
PhD thesis

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TIME-SIMULATION
OF
SHIP MOTIONS

by

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Submitted as a Thesis for the Degree of
Doctor of Philosophy
Department of Naval Architecture and Ocean Engineering
University of Glasgow

July 1984
To All my Teachers Throughout my Life,

Beginning With my Parents,

Through To my Supervisor, Mr N. S. Miller
SUMMARY

Recent accidents with small vessels have focused interest on the problems associated with extreme ship motions, with particular emphasis on capsizing problems.

This thesis is a theoretical investigation, using a numerical model, into ship motions in a seaway which aims at a better understanding of ship behaviour in waves.

A time-domain numerical simulation of the ship motions in regular sinusoidal waves in six-degrees of freedom has been developed to try to discover dangerous situations which may lead a ship to capsize. An investigation into the dynamic stability of the trawler GAUL, which disappeared in heavy seas in 1974, is used as a demonstration.

The basic approach of the simulation program involves the computation of the coefficients of the equations of motions at each step in time according to the exact wave and vessel position using strip theory.

The thesis describes the computational technique used for representing the instantaneous under-water shape of the hull for a ship advancing at a constant speed with arbitrary heading angle in waves, taking into account the shape of the wave as well as the resultant ship motions in the six degrees. Such a method makes it possible to calculate the exact restoring forces and moments acting on the ship during the motion and, therefore, is applicable to the studying of large amplitude motions.
The effect of wave shape, fifth order gravity waves and ship oscillatory motions on the fluctuations of the lever arm and righting moment curves for the GAUL, is presented and compared with those obtained in still water.

Analysis of the variation of the various hydrodynamic terms during the ship motion has been carried out, using the Frank close-fit technique. Some of these analyses are given, to provide a clear illustration of the non-linear behaviour of such terms.

Computed results relating to the effect of heading angle, wave characteristics, ship speed, loading condition and wind moment on the amplitudes of ship motions, are presented with a particular emphasis on the rolling motion. The results indicate that each of these parameters can contribute to the occurrence of excessive roll motion in certain conditions and a combination of these effects may cause dynamical instabilities.

The program was used also to investigate the effects of tethering on roll behaviour of a model with a bias in roll in regular beam seas. The results are presented and compared to those of model experiments.
ACKNOWLEDGEMENT

In the course of preparing this study, I have received substantial help from others, to whom I extend my thanks.

My greatest debt of gratitude is owed to my supervisor, Mr N S Miller, who gave me the initial stimulus for this study and offered all the help that made it possible. His constant encouragement, generosity with time, constructive comments and inspiring discussions, have been of immeasurable value to me.

My grateful thanks are given to Mrs Clare MacEachen for her continuous assistance during the development of the computer software routines. I am also much indebted to the other members of the staff for very useful discussions on various matters associated with this study, especially Dr Neil Bose, Dr Atilla Incecik and Mr Mehmet Atlar.

Mrs Patricia Peters has made a valuable contribution by typing the thesis so expertly. I owe her my sincere thanks.

I would like to express my heartfelt gratitude to my wife and my children, Amira and Aya. They have surrounded me with a warm and joyful atmosphere which allowed me to carry on my research.

Finally, I am deeply indebted to The Arab Maritime Transport Academy (AMTA) for the financial support to undertake this study. This support is gratefully acknowledged.

N M ELSIMILLAWY
Glasgow, July 1984
DECLARATION

Except where reference is made to the work of others, this thesis is believed to be original.
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APPENDIX A

(A.1) Profile and Bodyplan of the Trawler GAUL 185

TABLES:

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NOMENCLATURE

a(a) : Two-dimensional sectional added-mass coefficient
A_{IJ} : Added-mass coefficients (I,J = 1,2,..6)
b^{(m)} : Two-dimensional sectional damping coefficient
b_{BK} : Breadth of bilge keel
b_1 : Linear term of roll damping coefficient
b_2 : Non-linear term of roll damping coefficient
B : Ship beam
B_{44} : Total roll damping coefficient in equivalent linear form
B_{BK} : Bilge keel component in B_{44}
B_E : Eddy component in B_{44}
B_{Eo} : B_E at zero forward speed
B_F : Frictional component in B_{44}
B_{Fo} : B_F at zero forward speed
B_{IJ} : Damping coefficients
B_L : Lift component in B_{44}
B_N : Normal force component in B_{BK}
B_S : Hull surface pressure component in B_{BK}
B_V : Viscous roll damping
B_W : Wave component in B_{44}
C_D : Drag coefficient of bilge keel
C_f : Frictional coefficient
C_M : Midship-section coefficient
C_0 : Initial value of any of motion equation terms (at t = 0)
C_P : Pressure coefficient
C_R : Eddy-making force coefficient
C_t : Value of any of motion equation terms at any time (t = t)
d : Sectional draught
D : Water depth
E : Energy dissipated because of B_V
NOMENCLATURE (Cont'd)

\( f \) : Correction factor for the flow speed increase at bilge

\( f^{(m)} \) : Two-dimensional sectional exciting force

\( F_I \) : Wave-exciting force and moment

\( F^{(m)} \) : Hydrodynamic force

\( F_0 \) : Maximum amplitude of exciting force

\( F_N \) : Froude number

\( g \) : Gravitational acceleration

\( G \) : Length along the girth

\( G_Z \) : Righting arm

\( h \) : Wave height

\( i \) : \( \sqrt{-1} \) or subscript designating imaginary part

\( I \) : \((I = 1,2...6)\) for surge, sway, heave, roll, pitch and yaw respectively

\( I_{I} \) : Moment of inertia in \( I^{th} \) mode

\( J \) : Subscript designating the mode of oscillatory motion \((J = 1,2...6)\)

\( K \) : Wave number

\( K_B \) : Reduced frequency factor

\( K_G \) : Vertical distance of centre of gravity above the keel

\( K_n \) : Coefficient depends on \( C_M \) values

\( K_N \) : The slope constant of the lift coefficient

\( l \) : Lever moment of bilge keel

\( l_{BK} \) : Length of bilge keel

\( l_o \) : The vertical distance between the roll axis and the point at which the representative attack angle is assumed to act

\( l_R \) : The distance between the roll axis and the centre of action of the lift force on a rolling ship hull

\( L \) : Length between perpendiculars

\( m \) : Subscript designating the mode of excitation

\( M \) : Mass of ship

\( O \) : Origin of the body coordinate system

\( O_G \) : The vertical distance between the centre of gravity and the still water surface

\( p \) : Hydrodynamic pressure

\( r \) : Subscript designating real part

\( r_{BK} \) : Distance between the roll axis and the bilge keel

\( r_s \) : Mean radius of the wetted surface of the ship
NOMENCLATURE (Cont'd)

RI : Hydrostatic restoring coefficients
RE : Symbol means 'the real part of'
s : Contour length
S : Wetted surface of ship
SL : Lateral area (= L.T)
t : Time variable
T : Draught of ship
XG : Distance between the cross-section and CG of the ship
x, y, z : Moving coordinates on ship
X, Y, Z : Fixed coordinates in space
V : Ship speed

δE_f : Energy dissipated because of B_F
Δ : Ship displacement mass
V : Volume of ship displacement
ε (m) : Phase lag or lead
ζ : Wave elevation
ζ_o : Wave amplitude
λ : Wave length
n_I, n_I, n_I : amplitude, velocity and acceleration of oscillation
(They are given also as n(m))
n_a : Average roll amplitude
n_o (m) : Maximum amplitude of motion
μ : Incident wave angle
π : Constant 3.142
ρ : Water density
ω : Wave frequency
ω_e : Encounter frequency
Introduction and Overview
INTRODUCTION AND OVERVIEW

1. INTRODUCTION

The dynamic stability of marine vessels has received significant attention in recent years. This has encompassed the gamut of vessel types and sizes. The primary motivation is that ships, notably small vessels, are still being lost. Some ships have capsized while fully meeting the rules of the International Maritime Organisation (IMCO), while others are operating safely although not fulfilling these requirements, see Reference (1). Therefore, it is a recognised fact that the stability criteria, which are in use today, have proved to be inadequate in some circumstances. This presents naval architects with a pressing need to understand the precise reasons for the capsizing of an intact ship in a seaway in order to establish better stability criteria concerning the operating conditions of ships.

Although during the last three decades, following the initial efforts of Grim(2) and Wendel(3), a considerable amount of effort has been devoted to the various aspects of the problem, progress has by no means been satisfactory and some essential aspects of the problem seem to be insufficiently investigated, see Reference (4). However, the problem of determining a vessel's stability characteristics under the dynamic conditions of a seaway remains unsolved. In addition, the
determination of the minimum stability required to prevent capsizing remains unknown, as well as the best means for presenting this information.

So far, the treatment of this problem suffers from the difficulty of predicting ship motion at large amplitudes due to the considerable amount of computations involved as well as the lack of knowledge on the dynamics and hydrodynamics.

With the advent of high-speed, high-capacity digital computing machines, it is no longer necessary to restrict the treatment of motion problems to unrealistic, linearised cases having a small number of degrees of freedom. Some of these machines can solve systems of extremely complicated nonlinear, ordinary differential equations with great rapidity.

Meanwhile, the recently developed programs for calculating the hydrodynamic coefficients of motion equations for unsymmetrically submerged cross-sections of a ship's hull, permits the calculation of such coefficients in the frequency as well as the time domain.

Accordingly, better methods of investigating the dynamic stability of ships can be carried out, without making many of the assumptions most current researchers are compelled to make in order to make the problem tractable. Such an investigation must apply all the experience and judgement gathered from earlier work. Thus, it would be useful to review, briefly, the essential aspects of the problem which will eventually guide us to the determination of methods of stability assessment.
2. THE PROBLEM WITH STABILITY CRITERIA

Stability standards are used today in a form which was suggested in the 1960's by Rahola in Finland on a foundation of classical naval architecture. All criteria, however complicated, can always be transformed into a GM requirement, which the ship masters can easily handle.

Present criteria are mainly of two types as illustrated in fig. (1.1) from Reference (5). They both refer to the still water righting lever curve of GZ values versus heel angle. The first criterion defines initial tangent, location of peak value, single ordinates, maximum heel angles and the like, while the second type measures the vessel's capability by expressing the environmental demand by an equivalent wind heeling lever to be applied against the capability. All quantities are relevant to the vessel's capsize safety and relate to physical conditions.

In spite of the basically correct qualities in these criteria and the large amount of work involved in quantifying these (see Reference (6)), accidents do still occur even for ships which comply with the specifications. Consequently, if all relevant parameters of the stability criteria have been included, the most remarkable omission, which can thus be attributed to the capsizing of ships, seems to be the wave environment and the dynamical behaviour in waves. As a result, a modification of the criteria is needed and rational stability criteria are understood to be those that can take into account the physical phenomena occurring during the ship's service and all external forces exerted on it and, so, give credit to good sea-keeping designs.
The development of such rational criteria is a difficult task because there are a variety of conditions which might lead to the capsizing of a ship. Therefore, a qualitative examination of casualty records will certainly serve towards a better understanding of the capsize phenomenon.

Fig. (1.1) The Stability Criteria

3 CASUALTY RECORDS

A better picture of the capsizing phenomena of ships can be recognised by careful analysis of casualty records and their statistics. A glance at casualty records, which have appeared in recent literature(1, 6-9), indicates that most of the casualties occur in situations where several high waves occur in a group and cause excessive motion. It is also demonstrated that, more than half of these casualties were under the action of following or quartering seas as shown in Fig. (1.2), Reference(10).
Fig. (1.2) - Qualitative Analyses of Casualty Records
Therefore, it can be concluded that such wave conditions can influence the instability. Moreover, the presence of waves can also produce a resonant motion, i.e. the ship in a seaway may be subject to dynamic instability. Accordingly, capsising can be considered as a particular event of a ship motion, happening to an intact vessel due to the action of winds, waves and ship dynamics.

The importance of this consideration lies in the attempt to relate the stability of a ship to its motion and so the stability requirements might be based on a certain physical picture of ship’s behaviour in a variety of situations which are dangerous from the point of view of capsizing. As a result, the discovery of such situations and their probability of occurrence becomes an important stage in determining the sources which lead a ship into these dangerous situations and, in turn, understanding the factors that degrade stability.

Since ship-motion experiments are extremely expensive and time consuming and since they require a large basin with the capability of producing random seas from any direction, it is not usually feasible to perform these experiments for individual ship designs. Therefore, it was necessary to develop a theoretical and numerical method for predicting the actual ship responses.

4. SHIP MOTION

The connection between the ship motions and its stability was recognised a long time ago (see fig. (1.3), Reference(11)) and through the past three decades, considerable advances have been
Historical development of the theory of seakeeping.

(Dates in parentheses indicate major publications)
achieved in the theoretical prediction of ship motions in a seaway, see Reference (1). The two-dimensional 'strip theory' which was initially put forward by Korvin-Kroukovsky et al(12,13) and developed by other authors, such as Salvesen, Tuck and Faltinsen(14), has been proved by both model tests and full scale trials to predict some ship motions in a seaway with acceptable accuracy.

However, the advances in ship-motion prediction have been concerned with phenomena which are amenable to a linearised analysis and where the properties of the equations of motions remain constant during the motion. Consequently, the non-linearities as well as the coupling in terms of the equations of motions are omitted and so only approximate solutions are available(15) which do not cover the extreme conditions associated with capsize.

Fortunately, most ships operate in less than extreme conditions most of the time and, consequently, the results of such analyses find direct and useful application to many of the engineering, operational and economic problems involved in the ship design process.

However, investigations into the causes of capsizing accidents and ship survivability in extreme sea conditions requires knowledge of the ship's response to waves of large amplitude where the linearising assumptions are no longer permissible. The simple linear relationships between important motion parameters no longer exist when the non-linearities become effective in different ways, see Reference(16), where many of the parameters involved are highly non-linear and interact together in a very complex manner.
By far the most common approach adopted in studying ship capsizing seeks, at least implicitly, to formulate a mathematical description of the process of capsize and to examine the significance of various relevant parameters in the resulting non-linear theory.

In formulating a representative mathematical model of coupled large amplitude rolling motions, some of the investigators employed linear sea-keeping equations, of Bishop et al(17), some others included ad hoc non-linear corrections for damping and restoring terms, of Blagovechinsky(18), Odabasi(10), whereas another group retained only restoring and excitation terms, of Kuo and Welaya(19).

Although these attempts may be justified on the grounds of gradual build up of an appropriate mathematical model, the analyses of the variation of terms of the equations of motion, such as added mass, inertia, damping and wave excitation with the variation of ship position as well as wave configuration(20-22), provide a clear illustration of the inappropriateness of the linearity assumptions for any of these terms. All of these terms vary considerably with time as the ship moves in a seaway and depend on ship geometry, speed, heading and sea severity.

In the light of the brief review presented here, the only viable alternative for the accurate prediction of the behaviour of a ship in waves of large amplitude is the examination of the step by step history of her motion resulting from numerical integration simulation of a set of non-linear differential equations, where the causal relationship between the environmental seaway and the resulting behaviour of the ship can be established and where the non-linearities of all the terms of the equation of motion can be taken into account. Such time domain solutions permit the extension
of the linear solutions into somewhat more severe motion regimes and may reveal some of the phenomena of the dynamic motion instabilities which are not apparent from a linear analysis.

