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UTILITY THEORY AND ATTITUDES TOWARDS  
RISK IN MANAGEMENT DECISION MAKING.

by

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M. Litt. Thesis

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June, 1979

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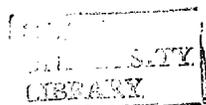
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## ACKNOWLEDGEMENTS

Few research studies are completed without involving the help and support of others and my own work is no exception. I acknowledge the assistance given by Professor Myles Dryden, Department of Management Studies, University of Glasgow who supervised this research and from whose wise counsel I have benefited in many ways.

The support of my employers who allowed this research to be carried out while still in full-time employment is acknowledged as is the help rendered by colleagues in reading and commenting on earlier drafts of this thesis.

A large debt of gratitude is owed to Professor Michael Murray, College of Business Administration, University of Iowa whose own work prompted this study in the first place and whose advice and stimulation over the past years cannot adequately be measured.

Seldom can a family have been more supportive than my own. Thanks are due to my mother who typed the entire manuscript and several earlier drafts with a conscientiousness not always apparent today. Finally to my wife and children who have lived, as I have, with this research and all its trials and tribulations, thank you.

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## ABSTRACT

A recognition of the fundamental and pervasive nature of decision making has resulted in the decision making process assuming an importance that is quite distinct from the detail of any particular problem. A logical and systematic approach developed over many years that shifted the emphasis away from the terminal act of selecting one course of action to an examination of the structure of the whole decision.

This decomposition of decisions highlighted problems associated with various components. One such difficulty was the manner by which the consequences of selecting a course of action were to be represented. With monetary outcomes the use of the real value of money does not always represent a person's true preference pattern and Utility Theory has been put forward as a substitute by which direct preferences and attitudes towards risk can be codified and transformed into a numerical scale.

In this study Utility Theory was viewed as a normative decision making aid and two sets of experiments, involving twenty-nine business managers, were carried out, firstly to create personal utility functions and secondly to gauge to what extent they were workable.

A self administered utility test was specially developed to obtain pound-utile co-ordinates and a least squares regression approach adopted to fit a quartic function to them. This method avoided many problems experienced by previous researchers, particularly the length of time demanded of

each subject and the mathematical description of the pound - utile relationship.

As a test of the workable nature of the derived functions a second set of experiments were carried out where each manager was presented with twenty-four hypothetical business decision problems, contained in narrative form. Testing the effectiveness of the utility functions could not be done simply by comparing actual and prescribed behaviour alone as the functions were being looked upon as normative and not necessarily predictive of what each subject would do. The managers were therefore assigned to one of three groups. Group one were given details of their utility prescriptions prior to tackling the decision problems. Group two were given their prescriptions after they had provided answers to the decisions and were then given the opportunity to make any alterations and group three were given no information about their prescriptions.

A significant variation was detected between the three groups with those having knowledge of their prescriptions displaying the smallest differences between actual and prescribed behaviour. While the timing of this knowledge did not seem to be important the managers, all mature executives, made considerable use of the information they were given. A wide monetary range was covered by the decision problems but for subjects, with knowledge of their prescriptions, the mean difference between actual and prescribed behaviour, over the twenty-four decisions, was as low as £492. For subjects with no knowledge this figure rose to £9,223.

As Utility Theory was being looked upon as representative of underlying risk taking attitudes it was decided to measure such attitudes as distinct from any measurement contained in the utility functions. A formula was derived to obtain a risk taking score for each manager, based on their response to the twenty-four decision problems, and these scores confirmed what had been visible from the utility curves, that the most prevalent attitude was risk aversion.

The prescriptive as opposed to predictive power of the functions was also confirmed when the only relationship between the measured attitude towards risk and the attitude described by the functions was found among those who had been given knowledge of their prescriptions.

The decision problems had been divided equally between those only involving degrees of loss and those only holding out the prospect of profit in order that the attitudes of managers with different backgrounds could be compared. Fifteen of the subjects were employed as insurance managers by their companies and dealt, daily, with situations involving potential losses. Rather than their experience leading them to be less cautious, their familiarity with loss led them to be significantly more risk averse than the remaining subjects and also far more homogeneous as a group. There was no detectable difference between management type where profit was involved, suggesting perhaps a greater ability to adapt to profit than to loss decisions.

