Essays on Real Business Cycle Modeling and the Public Sector

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Abstract

This thesis is composed of three core chapters on modern dynamic macroeconomics, which study different aspects of the public sector labor market in a large EU economy with significant public employment share and a non-trivial public sector wage premium over the private sector labor compensation. The study in this dissertation adds to earlier research by incorporating endogenous government hours and wages in the model framework and argues that the presence of a sizable public sector labor market in European economies generates significant interaction with the private sector labor and capital markets. In addition, the presence of interest groups (labor unions, government bureaucracy), as well as other labor market frictions in the public sector, is shown to be an important element of the analysis when discussing fiscal policy reforms.

Motivated by the highly-unionized public sectors, the high public shares in total employment, and the public sector wage premia observed in most post-WWII European economies, Chapter 1 examines the role of public sector unions in a general equilibrium framework. A strong union presence in a large non-market sector is shown to be relevant for both business cycle fluctuations and for the welfare effect of fiscal regime changes. To this end, an otherwise standard real-business-cycle (RBC) model is augmented with a public sector union optimization problem. The resulting theoretical setup generates cyclical behavior in government hours and wages that is consistent with data behavior in an economy with a highly-unionized public sector, namely Germany during the period 1970-2007. The main findings of Chapter 1 are: (i) the model with a public sector union performs reasonably well *vis-a-vis* data; (ii) overall, the public sector union model is a significant improvement over a similar model with exogenous public sector employment; (iii) endogenously-determined public wage and hours add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. Additionally, the union model requires greater changes

in tax rates to achieve a pre-specified increase in tax revenue compared to an equivalent model with exogenous public sector hours. Thus, endogenous public sector hours and wages in the setup are shown to be quantitatively important for public policy evaluation. Ignoring the positive co-movement between public and private hours and wages leads to a significant underestimation of the welfare effect of fiscal regime changes.

Chapter 2 characterizes optimal fiscal policy and evaluates it relative to the exogenous (observed) one. Motivated by the high public employment, and the public wage premia observed in the major European economies, a Real-Business-Cycle model, calibrated to German data (1970-2007), is set up with a richer government spending side, and an endogenous private-public sector labor choice. To illustrate the effects of fiscal policy on sectoral allocation of hours, public wage rate determination and the provision of labor-intensive public services, two regimes are compared and contrasted to one another - exogenous *vs.* optimal (Ramsey) policy case. The main findings from the computational experiments performed in Chapter 2 are: (i) The optimal steady-state capital tax rate is zero, as it is the most distortionary tax to use; (ii) A higher labor tax rate is needed in the Ramsey case to compensate for the loss in capital tax revenue; (iii) Under the optimal policy regime, public sector employment is lower, but government employees receive higher wages; (iv) The benevolent Ramsey planner provides the optimal amount of the public good, and substitutes labor for capital in the input mix for public services and private output; (v) The government wage bill is smaller, while public investment is three times higher than in the exogenous policy case.

Lastly, the thesis tries to delve into the hierarchical structure of public employment service and addresses the problem of rent-seeking in the public sector by government bureaucrats. Chapter 3 studies the wasteful effect of bureaucracy on the economy by addressing the link between rent-seeking behavior of government bureaucrats and the public sector wage bill, which is taken to represent the rent component. In particular, public officials are modeled as individuals competing for a larger share of those public funds. The theoretical model used

is calibrated to German data for the period 1970-2007. The analysis then extends to the other major EU economies as well. To illustrate the effects of fiscal policy on rent-seeking, the exogenous and the optimal (Ramsey) policy cases are compared and contrasted to one another. The main findings of Chapter 3 are: (i) Due to the existence of a significant public sector wage premium and the large public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses measured in terms of aggregate output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime, steady-state rent-seeking is smaller relative to the exogenous policy case. The benevolent government invests more in public capital, sets a higher public wage premium, but chooses much lower public employment, thus achieving a decrease in rent-seeking.

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Author's Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signature

Printed Name ALEKSANDAR VASILEV

Date May 22, 2013

"It is a capital mistake to theorize in advance of the facts."

Arthur Conan Doyle, recording the words of S. Holmes

"Our task as I see it ... is to write a FORTRAN program that will accept specific economic policy rules as 'input' and will generate as 'output' statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies."

1995 Nobel Prize Winner in Economics Robert Lucas, Jr.

"...[T]hese artificial economies are useful laboratories for studying the business cycle and for studying economic policy. The goal of this research is to better understand the behavior of actual economies by studying the equilibria of these synthetic economies. The realization of this goal requires a careful marriage of economic theory and empirical observation."

Edward C. Prescott

"...[G]iven that all models are false, how true are they?"

Patrick Minford

"All models are wrong, but some are useful."

G. E. Box

"That's all well and good in practice, but how does it work in theory?"

Shmuel Weinberger

"No assumptions about economic behavior are absolutely right and no theoretical conclusions are valid for all times and places, but would anyone seriously deny that in the matter of techniques and analytical constructs there has been progress in economics?"

Mark Blaug

"... Meticulous attention should be paid to the special relationships and obligations of public servants to the public itself and to the government. All Government employees should realize that the process of collective bargaining, as usually understood, cannot be transplanted into the public service. It has its distinct and insurmountable limitations ... The very nature and purposes of Government make it impossible for ... officials ... to bind the employer ... The employer is the whole people, who speak by means of laws enacted by their representatives ...

"Particularly, I want to emphasize my conviction that militant tactics have no place in the functions of any organization of government employees. Upon employees in the federal service rests the obligation to serve the whole people ... This obligation is paramount ... A strike of public employees manifests nothing less than an intent ... to prevent or obstruct ... Government ... Such action, looking toward the paralysis of Government ... is unthinkable and intolerable."

F.D.Roosevelt

Chapter 0

Thesis Introduction and Motivation

This thesis is composed of three core chapters on modern dynamic macroeconomics, which study different aspects of the public sector labor market in a large economy with significant public employment share and a non-trivial public sector wage premium over the private sector labor compensation. In addition, the behavior of the labor input is very important for aggregate economic fluctuations, as Cooley and Prescott (1995) and Kydland (1995) have pointed out. Despite this, real business cycle (RBC) theory so far has been predominantly focused on the private sector and has largely ignored the dynamic general-equilibrium effects of public sector labor choice. This thesis seeks to fill this gap by studying government employment- and wage determination, and argues that the presence of a sizeable public sector labor market in European economies generates significant interaction with the private sector labor and capital markets. If public sector labor choice is ignored, then important effects on cyclical fluctuations, as well as on welfare, due to fiscal regime changes, will be missed. This work also characterizes optimal government spending policy across consumption and investment categories. Lastly, this thesis presents a first attempt to examine the relevance of bureaucracy for the macroeconomy by examining the cost of rent-seeking by government bureaucrats in a general equilibrium setup, and by comparing it to the optimal time spent on wasteful activities.

The study uses the RBC framework to study the cyclical properties of European public

sector labor markets and to provide normative statements about fiscal policy, thus suggesting important reforms in the public sector. The RBC paradigm has established itself in the modern macroeconomic literature as a useful environment for studying aggregate fluctuations in developed economies, and an indispensable tool in the toolkit of the macroeconomist conducting quantitative research. In addition, the baseline RBC model performance improves significantly when extended to capture specific features of the economy of interest. Some examples include: distortionary taxation (McGrattan 1994), government spending (Christiano and Eichenbaum 1992), and productive public investment (Baxter and King 1993). The main focus of the computational experiments performed in those papers, however, was on the effect of government purchases, public investment and taxes. In addition, most of the extensions to the benchmark RBC model, which allow for public employment model public sector labor market variables predominantly as exogenous, *e.g.* Finn (1998), Cavallo (2005), and Linnemann (2009). Those models feature a representative household and two sectors - public and private - where labor hours can be supplied.

However, a shortcoming in all these models is that wage rates in the economy are identical, with public hours approximated by a stationary stochastic process. Therefore, these models, despite being an improvement over earlier vintages of RBC models, produce a good match *vis-a-vis* data along the public sector labor market dimension, *e.g.* public hours volatility, mostly by construction. The absence of an internal propagation mechanism for public employment is another limitation in this class of models, particularly when the research focus falls on the interactions between the two labor markets and their relation to business cycle fluctuations and optimal fiscal policy issues. With the exception of Ardagna (2007) and Fernandez-de-Cordoba *et al.* (2009, 2012), no previous studies had pursued a systematic study of government spending behavior in RBC models with endogenously-determined public employment and wages, as well as labor-intensive government services.[1] Nonetheless, the

[1] A discussion on how the work in this thesis differs in important ways from Ardagna (2007) and Fernandez-de-Cordoba *et al.* (2009, 2012) is provided on pp. 4 and 13.

greater part of government consumption in a country's national accounts consists of wage consumption, and the smaller part corresponds to government purchases of commodities. (OECD 2012).

The models presented in this thesis build on a very small number of existing Real-Business-Cycle (RBC) models with public employment, *e.g.* Finn (1998), Cavallo (2005), and Linnemann (2009). The extensions discussed in each of the three core chapters are consistent with an important feature in data, namely the high unionization rate in the public sector, the inherent difference between working in the public and private sectors, the bureaucracy in the government sector and the (anecdotal) evidence of rent-seeking by public officials in government administration. After all, central governments in EU countries are the biggest employers at a national level, with a high public share in total employment. In addition, government sectors in the major European Union (EU) member states are highly unionized, coordinated and centralized, and significantly more so than the respective private sectors. Therefore, the presence of well-organized interest groups operating in the public sector, as well as the existence of other labor market frictions, often imposes a significant constraint on the use of fiscal policy in Europe as a tool for economic stabilization, and thus accentuates cyclical fluctuations.

In addition, unlike other studies (Gomes 2012), this thesis also analyses the optimal fiscal policy setup in a framework with a competitive private labor market and endogenously-determined public sector hours and wages, both in the steady-state and in terms of responses in the face of unexpected technology shocks. Lastly, the thesis tries to delve into the internal competition within the public employment service and tries to address the problem of rent-seeking in the public sector by government bureaucrats. The focus is on government spending on wages and its potential to produce rent-seeking behavior. Importantly, this bloating process in the administration and the subsequent expansion of the public sector wage bill should raise concerns in policy makers, since larger governments tend to lose effi-

ciency progressively with size and when the cost of labor largely exceeds its marginal revenue product. After all, labor productivity in the public sector is difficult to measure as a quantity corresponding to government production is hard to define.

Research Questions and Contributions

The thesis opens the discussion by asking whether the benchmark Real-Business-Cycle (RBC) model, augmented with a public sector union, can match labor market fluctuations observed in a representative European economy (Germany) better than earlier/alternative models, and whether a strong union presence in the public sector is relevant for the welfare effect of fiscal regime changes. Answering these questions requires a careful calibration and simulation of an RBC model augmented with a public sector union optimization problem. This is pursued in Chapter 1 of the thesis. The setup in that chapter combines two elements used in earlier research to address new aspects of the economy and produce new results: it adopts the public sector union maximization problem from Fernandez-de-Cordoba *et al.* (2009, 2012) and incorporates it into a RBC model with richer tax structure and fiscal policy instruments, *i.e.* Finn (1998). Thus, the individual quantitative effect of union optimization can be assessed relative to Finn's (1998) setup with exogenous public hours and a single, competitive wage rate.

The study in Chapter 1 also takes a much wider scope than does Finn (1998) and at the same time is complementary to Fernandez-de-Cordoba *et al.* (2009, 2012). It includes a complete evaluation of an RBC model with an optimizing public sector union, following the widely-accepted methodology in the RBC literature. The model used in Chapter 1 successfully matches the cyclical fluctuations in the public and private sector labor markets. Additionally, it also compares well to the empirical autocorrelation and cross-correlation functions generated from an unrestricted Vector Auto Regression (VAR) estimation. While all these features are important for understanding the aggregate fluctuations, these aspects were not

addressed in the earlier studies on the dynamic general equilibrium effects of public sector unions. Lastly, endogenously-determined public wages and hours will be shown to add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. The union model requires larger changes in tax rates to achieve a pre-specified increase in tax revenue, compared to Finn's (1998) model with exogenous public sector hours. Thus, endogenous public hours are quantitatively important for fiscal policy evaluation. Ignoring the interaction between hours and wages leads to a significant underestimation of the welfare effect of tax regime changes.

However, Chapter 1 leaves outside its research scope interesting issues such as optimal tax-rate determination, public sector wage- and employment-setting possibilities. Thus, it does not adequately address either the household's choice at the margin between entering the public or the private sector, or the optimal policy framework, in which a benevolent government chooses all spending components (in addition to setting the two tax rates on labor and capital, respectively). All these structural/labor-market aspects of public finance and public policy represent a gap in the literature, which the research in Chapter 2 aims to fill.

An important limitation of the analysis in Chapter 1, and the thesis as a whole, is that in order to focus on the bargaining between the public sector union and the government, it abstracts away from issues such as public debt. If debt is included as a residually-determined fiscal instrument, and government wage and employment determined through (Nash-)bargaining, that would complicate the problem substantially. Depending on the initial condition on debt-to-GDP ratio, model dynamics would change in a substantial way. Initial conditions for debt would then matter, as they would affect the process of debt repayment, and/or possible default decisions. In addition, since there is no zero-profit constraint in the government sector, there is *a priori* no counteracting force in the model to oppose the upward pressure on wages and employment by public sector unions. Following the model logic, if debt is then included as a residually-determined fiscal instrument, new debt will

be always issued to satisfy the wage and employment demands of unions, thus leading the economy to collapse.[2] Thus, the presence debt in the model leads to a discussion of sovereign default issues.[3] These all interesting venues of research would be left for future work, as the focus of this thesis are models that feature a unique steady state under a realistic model calibration.

Chapter 2 of the thesis shifts gears to address new effects of labor markets for government spending and fiscal policy in general. In particular, it discusses the optimal size and composition of the public wage bill, the efficient level of government investment, and the optimal provision of a labor-intensive public good. To this end, optimal fiscal policy is characterized and evaluated against the exogenous (observed) fiscal stance. Addressing these research questions requires a general-equilibrium model with a richer and more realistic government spending side, and an endogenous private-public labor choice in particular. However, the presence of a public employment decision margin, as a separate labor market choice made by the household, has not been sufficiently investigated in the literature. Hence, an otherwise standard dynamic stochastic general equilibrium (DSGE) model will be augmented with a convex cost of working in the government sector to reflect the fact that wages in the private and the public sectors were determined within different institutional settings (thus linking the analysis with the study undertaken in Chapter 1). Chapter 2 argues that if public sector labor choice is ignored, then important effects on allocations and welfare, driven by government wage-setting and household's decisions on hours, will be missed.

[2]Forni and Giordano (2003) show that for the case of Greece, public sector wage bill and debt series are positively co-moving over time.

[3]Alternatively, the model may need to incorporate some credible fiscal rules on the budget deficit and public debt, so that such constraint would become binding at a certain level. However, the introduction of such rules, despite necessary, might not be sufficient to guarantee a stable solution. Simulations might need to be performed to endogenously-determine the debt threshold corresponding to certain initial conditions, after which the model dynamics becomes unstable, or the model could feature multiple equilibria, and/or indeterminacies. However, Evans and Honkapohja (2003) argue that casting a government policy rule that leads to indeterminacy is "not a good idea."

The second novelty in the framework presented in Chapter 2, which adds value to earlier studies, is the more interesting and meaningful role attributed to government employees. In particular, the study constructs in greater detail the mechanism of public good provision. The setup modeled the government as an employer, needing labor hours to provide public goods. In contrast to Cavallo (2005) and Linnemann (2009), labor is combined with government capital (instead of government purchases) to produce valuable government services. Therefore, government investment is a productive government spending category in the setup, and public sector wage consumption is not entirely wasteful. Importantly, when hiring workers, the government is able to set the public sector wage rate, an assumption which is consistent with data, *e.g.* Perez and Schucknecht (2003). Overall, the interaction between the two types of labor and capital stocks will be the driving force behind some of the new results obtained in Chapter 2.

Chapter 2 then proceeds to study public employment, wage rate and government investment determination, as well as the level of public services provision, are optimally chosen by a benevolent government. In particular, under the optimal (Ramsey) policy regime, the benevolent government will choose the socially optimal levels of both private allocations and the public good. The main findings from the computational experiments performed in Chapter 2 are consistent with earlier findings in the literature. First, as in Judd (1985), Chamley (1986) and Zhu (1992), the optimal steady-state capital tax rate is zero, as this is the most distortionary tax to use. Next, a higher labor tax rate is needed to compensate for the loss in capital tax revenue. The new result regarding optimal government spending is that under the optimal policy regime, public sector employment is lower. As a result, government employees are more valuable, and receive higher wages. Under the optimal policy regime, the benevolent Ramsey planner substitutes labor for capital in the production of both public services and private output. Furthermore, the government wage bill is smaller,

while public investment is three times higher than in the exogenous policy case.[4]

Given the overall reasonable performance of the model in Chapter 2, the effect of the organizational structure of the public sector labor market for the macroeconomy is investigated in a separate chapter, Chapter 3 in this thesis. Von Mises (1944), Parkinson (1957) and Tullock (1974) are one of the first to suggest that bureaucracy itself has been one of the most important factors affecting economic activity, mainly through the development and implementations of different legislative procedures, rules and regulations. In particular, civil servants are usually insulated from market forces in both the input and output markets: many government positions do not have a close equivalent in the private sector, and there is no direct way to measure performance. A closer analysis of bureaucrats' behavior, and their effect on aggregate fluctuations in European economies would be a logical extension to the work in Chapter 2, and is discussed at length in Chapter 3.

Chapter 3 aims to contribute to the body of both macroeconomic and political economy literature by focusing on the rent-seeking behavior of bureaucrats in the public sector within a general equilibrium setup. By incorporating an auctioning mechanism from game theory into the model framework utilized in Chapter 2, the augmented setup in Chapter 3 attempts to quantify the cost of rent-seeking that is produced as a result of the non-productive behavior of government bureaucrats competing for a larger share of a contestable transfer. In particular, the amount of the rent in Chapter 3 is assumed to be the wage bill. After all, public administration consists of a system of bureaus, and this type of organization in the government sector is shown to produce significant losses for the economy through the rent-seeking mechanism. The research argues that each bureau head could be perceived as a

[4]Regarding the limitations of the analysis resulting from the absence of debt in the Ramsey framework discussed in Ch. 2 (and later in Ch.3), Stockman (2001), Aiyagari, Marcet, Sargent and Seppala (2002), and Chari, Christiano and Kehoe (1994) show that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect (numerically) on the optimal policies in the full commitment case. Of course, the exact allocations will differ across cases, but the difference in magnitudes is tiny.

rational income-maximizing individual, who wants to attract more public funds for his/her agency. The contribution of this chapter is that very few economists, with the notable exception of Buchanan and Tullock (1962), Tullock (1965), and Niskanen (1971), have focused on the presence of a large bureaucracy and provided evidence of its importance for the economy, but only in a partial-equilibrium context. In addition, Chapter 3 in this thesis aims to fill this gap in the literature by motivating and presenting a more intricate channel used by bureaucrats to rent-seek and lobby for government funds.

To illustrate the processes taking place within public administration, the model setup in Chapter 3 describes a symmetric non-cooperative game that is played among government bureaucrats themselves to increase individual income at the expense of the other public officials earnings. The interaction between agents in the public sector generated strategic complementarities, as individual rent-seeking is positively related to opponents' choice of rent-seeking. Another novelty in Chapter 3 is that rent-seeking occurs in a non-competitive labor market, the public sector one, where the wage rate is set above private sector pay. This stimulates the entry of labor to the sector, and as a result, public employment eventually becomes too high. In particular, the high public wage and employment both stimulate rent-seeking by generating a positive benefit of engaging in wasteful activities.

In the model presented in Chapter 3, the positive amount of time dedicated to opportunistic activities in the steady-state is an efficient outcome from an individual worker's point of view, as all agents are fully rational and maximize their utility levels. Thus, in equilibrium, individual bureaucratic rent-seeking efforts adjust to the point where the value of additional resources spent per bureaucrat equals the benefit that accrues to that individual. In turn, a higher wage bill requires higher tax rates to finance government spending. In the private sector, high taxes reduce incentives to supply labor and accumulate capital, and decrease consumption and output. Thus rent-seeking has a negative impact on the economy, and Chapter 3 attempts to quantify the loss for the economy in a general-equilibrium frame-

work. In addition, the mechanism described in this chapter finds a strong empirical support in the face of different indices of institutional quality.

Finally, Chapter 3 characterizes the optimal fiscal policy regime, where steady-state rent-seeking is significantly smaller relative to the exogenous policy case. In addition to the zero capital tax rate, and the higher labor tax rate, the benevolent government planner produces the efficient level of the public good by substituting labor for capital in the production of government services, and relocating hours to the private sector. The government invests more in public capital, chooses a higher public wage premium and sets much lower public employment, thus achieving an overall decrease in the level of rent-seeking.

Chapter 1

Cyclical and welfare effects of the presence of strong public sector unions in a Real-Business-Cycle model

1.1 Introduction

The behavior of the labor input is very important for aggregate economic fluctuations, as Cooley and Prescott (1995) and Kydland (1995) have pointed out. In particular, changes in hours account for two-thirds of the movement in US output per person over the business cycle. Despite this, real business cycle (RBC) theory has been predominantly focused on the private sector and largely ignored the dynamic general-equilibrium effects of public sector labor choice. This chapter adds to earlier research by distinguishing between the two types of hours and argues that the presence of the public sector labor market in European economies generates significant interaction with the private sector labor and capital markets. If public sector labor choice is ignored, then important effects on cyclical fluctuations, as well as on welfare, due to fiscal regime changes, will be missed.

Furthermore, several stylized facts suggest that this labor market is driven by non-competitive arrangements: As reported in table 1.1 below, the public sectors in the major European Union (EU) member states are highly unionized, and significantly more so than the respective private sectors. Although the unionization rates for the EU countries in each sector were calculated and documented in Visser (2003) for only one year, the wide gap in union density indicates that the two labor markets operate in different settings. High unionization rates alone do not necessarily translate into strong unions, but the significance of unions in Europe can be inferred from the generally high coordination, centralization and in particular, the extensive coverage rate. Therefore, collective bargaining agreements are often used to set public wage rates and employment levels in European economies.

Table 1.1: Labor market facts 1970-2007

Country	Private sector union density (%)	Public sector union density (%)	Coverage rate (%) (2000)	Average publ./priv. compensation	Average publ./priv. employment
Euro Area (2001)	26	N/A	78	1.22	0.22
France (1993)	4	25	95	1.00	0.32
Germany (1997)	22	56	73	1.20	0.17
Italy (1997)	36	43	82	1.30	0.26
Spain (1997)	15	32	80	1.60	0.16
UK (2003)	18	59	36	1.08	0.27
US (2010)	7	35	15	1.08	0.16

Sources: BLS (2011), OECD (2011), Visser (2003)

Central governments in EU countries are the biggest employers at a national level, with a high public share in total employment, as documented in table 1.1, for the largest EU economies. The high share of public employees in total employment could in itself constitute a source of union power, and could explain the positive public sector wage premia over the

private wages, which are observed in most post-WWII European economies over the period 1970-2007. The Wage Dynamics Network's (WDN) 2010 Final Report[1] also emphasizes that wage bargaining institutions are an important determinant of the wage dynamics and wage structure in the EU countries, and the major reason for the existence of the public wage premia.[2]

Additionally, Forni *et al.* (2003) and Gomes (2010) reported that the compensation of public employees in OECD countries takes 60% of total government expenditure. Furthermore, Lane (2003) shows that the public wage bill in OECD countries is pro-cyclical, as opposed to government purchases, which are acyclical. Further empirical work from Lamo, Perez and Schuknecht (2007, 2008) concludes that pro-cyclical discretionary fiscal policy can have important effects on the economy through the unions. In particular, public sector unions act as organized groups that constantly press for an expansion in the government wage bill. Therefore, the presence of interest groups in the public sector imposes a significant constraint on the use of fiscal policy in Europe as a tool for economic stabilization, and thus accentuates cyclical fluctuations.

This chapter uses the RBC framework to study the cyclical properties of European public sector labor markets. The benchmark RBC model has established itself as a useful environment for studying aggregate fluctuations in developed economies. In addition, the baseline RBC model performance improves significantly when extended to capture specific features of the economy of interest. Some examples include: distortionary taxation (McGrattan 1994), government spending (Christiano and Eichenbaum 1992), and productive public investment

[1]"WDN is a research network consisting of economists from the European Central Bank (ECB) and the national central banks (NCBs) of the EU countries, which aims at studying in depth the features and sources of wage and labor cost dynamics and their implications for monetary policy in the euro area."(ECB 2011)

[2]Other reasons for the existence of a public sector wage premia, as documented in Ehrenberg and Schwartz (1986) can be due to skill and experience differences: on average, public employees are older and have higher qualification. In addition, females and employees belonging to a minority group receive higher labor compensation compared to the remuneration package for similar duties in the private sector.

(Baxter and King 1993). Most of the extensions to the benchmark RBC model, which allow for public employment, however, model public sector labor market variables predominantly as being exogenously-determined, *e.g.* Finn (1998), Cavallo (2005), and Linnemann (2009). Those models feature a representative household and two sectors - public and private - where labor hours can be supplied. A serious shortcoming in these models is that wage rates in the economy are identical, with public hours approximated by a stationary stochastic process. These models, despite being an improvement over earlier vintages of RBC models, produce a good match *vis-a-vis* data along the public sector labor market dimension, *e.g.* public hours volatility, mostly by construction. The absence of an internal propagation mechanism for public employment is a serious limitation in this class of models, particularly when the research focus falls on the interactions between the two labor markets and their relation to the business cycle.

There are few RBC models with endogenous public sector wages and employment. Ardagna (2007), for example, departs from the representative-agent assumption. Total population is split into capitalists and workers, with workers being either employed in the private or public sector, or unemployed. In addition, both sectors are unionized, and public sector wage is different from the private sector wage rate. Public wage and employment are obtained from the government's maximization problem, where the government profit function is augmented with an *ad hoc* quadratic term capturing equity considerations. However, a major limitation of Ardagna's (2007) setup is that it assigns each household to a sector and by default excludes further labor reallocation, which is the focus in this chapter.

Additionally, there are even fewer RBC models that incorporate endogenously-determined wage and hours in the public sector, and also reflect the importance of public sector unionization for the business cycles in EU countries: Fernandez-de-Cordoba *et al.* (2009, 2012) were the first to develop a Dynamic Stochastic General Equilibrium (DSGE) model with public and private wages being determined in different environments. The private sector wage is de-

termined within a competitive market framework, while the public sector wage is an optimal solution to the government sector union's optimization problem. In addition, the impulse response analysis in Fernandez-de-Cordoba *et al.* (2009, 2012) generates pro-cyclical public wage and hours. Another important finding is the positive, but only moderate, co-movement between the two wage rates, and public and private hours. These are all robust patterns that have been observed previously in the empirical work of Lamo, Perez and Schuknecht (2007, 2008) and Perez and Sanchez (2010).

The model in this chapter combines two ingredients used in earlier research to address new aspects of the economy and produce new results: it adopts the public sector union maximization problem from Fernandez-de-Cordoba *et al.* (2009, 2012) and incorporates it into a RBC model with richer tax structure and fiscal policy instruments, *i.e.* Finn (1998). Thus, the individual quantitative effect of union optimization can be assessed relative to Finn's (1998) setup with exogenous public hours and a single, competitive wage rate. In addition, the fiscal policy instruments will be the shares of government consumption and investment in output, which allows the government to react to output. The presence of a union in the public sector will crowd out the other types of the government spending at the expense of the public sector wage bill, an effect not present in Fernandez-de-Cordoba *et al.* (2009, 2012). Additionally, in contrast to Fernandez-de-Cordoba *et al.* (2009, 2012), who model public employment as output-enhancing, public employment in this chapter is a wasteful expenditure from a productive point of view. This modeling choice is used to reflect the view that the public sector bureaucracy's direct contribution to the national product in the economy is somewhat small. Moreover, the setup is consistent with Blanchflower (1991), who suggests that some governments use public sector jobs as a tool to fight unemployment and generate votes for re-election. Lastly, a government's completely wasteful public wage bill spending is expected to amplify fluctuations in hours, as there will be no direct substitutability/complementarity between private and public hours. In other words, the allocative efficiency will decrease significantly, as a wasteful hour spent working in the public sector

receives a higher return relative to a productive hour of work in the private sector.

The analysis in this chapter, as well as in the subsequent parts of the thesis, is done at the country level, as taxation and government spending decisions are still to a great extent country-specific for individual EU member states. Furthermore, based on their extensive compilation of case studies, Ebbinghaus and Visser (2000) and Visser (2003) conclude that international unionism is weak, *i.e.* the influence of labor unions in Europe tends to be constrained to the respective countries' borders. This approach differs from Fernandez-de-Cordoba *et al.* (2009, 2012), who analyze the Euro Area as a whole. Germany is the preferred choice for calibration in this chapter (and throughout the thesis), as it is the classical example of a large EU economy. Some of the features of the German economy include strong public sector unions, and a large and growing gap between public and private sector unionization, as reported in *The Economist* (2011). Additionally, Germany has a public sector wage premium and public/private employment ratio similar to the EU averages.

The study in this chapter takes a much wider scope relative to Finn (1998) and at the same time is complementary to Fernandez-de-Cordoba *et al.* (2009, 2012). It includes a complete evaluation of an RBC model with optimizing public sector union, following the widely-accepted methodology in the RBC literature. The model here matches the cyclical fluctuations in the public and private sector labor markets. Additionally, it also compares well against the empirical autocorrelation and cross-correlation functions generated from an unrestricted Vector Auto Regression (VAR). While all these features are important for understanding the aggregate fluctuations, these aspects were not addressed in the earlier studies on the dynamic general equilibrium effects of public sector unions. Lastly, endogenously-determined public wage and hours will be shown to add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. The government sector union model requires larger changes in tax rates to achieve a pre-specified increase in tax revenue, as compared to Finn's model with exogenous public sector hours. Thus,

endogenous public hours are quantitatively important for fiscal policy evaluation. Ignoring the interaction between hours and wages leads to a significant underestimation of the welfare effect of tax regime changes.

The chapter is organized as follows: Section 1.2 presents the model setup in the context of the relevant literature. Section 1.3 explains the data used and model calibration. Section 1.4 solves for the steady-state. Section 1.5 presents the model solution procedure, discusses the effects of different shocks and the impulse responses of variables across model. Section 1.6 simulates the competing models and evaluates their properties for the calibrations performed for Germany; it also computes the long-run welfare costs of exogenous tax regime changes, both across models and across countries. Section 1.7 concludes.

1.2 Model setup

1.2.1 Description of the model:

The model builds upon Finn (1998). There is a representative household, as well as a representative firm. The household owns the private physical capital and labor, which it supplies to the firm. Hours supplied in the public sector are decided via a collective agreement between a public sector union and the government. The perfectly-competitive firm produces output using labor, private and public capital. The government uses tax revenues from consumption expenditure, labor and capital income to finance: (1) government consumption (which is valued by the representative household), (2) government investment (public capital generates mild increasing returns to scale in the aggregate output production function), (3) government transfers, and (4) the public wage bill. The wage rate and hours supplied in the public sector are determined by a utility-maximizing public sector union, as in Fernandez-de-Cordoba *et al.* (2009, 2012), subject to a balanced government period budget constraint.

1.2.2 Households

There is an infinitely-lived representative household in the model economy, and no population growth. The household maximizes the following expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h, G_t^c, N_t^h), \tag{1.2.2.1}$$

where E_0 is the expectation operator as of period 0; C_t^h, G_t^c and N_t^h are household's consumption, per household consumption of government services, and hours worked by the household at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. Following Finn (1998), the CRRA form for the instantaneous utility is:

$$U(C_t^h, G_t, N_t^h) = \frac{[(C_t^h + \omega G_t^c)^{\psi}(1 - N_t^h)^{(1-\psi)}]^{(1-\alpha)} - 1}{1 - \alpha}, \tag{1.2.2.2}$$

where $(\alpha > 1)$. The parameter ψ is the weight of consumption in utility, $0 < \psi < 1$, and $0 < 1 - \psi < 1$ is the weight in the utility function that the household attaches to leisure. Government consumption is a substitute for private consumption, and the degree of substitutability is measured by ω, where $0 \leq \omega \leq 1$.

The household has an endowment of one unit of time in each period t, which is split between work, N_t^h and leisure, L_t^h, so that

$$N_t^h + L_t^h = 1. \tag{1.2.2.3}$$

The household can supply hours of work in the public sector, N_t^{gh}, or in the private one, N_t^{ph}, with $N_t^h = N_t^{ph} + N_t^{gh}$. The wage rates per hour of work in private and public sector are denoted by w_t^p and w_t^g, respectively. The household chooses N_t^{ph} only; public hours will be endogenously chosen jointly by the public sector union and government, so N_t^{gh} will be taken by the household as given, as in Fernandez-de-Cordoba *et al.* (2012). [3,4]

[3] This modeling choice also helps to keep the representation close to Finn (1998), where public employment is determined as a realization of a stochastic process, and thus taken as given by the household.

[4] An alternative way to think about the household's labor decision is that it chooses total hours worked (which is always positive because of the monotonicity of the household's utility function), and the split

The representative household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to private capital, and K_t^{ph} denotes private capital stock in the beginning of period t. As in Finn (1998), the household receives capital depreciation allowance in the amount of $\tau^k \delta^p K_t^{ph}$, where τ^k is the capital income tax rate and $0 < \delta^p < 1$ is the depreciation rate of private physical capital. In other words, capital income taxes are levied net of depreciation as in Prescott (2002, 2004) and in line with the methodology used in Mendoza, Razin, and Tesar (1994).

Finally, the household owns all firms in the economy, and receives all profit (Π_t^h) in the form of dividends. The household's budget constraint is

$$(1+\tau^c)C_t^h + I_t^h \leq (1-\tau^l)[w_t^p N_t^{ph} + w_t^g N_t^{gh}] + (1-\tau^k) r_t K_t^{ph} + \tau^k \delta^p K_t^{ph} + G_t^T + \Pi_t^h, \quad (1.2.2.4)$$

where τ^c, τ^l are the proportional tax rates on consumption and labor income, respectively, and G_t^t is the per household transfer from the government.

Household's private physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1-\delta^p) K_t^{ph}. \quad (1.2.2.5)$$

between private sector and public sector work are determined by the firm and the union, respectively. Monotonicity of the utility function, the Cobb-Douglas specification of the firm's production function, and union objective function will guarantee that in equilibrium both $N_t^{ph}, N_t^{gh} > 0$. Furthermore, in the model calibration, consumption utility weight parameter ψ would be set to correspond to the average share of time endowment spent working. In the private sector, the equilibrium private sector wage and interest rate will such that optimal $N_t^{ph} < N_t^h$. Next, government transfers share and the relative weight on wages in the union maximization problem will be set to match the average public-to-private employment and wage ratios in data. Furthermore, the way the transfers share and the relative weight on public wages in the union's objective are fixed will achieve $N_t^{gh} < N_t^h$. This is because out of the steady state, the hours ratio will fluctuate only in a small neighborhood around its steady state value. (This could be seen later from the impulse responses documented in Table 1.1 - 1.3.)

The representative household acts competitively by taking prices $\{w_t^p, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau^c, \tau^l, \tau^k\}$, policy variables $\{w_t^g, N_t^{gh}, G_t^c, G_t^i, G_t^T\}_{t=0}^{\infty}$ as given, (where G_t^i denotes government investment) and chooses allocations $\{C_t^h, N_t^{ph}, I_t^h, K_{t+1}^{ph}\}_{t=0}^{\infty}$ to maximize Eq. (1.2.2.1) subject to Eqs. (1.2.2.2)-(1.2.2.5), and initial condition for private physical capital, K_0^{hp}.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for private physical capital, are as follows[5]

$$C_t^h\colon \left[(C_t^h + \omega G_t^c)^{\psi}(1 - N_t^h)^{(1-\psi)}\right]^{-\alpha} \psi(C_t^h + \omega G_t^c)^{\psi-1}(1 - N_t^h)^{(1-\psi)} = \Lambda_t(1 + \tau^c) \tag{1.2.2.6}$$

$$N_t^{ph}\colon \left[(C_t^h + \omega G_t^c)^{\psi}(1 - N_t^h)^{(1-\psi)}\right]^{-\alpha} (1 - \psi)\left[\frac{C_t^h + \omega G_t^c}{1 - N_t^h}\right]^{\psi} = \Lambda_t(1 - \tau^l)w_t^p \tag{1.2.2.7}$$

$$K_{t+1}^{ph}\colon \beta E_t \Lambda_{t+1}\left[(1 - \tau^k)r_{t+1} + \tau^k\delta^p + (1 - \delta^p)\right] = \Lambda_t \tag{1.2.2.8}$$

$$\text{TVC}\colon \lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^{ph} = 0, \tag{1.2.2.9}$$

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Private hours are chosen so that the disutility of an hour work in the private sector at the margin equals the after-tax return to labor. Next, the Euler equation describes the optimal capital accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two contiguous periods. The last expression is the TVC, imposed to ensure that the value of the private physical capital that remains at the end of the optimization horizon is zero. This boundary condition guarantees that the model equilibrium is well-defined by ruling out explosive solution paths.

[5]Detailed derivation of the household's optimality conditions is provided in Appendix 1.8.1.

1.2.3 Firms

Following Finn (1998), there is also a representative private firm in the model economy. It produces a homogeneous final product using a production function that requires private and public physical capital, K_t^p, K_t^g respectively, and labor hours N_t^p. The production function is as follows

$$Y_t = A_t (N_t^p)^{\theta} (K_t^p)^{1-\theta} (K_t^g)^{\nu}, \tag{1.2.3.1}$$

where A_t measures the total factor productivity in period t; $0 < \theta, (1-\theta) < 1$ are the productivity of labor and private physical capital, respectively. Parameter $\nu \geq 0$ measures the degree of increasing returns to scale (IRS) that public capital has on output.

The representative firm acts competitively by taking prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and policy variables $\{\tau^c, \tau^k, \tau^l, w_t^g, N_t^g, G_t^c, G_t^i, G_t^T, K_{t+1}^g\}_{t=0}^{\infty}$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize firm's static aggregate profit,

$$\Pi_t = A_t (N_t^p)^{\theta} (K_t^p)^{1-\theta} (K_t^g)^{\nu} - r_t K_t^p - w_t^p N_t^p. \tag{1.2.3.2}$$

In equilibrium, profit is zero. In addition, labor and capital receive their marginal products, *i.e*[6]

$$w_t^p = \theta \frac{Y_t}{N_t^p}, \tag{1.2.3.3}$$

$$r_t = (1-\theta) \frac{Y_t}{K_t^p}. \tag{1.2.3.4}$$

1.2.4 Government budget constraint

The government purchases goods, G_t^c, invests in public capital G_t^i, distributes transfers G_t^T, hires labor N_t^g and sets the public sector wage rate w_t^g. Public capital evolves according to the following law of motion:

$$K_{t+1}^g = G_t^i + (1-\delta^g) K_t^g, \tag{1.2.4.1}$$

[6]Detailed derivation of the firm's optimality conditions is provided in Appendix 1.8.1.

where $0 < \delta^g < 1$ is the linear depreciation rate on government physical capital.

Total government expenditure, $G_t^c + G_t^i + w_t^g N_t^g + G_t^T$, is financed by levying proportional taxes on consumption, capital and labor income. Thus, the government budget constraint is

$$G_t^c + G_t^i + w_t^g N_t^g + G_t^T = \tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t^p + \tau^l \left[w_t^p N_t^p + w_t^g N_t^g \right]. \tag{1.2.4.2}$$

Government takes market prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and allocations $\{N_t^p, K_t^p\}_{t=0}^{\infty}$ as given.

The following six policy instruments, $\{\tau^c, \tau^k, \tau^l, \frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}, \frac{G_t^T}{Y_t}\}$, will be exogenously set for all t. In particular, shares of government consumption and investment in output, rather than the levels of the fiscal variables, will follow stochastic processes. Thus, public consumption and investment will respond to both exogenous shocks and output. (K_{t+1}^g will be exogenously determined as well, subject to the initial condition K_0^g and the law of motion for G_t^i.) Government transfers-to-output ratio $G^{Ty} \equiv \frac{G_t^T}{Y_t}$ will be fixed,[7] but the level of public transfers will vary with output (*i.e.* $G_t^T = G^{Ty} Y_t$). All three tax rates $\{\tau^c, \tau^k, \tau^l\}$ will be kept constant. Finally, the pair $\{N_t^g, w_t^g\}_{t=0}^{\infty}$ will be determined as an optimal solution from a collective bargaining problem between the government and a public sector union, which is described in the next subsection.

1.2.5 Government sector union objective function

In contrast to Finn's (1998) model, which features a single wage rate w_t and exogenous public employment, modeled as an AR(1) process, in this chapter the two variables will be obtained as optimal choices from an explicit objective function maximization, similar to the

[7] The fixed government transfers/output ratio is to be interpreted as an "implied" one, as it will be set so that the model matches the long-run wage and employment ratios, as it will be shown in the following sections. In this sense, it bears little correspondence to the average ratio in data.

one used in Fernandez-de-Cordoba *et. al* (2009, 2012):[8]

$$\max_{w_t^g, N_t^g} \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{1/\rho}, \tag{1.2.5.1}$$

where $\eta > 0$ is the relative weight put on wages, and ρ is the parameter determining the constant elasticity of substitution (CES) between wages and hours, $\frac{1}{1-\rho}$. Hence, the pair $\{N_t^g, w_t^g\}$ solves Eq. (1.2.5.1) s.t. Eqs. (1.2.4.1)-(1.2.4.2) and the processes for the other policy instruments.[9,10]

In the union literature, Doiron (1992) uses an equivalent representation to model union preferences over wages and employment between a private sector union and a firm.[11] Furthermore, the functional form of the objective function used in this chapter can be traced back to Oswald *et al.* (1984) and Alogoskoufis and Manning (1988). Recent studies by Demekas and Kontolemis (2000), and Forni and Giordano (2003) also use social welfare

[8]The difference in this chapter is that instead of having two separate weights, ϕ on wages, and $(1-\phi)$ on employment, only one relative weight will be used, mainly due to the large difference in the magnitudes of public sector hours and the wage rate.

[9]The public sector union should be taken as an aggregation of smaller unions who operate on federal and state/local levels, who maximize the same objective function over local government period budget constraint. The coalition of workers is large at a regional level, and thus able to influence the public sector wage rate. Sill, local unions are small relative to the size of the economy, hence w^p is taken as given. Nonetheless, both wage rates will be determined within the system, so there will be some feedback effect from public to private wage.

[10]In the model, the assumption that the government cannot influence private sector prices was used to make the government sector union constraint optimization problem well-defined, *i.e.*, that the public sector union can determine the public sector wage rate, but cannot affect the private sector wage rate. It is also a technical condition that allows for the DCE to be solved. Also, empirical evidence (WDN 2010) suggests that private sector is a wage leader, sets wages first, and then government follows to determine compensation in the public sector. The same effect is observed in the CCFs in Table 1.7 in Appendix 1.8.5. Regarding the point that the government should be treated as a "Stackelberg leader," this is indeed an important aspect of reality. Pursuing an extension of the model along that venue is left for future research, as the problem becomes computationally very intensive, since the whole history of the economy becomes a state variable, which makes the state space a complicated object to handle, and interferes with the model tractability.

[11]The equivalence is shown in Appendix 1.8.1.

functions where public sector wage and employment appear as separate arguments. Indeed, all those studies ignore the process of bargaining, and thus the objective function used here is not micro-founded as well.[12] Alternatively, deriving the reduced-form utility function using union aggregation over individual worker preferences is still an open question in the literature. Oswald (1982) shows in a simple static framework that if the union is utilitarian, *i.e.*, the union maximizes the expected utility of a representative worker, and if members are risk-averse, then there exists a well-behaved union utility function defined over both wage rate and employment. Since those conditions are assumed to hold in this chapter, Oswald's (1982) result is one way to at least partially rationalize the *ad hoc* union utility function used here. Additionally, a CES union utility function, which is concave and increasing in wage and employment, has proven to be a successful modeling choice in econometric studies.[13] In contrast, simple models such as wage bill maximization ($w_t^g N_t^g$) and rent maximization ($[w_t^g - w_t^a]N^g$, where w_t^a denotes the alternative wage at time t) have been rejected in many studies.[14] In what is to follow, it will be shown that the CES union maximization function is empirically relevant, and thus is a useful modeling device, similar to the household's utility function and the aggregate production function, which could help generate several new and more importantly, interesting, results.

The interaction between the public sector union and the government is as follows: the wage bill in the public sector, modeled as a residual spending item that balances the budget constraint in every period, is distributed between wages and hours according to the

[12]Despite researchers' claims that this representation is consistent with Nash bargaining, such statements are incorrect. The author is not aware of any studies that explicitly show how the union objective function can be obtained from a Nash bargaining procedure.

[13]Papers applying this approach are Akerlof (1969), Atherton (1973), Calmfors (1981), Carruth and Oswald (1981), Cartter (1959), Corden (1981), Hersoug (1978), Fellner (1949), Mulvey (1978), Oswald (1979), Rees (1977), and Warren-Boulton (1977), among others.

[14]A partial list of such studies is Brown and Ashenfelter (1986), Card (1986), Carruth and Oswald (1983), Eberts and Stone (1986), Farber (1978), MaCurdy and Pencavel (1986), and Martinello (1989).

union utility function (1.2.5.1) specified above.[15] Additionally, government period budget constraint serves the role of a labor demand function, which will be subject to shocks, resulting from innovations to total factor productivity and the fiscal shares. The balanced budget assumption is thus an important assumption in the model setup. Since the wage bill is a residual, if the wage rate is increased, then hours need to be decreased. Now the problem in the public sector is transformed into the standard representation used in union literature, where a labor union maximizes utility, constrained by a stochastic labor demand curve. However, in addition to producing endogenous public wage and public hours, this optimization problem generates a public sector wage that features a positive premium over the private sector one. Therefore, at least part of this premium can be justified by the gains from unionization in the public sector. In equilibrium, a positive linear relation exists between the public wage rate and public sector hours, which is obtained from the marginal rate of substitution between the two:[16]

$$N_t^g = \eta^{\frac{1}{\rho}} w_t^g. \tag{1.2.5.2}$$

There are several interpretations for Eq. (1.2.5.2): first, it can be recognized as a standard neoclassical labor supply curve. Hence, this model can be viewed as one emphasizing the relative importance of supply-side factors, *i.e.* unions, in the economy. Second, and more importantly, such a relationship is called a "contract curve" in the union literature, *e.g.*, Blanchard (1991). In particular, this curve defines the set of allocations $\{w_t^g, N_t^g\}$, generated as an outcome of the collective bargaining between the government and the union. Since the union optimizes over both the public wage and hours, the outcome is efficient. The solution pair is at the intersection point of the contract curve, and the labor demand curve (government budget constraint).

[15]The modeling choice is also consistent with Tullock's (1974) hypothesis, which states that bureaucrats first exert an effort to increase their number, and once staff has expanded, the bureaucrats will then use their newly-increased power to negotiate higher wages.

[16]Detailed derivation of the union's optimality conditions is provided in Appendix 1.8.1.

Next, Eq. (1.2.5.2) is plugged back into (1.2.4.2) to obtain a solution for the public sector wage:

$$w_t^g = \eta^{-\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^T - G_t^i}{1 - \tau^l} \right]^{\frac{1}{2}}. \tag{1.2.5.3}$$

Optimal public hours are obtained by substituting (1.2.5.3) into (1.2.5.2) to obtain

$$N_t^g = \eta^{\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^T - G_t^i}{1 - \tau^l} \right]^{\frac{1}{2}}. \tag{1.2.5.4}$$

Both public sector wage and hours will be negatively related to government consumption and investment, and positively related to tax revenue from consumption, capital income and private sector labor income. Public hours and the wage rate are directly affected by fiscal policy variables: a decrease in government consumption, for example, will have a direct positive effect on both public hours and wages, and thus on the household's income. Such effect are empirically observed in Lano, Perez, and Schuknecht (2008). In the model, the crowding out effect of government spending will generate important differences from earlier literature. This makes it relevant for the analysis of the impulse responses to fiscal shares shocks and for the long-run welfare effects of fiscal policy. These effects will be discussed at length in the following sections.

1.2.6 Stochastic processes for the policy variables

The exogenous stochastic variables are the total factor productivity A_t, and the policy instruments $\frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$, where $\frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$ denote the shares of government consumption (purchases) and government investment in output, respectively. Then assume that $A_t, \frac{G_t^c}{Y_t}, \frac{G_t^i}{Y_t}$ follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a, \tag{1.2.6.1}$$

where $A_0 = A > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected

changes in the total factor productivity process.

The stochastic process for the government consumption/output share $\{\frac{G_t^c}{Y_t}\}$ is

$$\ln\left(\frac{G_{t+1}^c}{Y_{t+1}}\right) = (1-\rho_c)\ln\left(\frac{G_0^c}{Y_0}\right) + \rho_c \ln\left(\frac{G_t^c}{Y_t}\right) + \epsilon_{t+1}^c, \tag{1.2.6.2}$$

or

$$\ln G_{t+1}^{cy} = (1-\rho_c)\ln G_0^{cy} + \rho_c \ln G_t^{cy} + \epsilon_{t+1}^c, \tag{1.2.6.3}$$

where $G_{t+1}^{cy} = \frac{G_{t+1}^c}{Y_{t+1}}$, and $\frac{G_0^c}{Y_0} = \frac{G^c}{Y} > 0$ is the steady-state public consumption/output ratio, $0 < \rho^c < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^c \sim iidN(0, \sigma_c^2)$ are random shocks to government consumption/output share. Hence, the innovations ϵ_t^c represent unexpected changes in government consumption/output share.

The stochastic process followed by the government investment/output share $\{\frac{G_t^i}{Y_t}\}$ is

$$\ln\left(\frac{G_{t+1}^i}{Y_{t+1}}\right) = (1-\rho_i)\ln\left(\frac{G_0^i}{Y_0}\right) + \rho_i \ln\left(\frac{G_t^i}{Y_t}\right) + \epsilon_{t+1}^i, \tag{1.2.6.4}$$

or

$$\ln G_{t+1}^{iy} = (1-\rho_i)\ln G_0^{iy} + \rho_i \ln G_t^{iy} + \epsilon_{t+1}^i, \tag{1.2.6.5}$$

where $G_{t+1}^{iy} = \frac{G_{t+1}^i}{Y_{t+1}}$, and $\frac{G_0^i}{Y_0} = \frac{G^i}{Y} > 0$ is the steady-state public investment/output ratio, $0 < \rho^i < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^i \sim iidN(0, \sigma_i^2)$ are random shocks to government investment/output share. Hence, the innovations ϵ_t^i represent unexpected changes in government investment/output share.

Additionally, in Finn (1998), public hours will also follow an AR(1) process:

$$\ln N_{t+1}^g = (1-\rho_n)\ln N_0^g + \rho_n \ln N_t^g + \epsilon_{t+1}^n, \tag{1.2.6.6}$$

where $N_0^g = N^g > 0$ is the steady-state public employment, $0 < \rho^n < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^n \sim iidN(0, \sigma_n^2)$ are random shocks to government employment. Hence, the innovations ϵ_t^n represent unexpected changes in government employment.

1.2.7 Decentralized competitive equilibrium

Given the fixed value of government transfers/output ratio G^{ty}, the exogenous processes followed by $\{A_t, G_t^{cy}, G_t^{iy}\}_{t=0}^{\infty}$ and initial conditions for the state variables $\{A_0, G_0^{cy}, G_0^{iy}, K_0^{ph}, K_0^{g}\}$, a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t^h, N_t^{ph}, N_t^{gh}, I_t^{ph}, K_{t+1}^{ph}, K_{t+1}^{g}\}$ $\forall h$, prices $\{r_t, w_t^p, w_t^g\}_{t=0}^{\infty}$ and the tax rates $\{\tau^c, \tau^l, \tau^k\}$ such that (i) the representative household maximizes utility; (ii) the stand-in firm maximizes profit every period; (iii) government sector union objective function is maximized $s.t$ the government budget constraint being satisfied in each time period; (iv) all markets clear.[17]

1.3 Data and model calibration

Both the model in this chapter and Finn (1998) are calibrated for German data at annual frequency to correspond to the collective bargaining frequency. The chapter follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate certain features of the reference economy.

1.3.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007, while the sub-period 1970-91 covers West Germany only. For Germany, data on real output per capita, household consumption per capita, gross fixed capital formation per capita, as well as government consumption and population were taken from the World Development Indicators (WDI) database. The OECD statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Public transfers ratio were calculated from the *CES-Ifo* DICE Database (2011). Public and private investment and capital stock series were obtained from EU Klems database (2009).

[17]The system of equations that characterizes the DCE is provided in Appendix 1.8.2.

German average annual real public compensation per employee was estimated by dividing the real government wage bill (OECD 2011) by the number of public employees. Due to data limitations on the average hours worked in each sector, employment statistics were used instead. To make empirical variables comparable with model variables, employment series in Germany were normalized by total population (obtained from WDI).

1.3.2 Calibrating model parameters to German data

In German data, the average public/private employment ratio over the period 1970-2007 is 0.17, and the average wage ratio in data equals 1.20. The relative weight attached to public wages, η, as well as government transfers/output ratio g^{ty}, will be set so that the steady-state wage and employment ratios in the model match the corresponding data averages. The curvature parameter of the union's CES maximization function, was set to a standard value, $\rho = -1$, as in Fernandez-de-Cordoba *et al.* (2012).[18] The average effective tax rates in EU countries were obtained from McDaniel's (2009) dataset. McDaniel's approach was preferred to the one used by Mendoza et al. (1984) and the subsequent updates due to the more careful treatment of property and import taxes. Over the period studied, German economy is characterized by a low average capital income tax rate, $\tau^k = 0.16$, and a relatively high labor income tax rate, $\tau^l = 0.409$. The labor share, $\theta = 0.71$, was computed as the average ratio of compensation of employees in total output.[19] Private and public capital depreciation rates, $\delta^p = 0.082$ and $\delta^g = 0.037$, respectively, were approximated from the EU Klems Database as the average ratio of gross fixed capital formation in constant 1995 prices and and the corresponding value of fixed capital stock in constant 1995 prices over the period 1970-2007.[20] The discount rate $\beta = 0.973$ was calibrated from the steady-state Euler

[18] A robustness check on the curvature parameter was performed with $\rho = [-5, -4, -3, -2, -0.5]$, which did not produce any significant difference in the results obtained, as parameter η adjusted accordingly.

[19] Alternatively, capital share, $1-\theta$, can be obtained as the mean ratio of gross private capital compensation in output from EU Klems.

[20] To impute public sector capital stocks and investment, the series for education, public administration, social security and health sectors were used.

equation (Eq. 1.8.2.18 in Appendix 1.8.2.). The parameter describing the curvature of the household's utility function was set to $\alpha = 2$, which is a typical value of relative risk aversion in the literature. As in Kydland (1995), the weight on consumption, $\psi = 0.296$, was set equal to the average steady-state total hours of work in data as a share of total hours available. The weight put on government consumption in the utility function, $\omega = 0.099$, was calibrated using the marginal rate of substitution (MRS) equation (Eq. 1.8.2.19 in Appendix 1.8.2.) and data averages. The implied substitutability between private and public consumption is therefore quite low. The public capital share in the production function, $\nu = 0.0233$, equals the average public investment/output ratio in German data. Persistence and innovation volatility of the stochastic processes, as well as the AR(1) process for public employment in Finn (1998), were estimated using OLS. Total factor productivity parameters, $\rho^a = 0.943$ and $\sigma^a = 0.013$, were estimated using the logged and linearly detrended Solow residual series, obtained from the model's aggregate production function and data. Table 1.2 on the next page summarizes the model parameters for Germany.

Table 1.2: Model Parameters

Param.	Value	Definition	Source
β	0.973	Discount factor	Calibrated
θ	0.710	Labor income share	Data average
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on government capital	Data average
α	2.000	Curvature parameter of the utility function	Set
ψ	0.296	Weight on consumption in utility	Set
ν	0.023	Degree of increasing returns to scale of public capital	Set
ρ	-1.000	Curvature parameter of the union's maximization function	Set
ω	0.099	Weight on government services in household's consumption	Calibrated
τ^c	0.148	Effective tax rate on consumption	Data average
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
g^{cy}	0.189	Steady-state government consumption as a share of output	Data average
g^{iy}	0.023	Steady-state government investment as a share of output	Data average
g^T/y	0.047	Steady-state government transfers as a share of output	Set
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
ρ^c	0.976	AR(1) parameter government consumption/output ratio	Estimated
ρ^i	0.853	AR(1) parameter government investment/output ratio	Estimated
ρ^n	0.915	AR(1) parameter government employment (Finn's model)	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated
σ_c	0.016	SD of government consumption/output share innovation	Estimated
σ_i	0.023	SD of government investment/output share innovation	Estimated
σ_n	0.016	SD of government employment innovation (Finn's model)	Estimated

1.4 Solving for the steady-state

Once model parameters were obtained, the unique steady-state of the system was computed numerically for the Germany-calibrated model. Results are reported in Table 1.3 below.

Table 1.3: Data averages and long-run solution

	Description	GE Data	Finn GE	Union GE
c/y	Consumption-to-output ratio	0.590	0.576	0.576
i/y	Investment-to-output ratio	0.210	0.212	0.212
g^c/y	Gov't consumption-to-output ratio	0.189	0.189	0.189
g^i/y	Gov't investment-to-output ratio	0.023	0.023	0.023
g^T/y	Gov't transfers-to-output ratio	0.170	0.047	0.047
k^p/y	Private capital-to-output ratio	2.350	2.350	2.350
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
$w^p n^p/y$	Priv. labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill-to-output ratio	0.130	0.146	0.145
rk^p/y	Capital share in output	0.290	0.290	0.290
n^g/n^p	Public-private employment ratio	0.170	0.205	0.170
w^g/w^p	Public-private employment ratio	1.200	1.000	1.200
n	Total employment	0.296	0.253	0.247
n^p	Private sector employment	0.253	0.210	0.211
n^g	Public sector employment	0.043	0.043	0.036
η	Relative weight on public wage rate	-	N/A	31.63
$\tilde{r}$	After-tax net return to capital	0.036	0.028	0.028

Note that the public transfers share, g^{Ty}, and the relative weight attached to public wages, η, are set so that the wage and hours ratios match the corresponding data averages.[21] In

[21] In this model, the implied η cannot be interpreted directly, but should rather be regarded as containing a scaling factor, as n^g and w^g differ in magnitude (due to the normalization of the time endowment to unity). Therefore, once this is accounted for, i.e. when η is normalized by w^g/n^g, the "corrected" parameter, $\bar{\eta}$,

addition, the steady-state values for hours in data are approximated by splitting the average hours, expressed as a share of total available hours of work, according to the average hours ratio.[22] In Finn (1998), public hours are set to match the corresponding data average.

Overall, the long-run solutions of both models are good approximations to the data averages. The steady-state real after-tax real interest rate, net of depreciation, delivered by the two models, *i.e.* $\tilde{r} = (1 - \tau^k)(r - \delta^p)$, is close to the average real interest rate on 10-year bonds, which is taken as a proxy for the return to private physical capital in the model. Both models capture the public wage bill share of GDP in Germany. Furthermore, public sector labor income is also a significant share relative to capital in Germany as well.

Across models, several important differences can be noted: in steady-state, Finn (1998) produces a slightly higher level of total hours and lower public sector wages, compared to the model with the public sector union. This is due to the additional constraints imposed in the union model on the steady-state public-private hours and wages ratio. Model dynamics out of the steady-state is investigated in the following section.

1.5 Model solution and impulse responses

Since there is no closed-form general solution for the model in this chapter, a typical approach followed in the RBC literature is to log-linearize the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The linearized DCE system[23] can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988):

$$\mathbf{A} E_t \hat{\mathbf{x}}_{\mathbf{t+1}} = \mathbf{B} \hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C} E_t \epsilon_{t+1} \tag{1.5.0.1}$$

equals 0.998 for Germany. In other words, wage rate and hours are equally-weighted in the generalized Stone-Geary union utility function, as typically assumed in the trade union literature.

[22] In this way hours/employment data averages are made comparable in magnitude with the corresponding theoretical variables in the union model.

[23] Detailed derivations in Appendix 1.8.3-1.8.4.

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\epsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{g}_t^{cy} & \hat{g}_t^{iy} & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^c & \hat{g}_t^i & \hat{g}_t^T & \widehat{rw}_t & \widehat{rl}_t \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model.

Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to either productivity, or government consumption. The effects of total factor productivity (TFP) and fiscal shocks to the government consumption and investment shares in a model with public sector union differ from Finn (1998), particularly when the behavior of labor market variables and the labor reallocation is given close scrutiny.

1.5.1 The effect of a positive productivity shock

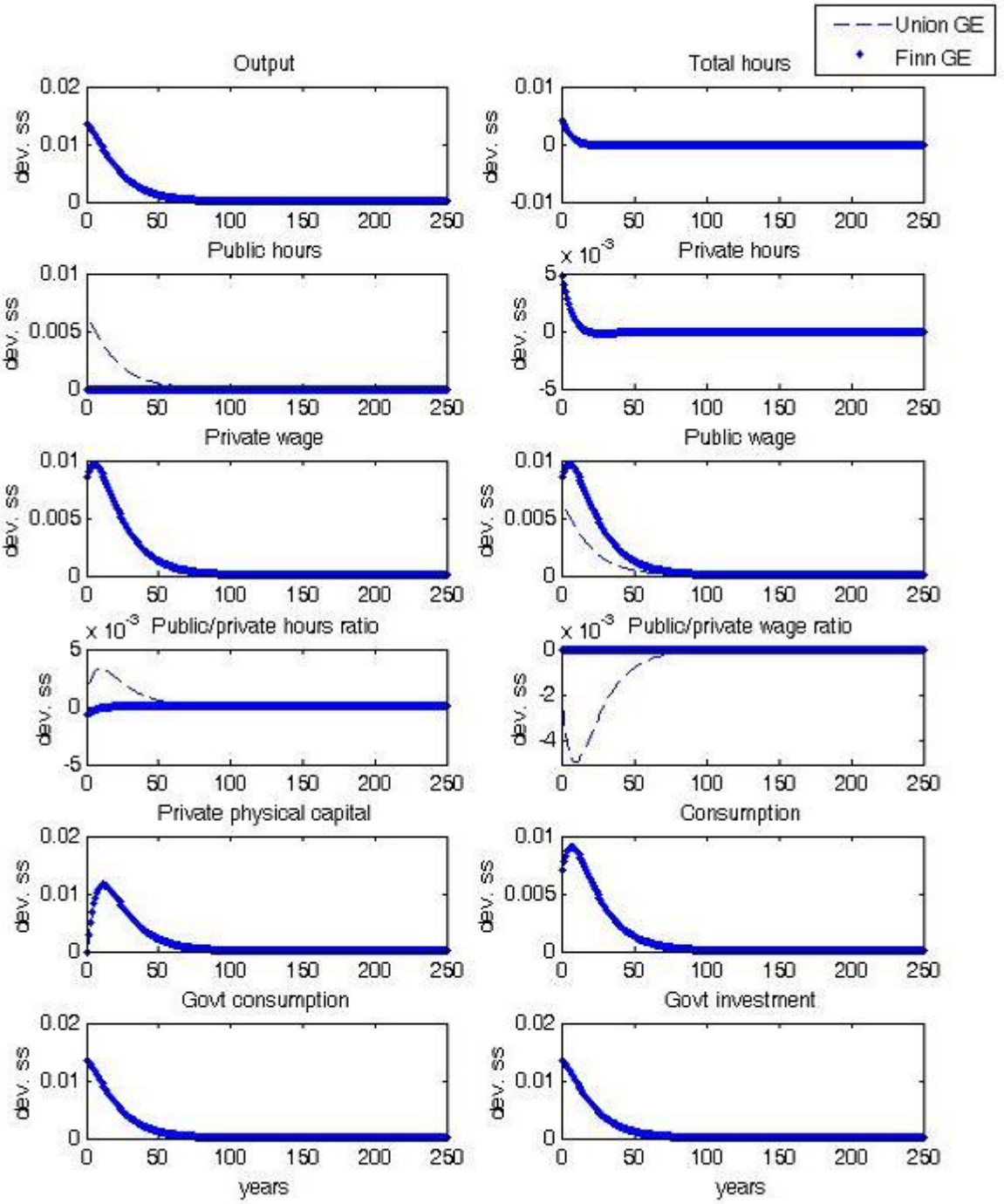
Figure 1.1 shows the impact of a 1% surprise TFP innovation on the economy with public sector union and Finn's setup. The impulse responses are expressed in log-deviation from the variables' original steady-states in the model economy calibrated to annual German data. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as investment. From the rules for the government spending, investment and transfers in levels, a higher output translates into higher level of expenditure in each of the three categories. In turn, there is also a feedback effect from government investment to output through public capital, which comes with a one-period lag. This indirect effect is quite small. Meanwhile, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not pictured) and the private wage rate increase. The household responds to the price signals and supplies more hours in the private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private consumption

and leisure. In terms of the labor-leisure trade-off, the income effect ("work more") produced by the increase in the private wage dominates the substitution effect ("work less"). Furthermore, the increase of private hours expands output even further, and thus both output and government spending categories increase more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

An observational equivalence is noted in the responses of most of the model variables across the two models. Public sector labor dynamics, however, is quite distinct: In Finn (1998), public hours stay fixed at their steady-state, and public wage transition is identical to the private wage one. In the model with public sector union, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. For example, higher income and consumption lead to larger tax revenue. In addition, the growth in government revenue exceeds the increase in the fiscal spending instruments, hence the additional funds available for the wage bill, which leads to an expansion in both public sector wage and hours. In turn, this expansion has a feedback effect on the household's income and consumption. The effect on total hours through the increase in public hours is quite small, though.[24] The model with public sector union in this chapter also generates an interesting dynamics in the wage and hours ratio, which is not present in Finn (1998). In particular, the two wage rates, as well as the two types of hours move together, making the model consistent with the empirical evidence presented in Lamo *et al.* (2007, 2008).

