Assessment of Fairness and Equity in Trajectory Based Air Traffic Management

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The candidate confirms that the work submitted is her own and that appropriate credit has been given where reference has been made to the work of others.

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Abstract

This thesis investigates the application of the concepts of fairness and equity in Air Traffic Management (ATM), specifically focusing on trajectory-based operations. One of the main objectives of these new type of operations, which are at the core of the two major ongoing ATM modernisation initiatives (SESAR in Europe and NextGen in the United States), is to enable Air Navigation Service Providers (ANSPs) to better accommodate the trajectories preferred by the different airspace users. In this context, it is pivotal to ensure that all airspace users are dealt with impartially by the ANSP responsible for monitoring, adjusting and clearing their trajectories. Thus, fairness and equity considerations need to be integrated in the definition and implementation of trajectory-based operations.

In view of the lack of a rigorous approach to the integration of the concepts of fairness and equity in trajectory-based operations, this thesis proposes fairness and equity metrics to assess the impact of the decisions of ANSPs on the distribution of cost penalties among different users. In this context, a cost penalty is defined as the increment in operational cost for the user that results from being cleared to fly a trajectory that is different from its preferred one. In addition, this thesis proposes methods to incorporate the aforementioned metrics to the decision-making process of ANSPs in the context of trajectory-base operations, so that the trajectories assigned to the users may be fair to all of them.

In a first step, this dissertation derives specifically for ATM the concepts of justice, fairness and equity from traditional disciplines such as Philosophy, Economics or Sociology. These theoretical notions are the foundations for developing the mathematical expressions to obtain quantitative values of fairness and equity in the context of future trajectory-based operations. These fairness and equity metrics are initially defined considering that each flight is an independent airspace user and later generalised to the possibility of airspace users simultaneously operating several flights in the same operational context, e.g. a scenario including several airlines where each one is operating several flights.

Secondly, the practical application and incorporation of the developed metrics into the existing ATM systems is explored. Two different methodologies to apply the proposed fairness and equity metrics in practice are presented, each one addressing a specific example where using such metrics could benefit future trajectory-based operations. Specifically, the proposed methodologies deal with how to incorporate fairness and equity considerations in the design and evaluation of so-called trajectory management algorithms, which will be used by future ATM systems to adjust the airspace users' preferred trajectories so that they remain conflict-free within a given time frame, e.g. during the arrival phase. The usability of the metrics according to the proposed methodologies is illustrated by means of specific and ATM-relevant examples. On the one hand, the integration of the fairness metric into a trajectory conflict resolution algorithm is presented with a view to enlarge the optimisation criteria of trajectory modifications, i.e. inclusion of fairness considerations during trajectory adjustments. On the other, a comparative assessment is described with regard to fairness and equity of three different conflict resolution algorithms for a given environment, i.e. operational context, route network and traffic situation.

To complement the above, a preliminary robustness analysis of the proposed fairness and equity metrics has been conducted. This analysis is an important part of this work, addressing explicitly the need of particular characteristics in the ATM environment for successfully making use of fairness and equity metrics, identifying situations that can affect the effective application of metrics and establishing the provisions to guarantee it. This study is based on concepts from the field of Decision Theory where the ATM system is modelled to reflect the de-confliction of trajectories based on user preferences in a two-player game with a fairness-oriented ANSP.

In summary, this thesis proposes a new framework to incorporate fairness and equity in future air traffic operations based on quantitative metrics. The framework includes methodologies to apply these metrics in practice with a view to enabling the fair or equitable distribution of cost penalties among users in trajectory-based operations.

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1 INTRODUCTION

1.1 Current Air Traffic Management

The Air Traffic Management (ATM) system is "*an essential infrastructure of systems, people, and procedures which, together with airports, enable air transport and other aerial movements to operate in a safe and expeditious manner*"[1]. The aim of ATM is to achieve a safe, efficient, and expeditious movement of aircraft in the airspace [2]. The current role of ATM is to deliver air navigation services directly to the airspace users [1]. Air Traffic Management (ATM) service is provided throughout the majority of the airspace and is available for commercial, military, and private airspace users. This service has two clearly defined ground-based processes: Air Traffic Control (ATC) and Traffic Flow Management (TFM) [2]. ATC is mainly a safe separation assurance service that gives guidance to aircraft, both on the ground and in the air, to maintain the prescribed separation minima and avoid collisions among aircraft or with terrain hazards. The traffic flow manager has to organise, expedite, and allocate traffic flow within the limited airspace capacity resources. The TFM is also responsible for the prevention of unsafe levels of traffic congestion organising the associated movements so that delays are equitably distributed among system users [2].

The services which make up the ATM are mainly provided by national Air Navigation Service Providers (ANSPs), typically one per State [1]. The responsibility of the International Civil Aviation Organisation (ICAO) is to establish a standardisation of air traffic management practices and ensure that an equitable ATM service is provided to all users [2]. This organisation is affiliated with the United Nations and provides technical standards for equipments and procedures to align countries and facilitate safe and efficient operations of international aircraft around the globe [2].

Nevertheless, there is still room for improvement as many current interactions within the ATM system lack of an explicit specification or agreement on the quality aspects of the services provided, their prices, and reciprocal obligation [1].

Currently, ANSPs base their service on the “first come, first served” (FCFS) principle, without an explicit commitment to maintain airline schedules. This FCFS principle is the industry-accepted view of fairness, [3] and [4]. But this way of providing the service inhibits the competitive strategies of many commercial airlines, which demand overall network optimisation to construct their schedules and cost strategies to satisfy their business models. Main requisite for this optimisation is the punctuality performance [1]. Regarding these demands of the airspace users, which are to a great extent the commercial airlines, the ANSPs show a positive movement towards a better alignment with the business imperatives of their clients [1].

Another important issue to be addressed is that of the recurrent concerns of airlines regarding the lack of a standard framework to establish and identify unfair practices. For example, a specific concern of commercial pilots it that they complain about air traffic controllers favouring local airlines [5][6].

In principle, regulatory authorities shall strive to ensure an equitable ATM service to all airspace users worldwide. ICAO, the Federal Aviation Administration (FAA) responsible for the US airspace and EUROCONTROL, aiming at coordinating the European airspace, care and are also responsible for the well-functioning of the ATM system, and are concerned about allocating in an equitable way the scarce resources.

Two main ATM modernisation programmes, Single European Sky ATM Research (SESAR) and Next Generation Air Transportation System (NextGen), have been launched to address and analyse the current constraints in ATM and adapt the system to facilitate future requirements. Thus, it seems to be now the adequate moment to address the fairness and equity issues concerning the ATM service, so that possible solutions can be taken into account by these modernisation programmes.

1.2 Towards the Future of Air Traffic Management

Nowadays, it is a fact that the number of aircraft requiring ATM services, and consequently the number of aircraft operations, is increasing. In Europe alone EUROCONTROL plans a 3-fold increase in air traffic movements for 2020 [1]. According to AIRBUS; the "*European air traffic growth cannot be sustained by the current air navigation services organisation and ageing ATM technologies*" [7]. This trend will inevitably force the current busiest controlled airspaces to their capacity limits, namely the ATM systems in Europe and in the US [8]. Main consequence will be the increase in delays and the associated extra costs for airlines and also for passengers [8].

NextGen in the United States and SESAR in Europe are addressing the future ATM requirements to accommodate the predicted growth in air traffic activity in the best suitable way for all users involved. These two programmes share common challenges in proposing solutions to the given bottlenecks to ensure a future ATM able to accommodate a three-fold growth in system capacity. Both NextGen and SESAR programmes are based on three main premises [9]:

- the airspace operations should be performance-based, giving more relevance on how the airspace users want to design their trajectories and fly them;
- the ATM service has to support aircraft of varying capabilities assuming that not all aircraft have the same systems on board,
- the aircraft operations can be enhanced by the sharing of common and timely information, having in mind that as the idea of how to fly the trajectory gains on relevance, the sharing of this information will be of extreme importance not only for the ATM service provider but also for the users.

The fundamental concept of both programmes is to build a "network-centric" system of operations, which will be based on three main assertions, these being [9]:

- an information management network for sharing data and services,
- new air-air, ground-ground and air-ground data communications systems,

- an increased reliance of airborne and ground-based automated support tools.

The intention of the “network-centric” system is to achieve an integrated ATM system, both in NextGen and SESAR [10], wherein automated tools, data network infrastructures, improved surveillance, navigation, and communication capabilities are synchronised to improve the management of events and their consequences [11].

According to the “ATM Target Concept”, as defined within SESAR, the requirements of the airspace users need to be better accommodated [11]. The main driving principle of this concept is that “*each single flight shall be executed as close as possible to the intention of its owner*” [11]. The user’s intentions with respect to a given flight are represented within SESAR by the Business Trajectory (Mission Trajectory for military purposes). Thus, the concept of operations becomes trajectory based, being the trajectory management the central piece of the future ATM Concept. Trajectory management is the “*process by which the Business or Mission Trajectory of the aircraft is planned, agreed, updated, and revised”* [12].

1.2.1 Moving Towards Strategic, Highly Automated Operations in ATM

Trajectory management is a key process of the SESAR concept. The sooner the trajectory is known and the better the accuracy of the trajectory, the earlier the de-confliction and optimisation of individual trajectories and of the traffic flow can be started. This ensures that airspace users are able to conduct their operations with minimum restrictions and maximum flexibility whilst meeting the safety targets of the ATM system, and simultaneously improving the capacity and efficiency of the system [11][12]. To achieve these objectives air traffic controllers require support from computer based systems to manage the amounts of information and decide on the best suitable trajectory amendments (if required) to improve operational efficiency, cost effectiveness, environmental impact, and meeting security and safety requirements [11].

The current ATM systems, both in the US and in Europe, are characterised by their reliance on the manual execution of tasks [13]. Nowadays, Air Traffic Controllers (ATC) have to do most, if not all, of these afore mentioned activities manually.

Based on the current prognoses for the growth of air traffic, which is predicted to more than double in the next decade, strategic planning of traffic flows and long-term prevention of potential traffic conflicts will play a major role for ATC [11]. Thus, there is an accrual need to move from tactical to strategic operations.

As a consequence, this move from tactical to strategic operations will include the shifts of [11]:

- controlling traffic through instantaneous observations to managing traffic based on predictions
- the manual execution of tasks to their execution supported by automation.

There is a need to develop automated management systems to support the human decision making process. The air traffic controllers alone without the help of automated systems will be unable to serve the future demand and, at the same time, improve efficiency and capacity of the current system while implementing the Trajectory-based Operation (TBO) concept proposed by SESAR and NextGen [11][14].

Automated support systems aim to shift the distribution of controller workload away from monitoring the separations of all aircraft in a sector and towards the management of traffic flow [15]. To facilitate the implementation of automated systems to assist the controllers and move towards strategic operations, the air traffic management system requires more orderly and predictable traffic patterns [16].

SESAR's proposal for the automation support addresses the controller task load issue, without incurring a significant increase in Air Navigation Service Provider costs, and presenting three lines of action [11]:

- Automation for the routine controller task load supported by better methods of data input and data management.
- Automation support to conflict/interaction detection, situation monitoring, and conflict resolution.
- A significant reduction in the need for controller tactical intervention, by (a) reducing the number of potential conflicts using a range of de-confliction methods, and (b) redistributing the tactical interventions to the pilots.

For the current ATM system to focus on the strategic management, optimisation, and de-confliction of trajectories, there is a need to reduce uncertainty in the predictions. The performance of the trajectory prediction tools is dependent on the accuracy with which the future positions of aircraft can be predicted. This is so due to the difficulty in estimating the influence of the weather on the trajectory, specially wind. Any step that reduces uncertainty of prediction, specially the uncertainty of the initial conditions upon which the prediction is based, will increase the usable prediction horizon and allow longer duration clearances; thus, enabling strategic operations [11].

There are many measures that can be taken to reduce uncertainty of ground-based as well as airborne-based trajectory prediction (for example more reliable weather forecasting, more accurate aircraft performance models). Not only automation but also current Communication, Navigation, and Surveillance (CNS) capabilities have to be improved both in the ground and in the air. That is why SESAR defines ATM Capability Levels to describe the on-going deployment of progressively more advanced CNS and Automation (CNS-A) systems for aircraft and ground systems.

The enhanced CNS-A capabilities for both ground and airborne systems will facilitate the flow of accurate information and enable the sharing of trajectory related data between the Airline Operation Centre (AOC), the aircraft system, and the ground system. This sharing of trajectories, contemplated within the trajectory management, permits a number of very significant advantages [11]:

- Reduce the uncertainty which in turn reduces the number of conflicts/interactions that need to be resolved.
- When combined with improved navigation performance (vertical, lateral, and in time), reduce the amount of ‘unusable’ airspace around each aircraft thus allowing more aircraft in the airspace.
- It is a source of accurate data which can be used by automated controller support tools.

The ATM modernisation programmes are based on a shift towards more strategic and automated operations. To this aim, the accuracy of the trajectory prediction tools needs to be improved, and the communication, navigation, and surveillance capabilities to be enhanced.

1.2.2 Strategic and Automated Support Tools for Trajectory Based Operations

According to the requirements foreseen for the future ATM and the needs identified in the two modernisation programmes, SESAR and NextGen, ANSPs are already looking for computer based systems to support the controller’s decision making process to improve the utilisation of the available resources.

The demand for these automated systems is increasing in such a rapid way that air service providers are already developing their own automated support tools adjusted to their national requirements. This is, for example, the case with SARA (Speed And Route Advisor) developed by LVNL (Air Traffic Control the Netherlands) [17], iFACTS (interim Future Area Control Tools Support) developed by NATS (UK National Air Traffic Services) [18] or FPCF (Flight Plan Conflict Function) developed by ASA (Air Services Australia) [19].

In a first development step, the automated support tools had to be able to predict trajectories with a certain accuracy, establish the aircraft sequence at a certain waypoint, assign required time over significant waypoints to each aircraft and give possible solutions to the controller in the form of different trajectories in compliance with the assigned times.

Main congested airspace areas are those around airports. Thus, the development of automated tools focused, in a first phase, on those areas. Many of the decision support tools (DSTs) used nowadays assist air traffic controllers in charge of air traffic around airports. In line with this first research focus, the approach of most of the current commercialised automated decision support tools is to consider the runway as the main limited resource defining the capacity restrictions for arrivals and departures. The sequences at certain waypoints and the support to the traffic flow activities is extrapolated from the time requirements imposed on the runway.

The original basic function of commercialised DSTs is to establish the sequence at the runway threshold, then assigned to each aircraft a landing time according to the departure and arrival intervals at the runway. These two functionalities are more commonly known as sequencing[1] and scheduling[2]. At the beginning in the early 80s, many commercialised systems just had the sequencing and scheduling function [20]. Nowadays, most of them also include the metering[3] function.

These basic functions of decision support systems can be complemented with two further functions, conflict detection[4] and conflict resolution[5], to assure that the predicted trajectory,

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[1] *Sequencing* is the process of establishing the order of arrivals at a constraint point (e.g. FIR exit fix or IAF) or at a runway threshold, while providing an optimum (smooth, economic, max. possible) sequenced traffic flow by taking into consideration allowed runway acceptance rates (airport /runway capacity) as well as minimum separation criteria (wake turbulence constraints) [21].

[2] *Scheduling* is the process of assigning a specific landing time to each individual aircraft [22].

[3] *Metering* is the calculation of target times (amounts of time to lose or to gain) to meet the calculated sequence times at defined constraint points based on a pre-defined sequence and on the arrival interval, i.e. the rate at which the corresponding aircraft will pass through that fix [15]. Metering involves a previous sequencing and scheduling process.

[4] *Conflict Detection* is the process of comparing the trajectories between two aircraft to predict if they will violate required spatial or temporal separation [25].

[5] *Conflict Resolution* is the process of proposing required amendments to a certain trajectory or trajectories to avoid a potential conflict between two or more aircraft.

which is in compliance with the assigned scheduled times, is also conflict free. DSTs including the conflict detection and resolution functionalities perform strategic separation assurance which indirectly contributes to optimise the traffic flow management process. Those DSTs can be used to assist air traffic controllers in various situations when trying to detect and resolve air traffic conflicts, e.g. in the en-route phase or in arrivals or departures.

DSTs which intend to be compliant with the future concepts proposed by SESAR and NextGen need to provide ATC with strategically planned trajectories meeting several objectives [23][24]:

- have minimum impact to user-preferences in terms of operating cost (time and fuel),
- have maximum efficiency in exploitation of airspace and runway capacity,
- and fairness in the distribution of deviations from user-preferred trajectories.

Research activities as NASA's research project "Center TRACON (Terminal Radar Approach Control) Automation System (CTAS)" [25][26][27], or the updated versions of MAESTRO [28] and COMPAS [29], have realised the importance of the CD&R functionalities and resources have been invested to propose strategic conflict detection and conflict resolution algorithms for automated and mainly centralised separation assurance systems. A detailed literature review on this subject can be found in Appendix A.

Among the automated decision support tools, the arrival management problem presents one of the most interesting challenges, as it has to handle different routes converging to one single constrained resource, namely the aerodrome, whilst optimising the overall efficiency of the whole network. The Terminal Manoeuvring Area (TMA or TRACON in the US) presents a particularly complicated and constrained airspace area. It is complicated mainly due to the converging routes to the airport and the complex activities taking place in the same airspace: arrivals, departures, and crossings [27]. Major constraints of a TMA are the runway operations and the environmental restrictions due to the proximity of the operations to ground level, where not only terrain hazards play a major role but also noise and emissions restrictions.

1.2.3 Evaluating the Performance of Decision Support Tools

Decision Support Tools (DST) including the conflict detection and resolution functionalities are key to enable strategic operations in the future scenarios described by SESAR and NextGen. But prior to the integration of DSTs in the ATM system, an evaluation should be made to assess their performance benefits as well as their impacts on the different ATM stakeholders, and on current and future operations.

One of the current challenges in ATM is to assess the performance of these new strategic optimisation and support tools including conflict detection and resolution algorithms and, subsequently, to evaluate and compare different automated separation assurance systems against given requirements. The focus of this section is not the evaluation of the technical characteristics of the automation tools such as algorithmic or computational performance, response times, etc, but the evaluation of the impact of these automation tools in trajectory-based operations. As stated in the previous section, the requirements are:

- Minimum impact on user preferences
- Maximum efficiency in exploiting scarce resources
- Fairness in the distribution of trajectory deviations

The parameters to be evaluated in these new automated systems regarding the afore-mentioned requirements still have to be determined to establish the different success levels of DST in the new operational concepts.

One wide extended problem when comparing different systems, is the inconsistency in approaches and terminology. Some efforts in recent years (e.g. SESAR Definition Phase D2 “The Performance Target” [30]) stand out as important contributions to the definition and standardisation of performance metrics of specific aspects within ATM.

Currently, the ATM system is in a situation where the number of ATM-related metrics is increasing but there is still a lack of standard methodology. The terminology used to describe metrics in ATM often leads to ambiguity and misunderstanding, because different studies and research communities may have for the same term definitions differing from each other. Depending on the ATM stakeholder defining the metric, one can find different terminologies addressing the same metric or identical terminologies referring to different metrics.

For instance, metrics related to the number of conflicts, typically used when assessing the effectiveness of conflict detection tools, vary depending on the algorithm or the system in use. The number of conflicts may be understood and measured as the number of the detected potential (probable and future) conflicts that are expected to take place with no intervention of the automated support tool or the number of actual conflicts that took place once the trajectories were flown with or without intervention of the automated support tools.

Metrics can be easily misinterpreted due to the complexity of measuring activities in the airspace and the lack of standardised definition of metrics [31]. In ATM there is a need for a common understanding of ATM performance. Due to this lack of standardisation, the comparison of results and conclusions from different research studies becomes a difficult task, if not impossible at the moment.

According to ICAO, care should be taken that the ATM metrics faithfully reflect the nature of the expectations. Metrics should be measurable directly by the ATM community and should be SMART, meaning to be Specific, Measurable, Accurate, Reliable, and Timely [32].

According to the definition of the word *metric* that can be found in the dictionary [33], a metric *represents a calculation guideline* and according to this a formula, which has to define units in which the measurement is to be expressed.

A performance metric is a standard definition of a measurable quantity that indicates some aspect of performance [34]. Performance metrics need to [34]:

- be measurable,
- have a clear definition including boundaries of measurement,

- indicate progress towards a performance goal,
- answer specific questions about the performance.

For a better understanding and to avoid confusion, the differences between performance objective and performance goal, as to be understood in this thesis, need to be clarified.

Performance objective is a general statement of the desired achievement (e.g. minimise fuel consumption). Performance goal is a specific statement (measurable) of the desired level of achievement [34] (e.g. reduce fuel consumption up to 20%).

In ATM, the ATM performance metrics are associated with ATM performance areas. Those performance areas identify the key areas where the ATM performance can be measured. Those areas have been defined by the three main ATM stakeholders, these being:

- Airline Associations
- Air Navigation Service Providers
- ATM Research Organisations

These three main stakeholders agree that there are 11 key performance areas in ATM. Those key performance areas are defined by ICAO in its document 9854-Global ATM Operational Concept [32] and were adopted by SESAR and NextGen:

- access and equity,
- capacity,
- cost effectiveness,
- efficiency,
- environmental sustainability,
- flexibility,
- global interoperability,
- participation by ATM community,

- predictability,
- safety,
- security.

However, the three main ATM stakeholders do not agree on what needs to be measured within those performance areas and how to measure it. This is made evident by the fact that methods and metrics used for comparison are often inconsistent with each other.

Some airline's associations understand flexibility as the performance area addressing the ANSPs ability to comply with airline's air traffic service (ATS) change requests. These airline's associations have defined a metric named flexibility as follows [35]:

Flexibility = ATS denials / ATS change requests (ATS=Air Traffic Service), measuring how many airline change requests (e.g. flight level, speed, routing, etc.) are denied by the ATS or air navigation service provider.

On the other hand, some ATM Research Organisations understand flexibility as the name of the key performance area focusing mainly on trajectory flexibility. Trajectory flexibility, so they have defined it, is measured with the help of two metrics, robustness and adaptability [36]. Robustness describes *the ability of the aircraft to keep its planned trajectory in response to the occurrence of a disturbance*, such as e.g. an air traffic conflict or changes in the aircraft state relative to the prediction due to imperfect wind forecast. Adaptability is *the ability of an aircraft to change its planned trajectory in response to the occurrence of a disturbance*. Main difference between those two metrics is that, given a disturbance, robustness describes the relative number of feasible trajectories while adaptability measures the total number of feasible trajectories [36]:

Robustness (given disturbance) = Number of feasible trajectories (given a disturbance) / total number of feasible trajectories (without disturbance)

Adaptability (given disturbance) = Number of feasible trajectories (given disturbance)

As mentioned before, it is also possible to find two metrics with the same name but describing different measurements.

Metric Name	**Metric Definition**	**Defined By**
Delay	actual flight time/optimum flight time	ATA[6], CAASD[7], EPRU[8] [35]
Delay	number of flights delayed/total number of flights	CANSO[9] [35]

At the current moment, there exists inconsistency in approaches and terminology when defining metrics. The lack of consensus between the three main ATM stakeholders in the definitions of terms and performance metrics is the main reason for this situation.

It is critical for SESAR and NextGen that metrics are applied uniformly across the whole ATM system. Specially in Europe, it is important that in a series of linked systems, as for example different national ANSPs operating in nearby sectors or regions, the applied performance metrics are the same, while the actual required level of performance may be variable [32].

Regarding the new automated systems enabling the future operational concepts for the ATM, and more precisely the DST to assist air traffic controllers, there is a need to develop within the modernisation programmes a common understanding to evaluate the performance of those tools towards their impact on the proposed new operations. Otherwise, it is impossible to neither compare automated systems against each other nor assess their effectiveness against requirements for the future operational concepts of SESAR or NextGen.

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[6] *ATA: Air Transport Association*

[7] *CAASD: MITRE Center for Advanced Aviation System Development*

[8] *EPRU: Eurocontrol Performance Review Unit*

[9] *CANSO: Civil Air Navigation Service Organization*

Dealing with the evaluation of DST within SESAR or NextGen, as mentioned before, key areas will be to assess the impact of those automated systems on maintaining user preferences, especially towards the airline's cost strategies, exploit the airspace resources, and ensure fairness among all users in their modified trajectories. The latter point, namely fairness, is a recurring concern of airspace users, especially commercial airlines. They are one of the principal airspace users and main consumers of air navigation service provided by ANSPs. Commercial airlines are concerned about ATC favouring local airlines [5][6] and incurring in unfair trajectory modifications to the rest of the airlines causing them extra delay and fuel consumption compared to local competitors, which turns out in a cost increase and competitive disadvantage. ANSPs need to demonstrate that their procedures are fair and objectively ensure their customers that no airline interests are being favoured.

It seems to be the right moment, within the modernisation programmes to invest effort in a framework to assess the performance of the required automation systems towards their impact on the future concept of operations and, more precisely, evaluate DST towards maintaining the user preferences while making the most out of the airspace capacity and, very important, ensuring fair procedures among all users.

1.3 Fairness in ATM

Fairness has been an important concept in other disciplines but so far not in ATM. Areas of study such as Philosophy, Sociology or Economics have recurrently analysed the idea of fairness.

In Philosophy, fairness is associated to the concept of justice and morality [37]. Its implications in political theories and law are deeply analysed and discussed, specially by John Rawls in his works *A Theory of Justice* [65] and *Justice as Fairness A Restatement* [63] and by George Klosko in his book *The Principle of Fairness and Political Obligation* [38]. Sociological studies have been carried out analysing the intrinsic conception of fairness that each individual has and how that conception is passed on to a group of people or society, usually associating fairness with ethics and uncertainty, for example in Anna Wierzbicka's book *English: Meaning and Culture* [58] and Kristina Diekmann et al. in their paper *Uncertainty, Fairness Perceptions, and Job Satisfaction: A Field Study* [39]. In Economics, fairness has been studied in various aspects: associated to the implications it has on the economic satisfaction of single individuals as well as on a group of them sharing the same economic interest [40], or how fairly scarce resources are distributed and shared [41][42].

The ATM, from an economical point of view, can be analysed very similarly to any given market. There are users and service providers, i.e. demand and supply. The users want to make the best economic profit for themselves, maximising their satisfaction, while the service providers want to achieve the same on their side, maximise their objectives, thus their own satisfaction. There are regulation authorities, as ICAO, controlling and supervising the well functioning of the system according to agreed standards and procedures. Still, it is surprising that little research has been done on the fairness issue within ATM. Having a service that is provided to different users and constituting those service providers the monopole of the service at national level, it appears interesting to analyse the fairness issues that may arise.

Together with the predicted traffic growth, different sources [8][43] assume an increase in delays that may undermine business strategies of many commercial airlines, as delays impose additional costs on them. Thus, it has to be guaranteed that potential reductions in delay costs are fairly distributed among airlines. New practices are required because formulations minimising the total delay costs of the system at the expense of some airlines will not be accepted in a highly competitive environment as the airlines industry [44]. Fairness implications have to be considered and introduced in the new ANSPs practices.

The two main modernisation programmes aim to overcome the foreseen bottlenecks for ATM by proposing new operational concepts and introducing the required modifications gradually in the current ATM system. These programmes integrate representatives from all stakeholders involved in ATM looking so for the maximum consensus when defining the future operations.

The new automated tools that will be needed to enable the new operational concepts, and the assessment and evaluation of these tools have gained importance; specially the assessment of the automated systems regarding their impact on the proposed operations. According to Geert Jonker et al. [45], the automation tools have the sufficient computational capacity to do the administration needed for fairness while human air traffic controllers have not. So, there is little doubt that automation systems can perfectly incorporate fairness in their considerations when supporting the decision making process of air traffic controllers.

The economic aspect is also acquiring relevance, especially in the current economic context of crisis. With the rising competitiveness among the airlines, maintaining their cost strategies is of important relevance to them. Given the trajectory management process proposed by SESAR and NextGen, the airlines are given the possibility to negotiate strategically with the corresponding ANSPs their trajectories and the subsequent modifications with certain flexibility. Thus, airlines are given a mechanism to obtain the trajectories that best fit into their cost strategies for their fleets. It is anticipated that fairness in ATM will become a major subject of discussion.

Fairness in ATM has been incipiently analysed in recent years. Gregory Carr, Heinz Erzberger and Frank Neuman wrote already in 1998 [46] that if user preferences were to be included in the arrival flow management process, the process had to be ultimately fair to all air carriers.

ICAO stated in 2008 that *a set of Principles, adopted by unanimity of Eurocontrol States and complying with ICAO recommendations, allowed putting in place adequate requirements in terms of user consultation, transparency, fair cost allocation, and cost-relatedness of en route charges* [47].

As previously indicated, not only has the current ATM a lack of standard metrics and corresponding standard frameworks to assess its performance, the definition of fairness in ATM is still ambiguous.

Papers talk at the same time about fairness and equity in ATM indistinctively [48][49], suggesting an equal definition for fairness and equity. According to Geert Jonker et al. in their paper *Efficiency and Fairness in Air Traffic Control* [45], the primary objective of fairness is maximising egalitarian welfare. M.J. Soomer et al. state in *Fairness in the Aircraft Landing Problem* [50] that absolute fairness is distributing scaled cost equally while relative fairness is measured according to a first-come-first-served schedule, a definition of fairness also shared by [51]. FCFS schedule is the industry-accepted view of fairness, after [3][4].

Fairness is also defined as the allocation of resources according to the predefined schedule, which is assumed to have been agreed between the airlines and the ANSPs. This is done for example by Steve L. Waslander et al., *Towards Efficient and Equitable Distributed Air Traffic Flow Control* [48], focusing on distributing fairly the arrival delay cost, and also by Nasim Pourtaklo et al in [49] proposing how to optimise the air traffic flow in the En-Route phase. In their paper *Contingency Plans for Air Traffic Management* [52] Karl Blomdahl et al. adopt a similar definition of fairness for improving air traffic flow management while reducing the incurred delay costs.

Some papers also start analysing the implications of fairness considerations on the efficiency of the air traffic [44][45][50]. Specially Dimitris Bertsimas et al. conclude in *The Price of Fairness* [53] that fairness considerations will have an impact on the efficiency of the operations, which may distribute cost equally among users but decrease the efficiency of the ATM system implying more overall delays.

Regarding the current economic situation, the evolution of the airline industry and the foreseen bottlenecks of the current ATM system, fairness in ATM has become an interesting aspect that has not been deeply analysed yet. It needs to be included in the new operational concepts. Among others, the automation systems required to implement those concepts are good candidates to include fairness in their processes assisting air traffic controllers in the decision making. A clear definition of fairness in ATM needs to be provided as well as a clear framework to analyse it.

1.4 Objectives and Contributions of the Research Described in this Thesis

The main objectives of the research described in this thesis are:

- to provide rigorous definitions of the concepts of justice, equity, and fairness in the ATM context
- to define mathematically sound metrics of fairness and equity in ATM
- to propose a methodology to evaluate those metrics in practice, focusing on assessing the fairness and equity implications of the DSTs that will enable the future TBO.

To achieve these aims, this thesis proposes a mathematical definition of the concept of fairness capturing the abstract concept of fairness as a concrete mathematical expression. This mathematical expression leads to the proposed fairness metric. To set this work apart from previous definitions of fairness, not only are the concepts of fairness and equity clearly differentiated, but an equity metric is also defined, exposing the unambiguity between the two concepts. To understand these two concepts, previously the concept of justice applied to the ATM context is introduced.

This thesis focuses on the fairness and equity aspects of the decision support tools required by the trajectory based operational concepts defined by SESAR and NextGen. The contribution of this work is not only to provide a definition of fairness and equity in ATM but also to propose a methodology to evaluate the fairness and equity of the automated systems at hand.

The defined methodology allows the evaluation of the trajectory modifications proposed by the automated decision support tools *a posteriori* and *a priori*. This means, the proposed methodology offers the possibility to incorporate a fairness or equity optimisation within the automated tools to achieve a higher fairness or equity of the solutions or the methodology focuses on analysing the solutions proposed by the automated tools and evaluates the fairness and/or equity of those solutions.

As it will be shown in the course of this thesis, fairness is a concept inherently relational, and fairness can be analysed from different points of view. One can analyse and compare the fairness of the air navigation service provided to different individual flights or to different airlines. An air navigation service provider can use the proposed framework to assess the fairness of different automation systems or improve an automated decision support tool with the proposed fairness optimisation process. An airline can evaluate the fairness of the trajectory based solutions obtained from different ANSPs. A regulation authority can objectively examine new automated decision support tools towards fairness when accommodating user's preferences as well as analyse the airlines' concerns complaining about ANSPs favouring local airlines.

The proposed methodology sets up the foundations for a standard to assess of fairness and equity in ATM. Thinking of future studies, the fairness and equity metrics defined here can be validated in more complex scenarios and finally, extrapolated to a standard method for the various and future ATM decision support tools specialised in different ATM problems (e.g. en-route management, flow management, or departure management). This will allow stakeholders to have a common and harmonised framework to evaluate the performance of DST against the fairness and equity requirements of SESAR and NextGen.

Regarding the safety aspects of ATM, those are not explicitly mentioned in this thesis. Important safety issues are assumed to be integrated in the considerations of automation tools responsible for the trajectory management process.

1.5 Outline of the Dissertation

Next chapter, chapter two, focuses on the definition of the concept of fairness in ATM. Together with the concept of fairness, related concepts such as justice and equity which may lead to a misunderstanding of the concept of fairness are also clarified. Starting from the research done in other disciplines on these three complex concepts, especially in Philosophy, Sociology and Economics, existing definitions are analysed and finally adapted to match the requirements of the ATM context.

Chapter three describes the lifecycle of a flight and its trajectory according to the trajectory management process defined by SESAR and NextGen. This is important as the methodologies proposed later to measure fairness and equity are based on this process.

Chapter four provides mathematical definitions of the concepts of fairness and equity in ATM proposed in chapter two. These mathematical definitions are metrics applicable in the context of trajectory based operations and based on a set of assumptions for the operating costs of airspace users. Based on a cost index-centred cost function defining the variable cost of a flight for an airline, a concrete example of a cost model is proposed. Associated to the assumptions and constraints of that model, a penalty function is determined. This penalty function and the corresponding relative penalty function are used to develop the definition of the fairness and equity metrics.

Chapter five defines the methodologies required to be able to apply those metrics in trajectory based operations according to the defined concepts of justice, fairness, and equity in ATM presented in chapter two. The two methodologies described, explain how to implement the metrics successfully for the assessment of automated tools in TBO.

Chapter six uses decision theory to analyse the robustness of the metrics and proof their consistency with the concepts presented in Chapter two. As a result, this chapter identifies the risks that could jeopardise the effective application of the metrics and the measures that can be taken to overcome those situations.

Chapter seven presents two examples where fairness and equity are evaluated based on the two methodologies described in chapter five. First, an optimisation algorithm is defined integrating the fairness metric. This algorithm can be included within any automated decision support tool which has to modify trajectories. The proposed algorithm helps to achieve, whenever possible, a higher fairness degree among the trajectory solutions resulting from a previous conflict resolution process. The fairness metric is applied to establish the fairest possible solution for a given set of modified trajectories taken into account user preferences. After that, an example is provided where three different conflict resolution algorithms are compared solving the same arrival management scenario with the same incoming traffic. Based on their different logics, each of the three algorithms modifies the trajectories of the incoming traffic to avoid potential conflicts. The solution proposed by each algorithm is evaluated and compared using the metrics of fairness and equity.

Chapter eight summarises the conclusions resulting from the work presented in this dissertation and critically reviews them; from the proposed concepts, to the metric definition, the proposed methodologies and the robustness analysis.

Chapter nine makes recommendations for future research.

The dissertation ends with two appendices; Appendix A gives an overview on the evolution of the automated decision support tools and Appendix B presents the numerical results from the decision theory analysis.

2 TOWARDS A DEFINITION OF THE CONCEPTS OF FAIRNESS AND EQUITY IN ATM

2.1 Main Air Traffic Management Stakeholders and their International Organisations

The ATM system has two main types of stakeholders: airspace users and air navigation service providers (ANSPs). Both sides have different business interests, and both sides need to define a way to cooperate in their best interest, as both sides need each other.

Regarding the airspace users, this work focuses on civil commercial airspace users, i.e. airlines carrying passengers, cargo or a combination of both.

The ANSP providing services to the civil airspace users are represented throughout the world by an organisation called Civil Air Navigation Services Organisation (CANSO) [54]. This organisation was founded in 1996 and its aim is to improve the global air navigation services on the ground and in the air worldwide [54]. CANSO also represents its member's view in major regulatory and industry forums, including at ICAO [54].

Similarly, most airlines are represented by an international industry trade group called International Air Transportation Association (IATA) [55]. IATA was founded in 1945 and represents today up to 93% of international scheduled air traffic [55].

These organisations, CANSO and IATA, lead, serve and represent the interests of the main two groups of stakeholders that can be found in the global ATM system.

To provide a safe and equitable ATM system that covers the needs of all its stakeholders, the International Civil Aviation Organisation (ICAO) was founded to oversee the global ATM system. ICAO codifies "*the principles and techniques of international air navigation and* *fosters the planning and development of international air transport to ensure safe and orderly growth*" [56].

The main task of ICAO is to establish a standardisation of air traffic management practices and ensure that an equitable ATM service is provided to all airspace users [2] by defining standards, procedures, functions and responsibilities for the ATM community to adopt. ICAO helps to preserve a safe and just ATM system for all stakeholders involved. However, a definition of a "just" ATM system does not exist. One commonly tends to associate the notion "just system", to the expression of "fair system". To move forward in the understanding of the concepts justice and fairness within the ATM system basic conceptual questions, such as the following, need to be answered:

- What characterises a just system and a fair system?
- What is the difference between a just and a fair system?
- How can justice and fairness be measured?
- How stands the concept of equity with regard to justice and fairness?

These are key questions that have motivated this work and whose answers inspire the development of the methodology and analysis detailed in chapters 4 and 5.

ICAO states in a recent document addressing the policy on charges in air navigation services: "*The crisis affecting the industry and in particular the aircraft operators since 2001 calls for significant actions by States and International Organisations, aiming at ensuring transparency, fairness, comparability and predictability of the costs of the air transport infrastructure*"[57].

In ATM, fairness is usually related to operational costs, availability of services, information use. Ensuring fairness of the ATM system is gaining more importance for the ATM community, particularly in the current economic climate.

It is not only of relevance to guarantee fairness of the ATM system but also to have a common definition of a fair ATM system, a common way to measure that fairness, to have a common basis to compare results and ensure the transparency of the ATM related processes.

2.2 Understanding the Basic Concepts

Before defining the concepts of justice, fairness, and equity specifically for ATM, it is necessary to understand the basic concepts. For that aim, a short review is provided of the definition of these concepts in other disciplines that have looked into their meaning in deep studies and discussions over the years; understanding not only the common use of these concepts in the English language but also the nuances their semantic allows.

In that sense, this section clarifies the meaning of the noun and the adjective of the three concepts at hand, namely:

- Justice and just
- Fairness and fair
- Equity and equitable

2.2.1 Justice and Just

According to Anna Wierzbicka, a recognised polish linguist at the Australian National University, in her book *English: Meaning and Culture*, the definition of justice is a *central piece of human ethics, morality and philosophy* [58]. The concept of justice is language independent and a universal human idea that can be discussed from a philosophical point of view without any special attention to the semantics of individual languages [58].

The concept and its foundations have been considered many times throughout history, from the Greeks philosophers, starting with Aristotle, until today, and from a variety of perspectives.

For instance, Aristotle first addressed the notion of justice stating that "*equals should be treated equally and unequals unequally*" [59]. According to the Cambridge dictionary of philosophy, justice is associated to the idea of "*each getting what she or he is due*" [37]. Following the Merriam Webster Dictionary, "just" is the attitude of "*acting or being in conformity with what is morally upright or an exact following of a standard of what is right*" [61]. The question however remains: how is "what is morally upright" or "a standard of what is right" defined?

According to the Stanford Encyclopaedia of Philosophy *justice is a concept of moral rightness* and, consequently, a just system is one that gives the *precise equivalent of what one has received, at an individual and universal level* [62]. This definition clarifies that justice defines what is morally right and wrong but it is still not clear how one can define justice and the meaning of justice at an individual and a universal level.

At this point, John Rawls's philosophy comes precisely at hand. John Rawls is an American philosopher that represented the leading figure in moral and political philosophy in the 20th century [37]. Given the inherent difficulty, if not impossibility, to give an exact account of moral rightness or standard of what is right, John Rawls suggests a theoretical approach by means of which a community or society can define the standards and laws of a just framework. In his two books "*A Theory of Justice*" and "*Justice as Fairness A Restatement*" Rawls focuses on exactly this question. He states that, to ensure justice among a community or society, the different stakeholders or society representatives need to start at the so called "original position" [63]. In this original position, no one is aware of her or his position or characteristics within their community or society and so is ignorant of her or his own incentives. Therefore, tendency toward selfish behaviour can be excluded.

The "original position" is the premise and the requirement to be applied to all stakeholders or representatives before agreeing upon standards and laws that will define a just framework for society. According to Rawls' philosophy, to ensure justice, the principles of justice have to be selected "*by all whom they apply under conditions preventing them from tailoring the principles to their own advantage*" [64]. By assuming this "original position" where no one knows its advantages and disadvantages, no one is aware of her or his position in society, and thus no one has an interest in defining principles or standards to benefit her or himself. This is the only way, so Rawls, to define a just framework, namely, through objective comparison and logical reasoning without self-regard, a just outcome can be achieved. The "original position" is also referred to as the veil of ignorance [65].

Michael Sandel, also an American philosopher and current professor at Harvard University, criticises John Rawl's veil of ignorance, stating that each individual is inevitable encumbered by certain ties that make it impossible to have, even hypothetically, the veil of ignorance or start at the original position [60]. John Rawl's premise for achieving a just society, so Sandel, is unrealisable, namely to become "*unencumbered individuals*" [60]. One example for such ties are the ones each individual has to its family members, which are not made by conscious choice but each individual is born with them attached [60]. Because of his philosophy, ideas, and statements Michael Sandel subscribes to the theory of communitarianism, which understands justice by the need to balance individual rights and interests with that of the community as a whole [62].

2.2.2 Fairness and Fair

The words justice and fairness are often used interchangeably because their meanings and usages are closely linked despite their distinct connotations [66]. Fairness or something fair does not have a one to one translation from English to other languages [58]. Conceptual differences exist between justice and fairness, even though there is not always a distinct word for the two concepts in all languages.

At the time of the Industrial Revolution, there was a shift in the understanding of the concept of justice focusing on relative rather than absolute morality and on mutually advantageous cooperation between individuals; reason, social cooperation, business advantage start gaining relevance. In this context, the notion of fairness emerges in relation to the raising notion of individual rights [58].

The concept of fairness and the word "fair" evolve as the result of certain situations that become more common during the Industrial Revolution. Those situations involve people trading off not only goods but also labour working hours against a salary, a place to live, namely tradeoffs in welfare between individuals. People start to develop a common thinking of the limits to how much one person is allowed to cost others to benefit her or himself [67]. This common thinking of the limit up to one person can take advantage of another person or a group of persons builds the basis for the concept of fairness.

Fairness implies achieving a balance between conflicting interests and represents a potential tension between someone's demands and detrimental consequences to others within a just framework [58]. It is inherently relational, as one's actions affect someone else [58].

With a view to better understand the concept of fairness it can be considered the so-called "ultimatum game", often used in economic thought-experiments. The standard version of this game consists of a sum of money which is to be divided between two players [66]. One player is randomly selected as the proposer and the other player as the responder. The proposer has the responsibility of proposing how to divide the sum between both players. The responder decides whether to accept the proposal or not. The rules of the game, known by both players, are the following:

- whenever the proposal is accepted by the responder the money is distributed according to the proposal among both players
- whenever the proposal is rejected by the responder both players get zero money.

If this game is played by computers, any offer will be accepted by the responder, as any percentage of the sum of money is better or at least equal to rejecting the proposal that is greater or equal zero. However, the outcome is different if the game is played by people. The most common result is the proposer offering 50% of the whole amount of money, because this is the amount expected to be accepted by the responder [66]. Studies show that the responder will most likely reject offers below 50% of the amount, considering that the proposer is acting not "fair", and cost both players the money. The different outcomes of this same game, when played by computers and people, show there is a specific human feature that is not taken into account when the ultimatum game is modelled for computers. According to studies, after [66], the inherent sense of fairness that all people have is the reason for this difference.

If the standard version of the game is modified in such a way that the proposer is selected out of a quiz, where both players compete for being the proposer, then the outcome is not quite the same. In this modified “ultimatum game”, the amount of money that is acceptable for the responder is less than 50%, as it is understood that the proposer deserves extra money for having won the quiz. Thus, it is acceptable or “fair” in this situation to let the proposer get more money than the responder [66].

Unconsciously each person has a sense for what is acceptable and what not, thus a sense of what is fair and not fair. Fair is related in meaning to just and acceptable limits for given circumstances. Within a just framework, fairness establishes a balance between divers and even conflicting interests of people. Thus, fairness is inherently relational.

As shown with the standard and the modified version of the ultimatum game, fairness does not mean that the balance between the diverse interests of people implies to be equal. Nevertheless, the concept of fairness is sometimes misunderstood as equity.

2.2.3 Equity and Equitable

According to the Merriam Webster Dictionary, equity implies equal treatment of all concerned, people or parties [61]. Equity may be a special case of fairness, as it will be shown later, but it does not have to be.

Equity has different connotations in various fields as economics, political philosophy, social contract theories; but all those definitions are very similar in the general concept, always involving same treatment of same individuals. Equity is concerned with the *proper distribution of resources, rights, duties, opportunities and obligations in society at large* [68].

Each state has a very simple and basic definition of equity implemented in their public finance system, namely taxes. People with similar incomes pay similar taxes, defined as horizontal equity [69]. People with higher or lower incomes pay higher or lower taxes accordingly, defined as vertical equity [69]. Thus, equity is, as it was stated for fairness before, inherently relational.

Equity may seem to be a simple concept, but it is, indeed, a complex idea that resists simple formulations [68]. For example, talking about gender equity the definition that comes to one's mind is equal opportunity, equal responsibilities and equal remuneration for equal work for women and men [70]. In this sense the word "equal" could perfectly be replaced by the word "same".

Within a just framework where equity is guaranteed, fairness distinguishes itself from equity by providing a refinement of the measures in place to guarantee equity. Recall the example given above with the public finance system, where people within the same range of income pay the same taxes. This is an equity measure. In case two persons have the same income but one lives alone and the other has to take care of a child, the latter person has the possibility to declare this situation and obtain a deduction from its taxes. This is a fairness measure. The public finance system usually contemplates certain types of situations that result in tax deduction. Thus, people within the same income range, may pay the same taxes depending on their special situation. Fairness takes into account, within recognised and by the society accepted situation, the individual needs while equity aims at distributing same treatment in a same way according to recognised and by the society accepted standards.

The concept of equity is related in meaning to justice and fairness [68]. To further clarify the differences between just, fair and equitable, the following example is proposed after reference [66].

As an example: consider two hungry friends, each of them wanting to buy a slice of pizza. They enter their preferred take away shop and, to their surprise, there is only one slice left. Neither wants to go to another shop so they decide to share the slice of pizza. In this case, both have decided a solution that is just. However, within this just solution, there are several ways of sharing the slice of pizza. The just solution both friends have agreed on represents a just framework. This framework has been agreed by the parties involved with no interest to tailor the framework to their own advantage.

Within the just framework, fairness and equity can co-exit. This example demonstrates the difference between fairness and equity by showing the equitable way of sharing the slice of pizza and the fair way of sharing that same slice:

- Equitable would be to divide the single slice of pizza in two identical portions, one for each of the two friends.
- Fair would be to take into account how hungry each of the two friends is and divide the slice according to each person's hunger.

The latter case relies on the honesty of the two friends. It assumes both friends have agreed a common method to measure and express hunger and both would not lie when they express their personal hunger. This manner of sharing is fair if these assumptions do not fail. If it can be proven that one of both friends was not honest and this gave him an advantage to get a bigger part of the slice, then the fairness of this sharing method is undermined.

Only if both friends had the very same hunger, then the fairest way of sharing the slice of pizza is also equitable. An equitable outcome is only the same as a fair outcome whenever all parties concerned are defined exactly the same, e.g. same circumstances, needs, preferences, etc.

2.3 Justice, Fairness and Equity in ATM

Prior to applying the basic concepts of justice, fairness and equity to ATM, the definitions provided in the previous sections are briefly summarised:

- Justice or something just is the quality of being or acting in conformity with what is morally upright by following standards of what is right. This assumes that those standards were defined or agreed by those whom they apply under conditions preventing those setting the standards from tailoring them to anyone's advantage.
- Fairness or something fair is the quality of achieving a balance between conflicting interests by means of a just procedure that takes into account the acceptance levels of all concerned and the satisfaction of the individuals. Fairness is inherently relational.

- Equity or something equitable is the quality of applying equal treatment to all concerned. Equity is also inherently relational. In case the particular circumstances of the individuals concerned are identical then, and only then, is equity a special case of fairness.

In the context of ATM, ICAO, which is assumed to be impartial according to its statutes, defines the standards of what is "right" and "wrong", what are the responsibilities of the different parties and the duties and obligations they have towards each other. These standards have to be agreed among the stakeholders of the ATM system affected by those standards and represented at ICAO. The agreed standards become then the legal basis and common measure to decide on controversies between ATM stakeholders (ANSP, Airlines, Airport, Government, etc). Thus, regarding the ATM system, justice at an international level is provided and taken care of by ICAO. As long as the ANSPs and airlines act accordingly to the standards defined by ICAO, they are acting justly. These standards were established without biased representing as close as possible Rawls "original position".

Considering that the ATM system has two active and key types of stakeholders, service providers and airspace users who receive the service, fairness and equity can be studied when comparing how the service is provided to the different users:

- When analysing a relation between one ANSP and one user, statements about whether the service is being provided in a just way or not can be made, that is within the standards defined by ICAO.
- When analysing the relation between one ANSP and several users, statements can be made regarding how fair or equitable the service was provided to those different users.
- When analysing the relation between several ANSPs and one user, statements can also be made comparing how fair or equitable those services were provided to that same user.

The airspace users can be represented by a single flight or by a set of flights from the same airline, or by different sets of flights from different airlines. The fairness and equity analysis proposed in this thesis goes from regarding the airspace user as a single flight to focusing on sets of those flights, namely one or more airlines.

Since, as explained before, fairness and equity are inherently relational, statements about fairness and equity can be made when comparing several elements within similar conditions. Related to ATM, fairness and equity can be measured for example when:

- the service provided by different ANSPs is compared,
- the service provided to different airlines is compared
- the service provided to different flights is compared.

The following chapters start the development of the fairness and equity metrics as well as the proposed analyses by considering the airspace user first in its simplest form, namely one single flight, and then adding complexity by describing the airspace user through different airlines operating several flights. Thus, it is important, in the context of this dissertation, to clarify a precise definition of a “flight”, and have a clear understanding of the flight’s preferences in terms of how they are expressed and how they are related to the airlines’ preferences.

3 THE DEVELOPMENT OF A FLIGHT IN THE TBO CONCEPT

The present chapter details the process of the creation and execution of a flight. The parties involved are airlines, ANSPs and other ATM organisations. This process starts with the airline's idea and finishes with the actual performance of the flight. At the end of the chapter, the elements and steps of the process having implications on justice, fairness, and equity are analysed.

The information contained in this chapter is based on interviews held with Air Services Australia, Emirates, QANTAS, and Virgin Blue within a Boeing Research &Technology Europe's internal project.

3.1 From the Idea to the Schedule

Commercial airlines design and implement their operations according to their respective business strategies. Within an airline, the so-called airline operation centre (AOC) makes the relevant decisions about flight planning taking into consideration airline preferences and cost structure to maximise the airline's profits [71].

At the very start of the process of developing a flight, the AOC identifies a route between two cities or regions that may be of interest for the airline's business if incorporated into their business portfolio, i.e. the idea.

To know if this flight idea matches requirements according to the airline's business strategy, a viability study is carried out by the airline. Flight details such as schedule, aircraft size and type, required crew, needed ground support and infrastructure in place, and other operating parameters are pre-defined and evaluated [72]. Outcomes of the viability study are, among other things, the exact airport pair which is going to be connected by the flight and the associated expected profitability for the airline.

After the expected profitability is deemed acceptable for the airline, the negotiation of the required resources starts. Depending on the operation parameters, approvals must be obtained from the government and/or from the airport regulator and corresponding ANSPs. As an example, consider an airline intending to open a route into a congested area such as London. Due to the restrictions concerning fuel emissions and noise in London airports, airlines require a government approval for opening a new route from or to London. The airline also needs to negotiate and get the approval from the airport regulator for ground support and slot times, especially for airports that are close to their maximum capacity. This process involving the negotiation being carried out by the airline, ANSP, airport regulator and government (if required) is called Collaborative Decision Making (CDM) in the TBO concept as described by SESAR [11], which is a main part of the TBO concept.

Once the required approvals and permits are obtained, the next step is to identify route options and specify a route choice according to the airline's cost structure or, in other words, airline's preferences. During route negotiation, the ANSP considers airline's preferences but its main objective during this negotiation is to ensure safety. When the route has been agreed between the airline defending its business strategy and the ANSP ensuring safety and availability of all required air navigation services, the airline publishes the new flight schedule, usually months ahead of the first flight execution.

Nevertheless, the specific route and its restrictions (altitude, speed) may not be fixed until the day of the flight, and they may be flexible even during the flight; as for example the flex tracks in Australia which are defined according to the latest wind prediction [73] or the free route concept being implemented in Portugal [74].

3.2 From the Schedule to the Flight Plan

To understand the process from establishing a preliminary schedule until it becomes an actual flight plan[10], a typical and representative example is explained.

As the day of the operation approaches, the AOC refines the schedule for the flight. Within the negotiated schedule and route assignment, the AOC specifies certain parameters concerning the flight, such as crew members, pilots, route specific constraints and preliminary fuel calculations. One week before the estimated time of departure (ETD), the AOC has a fairly precise schedule for the flight. Usually around three days before ETD, the schedule is introduced in the flight planning system. The flight planning system is a system most airlines use to introduce the required data to produce a flight plan that is interactive and can be shared with the ANSP, with the aircraft's FMS, and the pilots. Once the needed data is in the flight planning system, the airline has for the first time visibility of the updated flight plan.

The airline's flight plan includes the flight plan submitted to the ANSP usually according to ICAO standard [75], ETOPS (Extended Twin engine Operations) route plan, weather data, NOTAMs[11] (NOTices to AirMen) [75] and fuel load. Although the flight plan is not intended to be modified after being provided to the ANSP, updates cannot be excluded due to changing circumstances until the flight start. For example, pilots can introduce last minute modifications to the flight plan, particularly concerning fuel load. These changes need to be communicated to the rest of the system or users handling the same flight plan.

The updated details of the flight are entered directly by the pilot into the aircraft Flight Management System (FMS) which, under consideration of airline specific operating parameters (e.g. cost index CI), calculates a rate of climb and, in turn, a top of climb

[10] "*Specified information provided to air traffic services units, relative to an intended flight or portion of a flight of an aircraft*"[75].

[11] "*A notice distributed by means of telecommunication containing information concerning the establishment, condition or change in any aeronautical facility, service, procedure or hazard, the timely knowledge of which is essential to personnel concerned with flight operations*" [75].

according to the latest fuel load data. Based on this updated top of climb, the airline refines the flight trajectory and, consequently, the flight plan.

The ANSP receives the updated flight plan (usually in the standard ICAO format) for approval. If any incoherence is identified by the ANSP, e.g. comparing the flight plan agreed before, changes needed due to traffic congestion or arise safety issues that may affect the flight, amendments are proposed to the airline. In Europe for example, it is the Central Flow Management Unit[12] (CFMU) at EUROCONTROL receiving the flight plans and distributing them to the ANSPs that are going to be involved in the management of the flight [76].

The ANSP calculates the first predicted trajectory for that flight with the updated flight plan just received. That prediction for the aircraft trajectory is based on the updated flight plan, the ETD, TAS (True Air Speed) and the forecasted wind (among others: speed, direction, pressure and temperature) for the different flight levels [77].

At this stage, there are three planned trajectories in the system: the one planned by the AOC, the one computed by the ANSP (ground-based automation system), and the other computed by the FMS (airborne automation system) of the aircraft. Figure 1 shows the process described in this and the previous section; the development from the idea to the flight plan.

[12] Operational unit at EUROCONTROL with the mission: "*to enhance safety through coordinated management of the air traffic in Europe and to ensure congestion in the air does not occur and that available capacity is used effectively*" [76].

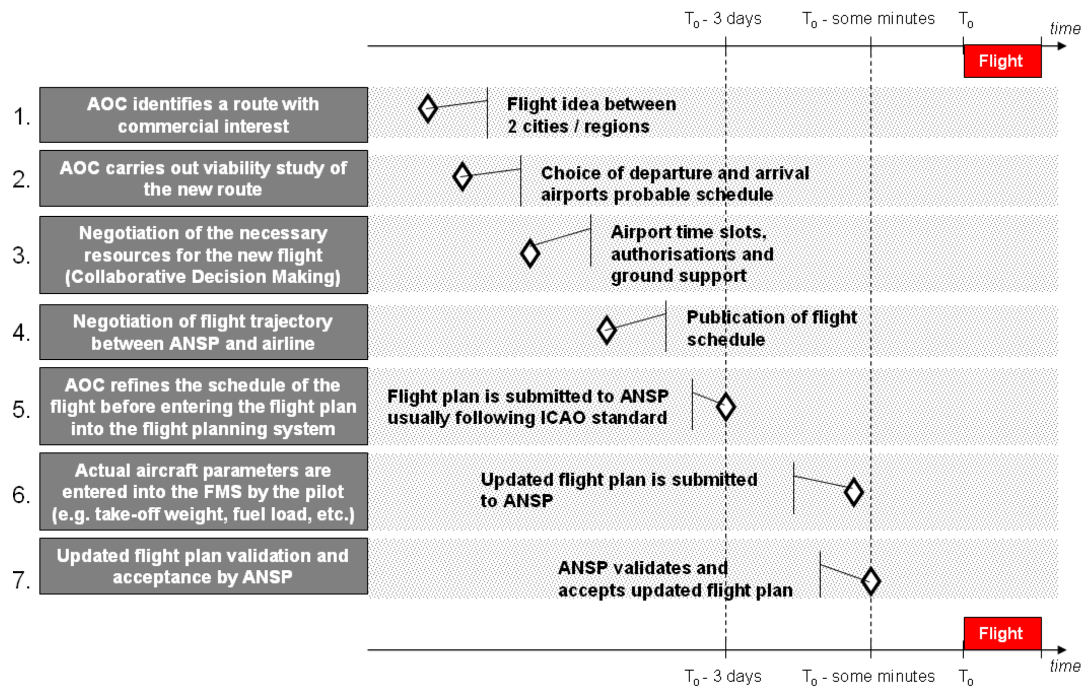


Figure 1 Flight development from the idea to the flight plan

3.3 Flight Plan Negotiation after Take-Off

After negotiations between the AOC and the ANSP to try to accommodate airline preferences when determining the schedule and defining the flight plan, it seems natural to continue doing the same during the execution of the flight.

Any modification to the initially planned trajectory at take-off may substantially alter the incurred costs of a given flight. Thus, the negotiation of modifications to the trajectory during the flight execution is extremely important. This way, the airline preferences can be accommodated and the preferred costs maintained for a given flight.

However, negotiation of trajectory modifications during flight does not effectively take place in conventional ATM. In fact, pilots perform the flight as close as possible to the FMS planned trajectory considering instructions and restrictions imposed by air traffic controllers. The only exchanges possible between pilot and air traffic controller take place via radio communication. It cannot be assumed that a negotiation between ANSP and airline about trajectory modifications can actually be performed by radio. [78]

To be able to negotiate based on trajectory information, there is a need to define common and standard formats to describe that trajectory information. This is one of the main objectives of the TBO concept as proposed by SESAR and NextGen, namely to facilitate the negotiation between the AOC and the ANSP during the whole flight by exchanging trajectory information and improving the strategic operations through increased predictability and better automation decision support tools.

3.4 Identifying Justice and Fairness in the Development and Execution of a Flight

According to the definition of justice provided in Chapter 2, justice is given when standards are followed assuming that those standards were defined under conditions preventing the parts involved to define them for their own advantage. Justice in the development and execution of a flight in the ATM system is given when the ANSPs and AOCs share information following the defined standards for the negotiation process as well as for how to express the trajectory related information, handle this information as it was agreed in the negotiation process, and facilitate the negotiation of trajectory modifications before and during the flight execution. These standards still need to be defined. SESAR and NextGen are dealing with this issue within the definition of the TBO operational concept [12].

Regarding fairness in the development and execution of a flight, it is important to recall the definition of fairness also provided in Chapter 2. Fairness is defined as the quality with which the balance between conflicting interests is achieved by means of a just procedure. Within this process of achieving the balance the acceptance levels of all concerned as well as the satisfaction of the individuals has to be taken into account. Thus, fairness is inherently relational.

A statement about fairness in the development and execution of a flight can be made in relation to the accommodation of the airline’s preferences. For the airlines, the most important objective during flight execution is to minimise their costs which are defined by their specific cost function. Any modification issued to the flight by the ANSP results into changes to the trajectory and, as the last instance, those changes have repercussions on the incurred costs of the airline.

Whenever the honesty of the airspace users communicating their preferences cannot be guaranteed, then the concept of fairness may be jeopardised. In those cases, equity can always be measured given a just framework is provided. In the development and execution process of a flight, equity assesses whether the modifications required to execute a flight compared to the rest of the flights where done in accordance with the equity concept.

Fairness and equity can be evaluated by comparing different flights and how they were treated by the ANSP together with the resulting impact on their corresponding airline's costs. Fairness and equity relate in this context to the cost impact of different modifications imposed to the whole trajectory or just a segment of it. The difference between the two concepts relies in fairness assessing how good the airline's preferences were accommodated, while equity evaluates whether the required modifications (and their cost implications) were distributed equitably. This can be done for the different steps during the negotiation process for flight development as well as for flight execution.

4 DEVELOPING THE METRICS

This chapter defines a cost model framework upon which the fairness and equity metrics are based. Associated to the cost model, a cost penalty is defined. Accordingly, the penalty function and relative penalty function are developed and specific examples for these two functions are presented. Those two functions play a fundamental role in the fairness and equity metric definition.

The cost model, the associated cost penalty as well as the proposed penalty and relative penalty functions should help as an example to illustrate the proposed metrics for fairness and equity in Trajectory Based Operations (TBO) and to obtain meaningful results that substantiate those metrics.

The fairness and equity metrics are independent of the specific form of the costs, the penalty or the relative penalty function which have to be compliant with the assumptions and conditions for such functions stated in this chapter. Both metrics capture the definition of fairness and equity in ATM as defined in Chapter 2.

4.1 The Flight Costs

When planning a flight for the first time, the airline operation centre (AOC) has to consider several factors for defining a whole trajectory that suits the business strategy of the airline. Thus, the airline needs to consider all factors that affect the costs of that flight, factors such as “fuel and personnel costs, alternate airports, route charges, traffic constraints, day of the year, infrastructure availability (e.g. NOTAMS), weather including warning for SIGnificant METeorological information (SIGMET)” [79] and implications with other flights of the same airline’s network, such as connecting flights.

The cost functions used by AOCs are usually complex and airline dependant. Those cost functions are not public, at least not in detail. Thus, a generic, simplified cost function is described here in line with the examples than can be found in the literature [80][81]. By means of this cost function, it can be understood the importance of defining the costs when deciding the trajectory, whether the preferred one or amendments, of a specific flight. The role the cost function plays is relevant nowadays as well as for the future TBO scenarios as proposed by SESAR and NextGen.

The cost function, also in its generic and simplified expression, has flight specific coefficients that are defined by each airline according to its strategy. To explain the meaning of these flight specific coefficients, the cost index is introduced. The role it plays in defining the trajectory of the flight is also explained.

Based on the relevance of the costs when defining a flight and its preferred trajectory according to the airline's strategy, a cost model framework is proposed. The model is based on the cost function described in the next subsection and captures the assumptions associated to the cost penalty (section 4.2). This model leads to the definition of the penalty function, which represents the cost penalty level incurred when deviating from the airline's defined costs for a specific flight. The penalty function is the key for defining the metric of fairness and the metric of equity, which is the main objective of this chapter.

4.1.1 The Cost Function

The total flight costs are reflected in the cost function. Each cost function has fixed costs and variable costs [79].

$$C = C_{fixed} + C_{variable} \tag{4.1}$$

The fixed costs C_{fixed} comprehend for example the insurance costs, the personal equipment costs, the costs for uniforms, and crew training's costs [79] and is by definition independent of the way the trajectory is flown and of the flight duration.

The variable costs $C_{variable}$ are a function of the flight time related costs and of the costs associated to the fuel consumption for a given flight [79]:

$$C_{variable} = C_T \cdot \Delta T + C_F \cdot \Delta F \qquad (4.2)$$

where C_T and C_F are flight specific coefficients integrated in the cost function for each flight according to the airline’s strategy. C_T defines the time-related cost per minute of flight and C_F defines the cost of fuel per kg.

The aim of the AOC is to minimise the total flight costs. In order to do so, the variable costs need to be minimised.

The variable costs are basically split into time-related costs and the fuel-related costs. Time-related costs are the costs associated to the maintenance of the aircraft, the costs of ownership or leasing of an aircraft, the crew costs per hour but also the repercussion of passenger satisfaction, the repercussion of missing connections and compensations, etc [79].

Fuel-related costs depend on the actual fuel price and the amount of fuel consumed. Fuel price varies from area to area and over time according to different factors such as national taxation rates, oil market price and airport servicing fees, among others [79]. How much fuel is consumed is associated to how the aircraft is flown and thus, to the trajectory. The fuel consumed depends on the aircraft type, the speed, altitude, descent and climb profiles, and also on the weather conditions during the flight, specially the winds.

As explained before, each flight has associated a specific cost function that has been defined according to the airline’s strategy and can be referred to as the user-defined cost function, being the airline the user.

The user-defined cost function is the one that defines the preferred total flight costs, corresponding to its airline’s cost strategy. The trajectory that results when considering all the factors in order to maintain those flight costs obtained from the user-defined cost function is the user preferred trajectory (UPT).

4.1.2 The Cost Index

As mentioned before, the total flight costs are composed of fixed and variable costs. The variable costs are in turn split into time-related and fuel-related costs. The airline's aim is to minimise the variable costs in order to minimise its total flight costs. To minimise the variable costs, the airline can essentially act upon two variables, which are not independent of each other:

- the flight duration, which would reduce time-related costs
- the fuel consumption, which would reduce fuel-related costs.

These two variables are closely interrelated. The cost index captures how the airline weighs the relative importance of reducing the time-related costs and the fuel-related costs for a specific flight. Thus, the ratio between C_T (for time-related costs) and C_F (for fuel-related costs) for a specific flight defines the cost index CI for that flight:

$$CI = \frac{C_T}{C_F} \qquad (4.3)$$

During flight execution, the Flight Management Systems (FMS) calculates the most efficient trajectory, namely the one that minimises the costs defined by the CI input by the pilot or the AOC. Typically, the FMS bases its cost calculations on the cost function detailed in the previous subsection 4.1.1.

The higher the cost index value, the more important are the time-related costs. There are two extreme cases for the cost index as interpreted by the FMS [80][81]:

- when the cost index is zero, which represents the minimum fuel consumption for maximum range;
- when the cost index is maximum, which represents the minimum flight duration for maximum speed (scales vary from FMS vendor to vendor, from 0-99 (min/kg) or 0-999 (h/lb) [80])

As an example, imagine two airlines, airline A and airline B, with different business strategies for the same aircraft type covering the same route between the same two city pair, take for example from Madrid to Marseille.

Airline A has built on its image on punctuality and gain the reliance of its clients. In order to maintain its status, it is extremely important for airline A to arrive on-time at its destination. When optimising the variable costs, airline A weighs the time-related costs relatively higher than the fuel-related costs.

Imagine now airline B whose image is based on selling cheap flight tickets. For this airline, the importance relies on maintaining its capacity of selling cheap flight tickets by constraining the operational trip cost. For that aim, the fuel-related costs are relatively more important. When optimising the variable costs, airline B weighs the fuel-related costs as more relevant than the time-related costs. Thus, the cost index of airline A has a higher value than the cost index of airline B.

4.1.3 Maintaining Flight Costs in a Trajectory Based Operation Environment

In a trajectory based operation environment, trajectory-related information will become the main piece of information being shared. This trajectory-related information has to contain an accurate description of how a specific trajectory is intended to be flown, specially regarding user preferences.

Within the TBO concept, the airline will have to provide the ANSP with its preferred trajectory for a given flight prior to that flight flying the assigned trajectory [11].

The preferred trajectory describes implicitly the preferred costs by defining the preferred flight duration and fuel consumption.

The preferred cost can only be achieved if the preferred trajectory is flown. This would be the ideal situation. Usually, the trajectory has to be modified due to unpredictable weather conditions, unexpected traffic congestions or due to any other modifications required by the ANSP to ensure the safety of the operations.

The deviation from the preferred trajectory is almost inevitable, but the ANSP can try to accommodate the airline's preferences regarding the preferred flight costs. This can be done by taking into account how the airline weighs against each other time-related and fuel-related costs to adjust the modified trajectory so that it matches as close as possible the preferred flight costs.

4.1.4 Two Possible Analysis Cases

Flight costs deviations from the user preferred costs can be analysed in two different cases:

- once the decisions have been made, the trajectories modified and flown (with *a posteriori* data),
- or during the modification process of the trajectories as a supporting process to decide which is the best possible modification according to the user preferences (with *a priori* data).

Further details on these two analysis cases are explained in Chapter 5. This subsection provides a brief description for the reader to have an indication of how the cost deviation analysis could be done.

In case *a posteriori* data is used for the analysis, this data refers to trajectories that have been flown. The cost information refers to the real incurred costs of the given flights following their final trajectories that resulted after all required modifications. The preferred trajectories and the associated preferred costs are both based on predictions. In this case, the analysis could be made by comparing the preferred costs based on the predicted preferred trajectories and the costs incurred based on the finally flown trajectories. For that aim, it has to be assumed that all trajectories handled within this analysis have been predicted with tools that provide if not the same at least similar accuracy. Otherwise the conclusions from such a comparative analysis are not significant.