From the practical point of view, the development of a computational method for the accurate prediction of the vessel's motion as well as the estimation of ship safety against capsizing in a seaway, is complicated and rather difficult for the following reasons:

a. Our present knowledge of the environment in severe sea conditions, in terms of physical description of breaking wave conditions and the nature of the wind conditions which may be associated with the waves is inadequate.

b. To perform a time simulation for a vessel's motion in six degrees of freedom is, by itself, difficult and requires a considerable amount of computations which require high-speed and high-capacity computers.

c. The choice of factors which may lead a ship to capsize in a seaway is broad and to include their effects in a time domain solution is not an easy task and will increase the amount of computations considerably.

d. It is still difficult to obtain expressions for the hydrodynamic coefficients such as added mass and damping and the programs available to calculate these coefficients are based on linear assumptions.
For the reasons stated above, it was necessary to minimise the amount of computations and, therefore, the influence of some interesting points such as green water on deck, steep and breaking waves and propeller action on the dynamic stability of the ship, were not taken into account in performing the time simulation. Initially, it was hoped to concentrate on the ship responses in regular sinusoidal waves where the programs available for calculating the hydrodynamic coefficients are expected to have a high level of accuracy. It would be desirable to be able to do the time simulation of the motion in an irregular seaway which is commonly thought of as a linear combination of sinusoidal waves. To do this directly leads to theoretical difficulties on the correct added mass and damping matrices to be associated with the motion at any one instant of time, although the exciting forces and moments can be obtained as the algebraic sum of the contributions from each individual wave. Alternatively, current non-linear spectral methods may be used to build up the response in the irregular seaway from the responses to individual waves but this is a complex operation open to considerable errors.

Although it was not expected from a time simulation of the vessel's motion with those limitations, to estimate the probability of capsize, it was necessary to perform such a simulation to discover, at least, the dangerous situations which may lead a ship to capsize and to advance existing knowledge about them. Consequently, the main objective of performing such a time simulation was the proper selection of these situations and so, to provide a more realistic means to understand the sources of occurrence of such situations. This understanding may then be used as a guidance for formulating mathematical models of capsizing as well as carrying out experiments on ship models.
The work presented here is an attempt to develop equations of ship motions that, in some respects, are more realistic and accurate in predicting the ship responses in seaway than those which have been formulated previously. Representative examples (23 to 33) of these latter papers, indicate that the variation in added mass, inertia and damping coefficients, are not taken into account, important parts of exciting forces and moments are omitted and, in some cases, the coupled terms of the combined motions are neglected. These examples generally neglect the relative motion of the ship in waves resulting from ship motion. These aspects, which are believed to have an important influence on the dynamic stability and ship behaviour, are taken into account in calculating the ship motion in five-degrees of freedom of a ship travelling in regular sinusoidal waves of any given length, height and direction by the computer program developed herein.

The development of the program was made general enough to be used for a wide variety of vessel types and sizes. Although the program does not include the effect of some factors such as green water on deck and rudder direction on the behaviour of the ship, it will be comparatively easy to extend the program to take account of these aspects.

With the aid of high speed computers, the direct computations of the terms of the equations of motion during the step-by-step integration were considered. The method used for programming mainly follows the ideas of Salveson, Tuck and Faltinsen (14), with slight modification to include viscous effects for the roll damping (34).
The outline of the theoretical computation procedure is as follows:

1. The ship's hull is divided into any odd number of sections and each section is represented by a set of offset values including the deck description.

2. The offsets representing the under-water shape of the hull are calculated during the motion taking into account the effect of wave configuration and the resultant ship motions on these offsets.

3. The instantaneous restoring forces and moments are calculated from the exact determination of the ship's displacement and its centre of buoyancy at each time step.

4. Considering the flow around each strip to be two-dimensional, the potential added-mass, inertia and damping coefficients are computed for each section by using the Frank close-fit method(35).

5. Exciting forces and moments caused by incident waves and diffraction effects are computed for the ship.

6. A modified form of the strip theory is used to account in the hydrodynamic coefficients of the equations of motion for the frequency, heading angle and ship speed dependencies(36).

7. The Runge-Kutta method is applied to solve the equations of motion through available routines in the computer library(37).
In order to build up the development of the computer program, it was necessary to obtain an accurate knowledge of the numerical methods available for calculating the many hydrodynamic factors appearing in the equations of ship motion. On the other hand, it was necessary to discover the order of importance of these factors so that by careful handling of the sensitive factors, while eliminating from consideration those terms which have little influence on the ship responses, the computing process could be speeded up as much as possible. Therefore, an analysis of the various hydrodynamic terms has been carried out and their classification according to importance and availability is considered to be one of the most significant aspects of the work presented herein because of its possible use as a guide in future dynamical stability research.

In performing these analyses, the determination of the numerical values of all the aspects presented here, was made for the trawler GAUL, a large modern stern trawler, Fig. (1.4), which disappeared in heavy seas off the North Cape of Norway, in February 1974. The analyses were made into this particular ship since it would appear that she was not lost as a result of inadequate intact stability or poor seakeeping qualities\(^\text{38}\), and it was concluded that the most probable cause of the loss was due to the effect of the severe waves. It was hoped that this study might disclose some reasons for this occurrence. Also, the many investigations carried out into her loss would provide the study with the necessary data and information.

The various chapters describe the investigation into each term in the equation of motion and how the complete data were built up into a full time simulation program. In Chapter 2, a computational
Fig. (1.4) General Arrangement of GAUL
technique is presented for calculating the instantaneous hydrostatic restoring coefficients for a ship advancing at a constant speed with arbitrary heading angle in regular sinusoidal waves, taking into account the resultant ship motion in six degrees of freedom. The co-ordinate systems adopted for this study are defined with the representation of ship's hull, ship oscillatory motions and wave surface. Some aspects of the influence of the relative motion of the ship among waves on the stability characteristics are discussed.

The analysis of the values of the potential added-mass, moment of inertia and damping coefficients for different ship cross-sections is illustrated in Chapter 3. Attention was focused on the variation of these values with the variation in draught and angle of heel.

Based on the analysis of the two-dimensional potential roll damping moments, it was decided to extend the work to introduce viscous roll damping. The viscous contribution to roll damping and the damping caused by bilge keels at differing draughts and roll amplitudes are described in Chapter 4 with the effect of ship forward speed and resulting damping due to lift.

Chapter 5 is devoted to the estimation of the amplitudes of the wave-exciting force and moment and the phasing of these excitations with the wave position along the hull of the ship. The effect of heading angle, wave length and oscillatory ship motion on these excitations are emphasised.

In Chapter 6, a computational method, developed by the author, to simulate the motions of the ship in regular sinusoidal
waves, is described. The method of approach and the mathematical model are presented. The influence of the different parameters on the motion of the ship is illustrated and computed results are presented. Also, an investigation into the effects of tethering on the roll behaviour of a model with a bias in roll in beam sea was carried out. The results are presented and compared to those of the model experiments. Finally, the conclusions achieved from this study are given in Chapter 7.

It should be mentioned that in the following chapters the graphs and tables appear at the end of each chapter as in some cases they are so numerous as to make reading difficult if presented in the text.

The computer programs developed to perform the calculations of the data used for the analyses presented in each chapter are described in separate reports with typical examples for running them (22, 39 to 42). The programs for individual variables are designed to run on the PDP11/40 computer in the Department of Naval Architecture and Ocean Engineering at the University of Glasgow but the computer time simulation program can only be run on a fairly large computer such as the ICL 2988 at the University of Glasgow.
CHAPTER 2

Restoring Forces & Moments
CHAPTER 2

RESTORING FORCES AND MOMENTS

1. INTRODUCTION

A complete analytical solution for the motion of a ship in waves requires first that the relationship between the environmental seaway and the resulting behaviour of the ship be established. This causal relationship is difficult to determine because the seaway, in the general case, defies simple description; ships do not have simple geometry and the resultant motions of the ship which are normally mutually coupled have complex effects.

A number of non-linear problems occur in various phenomena, related with the changed displacement of the ship and its centre of buoyancy during motion, in which the non-linearities of restoring forces and moments have to be taken into consideration\(^{(43)}\). It has been shown by Wendel, Paulling and others\(^{(44} \text{ to } 50)\) that there can be a significant variation in the roll restoring moment as a wave progresses along the ship's length as well as the change in this moment caused by large amplitude roll angles. On the other hand, the shape of the ship's hull may contribute to the stability of ships in a seaway. It is shown that there are small metacentric height changes at the crest and trough of a wave which are in opposite directions for Vee and Wall sided sections, see Reference\(^{(51)}\).
However, investigations of ship motions have heretofore been obtained only under the assumption of small motion amplitudes, in which case the restoring forces and moments are computed as though the instantaneous position of the ship differs but little from its mean position. Such an assumptions cannot be used in the present case where large deviations in position from mean are an essential feature of the phenomenon. Therefore, it was necessary, in starting the study of extreme ship motions, to establish a convenient procedure for calculating the exact position of the ship and its surrounding waves.

In this chapter, a computational method is presented to calculate, from the ship's hull offsets and the incoming wave characteristics, the wave shape intersection with the ship's hull in a time domain solution taking into account the oscillatory motion of the ship in six degrees of freedom as well as the ship speed and heading angle.

This method is used to calculate the instantaneous restoring forces and moments acting on a ship moving among waves as described by the author in Reference(41). The computational technique of this method is described herein briefly with the co-ordinate systems adopted for this study and the representation of ship's hull, ship oscillatory motions and wave surface.

Also, some aspects of the influence of the relative motions of the ship among waves on the stability characteristics are discussed. Interesting points regarding the effect of wave shape, oscillatory ship motion and heading angles on the stability variations are presented.
Furthermore, the investigation was extended to allow for fifth order gravity waves in order to explore the influence of wave shape on the stability of ships in a seaway.

2. SHIP'S HULL REPRESENTATION

As with any calculation on ship geometry, the hull must first be defined for the computer. The hull definition used in this study enables all the traditional shapes to be defined easily, with a relatively small number of points. The shape of the ship is represented by offsets taking into consideration the following:-

a. The offsets to be stored as input data should be independent of any ship loading condition, that is, any change in draught and trim should not require any further input data.

b. A system of hull definition be designed so that any kind of vessel can be suitably described by the waterlines and offsets.

c. The different hull features such as the rake and flatness of the keel and the presence of the bulwark along the sides of the ship, should be considered automatically from the hull offsets values.

The shape of the hull is represented by a set of offset points as follows:-

2.1 The hull is referred to a set of three orthogonal axes whose origin may be selected at any convenient points, provided that the
draught and centre of gravity of the ship are referred to these axes. Fig. (2.1) shows the co-ordinate system considered in this study with the plotting of the hull of the trawler GAUL.

2.2 The hull is divided into any odd number of sections, with equal intervals, and each section is represented by a set of offset values which may be taken at non-equidistant ordinates. Sections are assumed to be continuous round the contour but section slopes may be discontinuous at any interval end. Thus, all corners on the section, eg flat-keel, deck edges, etc must lie on interval ends. Fig. (2.2) illustrates the representation of the cross-section shape. The maximum number of the offset points for a cross-section is 25.

2.3 When there is no bulwark present on the whole length of the ship the last offset in each section is the deck edge offset. To allow for the presence of a bulwark, the following procedure is adopted:-

   a. For sections where the bulwark is present, the last two offset values apply to the deck edge and the top of the bulwark, ie the difference in the z-values

   b. For sections where there is no bulwark, the uppermost waterline which is at the deck edge height above base is repeated in the data (both z and y values), see fig. (2.2).

2.4 Within the framework of these rules, data can be fitted to represent any ship hull. The offset values of the ship cross-sections
are arranged and stored in data files with the following sequence:

SHIP LENGTH (LBP), SHIP BREADTH
NUMBER OF SECTIONS (odd number)

Then, for each of the equally spaced sections, starting at No 1 at A.P.
NUMBER OF OFFSET POINTS OF THE SECTION (max 25)
LONGITUDINAL DISTANCE OF THE CROSS SECTION FROM THE DEFINED ORIGIN
Z-COORDINATES
Y-COORDINATES
and so on.

A typical offset data file prepared following this sequence for the trawler GAUL is given in Appendix (A), with the plotting of the profile and the body plan corresponding to these offsets.

3. DEFINITION OF THE SHIP OSCILLATORY MOTIONS AND HEADING ANGLES

The oscillatory motions of the ship are defined by the frame G-xyz relative to the co-ordinate system considered for the ship's hull. The translatory displacements in the x, y and z directions with respect to the origin were considered to be \( n_1, n_2 \) and \( n_3 \), respectively, where \( n_1 \) is the surge, \( n_2 \) is the sway and \( n_3 \) is the heave displacement. Similarly, let the angular displacement of the rotational motion about the x, y and z axes be \( n_4, n_5 \) and \( n_6 \), respectively, where \( n_4 \) is the roll, \( n_5 \) is the pitch and \( n_6 \) is the yaw.
angle. The oscillatory motions relative to the co-ordinate system are shown in fig. (2.3). The heading angle (μ) from 0°-180° was defined relative to the co-ordinate system and is illustrated in fig. (2.4).

4. WAVE SURFACE REPRESENTATION

The relationship between the wave surface and the ship's hull in the seaway was established, taking into account the result of the oscillatory motion of the ship in the six degrees of freedom as well as the ship speed and heading angle. The influence of these parameters on this relationship is considered by including their variations in the equation of the wave surface altitude relative to the body co-ordinate as follows:

4.1 To begin with, let the ship be subjected to a regular sinusoidal wave train with crests lines parallel to Y-axis, i.e. a following sea, as shown in fig. (2.5). Let S and P be the intersection points between the wave surface and any arbitrary cross-section contour. The altitudes of points S and P relative to the origin O can be obtained by the following equation:

\[ \zeta(x,t) = \zeta_0 \cos(KX_G - \omega t) + T \]  \hspace{1cm} (2.1)

where \( \zeta_0 \): is the wave amplitude at the water surface

\( K \): is the wave number

\( X_G \): is the distance between the cross-section and CG of the ship

\( \omega \): is the angular wave frequency
4.2 Let the angle of incidence of the wave be designated by \( \mu \) and let the wave progress in the positive \( x \)-direction as shown in fig. (2.6). Then, apart from the ship oscillatory motions, the wave profile in the body co-ordinate is,

\[
\zeta(x,y,t) = \zeta_0 \cos(Kx \cos \mu - Ky \sin \mu - \omega t) + T \tag{2.2}
\]

The wave equation can be interpreted by noting that the term \( Kx \cos \mu \) determines the wave form in the two-dimensional domain and that the term \( Ky \sin \mu \) represents the phase shift of the wave at \( x = 0 \) and \( y = y \) from the crest of the incident wave at the origin.

4.3 Considering the rotation of the ship about the centre of gravity, equation (2.2) was modified to give the altitude of any point at the wave surface, relative to the origin \( 0 \), at any instant, taking into account the relative motion of the ship in the six degrees of freedom as follows – see figs. (2.5), (2.6) and (2.7):

\[
\zeta(x,y,t,\eta) = \zeta_0 \cos \left[ \frac{K}{(X_G + \eta_1) \cos (\mu - \eta_6) - (Y_O + \eta_2) \sin (\mu - \eta_6)} \right] - \omega e t + Y_O \tan \eta_4 - X_G \tan \eta_5 - \eta_3 \\
+ (T - KG) \left( \frac{1}{\cos \eta_4 - \eta_1} \right) + T \tag{2.3}
\]

where, \( I \) : mode of motion from 1 to 6

\( \omega e \) : encounter frequency = \( \omega(1 - \omega V \cos \mu / g) \)

\( V \) : ship speed

\( Y_O \) : \( Y \)-Coordinate of the intersected points
KG : Vertical distance of centre of gravity of the ship above the keel.

