Constructing Ancient Mechanics

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My thanks firstly to my supervisor, Ian Ruffell, whose patience and understanding has dragged me kicking and screaming to the point of submission. The flexibility he has offered me in presenting papers and pursuing publication has left me with a far richer postgraduate experience than expected.

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In the end, I think little describes my experience of writing, and relationship with, this thesis as well as the following lines from a song I have listened to countless times while preparing this work:

> My head on the ground
> with my mouth full of dirt
> your foot in my face
> is what keeps me alive
> when it's over,
> when it's over,
> when it's over,
> when this is over, I will sleep
>
> *Sleep!*, Big Black (1984), *Racer X*, Homestead Records.

Contents

1 Introduction

Newton opens his introduction to the first edition of the *Principia Mathematica* with the following passage:

> *Cum veteres mechanicam (uti auctor est Pappus) in rerum naturalium investigatione maximi fecerint; & recentiores, missis formis substantialibus & qualitatibus occultis, phaenomena naturae ad leges mathematicas revocare aggressi sint: Visum est in hoc tractatu mathesin excolere, quatenus ea ad philosophiam spectat. Mechanicam vero duplicem veteres constituerunt: rationalem, quae per demonstrationes accurate procedit, & practicam. Ad practicam spectant artes omnes manuales, a quibus utique mechanica nomen mutuata est.*[1]

These statements, made in the introduction to the work that would revolutionise and establish the science of mechanics, reflecting on the foundations upon which Newton's work is built, seem to be a legitimate starting point for investigation into ancient mechanics. This area, now oft-referred to as being 'the Cinderella of ancient science'[2] after Fraser's description of the field, is a significant collection of texts in itself, though the extant corpus is substantially smaller than the extant work on other subjects such as

[1] Newton 1972, p. 15
[2] Fraser 1972, p. 425

mathematics or medicine. Newton's assertion that there was a theoretical and practical aspect of mechanics is one that has been lifted directly from the work of Pappus, and is one that would seem to hold when compared with the content of the extant mechanical corpus, though this claim will be examined in far greater depth in chapter 2. It must be said, however, that our own viewpoint on the field is so strongly informed by the work of Newton and other modern scientists, and this interpretation of Pappus so closely reflects a modern viewpoint, that it is easy to assume that the ancient definition of theoretical mechanics is the same as our own. After all, we understand a clear division between the theoretical work of the physicist and the practical work of the engineer, and it is tempting to see this division in the audiences for ancient mechanical texts.

While we may see the division between theoretical and practical reflected to an extent in the ancient world, the field of ancient mechanics is such a disparate and diverse area that there is an inevitable breakdown in the similarity. Simply put, the aim of this thesis is to establish what the extent, and nature, of theoretical mechanics is in the ancient world. The basic outline of this thesis is as follows:

Chapter 2 deals with the scope of ancient mechanics in the ancient world by providing a chronological overview of the work of the main figures of the field. The subject matter of these works is summarised in an attempt to determine the sub-fields of ancient mechanics.

Chapter 3 looks at the definition and perception of mechanics in the ancient world by examining the way in which mechanics is represented in both mechanical texts and other ancient sources.

These two chapters should provide wider context for the remaining investigation into a number of topics that have been identified as being representative of the state of ancient mechanics in the ancient world. While the two chapters just mentioned will examine a wide variety of sources, these chapters will focus on the contents of a number of texts that have seemed to provide the most theoretical content. These are the Pseudo-Aristotelean *Mechanica*, Archimedes' *De Planorum Aequilibriis*, Heron's *Mechanica*, and the final book of Pappus' *Synagoge*

Chapter 4 discusses the principal mechanical theory that underpins the discussion in the Pseudo-Aristotelean *Mechanica*, that comes to subsequently play a significant role in the ancient theory of mechanics.

Chapter 5 examines the exploration of equilibrium found in Archimedes *De Planorum Aequilibris*, along with the axiomatic methodology it employs.

Chapter 6 analyses the quintessentially mathematical Delian problem in the context of ancient mechanics.

Chapter 7 examines examines the simple machines, with the exclusion of the lever, as outlined in Heron's *Mechanica*.

Chapter 8 examines the approach found in Heron's *Mechanica* and Pappus' *Synagoge* to force, friction and the inclined plane.

2 Scope

There is a certain difficulty in attempting to define a field such as mechanics in the ancient world. The relative paucity of extant textual evidence often makes definite statements on the field at a given point reliant upon a single text, which normally covers only a limited subsection of the field. To provide context for subsequent definitions of the field and examination of theoretical mechanics, I will in this chapter, provide a chronological account of the figures associated with mechanics in the period of time being covered in this thesis, roughly from the tail end of the fifth century BC till the fifth century AD. While the exact scope of the field across some nine hundred or so years was almost certainly not static, an overview of who was working on what, and when they were doing it, can provide us with some insight before attempting a definition of the entire field. Having completed this survey, some general comments will be made about the main fields that we have seen in the works of the extant mechanical authors, taken in tandem with the definition of the field found in the work of Pappus and Proclus, establishing the extent to which their accounts are accurate.

2.1 Archytas

Archytas of Tarentum, a statesman and philosopher of the fifth to fourth centuries BC, is generally attributed with the foundation of the field of mechanics in the ancient world.[1] There is, however, no real surviving work by him on the subject, and the attribution may have far more to do with the trend in the ancient doxographical tradition of attempting to find the 'first founders' of a field rather than the composition of a work that could be considered a *Mechanica*.[2] The main mechanical association in the work of Archytas seems to be his solution to the problem of finding the two mean proportionals, a topic that will be explored in depth in chapter 6. This essentially geometrical problem does not have an immediate association for us with mechanics, but it does become a significant element in other ancient works on the subject, and would certainly seem to be the main reason for Archytas' importance to mechanics. I will explore this attribution further in the following chapter.

2.2 Pseudo-Aristotelean *Mechanica*

The pseudo-Aristotelean *Mechanica* is the earliest extant, and, indeed, attested text on the subject of mechanics. The work examines a series of thirty-five problems deemed to be mechanical, proceeding with a question and answer structure that is familiar from the Aristotelean *Problemata*. The *Mechanica*, however, begins by exploring the underlying principles by means of which these mechanical phenomena can be explained, which is the subject of chapter 4. The focus of this text is entirely upon what we would now consider the field of statics, and is principally concerned with explaining mechanical phenomena, rather than providing descriptions of the construction of devices, which forms such a large part of the work of later mechanical authors. Authorship of this text has been attributed to Aristotle in the ancient world and early modern period, but it

[1] Diogenes Laertius and Plutarch cast Archytas in this role when talking about mechanics: see D.L. 8.83 and Plut. *Marc.* 14.5-6. Huffman provides much more extensive commentary on the subject, see Huffman 2005, pp. 77-83.

[2] Zhmud 2006, p. 176 and Berryman 2009, p. 88

is most likely to be the work of another peripatetic author.[3] For our purposes the fact that it was composed in this *milieu* is far more significant than ascribing authorship to one particular individual, as we will explore in the subsequent chapter.

A definition of the field of mechanics as a whole is not found in this text, but rather a more general statement about the content of this work; *περιέχεται δὲ τῶν ἀπορουμένων ἐν τῷ γένει τούτῳ τὰ περὶ τὸν μοχλόν*.[4] This association of mechanical problems back to the lever describes the vast majority of questions found in the *Mechanica*, but does not preclude the existence of other aspects of mechanics. Indeed, seems to suggest that the problems being dealt with in this text only constitute a subset of problems from a wider field. This particular text had a significant influence upon later work on mechanics, which can certainly be seen in the work of Heron, and certainly on later medieval work on the subject.[5]

2.3 Euclid

There are several medieval Latin texts on statics that are attributed to Euclid, which have titles such as *De levi et ponderoso*, *De gravi et levi*, *De ponderibus* or *De canonio*.[6] Some of these Latin texts are translations from Arabic texts, though *De canonio* is presumed to be a translation from a Greek source.[7] It is difficult to come to any conclusions as to

[3] Heath does not venture a suggestion as to the identity of the author, but does discuss the difference in terminology in the *Mechanica* from that of Aristotle, and the similarity of the terminology to Euclid (T. Heath 1921, p. 344). The authorship of the work is mentioned by Ross, who hesitantly ascribes the work to 'Strato or one of his pupils' (Ross 1995, p. 6) which is echoed by Drachmann (Drachmann 1963b, p. 10). Krafft cites a number of other authors who discuss Strato as the author, but he himself would rather ascribe authorship to Aristotle (Krafft 1970, p. 18). Winter has recently argued for Archytas as the author of the work, dismissing Ross' assertions as attribution by convenience before proceeding to do exactly the same thing himself (Winter 2007).

[4] Arist. *Mech.* 847b11

[5] Clagett 1959, pp. 3-4

[6] Pauly. *Euclid*

[7] Moody and Clagett 1960, pp. 58-59

whether these texts are representative of genuine works by Euclid on mechanics. There are no references in other Greek texts to any such works by Euclid,[8] and, at least as far as Heath is concerned, the conception of specific gravity found in *De levi et ponderoso* could not pre-date the work of Archimedes.[9] The lack of any definite conclusions about the date and source of these texts somewhat precludes their use as a source for ancient mechanics, and so they are certainly more useful as a source for medieval mechanics.

2.4 Ctesibius

There are no surviving works by the mathematician Ctesibius, who was working in Alexandria around 270 BC. He is, however, repeatedly mentioned by later mechanical authors, and it seems that these mechanicians owe a considerable debt to him, with all those who subsequently documented the construction of artillery devices citing his work.[10] While from this it can be established that he produced significant work on belopoeitics, he is also known to have produced some of the early work on pneumatics. His anaphoric water clock[11] and his keyboard-driven water organ[12] are both reported manifestations of his work on this subject.

2.5 Archimedes

Archimedes is, almost certainly, the first name that would come to mind when thinking of ancient mechanics. This association follows from, for most of us, the apocryphal stories recounted in later sources about his life. His exposed exit from the bathtub ex-

[8] Clagett 1959, p. 28

[9] T.L. Heath 1956, p. 18, and as Clagett points out, the nature of the Arabic *On the Balance* is Archimedean rather than Aristotelean.

[10] Marsden 1971, p. 2

[11] Oleson 2008, pp. 340-341

[12] Vitr. *De Arch.* 10.8, Hero. *Spir.* 76

claiming *εὕρηκα* is the first introduction that many have to hydrostatics,[13] while the statement δός *μοί ποῦ στῶ καὶ κινῶ τὴν γῆν* has become the maxim by which the power of the lever is expressed, though it is far more likely a statement on mechanics more generally.[14] This Syracusan mathematician lived during the third century, and is famously reputed to have died at the hands of a Roman soldier during the fall of the city.[15]

Of his extant works, the only two notable texts on mechanics are the *De planorum aequilibriis* and *De corporibus fluitantibus*, which deal respectively with statics and hydrostatics. There is no evidence to be found within these texts for Archimedes' own definition of mechanics; indeed, they begin without any introduction, and plunge straight into the Archimedean structure of postulates and propositions. Archimedes' *Ad Eratosthenem methodus*, conventionally known as 'The Method', is a work that would initially seem to be, by its reputation and the terminology used, one that deals with mechanics. However, it deals instead with the application of mechanical principles to solving the mathematical problem of calculating the area of given geometrical shape. The work on statics that is found in *De planorum aequilibriis* is discussed in greater depth in chapter 5.

Pappus, quoting Carpus of Antioch, mentions that Archimedes only produced one work on mechanics, which dealt with the construction of spheres (*περὶ σφαιροποιίας*) , that is, astrolabes and devices that recreated astronomical phenomena,[16] however, he does later mention that Archimedes had produced a *περὶ ζυγῶν*,[17] and a *κεντροβαρικά*

[13] For the original account see *Vitr.*9.10

[14] We are familiar with the statement recorded by Pappus (see Pappus.*Collectio*.8.10: 1060) but the story exists in a variety of forms. See Dijksterhuis 1987, pp. 14-21 a fuller exploration of these two tales and the biographical tradition about Archimedes.

[15] Dijksterhuis 1987, pp. 30-32

[16] Papp. 1026

[17] Papp. 1068

is mentioned by Simplicius.[18] Drachmann has argued that extracts of these two texts, along with another, perhaps titled *On Uprights*, are to be found in Heron's *Mechanica*.[19] It is the opinion of Heath that they are the missing works that would fully explain the material in *De planorum aequilibriis*.[20]

2.6 Biton

Biton is the author of a short text on siege equipment, who, given that he dedicated the work to a King Attalus, was likely in the employ of one of the Attalid dynasty. An association that dates him to between 230 BCand 133 BC,[21] though Marsden considers it most likely that he produced the work at some point in the middle of this period.[22] The work is therefore the earliest extant text on artillery construction, a field that forms a significant portion of extant ancient mechanical texts. The text is notable in as far as it describes the construction of catapults that do not use torsion engines.

2.7 Philon

Of the Hellenistic mechanical authors, Philon is the earliest for whom a substantial amount of writing is extant. He is reputed to have lived very shortly after Ctesibius, so is likely to have been active at the beginning of the second century BC. He was the author of a *Mechanike syntaxis* consisting of nine books;

1. *Isagoge* – Introduction
2. *Mochlica* – On Levers

[18] Simp. *In Cael.* 508a30
[19] Drachmann 1963a
[20] T. L. Heath 1897, pp. xxxvii-xxxviii
[21] *Pauly* s.v. Biton
[22] Marsden 1971, p. 61

3. *Limenopoeica* – On Harbour Construction
4. *Belopoeica* – The Artillery Manual
5. *Pneumatica* – Pneumatics
6. *Automatopoeica* – On Automata Making
7. *Parasceuastica* – Siege Preparations
8. *Poliorcetica* – Siegecraft
9. *Peri Epistolon* – On Strategems[23]

Of these nine original texts, the *Belopoeica*, *Parasceuastica* and *Poliorcetica* are all extant in Greek, while the *Pneumatica* is extant in Latin and in an Arabic translation. That he had produced a collection consisting of a number of books on different mechanical subjects is itself notable as this seems to have been the first work of this kind. Many of the subjects included in the work of Philon were later covered, and, if we are to believe him, improved by Heron. It is likely that this is because these topics represented in this text are typical of those worked on by mechanicians rather than Philon having produced a text that significantly influenced the course of writing on mechanics.

2.8 Athenaeus Mechanicus

Very little is known about the Athenaeus who composed the relatively short *Peri Mechanematon*. He dedicates the text to a Marcellus, who is likely to be the nephew and prospective heir of Augustus, dating him to the first century BC/AD.[24] The work once again deals with weapons of war, detailing the construction of a variety of siege machinery

[23] Marsden 1971, p. 156, which cites Orinsky, Neugebauer and Drachmann's entry on Philon in the *Realencyclopädie der Classischen Altertumswissenschaft*

[24] Whitehead and Blyth 2004, p. 18-19

such as siege towers and rams. He cites a number of other mechanical authors where he considers their descriptions of devices sufficient rather than his own.[25]

2.9 Vitruvius

The sole Latin author who produced work on the field of mechanics is the famous architect Marcus Vitruvius Pollio. Born in the early first century BC, the ten books of his *De Architectura* were likely published at some point between 30 and 20 BC.[26] The ten books all deal with architecture and related fields, included in which is the tenth book which deals with the construction of machines from the perspective of the Roman architect. This work discusses the fundamentals of the construction of machines (rather than the fundamentals of mechanics), the construction of cranes and weight lifting devices, water based devices (including the aforementioned water organ and water pump of Ctesibius) as well as the construction of siege machinery and catapults.

2.10 Heron

The extant corpus of Heron of Alexandria is fairly substantial, and includes the largest surviving group of works on mechanics by a single author in the ancient world. Heron, who is now generally accepted to be living and working in the first century BC,[27] is almost certainly most famous for his record of the aeolipile, a simple steam-engine, in the *Pneumatica*, the longest of his extant mechanical works. This text describes over seventy different devices, all of which work on some kind of water or compressed air power. As

[25] It is worth noting here that there are a number of other mechanical authors cited in this manner, not only in the work of Athenaeus but in other mechanical authors. I have not included them in this discussion as little can be gain from their names alone.

[26] Rowland and Howe 2002, p. 2

[27] Neugebauer 1938

Berryman has noted, to call this topic pneumatics in English does not strictly describe the extent of the field, which deals with more than just compressed gases, it is, however, a descriptive enough name for the topic.[28] Heron's *Automatopoeitica*, a work on automata making is, by his own account, an improvement on the homonymous work by Philon, and seems to deal with the same devices. This text describes the construction of mobile and stationary automata, going into significant detail, especially when compared with the *Pneumatica*, on the exact mechanisms that should be utilised to generate specific effects, along with some explanation of the underlying theoretical basis for the choice of these mechanisms. There are two extant works, of very different character, by Heron on the subject of artillery construction. His *Belopoeica* discusses the construction, in some detail, of a number of different artillery weapons that use torsion engines to drive the missile. He introduces the text with some comments on the importance and history of the field, and includes some historical comment throughout. The final sections of the text deal with scaling torsion engines, and the theorem of the two mean proportionals, which will be discussed further in chapter 6. The text is titled Ἥρωνος Κτησιβίου Βελοποιικά, which perhaps indicates, though it is by no means certain, that the text is building upon the work of Ctesibius. The similarities it shares with Philon's *Belopoeica*, are explained by Marsden as being due to the reliance of both authors on the work of Ctesibius.[29]His *Cheiroballistra* adopts a different and much drier tone, providing only a listing of the components that make up the device, along with their dimensions. Marsden has taken this as indicating that the work was intended for a more technical audience, perhaps describing a device that was a new design.[30] Finally, there is a *Mechanica* by Heron, of which only some sections are extant

[28]Berryman 2009, p. 155
[29]Marsden 1971, pp. 1-2
[30]Marsden 1971, pp. 2-3

in Greek, with the rest surviving in an Arabic translation produced by Qusṭâ ibn Lûqâ at some point between 862 AD and 866 AD.[31] The text begins with a description of a *barulkos*, an device for lifting heavy weights, which utilises a geared transmission to generate mechanical advantage. This description is also found in Heron's *Dioptra*,[32] a text that deals with surveying instruments, an appearance as incongruous as at the start of the *Mechanica*. Although related to the material later in the *Mechanica* it is generally accepted that this passage is from another work and has been added to these two texts at some point in the tradition.[33] The rest of the first book deals with a more generalised theory of mechanics and force (discussed in greater depth in chapter 8), while the second is where we first find the concept of the five simple machines (or powers) in mechanical literature.[34] The final book describes more complex devices that are related to the five simple machines.

2.11 Pappus of Alexandria

The mathematician Pappus of Alexandria can be positively dated to the fourth century AD by a solar eclipse that he mentions in his commentary on Ptolemy's *Almagest*, which was identified by Rome as occurring in 320 AD.[35] Pappus is also the author of a commentary on Euclid's *Elements*, but his *magnum opus* is his *Synagoge*, a collection of eight books on a variety of mathematical topics. The majority of this text is extant, with only the first book and the beginning of the second entirely missing. The remaining books deal with geometry, arithmetical and astronomical problems, but it is the eighth book,

[31] Drachmann 1963b, p. 21 I am relying on the English translation of Drachmann, and the German translation of Nix and W. Schmidt when dealing with the material in Arabic.

[32] Hero. *Dioptr.* 37

[33] Berryman 2009, pp. 134-135 and Drachmann 1963b, pp. 27-32

[34] While these devices appear in earlier texts, such as the Pseudo-Aristotelean *Mechanica*, this is the first point they are grouped and described as the fundamental building blocks of all mechanisms.

[35] Rome 1931, pp. x-xiii, see also Cuomo 2000, pp. 5-6

dealing with the subject of mechanics, which is of interest to us. Pappus does not devote much of this work to the construction of devices, describing only a weight lifting device, but instead focusses on three problem areas: the inclined plane, the duplication of a cube (the problem of the two mean proportionals) and matching cog wheels of given numbers of teeth. The inclined plane and the problem of the two mean proportionals had previously been covered in Heron's *Mechanica*, and the problem of the cog wheels has similarities with the problems presented in Heron's description of the *barulkos*.[36] However, Pappus' lengthy introduction to this book, with its provisioning of a definition of the field of mechanics, and a description of its sub-fields, provides us with a great deal of insight into the subject.

Pappus defines mechanics as follows:

> *στάσεως γὰρ καὶ φορᾶς σωμάτων καὶ τῆς κατὰ τόπον κινήσεως ἐν τοῖς ὅλοις θεωρηματικὴ τυγχάνουσα τὰ μὲν κινούμενα κατὰ φύσιν αἰτιολογεῖ, τὰ δ' ἀναγκάζουσα παρὰ φύσιν ἔξω τῶν οἰκείων τόπων εἰς ἐναντίας κινήσεις μεθίστησιν ἐπιμηχανωμένη διὰ τῶν ἐξ αὐτῆς τῆς ὕλης ὑποπιπτόντων αὐτῇ θεωρημάτων. τῆς δὲ μηχανικῆς τὸ μὲν εἶναι λογικὸν τὸ δὲ χειρουργικὸν οἱ περὶ τὸν Ἥρωνα μηχανικοὶ λέγουσιν καὶ τὸ μὲν λογικὸν συνεστάναι μέρος ἔκ τε γεωμετρίας καὶ ἀριθμητικῆς καὶ ἀστρονομίας καὶ τῶν φυσικῶν λόγων, τὸ δὲ χειρουργικὸν ἔκ τε χαλκευτικῆς καὶ οἰκοδομικῆς καὶ τεκτονικῆς καὶ ζωγραφικῆς καὶ τῆς ἐν τούτοις κατὰ χεῖρα ἀσκήσεως* [37]

Ascribing the idea that there is a division between the practical and theoretical aspects of mechanics to the followers of Heron, he describes the disciplines that constitute

[36]Cuomo 2000, p. 109
[37]Papp. 1022-1024

each side of this dichotomy. Geometry, arithmetic and physics have all played a clear role in the earlier works on mechanics, astronomy has not featured in other texts, but this may be explained by the mention of sphaeropoetics later in the introduction. The division he makes here between the theoretical and the practical is not one between different types of mechanical texts, but rather between the theory underpinning the design, and the art of constructing the devices themselves.