In case *a priori* data is used, all the statements are based on predicted trajectories. It has to be assumed that those trajectories are predicted by systems that guarantee the same or similar predictability and accuracy. Different possible trajectory modifications, i.e. what-if scenarios, can be compared to the preferred trajectories and analyse the costs incurred in each of those modifications. The additional costs incurred with each modification can be compared to the airline's preferred flight costs and, this way, establish which of the possibilities for the modified trajectories better accommodates the airline's preferences regarding the variable flight costs.

In each of the analysis cases, the deviation from the preferred trajectories and consequently the increase in the incurred costs are reflected in two main variables, relevant for the variable costs, namely the difference in the flight duration ΔT and the difference in the fuel consumption ΔF. These two differences are determined by comparing the preferred values for flight duration T_P and fuel consumption F_P against the values resulting from the modified trajectory for flight duration T_M and fuel consumption F_M.

$$\Delta T = T_M - T_P \tag{4.4}$$

$$\Delta F = F_M - F_P \tag{4.5}$$

4.2 The Proposed Cost Model and the Cost Penalty

The cost model used for the development of the metrics for fairness and equity is based on following assumptions:

1) The additional costs incurred by a flight are defined as the deviation from the airline's preferred variable costs for that flight. This is reflected in the difference between the modified (T_M and F_M) and the preferred values (T_P and F_P) for flight duration and fuel consumption. Thus, as these differences increase (equations 4.4 and 4.5) the costs incurred also increase (equation 4.2).

2) The airline does not incur in extra costs when the flight duration is shorter than the preferred duration or the fuel consumption is less than the preferred value or both.

This assumption has been made to simplify the proposed cost model and is based on a similar assumption made in [50] also to describe the extra incurred flight costs.

3) Each flight according to the airline's strategy has maximum acceptable costs which are correlated to specific values for the increase in flight duration and fuel consumption. The maximum acceptable costs are defined by a pair of reference values of flight duration and fuel consumption, T_{REF} and F_{REF} respectively. The acceptable delay is defined as the difference between T_{REF} and the preferred flight duration T_P. Similarly, the maximum tolerable increase in fuel consumption is defined as the difference between F_{REF} and the preferred fuel consumption F_P.

$$\begin{aligned} \Delta T_{REF} &= T_{REF} - T_P \\ \Delta F_{REF} &= F_{REF} - F_P \end{aligned} \tag{4.6}$$

The maximum tolerable increases in flight duration and fuel consumption (ΔT_{REF} and ΔF_{REF}) represent the acceptance limits of an airline for a specific flight. Because fairness has to take into account the acceptance limits and the satisfaction of the stakeholders concerned (according to the definition provided in Chapter 2), the maximum acceptable values determined by the airline are relevant to the metric.

The reference values T_{REF} and F_{REF} do not constrain T_M and F_M, which can trespass them, but they represent a pair of maximum acceptable limits as defined by the airline. Those maximum acceptable limits indicate the airline's maximum acceptable extra costs for a specific flight.

In an ideal situation, the airline provides this pair of maximum tolerable values T_{REF} and F_{REF} according to its strategy, the defined flight costs and their preferred trajectory. Nevertheless, with a view to avoid airline's bias, these reference values could also be provided by a neutral party; the reference values could be defined for example by SESAR, NextGen or Eurocontrol's Performance Review Commission.

4) The additional costs incurred cannot increase infinitely. The maximum additional costs are directly related to the feasible performance of the aircraft; i.e. neither T_M nor F_M can increase infinitely due to the limited quantity of fuel in the aircraft.

The cost penalty results from the deviation of the preferred flight costs. In contrast to the additional costs, the penalty cost has a maximum value that cannot be trespassed and is defined by the maximum tolerable values T_{REF} and F_{REF}.

In other words, the cost penalty fulfils assumptions 1 and 2, while the maximum tolerable costs described in assumption 3 define the maximum value of the cost penalty.

4.3 The Penalty Function

There is a need to define a function to characterise the cost penalty associated to the additional cost incurred when a flight is deviated from the preferred trajectory. That function has to comply with the proposed cost model, the associated cost penalty and the four assumptions described above. The function serves as the basis to measure fairness and equity of a process, system or method handling and modifying trajectory related information.

Let that function be called the penalty function, which is a function of the resulting values from the modified trajectory for flight duration T_M and fuel consumption F_M and the preferred values for flight duration T_P and fuel consumption F_P.

$$\mathrm{P} = f\left(T_M, F_M, T_P, F_P\right) \tag{4.7}$$

The values of T_M, F_M, T_P, and F_P have to fulfil the following properties:

A) T_P is the preferred flight duration for a given flight which can be either declared directly by the airline or deduced from the preferred trajectory for that flight. T_P is always a real, positive value, $T_P \in \Re^+$

B) F_P is the preferred fuel consumption for a given flight which can be either declared by the airline or deduced from the preferred trajectory for that flight F_P is always a real, positive value, $F_P \in \Re^+$

C) T_M is the flight duration of a given flight based on the modification(s) made to the preferred trajectory. The value of T_M can be obtained from actual trajectory data, after it has been flown, or from predictions made to, for example, create what-if scenarios required to plan the trajectory modification(s) before it is flown. T_M is always a real, positive value, $T_M \in \Re^+$

D) F_M is the fuel consumption of a given flight based on the modification(s) made to the preferred trajectory. The value of F_M can be obtained from the actual trajectory data, after it has been flown, or from predictions made to, for example, create what-if scenarios required to plan the trajectory modification(s) before it is flown. F_M is always a real, positive value, $F_M \in \Re^+$

E) T_{REF} defines (together with F_{REF} according to the cost penalty definition) the maximum cost penalty for a given flight. The values for T_{REF} are always positive and equal or greater than the values for T_P: $T_{REF} \geq T_P$

F) F_{REF} defines (together with T_{REF} according to the cost penalty definition) the maximum cost penalty for a given flight. The values for F_{REF} are always positive and equal or greater than the values for F_P: $F_{REF} \geq F_P$

As a result of the above properties of T_M, F_M, T_{REF}, F_{REF}, T_P, and F_P, and the assumptions of the cost model and the associated cost penalty, the penalty function has the following constraints:

i) No cost penalty is incurred when T_M is equal or less than T_P and F_M is equal or less than F_P. In such cases, the penalty function is equal zero (according to assumption 2).

$$P(T_M, F_M, T_P, F_P) = 0\Big|_{T_M \leq T_P, F_M \leq F_P} \tag{4.8}$$

ii) The penalty function adopts a strictly positive value for any value of F_M when T_M is greater than T_P (as a consequence of assumption 1)

$$P(T_M, F_M, T_P, F_P) > 0\Big|_{T_M > T_p} \tag{4.9}$$

iii) The penalty function adopts a strictly positive value for any value of T_M when F_M is greater than F_P (as a consequence of assumption 1)

$$P(T_M, F_M, T_P, F_P) > 0\Big|_{F_M > F_P} \tag{4.10}$$

iv) The cost penalty has an upper limit. The penalty function is saturated when that upper limit is reached. This limit is referred to as P_{SAT}.

v) The penalty function reaches P_{SAT} when T_M is equal T_{REF} and when F_M is equal F_{REF}:

$$P_{SAT} = P(T_M = T_{REF}, F_M = F_{REF}, T_P, F_P) \tag{4.11}$$

T_{REF} and F_{REF} are in turn used to define the maximum acceptable extra costs. P_{SAT} could be equal zero for the case where T_{REF}=T_P and F_{REF}=F_P. To avoid that case, either T_{REF} or F_{REF} (or both) must be strictly greater than T_P and F_P, respectively. This means, that at least one of the two following inequalities must be true:

$$T_{REF} > T_P \tag{4.12}$$

$$F_{REF} > F_P \tag{4.13}$$

The penalty function is also saturated for certain combinations of values for T_M and F_M. There are two special cases, where P_{SAT} is reached even if one of the modified values, T_M or F_M, is maintained equal to the preferred values, T_P or F_P respectively.

vi) For F_M equal F_P, P_{SAT} is reached for a value of T_M, denoted as T_{SAT}, which is implicitly defined as:

$$P(T_M = T_{SAT}, F_M = F_P, T_P, F_P) = P_{SAT} \tag{4.14}$$

Corollary to v) and vi):

$$T_{SAT} \geq T_{REF} \tag{4.15}$$

vii) For T_M equal T_P, P_{SAT} is reached for a value F_M, denoted as F_{SAT}, which is implicitly defined as:

$$P(T_M = T_P, F_M = F_{SAT}, T_P, F_P) = P_{SAT} \tag{4.16}$$

Corollary to v) and vii):

$$F_{SAT} \geq F_{REF} \tag{4.17}$$

As requested by assumption 1) of the proposed cost model, any increment of T_M and F_M with respect to T_P and F_P represents additional costs incurred in flying the modified trajectory. According to the cost model any additional costs represent a cost penalty to the airline, although the maximum value for the additional costs and the cost penalty may differ from each other. As a consequence of the cost penalty definition, the highest cost penalty value for the penalty function is P_{SAT}.

viii) The penalty function is a monotonically increasing function, which is comprehended between the values zero and P_{SAT}.

$$P \in [0, \ldots, P_{SAT}] \tag{4.18}$$

Any function P can be used as penalty function as long as the set of variable properties described in A) to F) and the set of mathematical requirements in i) to viii) are respected.

This general definition of the penalty function has taken into account the conclusions of the Performance Review Report issues by EUROCONTROL for the Calendar Year 2009 [83] evaluating the performance of the ATM system. In particular, chapter 7 of this report, *Environment*, states explicitly that there is a need to develop an important Air Navigation Service (ANS) related performance indicator, namely a fuel efficiency indicator, "*based on comparison of actual fuel burn with the fuel burn needed for the user-preferred trajectory*".

The penalty function includes in its properties a fuel efficiency indicator as well as a delay efficiency indicator (see equations 4.04 and 4.05). Those are integrated in the variable costs definition upon which the cost model is based, which in turn defines the assumptions for the penalty function definition.

4.3.1 The Proposed Penalty Function

A specific penalty function is proposed to serve as an example and illustrate the further development of the metrics of fairness and equity. The penalty function can be defined as the function depicting the relationship between the additional time-related costs and the additional fuel-related costs incurred with reference to the preferred time-related and fuel-related costs.

The proposed penalty function is defined as follows:

$$P = \begin{cases} \sqrt{C_T^2\left(T_M - T_P\right)^2 + C_F^2\left(F_M - F_P\right)^2}\Big|_{T_P \le T_M \le T_{REF};F_P \le F_M \le F_{REF}} \\ 0\Big|_{T_M < T_P;F_M < F_P} \end{cases} \tag{4.19}$$

where C_T and C_F are flight specific coefficients with fixed values defined by the airline for each flight according to the airline's strategy, as explained earlier in Section 4.1.1. This example of a penalty function complies with the assumptions, variable properties, and the constraints defined for the generic penalty function:

- According to i), for $T_M \le T_P$ and $F_M \le F_P$ then P=0.
- Condition v) leads to the following result:

$$P_{SAT} = \sqrt{C_T^2\left(T_{REF} - T_P\right)^2 + C_F^2\left(F_{REF} - F_P\right)^2} \tag{4.20}$$

- As a consequence of conditions vi) and vii):

$$\begin{aligned} T_{SAT} &= T_P + \frac{P_{SAT}}{C_T} \\ F_{SAT} &= F_P + \frac{P_{SAT}}{C_F} \end{aligned} \tag{4.21}$$

- Furthermore, it shall be contemplated a possible set of values T_M and F_M for which

$$\sqrt{C_T^2\left(T_M - T_P\right)^2 + C_F^2\left(F_M - F_P\right)^2} \ge P_{SAT} \tag{4.22}$$

  In such a case, and respecting the cost model and the cost penalty definition, P shall be limited to P_{SAT}. Following Figure 2 depicts those limits. The abnormal configuration of the axes has been chosen to show the analogy between the penalty values and the indifference curve used microeconomic theory. This will be explained later in this same section.

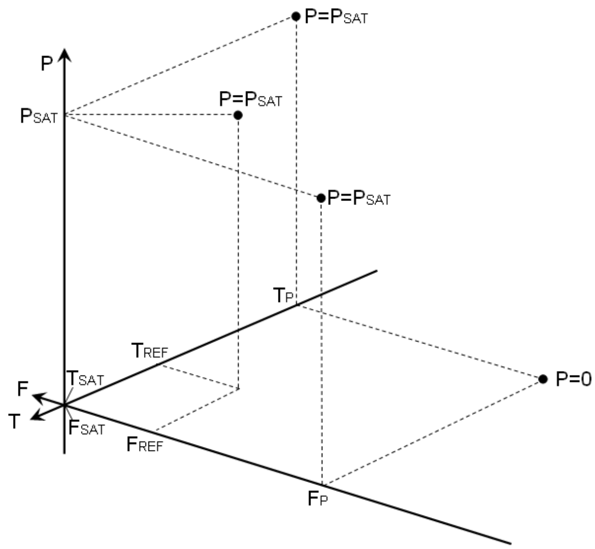


Figure 2 Penalty Value Limitations

The following table summarises the proposed penalty function values for different intervals of T_M and F_M:

Table 1 Values for the proposed penalty function

$P(T_M, F_M, T_P, F_P)$		F_M		
		$F_M \le F_P$	$F_P < F_M < F_{SAT}$	$F_M \ge F_{SAT}$
T_M	$T_M \le T_P$	0	$C_F \sqrt{(F_M - F_P)^2}$	P_{SAT}
	$T_P < T_M < T_{SAT}$	$C_T \sqrt{(T_M - T_P)^2}$	$\min\left\{\begin{matrix}\sqrt{C_T^2(T_M - T_P)^2 + C_F^2(F_M - F_P)^2} \\ P_{SAT}\end{matrix}\right\}$	P_{SAT}
	$T_M \ge T_{SAT}$	P_{SAT}	P_{SAT}	P_{SAT}

The following Figures 3 and 4 show a graphical representation of the proposed penalty function in 3D and 2D.

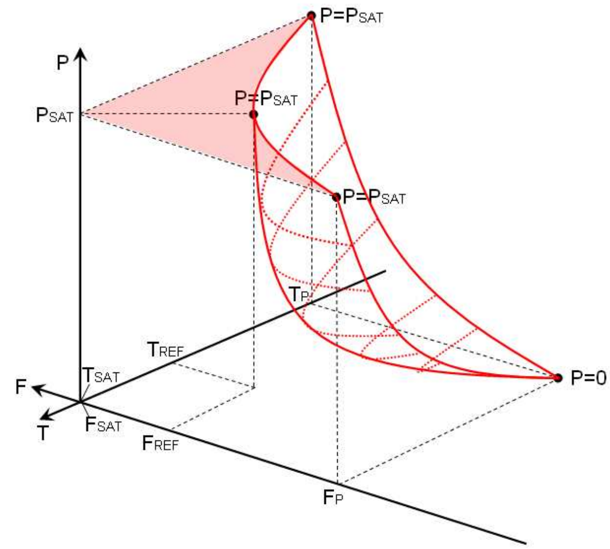


Figure 3 Proposed Penalty Function in 3D

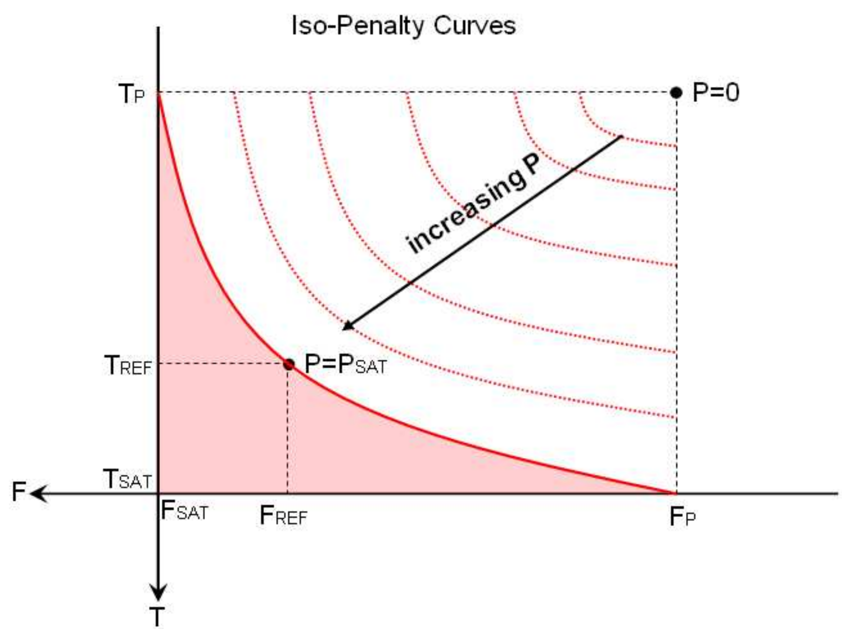


Figure 4 Proposed Penalty Function in 2D

The Iso-Penalty Curves are comparable to indifference curves in microeconomic theory [82]. An Iso-Penalty Curve corresponds to the set of pairs T_M and F_M for which the cost penalty takes the same value. Thus, the airline has no preference for any T_M, F_M pair on the same curve, as they all result in the same penalty cost.

Recall the example with airline A and B (Section 4.1.2). For both airlines, their strategy when minimising the variable costs and defining their preferred trajectory is completely different for the same aircraft type covering the same route.

The preferred flight duration and fuel consumption values will most probably be different due to the different strategies, although the maximum acceptable values for delay and extra fuel consumption T_{REF} and F_{REF} may be the same if those values are obtained according to the standards defined by a neutral party as SESAR.

Different airlines may prefer different strategies to accommodating the additional flight costs. In the example discussed in section 4.1.2, airline A weighs the time-related costs as more important than the fuel-related costs while for airline B, it is the other way round. Thus, if it needs to be penalised with extra costs, airline A prefers to incur in extra costs in fuel consumption while minimising the delay. On the other hand, airline B prefers to accept an increase in delay to minimise the fuel-related costs. The saturation value P_{SAT} of the corresponding penalty function for each flight varies depending on the selected cost index.

Assume that airline A selects CI=0,8 and airline B selects CI=0,4. According to the defined cost index, airline A will reach the saturation value of the penalty function P_{SAT} for a smaller increase of the flight duration than airline B, but tolerate a greater increase of the fuel consumption before reaching P_{SAT}. For airline B, it is the other way round. Airline B will reach saturation of the penalty function for a smaller increase in the fuel consumption, but it tolerates a greater increase in the flight duration than airline A before reaching P_{SAT}.

If ANSPs take into account the weight the airline gives to the time-related and fuel-related costs, the incurred costs could be maintained closer to the preferred costs. That is one important reason for including the cost index as part of the function describing the cost penalty.

A flight is executed in a specific period of time for which the fuel price can be assumed to be constant.

$$C_F\text{=const.}$$

Taken into account the definition for the cost index:

$$CI = \frac{C_T}{C_F} \text{ with } C_F\text{=const >0}$$

$$C_T = CI \cdot C_F \tag{4.23}$$

The penalty function can also be expressed as:

$$P = \sqrt{C_T^2 (T_M - T_P)^2 + C_F^2 (F_M - F_P)^2} = C_F \sqrt{CI^2 (T_M - T_P)^2 + (F_M - F_P)^2} \tag{4.24}$$

Figures 5 and 6 depict this example according to the proposed specific penalty function.

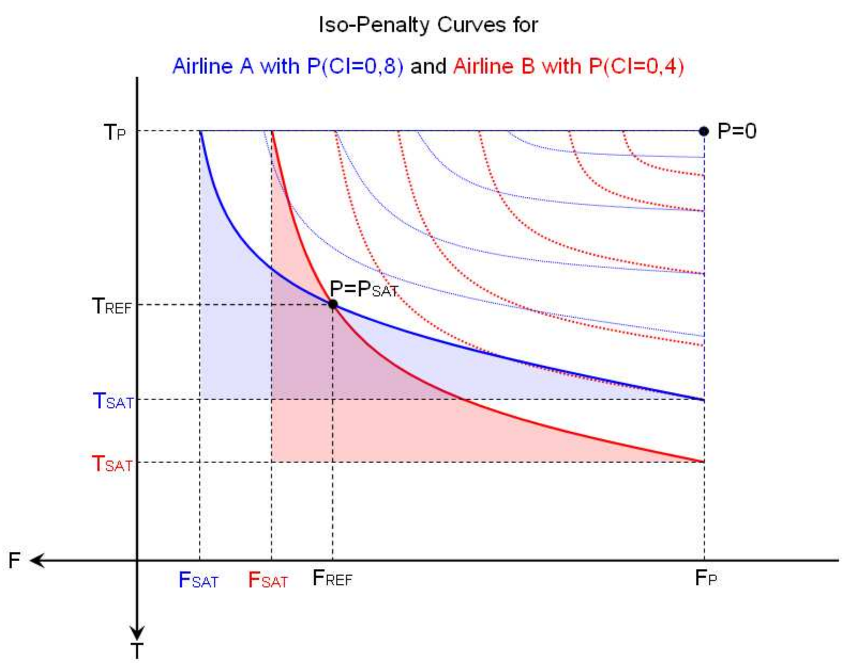


Figure 5 Iso-Penalty Curves for two flights with different Cost Index operating the same aircraft on the same route

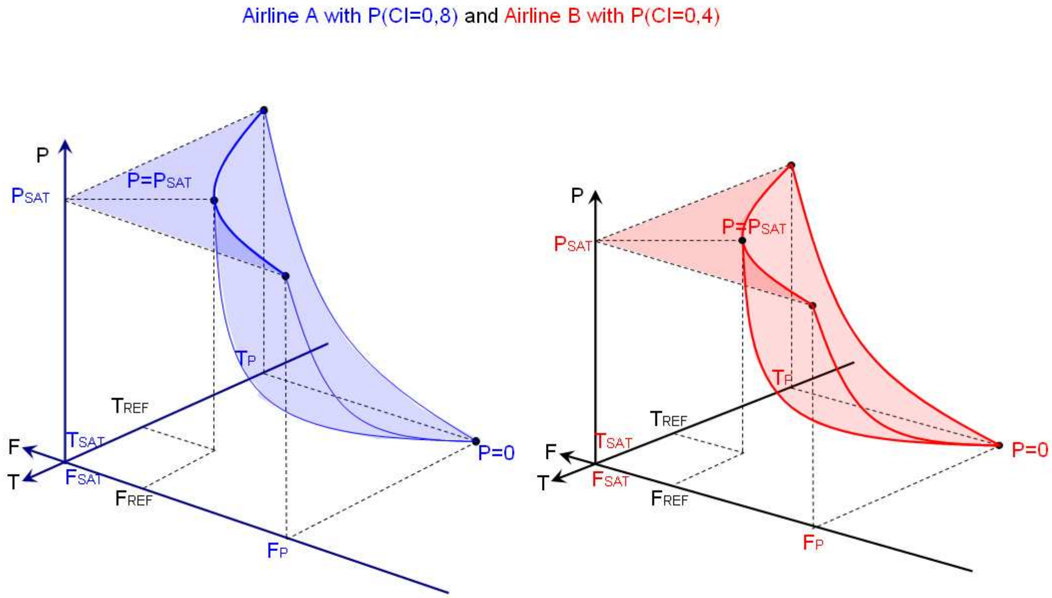


Figure 6 Penalty Function for two flights with different Cost Index operating the same aircraft on the same route

The proposed unit dimensions for the variables are:

$$P, P_{SAT} \text{ [€]}$$

$$T_M, T_P, T_{REF}, T_{SAT} \text{ [min]}$$

$$F_M, F_P, F_{REF}, F_{SAT} \text{ [kg]}$$

$$CI \text{ [kg/min]}$$

$$C_T \text{ [€/min]}$$

$$C_F \text{ [€/kg]}$$

When proposing this specific form of the penalty function, the study made by the University of Westminster related to “Dynamic Cost Indexing” was taken into account. The University of Westminster has studied, together with EUROCONTROL, the relevance of the CI for the airline cost strategies. In their paper [84], they highlight the advantages of using the CI parameter within a tool integrated in the FMS to optimise delay recovery during the flight as well as fuel conservation strategies.

4.3.2 The Relative Penalty Function

One of the key objectives of the work presented in this dissertation is to define metrics that are able to evaluate fairness and equity of the process by which the ATM system modifies the intended trajectories preferred by the airlines, focusing on the cost penalties of the airlines derived from the additional costs incurred due to those modifications.

The penalty function defined in Equation 4.24 reflects the extra costs incurred by one flight when the preferred costs determined by the airline's strategy cannot be maintained. As detailed in the preceding section, each flight has different values for the maximum penalty cost P_{SAT}, which depend on CI, $C_{F,}$ $T_{REF,}$ T_P, F_{REF}, and F_P. By comparing different resulting cost penalty values for the corresponding flights, it is difficult to draw any conclusions on whether one flight is relatively more or less penalised than another flight.

A common context needs to be provided to compare the values resulting from each penalty function for each flight. In such cases, using dimensionless variables paves the way to the required comparisons.

For that aim, a dimensionless penalty function is defined, which is referred to as the relative penalty function:

$$\wp = \frac{P}{P_{\max} + \kappa} = \frac{P}{P_{SAT} + \kappa} \qquad (4.25)$$

where κ is a real, positive value much smaller than P_{SAT}.

$$0 < \kappa << P_{SAT}$$

Adding κ prevents the relative penalty function to adopt the value "1" whenever the resulting cost penalty value is equal to P_{SAT}. This will prove itself as useful in the later metric definition.

The relative penalty function shows the percentage of the cost penalty that was incurred by a single flight compared to the maximum cost penalty value determined by the airline according to its strategy, namely P_{SAT}. This enables to compare different relative values for the incurred penalty among different flights.

The dimensionless penalty function has the following properties:

a) the dimensionless penalty function is maximal when the penalty function is also maximal:

$$\wp_{\max} = \frac{P_{SAT}}{P_{SAT} + \kappa} \tag{4.26}$$

b) the dimensionless penalty function is minimal when the penalty function is also minimal:

$$\wp_{\min} = \frac{0}{P_{SAT} + \kappa} = 0 \tag{4.27}$$

The relative penalty function is a monotonically increasing function and is contained between the values zero and one:

$$\wp \in [0,\ldots,1[$$

4.4 Brief Discussion on Previous Fairness Metrics Defined in the Literature

As detailed in the introduction (Section 1.3), papers in ATM talk about fairness when actually equity is meant [47][50]. As defined in chapter 2, equity is independent of the fairness concept. Nevertheless, equity may be a special case of fairness under certain circumstances. If all airlines had the same preferences and same cost index, then, if one wants to be fair when distributing the additional incurred costs, the result would be to distribute the cost penalties equally among all. Thus, equity is in this case a special case of fairness, which is given when the constraints and preferences are the same for everyone.

Papers describing a metric for fairness base their definition, mathematically speaking, on the arithmetic mean [47][48][49][50][51]. The weakness of those metrics towards evaluating fairness is that the arithmetic mean does not penalise the dispersion of the results. This is shown analysing following example with a metric called the satisfaction metric. First the metric needs to be defined. For that aim, the utility function is described. The proposed satisfaction metric is built upon this utility function.

4.4.1 The Utility Function

In microeconomics, utility functions are defined to measure the relative satisfaction [82]. Adopting this sense, a utility function is defined to measure the satisfaction of an airline regarding the costs incurred when a flight is deviated from the preferred trajectory, and thus, deviated from the preferred costs defined by the airline’s strategy.

The penalty and the utility are similar in the sense that they describe the same observation from a different perspective. The first one focuses on describing the additional incurred costs, namely the cost penalty, while the second one focuses on describing how far the additional incurred costs are from the maximal penalty cost, namely reflecting the satisfaction of the user.

From a strictly mathematical point of view, the introduction of the utility function does not add any new or particular insight to the analysis of the proposed cost model. It only serves to describe the satisfaction metric, which is used as a representative to identify the weaknesses of fairness metrics defined in the literature. Nevertheless, the notion of utility enjoys a widespread use in other fields of research which may be useful to the reader when interpreting and understanding the present work through analogies to other branches of knowledge.

According to the hypothesis and constraints mentioned in previous Section 4.2 for the cost model as well as Section 4.3 for the penalty function, following conditions must be fulfilled by the utility function:

I. The utility is maximal when the preferred trajectory is flown, i.e. the preferred costs are maintained. Thus, the utility is maximal when the penalty is minimal, when P takes the value zero, i.e. preferred trajectory is achieved (in the sense of $T_M=T_P$ and $F_M=F_P$).

$$U(P=0)=U_{\max} \tag{4.28}$$

II. The utility function is minimal when the penalty value is maximal.

$$U(P=P_{SAT})=U_{\min} \tag{4.29}$$

III. The utility decreases as the cost penalty value increases. The penalty function is a monotonically increasing function because, as explained before, for any increment of T_M and/or F_M with respect to the preferred values T_P and F_P, the costs incurred in flying a certain trajectory increase. This is the reason why the utility function is a monotonically decreasing function, as for any increment of T_M and/or F_M the additional costs incurred increase and, thus, the utility decreases.

This utility function always has positive values because the penalty function always results in positive values that are never greater than P_{SAT}. This means, the values of the utility function are always between zero and P_{SAT}:

$$U \in [P_{SAT},...,0]$$

4.4.2 The Dimensionless Utility Function

A dimensionless utility function needs to be defined to provide a common context to compare the utility of different flights. This dimensionless utility function or relative utility function reflects the percentage of utility that was achieved for a single flight, namely how far the cost penalty incurred is from the maximum cost penalty defined by the airline.

This enables to compare the achieved utility percentage among different flights. A function to reflect the relative utility is proposed:

$$u = 1 - \wp = 1 - \frac{P}{P_{SAT} + \kappa} \tag{4.30}$$

This relative utility function has the following properties:

i. the relative utility function is maximal when the relative penalty function is minimal

$$u_{\max} = 1 - \wp_{\min} = 1 - \frac{0}{P_{SAT} + \kappa} = 1 \tag{4.31}$$

ii. the dimensionless utility function is minimal when the relative penalty function is maximal

$$u_{\min} = 1 - \wp_{\max} = 1 - \frac{P_{SAT}}{P_{SAT} + \kappa} \text{ where } \frac{P_{SAT}}{P_{SAT} + \kappa} \xrightarrow[\kappa \to 0]{} 1 \tag{4.32}$$

$$u_{\min} \approx 0$$

The relative utility function is a monotonically decreasing function, which is comprehended between the values zero and one:

$$u \in [1, \ldots, 0[$$

4.4.3 The Satisfaction Metric

Assume a metric called the satisfaction metric S, which is defined as the arithmetic mean of the relative utility for a set of flights, where utility is defined as the utility function introduced in the previous subsection:

$$S = \frac{1}{n} \sum_{i=1}^{n} u_i \tag{4.33}$$

where i is an element of a given set of n flights.

This satisfaction metric reflects the overall, average satisfaction level achieved for a set of n flights based on the utility function. The metric reaches its maximum when the maximum satisfaction is guaranteed for all flights. The metric's properties are:

$$S_{\max} = 1, \text{ for all } u_{i,\max} \tag{4.34}$$

$$S_{\min} \approx 0, \text{ for all } u_{i,\min} \tag{4.35}$$

The weakness of this metric is that the overall satisfaction of a set of two flights would be the same in cases where both flights achieved the same level of utility as well as in situations where one flight achieves a much higher utility than the other. See the results shown in the following table:

Table 2 Values for overall satisfaction

Overall Satisfaction	Flight One	Flight Two
S=0,5	S_1=0,5	S_2=0,5
S=0,5	$S_1=S_{min}\approx 0$	$S_2= S_{max}=1$

4.4.4 Discussion of Results

From a mathematical point of view, the arithmetic mean has shortcomings when used to define fairness within the context presented in this thesis. According to the fairness definition provided in this dissertation, a fairness metric should penalise the dispersion of the utility values. For this, the geometric mean is a better tool.

By means of the following simple example, the differences between the geometric and arithmetic mean are illustrated. Imagine a set of three numbers, n=3, for example $x_1=0$, $x_2=4$ and $x_3=8$. The arithmetic mean for these three numbers is:

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{3}(0+4+8) = 4 \tag{4.36}$$

Whereas the geometric mean for the same three numbers is:

$$\left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} = (0 \cdot 4 \cdot 8)^{\frac{1}{3}} = 0 \tag{4.37}$$

Following table shows different set of numbers and compares the arithmetic mean and the geometric mean:

Table 3 Comparison Arithmetic and Geometric Mean

Set of Numbers	Arithmetic Mean	Geometric Mean
{4; 4; 4}	4	4
{3; 4; 5}	4	3,9
{2; 4; 6}	4	3,6
{0; 4; 8}	4	0

Comparing the results for the arithmetic and the geometric means, one can observe that, as the numbers are wider spread, the geometric mean penalises this dispersion, and the mean value decreases. The geometric mean equals the value for the arithmetic mean only when all the values are the same, thus when the dispersion is minimal.

When one finds a fairness metric in the literature that is mathematically expressed by an arithmetic mean, this metric cannot reflect the fairness concept as defined in Chapter 2.

Another way to define the fairness metric needs to be found to capture mathematically the features of the fairness concept for ATM described in this work.

4.5 The Fairness Metric

The fairness metric proposed in this work exploits the mathematical properties of the arithmetic and geometric means. The proposed fairness metric evaluates, similar to the satisfaction metric, whether the cost penalty has been distributed in an even manner but also penalises the dispersion in the distribution.

Recalling the definition of the fairness concept described in Chapter 2 (Section 2.3), the fairness metric has to reflect the quality of achieving a balance of conflicting interest by means of a just procedure; take into account the acceptance levels of all concerned and satisfaction of individuals.

The proposed fairness metric is expressed as follows:

$$\Phi = \frac{\left(\prod_{i=1}^{n}(1-\wp_i)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}(1-\wp_i)} \cdot n \tag{4.38}$$

To provide a just procedure to measure fairness, it has to be agreed by all concerned that fairness is to be measured according to the assumptions and constraints presented in this chapter (Sections 4.2 and 4.3). All stakeholders involved have to define without self-regard a penalty function that complies with those assumptions and constraints. Once this just framework is provided, the proposed fairness metric can be used as it covers the key features of the concept definition of fairness:

The proposed metric measures the extent to which conflicting interest of different airlines or flights have been balanced; namely to minimise their extra costs and try to maintain the flight costs as close as possible to the preferred ones.

The relative penalty function describes the cost penalty incurred relative to the maximum cost penalty P_{SAT}, which is defined by the reference values T_{REF} and F_{REF}. These reference values are an instrument at the airline's disposal for expressing the maximum cost penalty it can tolerate according to its strategy for a given flight. Thus, the metric considers the acceptance levels defined by the user.

The proposed fairness metric also describes how far the additional incurred costs are from the maximal cost penalty, namely reflecting the satisfaction of the user, by including the expression $(1-\wp)$.

The proposed fairness metric does not have to be based on the specific penalty function presented in Section 4.3.1. It can be based on any other specific penalty function that respects the assumptions and constraints describing the cost model framework and general characteristics of the penalty function presented in Section 4.2 and 4.3.

The fairness metric relies on the honesty of the airlines when providing their reference values T_{REF} and F_{REF}, which lead to P_{SAT}. If the airlines are untruthful when providing the data, then the correct measurement of fairness cannot be guaranteed. In such cases, the reference values have to be provided by a neutral third party.

The proposed fairness metric is maximal when the relative penalty is the same in all cases. In that case, fairness is 1.

$$\Phi_{\max} = 1 \tag{4.39}$$

The fairness metric is minimal when the relative penalty values are maximally spread. In that case, fairness goes towards the value zero:

$$\lim_{\kappa \to 0} \Phi_{\min} \to 0 \tag{4.40}$$

The fairness metric is comprehended between the values zero and one:

$$\Phi \in \left]0,...,1\right]$$

4.6 The Equity Metric

The equity metric proposed in this work evaluates the distribution of the cost penalty independently of the maximum penalty cost that has been defined by the airlines.

$$\mathrm{E} = \frac{\left(\prod_{i=1}^{n} (P_i + e)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n} (P_i + e)} \cdot n \tag{4.41}$$

where e is a real, positive value much smaller than P_i.

$$0 < e << P_i$$

The use of the variable e ensures that the equity metric is not zero when the cost penalty is zero for all flights of the set of n flights. For example, when for three flights there is no cost penalty, P1=P2=P3=0, the equity metric is maximal, as the same cost penalty, namely zero, has been distributed to all flights. If the value *e* was not included in the metric, the equity metric would result to be undefined; i.e. zero divided by zero.

It is important to notice that this metric provides a statement about the distribution of the incurred penalty costs and measures whether those have been distributed in an equal manner among all flights being analysed.

Equity, as defined in Chapter 2, may represent a special case of fairness. That case is given when the airlines' costs, costs strategies as well as their preferences are exactly the same (C_F, CI, T_P; F_P, T_{REF}, and F_{REF}). Consequently, all flights would have the same value for P_{SAT}. In such a situation, distributing the additional costs in the fairest manner, results in distributing the cost penalties equally among all. Fairness is defined to be maximal when the values for the different relative penalties are the same. Thus, in this case, the fairest distribution is also the most equitable.

Nevertheless, equity is independent of fairness and can be measured independently. The equity metric is maximal when the cost penalty, has been equally distributed among all, regardless its relation to each flight's P_{SAT}. In that case, equity is 1.

$$E_{max} = 1 \tag{4.42}$$

The equity metric is minimal when the cost penalty values have maximum dispersion. The value for the equity metric goes towards the value zero:

$$\lim_{e \to 0} E_{min} \to 0 \tag{4.43}$$

The equity metric is comprehended between the values zero and one:

$$E \in]0,...,1]$$

4.7 Proposed Theoretical Cases

With the objective to conveying a better understanding of the metrics introduced above, a closer look at a set of specific examples are presented together with a discussion on the interpretation of the metrics and information they provide.

All cases discussed here are valid for three flights with the same aircraft type covering the same route between the same two city pair. The penalty function and associated relative penalty function applied in all cases is according to the specific definition provided in equation 4.24. All flights are executed during the same time period, so it can be assumed that the coefficient C_F is constant for all flights. In all cases, the reference values have been defined by a neutral third party and not by the airline. T_{REF} and F_{REF} are the same for all three flights.

- **<u>Case A:</u>** the preferred trajectories, and consequently the preferred costs, are respected for all three flights; no cost penalty is incurred in any of the three flights.
    - $\wp_1 = \wp_2 = \wp_3 = 0$

  In this particular case, both metrics, Φ and E, are maximal because the preferences are fully respected, i.e. the preferred cost is achieved for all three flights and the penalty is distributed fairly and equitably. In this special case, the values for fairness and equity coincide although the preferences may not be the same:

$$\Phi_A = \frac{\sqrt[3]{(1-0)\cdot(1-0)\cdot(1-0)}}{(1-0)+(1-0)+(1-0)}\cdot 3 = \frac{1}{3}\cdot 3 = 1$$

  In order to calculate E, it can be taken advantage of the known relationship between the relative penalty and the cost penalty:

$$\wp_i = \frac{P_i}{P_{SAT}+\kappa} \Leftrightarrow P_i = \wp_i \cdot (P_{SAT}+\kappa)$$

  For $\wp_i = 0$, P_i will be zero for any value of P_{SAT}. Therefore,

$$\mathrm{E}_A = \frac{\sqrt[3]{(0+e)\cdot(0+e)\cdot(0+e)}}{(0+e)+(0+e)+(0+e)}\cdot 3 = \frac{e}{3\cdot e}\cdot 3 = 1$$

- **Case B:** for all three flights the percentage of the cost penalty incurred is the same positive value:
  - $\wp_1 = \wp_2 = \wp_3 = \wp_B$ with $0 < \wp_B < 1$

In this case, the fairness with which the additional cost was distributed among the three flights is maximal:

$$\Phi_B = \frac{\sqrt[3]{(1-\wp_B)\cdot(1-\wp_B)\cdot(1-\wp_B)}}{(1-\wp_B)+(1-\wp_B)+(1-\wp_B)}\cdot 3 = \frac{(1-\wp_B)}{3\cdot(1-\wp_B)}\cdot 3 = 1$$

The value of the equity metric depends on the strategy of the different airlines. If for all flights the preferences T_P and F_P as well as the cost index are the same, this results in all flights having the same values for P_{SAT}. In this case, the same amount of additional cost was distributed among the three flights (P_1=P_2=P_3=P_B), so the equity metric is maximal:

$$\mathrm{E}_{B1} = \frac{\sqrt[3]{(P_1+e)\cdot(P_2+e)\cdot(P_3+e)}}{(P_1+e)+(P_2+e)+(P_3+e)}\cdot 3 = \frac{(P_B+e)}{3\cdot(P_B+e)}\cdot 3 = 1$$

If the value of the preferences, e.g. the cost index, is different for any of the flights, then the amount of additional cost distributed among the three flights is not the same, resulting in a value for the equity metric less than 1.

Consider the case where CI_1=CI_2<CI_3. From the reasoning above it can be concluded that P_1=P_2(=P). According to the definition for P_{SAT} recalled below, it can be easily shown that for all other parameters assumed to be fixed, P_{SAT} grows as the cost index CI grows.

$$P_{SAT} = C_F\sqrt{CI^2(T_{REF}-T_P)^2+(F_{REF}-F_P)^2}$$

Therefore, knowing that P_{SAT3} is larger than P_{SAT1} (or P_{SAT2}), the value for P_3 must be higher than P_1 (or P_2) maintaining the same proportion $\wp_B$. As a result,

$$E_{B2} = \frac{\sqrt[3]{(P+e)\cdot(P+e)\cdot(P_3+e)}}{(P+e)+(P+e)+(P_3+e)}\cdot 3 = \frac{\sqrt[3]{(P+e)^2\cdot(P_3+e)}}{2\cdot P+P_3+3\cdot e}\cdot 3 < 1$$

Recalling the definition of the equity metric, the geometric mean in the numerator is smaller than the arithmetic mean in the denominator for any set of non-identical (positive) values, which leads to the inequality just above.

- **<u>Case C</u>**: The modified trajectories result in the maximum cost penalty for the three flights:
  - P_1=P_{SAT1}; P_2=P_{SAT2}; P_3=P_{SAT3}

Independent of each value of the maximum cost penalty, the fairness metric is maximal, because the three flights incur the same relative penalty:

$$\wp_1 = \wp_2 = \wp_3 = \wp_C \text{ where } \wp_C \approx 1 \text{ for } \kappa \to 0$$

$$\Phi_C = \frac{\sqrt[3]{(1-\wp_c)\cdot(1-\wp_c)\cdot(1-\wp_c)}}{(1-\wp_c)+(1-\wp_c)+(1-\wp_c)}\cdot 3 = 1$$

The value of the equity metric depends again on the different airline strategies. If the three flights have the same preferences and cost index values, then for all flights the maximum cost penalty is the same:

P_{SAT1}=P_{SAT2}=P_{SAT3}=P_{SAT}

In that case, the equity metric is maximal because the same cost penalty was distributed to the three flights:

$$E_{C1} = \frac{\sqrt[3]{(P_{SAT}+e)\cdot(P_{SAT}+e)\cdot(P_{SAT}+e)}}{(P_{SAT}+e)+(P_{SAT}+e)+(P_{SAT}+e)}\cdot 3 = 1$$

For different airline strategies, that means for different values of the cost index, the equity metric results in a value less than 1. Considering again the case where $CI_1=CI_2<CI_3$, it can be concluded that $P_{SAT1}=P_{SAT2}(=P_{SAT})$ and that P_{SAT3} is greater than P_{SAT1} (or P_{SAT2}). This results in the equity metric:

$$\mathrm{E}_{C2}=\frac{\sqrt[3]{(P_{SAT}+e)\cdot(P_{SAT}+e)\cdot(P_{SAT3}+e)}}{(P_{SAT}+e)+(P_{SAT}+e)+(P_{SAT3}+e)}\cdot 3=\frac{\sqrt[3]{(P_{SAT}+e)^2\cdot(P_{SAT3}+e)}}{2\cdot P_{SAT}+P_{SAT3}+3\cdot e}\cdot 3<1$$

Due to the same reasoning as for E_{B2}, the above inequality applies for E_{C2}. Additionally, it can be deduced that $E_{B2}= E_{C2}$ only if the preferences and cost indexes (CI_1, CI_2 and CI_3) are the same for both cases.

- **<u>Case D:</u>** for all three flights, different cost penalties are given. The preferred flight cost is respected for flight one, flight two reaches the maximum cost penalty and flight three accommodates a positive penalty value P_D.
  - $P_1=0$; $P_2=P_{SAT2}$; $P_3=P_D$ where $0<P_D<P_{SAT3}$

The relative penalties result to be all different:

$$\wp_1=0;\ \wp_2\approx 1 \text{ for } \kappa\to 0;\ \wp_3=\wp_D$$

Thus, the fairness metric results in value smaller than 1:

$$\Phi_D=\frac{\sqrt[3]{1\cdot(1-\wp_2)\cdot(1-\wp_D)}}{1+(1-\wp_2)+(1-\wp_D)}\cdot 3<1$$

Independently of the airline strategy, the equity metric also results in a value smaller than 1:

$$\mathrm{E}_D=\frac{\sqrt[3]{e\cdot(P_{SAT2}+e)\cdot(P_D+e)}}{e+(P_{SAT2}+e)+(P_D+e)}\cdot 3<1$$

With the help of these different cases used for this preliminary analysis of the metrics, the consistency of the metrics with the concept definition is confirmed.

4.8 Conclusions

In order to measure fairness or equity of a system, a just framework has to be provided. In Chapter 2 the requirements to ensure a just framework were introduced, namely to agree upon certain standards applicable to all concerned, under conditions preventing them from tailoring the principles to their own advantage.

Within a just framework, fairness and equity can be defined and assessed. Chapter 2 has established a definition of the concept of fairness and equity in ATM in the context of trajectory based operations (TBO).

According to the requirements of SESAR and NextGen concept of operations for the future TBO environment, a cost model framework has been proposed for the definition of the fairness and equity metrics. This cost model is based on assumptions already regarded within SESAR and NextGen.

The proposed cost model presents a comprehensive set of assumptions and constraints in order to define a penalty function as well as a relative penalty function. A specific penalty function and a relative penalty function are proposed in this chapter.

Nevertheless, it has to be clear that the cost model, as well as the functions and the developed metrics, have been defined in a generic way. The examples provided for the penalty as well as for the relative penalty function are intended to help the metric development. Although the metrics are based on these functions, the usefulness and application of both metrics are independent of the possible expressions of the penalty function and of the relative penalty function.

The fairness metric fully complies with the definition concept of fairness in ATM. The metric is based on the relative penalty, which shows the cost penalty incurred with respect to the maximum acceptable cost penalty defined by the airline or a neutral third party. The maximum acceptable cost penalty P_{SAT} may be defined by the airlines according to its cost strategy for a certain flight. This is an instrument for the airspace users to express their acceptance levels.

Fairness is based on the honesty of the individuals when expressing their acceptance levels and satisfaction. If the individuals, in this case the users, are untruthful, then the correct evaluation of fairness cannot be guaranteed. In those cases, the best solution is to evaluate the equity of the ATM system and make decision upon the results of the equity metric. The equity metric is independent of the maximum acceptable penalty cost P_{SAT}.

The equity metric is based on the penalty function. It evaluates the distribution of the absolute cost penalty values without taking into account the acceptance levels of the users, i.e. P_{SAT}. As long as a just framework is provided, equity can be measured.

The resulting metrics are intended to build the basis for a standard methodology to evaluate fairness and equity in ATM operations where the individual user-preferred trajectories need to be modified, e.g. to resolve potential conflicts. The metrics can also be considered to guide the design of automation tools that support such operations in future TBO.

5 THE FRAMEWORK AND METHODOLOGIES

This chapter details the framework required to apply the fairness and equity concepts defined in this dissertation and the methodologies that have to be followed to incorporate fairness and equity metrics in optimisation and evaluation processes.

The proposed metrics used in the appropriate framework with the appropriate methodology evaluate whether the additional incurred costs resulting from the proposed modifications to a certain trajectory have been done in a fair or/and an equitable manner compared to the rest of modified trajectories. Additionally, the metrics evaluate whether the user's preferences- a single flight or an airline operating more than a single flight- have been accommodated fairly or equitably with respect to the required trajectory modifications of the users.

5.1 The Framework

To apply the proposed metrics for fairness and equity in operational concepts as SESAR or NextGen, a certain framework is required. By framework shall be understood the set of assumptions, concepts and requirements that describe the ATM system to enable the analysis by means of the proposed methodologies.

The first requisite is to have a just system in which to apply the proposed metrics. According to the definitions provided in Chapter 2, more precisely in Section 2.3, it can be assumed with reasonable certainty that the ATM system nowadays displays the main features of a just system.

Next step is to define a particular mathematical expression for the penalty function and relative penalty function. Each ATM stakeholder can define it according to their requirements. Although each of those functions has to be compliant with the cost model, the definition of cost penalty, and the set of assumptions and constraints defined for the generic definition of the penalty and relative penalty function (according to Chapter 4). Nevertheless, to use the fairness and equity metrics universally in ATM, the stakeholders have to agree on a common specific expression for the penalty and relative penalty function. This way, fairness and equity can be assessed in the same way for the same processes independently of the stakeholder conducting the analysis.

Whatever the particular mathematical expressions for the penalty or the relative penalty functions are, it is required to agree among the ATM stakeholders on how to obtain the maximum acceptable cost penalty value P_{SAT}. That value is crucial to the relative penalty function and, by extension, to the fairness metric. As it will be shown in Chapter 6, the airlines have incentives to provide values for parameters impacting P_{SAT} that tailor the fairness considerations to their own advantage. Thus, it has to be guaranteed that the airlines communicate those values for each flight without manipulating the information or that P_{SAT} can be obtained through a neutral third party.

Either way, the value for the maximum acceptable cost penalty P_{SAT} has to be provided truthfully for each flight. If this cannot be guaranteed, then the effective applicability of the fairness metric is at risk. In this sense, the best solution would be then to apply only the equity metric. Otherwise, whenever it can be guaranteed that the P_{SAT} values are provided truthfully and no user can tailor those values to their own advantage, then the fairness metric can be applied.

5.2 The Methodologies

Two methodologies are proposed to introduce fairness and equity considerations in trajectory based operational concepts as SESAR or NextGen. These methodologies are indented to facilitate the use of the metrics based on the theory described in the previous chapter. The aim of these methodologies is to help in the process of:

1) introducing fairness and equity concepts in trajectory management
2) quantifying fairness and equity

The first methodology is called *a priori* and introduces fairness and/ or equity considerations in the optimisation process within the trajectory management process, for example within a conflict detection and resolution process. The other proposed methodology called *a posteriori* serves to compare and evaluate different trajectory management processes on their successfulness considering fairness and/or equity. This analysis within the *a posteriori* methodology can only be done after the optimisation process has been completed.

5.2.1 *A Priori* Methodology

The *a priori* methodology describes the steps to be followed to integrate the fairness and/or equity metric within the evaluation of the trajectory modifications based on user preferences. The aim of this methodology is to introduce fairness and/or equity considerations supporting the optimisation process that decides on the trajectory modifications.

Determining the User Preferences

First step is to gather the user preferred trajectories together with the user preferences indicated by the airline for each of its flights. Depending on the specific mathematical expression chosen for the penalty function, the required user preferences may vary accordingly.

Trajectory Management Process

The user preferred trajectories (UPTs) have to be analysed to establish whether deviations from the preferred path are needed due to, for example, potential conflicts with other aircraft, sector restrictions, congestions at airports, weather constraints.

In the TBO concept, each time modifications are required to the preferred trajectories of the flights, a specific automated tool handling the trajectory management process will support the decision making, according to SESAR and NextGen (see Chapter 1). This automated tool establishes different “what-if scenarios” with possible trajectory modifications. Here is where the fairness or the equity metric can be integrated, within those automated tools, providing additional criteria to decide on the best trajectory solution.

Fairness versus Equity

Whenever it can be guaranteed, that the P_{SATi} values of each flight have not been manipulated, then fairness as well as equity can be evaluated within the trajectory modification process. If it is not the case, then the concept of fairness may be easily jeopardized, and thus, only equity should be taken into account within the process assessment. Equity does not consider P_{SAT}, thus the manipulation of these values does not put the significance and validity of the equity metric into question. Whether only equity or both metrics, fairness and equity, may be considered depends on the established framework, and more concretely on the trustfulness of the source providing the P_{SAT} values. Following figure shows the described *a prior* methodology.

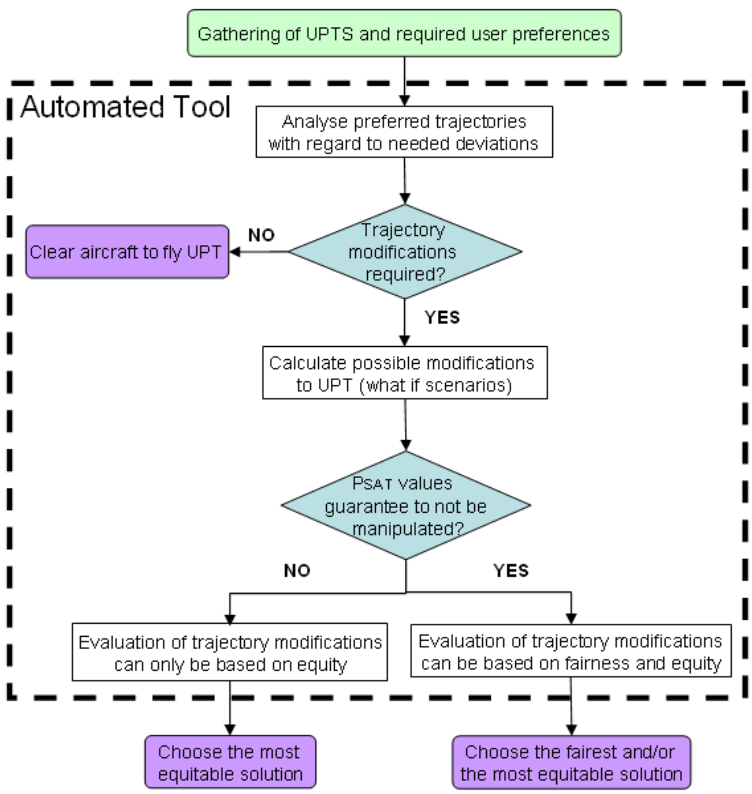


Figure 7 Methodology for the *a priori* integration of the metrics

5.2.2 *A Posteriori* Methodology

This methodology introduces fairness and equity metrics in the assessment of a certain trajectory management process. Main difference is that the *a posteriori* methodology applies the metrics after the trajectory management process is completed with a view to evaluate its outcome.

The assessment of a trajectory management process basically consists in comparing the input information, namely UPT and the required airline's preferences, and the output produced by the process itself, namely the modified trajectories. Evaluation criteria are fairness and/or equity of the process when accommodating the required trajectory modifications and the impact of the incurred costs on the airline's cost strategy. The fairness and/or equity of the processes are measured by means of the defined metrics.

Determining the User Preferences

The methodology starts by gathering the user preferred trajectories that served as input to the process under analysis, as well as the complementary user preferences. The required information on those preferences depends on the specific mathematical definition of the penalty function.

Trajectory Management Process

As mentioned previously, essential part of the current methodology is to gather the output of the process under consideration, namely the modified trajectories that resulted from that trajectory management process. This is the only piece of information required from the trajectory management process and it has to be obtained once the process is completed.

Fairness versus Equity

Again, depending on whether the P_{SAT} values can be guaranteed to not have been manipulated by the users to tailor the fairness concept to their own advantage, then, and only then, the fairness metric can be applied. Otherwise, only equity can be evaluated with the required certainty to not jeopardize the equity or the fairness concept defined in Chapter 2. The *a posteriori* methodology is detailed in the figure depicted below.

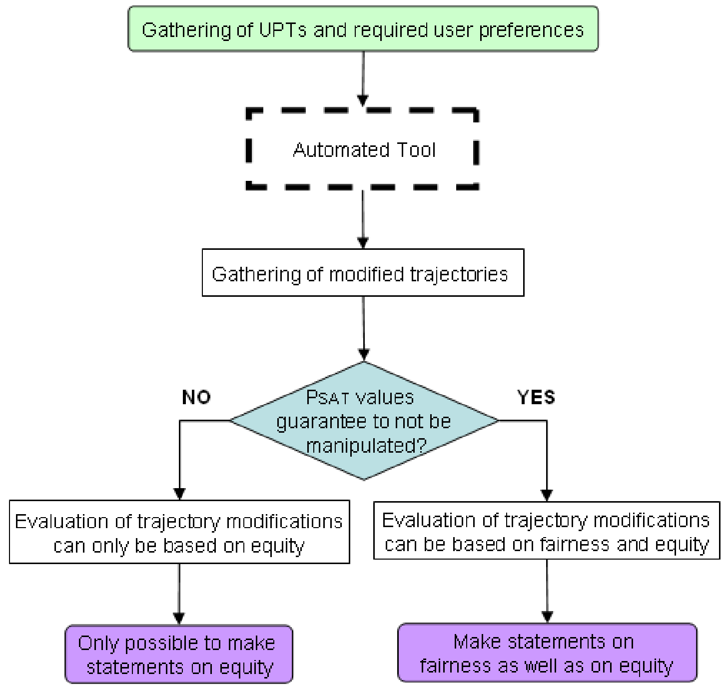


Figure 8 Methodology for the *a posteriori* consideration of the metrics

6 ANALYSING THE ROBUSTNESS OF THE FAIRNESS METRIC

The purpose of this chapter is to apply the fairness and equity metrics defined in Chapter 4 in a set of relevant cases to analyse the performance, robustness, and usability of the metrics, with special focus on the fairness metric. The cases to be studied are based on the *a priori* methodology (described in Chapter 5), i.e. the ATM system is a just framework where the fairness metric is used to drive traffic management decisions as part of the automated system's logic supporting the ANSP activity.

The stakeholders of the ATM system may, willingly or unwillingly, hide information or communicate erroneous information to other stakeholders of the system. It is even conceivable that, in the proposed framework to measure fairness, airlines have incentives to provide misleading data about their reference values (to define their P_{SAT}) to the ANSP in order to induce an outcome that would be beneficial to their interests. Exactly this is the aim of the proposed analysis, to identify the possible incentives airline may have to not communicate their reference values truthfully. Those values define the airline and flight specific P_{SAT} values which in turn are relevant for the fairness metric. The honesty of these values has to be guaranteed to be able to include fairness considerations in ATM.

Commercial competition means that the parameters of cost functions and cost preferences remain closely protected information that airlines do not usually disclose. This may lead to situations of imperfect information where ANSPs are not in a position to ensure fairness among airspace users as it cannot be certain about the impact of certain decisions on the airline's strategy and ultimately on the airline's costs.

The remainder of this chapter will focus on answering following questions:

a) do airlines have a benefit in providing untruthful information to fairness-oriented ANSPs?

b) if this is the case, what incentives do airlines have for doing so regarding what information or parameters?

c) to what extent would overall airlines' costs increase or decrease when they provide untruthful information?

To answer these questions, a simple decision game following the principles of decision theory is proposed as a means to predict and determine the airlines' behaviour within a given framework. With the help of this game, it can be analysed which information airlines would preferably communicate untruthfully to the ANSPs to minimise their cost penalty.

To guarantee fairness driven air traffic management, the fairness metric required parameters should be designed to prevent lying and promote sharing of required information as accurately as possible. According to the fairness concept defined in this dissertation and, as established in Chapter 5, only if honesty of all stakeholders involved can be assumed, then fairness can be applied. This chapter analyses the impact of fairness oriented ANSPs where honesty is not assumed and the measures that can be taken to overcome the possible negative consequences on fairness, i.e. improve robustness of the proposed fairness metric.

6.1 The Approach

The proposed decision game consists in analysing the behaviour of two airlines in a given framework, where the ANSP's objective is to maximise the fairness of the outcome, i.e. the modified aircraft trajectories. The fairness metric requires certain information that is provided by the airlines. The airlines decide whether this information is provided truthfully or not. This decision game aims to help understand the rationale behind the airlines' decision process and their incentives in providing values different from the true ones.

The two airlines, namely the two players, have at their disposal an ANSP model. With this model, each player can predict the behaviour of the ANSP based on the information provided. This is an important part of this decision game as it is assumed that the preferences of both airlines cannot be maintained due to air traffic conflicts.

Within the game, the players also have a finite number of possible strategy choices. So, each player can calculate its own expected cost penalty for any given combination of strategy choices of both players within the assumptions and constraints of the proposed decision game.

The decision game presented in this chapter consists of the following approach divided in five steps:

1) description of the decision game's framework, where framework is understood as the game's required assumptions and constraints: the players are presented as well as the framework's rules, constraints and limitations
2) description of the decision game: definition of the game, the objective of the players, their strategies and the rules of this game as well as the main assumptions
3) modelling based on decision theory: introduces the terminology outside and inside information and the required equations to obtain the payoff values for the analysis
4) solution based on equilibrium: the criterion selected to predict the equilibrium strategies is presented
5) analysis of the results: based on the concrete values resulting from this game, the analysis answers the three questions mentioned before.

The proposed decision game is a two-player game as this type of game captures the essential features of airlines' incentives and expected behaviour regarding the communication of airline specific information.

A multiple-player game (n-player game) could model with finer accuracy the fact that usually more than two airlines are competing in the ATM for the limited airspace resources. However, the complication implied by modelling n airlines (n>2), predicting their decisions, and analysing them has been deemed unnecessary and, in any case, over-proportional given the behavioural insights aimed in the present chapter.

6.2 Brief Introduction on Decision Theory

The principles of decision theory (or game theory) give guidance on how to capture mathematically the behaviour of individuals in strategic situations and predict their choices assuming that those individuals will always try to maximise their individual benefit [85]. This fundamental premise of decision theory is based on the assumption that human behaviour is guided by rational decisions and driven by self-interest [86] "*A player is said to be rational if she or he seeks to play in a manner which maximizes his own payoff*"[85].

It is important to note that the definition of a game must be precise and exhaustive to allow a mathematical treatment. Thus, the game model has to be similar to reality in those respects which are essential in the investigation [87]. Before proceeding to the details of the modelling, the main terms concerning the game definition have to be explained [87]:

a) A game is described by the totality of the rules that compose it. Those rules include the alternative options available to the players.

b) A play is a particular instance at which the game is played.

c) A move is the occasion of a choice between various alternatives.

d) A choice is a specific alternative selected during a play.

Thus, a game consists of a sequence of moves and a play of a sequence of choices. Each choice or strategy is selected freely by each player within the game rules. Usually, a two-person game consists of minimum two moves: one player chooses its strategy; the other player chooses its strategy. These moves can be done sequentially or simultaneously. The players may know exactly what the other is going to do (i.e. complete information) or not (i.e. incomplete information). The player's move can also be done in cooperation with each other or not, i.e. cooperative or non-cooperative [85].

According to decision game theory [87], the payoff-matrix or "normal form" is one of the main tools for performing the analysis of a two-player game. The payoff matrix form is the tool chosen in this chapter to depict the results of the proposed game. This matrix also helps to identify the game's solution.

In game theory, a solution concept is a formal rule for predicting how the game will be played assuming that players behave rationally [86][87]. Different solution concepts can be applied to a game in order to find out the set(s) of strategies chosen by each player (i.e. solution(s)), that is, to predict the outcome(s) of the game [88]. Since the first half of last century several authors have worked on the classification and formalisation of different types of games (e.g. static – dynamic, simultaneous – sequential, complete – incomplete information, cooperative – non-cooperative, zero sum – non zero sum, etc.) and developed different solution concepts for predicting their outcomes [87].

An account of the various kinds of games and the associated suitable solution concepts is beyond the scope of this work. The game proposed in this chapter is a static simultaneous non-cooperative game with incomplete information: static because the players only get to select a strategy once within the same play, i.e. they do not learn from previous moves; simultaneous because both players make their move at the same time; non-cooperative because the players do not jointly decide their strategies, incomplete because the player do not know the move of their opponent, i.e. they do not know the other player’s resulting outcome or payoff.

At a later point in this chapter, namely in Section 6.8, one can deduce that depending on the scenario and the case being analysed, the game can be described as a zero-sum or a non zero-sum game. According to von Stengel [85], a game is said to be zero-sum if *for any outcome, the sum of the payoffs to all players is zero*. This means that in the case of a two-player game, as the one at hand, a zero-sum game results only if one player’s gain is exactly the other player’s loss, *so their interests are diametrically opposed* [85].

In any case, for each game a solution has to be found. According to von Neumann and Morgenstern [87], the concept of solution in game theory is a set of rules indicating each participant how to behave rationally for the given situation.

In decision theory, the equilibrium is achieved when no player has the tendency to change its choice. At the equilibrium, the solution is the numerical statement defining how much the participant under consideration can benefit itself if she or he behaves rationally [87].

The solution concept applied in this chapter for the proposed game is the minimax criterion. The minimax criterion was first demonstrated mathematically by John von Neumann in [89]. It has been defined by John McDonald in his review of the book *Theory of Games and Economic Behavior*, A Theory of Strategy [87, pp. 692-711], as *the only theory that defines how to proceed rationally in what has classically been considered an irrational situation.* The minimax criterion is basically a strategy solution used for minimising the maximum loss. It can also be applied the other way around, being called then the maximin or max-min criterion. This solution concept was originally formulated for a two-player zero-sum game with perfect information [89]. This strategy solution evolved and was applied in two-player games with alternative moves, then games with simultaneous moves and finally also in complex games [89][90].

6.3 Description of the Game's Framework

The set of players considered is composed of two different airlines, airline A and airline B, responsible for two independent flights operating on similar arrival routes to the same destination airport. The two aircraft, which are the same aircraft type, are flying in a congested airspace, in this case the terminal manoeuvring area (TMA). Those two flights are in conflict with each other, thus the operational preferences of the respective airlines for an arrival trajectory cannot be fully satisfied.

The game's framework is based on the cost model proposed in Chapter 4. Applying the assumptions, constraints and functions described in that chapter, the cost-index based penalty function is represented as:

$$P = C_F \sqrt{CI^2 (T_M - T_P)^2 + (F_M - F_P)^2} \qquad (6.1)$$

The system's framework is built upon the Trajectory Based Operation (TBO) concept proposed by SESAR and NextGen. According to it, airlines will provide ANSPs with their preferred trajectories. A negotiation process would follow whereby ANSPs and airlines agree upon a trajectory, which the aircraft will be assigned to fly, and the ANSP will facilitate. That trajectory is called in this game the preferred trajectory. It is not the same as the user preferred trajectory.

This preferred trajectory accommodates as good as possible the airline's preferences, which were expressed in the user preferred trajectory. It is counter productive for the airlines not to provide, during this early stage of the negotiation, their operational preferences. Otherwise, they will be negotiating trajectories for their flights that do not suit their preferred business strategy straightforward. Thus, truthfulness on the preferred parameters T_P and F_P is assumed.

As mentioned before, the game has two players and now a new element of the framework is introduced, a centralised decision maker, which also manages the airline's information, namely the ANSP.

For the specific purpose of analysing the robustness of the fairness metric, the modelling of the ANSP has been simplified to capture only the essential behaviour required by the defined framework. Section 6.5 details the assumptions made.

It is important to remark that this simplified ANSP model used in this chapter is deemed sufficient for the analysis made here but in another contexts it my not be appropriate. Assumptions and simplifications associated to the ANSP would need to be enhanced and completed with a more complex logic, which has been left aside for the purpose of this analysis.

In the context of the current framework, the ANSP is fairness oriented and it is assumed that the ANSP allocates trajectories to the users so that always maximum fairness is achieved based on its knowledge of the users' strategies.

$$\Phi = 1 \tag{6.2}$$

Recalling the definition of the fairness metric, in cases where fairness is maximised, then the relative penalty costs are equal for all parts involved, in this game meaning both players, airline A and B:

$$\wp_a = \wp_b = \frac{P_a}{P_{SATa} + \kappa} = \frac{P_b}{P_{SATb} + \kappa} \qquad (6.3)$$

where κ is a real, strictly positive value much smaller than P_{SATa} or P_{SATb}; $0 < \kappa << P_{SATa}$ and $0 < \kappa << P_{SATb}$

The rules of the game, assuming a cost index based cost model as proposed in Chapter 4, require the airlines to provide following information to the ANSP:

i. their preferred values (T_P and F_P),
ii. their reference values (F_{REF} and T_{REF}) to obtain their maximum acceptable penalty cost P_{SAT}
iii. their cost index for the trajectory segment subject to analysis, in this case the arrival trajectory.

The cost index CI indicates to the ANSP which of both objectives, maintain preferred flight duration or preferred fuel consumption, has more weight in the airline's strategy. This is an important piece of information, as the ANSP has to maximise fairness according to the airline's strategies.

The players, airline A and B, are fully defined in this system through the above mentioned parameters, which are recalled herebelow:

Table 4 Parameters defining the airlines

	Airline A	**Airline B**
Preferred flight time	T_{Pa}	T_{Pb}
Maximum acceptable flight time	T_{REFa}	T_{REFb}
Preferred fuel consumption	F_{Pa}	F_{Pb}
Maximum acceptable fuel consumption	F_{REFa}	F_{REFb}
Cost Index for given flight segment	CI_a	CI_b

The maximum acceptable penalty cost P_{SAT} is calculated by means of the following equation:

$$P_{SAT} = C_F \sqrt{CI^2 (T_{REF} - T_P)^2 + (F_{REF} - F_P)^2} \qquad (6.4)$$

It is assumed that the airlines provide their data as required. Nevertheless, as mentioned at the beginning of this chapter, the airlines may provide false or untruthful information.

Whether the values are true or not, the ANSP has to base its decision on the information provided by the airlines. Thus, the ANSP assumes the information received is accurate and truthful and uses it as the basis to accommodate the required trajectory modifications in order to achieve maximum fairness. In this game, the proposed trajectory modifications are not subject to negotiation. As the ANSP always achieves maximum fairness, the airlines have to accept the resulting trajectory amendments.

By means of the cost values resulting form the trajectory modifications proposed by the ANSP, which are not necessarily based on true values, one can asses the impact of providing untruthful data one each airline's flight costs and on the overall costs.

