. The following intuition can give some insight about the potential usefulness of adopting threshold models to assess the predictive power of the yield spread. Suppose the economy is hit by a negative supply shock, like a sudden increase in the oil price; suppose
further the effects on inflation are expected to be long-lasting; hence, it follows a sharp and permanent increase in long term rates. The yield curve becomes steeper reflecting the rise in term premia. The shock affecting the dynamics of interest rates also increases the probability of switching regime. Forward-looking agents rationally expect the transition to the new regime and, in such enriched informative context, they can forecast theory-consistent variations in interest rates. Since large values of the spread anticipate upward movements in short rates, agents expect future rise in interest rates. In addition, rational agent can anticipate future policy tightening as long as the central bank takes action to calm down inflationary pressures. Threshold models for risk premia thus provide with a technical framework that works as uncertainty reducer augmenting the predictability of interest rates dynamics. Agents rationalize their attitude towards risk and process information more effectively. In this sense, we speculate there might be consistency between threshold models and the hypothesis of complete financial markets, where agents know the payoff in each state of the world.

In addition, we recall that one of the main benefits of employing threshold models to analyse EH is that they are immune to the criticism by Thornton (2004) and Kool and Thornton (2004). EH states that long term yields can be expressed as a maturity-weighted average of expected future spot rates:

$$y^m_t = \left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E_t q^m + t_{n,m}$$

where $E_t$ represents rational expectations conditional to the available information at time $t$. The long term rate $y^m_t$ has maturity $n$, while the short rate $y^m_t$ has maturity $m (m < n)$. The maturity ratio $(n/m)$ is an integer. The pure version of EH implies a null term premium $(t_{n,m} = 0)$; while the traditional version of EH assumes a constant term premium, i.e. the expected holding period returns are equal on bonds of all maturities. Subtracting the short rates from both sides, and readjusting, yields:

$$t_{n,m} = \left[ \left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E_t y^m_{t+q} - i^m_t \right] - \left[ y^n_t - y^m_t \right]$$

Expression (3.13) reveals that, when expectations are not rational, the term premium is simply computed as the deviations between the expected theoretical and the actual observable spreads, i.e. it is the unpredicted component of the yield spread. In addition, the term premium can be viewed as the sum of a liquidity premium and a risk premium, which capture respectively investors’ preferences and agents’ risk-aversion. Simple algebra leads to the fundamental EH equation:

$$y^n_t = \left( \frac{m}{n} \right) y^m_t + \left( \frac{n-m}{n} \right) E_t y^{n-m}_{t+m}$$

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where $y_{r,m}^{n,m}$ denotes the forward (implicit) rate prevailing from $m$ to $n$. According to EH, $y_{r,m}^{n,m}$ would be the yield associated to any bond issued at time $m$ with maturity $n$. Both sides of (3.14) must be equal to satisfy the no-arbitrage condition. Equation (3.14) simply states that a maturity strategy (LHS) must guarantee the same rate of return of a roll-over strategy (RHS).

The following equation captures the predictive ability of the spread to anticipate future movements in short yields over the life of the long term bond:

$$
\sum_{q=1}^{n-m} \left( \frac{n-mq}{n} \right) (y_{r+mq}^{n} - y_{r+m(q-1)}^{m}) = \alpha + \beta (y_{r}^{n} - y_{r}^{m}) + \varepsilon_{t}
$$

(3.15)

Or, equivalently:

$$
\left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E_{t} y_{r+mq}^{n} - y_{r}^{m} = \alpha + \beta (y_{r}^{n} - y_{r}^{m}) + \varepsilon_{t}
$$

(3.15')

Thornton (2004) calls the Campbell-Shiller equations (3.15) the conventional test of EH, since they usually return positive estimates of the slope coefficient $\beta$ for any pair $(m,n)$. The LHS represents the theoretical, or perfect foresight, spread. EH holds if coefficients $\alpha$ and $\beta$ are zero and one respectively, i.e. if the actual spread perfectly matches the theoretical spread. The $n$-period overlapping errors implied by (3.15) produce serially correlated (MA) least squares residuals. To correct for the presence of non-spherical disturbances, both Hansen and Hodrick (1980) and Newey and West (1987) have suggested a consistent estimate of the variance-covariance matrix.

In this Section we provide evidence that in favour of EH when the dynamics of the term premium is allowed to separate regimes:

$$
\left\{ \begin{array}{c}
\left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E_{t} y_{r+mq}^{n} - y_{r}^{m} = \alpha_{1} + \beta_{1} (y_{r}^{n} - y_{r}^{m}) + \varepsilon_{t}, \quad thr \leq \hat{\gamma} \\
\left( \frac{m}{n} \right) \sum_{q=0}^{n-m} E_{t} y_{r+mq}^{n} - y_{r}^{m} = \alpha_{2} + \beta_{2} (y_{r}^{n} - y_{r}^{m}) + \varepsilon_{t}, \quad thr > \hat{\gamma}
\end{array} \right.
$$

(3.16)

The threshold variable $thr$ is either the previous realization of the term premium ($tp_{r-i}^{n,m}$) or its absolute value. The term premium is a measure of monetary policy uncertainty and it captures agents’ attitude toward risk. Separating regimes on the basis of the unexpected component of the future monetary policy stance gives investors the opportunity of conditioning their aptitude to predict on the perception about future monetary policy uncertainty. A multiple regime framework for risk premia accounts also for the empirical regularity that time varying premia affect the
corroboration of EH. Finally, as discussed above, conditioning the determination of regimes upon the unexpected component of the yield spread allows to reduce aggregate uncertainty.

Threshold methodology allows us to match two salient features outlined in the empirical literature regarding EH. On the one hand, we allow for time variation in term premia; in particular, the term premium is also assumed to be regime-dependent \( tp(\hat{\gamma}, t, n, m) \). Threshold models are a special case of Markov switching models in which the probability of switching regime is known *ex-ante*. On the other hand, we follow Mankiw and Miron (1986), who suggest that different (monetary) regimes could determine considerably different results for EH.

Some more insight to justify the adoption of threshold modelling follow. Using U.S. yields data between 1964 and 2002 from the McCulloch-Kown and Fama-Bliss datasets (depending on maturity) we estimate the Campbell-Shiller equation (3.15) to predict future movements in short term interest rates. Regression (3.15) is estimated with a *rolling window* procedure to emphasize the time-varying pattern of the slope coefficient. Each regression is estimated by Newey-West corrected *least squares* with 60 monthly observations. Figure 3.1 shows the *rolling* estimation of the slope coefficient \( \hat{\beta} \) and the associated probability values (null hypothesis \( \hat{\beta} = 0 \)) for both couples of maturities \((120, 3)\) and \((60, 3)\).

**Figure 3.1**

*Rolling* estimates of the slope coefficient tend to oscillate around one. However, both in early-mid 1980s and late 1990s the slope coefficient is far from the EH-implied level. Hence, results might suggest the presence of non linearity in the empirical model used to test EH74. In particular, \( \beta \) estimates are not significant when both the level and the volatility of term premia are large. Figure 3.2 plots both the term premium and its volatility for the pairs of maturities \((120, 3)\) and \((60, 3)\); volatility is computed as the squares of the first differences.

---

74 Equation (3.15) has been estimated in the entire sample (between 1964 and 2002). In months characterized by high local volatility of the term premium the Chow breakpoint test could not reject the null of no structural break.
Therefore, our choice of setting the term premium as threshold variable allows obtaining a cluster of volatility as well, since the variability of term premia is strictly associated to the level. When the economy experiences high inflation, term premia are unusually large as it happened in the 1980s. In addition, weak GDP growth boosts uncertainty which is reflected in agents’ risk aversion. A similar conjuncture has important policy effect. A negative output gap could induce the monetary authority to reconsider its preferences, since it becomes socially optimal including unemployment among final targets. The enhanced complexity of the macro scenario coupled with uncertainty about monetary policy goals affect investors’ ability to anticipate future movements in interest rates, thus generating important downward bias in EH estimations of the slope.

The threshold methodology thus provides with a useful framework to separate periods with low uncertainty from periods characterized by high uncertainty. The term premium not only captures agents’ sentiment towards risk but it is also a proxy for (excess) bond returns. We need to forecast the threshold value since the term premium is not directly observable. Ideally, once investors know the threshold estimate, they are able to distinguish regimes with certainty; threshold modelling is thus appealing because it acts as an uncertainty reducer. Estimation results of model (3.16) are reported in Table 3.1.

Single-regime estimates of the traditional Campbell-Shiller equation are reported in the left panel of Table 3.1. The central panel shows estimates when the threshold variable is the term premium ($tp_{t-1}$); estimates in the right part refer to the threshold model whose regimes are determined by the absolute value of the term premium.

---

**Figure 3.2**

$tp (120,3)$: term premium ($n$:10 years; $m$: 3 months). $tp (60,3)$: term premium ($n$:5 years; $m$: 3 months). $tp$ volatility: volatility of the term premium (squared first difference).

---

75 Term premia reflect the future evolution of the stochastic discount factor (Rudebusch, Sack, and Swanson, 2007) and are perfectly correlated with (log) excess bond returns (Cochrane and Piazzesi, 2005). Campbell and Shiller (1991) point out that term premia represent the unexpected component of the yield spread thus reflecting market participants’ inability of anticipating future interest rates. Term premia capture the incapacity of anticipating the future stance of monetary policy as summarized by the yield spread (Laurent, 1988; Bernanke and Blinder, 1992).

76 Few methods are available to achieve the same objective: Markov switching models, structural change multiple-break models, and threshold models. The choice of threshold modelling is motivated by the fact that we wish to avoid constraining regimes to time-continuity; in this respect threshold frameworks allow for greater flexibility. As explained above, threshold modelling allows distinguishing the high uncertainty regime from the low uncertainty regime.

77 $\gamma$ is the estimate of the threshold variable. The joint goodness of fit ($j-R^2$) is computed as one minus the ratio between the sum of squares of both regimes regressions and the total sum of square of the single regime regression. obs denotes the number of observations. $R^2$ is the goodness of fit in each regime. $\beta$ the slope estimated coefficient with associated $p$-values of the $t$-test (in parenthesis). 1 and 2 denote regimes below and above the threshold respectively.
In the entire sample the estimated slope coefficient increase with the long term maturity $n$. At the very short end the TS spread is not particularly informative about future short rates. However, Rudebusch (1995) documents a significant predictive power of the spread for horizons lower than two months using data with different frequency. Our analysis is not comparable with his study since we do not consider maturities shorter than 3 months. At any horizon, the single-regime predictive power of the spread, as implied by the regression goodness of fit, is definitely poor. Results substantially improve in the threshold setting. Both regimes $\beta$ estimates get close to one and are statistically significant. The joint goodness of fit is much higher in the threshold model; moreover, $R^2$ is also higher in each sub-regime than in the single regime.

When the absolute value of the term premium discriminate regimes, at short-medium maturity the slope coefficient is statistically significant only in regime 1 characterized by moderate uncertainty; while, regime 2 estimates are not significant. Evidence seems to support the presence of an empirical asymmetric effect in the analysis of EH.

Our results can be interpreted consistently with the idea put forward by Mankiw and Miron (1986). Separating regimes on the basis of the term premium allows investors to identify two distinct states
of the world, each characterized by a contingent level of uncertainty; in particular, in both states the range of values assumed by the term premium is bounded and term premia volatility limited (Table 3.4). Hence, in each regime the low variability of term premia allows agents to make more accurate predictions. In Figure 3.3 we plot the estimated slope $\beta$ against maturity together with the 95% confidence interval bands. In left panel we report the slope estimates obtained with the traditional linear model; while, in the central and right panels we plot the slope coefficients against maturity in regime 1 and 2 respectively.

Despite single-equation evidence, threshold estimates reveal that TS is informative about future interest rates movements also at short horizons. Empirical evidence suggests that the predictive power of the spread increases with maturity in both the single and in the threshold regimes. In the single regime the slope coefficient never reaches one though. Threshold estimates of the slope coefficient are quite close to one; so that, in each regime the predictive power of the yield spread is substantial. In particular, the 95% confidence interval at medium-long horizons contains the expectations hypothesis value one (the horizontal dashed line).

Non linear investigation of EH allows obtaining theory-consistent empirical results since threshold modelling provides with a suitable framework to cluster term premia volatility, which is responsible of EH failure in linear models. Term premia volatility influences agents’ risk aversion thus affecting their rationality. The scatter diagrams in Figure 3.4 show the importance of analysing the predictive power of the spread, which reflects investors’ rational expectations, using econometric methods that allow for regime shifting. In all the diagrams the regression line captures the relationship between the theoretical and the actual spread, i.e. a measure of the $\beta$ estimates in the Campbell-Shiller equation (3.15).

Results in Figure 3.4 regards the pair $(120, 3)$. The steepness of the red line in both sub-regimes is closer to one than that displayed by the single-regime line. In addition, sub-regimes diagrams show

---

78 However, in regime 2 when the threshold is the absolute value of the term premium this result does not hold.
by large a better fit. A comparison between the second and the third plot reveal that high values of
the actual spread are associated to large values for term premia (regime 2).

Figure 3.4

In Figure 3.5 we report the scatter diagram for the couple of maturities (60, 3). Again results show
by large a better fit in threshold sub-regimes. In both sub-regimes the slope of the red line is close to
the 45-degree line; hence, the ability of the spread to anticipate future interest rates movements
improves in sub-regimes.

Figure 3.5

The advantages of threshold modelling can also be captured by the kernel density estimate\(^{79}\) of the
distribution of the variables of interest. The bottom diagrams of Figure 3.6 show how peaked the
density estimate are in regime 1 with respect to the single-regime plots reported in the top panels.
Density distributions concentrated around the mean are typical of mean-reverting and thus easily
predictable process.

\(^{79}\) The Epanechnikov kernel density estimation is performed with automatic bandwidth selection. These results are obtained by
focusing on the pair of maturities (60, 3); similar results hold focusing on other couples of maturities (n, 3). In the entire sample the
standard deviation of the 3-month yields is 2.62, the kurtosis is 5. In regime 1 (below the estimated threshold) the standard deviation
is 2.08 and the kurtosis is 7.18.
Regime 1 density estimate of the short rate process is definitely more concentrated around the mode than the distribution obtained with the entire-sample data. According to Mankiw and Miron (1986) short rates predictability is a crucial issue in the empirical assessment of EH. The empirical distribution in regime 1 is characterized by low values of the term premium. The standard deviation of all variables is much lower in regime 1 than in the entire sample; oppositely the kurtosis is higher in regime 1 than in the entire sample.

Table 3.2

Wald Test Ho: β = 1

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>th = tp,n,m</th>
<th>th =</th>
<th>Regime</th>
<th>(p-val)</th>
<th>Regime</th>
<th>(p-val)</th>
<th>Regime</th>
<th>(p-val)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(120,3)*</td>
<td>(0.0355)</td>
<td>1</td>
<td>(0.0002)</td>
<td>1</td>
<td>(0.3784)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>(0.6925)</td>
<td>2</td>
<td>(0.9896)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(60,3)</td>
<td>(0.0434)</td>
<td>1</td>
<td>(0.7948)</td>
<td>1</td>
<td>(0.0006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>(0.1901)</td>
<td>2</td>
<td>(0.9643)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(36,3)</td>
<td>(0.0085)</td>
<td>1</td>
<td>(0.0154)</td>
<td>1</td>
<td>(0.0005)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>(0.0897)</td>
<td>2</td>
<td>(0.0000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(24,3)</td>
<td>(0.0011)</td>
<td>1</td>
<td>(0.0002)</td>
<td>1</td>
<td>(0.0000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>(0.0892)</td>
<td>2</td>
<td>(0.0001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(12,3)</td>
<td>(0.0000)</td>
<td>1</td>
<td>(0.0010)</td>
<td>1</td>
<td>(0.0058)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
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<td>(0.0020)</td>
<td>2</td>
<td>(0.0000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample jan64-sep02; *jan64-mar97. p-values in parenthesis
Results of the Wald test are supportive of the better fit of EH equations in the threshold setting. The probability of rejecting the null hypothesis of $\beta = 1$ occurs less frequently.

Finally, as a further robustness check we run a rolling estimate of the Campbell-Shiller equation in both regimes. The time-varying behaviour of the slope coefficient in each regime (regime 1 in the left panel; regime 2 in the right panel). Estimates are obtained from rolling OLS (Newey-West corrected) on sequential samples of 50 observations. In both sub-regimes the plot of $\hat{\beta}$ over time is smooth and fluctuates closely around one (horizontal solid line). Furthermore, in each regime the slope coefficients are statistically significant as opposed to the rolling $\hat{\beta}$ estimates obtained in the single regime setting (Figure 3.1). Figure 3.7 is obtained for the maturity pair (60, 3); but results are similar for other couples of maturities (n, 3).

![Figure 3.7](image)

An additional way to compare single regime rolling $\hat{\beta}$ estimates with sub-regimes estimates is shown in Figure 3.8.

![Figure 3.8](image)
The left panel shows the distribution of the estimated coefficient in the single-regime setting (as of the right diagram of Figure 3.1); while, the central and the right panel show the estimated $\hat{\beta}_i$ coefficients in the first and in the second threshold sub-regime respectively. Results are striking. The estimates are definitely more concentrated around one in the threshold sub-regimes, in particular this is true for regime 1, i.e. for low values of the term premium $^80$.

Table 3.3 shows some descriptive statistics of the rolling estimated slope coefficient. The top part of the Table reports statistics worked out within each threshold sub-regime. In both regimes the mean of the estimated slope coefficient is very close to unity for any combination of maturities $(n, m)$. In addition, the variability of $\hat{\beta}_i$ is definitely low in the sub-samples. The standard deviation of the rolling $\hat{\beta}_i$ estimates in the sub-regimes is, in fact, approximately one-tenth of the standard deviation computed in the single regime model.

### Table 3.3

<table>
<thead>
<tr>
<th>long term maturity</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>regime 1 mean($\beta$)</td>
<td>0.9792</td>
<td>0.9814</td>
<td>0.9871</td>
<td>0.9954</td>
<td>1.0104</td>
</tr>
<tr>
<td>stdev($\beta$)</td>
<td>0.0496</td>
<td>0.0529</td>
<td>0.0518</td>
<td>0.0578</td>
<td>0.0434</td>
</tr>
<tr>
<td>regime 2 mean($\beta$)</td>
<td>0.9628</td>
<td>0.9930</td>
<td>0.9651</td>
<td>0.9748</td>
<td>0.9848</td>
</tr>
<tr>
<td>stdev($\beta$)</td>
<td>0.0530</td>
<td>0.0701</td>
<td>0.0776</td>
<td>0.0689</td>
<td>0.0494</td>
</tr>
<tr>
<td>linear mean($\beta$)</td>
<td>0.5671</td>
<td>0.6323</td>
<td>0.8385</td>
<td>0.9929</td>
<td>0.9401</td>
</tr>
<tr>
<td>stdev($\beta$)</td>
<td>0.4427</td>
<td>0.5608</td>
<td>0.7087</td>
<td>0.6486</td>
<td>0.5103</td>
</tr>
</tbody>
</table>

A financial interpretation rationalizes our empirical results. Campbell (1995) argues that a large bias downward of the estimated slope coefficient is due to a small variance of the rationally expected changes in short rates relative to the variance of the term premium. Hence, EH holds when investors are well informed about future movements in short rates. Equation (3.13) can be readjusted in order to express the actual spread in terms of the theoretical spread ($thsp_t$) and the term premium ($tp_t$).

Therefore, the variance of the spread depends on the variance of both its components plus twice their covariance $^81$:

$$
\text{var}\{y^n_t - y^m_t\} = \text{var}\{thsp^n_t, thsp^m_t\} + \text{var}\{y^n_t, y^m_t\} + 2 \cdot \text{cov}\{thsp^n_t, thsp^m_t\} + 2 \cdot \text{cov}\{y^n_t, y^m_t\}
$$

$$
(3.17)
$$

$$
= \text{var}\{y^n_t\} + \text{var}\{y^m_t\} - 2 \cdot \text{cov}\{y^n_t, y^m_t\}
$$

$^80$ Results for other pairs of maturities $(n, m)$ are very similar.

$^81$ The covariance between the theoretical spread and the term premium is negative. At short maturities ($n = 12, 6$) the covariance is close to zero, but still negative. The covariance between long and short term rates is positive. The covariance is a negative function of the distance between maturities $(n - m)$. The variance of short term yields is generally larger than the variance of long term yields.
Table 3.4 shows that in the single-regime model for \( n \leq 36 \), the standard deviation of the expectations-based component is lower than the standard deviation of the term premium; the slope estimate is thus far below unity. Conversely, in each threshold regime the variability of the term premium is much lower than the variability of the theoretical spread. Threshold estimates provide results consistent with the Campbell (1995) view that a large variance of rationally expected changes in short rates generates a small downward bias of the estimated slope coefficient. The augmented predictive power of the spread in both regimes follows directly from the lower level of uncertainty that characterizes each regime, as captured by term premia volatility. This story is also consistent with the idea put forward by Mankiw and Miron (1986), who suggest the low predictability of short rates affect the empirical corroboration of EH. In particular, they attribute the lack of empirical support for EH to the random walk behaviour of the short rate. They argue that weak empirical evidence in favour of EH is due to the policy of smoothing interest rates pursued by the Federal Reserve after its foundation in 1914.

<table>
<thead>
<tr>
<th>Standard Deviations</th>
<th>long term maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>regime 1 spread</td>
<td>0.4134</td>
</tr>
<tr>
<td>thsp</td>
<td>0.6723</td>
</tr>
<tr>
<td>tp</td>
<td>0.6394</td>
</tr>
<tr>
<td>regime 2 spread</td>
<td>0.5074</td>
</tr>
<tr>
<td>thsp</td>
<td>0.7307</td>
</tr>
<tr>
<td>tp</td>
<td>0.6663</td>
</tr>
<tr>
<td>linear spread</td>
<td>0.4656</td>
</tr>
<tr>
<td>thsp</td>
<td>0.9174</td>
</tr>
<tr>
<td>tp</td>
<td>0.9578</td>
</tr>
</tbody>
</table>


Table 3.4

Thornton (2003) points out that the corroboration of EH depends on the ratio between the variances of the short and the long rates. In line with Mankiw and Miron (1986) he believes that uncertainty largely depends upon the conduct of the Fed. In particular, the more volatile short rates are relative to long rates, the closer the estimated \( \beta \) to one. For any pair of maturities \( (n, 3) \) the values of the ratio between the short to long term rate variances are reported in Table 3.5. The relative variance increases with maturity \( n \) both in the entire sample and in each regime determined by the level of the term premium; while, in regimes split by the absolute value of the term premium the relative variance is increasing with maturity only below the threshold (regime 1).
In the final part of this Section we wish to test the conjecture put forward by Mankiw and Miron (1986). They suggest that, after the creation of the Federal Reserve, monetary policy inertia has lowered the predictability of short rates thus affecting the empirical examination of EH. In particular, the short rate has become a martingale due to the practice of U.S. monetary authority of smoothing interest rates. Mankiw and Miron argue, in fact, that the Fed was committed to “stabilize or even peg interest rates”. On the one hand, if the short rate follows a random walk process its dynamics over time is completely casual, and thus unpredictable. On the other hand, yield movements can be inferred if they display a mean-reverting pattern; hence EH should be respected. It thus follows a tricky experiment that allows checking to what extent the predictability of the fed funds affects EH tests. We compute the deviations between the effective fed funds and its moving averages, then we consider the absolute value of this mean-reverting measure as threshold variable discriminating regimes in (3.5). We expect that \( \beta \) estimates are close to one below the estimated threshold (regime 1). Mean-reversion implies that the federal funds rate is easily predictable as its level gets closer to its local mean, i.e. when deviations from its moving average are small. The rationale works as follows. If the fed funds is moving towards the mean it will soon catch it up; moreover, if the fed funds is departing from the mean, but it is still close to it, it will keep on moving off. Conversely, if the fed funds is far from its local mean, it will be difficult to foresee the exact turning point.