## CHAPTER ONE

### INTRODUCTION TO DECISION MAKING.

Decision making is one human process in which everyone acquires a good deal of experience. From early childhood, through school, career, marriage and into retirement we are faced with choice after choice. It is difficult to imagine an existence where no decisions have to be made and even if such a situation could be brought about, few of us would relish the curtailment of freedom and initiative that it would imply.

The process of how decisions are made is so fundamental to our progress and well being that it assumes an importance that is quite distinct from the detail of any particular problem. The recognition of this fact has prompted many, from a variety of disciplines to examine the way in which choices are made.

This present study has been similarly motivated and is concerned with managerial decision making in a business environment. Peter Drucker (1968) captures the pervasive nature of business decision making when he writes, " Whatever a manager does he does through decision making. Those decisions may be made as a matter of routine. Indeed, he may not even realise that he is making them.....But management is always a decision making process." (p. 419)

There is little doubt that decision making is intertwined with all

business activity. Whether such decisions are routine or of considerable significance they exist, and the manager is compelled to act in some way.

The term "decision making" will appear again and again throughout this study and it may be valuable to place a slightly different construction on it than normal. To the majority of people, decision making may imply a final step, the terminal act in selecting one option from a range of possible options.

If, however, decision making is looked upon in the sense of decision building then we can begin to see the importance of examining the decision process to ensure that each stage of the decision is constructed soundly. What this way of looking at decision making does, is to shift the emphasis away from the final choice to the broader process of constructing the decision itself.

This change in emphasis has taken place slowly and from the mid 1950's onwards, the work of Savage (1954), Luce and Raiffa (1957), Chernoff and Moses (1959) and Schlaifer (1959), helped to form the theoretical framework of what was coming to be known as "Decision Theory Analysis". These writers were certainly not the first to turn their attention to the question of decision making but they did mark a change in approach. Previous texts had dealt with the problem mainly from an economic standpoint, looking at how individuals behaved in the face of competing, mutually exclusive alternatives. What was now emerging was an analysis of the decision itself, with the decision being broken down into manageable parts.

The development of this systematic and logical approach to decision making continued and by the late 1960's the various theories and techniques that had appeared were given weight by the publication of works by Raiffa (1968) and Schlaifer (1969), that eventually were to be looked upon as classics in the field of decision analysis. By 1970 Brown (1970) was able to write, "Rare are the instances in which successful use of Decision Theory Analysis is held up through shortcomings in the purely technical state of the art" (p.89).

Although the consequent growth in the development and application of Decision Theory Analysis is largely attributable to the work of Raiffa and Schlaifer at Harvard Business School, it is important to remember the valuable part played by Howard (1968), Lindley (1970), Aitchison (1971), Moore (1972) and Brown et.al. (1974).

#### OUTLINE OF THESIS

The remainder of this chapter is devoted to an examination of the decision process as a discussion on the process of how decisions are made will enable us to identify the direction of this present study.

Chapter two concentrates on Utility Theory and following a brief account of its development as a decision making aid goes on to review the literature relating to normative utility theory, the measurement of utilities and the various applications that have been recorded. The chapter concludes by locating this present work in relation to previous research.

The research problem is stated in chapter three together with the hypotheses that are to be tested. Chapter four describes and justifies the methodology employed in testing the hypotheses while chapter five presents the results and an interpretation of them.

The final chapter provides the conclusions to the study in addition to suggesting scope for further research.

### THE DECISION PROCESS

The basic philosophy behind Decision Analysis is that most decision problems are so complex that they place too much of a cognitive strain on the decision maker. The decision maker is only able to deal with details of a decomposed problem, and it is argued that the total benefit of examining parts of the decision and then re-assembling the whole problem is greater than any benefit derived from tackling it as a whole.