6.4 Description of the Game's Actors

At this point, it is necessary to introduce the notion of inside and outside information to understand the proposed decision game.

As a matter of fact, there exists the possibility that airlines share values that are not truthful. In this game, the ANSP has no way to know whether that is the case. Because of this, a distinction is made between the parameters the airlines handle in their internal calculations and the parameters they share with external stakeholders, these being the ANSP and perhaps also other airlines.

- "Inside parameters" are the actual true values only known with certainty by each airline for T_{REF}, F_{REF}, P_{SAT}, CI.

- "Outside parameters" are the values made public by the airline for which external actors like ANSP or other airlines do not have any possibility to confirm its truthfulness. Those outside parameters are the only ones the ANSP has access to and are used by the ANSP to design the trajectory modifications to maximise fairness.

In the proposed game, the preferred values (T_P and F_P) are assumed to be provided truthfully whereas the reference values (T_{REF} and F_{REF}) and the cost index CI are not.

Outside parameters are distinguished from inside parameters by an apostrophe (T'_{REF}, F'_{REF}, CI'). Whenever the values of inside and outside parameters are identical, the airline is being truthful.

It is assumed that each player, i.e. airline, has at its disposal a finite set of strategies each corresponding to a set of values for the shared parameters. A set of 27 possible strategies per player are considered, which are the result of having three parameters, and three possible values per parameter; namely to declare the actual true values (i.e. inside values), to lie by giving a value below the true parameter's value or to lie providing a value greater than the true parameter's value.

The strategy tree below (Figure 9) illustrates all possible strategies available to each player. Starting with the decision on whether to provide the value of T_{REF} truthfully (middle) or to give a value smaller than the real one (left) or greater than the real one (right), this pattern is repeated through the other two parameters F_{REF} and CI.

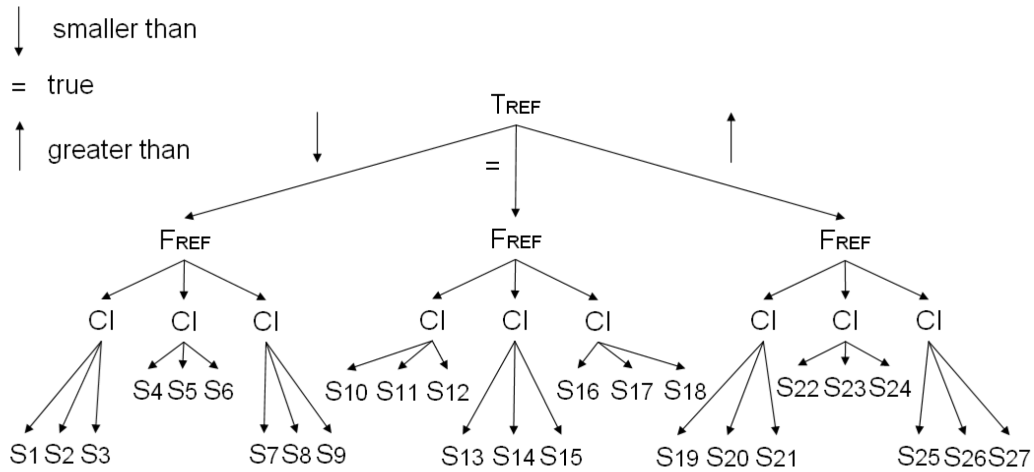


Figure 9 Strategy tree representing the set of all possible strategies

In the game at hand, the players, namely airline A and B, are represented by a single fight. Both players make their choices simultaneously in complete ignorance of each other's move. Once the choices have been made, the players get their resulting outputs (cost penalty) which are the payoffs of the game. Both players have the same interest, namely to reduce their resulting cost penalty. Both players have the same strategy tree at their disposal. Neither player has full control over the resulting payoffs. The resulting payoff values do not depend on one player's strategy only but on the joint choices of both players. The players cannot change their strategy during a play. Whenever the game proposed is played by exactly the same two players with their same individual preferences under the same conditions, the outcome will always be the same because their strategy choices, in exactly the same context, will be the same.

One can describe the proposed game as a static, simultaneous, non-cooperative game with incomplete information (Section 6.2) as the payoffs of the opponent are not known.

Each player has its own business strategy which can be the same or different from the other player's business strategy. Regardless of the other airline's decision, the driver for an airline's choice in this decision game is to minimise its cots penalty in the given framework. An example of a possible strategy for airline A is the following set of values, which corresponds to strategy S2 in Figure 9:

Table 5 Possible set of parameters for a given strategy for airline A

Airline A	Possible Strategy
T'_{REFa}	$T'_{REFa} < T_{REFa}$
F'_{REFa}	$F'_{REFa} < F_{REFa}$
CI'_a	$CI'_a = CI_a$

Based on the information provided by the airlines, the ANSP calculates the required modifications to the trajectories of the corresponding flights, resulting in a flight time and fuel consumption values that may be different from the preferred ones (T_{Ma}, T_{Mb}, F_{Ma}, F_{Mb}). These modified parameters are also the ones the airlines have to use to determine the impact of the ANSP's decision on their real cost penalty with their inside parameters. This way, the airlines can conclude the outcome of their choice of declared parameters.

The figure below summarises and provides an overview of the framework leading to the proposed game as well as the underlying relationships and assumptions.

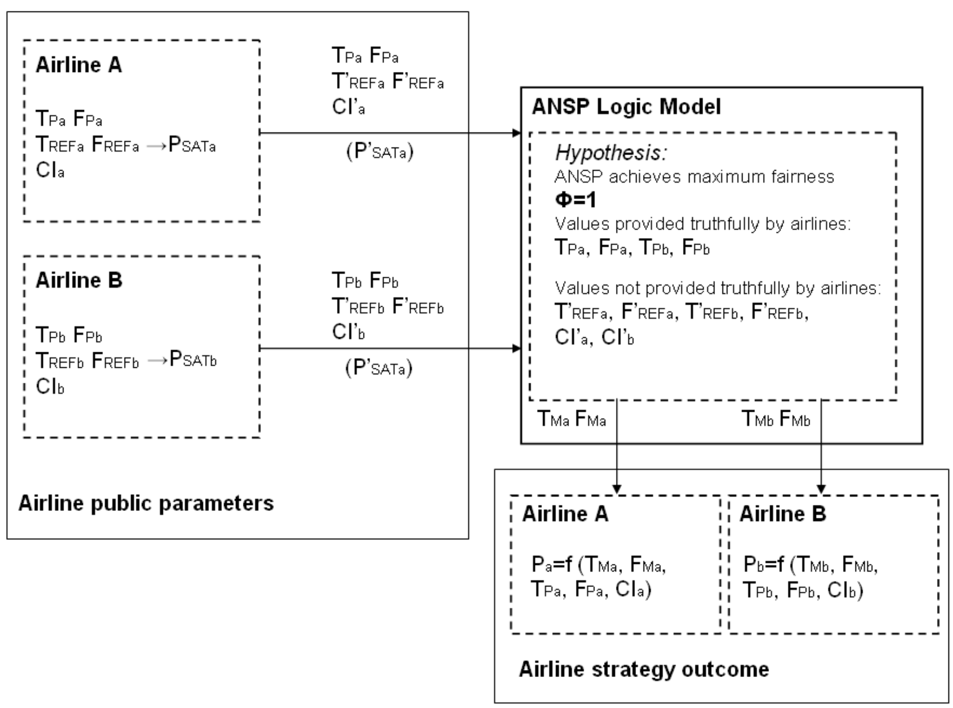


Figure 10 Game description

As stated above, in the proposed game the players decisions, i.e. declared values for T_{REF}, F_{REF} and CI, are aimed at obtaining an outcome from the ANSP, i.e. modified trajectories, that minimises their individual cost penalty.

The resulting cost penalty for each airline is a consequence of the actions of the ANSP based on the declared strategy combination of both players. Those penalty costs are the payoffs of the game, which can be represented in the so-called payoff matrix as depicted below.

Airline A choices for T'_{REFa}, F'_{REFa}, CI'_a
T_{Pa} and F_{Pa} provided truthfully

Airline B choices for T'_{REFb}, F'_{REFb}, CI'_b
T_{Pb} and F_{Pb} provided truthfully

Strategies	S_1^A	...	S_{n-1}^A	S_n^A
S_1^B	$(P_{1,1}^A; P_{1,1}^B)$	...	$(P_{1,n-1}^A; P_{1,n-1}^B)$	$(P_{1,n}^A; P_{1,n}^B)$
...	...	...	...	...
S_{n-1}^B	$(P_{n-1,1}^A; P_{n-1,1}^B)$	...	$(P_{n-1,n-1}^A; P_{n-1,n-1}^B)$	$(P_{n-1,n}^A; P_{n-1,n}^B)$
S_n^B	$(P_{n,1}^A; P_{n,1}^B)$	...	$(P_{n,n-1}^A; P_{n,n-1}^B)$	$(P_{n,n}^A; P_{n,n}^B)$

Figure 11 Example for a payoff-matrix

The payoff matrix form is the tool chosen to depict the results of the proposed game described in Section 6.8 and corresponding Appendix B.

6.5 Mathematical Model of the Game

The model of the proposed game is based on five equations (A1, A2, A3, A4, A5) which are presented and discussed in this section.

The central piece of this game's modelling is the cost index based cost model introduced in Chapter 4. That model uses the following function for the cost index based calculation of the airline's cost, which is assumed to be the same for airline A and B:

$$P = C_F \sqrt{CI^2 (T_M - T_P)^2 + (F_M - F_P)^2} \qquad \textbf{(A1)} \quad (6.5)$$

According to the game's rules, the airlines have to provide their true and preferred values for flight time and fuel consumption (T_P and F_P), as well as the required values corresponding to the chosen strategy for their cost index (CI'), their maximum acceptable flight duration and fuel consumption (T'_{REF} and F'_{REF}). It is important to recall the notation for inside and outside parameters, where outside parameters are characterised by an apostrophe.

The ANSP deduces the maximum acceptable penalty cost P'_{SAT} from the reference values provided by the airline using the following expressions:

$$P'_{SATa} = C_{Fa} \sqrt{CI'^{\,2}_a (T'_{REFa} - T_{Pa})^2 + (F'_{REFa} - F_{Pa})^2} \qquad (6.6a)$$

$$P'_{SATb} = C_{Fb} \sqrt{CI'^{\,2}_b (T'_{REFb} - T_{Pb})^2 + (F'_{REFb} - F_{Pb})^2} \qquad (6.6b)$$

As in chapter four, C_F is assumed to be constant for each flight. This value is airline dependent and defines the fuel cost rate each airline has negotiated with their fuel provider. Thus, each flight may have different values for their C_F. For the purpose of this game, C_{Fa} and C_{Fb} are assumed to be the same.

$$C_F = C_{Fa} = C_{Fb} \qquad (6.7)$$

The role of ANSP in ATM is key since it encompasses a number of diverse functions as: monitoring traffic in the airspace and airports, detecting and resolving air traffic conflicts whilst maintaining safety of the whole traffic, and ensuring that user preferences are considered throughout the traffic management decision making process. Modelling the whole range of activities and considerations of real ANSP is indeed a significant affair in terms of time, effort, and resources, which does certainly surpass the scope of the present chapter.

Aiming at the specific purpose of the present chapter, namely analysing the robustness of fairness and equity metrics, a simple ANSP model is considered in this game capturing the essential features required for the framework at hand:

- Fairness oriented ANSP: Airspace user preferences are considered by the ANSP when providing ATM services, which in this case are trajectory allocation for de-conflicting arrival management purposes.
- Airspace congestion: Airspace is congested to the extent that the flight of airline A is conflict with the flight of airline B. The preferences of all users cannot be fully satisfied. This feature is key for the present framework in order to be able to assess airlines' incentives to behave untruthfully when preferences are not achieved.
- Analytically explicit relationship between input and output parameters: Although not strictly mandatory from a theoretical point of view, the resulting relationship between ANSP input and output parameters is desirable in an analytically explicit way. In particular, recursive or implicit equations are to be avoided for the sake of computation practicality.

Making use of the modelling maxima "as complex as necessary and as simple as possible" the ANSP model adopted in this game is described by the following assumptions:

1) The ANSP always designs amended trajectories that achieve maximum fairness given the information received from the airline, thus fairness is always equal 1:

$$\Phi = 1 \qquad \textbf{(A2)} \qquad (6.8)$$

2) The ANSP has to modify the given planned trajectory to resolve the conflict between the two flights in their arrival phase to the same airport. The relationship given in this game between fuel consumption and flight time can be assumed to be proportional. That proportional relation is characterised by the ratio λ. This simplified trajectory modelling is described in the following equation, for which the condition applied for this specific case is: whenever the flight-time preference is maintained (T_M=T_P), then the fuel preference also has to be maintained (F_M=F_P).

$$F_M = F_P + \lambda \left(\frac{T_M - T_P}{T_P} \right) \quad \textbf{(A3)} \quad (6.9)$$

As mentioned before, both players, airline A and airline B, operate with the same aircraft type, thus the same value of λ applies for both airlines.

$$F_{Ma} = F_{Pa} + \lambda \left(\frac{T_{Ma} - T_{Pa}}{T_{Pa}} \right) \quad (6.9a)$$

$$F_{Mb} = F_{Pb} + \lambda \left(\frac{T_{Mb} - T_{Pb}}{T_{Pb}} \right) \quad (6.9b)$$

3) For the analysis of this game, the ANSP detects a conflict between both flights in their arrival phase. Thus, not all user preferences can be maintained. As detailed in Section 6.3 (description of the game's framework), the game's scenario is a terminal manoeuvring area where the players, namely the two airlines, declare their preferred arrival trajectories to the same airport. The two airlines operate each a single flight. Both flights are in conflict. The trajectory preferences of both players cannot be fully satisfied. Thus, the sum of the modified flight duration always has to be greater than the sum of the preferred flight duration. For the purpose of the present model, the factor c (c>1) is used to represent this assumption.

$$T_{Ma} + T_{Mb} = c(T_{Pa} + T_{Pb}) \quad (A4) \quad (6.10)$$

The assumptions described in equations A1 through A4, help establish an analytical relation between the input parameters (T_{Pa}, T_{Pb}, T'_{REFa}, T'_{REFb}, CI'_a, CI'_b) received by the ANSP and the output parameters (T_{Ma}, T_{Mb}, F_{Ma}, F_{Mb}) resulting as a consequence of the decisions taken by the ANSP.

The values for T_{Ma}, F_{Ma} T_{Mb} and F_{Mb} can be determined as well as the resulting payoff values. Those resulting payoff values are the true values for the cost penalty P_a and P_b incurred by the airlines after their preferences have been processed by the ANSP.

Applying the definition of the cost function (A1), P'_a is expressed as:

$$\frac{P_a'^2}{C_F{}^2} = CI_a'^2 (T_{Ma} - T_{Pa})^2 + (F_{Ma} - F_{Pa})^2 \tag{6.11}$$

The same applies for P'_b, as all equations are symmetric for airline A and B.

$$\frac{P_b'^2}{C_F{}^2} = CI_b'^2 (T_{Mb} - T_{Pb})^2 + (F_{Mb} - F_{Pb})^2 \tag{6.12}$$

If this definition of the cost function is combined with equation (A3), where it is stated that the fuel consumption is proportional to the flight duration, the equation described above results in the following expression by substituting F_M:

$$\frac{P_a'^2}{C_F{}^2} = CI'^2_a (T_{Ma} - T_{Pa})^2 + \left(F_{Pa} + \frac{\lambda}{T_{Pa}} (T_{Ma} - T_{Pa}) - F_{Pa} \right)^2$$

Simplifying the equation above results in the following one:

$$\frac{P_a'^2}{C_F{}^2} = CI'^2_a (T_{Ma} - T_{Pa})^2 + \left(\frac{\lambda}{T_{Pa}} (T_{Ma} - T_{Pa}) \right)^2$$

$$\frac{P_a'^2}{C_F{}^2} = CI'^2_a (T_{Ma} - T_{Pa})^2 + \frac{\lambda^2}{T_{Pa}^2} (T_{Ma} - T_{Pa})^2$$

$$\frac{P_a'^2}{C_F{}^2} = (T_{Ma} - T_{Pa})^2 \left(CI'^2_a + \frac{\lambda^2}{T_{Pa}^2} \right) \tag{6.13}$$

The same applies for airline B:

$$\frac{P_b'^2}{C_F{}^2} = (T_{Mb} - T_{Pb})^2 \left(CI'^2_b + \frac{\lambda^2}{T_{Pb}^2} \right) \tag{6.14}$$

According to this, the resulting function for calculating airline A’s modified flight duration is:

$$T_{Ma} = T_{Pa} + \frac{P_a'}{C_F} \frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T_{Pa}^2}}} \tag{6.15}$$

Identical equation applies for the modified flight duration of airline B.

$$T_{Mb} = T_{Pb} + \frac{P_b'}{C_F} \frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}} \tag{6.16}$$

As the values for P’$_a$ or P’$_b$ still have to be determined, those need to be calculated first, before obtaining T_{Ma} or T_{Mb}.

The framework of this game establishes that the preferences of the players cannot be maintained. Thus, the sum of the modified values has to be greater than the sum of the preferred values. Recalling equation (A4) and substituting T_{Ma} and T_{Mb} by the two equations described just above, the following expression is obtained:

$$T_{Pa} + \frac{P_a'}{C_F} \frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T^2_{Pa}}}} + T_{Pb} + \frac{P_b'}{C_F} \frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}} = c\left(T_{Pa} + T_{Pb}\right) \tag{6.17}$$

The main assumption in this game states the ANSP always achieves maximum fairness (A2), and so the relative cost penalty values are equal for both players. From the ANSP’s perspective, which can only handle outside parameters, the following relation applies:

$$\wp'_a = \wp'_b = \frac{P'_a}{P'_{SATa} + \kappa} = \frac{P'_b}{P'_{SATb} + \kappa} \qquad \textbf{(A5)} \tag{6.18}$$

The value of κ has to be a very small value, for which in this game it is assumed to be:

κ =0,001

Knowing that P’$_b$ can also be expressed as $P'_a \frac{P'_{SATb} + \kappa}{P'_{SATa} + \kappa}$ according to A5, then Equation 6.17 can also be expressed as:

$$T_{Pa} + \frac{P_a'}{C_F}\frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T^2_{Pa}}}} + T_{Pb} + \frac{P_a'}{C_F}\frac{P'_{SATb}+\kappa}{P'_{SATa}+\kappa}\frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}} = c\left(T_{Pa} + T_{Pb}\right)$$

$$\frac{P_a'}{C_F}\frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T^2_{Pa}}}} + \frac{P_a'}{C_F}\frac{P'_{SATb}+\kappa}{P'_{SATa}+\kappa}\frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}} = \left(c-1\right)\left(T_{Pa} + T_{Pb}\right) \tag{6.19}$$

This is the previous step before obtaining the equation for the ANSP to calculate the resulting cost penalty with the outside parameters provided by each airline. According to the parameters provided to the ANSP, the equation for determining the cost penalty P'_a of airline A is:

$$P'_a = \frac{\left(c-1\right)\left(T_{Pa} + T_{Pb}\right)C_F}{\frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T^2_{Pa}}}} + \frac{P'_{SATb}+\kappa}{P'_{SATa}+\kappa}\frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}}} \tag{6.20}$$

By symmetry, the following applies for airline B:

$$P'_b = \frac{\left(c-1\right)\left(T_{Pb} + T_{Pa}\right)C_F}{\frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T^2_{Pb}}}} + \frac{P'_{SATa}+\kappa}{P'_{SATb}+\kappa}\frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T^2_{Pa}}}}} \tag{6.21}$$

Note that here, P'_a and P'_b are the cost penalty values calculated by the ANSP and they are considered as outside parameter, thus P'_a and P'_b have an apostrophe.

At this stage, knowing P'_a and P'_b, the resulting values for the modified flight duration T_M and the modified fuel consumption F_M for players, airline A and airline B, can be determined. Note that T_M and F_M do not have an apostrophe. These parameters are not referred to as outside parameters but as inside parameters.

In any case, the resulting values for T_M and F_M, for both airlines, are the ones being calculated by the ANSP with the values the ANSP knows. Those values are the real true values resulting from the final trajectory amendments.

The airlines have to use those same values (T_M and F_M) to then recalculate their resulting true penalty cost P_a and P_b (no apostrophe here) according to their inside parameters, that is with the true values of T_{REF}, F_{REF}, CI and the resulting true value for P_{SAT}. The cost penalty calculated by the ANSP and the cost penalty calculated by each airline will not be the same in case the required information was not provided truthfully to the ANSP.

The resulting equations for T_M and F_M, were detailed before in Equations 6.9a, 6.9b, 6.15, and 6.16, but here they are written down again, now knowing that the value for P'_a and P'_b have been determined.

For airline A:

$$T_{Ma} = T_{Pa} + \frac{P_a'}{C_F} \frac{1}{\sqrt{CI'^2_a + \frac{\lambda^2}{T_{Pa}^2}}}$$

$$F_{Ma} = F_{Pa} + \lambda \left(\frac{T_{Ma} - T_{Pa}}{T_{Pa}} \right)$$

For airline B:

$$T_{Mb} = T_{Pb} + \frac{P_b'}{C_F} \frac{1}{\sqrt{CI'^2_b + \frac{\lambda^2}{T_{Pb}^2}}}$$

$$F_{Mb} = F_{Pb} + \lambda \left(\frac{T_{Mb} - T_{Pb}}{T_{Pb}} \right)$$

Based on these values, which correspond to the new trajectories defined by the ANSP to achieve maximum fairness based on the information it knows, airlines can compute their actual cost penalty using their respective inside parameters in order to determine the true impact on their costs.

$$P_a = C_F \sqrt{CI_a^{\,2} (T_{Ma} - T_{Pa})^2 + (F_{Ma} - F_{Pa})^2} \tag{6.22}$$

$$P_b = C_F \sqrt{CI_b^{\ 2} (T_{Mb} - T_{Pb})^2 + (F_{Mb} - F_{Pb})^2} \tag{6.23}$$

These resulting penalty costs represent the payoff values for the payoff-matrix.

6.6 Equilibrium and Game Solution

This section presents and discusses the possible outcome of the game modelled in the previous sections from a qualitative perspective. The focus is on understanding the strategies that the players would rationally choose given the game assumptions.

In the game proposed, it is assumed that the players make their decisions simultaneously, without any knowledge of what the other player will do and with no possibility to cooperate. They are assumed to be mutually aware of the set of strategies each player has at its disposal. Thus, each player can calculate its own payoff values for all possible strategy combinations. This last statement is fundamental to the game and for each player. To decide which strategy to select, each player can calculate ahead its own expected payoff values for each strategy combination and know the impact of each strategy combination to its business strategy. Each player can do this based on these three assumptions:

- the strategy tree at the disposal of each player is known by all players,
- the ANSP always achieves maximum fairness
- the ANSP model used in the game to calculate the modified values for the flight duration and fuel consumption (T_M and F_M) is known by both players.

For the game proposed, the solution concept applied is the minimax criterion. The payoffs represent the expected cost penalty. As stated before, each player can calculate their own resulting cost penalty for all possible strategy combinations. Applying the minimax criterion implies that each player, independently of what the others might decide to do and without knowledge of the opponent's payoffs, chooses the strategy that results in its best penalty cost (as minimum as possible) among its worst penalty costs (maximum possible).

In the game at hand, only two airlines are considered, airline A and B. By applying the minimax criterion the strategy chosen by both players can be predicted. The point where those two strategies cross in the matrix form is defined as the game's equilibrium. The equilibrium identifies the solution payoff values for each player. Because the solution is determined applying the minimax criterion, the payoff values of the solution are characterised by the subscript "*minimax*" as follows:

$P_{\min\max}^{A} = P_{I,J}^{A} = \min_{i}\left[\max_{j}\left(P_{i,j}^{A}\right)\right]$ for i,j=1…27 where i is airline A's strategy and j airline's B strategy.

$P_{\min\max}^{B} = P_{I,J}^{B} = \min_{j}\left[\max_{i}\left(P_{i,j}^{B}\right)\right]$ for i,j=1…27 where i is airline A's strategy and j airline's B strategy.

The minimax solution is then the set of strategies I and J for airlines A and B, respectively, which leads to penalty costs $P_{I,J}^{A}$ and $P_{I,J}^{B}$.

For the representation of the results, the payoff matrix form is used. Airline A choices are represented in the columns and airline B choices in the rows. Applying the minimax criterion, one can determine the strategy choice of each player:

- For airline A, its strategy choice is determined by selecting the minimum of the column maxima.
- For airline B, its strategy choice is determined by selecting the minimum of the row maxima.

Following example should clarify these statements (Figure 12). The two players of the following example have three possible strategies at their disposal. The aim of both players in this example depicted below is to minimize their resulting payoff values:

Player 1 choices

Strategies	S_1^1	S_2^1	S_3^1	row maxima for player 2
S_1^2	(1; -1)	(2; 1)	(-1; 2)	2
S_2^2	(-2; -1)	(0; 0)	(1; -1)	0
S_3^2	(1; 2)	(1; 1)	(2; 1)	2
column maxima for player 1	1	2	2	

Player 2 choices

Equilibrium according to minimax:
Player 1 selects strategy S_1
and Player 2 selects strategy S_2

Solution:
$P_{minimax}^1 = -2$
$P_{minimax}^2 = -1$

Figure 12 Example illustrating the minimax criterion

6.7 Simulation Set-Up

This section provides a detailed description of the simulation setup used to run the analysis, the methodology applied, the parameters defined to evaluate the results, and the cases studied. This simulation set up defines the required parameters to perform the calculations needed to obtain each player's payoff values. With the help of those values the two players decide on their best strategy choice.

6.7.1 Definition of the Game Parameters

The analysis is based on the following simulation set-up focusing on two given flights of two different airlines, airline A and B, during the period they fly through the terminal manoeuvring area (TMA) with a given, pre-defined set of standard terminal arrival routes (STARs) into the same runway. The flights are flown by the same aircraft type, namely a Boeing 737-800W26. The ANSP knows the preferred trajectory of each of those two flights as well as the value for CI, T_{REF} and F_{REF} as provided by each airline. The ANSP is able to predict with the help of an automation tool, for example an arrival management decision support tool, potential conflicts between the trajectories. In this game, the two flights, i.e.

their corresponding trajectories, are in conflict with each other. Thus, the ANSP has to modify those trajectories and design the required amendments according to the indicated preferences.

The simulation setup models the TMA of the Canary Islands and two Standard Terminal Arrival Routes (STARs) to Gran Canaria's airport as described in the Spanish Aeronautical Information Publication (AIP) [91]: TERTO 3C and RUSIK 3C for the same runway (RW03L). Following figure shows those two STARs:

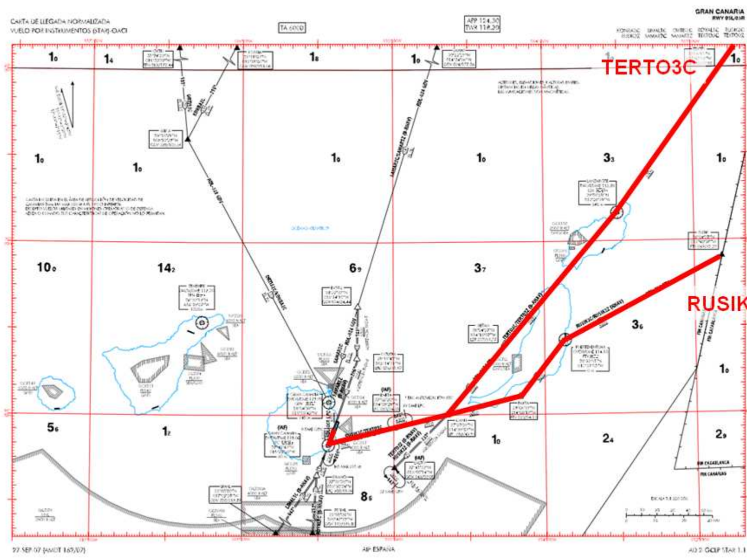


Figure 13 STAR TERTO 3C and RUSIK 3C for Gran Canaria's airport

a. The user preferences for flight duration and fuel consumption: T_P and F_P

The two flights can fly either one of those two routes. The user-preferred trajectory for both airlines is assumed to be a continuous descent approach (CDA). This CDA, either for the STAR RUSIK 3C or TERTO 3C, has been computed using Boeing Research & Technology Europe's (BR&TE) trajectory computation infrastructure (TCI) based on BADA 4.0 (Base of Aircraft DAta) aircraft performance models (APM)of the aircraft type considered.

BRT&E's TCI is a computer-implemented method producing a description of aircraft intent to predict aircraft trajectory, for example by air traffic management. *Rules are used in association with information provided to generate a set of instructions describing both the* *aerodynamic configuration of the aircraft and the motion of the aircraft. These instructions are checked to ensure that they describe unambiguously the aircraft's trajectory* [92]. This TCI uses the aircraft performance models of BADA 4.0 [93], which is the current latest version. This version is being developed by EUROCONTROL in collaboration with BR&TE [94]. BADA is a *kinetic, mass-varying* APM developed and managed by the Eurocontrol Experimental Center [94].

With this precise TCI, the preferred values for time duration T_P and fuel consumption F_P can be obtained for both flights depending on the STAR assigned to them. Those preferred values refer to the flight duration and fuel consumption from TMA entry until touch down at the airport's runway (RW03L).

For identical initial conditions at the beginning of each STAR, operating the two flights the same aircraft type, and both airlines preferred trajectory being a CDA, it can be assumed that the preferred values, T_P and F_P, are the same for both flights whenever they are assigned to fly the same STAR to Gran Canaria's airport. Under same initial conditions it is understood same aircraft state at the beginning of the STAR (i.e. same 3D position - latitude, longitude and altitude- and same aircraft weight) and same weather conditions. The preferred values T_P and F_P are shown in Table 6 according to the assigned STAR.

Table 6 Preferred values depending on the STAR assigned

	T_P	F_P
TERTO 3C	2212s = 36min 52s	1027,321kg
RUSIK 3C	1833s = 30min 33s	760,493kg

b. The fuel cost C_F

According to IATA's (International Air Transport Association) Jet Fuel Monitor published in its webpage [95], the average fuel price for 2010 was $88,1/b or 0,52€/kg.

Considering the exchange rate of [96]:

1$ = 0,758€

Knowing that:

1US b = 0,158987 m^3

According to [95], the Fuel Jet-A average density is: $\rho_{Jet-A}(15^o C) = 807{,}5 \frac{kg}{m^3}$

Thus, the Jet fuel price used in this simulation results to be:

$$88{,}1\frac{\$}{b} = 88{,}1\frac{0{,}758€}{0{,}158987m^3} = 420{,}033\frac{€}{m^3}$$

$$420{,}033\frac{€}{m^3} = 420{,}033\frac{€}{m^3} \cdot \frac{1}{807{,}5}\frac{m^3}{kg} = 0{,}52\frac{€}{kg}$$

In this specific game, the fuel price C_F is assumed to be the same for both airlines and have a value of 0,52€/kg.

c. The cost index CI

The range for cost index value for a Boeing 737-800 varies from 0 to 375 kg/min according to [81]. Typical values used in commercial aviation are following as expressed by Boeing's FMS simulation specialists:

$CI_{737\text{-}800}$ =9-100[kg/min]

For the proposed game, airline A and B will manage two possible CI, arbitrarily chosen within the range of the typically used commercial values, $CI_{737\text{-}800}$ =33 and $CI_{737\text{-}800}$ =70. The cases described later will specify which CI is used accordingly.

d. The reference values for flight duration and fuel consumption T_{REF} and F_{REF}

The airlines communicate to the ANSP the values for T_{REF} and F_{REF} according to their chosen strategy. Independently, a baseline has to be established defining the true values of those parameters, namely the ones assumed to be true in the proposed game. These reference values (i.e. T_{REF} and F_{REF}) for each airline cannot be deduced from the simulation setup, thus the time and fuel performance target values defined by SESAR for European/Continental flights [30] are assumed:

- maximum of 3min delay
- maximum of 5% increase in the expected fuel consumption.

T_{REF} and F_{REF} values of both flights are obtained by adding the above mentioned time and fuel performance targets to the preferred values, which depend on the STAR assigned to each flight.

Table 7 Reference values depending on the STAR assigned

	T_{REF}	F_{REF}
TERTO 3C	T_P + 180s = 2392s	F_P + 5% = 1078,687kg
RUSIK 3C	T_P + 180s = 2013s	F_P + 5% = 798,518kg

e. The airspace congestion value c

Recalling the assumption stating that the airspace is congested and that the trajectories of both flights are in conflict with each other, the time preferences of both airlines cannot be maintained. Thus, the sum of the modified time durations of both airlines has to be greater than the sum of the preferred flight durations:

$$T_{Ma} + T_{Mb} = c\left(T_{Pa} + T_{Pb}\right) \text{ for } c>1 \qquad (6.24)$$

In order to run the required simulations, the value for c is assumed to be 105% (=1,05). Although this percentage is arbitrarily chosen, it captures the essential intention, namely to have a congested airspace slightly above a traffic level where the preferences could be maintained within the given airspace.

f. The ratio between flight time and fuel consumption λ

The data extracted from several trajectory computations within this scenario set-up helped identify the value of λ, which characterises the relationship between time and fuel in this specific game and only under the assumptions introduced in Section 6.5.

The following table shows the numerical results for fuel consumption and time duration extracted from the different trajectory computations. This trajectory computation has been performed for the simulation set-up using the STARs TERTO 3C and RUSIK 3C and the trajectory computation infrastructure developed by BR&TE [92]. In each case, only offsets were instructed, limiting the trajectory modifications to the lateral track. The speed and altitude profile were adjusted to model how the aircraft would fly as efficient as possible given the re-route.

Table 8 Simulation results for modified trajectories for the STARs TERTO 3C, RUSIK 3C

	Modified Trajectories									
TERTO 3C	tm1	tm2	tm3	tm4	tm5	tm6	tm7	tm8	tm9	**User-Preferred Trajectory**
flight duration (s) T_M	2252	2333	2230,57	2267,09	2318,75	2230,46	2254,09	2218,09	2236,04	T_P=2212
fuel consumption (kg) F_M	1027,32	1056,5	1113,81	1041,06	1103,5	1040,97	1057,71	1032,22	1044,93	F_P=1027,321

	Modified Trajectories									
RUSIK 3C	tm1	tm2	tm3	tm4	tm5	tm6	tm7	tm8	tm9	**User-Preferred Trajectory**
flight duration (s) T_M	1907	1855	1919	1897,45	1916,09	1857,99	1861,08	1944,79	1861,22	T_P=1833
fuel consumption (kg) F_M	813,41	776,5	822,49	806,926	820,152	778,963	781,159	840,484	781,25	F_P=760,493

According to assumption A3 (Equation 6.9) in Section 6.5, these values can be represented in the table shown below, which facilitates the identification of the λ value.

Table 9 Values for λ according to assumption A3

		$(T_M-T_P)/T_P$	F_M-F_P
TERTO 3C	tm1	0,018083183	29,183
	tm2	0,054701627	86,486
	tm3	0,008395118	13,735
	tm4	0,024905063	39,596
	tm5	0,048259494	76,182
	tm6	0,008345389	13,653
	tm7	0,019028029	30,385
	tm8	0,002753165	4,896
	tm9	0,010867993	17,605
RUSIK 3C	tm1	0,040370977	52,917
	tm2	0,012002182	16,007
	tm3	0,046917621	61,997
	tm4	0,035160938	46,433
	tm5	0,04533006	59,659
	tm6	0,013633388	18,47
	tm7	0,015319149	20,666
	tm8	0,060987452	79,991
	tm9	0,015395526	20,757

λ is extracted from these resulting values through a lineal regression as depicted below:

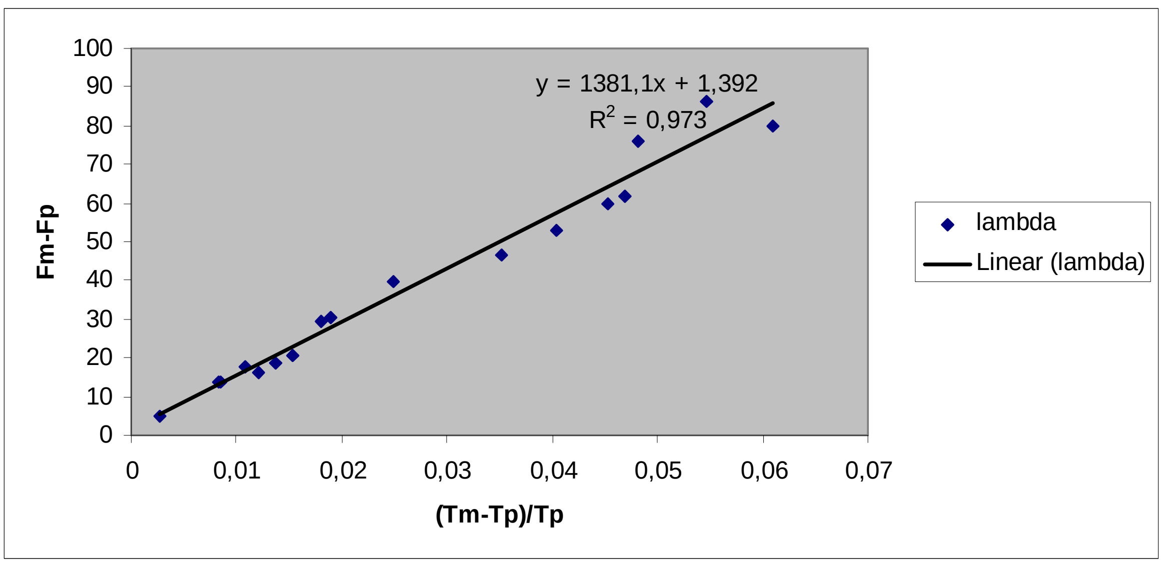


Figure 14 Linear regression for the λ value

As it can be seen, the relationship between fuel consumption and flight duration can be expressed, in this simulation set-up, as quasi-linear showing a deviation of 2,7% (R^2=0,973) with a λ value of 1381,1 (kg).

6.7.2 Methodology

Three scenarios are going to be analysed in order to assess the incentives that determine the best strategy choice between airlines with similar or different cost strategies and preferred trajectories:

- the first scenario, scenario α, consists of two flights from two different airlines, A and B, with identical cost strategies flying the same STAR;
- the second scenario, scenario β, consists of two flights from two different airlines, A and B, with different cost strategies flying the same STAR;
- the third scenario, scenario γ, consists of two flights from two different airlines, A and B, with different cost strategies flying different STARs.

The following table characterises each scenario. The parameters shown in the table below are the ones known by the airlines.

Table 10 Details of the three scenarios to be analysed

	T_P, F_P	T_{REF}, F_{REF}, CI	resulting P_{SAT}
Scenario α	$T_{Pa}=T_{Pb}$ $F_{Pa}=F_{Pb}$	$T_{REFa}=T_{REFb}$ $F_{REFa}=F_{REFb}$ $CI_a=CI_b$	$P_{SATa}=P_{SATb}$
Scenario β	$T_{Pa}=T_{Pb}$ $F_{Pa}=F_{Pb}$	$T_{REFa}=T_{REFb}$ $F_{REFa}=F_{REFb}$ $CI_a\neq CI_b$	$P_{SATa}\neq P_{SATb}$
Scenario γ	$T_{Pa}\neq T_{Pb}$ $F_{Pa}\neq F_{Pb}$	$T_{REFa}\neq T_{REFb}$ $F_{REFa}\neq F_{REFb}$ $CI_a\neq CI_b$	$P_{SATa}\neq P_{SATb}$

Within each scenario, different cases are defined limiting the game strategies the airlines can apply (the whole strategy tree is depicted in Figure 9 “Strategy Tree”). This will allow evaluating which are the relative incentives the players have for providing untruthful information on a specific parameter or on a specific combination of parameters.

- Case 1: airlines can only play (i.e. provide truthful or untruthful information) with the value of one parameter (T'_{REF}, F'_{REF}, or CI')
- Case 2: airlines can play with the values of two parameter (T'_{REF} and F'_{REF}, F'_{REF} and CI', or T'_{REF} and CI')
- Case 3: airlines can play with the value of all three parameter (no restriction apply to the solution tree)

Using the model based on game theory described in the previous sections, the outcome of the possible game strategies on each airline's penalty cost can be calculated. For the specific scenario and case, together with the max-min approach, it can be predicted which strategy will be chosen by each airline, which will always intend to maximise its benefit, or in other words, to minimise its cost penalty.

Thus, the airline may attempt to influence the outcome of the game, i.e. the ANSP's trajectory amendments, by tailoring the information it provides instead of providing truthful information. It is assumed that a player's strategy choice deciding the values of the information it has to provide depends on the player's model of how the ANSP is going to change its trajectory. That ANSP model is common for both players of the game at hand.

Figure 15 shows the methodology followed. It is important to define the analysis scenarios and the cases beforehand. The model defined in the previous sections, in order to emulate the relevant behaviour of the players and the ANSP, is used to calculate all payoff values. The minimax criterion is applied to predict the strategy choice of each player and obtain the values of the player's payoff at the equilibrium. Applying the proposed model for each scenario and case, it results in different output data that needs to be evaluated.

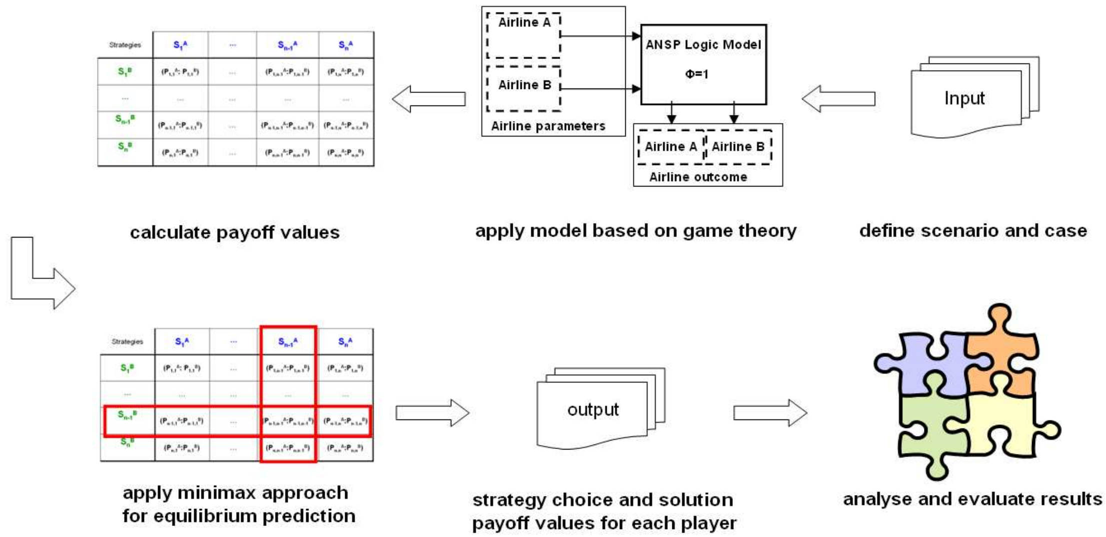


Figure 15 Methodology for the Analysis

In order to evaluate and analyse the results, two dimensionless auxiliary variables are defined in the next section. With the help of these variables, it will be possible to evaluate the implications of the chosen game strategy by the airlines on each airline's cost penalty and on the overall penalty cost of the system, i.e. the aggregated penalty cost of all players. This will also allow comparing the results of the different scenarios.

6.7.3 Auxiliary variables

The two dimensionless auxiliary variables describe here are intended to assist in the analysis of the resulting payoff values:

- one variable helps to evaluate the impact of the game strategy choice made by each airline on their respective resulting penalty cost, $\prod_{i,j}^{x}$
- the other variable helps to evaluate the impact of the game strategy choice of both airlines on the system's cost efficiency, $\eta_{i,j}$

I. **Percentage of player's benefit** $\prod_{i,j}^{x}$: this first variable is designed for the analysis of the airline's cost penalty.

$\prod_{i,j}^{x}$ is defined as follows:

$$\prod_{i,j}^{A} = \frac{P_{i,j}^{A} - P_{true}^{A}}{P_{true}^{A}} \tag{6.25a}$$

where $P_{i,j}^{A}$ represents the airline's A cost penalty resulting when airline A chooses strategy i and airline B chooses strategy j. P_{true}^{A} is the airline's cost penalty that would result for airline A if both airlines provide the ANSP with truthful information.

The same applies for airline B:

$$\prod_{i,j}^{B} = \frac{P_{i,j}^{B} - P_{true}^{B}}{P_{true}^{B}} \tag{6.25b}$$

This auxiliary variable evaluates the increase or decrease in percentage of the resulting payoff value for a given airline according to the set of strategies chosen by all players. The airline's resulting payoff value is always compared against the true payoff value of that same airline. The true payoff value is only given if all players provided all parameters truthfully to the ANSP. This last payoff value is characterised by the subscript "*true*".

Applying the minimax criterion to obtain the game solution or equilibrium, it can be identified which game strategy of the whole set of possible strategies is going to be chosen by each player. The strategy identified allows each player to achieve their best possible solution independently of the other's player strategy. As a reminder, the game at hand is a static simultaneous non-cooperative game with incomplete information. The airline specific penalty value resulting from that special set of strategies is characterised by the subscript "*minimax*" ($P^{A}_{minimax}$, $P^{B}_{minimax}$) According to it, the percentage difference of the penalty costs is also characterised by that same subscript $\prod_{\min i \max}^{x}$.

$$\prod_{\min i \max}^{A} = \frac{P_{\min i \max}^{A} - P_{true}^{A}}{P_{true}^{A}} ; \prod_{\min i \max}^{B} = \frac{P_{\min i \max}^{B} - P_{true}^{B}}{P_{true}^{B}} \tag{6.26}$$

II. **Percentage of system's benefit** $\eta_{i,j}$: the second variable to be described is the auxiliary variable designed for the analysis of the system's cost penalty.

Let the system cost penalty P^{system} be defined as the sum of the resulting cost penalties of both players:

$$P_{i,j}^{system} = P_{i,j}^{A} + P_{i,j}^{B} \tag{6.27}$$

Then, the auxiliary variable $\eta_{i,j}$ represents the percentage increase or decrease (if negative) of the system's incurred cost penalty compared to the situation where both airlines provide truthfully all parameters. This percentage increase or decrease is specific for a particular set of game strategy choices (i,j) made by each player.

$\eta_{i,j}$ is calculated by comparing the sum of the resulting penalty values for the set of strategies i and j chosen by both players against the true system cost penalty. The true system cost penalty results when both players, the airlines, provide all required information truthful to the ANSP. Again, the subscript "*true*" characterises the true system cost penalty.

$$\eta_{i,j} = \frac{\left(P_{i,j}^{A} + P_{i,j}^{B}\right) - \left(P_{true}^{A} + P_{true}^{B}\right)}{\left(P_{true}^{A} + P_{true}^{B}\right)} = \frac{P_{i,j}^{system} - P_{true}^{system}}{P_{true}^{system}} \tag{6.28}$$

By applying the minimax criterion to predict the game solution, the strategy choice of both players is determined. Those choices are the ones defining the equilibrium and its corresponding payoff values. That equilibrium is characterised by the subscript "*minimax*":

$$\eta_{\min i \max} = \frac{\left(P_{\min i \max}^{A} + P_{\min i \max}^{B}\right) - \left(P_{true}^{A} + P_{true}^{B}\right)}{\left(P_{true}^{A} + P_{true}^{B}\right)} \tag{6.29}$$

The auxiliary variables detailed here above, are key for the analysis. The incentives to provide untruthful information to the ANSP can be quantified in the present model by means of the marginal benefit obtained by each player when communicating values different from the inside parameter values, i.e. percentage of player's benefit . The impact of each player's choice on the system can as well be quantified by the marginal benefit described by the percentage of system's benefit.

In other words, with the help of these auxiliary variables ($\prod_{i,j}^{x}$ and $\eta_{i,j}$) and analysing their values in each scenario and case, it can be shown to what extent a player reduces its resulting cost penalty when untruthful information is provided to the ANSP and the impact it has on the overall system.

6.8 Analysis of results

The analysis consists of comparing the game results for the three different scenarios and the three different cases. For each combination of scenario and case, namely a game run, the equilibrium has to be determined in order to predict the strategy chosen by each player and calculate the solution payoff values.

6.8.1 Scenario Definition

The three scenarios and cases were defined in the Section 6.7.2. The tables below detail the values of the parameters for each scenario. As a reminder, the game is composed of two players, airline A and airline B, each represented by one single flight. The three scenarios only focus on the arrival phase of those flights.

In Scenario α both airlines cover the same STAR, in this case RUSIK3C, and have the same business strategy, which means that both players have the same cost index.

Table 11 Specification for Scenario α

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	30,5	760,49	33,5	798,52	0,52	33

	c [-]	λ [kg]
ANSP	1,05	1381,1

In Scenario β both airlines cover the same STAR, in this case RUSIK3C, and have different business strategies, i.e different cost indexes.

Table 12 Specification for Scenario β

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	30,5	760,49	33,5	798,52	0,52	70

	c [-]	λ [kg]
ANSP	1,05	1381,1

In Scenario γ both airlines cover different STARs, in this case RUSIK3C for airline A and TERTO 3C for airline B, and have different business strategies, i.e different cost indexes.

Table 13 Specification for Scenario γ

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	37	1027,32	40	1078,69	0,52	70

	c [-]	λ [kg]
ANSP	1,05	1381,1

Regardless of the scenarios at hand, the ANSP model is always the same: achieving maximum fairness and assuming the constant parameters c and λ describing the model.

Considering scenario α, both players are described by exactly the same characteristics; have exactly the same preferences. The peculiarity of this scenario is that both players want to achieve exactly the same outcome. As it can be deduced from the results (Appendix B), whenever one player gains the other player loses. The gains and losses are exactly balanced among the participants, thus, the sum of the gains and losses of the game is zero (recall definition given in Section 6.2). This adds to the definition of the game under scenario α the special characteristic of being also a zero sum game.

A zero-sum game can be easily identified by the values adopted by $\eta_{i,j}$, i.e. the system cost efficiency. Whenever the value is zero for all i and j, this denotes a zero-sum game. As one can observed from the results explained in the section below, only the game run with scenario α, independently of the cases considered, is a zero sum game. Scenario β and γ are non-zero sum games.

6.8.2 Case Definition

The three different cases, which are applied to each scenario, define different sets of possible game strategies at the disposal of each player.

In **case 1**, the players can choose their strategy among a set of strategies where they can only provide, for **one parameter**, a value that does not correspond to their inside parameter value. Following figure depicts the three possible solution trees for each player, where S2, S5 and S8 are the same:

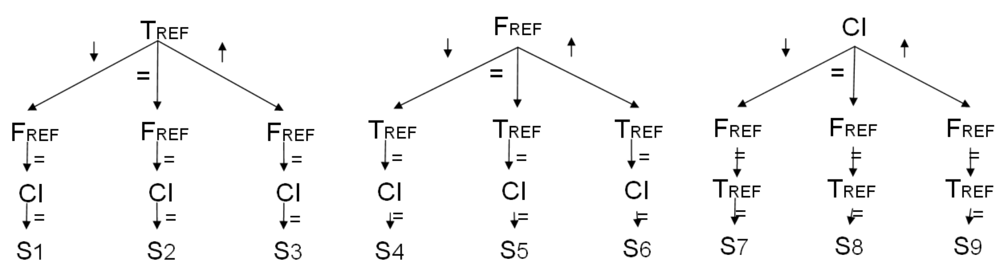


Figure 16 Three simplified strategy trees at the disposal of the players in Case 1

This case can be played for each scenario in two variants:

- case 1.1: only one player can provide a value different form the inside parameter values for only one parameter,
- case 1.2: both players can provide a value different from the inside parameter values for only one parameter.

Following tables describe the subcases of case 1.1 and case 1.2, detailing in each subcase in which parameter each player, airline A or B, is allowed to provide a value different from the inside parameter value.

Table 14 Detailed Subcases for Case 1.1

Subcase	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B
1.1 T_A	X					
1.1 F_A		X				
1.1 CI_A			X			
1.1 T_B				X		
1.1 F_B					X	
1.1 CI_B						X

Table 15 Detailed Subcases for Case 1.2

Subcase	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B
1.2 T_AT_B	X			X		
1.2 T_AF_B	X				X	
1.2 T_ACI_B	X					X
1.2 F_AT_B		X		X		
1.2 F_AF_B		X			X	
1.2 F_ACI_B		X				X
1.2 CI_AT_B			X	X		
1.2 CI_AF_B			X		X	
1.2 CI_ACI_B			X			X

In **case 2**, the players can choose their strategy among a set of strategies where they can provide, for any combination of **two parameters**, values that do not correspond to their inside parameter values. This case is only played in one variant, namely both players playing at the same time. The figures below show the possible strategy combinations for each of the three possible combination, where S5, S14 and S23 are the same.

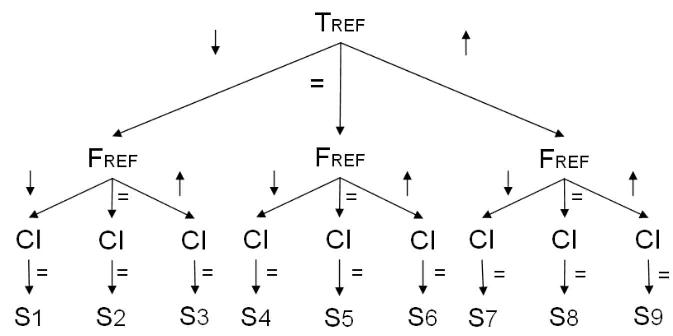


Figure 17 Strategies with combination of parameter values T_{REF} and F_{REF}

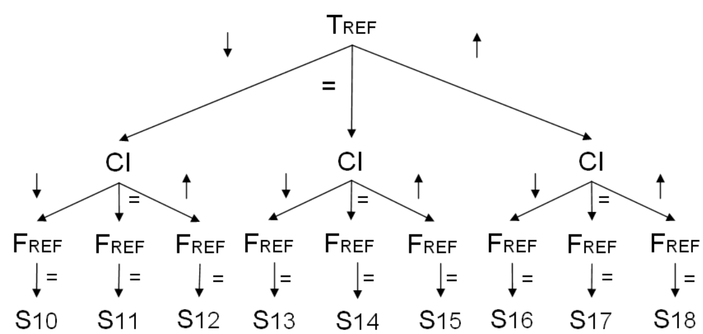


Figure 18 Strategies with combination of parameter values T_{REF} and CI

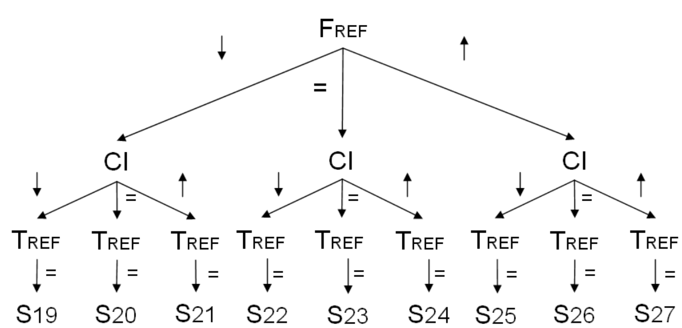


Figure 19 Strategies with combination of parameter values F_{REF} and CI

Table 16 shows the different subcases and details with which combination of parameters each airline is allowed to play.

Table 16 Detailed Subcases for Case 2

Subcase	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B
2 $T_AF_AT_BF_B$	**X**	**X**		**X**	**X**	
2 $T_AF_AT_BCI_B$	**X**	**X**		**X**		**X**
2 $T_AF_AF_BCI_B$	**X**	**X**			**X**	**X**
2 $T_ACI_AT_BF_B$	**X**		**X**	**X**	**X**	
2 $T_ACI_AT_BCI_B$	**X**		**X**	**X**		**X**
2 $T_ACI_AF_BCI_B$	**X**		**X**		**X**	**X**
2 $F_ACI_AT_BF_B$		**X**	**X**	**X**	**X**	
2 $F_ACI_AT_BCI_B$		**X**	**X**	**X**		**X**
2 $F_ACI_AF_BCI_B$		**X**	**X**		**X**	**X**

In **case 3**, the players can choose their strategy among the set of strategies where they can provide, for any combination of the **three parameters**, values that do not correspond to their inside parameter values. Each player can choose a strategy among the complete strategy tree, as presented in Section 6.4 and shown again in Figure 20.

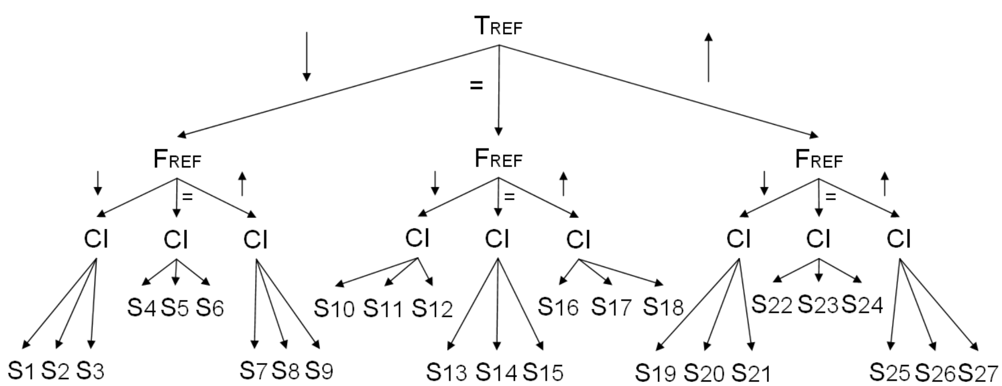


Figure 20 Complete strategy tree

Different as in the other cases, for case 3 there is only one subcase possible:

Table 17 Detailed Subcase for Case 3

Subcase	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B
3 $T_AF_ACI_AT_BF_BCI_B$	**X**	**X**	**X**	**X**	**X**	**X**

6.8.3 Analysis of Results for Case 1

The analysis starts with the simplest case, namely case 1. The game will be run for each scenario only allowing the players to provide untruthful information for one parameter.

6.8.3.1 Airline Incentives

The results for case 1, considering all three scenarios (see Appendix B) conclude that whenever the players provide only for one parameter a value different from their inside parameter value, they will always provide a value smaller than the inside parameter's one. The explanation for this relies on the fact that each player, i.e. airline, tries to reduce the value of the saturated cost penalty communicated to the ANSP. Recalling the definition of the cost penalty function, one has to remember that by definition the cost penalty function ***P*** can maximally reach P_{SAT} when the maximal acceptable costs are reached, although the actual incurred costs may trespass P_{SAT} (Chapter 4 Section 4.3).

To achieve maximum fairness, the ANSP will always try to distribute the relative penalty cost equally among all players, preferably trying not to reach the saturated penalty cost P_{SAT} of the different airlines.

Thus, in the game at hand, the objective of the airlines is to minimise their resulting cost penalty values, namely their actual cost penalty computed with their inside parameter values. Associated to that, each player intends to communicate to the ANSP a saturated penalty cost value (results from CI, T_{REF}, and F_{REF} according to equations 6.6a and 6.6b) that favours them to achieve their objective. So each airline has an incentive to communicate, whenever possible, to the ANSP a P_{SAT} value below their inside value.

Taking into account the game modelling, the airlines cannot provide directly to the ANSP their value for P_{SAT}. They can do it through the values of T_{REF}, F_{REF} or CI.

When the only strategy available is to decide on the value of one single parameter communicated to the ANSP, then the only way to minimise P_{SAT} is to communicate to the ANSP values for T_{REF} or F_{REF} or CI below their inside values.

$$\frac{\partial P_{SAT}}{\partial T_{REF}} > 0\,;\ \frac{\partial P_{SAT}}{\partial F_{REF}} > 0\,;\ \frac{\partial P_{SAT}}{\partial CI} > 0 \qquad (6.30)$$

A decrease in any of the values of the parameters T_{REF} or F_{REF} or CI goes along with a decrease of the value of P_{SAT}.

Thus, in this first analysis for each scenario and for each parameter, the airlines always prefer to communicate values below their real inside parameter's ones.

Within case 1, the analysis consists in having:

- only one player choosing a strategy from the available set of strategies while the other player can only provide its inside parameter values, namely case 1.1
- both players can choose a strategy from the available set of strategies, namely case 1.2.

Case 1.1 is very simple and can probably not be considered as a game *per se* but it clarifies whether each player in each scenario has an incentive to lie, and if so in which direction - value smaller or greater than the true value. The solution strategies for each parameter can be found by identifying those strategies that provide the maximum benefit for the given player.

The maximum benefit is in all scenarios associated to communicating to the ANSP a lower value for the parameter in question (see Appendix B). In all three scenarios, the parameter in which both airlines find the greatest incentive to provide a value below the inside parameter is T_{REF}. On the other hand, the CI is the one parameter that provides the lowest benefit. The tables below (Tables 18, 19, and 20) show the results for the auxiliary variable describing the percentage player's benefit. Whenever the value is negative, it demonstrates that the player has an incentive in providing a value for the parameter at hand that is below the inside parameter value. Further details on the numerical results can be found in Appendix B.

Table 18 Percentage player's benefit for scenario α case 1.1

	Π_A	Π_B
T_{REF}	-41,6	-41,6
F_{REF}	-2,9	-2,9
CI	-2,8	-2,8

Table 19 Percentage player's benefit for scenario β and case 1.1

	Π_A	Π_B
T_{REF}	-45	-52,3
F_{REF}	-3,4	-0,6
CI	-3,2	-1,3

Table 20 Percentage player's benefit for scenario γ case 1.1

	Π_A	Π_B
T_{REF}	-45,7	-76,4
F_{REF}	-3,5	-1,5
CI	-3,3	-1,1

6.8.3.2 Analysis of System Impact

The behaviour of the system's cost penalty depends on the scenario and the strategy chosen. For analysing this, the focus relies on case 1.2 because the characteristics defining case 1.1, -only one airline is allowed to provide information for one parameter different than the inside parameter value-, is too simple and the impact of the strategy choice on the system's penalty depends always on the win or loss of one single player.

In case 1.2, each airline can provide values different than the inside parameter values for only one parameter. Still, the combinatory allows nine different cases (three parameters for airline A multiplied by three parameters for airline B). The equilibrium in all cases is given when both airlines provide the minimum possible value for the parameter at hand.

For scenario α, as stated before it is a zero-sum game, the system neither looses nor gains ($\eta_{minimax}$=0), independently of the strategy chosen. As both airlines have identical aircraft models, cost strategies and preferences, the global system costs remains not annoyed as long as both airlines can lie to an equal extent.

For scenario β and scenario γ, both non zero-sum games, the system's cost penalty behaves similarly:

a) The system achieves its maximum gain ($\eta_{minimax}$ is maximised) when airline A chooses to provide its minimum value for F_{REF} and airline B chooses to provide its minimum value for T_{REF}. This strategy choices go in line with each airline's business strategies as it can be deduced from the cost index (CI_A=33 and CI_B=70). For airline A, fuel consumption related costs are more important than time related costs while for airline B it is the other way around. According to this, both airlines lie on the parameter value that has greater impact on the communicated P_{SAT} value. The losses and gains of each airline in terms of their resulting penalty cost do not compensate each other, resulting therefore in a non zero-sum game. Concretely, the gain of one airline is greater than the loss of the other, enabling the system to also achieve a gain.

b) The system almost achieves zero sum ($\eta_{minimax}$ close to zero) when both airlines choose to communicate their minimum values for CI. As stated before, lying in the CI value provides the lowest benefit for both players.

c) The system achieves its maximum loss ($\eta_{minimax}$ is minimised) when airline A chooses to provide its minimum value for T_{REF} and airline B chooses to provide its minimum value for F_{REF}. This strategy choices do not go in line with the airline's business strategies (CI_A=33 and CI_B=70). In this case, what is lost (in terms of *P*) by one airline is more than the gain of the other airline.

By this analysis of the system's cost impact it is shown that there may exist situations where both airlines provide values different form their inside parameter values that result in a positive impact on the system's cost.

6.8.4 Analysis of Results for Case 2

In case 2, there are three possible parameter combinations for each airline: T_{REF} and F_{REF}, T_{REF} and CI, and F_{REF} and CI. In total, this results in 9 analysis subcases. Within each analysis subcase, each parameter has three different values that each participant can choose to communicate: a value below the true one (inside parameter value), a value equal to the true one, or a value above the true one.

6.8.4.1 Airline Incentives

Different as for the results described in case 1, for case 2 the airlines do not always choose to provide the minimum value for the parameters involved in the strategy chosen. Independently of the preferred STAR and of the airline's business strategy, the minimax criterion applied to each of the nine analysed subcases results in the following equilibria (depicted in Table 21): the airlines choose to provide the minimum value possible for each parameter in the combination T_{REF} and F_{REF} and F_{REF} and CI, but not for T_{REF} and CI.

Table 21 Case 2 equilibria for each subcase

Subcase	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B
2 $T_AF_AT_BF_B$	↓	↓		↓	↓	
2 $T_AF_AT_BCI_B$	↓	↓		↓		↑
2 $T_AF_AF_BCI_B$	↓	↓			↓	↓
2 $T_ACI_AT_BF_B$	↓		↑	↓	↓	
2 $T_ACI_AT_BCI_B$	↓		↑	↓		↑
2 $T_ACI_AF_BCI_B$	↓		↑		↓	↓
2 $F_ACI_AT_BF_B$		↓	↓	↓	↓	
2 $F_ACI_AT_BCI_B$		↓	↓	↓		↑
2 $F_ACI_AF_BCI_B$		↓	↓		↓	↓

In the latter case, the airlines choose to provide the maximum value possible for CI and the minimum possible for T_{REF} (see Appendix B). By indicating higher CI values, the airlines communicate a relatively higher weighting of time related costs compared to fuel related costs, while at the same time the reference value for time related costs (T_{REF}) is lowered. On the other hand, the airlines communicate a lower CI indicating the fuel related cost are more relevant and, at the same time, the reference value for the fuel related cost is lowered. These two strategies denote the meaning of the cost index value has been correctly modelled.

All the equilibria are detailed in Appendix B where it can also be found the percentage difference $\prod_{\min\max}^{X}$ for airline A and B as well as the resulting payoff values for each solution.

6.8.4.2 Analysis of System Impact

Regarding the system's impact (all resulting values are also detailed in Appendix B), the conclusions made for scenario β and γ are similar, showing that the impact on the system's cost impact depends mainly on the airline's business strategy and not on the route flown. Remember, the difference between scenario β and γ is that airline B flies a different STAR than airline A. Scenario α is of no relevance for analysing the system's impact, as it is a zero sum game.

In scenarios β and γ the system gains the most ($\eta_{minimax}$ is maximised) when airline A is able to provide untruthful information on the parameters F_{REF} and CI while airline B does the same for the parameters T_{REF} and F_{REF}.

The system loses the most ($\eta_{minimax}$ is minimised) when the airlines choose those same strategies the other way around (results shown in Table 22).

Table 22 Case 2 system's percentage benefit for each subcase and scenario

Subcase	Scenario α	Scenario β	Scenario γ
2 $T_AF_AT_BF_B$	0	-0,7	-4,1
2 $T_AF_AT_BCI_B$	0	1,2	0,3
2 $T_AF_AF_BCI_B$	0	10,2	8,7
2 $T_ACI_AT_BF_B$	0	-5,6	-8,2
2 $T_ACI_AT_BCI_B$	0	-3,7	-4
2 $T_ACI_AF_BCI_B$	0	7	6
2 $F_ACI_AT_BF_B$	0	-12,1	-13,2
2 $F_ACI_AT_BCI_B$	0	-10,6	-10
2 $F_ACI_AF_BCI_B$	0	0,7	0,6

The system's cost remains nearly unchanged ($\eta_{minimax}$ close to zero) in scenario β for following two subcases and their corresponding equilibria:

I. airline A and airline B choose strategy T_{REF} and F_{REF} (-0,7%)

II. airline A and airline B choose strategy F_{REF} and CI (0,7%)

A system's cost that is almost zero denotes the equilibrium strategies achieve similar gains and losses for the corresponding players. The exact impact of those strategy choices for each subcase can be explained analysing those equilibria more deeply.

For scenario β, the equilibrium I implies a similar gain for B ($\Pi^{B}_{\min i \max} = -3{,}4\%$) as the loss of A ($\Pi^{A}_{\min i \max} = 4{,}6\%$) while the equilibrium II distributes the gains and losses the other way around. Due to the business strategy of airline B, the strategy T_{REF} and F_{REF} favours more its interest while strategy F_{REF} and CI clearly goes in line with airline A's business strategy.

For scenario γ, the system's cost is almost zero for following two subcases and their corresponding equilibria:

i. airline A chooses strategy T_{REF} and F_{REF} and airline B chooses strategy T_{REF} and CI (0,3%)

ii. airline A and airline B choose strategy F_{REF} and CI (0,6%)

For scenario γ, both equilibria result in a slightly better result for airline A. Although airline B's strategy for equilibrium i) goes in line with its business strategy, airline A's achieves more advantage with its strategy choice ($\Pi^{A}_{\min i \max} = -4{,}6\%; \Pi^{B}_{\min i \max} = 3{,}2\%$). In equilibrium ii), the strategy choice favours A against B as that choice mainly goes in line with airline A's business strategy ($\Pi^{A}_{\min i \max} = -2{,}1\%; \Pi^{B}_{\min i \max} = 1{,}4\%$).

6.8.5 Analysis of Results for Case 3

Considering case 3 for each of the three scenarios means that each airline has the complete game strategy tree at its disposal. Both players, at the same time, can provide for all three parameters values different from the inside parameter ones.

6.8.5.1 Airline Incentives

Analysing the equilibria obtained for this scenario, it is interesting to observe that the airlines always choose to communicate values different from the inside parameter ones.