Pappus' book on mechanics is unique amongst ancient sources on mechanics as it is intended to serve as a compendium of the most significant aspects of previous work on the field. It should then provide an overview of the field of mechanics that is not found in other extant works, and give a suitable indication of the constituent parts of the greater whole. Pappus describes the key sub fields of mechanics later in the introduction:

> Μάλιστα δὲ πάντων ἀναγκαιόταται τέχναι τυγχάνουσιν πρὸς τὴν τοῦ βίου χρείαν [μηχανικὴ προηγουμένη τῆς ἀρχιτεκτονικῆς] ἥ τε τῶν μαγγαναρίων, μηχανικῶν καὶ αὐτῶν κατὰ τοὺς ἀρχαίους λεγομένων (μεγάλα γὰρ οὗτοι βάρη διὰ μηχανῶν παρὰ φύσιν εἰς ὕψος ἀνάγουσιν ἐλάττονι δυνάμει κινοῦντες), καὶ ἡ τῶν ὀργανοποιῶν τῶν πρὸς τὸν πόλεμον ἀναγκαίων, καλουμένων δὲ καὶ αὐτῶν μηχανικῶν (βέλη γὰρ καὶ λίθινα καὶ σιδηρᾶ καὶ τὰ παραπλήσια τούτοις ἐξαποστέλλεται εἰς μακρὸν ὁδοῦ μῆκος τοῖς ὑπ' αὐτῶν γινομένοις ὀργάνοις καταπαλτικοῖς), πρὸς δὲ ταύταις ἡ τῶν ἰδίως πάλιν καλουμένων μηχανοποιῶν (ἐκ βάθους γὰρ πολλοῦ ὕδωρ εὐκολώτερον ἀνάγεται διὰ τῶν ἀντληματικῶν ὀργάνων ὧν αὐτοὶ κατασκευάζουσιν). καλοῦσι δὲ μηχανικοὺς οἱ παλαιοὶ καὶ τοὺς θαυμασιουργούς, ὧν οἱ μὲν διὰ πνευμάτων φιλοτεχνοῦσιν, ὡς Ἥρων πνευματικοῖς, οἱ δὲ διὰ νευρίων καὶ σπάρτων ἐμψύχων

> *κινήσεις δοκοῦσι μιμεῖσθαι, ὡς Ἥρων αὐτομάτοις καὶ ζυγίοις, ἄλλοι δὲ διὰ τῶν ἐφ' ὕδατος ὀχουμένων, ὡς Ἀρχιμήδης ὀχουμένοις, ἢ τῶν δι' ὕδατος ὡρολογίων, ὡς Ἥρων ὑδρείοις, ἃ δὴ καὶ τῇ γνωμονικῇ θεωρίᾳ κοινωνοῦντα φαίνεται. μηχανικοὺς δὲ καλοῦσιν καὶ τοὺς τὰς σφαιροποιΐας [ποιεῖν] ἐπισταμένους, ὑφ' ὧν εἰκὼν τοῦ οὐρανοῦ κατασκευάζεται δι' ὁμαλῆς καὶ ἐγκυκλίου κινήσεως ὕδατος.* [38]

Proclus, living a generation after Pappus, in the fifth century AD[39] produced a very similar listing of the fields of mechanics in his commentary on the first book of Euclid. It is useful to view this passage in tandem with that of Pappus, as the similarities between the two lists indicate the wider codification and acceptance of these definitions:

> *Πρὸς δὴ ταύταις ἡ μηχανικὴ καλουμένη τῆς περὶ τὰ αἰσθητὰ καὶ τὰ ἔνυλα πραγματείας μέρος ὑπάρχουσα, ὑπὸ δὲ ταύτην ἥ τε ὀργανοποιϊκὴ τῶν κατὰ πόλεμον ἐπιτηδείων ὀργάνων, οἷα δὴ καὶ Ἀρχιμήδης λέγεται κατασκευάσαι τῶν πολεμούντων τὴν Συράκουσαν ἀμυντικὰ ὄργανα, καὶ ἡ θαυματοποιϊκὴ τὰ μὲν διὰ πνῶν φιλοτεχνοῦσα, ὥσπερ καὶ Κτησίβιος καὶ Ἥρων πραγματεύονται, τὰ δὲ διὰ ῥοπῶν, ὧν τῆς μὲν κινήσεως τὴν ἀνισορροπίαν αἰτιατέον, τῆς δὲ στάσεως τὴν ἰσορροπίαν, ὥσπερ καὶ ὁ Τίμαιος διώρισεν, τὰ δὲ διὰ νεύρων καὶ σπάρτων ἐμψύχους ὁλκὰς καὶ κινήσεις ἀπομιμουμένων. ὑπὸ δὲ τὴν μηχανικήν ἐστιν καὶ ἡ τῶν ἰσορρόπων ὅλως καὶ τῶν λεγομένων κεντροβαρικῶν διάγνωσις, καὶ ἡ σφαιροποιΐα κατὰ μίμησιν τῶν οὐρανίων περιφορῶν, οἵαν καὶ Ἀρχιμήδης ἐπραγματεύσατο, καὶ ὅλως πᾶσα ἡ τῆς ὕλης κινητική.*[40]

[38]Papp. 1024-1026
[39]8 February 412 AD- 17 April 485 AD
[40]Procl. *in Euc.* 41-42 (Friedlein)

While this passage likely demonstrates Proclus' familiarity with the work of Pappus, that it is echoed without criticism demonstrates that this conception of the field of mechanics is a common one at this point. The difficulty with overt reliance on these definitions of the sub-fields of mechanics is that these sources are very late in the tradition of ancient mechanical writing. A field such as this is not static, certainly not over the course of some thousand years.

Of course, the question then is, do these definitions of the field of mechanics match what we find in the earlier sources? The ample extant sources on the creation of artillery and siege equipment certainly supports the inclusion of this field in both of these passages. Both sources talk of the construction of 'marvellous devices',[41] the description of which matches the *Pneumatica* of both Heron and Philon, as well as Heron's *Automatopoeitica*. Although there are a wide range of different techniques and mechanisms used to implement the devices described in these texts, the commonality is certainly the nature of these devices, in that they mostly seem to serve as objects for entertainment or amusement rather than any practical purpose.[42] 'Sphere construction' is not a field that is represented in the other mechanical texts,[43] however, as these devices are astrolabes and other astronomical tools, we might count the Antikythera mechanism as physical evidence of this field of mechanics.[44] Krafft has equated the devices for water lifting mentioned in Pappus listing with irrigation.[45] Aside from this, the other subjects mentioned, that is, centres of gravity, equilibrium and the lifting of weights all match the subject matter of the Pseudo-Aristotelean *Mechanica*, Heron's *Mechanica*

[41] *θαυματοποιϊκὴ* and *θαυμασιουργοὺς*.

[42] See Tybjerg 2003 for an alternate perspective on the purpose of automata-making.

[43] Ptolemy does mention it, so it is not unknown outside of these mentions of it, see Ptol. *Hyp.* 2.70

[44] See Freeth, Bitsakis, et al. 2006 and Freeth, Jones, et al. 2008 for the proposed functions of the Antikythera mechanism.

[45] *Pauly.* s.v. Mechanics

and Archimedes' *De Planorum Aequilibris*.

It is clear that from early in the tradition the construction of artillery and siege machinery was one of the principal sub-fields of mechanics. Otherwise we have mention of Pneumatics and Automaton-making, and all of these things can help confirm the taxonomic classifications made in Proclus' description of mechanics. Although this is a much later source on the subject, although attributed by Proclus to Geminus, a first century BC mathematician, it, along with the passage in Pappus, are the only sources of this kind we have.

From the evidence we have, it certainly seems that the extent of the field of mechanics was not particularly well defined in the ancient world. The work of Philon and Heron seems to have played some part in establishing the scope of the field, and the selection of topics that appear in their work seems to have played a part in the codification of the field that appear in the later texts.

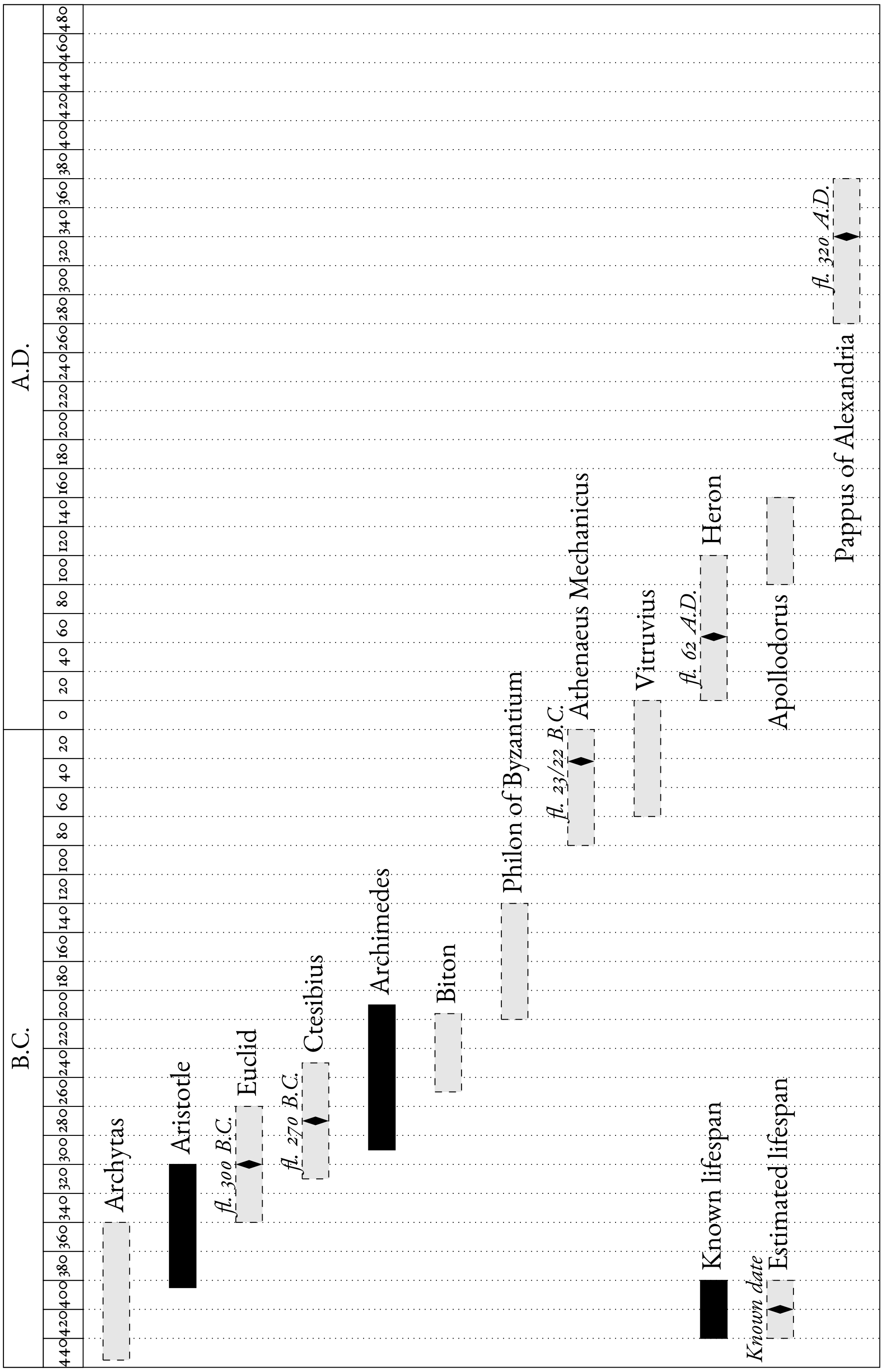

Figure 2.1: Timeline of mechanical authors.

3 Perception

The analysis of the extant mechanical sources in the previous chapter suggest the areas of knowledge that can be considered to be the constituent parts of mechanics as a field. It is this classification itself that has subsumed these distinct outputs under one topic heading rather than by virtue of a shard conceptual background. For example, we cannot say that every area of ancient mechanics has a conceptual reliance on mechanical advantage. However, this still leads to the question, what is this overarching classification that is mechanics?

In this chapter I will examine the internal and external representations of the field of mechanics to try and gain some insight into the way in which the field is perceived in the ancient world. Internal representations are those which we find in the mechanical texts, while the external representations are those which occur in other ancient philosophical and biographical texts. As the perception of mechanics in the ancient world is explored, the biases of both these sets of sources will be considered to ensure the clearest view of the nature of the field.

The introduction the Pseudo-Aristotelean *Mechanica* opens with an overarching statement on the nature of mechanics:

> *Θαυμάζεται τῶν μὲν κατὰ φύσιν συμβαινόντων, ὅσων ἀγνοεῖται τὸ αἴτιον, τῶν δὲ παρὰ φύσιν, ὅσα γίνεται διὰ τέχνην πρὸς τὸ συμφέρον*

> *τοῖς ἀνθρώποις. ἐν πολλοῖς γὰρ ἡ φύσις ὑπεναντίον πρὸς τὸ χρήσιμον ἡμῖν ποιεῖ ἡ μὲν γὰρ φύσις ἀεὶ τὸν αὐτὸν ἔχει τρόπον καὶ ἁπλῶς, τὸ δὲ χρήσιμον μεταβάλλει πολλαχῶς. ὅταν οὖν δέῃ τι παρὰ φύσιν πρᾶξαι, διὰ τὸ χαλεπὸν ἀπορίαν παρέχει καὶ δεῖται τέχνης. διὸ καὶ καλοῦμεν τῆς τέχνης τὸ πρὸς τὰς τοιαύτας ἀπορίας βοηθοῦν μέρος μηχανήν. καθάπερ γὰρ ἐποίησεν Ἀντιφῶν ὁ ποιητής, οὕτω καὶ ἔχει τέχνῃ γὰρ κρατοῦμεν, ὧν φύσει νικώμεθα.*[1]

This characterisation of mechanics could easily accompany the discussion of other similar subjects, and says more about the relationship between *τέχνη* and *φύσις* than mechanics specifically. It does, however, distinguish mechanics as a *τέχνη*, a statement that provides substantial information on the status of the field. There has been some debate over the exact meaning of the statement that mechanics produces effects *παρὰ φύσιν*. Krafft has stated that *παρὰ φύσιν* means 'contrary to nature', and that the goal of ancient mechanics was to 'trick nature';[2] however, Schiefsky has made a strong argument for the interpretation of this phrase as meaning 'beyond nature', and that mechanics accomplishes through *τέχνη* that which would be otherwise impossible.[3] This definition certainly seems more likely, as, although there is perhaps a tendency for other later mechanical authors to present mechanical effects as marvellous or supernatural, this text does not feature anything other than the exploration of observable, natural phenomena, and the presentation of them as being mechanical in nature. Alongside this there is an absence in this text of any laudatory comments on the subject matter in the introduction. The author is not attempting to convince us of the utility of anything in the text; it is merely being presented, and the explanatory, rather than instructive,

[1] Arist.*Mech.*847a11-21
[2] 'Überlistung der Natur', Krafft 1970, p. 27
[3] M.J. Schiefsky 2007, pp. 67-70

nature of the text reflects this. The very fact that this text seems to be serving a more investigative role may explain the absence of laudatory comments, as there is simply no need to boast of the importance of the subject matter when it is just one topic being investigated amongst many.

Outside of the Pseudo-Aristotelean *Mechanica*, we can see reference made to mechanics in contemporary philosophical texts. Aristotle, in his*Posterior Analytics*, mentions mechanics as part of a hierarchical classification of fields of knowledge. His discussion is focused around the relationship between field of knowledge that are, to an extent, entirely rational, and those that depend upon observation.

> *ἄλλον δὲ τρόπον διαφέρει τὸ διότι τοῦ ὅτι τῷ δι' ἄλλης ἐπιστήμης ἑκάτερον θεωρεῖν. τοιαῦτα δ' ἐστὶν ὅσα οὕτως ἔχει πρὸς ἄλληλα ὥστ' εἶναι θάτερον ὑπὸ θάτερον, οἷον τὰ ὀπτικὰ πρὸς γεωμετρίαν καὶ τὰ μηχανικὰ πρὸς στερεομετρίαν καὶ τὰ ἁρμονικὰ πρὸς ἀριθμητικὴν καὶ τὰ φαινόμενα πρὸς ἀστρολογικήν. σχεδὸν δὲ συνώνυμοί εἰσιν ἔνιαι τούτων τῶν ἐπιστημῶν, οἷον ἀστρολογία ἥ τε μαθηματικὴ καὶ ἡ ναυτική, καὶ ἁρμονικὴ ἥ τε μαθηματικὴ καὶ ἡ κατὰ τὴν ἀκοήν. ἐνταῦθα γὰρ τὸ μὲν ὅτι τῶν αἰσθητικῶν εἰδέναι, τὸ δὲ διότι τῶν μαθηματικῶν οὗτοι γὰρ ἔχουσι τῶν αἰτίων τὰς ἀποδείξεις, καὶ πολλάκις οὐκ ἴσασι τὸ ὅτι, καθάπερ οἱ τὸ καθόλου θεωροῦντες πολλάκις ἔνια τῶν καθ' ἕκαστον οὐκ ἴσασι δι' ἀνεπισκεψίαν.*[4]

Mechanics is classed here as being subordinate (*εἶναι θάτερον ὑπὸ θάτερον*) to stereometry (solid geometry).[5] Barnes, in his commentary on the Posterior Analytics, pro-

[4]Arist.*APo.*78b34-79a5

[5]At *Apo.* 76a24 we find it as subordinate to geometry, but this term can include stereometry.

vides a list of the general relations that Aristotle states exist between these pairs of fields,[6] and from this we can make a series of statements on the relationship expressed by Aristotle between mechanics and solid geometry. These are that:

- Mechanics is proven through solid geometry.
- The fact belongs to mechanics, the reason why to solid geometry.
- Mechanics is like particular knowledge, solid geometry like universal knowledge.
- Mechanics is studied by empirical scientists, solid geometry by mathematical scientists.
- Mechanics uses forms and solid geometry is about forms.

Although these statements of relationship are intended to apply to a number of fields in the original text, this focused list serves to highlight the Aristotelean conception of mechanics, defined as an empirical science in relation to the rational field of stereometry. Elsewhere in the Aristotelean corpus we find that he is of the opinion that mechanics is among a group that is the 'more mathematical of the natural sciences'.[7] Berryman has argued that it is exactly this which counteracts the claims that $\pi\alpha\rho\grave{\alpha}\ \phi\acute{\upsilon}\sigma\iota\nu$ in any way means 'against nature', highlighting that this particular interpretation of the term is one that is heavily influenced by the mechanical tradition which Galileo is refuting in his work.[8]

Given Archimedes reputation as a mechanician in the ancient world, we might well expect some kind of comment on the field in his work. Unfortunately, in the two extant works of his that we might consider to be mechanical, that is, *De planorum aequilibriis*

[6] Barnes 1994, pp. 158-159
[7] Barnes 1994, p. 159, see Arist. *Ph.* 194a10, *Metaph.* 997b15-21
[8] Berryman 2009, pp. 44-45

and *De corporibus fluitantibus*, which cover statics and hydrostatics respectively, there is no evidence to be found for Archimedes own perception of mechanics; indeed, they begin without any introduction, and plunge straight into the Archimedean structure of postulates and propositions. This itself is not typical of the work of Archimedes, as many of the other texts in his extant corpus have introductions consisting of letters to named correspondents. Rather than there being anything different about these texts, it seems rather that any introduction to these two texts has been lost in transmission. I will talk about the particular, axiomatic, form of these texts in chapter 5.

His '*Ad Eratosthenem methodus*', conventionally known as 'The Method', is a work that would initially seem to be, by its reputation and the terminology used, one that deals with mechanics. However, it deals instead with the application of mechanical principles to solving the mathematical problem of calculating the area of a given geometrical shape. This work does come with introductory material, and it is from this that we gain some insight into the utility of the mechanical methodology that will be set forth. Archimedes says:

> *Ὁρῶν δέ σε, καθάπερ λέγω, σπουδαῖον καὶ φιλοσοφίας προεστῶτα ἀξιολόγως καὶ τὴν ἐν τοῖς μαθήμασιν κατὰ τὸ ὑποπίπτον θεωρίαν τετιμηκότα ἐδοκίμασα γράψαι σοι καὶ εἰς τὸ αὐτὸ βιβλίον ἐξορίσαι τρόπου τινὸς ἰδιότητα, καθ' ὅν σοι παρεχόμενον ἔσται λαμβάνειν ἀφορμὰς εἰς τὸ δύνασθαί τινα τῶν ἐν τοῖς μαθήμασι θεωρεῖν διὰ τῶν μηχανικῶν. Τοῦτο δὲ πέπεισμαι χρήσιμον εἶναι οὐδὲν ἧσσον καὶ εἰς τὴν ἀπόδειξιν αὐτῶν τῶν θεωρημάτων. Καὶ γάρ τινα τῶν πρότερόν μοι φανέντων μηχανικῶς ὕστερον γεωμετρικῶς ἀπεδείχθη διὰ τὸ χωρὶς ἀποδείξεως εἶναι τὴν διὰ τούτου τοῦ τρόπου θεωρίαν* [9]

[9] Archim. *Eratosth.* 83.17-28

The attitude expressed here has some similarities with the Aristotelean classification of the relationships that we have previously examined. The mechanical method which he will describe can provide the facts of the problem. That is, that it can discover, through an empirical method, the area of a geometrical shape. It does not, however, establish a geometrical demonstration of the theorem, which must be supplied upon the basis of the results of the mechanical method. The aim of this text is to describe this specific mechanical method, and not to develop any more widely applicable mechanical theory. This is mechanics as it applies to mathematics rather than mathematics as it applies to natural phenomena. Extant testimonia on Archimedes offers a more instructive view of ancient attitudes and criticisms of mechanics than the perception of in what little of Archimedes work mentions such things. He is inevitably mentioned in Plutarch's *Marcellus* for the role that he played in the siege of Syracuse, where the deployment of war machines which he had constructed are said to have delayed the fall of the city. Plutarch says of these machines:

> *ἧς ἄρα λόγος οὐδεὶς ἦν Ἀρχιμήδει καὶ τοῖς Ἀρχιμήδους μηχανήμασιν. ὧν ὡς μὲν ἔργον ἄξιον σπουδῆς οὐδὲν ὁ ἀνὴρ προὔθετο, γεωμετρίας δὲ παιζούσης ἐγεγόνει πάρεργα τὰ πλεῖστα, πρότερον φιλοτιμηθέντος Ἱέρωνος τοῦ βασιλέως καὶ πείσαντος Ἀρχιμήδη τρέψαι τι τῆς τέχνης ἀπὸ τῶν νοητῶν ἐπὶ τὰ σωματικά, καὶ τὸν λόγον ἁμῶς γέ πως δι' αἰσθήσεως μείξαντα ταῖς χρείαις ἐμφανέστερον καταστῆσαι τοῖς πολλοῖς.*[10]

These mechanical devices are here presented as a mere by-product of Archimedes' mathematical output, and he has to be coaxed into using his knowledge to construct

[10] Plu. *Marc.* 14.7–9

these devices for the defence of the city. This portrayal of Archimedes as regarding the field of mechanics as banausic, and unworthy of serious attention, is explained by Plutarch as he goes on with further discussion of mechanics:

> *τὴν γὰρ ἀγαπωμένην ταύτην καὶ περιβόητον ὀργανικὴν ἤρξαντο μὲν κινεῖν οἱ περὶ Εὔδοξον καὶ Ἀρχύταν, ποικίλλοντες τῷ γλαφυρῷ γεωμετρίαν, καὶ λογικῆς καὶ γραμμικῆς ἀποδείξεως οὐκ εὐποροῦντα προβλήματα δι᾽ αἰσθητῶν καὶ ὀργανικῶν παραδειγμάτων ὑπερείδοντες, ὡς τὸ περὶ δύο μέσας ἀνὰ λόγον πρόβλημα καὶ στοιχεῖον ἐπὶ πολλὰ τῶν γραφομένων ἀναγκαῖον εἰς ὀργανικὰς ἐξῆγον ἀμφότεροι κατασκευάς. μεσογράφους τινὰς ἀπὸ καμπύλων γραμμ[ατ]ῶν καὶ τμημάτων μεθαρμόζοντες ἐπεὶ δὲ Πλάτων ἠγανάκτησε καὶ διετείνατο πρὸς αὐτούς, ὡς ἀπολλύντας καὶ διαφθείροντας τὸ γεωμετρίας ἀγαθόν, ἀπὸ τῶν ἀσωμάτων καὶ νοητῶν ἀποδιδρασκούσης ἐπὶ τὰ αἰσθητά, καὶ προσχρωμένης αὖθις αὖ σώμασι πολλῆς καὶ φορτικῆς βαναυσουργίας δεομένοις, οὕτω διεκρίθη γεωμετρίας ἐκπεσοῦσα μηχανική, καὶ περι ορωμένη πολὺν χρόνον ὑπὸ φιλοσοφίας, μία τῶν στρατιωτίδων τεχνῶν ἐγεγόνει.*[11]

I am not really convinced that this can be used as reasoning for the apparent lack of written work by Archimedes on the more practical aspects of mechanics, nor for his attitudes towards the field, despite this account's popularity. His attitude to the mechanical method, as we have seen, is not at all dismissive, but rather presented as something of great utility to a student of mathematics. Identifying Plutarch's own philosophical viewpoint as a Platonist is one of the key issues when attempting to evaluate this passage as reliable testimonia on the attitudes of Plato and Archimedes. There is a dismissal

[11] Plu. *Marc.* 9-12

of the corporeal in Platonic thought, which engenders the criticism of *τέχνη* in these comments on mechanics. Much of the criticism of mechanics as a science can be traced to this philosophical viewpoint, which, following the theory of forms, is dependent on rationalism rather than empiricism. This view may also be seen to an extent in the mention of mechanics in the *Posterior Analytics*, in its relative positioning with solid geometry. However, as I have mentioned, this text is concerned with documenting scientific methodology rather than making value judgements on the fields.