The basic idea of the wave altitude conversion to take into account the angular displacements of the ship is illustrated in fig. (2.7).

5. THE COMPUTATIONAL TECHNIQUE

Since the resultant motions of the ship are normally coupled, the ship's hull was considered stationary in space for each time step, and the equation to the wave surface was modified to take into account the oscillatory and drift motions of the ship in the six degrees of freedom at the relevant time step, including the effect of the ship's forward speed. The computational method can be briefly outlined as follows:-

5.1 The intersection points between the wave surface and cross-sections contours alongside the ship are determined by an iterative process between the known cross-section offsets and the equation of wave surface elevation and assuming the segment between the two offset points, below and above the intersection point, is a straight line. In the case when the deck edge, or bulwark, is immersed, a flat deck is assumed to find the intersection point at this side.

5.2 Provision was made for all foreseeable intersection cases between a normal ship's cross-section and the wave surface in a seaway. Moreover, the complete immersion and emergence of any ship cross-section as well as its corresponding position was taken into
consideration. Figure (2.8) illustrates the plotting of different intersection cases between the wave surface and different cross-section of the trawler GAUL.

5.3 In order to determine the intersected wave shape athwart the ship, the transverse distance between the intersection points for each cross-section contour is divided into 10 equally-spaced intervals, see fig. (2.9). These give the y-coordinates of the transverse wave representation points and on substitution into the equation of wave surface elevation, determine the z-coordinates of these points. Figure (2.10) shows the plotting of the intersection between the wave surface and the hull of the trawler GAUL for different heading angles.

5.4 The wave shape intersection with each cross-section and the immersed portion of that cross-section are defined by a set of points as shown in fig. (2.9). Trapezoidal rule integration through these points is used to calculate the cross-sectional area up to the wave trace and its centroid as well as the displacement of the ship and the centre of buoyancy. Figure (2.9) shows the plottings of the points representing the immersed portions of the cross-sections with the centroid marked for the cases presented in fig. (2.8). The procedure of calculating the cross-sectional area up to the wave trace is illustrated in fig. (2.11).

6. VARIATION OF RIGHTING ARMS OF STABILITY IN WAVES

The variation of the wave-induced righting arm is dependent upon the geometry of the ship and the position and size of the wave.
In order to emphasise the effect of this variation on the stability of the ship, calculations were made for the trawler GAUL both in still water and waves. A vertical shift was made to balance the ship in waves in order to give the corresponding displacement in still water in the upright condition. The results obtained from this investigation are illustrated below.

6.1 In examining the stability of the trawler GAUL in still water, the calculations were made for the stability condition at the time of her loss, similar to that given in Reference(38) for a displacement of 1547 tonnes, a KG of 5.45m and for draughts of 4.523m aft and 3.913m forward. Figure (2.12) shows the plotting of the calculated GZ-curve for this condition with the effect of considering the bulwark and part of the forecastle deck on this curve.

6.2 Considering following seas, the wave length and wave height were taken equal to the ship's length and 1/20 of the wave length respectively and the positions of the wave along the ship were considered for the crest and trough at amidships as shown in fig. (2.13.a). The GZ-curves obtained for each wave position with the curve for the still water condition are given in fig. (2.13.b).

6.3 Further calculations of the GZ-curves were carried out for the ship in beam seas for wave length equal to two times the beam with a height of 1/20 of the ship's length. The positions of the wave across the ship were considered for the crest and trough at the centreline of the ship as shown in fig. (2.14.a). The GZ-curves for these conditions with the curve for the still water condition are shown in fig. (2.14.b).
7. INFLUENCE OF OSCILLATORY MOTIONS ON RIGHTING MOMENT

Heaving and pitching are of special interest when investigating the influence of the oscillatory motions on stability in a seaway. However, at this stage the ship is considered to oscillate from its original position in still water by any arbitrary displacement to emphasise the influence of such displacement on the righting moment.

7.1 Heaving Motion

According to the co-ordinate system considered for the ship oscillatory motion, it is assumed that the ship is given a heaving moment of ± 0.5m along the Z-axis. The GZ-curves calculated for these positions with the curve for the original position are shown in fig. (2.15.a), from which it seems that the effect of heaving motion on ship stability is negligible but the righting moment is not a function of GZ only. Therefore, the effect of heaving motion on the righting moment were calculated and shown in fig. (2.15.b).

7.2 Pitching Motion

Similarly, the effect of pitching motion on the righting moment was calculated by considering an angular displacement of ± 10° from the equilibrium position of the ship in still water. Figures (2.16.a) and (2.16.b) illustrate the effect of pitching motion on GZ and righting moment curves. It should be noted that the ship was originally trimmed 0.61m by the stern in the equilibrium position.
8. INFLUENCE OF WAVE STEEPNESS ON THE VARIATION OF THE RIGHTING ARM

The amount of righting arm variation of a ship moving in waves depends mainly on the wave steepness, i.e., the wave height to wave length ratio \( h/\lambda \) and wave length. Perhaps the naval architect's standard wave of height \( \lambda/20 \) is the most popular one but regarding the capsizing phenomena of a ship in a seaway, a large value of wave steepness should be taken into account. Meanwhile, choosing only one wave height, even if taken at an extreme value, can only yield a rough estimate of the expected righting arm variations but will not cover the whole pattern, see Reference(28).

However, in fig. (2.17) a comparison is made between the following proposed values:

a. \( h = \frac{\lambda}{10 + 0.05\lambda} \) (m)

This formula is commonly used in Germany(28) and it yields larger wave steepness for shorter wave lengths. This shifts the maximum reduction in GZ towards smaller \( \lambda/L \) values.

b. \( h = 0.607 \sqrt{\lambda} \) (m)

This value was used by Miller et al(52) and by the US Navy for their vessels. They believe that it is a realistic assumption for small vessels that will normally operate in short and steep waves.

c. \( h = \lambda/7 \) (m)
This was recommended by Fraser et al. They also recommended that this should be used with a wave length equal to 0.75 times the waterline length of the ship.

It is obvious from fig. (2.17), in which a wave length to ship length ratio, \( \lambda / L \), equal to 1.0 is used, that the variation between different formulae is more pronounced when the crest is amidships.

Also, the effect of wave length on the GZ-curve variation was investigated for waves of steepness of \( \lambda / 7 \). The calculations were carried out for \( \lambda / L \) ratios of 0.75, 1.0, 1.25. Figure (2.18) illustrates the effect of these ratios on GZ-curve. It can be seen that the variation of the wave length has a slight effect on GZ-curve.

9. INFLUENCE OF WAVE SHAPE ON SHIP RIGHTING ARM

The effect of wave shape on the stability of the ship was explored by considering the shape of the fifth order gravity waves. A computer program has been developed to calculate the coefficients of the equation of the wave profile, see Reference(42). Figure (2.19) shows a comparison of wave forms between the sinusoidal and fifth order gravity waves for different water depth/wave length, \( D/\lambda \), ratios.

The computer program, developed to perform the calculations of restoring coefficients in sinusoidal waves, was modified to perform these calculations for fifth order waves. The
calculations were carried out for following seas, for wave positions along the ship similar to those considered for sinusoidal waves, with water depth of 10m. The influence of wave position on the righting arm curves is shown in fig. (2.20) with the GZ-curves for sinusoidal waves and still water conditions.

Figure (2.20) indicates that the differences in righting arm between the use of linear wave theory and fifth order theory, for the following sea case, are greatest for the trough amidships when the righting arm becomes less than the still water value for a range of heel angles between $30^\circ$ and $60^\circ$.

10. CONCLUDING REMARKS

10.1 Wave shape and dimensions and ship oscillatory motions produce a considerable variation of righting moment and the checking of stability in a seaway is essential especially for small ships in all loading conditions.

10.2 The influence of the wave on the restoring forces stems from its effect on the righting arm, while the influence of the ship oscillatory motions stems from the altered righting arm and the immersed volume of ship's hull.

10.3 Considering the bulwark shape in the stability calculations produces a slight increase in the righting moment before its immersion.
10.4 In the case of the GAUL, the GZ-curve in a following sea with a wave height equal to $\lambda/20$ and a wave length $L$, is reduced by the order of 30%. In a beam sea a similar order of reduction can occur for a crest at the centreline and a wave length of twice the beam.

10.5 The wave length and steepness have a considerable influence on the variation of GZ values. For a wave length equal to the length of the ship and the wave crest amidships a great reduction in GZ value can occur.

10.6 The difference in righting arm between the use of linear wave theory and fifth order wave theory, for the following sea case, are greatest for the trough amidships case when the righting arm becomes less than the still water value for a range of heel angles between 30° and 60°.
Fig. (2.1) Co-ordinate system of ship's hull

Fig. (2.2) Co-ordinates of the ship cross-section
Fig. (2.3) Definition of oscillatory motions

Fig. (2.4) Definition of the heading angle, $\mu$
Fig. (2.5) Wave representation

Fig. (2.6) Space and body coordinate system
Fig. (2.7) Effect of angular displacements on wave surface altitude relative to ship's hull
Fig. (2.8) Some intersection cases between wave surface and different cross-sections of the trawler GAUL
Cross-section No 17

Fig. (2.9) Representation of the immersed portion of the cross-section and its centroid
Fig. (2.10) The intersection between the wave surface and ship's hull
Fig. 12.11) The procedure of calculating the cross-sectional area up to the wave-surface.
\[ \Delta = 1547 \text{ tonnes} \]

\[ KG = 5.45 \]

\[ Trim = 0.61 \]

**Effect of Bulwark**

**Fig. (2.12) GZ-curve of the trawler GAUL**
Fig. (2.13.a) Wave position along the ship

Fig. (2.13.b) GZ-curves for various wave positions
Fig. (2.14.a) Wave positions across the ship

Fig. (2.14.b) GZ-curves in beam seas
Fig. (2.15.a) Effect of heaving motion on righting arm

Fig. (2.15.b) Effect of heaving motion on righting moment
Fig. (2.16.a) Effect of pitching motion on righting arm

Fig. (2.16.b) Effect of pitching motion on righting moment
Fig. (2.17) Effect of wave steepness on GZ-curve
Fig. (2.18) Effect of wave length on GZ-curve
Fig. (2.19) Comparison of wave forms.
Fig. (2.20) Effect of fifth order gravity wave position on GZ-curves
CHAPTER

3

The Hydrodynamic Coefficients
1. INTRODUCTION

Whenever a ship's responses to wave excitation are estimated the hydrodynamic actions of the sea have to be specified. A knowledge of the many hydrodynamic derivatives in the equations of motion, such as added-mass and damping coefficients, is a primary requirement for a study of the motions of the ship.

On the other hand, in order to perform a time simulation of the vessel's motion, it is necessary to have a complete record of the variation of the hydrodynamic coefficients with variation in draught, angle of heel and trim. Also, when considering the stability of the ship under the effect of a strong wind, the estimation of these coefficients in the heeled condition become essential, see Reference(55).

Meanwhile, there has been a suggestion (Bishop and Price)(16) that linear theory is adequate to predict possible instabilities and it is, therefore, of interest to see how far the linear assumptions hold true so far as added-mass and damping are concerned.

Accordingly, it was important to carry out a complete analysis of the various hydrodynamic terms and to classify them
according to importance and characteristics. This chapter gives some of these analyses regarding the effect of the variation in draught and angle of heel of the trawler GAUL on the values of the hydrodynamic coefficients for heave, sway and rolling motions.

Since ship motions were to be predicted by applying the strip method, which uses the two-dimensional characteristics of each cross-section of the ship, it was necessary to use one or other of the hydrodynamic theories to calculate the two-dimensional hydrodynamic coefficients. These coefficients depend on the ship's shape, the frequency of oscillation of the ship and the time history of the motion\(^{(56)}\). As the cross-sections of the ship are asymmetrical in some cases, it was also necessary to use a hydrodynamic method employing some suitable representation of the hull shape in heeled conditions.

However, the most common procedures, for calculating the frequency dependent 2D hydrodynamic coefficients, are the Lewis form technique and the Frank close-fit method, see Reference\(^{(57)}\). In the Lewis form technique the submerged cross-sectional curve is described by three parameters, i.e. the sectional beam, draught and area, see Reference\(^{(58)}\). The method is applicable for normal ship sections but has limitations, see Reference\(^{(59)}\) and does not deal with asymmetrical cross-sections. Therefore, the Frank close-fit method was used to perform the analysis presented in this chapter since it can be applied to any cross-sectional shape. This method only provides the potential hydrodynamic coefficients.
2. THE FRANK CLOSE-FIT METHOD

W Frank (1967) introduced a method in which the required velocity potential is represented by a distribution of sources over the submerged cross-section (35). The unknown function of the density of the sources along the cylinder contour is determined from the integral equations obtained by satisfying the kinematic boundary condition over the submerged cross-section. The hydrodynamic pressures are obtained from the velocity potential by using the linearised Bernoulli equation. Integration of these pressures over the immersed portion of the cylinder yields the hydrodynamic forces and moments.

The hydrodynamic force $F(m)$ induced by the motions of the ship has two components which are linearly proportional to acceleration and velocity of motion $\eta(m)$, respectively, the added mass $a(m)$ and the damping $b(m)$ coefficient as given by the following form:

$$f(m) = -a(m) \dot{\eta}(m) - b(m) \ddot{\eta}(m)$$  \hspace{1cm} (3.1)

The majority of the methods for calculating these coefficients use the strip theory which has been recognised as the most practical tool in terms of the application of the method to ship forms which can be assumed to be slender.

Salveson et al (14) employed the Frank close-fit method in calculating the two-dimensional hydrodynamic forces on a ship having a large bulbous bow and it was found that the amplitude and the phase of pitch and heave motions in head waves calculated by the strip theory agreed quite well with the experimental values.
3. **CALCULATIONS OF THE HYDRODYNAMIC COEFFICIENTS**

A computer program, utilising the Frank close-fit technique, developed by M Atlar\(^{60}\), was used to calculate the hydrodynamic coefficients. The program calculates the non-dimensional pressures, added mass, inertia and damping coefficients of a horizontal cylinder oscillating in heave, sway or roll while located in the free surface for a range of encounter frequencies designed to cover all conditions.

The analytical technique used in this program to calculate these coefficients involves distributing two-dimensional pulsating sources on the boundary of the ship section using Green's theorem\(^{61}\). This is applicable to any shape of cross-section. The method was used to obtain the motion-induced hydrodynamic forces and moments on the strip section in three modes of oscillation in calm water.

The three modes are, translational heave and sway and rotational roll and can be expressed by the following equation:

\[
\eta(m) = \eta_0(m) \cdot \sin(\omega t)
\]

where \(\eta_0(m)\) : is the maximum of the motion displacement \(\eta(m)\)

\(m\) : is mode of oscillation and takes 2, 3 and 4 for sway, heave and roll.

In order to use this program it was necessary to calculate the input data giving the shape of each ship cross-section relative to an axis parallel to and normal to the waterplane. Therefore, a
computer program has been developed to perform this operation at any
draught and angle of heel, see Reference(40), to give the analysis
presented herein.