[24]Nonetheless, the increase in hours is much greater in magnitude than the responses reported in Fernandez-de-Cordoba *et al.* (2009, 2012).

Figure 1.1: Impulse Responses to a positive 1% productivity shock in Germany

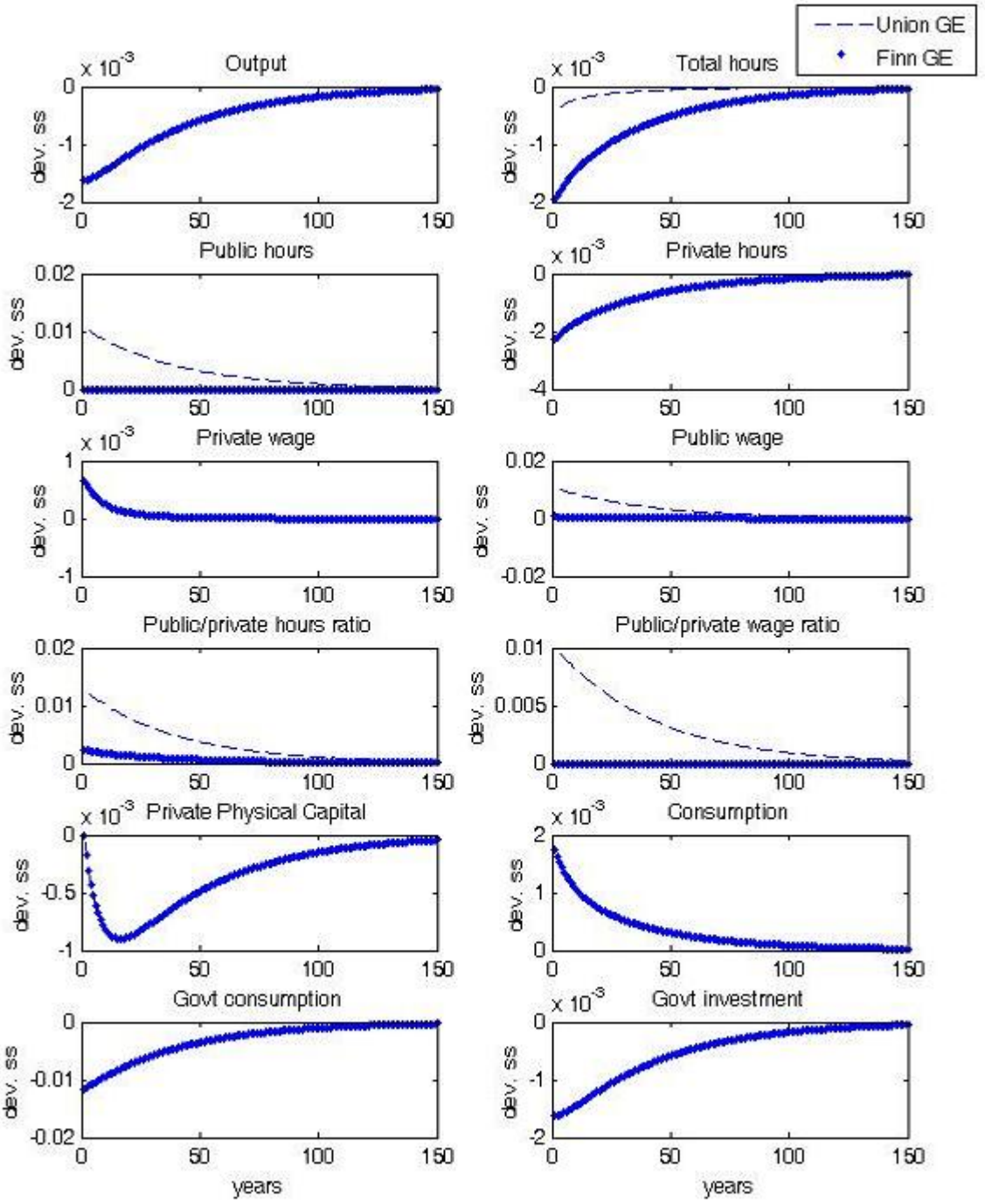


Overall, the endogenous public sector hours model shows an important difference in the composition of household's labor income with the public sector share increasing at the expense of private sector labor income. At aggregate level, however, this distributional effect disperses, as output and consumption dynamics are identical across models. Another important observation to make is that the TFP shocks, being the main driving force in the union model, induce pro-cyclical behavior in public wage and hours. In both model economies, the shock effects are smaller and variables reach their peak response much more rapidly. This means the impulse effect dies out much faster but the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks in the labor markets, and particularly on the wage- and hours ratios.

1.5.2 The effect of a negative government consumption share shock

The second scenario is an exogenous restrictive fiscal policy, which is an unexpected decrease in the government consumption/output ratio. The impulse response functions for this scenario are reported in Figure 1.2 on the next page. The results are similar to those obtained from a standard RBC model. The plots show that a negative government consumption shock partially crowds-in private consumption, as public consumption is only an imperfect substitute for private consumption from the household's point of view. This creates a significant positive welfare effect in the model economy as the decrease in the government consumption ratio frees additional resources that could be directed to private use. The increase in consumption at the expense of a drop in investment triggers a decrease in private sector hours through the marginal rate of substitution between consumption and leisure. In other words, the increase in consumption, resulting from the positive wealth effect, decreases the need to supply labor, so the household enjoys more leisure. The decrease in labor input leads to a fall in output, and an increase in the private wage. Since government expenditure categories follow output, public consumption, investment, and government transfers (not presented) also fall. Over time, all variables return to their old-steady states.

Figure 1.2: Impulse Responses to a negative 1% government consumption/output share shock in Germany



Even though those common responses are typical in the RBC literature but in the presence of a union in the public sector, the fall in labor supply leads to a lower tax revenue, while the increase in consumption increases the tax revenue. The other spending categories also decrease, thus leaving more funds available for the public sector wage bill. The effect on public hours is very pronounced, when total hours responses are compared across models. Furthermore, the model with public sector union generates a realistic labor reallocation from private to public sector meaning that in times of fiscal restraint, government jobs become more attractive. In a model with exogenous public employment, public sector hours stay fixed at their steady-state value and do not respond to fiscal shocks. The effect of a decrease in the government consumption/output ratio in Finn (1998) leads to a significant underestimation in total hours. Additionally, the model with public sector union could again address the relative labor income share evolution, which is the product of the public-private wage and employment ratios. The results in this subsection differ from those in Fernandez-de-Cordoba *et al.* (2009, 2012) in important ways: The negative shock to the fiscal instruments creates a new substitution effect and leads to the crowding-in of the public wage bill. In other words, even under a regime of fiscal tightening, public employment and the public wage are increased, *i.e.* shocks to the government consumption share make public wage and hours behave counter-cyclically.

1.5.3 The effect of a negative government investment share shock

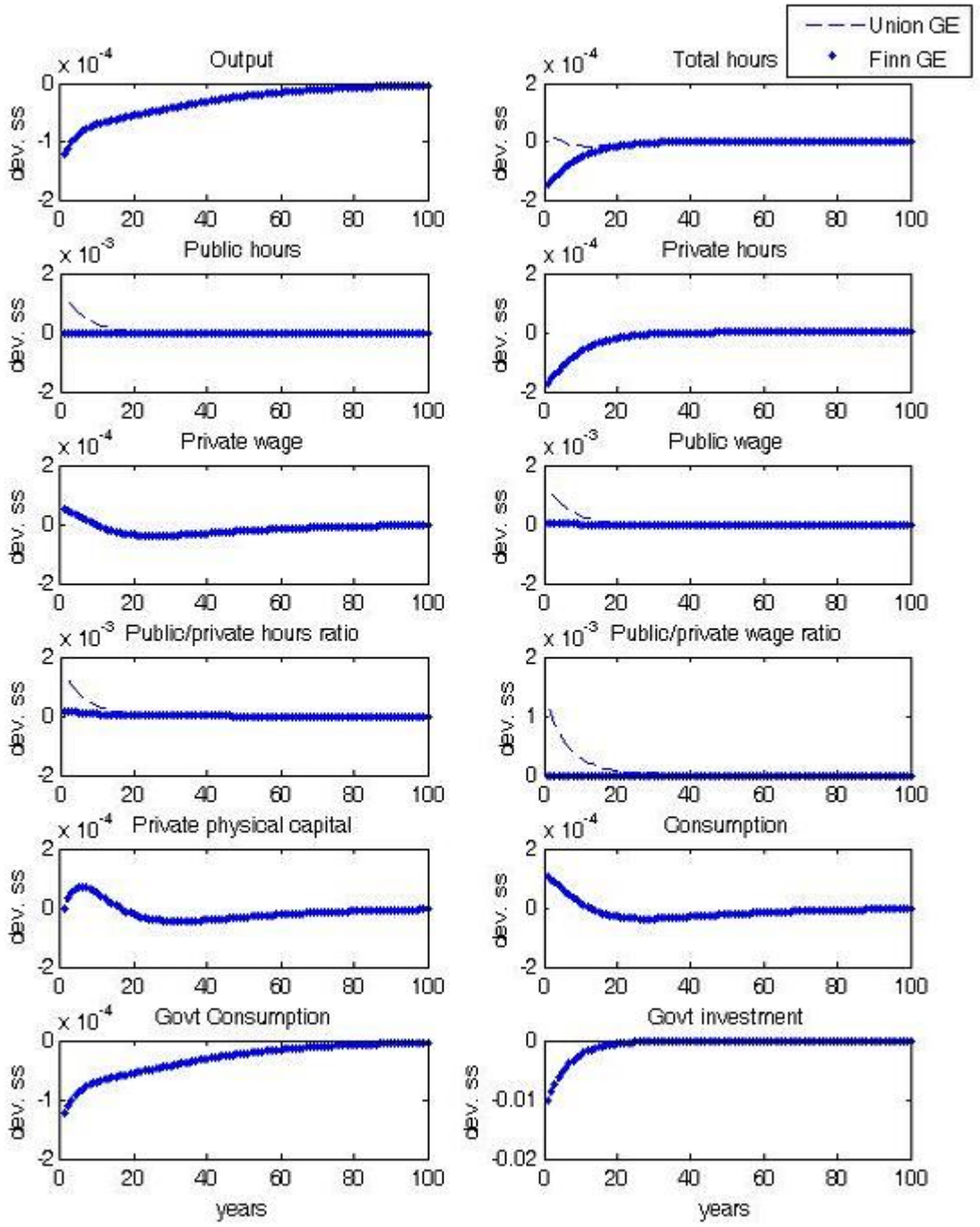
This experiment simulates the effect of a surprise negative innovation in the government investment/output ratio. The impulse response functions are reported in Figure 1.3. This scenario is very relevant in times of crisis, as public investment projects are small relative to the GDP, thus usually the first ones to be cut. The decrease in the government investment share has a direct negative effect due to the decrease in the public physical capital input in the aggregate production function. The magnitude of the shock effect depends on the degree of IRS, captured by the parameter ν. Public investment falls both because of the fall in the public investment/output ratio, and the fall of output itself. Following the output

fall, public consumption and government transfers also fall. On aggregate level, there is a positive welfare effect: output falls less compared to the fall in government consumption and investment. Therefore, the extra resources available now in the economy are used for private consumption and investment. Private physical capital increases, but the effect is short-lived as the marginal product of capital decreases fast, and capital even falls below the steady-state level along the transition path. At the same time, the positive wealth effect leads to a fall in the private sector hours supplied by the household, meaning that private wage increases; The subsequent transition behavior of the private sector wage is determined by the private physical capital dynamics. In the long run, all variables return to their old steady-state values.

The model with public sector union generates the expected additional positive effect on the public wage bill. As total tax revenue increases, and other spending items decrease, the additional revenue is allocated to raise private sector wages and hours. The total contemporaneous effect on hours changes from negative in Finn (1998) to slightly positive, with the overall impact on model variables being very small and short-lived. The model with public sector union, however, produces important transition in the wage and hours ratio and is present for almost 20 periods. In addition, the shocks to public investment share add to the counter-cyclical behavior of public hours and wage rate.

To investigate fully the forces that operate within the model and to study in detail the dynamic interaction among model variables, a complete simulation of the model is performed in the next subsection.

Figure 1.3: Impulse Responses to a negative 1% government investment/output share shock in Germany



1.6 Model simulation, goodness-of-fit, and the welfare effect of tax reforms

Using the model solutions, shock series were fed to the stochastic processes to produce simulated data series. The length of the draws for the series of innovations was 138, and the simulation was replicated 1000 times. Natural logarithms were taken from all series, and then the resulting logged series were run through the Hodrick-Prescott filter with a smoothing parameter equal to 100. The first 100 observations were then excluded to decrease any dependence on the initial realizations of the innovations. Average standard deviation of each variable and its correlation with output were estimated across the 1000 replications.[25] The large number of replications were implemented to average out sampling error across simulations, before comparing model moments to the ones obtained from data.

1.6.1 Relative second moments evaluation

This section compares the theoretical second moments of the simulated data series with their empirical counterparts, with special attention paid to the behavior of public sector hours and wages. Table 1.4 on the next page summarizes the empirical and simulated business cycle statistics for the two models calibrated for Germany.

In the German data, relative consumption volatility exceeds one, as the available series does not provide a breakdown into consumption of non-durables and consumption of durables.[26] Durable products behave like investment, and vary much more than non-durables, while model consumption corresponds to non-durable consumption. Since a major force in both models is consumption smoothing, as dictated by the Euler equation, both models under-predict consumption volatility and investment variability. Across models, private sector

[25] As an additional model check, the autocorrelation (ACFs) and the cross-correlation functions (CCFs) were also generated, and compared it to those computed from German data. The results of this computational experiment are presented in Appendix 1.8.5.

[26] Another possible reason could be the presence of strong habits in consumption.

Table 1.4: Business Cycle Statistics Germany, 1970-2007

	GE Data	Finn (1998)	Public Sector Union
$\sigma(y)$	0.0154	0.0165	0.0165
$\sigma(c)/\sigma(y)$	1.11	0.56 [0.49,0.62]	0.56 [0.49,0.62]
$\sigma(i)/\sigma(y)$	3.57	2.30 [2.24,2.36]	2.30 [2.24,2.36]
$\sigma(n^p)/\sigma(y)$	1.05	0.45 [0.40,0.50]	0.45 [0.40,0.49]
$\sigma(n^g)/\sigma(y)$	1.06	0.91 [0.69,1.13]	1.27 [0.98,1.56]
$\sigma(n)/\sigma(y)$	0.73	0.38 [0.33,0.43]	0.39 [0.38,0.40]
$\sigma(w^p)/\sigma(y)$	1.16	0.63 [0.59,0.68]	0.63 [0.59,0.68]
$\sigma(w^g)/\sigma(y)$	3.50	0.63 [0.59,0.68]	1.19 [0.92,1.47]
$corr(c,y)$	0.80	0.85 [0.79,0.92]	0.85 [0.79,0.92]
$corr(i,y)$	0.85	0.99 [0.98,0.99]	0.99 [0.98,0.99]
$corr(n^p,y)$	0.60	0.89 [0.84,0.93]	0.89 [0.85,0.94]
$corr(n^g,y)$	0.11	-0.05 [-0.29,0.20]	0.19 [0.04,0.43]
$corr(n,y)$	0.60	0.84 [0.78,0.91]	0.97 [0.97,0.98]
$corr(w^p,y)$	0.60	0.95 [0.92,0.97]	0.94 [0.93,0.97]
$corr(w^g,y)$	0.35	0.95 [0.92,0.97]	0.19 [0.04,0.43]
$corr(n,n^p)$	0.92	0.90 [0.86,0.95]	0.88 [0.79,0.92]
$corr(n,n^g)$	0.43	0.28 [0.06, 0.51]	0.27 [0.05,0.49]
$corr(n^p,n^g)$	0.12	-0.15 [-0.38,0.08]	-0.21 [-0.44,0.02]
$corr(n^p,w^p)$	0.21	0.70 [0.59,0.81]	0.71 [0.61,0.81]
$corr(n^g,w^g)$	-0.38	0.03 [-0.22,0.28]	1.00 [1.00,1.00]
$corr(n^g,w^p)$	0.20	0.03 [-0.22,0.28]	0.45 [0.26,0.64]
$corr(n^p,w^g)$	0.34	0.70 [0.59,0.81]	-0.21 [-0.44,0.02]
$corr(w^p,w^g)$	0.48	1.00 [1.00,1.00]	0.45 [0.26,0.65]

employment and private wage also vary less compared to data. Total employment in German data varies less than either private or public employment due to smaller variation in

the number of self-employed individuals. It is evident from Table 1.4 that the model with public sector union underestimates public wage volatility (but still significantly better than Finn (1998) along that dimension), but matches public employment quite well. Finn's model captures the volatility of public employment due to the fact that it is modeled as an exogenous stochastic process to mimic public hours time series behavior.

Both models capture the high contemporaneous correlations of main variables with output relatively well. Moreover, public sector variables are also pro-cyclical, but not as much as the models predict: Finn (1998) even predicts that public employment is countercyclical. Nevertheless, the model with the public union captures the co-movement between labor market variables, as well as their contemporaneous correlations with output quite well, compared to the alternative. The German data, as well as the model with public sector union, provide some support to the "private sector wage-leader" hypothesis. In other words, there is some evidence that public sector wages follows those in the private sector but only moderately so. The dimension where the union model fails, however, is the correlation between public sector hours and wages: in the German data, it is negative, while the public sector union model predicts a perfect positive linear relationship. The reason is that the empirical correlation can be interpreted as showing the net effect of supply and demand factors, while the model models concentrates exclusively on the supply-side forces. It is plausible to assume that due to the population aging, demand for public employees will also be high, particularly in healthcare, social security and senior care. The empirical correlation between wages is also well-captured by the model with public sector unions. In other words, empirical public sector wage follows the private sector wage to a much lesser degree. A failure of the model with public sector union is the predicted negative correlation between the two types of hours. To a certain extent, this is an artifact of the way fiscal instruments were specified. The prediction of the model along this dimension greatly improves if government consumption and investment follow AR(1) processes in levels, and thus do not react to output. Furthermore, it is a well-known fact (*e.g.* Prescott 1986, Hansen 1992) that the RBC model does not

capture private sector labor market dynamics very well.

Overall, the model with the public sector union captures the labor market dynamics in Germany, addressing dimensions that were ignored in earlier RBC models. Thus, an optimizing union in the public sector proves to be an important ingredient in RBC models when studying European labor markets with strong public sector unions. To assess the welfare cost of fiscal policy in the presence of public sector union, several fiscal experiments were performed and reported in the following subsection.

1.6.2 Welfare evaluation of fiscal regime changes

The goal of this section is to quantify the importance of endogenously-determined public sector hours for fiscal policy, relative to Finn's setup with exogenously-fixed public hours. Additionally, the explicit welfare analysis complements earlier studies in Finn (1998) and Fernandez-de-Cordoba *et al.* (2009, 2012). To understand the adjustment mechanisms after an exogenous change in fiscal policy, each tax rate in the two models is varied over the $[0, 1]$ interval. Since all three tax rates were exogenously-specified, Schmitt-Grohe and Uribe (1997) show that for a wide class of RBC models, and plausible values for model parameters, a unique long-run solution exists. When tax rates are plotted against tax revenues, Laffer curves (Laffer 1974, Schmidt-Grohe and Uribe 1997) appear: in both Finn (1998) and the public sector union model, presented in this chapter, an inverted U-shape relationship is observed between labor and capital income tax rates and total tax revenues. Thus, there are pairs of tax rates that generate the same level of tax revenue.[27] In general, increasing tax rates could lead to either an increase or a decrease in total tax revenue, depending on which side of the Laffer curve the economy is situated. For the German model economy, however, both setups place Germany on the left side of the labor and capital tax Laffer curve, as seen in Figs. 1.4-1.5. Furthermore, a change in a tax rate affects the tax receipts from other

[27]Sensitivity analysis of the effect of model parameters on the shape of the Laffer curves is performed in Appendix 1.8.6.

tax bases as well, by influencing steady-state allocations and prices. Therefore, to gain an additional insight of the effect of fiscal policy in the steady state, total tax revenue is broken down into individual tax revenues corresponding to the tax bases, and plotted as a function of each individual tax rate in Figs. 1.4-1.6, for both the public union model and Finn.

The shape of the capital tax Laffer curve, for example, presents an interesting case: an increase in τ^k leads to a negligible marginal increase in total tax revenue, since total tax revenue is essentially flat in the $\tau^k \in [0, 0.5]$ range, and for $\tau^k \in [0.5, 1]$ total revenue is negatively related to capital income tax rate.[28] The German economy features a low rate of capital income taxation, $\tau^k = 0.16$, thus the economy is situated safely away from the downward sloping segment of the Laffer curve. The reason for the flat Laffer curve is clearly seen from the breakdown in individual tax revenues as a function of capital income tax rate; all increases in capital income tax revenue are offset by corresponding decreases in labor income and consumption tax revenue. Since τ^c and τ^l are held fixed while τ^k is varied, the fall in labor income and consumption tax revenue is entirely driven by the shrinking tax bases. Across models, union framework features only slightly higher capital income and consumption revenue, and lower labor income tax revenue for each τ^k, as compared to Finn's setup.

On the other hand, labor income tax rate places Germany much closer to the peak of the labor tax Laffer curve, but still far away from the downward-sloping segment. Thus, the government could increase tax revenue by increasing τ^l. The computed total tax revenue-maximizing τ^l is approximately 50% in the union model, and 55% in Finn. As demonstrated in Fig. 1.5, the difference in computed total tax revenue with respect to labor income tax in the union model and Finn is due to the difference in the steady-state public and private hours, as well as the wage rates in the two models: Finn's model, featuring a single wage

[28] Uhlig and Trabandt (2010) find a similarly-shaped capital tax Laffer curve in an RBC model without public employment, calibrated to the EU-15 data.

rate and fixed public employment, generates both a higher total tax revenue and a higher labor income tax revenue Laffer curve, as compared to the union model.

Lastly, for the consumption tax rate, no Laffer curve is observed: within a realistic range, Fig. 1.6 shows no negative relationship between τ^c and tax revenue. The reason for this is as follows: In the model parameterizations the risk aversion value $\alpha > 1$, thus the income effect dominates the substitution effect: when τ^c increases, labor supply and capital stock increase while consumption falls.[29] As argued in Trabandt and Uhlig (2010), a consumption tax Laffer curve arises if $\alpha < 1$, so that after an increase in τ^c, the substitution effect dominates the income effect and hours and capital stock fall together with consumption. In the union model, public employment also falls, driven by the fall in tax revenue. In the borderline case, when $\alpha = 1$ (log-utility), the two effects offset one another. Again, no consumption tax Laffer curve occurs.

[29]Note that the increase in private hours and capital, driven by the increase in consumption tax rate does not translate into an increase in the corresponding tax revenue category. In addition, a higher τ^c leads to lower steady-state consumption, but a higher consumption revenue.

Figure 1.4: Capital tax Laffer curve

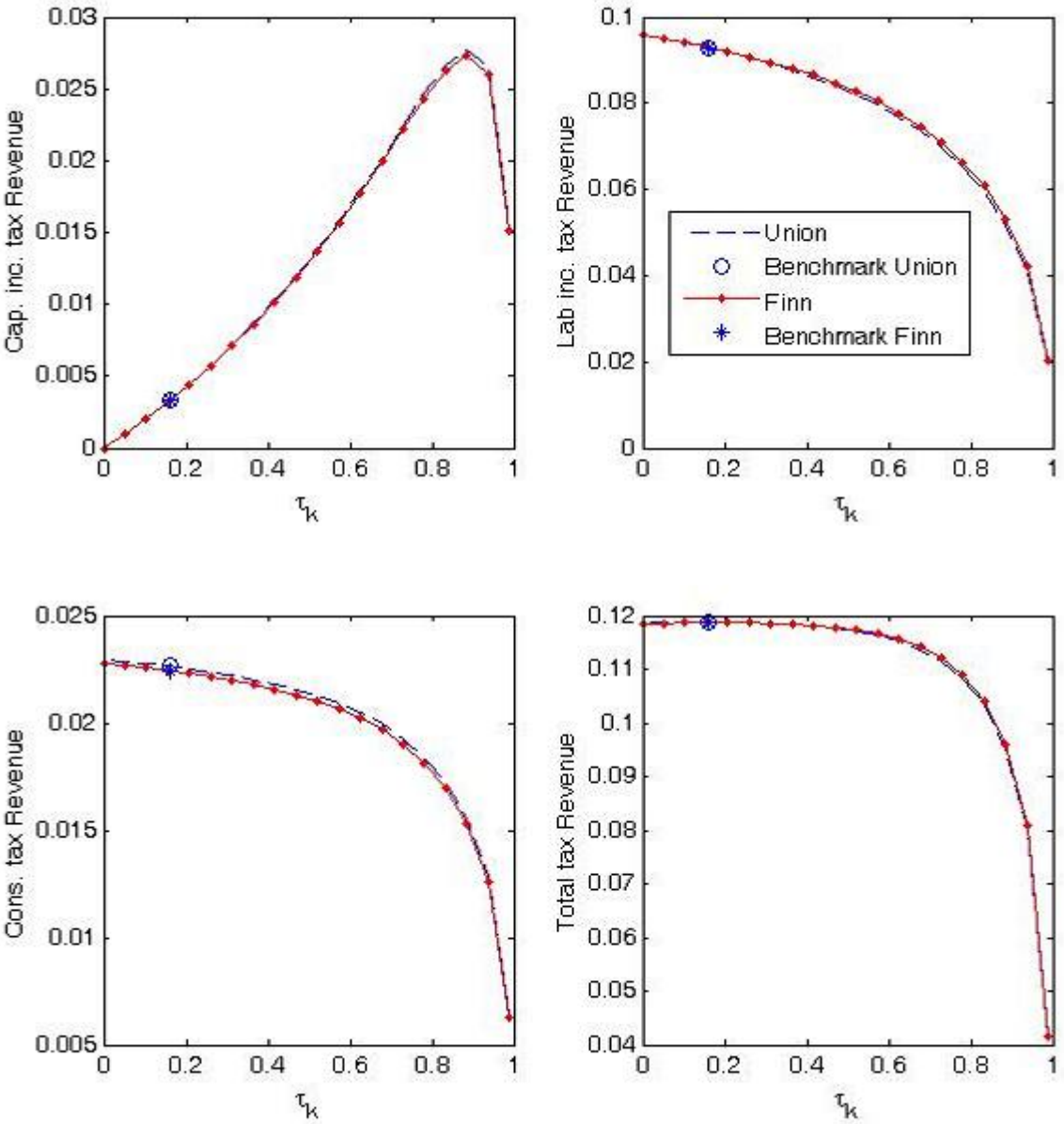


Figure 1.5: Labor tax Laffer curve

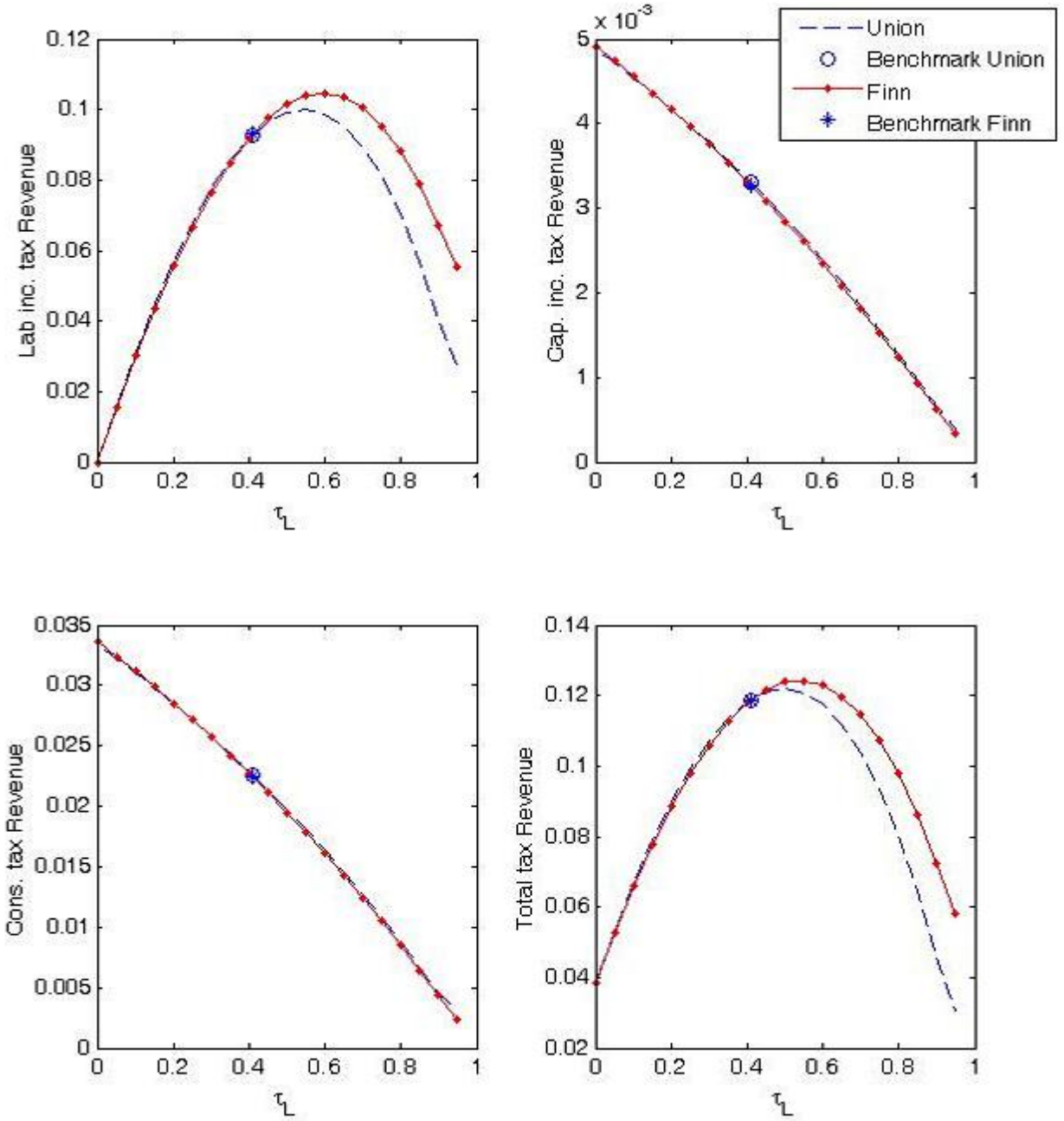
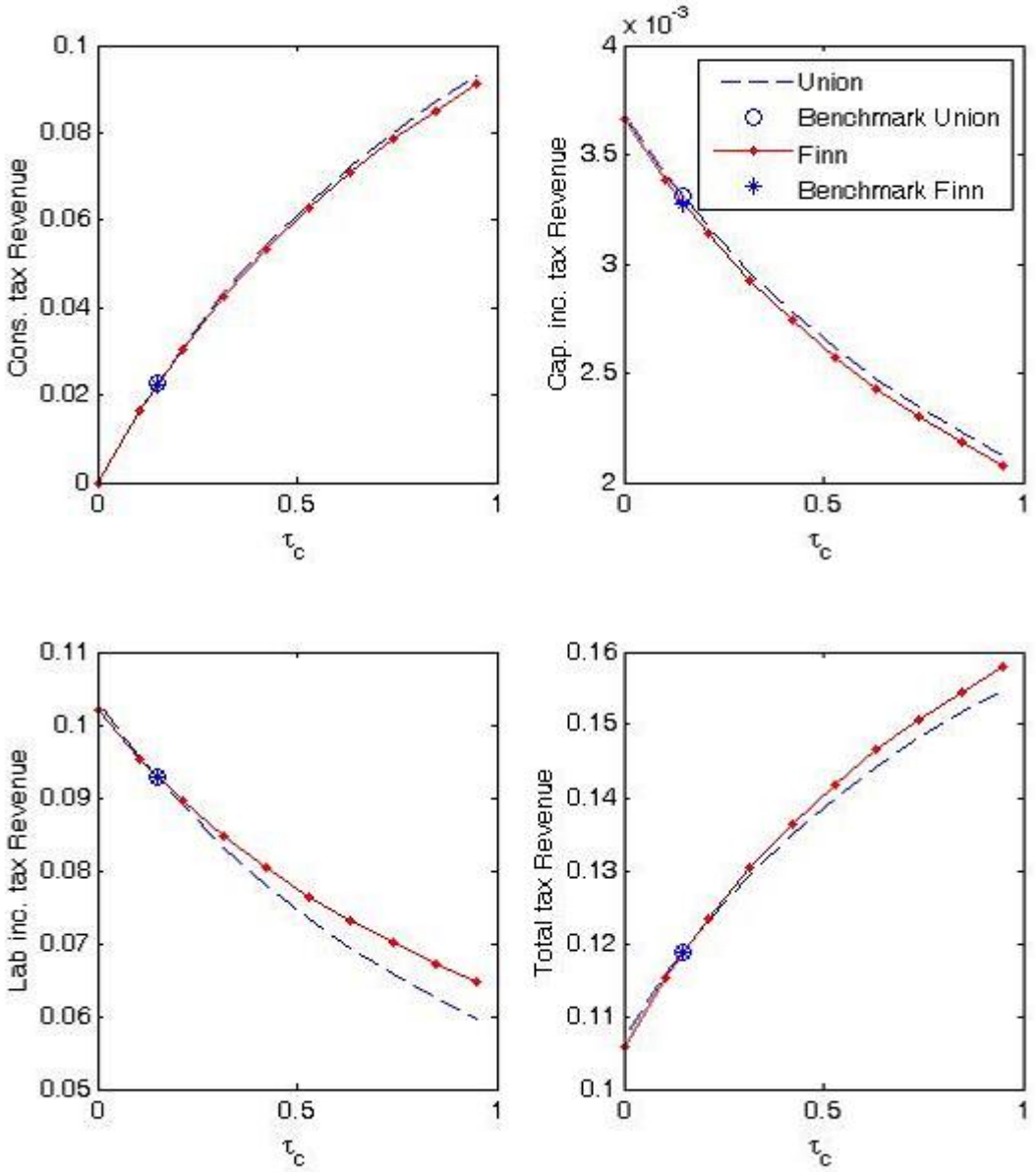


Figure 1.6: Consumption tax Laffer curve



Across models, the exogenous public hours in Finn (1998) produce a slightly flatter total tax revenue curve as a function of τ^c. In particular, the important difference across the setups is a steeper labor income tax revenue curve in the union model *vs.* a flatter labor income tax revenue curve in Finn's (1998) model. The slope of the labor tax revenue curve is determined by the elasticity of hours with respect to changes in the tax rate. In both models, a higher τ^c decreases the labor wedge, $(1-\tau^l)/(1+\tau^c)$. However, the response in hours is larger in the case of the union model, which features endogenous public sector hours, as compared to Finn's setup, where n^g is held fixed.

After characterizing and comparing the shapes of the Laffer curves in both models, this section proceeds to welfare-evaluate the effects of different tax regimes. This is achieved through several normalized fiscal policy experiments. In all of the experiments a combination of tax rate changes will be specified so that total tax revenue is kept constant.[30] The general usefulness of this approach is that it separates tax and spending issues. In the framework considered in this chapter, however, public sector labor income appears on both sides of the government budget constraint. In addition, the substitutability/complementarity of the capital and labor input in the Cobb-Douglas production function, the substitutability between consumption and labor, as well as the substitutability between consumption and investment implies that changes in a single tax rate will affect the tax revenue generated from the other two tax bases.

Following Lucas (1987), the approach taken is to compute the compensatory variation in consumption.[31] In other words, this section calculates the percentage of compensating consumption, ζ, that is to be given to the household to make it indifferent between the two regimes. The initial regime for Germany is as described in Section 1.2, with the calibration and steady state solution presented in Section 1.4. The value of ζ is calculated for all restrictive fiscal policy scenarios, where a positive (negative) value indicates a welfare gain (loss).

[30] As it will be seen in the next section, not all such combinations will be feasible.

[31] Detailed derivations are shown in Appendix 1.8.7.

Three different policies will be examined: a 1% increase in capital income, labor income and consumption tax rate will be considered. In order to keep total tax revenue constant, whenever a tax rate increases, one of the other two tax rates will be allowed to adjust, holding all other model parameters fixed.[32]

Revenue-neutral increase in capital income tax rate

This subsection discusses the steady-state effect of a 1% increase in τ^k, with results presented in Table 1.5 below.

Table 1.5: Welfare gains/costs of 1 % increase in τ^k in Germany

Model	τ^l fixed, τ^c adjusts	τ^c fixed, τ^l adjusts
	$\tau^c = 0.4033 \uparrow (25.52\%)$	$\tau^c = 0.1481$
	$\tau^l = 0.4085$	$\tau^l = 0.5535 \uparrow (14.50\%)$
Union	$\zeta = -0.2093$	$\zeta = -0.2425$
	$\tau^c = 0.3657 \uparrow (21.76\%)$	$\tau^c = 0.1481$
	$\tau^l = 0.3596$	$\tau^l = 0.5415 \uparrow (13.30\%)$
Finn	$\zeta = -0.1430$	$\zeta = -0.1745$

Higher capital income tax rate enters the Euler equation and thus decreases the steady state private capital-to-output ratio. Since total revenue with respect to τ^k is relatively flat in both models, the increase in capital income tax essentially does not change total revenue. Variations in labor income tax rate, or consumption tax rate, however, are very distortionary, as they operate through the marginal rate of substitution. A higher labor, or a higher consumption tax rate, lower private hours. From the complementarity of hours and capital in the production function, capital stock also falls. Lower levels of labor and capital inputs shrink output, which in turn decreases consumption. This change in steady-state allocation

[32]For example, η and g^{Ty} in the union model, and g^{Ty} in Finn, are held fixed at the values obtained in the original steady-state computation.

requires additional adjustment in the varying tax rate (τ^l or τ^c) to preserve revenue neutrality. The computational experiment performed shows that in either case, the adjusting tax rate has to change significantly to satisfy the revenue neutrality restriction. Across models, consumption tax is the less distortive instrument. Additionally, the computed welfare cost is higher in the union model by 6.63% (6.8% when τ^l varies) due to the endogenous response of public hours, which requires significantly larger tax rate increases in the union model.

Revenue-neutral increase in labor income tax rate

In this case, an increase in τ^l affects the marginal rate of substitution (MRS) between steady-state hours and consumption. As in the previous subsection, the analysis is split into two sub-cases, with results summarized in Table 1.6 below.

Table 1.6: Welfare gains/costs of 1% increase in τ^l in Germany

Model	τ^k fixed, τ^c adjusts	τ^c fixed, τ^k adjusts
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^c = 0.3862 \uparrow (23.81\%)$	$\tau^c = 0.1481$
Union	$\zeta = -0.2105$	$\zeta = N/A$
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^c = 0.35 \uparrow (20.19\%)$	$\tau^c = 0.1481$
Finn	$\zeta = -0.1444$	$\zeta = N/A$

When the consumption tax rate is the adjusting rate, a 23.81% increase in τ^c is required in the union model. Again, Finn's setup generates much smaller welfare cost as compared to the union model, as the setup with exogenous public sector hours requires consumption tax rate to increase by 20.19% to preserve the initial level of tax revenues.[33] In both models, the increase in the consumption tax rate relative to the increase in the labor income tax rate is larger. Therefore, the labor wedge, $(1 - \tau^l)/(1 + \tau^c)$, decreases in both cases, which leads to an increase in private hours. Since hours and private physical capital are complements in

[33]Note that the higher fall in a tax rate results in a lower level of distortions in the economy.

the production function, the increase in labor input raises the marginal product of private capital, hence real interest rate will increase as well. The higher return to capital encourages investment, and thus steady-state private capital stock expands. Following the expansion in capital input, output increases as well. In turn, higher output leads to higher consumption. The increase in consumption, however, is dominated by the increase in hours, so long-run welfare decreases relative to the one obtained in the initial steady-state. In addition, in the union model, there is an important feedback effect, which further increases welfare cost. This effect works to increase public hours, as a result of the higher tax revenue. In effect, endogenously-determined public hours add to the allocative distortions in the union model. Public hours enter the MRS condition, and thus necessitate a much larger adjustment in the union economy, as compared to Finn's framework. The presence of endogenously-determined public hours and wages adds 6.6% to the computed welfare loss.

In the second sub-case, when capital income tax rate varies in response to the increase in labor income tax, no reasonable level of τ^k (i.e. $\tau^k \in [-1, 1]$) exists that satisfies the revenue neutrality restriction. This is a straightforward consequence of the relatively flat Laffer curve with respect to the capital income tax rate, as demonstrated in the section on capital tax Laffer curve. Additionally, in both models the share of capital income tax revenue is less than 3% (note the model features depreciation allowance), which is very small when compared to consumption tax revenue share (22%) and labor income tax revenue share (75%). Thus, capital income tax rate is not a suitable instrument for fiscal adjustment, due to its limited ability to affect total tax revenue, while at the same time greatly distorting capital and labor decisions.

Revenue-neutral increase in consumption tax rate

The increase in τ^c affects the marginal rate of substitution between steady-state hours and consumption; hence, the effect on allocations is qualitatively similar to the one described in the previous section. In the first sub-case of this scenario (Table 1.7 below), when labor income tax rate changes to preserve the tax revenue, the labor income tax rate needs to increase by 12.73% and 16.96% in Finn and the union model, respectively.

Table 1.7: Welfare gains/costs of 1% increase in τ^c in Germany

Model	τ^k fixed, τ^l adjusts	τ^l fixed, τ^k adjusts
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^l = 0.5781 \uparrow (16.96\%)$	$\tau^l = 0.4085$
Union	$\zeta = -0.2404$	$\zeta = N/A$
	$\tau^k = 0.1603$	$\tau^k = N/A$
	$\tau^l = 0.5358 \uparrow (12.73\%)$	$\tau^l = 0.4085$
Finn	$\zeta = -0.1724$	$\zeta = N/A$

This upward change in the labor income tax rate is significantly larger than the increase in consumption tax rate. The resulting decrease in the effective labor wedge, $(1-\tau^l)/(1+\tau^c)$, affects labor supply and consumption decisions: the household responds to the dominating income effect and supplies more hours in the private sector. Next, the higher level of labor input in the production function raises both output and the interest rate. The higher return to private physical capital leads to an increase in investment, which adds to the capital stock and expands output. The positive wealth effect then translates into an increase in consumption. However, the higher consumption is offset by the increase in hours, so welfare decreases. Additionally, the increase in hours is higher in the union model, driven by the endogenously-determined public hours, which positively co-move with private hours. Thus the required increases in labor income tax rates produce nearly 6.8% larger welfare losses in the union model, a result attributed to the endogenously-determined public hours. The case when τ^k is the adjusting tax rate unravels exactly as the case when τ^l increased by 1%

and τ^k was the adjusting tax rate. Intuitively, both an increase in τ^c and τ^l decrease the effective labor wedge, thus the resulting adjustments through τ^k are qualitatively smilar. Again, there is no feasible capital income tax rate that preserves revenue neutrality.

Overall, the experiments performed in this section uncovered some important limitations of Finn's model with exogenous public hours. The presence of endogenously-determined public sector hours and wage rate was shown to generate important interactions, which add to the distortionary effect of taxes. If ignored, the long-run welfare cost of revenue-neutral tax increase policies could be significantly underestimated. To strengthen the results obtained so far, a robustness check in the next subsection will consider tax reform scenarios that depart from the revenue neutrality restriction.

Non-revenue-neutral tax rate increases

In contrast to revenue-neutral policy experiments, this section quantifies the welfare effect of a contractionary fiscal regime, when the increases in one tax rate are not offset by a change in another tax rate. The exogenously-specified common objective in both models is to increase total tax revenue by 10% and 5%, respectively, by allowing a single tax rate to vary, while keeping all other parameters fixed at their initial steady-state values.

Table 1.8: Welfare effect of 10%-tax-revenue increase

Model	τ^k	τ^l	τ^c
Union	N/A	0.6405 ↑(+23.20%)	0.5033 ↑(+35.52%)
ζ	N/A	-0.3432	-0.2406
Finn	N/A	0.6032 ↑(+19.47%)	0.4090 ↑(+25.09%)
ζ	N/A	-0.2332	-0.1787

As in the revenue neutral experiments, it is evident from Table 1.8 above that there is no feasible capital tax rate, which can satisfy the objective, a result which follows directly from the flatter Laffer curve with respect to τ^k. Next, if labor taxes are the instrument used to

achieve the targeted revenue revenue, the required increase in τ^l in Finn, is almost 4.3% smaller compared to the union model. This outcome is due to the exogenously-fixed public sector hours in Finn: the distortions caused by an increase in τ^l, and thus in the effective labor wedge, which appears in the MRS condition are smaller. In addition, in both models, the new level of τ^l places the German economy on the downward-sloping segment of the labor tax Laffer curve.[34]

As shown in Table 1.8 on the previous page, across both models, changing τ^c is the cheaper option to raise additional tax revenue, measured in terms of the welfare cost incurred. Additionally, the required change in consumption tax rates to achieve 10% increase in total revenue, is approximately 10% larger in the union model. The two models produce significant differences in terms of the magnitude of the tax rate changes required to achieve a pre-specified tax revenue increase. When public hours are considered to be endogenously-determined in the model, the tax rates increase by a significantly larger amount. This is a new result in the literature, with important policy implications.

For the 5% tax-revenue-increase objective scenario, the results reported in Table 1.9 below are qualitatively very similar to the outcomes in the 10% revenue increase scenario.

Table 1.9: Welfare effect of 5%-tax-revenue increase

Model	τ^k	τ^l	τ^c
Union	N/A	0.6065 ↑(+19.80%)	0.5033 ↑(+35.52%)
ζ	N/A	-0.2988	-0.2406
Finn	N/A	0.5728 ↑(+16.43%)	0.4300 ↑(+28.19%)
ζ	N/A	-0.2026	-0.1600

Again, there is no feasible capital income tax rate to achieve the new objective. However, when τ^l is the instrument used to increase total tax revenue, the two models generate differ-

[34]The computed revenue-maximizing τ^l is 50% in the union model, and 55% in Finn.

ent predictions: after the resulting increase in labor tax rate, the German economy is again situated on the slippery slope of the respective Laffer curve in the union model, while Finn (1998) places the economy on its upward segment. In terms of welfare loss, consumption tax rate is again the preferred instrument to achieve the 5% total tax revenue increase. Finally, the new tax rate levels, as well as the welfare costs are higher in the public sector union model, due to the additional allocative distortion caused by the endogenous adjustment of public hours.

1.7 Summary and Conclusions

Motivated by the highly-unionized public sectors, the high public shares in total employment, and public sector wage premia observed in most post-WWII European economies, this chapter examined the role of public sector unions in a DSGE framework. A strong union, operating in a largely non-market sector was shown to be relevant for business cycle fluctuations, and when evaluating the welfare effects of fiscal policy. Following Fernandez-de-Cordoba *et al.* (2009), an optimizing public sector union was incorporated in a real business cycle model with valuable government consumption and productive public investment. The RBC model generated cyclical behavior in hours and wages that is consistent with data behavior in an economy with highly-unionized public sector, Germany during the period 1970-2007. The main findings are: (i) the model with collective bargaining performs reasonably well *vis-a-vis* data; (ii) overall, the model with collective bargaining in the public sector is an improvement over a similar model with exogenous public employment, namely Finn (1998); (iii) endogenously-determined public wage and hours add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. In addition, the endogeneity of public hours in the union model generates greater changes in tax rates to achieve a pre-specified increase in tax revenue and produce significantly higher welfare losses compared to Finn's (1998) model with exogenous public sector hours. Thus, endogenous public hours are quantitatively important model ingredient when evaluating fiscal policy. In

particular, ignoring the positive co-movement between public and private wage and hours leads to a significant underestimation of the welfare effect of tax regime changes. Therefore, a model with endogenous public sector labor market choice made by the household is considered in Ch.2.

There are some limitations of the model setup: the dynamics of public hours and wages in the model is identical, which constraints the ability of the model to match well the behavior in the two variables simultaneously. Furthermore, the theoretical framework ignores household's increased demand for labor-intensive programs such as healthcare, education and social security, which would require additional employment in the public sector. This is quickly remedied in Chapters 2 and 3 in this thesis, where those issues are addressed and thoroughly investigated. Next, in reality public sector unions and government usually bargain over nominal wage increases, and against redundancies. They do not negotiate hours and the level of the real wage directly. Before engaging in negotiations, unions also take into consideration many macroeconomic indicators. Labor productivity in the private sector and the private wage, are often used as a leverage in the negotiations over the public wage. The simple public sector union objective used in this chapter ignores other possible demands by unions, such as job security, work conditions, health and safety considerations, government pensions, other non-monetary benefits, etc. Indeed, some of these factors can be incorporated in the union utility function and thus extend the basic model. Nevertheless, the importance of public sector unions is evident even from the reduced-form representation used in this chapter. In addition, this chapter suggests that the model with public sector unions could produce potentially useful insights regarding optimal taxation. The potentially interesting issue of public sector union power in the context of a Ramsey problem of setting tax rates in an optimal way is left for future research.

Given the overall reasonable performance of the model with public sector union, the organizational structure of public sector labor market deserves further and deeper investigation as

well. Von Mises (1944), Parkinson (1957) and Tullock (1974) are among the first to suggest that bureaucracy itself has been one of the most important factors affecting economic activity, mainly through the development and implementations of different legislative procedures, rules and regulations. In particular, civil servants are usually insulated from market forces in both the input and output markets: many government positions do not have a close equivalent in the private sector, and there is usually no direct way to measure performance. A closer analysis of bureaucrats' behavior, and their effect on aggregate fluctuations in European economies would be a logical extension to the work in this chapter, and thus discussed at length in Chapter 3.

1.8 Technical Appendix

1.8.1 Optimality conditions

Firm's problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} - r_t \quad = \quad 0 \tag{1.8.1.1}$$

where $(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu}$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, *i.e* the interest rate.

$$r_t \quad = \quad (1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} \tag{1.8.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} = r_t K_t^p \quad \text{or} \quad (1-\theta)Y_t = r_t K_t^p \tag{1.8.1.3}$$

because

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} = A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta}(K_t^g)^{\nu} \quad = \quad (1-\theta)Y_t$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1}(K_t^g)^{\nu} - w_t^p = 0 \text{ or } w_t^p = \theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1}(K_t^g)^{\nu} \tag{1.8.1.4}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, *i.e* the hourly

wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p \theta A_t (K_t^p)^{1-\theta} (N_t^p)^{\theta-1} (K_t^g)^{\nu} = w_t^p N_t^p \quad \text{or} \quad \theta Y_t = w_t^p N_t^p \tag{1.8.1.5}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \;=\; Y_t - r_t K_t^p - w_t^p N_t^p = Y_t - (1-\theta) Y_t - \theta Y_t = 0 \tag{1.8.1.6}$$

Indeed, in equilibrium, economic profits are zero.

Consumer problem

Set up the Lagrangian

$$\begin{aligned}\mathcal{L}(C_t, K_{t+1}^p, N_t^p; \Lambda_t) = E_0 \sum_{t=0}^{\infty} \Bigg\{ & \frac{\left[(C_t + \omega G_t^c)^{\psi} (1-N_t)^{(1-\psi)} \right]^{1-\alpha} - 1}{1-\alpha} + \\ & + \Lambda_t \Bigg[(1-\tau^l)(w_t^p N_t^p + w_t^g N_t^g) + (1-\tau^k) r_t K_t^p + \\ & + \tau^k \delta^p K_t^p - (1+\tau^c) C_t - K_{t+1}^p + (1-\delta) K_t^p + G_t^T \Bigg] \Bigg\}\end{aligned} \tag{1.8.1.7}$$

This is a concave programming problem, so the FOCs, together with the additional, boundary ("transversality") conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\begin{aligned}\beta^t \Bigg\{ & \frac{1-\alpha}{1-\alpha} \left[(C_t + \omega G_t^c)^{\psi} (1-N_t^h)^{(1-\psi)} \right]^{-\alpha} \times \\ & \psi (C_t + \omega G_t^c)^{\psi-1} (1-N_t^h)^{(1-\psi)} - \Lambda_t (1+\tau^c) \Bigg\} = 0\end{aligned} \tag{1.8.1.8}$$

Cancel the β^t and the $1-\alpha$ terms to obtain

$$\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}\psi(C_t+\omega G_t^c)^{\psi-1}(1-N_t)^{(1-\psi)}-\Lambda_t(1+\tau^c)=0 \quad (1.8.1.9)$$

Move Λ_t to the right so that

$$\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}\psi(C_t+\omega G_t^c)^{\psi-1}(1-N_t)^{(1-\psi)}=\Lambda_t(1+\tau^c) \quad (1.8.1.10)$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K_{t+1}^p (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K_{t+1}^p}=0$. That will result in the following expression

$$\beta^t\left\{-\Lambda_t+E_t\Lambda_{t+1}\left[(1-\tau^k)r_{t+1}+\tau^k\delta^p+(1-\delta^p)\right]\right\} = 0 \quad (1.8.1.11)$$

Cancel the β^t term to obtain

$$-\Lambda_t+\beta E_t\Lambda_{t+1}\left[(1-\tau^k)r_{t+1}+\tau^k\delta^p+(1-\delta^p)\right]=0 \quad (1.8.1.12)$$

Move Λ_t to the right so that

$$\beta E_t\Lambda_{t+1}\left[(1-\tau^k)r_{t+1}+\tau^k\delta^p+(1-\delta^p)\right]=\Lambda_t \quad (1.8.1.13)$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1}=(1-\theta)\frac{Y_{t+1}}{K_{t+1}^p}$$

$$\beta E_t\Lambda_{t+1}\left[(1-\tau^k)(1-\theta)\frac{Y_{t+1}}{K_{t+1}^p}+\tau^k\delta^p+(1-\delta^p)\right]=\Lambda_t \quad (1.8.1.14)$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N_t^p (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N_t^p}=0$. That will result in the following expression

$$\beta^t\left\{\frac{1-\alpha}{1-\alpha}\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}\times\right.$$
$$\left.(1-\psi)(C_t+\omega G_t^c)^{\psi}(1-N_t)^{-\psi}(-1)+\Lambda_t(1-\tau^l)w_t^p\right\} = 0 \quad (1.8.1.15)$$

Cancel the β^t and the $1-\alpha$ terms to obtain

$$\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}(1-\psi)\left[\frac{C_t+\omega G_t^c}{1-N_t}\right]^{\psi}(-1)+\Lambda_t(1-\tau^l)w_t^p=0 \quad (1.8.1.16)$$

Rearranging, one can obtain

$$\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}(1-\psi)(C_t+\omega G_t^c)^{\psi}(1-N_t)^{-\psi}=\Lambda_t(1-\tau^l)w_t^p \quad (1.8.1.17)$$

Plug in the expression for w_t^h, that is,

$$w_t^p=\theta\frac{Y_t}{N_t^p} \quad (1.8.1.18)$$

into the equation above. Rearranging, one can obtain

$$\left[(C_t+\omega G_t^c)^{\psi}(1-N_t)^{(1-\psi)}\right]^{-\alpha}(1-\psi)(C_t+\omega G_t^c)^{\psi}(1-N_t)^{-\psi}=\Lambda_t(1-\tau^l)\theta\frac{Y_t}{N_t^p} \quad (1.8.1.19)$$

Transversality conditions need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t\to\infty}\beta^t\Lambda_t K_{t+1}^p = 0 \quad (1.8.1.20)$$

The Objective Function of a Public Sector Union: Derivation

This subsection shows that the objective function in the government sector is a generalized version of Stone-Geary monopoly union utility function used in Dertouzos and Pencavel (1981) and Brown and Ashenfelter (1986). The utility function is

$$V(w^g,N^g)=(w^g-\bar{w}^g)^{\phi}(N^g-\bar{N}^g)^{(1-\phi)}, \quad (1.8.1.21)$$

where ϕ and $1-\phi$ are the weights attached to public wage and hours, respectively, and $\bar{w}^g$ and $\bar{N}^g$ denote subsistence wage rate and hours. Since there is no minimum wage in the model, $\bar{w}^g=0$. Additionally, as public hours are assumed to be unproductive, it follows that $\bar{N}^g=0$ as well. Therefore, the utility function simplifies to

$$V(w^g,N^g)=(w^g)^{\phi}(N^g)^{(1-\phi)}. \quad (1.8.1.22)$$

Doiron (1992) uses a generalized representation, which encompasses (1.11.1.22) as a special case when $\rho \to 0$.

$$\left[\phi(N^g)^{-\rho} + (1-\phi)(w^g - \bar{w})^{-\rho}\right]^{-1/\rho}, \tag{1.8.1.23}$$

when $\bar{w} = 0$, the function simplifies to

$$\left[\phi(N^g)^{-\rho} + (1-\phi)(w^g)^{-\rho}\right]^{-1/\rho}, \tag{1.8.1.24}$$

Union objective function used in the chapter is very similar to Doiron's (1992) simplified version:

$$\left[(N^g)^{\rho} + \eta(w^g)^{\rho}\right]^{1/\rho}, \tag{1.8.1.25}$$

can be transformed to

$$\left[(N^g)^{\rho} + \frac{\phi}{(1-\phi)}(w^g)^{\rho}\right]^{1/\rho}, \tag{1.8.1.26}$$

Collecting terms under common denominator

$$\left[\frac{(1-\phi)}{(1-\phi)}(N^g)^{\rho} + \frac{\phi}{(1-\phi)}(w^g)^{\rho}\right]^{1/\rho}, \tag{1.8.1.27}$$

Factoring out the common term

$$\left[\frac{1}{1-\phi}\right]^{1/\rho}\left[(1-\phi)(N^g)^{\rho} + \phi(w^g)^{\rho}\right]^{1/\rho}, \tag{1.8.1.28}$$

Note that the constant term $\left[\frac{1}{1-\phi}\right]^{1/\rho} > 0$ can be ignored, as utility functions are invariant to positive affine transformations. After rearranging terms, the equivalent function

$$\tilde{V} = \left[\phi(w^g)^{\rho} + (1-\phi)(N^g)^{\rho}\right]^{1/\rho}. \tag{1.8.1.29}$$

Take natural logarithms from both sides to obtain

$$\ln \tilde{V} = \frac{1}{\rho}\ln\left[\phi(w^g)^{\rho} + (1-\phi)(N^g)^{\rho}\right]. \tag{1.8.1.30}$$

Take the limit $\rho \to 0$

$$\lim_{\rho\to 0}\ln \tilde{V} = \lim_{\rho\to 0}\frac{\ln\left[\phi(w^g)^{\rho} + (1-\phi)(N^g)^{\rho}\right]}{\rho} \tag{1.8.1.31}$$

Apply L'Hopital's Rule on the right-hand-side (R.H.S.) to obtain

$$\lim_{\rho\to 0} \ln \tilde{V} = \lim_{\rho\to 0} \frac{\frac{\partial}{\partial\rho} \ln \left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]}{\frac{\partial\rho}{\partial\rho}} \tag{1.8.1.32}$$

Thus

$$\ln \tilde{V} = \lim_{\rho\to 0} \frac{\left[\phi(w_t^g)^\rho \ln w^g + (1-\phi)(N^g)^\rho \ln N^g\right] / \left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]}{1} \tag{1.8.1.33}$$

Simplify to obtain

$$\ln \tilde{V} = \frac{\lim_{\rho\to 0}\left[\phi(w_t^g)^\rho \ln w^g + (1-\phi)(N^g)^\rho \ln N^g\right]}{\lim_{\rho\to 0}\left[\phi(w^g)^\rho + (1-\phi)(N^g)^\rho\right]} = \frac{\phi \ln w^g + (1-\phi)\ln N^g}{\phi + (1-\phi)} \tag{1.8.1.34}$$

Therefore,

$$\ln \tilde{V} = \phi \ln w^g + (1-\phi)\ln N^g. \tag{1.8.1.35}$$

Exponentiate both sides of the equation to obtain

$$e^{\ln \tilde{V}} = e^{\phi \ln w^g + (1-\phi)\ln N^g}. \tag{1.8.1.36}$$

Thus

$$\tilde{V} = e^{\ln(w^g)^\phi + \ln(N^g)^{(1-\phi)}}. \tag{1.8.1.37}$$

or

$$\tilde{V} = e^{\ln(w^g)^\phi (N^g)^{(1-\phi)}}. \tag{1.8.1.38}$$

Finally,

$$\tilde{V} = (w^g)^\phi (N^g)^{(1-\phi)} \tag{1.8.1.39}$$

Furthermore, government period budget constraint indeed could be regarded as serving the role of a labor demand function. Additionally, the public sector demand curve will be subject

to shocks, resulting from innovations to the fiscal shares. The balanced budget assumption is thus important in the model setup to guarantee easy model tractability. Since public sector wage bill is a residual, if the government sector wage rate is increased, then public sector hours need to be decreased. Additionally, government period budget constraint can be expressed in the form $N^g = N^g(w^g)$ as

$$N^g = \frac{\tau^l w^p N^p + \tau^k (r - \delta^p) K^p + \tau^c C - G^c - G^i - G^T}{(1 - \tau^l) w^g} \tag{1.8.1.40}$$

Therefore, the problem in the government sector is reshaped in the standard formulation in the union literature:

$$\max_{w^g, N^g} V(w^g, N^g) \quad \text{s.t.} \quad N^g = N^g(w^g) \tag{1.8.1.41}$$

Since the public sector union optimizes over both the public wage and hours, the outcome is efficient. The solution pair is on the contract curve (obtained from the FOCs), at the intersection point with the labor demand curve (government budget constraint).