Within the many extant texts on the construction of war-machines we find a number of comments, particularly in the introductions to these works, which are illustrative of the attitude of Hellenistic mechanicians towards mechanics. Within these we may begin to detect the presence of a tradition of writing on the subject. This is a notable move away from the previous authors discussed, whose work by and large stands in isolation. In the introduction to his *Belopoeica*, Philon explains the reasons for dealing with the particular topics found in the text:

> *εἰ μὲν οὖν συνέβαινεν ὁμοίᾳ μεθόδῳ κεχρῆσθαι πάντας τοὺς πρότερον πεπραγματευμένους περὶ τοῦ μέρους τούτου, τάχα ἂν οὐθενὸς ἄλλου προσεδεόμεθα πλὴν τοῦ τὰς συντάξεις τῶν ὀργάνων ὁμολόγους οὔσας ἐμφανίζειν. ἐπεὶ δὲ διηνεγμένους ὁρῶμεν οὐ μόνον ἐν ταῖς πρὸς ἄλληλα τῶν μερῶν ἀναλογίαις, ἀλλὰ καὶ ἐν τῷ πρώτῳ καὶ ἡγουμένῳ στοιχείῳ, λέγω δὲ τῷ τὸν τόνον μέλλοντι δέχεσθαι τρήματι, καλῶς ἔχον ἐστὶν περὶ μὲν τῶν ἀρχαίων παρεῖναι, τὰς δὲ τῶν ὕστερον παραδεδομένας μεθόδους περὶ τῆς καθόλου τέχνης δυναμένας ἐπὶ τῶν ἔργων τὰ δέοντα ποιήσειν ταύτας ἐμφανίζειν.* [12]

[12] Ph. *Bel.* 49

Philon is here both acknowledging that his work is building upon the work of predecessors, but also presenting the approach he is adopting as an improvement upon the previous texts on the subjects. In contrast with the work of Archimedes, or the Pseudo-Aristotelean *Mechanica*, this text is being placed in the context of a tradition of literary work on the subject. This acknowledgement of tradition usually features the dismissal of the work of predecessors, and the emphasis on the idea that an improved approach to the subject will be found in a given text, and frequently appears in the introductory material of mechanical texts, particularly in the work of Heron. Rather than presentation of a static body of knowledge, the field is presented as an ongoing and evolving science. Philon's justification for writing the work, and updating the previous material on the subject is that previous authors have not expressed, or understood, the key theoretical underpinnings of the construction of siege-machinery, namely the application of the doubling of the cube.

When Plutarch mentioned *τὸ περὶ δύο μέσας ἀνὰ λόγον πρόβλημα*[13] in the previous extract, it was to this problem he was referring. In another telling of the same story, Plutarch states that the solutions to this problem were found through *ὀργανικὰς καὶ μηχανικὰς κατασκευὰς*[14]. Huffman explores criticism of Archytas' solution to the Delian problem at great length, examining all the different manners in which his working, 'dazzlingly abstract'[15] in its mathematical content, could be construed as mechanical. Ultimately, as a reason for this, he settles on the criticism stemming from the fact that the problems originated, and had applications, in the physical world.[16] There is another quick and practical method of deriving the mean proportionals using a sliding

[13] Plu.*Vit.Marc.*14.5
[14] Plu.*Mor.Quaest.conv.*718e
[15] Huffman 2005, p. 356, See chapter 6 for his solution.
[16] Huffman 2005, pp. 384-385

ruler, which, ironically, has been passed down attached to Plato. This solution to the problem can easily be conceived of as mechanical, and is another good example of the mechanical methodology previously discussed in its use of an intermediate mechanistic device to arrive at the result. Archytas' work on this particular problem is probably the root of the frequent references to him as the founder of the field of mechanics. In addition to this, Huffmann sees the split between Plato and Archytas on this subject as an etiological myth to explain the separation of mechanics from geometry, with each of these figures as an idealogical figurehead.[17]

This material on mechanical methodology is all very abstract, and there is a noticeable split in the character of the extant texts on mechanics between that which is practical and that which is theoretical. In a work dedicated to Marcellus, nephew of Augustus,[18] the author Athenaeus writes the following:

> *Τουτὶ γὰρ ἄν τις <εἰς> πραγμάτων λόγον ὠφεληθεὶς ἀπέλθοι, ἐπιμελῶς ἐπιστήσας ἑαυτόν, ἐκ τοῦ Δελφικοῦ ἐκείνου παραγγέλματος ἢ ἐκ τῶν Στράτωνος καὶ Ἑστιαίου καὶ Ἀρχύτου καὶ Ἀριστοτέλους καὶ τῶν ἄλλων τῶν παραπλήσια ἐκείνοις γεγραφότων.* Ν*εωτέροις μὲν γὰρ φιλομαθοῦσιν οὐκ ἄχρηστα εἴη <πρὸς ἕξιν> τοῦ στοιχειωθῆναι τοῖς δὲ βουλομένοις ἤδη τι πράττειν μακρὰν παντελῶς ἂν εἴη καὶ ἀπηρτημένα τῆς πραγματικῆς θεωρίας.*[19]

Although these criticism of earlier authors serves a number of purposes, it's principle role is in establishing that this text will be useful, in that it will provide actual plans and methods for the construction of this war machinery rather than conceptual me-

[17] Huffman 2005, p. 379
[18] Whitehead and Blyth 2004, pp. 18-19
[19] Ath.Mech.4-5

chanical discussion. If the text was to serve as a manual on siege machinery for a young Marcellus accompanying Augustus on campaign, as Whitehead and Blythe argue, then the emphasis would rightly be on the practical benefits of the text in real life scenarios rather than general educational benefits. Athenaeus may well be attempting to flatter the boy with emphasis on the action he will undertake. The mention of these names also serves to establish Athenaeus' familiarity with the intellectual basis of the field, to establish himself as a well-read authority on the subject, able to analyse and dismiss what is irrelevant. The material discussed is very similar to material on war-machines found in Vitruvius, and it is likely that both are derived from another, earlier source.

Another, radically different reasoning for the importance of the construction of war machines occurs in the introduction of Heron's Belopoieca, where he begins by saying:

> *Τῆς ἐν φιλοσοφίᾳ διατριβῆς τὸ μέγιστον καὶ ἀναγκαιότατον μέρος ὑπάρχει τὸ περὶ ἀταραξίας, περὶ ἧς πλεῖσταί τε ὑπῆρξαν ζητήσεις παρὰ τοῖς μεταχειριζομένοις τὴν σοφίαν καὶ μέχρι νῦν ὑπάρχουσιν καὶ νομίζω μηδὲ τέλος ποτὲ ἕξειν διὰ τῶν λόγων τὴν περὶ αὐτῆς ζήτησιν. μηχανικὴ δὲ ὑπερβᾶσα τὴν διὰ λόγων περὶ ταύτης διδασκαλίαν ἐδίδαξεν πάντας ἀνθρώπους ἀταράχως ζῆν ἐπίστασθαι δι' ἑνὸς καὶ ἐλαχίστου μέρους αὐτῆς, λέγω δὴ τοῦ κατὰ τὴν καλουμένην βελοποιίαν, δι' ἧς οὔτε ἐν εἰρηνικῇ καταστάσει ταραχθήσονταί ποτε ἐχθρῶν καὶ πολεμίων ἐπανόδοις, οὔτε ἐνστάντος πολέμου ταραχθήσονταί ποτε τῇ παραδιδομένῃ ὑπ' αὐτῆς διὰ τῶν ὀργάνων φιλοσοφίᾳ.*[20]

This apparently paradoxical statement is merely an early occurrence of a modified form of the adage *si vis pacem, para bellum*,[21] which can otherwise be found in Book

[20] Hero. *Bel.*1.1
[21] Marsden 1971, p. 44

Seven of Thucydides[22], or in the introduction to the work of Aeneas Tacticus.[23] It is not a particularly radical statement in this respect, save for this particular emphasis on machinery. However, the claim that mechanics will step in where philosophy has failed is a far more contentious sentiment. It serves to both elevate the importance of mechanics, and associate it with the intellectual heights of philosophical study. There is also here a criticism of the rejection of $\tau\acute{\epsilon}\chi\nu\eta$, and the contemporary attitude that it is banausic and unworthy of proper attention. By presenting the benefits of the study of the field, which are otherwise unavailable, Heron is making a case, through reason, for the appreciation of mechanics as a field on par with others for serious research and study.

The presentation of mechanics in the sole Latin source on the subject takes a different, yet apparently widespread, approach to the subject, whereby it is presented as the final area of knowledge necessary for a student of architecture. Vitruvius' *De Architectura*, itself anomalous as a work on architecture, is anomalous in its presentation of the mastery of a technical field as requiring a well-rounded education in the 'liberal arts', as well as knowledge of the field itself. Vitruvius sees architecture as a taking in both the built and mechanical environment, with proper practice requiring mastery of many areas of theoretical and practical knowledge.[24] For mechanics this is expressed when he states:

> *in his vero opus est prudentia diligens et ingenii doctissimi cogitata, quod nihil eorum perficitur sine machinatione studiorumque vario ac sollerti vigore.*[25]

[22] Th. 7.92.4
[23] Aen. Tact. praef.
[24] Rowland and Howe 2002, p. 13
[25] Vitr. *De Arch.* 10.pref.3

Vitruvius provides a basic description of the qualities of a machine, saying:

> *Machina est continens e materia coniunctio maximas ad onerum motus habens virtutes. ea movetur ex arte circulorum rotundationibus, quam Graeci κυκλικην κίνησιν appellant.*[26]

Although concerned with the nature of machines rather than mechanics, this statement allows us insight into his conception of the basic nature of mechanics, as *machina* does not have a specific meaning in Latin, so we do not need to consider this description to be one of a specific machine. This focus on rotational movement and the weight-lifting properties of mechanisms can also be seen in the emphasis on the circle in the Pseudo-Aristotelean *Mechanica*, and in the *Mechanica* of Heron, which deals, at least in part, with geared weight-lifting devices. Furthermore, as Vitruvius is writing in the context of architecture, the machine most associated with this field would be weight-lifting devices which the architect was also obliged to produce as part of their commission. Vitruvius also interestingly makes a distinction between different kinds of machines:

> *Ex his sunt quae μηχανικώς, alia ὀργάνικως moventur. inter machinas et organa id videtur esse discrimen, quod machinae pluribus operis vel vi maiore coguntur effectus habere, uti ballistae torculariorumque prela; organa autem unius operae prudenti tactu perficiunt quod est propositum, uti scorpionis seu anisocyclorum versationes.*[27]

This distinction and division of mechanisms seems to be based on the scale and number of people required to operate the device, rather than the particular function or mechanisms employed in the device. This particular division does not occur in other texts,

[26]Vitr. *De Arch.* 10.1.1

[27]Vitr. *De Arch.* 10.1.3

though we might consider it analogous to the division between different fields of mechanics made by Pappus and Proclus, as seen in the previous chapter.

Finally, Vitruvius here expresses his view on the relationship between nature and mechanics, stating:

> *Omnis autem est machinatio rerum natura procreata ac praeceptrice et magistra mundi versatione instituta.*[28]

If this is a statement that mechanics is devised from the observation of nature, it is unlike those which we have seen from other authors on the character of mechanics as being 'beyond nature', or in some way distinct from it. In this example mechanical effects are an innate part of the workings of the universe, though it may be that what is being suggested is inspiration from the workings of nature rather than a direct imitation.

Other extant works by Heron can give us insight into the perception of two other fields of mechanics that have not yet been discussed, pneumatics and automaton-building. Heron introduces his *Pneumatica* in the following fashion:

> Τῆς πνευματικῆς πραγματείας σπουδῆς ἠξιωμένης πρὸς τῶν παλαιῶν φιλοσόφων τε καὶ μηχανικῶν, τῶν μὲν λογικῶς τὴν δύναμιν αὐτῆς ἀποδεδωκότων, τῶν δὲ καὶ δι' αὐτῆς τῆς τῶν αἰσθητῶν ἐνεργείας, ἀναγκαῖον ὑπάρχειν νομίζομεν καὶ αὐτοὶ τὰ παραδοθέντα ὑπὸ τῶν ἀρχαίων εἰς τάξιν ἀγαγεῖν, καὶ ἃ ἡμεῖς δὲ προσευρήκαμεν εἰσθέσθαι οὕτως γὰρ τοὺς μετὰ ταῦτα ἐν τοῖς μαθήμασιν ἀναστρέφεσθαι βουλομένους ὠφελεῖσθαι συμβήσεται.[29]

Again we see in this passage an association being made between mechanics and philosophy. However, unlike the more negative parallels that are drawn in Athenaeus, the

[28]Vitr. *De Arch.* 10.1.4
[29]Hero *Spir.* 1.1

presentation of the relationship here is harmonious, and in line with the Aristotelean conception of the relationship between empirical and rational science described earlier. The exchange between these two specialisms on the subject of pneumatics is evident in the development of atomic theory and the importance of *πνεῦμα* in philosophical works. Later in the *Pneumatica*, Heron uses the phrase '*παρὰ φύσιν*' to describe the actions of pneumatics, both in creating a vacuum, and compressing gases. The work clearly demonstrates the use and manipulation of pneumatic effects through the tools and devices that are being described, and so agrees with the definition of *παρὰ φύσιν* discussed earlier. Pneumatics also seems to be the furthest removed theoretically from the other areas of mechanics that appear ancient technical texts. However, if mechanics as a field is conceived in the way Heron describes, pneumatics can be placed in the same category as other aspects of ancient mechanics. While Heron's *Automatatopoetike* begins as follows:

> Τῆς *αὐτοματοποιητικῆς πραγματείας ὑπὸ τῶν πρότερον ἀποδοχῆς ἠξιωμένης διά τε τὸ ποικίλον τῆς ἐν αὐτῇ δημιουργίας καὶ διὰ τὸ ἔκπληκτον τῆς θεωρίας. ἔστι γάρ, ὡς συνελόντι εἰπεῖν, πᾶν μέρος τῆς μηχανικῆς ἐν αὐτῇ τῇ αὐτοματοποιητικῇ παραλαμβανόμενον διὰ τῶν κατὰ μέρος ἐν αὐτῇ ἐπιτελουμένων.*[30]

Again we find an emphasis on this text as part of a tradition of writing on mechanics, along with specific laudatory comments about this particular topic within the field of mechanics. The emphasis on the complexity of automaton making, and the idea that it utilises all aspects of the field of ancient mechanics are particularly interesting, though within the text itself, we do not find any particular fields of mechanics that are not found

[30]Hero *Aut.* 1.1

elsewhere. That each aspect of mechanics is not an endpoint in itself, but could be incorporated as a component for different purposes within a device, is perhaps reflective of what has already been seen of mechanics, where the mechanician would be expected to produce a variety of devices.

Finally, we find comments on the nature of mechanics in the introduction to the eighth book of Pappus *Synagoge*, where he introduces the field to Hermodorus, to whom this book is addressed:

> Ἡ μηχανικὴ θεωρία, τέκνον Ἑρμόδωρε, πρὸς πολλὰ καὶ μεγάλα τῶ ἐν τῷ βίῳ χρήσιμος ὑπάρχουσα πλείστης εἰκότως ἀποδοχῆς ἠξίωται πρὸς τῶν φιλοσόφων καὶ πᾶσι τοῖς ἀπὸ τῶν μαθημάτων περισπούδαστός ἐστιν, ἐπειδὴ σχεδὸν πρώτη τῆς περὶ τὴν ὕλην τῶν ἐν τῷ κόσμῳ στοιχείων φυσιολογίας ἅπτεται.[31]

Where the attribution of interest in pneumatics to philosophers was understandable in Heron's introduction to the Pneumatica, this attribution of interest in the field of mechanics as a whole is a fairly radical change in viewpoint from that expressed in earlier texts. The expansion to include mathematicians among the interested parties is perhaps telling of the way in which the field is regarded at this later date. Pappus has included it as part of a compendium of mechanical knowledge, and the subject matter dealt with in this book is far more mathematical than that found elsewhere in earlier mechanical literature. In the section immediately after this, quoted in the previous chapter, we again find mention of the idea that mechanics is something that is *παρὰ φύσιν*. In addition, we here do not find the strict hierarchical division between fields of knowledge that is being enforced by Aristotle in his *Posterior Analytics*. The division between theoreti-

[31]Papp. 1022

cal and practical aspects of mechanics includes geometry as one of the disciplines that make up the theoretical aspects of mechanics. This could in part reflect a less specific definition of mechanics, which incorporates more heavily the particular skills required of someone practising in the field, rather than a specific definition of the field of knowledge. While this, along with other differences from other mechanical texts, could reflect that the text is intended as a training manual for students of architecture[32], that is, having a broader focus than just mechanics. However, it may just be that Pappus is less concerned with the strict divisions between fields of knowledge that are found in Aristotle, and more interested in all aspects of mathematics, among which we can count mechanics.

I think we can, therefore, make a number of general conclusions about the perception of mechanics in the ancient world. Firstly, with regards to the general nature of the field; In the Pseudo-Aristotelean *Mechanica* we find the idea that mechanics is *παρὰ φύσιν* in its character, and that all aspects of mechanical motion can be related to the circle. Aristotle, in the *Posterior Analytics*, sees it as being defined by its subordinate relationship to solid geometry. These two defining aspects are seen in some way or another in most of the mechanical texts from the middle of the chronological range, that is Philon, Athenaeus, Vitruvius and Heron.

Secondly, with regards to the praise directed at the field; The majority of the works examined, from Philon onwards, have introductions that make some kind of case for the worth of mechanics or the sub-field being discussed. These make a variety of arguments, which can take the form of an advertisement of the benefits of the field, or a dismissal of previous work on the subject. Frequently the work is defined in relation to philosophy, using it as a reference point to validate the legitimacy of mechanics,

[32]Downey 1948

or a straight declaration of superiority to it. The criticism visible in the passage from Plutarch, aside from the general sentiment that mechanics is in some way menial, and not an elevated area of study, is focused around Plato, and his criticism of the use of a mechanical methodology.

There will be an introductory chapter here that discusses the problems associated with defining the theoretical in ancient mechanics. It will lay out the scope of the following chapters and the reasons for focusing on the particular problems and theories discussed in the texts. It will discuss why these particular texts have been chosen as the focus for analysis.

When considering these aspects of ancient mechanics, the key question is not "is this correct", though I will mention if it is not, but rather "how is this being understood" or "why is this methodology used". The question of what mechanics is, particulary of what theoretical mechanics is, is not so much about the results, but about the method.

There is the conflict between empiricism and rationalism that is inherent in these works, and this is the issue when considering much of ancient science. The law of the lever, which much of the first two chapters here are concerned with, is of interest not so much for having been discerned, but rather for having been explained. It is the rationale that makes the theory, not the existence of the rule. We could conduct experiments with millions of combinations of arm lengths and weights to develop and demonstrate the law of the lever, but an attempt to describe the reasons for it and construct general principles of explain it.

4 Aristotelean Mechanics

The perpatetic *Mechanica* constitutes the genesis of theoretical mechanics, at least as far as the extant textual tradition is concerned. It was attributed to Aristotle in antiquity and transmitted as part of the Aristotelean corpus. The structure of the text is the same as that of the *Problemata*, another pseudo-Aristotelean text that consists of thirty-eight books on a wide variety of topics such as medicine, bodily functions, moral qualities and botany. The material found in the *Mechanica* consists of the examination of a series of problems deemed to be mechanical, proceeding with a question and answer structure. While this aspect of the text is the same as the *Problemata*, there is a significant difference in the relationship between the questions. In the *Mechanica*, the text begins by describing the mechanical principle to which the mechanical phenomena described in the problems can all be related.

It is this theoretical basis that I wish to explore in this chapter, considering these concepts in particular to constitute Aristotelean mechanics, in contrast to the Archimedean mechanics that will be investigated in the following chapter. It is worth noting here that there is other material in the Aristotelean corpus that we might deem to be mechanical, such as the treatment of dynamics that can be found in works such as the *Physica*. This material is not, however, described as being mechanics by Aristotle, and so cannot be included when discussing the ancient conception of the field of mechanics.

The theoretical aspects of the work begins in the relation of mechanical problems to the lever, and the attribution of the power of the lever to the circle.