4. THE ANALYSIS OF THE HYDRODYNAMIC COEFFICIENTS IN THE
UPRIGHT CONDITION

In order to emphasise the effect of the draught variation on
the hydrodynamic coefficients the computations were carried out for
the different cross-sections of the trawler GAUL with different
draughts starting from the 1m draught up to the maximum height of each
cross-section. The maximum heights for the different ship
cross-sections, including the bulwark, are shown in fig. (3.1).

4.1 The values of added mass, inertia and damping coefficients
for the midship section (cross-section No 5) are shown graphically in
figs. (3.2), (3.3) and (3.4) for heaving, swaying and rolling motions
for a range of encounter frequencies of 0.1 to 1.0 rad/sec. It can be
seen that for rolling motion, fig. (3.4.a), the added mass moment of
inertia becomes a very small value at the 5 metre draught.
Furthermore, fig. (3.4.b) indicates that the damping moment
coefficients reach zero at the same draught for all the encounter
frequencies.

4.2 The values of the hydrodynamic coefficients for heaving and
swaying motions are presented graphically along the length of the
ship in figs. (3.5.a) and (3.5.b) for a draught of 5 metres. For
rolling motion, it was noticed that there was a dramatic variation in
the distribution of the hydrodynamic coefficients along the ship
length when changing the draught by ±1 metre as shown in fig. (3.6). Again, it can be seen that for some cross-sections at each draught there are small values of added mass moment of inertia and zero damping and such cross-sections shift as the draught is changed.

4.3 Based on the above analysis, the variation of the hydrodynamic coefficients for rolling motion had a special interest. Therefore, the added inertia and damping moment coefficients for rolling motion were analysed for nine different cross-sections along the ship length for different draughts and the results are shown in fig. (3.7.a) and (3.7.b). From this data figs (3.8.a) and (3.8.b) have been produced. These show all the points of minimum added moment of inertia and zero damping moment coefficients. It can be seen that these points represent a shape which looks like a wave profile which the ship may be subjected to during its motion. It is to be remembered that this is only the potential damping and viscous effects will also be important but the fact that the damping can be zero for certain wave profiles may have a significant effect on its roll motion behaviour.

5. THE ANALYSIS OF THE HYDRODYNAMIC COEFFICIENTS IN THE HEELLOWED CONDITION

The variation in the potential added mass, inertia and damping coefficients with the variation in angle of heel was calculated for each ship cross-section at a number of draughts for heaving, swaying and rolling motions. This section gives some of these items for the midship-section (cross-section No 5) and for more data see Reference (21).
5.1 The computations were carried out for each cross-section with different angles of heel for different draughts. The variation of angle of heel was taken starting from \( 10^\circ \) to \( 40^\circ \) with interval \( 10^\circ \). The plottings of the under-water shapes of cross-sections No 2, 5 and 9 at different draughts and angles of heel are shown in fig. (3.9).

5.2 The values of the hydrodynamic coefficients for the midship-section for different heeling angles are shown graphically in figs. (3.10), (3.11) and (3.12) for heaving, swaying and rolling motions for a range of encounter frequencies of 0.1 to 1.0 rad/sec.

5.3 Figures (3.10.a) and (3.10.b) indicate that for heaving motion, the values of added mass and damping coefficients increase as the angle of heel increases, while figs. (3.11.a) and (3.11.b) indicate that for swaying motion the values of added mass increase and the values of damping decrease with the increase of heeling angle. For rolling motions, figs. (3.12.a) and (3.12.b) indicate a considerable increase in the added inertia and damping coefficients when the heeling angle exceeds \( 10^\circ \). Also, it can be seen that large angles of heel produce very large values of these quantities.

5.4 The broad conclusions from these analyses for the other cross-sections are the same as those for cross-section No 5 with some differences in the variation of added mass, inertia and damping coefficients due to the effect of the cross-section shape, see Reference(21).
6. EFFECTS OF DECK IMMERSION

The hydrodynamic coefficients values of the under-water shapes when the deck is immersed during the rolling motion also have been taken into account. Figure (3.13.a) shows the under-water shape of cross-section No 1 at 40° angle of heel considering the bulwark shape. In this case, the damping of the bulwark is similar to the effect of bilge keels, i.e., it is due primarily to viscosity and vortex shedding(62) which are not considered in the calculation of the potential hydrodynamic coefficients. Therefore, if the bulwark shape is included in the relevant ship's hull for some reason, the deck immersion of any cross-section will be considered without representing the bulwark shape as shown in fig. (3.13.b).

6.1 However, figs (3.14), (3.15) and (3.16) demonstrate the effect of deck immersion on the values of the hydrodynamic coefficients for heave, sway and rolling motions. It can be seen that the deck immersion causes a drop in these values for heaving and rolling motions, while in swaying motion the damping coefficients increase with the immersion of the deck.

7. INFLUENCE OF OSCILLATORY MOTIONS ON THE HYDRODYNAMIC COEFFICIENTS

The influence of heaving, rolling and pitching motions on the values of the added mass, inertia and damping coefficients was investigated for the trawler GAUL. The ship was considered to oscillate from its original position in still water by a considerable displacement and the new offsets representing the immersed hull of
the ship at the relevant displacement were calculated to emphasise
the influence of such displacements on the hydrodynamic coefficients.

7.1 Heaving Motion

According to the co-ordinate system considered for the ship
oscillatory motion, it is assumed that the ship is given a heaving
movement up to ± 1.2m with ± 0.3m intervals along the Z-axis. The
effect of these displacements on the hydrodynamic coefficients is
shown in fig. (3.17). It can be seen that the values of the
hydrodynamic coefficients for heaving and sway motions decrease as
the heaving displacement increases, while for rolling motion these
values increase.

7.2 Rolling Motion

The effect of rolling on the hydrodynamic coefficients was
calculated by considering an angular displacement up to ± 40° with
10° intervals from the equilibrium position of the ship in still
water. Figure (3.18) illustrates the effect of heeling angle on the
values of these coefficients. In this case, the rolling displacement
has a slight influence on the coefficients values for heaving and
swaying motions. For rolling motions these values rapidly decrease
once the ship is heeled from the upright position.

7.3 Pitching Motion

Similarly, the ship was given an angular displacement about
the y-axis up to ± 10° with ± 2.5° intervals and the effect of such
displacements on the hydrodynamic coefficients is shown in fig.
It can be seen that the values of these coefficients decrease with the pitching displacement of the ship to any direction.

8. **CONCLUDING REMARKS**

8.1 The variation in draught and angle of heel produce large changes in the added mass, inertia and damping coefficients and any theory of motion behaviour must take account of these variations.

8.2 For certain wave configurations the potential roll damping moment will be zero and the added moment of inertia becomes very small. This makes it necessary to calculate the instantaneous hydrodynamic coefficients for rolling motion in performing a time simulation of the ship's motion.

8.3 The immersion of the deck during ship motion has a considerable influence on the values of the hydrodynamic coefficients and must be taken into account.

8.4 Although the hydrodynamic coefficient for an individual section does not vary greatly with heave and sway but is very non-linear with roll, when integrated for the whole hull the reverse is true, ie there is a considerable variation in the hydrodynamic coefficients with heave and sway and a much smaller variation with roll.
Fig. (3.1) Maximum Deck at Side Heights of the Ship's Hull
Fig. (3.2.a) ADDED-MASS VALUES FOR HEAVING MOTIONS

Fig. (3.2.b) DAMPING COEFFICIENTS FOR HEAVING MOTION
Fig. (3.3.b) Damping Coefficients for Swaying Motion

Fig. (3.3.a) Added-Mass Values for Swinging Motions
Fig. (3.4.b) Damping Moment Coef. for Rolling Motion

Fig. (3.4.a) Added Mass Moment of Inertia for Rolling
Fig. (3.5.a) DISTRIBUTION OF ADD mass AND DAMPING COEFF. OF HEAVING MOTIONS ALONG THE SHIP LENGTH.

Fig. (3.5.b) DISTRIBUTION OF ADD mass AND DAMPING COEFF. OF SWAYING MOTIONS ALONG THE SHIP LENGTH.
Fig. (3.6.a) Added mass moment of inertia for rolling motion

Fig. (3.6.b) Damping moment coefficient for rolling motion
Fig. (3.7.a) Variation in the sectional hydrodynamic coefficients for rolling motion with the variation in draught
Fig. (3.7.b) Variation in the sectional hydrodynamic coefficients for rolling motion with the variation in draught
Fig. (3.8.a) POINTS OF MINIMUM ADDED-MASS MOMENT OF INERTIA FOR ROLLING MOTION.

Fig. (3.8.b) POINTS OF ZERO DAMPING MOMENT FOR ROLLING MOTION.
Fig. (3.9) Under-water shapes of different cross-sections
Fig. (3.10.a) ADDED-MASS VALUES FOR HEAVING MOTIONS
(5m Level draft, Units Tonnes m\(^{-1}\))

Fig. (3.10.b) DAMPING COEFFICIENTS FOR HEAVING MOTION
(5m Level draft, Units Tonnes m \(^{-1}\) s m\(^{-1}\))
Fig. (3.11 a) ADDED-MASS VALUES FOR SWAYING MOTIONS
(At 5.0m Level Draught - Units Tonnes m\(^{-1}\))

Fig. (3.11 b) DAMPING COEFFICIENTS FOR SWAYING MOTION
(At 5.0m Level Draught - Units F m\(^{-1}\) s m\(^{-1}\))
Fig. (3.12.a) ADDED-MASS MOMENT OF INERTIA FOR ROLLING
(at 5m Level Draft - Units Tonnes m² m⁻¹)

Fig. (3.12.b) DAMPING MOMENT COEFF. FOR ROLLING MOTION
(at 5m Level Draft - Units Tonnes m² rad s⁻¹ m⁻¹)
Fig. (3.13.a) The immersion of deck and bulwark

Fig. (3.13.b) Immersion of the deck
Fig. (3.16.a) ADDED-MASS MOMENT OF INERTIA FOR ROLLING

Fig. (3.16.b) DAMPING MOMENT COEFF. FOR ROLLING MOTION
Fig. (3.17) The effect of heaving motion on the hydrodynamic coefficients for heave, sway and roll motions.
Fig. (3.18) The effect of rolling motion on the hydrodynamic coefficients for heave, sway and rolling motions.
Fig. (3.19) The effect of pitching motion on the hydrodynamic coefficients for heave, sway and rolling motions.
CHAPTER 4

The Roll-Damping Moment
CHAPTER 4

THE ROLL DAMPING MOMENT

1. INTRODUCTION

Since rolling motion is one of the most important responses of a ship in waves and, if it becomes of large amplitude, may have serious consequences, some examination of the terms of the rolling equation is necessary. The degree to which current ship motion programs can be used to predict roll motions has been the subject of much debate in the past and, was recently discussed at the 14th ITTC.

The roll motion of a ship can be determined by analysing the various kinds of moments acting on the ship, inertia moment, roll damping moment, restoring moment, wave excitation and other moments caused by other modes of ship motion. Among these, the roll damping moment has been considered to be the most important term that should be correctly predicted. The designer must ensure through bilge keels or other means an adequate level of damping to secure the safety of a ship. There is a need to obtain a better understanding of the damping of ship motions in waves.

The recent development of the 'strip method' has made it possible to calculate almost all the terms in the equations of ship motions in waves with practical accuracy, except for the roll damping(63). Difficulties in predicting the roll damping of ships
arise from its non-linear characteristics, due to the effect of fluid viscosity, as well as from its strong dependence on the forward speed of the ship. Moreover, the presence of bilge-keels makes the problem even more complicated. Therefore, theories and experimental results are often combined to deal with roll damping.

Data on radiation forces acting on the ship hull, including roll damping, have been accumulated through forced oscillation tests carried out by Vugts, Fujii and others(63). These experiments have demonstrated that there are still considerable differences between measured values of roll damping and those predicted by existing methods.

In this period, much effort has been expended to obtain improved data on ship roll damping. For example, works by Bolton, Lofft and Lugovski et al(63), concerning the effect of bilge keels, Gersten's(63) studies on the viscous effects and free roll experiments by Takaishi et al(63) and Tanaka et al(63).

Extensive and systematic work in Japan has been carried out recently for the prediction of roll damping. Through experimental investigations a method for predicting the roll damping of an ordinary ship hull form has been proposed by a number of Japanese authors(64 to 67). In this method damping is divided into several components. Then the total damping is obtained by summing up these damping components predicted separately. This attempt appears to have had a certain success for ordinary ship hull forms and the values predicted by this method were found in fairly good agreement with the experimental ones for ship models(67). Empirical formulae are presented for each of the damping components except wavemaking. The
method also presents an empirical formula for correcting the roll damping components for a given forward speed.

It is important to mention that this method describes the expressions and formulae so that their values can be calculated promptly once the particulars of a ship are given. This makes the use of this method convenient for this study since many of the data required to obtain the roll damping components has already been calculated for the simulation program developed herein. Therefore, it was decided to use this method to estimate the roll damping terms in predicting the behaviour of a ship among waves.

In order to use a computer program (presented in Reference(63)) utilising this method and to check its performance before including it in the simulation program, it is necessary to calculate the form coefficients and particulars of the ship. Therefore, a computer program was developed to calculate this data from the hull offsets at any loading condition, see Reference(40).

However, in this chapter, the analysis of viscous contribution to roll damping as well as the damping moment caused by the bilge-keels for the trawler GAUL are presented. The calculations were carried out for different draughts, roll amplitudes and encounter frequencies, see Reference(68). Also, the effect of ship forward speed on the roll damping components is emphasised for a wide range of Froude Number (FN).
2. THE COMPONENTS OF THE ROLL DAMPING COEFFICIENTS

The damping term in the rolling equation comprises linear and non-linear terms which are expressed as \( b_1 \dot{\eta}_4 + b_2 |\dot{\eta}_4| \). Reference(69). It is assumed that the non-linear roll damping can be represented by an equivalent linear term \( B_{44} \dot{\eta}_4 \) and that the linear roll damping coefficient \( B_{44} \) is made up of five components: a) due to waves \( B_w \), b) due to the friction \( B_F \), c) due to eddies \( B_E \), d) due to bilge-keels \( B_{BK} \). The effect of forward speed may be taken into account by adding different correction factors to each component. The interaction among these components is ignored. So, we have the following expression for the total roll damping coefficients(65):

\[
B_{44} = B_w + B_F + B_E + B_L + B_{BK} \quad (4.1)
\]

The components \( B_F, B_E, B_L \) and \( B_{BK} \) are non-linear and are all caused by viscosity except for \( B_L \). The viscous roll damping components \( B_F, B_E \) and \( B_{BK} \) are evaluated by a linear approximation obtained by equating the energy dissipated by the non-linear viscous effect during one roll cycle to that dissipated by a linear damping term. If \( E \) is the energy dissipated because of the viscous effect then \( B_V \) is given by:

\[
B_V = \frac{E}{\pi \omega \eta_4^2} \quad (4.2)
\]

The evaluation of \( B_F, B_E \) and \( B_{BK} \) is done in this way.

Ikeda(67), linearised the non-linear components by the expression:

\[
b_2 |\dot{\eta}_4| = b_2 \frac{4}{3\pi} \eta_4 \omega \dot{\eta}_4
\]
where $\bar{\theta}_n$ is the average roll amplitude. He has non-dimensionalised each of the linear or linearised damping terms by dividing by,

$$\rho VB^2 \left( \frac{h}{c_B^2} \right)^{\frac{1}{2}},$$

which has units of tonnes $m^2 s^{-1}$.

