AN EMPIRICAL ANALYSIS OF INTEREST RATE SPREADS AND TERM STRUCTURES IN EURO AND STERLING MARKETS

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To My Parents

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INTRODUCTION

1 Background and Motivation

The introduction of the euro in January 1999 was a landmark event in European monetary integration and the bond market has, arguably, been the financial market segment where the influence of the single currency has been the most pronounced. By eliminating exchange risk between EMU member states, conditions were created for a substantially more integrated public debt market in the euro area. However, as monetary union would potentially allow members to free-ride on the common monetary policy by running excessive deficits and increasing debt ratios, all EMU member states were obliged to meet the fiscal convergence criteria as laid out in the Maastricht Treaty of 1993. This includes ensuring that the ratio of public deficit to GDP does not exceed 3 per cent and the ratio of public debt to GDP does not exceed 60 per cent.

A direct consequence of the introduction of the single European currency was that all new fungible debt by EMU member states should be issued in euros. Furthermore, outstanding stocks of government debt had to be redenominated into euros which not only added large volumes to the long end of the yield curve but also created markets for euro-denominated bonds at shorter maturities. However, euro-denominated bonds are not being placed by the currency area as a whole, but rather by individual countries. Thus a significant feature of the euro capital market is the absence of Federal European government debt whose yields would form the natural constituents of the euro term structure relationship. Thus, the individual euro area

countries still have national yield curves, and while spreads between them have narrowed considerably as a result of the single monetary policy and the elimination of exchange rate uncertainty, euro area government bonds are still not perfect substitutes as non-negligible differences in yield levels across countries remain. Yield spreads mainly reflect the market's assessment of the risk of default and, in the process, provide a measure of the extent of financial integration amongst EMU member states.

Monetary union may increase the default risk of member states, since they have surrendered their monetary sovereignty and, therefore, the possibility to monetise their debts. Moreover, the no-bailout clause of the Maastricht Treaty would indicate that other governments and the European Central Bank (ECB) would not be compelled to rescue governments in financial crisis. This altered risk profile makes euro area government bond markets distinct from that of the UK, which did not participate in EMU.

The differing degrees of credit risk that have emerged in UK and euro area government bonds, after EMU, provided the motivation to compare these two markets. One purpose of this thesis is to study interest rate linkages between UK and the euro area by analysing their respective fixed income markets post-EMU.

The process by which prices in fixed income markets respond to new information is rendered more efficient when market participants agree on certain instruments that can serve as references - or benchmarks - for pricing other securities. Traditionally, market participants have relied on government bond yield curves to assess the cost of funds at different borrowing horizons. The prices revealed in deep

and liquid government bond markets have been used by central banks to make inferences about future inflation. Market participants use government bonds for hedging interest rate risk. However, in recent years private sector debt instruments such as interest rate swaps have also emerged as benchmarks. The euro interest swap market is one of the largest and most liquid financial markets in the world and was among the first financial markets to become integrated following EMU quickly gaining benchmark status. The fragmented nature of the Euro denominated government securities markets induced the switch to swaps for speculation and for hedging interest rate movements. On the other hand, the government bond yield curve remains the benchmark in sterling markets. The market for 10-year benchmark bonds is the most liquid segment for sovereign debt and remains the focus for the analysis in this thesis. Its characteristics are sufficiently homogeneous across European government bond markets to permit valid cross-country comparisons.

2 Overview of the Thesis

A brief synopsis of the content of the four chapters is provided below.

The purpose of this Chapter 1 was to clearly define what is meant by the term structure of interest rates and to show how it is estimated in government bond and interest rate swap markets. It uses the Nelson and Siegel (1987) model for estimating the term structure of interest rates using government bonds and for interest rate swaps, a cubic smoothing spline specification proposed by Fisher, Nychka and Zervos (1995). The term structures for Euro-denominated German government bonds and UK Gilts are estimated as zero coupon yield curves on certain specific trade dates. The construction of a euro swap curve is also shown in view of its role as a

benchmark in the euro fixed income market. For the government bond yield curves the data that are used for the estimations are the closing mid-prices of bonds. In order to estimate the swap yield curve, short term rates are obtained from the Euro interbank market and medium and long-term rates from the fixed arm of a generic interest rate swap.

Chapter 2 presents evidence to suggest that the loss of monetary sovereignty, that would otherwise have given them the right to print money, has exposed EMU governments to credit risk. The covered interest parity condition was used as a starting point for an enquiry into interest rate linkages between euro and sterling markets. Cointegration analysis showed that the covered interest parity condition does not hold between UK and euro-denominated 10-year bonds issued by France, Germany and Italy. In view of the increased credit risk witnessed in EMU member states, deviations from covered interest rate parity are rationalised in terms of default risk. It is demonstrated that over the sample period 1999-2003, UK government bonds have a lower default probability as compared to euro-denominated bonds and may, therefore, be treated as a benchmark. A credit risk model was estimated on a panel data set of the three largest Euro area government bond markets, namely, France, Germany and Italy. Empirical results show that the credit spread between the UK and the three EMU member states can be attributed to the latter's fiscal performance. The credit risk spread increases with an increase in their deficit/GDP ratio and a higher ratio of net government interest payments to government receipts.

Chapter 3 provides an investigation into the linkages between euro and sterling swap spreads. The observed difference between the swap rate and the government bond yield of corresponding maturity is known as the swap spread. Swap spreads reflect the default risk of the interbank market quoting Libor/Euribor rates and those of the government treasury. This chapter examines the causal relationship between euro and sterling swap spreads during the period January, 1999 to March, 2003 with euro swap spreads proxied using German government bonds. Both the euro and sterling swap spreads are non-stationary across the term structure and follow a random walk. However, sterling swap spreads have been perceptibly wider than euro swap spreads since the launch of the single currency.

The absence of any correlation between changes in the two swap spreads would indicate that credit risk factors are country-specific. But euro swap spreads showed some correlation with the interest rate differentials between the two markets. Both spreads follow a GARCH process but sterling swap spreads reacted more intensely to market movements and were more volatile than their euro counterparts. There was evidence of mild volatility transmission from the sterling swap spreads to the euro swap spreads but the causality was one sided.

The purpose of chapter 4 was to see how the term structure of interest rates has evolved in the sterling and euro treasury bond markets over the period 1999-2003. German bonds have been again used as a proxy for euro-denominated bonds. A state-space representation for the single-factor Cox, Ingersoll and Ross (1985) model was employed to analyse the intertemporal dynamics of the term structure. The zero-coupon yields for UK Gilts and Euro-denominated German Bunds are used as inputs

for the estimation process. Closed form solutions for the prices of discount bonds are derived such that they are a function of the unobserved instantaneous spot rate and the model's parameters. Quasi-maximum likelihood estimates of the model parameters were obtained by using the Kalman filter to calculate the likelihood function. Results of the empirical analysis show that while the unobserved instantaneous interest rate exhibits mean reverting behaviour in both the UK and Germany, the mean reversion of the interest rate process has been relatively slower in the UK. The volatility component, which shocks the process at each step in time is also higher in the UK as compared to Germany.

CHAPTER 1

ESTIMATING THE TERM STRUCTURE OF INTEREST RATES

1.1 Introduction

The graphical depiction of the relationship between the yield on bonds of the same credit quality but different maturities is known as the *yield curve*. In order to construct a yield curve, market yields/prices of bonds with different maturities are usually used, although yield curves may also be constructed from certain interest rate derivative prices such as swaps. These securities have to fall in the same risk category in terms of default risk. In other words, there is a separate yield curve for each level of default risk. In practice, yield curves are usually constructed using price/yield data of a special risk category, namely government bonds, since this category has the largest number of instruments in a wide maturity range, traded on a liquid market. This treasury yield curve, which plots the yield of Treasury bonds against their maturity, is one of the most closely watched financial indicators. Its key function is to serve as a benchmark for pricing bonds and to set yields in all other sectors of the debt market.

The term structure is a particular yield curve - that for discount or zero-coupon Treasury securities. Zero coupon bonds do not pay any coupons. Instead, they are initially offered at a discount to their face value, so their yield is equal to the annualised yield return resulting from their conversion to face value. The term structure is also called the zero coupon yield curve (ZCYC) - every point on the curve

is a zero coupon security for that term. Modelled as a series of cash flows due at different points of time in the future, the market price of a bond is the net present value of the stream of cash flows associated with that bond. Each cash flow, has to be discounted using the interest rate for the associated time to maturity, and each interest rate is a point on the ZCYC. An upward (downward) sloping term structure indicates expansionary (contractionary) monetary policy because monetary policy affects interest rates primarily at the short end of the market.

The received literature on estimating the term structure of interest rates can be broadly divided into two basic categories – *theoretical* methods and *empirical* methods. Theoretical models posit an explicit structure for coupon bond prices, whose values depend on a set of parameters that govern the mean reversion and volatility of the so-called short interest rate. Theoretical models can be further categorised as being either *general equilibrium* models or *no-arbitrage* models.

The general equilibrium models include those of Vasicek (1977), Cox, Ingersoll & Ross (1985), Brennan & Shwartz (1979) and Longstaff & Schwartz (1992) which fit the term structure to a general equilibrium of interest rates. In these models, the market prices of the bonds are assumed to capture some equilibrium notion of the interest rates across all maturities. The no-arbitrage models include the binomial tree models of Ho and Lee (1986), the continuous time model of forward rate volatilities of Heath, Jarrow & Morton (1990) and Hull & White (1990). These models typically use continuous time processes to model interest rates, which are then discretised to give mathematical forms for the ZCYC.

Empirical methods compute spot interest rates using a cross-sectional analysis of bond market prices. Unlike the theoretical methods, the empirical methods are independent of any model of the term structure. Whereas the theoretical methods attempt to explain the typical features of the term structure, which may include how the term structure evolves through time, the empirical methods try to find a close representation of the term structure at any point in time, given some observed interest rate data.

Although government bonds have conventionally provided the yardstick for a risk-free asset, a benchmark yield curve need not necessarily be a risk-free curve. In the case of euro-denominated government bonds, the fragmented nature of the market has induced a switch to swaps for hedging interest rate movements. Prior to the introduction of the euro, growth in the swap market had been driven mainly by the arbitrage opportunities and hedging needs resulting from interest rate convergence. Following European monetary union, swaps gained benchmark status due to the surge in the market for euro-denominated corporate bonds.

The focus of this chapter is on empirical models and the purpose is to demonstrate the methodologies used for a cross sectional analysis of government bonds and interest rate swaps on any given trade date. The chapter is organised as follows. Section 2 provides a review of bond and interest rate swap notations. Section 3 examines the literature on empirical methods to estimate the term structure of interest rates. Section 4 describes the data and demonstrates the results of the estimation on certain selected trade dates.

1.2 A review of bond and interest rate swap notations

1.21 Bond Pricing

This section briefly reviews the terminology used as well as the concept of bond pricing. A bond is the obligation of the bond's issuer to provide a stream of future cash flows – the coupon and redemption payments – at predetermined dates in the future. These conventional straight bonds represent the major instrument in the government bond markets for which the term structure is estimated. However, these markets also contain bonds with special features such as call or put options. The empirical analysis in this thesis will be confined to conventional non-callable bonds.

The valuation of conventional government bonds is straightforward. The market price of a bond is the market valuation of the stream of cash flows associated with that bond. Each cash flow, in such a formulation has to be discounted using the interest rate for the associated term to maturity. If the ZCYC is known, then the market price of a bond maturing m years ahead in the future (i.e. at date $t+m$ if t is the present date) and paying coupon C annually is given by the present value of its cash flow, where the discount factors are calculated from the corresponding zero-coupon yields

$$P(m) = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \ldots\ldots\ldots\ldots\ldots + \frac{C+F}{(1+r_m)^m} \qquad (1.1)$$

In (1.1) F is the redemption payment, $P(m)$ is the price of the bond and $r_1, r_2, \ldots, r_m$ are spot interest rates obtained from the ZCYC. It follows, that in order to price coupon bonds, market participants are required to form some view about the ZCYC. As one cannot directly observe the discount rates that market participants attach to different maturities, the ZCYC will always have to be estimated, using price data for a set of coupon bonds. From the price equation of an individual bond, only the yield-to-maturity (YTM) can be calculated.

The relationship between spot rates and discount factors

The bond price equation (1.1) describes how the price of a bond can be calculated if all the spot rates r_t ($t = 1, \ldots\ldots, m$) for every future period are known. However, the above equation is non-linear in the interest rates, which makes for some complications in the estimation. This equation can be written in terms of *discount factors* so that the present value of each cash flow is written as the product of its nominal value and its discount factor:

$$P(m) = d_1 C + d_2 C + \ldots\ldots\ldots\ldots\ldots + d_n (C + F) \qquad (1.2)$$

which can be rewritten as

$$P(m) = C\sum_{t=1}^{m} d_t + d_m F \qquad (1.3)$$

where d_t is the discount factor for period t (t = 1,--------,m) and is simply a transformation of the tth period spot rate:

$$d_t = \frac{1}{(1+r_t)^t} \quad \text{where } t = 1, \ldots\ldots\ldots\ldots m \tag{1.4}$$

Using discount functions rather than the spot interest rates simplifies the estimation, since the price now becomes a linear function of the discount rates.

It is often useful to think of the continuous analogue to the set of discount factors, the discount function $\delta(t)$, as a continuous function that maps t to a discount factor. A set of discount factors d_t (t = 1,....., m) can therefore be thought of as discrete points on the continuous discount function $\delta(t)$

$$d_j = \delta(t_j)$$

where t_j is the time period at the end of the jth period. In terms of the discount function, the bond price equation becomes:

$$P(m) = c\sum_{j}^{m} \delta(t_j) + \delta(t_m)F \tag{1.5}$$

In the term structure literature, δ_m, the discount function, is transformed into a spot rate curve, using the relationship given by

$$r(m) \cong -\log(\delta_m)/m$$

In general practice, interest rates are compounded at discrete intervals. For example,

on a typical treasury bond with semi-annual compounding, it is assumed that payments earn interest for six months and then are "rolled over" for another six months. In this case, the compounding frequency is 2.

If $r(m,k)$ represents the spot rate of interest with maturity m, and compounding frequency k, the relationship between the price of a discount bond $\delta(m_j)$ and the spot rate is given by

$$\delta(m_j)\{1+r(m_j,k)/k\}^{mk}=1 \qquad (1.6)$$

The spot rate based on continuous compounding represents the relationship between the spot rate, $r(m)$, and time to maturity m. Continuous compounding assumes that payments are rolled over and earn interest at every instant in time. In contrast to numerous discretely compounded spot curves, there is only one continuously compounded spot curve.

With continuous compounding, the expression $\{1 + r(m_j,k)/k\}^{mk}$ becomes $e^{r(m)}$. In other words, $\{1 + r(m_j,k)/k\}^{mk}$ converges to $e^{r(m)}$ as k approaches infinity. The continuously compounded discount factor can similarly be expressed as $e^{-r(m)m}$. Therefore, the price of a discount bond using continuous compounding can be written as:

$$\delta(m_j)=e^{-r(m)m_j} \qquad (1.7)$$

If spot rates for payments at all dates in the future are known, then the price of a coupon bond $p(m_j)$ maturing in m periods can be equated to the present value of the future cash flows. The final payment c_j is assumed to include the redemption payment.

$$p(m_j) = \sum_{m=1}^{m_j} c_{jm} e^{r(m)} \tag{1.8}$$

The bond price equation (1.1) can also be written in terms of discount factors as follows:

$$p(m_j) = \sum_{m=1}^{m_j} c_{jm} e^{-r(m)m} \tag{1.9}$$

Clean prices and dirty prices

In order to estimate the term structure of interest rates, one needs observed bond prices and the terms of each bond which determine the timing and size of future cash flows. In bond markets prices are quoted as *clean prices*. If a transaction takes place, the seller also receives accrued interest for holding the bond over the period since the last coupon payment. While coupon payments on individual bonds are made at fixed dates, bonds can be traded on any given working day. Whenever a bond is traded on a day that is not a coupon payment date, the valuation of the bond will reflect the proximity of the next coupon date.

The price including accrued interest is called the *dirty price* which represents the market value of the bond.

The accrued interest A_t is calculated as a fraction of the coupon C foregone by the seller

$$A_t = a_t C$$

where $a_t = (1 - n_t / 365)$

and n_t is the number of days since the last coupon payment. The number of days in a standard year (taken here as 365) depends on day count conventions of different bond markets. The market price of a bond can therefore be decomposed into two components: the accrued interest and the clean price of the bond. It is important to note that the assumption of continuous compounding would imply that coupon payments are made continuously rather than at discrete points in time such that interest does not accrue.

The relationship between spot and forward rates

The relationship between spot and forward rates is an integral part of term structure estimation methods. The *forward rate* is the interest rate that will apply to an instrument commencing at some future date and can be derived from the spot rates of interest. For example, the forward rate on a one-year instrument one year hence is determined so that an investor is indifferent between purchasing a two-year instrument today and holding it to maturity or purchasing a one-year instrument today

and entering into a forward contract to purchase a one-year instrument one year from now. This equality is shown in Equation (1.10).

$$(1+r_2)^2 = (1+r_1).(1+f_{1,1}) \tag{1.10}$$

where

r_2 = spot rate for two-year instrument,

r_1 = spot rate for one-year instrument and

$f_{1,1}$ = one-year forward rate for one year instrument

In general, the forward rate for any future date and for any instruments of any maturity can be derived using Equation (1.11), provided the instruments with requisite maturities can be observed today.

$$f_{t,T-t} = [(1+r_T)^T / (1+r_t)^t]^{1/(T-t)} - 1 \tag{1.11}$$

where

$f_{t,T-t}$ = t-year forward rate for (T-t) year instrument,

r_T = spot rate for T-year instrument and

r_t = spot rate for t-year instrument

1.22 Swaps and the Swaps Yield Curve

A conventional interest swap is a contract between two parties in which one

party makes fixed interest payments, calculated on a notional amount, while the other party makes floating-rate interest payments. The fixed rate is set at the inception of the contract and the floating-rate is linked to an external reference such as Libor [1] during the life of the swap. In a generic (or "plain-vanilla") interest rate swap the present value of fixed payments is set equal to the present value of the floating payments. Consequently, at the origination date the value of the swap is zero and no cash transfers take place.

However, their value changes over time as interest rates fluctuate. In the case of a fixed-for-floating interest rate swap, as long as interest rates rise, the fixed rate payer benefits from being locked into the lower interest rate. Consequently, the value of the fixed side moves "in the money". By the same token, the floating rate payer receives flows that are lower than the changed interest rate would dictate, and hence, the value of the floating side "moves out of the money." The opposite occurs when long term interest rates decline. Default risk arises when the entity for which the swap is out-of -the-money is unable to meet its commitments to the counterparty for which the swap is in the money. [7] In an efficient market, one would expect market swap rates to incorporate the risk of default, if the counterparties rationally anticipate this possibility. Hence, one would expect swap rates to be sensitive to the credit ratings of the counterparties.

Interest rate swaps are traded over the counter (OTC), rather than, through an organised exchange. Similar to other OTC securities, swaps are characterised by the presence of credit and liquidity risks. Each of the two parties in an OTC transaction

[1] Since the advent of the euro swaps in the euro-zone have been based on Euribor instead of Libor

is exposed to the default risk of the other. Thus, to compensate for these risks, market swap rates are generally at a premium over the comparable government bond rates. This premium is usually termed as the *swap spread*. Swap spreads over government bonds reflect the supply and demand conditions of both swaps and government bonds, as well as the market's view of the credit quality of swap counterparties. There is considerable information content in the swap yield curve, much like that in the government bond yield curve. It will be argued in Chapter 3 that, during times of credit concerns in the market, the swap spread will increase and more so at longer maturities.

1.3 Estimation Issues

The term structure is defined as a continuous function of maturity. This allows for assigning spot rates to any maturity to price a payment at any date in the future. However, the term structure cannot be directly observed using bond price data. In practice, two problems must be solved in order to estimate the term structure. First, only finite numbers of bonds are traded at any one point in time and their maturities provide only a discrete set of points of the term structure. Second, the majority of bonds are coupon bonds that do not allow for a direct calculation of a unique set of spot rates.

[2] Eom , Subrahmanyam and Uno (2000)

However, the spot rates define a set of discount factors. The price of each bond is equal to the sum of each cash flow arising from that bond multiplied by the discount factor applicable to the date of that cash flow. It is necessary, therefore, to fit an approximate discounting function for the spot rates to obtain the rates for all possible maturities. The rationale for this approach is that a general functional specification of discount factors can explain all current bond prices as closely as possible. Thus the literature on estimating the term structure revolves around finding a suitable functional form for the discount factors. In the following sections I will analyse two distinct types of functions, one defined by exponential polynomials and another by splines.

Estimating a term structure involves three basic steps:

(i) Specifying a bond pricing equation that relates bond prices, $p(m)$, to the spot rate function, $r(m)$, via the stream of coupon payments and principal.

(ii) A functional form to be used to approximate the spot rate function, $r(m)$, or discount function, $\delta(m)$ and whose value is determined by a set of parameters.

(iii) Using observed prices of coupon bonds to estimate the parameters of the discount function.

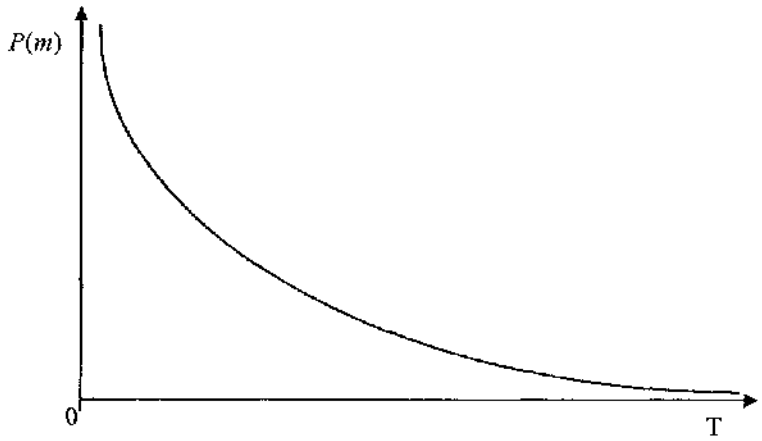
1.31 Approximating the discount function with polynomials and splines

The discount factor must fall between 0 and 1 and is a non-linear function of the term to maturity. It approaches 0 as the term to maturity approaches infinity, and it

approaches 1 as the term to maturity approaches 0. Figure 1.1 shows the typical shape of the discount function $p(m)$ from 0 to T.

Figure 1.1

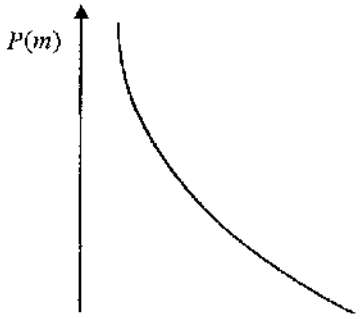
The discount function as a non-linear function of term to maturity

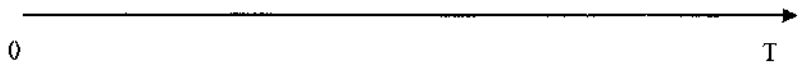


If a line were to be traced through the yields on pure discount government bonds as they relate to maturity, it is unlikely that this line would form a smooth curve. A continuous and complete discount function is, therefore, unobservable. That is, one does not observe prices of discount bonds at all possible maturities since coupon bonds will only yield a set of discrete discount bond prices. One approach to estimating a complete discount function is to find a polynomial that has a similar shape to the true discount function. Such a function would be defined over all maturities from 0 to time T. Figure 1.2 shows how the shape of an approximating polynomial may appear in comparison to the discount function.

Figure 1.2

Fitting a polynomial to approximate the discount function



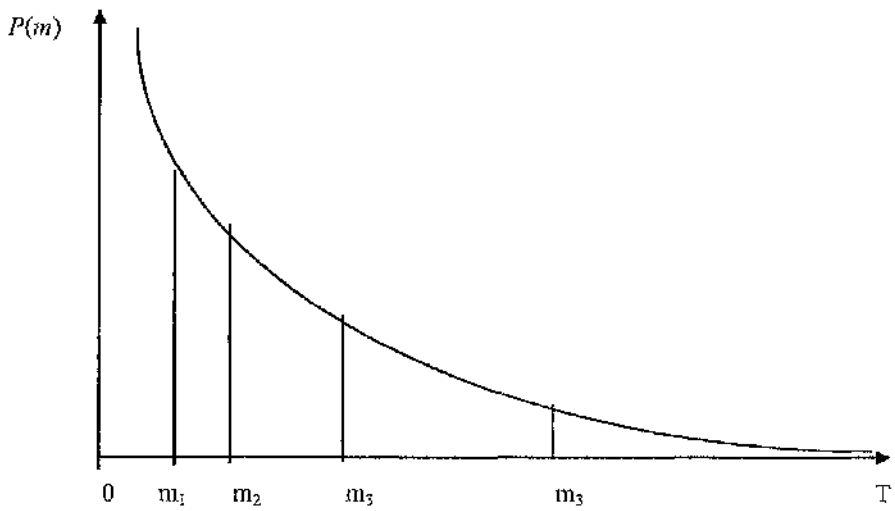


For a large number of unequally spaced observations, the problem related to polynomial interpolation is that the polynomial tends to be of exceedingly large degree. The criticism that most frequently emerges in connection with simple polynomial fitting is centred around the trade-off between goodness-of-fit and stability. The higher the degree of the applied polynomial, the less smooth is the resulting yield curve. If the order of the polynomial is too low on the other hand, the goodness-of-fit will not be satisfactory. Since the majority of instruments used for the estimation typically have short maturities, the goodness-of-fit is usually worse in the case of longer maturities. Any attempt to remedy this problem by increasing the order of the polynomial will make the curve more flexible in the longer horizon. But, in the process, the implied forward curve, instead of converging to some long-run level, may become steep, or start oscillating. Economic intuition suggests that beyond some time horizon nominal interest rate expectations of agents should converge to a certain level, as they have less and less information to distinguish between expected interest rates with maturity m and say, $m + 1$ as m increases. In the light of this intuition implied forwards at the longer end of the yield curve, which change very much as maturity increases are implausible.

Rather than using one polynomial, defined over the entire set of maturities, it may be more appropriate to approximate the shape of the discount function by applying a piecewise polynomial. That is, instead of approximating the function over the entire domain of maturities [0, T], one can first break up the maturities into segments. The next step is to find functions that locally describe the discount function over each of these segments. One can then fit a polynomial to each segment $[m_{j-1}, m_j]$ for $j = 1,----, n$ and $m_0 = 0$ and $m_n = T$. Finally one can attach each of these functions at their join points. This kind of a piecewise polynomial is known as a *polynomial spline* and is illustrated in Figure 1.3.

Figure 1.3

Spline smoothing of the discount function



This kind of a piecewise polynomial is known as a *polynomial spline*. The spline is this curve consisting of different polynomials, which yields satisfactory goodness-of-fit without the instability arising from fitting high-degree polynomials. The joint points are called *knot points* and the notion of smoothness means that at these points first and second derivatives of the curve exist. The original work on estimating the term structure of interest rates used splines to approximate the discount function $\delta(m)$. In the general form, the discount function at maturity m is defined as a linear combination of a set of k linearly independent, underlying basis functions given as:

$$\delta(m) = 1 + \sum_{j=1}^{k} a_j f_j(m) \qquad (1.12)$$

where $f_j(m)$ is the jth basis function, and a_j is the corresponding coefficient. There are k coefficients which have to be estimated, which are $a_1, a_2, ..., a_k$. Since the bond price is a linear function of the discount factors, the coefficients can be estimated using linear regression. In the general form, discount factors are estimated only for those points at which the cash flows occur. To smooth out the entire yield curve, the basis function $f_j(m)$ can take different mathematical functions depending upon the kind of discount function desired. The only restriction on the form of $f_j(m)$ is that the discounts should be positive, non-increasing (to avoid negative forward rates) and should be equal to one at $m = 0$.