From left to right, top panels of Figure 3.9 plot the fed funds together with its 3-, 12-, and 24-month moving averages, and the relative discrepancies. As the black semi-dashed line highlights, the fed funds rate tends to revert towards its local mean displaying a mean-reverting pattern\(^{82}\). The bottom panels show the respective absolute value of the deviations between the fed funds and the moving averages.

---

\(^{82}\) The shorter the horizon to compute the moving average, the higher the speed of adjustment towards the local mean. The dependent variable of the equation used to test the mean-reverting property of the fed funds is the difference between the one-period ahead fed funds and its current value. The explanatory variables of such equation are the current level of the fed funds and the first lag (and a constant). Statistical results are weekly supportive of the mean-reverting behaviour, since the goodness of fit is very low and the estimated (negative) coefficient is marginally significant. Mankiw and Miron (1986) adopt this specification as the discrete-time version of the short rate diffusion process by Vasicek (1977). Therefore, the qualitative analysis reported in Figure 3.9 would be considered as evidence of the mean-reverting behaviour of the effective federal funds rate. Both continuous-time simulations of the
Estimations are reported in Table 3.6. There is evidence suggesting that when the federal funds is close to its local mean (regime 1) the estimated slope coefficient is almost one. The Wald test does not reject the null hypothesis of unity coefficient below the threshold. The predictive power of the spread is thus much higher in the threshold model than in the single-regime equation, as suggested by both the joint and “individual” goodness of fit ($j-R^2$ and $R^2$). The spread seems to be particularly informative about future interest rate movements at medium-short maturities. Evidence provided in this final part of Section 3.4 thus support the theory advanced by Mankiw and Miron (1986). In Figure 3.9 we show the scatter diagrams of the relationship between the theoretical and the actual spreads both in the single-regime (left panels) and threshold sub-regimes (right panels) for the pairs of maturities (36, 3) and (24, 3) using threshold deviations computed respectively with the 24- and 12-month moving average.

<table>
<thead>
<tr>
<th>ffr: effective federal funds rate. MA: moving average. diff: difference between the effective federal funds rate and its moving average. abs: absolute value of the difference between the federal funds rate and its moving average (m stands for month).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3.9</td>
</tr>
</tbody>
</table>
Table 3.6

Sub-regime scatter plots denote a better fit than single regime ones. In particular, in regime 1 the distribution of observations is concentrated around the linear interpolant. The slope of the red curve is always steeper in regime 1 thus confirming evidence in favour of EH.
Finally, also a visual inspection at the distributions around the mode of the rolling estimated slope coefficients can be interpreted as evidence that EH is closer to be respected in regime 1 (Figure 3.11).

<table>
<thead>
<tr>
<th>Kernel Density - single regime</th>
<th>Kernel Density - regime 1</th>
<th>Kernel Density - regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Kernel Density - single regime" /></td>
<td><img src="image2" alt="Kernel Density - regime 1" /></td>
<td><img src="image3" alt="Kernel Density - regime 2" /></td>
</tr>
</tbody>
</table>


**Figure 3.11**

### 3.5 Monetary Policy Expectations and Asymmetric Inflation Dynamics

Expected inflation is a crucial macroeconomic variable since it contributes to the determination of real interest rates, and thus of aggregate demand through the effect on investments and consumption. Therefore forecasting inflation is an important goal for both policymakers and private agents. Expected inflation is not directly observable though, so that it is necessary to extract information about future price dynamics from observable factors. A natural candidate seems to be the term structure of interest rates.

Mishkin (1990a) has developed an empirical model to assess the ability of the yield spread to predict future inflation. He finds that at very short maturities TS is not informative about future inflation but it contains “a great deal of information” about the real TS; then he finds that TS at medium-long term maturities help predict inflation changes (1990b). On the other hand, the model also provides with an effective tool to test EH. Unfortunately, in this respect data evidence does not support EH.

As in the case of interest rate prediction, we believe that linear models might be inappropriate to describe the dynamic relationship between inflation and the term structure. We suggest that the informative content of the yield spread about future inflation partially depends on the volatility of the
effective federal funds rate (Figure 3.12) and, thus, on the ability of agents to forecast it. The variability of the fed funds, in fact, not only exerts important influence on the short end of TS, but also increases uncertainty regarding the nominal stance of monetary policy.

In this Section we thus provide evidence that the predictive power of spread improves substantially when inflation prediction is conditional to agents’ expectations regarding the effective stance of monetary policy, as captured by a strategic measure of the TS slope. Within a threshold model, we find a stronger link between inflation and the spread when the expected yield curve is either flat or inverted, as reflecting severe stance of monetary policy. Our findings thus imply an asymmetric effect running from monetary policy to inflation.

On one side, a recent contribution by Ravenna and Seppala (2006) has emphasized the role of monetary policy to explain EH rejection. Short-run strong monetary non-neutrality implies systematic reactions of the monetary authority to movements in endogenous variables. The conduct of the central bank, which affects the co-variation between nominal and real variables, influences TS dynamics thus generating the well-known EH anomalies. On the other side, we propose a refinement of the study by Tkacz (2004) who finds that “for U.S. the inflation-spread relationship is more pronounced when the yield curve is inverted” signalling an asymmetric effect of monetary policy on inflation; he also finds the Canadian asymmetry works in the opposite direction.

The volatility of the fed funds seems coupled with sharp variations in the long-short yield spread as shown in Figure 3.13 (left panel); moreover, when the economy experiences a two-digit inflation rate the volatility of the fed funds shoots exponentially (right panel). High and unpredictable inflation thus is related to uncertainty regarding the stance of monetary policy as captured by both the federal funds and the TS slope.

---

84 Figure 3.12 shows that whenever the volatility of the fed funds increases (bottom left panel), the rolling (60-month window) estimated slope coefficient in equation (3.18) becomes insignificant being very distant from one (p-value of the t-test plotted in the bottom part of right panel).

85 For an extensive review of the literature on the point, please refer to Section 1.4 of Chapter 1.
We thus propose to consider a non-linear model for inflation prediction in which regimes transition depends on a measure of the TS slope that explicitly incorporate agent’s expectations about the effective stance of future monetary policy.

The basic idea developed in this chapter is also consistent with the prevalent view that the unpredictable pattern of short rates affect the empirical analysis of EH (Mankiw and Miron, 1986; Campbell, 1995; Thornton 2004; Ravenna and Seppala, 2006). Unpredictability of short rates trivially implies indeterminacy of the yield spread, and thus uncertainty on monetary policy expectations.

Additionally, analysing EH within threshold models accounts for the criticism by Thornton (2004), and Kool and Thornton (2004) who attribute anomalous EH empirical findings to the presence of extreme observations. Clustering data into threshold sub-regimes reduces significantly the effect of outlier data.

We suggest constructing the threshold variable as the spread between the observed long rate and some measure of the expected policy rate $th_r = y^e_t - E_t(ffe_{t+1})$. The policy rate is the federal funds rate as inferred by computing a time-adjusting Taylor-type reaction function. The policy spread, rather than the market spread, incorporates updated agents’ expectations about the incoming stance of monetary policy. Our approach is in line with the view expressed by Favero, Kaminska, and Soderstrom (2005). With the aim of testing the predictive power of the spread, in fact, they argue that deriving a recursive prediction of the short rate exploiting real time information provides with a more realistic analysis, since it forces econometricians to use the same informative set available to

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The GMM rolling model for the fed funds rate is estimated over a sample of five years. The explanatory variables are the first lag of the policy rate (monetary policy inertia); past inflation, a measure of the output gap, either the HP filtered (log) IP series or unemployment. The instruments are the first lag of the regressors. Our approach is thus to estimate a rolling model for the fed funds at any point in time using historical available information, and then projecting (predicting) the short rate one-period ahead. The iterative procedure of updating and predicting allows obtaining a realistic measure of the market sentiment about future movements in the short rate. The method of estimating and simulating the model forward put the econometrician on the same level of market participants’ who elaborate real-time available information. Finally, the choice of estimating a rolling rather than a recursive model is motivated by the fact that the rolling methodology better captures the evolution of the economy without preserving an excessive long memory of past events.
market participants. Therefore, we believe that our measure of the policy spread, rather than the market spread, appropriately reflects investors view about imminent monetary policy.

Finally, the choice of considering the policy spread, rather than the mere market spread, derives from Bernanke (1992) “the interest rate on federal funds is extremely informative about future movements of real macroeconomic variables, ... the funds rate (or a measure based on it) is a good indicator of monetary policy, even for the period after 1979. The federal funds rate is a particularly informative variable”.

Constructing the policy spread with the actual observed long term rate and a predicted short rate is also consistent with the empirical regularity that long rates are smooth and persistent while short rates are quite volatile. So that, on the one hand, it seems superfluous to use a predicted series for long rates since their dynamics is particularly smooth; however, on the other hand, predicting short rates is necessary because it reflects effective investors behaviour and their changing view about the state of the economy.

According to Mishkin the expectations theory can be tested using the following equation:

\[ \pi^n_t - \pi^m_t = \alpha + \beta \left( y^n_t - y^m_t \right) + \varepsilon_t \]

(3.18)

Where, as before, \( n \) and \( m \) represent the long and the short maturity respectively; \( \pi \) is the inflation rate. EH holds if the estimated slope coefficient \( \beta \) is one and the intercept is zero. Newey-West corrected rolling estimations, on a window of 60 monthly observations, return the pattern of \( \beta \) over time as reported in Figure 3.12. The coefficient is often lower than one. In particular, the weak statistical significance of \( \beta \) suggests that model (3.18) may be mis-specified. We argue that misspecification derives from non linearity in the spread-inflation relationship in conjunction with the pattern of volatility displayed by the fed funds. We thus estimate the threshold model (3.19)

\[
\begin{align*}
\pi^n_t - \pi^m_t &= \alpha_1 + \beta_1 \left( y^n_t - y^m_t \right) + \varepsilon_t^{n,m} & \text{if } \text{thr} \leq \hat{\gamma} \\
\pi^n_t - \pi^m_t &= \alpha_2 + \beta_2 \left( y^n_t - y^m_t \right) + \varepsilon_t^{n,m} & \text{if } \text{thr} > \hat{\gamma}
\end{align*}
\]

(3.19)

Estimating the non linear model by using the market spread \( \text{thr}_t = y^n_t - y^m_t \) as threshold variable does not deliver any significant improvement, as reported in Tables 3.7 (a, c, e). In Tables 3.7 (b, d, f) we report threshold estimates when regimes are split on the basis of the policy spread \( \text{thr}_t = y^n_t - E_t \left( f_{h+1} \right) \).

We briefly comment on the empirical results. Hereafter, for sake of parsimony we refer to model (3.19) as the financial model when the threshold variable is the market spread; and we label model
(3.19) the policy model when the threshold variable is the policy spread\(^7\). As expected, in the policy model the estimated threshold increases with the maturity of the long term bond; this effect is absent in case of market spread. A significant implication is that the number of observations in the policy model threshold sub-samples are balanced compared to the sample size in sub-regimes of the financial model. Therefore in the financial model there seems to be a weak threshold effect; or more precisely, as in Tkacz (2004), the threshold methodology merely seems to remove extreme observations from the entire sample. Few observations are in fact clustered in regime 1, i.e. below the estimated market spread (right column of Tables 3.7 a, c, e).

\[\text{Table 3.7 (a)}\]

<table>
<thead>
<tr>
<th>(n, m)</th>
<th>obs</th>
<th>(\beta)</th>
<th>Wald ((p\text{-val}))</th>
<th>(\gamma)</th>
<th>obs</th>
<th>(\beta)</th>
<th>Wald ((p\text{-val}))</th>
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<td>528</td>
<td>1.2080</td>
<td>0.0858</td>
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<td>1</td>
<td>84</td>
<td>1.2642</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>504</td>
<td>1.3228</td>
<td>0.0014</td>
<td>-0.30</td>
<td>1</td>
<td>31</td>
<td>-0.6167</td>
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<td>473</td>
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</tr>
<tr>
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<td>0.0239</td>
<td>-1.10</td>
<td>1</td>
<td>14</td>
<td>-0.0791</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>430</td>
<td>1.1983</td>
</tr>
</tbody>
</table>

\[\text{Table 3.7 (b)}\]

| (n,m) | \(\gamma\) | regime | Obs | \(\beta\) | Wald \((p\text{-val})\) | \(\gamma\) | regime | Obs | \(\beta\) | Wald \((p\text{-val})\) |
|-------|-----------|-------|-----|---------|----------------|--------|-------|-----|---------|----------------|--------|
| (36,3)| 1.48      | 1     | 358 | 1.4923 | 0.0017 | 1.11   | 1     | 332 | 1.5363 | 0.0017 | 1.11   | 1     | 332 | 1.5363 | 0.0017 |
|       |           |       |     |         |         |       |       |     |         |         |       |       |     |         |         |
|       |           |       |     |         |         |       |       |     |         |         |       |       |     |         |         |
| (60,3)| 1.55      | 1     | 316 | 1.1887 | 0.1352 | 1.54   | 1     | 336 | 1.2833 | 0.0279 | 1.54   | 1     | 336 | 1.2833 | 0.0279 |
|       |           |       |     |         |         |       |       |     |         |         |       |       |     |         |         |
|       |           |       |     |         |         |       |       |     |         |         |       |       |     |         |         |
| (120,3)| 1.83     | 1     | 270 | 1.0541 | 0.6432 | 1.94   | 1     | 292 | 1.0775 | 0.5184 | 1.94   | 1     | 292 | 1.0775 | 0.5184 |

EH-consistent estimates of the slope coefficients \(\beta(\hat{\gamma})\) are by large better in the policy model than in the financial model. The improvement occurs at almost any combination of maturities \((n, m)\). In

\(^7\) We apologize for the eventual abuse of terminology in the latter case.
addition, results indicate that EH is close to be respected particularly when the long yield is associated to a long term bond (5 or 10 years). The long end of the yield curve seems to carry relevant information about future inflation, whereas little information can be extracted from the short end of the yield curve. This confirms previous evidence that the information extracted from the entire spectrum of TS maturities is relevant to infer the future state of the economy.

In the financial model the improvement usually occurs only in one regime, whereas in the policy model it occurs in both regimes. However, policy model parameter estimates are significant above the threshold rather than below.\(^{88}\)

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<tr>
<td></td>
<td>R²</td>
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Table 3.7 (c)

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<th>obs</th>
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<td>184</td>
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<td>(0.000)</td>
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</tr>
</tbody>
</table>

Table 3.7 (d)

More importantly, our results highlight an important asymmetric effect regarding the reaction of inflation to the expected policy spread. In the policy model estimates of the slope coefficients in

---

88 Although not reported, we find that estimates of the intercept turn out to be not significantly different from zero. This result is particularly evident when the policy spread is adopted as the regime determinant variable.
regime 1 are significantly higher than in regime 2. It means that in the regime characterized by low values of the policy spread the change of inflation is highly responsive to the yield spread. This can be interpreted as evidence that inflation strongly responds to monetary policy actions when the monetary authority is expected to be severe. Conversely, when the expected policy spread denotes loose monetary policy, i.e. the yield curve is significantly upward sloping, the effectiveness of monetary policy on inflation seems to be weaker. From the analysis it thus seems that emerges a threshold effect which is analogous to the so-called traditional Keynesian asymmetry\(^{89}\); it appears, in fact, that severe monetary policy is rapidly effective in influencing inflation, whereas accommodative monetary policy marginally affects inflation changes\(^ {90}\).

<table>
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<tr>
<th>(n,m)</th>
<th>regime obs</th>
<th>(\beta)</th>
<th>Wald (R^2)</th>
<th>(p-val)</th>
<th>regime obs</th>
<th>(\beta)</th>
<th>Wald (R^2)</th>
<th>(p-val)</th>
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<td>20</td>
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Table 3.7 (e)

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<th>(\beta)</th>
<th>Wald (R^2)</th>
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<td>0.000</td>
<td></td>
<td></td>
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<td>0.000</td>
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</tr>
</tbody>
</table>

Table 3.7 (f)

The asymmetric effect is clear from a visual inspection of Figure 3.14 (a). A symmetric outcome would imply strong similarity between regime 1 and regime 2 scatters. But, the concentration of observations in different regions of the charts gives a clue of how different the outcome of threshold

\(^{89}\) Only negative monetary policy shocks display significant effect on output.

\(^{90}\) Although not reported, estimates for short maturities of the long term rate \((n = 6-, 12-, 24-month)\) show that the slope coefficient is statistically significant only in regime 2, i.e. above the estimated threshold. In respect with these short horizons, monetary policy expectations imply a strong influence of the actual spread on inflation changes only when the steepness is significantly positive (even though on medium-short horizons). Thus there seems to exist a reverse Keynesian asymmetry effect at very short horizons, since monetary policy affects inflation when it is either expected or perceived accommodative.
equations may be. In Regime 1 observations tend to be concentrated in the mid-south area of the scatter; while, in Regime 2 data points are located in the north-east part. More specifically, in the left panel we report the scatter diagrams for regime 1 while in the left we report that of regime 2. The range of variation of the actual spread along the x-axis is almost identical for both scatter plots; however, only in regime 1 (left plot) inflation changes assumes important negative values, denoting a sensible reduction of inflation when the yield curve is either flat or inverted. In addition, the linear interpolant is steeper in regime 1; once again, a stronger reaction of inflation to monetary policy expectations, as captured by the actual spread, occurs when the policy spread highlights incoming severe actions. Figure 3.14 (b), instead, shows the scatter plots of the actual spread against inflation changes in threshold sub-regimes obtained by the financial model, i.e. when regimes discrimination is driven by the mere market long-short yield spread. As noted above, there is a clear concentration of observations in regime 2 (above the estimated threshold) which are symmetrically distributed below and above the zero value of the y-axis (inflation change). In this case it seems that a certain number of outlier data points are relegated in regime 1 once the threshold technique is applied.

We report some more evidence that a threshold model fit data better than a linear one. The rolling estimate of equation (3.18) is reported in Figure 3.12. Figure 3.15 shows the rolling estimated slope
coefficient in both threshold sub-sample for the pair \((60, 3)\). The pattern of the estimated coefficient closely fluctuates around one till data point corresponding to the mid-1990s. The value one (semi-dotted horizontal black line) is contained for most of the sample in the standard error bands. The statistical significance is not always guaranteed though, as long as sometimes also the zero line lies within the standard error bands.

![Figure 3.15](image)

Figure 3.15

Figure 3.16 reports the distribution of the estimated rolling \(\hat{\beta}_i\) coefficients. The single regime model (left diagram) denotes a clear bi-modal distribution, which constitutes strong evidence in favour of a non linear analysis. In addition, as stressed above, there is a significant difference between the distributions of the coefficients in the first and second threshold sub-regimes. Estimated coefficients in regime 1 are peaked around an higher values compared to coefficients in regime 2 confirming the relevant asymmetric effect.

![Figure 3.16](image)

Figure 3.16

Finally, we report also the quantile-quantile plot of estimated coefficients in both the linear model and in threshold sub-regimes. Consistently with the left diagrams of Figure 3.16, extreme actual
quantile-values of the single regime rolling estimated $\hat{\beta}$ denote both fat and long tails of the distribution. In regime 1, instead, there is an important concentration of empirical $\hat{\beta}$ quantiles around the theoretical normal quantiles. The large gaps at the edges of regime 2 $\hat{\beta}$ qq-plot (right panel) reflect the shape of the tails of the empirical $\hat{\beta}$ distribution.

The forecasting power of the yield spread is usually related to the predictability of short rates, and, as we argue in this chapter, to the dynamics of the federal funds rate which contributes to the determination of the policy spread thus reflecting expectations of the future effective monetary policy stance. For the combination of maturity (60, 3) we have obtained a kernel density estimate of the fed funds distributions in both the entire sample and in sub-regimes (threshold policy model).

The kernel density estimate of the federal funds rate in regime 2 is evidently peaked (right panel). This might reveal easy predictability, which is, in fact, consistent with the EH-implied slope coefficient estimate in regime 2 (Table 3.7 b). The variance of the estimated distribution is much larger in the entire sample (left panel).

In this chapter we have provided evidence that the spread is particularly informative about future inflation at medium-long horizons ($n = 120, 60, 36$). We can only speculate that this result is linked to the dichotomy between real and nominal variables. Aggregate demand spending and private
investments are related to the long end of the yield curve (Walsh, 2003). However, the opportunity
cost of holding money depends on short rates. We speculate that inflation dynamics might be
related to the TS maturity spectrum which lies between these extreme cases. The quantity theory of
money states that the price level is merely determined by the money stock in the long run; therefore
nominal variables do not influence real variables. However, the adjustment process in the medium-
short run displays real effects on the economy. Ravenna and Seppala (2006) relate the changing
correlation between nominal and real variables to monetary policy actions, i.e. money is not neutral
in the short run. We acknowledge that, in most modern countries today, short run stabilization
policies are carried out by the monetary authority rather than the fiscal one. Suppose there is a
permanent increase in the money growth. If prices are flexible enough, as reasonably expected at
medium term horizons, growing money supply reduces real money balances, since expected
inflation rises more than the change in the money stock. It follows an upward jump of nominal rates
which raises the opportunity cost of holding money reducing agents’ desired amount of real money
balances. If the price level increases more rapidly than money stock does, inflation exceeds the rate
of money growth. If prices are fully flexible this occurs when the central bank change the money
supply; if prices are not perfectly flexible, the adjustment is smoothed over a longer period. The
dynamics at medium TS maturities thus might reflect this process without affecting the long end of
the yield curve which drives aggregate spending decisions.

As a final robustness check, and consistently with methodology adopted above (the rolling
regressions), we prove that within threshold sub-samples there is no evidence of structural breaks.
While the Chow breakpoint test performed on the estimated EH equation over the entire sample
reveals the existence of several breaks, the Chow tests performed within threshold sub-regimes will
not detect any break. Hence, on the one hand, the Chow test suggests splitting the linear model into
sub-samples, on the other hand, the test supports the success of the threshold method.
We have performed a battery of rolling Chow breakpoint test to check for parameter stability within
threshold sub-regimes. In a threshold setting it seems appropriate to check whether the eventual
structural break occurs in correspondence of any observation, i.e. at any point in time. The Hansen
(2000) stability test has been inspired by a rolling application of the Chow test. The statistics for the
test is built as follows. Let $n_1$ and $n_2$ denote the number of observations in regime 1 and 2
respectively. First, we estimate a regression within the sub-regime computing the restricted sum of
squared residuals ($SSR_r$). Second, focusing on in regime 1 we allow for the possibility of a structural
break at any point of the $(n_1 - 60)$ central observations; this means that we split regime 1 (below
the estimated threshold) into two further sub-samples of variable size $m_1$ and $m_{n_1-1}$ respectively.
Moving $i$ from 30 to $(n_1 - 30)$ we estimate sequential regressions on the obtained sub-samples $A$
and $B$, being $i$ the point of structural break under examination. Residual sum of squares are computed for regressions in the two sub-sample, $SSR_A$ and $SSR_B$, respectively. We obtain $(n_i - 60)$ sequential tests and thus an $(n_i - 60)$-dimensional vector of probability values associated to the tests. The statistics takes the form:

$$\frac{[(SSR_A - (SSR_A + SSR_B))]/(k)]}{[(SSR_A + SSR_B)/(n_i - 2 \cdot k)]} \sim F_{k, n_i - 2k}$$

where $k$ is the number of parameters to be estimated in each regression; and the sample size in the threshold regime $l$ is $n_i = m_A + m_B$. The statistics, under the null hypothesis of no structural break, is distributed like an $F$ with $k$ and $(n_i - 2 \cdot k)$ degrees of freedom. The test is repeated for threshold sub-regime 2 to check coefficient stability also in the second threshold sub-regime (above the estimated threshold). In each diagram the probability values associated to the Chow test is approximately one; therefore null hypothesis of absence of structural break can never be rejected. The Chow tests provide evidence supporting that no structural breaks occur within the threshold sub-samples.