The advantages of decision decomposition were recognized long before formal procedures of decision analysis had been developed. Benjamin Franklin (1772) wrote, " My way of making decisions is to divide half a sheet of paper by a line into two columns, writing over one Pro and over the other Con. Then, during three or four days' consideration, I put down under the different heads short hints of the different motives, that at different times occur to me, for or against the measure.....and where I find two, one on each side, that seem equal I strike them both out.....I think I can judge better and am less liable to make a rash step, and in fact, I have found great advantage

from this kind of equation in what may be called moral or prudential algebra".(p.786.)

The study of the process by which decisions are made has come a long way since then and in doing so has cut across disciplines such as economics, psychology, sociology and statistics.

Toda (1976) looks upon the decision process as the sum of all the subprocesses related to the selection of a course of action and its execution. He goes on to say that, " Most of these subprocesses except some very well formalized ones like Decision Analysis are covert processes taking place exclusively in someone's mind, and are hitherto paid relatively little attention "(p.79). This emphasizes the role of Decision Analysis as a formal framework for the logical analysis of decisions and not just as another technique that might throw some light on any given problem.

Decision Analysis, therefore, starts by highlighting the decisional process itself, quite apart from the detail of any particular problem. When the various stages that make up a decision are brought into the open, aspects of the problem come to light that otherwise could easily be overlooked. This research is concerned with one such aspect but prior to identifying it we will look at the process itself as this will enable us to locate the subject matter of this study in the general framework of the business decision.

Many writers have provided descriptions of the decisional process. Hull, Moore and Thomas (1973), p.226, limit the process to identifying the alternative courses of action, using judgement to evaluate the consequences of each alternative and selecting the alternative considered to be preferable. Vlek and Wagenaar (1979) p.292 identify the same three phases but while these descriptions may be concise, their brevity conceals certain parts of the process that merit individual mention. A more expansive definition of the decision process is given by Elbing (1970) p.13, Thomas (1972) p.31 and Keeney and Raiffa (1976) p.5.

In order to identify that part of the process with which this research is concerned the following paradigm of decision making is put forward.

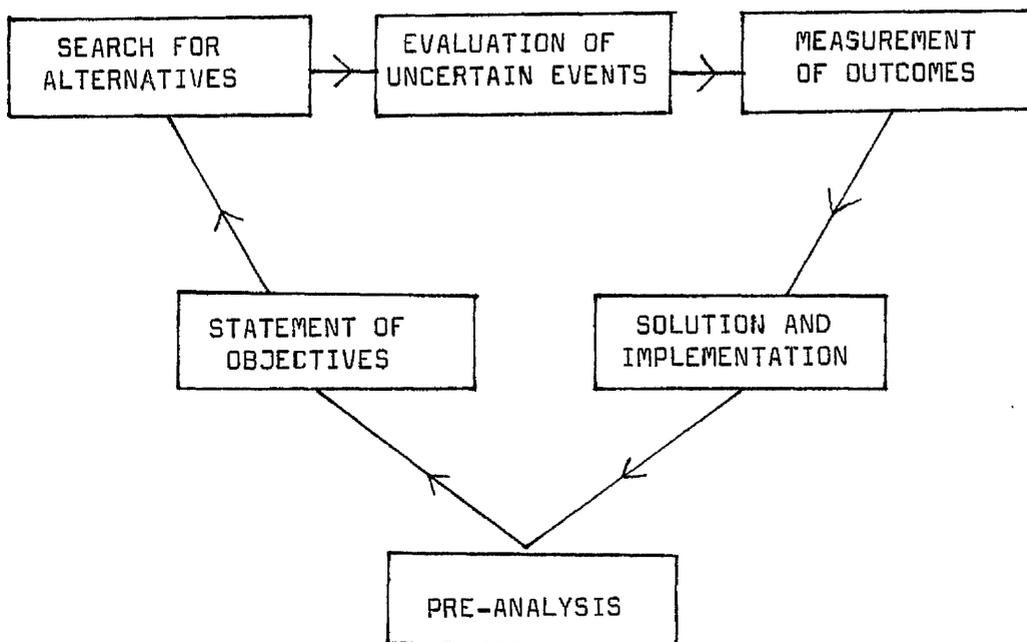


Fig.1

The diagnosis of elements or symptoms of a problem is followed by a statement of the decision objectives together with the attributes by which the degree of attainment of the objectives is to be measured. This leads on to the structural analysis of the problem in terms of alternative courses of action, evaluation of uncertain events and the measurement of outcomes. The solution strategy follows and finally the implementation stage ensures that the result of the decision is put into effect. Implementation also implies the continual monitoring of the system and the feeding back of new variables.