3. ANALYSIS OF ROLL DAMPING COMPONENTS

The computations of the roll damping components were carried out for a range of Froude number of 0.0 to 0.4, encounter frequencies of 0.1 to 1.0 rad/sec and roll amplitudes of 10° to 40°. The input data required for the computations was calculated from the ship's hull offsets and bilge-keel information for draughts of 4.0 and 5.0m, see Reference(40). The analyses were performed for each component of roll damping with the parameters upon which it may depend.

3.1 The Wave Making Component $B_w$

The wave roll damping at zero speed can be obtained accurately as stated in the previous chapter but it is quite difficult at the present time to establish a theoretical treatment for this component for an ordinary ship hull form at forward speed.

The effect of the advance speed on this component has been studied by assuming the simple mathematical flow model in which the ship roll motion is represented by two doublets located at the bow and stern of the ship(67).

However, the 2-D values of the component $B_w$ at zero speed, calculated previously by the Frank close-fit method, can be
integrated to give the value for the whole ship and by using the program provided in Reference (63), its value obtained for different Froude numbers (FN). The variation in $B_w$ with FN is shown in figs. (4.1.a) and (4.1.b) for ship draughts of 4.0 and 5.0m. It can be seen that $B_w$ increases with the increase of FN up to a certain limit and decreases again depending upon the frequency of encounter.

3.2 The Frictional Component $B_F$

Analysing the results of experiments on the oscillation of suspended cylinders wholly immersed in water, a formula (4.2) for estimating the energy dissipated by frictional damping during one roll period was obtained by Kato (70).

Using the mean radius $r_s$ of the wetted surface of the ship instead of the cylinder, the formula can be extended to suit ship forms:

$$\delta E_f = \frac{2}{3} S C_f \omega^2 r_s^3 \eta^3$$  \hspace{1cm} (4.4)

where $\delta E_f$ is the energy dissipated, $S$ is the wetted surface of the ship, and $C_f$ the frictional coefficient.

By means of (4.2), the frictional component at zero forward speed, $B_{F0}$, may be obtained as:

$$B_{F0} = \frac{2}{3\pi} \rho S C_f \omega r_s^3 \eta^3$$  \hspace{1cm} (4.5)

If the forward speed effect is taken into account, an empirical correction factor may be added to (4.5) (71), then we have:

$$B_F = B_{F0} \left[ 1 + 4.1 \left( \frac{V}{L_w} \right) \right]$$  \hspace{1cm} (4.6)
where \( V \) is the forward speed (m/sec) and \( L \) is the length of waterline (m).

The variation in this component with the variation in \( FN \) is shown in figs. (4.2.a) and (4.2.b). The graph shows that \( BF \) increases as \( FN \) increases.

### 3.3 The Eddy-making Component \( B_E \)

When a ship is rolling the pressure distribution on the hull varies due to the separation of flow and eddy-making from the hull surface, which leads to the eddy-making component of roll damping(66).

Based on forced roll experiments with two dimensional cylinders having various cross sections and the strip assumption, an empirical formula for the eddy-making component of the roll damping of the hull at zero forward speed is deduced.

\[
B_{EO} = \frac{4}{3\pi} \rho \omega \eta_4 \int_0^L d^4 \cdot C_R \cdot dL
\]

(4.7)

where \( d \) is the sectional draught and \( C_R \) is the eddy-making force coefficient.

It is found that the eddy-making component of the hull decreases rapidly with forward speed. This effect may be taken into account by the following formula, which is deduced from some theoretical consideration and experimental results(67).

\[
B_E = \frac{(0.04K_d)^2}{(0.04K_d^2) + 1} \cdot B_{EO}
\]

(4.8)

where \( K \) is the reduced frequency = \( L \cdot \omega / V \)
Since the component $B_E$ is affected by Froude number as well as roll amplitude, the values of $B_E$ were calculated for a wide range of $FN$ from 0.0 to 0.4 and are shown in fig. (4.3.a) for a roll amplitude of $20^\circ$. The effect of roll amplitude on $B_E$ is also emphasised in fig. (4.3.b) for a frequency of 0.5 rad/sec. It can be seen that $B_E$ decreases rapidly with a slight increase of $FN$ and will vanish with large values of $FN$.

3.4 The Lift Damping Component $B_L$

The hull may be considered as a vertical wing with low aspect ratio oscillating around the rolling axis and moving in the water.

Ikeda et al (65) deduced a prediction formula for the lift component as follows:

$$B_L = \frac{1}{2} \rho \cdot S_L \cdot V \cdot K_N \cdot l_0 \cdot l_1 \cdot (1 - 1.4 \frac{OG}{l_R} + 0.7 \frac{OG^2}{l_0 l_R}) \quad (4.9)$$

where, $S_L$ : represents the lateral area (L,T)

$K_N$ : the slope constant of the lift coefficient

$l_0$ : the vertical distance between the roll axis and the point at which the representative attack angle is assumed to act.

$l_R$ : the distance between the roll axis and the centre of action of the lift force on a rolling ship hull.

$OG$ : the vertical distance between the centre of gravity and the still water surface.
From the experimental results, \( l_0 \), \( l_R \) and \( K_N \) respectively, are expressed as follows:

\[
\begin{align*}
l_0 &= 0.3T \\
l_R &= 0.5T \\
K_N &= \frac{2\pi T}{L} + K_n (4.1 \frac{B}{L} - 0.045) \\
K &= \begin{cases} 
0 & C_M \leq 0.92 \\
0.1 & 0.92 < C_M \leq 0.97 \\
0.3 & 0.97 < C_M \leq 0.99 
\end{cases}
\end{align*}
\]

where, \( C_M \) is the midship section coefficient.

However, the effect of Froude Number on \( B_L \) was calculated for the trawler GAUL and is shown in fig. (4.4) which indicates that the component \( B_L \) becomes significant at large values of \( FN \).

### 3.5 The Bilge-keel Damping Component \( B_{BK} \)

The bilge-keel component \( B_{BK} \) is divided into two components; the component \( B_N \) due to the normal force of the bilge-keels and the component \( B_S \) due to the pressure on the hull surface created by the bilge-keels.

Ikeda et al. (67) deduced from experimental results with oscillating flat plates, the component \( B_N \) as follows:

\[
B_N = \frac{8}{3\pi} r_{BK}^2 \cdot \eta_{BK} \cdot g_{BK} \cdot f \cdot C_D 
\]  (4.10)
where $b_{BK}$ and $l_{BK}$ are the breadth and length of bilge-keel, $f$ is a correction factor for the flow speed increase at the bilge, $C_D$ is the drag coefficient and $r_{BK}$ is the distance between the roll axis and the bilge-keel.

The prediction formula for the coefficient $C_D$ of the normal force of a pair of the bilge keels can be expressed as follows:

$$C_D = 22.5 \frac{b_{BK}}{\pi \cdot r_{BK} \cdot \eta_4 \cdot f} + 2.4 \quad (4.11)$$

From experimental results the component $B_S$ also can be expressed as follows:

$$B_S = \frac{4}{3\pi} \cdot r_{BK}^2 \cdot f^2 \cdot \omega \cdot \eta_4 \cdot \int_C \frac{1}{G} \cdot dG \quad (4.12)$$

where $G$ is the length along the girth and $l$ the lever of the moment. The coefficient $C_p$ has been found to be about 1.2 empirically.

The roll damping component $B_{BK}$ for a pair of the bilge-keels can be obtained by summing up $B_N$ and $B_S$.

The evaluation of this component was carried out for the trawler GAUL. The location and dimensions of the bilge-keel are illustrated in fig. (4.5). Figure (4.6) shows the values of the this component for the differing frequencies, roll amplitudes and ship draughts. It can be seen that the increase of these variables increases the component $B_{BK}$. 
3.6. The Total Roll Damping Coefficient $B_{44}$

The damping components that were introduced in this section can be summed up to give the total damping of ship rolling.

In the absence of ship speed, since only the lift term $B_L$ becomes zero, the other terms are predicted by integrating sectional values along the length of the ship, as in the strip method. Therefore, the longitudinal distribution of the roll damping can also be obtained.

At forward speed, however, the modification to the individual components is made for the whole ship form, not for each ship section (67).

The values of the $B_F$, $B_E$, $B_L$ and $B_{BK}$ components for different roll amplitudes and FN of 0.3 are shown in Table (4.1) for a ship draught of 4.0m and in Table (4.2) for 5.0m draught.

Also, comparisons of these components are shown schematically in fig. (4.7) with the abscissa FN and in fig (4.8) with frequency. Figure (4.7) indicates that, at large values of FN, $B_W$ and $B_L$ represent the major part of the total roll damping moment, while $B_F$ and $B_E$ become, relatively, very small values and can be neglected. It can also be seen that at zero ship speed $B_{BK}$ becomes significant, while fig. (4.8) indicates that $B_W$ becomes a very considerable value at certain frequencies depending upon the value of FN.

Finally, the variation of the total roll damping moment is shown with the variation in FN and $\omega$ in fig. (4.9.a). Figure (4.9.b) shows this variation in a different way. It is obvious that the variation of encounter frequency and Froude number has a considerable effect on the values of the total roll damping moment.
4. TREATMENT OF ROLL DAMPING IN PREDICTION OF ROLL MOTION

In the light of the analyses of roll damping components, a computational technique is adopted to estimate the total roll damping during the rolling motion. Since each of these components is affected by certain parameters, the value of these components during ship motion may be calculated according to the variation of these parameters.

However, it was decided to neglect the component due to eddy formation for the ship at forward speed since it is a very small amount relative to the other components and its evaluation required a considerable computation.

The components $B_W$, $B_F$ and $B_L$ are ship speed dependent and may be evaluated once only in performing a time simulation of ship's motion since a constant ship speed is assumed in this study.

Since the roll damping component due to bilge-keels and eddy-making are dependant upon the roll amplitude value, which varies during the motion, it is necessary to recalculate the value of this component for each rolling cycle. Therefore, procedures are incorporated in the simulation program to estimate the average roll amplitude, $\eta_a$, for each roll cycle and to use this value to compute the new bilge-keel damping. Figure (4.10) shows the variations of $\eta_a$ during the rolling motion.
5. CONCLUDING REMARKS

5.1 The roll damping moment due to bilge-keels and lift represent a considerable amount of the total roll damping and must be taken into account in considering the roll motion.

5.2 The variation in Froude number, roll amplitude, encounter frequency, and ship draught produces a large variation in the total roll damping moment and must be considered in predicting the roll motion.

5.3 The components of damping due to friction and eddy decrease rapidly with forward speed and become very small amounts relative to the other components and so can be neglected.

5.4 The wave-making damping component, which varies considerably with the motion of the ship as shown in the previous chapter, represents a considerable percentage of the total roll damping and should be handled with care in rolling motion prediction.

5.5 The components of damping due to bilge-keels and eddy-making are roll amplitude dependent and their evaluation should be considered whenever the roll amplitude of the ship changed during its motion.

5.6 Much of the data used in the formulation of the damping terms is empirical and based on a limited number of experiments. It would be highly desirable to conduct these experiments on a wider range of forms and to attempt to assess any possible scale effects.
Fig. (4.3.a) EFFECT OF SHIP SPEED ON EDDY DAMPING.

Fig. (4.3.b) EFFECT OF ROLL AMPLITUDE ON EDDY DAMPING
LIFT DAMPING

FIG. (4.4) EFFECT OF SHIP SPEED ON LIFT DAMPING.
FIG. (4.5) DIMENSIONS AND LOCATION OF BILGE-KEELS
BILGE-KEELS DAMPING

Fig. (4.6) DAMPING MOMENT COEFFICIENTS OF BILGE KEELS
### Roll Damping Components of the Trawler Gaul

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*Table (4.1) Damping Moment Coefficients at 4.0 m Draught (ton.m/sec)*
### ROLL DAMPING COMPONENTS OF THE TRAWLER GAUL

**FREQ. NO. (FN) = 0.3**

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### ROLL DAMPING COMPONENTS OF THE TRAWLER GAUL

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**TABLE (4.2) DAMPING MOMENT COEFFICIENTS AT 5.0 M DRAUGHT (TON.M/SEC)**
FIG. (4.7) COMPONENTS OF DAMPING MOMENT COEFFICIENTS
ROLL DAMPING COMPONENTS

ENCOUNTER FREQ. = 0.7 rad/sec

FIG. (4.8) COMPONENTS OF DAMPING MOMENT COEFFICIENTS
Fig. (4.10) Variation of roll amplitude during ship motion
CHAPTER
5

Wave-Exciting Forces & Moments
1. INTRODUCTION

The accurate prediction of the behaviour of a ship in a seaway requires first the determination of the exciting forces and moments acting on the ship taking into account the effect of wave shape, encounter frequency, heading angle and the oscillatory ship motion.

The wave-exciting forces can be calculated in three different ways. Two methods use formulae derived from a relative motion concept and the Haskind-Newman(72) relationship, respectively, and the third method calculates directly the diffraction force satisfying the diffraction boundary condition on a hull surface(73).

The first and second approaches are quite similar in that the wave-exciting force is evaluated by using motion-induced added mass and damping coefficients in combination with incident wave characteristics such as wave number and wave heading, see Reference(73). The third approach was first used by Grim(74) and was later adopted by Kim and Chou(75).

Grim's method is based on the assumption that the disturbance of an incident wave caused by the ship's body is represented by the
potential used in describing the water flow around the body when the body is oscillating harmonically in the calm water surface. This potential, together with the incident wave potential, constitutes the potential that describes the flow around the body when it is restrained in waves.

Based on Grim's method, Kim and Chou (75) computed the two-dimensional radiation forces by the Frank close-fit method and extended this to evaluate the two-dimensional wave-exciting forces and moments acting on a ship in oblique waves. The three-dimensional radiation forces and the wave exciting forces and moments acting on the ship hull are calculated by conventional strip theory.

The degree of the reliability of this method was determined by comparing the theoretical predictions with Lalangas' (75) latest experimental results for a Series 60 model and with additional experimental results for a rectangular barge, see Reference (75). Theory and experiment were found in generally good agreement in both cases, as shown in fig. (5.1) from Reference (75).

The method enables us to compute the exciting forces and moments for widely varying configurations of ship cross-sections. Therefore, this method was used for the determination of the wave-exciting forces and moments in this study.

A computer program utilising this technique, Ref. (61), was used to calculate the wave-exciting forces and moments acting on the restrained trawler form in regular sinusoidal waves. The amplitude of these forces and their phase shifts were calculated for the trawler GAUL for different modes of motions, see Reference (23). Modifications
were made to the program to give, from the original ship's hull offsets, the instantaneous exciting forces and moments acting on the ship according to the incoming wave characteristics, heading angle and the resultant ship oscillatory motions.

In this chapter, interesting points regarding the effect of heading angle, wave shape and ship oscillatory motion on the exciting forces are analysed and presented.

2. THE ANALYTICAL TECHNIQUE FOR ESTIMATING THE WAVE EXCITING FORCES AND MOMENTS

2.1 The three-dimensional forces on the restrained ship, induced by the oblique waves, are determined approximately by summation over the ship's length of the two-dimensional elementary forces induced by the waves on an elemental transverse (cross-section) of the ship. The ship is split into an odd number of equally spaced strips along its length and any hydrodynamic interaction between adjacent cross-sections, in the longitudinal direction, is ignored. This implies a 2-dimensional hydrodynamically fixed body problem at each section.