Figure 3.19
3.6 Term Premia, Non Linearity and Future Economic Activity

A vast portion of economic literature has examined whether the informative content carried by the TS of interest rates helps to predict future economic activity. Following Stock and Watson (1989), who have included the yield spread among leading economic indicators, early contributions have examined the role of the yield spread in anticipating real variables. Harvey (1988, 1989) has shown that the real TS is informative about future consumption and output growth. Estrella and Hardouvelis (1991) provide evidence that the spread between the 10-year T-bond and the 3-month T-bill rates is a useful predictor of future GDP. Estrella and Mishkin (1997) essentially confirm previous evidence. Dueker (1997) employs a probit model reinforcing empirical findings “the yield curve slope remains the single best recession predictor in the examined set of variables”. Also Wright (2006) shows that the yield spread anticipates recessions with a relatively high probability; moreover, he finds that the predictive ability of the spread is robust and does not fade away after the inclusion of a policy variable (the effective fed funds).

Recently the literature has turned to investigate whether term premia are informative about the business cycle. In particular, researchers have focused on the possibility that a decomposition of the yield spread, into the expected change in short rates and a term premium, can be helpful to examine movements in future output. Hamilton and Kim (2002) find that the aforementioned decomposition of the spread is useful to predict future GDP. Coefficient estimates of both components turn out to be positive and statistically significant; but the Wald test rejects the null hypothesis that the effect of the term premium is as important as that of the expected change in short rates. Ang, Piazzesi, and Wei (2006) compare the predictive power of the spread with that of its decomposition. They find extremely useful the decomposition since it substantially increases the goodness of fit of the predictive model; however, the term premium turns out to be not significantly different from zero. Favero et al. (2005) claim that the aforementioned decomposition leads to a better understanding of the predictive model; they find a positive sign for the coefficient of the term premium indicating that a lower term premium predicts slower output growth. Reduced-form empirical analysis performed by Rudebusch, Sack, and Swanson (2007) suggests, instead, that a reduction in term premia stimulates economic activity thus challenging previous evidence.

In this Section we offer an innovative approach to exploit the decomposition of the spread for forecasting future economic activity. As argued in Section 3.4, we suggest that term premia introduce some noise in the linear analysis of the TS informative content; we thus propose to interpret evidence of time-variation in term premia as support of non linearity.

In Figure 3.20 we report the $\beta$ coefficient, together with standard error bands and the $t$-test probability value, of the following equation estimated with a rolling window of 60 observations:
\[ \Delta IP_{t+k} = \alpha + \beta (y_i^n - y_i^m) + \epsilon_i \]  

(3.21)

Where \( \Delta IP_{t+k} = (1200/k) \cdot \ln(IP_{t+k} / IP_t) \) is the monthly-adjusted rate of growth of industrial production \((k=12)\). Although the left panel shows that the spread carries substantial predictive power over time, the right diagram highlights that there is a clear connection between the lack of statistical significance of model (3.21) and the level of the term premium. A multiple regime model to examine the predictive ability of the spread can thus account for the aforesaid connection.

Term premia reflect agents’ risk aversion and provide with a measure of monetary policy innovation; according to equation (3.13), in fact, term premia represent the unanticipated component of the yield spread. We suggest that the ability of the TS slope to anticipate future output movements is inversely related to the level of term premia. \textit{Ceteris paribus}, on the one hand, given a value of the yield spread, lower term premia imply stronger expectations of accommodative monetary policy. On the other hand, low term premia also reveal little uncertainty regarding the expected future monetary policy stance. A combination of these complementary effects induce self-fulfilling activities, since agents not only expect the monetary authority to accommodate output growth, buy also they are confident about the conduct of the central bank.

In order to capture the dynamics described above, we outline the following framework where threshold regimes are determined by the first lag of the term premium \((thr = tp_{t-1}^{n,m})\).

\[
\begin{align*}
\Delta IP_{t+k} &= \alpha_1 + \beta_1 (y_i^n - y_i^m) + \epsilon_i, & thr \leq \gamma \\
\Delta IP_{t+k} &= \alpha_2 + \beta_2 (y_i^n - y_i^m) + \epsilon_i, & thr > \gamma
\end{align*}
\]

(3.22)

The following Tables report the empirical results. We test the ability of the spread to anticipate future output growth on both industrial production (left panel) and total capacity utilization (right panel). \(k\) represents the forecasting horizon length \((k \text{ months ahead})\). We consider only two pairs of maturities \((120, 3)\) -top panel- and \((60, 3)\) -low panel-. The threshold variable \((thr)\) is the term premium. The magnitude of the estimated slope coefficients is by large greater in regime \(1\) (below
the threshold estimate) than in regime 2. In addition, when term premia are lower than the threshold estimated value the goodness of fit of the predictive model substantially increases. The joint goodness of fit of the threshold model almost doubles with respect to the linear model; moreover, in regime 1 the empirical model fits data much better than in regime 2.

Output Growth Prediction

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<tr>
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<td></td>
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<tr>
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</tr>
<tr>
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</tbody>
</table>

Sample jan67-dec97, spread (n = 120; m = 3); *Sample: jan67-dec03, spread(n = 60; m = 3). The null hypothesis of the LM test is *"absence of threshold"*

Empirical results are clearly supportive of the inverse relationship between the predictive power of the spread and the level of term premia. Low term premia tend to be associated to faster output growth in the future. The huge threshold effect, i.e. the large gap between the magnitude of the estimated \( \beta \) coefficient in different regimes, can be interpreted as a sign of self-fulfilling expectations. This is particularly true when the forecasting horizon is between 12 and 24 months ahead. In a credible monetary regime, low term premia reflect agents’ accurate expectations of an accommodative stance of monetary policy as displayed by a positively sloped yield curve. The expected increase of money supply encourages individuals to raise consumption spending and private investments, and leads financial institutions to expand credit; the final impact on aggregate demand is thus substantial. Conversely, when term premia are high (regime 2) the greater uncertainty regarding the future stance of monetary policy prevents agents from increasing
consumption spending and borrowing. Figure 3.21 reports the scatter plots of the relationship between IP growth and the yield spread for the pair $(120, 3)$ when $k$ is 12 (top panels) and 24 (bottom panels) months respectively.

The regression line is definitely steeper in regime 1 (central panels) then in regime 2; in addition, regime 1 scatter plots display a closer concentration around the regression line.

Table 3.9 shows that previous results are robust to different model specifications. Following Wright (2006) we check whether the predictive power of the spread, i.e. the statistical significant of $\beta$, does not vanish after the introduction of the federal funds rate among explanatory variables.

$$
\begin{align}
\Delta IP_{t+j, k} &= \alpha_1 + \beta_1 \left(y_t^n - y_t^m\right) + \rho ff_{r_i} + \epsilon_i & \text{thr} \leq \hat{Y} \\
\Delta IP_{t+j, k} &= \alpha_2 + \beta_2 \left(y_t^n - y_t^m\right) + \rho ff_{r_i} + \epsilon_i & \text{thr} > \hat{Y}
\end{align}
$$

The estimated yield spread coefficient remains positive and it is higher in regime 1 than in regime 2. The negative coefficient of the federal funds rate reveals that high values of the policy rates are associated to reduction in future economic activity. The inclusion of the federal funds rate improves the fit in both regimes without affecting the significance of the yield spread coefficient.
### Output Growth Prediction

#### Industrial Production

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<tr>
<th>k</th>
<th>obs</th>
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<th>p</th>
<th>$\gamma$</th>
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<th>obs</th>
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<td>(p-val)</td>
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<td>$R^2$</td>
<td>(p-val)</td>
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</table>

Sample jan67-dec97, spread ($n = 120; m = 3$); *Sample: jan67-dec03, spread($n = 60; m = 3$). The null hypothesis of the LM test is "absence of threshold"

### 3.7 Concluding Remarks

Linear models have been largely adopted to test the predictive ability of the spread. However, the empirical failure of the expectations theory has encouraged researchers to find alternative ways to analyse the informative content of the slope of the term structure of interest rates. The goal of this chapter is thus to examine the predictive ability of the spread in threshold models.

Our interest in non linearity moves from the empirical failure of EH in linear models; in addition, the choice of a multiple regime frameworks accounts for the criticism by Thornton (2004) who attributes the eventual and unusual empirical support for EH to the presence of outlier observations. Time variation in risk premia have been advocated as a possible explanation of the expectations puzzle. We thus propose to associate threshold sub-regimes either to the dynamics of term premia or to the expectations about the future stance of monetary policy.

We find that the slope of the term structure is informative about future movement in short rates once the risk averse attitude of private agents is properly taken into account. We also provide...
evidence that future inflation prediction is conditioned to the expected stance of monetary policy. Finally we document an important inverse relationship between the level of term premia and the ability of the spread to anticipate future output growth.

To summarize, this chapter provides significant evidence of asymmetry affecting the predictive power of the yield spread.

**Appendix A3.I - Data**

All data employed in the analysis have monthly frequency. United States ZCB yields data from January 1964 and December 1998 are from either the McCulloch-Kown database (3-month, 6-month, and 10-year) or from the Fama-Bliss dataset (1-, 2-, 3-, 4-, 5-year)\(^1\). From January 1999 to June 2007 all yields data are from the Datastream database (ZCB yields). The effective federal funds rate series is from the Federal Reserve Economics Database (FRED). Below we plot the federal funds rate, the 3-month, and the 60-month yields from January 1964 to June 2002, the range over which the empirical analysis is performed. Rather than in yields’ level we are interested in the spreads. We compute the spread between long term yields and the 3-month yield. Shaded areas in the following figures indicate periods of recession\(^2\); the spreads tend to be negative, i.e. the yield curve either inverted or flat, immediately before recessions. This is consistent with the prevalent view that an inverted yield curve reflects agents’ expectations of a severe tightening in the monetary policy conduct.

![Figure 3.22](image)

*ffr*: effective federal funds rate. *y3m*: 3-month yield. *y60m*: 60-month yield. *spr (n, m)*: spread between the *n*-month yield and the *m*-month yield.

Campbell and Shiller (1991) show that the yield spread can be decomposed into the expected change in short term rates \(i_t^m\) and a term premium, according to the following:

\[
y_t^s - y_t^w = \left( \frac{m}{n} \sum_{q=0}^{n-m} E_t y_{t+mq}^w - y_t^w \right) + tp_t^{n,m}
\]

\(^{(3.24)}\)

---

\(^1\) McCulloch data are available from the Gregory R. Duffee web page; while the Fama-Bliss yields data are from Cochrane and Piazzesi (AER, 2005).

where \( n \) is the long term maturity; \( m \) is the short term maturity and \( t_{p_i}^{n,m} \) is the term premium associated to the combination of maturities \((n, m)\). The first element on the RHS is the expectational component, otherwise known as the theoretical, or perfect foresight, spread. The table below show some descriptive statistics about the spread and its two components \((m = 3)\). The sample ends in September 1997 which is the most recent available observation for the theoretical spread associated to the 10-year bond.

<table>
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<th>Spread</th>
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<td>(0.787)</td>
<td>(0.614)</td>
<td>(0.000)</td>
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</table>

*sample jan64-sep97

Table 3.10

The mean of both the spread and the term premium is increasing with maturity \((n)\); whereas the mean of the theoretical spread is almost constant. Similar results are obtained by Campbell (1995). The standard deviation of all variables is increasing with maturity. The null hypothesis of normality is almost invariably rejected (Jarque and Bera test).

According to Campbell, the empirical failure of the expectations theory can be rationalized by noting that the standard deviations of the expected changes in the interest rates (theoretical spread) are smaller relative to the standard deviations of the term premia. With our data this occurs for medium-short horizons \((n \leq 36)\). An implication of the magnitude of relative variance is the downward bias of the estimated slope coefficient (lower than unity) in the single equation model to test the expectations hypothesis. Term premia reflect the unexpected future changes in interest rates, i.e. the unpredictable evolution of the yield curve, which is regarded to be a measure of investors’ risk aversion; in addition, when agents are well informed about future movements in interest rates the variability of term premia should be lower than the variability of the perfect foresight spread. Consistently with this idea, Mankiw and Miron (1986), among others, suggest that the empirical failure of the expectations hypothesis can be attributed to the increased unpredictability of interest rates after the creation of the Federal Reserve System in 1914.

As shown in the following figure, the term premium tends to rise before recessions.
Figure 3.23

All the time series considered are covariance stationary. The results of both the Augmented Dickey-Fuller (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are reported below. The upper part of the table reports the probability values of the ADF test. In the bottom panel the LM statistics values of the KPSS test are displayed.

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<td>(0.000)*</td>
<td>(0.000)*</td>
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<td>(0.011)*</td>
<td>(0.017)*</td>
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<td>(0.001)*</td>
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<td>0.076**</td>
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<td>0.080**</td>
<td>0.206**</td>
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<td>tp(n,3) kpss</td>
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<td>0.490*</td>
<td>0.083**</td>
<td>0.143**</td>
<td>0.208**</td>
</tr>
</tbody>
</table>

Exogenous: *Intercept, **Intercept and Trend; *18 lags
sample jan64-sep02

Table 3.11

The ADF test leads to the rejection of the unit root hypothesis; whilst, the null hypothesis of stationarity cannot be rejected by the KPSS test\(^{93}\).

The rate of inflation has been calculated as the expected annual percentage \((log)\) variation in the Seasonally Adjusted Consumer Price Index (CPI all urban consumers, all items). The series has been obtained by the FRE-Database (originally from U.S. Department of Labour: Bureau of Labor Statistics).

\[
\pi_i' = \left(100 \times \frac{k}{j}\right) \cdot \log \left(\frac{CPI_{i+j}}{CPI_i}\right) \tag{3.25}
\]

In the above formula \(k = 12\) (monthly frequency); \(j\) is a time index that indicates the horizon extent. The change in inflation over time is given by the difference \(\pi_i^n - \pi_i^m\). The difference \(\pi_i^n - \pi_i^m\) indicates the evolution of the inflation rate over the horizon \(n-m\) (with \(n > m\)). In the analysis of the ability of the spread to

\(^{93}\) To match the monthly frequency of data, the rule of thumb selected number of lags in the auxiliary regression is 12; in few cases the number of lags is different but very close to 12. The automatic lag selection based on different criteria (Akaike, Schwarz, Hannan-Quinn) is consistent with our choice. Unit root test results obtained with the automatic lag selections are similar. The KPSS critical values are 0.739 (1%), 0.463 (5%), and 0.347 (10%) when the intercept is included in the model. The KPSS test critical values if also the trend is added are 0.216, 0.146, and 0.119 at 1%, 5%, and 10% significance levels respectively.
predict inflation we fix the short term maturity \((m)\) equal to 12 months. In the left panel of the following figure we plot the differences \(\pi_t^n - \pi_t^{12}\), while in the right panel we show the yield spreads \((n, 12)\).

![Figure 3.24](image)

Between 1964 and 2002 the mean is approximately zero for all the series, whereas the standard deviations rise with the long term maturity \((n)\). Both the ADF and the KPSS test suggest these series are stationary (integrated of order zero) as shown in the table\(^{94}\).

<table>
<thead>
<tr>
<th>sample jan64-sep02</th>
<th>long term maturity (n)</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>obs 405</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>infl((n,12))</td>
<td>stddev</td>
<td>0.4373</td>
<td>0.6925</td>
<td>0.8946</td>
<td>1.0945</td>
</tr>
<tr>
<td></td>
<td>adf</td>
<td>(0.000)*</td>
<td>(0.001)*</td>
<td>(0.009)*</td>
<td>(0.002)*</td>
</tr>
<tr>
<td></td>
<td>kpss</td>
<td>0.127*</td>
<td>0.152*</td>
<td>0.274*</td>
<td>0.664*</td>
</tr>
<tr>
<td>spread((n,12))</td>
<td>stddev</td>
<td>0.3919</td>
<td>0.6131</td>
<td>0.8810</td>
<td>1.1414</td>
</tr>
<tr>
<td></td>
<td>adf</td>
<td>(0.019)*</td>
<td>(0.038)*</td>
<td>(0.043)*</td>
<td>(0.044)*</td>
</tr>
<tr>
<td></td>
<td>kpss</td>
<td>0.092**</td>
<td>0.072**</td>
<td>0.076**</td>
<td>0.073**</td>
</tr>
</tbody>
</table>

Exogenous: *Intercept, **Intercept and Trend

\(^{94}\) See previous note.
Chapter 4

Forecasting Economic Activity using the Conditional Volatility of Term Premia

Abstract

The empirical rejection of the expectations hypothesis has often been attributed to time variation in term premia. Following recent developments in the literature, we believe that the informative content of term premia partially offsets the weak predictive power of the slope of the term structure. Deriving term premia as the difference between the ex-post observed and the ex-ante expected long term rates, we check for the time-varying nature of term premia; we then obtain the conditional variance of term premia by Kalman filtering an empirical macro-model. In this chapter we provide evidence suggesting that the conditional volatility of term premia is informative beyond term premia and the yield spread. We find significant evidence that the conditional variance of term premia, which we reckon as a measure of financial distress, is a significant predictor of future economic activity. In particular, high volatility of term premia is associated with slower output growth in the future. Finally, in line with Rudebusch, Sack, and Swanson (2007) we document an inverse correlation between term premia and the business cycle.
4.1 Introduction

The expectations hypothesis asserts that the yield spread is informative about future changes in interest rates. However, although appealing the expectations theory has found weak empirical support. A large portion of the empirical research has suggested the time-varying pattern of term premia as a possible explanation for the expectations puzzle (Mankiw and Miron, 1986; Fama, 1986; Cook and Hahn, 1989; Lee, 1995; Tzavalis and Wickens, 1997; Hejazi and Li, 2000). In this chapter we examine whether the variability of term premia, rather than the level, is informative about the economy.

Predicting future economic activity is a recurrent theme in empirical economics (Stock and Watson, 1989; Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1997; Hamilton and Kim, 2002, among others); in particular, the slope of the term structure is believed to anticipate business cycle fluctuations. Bond prices, in fact, summarize quite accurately agents’ expectations about the incoming stance of monetary policy since financial markets are efficient at distilling economic information.

A recent strand of the literature has focused on the possibility that the term premium, rather than the yield spread, is capable of predicting future economic activity (Hamilton and Kim, 2002; Favero, Kaminska, and Soderstrom, 2005; Ang, Piazzesi, and Wei, 2006; Rudebusch, Sack, and Swanson, 2008). In this chapter the term premium represents the gap between the observed yield spread and the theoretical, or perfect foresight, spread; alternatively the term premium is computed as the difference between the actual (observable) and the ex-ante (expected) long term rate. In principle, large values of the yield spread are due either to an expected accommodative stance of monetary policy or to the presence of large risk premia required by investors. These effects are complementary as long as, for a certain value of the yield spread, the larger the term premium the lower the rationally expected level for the long rate, and viceversa. Hence, the term premium captures the unexpected change in the monetary policy stance. The decomposition of the yield spread into an expectations-based component and a risk premium allows distinguishing the effect of risk aversion from that of monetary policy as implied by rational expectations.

In this chapter we augment traditional predictive models by allowing for the possibility that the variability of term premia can anticipate future output growth, suggesting that financial distress, rather than mere risk aversion, is informative future output fluctuations. To some extent, the inspiration of this work derives from the effort of the existing literature to improve the forecasting model by incorporating macro variables in reduced-form empirical models (Evans and Marshall 2001; Favero, Kaminska, and Soderstrom 2005; Rudebusch, Sack and Swanson, 2007).
In particular, Favero et al. (2005) claim that the decomposition of the spread into an expectational component and a term premium leads to a better understanding of the forecasting model since it allows separating the effect of future monetary policy from that of risk aversion. We thus propose to improve upon their suggestion by analysing the effect of term premia variability which reflects financial distress rather than simple agents’ risk-averse attitude. Moreover, although several studies have highlighted that the time-variation of term premia is a significant component of the yield spread, so far no one has proposed to investigate the dynamic properties associated to the time-varying nature of term premia for forecasting macroeconomic variables. Ang, Piazzesi, and Wei (2006) find that the aforementioned decomposition improve the forecasting performance of the predictive model; however, there is weak statistical evidence to support an active role for term premia.

Rather than focusing on term premia, we examine whether volatility is informative about the economy. In this chapter we thus find robust evidence that the conditional variance of term premia prediction errors, rather than term premia, helps to predict future economic activity. We follow Engle’s (1982) suggestion that the conditional variance, i.e. the variance conditional upon available information at the time of forecasting, rather than the unconditional variance, is what really influences agents’ behaviour.

Despite interest rate variability is an important determinant of both the term premium and the yield spread, Hamilton and Kim (2002) document it is not informative about future output. We provide a refinement of their model by considering the conditional variability of term premia, which, instead, appears to carry useful information for predicting future industrial production growth. We believe that future economic activity is related to term premia volatility rather than to interest rate volatility. We thus emphasize the role of both financial distress and risk aversion as opposed to the unpredictability of monetary policy.

The rest of the chapter is organized as follows. The next Section contains a brief survey of the literature. In Section 4.3 we discuss the macroeconomic foundation of term premia stressing out their non linear nature. In Section 4.4 we present empirical evidence about future output prediction. Section 4.5 concludes. All data are presented in Appendix A4.I. Finally, in Appendix A4.II we outline the Hansen method to check for coefficient stability.

4.2 Literature Review

In a seminal work Stock and Watson (1989) find that the yield spread can be considered a leading economic indicator for predicting future output changes. The predictive ability of the yield spread has found significant support afterwards. Estrella and Hardouvelis (1991) find that the slope of the
yield curve, i.e. the spread between the 10-year T-bond and the 3-month T-bill rates, is a good predictor of future real GDP growth. They document that the predictive accuracy of the spread for cumulative changes from 5 to 7 quarters ahead is quite impressive since the spread explains more than one-third of the variations of output changes. Although results differ across countries and across maturities, Estrella and Mishkin (1997) roughly confirm previous empirical results after extending the analysis to some European countries. A probit model returns significant positive probability of the yield spread to anticipate recessions. Consistently with Estrella and Mishkin (1997), both Dueker (1997) and Wright (2006) have shown that the yield spread is a relatively good predictor of recessions.

Generally speaking, the effect of the spread on future output growth is positive, since small values of the spread, reflecting agents’ expectations of tight monetary policy, tend to predict slower growth in economic activity. However, Feroli (2004) points out that the predictive ability of the spread is contingent on the monetary authority’s reaction function; in particular, the predictive power of the spread depends on the accuracy of the expectations about the future stance of monetary policy. Consistently with this view, in the previous chapter of the thesis we have, in fact, provided evidence of the asymmetric predictive power of the spread. Feroli (2004) develops a small macro model in which the informative content of the term structure depends on the parameters of the monetary policy reaction function. Simulation results show that, depending on parameters’ values, the model can account for the diminished predictive power of the spread after 1979. Also Ravenna and Seppala (2006) have explored the connection between monetary policy and the predictive ability of the spread. They find that short run monetary non-neutrality captured by a simple monetary policy rule can account for the empirical failure of EH, and thus can affect the predictive power of the yield spread.

After analysing the predictive power of the yield spread, researchers have focused on the role of term premia. Hamilton and Kim (2002) propose to decompose the yield spread using ex-post observed short rates data instead of ex-ante expected rates. The spread decomposition reflects the effect of expectations about future monetary policy, as captured by the expected future changes in short rates, and risk aversion capture a term premium. They show that both components help predict real GDP growth. The estimated effect of both components is significantly positive. On the contrary, Ang, Piazzesi and Wei (2006) find that the term premium is not statistically significant. However, they acknowledge that, on the one hand, the distinction of both components is important to obtain a clear understanding of the forecasting model, and on the other hand, it leads to a substantial improvement in the goodness of fit. Along the same line, Favero, Kaminska, and Soderstrom (2005) decompose the spread into an expectational component and a pure term
premium. They show that adding some macroeconomic variables in a reduced-form empirical model improves the forecasting ability of the spread. Using quarterly data, they find that the spread between 5-year and 3-month interest rates, and the term premia associated to those maturities, are reliable predictors of the GDP quarterly change. Consistently with previous findings, they provide evidence that a lower term premium predicts slower GDP growth. Kim and Wright (2005) employ a standard arbitrage-free dynamic latent factor term structure model to obtain a measure of risk premia. They ascribe the so-called conundrum, i.e. the decline in long term rates in response to a policy tightening action in 2004, to a fall in term premia. Wright (2006) finds that low term premia raise the probability of a recession in the future. The probit model associate to the yield spread a significant probability of future recessions. Moreover, the inclusion of the policy rate in the model improves the forecasting power both in- and out-of-sample. Also Hejazi (2000) adopts the aforementioned decomposition to analyse the informative content of the term structure. Differently from Hamilton and Kim (2002), his findings highlight that the variability in interest rates is a significant empirical determinant of the future changes in the industrial production index. High interest rate variability is associated to future contraction in economic activity.