> *περιέχεται δὲ τῶν ἀπορουμένων ἐν τῷ γένει τούτῳ τὰ περὶ τὸν μοχλόν. ἄτοπον γὰρ εἶναι δοκεῖ τὸ κινεῖσθαι μέγα βάρος ὑπὸ μικρᾶς ἰσχύος, καὶ ταῦτα μετὰ βάρους πλείονος ὃ γὰρ ἄνευ μοχλοῦ κινεῖν οὐ δύναταί τις, τοῦτο ταὐτὸ βάρος, προσλαβὼν ἔτι τὸ τοῦ μοχλοῦ βάρος, κινεῖ θᾶττον. πάντων δὲ τῶν τοιούτων ἔχει τῆς αἰτίας τὴν ἀρχὴν ὁ κύκλος. καὶ τοῦτο εὐλόγως συμβέβηκεν ἐκ μὲν γὰρ θαυμασιωτέρου συμβαίνειν τι θαυμαστὸν οὐδὲν ἄτοπον, θαυμασιώτατον δὲ τὸ τἀναντία γίνεσθαι μετ' ἀλλήλων. ὁ δὲ κύκλος συνέστηκεν ἐκ τοιούτων εὐθὺς γὰρ ἐκ κινουμένου τε γεγένηται καὶ μένοντος, ὧν ἡ φύσις ἐστὶν ὑπεναντία ἀλλήλοις.*[1]

It is this relationship with the circle that is dissected in the subsequent chapters. The expansion on the marvellous nature of the circle that occurs between 847b and 848a serves to emphasise the extent to which this form has properties that are conducive to explaining the extraordinary or *παρὰ φύσιν* nature of the lever. The author expands on this further, by examining a ritual device that uses circles that turn each other like gears. The key point is that circle can be used to explain the balance, the balance can explain the lever, and the lever can be used to explain the majority of mechanical phenomena.[2]

The phenomena associated with levers and balances being explored in the early parts of the text is normally described in modern terms as 'the law of the lever'. While the analysis of the balance that we find in Archimedes is, as we will see in the next chapter,

[1] Arist. *Mech.* 847b11-21

[2] *τὰ μὲν οὖν περὶ τὸν ζυγὸν γινόμενα εἰς τὸν κύκλον ἀνάγεται, τὰ δὲ περὶ τὸν μοχλὸν εἰς τὸν ζυγόν, τὰ δ' ἄλλα πάντα σχεδὸν τὰ περὶ τὰς κινήσεις τὰς μηχανικὰς εἰς τὸν μοχλόν.* Arist.*Mech.*848a. See page 72 for a notable failure of this analytical model.

much closer to our own conception of the machine, the principles that are being explored in the Aristotelean *Mechanica* are basically the same. The lever is composed of two fundamental parts: a rigid object serving as the lever itself, and a fulcrum around which it can pivot. The importance of this particular machine is the way in which it can serve as a model for the amplification of force found in machines more generally. This amplification of force, when quantified, is called the mechanical advantage of the device. The law of the lever describes the relationship between a force acting upon a lever, and the distance of that application of force from the fulcrum. We understand this chiefly through a statical model, where the system is in a state of equilibrium. The most basic expression of this is a balance upon which equal forces at equal distances are acting on opposing sides of the fulcrum. However, the key aspect of the relationship between the distance from the fulcrum and the force acting upon the lever is not expressed through this symmetrical system. It almost seems to be intuitively understood that if one of the two aforementioned forces acting upon the lever were to be increased, or the distance from the fulcrum increased, that side of the lever would descend. Essential in the importance of the lever is the fact that for the system to remain in equilibrium, it is not necessary for the force or distance to be increased in a symmetrically proportional way on the opposite side of the fulcrum. There is an inversely proportional relationship between force and distance on the opposite side of the fulcrum, that is, if the force acting upon on one side of the lever is increased, the distance from the fulcrum on the other side can be increased by an amount proportional to the increase in force, and the system will return to a state of equilibrium. It is this relationship that is being explored when we are discussing the law of the lever, and this is what the author of the *Mechanica* is attempting to explain.

The link between the balance and the circle is not immediately obvious. If we are at all familiar with a modern theoretical construct of the lever then we might imagine the lever placed across the fulcrum, with some demonstration of the forces acting upon either end. We are not encouraged to imagine the movement of the lever, nor to imagine the path that a point on the lever will trace when moving. Nor do either of these things play a part in standard analytical models of the lever. This, however, is what the author of the *Mechanica* goes on to do, or at least to explain circular motion, relating it back to the lever. The question is framed as the following:

> *Πρῶτον μὲν οὖν τὰ συμβαίνοντα περὶ τὸν ζυγὸν ἀπορεῖται, διὰ τίνα αἰτίαν ἀκριβέστερά ἐστι τὰ ζυγὰ τὰ μείζω τῶν ἐλαττόνων. τούτου δὲ ἀρχή, διὰ τί ποτε ἐν τῷ κύκλῳ ἡ πλεῖον ἀφεστηκυῖα γραμμὴ τοῦ κέντρου τῆς ἐγγὺς τῇ αὐτῇ ἰσχύϊ κινουμένης θᾶττον φέρεται τῆς ἐλάττονος;*

Like the rest of the mechanical problems in this *Mechanica*, the starting point of the investigation is resolutely physical and familiar. The phenomena is of course a generic one, the differing accuracy of balances has the same root cause as the force magnifying effects of a lever. That said, it seems that this example has been selected in particular because it is easily comprehensible by means of, and conformable to, the circular model that the author of the *Mechanica* is developing. Having established the specific instance of the problem, the author then goes on to develop the general theory, first of all establishing a general theory of compound motion, before he goes on to deal specifically with circular motion:

> *ὅταν μὲν οὖν ἐν λόγῳ τινὶ φέρηται, ἐπ' εὐθείας ἀνάγκη φέρεσθαι τὸ φερόμενον, καὶ γίνεται διάμετρος αὐτὴ τοῦ σχήματος ὃ ποιοῦσιν αἱ ἐν τούτῳ τῷ λόγῳ συντεθεῖσαι γραμμαί. ἔστω γὰρ ὁ λόγος ὃν*

> *φέρεται τὸ φερόμενον, ὃν ἔχει ἡ* AB *πρὸς τὴν* AΓ *καὶ τὸ μὲν* AΓ *φερέσθω πρὸς τὸ* B, *ἡ δὲ* AB *ὑποφερέσθω πρὸς τὴν* HΓ *ἐνηνέχθω δὲ τὸ μὲν* A *πρὸς τὸ* Δ, *ἡ δὲ ἐφ' ᾗ* AB *πρὸς τὸ* E. *εἰ οὖν ἐπὶ τῆς φορᾶς ὁ λόγος ἦν ὃν ἡ* AB *ἔχει πρὸς τὴν* AΓ, *ἀνάγκη καὶ τὴν* AΔ *πρὸς τὴν* AE *τοῦτον ἔχειν τὸν λόγον. ὅμοιον ἄρα ἐστὶ τῷ λόγῳ τὸ μικρὸν τετράπλευρον τῷ μείζονι, ὥστε καὶ ἡ αὐτὴ διάμετρος αὐτῶν, καὶ τὸ* A *ἔσται πρὸς* Z. *τὸν αὐτὸν δὴ τρόπον δειχθήσεται κἂν ὁπουοῦν διαληφθῇ ἡ φορά αἰεὶ γὰρ ἔσται ἐπὶ τῆς διαμέτρου. φανερὸν οὖν ὅτι τὸ κατὰ τὴν διάμετρον φερόμενον ἐν δύο φοραῖς ἀνάγκη τὸν τῶν πλευρῶν φέρεσθαι λόγον. εἰ γὰρ ἄλλον τινά, οὐκ οἰσθήσεται κατὰ τὴν διάμετρον.*[3]

This is the earliest expression of the idea of a parallelogram of forces,[4] a concept normally closely related with the field of kinematics, as it deals with the movement of a body, rather than the cause of the movement. This can be attributed to the 'simple proportionality between force and velocity that characterises the *Peripatetic* dynamics.'[5] Or rather as Duhem tells it, 'Aristotle admits in principle that the power of a weight suspended from a lever is proportional to the velocity at which this weight moves when the lever is turned.'[6] For the author of this text, the proportionality of force to veloc-

[3]Arist. *Mech.* 848b10-26, 'Now if the two displacements of a body are in any fixed proportion, the resulting displacement must necessarily be a straight line, and this line is the diagonal of the figure, made by the lines drawn in the proportion. Let the proportion of the two displacements be as AB to AC, and let A be brought to B, and the line AB to E; then if the proportion of the two displacements be maintained, AD must necessarily have the same proportion to AE as AB to AC. Therefore the small parallelogram is similar to the greater, and their diagonal is the same so that A will be at F. In the same way it can be shown, at whatever points the displacement be arrested, that the point A will in all cases be on the diagonal. Thus it is plain that, if a point be moved along the diagonal by two displacements, it is necessarily moved according to the proportion of the sides of the parallelogram; for otherwise it will not be moved along the diagonal.' Barnes 1984, pp. 1300-1

[4]For a clear explanation of the parallelogram of forces see Asimov 1966, pp. 40-1.

[5]Benvenuto 1985, p. 101

[6]Duhem 1991, p. 55 & Duhem 1905, p. 72

ity allows these two different physical quantities to function interchangeably. So this quintessentially kinematic conception of motion can provide a demonstration of both the component velocities of an object's movement, and the constituent forces that act upon an object. Once this has been established, it is easier to conceive of how motion can form the basis of the approach to a statical problem.

Having established this relationship for the component forces and velocities present in linear motion, the author then returns to the circle, considering how this approach to forces can be applied to circular motion. The author reasons that two component forces (*φορα*) must also be present in circular motion, as the point on the circumference, at the end of the radius describing the circle, moves from a point vertically above the centre of the circle, to one horizontal to the centre of the circle. This transition of the point from one place to another could be described by a parallelogram of forces, except that the movement does not occur in a fixed ratio with regards to each vector of the parallelogram. The author describes the two forces that are in action as follows:

> *ἐὰν δὲ δυοῖν φερομένοιν ἀπὸ τῆς αὐτῆς ἰσχύος τὸ μὲν ἐκκρούοιτο πλεῖον τὸ δὲ ἔλαττον, εὔλογον βραδύτερον κινηθῆναι τὸ πλεῖον ἐκκρουόμενον τοῦ ἔλαττον ἐκκρουομένου ὃ δοκεῖ συμβαίνειν ἐπὶ τῆς μείζονος καὶ ἐλάττονος τῶν ἐκ τοῦ κέντρου γραφουσῶν τοὺς κύκλους. διὰ γὰρ τὸ ἐγγύτερον εἶναι τοῦ μένοντος τῆς ἐλάττονος τὸ ἄκρον ἢ τὸ τῆς μείζονος, ὥσπερ ἀντισπώμενον εἰς τοὐναντίον, ἐπὶ τὸ μέσον βραδύτερον φέρεται τὸ τῆς ἐλάττονος ἄκρον. πάσῃ μὲν οὖν κύκλον γραφούσῃ τοῦτο συμβαίνει, καὶ φέρεται τὴν μὲν κατὰ φύσιν κατὰ τὴν περιφέρειαν, τὴν δὲ παρὰ φύσιν εἰς τὸ πλάγιον καὶ τὸ κέντρον. μείζω δ' ἀεὶ τὴν παρὰ φύσιν ἡ ἐλάττων φέρεται διὰ γὰρ τὸ ἐγγύτερον εἶναι τοῦ*

κέντρου τοῦ ἀντισπῶντος κρατεῖται μᾶλλον. [7]

The author then proceeds to make a fairly complex geometrical demonstration that purports to show the influence of the 'unnatural' force that causes points closer to the centre move slower than those further away. Two chords of equal length are drawn in each of the concentric circles, centred on, and perpendicular to, the same radial line. The distance between the intersection of the chord and the radial line and the intersection of the circle and the radial line is found to be greater in the smaller circle than the larger circle. As these two lengths can be considered the vertices of parallelograms of force that describe the arcs of the respective circles, the greater distance should effectively prove that the larger circle is moving faster than the smaller circle, and so the 'unnatural' force is having a greater effect on the smaller circle.

The main issue with the conception of circular motion that exists in the text is that it relies upon a flawed understanding of circular motion as it relates to the lever or balance. As Heath points out (though he is perhaps being generous towards the anonymous peripatetic), there seems to be some conflation of free circular motion and constrained circular motion.[8] The forces at work in constrained circular motion, as exemplified by the lever and balance, are the motive force and the constraining centripetal force, generating motion that is always tangential and perpendicular to the radius. For two component velocities to describe a circle the ratio between the two would have to be constantly changing. However, the author of the *Mechanica* does recognise this, stating that ἐὰν

[7] Arist. *Mech.* 849a9-16 '...if one of two displacements caused by the same forces is more interfered with and the other less, it is reasonable to suppose that the motion more interfered with will be slower than the motion less interfered with; which seems to happen in the case of the greater and less of the radii of circles. For on account of the extremity of the lesser radius being nearer the stationary centre than that of the greater, being as it were pulled in a contrary direction, towards the middle, the extremity of the lesser move more slowly. This is the case with every radius, and it moves in a curve, naturally along the tangent, and unnaturally towards the centre. And the lesser radius is always moved more in respect of its unnatural motion; for being nearer to the retarding centre it is more constrained.' Barnes 1984, p. 1301

[8] T. Heath 1949, p. 230

δὲ ἐν μηδενὶ λόγῳ φέρηται δύο φορὰς κατὰ μηδένα χρόνον, ἀδύνατον εὐθεῖαν εἶναι τὴν φοράν,[9] but, the distinction that the theory of equilibrium and the theory of motion are governed by separate principles is not made in this work. [10] Equilibrium involves the absence of movement rather than merely the balancing of forces.

For Schiefsky the analysis undertaken in the text, involving the reduction of the mechanical problems down to three analytical models, the circle, the lever and the balance, represents something of an inversion of an axiomatic model. Rather than progressing from the simple to the complex, the analysis of these mechanical problems explains the complex through these simpler fundamental models.[11] This is in contrast to the strict axiomatic method which we will see in the following chapter employed by Archimedes. The method of exploring the theory developed in the Pseudo-Aristotelean *Mechanica* applies the solution to observable phenomena.

The importance of this text for the field of ancient mechanics does not rely upon the efficacy of the analytical model, but rather the fact that an analytical model has been developed for the working of the lever. The expression of the relationship between the circle, the balance, and the lever, as well as the realisation that the lever can act as an exemplar for the working of all machines is the earliest theory of mechanics. Duhem said it best, 'Had Aristotle formulated only this single idea, he would deservedly have to be celebrated as the father of rational mechanics.'[12] The material found in this text would go on to be one of the main sources for later medieval work on the subject,[13] and this Aristotelean conception of the law of the lever would, unfortunately, be much more influential than that of Archimedes.

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[9]rist. *Mech.* 848b
[10]Duhem 1991, p. 11
[11]M. Schiefsky 2009, p. 53
[12]Duhem 1991, p. 13
[13]Clagett 1959, pp. xxiii-xxiv

5 Archimedean Mechanics

Despite this close association with mechanics, the extant Archimedean texts are more representative of the work of a pure mathematician than a mechanician. The works *De sphaera et cylindro*, *De conoidibus et sphaeroidibus* and *De lineis spiralibus* constitute the vast majority of his extant corpus, and deal with the volumes of different solids and other geometrical constructions. What Aristotelean work that we have on mechanics is represented by three extant texts. Two of these texts deal with statics and hydrostatics respectively, and the third is a mathematical work demonstrating the application of mechanical thought to problems such as calculating the area or volume of a given geometrical form. While *De Planorum Aequilibris*, as the extant work by Archimedes on statics, will be explored at length in this chapter, it is worth giving a brief overview of both *De corporibus fluitantibus* and *Ad Eratosthenem methodus*. The notable thing about these mechanical works of Archimedes is their engagement with the physical world. As is apparent from Plutarch's account of the mathematician, a disregard for the tangible aspects of his research was one of the main characteristics latterly attributed to Archimedes.[1] The field of hydrostatics did not offer quite the same universal range of applications as statics for an ancient audience.

The most significant text by Archimedes on mechanics is *De Planorum Aequilibris*.

[1]Plut.*Marc.*14.3-6

This text develops and explores from first principles the theory of equilibrium. Introduced in this text is not only a new methodological approach to the field of mechanics, but also a new theoretical approach to area of statics. These two changes represent a significant departure from the earlier work on the subject found in the Aristotelean *Mechanica*.

While the *Mechanica* uses a dynamic model of circular motion to develop a theory of the lever, the approach found in *De Planorum Aequilibris* is concerned with developing a static model for the behaviour of the lever, based on the study of this fundamental machine while in a state of equilibrium. The Aristotelean work did carry out a mathematisation of the problem, as we have seen in the preceding chapter. However, the approach taken by Archimedes is much closer to our expectations of what would constitute a mathematical approach to a physical phenomenon. Primarily, this is because he applied what became the principle means of mathematical deduction and proof, the codification of which by Euclid may be dated to the generation before Archimedes, namely, the axiomatic method.[2] An axiomatic method is one which follows a line of deductive reasoning, proceeding from a series of primitive statements known as axioms.[3] If a theorem is derived through a deductive system such as this, the axioms from which they are derived must be sufficient to provide proof of the theorem. As this method is being applied by Archimedes to the field of mechanics, it is to define it in strictly rational rather than empirical terms, and the success of this is largely hinged upon the quality of the initial axioms defined for the system. The postulates that Archimedes provides in *De Planorum Aequilibris* are as follows:

α . Αἰτούμεθα τὰ ἴσα βάρεα ἀπὸ ἴσων μακέων ἰσορροπεῖν, τὰ δὲ

[2]T.L. Heath 1956, pp. 1-2

[3]This definition of an axiom is essentially a modern one, the

ἴσα βάρεα ἀπὸ τῶν ἀνίσων μακέων μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος τὸ ἀπὸ τοῦ μείζονος μάκεος.

β . Εἴ κα βαρέων ἰσορροπεόντων ἀπό τινων μακέων ποτὶ τὸ ἕτερον τῶν βαρέων ποτιτεθῇ, μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος ἐκεῖνο, ᾧ ποτετέθη.

γ . Ὁμοίως δὲ καί, εἴ κα ἀπὸ τοῦ ἑτέρου τῶν βαρέων ἀφαιρεθῇ τι, μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος, ἀφ' οὗ οὐκ ἀφῃρέθη.

δ . Τῶν ἴσων καὶ ὁμοίων σχημάτων ἐπιπέδων ἐφαρμοζομένων ἐπ' ἄλλαλα καὶ τὰ κέντρα τῶν βαρέων ἐφαρμόζει ἐπ' ἄλλαλα.

ϵ . Τῶν δὲ ἀνίσων, ὁμοίων δέ, τὰ κέντρα τῶν βαρέων ὁμοίως ἐσσεῖται κείμενα. Ὁμοίως δὲ λέγομες σαμεῖα κέεσθαι ποτὶ τὰ ὁμοῖα σχήματα, ἀφ' ὧν ἐπὶ τὰς ἴσας γωνίας ἀγόμεναι εὐθεῖαι ποιέοντι γωνίας ἴσας ποτὶ τὰς ὁμολόγους πλευράς.

. Εἴ κα μεγέθεα ἀπό τινων μακέων ἰσορροπέωντι, καὶ τὰ ἴσα αὐτοῖς ἀπὸ τῶν αὐτῶν μακέων ἰσορροπήσει.

ζ . Παντὸς σχήματος, οὗ κα ἁ περίμετρος ἐπὶ τὰ αὐτὰ κοῖλα ᾖ, τὸ κέντρον τοῦ βάρεος ἐντὸς εἶμεν δεῖ τοῦ σχήματος.[4]

[4] Arch. *Aequil.* 2.80-81 Dijksterhuis translates the postulates as:
1. We postulate that equal weights at equal distances are in equilibrium, and that equal weights at unequal distances are not in equilibrium, but incline towards the weight which is at the greater distance.
2. That if, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium, but incline towards that weight to which something has been added.
3. Similarly that, if anything be taken away from one of the weights, they are not in equilibrium, but incline towards that weight from which nothing has been taken away.
4. When equal and similar figures are made to coincide, their centres of gravity likewise coincide.
5. In figures which are unequal, but similar, the centres of gravity will be similarly situated. We say that points are similarly situated in relation to similar figures if straight lines drawn from these points to the equal angles make equal angles with the homologous sides.
6. If magnitudes at certain distances be in equilibrium, other [magnitudes] equal to them will also be in equilibrium at the same distances.
7. In any figure whose perimeter is concave in the same direction the centre of gravity must be within the figure.

Of these seven postulates, the first three and the sixth can be considered as being related to the balance, and the remaining three postulates as being related to the exploration of the centres of gravity in plane figures, the subject of the latter half of this work. These four postulates must, then, provide a basis from which it is possible derive the theory of the lever and equilibrium. However, there has been debate over whether or not these postulates can adequately do this, prompted by Mach's criticism of the formal methodology found in the work. This criticism was focused on the first postulate; specifically, the extent to which it relies upon the readers existing knowledge or experience of the working of a lever or balance. Mach highlights the breadth of factors external to those stated by Archimedes in this postulate that may theoretically impinge upon the behaviour of a balance.[5] These examples, such as the colour of the opposite sides of the balance, obviously do not have an effect, but it is exactly this that leads both Mach, and Goe,[6] to the conclusion that the 'equal arms' axiom is dependent upon a pre-existing understanding of the behaviour of the lever and balance beam.

Goe relates the first postulate of Archimedes to Euclid's fifth postulate,[7] stating that Mach errs in ascribing to Archimedes the assertion that that the 'equal arms' axiom is self-evident.[8] Euclid's Fifth postulate is here used as an example of another notable example of an axiom that is necessary for the foundations of the work, but is not easily proven using the other provided axioms.

Dijksterhuis 1987, pp. 287-7

[5]Mach 1893, pp. 9-10

[6]Goe 1972, p. 330

[7]Also known as the parallel postulate, which states '...if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.'T.L. Heath 1956, p. 202. Although the defining axiom for what we think of as 'Euclidean Geometry', the fact that it could not be readily solved using Euclid's previous four postulates has led to much debate over its status as a postulate or theorem, as well as many attempts at a proof. For a concise examination of the postulate and the tradition surrounding it, see T.L. Heath 1956, pp. 202-220.