2.2 As explained in References (61) and (75), when small amplitude incident waves are encountered by the ship, they will be diffracted from it assuming that the ship is rigidly held. Therefore, the fluid potential will be the sum of the potentials due to the incident wave and the diffracted wave. Since the diffraction is a disturbance, by using Green's theorem and the Frank close-fit technique, the diffracted wave potential is represented by the source potentials
distributed along the wetted strip contour with unknown source densities which can be found from the kinematic boundary conditions. By applying the kinematic boundary condition on the fixed strip contour, the unknown source densities are evaluated in terms of the known incident wave characteristics at each section for the required mode of motion (ie heave, sway or roll).

2.3 Since the source densities are complex the application of the kinematic boundary conditions yields two sets of linear equations for the real and imaginary parts. This implies that the required potential and the pressures which are found by Bernoulli's equation, will have two components for the real and imaginary parts and the sectional wave exciting force or moment can be evaluated by integrating the pressure around the strip contour as follows:

\[ f(m) = \int p(m) \cos(m)(n,s) \, ds \]  \hspace{1cm} (5.1)

where \( p(m) \) = hydrodynamic pressure in the mode of (m)
\( \cos(m)(n,s) \) = directional cosine in the mode of (m)

The hydrodynamic pressure \( p(m) \) can be defined as:

\[ p(m) = \text{RE} [p(m) e^{-i\omega t}] \]

\[ = p(m) \cos \omega t + p(m) \sin \omega t \]  \hspace{1cm} (5.2)

The substitution of equation (5.2) into (5.1) will yield the sectional wave exciting force or moment in terms of \( \cos \) (or real) and \( \sin \) (or imaginary) parts as follows:

\[ f(m) = f_r(m) \cos \omega t + f_i(m) \sin \omega t \]  \hspace{1cm} (5.3)
In order to find the total force acting on the ship, the sectional force \( f(m) \) is integrated along the ship length considering its real and imaginary components as follows:

\[
F_r(m) = \int_{L} f_r(m) \, dx
\]

\[
F_i(m) = \int_{L} f_i(m) \, dx
\]

and the final expression of the total wave exciting force or moment can be written as:

\[
F(m) = F_0(m) \cos (\epsilon(m) - \omega t)
\]

where \( F_0(m) = \sqrt{F_r(m)^2 + F_i(m)^2} \)

\[
\epsilon(m) = \tan^{-1} \frac{F_i(m)}{F_r(m)}
\]

3. **THE EFFECT OF WAVE LENGTH AND HEADING ANGLE ON WAVE EXCITATION**

The amplitudes of the wave-exciting forces and moments and their phases were calculated for the trawler GAUL for five degrees of freedom, \( n_{2-6} \), taking account of the effects of the wave length to ship length ratio, \( (\lambda/L) \), and the heading angle, \( \mu \).

The calculations were carried out for a range of \( (\lambda/L) \) ratios between 0.25 and 2.0 (equivalent to 0.7 \( \rightarrow \) 2.0 rad/sec wave
frequency) and for values of heading angles of 0, 30, 60 and 90 degrees. The wave profile along the sides of the ship hull was taken into account for each value of $(\lambda/L)$ and $\omega$. The effect of $(\lambda/L)$ ratio on the underwater shape of the ship's hull is shown in fig. (5.2.a) for the trawler GAUL, while the effect of the heading angle ($\mu$) on such a shape is shown in fig. (5.2.b).

The non-dimensional values of the wave-exciting forces and moments and the phase angles are shown in figs. (5.3) to (5.7) for sway, heave, roll, pitch and yaw excitation, respectively. The figures demonstrate that the amplitude of the wave-exciting force or moment is strongly affected by the heading angle and the $(\lambda/L)$ ratio. It also can be seen that, for each mode of excitation, the phase angles for the different heading angles become essentially the same and vary slightly as the $(\lambda/L)$ ratio increases beyond the value of about 0.75.

4. **INFLUENCE OF OSCILLATORY MOTIONS ON WAVE-EXCITATIONS**

The influence of heaving, rolling and pitching motions on the wave-exciting forces and moments was investigated for the trawler GAUL in beam seas ($\mu = 90^\circ$). The ship was considered to oscillate from its original position in still water by a considerable displacement and the new offsets representing the immersed hull of the ship at the relevant displacement were calculated to emphasise the influence of such displacements on the wave excitations.
4.1 **Heaving Motion**

According to the coordinate system considered for the ship oscillatory motion, it is assumed that the ship is given a heaving movement up to $\pm 1.2$ m with $\pm 0.4$ m intervals along the Z-axis. The effect of this heaving motion on the exciting forces and moments and phase angles for the trawler GAUL in beam seas are shown in fig. (5.8.a), (5.8.b) and (5.8.c) for the sway, heave and roll motions, respectively.

The figures demonstrate that the sway and heave exciting forces decrease as the ship moves upward, while the roll-exciting moment increases. It also can be seen that heaving motion has no effect on the phase angles of these forces and moments. It is important to note that the variations of these forces and moments with the heaving motion are nearly linear.

4.2 **Rolling Motion**

The effect of rolling motion on the wave-exciting forces and moments was calculated by considering angular displacements up to $\pm 40^\circ$ at $10^\circ$ intervals from the equilibrium position of the ship in still water. Figures (5.9.a), (5.9.b) and (5.9.c) illustrate the effect of heeling angle on the forces, moments and phase angles for the different modes of motions. It can be seen that the effect of rolling motion on these quantities is very small.
4.3 **Pitching Motion**

The ship was given an angular displacement about the Y-axis up to ± 6° at 2° intervals and the effect of such displacements on the exciting forces and moments are shown in figs. (5.10.a), (5.10.b) and (5.10.c). The figures show that the roll-exciting moment is most affected by the pitching motion and it decreases considerably as the ship pitches in any direction.

5. **VARIATION OF WAVE-EXCITATION WITH VARIATION IN DRAUGHT**

In order to have a complete record of the relationship between the wave-exciting forces and moments and the relative ship position in waves, the investigation of the variation of these exciting forces and moments in a seaway was extended to analyse the variation of these quantities with variation of draught for each ship cross-section.

5.1 To begin with, the computations were carried out for the amidships section of the trawler GAUL with different draughts starting from the 2m draught up to 10m in the upright condition. Figure (5.11) shows the variation of the wave-exciting forces and moments with the variation in draught for heaving, swaying and rolling motions for a range of encounter frequencies between 0.6 and 1.4 rad/sec for this cross-section. It can be seen that the wave-exciting moment for rolling motion reaches zero at 5 metre draught, as has been theoretically deduced by Ursell(77).

5.2 The wave-exciting moment for rolling motion was analysed for nine different cross-sections along the ship length for different
draughts for a wave frequency of 1.0 rad/sec and the results are shown in fig. (5.12.a). From this data, fig. (5.12.b) has been produced which shows all the points of nearly zero exciting moments. These points represent a shape similar to the shape of zero-damping moment presented in Chapter 3.

6. CONCLUDING REMARKS

6.1 The amplitude of wave-exciting force or moment is strongly affected by the heading angle and wave length to ship length ratio.

6.2 The investigation carried out regarding the effect of ship oscillatory motions on wave-exciting forces and moments demonstrates that heaving and pitching motions produce a considerable variation in these values, while the variation in heeling angle produces only a slight variation. It has been shown also that the rate of change of the amplitude of wave-excitation with the amplitudes of the oscillatory ship motion, is nearly linear and this can be used in calculating these forces and moments during ship motion.

6.3 The investigation of the variation of the wave forces and moments with the variation in draught showed that the roll exciting moment acting on a ship may vanish at a certain water surface shape according to the shape of the hull. This makes it necessary to calculate the instantaneous roll-exciting moment in order to perform a time simulation of the ship's motion.
Fig. (5.1) The effect of $\mu$ and $\lambda/L$ on the wave-exciting forces and moments, (from ref. 75)
Fig. (5.2.a) The effect of heading angle on the under-water shape of the hull of the trawler GAUL.

Fig. (5.2.b) The effect of (l/l) ratio on the under-water shape of the hull of the trawler GAUL.
Fig. (5.3): Amplitude of sway-exciting Force and phase angle for the trawler GAUL
Fig. (5.4) Amplitude of Heave-exciting FORCE and phase angle for the trawler GAUL
Fig. (5.5) Amplitude of Roll-exciting moment and phase angle for the trawler GAUL
Fig. (5.6): Amplitude of Pitch-exciting moment and phase for the trawler GAUL.
Fig. (5.7) Amplitude of Yaw-exciting moment and phase angle for the trawler GAUL.
The Effect of Heaving Displacement on the Sway, Heave and Roll Exciting Forces and Moments
The Effect of Heeling Angle on the Sway, Heave and Roll Exciting Forces and Moments
The Effect of Pitching Angle on the Sway, Heave and Roll Exciting Forces and Moments
Fig. (5.11) Variation of Exciting Forces and Moments with Draught for the amidships section of the trawler GAUL
\( \omega = 1.0 \text{ rad/sec} \)

**Fig. (5.12.a)** Variation of Roll-exciting moment with draught for different cross-sections of the trawler GAUL

**Fig. (5.12.b)** Points of minimum exciting rolling moment
CHAPTER 6

Simulation of Ship Motions
1. INTRODUCTION

The all round understanding of the behaviour of a ship among waves may be attained by the theoretical treatment of ship motion performed on a mathematical model which reflects the physical situation and is computed with sufficient precision as to be practically satisfactory. Such treatment can be developed by taking into account the non-linearities of the various coefficients of the equations of ship motions. A number of non-linear effects are involved in considering the motion of the ship in waves of large amplitude such as added mass, damping and restoring terms. Moreover, the effects of the different parameters, such as wave shape, heading angle, ship speed and loading condition, on the behaviour of the ship must be considered. These effects can only be taken into account by performing the solution of the equations of motion in the time domain. In other words, the behaviour of the ship must be studied by examining the step by step history of the motions resulting from numerical integration simulation.

With the aid of high speed digital computers, solving non-linear motion equations in a time-step-integration procedure becomes possible. However, this method requires extensive calculations and the problem of evaluating a number of time series arises.
It has been shown by Stiansen et al (78) that a typical non-linear time domain simulation, for predicting the motions of tension-leg platforms, required 1000 times more computing time than is required for the linear solution, but the comparison between the two methods on the basis of computing time is not acceptable because the results obtained by the linear and non-linear analyses are different in their fundamental nature and usefulness. It is clear that the prediction of ship responses in large waves, where the non-linearities in the terms of the motion equations become significant, has to be carried out by time domain solutions. Consequently, it is essential to write a time simulation program in order to investigate the behaviour of the ship among waves.

However, the use of computers in studying the seakeeping performance of ships has been appreciated for some years. For instance, Swann and Vossers (79) pointed out the use of computer programs in investigating the influence of different parameters on the motion of ships among waves.

At the present time there already exists some programs based on the simulation approach. In 1969 (80) the simulation and statistical evaluation of a non-linear system of one degree of freedom with a random parametric excitation was introduced. A Runge-Kutta-Merson integration procedure (81) was used. Paulling and Wood (26) in 1974 set up a six-degree-of-freedom motion simulation program. They used the Frank close-fit method in order to incorporate the linear hydrodynamic parameters correctly. For large motion amplitudes, they rely on the assumption that at the time of a capsize, the roll motion itself tends to have an extremely large period or, conversely, a very small frequency and slow motion. From this assumption, one may introduce the hydrostatic non-linearities solely.
However, the application of such methods is not widespread in ship stability but is more common in the Ocean Engineering field especially for offshore structures, see Ref. (30). The application of these programs is limited because the time-variation of roll restoring moment was only performed for the following seas case as described in Refs. (23, 82).

This chapter describes a computational method, developed by the author, to simulate the motions of the ship in regular sinusoidal waves with arbitrary heading angle. The basic approach involves the computation of the coefficients of the equation of motions at each step in time according to the exact wave profile and vessel position.

The present method has been aimed at the problem of capsizing in extreme ship motion. It is difficult to determine which parameter contributes most to the capsizing phenomena since there is no readily available formula regarding the influence of any of the many parameters involved. Therefore, the time simulation program was used as a qualitative and comparative rather than a quantitative or predictional tool. The influence of some of the different parameters on the motions of the ship was calculated and is presented herein, in a sequence to explore the effect of these parameters on ship responses with a particular emphasis on rolling motion.

The program was used also to investigate the effects of tethering on roll behaviour of a model with a bias in roll in regular beam seas. The results are presented and compared to those of the model experiments.
The method of approach chosen for this study consists of a computerised time domain analysis of a ship's motions in waves of large amplitude. This method utilises the numerical integration of the equations of ship motion. The main features and assumptions inherent in this approach are summarised in the following paragraphs.

2.1 No restrictions are placed on the amplitudes of motions, as would be the case in a linear, frequency-based analysis. Therefore, the accurate time-varying representation of the restoring forces and moments becomes critical and the main effort was placed on the accurate computation of the static forces acting at each instant of time.

2.2 Five-degrees of freedom are included in the simulation program (surge is omitted) as they are considered essential in the determination of the immersed hull geometry, which is critical to the transverse stability.

2.3 The vessel is assumed to advance at a steady speed on a constant course with arbitrary heading in regular sinusoidal waves.

2.4 All viscous effects other than roll damping are neglected. Hence, the only damping considered is the damping due to the energy loss in creating free-surface waves.

2.5 It is assumed that the ship's length is much larger than either its beam or draught.
The equations of motion used to represent the physical situation are integrated time-wise by using the second order Runge-Kutta method, see Ref.(81). The method employs a straightforward extrapolation of the motions based on constant accelerations across each time step. This is accurate so long as the time step used is sufficiently small.

Within this framework, the mathematical model was formulated and a calculation procedure was developed.

3. MATHEMATICAL MODEL

The mathematical model is based on a time-step solution of a system of five coupled differential equations of motions. The equations consist of two sets of coupled differential equations with time-dependent coefficients. One set of equations is for heave-pitch motions and the other is for the sway-roll-yaw motions.