In this chapter we suggest that the decomposition of the spread into an expectations-based component and a risk factor can be further improved upon. We thus focus on the volatility of term premia, rather than on the variability of interest rates, with the aim of analysing whether the conditional variance of term premia helps to predict economic activity. Results are encouraging since we find significant evidence that the volatility of term premia is informative about the business cycle.

### 4.3 Term Premia and Macroeconomic Variables

The empirical rejection of EH is often attributed to time variation in term premia. Tzavalis and Wickens (1997), as well as Fama (1984) and Hardouvelis (1988), among others, provide evidence that the EH failure is due to the omission of a time-varying term premium, which is correlated with the term spread, from the predictive model. Also Cook and Hahn (1986) suggest that the weak support for EH can be attributed to small changes of the term premium over time.

We recall that the expectations theory is built on the crucial assumption of rationality. The traditional version of EH implies a constant term premium over time; while the pure EH theory implies the term premium equal to zero. In this chapter we consider the softer version of EH allowing for the presence of term premia. Adopting the notation introduced by Campbell and Shiller
We define term premia as the difference between the long term yield \( y_n \) and its EH-implied value, i.e. an average of expected short term yields \( y^m \):

\[
y_n = \left( \frac{m}{n} \sum_{q=0}^{n-m} E_t y^m_{t+mq} + tp^{n,m}_t \right)
\]

(4.1)

Where \( n \) represent the monthly long term maturity, while \( m \) is the short term maturity. \( E_t \) denotes expectations. In this chapter we consider 120 and 60 months as long maturities \( n \) and 3 months as short maturities \( m \).

Standard dynamic asset pricing theory emphasizes the time-varying nature of term premia. The fundamental asset pricing equation asserts that the price of a security can be seen as the discounted value of its expected future payoffs. The time \( t \) price of a \( n \)-period bond is simply the expected discounted value of its price one-period ahead. If the bond expires in period \( l \), the current discount factor is deterministic, i.e. it is known with certainty. On the other hand, if the maturity date is far into the future \( n > l \), the actual discount factor is stochastic since it depends upon the sequence of future one-period discount factors \( \{z_t, E_t(z_{t+1}), E_t(z_{t+2}), ..., E_t(z_{t+n})\} \). We recall that the iterated law of expectations (tower rule) implies \( E_t\{E_{t+1}(z_{t+n})\} = E_t(z_{t+n}) \). In asset pricing theory the stochastic discount factor \( (z_t) \) represents a convenient way to model uncertainty; the current price of a security can thus be expressed by appropriately discounting future state-contingent payoffs:

\[
p_t^n = E_t\left( z_{t+1} p_{t+1}^{n-1} \right)
\]

(4.2)

The price of the bond at time \( t+1 \) is the present value of its future payoff \( p_{t+1}^{n-1} = E_{t+1}\left( z_{t+2} p_{t+2}^{n-2} \right) \).

Substituting this expression in formula (4.2) yields:

\[
p_t^n = E_t\left[ z_{t+1} E_{t+1}\left( z_{t+2} p_{t+2}^{n-2} \right) \right]
\]

(4.3)

Using the law of iterated expectations, and iterating the process recursively forward yields:

\[
p_t^n = E_t\left( \prod_{q=1}^{n} z_{t+q} \right)
\]

(4.4)

The price of a unit of currency delivered at time \( t \) is trivially one dollar \( p_t^0 = 1 \) \(( \forall q \) \). The price of a bond thus depends upon the sequence of future stochastic discount factors along the entire life of the bond. The microeconomic foundation of the stochastic discount factor derives from an intertemporal optimization problem of resource allocation. Defining the marginal utility of

---

95 The empirical analysis is performed with data from January 1988 and June 2007 in different geographical areas. We consider United States, the Euro area, Canada, and the United Kingdom. Samples are automatically adjusted as imposed by equation (4.1); recent observations are thus lost due to the expectations of future short yields. Data are presented in Appendix A4.I.
consumption as the first derivative with respect to consumption of a standard Von Neumann-Morgenstern utility function, the stochastic discount factor is:

\[ z_{t+1} = \delta E_t \left( \frac{u'(c_{t+1})}{u'(c_t)} \right) \]  

(4.5)

Where \( \delta \) is the subjective discount factor, i.e. a parameter describing the temporal preferences of the representative agent. The lower \( \delta \), the lower the weight given to future consumption, and the more impatient the consumer. The utility function is increasing, so that it reflects the desire for more consumption (positive first derivative). The concave shape of the utility function \( u(\bullet) \) indicates risk aversion and justifies the desire for intertemporal substitution. The risk-averse attitude of households implies preference for a smooth stream of consumption both over time and across states of nature. Optimizing behaviour implies a steady state consumption level which depends on agents’ preferences and endowments. The stochastic discount factor in (4.5) represents the marginal rate of substitution, i.e. the condition describing the representative agent’s willingness to shift consumption between two points in time, from present to future or viceversa. In principle, the higher the savings rate, the lower the actual level of aggregate consumption, the greater the demand for financial securities. Therefore, the demand for assets depends upon the relative convenience of saving to consuming. In absence of shocks and with homogeneous preferences, individual preferences are consistently reflected at an aggregate level; but, if a shock hits the economy the following happens. In a complete contingent claim market, risk sharing implies that agents’ consumption moves together, since only aggregate risk matters. In complete markets the marginal rate of substitution of any investor equals the contingent claim price ratio, so that all individuals share all risks equally. Any shock hitting the economy thus affects all people equally; shocks to consumption are perfectly correlated across individuals. Risk sharing does not imply that all agents’ will afford the same level of consumption though, since it depends on individual initial endowments; hence, consumption smoothing is possible only to the extent that there is no uncertainty regarding endowments (risk sharing is also Pareto optimal). The stochastic discount factor model of risk sharing is similar to the insurance principle: financial markets allow people to make consumption independent of contingent income volatility.

From (4.1) term premia can be seen as the difference between the long rate and its EH-consistent rationally expected value. Exploiting the famous inverse relationship between bond prices and yields \( (y^n = -\frac{1}{ln}(p^n)) \), we can highlight the dependence of term premia on the summation of the future path of stochastic discount factors:
The stochastic discount factor is a useful device that allows to determine the price level of a security from the future path of expected payoffs. The stochastic discount factor, or pricing kernel, is a random variable that features the law of one price, i.e. the absence of arbitrage opportunities.

The stochastic discount factor may respond to a great variety of shocks that hit the economy, such as monetary and fiscal shocks; in addition, also technological and institutional changes affect the dynamics of the stochastic discount factor. There is large evidence of inverse correlation between excess returns and the cyclical behaviour of the economy; this result holds both for stocks (Lettau and Ludvigson, 2001) and for bonds (Cochrane and Piazzesi, 2005). The variability of the discount factor over the business cycle is documented both by Constantinides and Duffie (1996), who argue that risk is high during recessions, and by Campbell and Cochrane (1999), who suggest that risk-aversion increases in bad times. In light of these considerations, the derivation of the term premium in (4.6) provides a theoretical justification for modelling term premia by means of a multifactor time-varying model.

We assume term premia are functions of variables acknowledged to be important macroeconomic determinants in the macro-finance empirical literature. It follows a qualitative analysis.

\[
\frac{t}{p_{x,y}^{m}} = y_{t}^{x} - \left( \frac{m}{n} \sum_{q=0}^{n-w} E_{t} y_{t+q}^{m} \right) = \\
= - \frac{1}{n} \log p_{t}^{x} - \left( \frac{m}{n} \sum_{q=0}^{n-w} E_{t} \left[ - \frac{1}{m} \log p_{t+q}^{m} \right] \right) = \\
= - \frac{1}{n} \log E_{t} \left[ \prod_{q=1}^{n} z_{t+q} \right] + \frac{1}{n} E_{t} \left[ \sum_{q=1}^{n-w} \log E_{t+q}^{m} z_{t+q} \right]
\]

(4.6)

U.S. data. Scatter plot diagrams of the term premium (Term Premium) against macroeconomic variables: the effective federal funds rate (ffr), unemployment rate (unemp), inflation, and the yield spread (spread). Term premium computed from the 120-month and the 3-month maturities (top panels). Term premium computed from the 60-month and the 3-month maturities (bottom panels).

Figure 4.1 (a)
U.S. scatter plots between the term premium and macro variables are reported in Figure 4.2 (a). From left to right the term premium is plotted against the effective fed funds, unemployment, inflation, and the slope of the yield curve\(^6\). The term premium seems positively related to the federal funds and inflation; while the greater dispersion of the observations around the regression line tends to suggest a weaker effect of both the unemployment rate and the yield spread. Figure 4.2 (b) plots the scatter diagrams of the term premium against macroeconomic variables for Canada (top panels), U.K. (middle panels), and the Euro Area (bottom panels). The plot of Canadian observations are not so concentrated around the interpolant line. In particular, there seems to be an awkward negative effect of inflation on the term premium. However, we point out that the term premium is always positive and the fit is definitely weak. We might speculate this is a consequence of the successful Canadian inflation targeting regime. A visual inspection of U.K. scatter diagrams suggest that results for U.K. are in between the U.S. and the Canadian ones. U.K. inflation has a

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\(^6\) The term premium is derived applying equation (4.1). In the top panels the term premium is computed rolling the 3-month rate on the 120-month horizon. In the bottom panels the term premium is obtained rolling the 3-month yield on the 60-month horizon. In order to avoid any multicollinearity issue, the yield spread is the (60, 3) in the former case, and the (120, 3) in the latter case.
positive effect on the term premium. Also, in the Euro zone the term premium appears to be positively affected by the inflation rate. Delivering price stability is, in fact, the primary objective of the European Central Bank, so that investors require a positive liquidity premium when the monetary authority fails; in addition, consistently with this story, the positive relationship between the term premium and the policy rate reflects the attempt of monetary policy to calm inflation down when the price dynamics tends to move out of control.

We specify the following multifactor model for term premia:

$$tp_{n,m}^t = \beta_0 + \beta_1 ffr_t + \beta_2 un_t + \beta_3 \pi_{t}^{exp} + \beta_4 \left(y_t^n - y_t^m\right) + e_{n,m}^t$$  \hspace{1cm} (4.7)

The above equation has been estimated for different combination of maturities \((n, m)\) and for all countries \((U.S., the Euro zone, Canada, and U.K.). Different statistical and econometric tests have been performed on the linear equation (4.7) in order to detect instability. Evidence of non linearity is provided in Appendix A4.II.

The idea of deriving the dynamic properties of term premia from a time-varying model and then to gauge the ability to predict the future level of economic activity finds inspiration in Rudebusch Sack, and Swanson (2007). They observe that both the effect of term premia on output and the ability of term premia to predict future activity depend on the nature of shocks affecting term premia. A time-varying approach thus might be particularly effective since it deals with the unpredictable nature of disturbances that influence both macro and financial variables. The conditional variance of term premia forecast errors, in fact, emphasizes the joint effect of two sources of uncertainty. On the one hand we consider the noise associated to idiosyncratic disturbances; on the other hand, uncertainty derives from the evolutionary behaviour of parameters in (4.7). The idea of allowing for time-varying coefficients seems to be consistent with the large evidence attributing EH failure to variation of term premia over time. Kalman filtering is a recursive method widely employed in economics and finance which suits for our purposes, since it represents a convenient procedure to describe how agents process available information. The main advantage of the filter is to generate expectations in a continuously updated Bayesian fashion. Kalman filtering thus matches agents’ rational behaviour since, at any point in time, the up-to-date filtration process becomes part of the current informative set.

In this chapter, for different pairs of maturities \((n, m)\) we derive term premia as a residual component of the yield spread, which can be decomposed into an expectations-based component (first RHS term) and a term premium (second RHS element):

---

97 According to (4.7) term premia depend on a constant, the policy rate or a proxy of it, the rate of unemployment, inflation, and a measure of the yield curve slope.

98 The policy rates are the following. The effective fed funds rate for U.S.; the overnight rate for the Canadian economy; the Libor and the Fibor/Euribor for U.K. and the Euro area respectively.
\[ y_t^m - y_t^n = \left( \frac{m}{n} \sum_{q=0}^{n-m} E_t y_{t+q}^m - y_t^m \right) + \left( y_t^n - \frac{m}{n} \sum_{q=0}^{n-m} E_t y_{t+q}^n \right) \] (4.8)

The term premium in (4.8) is also consistent with the term premium obtained from the Campbell and Shiller (1991) equation (4.1). In general, term premia can be viewed as the sum of a liquidity and a risk premium (Hamilton and Kim, 2002; Favero, Kaminska, and Soderstrom, 2005).

We recall that our final goal is to examine whether the dynamic properties of term premia can be exploited to infer future output growth. In this chapter we thus propose a time-varying multifactor model for risk premia. Term premia are assumed to be a dynamic function of the policy rate, the rate of unemployment, the inflation rate, and a measure of the term structure slope, i.e. the yield spread\(^99\). Risk premia are associated to the monetary policy stance as captured by the policy rate\(^100\).

The level of the policy rate is usually tied to its variability, hence this model accounts also for the conjecture by Hamilton and Kim (2002) who argue that the interest rate variability is a determinant of term premia. Unemployment affects term premia through risk aversion. Backus and Wright (2007) provide evidence of the cyclical behaviour of term premia reaching high levels when the unemployment rate raises. According to Cochrane (2005) “an asset that does badly in states of nature like a recession, in which the investor feels poor and is consuming little, is less desirable than an asset that performs badly in states of nature like a boom in which the investor feels wealthy and is consuming a great deal”. Inflation is another important determinant of term premia as long as economic agents aim at preserving the real value of their assets. Ang and Bekaert (2002, 2006) show that the positive slope of the yield curve is due to a inflation risk premium indeed. Finally, Lee (1995) emphasizes the role of the yield spread in explaining the magnitude and the variability of risk premia. The state space form of the Kalman filter is represented by two basic equations. The observation equation, or the measurement equation, is:

\[ tp_{t,j}^{n,m} = d_j + x_{t,j} \beta_{t,j} + e_{t,j}^{n,m} \] (4.9)

The observation equation relates the dependent variable to the explanatory variables; the subscript \( j \) indicates the country. \( e_{t,j} \) is a country specific stochastic disturbance \( i.i.d.(0, \sigma_e) \). The state, or transition, equation captures the evolution of coefficients over time:

\[ \beta_{t,j} = \mu_j + F_{j} \beta_{t-1,j} + v_{t,j} \] (4.10)

\(^99\) Since the influence of both CPI-inflation and the yield spread on U.S. term premia seems statistically weak, these variables have been replaced by the PPI-inflation and the effective exchange rate.

\(^100\) In this framework we do not explicitly allow for the credibility of the monetary regime or the reputation of the monetary authority, although both aspects are regarded to affect risk premia on international financial markets. We believe that the time-varying pattern of coefficients implicitly captures both effects.
\( \nu_{i,j} \) is an idiosyncratic noise \( i.i.d(0, \sigma^2) \). It is common practice to assuming a random walk behaviour for regressing coefficients (Kim and Nelson, 1998; Kim and Nelson, 2006; Boivin, 2006); matrix \( F \) in equation (4.10) is thus the identity matrix. The Kalman filter is an iterative algorithm which we summarize here by means of by the following expressions:

\[
P_{\beta_{t-1}} = E \left[ (\beta_t - \beta_{t-1}) (\beta_t - \beta_{t-1})' \right] \quad (4.11)
\]

Equation (4.11) represents the variance-covariance matrix of the coefficients conditional on information up to \( t-1 \); equation (4.12) is the forecast of the term premium based on information available up to time \( t-1 \); equation (4.13) represents the prediction error, while equation (4.14) is its conditional variance.

\[
\eta_{t|t-1} = x't \beta_{t-1} \quad (4.12)
\]

\[
\eta_{t|t-1} = \eta_t - x't \beta_{t-1} = \eta_t - \eta_{t|t-1} \quad (4.13)
\]

\[
h_{t|t-1} = E[\eta_{t|t-1}^2] = x't P_{\eta_{t|t-1}} x_t + \sigma^2 \quad (4.14)
\]

Briefly focus on expression (4.14). Kalman filtering implies that two sources of uncertainty characterize the conditional variance of the forecast error \( h_{t|t-1} \): one form of uncertainty is due to the evolutionary behaviour of estimated coefficients, the other is a random noise associated to future unpredictable disturbances, such as political, institutional, or technological shocks. Risk premia are a function of the expected path of the stochastic discount factor, which is regarded to respond to a variety of shocks. Hence, the assumption of a constant variance of nominal shocks to term premia within a country over time does not seem realistic; the variance conditional upon available information at the time of forecasting is assumed to be time-varying due, for instance, to a continuously changing regime, as captured by evolutionary behaviour of \( \beta \) coefficients, or to some unpredictable shocks that hit the economy, as captured by the stochastic noise. The main advantage of Kalman filtering is that expectations are continuously updated over time depending on the state of the economy. For each country, the Kalman filter estimation of the time-varying parameter model (4.7) returns, in fact, by large a better fit that linear OLS estimations. Linear estimations of equation (4.7) return serially correlated residuals, whereas the time-varying coefficient model generates a more suitable pattern for disturbances. High residual autocorrelation implies a predictable pattern for disturbances thus affecting estimation efficiency. Residuals predictability is, on the one hand, a sign of model mis-specification, and, on the other hand, provides with a clue to improve forecasts of the dependent variable. Residuals are a core variable in this study, since we exploit the dynamic properties of disturbances to make inference about the future level of economic
activity. We claim that the less informative residuals are, the more robust our empirical results should be. The conditional variance of a serially correlated series may be trivially informative since it is obtained by processing data which are themselves informative. Obtaining the conditional variance from a plain and uninformative series, instead, makes the empirical analysis more robust since it prevents the predictive model from being redundant.

In the following Section we present empirical evidence highlighting that the conditional variance of term premia forecast errors is informative beyond the yield spread and the term premium about future business cycle fluctuations.

### 4.4 Empirical Evidence

In this Section we report empirical results showing that not only term premia but also their conditional variances are informative about the business cycle. Let $IP_{t,j}$ denote the time $t$ level of the seasonally adjusted industrial production index in country $j$ (U.S., the Euro area, Canada, and U.K.); $\Delta IP_{t+T,j}$ represents the annualized rate of growth of $IP$ over the period $t$ to $t+T$:

$$
\Delta IP_{t+T,j} = \frac{12}{T} \log \left( \frac{IP_{t+T,j}}{IP_{t,j}} \right) \times 100 \quad (4.15)
$$

$T$ indicates the length of the forecast horizon (6, 12, 18, 24, 36 months). Four different specifications of the predictive model will be estimated:

$$
\Delta IP_{t+T,j} = \alpha_0 + \alpha_4 \Delta IP_{t,j} + u_{t,j} \quad (4.16)
$$

$$
\Delta IP_{t+T,j} = \alpha_0 + \alpha_1 h_{t+1-j} + \alpha_2 h_{t+1-j} + \alpha_3 \Delta IP_{t,j} + u_{t,j} \quad (4.17)
$$

$$
\Delta IP_{t+T,j} = \alpha_0 + \alpha_3 \Delta t_{n} + \alpha_3 h_{t+1-j} + \alpha_4 \Delta IP_{t,j} + u_{t,j} \quad (4.18)
$$

$$
\Delta IP_{t+T,j} = \alpha_0 + \alpha_3 h_{t+1-j} + \alpha_3 t_{n} + \alpha_3 h_{t+1-j} + \alpha_4 \Delta IP_{t,j} + u_{t,j} \quad (4.19)
$$

Where, according to equation (4.8) the $spread_{t,j}^{n,m}$ between the long ($n$) and the short ($m$) term yield can be decomposed into the term premium ($tp_{t,j}^{n,m}$) and the expectations-based component ($thsp_{t,j}^{n,m}$), i.e. the theoretical, or perfect foresight, spread; $h_{t+1-j}$ is the conditional variance of term premia prediction errors and $\eta_{t+1-j}$ is the term premia forecast errors. The actual value of the industrial production growth ($\Delta IP_{t,j}$) has been included in order to show that the predictive ability (i.e. the statistical significance) of the financial indicators is robust to the inclusion of a real
variable. All equations have been estimated imposing the Newey-West (1987) correction to handle with overlapping nonspherical disturbances.

Mishkin (1982) and Pagan (1984) pointed out that generated regressors in the above equation might influence the distribution of test statistics, and, consequently, invalidate the inference procedure to verify parameters’ significance. In order to prove our results are robust we have estimated quite a few augmented specifications of the above regressions. These equations include additional explanatory variables such as the policy rate, the effective exchange rate, the nominal bilateral exchange rates between two economies among the four considered. Results are definitely robust to different model specifications. In addition, we can count on a sufficiently large number of available observations. Third, the functional form of coefficient $\alpha_2$ has also been chosen to avoid any potential multicollinearity problem in equations (4.17) and (4.19). Following Kim and Nelson (1989) coefficient $\alpha_2$ has been set to be a function of the term premium conditional variance: $\alpha_{2,j} = \phi_{0,j} + \phi_{1,j} \ln(h_{1-j})$. Both the two-step estimation procedure and the joint estimation confirm results are robust. Finally, as shown in the following tables, we carry out the instrumental variables estimations in order to back generated regressors with observable variables. Once again results are encouraging, the estimated coefficient of the term premia conditional variance remains statistically significant and preserve the negative sign. Empirical results are presented in the following Sections.

It follows a deeper overview about the method adopted in this chapter to derive the term premium, explaining also the econometric methodology employed to estimate equations (4.16) – (4.19). Equations (4.1) and (4.8) imply that the yield spread can be decomposed into two terms. One is the difference between short-term rate expected over the next $n$ periods and the current interest rate; the other is the time-varying term premium, which can be viewed as the summation of a liquidity premium and a pure risk premium (Kim, 2000). After adjusting for a constant scaling factor, the term premium implied by the Campbell and Shiller equation (4.1) is exactly the same obtained by Cochrane and Piazzesi (2005). Thus, if a fall in the yield spread is expected to anticipate recessions it might be that this is due either to a temporary high short rate indicating an incoming slowdown in economic activity or to a fall in the premium on long bonds (relative to short bonds) suggesting an imminent recession. Since short rates rise relative to long rates before a recession, “to what extent is this because future short rates are rationally expected to fall (the simple EH), to what extent is it because the forecastable excess yield from holding long term bonds has fallen (which must be a risk

---

101 Depending on the pair of maturities considered ($n, m$) the lowest number of observation is 117, so that statistical inference is based on distributions with 112 degrees of freedom. When the long term maturity is 60-month inference is based on statistics with 177 degrees of freedom.

102 Also dropping the forecast errors term from the equations does not affect the significance of term premia conditional variance.