Table 23 Equilibria for Case 3

3 $T_AF_ACI_AT_BF_BCI_B$	$\mathbf{T_{REFa}}$	$\mathbf{F_{REFa}}$	$\mathbf{CI_A}$	$\mathbf{T_{REFb}}$	$\mathbf{F_{REFb}}$	$\mathbf{CI_B}$
Scenario α	↓	↓	↓	↓	↓	↓
Scenario β	↓	↓	↓	↓	↓	↓
Scenario γ	↓	↓	↓	↓	↓	↑

By analysing the results shown in Table 23 in more detail, a certain peculiarity can be observed. Once the equilibriums have been determined, the strategy choice at each equilibrium seems to be dependent on the STAR flown, and not directly on the airline's business strategy.

Thus, in order to analyse this peculiarity more deeply, variants of the previously defined scenarios are introduced (see Tables 24, 25, and 26).

- Case 3a shows the impact of case 3 on the nominal description of all scenarios as defined in Section 6.7.2
- Case 3b shows the impact of case 3 on similar 3 scenarios. The difference relies on the STAR choice, no other modification is undertaking, the CI remain the same.
  - ⇒ in scenario α_b both flights fly STAR TERTO 3C (instead of both flying RUSIK 3C)
  - ⇒ in scenario β_b both fly the same STAR TERTO 3C (instead of both flying RUSIK 3C)
  - ⇒ in scenario γ_b airline A flies STAR TERTO 3C and airline B STAR RUSIK 3C (the other way around as originally defined)
- Case 3c analysis the impact of case 3 on the modified scenarios. The scenarios maintain the STAR choice as proposed by case 3b but in here also the CI are modified:
  - ⇒ in scenario α_c both flights fly STAR TERTO 3C (instead of both flying RUSIK 3C) and both have the cost index CI=70 (instead of CI=33)
  - ⇒ in scenario β_c both flights fly STAR TERTO 3C (instead of both flying RUSIK 3C) and have different cost indexes, CI_A=70 (instead of CI_A=33) and CI_B=33 (instead of CI_B=70)
  - ⇒ in scenario γ_c airline A's flight flies TERTO 3C has the cost index CI_A=70 and airline B's flight flies STAR RUSIK 3C and has the cost index CI_B=33

Table 24 Variants a, b and c for scenario α

Scenario α	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
α_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	RUSIK	30,5	760,49	33,5	798,52	33
α_b	A	TERTO	37	1027,32	40	1078.69	33
	B	TERTO	37	1027,32	40	1078.69	33
α_c	A	TERTO	37	1027,32	40	1078.69	70
	B	TERTO	37	1027,32	40	1078.69	70

Table 25 Variants a, b and c for scenario β

Scenario β	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
β_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	RUSIK	30,5	760,49	33,5	798,52	70
β_b	A	TERTO	37	1027,32	40	1078.69	33
	B	TERTO	37	1027,32	40	1078.69	70
β_c	A	TERTO	37	1027,32	40	1078.69	70
	B	TERTO	37	1027,32	40	1078.69	33

Table 26 Variants a, b and c for scenario γ

Scenario γ	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
γ_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	TERTO	37	1027,32	40	1078.69	70
γ_b	A	TERTO	37	1027,32	40	1078.69	33
	B	RUSIK	30,5	760,49	33,5	798,52	70
γ_c	A	TERTO	37	1027,32	40	1078.69	70
	B	RUSIK	30,5	760,49	33,5	798,52	33

Analysing all the equilibria for the different cases, following conclusions can be made regarding the strategy choice of the airlines:

- with independence of the business strategy, whenever the airline's flight covers STAR TERTO 3C, the strategy chosen is to provide the maximum possible value for CI and the minimum possible value for T_{REF} and F_{REF}.
- with independence of the business strategy, whenever the airline's flight covers RUSIK 3C, the strategy chosen is to provide the minimum possible value for all three parameters CI, T_{REF} and F_{REF}.

The standard terminal arrival route TERTO is longer than RUSIK. Depending on the type of route to be flown, the airline's communicated values vary. On a longer route as TERTO, the airlines prefer to favour the time flown by increasing the value of the communicated cost index while the communicated values for target maximum flight duration and maximum fuel consumption are decreased (T_{REF} and F_{REF}). On a shorter route as RUSIK, the airlines prefer to favour the fuel consumption by decreasing the value of the communicated cost index as well as T_{REF} and F_{REF}. The value of the cost index serves in this case to emphasis the importance of the time flown or the fuel consumption. It is interesting to realise that this behaviour is only route dependent. Further details on the results can be observed in Appendix B.

6.8.5.2 Analysis of System Impact

Scenario α is always, in any of its variants a zero sum game (see Table 27). For analysing the impact of the airlines strategy choices on the system, the focus stays on scenario β and γ and all its variants for this case 3.

Considering all the variants of scenario β (Table 28), the system gains the most in the equilibrium found for scenario β_b and scenario β_c (both times $\eta_{minimax}$= -3,4), and the system reaches almost zero sum in the equilibrium found for scenario β_a ($\eta_{minimax}$= -0,6). Comparing scenarios β_b and β_c, the only difference is the business strategy of both airlines. Thus the system remains with the same percentage benefit while the values for $\Pi^A_{minimax}$ and $\Pi^B_{minimax}$ are interchanged (see Appendix B). In all subcases analysed for scenario β the system's cost is either negative, meaning the system gains, or almost zero, meaning the system stays undisturbed. There is no variant where the system's cost is positive, meaning the system does not lose with any of the strategy choices of the airlines.

Considering all the variants of scenario γ (Table 29), the system gains the most in the equilibrium found for scenario γ_a and scenario γ_c (both times $\eta_{minimax}$= -3,9). In those two scenario variants, the only difference is again the business strategy of both airlines. Thus the system remains with the same percentage benefit while the values for $\Pi^A{}_{minimax}$ and $\Pi^B{}_{minimax}$ are interchanged (see Appendix B). Again, there is no variant for scenario γ_b where the system's cost is positive, meaning the system does not lose with any of the strategy choices of the airlines.

Table 27 Equilibria for scenario α, case 3, all variants

Scenario α	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B	$\eta_{minimax}$
α_a	↓	↓	↓	↓	↓	↓	0
α_b	↓	↓	↑	↓	↓	↑	0
α_c	↓	↓	↑	↓	↓	↑	0

Table 28 Equilibria for scenario β, case 3, all variants

Scenario β	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B	$\eta_{minimax}$
β_a	↓	↓	↓	↓	↓	↓	-0,6
β_b	↓	↓	↑	↓	↓	↑	-3,4
β_c	↓	↓	↑	↓	↓	↑	-3,4

Table 29 Equilibria for scenario γ, case 3, all variants

Scenario γ	T_{REFa}	F_{REFa}	CI_A	T_{REFb}	F_{REFb}	CI_B	$\eta_{minimax}$
γ_a	↓	↓	↓	↓	↓	↑	-3,9
γ_b	↓	↓	↑	↓	↓	↓	1,2
γ_c	↓	↓	↑	↓	↓	↓	-3,9

6.9 Conclusions

At the beginning of this chapter, three questions were raised that had to be answered:

a) do airlines have a benefit in providing untruthful information to fairness-oriented ANSPs?

b) if this is the case, what incentives do airlines have for doing so regarding what information or parameters?

c) to what extent would overall airlines' costs increase or decrease when they provide untruthful information?

Answer to a)

Within the game model proposed here, whenever the airlines can provide a value for T_{REF}, F_{REF} or CI different than the inside parameter value they do.

Applying the minimax approach to all possible combinations for each scenario and case, the following conclusions can be made:

If the airlines can only choose the value of a single parameter, they will always choose to communicate the minimum value possible.

If the airlines can choose the values of a combination of two parameters, independently of the route chosen, they will always communicate the minimum value possible for both parameters of the combination T_{REF} and F_{REF}. When the combination includes the cost index, the values chosen, with independence of the route (i.e. STAR), will be provided according to the cost index logic. For the combination CI and F_{REF}, the airlines communicate the minimum value possible, indicating that the fuel related cost gain in relevance and at the same time lowering the reference value for those fuel related cost. For the combination CI and T_{REF}, the airline communicate the maximum possible value for the CI, indicating that the time related cost gain in relevance and at the same time lowering the reference value of those time related cost.

If the airlines can choose the values of all three parameters, the strategy choice depends on the route to be flown. For a shorted route, the airlines choose to communicate for all parameters the minimum values possible, indicating that the fuel related cost have more relevance and at the same time lowering all the reference values. For a longer route, the airlines choose to communicate the maximum possible value for the cost index, indicating that the time related cost have more relevance and, at the same time, lowering the two reference values.

Answer to b)

The incentives to provide untruthful information to the ANSP is quantified in the present model by means of the auxiliary variable showing the percentage benefit for each player.

Airlines have clearly an incentive to provide in all three parameters (CI, T_{REF} and F_{REF}) values different from their inside parameter ones because those parameters directly affect the value of the saturated penalty cost they communicate to the ANSP. Following tables show the percentage benefit of each player in the different equilibria according to each case, scenario and corresponding subcase analysed. The answer to b) can be read directly from the tables detailed below. The airline's incentive to provide values different from the inside parameter ones are quantified by auxiliary variable Π. The airlines prefer to provide untruthful information for that parameter or parameter combination which in each scenario has the lowest value for Π.

Table 30 Player's incentives for equilibria in each subcase of case 1.1

Subcase	Scenario α	Scenario β	Scenario γ
1.1 T_A	Π^A= -41,6	Π^A= -45	Π^A= -45,7
1.1 F_A	Π^A= -2,9	Π^A= -3,4	Π^A= -3,5
1.1 CI_A	Π^A= -2,8	Π^A= -3,2	Π^A= -3,3
1.1 T_B	Π^B= -41.6	Π^B= -52,3	Π^B= -76,4
1.1 F_B	Π^B= -2,9	Π^B= -0,6	Π^B= -1,5
1.1 CI_B	Π^B= -2,8	Π^B= -1,3	Π^B= -1,1

In case 1.1 (see Table 30), both airlines, A and B, clearly prefer to provide for the parameter T_{REF} a value below their inside parameter one.

Table 31 Player's incentives for equilibria in each subcase of case 1.2

Subcase	Scenario α		Scenario β		Scenario γ	
	Π^A	Π^B	Π^A	Π^B	Π^A	Π^B
1.2 T_AT_B	0	0	22,8	-16,9	26,5	-18,4
1.2 T_AF_B	-39,1	39,1	-44,4	32,9	-44,6	31
1.2 T_ACI_B	-39,2	39,2	-43,8	32,4	-44,9	31,2
1.2 F_AT_B	39,1	-39,1	68	-50,3	73,5	-51
1.2 F_AF_B	0	0	-2,6	1,9	-2	1,4
1.2 F_ACI_B	-0,1	0,1	-1,7	1,2	-2,4	1,6
1.2 CI_AT_B	39,2	-39,2	68,1	-50,4	73,6	-51,1
1.2 CI_AF_B	0,1	-0,1	-2,4	1,8	-1,8	1,3
1.2 CI_ACI_B	0	0	-1,5	1,1	-2,2	1,5

As depicted in table above, in all scenarios, airline A prefers to provide a value below the inside parameter value for T_{REF}; airline A gains the most with this strategy choice and loses also the minimum. The results, on whether airline A gains or loses with this decision depend on airline B's strategy choice. Airline B also prefers, in all scenarios, to provide a value below its inside parameter one for T_{REF}, because it is with this strategy choice that B achieves its greates gain and smallest loss.

Table 32 Player's incentives for equilibria in each subcase of case 2

Subcase	Scenario α		Scenario β		Scenario γ	
	Π^A	Π^B	Π^A	Π^B	Π^A	Π^B
2 $T_AF_AT_BF_B$	0	0	4,6	-3,4	29,8	-20,7
2 $T_AF_AT_BCI_B$	-25,3	25,3	-7,2	5,4	-2,1	1,4
2 $T_AF_AF_BCI_B$	-56,7	56,7	-62,7	46,4	-63,2	43,9
2 $T_ACI_AT_BF_B$	25,3	-25,3	-25,8	34,6	60,1	-41,7
2 $T_ACI_AT_BCI_B$	0	0	22,8	-16,9	29,2	-20,3
2 $T_ACI_AF_BCI_B$	-36,6	36,6	-43,5	32,1	-44	30,6
2 $F_ACI_AT_BF_B$	56,7	-56,7	74,9	-55,3	96,2	-66,9
2 $F_ACI_AT_BCI_B$	36,6	-36,6	65,1	-48,2	72,8	-50,5
2 $F_ACI_AF_BCI_B$	0	0	-4,6	3,4	-4,6	3,2

Analysing the player's percentage benefit for case 2 (see Table 32), in all scenarios, airline A prefers to provide untruthful information for the parameter combination T_{REF} and F_{REF} independently of airline B's strategy choice.

In all scenarios, airline B has a greater incentive in providing untruthful information for the parameter combination T_{REF} and F_{REF}. In scenario α and γ, airline B's loss is also less with this strategy choice. Airline B's final result on whether it gains or loses depends on airline A's strategy choice. In scenario β, airline B's loss is less if the parameter combination T_{REF} and CI is chosen but, on the other hand, the gains are also a bit less compared to the ones that may be obtained from choosing T_{REF} and F_{REF}, as shown in the following table:

Table 33 Gains and losses for B's strategy choice T_{REF} and F_{REF} compared to T_{REF} and CI

Π_B for scenario β	T_AF_A	T_ACI_A	F_ACI_A
T_BF_B	-3,4	34,6	-55,3
T_BCI_B	5,4	-16,9	-48,2

According to the game definition, the players choose the strategy that minimises their worst results (minimax criterion). Thus, for scenario β, airline B chooses the parameter combination T_{REF} and CI.

Table 34 Player's incentives for equilibria in each variant of case 3

Variant	Scenario α		Scenario β		Scenario γ	
	Π^A	Π^B	Π^A	Π^B	Π^A	Π^B
Variant a	0	0	3,8	-2,8	28,2	-19,6
Variant b	0	0	17	-14	-5	4,4
Variant c	0	0	-14	17	-19,6	28,2

In case 3, each airline provides values for the three parameter T_{REF}, F_{REF} and CI different than their inside parameter values (see Table 34 above).

Following behaviour can be observed, whenever the preferences of the airline are the same and they are flying the same route (scenario α), then the incentives to provide untruthful information will be the same. In these situations, which can be observed in all cases analysed, the resulting percentage benefit of each player is equal to telling the truth.

The incentive of each player to provide values different than the inside parameter values depends on the corresponding case and scenario. In case 3, it also depends on the corresponding variant. Following tendencies can be deduced from the game results:

- whenever the preferred flight time T_P and fuel consumption F_P are the same for both airlines, then the airline with the highest cost index is more benefited from providing untruthful information.

- whenever the preferences are completely different, then the airline flying the longest route is more benefited from providing untruthful information

Fairness oriented ANSPs have to accommodate the required modifications to the airlines' preferred trajectories and the associated extra costs in the fairest possible way. In order to do so, the ANSP takes into account the saturated penalty costs for achieving similar relative penalty costs among all the airspace users involved.

If the fairness of a system has to be guaranteed, then it has to be made sure, that lying on the parameters that impact the strongest on P_{SAT} is not possible.

Remember the example of the slice of pizza given in Chapter 2 for illustrating the meaning of fairness. Both students are hungry but there is only one single slice of pizza left. On the one hand, they can divide it into two exact pieces. This way equity would be achieved. On the other hand, both students can agree on how to measure their hunger and divide the slice of pizza according to it. This way, the student that is hungrier would get a bigger part than the student that it not as hungry. This is a fair partition as long as both students define their hunger according to the agreed measure and provide the value truthfully.

In the game proposed, the rules imply that the airlines have to communicate their values for CI, T_{REF} and F_{REF}. The ANSP, among other things, calculates the resulting P_{SAT} for each airline. The penalty cost resulting from trajectory modifications will be compared relatively to the saturated penalty cost, as indicated by each airline. If the airlines have an incentive to not provide the required values truthfully, as it is the case, then the ANSP cannot guarantee the distribution of the incurred cost fairly among all airlines by using the saturated penalty cost value. Because of that, a fairness oriented ANSP is not able to effectively fulfil its function when airlines can communicate values different from the inside parameter ones for T_{REF} and F_{REF}.

Answer to c)

The equilibrium strategies predicted for each scenario and case do usually imply a gain or loss for the system's cost. Only the equilibria for scenario α do not have an impact on the system because that scenario is a zero sum game.

Communicating values different form the inside parameter values does not necessarily impact negatively on the system, it may as well be of benefit. The only thing that cannot be guarantee is the fairness of such a system or the fairness with which the ANSP act when deciding on the required trajectory modifications, as the ANSP does not have the mechanism to know whether truth is being said or not.

Following tables (Tables 35, 36, 37, and 38) show the system's gain or loss by quantifying the value of the system's percentage benefit $\eta_{minimax}$ at the equilibrium of each case, scenario

β and γ and corresponding subcases. Scenario α is not shown because for any case or subcase the systems percentage benefit is always zero (($\eta_{minimax}$ =0).

Table 35 System's percentage benefit for equilibria in each subcase of case 1.1

Subcase	Scenario β	Scenario γ
1.1 T_A	η= 7,3	η= 6,3
1.1 F_A	η= 0,5	η= 0,5
1.1 CI_A	η= 0,3	η= 0,5
1.1 T_B	η= -11,5	η= -10,5
1.1 F_B	η= -0,1	η= -0,2
1.1 CI_B	η= -0,3	η= -0,2

In case 1.1, the greatest impact on the system's cost is achieved when the player provide values different than their inside parameter value for T_{REF}. The system gains the most if airline B provides untruthful information for T_{REFb} and loses the most when airline A provides untruthful information for T_{REFa}.

Table 36 System's percentage benefit for equilibria in each subcase of case 1.2

Subcase	Scenario β	Scenario γ
1.2 T_AT_B	η= -3,7	η= -3,6
1.2 T_AF_B	η= 7,2	η= 6,1
1.2 T_ACI_B	η= 7,1	η= 6,1
1.2 F_AT_B	η= -11	η= -10,1
1.2 F_AF_B	η= 0,4	η= 0,3
1.2 F_ACI_B	η= 0,3	η= 0,3
1.2 CI_AT_B	η= -11	η= -10
1.2 CI_AF_B	η= 0,4	η= 0,3
1.2 CI_ACI_B	η= 0,2	η= 0,3

Between case 1.1 and case 1.2 the only difference is that both players can provide values different than their inside parameter values for only one parameter at the same time. The impact on the system's cost is similar as in the previous case 1.1, the system gains the most when airline B provides untruthful information for T_{REFb} and loses the most when airline A provides untruthful information for T_{REFa}.

Table 37 System's percentage benefit for equilibria in each subcase of case 2

Subcase	Scenario β	Scenario γ
2 $T_AF_AT_BF_B$	η= -0,7	η=-4,1
2 $T_AF_AT_BCI_B$	η= 1,2	η= 0,3
2 $T_AF_AF_BCI_B$	η= 10,2	η= 8,7
2 $T_ACI_AT_BF_B$	η=-5,6	η= -8,2
2 $T_ACI_AT_BCI_B$	η= -3,7	η= -4
2 $T_ACI_AF_BCI_B$	η= 7	η= 6
2 $F_ACI_AT_BF_B$	η= -12,1	η= -13,2
2 $F_ACI_AT_BCI_B$	η= -10,6	η= -10
2 $F_ACI_AF_BCI_B$	η= 0,7	η= 0,6

As shown in the table above, in both scenarios, the system gains the most for subcase $F_ACI_AT_BF_B$ and loses the most for subcase $T_AF_AF_BCI_B$. The system remains almost undisturbed when neither airline A or B provide a value different from their inside parameter value for T_{REF}, i.e. for subcase $F_ACI_AF_BCI_B$.

Table 38 System's percentage benefit for equilibria in each variant of case 3

Variant	Scenario β	Scenario γ
Variant a	η= -0,6	η= -3,9
Variant b	η= -3,4	η= 1,2
Variant c	η= -3,4	η= -3,9

In case 3, according to the results detailed in Table 38, the impact on the system's percentage benefit is substantially smaller than in the other cases analysed (case 1.1, 1.2 and 2). The greatest system's gain in case 3 is approximately three times smaller than the ones detailed for cases 1.1, 1.2, and 2. On the other hand, the greatest system's loss is approximately six to eight times less than in the other cases analysed.

Critical Discussion

For the game analysis as proposed in this chapter, it can be concluded that providing values different than the ones of the inside parameter values can be of benefit for the players involved as well as for the system. But it jeopardises the correct functioning of the ANSP as "ensurer" of fairness in the system.

The perturbation of having the players provide untruthful information on key parameters (T_{REF} and F_{REF}) that directly impact on the communicated P_{SAT} value has to be minimised to comply with the fairness definition. It has been proven that airlines do have incentives to provide values different from their inside parameter values, specially for the parameters T_{REF} and F_{REF}.

Recalling the example detailed in chapter two with the slice of pizza, in order to apply fairness, both friends have to agree on how to measure their hunger and provide their corresponding "hunger value" truthfully. If they cannot trust each other, then fairness cannot be guaranteed.

One of the main parameters for the fairness metric is the P_{SAT} value (see Chapter 4, Section 4.5). If it cannot be trusted that the P_{SAT} value handled by the ANSP to ensure fairness among all airspace user has been truthfully communicated by those same airspace users, then fairness in the ANSP's actions cannot be guaranteed.

As it can be concluded from the results summed in the tables detailed above, the impact of proving values for the cost index CI that are not equal to the inside parameter value is not significantly beneficial neither to the airlines nor to the system.

Main conclusion from the results obtained in this game analysis is following: to maintain the fairness of the system, the players cannot be allowed to communicate their reference values for T_{REF} and F_{REF}.

The conclusions gained from this analysis using game theory can be of value when fairness is to be applied to the current and future ATM system.

To achieve a fair system, a just framework has to be provided and a way agreed by all users on how to measure fairness. If it cannot be guaranteed that any user can use the system to her or his own benefit at the cost of others, then fairness cannot be considered.

The conclusions from the analysis presented for the proposed game can be extrapolated in their general sense to the ATM system. It can be concluded that the proposed fairness concept represented in the fairness metric defined in this work and the associated cost penalty function with its comprehensive set of assumptions and constraints, can be applied in the ATM system if following conditions can be ensured:

- airlines communicate to the ANSP for each flight their values for T_P, F_P and CI, which correspond to the airline preferences;
- a mean has to be defined by which the ANSPs can obtain the saturation cost P_{SAT} of each airline for each flight, i.e. the values for T_{REF} and F_{REF}, truthfully or at least objectively.

The analysis results show that the first condition can be achieved while the second condition is more complicated to fulfil as airlines clearly have incentives to not communicate their true values. Two options are possible to overcome this situation:

a) the airlines can be forbidden to lie on T_{REF} and F_{REF}
b) T_{REF} and F_{REF} can be defined by an neutral party taking into account certain criteria, e.g. route, flight time, type of aircraft.

Assuming an environment where airlines are forced to tell their true values is currently not as realistic as assuming that the ANSP has to get those values for T_{REF} and F_{REF} from the operational context for each aircraft model and each route covered; as referred to in SESAR [30].

Providing the reference values this way, enables organisations as EUROCONTROL or the FAA to define them according to objective criteria.

Regarding the operational concepts proposed by SESAR and NextGen, the aircraft will be required to update their communication, navigation, surveillance, and automation capabilities. According to the equipment of the aircraft, in the operational context different target maximum delays and extra fuel consumption can be defined.

So, for example, an aircraft with better capabilities could have values for the maximum acceptance levels that are lower as for aircraft with older equipment. Those older equipment may have worst suited capabilities for the new operational concepts. For these aircraft, achieving the same values for target minimum delay or extra fuel consumption is going to be more complicated as for aircraft with the adapted capabilities for the TBO. The values for the maximum acceptance levels, which ultimately result in lower or higher P_{SAT}, may be defined taken into account the equipment of the aircraft.

In other words, the investments made by each airline to modernise their fleet and adapt their aircraft to the new ATM system, could impact directly their value of P_{SAT} to be considered by the ANSP.

This way, authorities as EUROCONTROL and FAA can incentivise airlines to modernise their fleet by defining lower values for their reference values and recognise their investments.

7 ASSESSMENT OF FAIRNESS AND EQUITY IN TRAJECTORY BASED OPERATIONS

A brief review of the work described until this point may be of utility to present and better understanding of two examples for the assessment of fairness and equity in trajectory based operations. Those two examples have been defined and analysed according to the framework and methodologies described in Chapter 5. The first example is focused on the *a priori* assessment of fairness and equity while the second example evaluates, based on the *a posterior* methodology, three different conflict detection and resolution algorithms towards fairness and equity.

7.1 From the Concept to the Mathematical Expressions of Fairness and Equity

First step was to derive in Chapter 2 from other disciplines, such as philosophy, economics and sociology, the main features of the concepts of justice, fairness and equity that may be applied in ATM. Accordingly and under consideration of the specificities of ATM, the definitions for justice, fairness and equity in ATM were established.

Chapter 3 detailed the lifecycle of a flight from the idea to the flight plan execution for a better understanding of the relevance of the flight's costs on the airline's cost strategy. In Chapter 4, parting from the abstract concept definition, a concrete formalisation for fairness and equity was proposed and captured in a mathematical expression, namely the proposed metric for fairness and equity. Special care has been taken to ensure that metrics contain the essential elements of the conceptual definitions. Herefor, the properties of fairness and equity had to be identified, those properties captured mathematically to determine the required parameters, their concrete definition, assumptions their based on, constraints and boundaries.

In this sense, to develop the metrics a definition of cost penalty was provided based on a consistent cost model. After that, a comprehensive set of assumptions and constraints needed to be described to define the generic penalty function. Main requisite was that the penalty function integrated the airline's cost preferences for each single flight.

Associated to the penalty function, a comprehensive set of assumptions and constraints was described to frame the general model of a relative penalty function. The main difference between the absolute penalty function -or simply penalty function- and the relative penalty function is that the first one only takes into account the incurred cost penalty while the relative penalty function compares the incurred cost penalty to the airline's maximum acceptable costs.

Accordingly, main difference between the mathematical expression of equity and fairness is the use of the absolute or the relative penalty function respectively.

The equity metric is based on the penalty function aiming a similar distribution of the absolute penalty costs among all flights and/ or airlines analysed and penalising the dispersion of these costs. The equity metric is compliant with the concept of equity in ATM (Chapter 2).

On the other hand, the fairness metric is based on the relative penalty function aiming a similar distribution of the relative penalty costs among all flights and/ or airlines analysed and penalising its dispersion. This, again, is compliant with the concept of fairness in ATM (Chapter 2).

The cost model and the metrics have been defined in a generic way independent of the expressions of the penalty function and of the relative penalty function. Thus, the development of the metrics is self-contained.

7.2 Analysing and Applying Fairness and Equity Metrics

A particular penalty function was proposed as well as the associated relative penalty function. This way, results could be obtained to evaluate the appropriateness of the fairness and equity metric in specific examples. Other expressions of a penalty function could be used as long as they are compliant with the mathematical properties defined for the generic penalty function and the cost model in Chapter 4.

By means of the proposed expression for the penalty and the relative penalty functions, in Chapter 6 the behaviour of the airlines was analysed; namely their incentives in providing untruthful information in certain parameters that are key for calculating the airline's maximum acceptable cost penalty P_{SAT}. This also enabled to evaluate the impact on the system when information of those key parameters is manipulated.

From this analysis it was deduced which are the parameters that have to be specially monitored by the entities ensuring a just framework in ATM. Particularly for those parameters, airlines cannot be allowed to provide the values because they have incentives to manipulate the provided information to tailor the fairness considerations to their own advantage.

Until that point in the dissertation (i.e. chapter 6), each actor or airline represented in the conducted analyses operated a single flight. In this chapter, namely in the following Section 7.3, this vision is amplified, allowing each actor or airline to operate more than one flight. The general definitions of the penalty function as well as the relative penalty function had to be expanded to capture this feature. This enables to evaluate fairness between different flights as well as between different airlines.

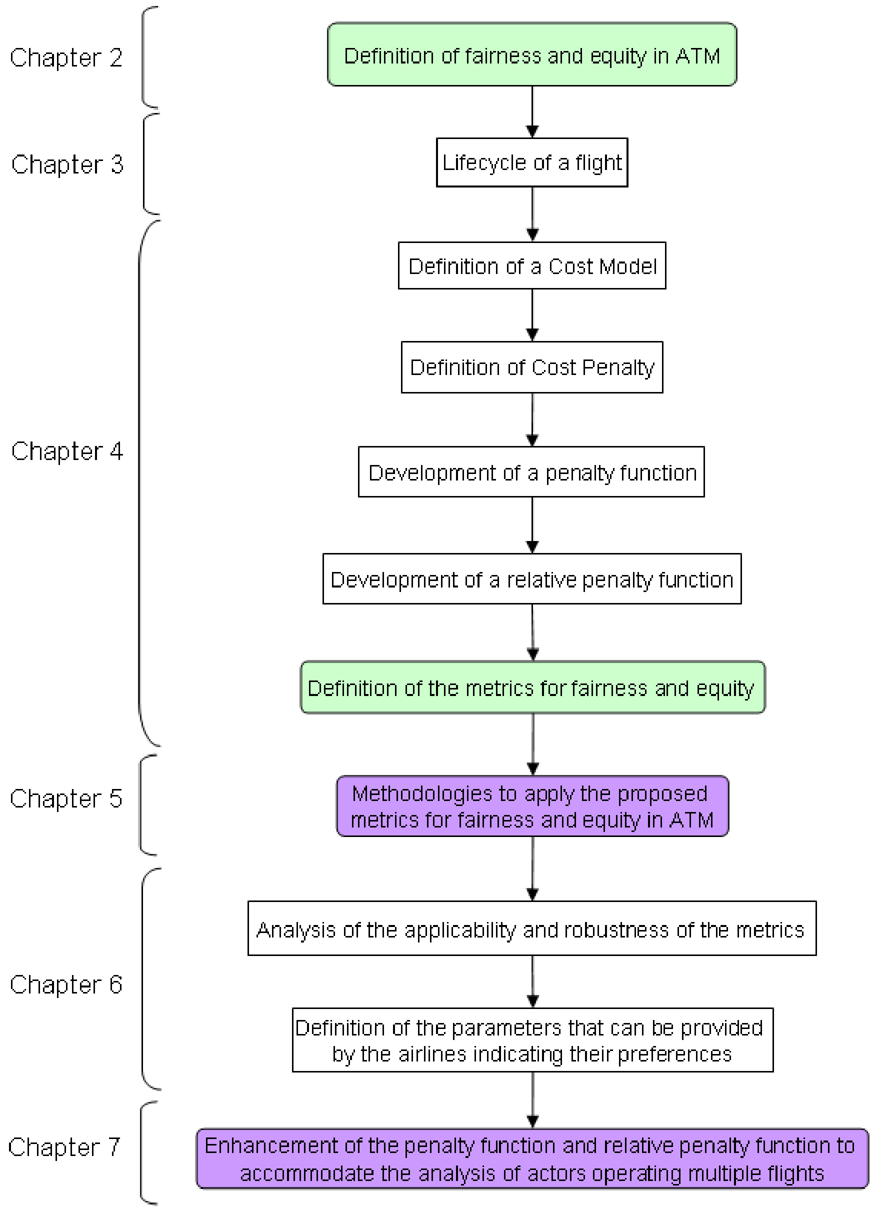


Figure 21 Development described in the dissertation up to this chapter

7.3 Required Enhancements to the Penalty Function

Assuming the mathematical expression of the penalty function proposed in this thesis (Equation 4.24), and to ensure the fairness concept described in this work, the airspace users (e.g. airlines) are required to provide to the ANSP certain values according to their preferences:

- the cost index CI for each flight, indicating their preferred weight between time and fuel related costs
- the preferred flight time T_P and preferred fuel consumption F_P for each flight.

These last two values can be provided directly by the user, or they can be deduced by the ANSP from the preferred trajectory of each flight. For this latter case, the airspace user has to facilitate the preferred trajectory for each of its flights.

Until now, the airspace user has been analysed in this work in a simplified way, namely represented by one single flight. But users as commercial and cargo airlines do usually operate several flights in one or more routes. The cost strategy of the airline is built up considering the cost strategy of each flight and the impact of that single flight costs on the overall airline’s cost strategy.

Thus, the penalty function and the relative penalty function, as developed in Chapter 4, need to be enhanced to capture the implications of several individual flight penalty functions on a final and global airline penalty function.

The extension of the penalty function from a “flight” level up to an “airline” level offers the airspace user the possibility to prioritise among its flights. The consideration of this airline specific “flight priority” by the ANSP can be beneficial for airlines, since it may lead to a reduction in the overall airline costs.

This section describes, the essential enhancements to consider cumulated flight costs at airline level.

7.3.1 The Enhanced Penalty Function

The enhancement proposed to the penalty function as described in Chapter 4 assume that each flight operated by an airline contributes to the airline total costs by a weighted summation of each flight costs.

a) Flight Weight w_i

For a given airline j operating n_j flights, the relative importance of each flight i on the airline's cost strategy is described by the weight $\boldsymbol{w_i}$, such that following condition applies:

$$\sum_{i=1}^{n_j} w_i = 1 \text{ where } i \text{ represents each single flight of airline j} \qquad (7.1)$$

This way, it is ensured that each airline can fix the corresponding flight's w_i which defines the impact of that flight's costs on the global costs of the airline.

The weight of each flight on the airline's cost strategy should be communicated to the ANSP together with CI, T_P and F_P indicating the airline's preferences. As it will be seen in the next section (7.4), w_i is an important indicator for the ANSP.

Fairness oriented ANSPs have the objective to maintain the indicated preferences as good as possible and distribute the penalty costs fairly among all parties involved, considering the safety of the whole traffic and the feasibility of the proposed solutions. The airlines have an incentive to provide these values (CI, T_P, F_P and w_i) to ensure that ANSPs receive their preferences and take them into account.

b) Cost Penalty of an Airline

The cost penalty of an airline j is equal to the sum of each cost penalty from each individual flight i of that same airline j multiplied by its corresponding weight on the airline's cost structure

$$P_j^{ARL} = \sum_{i=1}^{n_j} w_i P_i \text{ where } P_i \text{ is the cost penalty incurred by flight } i. \qquad (7.2)$$

c) Saturated Penalty of an Airline

Same way, the saturated penalty value of an airline j is defined as the weighted sum of all saturated penalty values of the airline's flights:

$$P_{SAT,j}^{ARL} = \sum_{i=1}^{n_j} w_i P_{SAT,i} \quad \text{where } P_{SAT,i} \text{ is the saturated penalty value of flight } i. \tag{7.3}$$

7.3.2 The Enhanced Relative Penalty Function

According to the definition of the airline's penalty cost and the associated saturated penalty, the airline's relative penalty is deduced as follows:

$$\varphi_j^{ARL} = \frac{P_j^{ARL}}{P_{SAT,j}^{ARL} + \kappa} = \frac{\sum_{i=1}^{n_j} w_i P_i}{\left(\sum_{i=1}^{n_j} w_i P_{SAT,i}\right) + \kappa} \tag{7.4}$$

where κ is a strictly positive value, $0 < \kappa << P_{SAT,j}^{ARL}$

7.4 *A Priori* Assessment of Fairness and Equity within an optimisation algorithm for ANSPs

The effort of communicating to the ANSP their preferences on the side of the airspace users would be of no help if, on the other side, the ANSP does not consider fairness in its optimisation process when deciding how to modify certain trajectories to ensure the safety of the ATM system.

Based on the proposed enhancements to the penalty function, an algorithm is presented explaining how the ANSP can include the fairness and/ or equity concept to optimise trajectory modifications while maximising fairness. This algorithm allows an *a priori* assessment of the possible amendments to the trajectories based on the fairness and equity metrics.

7.4.1 Integration of the Algorithm in an Automated System

According to the methodology described in Chapter 5 Section 5.2, the proposed algorithm describes a fairness oriented optimisation whenever it can be assumed that the values for P_{SAT} come from a trustful source, otherwise fairness considerations have to be substituted by equity based optimisation, i.e. use the equity metric instead of the fairness metric.

This algorithm is not tool-specific but applicable to any centralised and automated ground based system aimed at supporting the ANSP on the process of modifying trajectories. Examples of those systems are decision support tools (DSTs) for conflict detection and resolution to be applied in any flight phase as the ones described in the introduction or Appendix A (SARA [17], iFACTS [18], FPCF [19], MAESTRO [28], CTAS [27], etc). Those tools can integrate the proposed fairness optimisation algorithm after the resolution process to evaluate the proposed trajectory modifications, optimise those towards fairness, whenever possible, and achieve the fairest solution for all users involved.

Following figure shows where the fairness oriented optimisation algorithm is allocated within one typical DST including a process that modifies trajectories, as for example a Conflict Detection and Resolution (CD&R) process:

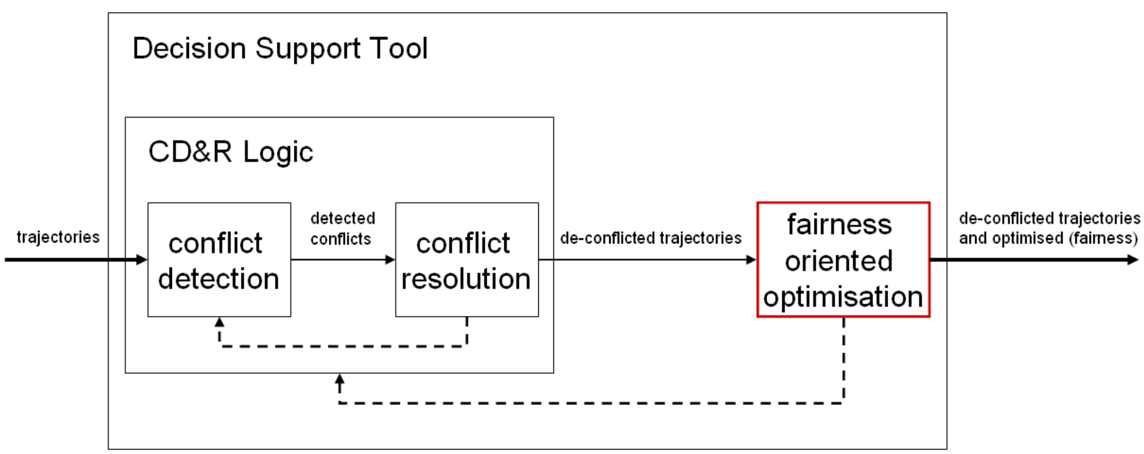


Figure 22 Proposal of the fairness oriented optimisation algorithm included in an automated ANSP tool

The fairness oriented algorithm incorporates the fairness concept as described in this dissertation. It optimises the fairness of a set of modified trajectories, with regard to the indicated user preferences, based on the fairness metric defined in Chapter 4.

7.4.2 Understanding the proposed Algorithm

Decision support tools, as the one presented in the figure above (Figure 22), take as their main input trajectories, specially for the conflict detection and resolution process. Those input trajectories are assumed to be the user-preferred trajectories at that very moment. Within the conflict detection and resolution process (CD&R), those trajectories are compared among each other searching for possible air traffic conflicts to predict those potential conflicts in advanced. Air traffic conflicts or aircraft conflicts are given when two or more aircraft violate or are predicted to violate the prescribed separation minima at any probable position in space or on the airport ground [97]. The separation minima are defined in time or in lateral, longitudinal or vertical separation [75]. If a conflict is detected, the resolution process, according to its internal logic, defines a new or modified set of trajectories for the aircraft involved to avoid the detected conflict or conflicts. In a typical decision support tool with a CD&R process, the output would be that new or modified set of trajectories.

The proposed fairness oriented optimisation algorithm is to be applied after the resolution process and before the decision support tool gives its output. Having as input the output trajectories of the CR process, the fairness oriented optimisation algorithm determines the required modifications to that set of trajectories in order to improve the resulting fairness among the airspace users involved and, consequently, also the fairness of the decision support tool.

The set of trajectories that represent the input to the decision support tool depend on the time window, also called look ahead time, determined for the CD&R process. That time window is established by the own decision support tool depending on the performance requirements. The time window or look ahead time can be seconds, minutes, hours or even moths [23].

The proposed fairness oriented optimisation algorithm can be used in any strategic automated tool that modifies trajectories and it is the strategic tool that determines with how much time in advance the optimisation is made. Thus, the set of trajectories to be analysed is defined inevitable by the time window determining the number of flights to be optimised and their corresponding airlines.

Altogether, the time window of each decision support tool defines the “system” that is going to be optimised by the proposed algorithm, meaning by “system” the number of flights, their trajectories and their corresponding airlines.

In order to understand the algorithm following sets have to be defined:

A) Set of Airlines

The set of airlines is the set composed of all airlines of the system. Each airline is represented by its element a_j. The total number of airlines is *m.*

$$\tilde{A} = \left\{ \forall a_j \Big|_{j=1\cdots m} \right\} = \bigcup_{j=1}^{m} a_j \qquad (7.5)$$

B) Set of Reviewed Airlines

The set of reviewed airlines is characterised by A.

$$A \subseteq \tilde{A} \qquad (7.6)$$

C) Set of all Airline’s Flights

The set of all flights of airline j is the set composed of all flights i of the airline j. Each flight is an element defined as:

$f_i^{\,j}$ is the flight i of airline j. This element represents a single flight for each airline j (with j=1…m) and within each airline j for all flights i (with i=1…n_j)

$$F_j^{ARL} = \left\{ \forall f_i^{\,j} \Big|_{i=1\cdots n_j} \right\} \qquad (7.7)$$

D) Set of all System's Flights

The set of all flights of the system is the set composed of all flights I of the given system currently under analysis.

$$\tilde{F} = \left\{ \forall f_i^{\,j} \Big|_{i=1\cdots n_j,\, j=1\cdots m} \right\} = \bigcup_{j=1}^{m} F_j^{ARL} \quad (7.8)$$

E) Set of Reviewed Flights

The set of reviewed flights is characterised as F.

$$F \subseteq \tilde{F} \quad (7.9)$$

For further comprehension of the algorithm, one has to take into account following definitions:

α) Relative Penalty of the System

The system's relative penalty is defined as the average relative penalty of all m airlines and is calculated as:

$$\overline{\varphi}^{SYS} = \frac{\sum_{j=1}^{m} \varphi_j^{ARL}}{m} \quad (7.10)$$

β) Airline's Deviation

The deviation of the relative penalty of airline j with respect to the system's relative penalty is defined as:

$$d_j = \left| \overline{\varphi}^{SYS} - \varphi_j^{ARL} \right| \quad (7.11)$$

γ) Flight's Deviation

The deviation of the relative penalty of flight i with respect to the airline's relative penalty is defined as:

$$d_i = \left| \varphi_j^{ARL} - w_i \varphi_i \right| \quad (7.12)$$

7.4.3 Detailed Description of the Algorithm

The proposed algorithm starts gathering the de-conflicted trajectories as proposed by the CD&R tool together with the airline preferences indicated for each flight (CI, T_P, F_P, w_i). The values required to calculate P_{SAT} from each flight are extracted from the operational concept as provided by EUROCONTROL [30].

As it can be seen in the figure depicted below (Figure 23), the first step is to determine the fairness of the preliminary solution, as it came out of the CD&R process. Only when the fairness of that solution is not already at its maximum, the algorithm can continue. Otherwise, there is no room for fairness improvement.

Next step is to select the airline which shows the greatest deviation from the system's relative penalty, i.e. greatest d_j. Accordingly, within that same airline, the algorithm searches for the flight which has the greatest deviation of its relative penalty value compared with its airline's relative penalty, i.e. greatest d_i.

The relative penalty is the fundamental parameter of the fairness metric. Because of that, by selecting the airline with the worst relative penalty ratio and, within that airline, the flight with the worst relative penalty ratio, the improvement of that single flight has a significant impact on the overall fairness of the solution. Fairness decreases with the inequalities in the relative penalty and increases as the relative penalty approach similar values.

In the fairness oriented optimisation process, only the trajectory of that single flight is modified ensuring that the new trajectory remains conflict-free and improves its relative penalty value, thus reducing its deviation d_i. If a new trajectory with these constraints, conflict free and reduced relative penalty value, is found, then the fairness of the whole solution is calculated taking into account this new modified trajectory.

Otherwise, if it is not possible to find a new trajectory that remains conflict free while optimising its relative penalty value, then the flight currently under analysis is added to the set of reviewed flights. As long as the set of reviewed flights does not equal the set of all flights of the system, the optimisation process can continue.

If the set of reviewed flights is different than the set of all flights of the airline at hand, then a new flight of that same airline is selected, namely exactly the flight with the next greatest deviation d_i or, in other words, with the worst relative penalty ratio.

If the set of reviewed flights equals the set of all flights of the airline at hand, then a new airline is selected. The criterion for the selection is the same as before, to choose the airline with the next greatest deviation d_j. As long as the set of reviewed airlines does not equal the set of all airlines of the system, the optimisation process can continue.

Whenever a feasible trajectory is found for the selected flight, then the sets of reviewed flights and reviewed airlines are reset.

To use this algorithm for equity based optimisation instead of for fairness based optimisation, the algorithm has to be adapted. The equity metric is to be used instead of the fairness metric and the corresponding values resulting from the absolute or simply penalty function instead of the relative penalty function.

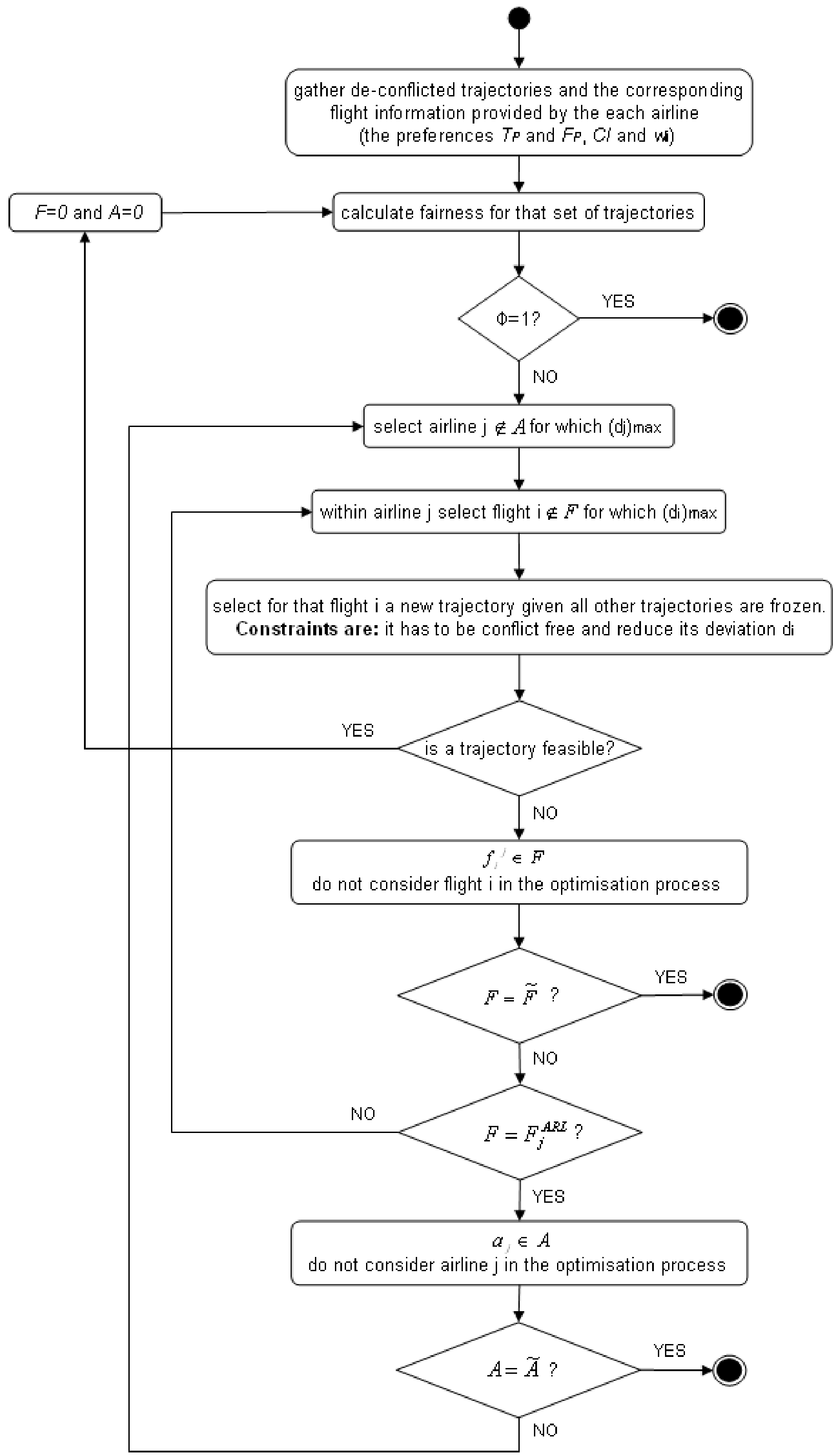


Figure 23 Fairness oriented optimisation algorithm for ANSP

7.5 *A Posteriori* Assessment of Fairness and Equity within the Evaluation of three CD&R Algorithms

This section provides an example on how the *a posteriori* methodology could be implemented for the analysis and evaluation of different conflict detection and resolution tools. The intention here is not to assess the effectiveness of those tools detecting and resolving potential conflicts, neither to evaluate them based on their performance. The aim is to analyse the resolution trajectories they produced based on fairness and equity.

Three different algorithms resolving the same traffic situation in the same scenario are compared. Those three algorithms were developed for the ATLANTIDA Project, "*Application of Leading Technology to Unmanned Aerial Vehicles for Research and Development in ATM*". This three year project was sponsored by the Spanish Government within the CENIT Programmes (CEN 20072008) under the leadership of BR&TE.

7.5.1 The Three Algorithms under Analysis

The three algorithms for conflict detection and resolution have been developed within the ATLANTIDA project by three different partners. One of the algorithms was developed by INDRA, a Spanish company specialised in ATC systems [98]. The second algorithm was developed by the Engineering School of the Universidad Autónoma de Barcelanoa (UAB) together with Boeing Research and Technology Europe (BR&TE) and published at the 4th International Conference on Research in Air Transportation conference [99] and is expected to be published also in the journal *Transportation Research Part C: Emerging Technologies*. The third and last algorithm was developed by the Engineering School of the Universidad de Sevilla (AICIA) together with Boeing Research and Technology Europe (BR&TE). An abstract on this algorithm was submitted to the AIAA ATIO 2011 conference [100] and publication is still pending.

All three conflict detection and resolution (CD&R) algorithms have different CD&R logics. These logics are not going to be explained in detail since the logic of each algorithm does not add any additional information for the evaluation regarding fairness or equity. Only the common characteristics are described in this section.

The input information, detailed in following Section 7.5.3, is the same for the three algorithms as well as the scenario and traffic situation. Subsequent sections describe in detail the scenario applied as well as the input information. The user preferred trajectories (UPTs) are known twenty minutes before the aircraft enter the TMA through the assigned TMA entry points. The TMA entry point determines the standard terminal arrival route (STAR) that will be flown.

In those twenty minutes where the UPT is known, the algorithms have to perform their conflict detection and resolution logic. All algorithms use the same separation minima to predict the potential conflicts among the UPTs given the TMA entry times specified in the input file. Those separation minima are detailed in the table below and have been extracted from ICAO's document 4444 [75]:

Table 39 separation minima applied by all three algorithms

Separation minima [nm]		Aircraft preceding		
		Heavy	**Medium**	**Light**
Aircraft following	**Heavy**	4	3	3
	Medium	5	3	3
	Light	6	4	3

Because the three algorithms use the same trajectory computation infrastructure (TCI) developed by BR&TE, managing so the same trajectory information as well as the same separation minima, this leads to all three algorithm detecting the same potential conflicts. Figure 24 depicts the interactions of the CD&R module with the TCI. That CD&R module can be substituted by any of the three CD&R algorithms under analysis.

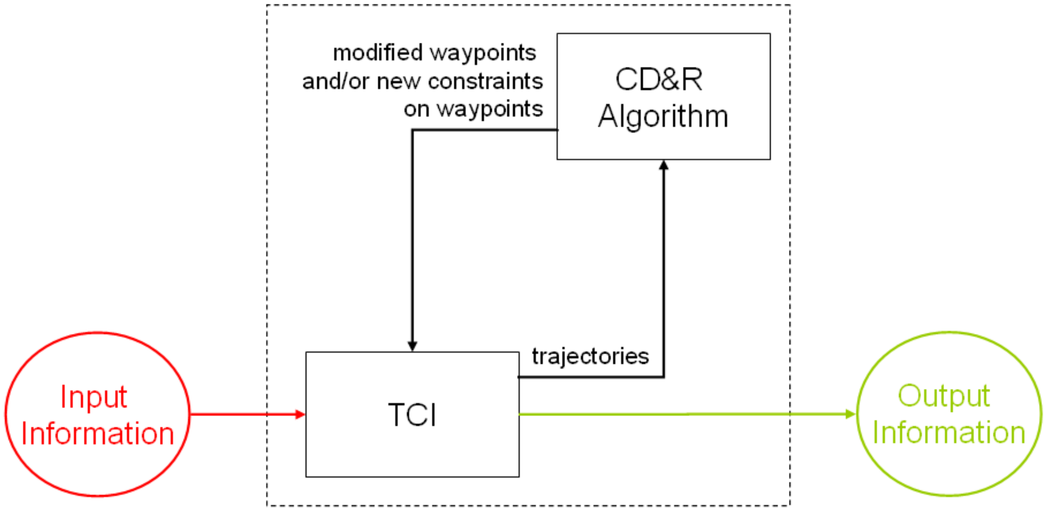


Figure 24 Interaction between TCI and CD&R algorithm(s)

The resolution manoeuvres proposed by each of the algorithms to resolve the potential detected conflicts have to be expressed in the same format, to be computed by BR&TE's TCI. Thus, the output information of the three algorithms is the same. That output format includes modified or new waypoints described by their latitude and longitude coordinates, speed and altitude constraints on waypoints already contained in UPTs or on the new or modified waypoints as results of the resolution manoeuvres.

As part of the scenario (Section 7.5.2), a restriction is imposed to all three algorithms, namely all trajectory modifications resulting from the resolution logic have to be comprehended within the trajectory segment starting at the assigned TMA entry point until ENETA, where the approach begins.

All three algorithms have as one of their optimisation objectives to deviate the flights the least possible from their preferred trajectories.

7.5.2 Scenario Definition

The scenario used for this analysis is similar to the simulation set-up described for the decision theory analysis in Chapter 6 (Section 6.7 *Simulation Set-Up*). Based on the information obtained from the Spanish AIP [91] for Gran Canaria's TMA, the aircraft fly two STARs that are defined therein, namely TERTO3C and RUSIK3C. A new STAR was defined to add complexity to the scenario for conflict detection and resolution. That STAR is called NWPT3C. For this aim, the TMA of Canarias is modified with an additional entry point (NWPT), placed South-East of waypoint FAYTA, at φ =27º40'00.00"N and λ=13º30'00.00"W. With this set-up, the scenario has two merging points, FAYTA between the STARs RUSIK3C and NWPT3C and CANIS between TERTO3C and the merged route RUSIK3C-NWPT3C.

Following figure shows schematically the three STARs with their corresponding merging points, FAYTA and CANIS, as well as ENETA, the initial approach fix (IAF):

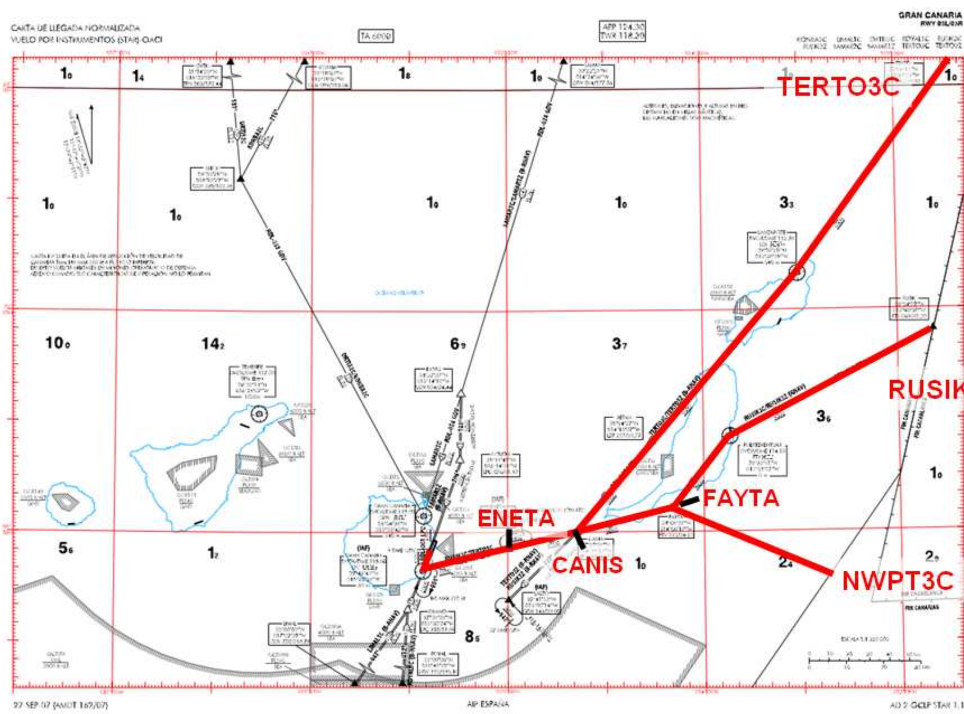


Figure 25 Scenario Set-Up

The approach procedure to RWY 03L is the only and same for all three STARs, starting at ENETA, the immediate waypoint after CANIS, where all three STARs converge. Figure 26 details the approach starting at ENETA.

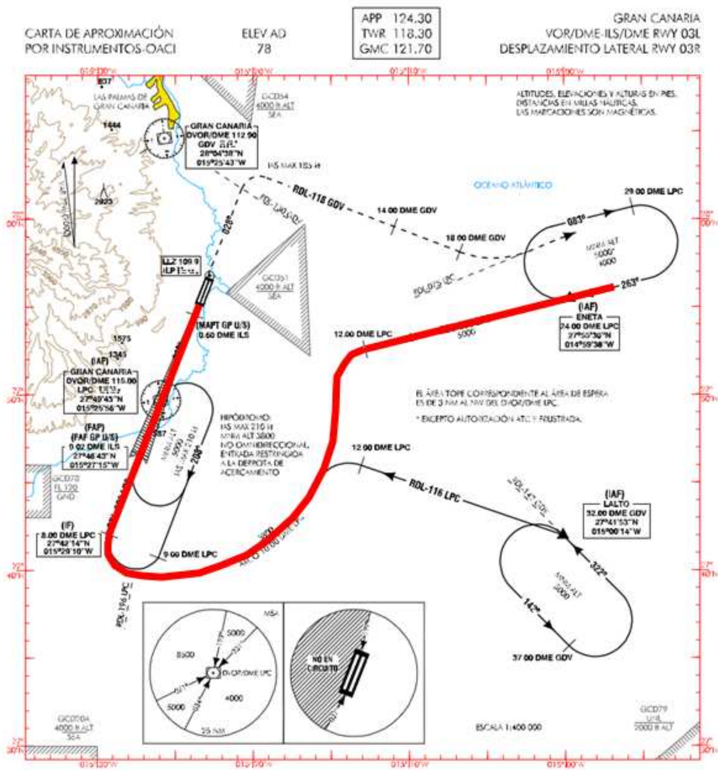


Figure 26 Approach procedure for all three STARs

The comparison of the fairness and equity of the resolution trajectories is based on the proposed penalty function and relative penalty function as described in Chapter 4, Section 4.3.1 *The Proposed Penalty Function*, Equations 4.24 and 4.25.

$$P = C_F \sqrt{CI^2 (T_M - T_P)^2 + (F_M - F_P)^2} \tag{7.13}$$

$$\wp = \frac{P}{P_{\max} + \kappa} = \frac{P}{P_{SAT} + \kappa} \tag{7.14}$$

where κ is a strictly positive value much smaller than P_{SAT}; $0 < \kappa << P_{SAT}$

The saturated penalty value is calculated according to Equation 4.20 also detailed in Chapter 4, Section 4.3.1 *The Proposed Penalty Function*.

$$P_{SAT} = C_F \sqrt{CI^2 (T_{REF} - T_P)^2 + (F_{REF} - F_P)^2} \tag{7.15}$$

As proposed in Chapter 6, Section 6.7.1 *Definition of Parameters* under point d), the reference values for T_{REF} and F_{REF} are obtained from the time and fuel performance target values defined by SESAR for European/continental flights [30]:

- maximum of 3minutes delay with regard to preferred flight time T_P
- maximum of 5% increase in the preferred fuel consumption F_P.

Also described in the same section but under point b) fuel price C_F is assumed to be the same for all airlines, and corresponding flights, and has the value of 0,52€/kg.

7.5.3 The Input Information

The input information for the three algorithms described in this section is extracted from the ATLANTIDA project. The input file detailed here was executed by the three algorithms in the same scenario with the same constraints. Therefore, it represents the basis to compare *a posteriori* the results of the three algorithms under the same conditions.

The input file handled by all three CD&R algorithms is the same and consists of twenty flights from five different airlines arriving through the three afore mentioned possible STARs to Gran Canaria's airport. All user preferred trajectories (UPTs) are Continuous Descent Approaches (CDAs) for each of the possible STARs. The UPTs are known to all three algorithms beforehand.

All trajectories, i.e. the input trajectories, these being the UPTs, and the trajectories resulting from the proposed modifications of each algorithm, are computed by the same trajectory computation infrastructure (TCI). That TCI [92] is Boeing Research and Technology Europe's own developed TCI based on the aircraft performance model described in BADA 4.0 (Base of Aircraft Data).

From the data contained in the UPTs, the preferred flight time and fuel consumption can be extracted, i.e. T_P and F_P, for each flight according to the STAR flown. Those preferred values only refer to the flight duration and fuel consumption from TMA entry until touch down at the airport's runway (RW03L).

The aircraft models simulated are all Boeing models, 737-800 W26 and 777-300 PW90. In the scenario at hand, STAR NWPT3C can only be flown by 777-300 PW90 aircraft models. STARs TERTO3C and RUSIK3C can only be flown by 737-800 W26 aircraft models. Thus, in this scenario, the traffic is segregated depending on the aircraft model.

At the same TMA entry points, i.e. TERTO, RUSIK or NWPT, the flights have the same initial conditions describing the aircraft state at those exact points, namely their 4D position. The relevant parameters of the initial conditions are summarised below:

Table 40 Initial condition for each of the TMA entry points

Initial Conditions	TERTO (737-800)	RUSIK (737-800)	NWPT (777-300)
Mass [kg]	51971	51971	180000
Mach [-]	0,78	0,78	0,78
Pressure Altitude H_P [m]	9144	9144	9144

The time at which each of the twenty flights reaches its corresponding TMA entry point is extracted from the input file (Table 41).

Table 41 The input file

Callsign	Airline	Sequence at TMA entry	Scheduled Time at TMA entry point hh:mm:ss	TMA entry point
ATLS001	EDW	1	0:09:56	RUSIK
ATLS002	DE	2	0:10:36	TERTO
ATLS003	JK	3	0:13:46	TERTO
ATLS004	TOM	4	0:20:06	NWPT
ATLS005	EDW	5	0:20:06	RUSIK
ATLS006	EDW	6	0:22:56	RUSIK
ATLS007	DE	7	0:24:56	TERTO
ATLS008	EDW	8	0:25:36	RUSIK
ATLS009	JK	9	0:27:16	TERTO
ATLS010	EDW	10	0:31:46	NWPT
ATLS011	DE	11	0:38:36	TERTO
ATLS012	JK	12	0:39:56	NWPT
ATLS013	DE	13	0:43:06	TERTO
ATLS014	TOM	14	0:46:26	RUSIK
ATLS015	JK	15	0:50:26	RUSIK
ATLS016	TOM	16	0:52:16	TERTO
ATLS017	IB	17	0:55:36	TERTO
ATLS018	TOM	18	0:59:16	TERTO
ATLS019	IB	19	0:59:56	NWPT
ATLS020	TOM	20	1:01:46	RUSIK

Following table shows the preferred values and the reference values for each of the STARs. The values for TERTO3C and RUSIK3C are the same as those presented in Chapter 6, Section 6.7.1.The values for NWPT3C have been computed accordingly; assuming the preferred trajectory for the given initial conditions at NWPT is a CDA.

Table 42 Preferred and reference values depending on the STAR assigned

STAR	T_P	F_P	T_{REF}	F_{REF}
TERTO3C (737-800)	2212s = 36min 52s	1027,321kg	T_P + 180s = 2392s	F_P + 5% = 1078,687kg
RUSIK3C (737-800)	1833s = 30min 33s	760,493kg	T_P + 180s = 2013s	F_P + 5% = 798,518kg
NWPT3C (777-300)	1518s = 25min 18s	1041,901kg	T_P + 180s = 1698s	F_P + 5% = 1093,996kg

Still missing is the information concerning the Cost Index of each flight. This information is not needed by any of the three algorithms as input information. Thus, it is added later in the following Section 7.4.4 *Analysis Result*. As it will be described there, the algorithms do not include the proposed fairness concept or any fairness consideration in their resolution algorithms (i.e. no a priori consideration of fairness).

7.5.4 Analysis Results

To start the analysis, the output data of the three CD&R algorithms needs to be gathered. That output contains the information on which trajectories have been modified and the required amendments to solve the predicted conflicts. Those amendments are expressed in form of speed and/or altitude restrictions on waypoints as well as modified waypoint coordinates or new waypoints that have to be added to the nominal path. The trajectories that have to be modified and the actual modifications depend on the resolution logic implemented in each of the algorithms.

The output data from each conflict resolution algorithm is passed to the trajectory computation infrastructure (BR&TE's TCI) which interprets the proposed amendments and recalculates the trajectory. That new trajectory is referred to as resolution trajectory.

The preferred values T_P and F_P result from user preferred trajectory and are included in the input file. In the following analysis, the user preferred trajectory is referred to as the nominal trajectory in contrast to the resolution trajectory which is the one resulting from the CD&R process.

7.5.4.1 The Cost Index Values

Neither algorithm takes into account the cost index of each flight but it is necessary to define the cost index of each flight to calculate the cost penalty.

As already mentioned in Chapter 6, Section 6.7.1 *Definition of Parameters* under point c), typical cost index values for a 737-800 are between 9kg/min and 100kg/min. According to [81] typical CI values for a 777-300 are around 60kg/min and 120kg/min. Among the airlines specified in the input file, there are three charter airlines and two regular airlines. Details are shown in Table 43 below:

Table 43 Charter and Regular Airlines

Code	Airline	Type	Nation
EDW	Edelweiss	Charter	Switzerland
DE	Condor	Charter	Germany
TOM	Thomas Cook	Charter	United Kingdom
JK	Spanair	Regular	Spain
IB	Iberia	Regular	Spain

For the charter airlines the cost index values have been defined as detailed below, assuming these charter airlines operate their flights in a similar way:

CI_{737}=33[kg/min] and CI_{777}=85[kg/min]

The regular airlines have slightly higher CI values for the same aircraft models.

CI^{JK}_{737}=36[kg/min] and 76[kg/min] and CI^{JK}_{777}=96[kg/min]

CI^{IB}_{737}=76[kg/min] and CI^{IB}_{777}=110[kg/min]

Those exact values for the CI are arbitrarily chosen based on the typical values described in [81] and following the rational that regular airlines are usually stronger penalized by time delay than charter airlines, i.e. $CI_{regular} > CI_{charter}$.

The table shown below details for each flight it's indicated preferences that are taken into account for calculating the corresponding cost penalty.

Table 44 Flight preferred values and indicated cost index

Callsign	Airline	CI	Tp (min)	Fp (kg)
ATLS001	EDW	33	30,5	760,49
ATLS002	DE	33	37	1027,32
ATLS003	JK	36	37	1027,32
ATLS004	TOM	85	25,3	1041,9
ATLS005	EDW	33	30,5	760,49
ATLS006	EDW	33	30,5	760,49
ATLS007	DE	33	37	1027,32
ATLS008	EDW	33	30,5	760,49
ATLS009	JK	36	37	1027,32
ATLS010	EDW	85	25,3	1041,9
ATLS011	DE	33	37	1027,32
ATLS012	JK	96	25,3	1041,9
ATLS013	DE	33	37	1027,32
ATLS014	TOM	33	30,5	760,49
ATLS015	JK	76	30,5	760,49
ATLS016	TOM	33	37	1027,32
ATLS017	IB	76	37	1027,32
ATLS018	TOM	33	37	1027,32
ATLS019	IB	110	25,3	1041,9
ATLS020	TOM	33	30,5	760,49

7.5.4.2 Calculating Fairness and Equity

First step is to calculate the resulting penalty cost for each flight. According to the reference values T_{REF} and F_{REF} detailed for each STAR in Table 42, the P_{SAT} values for each flight can be determined applying equation (7.15). Together with the output file of each algorithm, the resolution trajectories are computed with BR&TE's TCI. From that resolution trajectory, the modified values for flight duration and fuel consumption, T_M and F_M are obtained. Inserting those values in equation (7.13) provides the cost penalty values. If the calculated cost penalty of a certain flight is greater than the corresponding P_{SAT} value, then the P_{SAT} value has to be chosen as the final cost penalty value. This is compliant with the assumptions and constraints defined for the cost penalty and the penalty function in Chapter 4.

Next step is to obtain for each flight the relative penalty value according to equation (7.14). In this analysis the value of κ, which has to be much smaller than the P_{SAT} value, is assumed for all flights to be κ=0,01.