To improve the fit between modelled and observed yields, it was McCulloch (1971) who proposed approximating the discount function by a quadratic polynomial spline fitted to price data. Since then, estimation of the yield curve has been dominated by splining models. Notwithstanding its advantages, estimation of the yield curve with splines is not without its own problems. Although it improves the trade-off between goodness-of-fit and stability, spline-based estimation may result in implausible behaviour of implied forwards at longer horizons. The other major problem is that the number and location of the knot points is chosen usually arbitrarily or at best according to some rules of thumb. The number of knot points determines, inter-alia, the flexibility of the spline. Too few knot points give rise to a bad fit while with too many knot points the estimated curve will adjust to outliers too readily, a trade-off similar to that observed in the case of fitting simple polynomials. On account of these deficiencies, other variants were experimented with in the academic literature.

McCulloch (1975) proposed using regression cubic splines to the discount function. Schaefer (1981) extends the analysis and suggests that the spline function should be constrained so that the discount function is everywhere negatively sloping. Shea (1984) summarises the first attempts with spline techniques. He demonstrates that most of the spline models failed because they did not specify adequate constraints. A *cubic polynomial spline* would model the discount function $\delta(m_j)$ as

$$\delta(m_j) = 1 + \beta_1 + \beta_2 m_j + \beta_3 m^2{}_j + \beta_4 m^3{}_j$$

where $\beta_1, \beta_2, \beta_3$ and β_4 are parameters which are estimated from observed bond prices.

Vasicek and Fong (1982) have recommended *exponential splines* as an alternative to polynomial splines on the grounds that polynomial functions weave around the discount function resulting in highly unstable forward rates. They fit a third order exponential spline to US Treasury securities. The function specification would be as follows:

$$\delta(m_j) = \beta_1 + \beta_2 \exp(-\alpha m_j) + \beta_3 \exp(-2\alpha m_j) + \beta_4 \exp(-3\alpha m_j)$$

where $\beta_1, \beta_2, \beta_3$ and β_4 are parameters and α is the instantaneous forward rate.

Vasicek and Fong simply propose this model and suggest a methodology to estimate the parameters. The authors do not fit the model to any data. Shea (1985), however, concluded that the estimation of exponential splines does not offer any decisive advantages vis-à-vis estimating with polynomial splines. In fact, the estimation is more difficult because the model is nonlinear rather than linear. Shea, therefore, recommends the use of ordinary splines rather than exponential ones.

Fisher, Nychka, and Zervos (1995), proposed using a cubic spline with a *roughness penalty* to extract the forward rate curve. The roughness penalty stiffens the spline, which reduces the oscillatory behaviour, but also reduces the fit. Bliss (1996) compares five models including the approach by McCulloch (1975) and Fisher et al (1995). He found that while McCulloch's method accurately prices bonds, the

forward rate curves it produces often tend to oscillate. And the approach by Fisher et al tends to misprice short maturity securities. Waggoner (1997) modified Fisher's method, using a *variable roughness penalty*. He demonstrates that using a small roughness penalty on the short end of the term structure and a larger penalty on the long end allows the flexibility to price short term securities well without the giving up the desirable oscillation damping on the long end. The estimation technique used in this chapter is based on the approach presented by Fisher, Nychka and Zervos (1995).

Fitting spot rates with smoothing cubic splines

A cubic spline fits a cubic polynomial between each pair of adjacent knots, subject to constraints that guarantee a smooth function. Cubic splines require that the spline passes through all observations exactly. Outside the interval that is used for the spline, the polynomials deviate heavily from the observed shape. This explains why extrapolating a cubic spline is problematic. A cubic smoothing spline relaxes the assumption that the spline is required to pass through all observations.

This subsection describes how cubic smoothing splines work. As with the parametric models the main objective is to choose the parameters so as to minimise the difference between actual and fitted values. A cubic spline h minimises the expression:

$$\sum_{i=1}^{N}(P_i - \hat{P}_i(h))^2 \qquad (1.13)$$

where $h(t)$ is the function used to compute the fitted bond prices, $\hat{P}_i$, and may be the discount rate, spot rate, or forward rate function. The problem with an unconstrained spline, such as the one in equation (1.13) is that it would actually interpolate the data and be far too flexible. When a curve moves too much, or is said to "overfit" the data, it will inevitably fail to identify securities that have been mispriced by the market place. Conversely, if the curve is too smooth and not fit the data points well it will not price bonds in close accordance with observed market prices and the number of supposedly mispriced bonds that the curve will identify will be many. To control the trade-off between goodness of fit (flexibility) and the smoothness of the curve, a roughness penalty can be included to penalise excessive curvature. The problem now consists of minimising the residual sum of squares plus the penalty. So the function, $h(m)$, is now chosen to minimise the objective function:

$$\min \sum_{i=1}^{N} \lambda.[P_i - \hat{P}_i(h)]^2 + (1-\lambda).\int_0^T [h''(m)]^2 \, dm \qquad (1.14)$$

This roughness penalty is given by the integral of the squared second derivative of the function. The second derivative measures the rate of change of the gradient of a curve, i.e, how the slope of the curve changes as the independent variable (maturity, in this case) changes. Hence, the closer the second derivative is to zero, the more smooth the curve. Smoothness also requires that the second derivative is small at each point in time from the beginning (time 0) to the end (time T). Therefore, it is desirable that the sum (or integral) of the squared deviations of the second derivative $h''(m)$ from zero is as small as possible. The square on $h''(m)$ in equation (1.14) avoids that negative and positive values balance each other, by

making all them positive values. As the functions variability increases, the integral of the squared second derivative value will increase too, increasing the roughness measure. This explains how smoothing splines are different from regression splines. Smoothing splines have a penalty for excess roughness and a single parameter that controls the size of the penalty.

Minimising the expression given in equation (1.14) is a trade off between minimising the first term, which measures the goodness of fit, and the second term, which measures smoothness. The positive constant λ is the smoothing parameter. As both weights (λ and $(1-\lambda)$) sum up to 1, one can see the trade off between getting closer to the data as against getting smoother functions. Setting the smoothing parameter $\lambda = 1$ in equation (1.14), the resultant function would actually interpolate the data, since the roughness penalty would be multiplied by zero. On the other hand, setting $\lambda = 0$ would only force the function to become smooth to the point of being linear giving rise to a straight line that minimises the sum of squares. This method offers flexibility whereby setting the smoothing parameter between value between 0 and 1 enables one to handle the trade off between smoothing and local variability. The choice of an "optimal value" of λ is a subtle problem. It is a compromise between extracting as much information from the data as possible and eliminating noise.

Fisher, Nychka and Zervos (1995) assume that the smoothing parameter is invariant to maturity but variable over time. They then use a procedure known as

generalised cross validation [3] to choose λ on a daily basis. Waggoner (1997) who used a variable roughness penalty approach allows the smoothing parameter to vary across maturity but keeps it constant over time. He observed that cubic splines tend to oscillate excessively on long-term maturities, while failing to fit short term observations. To address this problem, he formulated a technique that ascribes more flexibility at the short end than at the long end of the yield curve. If a roughness penalty is incorporated, the flexibility of a cubic spline depends not only on the number of knot points and their spacing, but also on the value of λ. Waggoner argued that as λ increases, the number and spacing of the knot points becomes less important. Thus for large values of λ, the flexibility of the spline is approximately the same across all regions. This he views as being problematic as the spline should be more flexible on the short end than on the long end. To solve this, he proposed a variable smoothing parameter, $\lambda(m)$, decreasing on maturity, transforming the roughness penalty term on equation (1.14), into a variable roughness penalty. Therefore, the objective function to be minimised would be (1.15). This equation is basically the same as (1.14), except that the smoothing parameter is now dependent on the time to maturity m.

$$\min \sum_{i=1}^{N} \lambda(m).(P_i - \hat{P}_i(h))^2 + [1-\lambda(m)].\int_0^T [h''(m)]^2\, dm \qquad (1.15)$$

It is evident that more flexibility is likely to be needed at shorter maturities as expectations are better informed and more subject to revision as news reaches the market. It is possible to fit splines that are flexible in short maturities and smoother in

[3] The basic principle of cross-validation is to leave the data points out one at a time and to choose that

longer maturities using the model developed by Fisher, Nychka and Zervos (1995). This can be done by assigning a weight w_i to each individual data point. In order to fit a spline avoiding long term oscillations, but fitting short term observations well, w could be set equal to the inverse of the duration squared.[4] The objective function to be minimised in order to obtain a smoothing cubic spline would then be:

$$\min \sum_{i=1}^{N} \lambda . w_i . [P_i - \hat{P}_i(h)]^2 + (1-\lambda) . \int_0^T [h''(m)]^2 \, dm \qquad (1.16)$$

This equation is almost the same as equation (1.14). But by incorporating the weight parameter, w_i, it allows the estimation process to differentiate between the level of importance assigned to different observations in the sample. It is another way of reducing the oscillatory behaviour of long term rates that Waggoner highlighted. By assigning smaller weights on long-term observations, the resulting spline would fit rates at the short end better than those at the long-end thereby avoiding excessive long term oscillations. Waggoner chose a three-tiered step function for his smoothing parameter with steps at one and ten years in maturity. This was based on the natural segmentation of the US market into bills, notes and bonds. However, the Euro-denominated government bond markets cannot be naturally divided in the same way.

1.32 Parsimonious models of the yield curve

value of λ under which the missing data points are best predicted by the remainder of the data.

[4] Duration is a measure of the weighted average term-to-maturity of a bond, where the weights are the present value of the cash flow.

While spline functions belong to the class of highly parameterised functions, the so-called parsimonious models of the yield curve work with a lower number of parameters. This parsimonious representation, defined by an exponential decay term, was developed by Nelson and Siegel (1987) and extended by Svensson (1994). Unlike the spline techniques that model the discount function, this technique explicitly models the forward curve. Nelson and Siegel (1987) concluded that it was most straightforward to start from the functional form of an instantaneous forward yield curve, and then, using the simple dependence between forward and spot rates, to derive the functional form of the zero-coupon yield curve.

The functional form can be derived from their assumption that the path of instantaneous forwards is described by the solution to a second-order differential equation:

$$f(m,\beta) = \beta_0 + \beta_1 * e^{(-m/\tau)} + \beta_2 * (\frac{m}{\tau}) * e^{(-m/\tau)} \qquad (1.17)$$

where f (m,β) is the instantaneous forward rate for the period m periods ahead and $\beta = (\beta_0, \beta_1, \beta_2, \tau)$ is the vector of parameters affecting the shape of the curve. The notation $f(m,\beta)$ is used to emphasise the functional dependence of the forward rate on maturities and on parameters. Forward rates are represented as a sequence of exponential terms. Nelson and Siegel have based their model on the premise that exponential functions are capable of capturing most shapes of the term structure. β_0 is a constant, the exponential term $\beta_1 e^{(-m/\tau)}$ is monotonically decreasing (increasing) with time to maturity m if β_1 is positive (negative). The second exponential term

$\beta_2(\frac{m}{\tau})e^{(-m/\tau)}$ produces a hump (trough) if β_2 is positive (negative). If the time to maturity converges to infinity both exponential functions become zero and the limiting value of equation (1.17) is β_0. If the time to maturity approaches zero the exponential functions become 1, but the β_2 term drops out as it includes the fraction (m/τ). Hence, the result is $\beta_0 + \beta_1$.

From this forward rate equation one can derive algebraic expressions for both the spot curve and the discount function. The spot rate, denoted $r(m)$ can be represented as the average of the forward rates. In continuous time this turns out to be the definite integral of the instantaneous forward rate with limits of integration of 0 and m, divided by m. Integrating equation (1.17) from 0 to m and dividing by m gives the spot interest rate for maturity m. The resulting function is:

$$r(m) = \beta_0 + (\beta_1 + \beta_2) * [\frac{1 - e^{-m/\tau}}{m/\tau}] + \beta_2 * [\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau}] \qquad (1.18)$$

which is linear in coefficients, given τ.

This specification of the spot rate function can be used to obtain the discount function, $e^{-r(m)m}$. Substituting the specification of the spot rate function as given by equation (1.18) into equation (1.7) provides the specification of the discount function as given below.

$$\delta(m) = \exp\{-m[\beta_0 + \beta_1[\frac{1 - e^{-m/\tau}}{m/\tau}] + \beta_2[\frac{1 - e^{-m/\tau}}{m/\tau} - e^{-m/\tau}]\}} \qquad \text{..... (1.19)}$$

The discount function is the vehicle used to determine the price of a set of bonds because the present value of a cash flow is calculated by taking the product of this cash flow and its corresponding discount factor. The application of the discount factor function to all the coupon and principal payments that comprise a bond provides an estimate of the price of the bond. The discount function, therefore, is the critical element of the model that links the forward rate and bond prices.

A question that needs to be addressed, at this stage, is what are the advantages of focussing directly on the forward curve instead of the discount function ? The major benefit is that there are some a priori assumptions about the shape of the forward curve on the basis of which some estimated yield curves and the implied forward curves can be rejected as implausible. The most significant such a priori assumption is that forwards on longer horizons should converge to some asymptotic value, as agents have less and less information to discriminate between expected rates as the horizon of these rates get longer. Nelson and Siegel (1987) ensured that this is always satisfied when they assumed the functional form (1.18) for the yield curve, which is sufficiently flexible to provide the typical (increasing, inverted and hump-shaped) forms of the yield curve while satisfying most of the a priori assumptions.

Another advantage of this specification is that the parameters can be interpreted. Since

$$\lim_{m \to \infty} r(m) = \beta_0$$

$$\lim_{m \to 0} r(m) = \beta_0 + \beta_1$$

β_0 is the limit of the spot rate as the maturity tends to infinity. In other words, it is the long-term interest rate (in the limit forward and spot rates coincide). As the curve asymptotes to a value of β_0, this long-term component is a constant and does not decay to zero in the limit. If the maturity tends to zero the spot-rate converges to the sum $\beta_0 + \beta_1$. This can be interpreted as the instantaneous spot rate.[5] This further implies that ($-\beta_1$) can be interpreted as the spread between long and short-term interest rates. The parameters β_2 and m determine the shape of the curve, there is no direct economic interpretation for them. τ is the time constant associated with the positioning of humps in the curve; it determines the rate of decay toward the long-term rate.

Svensson (1994) has proposed an extension of the Nelson and Siegel model that allows for more flexibility. While the Nelson and Siegel model can have only one hump (a local maximum or minimum), the Svensson model allows for two humps by adding a fourth term, a hump-shaped (or U-shape), $\beta_3 * (\frac{m}{\tau_2}) e^{(-m/\tau_2)}$, with two additional parameters, β_3 and τ_2. The forward rate function is then

$$f(m, \beta) = \beta_0 + \beta_1 * e^{(-m/\tau_1)} + \beta_2 * (\frac{m}{\tau_1}) * e^{(-m/\tau_1)} + \beta_3 * (\frac{m}{\tau_2}) * e^{(-m/\tau_2)} \qquad (1.20)$$

where $\beta = (\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2)$.

The corresponding spot rate function in the Svensson model is then given by:

$$r(m,\beta) = \beta_0 + \beta_1 * [\frac{1-e^{-m/\tau_1}}{m/\tau_1}] + \beta_2 * [\frac{1-e^{-m/\tau_1}}{m/\tau_1} - e^{-m/\tau_1}] + \beta_3 * [\frac{1-e^{-m/\tau_2}}{m/\tau_2} - e^{-m/\tau_2}]$$

(1.21)

Econometric methodology

The key to the estimation process is that for a given set of parameters, cash flows c_j and payment intervals m_j, the Nelson and Siegel model implies a theoretical price $\hat{P}$ where:

$$\hat{P} = \sum_{m=1}^{m_j} c_{jm} e^{r(m)} \qquad (1.22)$$

where $r(m)$ is the spot-rate for maturities m (or cash flows due m periods in the future) implied by the parameters of the model. Every different set of parameter values in the discount function translates into different discount factors and thus different theoretical bond prices. Based on the principle of least squares, the parameters can be chosen such that the sum of squared differences between observed and theoretical prices for all observed bonds is minimised:

$$\min \sum_{i=1}^{n} (P_i - \hat{P}_i)^2 \qquad (1.23)$$

where P_i is the i-th out of n bonds on a particular trading day.

[5] In some empirical work, the instantaneous spot rate is used as an approximation of the overnight rate.

There is no analytical solution and the equation must be solved numerically.

An alternative would be to minimise the sum of squared differences between observed yields and estimated yields (yield to maturity) calculated at each iteration from the estimated prices. Since there is a non-linear relationship between coupon bond prices and yields, the result of this procedure is not equivalent to that of the price-error minimisation procedure. The yield y of a single bond can be calculated from the following equation using an iterative search procedure.

$$P = \sum_{m=1}^{m_j} C_{jm} e^{(-m_j y)} \qquad (1.24)$$

Since the term structure model implies a theoretical price $\hat{P}$, the corresponding theoretical yield $\hat{y}$ can be obtained from the following equation:

$$\hat{P} = \sum_{m=1}^{m_j} C_{jm} e^{(-m_j \hat{y})} \qquad (1.25)$$

It is important to note that $\hat{P}$ depends on the parameters of the spot rate function. In other words, during the parameter estimation process a theoretical yield $\hat{y}$ is determined for each bond and each set of parameters.

Given the observed yields $\hat{y}$ and theoretical yields $\hat{y}_i$ for a set of n bonds, the parameters of the term structure model can be estimated by minimising the sum of squared yield errors:

$$\min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \qquad (1.26)$$

The duration of instruments with short remaining maturities, and thus the elasticity of their prices with respect to yield changes is smaller than that of instruments with long maturities. This implies that the price-error minimisation method implicitly places less weight on shorter maturities relative to the yield-error minimisation estimation. Thus, the former will perform relatively better at longer horizons while the latter is superior at short horizons. The choice between price or yield-error minimising should be driven by the motivation of the estimation recognising however that minimising yield errors requires much more computer time, since at each iteration, the yield to maturities of all the bonds have to be calculated - again by an iterative procedure.

Construction of Yield Curves for the Interest Rate Swap Market

The swap curve depicts the relationship between the term structure and swap rates. The methodology for deriving the swap term structure can be found in Meier(2000). As swap data begins with the two-year rate, it is necessary to use interbank rates for the short end of the curve. So in order to estimate the swap term structure data is sourced from two markets. Short-term rates are obtained from the Euro area interbank market with 1,3,6 and 12 months time to maturity measured as the average of the bid and ask prices (middle rate) from Euribor. Medium and long-term rates (between 2 and 10-year maturities) are obtained from the fixed arm of a generic interest rate swap. The term structure is made up of ten vertices on a given trade date. Quotes for

daily interest rates from the money markets and interest rate swap markets were obtained from *Datastream*. For maturities (6, 8 and 9 years) not collected in the database, linear interpolation is used.

Bootstrapping Spot Rates

Although, the euro interbank market rates are essentially zero coupon rates, they need to be converted to the euro swap rate compounding frequency and day count convention. The day convention in the interbank market is actual days/360 whereas the euro swap markets are quoted on a 30/360 day-count convention. In order to make the rates comparable, the interbank rates are multiplied by 365/360. The n-month observable interbank rate is as $r_d(t)$ with $t = n/12$. Given this discrete value of the interbank rate, the following equation is solved to compute the continuously compounded zero swap rate $r_c(t)$ with t measured in years.

$$r_c(t) = \frac{1}{t}\ln[1 + tr_d(t)\frac{365}{360}] \qquad (2.11)$$

The long end of the swap curve out to ten years is derived directly from observable coupon swap rates. The fixed swap rates are quoted as par rates and are compounded annually for the euro. [6] Annualised zero-coupon rates are derived from swap par rates by the method of bootstrapping. The starting point for the bootstrapping process is the discrete time one-year euro interbank rate $r_d(t) * \frac{365}{360}$.

1.4 Data and Results

This section demonstrates the estimation of a government bond yield curve using the Nelson and Siegel (1987) model and an interest rate swap curve using the cubic smoothing splines approach of Fisher, Nychka and Zervos (1995).

1.41 Construction of a ZCYC using Government bonds

As a first step, the term structure of interest rates for Germany on the settlement date 29th January 1999 has been estimated using the Nelson-Siegel model. This would serve as an indication of what the German yield curve looked like after the introduction of the euro. The parameters of the model have been estimated by minimising the sum of squared yield errors in a non-linear optimisation procedure called BFGS. This involved estimating the function specified in equation (1.19). The calculations have been done using the GAUSS programming language.

The ZCYC has been estimated by the closing mid-prices obtained from *Datastream*. The database contains market data on dirty price, coupon, yield to maturity, and amount outstanding per bond on the settlement date. Euro-denominated government bonds pay a fixed annual interest rate and have a fixed maturity date.

An appropriate measure of goodness-of-fit is the Root Mean Squared Error (RMSE) which could be the sum of squared differences of estimated and observed yields/prices divided by the number of instruments used for the estimation.

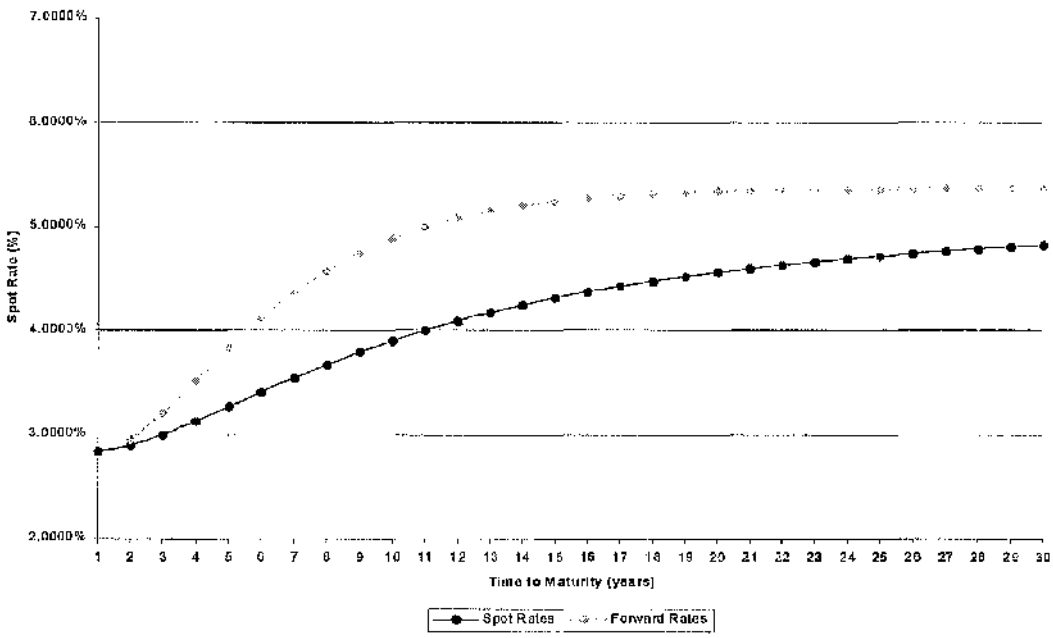
[6] Sterling and US dollar swap rates are compounded semi-annually

$$RMSE = \frac{\sum_{i=1}^{n}(P_{obs,i} - P_{est,i})^2}{n}$$

$$RMSE = \frac{\sum_{i=1}^{n}(Y_{obs,i} - P_{est,i})^2}{n} \qquad (1.27)$$

The RMSPE and RMSYE for the non-linear estimation on the trade date 29th January, 1999 are 0.2244 and 0.00372. The ZCYC is portrayed in Figure 1.1.

Figure 1.1 **German yield curve on 29th January 1999**

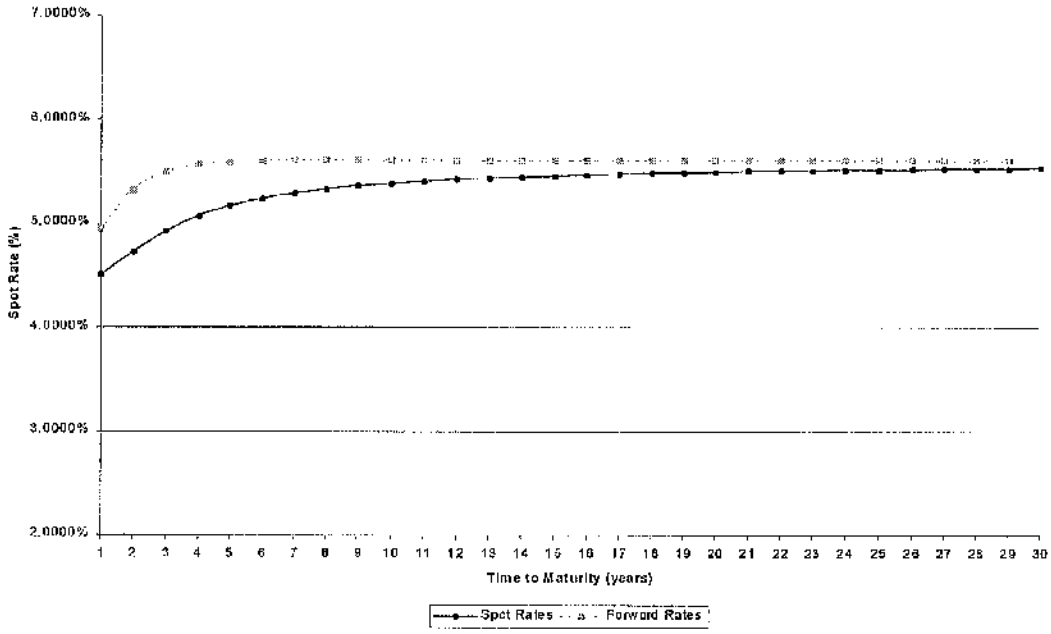


The shape of the yield curve, the difference between short and long-term maturity government bonds is traditionally considered a serious economic forecasting tool

among professional investors. When the yield curve slopes upwards with a spread of about 130 to 200 basis points, the case of Germany in 1999, investors expect normal future economic growth in the range of 2-3 per cent per year. In this particular case, on 29th January 1999, the spread was 198 basis points.

When long-term rates begin to fall relative to short-term rates, markets expect the future rate of inflation to go down and therefore long-term bonds become a more attractive investment proposition. This was roughly the situation prevaling in Germany on 28th April, 2000. As shown in Figure 1.2, the ZCYC for that date had a more gentle upward slope with a yield spread of 102 basis points.

Figure 1.2 **German yield curve on 28th April 2000**



When short term rates exceed the rates at the long end of the curve the spread becomes negative giving rise to an inverted yield curve. This was witnessed in the UK Treasury bond market on April 28 2000, as shown in Figure 1.3, where there was

a negative yield spread of 197 basis points. When long-term rates drop relative to short-term rates investors normally expect the economy will go into recession. This often induces them to exit the stock market and transfer funds to long-term bonds. It will be discussed in Chapter 3, that the inversion of the UK Treasury yield curve in April 2000, could be partially explained by the scarcity of long bonds leading to a decline in yields.