Public sector union optimization problem

The union solves the following problem:

$$\max_{w_t^g, N_t^g} \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{1/\rho} \tag{1.8.1.42}$$

s.t

$$G_t^c + G_t^T + G_t^i + w_t^g N_t^g = \tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l [w_t^p N_t^p + w_t^g N_t^g] \tag{1.8.1.43}$$

Setup the Lagrangian

$$\mathcal{V}(w_t^g, N_t^g; \nu_t) = \max_{w_t^g, N_t^g} \Bigg\{ \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{1/\rho}$$
$$-\nu_t \Big[G_t^c + G_t^T + G_t^i + w_t^g N_t^g - \tau^c C_t - \tau^k r_t K_t^p + \tau^k \delta^p K_t - \tau^l [w_t^p N_t^p + w_t^g N_t^g] \Big] \Bigg\} \tag{1.8.1.44}$$

Optimal public employment is obtained, when the derivative of the government Lagrangian is set to zero, i.e $\mathcal{V}_{N_t^g} = 0$

$$(1/\rho) \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} \rho (N_t^g)^{\rho-1} - (1-\tau^l) \nu_t w_t^g = 0 \tag{1.8.1.45}$$

or, when ρ is canceled out and $(1-\tau^l)\nu_t w_t^g$ put to the right

$$\left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} (N_t^g)^{\rho-1} = (1-\tau^l) \nu_t w_t^g \tag{1.8.1.46}$$

Optimal public wage is obtained, when the derivative of the government Lagrandean is set to zero, i.e $\mathcal{V}_{w_t^g} = 0$

$$(1/\rho) \left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} \eta \rho (w_t^g)^{\rho-1} - (1-\tau^l) \nu_t N_t^g = 0 \tag{1.8.1.47}$$

or, when ρ is canceled out and$(1-\tau^l)\nu_t N_t^g$ term put to the right

$$\left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} \eta (w_t^g)^{\rho-1} = (1-\tau^l) \nu_t N_t^g \tag{1.8.1.48}$$

Divide (1.8.1.46) and (1.8.1.48) side by side to obtain

$$\frac{\left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} (N_t^g)^{\rho-1}}{\left[(N_t^g)^\rho + \eta (w_t^g)^\rho \right]^{(1/\rho)-1} \eta (w_t^g)^{\rho-1}} = \frac{(1-\tau^l) \nu_t w_t^g}{(1-\tau^l) \nu_t N_t^g} \tag{1.8.1.49}$$

Cancel out the common terms

$$\frac{(N_t^g)^{\rho-1}}{\eta(w_t^g)^{\rho-1}} = \frac{w_t^g}{N_t^g} \tag{1.8.1.50}$$

Now cross-multiply to obtain

$$\frac{(N_t^g)^{\rho}}{\eta} = (w_t^g)^{\rho} \tag{1.8.1.51}$$

Hence

$$w_t^g = \left(\frac{1}{\eta}\right)^{1/\rho} N_t^g \tag{1.8.1.52}$$

The wage bill expression, which is obtained after simple rearrangement of the government budget constraint, is as follows

$$w_t^g N_t^g = \frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^T - G_t^i}{1-\tau^l} \tag{1.8.1.53}$$

Use the wage bill equation and the relationship between public wage and employment in order to obtain

$$w_t^g = \eta^{-\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^T - G_t^i}{1-\tau^l}\right]^{\frac{1}{2}} \tag{1.8.1.54}$$

and

$$N_t^g = \eta^{\frac{1}{2\rho}} \left[\frac{\tau^c C_t + \tau^k r_t K_t^p - \tau^k \delta^p K_t + \tau^l w_t^p N_t^p - G_t^c - G_t^T - G_t^i}{1-\tau^l}\right]^{\frac{1}{2}} \tag{1.8.1.55}$$

1.8.2 Per capita stationary DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, i.e $Z_t^h = Z_t = z_t$. The system of equations that describes the DCE is as follows:

$$y_t = a_t (k_t^p)^{1-\theta} (n_t^p)^{\theta} (k_t^g)^{\nu} \tag{1.8.2.1}$$

$$y_t = c_t + g_t^c + g_t^i + k_{t+1}^p - (1-\delta^p) k_t^p \tag{1.8.2.2}$$

$$\psi (c_t + \omega g_t^c)^{\psi(1-\alpha)-1} (1 - n_t^p - n_t^g)^{(1-\alpha)(1-\psi)} = (1+\tau^c)\lambda_t \tag{1.8.2.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[1 - \delta^p + (1-\tau^k)(1-\theta) \frac{y_{t+1}}{k_{t+1}^p} + \tau^k \delta^p \right] \tag{1.8.2.4}$$

$$(1-\psi)(c_t + \omega g_t^c) = \psi (1 - n_t^p - n_t^g) \frac{(1-\tau^l)}{(1+\tau^c)} \theta \frac{y_t}{n_t^p} \tag{1.8.2.5}$$

$$k_{t+1}^p = i_t + (1-\delta^p) k_t^p \tag{1.8.2.6}$$

$$k_{t+1}^g = g_t^i + (1-\delta^g) k_t^g \tag{1.8.2.7}$$

$$g_t^i = g_t^{iy} y_t \tag{1.8.2.8}$$

$$g_t^c = g_t^{cy} y_t \tag{1.8.2.9}$$

$$g_t^T = g^{Ty} y_t \tag{1.8.2.10}$$

$$w_t^p = \theta \frac{y_t}{n_t^p} \tag{1.8.2.11}$$

$$r_t = (1-\theta) \frac{y_t}{k_t^p} \tag{1.8.2.12}$$

$$w_t^g = \eta^{-\frac{1}{2\rho}} \left[\frac{\tau^c c_t + \tau^k r_t k_t^p - \tau^k \delta^p k_t^p + \tau^l w_t^p n_t^p - g_t^c - g_t^t - g_t^i}{1 - \tau^l} \right]^{\frac{1}{2}} \tag{1.8.2.13}$$

$$n_t^g = \eta^{\frac{1}{\rho}} w_t^g. \tag{1.8.2.14}$$

Therefore, the DCE is summarized by Equations (1.8.2.1)-(1.8.2.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, g_t^c, g_t^i, g_t^T, k_t^p, k_t^g, n_t^p, n_t^g, \lambda_t, w_t^g, w_t^p, r_t\}_{t=0}^{\infty}$ given the paths of technology $\{a_t\}$, the fixed level of implied government transfers/output ratio $\{g^{Ty}\}$, $\forall t$, and the exogenously set stationary government spending/output and government investment/output ratio processes, $\{g_t^{cy}, g_t^{iy}\}_{t=0}^{\infty}$, whose motion was determined in the previous subsection.[35]

[35]Note that Eqs. (1.3.0.13)-(1.3.0.14) imply the government budget constraint.

Steady-state system

In steady-state, there is no uncertainty, and $z_{t+1} = z_t = z$. Thus, expectations operators and time subscripts can be removed to obtain

$$y = a(k^p)^{1-\theta}(n^p)^{\theta}(k^g)^{\nu} \tag{1.8.2.15}$$

$$y = c + g^c + g^i + \delta^p k^p \tag{1.8.2.16}$$

$$\psi(c + \omega g^c)^{\psi(1-\alpha)-1}(1 - n^p - n^g)^{(1-\alpha)(1-\psi)} = (1 + \tau^c)\lambda \tag{1.8.2.17}$$

$$1 = \beta\left[1 - \delta^p + (1 - \tau^k)(1 - \theta)\frac{y}{k^p} + \tau^k \delta^p\right] \tag{1.8.2.18}$$

$$(1 - \psi)(c + \omega g^c) = \psi(1 - n^p - n^g)\frac{(1 - \tau^l)}{(1 + \tau^c)}\theta\frac{y}{n^p} \tag{1.8.2.19}$$

$$i = \delta^p k^p \tag{1.8.2.20}$$

$$g^i = \delta^g k^g \tag{1.8.2.21}$$

$$g^i = g^{iy} y \tag{1.8.2.22}$$

$$g^c = g^{cy} y \tag{1.8.2.23}$$

$$g^T = \quad g^{Ty} y \tag{1.8.2.24}$$

$$w^p = \theta\frac{y}{n^p} \tag{1.8.2.25}$$

$$r = (1 - \theta)\frac{y}{k^p} \tag{1.8.2.26}$$

$$n^g = \eta^{\frac{1}{\rho}} w^g \tag{1.8.2.27}$$

$$w^g = \eta^{-\frac{1}{2\rho}}\left[\frac{\tau^c c + \tau^k(r - \delta^p)k^p + \tau^l w^p n^p - g^c - g^T - g^i}{1 - \tau^l}\right]^{\frac{1}{2}} \tag{1.8.2.28}$$

1.8.3 Log-linearized model equations

Linearized market clearing

$$c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t = y_t \tag{1.8.3.1}$$

Take logs from both sides to obtain

$$\ln[c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t] = \ln(y_t) \tag{1.8.3.2}$$

Totally differentiate with respect to time

$$\frac{d\ln[c_t + k^p_{t+1} + g^c_t + g^i_t - (1-\delta^p)k^p_t]}{dt} = d\ln(y_t) \tag{1.8.3.3}$$

$$[\frac{1}{c+g^c+g^i+\delta^p k^p}][\frac{dc_t}{dt}\frac{c}{c} + \frac{dg^c_t}{dt}\frac{g}{g} + \frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p} - (1-\delta^p)\frac{dk^p_t}{dt}\frac{k^p}{k^p}] = \frac{dy_t}{dt}\frac{1}{y} \tag{1.8.3.4}$$

Define $\hat{z} = \frac{dz_t}{dt}\frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y}[\hat{c}_t c + \hat{g}^c_t g^c + \hat{g}^i_t g^i + \hat{k}^p_{t+1}k^p - (1-\delta^p)\hat{k}^p_t k^p] = \hat{y}_t \tag{1.8.3.5}$$

$$\hat{c}_t c + \hat{g}^c_t g^c + \hat{g}^i_t g^i + \hat{k}^p_{t+1}k^p - (1-\delta^p)\hat{k}^p_t k^p = y\hat{y}_t \tag{1.8.3.6}$$

$$k^p\hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t - g^c\hat{g}^c_t - g^i\hat{g}^i_t + (1-\delta^p)k^p\hat{k}^p_t \tag{1.8.3.7}$$

Linearized production function

$$y_t = a_t(k^p_t)^{1-\theta}(n^p_t)^{\theta}(k^g_t)^{\nu} \tag{1.8.3.8}$$

Take natural logs from both sides to obtain

$$\ln y_t = \ln a_t + (1-\theta)\ln k^p_t + \theta\ln n^p_t + \nu\ln k^g_t \tag{1.8.3.9}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln y_t}{dt} = \frac{d\ln a_t}{dt} + (1-\theta)\frac{d\ln k^p_t}{dt} + \theta\frac{d\ln n^p_t}{dt} + \nu\frac{d\ln k^g_t}{dt} \tag{1.8.3.10}$$

$$\frac{1}{y}\frac{dy_t}{dt} = \frac{1}{a}\frac{da_t}{dt} + \frac{1-\theta}{k^p}\frac{dk^p_t}{dt} + \frac{\theta}{n^p}\frac{dn^p_t}{dt} + \frac{\nu}{k^g}\frac{dk^g_t}{dt} \tag{1.8.3.11}$$

Pass to log-deviations to obtain

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta\hat{n}^p_t + \nu\hat{k}^g_t \tag{1.8.3.12}$$

Linearized FOC consumption

$$[(c_t+\omega g_t^c)^{\psi}(1-n_t)^{(1-\psi)}]^{-\alpha}\psi(c_t+\omega g_t^c)^{\psi-1}(1-n_t)^{(1-\psi)} = (1+\tau^c)\lambda_t \quad (1.8.3.13)$$

Simplify to obtain

$$\psi(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = (1+\tau^c)\lambda_t \quad (1.8.3.14)$$

Take natural logs from both sides to obtain

$$\ln\psi(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = \ln(1+\tau^c)+\ln\lambda_t \quad (1.8.3.15)$$

$$\ln(c_t+\omega g_t^c)^{\psi-1-\alpha\psi}(1-n_t)^{(1-\alpha)(1-\psi)} = \ln(1+\tau^c)+\ln\lambda_t \quad (1.8.3.16)$$

$$(\psi-1-\alpha\psi)\ln(c_t+\omega g_t^c)+(1-\alpha)(1-\psi)\ln(1-n_t) = \ln(1+\tau^c)+\ln\lambda_t \quad (1.8.3.17)$$

Totally differentiate with respect to time to obtain

$$(\psi-1-\alpha\psi)\frac{d\ln(c_t+\omega g_t^c)}{dt}+(1-\alpha)(1-\psi)\frac{d\ln(1-n_t)}{dt} = \\ = \frac{d\ln(1+\tau^c)}{dt}+\frac{d\ln\lambda_t}{dt} \quad (1.8.3.18)$$

$$(\psi-1-\alpha\psi)\frac{1}{c+\omega g^c}(\frac{dc_t}{dt}+\omega\frac{dg_t^c}{dt})+(1-\alpha)(1-\psi)\frac{-1}{1-n}\frac{dn_t}{dt} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \quad (1.8.3.19)$$

$$\frac{(\psi-1-\alpha\psi)}{c+\omega g^c}\frac{dc_t}{dt}\frac{c}{c}+\frac{\omega(\psi-1-\alpha\psi)}{c+\omega g^c}\frac{dg_t^c}{dt}\frac{g^c}{g^c}+ \\ -(1-\alpha)(1-\psi)\frac{1}{1-n}\frac{dn_t}{dt}\frac{n}{n} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \quad (1.8.3.20)$$

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{g}_t^c-(1-\alpha)(1-\psi)\frac{n}{1-n}\hat{n} = \hat{\lambda}_t \quad (1.8.3.21)$$

Since

$$\hat{n} = \frac{n^p}{n^p+n^g}\hat{n}^p+\frac{n^g}{n^p+n^g}\hat{n}^g = \frac{n^p}{n}\hat{n}^p+\frac{n^g}{n}\hat{n}^g, \quad (1.8.3.22)$$

and consumers choose n^p only, pass to log-deviations to obtain

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c^c+\omega g}\hat{g}^c_t-(1-\alpha)(1-\psi)\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p = \hat{\lambda}_t \tag{1.8.3.23}$$

Since $n=n^p+n^g$, it follows that

$$\frac{c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{c}_t+\frac{\omega g^c(\psi-1-\alpha\psi)}{c+\omega g^c}\hat{g}^c_t-(1-\alpha)(1-\psi)\frac{n^p}{1-n}\hat{n}^p = 0 \tag{1.8.3.24}$$

Linearized no-arbitrage condition for capital

$$\lambda_t = \beta E_t\lambda_{t+1}[(1-\tau^k)r_{t+1}+\tau^k\delta^p+(1-\delta^p)] \tag{1.8.3.25}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t = \beta E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p+1-\delta^p)] \tag{1.8.3.26}$$

Take natural logs from both sides of the equation to obtain

$$\ln\lambda_t = \ln E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p+1-\delta^p)] \tag{1.8.3.27}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\lambda_t}{dt}=\frac{d\ln E_t[\lambda_{t+1}((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p+1-\delta^p)]}{dt} \tag{1.8.3.28}$$

$$\begin{aligned}\frac{1}{\lambda}\frac{d\lambda_t}{dt}=E_t\Bigg\{&\frac{1}{\lambda((1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p+\tau^k\delta^p}\times\\&\Bigg[((1-\tau^k)(1-\theta)\frac{y}{k^p}+\tau^k\delta^p+1-\delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda}\\&+\frac{\lambda(1-\tau^k)(1-\theta)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y}-\left[\frac{\lambda(1-\tau^k)(1-\theta)y}{(k^p)^2}\right]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\Bigg]\Bigg\}\end{aligned} \tag{1.8.3.29}$$

Pass to log-deviations to obtain

$$\begin{aligned}\hat{\lambda}_t=E_t\Bigg\{\hat{\lambda}_{t+1}+\Bigg[&\frac{(1-\tau^k)(1-\theta)y}{((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p+1-\delta^p)k^p}\hat{y}_{t+1}\\&-\frac{(1-\tau^k)(1-\theta)y}{((1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+\tau^k\delta^p+1-\delta^p)k^p}\hat{k}^p_{t+1}\Bigg]\Bigg\}\end{aligned} \tag{1.8.3.30}$$

Observe that

$$(1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + \tau^k\delta^p + 1 - \delta^p \quad = \quad 1/\beta \tag{1.8.3.31}$$

Plug it into the equation to obtain

$$\hat{\lambda}_t \quad = \quad E_t\left[\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{k}^p_{t+1}\right] \tag{1.8.3.32}$$

$$\hat{\lambda}_t \quad = \quad E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{1.8.3.33}$$

Linearized MRS

$$(1-\psi)(c_t + \omega g^c_t) \quad = \quad \psi(1-n_t)\frac{(1-\tau^l)}{(1+\tau^c)}\theta\frac{y_t}{n^p_t} \tag{1.8.3.34}$$

Take natural logs from both sides of the equation to obtain

$$\ln(1-\psi)(c_t + \omega g^c_t) \quad = \quad \ln\psi(1-n_t)\frac{(1-\tau^l)}{(1+\tau^c)}\theta\frac{y_t}{n^p_t} \tag{1.8.3.35}$$

$$\ln(c_t + \omega g^c_t) \quad = \quad \ln(1-n_t) + \ln y_t - \ln n^p_t \tag{1.8.3.36}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln(c_t + \omega g^c_t)}{dt} \quad = \quad \frac{d\ln(1-n_t)}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln n^p_t}{dt} \tag{1.8.3.37}$$

$$\frac{1}{c+\omega g^c}(\frac{dc_t}{dt} + \omega\frac{dg^c_t}{dt}) \quad = \quad -\frac{1}{1-n}\frac{dn_t}{dt} + \frac{1}{y}\frac{dy_t}{dt} - \frac{1}{n^p}\frac{dn^p_t}{dt} \tag{1.8.3.38}$$

$$\frac{1}{c+\omega g^c}\frac{dc_t}{dt}\frac{c}{c} + \frac{\omega}{c+\omega g^c}\frac{dg^c_t}{dt}\frac{g^c}{g^c} \quad = \quad -\frac{1}{1-n}\frac{dn_t}{dt}\frac{n}{n} + \frac{1}{y}\frac{dy_t}{dt} - \frac{1}{n^p}\frac{dn^p_t}{dt} \tag{1.8.3.39}$$

$$\frac{c}{c+\omega g^c}\frac{dc_t}{dt}\frac{1}{c} + \frac{\omega g^c}{c+\omega g^c}\frac{dg^c_t}{dt}\frac{1}{g^c} \quad = \quad -\frac{n}{1-n}\frac{dn_t}{dt}\frac{1}{n} + \frac{1}{y}\frac{dy_t}{dt} - \frac{1}{n^p}\frac{dn^p_t}{dt} \tag{1.8.3.40}$$

Pass to log-deviations to obtain

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g}\hat{g}^c_t \quad = \quad -\frac{n}{1-n}\hat{n} + \hat{y}_t - \hat{n}^p_t \tag{1.8.3.41}$$

Since

$$\hat{n} = \frac{n^p}{n^p + n^g}\hat{n}^p + \frac{n^g}{n^p + n^g}\hat{n}^g, \tag{1.8.3.42}$$

and noting that consumers are only choosing n^p, then

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g^c}\hat{g}^c_t = -\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p + \hat{y}_t - \hat{n}^p_t \tag{1.8.3.43}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g^c}\hat{g}^c_t = -\frac{n}{1-n}\frac{n^p}{n^p+n^g}\hat{n}^p + \hat{y}_t - \hat{n}^p_t \tag{1.8.3.44}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g^c}\hat{g}^c_t = -\left(1 + \frac{n}{1-n}\frac{n^p}{n^p+n^g}\right)\hat{n}^p + \hat{y}_t \tag{1.8.3.45}$$

Since $n = n^p + n^g$, it follows that

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g^c}\hat{g}^c_t = -\left(1 + \frac{n^p}{1-n}\right)\hat{n}^p + \hat{y}_t \tag{1.8.3.46}$$

$$\frac{c}{c+\omega g^c}\hat{c}_t + \frac{\omega g^c}{c+\omega g^c}\hat{g}^c_t + \left(1 + \frac{n^p}{1-n}\right)\hat{n}^p - \hat{y}_t = 0 \tag{1.8.3.47}$$

Linearized private capital accumulation

$$k^p_{t+1} = i_t + (1-\delta^p)k^p_t \tag{1.8.3.48}$$

Take natural logs from both sides of the equation to obtain

$$\ln k^p_{t+1} = \ln(i_t + (1-\delta^p)k^p_t) \tag{1.8.3.49}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^p_{t+1}}{dt} = \frac{1}{i + (1-\delta^p)k^p}\frac{d(i_t + (1-\delta^p)k^p_t)}{dt} \tag{1.8.3.50}$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that } \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \tag{1.8.3.51}$$

Then

$$\frac{dk^p_{t+1}}{dt}\frac{1}{k^p} = \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i + (1-\delta^p)k^p_t}\frac{dk^p_t}{dt}\frac{k^p}{k^p} \tag{1.8.3.52}$$

Pass to log-deviations to obtain

$$\hat{k}^p_{t+1} = \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}^p_t \tag{1.8.3.53}$$

$$\hat{k}^p_{t+1} = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{1.8.3.54}$$

Linearized government capital accumulation

$$k^g_{t+1} = g^i_t + (1-\delta^g)k^g_t \tag{1.8.3.55}$$

Take natural logs from both sides to obtain

$$\ln k^g_{t+1} = \ln(g^i_t + (1-\delta^g)k^g_t) \tag{1.8.3.56}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^g_{t+1}}{dt} = \frac{1}{g^i + (1-\delta^g)k^g}\frac{d(g^i_t + (1-\delta^g)k^g_t)}{dt} \tag{1.8.3.57}$$

Observe that since

$$g^i = \delta^g k^g, \tag{1.8.3.58}$$

it follows that

$$g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \tag{1.8.3.59}$$

Hence,

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{x+(1-\delta^g)}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{1.8.3.60}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^g)k^g}{k^g}\hat{k}^g_t \tag{1.8.3.61}$$

Cancel out the k^g terms to obtain

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{1.8.3.62}$$

Public wage rate rule

$$w^g_t = \eta^{-\frac{1}{2\rho}}\left[\frac{\tau^c c_t + \tau^k r_t k^p_t - \tau^k \delta^p k^p_t + \tau^l w^p_t n^p_t - g^c_t - g^T_t - g^i_t}{1-\tau^l}\right]^{\frac{1}{2}} \tag{1.8.3.63}$$

Take logs from both sides to obtain

$$\ln w_t^g = -\frac{1}{2\rho}\ln\eta - \frac{1}{2}\ln(1-\tau^l) + \frac{1}{2}\ln\left\{\tau^c c_t + \tau^k r_t k_t^p - \tau^k\delta^p k_t^p + \tau^l w_t^p n_t^p - g_t^c - g_t^T - g_t^i\right\} \tag{1.8.3.64}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln w_t^g}{dt} = \frac{1}{2}\frac{d}{dt}\ln\left\{\tau^c c_t + \tau^k r_t k_t^p - \tau^k\delta^p k_t^p + \tau^l w_t^p n_t^p - g_t^c - g_t^T - g_t^i\right\} \tag{1.8.3.65}$$

Observe that

$$\begin{aligned}\tau^k r_t k_t^p - \tau^k\delta^p k_t + \tau^l w_t^p n_t^p &= \tau^k(1-\theta)y_t + \tau^l\theta y_t - \tau^k\delta^p k_t^p = \\ &= \left[\tau^k(1-\theta) + \tau^l\theta\right]y_t - \tau^k\delta^p k_t^p\end{aligned} \tag{1.8.3.66}$$

Also

$$(1-\tau^l)w^g n^g = \tau^c c + [\tau^k(1-\theta) + \tau^l\theta]y - \tau^k\delta^p k^p - g^c - g^i - g_t^T \tag{1.8.3.67}$$

Thus

$$\frac{dw_t^g}{dt}\frac{1}{w^g} = \frac{1}{2}\frac{1}{(1-\tau^l)w^g n^g}\left\{\tau^c\frac{dc_t}{dt} + [\tau^k(1-\theta)+\tau^l\theta]\frac{dy_t}{dt} - \tau^k\delta^p\frac{dk_t^p}{dt} - \frac{dg_t^c}{dt} - \frac{dg_t^i}{dt} - \frac{dg_t^T}{dt}\right\} \tag{1.8.3.68}$$

$$\frac{dw_t^g}{dt}\frac{1}{w^g} = \frac{1}{2}\frac{1}{(1-\tau^l)w^g n^g}\times \left\{\tau^c\frac{dc_t}{dt}\frac{c}{c} + \left[\tau^k(1-\theta)+\tau^l\theta\right]\frac{dy_t}{dt}\frac{y}{y} - \tau^k\delta^p\frac{dk_t^p}{dt}\frac{k^p}{k^p} - \frac{dg_t^c}{dt}\frac{g^c}{g^c} - \frac{dg_t^i}{dt}\frac{g^i}{g^i} - \frac{dg_t^T}{dt}\frac{g^T}{g^T}\right\} \tag{1.8.3.69}$$

$$\begin{aligned}\frac{dw_t^g}{dt}\frac{1}{w^g} = &\frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\frac{dc_t}{dt}\frac{1}{c} + \frac{(1/2)\left[\tau^k(1-\theta)+\tau^l\theta\right]y}{(1-\tau^l)w^g n^g}\frac{dy_t}{dt}\frac{1}{y} \\ &-\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\frac{dk_t^p}{dt}\frac{1}{k^p} - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\frac{dg_t^c}{dt}\frac{1}{g^c} \\ &-\frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\frac{dg_t^i}{dt}\frac{1}{g^i} - \frac{(1/2)g^T}{(1-\tau^l)w^g n^g}\frac{dg_t^T}{dt}\frac{1}{g^T}\end{aligned} \tag{1.8.3.70}$$

Pass to log-deviations to obtain

$$\hat{w}_t^g = \frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\hat{c}_t + \frac{(1/2)\Big[\tau^k(1-\theta)+\tau^l\theta\Big]y}{(1-\tau^l)w^g n^g}\hat{y}_t$$
$$-\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\hat{k}_t - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\hat{g}_t^c - \frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\hat{g}_t^i - \frac{(1/2)g^t}{(1-\tau^l)w^g n^g}\hat{g}_t^T \quad (1.8.3.71)$$

Public hours/employment rule

$$n_t^g = \eta^{\frac{1}{\rho}} w_t^g \quad (1.8.3.72)$$

Take logs from both sides to obtain

$$\ln n_t^g = \frac{1}{\rho}\ln\eta + \ln w_t^g \quad (1.8.3.73)$$

Totally differentiate both sides to obtain

$$\frac{d\ln n_t^g}{dt} = \frac{d\ln w_t^g}{dt} \quad (1.8.3.74)$$

$$\frac{dn_t^g}{dt}\frac{1}{n^g} = \frac{dw_t^g}{dt}\frac{1}{w^g} \quad (1.8.3.75)$$

Pass to log-deviations to obtain

$$\hat{n}_t^g = \hat{w}_t^g \quad (1.8.3.76)$$

Total hours/employment

$$n_t = n_t^g + n_t^p \quad (1.8.3.77)$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \quad (1.8.3.78)$$

Totally differentiate to obtain

$$\frac{d\ln n_t}{dt} = \frac{d\ln(n_t^g + n_t^p)}{dt} \quad (1.8.3.79)$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{1.8.3.80}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{1.8.3.81}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{1.8.3.82}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{1.8.3.83}$$

Linearized private wage rate

$$w_t^p = \theta\frac{y_t}{n_t^p} \tag{1.8.3.84}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln\theta + \ln y_t - \ln n_t^p \tag{1.8.3.85}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln w_t^p}{dt} = \frac{d\ln\theta}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln n_t^p}{dt} \tag{1.8.3.86}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{1.8.3.87}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{1.8.3.88}$$

Linearized real interest rate

$$r_t = \theta\frac{y_t}{k_t^p} \tag{1.8.3.89}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln \theta + \ln y_t - \ln k_t^p \tag{1.8.3.90}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln r_t}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln k_t^p}{dt} \tag{1.8.3.91}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{1.8.3.92}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{1.8.3.93}$$

Public/private wage ratio

$$rw_t = w_t^g / w_t^p \tag{1.8.3.94}$$

Take logs from both sides of the equation

$$\ln rw_t = \ln w_t^g - \ln w_t^p \tag{1.8.3.95}$$

Totally differentiate to obtain

$$\frac{d \ln rw_t}{dt} = \frac{d \ln w_t^g}{dt} - \frac{d \ln w_t^p}{dt} \tag{1.8.3.96}$$

$$\frac{drw_t}{dt}\frac{1}{rw} = \frac{dw_t^g}{dt}\frac{1}{w^g} - \frac{dw_t^p}{dt}\frac{1}{w^p} \tag{1.8.3.97}$$

Pass to log-deviations to obtain

$$\hat{rw}_t = \hat{w}_t^g - \hat{w}_t^p \tag{1.8.3.98}$$

Public/private hours/employment ratio

$$rl_t = n_t^g / n_t^p \tag{1.8.3.99}$$

Take logs from both sides of the equation

$$\ln rl_t = \ln n_t^g - \ln n_t^p \tag{1.8.3.100}$$

Totally differentiate to obtain

$$\frac{d \ln rl_t}{dt} = \frac{d \ln n_t^g}{dt} - \frac{d \ln n_t^p}{dt} \tag{1.8.3.101}$$

$$\frac{drl_t}{dt}\frac{1}{rl} = \frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{1.8.3.102}$$

Pass to log-deviations to obtain

$$\hat{rl}_t = \hat{n}_t^g - \hat{n}_t^p \tag{1.8.3.103}$$

Linearized technology shock process

$$\ln a_{t+1} = \rho_a \ln a_t + \epsilon_{t+1}^a \tag{1.8.3.104}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln a_{t+1}}{dt} = \rho_a \frac{d \ln a_t}{dt} + \frac{d\epsilon_{t+1}^a}{dt} \tag{1.8.3.105}$$

$$\frac{da_{t+1}}{dt} = \rho_a \frac{da_t}{dt} + \epsilon_{t+1}^a \tag{1.8.3.106}$$

where for $t = 1$ $\frac{d\epsilon_{t+1}^a}{dt} \approx \ln(e^{\epsilon_{t+1}^a}/e^{\epsilon^a}) = \epsilon_{t+1}^a - \epsilon^a = \epsilon_{t+1}^a$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \epsilon_{t+1}^a \tag{1.8.3.107}$$

Linearized stochastic process for government consumption/output share

$$\ln g_{t+1}^{cy} = (1-\rho^g)\ln g^{cy} + \rho^g \ln g_t^{cy} + \epsilon_{t+1}^c \tag{1.8.3.108}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_{t+1}^{cy}}{dt} = (1-\rho^g)\frac{d\ln g^{cy}}{dt} + \rho^g \frac{d\ln g_t^{cy}}{dt} + \frac{d\epsilon_{t+1}^c}{dt} \tag{1.8.3.109}$$

$$\frac{dg_{t+1}^{cy}}{dt} = \rho_g \frac{dg_t^{cy}}{dt} + \epsilon_{t+1}^c \tag{1.8.3.110}$$

where for $t=1$ $\frac{d\epsilon_{t+1}^c}{dt} \approx \ln(e^{\epsilon_{t+1}^c}/e^{\epsilon^c}) = \epsilon_{t+1}^c - \epsilon^c = \epsilon_{t+1}^c$ since $\epsilon^c = 0$. Pass to log-deviations to obtain

$$\hat{g}_{t+1}^{cy} = \rho_g \hat{g}_t^{cy} + \epsilon_{t+1}^c \tag{1.8.3.111}$$

Linearized level of government consumption

$$g_t^c = g_t^{cy} y_t \tag{1.8.3.112}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^c = \ln g_t^{cy} + \ln y_t \tag{1.8.3.113}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_t^c}{dt} = \frac{d\ln g_t^{cy}}{dt} + \frac{d\ln y_t}{dt} \tag{1.8.3.114}$$

$$\frac{dg_t^c}{dt}\frac{1}{g^c} = \frac{dg_t^{cy}}{dt}\frac{1}{g^c} + \frac{dy_t}{dt}\frac{1}{y} \tag{1.8.3.115}$$

Pass to log-deviations to obtain

$$\hat{g}_t^c = \hat{g}_t^{cy} + \hat{y}_t \tag{1.8.3.116}$$

Linearized stochastic process for the government investment/output ratio

$$\ln g_{t+1}^{iy} = (1-\rho^i)\ln g^{iy} + \rho^i \ln g_t^{iy} + \epsilon_{t+1}^i \tag{1.8.3.117}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_{t+1}^{iy}}{dt} = (1-\rho^i)\frac{d\ln g^{iy}}{dt} + \rho^i \frac{d\ln g_t^{iy}}{dt} + \frac{d\epsilon_{t+1}^i}{dt} \tag{1.8.3.118}$$

$$\frac{dg_{t+1}^{iy}}{dt} = \rho_g \frac{dg_t^{iy}}{dt} + \epsilon_{t+1}^i \tag{1.8.3.119}$$

where for $t=1$ $\frac{d\epsilon_{t+1}^i}{dt} \approx \ln(e^{\epsilon_{t+1}^i}/e^{\epsilon^i}) = \epsilon_{t+1}^i - \epsilon^i = \epsilon_{t+1}^i$ since $\epsilon^i = 0$. Pass to log-deviations to obtain

$$\hat{g}_{t+1}^{iy} = \rho_i \hat{g}_t^{iy} + \epsilon_{t+1}^i \tag{1.8.3.120}$$

Linearized level of government investment

$$g_t^i = g_t^{iy} y_t \tag{1.8.3.121}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g_t^{iy} + \ln y_t \tag{1.8.3.122}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_t^i}{dt} = \frac{d\ln g_t^{iy}}{dt} + \frac{d\ln y_t}{dt} \tag{1.8.3.123}$$

$$\frac{dg_t^i}{dt}\frac{1}{g^i} = \frac{dg_t^{iy}}{dt}\frac{1}{g^i} + \frac{dy_t}{dt}\frac{1}{y} \tag{1.8.3.124}$$

Pass to log-deviations to obtain

$$\hat{g}_t^i = \hat{g}_t^{iy} + \hat{y}_t \tag{1.8.3.125}$$

Linearized level of government transfers

$$g_t^T = g^{Ty} y_t \tag{1.8.3.126}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^T = \ln g^{Ty} + \ln y_t \tag{1.8.3.127}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^T}{dt} = \frac{d \ln g^{Ty}}{dt} + \frac{d \ln y_t}{dt} \tag{1.8.3.128}$$

$$\frac{dg_t^T}{dt} \frac{1}{g^t} = \frac{dy_t}{dt} \frac{1}{y} \tag{1.8.3.129}$$

Pass to log-deviations to obtain

$$\hat{g}_t^T = \hat{y}_t \tag{1.8.3.130}$$

1.8.4 The log-linearized system of equations

The log-linearized system of model equations is as below[36] ($\hat{rw}_t$ and $\hat{rl}_t$ denote the log-deviations in the wage and hours ratios, respectively):

$$k^p \hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t - g^c \hat{g}^c_t - g^i \hat{g}^i_t + (1-\delta^p)k^p \hat{k}^p_t \tag{1.8.4.1}$$

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta \hat{n}^p_t + \nu \hat{k}^g_t \tag{1.8.4.2}$$

$$\frac{c(\psi - 1 - \alpha\psi)}{c+\omega g}\hat{c}_t + \frac{\omega g(\psi - 1 - \alpha\psi)}{c+\omega g}\hat{g}^c_t - (1-\alpha)(1-\psi)\frac{n^p}{1-n}\hat{n}^p - \hat{\lambda}_t = 0 \tag{1.8.4.3}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{1.8.4.4}$$

$$\frac{c}{c+\omega g}\hat{c}_t + \frac{\omega g}{c+\omega g}\hat{g}_t + (1 + \frac{n^p}{1-n})\hat{n}^p - \hat{y}_t = 0 \tag{1.8.4.5}$$

$$\hat{k}^p_{t+1} = \delta^p \hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{1.8.4.6}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{1.8.4.7}$$

$$\hat{a}_{t+1} = \rho^a \hat{a}_t + \epsilon^a_{t+1} \tag{1.8.4.8}$$

$$\hat{g}^{cy}_{t+1} = \rho_c \hat{g}^{cy}_t + \epsilon^c_{t+1} \tag{1.8.4.9}$$

$$\hat{g}^{iy}_{t+1} = \rho_i \hat{g}^{iy}_t + \epsilon^i_{t+1} \tag{1.8.4.10}$$

$$\hat{g}^c_t = \hat{g}^{cy}_t + \hat{y}_t \tag{1.8.4.11}$$

$$\hat{g}^i_t = \hat{g}^{iy}_t + \hat{y}_t \tag{1.8.4.12}$$

[36] Detailed derivations in Appendix 11.2.1-12.2.19

$$\hat{w}_t^g = \frac{(1/2)\tau^c c}{(1-\tau^l)w^g n^g}\hat{c}_t + \frac{(1/2)[\tau^k(1-\theta)+\tau^l\theta]y}{(1-\tau^l)w^g n^g}\hat{y}_t$$
$$-\frac{(1/2)\tau^k\delta^p k^p}{(1-\tau^l)w^g n^g}\hat{k}_t - \frac{(1/2)g^c}{(1-\tau^l)w^g n^g}\hat{g}_t^c - \frac{(1/2)g^i}{(1-\tau^l)w^g n^g}\hat{g}_t^i - \frac{(1/2)g^T}{(1-\tau^l)w^g n^g}\hat{g}_t^T \quad (1.8.4.13)$$

$$\hat{n}_t^g = \hat{w}_t^g \quad (1.8.4.14)$$

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \quad (1.8.4.15)$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \quad (1.8.4.16)$$

$$\hat{n}_t = \frac{n^p}{(n^p+n^g)}\hat{n}_t^p + \frac{n^g}{(n^p+n^g)}\hat{n}_t^g \quad (1.8.4.17)$$

$$r\hat{w}_t = \hat{w}_t^g - \hat{w}_t^p \quad (1.8.4.18)$$

$$\hat{rl}_t = \hat{n}_t^g - \hat{n}_t^p \quad (1.8.4.19)$$

1.8.5 Auto- and cross-correlation functions

As an additional test of model fit, this appendix compares auto- and cross-correlation functions generated from the model with collective bargaining and Finn (1998) calibrated for Germany, with their empirical counterparts. The main emphasis in this subsection is on the ACFs and CCFs of labor market variables. In particular, close attention is paid to cyclical properties of public and private wage rates and hours. To establish 95% confidence intervals for the theoretical ACFs and CCFs, as in Gregory and Smith (1991), the simulated time series are used to obtain 1000 ACFs and CCFs. The mean ACFs and CCFs are computed by averaging across simulations, as well as the corresponding standard error across simulations. Those moments allow for the lower and upper bounds for the ACFs confidence intervals to be estimated. The empirical ACFs and CCFs are then plotted, together with the theoretical ones. If empirical ACFs lie within the confidence region, this means that the calibrated model fits data well.

Empirical ACFs and CCFs were generated from a Vector Auto-Regressive (VAR) process of order 1. Since ACFs and CCFs are robust to identifying restrictions (Canova (2007, Ch.7)), the VAR(1) was left unrestricted. The figures on the following pages display empirical ACFs (solid line), together with the simulated average ACFs (dashed line) and the corresponding stochastic error bounds (dotted lines). This is done for the public sector union model first, and then for the calibration using Finn's (1998) framework.

The model with the public sector union Germany outperforms Finn (1998), especially in the prediction of the dynamic behavior of labor market variables. In terms of capturing the autocorrelation structure of the variables, the union model fits data quite well. One exception is the public sector wage: in data, it is highly autocorrelated, while the model generates low persistence.[37]

[37] A possible explanation could be that the public union puts weight also on last year's public sector wage level, *i.e.* the union bargains over the public wage increase rate, and not just the wage level.

Figure 1.7: Theoretical and empirical ACFs for Germany: Union

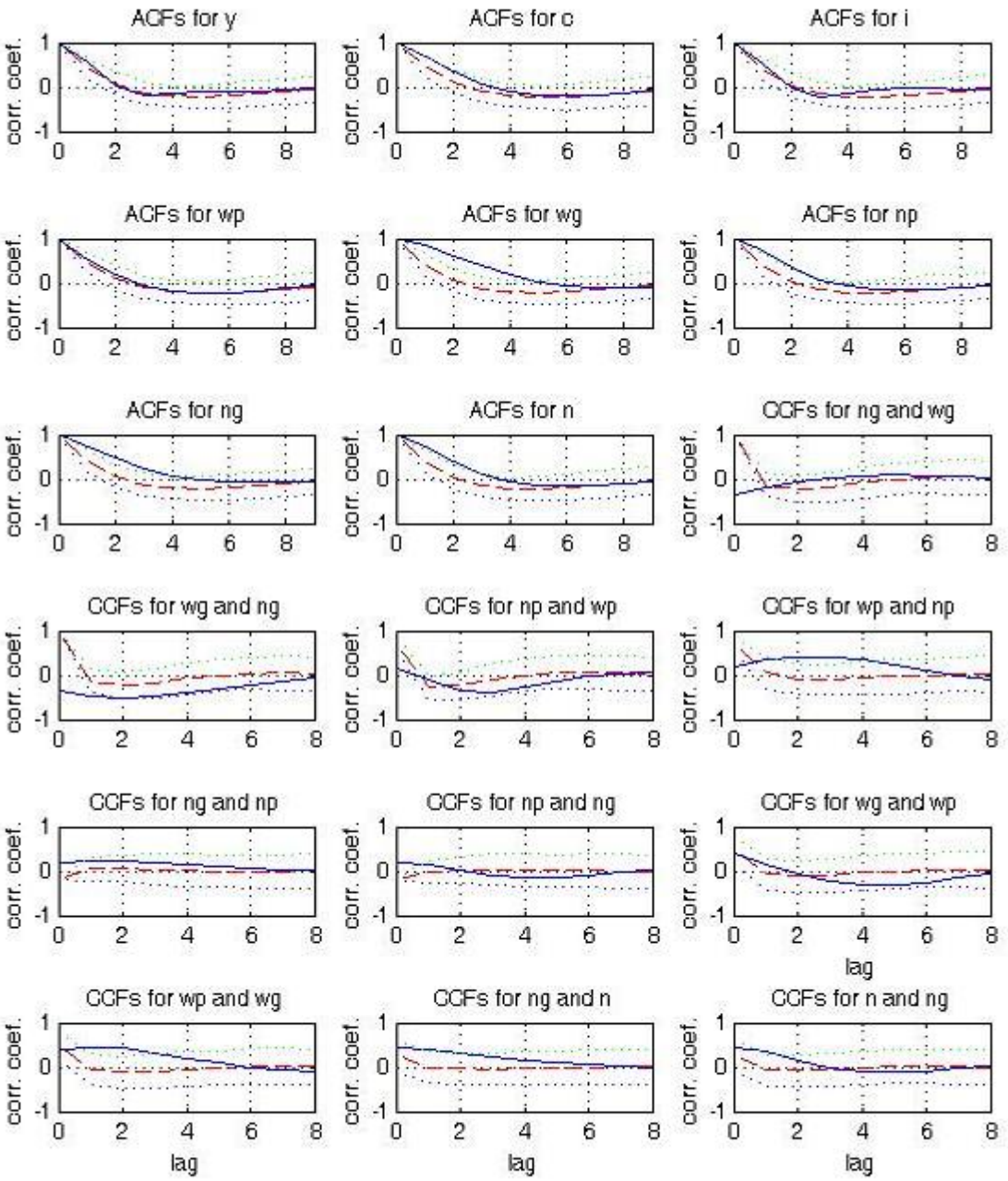


Figure 1.8: Theoretical and empirical ACFs for Germany: Union

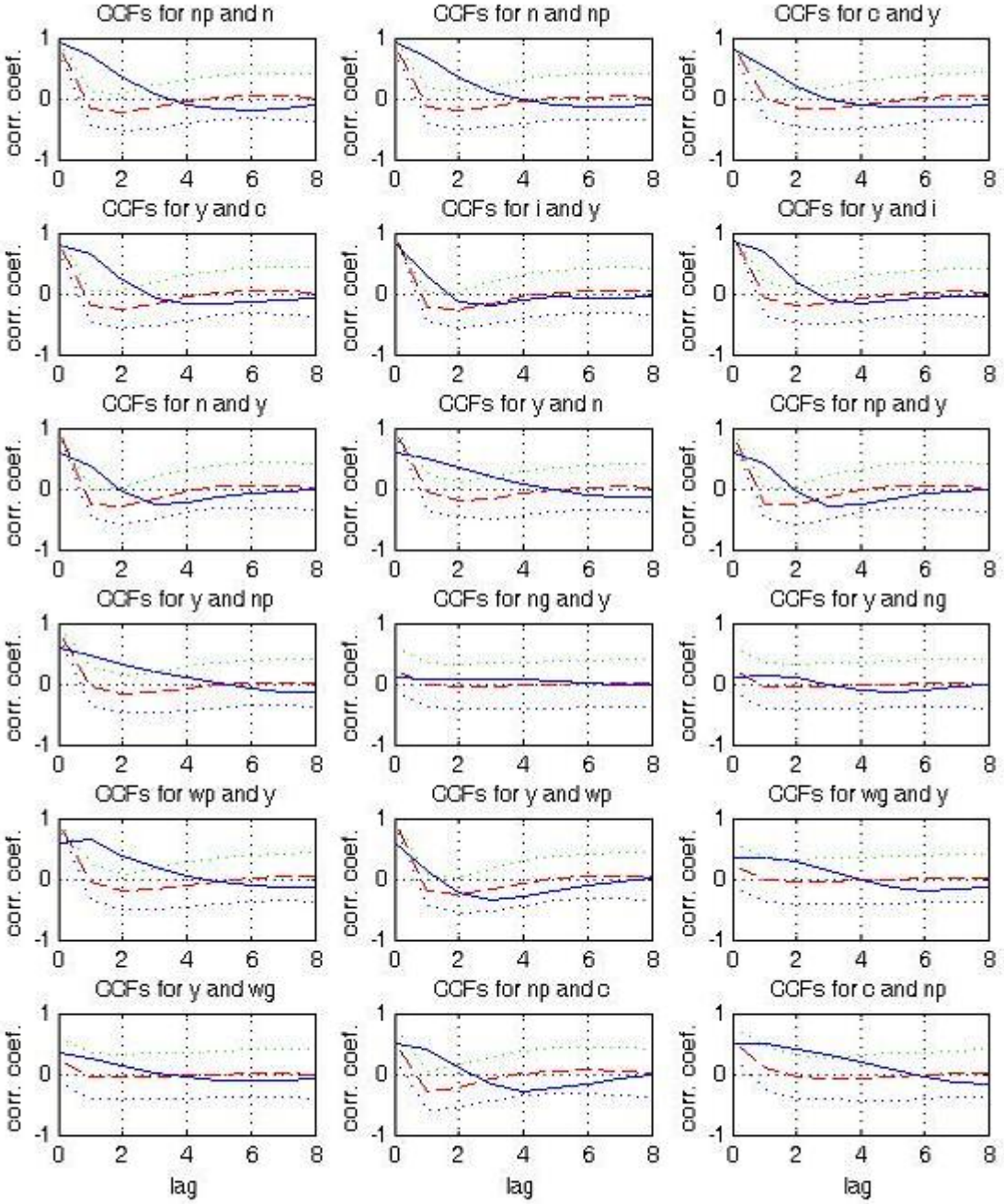


Figure 1.9: Theoretical and empirical ACFs for Germany: Union

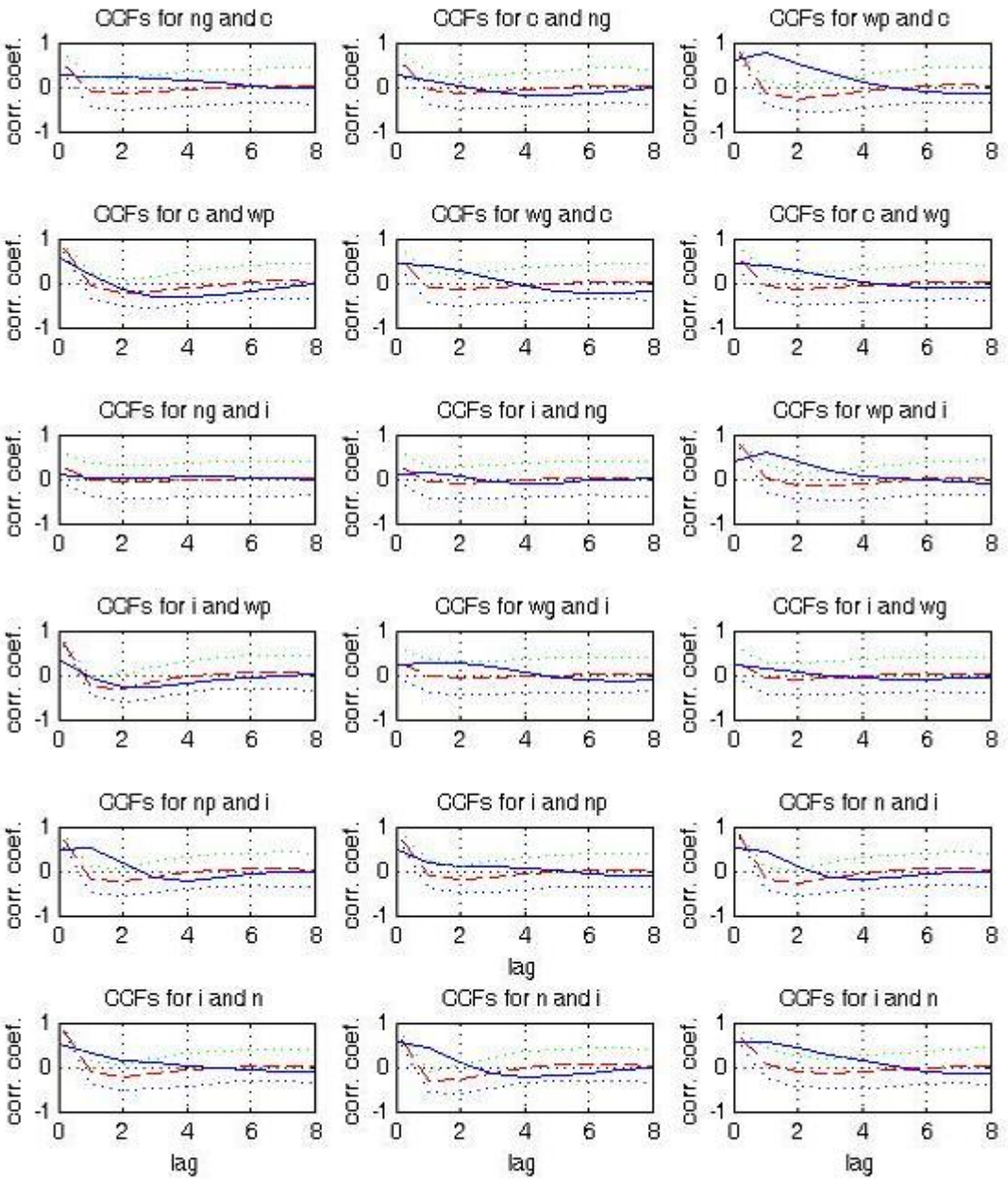


Figure 1.10: Theoretical and empirical ACFs for Germany: Finn

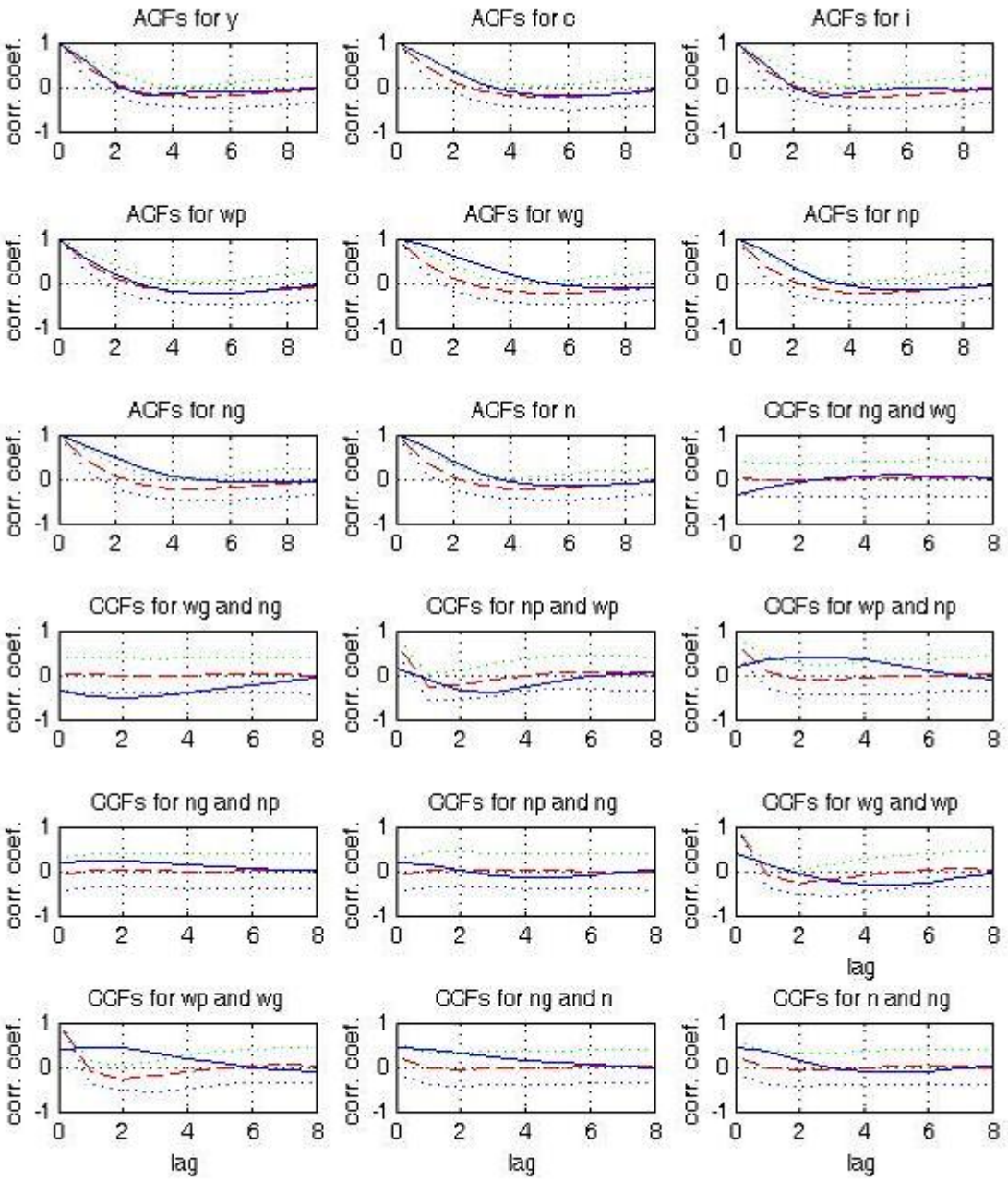


Figure 1.11: Theoretical and empirical ACFs for Germany: Finn

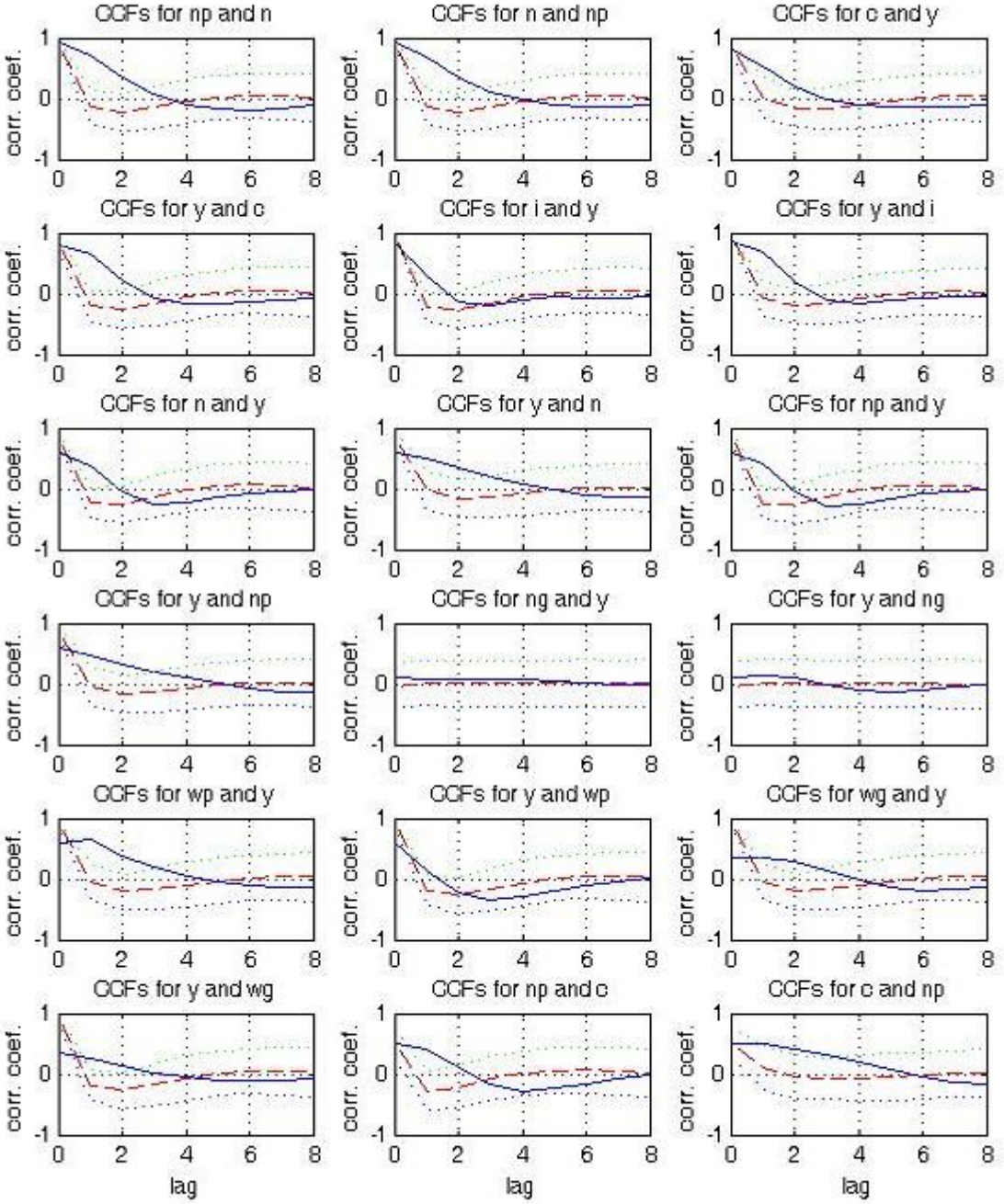
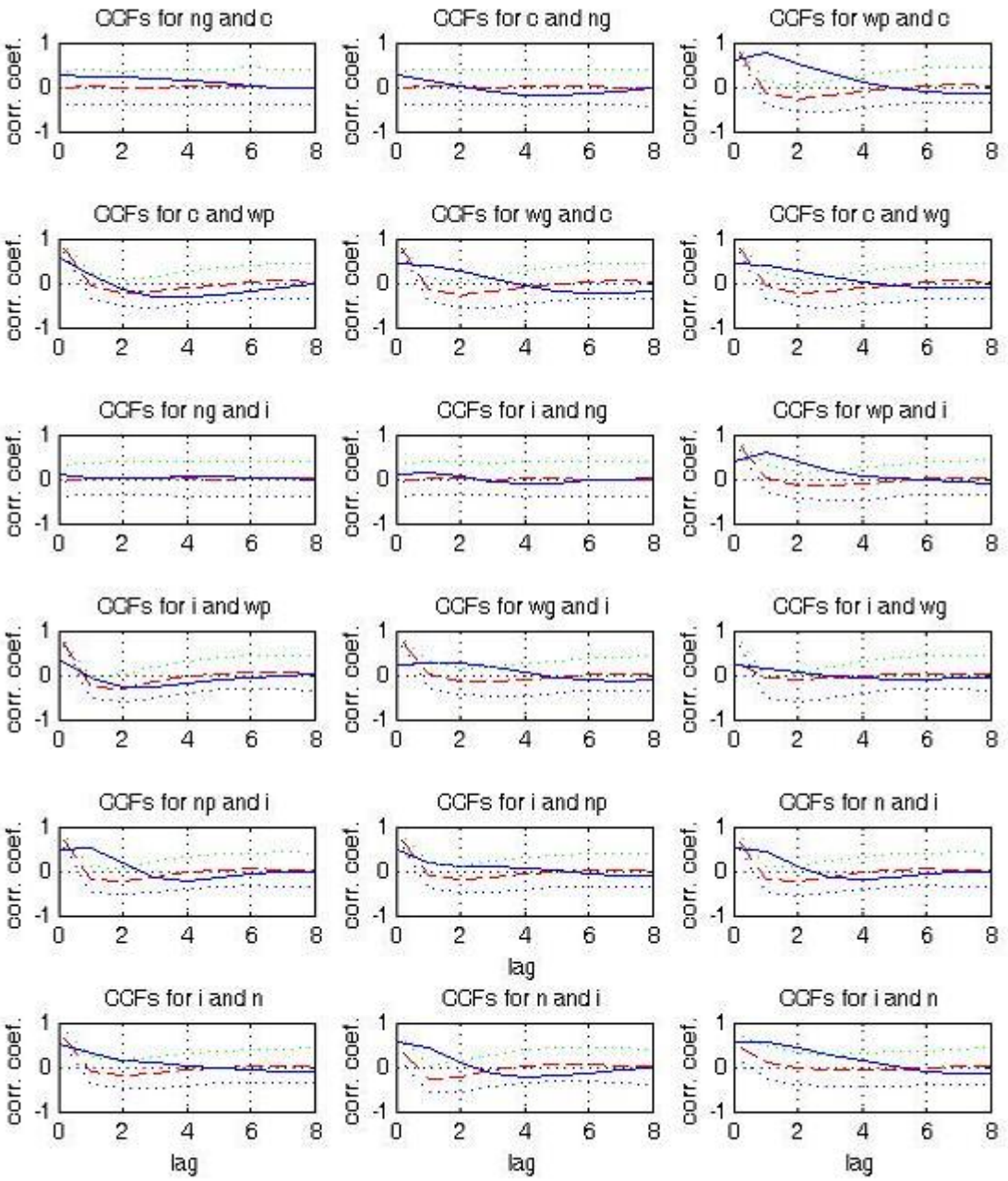


Figure 1.12: Theoretical and empirical ACFs for Germany: Finn



Public and total hours are also borderline cases, as employment rates in data were used instead. In addition, the public sector union model predicts perfect positive contemporaneous correlation between public wages and hours, while in data, it is negative. Overall, the model with public sector union calibrated for Germany captures the dynamic co-movement of hours and wages with output, consumption and investment. In addition, the public sector union model is able to address and match some new dimensions such as the dynamic correlation of the two wage rates and the pair of hours worked.