The coupled equations of motion for heave, pitch, sway, roll and yaw, respectively, can be written in the following form:

\[(M + A_{33})\ddot{n}_3 + B_{33}\dot{n}_3 + R_3 + A_{35}\ddot{n}_5 + B_{35}\dot{n}_5 = F_3 \quad (6.1)\]
\[(I_5 + A_{55})\ddot{n}_5 + B_{55}\dot{n}_5 + R_5 + A_{53}\ddot{n}_3 + B_{53}\dot{n}_3 = F_5 \quad (6.2)\]
\[(M + A_{22})\ddot{n}_2 + B_{22}\dot{n}_2 + A_{24}\ddot{n}_4 + B_{24}\dot{n}_4 + A_{26}\ddot{n}_6 + B_{26}\dot{n}_6 = F_2 \quad (6.3)\]
\[(I_4 + A_{44})\ddot{n}_4 + B_{44}\dot{n}_4 + R_4 + A_{42}\ddot{n}_2 + B_{42}\dot{n}_2 + A_{46}\ddot{n}_6 + B_{46}\dot{n}_6 = F_4 \quad (6.4)\]
\[(I_6 + A_{66})\ddot{n}_6 + B_{66}\dot{n}_6 + A_{62}\ddot{n}_2 + B_{62}\dot{n}_2 + A_{64}\ddot{n}_4 + B_{64}\dot{n}_4 = F_6 \quad (6.5)\]
where $M$ is the mass of ship, $I_I$ the moment of inertia in $I^{th}$ mode, $A_{II}$ and $B_{II}$ are the added-mass and damping coefficients, $R_I$ and $F_I$ are the restoring and the exciting forces and moment. Their expressions are given as follows:

\[
\begin{align*}
A_{22} &= \int a_{22} \, dx \\
B_{22} &= \int b_{22} \, dx \\
A_{33} &= \int a_{33} \, dx \\
B_{33} &= \int b_{33} \, dx \\
A_{44} &= \int a_{44} \, dx \\
B_{44} &= \int b_{44} \, dx + B_E + B_F + B_{BK} + B_L \\
A_{55} &= \int x^2 a_{33} \, dx \\
B_{55} &= \int x^2 b_{33} \, dx \\
A_{66} &= \int x^2 a_{22} \, dx \\
B_{66} &= \int x^2 b_{22} \, dx
\end{align*}
\]

$A_{24} = A_{42} = \int a_{24} \, dx$

$B_{24} = B_{42} = \int b_{24} \, dx$

$A_{26} = \int x a_{22} \, dx$

$B_{26} = \int x b_{22} \, dx$

$A_{35} = -\int x a_{33} \, dx$

$B_{35} = -\int x b_{33} \, dx$

$A_{53} = -\int x a_{33} \, dx$

$B_{33} = -\int x b_{33} \, dx$

$A_{46} = \int x a_{24} \, dx$

$B_{46} = \int x b_{24} \, dx$

$A_{62} = \int x a_{22} \, dx$

$B_{62} = \int x b_{22} \, dx$

$A_{64} = \int x a_{24} \, dx$

$B_{64} = \int x b_{24} \, dx$
The five second order differential equations of motion were solved using a numerical technique. The NAG Library routines(37) cover the solution of coupled single order ordinary differential equations in some depth, utilising different methods for the various types of problem.

The equations presented here formed an initial value problem where the solution was obtained starting from initial values of the dependent variables \((n_I, \dot{n}_I)\) and integrating with respect to time in a step-by-step manner. Various methods for solving this type of problem were studied(81,83,84) but in order to classify the problem, the initial calculations were carried out using the Runge-Kutta Merson routine as suggested by the NAG Library manual. After many checks, it was decided that DOBBF routine(39) be used and incorporated into the computer program.

It was first necessary to convert the five second order equations - equations (6.1) to (6.5) into ten coupled first order equations(84). Writing \(y_1 = n_3, y_2 = \dot{n}_3, y_3 = n_5, y_4 = \dot{n}_5, \) and so on to \(y_9 = n_6, y_{10} = \dot{n}_6\) and solving the five simultaneous equations in \(\dot{n}_I, \) where \(I = 2\) to 6, these become:

\[
\begin{align*}
\dot{y}_1 &= y_2 \ (= \dot{n}_3) \\
\dot{y}_2 &= \ldots \\
\dot{y}_3 &= \ldots \\
\dot{y}_8 &= \ldots \\
\dot{y}_9 &= y_{10} \ (= \dot{n}_6)
\end{align*}
\]
The expressions for $y_2$, $y_4$, ..., and $y_{10}$ can be obtained from the two sets of coupled differential equations. Using equations (6.1) and (6.2) for the first set and equations (6.3), (6.4) and (6.5) for the second, the accelerations of heave ($\eta_3$) and roll ($\eta_4$) can be obtained from the following equations:

\[
\eta_3 = \left[ \frac{(I_5 + A_{55})}{(I_5 + A_{55}) \cdot (M + A_{33}) - A_{35} \cdot A_{53}} \right] \\
\times \left[ F_3 - B_{33} \dot{h}_3 - R_3 - \frac{A_{35}}{(I_5 + A_{55})} \cdot (F_5 - B_{53} \dot{h}_3 - B_{55} \dot{h}_5 - R_5) - B_{35} \dot{h}_5 \right] \\
= \dot{y}_2 \\
(6.6)
\]

\[
\eta_4 = \left[ \frac{(I_6 + A_{66}) \cdot (H + A_{22}) - A_{62} \cdot A_{62} \cdot (I_6 + A_{66})}{(I_6 + A_{44}) \cdot (I_6 + A_{66}) - A_{46} \cdot A_{66} \cdot (I_6 + A_{66}) \cdot (I_6 + A_{22}) - A_{62} \cdot A_{62} \cdot (I_6 + A_{66})} \right] \\
\times \left[ F_4 - B_{42} \dot{h}_2 - B_{44} \dot{h}_4 - B_4 - B_{46} \dot{h}_6 - \frac{A_{46}}{(I_6 + A_{66}) \cdot (I_6 + A_{22})} \cdot (F_6 - B_{62} \dot{h}_2 - B_{64} \dot{h}_4 - B_{66} \dot{h}_6) \right] \\
= \dot{y}_6 \\
(6.7)
\]

Similarly, the accelerations $\dot{\eta}_2$, $\dot{\eta}_5$, and $\dot{\eta}_6$ (i.e. $\dot{y}_8$, $\dot{y}_4$, and $\dot{y}_{10}$) can be derived.
5. THE COMPUTATIONAL PROCEDURE

The calculation procedure involves the numerical integration of the equations of motion through time. Briefly, the overall procedure utilised to obtain the vessel motions in the time domain is described by the following steps:

1. Using the exact ship and wave position, inertial, damping, restoring and exciting forces are computed by earlier subroutines.

2. All the forces are combined to find vessel accelerations, see equations (6.6) to (6.7).

3. The vessel velocities and displacements are found by integrating the previous accelerations over the time step by applying the Runge-Kutta method through available routines in the computer library.

4. The entire procedure is then repeated for the next time step to find new vessel accelerations, velocities and displacements.

The NAG routine D02BBF was implemented from the main program (MOTION). The subroutine (FCN) returns the values of $y_1$, $y_2$, ..., and $y_{10}$ and time. Figure (6.1) shows the block diagram for program MOTION and the different subroutine used for the solution of the non-linear equations of motion. The details of each subroutine are described in separate reports as mentioned previously.
6. SPEEDING UP THE COMPUTATION

The initial version of the time simulation program has been designed for full size vessels and assumes that all the coefficients of the motion equations are computed at each time step. This method requires a considerable computational time even with the use of high speed and high capacity machines such as the 2988 computer at the University of Glasgow. Limitations on computer times, however, do not allow the use of such programs for the many cases required. Therefore, simplifications were made to the program, using the analysis presented in the previous chapters, to enable us to retreat from the necessity of determining all of the coefficients of the equations of motions at each time step.

It has been found that the calculations of the hydrodynamic coefficients and wave excitations, using the Frank close-fit technique, are the most computationally time consuming. Meanwhile, the analysis of these terms illustrate that the rate of change of the amplitudes of these terms for sway motion are nearly constant or linear with the amplitudes of oscillatory ship motions, see figs. (3.17), (3.18), (3.19), (5.8), (5.9) and (5.10). Therefore, the hydrodynamic coefficients and wave-exciting force for sway motion is calculated once in the first time step and modified to the following time steps using the following equations:

\[ C_t = C_0(1 - (\pi_3)t/T) \]  

where, \( C_0 \) = Initial value of any of motion equation terms  
(at \( t = 0 \))  
\( C_t \) = The value of any of motion equation terms at any time (\( t = t \))
The amplitude of heaving motion at Time \( t = t \)

\[ n_3(t) = \text{The amplitude of heaving motion at Time } (t = t) \]

\[ T = \text{Mean draught} \]

The advantage of this simplification is to save computational time as well as to take into account the effect of relative ship motion in the motion equations of sway and yaw. For heave, pitch and roll motions, this effect already exists in the restoring terms.

It is important to mention that this simplification speeds up the running of the program by 70 times compared to the initial version of the program, without significant loss in accuracy. Therefore, it was possible to carry out an investigation into the effects of the many parameters on the behaviour of the ship among waves.

7. **EFFECTS OF THE VARIABLES ON SHIP MOTIONS**

The computer program was applied to study the influence of different parameters on ship responses in regular sinusoidal waves. Since the choice of variables is broad, a procedure was used for this study to minimise the number of program runs as well as to explore the effects of the various parameters on ship motions.

Briefly, the procedure is to start the investigation by considering the effect of a certain parameter on the behaviour of the ship, after the motion has settled down. Then, applying a second parameter to the outcome of the motion response from the previous parameter, and so on. This procedure may lead to a combination of these parameters which degrades the dynamical stability of the ship. However, the procedure considered in this study can be summarised by
the following steps:-

a. To begin with, the effect of heading angle (\( \mu \)) was considered using a wave of (\( \lambda/L \)) ratio of 1.0.

b. Then, the influence of wave frequency was applied to certain \( \mu \) values chosen from the first step. A wave height of (\( \lambda/20 \)) was assumed for the different waves in order to reduce the number of variables.

c. The influence of ship forward speed was taken into consideration for some cases. This effect may be considered, also, as the effect of encounter frequency on ship motions.

d. Different loading conditions of the ship were also included in this investigation. This parameter includes the effect of ship displacement, trim and vertical centre of gravity on ship response.

e. Wind moment and bias in roll were considered for beam sea conditions.

f. Finally, cases were examined under the effects of a combination of the above parameters, which provided the greatest influence on rolling motion.

According to this procedure, the computer program was run many times for each of these parameters. Selected results from this investigation are presented and discussed in detail. Table (6.1) summarises the parametric variations which have been carried out.
7.1 **Heading Angle ($\mu$)**

The computations were carried out for different heading angles for the ship loading condition given in Table (6.1). To begin with, heading angles were considered starting from $15^\circ$ up to $165^\circ$ with $15^\circ$ intervals. The effect of heading angles $15^\circ$, $45^\circ$, $90^\circ$ and $135^\circ$ on ship responses are presented in figs. (6.2), (6.3), (6.4) and (6.5). A wave of $1.0$ rad/sec frequency and $1.54$ wave amplitude (equal to $\lambda/20$) is considered for all the cases, see Table (6.1). Wave and ship loading data for each relevant case is stated at the top of the figures. These show the wave configuration followed by the amplitude of heave, pitch, roll, sway and yaw, respectively. Particular emphasis has been placed on the rolling motion by visualising the intersection between the ship's hull and the wave configuration at the instants of maximum, zero and minimum rolling amplitudes. Moreover, plots of different intersection cases between the wave surface and different cross-sections, with the centroid of sectional-area marked, are shown to illustrate the causes of the variation in the restoring moments during the ship motion among waves.

Figures (6.2), (6.3) and (6.4) indicate a bias on roll behaviour in that the mean roll angle varies between $1^\circ$ and $5^\circ$. This phenomena could affect the dynamical stability of the ship. Therefore, the occurrence of such bias is explained in fig. (6.6) for the condition presented in fig. (6.3), $\mu = 45^\circ$. Figure (6.6) explains this phenomena by plotting the values of $GZ$, displacement and righting moment during the ship motion. It can be seen that both lines 1 and 2, which pass through the maximum and minimum roll amplitudes have the same righting moment value although the values of $GZ$ at these positions are different. This shows the influence of the ship's
oscillatory motion which is reflected in the variations of ship displacement due mainly to heaving motions as shown in fig. (6.6). Following line 3, which passes through a point of zero righting moment, line 4 can be obtained which gives the steady angle about which the ship is rolling. As a result the bias on roll behaviour can be referred to the wave configuration and ship's oscillatory motion in the computations. For instance, fig. (6.3) shows that at certain ship positions, in the wave, cross-section No 2 is subjected to a righting moment opposite to that for cross-section No 17. Meanwhile, fig. (6.4) indicates that the ship may be subjected to a righting moment even when the ship is in the upright position (zero roll amplitude).

The results for swaying motion, indicates that the ship drifts with the wave direction for heading angles between 60° and 120°, see fig. (6.3), while she drifts slightly in the opposite direction for the other heading angles as shown in figs. (6.2), (6.3) and (6.5). The drift of the ship occurs because the relative ship motion was taken into account in calculating the hydrodynamic coefficients and the wave-exciting forces for the sway motion. In this study, these quantities are referred to the heaving motion, as mentioned in Chapter 5 and, therefore, the sway amplitude and the direction of drift is governed by the heaving amplitude and the phase difference between the heaving and swaying motions. However, the heading angle has considerable influence on the amplitudes of ship motions as illustrated in figs. (6.2), (6.3) and (6.5). The maximum roll and heave amplitudes occur in beam seas and decrease as the heading angle deviates from this position and vice versa for pitching motion and the sway and yaw motions are dependent upon the heaving amplitude.
7.2 Wave Frequency ($\omega$)

The effect of wave frequency on ship responses were calculated for heading angles of $15^\circ$ and $45^\circ$. The wave amplitude was taken as $(\lambda/20)$ for all the cases to limit the number of variables and wave frequencies of 0.6 and 1.4 rad/sec were considered. Their influences on ship motions are illustrated in figs. (6.7) and (6.8) for $15^\circ$ heading angle and in figs. (6.9) and (6.10) for $45^\circ$ heading angle. It can be seen that, for the GAUL, the smaller wave frequency produces the larger amplitudes of motions. It is important also to note that the variation of wave frequency affects the amount and direction of roll under tethered conditions as demonstrated in figs. (6.9) and (6.10).

7.3 Ship Speed ($V$)

The effect of ship speed on ship behaviour is considered as the effect of encounter frequency. The hydrodynamic coefficients and the wave-exciting forces and moments are calculated according to the encounter frequency.

Figures (6.11) to (6.14) illustrate the ship responses for the cases considered. In figs. (6.11) and (6.12), where a heading angle of $15^\circ$ is considered, it can be seen that the ship speed increases the amplitudes of motions and causes irregularities in rolling motion. These irregularities always arise when the encounter frequency becomes less than 0.4 rad/sec, in which the oscillatory ship motion and the wave shape greatly affect the restoring term as explained later in fig. (6.21).
When a heading angle of 45° is considered, the effect of ship speed on rolling amplitude becomes considerable, as shown in fig. (6.13).

7.4 Loading Condition

The effect of different loading conditions on the behaviour of the GAUL was investigated using the conditions given by IMCO, see Appendix B. The results are presented in figs. (6.15) to (6.18). The figures indicate that the larger displacements (mean draught) experiences a slightly larger roll amplitude.

7.5 Wind Moment

The effect of wind moment on the response of the ship in a seaway was next considered. The following formula, recommended by Wendel(85), is added to the wave-exciting moment for rolling motion:

\[ M_w(\eta_y) = M_w(0) [0.25 + 0.75 \cos^3\eta_y] \]  

(6.9)

where \( M_w(0) \) : is the wind heeling moment in the upright position

\( M_w(\eta_y) \) : is the wind heeling moment in the heeled position.

The constant term in equation (6.9) shows that a percentage of the wind moment is still acting at a heel of 90°. The calculation of \( M_w(0) \) for the trawler GAUL in the relevant condition is given in Appendix C. The wind is assumed to act in the beam direction and the wind moment was applied to the ship for two cases, following and beam seas. The following sea (\( \mu = 0^\circ \)) condition is considered to emphasise the effect of wind moment only on the rolling motion since, in this
case, the wave-exciting roll moment is zero so long as the vessel is upright. Under the effect of wind moment, the computations were carried out for the following and beam seas conditions.