Following Tables 45, 46, and 47 show the resulting values for the three algorithms:

- the saturated cost penalty value for each flight (P_{SATi}), which is the same for the three algorithms,
- the resulting cost penalty value for each flight (P_i), which is dependent on the algorithm's output
- the corresponding relative penalty value for each flight ($\wp_i$), which is dependent on the algorithm's output

Table 45 Results for AICIA's algorithm

Callsign	Airline	P_{SAT}	P	φ
ATLS001	EDW	55,148	0,000	0,0000
ATLS002	DE	57,998	25,218	0,4347
ATLS003	JK	62,189	0,000	0,0000
ATLS004	TOM	135,339	0,000	0,0000
ATLS005	EDW	55,148	2,699	0,0489
ATLS006	EDW	55,148	29,465	0,5342
ATLS007	DE	57,998	16,136	0,2782
ATLS008	EDW	55,148	55,148	0,9998
ATLS009	JK	62,189	0,000	0,0000
ATLS010	EDW	135,339	135,339	0,9999
ATLS011	DE	57,998	57,998	0,9998
ATLS012	JK	152,191	152,191	0,9999
ATLS013	DE	57,998	57,998	0,9998
ATLS014	TOM	55,148	0,000	0,0000
ATLS015	JK	120,198	120,198	0,9999
ATLS016	TOM	57,998	57,998	0,9998
ATLS017	IB	121,532	11,171	0,0919
ATLS018	TOM	57,998	48,778	0,8409
ATLS019	IB	173,725	62,305	0,3586
ATLS020	TOM	55,148	55,148	0,9998

Table 46 Results for INDRA's algorithm

Callsign	Airline	PSAT	P	φ
ATLS001	EDW	55,148	0,000	0,0000
ATLS002	DE	57,998	0,000	0,0000
ATLS003	JK	62,189	62,189	0,9998
ATLS004	TOM	135,339	0,000	0,0000
ATLS005	EDW	55,148	0,000	0,0000
ATLS006	EDW	55,148	55,148	0,9998
ATLS007	DE	57,998	0,000	0,0000
ATLS008	EDW	55,148	55,148	0,9998
ATLS009	JK	62,189	0,000	0,0000
ATLS010	EDW	135,339	135,339	0,9999
ATLS011	DE	57,998	0,000	0,0000
ATLS012	JK	152,191	152,191	0,9999
ATLS013	DE	57,998	0,000	0,0000
ATLS014	TOM	55,148	0,000	0,0000
ATLS015	JK	120,198	0,000	0,0000
ATLS016	TOM	57,998	0,000	0,0000
ATLS017	IB	121,532	121,532	0,9999
ATLS018	TOM	57,998	57,998	0,9998
ATLS019	IB	173,725	0,000	0,0000
ATLS020	TOM	55,148	0,000	0,0000

Table 47 Results for UAB's algorithm

Callsign	Airline	PSAT	P	φ
ATLS001	EDW	55,148	21,368	0,3874
ATLS002	DE	57,998	3,753	0,0647
ATLS003	JK	62,189	62,189	0,9998
ATLS004	TOM	135,339	81,136	0,5995
ATLS005	EDW	55,148	21,368	0,3874
ATLS006	EDW	55,148	16,772	0,3041
ATLS007	DE	57,998	3,753	0,0647
ATLS008	EDW	55,148	22,404	0,4062
ATLS009	JK	62,189	3,934	0,0632
ATLS010	EDW	135,339	135,339	0,9999
ATLS011	DE	57,998	3,753	0,0647
ATLS012	JK	152,191	152,191	0,9999
ATLS013	DE	57,998	3,753	0,0647
ATLS014	TOM	55,148	22,404	0,4062
ATLS015	JK	120,198	32,816	0,2730
ATLS016	TOM	57,998	3,753	0,0647
ATLS017	IB	121,532	121,532	0,9999
ATLS018	TOM	57,998	45,230	0,7797
ATLS019	IB	173,725	104,182	0,5997
ATLS020	TOM	55,148	21,368	0,3874

The fairness and equity of the three solutions can be determined by applying the fairness and equity metric defined in Chapter 4 (Section 4.5 and Section 4.6, Equations 4.38 and 4.41) based on the calculated values for P_i and $\wp_i$.

$$\Phi = \frac{\left(\prod_{i=1}^{n}(1-\wp_i)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}(1-\wp_i)} \cdot n \tag{7.16}$$

$$\mathrm{E} = \frac{\left(\prod_{i=1}^{n}(P_i+e)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}(P_i+e)} \cdot n \tag{7.17}$$

Fairness and equity are determined comparing the set of resolution trajectories that are the solution proposed by each algorithm for the twenty flights defined in the input file. To compute the equity value, the value of e, which has to be much smaller than the penalty cost value, is assumed for all flight in all three algorithms to be e=0,01.

As depicted in the next table, the algorithm achieving the fairest solution is the one proposed by UAB, which also has the most equitable solution. The algorithms proposed by AICIA and INDRA score very low in fairness. Regarding equity, Indra's algorithm has the lowest value.

Table 48 Fairness and Equity results for each algorithm

	AICIA	**INDRA**	**UAB**
Fairness	0,048	0,065	0,198
Equity	0,116	0,007	0,489

7.5.5 Discussion of Results

The three algorithms analysed achieve very low values for both their equity and fairness. Considering all three cases, fairness reaches a maximum of 20% and a minimum of 5%, while equity results slightly above reaching a maximum of 50% and a minimum of 1%.

The algorithm proposed by UAB is the one that better distributes the cost penalty (equity value), but it does not take into account the saturated cost penalty of each flight, which forces the resulting fairness value to be lower then the equity value. Regarding Table 47, the values clearly show that all flights are penalised, although some flights suffer very low cost penalties, 4 flights out of 20 reach their saturated penalty cost. Disregarding these inequalities, this algorithm is still the one that proposes the better solution regarding fairness and equity.

The algorithm proposed by AICIA results in an extremely low fairness while the equity value is slightly better. This denotes that the penalty cost values are distributed among the flights but the saturated penalty cost values are not taken into account. Recalling Table 45, out of 20 flights the algorithm penalises 15 flights while 5 flights are able to fly their preferred trajectory without any disturbance. Out of the 15 flights that incurred in penalty cost, 8 reach their saturated penalty cost value.

The algorithm proposed by INDRA has the lowest equity value, almost zero (E=0,007), and the fairness value is also extremely low. This algorithm tries to maintain the preferred trajectory for as many flights as possible. In this sense, it demonstrates to be the best out of the three algorithms. As shown in Table 46, out of 20 flights 13 are able to execute their preferred trajectory with zero penalty cost. On the other hand, the remaining 7 flights reach their saturated penalty cost. This leads to extreme inequities among the set of flights which results in an equity value very close to zero. The fairness value is slightly better due to the flights incurring in no cost.

Applying the fairness and equity metrics helps identify the characteristics that can be improved to better accommodate the user preferences as well as distributing the extra cost homogeneous among all flights. It sets the bases for an objective comparison in terms of equity and fairness, two performances areas still to be exploited in the design of future algorithms. Especially in those algorithms that will be implemented in automation tools that have to support the decision making process of modifying user preferred trajectories in the future operational concepts proposed by SESAR and NextGen.

7.6 Conclusions

Aligned with the *a posteriori* assessment of fairness and equity presented in the previous section, it has to be remarked that the present assessment could be improved if the algorithms are compared in several scenarios with different traffic densities, input files, preferred trajectories and different airspace characteristics.

The computation effort required for using the fairness and equity metrics in this type of assessment (i.e. *a posteriori*) is relatively low, although it depends on the complexity of the penalty function.

Considering fairness and equity metrics in this context is essentially for comparison purposes to analyse different algorithms under the same circumstances. This enables to evaluate their outcomes in those same situations and compare those outcomes. In this sense, comparing different algorithms not only in one scenario but in a set of different scenarios reduces the possibility that a certain type of scenario might favour a certain algorithm upon others.

A standard set of scenarios and their corresponding input files should be defined to assess different types of algorithms. These scenarios and input files should be adapted to analyse the features of the algorithms and test their response to nominal as well as extreme traffic situations. Only this way, the assessment can be based on a wide enough bunch of possible traffic situations enabling a thorough comparison of similar algorithms and their characteristics. The fairness and equity evaluation based on such a set of scenarios adds meaningful information to the algorithms' performance towards fairness and equity. It also constitutes an additional criterion for the selection of appropriate algorithms for their integration in trajectory based operations.

8 CONTRIBUTION AND CRITICAL DISCUSSION

The work presented in this thesis has provided definitions of the concepts of justice, fairness, and equity for Air Traffic Management, has derived metrics of fairness and equity according to those definitions and has established two possible methodologies to apply those metrics in the context of future trajectory-based operations. The proposed metrics take into account the cost implications of each single flight as well as their aggregated impact on the overall cost strategies of airlines operating several flights.

In addition, this thesis has specified the limitations and restrictions required for effectively applying the proposed metrics in specific contexts, analysing types behaviour of the actors involved that may jeopardise the correct functioning of fairness-oriented ATM systems and suggesting procedures to prevent the negative consequences of such types of behaviour.

8.1 Concept Definitions

The concepts of justice, fairness, and equity have not been clearly defined for the air traffic management before. The concept definition has been deduced applying the knowledge gained in other disciplines as philosophy, economics or sociology. Those disciplines have thoroughly analysed the notions of justice, fairness, and equity over the years. The insight gained from those studies builds the basis for the definition of justice, fairness, and equity in ATM (Chapter 2, Section 2.3).

According to its generic definition (Section 2.2.1), justice is the quality of being or acting in conformity with what is morally upright by following standards of what is right. These standards are assumed to be defined and agreed by those whom they apply under such conditions that tailoring the standards to anyone’s advantage is prevented. In ATM, the stakeholders are represented in the ICAO committee defining the international standards, procedures and directives to regulate the well functioning of the ATM system. ICAO is assumed to act as a neutral organisation taking care of justice in ATM at an international level, i.e. ensuring that ATM standards are defined without bias towards a particular group of interest. As long as ATM stakeholders operate according to the standards and procedures defined by ICAO, they are acting justly; this is, within the boundaries of a just framework (Chapter 5, Section 5.1).

Putting it short, justice is a general concept including standards of what is right and wrong. Fairness and equity, on the other hand, focus on the characteristics in distributing welfare according to different criteria within a just framework.

Fairness is the quality of achieving a distribution according to individual acceptance levels of satisfaction. These acceptance levels of satisfaction vary from individual to individual and fairness takes them into consideration when setting up the “fair” distribution (Chapter 2, Section 2.2.2). The essential particularity of fairness is the necessity to agree on a common way to measure individual acceptance levels of satisfaction. The application of the concept of fairness in ATM addresses the success in distributing additional costs to a set of flights in accordance with the maximum acceptance level of additional costs of each flight (Chapter 4, Section 4.3).

Equity is the quality of applying equal treatment to all concerned, independently of their individual acceptance levels of satisfaction. According to the present dissertation, equity in ATM would be achieved when additional costs are distributed among a set of flights in an equalitarian way, regardless of the acceptance levels of additional costs of each flight (Chapter 2, Section 2.2.3).

Following the different peculiarities and meanings of fairness and equity, it is an error to use these two concepts indistinctively. Nevertheless, this is a frequent mistake made by researchers and professionals in the field of ATM (Chapter 1, Section 1.3).

8.2 Metrics associated to Fairness and Equity

Currently, there is a lack of a standard set of metrics and methodologies for rigorously measuring performance in ATM (Section 1.2.3). This thesis addresses this need by proposing metrics to measure fairness and equity as well as associated methodologies to apply them in accordance with the defined concepts of justice, fairness and equity in ATM.

For the development of these metrics, the relevance and structure of flight costs are first analysed (Chapter 3). Then, in Chapter 4, a commonly used cost model (Section 4.2) is taken as the starting point to derive the notions of cost penalty (section 4.2), maximum acceptable cost penalty P_{SAT} (Section 4.3), penalty function P (Section 4.3.1) and relative penalty function $\wp$ (Section 4.3.2). The metrics of fairness and equity are built upon these concepts and mathematically formalised. It is worth mentioning, even though they have been derived and applied considering a specific cost model, the metrics proposed in this dissertation are generic and consequently, they can in principle be applied in other scenarios regardless of the specific mathematical form of the cost model used by the airlines.

The fairness metric Φ evaluates whether the additional incurred costs (also called penalty cost) have been distributed according to the maximum acceptance levels of additional costs of each flight (Section 4.5):

$$\Phi = \frac{\left(\prod_{i=1}^{n}\left(1-\wp_i\right)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(1-\wp_i\right)} \cdot n$$

The equity metric E evaluates whether the additional incurred costs have been distributed equally among all flights (Section 4.6):

$$\mathrm{E} = \frac{\left(\prod_{i=1}^{n}\left(P_i+e\right)\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(P_i+e\right)} \cdot n \text{ ; where } 0<e<<P$$

The main difference between these two metrics is that the equity metric, according to its conceptual definition, only takes into account the additional incurred costs, represented in the cost penalty P_i for each flight. The fairness metric also considers the maximum acceptance level of additional costs for each flight, represented by the maximum acceptable cost penalty P_{SATi} and integrated in the relative cost penalty $\wp_i$ (Section 4.3.2 for details).

For the mathematical definition of the metrics, the arithmetic mean was considered inadequate to evaluate the distribution of additional costs for both the fairness and the equity metric. As stated in Section 4.4.4, a fairness or equity metric has to penalise the dispersion in the distribution of additional costs. The geometric mean captures mathematically this requirement and has therefore been adopted for the formulation of the metrics.

It was also identified that the penalty and the relative penalty functions had to be enhanced to become applicable not only to the cost penalty and maximum acceptable cost penalty of a single flight but also to the cost penalty and maximum acceptable cost penalty of an airline operating several flights simultaneously (Chapter 7, Section 7.3). Each flight operated by a given airline has associated a relative importance or weight w_i on the airline's cost strategy. The total cost penalty of an airline is an aggregate of the different cost penalties associated to each of its flights according to the relative importance they have in the airline's cost strategy:

$$P_j^{ARL} = \sum_{i=1}^{n_j} w_i P_i$$

The maximum acceptable cost penalty of an airline is composed of the sum of the maximum acceptable cost penalties of all of its individual flights; each multiplied by its relative importance in the airline's cost strategy:

$$P_{SAT,j}^{ARL} = \sum_{i=1}^{n_j} w_i P_{SAT,i}$$

In the literature, other metrics can be found for fairness and equity (Chapter 1, Section 1.3). Fairness metrics have been proposed elsewhere to evaluate whether the distribution of delay costs has been done equally among airspace users [51] or to capture the deviations from the First Come First Serve order [50], which is the industry accepted standard for a fair sequence. These alternative fairness metrics only partially comply with the conceptual definition of fairness in ATM established in this work. They are not based on a concrete cost model, as proposed here, but on loose elements and features from different sources. For example, these metrics do not take into account airline preferences regarding individual flight costs or the maximum acceptable cost penalty, which are key elements to take into account the concept of fairness is to be rigorously applied in the ATM context.

The concepts of fairness and equity are usually considered to be the same and used indistinctively, which is due to the fact that they are not based on solid concept definitions as the ones presented in this thesis. This makes evident the relevance of clarifying the notions of fairness and equity as a pre-requisite for building up metrics that effectively contain the necessary features. The metrics proposed in this dissertation capture all the features of the established concepts of fairness and equity in ATM and are based on a consistent cost model and the associated conditions to capture the cost penalty, i.e. additional incurred costs.

8.3 Robustness of the Concepts and Metrics

To analyse the robustness of the proposed metrics and associated concepts, a decision game based on decision theory was designed (Chapter 6). Special attention was given to the robustness assessment of the fairness metric; focusing on the implications of airlines providing their maximum acceptable cost penalty values to an ANSP in charge of managing their flights.

The game assumptions simplified the complexities of reality capturing the essential features that were considered relevant for the game at hand (Sections 6.5). These were mainly: a fairness-oriented ANSP, two airlines whose preferences cannot be fulfilled, and the possibility for the airlines' to decide what information is shared with the ANSP (Section 6.4).

The main conclusion of this game is that, in the proposed framework to integrate fairness in the ATM context, the airlines may have incentives to provide untruthful information regarding key parameters about their preferences that would impact directly the applicability of the fairness metric (Section 6.9). This thesis proposes solutions to deal with this issue and eliminate the incentive for the airlines to manipulate the information they share with the ANSP to influence its decisions in order to obtain an advantage.

In the proposed framework, it is in the airlines' interest to provide the parameters that define their preferred trajectory truthfully (e.g. cost index, preferred flight duration, preferred fuel consumption, and relative relevance of each of its flights on its cost strategy). However, it is shown that airlines have an incentive to share misleading information with a fairness-oriented ANSP regarding the parameters describing the maximum acceptable cost penalty. Thus, an airline may share a false maximum acceptable cost penalty to indirectly influence the outcome of the ANSP's decisions to its own advantage (Section 6.9).

To achieve a truly fair distribution of costs, the honesty of the airlines declaring their individual maximum acceptance level of additional costs is required. The results of the proposed decision game conclude that airlines have incentives not to act honestly. This behaviour jeopardises the effective application of the fairness metric.

Two possible solutions are proposed to overcome this issue (Section 6.9):

- As part of the operational regulations in place, airlines are requested to provide truthful information on the parameters defining their maximum acceptable cost penalty (they would be penalized if they are found to violate this regulation by the relevant overseeing authority).
- The relevant overseeing authority (assumed neutral) is responsible for defining the value of the maximum acceptable cost penalty for all airlines.

Airlines or individual flights may then be prevented from providing untruthful information by, means of fines, penalties or restrictions on the service they receive if they are found to not comply. This solution could be found hard to implement or ineffective as it may be very difficult to identify when the airline has provided truthful information or not.

On the other hand, a neutral authority could define the values of the maximum acceptance cost penalties on behalf of the airlines according to certain pre-established and agreed objective criteria, such as the type of aircraft, type of route, etc. Candidates to act as such neutral authority could be EUROCONTROL for the European ATM or the FAA for the US airspace. The proposed new operational contexts leave the door open to this possibility. The maximum acceptance levels for delay and extra fuel consumption can already be stated as part of the operational contexts of SESAR and NextGen, as suggested in [30].

Applying decision theory is not commonly used to analyse airline's behaviour in ATM. In the context of this thesis, it was the appropriate way to analyse the possible strategies available to the airlines when deciding which information to communicate to a fairness-oriented ANSPs.

Given the assumptions made, it can be anticipated that the quantitative estimation of airline's incentives to provide untruthful information on certain parameters may vary or be inaccurate compared to reality, whereas the qualitative conclusion remains valid; namely that airlines expect a potential cost saving when they choose to manipulate the information about certain parameters they provide to fairness oriented ANSPs and, therefore, have incentives to do so.

To apply the fairness metric, the honesty of the actors involved has to be assumed. This dissertation concludes that whenever this cannot be guaranteed through one of the two solutions proposed in this work or any other, then only the equity metric can be applied effectively (Chapter 5, Section 5.1). This is so because the equity metric does not include the maximum acceptable cost penalty in its definition. This metric evaluates whether the distribution of absolute additional incurred cost (i.e. cost penalty P_i) has been distributed equally among all involved flights or corresponding airlines, independently of their P_{SATi} values.

8.4 Assessment of Fairness and Equity

One further contribution of the present dissertation is the two methodologies presented in Chapter 5. Those methodologies structure and serve as guideline for the application of the proposed metrics within a trajectory based operational context.

The *a priori* methodology describes how to include the metrics in a trajectory optimisation process. A specific algorithm is proposed in Chapter 7 (Section 7.4) which includes the fairness metric and is adapted for its integration into any automated tool assisting in the conflict detection and resolution activities of ATC.

The *a priori* methodology may as well assist any air navigation service provider in incorporating fairness and equity considerations into their optimisation processes. It also serves as guideline for any supplier of ATM automated tools who intends to integrate fairness and equity objectives in the optimisation processes of their products (Section 5.2.1).

The *a posteriori* methodology describes how to include the metrics within the assessment of a trajectory optimisation process once the solution has been determined. This methodology combined with fairness or equity metrics is of particular interest in the field of comparative analysis for which no standard framework exists today in ATM (Section 1.2.3). To illustrate this, a practical example is developed in Chapter 7 (Section 7.5): Three different conflict resolution algorithms are compared in the same simulation scenario with a given air traffic input. Following the steps of this methodology (Section 5.2.2), the equity and fairness of the proposed solutions by the three different algorithms can be evaluated and compared quantitatively with each other. The results of this example show the possibility to establish which of the three algorithms is the fairest or the most equitable for the given scenario and traffic input.

The *a posteriori* methodology can be used by any air navigation service provider (e.g. FAA (USA), EUROCONTROL (Europe), AENA (Spain), ASA(Australia), NLR (Netherland), DFS (Germany) and many more) to evaluate and compare the fairness and equity of the service provided (Section 5.2.2), for example:

- between different ANSPs,
- between different sectors where the same ANSP is in charge of
- between different time frames within the same sector,
- etc.

It can as well be used by ANSP customers (such as airlines) or supervisors of ANSPs (such as EUROCONTROL or FAA) to assess the performance towards fairness and equity of any automated system supporting the activities of air navigation service providers (Section 7.6).

The proposed methodologies are intended to build the basis to define a standard way to assess fairness and equity in the new ATM operational concepts and to be used by all ATM stakeholders and actors.

SESAR and NextGen are in a position to incorporate the consideration and improvement of fairness and equity as part of their target performance objectives, and include the proposed metrics together with the two proposed methodologies as means to apply and measure fairness and equity in ATM.

The proposed *a priori* and *a posteriori* methodologies are focused on assessing fairness and equity by means of the proposed metrics and in accordance with the described concepts. Even if other methodologies may be developed and used, these two methodologies detailed in this dissertation aim at providing two basic examples (Chapter 7, Sections 7.4 and 7.5) of how to guide the performance assessment to include the fairness and equity metrics described herein.

9 FUTURE WORK

This dissertation has focused its research on answering specific questions regarding how to define the concepts of fairness and equity, how to measure those concepts and how to assess those concepts within the ATM system. Along the way, as progress has been achieved in the subject at hand, new questions and areas of investigation have been opened and identified. This chapter summarises those new lines of research which could possibly be considered enriching and of interest for future works.

The metrics for fairness and equity derived in this dissertation as well as the methodologies to apply those metrics are only applicable whenever justice is guaranteed. The thesis assumes that the ATM system represents a just system where authorities as ICAO act as a neutral organisation ensuring justice at an international level.

According to the definition of justice deduced in this work (Chapter 2, Section 2.2.1) and based on John Rawls's statements on justice, the standards of what is right and wrong that guide a just behaviour have to be agreed by those whom they apply under certain circumstances. Those circumstances are a direct consequence of John Rawls's definition of the "veil of ignorance"; the agreement has to be made under such conditions that prevent anyone from tailoring those standards to their own advantage.

This thesis has focused on the civil airspace users and thus, established that ICAO is a neutral authority where the ATM stakeholders are sufficiently represented whenever new standards are agreed or modified, and can be held responsible for ensuring justice in the ATM system regarding civil use. Further studies could deal with analysing how ICAO ensures a just system, the mechanism to prevent certain standards from favouring specific stakeholder groups and identify another neutral authority, similar to ICAO, assuming the same role for military airspace users and how those two authorities ensuring justice in ATM could collaborate.

A further line of research is to refine the cost model upon which the definition of the penalty and relative penalty functions are based. The cost model described in this dissertation is based on a commonly used cost function [79] capturing the basic cost structure of a flight and, ultimately, of an airline's cost network. By gaining insight into a more precise cost function, it could be possible to study whether the refined cost model has an impact on the assumptions and constrains defined for the penalty and the relative penalty functions. Eventually new parameters impacting on costs may play a non negligible role defining the maximum acceptable cost penalty. In that case, the robustness analysis proposed in this thesis would help evaluating the incentives airlines may have to communicate certain information truthfully or not on those new parameters.

Regarding the decision game upon which the robustness analysis is based, this dissertation proposes a two player game. For further research studies, the two player game could be modified to become a multi player game. This change of game specification would not add additional information on the airline's incentives to communicate truthful or untruthful information for specific parameters to fairness oriented ANSPs. But this game variation may provide insight on the implication of strategy choices on the different players as well as which strategy is more often chosen by the players and clarify the reason for that.

As stated in this thesis, the effective application of the fairness metric is jeopardised in situations where airlines have incentives to provide untruthful information on key parameters of special relevance to the fairness metric. To mitigate such behaviour, one of the measures contemplated in this dissertation is to let a neutral authority define the values for airlines maximum acceptable cost penalty according to pre-establish, objective criteria on behalf of the airlines themselves.

As a suggestion for further research on this measure, one of these criteria could be the equipment capabilities of the aircraft. This way, authorities as for example EUROCONTROL or the FAA can incentivise airlines to invest in modernising their fleet with avionic systems better suited to meet the performance targets of SESAR and NextGen. This favours a reduction of the maximum acceptable cost penalty for delay and extra fuel consumption of those aircraft. By doing so, fairness oriented ANSPs trying to maintain user preferences and, if not possible, trying to remain within the maximum acceptable cost penalty of each flight, penalise according to the values assigned for maximum acceptable cost penalty. Thus, airlines with better suited equipments on their aircraft would be less penalised, encouraging so the investments in adapting their fleets to meet the requirements of the future operational concepts.

Together with the proposed metrics for fairness and equity this dissertation introduces a methodology (*a posteriori*) for evaluating the performance regarding fairness and equity of ANSPs focusing on the service they provide and on the automated tools supporting that service. Chapter 7 presents the results of applying this methodology to compare fairness and equity of the trajectory solutions resulting from three different conflict resolution algorithms for the same scenario and traffic situation. As mentioned in the conclusions of that chapter, Section 7.6, this study could be complemented by defining a set of scenarios to analyse resolution algorithms in any relevant ATM situation with different traffic densities and different airspace configurations. So, the specificity linked to a certain operational context, route network or traffic situation is avoided, being able to make an assessment on the fairness and equity of the algorithms not associated to a specific scenario.

Based on the concepts and metrics of fairness and equity as well as the associated methodologies, the ATM stakeholders defining the requisites for SESAR and NextGen can standardise the assessment of a fair and equitable use of user-preferred information for the trajectory based operational ATM system they are proposing. The proposed methodologies could be extended to standard methodologies for assessing fairness and equity of automated tools and air navigation services. This is a critical point worth being considered and included as part of the modernisation programmes of SESAR and NextGen because of the current lack of standard metrics and methodologies (Chapter 1, Section1.2.3).

Guaranteeing the fair and equitable use of user preferred information is anticipated to become more and more relevant as the new operational concepts are implemented and airlines are required to communicate their preferences.

The lines of further research addressed in this chapter are best suited to be undertaken by the ATM stakeholders currently defining and implementing the requirements and concepts of the modernisation programmes intended to improve their ATM systems.

APPENDIX A – EVOLUTION OF DECISION SUPPORT TOOLS

1 DEVELOPMENT IN EUROPE

The evolution of the strategic and centralised Decision Support Tools (DSTs) in Europe can be followed analysing the history of two of the most commercialised DST for arrival management, COMPAS (or 4D Planner and 4D CARMA in its updated versions) and MAESTRO.

1.1 Computer Oriented Metering Planning and Advisory System (COMPAS)

COMPAS (Computer Oriented Metering Planning and Advisory System) is the forefather of the automated arrival management systems in Europe. It was developed in the early 80's by DLR (Deutsches Zentrum für Luft- und Raumfahrt) in co-operation with the German Air Navigation Services, DFS (Deutsche Flugsicherung). The aim of this system is to assist controllers in a smooth and efficient handling of the arrivals. COMPAS has been designed to estimate arrival times based on flight plan data, radar data, aircraft performance data, and wind data [29]:

- to determine the required separation between aircraft of the same or different weight classes,
- to plan an optimal sequence of aircraft with regard to a smooth integration of traffic from the different arrival routes,
- to make best use of the existing runway capacity.

Simulations and field trials were issued before introducing COMPAS as a decision support tool for controllers in the Frankfurt TMA. The major real-time simulation exercise was done as part of the Programme for Harmonised Air Traffic Management Research in EUROCONTROL (PHARE), particularly the PHARE Demonstration 2 Simulations (1997) [101]. It also served to test functionalities for the COMPAS successor system, the 4D Planner. The task of the field trials was to assess traffic handling performance, controller workload, and controller acceptance. The trials demonstrated that COMPAS reduces the coordination effort between different air traffic control centres and that the approach sequences established show less frequent violations of the first-come-first-served rule. In the TMA in particular, a more direct vectoring was observed from the radar plots, together with a significant reduced communication load and a significant decrease of average flight time [29].

The output of COMPAS is an arrival sequence with passive advisories, which means assigned Scheduled Time of Arrivals (STA) at runway threshold and other relevant waypoints. Time based information shown to the controller is called time advisory. Those are defined as passive advisories compared to speed, altitude or heading advisories, which are active. Main difference between both advisories is basically that active advisories define how the trajectory should look like, while passive advisories rely on the controllers to take those decisions.

Main disadvantage of COMPAS is that the proposed sequence is frozen at an early stage, and any changes the controller wants to introduce to the sequence have to be done manually [29].

1.1.1 4D Planner

The 4D Planner, which was also developed cooperatively by DLR and DFS, replaced the COMPAS System in Frankfurt in 1999 and it has also been introduced in Munich. The 4D Planner also provides time-based arrival planning as well as information to the controller on time-based guidance of inbound aircraft. In contrast to its predecessor, it continuously monitors the aircraft position, detects off-plan deviations, and is able to include planning updates if necessary, which is a functionality that goes beyond the ones from COMPAS. The 4D Planner does not freeze the sequence. It also generates time-based advisories (STA at fixes and runway threshold) and it assigns runway allocation for optimising the runway throughput [102][103].

COMPAS as well as 4D Planner computation takes into account the arrival flow at the entry points or fixes and estimates their arrival time at the gate. This is used as input to establish the optimised landing sequence and recalculate backwards the schedule time of arrival at predetermined fixes along the route down to the runway threshold. The main delay is intended to be gained or should be gained before the TMA entry fix. The fly-time from TMA entry fix up to the runway threshold varies in general between 30 and 20 minutes. After the TMA entry fix only fine tuning is allowed to gain normally up to three minutes of maximum delay [20].

COMPAS introduced the time line as controller interface to display the arrival sequence and planned landing times; however, it fixes the sequence at a very early stage. The 4D Planner improves the sequence planning task by constantly considering the actual radar data, it is able to adapt the schedule of arrivals at any ATC control action.

1.1.2 4D Cooperative Arrival Manager (4D CARMA)

The 4D CARMA system (4 Dimensional Cooperative Arrival Manager), which is a further development of the 4D Planner, is intended to be able to generate trajectories. Thus, this system provides active advisories, from aircraft position to the runway threshold, taking into account weather conditions and aircraft performance data from the BADA (Base of Aircraft DatA) model. It generates guidance instructions for voice or data link communication. The trajectory based guidance has to comprise features for conformance monitoring, conflict detection and resolution. The sequence generation is based on optimisation of several criteria, e.g. comparison of two sequences with respect to their noise profile in the TMA [104].

Main research focus of 4D CARMA is the integration between the DSTs for arrival and departure management (AMAN and DMAN respectively). This integration should be provided in the next update through a new system, namely an Arrival Departure Coordinator (ADCO) [104].

1.2 Moyen d'Aide à l'Ecoulement Séquencé du Trafic avec Recherche d'Optimisation (MAESTRO)

The most commercialised DST up to date, MAESTRO (Moyen d'Aide à l'Ecoulement Séquencé du Trafic avec Recherche d'Optimisation), was launched in 1985 by the French Research Centre (CENA). It was first installed in Paris Orly and Paris Air traffic Control Centre (ACC) in 1990. At this early stage of the development, MAESTRO was a DST only providing runway sequences and schedules for the runway threshold, freezing the sequence also very early. In 1992, a new design was developed with multi runway and multi airport capacity [105].

MAESTRO is now a multi-airport and multi-runway decision making tool for airport terminal area management. The system aims to minimise delays and excessive fuel consumption. It provides a graphical view of the computed sequence and the control actions required to expedite the traffic. MAESTRO enables manual changes to test various sequencing options [105].

This system includes separately an AMAN (Arrival Manager) and a DMAN (Departure Manager). Both systems share runway information to know how to allocate the flights and optimise the runway occupancy times, but there is no integration between the two systems.

The MAESTRO AMAN handles flight plans and radar tracks from local Flight Data Processing System (FDPS) and Radar Data Processing System (RDPS), allocates each incoming aircraft to a destination runway taking into account runway allocation rules, runway restrictions for handling noise abatement procedures, selected TMA configuration and runway separation and flight priorities. It calculates the STA at the corresponding TMA entry fix and at the runway threshold, and the delay to be absorbed. The system then optimises the overall sequence to minimise the global delay and reduce holding pattern situations. That sequence is displayed and passive advisories are supplied. MAESTRO shows a time record of minutes to lose and minutes already lost for each aircraft. It also generates automatic coordination messages to relevant positions when the sequence is modified manually by one of the controllers [28].

The aim of MAESTRO is to optimise the sequence to provide pressure on the TMA entry points, which results in a tight sequence without gaps.

The sequence passes through different stages until it is frozen 20 to 15 min prior to the scheduled landing time. At first, the sequence is unstable, many changes occur as additional aircraft are added to the sequence. Secondly, the sequence is stable and only minor changes are allowed to fine-tune the sequence. The last step consists in freezing the sequence, providing a final sequence for aircraft to enter the Initial Approach Fix (IAF) in the TMA [28][105].

MAESTRO is the most used DST for arrival management up to date. It has been introduced in several airports over the world (Paris Orly, Paris Charles de Gaulle, Copenhagen, Melbourne, Brisbane, Sydney, Johannesburg, Cape Town) [105].

MAESTRO accounts amongst its strengths a graphical view of computed sequence and control actions, it enables manual changes to test sequencing options, is relative flexible to be adapted to operational requirements, and to different Air Traffic Controller (ATC) environments. The innovative idea incorporated in this DST relies in having a system's logic for determining the most desirable runways to maintain proper airport balancing.

2 DEVELOPMENT IN THE USA

In the USA, the whole controlled airspace is managed by a single air navigation service provider, the Federal Aviation Administration (FAA). That is the main reason why most of the development that has been done towards automation of the air traffic management has been focused into one single effort, the Center-TRACON Automation System, (CTAS) conducted by NASA Ames Research Center. In Europe, as each country has its own air navigation service provider, some of those have developed their own automation tools with different partners to cover, in general, similar needs.

CTAS is a system that is been developed by NASA since the early 1990's and aims to include several decision support tools for arrival management from en-route to terminal airspace traffic management [27]. Thus, the system includes in its original version an En-route Descent Advisor (EDA), a Traffic Management Advisor (TMA) and a Final Approach Spacing Tool (FAST). Since the original architecture was developed, other tools and also improvements to the original tools have been added to the system.

2.1 En route Descent Advisor (EDA)

The En-route Descent Advisor (EDA or DA) provides the required advisories to the aircraft to meet the determined schedule by the TMA tool. The advisories, i.e. speed, altitude, and heading, are continuously being calculated and are intended to meet the meter fix schedules, to provide high fuel efficiency, and to be as close as possible to the airline's preferred descent speed, route, altitude and entire 4D trajectory [106].

2.2 Traffic Management Advisor (TMA)

The Traffic Management Advisor (TMA) is a strategic decision support tool for long-term planning and optimisation. The TMA does scheduling and sequencing at different waypoints as well as at the runway threshold.

The basic schedule principle used in the original version of TMA is FCFS. This strategy is commonly accepted as a fair schedule when delays must be absorbed. For a simple algorithm, the FCFS rule is a starting point to build a sequence with.

NASA identified that there are two types of scheduling constraints in case of arrival management: in trail distance separation constraints and sequence order constraints [16]:

- The in trial distance separation constraint considers aircraft weight, landing order and wake vortex rules according to FAA. For optimisation purposes this constraint has to be kept to the minimum.
- Sequence order constraints are defined by ATC. The points where sequence order constraints are frequently enforced are runway threshold and meter fixes.

A FCFS order is a standard sequence order. Once the first sequence order is established, then the sequence can be optimised and it should be optimised by position-shifting. According to NASA, a positive position shift means advancing an aircraft relative to the FCFS order; a negative position shift means delaying an aircraft relative to the FCFS order. The position shifting technique was developed by NASA for reducing delays by optimizing the landing sequence [107] and incorporated in the TMA tool.

In the subsequent version of TMA, the algorithm was based on branch and bound technique for scheduling the meter fix STAs [22]. The optimisation algorithm also considered a cost function. That function consisted of a weighted combination of delays and fuel consumption. During the sequencing and scheduling process, the aircraft's STA is constantly being updated until the aircraft's meter fix estimated time of arrival (ETA) is less than or equal to 19 minutes in the future. After that, the STA is frozen [22].

Input to this algorithm is the information required to calculate the estimated time of arrival to meter fixes (e.g. runway threshold) and the output is the scheduled time of arrival for each aircraft to those meter fixes or predetermined reference points.

2.3 Final Approach Spacing Tool (FAST)

The Final Approach Spacing Tool (FAST) schedules and sequences traffic at the runway threshold; it generates advisories, has a 4D trajectory synthesiser, and a graphical interface which displays information to the controllers. The Human Machine Interface (HMI) shows a time line where the bottom of the time line corresponds to the current time at the runway threshold. The time line extends into the future and the aircraft are placed on it in relation to the time they are scheduled to arrive at the runway threshold [108].

The evolution of the FAST tool is very similar to the evolution of other European AMAN DST, e.g. COMPAS or MAESTRO. The first step was to develop the so-called passive FAST, or pFAST, as it only provided passive advisories, in other words, time advisories for each aircraft at the runway threshold. The main function of this tool was to schedule the arrivals and give runway assignment.

The pFAST tool was developed for short term planning and optimisation processes. It is a tactical decision support tool that splits the traffic merging problem into merges at multiple points. The scheduling process is mathematically single-objective and the optimisation is carried out switching aircraft to alternative runways if this is found to reduce delays. The first FAST design had the minimum required functionality of a tactical decision support tool, those being: give sequence number and assigned runway advisor regarding trajectory based constraints [26].

NASA identified that they needed to add a separation optimisation algorithm to improve the pFAST tool. For that aim, the algorithm had to consider a new parameter: trajectory based spatial constraint satisfaction. The motivation for this improvement resulted from the analysis of two NASA Ames researchers, Krzeczowski and Neuman [107].

The improved pFAST scheduling algorithm implements this spatial constraint satisfaction concept by performing sequencing and conflict resolution serially. Conflict Resolution occurs at the conclusion of the scheduling process.

As well as it happened in Europe, it was identified that passive advisories only do not optimised the arrival management bottleneck and the controllers workload could be reduced increasingly by providing active advisories (speed, heading and altitude advisories) supporting ATC on deciding how to meet the scheduled times. Thus, a new FAST design was developed, the active FAST or aFAST.

This version of FAST refines and enhances the spatial constraint concept which is the foundation of the scheduling algorithm. The landing sequence is determined in two steps, ordering the aircraft on each flight segment and repeatedly merging these aircraft into a consistent final sequence at the runway threshold. The first step is always ordering then merging. For this purpose, the implemented algorithm applies ordering and merging heuristics for sequencing a particular pair of aircraft [26][106].

In contrast to pFAST, aFAST does not postpone the conflict resolution process until all sequencing decisions are made. Instead, it performs conflict resolution immediately following each individual sequencing decision. At every point, the system gives the best possible ETA. For the merge location, a FCFS strategy is favoured with special consideration made for grouping aircraft by weight class [106].

3 CONCLUSIONS

Decision support tools in general and arrival management support tools in particular, reduce the air traffic controller’s workload by minimising significantly the communications load as well as the monitoring activity [29][101]. The benefits of introducing assistance tools in the ATM environment are clearly visible; DST help to minimise delays, reduce fuel consumption, improve coordination between air traffic controllers and effectively distribute the controller's workload whilst optimising airspace use and runway capacity [29]. Thus, the main hub airports are demanding and driving the research and development activities towards improving their DST systems.

Current research activities focusing on arrival management have identify that common functionalities as sequencing, scheduling and metering providing time-based advisories will not be sufficient to accomplish the requirements of new ATM concepts.

DSTs have to extend their scope beyond the limits of the current busy TMA area into areas currently considered as En-Route [109]. According to the predicted air traffic grow and the resulting demand, decision support tools have to be able to detect potential conflicts, generate what if scenarios with possible solutions, and display the optimal solution to the controller via active advisories; as speed, heading and altitude advisories.

The main challenge of future automated assistance tools is to provide not only a sequence with schedule times for predetermine fixes, but to strategically plan air traffic flow movements within its area of responsibility, detect conflicts and resolve them whilst optimising efficiency of resources and respecting user preferences.

APPENDIX B – SCENARIOS AND CASES

1 BRIEF DESCRIPTION OF SCENARIOS AND CASES

Scenario α

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	30,5	760,49	33,5	798,52	0,52	33

	c [-]	λ [kg]
ANSP	1,05	1381,1

Scenario β

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	30,5	760,49	33,5	798,52	0,52	70

	c [-]	λ [kg]
ANSP	1,05	1381,1

Scenario γ

	T_P [min]	F_P [kg]	T_{REF} [min]	F_{REF} [kg]	C_F [€/kg]	CI [kg/min]
Airline A	30,5	760,49	33,5	798,52	0,52	33
Airline B	37	1027,32	40	1078,69	0,52	70

	c [-]	λ [kg]
ANSP	1,05	1381,1

For each of the scenarios detailed above following cases apply:

Case 1

Only the value of one single parameter (T_{REF}, F_{REF} or CI) can be chosen to be different from the inside parameter value.

- Case 1.1 only one player can provide a value for one single parameter different from the inside parameter value
- Case 1.2. both players can provide simultaneously a value for one single parameter different from the inside parameter value

Case 2

Both players, at the same time, can provide for a combination of two parameters values different from their inside parameter ones.

Case 3

Both players, at the same time, can provide for all three parameters values different from the inside parameter ones. As a certain peculiarity can be observed from the results, modifications to the previously defined scenarios are introduced to analyse this peculiarity, namely variants a, b, and c to the three scenarios α, β, γ:

Table 49 Variants a, b and c for scenario α

Scenario α	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
α_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	RUSIK	30,5	760,49	33,5	798,52	33
α_b	A	TERTO	37	1027,32	40	1078.69	33
	B	TERTO	37	1027,32	40	1078.69	33
α_c	A	TERTO	37	1027,32	40	1078.69	70
	B	TERTO	37	1027,32	40	1078.69	70

Table 50 Variants a, b and c for scenario β

Scenario β	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
β_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	RUSIK	30,5	760,49	33,5	798,52	70
β_b	A	TERTO	37	1027,32	40	1078.69	33
	B	TERTO	37	1027,32	40	1078.69	70
β_c	A	TERTO	37	1027,32	40	1078.69	70
	B	TERTO	37	1027,32	40	1078.69	33

Table 51 Variants a, b and c for scenario γ

Scenario γ	Airline	STAR	$T_{P\ [min]}$	$F_{P\ [kg]}$	$T_{REF\ [min]}$	$F_{REF\ [kg]}$	CI $_{[kg/min]}$
γ_a	A	RUSIK	30,5	760,49	33,5	798,52	33
	B	TERTO	37	1027,32	40	1078.69	70
γ_b	A	TERTO	37	1027,32	40	1078.69	33
	B	RUSIK	30,5	760,49	33,5	798,52	70
γ_c	A	TERTO	37	1027,32	40	1078.69	70
	B	RUSIK	30,5	760,49	33,5	798,52	33

2 EQUILIBRIUM RESULTS FOR EACH CASE

2.1 Case 1.1

For each scenario analysed under case1.1, the depicted values characterise the slope of the curve showing the percentage difference $\prod^{X}$ for airline A and B. Both airlines always prefer to communicate a value for any single parameter below their inside parameter value.

Π^{A} corresponds to the slope given the case where only airline A can provide one parameter value different from its inside parameter value. Π^{B} shows the equivalent for airline B.

Scenario α

Table 52 Curve slopes for scenario α case 1.1

	Π A	Π B
T_{REF}	-5,48	-5,48
F_{REF}	-1,44	-1,44
CI	-0,27	-0,27

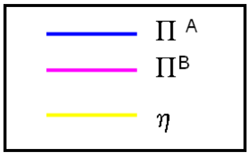
These slopes correspond to the figures depicted in the table below (Table 53). Those figures represent the marginal benefit of each airline, given they can only provide a value different from the true one for one parameter while the other player has to communicate all true values.

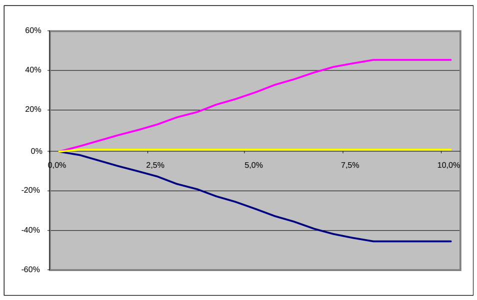
In any case, the values provided cannot be less than the values indicated for T_P and F_P. Thus, as it will be seen, for T'_{REF} and F'_{REF} the values provided can only be lowered up to:

- - 7% for T'_{REF}
- - 3% for F'_{REF}

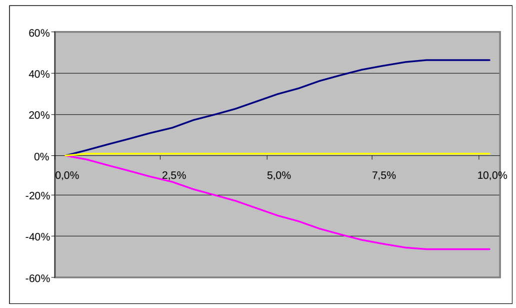
This is also the case for scenario β and γ.

Table 53 Figures showing the corresponding curve slopes for each parameter and airline in scenario α case 1.1

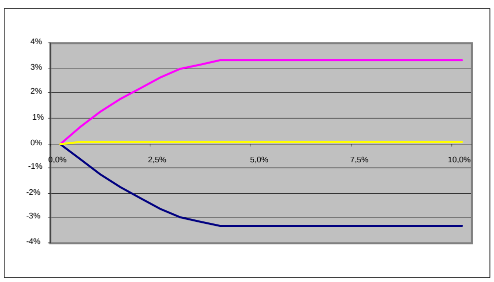




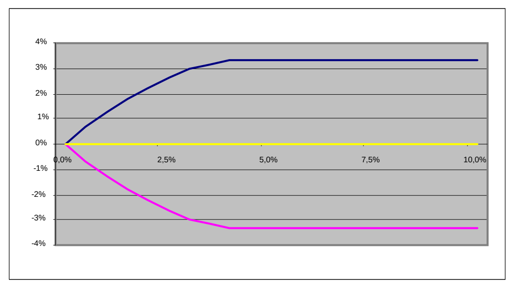
T^{A}_{REF} varying vs. T^{B}_{REF} constant at true value



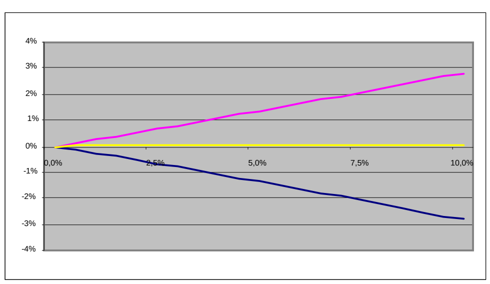
T^{B}_{REF} varying vs. T^{A}_{REF} constant at true value



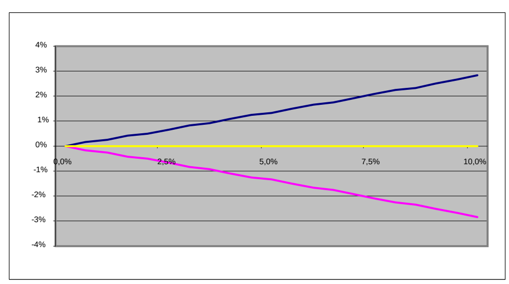
F^{A}_{REF} varying vs. F^{B}_{REF} constant at true value



F^{B}_{REF} varying vs. F^{A}_{REF} constant at true value



CI^{A} varying vs. CI^{B} constant at true value



CI^{A} varying vs. CI^{B} constant at true value

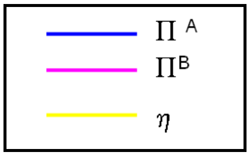
Scenario β

As a quick remainder, in this scenario both flights fly the same route but with different cost strategies. Following tables and figures show the results for this scenario β.

Table 54 Curve slopes for scenario β and case 1.1

	Π A	Π B
T_{REF}	-6,14	-5,65
F_{REF}	-1,31	-0,23
CI	-0,31	-0,12

Table 55 Figures showing the corresponding curve slopes for each parameter and airline in scenario β case 1.1



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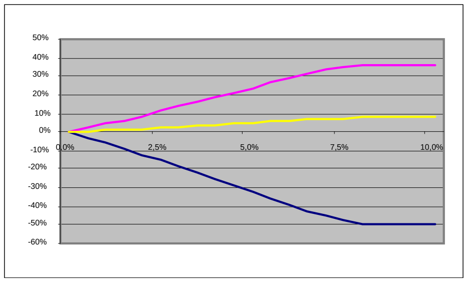
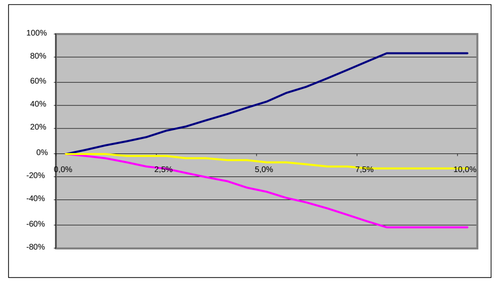
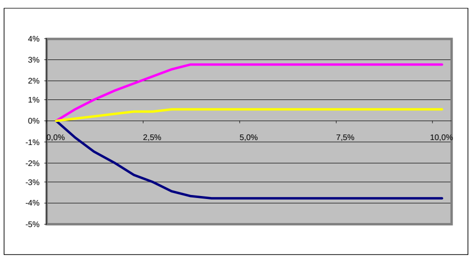
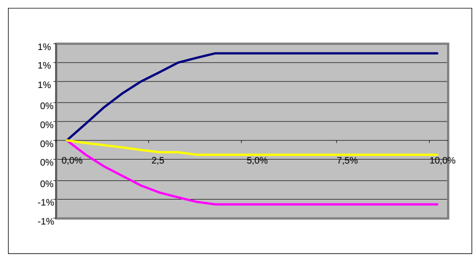
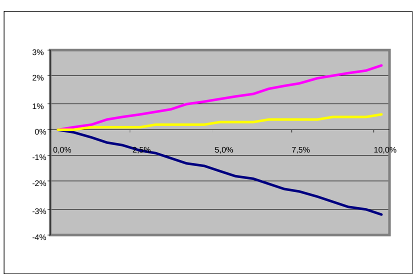
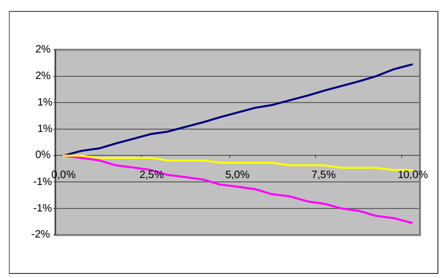
<td>

T^{A}_{REF} varying vs. T^{B}_{REF} constant at true value</td>
<td>

T^{B}_{REF} varying vs. T^{A}_{REF} constant at true value</td>
</tr>
<tr>
<td>

F^{A}_{REF} varying vs. F^{B}_{REF} constant at true value</td>
<td>

F^{B}_{REF} varying vs. F^{A}_{REF} constant at true value</td>
</tr>
<tr>
<td>

CI^{A} varying vs. CI^{B} constant at true value</td>
<td>

CI^{A} varying vs. CI^{B} constant at true value</td>
</tr>
</table>

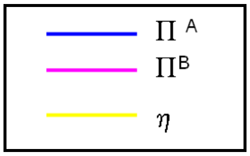
Scenario γ

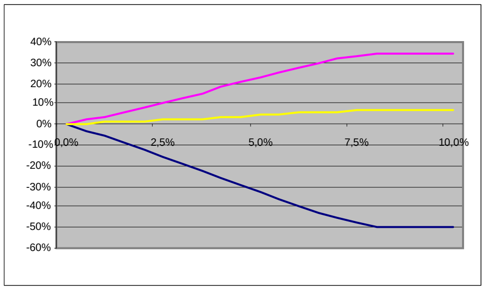
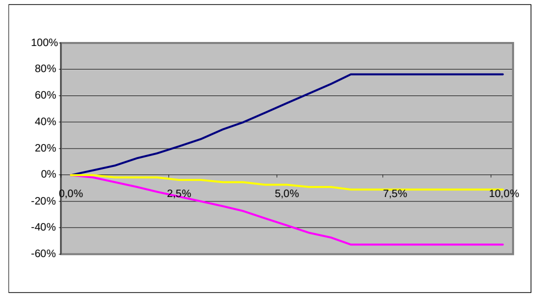
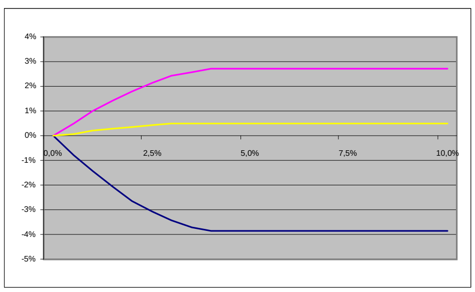
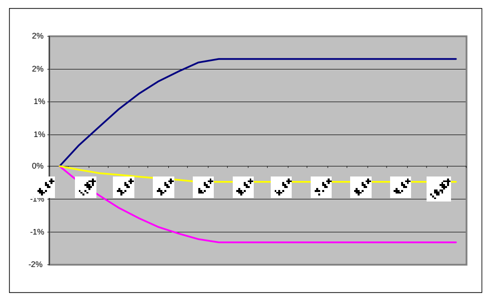
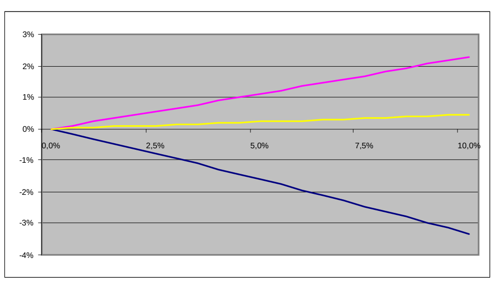
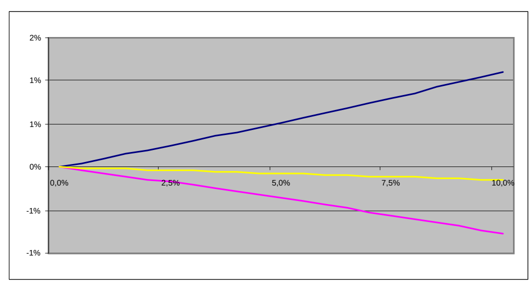
As a quick remainder, in this scenario each flight flies different routes with different cost strategies. Following tables and figures detail the results for this scenario γ.

Table 56 Curves slopes for scenario γ case 1.1

	Π A	Π B
T_{REF}	-6,27	-6,53
F_{REF}	-0,81	-0,38
CI	-0,32	-0,07

Table 57 Figures for the corresponding curve slopes for each parameter and airline in scenario γ case 1.1



 T^A_{REF} varying vs. T^B_{REF} constant at the true value	 T^B_{REF} varying vs. T^A_{REF} constant at the true value
 F^A_{REF} varying vs. F^B_{REF} constant at the true value	 F^B_{REF} varying vs. F^A_{REF} constant at the true value
 CI^A varying vs. CI^B constant at the true value	 CI^A varying vs. CI^B constant at the true value

2.2 Case 1.2

Both airlines can provide values different from their inside parameter value for only one parameter. Following tables detail all the possible analysis cases. The equilibrium is determined by applying the minimax criterion. The strategy choices of each airline are predicted. The red cell shows the solution payoff values for each airline, first the value for airline A, then the value for airline B.

For each analysis case, independently of the scenario, the solution strategies are always those facilitating the lowest possible value for the parameter at hand.

The value for the percentage of system's benefit, the percentage benefit of airline A and that of airline B in the equilibrium are detailed alongside the payoff matrix.

Scenario α

Scenario α is a special case, as it results a zero sum game. Thus, the system's cost benefit is always zero, and the gains and losses of both airlines are balanced.

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
T'_REF_B	-7	44,43 / 44,43	62,92 / 25,95	71,53 / 17,34	η	0,00%
	0	25,95 / 62,92	44,43 / 44,43	55,97 / 32,9	Π_A	0,00%
	7	17,34 / 71,53	32,9 / 55,97	44,43 / 44,43	Π_B	0,00%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
T'_REF_B	-7	61,82 / 27,04	62,92 / 25,95	64,65 / 24,21	η	0,00%
	0	43,13 / 45,74	44,43 / 44,43	46,58 / 42,29	Π_A	39,10%
	7	31,69 / 57,18	32,9 / 55,97	34,92 / 53,95	Π_B	-39,10%

		CI'_A			Equilibrium minimax	
		-10	0	10		
T'_REF_B	-7	61,87 / 27,0	62,92 / 25,95	63,8 / 25,06	η	0,00%
	0	43,18 / 45,69	44,43 / 44,43	45,52 / 43,35	Π_A	39,20%
	7	31,74 / 57,13	32,9 / 55,97	33,92 / 54,95	Π_B	-39,20%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
F'_REF_B	-3	27,04 / 61,82	45,47 / 43,13	57,18 / 31,69	η	0,00%
	0	25,95 / 62,92	44,43 / 44,43	55,97 / 32,9	Π_A	-39,10%
	3	24,21 / 64,65	42,29 / 46,58	53,95 / 34,92	Π_B	39,10%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
F'_REF_B	-3	44,43 / 44,43	45,74 / 43,13	47,88 / 40,99	η	0,00%
	0	43,13 / 45,74	44,43 / 44,43	46,58 / 42,29	Π_A	0,00%
	3	40,99 / 47,88	42,29 / 46,58	44,43 / 44,43	Π_B	0,00%

		CI'_A			Equilibrium minimax	
		-10	0	10		
F'_REF_B	-3	44,49 / 44,38	45,74 / 43,13	46,82 / 42,04	η	0,00%
	0	43,18 / 45,69	44,43 / 44,43	45,52 / 43,35	Π_A	0,10%
	3	41,04 / 47,82	42,29 / 46,58	43,37 / 45,49	Π_B	-0,10%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
CI'_B	-10	27,0 / 61,87	45,69 / 43,18	57,13 / 31,74	η	0,00%
	0	25,95 / 62,92	44,43 / 44,43	55,97 / 32,9	Π_A	-39,20%
	10	25,06 / 63,8	43,35 / 45,52	54,95 / 33,92	Π_B	39,20%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
CI'_B	-10	44,38 / 44,49	45,69 / 43,18	47,83 / 41,04	η	0,00%
	0	43,13 / 45,74	44,43 / 44,43	46,58 / 42,29	Π_A	-0,10%
	10	42,04 / 46,82	43,35 / 45,52	45,49 / 43,37	Π_B	0,10%

		CI'_A			Equilibrium minimax	
		-10	0	10		
CI'_B	-10	44,43 / 44,43	45,69 / 43,18	46,77 / 42,1	η	0,00%
	0	43,18 / 45,69	44,43 / 44,43	45,52 / 43,35	Π_A	0,00%
	10	42,1 / 46,77	43,35 / 45,52	44,43 / 44,43	Π_B	0,00%

Scenario β

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
T'_REF_B	-7	46,4 / 63,19	64,51 / 36,24	72,73 / 24,01	η	-3,70%
	0	20,77 / 101,32	37,78 / 76,02	49,51 / 58,56	Π_A	22,80%
	7	13,11 / 112,72	26,27 / 93,14	37,02 / 77,15	Π_B	-16,90%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
T'_REF_B	-7	63,46 / 37,8	64,51 / 36,24	66,18 / 33,75	η	-11,00%
	0	36,50 / 77,91	37,78 / 76,02	39,89 / 72,88	Π_A	68,00%
	7	25,19 / 94,74	26,27 / 93,14	28,09 / 90,43	Π_B	-50,30%

		CI'_A			Equilibrium minimax	
		-10	0	10		
T'_REF_B	-7	63,5 / 37,74	64,51 / 36,24	65,37 / 34,97	η	-11,00%
	0	36,56 / 77,83	37,76 / 76,02	38,84 / 74,43	Π_A	68,10%
	7	25,23 / 94,68	26,27 / 93,14	27,18 / 91,78	Π_B	-50,40%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
F'_REF_B	-3	21,0 / 101,0	39,08 / 75,57	49,81 / 58,11	η	7,20%
	0	20,77 / 101,32	37,78 / 76,02	49,51 / 58,56	Π_A	-44,40%
	3	20,36 / 101,93	37,22 / 76,84	48,95 / 59,4	Π_B	32,90%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
F'_REF_B	-3	36,8 / 77,46	38,08 / 75,57	40,19 / 72,42	η	0,40%
	0	36,50 / 77,91	37,78 / 76,02	39,89 / 72,88	Π_A	-2,60%
	3	35,95 / 78,73	37,22 / 76,84	39,32 / 73,71	Π_B	1,90%

		CI'_A			Equilibrium minimax	
		-10	0	10		
F'_REF_B	-3	36,86 / 77,39	38,08 / 75,57	39,14 / 73,98	η	0,40%
	0	36,56 / 77,83	37,76 / 76,02	38,84 / 74,43	Π_A	-2,40%
	3	36,01 / 78,65	37,22 / 76,84	38,28 / 75,27	Π_B	1,80%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
CI'_B	-10	21,55 / 100,61	38,42 / 75,05	50,16 / 57,59	η	7,10%
	0	20,77 / 101,32	37,78 / 76,02	49,51 / 58,56	Π_A	-43,80%
	10	20,4 / 101,88	37,27 / 76,78	49,0 / 59,33	Π_B	32,40%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
CI'_B	-10	37,15 / 76,95	38,42 / 75,05	40,54 / 71,9	η	0,30%
	0	36,50 / 77,91	37,78 / 76,02	39,89 / 72,88	Π_A	-1,70%
	10	36,0 / 78,66	37,27 / 76,78	39,37 / 73,65	Π_B	1,20%

		CI'_REF_A			Equilibrium minimax	
		-10	0	10		
CI'_B	-10	37,2 / 76,88	38,42 / 75,05	39,49 / 73,46	η	0,20%
	0	36,56 / 77,83	37,76 / 76,02	38,84 / 74,43	Π_A	-1,50%
	10	36,05 / 78,58	37,27 / 76,78	38,33 / 75,2	Π_B	1,10%

Scenario γ

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
T'_REF_B	-7	50,98 / 67,05	71,1 / 38,57	80,26 / 25,59	η	-3,60%
	0	21,9 / 108,22	40,3 / 82,16	53,26 / 63,82	Π_A	26,50%
	7	12,93 / 120,92	26,4 / 101,85	37,79 / 85,72	Π_B	-18,40%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
T'_REF_B	-7	69,92 / 40,23	71,1 / 38,57	72,96 / 35,93	η	-10,10%
	0	38,91 / 84,13	40,3 / 82,16	42,62 / 78,88	Π_A	73,50%
	7	25,28 / 103,44	26,4 / 101,85	28,3 / 99,16	Π_B	-51,00%

		CI'_A			Equilibrium minimax	
		-10	0	10		
T'_REF_B	-7	69,97 / 40,16	71,1 / 38,57	72,05 / 37,22	η	-10,00%
	0	38,97 / 84,05	40,3 / 82,16	41,47 / 80,51	Π_A	73,60%
	7	25,32 / 103,38	26,4 / 101,85	27,35 / 100,5	Π_B	-51,10%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
F'_REF_B	-3	22,33 / 107,62	40,9 / 81,32	53,88 / 62,95	η	6,10%
	0	27,897 / 108,22	40,3 / 82,16	53,26 / 63,815	Π_A	-44,60%
	3	21,15 / 109,29	39,25 / 83,66	52,168 / 65,363	Π_B	31,00%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
F'_REF_B	-3	39,5 / 83,3	40,9 / 81,32	43,22 / 78,03	η	0,30%
	0	38,91 / 84,13	40,3 / 82,16	42,62 / 78,88	Π_A	-2,00%
	3	37,87 / 85,61	39,25 / 83,66	41,54 / 80,41	Π_B	1,40%

		CI'_A			Equilibrium minimax	
		-10	0	10		
F'_REF_B	-3	39,56 / 83,22	40,9 / 81,32	42,07 / 79,66	η	0,30%
	0	38,97 / 84,05	40,3 / 82,16	41,47 / 80,51	Π_A	-1,80%
	3	37,92 / 85,53	39,25 / 83,66	40,40 / 82,02	Π_B	1,30%

		T'_REF_A			Equilibrium minimax	
		-7	0	7		
CI'_B	-10	22,22 / 107,77	40,75 / 81,53	53,72 / 63,17	η	6,10%
	0	21,9 / 108,22	40,3 / 82,16	53,26 / 63,82	Π_A	-44,90%
	10	21,65 / 108,57	39,96 / 82,65	52,9 / 64,32	Π_B	31,20%

		F'_REF_A			Equilibrium minimax	
		-3	0	3		
CI'_B	-10	39,35 / 83,51	40,75 / 81,53	43,07 / 78,24	η	0,30%
	0	38,91 / 84,13	40,3 / 82,16	42,62 / 78,88	Π_A	-2,40%
	10	38,57 / 84,62	39,96 / 82,65	42,27 / 79,38	Π_B	1,60%

		CI'_A			Equilibrium minimax	
		-10	0	10		
CI'_B	-10	39,41 / 83,43	40,75 / 81,53	41,92 / 79,87	η	0,30%
	0	38,97 / 84,05	40,3 / 82,16	41,47 / 80,51	Π_A	-2,20%
	10	38,63 / 84,54	39,96 / 82,65	41,12 / 81,0	Π_B	1,50%

2.3 Case 2

Case 2 allows the combination of two parameters. Different as in case 1.2, the resulting equilibrium strategy for each player does not always involve proving the minimum value possible. But in any case, communicating the inside parameter value does not appear as the best choice.

Following table describes the different analysis cases. Again, with the minimax criterion, the equilibrium is determined for each analysis case. Alongside the payoff matrix, the corresponding values for the percentage benefit of both airlines as well as the resulting percentage of system's benefit are shown. Those values are given considering the equilibrium strategies and resulting solution (minimax) payoff values.