1.8.6 Sensitivity analysis

To evaluate the effect of structural parameters on the shape of the Laffer curves, this section performs sensitivity analysis for different values of model parameters and how those affect tax revenues. The two parameters of interest are the curvature parameter of household's Cobb-Douglas utility function α, as well as the weight on composite consumption, ψ. Interestingly, as α is allowed to vary, steady-state revenues are essentially unchanged. Even an implausibly high value, $\alpha = 50$, does not produce any difference in steady state tax revenues. In both models considered in this chapter, the preference parameter is not important for steady-state fiscal policy effect. This result is not surprising in the literature, as Trabandt and Uhlig (2010) obtain a very similar finding in their paper.[38]

In contrast, changes in the second parameter, ψ, yield significant differences. Both the capital and labor tax Laffer curves, and the responses of the other tax bases to capital and labor income tax rate are affected when ψ is allowed to vary.[39] Higher values of ψ shift up the Laffer curve and make it steeper, without significant change in its peak. The difference between Finn and the model with endogenous public employment becomes significant for implausibly high values of ψ, i.e. $\psi > 0.5$. (As explained in the calibration section, parame-

[38]Parameter α is important for model dynamics, though.

[39]Consumption tax Laffer curve proves to be very sensitive to ψ parameter. In the majority of the cases it breaks down for values outside the benchmark value. This is also a typical result in the literature, e.g. Trabandt and Uhlig (2010).

ter $\psi = 0.296$, describing household's preference was calculated as the ratio of hours of work out of total potential hours in the model.) Intuitively, a higher ψ corresponds to a lower weight to leisure, $(1 - \psi)$, in the household's utility function. In other words, a higher ψ decreases the elasticity of private labor supply. Intuitively, when labor tax rate increases, or equivalently, after tax private wage falls, private hours respond less, thus increasing labor income tax revenue, as well as total tax revenue.

The effect of higher ψ on capital tax Laffer curve is similar to ψ's effect on the labor tax Laffer curve above. When τ^k is allowed to vary, a higher weight attached to consumption in household's utility function, together with the optimality condition for the marginal rate of substitution between consumption and hours require private higher capital stock to finance private consumption. Therefore, a higher ψ shifts the capital tax Laffer curve upward as well.

Figure 1.13: Sensitivity analysis: capital income tax Laffer curve (Union)

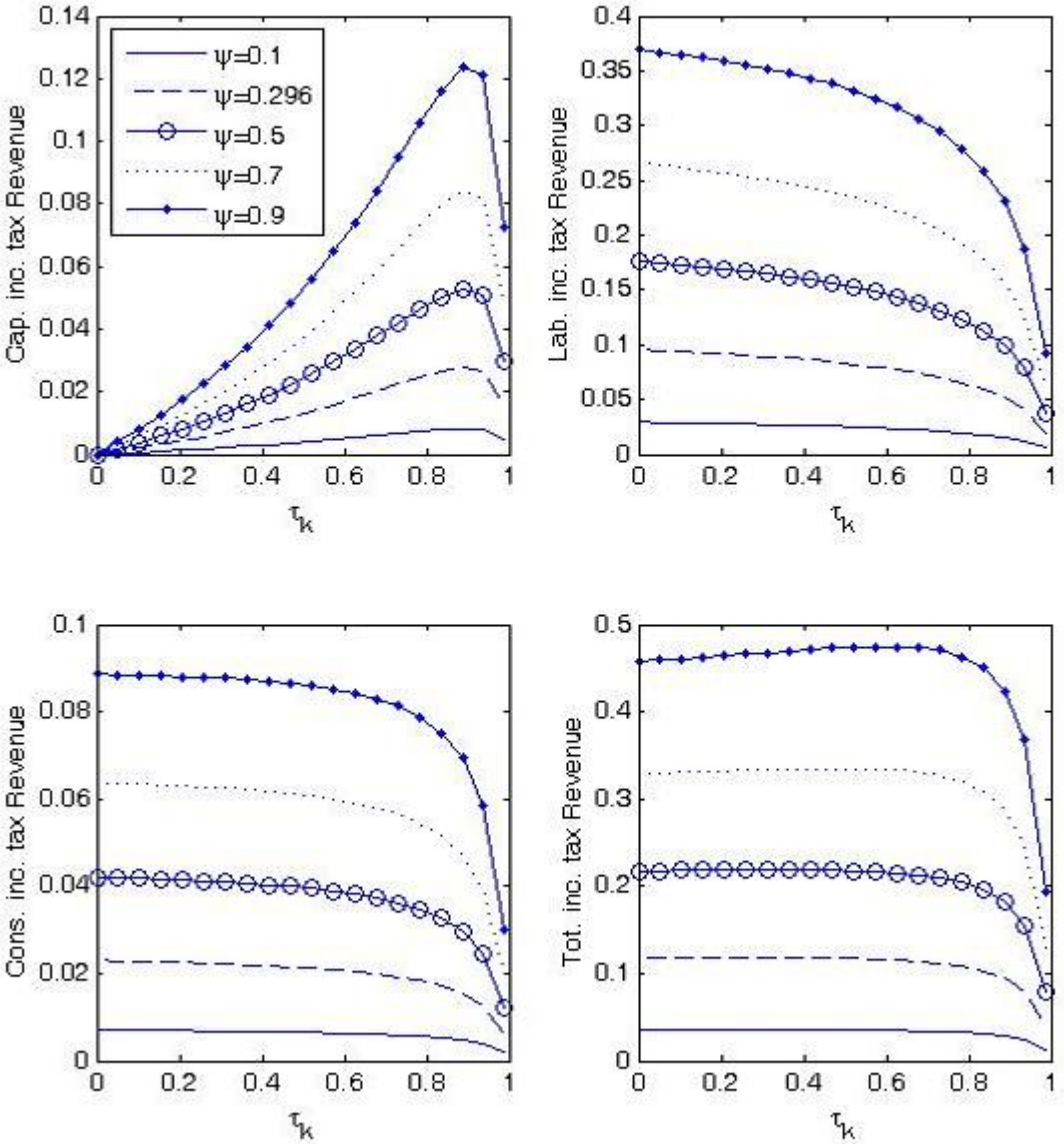


Figure 1.14: Sensitivity analysis: labor income tax Laffer curve (Union)

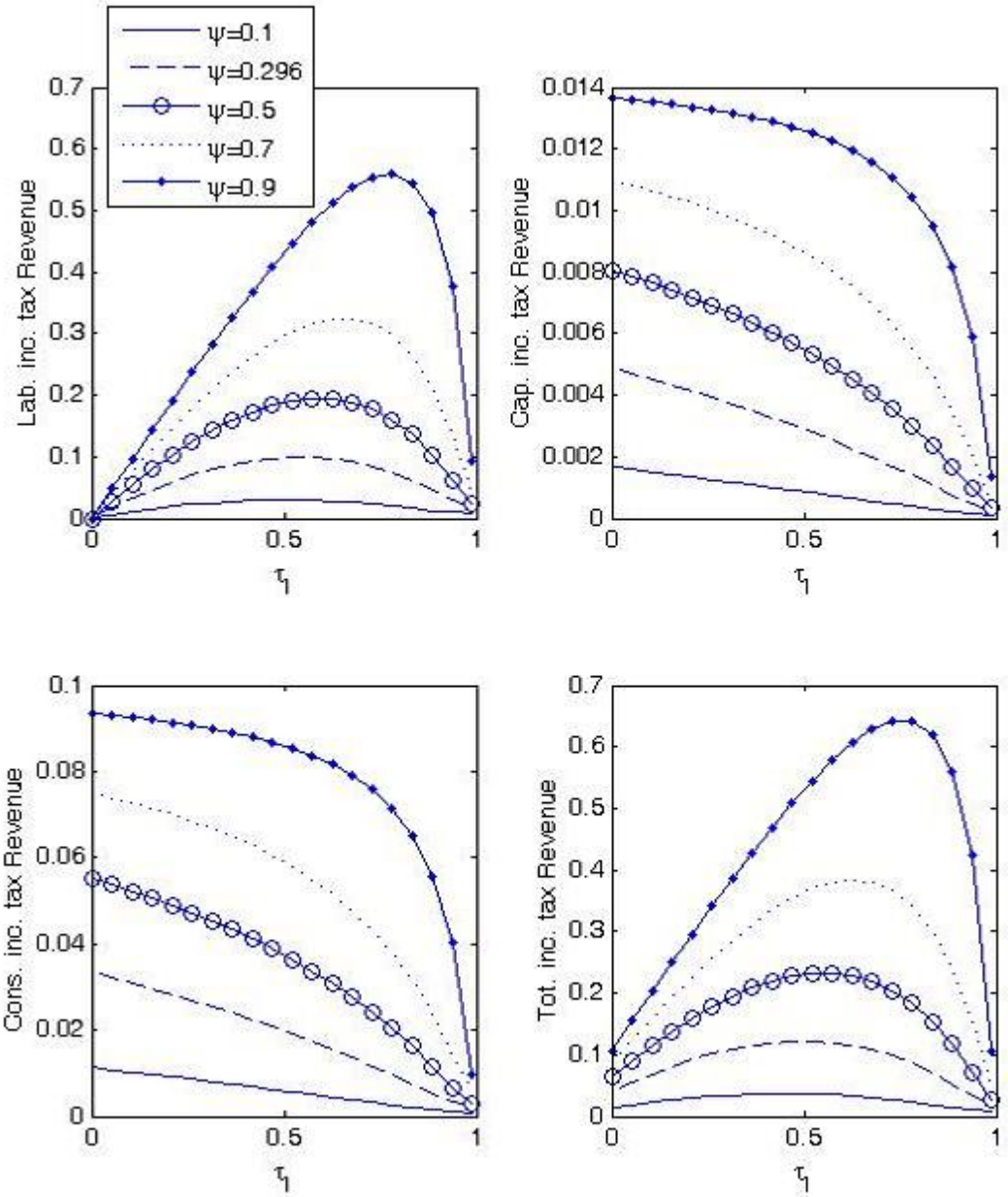


Figure 1.15: Sensitivity analysis: capital income tax Laffer curve (Finn)

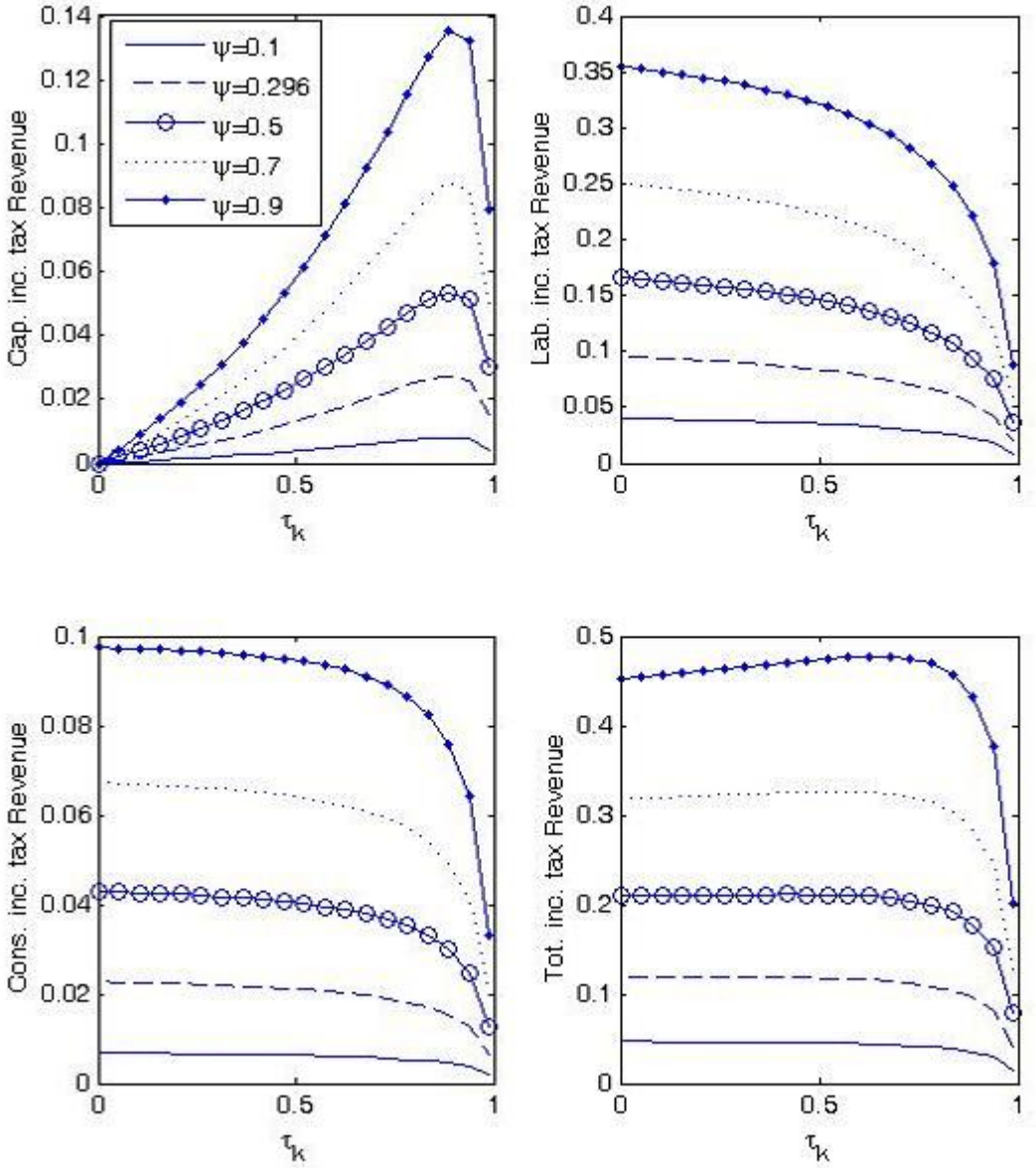
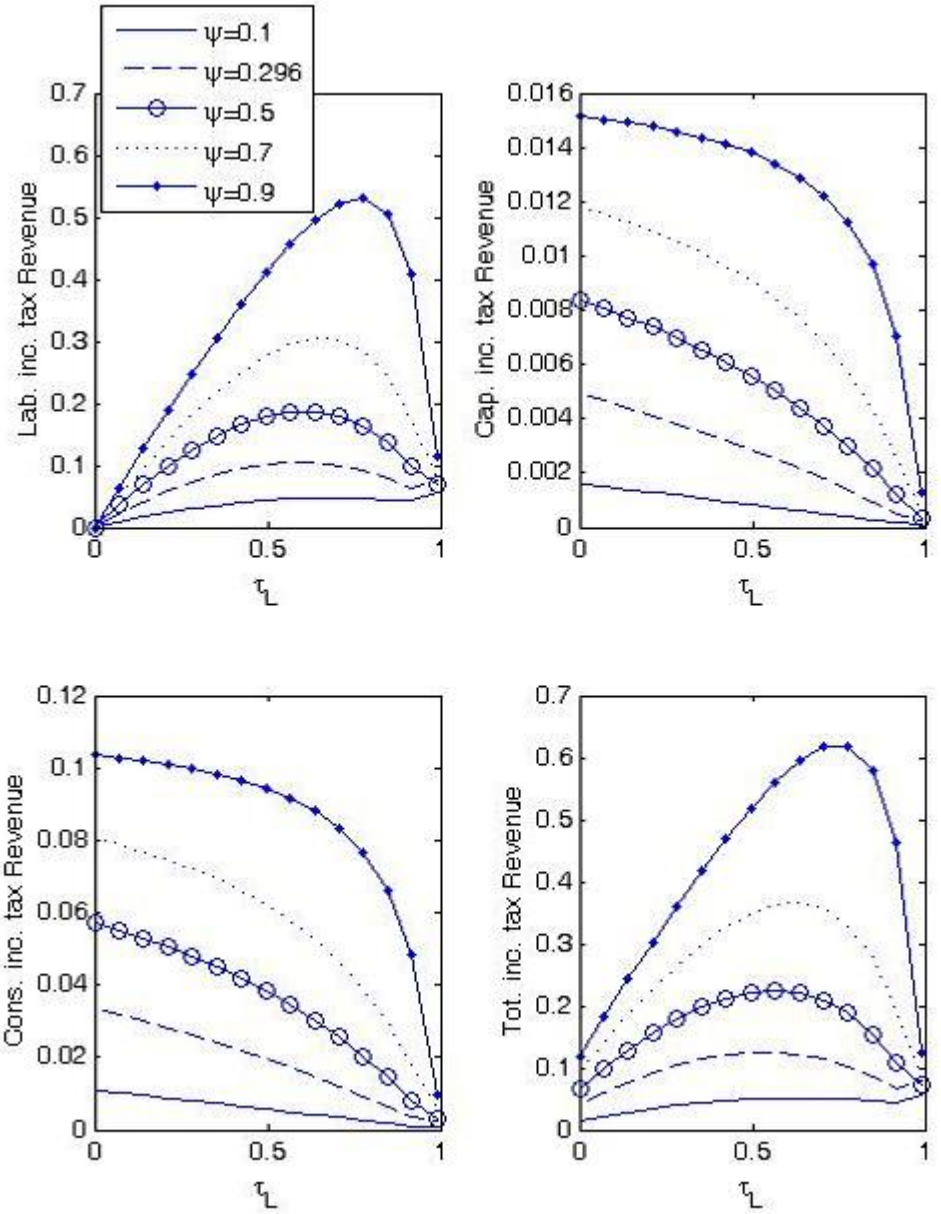


Figure 1.16: Sensitivity analysis: labor income tax Laffer curve (Finn)



1.8.7 Measuring conditional welfare

In steady state

$$u(c, g^c, 1-n) = \frac{[(c+\omega g^c)^{\psi}(1-n)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} \tag{1.8.7.1}$$

Let A and B denote two different regimes. The welfare gain, ζ, is the fraction of consumption that is needed to complement household's steady-state consumption in regime B so that the household is indifferent between the two regimes. Thus

$$\frac{[(c^A+\omega g^{c,A})^{\psi}(1-n^A)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} = \frac{[((1+\zeta)c^B+\omega g^{c,B})^{\psi}(1-n^B)^{(1-\psi)}]^{(1-\alpha)} - 1}{1-\alpha} \tag{1.8.7.2}$$

Multiply both sides by $(1-\alpha)$ to obtain

$$[(c^A+\omega g^{c,A})^{\psi}(1-n^A)^{(1-\psi)}]^{(1-\alpha)} - 1 = [((1+\zeta)c^B+\omega g^{c,B})^{\psi}(1-n^B)^{(1-\psi)}]^{(1-\alpha)} - 1 \tag{1.8.7.3}$$

Cancel -1 terms at both sides to obtain

$$[(c^A+\omega g^{c,A})^{\psi}(1-n^A)^{(1-\psi)}]^{(1-\alpha)} = [((1+\zeta)c^B+\omega g^{c,B})^{\psi}(1-n^B)^{(1-\psi)}]^{(1-\alpha)} \tag{1.8.7.4}$$

Raise both sides to the power $\frac{1}{1-\alpha}$ to obtain

$$(c^A+\omega g^{c,A})^{\psi}(1-n^A)^{(1-\psi)} = ((1+\zeta)c^B+\omega g^{c,B})^{\psi}(1-n^B)^{(1-\psi)} \tag{1.8.7.5}$$

Divide throughout by $(1-n^B)^{(1-\psi)}$ to obtain

$$((1+\zeta)c^B+\omega g^{c,B})^{\psi} = (c^A+\omega g^{c,A})^{\psi}\left(\frac{1-n^A}{1-n^B}\right)^{(1-\psi)}$$

Raise both sides to the power $1/\psi$ to obtain

$$(1+\zeta)c^B+\omega g^{c,B} = (c^A+\omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} \tag{1.8.7.6}$$

Move $\omega g^{c,B}$ term to the right to obtain

$$(1+\zeta)c^B = (c^A+\omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B} \tag{1.8.7.7}$$

Divide both sides by c^B to obtain

$$1+\zeta = \frac{1}{c^B}\left\{(c^A+\omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B}\right\} \tag{1.8.7.8}$$

Thus

$$\zeta = \frac{1}{c^B}\left\{(c^A+\omega g^{c,A})\left(\frac{1-n^A}{1-n^B}\right)^{\frac{(1-\psi)}{\psi}} - \omega g^{c,B}\right\} - 1 \tag{1.8.7.9}$$

Note that if $\zeta > 0 (< 0)$, there is a welfare gain (loss) of moving from B to A. In this chapter B is the initial scenario, while A will be the fiscal regime change.

Chapter 2

Fiscal policy in a Real-Business-Cycle model with labor-intensive government services and endogenous public sector wages and hours

2.1 Introduction

Since the early 1990s, many macroeconomic studies have investigated the effects of fiscal policy in general equilibrium setups, *e.g.* Christiano and Eichenbaum (1992), Baxter and King (1993), MacGrattan (1994), Mendoza and Tesar (1998), Chari, Christiano and Kehoe (1994, 1999), and Kocherlakota (2010).[1] The main focus of the computational experiments performed, however, has been predominantly on the effects of government purchases, public investment and taxes. With the exception of Ardagna (2007) and Fernandez-de-Cordoba *et al.* (2009, 2012), no previous studies had pursued a systematic study of government spending behavior in RBC models with endogenously-determined public employment and wages, as

[1] This chapter abstracts away from nominal rigidities. Readers interested in studying the effects of fiscal policy in frameworks with price stickiness should consult Burnside *et al.* (2004), Pappa (2004), Gali *et al.* (2007), and the references therein.

well as labor-intensive government services. Nonetheless, most of government consumption in national accounts consists of wage consumption, and only a small part actually corresponds to government purchases. (OECD 2012).[2]

This chapter characterizes optimal fiscal policy and then evaluates it relative to the exogenous (observed) one. To this end, a Dynamic Stochastic General Equilibrium (DSGE) model will be set up with a richer government spending side, and an endogenous private-public sector labor choice in particular. The presence of a public employment decision margin, as a separate labor market choice made by the household, has not been sufficiently investigated in such setups properly. In addition, very few books on labor economics, with the exception of Bellante and Jackson (1979), adequately discuss public sector labor markets. This is regrettable, as governments, as the largest employers in Europe, usually have control over wages, at least those in the public sector. In periods of fiscal contraction, wage and employment cuts are often used as instruments to control wage bill spending. When incorporated in an RBC model, such wage-setting powers of the government could produce additional interactions between model variables. Thus, the chapter argues that if public sector labor choice is ignored, then important effects on allocations and welfare, driven by government wage-setting and household's decisions on hours, will be missed. In addition, it will be also shown that aggregate labor market policies are important for public finance management. The interplay between those two is the niche in the fiscal policy literature where this chapter aims to position itself.

This chapter goes on to propose optimal public/private employment- and wage ratios. These labor market dimensions are relevant for policy-makers because many economists since the

[2]Rogoff (2010) also asserts that "it has to matter greatly what the government is spending money on." Reis (2010) adds to that that "the mechanism by which government policy stimulates the economy in standard models is a caricature of reality at best... Thinking harder about how is it that government consumption affects decisions to invest and work will require a transformation in these models, but the toolkits that macroeconomists use leave much room for creativity."

1970s have claimed that public administration is bloated, and have argued that government wages in continental Europe are too generous relative to measured productivity in the private sector. Since there are no other studies that address these labor aspects of fiscal policy, the work in this chapter aims to contribute to the considerable debate on the effects of fiscal policy on aggregate variables. This is followed by a close study of the interactions generated by the presence of a public labor market in a general-equilibrium setup. In particular, the question of the real effects of exogenous and optimal fiscal policy is addressed using a representative-agent framework, where the household chooses hours worked in both sectors. To this end, an otherwise standard Real-Business-Cycle (RBC) model is augmented with a convex cost of working in the government sector. This new component is a useful device in the model, as it produces endogenously-determined public employment and wage rates. The quantitative importance of this transaction cost for the values of allocations in steady-state and their cyclical fluctuations is then studied thoroughly. In addition, the chapter investigates how fiscal policy instruments should optimally react to technology shocks, and how the responses differ between the exogenous and Ramsey (optimal) policy framework.

The novelty in the otherwise standard setup, the introduction of this government hours friction, is a plausible assumption. Mechanically, the cost representation helps the model to incorporate an important stylized fact: Ehrenberg and Schwarz (1986) and Gregory and Borland (1999) show that in the major EU countries, public sector wages feature a significant positive net mark-up above private sector wages. Furthermore, the convex cost function for varying public hours is broadly consistent with the view that the nature of work in the public sector is inherently different from supplying labor services in the private sector. As a result, the modeling approach generates a different disutility of an hour of work across the two sectors.

The inclusion of such a cost in the otherwise standard RBC model with (exogenous) public employment, *e.g.* Finn (1998) has not been used in the literature before. Nevertheless, it can

be justified on many different grounds. For practical purposes, the friction can be interpreted here as a transaction (resource) cost that arises from working in the government sector. This real rigidity is due to the fact that for a worker in the public administration, the employment process and the general organization of work in the public sector are different from job seeking-strategy and the nature of working in the private sector. For an aspiring government bureaucrat, a strict pre-specified sequence of career steps should be taken in order to guarantee advance in the service by achieving regular promotions in the hierarchy. This may involve preparing for a civil service entry exam, which is costly in terms of both time and effort, as it may not be directly related to one's university degree. Another negative effect is that this process often leads to employing overqualified personnel. Additionally, in the public administration there is an over-emphasis on status and hierarchy (OECD 1993), as well as anonymity of the individual bureaucrat, who is merely an instrument of the public administration. Furthermore, once employed in the service, the newly-appointed civil servant starts at the lowest pay grade. Promotion in public administration is usually a wait-in-line process, or is automatically triggered with seniority, and thus not directly based on performance.

Furthermore, employees in strategic sectors such as law and order, and security (police officers, fire fighters), do not enjoy certain civil liberties, *i.e.* soldiers and police officers are not allowed to be members of a political party and/or go on strike. In addition, those employees have to be always prepared to act on call in emergency situations outside normal working hours. Moreover, magistrates, such as prosecutors and judges could be exposed to threats and attacks from the criminal underworld. Those professionals are often exposed to higher levels of stress, and face life-threatening situations, especially in countries where the crime rate is high. Furthermore, Donahue (2008) points out that the private sector counterparts of the aforementioned professions, *e.g.* private detectives, security consultants, and notaries, generally enjoy a much more peaceful life, at the expense of a significantly lower wage rate (pp. 55-56).

In contrast, in the private sector, there is strong emphasis on personal contribution and performance; In terms of working patters, statutory working hours for government employees are usually a fixed full 8-hour shift as compared to flexible working time in the private sector. Additionally, private sector also offers more part-time employment opportunities, which can be the preferred option for female workers and single mothers, who want to spend more time with their children. Importantly, tasks in the civil service are to follow regulations and legislation in an impartial and strict manner. Furthermore, the span of power and job boundaries are clearly defined. This way of organizing work, however, is different from the approach adopted in the private sector, where involvement with a problem, customer-based individual approach, initiative-taking and originality in problem-solving are highly-valued.[3]

In effect, the aforementioned aspects of government work could be viewed as generating hidden per-period resource- or transaction costs. However, the welfare effect of these costs is difficult to measure directly. Thus, the chapter will adopt Balestrino's (2007) modeling approach, who includes such costs as a term that directly decreases household's utility. In particular, in the model framework, utility costs of working in the public sector will decrease the amount of leisure enjoyed by the household, in a manner reminiscent to Kydland and Prescott (1982).[4,5] Given the case-study evidence in Box (2004) on the hidden costs of working in the public sector in the US, the difference in wages across sectors could be justified, at

[3]Independently from the arguments presented above, the time cost introduced in this chapter is reminiscent of the psychic cost interpretation discussed in Tinbergen (1985). He argues that there is a strong negative non-monetary benefit of working in the public sector. In particular, job satisfaction experienced by government workers is usually low. Therefore, the modeling choice in this chapter could be regarded as incorporating the psychic cost theory, *e.g.* as in Corbi *et al.* (2008), into the RBC framework, and applying it to the public sector labor choice context.

[4]In contrast to Kydland and Prescott (1982), transaction cost will be modeled as a quadratic function of current-period hours, and not as a polynomial function of lagged hours, as is the case in the original article.

[5]An alternative specification, *e.g.* modeling the transaction cost as a consumption/output cost is also possible. In that case, however, the disutility of an hour worked in the model would have been the same across sectors. Since this assumption is in contrast with the evidence provided above, this approach was not pursued further.

least partially, as a compensation for the additional transaction cost incurred from government work. However, in contrast to earlier literature, *e.g.* Alesina *et al.* (2002), Algan *et al.* (2002), Demekas and Kontolemis (2000), Forni and Giordano (2003) and Ardagna (2007), in this chapter the wage rate difference will persist despite the free mobility of labor between sectors.[6]

Additionally, the specific focus on government spending categories in this chapter, and public hours choice in particular, provides a new explanation for the macroeconomic importance of government employment, as a rational supply decision made by the household in a clearing labor market. After all, the strong positive trend in public employment is a common stylized pattern observed in the post-WWII data series in the major European countries (OECD 2011). Earlier studies by Finn (1998), Cavallo (2005) and Linnemann (2009), however, circumvent the problem of optimal choice of hours between private and public sector by modeling public employment as a stochastic process that approximates data behavior.

The recent study by Fernandez-de-Cordoba *et al.* (2009, 2012) takes an alternative approach: Their model endogeneizes public hours using political-economy arguments as a major factor behind public employment dynamics. In their setup, employment in the government sector is an optimal decision made by a monopolistic public sector union, and not by the household, though. The union, being a special-interests group, is assumed to optimize over a weighted average of the public wage rate and public employment. In addition, the union's objective function is maximized subject to the government budget constraint, where government budget is balanced in every period. In their model, spending on government wages is productive, as public employment produces a positive externality on aggregate output. However, Fernandez-de-Cordoba *et al.* (2009, 2012) consider exogenous income tax rate shocks only, and leave outside the scope of their study interesting issues such as optimal

[6]There are other modeling approaches, *e.g.* setups that emphasize directed jobs search mechanisms, which are not going to be covered in this chapter. Interested readers should consult Quadrini and Trigari (2007), Gomes (2009, 2012) and the references therein.

tax-rate determination, public-wage- and employment-setting possibilities. Thus, previous literature has not adequately addressed either the household's choice at the margin between entering the public or the private sector, or the optimal policy framework, in which a benevolent government chooses all spending components in addition to the tax rates.[7] All these structural aspects of public sector labor markets and their relevance for public finance and public policy represent a gap in the literature, which the research in this chapter aims to fill.

The second novelty in the framework, which adds value to earlier studies, is the more interesting and meaningful role attributed to government employees. In particular, the study models in greater detail the mechanism of public good provision. The setup models the government as an employer, needing labor hours to provide public goods. In contrast to Cavallo (2005) and Linnemann (2009), labor here is combined with government capital (instead of government purchases) to produce valuable government services. Therefore, government investment is a productive government spending category in the setup, and public sector wage consumption is not entirely wasteful. Importantly, when hiring workers, the government will be able to set the public sector wage rate, an assumption which is consistent with data, *e.g.* Perez and Schucknecht (2003).

As pointed out in Ross (1985), many government programs are indeed labor-intensive, such as law and order, public administration, education, and health- and social care.[8] Moreover, public administration, which is in charge of enforcing various rules and regulations, represents the biggest category of public employees in data that provides labor-intensive services. The other input in the public good production, government capital, will be an aggregate category that includes hospitals, schools, administrative buildings, infrastructure and equip-

[7]The model in Fernandez-de-Cordoba *et al.* (2009, 2012) does not satisfy Hagedorn's (2010) conditions for the existence of a "sensible" Ramsey equilibrium, *i.e.* one that satisfies the non-negativity conditions (and the one for public employment in particular). Strictly speaking, no direct mapping exists between the exogenous policy case and the optimal policy framework in their model.

[8]In addition, as pointed out in Downs (1967), a big part of government services is non-market output.

ment, etc. Thus, the chapter expands on earlier research by endogeneizing public goods provision, as well as the determination of the wage rate and employment in the public sector. Furthermore, the presence of both public hours and government investment as inputs in the production of government services will have important external effects on the economy: In competitive equilibrium, the household will ignore the utility-enhancing effect of more public hours supplied, which increases the level of public good provided. The same outcome occurs with public investment, which is taken by households as being set exogenously.

After presenting and discussing the benchmark exogenous policy case, Chapter 2 proceeds to study public employment, wage rate and government investment determination, as well as the level of public services provision, when all policy variables are optimally chosen by a benevolent government. In particular, under the optimal (Ramsey) policy regime, the benevolent government will choose the socially optimal levels of both private allocations and the public good. However, in a second-best world, the benevolent government will still finance its expenditure using proportional taxation, and remedy the existing distortions at the cost of introducing new ones. The comparison across the two regimes focuses on both the long-run behavior of government spending and the transitional dynamics of fiscal policy instruments and the other allocations in the economy. The model is calibrated to German data, which feature both a significant public wage rate premium, and a large public employment share, and thus are a good testing ground for the theory.

Similar to earlier literature, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004), and Kocherlakota (2010), allowing for fiscal interventions in an RBC framework creates interesting trade-offs. On the one hand, public wage bill tends to increase welfare by providing higher consumption. In addition, higher public employment, government investment and public capital increase the level of utility-enhancing productive government services. On the other hand, more hours spent in the public sector, together with the transaction costs associated with government work, decrease household's utility

from leisure. Furthermore, the proportional taxes on labor and capital are known to distort incentives to supply labor in the private and public sectors, and to accumulate physical capital. Therefore, higher taxes reduce consumption, which in turn lowers welfare. Lastly, despite the presence of public employment and government services in the model, the public finance problem is still to choose labor and capital tax rates to finance total government expenditure, while at the same time minimizing the allocative distortions created in the economy, as a result of the presence of proportional taxation.

The main findings from the computational experiments performed in this chapter are: (i) as in Judd (1985), Chamley (1986) and Zhu (1992), the optimal steady-state capital tax rate is zero, as it is the most distortionary tax to use; (ii) A higher labor tax rate is needed to compensate for the loss in capital tax revenue; (iii) Under the optimal policy regime, public sector employment is lower. As a result, government employees are more valuable, and receive higher wages. (iv) The government wage bill is smaller, while public investment is three times higher than in the exogenous policy case. In other words, the model predicts that on average, public employment in Germany is too high, government employees are underpaid, and too little is invested in public capital; (v) The benevolent Ramsey planner substitutes labor for capital in the production of both public services and private output.

The rest of Chapter 2 is organized as follows: Section 2.2 describes the model framework, Section 2.3 lays out the equilibrium system, Section 2.4 present the calibration, the steady-state model solution and some comparative statics. Sections 2.5 provides the model solution and discusses transitional dynamics of the model variables in response to technological innovations. Sections 2.6 proceeds with the optimal taxation (Ramsey) policy problem. Section 2.7 evaluates both the transitional dynamics and the long-run effects on the economy. Section 2.8 acknowledges the limitations of the study, and section 2.9 concludes the chapter.

2.2 Model setup

The model features a representative household, as well as a representative firm. The household owns the capital, which it supplies to the firm. Next, the household's unit endowment of time can be supplied to the private sector (firm), public sector, or enjoyed in the form of leisure. Due to the presence of additional transaction costs associated with government work, hours supplied in the public sector impose an additional utility cost on the household. The perfectly-competitive firm produces output using labor and capital, while the government hires labor and combines it with public capital to produce valuable government services. To finance the public wage bill, government investment and transfers, tax revenues from labor and capital income are collected, and the wage rate in the public sector is determined residually to balance the government budget in every period.

2.2.1 Households

There is an infinitely-lived representative household in the model economy, and no population growth. The household maximizes an expected utility function, as in Klein *et al.* (2008):

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^h, L_t^h, S_t^g), \tag{2.2.1.1}$$

where E_0 is the expectation operator as of period 0; C_t^h, L_t^h and S_t^g are household's consumption, leisure and per household consumption of government services enjoyed at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. The particular form chosen for utility is:

$$U(C_t^h, L_t^h, S_t^g) = \psi_1 \ln C_t^h + \psi_2 \ln L_t^h + \psi_3 \ln S_t^g \tag{2.2.1.2}$$

where the parameters ψ_1, ψ_2 and $\psi_3 \equiv 1 - \psi_1 - \psi_2$ denote the weights attached to the utility of consumption, leisure and government services (public good consumption), respectively, $0 < \psi_1, \psi_2, \psi_3 < 1$. The level of government services is taken as given by the household.[9]

[9]The logarithmic specification for consumption can be interpreted as households pooling resources from public and private sector together, as in Merz (1995). The logarithmic form for leisure is in line with the

The household has an endowment of one unit of time in each period t, which is split between work, N_t^h and leisure, L_t^h, so that

$$N_t^h + L_t^h = 1. \tag{2.2.1.3}$$

The household can supply hours of work in the public sector, N_t^{gh}, or in the private one, N_t^{ph}.[10] The wage rate per hour of work in public sector is w_t^g, and w_t^p in the private sector, which will be allowed to differ from one another. In particular, when the household chooses N_t^{gh}, it incurs an additional convex transaction cost, $\gamma(N^{gh})^2$, measured in terms of time, which will also depend on the level of public employment (where $\gamma > 0$).[11] Thus, the effective leisure for the household becomes

$$L_t = 1 - N_t^{ph} - N_t^{gh} - \gamma(N_t^{gh})^2. \tag{2.2.1.4}$$

Observe that in the polar case, when there is no transaction cost, or $\gamma = 0$, the two wage rates are equal $w_t^p = w_t^g$, because the disutility of an hour worked is equalized across sectors. This special case of the model collapses to the representation used in Finn (1998), Cavallo (2005) and Linnemann (2009). Furthermore, Tinbergen (1985) argues that public sector employees suffer from lower job satisfaction, compared to their private sector counterparts, and incur a per-period psychic (non-monetary) cost of working for the government (p.36). Therefore, the usefulness of this particular modeling choice is that it generates a friction between the marginal disutility of an hour worked in the public and private sector, which is generally consistent with the evidence and also helps the framework to accommodate the

specification used in Cavallo (2009) and Gomes (2012).

[10] In this chapter, due to the normalization of total population to unity, "hours" and "employment" will be used interchangeably.

[11] This is consistent with the evidence that labor flows from the public to the private sector are much smaller than those from the private to the public sector.

different wage rates in the two labor markets.[12,13]

In addition to the labor income generated from supplying hours in the two sectors, the representative household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to private capital, and K_t^{ph} denotes private capital stock in the beginning of period t.

Finally, the household owns all firms in the economy, and receives all profit (Π_t^h) in the form of dividends. Household's budget constraint is

$$C_t^h + I_t^h \leq (1-\tau_t^l)[w_t^p N_t^{ph} + w_t^g N_t^{gh}] + (1-\tau_t^k) r_t K_t^{ph} + G_t^T + \Pi_t^h, \tag{2.2.1.5}$$

where τ_t^l, τ_t^k are the proportional tax rates on labor and capital income, respectively, and G_t^T denotes the level of lump-sum government transfers per household.

Household's private physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1-\delta^p) K_t^{ph}, \tag{2.2.1.6}$$

where $0 < \delta^p < 1$ denotes the depreciation rate on private physical capital.

The representative household acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$, policy variables $\{K_{t+1}^g, S_t^g, G_t^T\}_{t=0}^{\infty}$ as given, and chooses allocations $\{C_t^h, N_t^{ph}, N_t^{gh}, I_t^h, K_{t+1}^{ph}\}_{t=0}^{\infty}$ to maximize Eq. (2.2.1.1) subject to Eqs. (2.2.1.2)-(2.2.1.6), and initial conditions for private and public physical capital stocks, $\{K_0^{ph}, K_0^g\}$.

The optimality conditions from the household's problem, together with the transversality

[12]In addition, when both public and private hours are chosen by the households, the model will be consistent with the findings that public employment crowds out private employment, *e.g.* Malley and Moutos (1998) on Sweden, and Algan *et al.* (2002) on OECD countries.

[13]On a more abstract level, the convex cost could be interpreted as a short-cut, which substitutes for an explicit government optimization problem. Examples of the latter are Niskanen (1971) and Ardagna (2007).

condition (TVC) for physical capital, are as follows: [14]

$$C_t^h\colon \frac{\psi_1}{C_t^h} = \Lambda_t \tag{2.2.1.7}$$

$$N_t^{ph}\colon \frac{\psi_2}{1 - N_t^{ph} - N_t^{gh} - \gamma(N_t^{gh})^2} = \Lambda_t(1-\tau_t^l)w_t^p \tag{2.2.1.8}$$

$$N_t^{gh}\colon \frac{\psi_2}{1 - N_t^{ph} - N_t^{gh} - \gamma(N_t^{gh})^2}[1 + 2\gamma N_t^{gh}] = \Lambda_t(1-\tau_t^l)w_t^g \tag{2.2.1.9}$$

$$K_{t+1}^{ph}\colon \beta E_t \Lambda_{t+1}\Big[(1-\tau_{t+1}^k)r_{t+1} + (1-\delta^p)\Big] = \Lambda_t \tag{2.2.1.10}$$

$$\text{TVC}\colon \lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^{ph} = 0, \tag{2.2.1.11}$$

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Private and public hours are chosen so that the disutility of an hour of work in each sector at the margin equals the after-tax return to labor in the corresponding sector. Next, the Euler equation describes the optimal private capital accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two adjacent periods. The last expression is the TVC, imposed to ensure that the value of the physical capital that remains at the end of the optimization horizon is zero. This boundary condition ensures that the model equilibrium is well-defined by ruling out explosive solution paths.

2.2.2 Firms

There is also a representative private firm in the model economy as well. It produces a homogeneous final product using a production function that requires private physical capital, K_t^p, and labor hours N_t^p. The production function is as follows

$$Y_t = A_t(N_t^p)^\theta (K_t^p)^{1-\theta}, \tag{2.2.2.1}$$

[14] Detailed derivation of household's optimality conditions is provided in Appendix 2.10.1.

where A_t measures the level of total factor productivity at time t; $0 < \theta, (1-\theta) < 1$ are the productivity of labor and capital, respectively.

The representative firm acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ and policy variables $\{\tau_t^k, \tau_t^l, K_{t+1}^g, S_t^g, G_t^T\}_{t=0}^{\infty}$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize the firm's static aggregate profit,

$$\Pi_t = A_t (N_t^p)^{\theta} (K_t^p)^{1-\theta} - r_t K_t^p - w_t^p N_t^p. \tag{2.2.2.2}$$

In equilibrium, profit is zero. In addition, labor and capital receive their marginal products, *i.e* [15]

$$w_t^p = \theta \frac{Y_t}{N_t^p} \tag{2.2.2.3}$$

$$r_t = (1-\theta) \frac{Y_t}{K_t^p}. \tag{2.2.2.4}$$

2.2.3 Government budget constraint

The government distributes transfers G_t^T to the household, invests G_t^i in public capital K_t^g, and hires labor N_t^g at the public sector wage w_t^g. Public employment and government capital are then combined to provide utility-enhancing government services, S_t^g, according to the following constant-returns-to-scale production function, as in Cavallo (2005), Linnemann (2009), and Economides *et al* (2011):

$$S_t^g = (N_t^g)^{\alpha} (K_t^g)^{(1-\alpha)}, \tag{2.2.3.1}$$

where α and $1-\alpha$ denote labor and capital share in government services, respectively, and $0 < \alpha, 1-\alpha < 1$. Since the household takes the level of government services as given, in competitive equilibrium there will be an externality arising from the presence of public employment and investment in the government services production function: more hours in the public sector generate more government services (a higher level of the public good available

[15] Detailed derivation of firm's optimality conditions is provided in Appendix 2.10.1.

for public consumption), which increase household's utility directly. In addition, holding all else equal, an increase in public employment raises welfare indirectly by increasing the after-tax public sector labor income, and hence consumption. Lastly, more hours spent in the public sector decrease the amount of leisure the household can enjoy in a certain period, and thus lower welfare.

Next, total government expenditure, $G_t^T + G_t^i + w_t^g N_t^g$, is financed by levying proportional taxes on capital and labor income. Thus, the government budget constraint is[16]

$$G_t^T + G_t^i + w_t^g N_t^g = \tau_t^k r_t K_t^p + \tau_t^l \left[w_t^p N_t^p + w_t^g N_t^g \right]. \tag{2.2.3.2}$$

Next, public capital accumulates according to the following law of motion:

$$K_{t+1}^g = G_t^i + (1 - \delta^g) K_t^g, \tag{2.2.3.3}$$

where $0 < \delta^g < 1$ denotes the depreciation rate on public capital.

The government takes market prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and allocations $\{N_t^p, N_t^g, K_t^p\}_{t=0}^{\infty}$ as given.[17] It will be assumed that the government chooses the public investment and transfers shares, where $G_t^{iy} = \frac{G_t^i}{Y_t}$ and $G_t^{Ty} = \frac{G_t^T}{Y_t}$. Thus, the level of government investment $G_t^i = G_t^{iy} Y_t$, and government transfers $G_t^T = G_t^{Ty} Y_t$ will both respond to output.

Only four of the following five policy instruments, $\{\tau_t^k, \tau_t^l, w_t^g, \frac{G_t^i}{Y_t}, \frac{G_t^T}{Y_t}\}_{t=0}^{\infty}$, will be exogenously

[16]Since public sector wage bill appears both as a revenue and expenditure category, the representation above is equivalent to a setup in which the government pays public wages net-of-taxes directly.

[17]In the model, the assumption that the government cannot influence private sector prices in the exogenous policy case is a technical condition that allows for the DCE to be solved. Later, in the optimal policy case, the government will choose optimally all allocations, subjects to the constraints imposed by the DCE. First, by choosing the tax rates on labor and capital, the government can set the after-tax returns of the factors of production. In addition, by choosing the levels of private sector hours worked and the private physical capital, the benevolent government implicitly determines prices, using the constraint from the decentralized competitive equilibrium (DCE) system that in equilibrium private factors of production receive their marginal product.

set. First, the government transfers share will be set to match the average public/private employment ratio in data. Next, the government investment share, $\frac{G_t^i}{Y_t}$, as well as capital and labor income tax rates will be set equal to the average rates in data. Lastly, the public sector wage rate will be determined as a residual to ensure that the government runs a balanced budget in every period. Thus, the government still acts to a certain degree as a regulator of the labor supplied in the public sector.[18]

2.2.4 Stochastic processes for the exogenous variables

The exogenous stochastic variable is total factor productivity A_t, which is assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a, \tag{2.2.4.1}$$

where $A_0 = A > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in the total factor productivity process.

2.2.5 Decentralized competitive equilibrium

Given the fixed values of capital and labor income tax rates, government transfers/output and government investment/output ratios $\{\tau^k, \tau^l, G^{Ty}, G^{iy}\}$, the exogenous process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$, the initial conditions for the state variables $\{A_0, K_0^{ph}, K_0^g\}$, a decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t^h, G_t^i, N_t^{ph}, N_t^{gh}, I_t^h, K_{t+1}^{ph}, K_{t+1}^g, S_t^g, G_t^T\}$ for all h, and prices $\{r_t, w_t^p, w_t^g\}_{t=0}^{\infty}$ such that (i) the representative household maximizes utility; (ii) the stand-in firm maximizes profit every

[18]Note that in general equilibrium, the two wage rates will be inter-related, which is in line with the empirical study in Lamo, Perez and Schucknecht (2007, 2008). Thus, the level of government investment will respond to output. (K_{t+1}^g will be exogenously determined as well, subject to the initial condition K_0^g and the law of motion for G^i.) Note also that the public sector wage rate schedule implicitly determines government's endogenous demand for public labor.

period; (iii) government budget constraint is satisfied in each time period, and (iv) all markets clear.[19]

2.3 Data and model calibration

The model in this chapter is calibrated for German data at annual frequency for consistency reasons. The choice of this particular economy was made, as in Chapter 1, based on the large public employment share, as well as the significant public wage premium observed in this country. Since there is no EU-wide fiscal authority, an individual country was chosen, rather than calibrating the model for the EU Area as a whole. In addition, payment in the public sector in the model is determined not by marginal productivity of labor, but rather by budgetary considerations and thus most likely by non-market factors (as the price of the public good is zero). Lastly, given the importance of government transfers in matching average employment ratio in data by the model, the calibration for a particular country is preferable as transfers are to a great extent driven by political considerations and are also determined at national-, and not at EU level.

The chapter follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate relevant long-run moments of the reference economy for the question investigated in this chapter.

[19]The system of equations that characterizes the DCE is provided in Appendix 2.10.2.

2.3.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007 only, while the sub-period 1970-91 covers West Germany only.[20,21,22] For Germany, data on real output per capita, household consumption per capita, government transfers and population was taken from the World Development Indicators (WDI) database. The OECD statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Investment and capital stock series were obtained from the EU Klems database (2009). German average annual real public compensation per employee was estimated by dividing the real government wage bill (OECD 2011) by the number of public employees.

2.3.2 Calibrating model parameters to German data

In German data, the average public/private employment ratio over the period 1970-2007 is $n^g/n^p = 0.17$, and the average public/private wage ratio is $w^g/w^p = 1.20$. Next, the average effective tax rates on labor and physical capital, obtained from McDaniel's (2009) dataset are $\tau^l = 0.409$ and $\tau^k = 0.16$, respectively. As in Chapter 1, McDaniel's approach was preferred to that used by Mendoza et al. (1984) and the subsequent updates, *e.g.* Martinez-Mongay (2000), Carey and Tchilinguirian (2000) and Carey and Rabesona (2002), due to the more careful treatment of property and import taxes in the former. The labor share, $\theta = 0.71$, was computed as the average ratio of compensation of employees in total output. Alterna-

[20]The time period is particularly suitable for the study of public employment, and government wage bill spending; Hughes (1994), for example, argues that "[i]n the 1970s, intellectual arguments were mounted by conservative economists that government was <u>the</u> economic problem restricting economic growth and freedom."(p. 11)

[21]The values of the model parameters used in the calibration do not differ significantly from those computed for the periods 1970-91, and 1991-2007 separately.

[22]Despite the fact that Germany is an open economy, here the focus is on the closed-economy case, as fiscal policy mainly serves domestic interests. Thus, the open-economy dimension is left for future research.

tively, average capital share, $1-\theta = 0.29$, can be obtained as the mean ratio of gross capital compensation in output from EU Klems Database (2009). Private capital depreciation rate was found to be $\delta^p = 0.082$, while public capital depreciation rate is $\delta^p = 0.037$ over the period.

The discount rate $\beta = 0.979$ was calibrated from the steady-state consumption Euler equation to match the average private capital-to-output ratio in data. Parameter $\alpha = 0.62$, which measures the weight on public sector hours in the public good production is obtained as the average ratio of public sector wage bill to total government expenditure less transfers and subsidies, as in Cavallo (2005) and Linnemann (2009). The value is consistent with OECD (1982) estimates for the period 1960-78 for Germany, which was obtained from a log-linear regression estimation. Additionally, the calibrated value of public capital elasticity, $1-\alpha = 0.38$, is consistent with the government capital effect estimated in Aschauer (1989) and Hjerppe *et al.* (2006). In the exogenous policy setup, parameter α does not affect allocations, since the household ignores the externality. Thus, the level of government services will be residually determined. Nevertheless, there is a negative monotone relationship between public hours elasticity and welfare.

Next, using the estimate obtained in Finn (1994), the weight attached to productive government services in utility is set equal to $\psi_3 = 0.16$.[23] This value is consistent with the one used in Klein *et al.* (2008) for the weight attached by the household to utility derived from the consumption of the public good. The weight on private consumption in the household's utility function in this chapter was then set equal to $\psi_1 = 0.31$. This produces a ratio $\psi_1/\psi_3 = 1.94$, which is also consistent with the ratios in Bouakez and Rebel (2007), Leeper *et al.* (2009) and Conesa *et al.* (2009), who argue that the private consumption good is on average twice more valuable for the household, compared to the public good. Next, the

[23]Robustness checks with $\psi_3 = 0.1$ and $\psi_3 = 0.2$ to reflect the standard error of the estimation performed in Finn (1994) do not significantly affect either steady-state results, or transitional dynamics of the model.

weight on utility is determined residually as $\psi_2 = 1 - \psi_1 - \psi_3$. The calibrated value for ψ_2 is also in line with earlier RBC studies, which usually attach a weight on leisure, which is twice as large as the weight attached to private consumption in the household's utility function. In this model, the ratio in question is $\psi_2/\psi_1 = 1.8$. Note that in contrast to Kydland (1995), ψ_1 was set slightly higher than the average steady-state total hours of work in data as a share of total hours available, $n = 0.296$, to account for the presence of transaction costs, which decrease the effective leisure.[24] Nevertheless, total employment is consistent with the estimates of the fraction of time spent working in Ghez and Becker (1975) and Juster and Stafford (1991). Together with the employment ratio, this yields the model-consistent steady-state values for private and public hours, $n^p = 0.253$ and $n^g = 0.043$, respectively.

Next, the scale parameter $\gamma = 2.576$ in the public employment convex utility cost was set to match the average public/private wage ratio from data in steady-state. The wage ratio was chosen as a target, as the higher average wage in the public sector is viewed to a certain degree as a compensation for the transaction costs incurred from government work. In line with the RBC literature, the steady-state level of technology, A, is normalized at unity.

Total factor productivity moments, $\rho^a = 0.9427$ and $\sigma^a = 0.0131$, were obtained in several steps: First, using the model's aggregate production function specification and data series for physical capital and labor, Solow residuals (SR) were computed in the following way:

$$\ln SR_t = \ln y_t - (1-\theta)\ln k_t^p - \theta \ln n_t^p. \tag{2.3.2.1}$$

The logged series are then regressed on a linear trend ($b > 0$) to obtain

$$\ln SR_t = bt + \epsilon_t^{SR}. \tag{2.3.2.2}$$

[24] In this setup with two types of endogenously-determined hours and public hours transaction costs, it is not possible to derive labor supply functions explicitly.

Observe that the residuals from the regression above,

$$\epsilon_t^{SR} = \ln SR_t - bt \equiv \ln a_t, \tag{2.3.2.3}$$

represent the stationary, or detrended, component of the logged TFP series.

Next, the AR(1) regression

$$\ln a_t = \beta_0 + \beta_1 \ln a_{t-1} + \epsilon_t^a \tag{2.3.2.4}$$

was run using ordinary least squares (OLS) to produce the estimates (denoted by the "hat" symbol) for the persistence and standard deviation parameters of the total factor productivity process to be used in the calibration of the model. In particular,

$$\hat{\beta}_1 = \rho^a \tag{2.3.2.5}$$

$$\hat{\epsilon}_t^a \sim N(0, \sigma_a^2). \tag{2.3.2.6}$$

Table 2.1 on the next page summarizes all the model parameters used in the calibration.

Table 2.1: Model Parameters

Param.	Value	Definition	Source
β	0.979	Discount factor	Calibrated
θ	0.710	Labor income share	Data average
$1-\theta$	0.290	Capital income share	Calibrated
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on government capital	Data average
ψ_1	0.310	Weight on consumption in utility	Set
ψ_2	0.530	Weight on leisure in utility	Calibrated
ψ_3	0.160	Weight on government services in utility	Set
γ	2.5762	Scale parameter for public hours transaction cost	Calibrated
α	0.620	Labor share in public services production	Data average
$1-\alpha$	0.380	Govt. capital share in public services production	Calibrated
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
G^{iy}	0.023	Government investment-to-output ratio	Data average
G^{Ty}	0.228	Government transfers-to-output ratio	Set/Calibrated
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated

2.4 Steady state results

Once model parameters were obtained, the unique steady-state of the system was computed numerically[25] for the Germany-calibrated model. Results are reported in Table 2.2 on the next page. The net returns to public and private labor,

$$\tilde{w}^p = (1 - \tau^l)w^p \quad \text{and} \tag{2.4.0.7}$$

$$\tilde{w}^g = (1 - \tau^l)w^g, \tag{2.4.0.8}$$

respectively, as well as the after-tax net of depreciation real return to capital,

$$\bar{r} = (1 - \tau^k)(r - \delta^p), \tag{2.4.0.9}$$

are also reported. These prices will be useful for comparisons across tax regimes in the analysis that follows. Finally, social welfare U is the discounted stream of instantaneous utilities evaluated at the steady-state allocations of consumption, hours and government services in every period.

The model performs relatively well *vis-a-vis* data. It slightly overestimates average consumption and underestimates the investment shares in output. This mismatch is due to the fact that the model treats government wage bill consumption as a transfer payment, and not as final public consumption, as is the case in the national accounts. This is not an issue here as the main objective of the model is to replicate the stylized facts in the labor markets. However, the model accurately captures the long-run after-tax capital return, where the latter is proxied by the average return on 10-year generic bonds net of CPI inflation. Moreover, the imputed government services is also predicted to make a significant share of output.[26]

Along the labor market dimension, the average time spent working is also close to its empirical counterpart over the period. By construction, the model was set to match the wage and

[25] Appendix 2.10.3 summarizes the steady-state DCE system.

[26] In addition, this figure is close to the average government consumption-to-output ratio in German data (0.20).

Table 2.2: Data averages and long-run solution: exogenous policy

	GE Data	Exogenous
c/y	0.590	0.784
i/y	0.210	0.192
g^i/y	0.023	0.023
k^p/y	2.350	2.346
k^g/y	0.630	0.630
s^g/y	0.196	0.224
g^T/y	0.170	0.228
$w^p n^p/y$	0.710	0.710
$w^g n^g/y$	0.130	0.145
rk/y	0.290	0.290
w^g/w^p	1.200	1.200
n	0.296	0.266
n^p	0.253	0.227
n^g	0.043	0.039
n^g/n^p	0.17	0.170
$\gamma(n^g)^2/n^g$	N/A	0.099
$\bar{r}$	0.036	0.035
U	N/A	-47.91

employment ratios in data. Given the focus on the labor effects of fiscal policy, exogenous policy framework was calibrated to reproduce those stylized facts in the steady state, as this framework will provide an important benchmark for fiscal policy experiments in the later sections. Next, the ratio of the time cost of working in the public sector relative to public labor supply is a non-trivial figure in steady-state, $\frac{\gamma(n^g)^2}{n^g} = 0.10$. Thus, transaction costs in the government sector are likely to be an important factor for a worker who is considering whether to enter the public sector labor market or the private sector one. In other words,

these additional transaction costs should be incorporated in a(n) (equilibrium) wage offer accepted by a rational worker, who decides to supply labor services in the public sector.[27]

In the next section, the public/private wage ratio target is relaxed in order to perform an important comparative-statics exercise. The focus of the analysis will fall on the new parameter, γ, as its properties within the otherwise standard model framework have not been investigated so far. In particular, the computational experiments in the following section will aim to uncover whether a systematic relationship exists between transaction cost scale parameter γ and the steady-state values of the other model variables.

2.4.1 Comparative Statics: changes in transaction cost parameter

This section investigates the dependence of the steady-state results obtained in the previous section on the value of γ. Making jobs between sectors more similar corresponds to a decrease in the value of parameter γ, and thus a decrease in the level of the transaction costs of government work. In particular, the changes in this new parameter could be mapped to concrete institutional reforms in the government administration. Reform measures could include, but are not limited to, new human resource practices in the public sector that try to follow as closely as possible the practices used in the private sector. For example, the measures could involve the introduction of flexible hours and performance pay for government employees, as well as a "fast track career path" in the public sector. Other possible measures include decentralization, job evaluation, the breakdown of old established work demarcations and job boundaries, and greater use of public relationships (PR) and informational technology (IT) (OECD 1993).

In the literature, this move toward market-based labor relationships in the 1990s was called "New Public Management"(NPM) and discussed in Hughes (1991), Ferlie *et al.* (1996) and

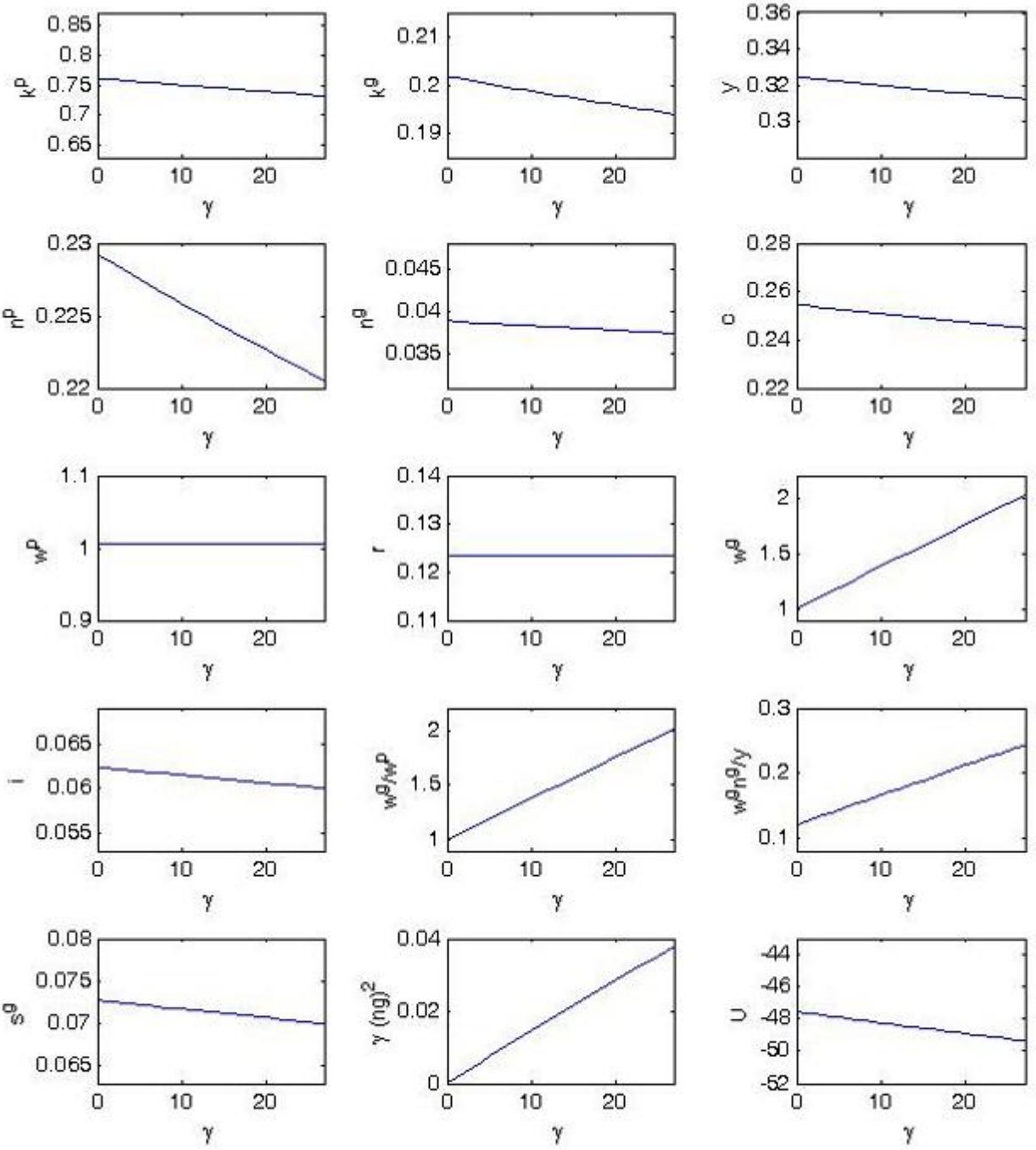
[27]The average magnitude of the transaction cost is comparable to the size of another type of transaction costs in labor economics literature, namely the cost of commuting, as shown in Ehrenberg and Smith (2003).

Nolan (2001). In particular, the major Western governments were trying to apply the best business practices used in the private sector into the public sector. The NPM approach proposed that public managers should be put in charge of the size, evaluation and qualification level of the workforce in their respective government agencies. The expected effect of the NPM was to empower heads of agencies to adjust employment size and skill level and implement best business practices, or the so-called "market-based public administration." NPM were supposed to make the public sector more efficient and in line with the practices in the private sector. Such administrative reforms would make public sector jobs more similar to private sector ones, and rationalize a reduction in transaction costs in the model.