[8]Goe 1972, p. 330

> *πᾶσα γὰρ ἀποδεικτικὴ ἐπιστήμη περὶ τρία ἐστίν, ὅσα τε εἶναι τίθεται (ταῦτα δ' ἐστὶ τὸ γένος, οὗ τῶν καθ' αὑτὰ παθημάτων ἐστὶ θεωρητική), καὶ τὰ κοινὰ λεγόμενα ἀξιώματα, ἐξ ὧν πρώτων ἀποδείκνυσι, καὶ τρίτον τὰ πάθη, ὧν τί σημαίνει ἕκαστον λαμβάνει. ἐνίας μέντοι ἐπιστήμας οὐδὲν κωλύει ἔνια τούτων παροραν, οἷον τὸ γένος μὴ ὑποτίθεσθαι εἶναι, ἂν ᾖ φανερὸν ὅτι ἔστιν (οὐ γὰρ ὁμοίως δῆλον ὅτι ἀριθμὸς ἔστι καὶ ὅτι ψυχρὸν καὶ θερμόν), καὶ τὰ πάθη μὴ λαμβάνειν τί σημαίνει, ἂν ᾖ δῆλα* [9]

In accordance with this criteria, *De Planorum Aequilibris's* postulates need only serve as basis for the exploration of the subject in the terms defined. In the same way that the non-convergence of parallel lines is a defining feature of what is now known as Euclidean geometry, the 'proper' axioms defined by Archimedes serve only as the basis for this particular demonstration of the principle of equilibrium. In essence this is what divides the theoretical from the actual, a system is established that models the physical to some degree, and from this we can draw some conclusion or effect some demonstration. The system need only be internally consistent rather than consistent with all physical actualities. If this were to be taken as a more universal mechanical work, criticism may also be levelled at the absence of a plane and fulcrum upon which the two equal weights are placed in the first postulate. Mach has had to supply 'from their point of support' in his rendering of the postulates,[10] and the fact that this is included in the described system is only apparent from the propositions. Mechanics is a physical science, and it is

[9] Arist. *APo.* 76b11-20Of the items used in the demonstrative sciences some are proper to each science and others common — but common by analogy, since they are only useful in so far as they bear on the kind under the science. Proper: e.g. that a line is *such-and-such*, and straight so-and-so. Common: e.g. that if equals are removed from equals, the remainders are equal. It is sufficient to assume each of these in so far as it bears on the kind; for it will produce the same results even if it is assumed as holding not of everything but only for magnitudes (or, for arithmeticians, for numbers). Barnes 1994, p. 15

[10] Mach 1893, pp. 8-9

perhaps not surprising that the foundations of the text are, at least to some extent, based upon empirical or casual observation. That there has been some reliance upon pre-existing knowledge, and that the theory of equilibrium has not been been developed *ab initio*, perhaps leaves some ambiguities in the system. However, this does not mean that it cannot serve as appropriate grounds upon which the theory of equilibrium can be explored.

While the postulates form the basis of the work, the propositions build upon this to define the theory of equilibrium that can be developed from them. The abstract postulates are applied to scenarios developed to test and explore them. This is where the balance becomes the proving ground for the development of this theory of equilibrium. It is undoubtedly used because it is the most fundamental and readily comprehensible model for the theory. The theory of equilibrium is, of course, one which is applicable far more universally than is explored in this text. However, the importance of the balance, and the lever as its theoretical forebear, as has been seen in the Aristotelean *Mechanica*, is that the behaviour of all other weight-lifting machinery can be derived from the theory of equilibrium. More importantly, as has already been mentioned, much of the understanding of equilibrium and the law of the lever must have been derived from the observation of the balance. The early propositions in the work all deal with various states of equilibrium that bodies may be in, such as two equal weights at equal distances from one another, and these are proven through reasoning based upon the first four postulates. The sixth and seventh propositions are of considerably greater interest to us, as it is here that the law of the lever is most readily encapsulated. This is achieved through the exploration and proof of the fact that unequal weights are in equilibrium on a balance at a distance to the fulcrum that is inversely proportional to their weight. The seventh proposition deals with the proof of this for magnitudes that are irrational, but it is the sixth that deals with rational numbers that will be focused upon here. The sixth proposition is as follows:

> . Τὰ *σύμμετρα μεγέθεα ἰσορροπέοντι ἀπὸ μακέων ἀντιπεπονθότως τὸν αὐτὸν λόγον ἐχόντων τοῖς βάρεσιν.* Ἔστω *σύμμετρα μεγέθεα τὰ* Α, Β, *ὧν κέντρα τὰ* Α, Β, *καὶ μᾶκος ἔστω τι τὸ* ΕΔ, *καὶ ἔστω ὡς τὸ* Α *ποτὶ τὸ* Β, *οὕτως τὸ* ΔΓ *μᾶκος ποτὶ τὸ* ΓΕ *μᾶκος δεικτέον ὅτι τοῦ ἐξ ἀμφοτέρων τῶν* Α, Β *συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος τὸ* Γ. Ἐπεὶ *γάρ ἐστιν, ὡς τὸ* Α *ποτὶ τὸ* Β, *οὕτως τὸ* ΔΓ *ποτὶ τὸ* ΓΕ, *τὸ* δὲ Α *τῷ* Β *σύμμετρον, καὶ τὸ* ΓΔ *ἄρα τῷ* ΓΕ *σύμμετρον, τουτέστιν εὐθεῖα τᾷ εὐθείᾳ ὥστε τῶν* ΕΓ, ΓΔ *ἐστὶ κοινὸν μέτρον.* Ἔστω *δὴ τὸ* Ν, *καὶ κείσθω τᾷ μὲν* ΕΓ *ἴσα ἑκατέρα τᾶν* ΔΗ, ΔΚ, *τᾷ* δὲ ΔΓ *ἴσα ἁ* ΕΛ. Καὶ *ἐπεὶ ἴσα ἁ* ΔΗ *τᾷ* ΓΕ, *ἴσα καὶ ἁ* ΔΓ *τᾷ* ΕΗ *ὥστε καὶ ἁ* ΛΕ *ἴσα τᾷ* ΕΗ. Διπλασία *ἄρα ἁ μὲν* ΛΗ *τᾶς* ΔΓ, *ἁ* δὲ ΗΚ *τᾶς* ΓΕ *ὥστε τὸ* Ν *καὶ ἑκατέραν τᾶν* ΛΗ, ΗΚ *μετρεῖ, ἐπειδήπερ καὶ τὰ ἡμίσεα αὐτᾶν.* Καὶ *ἐπεί ἐστιν, ὡς τὸ* Α *ποτὶ τὸ* Β, *οὕτως ἁ* ΔΓ *ποτὶ* ΓΕ, *ὡς* δὲ *ἁ* ΔΓ *ποτὶ* ΓΕ, *οὕτως ἁ* ΛΗ *ποτὶ* ΗΚ *διπλασία γὰρ ἑκατέρα ἑκατέρας καὶ ὡς ἄρα τὸ* Α *ποτὶ τὸ* Β, *οὕτως ἁ* ΛΗ *ποτὶ* ΗΚ. Ὁσαπλασίων *δέ ἐστιν ἁ* ΛΗ *τᾶς* Ν, *τοσαυταπλασίων ἔστω καὶ τὸ* Α *τοῦ* Ζ *ἔστιν ἄρα ὡς ἁ* ΛΗ *ποτὶ* Ν, *οὕτως τὸ* Α *ποτὶ* Ζ. Ἔστι δὲ *καὶ ὡς ἁ* ΚΗ *ποτὶ* ΛΗ, *οὕτως τὸ* Β *ποτὶ* Α *δι' ἴσου ἄρα ἐστὶν ὡς ἁ* ΚΗ *ποτὶ* Ν, *οὕτως τὸ* Β *ποτὶ* Ζ *ἰσάκις ἄρα πολλαπλασίων ἐστὶν ἁ* ΚΗ *τᾶς* Ν *καὶ τὸ* Β *τοῦ* Ζ. Ἐδείχθη δὲ *τοῦ* Ζ *καὶ τὸ* Α *πολλαπλάσιον ἐόν ὥστε τὸ* Ζ *τῶν* Α, Β *κοινόν ἐστι μέτρον.* Διαιρεθείσας *οὖν τᾶς μὲν* ΛΗ *εἰς τὰς τᾷ* Ν *ἴσας, τοῦ* δὲ Α *εἰς τὰ τῷ* Ζ *ἴσα, τὰ ἐν τᾷ* ΛΗ *τμάματα ἰσομεγέθεα τᾷ* Ν

ἴσα ἐσσεῖται τῷ πλήθει τοῖς ἐν τῷ Α *τμαμάτεσσιν ἴσοις ἐοῦσιν τῷ* Ζ. *Ὥστε, ἂν ἐφ' ἕκαστον τῶν τμαμάτων τῶν ἐν τᾷ* ΛΗ *ἐπιτεθῇ μέγεθος ἴσον τῷ* Ζ *τὸ κέντρον τοῦ βάρεος ἔχον ἐπὶ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐντὶ τῷ* Α, *καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον ἐσσεῖται τοῦ βάρεος τὸ* Ε *ἄρτιά τε γάρ ἐστι τὰ πάντα τῷ πλήθει, καὶ τὰ ἐφ' ἑκάτερα τοῦ* Ε *ἴσα τῷ πλήθει διὰ τὸ ἴσαν εἶμεν τὰν* ΛΕ *τᾷ* ΗΕ. *Ὁμοίως δὲ δειχθήσεται ὅτι κἄν, εἴ κα ἐφ' ἕκαστον τῶν ἐν τᾷ* ΚΗ *τμαμάτων ἐπιτεθῇ μέγεθος ἴσον τῷ* Ζ *κέντρον τοῦ βάρεος ἔχον ἐπὶ τοῦ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐσσεῖται τῷ* Β, *καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον τοῦ βάρεος ἐσσεῖται τὸ* Δ *ἐσσεῖται οὖν τὸ μὲν* Α *ἐπικείμενον κατὰ τὸ* Ε, *τὸ δὲ* Β *κατὰ τὸ* Δ. *Ἐσσεῖται δὴ μεγέθεα ἴσα ἀλλάλοις ἐπ' εὐθείας κείμενα, ὧν τὰ κέντρα τοῦ βάρεος ἴσα ἀπ' ἀλλάλων διέστακεν,* [*συγκείμενα*] *ἄρτια τῷ πλήθει δῆλον οὖν ὅτι τοῦ ἐκ πάντων συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος ἁ διχοτομία τᾶς εὐθείας τᾶς ἐχούσας τὰ κέντρα τῶν μέσων μεγεθέων. Ἐπεὶ δ' ἴσαι ἐντὶ ἁ μὲν* ΛΕ *τᾷ* ΓΔ, *ἁ δὲ* ΕΓ *τᾷ* ΔΚ, *καὶ ὅλα ἄρα ἁ* ΛΓ *ἴσα τᾷ* ΓΚ *ὥστε τοῦ ἐκ πάντων μεγέθεος κέντρον τοῦ βάρεος τὸ* Γ *σαμεῖον. Τοῦ μὲν ἄρα* Α *κειμένου κατὰ τὸ* Ε, *τοῦ δὲ* Β *κατὰ τὸ* Δ, *ἰσορροπησοῦντι κατὰ τὸ* Γ.[11]

[11] Arch. *Aequil.* 85-86 Dijkesterhuis translates this passage as:
Proposition 6.
Commensurable magnitudes are in equilibrium at distances reciprocally proportional to the weights.
Let the commensurable magnitudes be A and B, of which A and B are the centres, and let ΕΔ be a given distance, and let the distance ΔΓ be to the distance ΓΕ as A to B. It has to be proved that the centre of gravity of the magnitude composed of A and B is Γ.
Since A and B are commensurable, so are ΔΓ and ΓΕ.
Let N be a common measure of these two distances. Make ΔΗ=ΔΚ=ΕΓ and ΕΛ=ΔΓ. Apparently ΕΗ is also equal to ΔΓ. Since ΗΛ=2.ΔΓ and ΗΚ=2.ΕΓ, we also have
A:B = ΛΗ:ΗΚ.
Now let the magnitude Z be contained as many times in A as the distance N in ΛΗ, whence also as many

Archimedes here is drawing upon the earlier propositions, which had proved that the centre of gravity of two equal weights will occur at a point equidistant between the centres of gravity of the two individual weights, to provide a proof of the sixth proposition. The more complex problem of unequal weights at unequal distances is transmuted to the simpler problem of equal weights at equal distances. In comparison with the approach found in the Aristotelean *Mechanica* we here have a simple, elegant, and most importantly, quantifiable proof of the law of the lever. There is however another issue to be found with these postulates and this proof, which again formed part of Mach's criticism of the work, namely that a concrete definition of what is meant by 'centre of gravity'[12] is not found in this work.

Given that this concept occupies such a central position in the development of the theories discussed both here and later in the work when plane figures are discussed it seems to be a rather troubling omission. The significance of the concept to the sixth proposition is that in this model the effect that a weight suspended upon a balance has is entirely dependent upon its centre of gravity, and that the proof of the theory is reliant upon the ability to replace this weight with a number of others with the same centre of gravity. Two differing theories have been proposed for the absence of this

times in B in N in HK. Divide ΛH and HK each into equal parts N, A and B each into equal parts Z. Place on each of the line segments N a magnitude Z, so that in each case the centre of gravity of Z is the middle point of N, then the centre of gravity of all the magnitudes Z placed on the parts of ΛH will be the point E, while in the same way the centre of gravity of all the magnitudes Z placed on the parts of HK will be the point Δ. Now therefore A will be at E and B at Δ. There will now be equal magnitudes on a straight line, the centres of gravity of which are equidistant from one another and the number of which is even. It is now obvious that of the magnitude composed of all the magnitudes the middle point of the straight line bounded by the centres of the middle magnitudes will be the centre of gravity. So that the centre of gravity of the magnitude composed of all the magnitudes is the point Γ. If therefore A is at E and B at Δ, they will be in equilibrium about Γ.' Dijksterhuis 1987, p.289-90 Although T. L. Heath 1897 is the classic translation of the work of Archimedes, and by rendering large parts of the propositions in modern notion he renders the text more readily comprehensible, I have preferred the translation found in Dijksterhuis due to its stricter adherence to the original text.

[12] *κέντρον τοῦ βάρεος*

definition from the work, and both have been thoroughly examined by Dijksterhuis. The first is that the concept of centres of gravity is a familiar one, either from the other work by Archimedes, or as a more commonly defined term that was generally known. Alternatively, the definition of the centres of gravity is intended to be implicit from the postulates of this work. The fact that implicit definitions of terms are not present in Euclid's *Elements* is highlighted by Dijksterhuis as evidence that it is unlikely that an ambiguous approach such as this would have been adopted by Archimedes. He reasons that the model for our understanding of these ideas of equilibrium, inclination and weight is the observation of the lever and balance. That rather than there being an implicit definition of these concepts in the postulates, there is the implicit understanding that these things are comprehensible from observation and prior knowledge of their working.[13] The notion of centres of balance is not as intuitive, and so it remains that the concept had been explored in another work on mechanics or statics by Archimedes.

A number of other works on mechanics have been attributed to Archimedes in antiquity, and it is in these that the concept of a centre of gravity may have been established theoretically initially. An 'Elements of Mechanics' is referred to in *De corporibus fluitantibus*,[14], as well as a 'Mechanica' in *On the Quadrature of the Parabola*[15]. We also have reference to an 'Equilibria' which is likely the same work, if not on a similar subject.[16] Dijksterhuis believes *De Planorum Aequilibris* was a constituent part of a larger *Mechanica* that may be what these works refer to.[17] Drachmann has carried out a fairly

[13]Dijksterhuis 1987, pp. 295-298

[14]Στοιχεία τῶν μηχανικῶν in Archim. *Fluit.* 2.2.

[15]Archim. *Quad. Parab.* 6 & 10

[16]Archim. *Fluit.* 2.2., but also at *Meth.*1. as τά Ἰσορροπικά.

[17]Dijksterhuis 1987, pp. 47-48 For completeness it is worth mentioning that there are a couple of other titles attributed to Archimedes that were undoubtedly mechanical in nature. Pappus refers to a work titled 'On Balances', (περί ζυγῶν in Papp. *1068.*) while Heron refers to a work 'On Supports' (Hero. *Mech.* 1.25, extant in Arabic only, translated by Nix as 'Buch der Stützen' Nix and W. Schmidt 1976, p. 70). For more on both of these see the latter two sections of Drachmann 1963a, pp. 114-143.

extensive survey of these works while pursuing the thesis that some of Heron's *Mechanica* are excerpts of a lost Archimedean work.[18] The conclusions of both of these authors has been that *De Planorum Aequilibris* is likely the second section of a larger work, the first part of which would have explored and defined the concept of centres of gravity.

This does not, unfortunately, bring us any closer to a solid answer for the absence a definition of this concept in the work. In the apparent absence of any other work on Mechanics of the same era or earlier it seems that the concept must indeed be attributed to Archimedes. The texts attributed to him form a more likely list than some of the devices that make up the catalogue of his attested achievements as an inventor, and certainly the mode of analysis and approach taken by Archimedes in *De Planorum Aequilibris* and his other works would suggest that some definition of the centre of gravity would appear elsewhere in his work. The theoretical gaps have been covered by Drachmann and Dijksterhuis, and in addition to this Olaf Schmidt has undertaken the task of producing a complete proof of the theory of equilibrium and centres of gravity using the Archimedean methodology.[19]

Although we might not consider *De Planorum Aequilibris* to represent a complete mathematisation of the theory of equilibrium, it is, nevertheless, a far more elegant and definitive approach to the problem than had previously been undertaken. It may be important to note here that the conclusions of this Archimedean work are no different to those found in the Aristotelean work, the real difference is in the static rather semi-dynamic method used in the proof. It is safe to say that the law of the lever was understood long before either of these authors set out to explain it theoretically. The approach taken by Archimedes, although abstract, allows us to immediately relate the

[18] Drachmann 1963a
[19] O. Schmidt 1975

geometrical lines and magnitudes to a balance with weights suspended upon it. This is an attestation in itself of its efficacy. The fact that there is no real connection made with a physical object in *De Planorum Aequilibris*, that it is an entirely mathematical abstraction of the problem, places it in a unique position among ancient texts on 'theoretical mechanics'. It is the only text that neither sets out to apply itself in any way to practical applications, nor to serve to directly explain physical phenomena.

Without an introductory letter explaining it, as is found accompanying some of the other works of Archimedes, it seems that the motivation for this research is somewhat obscure. Drachmann has made a compelling argument for the root of Archimedes investigations into equilibrium. Using the material found in *Ad Eratosthenem methodus* he has explored the idea that Archimedes' investigations into statics and equilibrium were the result of his use of mechanical techniques to explore mathematical problems.[20] The utility of the lever for exploring the areas and volumes of geometrical figures as described in the method seems like an obvious and attractive reason for the investigation and codification of this subject by Archimedes. There is one particularly telling comment from *Ad Eratosthenem methodus* that illustrates the relationship between Archimedes' mechanical and mathematical investigation, where he says in his introduction to the work:

> *Καὶ γάρ τινα τῶν πρότερόν μοι φανέντων μηχανικῶς ὕστερον γεωμετρικῶς ἀπεδείχθη διὰ τὸ χωρὶς ἀποδείξεως εἶναι τὴν διὰ τούτου τοῦ τρόπου θεωρίαν* [21]

As Archimedes has developed a specific mechanical methodology for the study of volume and area it seems that this is a much more likely source for the codification of the

[20] Drachmann 1967, pp. 5-7

[21] Archim. *Meth.* 1

study of these subjects than the understanding of the mechanical phenomenon alone. Although he cannot admit these methods as formal mathematical proof, this does not preclude their use in the investigation of these mathematical problems. While a full understanding of the law of the lever does lend itself to the use of the device as a physical tool, the kind of mathematical abstraction undertaken by Archimedes in *De Planorum Aequilibris* enables it to be fully utilised as a rational tool. A heuristic approach to these mathematical problems could be adopted, and one which could be carried out upon a solid theoretical basis. The work is, as Drachmann has succinctly put it, the mathematical proof of mechanical perception which was necessitated by the need to define the act of weighing.[22] As far as the mechanician is concerned, Duhem may well have been correct when he said that the theories expressed in *De Planorum Aequilibris* 'do not reveal any novel insight into questions of importance to him'.[23] This is certainly true in that they do not represent some new development in the practice of mechanics. The thorough approach taken did, however, leave a lasting impact on the theory of the field. This is visible from Heron's referencing of Archimedes and attestation that his work on the centre of gravity gives a 'more precise definition' of the concept.[24]

[22]Drachmann 1967, but see also Knorr 1978, who has come to a similar conclusion that Archimedes' 'mechanical' works have been produced with little or no interest in mechanics itself.

[23]Duhem 1991, p. 14

[24]Hero. *Mech.* 1.24, Drachmann 1963a, p. 100

6 The Delian Problem

In the introduction to the eighth book of his *Collection*, which deals with mechanics, Pappus sets out a programme for the material he will cover on mechanics, emphasising three theorems in particular which he states are the most essential for the movement of weights.[1] Among these we find the following description of a theorem:

> *δύο δοθεισῶν εὐθειῶν ἀνίσων δύο μέσας ἀνάλογον εὑρεῖν ἐν συνεχεῖ ἀναλογίᾳ (διὰ γὰρ τοῦ θεωρήματος τούτου πᾶν τὸ δοθὲν στερεὸν σχῆμα κατὰ τὸν δοθέντα λόγον αὔξεταί τε καὶ μειοῦται)*[2]

This problem appears frequently in ancient texts, described in a number of different ways, and is of particular importance in the history of ancient mathematics. It is commonly referred to as the Delian problem, and deals with doubling the size of a cube. I will explore in this chapter how this mathematical problem becomes so closely associated with mechanics, with which it does not seem to have an immediately obvious relationship.

[1] "*ἀναγκαιότατα περὶ τὴν τῶν βαρῶν κίνησιν*" Papp.1028

[2] Papp. 1028, 'Given two unequal straight lines to find two mean proportionals in continued proportion. By this theorem every solid figure may be augmented or decreased in any given ratio.'Cohen and Drabkin 1948, p. 185

Along with squaring the circle and trisecting a given angle, the Delian problem is one of the three classical problems of ancient mathematics.[3] These three problems are characterised by being unsolvable using only compass and straightedge construction, which was the primary means of calculating non-integer values in antiquity, and the most practical way of deriving meaningful results from such calculations.[4] There is a relationship between the problem of doubling the cube and doubling the square, famous from Socrates' didactic demonstration in the *Meno*[5] but while the former presents some difficulty, the latter is easily derived with a compass.[6]

The Delian problem itself can be simply stated as follows: given a cube with side x, where the volume $V = x^3$, construct another cube of $2V$. While this may initially seem to be relatively straightforward, we would quickly find ourselves in a far greater quandary than the anonymous $\pi\alpha\hat{\imath}\varsigma$ of the Socratic dialogue when asked to calculate the exact length of the cube's sides, as their lengths are equal to $x\sqrt[3]{2}$, a value far more difficult to calculate than for the square. The name of the problem comes from its reputed origin; the story goes that the Delians had consulted the oracle of Apollo as to how they might escape from a plague, and had been told that they must construct an altar of double the size of their previous one. In some sources they make a variety of different attempts at doubling the size of the altar, including constructing an identical altar and placing it on top of the first,[7] or doubling each of the sides of the altar.[8] The story itself, and these attempted solutions, are really focused around Plato rather than

[3]Although the construction of a regular heptagon is also frequently included as a fourth problem in the set.

[4]Russo 2004, pp. 41-2. The fact that a solution cannot be derived by compass and straight edge construction was not proven till 1837 by Wantzel (see Dörrie 1965, pp. 174-7 for a proof in English).

[5]Pl.*Meno*.82b9-85b7, Huffman 2005, p. 360.

[6]The diagonal of the original square is, of course, equal to the sides of a square of twice the size.