Figure 1.3 **UK Treasury yield curve on April 28, 2000**

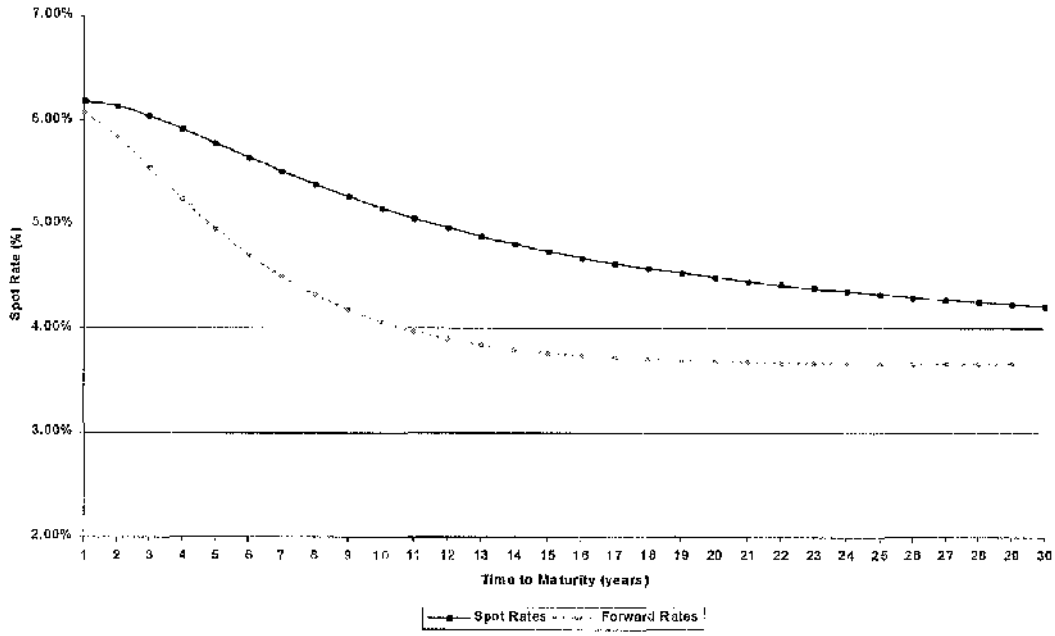
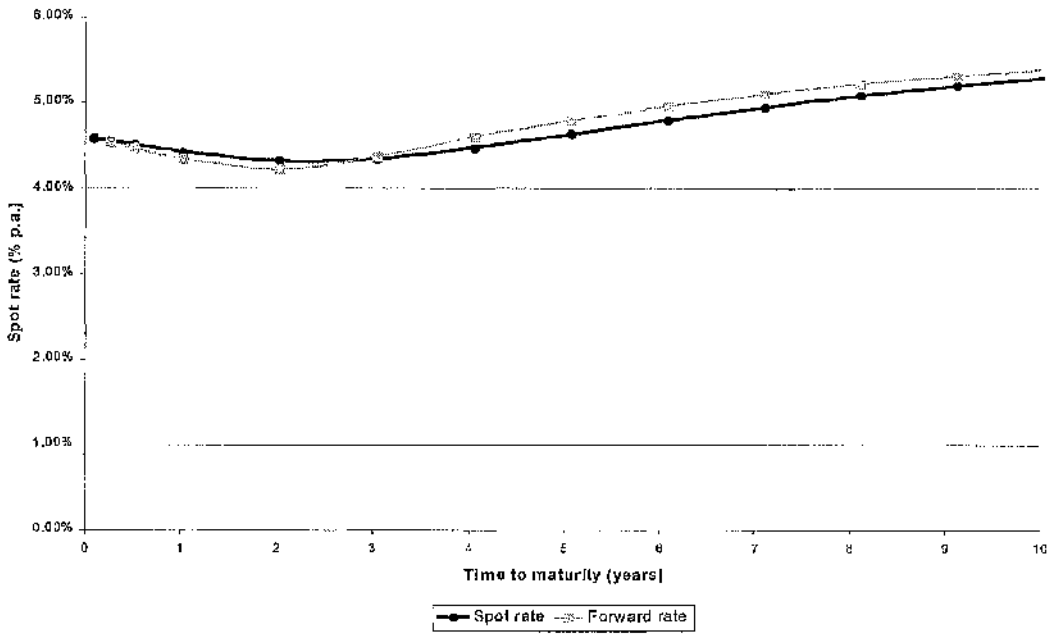


Figure 1.4 depicts the euro swap curve estimated for the trade date July 27, 2001. To begin with, zero-coupon swap rates are derived from quoted swap par rates using the method of bootstrapping. The zero swap rates and euro interbank rates are then interpolated using a cubic smoothing spline as given by equation (1.14). In order to arrive at an appropriate smoothing parameter value in smoothing splines, the generalised cross validation method is employed. For the 13 interbank market and zero swap rates on July 27, 2001 the value of the smoothing parameter, λ, was equal to 0.91. The resultant swap yield curve is shown below:

Figure 1.4 **Euro swap curve on July 27, 2001**



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CHAPTER 2

CREDIT SPREADS BETWEEN UK AND EURO AREA GOVERNMENT BONDS

2.1 Introduction

The introduction of the euro in January 1999 and the associated elimination of foreign exchange risk fundamentally altered the structure of the European bond market. Governments having joined the EMU have lost their monetary sovereignty, that is the right to print money to pay off domestic currency debt. Before EMU, differences in credit ratings were partially accounted for by the foreign exchange factor since governments had the option of preventing default by monetising debt denominated in their own currency, leading to inflation and devaluation of the national currency. But the loss of monetary sovereignty, that would otherwise have given them the right to print money and, the infeasibility of exchange rate devaluation, exposes EMU governments to credit risk.

Credit risks have now replaced market risks, caused by variations in exchange rates, as the principal source of relative risk in euro-denominated government bond markets. In the context of government bonds, credit risk or default risk refers to the probability that a country is unable or unwilling to make timely principal and/or interest payments. A government that defaults on its debt will lose its reputation in terms of creditworthiness and this will impinge on its ability to access the private capital market in future. An actual default by a EMU member state is very unlikely. Nevertheless, yield spreads on government bonds reflect market perceptions of the risks of default and the endeavour to create an integrated bond market in the euro area

has focused much attention on the yield spreads between different sovereign states.

There are similarities in the monetary policy frameworks of the UK and the euro area in that interest rate decisions are taken by an independent central bank with a statutory mandate to ensure price stability.[1] Both the UK's fiscal framework and the EU's Stability and Growth Pact are designed to ensure sound public finances, as a pre-requisite to achieving stable long-term economic growth. However, policy coordination is more complex in the euro area where there is a single monetary authority but multiple fiscal authorities. As a result 12 different issuers participate in the euro government bond market with varying degrees of credit risk. As an EU Member State, the UK is not obliged to adopt the single currency but has the option to join EMU if it declares its willingness to do so and fulfills the necessary conditions.[2] Were the UK to join EMU, it would adopt the euro area's monetary policy framework with interest rates set by the European Central Bank (ECB) for the euro area as a whole, including the UK. Fiscal policy remains the responsibility of Member States, in or out of EMU. This special status that the UK has with respect to EMU has provided the motivation for a study of credit risk in UK and euro area government bond markets.

The literature on sovereign credit risk draws a distinction between market-based and rules-based fiscal discipline. Market discipline is based on the premise that financial markets effectively restrict the public borrowers' ability to raise debt by imposing a risk premium as a compensation for increased sovereign credit risk. Such market-based fiscal discipline would initially take the form of rising yields on the

[1] HM Treasury (2003)
[2] European Central Bank : Convergence Report (2002)

debt of a country running large fiscal deficits. If these deficits persisted the default premium would increase at an increasing rate and eventually the governments would encounter credit rationing.

The Stability and Growth Pact, which puts limits on government borrowing for euro area countries, is meant to enforce the principle of rule-based fiscal discipline. It stems from the Maastricht treaty commitment that countries using the euro must keep their budget deficits within 3 per cent of gross domestic product, and calls on all EU members to keep their budgets "close to balance or in surplus" in the medium term. The rationale is to prevent "free riding" whereby countries borrow heavily while getting the benefit of the eurozone's common interest rate. The no-bailout clause in the Maastricht Treaty is aimed at reducing the bailout expectations of profligate governments. The objective of European fiscal policy, is to ensure the sustainability of public finances, since high or rapidly rising debt levels in one country could have externalities on others. The rules governing the Stability Pact are enforced by the European Commission. Its directorate of economic and financial policy decides whether a country has breached the pact, and recommends measures to correct the problem. If persistent breaches occur, it can recommend fines - which in theory can be a large proportion of government revenue - to the council of European finance ministers. The UK is covered by many of the requirements of the EU fiscal policy framework and must also endeavour to avoid excessive budget deficits.

The credibility of the Stability Pact ultimately refers to its ability to prevent unsustainable fiscal policies that could eventually lead to the risk of default, financial crisis and a possible bailout by the ECB. Afonso and Strauch (2004) discuss how the

fiscal events that occurred in 2002 challenged the credibility of the European fiscal framework. The developments in 2002 started with the European Commission's recommendations for an early warning once it became apparent that Germany and Portugal would be close to the 3% of GDP limit for the deficit. Since then, the credibility of the Stability Pact has been subjected to further doubts with both Germany and France, missing their 3 percent deficit for two consecutive years. This can be explained by the slowdown in their GDP growth that, in turn, worsened the deficits through the mechanism of automatic stabilizers. The government budget deficits increase as GDP falls because tax revenues fall, and some transfer payments, especially unemployment-related benefits, rise. On its part, the European Commission is finding it difficult to impose any fines as this would directly add to the deficit and exacerbate the situation. Against this backdrop, proposals for redrafting the Stability Pact are being presented by various sides. Germany and France want to see considerations such as inflation and employment included in addition to the budget deficit. Connolly and Whittaker (2003) argue that default premia on eurozone government debts have remained low as lenders see eurozone government debts as collectively underwritten by the EU. At the same time, inflation premia have been brought down by the success in holding down inflation expectations. They take the view that membership of the EMU dilutes the financial discipline that would be faced by an independent government. In the context of the vulnerability of banks to default risk in EMU, Arnold and Lemmen (2002) point out that this depends not only on their total amount of public debt and on the number of different public sector debtors, but also on the covariance structure of default risk. They recommend that if banks spread their government bond holdings across countries this would increase the stability of the EMU financial system.

The credit rating agency Standard and Poor's (S&P) has recognized the increase in credit risk by downgrading domestic currency debt from the governments in the Eurozone from the standard AAA rating for domestic currency debt in the pre-EMU era to the lower foreign currency ratings that have prevailed post-EMU. Governments generally seek credit ratings in order to ease their access to international capital markets, where investors typically prefer rated securities over unrated securities of apparently similar credit risk. In the past, governments tended to seek ratings on their foreign currency obligations exclusively, because foreign currency bonds were more likely to be placed with international investors than domestic currency offerings. But in recent years, as international investors have increased their demand for bonds issued in currencies other than the traditional global currencies, more sovereigns have been obtaining domestic currency bond ratings as well. However, credit ratings on a sovereign's foreign currency bonds, as a rule, do not exceed the ratings on domestic currency obligations. This is based on the premise that since governments have the power to print domestic currency, they may be in a better position to fulfill domestic currency obligations. But with the loss of monetary sovereignty this premise no longer holds for euro area government bond markets.

This chapter analyses credit risk spreads between sterling and euro denominated ten-year government bonds over the period 1999-2003 using the interest parity condition. The theory of interest rate parity proposes that given perfect capital mobility, fixed exchange rates and perfect capital markets, interest rates will be equal across countries. This situation however is reflected only in a perfect world where there exists a single global market and no market imperfections. The reality of

imperfect capital mobility and floating exchange rates implies that interest differentials across countries will persist. Therefore, the interest rate parity condition provides a starting point on which to base an enquiry into interest rate linkages between euro and sterling markets. The analysis in this chapter demonstrates that covered interest parity does not hold between UK and Euro-denominated ten-year bonds over the period of observation. In view of the increased credit risk witnessed in the EMU countries, deviations from the covered interest parity condition (CIP) are rationalised in terms of default risk. It is shown that over the sample period, UK government bonds have a lower default probability compared to euro-denominated bonds and may, therefore, be treated as a benchmark. A credit risk model is then estimated on a panel data set of the three largest Eurozone government bond markets, namely, France, Germany and Italy using a fixed effects model. The model is used to estimate the effects of fiscal variables on sovereign credit spreads between the UK and the three EMU countries.

The contribution of the chapter to this area of research is that it would be the first to analyse sovereign credit spreads between the UK and the euro area since the launch of the single currency. Studies exploring the difference in long-term bond yields of European governments have confined their analysis to EMU countries alone. The chapter is organised as follows. Section II provides a literature review on measuring government default risk. Section III tracks movements in yields differentials on euro area government bonds and measures credit risk. Section IV shows the formulation of the model based on the covered interest parity condition to estimate sovereign credit spreads. Section V outlines the data used in the estimations that follow. The empirical analysis in Section VI has two parts. The first part uses

cointegration analysis to test whether the interest parity condition holds between UK and euro area government bonds issued by Germany, France and Italy. The second part estimates the sovereign credit spreads for these countries using UK as the benchmark. Section VII concludes the chapter.

2.2 Review of literature on measuring credit risk

The literature on credit risk offers different approaches in terms of its actual measurement. However, a number of studies dealing with the default risk in bond yields focus on the spread between holding government debt and highly rated private debt of equivalent maturity and denominated in the same currency.

Alesina, De Broeck, Prati and Tabellini (1992) measure government default risk by the difference between corporate and government bond yields of similar maturity denominated in the same currency. They compare the interest on public and private financial instruments denominated in the same currencies in 12 OECD countries. But in doing so, they apply a variety of definitions for public and private yields, which hampers comparability between different yield measures. Their analysis reveals a strong correlation between the degree of public indebtedness and the interest rate spread between private and public rates of return, which they interpret as proof of the existence of a small but significant default premium on public debt. For a bond issued in a national currency the risk involved is related both to the possibility that government stops payments and to the possibility of monetization of the debt, leading to inflation and devaluation of the national currency. By comparing private and public bonds issued in the same currency, Alesina et al. (1992) avoid premia due to

devaluation expectations. On the other hand, it can be hard to tell whether variation in the discrepancies between the interest rate on public and private bonds is due to variation in public or in private interest rates. Therefore, using corporate debt as a benchmark complicates the identification of the variation in government default risk.

Favero et. al (1997) measure government default risk in Europe as $[(i - i^{ge}) - (i^{swap} - i^{ge,swap})]$, which is the difference between the total yield (i) differential and swap yield (i^{swap}) differential with Germany (denoted by ge) as the benchmark country. This was based on the reasoning that German yields have been the lowest on all maturities and German bonds provide the yardstick of a risk-free asset. The total yield differential is defined as the yield on a 10-year government bond in a particular currency minus the yield on a 10-year German government bond denominated in Deutsch mark. The swap yield differential is defined as the difference between the fixed interest on a 10-year swap contract in a particular currency minus the fixed interest on a 10-year swap contract in Deutsch mark. The spread on fixed interest rate swaps is an indicator of the exchange rate component of the total yield differential. After the introduction of the euro in 1999, swap differentials amongst EMU member states converged to zero and total yield differentials converged to differences between bond yields.

Another measure applied by Lemmen and Goodhart (1999), denoted as $i - i^{swap}$ is the spread of 10-year benchmark government bond yields over the corresponding swap yield of the same 10-year maturity denominated in the same currency. The measure is based on the assumption that the spreads reflect the credit risk of prime banks operating in the interbank market vis-à-vis the government

treasury. It further assumes that swap rates do not vary greatly from currency to currency as major banks post interest rate swap yields in a variety of currencies (McCauley 1996). Moreover, the private risks entailed in interest rate swap yields are much less than the risks in corporate bonds, as there is no principal at risk in an interest rate swap. Consequently a measure of default risk that uses interest rate swaps as opposed to corporate debt as the benchmark, is less sensitive to significant changes in private risk. Using this measure, Lemmen and Goodhart (1999) find a significant positive correlation between the first differenced government debt ratio and sovereign credit spreads in a sample of 13 EU member countries over the period 1987-1996. However, in the European context, this relationship may be a combination of market-based fiscal discipline and the rule-based fiscal convergence criteria imposed by the Maastricht Treaty.

D'Amato and Pistoresi (2001) study the determinants of the long term yield spread between Italian and German government bonds using daily observations over the period 1997-1999. The total spread is split into two main factors: an exchange rate factor, which is approximated by a differential on swap contracts and a default factor, which is treated as a residual. They use cointegration analysis to test if the interest parity condition holds in the period considered. Their main result is that although the usual uncovered interest parity condition does not hold, it may not be rejected if the relationship is augmented by the German short term interest rate.

Landschoot (2004) examines 7 EMU-countries over the 1990s and shows that governments can influence the credit spread on their long-term debt by changing the composition of the budget balance. As in Favero et al. (1997), Landschoot (2004)

models sovereign credit risk by assuming the covered interest parity condition. To the extent that sovereign credit risk depends on fiscal policy, a government may be able to lower the yield on its long-term debt by altering the balance between government consumption, government investment and social security expenditure and subsidies. Landschoot (2004) demonstrates that governments that invest more and spend less of their budget on consumer goods have significantly lower credit spreads.

Studies on the US have focused on the difference in borrowing conditions of states within the federal union. This relationship is similar to that currently prevailing amongst EMU member states as one can ignore inflation risk and expectations of exchange rate changes. Goldstein and Woglom (1991) use a data set of 41 US states and conclude that states which follow a more prudent fiscal policy are perceived by the market as having lower credit risk and are therefore able to reap the benefits of lower borrowing costs. Bayoumi et al. (1995) conclude that credit markets do appear to provide incentives for sovereign borrowers to restrain borrowing. They find a non-linear relationship between the government debt ratio and the spread on twenty-year bonds of US states, relative to the yield on a comparable New Jersey twenty-year state bond.

2.3 Credit risk in euro area government bonds

As shown in Figure 2.1, yields spreads of 10-year benchmark bonds issued by EMU member countries relative to German 10-year benchmark bonds narrowed considerably in the run up to the launch of the single European currency. Greece has not been included in the figure since in 1999, the convergence process to EMU was

still ongoing for Greece. Luxemburg is not included in the analysis on account of its tiny bond market.

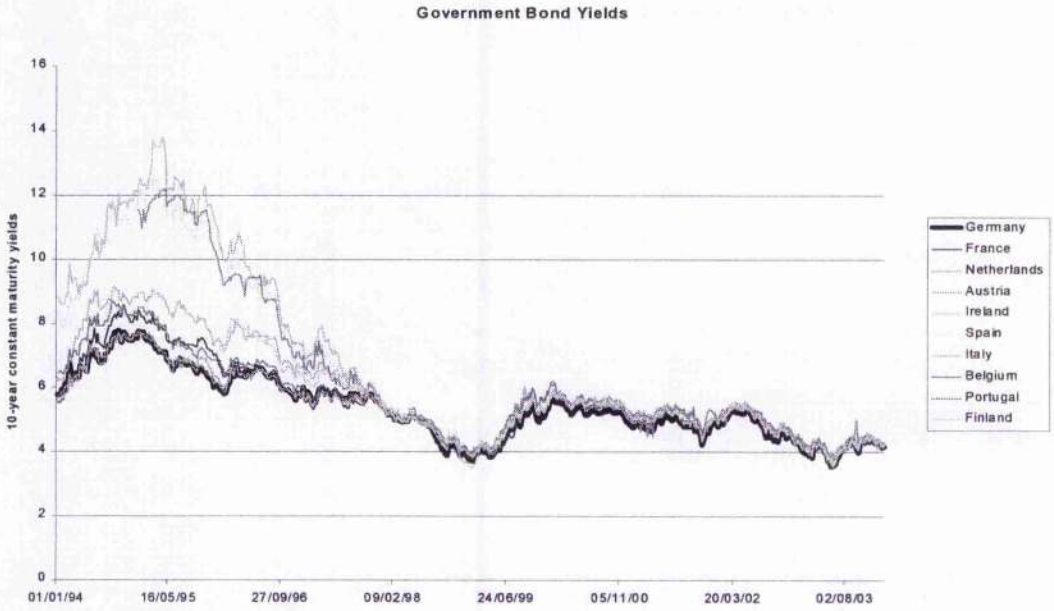

Figure 2.1 10-year government bond yields in the euro-zone

Table 2.1 shows that yield spreads on euro area government bonds converged to less than 50 basis points by 1998. Countries like Austria, Belgium, France and the Netherlands had relatively low yield spreads relative to Germany over the second half of the 1990s. Both Austria and the Netherlands had pegged their currencies to the Deutsche Mark and were effectively subject to German monetary policy. But what is noteworthy is that the former high inflation countries like Italy, Portugal and Spain experienced a dramatic reduction in bond yields. As soon as it became clear that these countries would be joining EMU, inflation expectations fell sharply, leading to a reduction in interest rates and a large reduction in the cost of servicing government debt.

Table 2.1
Pre-EMU 10-year government bond yield spread vis-à-vis Germany
Figures in basis points

	1994	1995	1996	1997	1998
AUSTRIA	17.5	31.5	10	1.8	15.4
BELGIUM	90	66.8	27.2	10.7	18.9
FINLAND	218.3	194.5	81.6	27.6	21.3
FRANCE	35.7	70.9	10.1	-9.2	8
IRELAND	109.4	143.6	107	63.1	22.2
ITALY	370.4	540.3	321.3	118.1	33.3
NETHERLANDS	0.98	7.6	-6.7	-7.8	6.4
PORTUGAL	359	463	235.4	70.2	26.3
SPAIN	317	447.5	252.8	73.8	26.8

Source: Datastream

Table 2.2 shows that despite such convergence, integration in euro area government bond markets is not complete as non-negligible differences in yield levels across government bonds persist . Average spreads of more than 10 basis points have separated French and Austrian 10-year yields from that of Germany even though they all share the same AAA rating. During the course of 2002 and 2003 German government bond yields have remained below those of other EMU member states such as Austria that have had better budgetary positions. The lower bond yields for German bonds may be attributed to their significantly greater liquidity that allows them to command a premium vis-à-vis smaller sovereign issues. The well developed Bund futures market enhances liquidity of the underlying German government bonds

and serves to increase the demand for such securities pushing up prices and lowering yields.

Table 2.2
Post-EMU 10-year government bond yield spread vis-à-vis Germany
Figures in basis points

	1999	2000	2001	2002	2003
AUSTRIA	30.4	24.7	23.7	14.4	7.6
BELGIUM	38.2	35	32.4	15.56	1.43
FINLAND	20.5	29.1	19.7	12.5	5.12
FRANCE	3.7	14.8	14.2	12	7.85
GREECE			51.9	31.4	18.1
IRELAND	14.2	21.6	30	22.4	18.1
ITALY	19.5	35	38.9	25.3	17.9
NETHERLANDS	13.5	12.6	14.5	9	6
PORTUGAL	31.3	32.4	29.4	22	18.9
SPAIN	21.8	27.2	34	15.12	5.2

Source: Datastream

Before the launch of the euro, cross-country yield spreads within participating member states could be explained by three basic factors: currency risk, credit risk and liquidity risk. The introduction of the euro eliminated the first factor (i.e. risks arising out of exchange rate fluctuations) creating conditions for a substantially more integrated debt market for the euro area. Although the anticipation of EMU had reduced exchange rate volatility among a few European Monetary System (EMS) member states in the second half of the 1990s to very low levels, exchange rate risk had been an important component of intra-European market risk in the 1990s. The

study by Favero et al (1997) finds that yield differentials between German and European high interest countries is largely determined by exchange rate factors and low default risk premia.

Blanco (2001) has broken down spreads over German yields at the 10-year maturity between foreign exchange and other factors, which he identifies with credit risk and market microstructure characteristics, in particular liquidity. He finds that for those countries with wide pre-1999 spreads, the main component was exchange-rate risk. Moreover, taking that factor out, spreads have in fact widened significantly for all countries since the introduction of the euro. Blanco (2001) states that this might partly reflect a change in price assigned by the market to credit and liquidity factors due to the higher degree of market integration. The argument being that before EMU differences in credit and liquidity were not completely priced due to market segmentation.

In a recent study Codogno (2003) et. al provide evidence that the movements in yield differentials on euro area government bonds are mostly explained by changes in international risk factors, as measured by US swap and corporate bond spreads relative to US Treasury yields. The international factors affect spreads because they change the perceived default risk of government bonds in the euro area. In their findings, liquidity factors play only a smaller role.

The spread between the 10-year benchmark government bond yields and the corresponding swap yield of the same 10-year maturity is now used to measure credit risk in 11 EMU member states and UK. A major determinant of the level of swap

rates relative to underlying bond yields is the perceived risk premium between sovereign borrowers and the interbank market. Swap spreads, therefore, reflect the difference between the default risk of the interbank market quoting Libor/Euribor rates and that of the government treasury. As swap yields are generally at a premium over the comparable government bond yields, $i - i_{swap}$ would be a negative magnitude and a narrower spread would be indicative of greater government default risk. As the interest rate swap yields for the EMU member states are all denominated in euros, fluctuations in this differential primarily reflect shifts in the credit risk of the country in question. As in Lemmen and Goodhart (1999), it is assumed that variations in yield differentials stem from credit risks and that variations in liquidity are negligible.

For 11 EMU member states: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain, the credit risk measure $i - i^{swap}$ is tracked over the period January 1999 to January 2004. In the case of Greece, which was formally admitted to the EMU in June 2000, the period of observation is January 2001 to January 2004. Daily data on 10-year benchmark government bond yields and interest rate swap yields have been obtained from *Datastream*. A graphical depiction of the time series of these spreads can be seen in Appendix 2.1. Two inferences can be made from these graphs. First, government default risk is higher in EMU member states as compared to the UK. Second, credit risk conditions in the UK and euro area government bond markets have deteriorated after 2002. Within the euro area there are significant variations in the magnitude of credit risk. In the case of Greece the spread has remained positive for most of the period while for Italy and Portugal it has remained positive over prolonged phases. What is noteworthy is the marked narrowing of the spread for AAA rated France and Germany.

The empirical analysis in subsequent sections will narrow down the coverage to credit risk in the three largest euro area government bond markets: France, Germany and Italy. These countries represent more than 70% of the European market for public debt (Mathieson and Schinasi (2001) and are known to possess the most liquid bond markets in Europe. Liquidity premiums in their prices are, therefore, not expected to play a significant role. The distinction between the pure credit spread and liquidity premium will not be made because of the difficulty in disentangling them. Liquidity in the over-the-counter Government securities markets is associated with daily market trading and is, in essence, supplied by dealers. Liquidity-related variables affect yields at high frequencies, while credit risk-related variables reflect slow-moving economic fundamentals (Codogno et al (2003)). An analysis of liquidity risk would involve examining intra-day observations whereas credit risk would be more discernible by looking at quarterly or monthly data.

2.4 Modelling sovereign credit risk

To explain sovereign credit risk in the context of government bonds one has to account for both default risk and recovery risk. Recovery risk concerns the uncertainty about the value of the bond that is faced once the bond defaults. For government bonds this would depend on the probability of a public bailout. Default risk refers to the probability that a country is unable or unwilling to pay its interest charges in a timely manner. Credit risk subsumes the risk of default as well as the risk of an adverse rating change. In other words, though the prospect of default is remote, there may be a more immediate possibility of a rating downgrade.

The credit risk model used in this chapter follows Favero et al. (1997), D'Amato and Pistoresi (2001), Landschoot (2004) by using the covered interest parity condition which describes a relationship between the spreads of bonds issued in different currencies but that are otherwise equivalent. It would, therefore, provide the link between yields on sterling and euro-denominated government bonds. Covered interest parity is an equilibrium condition which states, that under full integration, capital flows should equalise the returns on any two assets that differ only in their country of issue and currency of denomination, while being identical in terms of maturity, liquidity and default risk. In equilibrium, the interest differential on the two assets is equal to the forward premium or discount.

Empirical evidence shows that the interest parity condition is not always satisfied. Various studies have reported deviations from covered interest parity for a number of assets and currencies, suggesting unexploited profit opportunities. These potential profits, however, are reduced by causes of deviations such as default risk, taxation, capital market imperfections and transaction costs. Stoll (1972) and Adler and Dumas (1976) analysed deviations in terms of default risk. Frankel and Levich (1975) tested the parity conditions in terms of transaction costs. Levi (1977) highlights that deviations from covered interest parity may occur due to tax advantages. More recently, Taylor (1989) reported the presence of profitable opportunities from covered interest arbitrage after taking into account transaction costs based on bid and ask quotes used by traders. However, these tests were performed during periods when there was turbulence in the markets.