Therefore, in the model setup, all those measures listed above would effectively decrease the hidden costs of working in the government for any amount of public hours chosen by the household. The fall in the transaction cost is expected to affect consumption, private hours, capital, and output in a positive way. In addition, a fall in γ is expected to drive down the public wage premium, as well as the public wage bill share in output. As shown in Fig. 2.1 on the next page, the model is consistent with the economic intuition. The computational experiment performed shows a negative monotone relationship between the transaction cost scale parameter γ (or equivalently, total transaction cost $\gamma(n^g)^2$) and the model variables. Only public/private wage ratio and the public wage bill share in output positively co-move with the cost parameter. Clearly, a reduction in the waste, embodied in the government-work friction, is welfare-improving.

In the next section, model's behavior outside of the steady-state is investigated. In particular, the transitional dynamics of the model economy and the responses of the variables in the face of a surprise technological innovation is presented and discussed. Moreover, a robustness check will be performed for the effect of γ on the cyclical fluctuations exhibited by the model variables as well.

Figure 2.1: Comparative Statics: changes in γ and its effect on the model economy



2.5 Model solution and impulse responses

Since there is no closed-form general solution for the model in this chapter, a typical approach followed in the RBC literature is to log-linearize the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The log-linearized system of model equations is derived and summarized in Appendix 2.10.4-2.10.5. The linearized DCE system can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988):

$$\mathbf{A}E_t\hat{\mathbf{x}}_{\mathbf{t+1}} \quad = \quad \mathbf{B}\hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C}E_t\varepsilon_{\mathbf{t+1}}, \tag{2.5.0.1}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^T & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model. Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to productivity.[28]

2.5.1 The Effect of a positive productivity shock

Figure 2.2 shows the impact of a 1% surprise TFP innovation on the model economy. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as investment. From the rules for the government investment and transfers in levels, a higher output translates into higher level of expenditure in each of the two categories (not pictured). Next, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not pictured) and the private wage rate increase. The household responds to the price signals and supplies more hours in the

[28]Sensitivity of IRFs for different values of γ, and the dependence of second moments on the magnitude of the transaction cost parameter, and utility weights are presented in Appendix 2.10.6.

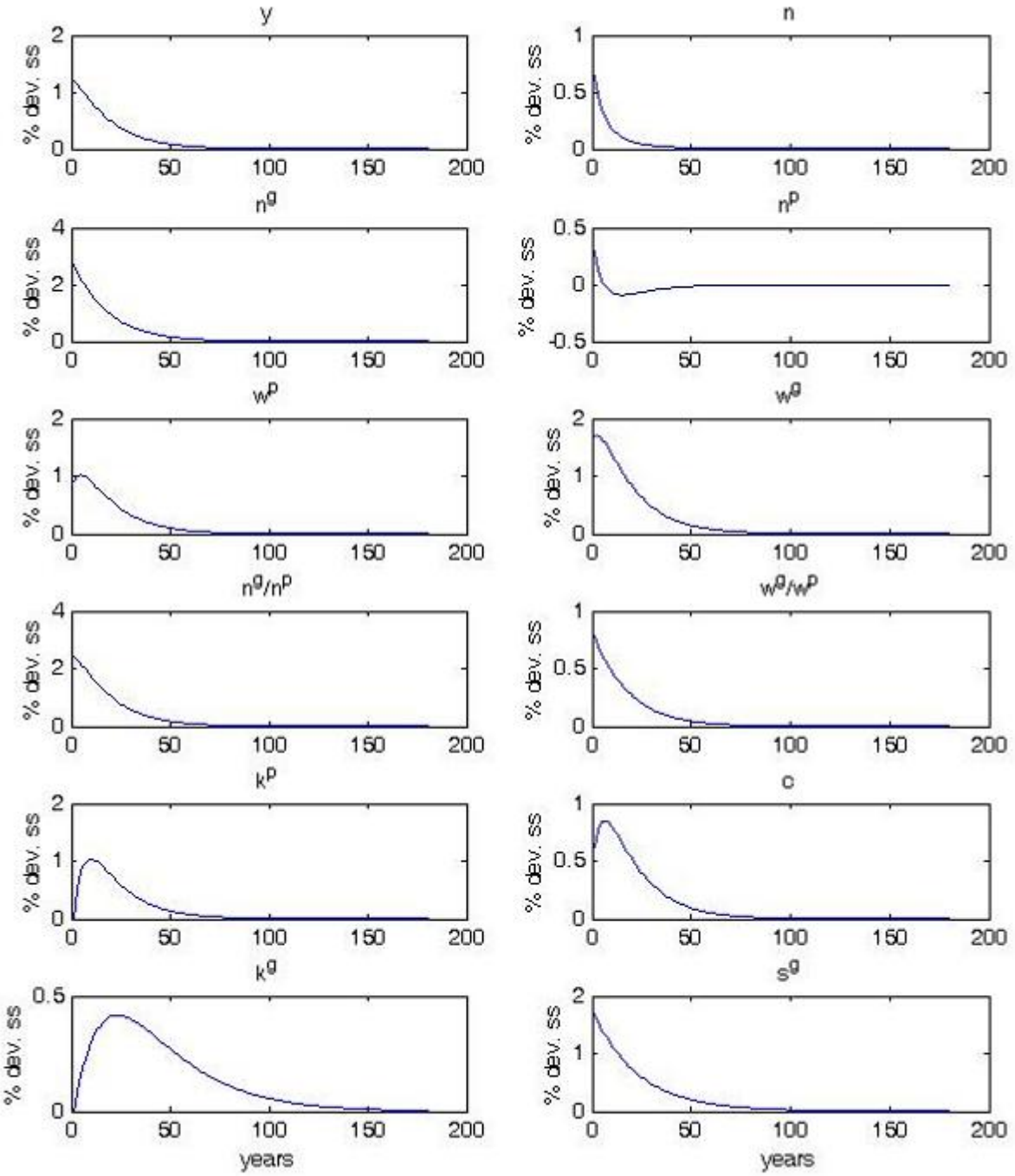
private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private consumption and leisure. In terms of the labor-leisure trade-off, the income effect ("work more") produced by the increase in the private wage dominates the substitution effect ("work less"). Furthermore, the increase of private hours expands output even further, thus both output and government spending categories increase more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

With regard to public sector labor dynamics, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. Higher income and consumption lead to larger tax revenue. In particular, the growth in government revenue exceeds the increase in the fiscal spending instruments. As a result, the additional funds available are spent on government investment, transfers and the wage bill. In turn, the increase in the latter leads to an expansion in both public sector wage and hours.[29] In addition, the model in this chapter generates an interesting dynamics in the wage and hours ratio, which is not present in models with stochastic public employment, such as Finn (1998), Cavallo (2005) or Linnemann (2009). The two wage rates, as well as the two types of hours move together, making the model consistent with the empirical evidence presented in Lamo, Perez and Schuknecht (2007, 2008).

Overall, a positive innovation to total factor productivity has a positive effect on the allocations and prices in the economy. The novelty is that the endogenous public sector hours

[29]The effect on total hours in Germany is very small. Nonetheless, the increase in hours is much larger in magnitude than the responses reported in Fernandez-de-Cordoba *et al.* (2009, 2012) and the model with public sector union presented in Ch. 1.

Figure 2.2: Impulse Responses to a positive 1% productivity shock in Germany



model generates an important difference in the composition of household's labor income with the public sector share increasing at a much faster rate than the private sector labor income. Another important observation to make is that the TFP shocks, being the main driving force in the model, induce pro-cyclical behavior in public wage and hours. The shock effects are smaller and variables reach their peak very quickly. This means the impulse effect dies out relatively fast. Nonetheless, the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks on the wage- and hours ratios.

However, an important limitation of the exogenous policy analysis performed so far is that tax rates were fixed. In addition, government investment share was exogenously set, and public wage rate was a residually-determined instrument that always adjusted accordingly to balance the budget. In effect, by construction all interaction between the two tax rates was precluded, by fixing each to the corresponding average effective rate in data over the chosen period of study. These restrictions will be lifted in the next section, and the optimal fiscal policy framework will be considered in an environment, in which the two tax rates, government investment, public employment (and hence also government services) and public sector wage rate, will be chosen jointly by a benevolent government, whose preferences are perfectly aligned with the household's utility function.

2.6 The Ramsey problem (Optimal fiscal policy under full commitment)

In this section, the government will assume the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interests, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital (hence public investment), public employment and public sector wage rate. Government transfers will be held fixed at the level from the exogenous policy case. It is assumed that only linear taxes are allowed,

and that the government can credibly commit to those. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus can be taken as a better approximation to the environment in which policymakers decide on a particular fiscal policy.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of generating a different competitive equilibrium allocation, i.e. allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the chapter, is that in Ramsey framework, the government chooses all instruments, instead of taking them as being exogenous. At the same time, the government also optimally chooses the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[30] It is also assumed that the government discounts time at the same rate as the representative household. The constraints which the government takes into account when maximizing the household's welfare include the government budget constraints, and the behavioral responses of both the household, and the firm. These are summarized in the DCE of the exogenous fiscal policy case.[31,32] In other words, in the dual approach to the Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[33] The initial conditions for the state variable $\{A_0, K_0^p, K_0^g\}$, as well as the sequence of government transfers $\{G^T\}_{t=0}^{\infty}$ and the process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$ are taken as

[30]In contrast, in the primal approach all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

[31]The DCE system is summarized in Appendix 2.10.3.

[32]Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[33]Note that by choosing next-period public capital, the planner is choosing public investment optimally.

given.[34]

Following the procedure in Chamley (1986) and Ljungqvist and Sargent (2004), the Ramsey problem will be transformed and simplified, so that the government chooses after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1-\tau_t^k)r_t \tag{2.6.0.1}$$

$$\tilde{w}_t^p \equiv (1-\tau_t^l)w_t^p \tag{2.6.0.2}$$

$$\tilde{w}_t^g \equiv (1-\tau_t^l)w_t^g. \tag{2.6.0.3}$$

Thus, the transformed government budget constraint becomes

$$A_t(N_t^p)^\theta(K_t^p)^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^T. \tag{2.6.0.4}$$

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed Ramsey problem then becomes:[35]

$$\max_{C_t,N_t^p,N_t^g,K_{t+1}^p,K_{t+1}^g,\tilde{w}_t^p,\tilde{w}_t^g,\tilde{r}_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln \left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] + (1-\psi_1-\psi_2)\ln\left[(N_t^g)^\alpha (K_t^g)^{1-\alpha}\right] \Bigg\} \tag{2.6.0.5}$$

[34]In reality government often acts as a "Stackelberg leader." Such an assumption could change the results in important ways, as it matters whether the government chooses sequentially, or whether the government chooses once and for all. Here (and in Ch.3 of the thesis) the focus is on the latter case: it is assumed that there is a perfect commitment device used by the government, and no (profitable) deviations from the pre-announced plan are possible. The time-consistent case, or the political competitive equilibrium, is left for future research: The interested reader should consult Marcet and Marimon's (1992, 1999) work on recursive contracts, which is closely related to such a setup.

[35]Detailed derivations in Appendix 2.10.8.

s.t

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left[1 - \delta^p + (1 - \tau^k_{t+1})(1 - \theta) \frac{Y_{t+1}}{K^p_{t+1}} \right] \tag{2.6.0.6}$$

$$\psi_2 C_t = \psi_1 \left[1 - N^p_t - N^g_t - \gamma (N^g_t)^2 \right] (1 - \tau^l_t) w^p_t \tag{2.6.0.7}$$

$$\psi_2 C_t [1 + 2\gamma N^g_t] = \psi_1 \left[1 - N^p_t - N^g_t - \gamma (N^g_t)^2 \right] (1 - \tau^l_t) w^g_t \tag{2.6.0.8}$$

$$A_t (N^p_t)^\theta (K^p_t)^{(1-\theta)} = C_t + K^g_{t+1} - (1 - \delta^g) K^g_t + K^p_{t+1} - (1 - \delta^p) K^p_t \tag{2.6.0.9}$$

$$A_t (N^p_t)^\theta (K^p_t)^{1-\theta} - \tilde{r}_t K^p_t - \tilde{w}^p_t N^p_t = \tilde{w}^g_t N^g_t + K^g_{t+1} - (1 - \delta^g) K^g_t + G^T_t \tag{2.6.0.10}$$

$$K^p_{t+1} = I_t + (1 - \delta^p) K^p_t \tag{2.6.0.11}$$

$$r_t = (1 - \theta) \frac{Y_t}{K^p_t} \tag{2.6.0.12}$$

$$w^p_t = \theta \frac{Y_t}{N^p_t} \tag{2.6.0.13}$$

$$S^g_t = (N^g_t)^\alpha (K^g_t)^{(1-\alpha)} \tag{2.6.0.14}$$

$$K^g_{t+1} = G^i_t + (1 - \delta^g) K^g_t \tag{2.6.0.15}$$

After numerically solving for the unique steady-state,[36] the full characterization of the long-run Ramsey equilibrium is summarized in Table 2.3 on the next page, where the same values for the parameters from the exogenous policy section (see Table 2.1) were used.

As in Lucas (1990), Cooley and Hansen (1992) and Ohanian (1997), parameter ξ is introduced to measure the consumption-equivalent long-run welfare gain of moving from the

[36] Appendix 2.10.8-2.10.9 contains the detailed derivation of the Ramsey problem and steady-state system representation.

steady-state allocations in the exogenous policy case to the equilibrium values obtained under Ramsey policy. In other words, the value of ξ measures the share of steady-state consumption under the exogenous policy that the household has to be compensated with, in order to achieve the same level of utility as the one under the Ramsey policy. A fraction $\xi > 0$, which is the case reported in Table 2.3 on the next page, demonstrates that the agent is better-off under Ramsey, while $\xi < 0$ would have implied that the agent is worse-off under Ramsey.[37] The derivation of the analytic expression for ξ is presented in Appendix 2.10.10.

There are several additional important findings in the Ramsey equilibrium that can be seen in Table 2.3 on the previous page. First, as expected, total discounted welfare is higher under the Ramsey regime. Next, private consumption share is lower, while private capital- and investment shares are higher, and thus the interest rate is lower. The model generates a zero steady-state optimal capital tax, and a higher labor tax rate. All these results are consistent with the findings in earlier studies, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004) and Kocherlakota (2010). In addition, earlier studies that use the representative-agent setup, *e.g.* Lucas (1990), Cooley and Hansen (1992), have shown that tax reforms which abolish capital taxation, even at the expense of higher tax burden on labor, still produce significant welfare gains for the society.

Next, due to the presence of a second labor market, as well as an endogenous public sector hours, sophisticated labor market interactions are generated. In the framework presented in this chapter, the labor market structure allows for labor flows between sectors. Furthermore, the government internalizes the public services in its choice. Thus, it selects the socially optimal levels of public hours and capital stock to provide the optimal level of the public consumption good. In addition, the benevolent planner chooses a different mix between the inputs used in the provision of government services: a higher level of government investment is undertaken, while fewer public hours are employed than the in the DCE solution. As a

[37] However, a case with $\xi < 0$ can never occur under optimal policy.

Table 2.3: Data averages and long-run solution: exogenous *vs*. optimal policy

	GE Data	Exogenous	Ramsey
c/y	0.590	0.784	0.709
i/y	0.210	0.192	0.229
g^i/y	0.023	0.023	0.062
k^p/y	2.350	2.346	2.793
k^g/y	0.630	0.630	1.669
s^g/y	0.193	0.224	0.289
g^T/y	0.170	0.228	0.221
$w^p n^p/y$	0.710	0.710	0.710
$w^g n^g/y$	0.130	0.145	0.143
rk/y	0.290	0.290	0.290
w^g/w^p	1.200	1.200	1.339
w^p	N/A	1.006	1.079
w^g	N/A	1.207	1.445
$\tilde{w}^p$	N/A	0.595	0.541
$\tilde{w}^g$	N/A	0.714	0.724
n	0.296	0.266	0.251
n^p	0.253	0.227	0.218
n^g	0.043	0.039	0.033
n^g/n^p	0.170	0.170	0.150
$\gamma(n^g)^2/n^g$	N/A	0.099	0.085
$\bar{r}$	0.036	0.035	0.022
τ^k	0.160	0.160	0.000
τ^l	0.409	0.409	0.499
U	N/A	-47.91	-46.22
ξ	N/A	0	0.123

result, public investment (and thus public capital) share almost triples, which increases the amount of the public good produced relative to output. The same substitution of labor for capital is observed in the production of the private consumption good. Furthermore, the Ramsey planner finds it optimal to produce a higher level of the private output using more capital and fewer private hours. As a result, total employment also decreases.

In terms of the relative price of labor in the two labor markets, the after-tax private wage decreases after the higher labor tax is levied, while the after-tax public wage slightly increases. The higher public/private wage ratio, and thus the higher public wage premium in the optimal policy case overcompensate for the increase in the labor tax. Furthermore, the public/private hours ratio is lower, due to the substitution away from labor in the government sector. In other words, the increase in the public wage premium is driven by budgetary considerations, as the public wage is the residually-determined fiscal instrument that balances the per-period government budget constraint. In addition, the result is consistent with economic logic and the scarcity argument: relatively fewer hours are employed in the public sector, thus the steady-state public wage rate is higher. Furthermore, the optimal government wage consumption is a little lower.[38] In addition, given the substantial increase in public investment, the fixed level of government transfers, and the loss of capital income tax revenue, steady-state labor tax is 10% higher relative to the exogenous policy case. Overall, these changes in the distribution of spending are new results in the optimal policy literature. As seen from Table 2.3, if those aspects are ignored, important adjustment mechanisms are missed.

[38]The result that cuts in the wage bill have an expansionary effect on the economy is not new to the empirical macroeconomic studies, *e.g.* Algan *et al.* (2002), Alesina (1997), Alesina *et al.* (2001), Alesina *et al.* (2002), and Giavazzi and Pagano (1990). However, the optimal public wage and employment aspects in the analysis are novel in the modern macroeconomic literature, given the predominance of setups with single wage rates, and exogenously-determined public employment. Lastly, the results are robust: changes in the relative utility weights do not significantly affect the results obtained here.

The value-added of the model with endogenous hours and wages in this chapter is that it generates new predictions about the long-run effects of fiscal policy on the labor markets, such as the wage and employment ratios, the optimal composition of the government wage bill consumption, and the distribution of spending across government expenditure categories. These results, generated from the incorporation of a richer government spending side, are new and interesting for policy makers, as previous research had ignored those important dimensions.

In the next section, the analysis is extended to the behavior of the Ramsey economy outside of the steady-state. The transitional dynamics of model variables under optimal policy setup is also analyzed. In particular, the optimal responses of the fiscal instrument and the other prices and allocations to positive shocks to TFP is presented and discussed.

2.7 Optimal reaction of fiscal policy instruments to productivity shocks

The optimal policy model is now solved using the first-order linearization procedure from Schmitt-Grohe and Uribe (2004) to study the dynamics of prices and allocations outside the steady-state.[39] The model solution is then used to study transitional behavior in response to a surprise innovation in total factor productivity. Under Ramsey, endogenous variables would generally behave differently, as compared to the responses to a positive technology shock under the exogenous fiscal policy case. Fig. 2.3 summarizes all responses to a 1% surprise innovation to total factor productivity. To highlight differences across regimes, Fig. 2.4 plots on the same graph both the IRFs from the exogenous policy case and the optimal ones. The new variables in the system are the five fiscal policy instruments - capital and labor taxes, as well as public investment (hence public capital), public wage rate and

[39] Given the absence of curvature in the model, the second-order approximation to the equilibrium system of equations did not change results significantly.

Figure 2.3: Impulse Responses to a positive 1% productivity shock under Ramsey policy

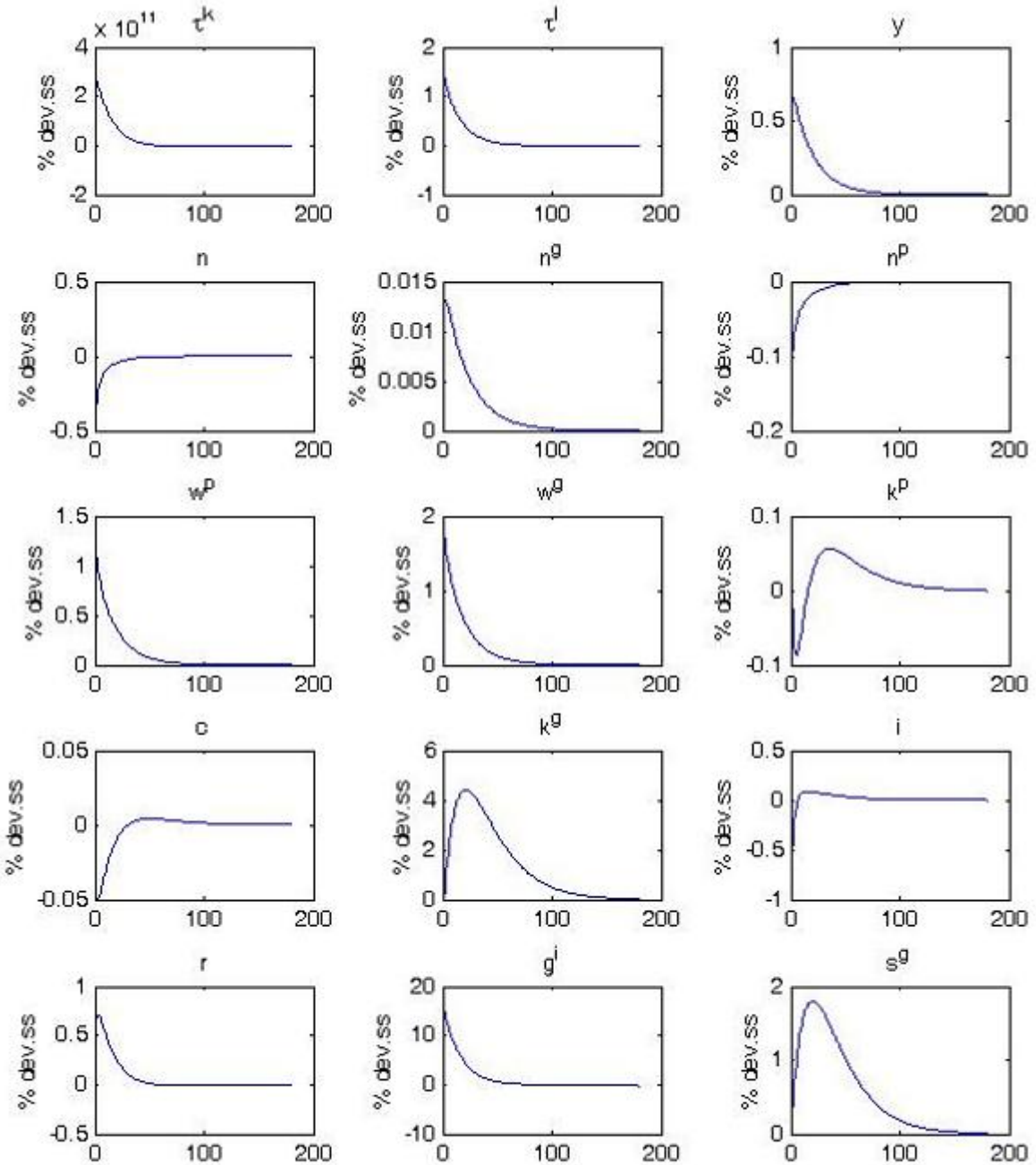
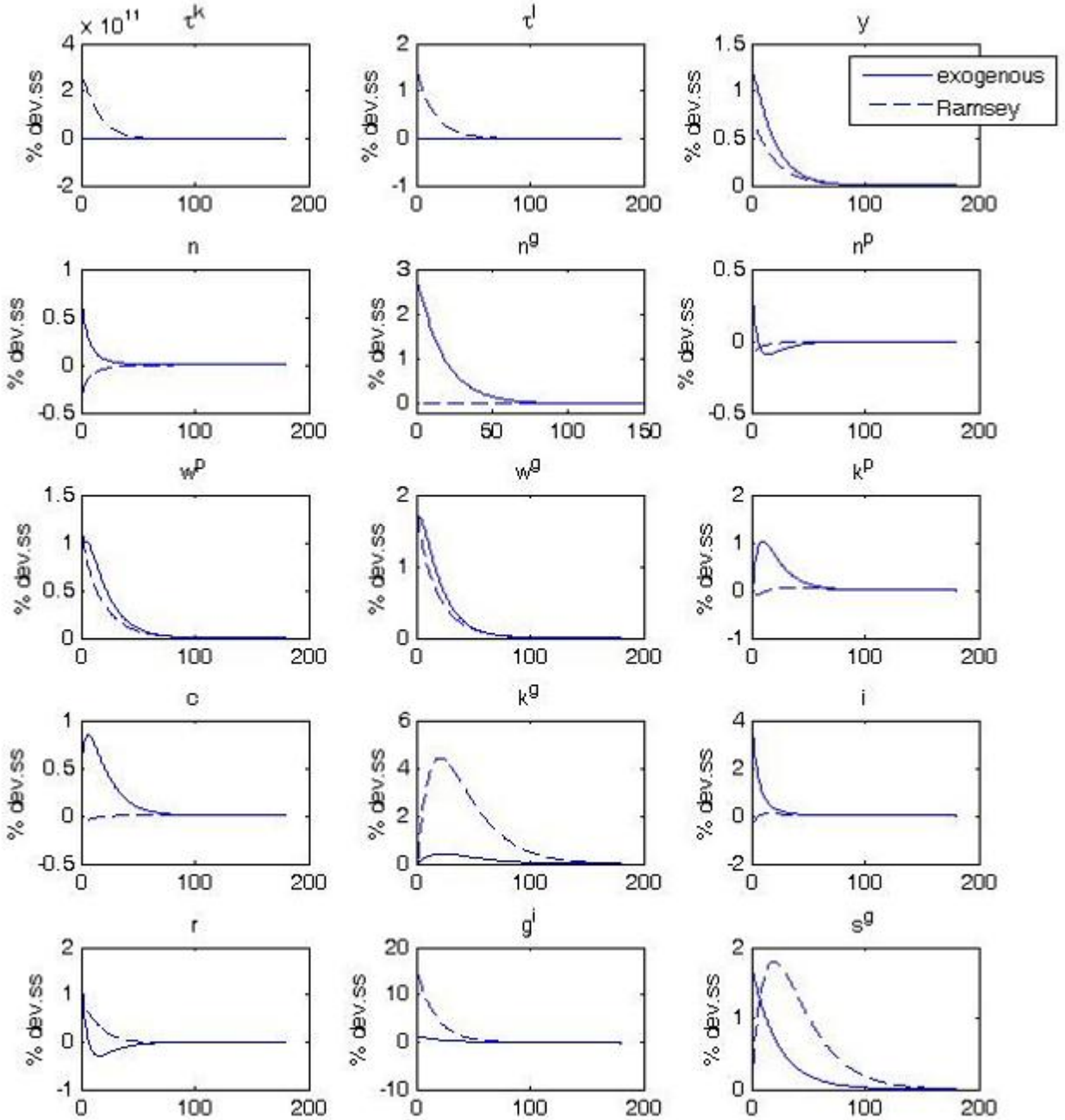


Figure 2.4: Impulse Responses to a positive 1% productivity shock under exogenous and Ramsey policy



public employment. By intervening in the public sector labor market, the benevolent government can influence the private sector labor market, and thus affect the course of the economy.

In period 0, after the realization of the unexpected technology innovation, capital tax remains unchanged.[40] This result is in line with previous findings in the literature, *e.g.* Chari and Kehoe (1994, 1999) who show that in a standard RBC model capital tax rate does not respond to productivity shocks. In other words, the benevolent government would not deviate from the optimal zero steady-state capital tax rate even in the face of uncertain productivity shocks.[41] Next, labor income tax rate increases upon the impact of the positive surprise innovation in TFP and then slowly returns to its old steady-state; the substantial persistence observed is in line with earlier studies (Chari and Kehoe, 1994, 1999). However, due to the richer structure of the and endogenously-determined government spending, the magnitude of the response in the labor tax is greater.

Furthermore, given that public spending categories are optimally chosen in this framework, the setup generates much more interaction among the variables than does the standard RBC model. For example, public investment increases substantially, as the government under the Ramsey regime also chooses public capital and government services optimally. Next, as in the exogenous policy case, public sector wages will increase more than the private sector wage. The higher volatility in public wages, as discussed in earlier sections, is an artifact of the presence of transaction costs from government work. The change in the public/private wage ratio in turn triggers a reallocation of labor resources from the private to the public sector. The net effect on labor supply, however, is negative, as the transaction cost friction depresses the response of public hours. Next, the outflow of hours from the firm leads to

[40] At first glance the huge percentage deviation from the steady-state for capital tax can be misleading. However, noting that the steady-state capital tax rate under Ramsey is $\tau^k = 2.58 \times 10^{-12} \approx 0$, it follows that the log-deviation from the steady-state is an extremely large number, as the denominator is close to zero, even though the absolute value of change is tiny.

[41] This is also a result of the logarithmic specification of the household's utility of consumption.

an increase in the marginal product of capital, and hence the real interest rate increases. However, from the complementarity between private labor and capital in the Cobb-Douglas production function, private capital decreases. Therefore, due to the fall in the levels of the two private inputs, output increases by less than the size of the technology shock.

In addition, given the jump in government investment, private consumption and investment fall upon the impact of the shock. Overall, the difference in the dynamics in the main model variables under the Ramsey regime is due to the fact that the government chooses the optimal levels of public hours and capital (and hence also public investment). Over time, attracted by the above-steady-state real interest rate, more private investment is undertaken by the government. In turn, private capital accumulation increases, and the usual hump-shape dynamics appears. Higher capital input increases the marginal productivity of labor, and private labor starts slowly to recover to its old steady-state. As time passes, private consumption response also turns positive, and the shape if its response follows the dynamic path of private capital. In general, positive innovations to TFP have a positive effect on the economy. Additionally, there is a long-lasting internal propagation effect in the economy. This is due to the fact that there are two labor markets featuring different wage rates, and labor can flow between sectors in response to changes in the relative wage. Moreover, there is complementarity between public hours and the public capital, which reinforces the complementarity between private and public consumption in the household's utility. The quantitative effect of public sector labor market, however, completely dominates capital response in terms of the initial dynamics. Nevertheless, in the long-run, private capital accumulation effects becomes the dominant one, as dictated by the standard neoclassical RBC model.

An interesting result is that a significant portion of the private gains are channeled to the public sector in the form of higher public wage bill spending and public investment. Indeed, in this model both categories are productive expenditures, as labor and capital are

combined in the provision of the public good. Even though the household suffers a little from the lower private consumption, this negative effect is overcompensated for through the increase in leisure (as private hours fall by much more relative to the increase in private hours), and a higher level of public good consumption. Overall, it takes more than 100 years for all the model variables to return to their old steady states.

2.8 Limitations of the study

The analysis performed in this section was based on the strong assumption that the government budget is balanced every period and that the household can work in both sectors. However, labor supply decision is done sequentially in the real world. A worker usually decides on a sector first, and only then on the number of hours worked in the selected sector. Furthermore, it is a stylized fact in labor data that most of the variations in hours worked in data are driven by changes in employment rates rather than by changes in hours worked per person. Thus, the model is too simple and cannot distinguish between employment and hours per person in the two sectors. A possible extension, left for future research, is to setup a model with heterogeneous-agents, who search for work according to a directed search process,[42] similar to that used in Gomes (2009, 2012).

Next, the model setup presented in this chapter abstracted away from debt issues, which are important for modern economies. In models with full commitment, however, Stockman (2001) has shown the presence of debt not to be relevant. Nevertheless, as a possible extension of the model, government bonds can be introduced, together with a long-run target debt/GDP ratio that has to be met in the long-run. Studying the transitional dynamics of the economic variables and the adjustment in the different of public finance categories, in the presence of endogenous public hours and wages, is another promising avenue for future work.

[42] In particular, every agent searches for work only in one of the two sectors, and the sector is determined from the realization of a stochastic process.

Additionally, since the analysis focused on the long-run full commitment case, the setup abstracted away from electoral uncertainty and thus ignores possible departures from the full-commitment case. Across the political spectrum in most democratic societies, there are different parties with diverse objectives that compete for the popularity vote at parliamentary/presidential elections. In the model setup in this chapter, the benevolent government's utility function was assumed to coincide with that of the household. In reality, however, a party's utility function can be quite different when the party is in office, as compared to the case when the party is in opposition, as suggested in Philippopoulos, Economides and Malley (2004) and Malley, Philippopoulos and Woitek (2007). Different parties might have different preferences for the level of public employment. In other words, jobs can be created in the public sector to generate political support and increase the chances of re-election. However, such considerations, as well as possible departures from the full commitment case, and a focus on "loose commitment" as in Debortoli and Nunes (2010), or time-consistent policies as in Klein and Rios-Rull (2003), Ortigueira (2006), Klein *et al.* (2008) and Martin (2010), will be put on the agenda for future research.

Lastly, aside from political considerations, the observed premium at macroeconomic level is also likely to be a by-product of aggregation of microeconomic data. In the German Socio-Economics Panel (SOEP), as well as in the US Panel Study of Income Dynamics (PSID) database, for example, the age-skill profile of public employees is skewed to the left: the average public employee is older, more skilled, more experienced, and tends to occupy managerial positions as compared to his/her private sector counterpart. However, such distributional and occupational dimensions are outside the scope of a simple RBC model with a representative household. Therefore, further work to endogeneize public wage premium, perhaps within a heterogeneous-agents framework, needs to be undertaken.

2.9 Summary and Conclusions

This chapter characterized optimal fiscal policy and evaluated it relative to the exogenous (observed) one. The focus was on the labor market effects of fiscal policy in a model with endogenously-determined public wages and hours, as well as labor-intensive government services. To this end, a Real-Business-Cycle model, calibrated to German data (1970-2007), was set up with a richer government spending side, and an endogenous private-public sector labor choice. The latter was achieved by the inclusion of a transaction cost of government work, generating a wedge between the marginal disutility of an hour worked in the public and private sector, which is generally consistent with the evidence. This friction also helped the framework to accommodate the different wage rates in the two labor markets. To illustrate the effects of fiscal policy, two regimes were compared and contrasted to one another - exogenous *vs.* optimal (Ramsey) policy case, in terms of their long-run effects, as well as in terms of the dynamics of the economy in response to technology shocks. The interaction between the two labor markets was shown to be quantitatively important for both the short-run model dynamics, as well as for the steady-state values of the allocations.

The main findings from the computational experiments performed in this chapter were: (i) as in Judd (1985), Chamley (1986) and Zhu (1992), the optimal steady-state capital tax rate is zero, as it is the most distortionary tax to use; (ii) Given a fixed level of government transfers, a higher labor tax rate is needed to compensate for the loss in capital tax revenue, despite the fact that the Ramsey planner chooses both components of the government wage bill, as well as the level of investment optimally; (iii) Under the optimal policy regime, public/private employment ratio is lower, while public/private wage rate is higher. In other words, employment in the public sector should be diminished, but government employees should receive better renumeration. (iv) The government wage bill is smaller, while public investment is three times higher than in the exogenous policy case; (v) The government chooses the optimal level of government services provided. Additionally, it substitutes labor for capital in the input mix of both public good production and private output. In turn,

this substitution makes government hours more valuable. All these result are in line with Alesina (1997) and Alesina *et al.* (2002) who found that for OECD countries, fiscal adjustments that rely on cuts in government wage bill produce a stimulus to the economy through higher investment, and thus a leaner public sector creates an expansionary effect for the economy.

2.10 Technical Appendix

2.10.1 Optimality conditions

Firm's problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} - r_t \quad = \quad 0 \tag{2.10.1.1}$$

where $(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta}$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, i.e the interest rate.

$$r_t \quad = \quad (1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} \tag{2.10.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} = r_t K_t^p \quad \text{or} \quad (1-\theta)Y_t = r_t K_t^p \tag{2.10.1.3}$$

because

$$K_t^p(1-\theta)A_t(K_t^p)^{-\theta}(N_t^p)^{\theta} = A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta} \quad = \quad (1-\theta)Y_t$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1} - w_t^p = 0 \text{ or } w_t^p = \theta A_t(K_t^p)^{1-\theta}(N_t^p)^{\theta-1} \tag{2.10.1.4}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, i.e the hourly

wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p \theta A_t (K_t^p)^{1-\theta} (N_t^p)^{\theta-1} = w_t^p N_t^p \quad \text{or} \quad \theta Y_t = w_t^p N_t^p \tag{2.10.1.5}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \;=\; Y_t - r_t K_t^p - w_t^p N_t^p = Y_t - (1-\theta) Y_t - \theta Y_t = 0 \tag{2.10.1.6}$$

Indeed, in equilibrium, economic profits are zero.

Consumer problem

Set up the Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln \left[1 - N_t^p - N_t^g - \gamma (N_t^g)^2 \right] + \psi_3 \ln S_t^g$$
$$+ \Lambda_t \left[(1-\tau_t^l)(w_t^p N_t^p + w_t^g N_t^g) + (1-\tau_t^k) r_t K_t^p + G_t^T - C_t - K_{t+1}^p + (1-\delta) K_t^p \right] \Bigg\} \tag{2.10.1.7}$$

This is a concave programming problem, so the FOCs, together with the additional, boundary ("transversality") conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\beta^t \left\{ \frac{\psi_1}{C_t} - \Lambda_t \right\} = 0 \quad \text{or} \quad \frac{\psi_1}{C_t} = \Lambda_t \tag{2.10.1.8}$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K^p_{t+1} (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K^p_{t+1}} = 0$. That will result in the following expression

$$\beta^t \left\{ -\Lambda_t + E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + (1-\delta^p) \right] \right\} = 0 \tag{2.10.1.9}$$

Cancel the β^t term to obtain

$$-\Lambda_t + \beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \right] = 0 \tag{2.10.1.10}$$

Move Λ_t to the right so that

$$\beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + (1-\delta^p) \right] = \Lambda_t \tag{2.10.1.11}$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1} = (1-\theta)\frac{Y_{t+1}}{K^p_{t+1}}$$

$$\beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1})(1-\theta)\frac{Y_{t+1}}{K^p_{t+1}} + (1-\delta^p) \right] = \Lambda_t \tag{2.10.1.12}$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N^p_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^p_t} = 0$. That will result in the following expression

$$\beta^t \left\{ -\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)w^p_t \right\} = 0 \tag{2.10.1.13}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} + \Lambda_t(1-\tau^l_t)w^p_t = 0 \tag{2.10.1.14}$$

Rearranging, one can obtain

$$\frac{\psi_2}{1-N^p_t-N^g_t-\gamma(N^g_t)^2} = \Lambda_t(1-\tau^l_t)w^p_t \tag{2.10.1.15}$$

Plug in the expression for w^h_t, that is,

$$w^p_t = \theta\frac{Y_t}{N^p_t} \tag{2.10.1.16}$$

into the equation above. Rearranging, one can obtain

$$\frac{\psi_2}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2} + \Lambda_t(1 - \tau_t^l)\theta\frac{Y_t}{N_t^p} \tag{2.10.1.17}$$

Take now the derivative of the Lagrangian w.r.t N_t^g (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N_t^g} = 0$. That will result in the following expression

$$\beta^t\left\{ - \frac{\psi_2(1 + 2\gamma N_t^g)}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2} + \Lambda_t(1 - \tau_t^l)w_t^g \right\} \quad = \quad 0 \tag{2.10.1.18}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2(1 + 2\gamma N_t^g)}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2} + \Lambda_t(1 - \tau_t^l)w_t^g = 0 \tag{2.10.1.19}$$

Rearranging, one can obtain

$$\frac{\psi_2(1 + 2\gamma N_t^g)}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2} = \Lambda_t(1 - \tau_t^l)w_t^g \tag{2.10.1.20}$$

Transversality conditions need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^p \quad = \quad 0 \tag{2.10.1.21}$$

2.10.2 Per capita stationary DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, i.e $Z_t^h = Z_t = z_t$. The system of equations that describes the DCE is as follows:

$$y_t = a_t(k_t^p)^{1-\theta}(n_t^p)^{\theta} \tag{2.10.2.1}$$

$$y_t = c_t + k_{t+1}^p - (1-\delta^p)k_t^p + g_t^i \tag{2.10.2.2}$$

$$\frac{\psi_1}{c_t} = \lambda_t \tag{2.10.2.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left[1 - \delta^p + (1-\tau^k)(1-\theta)\frac{y_{t+1}}{k_{t+1}^p}\right] \tag{2.10.2.4}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - \gamma(n_t^g)^2} = \frac{\psi_1}{c_t}(1-\tau^l)\theta\frac{y_t}{n_t^p} \tag{2.10.2.5}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - \gamma(n_t^g)^2}[1 + 2\gamma n_t^g] = \frac{\psi_1}{c_t}(1-\tau^l)w_t^g \tag{2.10.2.6}$$

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{2.10.2.7}$$

$$r_t = (1-\theta)\frac{y_t}{k_t^p} \tag{2.10.2.8}$$

$$w_t^p = \theta\frac{y_t}{n_t^p} \tag{2.10.2.9}$$

$$g_t^T + g_t^i + w_t^g n_t^g = \tau^k r_t k_t^p + \tau^l\left[w_t^p n_t^p + w_t^g n_t^g\right]. \tag{2.10.2.10}$$

$$k_{t+1}^g = g_t^i + (1-\delta^g)k_t^g \tag{2.10.2.11}$$

$$g_t^i = g_t^{iy} y_t \tag{2.10.2.12}$$

$$g_t^T = g_t^{Ty} y_t \tag{2.10.2.13}$$

$$s_t^g = (n_t^g)^\alpha (k_t^g)^{1-\alpha} \tag{2.10.2.14}$$

Therefore, the DCE is summarized by Equations (2.10.2.1)-(2.10.2.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, k_t^p, k_t^g, g_t^i, g_t^t, n_t^p, n_t^g, s_t^g, w_t^p, w_t^g, r_t, \lambda_t\}_{t=0}^\infty$ given the process followed by total factor productivity $\{a_t\}_{t=0}^\infty$, the values of government investment and government transfers shares g^{iy}, g^{Ty}, and the fixed capital and labor tax rates $\{\tau^k, \tau^l\}$.

2.10.3 Steady-state

In steady-state, there is no uncertainty and variables do not change. Thus, eliminate all stochasticity and time subscripts to obtain

$$y = a(k^p)^{1-\theta}(n^p)^\theta \tag{2.10.3.1}$$

$$y = c + \delta^p k^p + g^i \tag{2.10.3.2}$$

$$\frac{\psi_1}{c} = \lambda \tag{2.10.3.3}$$

$$1 = \beta\left[1 - \delta^p + (1-\tau^k)(1-\theta)\frac{y}{k^p}\right] \tag{2.10.3.4}$$

$$\frac{\psi_2}{1 - n^p - n^g - \gamma(n^g)^2} = \frac{\psi_1}{c}(1-\tau^l)\theta\frac{y}{n^p} \tag{2.10.3.5}$$

$$\frac{\psi_2}{1 - n^p - n^g - \gamma(n^g)^2}[1 + 2\gamma n^g] = \frac{\psi_1}{c}(1-\tau^l)w^g \tag{2.10.3.6}$$

$$i^p = \delta^p k^p \tag{2.10.3.7}$$

$$r = (1-\theta)\frac{y}{k^p} \tag{2.10.3.8}$$

$$w^p = \theta\frac{y}{n^p} \tag{2.10.3.9}$$

$$g^T + g^i + w^g n^g = \tau^k r k^p + \tau^l \Big[w^p n^p + w^g n^g \Big]. \tag{2.10.3.10}$$

$$g^i = \delta^g k^g \tag{2.10.3.11}$$

$$g^i = g^{iy} y \tag{2.10.3.12}$$

$$g^T = g^{Ty} y \tag{2.10.3.13}$$

$$s^g = (n^g)^\alpha (k^g)^{1-\alpha} \tag{2.10.3.14}$$

2.10.4 Log-linearization

Log-linearized production function

$$y_t = a_t (k_t^p)^{1-\theta} (n_t^p)^{\theta} \tag{2.10.4.1}$$

Take natural logs from both sides to obtain

$$\ln y_t = \ln a_t + (1-\theta) \ln k_t^p + \theta \ln n_t^p \tag{2.10.4.2}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln y_t}{dt} = \frac{d \ln a_t}{dt} + (1-\theta) \frac{d \ln k_t^p}{dt} + \theta \frac{d \ln n_t^p}{dt} \tag{2.10.4.3}$$

$$\frac{1}{y} \frac{dy_t}{dt} = \frac{1}{a} \frac{da_t}{dt} + \frac{1-\theta}{k^p} \frac{dk_t}{dt} + \frac{\theta}{n^p} \frac{dn_t^p}{dt} \tag{2.10.4.4}$$

Pass to log-deviations to obtain

$$0 = -\hat{y}_t + (1-\theta) \hat{k}_t^p + \hat{a}_t + \theta \hat{n}_t^p \tag{2.10.4.5}$$

Linearized market clearing

$$c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i = y_t \tag{2.10.4.6}$$

Take logs from both sides to obtain

$$\ln[c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i] = \ln(y_t) \tag{2.10.4.7}$$

Totally differentiate with respect to time

$$\frac{d \ln[c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i]}{dt} = d \ln(y_t) \tag{2.10.4.8}$$

$$\frac{1}{c + \delta k^p + g^c} \left[\frac{dc_t}{dt} \frac{c}{c} + \frac{dk_{t+1}^p}{dt} \frac{k^p}{k^p} - (1-\delta^p) \frac{dk_t^p}{dt} \frac{k^p}{k^p} + \frac{dg_t^i}{dt} \frac{g^i}{g^i} \right] = \frac{dy_t}{dt} \frac{1}{y} \tag{2.10.4.9}$$

Define $\hat{z} = \frac{dz_t}{dt} \frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y} \left[\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p) \hat{k}_t^p k^p + g^i \hat{g}_t^i \right] = \hat{y}_t \tag{2.10.4.10}$$

$$\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p) \hat{k}_t^p k^p + g^i \hat{g}_t^i = y \hat{y}_t \tag{2.10.4.11}$$

$$k^p \hat{k}_{t+1}^p = y \hat{y}_t - c \hat{c}_t + (1-\delta) k^p \hat{k}_t^p - g^i \hat{g}_t^i \tag{2.10.4.12}$$

Linearized FOC consumption

$$\frac{\psi_1}{c_t} = \lambda_t \tag{2.10.4.13}$$

Take natural logarithms from both sides to obtain

$$\ln \psi_1 - \ln(c_t) = \ln \lambda_t \tag{2.10.4.14}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \psi_1}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{2.10.4.15}$$

or

$$-\frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{2.10.4.16}$$

$$-\frac{dc_t}{dt}\frac{1}{c} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \tag{2.10.4.17}$$

Pass to log-deviations to obtain

$$-\hat{c}_t = \hat{\lambda}_t \tag{2.10.4.18}$$

Linearized no-arbitrage condition for capital

$$\lambda_t \quad = \quad \beta E_t \lambda_{t+1}[(1-\tau^k_{t+1})r_{t+1} + (1-\delta^p)] \tag{2.10.4.19}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t \quad = \quad \beta E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{2.10.4.20}$$

Take natural logs from both sides of the equation to obtain

$$\ln \lambda_t \quad = \quad \ln E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{2.10.4.21}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \lambda_t}{dt} = \frac{d \ln E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + (1-\delta^p)]}{dt} \tag{2.10.4.22}$$

$$\frac{1}{\lambda}\frac{d\lambda_t}{dt} = E_t\Bigg\{\frac{1}{\lambda((1-\tau^k_{t+1})(1-\theta)\frac{y}{k^p}+1-\delta^p}\Bigg[((1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda} + \frac{\lambda(1-\tau^k)(1-\theta)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y} - \Bigg[\frac{\lambda(1-\tau^k)(1-\theta)y}{(k^p)^2}\Bigg]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\Bigg]\Bigg\} \tag{2.10.4.23}$$

Pass to log-deviations to obtain

$$\hat{\lambda}_t = E_t\Bigg\{\hat{\lambda}_{t+1} + \Bigg[\frac{(1-\tau^k)(1-\theta)y}{((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{y}_{t+1} - \frac{(1-\tau^k)(1-\theta)y}{((1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{k}^p_{t+1}\Bigg]\Bigg\} \tag{2.10.4.24}$$

Observe that

$$(1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p = 1/\beta \tag{2.10.4.25}$$

Plug it into the equation to obtain

$$\hat{\lambda}_t = E_t\Bigg[\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{k}^p_{t+1}\Bigg] \tag{2.10.4.26}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{2.10.4.27}$$

Linearized MRS(c_t, n^p_t)

$$\psi_2 c_t = \psi_1[1-n^p_t-n^g_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{2.10.4.28}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n^p_t-n^g_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{2.10.4.29}$$

$$\ln\psi_2 + \ln c_t = \ln\psi_1 + \ln[1-n^p_t-n^g_t-\gamma(n_t)^2] + \ln(1-\tau^l_t) + \ln y_t - \ln n^p_t \tag{2.10.4.30}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt} + \frac{d\ln c_t}{dt} = \frac{d\ln\psi_1}{dt} + \frac{d\ln[1-n^p_t-n^g_t-\gamma(n_t)^2]}{dt} + \frac{d\ln(1-\tau^l_t)}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln n^p_t}{dt} \tag{2.10.4.31}$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-\gamma(n^g)^2}\frac{d\left[n_t^p+n_t^g+\gamma(n_t)^2\right]}{dt} - \frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l} + \frac{1}{y}\frac{dy_t}{dt} - \frac{1}{n^p}\frac{dn_t^p}{dt} \tag{2.10.4.32}$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p} - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l} + \frac{1}{y}\frac{dy_t}{dt} - \frac{1}{n^p}\frac{dn_t^p}{dt} \tag{2.10.4.33}$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l + \hat{y}_t - \hat{n}_t^p \tag{2.10.4.34}$$

Group terms to obtain

$$\hat{c}_t = -\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l + \hat{y}_t \tag{2.10.4.35}$$

Linearized MRS(c_t, n_t^g)

$$\psi_2 c_t = \psi_1[1-n_t^p-n_t^g-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{2.10.4.36}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n_t^p-n_t^g-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{2.10.4.37}$$

$$\ln\psi_2 + \ln c_t = \ln\psi_1 + \ln[1-n_t^p-n_t^g-\gamma(n_t)^2] + \ln(1-\tau_t^l) + \ln w_t^g \tag{2.10.4.38}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt} + \frac{d\ln c_t}{dt} = \frac{d\ln\psi_1}{dt} + \frac{d\ln[1-n_t^p-n_t^g-\gamma(n_t)^2]}{dt} + \frac{d\ln(1-\tau_t^l)}{dt} + \frac{d\ln w_t^g}{dt} \tag{2.10.4.39}$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-\gamma(n^g)^2}\frac{d\left[n_t^p+n_t^g+\gamma(n_t)^2\right]}{dt} - \frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l} + \frac{dw_t^g}{dt}\frac{1}{w^g} \tag{2.10.4.40}$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p}$$
$$-\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l} + \frac{1}{w^g}\frac{dw_t^g}{dt} \tag{2.10.4.41}$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^p - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l + \hat{w}_t^g \tag{2.10.4.42}$$

Linearized private physical capital accumulation

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{2.10.4.43}$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^p = \ln(i_t + (1-\delta^p)k_t^p) \tag{2.10.4.44}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}^p}{dt} = \frac{1}{i+(1-\delta)k^p}\frac{d(i_t+(1-\delta^p)k_t^p)}{dt} \tag{2.10.4.45}$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that } \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \tag{2.10.4.46}$$

Then

$$\frac{dk_{t+1}^p}{dt}\frac{1}{k^p} = \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i+(1-\delta^p)k_t^p}\frac{dk_t^p}{dt}\frac{k^p}{k^p} \tag{2.10.4.47}$$

Pass to log-deviations to obtain

$$\hat{k}_{t+1}^p = \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}_t^p \tag{2.10.4.48}$$

$$\hat{k}_{t+1}^p = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}_t^p \tag{2.10.4.49}$$

Linearized government physical capital accumulation

$$k^g_{t+1} = g^i_t + (1-\delta^g)k^g_t \tag{2.10.4.50}$$

Take natural logs from both sides of the equation to obtain

$$\ln k^g_{t+1} = \ln(g^i_t + (1-\delta^g)k^g_t) \tag{2.10.4.51}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^g_{t+1}}{dt} = \frac{1}{g^i + (1-\delta^g)k^g}\frac{d(i_t + (1-\delta^g)k^g_t)}{dt} \tag{2.10.4.52}$$

Observe that since

$$g^i = \delta^g k^g, \text{ it follows that } \quad g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \tag{2.10.4.53}$$

Then

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{{}^i + (1-\delta^g)k^g_t}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{2.10.4.54}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^p)k^g}{k^g}\hat{k}^g_t \tag{2.10.4.55}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{2.10.4.56}$$

Linearized government budget constraint

$$(1-\tau^l_t)w^g_t n^g_t + g^i_t = \tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t. \tag{2.10.4.57}$$

Take natural logarithms from both sides to obtain

$$\ln\left[(1-\tau^l_t)w^g_t n^g_t + g^i_t\right] = \ln\left[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\right]. \tag{2.10.4.58}$$

Totally differentiate with respect to time to obtain

$$\frac{d}{dt}\ln\left[(1-\tau^l_t)w^g_t n^g_t + g^i_t\right] = \frac{d}{dt}\ln\left[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\right]. \tag{2.10.4.59}$$

or

$$\frac{1}{(1-\tau^l)w^g n^g + g^i}\frac{d\Big[(1-\tau_t^l)w_t^g n_t^g + g_t^i\Big]}{dt} = \frac{1}{\Big[\tau^k rk + \tau^l w^p n^p\Big]}\frac{d\Big[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\Big]}{dt}. \tag{2.10.4.60}$$

Note that

$$(1-\tau^l)w^g n^g + g^i = \tau^k rk + \tau^l w^p n^p \tag{2.10.4.61}$$

Hence

$$\frac{d\Big[(1-\tau_t^l)w_t^g n_t^g + g_t^i\Big]}{dt} = \frac{d\Big[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\Big]}{dt}. \tag{2.10.4.62}$$

or

$$\begin{aligned} &-w^g n^g \frac{d\tau_t^l}{dt}\frac{\tau^l}{\tau^l} + (1-\tau^l)n^g\frac{dw_t^g}{dt}\frac{w^g}{w^g} + (1-\tau^l)w^g\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dg_t^i}{dt}\frac{g^i}{g^i} \\ &= rk\frac{d\tau_t^k}{dt}\frac{\tau^k}{\tau^k} + \tau^k k\frac{dr_t}{dt}\frac{r}{r} + \tau^k r\frac{dk_t}{dt}\frac{k}{k} + w^p n^p\frac{d\tau_t^l}{dt} + \tau^l n^p\frac{dw_t^p}{dt}\frac{w^p}{w^p} + \tau^l w^p\frac{dn_t^p}{dt}\frac{n^p}{n^p} \end{aligned} \tag{2.10.4.63}$$

Pass to log-deviations to obtain

$$\begin{aligned} &-\tau^l w^g n^g \hat{\tau}_t^l + (1-\tau^l)w^g n^g \hat{w}_t^g + (1-\tau^l)w^g n^g \hat{n}_t^g + g^i \hat{g}_t^i \\ &= \tau^k rk\hat{\tau}_t^k + \tau^k rk\hat{r}_t + \tau^k rk\hat{k}_t + \tau^l w^p n^p \hat{\tau}_t^l + \tau^l w^p n^p \hat{w}_t^p + \tau^l w^p n^p \hat{n}_t^p \end{aligned} \tag{2.10.4.64}$$

Total hours/employment

$$n_t = n_t^g + n_t^p \tag{2.10.4.65}$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \tag{2.10.4.66}$$

Totally differentiate to obtain

$$\frac{d\ln n_t}{dt} = \frac{d\ln(n_t^g + n_t^p)}{dt} \tag{2.10.4.67}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{2.10.4.68}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{2.10.4.69}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{2.10.4.70}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{2.10.4.71}$$

Linearized private wage rate

$$w_t^p = \theta\frac{y_t}{n_t^p} \tag{2.10.4.72}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln\theta + \ln y_t - \ln n_t^p \tag{2.10.4.73}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln w_t^p}{dt} = \frac{d\ln\theta}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln n_t^p}{dt} \tag{2.10.4.74}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{2.10.4.75}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{2.10.4.76}$$

Linearized real interest rate

$$r_t = \theta\frac{y_t}{k_t^p} \tag{2.10.4.77}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln \theta + \ln y_t - \ln k_t^p \tag{2.10.4.78}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln r_t}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln k_t^p}{dt} \tag{2.10.4.79}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{2.10.4.80}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{2.10.4.81}$$

Linearized government investment

$$g_t^i = g^{iy} y_t \tag{2.10.4.82}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g^{iy} + \ln y_t \tag{2.10.4.83}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^i}{dt} = \frac{d \ln g^{iy}}{dt} + \frac{d \ln y_t}{dt} \tag{2.10.4.84}$$

or

$$\frac{dg_t^i}{dt}\frac{1}{g^i} = \frac{dy_t}{dt}\frac{1}{y} \tag{2.10.4.85}$$

Passing to log-deviations

$$\hat{g}_t^i = \hat{y}_t \tag{2.10.4.86}$$

Linearized government transfers

$$g_t^T = g^{Ty} y_t \tag{2.10.4.87}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^T = \ln g^{Ty} + \ln y_t \tag{2.10.4.88}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^T}{dt} = \frac{d \ln g^{Ty}}{dt} + \frac{d \ln y_t}{dt} \tag{2.10.4.89}$$

or

$$\frac{dg_t^T}{dt}\frac{1}{g^T} = \frac{dy_t}{dt}\frac{1}{y} \tag{2.10.4.90}$$

Passing to log-deviations

$$\hat{g}_t^T = \hat{y}_t \tag{2.10.4.91}$$

Linearized government services

$$s_t^g = (n_t^g)^\alpha (k_t^g)^{(1-\alpha)} \tag{2.10.4.92}$$

Take natural logarithms from both sides to obtain

$$\ln s_t^g = \alpha \ln n_t^g + (1-\alpha) \ln k_t^g \tag{2.10.4.93}$$

Totally differentiate with respect to time to obtain

$$\frac{ds_t^g}{dt}\frac{1}{s^g} = \alpha \frac{dn_t^g}{dt}\frac{1}{n^g} + (1-\alpha)\frac{dk_t^g}{dt}\frac{1}{k^g} \tag{2.10.4.94}$$

$$\hat{s}_t^g = \alpha \hat{n}_t^g + (1-\alpha)\hat{k}_t^g \tag{2.10.4.95}$$

Linearized technology shock process

$$\ln a_{t+1} = \rho^a \ln a_t + \epsilon^a_{t+1} \tag{2.10.4.96}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln a_{t+1}}{dt} = \rho^a \frac{d \ln a_t}{dt} + \frac{d\epsilon^a_{t+1}}{dt} \tag{2.10.4.97}$$

$$\frac{da_{t+1}}{dt} = \rho^a \frac{da_t}{dt} + \epsilon^a_{t+1} \tag{2.10.4.98}$$

where for $t = 1$ $\frac{d\epsilon^a_{t+1}}{dt} \approx \ln(e^{\epsilon^a_{t+1}}/e^{\epsilon^a}) = \epsilon^a_{t+1} - \epsilon^a = \epsilon^a_{t+1}$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} = \rho^a \hat{a}_t + \epsilon^a_{t+1} \tag{2.10.4.99}$$

2.10.5 Log-linearized DCE system

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta \hat{n}^p_t \tag{2.10.5.1}$$

$$k^p \hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t + (1-\delta^p)k^p\hat{k}^p_t - g^i\hat{g}^i_t \tag{2.10.5.2}$$

$$-\hat{c}_t = \hat{\lambda}_t \tag{2.10.5.3}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{2.10.5.4}$$

$$\hat{c}_t = -\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^p_t - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^g_t + \hat{y}_t \tag{2.10.5.5}$$

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^p_t - \frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^g_t + \hat{w}^g_t \tag{2.10.5.6}$$

$$\hat{k}^p_{t+1} = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{2.10.5.7}$$

$$\hat{k}^g_{t+1} = \delta^g\hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{2.10.5.8}$$

$$-\tau^l w^g n^g \hat{\tau}_t^l + (1-\tau^l) w^g n^g \hat{w}_t^g + (1-\tau^l) w^g n^g \hat{n}_t^g + g^i \hat{g}_t^i$$
$$= \tau^k rk \hat{\tau}_t^k + \tau^k rk \hat{r}_t + \tau^k rk \hat{k}_t + \tau^l w^p n^p \hat{\tau}_t^l + \tau^l w^p n^p \hat{w}_t^p + \tau^l w^p n^p \hat{n}_t^p \tag{2.10.5.9}$$

$$\hat{g}_t^i = \hat{y}_t \tag{2.10.5.10}$$

$$\hat{g}_t^t = \hat{y}_t \tag{2.10.5.11}$$

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{2.10.5.12}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{2.10.5.13}$$

$$\hat{s}_t^g = \alpha \hat{n}_t^g + (1-\alpha) \hat{k}_t^g \tag{2.10.5.14}$$

$$\hat{a}_{t+1} \quad = \quad \rho^a \hat{a}_t + \epsilon_{t+1}^a \tag{2.10.5.15}$$

The model can be now solved by representing it in the following matrix form

$$\mathbf{A}E_t\hat{\mathbf{x}}_{\mathbf{t+1}} = \mathbf{B}\hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C}E_t\varepsilon_{\mathbf{t+1}}, \tag{2.10.5.16}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{g}_t^T & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue ("Schur") decomposition algorithm was used to solve the model. The MATLAB function to solve the above linear system is *solab.m*. The inputs are matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ defined above and $nk = 3$, which is the number of state variables. The outputs are the coefficient matrices $\mathbf{M}$ and $\mathbf{\Pi}$ which solve the linearized system. A solution to an RBC model is in the form of (approximate) policy, or transition rule, which describes the evolution of each variable. In particular, the predetermined and non-predetermined variables can be represented in the following form:

$$E_t\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} \tag{2.10.5.17}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = M\hat{\mathbf{s}}_{\mathbf{t}} \tag{2.10.5.18}$$

To simulate the model, one requires a sequence of normally distributed disturbances, $\{\epsilon_t\}_{t=0}^{\infty}$ for the three exogenous shocks with sample size T, the initial values of the endogenous predetermined variables, $\{k_0^p, k_0^g, a_0\}$ ($a_0 = 1$), and the evolution of the endogenous non-predetermined variables in model solution form

$$\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} + \mathbf{D}\varepsilon_{\mathbf{t+1}} \tag{2.10.5.19}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = \mathbf{M}\hat{\mathbf{s}}_{\mathbf{t}}, \tag{2.10.5.20}$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.10.5.21}$$

Based on the above representation, MATLAB code was written to simulate the model. The computation of impulse responses using the linearized model solution is straightforward.