103 Each regression has been estimated using the first lag of explanatory variables as instruments.
premium or a liquidity premium)?” (Hamilton and Kim, 2002). Analyzing this effect directly from the data yields to the following reasoning. The spread between the n-period yield and the 1-period yield is:

\[ y_t^n - y_t^1 = \left( \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^1 - y_t^1 \right) + \left( \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^n - y_t^n \right) \]  

(4.20)

After ruling out the contemporaneous real variable in equation (4.16) we substitute (4.20):

\[ \Delta P_{t+T,j} = \alpha_0 + \alpha_3 \left( y_t^n - \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^1 \right) + \alpha_3 \left( y_t^n - \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^n \right) + \omega_t \]  

(4.21)

In equation (4.21) the forecasting contribution of the term premium and of the rationally expected change in short rates is clearly distinct. A generalization of equation (4.21) would imply this different terms to have a different impact on future output; we thus should estimate the following:

\[ \Delta P_{t+T,j} = \alpha_0 + \alpha_{3a} \left( y_t^n - \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^1 \right) + \alpha_{3b} \left( \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^n - y_t^n \right) + \omega_t \]  

(4.22)

Let \( \xi_{t+u} \) denote the error in forecasting the future short rate:

\[ \xi_{t+u} = \left( \frac{1}{n} \sum_{q=0}^{n-1} y_{t+q}^1 \right) - \left( \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^1 \right) \]  

(4.23)

Actually (4.22) can be written as:

\[ \Delta P_{t+T,j} = \alpha_0 + \alpha_{3a} \left( y_t^n - \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^1 \right) + \alpha_{3b} \left( \frac{1}{n} \sum_{q=0}^{n-1} E_r y_{t+q}^n - y_t^n \right) + u_t \]  

(4.24)

We now focus on the disturbance term \( u_t \) which is a combination of both the output prediction error \( \omega_t \) and of the interest rate forecast noise \( \xi_{t+u} \) according to the following:

\[ u_t = \omega_t + (\alpha_{3b} - \alpha_{3a}) \xi_{t+u} \]  

(4.25)

Under rational expectations, the error term \( u_t \) should be uncorrelated with any know variable at time \( t \). Thus (4.24) can be estimated using the instrumental variables technique with any variable dated at \( t \) or earlier as instruments. Equation (4.24) is the basic empirical version of the models we are going to estimate in the following Sections. Favero, Kaminska, and Soderstrom (2005) criticize the approach by Hamilton and Kim (2002) providing with a decomposition of the spread using ex-post observed interest rate data to substitute for ex-ante expected values. Favero et al. (2005) try to improve upon also the model by Ang, Piazzesi and Wei (2006) who estimate a VAR and then projects expectations for the short term rate. Favero, Kaminska and Soderstrom (2005) thus suggest estimating and simulating forward a reduced macro-finance model to derive reliable “real-time”
short term rate expectations to substitute in (4.20) in order to derive a realistic measure of the term
premium. In this chapter we construct the term premium following Hamilton and Kim, but we also
improve upon their model to assess the predictive ability of the conditional variance of the term
premium, which is interpreted as a sign of financial distress, which is, in turn, a symptom of
incoming financial fragility.

4.4.1 Evidence for U.S.

Figure 4.2 shows the dynamics of both the yield spread, the term premium, and the conditional
variance of the term premium prediction errors before NBER recessions (grey shaded areas). The
yield spreads decrease, and eventually become negative, before recessions (left panel). The term
premium appears to anticipate a decline in real activity as well since it rises substantially before
recessions (central panel). More importantly, the clear spike of term premia conditional variance
which occurs either immediately before or at the beginning of slowdowns seems to be quite
informative about business cycle fluctuations (left panel).

spread \((n, m)\): yield spread between the \(n\)-month and the \(m\)-month yields. \(tp \ (n, m)\): term premium computed from
the \(n\)-month and the \(m\)-month maturities. \(U.S. \ tp \ cvar \ (n, m)\): U.S. term premium conditional variance.

U.S. empirical results suggest that the conditional variance of term premia is a powerful predictor
of the future growth in the industrial production index. Negative values of \(\alpha_i\) in equations (4.17) and
(4.19) indicate that high term premia conditional variance predicts slower economic growth in the
future. Therefore, financial distress, as reflected in excess volatility of term premia, tends to
anticipate future slowdown in economic activity. Data evidence seems thus to support the
hypothesis that the conditional variance of term premia forecast errors is informative beyond the
yield spread and term premia about future business cycle fluctuations. The adjusted-R\(^2\), in fact,
increases substantially when we include a measure of the volatility of term premia. We interpret the
conditional variance of term premia as a sign of financial fragility which reflects uncertainty about
the future evolution of the economy. The rational works as follows. A typical Keynesian equation
implies that actual investments depend negatively on the (real) interest rate and positively by
expected level of income. Financial distress reflected in high term premia volatility exerts direct influence on interest rates variability and, moreover, it affects expectations about future GDP. A primary effect on future aggregate demand thus derives from the channel of uncertainty regarding capital accumulation. Secondly, as highlighted above, uncertainty reduces aggregate consumption through the diminishing marginal propensity to consume since rational agents shift to precautionary savings. The perverse effect is amplified in a traditional dynamic multiplier Keynesian framework explaining why financial distress may anticipate a weaker economic growth.

Results suggest that also coefficient $\alpha_2$ tends to be negative and statistically significant; business cycle movements are thus inversely related, not only to the volatility of term premia as captured by the conditional variance, but also to the magnitude of prediction errors.

The second important result is that the level of term premia is inversely related to the future level of real activity (Rudebusch, Sack, Swanson, 2007). Coefficient $\alpha_3$ is negative in equations (4.18) and (4.19).

In line with Favero et al. (2005), our results confirm that splitting the yield spread into the term premium and the theoretical spread leads to a better understanding of the forecasting model. If we compare, in fact, the goodness of fit from regressions on the left with that of the ones on the right the improvement occurs particularly at long horizons. Table 4.1 report results for U.S. when term premia are computed from term structure maturities ($m = 3, n = 120$).

Term premia forecast errors and the associated conditional variance can anticipate movements in real activity up to three years ahead.

Many authors have documented that lower term premia tend to predict slower GDP growth, since the estimated $\alpha_3$ coefficient turn out to be positive$^{104}$. We claim that this is contrary to common wisdom, as long as risk aversion should exert a negative effect on output. Consistently with Rudebusch, Sack, and Swanson (2007), our results suggest an inverse correlation between actual term premia and future economic activity. Interestingly, the expected component of the yield spread ($\alpha_{3b}$) seems to be uninformative about future movements in the business cycle. Finally, the coefficient of the yield spread ($\alpha_3$) tends to be positive, but its statistical significance is not robust to different empirical specifications. Large values of the spread are typically associated to accommodative stance of monetary policy and stimulus to real economic activity.

---

$^{104}$ Hamilton and Kim (2002); Favero, Kaminska, and Soderstrom (2005); Ang, Piazzesi, and Wei (2006).
The table below shows the results for U.S. future industrial production growth with different forecast horizons. The table includes the coefficients ($\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$) and their p-values ($R^2$) for various forecast periods. The results are statistically significant, indicating a negative correlation with economic activity.

### U.S. Future Industrial Production Growth - (120, 3)

<table>
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<th>Horizon</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
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<th>$\alpha_4$</th>
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<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
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In Table 4.2 we report results for U.S. when the term premium is computed using the pair of maturities ($n = 60, m = 3$).

As far as the conditional volatility of term premia is concerned, results are similar to those obtained from the couple of maturities ($120, 3$). Coefficient $\alpha_1$ is always statistically significant. The variability of financial markets’ sentiment thus seems to display a significant negative effect on the economic conjuncture ($\alpha_1 < 0$; $\alpha_2 < 0$). Term premia forecast errors ($\alpha_2$) are not robust to the instrumental variables specification though. Again the level of term premia tends to be inversely related to the business cycle, but the statistical significance is robust only for prediction of economic activity 18 to 24 months ahead; over those forecasting horizons a decline in term premia tends to be a stimulus to economic activity.
We remark our main result that the effect of term premia conditional variance is robust to different model specifications, and it does not vanish after the inclusion of the current level of output growth\textsuperscript{105}.

Equations (4.17) and (4.19) have also been estimated for the sample between the two recessions: from April 1991 to December 2000. The coefficients of the conditional variance remain statistically significant but decrease in magnitude. This result may reflect the slowdown in industrial production occurred in the mid 1990s.

We recall that our goal is not to prove a direct influence running from financial markets to the real economy; we rather aim to detect whether distress on bond markets, as captured by the conditional volatility of term premia, is informative about business cycle fluctuations. We thus provide

\textsuperscript{105} Our results are comparable with those by Schwert (1989), who analyses the relation between stock market volatility and the business cycle during financial crisis. He finds that stock market volatility increases substantially after stock prices drop; in particular, financial markets volatility remains high, or increases, during recessions. Empirical evidence thus tends to support the popular view that stock market volatility is a significant cyclical indicator. Our analysis complements that by Schwert as long as we focus on bond markets rather than on stock markets. We find evidence that the conditional volatility of bond risk premia is an important indicator of future fluctuations in economic activity.

Table 4.2

<table>
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<th>$a_2$ (p-val)</th>
<th>$a_{3a}$ (p-val)</th>
<th>$a_{3b}$ (p-val)</th>
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<th>$a_1$ (p-val)</th>
<th>$a_2$ (p-val)</th>
<th>$a_3a$ (p-val)</th>
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sample 1988 - 2005
evidence suggesting that financial markets may anticipate future movements in real activity. In this chapter we thus emphasize the signalling role of uncertainty without insinuating the existence of any causality implication. We may only suggest that agents heavily discount expected future events in current prices through the stochastic discount factor when great uncertainty is reflected in bond markets. In addition, we may observe that current uncertainty might encourage further perverse behaviours, such as adverse selection and moral hazard, which, in turn, would contribute to worsening the expectations about future economic conjuncture. The adverse effects on the economy thus mirror rational agents’ concern of bearing an excessive, or unnecessary, risk.

So far we have examined the effect on output exerted by term premia obtained from the entire length of maturity spectrum of the term structure ($n = 120, 60; m = 6, 3$). If we focus on the medium and short end of the yield curve ($n = 36, 24$) results are not so encouraging. In particular, when the long term rate is $n = 36$, the estimated coefficient $\alpha_1$, which describes the effect on output by the conditional variance of term premia, is informative about business cycle only over short forecasting horizon, i.e. from one to two quarters.

### 4.4.2 Evidence for the Euro Area

Also in the Euro area the conditional variance of term premia turns out to be a useful predictor of future growth in industrial production, which is acknowledged to be an important engine that moves the overall economy. Our empirical findings highlight that the forecasting power is significant from 6 to 18 months ahead when the conditional variance is extracted from term premia on bonds with maturities ($60, 3$); while, for predicting movements of industrial production over longer horizons (from 24 to 36 months ahead) it is useful to exploit information from longer maturities of the term structure ($n = 120$).

The forecasting ability of term premia conditional variance is robust to different specifications of the predictive model as described above. In addition, instrumental variables estimations suggest that the statistical significance of the generated variables can be considered robust.\(^{106}\)

Two further results needs mentioning. First, including the conditional variance of term premia in the predictive model substantially increases the goodness of fit. Second, results suggest that future output growth tends to be inversely related to the actual level of term premia; coefficient $\alpha_{3a}$ is negative but not always statistically significant. In particular, differently from U.S. estimates, EMU results highlight that the decomposition of the spread into a term premium and an expectational component improves the forecasting ability of the model mainly because of the effect of the

\(^{106}\) We instruments the explanatory variables with their first lag; instruments and regressors are thus highly correlated.
expected component of the yield spread. The adjusted goodness of fit of equation 4.18 (left panel of Tables 4.3 and 4.4) is generally greater than the adjusted fit of equation 4.16 (right panel).

### Table 4.3

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sample 1988 – 1999

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sample 1988 – 2005
### 4.4.3 Evidence for the Canadian Economy

Estimations for the Canadian economy return similar results as well. The predictive ability of term premia conditional variance is negative and statistically significant. Surprisingly, Canada results return a positive coefficient estimate for the level of term premia ($\alpha_3$); term premia thus seem to anticipate faster, rather than weaker, growth in industrial production.

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<td>$T$ (p-val)</td>
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<tr>
<td>(+12)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>(+18)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>(+24)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>(+36)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Table 4.5
The decomposition of the spread into a term premium and an expectations-based component does not seem to improve the forecasting ability; the adjusted goodness of fit remains unchanged passing from equation (4.16) to equation (4.18). The forecasting ability improves substantially only for very long forecasting horizons ($T = 36$ months ahead). This result holds in spite of the derivation of term premia from the $(120, 3)$ rather than the $(60, 3)$ maturity field.

![Figure 4.4](image)

**Figure 4.4**

<table>
<thead>
<tr>
<th>CAN Future Industrial Production Growth - (60,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizon $T$</td>
</tr>
<tr>
<td>(+6)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td>(+12)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td>(+18)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td>(+24)</td>
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<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
<tr>
<td>(+36)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>IV</td>
</tr>
</tbody>
</table>

Table 4.6

Canadian data. Scatter plot diagrams between the industrial production future growth rate (IP grw), or the unemployment rate (right panel), and the conditional variance of the term premium (cVar) computed from the $n$-month and the $m$-month maturities.
4.4.4 Evidence for U.K.

Finally, we analyse whether the conditional variance of bond term premia is informative about future fluctuations in economic activity also for the British economy. Results for U.K. are not so encouraging. Firstly, the conditional variance of term premia seems to be informative about future economic activity only if information about term premia is extracted from the entire yield curve, i.e. when the long term maturity is 10 years ($m = 120$). The conditional variance for term premia obtained from the pair of maturities $(60, 3)$ does not help to predict future industrial production. Second, the predictive ability of conditional variance is not significant for short forecasting horizons (less than 18 months). Finally, although the fit is weak a visual inspection of Figure 4.5 reveals that the future growth in the industrial production index (12 months ahead) seems to be positively related to the conditional variance of term premia, which clearly contradicts previous evidence.

<table>
<thead>
<tr>
<th>Horizon $T$</th>
<th>$a_1$ (p-val)</th>
<th>$a_2$ (p-val)</th>
<th>$a_3a$ (p-val)</th>
<th>$a_3b$ (p-val)</th>
<th>$a_4$ (p-val)</th>
<th>$a-R^2$ (p-val)</th>
<th>$a_1$ (p-val)</th>
<th>$a_2$ (p-val)</th>
<th>$a_3$ (p-val)</th>
<th>$a_4$ (p-val)</th>
<th>$a-R^2$ (p-val)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+18)</td>
<td>-0.0064 (0.833)</td>
<td>0.0521 (0.005)</td>
<td>-0.4838 (0.001)</td>
<td>0.76 (0.001)</td>
<td>0.5434 (0.011)</td>
<td>-0.4738 (0.000)</td>
<td>0.73 (0.000)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.1779 (0.957)</td>
<td>-0.0024 (0.978)</td>
<td>-0.0007 (0.930)</td>
<td>0.5293 (0.000)</td>
<td>0.82 (0.000)</td>
<td>-0.2033 (0.687)</td>
<td>0.2813 (0.064)</td>
<td>0.5355 (0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.2948 (0.569)</td>
<td>-0.6591 (0.586)</td>
<td>-0.0965 (0.980)</td>
<td>-0.7652 (0.682)</td>
<td>0.00 (0.001)</td>
<td>-0.2803 (0.615)</td>
<td>-0.1855 (0.000)</td>
<td>-0.1452 (0.000)</td>
<td>-0.7450 (0.000)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| IV          | -0.0021 (0.947) | 0.0329 (0.000) | -0.5707 (0.000) | 0.73 (0.000) | 0.9739 (0.000) | 0.0548 (0.625) | 0.56 (0.000) |
|             | -0.2215 (0.905) | 0.0038 (0.432) | 0.0171 (0.087) | 0.000 (0.000) | 0.85 (0.000) | -0.2719 (0.605) | 0.3376 (0.058) | -0.2353 (0.027) | 0.68 (0.000) |
|             | -0.2752 (0.655) | -0.7682 (0.707) | -0.0775 (0.987) | -0.9718 (0.300) | 0.00 (0.000) | 0.4707 (0.954) | -0.5545 (0.166) | 0.2661 (0.000) | -0.5428 (0.000) | 0.02 (0.000) |

| (+24)       | 0.0723 (0.014) | 0.0667 (0.007) | 0.1552 (0.346) | 0.18 (0.346) | 0.6749 (0.002) | 0.1597 (0.308) | 0.19 (0.000) |
|             | -0.4613 (0.898) | 0.0059 (0.000) | -0.0004 (0.975) | -0.0277 (0.000) | 0.79 (0.000) | -0.3887 (0.000) | 0.0404 (0.228) | 0.1753 (0.026) | -0.1480 (0.000) | 0.70 (0.000) |
|             | -0.5448 (0.916) | -0.0446 (0.173) | -0.0639 (0.081) | -0.4576 (0.040) | 0.75 (0.041) | -0.5915 (0.627) | -0.5816 (0.591) | -0.2085 (0.330) | 0.6313 (0.000) | 0.00 (0.000) |

Table 4.7

<table>
<thead>
<tr>
<th>U.K. Future Industrial Production Growth - (120, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$ (p-val)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>(+18)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

| IV        | -0.0021 (0.947) | 0.0329 (0.000) | -0.5707 (0.000) | 0.73 (0.000) | 0.9739 (0.000) | 0.0548 (0.625) | 0.56 (0.000) |
|           | -0.2215 (0.905) | 0.0038 (0.432) | 0.0171 (0.087) | 0.000 (0.000) | 0.85 (0.000) | -0.2719 (0.605) | 0.3376 (0.058) | -0.2353 (0.027) | 0.68 (0.000) |
|           | -0.2752 (0.655) | -0.7682 (0.707) | -0.0775 (0.987) | -0.9718 (0.300) | 0.00 (0.000) | 0.4707 (0.954) | -0.5545 (0.166) | 0.2661 (0.000) | -0.5428 (0.000) | 0.02 (0.000) |

| (+24)     | 0.0723 (0.014) | 0.0667 (0.007) | 0.1552 (0.346) | 0.18 (0.346) | 0.6749 (0.002) | 0.1597 (0.308) | 0.19 (0.000) |
|           | -0.4613 (0.898) | 0.0059 (0.000) | -0.0004 (0.975) | -0.0277 (0.000) | 0.79 (0.000) | -0.3887 (0.000) | 0.0404 (0.228) | 0.1753 (0.026) | -0.1480 (0.000) | 0.70 (0.000) |
|           | -0.5448 (0.916) | -0.0446 (0.173) | -0.0639 (0.081) | -0.4576 (0.040) | 0.75 (0.041) | -0.5915 (0.627) | -0.5816 (0.591) | -0.2085 (0.330) | 0.6313 (0.000) | 0.00 (0.000) |

<table>
<thead>
<tr>
<th>sample 1988 – 1998</th>
</tr>
</thead>
</table>

Table 4.7

U.K. data. Scatter plot diagrams between the industrial production future growth rate (IP grw), or the unemployment rate (right panel), and the conditional variance of the term premium (cVar) computed from the $n$-month and the $m$-month maturities.

Figure 4.5
In this chapter we have developed an innovative method to extract valuable information from financial markets. Our approach highlights the role of term premia volatility in forecasting future business cycle fluctuations. Results are robust across countries and our findings suggest a statistically significant predictive power of term premia conditional variance over different horizon lengths.

4.5 Concluding Remarks

In this chapter we provide evidence confirming that the informative content of the term structure helps to predict future economic activity. In particular, we suggest that the conditional variance of term premia forecast errors is informative beyond both the yield spread and term premia about future output fluctuations.

After decomposing the yield spread into an expectations-based component and a term premium we examine the time-varying pattern of term premia. First, consistently with other studies we document that the aforementioned decomposition leads to a better understanding of the predictive model and allows obtaining more accurate forecasts of future economic activity. Hence, we separate the effect of monetary policy expectations and risk aversion in the prediction of future output. In contrast with previous evidence but in line with Rudebusch, Sack, and Swanson (2007), our findings highlight an inverse correlation between term premia and the business cycle; therefore declining term premia tend to anticipate faster output growth, and vice versa.

Our core analysis regards the predictive ability of term premia conditional variance which we interpret as a sign of financial distress. Data evidence seems to support the hypothesis that adding the conditional variance of term premia to the traditional predictive equation for real economic activity leads to a considerable improvement in the forecasting model. Our main result is that high values of term premia conditional variance anticipate slower output growth in the future. This result seems robust across countries and to different model specifications. However, we do not argue that exists a direct influence running from financial markets to the real economy; we simply examine whether distress on bond markets, as captured by the conditional volatility of term premia, is informative about future business cycle fluctuations.

In this chapter we thus focus on the signalling role of uncertainty. We may speculate that uncertainty might encourage perverse behaviours, such as moral hazard and adverse selection, which, in turn, could contribute to worsening the expectations about future economic conjuncture.
Appendix A4.1 - Data

All data have monthly frequency; the sample starts in January 1987. The core econometric analysis, after Kalman filtering, is thus performed from January 1988 since we rule out the first 12 observations.

**Industrial Production and Unemployment.** The U.S. series of seasonally adjusted industrial production is from the FRED database (Federal Reserve Economic Data). The European industrial production growth index is from Bloomberg (ticker: EUIPEMUY). Both the Canadian and the U.K. series of seasonally adjusted industrial production is from the IMF database (available from Datastream). The U.S. seasonally adjusted unemployment rate series (civilian unemployment) is from the FRED database; the source is the U.S. Department of Labour (Bureau of Labour Statistics). The European rate of unemployment is from Bloomberg (UMRTEMU). The Canadian unemployment rate series (seasonally adjusted percentage of civilian labour force) is from the OECD database (available from Datastream). The U.K. unemployment rate series is from IMF financial statistics (Datastream).

The log-industrial production growth and the unemployment rate are covariance stationary as suggested by both the augmented Dickey-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin test.

<table>
<thead>
<tr>
<th>Stationarity</th>
<th>U.S.</th>
<th>EMU</th>
<th>CAN</th>
<th>U.K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP grw</td>
<td>ADF</td>
<td>(0.050)*</td>
<td>(0.030)*</td>
<td>(0.064)*</td>
</tr>
<tr>
<td></td>
<td>KPSS</td>
<td>0.159*</td>
<td>0.154*</td>
<td>0.179*</td>
</tr>
<tr>
<td>Unemp</td>
<td>ADF</td>
<td>(0.019)*</td>
<td>(0.103)*</td>
<td>(0.027)*</td>
</tr>
<tr>
<td></td>
<td>KPSS</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.8

The ADF test rejects the null hypothesis of unit root; while the null hypothesis of stationarity cannot be rejected by the KPSS test. To match the monthly frequency of data, the rule of thumb selected number of lags in the auxiliary regression is either 11 or 12. The automatic lag selection based on different criteria (Akaike, Schwarz, Hannan-Quinn) is consistent with our choice. Unit root test results obtained with the automatic lag selections are similar. The critical values of the KPSS test are 0.739 (1%), 0.463 (5%), and 0.347 (10%) when the intercept is included in the auxiliary model. The compute KPSS statistics never falls in the critical region. The KPSS test tends to reject the null of stationarity for the unemployment series.

**Interest Rates.** U.S. yields data are from different sources. Before January 1999 the 3-and 6-month, and the 10-year yields are from the McCulluch database, while the yields associated to the remaining maturities (1-, 2-, 3-, 5-year) are from the Fama and Bliss CRSP database, as reported by Cochrane and Piazzesi (2005). After 1999 U.S. data series are the ZCB yield from Datastream. The U.S. effective federal funds rate is from the FRED database. Yields data for Canada are from the central Bank of Canada. Yields data for U.K. are from the Bank of England, while yield data for Europe are either from the European Central Bank (after 2006) or from Bloomberg. The Fibor (before January 1999) and the Euribor (afterwards) is from Datastream. Precisely, Fibor is the Germany Interbank 3-month offered rate; while Euribor is the 3-month offered rate. In the Figure below we plot the series of the U.S. federal funds, the Canadian overnight rate, and the Fi-Euribor.
Spreads. According to both the ADF and the KPSS tests the yield spreads are stationary. KPSS critical values when both the intercept and the trend are included in the regression are 0.216 (1%), 0.146 (5%), and 0.119 (10%).

<table>
<thead>
<tr>
<th></th>
<th>Stationarity</th>
<th></th>
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<th></th>
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</tr>
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<td></td>
<td>spread U.S.</td>
<td>EMU</td>
<td>CAN</td>
<td>U.K.</td>
<td></td>
</tr>
<tr>
<td>(120, 3)</td>
<td>(0.032)*</td>
<td>(0.017)*</td>
<td>(0.127)**</td>
<td>(0.093)</td>
<td></td>
</tr>
<tr>
<td>ADF</td>
<td>0.213</td>
<td>0.676</td>
<td>0.089**</td>
<td>0.131**</td>
<td></td>
</tr>
<tr>
<td>KPSS</td>
<td>(0.068)*</td>
<td>(0.020)**</td>
<td>(0.076)</td>
<td>(0.035)</td>
<td></td>
</tr>
<tr>
<td>(60, 3)</td>
<td>0.124*</td>
<td>0.099**</td>
<td>0.534*</td>
<td>0.338*</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9

Exchange Rates. The nominal bilateral exchange rates series between U.S. Dollar, and both the Canadian and U.K. currencies are from the FRED database. The nominal bilateral exchange rate between the Canadian Dollar and the U.K. Sterling has been derived from the two above series. The nominal exchange rate between U.S. and the Euro area is from Bloomberg. Selecting an appropriate number of lags, and including either the intercept both the intercept and a trend the ADF test suggests that these series are stationary. Also the KPSS test supports stationary (empirical statistics are lower than the critical values).