[7]Philoponus, *On the Posterior Analytics - CAG* XIII.3 102.12-22

[8]Plut. *De gen. Socr.* 579b-d

the problem, serving, as Zhmud puts it, to cast him as "the hero of 'historico-scientific' legends", reflecting the fact that he is regarded in the ancient tradition as "an 'architect of *mathēmata*'".[9] The story far more likely represents the point, rather than the circumstances in which, the problem became a subject of investigation. Nevertheless, the fact that there is a creation myth associated with the Delian problem at all is indicative of the wider importance of the problem. The sheer number of solutions to the problem, reproduced in works by, or attributed to, many ancient mathematicians also attests to its status as one of the key problems of ancient mathematics. It is far beyond the scope of this chapter to attempt any real discussion of the solutions to the problem and the evolution of the approaches taken to it.[10]

However, the problem is not typically referred to as 'the Delian Problem' (*τὸ Δηλιακὸν πρόβλημα*) in the vast majority of its appearances in the technical corpus, nor indeed is the problem of doubling a cube typically found as the main subject of discussion or proof. Instead what we find discussed are methods of 'finding the two mean proportionals' (δ*ύο μέσας ἀνάλογον*), which can be used to allow for the scaling up and down of solid forms, a much more useful generalisation of the problem that became the main focus of work on the subject. The reduction of the problem to these terms is attributed to Hippocrates of Chios, though as Eratosthenes points out in his account of the problem, it does not make the problem an simpler to deal with.[11] Finding the two mean proportionals can be described as follows; Given two numbers, the two mean proportionals are two values that exist in continued proportion between these numbers. The relationship between them can be described as $a : x = x : y = y : b$, where x and y are

[9]Zhmud 2006, p. 83

[10]An overview of the problem can be found in T. Heath 1921, pp. 244-270, but the best account can be found in the exhaustive chapter in Huffman 2005, pp. 342-401.

[11]*ὥστε τὸ ἀπόρημα αὐτῷ εἰς ἕτερον οὐκ ἔλασσον ἀπόρημα κατέστρεφεν* Heiberg 1913, p. 88

the mean proportionals between the lengths a and b. This is reducible to the expression $\frac{a^3}{x^3} = \frac{a}{b}$, which demonstrates that since $b = 2a$, the length x is that of the side of a cube of double the size of a. With this value it is either possible to use it as a scaling factor for all sides of the solid that is being doubled, that is, multiply each side by $\sqrt[3]{2} = (1.259)$, or with a easily reproducible methodology, geometrically derive the lengths of each side from the original lengths.

A solution to the problem of the two mean proportionals appears in two different works on mechanics by Heron of Alexandria, his *Mechanica* and his *Belopoeica*. The version of the solution that is present in the *Mechanica* is also found in the third book of Pappus' *Collection*, and the substantial similarities between the extant Arabic text of the *Mechanica* and the Greek text of Pappus indicates that the *Mechanica*, rather than the *Belopoeica* was the source used by Pappus.[12] However, the differences between these two versions are in wording rather than the actual method used. The solution provided in the *Belopoeica* is as follows:

> *Ὡς δὲ δεῖ, δύο δοθεισῶν εὐθειῶν, δύο μέσας ἀνὰ λόγον λαβεῖν, ἑξῆς ἐροῦμεν. Ἔστωσαν αἱ δύο δοθεῖσαι εὐθεῖαι αἱ* ΑΒ, ΒΓ *πρὸς ὀρθὰς* <*ἀλλήλοις*> *κείμεναι. ὧν δεῖ δύο μέσας ἀνὰ λόγον εὑρεῖν καὶ συμπεπληρώσθω τὸ* ΑΒΓΔ *παραληλλόγραμμον. καὶ ἐπεζεύχθωσαν αἱ* ΑΓ, ΒΔ. *καὶ ἐκβεβλήσθωσαν αἱ* ΔΓ, ΔΑ *καὶ παρακείσθω παρὰ τὸ* Β *σημεῖον κανὼν τέμνων τὰς ἐκβαλλομένας εὐθείας, καὶ κινείσθω ὁ εἰρημένος κανὼν περὶ τὸ* Β *σημεῖον, ἄχρις ἂν αἱ ἀπὸ τοῦ* Ε *ἐπὶ τὰς τομὰς ἐπιζευγνύμεναι ἴσαι ἀλλήλαις ὦσι καὶ ἔστω ὁ μὲν κανὼν θέσιν εἰληφὼς οἵαν ἔχει ἡ* ΖΒΗ *εὐθεῖα αἱ δὲ ἄλλαι δύο εὐθεῖαι αἱ* ΕΖ, ΕΗ. *λέγω ὅτι τῶν* ΑΒ, ΒΓ *εὐθειῶν αἱ μέσαι ἀνὰ λόγον εἰσὶν αἱ* ΑΖ, ΓΗ *καὶ πρώτης*

[12] Knorr 1989, pp. 11-13 & Heiberg 1913, 59 n.1

> *οὔσης τῆς* AB, *δευτέρα μὲν ἔσται ἡ* AZ, *τρίτη δὲ ἡ* ΓH, *τετάρτη δὲ ἡ* BΓ. <*ἐπεὶ γὰρ διαγώνιόν ἐστιν τὸ* ABΓΔ *παραλληλόγραμμον, αἱ τέσσαρες εὐθεῖαι αἱ* ΔE, EA, EB, EΓ *ἴσαι ἀλλήλαις εἰσίν*> *ἐπεὶ γὰρ ἴση ἐστὶν ἡ* AE *τῇ* EΔ *καὶ διῆκται ἡ* EZ, *τὸ ἄρα ὑπὸ* ΔZA *μετὰ τοῦ* <*ἀπὸ*> AE *ἴσον ἐστὶν τῷ ἀπὸ τοῦ* EZ. *διὰ τὰ αὐτὰ δὴ καὶ τὸ ὑπὸ* ΔHΓ *μετὰ τοῦ ἀπὸ* ΓE *ἴσον ἐστὶν τῷ ἀπὸ* EH *καὶ ἔστιν ἴση ἡ μὲν* AE *τῇ* EΓ, *ἡ δὲ* EZ *τῇ* EH. *ἔσται ἄρα καὶ τὸ ὑπὸ* ΔZA *ἴσον τῷ ὑπὸ* ΔHΓ. *ὡς ἄρα ἡ* HΔ *πρὸς* ΔZ, *οὕτως ἔστιν ἡ* AZ *πρὸς* ΓH. *ἀλλ' ὡς ἡ* HΔ *πρὸς* ΔZ, *ἥτε* AB *πρὸς* AZ, *καὶ ἡ* ZA *πρὸς* ΓH, *καὶ ἡ* HΓ *πρὸς* ΓB *ἔσται ἄρα καὶ ὡς ἡ* BA *πρὸς* AZ, *οὕτως καὶ ἡ* HΓ *πρὸς* ΓB *τῶν ἄρα* AB, BΓ *δύο μέσαι ἀνὰ λόγον εἰσὶν αἱ* AZ, ΓH.[13]

An almost identical method of calculating the two mean proportionals appears in Philon's *Belopoeica*, which I will not reproduce here as it adds little to the solution we have already seen from Heron.[14] Beyond these three works we find mean proportionals being mentioned in passing by Vitruvius in the tenth book of his *De Architectura*, where, rather than providing the means for the reader to calculate the required values, he instead provides tables of values derived from these calculations. What is really no-

[13]Hero. *Bel.* 33-34, Marsden's translation of the passage reads: We shall now explain how you must find the two mean proportionals between two straight lines. Set two given straight lines AB and BΓ at right angles. It is required to find the two mean proportionals between these. Complete the rectangle ABΓΔ. Join AΓ, BΔ; extend ΔΓ, ΔA. Lay a ruler through point B, crossing these extensions, and move the ruler around point B until lines joining E to the points of intersection are equal to each other. Suppose the ruler has asssumed the position represented by the straight line ZBH. The other straight lines are EZ, EH. I affirm that the two mean proportionals (of AB, BΓ) are AZ, ΓH. If AB is first, second will be AZ, third ΓH, fourth BΓ. Since AE equals EΔ and EZ has been drawn, the product of ΔZ times ZA plus the square on AE equals the square on EZ. Similarly, the product of ΔH times HΓ with the square on ΓE equals the square on EH. And AE is equal to EΓ, EZ to EH. Therefore, ΔZ times ZA will equal ΔH times HΓ. As HΔ is to ΔZ, so is AZ to HΓ. But HΔ is to ΔZ as AB to AZ, ZA to ΓH and HΓ to ΓB;therefore, BA will be to AZ as HΓ to ΓB; therefore, the two mean proportionals of AB and BΓ are AZ and ΓH. Marsden 1971, pp. 41-43

[14]See T. Heath 1921, pp. 262-4 for an overview and comparison of Philon, Heron and Apollonius' solutions.

table about this particular method of calculating the two mean proportionals is that it is simple to effect using just a compass and a straight edge, though there is something of a trade-off in the lack of accuracy, as the fact that it involves some estimation precludes it from being considered a mathematical proof.

We might contrast this with the other approaches that have been taken to calculating a solution to the problem. Archytas' method, ostensibly criticised by Plato for its mechanical nature (see chapter 3) is far more difficult to utilise in any practical context. This method involves finding the intersection, in three dimensional space, of the surfaces of a right cone, a cylinder and a torus with an inner product of zero,[15] something that was not easily or quickly done. This was only one of a number of proposed solutions to this problem, though the only other that had potentially the same ease of use as that found in the mechanical texts is one attributed to Plato by Eutocius which uses a tool, similar to a steel square except with an additional adjustable arm forming a U shape, to generate the mean proportionals for a given shape.[16]

Why then is this particular mathematical problem included in such a significant number of the extant sources on mechanics? It is easy to appreciate how the ability to increase and decrease the sized of a solid in a fixed ratio is useful, but the reason that this was of such singular importance for the field of mechanics is not as immediately obvious to the modern audience. Philon recounts the reason near the beginning of the *Belopoieca*:

> *ἐπεὶ φὰρ τῶν ἀρχαίων τινὲς ηὕρισκον στοιχεῖον ὑπάρχον καὶ ἀρχὴν καὶ μέτρον τῆς τῶν ὀργάνων κατασκευῆς τὴν τοῦ τρήματος διάμετρον ταύτην δ' ἔδει μὴ ἀπὸ τύχης μηδὲ εἰκῇ λαμβάνεσθαι, μεθόδῳ δέ*

[15]See T. Heath 1921, pp. 246-247 and Huffman 2005, pp. 349-360
[16]See Knorr 1986, pp. 58-60

> *τινι ἑστηκυίᾳ καὶ ἐπὶ πάντων τῶν μεγεθῶν δυναμένῃ τὸ ἀνὰ λόγον ὁμοίως ποιεῖν.*[17]

The point at which it was discovered that the size of the hole (*τό τρῆμα*) in which the spring of the torsion engine was mounted was the determining factor in the efficacy of these pieces of artillery is not known, but this became one of the key principles of artillery construction. For example, it was known that a stone thrower with holes with a diameter of eleven dactyls could effectively throw a missile weighing ten minae.[18] The discovery of these guiding principles of artillery construction is stated by Philon and Heron as having occurred through experimentation. Given that the size of this component is the determining factor in the success of one of these machines, this was the part by which the rest of the machine was scaled. This allowed for a process of experimental design, where scale models of artillery could be built and tested, and those designs which were the most successful could then be scaled up to full size. By first scaling up the size of the hole for the torsion engine, using the theorem of the two mean proportionals, the rest of the machine could be scaled up in the same proportions as this component.

Given that we have already seen the importance that artillery construction had in the field of ancient mechanics, it is not at all surprising that this particular theorem that is so important for the effective design of artillery is included in these mechanical texts. That all three texts mentioned, that is, Philon's *Belopoeica*, Philon's *Belopoeica* and Pappus' *Synagoge*, all describe the same method of determining the two mean proportionals is, I think, easily explicable. The simplicity of this particular technique makes it a very practical solution for the practising ancient mechanician, given that it allows a suitably accurate value to be calculated from real world measurements without much

[17] Philo *Bel.* 50.14-17

[18] Philo *Bel.* 51 The mina was a measure of weight varying between around 430g and 654g, see Pauly *Mina*

hassle. This in particular would have been the defining factor in the codification of this method in works on mechanics. The practical application of this particular theory was far more important than mathematical correctness of the results, as the resulting measurements would certainly be good enough for this purpose, and could be quickly and easily calculated there and then.

7 The Simple Machines

The simple machines are a collection of mechanical devices that, on their most basic level, change the direction, or the magnitude of a force applied to them. The nature of the simple machines is ostensibly such that their working cannot be broken down into simpler machines, they serve as usable devices in their own right, but also as a series of building blocks from which other more complex machines can be created.[1]

The simple machines are first introduced as a conceptual collection in Heron's *Mechanica*. While they appear individually in the Pseudo-Aristotelean *Mechanica*, they are primarily utilised in that text as examples of the application of the law of the lever to explain other mechanical devices. The majority of the second book of Heron's *Mechanica* is concerned with the five simple machines, beginning with descriptions of each of the machines and the basics of their construction, before investigating their theoretical underpinnings more fully, dealing with both issues arising from each of the five machines, and undertaking analysis of their working using the Archimedean statical model. They are introduced in the following passage:

> *Πέντε τοίνυν οὐσῶν δυνάμεων δι' ὧν τὸ δοθὲν βάρος τῇ δοθείσῃ βίᾳ κινεῖται, ἀναγκαῖόν ἐστιν τά τε σχήματα αὐτῶν καὶ τὰς χρείας, ἔτι*

[1] For an elegant and straightforward description of the simple machines see Asimov 1966, p. 88, which is far better than those found in mechanical textbooks.

> *δὲ καὶ τὰ ὀνόματα ἐκθέσθαι. ἀποδέδοται δὲ ὑπὸ τοῦ Ἥρωνος καὶ Φίλωνος καὶ διότι αἱ προειρημέναι δυνάμεις εἰς μίαν ἄγονται φύσιν, καίτοι παρὰ πολὺ διαλλάσσουσαι τοῖς σχήμασιν. ὀνόματα μὲν οὖν ἐστιν τάδε ἄξων ἐν περιτροχίῳ, μοχλός, πολύσπαστον, σφὴν καὶ πρὸς τούτοις ὁ καλούμενος ἄπειρος κοχλίας.*[2]

The concept being expressed in this passage is a significant one, that these five machines are all governed by the same principles and share the same basic nature. Although not a step away from the Aristotelean position that all these devices could potentially be explained by means of the lever, it does represent a classification of machines that distils their working down to their base principles. The significance of this was stated by Reuleaux: 'In the history of machine-development the simple machines formed the first experiment at a scientific arrangement of existing material; the same train of ideas which governed its phenomena as a whole repeated itself upon a smaller scale in the early attempts at the scientific explanation of what had been empirically determined.'[3]

The idea that the five simple machines represent the reduction of mechanisms to their most elemental components was one that was seized upon during the renaissance, and became the basis of much later work on mechanics. The inclined plane, which will be discussed at length in the following chapter, is not considered a simple machine itself in the ancient texts, but was subsequently included among the simple machines by later scholars. This remained the fundamental means of mechanistic analysis until

[2]This fragment of the Greek text is found in Pappus 1116.7-15, it can however, be considered to be very close to Hero. *Mech.* 2.1 as it closely echoes the extant Arabic translation of the text. Drachmann's translation of the Arabic reads 'Since the powers by which a given burden is moved by a given power are five, we must of necessity present their form and their theory and their names, because these powers are all related to the same natural principle, though they are very different in form; and as for their names they are as follows: the axle going through a wheel (the windlass), the lever, the pulley, the wedge, the screw.' Drachmann 1963b, p. 50

[3]Reuleaux 1876, p. 282

the seminal work of Franz Reuleaux in the late nineteenth century in developing the field of kinematics.[4]

We must also consider what purpose the listing and categorisation of the simple machines served; Heron is concerned with combining them into more complex mechanisms that would serve a useful purpose in lifting weights. What is notable about the presentation of the five simple machines in this text is that there is not an assessment of the relative merits or uses of these machines with regards to the situations in which they are useful or applicable. It does seem however that these five devices are included here can easily be appreciated as being machines, but also are not reductions of the concept to such a point that they do not serve a practical purpose.

I will proceed through some of these simple machines, examining the descriptions of the devices found in the extant Greek text, and the theoretical underpinnings described in the Arabic text. I will not examine the lever as it appears in Heron, because any exploration of this device can add little to what we have already seen in the earlier chapters discussing Aristotelean and Archimedean mechanics.

7.1 The Wheel and Axle

The wheel and axle, also known as the windlass when listed with the other simple machines, is the first of the machines mentioned in Herons list. Before proceeding to the discussion of the use of the wheel and axle, Heron begins with the construction of the machine, recounting a generic method of construction and describing the general form of the wheel and axle assembly.

[4] Reuleaux sees the three important simple machines as being the lever, the inclined plane and the screw, which he categorises as being representative of three lower-pair kinematic linkages (*R*-type (revolute), *P*-type (prismatic) and *S*-type (spherical) respectively), with the pulley representative of a higher pair linkage (*R,T*).

> *Ὁ μὲν οὖν ἄξων ὁ ἐν τῷ περιτροχίῳ κατασκευάζεται οὕτως. ξύλον δεῖ λαβεῖν εὔτονον τετράγωνον καθάπερ δοκίδα καὶ τούτου τὰ ἄκρα σιμώσαντα στρογγύλα ποιῆσαι καὶ χοινικίδας περιθεῖναι χαλκᾶς συναραρυίας τῷ ἄξονι, ὥστε ἐμβληθείσας αὐτὰς εἰς τρήματα στρογγύλα ἐν ἀκινήτῳ τινὶ πήγματι εὐλύτως στρέφεσθαι, τῶν τρημάτων τριβεῖς χαλκοῦς ἐχόντων ὑποκειμένους ταῖς χοινικίσι. καλεῖται δὲ τὸ εἰρημένον ξύλον ἄξων. περὶ δὲ μέσον τὸν ἄξονα περιτίθεται τύμπανον ἔχον τρῆμα τετράγωνον ἁρμοστὸν τῷ ἄξονι, ὥστε ἅμα στρέφεσθαι τόν τε ἄξονα καὶ τὸ περιτρόχιον.*[5]

The use of the wheel and axle is obviously presented with practical applications in mind, and the mention of the purpose of the machine as being *μεγάλα βάρη κινεῖν ἐλάσσονι βίᾳ*, is an obvious indicator of this. It is notable, as we will see, that Heron avoids making any mention of specific uses of the machines, or any assessment of the relative advantages and disadvantages of the five machines for the purpose of lifting weights, for which they are obviously intended.

There is not a distinct theoretical explanation of the working of the wheel and axle in chapters 7–19 of the Arabic text, as these begin, naturally enough, with the theory of the lever. There is however discussion of the relationship between the wheel and axle sizes in chapter 22, which discusses the use of a series of wheels and axles to effect a transmission. Although the principles discussed here also deal with a gear chain, the nature of the wheel and axle assembly is such that it is always going to deal with the ratio between the point at which the force is being applied and the point at which the force is being expressed. The general conception of the wheel and axle is one that seems to be subsumed in Heron's *Mechanica* into this more general idea surrounding gear trains,

[5] Hero *Mech.* 2.1

that is utilised to great effect in the *barulkos*.

7.2 The Wedge

Of all the mechanical devices explored in ancient mechanical texts, the wedge is the machine which has been manufactured for the longest period of time, and thus the earliest machine for which we have evidence. Initially utilised in the form of stone choppers and biface tools from around 1.9 million years ago[6], the wedge is the machine at work in all bladed cutting and splitting tools. The study of these common tools does not seemed to have formed a significant portion of the work on mechanics in the ancient world, but analysis of the wedge in various forms does appear in a few of the extant mechanical texts.

As discussed about, the wedge (*ὁ σφήν*) was included by Heron of Alexandria in his list of the five simple machines.[7] The wedge is not however strictly irreducible, as it is a compound machine consisting of two inclined planes.[8] The wedge is generally triangular in cross-section, with two the faces functioning as inclined planes and a third as the point of application of force. Force applied to to the wedge will be transformed into forces perpendicular to the angle of the two inclined planes. For example, in the case of a splitting wedge, a vertical force applied to the wedge will be transformed into lateral force, splitting the wood. Besides cutting and splitting, the wedge is also used to lift weights or hold objects in place.

The wedge forms part of the investigation carried out by the author of the Aristotelean *Mechanica*, where is is analysed using the law of the lever established earlier in the text:

[6]See Leakey 1971, p. 258 for date, 262-75 for tools.
[7]Pappus 1116, Hero. *Mech.* 2.1
[8]Asimov 1966, p. 88

> *Διὰ τί τῷ σφηνὶ ὄντι μικρῷ μεγάλα βάρη διίσταται καὶ μεγέθη σωμάτων, καὶ θλῖψις ἰσχυρὰ γίνεται; ἢ διότι ὁ σφὴν δύο μοχλοί εἰσιν ἐναντίοι ἀλλήλοις, ἔχει δὲ ἑκά τερος τὸ μὲν βάρος τὸ δὲ ὑπομόχλιον, ὃ καὶ ἀνασπᾷ ἢ πιέζει. ἔτι δὲ ἡ τῆς πληγῆς φορὰ τὸ βάρος, ὃ τύπτει καὶ κινεῖ, ποιεῖ μέγα καὶ διὰ τὸ κινούμενον κινεῖν τῇ ταχυτῆτι ἰσχύει ἔτι πλέον. μικρῷ δὲ ὄντι μεγάλαι δυνάμεις ἀκολουθοῦσι διὸ λανθάνει κινῶν παρὰ τὴν ἀξίαν τοῦ μεγέ θους. ἔστω σφὴν ἐφ' ᾧ* ΑΒΓ, *τὸ δὲ σφηνούμενον* ΔΕΗΖ. *μοχλὸς δὴ γίνεται ἡ* ΑΒ, *βάρος δὲ τὸ τοῦ* Β *κάτωθεν, ὑπομόχλιον δὲ τὸ* ΖΔ. *ἐναντίος δὲ τούτῳ μοχλὸς τὸ* ΒΓ. *ἡ δὲ* ΑΓ *κοπτομένη ἑκατέρᾳ τούτων χρῆται μοχλῷ ἀνα σπᾷ γὰρ τὸ* Β.[9]

The author continues to adhere to the analytical model that is developed in the text, despite the difficulty in using the lever to explain the forces present in the wedge. The problem in attempting to understand the working of the wedge by means of the lever is that in the conception of the machine that the author puts forward, the two sides of the wedge which are imagined as functioning in the same way as levers cannot move, and cannot be conceived of as moving. This may seem like a minor point, but I don't think that it is merely a failure to understand the nature of the model. The triangular and immutable shape of the wedge is such that the distance between two points on the lines AB and BΓ will not change as force is applied to the wedge. Attempts to understand the wedge by means of the lever fail as it is not a comparison of like with like. In modern terms, that is, when considered as kinematic pairs, the wedge is an example of a prismatic joint, as is the inclined plane, and the lever is an example of a revolute pair.[10] Although

[9] Arist. *Mech.* 853b20-31
[10] Hartenberg and Denavit 1964, pp. 33-34

both pairs operate with a single degree of freedom, they embody two fundamentally different kinds of movement.[11]

The introduction to the wedge that Heron gives as part of his list of five simple machines does not shed much light on his understanding of the machine. It outlines the main applications of the wedge, focusing upon its use in quarrying blocks of stone.[12] He emphasises that the characteristic that makes the wedge useful for this purpose is that force does not need to be constantly applied to the machine for it to exert force.[13] Heron displays understanding of the relationship between the angle of the wedge and the mechanical advantage of the wedge, though this is expressed in terms of the force required to operate the wedge rather than force exerted by the wedge. [14]

As with the other simple machines that Heron lists in his *Mechanica*, the examination of the theory of the wedge, that is, exploration of the cause of the mechanical effect seen in the device,[15] takes place separately, and is extant only in Arabic.