Each aspect of the process described above has been the subject of study but by far the greatest volume of work has been in the evaluation of uncertain events and measurement of outcomes. This research is concerned with the latter of these two areas, the measurement of outcomes, and wherever decisions are posed in this study no search will be required on the part of managers to discover alternative actions, uncertain events or outcomes.<sup>1</sup>

To this extent we are not directly concerned with the work of those like Simon (1957), Miller and Starr (1967) and MacCrimmon (1969) who have directed their attention at times, to the more qualitative aspects of decision making.

#### Evaluation Of Uncertain Events.

---

1

No mention has been made of decision making under certainty, i.e., where each alternative is known to result invariably in a specific consequence as the decisions posed later in this study involve problems where each alternative leads to one of a set of possible consequences, each consequence occurring with a known probability.

The evaluation of the uncertain events that may play upon any problem is not central to the main theme of this study but a short description of what is involved is a necessary lead in to the measurement of outcomes.

Decision making under uncertainty can be likened to a game against nature, where nature is looked upon as some mechanism that produces events that occur in the real world. For this reason uncertain events are often referred to as states of nature. These states of nature will then play upon whatever alternative course of action is adopted.

In a simplified insurance purchasing decision a manager may be considering the two alternatives, buy or not to buy insurance against fire damage. The states of nature or uncertain events in such a problem could be the occurrence or non-occurrence of a fire.

The evaluation of the likelihood that a particular state of nature will occur has been the subject of much work. Where some relative frequency probability is calculable or a ratio of favourable events to the total number of equally likely events is known, such a number could be used to evaluate the likelihood of states of nature. Some would limit the application of probability theory to these two instances and we could refer to such people as objectivists.

In the business world there are very few situations that would satisfy completely the objectivists criteria and in the majority of cases, therefore, we are still left with the problem of measuring

uncertain events.<sup>2</sup> Faced with this problem, the Bayesian or subjectivist emerged with his solution.

The subjectivist maintains that probabilities measure degrees of belief in the likelihood of a particular event occurring, and he tries to bring to the surface his deepest feelings about each uncertain event. While the objectivist deals only with real, observable, countable facts, the subjectivist deals in the subjective assessment of reality.

Subjective probabilities can be used on a whole range of problems wherever some judgement has to be made as to the likelihood of an event occurring or of a proposition about the future being true.

The fact that so few real world situations satisfy the objectivists' criteria has prompted many to consider the use of subjective assessment techniques. Howard Raiffa (1968) justifies his own subjectivist stance when he writes, " I for one, adopted the subjectivist, Bayesian platform (use of subjective probabilities and utilities) gradually and begrudgingly in the early 1950's. It is not easy to give up an identification with scientific objectivity. But my intellectual conversion from the objectivist to the subjectivist school did not carry any emotional convictions until I began working with Robert

---

2

A distinction is sometimes drawn between decision making under risk and decision making under uncertainty with the latter referring to those situations where no probability estimate is available. In this study we will not maintain such a distinction and will look upon the terms as interchangeable, both implying that some probability estimate is used regardless of how it was calculated. A more apt description of those decisions where no probabilities are either available or meaningful has been used by Harrison (1977) when he referred to decision making in conditions of extreme uncertainty.

Schlaifer on concrete decision problems in business ". (p.278).

The reliability and validity associated with placing some probability estimate on our degree of belief about the future revolves around the methodology adopted to measure this belief. Among the main contributors to the theory of measuring subjective probabilities are Winkler (1967a 1967b), Good (1965), Smith (1967), Raiffa and Alpert (1969) and Stael von Holstein (1970).