The responses of the ship for heaving, pitching and rolling motions are presented in figs. (6.19) and (6.20) with the variation in GZ and displacement during these motions. It can be seen in fig. (6.19) that the wind moment heeled the ship to about 3.5°. Consequently, this effect is added to the rolling motion due to wave-excitation in the beam sea condition, as shown in fig. (6.20).

When a ship speed of 15 knots is considered in the following sea condition, the wind moment effect becomes considerable as illustrated in fig. (6.21). In this figure, the irregularity in rolling behaviour is explained by examining the GZ and displacement curves as well as the plotting of the wave intersection with ship's hull at positions 1 and 2. These positions represent the conditions when the ship is at the higher and lower peaks of rolling motion. Lines 1 and 2 demonstrate that the ship possesses, in both positions, the same restoring moment because of the fluctuation of GZ and displacement, caused by the heaving motion and the wave movement on the ship. The plotting of the wave intersection with the ship emphasises this effect. It can be seen that in position 1, the displacement is small because the ship is in upward motion. Also, the wave crest is amidship which produces a relatively small GZ value. In position 2 the displacement is large and the trough is amidship. This demonstration shows that the restoring moment is not purely heeling angle-dependent and must be considered as time-dependent in predicting the motions of a ship moving among waves.
8. COMPARISON WITH EXPERIMENTAL RESULTS (BIAS EFFECT)

The program was used to measure the roll response of the model used in Ref. (86), see fig. (6.22.a). The rounded deck edge of the model has been changed into a right angle corner, as shown in fig. (6.22.b) because the computer program assumes that the higher offsets will be constant or increasing.

However, the computation was carried out for the two cases presented in Ref (86), (case 65A and 58B). Both cases were investigated including the bilge keels and a bias in roll as given in Ref. (86). The resulting behaviour of the model for these cases is shown in figs. (6.23) and (6.24) representing the conditions (65A) and (58B) in Ref. (86), respectively. Despite the bias effect on the roll response there is no tendency to capsize.

When these cases are repeated without including the bilge keels, capsize occurs in both conditions in the direction of the bias angle as occurred in the experiment tests, as shown in figs. (6.25) and (6.26). The capsize occurs more quickly in the theoretical model than in the physical which is probably due to some slight difference in the damping or in the physical wave conditions.

The bias effect on the rolling motion of the trawler GAUL was also calculated and the results are presented in fig. (6.27). In this figure, Case B shows that bias in roll can heel the ship to a large angle.
9. EFFECT OF COMBINED PARAMETERS

Since the ship in a seaway may be subject to the effects of a combination of the parameters examined earlier, an investigation was carried out to predict the ship responses under a combination of these conditions.

Wind moment and bias in roll were applied to the ship with forward speed for different heading angles and wave frequencies, as given in Table (6.1). The results of this investigation are presented in figs. (6.28) to (6.31).

It can be seen that the ship experiences a considerable angle of heel in all the cases. Among these cases, the condition of quartering sea, fig. (6.31), appears as the most dangerous situation. Such a situation occurs because in this case the wave-exciting moment is considerable and the ship speed reduces the encounter frequency which, in turn, affects the values of the hydrodynamic coefficients. When this case is considered with a different wave frequency, extreme ship motions occur and the ship nearly reached the capsizing situation as shown in fig. (6.32).

These examples demonstrate that, in certain conditions, a combination of the effect of the different parameters may cause dynamical instabilities.
10. **THE EFFECT OF SHIP TURNING ON MOTION AMPLITUDES**

The effect of a ship turning on the amplitudes of her motions was investigated to show the possibility of modifying the computer program to include the rudder effect and so to investigate the manoeuvring of ships.

However, a rough estimation of the rudder moment is included in the equation of yaw motion. The resulting behaviour of the ship under this effect is shown in fig. (6.33) which shows that the ship made four complete turns.

It can be seen that heave, pitch and roll amplitudes vary according to the rotation of the ship, i.e. with the heading angle. The amplitudes of heave and roll become maximum where the ship becomes in beam sea conditions, while the amplitude of pitching motion becomes minimum.
11. **CONCLUDING REMARKS**

Certain conclusions regarding the effect of the different parameters on the motions of the GAUL can be drawn from this investigation.

11.1 The dynamical stability of the ship in a seaway is affected considerably by some parameters, such as heading angle and encounter frequency in certain conditions and a combination of the effects of these parameters may lead the ship to capsize.

11.2 The restoring term has the most influence on the behaviour of the ship in waves. Bias and non-linearities, which may appear in roll behaviour, occur due to the fluctuation of the lever arm and the displacement of the ship during motion.

11.3 The investigation has illustrated that the conditions where the ship experiences a small encounter frequency, ie the relative speed between the ship and waves is small, cause dangerous situations due to the effect of encounter frequency on damping coefficient values.

11.4 Theoretical and experimental results of the model behaviour in beam seas demonstrate that bilge keels are of great importance to the dynamical stability.

11.5 The different loading conditions and wind moment have a slight influence, for this particular ship, on the amplitudes of motions and the mean roll angle but their effects could be considerable if combined and added to other effects such as the effect of bias and small encounter frequencies.
Fig. (6.1) Block diagram showing the structure of the Time Simulation Computer Program
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation</th>
<th>Condition of the Other Parameters</th>
<th>Figure Nos</th>
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<tr>
<td></td>
<td></td>
<td>$\mu$</td>
<td>$\omega$</td>
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<td>Heading Angle</td>
<td>$\mu = 15^\circ$, 45^\circ$</td>
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<td></td>
<td>$90^\circ$ and 135$^\circ$</td>
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<td>$T$</td>
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Table (6.1) Parametric Variations
FIG. (6.3) THE EFFECT OF HEADING ANGLE ON SHIP MOTIONS
FIG. (6.5) THE EFFECT OF HEADING ANGLE ON SHIP MOTIONS

\[ \mu = 135^\circ \]
Fig. (6.6) Variation of GZ, displacement and righting moment during ship motion
FIG. 15 - 8. THE EFFECT OF WAVE FREQUENCY ON SHIP MOTIONS

FIG. 15 - 9. THE EFFECT OF WAVE FREQUENCY ON SHIP MOTIONS
<table>
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<th>Roll</th>
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**Fig. 6.10** The effect of wave frequency on ship motions.

**Fig. 6.91** The effect of wave frequency on ship motions.
FIG. (6.15) THE EFFECT OF LOADING CONDITION ON SHIP MOTIONS

FIG. (6.16) THE EFFECT OF LOADING CONDITION ON SHIP MOTIONS
### Model details

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**Fig. (6.22.a) Bodyplan of the model**

**Fig. (6.22.b) Bodyplan of the model after changes**
The Effect of Bias on Model Motions

Model Test
Case 58B
Fig. (6.27) The effect of bias on ship motions.
FIG. 6.29 THE EFFECT OF SOME PARAMETERS ON SHIP MOTIONS

FIG. 6.28 THE EFFECT OF SOME PARAMETERS ON SHIP MOTIONS
FIG. (6.32) THE EFFECT OF SOME PARAMETERS ON SHIP MOTIONS
FIG. (6.33) THE EFFECT OF SOME PARAMETERS ON SHIP MOTIONS
Conclusions
CHAPTER 7

CONCLUSIONS

OVERALL CONCLUSIONS

In this thesis conclusions have been presented for each chapter. Many of these are significant in relation to the problem discussed in that chapter, e.g. in Chapter 2, the variation of GZ in different wave profiles has a considerable importance to the quasi-static analysis usually carried out to investigate accidents such as the loss of the GAUL.

However, the results of the time-simulation program enables one to assess the individual effects in perspective and this chapter attempts to draw broad conclusions from the complete study.

Many of these depend on the validity of the numerical modelling of the ship's behaviour and, within the time available, every effort has been made to verify each part of the program against either published computations or model tests. It is believed that the agreement between the model tests reported in Ref. (86) and the program results for the same conditions demonstrates the validity and utility of the numerical model.

As the calculations have proceeded through the analysis of righting moments, effects of wave shape, hydrodynamic coefficients,
viscous roll damping and wave-exciting forces and moments, it has been observed on many occasions that the behaviour of individual characteristics are non-linear, particularly those associated with rolling. However, in the time-simulation of the motion it has been demonstrated that by far the most important non-linearity is associated with the restoring moment and that the variation of many of the hydrodynamic terms with time, for the ship as a whole, can be linearly associated with the heave. This is not true for the hydrodynamic coefficients for roll which have shown a clear non-linearity in their quantities as the ship moves in waves. Circumstances have been illustrated in which these coefficients may vanish showing the necessity to deal with all the terms of the rolling equation in a time-domain manner.

It is important to use the above observations about the relationship of the hydrodynamic coefficients with heave as the time simulation can be speeded up by a factor of 100 to 200 and makes possible the practical use of such programs. As computer facilities improve (and at present computers are available which have computing powers 100 times the computer available in this project) it will be possible to do extended runs in real seaways with human interaction in terms of control of rudder angle, etc.

To do this the program needs to be extended to include random seas with a particular concentration on the accurate calculation of the 'restoring term', which cannot be presented by a mathematical form, since it is strongly dependent upon the relative motions of the ship and waves. This term has been shown to be the biggest source of the non-linearity of ship motions when the ship's oscillatory
motions, which are usually neglected in the theoretical analysis are considered. Accordingly, the examination of dynamical stability must take the variation of both GZ and ship displacement during the motion into account. This variation cannot be represented in either the static or quasi-static analysis and its accurate determination has to be one of the features of dynamical analysis.

The limited parametric studies, which have been achieved for complete vessel motions, have indicated that the most dangerous situation for the ship is in a quartering sea condition when the wave speed corresponds approximately to the ship's speed. Such a situation gives the ship a small encounter frequency leading to a small damping coefficient and, consequently, the ship experiences large amplitude motions which may result in a small restoring moment at large heeling angles. However, it will be necessary to perform further investigations into the hydrodynamic and hydrostatic effects of following seas upon the stability of ships in order to ascertain the causes of the dangerous situations and to specify their circumstances.

On the other hand, with regard to the selective parameters studied, the effect of wind moment on the dynamical stability of the ship is not a decisive factor as suggested by the normal stability criteria. However, the effect of bilge keels, which is not included in any of these criteria, has the most influence on the dynamical stability as shown by both the theoretical and experimental investigation of a simple model. Thus the designer must ensure, through bilge keels or similar devices, an adequate level of damping to secure the safety of the ship.
The investigation into the hydrodynamic features of different ship cross-sections has indicated that different shapes produce different hydrodynamic features in magnitude and character; e.g., the hydrodynamic values for rolling motion reach zero at certain draughts for U-type cross-sections while they increase with draught for V-type sections. This highlights another area for future use of this program as a means of comparing the seakeeping qualities for different hull forms and to look at the design of hull forms from the viewpoint of dynamical stability.

Before doing such work, the program needs to be extended to cover the following situations:

1. Unidirectional random seaway.

2. Multi-directional random seaway.

3. Random wind gusting.

4. Operation of helm to represent realistic full-scale behaviour.

At one time it was thought that the extension to random seas would present theoretical difficulties due to the difficulty of identifying a specific frequency to associate with the determination of the hydrodynamic coefficients but the current work suggests that many of these coefficients are insensitive to frequency for the ship as a whole and that it is now possible to extend the program to the random sea situation. It will be important to have some good model results to compare with the computed motions but once the validity of
such a program has been established, it will eventually be much cheaper to carry out extended simulated motion studies rather than run models in experiment tanks. The physical understanding of the behaviour which can be obtained from the computation is greater than can be achieved by observing and measuring the vessel motion in waves since it is difficult to assess how the righting moment and other factors are behaving.

It is thus felt that the completion of this time simulation program represents a significant step forward towards an improved understanding of ship dynamics in severe seas and to eventually improve ship design.
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Appendices
APPENDIX A

SHIP'S HULL OFFSETS

The ship dimensions and offset values of the hull are stored in a data file with a sequence as described earlier. A typical offset data file for the hull of the trawler GAUL is given as follows under the name of 'GAUL.DAT' without considering the bulwark height.

Checks are made, for mis-typing an offset value or incorrectly reading a value from the drawing, by plotting the body plan of the relevant ship. The plottings of the profile and the body plan corresponding to the following values is shown in fig. (A.1).

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Fig. (A.1) Profile and bodyplan of the trawler GAUL
## APPENDIX B

### DIFFERENT LOADING CONDITIONS FOR THE TRAWLER GAUL

#### Condition IMCO 1

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<tr>
<th>DEADWEIGHT ITEMS</th>
<th>TANKS</th>
<th>KG (MT)</th>
<th>VERTICAL MOMENTS</th>
<th>LCG (CM)</th>
<th>LONGITUDINAL MOMENT</th>
<th>FREEBOARD (CM)</th>
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<tr>
<td>D.O. FORWARD DEEP TANK</td>
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<td>3.66</td>
<td>326.8</td>
<td>20.71</td>
<td>16.76</td>
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<td>9.36</td>
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<td>4.42</td>
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<td>316.39</td>
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<td>0.81</td>
<td>64.7</td>
<td>16.73</td>
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<td>L.O. NO. 3 DOUBLE BTH, P.A.</td>
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<td>OIL TANKS AFT</td>
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<tr>
<td>F.W. DRINKING P.E.S.</td>
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<td>4.36</td>
<td>98.1</td>
<td>174.8</td>
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<td>2.37</td>
<td>39.7</td>
<td>164.6</td>
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<th>LONGITUDINAL MOMENT</th>
<th>FREEBOARD (CM)</th>
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<td>4.42</td>
<td>276.1</td>
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<td>316.39</td>
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<td>0.81</td>
<td>64.7</td>
<td>16.73</td>
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<tr>
<td>L.O. NO. 3 DOUBLE BTH, P.A.</td>
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<td>OIL TANKS AFT</td>
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<td>7.32</td>
<td>37.6</td>
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</table>

**Note:** The table includes various items such as deadweight, light weight, displacement, draughts, and mean. Each condition (IMCO 1 and IMCO 2) lists different loading conditions for the trawler Gaul.
**DIFFERENT LOADING CONDITIONS FOR THE TRAWLER GAUL**

### Condition IMCO 3

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<th>VERTICAL MOMENTS</th>
<th>L.C.G.</th>
<th>LONGITUDINAL M.M.</th>
<th>DEADWEIGHT</th>
<th>GENERAL EG.</th>
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### Condition IMCO 4

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<th>GENERAL EG.</th>
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### DRAUGHTS

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<tr>
<td>@ A.P.</td>
<td>4.56 M.</td>
<td>Fluor</td>
<td>0.05</td>
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APPENDIX C:

CALCULATION OF WIND MOMENT

The particulars of the ship are as follows:

L = 56.85 metres
B = 12.19 metres
Δ = 1529 Tonnes
KG = 5.45 metres
T = 4.21 metres

Projected windage area:

A_w = 420m^2
a = 2.9

Wind moment arm: (see fig. (C.1))

h_w = T + a - KG = 1.51 metres

The wind pressure is taken as:

p = 0.48 KN/m^2

Then the steady wind moment can be found as

M_w = pA_wh/g, g = 9.81 m/sec^2

M_w = 30.9 tonne.metre