Term Premia. Term premia as obtained by equation (4.1) can be considered a proxy for excess bond returns. We discuss some descriptive statistics about term premia. Keeping the maturity $m$ of the short term yield constant (either 3, or 6, or 12 months) the mean is increasing with the maturity $n$ of the long term yield; similarly, given the maturity $n$ of the long term yield, the mean diminishes with the increase of the short term maturity $m$. The standard deviation of term premia is higher at shorter horizons. With few exceptions, the highest standard deviation is displayed by term premia whose longer maturity is 36 months. These descriptive statistics are consistent with some stylized facts in bond pricing. Firstly, at long horizons investors require a positive liquidity premium, which is increasing with maturity. Secondly, the medium-short end of the yield curve is more volatile than the long end. Yields are quite volatile at short maturities; whereas, long term rates tend to be smooth and persistent.

Appendix A4.11 - Testing for Parameter Constancy

In this Appendix we outline a testing procedure outlined by Hansen (1992) who puts forward a test which has larger applications than the traditional Chow breakpoint test since no prior knowledge about the structural break is required. Furthermore, the Hansen test overcomes the drawbacks of both the CUSUM and CUSUM of squares tests. In particular, the former has been criticized for merely detecting instability of the intercept; the

107 After adjusting for a scaling factor, term premia implied by (4.1) are identical to bond risk premia, or excess log returns, as in Cochrane and Piazzesi (2005).
latter, instead, suffers from poor asymptotic power while the Hansen test has locally optimal power. We start by estimating parameters \( \{\hat{\beta}^l, \hat{\sigma}^2\} \) by applying ordinary least squares to equation (4.7):

\[
\begin{align*}
tp_t^{n,m} &= \beta^l x_t + e_t^{n,m} \\
\text{(4.7')} \end{align*}
\]

where \( x_t \) is the usual matrix of explanatory variables, which are assumed to be weakly dependent process (no deterministic or stochastic trend are allowed). Residuals have to be stationary as well. In addition, usual conditions hold: a zero-mean disturbance term \( E(e_t | x_t) = 0 \); a constant second moment \( E(e_t^2) = \sigma^2 \); null covariance between noise and regressors \( E(x_t' e_t) = 0 \). Residuals are computed as follows:

\[
\hat{e}_t^{n,m} = \hat{e}_t = tp_t^{n,m} - x_t' \hat{\beta} \tag{4.21}
\]

Rewriting the first-order conditions in a slightly different way yields:

\[
\sum_{t=1}^{T} x_t \hat{e}_t = 0 \quad \text{and} \quad \sum_{t=1}^{T} (\hat{e}_t^2 - \hat{\sigma}^2) = 0 \tag{4.22}
\]

Defining a new variable \( f_{it} \):

\[
f_{it} = \begin{cases} 
    x_t \hat{e}_t, \\
    \hat{e}_t^2 - \hat{\sigma}^2 
\end{cases} \tag{4.23}
\]

Expressions (4.22) is equivalent to:

\[
\sum_{i=1}^{T} f_{it} = 0 \tag{4.24}
\]

the variables \( f_{it} \) are the first-order conditions, and are akin to the score in the maximum likelihood estimation. The Hansen test statistics are based on the cumulative sums of the \( f_{it} \), namely:

\[
S_{it} = \sum_{t=1}^{T} f_{it} \tag{4.25}
\]

Two versions of the tests are available. To check for individual parameter stability the test is based on the following statistics:

\[
L_i = \frac{1}{TV_i} \sum_{t=1}^{T} S_{it}^2 \tag{4.26}
\]

where \( V_i \) is the cumulative sum of \( f_{it}^2 \). Asymptotic critical values for the individual parameter stability test are given by Hansen (1992). At 5% significance level the critical value is 0.47; the 10% critical value is 0.353. Large values of the test statistics (\( Li \)) implies a violation of the first-order conditions, and thus suggest rejection of the null hypothesis of parameter stability. The \( Li \) test by proposed by Hansen is similar to the \( t \)-test to assess significance of individual parameter of an OLS regression. The test statistics to assess joint parameter stability is:
where \( s_t = (S_{t1}, S_{t2}, ..., S_{tk+1}) \), \( f_t = (f_{t1}, f_{t2}, ..., f_{tk+1}) \), and \( V = \sum_{t=1}^{T} f_t f_t' \). Under the null hypothesis of parameter constancy, the first-order conditions are mean zero, thus the cumulative sum tend to be distributed around zero. Under the alternative hypothesis of parameter instability, the cumulative sum does not have zero mean and the test statistics tends to assume large values. Therefore, the distribution is not standard and is tabulated by Hansen (1992). There are six explanatory variables in model (4.7) including both the constant and the errors variance. At 5% significance level the critical value is 1.68, while the 10% critical value is 1.49. The null hypothesis of joint parameter stability is rejected if the test statistics exceeds the critical values. The Hansen joint test for parameter stability reminds of the \( F \)-test to assess the joint significance of parameters in an ordinary least squares regression. Hansen reveals “if a large number of parameters are estimated,..., the joint significance test is a more reliable guide”. Results are supportive of parameters instability.

<table>
<thead>
<tr>
<th>U.S. - (120, 3)</th>
<th>coef</th>
<th>Li</th>
<th>EMU - (120, 3)</th>
<th>coef</th>
<th>Li</th>
<th>CAN - (120, 3)</th>
<th>coef</th>
<th>Li</th>
<th>U.K. - (120, 3)</th>
<th>coef</th>
<th>Li</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>0.3583</td>
<td>0.233</td>
<td>( \beta_1 )</td>
<td>0.4007</td>
<td>0.278</td>
<td>( \beta_1 )</td>
<td>0.6395</td>
<td>1.950</td>
<td>( \beta_1 )</td>
<td>0.4952</td>
<td>0.228</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.1403</td>
<td>0.178</td>
<td>( \beta_2 )</td>
<td>-0.3239</td>
<td>0.213</td>
<td>( \beta_2 )</td>
<td>0.1152</td>
<td>1.993</td>
<td>( \beta_2 )</td>
<td>-0.6573</td>
<td>0.511</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.0394</td>
<td>0.191</td>
<td>( \beta_3 )</td>
<td>4.0527</td>
<td>0.280</td>
<td>( \beta_3 )</td>
<td>-0.2464</td>
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<td>( \beta_3 )</td>
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<tr>
<td>( \beta_4 )</td>
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<td>( \text{var} )</td>
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<td>( \text{var} )</td>
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<td>0.640</td>
<td>( \text{var} )</td>
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<td>0.2048</td>
<td>0.182</td>
</tr>
<tr>
<td>( Lc )</td>
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<td></td>
<td>( Lc )</td>
<td>2.957</td>
<td></td>
<td>( Lc )</td>
<td>7.379</td>
<td></td>
<td>( Lc )</td>
<td>3.389</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>U.S. - (60, 3)</th>
<th>coef</th>
<th>Li</th>
<th>EMU - (60, 3)</th>
<th>coef</th>
<th>Li</th>
<th>CAN - (60, 3)</th>
<th>coef</th>
<th>Li</th>
<th>U.K. - (60, 3)</th>
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<tbody>
<tr>
<td>( \beta_1 )</td>
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<td>1.849</td>
<td>( \beta_1 )</td>
<td>0.8860</td>
<td>0.530</td>
<td>( \beta_1 )</td>
<td>0.2835</td>
<td>1.944</td>
<td>( \beta_1 )</td>
<td>0.5987</td>
<td>1.074</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.3267</td>
<td>1.862</td>
<td>( \beta_2 )</td>
<td>-0.5189</td>
<td>0.335</td>
<td>( \beta_2 )</td>
<td>-0.0559</td>
<td>1.622</td>
<td>( \beta_2 )</td>
<td>-0.4419</td>
<td>2.207</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.0283</td>
<td>0.731</td>
<td>( \beta_3 )</td>
<td>1.5821</td>
<td>0.469</td>
<td>( \beta_3 )</td>
<td>-0.1290</td>
<td>2.230</td>
<td>( \beta_3 )</td>
<td>0.1975</td>
<td>2.111</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>2.3116</td>
<td>1.955</td>
<td>( \beta_4 )</td>
<td>0.9369</td>
<td>0.681</td>
<td>( \beta_4 )</td>
<td>0.3505</td>
<td>0.330</td>
<td>( \beta_4 )</td>
<td>0.5786</td>
<td>0.403</td>
</tr>
<tr>
<td>( \text{var} )</td>
<td>0.6508</td>
<td>1.230</td>
<td>( \text{var} )</td>
<td>0.0971</td>
<td>2.291</td>
<td>( \text{var} )</td>
<td>0.9492</td>
<td>3.117</td>
<td>( \text{var} )</td>
<td>0.7354</td>
<td>1.529</td>
</tr>
<tr>
<td>( Lc )</td>
<td>10.27</td>
<td></td>
<td>( Lc )</td>
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<td></td>
<td>( Lc )</td>
<td>8.603</td>
<td></td>
<td>( Lc )</td>
<td>11.73</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.14
Chapter 5

An Empirical Investigation of the Lucas Hypothesis:
Non Linearity in the Money-Output Relationship

Abstract

Existing evidence about the effectiveness of money growth to stimulate economic activity has been criticized from different perspectives. In addition, high correlation between money and output is not helpful to detect the direction of causality. From a policy perspective, in fact, positive correlation may arise from two opposite conducts: either the monetary authority sets the supply of money to influence future output fluctuations, or the central bank controls money growth reacting to the past evolution of macroeconomic variables. In this chapter the relationship between money and output is analysed within a non linear framework that ascribes a primary role to expectations. In particular, we find evidence that the Lucas (1973) hypothesis, that there is an inverse correlation between the variance of nominal shocks and the magnitude of output response to nominal shocks, is supported by data evidence when the yield curve is either flat or downward sloping. We also provide evidence suggesting that the Friedman (1977) hypothesis, that the variability of inflation exerts a negative effect on the natural level of output, holds when a positive risk premium is incorporated in an upward sloping term structure of interest rates.
5.1 Introduction

In previous chapters we have always paid attention to the link between macroeconomics and finance, focusing on the informative content carried by the term structure of interest rates about real output. In particular, we have investigated whether the curvature of the yield curve reflects the evolution of economic activity; secondly, we have examined to what extent the yield spread can anticipate future output growth. Ultimately, in Chapter 4 we have also found the conditional volatility of term premia helps to anticipate future output fluctuations.

In this last chapter of the thesis we would like to investigate whether there is a direct relationship between money, or a measure of it, and output. We would like to address the issue whether or not the stock of money itself, without being filtrated by movements of the term structure of interest rates through agents’ expectations regarding the incoming stance of monetary policy, is informative about the business cycle. Actually, in chronological terms, we move back in time by focusing on the starting point of the research in monetary economics (Friedman and Schwartz, 1963).

Investigating the relationship between money and output has always been a major concern of macroeconomists. The classical dichotomy about whether money influences the future level of output or, vice versa, whether output fluctuations influence money supply, is still an unresolved puzzle. Economists affiliated to the monetarist school believe that money growth will be merely reflected in the future price level. However, the monetarist view that money does not affect real output seems to be a weak argument in the short run. On the other hand, Keynesian economists believe that short run policies may well influence the level of economic activity.

In this chapter we analyse the relationship between money and output within a non linear empirical framework that allows for rational expectations. In particular, we investigate whether the Lucas (1973) hypothesis, that there is a negative relationship between the variance of nominal shocks and the magnitude of output response to nominal shocks, is supported by data evidence. The contrarian argument, that the conditional variance of money forecast errors negatively affects the natural level of output, has been proposed by Friedman (1977).

We find evidence that conditioning the examination of the money-output relation to the shape of the yield curve gives the opportunity of reconciling the aforementioned opposite views. Evidence suggests that the variability of inflation, captured by the conditional variance of money forecast errors, exerts a negative influence on output when the yield curve is upward sloping. There is also some evidence that the Friedman hypothesis holds when the linear model is estimated over the entire sample (from 1967 to 2007). However, in the regime characterized by a flat or downward

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108 Estrella and Mishkin (1997), as well as Estrella and Hardouvelis (1991) and Wright (2006), show, in fact, that a negative spread increases the likelihood of a recession in the near future.
sloping yield curve the Lucas hypothesis seems to prevail. We thus provide evidence supporting the Lucas hypothesis, which is usually rejected by data analysis (Kim and Nelson, 1989).

The weaknesses of traditional models (King and Plosser, 1984; Ravn and Sola, 2004) coupled with the difficulty of detecting a unique direction for causality call for an approach that emphasizes the role of expectations. Hence, the contribution of this chapter is also methodological. We propose to examine expectations exploiting a two-level structure. At a micro level, the bottom level, agents’ expectations focus on the central bank operational procedure regarding the supply of money. The micro mechanism of processing available information is based on the Kalman filter, which implies a continuous refinement of expectations on the basis of past prediction errors, i.e. deviations between ex ante expected and ex post observed values of the money stock. Hence, agents form expectations according to a Bayesian iterative sequence that combines the re-elaboration of past prediction errors with the analysis of new flows of information. Moreover, what is peculiar in the micro analysis of expectations is that Kalman filtering allows separating the expected from the unexpected component of money growth. The adoption of a time-varying approach for the monetary policy rule is also consistent with recent evidence. Cogley and Sargent (2006) as well as Boivin (2006) document important time variation in the response of the monetary authority to the state of the economy. Also Sims and Zha (2006) point out that the changing view of the Fed about the economy has been gradual; they argue it could be attributed to the changes of shocks’ variance. Finally, Primiceri (2008) provides evidence that the reaction of monetary policy to the changes in both inflation and unemployment has become more and more aggressive in the last decades. Last, but certainly not least, we focus on a policy rule expressed in terms of money supply since our sample is characterized by periods of inflation instability; Bernanke and Mishkin (1993) argue, in fact, that central banks tend to adopt targets in terms of money growth when the inflation rate threatens to be out of control.

At a macro level, the top level, expectations focus on the future economic outlook, as reflected by the dynamics of the term structure of interest rates. The macro perspective captures the sentiment regarding the future evolution of key macroeconomic variables as well as institutional or socio-political factors, or, finally, technological changes. There is large evidence that the slope of the term structure could be used to make inference about the future state of the economy (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1997; Ang, Piazzesi and Wei, 2006).

Our methodology also partially accounts for the criticism moved by Amato and Swanson (2000). Despite some evidence suggests that monetary aggregates help to predict future output (Stock and Watson, 1989; Beckett and Morris, 1992; Feldstein and Stock, 1994), Amato and Swanson (2000) point out that such evidence might be contingent upon the nature of the dataset. Using real time,
rather than revised, data they document a substantial reduction of the marginal predictive power of money. The threshold approach adopted in this chapter implies non linearity in the dataset thus breaking time continuity; for this reason, it allows reducing the impact of the historical track.

The rest of the chapter is organized as follows. The next Section contains a brief survey of the literature and discusses motivations. In Section 5.3 we present some evidence about causality. In Section 5.4 we outline the structure of expectations at the micro level. Empirical evidence is discussed in Section 5.5. Finally, Section 5.6 concludes. Data are presented in Appendix A5.I. In Appendix A5.II we report auxiliary estimations.

5.2 Motivations and Literature Review

Whether and to what extent money growth is capable of contributing to the determination of real output is still an unresolved puzzle of macroeconomics. The monetarist view that money growth induces a proportional change in the price level leaving real output unaffected is acknowledged to work in the medium-long run. However, as Keynesian theory suggests, monetary disturbances are believed to have some real effects in the short run. Although the contribution of monetary shocks is limited, or absent, on permanent income growth, managing money supply is a useful instrument under the control of the monetary authority for stabilizing or stimulating the economy across the business cycle.

In this Section we investigate the relationship between money and output for the U.S. economy. The classical dichotomy of money neutrality, i.e. nominal variables are unable to affect real variables, has been initially investigated by means of the following equation, which is known as the Saint Louis equation since it has been introduced by economists working in that Federal Reserve District:

\[ \Delta IP_{t,3} = \alpha + \beta_0 \Delta M_{t,3} + \beta_1 \Delta M_{t-3,6} + \beta_2 \Delta M_{t-6,9} + \beta_3 \Delta M_{t-9,12} + \beta_4 trend + \epsilon_i \]

The LHS variable is the quarterly change in the industrial production index; while, the quarterly changes of the money stock over the last year are explanatory variables. The above regression also includes a constant and a time trend (to account for eventual trend in money and output growth). Different monetary aggregates have been considered: M1, M2, and the U.S. Fed Board of Governors monetary base (MB). The analysis is performed on U.S. monthly data from 1967 to 2007. Empirical results are reported in Table 5.1. The contemporaneous effect on output exerted by M2 turns out to be negative (\( \beta_6 < 0 \)); however, more generally, results suggest that the rate of growth of M2 over the last three quarters have a positive influence on IP growth (\( \beta_1, \beta_2 > 0 \)). The rate of growth of M1 has a marginal, though significant, effect on the current growth of the IP index.
In order to see whether there is a significant influence of money on output we have looked at the jointly significance of estimated coefficients in each equation. In all the equations, coefficients turn out to be jointly significant supporting the influence of money on output. In addition, we have also run a Wald test to check the following restrictions: \( \sum_{i=0}^{3} \beta_i = 0 \), i.e. to check whether money growth does explain output. The null hypothesis is rejected in two cases since the probability value associated to the test is zero for both M2 and MB equations, thus suggesting the influence of these monetary aggregates on real activity. M1 growth, instead, does not seem informative about output.

As a forecasting exercise we have estimated the above regression using the future determination of output in the LHS:

\[
\Delta IP_{t+3,t} = \alpha + \beta_0 \Delta M_{t-3,t-3} + \beta_1 \Delta M_{t-3,t-6} + \beta_2 \Delta M_{t-6,t-9} + \beta_3 \Delta M_{t-9,t-12} + \beta_4 \text{trend} + \epsilon_i \tag{5.2}
\]

Results reported in Table 5.2 suggest that M2, rather than M1 and the monetary base, is effective in influencing both the current and the future level of industrial production. In the equation for M2 both coefficients \( \beta_0 \) and \( \beta_1 \) are statistically significant indicating that future output is influenced by money growth up to six months before. The goodness of fit is definitely poor; however, the M2 equation returns a much better fit than the other equations. The dynamics of M1 over the most recent quarter has a marginal impact on the IP growth. Coefficients are jointly significant (\( F \)-test) in equations M1 and M2. We repeat the Wald test to check for the following restriction: \( \sum_{i=0}^{3} \beta_i = 0 \).

The null hypothesis is definitely rejected in all the three cases supporting the influence of money (M1, M2, and MB) on future economic activity.
The macroeconomic debate has further focused on the asymmetric effect of monetary policy on output. Models with sticky prices or financial constraints suggest that interest rate changes generate greater effect on real activity during recessions. Similarly to Romer and Romer (1994), Garcia and Schaller (1999) find evidence in line with this conjecture arguing that monetary policy is more effective during recessions. Ravn and Sola (2004) find evidence corroborating the hybrid traditional Keynesian asymmetry, that is only small negative monetary policy shocks tend to influence real output. In order to account for this effect we estimate the above equations including dummy variables to distinguish the effect of positive rather than negative money growth rates.

\[
\Delta IP_{t, -3} = \alpha + \beta_0 D^{(+)} \Delta M_{t, -3} + \beta_1 D^{(+)} \Delta M_{t-3,t-6} + \beta_2 D^{(+)} \Delta M_{t-6,t-9} + \beta_3 D^{(+)} \Delta M_{t-9,t-12} + \epsilon, 
\]

(5.3)

\[
\Delta IP_{t, -3} = \alpha + \beta_0 D^{(-)} \Delta M_{t, -3} + \beta_1 D^{(-)} \Delta M_{t-3,t-6} + \beta_2 D^{(-)} \Delta M_{t-6,t-9} + \beta_3 D^{(-)} \Delta M_{t-9,t-12} + \epsilon, 
\]

(5.4)

Where \(D^{(+)}\) indicates a positive quarterly growth rate of the monetary aggregate, while \(D^{(-)}\) indicates a negative growth rate. Results strongly support the hypothesis advanced by Romer and Romer (1994). Coefficients are statistically significant only in equation (5.3) (top panel of Table 5.3) suggesting that only stimulus to economic activity seem to be effective. Moreover, the goodness of fit (0.033) of equation (5.3) is much larger than that (0.006) of equation (5.4) (bottom panel of Table 5.3). The \(F\)-test suggests coefficients are jointly significance only in equation (5.3). The Wald test confirms that the null hypothesis \(\sum_{i=0}^{3} \beta_i = 0\) cannot be rejected for equation (5.3) solely.

<table>
<thead>
<tr>
<th>St. Louis Equation - IP grw (+3)</th>
<th>(\alpha)</th>
<th>(\beta_0)</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1</strong></td>
<td>0.0228</td>
<td>0.0304</td>
<td>0.0190</td>
<td>0.0215</td>
<td>-0.0181</td>
<td>0.0005</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.2877)</td>
<td>(0.0796)</td>
<td>(0.2883)</td>
<td>(0.2321)</td>
<td>(0.2941)</td>
<td>(0.3033)</td>
</tr>
<tr>
<td>NW</td>
<td>(0.6378)</td>
<td>(0.2752)</td>
<td>(0.4457)</td>
<td>(0.2973)</td>
<td>(0.4842)</td>
<td>(0.6076)</td>
</tr>
<tr>
<td>White</td>
<td>(0.3499)</td>
<td>(0.0812)</td>
<td>(0.2828)</td>
<td>(0.1945)</td>
<td>(0.2622)</td>
<td>(0.3162)</td>
</tr>
<tr>
<td><strong>M2</strong></td>
<td>-0.0074</td>
<td>0.2269</td>
<td>0.2916</td>
<td>0.0989</td>
<td>-0.0191</td>
<td>0.0001</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.0101)</td>
<td>(0.0223)</td>
<td>(0.0068)</td>
<td>(0.3563)</td>
<td>(0.8461)</td>
<td>(0.0081)</td>
</tr>
<tr>
<td>NW</td>
<td>(0.2359)</td>
<td>(0.1031)</td>
<td>(0.0492)</td>
<td>(0.4050)</td>
<td>(0.8891)</td>
<td>(0.1906)</td>
</tr>
<tr>
<td>White</td>
<td>(0.0197)</td>
<td>(0.0126)</td>
<td>(0.0101)</td>
<td>(0.3488)</td>
<td>(0.8458)</td>
<td>(0.0112)</td>
</tr>
<tr>
<td><strong>MB</strong></td>
<td>-0.0011</td>
<td>0.1584</td>
<td>0.1056</td>
<td>0.1041</td>
<td>0.0086</td>
<td>0.0004</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.7192)</td>
<td>(0.0771)</td>
<td>(0.2396)</td>
<td>(0.2474)</td>
<td>(0.9238)</td>
<td>(0.4298)</td>
</tr>
<tr>
<td>NW</td>
<td>(0.8391)</td>
<td>(0.1954)</td>
<td>(0.2347)</td>
<td>(0.3739)</td>
<td>(0.9369)</td>
<td>(0.6890)</td>
</tr>
<tr>
<td>White</td>
<td>(0.6976)</td>
<td>(0.0422)</td>
<td>(0.0674)</td>
<td>(0.1437)</td>
<td>(0.9073)</td>
<td>(0.4349)</td>
</tr>
</tbody>
</table>

p-values in parenthesis
\begin{table}
\centering
\begin{tabular}{|c|cccc|c|}
\hline
Dummies D(+) - IP grw (3) & & & & \\
\hline
\(\alpha\) & \(\beta_0\) & \(\beta_1\) & \(\beta_2\) & \(\beta_3\) & \(R^2\) \\
\hline
M1 & 1.5221 & -0.0924 & 0.1143 & 0.1369 & 0.1591 & 0.033 \\
t-stat & (0.0009) & (0.1307) & (0.0538) & (0.0212) & (0.0094) & \\
p-values in parenthesis \\
\hline
Dummies D(-) - IP grw (3) & & & & \\
\hline
\(\alpha\) & \(\beta_0\) & \(\beta_1\) & \(\beta_2\) & \(\beta_3\) & \(R^2\) \\
\hline
M1 & 2.2632 & 0.1056 & 0.0615 & 0.0301 & -0.0415 & 0.006 \\
t-stat & (0.0000) & (0.1877) & (0.4278) & (0.6977) & (0.6020) & \\
p-values in parenthesis \\
\hline
\end{tabular}
\caption{Table 5.3}
\end{table}

The \(F\)-test reveals that coefficients are jointly not significant in equation (5.4); moreover, the Wald test to check for \(\sum_{i=0}^{3} \beta_i = 0\) does not allow to reject the null hypothesis.