The work of these scholars is continually being re-examined with the effect that the techniques available for the measurement of uncertain events are becoming less fraught with problems and increasingly more sophisticated. The work of Vlek (1973), Pitz (1974), Selvidge (1975) and Spetzler and Von Holstein (1975) indicate the depth to which the study of subjective probabilities has gone. One major difficulty still remains in that while the techniques of arriving at subjective probabilities are being studied very little work has been done on the question of making managers familiar with basic concepts and, more importantly, having them consider their application.

Having assigned a probability estimate to each uncertain event the decision maker can gauge the attractiveness of the alternative courses of action by calculating the expected monetary value (EMV) of these alternatives.

The E M V is found by adding the monetary consequences of each alternative after having multiplied each consequence by the corresponding probability of it occurring.

Measurement Of Outcomes.

In the insurance purchasing decision such a procedure would not yield an intuitively appealing solution. Let us assume that there are only two possible states of nature, a total loss or no loss. The cost associated with a total loss is £10,000 for which the fire insurance premium is £35. The probability of a 'total' loss will be very low, say 0.001. The decision is whether or not to purchase insurance and the figures are displayed in the decision matrix below.

	Total loss (0.001)	No loss (0.999)
Insure	-£35	-£35
Not Insure	-£10,000	0

The E M V of the insure option is - £35 and for the non-insurance alternative is - £10.

Basing a decision solely on these calculations the manager would not insure but such a decision is unrealistic as there are few people who would not pay a £35 premium to purchase insurance protection against a risk of £10,000. The combination of probabilities and monetary outcomes produced a result that had little or no intuitive appeal. This combination did not reflect the managers' true preferences and

it is this realisation that has led to the replacement of the real value of money by a substitute that more adequately reflects a person's attitude towards risk and his preference ordering for different monetary values.

One such substitute is found in Utility Theory and it is on this theory that the remainder of this thesis is based.

Vlek and Wagenaar (1979) define utility as, " a psychological quantity which indicates a person's strength of preference for something ". (p.272). In the insurance example the decision maker would normally have a preference for no loss but when faced with the chance of loss would be prepared to pay more than the expected value of the loss to avoid the risk.<sup>3</sup> In other words while he preferred no loss he was willing to experience a small but certain loss, the insurance premium, in order to be relieved of the possibility of incurring a larger one.

---

<sup>3</sup>

This view of Utility Theory is an important factor in explaining the economic growth of insurance. Without the concept of utility why should a person pay £35 for insurance against a loss that has an E M V of £10 (£10,000 x 0.001). The answer is that to most people the potential loss of £10,000 is so great that they are willing to pay much more than £10 to avoid it.

## CHAPTER 2

### UTILITY THEORY : PREVIOUS RESEARCH AND RELEVANCE OF THIS STUDY.

This chapter opens with a brief account of the historical development of Utility Theory and then concentrates on the literature relating to normative utility theory, the measurement of utilities and reported applications of the theory. It concludes by identifying some unresolved problems and indicating the relevance of this present research.

What has been suggested in the previous chapter is that the decision maker has various possible alternatives open to him; that he has one or more objectives which he hopes to achieve; and that states of nature will occur which, for each of the alternatives, will result in a range of outcomes or consequences. As was evidenced by the insurance purchasing example on page 11, the real value of each alternative and state of nature combination may not represent a manager's true preference pattern over the range of outcomes.

Using the real value of money as a measure of each outcome resulted, in that example, in a solution to the decision that neither described how the manager normally acted in similar situations nor how he would act in the future.

In a general decision making problem preferences are of two types, as

explained by Hull, Moore and Thomas (1973) p.226.

- a) The direct preference of one outcome over the other, e.g., the preference for no loss over a loss of £10,000 should the factory be destroyed by fire.
- b) Attitudes to risk where a person prefers a certain outcome to the chance of securing one of two possible outcomes at given probabilities, e.g., the preference for the certain loss of £35, being the insurance premium, rather than the chance of losing £10,000 if the factory burns down or breaking even if there is no fire.