What distinguishes this chapter from most studies on covered interest parity is that it uses long-term bonds as opposed to short-term instruments. The covered interest parity condition between two default-free financial assets is given by:

$$(1+i_{j,t}) = (1+i_{k,t})\left(\frac{F_{t+m}}{S_t}\right)^{1/m} \qquad (2.1)$$

where $i_{j,t}$ and $i_{k,t}$ are respectively the annualised interest rates on sterling and euro treasury bonds issued at time t (maturing at time $t+m$). S_t is the spot exchange rate at time t, which is measured as the home (sterling) price of one unit of the foreign (euro) currency. F_{t+m} refers to the forward value of the spot exchange rate S for a contract expiring at $t+m$. The investor buying the euro-denominated bond at time t converts pounds into euros at the spot exchange rate and, the proceeds expected when the bond matures at time $t+m$ are simultaneously sold at the forward rate F_{t+m}.[3] To begin with, the interest parity condition was applicable to short-term financial assets such as treasury bills that are potentially switchable between currencies. This was due to the fact that forward currency markets have a limited time frame. However, in the 1980s the development of currency swap markets has enabled the hedging of risk at longer maturities.

[3] Hallwood and MacDonald (2000)

Taking the natural logarithm of the above equation gives[4]

$$\ln(1+i_{j,t}) = \ln(1+i_{k,t}) + \frac{1}{m}[\ln F_{t+m} - \ln S_t]$$

or $$i_{j,t} = i_{k,t} + \frac{1}{m}[f_{t+m} - s_t]$$

$$i_{j,t} - i_{k,t} = \frac{1}{m}[f_{t+m} - s_t] \qquad (2.2)$$

The logarithm of F and S are indicated with lower case letters (f and s). Equation (2.2) is the logarithmic transformation of the CIP condition in equation (2.1). The CIP condition requires perfect capital mobility and the absence of default risk. These conditions do not hold in all circumstances, and this may explain the persistence of interest rate differentials that are not explained by forward/short rate premia. This means that the underlying assets $i_{j,t}$ and $i_{k,t}$ are not fully comparable. The literature has accounted for this imperfect asset substitutability leading to deviations from CIP by including factors such as default risk, tax rate differentials, transaction costs and capital market imperfections.

In the context of increased credit risk witnessed in the Eurozone the analysis presented here tries to rationalise deviations from CIP over the 1999-2003 period in terms of default risk. By introducing credit risk and assuming risk neutral creditors, the covered interest parity condition given by equation (1) becomes

[4] $\ln(1+x) \approx x$ for small values of x.

$$[1-p_j(x_j)](1+i_j)+p_j(x_j)(1-\alpha_j)(1+i_j)=[1-p_k(x_j)](1+i_k)(\frac{F_{t+m}}{S_t})^{\frac{1}{m}}+p_k(x_k)(1-\alpha_k)(1+i_{k,t})(\frac{F_{t+m}}{S_t})^{1/m}$$

(2.3)

where p_j, p_k represents the probability of default of the sterling bond and the euro-denominated bond respectively. x_j, x_k measures the determinants of default. α_j, α_k represents the fraction of the cost to the creditor due to default. $(1-\alpha_j)$ and $(1-\alpha_k)$ can, therefore, be viewed as the degree of recovery in the event of default on sterling and euro denominated bonds respectively. Equation follows Favero et al. (1997) where the existence of a spread between the yields on government bonds is attributed to an exchange rate factor and to a default risk factor.

Upon rearrangement, equation (3) can be written as:

$$(1+i_{j,t})[1-\alpha_j p_j]=(1+i_{k,t})[1-\alpha_k p_k](\frac{F_{t+m}}{S_t})^{\frac{1}{m}} \qquad (2.4)$$

As defined in Edwards (1986) and Favero et al. (1997) cr_t is a measure of the incidence of default at time t for a government bond with time to maturity m, which takes account of the cost to the creditor in the case of default.

$$cr_t=\frac{\alpha p(x)}{1-p(x)} \qquad (2.5)$$

cr will be zero when both α and $p(x)$ are zero and will become infinity when both α and p are equal to one.

Substituting the measure of the incidence of default into equation (2.4) gives:

$$(1+i_{j,t})[1-(1-\alpha_j p_j(x_j)cr_{j,t}] = (1+i_{k,t})[1-(1-\alpha_k p_k(x_j)cr_k p_k](\frac{F_{t+m}}{S_t})^{\frac{1}{m}}$$

$$(1+i_{j,t})[1-cr_{j,t}+cr_{j,t}\alpha_j p_j(x_j)] = (1+i_{k,t})[1-cr_{k,t}+cr_{k,t}\alpha_k p_k(x_j)](\frac{F_{t+m}}{S_t})^{\frac{1}{m}}$$

Now taking natural logarithms[5] the yield spread can be decomposed into two components: (i) the expected exchange rate change and (ii) the credit spread between country *j* and country *k*.

$$\ln(1+i_{j,t})+\ln[1-cr_{j,t}+cr_{j,t}\alpha_j p_j(x_j)] = \ln(1+i_{k,t})+\ln[1-cr_{k,t}+cr_{k,t}\alpha_k p_j(x)]+\frac{1}{m}\ln[F_{t+m}-S_t]$$

or, $$i_{j,t}-cr_{j,t} = i_{k,t}-cr_{k,t}+\frac{1}{m}(f_{t+m}-s_t)$$

$$i_{j,t}-i_{k,t} = cr_{j,t}-cr_{k,t}+\frac{1}{m}(f_{t+m}-s_t) \qquad (2.6)$$

Equation (2.6) can be rewritten as

$$Sp_t = Er_t + Dr_t \qquad (2.7)$$

where

$Sp_t = i_{j,t} - i_{k,t}$ is the total spread between sterling and euro-denominated bonds

$Er_t = \frac{1}{m}(f_{t+m} - s_t)$ is the expected exchange rate change

$Dr_t = cr_{j,t} - cr_{k,t}$ is the credit spread

The main problem in testing this equilibrium condition is that the two terms on the right hand side of equation (2.7) are not observable. A way out is to try and measure one of the terms independently and treat the other as a residual. As in Favero et. al. (1997) and D'Amato and Pistoresi (2001) the expected exchange rate change Er_t is measured as the interest rate differential on swap contracts denominated in currencies j and k. In that case,

$$Er_t = i^s_{j,t} - i^s_{k,t}$$

where, in this case, $i^s_{j,t}$ and $i^s_{k,t}$ are the fixed rate payment stream of sterling and euro interest swap with the same maturity. If the exchange rate component is measured as the spread between the offer rate on the fixed income side of swap contracts with the same maturity, the sovereign credit spread of country j compared to country k, $[cr_{j,t} - cr_{k,t}]$, can be estimated as the yield spread between country j and country k, $[i_{j,t} - i_{k,t}]$, minus the exchange rate component of the spread, Er_t.

[5] It is assumed that cr is small enough such that ln (1+cr)=0. In the context of EU countries it is not an unreasonable assumption to make.

$$cr_{j,t} - cr_{k,t} = (i_{j,t} - i_{k,t}) - ER_t \qquad (2.8)$$

Incorporating credit risk spreads into the model would necessitate assumptions to be made about the functional form of the probability of default and the loss rate in the case of default. For simplicity, it is assumed that in the case of default, the borrower repays nothing (α =1). In accordance with Edwards (1986) and Bayoumi et al. (1995), it has been assumed that the probability of default has a logistic form:[6]

$$p(x) = \frac{\exp \sum \beta_i x_i}{1 + \exp \sum \beta_i x_i} \qquad (2.9)$$

where the x_i s are independent determinants of the probability of default (including the level of indebtedness) and the β_i s are the corresponding coefficients. Substituting the probability of default into equation (2.5), and taking logarithms or logit form, the sovereign credit spread of country j can be written as

$$\ln cr_t = \ln\left(\frac{p(x)}{1-p(x)}\right) = \ln p(x) - \ln[1 - p(x)]$$

$$= \ln\left[\frac{\exp \sum \beta_j x_j}{1 + \exp \sum \beta_j x_j}\right] - \ln\left[\frac{1}{1 + \exp \sum \beta_j x_j}\right]$$

$$= \sum \beta_j x_j \qquad (2.10)$$

Similarly the credit risk of country k can be written as $\sum \beta_k x_k$

[6] Logit $p(x) = \ln[\frac{p(x)}{1-p(x)}] = \sum \beta_i x_i$

Regarding the determinants of sovereign credit a number of variables have been discussed in the literature. In the context of 13 EU member countries over the period 1987-1997, Lemmen and Goodhart (1999) have identified the determinants of government default risk to include the government's tax raising capability, government's ability to control spending, government's debt management policies. Given the time frame from 1999-2003 and the fact that I am considering countries following a common monetary policy, only the fiscal determinants of default risk would be appropriate in this analysis. In order to differentiate between the different degrees of default risk in Germany, France and Italy, the two variables considered are the deficit/GDP ratio and the ratio of net government interest payments to government receipts. The former is in conformity with the Maastricht Treaty commitment that countries using the euro must keep their budget deficits within 3 per cent of gross domestic product. The latter is akin to the notion of credit worthiness in corporate finance which is measured as a measured as the ratio of debt service to cash flows. Large current and previous borrowing means that the country has a debt service burden which might increase the possibility of default.

2.5 Data

The empirical analysis is confined to four EU member countries: UK, Germany, France and Italy over the period January 1999 to January 2004. Daily data on 10-year benchmark government bond yields and the fixed rate offer yield of 10-year interest rate swaps have been obtained from *Datastream*. 10-year euro swap rates are based on 6-month euribor while 10-year sterling swap rates are based on 6-month

sterling libor. For the purposes of modelling credit risk quarterly averages of the daily data were calculated. **Table 2.3** provides summary statistics of the time series of bond and interest swap yields.

Table 2.3 Summary statistics of bond and swap yields

10-year Benchmark Government Bond Yields (%)		
	Mean	Std. Dev
UK	4.90	0.39
Germany	4.68	0.50
France	4.79	0.54
Italy	4.96	0.56

10-year Sterling & Euro Interest Rate Swap Yields (%)		
	Mean	Std. Dev
Sterling	5.56	0.64
Euro	5.04	0.61

Table 2.4 shows the mean and standard deviation of yield spreads and credit spreads of the three EMU countries compared to UK 10-year government bonds. The mean of the credit spreads was negative for all the three countries and ranged from -31 basis points for Germany to -58 basis points for Italy. The negative sign for credit spreads indicate that, on average, British government bonds had a lower default probability than German, French and Italian bonds during the period of observation.

Table 2.4 Summary statistics of yield and credit spreads

	Yield Spread*		Credit Spread*	
	mean	st.dev	mean	st.dev
Germany	21	23	-31	20
France	10	26	-42	23
Italy	-6	29	-58	24

* Figures are in basis points(bp) [1 bp = .01 per cent]

Data on the components of the two fiscal variables, government sector deficit/surplus as a percentage of GDP and net debt interest payments as a percentage of government total receipts have also been sourced from *Datastream* on a quarterly basis.

2.6 Empirical Analysis

2.61 Testing for covered interest rate parity

The interest rate parity condition given by equation (2.7) would hold as an equilibrium condition if a cointegrating relationship exists between *Sp* and *Er*. Cointegration is based on the idea that while a set of variables are individually nonstationary, a linear combination of the variables might be stationary. The intuition is that economic forces should avoid persistent long run deviations from equilibrium conditions although short run deviations may be observed. This implies that although the government bond yield spreads and swap spreads are individually unbounded, they cannot drift too far apart arbitrarily as there may exist a long-run relationship linking them. The essence of the cointegration relationship among two variables is that they share a common unit root process. When this occurs it is possible to construct a stationary variable from the linear combination of the two-non stationary variables. So before performing the cointegration test, the series of interest rate spreads are tested for a unit root.

An example of a stationary time series is the process generated by an autoregressive model of order 1, the AR(1) model. I consider a simple version of the AR(1) model for credit spreads, denominated as y_t, and given by the equation,

$$y_t = \alpha y_{t-1} + \varepsilon_t \qquad (2.11)$$

where ε_t is an independent and identically distributed (i.i.d) random disturbance term with mean 0 and variance σ^2 for all t written $\varepsilon_t \sim$ i.i.d $(0, \sigma^2)$. Statistical tests of the null hypothesis that a time series is non-stationary against the alternative hypothesis that it is stationary are called unit root tests. The AR(1) model (2.11) is non-stationary if $\alpha = 1$. When $\alpha = 1$ the AR(1) becomes the random walk model,

If $|\alpha| < 1$, the AR(1) process is stationary. We can test for nonstationarity by testing the null hypothesis that $\alpha = 1$ against the alternative hypothesis that $|\alpha| < 1$, or simply $\alpha<1$. To test that $\alpha = 1$, it is not sufficient to estimate α and then use a simple t-test since these are biased in the case of a unit root. But using the first difference operator by subtracting y_{t-1} from both sides of equation (2.11) the AR(1) model can be rewritten as equation (2.12).

$$y_t - y_{t-1} = \alpha y_{t-1} - y_{t-1} + \varepsilon_t$$

$$\Delta y_t = (\alpha - 1) y_{t-1} + \varepsilon_t \qquad (2.12)$$

$$\Delta y_t = \beta y_{t-1} + \varepsilon_t \qquad (2.13)$$

where $\Delta y_t = y_t - y_{t-1}$ and $\beta = \alpha - 1$. Then

$$\begin{aligned} H_0 &: \alpha = 1 \leftrightarrow H_0 : \beta = 0 \\ H_1 &: \alpha \prec 1 \leftrightarrow H_1 : \beta \prec 0 \end{aligned} \qquad (2.14)$$

Dickey and Fuller (1979) showed that standard t-ratios based on (2.12) are biased and that appropriate critical values have to be increased by an amount that depends on the sample size. The test using these critical values is called a Dickey-Fuller (DF) test.

If y_t follows a random walk, then $\beta = 0$ and

$$\Delta y_t = y_t - y_{t-1} = \varepsilon_t \qquad (2.15)$$

Series like y_t which can be made stationary by taking the first difference are said to be integrated of order 1, and denoted I(1). A random walk is an integrated process of order 1. Stationary series are said to be integrated of order zero, I(0).

In addition to testing if a series is a random walk, Dickey and Fuller (DF) also developed critical values for the presence of a unit root in the presence of a drift.

$$\Delta y_t = c + \beta y_{t-1} + \varepsilon_t \qquad (2.16)$$

The inclusion of this constant term, c, allows for a trend and is important as asset prices and other financial variables often exhibit trends.

To control for the possibility that the error term in the DF regression equation (2.16) is autocorrelated, lagged dependent variables can be added. The number of lags included should be just sufficient to remove any autocorrelation in the errors, so that

OLS will give an unbiased estimate of the coefficients of y_{t-1}. The modified model is

$$\Delta y_t = c + \beta y_{t-1} + a_1 \Delta y_{t-1} + ... + a_m \Delta y_{t-m} + \varepsilon_t$$

which can be written as

$$\Delta y_t = c + \beta y_{t-1} + \sum_{i=1}^{m} a_i \Delta y_{t-i} + \varepsilon_t \qquad (2.17)$$

Testing the null hypothesis that $\beta = 0$ in the context of the model given by equation (2.17) is known as the Augmented Dickey-Fuller (ADF) test. The test critical values are the same as for the DF test.

Table 2.5 shows the results of the unit root tests on the time series of bond yield (*Sp*) and swap spreads (*Er*) for Germany, France and Italy with respect to UK. The interest rate swap differential between 10-year sterling and euro swap spreads (*Er*) would be the same for all the three EMU countries. The ADF statistic is the *t*-ratio on the lag coefficient y_{t-1} which, in all four cases, is too small to reject the null hypothesis that the series are non-stationary. The 5% critical value of the ADF distribution is -2.86 and this exceeds the *t*-statistics of -1.86, -1.33 , -1.14, -1.83 obtained for German, French and Italian bond yield spreads and sterling-euro swap spreads respectively. Thus the bond yield spreads (*Sp*) and interest rate swap spreads (*Er*) both have a unit root allowing for the possibility of cointegration which is tested for using the Johansen (1988) methodology for cointegration. The non-stationarity of the both the spreads may be attributed to the wide variability of UK bond and swap yields as there was no evidence of any structural breaks on the lines of the UK's departure from the

EMS in 1992 or the German re-unification in 1991.

Table 2.5 ADF Unit Root Tests

	ADF regression coefficients			
	Sp (Germany)	*Sp* (France)	*Sp*(Italy)	*Er*
y_{t-1}	-0.01	-0.008	-0.004	-0.005
	(-1.86)	(-1.33)	(-1.14)	(-1.83)
c	0.002	0.001	-0.00002	0.003

The figures in parentheses indicate the ADF test statistic

Generalization of the unit root tests described above for a VAR(1) process motivates the Johansen tests for a common stochastic trend or cointegration in the credit spread series.[7] A vector autoregression of order 1 on a bivariate system is

$$y_{1,t} = \alpha_{10} + \alpha_{11} y_{1,t-1} + \alpha_{12} y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = \alpha_{20} + \alpha_{21} y_{1,t-1} + \alpha_{22} y_{2,t-1} + \varepsilon_{2,t}$$

The equations can be represented in matrix notation as

$$y_t = \alpha_0 + A y_{t-1} + \varepsilon_t \qquad (2.18)$$

where $y_t = (y_{1,t}, y_{2,t})', \alpha_0 = (\alpha_{10}, \alpha_{20})', \varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$ and

$$\mathrm{A} = \begin{pmatrix} \alpha_{11} \alpha_{12} \\ \alpha_{21} \alpha_{22} \end{pmatrix}$$

[7] Enders (1995), Alexander (2001).

In a VAR process, each variable is expressed as a linear combination of lagged values of itself and lagged values of all other variables in the group. The basic form of VAR treats all variables symmetrically without making reference to the issue of dependence versus independence. The behaviour of the y's will depend on the properties of the A matrix.

The VAR(1) model (2.18) may be rewritten with Δy_t as the dependent variable in a regression on y_{t-1} such that

$$\Delta y_t = \alpha_0 + (A - I) y_{t-1} + \varepsilon_t \qquad (2.19)$$

Now if each variable in y is $I(1)$ then each equation in (2.19) has a stationary variable on the left-hand side. The errors are stationary and therefore each term in $(A-I)y_{t-1}$ must be stationary for the equation to be balanced. If $(A-I)$ consists of all zeros, so that rank of $(A-I) = 0$, nothing can be concluded about the relationship between the y variables. But if $A-I$ has rank $r>0$, then there are r independent linear relations between the y variables that must be stationary. Therefore the $I(1)$ variables in y will have a common stochastic trend - that is, they will be cointegrated - if the rank of $A-I$ is non-zero; the number of cointegrating vectors is the rank of $A-I$. In the two variable case, there can be at most one linear combination of $y_{1,t}$ and $y_{2,t}$ that is stationary. If the interest rate parity condition given by equation (2.7) holds, there should be one cointegrating relationship between Sp and Er. The rank of a matrix is given by the number of non-zero eigenvalues, so the Johansen procedure based on (2.19) tests for the number of non-zero eigenvalues in $A-I$.

In practice the VAR equations may be expanded to include deterministic time trends and other exogenous variables. If a higher-order VAP(p) model is used to motivate the Johansen tests, the first difference formulation becomes

$$\begin{aligned}\Delta y_t = a_0 &+ (A_1 - I)\Delta y_{t-1} + (A_1 + A_2 - 1)\Delta y_{t-2} + \ldots \\ &+ (A_1 + A_2 + \ldots + A_{p-1} - I)\Delta y_{t-p-1} \\ &+ (A_1 + A_2 + \ldots + A_p - I) y_{t-p} + \varepsilon_t \qquad (2.20)\end{aligned}$$

and the Johansen method is a test for the number of non-zero eigenvalues of the matrix

$$\Pi = A_1 + A_2 + \ldots + A_p - I$$

Johansen (1988) shows that the number of cointegrating vectors, R, equals the rank of Π given by r. He provides two likelihood ratio tests for determining r based on the number of nonzero eigenvalues in Π. The first test, the maximal eigenvalue test, is really a sequence of tests. After sorting the estimated eigenvalues of Π in descending order, the R-th statistic provides a test of the null hypothesis that $r = R$ against the alternative that $A - I$. The second test statistic, the trace statistic is the running sum of the maximum eigenvalue statistics. The R-th trace statistic provides a test of the null hypothesis that $r \leq R$ against the alternative that $r \succ R$. Critical values of these test statistics are given in Osterwald-Lenum (1992).

Table 2.6 shows the results of using the Johansen procedure to test for cointegration between bond yield and swap spreads for Germany, France, and Italy with respect to the UK. For all the three countries the trace statistics are not significant at the 5% level. The trace statistics of 10.33, 11.01 and 10.37 for Germany, France and Italy respectively, fail to reject the null hypothesis that there are no cointegrating vectors. The results for all the three series are based on using four lags of the data in the estimation. The lag length was determined by the Schwartz Information Criterion

Table 2.6 Johansen Cointegration test for bond and swap spreads

	Trace Statistic
Germany	10.33
France	11.01
Italy	10.37

Johansen likelihood ratio tests fail to reject the null hypothesis of no cointegrating relation at the 5% significance level (5% critical value = 15.41)

The cointegration tests reveal that the interest parity condition describing a long-run relationship between yields on government bonds issued by the UK and the three EMU countries does not hold.

2.62 Estimating the sovereign credit risk model

In order to make use of the panel characteristics of the data, the sovereign credit risk model is estimated for a panel data set of the three EMU countries using a one-way fixed effects model. The credit spread between UK and the EMU countries is the dependent variable and the two fiscal variables are the independent variables. The model can be specified as follows:

$$cr_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$$

where i is the cross-section dimension and represents the countries in the sample, and t is the time series dimension. x_i refers to the default probability of government bonds in country i. β are the coefficients of the explanatory variables x_{it}. ε_{it} is the error term for country i in period t. The intercept term α_i is called an unobserved individual effect that varies across countries or the cross section unit but is constant across time. It controls for country-specific omitted variables such as differences in the taxation system across these EMU member states and the gamut of political economy factors affecting the credit spread which are assumed to be constant through time. If the α_i's are assumed to be fixed unknown parameters, the model is referred to as a fixed effects model, which has the advantage that it does not impose any restrictions on the relation between explanatory variables and the fixed effects. If the α_i's are assumed to be random, i.e. drawn from a distribution with mean zero and a

variance σ_{α}^{2}, the model is referred to as a random effects model. In that case, the α_i's should be uncorrelated with the explanatory variables. The one-way fixed effects model, estimated here, implies that all countries have the same β coefficients for the explanatory variables but that the intercepts vary across countries. The two-way fixed-effects model, allows for separate β coefficients of the explanatory variables for different time intervals to see whether the β coefficients change over time. A two-way fixed effects model was not considered appropriate given the limited period of observation.

The dataset consists of a balanced panel of 3 Eurozone member countries (Germany, France, Italy) over the period 1999-2003. The results of the one-way fixed effects estimates in which UK 10-year bonds are treated as the benchmark are reported in **Table 2.7**. To overcome the problem associated with heteroskedasticity in the residuals, all standard errors are computed using White's method. White (1980) has derived a heteroskedasticity consistent covariance matrix estimator which provides correct estimates of the coefficient covariances in the presence of heteroskedasticity. The F-test on the absence of fixed effects is rejected in the regression.

An obvious limitation of this estimation is the size of the cross-sectional dimension which could have been made larger by including more EMU member states. The advantage of panel analysis is that it allows for more observations by the pooling of cross section and time series data that leads to more degrees of freedom. Including more Euro area countries in the panel was largely constrained by the availability of uniform data and this issue will be addressed if a more disaggregated

data set can be accessed.

Table 2.7

The table represents the results of one-way fixed effects estimates on a panel data set of 3 EMU-countries over the observation period 1999-2003 using quarterly data. The dependent variable comprises the sovereign credit spreads with respect to UK 10-year governments bonds for German, French and Italian government bonds of the same maturity. The explanatory variables are government deficit/GDP ratio (DEF) and the ratio of net government interest payments to government receipts (DSR). Pooled least squares regressions are estimated with White standard errors and associated t-statistics.

	Regression
Explanatory variables	
β_{DEF}	0.04 (0.0094) (4.39)
β_{DSR}	0.20 (0.0337) (5.96)
Fixed Effects	
α_{BD}	-0.86
α_{FR}	-0.59
α_{IT}	-1.72
Adj. R^2	0.52
S.E. of regression	0.17
N	60
F-test for fixed effects	17.13
(p-value)	(0.00)

The regression results in **Table 2.7** show the coefficients of explanatory variables have the expected sign and are significant at the 1% level. As this is a fixed effects regression, the partial impact stays constant over time and across countries. A higher deficit/GDP ratio and a higher ratio of net government interest payments to government receipts in EMU member countries would increase the magnitude of their credit spreads with respect to the UK. The coefficient of 0.04 suggests that if the budget deficit in the EMU governments was to increase by 1%, the credit spread

would increase by 4 basis points (100 basis points = 1%). More significantly, if the ratio of net government interest payments to government receipts were to increase by 1%, the credit spread would increase by 20 basis points. This conforms to the results obtained by Goldstein and Woglom (1991), Alesina et al. (1992), Bayoumi et al. (1995), Lemmen and Goodhart (1999), Landschoot (2004), that an increase in the government debt ratio significantly increases sovereign credit spreads.

2.7 Conclusion

This chapter has examined how the loss of monetary sovereignty coupled with the "No bail-out clause" of the Maastricht Treaty has exposed euro area government bond markets to increased credit risk as compared to the UK, which by virtue of being outside EMU retains its independent monetary policy.

EMU has altered the risk profile of public debt as governments have surrendered their monetary sovereignty and along with it the right to print money to pay off domestic currency debt. Studies have shown that before the launch of the euro, cross-country yield spreads within participating member states were largely determined by exchange rate factors. With the elimination of risks arising out of exchange rate fluctuations credit risk is the major source of risk in euro-denominated government bonds.

Credit risk has been measured as the spread of the 10-year benchmark government bond yields over the corresponding interest rate swap yield. As swap yields for the EMU member states are all denominated in euros, fluctuations in this differential would primarily reflect shifts in credit risk. Tracking this measure over the period January 1999 to January 2004 shows that government default risk is higher in the euro area as compared to the UK and that the magnitude of risk has increased after 2002.

Credit spreads between the UK and the euro area are then analysed using a credit risk model based on the covered interest parity condition. Cointegration

analysis shows that the covered interest parity condition does not hold between the UK and three major euro-denominated government bond markets of Germany, France and Italy. In view of the increased credit risk witnessed within EMU member states deviations from interest parity have been rationalized by the default risk factor.

Empirical results show that the credit spread between the UK and the three EMU member states can be attributed to the latter's fiscal performance. The credit risk spread increases with an increase in their deficit/GDP ratio and a higher ratio of net government interest payments to government receipts. This is consistent with the proposition that although default by an EMU member may be a remote possibility, financial markets can effectively restrict a government's ability to raise debt by imposing a risk premium as a compensation for increased sovereign credit risk.