Results are similar when replacing the dependent variable \(\Delta IP_{t-3}\) with its future realization \(\Delta IP_{t+3}:\)

\begin{table}
\centering
\begin{tabular}{|c|cccc|c|}
\hline
Dummies D(+) - IP grw (+3) & & & & \\
\hline
\(\alpha\) & \(\beta_0\) & \(\beta_1\) & \(\beta_2\) & \(\beta_3\) & \(R^2\) \\
\hline
M1 & 0.3606 & 0.0352 & 0.0335 & 0.0323 & -0.0157 & 0.032 \\
t-stat & (0.0019) & (0.0220) & (0.0247) & (0.0301) & (0.3046) & \\
p-values in parenthesis \\
\hline
Dummies D(-) - IP grw (+3) & & & & \\
\hline
\(\alpha\) & \(\beta_0\) & \(\beta_1\) & \(\beta_2\) & \(\beta_3\) & \(R^2\) \\
\hline
M1 & 0.5663 & 0.0910 & 0.0747 & -0.0459 & 0.0242 & 0.005 \\
t-stat & (0.0000) & (0.6507) & (0.7008) & (0.8131) & (0.2246) & \\
p-values in parenthesis \\
\hline
\end{tabular}
\caption{Table 5.4}
\end{table}

Although not reported, and coherently with the estimations above, the joint estimation of an equation including all dummy variables, both \(D^{(+)}\) and \(D^{(-)}\), returns significant coefficients only for dummies \(D^{(+)}\) denoting an increase of the monetary aggregate.

The main drawback of the above equations is that they are not sufficient to establish any causality relation running from money to output. King and Plosser (1984) observe that monetary aggregates such as M1 and M2 are determined by the interaction between the high-powered money, a liability of the central bank, the behaviour of both firms and households, and the efficiency of the financial system through the strategies of the banking sector. Therefore it is possible to observe changes in the money stock that anticipate output movements without causing them. Endogeneity is the second problem associated with both equations (5.1) and (5.2) and equations (5.3) and (5.4). The high
correlation eventually captured by the coefficients of the equations may well derive from the conduct of the monetary authority that sets the future supply of money in response to past output fluctuations. The chronological sequence of a tight monetary policy which follows growing GDP, like a reduction of the rate of money growth whose final goal is to curb economic activity, and of an accommodative policy to tackle falling GDP preserves high correlation between money and output but with important implications for reverse causation. In addition, from a policy point of view, it is impossible to ascribe to monetary policy the effect of money on output without simultaneously considering the effect on GDP exerted by fiscal policies. The poor good of fit obtained for the above regressions is, in fact, a sign of misspecification; in particular, some relevant variables may be omitted. Finally, the time series analysis performed by estimating the above equations might be affected by shifts in money demand since financial innovations contributes to changing agents’ preferences. In particular, as Ravn and Sola (2004) argue, the instability of M1 demand may underlie the poor fit of the M1 equation; furthermore, and specifically in this analysis, the monthly frequency of data may, in principle, accentuate the effect of M1 volatility.

The aforementioned intrinsic difficulties of detecting the effect of money on output coupled with the devastating effect of the Lucas critique call for an empirical method based on dynamic expectations as that implied by Kalman filtering. Expectations are subject to continuous refinement as long as new information becomes available; in addition, agents revisit their expectations on the base of past prediction errors. So far, in fact, we have not discriminated between anticipated and unanticipated money growth which is a core distinction in economics. In this vein, prediction errors work like a proxy for unanticipated money supply. Our approach will be deeply motivated later.

Before presenting in details the methodology adopted in this chapter, next Section provides some more evidence regarding the money-output relation.

5.3 Preliminary Evidence on Causality

In this Section we focus on the causality issue characterizing the empirical relationship between real variables and monetary aggregates. Using U.S. monthly data from January 1967 to December 2007, we start by looking at dynamic short-run correlations. Each panel of Figure 5.1 shows correlations between a measure of real activity and different monetary aggregates (M1, M2, and MB, the monetary base). The top-left diagram indicates that all monetary aggregates are positively correlated with the Hodrick-Prescott detrended series of industrial production (IP) at lags, but negatively correlated at leads. Hence, booms (high IP relative to trend) tend to be preceded by high values of money growth; while positive values of IP relative to its HP trend tend to be followed by low values of money growth. Positive values of the HP filtered IP give signals of a thriving
economy; a positive IP gap means that resources are employed above the natural long-run equilibrium level. Hence, if important rates of money growth tend to anticipate positive HP gap there is some evidence that the causality relationship runs from money to output. This evidence is in line with the idea that money supply acts as a stimulus to real economic activity; while in response to fast-growing economy, and to the associated threat of mounting inflation, the monetary authority inverts the sign of the monetary policy conduct. The bottom-left diagram shows the correlations between monetary aggregates and the annual change in the unemployment rate. Consistently with the above story, all monetary aggregates are negatively correlated at lags with the growth in unemployment, i.e. a reduction in unemployment tends to be preceded by high values of money supply; on the other hand, monetary aggregates are positively correlated with the increase of unemployment at leads.

The top-right diagram of Figure 5.1 shows the pattern of short run correlations between the change in real consumption expenditures and monetary aggregates. Private consumption seems to be positively related to money growth in the short run at both leads and lags. The smoother pattern of the monetary base correlations is consistent with the theory put forward by King and Plosser (1984); they find that inside money, i.e. the internal monetary measures which represent the liabilities of the banking sector as a component of monetary aggregates, rather than outside money, i.e. the external monetary measures which represent the liabilities of the central bank, are positively correlated with real activity.

Figure 5.1
Finally, although lower with respect to other real indicators, the bottom-right panel shows that the correlation between monetary aggregates and the \((\log)\) total capacity utilization is positive at both lags and leads; thus both past money supply and expectations of future important money supply tend to positively affect the employment of the factors of production.

Previous evidence is provided by Friedman and Schwartz (1963) in a classical contribution about the monetary history of the United States; they find that money growth rate changes lead changes in real GDP. The left diagram of Figure 5.2 shows that the rate of growth of M1 systematically anticipates business cycle movements between 1967 and the mid 1980s. Falling money growth precedes slowdowns in economic activity; while increasing money stocks anticipate both recoveries and booms. However, more recent evidence presented in the right diagram is more controversial: starting from 1985, in fact, the relationship between money and output is not as close as before, both the length and the magnitude of cyclical fluctuations do not reproduce the preceding dynamics of the monetary aggregate. The different pattern of the relationship between M1 and the IP gap might be due to financial innovations which affected the demand for money. The greater variability of the rate of growth of M1 might also reflect greater difficulty of the money stock to influence output from 1986 and 1997. In addition, Choudhry (2002) finds that the stated monetary act of 1980 considerably affected the income and interest rate demand elasticities of both M1, M2 and their components in U.S. Moreover, he argues that the fall in the M1 interest rate elasticity may well indicate M1 as possibly a more effective monetary policy tool after 1980.

![Figure 5.2](image)

In line with the above evidence, the following scatter diagrams suggest a stronger effect displayed on both current and future output (detrended IP) by M2 rather than by M1 or MB\(^{109}\). The top diagrams show the scatter between the IP gap and the contemporaneous growth rate of the monetary aggregates. In the bottom panels we report the scatter plots between the actual HP-detrended IP and the rate of money growth 12 months before.

\(^{109}\) The regressing line associated to M2 is, in fact, always steeper. The only exception occurs in the mid-bottom panel, the regressing line associated to MB turns out to be marginally steeper than the one associated to M2. However, in the former case (MB) there is a greater vertical dispersion of observations around the regressing line; in the latter case (M2), instead, observations more closely concentrated around the regressing line along its entire length.
Scatter plot diagrams. **IP gap (HP)**: industrial production gap obtained with the Hodrick-Prescot filter. **M1 grw**: annual growth of monetary aggregate M1. **M2 grw**: annual growth of monetary aggregate M2. **MB grw**: annual growth of the monetary base. **(-12)** means the twelfth lag (monthly frequency of data).

**Figure 5.3**

As Walsh (2003) points out “while suggestive, evidence based on time patterns and simple correlations may not indicate the true casual role of money. Since the Federal Reserve and the banking sector respond to economic developments, movements in the monetary aggregates are not exogenous, and the correlation patterns need not reflect any casual effect of monetary policy on economic activity”. Notwithstanding the above mentioned stronger influence on real variables exerted by M2 than other aggregates, the Granger causality tests suggest M1 being the only source capable of affecting the future level of both industrial production and total capacity utilization. The null hypothesis that the rate of growth of M1 does not cause the IP gap cannot be rejected, as well as the null that M1 does not cause (**log**) TCU. The real personal consumption expenditure seems to be caused in the Granger sense by all monetary aggregates. Results are reported in Table 5.5.

The Granger tests are also employed to investigate whether lagged levels of the real variables help to predict the future path of monetary aggregates. Results are significantly supportive in this respect, as shown in Table 5.6. Such statistical evidence about causality is compatible with a Taylor-type monetary policy reaction function implying the monetary authority to raise the policy rate when the pace of economic growth is as fast as to create undesired inflationary pressures.
We sum up the preliminary evidence discussed in this Section by saying that results regarding the effect of money on output turn out to be somehow ambiguous. Short run correlations tend to suggest a positive influence of lagged money on actual output; however, the Granger tests partially contradict this evidence by suggesting a causality relationship working in the opposite direction. In addition, despite the existing evidence suggesting that money helps to predict future output (Stock and Watson, 1989; Becketti and Morris, 1992; Feldstein and Stock, 1994), Amato and Swanson (2000) argue that results are someway misleading because they crucially depend on revised, rather than real time, monetary aggregates data.

The aforementioned ambiguity can be dealt with by introducing a new element in the analysis; we thus attribute a role of primary importance to agents’ expectations and, in particular, to the...
associated expectations errors. To conclude, we recall that the choice of M1 as the benchmark reference aggregate for the monetary policy rule in the following analysis hinges on the results of the Granger tests reported in Table 5.5.

5.4 Empirical Methods for Expectations

In this Section we summarize the approach employed to derive agents’ expectations about the future stance of monetary policy as captured by the rate of growth of M1. King and Plosser (1984), in fact, suggest inside money, a component of M1 representing the liabilities of the banking sector, being highly correlated with business cycle movements. In addition, Bernanke and Mishkin (1993) argue that the monetary authority tends to define targets in terms of money growth if there is a concrete likelihood that inflation gets out of control. Our sample is characterized by periods of high and volatile inflation. Finally, we justify the time-varying approach by observing that there is substantial evidence highlighting that both the monetary policy conduct and the variance of nominal shocks have changed over time (Cogley and Sargent, 2006; Boivin, 2006; Sims and Zha, 2006; Primiceri 2008).

We thus compute expectations by applying the Kalman filter, since it provides with an effective formalisation of the mechanism through which agents form expectations rationally. Moreover, the Kalman approach gives the opportunity of deriving a measure of innovations which overcomes the criticism traditionally moved to the VAR approach. In what follows we briefly outline the main features of Kalman filtering.

The observation equation, or measurement equation, of the state-space system is:

$$\Delta M_t = a + x_{t-1} \beta_t + u_t$$  \hspace{1cm} (5.5)

Actual quarterly money growth is a function of the changes of the T-bill rate, of the price level, and of the money stock over the previous quarter; $u_t$ is a stochastic $i.i.d. \left(0, \sigma_u \right)$ noise. The specification of the money equation come from Mishkin (1982) and Weintraub (1980); it has been successively considered by Kim and Nelson (1989). The only difference is that we rule out the fiscal variable, because of the superior independence achieved by the monetary authority in recent times. The state, or transition, equation captures the evolution of coefficients over time:

$$\beta_t = \mu + F \beta_{t-1} + v_t$$  \hspace{1cm} (5.6)

Where $v_t$ is an idiosyncratic disturbance $i.i.d. \left(0, \sigma_v \right)$. Following standard practice, we impose matrix $F$ to be the identity matrix since we assume that the regressing coefficients follow random
walk processes (Kim and Nelson, 1998; Kim and Nelson, 2006; Boivin, 2006). The Kalman filter is an iterative algorithm based on updating the informative set with most recent available information and predicting future movements of the variable under examination. The coefficients covariance matrix conditional on information available up to time $t-1$ is:

$$ P_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'] $$  \hspace{1cm} (5.7)

Equation (5.8) provides the prediction of money growth based on information available up to time $t$ given that economists know the econometric relationship linking the core variable to the explanatory variables till time $t-1$.

$$ \Delta M_{t|t-1} = x_t \beta_{t|t-1} $$  \hspace{1cm} (5.8)

Once the actual contemporaneous value of the core variable is observed, agents can compute the prediction error according to the following

$$ \eta_{t|t-1} = \Delta M_t - x_t \beta_{t|t-1} = \Delta M_t - \Delta M_{t|t-1} $$  \hspace{1cm} (5.9)

Finally, equation (5.10) represents the conditional variance of money growth prediction errors:

$$ h_{t|t-1} = E[\eta_{t|t-1}^2] = x_t P_{t|t-1} x_t' + \sigma^2 $$  \hspace{1cm} (5.10)

According to (5.10) Kalman filtering allows two sources of uncertainty generating the conditional variance of the forecast error ($h_{t|t-1}$). One source depends on the evolutionary behaviour of estimated coefficients through the coefficients covariance matrix, thus capturing the gradual change of the policy regime over time; the other source is a random noise related to future disturbances, like unpredictable institutional or technological shocks. The assumption of a constant variance of nominal shocks to money growth seems too severe since aggregate M1 is regarded to respond on a great variety of shocks. First, M1 trivially depends upon the monetary policy conduct through the money supply (high-powered money). Second, M1 is affected by the interaction between money supply and money demand, so that a demand shock, rather than a supply shock, may influence aggregate M1. For instance financial innovations as well as deregulation may affect M1 in the medium-short run. Finally, M1 depends also on the strategic decisions of the banking system and on the credit market conditions. Therefore, a measure of variance which is conditional to the state of the economy provides with a more realistic picture of aggregate risk.
An alternative method to compute the time-varying conditional variance is to estimate an autoregressive model for money growth (either AR or VAR, depending on the nature of the analysis), and then to compute the squared of fitted residuals (Piazzesi, 2003).

5.5 Empirical Results

In this Section we provide evidence that the Lucas hypothesis, i.e. that exists a negative relationship between the variance of nominal shocks and the magnitude of output response to nominal shocks, usually rejected in linear model, holds when the likelihood of a slowdown in economic activity is warned by financial indicators. Evidence also suggests that the alternative Friedman hypothesis, that the augmenting variability of inflation exerts a negative effect on output, tends to hold when financial indicators anticipate a thriving pace of economic growth.

A crucial issue involved in testing the Lucas hypothesis is the examination of the conditional variance of nominal shocks over time. Hence, the analysis starts with the estimation of time-varying monetary policy function expressed in terms of money rather than in terms of the rate of interest. The inverse relationship tying money supply and interest rates is trivially respected in any modern economy; moreover, the extensive sample period covered in this analysis calls for a generic version of the monetary policy rule.

After estimating a time-varying specification of equations (5.5) and (5.6), we obtain a series of prediction errors \( \eta_{t-1} \) and a measure of forecast errors’ conditional volatility \( h_{t-1} \). The basic equation to test the Lucas versus the Friedman hypothesis is the following:

\[
gap_t = \alpha_0 + \alpha_1 \eta_{t-1} + \alpha_2 h_{t-1} + \alpha_3 \text{gap}_{t-1} + \nu_t \tag{5.11}
\]

Where the output gap is the HP de-trended series of \( \log \) IP. In addition, coefficient \( \alpha_1 \) is set equal to \( \gamma_0 + \gamma_1 h_{t-1} \). The functional form of \( \alpha_1 \) is motivated with the aim of reducing the effect of multi-collinearity in the OLS regression. Equation (5.11) thus becomes

\[
gap_t = \alpha_0 + \gamma_0 \eta_{t-1} + \gamma_1 (\eta_{t-1} \cdot h_{t-1}) + \alpha_2 h_{t-1} + \alpha_3 \text{gap}_{t-1} + \nu_t \tag{5.11’}
\]

The inclusion of the first lag of the dependent variable, which is highly correlated with its actual level, certifies the robustness of other coefficients estimates. In addition, different computational methods for the variance-covariance matrix have been employed in order to obtain consistent

\[\text{Appendix A5.II}\] we provide with a comparison between the conditional variances of money growth prediction errors obtained both by Kalman filtering and by autoregressive modeling.
estimates (White, 1980; Hansen and Hodrick, 1980; Newey and West, 1987; and, finally, the simplified Hansen and Hodrick).

The theory advanced by Lucas is satisfied when both $\gamma_0 > 0$ and $\gamma_1 < 0$; while, testing the Friedman hypothesis is equivalent to detecting whether coefficient $\alpha_2$ is negative ($\alpha_2 < 0$)$^{111}$. The assumption here is that the conditional variance of money forecast errors acts as a proxy for the variability of inflation. The original idea put forward by Friedman (1977), in fact, is that the variability of inflation, rather than that of money, reduces the natural level of output since it disturbs the allocation efficiency of the price system.

The linear model (5.11’), estimated over the entire sample 1967-2007, does not reveal any particular information about the way nominal shocks affect business cycle fluctuations. Table 5.7 shows estimation results for different measures of the business conditions. The dependent variable in the top panel is the Hodrick-Prescott measure of the IP gap; (from the top to the bottom) in the second panel the dependent variable is the log total capacity utilization; in the third panel the dependent variable is the rate of unemployment; finally, in the bottom panel, the dependent variable is the rate of change of unemployment$^{112}$.

There is weak evidence supporting the Friedman hypothesis. Although coefficient $\alpha_2$ is inversely related to the dynamics of real variables, it is either marginally or not significant with the only exception holding for unemployment. The Lucas hypothesis is definitely rejected. Coefficient $\gamma_0$ is not statistically significant. In two cases coefficient $\gamma_1$ turns out to be significantly positive thus contradicting the Lucas hypothesis$^{113}$.

The linear estimation of equation (5.11’) is not entirely reliable though, since the pattern of residuals series is affected by heteroscedasticity in all cases. In addition, a strong ARCH effect is found after performing the Engle (1982) test. The recursive residuals and the CUSUM square of residuals reveal the instability of coefficient estimates as reported in Figure 5.4$^{114}$. Finally, also the Hansen tests (1992, 2000) highlight the presence of non-linearity.

$^{111}$ Trivially it holds the opposite sign of coefficients when the dependent variable is either the unemployment or its rate of change.

$^{112}$ Coefficient $\alpha_3$ in equation (5.11’) multiplies the first lag of the respective dependent variable.

$^{113}$ In Appendix A5.II we report the estimation of equation (5.11’) using an alternative measure of the conditional variance of money growth forecast errors leading to similar results. In particular, after estimating an unrestricted VAR (9) model of money growth, inflation, and the change in the 3-month T-bill rate, we obtain the conditional variance of money forecast errors as the squares of residuals from the money growth equation.

$^{114}$ Figure 5.4 reports tests when the dependent variable is the HP filtered IP series; tests for other equations with different real variable offer very similar results.
Therefore, we consider a non linear version of model (5.11’) allowing for two different regimes
determined by slope of the term structure, i.e. the difference between the 10-year and the 3-month
yields. The slope of the yield curve is believed to reflect agents’ expectations about the future stance of monetary policy and, thus, it is thought to anticipate business cycle movements (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1997). Conditioning the test of the Lucas hypothesis to the slope of the term structure means to relating agent’s expectations to a leading economic indicator (Stock and Watson, 1989). In particular, the yield curve represents a link between monetary policy, the financial sector and the real economy. We recall that the peculiar aspect of this methodology is to consider expectations on a double level. At the first level, the micro level, expectations are modelled through Kalman filtering the money supply function in order to isolate prediction errors (Section 5.4). At the second level, the macro level, expectations have a forward-looking nature in that they are intended to capture the future evolution of the economy as reflected by the dynamics of the term structure of interest rates. Macro expectations interpret a perspective sentiment present throughout the economy, a broad view regarding the economic conditions, including political as well as institutional, or technological, factors.

A crucial issues to be pointed out is that the micro level analysis of expectations is performed on a monetary policy function expressed in terms of money supply. While, the macro level expectations are inferred by the evolution of the yield curve, whose dynamics depends not only on the determination of the policy rate, or a measure closely related to it as it may be the effective federal funds rate, but also on the abovementioned factors. The choice of the monetary policy function in terms of the money stock is thus intended to separate two different levels of expectations’ analysis. We aim at distinguishing the expectations regarding the operational procedure of the monetary authority in setting the money supply from the overall movements displayed by the yield curve.

Although there exists an unquestionable inverse relationship linking money supply and short rates, the evolution of the yield curve, as well as the determination of expectations at a macro level, depend on a greater variety of factors. So that we believe our approach is immune from the criticism that the micro and macro structures for expectations share a common root.

The threshold methodology implies that the same equation is estimated in two different regimes depending on the values assumed by a predetermined variable (τ), i.e. the yield spread which is a measure of the slope of the term structure.

\[
\begin{align*}
gap_\tau &= \alpha_0 + \gamma_0 \eta_{t-1} + \gamma_1 (\eta_{t-1} \cdot h_{t-1}) + \alpha_2 h_{t-1} + \alpha_3 \gap_{t-1} + \nu_t & \text{if } \tau \leq \hat{\tau} \\
gap_\tau &= \alpha_0 + \gamma_0 \eta_{t-1} + \gamma_1 (\eta_{t-1} \cdot h_{t-1}) + \alpha_2 h_{t-1} + \alpha_3 \gap_{t-1} + \nu_t & \text{if } \tau > \hat{\tau}
\end{align*}
\]

(5.12)

115 Similar results are obtained if the threshold variable is the spread between the 5-year and the 3-month yields.

116 If we had chosen to apply the Kalman filter to a monetary policy rule (Section 5.4) expressed in terms of the policy rate the criticism might have been appropriate.
Regime 1 is determined by values of the yield spread below the estimated threshold ($\hat{\tau}$); hence, the first regime is characterized by a flat or downward sloping yield curve. The conventional view tends to associate such a regime to an imminent slowdown in economic activity. On the other hand, regime 2 is defined on high values of the spread (positive slope of the yield curve) reflecting an accommodative stance of monetary policy. Estrella and Hardouvelis (1991), as well as Estrella and Mishkin (1997), present important evidence that the yield spread is related to the future evolution of real activity. In particular, a downward sloping or a flat yield curve augments the odds of a recession in the near future. Along the same line Wright (2006) finds a link between the shape of the yield curve and the probability of future economic slowdowns.

Estimation results for regime 1 (below the estimated threshold) are shown in Table 5.8.