While direct preferences of the type shown in (a) are less of a problem when monetary outcomes are used, most people preferring progressively larger amounts, the type (b) preferences are no less complex when money is the measurement being employed. In our insurance purchasing example it was the type (b) preference that was not taken account of when only the real value of the monetary outcomes was considered.

When considering the question of decision making under risky conditions we are concerned with both forms of preference and if a theory of choice behaviour in such situations is to be established then it must reflect preferences about the outcomes in a certain given situation. In this way it will reflect not only how the manager feels about the outcomes per se but also how he feels about them in the context, with the associated probabilities, of a particular problem.

## HISTORICAL PERSPECTIVE.

The criterion that historically had been developed to deal with risky choices was the maximisation of mathematical expectation. It had been developed during the 16th and 17th centuries by those concerned with dice, cards, roulette, and other games of chance. The outcomes of each alternative were multiplied by their respective frequency of occurrence and then summed. The alternative having the largest value was then selected as the most attractive.

During the early part of the 18th century people gradually realised that the use of mathematical expectation did not, in fact, produce results that conformed with the experience of everyday life. Daniel Bernoulli<sup>1</sup> and Gabriel Cramer were among those who questioned the use of money values as a valid method of explaining choice behaviour and Bernoulli (1738) in his explanation of the St. Petersburg Paradox suggests the concept of "moral expectation".

The St. Petersburg Paradox highlighted the inadequacies of the mathematical expectation theory. Consider a situation where A has entered into an agreement with B. whereby A is to toss a coin and if it lands heads up he is to receive £1 from B, further that if a head does not appear until the second toss he is to receive £2, if a head does not appear until the third toss he is to receive £4 and so on. What, then, is the mathematical expectation of A's gamble? As the outcomes increase with each subsequent flip of the coin the probability of maintaining a stream of tails get progressively lower. The result is as shown below:

---

1

It is interesting to note that Bernoulli used, as an example, the marine insurance purchasing behaviour of merchants.

1st Flip	2nd Flip	3rd Flip	4th Flip	..Nth Flip
.5 x £1	.25 x £2	.125 x £4	.0625 x £8	P x £R

Where N = an infinitely high number of flips.

P = the probability of gaining a head on the Nth flip.

R = the amount offered if a head does not appear until the Nth flip.

The expected value of this gamble is infinitely great and on this basis A should be willing to pay a large sum of money to be allowed to play. In fact most people would only pay a relatively low amount. The paradox is, therefore, that the mathematical expectation did not yield a result that conformed with common sense.

Bernoulli explained A's unwillingness to take advantage of the gamble offered to him by suggesting that a given amount of money is not of equal importance to people having different amounts of wealth to start with. Utility for money increases at a decreasing rate.

Bernoulli went on to suggest that the logarithm of income is an appropriate utility function and many people look upon this aspect of his work as being the most important. Bernoulli held that the relative value of an increment of money varies directly with the increment and inversely with the original sum possessed. i.e.,