2.10.6 The effect of a positive TFP shock: robustness check

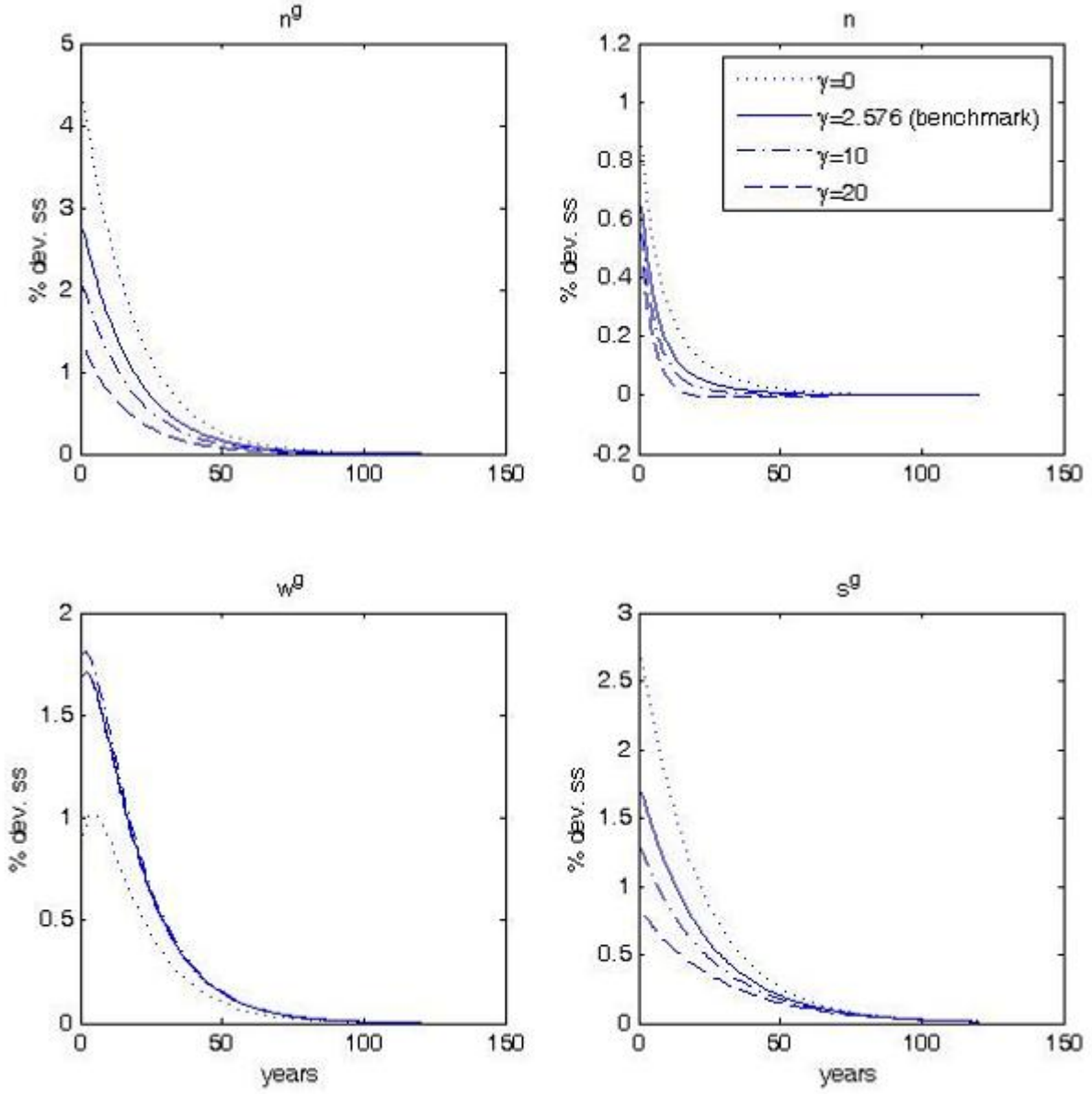
Given the quantitative importance of the new parameter γ for the steady-state values of the model variables, this subsection will examine the relevance of the transaction cost scale parameter for the transitional dynamics. The experiment in the next subsection will focus on the effect of a change in the transaction cost parameter γ on impulse responses.[43] When the value of the transaction cost parameter is changed, public sector labor variables are affected significantly. Other variables show no visible difference in terms of their impulse responses, and are thus not pictured in Fig. 2.5 on the next page. A higher transaction cost in the government sector is quantitatively important, as it affects the shape of the reaction function of public employment (hence total employment as well) and wage rate, as well as the level of public services provided, to unexpected technology innovation.

In particular, a higher value of γ significantly dampens public employment response in the face of a surprise TFP shock. For example, a higher transaction cost increases the disutility of work in the public section very quickly, which makes the household unwilling to reallocate hours between government work and leisure, as well as between the public and private sector work. Total employment, being a sum of private and public employment, is affected in a similar fashion. However, given the fact that private employment drives the dynamics of total employment, the quantitative effect of γ on the latter is rather small. Still, doubling the value of γ relative to the benchmark case produces a notable difference in the impulse responses that is present for more than 40 model years.

Next, through the mechanics of the public good production function, the dynamics of government services will be depressed as well. Since the impulse response of the public good provision depends on public hours and public capital, and given the fact that capital does not respond to changes in γ, the reaction in government services is proportional to the reaction

[43] Note that different values of γ produce different steady-states. Thus, in what follows, variables fluctuate around different equilibria.

Figure 2.5: Impulse Responses: Sensitivity Analysis



in public hours (artifact of the linearization procedure implemented to solve the model). In particular, the coefficient of proportionality equals the labor share α in the public sector.

Finally, the increase in transaction cost parameter γ above the benchmark calibration value strengthens the response of public wages upon the impact of the total factor productivity shock. The quantitative effect, however, is not large. Nevertheless, by driving the scale parameter down to zero, the government could decrease public wage volatility by a factor of two.

Overall, the small experiment performed in this section suggests that changes in γ parameter could significantly affect relative volatilities of public sector labor market variables as well. In particular, an increase in the transaction cost is expected to increase the relative volatility of the public wage rate, and decrease public employment variability. These conjectures are investigated further in the simulation section that follows.

2.10.7 Model simulation and goodness-of-fit

Using the model solutions, shock series were added to produce simulated data series. The length of the draws for the series of innovations is 138, and the simulation is replicated 1000 times. Natural logarithms are taken, and then all series are run through the Hodrick-Prescott filter with a smoothing parameter equal to 100. The first 100 observations are then excluded to decrease any dependence on the initial realizations of the innovations. Average standard deviation of each variable and its correlation of output of are estimated across the 1000 replications.The large number of replications implemented is to average out sampling error across simulations, before comparing model moments to the ones obtained from data.

Relative second moments evaluation

This section compares the theoretical second moments of the simulated data series with their empirical counterparts, with special attention paid to the behavior of public sector hours and wages. Table 2.4 on the next page summarizes the empirical and simulated business

cycle statistics for the model calibrated for Germany.

Table 2.4: Business Cycle Statistics Germany, 1970-2007

	GE Data	Model
$\sigma(y)$	0.0154	0.0144
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.00 [2.94,3.07]
$\sigma(n^p)/\sigma(y)$	1.05	0.26 [0.25,0.28]
$\sigma(n^g)/\sigma(y)$	1.06	2.25 [2.24,2.26]
$\sigma(n)/\sigma(y)$	0.73	0.54 [0.53,0.55]
$\sigma(w^p)/\sigma(y)$	1.16	0.77 [0.76,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	1.41 [1.40,1.42]
$corr(c, y)$	0.80	0.96 [0.95,0.97]
$corr(i, y)$	0.85	0.98 [0.97,0.99]
$corr(n^p, y)$	0.60	0.90 [0.86,0.94]
$corr(n^g, y)$	0.11	1.00 [1.00,1.00]
$corr(n, y)$	0.60	0.98 [0.97,0.99]
$corr(w^p, y)$	0.60	0.99 [0.99,0.99]
$corr(w^g, y)$	0.35	1.00 [1.00,1.00]
$corr(n, n^p)$	0.92	0.97 [0.96,0.98]
$corr(n, n^g)$	0.43	0.99 [0.98, 0.99]
$corr(n^p, n^g)$	0.12	0.92 [0.88,0.95]
$corr(n^p, w^p)$	0.21	0.83 [0.76,0.89]
$corr(n^g, w^g)$	-0.38	0.99 [0.99,1.00]
$corr(w^p, w^g)$	0.48	1.00 [1.00,1.00]

In the German data, relative consumption volatility exceeds one, as the available series does not provide a breakdown into consumption of non-durables and consumption of durables.

Another possible reason could be the presence of strong habits in consumption. Durable products behave like investment, and vary much more than non-durables, while model consumption corresponds to non-durable consumption. Since a major force in all the three models is consumption smoothing, as dictated by the Euler equation, the model under-predicts consumption volatility and investment variability. The lower variability in the model obtained is a new result in the literature. It is due to the fact that labor markets interaction are much more important quantitatively for the short-run dynamics of the model. After all, the simulation horizon in the annual model is only 38 periods, given the span of data.

In terms of labor market fluctuations, private sector employment and private wage in the model also vary less compared to data. Total employment in German data varies less than either private or public employment due to smaller variation in the number of self-employed individuals. It is evident from Table 2.4 on the previous page that the model underestimates public wage volatility. Still, this simple model generates public wage that varies twice as much as the private sector wage. Therefore, the introduction of the transaction cost in this chapter is definitely a step in the right direction. However, public employment in the model varies too much in the model, as compared to the volatility exhibited in German data. Overall, the transaction cost mechanism presented in this chapter seems to have an important quantitative effect in the German economy, especially when describing public sector labor market fluctuations.

The model also captures relatively well the high contemporaneous correlations of main variables with output. Moreover, public sector variables are also pro-cyclical, but not as much as the models predict. Lastly, the model captures quite well the co-movement between labor market variables. The dimension where the model fails, however, is the correlation between public sector hours and wages: in German data, it is negative, while the model predicts an almost perfect positive linear relationship. A possible reason could be that the government may have a target for the wage bill share in output, so any increase in employment has to be

matched with a corresponding decrease in the public sector wage rate. A fiscal rule of that sort (*e.g.* as in Economides *et al.* 2011) could generate the observed negative correlation in data.[44]

Overall, the model with transaction cost captures relatively well the labor market dynamics in Germany. Furthermore, the setup addresses dimensions that were ignored in earlier RBC models. Thus, the existence of such frictions in the public sector proves to be an important ingredient in RBC models when studying German labor markets.

Sensitivity Analysis

The results from the analysis performed so far are all contingent on the particular values of the parameters in the model calibration. Even though the benchmark calibration was justified either from previous studies, or from data averages, there might be still concerns that the results are sensitive to particular values chosen for the parameters. First, there are no previous studies and estimates of the scale parameter of the transaction cost. The problem in the model in this chapter was avoided by setting γ to match the average public/private wage ratio. It was shown that a change in the transaction cost parameter affects impulse responses of the public sector labor variables. Thus, it will be investigate further whether this result shows up in the relative second moments of public wage and hours as well.

Second, there might be uncertainties about the relative weights on different components of utility. Since there are limited micro econometric studies and no reliable estimates, the model was made consistent with the assumption used in the RBC literature that private consumption is twice more valuable than public consumption in household's utility, and that the household puts twice higher weight on utility from leisure as compared to utility derived

[44]However, Economides *et al.* (2011) model government wage bill share in output as a stochastic process. In addition, public employment is also fixed. Public sector workers choose their working hours, and public sector wage is determined residually from the wage bill. Thus, the correlation between hours and wages is matched by construction, and not due to any interesting mechanism in the model.

from private consumption. Again, the empirical evidence for those assumptions is scarce.

In light of the limited empirical evidence and/or conflicting studies, those parameters will be allowed to deviate significantly from their benchmark values. In the following, robustness checks will be performed with the value of the transaction cost parameter, as well as with the utility weights attached to leisure and public services. Relative second moments and contemporaneous correlations with output will be compared across cases, and compared against the benchmark calibration and data.

The simulated sample second moments generated by a lower ($\gamma = 0$) and higher ($\gamma = 10$) transaction cost scale parameter are reported in Table 2.5 on the next page against German data. As expected, the only significant changes are in the relative volatility of public employment and government wages. In line with the evidence from the earlier section dealing with impulse responses, higher transaction costs lead to lower variability in public hours, but higher volatility in government wages. In both cases, a higher γ brings simulated moments a bit closer to the moments in data. Nevertheless, no significant changes in terms of correlation are observed.

Table 2.5: Moments of the model and data (alternative transaction cost parameter)

	GE Data	$\gamma = 0$	$\gamma = 2.5762$	$\gamma = 10$
$\sigma(y)$	0.0154	0.0142	0.0144	0.0145
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.57]	0.55 [0.52,0.58]	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.01 [2.94,3.07]	3.00 [2.94,3.07]	3.00 [2.93,3.06]
$\sigma(n^p)/\sigma(y)$	1.05	0.24 [0.24,0.25]	0.26 [0.25,0.28]	0.27 [0.26,0.27]
$\sigma(n^g)/\sigma(y)$	1.06	3.61 [3.59,3.62]	2.25 [2.24,2.26]	1.68 [1.67,1.69]
$\sigma(n)/\sigma(y)$	0.73	0.72 [0.70,0.73]	0.54 [0.53,0.55]	0.46 [0.45,0.47]
$\sigma(w^p)/\sigma(y)$	1.16	0.79 [0.77,0.81]	0.77 [0.76,0.79]	0.77 [0.75,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	0.79 [0.77,0.81]	1.41 [1.40,1.42]	1.49 [1.48,1.50]
$corr(c, y)$	0.80	0.96 [0.95,0.97]	0.96 [0.95,0.97]	0.96 [0.95,0.97]
$corr(i, y)$	0.85	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(n^p, y)$	0.60	0.88 [0.84,0.93]	0.90 [0.86,0.94]	0.90 [0.87,0.94]
$corr(n^g, y)$	0.11	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, y)$	0.60	0.99 [0.98,0.99]	0.98 [0.97,0.99]	0.97 [0.96,0.98]
$corr(w^p, y)$	0.60	0.99 [0.98,0.99]	0.99 [0.99,0.99]	0.99 [0.99,0.99]
$corr(w^g, y)$	0.35	0.99 [0.98,0.99]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, n^p)$	0.92	0.95 [0.93,0.97]	0.97 [0.96,0.98]	0.98 [0.97,0.99]
$corr(n, n^g)$	0.43	0.99 [0.99,0.99]	0.99 [0.98, 0.99]	0.98 [0.98, 0.99]
$corr(n^p, n^g)$	0.12	0.90 [0.86,0.94]	0.92 [0.88,0.95]	0.92 [0.89,0.95]
$corr(n^p, w^p)$	0.21	0.81 [0.74,0.87]	0.83 [0.76,0.89]	0.83 [0.77,0.89]
$corr(n^g, w^g)$	-0.38	0.98 [0.98,0.99]	0.99 [0.99,1.00]	1.00 [0.99,1.00]
$corr(w^p, w^g)$	0.48	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]

Next, the results from simulations using higher- ($\psi_1/\psi_3 = 3$) and lower ratios ($\psi_1/\psi_3 = 1.5$) are reported in Table 2.6 on the next page. These two alternative calibrations are compared against the benchmark values and data. As ψ_3 increases, public good consumption becomes more valuable relative to the private consumption good. In addition, given the fixed value of the utility weight attached to the latter (ψ_1), the weight attached to leisure (ψ_2) decreases. The fall in ψ_2 makes private consumption more valuable relative to leisure. Therefore, hours will be expected to vary less, as leisure becomes a less important component of household's welfare. Indeed, as a result of the change in utility parameters, absolute output volatility, as well as relative public and total hours slightly fall, while private and public wage volatility increase a bit. However, the differences in volatilities are minute. Furthermore, there is no change in the other variables: consumption and investment vary the same across the three cases, and all contemporaneous correlations are virtually the same.

In summary, the sensitivity analysis performed above considered significant variations in the values chosen for some of the model parameters, *i.e.*, varying the scale parameter of the transaction cost function, or changing the weights on household's utility and thus affecting the preference for the mix of the private and the public consumption good. It was shown that the transaction cost scale parameter, γ, has important quantitative role in the model, as an increase in the level of transaction costs increases the relative volatility of public, and total hours, while depressing public wage rates. However, since γ was calibrated to match the average public/private wage ratio in data, those effects are not relevant in the current setup. In addition, changes in utility weights proved to be of no quantitative importance. Thus the benchmark model could be considered a plausible case which adequately approximates household's preferences in the real world. Furthermore, the model was shown to be robust to such changes in the preferences for consumption relative to leisure. In other words, there is no significant undervaluation or overvaluation of components of utility, and the public good in particular.

Table 2.6: Moments of the model and data (alternative utility parameters)

	Germany	$\psi_2 = 0.59$	$\psi_2 = 0.53$	$\psi_2 = 0.49$
	Data	$\psi_3 = 0.10$	$\psi_3 = 0.16$	$\psi_3 = 0.20$
$\sigma(y)$	0.0154	0.0147	0.0144	0.0144
$\sigma(c)/\sigma(y)$	1.11	0.55 [0.52,0.57]	0.55 [0.52,0.58]	0.55 [0.52,0.58]
$\sigma(i)/\sigma(y)$	3.57	3.01 [2.94,3.08]	3.00 [2.94,3.07]	3.00 [2.94,3.07]
$\sigma(n^p)/\sigma(y)$	1.05	0.28 [0.27,0.29]	0.26 [0.25,0.28]	0.26 [0.25,0.28]
$\sigma(n^g)/\sigma(y)$	1.06	2.27 [2.25,2.28]	2.25 [2.24,2.26]	2.25 [2.24,2.26]
$\sigma(n)/\sigma(y)$	0.73	0.56 [0.54,0.57]	0.54 [0.53,0.55]	0.54 [0.53,0.55]
$\sigma(w^p)/\sigma(y)$	1.16	0.76 [0.74,0.77]	0.77 [0.76,0.79]	0.77 [0.76,0.79]
$\sigma(w^g)/\sigma(y)$	3.50	1.40 [1.38,1.41]	1.41 [1.40,1.42]	1.41 [1.40,1.42]
$corr(c, y)$	0.80	0.96 [0.95,0.97]	0.96 [0.95,0.97]	0.96 [0.95,0.97]
$corr(i, y)$	0.85	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(n^p, y)$	0.60	0.91 [0.87,0.94]	0.90 [0.86,0.94]	0.90 [0.86,0.94]
$corr(n^g, y)$	0.11	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, y)$	0.60	0.98 [0.97,0.99]	0.98 [0.97,0.99]	0.98 [0.97,0.99]
$corr(w^p, y)$	0.60	0.99 [0.99,0.99]	0.99 [0.99,0.99]	0.99 [0.99,0.99]
$corr(w^g, y)$	0.35	1.00 [1.00,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]
$corr(n, n^p)$	0.92	0.97 [0.96,0.98]	0.97 [0.96,0.98]	0.97 [0.96,0.98]
$corr(n, n^g)$	0.43	0.99 [0.98, 0.99]	0.99 [0.98, 0.99]	0.99 [0.98, 0.99]
$corr(n^p, n^g)$	0.12	0.92 [0.88,0.95]	0.92 [0.88,0.95]	0.92 [0.88,0.95]
$corr(n^p, w^p)$	0.21	0.83 [0.77,0.89]	0.83 [0.76,0.89]	0.83 [0.76,0.89]
$corr(n^g, w^g)$	-0.38	0.99 [0.99,1.00]	0.99 [0.99,1.00]	0.99 [0.99,1.00]
$corr(w^p, w^g)$	0.48	1.00 [0.99,1.00]	1.00 [1.00,1.00]	1.00 [1.00,1.00]

2.10.8 Ramsey problem derivation

In this section, the public finance approach, pioneered by Atkinson and Stiglitz (1980) in a static context with many consumption goods, is applied here in a dynamic setting with a single consumption good. In particular, the intertemporal framework, as first outlined in Ramsey (1927), will consist of a social planner (government), who desires to maximizes agent's utility subject to the constraints that describe the competitive economy. In other words, in the optimal fiscal policy (Ramsey) framework, the government assumes the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interest, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital, public employment and public sector wage rate. It is assumed that only linear taxes are allowed, and that the government can credibly commit to those. Importantly, lump-sum taxation is prohibited, as the professional literature assumes that the government cannot redistribute income lump-sum. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus can be taken as a better approximation to the environment in which policymakers decide on a particular fiscal policy.

Another assumption crucial to the Ramsey approach is that the activities of all agents in the economy are observable, as information in this setup is perfect. Lastly, the government has access to a commitment technology, which effectively will tie its own hands and prevent it from reneging on an initially-made promise at a later point in time. Thus, in the full-commitment scenario, the time inconsistency problem, as described by Kydland and Prescott (1977), cannot arise; The government will made a "once-and-for-all" decision in period 0 and no further re-optimization will be allowed. In fact, under full commitment, such re-optimization will never be desired by the government: whatever policy is announced by the government in period 0 is the one that is implemented afterwards.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of generating a different competitive equilibrium allocation, i.e. allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the chapter, is that in Ramsey framework, the government chooses all instruments, instead of taking them as being exogenous. At the same time, the government also picks optimally the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[45] It will also be assumed that the government discounts time at the same rate as the representative household. The constraints which the government takes into account when maximizing household's welfare include the government budget constraints, and the behavioral responses of both the household, and the firm. These are summarized in the DCE of the exogenous fiscal policy case (2.10.2.1)-(2.10.2.14).[46] In other words, in the dual approach of Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[47] The initial conditions for the state variable $\{K_0^p, K_0^g\}$ is taken as given.

Following the procedure in Chamley (1986) and Ljungqvist and Sargent (2004), the Ramsey problem will be transformed and simplified, so that the government chooses after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1 - \tau_t^k) r_t \tag{2.10.8.1}$$

$$\tilde{w}_t^p \equiv (1 - \tau_t^l) w_t^p \tag{2.10.8.2}$$

$$\tilde{w}_t^g \equiv (1 - \tau_t^l) w_t^g. \tag{2.10.8.3}$$

[45] In contrast, the primal approach all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

[46] Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[47] Note that by choosing next-period public capital, the planner is choosing public investment optimally.

Next, the government budget constraint is rewritten as follows

$$\begin{aligned} \tau_t^k r_t K_t^p + \tau_t^l w_t^p N_t^p = \\ r_t K_t^p - (1-\tau_t^k) r_t K_t^p + w_t^p N_t^p - (1-\tau_t^l) w_t^p N_t^p = \\ (1-\tau_t^l) w_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^T. \end{aligned} \tag{2.10.8.4}$$

From the constant-returns-to-scale production function it follows that

$$r_t K_t^p + w_t^p N_t^p = A_t (N_t^p)^\theta (K_t^p)^{1-\theta}. \tag{2.10.8.5}$$

Substitute out the identities above into the government budget constraint and using the after-tax prices, the transformed government budget constraint becomes

$$A_t (N_t^p)^\theta (K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^T. \tag{2.10.8.6}$$

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed Ramsey problem then becomes:

$$\begin{aligned} \max_{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{r}_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \ln C_t + \psi_2 \ln \left[1 - N_t^p - N_t^g - \gamma (N_t^g)^2 \right] \\ + (1-\psi_1-\psi_2) \ln \left[(N_t^g)^\alpha (K_t^g)^{1-\alpha} \right] \Bigg\} \end{aligned} \tag{2.10.8.7}$$

s.t

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left[1 - \delta^p + (1-\tau_{t+1}^k)(1-\theta) \frac{Y_{t+1}}{K_{t+1}^p} \right] \tag{2.10.8.8}$$

$$\psi_2 C_t = \psi_1 \left[1 - N_t^p - N_t^g - \gamma (N_t^g)^2 \right] (1-\tau_t^l) w_t^p \tag{2.10.8.9}$$

$$\psi_2 C_t [1 + 2\gamma N_t^g] = \psi_1 \left[1 - N_t^p - N_t^g - \gamma (N_t^g)^2 \right] (1-\tau_t^l) w_t^g \tag{2.10.8.10}$$

$$A_t (N_t^p)^\theta (K_t^p)^{(1-\theta)} = C_t + K_{t+1}^g - (1-\delta^g) K_t^g + K_{t+1}^p - (1-\delta^p) K_t^p \tag{2.10.8.11}$$

$$A_t(N_t^p)^\theta(K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^T \quad (2.10.8.12)$$

$$K_{t+1}^p = I_t + (1-\delta^p)K_t^p \quad (2.10.8.13)$$

$$r_t = (1-\theta)\frac{Y_t}{K_t^p} \quad (2.10.8.14)$$

$$w_t^p = \theta\frac{Y_t}{N_t^p} \quad (2.10.8.15)$$

$$S_t^g = (N_t^g)^\alpha(K_t^g)^{(1-\alpha)} \quad (2.10.8.16)$$

$$K_{t+1}^g = G_t^i + (1-\delta^g)K_t^g \quad (2.10.8.17)$$

The Lagrangian function of the government thus becomes

$$\begin{aligned}
&\mathcal{L}^g(C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{r}_t, \lambda_t^1, \lambda_t^2, \lambda_t^3, \lambda_t^4, \lambda_t^5) = \\
&= E_0\sum_{t=0}^{\infty}\beta^t\Bigg\{\psi_1 \ln C_t + \psi_2 \ln\left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] \\
&+(1-\psi_1-\psi_2)\ln\left[(N_t^g)^\alpha(K_t^g)^{1-\alpha}\right] \\
&+\lambda_t^1\Big[-C_{t+1} + \beta C_t(1+\tilde{r}_{t+1}-\delta^g)\Big] \\
&+\lambda_t^2\Big[\psi_2 C_t - \psi_1[1 - N_t^p - N_t^g - \gamma(N_t^g)^2]\tilde{w}_t^p\Big] \\
&+\lambda_t^3\Big[\psi_2 C_t[1+2\gamma N_t^g] - \psi_1[1 - N_t^p - N_t^g - \gamma(N_t^g]^2)\tilde{w}_t^g\Big] \\
&+\lambda_t^4\Big[A_t(N_t^p)^\theta(K_t^p)^{(1-\theta)} + (1-\delta^p)K_t^p - C_t - K_{t+1}^p + (1-\delta^p)K_t^g - K_{t+1}^g\Big] \\
&+\lambda_t^5\Big[A_t(N_t)^\theta(K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p - \tilde{w}_t^g N_t^g - K_{t+1}^g + (1-\delta^g)K_t^g + G_t^T\Big]\Bigg\}
\end{aligned} \quad (2.10.8.18)$$

where $\lambda_t^i, i = 1,2,3,4,5$ is the Lagrangian multiplier associated with each constraint. FOCs:

$$C_t : \frac{\psi_1}{C_t} + \lambda_t^1\beta(1+\tilde{r}_{t+1}-\delta^p) + \frac{\lambda_{t-1}^1}{\beta} + \lambda_t^2\psi_2 + \lambda_3\psi_2[1+2\gamma N_t^g] - \lambda_t^4 = 0 \quad (2.10.8.19)$$

$$N_t^p : \frac{\psi_2}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2} = \lambda_t^2 \psi_1 \tilde{w}_t^p + \lambda_t^3 \psi_1 \tilde{w}_t^g + \lambda_t^4 \theta \frac{Y_t}{N_t^p} + \lambda_t^5 [\theta \frac{Y_t}{N_t^p} - \tilde{w}_t^p] \tag{2.10.8.20}$$

$$N_t^g : \frac{\psi_2}{1 - N_t^p - N_t^g - \gamma(N_t^g)^2}[1 + 2\gamma N_t^g] =$$
$$\lambda_t^2 \psi_1 (1 + 2\gamma N_t^g) \tilde{w}_t^p + 2\gamma \lambda_t^3 \psi_2 C_t + \lambda_t^3 \psi_1 (1 + 2\gamma N_t^g) \tilde{w}_t^g - \lambda_t^5 \tilde{w}_t^g \tag{2.10.8.21}$$

$$K_{t+1}^p : -\lambda_{t-1}^4 + \beta \lambda_t^4 [(1-\theta)\frac{Y_t}{K_t^p} + 1 - \delta^p] + \beta \lambda_t^5 [(1-\theta)\frac{Y_t}{K_t^p} - \tilde{r}_t] = 0 \tag{2.10.8.22}$$

$$K_{t+1}^g : -\frac{\beta(1 - \psi_1 - \psi_2)(1-\alpha)}{K_{t+1}^g} - \lambda_t^4 - \lambda_t^5 + \beta(1-\delta^g)(\lambda_{t+1}^4 + \lambda_{t+1}^5) = 0 \tag{2.10.8.23}$$

$$\tilde{w}_t^p : -\lambda_t^2 \psi_1 \left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] - \lambda_t^5 N_t^p = 0 \tag{2.10.8.24}$$

$$\tilde{w}_t^g : -\lambda_t^3 \psi_1 \left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] - \lambda_t^5 N_t^g = 0 \tag{2.10.8.25}$$

$$\tilde{r}_t : \lambda_{t-1}^1 C_{t-1} = \lambda_t^5 K_t^p \tag{2.10.8.26}$$

$$\lambda_t^1 : \frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left[1 - \delta^p + \tilde{r}_{t+1}\right] \tag{2.10.8.27}$$

$$\lambda_t^2 : \psi_2 C_t = \psi_1 (1 - N_t^p - N_t^g - \gamma(N_t^g)^2) \tilde{w}_t^p \tag{2.10.8.28}$$

$$\lambda_t^3 : \psi_2 C_t [1 + 2\gamma N_t^g] = \psi_1 \left[1 - N_t^p - N_t^g - \gamma(N_t^g)^2\right] \tilde{w}_t^g \tag{2.10.8.29}$$

$$\lambda_t^4 : A_t (N_t^p)^\theta (K_t^p)^{(1-\theta)} = C_t + K_{t+1}^g - (1-\delta^g) K_t^g + K_{t+1}^p - (1-\delta^p) K_t^p \tag{2.10.8.30}$$

$$\lambda_t^5 : A_t (N_t^p)^\theta (K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^T \tag{2.10.8.31}$$

$$I_t = K_{t+1}^p - (1-\delta) K_t^p \tag{2.10.8.32}$$

$$r_t = (1-\theta)\frac{Y_t}{K_t^p} \tag{2.10.8.33}$$

$$w_t^p = \theta \frac{Y_t}{N_t^p} \tag{2.10.8.34}$$

$$S_t^g = (N_t^g)^{\alpha}(K_t^g)^{(1-\alpha)} \tag{2.10.8.35}$$

$$G_t^i \quad = \quad K_{t+1}^g - (1-\delta^g)K_t^g \tag{2.10.8.36}$$

In contrast to previous studies, *e.g.* Chari, Christiano and Kehoe (1994, 1999), who investigate the behavior of taxes over the business cycle, next section will concentrate on the steady-state values of the fiscal policy instruments, as in Judd (1985), Chamley (1986) and Ljungqvist and Sargent (2004).[48]

[48]In addition, in business cycles literature, capital tax rate is on average zero, and labor income tax rate is essentially constant over the cycle. As noted by Wyplosz (2001), since the 1980s, the countercyclical use of fiscal policies in OECD countries has declined.

2.10.9 Steady state analysis of Ramsey equilibrium

In this section, the focus is on the long-run effect of optimal government behavior. Thus, all time subscripts and uncertainty are eliminated to obtain:

$$\frac{\psi_1}{c} + \lambda^1\beta(1+\tilde{r}-\delta) + \frac{\lambda^1}{\beta} + \lambda^2\psi_2 + \lambda_3\psi_2[1+2\gamma n^g] - \lambda^4 = 0 \tag{2.10.9.1}$$

$$\frac{\psi_2}{1-n^p-n^g-\gamma(n^g)^2} = \lambda^2\psi_1\tilde{w}^p + \lambda^3\psi_1\tilde{w}^g + \lambda^4\theta\frac{y}{n^p} + \lambda^5[\theta\frac{y}{n^p} - \tilde{w}^p] \tag{2.10.9.2}$$

$$\begin{aligned}\frac{\psi_2}{1-n^p-n^g-\gamma(n^g)^2}[1+2\gamma n^g] &= \lambda_t^2\psi_1(1+2\gamma n^g)\tilde{w}^p \\ &+2\gamma\lambda^3\psi_2 c + \lambda^3\psi_1(1+2\gamma n^g)\tilde{w}^g - \lambda^5\tilde{w}^g\end{aligned} \tag{2.10.9.3}$$

$$-\lambda^4 + \beta\lambda^4[(1-\theta)\frac{y}{k} + 1 - \delta] + \beta\lambda^5[(1-\theta)\frac{y}{k} - \tilde{r}] = 0 \tag{2.10.9.4}$$

$$-\frac{\beta(1-\psi_1-\psi_2)(1-\alpha)}{k^g} - \lambda^4 - \lambda^5 + \beta(1-\delta^g)(\lambda^4+\lambda^5) = 0 \tag{2.10.9.5}$$

$$-\lambda^2\psi_1[1-n^p-n^g-\gamma(n^g)^2] - \lambda^5 n^p = 0 \tag{2.10.9.6}$$

$$-\lambda^3\psi_1[1-n^p-n^g-\gamma(n^g)^2] - \lambda^5 n^g = 0 \tag{2.10.9.7}$$

$$\lambda^1 c = \lambda^5 k^p \tag{2.10.9.8}$$

$$1 \;=\; \beta[1-\delta^p+\tilde{r}] \tag{2.10.9.9}$$

$$\psi_2 c = \psi_1[1-n^p-n^g-\gamma(n^g)^2]\tilde{w}^p \tag{2.10.9.10}$$

$$\psi_2 c[1+2\gamma n^g] = \psi_1[1-n^p-n^g-\gamma(n^g)^2]\tilde{w}^g \tag{2.10.9.11}$$

$$y = c + \delta^g k^g + \delta^p k^p \tag{2.10.9.12}$$

$$y - \tilde{r}k^p - \tilde{w}^p n^p = \tilde{w}^g n^g + \delta^g k^g + g^T \tag{2.10.9.13}$$

$$i = \delta^p k^p \tag{2.10.9.14}$$

$$r = (1-\theta)\frac{y}{k^p} \tag{2.10.9.15}$$

$$w^p = \theta\frac{y}{n^p} \tag{2.10.9.16}$$

$$g^i = \delta^g k^g \tag{2.10.9.17}$$

Next, an important analytical result in the optimal policy framework will be demonstrated: First, simplify the FOC for private capital (2.10.9.4) to obtain

$$\lambda^4 = \beta\lambda^4[(1-\theta)\frac{y}{k^p} + 1 - \delta^p] + \beta\lambda^5[(1-\theta)\frac{y}{k^p} - \tilde{r}_t]. \tag{2.10.9.18}$$

Next, substitute out the expression for the interest rate from (2.10.9.9) into (2.10.9.18) to obtain

$$\lambda_4 = \beta\lambda^4[r + (1-\delta)] + \beta\lambda^5[r - \tilde{r}]. \tag{2.10.9.19}$$

Now plug in the expression for $(1-\delta)$ from (2.10.9.9) into (2.10.9.19) to obtain

$$\lambda_4[1 - \beta(r + \frac{1}{\beta} - \tilde{r})] = \beta\lambda^5[r - \tilde{r}], \tag{2.10.9.20}$$

or

$$-\lambda_4\beta[r - \tilde{r}] = \beta\lambda^5[r - \tilde{r}]. \tag{2.10.9.21}$$

Hence

$$(\lambda^4 + \lambda_5)[r - \tilde{r}] = 0. \tag{2.10.9.22}$$

Since $\lambda^4, \lambda^5 > 0$ by construction, it follows that $\tilde{r} = r$. Therefore, in steady-state Ramsey equilibrium, the optimal steady-state capital tax is zero. This result is consistent with earlier findings in Judd (1985), Chamley (1986), Zhu (1992), Ljungqvist and Sargent (2004, 2013) and Kocherlakota (2010).

2.10.10 Measuring conditional welfare

In steady state

$$u(c, 1-n) = \psi_1 \ln c + \psi_2 \ln[1 - n^p - n^g - \gamma(n^g)^2] + \psi_3 \ln(n^g)^\alpha (k^g)^{(1-\alpha)}. \quad (2.10.10.1)$$

Let A and B denote two different regimes. The welfare gain, ξ, is the fraction of consumption that is needed to complement household's steady-state consumption in regime B so that the household is indifferent between the two regimes. Thus

$$\psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha + \psi_3 \ln(k^{g,A})^{(1-\alpha)} =$$
$$\psi_1 \ln[(1+\xi)c^B] + \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2) + \psi_3 \ln(n^{g,B})^\alpha + \psi_3 \ln(k^{g,B})^{(1-\alpha)}, \quad (2.10.10.2)$$

where $n^A = n^{p,A} + n^{g,A}$ and $n^B = n^{p,B} + n^{g,B}$.

Next, expand the right-hand side to obtain

$$\psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha + \psi_3 \ln(k^{g,A})^{(1-\alpha)} = \psi_1(1+\xi)$$
$$+\psi_1 \ln c^B + \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2) + \psi_3 \ln(n^{g,B})^\alpha + \psi_3 \ln(k^{g,B})^{(1-\alpha)}. \quad (2.10.10.3)$$

Rearrange to obtain

$$\psi_1 \ln(1+\xi) = \psi_1 \ln c^A + \psi_2 \ln(1 - n^A - \gamma(n^{g,A})^2) + \psi_3 \ln(n^{g,A})^\alpha$$
$$+\psi_3 \ln(k^{g,A})^{(1-\alpha)} - \psi_1 \ln c^B - \psi_2 \ln(1 - n^B - \gamma(n^{g,B})^2)$$
$$-\psi_3 \ln(n^{g,B})^\alpha - \psi_3 \ln(k^{g,B})^{(1-\alpha)}. \quad (2.10.10.4)$$

Divide throughout by ψ_1 to obtain

$$\ln(1+\xi) = \ln c^A + \frac{\psi_2}{\psi_1} \ln(1 - n^A - \gamma(n^{g,A})^2) + \frac{\psi_3}{\psi_1} \ln(n^{g,A})^\alpha + \frac{\psi_1}{\psi_3} \ln(k^{g,A})^{(1-\alpha)}$$
$$- \ln c^B - \frac{\psi_2}{\psi_1} \ln(1 - n^B - \gamma(n^{g,B})^2) - \frac{\psi_3}{\psi_1} \ln(n^{g,B})^\alpha - \frac{\psi_3}{\psi_1} \ln(k^{g,B})^{(1-\alpha)}. \quad (2.10.10.5)$$

Raise to the exponent both sides of the equation and rearrange terms to obtain

$$(1+\xi) = \frac{c^A}{c^B}\left[\frac{1-n^A-\gamma(n^{g,A})^2}{1-n^B-\gamma(n^{g,B})^2}\right]^{\frac{\psi_2}{\psi_1}}\left[\frac{n^{g,A}}{n^{g,B}}\right]^{\frac{\alpha\psi_3}{\psi_1}}\left[\frac{k^{g,A}}{k^{g,B}}\right]^{\frac{(1-\alpha)\psi_3}{\psi_1}}. \tag{2.10.10.6}$$

Thus

$$\xi = \frac{c^A}{c^B}\left[\frac{1-n^A-\gamma(n^{g,A})^2}{1-n^B-\gamma(n^{g,B})^2}\right]^{\frac{\psi_2}{\psi_1}}\left[\frac{n^{g,A}}{n^{g,B}}\right]^{\frac{\alpha\psi_3}{\psi_1}}\left[\frac{k^{g,A}}{k^{g,B}}\right]^{\frac{(1-\alpha)\psi_3}{\psi_1}} - 1. \tag{2.10.10.7}$$

Note that if $\xi > 0 (< 0)$, there is a welfare gain (loss) of moving from B to A. In this chapter E is the exogenous policy case, while A will be the Ramsey policy scenario.

Chapter 3

On the cost of rent-seeking by government bureaucrats in a Real-Business-Cycle model

3.1 Introduction

The social cost of rent-seeking and administrative corruption in Europe can cause a significant loss for the economy, as argued in Rose-Akerman (1999). Rent-seeking behavior, however, can take many forms and in many instances corruption schemes are not obvious. In particular, this chapter focuses on the non-productive activities that occur inside public administration and models them in a dynamic general-equilibrium setting. This is achieved by adding public employment and rent-seeking by government officials to an otherwise standard RBC model. As in Wallenius and Prescott (2011), public sector labor choice is shown to be important not only for fiscal policy, but also for political economy issues. The framework in this chapter is then used to generate a theory-based measure for the cost of the waste imposed on the economy, and proceeds to compare and contrast it with the value obtained from the optimal fiscal policy case.

In line with Eliott (1997), Goel and Nelson (1998), and Persson and Tabellini (2000, p.8.), the focus in this chapter will be on particular types of government expenditure, namely spending on wages, and its potential to produce rent-seeking behavior. In particular, the sharp increase in public sector employment observed in EU member states in the post-WWII era, together with the existence of a significant public wage premium, could be driven by the tendency for bureaucracy to self-breed and expand independently. Borcherding (1977) was the first to provide some evidence for such a hypothesis by demonstrating that only half of the increase in real government spending can be rationalized with changes in relative prices, or demand factors such as increase in real income, and population growth. In addition, LaPalombara (1994) finds that the size of government budget relative to output is positively correlated with levels of corruption (p.338).

Importantly, this bloating process in the administration and the subsequent expansion of the public sector wage bill should raise concerns in policy makers, since larger governments tend to lose efficiency progressively with size, an issue first expressed in Parkinson (1962). Furthermore, a classic study by von Mises (1944) provides a possible justification for this claim by elaborating on some of the important differences between a bureaucrat and a private sector worker. First, labor productivity in the public sector is difficult to measure. Moreover, a quantity corresponding to government production is also hard to define. This is driven by the fact that public services are to a great extent composed of non-market output. Lastly, a government employee often comes into office through the political system - by election or by appointment of other bureaucrats, usually from the same party. Therefore, a bureaucrat is both an employer and an employee. Once organized, bureaucrats concentrate on increasing in numbers, keeping the *status quo*, and resisting change.[1]

In the literature on bureaucracy, the studies by von Mises (1944), Parkinson (1957), Niskanen

[1]Often, the expansion in bureaucracy is justified by bureaucrats themselves, who claim there is an increase in the perceived demand for public services and thus there is need for more regulation, hence larger administration.

(1971), Warwick (1975), Tullock (1976) and Tinbergen (1985) all focus on the strong competition for advancement within bureaus and the inter-unit conflicts. In particular, as noted in Box (2004), "public sector bureaucrats want their agencies to grow so that their status and freedom to act are increased," and thus each government official has a vested interest in promotion. Furthermore, the contemporary bureaucrat is not the ideal type envisioned and described in Weber (1963), the impartial official and the competent professional who shows a strong *ethos* for working in the public administration,[2] but rather a self-interested individual. In particular, Peter (1969) describes the process employed by a government worker to obtain promotions as "acquiring a pull" (p.48). This can occur when the employee finds a patron, a person superior in the hierarchy, who can help with the employee's promotion.[3] In some instances, multiple patrons may be chosen, producing a network effect, as individual patrons talk to one another about the employee's career prospects and advancement. In his monograph, Klitgaard (1991) pays particular attention to bureaucratic corruption, and investigates different cases that come as a result from bureaucratic employment. In particular, his study argues that administrative corruption, and often the use of patronage allow bureaucrats to supplement salaries with public funds.

In an important study, Rose-Akerman (1999) also argues that corruption, or rent-seeking behavior, is embedded in the hierarchical structure of public administration. For example, subordinates in the administration are treated as "family," and some of the gains obtained by their superiors through rent-seeking are shared with staff members, who are lower in the hierarchy. Thus, Rose-Akerman (1978) not only distinguishes between high-level and low-level bureaucrats, but also emphasizes inter-official competition.[4] In addition, as observed in Parkinson (1957), officials can increase and multiply by making work for each other through the redundancy of repetitive tasks and overlapping authorities and responsibilities. In this sense, as in McKenzie and Tullock (1978) and Reisman (1990), public professionals can be

[2]After all, Fry (1989) reminds that "Weber's ideal type is <u>not</u> a description of reality." (p. 21)

[3]Another author, who discusses the patronage system is Gortner (1977).

[4]Promotions and prizes in bureaucracy will be treated as rent-seeking activities in the model as well.

regarded as self-interested maximizers of their position in a bureaucratic world, who pursue career advancement, financial security, and try to use the organization where they work to serve their personal interests. In a more recent study, Lambsdorff (2007) claims that if corruption involves a rent-seeking government whose members attempt to enrich themselves, then the size of the government itself should be significantly decreased (p.4). Similar views are presented in Rose-Akerman (1978), who associates bureaucratic corruption with bloated agency budgets.[5]

Very few economists, with the notable exception of Buchanan and Tullock (1962), Tullock (1965), and Niskanen (1971), have focused on the presence of a large bureaucracy and provided evidence of its importance in the macroeconomic context. In addition, a very small economic literature exists on the internal organization of the state and the incentives of government bureaucracy, *e.g.*, Acemoglu and Verdier (1998), Acemoglu (2005), and Becker and Mulligan (2003). Guriev (2004), Dixit (2006, 2010), and Dodlova (2013) also point out to the multi-tier structure of the government bureaucracy, the principal-bureaucrat-agent hierarchy, as the main culprit for state inefficency due to the agency problem generated within such an organizational arrangement. More specifically, elected politicians (agents) need to elect experts (bureaucrats) to implement policies in the interest of the citizens (principal). Thus, as argued in Tirole (1994) and Aghion and Tirole (1997), bureaucrats possess the real authority in the government, as they have an "effective control over decisions" (even though they don't have the "formal authority"). In addition, Niskanen (1971), Bendor *et al.* (1985), and Horn (1995) argue that bureaucrats use their superior information when the budget is decided to inflate the costs of labor input.[6]

The focus in those studies also fall on the emergence of inefficient states, where the inefficiency is measured in terms of extracted rents and the excessive spending on wages of

[5]Bhagwati (1982) names the process "internal corruption," and rent-seeking activities "directly unproductive."

[6]For literature on the incentives of bureaucrats, see Laffont (2000), and Laffont and Martimort (2002).

bureaucrats. Gregory and Lazarev (2003) and Acemoglu *et al.* (2007) also documents cases of overemployment of bureaucrats to boost encumbant's party's votes.[7] Furthermore, government workers could be compensated with higher wages, as a favor by policymakers in exchange of bureaucrats' loyalty, or an an efficiency wage aiming to solve monitoring problems (Acemoglu and Verdier 2000). Makris (2006) also emphasizes the importance of the high cost of production of government services for the budget, while Migue and Belanger (1974) argue that bureaucrats oversupply public services, while at the same time enjoying an excessive budget, the size of which is unrelated to their labor productivity.[8] The latter could be rationalized, at least partially, with the moral hazard problem in the government employment sector, as bureaucrats have a better information about the level of the effort exerted, relative to the superiors.[9]

Despite being a possibly better description of the internal organization of the state and the incentives of the bureaucracy, the modeling choice in those partial-equilibrium setups cannot be easily translated and adopted in general equilibrium. Possible extensions of the work in this chapter along those lines is left for future research, as it does not serve our current purpose of quantifying the cost of rent seeking for the aggregate economic activity. This chapter would therefore present an alternative channel used by bureaucrats to rent-seek and lobby for government funds.[10] Bureaucracy in the setup will be implicitly modeled as

[7]Egorov and Sonin (2009) point out that often policy makers would prefer a loyal bureaucrat to a capable one.

[8]Roett (1999) points out that there is a public perception that public sector workers are overpaid and underworked.

[9]See Dewatripont *et al.* (1999), Francois (2000), Dixit (2002), Prendergast (2002) and Besley and Ghatak (2005) for the implications of moral hazard in government non-profit organizations. In addition, Tarschys (1975) and Wildavsky (1974) argue that government bureaucracies develop internal pressures for self-aggrandizement and expansion. Lastly, interaction among bureaucrats for budget appropriation are discussed in Miller and Moe (1983), Moe (1997), Bendor et al. (1985), Bos (2001), and Casas-Pardo and Puchades-Navarro (2001).

[10]Indeed, most of government consumption spending is on the wage bill for staff compensation (OECD 1982).

a collection of competing bureaus, as in Niskanen (1971): every bureaucrat, in each bureau, would want to control more and more subordinates. In addition, the bureaucrat's preference for power and prestige will bring utility through higher labor income, and thus higher consumption.

To illustrate the processes taking place within public administration, the model setup in this chapter incorporates a symmetric non-cooperative game that is played among government bureaucrats themselves to increase individual income at the expense of the other public officials earnings.[11] The symmetric rent-seeking process is modeled as in Murphy *et al.* (1991)[12]: Each individual allocates time optimally between both productive activities and rent-seeking. Additionally, the interaction between agents in the public sector generates strategic complementarities, as individual rent-seeking is positively related to opponents' choice of rent-seeking. As in Burnside and Eichenbaum (1993, 1996), rent-seeking increases one's own capacity and at the same time decreases others' capacity level. In reality, as pointed out in Tinbergen (1985), this correspond to public employees redistributing residual bureau funds, expropriating vacation money (Mieczkowski 1984, p. 164), or applying bonus schemes methodology according to rank and experience. The benefit from engaging in rent-seeking comes at the expense of a cost incurred, which is measured in terms of time, similar to the approach used in Angelopoulos *et al.* (2009).

[11]Note that when a large number of bureaucrats attempt to maximize identical objective functions, which are subject to the same set of constraints, this results in a theory covering bureaucracy as a whole, as was first attempted in Niskanen (1971).

[12]An incomplete list of the extraction literature that uses rent-seeking mechanism is: Krueger (1974), Posner (1974, 1975), Tullock (1975, 1980), Congleton (1980), Bhagwati and Srinivasan (1980), Corcoran and Karels (1985), Appelbaum and Katz (1987), Hillman and Riley (1989), Murphy et al. (1991, 1993), Perez-Castrillo and Verdier (1992), Nitzan (1994), Tirole (1986), Khan and Sundaran (2000), Mohtadi and Roe (1998, 2003) and Mauro (2004). Park et al. (2005) for similar setups in which rent-seekers compete with each other for fiscal favors. For surveys of the literature of rent-seeking see Drazen (2000, Ch.11), Persson and Tabellini (2000, Ch. 14.4) and Mueller (2003).

The value-added of this chapter is the focus on the link between the rent-seeking behavior in bureaucracy and the government wage bill, and the resulting cost imposed on the economy as a result of the non-productive activities taking place in the public sector. Another novelty here is that rent-seeking occurs in a non-competitive labor market, the public sector one, where wage rate is set above private sector pay. This stimulates entry of labor in the sector, and as a result, public employment eventually becomes too high. In particular, both the high public wage and employment stimulate rent-seeking by generating a positive benefit of engaging in wasteful activities. In turn, a higher wage bill requires higher tax rates to finance government spending. In the private sector, high taxes reduce incentives to supply labor and accumulate capital, and decrease consumption and output. Thus rent-seeking has a negative impact on the economy, and this chapter attempts to quantify the loss for the economy in a general-equilibrium framework.

The study in this chapter could be also considered complementary to Park *et. al* (2005), and Angelopoulos *et al.* (2009, 2011), who all address rent-seeking issues using DSGE models.[13] Their focus, however, falls on problems of tax collection and/or protection of property rights, while this study concentrates on the inefficiencies on the government spending side, and the wage bill in particular. In addition, earlier studies consider exogenous policy only, and focus on the non-cooperative Nash (1953) solution of the rent-seeking game, while only briefly discussing the existence (and attainability) of other subgame-perfect equilibria.[14]

Lastly, the rent-seeking process in the public sector can be also viewed as a "coordination failure" problem, as first described in Cooper and John (1988) and Cooper (1999). There is a bad equilibrium, from the non-cooperative game, but also a good equilibrium, where all agents coordinate on the zero rent-seeking. A positive value of rent-seeking time chosen

[13]In addition, Park *et al.* (2004), Economides *et al.* (2007) and Angelopoulos and Economides (2008) address rent-seeking in models with electoral uncertainty.

[14]This infinitely-repeated game can generate a very rich class of subgame-perfect equilibria, which are all sustainable (Mas-Colell *et al.* 1995).

by bureaucrats is socially costly, as the return comes not from productive effort, but rather from the distribution of public funds. At the same time, the positive amount of time dedicated to opportunistic activities is an efficient outcome from an individual worker's point of view, as all agents are fully rational and maximize their utility levels. Thus, in equilibrium, individual bureaucratic rent-seeking efforts will adjust to the point where the value of additional resources spent per bureaucrat equals the benefit that accrues to that individual.[15,16] Moreover, the higher the level of output, the higher the tax revenue, and thus the larger the pie available for redistribution.

This chapter also finds non-trivial welfare gains of cooperation in the exogenous policy case, which is not considered in previous studies. The increase in welfare is possible if all agents agree to coordinate on zero rent seeking, which in that case is a sustainable equilibrium. In addition, a comparative statics exercise is performed to show that significant gains can be realized when the economy moves to a steady-state in which wages are equalized across sectors, and thus the main reason for the existence of rent-seeking is abolished.

Next, using the model-generated measure, rent-seeking across the EU-12 is compared to indices of institutional quality in the government sector. Angelopoulos *et al.* (2009, 2011)

[15]Note that the first-best solution (the jointly optimal level) in the model results in zero rent-seeking.

[16]As it will be shown later in the chapter, the cost of rent-seeking can be significant at aggregate level. However, the strategic complementarity that occurs as a result of rent-seeking by individual bureaucrats is rather small, as compared to the aggregate. In technical terms, individual rent-seeking does not seem to influence the other aggregate allocations (compared to the steady-state allocations obtained in Ch.2 for a very similar model without rent-seeking). That is partially due to the fact that a particular equilibrium outcome from the rent-seeking game was chosen, the non-cooperative Nash equilibrium outcome from the static stage game that is repeatedly played over the infinite horizon of the game. Essentially, for simplicity and tractability, the players are assumed to ignore the history of the game (and thus possible multiplicity of equilibria is abstracted away from). In this simple static class of rent-seeking dames, there is a unique equilibrium. The structure of the rent-seeking game corresponds to those static models of rent-seeking, so the uniqueness of equilibria is directly translatable. Of course, this is a rather strong assumption in the theoretical setup, but one that ensures that the macroeconomic model will feature a unique equilibrium.

use the ICRG index as a proxy for rent-seeking, while this chapter considers additional indicators that specifically focus on public administration quality and government spending efficiency. In general, rent-seeking from the government budget is expected to be associated with low quality of government, high corruption, and heavy bureaucratization.[17]

The study in this chapter then proceeds to discuss the optimal fiscal policy, where not only tax rates, but also different categories of government spending, as well as rent-seeking[18] (from the non-cooperative Nash equilibrium) are optimally chosen by a benevolent Ramsey planner. Due to rent-seeking, public investment is lower, and government consumption (wage bill) is excessive. Furthermore, the rent-seeking estimates can be evaluated against findings from studies using static models with rent-seeking of tariff revenues, monopoly profits, and regulations, as well as the costs computed in Angelopoulos *et al.* (2009) in a general-equilibrium framework. Thus, using tools of modern dynamic economics, the study in this chapter contributes to the understanding of the wasteful effect of bureaucracy for the economy. It also provides an integrated framework to address both public economics and political economy issues, such as public sector labor supply in a non-competitive market, as well as the optimal production and provision of congestible government services.

The main findings of the study are that: (i) Due to the existence of a significant public sector wage premium and the high public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses in terms of output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime, steady-state rent-seeking is smaller relative to the exogenous policy case, as

[17] The ICRG index from the IRIS dataset has been used in, among others, Knack and Kiefer (1995), Barro (1997), Hall and Jones (1999), Rodrick (1999), Acemoglu et al. (2001, 2002), Fisman and Gatti (2002) and Persson et al. (2003).

[18] Rent-seeking is determined residually, as the government will influence its level only indirectly by optimally setting public sector employment and wages.

the government chooses a higher public wage premium, but sets a much lower public employment, thus achieving a decrease in rent-seeking.

The chapter is organized as follows: Section 3.2 presents the model setup in the context of the relevant literature. Section 3.3 explains the data used and model calibration. Section 3.4 solves for the steady-state, and section 3.5 calculates the cost of rent-seeking on the economy; Section 3.6 presents the model solution procedure, discusses the effect of technology shocks and the impulse responses of variables. Section 3.7 discusses the optimal policy (Ramsey) framework and solves for the steady-state. Section 3.8 presents the optimal reaction of fiscal policy instruments to technology shocks, and compares and contrasts it to the exogenous policy case. Section 3.9 acknowledges the limitations of the study, and Section 3.10 concludes.

3.2 Model Setup

3.2.1 Description of the model

There are H_t households, as well as a representative firm. Each household owns physical capital and labor, which it supplies to the firm. Household's time endowment (normalized to unity) can be spent working in the private and/or public sector, rent-seeking, or dedicated to leisure. Working in the government sector imposes an additional convex transaction cost, which decreases leisure time. The perfectly-competitive firm produces output using labor and capital. The government uses tax revenues from labor and capital income to finance: (1) government transfers, (2) government investment, and (3) public wage bill.

The public sector wage in this chapter is modeled as featuring a time-variant mark-up over the private sector wage. Next, individual hours supplied in the public sector can be augmented by using rent-seeking time, and the coefficient of proportionality positively depends on one's own rent-seeking time and negatively related to other households' rent-seeking time.

This is similar to the setup in Angelopoulos *et al.* (2009, 2011), in which households were able to extract part of the tax revenues, or output directly, respectively. In this chapter the resource extraction is slightly more sophisticated, as public wages are financed from government tax revenues, and therefore government resources are only indirectly expropriated.

3.2.2 Households

There are H_t homogenous households in the model economy, who are infinitely-lived.[19] There is no population growth. The household h maximizes the following expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^{ph}, S_t^g, L_t^h), \tag{3.2.2.1}$$

where E_0 is the expectation operator as of period 0; C_t^{ph}, S_t^g and L_t^h are household's private consumption, per household consumption of government services, and leisure enjoyed by household h at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. The particular functional form for instantaneous utility used is as follows:

$$U(C_t^{ph}, S_t^g, L_t^h) = \psi_1 \frac{(C_t^{ph})^{1-\sigma_c}}{1-\sigma_c} + \psi_2 \frac{(L_t^h)^{1-\sigma_l}}{1-\sigma_l} + \psi_3 \frac{(S_t^g)^{1-\sigma_s}}{1-\sigma_s} \tag{3.2.2.2}$$

where $\sigma_c, \sigma_l, \sigma_s > 0$ are the curvature parameters of private consumption, leisure, and government services utility component, respectively. Parameters ψ_1, ψ_2, ψ_3 are the weight attached to private consumption, leisure and public services components in utility, respectively, where $0 < \psi_1, \psi_2, \psi_3 < 1$, and $\psi_1 + \psi_2 + \psi_3 = 1$.

Total time available to each household is split between work, N_t^h, rent-seeking in the public sector, RS_t^h, and leisure, L_t^h. Households can supply hours of work in the public sector, N_t^{gh}, in the private one, N_t^{ph}, with $N_t^h = N_t^{ph} + N_t^{gh}$. Given a positive public sector wage, every household will optimally choose to supply a positive amount of hours in the public sector. Thus, the model allows everyone to engage in public sector rent-seeking.

[19] This number is countably infinite, and the households could be thought of being uniformly distributed on the $[0, 1]$ interval.

In addition, it will be assumed that the household incurs a quadratic transaction cost from government work, $\gamma(N_t^{gh})^2$, where $\gamma > 0$. The modeling choice tries to capture some of the market imperfections existing in the public sector labor markets, such as the high unionization, and the monopsony situation,as well as to help to accommodate public hours labor choice in the framework. This assumption is also in line with evidence from case studies in Box (2004), which shows that working in the public sector is different from working in the private sector, as the two sectors operate under different institutional settings. Similar to the approach adopted in Cho and Cooley (1994), and Hayashi and Prescott (2007), in this framework contracted and effective public hours enter the household's utility function through different functional forms. The wage rates per efficiency unit of labor in the private and the public sector are denoted by w_t^p and w_t^g, respectively. In the private sector, efficiency level is constant and for convenience will be normalized to unity, while utilization rate in the public sector can vary because of the rent-seeking. In addition, public wage rate will carry a premium over the private wage rate, which is allowed to vary over time. As pointed out in Bellante and Jackson (1979), the overpayment of public employees could be regarded as rent as it is "a pay level higher necessary to attract the requisite quality and quantity of labor in the public sector (p. 248).[20]

Next, after joining the public sector, rent-seeking occurs, as it brings a positive benefit from engaging in opportunistic behavior. The form of the corruption is a non-transaction type, and can be interpreted as an abuse of power for personal advantage, or putting one's own interests first in the performance of a public duty. In particular, by using his or her own

[20]This is in line with the evidence that public sector employees are more skilled than the private sector ones. On the other hand, Beaumont (1980) argues that public unions usually maintain that "a public employer should be the best employer, that its wage policy should be based on the highest rates being paid for comparable work in the private sector" (p. 32). In addition, Bender (1998, 2003) provides empirical support that part of the public wage premium is due to political economy factors, *e.g.*, public employees vote more often than the average private sector worker (Jensen *et al.* 2009).

rent-seeking time, an individual's public sector labor income can be augmented by increasing the effective hours worked in the government: By supplying N_t^{gh} contract hours in the public sector, and spending RS_t^h hours on rent-seeking, each households generates $\frac{RS_t^h}{RS_t} N_t^{gh}$ of "efficiency units of labor," as in Burnside and Eichenbaum (1993, 1996), hence total public sector labor income becomes $w_t^g \frac{RS_t^h}{RS_t} N_t^{gh}$.[21] At the same time, predatory behavior decreases the capacity utilization of labor of the other workers in the public sector. Note that each household is atomistic, so it takes the aggregate quantity of rent-seeking RS_t as given. (In equilibrium, $RS_t = \sum_h RS_t^h$.) Thus, even though total public employment is exogenous for each household,[22] the individual public hours are endogenous.

As in Burnside and Eichenbaum (1993, 1996), it is assumed that each household cares about effective hours of work only. Thus, the time constraint that each household faces in each period (in efficiency terms) is as follows:

$$N_t^{ph} + \frac{RS_t^h}{RS_t} N_t^{gh} + \gamma (N_t^{gh})^2 + RS_t^h + L_t^h = 1. \tag{3.2.2.3}$$

The rent-seeking technology described is a special case of a standard symmetric contestable prize function used in the literature.[23] This approach models rent-seeking as an optimal choice made by each government bureaucrat. In addition, the size of the total pie available to government workers will be endogenously determined, as each bureaucrat chooses individual public hours optimally. Thus each official has an incentive to choose the optimal

[21] Thus $\frac{RS^h}{RS}$ can be interpreted as a shift parameter of the rent-seeking extraction technology. As in Grossman (2002), this technology representation is a useful short-cut to model rent-seeking. In addition, note that at the aggregate level, the efficiency issues disappears, as $\sum_h \frac{RS^h}{RS} N^{gh} = \sum_h \frac{RS}{RS} N^{gh} = N^g$ by first applying the symmetry, and then aggregating over households.

[22] The mechanism has also empirical foundations: as established in Staaf (1977), in the US public education sector, the supervisory salaries are correlated with the number of subordinates (teachers in the district).

[23] An inexhaustive list of studies includes Krueger (1974), Posner (1974, 1975), Tullock (1975, 1980), Congleton (1980), Bhagwati and Srinivasan (1980), Corcoran and Karels (1985), Appelbaum and Katz (1987), Hillman and Riley (1989), Castrillo and Verdier (1992), Nitzan (1994) and Khan and Sundaran (2000). For an extensive review of "extraction technologies" the interested reader should consult Mueller (2003) Ch. 15.

size of their effective "slice." The only modeling difference in this chapter from the earlier general-equilibrium studies on rent-seeking, *e.g.* Angelopoulos *et al.* (2009, 2011), is that the cost of resources spent on influencing the probability of winning, $\frac{RS_t^h}{RS_t}$, is measured in terms of time and thus in utility of leisure terms instead of output/income directly. The specification used in the current model setup can also be interpreted as an auction in which competing bureaus lobby for a larger share of the contestable transfer, and the endogenous sharing rule defines the rent-seeking technology. Moreover, a larger share of the pie means higher effective public hours, which can be associated with promotion in the hierarchical structure, higher prestige, more subordinates, power by entrenchment in an organization and thus achievement of security and convenience.

In addition to the labor income received, each household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to physical capital and K_t^{ph} denotes physical capital stock in the beginning of period t.

The household's physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1 - \delta^p) K_t^{ph}, \tag{3.2.2.4}$$

where $0 < \delta^p < 1$ is the depreciation rate of private physical capital.