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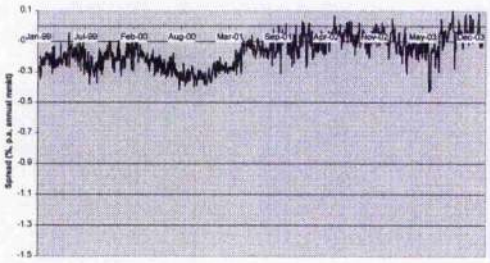
Appendix 2.1

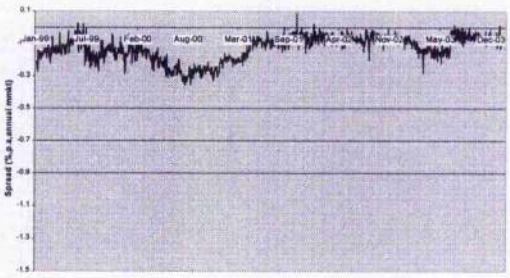
Spreads of 10-year Benchmark Government Bond Yields

over 10-year Interest Rate Swap Yields ($i - i^{swap}$)

AUSTRIA



BELGIUM



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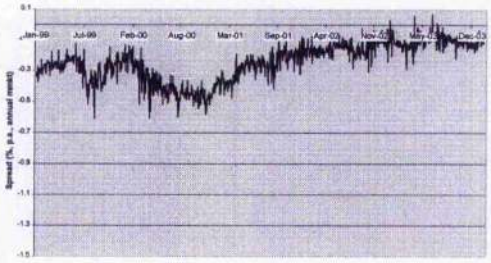

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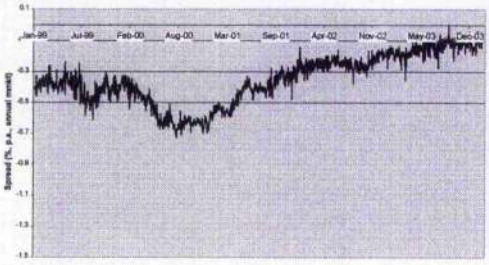

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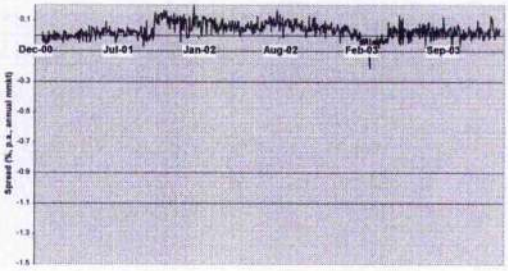

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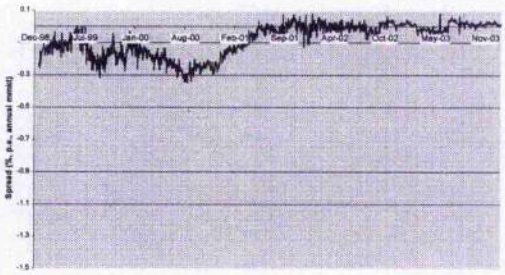
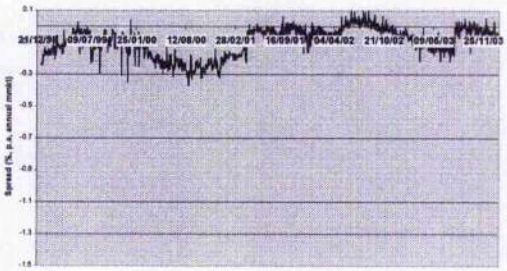


NETHERLANDS

PORTUGAL



SPAIN

UNITED KINGDOM

CHAPTER 3

AN INVESTIGATION INTO THE LINKAGES BETWEEN EURO AND STERLING SWAP SPREADS

3.1 Introduction

The observed difference between the swap rate and the government bond yield of corresponding maturity is known as the swap spread. If swap rates incorporate the risk of default they would be sensitive to the credit ratings of the counterparties. A swap dealer who pays a floating rate and receives fixed payments in exchange would require a BBB-rated counterparty to pay a higher fixed rate compared to a AAA-rated counterparty. Conversely, the dealer would be willing to make a lower fixed payments in exchange for floating rate payments to a BBB-rated counterparty than a AAA-rated counterparty. Hence, the swap spread, should be greater for the BBB-rated counterparty than the AAA-rated counterparty. As explained in Chapter 1, swap spreads reflect the default risk of the interbank market quoting Libor/Euribor rates and of the government treasury.[1] Fixed income securities, including corporate bonds and mortgage-backed securities use interest rate swap spreads as a key benchmark for pricing and hedging.

The importance of interest swap spreads derives from the dramatic recent growth in the notional amount of interest rate swaps outstanding relative to the government bond markets. After the introduction of the single currency, the euro

[1] For sterling fixed/floating swaps, the reference rate is by convention GBP 6-month Libor. For euro fixed/floating swaps, the reference rate is EUR 6-month Euribor.

swap market has nearly doubled in size and grown much faster than the bond market.[2] This can be attributed to the lack of homogeneity in the euro-denominated government securities market inducing a shift to interest rate swaps for hedging and positioning activity.

Swap spreads can be volatile and this has been very much in evidence during recent years. The Russian debt crisis in the autumn of 1998 and the subsequent near collapse of Long-Term Capital Management (LTCM), [3] resulted in a flight to UK, US, and German government bonds which lowered yields and widened swap spreads. This "flight-to-quality" caused by concerns about a systematic meltdown in the financial sector, had a profound effect on the importance of the swap market. A Treasury yield does not incorporate the risk premium that characterises a swap spread. Traditionally, it was the risk-free nature of the Treasury yield curve that necessitated its choice as a benchmark. During the 1998 financial crisis, the flight-to-quality bid that occurred in Treasury bonds, depressed their yields below "true" nominal risk-free rates and resulted in a steep increase in risk premiums. This impinged on the efficacy of Treasury bonds as benchmarks. As the market for Treasury bonds decoupled from other asset classes, market participants who hedged their portfolios with Treasury securities found themselves being adversely affected.

In the literature, swap spreads have been attributed mainly to two factors: the credit risk of counterparties giving rise to a default premium and, the liquidity of the swap market relative to the government securities market giving rise to a liquidity premium. Sun, Sundaresan and Wang (1993), Sorensen and Bollier (1994), Brown,

[2] Remolona and Wooldridge (2003)
[3] Edwards (1999)

Harlow and Smith (1994) are among those arguing in favour of default risk as a primary determinant of swap spread changes. On the other hand, Grinblatt (1995) and Liu, Longstaff and Mandell (2002) support the view that liquidity risk is a more plausible determinant of swap spreads than credit risk. Duffie and Singleton (1997) find that both credit and liquidity risk affect the behaviour of swap spreads but at different time horizons. Liquidity factors are more important in short horizons while credit shocks are more significant over long horizons.

The purpose of this chapter is not to analyse the determinants of swap spreads, but rather the dynamic behaviour of swap spreads. In particular, I am focussing on the transmission of information across the euro and sterling fixed income markets and exploring volatility interdependencies. Time series models of asset returns have emphasised stylised facts in the form of volatility clustering, whereby one period of high volatility is followed by more of the same, and then successive episodes of low volatility. Generalised autoregressive conditional heteroskedastic (GARCH) processes which parameterise time-varying conditional variances are able to capture this behaviour.

There have been several studies that have employed GARCH models for examining how news from one international market influences other markets' volatility process. For stock markets, Hamao, Masulis and Ng (1990) use the GARCH-M model to show that volatility spillovers exist from New York to Tokyo, London to Tokyo and New York to London. For currency markets, Engle, Ito and Lin (1990) use a GARCH model to find that Japanese news has the largest impact on the volatility spillovers of the yen/dollar exchange rates.

In the context of fixed income markets, Tse and Booth (1996) use US Treasury bill and Eurodollar futures to investigate volatility spillovers between US and Eurodollar interest rates. A bivariate EGARCH model that allows for the asymmetric volatility influence of the interest differential between markets (Eurodollar minus Treasury rate or the TED spread) as well as that of the domestic market, is used to analyse the volatility spillovers between markets. The results show that although the cross-market volatility effects are insignificant, the lagged TED spread is the driving force of the volatility process.

Eom, Subrahmanyam and Uno (2002) analyse the transmission of credit risk between Japanese yen and U.S. dollar interest rate swap markets between 1990 and 2000. Although they observed low correlations between yen and dollar interest rate swap spreads, they found that dollar interest rate swap spreads "Granger-cause" the changes in the yen swap spreads, for the 10-year maturities. Using a GJR-GARCH model to capture the asymmetric effects in the volatility process, they show that there is a strong transmission of volatility from the dollar swap spread to the yen swap spread.

The methodology used in this chapter follows that originally employed by Hamas, Masulis and Ng (1990) and also draws on the framework adopted by Eom, Subrahmanyam and Uno (2002).

The motivation for this chapter is the consideration that a comprehensive study on the linkages between euro and sterling swap markets has not been

undertaken so far. An investigation of the euro and sterling swap markets would promote a better understanding of the degree of integration, if any, between the fixed income segments of their respective financial markets. The flow of information between financial markets is an issue that has attracted considerable attention in the financial economics literature. Research in this area examines the extent to which a price shock in one market affects returns and volatilities in other markets. However, most of these studies focus on inter-linkages between equity markets rather than fixed income markets. Although a lot of research has been devoted to the determinants of swap spreads, the issue of international linkages between them has not been so well addressed.

The chapter is organised as follows. Section II provides a description of the data used and makes some inferences about the term structure of euro and sterling swap spreads. Section III attempts to trace the variability in these swap spreads to important economic events affecting the euro and sterling fixed income markets. Section IV examines the contemporaneous and causal relationship between euro and sterling interest rate swap spreads. Section V estimates the volatility in euro and sterling swap spreads and investigates the possibility of volatility spillovers between these markets. Section VI concludes the chapter.

3.2 Data and summary statistics

The euro swap rates used in this study are quoted rates from the fixed interest branch of a generic interest rate swap of 2-, 3-, 5-, 7-, and 10-years. Daily quoted rates were obtained from *Datastream* which are the average of bid and ask rates. These data cover the period from January 29, 1999 to March 28, 2003. The euro

swap spread is calculated by subtracting the swap rate from constant maturity yields of German government bonds with corresponding maturities, which were also obtained from *Datastream*. Yields on government securities at constant maturity are constructed by a country's treasury department for each business day, based on yields on actively traded marketable treasury securities. The dataset consists of 218 weekly observations and 1086 daily observations.

Table 3.1 Summary Statistics of the Euro Swap Spreads

Euro swap spreads defined as the difference between euro swap rates
and constant maturity yields of German sovereign bonds with the corresponding maturity.
Panel A provides the mean, standard deviation, skewness, kurtosis and ADF test for
Non-Stationarity where the critical t-ratio at the 5% level of significance is -2.87.
The tests for integration of order zero or, or I(0), are carried out on the levels of the variables
and the tests for integration of order one, or I(1), are carried out on their first differences.
Daily data are used from 29 January 1999 to 28 March 2003 (total 1086 observations).
Panel B provides the same summary statistics for weekly Euro swap rates
with 218 observations.

Panel A: Daily Observations

Maturity	Mean	Std. Dev	Skewness	Kurtosis	ADF t-stat for I(0) test	ADF t-stat for I(1) test
2 year	0.16	0.06	0.38	3.03	-2.04	-18.36
3 year	0.20	0.07	0.38	2.75	-2.36	-17.99
5 year	0.24	0.10	0.59	2.33	-2.28	-21.23
7 year	0.29	0.12	0.60	2.36	-1.67	-20.80
10 year	0.37	0.15	0.32	2.06	-1.47	-22.08

Panel B: Weekly Observations

Maturity	Mean	Std. Dev	Skewness	Kurtosis	ADF t-stat for I(0) test	ADF t-stat for I(1) test
2 year	0.16	0.06	0.46	2.85	-2.41	-16.94
3 year	0.20	0.07	0.41	2.67	-2.62	-17.19
5 year	0.24	0.10	0.57	2.34	-1.93	-16.94
7 year	0.29	0.12	0.61	2.39	-1.41	-16.86
10 year	0.38	0.15	0.32	2.09	-1.14	-15.92

Table 3.1 Panel A reports the summary statistics for the daily euro swap spreads on yield basis. Panel B provides the same statistics for the weekly observations in the euro swap spreads. As the table shows, the average spreads of the euro interest rate swaps over the corresponding German government bonds is upward sloping with maturity. The standard deviations of swap spreads increase as the swap maturity increases. Symmetric distributions, such as the normal distribution have a skewness of zero. Kurtosis measures the thickness of the tails and is equal to 3 for a normal distribution. Euro swap spreads show positive skewness across the term structure but, relatively close to normal kurtosis for the lower maturities. The Augmented Dickey-Fuller (ADF) test is performed to determine whether the various time series of swap rates are non-stationary. This is based on the null hypothesis of non-stationarity. The ADF statistics show that we cannot reject the null-hypothesis at the 5% level of significance. This suggests that euro swap spreads across all maturities are non-stationary.

Table 3.2 Panel A provides the summary statistics for the sterling swap spreads on a daily basis. Panel B provides the same statistics for the weekly sterling swap rates. As the table shows, the average sterling interest rate swaps slopes upward initially and then flattens out. It is interesting to note that the average swap spreads of sterling interest rate swaps are much larger than those of euro interest rate swaps. This difference can be accounted for by several factors and is discussed in the following section. The average standard deviations of the sterling swap spreads are also larger than those of the euro swap spreads for all maturities. We reject the stationarity of the sterling swap spread and conclude that they follow a random walk.

Table 3.2 Summary Statistics of the Sterling Swap Spreads

Sterling spreads defined as the difference between sterling swap rates and constant maturity yields of UK Treasury bonds with the corresponding maturity. Panel A provides the mean, standard deviation, skewness, kurtosis and ADF test for Non-Stationarity where the critical t-ratio at the 5% level of significance is -2.87. The tests for integration of order zero or, or I(0), are carried out on the levels of the variables and the tests for integration of order one, or I(1), are carried out on their first differences. Daily data are used from 29 January 1999 to 28 March 2003 (total 1086 observations). Panel B provides the same summary statistics for weekly pound swap spreads with 218 observations.

Panel A: Daily Observations

Maturity	Mean	Std. Dev	Skewness	Kurtosis	ADF t-stat for I(0) test	ADF t-stat for I(1) test
2 year	0.41	0.11	0.15	2.44	-2.13	-23.27
3 year	0.54	0.15	0.20	2.46	-1.21	-23.64
5 year	0.61	0.18	0.03	1.82	-0.93	-22.98
7 year	0.62	0.22	0.04	1.81	-0.77	-22.44
10 year	0.65	0.27	0.20	2.00	-0.99	-35.50

Panel B: Weekly Observations

Maturity	Mean	Std. Dev	Skewness	Kurtosis	ADF t-stat for I(0) test	ADF t-stat for I(1) test
2 year	0.40	0.11	0.05	2.33	-1.53	-18.16
3 year	0.54	0.15	0.16	2.38	-0.90	-17.84
5 year	0.61	0.18	0.01	1.82	-0.63	-18.90
7 year	0.62	0.22	0.05	1.81	-0.45	-18.33
10 year	0.65	0.28	0.20	2.00	-0.41	-18.10

3.3 Developments in swap spreads

This section focuses on developments in the sterling and euro swap spreads. Figure 3.1 shows a time series of 10-year euro and sterling swap spreads using daily observations from January 29, 1999 to March 28, 2003. The figure depicts that the sterling swap spreads were perceptibly wider than euro swap spreads since the launch of the single currency. During the period of observation, the average sterling swap spread was 65.07 basis points as compared to 37.28 basis points for the euro swap spread.

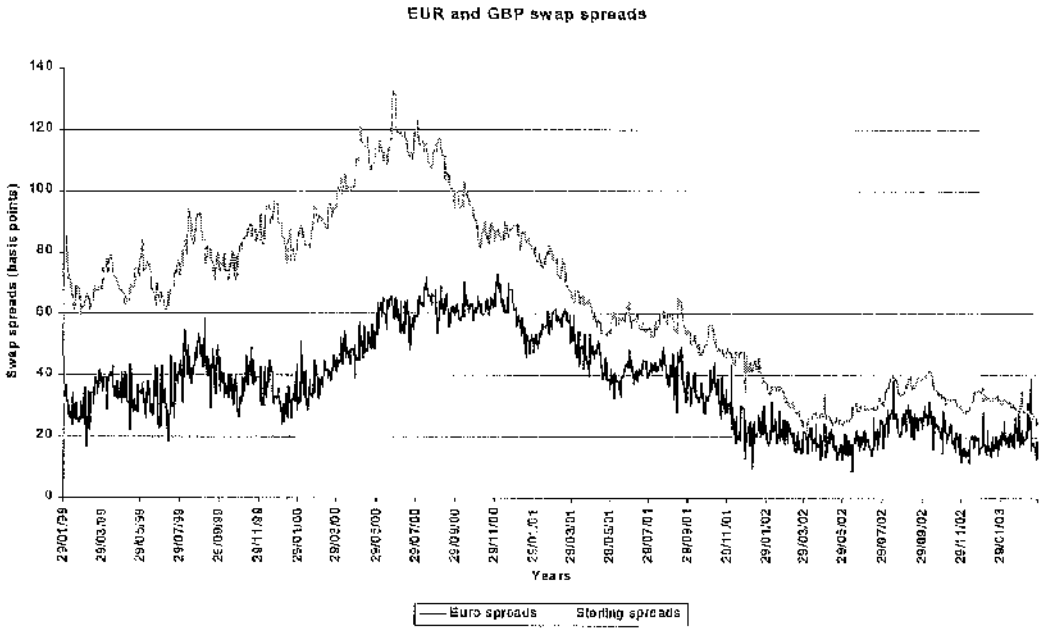


Figure 3.1 Euro and Sterling Swap Spreads

Although a number of factors may be cited to explain this divergence, the most significant relates to the issuance of bonds by British and other European government bond markets as necessitated by their differing budgetary positions. Cooper and Scholtes (2001) examined the link between swap spreads and net supply of government bonds in the UK and US markets and found mixed results. In both markets, a very simple regression between these variables suggested a strong negative relationship. But when they incorporated other variables, in particular the slope of the yield curve, net issuance ceased to be statistically significant.

Brooke, Clare and Lekkos (2000) have cited a number of UK-specific supply and demand-side factors that have influenced the shape of the gilt yield curve over the few years prior to 2000. On the supply side, net borrowing by the UK government had been negative between 1998 and 2000 and the outstanding stock of gilts had, therefore, contracted. The heavy demand for gilts from pension funds and insurance

companies increased strongly during that phase causing further downward pressure on government bond yields. Pension funds were obliged to buy gilts to comply with the Minimum Funding Requirement (MFR) of the Pensions Act, 1995, designed to ensure that pension fund managers do not take excessive risks with their investments.

Moreover, as yields continued to decline markedly, the UK treasury yield curve inverted. As an illustration, Figure 3.2 shows the inverted nature of the UK Treasury yield curve on April 28, 2000 where the spot and forward interest rates have been estimated using the Nelson and Siegel (1987) model.

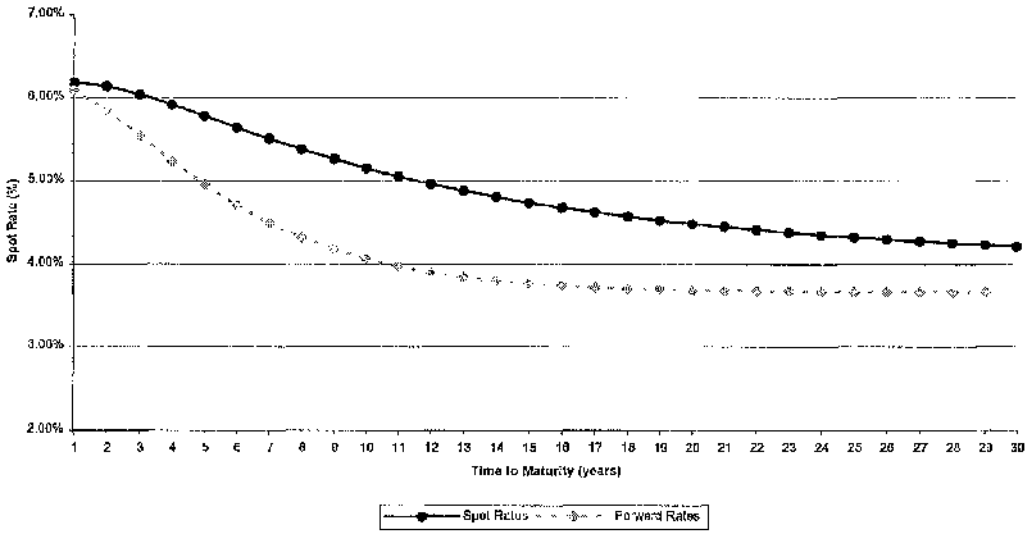


Figure 3.2 UK Treasury Yield Curve on 28th April 2000

Another factor that may have impacted on the yields on gilts relates to convergence plays associated with expectations about the United Kingdom joining EMU. Prior to the end of the year 2000 financial markets may have expected that the UK would adopt the single European currency in the near future. Although

government bond markets in the Eurozone have remained segmented, integration has been particularly strong at the short-term end of the yield curve. As a result, there is only one short-term interest rate for all EMU member countries, set by the European Central Bank. Thus, a corollary of the UK joining EMU would be the eventual convergence of UK short-term interest rates to the levels prevailing in the Eurozone. According to the expectations theory of the term structure, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. Based on the premise that all bonds will generate a riskless return and ignoring liquidity premia, convergence in future short-term interest rates would entail convergence in long-term bond yields. Therefore, the activities of hedge funds and other market participants betting on the convergence between gilt and bund yields would serve to reduce long-term gilt yields, further inverting the gilt yield curve.

By the year 2001, the UK budget position had moved away from surpluses to deficits with increased spending on public services. The consequent increase in the supply of gilts increased long-term bond yields. Following the release of the Myners' Report,[4] it was announced that the MFR would be abolished. Removing this artificial demand shifted pension fund investment away from gilts to UK corporate debt and with the consequent narrowing of the spread between 5-to 20-year gilts the yield curve flattened. With a high balance of opinion against EMU entry it became apparent that the prospect of the UK joining the single currency in the near future was remote.

[4] The Myners' Report was commissioned to identify the institutionalised obstacles distorting the investment process. In particular, Paul Myners was to determine what prevents the flow of long term savings into the growth points of the economy - namely, new ventures (private equity) and smaller companies.

This may have also contributed to the straightening out of the long end of the yield curve.

Euro swap spreads did not widen to the same degree as did sterling swap spreads. While in the UK budget surpluses caused the net issuing volume of Treasury bonds to decline in 1999 and 2000, in Europe the issuing activity of governments remained stable due to persistent budget deficits. There has also been a surge in issuance of corporate bonds denominated in euros since the introduction of the single currency. Although, the budgetary situation was not so comfortable in the main Euro-zone countries of France, Germany and Italy they all had upward sloping yield curves. Figure 3 shows the yield curve for German sovereign bonds on 28 April 2000.

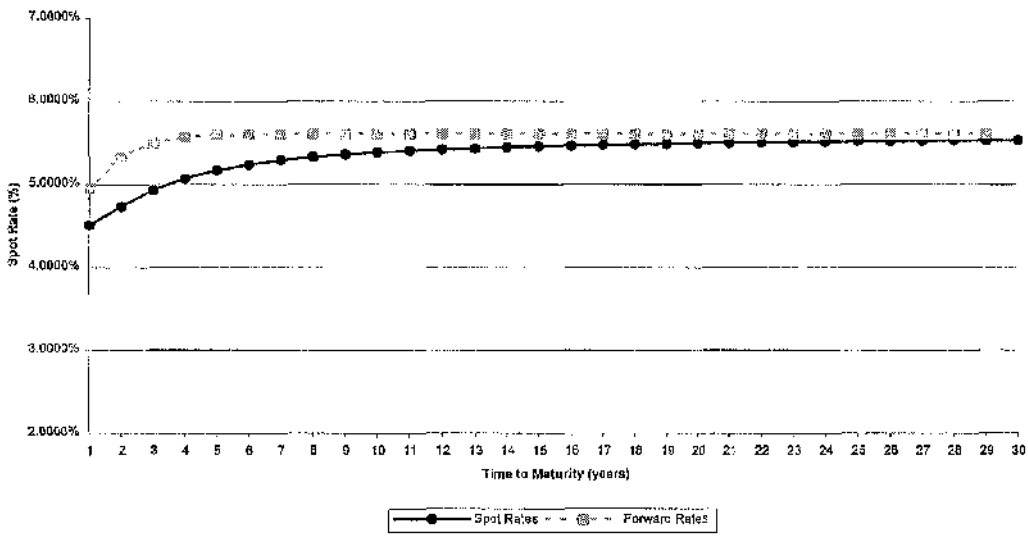


Figure 3.3 German government Yield Curve on 28th April 2000

From year 2001 onwards sterling swap spreads trended lower and fell more sharply than euro swap spreads. The UK budget position had also moved away from

surpluses to deficits with the increased spending on public services. The consequent increase in the supply of gilts coupled with the increased pension fund demand for UK corporate debt have acted as forces pulling sterling swap spreads lower. In the years 2002 and 2003 public sector borrowing requirements increased and the return to large-scale government debt issuance normalised the longer end of the sterling yields curve, while also helping to narrow spreads between government bonds and interest rate swaps.

The French, German, Italian, Spanish and Dutch governments have all used swaps to reduce the average maturity of their debt. [5] However, the large budget deficits in the main euro-zone countries of France, Germany and Italy have resulted in a narrowing of the spread between euro swaps and their respective government bonds in 2001 and 2002.

3.4 Relationship between swap spreads

This section examines the relationship between euro and sterling swap spreads. Table 3.3 shows the correlation coefficients between the changes in euro swap spreads, the changes in sterling swap spreads, and the changes in interest rate differentials between the U.K. and Germany. As indicated in the preceding section, both the euro and sterling swap spreads are non-stationary. Correlations between such time series data can be partly spurious if they exhibit consistent trends. However, both the variables are stationary if first differences are considered. So in order to avoid spurious correlations, the correlations are analysed for the first differences in these

[5] BIS Quarterly Review, March 2003

variables and not their levels. Given that swap spreads are a measure of interbank risk and the fact that most international banks have global operations it would be reasonable to expect swap spreads in euros and sterling to be highly correlated. But the coefficients in Table 3.3 reveal that this correlation is negligible. The correlation across 2-10 year vertices ranges from -0.04 to 0.17.

Table 3.3 Correlation between Euro and Sterling Swap Spreads

The table indicates the correlation coefficients among changes in the euro interest swap spreads, EURsp, changes in Sterling swap spreads, and the changes in interest rate differentials between UK and Germany (UK-GER) between 29 January 1999 to 28 March 2003.
The interest rate differentials are given by the constant maturity yields of government bonds with the same maturity as the swaps.

Maturity	Corr(EURsp, GBPsp)	Corr(EURsp,UK-GER)	Corr(GBPsp,UK-GER)
2 year	-0.04	0.37	0.08
3 year	0.01	0.46	0.22
5 year	0.05	0.55	0.15
7 year	0.08	0.57	-0.01
10 year	0.17	0.56	-0.1

However, the first differences in euro swap spreads are more correlated with the first differences in interest rate differentials between sterling and euro-denominated government bonds. The correlation coefficient between euro interest rate swap spread and the interest rate differentials given by the differences in yields of constant maturity UK and German Treasury bonds has ranged from 0.38 to 0.57. But the sterling swap spread has displayed negligible correlation with these interest rate differentials as indicated by the correlation coefficients ranging from -0.01 to 0.22.

A possible explanation for the higher correlation between the changes in euro interest rate swap spread and the interest rate differential is that arbitrageurs go long

in euro interest rate swaps and go short in sterling interest rate swaps to construct a spread position between the government bonds in the two countries. Such a spread position is constructed to take advantage of the differential between the low long-term yields of German sovereign bonds and the high long term yields of UK gilts. Eom, Subrahmanyam and Uno (2000) came to a similar conclusion on observing that changes in yen swap spreads were correlated with the interest differentials between US and Japanese treasury bond yields.

Correlation is intrinsically a short-run measure of co-dependency and reflects the contemporaneous relationship between interest rate swap spreads. The analysis of correlation is significant, in terms of depicting the degree of integration between the swap markets. Additionally, a lead-lag relationship can also be expected if there is some degree of co-dependency in interest rate swap markets. Vector autoregressive (VAR) models can be used to investigate any lead-lag behaviour between interest rate swap spreads. Granger causality tests are then conducted to see if lagged changes in the spreads for sterling interest rate swaps cause changes in the spreads of euro interest rate swaps.