$$y = k \log x + c$$

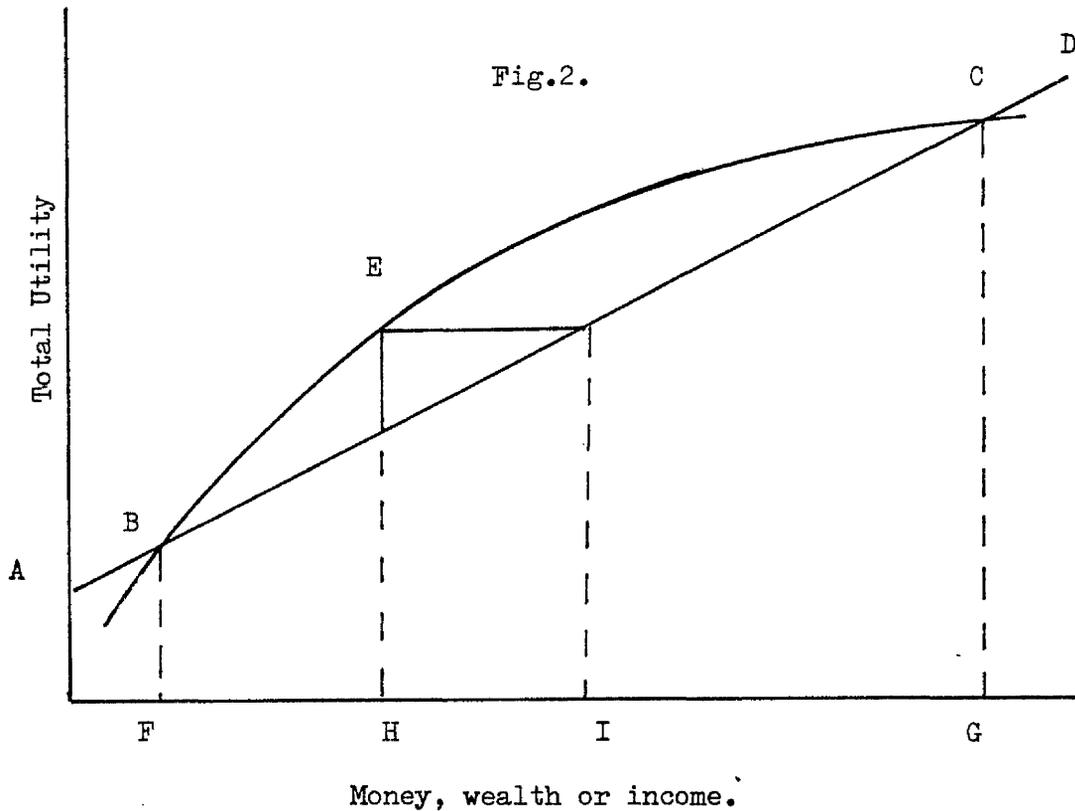
Where y = the total utility of wealth

k = a constant

x = the original sum of money

c = the constant of integration.

This theory can be represented graphically and is shown in Fig.2.



The line A D represents the mathematical expectation that each increase in wealth yields an equal utility to the individual. Line B E C is the total utility curve representing Bernoulli's moral expectation. For values between F and G the expected utility of any given amount of money exceeds its actuarial value and as a result the individual has expressed a preference for a greater degree of certainty than is held out by the probabilities upon which the mathematical expectation is based. In other words the individual used in Fig. 2 would accept H rather than take the chance of gaining the larger amount I.

As was said earlier, most people remember Bernoulli's work for the logarithmic transformation of the utility function, but a second aspect of his work could be said to be of equal importance and that is his view that the mathematical expectation of utility should be used in place of simple mathematical expectation when considering choices

involving risk.

Later in the same century Bayes (1763) touched upon the same issue of utility and probability in choices involving risk but it was not until 1931 that the two concepts of utility and personal probability were finally brought together by Ramsey (1931). Ramsey developed what he called " a logic of partial belief " and in so doing laid the foundations upon which a criterion of maximising expected utility could be developed.