> As for the wedge, the blow must move it during a given time, for there can be no movement without time, and this blow works by a mere touch, which does not stay with the wedge, not even [for] the shortest time. And it is evident to us from this that the wedge moves on after the blow has stopped. And we learn this also in another way: during a certain time after the blow there comes from the wedge noises and splinters from the

[11] It is worth noting that although the law of the lever may be considered the model for calculations of the mechanical advantage of a machine, the comparison discussed here is not the same as this. Mechanical advantage represents a quantification of the effect of the machine, rather than a description of the mode of operation.

[12] Hero.*Mech.* 2.4

[13] Greek

[14] *ὅσῳ δ' ἂν ἡ τοῦ σφηνὸς γωνία ἐλάσσων γίνηται, τοσούτῳ εὐχερέστερον ἐνεργεῖ, τουτέστιν δι' ἐλάσσονος πληγῆς*

[15] "*τίς δέ ἐστιν ἡ αἰτία, δι' ἣν δι' ἑκάστης αὐτῶν μεγάλα βάρη κινεῖται μικρᾷ παντάπασι δυνάμει…*" Hero.*Mech.* 2.7

> splitting by its edge.[16]

Heron goes on to compare the wedge to an arrow flung by a bow, or a stone thrown by hand, in that the duration of the initial impulse is short, but the effect of the impulse continues for some time after it occurs. This analogy is correct, as Drachmann points out,[17] but the rationale for this conclusion betrays that Heron has incorrectly interpreted the behaviour of the wedge that he has observed. The noises and splinters he describes as coming from the wedge after the initial impulse are not evidence of continued movement of the wedge (in the direction of the applied force), but rather the exertion of the force that has been transformed by the wedge.

Heron moves from this statement to attempt a geometrical demonstration of the relationship between the force applied to the wedge, the distance that the wedge will be moved by this force, and the angle of the wedge.

> Let us imagine a wedge whose edge is at the sign A, and let its head be the line DM. And let the blow that moves it be BĞ, and let its distance be AD. And let it be possible to move by a slight blow, and let us take away from the blow BĞ a blow that is the blow BH, and this is less than all known blows. Then I say that the blow BH by itself will drive in a certain part of the wedge. The proof this is that the blow BĞ moves the distance AD, and HĞ moves a distance less than AD, let it move the distance AZ, and then, if the blow BH is added, the distance will be AD, which is moved by the blow BĞ. And thus the blow BH by itself moves the distance DZ.
> ...four wedges, whose edges are at the point A, and their heads are the lines MF, FQ, QR, RD, and each of them is moved by a blow equal to the blow

[16] Drachmann 1963b, p. 72
[17] Drachmann 1963b, p. 72

> BH a distance equal to the line AD, and it is the same if we say that the blow BH drives the whole wedge the distance DZ...
> ...the smaller is the angle of the wedge, the further will the wedge penetrate by a smaller power than the power that drives in the whole wedge.[18]

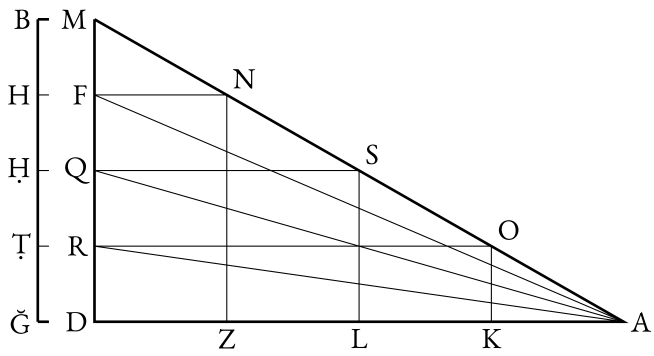

Figure 7.1: The Wedge - Diagram illustrating the theory of the wedge outlined in 2.15, modified from Dra63

Heron goes on at some length establishing the working of the theory, and the above represents only about a quarter of this chapter. He is mostly establishing a set of conditions whereby the relationship between the force applied to the wedge, the angle of the wedge, and the distance the wedge will be moved by this blow is easily quantifiable. This chapter functions as a demonstration of the relationship rather than offering a means of calculating the values that could be derived from the relationship. A significant constraint upon investigation of these issues for an ancient author is that there is an absence of a defined concept of force, and no real means of giving a value to the force that is being applied. Heron's geometrical proof functions within these constraints by representing the value of the force as a whole that has a direct and equal relationship with the other aspects under consideration in the wedge.

While the considerations of his working, namely the distance the wedge is driven by a given force, are different from what we would expect in modern problems related to the wedge, the theory that he espouses is correct. Heron demonstrates an understanding that there is a proportional relationship between the angle of the wedge and the force required to move the wedge, or, in modern terms, a proportional relationship

[18] Drachmann 1963b, pp. 72-3

between the angle of the wedge and its mechanical advantage.

There does not seem to me to be a situation where the understanding of this particular working of the wedge would be anything other than an intellectual concern. I do not believe that there would have been a situation where the architect of a project would be called upon to calculate the exact size of wedge needed for a task. The particular size and shape of a wedge used would be based on experience, rather than any rational approach utilising the mathematical techniques found in these texts. As much as there are issues with calculations involving the wedge, it is this lack of necessity that prevents extrapolation of methods of deriving values from the theories found in the Aristotelean *Mechanica* and Heron's *Mechanica*. These texts present reasons for the working of the wedge rather than methods to calculate real-world examples.

8 The Inclined Plane

It may appear on first inspection that the inclined plane is a concept so basic and intuitively understood that it does not require any explanation. It seems obvious to us that the longer, shallower route up a hill will be easier to climb than the shorter, steeper route. Conceptually the inclined plane would appear to be a degree simpler again than other simple machines, even although analysis would reveal that it is performing the same basic function. It is the realisation that the inclined plane constitutes a machine, and the appearance of analysis of the inclined plane, that perhaps most succinctly demonstrates the arrival of the mathematisation of the mechanics and the deepening understanding of the field.

The most fundamental definition of the inclined plane is *a plane which lies at an angle relative to the horizontal*. That is, a plane at an angle greater than 0° and less than 90° above the horizontal, with a body on a plane at 0° being at rest, and a body upon a plane at 90° being in free fall. Between these two extremes the inclined plane will fulfil the basic function of a machine by converting vertical force to horizontal force, and vice versa. The physical implementation of this machine is typically a ramp between higher and lower levels or a gradient cut into an elevation to allow for the raising and lowering of loads.

There is no mention of the inclined plane in the Aristotelean *Mechanica*, nor is it a topic that Archimedes devotes any time to analysis of. The inclined plane makes its first appearance in Heron's *Mechanica*, where its inclusion is characterised by Drachmann as representing a 'theory of mechanics in general'.[1] While this description is perhaps a little more prescriptive than is warranted, these chapters do constitute an approach to the inclined plane that is more expansive than one that deals solely with the use of the inclined plane as a simple machine. The inclined plane is really a specific subset of circumstances that can be subsumed in more general considerations of planes and forces, and, in ancient terms, the moving of weights.

Heron begins exploring the issue by attempting to describe the 'force' or 'power'[2] required to move a weight placed upon a level plane.

> "So let us explain that burdens placed in the way described are moved by a power smaller than any known power, and we shall explain why this is not evident in practice. Let us imagine a burden lying flat, and let it be regular, smooth and let its parts be coherent with each other. And let the surface on which the burden lies be able to be inclined to both sides, I mean to the right and the left. And let it be inclined first towards the right. Then it is evident to us that the supposed burden will incline towards the right side, because the nature of the burdens is to move downwards, if nothing holds them and hinders them from movement; and again if the inclined side is lifted to a horizontal position and comes into equilibrium, the burden will come to rest in this position. ... And the burden that is ready to go to every side, how can it fail to need to move it a very small power of the size of

[1] Drachmann 1963b, p. 46

[2] As these chapters are extant only in Arabic, these are the translations of the *Mechanica* found in Cohen and Drabkin and Drachmann, which should not be considered to directly reflect either the Greek term, nor to correspond exactly with a modern conception of 'force'.

> the power that will incline it? And so the burden in moved by any small power."[3]

The plane thus conceptualised may constitute the earliest expression of the idea that the force required to move a given weight upon a horizontal plane is minimal. I would specifically avoid any interpretation of this passage as demonstrating a conception of the idea of a 'frictionless plane', as we would expect in modern conceptual models. Russo, through misrepresentation of this quote and explicatory comment, would have us believe that this statement of Heron's represents an appreciable understanding of friction which would contribute to a heretofore unknown ancient understanding of inertia.[4] Heron's discussion of the problem is rooted in empiricism; while setting out the parameters of his hypothetical plane in the passage above, his conception of the circumstances is obviously rooted in an idealised set of those circumstances that would reduce friction in reality. If his conception was of a frictionless plane, or perhaps if he had fully understood the implications of a frictionless plane, then he would have concluded that the inclination of the plane at any angle greater 0° would cause the weight to move.[5] I think that it is safe to say, despite the problems with the transmission of this text, that Heron does not here quite make the leap to a full rationalisation of the problem, indeed, it is not until an early work of Galileo, the *De motu*, that we find a frictionless plane as the basis for a mathematical approach to the inclined plane.[6]

[3]Drachmann 1963b, p. 46 I have preferred this translation to the one found in Cohen and Drabkin 1948, as it is directly from the Arabic, rather than adapted from the German translation of Nix and W. Schmidt 1976.

[4]Russo 2004, p. 289 and also 352, where he quotes this statement as, "We demonstrate that a weight in this situation [that is, on a horizontal, frictionless plane] can be moved by a force less than any given force." In no other translation from the Arabic is this a complete sentence, nor do we find an editorial comment that the environment is 'frictionless' cf. Nix and W. Schmidt 1976, p. 54, Cohen and Drabkin 1948, p. 197 & Drachmann 1963b, p. 46.

[5]See Cohen and Drabkin 1948, 198 n.1 for further explanation of Heron's model.

[6]Drake 1973, p. 293, the passage he is referencing reads: *'Quae omnia si ita disposita fuerint, quod-*

In the following chapter Heron goes on to expand further upon the role that friction plays in maintaining a weight on the plane in position. He uses water as an example of a substance that is apparently not affected by friction as 'its parts are not strongly cohesive but are easily separable.'[7] He describes the surfaces of solids as being like 'teeth' which mesh with one another and require a great power to move over one another. He once again demonstrates his interest in the practical solutions to the problem of friction by describing a variety of methods that are used to reduce its effects. It is, however, obvious that the presence, or absence, of friction does not play a part in any analytical model created by Heron for dealing with the problem of the inclined plane.

Heron is again specifically concerned with the force required to move the weight on the inclined plane rather than other associated problems.[8] This is of course a statical problem, and Heron approaches it as such, first of all establishing the force required to raise a weight vertically. This is done by imagining two equal weights suspended upon either end of a rope which has been passed over a pulley. Heron states that they will remain in a state of equilibrium, or rather, specifically that neither of the weights will 'overcome' the other, nor will the pulley 'incline to either side'.[9] This state will change if one of the two weights is added to, causing the other weight to be drawn upwards. Heron presents this as being proof of the idea that a weight requires a power equal to it to raise it above its current position.

Having reiterated this conceptual cornerstone of the field of statics, Heron then goes on to examine the case of a cylinder on the inclined plane as a proof of the afore-

---

cunque mobile super planum horizonti aequidistans a minima vi movebitur, imo et a vi minori quam quaevis alia vis.' Galilei 1890, p. 299

[7] Cohen and Drabkin 1948, p. 198

[8] Such as either the mechanical advantage of the plane or the motion of a body down a inclined plane. The latter famously being the central point of investigation in the writings of Galileo.

[9] Drachmann 1963b, p. 47

mentioned statement on the required power to raise a weight:

> So that our explanation may be proved to be true, we will explain it for a given cylinder. Because the cylinder does not touch the ground with a great part of itself, it is its nature to roll downwards. Now let us imagine a plane going through the line that touches the surface and at right angles to that surface, and it is evident to us that the plane will pass through the axis of the cylinder and divide it into two halves, because if there is a circle and a line touches it and a line is drawn from the point of touch at right angles, then this line will go through the centre of the circle; and we will also draw through this line, I mean the line on the cylinder, another plane at right angles to the horizon, and this will not be the plane first drawn, and it will divide the cylinder into two unequal parts, of which the smaller will be towards the upper part, and the greater towards the lower part, and the greater will overcome the smaller part since it is greater than the other, and the cylinder will roll. But if we imagine on the other side of the intersecting plane that is at right angles to the horizon that there is taken away from the greater part as much as its excess over the smaller part, then the two parts will be in equilibrium, and the whole burden will be at rest on the line that touches the ground and it will not incline to either side, I mean neither upwards nor downwards. So we need a power equivalent to this to withstand it, and if a small increase is added to this power, it will overcome the burden.[10]

This specifically geometrical demonstration of the theory represents an almost com-

[10] Cohen and Drabkin 1948, p. 48

pletely mathematical approach to the issue. It is divorced from the actual business of calculating the force required to maintain a given body at rest upon a given plane, and thus the force required to raise the body to a higher point on the plane. This demonstration relies upon deriving a lenticular form from the portion of the circular cross section of the cylinder that lies directly above the point of intersection of the circle and the plane (See fig.8.1). The idea being that this lenticular form represents the portion of the cylinder that could considered to be in a state of equilibrium resting upon this point, and so can be discounted from any consideration of the force required to maintain the body at rest. The force required to maintain the position of the cylinder is, therefore, derived from the remaining area of the circular cross section once the lenticular form has been subtracted.[11]

While we find later in Heron's *Mechanica* the first mention of the simple machines as a conceptual grouping, he deals with it as he would a cylinder upon a horizontal plane. That is, he only deals with the vertical force acting upon the weight, rather than the horizontal force also.

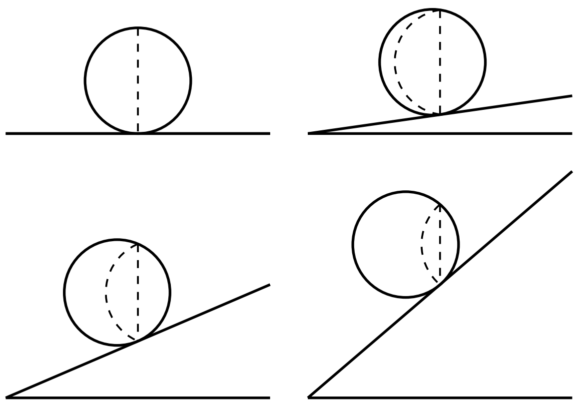

Figure 8.1: The Inclined Plane - Heron's geometrical demonstration from Book 1.23 of the *Mechanica*.

The other extant work which deals with the inclined plane is Book 8 of Pappus' *Collection*, where it is cited in the introduction as one of the most important mechanical theorems.[12] Pappus proceeds from a fundamentally different premise than Heron, in that his initial definition of the problem is that Βάρους δοθέντος ὑπὸ δοθείσης ἀγομένου

[11]This area can be found via the formula $A = \pi r^2 - r^2(\theta - \sin\theta)$ with area *A*, radius *r*, and central angle θ. For a complete formulation of how to derive the required force in modern terms from Heron's theory see Cohen and Drabkin 1948, 200 n.1, though this is an explicatory extrapolation from the basis of Heron's theory rather than reflecting the content of the text.

[12]Pappus 1028, cross-reference

δυνάμεως ἐν τῷ παρὰ τὸν ὁρίζοντα ἐπιπέδῳ...[13] That is, in direct contrast to Heron, he assumes that the force required to move a body on a plane is directly proportional to the weight of the body. This conception of the issue is not an innovation on Pappus' part. Heron acknowledged this viewpoint in introducing his theory of the inclined plane, saying 'There are those that think that burdens lying flat are moved by an equal power [only], wherein they hold wrong opinions.'[14] Pappus is therefore making an apparently retrograde step in using this as the basis of his analysis of the inclined plane, and has actively decided to adopt this approach rather than Heron's. Pappus divides his discussion of the inclined plane into two components, a mathematical demonstration of the issue, then a demonstration of the application of the theory to a specific example. Cuomo sees the mechanics of Pappus as operating 'within a mathematical universe of reference' whereas Heron's mechanics is 'seen against a physical background'.[15] Although Cuomo is not strictly setting the two authors up as having contrasting paradigms for analysis, it is important to recognise that they are not in opposition. Both are attempting a rationalisation of the problem, although both are taking slightly different approaches. When Heron attempts to remove friction as far as possible from his consideration of the problem, it involves a description that calls upon experiential knowledge to fully explain the details of the scenario. Pappus instead subsumes friction into a more generalised idea of a 'given force', which does not play a specific part in his model, but is allowed for in a general formula for the inclined plane that can be applied to real-world scenarios.

Pappus' initial mathematical exploration of the inclined plane not only begins from

[13] Papp. 1054 'A given force is needed to draw a given weight along a horizontal plane.' Cohen and Drabkin 1948, p. 194

[14] Drachmann 1963b, p. 46

[15] Cuomo 2000, p. 116

a different basis than Heron, but also proceeds to use a very different, but still classically statical, method to determine the force required to raise the weight.

> *ι* . *Βάρους δοθέντος ὑπὸ δοθείσης ἀγομένου δυνάμεως ἐν τῷ παρὰ τὸν ὁρίζοντα ἐπιπέδῳ καὶ ἑτέρου ἐπιπέδου κεκλιμένου πρὸς τὸ ὑποκείμενον δοθεῖσαν γωνίαν ὑποτιθέντος, εὑρεῖν τὴν δύναμιν ὑφ' ὅσης ἀχθήσεται τὸ βάρος ἐν τῷ κεκλιμένῳ ἐπιπέδῳ. Ἔστω τὸ μὲν διὰ τῆς* ΜΝ *εὐθείας ἐπίπεδον τὸ ὑποκείμενον, τὸ δὲ διὰ τῆς* ΜΚ *κεκλιμένον πρὸς αὐτὸ γωνίαν δοθεῖσαν τὴν ὑπὸ* ΚΜΝ *ὑποτιθέν, βάρος δέ τι τὸ* Α *κινείσθω ὑπὸ δυνάμεως τῆς* Γ *ἐπὶ τοῦ ὑποκειμένου ἐπιπέδου, καὶ νοείσθω τῷ* Α *ἰσοβαρὴς σφαῖρα ἡ περὶ κέντρον τὸ* Ε, *καὶ κείσθω ἐπὶ τοῦ διὰ τῶν* Μ Κ *ἐπιπέδου ψαύουσα αὐτοῦ κατὰ τὸ* Λ *σημεῖον, ὡς ἔστιν σφαιρικῶν γ θεωρήματι· ἡ ἄρα* ΕΛ *ἐπιζευχθεῖσα κάθετος ἔσται ἐπὶ τὸ ἐπίπεδον (καὶ τοῦτο γὰρ δέδεικται θεωρήματι δ σφαιρικῶν), ὥστε καὶ πρὸς τὴν* ΚΜ *κάθετός ἐστιν ἡ* ΕΛ. *ἐκβεβλήσθω τὸ διὰ τῶν* ΚΜ ΕΛ *ἐπίπεδον καὶ ποιείτω τομὴν ἐν τῇ σφαίρᾳ κύκλον τὸν* ΛΗΞ, *καὶ ἤχθω διὰ τοῦ* Ε *κέντρου τῇ* ΜΝ *παράλληλος ἡ* ΕΘ, *καὶ κάθετος ἐπ' αὐτὴν ἀπὸ τοῦ* Λ *ἡ* ΛΖ. *ἐπεὶ οὖν δοθεῖσά ἐστιν ἡ ὑπὸ* ΕΘΛ *γωνία (ἴση γάρ ἐστιν τῇ ὑπὸ* ΚΜΝ *δοθείσῃ [ὀξείᾳ] γωνίᾳ), δοθεῖσα ἄρα καὶ ἡ ὑπὸ* ΕΛΖ *ἴση οὖσα τῇ ὑπὸ* ΕΘΛ *(ἰσογώνιον γάρ ἐστιν τὸ* ΕΘΛ *τῷ* ΕΛΖ *τριγώνῳ)· δοθὲν ἄρα τὸ* ΕΛΖ *τρίγωνον τῷ εἴδει· λόγος ἄρα τῆς* ΕΛ, *τουτέστιν τῆς* ΕΗ, *πρὸς* ΕΖ *δοθείς· καὶ λοιπῆς ἄρα τῆς* ΖΗ *πρὸς* ΕΖ *λόγος ἐστὶν δοθείς. πεποιήσθω οὖν ὡς ἡ* ΗΖ *πρὸς* ΖΕ, *οὕτως τὸ μὲν* Α *βάρος πρὸς τὸ* Β, *ἡ δὲ* Γ *δύναμις πρὸς τὴν* Δ. *καὶ ἔστιν τοῦ* Α *δύναμις ἡ* Γ· *καὶ τοῦ* Β *ἄρα δύναμις ἐν τῷ αὐτῷ ἐπιπέδῳ ἔσται ἡ* Δ. *καὶ ἐπεί ἐστιν ὡς ἡ* ΗΖ *εὐθεῖα πρὸς τὴν* ΖΕ,

> *οὕτως τὸ* Α *βάρος πρὸς τὸ* Β, *ἂν τεθῇ τὰ* Α Β *βάρη περὶ κέντρα τὰ* Ε Η, *ἰσορροπήσει ἀρτώμενα ἀπὸ τοῦ* Ζ *σημείου* [*ἢ ἐπὶ ὑποθέματος κείμενα τοῦ* ΛΖ *ὀρθοῦ πρὸς τὸν ὁρίζοντα*]. *κεῖται δὲ τὸ* Α *βάρος περὶ κέντρον τὸ* Ε (*ἀντ' αὐτοῦ γὰρ ἡ σφαῖρα*)· *τεθὲν ἄρα τὸ* Β *βάρος περὶ κέντρον τὸ* Η *ἰσορροπήσει τῇ σφαίρᾳ, ὥστε μὴ καταφέρεσθαι τὴν σφαῖραν διὰ τὴν κλίσιν τοῦ ἐπιπέδου, ἀλλ' ἐφεστάναι ἀρρεπῆ, ὡς εἰ καὶ ἐπὶ τοῦ ὑποκειμένου ἑστῶσα ἐτύγχανεν. ἐκινεῖτο δὲ ἐν τῷ ὑποκειμένῳ ἐπιπέδῳ ὑπὸ τῆς* Γ *δυνάμεως· κινηθήσεται ἄρα ἐν τῷ κεκλιμένῳ ἐπιπέδῳ πρὸς συναμφοτέρου τῆς τε* Γ *δυνάμεως καὶ τῆς τοῦ* Β *βάρους, τουτέστιν τῆς* Δ *δυνάμεως. καὶ ἔστιν δοθεῖσα ἡ* Δ *δύναμις.*[16]

The approach adopted here by Pappus to derive the force required to move a given weight, where that weight is movable upon a horizontal plane with a given force, up a given inclined plane, is to construct a geometrical balance. This balance can be used

[16]Papp. 1054-1056 'It is required to the find the force needed to draw the weight up another plane inclined at a given angle to the horizontal plane. Let the horizontal plane pass through MN, and let the plane inclined to the horizontal at the given angle, KMN, pass through MK. Let A be the weight and C the force required to move it over the horizontal plane. Consider a sphere with center E and weight equal to that of A. Place this sphere on the inclined plane passing through M and K. The sphere will be tangent to the plane at L, as is shown in the third theorem of the *Spherics*. EL will therefore be perpendicular to the plane (for this is also shown in the *Spherics*, Theorem IV), and also to KM. Pass a plane through KM and EL cutting the sphere in circle LHX. Draw ET through center E parallel to MN, and draw LZ, from L, perpendicular to ET. Now since the angle ETL is given (for it is equal to the given angle KMN), the angle ELZ is also given, for the angle ELZ is equal to the angle ETL (since triangles ETL and ELZ are similar). Therefore the triangle ELZ is given in form. Hence the ratio EL:EZ, that is EH:EZ, is known, as is also(EH - EZ):EZ, that is ZH:EZ. Let weight A be to weight B and force C to force D, as HZ is to ZE. Now C is the force required to move A. Therefore the force required to move B on the same plane will be D. Since weight A : weight B is equal to HZ:ZE it follows that if E and H are the centers of gravity of weights A and B, respectively, the weights will be in equilibrium if balanced at point Z. But weight A has its center of gravity at E (for the sphere represents A). Therefore, if weight B is placed so that its center is at H, it will so balance the sphere that the latter will not move down because of the slope of the plane, but will remain unmoved, as if it were on the horizontal plane. But weight A required force C to move it in the horizontal plane. Therefore, to be moved up the inclined plane it will require a force which is the sum of the forces C and D, where D is the force required to move the weight B in the horizontal plane. Force D, moreover, is given.' Cohen and Drabkin 1948, pp. 194-6

with the law of the lever and the given values to calculate a motive force that is proportional to the angle of inclination of the plane. The fulcrum of the conceptual balance is above the point intersection of the sphere and the plane, and the beam of this balance extends horizontally from the centre of gravity of the sphere, representing the weight to be moved, to the point of intersection with the edge of the sphere, on the opposite side of the fulcrum. Since the length of the balance beam on either side of the fulcrum can be found, and the beam is considered to be in a state of equilibrium, then the ratio of these two lengths will be the same as the ratio of the two weights, and thus the ratio of the forces required to move the weights. Therefore, the motive force that is derived from this calculation is equal to the sum of the force required to maintain the position of the weight on the inclined plane and the force required to move the weight upon a horizontal plane. The problem has been stated, in terms that are understood, as a statical problem. Pappus has created a construct whereby force can be calculated by means of a known methodology rather than developing a new one that explores the problem as one that is distinct from other mechanical problems. However, Pappus' concern is not purely theoretical, and he goes on to provide an example (*παραδείγμα*) of an application of this theory in what would seem to be a real-world scenario.