To illustrate this, let x_t be the first differences in 10-year euro swap spreads and let y_t be the first differences in 10-year sterling swap spreads. As indicated in Tables 3.1 and 3.2 respectively, both the euro swap spreads and sterling swap spreads are I(1) variables such that their first differences are stationary. This guards against the possibility of the bivariate VAR model being misspecified on account of the underlying series being cointegrated. A bivariate VAR(2) model is described below:

$$x_t = c_1 + a_{11}x_{t-1} + a_{12}x_{t-2} + b_{11}y_{t-1} + b_{12}y_{t-2} + e_{1t} \quad (3.1)$$

$$y_t = c_2 + a_{21}x_{t-1} + a_{22}x_{t-2} + b_{21}y_{t-1} + b_{22}y_{t-2} + e_{2t} \quad (3.2)$$

The test for Granger causality from x to y is an F-test for the joint significance of a_{21} and a_{22}, in an OLS regression. Similarly, the test for Granger causality from y to x is an F-test for the joint significance of b_{11} and b_{12}.

Using 216 weekly observations over the sample period from January 29, 1999 to March 28, 2003, each equation has been estimated separately using OLS. Table 3.4 shows the results of the estimation.

TABLE 3.4

Bivariate VAR(2) Model using first differences in 10-year swap spreads

Sample period: January 29, 1991 - March 28, 2003

Equation (3.1)			Equation (3.2)		
F-statistic	20.07		F-statistic	2.53	
	Coeff.	t-stat		Coeff.	t-stat
c_1	-0.0014	-0.37	c_2	-0.0035	-1.11
a_{11}	-0.6002	-8.70	a_{21}	0.0569	0.98
a_{12}	-2.2604	-3.79	a_{22}	0.0018	0.03
b_{11}	0.2707	3.15	b_{21}	-0.2280	-3.07
b_{12}	0.0513	0.59	b_{22}	-0.0334	-0.45

The t-statistics (in parentheses) indicate that the model coefficients are more significant where the dependent variable is the change in 10-year euro swap spread.

The $F_{4,211}$ statistic for goodness of fit is 20.1 for the euro swap spread equation, and this is significant at the 5% level ($F_{4,211}$= 2.37). The F-statistic for the euro swap spread to sterling swap spread causality is only 2.53. Although this is just about significant at the 5% level, it is much weaker than the causality from the sterling to euro swap spreads. The results indicate that last week's changes in the 10-year sterling swap spread can have a predictive impact on this week's changes in 10-year euro swap spreads.

Table 3.5 Co-dependency between Euro and Sterling Swap Spreads

The table represents the results of bivariate "Granger causality" tests among changes in Euro swap spreads (EURsp), changes in Pound swap spreads (GBPsp) and the lagged changes in interest rate differentials between the Euro and the Pound (UK-GER). The numbers in the table are values of the F-statistic of the Granger causality test which have been performed for 2 lags.
Weekly data of changes is swap spreads are from 29 January 1999 to 28 March 2003 providing for a total of 219 observations. The quotations of swap rates and the corresponding constant maturity government bond yields were obtained from Datastream.

Maturity	EURsp to GBPsp	GBPsp to EURsp
2 year	4.31717	0.45855
3 year	4.5514	1.25617
5 year	3.23698	1.5803
7 year	1.84414	3.70722
10 year	0.61348	4.97335

Maturity	EURsp to UK-GER	UK-GER to EURsp
2 year	3.8187	0.83146
3 year	4.62935	0.36184
5 year	6.5062	0.81087
7 year	7.47418	0.06039
10 year	3.29147	0.76098

Maturity	GBPsp to UK-GER	UK-GER to GBPsp
2 year	0.91549	0.32706
3 year	0.32628	0.03602
5 year	0.38917	0.38917
7 year	1.6042	1.6042
10 year	0.41198	0.41198

Table 3.5 reports the Granger causality tests reflecting the lead-lag relationship among changes in euro and sterling swap spreads across the maturities under consideration. Granger causality tests are sensitive to the choice of the number of lags. These tests were performed using 2,3 and 4 lags which all produced qualitatively similar results. The results reported in Table 3.5 are for 2 lags. As revealed in the table, the nature of the causality depends on whether one is considering the short or long-end of the swap curve. The F-value of the Granger causality test for changes in the 10-year sterling swap spread to changes in the 10-year euro swap spread is 4.97, which is statistically significant at the 5% level. This indicates that lagged changes in the sterling swap spreads Granger cause changes in the euro interest swap spread at the 10-year maturity. But this causality is one-sided and does not transmit itself the other way. Lagged changes in 10-year euro swap spreads do not have any significant impact on changes in sterling swap spreads of the same maturity. A similar result emerges for the 7-year maturity. But at the short end of the swap curve the causality again reverses itself. At the 2-, 3- and 5-year maturities, euro-swap spreads Granger cause sterling swap spreads but the causality does not run the other way.

3.5 Volatility in swap spreads

In this section we examine the dynamic behaviour of volatility in the euro and sterling swap spreads. We make use of a GARCH framework to capture the time variation and persistence in volatility. The analysis is carried out on the 10-year swap spreads in euro and sterling markets using daily observations over the period January 29, 1999 to March 28, 2003.

GARCH Models

The GARCH (p,q) model expresses the conditional variance of a given time series (σ_t^2) as a linear function of p lagged squared errors and q lagged variances.

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-q}^2 \qquad (3.3)$$

$$\omega > 0, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q \geq 0$$

In the context of this analysis , the GARCH (1,1) model would consist of two equations:

$$y_t = c + x_t \delta + \varepsilon_t \qquad \varepsilon_t / I_{t-1} \sim N(0, \sigma_t) \qquad (3.4)$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \qquad \omega > 0, \alpha, \beta \geq 0 \qquad (3.5)$$

in which the conditional mean equation is (3.4), y_t represents the swap spread at time t, and ε_t their unanticipated component distributed independently over time and assumed to follow a normal distribution with zero mean and conditional variance σ_t^2,

x_t as the swap spread of the other currency. The conditional variance equation (3.5) is a function of the constant term, ω, news about volatility from the previous period, measured as the lag of the squared residual from the conditional mean equation, ε_{t-1}^2 and the previous period's forecast variance, σ_{t-1}^2.

Testing for ARCH effects

Various methods are available to test for the existence of autoregressive conditional heteroskedasticity (ARCH). A test based on the Lagrange multiplier (LM) principle formulated by Engle (1982) is applied here. Let y_t denote the swap spread of one country at time t and x_t the swap spread of the other country at time t. The process begins by running an OLS regression of y_t on x_t of the following form:

$$\hat{y}_t = a + bx_t \qquad (3.6)$$

Now the residuals from this preliminary OLS estimation can be tested for ARCH behaviour. The test proposed in Engle (1982) is to regress the squared residuals, e_t^2 (where $e_t = y_t - \hat{y}_t$) on a constant and p lagged values of the squared residuals:

$$e_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 e_{t-1}^2 + \ldots + \hat{\alpha}_{t-p}^2 + \upsilon_t \qquad (3.7)$$

where υ_t is the error term.

From the results of this auxiliary regression in residuals, the LM test statistic is calculated as $(T\text{-}p)*R^2$ where T is the number of observations. As explained in

Bollerslev (1986), the LM statistic has an asymptotic chi-square (χ^2) distribution with p degrees of freedom under the null hypothesis of no ARCH effects. If the LM statistic, evaluated under the null hypothesis, exceeds the critical value from a chi-square distribution with q degrees of freedom, the null hypothesis is rejected.

The results of the auxiliary regression, for one lag, are shown in Table 3.6. There is strong evidence to reject the null hypothesis of no ARCH effects as the resulting LM test statistic of 519.14 far exceeds the critical value of $\chi^2_{0.95}(1) = 3.84$. A regression residual series was generated by increasing the number of lags to five. But the results for increased lag lengths were not qualitatively different from that obtained for one lag and are not reported here.

TABLE 3.6

ARCH LM Tests on 10-year swap spreads

Sample period: January 29, 1991 - March 28, 2003

Included observations: 1085 after adjusting end points

	Euro Swap Spreads		Sterling Swap Spreads	
	Coeff.	*t*-stat	Coeff.	*t*-stat
α_0	0.0023	8.32	0.0089	10.008
α_1	0.6915	31.52	0.6292	26.67
F-stat	993.59	(0.0000)	711.47	(0.0000)
LM-stat	519.14	(0.0000)	430.18	(0.0000)

Figures in parenthesis show probabilities

The 10-year sterling swap spreads demonstrated similar ARCH effects where the squared residual series for one lag returned an LM test statistic of 430.18. Increasing the number of lags to five did not change the results in so far as the existence of ARCH was concerned.

Testing for an asymmetric effect on volatility

Several studies on the volatility dynamics of asset markets have shown evidence of asymmetry in the response of conditional variances to the type of news revealed to the markets. This is also referred to as the leverage effect in volatility and is often observed in equity markets where downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. In the context of swap spreads the leverage effect would arise if, for instance, the volatility of the swap spread increases more when there is a positive shock, which increases the swap spread, than when there is a negative shock.

The GARCH model specified in equation (3.5) cannot capture any asymmetric effect, since the conditional variance is a function only of the magnitudes of the lagged residuals and not their signs. The residuals ε_t are specified as a square and so it makes no difference whether they are positive or negative. Before proceeding to use a symmetric GARCH model as specified in equation (3.5) it is imperative to test for the existence of any asymmetric effects in swap spread volatilities.

In the exponential EGARCH model of Nelson (1991), σ_t^2 depends on both the size and the sign of lagged residuals. The purpose of this EGARCH specification is to try and build in some asymmetry, so that the sign of ε_t matters. The conditional variance equation in the EGARCH model is defined in terms of the standard normal variate z_t:

$$\ln \sigma_t^2 = \omega + g(z_{t-1}) + \beta \ln \sigma_{t-1}^2 \qquad (3.8)$$

where $g(.)$ is an asymmetric response function defined by

$$g(z_t) = \gamma z_t + \alpha(|z_t| - \sqrt{2/\pi})$$

The left-hand side of equation (3.8) shows the log of the conditional variance. This implies that the leverage effect is exponential, rather than the quadratic, and that the forecasts of the conditional variance are guaranteed to be nonnegative. The standard normal variable z_t is the standardized residual ε_t / σ_t. When $\alpha > 0$ and $\gamma < 0$, negative shocks to returns ($z_{t-1} < 0$) induce larger conditional variance responses than positive shocks. Therefore, the presence of asymmetric effects can be tested by the hypothesis that $\gamma < 0$. The impact is asymmetric if $\gamma \neq 0$. Formulae for higher order lags in ε_t can be found in Nelson (1991).

To test for the possible existence of this leverage effect in swap spread volatility the EGARCH model was applied to standardized residuals of the conditional mean model using one swap spread as the dependent variable and the swap spread of the other currency as the exogenous variable. The EGARCH model

was applied to the swap spreads of both currencies over the sample period. The results are shown in Table 3.7.

With the 10-year euro swap spread as the dependent variable and the corresponding sterling swap spread as the exogenous variable the asymmetric effect term (γ), is positive and equal to 0.0136. The z-statistic is equal to 0.98 which is not statistically different from zero at the 5% level of significance given by 1.645. It was, therefore, concluded that the volatility in 10-year euro swap spreads does not display asymmetric effects. Performing an identical operation with the sterling swap spreads as the dependent variable and the euro swap spread as the exogenous variable revealed similar results. The asymmetric effect term (γ) was again positive at 0.027. It was also not statistically significant from zero with the z-statistic equal to 0.92.

Eom, Subrahmanyam and Uno (2002) employed a GJR-GARCH model and found that there is an asymmetric volatility effect of dollar swap spreads on yen swap spreads, while the asymmetric effect of the shock on the yen swap spread is insignificant. In their analysis of the swap spreads in Australia, Brown, In and Fang (2002) used an EGARCH approach and found that the asymmetric effects are statistically significant for 3 and 5-year swaps but not for 10-year swaps.

With these tests demonstrating the absence of any asymmetric volatility effect of the shock on 10-year euro and sterling swap spreads, it would be appropriate to confine the analysis to symmetric GARCH models for modelling volatility.

TABLE 3.7
Testing for the Asymmetric Effect on Volatility

EGARCH Model:

$$\ln \sigma_t^2 = \omega + g(z_{t-1}) + \beta \ln \sigma_{t-1}^2$$

where $g(.)$ is an asymmetric response function defined by

$$g(z_t) = \gamma z_t + \alpha(|z_t| - \sqrt{2/\pi})$$

	Euro Swap Spread			Sterling Swap Spread		
	Coeff.	z-stat.	Prob.	Coeff.	z-stat.	Prob.
EGARCH Model						
Asymmetric effect parameter						
(H_0: $\gamma < 0$)	0.01	0.98	0.3255	0.027	0.93	0.3526

Estimating the GARCH (1,1) model

To assess the appropriateness of the GARCH specification for daily swap spreads, a GARCH (1,1) model based on equations (3.4) and (3.5) was used. This specification was found to be the most appropriate for modelling volatility in both euro and sterling 10-year swap spreads.

The model specification also includes a dummy variable for the trading day following a weekend, i.e. Monday, in the conditional variance equation to capture potential "day of the week" effects. The model now has the following form:

$$y_t = c + ax_t + \varepsilon_t \qquad (3.9)$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta D_t \qquad (3.10)$$

where D_t represents a dummy variable that takes the value of 1 on Mondays and is 0 otherwise. Panel A of Table 8 shows the results of the estimation of the GARCH(1,1) model for euro-swap spreads. There are no indications of any serious model misspecification.

TABLE 3.8
Estimation of GARCH(1,1) model using 10-year swap spreads

Sample period: January 29, 1991 - March 28, 2003

	PANEL A		**PANEL B**	
	Euro Swap Spread		Sterling Swap Spread	
Number of obs.	1086		1086	
Log-likelihood	1408.946		897.005	
	Coeff.	*z*-stat	Coeff.	*z*-stat
Conditional Mean				
c	0.065771	17.46	0.085852	17.08
a	0.441450	70.72	1.204970	84.22
Conditional Variance				
ω	0.000282	3.77	0.000513	4.07
α	0.182550	6.17	0.310197	4.92
β	0.799298	30.53	0.661948	10.53
δ	-0.000838	-2.61	-0.001146	-2.75
Residual Tests	Statistic	Prob.	Statistic	Prob.
Skewness	-0.17		-0.84	
Kurtosis	2.71		3.21	
Jarque-Bera	8.98	(0.011249)	129.78	(0.000000)
LM test statistic	0.004	(0.947156)	2.49	(0.114284)

The parameter estimates for the conditional variance equation (3.10) correspond to $\alpha = 0.1826$, $\beta = 0.7993$, $\omega = 0.000282$ and $\delta = -0.000838$. The z-statistics reveal that all coefficients are statistically significant. The coefficient of the dummy variable is negative indicating the influence of more subdued trading in government securities on a Monday.

By putting $\sigma_t^2 = \sigma^2$ for all t in equation (3.10) above, one gets an expression for the long-term steady state variance in a GARCH (1,1) model:

$$\sigma^2 = \omega / (1 - \alpha - \beta) \qquad (3.11)$$

or $$\sigma^2 = \omega / [1 - (\alpha + \beta)]$$

Equation (3.11) can then be rewritten as:

$$V = \omega / \gamma \qquad (3.12)$$

where V is the long-term variance which can be calculated as ω/γ. A stable GARCH (1,1) process requires that the sum $\alpha + \beta$ be less than 1. Only then will the GARCH volatility term structures converge to a long-term average level of volatility that is determined by (3.12). In this estimation the sum of the α and β is equal to 0.981848 which is less than one indicating that volatilities of the 10-year euro swap spread converge to some long-term average level of volatility.

Since $\gamma = 1 - \alpha - \beta$, it follows that $\gamma = 0.081152$. And since $\omega = \gamma V$, it follows that $V = 0.0155354$. In other words, the long-run average variance per day implied by the model is 0.0155354. This corresponds to a volatility of $\sqrt{.0155354} = 0.1246414$ or 12.46 % per day.

The residual tests display descriptive statistics of the standardised residuals, ε_t / σ_t. Under the null hypothesis of a normal distribution, the observed value of the Jarque-Bera test statistic of 8.975 exceeds the critical value of $\chi^2_{0.95}(2) = 5.99$. So the standardised residuals are not normally distributed. However, one cannot reject the null hypothesis of no ARCH effects in the standardised residuals as the observed LM test statistic of 0.004 is well short of the critical value of $\chi^2_{0.95}(1) = 3.84$. This clearly indicates that there are no ARCH effects left in the standardised residuals.

The same GARCH (1,1) was then employed to estimate volatility in the 10-year sterling swap spreads. Panel B of Table 3.8 shows the results of the estimation. In the conditional variance equation, $\alpha = 0.310197$, $\beta = 0.661948$, $\omega = 0.000513$ and $\delta = -0.001146$. As revealed by the z-statistics, all coefficients are statistically significant. The sum of the GARCH coefficients is given by $\alpha + \beta = 0.972145$, which being very close to one indicates that volatility shocks are quite persistent. The value of the coefficient α in the case of sterling swap spreads is much higher than that for euro swap spreads. Large GARCH error coefficients α mean that volatility reacts quite intensely to market movements, and so if α is relatively high and β is relatively low then volatilities tend to be more spiky. Using equations (3.11) and (3.12) above, the long-term variance V works out to 0.0184168. This means a

volatility of $\sqrt{0.0184168}$ = 0.1357085 or 13.57% per day. So we find the volatility of the sterling swap spread to be somewhat higher than that of the euro swap spread.

Although both the euro and sterling swap spreads are themselves non-stationary, we observed that their volatility forecasts are stationary and converge to the long-run average volatility level. But if the volatilities were also random walks the stationary GARCH model used above would no longer be applicable. When $\alpha + \beta = 1$ we can put $\beta = \lambda$ and rewrite the GARCH model given by equation (3.5) as

$$\sigma_t^2 = \omega + (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \qquad \text{where } 0 \le \lambda \le 1 \qquad (3.13)$$

It is important to note that the unconditional variance given by equation (3.11) is longer defined and the swap spread forecasts do not converge. Since in this case the variance process is non-stationary, (3.13) is called the integrated GARCH or I-GARCH model.

In the case of both the euro and sterling swap spreads, the distribution of the standardised residuals does not follow a normal distribution. However, the distribution of euro swap spread residuals is relatively closer to a normal distribution, whereas the sterling swap spreads exhibit a much more asymmetric and considerably broader distribution. Accordingly, the sterling swap spreads are more volatile than their euro counterparts.

Volatility Spillovers

Having estimated the volatilities of both the euro and sterling swap spreads over the sample period I now examine the possibility of a transmission of volatility between them. Although the GARCH (1,1) specification used above was descriptively accurate for estimating volatility in individual markets it did not incorporate the spillover effects from other markets. It is thus necessary to introduce an exogenous variable into the conditional variance equation that captures the potential spillover effect from one market into the other. The squared residual from one market is interpreted as a "volatility surprise" and is included in the other market's conditional variance specification:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \xi_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta D_t \qquad (3.14)$$

where ε_{t-1}^2 is the lagged squared residual of the domestic swap spread and ξ_{t-1}^2 is the lagged squared shock arising from the foreign market's swap spread.

The results of estimating this model for both the euro and sterling swap spreads are shown in Table 3.9

TABLE 3.9

Volatility spillovers between swap spreads

Sample period: January 29, 1991 - March 28, 2003

	PANEL A		PANEL B	
	Euro Swap Spread		Sterling Swap Spread	
Number of obs.	1086		1086	
Log-likelihood	1373.686		876.2183	
	Coeff.	z-stat	Coeff.	z-stat
Conditional Mean				
c	0.065911	17.02	0.084630	16.62
a	0.441040	61.92	1.209475	79.39
Conditional Variance				
ω	0.000300	3.91	0.000482	3.96
α_1	0.219476	5.79	0.307258	4.85
α_2	0.002997	2.04	0.006063	0.80
β	0.734628	19.78	0.661962	10.50
δ	-0.000766	-2.74	-0.001150	-2.85
Residual Tests	Statistic	Prob.	Statistic	Prob.
Skewness	-0.14		-0.84	
Kurtosis	2.64		3.21	
Jarque-Bera	9.27	(0.009724)	129.70	(0.000000)
LM test statistic	0.67	(0.412656)	2.58	(0.108109)

Panel A of Table 3.9 shows there is evidence of an element of volatility spillover from the sterling swap spreads to the euro swap spreads. The parameter estimate on the sterling swap spread volatility surprise ξ_{t-1} is positive and statistically significant at the 5% level. Therefore, the null hypothesis of no foreign volatility surprise is rejected at the 5% significance level, indicating that there are mild volatility transmissions from the sterling swap spreads to euro swap spreads. However, Panel B

of Table 3.9 shows that there is no such volatility spillover from euro swap spreads to sterling swap spreads as the parameter estimate is not statistically significant. These volatility spillover effects are consistent with the findings on Granger causality tests for 10-year swap spreads in Table 3.5.

3.6 Conclusions

This chapter empirically examines the case of market integration between euro and sterling swap spreads during the period January, 1999 to March, 2003. The swap spreads represent the difference between the swap rates and the constant maturity yields of government bonds with corresponding maturity. Euro swap spreads have been proxied using German sovereign bonds.

Initially, the main characteristics of the term structure of swap spreads in both the euro and sterling markets were examined. Both swap spreads are non-stationary across the term structure and follow a random walk. However, sterling swap spreads have been perceptibly wider than euro swap spreads since the launch of the single currency. This largely relates to the net supply of government bonds in British and European markets as driven by their respective budgetary positions.

While in the UK, budget surpluses caused the volume of Treasury bonds to decline in 1999 and 2000, in the main European markets of France, Germany and Italy the issuing activity of governments remained stable due to persistent budget deficits. The sterling swap spreads subsequently trended lower due to the UK budget

position moving away from surpluses to deficits and the shift in demand of UK pension funds from gilts to corporate debt.

The correlation coefficient between changes in euro swap spreads and changes in sterling swap spreads is negligible indicating that credit risk can be attributed to country specific factors as opposed to global influences. However, the changes in euro swap spreads are correlated, to some degree, with changes in interest rate differentials between sterling and euro-denominated government bonds. Moreover, no evidence is found of sterling swap spreads being correlated with the interest rate differentials. A plausible interpretation for the correlation between the euro swap spread and the interest differential is that arbitrageurs go long in euro interest rates swaps and go short in sterling interest rates swaps to construct a spread position between the government bonds in the two countries. Such a spread is constructed to take advantage of the low long-term yields of German bunds and the high long term yields of UK gilts.

Granger causality tests, analysing the lead-lag relationship among changes in euro and sterling swap spreads reveal that the causality depends on whether one is considering the short or long-end of the swap curve. Lagged changes in sterling swap spreads Granger cause changes in euro interest swap spreads at the 10-year maturity, but there is no evidence to suggest that euro swap spreads Granger cause sterling swap spreads at the 10-year maturity. At the short end of the swap curve the causality again reverses itself. For the 2-, 3- and 5-year maturities, euro swap spreads Granger cause sterling swap spreads but there is no causality in the reverse direction. The notion of market efficiency dictates that it should not be possible to predict swap

spreads in one market using lagged information generated in another market. To the extent that lagged changes in the spreads for sterling interest rates swaps cause changes in the spreads of euro interest swaps, the latter could be characterised as being informationally inefficient. However, interest rate differentials between these two markets do not Granger cause swap spreads in either of the markets.

The analysis of the causal relationship between swap spreads was then extended to the dynamic behaviour of volatility in 10-year euro and sterling swap markets. The time series of both the euro and sterling swap spreads show volatility clustering and reveal strong ARCH effects. An EGARCH model was employed to test for the existence of any asymmetric response in the volatility of 10-year swap spreads. But the volatilities did not display asymmetric effects for either of the swap spread markets.

The GARCH (1,1) specification was found to be the most appropriate for modelling volatility in 10-year swap spreads for both the markets. Volatility shocks were found to be quite persistent in both the markets. However, volatility in the sterling swap spreads reacted more intensely to market movements and was more volatile than their euro counterparts although, both volatility term structures converged to a long-run average level of volatility.

The possibility of volatility spillover effects between 10-year euro and sterling swap spreads was also examined. There was evidence of mild volatility transmission from the sterling swap spreads to euro swap spreads but no spillover

effects the other way round. This observation was consistent with the findings on Granger causality.

This investigation into the causal relationship between euro and sterling swap spreads could contribute to an understanding of the degree of financial market integration between the UK and the Eurozone. An awareness of the nature of volatility spillover across the markets could be of importance to economic policy makers from a financial stability perspective. Given our findings that there is no volatility transmission from the euro swap spreads to sterling swap spreads, it seems unlikely that a credit risk shock in the euro fixed income market would have a destabilising effect on the sterling fixed income market. However, the more general conclusions that can be drawn from this paper are somewhat tentative because of the limited period of observation.

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CHAPTER 4

APPLICATION OF THE KALMAN FILTER FOR ESTIMATING CONTINUOUS TIME TERM STRUCTURE MODELS: EVIDENCE FROM THE UK AND GERMANY

4.1 Introduction

Term structure modelling can explore two distinct, but related aspects. The first involves the fitting of a zero-coupon yield curve to a set of cross-sectional bond price observations on any given trading day. This relationship between the zero-coupon yields or spot interest rates and their term to maturity was the subject of discussion in Chapter 1. The second aspect, which is the focus of this chapter, relates to the specification of the intertemporal dynamics of the term structure and addresses the issue of how bond yields evolve over time. In this respect, it is useful to recall certain characteristics of bond yields that need to be considered while analysing their movements over time. Estimating the term structure is based on the premise that bonds with different maturities are traded at the same time. Bonds with long maturities are risky when held over short horizons and risk-averse investors demand compensation for bearing such risk. Arbitrage opportunities in these markets exist unless long-yields are risk-adjusted expectations of average future short rates. Restrictions are therefore imposed on inter-temporal interest rate behaviour by using the no-arbitrage argument. The absence of arbitrage, would ensure that movements of the term structure do not permit conditions to occur under which market participants may earn risk-free profits.

As interest rates are stochastic processes, models rely on the reduction of interest rate uncertainty and attempt to provide a parsimonious characterisation of the dynamics of the term structure. There exist various specifications that differ with respect to the number of underlying state variables and the type of the stochastic process. Affine term structure models are constructed by assuming that bond yields are a linear function of the underlying state variables that provide uncertainty to the model. Most modelling approaches are based on the concept that although interest rates change randomly over time, it is possible to divide the changes into two parts using a stochastic differential equation. The first part is a non-random, deterministic component, called the drift of the process, and the second is the random or noise part which entails the volatility component of the process. Examples are the one-factor Vasicek (1977) model with constant volatility, the Cox-Ingersoll-Ross (1985) model with square-root volatility and the two-factor model of Longstaff and Schwartz (1992). Stochastic differential equations have, in recent years, been increasingly used to model financial data. However, the process specified by a stochastic differential equation is defined in continuous time, while the observed data are sampled at discrete time intervals. As discussed in Durham and Gallant (2002) the resulting estimation problem turns out to be nontrivial, and research has focussed on developing computationally and statistically efficient estimation schemes. Although maximum likelihood is typically the estimator of choice, the transition density is generally unknown and has to be approximated.

The Vasicek (1977) model is a one-factor partial equilibrium model and starts out with the specification of a time series process for the instantaneous spot interest rate which is treated as the only factor of uncertainty. The no-arbitrage restriction then permits the derivation of a bond pricing formula whereby the bond price is a function of the unobserved instantaneous spot rate and the model's parameters. The approach was extended to include a second factor of uncertainty. Besides the real rate of interest, Richard (1978) chose the expected rate of inflation as the second source of uncertainty. Brennan and Schwartz (1979) model the long rate as the second factor and assume that the short rate is mean reverting to the long rate.