Scenario α

				T'_REF_A								
				-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A		
				-	=	+	-	=	+	-	=	+
T'_REF_B	-	F'_REF_B	-	44,43	55,90	63,80	70,64	71,48	72,79	77,56	77,75	78,09
				44,43	32,96	25,07	18,22	17,38	16,07	11,31	11,11	10,77
			=	32,96	44,43	53,33	61,82	62,91	64,65	71,25	71,53	72,02
				55,90	44,43	35,54	27,04	25,95	24,21	17,61	17,34	16,85
			+	25,07	35,54	44,43	53,65	54,89	56,89	64,82	65,16	65,77
				63,80	53,33	44,43	35,22	33,98	31,97	24,05	23,71	23,09
	=	F'_REF_B	-	18,22	27,04	35,22	44,43	45,74	47,88	56,78	57,18	57,90
				70,64	61,82	53,65	44,43	43,13	40,99	32,09	31,69	30,97
			=	17,38	25,95	33,98	43,13	44,43	46,58	55,56	55,97	56,70
				71,48	62,91	54,89	45,74	44,43	42,29	33,30	32,90	32,16
			+	16,07	24,21	31,97	40,99	42,29	44,43	53,53	53,94	54,69
				72,79	64,65	56,89	47,88	46,58	44,43	35,34	34,92	34,17
	+	F'_REF_B	-	11,31	17,61	24,05	32,09	33,30	35,34	44,43	44,87	45,66
				77,56	71,25	64,82	56,78	55,56	53,53	44,43	44,00	43,21
			=	11,11	17,34	23,71	31,69	32,90	34,92	44,00	44,43	45,22
				77,75	71,53	65,16	57,18	55,97	53,94	44,87	44,43	43,64
			+	10,77	16,85	23,09	30,97	32,16	34,17	43,21	43,64	44,43
				78,09	72,02	65,77	57,90	56,70	54,69	45,66	45,22	44,43

Equilibrium minimax	
η	0,00%
Π A	0,00%
Π B	0,00%

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	69,62	70,68	72,27	70,64	71,48	72,79	71,47	72,15	73,25	Equilibrium minimax	
				19,24	18,19	16,60	18,22	17,38	16,07	17,39	16,71	15,62		
			=	60,51	61,87	63,95	61,82	62,91	64,65	62,91	63,80	65,27	η	0,00%
				28,36	27,00	24,91	27,04	25,95	24,21	25,96	25,06	23,60	Π A	56,70%
			+	52,17	53,70	56,08	53,65	54,89	56,89	54,88	55,91	57,61	Π B	-56,70%
				36,69	35,17	32,78	35,22	33,98	31,97	33,98	32,96	31,26		
	=	F'_REF_B	-	42,90	44,49	47,01	44,43	45,74	47,88	45,73	46,82	48,65		
				45,96	44,38	41,86	44,43	43,13	40,99	43,13	42,04	40,21		
			=	41,60	43,18	45,70	43,13	44,43	46,58	44,42	45,52	47,35		
				47,27	45,69	43,16	45,74	44,43	42,29	44,44	43,35	41,51		
			+	39,47	41,04	43,56	40,99	42,29	44,43	42,28	43,37	45,21		
				49,40	47,82	45,31	47,88	46,58	44,43	46,58	45,49	43,65		
	+	F'_REF_B	-	30,69	32,14	34,50	32,09	33,30	35,34	33,29	34,33	36,09		
				58,18	56,73	54,36	56,78	55,56	53,53	55,57	54,54	52,78		
			=	30,30	31,74	34,09	31,69	32,90	34,92	32,89	33,91	35,67		
				58,57	57,13	54,77	57,18	55,97	53,94	55,98	54,95	53,20		
			+	29,59	31,02	33,35	30,97	32,16	34,17	32,16	33,17	34,91		
				59,27	57,85	55,52	57,90	56,70	54,69	56,71	55,69	53,95		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	56,12	70,68	77,09	55,90	71,48	77,75	55,69	72,15	78,29	Equilibrium minimax	
				32,75	18,19	11,77	32,96	17,38	11,11	33,17	16,71	10,58		
			=	44,66	61,87	70,58	44,43	62,91	71,53	44,21	63,80	72,30	η	0,00%
				44,20	27,00	18,28	44,43	25,95	17,34	44,66	25,06	16,57	Π A	25,30%
			+	35,76	53,70	63,99	35,54	54,89	65,16	35,32	55,91	66,12	Π B	-25,30%
				53,11	35,17	24,87	53,33	33,98	23,71	53,55	32,96	22,74		
	=	F'_REF_B	-	27,24	44,49	55,82	27,04	45,74	57,18	26,85	46,82	58,32		
				61,63	44,38	33,05	61,82	43,13	31,69	62,01	42,04	30,55		
			=	26,14	43,18	54,59	25,95	44,43	55,97	25,76	45,52	57,13		
				62,72	45,69	34,28	62,91	44,43	32,90	63,10	43,35	31,74		
			+	24,40	41,04	52,53	24,21	42,29	53,94	24,03	43,37	55,13		
				64,47	47,82	36,33	64,65	46,58	34,92	64,83	45,49	33,73		
	+	F'_REF_B	-	17,76	32,14	43,40	17,61	33,30	44,87	17,47	34,33	46,12		
				71,10	56,73	45,47	71,25	55,56	44,00	71,39	54,54	42,75		
			=	17,48	31,74	42,96	17,34	32,90	44,43	17,20	33,91	45,68		
				71,38	57,13	45,90	71,53	55,97	44,43	71,67	54,95	43,18		
			+	16,99	31,02	42,18	16,85	32,16	43,64	16,71	33,17	44,89		
				71,87	57,85	46,69	72,02	56,70	45,22	72,15	55,69	43,97		

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	F'_REF_B	-	19,24	28,36	36,69	45,96	47,27	49,40	58,18	58,57	59,27	Equilibrium minimax	
				69,62	60,51	52,17	42,90	41,60	39,47	30,69	30,30	29,59		
			=	18,19	27,00	35,17	44,38	45,69	47,82	56,73	57,13	57,85	η	0,00%
				70,68	61,87	53,70	44,49	43,18	41,04	32,14	31,74	31,02	Π A	-56,70%
			+	16,60	24,91	32,78	41,86	43,16	45,31	54,36	54,77	55,52	Π B	56,70%
				72,27	63,95	56,08	47,01	45,70	43,56	34,50	34,09	33,35		
	=	F'_REF_B	-	18,22	27,04	35,22	44,43	45,74	47,88	56,78	57,18	57,90		
				70,64	61,82	53,65	44,43	43,13	40,99	32,09	31,69	30,97		
			=	17,38	25,95	33,98	43,13	44,43	46,58	55,56	55,97	56,70		
				71,48	62,91	54,89	45,74	44,43	42,29	33,30	32,90	32,16		
			+	16,07	24,21	31,97	40,99	42,29	44,43	53,53	53,94	54,69		
				72,79	64,65	56,89	47,88	46,58	44,43	35,34	34,92	34,17		
	+	F'_REF_B	-	17,39	25,96	33,98	43,13	44,44	46,58	55,57	55,98	56,71		
				71,47	62,91	54,88	45,73	44,42	42,28	33,29	32,89	32,16		
			=	16,71	25,06	32,96	42,04	43,35	45,49	54,54	54,95	55,69		
				72,15	63,80	55,91	46,82	45,52	43,37	34,33	33,91	33,17		
			+	15,62	23,60	31,26	40,21	41,51	43,65	52,78	53,20	53,95		
				73,25	65,27	57,61	48,65	47,35	45,21	36,09	35,67	34,91		

				CI'_A								
				-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	F'_REF_B	-	44,43	46,02	48,53	45,96	47,27	49,40	47,26	48,35	50,16
				44,43	42,85	40,33	42,90	41,60	39,47	41,61	40,52	38,70
			=	42,85	44,43	46,96	44,38	45,69	47,82	45,68	46,77	48,60
				46,02	44,43	41,91	44,49	43,18	41,04	43,19	42,10	40,27
			+	40,33	41,91	44,43	41,86	43,16	45,31	43,15	44,25	46,08
				48,53	46,96	44,43	47,01	45,70	43,56	45,71	44,62	42,78
	=	F'_REF_B	-	42,90	44,49	47,01	44,43	45,74	47,88	45,73	46,82	48,65
				45,96	44,38	41,86	44,43	43,13	40,99	43,13	42,04	40,21
			=	41,60	43,18	45,70	43,13	44,43	46,58	44,42	45,52	47,35
				47,27	45,69	43,16	45,74	44,43	42,29	44,44	43,35	41,51
			+	39,47	41,04	43,56	40,99	42,29	44,43	42,28	43,37	45,21
				49,40	47,82	45,31	47,88	46,58	44,43	46,58	45,49	43,65
	+	F'_REF_B	-	41,61	43,19	45,71	43,13	44,44	46,58	44,43	45,53	47,36
				47,26	45,68	43,15	45,73	44,42	42,28	44,43	43,34	41,50
			=	40,52	42,10	44,62	42,04	43,35	45,49	43,34	44,43	46,27
				48,35	46,77	44,25	46,82	45,52	43,37	45,53	44,43	42,59
			+	38,70	40,27	42,78	40,21	41,51	43,65	41,50	42,59	44,43
				50,16	48,60	46,08	48,65	47,35	45,21	47,36	46,27	44,43

Equilibrium minimax	
η	0,00%
Π_A	0,00%
Π_B	0,00%

				CI'_A								
				-			=			+		
				T'_REF_A			T'_REF_A			T'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	F'_REF_B	-	28,56	46,02	57,24	28,36	47,27	58,57	28,16	48,35	59,68
				60,31	42,85	31,63	60,51	41,60	30,30	60,70	40,52	29,18
			=	27,19	44,43	55,77	27,00	45,69	57,13	26,81	46,77	58,27
				61,67	44,43	33,10	61,87	43,18	31,74	62,06	42,10	30,60
			+	25,10	41,91	53,37	24,91	43,16	54,77	24,73	44,25	55,95
				63,77	46,96	35,49	63,95	45,70	34,09	64,14	44,62	32,92
	=	F'_REF_B	-	27,24	44,49	55,82	27,04	45,74	57,18	26,85	46,82	58,32
				61,63	44,38	33,05	61,82	43,13	31,69	62,01	42,04	30,55
			=	26,14	43,18	54,59	25,95	44,43	55,97	25,76	45,52	57,13
				62,72	45,69	34,28	62,91	44,43	32,90	63,10	43,35	31,74
			+	24,40	41,04	52,53	24,21	42,29	53,94	24,03	43,37	55,13
				64,47	47,82	36,33	64,65	46,58	34,92	64,83	45,49	33,73
	+	F'_REF_B	-	26,15	43,19	54,60	25,96	44,44	55,98	25,77	45,53	57,13
				62,72	45,68	34,27	62,91	44,42	32,89	63,09	43,34	31,73
			=	25,25	42,10	53,55	25,06	43,35	54,95	24,88	44,43	56,12
				63,62	46,77	35,31	63,80	45,52	33,91	63,99	44,43	32,74
			+	23,78	40,27	51,78	23,60	41,51	53,20	23,42	42,59	54,39
				65,09	48,60	37,09	65,27	47,35	35,67	65,44	46,27	34,47

Equilibrium minimax	
η	0,00%
Π_A	-36,60%
Π_B	36,60%

				T'_REF_A								
				-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	T'_REF_B	-	**32,75**	44,20	53,11	61,63	62,72	64,47	71,10	71,38	71,87
				56,12	44,66	35,76	27,24	26,14	24,40	17,76	17,48	16,99
			=	18,19	27,00	35,17	44,38	45,69	47,82	56,73	57,13	57,85
				70,68	61,87	53,70	44,49	43,18	41,04	32,14	31,74	31,02
			+	11,77	18,28	24,87	33,05	34,28	36,33	45,47	45,90	46,69
				77,09	70,58	63,99	55,82	54,59	52,53	43,40	42,96	42,18
	=	T'_REF_B	-	32,96	44,43	53,33	61,82	62,91	64,65	71,25	71,53	72,02
				55,90	44,43	35,54	27,04	25,95	24,21	17,61	17,34	16,85
			=	17,38	25,95	33,98	43,13	44,43	46,58	55,56	55,97	56,70
				71,48	62,91	54,89	45,74	44,43	42,29	33,30	32,90	32,16
			+	11,11	17,34	23,71	31,69	32,90	34,92	44,00	44,43	45,22
				77,75	71,53	65,16	57,18	55,97	53,94	44,87	44,43	43,64
	+	T'_REF_B	-	33,17	44,66	53,55	62,01	63,10	64,83	71,39	71,67	72,15
				55,69	44,21	35,32	26,85	25,76	24,03	17,47	17,20	16,71
			=	16,71	25,06	32,96	42,04	43,35	45,49	54,54	54,95	55,69
				72,15	63,80	55,91	46,82	45,52	43,37	34,33	33,91	33,17
			+	10,58	16,57	22,74	30,55	31,74	33,73	42,75	43,18	43,97
				78,29	72,30	66,12	58,32	57,13	55,13	46,12	45,68	44,89

Equilibrium minimax	
η	0,00%
Π_A	-25,30%
Π_B	25,30%

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	**60,31**	61,67	63,77	61,63	62,72	64,47	62,72	63,62	65,09	Equilibrium minimax	
				28,56	27,19	25,10	27,24	26,14	24,40	26,15	25,25	23,78		
			=	42,85	44,43	46,96	44,38	45,69	47,82	45,68	46,77	48,60	η	0,00%
				46,02	44,43	41,91	44,49	43,18	41,04	43,19	42,10	40,27	Π A	36,60%
			+	31,63	33,10	35,49	33,05	34,28	36,33	34,27	35,31	37,09	Π B	-36,60%
				57,24	55,77	53,37	55,82	54,59	52,53	54,60	53,55	51,78		
	=	T'_REF_B	-	60,51	61,87	63,95	61,82	62,91	64,65	62,91	63,80	65,27		
				28,36	27,00	24,91	27,04	25,95	24,21	25,96	25,06	23,60		
			=	41,60	43,18	45,70	43,13	44,43	46,58	44,42	45,52	47,35		
				47,27	45,69	43,16	45,74	44,43	42,29	44,44	43,35	41,51		
			+	30,30	31,74	34,09	31,69	32,90	34,92	32,89	33,91	35,67		
				58,57	57,13	54,77	57,18	55,97	53,94	55,98	54,95	53,20		
	+	T'_REF_B	-	60,70	62,06	64,14	62,01	63,10	64,83	63,09	63,99	65,44		
				28,16	26,81	24,73	26,85	25,76	24,03	25,77	24,88	23,42		
			=	40,52	42,10	44,62	42,04	43,35	45,49	43,34	44,43	46,27		
				48,35	46,77	44,25	46,82	45,52	43,37	45,53	44,43	42,59		
			+	29,18	30,60	32,92	30,55	31,74	33,73	31,73	32,74	34,47		
				59,68	58,27	55,95	58,32	57,13	55,13	57,13	56,12	54,39		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	**44,43**	61,67	70,43	44,20	62,72	71,38	43,98	63,62	72,16	Equilibrium minimax	
				44,43	27,19	18,43	44,66	26,14	17,48	44,89	25,25	16,71		
			=	27,19	44,43	55,77	27,00	45,69	57,13	26,81	46,77	58,27	η	0,00%
				61,67	44,43	33,10	61,87	43,18	31,74	62,06	42,10	30,60	Π A	0,00%
			+	18,43	33,10	44,43	18,28	34,28	45,90	18,13	35,31	47,15	Π B	0,00%
				70,43	55,77	44,43	70,58	54,59	42,96	70,73	53,55	41,72		
	=	T'_REF_B	-	44,66	61,87	70,58	44,43	62,91	71,53	44,21	63,80	72,30		
				44,20	27,00	18,28	44,43	25,95	17,34	44,66	25,06	16,57		
			=	26,14	43,18	54,59	25,95	44,43	55,97	25,76	45,52	57,13		
				62,72	45,69	34,28	62,91	44,43	32,90	63,10	43,35	31,74		
			+	17,48	31,74	42,96	17,34	32,90	44,43	17,20	33,91	45,68		
				71,38	57,13	45,90	71,53	55,97	44,43	71,67	54,95	43,18		
	+	T'_REF_B	-	44,89	62,06	70,73	44,66	63,10	71,67	44,43	63,99	72,43		
				43,98	26,81	18,13	44,21	25,76	17,20	44,43	24,88	16,43		
			=	25,25	42,10	53,55	25,06	43,35	54,95	24,88	44,43	56,12		
				63,62	46,77	35,31	63,80	45,52	33,91	63,99	44,43	32,74		
			+	16,71	30,60	41,72	16,57	31,74	43,18	16,43	32,74	44,43		
				72,16	58,27	47,15	72,30	57,13	45,68	72,43	56,12	44,43		

Scenario β

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	39,50	51,16	59,60	67,20	68,15	69,65	75,17	75,40	75,80	Equilibrium minimax	
				73,45	56,10	43,54	32,23	30,82	28,60	20,38	20,04	19,44		
			=	34,82	46,40	55,20	63,46	64,51	66,18	72,47	72,73	73,19	η	-0,70%
				80,42	63,19	50,09	37,80	36,24	33,75	24,40	24,01	23,32	Π A	4,60%
			+	29,53	40,68	49,66	58,53	59,70	61,56	68,74	69,04	69,58	Π B	-3,40%
				88,28	71,70	58,33	45,13	43,40	40,63	29,95	29,50	28,69		
	=	F'_REF_B	-	13,71	20,99	28,17	36,80	38,08	40,19	49,39	49,81	50,59		
				111,83	100,99	90,31	77,46	75,57	72,42	58,74	58,10	56,95		
			=	13,54	20,77	27,90	36,50	37,78	39,89	49,08	49,51	50,29		
				112,07	101,32	90,71	77,91	76,02	72,88	59,20	58,56	57,40		
			+	13,25	20,36	27,41	35,95	37,22	39,32	48,52	48,95	49,73		
				112,50	101,93	91,43	78,73	76,84	73,71	60,04	59,40	58,24		
	+	F'_REF_B	-	8,26	13,16	18,39	25,27	26,35	28,17	36,69	37,11	37,88		
				119,93	112,64	104,87	94,62	93,02	90,31	77,63	77,01	75,86		
			=	8,23	13,11	18,32	25,19	26,27	28,09	36,59	37,02	37,78		
				119,98	112,72	104,96	94,74	93,14	90,43	77,78	77,15	76,00		
			+	8,17	13,02	18,20	25,04	26,11	27,93	36,41	36,83	37,60		
				120,07	112,86	105,14	94,97	93,37	90,67	78,04	77,42	76,28		

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	66,05	67,24	69,05	67,20	68,15	69,65	68,14	68,92	70,17	Equilibrium minimax	
				33,94	32,17	29,49	32,23	30,82	28,60	30,83	29,68	27,82		
			=	62,19	63,50	65,51	63,46	64,51	66,18	64,50	65,37	66,77	η	-12,10%
				39,69	37,74	34,75	37,80	36,24	33,75	36,25	34,96	32,88	Π A	74,90%
			+	57,14	58,58	60,81	58,53	59,70	61,56	59,69	60,65	62,22	Π B	-55,30%
				47,20	45,06	41,75	45,13	43,40	40,63	43,41	41,99	39,65		
	=	F'_REF_B	-	35,33	36,86	39,33	36,80	38,08	40,19	38,07	39,14	40,97		
				79,66	77,39	73,71	77,46	75,57	72,42	75,58	73,98	71,27		
			=	35,03	36,56	39,02	36,50	37,78	39,89	37,77	38,84	40,66		
				80,10	77,83	74,16	77,91	76,02	72,88	76,03	74,43	71,73		
			+	34,49	36,01	38,46	35,95	37,22	39,32	37,21	38,28	40,09		
				80,91	78,65	74,99	78,73	76,84	73,71	76,86	75,27	72,57		
	+	F'_REF_B	-	24,04	25,31	27,42	25,27	26,35	28,17	26,34	27,26	28,85		
				96,45	94,56	91,42	94,62	93,02	90,31	93,03	91,66	89,29		
			=	23,97	25,23	27,34	25,19	26,27	28,09	26,26	27,18	28,77		
				96,57	94,68	91,55	94,74	93,14	90,43	93,15	91,78	89,42		
			+	23,82	25,08	27,18	25,04	26,11	27,93	26,10	27,02	28,60		
				96,78	94,90	91,78	94,97	93,37	90,67	93,38	92,02	89,67		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	51,39	67,24	74,62	51,16	68,15	75,40	50,94	68,92	76,03	Equilibrium minimax	
				55,76	32,17	21,19	56,10	30,82	20,04	56,43	29,68	19,10		
			=	46,63	63,50	71,84	46,40	64,51	72,73	46,17	65,37	73,46	η	-5,60%
				62,84	37,74	25,34	63,19	36,24	24,01	63,52	34,96	22,92	Π A	-25,80%
			+	40,91	58,58	68,00	40,68	59,70	69,04	40,45	60,65	69,89	Π B	34,60%
				71,36	45,06	31,04	71,70	43,40	29,50	72,03	41,99	28,23		
	=	F'_REF_B	-	21,16	36,86	48,36	20,99	38,08	49,81	20,83	39,14	51,04		
				100,75	77,39	60,27	100,99	75,57	58,10	101,23	73,98	56,28		
			=	20,93	36,56	48,06	20,77	37,78	49,51	20,61	38,84	50,74		
				101,08	77,83	60,72	101,32	76,02	58,56	101,56	74,43	56,73		
			+	20,53	36,01	47,49	20,36	37,22	48,95	20,20	38,28	50,18		
				101,68	78,65	61,56	101,93	76,84	59,40	102,16	75,27	57,56		
	+	F'_REF_B	-	13,28	25,31	35,69	13,16	26,35	37,11	13,05	27,26	38,33		
				112,47	94,56	79,12	112,64	93,02	77,01	112,81	91,66	75,19		
			=	13,23	25,23	35,60	13,11	26,27	37,02	13,00	27,18	38,24		
				112,54	94,68	79,26	112,72	93,14	77,15	112,88	91,78	75,33		
			+	13,13	25,08	35,42	13,02	26,11	36,83	12,90	27,02	38,05		
				112,68	94,90	79,53	112,86	93,37	77,42	113,02	92,02	75,60		

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	F'_REF_B	-	14,09	21,52	28,81	37,52	38,80	40,92	50,11	50,53	51,31	Equilibrium minimax	
				111,26	100,20	89,36	76,40	74,50	71,34	57,67	57,03	55,89		
			=	13,89	21,25	28,47	37,15	38,42	40,54	49,73	50,16	50,93	η	10,20%
				111,56	100,61	89,86	76,95	75,05	71,90	58,23	57,59	56,44	Π A	-62,70%
			+	13,53	20,74	27,87	36,47	37,74	39,85	49,05	49,48	50,25	Π B	46,40%
				112,10	101,36	90,75	77,96	76,07	72,93	59,24	58,61	57,45		
	=	F'_REF_B	-	13,71	20,99	28,17	36,80	38,08	40,19	49,39	49,81	50,59		
				111,83	100,99	90,31	77,46	75,57	72,42	58,74	58,10	56,95		
			=	13,54	20,77	27,90	36,50	37,78	39,89	49,08	49,51	50,29		
				112,07	101,32	90,71	77,91	76,02	72,88	59,20	58,56	57,40		
			+	13,25	20,36	27,41	35,95	37,22	39,32	48,52	48,95	49,73		
				112,50	101,93	91,43	78,73	76,84	73,71	60,04	59,40	58,24		
	+	F'_REF_B	-	13,41	20,58	27,67	36,25	37,52	39,62	48,82	49,25	50,02		
				112,27	101,61	91,05	78,29	76,40	73,27	59,59	58,95	57,79		
			=	13,28	20,40	27,45	36,00	37,27	39,37	48,56	48,99	49,77		
				112,47	101,88	91,38	78,66	76,78	73,64	59,97	59,33	58,17		
			+	13,04	20,06	27,05	35,54	36,80	38,90	48,09	48,52	49,30		
				112,83	102,37	91,98	79,34	77,46	74,34	60,67	60,03	58,86		

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	F'_REF_B	-	36,03	37,57	40,05	37,52	38,80	40,92	38,79	39,87	41,69	Equilibrium minimax	
				78,62	76,33	72,63	76,40	74,50	71,34	74,51	72,90	70,19		
			=	35,66	37,20	39,68	37,15	38,42	40,54	38,41	39,49	41,32	η	0,70%
				79,16	76,88	73,19	76,95	75,05	71,90	75,07	73,46	70,75	Π A	-4,60%
			+	35,00	36,52	38,99	36,47	37,74	39,85	37,73	38,81	40,63	Π B	3,40%
				80,15	77,88	74,21	77,96	76,07	72,93	76,08	74,48	71,78		
	=	F'_REF_B	-	35,33	36,86	39,33	36,80	38,08	40,19	38,07	39,14	40,97		
				79,66	77,39	73,71	77,46	75,57	72,42	75,58	73,98	71,27		
			=	35,03	36,56	39,02	36,50	37,78	39,89	37,77	38,84	40,66		
				80,10	77,83	74,16	77,91	76,02	72,88	76,03	74,43	71,73		
			+	34,49	36,01	38,46	35,95	37,22	39,32	37,21	38,28	40,09		
				80,91	78,65	74,99	78,73	76,84	73,71	76,86	75,27	72,57		
	+	F'_REF_B	-	34,78	36,30	38,76	36,25	37,52	39,62	37,51	38,58	40,39		
				80,48	78,22	74,55	78,29	76,40	73,27	76,42	74,82	72,12		
			=	34,53	36,05	38,51	36,00	37,27	39,37	37,26	38,33	40,14		
				80,84	78,58	74,92	78,66	76,78	73,64	76,79	75,20	72,50		
			+	34,08	35,59	38,04	35,54	36,80	38,90	36,80	37,86	39,67		
				81,51	79,26	75,62	79,34	77,46	74,34	77,48	75,89	73,20		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	F'_REF_B	-	21,69	37,57	49,09	21,52	38,80	50,53	21,36	39,87	51,75	Equilibrium minimax	
				99,95	76,33	59,19	100,20	74,50	57,03	100,45	72,90	55,22		
			=	21,41	37,20	48,71	21,25	38,42	50,16	21,08	39,49	51,39	η	7,00%
				100,36	76,88	59,75	100,61	75,05	57,59	100,86	73,46	55,77	Π A	-43,50%
			+	20,91	36,52	48,02	20,74	37,74	49,48	20,58	38,81	50,71	Π B	32,10%
				101,11	77,88	60,77	101,36	76,07	58,61	101,60	74,48	56,78		
	=	F'_REF_B	-	21,16	36,86	48,36	20,99	38,08	49,81	20,83	39,14	51,04		
				100,75	77,39	60,27	100,99	75,57	58,10	101,23	73,98	56,28		
			=	20,93	36,56	48,06	20,77	37,78	49,51	20,61	38,84	50,74		
				101,08	77,83	60,72	101,32	76,02	58,56	101,56	74,43	56,73		
			+	20,53	36,01	47,49	20,36	37,22	48,95	20,20	38,28	50,18		
				101,68	78,65	61,56	101,93	76,84	59,40	102,16	75,27	57,56		
	+	F'_REF_B	-	20,74	36,30	47,79	20,58	37,52	49,25	20,42	38,58	50,48		
				101,36	78,22	61,12	101,61	76,40	58,95	101,84	74,82	57,12		
			=	20,56	36,05	47,54	20,40	37,27	48,99	20,24	38,33	50,23		
				101,63	78,58	61,49	101,88	76,78	59,33	102,11	75,20	57,49		
			+	20,22	35,59	47,06	20,06	36,80	48,52	19,90	37,86	49,76		
				102,13	79,26	62,20	102,37	77,46	60,03	102,61	75,89	58,19		

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	34,56	46,13	54,94	63,24	64,30	65,97	72,30	72,57	73,03	Equilibrium minimax	
				80,80	63,59	50,47	38,13	36,56	34,06	24,64	24,25	23,56		
			=	13,89	21,25	28,47	37,15	38,42	40,54	49,73	50,16	50,93	η	1,20%
				111,56	100,61	89,86	76,95	75,05	71,90	58,23	57,59	56,44	Π A	-7,20%
			+	8,47	13,48	18,80	25,78	26,87	28,71	37,29	37,71	38,49	Π B	5,40%
				119,62	112,17	104,26	93,87	92,25	89,51	76,74	76,11	74,96		
	=	T'_REF_B	-	34,82	46,40	55,20	63,46	64,51	66,18	72,47	72,73	73,19		
				80,42	63,19	50,09	37,80	36,24	33,75	24,40	24,01	23,32		
			=	13,54	20,77	27,90	36,50	37,78	39,89	49,08	49,51	50,29		
				112,07	101,32	90,71	77,91	76,02	72,88	59,20	58,56	57,40		
			+	8,23	13,11	18,32	25,19	26,27	28,09	36,59	37,02	37,78		
				119,98	112,72	104,96	94,74	93,14	90,43	77,78	77,15	76,00		
	+	T'_REF_B	-	35,04	46,63	55,42	63,65	64,70	66,36	72,61	72,87	73,33		
				80,09	62,84	49,77	37,52	35,96	33,49	24,19	23,81	23,12		
			=	13,28	20,40	27,45	36,00	37,27	39,37	48,56	48,99	49,77		
				112,47	101,88	91,38	78,66	76,78	73,64	59,97	59,33	58,17		
			+	8,04	12,83	17,96	24,74	25,80	27,60	36,05	36,47	37,23		
				120,26	113,13	105,51	95,42	93,84	91,15	78,59	77,96	76,82		

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	61,96	63,28	65,30	63,24	64,30	65,97	64,29	65,15	66,57	Equilibrium minimax	
				40,03	38,07	35,06	38,13	36,56	34,06	36,57	35,28	33,18		
			=	35,66	37,20	39,68	37,15	38,42	40,54	38,41	39,49	41,32	η	-10,60%
				79,16	76,88	73,19	76,95	75,05	71,90	75,07	73,46	70,75	Π A	65,10%
			+	24,53	25,82	27,95	25,78	26,87	28,71	26,86	27,79	29,40	Π B	-48,20%
				95,72	93,80	90,63	93,87	92,25	89,51	92,26	90,87	88,48		
	=	T'_REF_B	-	62,19	63,50	65,51	63,46	64,51	66,18	64,50	65,37	66,77		
				39,69	37,74	34,75	37,80	36,24	33,75	36,25	34,96	32,88		
			=	35,03	36,56	39,02	36,50	37,78	39,89	37,77	38,84	40,66		
				80,10	77,83	74,16	77,91	76,02	72,88	76,03	74,43	71,73		
			+	23,97	25,23	27,34	25,19	26,27	28,09	26,26	27,18	28,77		
				96,57	94,68	91,55	94,74	93,14	90,43	93,15	91,78	89,42		
	+	T'_REF_B	-	62,38	63,69	65,69	63,65	64,70	66,36	64,69	65,55	66,94		
				39,40	37,46	34,48	37,52	35,96	33,49	35,97	34,70	32,62		
			=	34,53	36,05	38,51	36,00	37,27	39,37	37,26	38,33	40,14		
				80,84	78,58	74,92	78,66	76,78	73,64	76,79	75,20	72,50		
			+	23,52	24,78	26,86	24,74	25,80	27,60	25,79	26,70	28,27		
				97,22	95,36	92,26	95,42	93,84	91,15	93,85	92,49	90,15		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	46,36	63,28	71,67	46,13	64,30	72,57	45,90	65,15	73,30	Equilibrium minimax	
				63,25	38,07	25,59	63,59	36,56	24,25	63,92	35,28	23,16		
			=	21,41	37,20	48,71	21,25	38,42	50,16	21,08	39,49	51,39	η	-3,70%
				100,36	76,88	59,75	100,61	75,05	57,59	100,86	73,46	55,77	Π A	22,80%
			+	13,59	25,82	36,29	13,48	26,87	37,71	13,36	27,79	38,94	Π B	-16,90%
				112,00	93,80	78,23	112,17	92,25	76,11	112,35	90,87	74,28		
	=	T'_REF_B	-	46,63	63,50	71,84	46,40	64,51	72,73	46,17	65,37	73,46		
				62,84	37,74	25,34	63,19	36,24	24,01	63,52	34,96	22,92		
			=	20,93	36,56	48,06	20,77	37,78	49,51	20,61	38,84	50,74		
				101,08	77,83	60,72	101,32	76,02	58,56	101,56	74,43	56,73		
			+	13,23	25,23	35,60	13,11	26,27	37,02	13,00	27,18	38,24		
				112,54	94,68	79,26	112,72	93,14	77,15	112,88	91,78	75,33		
	+	T'_REF_B	-	46,86	63,69	71,98	46,63	64,70	72,87	46,40	65,55	73,59		
				62,50	37,46	25,12	62,84	35,96	23,81	63,18	34,70	22,73		
			=	20,56	36,05	47,54	20,40	37,27	48,99	20,24	38,33	50,23		
				101,63	78,58	61,49	101,88	76,78	59,33	102,11	75,20	57,49		
			+	12,94	24,78	35,06	12,83	25,80	36,47	12,72	26,70	37,68		
				112,97	95,36	80,06	113,13	93,84	77,96	113,30	92,49	76,15		

Scenario γ

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	52,31	64,75	73,07	80,15	81,00	82,34	87,16	87,35	87,69	Equilibrium minimax	
				65,16	47,55	35,76	25,75	24,54	22,65	15,82	15,55	15,07		
			=	38,18	50,98	60,74	69,92	71,09	72,95	79,97	80,26	80,78	η	-4,10%
				85,16	67,05	53,23	40,23	38,57	35,93	26,00	25,59	24,86	Π A	29,80%
			+	28,92	40,72	50,61	60,74	62,09	64,27	72,85	73,22	73,87	Π B	-20,70%
				98,28	81,58	67,57	53,23	51,31	48,23	36,08	35,56	34,63		
	=	F'_REF_B	-	14,52	22,33	30,08	39,50	40,90	43,22	53,39	53,87	54,73		
				118,67	107,62	96,63	83,30	81,32	78,03	63,63	62,95	61,73		
			=	14,21	21,90	29,56	38,91	40,30	42,62	52,78	53,26	54,13		
				119,11	108,22	97,37	84,13	82,16	78,88	64,49	63,82	62,59		
			+	13,68	21,15	28,65	37,87	39,25	41,54	51,69	52,17	53,04		
				119,86	109,29	98,66	85,61	83,66	80,41	66,04	65,36	64,13		
	+	F'_REF_B	-	8,11	13,00	18,30	25,40	26,53	28,44	37,50	37,95	38,78		
				127,75	120,82	113,31	103,26	101,67	98,96	86,13	85,49	84,31		
			=	8,06	12,93	18,20	25,28	26,40	28,30	37,34	37,79	38,62		
				127,82	120,92	113,46	103,44	101,85	99,16	86,36	85,72	84,54		
			+	7,96	12,78	18,01	25,03	26,15	28,04	37,04	37,49	38,32		
				127,95	121,13	113,73	103,78	102,20	99,52	86,78	86,14	84,97		

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	79,10	80,18	81,81	80,15	81,00	82,34	81,00	81,69	82,80	Equilibrium minimax	
				27,23	25,70	23,40	25,75	24,54	22,65	24,55	23,57	21,99		
			=	68,51	69,97	72,21	69,92	71,09	72,95	71,09	72,05	73,61	η	-13,20%
				42,23	40,16	36,99	40,23	38,57	35,93	38,58	37,22	35,01	Π A	96,20%
			+	59,12	60,79	63,39	60,74	62,09	64,27	62,08	63,20	65,05	Π B	-66,90%
				55,52	53,15	49,47	53,23	51,31	48,23	51,33	49,74	47,13		
	=	F'_REF_B	-	37,88	39,56	42,27	39,50	40,90	43,22	40,89	42,07	44,08		
				85,59	83,22	79,37	83,30	81,32	78,03	81,33	79,66	76,82		
			=	37,30	38,97	41,67	38,91	40,30	42,62	40,29	41,47	43,47		
				86,41	84,05	80,22	84,13	82,16	78,88	82,17	80,51	77,68		
			+	36,27	37,92	40,60	37,87	39,25	41,54	39,24	40,40	42,39		
				87,87	85,53	81,74	85,61	83,66	80,41	83,67	82,02	79,21		
	+	F'_REF_B	-	24,13	25,45	27,65	25,40	26,53	28,44	26,52	27,48	29,15		
				105,07	103,19	100,07	103,26	101,67	98,96	101,68	100,31	97,95		
			=	24,00	25,32	27,52	25,28	26,40	28,30	26,39	27,35	29,01		
				105,24	103,38	100,27	103,44	101,85	99,16	101,86	100,50	98,15		
			+	23,77	25,08	27,26	25,03	26,15	28,04	26,14	27,10	28,75		
				105,57	103,72	100,63	103,78	102,20	99,52	102,22	100,86	98,52		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
T'_REF_B	-	F'_REF_B	-	64,98	80,18	86,69	64,75	81,00	87,35	64,52	81,69	87,89	Equilibrium minimax	
				47,23	25,70	16,49	47,55	24,54	15,55	47,87	23,57	14,79		
			=	51,23	69,97	79,26	50,98	71,09	80,26	50,73	72,05	81,08	η	-8,20%
				66,69	40,16	27,00	67,05	38,57	25,59	67,40	37,22	24,43	Π A	60,10%
			+	40,96	60,79	71,96	40,72	62,09	73,22	40,47	63,20	74,25	Π B	-41,70%
				81,23	53,15	37,34	81,58	51,31	35,56	81,92	49,74	34,09		
	=	F'_REF_B	-	22,50	39,56	52,26	22,33	40,90	53,87	22,15	42,07	55,24		
				107,36	83,22	65,24	107,62	81,32	62,95	107,86	79,66	61,02		
			=	22,07	38,97	51,64	21,90	40,30	53,26	21,72	41,47	54,63		
				107,97	84,05	66,11	108,22	82,16	63,82	108,47	80,51	61,87		
			+	21,32	37,92	50,55	21,15	39,25	52,17	20,98	40,40	53,54		
				109,04	85,53	67,66	109,29	83,66	65,36	109,52	82,02	63,42		
	+	F'_REF_B	-	13,12	25,45	36,42	13,00	26,53	37,95	12,89	27,48	39,27		
				120,65	103,19	87,65	120,82	101,67	85,49	120,98	100,31	83,62		
			=	13,04	25,32	36,27	12,93	26,40	37,79	12,81	27,35	39,11		
				120,76	103,38	87,88	120,92	101,85	85,72	121,09	100,50	83,85		
			+	12,90	25,08	35,97	12,78	26,15	37,49	12,67	27,10	38,81		
				120,96	103,72	88,29	121,13	102,20	86,14	121,29	100,86	84,28		

				T'_REF_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	F'_REF_B	-	14,82	22,75	30,59	40,08	41,48	43,81	53,99	54,46	55,32	Equilibrium minimax	
				118,24	107,02	95,91	82,48	80,49	77,19	62,79	62,11	60,89		
			=	14,44	22,22	29,95	39,35	40,75	43,07	53,24	53,72	54,58	η	8,70%
				118,78	107,77	96,82	83,51	81,53	78,24	63,84	63,17	61,95	Π A	-63,20%
			+	13,79	21,30	28,84	38,09	39,47	41,77	51,92	52,40	53,27	Π B	43,90%
				119,70	109,06	98,39	85,30	83,34	80,08	65,71	65,03	63,81		
	=	F'_REF_B	-	14,52	22,33	30,08	39,50	40,90	43,22	53,39	53,87	54,73		
				118,67	107,62	96,63	83,30	81,32	78,03	63,63	62,95	61,73		
			=	14,21	21,90	29,56	38,91	40,30	42,62	52,78	53,26	54,13		
				119,11	108,22	97,37	84,13	82,16	78,88	64,49	63,82	62,59		
			+	13,68	21,15	28,65	37,87	39,25	41,54	51,69	52,17	53,04		
				119,86	109,29	98,66	85,61	83,66	80,41	66,04	65,36	64,13		
	+	F'_REF_B	-	14,28	22,00	29,69	39,06	40,45	42,77	52,93	53,41	54,28		
				119,00	108,07	97,19	83,93	81,96	78,67	64,28	63,60	62,38		
			=	14,03	21,65	29,26	38,57	39,96	42,27	52,43	52,90	53,77		
				119,36	108,57	97,79	84,62	82,65	79,38	65,00	64,32	63,10		
			+	13,59	21,02	28,50	37,69	39,07	41,36	51,50	51,98	52,85		
				119,99	109,46	98,88	85,86	83,91	80,66	66,30	65,62	64,39		

				CI'_A								
				-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	F'_REF_B	-	38,45	40,14	42,86	40,08	41,48	43,81	41,47	42,66	44,67
				84,78	82,40	78,54	82,48	80,49	77,19	80,51	78,83	75,98
			=	37,74	39,41	42,12	39,35	40,75	43,07	40,74	41,92	43,92
				85,79	83,43	79,58	83,51	81,53	78,24	81,54	79,87	77,04
			+	36,49	38,14	40,83	38,09	39,47	41,77	39,46	40,63	42,62
				87,56	85,22	81,42	85,30	83,34	80,08	83,36	81,70	78,89
	=	F'_REF_B	-	37,88	39,56	42,27	39,50	40,90	43,22	40,89	42,07	44,08
				85,59	83,22	79,37	83,30	81,32	78,03	81,33	79,66	76,82
			=	37,30	38,97	41,67	38,91	40,30	42,62	40,29	41,47	43,47
				86,41	84,05	80,22	84,13	82,16	78,88	82,17	80,51	77,68
			+	36,27	37,92	40,60	37,87	39,25	41,54	39,24	40,40	42,39
				87,87	85,53	81,74	85,61	83,66	80,41	83,67	82,02	79,21
	+	F'_REF_B	-	37,44	39,11	41,82	39,06	40,45	42,77	40,44	41,62	43,62
				86,21	83,85	80,02	83,93	81,96	78,67	81,97	80,30	77,47
			=	36,96	38,62	41,32	38,57	39,96	42,27	39,95	41,12	43,11
				86,89	84,54	80,72	84,62	82,65	79,38	82,67	81,01	78,18
			+	36,10	37,75	40,42	37,69	39,07	41,36	39,06	40,22	42,21
				88,11	85,78	81,99	85,86	83,91	80,66	83,92	82,27	79,47

Equilibrium minimax	
η	0,60%
Π A	-4,60%
Π B	3,20%

				CI'_A								
				-			=			+		
				T'_REF_A			T'_REF_A			T'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	F'_REF_B	-	22,93	40,14	52,85	22,75	41,48	54,46	22,57	42,66	55,83
				106,76	82,40	64,39	107,02	80,49	62,11	107,27	78,83	60,18
			=	22,40	39,41	52,10	22,22	40,75	53,72	22,04	41,92	55,09
				107,52	83,43	65,45	107,77	81,53	63,17	108,01	79,87	61,23
			+	21,48	38,14	50,78	21,30	39,47	52,40	21,13	40,63	53,78
				108,82	85,22	67,33	109,06	83,34	65,03	109,30	81,70	63,09
	=	F'_REF_B	-	22,50	39,56	52,26	22,33	40,90	53,87	22,15	42,07	55,24
				107,36	83,22	65,24	107,62	81,32	62,95	107,86	79,66	61,02
			=	22,07	38,97	51,64	21,90	40,30	53,26	21,72	41,47	54,63
				107,97	84,05	66,11	108,22	82,16	63,82	108,47	80,51	61,87
			+	21,32	37,92	50,55	21,15	39,25	52,17	20,98	40,40	53,54
				109,04	85,53	67,66	109,29	83,66	65,36	109,52	82,02	63,42
	+	F'_REF_B	-	22,18	39,11	51,79	22,00	40,45	53,41	21,83	41,62	54,78
				107,82	83,85	65,89	108,07	81,96	63,60	108,32	80,30	61,67
			=	21,82	38,62	51,28	21,65	39,96	52,90	21,48	41,12	54,28
				108,33	84,54	66,61	108,57	82,65	64,32	108,82	81,01	62,38
			+	21,19	37,75	50,36	21,02	39,07	51,98	20,85	40,22	53,36
				109,22	85,78	67,92	109,46	83,91	65,62	109,70	82,27	63,68

Equilibrium minimax	
η	6,00%
Π A	-44,00%
Π B	30,60%

				T'_REF_A								
				-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A		
				-	=	+	-	=	+	-	=	+
CI'_B	-	T'_REF_B	-	36,78	49,50	59,33	68,69	69,89	71,80	79,05	79,35	79,89
				87,15	69,15	55,23	41,98	40,27	37,57	27,30	26,88	26,11
			=	14,44	22,22	29,95	39,35	40,75	43,07	53,24	53,72	54,58
				118,78	107,77	96,82	83,51	81,53	78,24	63,84	63,17	61,95
			+	8,23	13,19	18,55	25,72	26,85	28,78	37,88	38,34	39,17
				127,57	120,55	112,96	102,81	101,21	98,48	85,59	84,94	83,76
	=	T'_REF_B	-	38,18	50,98	60,74	69,92	71,09	72,95	79,97	80,26	80,78
				85,16	67,05	53,23	40,23	38,57	35,93	26,00	25,59	24,86
			=	14,21	21,90	29,56	38,91	40,30	42,62	52,78	53,26	54,13
				119,11	108,22	97,37	84,13	82,16	78,88	64,49	63,82	62,59
			+	8,06	12,93	18,20	25,28	26,40	28,30	37,34	37,79	38,62
				127,82	120,92	113,46	103,44	101,85	99,16	86,36	85,72	84,54
	+	T'_REF_B	-	39,47	52,33	62,00	71,02	72,16	73,97	80,78	81,06	81,56
				83,34	65,14	51,44	38,68	37,05	34,49	24,86	24,46	23,76
			=	14,03	21,65	29,26	38,57	39,96	42,27	52,43	52,90	53,77
				119,36	108,57	97,79	84,62	82,65	79,38	65,00	64,32	63,10
			+	7,92	12,72	17,93	24,93	26,04	27,94	36,92	37,37	38,20
				128,01	121,21	113,84	103,92	102,35	99,67	86,95	86,31	85,15

Equilibrium minimax	
η	0,30%
Π A	-2,10%
Π B	1,40%

				CI'_A										
				-			=			+				
				F'_REF_A			F'_REF_A			F'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	67,24	68,74	71,03	68,69	69,89	71,80	69,88	70,87	72,48	Equilibrium minimax	
				44,03	41,91	38,65	41,98	40,27	37,57	40,28	38,89	36,61		
			=	37,74	39,41	42,12	39,35	40,75	43,07	40,74	41,92	43,92	η	-10,00%
				85,79	83,43	79,58	83,51	81,53	78,24	81,54	79,87	77,04	Π A	72,80%
			+	24,43	25,76	27,98	25,72	26,85	28,78	26,84	27,82	29,50	Π B	-50,50%
				104,64	102,75	99,61	102,81	101,21	98,48	101,22	99,84	97,46		
	=	T'_REF_B	-	68,51	69,97	72,21	69,92	71,09	72,95	71,09	72,05	73,61		
				42,23	40,16	36,99	40,23	38,57	35,93	38,58	37,22	35,01		
			=	37,30	38,97	41,67	38,91	40,30	42,62	40,29	41,47	43,47		
				86,41	84,05	80,22	84,13	82,16	78,88	82,17	80,51	77,68		
			+	24,00	25,32	27,52	25,28	26,40	28,30	26,39	27,35	29,01		
				105,24	103,38	100,27	103,44	101,85	99,16	101,86	100,50	98,15		
	+	T'_REF_B	-	**69,64**	71,06	73,25	71,02	72,16	73,97	72,15	73,09	74,61		
				40,63	38,61	35,52	38,68	37,05	34,49	37,07	35,74	33,59		
			=	36,96	38,62	41,32	38,57	39,96	42,27	39,95	41,12	43,11		
				86,89	84,54	80,72	84,62	82,65	79,38	82,67	81,01	78,18		
			+	23,67	24,98	27,16	24,93	26,04	27,94	26,04	26,99	28,64		
				105,71	103,86	100,78	103,92	102,35	99,67	102,36	101,01	98,67		

				CI'_A										
				-			=			+				
				T'_REF_A			T'_REF_A			T'_REF_A				
				-	=	+	-	=	+	-	=	+		
CI'_B	-	T'_REF_B	-	49,75	68,74	78,32	49,50	69,89	79,35	49,25	70,87	80,20	Equilibrium minimax	
				68,79	41,91	28,34	69,15	40,27	26,88	69,50	38,89	25,68		
			=	22,40	39,41	52,10	22,22	40,75	53,72	22,04	41,92	55,09	η	-4,00%
				107,52	83,43	65,45	107,77	81,53	63,17	108,01	79,87	61,23	Π A	29,20%
			+	13,31	25,76	36,81	13,19	26,85	38,34	13,08	27,82	39,67	Π B	-20,30%
				120,38	102,75	87,11	120,55	101,21	84,94	120,71	99,84	83,07		
	=	T'_REF_B	-	51,23	69,97	79,26	50,98	71,09	80,26	50,73	72,05	81,08		
				66,69	40,16	27,00	67,05	38,57	25,59	67,40	37,22	24,43		
			=	22,07	38,97	51,64	21,90	40,30	53,26	21,72	41,47	54,63		
				107,97	84,05	66,11	108,22	82,16	63,82	108,47	80,51	61,87		
			+	13,04	25,32	36,27	12,93	26,40	37,79	12,81	27,35	39,11		
				120,76	103,38	87,88	120,92	101,85	85,72	121,09	100,50	83,85		
	+	T'_REF_B	-	52,58	71,06	80,09	52,33	72,16	81,06	**52,08**	73,09	81,84		
				64,78	38,61	25,82	65,14	37,05	24,46	**65,49**	35,74	23,35		
			=	21,82	38,62	51,28	21,65	39,96	52,90	21,48	41,12	54,28		
				108,33	84,54	66,61	108,57	82,65	64,32	108,82	81,01	62,38		
			+	12,84	24,98	35,85	12,72	26,04	37,37	12,61	26,99	38,68		
				121,05	103,86	88,46	121,21	102,35	86,31	121,37	101,01	84,46		

2.4 Case 3

Each airline can provide values different form their inside parameter values for any of the three possible parameters.

The equilibria result from applying the minimax criterion. The red cell shows the payoff value for each airline. In the upper left corner the percentage of system's benefit, as well as the corresponding airline's percentage benefits, are detailed for the equilibrium strategies.

Scenario α_a Case 3a

Both flights cover STAR RUSIK 3C and have the same cost index, CI=33

Equilibrium minimax	
η	0,00%
Π_A	0,00%
Π_B	0,00%

CI'_A = -

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	44,43	56,89	64,88	70,18	71,21	72,77	77,23	77,47	77,89
			44,43	31,98	23,98	18,68	17,65	16,10	11,64	11,40	10,97
-	-	=	31,98	44,43	53,61	60,31	61,67	63,77	70,08	70,43	71,05
			56,89	44,43	35,25	28,56	27,19	25,10	18,78	18,43	17,81
-	-	+	23,98	35,25	44,43	51,66	53,19	55,59	63,13	63,57	64,34
			64,88	53,61	44,43	37,21	35,67	33,27	25,73	25,30	24,53
-	=	-	18,68	28,56	37,21	44,43	46,02	48,53	56,75	57,24	58,11
			70,18	60,31	51,66	44,43	42,85	40,33	32,12	31,63	30,76
-	=	=	17,65	27,19	35,67	42,85	44,43	46,96	55,27	55,77	56,66
			71,21	61,67	53,19	46,02	44,43	41,91	33,60	33,10	32,21
-	=	+	16,10	25,10	33,27	40,33	41,91	44,43	52,86	53,37	54,29
			72,77	63,77	55,59	48,53	46,96	44,43	36,00	35,49	34,58
-	+	-	11,64	18,78	25,73	32,12	33,60	36,00	44,43	44,97	45,92
			77,23	70,08	63,13	56,75	55,27	52,86	44,43	43,90	42,94
-	+	=	11,40	18,43	25,30	31,63	33,10	35,49	43,90	44,43	45,39
			77,47	70,43	63,57	57,24	55,77	53,37	44,97	44,43	43,48
-	+	+	10,97	17,81	24,53	30,76	32,21	34,58	42,94	43,48	44,43
			77,89	71,05	64,34	58,11	56,66	54,29	45,92	45,39	44,43
=	-	-	43,60	56,12	64,22	69,62	70,68	72,27	76,84	77,09	77,52
			45,27	32,75	24,64	19,24	18,19	16,60	12,02	11,77	11,34
=	-	=	32,19	44,66	53,83	60,51	61,87	63,95	70,23	70,58	71,20
			56,68	44,20	35,03	28,36	27,00	24,91	18,63	18,28	17,66
=	-	+	24,40	35,76	44,96	52,17	53,70	56,08	63,56	63,99	64,76
			64,47	53,11	43,91	36,69	35,17	32,78	25,30	24,87	24,11
=	=	-	17,68	27,24	35,72	42,90	44,49	47,01	55,32	55,82	56,71
			71,18	61,63	53,14	45,96	44,38	41,86	33,55	33,05	32,16
=	=	=	16,87	26,14	34,47	41,60	43,18	45,70	54,08	54,59	55,49
			72,00	62,72	54,39	47,27	45,69	43,16	34,78	34,28	33,37
=	=	+	15,59	24,40	32,46	39,47	41,04	43,56	52,02	52,53	53,46
			73,28	64,47	56,40	49,40	47,82	45,31	36,85	36,33	35,41
=	+	-	10,94	17,76	24,46	30,69	32,14	34,50	42,87	43,40	44,35
			77,92	71,10	64,40	58,18	56,73	54,36	46,00	45,47	44,51
=	+	=	10,75	17,48	24,12	30,30	31,74	34,09	42,43	42,96	43,92
			78,11	71,38	64,75	58,57	57,13	54,77	46,43	45,90	44,94
=	+	+	10,42	16,99	23,50	29,59	31,02	33,35	41,65	42,18	43,13
			78,44	71,87	65,36	59,27	57,85	55,52	47,22	46,69	45,73
+	-	-	42,85	55,42	63,62	69,11	70,19	71,81	76,49	76,74	77,19
			46,01	33,45	25,25	19,75	18,68	17,06	12,37	12,12	11,68
+	-	=	32,40	44,89	54,05	60,70	62,06	64,14	70,38	70,73	71,34
			56,47	43,98	34,82	28,16	26,81	24,73	18,48	18,13	17,52
+	-	+	24,82	36,27	45,49	52,68	54,20	56,58	63,99	64,42	65,17
			64,04	52,60	43,38	36,18	34,66	32,29	24,87	24,45	23,69
+	=	-	16,87	26,15	34,48	41,61	43,19	45,71	54,09	54,60	55,50
			71,99	62,72	54,38	47,26	45,68	43,15	34,77	34,27	33,37
+	=	=	16,21	25,25	33,45	40,52	42,10	44,62	53,04	53,55	54,47
			72,66	63,62	55,42	48,35	46,77	44,25	35,82	35,31	34,40
+	=	+	15,14	23,78	31,74	38,70	40,27	42,78	51,26	51,78	52,70
			73,72	65,09	57,12	50,16	48,60	46,08	37,61	37,09	36,16
+	+	-	10,38	16,93	23,42	29,50	30,92	33,25	41,55	42,08	43,03
			78,48	71,94	65,44	59,36	57,94	55,61	47,32	46,79	45,83
+	+	=	10,23	16,71	23,14	29,18	30,60	32,92	41,19	41,72	42,67
			78,63	72,16	65,72	59,68	58,27	55,95	47,68	47,15	46,19
+	+	+	9,97	16,31	22,64	28,60	30,00	32,30	40,53	41,06	42,01
			78,90	72,56	66,23	60,26	58,86	56,56	48,33	47,81	46,85

CI'_A = =

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	45,27	56,68	64,47	71,18	72,00	73,28	77,92	78,11	78,44
			43,60	32,19	24,40	17,68	16,87	15,59	10,94	10,75	10,42
-	-	=	32,75	44,20	53,11	61,63	62,72	64,47	71,10	71,38	71,87
			56,12	44,66	35,76	27,24	26,14	24,40	17,76	17,48	16,99
-	-	+	24,64	35,03	43,91	53,14	54,39	56,40	64,40	64,75	65,36
			64,22	53,83	44,96	35,72	34,47	32,46	24,46	24,12	23,50
-	=	-	19,24	28,36	36,69	45,96	47,27	49,40	58,18	58,57	59,27
			69,62	60,51	52,17	42,90	41,60	39,47	30,69	30,30	29,59
-	=	=	18,19	27,00	35,17	44,38	45,69	47,82	56,73	57,13	57,85
			70,68	61,87	53,70	44,49	43,18	41,04	32,14	31,74	31,02
-	=	+	16,60	24,91	32,78	41,86	43,16	45,31	54,36	54,77	55,52
			72,27	63,95	56,08	47,01	45,70	43,56	34,50	34,09	33,35
-	+	-	12,02	18,63	25,30	33,55	34,78	36,85	46,00	46,43	47,22
			76,84	70,23	63,56	55,32	54,08	52,02	42,87	42,43	41,65
-	+	=	11,77	18,28	24,87	33,05	34,28	36,33	45,47	45,90	46,69
			77,09	70,58	63,99	55,82	54,59	52,53	43,40	42,96	42,18
-	+	+	11,34	17,66	24,11	32,16	33,37	35,41	44,51	44,94	45,73
			77,52	71,20	64,76	56,71	55,49	53,46	44,35	43,92	43,13
=	-	-	44,43	55,90	63,80	70,64	71,48	72,79	77,56	77,75	78,09
			44,43	32,96	25,07	18,22	17,38	16,07	11,31	11,11	10,77
=	-	=	32,96	44,43	53,33	61,82	62,91	64,65	71,25	71,53	72,02
			55,90	44,43	35,54	27,04	25,95	24,21	17,61	17,34	16,85
=	-	+	25,07	35,54	44,43	53,65	54,89	56,89	64,82	65,16	65,77
			63,80	53,33	44,43	35,22	33,98	31,97	24,05	23,71	23,09
=	=	-	18,22	27,04	35,22	44,43	45,74	47,88	56,78	57,18	57,90
			70,64	61,82	53,65	44,43	43,13	40,99	32,09	31,69	30,97
=	=	=	17,38	25,95	33,98	43,13	44,43	46,58	55,56	55,97	56,70
			71,48	62,91	54,89	45,74	44,43	42,29	33,30	32,90	32,16
=	=	+	16,07	24,21	31,97	40,99	42,29	44,43	53,53	53,94	54,69
			72,79	64,65	56,89	47,88	46,58	44,43	35,34	34,92	34,17
=	+	-	11,31	17,61	24,05	32,09	33,30	35,34	44,43	44,87	45,66
			77,56	71,25	64,82	56,78	55,56	53,53	44,43	44,00	43,21
=	+	=	11,11	17,34	23,71	31,69	32,90	34,92	44,00	44,43	45,22
			77,75	71,53	65,16	57,18	55,97	53,94	44,87	44,43	43,64
=	+	+	10,77	16,85	23,09	30,97	32,16	34,17	43,21	43,64	44,43
			78,09	72,02	65,77	57,90	56,70	54,69	45,66	45,22	44,43
+	-	-	43,69	55,20	63,19	70,15	71,01	72,34	77,22	77,42	77,77
			45,18	33,66	25,68	18,71	17,86	16,52	11,64	11,45	11,10
+	-	=	33,17	44,66	53,55	62,01	63,10	64,83	71,39	71,67	72,15
			55,69	44,21	35,32	26,85	25,76	24,03	17,47	17,20	16,71
+	-	+	25,50	36,05	44,96	54,15	55,39	57,38	65,23	65,57	66,18
			63,37	52,82	43,90	34,71	33,48	31,49	23,63	23,29	22,69
+	=	-	17,39	25,96	33,98	43,13	44,44	46,58	55,57	55,98	56,71
			71,47	62,91	54,88	45,73	44,42	42,28	33,29	32,89	32,16
+	=	=	16,71	25,06	32,96	42,04	43,35	45,49	54,54	54,95	55,69
			72,15	63,80	55,91	46,82	45,52	43,37	34,33	33,91	33,17
+	=	+	15,62	23,60	31,26	40,21	41,51	43,65	52,78	53,20	53,95
			73,25	65,27	57,61	48,65	47,35	45,21	36,09	35,67	34,91
+	+	-	10,73	16,79	23,02	30,88	32,07	34,07	43,11	43,54	44,33
			78,13	72,08	65,85	57,99	56,79	54,79	45,76	45,32	44,53
+	+	=	10,58	16,57	22,74	30,55	31,74	33,73	42,75	43,18	43,97
			78,29	72,30	66,12	58,32	57,13	55,13	46,12	45,68	44,89
+	+	+	10,30	16,17	22,24	29,96	31,13	33,11	42,09	42,52	43,31
			78,56	72,69	66,62	58,91	57,73	55,75	46,78	46,34	45,55

CI'_A = +

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	46,01	56,47	64,04	71,99	72,66	73,72	78,48	78,63	78,90
			42,85	32,40	24,82	16,87	16,21	15,14	10,38	10,23	9,97
-	-	=	33,45	43,98	52,60	62,72	63,62	65,09	71,94	72,16	72,56
			55,42	44,89	36,27	26,15	25,25	23,78	16,93	16,71	16,31
-	-	+	25,25	34,82	43,38	54,38	55,42	57,12	65,44	65,72	66,23
			63,62	54,05	45,49	34,48	33,45	31,74	23,42	23,14	22,64
-	=	-	19,75	28,16	36,18	47,26	48,35	50,16	59,36	59,68	60,26
			69,11	60,70	52,68	41,61	40,52	38,70	29,50	29,18	28,60
-	=	=	18,68	26,81	34,66	45,68	46,77	48,60	57,94	58,27	58,86
			70,19	62,06	54,20	43,19	42,10	40,27	30,92	30,60	30,00
-	=	+	17,06	24,73	32,29	43,15	44,25	46,08	55,61	55,95	56,56
			71,81	64,14	56,58	45,71	44,62	42,78	33,25	32,92	32,30
-	+	-	12,37	18,48	24,87	34,77	35,82	37,61	47,32	47,68	48,33
			76,49	70,38	63,99	54,09	53,04	51,26	41,55	41,19	40,53
-	+	=	12,12	18,13	24,45	34,27	35,31	37,09	46,79	47,15	47,81
			76,74	70,73	64,42	54,60	53,55	51,78	42,08	41,72	41,06
-	+	+	11,68	17,52	23,69	33,37	34,40	36,16	45,83	46,19	46,85
			77,19	71,34	65,17	55,50	54,47	52,70	43,03	42,67	42,01
=	-	-	45,18	55,69	63,37	71,47	72,15	73,25	78,13	78,29	78,56
			43,69	33,17	25,50	17,39	16,71	15,62	10,73	10,58	10,30
=	-	=	33,66	44,21	52,82	62,91	63,80	65,27	72,08	72,30	72,69
			55,20	44,66	36,05	25,96	25,06	23,60	16,79	16,57	16,17
=	-	+	25,68	35,32	43,90	54,88	55,91	57,61	65,85	66,12	66,62
			63,19	53,55	44,96	33,98	32,96	31,26	23,02	22,74	22,24
=	=	-	18,71	26,85	34,71	45,73	46,82	48,65	57,99	58,32	58,91
			70,15	62,01	54,15	43,13	42,04	40,21	30,88	30,55	29,96
=	=	=	17,86	25,76	33,48	44,42	45,52	47,35	56,79	57,13	57,73
			71,01	63,10	55,39	44,44	43,35	41,51	32,07	31,74	31,13
=	=	+	16,52	24,03	31,49	42,28	43,37	45,21	54,79	55,13	55,75
			72,34	64,83	57,38	46,58	45,49	43,65	34,07	33,73	33,11
=	+	-	11,64	17,47	23,63	33,29	34,33	36,09	45,76	46,12	46,78
			77,22	71,39	65,23	55,57	54,54	52,78	43,11	42,75	42,09
=	+	=	11,45	17,20	23,29	32,89	33,91	35,67	45,32	45,68	46,34
			77,42	71,67	65,57	55,98	54,95	53,20	43,54	43,18	42,52
=	+	+	11,10	16,71	22,69	32,16	33,17	34,91	44,53	44,89	45,55
			77,77	72,15	66,18	56,71	55,69	53,95	44,33	43,97	43,31
+	-	-	44,43	54,99	62,75	71,00	71,69	72,81	77,81	77,97	78,25
			44,43	33,88	26,12	17,87	17,17	16,05	11,05	10,90	10,61
+	-	=	33,88	44,43	53,04	63,09	63,99	65,44	72,22	72,43	72,83
			54,99	44,43	35,83	25,77	24,88	23,42	16,65	16,43	16,04
+	-	+	26,12	35,83	44,43	55,38	56,40	58,09	66,25	66,53	67,02
			62,75	53,04	44,43	33,48	32,46	30,78	22,61	22,34	21,84
+	=	-	17,87	25,77	33,48	44,43	45,53	47,36	56,80	57,13	57,74
			71,00	63,09	55,38	44,43	43,34	41,50	32,06	31,73	31,13
+	=	=	17,17	24,88	32,46	43,34	44,43	46,27	55,79	56,12	56,74
			71,69	63,99	56,40	45,53	44,43	42,59	33,08	32,74	32,13
+	=	+	16,05	23,42	30,78	41,50	42,59	44,43	54,05	54,39	55,02
			72,81	65,44	58,09	47,36	46,27	44,43	34,82	34,47	33,85
+	+	-	11,05	16,65	22,61	32,06	33,08	34,82	44,43	44,79	45,45
			77,81	72,22	66,25	56,80	55,79	54,05	44,43	44,07	43,41
+	+	=	10,90	16,43	22,34	31,73	32,74	34,47	44,07	44,43	45,09
			77,97	72,43	66,53	57,13	56,12	54,39	44,79	44,43	43,77
+	+	+	10,61	16,04	21,84	31,13	32,13	33,85	43,41	43,77	44,43
			78,25	72,83	67,02	57,74	56,74	55,02	45,45	45,09	44,43

Scenario α_b Case 3b

Both flights cover the same STAR, TERTO3C, and have the same cost index, CI=33

Equilibrium minimax	
η	0,00%
Π_A	0,00%
Π_B	0,00%

CI'_A: -

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	47,93	67,60	76,08	77,24	78,99	81,27	85,08	85,42	86,00
			47,93	28,26	19,78	18,62	16,86	14,59	10,78	10,43	9,86
-	-	=	28,26	47,93	59,10	60,80	63,45	67,05	73,56	74,18	75,23
			67,60	47,93	36,76	35,06	32,41	28,81	22,30	21,68	20,63
-	-	+	19,78	36,76	47,93	49,75	52,64	56,70	64,45	65,22	66,53
			76,08	59,10	47,93	46,11	43,22	39,16	31,41	30,64	29,33
-	=	-	18,62	35,06	46,11	47,93	50,83	54,93	62,83	63,62	64,96
			77,24	60,80	49,75	47,93	45,03	40,93	33,03	32,24	30,90
-	=	=	16,86	32,41	43,22	45,03	47,93	52,06	60,16	60,97	62,37
			78,99	63,45	52,64	50,83	47,93	43,79	35,70	34,89	33,49
-	=	+	14,59	28,81	39,16	40,93	43,79	47,93	56,21	57,05	58,51
			81,27	67,05	56,70	54,93	52,06	47,93	39,65	38,81	37,35
-	+	-	10,78	22,30	31,41	33,03	35,70	39,65	47,93	48,81	50,32
			85,08	73,56	64,45	62,83	60,16	56,21	47,93	47,05	45,54
-	+	=	10,43	21,68	30,64	32,24	34,89	38,81	47,05	47,93	49,45
			85,42	74,18	65,22	63,62	60,97	57,05	48,81	47,93	46,41
-	+	+	9,86	20,63	29,33	30,90	33,49	37,35	45,54	46,41	47,93
			86,00	75,23	66,53	64,96	62,37	58,51	50,32	49,45	47,93
=	-	-	48,29	67,90	76,31	77,47	79,20	81,45	85,22	85,56	86,13
			47,57	27,96	19,55	18,39	16,66	14,41	10,64	10,30	9,73
=	-	=	29,03	48,85	59,96	61,65	64,27	67,82	74,21	74,82	75,84
			66,83	47,01	35,90	34,21	31,59	28,04	21,65	21,04	20,02
=	-	+	20,44	37,71	48,92	50,74	53,62	57,66	65,32	66,08	67,37
			75,42	58,15	46,93	45,12	42,24	38,20	30,54	29,78	28,49
=	=	-	17,77	33,79	44,73	46,55	49,45	53,57	61,57	62,37	63,74
			78,09	62,07	51,13	49,31	46,41	42,29	34,29	33,49	32,12
=	=	=	16,35	31,61	42,33	44,13	47,03	51,17	59,31	60,14	61,55
			79,51	64,25	53,53	51,73	48,83	44,69	36,55	35,72	34,31
=	=	+	14,41	28,50	38,81	40,57	43,43	47,57	55,85	56,70	58,16
			81,45	67,36	57,05	55,29	52,43	48,29	40,01	39,16	37,70
=	+	-	10,21	21,28	30,14	31,73	34,35	38,25	46,48	47,35	48,87
			85,65	74,58	65,72	64,13	61,51	57,61	49,38	48,51	46,99
=	+	=	9,94	20,79	29,52	31,10	33,70	37,57	45,76	46,64	48,16
			85,92	75,07	66,33	64,76	62,16	58,29	50,10	49,22	47,70
=	+	+	9,48	19,94	28,46	30,00	32,55	36,37	44,50	45,37	46,89
			86,38	75,92	67,40	65,86	63,31	59,49	51,36	50,49	48,97
+	-	-	48,63	68,18	76,53	77,68	79,40	81,62	85,36	85,69	86,26
			47,23	27,68	19,33	18,18	16,46	14,24	10,50	10,17	9,60
+	-	=	29,81	49,77	60,82	62,49	65,08	68,58	74,85	75,44	76,44
			66,05	46,09	35,04	33,37	30,78	27,28	21,01	20,42	19,41
+	-	+	21,12	38,67	49,93	51,74	54,61	58,61	66,19	66,93	68,20
			74,74	57,19	45,93	44,12	41,25	37,24	29,67	28,93	27,66
+	=	-	17,08	32,74	43,59	45,40	48,30	52,43	60,50	61,31	62,70
			78,78	63,12	52,27	50,46	47,56	43,43	35,36	34,54	33,16
+	=	=	15,91	30,92	41,56	43,36	46,25	50,39	58,58	59,40	60,83
			79,95	64,94	54,29	52,50	49,61	45,47	37,28	36,45	35,03
+	=	+	14,24	28,23	38,49	40,25	43,11	47,24	55,53	56,38	57,85
			81,62	67,63	57,37	55,61	52,75	48,62	40,33	39,47	38,01
+	+	-	9,76	20,45	29,11	30,67	33,25	37,10	45,27	46,14	47,66
			86,10	75,41	66,75	65,19	62,61	58,76	50,59	49,72	48,20
+	+	=	9,55	20,06	28,61	30,15	32,71	36,53	44,68	45,55	47,07
			86,31	75,80	67,25	65,71	63,15	59,32	51,18	50,31	48,79
+	+	+	9,17	19,36	27,72	29,24	31,76	35,53	43,62	44,49	46,00
			86,69	76,50	68,14	66,62	64,10	60,33	52,24	51,37	49,86

CI'_A: =

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	47,57	66,83	75,42	78,09	79,51	81,45	85,65	85,92	86,38
			48,29	29,03	20,44	17,77	16,35	14,41	10,21	9,94	9,48
-	-	=	27,96	47,01	58,15	62,07	64,25	67,36	74,58	75,07	75,92
			67,90	48,85	37,71	33,79	31,61	28,50	21,28	20,79	19,94
-	-	+	19,55	35,90	46,93	51,13	53,53	57,05	65,72	66,33	67,40
			76,31	59,96	48,92	44,73	42,33	38,81	30,14	29,52	28,46
-	=	-	18,39	34,21	45,12	49,31	51,73	55,29	64,13	64,76	65,86
			77,47	61,65	50,74	46,55	44,13	40,57	31,73	31,10	30,00
-	=	=	16,66	31,59	42,24	46,41	48,83	52,43	61,51	62,16	63,31
			79,20	64,27	53,62	49,45	47,03	43,43	34,35	33,70	32,55
-	=	+	14,41	28,04	38,20	42,29	44,69	48,29	57,61	58,29	59,49
			81,45	67,82	57,66	53,57	51,17	47,57	38,25	37,57	36,37
-	+	-	10,64	21,65	30,54	34,29	36,55	40,01	49,38	50,10	51,36
			85,22	74,21	65,32	61,57	59,31	55,85	46,48	45,76	44,50
-	+	=	10,30	21,04	29,78	33,49	35,72	39,16	48,51	49,22	50,49
			85,56	74,82	66,08	62,37	60,14	56,70	47,35	46,64	45,37
-	+	+	9,73	20,02	28,49	32,12	34,31	37,70	46,99	47,70	48,97
			86,13	75,84	67,37	63,74	61,55	58,16	48,87	48,16	46,89
=	-	-	47,93	67,13	75,66	78,31	79,71	81,63	85,78	86,05	86,50
			47,93	28,73	20,20	17,55	16,15	14,22	10,08	9,81	9,35
=	-	=	28,73	47,93	59,03	62,91	65,06	68,12	75,21	75,69	76,52
			67,13	47,93	36,83	32,95	30,80	27,74	20,65	20,17	19,34
=	-	+	20,20	36,83	47,93	52,11	54,51	58,00	66,57	67,18	68,23
			75,66	59,03	47,93	43,74	41,35	37,85	29,29	28,68	27,63
=	=	-	17,55	32,95	43,74	47,93	50,35	53,93	62,89	63,54	64,66
			78,31	62,91	52,11	47,93	45,51	41,93	32,96	32,32	31,20
=	=	=	16,15	30,80	41,35	45,51	47,93	51,53	60,68	61,34	62,49
			79,71	65,06	54,51	50,35	47,93	44,33	35,18	34,52	33,36
=	=	+	14,22	27,74	37,85	41,93	44,33	47,93	57,26	57,95	59,15
			81,63	68,12	58,00	53,93	51,53	47,93	38,60	37,91	36,71
=	+	-	10,08	20,65	29,29	32,96	35,18	38,60	47,93	48,65	49,91
			85,78	75,21	66,57	62,89	60,68	57,26	47,93	47,21	45,95
=	+	=	9,81	20,17	28,68	32,32	34,52	37,91	47,21	47,93	49,19
			86,05	75,69	67,18	63,54	61,34	57,95	48,65	47,93	46,67
=	+	+	9,35	19,34	27,63	31,20	33,36	36,71	45,95	46,67	47,93
			86,50	76,52	68,23	64,66	62,49	59,15	49,91	49,19	47,93
+	-	-	48,27	67,42	75,89	78,51	79,90	81,81	85,91	86,17	86,62
			47,59	28,44	19,97	17,35	15,96	14,05	9,95	9,68	9,23
+	-	=	29,51	48,85	59,89	63,74	65,86	68,87	75,83	76,30	77,11
			66,35	47,01	35,96	32,12	30,00	26,99	20,03	19,56	18,75
+	-	+	20,88	37,79	48,93	53,11	55,49	58,96	67,42	68,01	69,04
			74,98	58,07	46,93	42,75	40,37	36,90	28,44	27,85	26,82
+	=	-	16,87	31,92	42,60	46,78	49,20	52,79	61,85	62,50	63,64
			78,99	63,94	53,26	49,08	46,66	43,07	34,01	33,36	32,22
+	=	=	15,72	30,12	40,59	44,73	47,15	50,76	59,95	60,62	61,78
			80,14	65,73	55,27	51,13	48,71	45,10	35,91	35,24	34,07
+	=	+	14,06	27,47	37,54	41,61	44,00	47,60	56,94	57,63	58,84
			81,80	68,39	58,32	54,25	51,86	48,26	38,92	38,23	37,02
+	+	-	9,63	19,84	28,27	31,88	34,07	37,44	46,72	47,44	48,70
			86,23	76,02	67,59	63,98	61,79	58,42	49,14	48,42	47,16
+	+	=	9,42	19,45	27,78	31,36	33,53	36,88	46,13	46,84	48,11
			86,44	76,40	68,08	64,50	62,33	58,98	49,73	49,02	47,75
+	+	+	9,05	18,77	26,91	30,42	32,56	35,87	45,06	45,78	47,04
			86,81	77,09	68,95	65,43	63,30	59,99	50,80	50,08	48,82