Finally, consumers are owners of the firms in the economy, and receive equal share of the profit (Π_t^h) in the form of dividends. The budget constraint for each household is

$$C_t^{ph} + I_t^h \leq (1 - \tau_t^l) \left[w_t^p N_t^{ph} + w_t^g \frac{RS_t^h}{RS_t} N_t^{gh} \right] + (1 - \tau_t^k) r_t K_t^{ph} + \Pi_t^h + G_t^{Th}, \tag{3.2.2.5}$$

where τ_t^l, τ_t^k are the proportional tax rates on labor and capital income, respectively, and G_t^{Th} denotes the level of per household lump-sum government transfer.

Each household h acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau_t^l, \tau_t^k\}$ and

policy variables $\{G_t^i, S_t^g, G_t^T, K_{t+1}^g\}_{t=0}^{\infty}$ as given, it chooses allocations $\{C_t^{ph}, N_t^{ph}, N_t^{gh}, RS_t^h, I_t^h, K_{t+1}^h\}_{t=0}^{\infty}$ to maximize Eq. (3.2.2.1) subject to Eqs. (3.2.2.2)-(3.2.2.5), and the initial condition for physical and public capital stocks $\{K_0^{ph}, K_0^g\}$.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for the private physical capital stock, are as follows:[24]

$$C_t^{ph} : \frac{\psi_1}{C_t^{ph}} = \Lambda_t \tag{3.2.2.6}$$

$$K_{t+1}^{ph} : \Lambda_t = \beta E_t \Lambda_{t+1} \left[(1 - \tau_{t+1}^k) r_{t+1} + (1 - \delta^p) \right] \tag{3.2.2.7}$$

$$N_t^{ph} : \frac{\psi_2}{L_t} = \Lambda_t (1 - \tau_t^l) w_t^p \tag{3.2.2.8}$$

$$N_t^{gh} : \frac{\psi_2}{L_t} \left[\frac{RS_t^h}{RS_t} + 2\gamma N_t^{gh} \right] = \Lambda_t (1 - \tau_t^l) w_t^g \frac{RS_t^h}{RS_t} \tag{3.2.2.9}$$

$$RS_t^h : \frac{\psi_2}{L_t} \left[1 + \frac{N_t^{gh}}{RS_t} \right] = \Lambda_t (1 - \tau_t^l) w_t^g \frac{N_t^{gh}}{RS_t} \tag{3.2.2.10}$$

$$\lim_{t \to \infty} \beta^t \Lambda_t K_{t+1}^{ph} = 0, \tag{3.2.2.11}$$

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Next, the Euler equations describes the optimal capital and bond accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two neighboring periods. Private hours are chosen so that the disutility of an hour work in the private sector at the margin equals the after-tax return to labor. The disutility of an hour of rent-seeking time equals the marginal increase in after-tax public sector labor income. At the margin, the benefit of engaging in rent-seeking equals the utility cost of doing so. The last expressions, (3.2.1.11), is the so-called "transversality condition" (TVC), imposed to ensure that the value of the private physical capital that remains after the optimization horizon is zero.

[24] Detailed derivation of the household's optimality condition is provided in Appendix 3.11.1.

This boundary conditions guarantees that the model equilibrium is well-defined by ruling out explosive solution paths.

Divide (3.2.2.9) by (3.2.2.8), and impose symmetry (hence $RS_t^h = RS_t$, $N_t^{gh} = N_t^g$) to obtain

$$1 + 2\gamma N_t^g = \frac{w_t^g}{w_t^p}. \tag{3.2.2.12}$$

Eq. (3.2.2.12) is a typical labor supply relationship, and characterized in this framework by a positive relationship between total public hours and the public/private wage ratio.[25] Next, divide (3.2.2.10) by (3.2.2.8) to obtain

$$\frac{w_t^g N_t^{gh}}{w_t^p RS_t} = 1 + \frac{N_t^{gh}}{RS_t}. \tag{3.2.2.13}$$

After some rearrangement, and by imposing symmetry once again, it can be shown that

$$RS_t = \left[\frac{w_t^g}{w_t^p} - 1\right] N_t^g. \tag{3.2.2.14}$$

Optimality condition (3.2.2.14) is new in the literature on rent-seeking. As seen from above, rent-seeking time is a product of public employment, N_t^g, and the net wage premium, $\frac{w_t^g}{w_t^p} - 1$. In other words, the corruption problem in this framework could be split into two parts: the high public employment ("extensive margin"), and the high public wage premium ("intensive margin").[26] Therefore, Eq. (3.2.2.14) suggests that cuts in the public wage bill are important for curbing the size of the contestable prize and thus effectively restraining the rent-seeking behavior of government bureaucrats.[27]

3.2.3 Firms

There is also a representative private firm. It produces a homogeneous final product using a production function that requires physical capital, K_t and labor hours N_t^p. The production

[25] Alternatively, the equation can be interpreted as a "wage curve" equation, similar to the one described in Blanchflower and Oswald (1996).

[26] Hence $RS_t = 0$ when $w_t^p = w_t^g$, and/or $N_t^g = 0$.

[27] Another instance when equilibrium rent-seeking is zero, is when all households decide to play the cooperative solution.

function is as follows

$$Y_t = A_t(N_t^p)^\theta(K_t^p)^{1-\theta}, \tag{3.2.3.1}$$

where A_t measures the total factor productivity in period t; $0 < \theta, (1-\theta) < 1$ are the productivity of labor and private physical capital, respectively.

The representative firm acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^\infty$ and policy variables $\{\tau_t^k, \tau_t^l, G_t^i, G_t^T, K_t^g\}_{t=0}^\infty$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize static aggregate profit,

$$\Pi_t = A_t(N_t^p)^\theta(K_t^p)^{1-\theta} - r_t K_t^p - w_t^p N_t^p. \tag{3.2.3.2}$$

In equilibrium, capital and labor receive their marginal products, *i.e.*[28]

$$r_t = (1-\theta)\frac{Y_t}{K_t^p}, \tag{3.2.3.3}$$

$$w_t^p = \theta\frac{Y_t}{N_t^p}. \tag{3.2.3.4}$$

Hence, equilibrium per-period profits are zero.

3.2.4 Government

Government invests in capital, G_t^i, which is used in the provision of the utility-enhancing government services. In addition, government hires labor N_t^g at a wage level w_t^g to produce public consumption goods and distributes transfers $G_t^t(\equiv \sum_h G^{th})$. The production function for public consumption is as in Cavallo (2005), Linnemann (2009) and Economides *et al.* (2011):

$$S_t^g = (N_t^g)^\alpha(K_t^g)^{(1-\alpha)}, \tag{3.2.4.1}$$

where $0 < \alpha < 1$ is the share of public employment. Since the household takes the level of government services as given, in competitive equilibrium there will be externality arising

[28]Detailed derivation of the firm's optimality condition is provided in Appendix 3.11.1.

from the presence of public employment and investment in the government services production function: More hours in the public sector generate more government services (a higher level of the public good available for public consumption), which increases directly utility. In addition, holding all else equal, an increase in public employment raises welfare indirectly by increasing the after-tax public sector labor income, and hence consumption. Lastly, more hours spent in the public sector decrease the amount of leisure the household can enjoy in a certain period (and also increase rent-seeking), and thus lower welfare. The quantitative effect will be determined by the value of the curvature parameter σ_s in the household's utility function.

Total government expenditure, $G_t^T + G_t^i + w_t^g N_t^g$, is financed by levying proportional taxes on capital and labor income. Thus, the government budget constraint is as follows:

$$G_t^T + G_t^i + w_t^g N_t^g = \tau_t^k r_t K_t^p + \tau_t^l \left[w_t^p N_t^p + w_t^g N_t^g \right]. \tag{3.2.4.2}$$

Next, the law for government capital accumulation is as follows:

$$K_{t+1}^g = G_t^i + (1 - \delta^g) K_t^g, \tag{3.2.4.3}$$

where $0 < \delta^g < 1$ is the depreciation rate of public capital.

Government takes market prices $\{w_t^p, r_t\}_{t=0}^{\infty}$ and allocations $\{N_t^p, N_t^g, K_t^p\}_{t=0}^{\infty}$ as given.[29] Finally, only four of the five policy instruments, $\{\tau_t^l, \tau_t^k, w_t^g, G_t^i, G_t^T\}_{t=0}^{\infty}$, can be exogenously set. Government investment share in output, $G_t^{iy} = \frac{G_t^i}{Y_t}$, as well as the two tax rates $\{\tau_t^l, \tau_t^k\}$

[29] In the model, the assumption that the government cannot influence private sector prices in the exogenous policy case is a technical condition that allows for the DCE to be solved. Later, in the optimal policy case, the government will choose optimally all allocations, subjects to the constraints imposed by the DCE. First, by choosing the tax rates on labor and capital, the government can set the after-tax returns of the factors of production. In addition, by choosing the levels of private sector hours worked and the private physical capital, the benevolent government implicitly determines prices, using the constraint from the decentralized competitive equilibrium (DCE) system that in equilibrium private factors of production receive their marginal product.

will be fixed to their corresponding data average in all time periods; Thus, the level of government investment will react to private output. Note that public capital stock series will be residually determined from a given initial stock, and public investment sequence. Next, government transfers $\{G_t^T\}_{t=0}^{\infty}$ will be set to match the employment ratio in data. Lastly, the public wage rate will be determined residually to ensure that the government budget constraint is satisfied in every period.

In other words, the government controls the labor demand in the public sector, and facing a supply schedule for labor services in the government sector, sets the price of labor to clear the market. Despite the market-clearing property in this market, however, the situation is one of imperfect competition, as the price of labor is decoupled from the marginal productivity in the public sector and, rather, determined by budgetary considerations. In a sense, public sector labor markets will operate inside the production possibilities frontier (as the cost of labor in the public sector exceeds its marginal revenue product).

3.2.5 Stochastic processes for the policy variables

Total factor productivity, A_t, will be assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1-\rho^a)\ln A_0 + \rho^a \ln A_t + \epsilon_{t+1}^a, \tag{3.2.5.1}$$

where $A_0 = A > 0$ is steady-state level value of the total factor productivity process, $0 < \rho^a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in total factor productivity process.

3.2.6 Symmetric Decentralized Competitive Equilibrium

Given the fixed values of capital and labor income tax rates, government transfers/output and government investment/output ratios $\{\tau^k, \tau^l, G^{Ty}, G^{iy}\}$, the exogenous process followed by total factor productivity, $\{A_t\}_{t=0}^{\infty}$, and initial conditions for the state variables $\{A_0, K_0^{ph}, K_0^g\}$,

a symmetric decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t^{ph}, N_t^{ph}, N_t^{gh}, RS_t^h, I_t^h, K_{t+1}^{ph}, K_{t+1}^g\}_{t=0}^{\infty}$, $\forall h$, and prices $\{r_t, w_t^p, w_t^g\}_{t=0}^{\infty}$ so that (i) all households maximize utility; (ii) firms maximize profits; (iii) the government budget constraint is satisfied in each time period, and (iv) all markets clear.[30]

3.3 Data and model calibration

The model in this last core chapter is calibrated to German data at annual frequency. The choice of the particular economy was made based on the large public employment share, as well as the significant public wage premium observed in this country. Since there is no EU-wide fiscal authority, an individual country was chosen, instead of calibrating the model for the EU Area as a whole. In addition, payment in the public sector in the model is determined not by marginal productivity of labor, but rather by factors outside the model.

The chapter follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate relevant long-run moments of the reference economy for the question investigated in this chapter.

3.3.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007 only, while the sub-period 1970-91 covers West Germany only.[31] For Germany, data on real output per capita, household consumption per capita, government transfers and population

[30]The symmetric DCE system of equations for the general case, as well as the steady-state system are presented in Appendices 3.11.2 and 3.11.3.

[31]The time period is particularly suitable for the study of public employment, and government wage bill spending; Hughes (1994), for example, argues that "[i]n the 1970s, intellectual arguments were mounted by conservative economists that government was <u>the</u> economic problem restricting economic growth and freedom."(p. 11)

was taken from the World Development Indicators (WDI) database. The Organization for Economic Co-operation and Development (OECD) statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Investment and capital stock series were obtained from the EU Klems database (2009). The German average annual real public compensation per employee was estimated by dividing the real government wage bill (OECD 2011) by the number of public employees.

3.3.2 Calibrating model parameters to the German data

In the German data, the average public/private employment ratio over the period 1970-2007 is $n^g/n^p = 0.17$, and the average public/private wage ratio is $w^g/w^p = 1.20$. Next, the average effective tax rates on labor and physical capital, obtained from McDaniel's (2009) dataset are $\tau^l = 0.409$ and $\tau^k = 0.16$, respectively. McDaniel's approach was preferred to the one used by Mendoza et al. (1984) and the subsequent updates, *e.g.* Martinez-Mongay (2000), Carey and Tchilinguirian (2000) and Carey and Rabesona (2002), due to its more careful treatment of property and import taxes in the former. The labor share, $\theta = 0.71$, was computed as the average ratio of compensation of employees in total output. Alternatively, average capital share, $1-\theta = 0.29$, was obtained as the mean ratio of gross capital compensation in output from the EU Klems Database (2009). The private capital depreciation rate was found to be $\delta^p = 0.082$, while public capital depreciation rate was $\delta^g = 0.037$ over the period.

The discount rate $\beta = 0.979$ was calibrated from the steady-state consumption Euler equation to match the average private capital-to-output ratio in data. Next, parameter $\alpha = 0.62$, which measures the weight on public sector hours in the public good production was obtained as the average ratio of public sector wage bill to total government expenditure less transfers and subsidies, as in Cavallo (2005) and Linnemann (2009). The value is consistent with OECD (1982) estimates for the period 1960-78, which was obtained from a log-linear regression estimation. Additionally, the calibrated value of public capital elasticity, $1 - \alpha = 0.38$,

is consistent with the government capital effect estimated in Aschauer (1989) and Hjerppe *et al.* (2006).

As in Cavallo (2005) and Linnemann (2009), a logarithmic specification is chosen for the utility of private consumption, namely $\sigma_c = 1$.[32] This follows Merz (1995) and Gomes (2012), who argue that workers from the private and public sector are able to pool their resources together, and thus achieve complete insurance.[33] Similarly, as in Gali (2008), Cavallo (2009), and Gomes (2012), the inverse of the Frisch elasticity of labor supply is assumed to be approximately unity, *i.e.*, $\sigma_l = 1$. In addition, the logarithmic form for the utility of leisure has empirical support, *e.g.* Asch and Heaton (2008) and Falsch (2008), who find that public labor supply elasticity in two representative sectors, secondary education and defense, does not differ significantly from unity. The logarithmic specification for private consumption and leisure may appear restrictive at first sight, but it assists greatly in matching hours across sectors, which is the dimension of interest in this chapter.[34]

Next, as in Chatterjee and Ghosh (2009), the curvature on government services utility component was set to $\sigma_s = 0.95$ to reflect the "degree of relative congestibility associated with the utility benefits derived from the public goods." Alternatively, $1/\sigma_s$ measures the intertemporal elasticity of substitution of government services, or how responsive is the median household (voter) to growth in public services with respect to the changes in the median household's income.[35] Given that government services are modeled as a non-market output, and the normalization of private consumption good to unity, total income is a good proxy for the willingness of pay, as it represents the tax base, on which the government levies

[32] Hansen and Singleton (1983) obtain this value from their econometric estimation for the US as well.

[33] A similar claim is made in Blundell *et al.* (2012) in the context of family labor supply and consumption smoothing.

[34] In addition, allowing for a general CRRA representation of the utility of consumption and leisure leads to non-convergence in the optimal policy framework.

[35] Since all households are the same, the median household is the same as the average, or the representative one in the model.

taxes to finance the provision of public services. The value for σ_s used in the calibration is in line with the findings in Falvey and Gemmel (1996), who estimate the elasticity $\frac{1}{\sigma_s}$ to fall in the $[1.03, 1.07]$ interval for general government services (*i.e.* $\sigma_s \in [0.93, 0.97]$), and Gibson (1980), who estimates that the income elasticities for public services, such as social care, education, pollution control, parks and recreational areas, as well as highway construction and maintenance, are slightly higher than unity. In the exogenous policy setup, parameter σ_s does not affect allocations (but it affects the level of utility), since the household ignores the externality. Thus, the level of government services will be residually determined given the steady-state public employment and government capital stock.[36]

The average steady-state total hours of work in data as a share of total hours available is $n = 0.296$, hence total employment in the model is consistent with the estimates in Ghez and Becker (1975) and Juster and Stafford (1991) of the fraction of time spent working. Together with the public/private employment ratio, this yields the model-consistent steady-state values for private and public hours, $n^p = 0.253$ and $n^g = 0.043$, respectively. Steady-state public hours in the model are set to the value in data. The weight on utility from government services was set to $\psi_3 = 0.15$, which is consistent with the value used in Finn (1994). Next, the weights attached to private consumption $\psi_1 = 0.35$ and $\psi_2 = 0.50$ are set to match exactly both types of hours in data.[37] Note that ψ_1 is set larger than the average time spent working, as suggested in Kydland (1995), due to the presence of government work transaction cost, and the existence of rent-seeking in the public sector. On the other hand, the model is roughly consistent with Bouakez and Rebel (2007), Leeper *et al.* (2009), and Conesa *et al.* (2009), who argue that private consumption good is on average twice as valuable as government services, as $\psi_1/\psi_3 = 2.33$, and leisure is twice as valuable

[36]In the optimal policy case, the congestibility condition for public services, $\sigma_s < 1$, turns out to be a necessary and sufficient to produce a decrease in steady-state decrease in the rent-seeking time relative to the exogenous policy case.

[37]Observe that given the pre-set value for ψ_3 and the fact that $\psi_1 + \psi_2 + \psi_3 = 1$, by setting ψ_1, ψ_2 will be residually determined.

as the private consumption good, as $\psi_1/\psi_1 = 1.43$. The scale parameter of the transaction cost associated with government work, $\gamma = 2.318$, is calibrated to match the average public/private wage ratio in the data.

Total factor productivity moments, $\rho^a = 0.943$ and $\sigma^a = 0.013$, were obtained in several steps: First, using the model's aggregate production function specification and data series for physical capital and labor, Solow residuals (SR) were computed in the following way:

$$\ln SR_t = \ln y_t - (1-\alpha)\ln k_t^p - \alpha \ln n_t^p. \tag{3.3.2.1}$$

The logged series were then regressed on a linear trend ($b > 0$) to obtain

$$\ln SR_t = bt + \epsilon_t^{SR}. \tag{3.3.2.2}$$

Observe that the residuals from the regression above,

$$\epsilon_t^{SR} = \ln SR_t - bt \equiv \ln a_t, \tag{3.3.2.3}$$

represent the stationary, or detrended, component of the logged TFP series.

Next, the AR(1) regression

$$\ln a_t = \beta_0 + \beta_1 \ln a_{t-1} + \epsilon_t^a \tag{3.3.2.4}$$

was run using ordinary least squares (OLS) to produce the estimates (denoted by the "hat" symbol) for the persistence and standard deviation parameters of the total factor productivity process to be used in the calibration of the model. In particular,

$$\hat{\beta}_1 = \rho^a \tag{3.3.2.5}$$

$$\hat{\epsilon}_t^a \sim N(0, \sigma_a^2). \tag{3.3.2.6}$$

Table 3.1 on the next page summarizes all model parameters used in the calibration.

Table 3.1: Model Parameters

Param.	Value	Definition	Source
β	0.979	Discount factor	Calibrated
θ	0.710	Labor income share	Data average
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on public capital	Data average
ψ_1	0.350	Weight on consumption in utility	Calibrated
ψ_2	0.500	Weight on leisure in utility	Calibrated
ψ_3	0.150	Weight on government services in utility	Set
σ_c	1.000	Curvature parameter of private consumption utility	Set
σ_l	1.000	Curvature parameter of leisure utility	Set
σ_s	0.950	Curvature parameter of the government services utility	Set
γ	2.318	Scale parameter of government work transaction cost	Calibrated
α	0.620	Labor share in public services production	Data average
$1-\alpha$	0.380	Capital share in public services production	Calibrated
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
G^{iy}	0.023	Government investment-to-output ratio	Data average
G^{Ty}	0.228	Government transfers-to-output ratio	Calibrated
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated

3.4 Steady state results

Given the model parameters, the unique steady-state of the system was computed numerically for the Germany-calibrated model. Results are reported in Table 3.2 below, where $\bar{r} = (1 - \tau^k)(r - \delta^p)$ denotes the after-tax net of depreciation real return to private capital.

Table 3.2: Data averages and long-run solution

	Description	GE Data	Model
c/y	Consumption-to-output ratio	0.590	0.784
i/y	Private investment-to-output ratio	0.210	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225
g^T/y	Public transfers-to-output	0.170	0.228
$w^p n^p/y$	Private labor share in output	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145
rk/y	Private capital share in output	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200
w^p	Private sector wage rate	-	1.006
w^g	Public sector wage rate	-	1.207
$\tilde{w}^p$	After-tax private wage rate	-	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714
n	Total employment	0.296	0.296
n^p	Private employment level	0.253	0.253
n^g	Public employment level	0.043	0.043
rs	Rent-seeking level	-	0.009
n^g/n^p	Public/private employment ratio	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035

The model performs relatively well *vis-a-vis* data. It slightly overestimates average consumption and underestimates the investment shares in output. This mismatch is due to the fact that the model treats government wage bill consumption as a transfer payment, and not as final public consumption, as is the case in the national accounts. This is not an issue here as the main objective of the model is to replicate the stylized facts in the labor markets. However, the model accurately captures the long-run after-tax capital return, where the latter is proxied by the average return on 10-year generic bonds net of CPI inflation. Moreover, the imputed government services is also predicted to make a significant share of output.

Along the labor market dimension, the model was calibrated to match the average time spent working, and the wage and employment ratios in data. Given the focus on the effects of rent-seeking in the public sector, the framework was calibrated to reproduce those stylized facts in the steady state, as this framework will provide an important benchmark for the measures use to quantify the loss from rent-seeking activities on the economy. Next, the ratio of time spent rent-seeking to public employment is a non-trivial figure in steady-state: Using $p \equiv \frac{w^g}{w^p}$ to denote the steady-state wage ratio, one can obtain $\frac{rs}{n^g} = \frac{(p-1)n^g}{n^g} = p - 1 = 0.20$. This value is consistent with the results obtained in Angelopoulos *et al.* (2009), who found that 18 % of working time in Germany is spent rent-seeking. Thus, the non-productive rent-seeking in the public sector is also likely to generate a significant waste on aggregate level.[38]

3.5 Long-Run Cost of Rent-seeking

The model in this chapter naturally suggests estimates of rent-seeking time. It also provides estimates that aim to quantify the loss from rent-seeking in terms of output. In turn, given the calibration objective in this chapter to match hours in each sector, the values for

[38]Note that given the structure of the problem, and the symmetry imposed, a first-best solution is to set rent-seeking to zero, as this results in a higher welfare. This is investigated Appendix 3.11.4. The other special case, when the government sets equal wages in the two sectors, is also presented in Appendix 3.11.4.

other EU countries can be easily obtained from data averages after some transformations. Given the calibrated values for different countries in EU, a ranking can be constructed for different countries. Finally, the model-based estimates are compared to empirical measures of institutional quality. One such index is the compound International Country Risk Guide (ICRG), from which the values for a selected set of European countries were obtained by Angelopoulos *et al.* (2009).[39] Additionally, a second set of indicators, the Worldwide Governance Indicators (WGI) were extracted from the WDI database. A detailed description of the indices used in this chapter is provided in Appendix 3.11.5. The chosen indices reflected government size, control of corruption, expenditure effectiveness of public funds, government effectiveness, and the efficiency of public administration.[40]

The first measures to be used in the comparison with indices is the steady-state rent-seeking time itself, which was computed as:

$$rs = (p-1)n^g. \tag{3.5.0.7}$$

Second, rent-seeking time is also expressed in relative terms as a share of public hours[41] to obtain:

$$\frac{rs}{n^g} = \frac{(p-1)n^g}{n^g} = p-1. \tag{3.5.0.8}$$

Next, several estimates of the loss imposed on the economy, in terms of output, were also calculated. The first such expression is named "wasteful lobbying cost," as it represents the opportunity cost of using time to engage in rent-seeking activities, which is not directly productive (but only indirectly increases the probability of winning the contestable prize, and/or increases the labor income from government work), instead of using the time to

[39]For more detailed discussion of this index, interested readers should consult Knack and Keefer (1995)

[40]World Values Studies compute a "general trust" measure, which is also highly correlated with indices of corruption and institutional quality (La Porta *et al.* 1997) and where the measure aims to capture the level of social trust and confidence in the government.

[41]Since rent-seeking occurs only in the public sector, it does not make much sense to express it relative to the total labor supply.

produce public services, which have utility-generating effect. The analytical representation of this cost is as follows:

$$\frac{w^g rs}{y} = \frac{pw^p(p-1)n^g}{y} = \frac{(p-1)pn^g}{n^p}\frac{w^p n^p}{y} = \theta(p-1)p\frac{n^g}{n^p}. \tag{3.5.0.9}$$

Furthermore, the value of the contestable transfer, the government wage bill, could also be regarded as a wasteful expenditure. This is because public sector wage and employment are determined in a non-competitive market, and public consumption is valued much less than the private consumption good. In other words, the wage payed to government employees is unrelated to productivity of labor in the government services production function. Moreover, the share of government wages in output can also be represented as a product of "primitives" as follows:

$$\frac{w^g n^g}{y} = \frac{pw^p n^p}{y}\frac{n^g}{n^p} = p\theta\frac{n^g}{n^p}. \tag{3.5.0.10}$$

Therefore, the total waste in the economy is the sum of the lobbying cost and the wage bill share. Given that government employees are not entirely wasteful, the combined measure presented below could be regarded as the upward bound of the total loss in the economy from rent-seeking, expressed relative to output. The analytical representation obtained is as follows:

$$\frac{w^g rs}{y} + \frac{w^g n^g}{y} = \theta(p-1)p\frac{n^g}{n^p} + p\theta\frac{n^g}{n^p} = \theta p^2\frac{n^g}{n^p}. \tag{3.5.0.11}$$

As seen from above, in the long run, the cost of rent seeking as a share in output depends only on the private labor share in output, the gross public wage mark-up p (*i.e.* the average public/private wage ratio), as well as the average public/private employment ratio. Thus, the model predicts that countries with a high labor share in aggregate production function, high public employment share in total, and high wages in the public relative to the private sector, will feature the highest losses.

Since the model was constructed to match those dimensions in data, estimates of the measures above were easily computed from OECD (2011) data for a cross-section of EU countries,

without explicitly calibrating the model for all the countries, but rather by simply computing the required averages for the corresponding country from OECD data directly. Following Angelopoulos *et al.* (2009, 2011), all measures are presented and ranked in Table 3 on the next page, together with the ICRG index first. A lower rent-seeking cost corresponds to a higher ranking. A higher value of the ICRG index reflects better institutions, and a higher ranking for the country.[42]

Table 3.3: Rent-seeking results in EU member countries

Country	rs	p	n^g/n^p	rs/n^g	$\frac{w^g rs}{y}$	$\frac{w^g n^g}{y}$	$\frac{w^g(n^g+rs)}{y}$	ICRG
Austria	0.016(5)	1.28(5)	0.207(4)	0.28(5)	0.050(5)	0.180(5)	0.23(5)	47.22(5)
Belgium	0.021(7)	1.28(6)	0.285(9)	0.28(6)	0.066(7)	0.231(10)	0.30(7)	47.46(4)
Finland	0.003(2)	1.03(2)	0.353(11)	0.03(2)	0.008(2)	0.226(9)	0.23(6)	48.76(3)
France	0.001(1)	1.01(1)	0.320(10)	0.01(1)	0.002(1)	0.204(7)	0.21(3)	46.62(6)
Germany	0.010(3)	1.20(3)	0.170(2)	0.20(3)	0.029(3)	0.145(1)	0.17(1)	48.92(2)
Greece	0.038(11)	1.41(9)	0.260(7)	0.41(9)	0.090(9)	0.220(8)	0.31(11)	34.36(11)
Ireland	0.015(4)	1.22(4)	0.236(6)	0.22(4)	0.036(4)	0.169(2)	0.21(2)	44.37(7)
Italy	0.025(8)	1.30(7)	0.266(8)	0.30(7)	0.070(8)	0.232(11)	0.30(10)	40.90(8)
Netherl.	0.028(9)	1.69(11)	0.166(1)	0.69(11)	0.118(11)	0.171(3)	0.29(8)	49.40(1)
Portugal	0.020(6)	1.30(8)	0.217(5)	0.30(8)	0.052(6)	0.175(4)	0.23(4)	40.13(10)
Spain	0.034(10)	1.60(10)	0.195(3)	0.60(10)	0.112(10)	0.187(6)	0.30(9)	40.40(9)

Results in Table 3.3 above show that the cost of lobbying is 2.9 % of GDP for Germany, but can reach 9 % of GDP in Greece, 11.8 % in the Netherlands, and 11.32 % in Spain. The magnitude of these is in line with Magee *et al.* (1989), who show in a static model that 5-15 % of an economy's capital and labor is lost in predatory lobbying. Next, when the share of the public wage bill is added, the costs rise significantly. Germany is still the

[42]Overall, countries with larger shares of the government wage bill in output also feature higher tax rates. However, since this chapter focuses on the relationship between rent-seeking and the government wage bill, and not on the effect of rent seeking on tax revenues, this stylized fact in data is not discussed.

leader with the lowest loss (17 %), while Greece features the highest figure (31 %), followed immediately by Belgium (30 %), Italy (30 %), and Spain (30 %). These values are also comparable with earlier studies, *e.g.*, using a static framework, Mohammad and Whalley (1994) compute redistributive activity costs to be 25-40 % of Indian GNP, while Ross (1984) calculates it as 38 % of Kenyan GNP.[43] Lastly, the size of the rent-seeking cost in terms of output is comparable to the estimates in Angelopoulos *et al.* (2009), who use a DSGE framework with a rent-seeking extraction of the government tax revenue to calculate the cost to be in the range of 0-16 % of GDP across the EU-12 countries.

Next, as documented in Table 3.4 on the next page, rent-seeking estimates and loss measures are found to be moderately- to highly-correlated to other indices of institutional quality. As expected, rent-seeking time in steady-state is very strongly negatively related to the indices of bureaucratic efficiency, where the values range between -0.50 and -0.73. The public wage premium is also moderately negatively related to institutional quality. This could be an indicator that public sector wages are indeed determined within a political economy environment. Lastly, the public/private employment ratio is essentially uncorrelated with the index values. The two loss measures, the lobbying cost and government wage bill as shares in output, are moderately to strongly negatively correlated with different indicators of bureaucratic efficiency.[44]

[43]These studies, however, focus on bureaucrats whose rent-seeking activity is tariff revenue extraction.

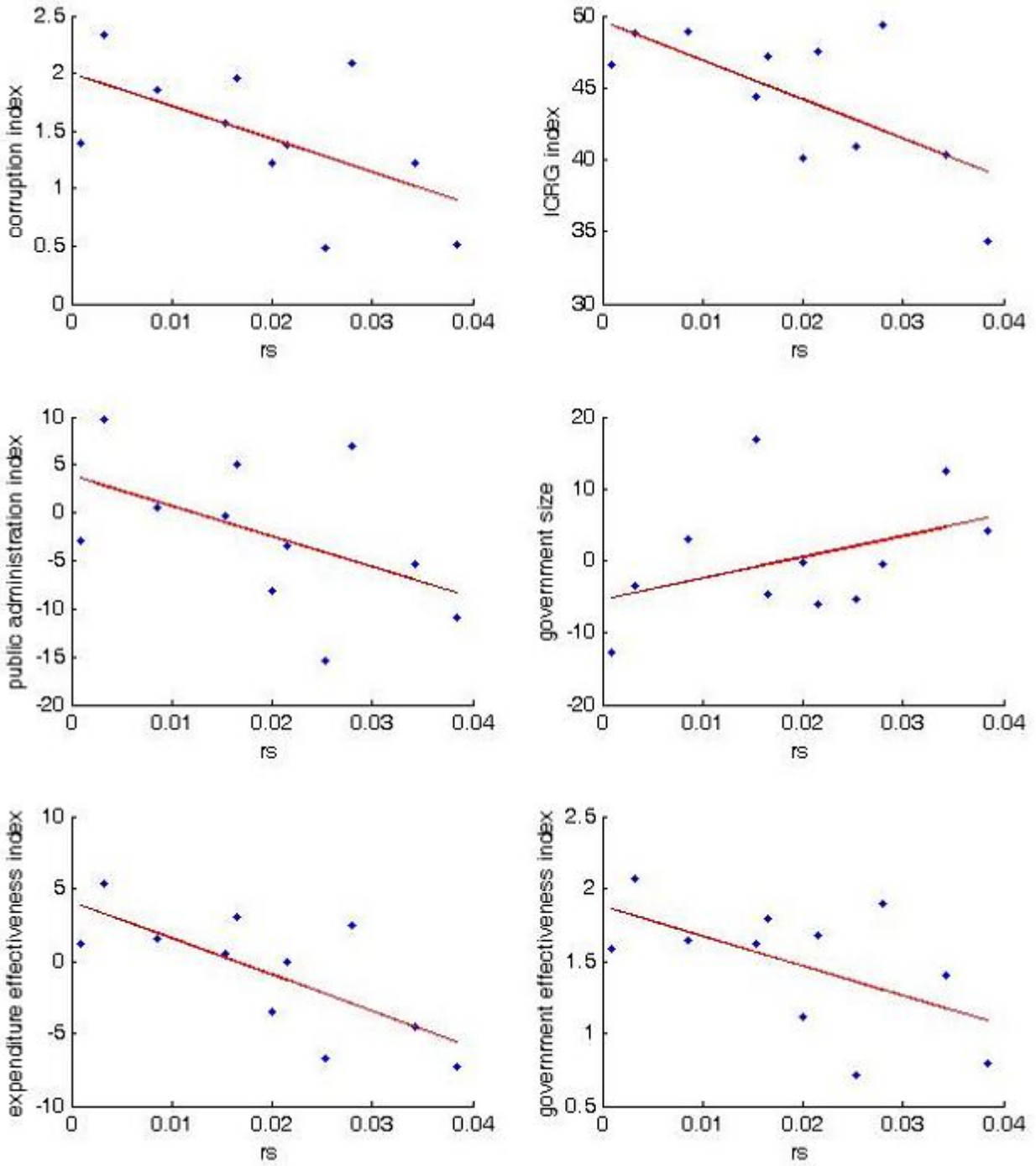
[44]Interestingly, the two loss measures produce the same correlations with the indices. This effect, however, is an almost direct consequence of how the two measures were constructed, as the lobbying cost is proportional to the government wage bill, with the coefficient of proportionality equal to $(p-1)$.

Table 3.4: Correlation Matrix

Index	rs	p	n^g/n^p	rs/n^g	$\frac{w^g rs}{y}$	$\frac{w^g n^g}{y}$	$\frac{w^g(n^g+rs)}{y}$
ICRG index	-0.68	-0.27	0.01	-0.27	-0.39	-0.39	-0.39
control of corruption index	-0.58	-0.16	-0.09	-0.16	-0.32	-0.32	-0.32
public administration index	-0.50	-0.12	-0.02	-0.12	-0.26	-0.26	-0.26
expenditure effectiveness index	-0.73	-0.37	0.12	-0.37	-0.50	-0.50	-0.50
government effectiveness index	-0.57	-0.19	0.04	-0.19	-0.31	-0.31	-0.31

In the next step in the analysis, rent-seeking time was plotted against the indices. As seen from Fig. 3.1 on the next page, this generated a good fit in a cross-section of EU countries. There is a clear negative relationship with the indices of institutional quality, and a positive one with the size of government. In the next section, the behavior of the model outside of the steady-state is investigated. In particular, the transitional dynamics of the model economy and the responses of the variables in the face of a surprise technological innovation are presented and discussed.

Figure 3.1: Rent-seeking time *vs.* indices of institutional quality



3.6 Model solution and impulse responses

Since there is no closed-form general solution for the model in this chapter, a typical approach followed in the RBC literature is to log-linearize the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The log-linearized system of model equations is derived and summarized in Appendices 3.11.6 and 3.11.7. The linearized DCE system can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988, 1999):

$$\mathbf{A}E_t\hat{\mathbf{x}}_{\mathbf{t+1}} \quad = \quad \mathbf{B}\hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C}E_t\varepsilon_{\mathbf{t+1}}, \tag{3.6.0.12}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{rs}_t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model. Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to productivity.

3.6.1 The Effect of a positive productivity shock

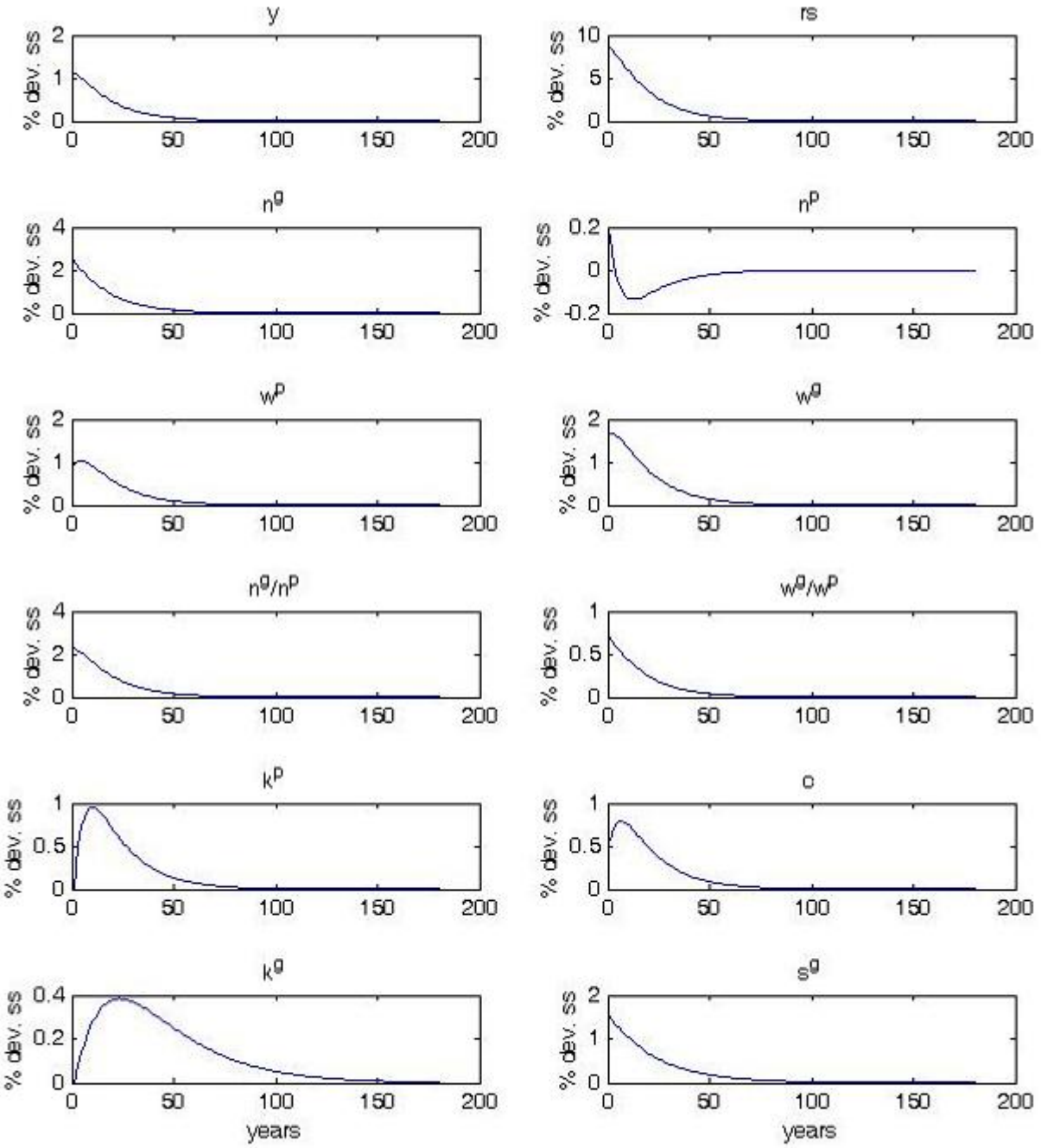
Figure 3.2 shows the impact of a 1% surprise TFP innovation on the model economy. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as for investment. From the rule for the government investment in levels, a higher output translates into a higher level of expenditure in that category (not pictured, identical to output response). Next, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not illustrated) and the private wage rate increase. The household responds to the price signals and supplies more hours in the private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private

consumption and leisure. In terms of the labor-leisure trade-off, the income effect ("work more") produced by the increase in the private wage dominates the substitution effect ("work less"). Furthermore, the increase of private hours expands output further, thus both output and government spending categories increase slightly more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, the marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

With regard to public sector labor dynamics, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. Higher income and consumption lead to greater tax revenue. In particular, the growth in government revenue exceeds the increase in the fiscal spending instruments. As a result, the additional funds available are spent on government investment and the wage bill. In turn, the increase in the latter leads to an expansion in both public sector wages and hours. In addition, the model in this chapter generates an interesting dynamic in the wage and hours ratio, which is not present in models with stochastic public employment, such as Finn (1998), Cavallo (2005) or Linnemann (2009). The two wage rates, as well as the two types of hours move together, but less than perfectly so, thus making the model consistent with the empirical evidence presented in Lamo, Perez and Schuknecht (2007, 2008). In addition, as in the data, public sector labor variables react more strongly to positive technological innovations than do their private sector counterparts.

Given that both public wages and hours react strongly and positively to technological improvements, the new variable in the model, rent-seeking time also increases. The intuition behind this result is that during unexpectedly good times, tax revenues are larger than usual, increasing the amount of funds "up for grabs," which are expropriated by government bureaucrats in the form of excessive salaries to government bureaucrats.

Figure 3.2: Impulse Responses to a positive 1% productivity shock in Germany



Rent-seeking time in the model thus responds very strongly to the dynamics of output, as it is related to the tax base in the labor income generated in the economy.

Overall, a positive innovation to total factor productivity has a positive effect on the allocations and prices in the economy. The novelty is that the endogenous public sector hours model generates an important difference in the composition of household's labor income with the public sector share increasing at a much faster rate than the private sector labor income. Another important observation to make is that the TFP shocks, being the main driving force in the model, induce pro-cyclical behavior in public wages and hours. The shock effects are smaller and variables reach their peak very quickly. This means that the impulse effect dies out relatively fast. However, the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks on the wage- and hours ratios.

However, an important limitation of the exogenous policy analysis performed so far is that both tax rates were taken to be fixed. In addition, government investment share was exogenously set, and public wage rate was a residually-determined instrument that always adjusted accordingly to balance the budget. In effect, by construction all interaction between the two tax rates was precluded, by fixing each to the corresponding average effective rate in data over the chosen period of study. These restrictions will be lifted in the next section, and the optimal fiscal policy framework will be considered in an environment, in which the two tax rates, government investment, public employment (and hence also government services as well), public sector wage rate, and thus effectively the optimal rent-seeking time, are chosen jointly by a benevolent government, whose preferences are perfectly aligned with the household's utility function.

3.7 The Ramsey problem (Optimal fiscal policy under full commitment)

In this section, the government assumes the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interest, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital (and hence public investment), public employment and public sector wage rate (thus the Ramsey planner effectively determines rent-seeking time). Government transfers are held fixed at the level from the exogenous policy case. It is assumed that only linear taxes are allowed, and that the government can credibly commit to those. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus can be taken as a better approximation of the environment in which policymakers decide on a particular fiscal policy.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of as generating a different competitive equilibrium allocation, *i.e.* allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the chapter, is that in the Ramsey framework, the government chooses all fiscal instruments, instead of taking them as being exogenous. At the same time, the government also selects optimally the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[45] It is also assumed that the government discounts time at the same rate as the households, and treats each household the same. The constraints which the government takes into account when maximizing household's welfare include the

[45]In contrast, the primal approach all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

government budget constraints, and the behavioral responses of both the household, and the firm. These are summarized in the symmetric DCE of the exogenous fiscal policy case.[46,47] In other words, in the dual approach of Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[48,49] The initial conditions for the state variable $\{A_0, K_0^p, K_0^g\}$, as well as the sequence of government transfers $\{G_t^t\}_{t=0}^{\infty}$ and the process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$ are taken as given.

Following the procedure in Chamley (1986) and Sargent and Ljungqvist (2004), the Ramsey problem will be transformed and simplified, so that the government chooses the after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1 - \tau_t^k) r_t, \tag{3.7.0.1}$$

$$\tilde{w}_t^p \equiv (1 - \tau_t^l) w_t^p, \tag{3.7.0.2}$$

$$\tilde{w}_t^g \equiv (1 - \tau_t^l) w_t^g. \tag{3.7.0.3}$$

Thus, the transformed government budget constraint becomes

$$A_t (N_t^p)^{\theta} (K_t^p)^{1-\theta} - \tilde{r}_t K_t^p - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1 - \delta^g) K_t^g + G_t^T. \tag{3.7.0.4}$$

[46]The DCE system is summarized in Appendix 3.11.5.

[47]Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[48]Note that by choosing next-period public capital, the planner is choosing public investment $\{G_t^i\}_{t=0}^{\infty}$ optimally. Similarly, by choosing public employment and the wage ratio optimally, the government chooses rent-seeking time $\{RS_t\}_{t=0}^{\infty}$ optimally as well.

[49]In reality government often acts as a "Stackelberg leader." Such an assumption could change the results in important ways, as it matters whether the government chooses sequentially, or whether the government chooses once and for all. Here the focus is on the latter case: it is assumed that there is a perfect commitment device used by the government, and no (profitable) deviations from the pre-announced plan are possible. The time-consistent case, or the political competitive equilibrium, is left for future research: The interested reader should consult Marcet and Marimon's (1992, 1999) work on recursive contracts, which is closely related to such a setup.

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed symmetric Ramsey problem (rent-seeking is substituted out) then becomes:

$$\max_{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, \tilde{w}_t^p, \tilde{w}_t^g, \tilde{r}_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \ln C_t$$
$$+\psi_2 \ln \left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \gamma (N_t^g)^2 \right]$$
$$+ \frac{\psi_3}{1-\sigma_s} \left[(N_t^g)^{\alpha} (K_t^g)^{1-\alpha} \right]^{(1-\sigma_s)} \Bigg\} \tag{3.7.0.5}$$

s.t

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left[1 - \delta^p + (1 - \tau_{t+1}^k)(1-\theta) \frac{Y_{t+1}}{K_{t+1}^p} \right] \tag{3.7.0.6}$$

$$\psi_2 C_t = \psi_1 \left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \gamma (N_t^g)^2 \right] (1 - \tau_t^l) w_t^p \tag{3.7.0.7}$$

$$\psi_2 C_t [1 + 2\gamma N_t^g] = \psi_1 \left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \gamma (N_t^g)^2 \right] (1 - \tau_t^l) w_t^g \tag{3.7.0.8}$$

$$RS_t = N_t^g \left[(\tilde{w}_t^g/\tilde{w}_t^p) - 1 \right] \tag{3.7.0.9}$$

$$A_t (N_t^p)^{\theta} K_t^{(1-\theta)} = C_t + K_{t+1}^g - (1-\delta^g) K_t^g + K_{t+1}^p - (1-\delta^p) K_t^p \tag{3.7.0.10}$$

$$A_t (N_t^p)^{\theta} (K_t^p)^{1-\theta} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g) K_t^g + G_t^T \tag{3.7.0.11}$$

$$K_{t+1}^p = I_t + (1-\delta^p) K_t^p \tag{3.7.0.12}$$

$$r_t = (1-\theta) \frac{Y_t}{K_t^p} \tag{3.7.0.13}$$

$$w_t^p = \theta \frac{Y_t}{N_t^p} \tag{3.7.0.14}$$

$$S_t^g = (N_t^g)^\alpha (K_t^g)^{(1-\alpha)} \tag{3.7.0.15}$$

$$K_{t+1}^g = G_t^i + (1-\delta^g)K_t^g. \tag{3.7.0.16}$$

After numerically solving for the unique steady-state, the full characterization of the long-run Ramsey equilibrium is summarized in Table 3.5 on the next page, where the same values for the parameters from the exogenous policy section (see Table 3.1) were used.

As in Lucas (1990), Cooley and Hansen (1992) and Ohanian (1997), parameter ξ is introduced to measure the consumption-equivalent long-run welfare gain of moving from the steady-state allocations in the exogenous policy case to the equilibrium values obtained under Ramsey policy. In other words, the value of ξ measures the share of steady-state consumption under the exogenous policy that the household has to be compensated with, in order to achieve the same level of utility as the one under the Ramsey policy. A fraction $\xi > 0$, which is the case reported in Table 3.5 on the next page, demonstrates that the agent is better-off under Ramsey, while $\xi < 0$ would have implied that the agent is worse-off under Ramsey.[50]

[50]However, a case with $\xi < 0$ can never occur under optimal policy.

Table 3.5: Data averages and long-run solution: exogenous *vs.* optimal policy

	Description	GE Data	Exogenous	Ramsey
c/y	Consumption-to-output ratio	0.590	0.784	0.718
i/y	Private investment-to-output ratio	0.210	0.192	0.229
g^i/y	Public investment-to-output ratio	0.023	0.023	0.053
k^p/y	Private capital-to-output ratio	2.350	2.346	2.793
k^g/y	Public capital-to-output ratio	0.630	0.630	1.442
s^g/y	Public services-to-output ratio	0.193	0.225	0.229
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.102
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.277
w^p	Private sector wage rate	-	1.006	1.080
w^g	Public sector wage rate	-	1.207	1.379
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.603
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.770
n	Total employment	0.296	0.296	0.294
n^p	Private employment level	0.253	0.253	0.264
n^g	Public employment level	0.043	0.043	0.030
rs	Rent-seeking time	-	0.009	0.008
n^g/n^p	Public/private employment ratio	0.170	0.170	0.112
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.022
τ^k	Capital income tax rate	0.160	0.160	0.000
τ^l	Labor income tax rate	0.409	0.409	0.442
U	Total discounted welfare	-	95.02	96.47
ξ	Welfare gain	-	0	0.095

There are several additional important findings in the Ramsey equilibrium that can be seen in Table 3.5 on the previous page. First, as expected, total discounted welfare is higher under the Ramsey regime.[51] Next, private consumption share is lower, while private capital- and investment shares are higher, thus the interest rate is lower. The model generates a zero steady-state optimal capital tax, and a higher labor tax rate. All these results are consistent with the findings in earlier studies, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Sargent and Ljungqvist (2004) and Kocherlakota (2010). In addition, earlier studies that use the representative-agent setup, *e.g.* Lucas (1990) and Cooley and Hansen (1992), have shown that tax reforms which abolish capital taxation, even at the expense of a higher tax burden on labor, still produce significant welfare gains for the society.

Next, due to the presence of a second labor market, as well as the endogenous public sector hours, sophisticated labor market interactions are generated. In the framework presented in this chapter, the labor market structure allows for labor flows between sectors. Furthermore, the government internalizes the public services externality in its choice. Thus, it picks the socially optimal levels of public hours and capital stock to provide the optimal level of the public consumption good. In addition, the planner chooses a different mix between the inputs used in the provision of government services: a higher level of government investment is undertaken, while fewer public hours are employed than the in the DCE solution. As a result, public investment (and thus public capital) share is more than double than that of the exogenous policy case. As a result, the amount of the public good produced relative to output is also slightly higher. In addition, public hours are substituted for private hours, keeping the total virtually unchanged.

In terms of the relative price of labor in the two labor markets, both the after-tax private and public wage rates increase slightly. The higher public/private wage ratio, and thus

[51] The positive values of utility are due to the domination of the government services term, given that $\sigma_s < 1$ (but close to unity).

the higher public wage premium in the optimal policy case overcompensate for the increase in the labor tax. Furthermore, the public/private hours ratio is lower, due to the substitution away from labor in the government sector. In other words, the increase in the public wage premium is driven by budgetary considerations, as the public wage is the residually-determined fiscal instrument that balances the per-period government budget constraint. In addition, the result is consistent with economic logic and scarcity argument: relatively fewer hours are employed in the public sector, thus the steady-state public wage rate is higher. Furthermore, optimal government wage consumption is significantly lower. In turn, optimally chosen rent-seeking level is also lower, as the public employment effect dominated the public wage premium effect.[52,53,54]

The value-added of the rent-seeking model with endogenous public hours and wages is that it generates predictions about the long-run effects of fiscal policy through labor markets, as

[52]This effect is due to the fact that public goods are congestible, or that $0 < \sigma_s < 1$. For a case when $\sigma_s \geq 1$, a case not supported by empirical data, optimal rent-seeking is higher than in the exogenous case, which is counterintuitive, and thus not considered here. In particular, a value higher than unity for σ_s results in a higher public wage premium and government investment, but dampens the negative effect on public employment.

[53]Alternatively, the rent-seeking chosen in the exogenous policy case can be interpreted as being "third-best," as households ignore the utility effect of public hours working through the government services production function, and thus the DCE choice is inferior to the second-best choice made by the benevolent Ramsey government.

[54]In the model, the transaction utility cost of working in the public sector is what generates the public sector wage premium. Obviously, a better compensation as a government employee attracts entry in the sector. In addition, since the wage rate is assumed to be paid per efficiency units of public sector hours of work, it is always in the interest of a worker in the public sector to invest in efficiency. This is done by allocating some time to rent-seeking. In equilibrium, a positive amount of rent-seeking would exist even in the optimal fiscal policy scenario, as rent-seeking is a product of public sector employment level and the public wage premium. In the presence of transaction costs, the wage premium still persists, and given that a positive number of government employees are demanded, rent-seeking is still positive in the optimal policy case. Such level of rent-seeking is second-best, as the government cannot use lump-sum instruments to achieve the first-best level of rent-seeking, which is indeed zero.

well as the level of rent-seeking in the government sector, which is in line with earlier studies. In particular, the benevolent Ramsey planner corrects two inefficiencies in the government sector, the excessive employment and the scarcity of public capital.[55] Moreover, the wage- and employment ratios, the optimal composition of the government wage bill consumption, as well as the distribution of spending across government expenditure categories were all important elements of the analysis on the optimal amount of rent-seeking activity within the public administration. The novel results obtained in this chapter were generated from the incorporation of a richer government spending side, are new and interesting for policy makers, as previous research had ignored these important dimensions.

The result that cuts in the wage bill have expansionary effect on the economy is not new to the empirical macroeconomic studies, *e.g.* Algan *et al.* (2002), Alesina (1997), Alesina *et al.* (2001), Alesina *et al.* (2002), and Giavazzi and Pagano (1990). However, the optimal public wage and employment aspects in the analysis are novel in the modern macroeconomic literature, given the predominance of setups with single wage rates, and exogenously-determined public employment. In addition, given the doubling in public investment share, the fixed level of government transfers, and the reduction of the public wage share in output, the loss of capital income tax revenue requires steady-state labor tax to increase by only 3.3% relative to the rate used in the exogenous policy case. The changes in the distribution of spending, as well as the optimal amount of rent-seeking, are new results in both the optimal policy and political economy literatures. As seen in Table 3.5, if those aspects are ignored, important public finance aspects are missed.

Finally, note that the restriction $\sigma_s < 1$ is a necessary and sufficient condition to generate lower rent-seeking under Ramsey. This value does not deal with rent-seeking theory *per se*, but rather captures an important characteristic of public goods, namely their congestible nature.[56] This plausible assumption can be viewed as a technical condition: In the general

[55]These features of the public sector are first noted in Baumol (1965).

[56]In addition, $1/\sigma_s$ is the intertemporal elasticity of substitution of government services, which in the data

case, with CRRA utility for government services, optimal public wages are higher, and optimal public employment is lower. However, only when $\sigma_s < 1$, the compositional effect on the wage bill is such that optimal rent-seeking is lower than the value in the exogenous policy case.

In the next section, the analysis is extended to the behavior of the Ramsey economy outside of the steady-state. The transitional dynamics of model variables, and rent-seeking in particular, under optimal policy setup is also analyzed. In particular, the optimal responses of the fiscal instruments and the other prices and allocations to positive shocks to TFP is presented and discussed.

3.8 Optimal reaction of fiscal policy instruments to productivity shocks

The optimal policy model is now solved using the first-order linearization procedure from Schmitt-Grohe and Uribe (2004) to study the dynamics of prices and allocations outside the steady-state.[57] The model solution is then used to study transitional behavior in response to a surprise innovation in total factor productivity. Under the optimal policy (Ramsey) regime, endogenous variables would generally behave differently to the responses to a positive technology shock under the exogenous fiscal policy case. Fig. 3.3 summarizes all responses to a 1% surprise innovation to total factor productivity. To highlight differences across regimes, Fig. 3.4 plots on the same graph both the IRFs from the exogenous policy case and the optimal ones. The new variables in the system are the five fiscal policy instruments - capital and labor taxes, as well as public investment (hence public capital), public wage rate and public employment. Note that by choosing the two wage rates, and employment in the public sector, the planner determines the optimal amount of rent-seeking time. Therefore,

is also slightly higher than unity, $1/\sigma_s \in [1.03, 1.07]$, hence $\sigma_s \in [0.93, 0.97]$.

[57] Given the absence of curvature in the model, the second-order approximation to the equilibrium system of equations did not change results significantly.

Figure 3.3: Impulse Responses to a positive 1% productivity shock under Ramsey policy

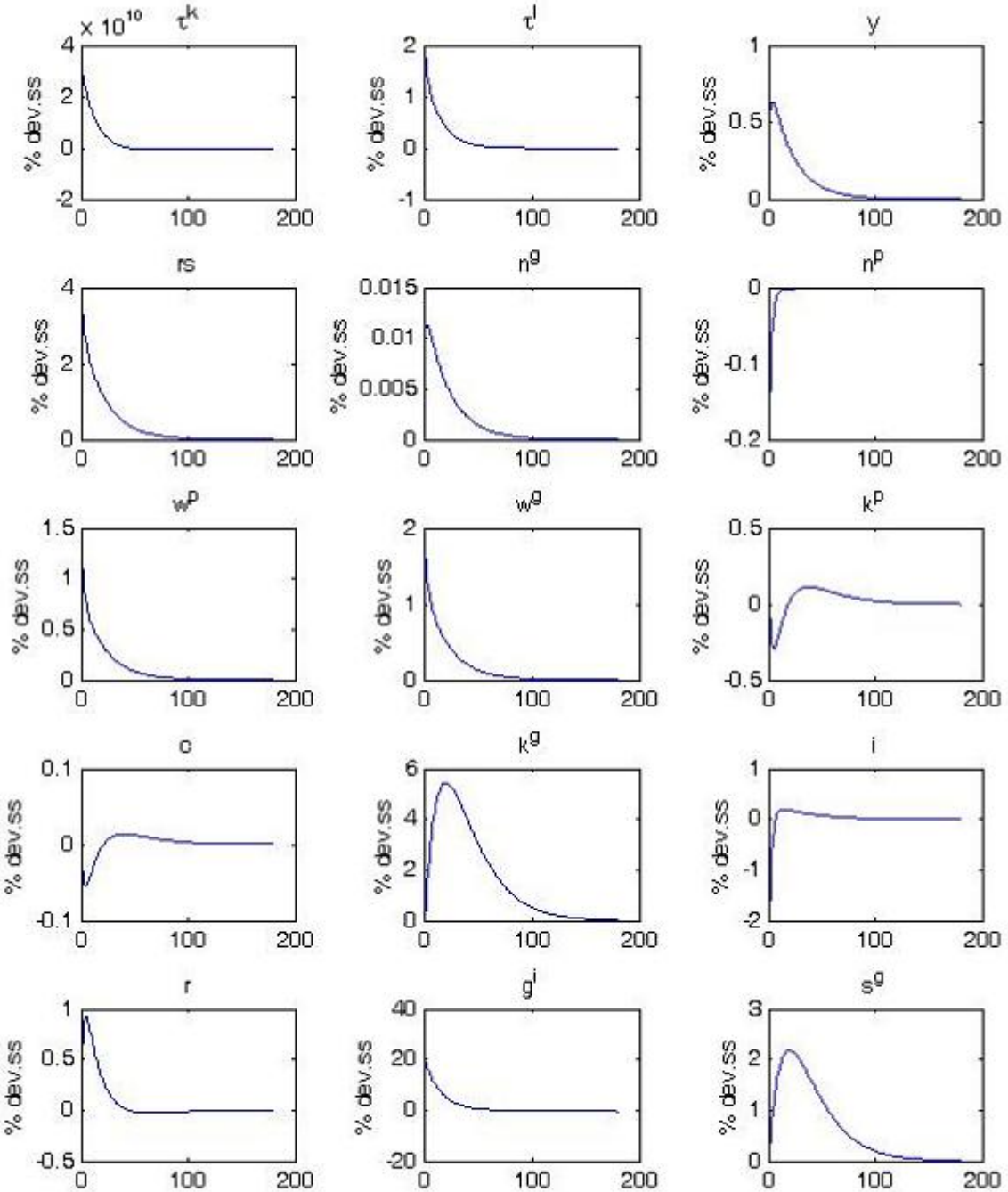
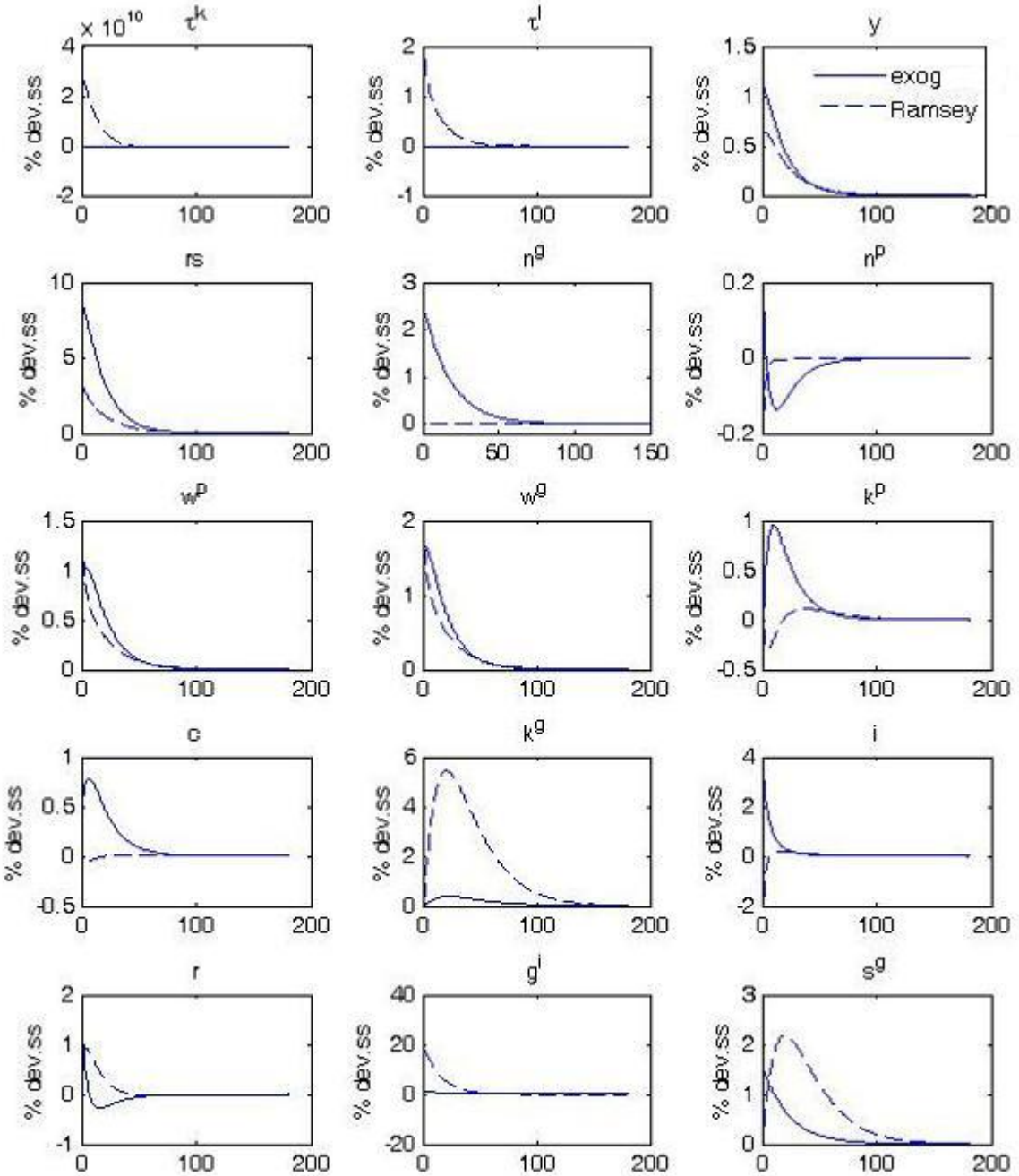


Figure 3.4: Impulse Responses to a positive 1% productivity shock under exogenous and Ramsey policy



by intervening in the public sector labor market, the benevolent government can influence the private sector labor market, and thus affect the course of the economy. In addition, the government can use the available fiscal instruments at its disposal to affect rent-seeking among bureaucrats, and thus reduce the loss due to these counter-productive activities.