<table>
<thead>
<tr>
<th>REGIME 1 - IP gap (HP)</th>
<th>$\alpha_0$</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>0.4327</td>
<td><strong>0.0291</strong></td>
<td><strong>-0.0118</strong></td>
<td><strong>0.0080</strong></td>
<td>0.9039</td>
<td>0.860</td>
</tr>
<tr>
<td>t-stat</td>
<td>[1.603]</td>
<td>[1.524]</td>
<td>[-1.809]</td>
<td>[2.246]</td>
<td>[19.60]</td>
<td></td>
</tr>
<tr>
<td>white</td>
<td>[2.083]</td>
<td>[2.245]</td>
<td>[-1.893]</td>
<td>[2.358]</td>
<td>[21.60]</td>
<td>obs 74</td>
</tr>
<tr>
<td>HH (12)</td>
<td>[2.857]</td>
<td>[2.644]</td>
<td>[-1.838]</td>
<td>[5.067]</td>
<td>[19.49]</td>
<td></td>
</tr>
<tr>
<td>NW (12)</td>
<td>[2.222]</td>
<td>[2.004]</td>
<td>[-1.693]</td>
<td>[2.533]</td>
<td>[18.95]</td>
<td></td>
</tr>
<tr>
<td>sHH</td>
<td>[0.912]</td>
<td>[1.975]</td>
<td>[-1.557]</td>
<td>[1.630]</td>
<td>[12.74]</td>
<td></td>
</tr>
<tr>
<td>REGIME 1 – TCU</td>
<td>$\alpha_0$</td>
<td>$\gamma_0$</td>
<td>$\gamma_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>OLS</td>
<td>-0.6857</td>
<td><strong>0.0242</strong></td>
<td><strong>-0.0119</strong></td>
<td>0.0017</td>
<td>0.9844</td>
<td>0.968</td>
</tr>
<tr>
<td>t-stat</td>
<td>[-1.584]</td>
<td>[2.651]</td>
<td>[-2.632]</td>
<td>[0.971]</td>
<td>[100]</td>
<td></td>
</tr>
<tr>
<td>white</td>
<td>[-1.531]</td>
<td>[2.373]</td>
<td>[-2.188]</td>
<td>[1.017]</td>
<td>[97.12]</td>
<td>obs 355</td>
</tr>
<tr>
<td>HH (12)</td>
<td>[-1.474]</td>
<td>[2.134]</td>
<td>[-1.993]</td>
<td>[1.448]</td>
<td>[93.61]</td>
<td></td>
</tr>
<tr>
<td>NW (12)</td>
<td>[-1.463]</td>
<td>[2.256]</td>
<td>[-2.080]</td>
<td>[1.194]</td>
<td>[92.99]</td>
<td></td>
</tr>
<tr>
<td>sHH</td>
<td>[-0.809]</td>
<td>[2.928]</td>
<td>[-2.967]</td>
<td>[0.777]</td>
<td>[51.40]</td>
<td></td>
</tr>
<tr>
<td>REGIME 1 – Unemployment</td>
<td>$\alpha_0$</td>
<td>$\gamma_0$</td>
<td>$\gamma_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>OLS</td>
<td>0.2949</td>
<td><strong>-0.0472</strong></td>
<td><strong>0.0232</strong></td>
<td>-0.0063</td>
<td>0.9967</td>
<td>0.985</td>
</tr>
<tr>
<td>t-stat</td>
<td>[0.071]</td>
<td>[-2.188]</td>
<td>[2.177]</td>
<td>[-1.547]</td>
<td>[146]</td>
<td></td>
</tr>
<tr>
<td>white</td>
<td>[0.073]</td>
<td>[-1.914]</td>
<td>[1.790]</td>
<td>[-1.611]</td>
<td>[145]</td>
<td>obs 362</td>
</tr>
<tr>
<td>HH (12)</td>
<td>[0.081]</td>
<td>[-1.761]</td>
<td>[1.708]</td>
<td>[-1.842]</td>
<td>[129]</td>
<td></td>
</tr>
<tr>
<td>NW (12)</td>
<td>[0.084]</td>
<td>[-1.875]</td>
<td>[1.812]</td>
<td>[-1.692]</td>
<td>[143]</td>
<td></td>
</tr>
<tr>
<td>sHH</td>
<td>[0.033]</td>
<td>[-2.399]</td>
<td>[2.409]</td>
<td>[-1.356]</td>
<td>[70.28]</td>
<td></td>
</tr>
<tr>
<td>REGIME 1 - Unemployment grw</td>
<td>$\alpha_0$</td>
<td>$\gamma_0$</td>
<td>$\gamma_1$</td>
<td>$\alpha_2$</td>
<td>$\alpha_3$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>OLS</td>
<td>-0.1966</td>
<td><strong>-0.0126</strong></td>
<td><strong>0.0622</strong></td>
<td>0.0174</td>
<td>1.0198</td>
<td>0.913</td>
</tr>
<tr>
<td>t-stat</td>
<td>[-2.558]</td>
<td>[-2.175]</td>
<td>[2.148]</td>
<td>[1.553]</td>
<td>[47.98]</td>
<td></td>
</tr>
<tr>
<td>white</td>
<td>[-2.615]</td>
<td>[-2.551]</td>
<td>[2.484]</td>
<td>[1.566]</td>
<td>[43.52]</td>
<td>obs 248</td>
</tr>
<tr>
<td>HH (12)</td>
<td>[-6.869]</td>
<td>[-2.325]</td>
<td>[2.147]</td>
<td>[4.158]</td>
<td>[37.65]</td>
<td></td>
</tr>
<tr>
<td>NW (12)</td>
<td>[-3.198]</td>
<td>[-2.257]</td>
<td>[2.189]</td>
<td>[1.843]</td>
<td>[43.36]</td>
<td></td>
</tr>
<tr>
<td>sHH</td>
<td>[-1.532]</td>
<td>[-2.305]</td>
<td>[2.357]</td>
<td>[1.217]</td>
<td>[39.38]</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8

t-statistics in square brackets.
The Lucas hypothesis seems to be respected regardless the variable used to measure the business cycle. The conditional variance of money growth affects real variables through the coefficients of the prediction-error term ($\gamma_1$). The direct influence of the conditional variance implied by the Friedman hypothesis is not significant with the only exception for the IP gap equation, where surprisingly the effect of the conditional variance appears to work in the opposite direction. However, this is far from being paradoxical as long as when the economy is going toward a recession, a peak in the variability of inflation, captured by the conditional variance of money growth, might act as a stimulus to economic activity, or might be interpreted as a sign that the recession is neither severe nor long-lasting.

Generated variables in the above regression might invalidate inference procedures. To handle with it not only we propose alternative measures of the standard errors (White, 1980; Hansen and Hodrick, 1980; Newey and West, 1987; the simplified Hansen and Hodrick), but also we perform a Monte Carlo simulation. Results show a clear convergence of both Lucas parameters towards the true values just after few thousands replications ($\gamma_0$ in the left panel; $\gamma_1$ in the right panel). The top diagrams show the simulation with 15000 replications, while in the bottom diagrams we run 50000 replications.

![Monte Carlo simulations][1]

The estimation of regime 1 has also been performed after ruling out the conditional variance of money prediction errors. Results are reported in Table 5.9. There is a clear confirmation that the Lucas hypothesis is not rejected by the data when the yield curve is either flat or downward sloping.

---

[1]: Figure 5.5
Regime 2 is characterized by high, and positive, values of the yield spread. Regime 2 estimates are reported in Table 5.10.

The monetary accommodation reflected in the upward sloping term structure is usually expected to stimulate economic activity thus pushing the economy on an expansionary path. On the other hand, a positive slope of the yield curve implies a positive risk premium required by investors to move to longer horizons. In particular, Ang, Bekaert and Wei (2008) find evidence that the slope of the nominal term structure is due to a positive inflation risk premium. In case of perfect foresight about future spot rates, in fact, the arbitrage mechanism would equalize holding period returns along the entire spectrum of maturities implying a flat yield curve. In a context characterized by imperfect information, uncertainty causes the term structure to deviate from its risk-neutral implied shape.
Table 5.10

The joint effect of uncertainty and economic growth is reflected in a greater variability of expected inflation, and, thus, in the dynamics of the conditional variance of money growth (Barro, 1976; Friedman, 1977). Therefore, the threshold estimation of regime 2 should return a significant coefficient $\alpha_2$ stressing out the inverse relationship between the money conditional variance and the economic cycle. Coefficient $\alpha_2$ is, in fact, negative in the equations expressed in terms of the IP gap and the total capacity utilization; while, coefficient $\alpha_2$ turns out to be positive when the dependent variable is unemployment or its rate of change. As Rudebusch, Sack and Swanson (2007) have found, this result can be interpreted in line with our previous findings (Section 4.4.1, Chapter 4) that there exists an inverse correlation between term premia and output growth.
5.6 Concluding Remarks

In this chapter we propose and implement an innovative method for expectations in order to investigate the relationship between money and output. Previous evidence tends to support the view that money is effective in stimulating real economic activity; however, some contrarian evidence suggests the issue is still controversial. In particular, the high correlation between money and output does not reveal an unambiguous direction of causality. From a policy perspective, in fact, the aforementioned positive correlation can derive from two opposite phenomena. On the one hand, the monetary authority can govern the supply of money to influence the future economic conjuncture; on the other hand, the central bank can manage the dynamics of monetary aggregates in response of past macroeconomic conditions. In addition, the weaknesses associated to traditional approaches call for an effective role of expectations.

After estimating a time-varying monetary policy rule where expectations are analysed at a micro level, we condition the examination of the money-output relationship to the shape of the yield curve. In particular, in this chapter we test the Lucas (1973) hypothesis against Friedman’s (1977). Within a non linear approach, we find evidence that the conditional variance of money growth affects real output through the coefficients on the forecast error term in the Lucas type output equation if the shape of the term structure reflects expectations of a slowdown in economic activity. The Lucas hypothesis implies a negative relationship between the variance of nominal shocks, quite low when a flat, or inverted, yield curve reflects a clear and severe monetary policy tightening, and the magnitude of output response to nominal shocks, quite large when policy tightens according to the traditional keynesian asymmetry.

Moreover, the conditional variance of money growth, which is used as a proxy for inflation variability, appears to affect directly output when the term structure is upward sloping, i.e. investors require a positive term premium. The Friedman hypothesis implies that high inflation tends to reduce the natural level of output since the re-adjusting preferences of the policy makers will distress the real interest rate thus reallocating resources and pushing the economy out of the steady-state equilibrium level.
Appendix A5.1 - Data

All data have monthly frequency; the sample starts in January 1966. The core econometric analysis, after Kalman filtering, is thus performed from January 1967 since we rule out the first 12 observations. The U.S. series of seasonally adjusted industrial production is from the FRED database (Federal Reserve Economic Data). The seasonally adjusted unemployment rate series (civilian unemployment), as well as the total capacity utilization index, are from the FRED database; the source is the U.S. Department of Labour (Bureau of Labour Statistics). The series are covariance stationary as suggested by both the augmented Dickey-Fuller test and the Kwiatkowski-Phillips-Schmidt-Shin test. (as usual grw stands for “rate of growth”). Also results from the Phillips and Perron test is reported in the Table.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Stationarity</th>
<th>ADF (AIC) [lag]</th>
<th>ADF (SIC) [lag]</th>
<th>ADF (HQ) [lag]</th>
<th>PP ( Bai )</th>
<th>PP (q)</th>
<th>KPSS (B)</th>
<th>KPSS (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ffr</td>
<td></td>
<td>(0.0253)* [16]</td>
<td>(0.0942)* [2]</td>
<td>(0.0883)* [13]</td>
<td></td>
<td></td>
<td>0.2652*</td>
<td>0.2161*</td>
</tr>
<tr>
<td>y3m grw3</td>
<td></td>
<td>(0.0001)* [17]</td>
<td>(0.0000)* [7]</td>
<td>(0.0002)* [16]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 grw3</td>
<td></td>
<td>(0.0548)* [17]</td>
<td>(0.0452)* [12]</td>
<td>(0.0452)* [12]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1 grw</td>
<td></td>
<td>(0.0043)* [12]</td>
<td>(0.0210)* [13]</td>
<td>(0.0043)* [12]</td>
<td></td>
<td></td>
<td>0.7103*</td>
<td>0.9431*</td>
</tr>
<tr>
<td>M2 grw</td>
<td></td>
<td>(0.0036)* [17]</td>
<td>(0.1160)* [13]</td>
<td>(0.1160)* [13]</td>
<td>(0.0465)*</td>
<td>(0.0440)*</td>
<td>0.9783*</td>
<td>1.3169*</td>
</tr>
<tr>
<td>MB grw</td>
<td></td>
<td>(0.0879)* [13]</td>
<td>(0.0879)* [13]</td>
<td>(0.0879)* [13]</td>
<td>(0.0088)*</td>
<td>(0.0163)*</td>
<td>0.4564*</td>
<td>0.5296*</td>
</tr>
<tr>
<td>infl(3)</td>
<td></td>
<td>(0.0751)* [16]</td>
<td>(0.0729)* [13]</td>
<td>(0.0729)* [13]</td>
<td>(0.0000)*</td>
<td>(0.0001)*</td>
<td>0.5873*</td>
<td>0.5244*</td>
</tr>
<tr>
<td>IP gap (HP)</td>
<td></td>
<td>(0.0000)* [9]</td>
<td>(0.0000)* [3]</td>
<td>(0.0000)* [3]</td>
<td>(0.0001)*</td>
<td>(0.0000)*</td>
<td>0.3804*</td>
<td>0.4213*</td>
</tr>
<tr>
<td>unemp</td>
<td></td>
<td>(0.0494)* [4]</td>
<td>(0.0494)* [4]</td>
<td>(0.0494)* [4]</td>
<td>(0.1347)*</td>
<td>(0.1429)*</td>
<td>0.1849*</td>
<td>0.2589*</td>
</tr>
<tr>
<td>unemp grw</td>
<td></td>
<td>(0.0001)* [16]</td>
<td>(0.0058)* [12]</td>
<td>(0.0001)* [16]</td>
<td>(0.0011)*</td>
<td>(0.0007)*</td>
<td>0.2707*</td>
<td>0.3901*</td>
</tr>
<tr>
<td>log(tcu)</td>
<td></td>
<td>(0.0006)* [9]</td>
<td>(0.0066)* [3]</td>
<td>(0.0066)* [3]</td>
<td>(0.0176)*</td>
<td>(0.0189)*</td>
<td>0.4238*</td>
<td>0.5549*</td>
</tr>
</tbody>
</table>


Table 5.11

The ADF test rejects the null hypothesis of unit root; while the null hypothesis of stationarity cannot be rejected by the KPSS test. To match the monthly frequency of data, the rule of thumb selected number of lags in the auxiliary regression is either 11 or 12. The automatic lag selection based on different criteria (Akaike, Schwarz, Hannan-Quinn) is consistent with our choice. Unit root test results obtained with the automatic lag selections are similar. The critical values of the KPSS test are 0.739 (1%), 0.463 (5%), and
0.347 (10%) when the intercept is included in the auxiliary model. The compute KPSS statistics never falls in the critical region. The KPSS test tends to reject the null of stationarity for the unemployment series. In Table 5.12 the stationarity tests are carried out on both the forecast error and the conditional variance series obtained from the monetary policy function expressed in terms of money growth. As mentioned above in the text both the time-varying parameter model estimated by Kalman filtering and the vector autoregressive model of order 9 have been employed in this chapter.

<table>
<thead>
<tr>
<th></th>
<th>adf (aic) [lag]</th>
<th>adf (sic) [lag]</th>
<th>adf (hq) [lag]</th>
<th>pp (b)</th>
<th>pp (q)</th>
<th>kpss (b)</th>
<th>kpss (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>forecast error</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>0.2636*</td>
<td>0.2681*</td>
</tr>
<tr>
<td>conditional variance VAR(9)</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>0.3061*</td>
<td>0.3221*</td>
</tr>
<tr>
<td>forecast error</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>0.0733*</td>
<td>0.1093*</td>
</tr>
<tr>
<td>(kf)</td>
<td>[6]</td>
<td>[4]</td>
<td>[4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conditional variance (kf)</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>(0.0000)*</td>
<td>0.1686*</td>
<td>0.1728*</td>
</tr>
<tr>
<td></td>
<td>[3]</td>
<td>[3]</td>
<td>[3]</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Appendix A5.II

In Table 5.13 we report the estimations of equation (5.11') after deriving the prediction error and the respective conditional variance series from a vector autoregressive model of order 9. The conditional variance series is obtained computing the square of the residuals in the money equation of the VAR(9) system (Piazzesi, 2003). Money growth, the inflation rate, and the quarterly change of the T-Bill rate are the endogenous variables; the constant is the only exogenous variable. The number of lags has been selected on the basis of the Akaike and Schwarz criteria. Results tend to support the Friedman hypothesis rather than the Lucas’ one. The dynamics of the real variables seems to be lowered by the conditional variance of money growth, which is regarded to be a proxy for price volatility.

<table>
<thead>
<tr>
<th>joint estimation - IP gap (HP) - VAR(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>t-stat</td>
</tr>
<tr>
<td>white</td>
</tr>
<tr>
<td>HH (12)</td>
</tr>
<tr>
<td>NW (12)</td>
</tr>
<tr>
<td>sHH</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>joint estimation - TCU - VAR(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>t-stat</td>
</tr>
<tr>
<td>white</td>
</tr>
<tr>
<td>HH (12)</td>
</tr>
<tr>
<td>NW (12)</td>
</tr>
<tr>
<td>sHH</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>joint estimation - Unemployment - VAR(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>t-stat</td>
</tr>
<tr>
<td>white</td>
</tr>
<tr>
<td>HH (12)</td>
</tr>
<tr>
<td>NW (12)</td>
</tr>
<tr>
<td>sHH</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>joint estimation - Unemployment grw - VAR(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
</tr>
<tr>
<td>OLS</td>
</tr>
<tr>
<td>t-stat</td>
</tr>
<tr>
<td>white</td>
</tr>
<tr>
<td>HH (12)</td>
</tr>
<tr>
<td>NW (12)</td>
</tr>
<tr>
<td>sHH</td>
</tr>
</tbody>
</table>

T-statistics in square brackets.
Conclusion

Economic theory has been remarkably innovative during the second half of the last century. Similarly, there has also been an outstanding development of technical methods to measure scientifically social phenomena. For instance, advancements in biometrics as well as in econometrics have contributed to the improvement of medical and economic analysis respectively. Furthermore, the classical interest of exploiting interrelations among sciences has renewed and developed further, emphasizing the multi-disciplinary nature of each field of research particularly in social sciences. The multi-disciplinary approach was already familiar to Adam Smith (1723 – 1790) who was a moral philosopher before being a social scientist and dedicating his efforts to the science of economics. As a matter of fact, the merger between microeconomics and macroeconomics has given life to micro-founded macro models; cognitive sciences have been coupled with both microeconomics and finance generating experimental economics and behavioral finance. Economic theory has progressively increased the interaction with statistics and mathematics; in that respect an independent discipline has emerged: econometrics. Econometrics has further specialized in macro- and micro-econometrics; more recently spatial econometrics, a new branch, has appeared. The advancements in computer technology have allowed economists to count on sophisticated, and computationally feasible, techniques shortening the step from theoretical to applied econometrics.

In this thesis I extensively use econometric methods for the empirical analysis of macroeconomics and finance; in particular, I deal with several issues recently arisen in applied economic research. The term structure of interest rates, which is a core issue throughout the thesis, represents the bridge between macroeconomics and finance. The short end of the term structure is directly influenced by announcements of the monetary authority regarding the level of the policy rate; empirical evidence suggests, in fact, that the short end is mostly affected by the evolution of macro factors (Ang and Piazzesi, 2003) which are observed by the monetary authority before policy decisions. Movements of the long end of the term structure are more closely related to the effect of expectations and financial factors. The long end of the term structure also captures the dynamics of term premia anchored to expected inflation. For these and other reasons, the shape of the term structure is considered a leading economic indicator which contains valuable information about the future evolution of both inflation and output.

In Chapter 2 we provide evidence that exists a close association between macroeconomic variables and the financial factors underlying the term structure of interest rates. In particular, consistently with what stressed above, we find that the slope of the term structure reflects the changes in the policy rate as implied by a Taylor-type central bank reaction function. This interpretation is also
theory-consistent since the slope is computed as the difference between smooth and persistent long yields and extremely dynamic short yields. The level of the term structure is associated to the rate of inflation targeted by the monetary authority. In this respect, since it is measured along the entire spectrum of term structure maturities, the level may well be informative about eventual inflation risk premia. Finally, we provide substantial evidence that curvature, which describes the term structure at medium maturities, is related to the cyclical conditions of the economy. Although few attempts have been proposed, the interpretation of curvature is still controversial in the economic literature. Dewachter and Lyrio (2002) put forward the idea that curvature represents an independent monetary policy factor. Similarly, Bekaert, Cho, and Moreno (2005) relate curvature to monetary policy shocks. Dewachter, Lyrio and Maes (2006) believe curvature is more closely related to the real stance of monetary policy. Giese (2008) argue that curvature is informative about the future path of interest rates; while, Hordhal, Tristani and Vestin (2006) find that both inflation and output display important effect on medium term maturities. Future research needs to concentrate on the interpretation of curvature corroborating the interpretation with more robust evidence. Nevertheless, we believe that our interpretation of curvature as a cyclical indicator might be consistent with all above suggestions. We think, in fact, that yields’ dynamics at medium term maturities represents an intermediate step of the transmission of monetary policy actions along the entire length of the yield curve thus synthesizing the evolutionary shape of the yield curve over the business cycle. We feel our analysis is robust since we also document that curvature from the real term structure reflects business cycle fluctuations.

Examining the term structure of interest rates primarily means to figure out how long term yields evolve after observing short yield movements. The expectations hypothesis might be helpful in this respect since it implies that long yields are weighted averages of risk-adjusted short term yields. Unfortunately, from times to times, the expectations theory has found little empirical support. In Chapter 3 we propose a non linear approach to assess the informative content of the yield spread. We find that the predictive ability of the yield spread increases substantially if both agents’ expectations about the incoming stance of monetary policy and individuals’ perception of risk are explicitly considered. Future research may focus on whether there is a correlation between the corroboration of the expectations hypothesis and some phases of the business cycle. Moreover, along the path traced Mankiw and Miron (1986) there is further space to investigate the expectations hypothesis in relation to different monetary regimes. In particular, it would be interesting to gauge whether testing the expectations hypothesis before and after the European monetary union leads to different results. Finally, it would be appealing to figure out whether either
financial integration or financial innovation have affected the relationship between long and short yields in transition economies, for instance in east European countries. Actually, a refinement in the analysis of the predictive power of the spread has been already proposed. The yield spread, in fact, has been decomposed into a term premium and an expectations based component; such a decomposition allows examining separately the effect of risk and that of expectations regarding the future stance of monetary policy. Predicting future output using the aforementioned decomposition has been useful; however, whether term premia play an active role in the predictive model is still controversial (Ang, Piazzesi and Wei, 2006). Hamilton and Kim (2002) find weak evidence that interest rates volatility can be regarded as a potential determinant of future output growth. In Chapter 4 we focus on the predictive content carried by the conditional volatility of term premia, which we consider a measure of financial distress. Data evidence tends to suggest that term premia variability is inversely correlated with future economic activity. Future empirical research based on survey data could provide with a deeper analysis of the predictive power of term premia; basically different sides of risk could be worked out from a single measure of term premia. From a theoretical perspective, instead, along the line highlighted by Estrella (2004) structural DSGE models, as the one by Rudebusch, Sack, and Swanson (2008) could be augmented in order to work out a link between term premia and economic activity.

Finally, we challenge to read the neoclassical synthesis under a new perspective. In Chapter 5, in fact, we examine whether money, or some measures of money, helps to anticipate the business cycle. Specifically, we condition the test of the Lucas (1973) hypothesis to the informative content carried by the slope of the term structure. The Lucas hypothesis, that there is an inverse correlation between the variance of nominal shocks and the magnitude of output response to nominal shocks, tends to prevail when the yield curve is either flat or negatively sloped. While, the alternative hypothesis that inflation variability reduces the natural level of output (Friedman, 1977) dominates when the yield curve is upward sloping.
References


Feldstein M., Stock J.H., 1994, The Use of a Monetary Aggregate to Target Nominal GDP, in G.N. Mankiw, Monetary Policy, Chicago University Press.