CI'_A: +

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	47,23	66,05	74,74	78,78	79,95	81,62	86,10	86,31	86,69
			48,63	29,81	21,12	17,08	15,91	14,24	9,76	9,55	9,17
-	-	=	27,68	46,09	57,19	63,12	64,94	67,63	75,41	75,80	76,50
			68,18	49,77	38,67	32,74	30,92	28,23	20,45	20,06	19,36
-	-	+	19,33	35,04	45,93	52,27	54,29	57,37	66,75	67,25	68,14
			76,53	60,82	49,93	43,59	41,56	38,49	29,11	28,61	27,72
-	=	-	18,18	33,37	44,12	50,46	52,50	55,61	65,19	65,71	66,62
			77,68	62,49	51,74	45,40	43,36	40,25	30,67	30,15	29,24
-	=	=	16,46	30,78	41,25	47,56	49,61	52,75	62,61	63,15	64,10
			79,40	65,08	54,61	48,30	46,25	43,11	33,25	32,71	31,76
-	=	+	14,24	27,28	37,24	43,43	45,47	48,62	58,76	59,32	60,33
			81,62	68,58	58,61	52,43	50,39	47,24	37,10	36,53	35,53
-	+	-	10,50	21,01	29,67	35,36	37,28	40,33	50,59	51,18	52,24
			85,36	74,85	66,19	60,50	58,58	55,53	45,27	44,68	43,62
-	+	=	10,17	20,42	28,93	34,54	36,45	39,47	49,72	50,31	51,37
			85,69	75,44	66,93	61,31	59,40	56,38	46,14	45,55	44,49
-	+	+	9,60	19,41	27,66	33,16	35,03	38,01	48,20	48,79	49,86
			86,26	76,44	68,20	62,70	60,83	57,85	47,66	47,07	46,00
=	-	-	47,59	66,35	74,98	78,99	80,14	81,80	86,23	86,44	86,81
			48,27	29,51	20,88	16,87	15,72	14,06	9,63	9,42	9,05
=	-	=	28,44	47,01	58,07	63,94	65,73	68,39	76,02	76,40	77,09
			67,42	48,85	37,79	31,92	30,12	27,47	19,84	19,45	18,77
=	-	+	19,97	35,96	46,93	53,26	55,27	58,32	67,59	68,08	68,95
			75,89	59,89	48,93	42,60	40,59	37,54	28,27	27,78	26,91
=	=	-	17,35	32,12	42,75	49,08	51,13	54,25	63,98	64,50	65,43
			78,51	63,74	53,11	46,78	44,73	41,61	31,88	31,36	30,42
=	=	=	15,96	30,00	40,37	46,66	48,71	51,86	61,79	62,33	63,30
			79,90	65,86	55,49	49,20	47,15	44,00	34,07	33,53	32,56
=	=	+	14,05	26,99	36,90	43,07	45,10	48,26	58,42	58,98	59,99
			81,81	68,87	58,96	52,79	50,76	47,60	37,44	36,88	35,87
=	+	-	9,95	20,03	28,44	34,01	35,91	38,92	49,14	49,73	50,80
			85,91	75,83	67,42	61,85	59,95	56,94	46,72	46,13	45,06
=	+	=	9,68	19,56	27,85	33,36	35,24	38,23	48,42	49,02	50,08
			86,17	76,30	68,01	62,50	60,62	57,63	47,44	46,84	45,78
=	+	+	9,23	18,75	26,82	32,22	34,07	37,02	47,16	47,75	48,82
			86,62	77,11	69,04	63,64	61,78	58,84	48,70	48,11	47,04
+	-	-	**47,93**	66,64	75,21	79,19	80,33	81,97	86,35	86,56	86,93
			47,93	29,21	20,64	16,67	15,53	13,89	9,51	9,30	8,93
+	-	=	29,21	47,93	58,95	64,76	66,52	69,14	76,62	76,99	77,66
			66,64	47,93	36,91	31,10	29,34	26,72	19,24	18,86	18,20
+	-	+	20,64	36,91	47,93	54,25	56,25	59,27	68,41	68,90	69,75
			75,21	58,95	47,93	41,61	39,61	36,59	27,44	26,96	26,11
+	=	-	16,67	31,10	41,61	47,93	49,98	53,11	62,94	63,48	64,43
			79,19	64,76	54,25	47,93	45,88	42,74	32,92	32,38	31,43
+	=	=	15,53	29,34	39,61	45,88	47,93	51,08	61,07	61,62	62,59
			80,33	66,52	56,25	49,98	47,93	44,78	34,79	34,24	33,26
+	=	+	13,89	26,72	36,59	42,74	44,78	47,93	58,10	58,67	59,68
			81,97	69,14	59,27	53,11	51,08	47,93	37,76	37,19	36,18
+	+	-	9,51	19,24	27,44	32,92	34,79	37,76	47,93	48,53	49,59
			86,35	76,62	68,41	62,94	61,07	58,10	47,93	47,33	46,27
+	+	=	9,30	18,86	26,96	32,38	34,24	37,19	47,33	47,93	49,00
			86,56	76,99	68,90	63,48	61,62	58,67	48,53	47,93	46,86
+	+	+	8,93	18,20	26,11	31,43	33,26	36,18	46,27	46,86	47,93
			86,93	77,66	69,75	64,43	62,59	59,68	49,59	49,00	47,93

Scenario α_c Case 3c

Both flights cover the same STAR, TERTO3C, and have the same cost index, CI=70

Equilibrium minimax	
η	0,00%
Π_A	0,00%
Π_B	0,00%

				CI'_A																										
				-									=									+								
				T'_REF_A									T'_REF_A									T'_REF_A								
				-			=			+			-			=			+			-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A		
CI'_B	T'_REF_B	F'_REF_B		-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+
-	-	-	-	76,32	99,55	113,39	131,77	132,31	133,24	141,04	141,13	141,29	75,78	97,44	111,27	132,20	132,64	133,40	141,30	141,37	141,51	75,33	95,49	109,23	132,53	132,89	133,53	141,51	141,56	141,67
				76,32	53,08	39,24	20,86	20,32	19,39	11,59	11,50	11,34	76,86	55,19	41,37	20,43	19,99	19,23	11,33	11,26	11,12	77,30	57,14	43,41	20,10	19,74	19,10	11,13	11,07	10,96
			=	53,08	76,32	92,56	117,68	118,50	119,91	132,24	132,39	132,67	52,59	74,01	89,93	118,33	118,99	120,15	132,68	132,80	133,02	52,19	71,92	87,45	118,83	119,38	120,34	133,02	133,11	133,29
				99,55	76,32	60,07	34,95	34,13	32,72	20,39	20,24	19,96	100,04	78,62	62,70	34,30	33,64	32,48	19,95	19,83	19,61	100,44	80,71	65,18	33,80	33,25	32,29	19,62	19,52	19,34
			+	39,24	60,07	76,32	104,72	105,72	107,45	123,33	123,53	123,90	38,83	57,89	73,58	105,51	106,32	107,75	123,92	124,08	124,38	38,49	55,93	71,05	106,13	106,80	107,99	124,37	124,50	124,75
				113,39	92,56	76,32	47,91	46,91	45,18	29,30	29,10	28,73	113,80	94,74	79,05	47,12	46,31	44,88	28,71	28,55	28,25	114,14	96,70	81,59	46,50	45,84	44,64	28,26	28,13	27,88
	=	=	-	20,86	34,95	47,91	76,32	77,48	79,54	100,47	100,76	101,30	20,61	33,35	45,59	77,25	78,20	79,90	101,33	101,56	102,00	20,40	31,94	43,49	77,97	78,76	80,19	101,99	102,18	102,54
				131,77	117,68	104,72	76,32	75,15	73,09	52,16	51,87	51,33	132,02	119,28	107,04	75,39	74,43	72,73	51,30	51,07	50,63	132,23	120,69	109,14	74,67	73,87	72,44	50,64	50,45	50,09
			=	20,32	34,13	46,91	75,15	76,32	78,37	99,42	99,71	100,25	20,07	32,56	44,62	76,08	77,03	78,74	100,28	100,52	100,96	19,87	31,17	42,54	76,80	77,59	79,02	100,95	101,14	101,50
				132,31	118,50	105,72	77,48	76,32	74,26	53,22	52,92	52,38	132,56	120,07	108,01	76,55	75,60	73,90	52,35	52,12	51,67	132,76	121,46	110,09	75,83	75,04	73,61	51,69	51,49	51,13
			+	19,39	32,72	45,18	73,09	74,26	76,32	97,53	97,83	98,38	19,15	31,20	42,93	74,02	74,97	76,68	98,41	98,65	99,10	18,95	29,86	40,91	74,74	75,53	76,97	99,09	99,28	99,65
				133,24	119,91	107,45	79,54	78,37	76,32	55,10	54,80	54,25	133,48	121,44	109,70	78,61	77,66	75,95	54,22	53,98	53,53	133,68	122,78	111,72	77,89	77,10	75,67	53,55	53,35	52,98
	+	+	-	11,59	20,39	29,30	52,16	53,22	55,10	76,32	76,64	77,24	11,44	19,34	27,64	53,00	53,87	55,44	77,27	77,53	78,02	11,32	18,44	26,16	53,65	54,38	55,70	78,01	78,23	78,64
				141,04	132,24	123,33	100,47	99,42	97,53	76,32	75,99	75,39	141,19	133,29	124,99	99,63	98,76	97,20	75,36	75,10	74,61	141,31	134,20	126,47	98,98	98,25	96,93	74,62	74,40	74,00
			=	11,50	20,24	29,10	51,87	52,92	54,80	75,99	76,32	76,92	11,35	19,20	27,45	52,71	53,57	55,14	76,95	77,21	77,70	11,23	18,30	25,98	53,36	54,08	55,40	77,69	77,91	78,31
				141,13	132,39	123,53	100,76	99,71	97,83	76,64	76,32	75,71	141,28	133,43	125,18	99,92	99,06	97,49	75,68	75,42	74,93	141,40	134,33	126,65	99,27	98,55	97,23	74,94	74,73	74,32
			+	11,34	19,96	28,73	51,33	52,38	54,25	75,39	75,71	76,32	11,19	18,94	27,09	52,16	53,03	54,58	76,35	76,61	77,10	11,07	18,05	25,64	52,81	53,53	54,85	77,09	77,30	77,71
				141,29	132,67	123,90	101,30	100,25	98,38	77,24	76,92	76,32	141,44	133,69	125,54	100,47	99,61	98,05	76,28	76,02	75,53	141,56	134,58	126,99	99,82	99,10	97,78	75,54	75,33	74,92
=	-	-	-	76,86	100,04	113,80	132,02	132,56	133,48	141,19	141,28	141,44	76,32	97,94	111,69	132,45	132,88	133,64	141,45	141,52	141,65	75,87	96,00	109,66	132,78	133,14	133,77	141,65	141,71	141,82
				75,78	52,59	38,83	20,61	20,07	19,15	11,44	11,35	11,19	76,32	54,69	40,94	20,18	19,75	18,99	11,18	11,11	10,98	76,76	56,63	42,97	19,85	19,50	18,87	10,98	10,92	10,82
			=	55,19	78,62	94,74	119,28	120,07	121,44	133,29	133,43	133,69	54,69	76,32	92,15	119,91	120,55	121,67	133,71	133,82	134,03	54,28	74,22	89,70	120,40	120,92	121,86	134,03	134,12	134,29
				97,44	74,01	57,89	33,35	32,56	31,20	19,34	19,20	18,94	97,94	76,32	60,48	32,72	32,08	30,96	18,93	18,81	18,60	98,35	78,41	62,93	32,23	31,71	30,78	18,61	18,51	18,34
			+	41,37	62,70	79,05	107,04	108,01	109,70	124,99	125,18	125,54	40,94	60,48	76,32	107,82	108,60	109,99	125,56	125,71	125,99	40,59	58,49	73,77	108,41	109,06	110,22	125,99	126,11	126,34
				111,27	89,93	73,58	45,59	44,62	42,93	27,64	27,45	27,09	111,69	92,15	76,32	44,81	44,03	42,64	27,07	26,92	26,64	112,04	94,14	78,86	44,22	43,57	42,41	26,65	26,52	26,29
	=	=	-	20,43	34,30	47,12	75,39	76,55	78,61	99,63	99,92	100,47	20,18	32,72	44,81	76,32	77,27	78,97	100,50	100,73	101,17	19,98	31,33	42,73	77,04	77,83	79,26	101,16	101,35	101,72
				132,20	118,33	105,51	77,25	76,08	74,02	53,00	52,71	52,16	132,45	119,91	107,82	76,32	75,36	73,66	52,14	51,90	51,46	132,66	121,30	109,90	75,60	74,80	73,37	51,47	51,28	50,92
			=	19,99	33,64	46,31	74,43	75,60	77,66	98,76	99,06	99,61	19,75	32,08	44,03	75,36	76,32	78,02	99,63	99,87	100,32	19,55	30,71	41,97	76,08	76,88	78,31	100,30	100,50	100,87
				132,64	118,99	106,32	78,20	77,03	74,97	53,87	53,57	53,03	132,88	120,55	108,60	77,27	76,32	74,61	53,00	52,76	52,32	133,09	121,92	110,66	76,55	75,76	74,32	52,33	52,13	51,77
			+	19,23	32,48	44,88	72,73	73,90	75,95	97,20	97,49	98,05	18,99	30,96	42,64	73,66	74,61	76,32	98,08	98,32	98,77	18,80	29,63	40,62	74,38	75,17	76,60	98,76	98,95	99,33
				133,40	120,15	107,75	79,90	78,74	76,68	55,44	55,14	54,58	133,64	121,67	109,99	78,97	78,02	76,32	54,55	54,31	53,86	133,83	123,00	112,01	78,25	77,46	76,03	53,88	53,68	53,31
	+	+	-	11,33	19,95	28,71	51,30	52,35	54,22	75,36	75,68	76,28	11,18	18,93	27,07	52,14	53,00	54,55	76,32	76,58	77,07	11,06	18,03	25,62	52,79	53,51	54,82	77,06	77,27	77,68
				141,30	132,68	123,92	101,33	100,28	98,41	77,27	76,95	76,35	141,45	133,71	125,56	100,50	99,63	98,08	76,32	76,05	75,56	141,57	134,60	127,01	99,85	99,13	97,81	75,58	75,36	74,95
			=	11,26	19,83	28,55	51,07	52,12	53,98	75,10	75,42	76,02	11,11	18,81	26,92	51,90	52,76	54,31	76,05	76,32	76,81	10,99	17,93	25,48	52,55	53,27	54,58	76,79	77,01	77,42
				141,37	132,80	124,08	101,56	100,52	98,65	77,53	77,21	76,61	141,52	133,82	125,71	100,73	99,87	98,32	76,58	76,32	75,82	141,64	134,71	127,15	100,08	99,36	98,05	75,84	75,62	75,21
			+	11,12	19,61	28,25	50,63	51,67	53,53	74,61	74,93	75,53	10,98	18,60	26,64	51,46	52,32	53,86	75,56	75,82	76,32	10,86	17,72	25,20	52,11	52,82	54,13	76,30	76,52	76,93
				141,51	133,02	124,38	102,00	100,96	99,10	78,02	77,70	77,10	141,65	134,03	125,99	101,17	100,32	98,77	77,07	76,81	76,32	141,77	134,91	127,43	100,52	99,81	98,50	76,33	76,11	75,70
+	-	-	-	77,30	100,44	114,14	132,23	132,76	133,68	141,31	141,40	141,56	76,76	98,35	112,04	132,66	133,09	133,83	141,57	141,64	141,77	**76,32**	96,42	110,03	132,98	133,33	133,96	141,77	141,83	141,93
				75,33	52,19	38,49	20,40	19,87	18,95	11,32	11,23	11,07	75,87	54,28	40,59	19,98	19,55	18,80	11,06	10,99	10,86	**76,32**	56,22	42,61	19,65	19,30	18,67	10,86	10,81	10,70
			=	57,14	80,71	96,70	120,69	121,46	122,78	134,20	134,33	134,58	56,63	78,41	94,14	121,30	121,92	123,00	134,60	134,71	134,91	56,22	76,32	91,72	121,77	122,28	123,18	134,90	134,99	135,16
				95,49	71,92	55,93	31,94	31,17	29,86	18,44	18,30	18,05	96,00	74,22	58,49	31,33	30,71	29,63	18,03	17,93	17,72	96,42	76,32	60,91	30,86	30,35	29,45	17,73	17,64	17,47
			+	43,41	65,18	81,59	109,14	110,09	111,72	126,47	126,65	126,99	42,97	62,93	78,86	109,90	110,66	112,01	127,01	127,15	127,43	42,61	60,91	76,32	110,48	111,11	112,23	127,42	127,54	127,76
				109,23	87,45	71,05	43,49	42,54	40,91	26,16	25,98	25,64	109,66	89,70	73,77	42,73	41,97	40,62	25,62	25,48	25,20	110,03	91,72	76,32	42,16	41,53	40,40	25,21	25,09	24,87
	=	=	-	20,10	33,80	46,50	74,67	75,83	77,89	98,98	99,27	99,82	19,85	32,23	44,22	75,60	76,55	78,25	99,85	100,08	100,52	19,65	30,86	42,16	76,32	77,11	78,54	100,51	100,71	101,07
				132,53	118,83	106,13	77,97	76,80	74,74	53,65	53,36	52,81	132,78	120,40	108,41	77,04	76,08	74,38	52,79	52,55	52,11	132,98	121,77	110,48	76,32	75,52	74,09	52,12	51,92	51,56
			=	19,74	33,25	45,84	73,87	75,04	77,10	98,25	98,55	99,10	19,50	31,71	43,57	74,80	75,76	77,46	99,13	99,36	99,81	19,30	30,35	41,53	75,52	76,32	77,75	99,80	99,99	100,36
				132,89	119,38	106,80	78,76	77,59	75,53	54,38	54,08	53,53	133,14	120,92	109,06	77,83	76,88	75,17	53,51	53,27	52,82	133,33	122,28	111,11	77,11	76,32	74,88	52,83	52,64	52,27
			+	19,10	32,29	44,64	72,44	73,61	75,67	96,93	97,23	97,78	18,87	30,78	42,41	73,37	74,32	76,03	97,81	98,05	98,50	18,67	29,45	40,40	74,09	74,88	76,32	98,49	98,69	99,06
				133,53	120,34	107,99	80,19	79,02	76,97	55,70	55,40	54,85	133,77	121,86	110,22	79,26	78,31	76,60	54,82	54,58	54,13	133,96	123,18	112,23	78,54	77,75	76,32	54,14	53,94	53,57
	+	+	-	11,13	19,62	28,26	50,64	51,69	53,55	74,62	74,94	75,54	10,98	18,61	26,65	51,47	52,33	53,88	75,58	75,84	76,33	10,86	17,73	25,21	52,12	52,83	54,14	76,32	76,53	76,94
				141,51	133,02	124,37	101,99	100,95	99,09	78,01	77,69	77,09	141,65	134,03	125,99	101,16	100,30	98,76	77,06	76,79	76,30	141,77	134,90	127,42	100,51	99,80	98,49	76,32	76,10	75,69
			=	11,07	19,52	28,13	50,45	51,49	53,35	74,40	74,73	75,33	10,92	18,51	26,52	51,28	52,13	53,68	75,36	75,62	76,11	10,81	17,64	25,09	51,92	52,64	53,94	76,10	76,32	76,72
				141,56	133,11	124,50	102,18	101,14	99,28	78,23	77,91	77,30	141,71	134,12	126,11	101,35	100,50	98,95	77,27	77,01	76,52	141,83	134,99	127,54	100,71	99,99	98,69	76,53	76,32	75,91
			+	10,96	19,34	27,88	50,09	51,13	52,98	74,00	74,32	74,92	10,82	18,34	26,29	50,92	51,77	53,31	74,95	75,21	75,70	10,70	17,47	24,87	51,56	52,27	53,57	75,69	75,91	76,32
				141,67	133,29	124,75	102,54	101,50	99,65	78,64	78,31	77,71	141,82	134,29	126,34	101,72	100,87	99,33	77,68	77,42	76,93	141,93	135,16	127,76	101,07	100,36	99,06	76,94	76,72	76,32

Scenario β_a Case 3a

Both flights cover the same STAR, RUSIK3C, and have different cost indexes, CI_A=33 and CI_B=70

Equilibrium minimax	
η	-0,60%
Π A	3,80%
Π B	-2,80%

CI'_A = -

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	39,19 73,91	51,89 55,01	60,52 42,18	66,45 33,35	67,62 31,61	69,41 28,95	74,62 21,20	74,90 20,78	75,40 20,03
-	-	=	33,77 81,97	46,36 63,25	55,44 49,73	61,96 40,03	63,28 38,07	65,30 35,06	71,33 26,09	71,67 25,59	72,26 24,71
-	-	+	28,08 90,44	40,09 72,58	49,37 58,77	56,38 48,34	57,84 46,17	60,09 42,81	67,01 32,52	67,40 31,93	68,10 30,90
-	=	-	13,65 111,91	21,69 99,95	29,27 88,67	36,03 78,62	37,57 76,33	40,05 72,63	48,56 59,97	49,09 59,19	50,03 57,78
-	=	=	13,46 112,20	21,41 100,36	28,93 89,17	35,66 79,16	37,20 76,88	39,68 73,19	48,18 60,53	48,71 59,75	49,66 58,34
-	=	+	13,10 112,73	20,91 101,11	28,33 90,07	35,00 80,15	36,52 77,88	38,99 74,21	47,49 61,56	48,02 60,77	48,97 59,36
-	+	-	8,23 119,98	13,66 111,90	19,23 103,61	24,63 95,57	25,92 93,65	28,06 90,48	35,89 78,82	36,40 78,06	37,33 76,68
-	+	=	8,19 120,04	13,59 112,00	19,15 103,73	24,53 95,72	25,82 93,80	27,95 90,63	35,77 79,00	36,29 78,23	37,21 76,85
-	+	+	8,11 120,15	13,48 112,17	18,99 103,96	24,35 95,99	25,63 94,08	27,75 90,93	35,55 79,33	36,07 78,56	36,99 77,18
=	-	-	38,68 74,67	51,39 55,76	60,06 42,86	66,05 33,94	67,24 32,17	69,05 29,49	74,33 21,62	74,62 21,19	75,13 20,44
=	-	=	34,03 81,59	46,63 62,84	55,69 49,36	62,19 39,69	63,50 37,74	65,51 34,75	71,50 25,83	71,84 25,34	72,42 24,47
=	-	+	28,80 89,37	40,91 71,36	50,18 57,56	57,14 47,20	58,58 45,06	60,81 41,75	67,62 31,62	68,00 31,04	68,68 30,03
=	=	-	13,28 112,47	21,16 100,75	28,63 89,63	35,33 79,66	36,86 77,39	39,33 73,71	47,83 61,05	48,36 60,27	49,31 58,86
=	=	=	13,12 112,70	20,93 101,08	28,36 90,03	35,03 80,10	36,56 77,83	39,02 74,16	47,53 61,51	48,06 60,72	49,00 59,31
=	=	+	12,84 113,13	20,53 101,68	27,86 90,76	34,49 80,91	36,01 78,65	38,46 74,99	46,96 62,35	47,49 61,56	48,44 60,15
=	+	-	7,99 120,34	13,28 112,47	18,73 104,35	24,04 96,45	25,31 94,56	27,42 91,42	35,18 79,88	35,69 79,12	36,61 77,75
=	+	=	7,95 120,39	13,23 112,54	18,67 104,45	23,97 96,57	25,23 94,68	27,34 91,55	35,09 80,02	35,60 79,26	36,52 77,89
=	+	+	7,89 120,48	13,13 112,68	18,55 104,63	23,82 96,78	25,08 94,90	27,18 91,78	34,91 80,28	35,42 79,53	36,34 78,16
+	-	-	38,27 75,28	50,98 56,37	59,70 43,40	65,73 34,42	66,93 32,63	68,76 29,92	74,10 21,96	74,40 21,53	74,91 20,76
+	-	=	34,25 81,27	46,86 62,50	55,91 49,03	62,38 39,40	63,69 37,46	65,69 34,48	71,65 25,62	71,98 25,12	72,56 24,26
+	-	+	29,45 88,41	41,64 70,27	50,91 56,48	57,82 46,20	59,24 44,08	61,44 40,80	68,15 30,82	68,53 30,26	69,19 29,27
+	=	-	12,99 112,90	20,74 101,36	28,13 90,37	34,78 80,48	36,30 78,22	38,76 74,55	47,26 61,90	47,79 61,12	48,74 59,70
+	=	=	12,86 113,09	20,56 101,63	27,90 90,70	34,53 80,84	36,05 78,58	38,51 74,92	47,01 62,28	47,54 61,49	48,49 60,08
+	=	+	12,63 113,44	20,22 102,13	27,50 91,31	34,08 81,51	35,59 79,26	38,04 75,62	46,53 62,99	47,06 62,20	48,01 60,78
+	+	-	7,80 120,62	12,98 112,90	18,35 104,92	23,59 97,13	24,84 95,26	26,93 92,16	34,63 80,70	35,13 79,95	36,05 78,58
+	+	=	7,77 120,66	12,94 112,97	18,30 105,00	23,52 97,22	24,78 95,36	26,86 92,26	34,55 80,82	35,06 80,06	35,97 78,70
+	+	+	7,72 120,73	12,87 113,08	18,20 105,15	23,40 97,40	24,65 95,54	26,73 92,45	34,40 81,04	34,91 80,28	35,82 78,92

CI'_A = =

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	40,02 72,68	51,67 55,34	60,06 42,86	67,58 31,66	68,52 30,27	70,00 28,08	75,44 19,97	75,66 19,65	76,06 19,06
-	-	=	34,56 80,80	46,13 63,59	54,94 50,47	63,24 38,13	64,30 36,56	65,97 34,06	72,30 24,64	72,57 24,25	73,03 23,56
-	-	+	28,81 89,36	39,86 72,91	48,85 59,54	57,79 46,24	58,96 44,49	60,85 41,68	68,15 30,82	68,46 30,36	69,01 29,54
-	=	-	14,09 111,26	21,52 100,20	28,81 89,36	37,52 76,40	38,80 74,50	40,92 71,34	50,11 57,67	50,53 57,03	51,31 55,89
-	=	=	13,89 111,56	21,25 100,61	28,47 89,86	37,15 76,95	38,42 75,05	40,54 71,90	49,73 58,23	50,16 57,59	50,93 56,44
-	=	+	13,53 112,10	20,74 101,36	27,87 90,75	36,47 77,96	37,74 76,07	39,85 72,93	49,05 59,24	49,48 58,61	50,25 57,45
-	+	-	8,51 119,55	13,54 112,08	18,88 104,14	25,88 93,72	26,97 92,10	28,82 89,35	37,41 76,56	37,83 75,93	38,61 74,78
-	+	=	8,47 119,62	13,48 112,17	18,80 104,26	25,78 93,87	26,87 92,25	28,71 89,51	37,29 76,74	37,71 76,11	38,49 74,96
-	+	+	8,39 119,73	13,36 112,35	18,64 104,48	25,59 94,15	26,67 92,54	28,51 89,80	37,07 77,07	37,49 76,44	38,26 75,29
=	-	-	39,50 73,45	51,16 56,10	59,60 43,54	67,20 32,23	68,15 30,82	69,65 28,60	75,17 20,38	75,40 20,04	75,80 19,44
=	-	=	34,82 80,42	46,40 63,19	55,20 50,09	63,46 37,80	64,51 36,24	66,18 33,75	72,47 24,40	72,73 24,01	73,19 23,32
=	-	+	29,53 88,28	40,68 71,70	49,66 58,33	58,53 45,13	59,70 43,40	61,56 40,63	68,74 29,95	69,04 29,50	69,58 28,69
=	=	-	13,71 111,83	20,99 100,99	28,17 90,31	36,80 77,46	38,08 75,57	40,19 72,42	49,39 58,74	49,81 58,10	50,59 56,95
=	=	=	13,54 112,07	20,77 101,32	27,90 90,71	36,50 77,91	37,78 76,02	39,89 72,88	49,08 59,20	49,51 58,56	50,29 57,40
=	=	+	13,25 112,50	20,36 101,93	27,41 91,43	35,95 78,73	37,22 76,84	39,32 73,71	48,52 60,04	48,95 59,40	49,73 58,24
=	+	-	8,26 119,93	13,16 112,64	18,39 104,87	25,27 94,62	26,35 93,02	28,17 90,31	36,69 77,63	37,11 77,01	37,88 75,86
=	+	=	8,23 119,98	13,11 112,72	18,32 104,96	25,19 94,74	26,27 93,14	28,09 90,43	36,59 77,78	37,02 77,15	37,78 76,00
=	+	+	8,17 120,07	13,02 112,86	18,20 105,14	25,04 94,97	26,11 93,37	27,93 90,67	36,41 78,04	36,83 77,42	37,60 76,28
+	-	-	39,09 74,06	50,76 56,70	59,23 44,09	66,89 32,69	67,85 31,27	69,36 29,02	74,95 20,70	75,18 20,36	75,59 19,76
+	-	=	35,04 80,09	46,63 62,84	55,42 49,77	63,65 37,52	64,70 35,96	66,36 33,49	72,61 24,19	72,87 23,81	73,33 23,12
+	-	+	30,19 87,30	41,41 70,61	50,39 57,25	59,19 44,15	60,34 42,44	62,18 39,70	69,25 29,19	69,55 28,75	70,08 27,95
+	=	-	13,41 112,27	20,58 101,61	27,67 91,05	36,25 78,29	37,52 76,40	39,62 73,27	48,82 59,59	49,25 58,95	50,02 57,79
+	=	=	13,28 112,47	20,40 101,88	27,45 91,38	36,00 78,66	37,27 76,78	39,37 73,64	48,56 59,97	48,99 59,33	49,77 58,17
+	=	+	13,04 112,83	20,06 102,37	27,05 91,98	35,54 79,34	36,80 77,46	38,90 74,34	48,09 60,67	48,52 60,03	49,30 58,86
+	+	-	8,07 120,22	12,87 113,07	18,01 105,43	24,80 95,32	25,87 93,74	27,67 91,05	36,13 78,47	36,55 77,85	37,31 76,71
+	+	=	8,04 120,26	12,83 113,13	17,96 105,51	24,74 95,42	25,80 93,84	27,60 91,15	36,05 78,59	36,47 77,96	37,23 76,82
+	+	+	7,99 120,33	12,75 113,25	17,86 105,66	24,61 95,60	25,67 94,03	27,47 91,35	35,90 78,81	36,32 78,19	37,08 77,05

CI'_A = +

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A - / F'_REF_A -	T'_REF_A - / F'_REF_A =	T'_REF_A - / F'_REF_A +	T'_REF_A = / F'_REF_A -	T'_REF_A = / F'_REF_A =	T'_REF_A = / F'_REF_A +	T'_REF_A + / F'_REF_A -	T'_REF_A + / F'_REF_A =	T'_REF_A + / F'_REF_A +
-	-	-	40,76 71,58	51,45 55,67	59,59 43,56	68,52 30,28	69,28 29,14	70,51 27,31	76,11 18,98	76,28 18,72	76,60 18,25
-	-	=	35,27 79,74	45,90 63,92	54,44 51,22	64,29 36,57	65,15 35,28	66,57 33,18	73,09 23,47	73,30 23,16	73,68 22,59
-	-	+	29,46 88,38	39,64 73,25	48,32 60,32	58,96 44,50	59,93 43,06	61,52 40,69	69,08 29,43	69,33 29,06	69,78 28,39
-	=	-	14,49 110,66	21,36 100,45	28,34 90,05	38,79 74,51	39,87 72,90	41,69 70,19	51,40 55,74	51,75 55,22	52,40 54,26
-	=	=	14,29 110,96	21,08 100,86	28,01 90,54	38,41 75,07	39,49 73,46	41,32 70,75	51,03 56,29	51,39 55,77	52,03 54,81
-	=	+	13,92 111,52	20,58 101,60	27,42 91,43	37,73 76,08	38,81 74,48	40,63 71,78	50,35 57,30	50,71 56,78	51,35 55,81
-	+	-	8,78 119,16	13,42 112,25	18,52 104,66	26,96 92,11	27,90 90,72	29,50 88,32	38,70 74,63	39,06 74,11	39,71 73,14
-	+	=	8,73 119,23	13,36 112,35	18,44 104,78	26,86 92,26	27,79 90,87	29,40 88,48	38,59 74,81	38,94 74,28	39,59 73,31
-	+	+	8,65 119,35	13,24 112,52	18,29 105,01	26,67 92,55	27,59 91,17	29,19 88,79	38,36 75,15	38,71 74,62	39,37 73,65
=	-	-	40,24 72,35	50,94 56,43	59,13 44,24	68,14 30,83	68,92 29,68	70,17 27,82	75,85 19,37	76,03 19,10	76,35 18,62
=	-	=	35,53 79,35	46,17 63,52	54,70 50,84	64,50 36,25	65,37 34,96	66,77 32,88	73,25 23,23	73,46 22,92	73,83 22,37
=	-	+	30,20 87,29	40,45 72,03	49,14 59,11	59,69 43,41	60,65 41,99	62,22 39,65	69,65 28,59	69,89 28,23	70,33 27,58
=	=	-	14,10 111,25	20,83 101,23	27,71 90,99	38,07 75,58	39,14 73,98	40,97 71,27	50,69 56,80	51,04 56,28	51,69 55,32
=	=	=	13,94 111,49	20,61 101,56	27,45 91,39	37,77 76,03	38,84 74,43	40,66 71,73	50,39 57,25	50,74 56,73	51,39 55,77
=	=	+	13,64 111,93	20,20 102,16	26,96 92,11	37,21 76,86	38,28 75,27	40,09 72,57	49,83 58,09	50,18 57,56	50,83 56,59
=	+	-	8,52 119,55	13,05 112,81	18,04 105,38	26,34 93,03	27,26 91,66	28,85 89,29	37,98 75,71	38,33 75,19	38,98 74,22
=	+	=	8,48 119,60	13,00 112,88	17,98 105,48	26,26 93,15	27,18 91,78	28,77 89,42	37,88 75,86	38,24 75,33	38,89 74,37
=	+	+	8,42 119,70	12,90 113,02	17,86 105,65	26,10 93,38	27,02 92,02	28,60 89,67	37,70 76,13	38,05 75,60	38,70 74,64
+	-	-	39,83 72,96	50,53 57,03	58,76 44,80	67,85 31,28	68,63 30,11	69,89 28,23	75,64 19,68	75,82 19,41	76,15 18,92
+	-	=	35,76 79,02	46,40 63,18	54,92 50,51	64,69 35,97	65,55 34,70	66,94 32,62	73,38 23,03	73,59 22,73	73,96 22,17
+	-	+	30,87 86,30	41,19 70,94	49,87 58,03	60,34 42,45	61,28 41,04	62,83 38,74	70,14 27,85	70,38 27,50	70,81 26,86
+	=	-	13,79 111,70	20,42 101,84	27,22 91,72	37,51 76,42	38,58 74,82	40,39 72,12	50,12 57,64	50,48 57,12	51,13 56,15
+	=	=	13,66 111,90	20,24 102,11	27,00 92,05	37,26 76,79	38,33 75,20	40,14 72,50	49,87 58,02	50,23 57,49	50,87 56,53
+	=	+	13,42 112,26	19,90 102,61	26,60 92,64	36,80 77,48	37,86 75,89	39,67 73,20	49,40 58,72	49,76 58,19	50,41 57,22
+	+	-	8,32 119,85	12,76 113,24	17,67 105,94	25,86 93,75	26,77 92,39	28,34 90,05	37,41 76,56	37,76 76,03	38,41 75,07
+	+	=	8,29 119,89	12,72 113,30	17,62 106,01	25,79 93,85	26,70 92,49	28,27 90,15	37,33 76,68	37,68 76,15	38,33 75,19
+	+	+	8,24 119,97	12,64 113,41	17,52 106,16	25,66 94,04	26,57 92,69	28,14 90,35	37,18 76,90	37,53 76,38	38,18 75,42

Scenario β_b Case 3b

Both flights cover the same STAR, TERTO3C, and have different cost indexes, CI_A=33 and CI_B=70

Equilibrium minimax	
η	-3,40%
Π_A	17,00%
Π_B	-14,00%

CI'_A = -

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A -			T'_REF_A =			T'_REF_A +		
			F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +
-	-	-	50,66	69,82	77,81	78,89	80,52	82,62	86,12	86,44	86,96
			71,97	41,46	28,74	27,02	24,42	21,08	15,50	15,00	14,16
-	-	=	35,85	56,40	66,79	68,31	70,63	73,71	79,09	79,59	80,43
			95,54	62,82	46,28	43,87	40,18	35,27	26,70	25,91	24,57
-	-	+	26,79	46,14	57,39	59,12	61,82	65,52	72,25	72,90	73,99
			109,98	79,17	61,26	58,50	54,19	48,31	37,58	36,56	34,82
-	=	-	14,45	28,57	38,88	40,65	43,51	47,64	55,93	56,78	58,24
			129,63	107,14	90,72	87,91	83,35	76,77	63,58	62,23	59,91
-	=	=	14,08	27,96	38,18	39,93	42,78	46,91	55,21	56,07	57,53
			130,22	108,12	91,84	89,05	84,51	77,94	64,72	63,36	61,02
-	=	+	13,44	26,90	36,95	38,68	41,51	45,62	53,94	54,80	56,29
			131,23	109,80	93,80	91,04	86,54	80,00	66,74	65,37	63,01
-	+	-	8,09	17,31	25,08	26,51	28,90	32,50	40,36	41,21	42,71
			139,76	125,07	112,70	110,42	106,62	100,88	88,37	87,01	84,63
-	+	=	8,02	17,19	24,92	26,35	28,73	32,32	40,16	41,01	42,51
			139,86	125,26	112,95	110,68	106,89	101,17	88,69	87,33	84,94
-	+	+	7,91	16,97	24,63	26,05	28,41	31,98	39,79	40,64	42,14
			140,04	125,61	113,41	111,16	107,40	101,71	89,27	87,92	85,54
=	-	-	50,99	70,09	78,01	79,09	80,70	82,78	86,25	86,56	87,08
			71,44	41,04	28,42	26,70	24,14	20,83	15,31	14,81	13,98
=	-	=	37,22	57,80	68,00	69,48	71,73	74,72	79,91	80,39	81,19
			93,37	60,60	44,35	42,00	38,42	33,66	25,40	24,63	23,35
=	-	+	28,19	47,85	59,02	60,73	63,38	66,99	73,51	74,13	75,17
			107,74	76,44	58,65	55,94	51,71	45,97	35,59	34,60	32,93
=	=	-	14,15	28,08	38,32	40,08	42,93	47,06	55,36	56,21	57,68
			130,10	107,92	91,61	88,81	84,27	77,70	64,49	63,13	60,79
=	=	=	13,85	27,59	37,75	39,50	42,34	46,46	54,77	55,63	57,10
			130,58	108,70	92,52	89,74	85,21	78,65	65,42	64,06	61,71
=	=	+	13,33	26,72	36,73	38,46	41,29	45,39	53,72	54,58	56,07
			131,41	110,09	94,15	91,39	86,89	80,36	67,09	65,72	63,36
=	+	-	7,90	16,96	24,62	26,03	28,39	31,97	39,77	40,63	42,12
			140,05	125,63	113,43	111,18	107,42	101,73	89,30	87,95	85,57
=	+	=	7,85	16,86	24,49	25,90	28,26	31,82	39,61	40,47	41,96
			140,13	125,78	113,63	111,39	107,64	101,97	89,56	88,20	85,83
=	+	+	7,76	16,69	24,26	25,66	28,00	31,55	39,31	40,16	41,65
			140,28	126,06	114,01	111,78	108,05	102,40	90,03	88,68	86,31
+	-	-	51,27	70,31	78,18	79,25	80,85	82,91	86,35	86,66	87,17
			70,99	40,69	28,14	26,44	23,90	20,61	15,15	14,65	13,84
+	-	=	38,48	59,05	69,07	70,51	72,71	75,61	80,63	81,09	81,86
			91,37	58,61	42,65	40,35	36,86	32,24	24,26	23,52	22,28
+	-	+	29,54	49,45	60,52	62,20	64,80	68,31	74,63	75,23	76,23
			105,60	73,90	56,26	53,60	49,46	43,86	33,80	32,85	31,25
+	=	-	13,92	27,71	37,89	39,64	42,49	46,61	54,92	55,77	57,24
			130,46	108,51	92,30	89,51	84,98	78,42	65,19	63,83	61,49
+	=	=	13,68	27,30	37,41	39,16	41,99	46,11	54,43	55,29	56,76
			130,85	109,16	93,06	90,28	85,77	79,21	65,97	64,60	62,25
+	=	+	13,24	26,57	36,56	38,29	41,11	45,21	53,54	54,40	55,89
			131,54	110,32	94,42	91,66	87,17	80,64	67,38	66,01	63,64
+	+	-	7,76	16,69	24,26	25,66	28,01	31,55	39,32	40,17	41,66
			140,27	126,06	114,00	111,77	108,04	102,39	90,02	88,67	86,30
+	+	=	7,72	16,61	24,16	25,56	27,89	31,43	39,19	40,04	41,53
			140,34	126,18	114,16	111,94	108,22	102,58	90,23	88,88	86,51
+	+	+	7,65	16,47	23,97	25,36	27,68	31,21	38,94	39,79	41,27
			140,46	126,41	114,47	112,26	108,55	102,94	90,63	89,28	86,91

CI'_A = =

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A -			T'_REF_A =			T'_REF_A +		
			F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +
-	-	-	50,30	69,08	77,19	79,68	81,00	82,79	86,64	86,89	87,31
			72,54	42,63	29,73	25,76	23,66	20,81	14,68	14,29	13,62
-	-	=	35,52	55,51	65,95	69,43	71,32	73,96	79,91	80,31	80,98
			96,08	64,25	47,63	42,09	39,08	34,86	25,39	24,76	23,69
-	-	+	26,50	45,22	56,43	60,42	62,64	65,83	73,32	73,83	74,71
			110,44	80,63	62,79	56,43	52,89	47,81	35,89	35,08	33,67
-	=	-	14,26	27,80	37,93	42,01	44,40	48,01	57,33	58,02	59,22
			129,92	108,36	92,24	85,75	81,93	76,19	61,34	60,25	58,34
-	=	=	13,90	27,20	37,23	41,28	43,68	47,27	56,63	57,32	58,53
			130,51	109,32	93,35	86,90	83,09	77,36	62,47	61,37	59,44
-	=	+	13,27	26,16	36,01	40,02	42,40	45,98	55,37	56,07	57,29
			131,51	110,97	95,30	88,91	85,13	79,42	64,47	63,36	61,41
-	+	-	7,98	16,77	24,32	27,63	29,66	32,83	41,78	42,49	43,74
			139,93	125,92	113,91	108,64	105,41	100,36	86,11	84,98	82,99
-	+	=	7,91	16,66	24,16	27,46	29,49	32,65	41,58	42,29	43,54
			140,03	126,11	114,16	108,90	105,68	100,65	86,42	85,30	83,31
-	+	+	7,80	16,44	23,88	27,15	29,16	32,31	41,21	41,91	43,16
			140,21	126,45	114,61	109,39	106,19	101,19	87,01	85,89	83,91
=	-	-	50,64	69,36	77,40	79,87	81,17	82,95	86,76	87,00	87,42
			72,01	42,20	29,39	25,45	23,38	20,55	14,49	14,10	13,44
=	-	=	36,88	56,91	67,18	70,57	72,40	74,97	80,70	81,08	81,72
			93,91	62,01	45,67	40,27	37,35	33,26	24,14	23,54	22,51
=	-	+	27,90	46,93	58,08	62,00	64,18	67,29	74,53	75,02	75,87
			108,22	77,90	60,16	53,91	50,44	45,49	33,96	33,18	31,83
=	=	-	13,97	27,33	37,37	41,43	43,82	47,42	56,77	57,46	58,67
			130,39	109,12	93,13	86,66	82,85	77,12	62,24	61,14	59,22
=	=	=	13,68	26,84	36,80	40,85	43,23	46,82	56,19	56,88	58,10
			130,86	109,90	94,03	87,60	83,80	78,07	63,16	62,06	60,12
=	=	+	13,16	25,98	35,80	39,80	42,17	45,75	55,15	55,85	57,07
			131,68	111,26	95,64	89,26	85,48	79,78	64,82	63,71	61,76
=	+	-	7,79	16,43	23,86	27,14	29,15	32,29	41,19	41,89	43,14
			140,22	126,47	114,63	109,42	106,22	101,22	87,04	85,92	83,94
=	+	=	7,75	16,34	23,74	27,01	29,01	32,14	41,03	41,73	42,98
			140,30	126,62	114,83	109,63	106,44	101,45	87,30	86,18	84,19
=	+	+	7,65	16,16	23,51	26,76	28,75	31,87	40,73	41,43	42,68
			140,44	126,90	115,19	110,03	106,85	101,89	87,78	86,66	84,68
+	-	-	50,92	69,58	77,58	80,03	81,32	83,08	86,85	87,09	87,51
			71,56	41,84	29,11	25,21	23,15	20,35	14,34	13,95	13,30
+	-	=	38,13	58,18	68,27	71,58	73,36	75,85	81,39	81,75	82,37
			91,91	60,00	43,93	38,67	35,82	31,86	23,04	22,46	21,47
+	-	+	29,23	48,53	59,59	63,45	65,58	68,61	75,61	76,09	76,90
			106,09	75,36	57,75	51,61	48,21	43,39	32,23	31,48	30,18
+	=	-	13,75	26,96	36,94	40,99	43,38	46,97	56,33	57,03	58,24
			130,74	109,71	93,81	87,37	83,57	77,84	62,93	61,83	59,90
+	=	=	13,50	26,56	36,47	40,50	42,88	46,47	55,85	56,55	57,76
			131,13	110,35	94,56	88,14	84,35	78,63	63,70	62,60	60,66
+	=	+	13,07	25,84	35,63	39,63	41,99	45,57	54,97	55,67	56,90
			131,81	111,48	95,91	89,54	85,77	80,07	65,10	63,99	62,04
+	+	-	7,66	16,17	23,52	26,76	28,76	31,88	40,74	41,44	42,68
			140,44	126,89	115,18	110,02	106,84	101,88	87,77	86,65	84,67
+	+	=	7,62	16,09	23,42	26,65	28,64	31,76	40,60	41,30	42,55
			140,50	127,01	115,34	110,19	107,03	102,07	87,98	86,86	84,88
+	+	+	7,54	15,95	23,23	26,45	28,43	31,53	40,35	41,05	42,30
			140,62	127,24	115,64	110,52	107,37	102,43	88,38	87,26	85,29

CI'_A = +

CI'_B	T'_REF_B	F'_REF_B	T'_REF_A -			T'_REF_A =			T'_REF_A +		
			F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +
-	-	-	49,96	68,33	76,55	80,32	81,40	82,95	87,05	87,25	87,59
			73,09	43,83	30,74	24,74	23,02	20,56	14,02	13,71	13,16
-	-	=	35,20	54,61	65,08	70,34	71,91	74,19	80,57	80,89	81,44
			96,58	65,68	49,01	40,64	38,14	34,49	24,34	23,84	22,96
-	-	+	26,23	44,30	55,45	61,49	63,35	66,12	74,17	74,59	75,32
			110,87	82,10	64,34	54,73	51,77	47,36	34,53	33,87	32,71
-	=	-	14,09	27,05	36,97	43,15	45,18	48,34	58,49	59,05	60,06
			130,20	109,56	93,76	83,93	80,69	75,67	59,50	58,60	57,00
-	=	=	13,73	26,46	36,28	42,42	44,45	47,60	57,79	58,36	59,37
			130,77	110,50	94,86	85,09	81,86	76,84	60,62	59,71	58,10
-	=	+	13,10	25,44	35,07	41,15	43,17	46,31	56,54	57,12	58,14
			131,77	112,13	96,79	87,11	83,90	78,90	62,60	61,68	60,05
-	+	-	7,87	16,25	23,56	28,59	30,33	33,12	42,97	43,56	44,62
			140,10	126,76	115,11	107,12	104,34	99,89	84,21	83,27	81,58
-	+	=	7,81	16,13	23,42	28,42	30,15	32,94	42,77	43,36	44,42
			140,19	126,94	115,35	107,39	104,62	100,18	84,53	83,59	81,90
-	+	+	7,70	15,92	23,14	28,10	29,83	32,60	42,40	42,99	44,04
			140,37	127,28	115,79	107,89	105,14	100,72	85,12	84,18	82,50
=	-	-	50,29	68,61	76,77	80,50	81,57	83,10	87,16	87,36	87,70
			72,55	43,38	30,39	24,45	22,75	20,31	13,84	13,53	13,00
=	-	=	36,56	56,02	66,33	71,45	72,97	75,19	81,33	81,64	82,17
			94,43	63,43	47,02	38,86	36,44	32,91	23,13	22,65	21,80
=	-	+	27,61	46,01	57,12	63,05	64,87	67,57	75,35	75,75	76,45
			108,66	79,37	61,69	52,24	49,34	45,05	32,65	32,02	30,90
=	=	-	13,80	26,58	36,42	42,57	44,60	47,75	57,93	58,50	59,51
			130,66	110,31	94,64	84,85	81,62	76,60	60,39	59,48	57,88
=	=	=	13,51	26,10	35,86	41,98	44,00	47,15	57,36	57,93	58,94
			131,12	111,07	95,54	85,79	82,57	77,55	61,30	60,39	58,78
=	=	+	13,00	25,26	34,86	40,93	42,94	46,08	56,32	56,90	57,92
			131,94	112,41	97,12	87,47	84,26	79,26	62,95	62,03	60,40
=	+	-	7,69	15,91	23,12	28,09	29,81	32,58	42,38	42,97	44,03
			140,38	127,29	115,82	107,91	105,16	100,75	85,15	84,22	82,53
=	+	=	7,64	15,82	23,00	27,95	29,67	32,44	42,22	42,81	43,86
			140,46	127,44	116,01	108,13	105,39	100,98	85,41	84,47	82,79
=	+	+	7,55	15,65	22,78	27,70	29,41	32,16	41,91	42,50	43,56
			140,60	127,71	116,36	108,53	105,81	101,42	85,89	84,96	83,28
+	-	-	**50,58**	68,84	76,95	80,65	81,72	83,23	87,26	87,45	87,78
			72,10	43,02	30,11	24,21	22,52	20,11	13,70	13,39	12,86
+	-	=	37,80	57,29	67,44	72,44	73,92	76,07	82,00	82,29	82,80
			92,44	61,41	45,26	37,29	34,94	31,51	22,07	21,61	20,80
+	-	+	28,94	47,61	58,64	64,47	66,25	68,88	76,41	76,79	77,46
			106,55	76,83	59,26	49,98	47,15	42,96	30,97	30,36	29,29
+	=	-	13,58	26,22	36,00	42,12	44,15	47,30	57,50	58,07	59,08
			131,01	110,88	95,32	85,56	82,34	77,32	61,08	60,17	58,56
+	=	=	13,34	25,83	35,53	41,63	43,65	46,80	57,02	57,59	58,61
			131,39	111,51	96,06	86,34	83,12	78,11	61,84	60,93	59,31
+	=	+	12,91	25,12	34,69	40,75	42,76	45,90	56,15	56,73	57,75
			132,07	112,63	97,39	87,75	84,54	79,54	63,23	62,31	60,68
+	+	-	7,56	15,66	22,78	27,70	29,41	32,17	41,92	42,51	43,56
			140,60	127,70	116,35	108,52	105,80	101,41	85,88	84,95	83,27
+	+	=	7,52	15,58	22,69	27,59	29,30	32,05	41,79	42,37	43,43
			140,66	127,82	116,51	108,70	105,98	101,61	86,09	85,16	83,48
+	+	+	7,44	15,44	22,50	27,38	29,08	31,82	41,54	42,12	43,18
			140,78	128,04	116,81	109,04	106,33	101,97	86,50	85,56	83,89

Scenario β_c Case 3c

Both flights cover the same STAR, TERTO3C, and have different cost indexes, CI_A=70 and CI_B=33

Equilibrium minimax	
η	-3,40%
Π_A	-14,00%
Π_B	17,00%

				CI'_A																										
				-									=									+								
				T'_REF_A									T'_REF_A									T'_REF_A								
				-			=			+			-			=			+			-			=			+		
				F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A		
CI'_B	T'_REF_B	F'_REF_B		-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+
-	-	-	-	71,97	95,54	109,98	129,63	130,22	131,23	139,76	139,86	140,04	71,44	93,37	107,74	130,10	130,58	131,41	140,05	140,13	140,28	70,99	91,37	105,60	130,46	130,85	131,54	140,27	140,34	140,46
				50,66	35,85	26,79	14,45	14,08	13,44	8,09	8,02	7,91	50,99	37,22	28,19	14,15	13,85	13,33	7,90	7,85	7,76	51,27	38,48	29,54	13,92	13,68	13,24	7,76	7,72	7,65
			=	41,46	62,82	79,17	107,14	108,12	109,80	125,07	125,26	125,61	41,04	60,60	76,44	107,92	108,70	110,09	125,63	125,78	126,06	40,69	58,61	73,90	108,51	109,16	110,32	126,06	126,18	126,41
				69,82	56,40	46,14	28,57	27,96	26,90	17,31	17,19	16,97	70,09	57,80	47,85	28,08	27,59	26,72	16,96	16,86	16,69	70,31	59,05	49,45	27,71	27,30	26,57	16,69	16,61	16,47
			+	28,74	46,28	61,26	90,72	91,84	93,80	112,70	112,95	113,41	28,42	44,35	58,65	91,61	92,52	94,15	113,43	113,63	114,01	28,14	42,65	56,26	92,30	93,06	94,42	114,00	114,16	114,47
				77,81	66,79	57,39	38,88	38,18	36,95	25,08	24,92	24,63	78,01	68,00	59,02	38,32	37,75	36,73	24,62	24,49	24,26	78,18	69,07	60,52	37,89	37,41	36,56	24,26	24,16	23,97
		=	-	27,02	43,87	58,50	87,91	89,05	91,04	110,42	110,68	111,16	26,70	42,00	55,94	88,81	89,74	91,39	111,18	111,39	111,78	26,44	40,35	53,60	89,51	90,28	91,66	111,77	111,94	112,26
				78,89	68,31	59,12	40,65	39,93	38,68	26,51	26,35	26,05	79,09	69,48	60,73	40,08	39,50	38,46	26,03	25,90	25,66	79,25	70,51	62,20	39,64	39,16	38,29	25,66	25,56	25,36
			=	24,42	40,18	54,19	83,35	84,51	86,54	106,62	106,89	107,40	24,14	38,42	51,71	84,27	85,21	86,89	107,42	107,64	108,05	23,90	36,86	49,46	84,98	85,77	87,17	108,04	108,22	108,55
				80,52	70,63	61,82	43,51	42,78	41,51	28,90	28,73	28,41	80,70	71,73	63,38	42,93	42,34	41,29	28,39	28,26	28,00	80,85	72,71	64,80	42,49	41,99	41,11	28,01	27,89	27,68
			+	21,08	35,27	48,31	76,77	77,94	80,00	100,88	101,17	101,71	20,83	33,66	45,97	77,70	78,65	80,36	101,73	101,97	102,40	20,61	32,24	43,86	78,42	79,21	80,64	102,39	102,58	102,94
				82,62	73,71	65,52	47,64	46,91	45,62	32,50	32,32	31,98	82,78	74,72	66,99	47,06	46,46	45,39	31,97	31,82	31,55	82,91	75,61	68,31	46,61	46,11	45,21	31,55	31,43	31,21
		+	-	15,50	26,70	37,58	63,58	64,72	66,74	88,37	88,69	89,27	15,31	25,40	35,59	64,49	65,42	67,09	89,30	89,56	90,03	15,15	24,26	33,80	65,19	65,97	67,38	90,02	90,23	90,63
				86,12	79,09	72,25	55,93	55,21	53,94	40,36	40,16	39,79	86,25	79,91	73,51	55,36	54,77	53,72	39,77	39,61	39,31	86,35	80,63	74,63	54,92	54,43	53,54	39,32	39,19	38,94
			=	15,00	25,91	36,56	62,23	63,36	65,37	87,01	87,33	87,92	14,81	24,63	34,60	63,13	64,06	65,72	87,95	88,20	88,68	14,65	23,52	32,85	63,83	64,60	66,01	88,67	88,88	89,28
				86,44	79,59	72,90	56,78	56,07	54,80	41,21	41,01	40,64	86,56	80,39	74,13	56,21	55,63	54,58	40,63	40,47	40,16	86,66	81,09	75,23	55,77	55,29	54,40	40,17	40,04	39,79
			+	14,16	24,57	34,82	59,91	61,02	63,01	84,63	84,94	85,54	13,98	23,35	32,93	60,79	61,71	63,36	85,57	85,83	86,31	13,84	22,28	31,25	61,49	62,25	63,64	86,30	86,51	86,91
				86,96	80,43	73,99	58,24	57,53	56,29	42,71	42,51	42,14	87,08	81,19	75,17	57,68	57,10	56,07	42,12	41,96	41,65	87,17	81,86	76,23	57,24	56,76	55,89	41,66	41,53	41,27
=	-	-	-	72,54	96,08	110,44	129,92	130,51	131,51	139,93	140,03	140,21	72,01	93,91	108,22	130,39	130,86	131,68	140,22	140,30	140,44	71,56	91,91	106,09	130,74	131,13	131,81	140,44	140,50	140,62
				50,30	35,52	26,50	14,26	13,90	13,27	7,98	7,91	7,80	50,64	36,88	27,90	13,97	13,68	13,16	7,79	7,75	7,65	50,92	38,13	29,23	13,75	13,50	13,07	7,66	7,62	7,54
			=	42,63	64,25	80,63	108,36	109,32	110,97	125,92	126,11	126,45	42,20	62,01	77,90	109,12	109,90	111,26	126,47	126,62	126,90	41,84	60,00	75,36	109,71	110,35	111,48	126,89	127,01	127,24
				69,08	55,51	45,22	27,80	27,20	26,16	16,77	16,66	16,44	69,36	56,91	46,93	27,33	26,84	25,98	16,43	16,34	16,16	69,58	58,18	48,53	26,96	26,56	25,84	16,17	16,09	15,95
			+	29,73	47,63	62,79	92,24	93,35	95,30	113,91	114,16	114,61	29,39	45,67	60,16	93,13	94,03	95,64	114,63	114,83	115,19	29,11	43,93	57,75	93,81	94,56	95,91	115,18	115,34	115,64
				77,19	65,95	56,43	37,93	37,23	36,01	24,32	24,16	23,88	77,40	67,18	58,08	37,37	36,80	35,80	23,86	23,74	23,51	77,58	68,27	59,59	36,94	36,47	35,63	23,52	23,42	23,23
		=	-	25,76	42,09	56,43	85,75	86,90	88,91	108,64	108,90	109,39	25,45	40,27	53,91	86,66	87,60	89,26	109,42	109,63	110,03	25,21	38,67	51,61	87,37	88,14	89,54	110,02	110,19	110,52
				79,68	69,43	60,42	42,01	41,28	40,02	27,63	27,46	27,15	79,87	70,57	62,00	41,43	40,85	39,80	27,14	27,01	26,76	80,03	71,58	63,45	40,99	40,50	39,63	26,76	26,65	26,45
			=	23,66	39,08	52,89	81,93	83,09	85,13	105,41	105,68	106,19	23,38	37,35	50,44	82,85	83,80	85,48	106,22	106,44	106,85	23,15	35,82	48,21	83,57	84,35	85,77	106,84	107,03	107,37
				81,00	71,32	62,64	44,40	43,68	42,40	29,66	29,49	29,16	81,17	72,40	64,18	43,82	43,23	42,17	29,15	29,01	28,75	81,32	73,36	65,58	43,38	42,88	41,99	28,76	28,64	28,43
			+	20,81	34,86	47,81	76,19	77,36	79,42	100,36	100,65	101,19	20,55	33,26	45,49	77,12	78,07	79,78	101,22	101,45	101,89	20,35	31,86	43,39	77,84	78,63	80,07	101,88	102,07	102,43
				82,79	73,96	65,83	48,01	47,27	45,98	32,83	32,65	32,31	82,95	74,97	67,29	47,42	46,82	45,75	32,29	32,14	31,87	83,08	75,85	68,61	46,97	46,47	45,57	31,88	31,76	31,53
		+	-	14,68	25,39	35,89	61,34	62,47	64,47	86,11	86,42	87,01	14,49	24,14	33,96	62,24	63,16	64,82	87,04	87,30	87,78	14,34	23,04	32,23	62,93	63,70	65,10	87,77	87,98	88,38
				86,64	79,91	73,32	57,33	56,63	55,37	41,78	41,58	41,21	86,76	80,70	74,53	56,77	56,19	55,15	41,19	41,03	40,73	86,85	81,39	75,61	56,33	55,85	54,97	40,74	40,60	40,35
			=	14,29	24,76	35,08	60,25	61,37	63,36	84,98	85,30	85,89	14,10	23,54	33,18	61,14	62,06	63,71	85,92	86,18	86,66	13,95	22,46	31,48	61,83	62,60	63,99	86,65	86,86	87,26
				86,89	80,31	73,83	58,02	57,32	56,07	42,49	42,29	41,91	87,00	81,08	75,02	57,46	56,88	55,85	41,89	41,73	41,43	87,09	81,75	76,09	57,03	56,55	55,67	41,44	41,30	41,05
			+	13,62	23,69	33,67	58,34	59,44	61,41	82,99	83,31	83,91	13,44	22,51	31,83	59,22	60,12	61,76	83,94	84,19	84,68	13,30	21,47	30,18	59,90	60,66	62,04	84,67	84,88	85,29
				87,31	80,98	74,71	59,22	58,53	57,29	43,74	43,54	43,16	87,42	81,72	75,87	58,67	58,10	57,07	43,14	42,98	42,68	87,51	82,37	76,90	58,24	57,76	56,90	42,68	42,55	42,30
+	-	-	-	73,09	96,58	110,87	130,20	130,77	131,77	140,10	140,19	140,37	72,55	94,43	108,66	130,66	131,12	131,94	140,38	140,46	140,60	**72,10**	92,44	106,55	131,01	131,39	132,07	140,60	140,66	140,78
				49,96	35,20	26,23	14,09	13,73	13,10	7,87	7,81	7,70	50,29	36,56	27,61	13,80	13,51	13,00	7,69	7,64	7,55	**50,58**	37,80	28,94	13,58	13,34	12,91	7,56	7,52	7,44
			=	43,83	65,68	82,10	109,56	110,50	112,13	126,76	126,94	127,28	43,38	63,43	79,37	110,31	111,07	112,41	127,29	127,44	127,71	43,02	61,41	76,83	110,88	111,51	112,63	127,70	127,82	128,04
				68,33	54,61	44,30	27,05	26,46	25,44	16,25	16,13	15,92	68,61	56,02	46,01	26,58	26,10	25,26	15,91	15,82	15,65	68,84	57,29	47,61	26,22	25,83	25,12	15,66	15,58	15,44
			+	30,74	49,01	64,34	93,76	94,86	96,79	115,11	115,35	115,79	30,39	47,02	61,69	94,64	95,54	97,12	115,82	116,01	116,36	30,11	45,26	59,26	95,32	96,06	97,39	116,35	116,51	116,81
				76,55	65,08	55,45	36,97	36,28	35,07	23,56	23,42	23,14	76,77	66,33	57,12	36,42	35,86	34,86	23,12	23,00	22,78	76,95	67,44	58,64	36,00	35,53	34,69	22,78	22,69	22,50
		=	-	24,74	40,64	54,73	83,93	85,09	87,11	107,12	107,39	107,89	24,45	38,86	52,24	84,85	85,79	87,47	107,91	108,13	108,53	24,21	37,29	49,98	85,56	86,34	87,75	108,52	108,70	109,04
				80,32	70,34	61,49	43,15	42,42	41,15	28,59	28,42	28,10	80,50	71,45	63,05	42,57	41,98	40,93	28,09	27,95	27,70	80,65	72,44	64,47	42,12	41,63	40,75	27,70	27,59	27,38
			=	23,02	38,14	51,77	80,69	81,86	83,90	104,34	104,62	105,14	22,75	36,44	49,34	81,62	82,57	84,26	105,16	105,39	105,81	22,52	34,94	47,15	82,34	83,12	84,54	105,80	105,98	106,33
				81,40	71,91	63,35	45,18	44,45	43,17	30,33	30,15	29,83	81,57	72,97	64,87	44,60	44,00	42,94	29,81	29,67	29,41	81,72	73,92	66,25	44,15	43,65	42,76	29,41	29,30	29,08
			+	20,56	34,49	47,36	75,67	76,84	78,90	99,89	100,18	100,72	20,31	32,91	45,05	76,60	77,55	79,26	100,75	100,98	101,42	20,11	31,51	42,96	77,32	78,11	79,54	101,41	101,61	101,97
				82,95	74,19	66,12	48,34	47,60	46,31	33,12	32,94	32,60	83,10	75,19	67,57	47,75	47,15	46,08	32,58	32,44	32,16	83,23	76,07	68,88	47,30	46,80	45,90	32,17	32,05	31,82
		+	-	14,02	24,34	34,53	59,50	60,62	62,60	84,21	84,53	85,12	13,84	23,13	32,65	60,39	61,30	62,95	85,15	85,41	85,89	13,70	22,07	30,97	61,08	61,84	63,23	85,88	86,09	86,50
				87,05	80,57	74,17	58,49	57,79	56,54	42,97	42,77	42,40	87,16	81,33	75,35	57,93	57,36	56,32	42,38	42,22	41,91	87,26	82,00	76,41	57,50	57,02	56,15	41,92	41,79	41,54
			=	13,71	23,84	33,87	58,60	59,71	61,68	83,27	83,59	84,18	13,53	22,65	32,02	59,48	60,39	62,03	84,22	84,47	84,96	13,39	21,61	30,36	60,17	60,93	62,31	84,95	85,16	85,56
				87,25	80,89	74,59	59,05	58,36	57,12	43,56	43,36	42,99	87,36	81,64	75,75	58,50	57,93	56,90	42,97	42,81	42,50	87,45	82,29	76,79	58,07	57,59	56,73	42,51	42,37	42,12
			+	13,16	22,96	32,71	57,00	58,10	60,05	81,58	81,90	82,50	13,00	21,80	30,90	57,88	58,78	60,40	82,53	82,79	83,28	12,86	20,80	29,29	58,56	59,31	60,68	83,27	83,48	83,89
				87,59	81,44	75,32	60,06	59,37	58,14	44,62	44,42	44,04	87,70	82,17	76,45	59,51	58,94	57,92	44,03	43,86	43,56	87,78	82,80	77,46	59,08	58,61	57,75	43,56	43,43	43,18