In period 0, after the realization of the unexpected technology innovation, capital tax stays unchanged.[58] This result is in line with previous findings in the literature, *e.g.* Chari and Kehoe (1994, 1999) who show that in a standard RBC model capital tax rate does not respond to productivity shocks. In other words, the benevolent government would not deviate from the optimal zero steady-state capital tax rate even in the face of uncertain productivity shocks.[59] Next, labor income tax rate increases upon the impact of the positive surprise innovation in TFP and then slowly returns to its old steady-state; the substantial persistence observed in line with earlier studies (Chari and Kehoe, 1994, 1999). However, due to the richer structure of the and endogenously-determined government spending, the magnitude of the response in the labor tax is higher.

Furthermore, given that public spending categories are optimally chosen in this framework, the setup generates considerably more interaction among the variables than does the standard RBC model. For example, public investment increases substantially, as the government under the Ramsey regime chooses also public capital and government services optimally. Next, as in the exogenous policy case, public sector wages will increase more relative to the private sector wage. The higher volatility in public wages, as discussed in earlier sections, is an artifact of the presence of transaction costs from government work. The change in the public/private wage ratio in turn triggers a reallocation of labor resources from the private

[58] At first glance the huge percentage deviation from the steady-state for capital tax can be misleading. However, noting that the steady-state capital tax rate under Ramsey is of order $10^{-10} \approx 0$, it follows that the log-deviation from the steady-state is a very very large number indeed, as the denominator is close to zero, although the absolute value of the change is minute.

[59] This is also a result of the logarithmic specification of the household's utility of consumption.

to the public sector. Next, the outflow of hours from the firm leads to an increase in the marginal product of capital; hence, the real interest rate increases. However, from the complementarity between private labor and capital in the Cobb-Douglas production function, private capital decreases. Therefore, due to the fall in the levels of the two private inputs, output increases by less than the size of the technology shock.

In addition, given the jump in government investment, private consumption and investment fall upon the impact of the shock. Overall, the difference in the dynamics in the main model variables under the Ramsey regime is due to the fact that the government chooses the optimal levels of public hours and capital (and hence also public investment). Over time, attracted by the above-steady-state real interest rate, more private investment is undertaken by the government. In turn, private capital accumulation increases, and the usual hump-shape dynamics appears. Higher capital input increases the marginal productivity of labor, and private labor starts slowly to recover to its old steady-state. As time passes, private consumption response also turns positive, and the shape if its response follows the dynamic path of private capital. Lastly, the benevolent Ramsey planner chooses to suppress the positive response of rent-seeking to technological improvements. This follows directly from the fact that in the optimal policy case, the government optimally chooses both public employment and the public/private wage ratio.

Overall, the positive innovations to TFP have a positive effect on the economy. Additionally, there is a long-lasting internal propagation effect on the economy. This is due to the fact that there are two labor markets featuring different wage rates, and labor can flow between sectors in response to changes in the relative wage. Moreover, there is complementarity between public hours and the public capital, which reinforces the complementarity between private and public consumption in the household's utility. The quantitative effect of public sector labor market, however, completely dominates capital response in terms of the initial dynamics. Nevertheless, in the long-run, the private capital accumulation effects becomes

the dominant one, as dictated by the standard neoclassical RBC model.

An interesting result is that a significant portion of the private gains are channeled to the public sector in the form of higher spending on public wages and public investment. Indeed, in this model both categories are productive expenditures, as labor and capital are combined in the provision of the public good. Even though the household suffers a little from the lower private consumption, this negative effect is overcompensated for by the increase in leisure (as there is a greater fall in private hours fall relative to the increase in private hours), the decrease in rent-seeking, and a higher level of public good consumption. Overall, it takes more than 100 years for all the model variables to return to their old steady states.

3.9 Limitations of the study and suggestions for future research

This section analyses the key assumptions of the model and proposes some possible extensions to the current framework. First, it was assumed that the government sector wage bill is the pool of public resources, which are up for grabs. In reality a much larger rent component is the tax revenues, an avenue pursued in Angelopoulos *et al.* (2009), and thus not discussed here. In the model in this chapter, a positive rent exist because of the higher public wages, and the high public employment. In addition, given the individual decision making in a DCE, a positive amount of rent-seeking time is chosen by each government bureaucrat. Rent-seeking in the model disappears if wages are equalized across sectors, or in case all agents decide to play the cooperative solution to the rent-seeking game.

Second, the model assumed that each individual could work in both the private and the public sectors, or equivalently, that workers from different sectors could safely pool together their resources and thus achieve complete insurance against variations in consumption. A possible extension is to model government officials and private sector workers separately,

as their preferences, and their attitude to risk might differ. This modeling choice, however, would complicate the algebra too much with limited promise of providing analytically tractable and interesting results. Still, this line of research is left on the agenda for future work.

Third, it was assumed that only public bureaucrats were allowed to engage in rent-seeking, and the only rents available were the funds from the wage bill. In reality, a much larger flow of funds are tax revenues, which can be expropriated for private gains by either public bureaucrats, or business people. A model along these lines, but with private sector individuals, is presented in Angelopoulos *et al.* (2009, 2011). Furthermore, such schemes are usually organized and jointly implemented by public bureaucrats and firm-owners. There will be insiders and outsiders to the scheme, both in the private and in the government sector, or honest and corrupt individuals. However, in the model presented in this chapter, all consumers own shares in the firm, and work as bureaucrats at the same time, so there are no outsiders. A very simple attempt in a partial-equilibrium setting is considered in Hillman and Ursprung (2000), but in the current setup, it greatly increases the complexity of the problem, and thus is left for further research.

Fourth, the model did not elaborate on the rent-seeking function. The simplest possible form of the contestable logit function was chosen to abstract away from possible non-linearities. The framework ignores rent-seeking in groups, and possible asymmetries in the distribution of the "prize." Such extensions are possible (see Congleton *et al.* 2008), but make the model cumbersome, and thus were not considered in this chapter. The simpler, and much more elegant representation of the auctioning mechanism was preferred instead in order to preserve model elegance and ensure analytical tractability.

Lastly, a very small economic literature exists on the internal organization of the state and the incentives of government bureaucracy, *e.g.*, Acemoglu and Verdier (1998), Acemoglu

(2005), and Becker and Mulligan (2003). Guriev (2004), Dixit (2006, 2010), and Dodlova (2013) also point out to the multi-tier structure of the government bureaucracy, the principal-bureaucrat-agent hierarchy, as the main culprit for state ineffiency, due to the agency problem generated within such an organization. More specifically, elected politicians (agents) need to elect experts (bureaucrats) to implement policies in the interest of the citizens (principal). Thus, as argued in Tirole (1994) and Aghion and Tirole (1997), bureaucrats possess the real authority in the government, as they have an "effective control over decisions" (even though they don't have the "formal authority"). In addition, Niskanen (1971), Bendor *et al.* (1985), and Horn (1995) argue that bureaucrats use their superior information when the budget is decided to inflate the costs of labor input.[60] In addition, government workers could be compensated with higher wages, as a favor by policymakers in exchange of bureaucrats' loyalty, or an an efficiency wage aiming to solve monitoring problems (Acemoglu and Verdier 2000). The latter could be rationalized, at least partially, with the moral hazard problem in the government employment sector, as bureaucrats have a better information about the level of the effort exerted, relative to the superiors.[61] In spite of being a possibly better description of the internal organization of the state and the incentives of the bureaucracy, the modeling choice in those partial-equilibrium setups cannot be easily translated and adopted in general equilibrium. Possible extensions of the work in this chapter along those lines is thus left for future research.

[60]For literature on the incentives of bureaucrats, see Laffont (2000), and Laffont and Martimort (2002).

[61]See Dewatripont *et al.* (1999), Francois (2000), Dixit (2002), Prendergast (2002) and Besley and Ghatak (2005) for the implications of moral hazard in government non-profit organizations. In addition, Tarschys (1975) and Wildavsky (1974) argue that government bureaucracies develop internal pressures for self-aggrandizement and expansion. Lastly, interaction among bureaucrats for budget appropriation are discussed in Miller and Moe (1983), Moe (1997), Bendor et al. (1985), Bos (2001), and Casas-Pardo and Puchades-Navarro (2001).

3.10 Conclusions

This chapter studied the wasteful effect of bureaucracy on the economy by addressing the link between the rent-seeking behavior of government bureaucrats and the public sector wage bill, which was taken to represent the rent component. In particular, public officials were modeled as individuals competing for a larger share of those public funds. The rent-seeking extraction technology in the government administration was modeled as in Murphy *et al.* (1991) and incorporated in an otherwise standard Real-Business-Cycle (RBC) framework with public sector. The model was calibrated to German data for the period 1970-2007. The main findings are: (i) Due to the existence of a significant public sector wage premium and the high public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses in terms of output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime, steady-state rent-seeking is smaller relative to the exogenous policy case, as the government chooses a higher public wage premium, but sets a much lower public employment level, thus achieving a decrease in rent-seeking.

3.11 Technical Appendix

3.11.1 Optimality Conditions

Firm's Problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\alpha)A_t(N_t^p)^{\theta}(K_t^p)^{-\theta} - r_t \quad = \quad 0 \tag{3.11.1.1}$$

where $(1-\theta)A_t(N_t^p)^{\theta}(K_t^p)^{-\theta} - r_t$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, i.e the interest rate.

$$r_t \quad = \quad (1-\theta)A_t(N_t^p)^{\theta}(K_t^p)^{-\theta} \tag{3.11.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$(K_t^p)(1-\theta)A_t(N_t^p)^{\theta}(K_t^p)^{-\theta} = r_t(K_t^p) \quad \text{or} \quad (1-\theta)Y_t = r_t(K_t^p) \tag{3.11.1.3}$$

because

$$(K_t^p)(1-\theta)A_t(N_t^p)^{\theta}(K_t^p)^{-\theta} = (1-\theta)A_t(N_t^p)^{\theta}(K_t^p)^{1-\theta} \quad = \quad (1-\theta)Y_t \tag{3.11.1.4}$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\theta A_t(N_t^p)^{\theta-1}(K_t^p)^{1-\theta} - w_t^p = 0 \text{ or } w_t^p = \theta A_t(N_t^p)^{\theta-1}(K_t^p)^{1-\theta} \tag{3.11.1.5}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, i.e the hourly

wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p \theta A_t (N_t^p)^{\theta-1} (K_t^p)^{1-\theta} = w_t^p N_t^p \quad \text{or} \quad \theta Y_t = w_t^p N_t^p \tag{3.11.1.6}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \;=\; Y_t - r_t K_t^p - w_t^p N_t^p = Y_t - (1-\theta) Y_t - \theta Y_t = 0 \tag{3.11.1.7}$$

Indeed, in equilibrium, economic profits are zero.

Consumer problem

Set up the Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \left\{ \psi_1 \ln C_t + \psi_2 \ln \left[1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2 \right] + \psi_3 \ln S_t^g \right.$$
$$\left. + \Lambda_t \left[(1-\tau_t^l)(w_t^p N_t^p + w_t^g \frac{RS_t^h}{RS_t} N_t^{gh}) + (1-\tau_t^k) r_t K_t^p + G_t^T - C_t - K_{t+1}^p + (1-\delta) K_t^p \right] \right\} \tag{3.11.1.8}$$

This is a concave programming problem, so the FOCs, together with the additional, boundary ("transversality") conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\beta^t \left\{ \frac{\psi_1}{C_t} - \Lambda_t \right\} = 0 \quad \text{or} \quad \frac{\psi_1}{C_t} = \Lambda_t \tag{3.11.1.9}$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K^p_{t+1} (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K^p_{t+1}} = 0$. That will result in the following expression

$$\beta^t \left\{ -\Lambda_t + E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + (1-\delta^p) \right] \right\} = 0 \tag{3.11.1.10}$$

Cancel the β^t term to obtain

$$-\Lambda_t + \beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \right] = 0 \tag{3.11.1.11}$$

Move Λ_t to the right so that

$$\beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1}) r_{t+1} + (1-\delta^p) \right] = \Lambda_t \tag{3.11.1.12}$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1} = (1-\theta) \frac{Y_{t+1}}{K^p_{t+1}}$$

$$\beta E_t \Lambda_{t+1} \left[(1-\tau^k_{t+1})(1-\theta) \frac{Y_{t+1}}{K^p_{t+1}} + (1-\delta^p) \right] = \Lambda_t \tag{3.11.1.13}$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N^p_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^p_t} = 0$. That will result in the following expression

$$\beta^t \left\{ -\frac{\psi_2}{1 - N^p_t - \frac{RS^h_t}{RS_t} N^g_t - RS^h_t - \gamma (N^{gh}_t)^2} + \Lambda_t (1-\tau^l_t) w^p_t \right\} = 0 \tag{3.11.1.14}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2}{1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \gamma (N^{gh}_t)^2} + \Lambda_t (1-\tau^l_t) w^p_t = 0 \tag{3.11.1.15}$$

Rearranging, one can obtain

$$\frac{\psi_2}{1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \gamma (N^{gh}_t)^2} = \Lambda_t (1-\tau^l_t) w^p_t \tag{3.11.1.16}$$

Plug in the expression for w^h_t, that is,

$$w^p_t = \theta \frac{Y_t}{N^p_t} \tag{3.11.1.17}$$

into the equation above. Rearranging, one can obtain

$$\frac{\psi_2}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} + \Lambda_t (1 - \tau_t^l) \theta \frac{Y_t}{N_t^p} \tag{3.11.1.18}$$

Take now the derivative of the Lagrangian w.r.t N_t^g (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N_t^g} = 0$. That will result in the following expression

$$\beta^t \left\{ - \frac{\psi_2 (1 + 2\gamma N_t^{gh})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} + \Lambda_t (1 - \tau_t^l) w_t^g \frac{RS_t^h}{RS_t} \right\} = 0 \tag{3.11.1.19}$$

Cancel the β^t term to obtain

$$- \frac{\psi_2 (1 + 2\gamma N_t^{gh})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} + \Lambda_t (1 - \tau_t^l) w_t^g \frac{RS_t^h}{RS_t} = 0 \tag{3.11.1.20}$$

Rearranging, one can obtain

$$\frac{\psi_2 (1 + 2\gamma N_t^{gh})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} = \Lambda_t (1 - \tau_t^l) w_t^g \frac{RS_t^h}{RS_t} \tag{3.11.1.21}$$

Take now the derivative of the Lagrangian w.r.t RS_t^h (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{RS_t^h} = 0$. That will result in the following expression

$$\beta^t \left\{ - \frac{\psi_2 (1 + \frac{N_t^{gh}}{RS_t})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} + \Lambda_t (1 - \tau_t^l) w_t^g \frac{N_t^{gh}}{RS_t} \right\} = 0 \tag{3.11.1.22}$$

Cancel the β^t term to obtain

$$- \frac{\psi_2 (1 + \frac{N_t^{gh}}{RS_t})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} + \Lambda_t (1 - \tau_t^l) w_t^g \frac{N_t^{gh}}{RS_t} = 0 \tag{3.11.1.23}$$

Rearranging, one can obtain

$$\frac{\psi_2 (1 + \frac{N_t^{gh}}{RS_t})}{1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2} = \Lambda_t (1 - \tau_t^l) w_t^g \frac{N_t^{gh}}{RS_t} \tag{3.11.1.24}$$

Lastly, a transversality condition need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t \to \infty} \beta^t \Lambda_t K_{t+1}^p = 0 \tag{3.11.1.25}$$

3.11.2 Per capita stationary symmetric DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, but only impose symmetry *i.e.* $Z_t^h = Z_t = z_t$. Thus, the system of equations that describes the DCE is as follows:

$$y_t = a_t (k_t^p)^{1-\theta} (n_t^p)^{\theta} \tag{3.11.2.1}$$

$$y_t = c_t + k_{t+1}^p - (1-\delta^p) k_t^p + g_t^i \tag{3.11.2.2}$$

$$\frac{\psi_1}{c_t} = \lambda_t \tag{3.11.2.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left[1 - \delta^p + (1-\tau^k)(1-\theta) \frac{y_{t+1}}{k_{t+1}^p} \right] \tag{3.11.2.4}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - rs_t - \gamma (n_t^g)^2} = \frac{\psi_1}{c_t} (1-\tau^l) \alpha \frac{y_t}{n_t^p} \tag{3.11.2.5}$$

$$\frac{\psi_2}{1 - n_t^p - n_t^g - rs_t - \gamma (n_t^g)^2} [1 + 2\gamma n_t^g] = \frac{\psi_1}{c_t} (1-\tau^l) w_t^g \tag{3.11.2.6}$$

$$k_{t+1}^p = i_t + (1-\delta^p) k_t^p \tag{3.11.2.7}$$

$$r_t = (1-\theta) \frac{y_t}{k_t^p} \tag{3.11.2.8}$$

$$w_t^p = \theta \frac{y_t}{n_t^p} \tag{3.11.2.9}$$

$$g_t^T + g_t^i + w_t^g n_t^g = \tau^k r_t k_t^p + \tau^l \left[w_t^p n_t^p + w_t^g n_t^g \right]. \tag{3.11.2.10}$$

$$k_{t+1}^g = g_t^i + (1-\delta^g) k_t^g \tag{3.11.2.11}$$

$$g_t^i = g_t^{iy} y_t \tag{3.11.2.12}$$

$$rs_t = n_t^g \left[\frac{w_t^g}{w_t^p} - 1 \right] \tag{3.11.2.13}$$

$$s_t^g = (n_t^g)^\alpha (k_t^g)^{1-\alpha} \tag{3.11.2.14}$$

Therefore, the DCE is summarized by Equations (3.11.2.1)-(3.11.2.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, k_t^p, k_t^g, g_t^i, rs_t, n_t^p, n_t^g, s_t^g, w_t^p, w_t^g, r_t, \lambda_t\}_{t=0}^\infty$ given the process followed by total factor productivity $\{a_t\}_{t=0}^\infty$, the values of government investment shares g^{iy}, the fixed level of government transfers g^T and capital and labor tax rates $\{\tau^k, \tau^l\}$.

3.11.3 Steady-state

In steady-state, there is no uncertainty and variables do not change. Thus, eliminate all stochasticity and time subscripts to obtain

$$y = a(k^p)^{1-\theta}(n^p)^\theta \tag{3.11.3.1}$$

$$y = c + \delta^p k^p + g^i \tag{3.11.3.2}$$

$$\frac{\psi_1}{c} = \lambda \tag{3.11.3.3}$$

$$1 = \beta \left[1 - \delta^p + (1-\tau^k)(1-\theta)\frac{y}{k^p} \right] \tag{3.11.3.4}$$

$$\frac{\psi_2}{1 - n^p - n^g - rs - \gamma(n^g)^2} = \frac{\psi_1}{c}(1-\tau^l)\theta\frac{y}{n^p} \tag{3.11.3.5}$$

$$\frac{\psi_2}{1 - n^p - n^g - rs - \gamma(n^g)^2}[1 + 2\gamma n^g] = \frac{\psi_1}{c}(1-\tau^l)w^g \tag{3.11.3.6}$$

$$i^p = \delta^p k^p \tag{3.11.3.7}$$

$$r = (1-\theta)\frac{y}{k^p} \tag{3.11.3.8}$$

$$w^p = \theta\frac{y}{n^p} \tag{3.11.3.9}$$

$$g^T + g^i + w^g n^g = \tau^k r k^p + \tau^l \left[w^p n^p + w^g n^g \right]. \tag{3.11.3.10}$$

$$g^i = \delta^g k^g \tag{3.11.3.11}$$

$$g^i = g^{iy} y \tag{3.11.3.12}$$

$$rs = n^g \left[\frac{w^g}{w^p} - 1 \right] \tag{3.11.3.13}$$

$$s^g = (n^g)^\alpha (k^g)^{1-\alpha} \tag{3.11.3.14}$$

3.11.4 Nash equilibria with zero rent-seeking

Cooperative solution

This subsection compares the non-cooperative Nash equilibrium solution to a case where all households coordinate on the first-best solution for rent-seeking. In particular, in every period, before choosing allocations, all households meet and discuss the possibility of engaging in rent-seeking activities. Throughout the discussion, they realize that if rent-seeking, they bargain again themselves, and thus agree not to rent-seek, as that would be jointly socially optimal outcome. Indeed, such a pre-communication results in sizable welfare gains, as shown in Table 3.6 on the next page. Aside from some slight differences in hours, there are no significant differences between the allocations in the tho equilibria. Note also that the transaction cost parameter in the cooperative solution is slightly lower.

Table 3.6: Data averages and long-run solution: non-cooperative vs. cooperative equilibrium

	Description	GE Data	Non-cooperative	Cooperative
c/y	Consumption-to-output ratio	0.590	0.784	0.784
i/y	Private investment-to-output ratio	0.210	0.192	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225	0.225
g^T/y	Public transfers-to-output ratio	0.170	0.228	0.228
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.145
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.200
w^p	Private sector wage rate	-	1.006	1.006
w^g	Public sector wage rate	-	1.207	1.207
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.714
n	Total employment	0.296	0.296	0.300
n^p	Private employment level	0.253	0.253	0.256
n^g	Public employment level	0.043	0.043	0.044
rs	Rent-seeking time	-	0.009	0.000
n^g/n^p	Public/private employment ratio	0.170	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.035
γ	Transaction cost parameter	-	2.318	2.298
U	Total discounted welfare	-	95.02	95.42
ξ	Welfare gain	-	0	0.025

Wage equalization across sectors

The second special case is when the government sets equal wages across sectors, and thus eliminates rent-seeking. This is still a non-cooperative Nash solution, in which transaction costs from working in the government are no longer present, as $\gamma = 0$ (there is no public wage premium). Again, as seen in Table 3.7, with the exception of hours, there are no other differences in steady-state allocations across equilibria. Still, the welfare gains of setting wages equal across sectors brings substantial gains in the exogenous policy case.

Table 3.7: Data averages and long-run solution: non-cooperative vs. equal-wages equilibrium

	Description	GE Data	Non-cooperative	Equal-wages eq.
c/y	Consumption-to-output ratio	0.590	0.784	0.784
i/y	Private investment-to-output ratio	0.210	0.192	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225	0.225
g^T/y	Public transfers-to-output ratio	0.170	0.228	0.242
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.121
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.000
w^p	Private sector wage rate	-	1.006	1.006
w^g	Public sector wage rate	-	1.207	1.006
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.595
n	Total employment	0.296	0.296	0.301
n^p	Private employment level	0.253	0.253	0.257
n^g	Public employment level	0.043	0.043	0.044
rs	Rent-seeking time	-	0.009	0.000
n^g/n^p	Public/private employment ratio	0.170	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.035
γ	Transaction cost parameter	-	2.318	0.000
U	Total discounted welfare	-	95.02	95.82
ξ	Welfare gain	-	0	0.052

3.11.5 Data description

ICRG: The ICRG index is based on annual values for indicators of the quality of governance, corruption and violation of property rights over the period 1982-1997. It has been constructed by Stephen Knack and the IRIS Center, University of Maryland, from monthly ICRG data provided by Political Risk Services. This index takes values within the range 0-50, with higher values indicating better institutional quality. The reported numbers are the averages over 1982-1997, and are taken from Angelopoulos *et al.* (2009). Knack and Keefer (1995) explain in detail the how the index was constructed.

Control of corruption index: Control of Corruption index measures perceptions of the extent to which public power is exercised for private gain, including both petty and grand forms of corruption, as well as "capture" of the state by elites and private interests. The index is obtained from the World Bank Worldwide Governance Indicators (WGI). The units in which the control of corruption is measured follow a normal distribution with a mean of zero and a standard deviation of one in each period. This implies that virtually all scores lie between -2.5 and 2.5, with higher scores corresponding to better outcomes. The values are averaged over the 1996-2009 period.

Quality of Public Finances (Size of government, Public Administration, Government expenditure effectiveness): The Quality of Public Finances (QPF) composite index is composed of sub-indices which distinguish between five dimensions through which public finances can impact long-term economic growth drawing on the theoretical and empirical literature on the links between public finances and long-term economic growth. The dimensions considered in this chapter are: (i) the size of government (dimension QPF 1),(ii) the composition, efficiency and effectiveness of expenditure (dimension QPF 3), and (iv) the structure and efficiency of the public administration (dimension QPF 4) QPF is defined as all fiscal policy arrangements and operations that support achieving macroeconomic goals of fiscal policy, in particular long-term economic growth. Scores range from -30 to +30 with an

EU-15 average of 0. Assuming a normal distribution a value between -10 and -30 is deemed as very poor, between -4 and -10 as poor, between -4 and +4 as average, between +4 and +10 as good, and between +10 and +30 as very good. Scores were calculated using linear unweighted average. More information on the index is contained in the European Commission (2009) report.

Government effectiveness index: The scores lie between -2.5 and 2.5 (distributed according to a standard normal distribution), with higher scores corresponding to better outcomes. In "Government Effectiveness" category the quality of public service provision, the quality of the bureaucracy, the competence of civil servants, the independence of the civil service from political pressures, and the credibility of the governments commitment to policies is combined to one index. The main focus of this index is on "inputs" required for the government to be able to produce and implement good policies and deliver public goods. The values are averaged over the 1996-2009 period.

3.11.6 Log-linearization

Log-linearized production function

$$y_t = a_t (k_t^p)^{1-\theta} (n_t^p)^{\theta} \tag{3.11.6.1}$$

Take natural logs from both sides to obtain

$$\ln y_t = \ln a_t + (1-\theta) \ln k_t^p + \theta \ln n_t^p \tag{3.11.6.2}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln y_t}{dt} = \frac{d \ln a_t}{dt} + (1-\theta) \frac{d \ln k_t^p}{dt} + \theta \frac{d \ln n_t^p}{dt} \tag{3.11.6.3}$$

$$\frac{1}{y} \frac{dy_t}{dt} = \frac{1}{a} \frac{da_t}{dt} + \frac{1-\theta}{k^p} \frac{dk_t}{dt} + \frac{\theta}{n^p} \frac{dn_t^p}{dt} \tag{3.11.6.4}$$

Pass to log-deviations to obtain

$$0 = -\hat{y}_t + (1-\theta) \hat{k}_t^p + \hat{a}_t + \theta \hat{n}_t^p \tag{3.11.6.5}$$

Linearized market clearing

$$c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i = y_t \tag{3.11.6.6}$$

Take logs from both sides to obtain

$$\ln[c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i] = \ln(y_t) \tag{3.11.6.7}$$

Totally differentiate with respect to time

$$\frac{d \ln[c_t + k_{t+1}^p - (1-\delta) k_t^p + g_t^i]}{dt} = d \ln(y_t) \tag{3.11.6.8}$$

$$\frac{1}{c + \delta k^p + g^c} \left[\frac{dc_t}{dt} \frac{c}{c} + \frac{dk_{t+1}^p}{dt} \frac{k^p}{k^p} - (1-\delta^p) \frac{dk_t^p}{dt} \frac{k^p}{k^p} + \frac{dg_t^i}{dt} \frac{g^i}{g^i} \right] = \frac{dy_t}{dt} \frac{1}{y} \tag{3.11.6.9}$$

Define $\hat{z} = \frac{dz_t}{dt} \frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y} \left[\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p) \hat{k}_t^p k^p + g^i \hat{g}_t^i \right] = \hat{y}_t \tag{3.11.6.10}$$

$$\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p) \hat{k}_t^p k^p + g^i \hat{g}_t^i = y \hat{y}_t \tag{3.11.6.11}$$

$$k^p \hat{k}_{t+1}^p = y \hat{y}_t - c \hat{c}_t + (1-\delta) k^p \hat{k}_t^p - g^i \hat{g}_t^i \tag{3.11.6.12}$$

Linearized FOC consumption

$$\frac{\psi_1}{c_t} = \lambda_t \tag{3.11.6.13}$$

Take natural logarithms from both sides to obtain

$$\ln \psi_1 - \ln(c_t) = \ln \lambda_t \tag{3.11.6.14}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \psi_1}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{3.11.6.15}$$

or

$$-\frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{3.11.6.16}$$

$$-\frac{dc_t}{dt}\frac{1}{c} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \tag{3.11.6.17}$$

Pass to log-deviations to obtain

$$-\hat{c}_t = \hat{\lambda}_t \tag{3.11.6.18}$$

Linearized no-arbitrage condition for capital

$$\lambda_t \quad = \quad \beta E_t \lambda_{t+1}[(1-\tau^k_{t+1})r_{t+1} + (1-\delta^p)] \tag{3.11.6.19}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t \quad = \quad \beta E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{3.11.6.20}$$

Take natural logs from both sides of the equation to obtain

$$\ln \lambda_t \quad = \quad \ln E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{3.11.6.21}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \lambda_t}{dt} = \frac{d \ln E_t[\lambda_{t+1}((1-\tau^k_{t+1})(1-\theta)\frac{y_{t+1}}{k^p_{t+1}} + (1-\delta^p)]}{dt} \tag{3.11.6.22}$$

$$\frac{1}{\lambda}\frac{d\lambda_t}{dt} = E_t\Bigg\{\frac{1}{\lambda((1-\tau^k_{t+1})(1-\theta)\frac{y}{k^p}+1-\delta^p}\Bigg[((1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda} + \frac{\lambda(1-\tau^k)(1-\theta)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y} - \Bigg[\frac{\lambda(1-\tau^k)(1-\theta)y}{(k^p)^2}\Bigg]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\Bigg]\Bigg\} \tag{3.11.6.23}$$

Pass to log-deviations to obtain

$$\hat{\lambda}_t = E_t\Bigg\{\hat{\lambda}_{t+1} + \Bigg[\frac{(1-\tau^k)(1-\theta)y}{((1-\tau^k)(1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{y}_{t+1} - \frac{(1-\tau^k)(1-\theta)y}{((1-\theta)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{k}^p_{t+1}\Bigg]\Bigg\} \tag{3.11.6.24}$$

Observe that

$$(1-\tau^k)(1-\theta)\frac{y}{k^p}+1-\delta^p = 1/\beta \tag{3.11.6.25}$$

Plug it into the equation to obtain

$$\hat{\lambda}_t = E_t\Bigg[\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}\hat{k}^p_{t+1}\Bigg] \tag{3.11.6.26}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p}E_t\hat{k}^p_{t+1} \tag{3.11.6.27}$$

Linearized MRS(c_t, n^p_t)

$$\psi_2 c_t = \psi_1[1-n^p_t-rs_t-n^g_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{3.11.6.28}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n^p_t-n^g_t-rs_t-\gamma(n_t)^2](1-\tau^l)\theta\frac{y_t}{n^p_t} \tag{3.11.6.29}$$

$$\ln\psi_2 + \ln c_t = \ln\psi_1 + \ln[1-n^p_t-n^g_t-rs_t-\gamma(n_t)^2] + \ln(1-\tau^l_t) + \ln y_t - \ln n^p_t \tag{3.11.6.30}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt}+\frac{d\ln c_t}{dt}=\frac{d\ln\psi_1}{dt}+\frac{d\ln[1-n_t^p-n_t^g-rs_t-\gamma(n_t)^2]}{dt}$$
$$+\frac{d\ln(1-\tau_t^l)}{dt}+\frac{d\ln y_t}{dt}-\frac{d\ln n_t^p}{dt} \tag{3.11.6.31}$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right]=-\frac{1}{1-n^p-n^g-rs_t-\gamma(n^g)^2}\frac{d\left[n_t^p+n_t^g+rs_t+\gamma(n_t)^2\right]}{dt}$$
$$-\frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{3.11.6.32}$$

$$\frac{dc_t}{dt}\frac{1}{c}=-\frac{n^p}{1-n^p-n^g-rs-\gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p}$$
$$-\frac{rs}{1-n^p-n^g-rs-\gamma(n^g)^2}\frac{drs_t}{dt}\frac{1}{rs}$$
$$-\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-rs-\gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g}-\frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \tag{3.11.6.33}$$

Pass to log-deviations to obtain

$$\hat{c}_t=-\frac{n^p}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}_t^p-\frac{rs}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{rs}_t$$
$$-\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}_t^g-\frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t-\hat{n}_t^p \tag{3.11.6.34}$$

Group terms to obtain

$$\hat{c}_t=-\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}_t^p$$
$$-\frac{rs}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{rs}_t-\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}_t^g-\frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t \tag{3.11.6.35}$$

Linearized MRS(c_t, n_t^g)

$$\psi_2 c_t = \psi_1[1-n_t^p-n_t^g-rs_t-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{3.11.6.36}$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n_t^p-n_t^g-rs_t-\gamma(n_t)^2](1-\tau^l)w_t^g \tag{3.11.6.37}$$

$$\ln \psi_2 + \ln c_t = \ln \psi_1 + \ln[1 - n_t^p - n_t^g - rs_t - \gamma(n_t)^2] + \ln(1 - \tau_t^l) + \ln w_t^g \tag{3.11.6.38}$$

Totally differentiate with respect to time to obtain

$$\begin{aligned} \frac{d \ln \psi_2}{dt} + \frac{d \ln c_t}{dt} &= \frac{d \ln \psi_1}{dt} + \frac{d \ln[1 - n_t^p - n_t^g - rs_t - \gamma(n_t)^2]}{dt} \\ + \frac{d \ln(1 - \tau_t^l)}{dt} + \frac{d \ln w_t^g}{dt} & \end{aligned} \tag{3.11.6.39}$$

$$\begin{aligned} \frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1 - n^p - n^g - rs - \gamma(n^g)^2} \frac{d\left[n_t^p + n_t^g + rs_t + \gamma(n_t)^2\right]}{dt} \\ - \frac{d\tau_t^l}{dt}\frac{1}{1 - \tau^l} + \frac{dw_t^g}{dt}\frac{1}{w^g} \end{aligned} \tag{3.11.6.40}$$

$$\begin{aligned} \frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1 - n^p - n^g - rs - \gamma(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p} - \frac{rs}{1 - n^p - n^g - rs - \gamma(n^g)^2}\frac{drs_t}{dt}\frac{1}{rs} \\ - \frac{n^g(1 + 2\gamma n^g)}{1 - n^p - n^g - rs - \gamma(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{\tau^l}{1 - \tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l} + \frac{1}{w^g}\frac{dw_t^g}{dt} \end{aligned} \tag{3.11.6.41}$$

Pass to log-deviations to obtain

$$\begin{aligned} \hat{c}_t = -\frac{n^p}{1 - n^p - n^g - rs - \gamma(n^g)^2}\hat{n}_t^p - \frac{rs}{1 - n^p - n^g - rs - \gamma(n^g)^2}\hat{rs}_t \\ - \frac{n^g(1 + 2\gamma n^g)}{1 - n^p - n^g - rs - \gamma(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1 - \tau^l}\hat{\tau}_t^l + \hat{w}_t^g \end{aligned} \tag{3.11.6.42}$$

Linearized private physical capital accumulation

$$k_{t+1}^p = i_t + (1 - \delta^p)k_t^p \tag{3.11.6.43}$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^p = \ln(i_t + (1 - \delta^p)k_t^p) \tag{3.11.6.44}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln k_{t+1}^p}{dt} = \frac{1}{i + (1 - \delta)k^p}\frac{d(i_t + (1 - \delta^p)k_t^p)}{dt} \tag{3.11.6.45}$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that } \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \tag{3.11.6.46}$$

Then

$$\frac{dk^p_{t+1}}{dt}\frac{1}{k^p} = \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i + (1-\delta^p)k^p_t}\frac{dk^p_t}{dt}\frac{k^p}{k^p} \tag{3.11.6.47}$$

Pass to log-deviations to obtain

$$\hat{k}^p_{t+1} = \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}^p_t \tag{3.11.6.48}$$

$$\hat{k}^p_{t+1} = \delta^p \hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{3.11.6.49}$$

Linearized government physical capital accumulation

$$k^g_{t+1} = g^i_t + (1-\delta^g)k^g_t \tag{3.11.6.50}$$

Take natural logs from both sides of the equation to obtain

$$\ln k^g_{t+1} = \ln(g^i_t + (1-\delta^g)k^g_t) \tag{3.11.6.51}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^g_{t+1}}{dt} = \frac{1}{g^i + (1-\delta^g)k^g}\frac{d(i_t + (1-\delta^g)k^g_t)}{dt} \tag{3.11.6.52}$$

Observe that since

$$g^i = \delta^g k^g, \text{ it follows that } \quad g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \tag{3.11.6.53}$$

Then

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{{}^i + (1-\delta^g)k^g_t}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{3.11.6.54}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^p)k^g}{k^g}\hat{k}^g_t \tag{3.11.6.55}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{3.11.6.56}$$

Linearized government budget constraint

$$(1-\tau_t^l)w_t^g n_t^g + g_t^i = \tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p. \tag{3.11.6.57}$$

Take natural logarithms from both sides to obtain

$$\ln\left[(1-\tau_t^l)w_t^g n_t^g + g_t^i\right] = \ln\left[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\right]. \tag{3.11.6.58}$$

Totally differentiate with respect to time to obtain

$$\frac{d}{dt}\ln\left[(1-\tau_t^l)w_t^g n_t^g + g_t^i\right] = \frac{d}{dt}\ln\left[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\right]. \tag{3.11.6.59}$$

or

$$\frac{1}{(1-\tau^l)w^g n^g + g^i}\frac{d\left[(1-\tau_t^l)w_t^g n_t^g + g_t^i\right]}{dt} = \frac{1}{\left[\tau^k r k + \tau^l w^p n^p\right]}\frac{d\left[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\right]}{dt}. \tag{3.11.6.60}$$

Note that

$$(1-\tau^l)w^g n^g + g^i = \tau^k r k + \tau^l w^p n^p \tag{3.11.6.61}$$

Hence

$$\frac{d\left[(1-\tau_t^l)w_t^g n_t^g + g_t^i\right]}{dt} = \frac{d\left[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\right]}{dt}. \tag{3.11.6.62}$$

or

$$\begin{aligned} &-w^g n^g \frac{d\tau_t^l}{dt}\frac{\tau^l}{\tau^l} + (1-\tau^l)n^g\frac{dw_t^g}{dt}\frac{w^g}{w^g} + (1-\tau^l)w^g\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dg_t^i}{dt}\frac{g^i}{g^i} \\ &= rk\frac{d\tau_t^k}{dt}\frac{\tau^k}{\tau^k} + \tau^k k\frac{dr_t}{dt}\frac{r}{r} + \tau^k r\frac{dk_t}{dt}\frac{k}{k} + w^p n^p\frac{d\tau_t^l}{dt} + \tau^l n^p\frac{dw_t^p}{dt}\frac{w^p}{w^p} + \tau^l w^p\frac{dn_t^p}{dt}\frac{n^p}{n^p} \end{aligned} \tag{3.11.6.63}$$

Pass to log-deviations to obtain

$$\begin{aligned} &-\tau^l w^g n^g \hat{\tau}_t^l + (1-\tau^l)w^g n^g \hat{w}_t^g + (1-\tau^l)w^g n^g \hat{n}_t^g + g^i \hat{g}_t^i \\ &= \tau^k r k \hat{\tau}_t^k + \tau^k r k \hat{r}_t + \tau^k r k \hat{k}_t + \tau^l w^p n^p \hat{\tau}_t^l + \tau^l w^p n^p \hat{w}_t^p + \tau^l w^p n^p \hat{n}_t^p \end{aligned} \tag{3.11.6.64}$$

Total hours/employment

$$n_t = n_t^g + n_t^p \tag{3.11.6.65}$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \tag{3.11.6.66}$$

Totally differentiate to obtain

$$\frac{d \ln n_t}{dt} = \frac{d \ln(n_t^g + n_t^p)}{dt} \tag{3.11.6.67}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{3.11.6.68}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{3.11.6.69}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{3.11.6.70}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{3.11.6.71}$$

Linearized private wage rate

$$w_t^p = \theta \frac{y_t}{n_t^p} \tag{3.11.6.72}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln \theta + \ln y_t - \ln n_t^p \tag{3.11.6.73}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln w_t^p}{dt} = \frac{d \ln \theta}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln n_t^p}{dt} \tag{3.11.6.74}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{3.11.6.75}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{3.11.6.76}$$

Linearized real interest rate

$$r_t = (1-\theta)\frac{y_t}{k_t^p} \tag{3.11.6.77}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln(1-\theta) + \ln y_t - \ln k_t^p \tag{3.11.6.78}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln r_t}{dt} = \frac{d\ln(1-\theta)}{dt} + \frac{d\ln y_t}{dt} - \frac{d\ln k_t^p}{dt} \tag{3.11.6.79}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{3.11.6.80}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{3.11.6.81}$$

Linearized government investment

$$g_t^i = g^{iy} y_t \tag{3.11.6.82}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g^{iy} + \ln y_t \tag{3.11.6.83}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln g_t^i}{dt} = \frac{d\ln g^{iy}}{dt} + \frac{d\ln y_t}{dt} \tag{3.11.6.84}$$

or

$$\frac{dg_t^i}{dt}\frac{1}{g^i} = \frac{dy_t}{dt}\frac{1}{y} \tag{3.11.6.85}$$

Passing to log-deviations

$$\hat{g}_t^i = \hat{y}_t \tag{3.11.6.86}$$

Linearized government services

$$s_t^g = (n_t^g)^\alpha (k_t^g)^{(1-\alpha)} \tag{3.11.6.87}$$

Take natural logarithms from both sides to obtain

$$\ln s_t^g = \alpha \ln n_t^g + (1-\alpha) \ln k_t^g \tag{3.11.6.88}$$

Totally differentiate with respect to time to obtain

$$\frac{ds_t^g}{dt}\frac{1}{s^g} = \alpha \frac{dn_t^g}{dt}\frac{1}{n^g} + (1-\alpha)\frac{dk_t^g}{dt}\frac{1}{k^g} \tag{3.11.6.89}$$

$$\hat{s}_t^g = \alpha \hat{n}_t^g + (1-\alpha)\hat{k}_t^g \tag{3.11.6.90}$$

Linearized rent-seeking rule

$$rs_t = n_t^g \left[\frac{w_t^g}{w_t^p} - 1\right] \tag{3.11.6.91}$$

Take natural logarithms from both sides to obtain

$$\ln rs_t = \ln n_t^g + \ln \left[\frac{w_t^g}{w_t^p} - 1\right] \tag{3.11.6.92}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln rs_t}{dt} = \frac{d \ln n_t^g}{dt} + \frac{d}{dt} \ln \left[\frac{w_t^g}{w_t^p} - 1\right] \tag{3.11.6.93}$$

$$\frac{drs_t}{dt}\frac{1}{rs} = \frac{dn_t^g}{dt}\frac{1}{n^g} + \frac{d\left[\frac{w_t^g}{w_t^p} - 1\right]}{dt}\frac{1}{\left[\frac{w^g}{w^p} - 1\right]} \tag{3.11.6.94}$$

$$\hat{rs}_t = \hat{n}_t^g + \hat{w}_t^g \frac{w^g}{w^p}\frac{n^g}{rs} + \hat{w}_t^p \frac{n^g}{rs}(w^g - 2) \tag{3.11.6.95}$$

Linearized technology shock process

$$\ln a_{t+1} = \rho^a \ln a_t + \epsilon^a_{t+1} \tag{3.11.6.96}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln a_{t+1}}{dt} = \rho^a \frac{d \ln a_t}{dt} + \frac{d\epsilon^a_{t+1}}{dt} \tag{3.11.6.97}$$

$$\frac{da_{t+1}}{dt} = \rho^a \frac{da_t}{dt} + \epsilon^a_{t+1} \tag{3.11.6.98}$$

where for $t = 1$ $\frac{d\epsilon^a_{t+1}}{dt} \approx \ln(e^{\epsilon^a_{t+1}}/e^{\epsilon^a}) = \epsilon^a_{t+1} - \epsilon^a = \epsilon^a_{t+1}$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} = \rho^a \hat{a}_t + \epsilon^a_{t+1} \tag{3.11.6.99}$$

3.11.7 Log-linearized DCE system

$$0 = -\hat{y}_t + (1-\theta)\hat{k}^p_t + \hat{a}_t + \theta \hat{n}^p_t \tag{3.11.7.1}$$

$$k^p \hat{k}^p_{t+1} = y\hat{y}_t - c\hat{c}_t + (1-\delta^p)k^p \hat{k}^p_t - g^i \hat{g}^i_t \tag{3.11.7.2}$$

$$-\hat{c}_t = \hat{\lambda}_t \tag{3.11.7.3}$$

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\theta)y}{k^p} E_t \hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\theta)y}{k^p} E_t \hat{k}^p_{t+1} \tag{3.11.7.4}$$

$$\begin{aligned}\hat{c}_t = -\frac{1-n^g-\gamma(n^g)^2}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}^p_t - \frac{rs}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{rs}_t \\ -\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-\gamma(n^g)^2}\hat{n}^g_t + \hat{y}_t\end{aligned} \tag{3.11.7.5}$$

$$\begin{aligned}\hat{c}_t = -\frac{n^p}{1-n^p-rs-n^g-\gamma(n^g)^2}\hat{n}^p_t - \frac{rs}{1-n^p-rs-n^g-\gamma(n^g)^2}\hat{rs}_t \\ -\frac{n^g(1+2\gamma n^g)}{1-n^p-n^g-rs-\gamma(n^g)^2}\hat{n}^g_t + \hat{w}^g_t\end{aligned} \tag{3.11.7.6}$$

$$\hat{k}^p_{t+1} = \delta^p \hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{3.11.7.7}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{3.11.7.8}$$

$$\begin{aligned} &-\tau^l w^g n^g \hat{\tau}^l_t + (1-\tau^l) w^g n^g \hat{w}^g_t + (1-\tau^l) w^g n^g \hat{n}^g_t + g^i \hat{g}^i_t \\ &= \tau^k rk\hat{\tau}^k_t + \tau^k rk\hat{r}_t + \tau^k rk\hat{k}_t + \tau^l w^p n^p \hat{\tau}^l_t + \tau^l w^p n^p \hat{w}^p_t + \tau^l w^p n^p \hat{n}^p_t \end{aligned} \tag{3.11.7.9}$$

$$\hat{g}^i_t = \hat{y}_t \tag{3.11.7.10}$$

$$\hat{rs}_t = \hat{n}^g_t + \hat{w}^g_t \frac{w^g}{w^p}\frac{n^g}{rs} + \hat{w}^p_t \frac{n^g}{rs}(w^g - 2) \tag{3.11.7.11}$$

$$\hat{w}^p_t = \hat{y}_t - \hat{n}^p_t \tag{3.11.7.12}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}^p_t \tag{3.11.7.13}$$

$$\hat{s}^g_t = \alpha \hat{n}^g_t + (1-\alpha)\hat{k}^g_t \tag{3.11.7.14}$$

$$\hat{a}_{t+1} = \rho^a \hat{a}_t + \epsilon^a_{t+1} \tag{3.11.7.15}$$

The model can be now solved by representing it in the following matrix form

$$\mathbf{A}E_t\hat{\mathbf{x}}_{\mathbf{t+1}} = \mathbf{B}\hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C}E_t\varepsilon_{\mathbf{t+1}}, \tag{3.11.7.16}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{rs}_t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue ("Schur") decomposition algorithm was used to solve the model. The MATLAB function to solve the above linear system is *solab.m*. The inputs are matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ defined above and $nk = 3$, which is the number of state variables. The outputs are the coefficient matrices $\mathbf{M}$ and $\mathbf{\Pi}$ which solve the linearized system. A solution to an RBC model is in the form of (approximate) policy, or transition rule, which describes the evolution of each variable. In particular, the predetermined and non-predetermined variables can be represented in the following form:

$$E_t\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} \tag{3.11.7.17}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = M\hat{\mathbf{s}}_{\mathbf{t}} \tag{3.11.7.18}$$

To simulate the model, one requires a sequence of normally distributed disturbances, $\{\epsilon_t\}_{t=0}^{\infty}$ for the three exogenous shocks with sample size T, the initial values of the endogenous predetermined variables, $\{k_0^p, k_0^g, a_0\}$ ($a_0 = 1$), and the evolution of the endogenous non-predetermined variables in model solution form

$$\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} + \mathbf{D}\varepsilon_{\mathbf{t+1}} \tag{3.11.7.19}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = \mathbf{M}\hat{\mathbf{s}}_{\mathbf{t}}, \tag{3.11.7.20}$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.11.7.21}$$

Based on the above representation, MATLAB code was written to simulate the model. The computation of impulse responses using the linearized model solution is straightforward.

Chapter 4

Thesis Summary, Conclusions and Future Research

This thesis has undertaken a thorough analysis of the effects of a large public employment sector on business cycles and welfare, and characterized optimal government spending across the wage bill and public investment categories. The study in this dissertation also presented a first attempt to explore the relevance of bureaucracy for the macroeconomy by examining the cost of rent-seeking by government bureaucrats on aggregate activity. Chapter 1 constructed an RBC model augmented by a public sector union maximization problem and conducted a thorough assessment of the model's fit relative to a model with exogenous public employment and a single wage rate prevailing in the economy. In addition, several important fiscal regime reforms were also considered. Next, Chapter 2 characterized optimal fiscal policy in an RBC model with endogenous public wages and hours, and evaluated it relative to the exogenous (observed) one. Lastly, Chapter 3 computed the welfare cost resulting from the presence of rent-seeking bureaucrats in the public administration. This was achieved by modeling bureaucrats as agents competing with one another for a contestable transfer, *i.e.* the government wage bill. In addition, Chapter 3 compared and contrasted the rent-seeking time measure with the amount of wasteful activity occurring under a benevolent government.

Main findings and contributions (Chapter 1)

Chapter 1 attempted to answer the question as to whether the benchmark Real-Business-Cycle (RBC) model, augmented with a union in the public sector, could match labor market fluctuations observed in a representative European economy (Germany) better than earlier/alternative models, and whether a strong union presence in the public sector was relevant for the welfare effect of fiscal regime changes. Answering these questions required a careful calibration and simulation of a relatively standard RBC model augmented with a public union optimization problem. To this end, the setup in Chapter 1 combined two elements used in earlier research to address new aspects of the economy and produce new results: it adopted the public sector union maximization problem from Fernandez-de-Cordoba *et al.* (2009, 2012) and incorporated it into a RBC model with government employment, *i.e.* Finn (1998). Furthermore, the model with the public sector union in this chapter also featured a much richer public finance structure, in both tax revenue and expenditure categories. While all these features are important for understanding the aggregate fluctuations, these aspects were not addressed in the earlier studies on the dynamic general equilibrium effects of public sector unions. Thus, the individual quantitative effect of union optimization was assessed relative to Finn's (1998) setup with exogenous public hours and a single wage rate.

Compared to earlier models with public sector unions in an RBC framework, the study in Chapter 1 also offered a much more detailed evaluation of a general equilibrium model by conducting a full characterization of the model's fit. In addition to presenting relative volatilities and contemporaneous correlations of model variables with output, the auto- and cross-correlation functions were also computed and compared to their empirical counterparts, where the latter were obtained from an unrestricted VAR. Along all those dimensions, the study was shown to be superior to a comparable model with exogenous public hours and a single competitive wage rate (Finn 1998; Cavallo 2005; Linnemann 2009).

Overall, the public sector union model proved to be useful for studying fiscal policy in Ger-

many. The impulse responses showed that a shock to government consumption share has a significant negative wealth effect. In contrast, the shock to public investment ratio produced positive wealth effects, which had a long-term impact on output. Lastly, endogenously-determined public wage and hours were shown to add to the distortionary effect of contractionary tax reforms and produce significantly higher welfare losses. In particular, the union model required larger changes in tax rates to achieve a pre-specified increase in tax revenue, as compared to Finn's (1998) model with exogenous public sector hours. Thus, endogenous public hours were shown to be quantitatively important for fiscal policy evaluation. Ignoring the interaction between hours and wages led to a significant underestimation of the welfare effect of tax regime changes.

However, despite its good performance *vis-a-vis* data, several aspects of the model used in Chapter 1 failed to capture important features of the real economy. For example, although the model captured the dynamic correlation between major variables relatively well, it failed to capture the contemporaneous correlation between public hours and wages, as well as the contemporaneous correlation between public and private hours. This implies that the simple public sector union maximization function should be extended further to capture those aspects in data.

Main findings and contributions (Chapter 2)

Next, Chapter 2 addressed new effects of fiscal policy. To this end, Chapter 2 calibrated a DSGE model with realistic public sector taxation and spending categories to study the labor market effects of fiscal policy in Germany over the period 1970-2007. Two fiscal regimes were compared and contrasted in the chapter - the exogenous (observed) fiscal policy case and the optimal policy one. Chapter 2 attempted to compute the optimal size (and composition) of the public wage bill, the efficient level of government investment, as well as the optimal level of provision for the labor-intensive public good. Addressing these research questions

required an RBC model with a richer and more realistic government spending side, and an endogenous private-public labor choice in particular. However, the presence of a public employment decision margin, as a separate labor market choice made by the household, had not been sufficiently investigated in the literature. Chapter 2 argued that when public sector labor choice was ignored, then important effects on allocations and welfare, driven by government wage-setting and household's decisions on hours, would be missed. Hence, an otherwise standard general-equilibrium model was thus augmented with a convex cost of working in the government sector to reflect the fact that wages in the private and the public sector were determined within different institutional settings (thus linking the analysis with the study undertaken in Chapter 1). Chapter 2 argued that if public sector labor choice is ignored, then important effects on allocations and welfare, driven by government wage-setting and household's decisions on hours, will be missed.

The second novelty in the framework presented in Chapter 2, which added value to earlier studies, was the more interesting and meaningful role attributed to government employees. In particular, the study modeled in greater detail the mechanism of public good provision. The setup modeled the government as an employer, needing labor hours to provide public goods. In contrast to Cavallo (2005) and Linnemann (2009), labor was combined with government capital (instead of government purchases) to produce valuable government services. Therefore, government investment was a productive government spending category in the setup, and public sector wage consumption was not entirely wasteful. Importantly, when hiring workers, the government was able to set the public sector wage rate, an assumption which is consistent with data, *e.g.* Perez and Schucknecht (2003). Overall, the interaction between the two types of labor and capital stocks were the major driving force behind the new results obtained in Chapter 2.

In line with earlier findings in the literature, the optimal steady-state capital tax was zero, as it was the most distortionary tax to use. In addition, a higher steady-state labor tax was

needed to compensate for the loss in capital income tax revenue. The new results in Chapter 2 were that under the optimal policy regime, public employment was lower, while government employees were better compensated. These results occurred because the benevolent Ramsey planner chose to provide the optimal amount of the public good, substituted labor for capital in the input mix for both public services production and private output. As a result, the government wage bill decreased, while public investment was three times higher than in the exogenous policy case.

Main findings and contributions (Chapter 3)

Chapter 3 analyzed the effect of a large public administration on the on aggregate economy in European countries. Furthermore, the research in this chapter contributed to the body of both macroeconomic and political economy literature by focusing on the rent-seeking behavior of bureaucrats in the public sector. By incorporating an auctioning mechanism from game theory into an otherwise standard RBC model, the setup tried to quantify the cost of rent-seeking that is produced as a result of the non-productive behavior of government bureaucrats competing for a larger share of a contestable transfer, which in Chapter 3 was assumed to be the wage bill. After all, public administration consists of a system of bureaus, and this type of organization in the government sector is shown to produce significant losses for the economy through the rent-seeking mechanism.

The research in this chapter argued that each bureau head could be perceived as a rational income-maximizing individual, who wants to attract more public funds for his/her agency. Another contribution of this chapter was that very few economists, with the notable exception of Buchanan and Tullock (1962), Tullock (1965), and Niskanen (1971), focused on the presence of a large bureaucracy and provided evidence of its importance for the economy, but only in a partial-equilibrium context. In addition, Chapter 3 in this thesis aimed to fill this gap in the literature by motivating and presenting a more intricate channel used by

bureaucrats to rent-seek and lobby for government funds.

To illustrate the processes taking place within public administration, the model setup in Chapter 3 incorporated a symmetric non-cooperative game that was played among government bureaucrats themselves to increase individual income at the expense of the other public officials earnings. The interaction between agents in the public sector generated strategic complementarities, as individual rent-seeking is positively related to opponents' choice of rent-seeking. Another novelty in Chapter 3 was that rent-seeking occurred in a non-competitive labor market, that of the public sector one, where the wage rate was set above private sector pay. This stimulated the entry of labor in the sector, and as a result, public employment eventually became too high. In particular, the high public wage and employment both stimulated rent-seeking by generating a positive benefit of engaging in wasteful activities.

In the model presented in Chapter 3, the positive amount of time dedicated to opportunistic activities was an efficient outcome from an individual worker's point of view, as all agents were fully rational and maximized their utility levels. Thus, in equilibrium, individual bureaucratic rent-seeking efforts adjusted to the point where the value of additional resources spent per bureaucrat equals the benefit that accrues to that individual. In turn, a higher wage bill required higher tax rates to finance government spending. In the private sector, high taxes reduced incentives to supply labor and accumulate capital, and decreased consumption and output. Thus rent-seeking had a negative impact on the economy, and Chapter 3 attempted to quantify the loss for the economy in a general-equilibrium framework.

The model in Chapter 3 with rent-seeking by government bureaucrats also performed well *vis-a-vis* data in the relevant dimensions. The main findings from the study were that due to the existence of a public sector wage premium and the high public sector employment,

a significant amount of working time was spent rent-seeking. In addition, the mechanism described in Chapter 3 found strong empirical support in the face of different indices of institutional quality. The model predicted that rent-seeking in the public sector led to significant losses in terms of output, with Germany featuring the lowest cost, and Greece, the highest, followed by Belgium, Italy, and Spain. In addition, the model-generated cost measures of rent seeking in a cross-section of EU countries were highly correlated with indices of institutional quality.

Finally, Chapter 3 characterized the optimal fiscal policy regime under rent-seeking. To this end, the exogenous (observed) fiscal policy case was compared and contrasted to the optimal policy one. Under the optimal policy regime, and with congestible public goods, steady-state rent-seeking was significantly smaller relative to the exogenous policy case. In addition to the zero capital tax rate, and the higher labor tax rate, the benevolent government planner chose to invest more in public capital, and set a higher public wage premium but a much lower public employment, thus achieving an overall decrease in the level of rent-seeking relative to the value obtained in the exogenous case.

Limitations of the study and directions for future research

In this final section of the thesis, the limitations of the study are properly acknowledged, and presented to motivate several directions for future research. Overall, the analysis utilized a representative-agent framework (or simplifying calculations by assuming a symmetric decentralized equilibrium), and thus abstracted from possible consumer/firm heterogeneities and distributional effects. In addition, the setup adopted the perfect private capital and labor market formulation to focus on the public sector labor market modeling. Furthermore, since the models focused on the real effects on the economy, the setups described did not feature money and/or nominal rigidities.

In the model setup in Chapter 1, the union optimization problem generated identical dynamics of public sector hours and wages, which constrained the ability of the model to match the behavior in the two variables well simultaneously. Moreover, in reality public sector unions and government usually bargain over nominal wage increases, and against redundancies. They do not negotiate hours and the level of the real wage directly. For example, before engaging in negotiations, unions take into consideration labor productivity in the private sector and the private wage, which are then used as leverage in negotiations over the public wage. Finally, the simple union objective function used in this chapter ignored other possible demands by unions, such as job security, work conditions, government pensions, other non-monetary benefits, etc. Indeed, as a possible direction for future research, some of these factors could be incorporated to extend the basic model.

Next, the analysis performed in Chapter 2 was based on the strong assumption that the government budget is balanced every period and that the household can work in both sectors. This assumption is not very realistic, as labor supply decisions are made sequentially in the real world. A worker usually decides on a sector first, and only then on the number of hours worked in the selected sector. Furthermore, it is a stylized fact in labor data that most of the variations in hours worked in data are driven by changes in employment rates rather than by changes in hours worked per person. Thus, the model is too simple and cannot distinguish between employment and hours per person in the two sectors. A possible extension, left for future research, would be to setup a model with heterogeneous-agents, who search for work according to a directed search process, similar to that used in Gomes (2009, 2012).

Another drawback of the model setups presented in this thesis was that they abstracted away from debt issues, which are important for EU economies. However, in the setups discussing optimal fiscal policy with full commitment, however, Stockman (2001) has shown the pres-

ence of debt not to be relevant. Nevertheless, as a possible extension of the model in Chapter 2, government bonds could be introduced, together with a long-run target debt/GDP ratio that has to be met in the long-run. Studying the transitional dynamics of the economic variables and the adjustment in the different of public finance categories, in the presence of endogenous public hours and wages, could be a possible avenue for future work.

Additionally, since the analysis in Chapter 2 focused on the long-run full commitment case, the setup abstracted away from electoral uncertainty and thus ignored possible departures from the full-commitment case. Across the political spectrum in most democratic societies, there are different parties with diverse objectives that compete for the popularity vote at parliamentary/presidential elections. In particular, different parties might have different preferences for the level of public employment. Government jobs can be created in the public sector to generate political support and increase the chances of re-election. However, such considerations, as well as possible departures from the full commitment case, and a focus on "loose commitment" as in Debortoli and Nunes (2010), or time-consistent policies as in Klein and Rios-Rull (2003), Ortigueira (2006), Klein *et al.* (2008) and Martin (2010), will be put on the agenda for future research.

Aside from political considerations, the observed premium at macroeconomic level is also likely to be a by-product of aggregation of microeconomic data. In the German Socio-Economics Panel (SOEP), as well as in the US Panel Study of Income Dynamics (PSID) database, for example, the age-skill profile of public employees is skewed to the left: the average public employee is older, more skilled, more experienced, and is more likely to occupy managerial positions as compared to his/her private sector counterpart. However, such distributional and occupational dimensions are outside the scope of a simple RBC model with a representative household. Therefore, further work to endogeneize the public wage premium, perhaps within a heterogeneous-agents framework, needs to be undertaken.

Finally, the limitations of the model used in Chapter 3 are also acknowledged: The theoretical setup assumed that each individual could work in both the private and the public sectors, or equivalently, that workers from different sectors could safely pool together their resources and thus achieve complete insurance against variations in consumption. A possible extension is to model government officials and private sector workers separately, as their preferences, and their attitude to risk might differ. This modeling choice, however, would complicate the algebra too much with limited promise of providing analytically tractable and interesting results. Therefore, this line of research is left on the agenda for future work.

In addition, the setup in Chapter 3 allowed only public bureaucrats to engage in rent-seeking, and the only rent available was the wage bill. In reality, tax revenues represent a much larger flow of funds, which can be expropriated for private gains by either public bureaucrats, or business people. Furthermore, such schemes are usually organized and jointly implemented by public bureaucrats and firm-owners. Lastly, the model discussed in Chapter 3 did not elaborate on the rent-seeking function, and ignores aspects covered in microeconomic and game theory literature, such as rent-seeking in groups, and possible asymmetries in the distribution of the "prize". Possible model extensions along those lines are left for future research.

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