> Ἡ *μὲν οὖν γεωμετρικὴ τοῦ προβλήματος ἀνάλυσις ὑποδέδεικται, ἵνα δὲ καὶ ἐπὶ παραδείγματος ποιησώμεθα τήν* (1058.) *τε κατασκευὴν καὶ τὴν ἀπόδειξιν, ἔστω τὸ μὲν* Α *βάρος ταλάντων, εἰ τύχοι,* ς *ἀγόμενον ἐν τῷ παραλλήλῳ ὁρίζοντι ἐπιπέδῳ ὑπὸ τῆς* Γ *κινούσης δυνάμεως, τουτέστιν οἱ κινοῦντες ἔστωσαν ἄνθρωποι μ , ἡ δὲ ὑπὸ* ΚΜΝ *γωνία, τουτέστιν ἡ ὑπὸ* ΕΘΛ, *διμοίρου ὀρθῆς· καὶ λοιπὴ ἄρα ἡ ὑπὸ* ΖΛΘ *τρίτου ὀρθῆς. καὶ ἔστιν ὀρθὴ ἡ ὑπὸ* ΕΛΘ· *διμοίρου ἄρα καὶ ἡ ὑπὸ* ΕΛΖ· *οἵων ἄρα αἱ δ ὀρθαὶ τξ τοιούτων ξ ἡ ὑπὸ* ΕΛΖ, *καὶ*

τοῦ περιγραφομένου ἄρα περὶ τὸ ΕΖΛ τρίγωνον ὀρθογώνιον κύκλου ἡ μὲν ἐπὶ τῆς ΕΖ περιφέρεια τοιούτων ἔσται ρκ οἵων ὁ κύκλος τξ , αὐτὴ δὲ ἡ ΕΖ τοιούτων ρδ ἔγγιστα οἵων ἡ ΕΛ τοῦ κύκλου διάμετρος ρκ · ταῦτα γὰρ δῆλα ἐκ τοῦ κανόνος τῶν ἐγκυκλίων εὐθειῶν τοῦ κατὰ Πτολεμαῖον [ὄντος] κειμένου ἐν τῷ α τῶν μαθηματικῶν. λόγος ἄρα τῆς ΕΛ, τουτέστιν τῆς ΕΗ, πρὸς ΕΖ, ὃν ρκ πρὸς ρδ · καὶ λοιπῆς ἄρα τῆς ΗΖ πρὸς ΖΕ λόγος ὃν ι πρὸς ρδ . τούτῳ δὲ ὁ αὐτός ἐστιν ὁ τοῦ Α βάρους πρὸς τὸ Β, καὶ τῆς Γ δυνάμεως πρὸς τὴν Δ, καὶ ἔστιν τὸ μὲν Α βάρος ταλάντων ϛ , ἡ δὲ κινοῦσα δύναμις ἀνδρῶν μ · ἔσται ἄρα καὶ τὸ μὲν Β βάρος ταλάντων ατ , ἡ δὲ Δ δύναμις ἀνθρώπων σξ (ὡς γὰρ ι πρὸς ρδ , οὕτως ϛ πρὸς ατ καὶ μ πρὸς σξ)· τοῦ ἄρα Α βάρους ταλάντων ϛ κινουμένου ἐν παραλλήλῳ τῷ ὁρίζοντι ἐπιπέδῳ ὑπὸ τῶν μ ἀνδρῶν, τὸ αὐτὸ βάρος κινηθήσεται ὑπὸ συναμφοτέρων τῶν προειρημένων ἀνθρώπων, τουτέστιν ὑπὸ τ ὅλων, ἐν ἐπιπέδῳ κεκλιμένῳ πρὸς τὸν ὁρίζοντα, τῆς ὑπὸ ΚΜΝ γωνίας διμοίρου ὀρθῆς ὑποκειμένης. [17]

[17]Papp. 1056-1058 'The geometrical solution of the problem has been indicated. However, to set forth the method and proof in a typical case, let weight A be, say, 200 talents, and let the moving force, C, required to draw the weight in a horizontal plane be equal to the force of 40 men. Let the angle KMN (that is, the angle ETL), be ⅔ of a right angle. The angle ZLT is therefore ⅔ of a right angle and, since the angle ELT is a right angle, the angle ELZ is also ⅔ of a right angle. Of the 360 equal parts into which four right angles are divided the angle ELZ contains 60. Therefore, if a circle be circumscribed about the right angled triangle EZL, the arc subtended by chord EZ will contain 120 of the 360 parts of the circumference, and chord EZ will itself be almost 104/120 of EL, the diameter of that circle. This is clear from the table of chords in the first book of the Mathematica of Ptolemy. Therefore EL/EZ = EH/EZ = 120/104' and HZ/ZE = 16/104 = weight A/weight B = force C/force D. But weight A is 200 talents, and the moving force, C, 40 men. Therefore weight B will be 1300 talents, and moving force, D, 260 men (for 16:104 = 200:1300 = 40:260). Hence if 40 men are required to move a weight, A, of 200 talents on a plane parallel to the horizon, it follows that the sum of 40 and 260, that is, 300 men, will be required to move the same weight up a plane inclined to the horizon at angle KMN, ⅔ of a right angle.' Cohen and Drabkin 1948, p. 196

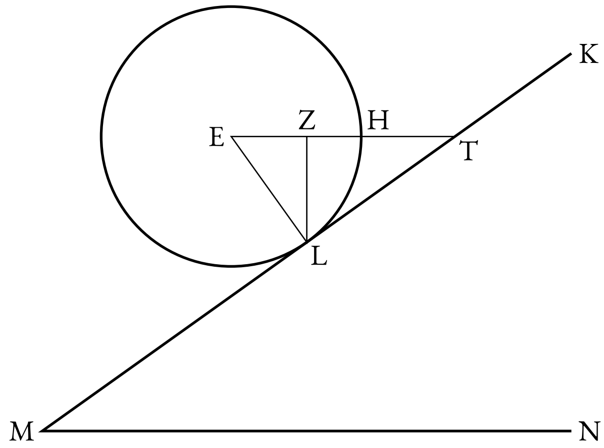

Figure 8.2: The Inclined Plane - Pappus' geometrical demonstration from Book 8 of the *Collection*.

This example provides a contrast with the rather abstract and mathematical approach found in Heron's *Mechanica*. If it were used for the same purpose it would rely upon finding the area of a lune to derive the force required to maintain the position of the weight upon the inclined plane. On the other hand, the method described by Pappus provides a means of calculating the force required to draw a given weight that is relatively feasible and usable for the sorts of calculations he describes in this demonstration. This is probably more of a reflection of the concerns of the two authors than anything else. Heron seems to be specifically exploring this underlying aspect of mechanical theory, and so it is not necessary for the passage to provide a practical application of the ability to calculate the force required to hold the weight in place on the slope. Heron is once again pointing out that he is correcting an error that is prevalant in earlier authors, the correcting of such an error does not necessarily require a corrected method of applying the theory. Pappus on the other hand is later in the tradition, and the application of the theory that he describes may represent the exact tradition that Heron was criticising. However, if taken from another source in the earlier mechanical tradition, it could be that its utility won out over the theory expressed.

9 Conclusion

In analysing theoretical mechanics in the preceding chapters an overarching, etic view of the field has been adopted. I've focused on the four main texts that deal with the elements of mechanics that we would expect to be considered theoretical in a modern text dealing with the field. However, there is a danger that when selecting texts in this manner, with the aim of investigating the status of a specific subject in the ancient world, that the selection will reflect an anachronistic notion of that subject that closely aligns with our own preconceived conception of the field rather than an emic reading of the materials. The main concern of this thesis has been to examine whether or not theoretical mechanics exists as a distinct aspect of mechanics in the ancient world, and while the texts that have been examined in the preceding chapters; the Pseudo-Aristotelean *Mechanica*, Archimedes' *De Planorum Aequilibriis*, Heron's *Mechanica*, and the final book of Pappus' *Synagoge*, have been selected with that aim, it does not follow that these particular texts are themselves separate and purely theoretical from an ancient viewpoint.

The commonality of these four texts is their dealings with the elements of mechanics that are, at least to some degree, divorced from the practicalities of the construction of actual mechanical devices. Heron's *Mechanica* is the closest of these texts to actually describing the construction of devices, and even then, the theoretical elements exist alongside these practical descriptions.[1] All of these texts discuss problems related to mechanical advantage to some degree. These texts can be contrasted with the majority of other ancient texts on mechanics, which deal with descriptions of mechanical devices and guides to their construction. The question that then arises is; Are these texts dealing with ancient, rather than modern, theoretical mechanics?

To answer this, we must first have some kind of idea about what ancient theoretical mechanics is. There is perhaps no one strict definition that we could apply to the entire chronological range of the texts that have been examined, but there is not the need for this kind of analysis. The concept of theory in contrast with practice is one that is receptive to a range of different practices that are not strictly limited to grand unifying theories that cover all aspects of a field. While we may wish to accommodate such a concept to fit with the rhetoric surrounding modern science, we find even there that theory does not need to be universal to be useful. Modern physics can encompass classical, quantum and relativistic mechanics, which are all applicable in their own specific domains. We should not be thinking so much of 'theoretical mechanics' in the ancient world, but rather theory which applies to mechanics in the ancient world.

What has become clear from these texts is that when examining theory in ancient mechanical texts is that we must be willing to accommodate more than our own relatively narrow definition of theoretical mechanics. The distinction that we find in Pappus between theoretical and practical aspects of mechanics allows for the inclusion of a number of different fields under the 'theoretical' umbrella, that may not seem to immediately belong.

All of these texts discuss problems related to mechanical advantage. The *Mechan-*

[1]Although, it is debatable whether or not the description of the *barulkos* should even be included as part of the *Mechanica*: See Drachmann 1963b, p. 22

ica of the ancient world is not a work that deals with a wide range of mechanical topics, but one that is fairly narrowly focused on statics, rather than covering any other related fields that we may expect in a modern work such as dynamics and kinematics. Common to all these works is the tangibility of the subject matter. While each author is either expounding a new theoretical basis for a particular mechanical phenomena, or following one that had previously been described. The mechanical effects that are being investigated are demonstrable, and so could be observed and all would be familiar to the reader. That is, there is not extrapolation from the grounds that are established in these theories, mechanics in the ancient world is a process of explanation rather than discovery. Mathematics is the mode of explanation being utilised, both due to the light that the it can shed upon this material, as well as a rhetorical tool that can be used to convince the audience of the validity of these particular explanations. The real linking factor between the four texts is not that they explore the basis for mechanical phenomena and the workings of mechanical devices, but rather the utilisation in all of these texts of an analytical paradigm which involves the application of a mathematical deductive approach to mechanics.

This is a field of human knowledge that does not exist in isolation. The unifying factor in all the theoretical aspects of mechanics that have been examined is that they have their roots in the practical applications which they aid. Ancient mechanics is a field where theory is driven by application. These authors recognise that they are engaging with a pre-existing tradition where these techniques are applied to these particular problems.

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Bibliography

[1] I. Asimov. *Understanding Physics: Motion, Sound, and Heat*. Vol. 1. London: George Allen & Unwin Ltd, 1966.

[2] J. Barnes. *Aristotle: Posterior Analytics*. Oxford, Clarendon Press: Oxford University Press, 1994.

[3] J. Barnes, ed. *The Complete Works of Aristotle: The Revised Oxford Translation*. Vol. 2. Princeton, New Jersey: Princeton University Press, 1984.

[4] E. Benvenuto. “The Parallelogram of Forces”. In: *Meccanica* 20.2 (1985), pp. 99–109.

[5] S. Berryman. *The Mechanical Hypothesis in Ancient Greek Natural Philosophy*. University Press, Cambridge: Cambridge University Press, 2009.

[6] M. Clagett. *The Science of Mechanics in the Middle Ages*. Oxford University Press, 1959.

[7] M.R. Cohen and I.E. Drabkin. *A Source Book in Greek Science*. New York: McGraw-Hill Book Company, Inc., 1948.

[8] S. Cuomo. *Pappus of Alexandria and the Mathematics of Late Antiquity*. University Press, Cambridge: Cambridge University Press, 2000.

[9] E. J. Dijksterhuis. *Archimedes*. Princeton, New Jersey: Princeton University Press, 1987.

[10] H. Dörrie. *100 Great Problems of Elementary Mathematics: Their History and Solution*. New York: Dover Publications, 1965.

[11] G. Downey. "Pappus of Alexandria on architectural studies". In: *Isis* 38.3/4 (Feb. 1948), pp. 197–200.

[12] A.G. Drachmann. "Archimedes and the Science of Physics". In: *Centaurus* 12.1 (Mar. 1967), pp. 1–11.

[13] A.G. Drachmann. "Fragments from Archimedes in Heron's Mechanics". In: *Centaurus* 8 (Mar. 1963), pp. 91–146.

[14] A.G. Drachmann. *The Mechanical Technology of Greek and Roman Antiquity*. Copenhagen: Munksgaard, 1963.

[15] S. Drake. "Galileo's Experimental Confirmation of Horizontal Inertia: Unpublished Manuscripts (Galileo Gleanings XXII)". In: *Isis* 64.3 (Sept. 1973), pp. 290–305.

[16] P. Duhem. *Les Origines de la Statique*. Paris: Librairie Scientifique A. Hermann, 1905.

[17] P. Duhem. *The Origins of Statics: The Sources of Physical Theory*. Trans. by G. F. Leneaux, V. N. Vagliente, and G. H. Wagener. Dordrecht: Kluwer Academic Publishers, 1991.

[18] P. M. Fraser. *Ptolemiac Alexandria*. Vol. 1. Oxford, Clarendon Press: Oxford University Press, 1972.

[19] T. Freeth, Y. Bitsakis, et al. "Decoding the ancient Greek astronomical calculator known as the Antikythera Mechanism". In: *Nature* 444.7119 (Nov. 2006), pp. 587–591. ISSN: 0028-0836. URL: `http://dx.doi.org/10.1038/nature05357`.

[20] T. Freeth, A. Jones, et al. "Calendars with Olympiad display and eclipse prediction on the Antikythera Mechanism". In: *Nature* 454.7204 (July 2008), pp. 614–617. ISSN: 0028-0836. URL: `http://dx.doi.org/10.1038/nature07130`.

[21] G. Galilei. *Le Opere di Galileo Galilei: Edizione Nazionale*. Ed. by A. Favaro. Vol. 1. Florence: G. Barbèra, 1890.

[22] G. Goe. "Archimedes' Theory of the Lever and Mach's Critique". In: *Studies in History and Philosophy of Science* 2.4 (1972), p329–45.

[23] R. S. Hartenberg and J. Denavit. *Kinematic Synthesis of Linkages*. New York: McGraw-Hill, 1964.

[24] T. Heath. *A History of Greek Mathematics: Vol. 1 From Thales to Euclid*. Oxford, Clarendon Press: Oxford University Press, 1921.

[25] T. Heath. *Mathematics in Aristotle*. Oxford, Clarendon Press: Oxford University Press, 1949.

[26] T. L. Heath. *The Works of Archimedes*. London: Cambridge University Press, 1897.

[27] T.L. Heath. *The Thirteen Books of Euclid's Elements.* Vol. 1. New York, USA: Cambridge University Press, 1956.

[28] J.L. Heiberg. *Archimedes: Opera Omnia Cum Commentariis Eutocii*. Vol. 2. Leipzig: Teubner, 1913.

[29] C. A. Huffman. *Archytas of Tarentum*. University Press, Cambridge: Cambridge University Press, 2005.

[30] W. R. Knorr. "Archimedes' Lost Treatise on the Centers of Gravity of Solids". In: *The Mathematical Intelligencer* 1.2 (June 1978), pp. 102–109.

[31] W. R. Knorr. *Textual Studies in Ancient and Medieval Geometry*. Boston: Birkhäuser, 1989.

[32] W. R. Knorr. *The Ancient Tradition of Geometric Problems*. Stuttgart: Birkhäuser, 1986.

[33] F. Krafft. *Dynamische und Statische Betrachtungsweise in der Antike Mechanik*. Wiesbaden: Franz Steiner Verlag GMBH, 1970.

[34] M. D. Leakey. *Olduvai Gorge Vol. 3: Excavations in Beds I and II, 1960-193*. Cambridge: Cambridge University Press, 1971.

[35] E. Mach. *The Science of Mechanics*. Chicago, USA: The Open Court Publishing Company, 1893.

[36] E.W. Marsden. *Greek and Roman Artillery: Technical Treatises*. Oxford, Clarendon Press: Oxford University Press, 1971.

[37] E. A. Moody and M. Clagett. *The Medieval Science of Weights (Scientia de ponderibus)*. University of Wisconsin Press, 1960.

[38] O. Neugebauer. "Uber eine Methode zur Distanzbestimmung Alexandria-Rom bei Heron". In: *ongelige Danske Viden-skabernes Selskab* 26.2 (1938), pp. 21–24.

[39] I. Newton. *Isaac Newton's Philosophiae Natural Principia Mathematica*. Ed. by A. Koyré, I.B. Cohen, and A. Whitman. Vol. 1. Cambridge University Press, 1972.

[40] L. Nix and W. Schmidt. *Heronis Alexandrini Opera Quae Supersunt Omnia Vol. II: Mechanica et Catoptrica*. Ed. by L. Nix and W. Schmidt. Stuttgart: Teubner, 1976.

[41] J. P. Oleson, ed. *The Oxford Handbook of Engineering and Technology in the Classical World*. New York: Oxford University Press, 2008.

[42] F. Reuleaux. *The Kinematics of Machinery: Outlines of a Theory of Machines*. Ed. by A.B.W Kennedy. London: MacMillan and Co., 1876.

[43] A. Rome. *Commentaires de Pappus et Théon d'Alexandrie sur l'Almageste: Pappus d'Alexandrie Commentaire sur les Livres 5 et 6 de l'Almageste*. Vol. 1. Biblioteca Apostolica Vaticana, 1931.

[44] D. Ross. *Aristotle*. London: Routledge, 1995.

[45] I.D. Rowland and T.N. Howe. *Vitruvius: Ten Books on Architecture*. Cambridge: Cambridge University Press, 2002.

[46] L. Russo. *The Forgotten Revolution: How Science Was Born in 300 BC and Why It Had to Be Reborn*. Berlin: Springer, 2004.

[47] M. Schiefsky. "Structures of Argument and Concepts of Force in the Aristotelian Mechanical Problem". In: *Early Science and Medicine* 14 (2009), pp. 43–67.

[48] M.J. Schiefsky. "Art and Nature in Ancient Mechanics". In: *The Artificial and the Natural: An Evolving Polarity*. Ed. by B. Bensaude-Vincent and W.R. Newman. Cambridge, Massachusetts: MIT Press, 2007. Chap. 4, pp. 67–108.

[49] O. Schmidt. "A System of Axioms for the Archimedean Theory of Equilibrium and Centre of Gravity". In: *Centaurus* 19 (Mar. 1975), pp. 1–35.

[50] Karin Tybjerg. “Wonder-making and Philosophical Wonder in Hero of Alexandria”. In: *Studies in History and Philosophy of Science* (2003).

[51] M. L. Wantzel. “Recherches sur les moyens de reconnaître si un Problème de Géométrie peut se résoudre avec la règle et le compas.” In: *Journal de Mathématiques Pures et Appliquées* 1.2 (1837), pp. 366–372.

[52] D. Whitehead and P. H. Blyth. *Athenaeus Mechanicus, On Machines*. Stuttgart: Franz Steiner Verlag, 2004.

[53] T.N. Winter. *The Mechanical Problems in the Corpus of Aristotle*. 2007. URL: `http://digitalcommons.unl.edu/classicsfacpub/68`.

[54] L. Zhmud. *The Origin of the History of Science in Classical Antiquity*. Berlin: Walter de Gruyter, 2006.