Scenario γ_a Case 3a

Airline A’s flight covers STAR RUSIK3C and has the cost index $CI_A=33$

Airline B’s flight covers STAR TERTO3C and has the cost index $CI_B=70$

Equilibrium minimax	
η	-3,90%
Π_A	28,20%
Π_B	-19,60%

CI'_A: -

CI'_B	T'_REF_B	F'_REF_B		T'_REF_A - / F'_REF_A -			T'_REF_A = / F'_REF_A -			T'_REF_A + / F'_REF_A -		
				-	=	+	-	=	+	-	=	+
-	-	-	F'_REF_B -	51,05	64,66	73,25	78,88	79,97	81,61	86,29	86,54	86,98
				66,95	47,67	35,51	27,54	26,00	23,68	17,05	16,70	16,07
		=		35,92	49,75	59,88	67,24	68,74	71,03	77,93	78,32	79,00
				88,36	68,79	54,44	44,03	41,91	38,65	28,88	28,34	27,38
		+		26,74	39,26	49,43	57,42	59,11	61,76	70,07	70,55	71,40
				101,36	83,64	69,24	57,93	55,53	51,78	40,01	39,33	38,13
	=	-		14,35	22,93	31,10	38,45	40,14	42,86	52,27	52,85	53,90
				118,90	106,76	95,20	84,78	82,40	78,54	65,23	64,39	62,91
		=		13,98	22,40	30,45	37,74	39,41	42,12	51,52	52,10	53,16
				119,43	107,52	96,11	85,79	83,43	79,58	66,29	65,45	63,96
		+		13,35	21,48	29,33	36,49	38,14	40,83	50,19	50,78	51,84
				120,33	108,82	97,70	87,56	85,22	81,42	68,16	67,33	65,83
	+	-		8,01	13,41	19,04	24,59	25,92	28,15	36,45	37,00	38,00
				127,88	120,24	112,27	104,42	102,52	99,37	87,62	86,84	85,42
		=		7,95	13,31	18,91	24,43	25,76	27,98	36,26	36,81	37,80
				127,96	120,38	112,45	104,64	102,75	99,61	87,89	87,11	85,70
		+		7,84	13,13	18,67	24,14	25,46	27,67	35,89	36,44	37,44
				128,13	120,64	112,79	105,04	103,17	100,05	88,40	87,63	86,22
=	-	-		51,39	64,98	73,52	79,10	80,18	81,81	86,44	86,69	87,12
				66,46	47,23	35,14	27,23	25,70	23,40	16,84	16,49	15,87
		=		37,31	51,23	61,29	68,51	69,97	72,21	78,89	79,26	79,92
				86,40	66,69	52,45	42,23	40,16	36,99	27,52	27,00	26,07
		+		28,16	40,96	51,19	59,12	60,79	63,39	71,49	71,96	72,78
				99,35	81,23	66,75	55,52	53,15	49,47	38,00	37,34	36,18
	=	-		14,06	22,50	30,58	37,88	39,56	42,27	51,67	52,26	53,31
				119,32	107,36	95,93	85,59	83,22	79,37	66,07	65,24	63,75
		=		13,76	22,07	30,06	37,30	38,97	41,67	51,06	51,64	52,70
				119,74	107,97	96,67	86,41	84,05	80,22	66,94	66,11	64,61
		+		13,24	21,32	29,13	36,27	37,92	40,60	49,96	50,55	51,60
				120,48	109,04	97,98	87,87	85,53	81,74	68,49	67,66	66,16
	+	-		7,83	13,12	18,66	24,13	25,45	27,65	35,88	36,42	37,42
				128,14	120,65	112,81	105,07	103,19	100,07	88,43	87,65	86,25
		=		7,78	13,04	18,55	24,00	25,32	27,52	35,72	36,27	37,26
				128,21	120,76	112,96	105,24	103,38	100,27	88,65	87,88	86,47
		+		7,69	12,90	18,36	23,77	25,08	27,26	35,43	35,97	36,96
				128,34	120,96	113,23	105,57	103,72	100,63	89,06	88,29	86,89
+	-	-		**51,68**	65,23	73,73	79,28	80,36	81,97	86,56	86,81	87,24
				66,05	46,86	34,83	26,97	25,45	23,17	16,67	16,32	15,71
		=		38,59	52,58	62,55	69,64	71,06	73,25	79,74	80,09	80,73
				84,59	64,78	50,67	40,63	38,61	35,52	26,33	25,82	24,93
		+		29,52	42,56	52,82	60,68	62,32	64,88	72,77	73,23	74,02
				97,43	78,96	64,44	53,31	50,98	47,37	36,19	35,55	34,42
	=	-		13,83	22,18	30,18	37,44	39,11	41,82	51,21	51,79	52,85
				119,64	107,82	96,49	86,21	83,85	80,02	66,73	65,89	64,40
		=		13,59	21,82	29,75	36,96	38,62	41,32	50,70	51,28	52,34
				119,99	108,33	97,10	86,89	84,54	80,72	67,45	66,61	65,12
		+		13,15	21,19	28,98	36,10	37,75	40,42	49,77	50,36	51,42
				120,60	109,22	98,20	88,11	85,78	81,99	68,76	67,92	66,42
	+	-		7,69	12,90	18,37	23,77	25,08	27,27	35,44	35,98	36,97
				128,33	120,96	113,22	105,56	103,71	100,62	89,05	88,28	86,88
		=		7,65	12,84	18,28	23,67	24,98	27,16	35,31	35,85	36,84
				128,39	121,05	113,34	105,71	103,86	100,78	89,24	88,46	87,07
		+		7,58	12,72	18,12	23,48	24,78	26,95	35,07	35,61	36,59
				128,50	121,22	113,57	105,98	104,14	101,07	89,58	88,81	87,42

CI'_A: =

CI'_B	T'_REF_B	F'_REF_B		T'_REF_A - / F'_REF_A -			T'_REF_A = / F'_REF_A -			T'_REF_A + / F'_REF_A -		
				-	=	+	-	=	+	-	=	+
-	-	-		51,97	64,43	72,81	79,94	80,80	82,15	87,02	87,21	87,56
				65,65	48,00	36,14	26,05	24,83	22,92	16,02	15,75	15,26
		=		36,78	49,50	59,33	68,69	69,89	71,80	79,05	79,35	79,89
				87,15	69,15	55,23	41,98	40,27	37,57	27,30	26,88	26,11
		+		27,48	39,02	48,85	59,06	60,44	62,66	71,47	71,85	72,53
				100,32	83,98	70,06	55,61	53,66	50,51	38,03	37,49	36,53
	=	-		14,82	22,75	30,59	40,08	41,48	43,81	53,99	54,46	55,32
				118,24	107,02	95,91	82,48	80,49	77,19	62,79	62,11	60,89
		=		14,44	22,22	29,95	39,35	40,75	43,07	53,24	53,72	54,58
				118,78	107,77	96,82	83,51	81,53	78,24	63,84	63,17	61,95
		+		13,79	21,30	28,84	38,09	39,47	41,77	51,92	52,40	53,27
				119,70	109,06	98,39	85,30	83,34	80,08	65,71	65,03	63,81
	+	-		8,30	13,29	18,68	25,88	27,02	28,95	38,08	38,54	39,37
				127,48	120,41	112,78	102,58	100,97	98,24	85,31	84,66	83,48
		=		8,23	13,19	18,55	25,72	26,85	28,78	37,88	38,34	39,17
				127,57	120,55	112,96	102,81	101,21	98,48	85,59	84,94	83,76
		+		8,11	13,01	18,31	25,42	26,54	28,46	37,52	37,97	38,80
				127,74	120,80	113,30	103,24	101,64	98,94	86,11	85,46	84,29
=	-	-		52,31	64,75	73,07	80,15	81,00	82,34	87,16	87,35	87,69
				65,16	47,55	35,76	25,75	24,54	22,65	15,82	15,55	15,07
		=		38,18	50,98	60,74	69,92	71,09	72,95	79,97	80,26	80,78
				85,16	67,05	53,23	40,23	38,57	35,93	26,00	25,59	24,86
		+		28,92	40,72	50,61	60,74	62,09	64,27	72,85	73,22	73,87
				98,28	81,58	67,57	53,23	51,31	48,23	36,08	35,56	34,63
	=	-		14,52	22,33	30,08	39,50	40,90	43,22	53,39	53,87	54,73
				118,67	107,62	96,63	83,30	81,32	78,03	63,63	62,95	61,73
		=		14,21	21,90	29,56	38,91	40,30	42,62	52,78	53,26	54,13
				119,11	108,22	97,37	84,13	82,16	78,88	64,49	63,82	62,59
		+		13,68	21,15	28,65	37,87	39,25	41,54	51,69	52,17	53,04
				119,86	109,29	98,66	85,61	83,66	80,41	66,04	65,36	64,13
	+	-		8,11	13,00	18,30	25,40	26,53	28,44	37,50	37,95	38,78
				127,75	120,82	113,31	103,26	101,67	98,96	86,13	85,49	84,31
		=		8,06	12,93	18,20	25,28	26,40	28,30	37,34	37,79	38,62
				127,82	120,92	113,46	103,44	101,85	99,16	86,36	85,72	84,54
		+		7,96	12,78	18,01	25,03	26,15	28,04	37,04	37,49	38,32
				127,95	121,13	113,73	103,78	102,20	99,52	86,78	86,14	84,97
+	-	-		52,60	65,01	73,29	80,32	81,17	82,50	87,27	87,46	87,80
				64,75	47,19	35,45	25,50	24,30	22,43	15,66	15,39	14,91
		=		39,47	52,33	62,00	71,02	72,16	73,97	80,78	81,06	81,56
				83,34	65,14	51,44	38,68	37,05	34,49	24,86	24,46	23,76
		+		30,30	42,31	52,24	62,27	63,60	65,74	74,09	74,44	75,08
				96,32	79,32	65,26	51,06	49,17	46,15	34,33	33,83	32,93
	=	-		14,28	22,00	29,69	39,06	40,45	42,77	52,93	53,41	54,28
				119,00	108,07	97,19	83,93	81,96	78,67	64,28	63,60	62,38
		=		14,03	21,65	29,26	38,57	39,96	42,27	52,43	52,90	53,77
				119,36	108,57	97,79	84,62	82,65	79,38	65,00	64,32	63,10
		+		13,59	21,02	28,50	37,69	39,07	41,36	51,50	51,98	52,85
				119,99	109,46	98,88	85,86	83,91	80,66	66,30	65,62	64,39
	+	-		7,96	12,79	18,01	25,04	26,15	28,05	37,05	37,50	38,33
				127,95	121,12	113,72	103,77	102,20	99,51	86,77	86,13	84,96
		=		7,92	12,72	17,93	24,93	26,04	27,94	36,92	37,37	38,20
				128,01	121,21	113,84	103,92	102,35	99,67	86,95	86,31	85,15
		+		7,84	12,60	17,77	24,74	25,84	27,72	36,67	37,12	37,95
				128,12	121,38	114,06	104,20	102,64	99,98	87,30	86,67	85,50

CI'_A: +

CI'_B	T'_REF_B	F'_REF_B		T'_REF_A - / F'_REF_A -			T'_REF_A = / F'_REF_A -			T'_REF_A + / F'_REF_A -		
				-	=	+	-	=	+	-	=	+
-	-	-		52,79	64,21	72,35	80,79	81,49	82,62	87,60	87,75	88,03
				64,48	48,32	36,78	24,83	23,84	22,25	15,20	14,98	14,59
		=		37,56	49,25	58,76	69,88	70,87	72,48	79,96	80,20	80,64
				86,05	69,50	56,03	40,28	38,89	36,61	26,02	25,68	25,06
		+		28,15	38,78	48,26	60,43	61,57	63,45	72,62	72,93	73,48
				99,37	84,32	70,90	53,67	52,06	49,39	36,41	35,97	35,18
	=	-		15,25	22,57	30,09	41,47	42,66	44,67	55,43	55,83	56,54
				117,63	107,27	96,62	80,51	78,83	75,98	60,74	60,18	59,17
		=		14,86	22,04	29,46	40,74	41,92	43,92	54,69	55,09	55,81
				118,19	108,01	97,52	81,54	79,87	77,04	61,79	61,23	60,21
		+		14,19	21,13	28,36	39,46	40,63	42,62	53,38	53,78	54,50
				119,13	109,30	99,08	83,36	81,70	78,89	63,65	63,09	62,06
	+	-		8,55	13,17	18,32	27,01	27,98	29,67	39,48	39,87	40,57
				127,11	120,58	113,29	100,99	99,60	97,22	83,33	82,78	81,78
		=		8,49	13,08	18,19	26,84	27,82	29,50	39,28	39,67	40,37
				127,21	120,71	113,47	101,22	99,84	97,46	83,61	83,07	82,07
		+		8,37	12,90	17,96	26,54	27,50	29,17	38,91	39,29	40,00
				127,38	120,96	113,80	101,65	100,29	97,92	84,14	83,59	82,60
=	-	-		53,13	64,52	72,62	81,00	81,69	82,80	87,73	87,89	88,16
				63,99	47,87	36,40	24,55	23,57	21,99	15,01	14,79	14,40
		=		38,97	50,73	60,18	71,09	72,05	73,61	80,84	81,08	81,49
				84,05	67,40	54,02	38,58	37,22	35,01	24,76	24,43	23,84
		+		29,61	40,47	50,02	62,08	63,20	65,05	73,96	74,25	74,79
				97,30	81,92	68,40	51,33	49,74	47,13	34,51	34,09	33,33
	=	-		14,94	22,15	29,59	40,89	42,07	44,08	54,84	55,24	55,96
				118,08	107,86	97,33	81,33	79,66	76,82	61,57	61,02	60,00
		=		14,62	21,72	29,07	40,29	41,47	43,47	54,24	54,63	55,35
				118,52	108,47	98,06	82,17	80,51	77,68	62,43	61,87	60,85
		+		14,08	20,98	28,17	39,24	40,40	42,39	53,15	53,54	54,27
				119,30	109,52	99,35	83,67	82,02	79,21	63,98	63,42	62,39
	+	-		8,36	12,89	17,95	26,52	27,48	29,15	38,89	39,27	39,98
				127,39	120,98	113,81	101,68	100,31	97,95	84,16	83,62	82,63
		=		8,31	12,81	17,85	26,39	27,35	29,01	38,73	39,11	39,81
				127,46	121,09	113,96	101,86	100,50	98,15	84,39	83,85	82,86
		+		8,21	12,67	17,66	26,14	27,10	28,75	38,43	38,81	39,51
				127,60	121,29	114,22	102,22	100,86	98,52	84,82	84,28	83,29
+	-	-		53,42	64,78	72,85	81,16	81,85	82,96	87,84	88,00	88,27
				63,59	47,50	36,09	24,31	23,34	21,77	14,85	14,64	14,25
		=		40,27	52,08	61,46	72,15	73,09	74,61	81,62	81,84	82,25
				82,21	65,49	52,21	37,07	35,74	33,59	23,67	23,35	22,78
		+		31,01	42,07	51,66	63,59	64,69	66,50	75,16	75,44	75,96
				95,32	79,66	66,08	49,19	47,63	45,07	32,81	32,41	31,67
	=	-		14,70	21,83	29,20	40,44	41,62	43,62	54,39	54,78	55,50
				118,41	108,32	97,89	81,97	80,30	77,47	62,22	61,67	60,65
		=		14,44	21,48	28,77	39,95	41,12	43,11	53,88	54,28	55,00
				118,78	108,82	98,49	82,67	81,01	78,18	62,94	62,38	61,36
		+		13,99	20,85	28,01	39,06	40,22	42,21	52,96	53,36	54,08
				119,42	109,70	99,56	83,92	82,27	79,47	64,24	63,68	62,65
	+	-		8,21	12,67	17,67	26,15	27,10	28,76	38,44	38,82	39,52
				127,60	121,28	114,21	102,21	100,85	98,51	84,81	84,27	83,28
		=		8,17	12,61	17,58	26,04	26,99	28,64	38,30	38,68	39,38
				127,66	121,37	114,33	102,36	101,01	98,67	84,99	84,46	83,47
		+		8,09	12,49	17,43	25,83	26,78	28,43	38,05	38,43	39,13
				127,77	121,54	114,55	102,65	101,31	98,98	85,35	84,81	83,82

Scenario γ_b Case 3b

Airline A's flight covers STAR TERTO3C and has the cost index CI_A=33

Airline B's flight covers STAR RUSIK3C and has the cost index CI_B=70

Equilibrium minimax	
η	1,20%
Π_A	-5,00%
Π_B	4,40%

			CI'_A -									CI'_A =									CI'_A +								
			T'_REF_A -			T'_REF_A =			T'_REF_A +			T'_REF_A -			T'_REF_A =			T'_REF_A +			T'_REF_A -			T'_REF_A =			T'_REF_A +		
CI'_B	T'_REF_B	F'_REF_B	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +	F'_REF_A -	F'_REF_A =	F'_REF_A +
-	-	-	39,37	57,90	66,37	67,56	69,36	71,72	75,73	76,09	76,70	39,05	57,14	65,70	68,43	69,89	71,91	76,33	76,61	77,11	38,74	56,37	65,01	69,14	70,34	72,08	76,81	77,04	77,44
			80,43	49,43	35,25	33,26	30,25	26,31	19,60	18,99	17,96	80,97	50,70	36,38	31,80	29,36	25,98	18,59	18,11	17,29	81,49	51,98	37,53	30,62	28,60	25,69	17,79	17,40	16,73
		=	34,01	52,78	62,08	63,42	65,47	68,20	72,92	73,36	74,09	33,69	51,97	61,32	64,41	66,09	68,42	73,64	73,99	74,58	33,40	51,16	60,55	65,22	66,61	68,63	74,22	74,49	74,98
			89,41	58,00	42,44	40,19	36,75	32,20	24,29	23,56	22,33	89,93	59,35	43,70	38,53	35,73	31,82	23,09	22,51	21,53	90,43	60,71	45,00	37,18	34,86	31,48	22,12	21,66	20,85
		+	28,34	46,72	56,70	58,19	60,51	63,62	69,17	69,69	70,57	28,06	45,88	55,87	59,31	61,20	63,88	70,03	70,44	71,15	27,78	45,04	55,02	60,22	61,79	64,11	70,72	71,05	71,64
			98,89	68,13	51,44	48,94	45,07	39,86	30,57	29,70	28,23	99,37	69,53	52,83	47,07	43,91	39,42	29,13	28,44	27,25	99,82	70,94	54,25	45,55	42,91	39,03	27,97	27,42	26,44
	=	-	13,86	27,17	36,74	38,37	40,99	44,77	52,28	53,05	54,36	13,69	26,46	35,86	39,61	41,81	45,10	53,55	54,17	55,25	13,53	25,75	34,98	40,66	42,52	45,40	54,59	55,10	56,00
			123,11	100,85	84,83	82,11	77,72	71,40	58,82	57,54	55,34	123,41	102,04	86,31	80,03	76,35	70,84	56,70	55,67	53,86	123,68	103,22	87,78	78,28	75,16	70,34	54,96	54,11	52,60
		=	13,67	26,85	36,38	38,00	40,62	44,40	51,93	52,69	54,01	13,49	26,14	35,50	39,24	41,44	44,73	53,20	53,82	54,90	13,33	25,44	34,62	40,29	42,15	45,03	54,24	54,75	55,66
			123,45	101,38	85,44	82,73	78,34	72,02	59,43	58,14	55,93	123,73	102,57	86,91	80,65	76,97	71,47	57,30	56,26	54,44	124,00	103,74	88,38	78,90	75,78	70,97	55,55	54,70	53,18
		+	13,31	26,27	35,72	37,33	39,94	43,72	51,26	52,04	53,37	13,14	25,57	34,84	38,57	40,76	44,05	52,54	53,17	54,26	12,98	24,88	33,97	39,61	41,47	44,35	53,60	54,11	55,02
			124,04	102,35	86,55	83,85	79,47	73,16	60,53	59,24	57,02	124,32	103,52	88,01	81,77	78,11	72,61	58,39	57,34	55,51	124,59	104,68	89,47	80,03	76,92	72,11	56,63	55,77	54,24
	+	-	8,38	17,68	25,32	26,70	29,00	32,45	39,82	40,62	42,00	8,26	17,15	24,58	27,79	29,74	32,76	41,14	41,79	42,95	8,16	16,62	23,84	28,71	30,38	33,04	42,24	42,79	43,76
			132,29	116,72	103,95	101,63	97,78	92,02	79,68	78,35	76,03	132,48	117,62	105,19	99,82	96,55	91,50	77,47	76,38	74,45	132,66	118,50	106,42	98,28	95,48	91,03	75,63	74,72	73,09
		=	8,34	17,61	25,22	26,60	28,90	32,34	39,71	40,50	41,88	8,22	17,07	24,48	27,68	29,63	32,65	41,02	41,68	42,83	8,12	16,55	23,75	28,60	30,27	32,93	42,12	42,67	43,64
			132,36	116,85	104,11	101,80	97,96	92,20	79,87	78,55	76,23	132,55	117,74	105,35	99,99	96,73	91,69	77,67	76,58	74,65	132,73	118,62	106,57	98,46	95,66	91,22	75,83	74,92	73,29
		+	8,26	17,46	25,03	26,41	28,70	32,13	39,48	40,27	41,66	8,15	16,93	24,30	27,49	29,43	32,44	40,80	41,45	42,60	8,04	16,41	23,57	28,40	30,06	32,72	41,90	42,44	43,41
			132,49	117,09	104,42	102,12	98,29	92,56	80,25	78,92	76,61	132,68	117,98	105,65	100,32	97,07	92,04	78,04	76,95	75,03	132,86	118,85	106,87	98,79	96,01	91,57	76,20	75,30	73,67
=	-	-	38,87	57,44	65,99	67,20	69,02	71,41	75,49	75,86	76,48	38,54	56,67	65,31	68,08	69,56	71,61	76,10	76,39	76,89	38,23	55,90	64,62	68,80	70,02	71,79	76,59	76,82	77,23
			81,28	50,21	35,88	33,87	30,82	26,82	20,00	19,38	18,34	81,82	51,48	37,02	32,39	29,91	26,49	18,98	18,49	17,65	82,34	52,77	38,19	31,19	29,15	26,19	18,16	17,77	17,08
		=	34,26	53,03	62,30	63,63	65,67	68,38	73,07	73,50	74,23	33,95	52,23	61,55	64,62	66,28	68,60	73,78	74,12	74,71	33,65	51,41	60,78	65,42	66,80	68,80	74,35	74,63	75,11
			88,99	57,57	42,07	39,83	36,42	31,90	24,05	23,32	22,10	89,51	58,92	43,33	38,18	35,40	31,52	22,85	22,28	21,30	90,00	60,28	44,62	36,84	34,54	31,18	21,90	21,44	20,64
		+	29,06	47,53	57,44	58,91	61,19	64,26	69,70	70,21	71,07	28,77	46,69	56,61	60,01	61,88	64,51	70,54	70,94	71,64	28,49	45,85	55,77	60,91	62,46	64,74	71,22	71,54	72,11
			97,69	66,78	50,20	47,73	43,92	38,79	29,69	28,83	27,39	98,17	68,18	51,58	45,90	42,77	38,36	28,28	27,60	26,44	98,64	69,58	52,99	44,39	41,80	37,98	27,14	26,60	25,64
	=	-	13,48	26,56	36,04	37,66	40,28	44,05	51,59	52,36	53,68	13,31	25,85	35,17	38,90	41,09	44,38	52,87	53,49	54,58	13,15	25,16	34,29	39,94	41,80	44,68	53,91	54,43	55,34
			123,75	101,87	86,00	83,30	78,92	72,60	59,99	58,70	56,48	124,03	103,05	87,47	81,22	77,55	72,05	57,85	56,81	54,99	124,30	104,22	88,94	79,47	76,36	71,55	56,10	55,24	53,72
		=	13,33	26,30	35,75	37,36	39,98	43,75	51,30	52,07	53,40	13,16	25,60	34,87	38,60	40,79	44,08	52,58	53,20	54,29	13,00	24,91	34,00	39,64	41,50	44,38	53,63	54,14	55,05
			124,01	102,30	86,49	83,79	79,42	73,11	60,48	59,19	56,96	124,30	103,48	87,96	81,72	78,05	72,55	58,34	57,29	55,46	124,56	104,63	89,42	79,98	76,87	72,05	56,58	55,72	54,19
		+	13,04	25,83	35,21	36,81	39,42	43,19	50,75	51,53	52,86	12,87	25,14	34,34	38,05	40,24	43,52	52,04	52,66	53,76	12,72	24,46	33,47	39,09	40,94	43,82	53,09	53,61	54,53
			124,49	103,09	87,40	84,71	80,35	74,05	61,39	60,09	57,86	124,77	104,25	88,85	82,64	78,99	73,49	59,24	58,19	56,35	125,03	105,39	90,31	80,91	77,80	72,99	57,47	56,61	55,07
	+	-	8,13	17,22	24,72	26,09	28,36	31,77	39,10	39,89	41,27	8,02	16,69	23,99	27,16	29,09	32,08	40,42	41,07	42,22	7,92	16,18	23,27	28,07	29,72	32,36	41,52	42,06	43,03
			132,71	117,50	104,94	102,66	98,85	93,15	80,88	79,56	77,25	132,89	118,38	106,17	100,87	97,64	92,63	78,68	77,59	75,67	133,07	119,24	107,37	99,35	96,58	92,17	76,84	75,94	74,31
		=	8,10	17,16	24,64	26,01	28,28	31,68	39,01	39,80	41,18	7,99	16,63	23,92	27,08	29,00	31,99	40,32	40,97	42,12	7,88	16,12	23,20	27,98	29,63	32,27	41,42	41,96	42,93
			132,77	117,60	105,08	102,79	99,00	93,30	81,04	79,72	77,41	132,95	118,48	106,29	101,01	97,79	92,78	78,84	77,75	75,83	133,12	119,34	107,50	99,49	96,73	92,32	77,01	76,10	74,47
		+	8,03	17,04	24,50	25,86	28,12	31,51	38,83	39,62	40,99	7,93	16,52	23,77	26,92	28,84	31,82	40,14	40,79	41,94	7,82	16,01	23,05	27,82	29,47	32,10	41,24	41,78	42,75
			132,87	117,80	105,32	103,05	99,26	93,58	81,35	80,02	77,72	133,05	118,67	106,54	101,27	98,06	93,07	79,15	78,06	76,13	133,22	119,52	107,74	99,76	97,00	92,61	77,31	76,40	74,78
+	-	-	38,46	57,06	65,69	66,90	68,75	71,16	75,29	75,67	76,30	38,14	56,30	65,00	67,80	69,29	71,36	75,91	76,21	76,71	37,83	55,52	64,30	68,52	69,76	71,54	76,41	76,64	77,06
			81,96	50,83	36,39	34,36	31,28	27,23	20,33	19,70	18,64	82,50	52,11	37,54	32,87	30,36	26,90	19,29	18,79	17,95	83,01	53,40	38,72	31,66	29,59	26,60	18,46	18,06	17,37
		=	34,48	53,25	62,48	63,81	65,84	68,53	73,19	73,62	74,35	34,16	52,45	61,73	64,79	66,45	68,76	73,90	74,24	74,82	33,87	51,63	60,97	65,59	66,96	68,96	74,47	74,74	75,22
			88,62	57,21	41,76	39,53	36,13	31,64	23,84	23,12	21,91	89,15	58,56	43,01	37,89	35,12	31,26	22,65	22,09	21,11	89,64	59,91	44,29	36,56	34,26	30,93	21,70	21,25	20,45
		+	29,71	48,25	58,09	59,55	61,80	64,82	70,16	70,66	71,51	29,41	47,41	57,27	60,63	62,47	65,07	70,99	71,38	72,06	29,13	46,58	56,44	61,52	63,05	65,30	71,65	71,97	72,53
			96,61	65,58	49,12	46,67	42,91	37,85	28,91	28,07	26,66	97,10	66,97	50,48	44,86	41,78	37,43	27,53	26,86	25,73	97,56	68,37	51,87	43,37	40,81	37,05	26,42	25,88	24,95
	=	-	13,19	26,08	35,50	37,11	39,72	43,49	51,04	51,82	53,15	13,02	25,38	34,62	38,34	40,53	43,82	52,32	52,95	54,05	12,87	24,70	33,75	39,38	41,24	44,12	53,38	53,89	54,81
			124,24	102,67	86,92	84,22	79,86	73,55	60,90	59,61	57,38	124,52	103,84	88,38	82,15	78,49	72,99	58,76	57,71	55,88	124,78	104,99	89,84	80,41	77,31	72,49	57,00	56,13	54,60
		=	13,06	25,87	35,25	36,86	39,47	43,23	50,80	51,57	52,91	12,90	25,18	34,38	38,10	40,28	43,57	52,08	52,71	53,81	12,74	24,49	33,51	39,13	40,99	43,86	53,14	53,65	54,57
			124,45	103,02	87,32	84,64	80,27	73,97	61,31	60,02	57,78	124,73	104,18	88,78	82,57	78,91	73,41	59,16	58,11	56,28	124,99	105,33	90,24	80,83	77,73	72,91	57,40	56,53	55,00
		+	12,83	25,48	34,80	36,40	39,00	42,77	50,34	51,12	52,46	12,66	24,80	33,94	37,64	39,82	43,10	51,63	52,26	53,36	12,51	24,12	33,07	38,67	40,52	43,40	52,69	53,21	54,13
			124,85	103,67	88,08	85,40	81,05	74,75	62,08	60,78	58,53	125,12	104,82	89,53	83,34	79,69	74,20	59,92	58,86	57,02	125,38	105,96	90,97	81,60	78,51	73,70	58,14	57,28	55,73
	+	-	7,94	16,86	24,26	25,61	27,86	31,24	38,54	39,33	40,70	7,83	16,34	23,54	26,67	28,58	31,55	39,85	40,50	41,65	7,73	15,84	22,83	27,57	29,21	31,83	40,95	41,49	42,46
			133,03	118,10	105,72	103,45	99,69	94,03	81,83	80,51	78,20	133,21	118,97	106,92	101,68	98,49	93,52	79,63	78,54	76,62	133,38	119,81	108,12	100,18	97,44	93,06	77,80	76,89	75,27
		=	7,91	16,81	24,20	25,55	27,79	31,17	38,46	39,25	40,62	7,80	16,30	23,48	26,60	28,51	31,48	39,77	40,42	41,57	7,70	15,79	22,77	27,50	29,14	31,75	40,87	41,41	42,38
			133,07	118,18	105,82	103,56	99,81	94,15	81,96	80,64	78,34	133,25	119,05	107,03	101,80	98,61	93,64	79,76	78,68	76,76	133,42	119,89	108,22	100,29	97,56	93,18	77,93	77,02	75,40
		+	7,86	16,72	24,08	25,42	27,66	31,03	38,31	39,10	40,47	7,76	16,20	23,36	26,47	28,38	31,34	39,62	40,27	41,42	7,65	15,70	22,65	27,37	29,00	31,61	40,71	41,26	42,23
			133,16	118,34	106,03	103,77	100,03	94,39	82,21	80,89	78,59	133,34	119,20	107,23	102,01	98,83	93,88	80,02	78,93	77,01	133,50	120,04	108,42	100,51	97,78	93,42	78,18	77,28	75,65

Scenario γ_C Case 3c

Airline A's flight covers STAR TERTO3C and has the cost index CI_A=70

Airline B's flight covers STAR RUSIK3C and has the cost index CI_B=33

Equilibrium minimax	
η	-3,90%
Π_A	-19,60%
Π_B	28,20%

						CI'_A																										
						-									=									+								
						T'_REF_A									T'_REF_A									T'_REF_A								
						-			=			+			-			=			+			-			=			+		
						F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A			F'_REF_A		
						-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+	-	=	+
CI'_B	-	T'_REF_B	-	F'_REF_B	-	66,95	88,36	101,36	118,90	119,43	120,33	127,88	127,96	128,13	66,46	86,40	99,35	119,32	119,74	120,48	128,14	128,21	128,34	66,05	84,59	97,43	119,64	119,99	120,60	128,33	128,39	128,50
						51,05	35,92	26,74	14,35	13,98	13,35	8,01	7,95	7,84	51,39	37,31	28,16	14,06	13,76	13,24	7,83	7,78	7,69	51,68	38,59	29,52	13,83	13,59	13,15	7,69	7,65	7,58
					=	47,67	68,79	83,64	106,76	107,52	108,82	120,24	120,38	120,64	47,23	66,69	81,23	107,36	107,97	109,04	120,65	120,76	120,96	46,86	64,78	78,96	107,82	108,33	109,22	120,96	121,05	121,22
						64,66	49,75	39,26	22,93	22,40	21,48	13,41	13,31	13,13	64,98	51,23	40,96	22,50	22,07	21,32	13,12	13,04	12,90	65,23	52,58	42,56	22,18	21,82	21,19	12,90	12,84	12,72
					+	35,51	54,44	69,24	95,20	96,11	97,70	112,27	112,45	112,79	35,14	52,45	66,75	95,93	96,67	97,98	112,81	112,96	113,23	34,83	50,67	64,44	96,49	97,10	98,20	113,22	113,34	113,57
						73,25	59,88	49,43	31,10	30,45	29,33	19,04	18,91	18,67	73,52	61,29	51,19	30,58	30,06	29,13	18,66	18,55	18,36	73,73	62,55	52,82	30,18	29,75	28,98	18,37	18,28	18,12
			=	F'_REF_B	-	27,54	44,03	57,93	84,78	85,79	87,56	104,42	104,64	105,04	27,23	42,23	55,52	85,59	86,41	87,87	105,07	105,24	105,57	26,97	40,63	53,31	86,21	86,89	88,11	105,56	105,71	105,98
						78,88	67,24	57,42	38,45	37,74	36,49	24,59	24,43	24,14	79,10	68,51	59,12	37,88	37,30	36,27	24,13	24,00	23,77	79,28	69,64	60,68	37,44	36,96	36,10	23,77	23,67	23,48
					=	26,00	41,91	55,53	82,40	83,43	85,22	102,52	102,75	103,17	25,70	40,16	53,15	83,22	84,05	85,53	103,19	103,38	103,72	25,45	38,61	50,98	83,85	84,54	85,78	103,71	103,86	104,14
						79,97	68,74	59,11	40,14	39,41	38,14	25,92	25,76	25,46	80,18	69,97	60,79	39,56	38,97	37,92	25,45	25,32	25,08	80,36	71,06	62,32	39,11	38,62	37,75	25,08	24,98	24,78
					+	23,68	38,65	51,78	78,54	79,58	81,42	99,37	99,61	100,05	23,40	36,99	49,47	79,37	80,22	81,74	100,07	100,27	100,63	23,17	35,52	47,37	80,02	80,72	81,99	100,62	100,78	101,07
						81,61	71,03	61,76	42,86	42,12	40,83	28,15	27,98	27,67	81,81	72,21	63,39	42,27	41,67	40,60	27,65	27,52	27,26	81,97	73,25	64,88	41,82	41,32	40,42	27,27	27,16	26,95
			+	F'_REF_B	-	17,05	28,88	40,01	65,23	66,29	68,16	87,62	87,89	88,40	16,84	27,52	38,00	66,07	66,94	68,49	88,43	88,65	89,06	16,67	26,33	36,19	66,73	67,45	68,76	89,05	89,24	89,58
						86,29	77,93	70,07	52,27	51,52	50,19	36,45	36,26	35,89	86,44	78,89	71,49	51,67	51,06	49,96	35,88	35,72	35,43	86,56	79,74	72,77	51,21	50,70	49,77	35,44	35,31	35,07
					=	16,70	28,34	39,33	64,39	65,45	67,33	86,84	87,11	87,63	16,49	27,00	37,34	65,24	66,11	67,66	87,65	87,88	88,29	16,32	25,82	35,55	65,89	66,61	67,92	88,28	88,46	88,81
						86,54	78,32	70,55	52,85	52,10	50,78	37,00	36,81	36,44	86,69	79,26	71,96	52,26	51,64	50,55	36,42	36,27	35,97	86,81	80,09	73,23	51,79	51,28	50,36	35,98	35,85	35,61
					+	16,07	27,38	38,13	62,91	63,96	65,83	85,42	85,70	86,22	15,87	26,07	36,18	63,75	64,61	66,16	86,25	86,47	86,89	15,71	24,93	34,42	64,40	65,12	66,42	86,88	87,07	87,42
						86,98	79,00	71,40	53,90	53,16	51,84	38,00	37,80	37,44	87,12	79,92	72,78	53,31	52,70	51,60	37,42	37,26	36,96	87,24	80,73	74,02	52,85	52,34	51,42	36,97	36,84	36,59
	=	T'_REF_B	-	F'_REF_B	-	65,65	87,15	100,32	118,24	118,78	119,70	127,48	127,57	127,74	65,16	85,16	98,28	118,67	119,11	119,86	127,75	127,82	127,95	64,75	83,34	96,32	119,00	119,36	119,99	127,95	128,01	128,12
						51,97	36,78	27,48	14,82	14,44	13,79	8,30	8,23	8,11	52,31	38,18	28,92	14,52	14,21	13,68	8,11	8,06	7,96	52,60	39,47	30,30	14,28	14,03	13,59	7,96	7,92	7,84
					=	48,00	69,15	83,98	107,02	107,77	109,06	120,41	120,55	120,80	47,55	67,05	81,58	107,62	108,22	109,29	120,82	120,92	121,13	47,19	65,14	79,32	108,07	108,57	109,46	121,12	121,21	121,38
						64,43	49,50	39,02	22,75	22,22	21,30	13,29	13,19	13,01	64,75	50,98	40,72	22,33	21,90	21,15	13,00	12,93	12,78	65,01	52,33	42,31	22,00	21,65	21,02	12,79	12,72	12,60
					+	36,14	55,23	70,06	95,91	96,82	98,39	112,78	112,96	113,30	35,76	53,23	67,57	96,63	97,37	98,66	113,31	113,46	113,73	35,45	51,44	65,26	97,19	97,79	98,88	113,72	113,84	114,06
						72,81	59,33	48,85	30,59	29,95	28,84	18,68	18,55	18,31	73,07	60,74	50,61	30,08	29,56	28,65	18,30	18,20	18,01	73,29	62,00	52,24	29,69	29,26	28,50	18,01	17,93	17,77
			=	F'_REF_B	-	26,05	41,98	55,61	82,48	83,51	85,30	102,58	102,81	103,24	25,75	40,23	53,23	83,30	84,13	85,61	103,26	103,44	103,78	25,50	38,68	51,06	83,93	84,62	85,86	103,77	103,92	104,20
						79,94	68,69	59,06	40,08	39,35	38,09	25,88	25,72	25,42	80,15	69,92	60,74	39,50	38,91	37,87	25,40	25,28	25,03	80,32	71,02	62,27	39,06	38,57	37,69	25,04	24,93	24,74
					=	24,83	40,27	53,66	80,49	81,53	83,34	100,97	101,21	101,64	24,54	38,57	51,31	81,32	82,16	83,66	101,67	101,85	102,20	24,30	37,05	49,17	81,96	82,65	83,91	102,20	102,35	102,64
						80,80	69,89	60,44	41,48	40,75	39,47	27,02	26,85	26,54	81,00	71,09	62,09	40,90	40,30	39,25	26,53	26,40	26,15	81,17	72,16	63,60	40,45	39,96	39,07	26,15	26,04	25,84
					+	22,92	37,57	50,51	77,19	78,24	80,08	98,24	98,48	98,94	22,65	35,93	48,23	78,03	78,88	80,41	98,96	99,16	99,52	22,43	34,49	46,15	78,67	79,38	80,66	99,51	99,67	99,98
						82,15	71,80	62,66	43,81	43,07	41,77	28,95	28,78	28,46	82,34	72,95	64,27	43,22	42,62	41,54	28,44	28,30	28,04	82,50	73,97	65,74	42,77	42,27	41,36	28,05	27,94	27,72
			+	F'_REF_B	-	16,02	27,30	38,03	62,79	63,84	65,71	85,31	85,59	86,11	15,82	26,00	36,08	63,63	64,49	66,04	86,13	86,36	86,78	15,66	24,86	34,33	64,28	65,00	66,30	86,77	86,95	87,30
						87,02	79,05	71,47	53,99	53,24	51,92	38,08	37,88	37,52	87,16	79,97	72,85	53,39	52,78	51,69	37,50	37,34	37,04	87,27	80,78	74,09	52,93	52,43	51,50	37,05	36,92	36,67
					=	15,75	26,88	37,49	62,11	63,17	65,03	84,66	84,94	85,46	15,55	25,59	35,56	62,95	63,82	65,36	85,49	85,72	86,14	15,39	24,46	33,83	63,60	64,32	65,62	86,13	86,31	86,67
						87,21	79,35	71,85	54,46	53,72	52,40	38,54	38,34	37,97	87,35	80,26	73,22	53,87	53,26	52,17	37,95	37,79	37,49	87,46	81,06	74,44	53,41	52,90	51,98	37,50	37,37	37,12
					+	15,26	26,11	36,53	60,89	61,95	63,81	83,48	83,76	84,29	15,07	24,86	34,63	61,73	62,59	64,13	84,31	84,54	84,97	14,91	23,76	32,93	62,38	63,10	64,39	84,96	85,15	85,50
						87,56	79,89	72,53	55,32	54,58	53,27	39,37	39,17	38,80	87,69	80,78	73,87	54,73	54,13	53,04	38,78	38,62	38,32	87,80	81,56	75,08	54,28	53,77	52,85	38,33	38,20	37,95
	+	T'_REF_B	-	F'_REF_B	-	64,48	86,05	99,37	117,63	118,19	119,13	127,11	127,21	127,38	63,99	84,05	97,30	118,08	118,52	119,30	127,39	127,46	127,60	63,59	82,21	95,32	118,41	118,78	119,42	127,60	127,66	127,77
						52,79	37,56	28,15	15,25	14,86	14,19	8,55	8,49	8,37	53,13	38,97	29,61	14,94	14,62	14,08	8,36	8,31	8,21	53,42	40,27	31,01	14,70	14,44	13,99	8,21	8,17	8,09
					=	48,32	69,50	84,32	107,27	108,01	109,30	120,58	120,71	120,96	47,87	67,40	81,92	107,86	108,47	109,52	120,98	121,09	121,29	47,50	65,49	79,66	108,32	108,82	109,70	121,28	121,37	121,54
						64,21	49,25	38,78	22,57	22,04	21,13	13,17	13,08	12,90	64,52	50,73	40,47	22,15	21,72	20,98	12,89	12,81	12,67	64,78	52,08	42,07	21,83	21,48	20,85	12,67	12,61	12,49
					+	36,78	56,03	70,90	96,62	97,52	99,08	113,29	113,47	113,80	36,40	54,02	68,40	97,33	98,06	99,35	113,81	113,96	114,22	36,09	52,21	66,08	97,89	98,49	99,56	114,21	114,33	114,55
						72,35	58,76	48,26	30,09	29,46	28,36	18,32	18,19	17,96	72,62	60,18	50,02	29,59	29,07	28,17	17,95	17,85	17,66	72,85	61,46	51,66	29,20	28,77	28,01	17,67	17,58	17,43
			=	F'_REF_B	-	24,83	40,28	53,67	80,51	81,54	83,36	100,99	101,22	101,65	24,55	38,58	51,33	81,33	82,17	83,67	101,68	101,86	102,22	24,31	37,07	49,19	81,97	82,67	83,92	102,21	102,36	102,65
						80,79	69,88	60,43	41,47	40,74	39,46	27,01	26,84	26,54	81,00	71,09	62,08	40,89	40,29	39,24	26,52	26,39	26,14	81,16	72,15	63,59	40,44	39,95	39,06	26,15	26,04	25,83
					=	23,84	38,89	52,06	78,83	79,87	81,70	99,60	99,84	100,29	23,57	37,22	49,74	79,66	80,51	82,02	100,31	100,50	100,86	23,34	35,74	47,63	80,30	81,01	82,27	100,85	101,01	101,31
						81,49	70,87	61,57	42,66	41,92	40,63	27,98	27,82	27,50	81,69	72,05	63,20	42,07	41,47	40,40	27,48	27,35	27,10	81,85	73,09	64,69	41,62	41,12	40,22	27,10	26,99	26,78
					+	22,25	36,61	49,39	75,98	77,04	78,89	97,22	97,46	97,92	21,99	35,01	47,13	76,82	77,68	79,21	97,95	98,15	98,52	21,77	33,59	45,07	77,47	78,18	79,47	98,51	98,67	98,98
						82,62	72,48	63,45	44,67	43,92	42,62	29,67	29,50	29,17	82,80	73,61	65,05	44,08	43,47	42,39	29,15	29,01	28,75	82,96	74,61	66,50	43,62	43,11	42,21	28,76	28,64	28,43
			+	F'_REF_B	-	15,20	26,02	36,41	60,74	61,79	63,65	83,33	83,61	84,14	15,01	24,76	34,51	61,57	62,43	63,98	84,16	84,39	84,82	14,85	23,67	32,81	62,22	62,94	64,24	84,81	84,99	85,35
						87,60	79,96	72,62	55,43	54,69	53,38	39,48	39,28	38,91	87,73	80,84	73,96	54,84	54,24	53,15	38,89	38,73	38,43	87,84	81,62	75,16	54,39	53,88	52,96	38,44	38,30	38,05
					=	14,98	25,68	35,97	60,18	61,23	63,09	82,78	83,07	83,59	14,79	24,43	34,09	61,02	61,87	63,42	83,62	83,85	84,28	14,64	23,35	32,41	61,67	62,38	63,68	84,27	84,46	84,81
						87,75	80,20	72,93	55,83	55,09	53,78	39,87	39,67	39,29	87,89	81,08	74,25	55,24	54,63	53,54	39,27	39,11	38,81	88,00	81,84	75,44	54,78	54,28	53,36	38,82	38,68	38,43
					+	14,59	25,06	35,18	59,17	60,21	62,06	81,78	82,07	82,60	14,40	23,84	33,33	60,00	60,85	62,39	82,63	82,86	83,29	14,25	22,78	31,67	60,65	61,36	62,65	83,28	83,47	83,82
						88,03	80,64	73,48	56,54	55,81	54,50	40,57	40,37	40,00	88,16	81,49	74,79	55,96	55,35	54,27	39,98	39,81	39,51	88,27	82,25	75,96	55,50	55,00	54,08	39,52	39,38	39,13

ABBREVIATIONS AND ACRONYMS

2D	Two Dimensional
3D	Three Dimensional
4D	Four Dimensional
4D CARMA	Four Dimensional Cooperative Arrival Manager
ACC	Air Traffic Control Centre
ADCO	Arrival Departure Coordinator
AENA	Aeropuertos Españoles y Navegación Aérea
AICIA	Asociación de Investigación y Cooperación Industrial de Andalucía
AOC	Airline Operation Centre
AMAN	Arrival Manager
ANS	Air Navigation Service
ANSP(s)	Air Navigation Service Provider(s)
APM	Aircraft Performance Model
ASA	Air Services Australia
ASTRA	Australian Strategic Air Traffic Management Group
ATA	Air Transportation Association
ATC	Air Traffic Control
ATCO	Air Traffic Controller
ATLANTIDA	Aplicación de Tecnologías Líder a Aeronaves No Tripuladas para la Investigación y Desarrollo de ATM
ATM	Air Traffic Management
ATS	Air Traffic Service(s)
BADA	Base of Aircraft Data
BR&TE	Boeing Research and Technology Europe
CAASD	MITRE Centre for Advanced Aviation System Development
CANSO	Civil Air Navigation Service Organisation

CD	Conflict Detection
CDA	Continuous Descent Approach
CD&R	Conflict Detection and Resolution
CENA	Centre d'Études de la Navigation Aérienne
CI	Cost Index
CFMU	Central Flow Management Unit
CNS	Communication Navigation Surveillance
CNS-A	Communication Navigation Surveillance and Automation
CR	Conflict Resolution
COMPAS	Computer Oriented Metering Planning and Advisory System
CTAS	Center TRACON Automation System
C_F	Coefficient for fuel related cost
C_{fixed}	Fixed Cost
$C_{variable}$	Variable Cost
C_T	Coefficient for time related cost
DFS	Deutsche Flugsicherung
DLR	Deutshes Zentrum für Luft- und Raumfahrt
DMAN	Departure Manager
DST	Decision Support Tools
EDA	En-route Descent Advisor
EPRU	Eurocontrol Performance Review Unit
ETA	Estimated Time of Arrival
ETD	Estimated Time of Departure
ETOPS	Extended Twin Engine Operations
EUROCONTROL	European Organization for the Safety of Air Navigation
FAA	Federal Aviation Administration
FAST	Final Approach Spacing Tool
FCFS	First Come First Served
FDPS	Flight Data Processing System
FIR	Flight Information Region
FL	Flight Level
FMS	Flight Management System

FPCF	Flight Plan Conflict Function
F_M	Modified Fuel Consumption
F_P	Preferred Fuel Consumption
F_{REF}	Reference Fuel Consumption
F_{SAT}	Saturated Fuel Consumption
HMI	Human Machine Interface
IAF	Initial Approach Fix
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
iFACTS	Interim Future Area Control Tools Support
KPI	Key Performance Indicator
LVNL	Luchtverkeersleiding Nederland
MAESTRO	Moyen d'Aide à l'Ecoulement Séquencé du Trafic avec Recherche d'Optimisation
MEL	Minimum Equipment List
METAR	Meteorological Aeronautical Information
NASA	National Aeronautics and Space Administration
NATS	UK National Air Traffic Services
NextGen	Next Generation Air Transportation System
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium
NOTAM(s)	Notice(s) to Airmen
PHARE	Programme for Harmonised Air Traffic Management Research in EUROCONTROL
P	Cost Penalty
P_{SAT}	Saturated Cost Penalty
RDPS	Radar Data Processing System
SARA	Speed And Route Advisor
SESAR	Single European Sky ATM Research
SIGMET	Significant Meteorological Information
SMART	Specific, Measurable, Achievable, Realistic, Timely
STA	Scheduled Time of Arrival
STAR	Standard Terminal Arrival Route
S	Satisfaction

TAS	True Air Speed
TBO	Trajectory Based Operations
TCI	Trajectory Computation Infrastructure
TFM	Traffic Flow Management
TMA	Terminal Manoeuvring Area
TMA	Traffic Management Advisor (related to CTAS)
TRACON	Terminal Radar Approach Control
T_M	Modified Flight Duration
T_P	Preferred Flight Duration
T_{REF}	Reference Flight Duration
T_{SAT}	Saturated Flight Duration
UAB	Universidad Autónoma de Barcelona
UIR	Upper Flight Information Region
UPT	User Preferred Trajectory
URL	Universal Resource Locator
U	Utility
u	Relative Utility
E	Equity
Φ	Fairness
$\wp$	Relative Penalty Function

REFERENCES

1 SESAR Consortium, *Air Transport Framework, The Current Situation,* Milestone Deliverable 1 issue for the SESAR Definition Phase, Version 3.0, July 2006

2 Kayton, M., Fried, W. R., *Avionics Navigation Systems*, 2^{nd} edition, 1997, New York: John Wiley & Sons, Inc., ISBN 0-471-54795-6

3 Fearing, Douglas, Cynthia Barnhart, Dimitris Bertsimas, Constantine Caramanis, *Equitable and Efficient Coordination of Traffic Flow Management Programs*, International Symposium for Mathematical Programming in Chicago, IL. August 24, 2009

4 Vossen, Thomas, Michael Bell, Robert Hoffman, Michael Wambsganss , *General Approach to Equity in Traffic Flow Management and its Application to Mitigating Exemptions Bias in Ground Delay Programs*, Air Traffic Control Quarterly 11:277-292 (2010)

5 Professional Pilots Network- ATC Issues URL (30.01.2011): http://www.pprune.org/atc-issues/437485-fco-atc.html

6 Professional Pilots Network- ATC Issues URL (30.01.2011): http://www.pprune.org/atc-issues/416995-spanish-atc-7.html

7 Airbus Statements on Air traffic Management, A new Solution, URL (01.02.2011): http://www.airbus.com/innovation/eco-efficiency/operations/air-traffic-management/

8 Anonymous, *A jam at 32,000 feet*, Published in *the Economist*, February 5, 2000

9 Joint Planning and Development Office, "A comparative Assessment of the NextGen and SESAR Operational Concepts", May 2008.

10 Ulfbratt, Elinor, Jay McConville, *Comparison of the SESAR and NextGen Concepts of Operations*, NCOIC AviationI PT-SESAR-NextGen 20090317, May 2008

11 SESAR Consortium, *The ATM Target Concept*, Milestone Deliverable 3 issue for the SESAR Definition Phase, September 2007

12 SESAR Joint Undertaking, *SESAR Trajectory Management Document*, Edition 00.02.90, Published September 2010

13 Haraldsdottis, Aslaug, Julien Scharl, Matthew E. Berger, Ewald G. Schoemig, Michael L. Coats, *Arrival Management with Required Navigation Performance and 3D Paths*, Presented at the 7th US/Europe ATM R&D Seminar, Barcelona, Spain, July 2-5, 2007

14 Joint Planning and Development Office, *Concept of Operations for the Next Generation Air Transportation System*, Version 2.0, June 2007

15 Erzberger, Heinz, Russel A. Paielli, *Concept for Next Generation Air Traffic Control System*, NASA Ames Research Center, Moffet Field, CA , Air Traffic Control Quarterly, Vol. 10(4), pp 355-378, 2002.

16 Robinson III, John E., Thomas J. Davis, Douglas R. Isaacson, *Fuzzy reasoning-based sequencing of arrival aircraft in the terminal area*, NASA Ames Research Center, Moffet Field, CA, Copyright 1997 by the American Institute of Aeronautics and Astronautics Inc.

17 Speed and Route Advisor (SARA) electronic Brochure, Knowledge & Development Center (KDC) Research Agenda, KDC/2007/0102, Version 1.7, URL (05.04.2011): www.kdc-mainport.nl

18 David Learmount, *iFACTS poised to transform ATC*, electronic publication on *Flightglobal Air Transport* 13/03/2007, URL (05.04.2011): http://www.flightglobal.com/articles/2007/03/13/212553/ifacts-poised-to-transform-atc.html

19 Curran, Peter, *Investigating in the future –Technology*, Speech held at the Industry Forum Waypoint June 2008, Airservices Australia, URL (05.04.2011): http://www.airservicesaustralia.com/projectsservices/industryforums/waypoint/2008/docs/day2_speech_peter_curran.pdf

20 Brockhaus, Rudolf, *Flugregelung von Rudolf Brockhaus*, Chapter 11.3 Flugbahnkriterien, Springer Verlag, Published 2001, ISBN-3540418903

21 Operational Requirements Document for EATCHIP Phase III ATM Added Functions, Volume 3 Arrival Manager (CIP Advanced Level), Edition 2.0, 1999

22 Vormer, Frizo Jan-Peter, *Theoretical and Operational Aspects of Optimal Airport Arrival Trajectories* by Frizo Jan-Peter Vormer, University of Delft, PhD Thesis, Chapter Two, Background and Related Research, 2005

23 EPISODE 3 Partners, *SESAR Detailed Operational Description* - *Arrival and Departure* - *High and Medium/Low Density Operations* - *E5*, EPISODE 3/WP2, E3-D2.2-027, January 2008

24 EPISODE 3 Partners, *SESAR Detailed Operational Description* - *Conflict Management in En-Route* - *High & Medium/Low Density Operations* - *E6*, EPISODE 3/WP2, E3-D2.2-028, January 2008

25 Erzberger, Heinz; Paielli, Russell A.; Isaacson, Douglas R.; Eshow, Michelle, M.: *Conflict Detection and Resolution in the Presence of Prediction Error*; presented at 1st USA/Europe Air Traffic R&D Seminar, Saclay, France, June 1997

26 Robinson III, John E.; Isaacson, D. R., *A Concurrent Sequencing and Deconfliction Algorithm for Terminal Area Air Traffic Control*; AIAA Guidance, Navigation, and Control Conference, paper no. AIAA-2000-4473, Denver, CO, August 2000

27 Erzberger, Heinz, *The Automated Airspace Concept*, Paper prepared for the 4th USA/Europe Air Traffic Management R&D Seminar, Santa Fe, New Mexico, December 2001

28 Sofrèavia, *MAESTRO sequencing system for arrival and departure flights*, (2005), URL (12.04.2007): www.sofreavia.com/uk/3_products/2_maestro/maestro.htm

29 Anonymous, *Flugfuehrungssystem COMPAS jetzt im Einsatz*, URL (22.01.2008): http://www.trust-us.ch/chalisti/chaliti_08/004_HWJ4.html

30 SESAR Consortium, *Air Transport Framework* - *The Performance Target*, Milestone Deliverable 2 issue for the SESAR Definition Phase, December 2006

31 Breunig, Tamara, Steve Bradford, Diana Liang, *Standardising Performance Metrics*, 5th USA/Europe Air Traffic Management R&D Seminar, June 2003

32 International Civil Aviation Organization, *Global Air Traffic Management Operational Concept*, First Edition 2005

33 Hornby, Albert Sidney, *Oxford Advanced Learner's Dictionary of Current English*, Oxford University Press, Fifth Edition, 1995, ISBN 0 19 431180 5

34 Deru, M., P. Torcellini, *Performance Metrics Research Project*, National Renewable Energy Laboratory, Technical Report, October 2005

35 Air Traffic Services Performance Focus Group and CNS/ATM Focused Team, *Airline Metric Concepts for Evaluating Air Traffic Services Performance*, February 1999

36 Idris, Husni; Robert Vivona, José-Luis García-Chico and David Wing, *Distributed Traffic Complexity Management by Preserving Trajectory Flexibility*, IEEE 2007

37 Cambridge University Press, *The Cambridge Dictionary of Philosophy*, Edited by Robert Audi, Second Edition, 1999, ISBN 0 521 63722 8 paperback

38 Klosko, George, *The Principles of Fairness and Political Obligation*, Published by Rowman & Littlefield Publishers, Inc., ©2004, ISBN 0-7425-3375-1

39 Diekmann, Kristina A., Zoe I. Barsness, Harris Sondak, *Uncertainty, Fairness Perceptions, and Job Satisfaction: A Field Study*, Published in Social Justice Research, Vol.17, No 3, September 2004

40 Fehr, Ernst, Klaus M. Schmidt, *A Theory of Fairness, Competition and Cooperation*, Published in: 'The Quarterly Journal of Economics', pp. 817-868, August 1999

41 Brams, S. J., A.D. Taylor, *Fair Division: from Cake-Cutting to Dispute Resolution*, Cambridge University Press, 1996.

42 Brams S. J., A.D. Taylor, *The Win-Win Solution: Guaranteeing Fair Shares to Everybody*, Norton & Co. 1999.

43 Kostiuk, Peter F., Eric M. Gaier, Dou Long, *The Economic Impact of Air Traffic Congestion*, Air Traffic Control Quarterly v.7., n. 2, pp. 123-145, 1999

44 Bertsimas, Dimitris, Vivek F. Farias, Nikolaos Trichakis, *Fairness in the Air Traffic Flow Management Problem*, Massachusetts Institute of Technology, 2009

45 Jonker, Geert, John-Jules Meyer, Frank Dignum, *Efficiency and Fairness in Air Traffic Control*, Utrecht University, Research supported by the Technology Foundation STW, applied science division of NOW and the technology programme of the Ministry of Economic Affairs, Project DIT5780

46 Carr, Gregory C, Heinz Erzberger, Frank Neuman, *Airline Arrival Prioritization In Sequencing & Scheduling*, 2nd USA/EUROPE Air Traffic Management R&D Seminar Orlando, USA, December 1998

47 International Civil Aviation Organization, *Europe: a "Total System" Approach to Transparency and Predictability of Air Transport Infrastructure Costs*, Conference on the Economics of Airports and Air Navigation Services, Working Paper, Montréal, September 2008

48 Waslander, Steve L., Robin L. Raffard, Claire J. Tomlin, *Towards Efficient and Equitable Distributed Air Traffic Flow Control*, Proceedings of the 2006 American Control Conference, Minneapolis, Minnesota, USA, June 14-16, 2006

49 Pourtaklo, Nasim Vakili, Michael Ball, *Equitable Allocation of Enroute Airspace Resources*, Eighth USA/Europe Air Traffic Management Research and Development Seminar, ATM2009

50 Soomer, M.J., G.M. Koole, *Fairness in the Aircraft Landing Problem*, VU University Amsterdam, published in The AGIFORS Airline Operations 2008 Amsterdam, Holland, 2008

51 Bertsimas, Dimitris, Shubham Gupta, *Fairness in Air Traffic Flow Management*, Massachusetts Institute of Technology, October 29, 2009

52 Blomdahl, Karl Sundequist, Pierre Flener, and Justin Pearson, *Contingency Plans for Air Traffic Management*, CP'10 Proceedings of the 16th International Conference on

Principles and Practice of Constraint Programming, pp 643-657, Springer-Verlag Berlin, Heidelberg ©2010

53 Bertsimas, Dimitris, Vivek F. Farias, Nikolaos Trichakis, *The Price of Fairness*, OPERATIONS RESEARCH, Published online in *Articles in Advance*, DOI: 10.1287/opre.1100.0865, February 8, 2011

54 Civil Air Navigation Services Organisation (CANSO) URL (09.06.2009): www.canso.org

55 International Air Transportation Association (IATA) URL (09.06.2009): www.iata.org

56 International Civil Aviation Organisation (ICAO) URL (09.06.2009): www.icao.int

57 International Civil Aviation Organisation, *ICAO'S Policies on Charges for Airports and Air Navigation Services*, Seventh Edition, 2004

58 Wierzbicka, Anna, *English: Meaning and Culture*, Chapter 5 *Being Fair Another Key Anglo Value and Its Cultural Underpinnings*, Copyright 2006 by Oxford University Press, Inc., ISBN 0-19-517474-7

59 Velasquez, Manuel, Claire Andre, Thomas Shanks, and Michael J. Meyer, *Thinking Ethically: A Framework for Moral Decision Making*, Markkula Center for Applied Ethics, Santa Clara University, Original Publication in *Issues in Ethics* V7 N1 (Winter 1996), URL (06.04.2011): http://www.scu.edu/ethics/practicing/decision/thinking.html

60 Sandel, Michael J., *Justice What's the Right Thing to Do?*, Chapter 6 *The Case for Equality/ John Rawls*, Published by the Penguin Group, Copyright Michael J. Sandel 2009, ISBN 978-1-846-14213-0

61 On-Line Dictionary and Thesaurus Merriam Webster URL (09.06.2009): www.merriam-webster.com

62 Stanford Encyclopaedia of Philosophy, URL (10.06.2009): http://plato.stanford.edu/

63 Rawls, John, *Justice as Fairness: A Restatement*, Section 6 *The Idea of the Original Position*, Copyright by Harvard University Press, ISBN 0-474-0511-2

64 Benjamin, Martin, *Book Review: Justice as Fairness: A Restatement from John Rawls*, Essays in Philosophy, A Biannual Journal, Vol. 4 N.1 January 2003, Copyright 2003 Humboldt State University

65 Rawls, John, *A Theory of Justice*, Revised Edition, Section 24 "The veil of ignorance", Sixth Printing 2003, Copyright by Harvard University Press, ISBN 0-674-00078-1

66 Schurter, Karl, Bart J. Wilson, *Justice and Fairness in the Dictator Game*, University of Virgina and Interdisciplinary Center of Economics Science George Mason University respectively, July 2007

67 Wilson, Bart J., *Fair's Fair*, Published January 25 2009, at *the Atlantic* electronic edition, URL (15.06.2009): http://business.theatlantic.com/2009/01/fairs_fair.php

68 Young, H. Peyton, *Equity: In Theory and Practice* , Preface and Chapter 1 *Overview*, A Russell Sage Foundation Book, Princeton University Press, Princeton, New Jersey, 1994

69 Duclos, Jean-Yves, *Horizontal and Vertical Equity*, The New Palgrave Dictionary of Economics, 2nd Edition, 2008

70 UNICEF, *Gender Equality*, URL (26.06.2010): www.unicef.org/gender/

71 Dareing, Stephen W., Debra Hoitmot, *Traffic Management and Airline Operations*, Proceedings of the American Control Conference, Anchorage, AK, May, 2008

72 Smyth, Martin, Brian Pearce, *IATA Economic Briefing Nº5: Airline Cost Performance*, IATA, July 2006

73 Australian Strategic Air Traffic Management Group (ASTRA), *Concept of Operations for Flex Tracks*, Version 1.0, December, 2003

74 NAV Portugal, *Implementation of free route airspace within Lisbon FIR/UIR (FRAL) above FL245*, AIC 002-2009, Published January, 2009

75 International Civil Aviation Organization, ICAO, *Procedures for Air Navigation Services*, DOC 4444 ATM/501, Fourteenth Edition, 2001

76 Central Flow Management Unit (CFMU), EUROCONTROL, URL (18.04.2011): http://www.cfmu.eurocontrol.int/cfmu/public/subsite_homepage/homepage.html

77 Internet Weather Source, Meteorological Aeronautical Report, URL (18.04.2011): http://weather.noaa.gov/weather/metar.shtml

78 System Wide Information Management, Federal Aviation Administration, URL (18.04.2011): http://www.faa.gov/about/office_org/headquarters_offices/ato/service_units/techops/atc_comms_services/swim/

79 Manzi, Patrick, Lars Lindberg, Hanyo Vera Anders, Per Ahl, Bengt Nilsson, Francisco A. Navarro, *COURAGE UPTs in Unconstrained ATM Environment*, submitted to EUROCONTROL, COURAGE Deliverable D1.1, 13.06.2005

80 Airbus Customer Services, *Flight Operations Support & Line Assistance, Getting to Grips with the Cost Index*, STL 945.2369/96, Issue II- May 1998

81 Roberson, Bill, *Fuel Conservation Strategies*, Boeing Commercial Airlines, Aeromagazine, Aero Quarterly QTR_02/07

82 Mankiw, Gregory, *Principles of Economics*, Part VII, Chapter 21 *The Theory of Consumer Choice*, *Preferences: What the Consumer Wants*, pp. 459-464, Fifth Edition, 2009 South-Western Cengage Learning, ISBN-10: 0-324-58997-2

83 EUROCONTROL, *Performance Review Report, An Assessment of Air Traffic Management in Europe during the Calendar Year 2009*, PRR 2009, EUROCONTROL, May 2010

84 Cook, A., G. Tanner, V. Williams and G. Meise, *Dynamic Cost Indexing*, 6th EUROCONTROL Innovative Research Workshops & Exhibition, December 2007

85 Von Stengel, Bernhard, Theodore L Turocy, *Game Theory*, Encyclopaedia of Information Systems, Elsevier Science, Vol.2 pp. 403-420, ISBN 9780122272400

86 Yildiz, Muhamet, *14.12 Game Theory Lecture Notes Solution Concepts - Dominance and Rationalizability*, MIT Department of Economics, Game Theory Course as taught in Spring 2010

87 Von Neumann, John, Oskar Morgenstern, *Theory of Games and Economic Behavior*, Sixtieth Anniversary Edition, 2004 Princeton University Press, ISBN 0-691-11993-7

88 Hansson, Sven Ove, *Decision Theory- A brief Introduction*, Department of Philosophy and History of Technology, Royal Institute of Technology, KTH, Stockholm, first published August 1994, minor revision August 2005

89 von Neuman, John, *Zur Theorie der Gesellschaftsspiele*, Math. Annalen, Vol. 100, pp. 295-320, 1928

90 von Neumann, John, *Über ein ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwer'schen Fixpunktsatzes*, Ergebnisse eines Math. Kolloquiums, Vol. 8, pp.73-83, 1937

91 Spanish AIP online, Aeronautical Information Publication Updated 24-february-2011, direct link for Gran Canaria's STARs URL (14.03.2011): http://www.aena.es/csee/ccurl/428/372/LE_AD_2_GCLP_STAR_1_en.pdf

92 Navarro, Francisco, Miguel Vilaplana, Carlos Querejeta, Eduardo Gallo, Javier López, *Predicting Aircraft Trajectory*, WPO Patent Application WO/2009/042405, The Boeing Company

93 Base of Aircraft Data, EUROCONTROL (04.04.2011) URL: http://www.eurocontrol.int/eec/public/standard_page/proj_BADA.html

94 Nuic, Angela, Chantal Poinsot, Mihai-George Iagaru, Eduardo Gallo, Francisco Navarro, Carlos Querejeta, *Advanced Aircraft Performance Modeling for ATM: Enhancements to the BADA Model*, Presented at 24th Digital Avionics System Conference, Washington D.C., November 2005

95 International Air Transport Association (IATA) Jet Fuel Monitor, URL (09.08.10): http://www.iata.org/whatwedo/economics/fuel_monitor/Pages/index.aspx

96 OANDA, The Currency Site for Foreign Exchange Services and Trading, URL (09.08.10): www.oanda.com

97 EUROCONTROL Glossary of terms (20.03.2011) URL: http://www.eurocontrol.int/fasti/public/site_preferences/display_glossary_list.html

98 Asensio, J.M., M. Martínez, M.A. Pérez and J.L. Besada, *Conflict Detection and Resolution Algorithms*, Informe General de Proyecto, ATLANTIDA, CENIT Programmes (CEN20072008) sponsored by the Centre for Technical and Industrial Development (CDTI), Spanish Ministry of Science and Innovation, to be published in December 2011

99 Zúñiga C.A., Miquel Angel Piera, Sergio Ruiz and Isabel del Pozo de Poza, *A TMA 4DT CD/CR causal model based in path shortening/path stretching techniques*, Best Paper Award at the *4th International Conference on Research in Air Transportation (ICRAT 2010),* June 01-04, Budapest, Hungary, 2010

100 Valenzuela, Alfonso, Damian Rivas, Rafael Vázquez, Isabel del Pozo and Miguel Vilaplana, *Conflict Resolution with Time Constraints in Trajectory Based Arrival Management*, submitted to the 11th AIAA Aviation Technology, Integration, and Operations (*ATIO*)

101 Van Gool, Mick, Hemut Schröter, *PHARE: Lessons Learned from the Organisation of Multi-National R&D on Air Traffic Management*, 2nd USA/Europe Air Traffic Management R&D Seminar, Orlando, December 1998

102 Schick, Fred, *Methods and Measurements for the Evaluation of ATM Tools in Real-Time Simulations and Field Tests*, 2nd USA/Europe Air Traffic Management R&D Seminar, Orlando, December 1998

103 Geissler, Carmen, Oliver Hassa, *4D-Planer – Nutzung des Forschungssimulators für Test und Prototyping*, Informationen aus dem Bereich Forschung & Entwicklung, DFS Deutsche Flugsicherung, Ausgabe 2/02, Seite 8-14, 2002

104 4 Dimensional Cooperative Arrival Manager (4D-CARMA), Brochure Published by AT-One the ATM Research Alliance, 2007

105 Garcia, Jean-Louis, *MAESTRO Concept*, Presentation prepared for MAESTRO Training in Thailand, Sofreavia, 2006

106 Erzberger, Heinz, *Design Principles and Algorithms for Automated Air Traffic Management*, NASA Ames Research Center, Paper presented at the Mission Systems Panel of AGARD and the Consultant and Exchange Program of AGARD, LS 200, 1995

107 Neuman, Frank, Heinz Erzberger, *Analysis of Delay Reducing and Fuel Saving Sequencing and Spacing Algorithms for Arrival Traffic*, NASA Ames Research Center California, NASA Technical Memorandum 103880, October 1991

108 Halverson, Christine A., Kelly Harwood, Thomas J. Davis, Christopher R. Briton, *A System Approach to Design: Developing a Usable Automation Tool for Air Traffic Control*, IEEE 0-7803-0820-4/92, pp 167- 173, 1992,

109 EPISODE 3 Partners, *SESAR Detailed Operational Description – General PurposeDOD -G*, EPISODE 3/WP2, E3-D2.2-020, January 2008