Cox, Ingersoll and Ross (1985, CIR hereafter) develop a general equilibrium asset pricing model that allows the derivation of the term structure of interest rates. The model is set up as a single-good, continuous time economy with a single state variable. A multivariate version was developed by Longstaff and Schwartz (1992) in which the two-factors were the short-term interest rate and the variance of changes in the short-term interest rate. Duffie and Kan (1996) define a general class of multifactor affine models of the term structure that allows for the nesting of some of the aforementioned term structure models such as Vasicek (1978), CIR (1985) and Longstaff and Schwartz (1992).

The literature would suggest that three state variables are adequate to explain most of the variability in bond yields. For example, Litterman and Scheinkman (1991) show that this can be captured by the level, the steepness and the curvature of the term structure. This paper focuses on the one-factor CIR model as the empirical estimation showed that the inclusion of additional factors did not increase the

performance of the model for either country. A plausible explanation for this could be the limited period of observation. Most studies have concluded that the level is the most important factor in explaining interest variation over time. In fact, Litterman and Scheinkman (1991) have demonstrated that three factors notwithstanding, almost 90 percent of the variation in US Treasury rates is attributable to the variation in the first factor, which is considered to correspond to the level of interest rates. Thus from an empirical point of view a one-factor CIR model can be considered acceptable.

The purpose of this chapter is to explore how the term structure has evolved in the sterling and euro treasury bond markets between January 1999 and January 2004. German bonds have been used to represent euro-denominated bonds as they are seen by market participants as the main component of the euro yield curve. Although there exists a considerable literature on empirically estimating the CIR model, most of the tests have been performed on US data. The few studies that have focussed on the UK and European markets relate to the pre-EMU period. Steeley (1997) has modelled the forward premium in the UK gilt-edged market over the period 1982-96 using a two-factor general equilibrium model of the term structure of interest rates. Nath and Nowman (2001) estimate multi-factor versions of the CIR model using the UK Gilt-edged market data over the period 1982-97.

It is believed that, this is the first study that estimates this model for the UK and Euro-denominated bond data since the launch of the single currency. By bringing together the empirical findings for the euro and sterling treasury bond markets an attempt is made to compare the dynamics of their respective term structures. This

investigation into the intertemporal behaviour of the euro and sterling term structure may provide evidence on whether there exists any common factors.

The rest of the chapter is organised as follows. Section II provides the theoretical framework that discusses in detail the one-factor CIR model for the instantaneous interest rate. Section III provides an overview of the different estimation methods. In Section IV the state space representation of the CIR model is formulated and, in Section V the Kalman filter algorithm is employed. Section VI presents the data and results. Finally, Section VII concludes.

4.2 Theoretical Framework

It is useful to begin by outlining some of the key bond pricing relationships in a continuous-time framework. A pure discount bond is defined as a contract that pays one unit of currency at its maturity date and its value is denoted by the function $P(t,T)$. The first argument, t, refers to the current time, while the second argument, T, represents the pure discount bond's maturity date. It follows that $t < T$. Given the contractual nature of the pure discount bond, $P(T,T) = 1$.

Given the pure discount bond price for any given maturity, the associated spot rate of interest for that date can be determined. The spot rate, denoted as $R(t,T)$, is the continuously compounded rate of return that generates the observed prices of the discount bond. The spot rate can then be solved for as follows:

$$P(t,T)e^{(T-t)R(t,T)} = 1, \qquad (4.1)$$

$$\ln(P(t,T)e^{(T-t)R(t,T)}) = 0,$$

$$\ln(e^{(T-t)R(t,T)}) = -\ln P(t,T)$$

$$R(t,T) = -\frac{\ln P(t,T)}{T-t} \tag{4.2}$$

As interest rate processes are stochastic processes, developing an affine term structure model involves a specification of a stochastic process for the state variables, or factors, that drive the dynamics of the term structure. In a one-factor term structure model, the factor is generally taken to be the instantaneous spot rate of interest. It is possible to divide the change in its value into two parts, the first is a non-random deterministic component, called the drift of the process, and the second is a diffusion term or random part, which is the variance of the process. This involves the assumption, that the interest rate process is generated by a standard Brownian motion[1], also known as a Wiener process, and that its dynamics can be described by the following first-order stochastic differential equation:

$$dr = \mu(r,t)dt + \sigma(r,t)dW \tag{4.3}$$

where dW is a Wiener process. The stochastic process described in equation (4.3) is also known as a *Ito process* which is a generalised Brownian process where the parameters μ and σ are functions of the underlying variable, r, and time, t. Both the

[1] A Brownian Motion is a stochastic process where the change in a variable during each short period of time Δt has a normal distribution with mean equal to zero and a variance that is proportional to time.

expected drift rate and variance rate of an Ito process are liable to change over time. The variable r has a drift rate of μ and a variance rate of σ^2.

The model can be used to derive a relationship linking the expected rates of returns of pure discount bonds of all maturities. Let $P(r,t,T)$ denote the price of a pure discount bond at time t which at maturity equals unity, since the bond is default free. Ito's Lemma shows that a function, P, of r and t follows the process

$$dP = (\frac{\partial P}{\partial r}\mu + \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma^2)dt + \frac{\partial P}{\partial r}\sigma dW \qquad (4.4)$$

where dW is the same Wiener process as in equation (4.3). Thus, P also follows an Ito process. It has a drift rate of $\frac{\partial P}{\partial r}\mu + \frac{\partial P}{\partial t} + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}\sigma^2$

and a variance rate of $\left(\frac{\partial P}{\partial r}\right)^2 \sigma^2$.

Denoting the partial derivatives as $P_r = \frac{\partial P}{\partial r}$, $P_t = \frac{\partial P}{\partial t}$, $P_{rr} = \frac{\partial^2 P}{\partial r^2}$ and dividing (4.4) by P gives the following:

$$\frac{dP}{P} = \frac{(\mu P_r + P_t + \frac{1}{2}\sigma^2 P_{rr})dt}{P} + \frac{P_r \sigma}{P} dW \qquad (4.5)$$

This can be written as

$$\frac{dP}{P} = \alpha_p dt + \sigma_p dW \tag{4.6}$$

where $\alpha_p = \dfrac{P_t + \mu P_r + \frac{1}{2}\sigma^2 P_{rr}}{P}$ and $\sigma_p = \dfrac{\sigma P_r}{P}$

To eliminate the random component in (4.6), one can build a portfolio of two bonds of value V with a proportional investment in each bond.

In this regard, two bonds of different maturities are considered: $P^T = P(r,t,T)$ and $P^S = P(r,t,S)$. A portfolio is now constructed with x of the first bond and y of the second. Then the value of this portfolio is,

$$V = xP^T + yP^S \tag{4.7}$$

and the proportionate change in the value of the portfolio over a short time interval is as follows,

$$\frac{dV}{V} = \frac{xP^T}{V}\frac{dP^T}{P^T} + \frac{yP^S}{V}\frac{dP^S}{P^S}$$

This can be written as,

$$\frac{dV}{V} = \omega_T \frac{dP^T}{P^T} + \omega_s \frac{dP^s}{P^s} \tag{4.8}$$

where $\omega_T = \frac{xP^T}{V}$ and $\omega_S = \frac{yP^S}{V}$ indicate the proportion of investment in each bond.

Both $\frac{dP^T}{P^T}$ and $\frac{dP^S}{P^S}$ can be expanded using equation (4.6) so that

$$\frac{dV}{V} = \omega_T(\alpha_T dt + \sigma_T dW) + \omega_S(\alpha_S dt + \sigma_s dW)$$

By rearranging this can be rewritten as,

$$\frac{dV}{V} = (\omega_T\alpha_T + \omega_S\alpha_S)dt + (\omega_T\sigma_T + \omega_S\sigma_S)dW \qquad (4.9)$$

The portfolio weights are adjusted continuously so that each source of uncertainty vanishes and at each point in time the change in the value of the portfolio is known. This implies,

$$(\omega_T\sigma_T + \omega_S\sigma_S)dW = 0 \qquad (4.10)$$

As the sum of the portfolio weights equals one, the appropriate weights can be found by solving the simultaneous equations:

$$\omega_T\sigma_T + \omega_S\sigma_S = 0$$
$$\omega_T + \omega_S = 1$$

This gives,

$$\omega_T = -\frac{\sigma_S}{\sigma_T - \sigma_S}; \omega_S = \frac{\sigma_T}{\sigma_T - \sigma_S}$$

By substituting these values, it follows that

$$\frac{dV}{V} = (\omega_T \alpha_T + \omega_S \alpha_S)dt = \left(\frac{\alpha_S \sigma_T - \alpha_T \sigma_S}{\sigma_T - \sigma_S} \right) dt$$

The arbitrage opportunities are precluded only if the portfolio earns the same return as the short rate at time t, where the rate corresponds to the return on a pure discount bond of infinitesimally short maturity. By specifying no arbitrage, it then follows that

$$\frac{\alpha_S \sigma_T - \alpha_T \sigma_S}{\sigma_T - \sigma_S} = r$$

The absence of arbitrage would, intuitively, mean that assets which exhibit the same risk should earn exactly the same (excess) return. Therefore, the return/risk ratio should be the same for all assets as given by, say, λ. So, by definition

$$\frac{\alpha_T - r}{\sigma_T} = \frac{\alpha_S - r}{\sigma_S} = \lambda(r,t)$$

Thus in an arbitrage-free market, bonds of all maturities have the same market price of risk, which does not depend on maturity. The subscripts S and T can therefore be dropped and the equation becomes

$$\frac{\alpha_p - r}{\sigma_p} = \lambda(r,t) \tag{4.11}$$

Equation (4.11) implies a direct linear return-risk relationship so that,

$$\alpha_p = r + \lambda(r,t)\sigma_p \tag{4.12}$$

where λ is independent of maturity. In (4.12), the expected rates of return on all assets dependent upon the state variable are equal to the risk free rate plus a term premium, which compensates the investor for one unit of risk associated with the underlying state variable r. Therefore, λ is the market price of risk corresponding to the source of uncertainty. [2] By combining (4.6) and (4.12) and rearranging one can arrive at the fundamental partial differential equation for pricing pure discount bonds.

$$P_t + \frac{1}{2}\sigma^2 P_{rr} + (\mu - \lambda\sigma)P_r - rP = 0 \tag{4.13}$$

Until now, the instantaneous drift μ and instantaneous volatility σ have been left unspecified. However, for the pricing and hedging of fixed income securities it is necessary clearly specify the form of the instantaneous drift and volatility. For pricing

[2] where there are n state variables, λ would be a vector containing the market price of risk corresponding to each individual source of uncertainty.

the underlying fixed income security one needs to clearly apply the appropriate boundary condition and solve (4.13) using numerical methods. In the case of pure discount bonds the boundary condition is $P(r,T,T) = 1$, since at maturity the price of a pure discount bond is unity.

Therefore the problem remains that although the bond price equation is a standard partial differential equation, $\lambda(r,t)$ is not determined within the model. For this reason, most term structure models including Vasicek (1977) and CIR (1985) directly specify $\alpha - \lambda\sigma$, the risk-adjusted drift.

Affine term structure models

Affine term-structure models are constructed by assuming that bond yields are a linear function of the underlying state variables that provide uncertainty in the model. In a single factor model, the instantaneous interest rate, r, is taken to be the only state variable. An affine function is a constant plus a linear function. Therefore, the price of a pure discount bond, $P(t,T)$, in an affine term structure model would have the following functional form:

$$P(t,T) = \exp(A(\tau) - B(\tau)X) \qquad (4.14)$$

where X is the state vector. The coefficients $A(\tau)$ and $B(\tau)$ are functions of the time to maturity, $\tau = T - t$, the parameters of the interest rate process and the market price of interest rate risk.

The set of prices of zero-coupon bonds as a function of time to maturity, $\tau = T - t$ will define the zero-coupon yield curve $R(t,T)$, where

$$R(t,T) = -\frac{1}{\tau}\ln[P(t,T)] = \frac{B(\tau)X - \ln A(\tau)}{\tau} \qquad (4.15)$$

The affine yield class property is displayed in equation (4.15). The zero-coupon yields are affine functions of the underlying factors, in this example the instantaneous short rate. For models where both the drift and volatility specifications are affine in r, it is possible to have closed form formulae for $A(\tau)$ and $B(\tau)$. Both the Vasicek (1977) and CIR (1985) models fulfil this criterion resulting in closed form solutions for the prices of pure discount bonds.

In the Vasicek (1977) model, the risk-neutral process for r can be described by the following first-order stochastic differential equation:

$$dr = k(\theta - r)dt + \sigma dw \qquad (4.16)$$

where w is a standard scalar Wiener (Gaussian) process while k, θ and σ are constants. This and other affine models use the assumption that the interest process is a Markov process where only the current value of a variable is relevant for predicting the future. When Markov processes are considered, the variances of successive time periods are additive. A Wiener process is a particular type of Markov process with a mean change of zero and a variance of 1.0 per period (Hull 2000). Interest rates appear to be pulled back to some long-term average level over time and this

phenomenon is known as mean reversion. Therefore, the drift term includes a long-term mean parameter, defined as θ, and a mean reversion parameter denoted k. When the short rate deviates from its long-term mean, θ, it will revert back to this mean at a speed governed by the parameter k. This process is hampered in its ability to revert back to its mean level by the diffusion term which essentially shocks the process at each step in time. In the Vasicek model, the diffusion term σdw is Gaussian or normally distributed. It is important to note that in this process the volatility parameter σ is independent of the level of interest rates.

Following Vasicek, numerous versions of the dynamics of the short rate process have been proposed. The stochastic differential equation (4.16) can be generalised to the following form

$$dr = k(\theta - r)dt + \sigma r^{\gamma} dw \qquad (4.17)$$

The expression in (4.17) can capture most of these versions as special cases. The additional parameter γ measures the degree to which the volatility σ of the short rate depends on the level r. The higher the γ, the more sensitively the volatility reacts. CIR assume that the volatility of the changes in the short rate is sensitive to the square root of the level of the rates, thus $\gamma = 1/2$. Chan, Karolyi, Longstaff and Sanders (1992) have estimated a discrete-time version of the stochastic differential equation (4.17).

The Cox, Ingersoll, and Ross (1985) Model

The CIR model is characterised by one factor, the instantaneous interest rate r, that evolves in continuous time as described by the following first-order differential equation,

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dw \qquad (4.18)$$

This has the same mean reverting drift as Vasicek, but the standard deviation is proportional to $\sqrt{r}$. This model is time homogeneous in the sense that neither the drift nor volatility terms are a function of time. By virtue of the square root process interest rates are prevented from becoming negative and are conditionally heteroskedastic i.e. the volatility of the short-term interest rates increases with an increase in the level of short-term interest rates. dw is a Wiener process. Gaussian processes like the Vasicek (1977) model and the square-root processes as proposed in the CIR (1985) model are the most popular versions of affine diffusions. While Gaussian processes have a constant variance matrix, square root processes introduce conditional heteroskedasticity by allowing σ to depend on the state. However, given the apparent stochastic properties of the volatility of interest rates, Gaussian or constant volatility models imply an element of simplification. In this study the movements in bond yields are estimated using the square root processes of the CIR model.

Using risk-adjusted processes consistent with the absence of arbitrage, the effect of the market price of risk on the level of the short can be incorporated in the model. Therefore, the CIR process given by equation (4.18) can be represented as:

$$dr = (k(\theta - r) - \lambda r)dt + \sigma\sqrt{r}dw \tag{4.19}$$

where λ is the market value of risk.

For the one-factor CIR model, the solution for the nominal price of a pure discount bond is given by

$$P(t,T) = A(t,T)e^{-B(t,T)r} \tag{4.20}$$

where, after incorporating the market value of risk, λ,

$$A(t,T) = [\frac{2\gamma e^{(k+\gamma+\lambda)(T-t)/2}}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma}]^{2k\theta/\sigma^2} \tag{4.21}$$

$$B(t,T) = \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+k+\lambda)(e^{\gamma(T-t)}-1)+2\gamma} \tag{4.22}$$

$$\gamma = \sqrt{(k+\lambda)^2 + 2\sigma^2} \tag{4.23}$$

The continuously compounded yield for discount bonds is given by:

$$R(t,T) = -\frac{\log P(t,T)}{T-t} \tag{4.24}$$

Using (4.20), this can be rewritten as:

$$R(t,T) = \frac{-\log A(t,T) + B(t,T)}{T-t} \tag{4.25}$$

Jump Diffusion Models

Modelling short-term interest rates necessitates incorporating several relevant features in a single specification. The significance of jump-diffusion processes arises from the failure of single factor Gaussian models to capture large variation in short term interest rates. Jump diffusion models have been proposed in the context of the high short-term interest rate volatility and the wide range of skewness and kurtosis observed in consecutive interest rate changes. The CIR (1985) model is able to capture features such as the strong mean persistence (short-term non-stationarity), the long-run mean reversion (stationarity) and the level-dependence of changes or volatility in the short rate. But it is unable to capture the high volatility and volatility persistence of the short rate. In view of this limitation a mixed model of a continuous Brownian motion and a discrete Poisson jump may be able to capture the large variation and the wide range of skewness and kurtosis effects. With a process of this type, both the smooth Gaussian behaviour and the large and infrequent jumps associated with interest rates can be simultaneously accommodated.

The foundations of this Gaussian-Poisson process was first formulated by Ahn and Thompson (1988) who have shown that it is possible to extend the CIR model to the following square root jump diffusion process

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dw + \delta dY \qquad (4.26)$$

where, δ denotes the jump, dW is the standard Brownian motion and dY denotes an independent Poisson process with a jump intensity equal to πr. If there are no jumps (δ = 0), equation (4.26) reduces to the benchmark square-root model of CIR (1985). If the focus of the modelling is to simply price interest rate securities and there is a concern about jumps of the actual interest rate series then the Gaussian-Poisson process provides a useful framework. But if the focus is on capturing most of the variation in bond returns, then this framework may be considered inflexible in terms of incorporating most of the information in the term structure of interest rates.

4.3 Estimating the CIR model

A variety of methods have been developed in the finance literature for the estimation of CIR-type models. The two basic approaches may be characterised as the cross section approach and the time series approach.

In the cross-section approach, only information on the yields of bonds with different maturities at a point in time is used in the estimation process. This generates a different set of parameters for each time period. The state variable r_t, treated as an additional unknown parameter, is estimated jointly with the structural parameters.

This solution is chosen when the purpose of the econometric analysis is to price derivative assets. The disadvantage of this approach is that the risk premium parameters cannot be identified because they are subsumed in the drift term. Moreover, if the estimation is carried out sequentially at different points in time with different cross sections of rates, the estimated parameters can vary with sudden jumps when the observations have to contend with temporary shocks.

The time series approach, on the other hand, focuses on the dynamic implications of the model and ignores the cross-sectional information. A univariate time series approach is based on fitting equation (4.18) to estimate the parameters, using short-term observable data (e.g. the yield of one-month Treasury bills or money market rates) as an approximation of the unknown parameter estimates. In order to properly capture the information contained in the observed interest rates it would be necessary to use these rates across a range of maturities. However, if multivariate time series data are used it would give rise to an identification problem. The CIR model implies that any cross section of rates observed at time t is a function of the parameters (which are constant over time) and the value of the risk factors at time t. Therefore, using more interest rates than risk factors would result in the model becoming underidentified whereby its parameters cannot be consistently estimated. One solution is to allow for discrepancies between observed rates and the theoretical rates i.e. to introduce measurement errors in the relationship between observed rates and the state factors. These deviations can be explained by actual market features such as bid-ask spreads, rounding of prices, differences in the timing of observing financial variables and non-synchronous trading. In a modelling context this can be done by assuming that observed rates are affected by temporary shocks which are

Gaussian white noise errors. Therefore equation (4.15) which is treated as an exact relationship between factors and yields would now read as:

$$R(\tau) = \frac{B(\tau)X}{\tau} - \frac{\ln A(\tau)}{\tau} + \varepsilon_t$$

Although the model is affine in the state vector X, the functions $A(\tau)$ and $B(\tau)$ are non-linear functions of the underlying parameters. So when this assumption about measurement errors is made, maximum likelihood estimation is no longer feasible, because the density of the yields is not available in closed form. Depending on the structure of the variance-covariance matrix of measurement errors, different estimation methods have been proposed using a panel-data approach.

A basic approach to resolving this estimation problem is to select as many different yields as factors and obtain the factors by inverting the model. Pearson and Sun (1994) followed this approach by formulating a likelihood function for a two-factor CIR model on the basis of the conditional density of the underlying factors. The model is estimated by replacing the two factors by two zero-coupon yields that are observed without error. Chen and Scott (1993) estimate a model with two factors and four maturities. In this case, the variance-covariance matrix of measurement errors has less than full rank. They assume that two yields are observed without error so that the model for these two maturities can be inverted directly to obtain the factors. The other yields are assumed to be measured with a normally distributed measurement error. The state variables can be uniquely determined and the inversion approach can be used to obtain the joint density function and therefore the log-likelihood function.

In the case where the variance-covariance matrix of measurement errors is assumed to be full rank, a quasi-maximum likelihood estimator based on the linear Kalman filter is a common technique. The Kalman filter has been used in a series of papers dealing with the estimation of exponential affine term structure models. The Kalman filter is a linear estimation method and makes use of the assumption of an affine relationship between bond yields and state variables to subsequently estimate the parameter set. The main advantage of this technique stems from the fact that it allows the state variables to be unobserved magnitudes.

The nature of the application of the Kalman filter depends on whether the term structure model is Gaussian such as the Vasicek model or non-Gaussian such as the CIR model. A Gaussian distribution is fully characterised by its first two moments and the exact likelihood function is obtained as a by-product of the Kalman filter algorithm. An example of the Gaussian case is provided in Babbs and Nowman (1999), who estimated a two-factor generalised Vasicek model. Babbs and Nowman (1999) observed eight spot rates with maturities between one and ten years. When using non-Gaussian models the exact likelihood function is not available in closed-form, however a quasi-maximum likelihood estimator can be constructed from the first and second conditional moments of the state variables. Examples of the non-Gaussian CIR model, may be found in Duan and Simonato (1995), Lund (1997) and Geyer and Pichler (1999). De Jong (2000) provides an empirical analysis of the affine class of term structure models proposed by Duffie and Kan (1996) using a quasi-maximum likelihood estimator.

Markov chain Monte Carlo estimation is an alternative to the Quasi Maximum Likelihood approach and has recently been proposed by Lamoureax and Witte (2002). The main drawback of this approach is that it turns out to be computationally extremely time consuming because the state variables evolve very slowly. Lamoureaux and Witte (2002) report that it takes more than five days on a very sophisticated machine to obtain a sufficient number of iterations for a two-factor model.

In this chapter, a panel-data estimation of the CIR model is presented from multivariate time series data. Combined use of time series and cross section data as entailed in the panel data approach allows for the identification of the market price of interest-rate risk, which is not identified from each dimension separately. Panel data estimation also provides an effective specification of the model. Its cross section dimension captures the restrictions imposed by the model on the parameters of the bond pricing equations and its time series dimension captures the dynamic model for the state variables.

The approach is based on a state-space representation of the term structure model where the underlying state variable(s) is treated as unobservable. This obviates the need to employ proxies for the unobserved factors. The yields are affine in the underlying state variables and the model explicitly allows for measurement errors. Quasi-maximum likelihood estimates of the model parameters are obtained by using an approximate Kalman filter to calculate the likelihood function.

4.4 The state space representation

This section demonstrates the reformulation of the CIR model given by equation (4.19) in the state space form and draws on the explanations provided in Harvey (1992). This formulation includes a *measurement equation* that relates the observable, or measurable bond yields to the unobservable state variables. The unobservable state variables are, in turn, assumed to follow a Markov process described by the *transition equation.*

Let the state vector X be a Markov process with $X_0 \sim p(X_0)$ and $X_t | X_{t-1} \sim p(X_t | X_{t-1})$. $P(X_0)$ is the density of the initial state and $P(X_t | X_{t-1})$ is the transition density. The exact transition density of the state variable for the CIR model is a non-central chi-square, $\chi^2[2cX_t; 2q+2, 2u]$, with $2q+2$ degrees of freedom and noncentrality parameter, $2u$. (CIR 1985). Estimation of the unobservable state variables by the Kalman filter coupled with a quasi-maximum likelihood estimation of the model parameters can be accomplished by substituting the exact transition density by a Gaussian or normal density. Therefore, the probability density of the state vector at time t, conditional on its value at time, t-1, should be distributed in a manner such that:

$$X_t | X_{t-1} \sim N(\mu_t, Q_t)$$

where μ_t and Q_t are distributed in such a way that the two moments of the approximate normal and exact transition density are equal. The elements of a j x 1 vector μ_t would be defined as

$$\mu_{t,j} = \theta_j[1 - e^{-k\Delta t}] + X_{t-1,j}e^{-k_j\Delta t} \qquad (4.27)$$

where $\Delta t =$ the time interval between t and t-1.

The matrix Q_t is diagonal and is dependent on the state of the process. For a three-factor model, the conditional variance of the transition system would have the following form:

$$Q_{t_t} = \begin{bmatrix} \xi_1 00 \\ 0\xi_2 0 \\ 00\xi_3 \end{bmatrix},$$

where $\xi_j = \frac{\theta_j\sigma_j^2}{2k_j}(1 - e^{-k_j\Delta t})^2 + \frac{\sigma_j^2}{k_j}(e^{-k_j\Delta t} - e^{-2k_j\Delta t})X_j(t_{t-1})$ (4.28)

for $j = 1, 2, 3$.

Yields on zero-coupon bonds are the inputs to the estimation process. Eight maturities have been chosen that span the yield curve from 2 years to 25 years in order to incorporate information affecting trading at the short, medium and long ends of the yield curve.

In the CIR model, the measurement equation represents the affine relationship between zero coupon bond yields and the state variables. Under the assumption that measurement errors in bond yields are additive and normally distributed, the measurement equation for observed yields is given by:

$$R_t = Z(\psi)X_t + d(\psi) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H) \qquad (4.29)$$

where $\psi = (\theta, \kappa, \sigma, \lambda, h_{1\ldots N})$ is a vector of hyperparameters which contains the unknown parameters of the model including the parameters from the distribution of measurement errors. R_t is the n x 1 vector of observations, X_t is the unobservable j x 1 state vector at time t, Z is an n x j matrix, d is an n x 1 vector, ε_t is an n x 1 vector of measurement errors. H is the variance-covariance matrix of ε_t. In this estimation the number of observed bonds and the associated maturities do not change over time. Therefore, H has a constant dimension of n x n and is assumed to be a diagonal matrix. As 8 different maturities are considered in this estimation, the variance-covariance matrix of the measurement errors, H, is an 8 x 8 diagonal matrix.

$$H = \begin{bmatrix} h_1^2 0 \cdots 0 \\ 0 h_2^2 \cdots 0 \\ \vdots\vdots \ddots \vdots \\ 00 \ldots h_8^2 \end{bmatrix}$$

The values in the diagonal would differ implying that the variance of measurement errors will depend on the maturities under consideration. This can be

justified on the grounds that trading activity and, therefore, bid-ask spreads are not equally distributed across maturities. In the case of a one-factor affine term structure model, equation (4.28) would read as:

$$R_t = \frac{B(t,T)}{T-t} X_t - \frac{\log A(t,T)}{T-t} + \varepsilon_t \qquad \varepsilon_t \sim N(0,H)$$

The stochastic differential equation (4.19) represents the dynamics of the state variable as specified in continuous time. As the transition equation captures the discrete dynamics of the state variable, it corresponds to the discrete time version of equation (4.19). This, along with a first order autoregression model, is used to formulate the transition equation,

$$X_t = \phi(\psi) X_{t-1} + c(\psi) + \eta_t, \;\; E(\eta_t | \Im_{t-1}) = 0, \;\; \text{var}(\eta_t | \Im_{t-1}) = Q_t \qquad (4.30)$$

where

$c = \theta_j (1 - e^{-k_j \Delta t})$ is j x 1 vector and $\phi = e^{-k_j \Delta t}$ is a j x j diagonal matrix

$\Delta t =$ the time interval in the discrete sample (here 1 week)

and so the discretisation step $\Delta t = 1/52$ for weekly data.

η_t is g x 1 vector of disturbance terms with mean zero and variance-covariance matrix Q_t and where $\Im_{t-1}$ represents the information available at time t - 1.

It is further assumed that the error terms of the measurement (ε_t) and transition equations (η_t) are not correlated.

4.5 The Kalman Filter

Now that the model in (4.19) has been put in state space form, as defined in equations (4.29) and (4.30) and summarised below, the Kalman filter can be used to obtain information about X_t from the observed zero coupon yields.

Measurement Equation:

$$R_t = Z(\psi)X_t + d(\psi) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H)$$

Transition Equation:

$$X_t = \phi(\psi)X_{t-1} + c(\psi) + \eta_t, \;\; E(\eta_t | \Im_{t-1}) = 0, \quad \text{var}(\eta_t | \Im_{t-1}) = Q_t$$

where $\psi = (\theta, \kappa, \sigma, \lambda, h_{1,\ldots N})$ is a vector of hyperparameters which contains the unknown parameters of the model.

A detailed explanation of the Kalman filter can be found in Harvey (1992) and Lutkepohl (1991). The Kalman filter recursion is a set of equations which allows an estimator to be updated once a new observation becomes available. It first forms an optimal predictor of the unobserved state variable vector given its previously estimated value. This prediction is obtained using the distribution of the unobserved state variables, conditional on the previous estimated values. These estimates for the unobserved state variables are then updated using the information provided by the observed variables. Although the Kalman filter relies on the normality assumption of the measurement error and initial state vector, it can calculate the likelihood function by decomposing the prediction error.

Consider the conditional distribution of the state vector X_t given information at time s. The mean and covariance matrix of this distribution can be defined as

$$\hat{X}_{t|s} = E_s(X_t) \tag{4.31}$$

$$P_{t|s} = E_s([(X_t - \hat{X}_{t|s})(X_t - \hat{X}_{t|s})']) \tag{4.32}$$

where the expectations operator indicates that expectations are formed using the conditional distribution for that period.

To obtain the one-step ahead mean, $\hat{X}_{t|t-1}$ and covariance, $P_{t|t-1}$ of X_t we use the conditional distribution implied by setting $s = t-1$. This yields the following *prediction equations*

$$\hat{X}_{t|t-1} = E_{t-1}(X_t) = \phi(\psi)X_{t-1} + c(\psi) \tag{4.33}$$

where $\hat{X}_{t|t-1} = E_{t-1}(X_t)$

$$P_{t|t-1} = \phi(\psi)P_{t-1}\phi(\psi)' + Q_t \tag{4.34}$$

where $P_{t|t-1} = E_{t-1}[(X_t - \hat{X}_{t|t-1})(X_t - \hat{X}_{t|t-1})']$

To calculate the prediction equations we need to assume initial values for the elements of the state vector in the previous period, $\hat{X}_{t-1}$ and the system matrices $\phi(\psi)$, $c(\psi)$ and $Q(\psi)$. Starting values of X_0 and P_0 are provided.

The second step in calculating the Kalman filter is to revise the estimation from step-one using the updating equations that are actual observations which are based on actual observations of R available at time t. The updating equations are given by

$R_{t\|t-1} = Z\hat{X}_{t\|t-1} + d$;	estimation of R_t	(4.35)
$v_t = R_t - R_{t\|t-1}$;	observation vector estimation error	(4.36)
$F_t = ZP_{t\|t-1}Z_t' + H$;	covariance matrix of $R_{t\|t-1}$	(4.37)
$K_t = P_{t\|t-1}Z_t'F_t^{-1}$;	Kalman gain	(4.38)
$\hat{X}_t = \hat{X}_{t\|t-1} + K_t v_t$;	updating of the state vector	(4.39)
$P_t = P_{t\|t-1} - K_t Z_t P_{t\|t-1}$;	updating of state covariance matrix	(4.40)

The prediction and update steps must be repeated for each discrete-time step in the data sample. For the analysis in this chapter , weekly observations over a period of five years were used.

The intuition underlying the Kalman filter is that $\hat{X}_t$ is the best linear approximation of the true state vector X_t, if the state vector estimation error, $(X_t - \hat{X}_t)$ is independent of past and present observations R_t, i.e.

$$Cov[(X_t - \hat{X}_t), R_s] = 0\,; \quad s = 1, \ldots, t. \qquad (4.41)$$

The Kalman gain, K_t defined in equation (4.38) is derived to ensure that the above condition holds. In order to elaborate on this, one starts by assuming that the state

vector estimation error, $(X_t - \hat{X}_t)$ is equal to the difference between the true state vector, X_t and the prediction of the state vector based on information in the previous period, $\hat{X}_{t|t-1}$ net of a proportion, K_t of the observation vector estimation error, $(R_t - R_{t|t-1})$, i.e.

$$(X_t - \hat{X}_t) = (X_t - \hat{X}_{t|t-1}) - K_t(R_t - R_{t|t-1}). \qquad (4.42)$$

Equation (4.41) implies the state updating equation given by equation (4.39) i.e.

$$\hat{X}_t = \hat{X}_{t|t-1} + K_t v_t$$

where $v_t = (R_t - R_{t|t-1})$ which was defined in equation (4.36).

The above discussion implies that for the observations R_s, s = 1, ..., t-1 and any arbitrary matrix K_t the following condition must hold

$$[Cov[X_t - \hat{X}_t, R_s]] = Cov[\{(X_t - \hat{X}_{t|t-1}) - K_t(R_t - R_{t|t-1})\}, R_s] = 0$$

$$= Cov[(X_t - X_{t|t-1}), R_s] - K_t Cov[(R_t - R_{t|t-1}), R_s] = 0 \qquad (4.43)$$

$$s = 1, \ldots, t-1.$$

As discussed in Duan and Simonato (1998), when the state space model is Gaussian, the Kalman filter provides an optimal solution to predicting, updating and evaluating the likelihood function. When the state-space model is non-Gaussian, the Kalman filter can still be applied to obtain approximate first and second moments of the model and the resulting filter is quasi-optimal. The use of this quasi-optimal filter yields an

approximate quasi-likelihood function with which the parameter estimation can be carried out.

Quasi-Maximum likelihood estimation

In the state space form described above it is not possible to write the density of the observations $R_1,...,R_n$ directly, because the conditional density is assumed. The joint density function of the n x 1 vector of observations is given by

$$\ln L(R_1,...R_n;\psi)=\prod_{t=1}^{n} p(R_t|\Im_{t-1}),$$

where ψ is a vector of hyperparameters and $p(R_t|\Im_{t-1})$ is the distribution of R_t conditional on the information set, $\Im$ at time t - 1. Given the information set $\Im_{t-1}$, the true state vector is normally distributed with mean $\hat{X}_{t|t-1}$ and covariance matrix P_t. Hence, R_t is also normally distributed with mean $R_{t|t-1}=Z_t\hat{X}_{t|t-1}+d_t$ and error variance-covariance matrix F_t.

Assuming that the prediction errors are normally distributed, the log-likelihood function is given by,

$$\log L(R_1,...,R_n;\psi)=-\frac{n}{2}\log 2\pi-\frac{1}{2}\sum_{t=1}^{n}\log|F_t|-\frac{1}{2}\sum_{t=1}^{T}v_t'F_t^{-1}v_t \quad \ldots (4.44)$$

Since the prediction error is Gaussian, equation (4.44) is the quasi maximum likelihood estimator which best explains the observed values of R_t. Both F_t and v_t depend upon the parameter set given by ψ. Therefore, ψ is chosen so as to maximise the likelihood function $\log L$.

4.6 Data and Estimation Results

Data description

The data comprises 265 weekly observations of zero-coupon yields for UK and German Treasury bonds from January 6 1999 to January 28, 2003. These observations were sampled every Wednesday to take advantage of high liquidity and avoid beginning and end of week effects. The data sets have a panel data structure with a time dimension and a cross-sectional (maturity) dimension. For the UK, the data set used here are zero coupon yields available in the Bank of England public domain yield curve database. In the case of Germany, zero coupon yields on euro-denominated bonds have been sourced from Reuters. Eight different maturities that would broadly cover the maturity spectrum of the yield curve are considered; they are 2-, 3-, 5-, 7-, 10-, 15-, 20- and 25-year bonds. Table 4.1 shows the structure of the panel using a sample of UK Treasury zero coupon bond yields.

Table 4.1 Balanced Panel: Time Series and Cross Section Dimension

TIME				CROSS SECTION				
SERIES	2Y	3Y	5Y	7Y	10Y	15Y	20Y	25Y
06.01.99	4.84	4.63	4.4	4.31	4.29	4.36	4.37	4.28
13.01.99	4.66	4.48	4.3	4.25	4.27	4.4	4.43	4.34
20.01.99	4.69	4.5	4.32	4.25	4.25	4.34	4.35	4.25
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
14.01.04	4.19	4.34	4.51	4.6	4.67	4.67	4.62	4.55
21.01.04	4.27	4.41	4.58	4.67	4.74	4.74	4.67	4.62
28.01.04	4.45	4.5	4.66	4.74	4.79	4.77	4.7	4.62

Table 4.2 provides the summary statistics for the estimated zero coupon yields.

Table 4.2
Summary statistics of zero coupon yields: Germany and UK (Jan 1999 to Jan 2004)

Maturity	Mean Yield		Standard Deviation	
years	GER	UK	GER	UK
2	4.18	4.89	0.87	0.86
3	4.36	4.97	0.79	0.78
5	4.61	5.00	0.67	0.64
7	4.83	4.97	0.56	0.52
10	5.05	4.87	0.51	0.35
15	5.12	4.73	0.50	0.21
20	5.55	4.60	0.39	0.22
25	5.56	4.48	0.39	0.24

Figure 4.1 shows the dynamic path of the UK term structure between January, 1999 and January, 2004. Similarly, Figure 4.2 shows the dynamic path of the German term structure over the aforesaid period.

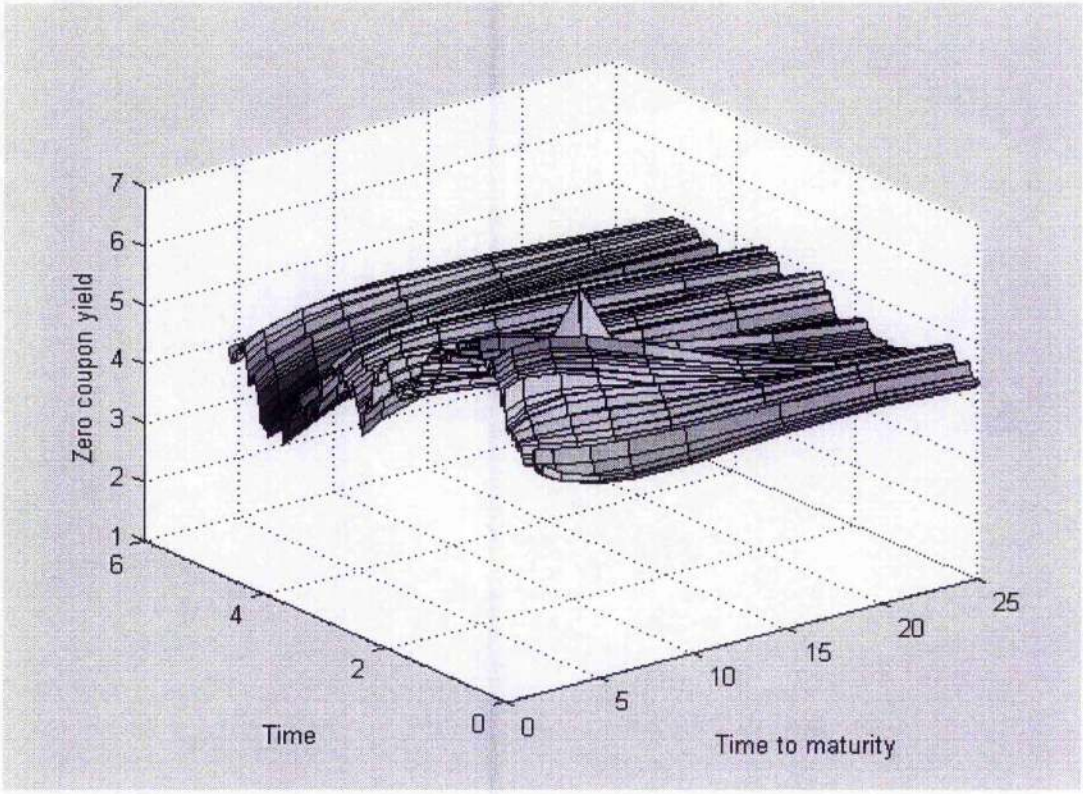

Figure 4.1 Dynamic path of the UK Term Structure (Jan'99 -Jan'04)

In contrast to the UK, the German term structure has evolved in a steady manner with no dramatic changes. It has maintained an upward slope during the five period since the launch of the euro.

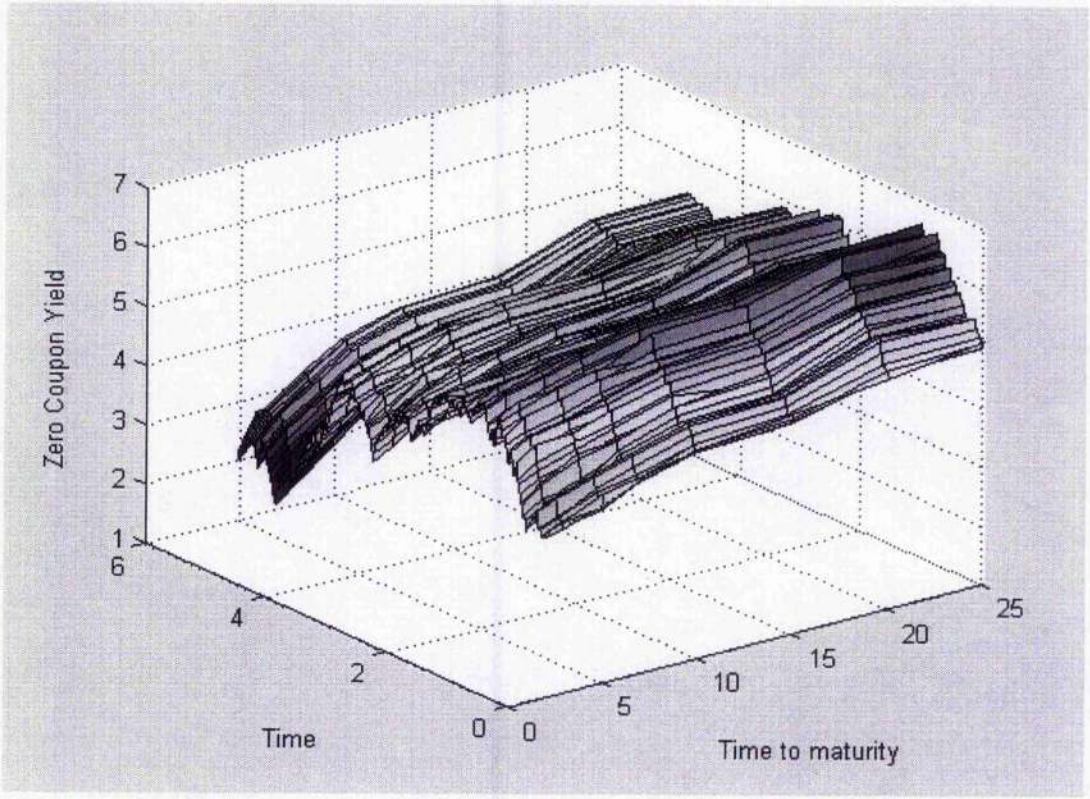

Figure 4.2 Dynamic path of the German term structure (Jan'99 -Jan'04)

Parameter Estimation

The Kalman filter was used to estimate the one-factor CIR model using data on the UK and German term structure of interest rates. The objective was to estimate the parameters of the processes that are posited to drive interest rate changes.

The standard errors of the parameter vector $\psi = (\kappa, \theta, \sigma, \lambda, h_1, \cdots, h_8)$ can be computed by using the result shown by White (1982). He showed that the covariance matrix for $\sqrt{n}(\hat{\psi} - \psi)$ converges to

$$\left[E\left(\frac{\partial^2 L}{\partial \psi_i \partial \psi_j}\right)\right]^{-1} E\left(\frac{\partial L}{\partial \psi_i}\frac{\partial L}{\partial \psi_i}\right)\left[E\left(\frac{\partial^2 L}{\partial \psi_i \partial \psi_j}\right)\right]^{-1}$$

where L is the log-likelihood function. The standard errors are given by the diagonals of the above matrix result. Thus for each observation, the partial derivatives of the likelihood with respect to the twelve parameters $\psi = (\kappa, \theta, \sigma, \lambda, h_1, \cdots, h_8)$ were numerically determined, evaluated at the maximum likelihood estimate $\hat{\psi}$.

The elements of $\frac{\partial L}{\partial \psi_i}$ can be computed by using the symmetric central difference method.

$$\frac{\partial L}{\partial \psi_i} = \frac{L(\psi_i + \delta_i) - L(\psi_i - \delta_i)}{2\delta_i}$$

Diagonal elements of $\dfrac{\partial^2 L}{\partial\psi_i\partial\psi_j}, (i=j)$

These are $\dfrac{\partial^2 L}{\partial\psi_i^2}$ and can be computed using the symmetric central difference method.

$$\frac{\partial^2 L}{\partial\psi_i^2} = \frac{\dfrac{L(\psi_i+\delta_i)-L(\psi_i)}{\delta_i} - \dfrac{L(\psi_i)-L(\psi_i-\delta_i)}{\delta_i}}{\delta_i}$$

or

$$\frac{\partial^2 L}{\partial\psi_i^2} = \frac{L(\psi_i+\delta_i)-2L(\psi_i)+L(\psi_i-\delta_i)}{\delta_i^2}$$

Off-Diagonal elements of $\dfrac{\partial^2 L}{\partial\psi_i\partial\psi_j}, (i\neq j)$

These are $\dfrac{\partial^2 L}{\partial\psi_i\partial\psi_j}$ and can be computed along each axis (i or j) in turn so that

$$\frac{\partial^2 L}{\partial\psi_i\partial\psi_j} = \frac{\dfrac{L(\psi_i+\delta_i,\psi_j+\delta_j)-L(\psi_i-\delta_i,\psi_j+\delta_j)}{2\delta_i} - \dfrac{L(\psi_i+\delta_i,\psi_j-\delta_j)-L(\psi_i-\delta_i,\psi_j-\delta_j)}{2\delta_i}}{2\delta_j}$$

$$\frac{\partial^2 L}{\partial\psi_i\partial\psi_j} = \frac{L(\psi_i+\delta_i,\psi_j+\delta_j)-L(\psi_i-\delta_i,\psi_j+\delta_j)-L(\psi_i+\delta_i,\psi_j-\delta_j)+L(\psi_i-\delta_i,\psi_j-\delta_j)}{4\delta_i\delta_j}$$

Estimation Results

In keeping with the different dynamics of the term structure observed in the two markets different starting values are chosen. For the UK term structure, the true values or initial starting values chosen for the parameters were $\kappa = 0.15$, $\theta = 0.05$, $\sigma = 0.1$, $\lambda = -0.1$. Results of the parameter estimation using the Kalman filter over the entire observation period from January, 1999 to January, 2004 are shown in Table 4.3 . Figures in parenthesis indicate t-values.

Table 4.3 The Kalman Filter estimates of the one-factor CIR model for UK Treasury bond yields from 06.01.1999 to 28.01.2004

κ	θ	σ	λ
0.1443	0.0879	0.0801	-0.1176
(3.45)	(3.46)	(3.76)	(2.53)

Significant parameter estimates were obtained for all the parameters at the 5% level The significant mean reversion parameter of 0.1443 implies mean reversion in the underlying interest rate. The estimate of 0.1443 indicates a mean half life of 4.8 years which is the expected time for the short rate to return halfway to its long-run average mean, θ.[3] Half-life gives the *slowness* of the mean reversion process and a value of 4.8 years would indicate slow mean reversion for interest rates. Accordingly, this process is also characterised by a low but significant volatility estimate (σ = 0.0801).

The market price of risk (λ = -0.1176) is negative, a necessary condition for positive risk premia. The result implies that the risk premium for holding long term bonds is positive.

In the case of the German term structure, the initial starting values chosen for the parameters were $\kappa = 0.15$, $\theta = 0.04$, $\sigma = 0.05$, $\lambda = -0.1$. Results of the parameter estimation using the Kalman filter are shown in Table 4.4. Figures in parenthesis indicate t-values.

Table 4.4 The Kalman Filter estimates of the one-factor CIR model for German Treasury bond yields from 06.01.1999 to 28.01.2004

κ	θ	σ	λ
0.1579	0.0646	0.0556	-0.00095
(20.83)	(15.1)	(2.37)	(0.12)

Parameter estimates are significant for all the parameters except the market price of risk. This would suggest that this variable has not been priced by the market. In accordance with the lower level of short-term yields for German Treasury bonds, the long-term mean parameter is 6.46 per cent as compared to 8.79 per cent for the UK Treasury. The mean reversion of 0.1579 implies a mean half-life of 4.38 years and this is somewhat smaller than that obtained for the UK term structure. However, the

[3] The half life is given by $e^{-kt} = 0.5$. This implies $t = -\ln(0.5)/k$

volatility parameter given by 0.0556 is significantly smaller than that obtained for the UK term structure.

4.7 Conclusion

In this chapter a single-factor CIR model has been estimated for the UK and German term structure for the period January, 1999 to January, 2004. Modelling continuous time term structure models, started with the specification of a time series process for the instantaneous spot interest rate. The no-arbitrage condition then permits the derivation of a bond pricing formula whereby the bond price is a function of the unobserved instantaneous spot rate and the model's parameters. These parameters are the long-run mean, the speed of adjustment towards the long-run mean, the volatility of the short-term interest rate and the market price of risk. The model was estimated for a single factor using a quasi maximum likelihood approach based on the Kalman filter. The Kalman filter algorithm uses observable data on bonds to extract values for the unobserved state variables. It combines both the cross section and time series information in the term structure.

Yields on zero-coupon bonds were used as inputs for the estimation process. The empirical analysis was based on weekly observations of UK and German Treasury zero coupon bonds over the period January 1999 to January 2004. Eight maturities were chosen that spanned the yield curve from 2 years to 25 years and were expected to incorporate influences on the short, medium and long end of the term structure. The parameters of the model and their standard errors were estimated.

Results of the empirical analysis showed that the unobserved instantaneous interest rate exhibits mean reverting behaviour in both the UK and German term structure. However, the mean reversion of the interest rate process has been relatively slower in the UK as compared to Germany since the introduction of the euro. Accordingly, the volatility component, which shocks the process at each step in time was also higher in the UK as compared to Germany. The results indicated that the one-factor CIR model provides a good representation of the UK Gilt-Edged market. However, its inability to meaningfully account for the market price of risk has impinged on its efficacy in capturing the dynamics of the German term structure.

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SUMMARY OF FINDINGS AND SUGGESTIONS FOR FURTHER RESEARCH

This thesis has addressed some empirical questions regarding the interest rate spreads and term structure dynamics of euro and sterling fixed income markets. The analysis has provided an insight into the heterogeneity in the euro-denominated government bond market notwithstanding the issuance of bonds in a single currency. It has also demonstrated that credit risks have now replaced market risks as the principal source of relative risk in euro denominated government bond markets. The loss of monetary sovereignty, that would otherwise have given them the right to print money, and the infeasibility of exchange rate devaluation has exposed EMU governments to credit risk. This changed scenario with regard to credit risk in euro area countries gave added interest to the analysis of linkages with the sterling market.

Comparisons between the euro and sterling markets were not confined to the bond market but also included the interest rate swap market. Both the UK and the larger EMU member states such as France, Germany and Italy have well established bond markets. However, the lack of homogeneity in the euro area government bond market has seen the euro interest rate swaps market assuming benchmark status. A major determinant of the level of swap rates relative to the underlying government bond yields is the perceived risk premium between sovereign borrowers and the interbank market. As swap yields are generally at a premium over comparable government bond yields, the difference between government bond yields and swap yields would be a negative magnitude and a narrower spread would be indicative of greater government default risk. According to this measure, credit risk is higher in EMU member states as

compared to the UK. This finding provided the motivation for the modelling of credit spreads between UK and EMU member states and for examining the degree of market integration between euro and sterling swap spreads. Although both the euro and sterling swap spreads are non-stationary, sterling swap spreads have been perceptibly wider and more volatile. The dynamic path of the UK term structure has also been subjected to a lot of upheaval as compared to the German term structure which has evolved in a more gradual manner. The mean reversion process for the instantaneous short term interest rate has also been found to be slower in the UK as compared to Germany. Accordingly the volatility in the short rate is also higher in the UK.

In Chapter 1, the thesis developed a highly parameterised spline-based estimation process, as well as reviewing models of the yield curve that work with a lower number of parameters. Although splining models improve the trade-off between goodness-of-fit and stability, they tend to oscillate excessively at long maturities, while failing to fit short-term observations. This may result in implausible behaviour of implied forwards at longer horizons. The parsimonious models make the a priori assumptions that forwards at longer horizons could converge to some asymptotic value, as agents have less and less information. Which methodology to choose would depend on the requirements of the estimation process. For example, for monetary policy purposes which require greater accuracy at the short end, the parsimonious model might be more appropriate; whereas for the pricing of financial instruments, which requires greater accuracy at the longer end, the spline-based method may be more effective.

Chapter 2 provides evidence that government default risk is higher in the euro area as compared to the UK and that the magnitude of risk has increased after 2002. Empirical results indicate that the credit spread between the UK and three EMU member states, namely, France, Germany and Italy could be attributed to the latter's fiscal performance. The credit spread increases with an increase in their deficit/GDP ratio and a ratio of net government interest payments to government receipts. In this case, a credit risk model was formulated by introducing default risk into the covered interest parity condition. The behaviour of default risk could also be captured by using an alternative credit risk model. In the context of the continuous time term structure models analysed in Chapter 4, a future area of research could be to try and fit an affine yield model to prices of bonds of different credit risks and varying maturities to estimate parameters describing the behaviour of default risk over time and across maturities.

Chapter 3 examines the case of market integration between euro and sterling swap spreads. The correlation coefficient between changes in euro swap spreads and sterling swap spreads is negligible indicating that credit risk can be attributed to country specific factors as opposed to global influences. The time series of both the euro and sterling swap spreads show volatility clustering and reveal strong ARCH effects. However, the absence of any asymmetric volatility effect of sterling swap spreads on euro swap spreads and vice versa necessitated confining the analysis to symmetric GARCH models. There was also evidence of mild volatility transmission from the sterling swap spreads to euro swap spreads but no spillover effects the other way round. Given the more obvious linkages with US dollar swap spreads for both the markets, it would be appropriate to incorporate this variable into the model and extend

the analysis. Further work in this area might extend the data set and try fitting other GARCH models that would better capture volatility spillovers if, indeed, they do exist.

In chapter 4 a single-factor CIR (1985) model was estimated for the UK and German term structure for the period January, 1999 to January, 2004. Results of the empirical analysis showed that the unobserved interest rate exhibits mean reverting behaviour in both the UK and German term structure. However, the mean reversion process has been relatively slower in the UK as compared to Germany. Accordingly, the volatility component which shocks the process at each step in time was also higher in the UK as compared to Germany. The literature suggests that three state variables are adequate to explain most of the variability in bond yields. However, the chapter focuses on the one-factor CIR model, as the empirical estimation showed that the inclusion of additional factors did not increase the statistical performance of the model for either country. A plausible explanation for this could be the limited period of observation. However, the defining feature of theoretical term structure models based on the pioneering work of Vasicek (1997) and CIR (1985) is that they permit more than one risk source or factor. This sets them apart from models based on the unbiased expectations hypothesis where the term premia usually depend on a single measure of interest rate risk. Extending the period of observation may enable a meaningful estimation of the increased number of parameters that a two-factor model would entail.