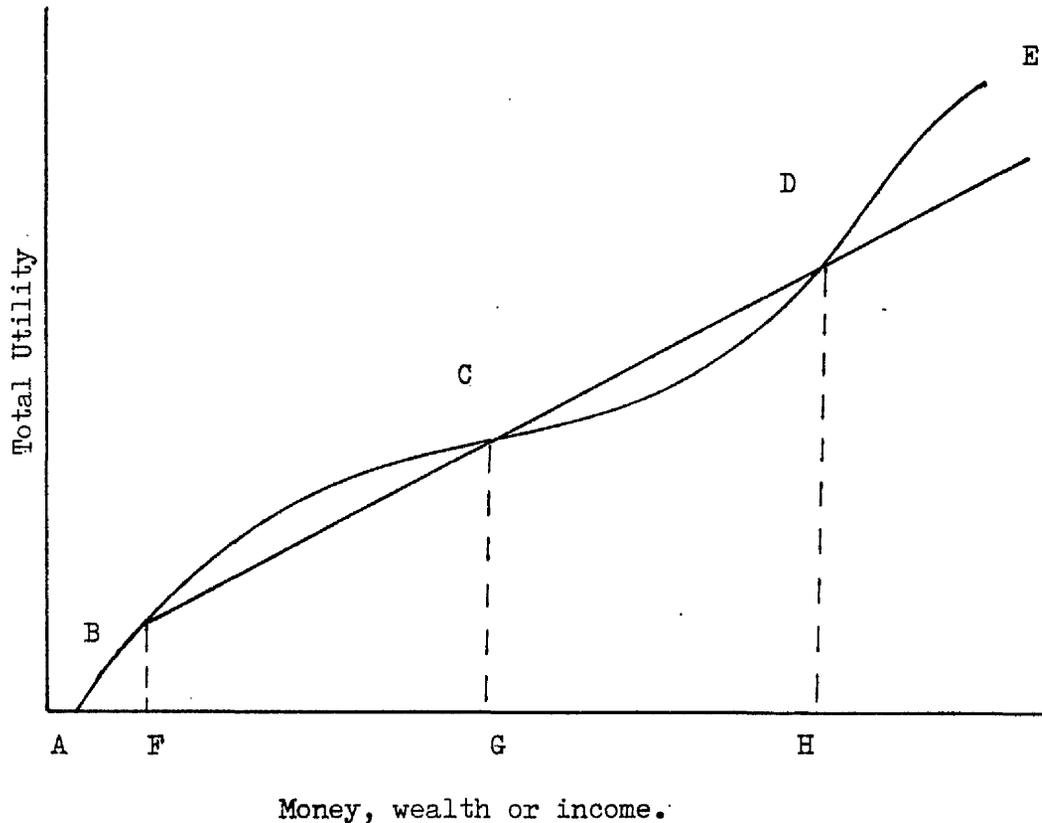
Although the theory of risky choices had been enhanced greatly by the work of those scholars it was certainly a long way from being free of paradox. Bernoulli's St. Petersburg Paradox and his theory of moral expectation, in itself, gave rise to a paradox. This one was not accorded the respectability of a name but could easily have been termed the " Insuring Gamblers' Paradox ".

Friedman and Savage (1948) deal with it by suggesting a utility curve that is both concave and convex over different income amounts for the same individual. The problem arises because of Bernoulli's assumption of diminishing marginal utility which certainly accords with the actions of a person prepared to pay a definite amount of money by way of insurance to be released from his uncertainty about some larger amount he may or may not have to pay out in the future, it does not, however, describe the actions of that same person should he purchase a lottery ticket and in so doing risk the loss of income in the hope of a large gain.

The moral expectation curve of the person willing to purchase a lottery

ticket must be below the straight line of mathematical expectation. In Fig.2 a concave curve would have to be drawn to represent the gambler. Fig 3 shows Friedman and Savage's solution to the insuring gambler's paradox.

Fig. 3



In words Friedman and Savage conclude that an individual exhibits diminishing marginal utility of money income for incomes below some income (below income level G), increasing marginal utility of money for incomes between that income and some larger income (between incomes G and H) and diminishing marginal utility of money income for all higher incomes (incomes greater than H).

#### NORMATIVE UTILITY THEORY.

The debate on the shape of utility curves continued but it was John Von Neumann and Oskar Morgenstern (1947) who paved the way for the

eventual application of utility theory in the field of decision making<sup>2</sup>. Their approach to utility theory differed significantly from earlier writers and the fact that their treatment of it did differ may be related to the main body of their book, which was concerned with the theory of games. Rather than dealing with utility as a descriptive concept they looked upon it as a prescriptive theory i.e., it suggested ways of acting in order to arrive at an optimal solution.

Von Neumann and Morgenstern suggested that if a person was able to express preferences between pairs of gambles where the gambles incorporated basic outcomes, then it was possible to introduce utility equivalents of these basic outcomes in such a way that if a person was guided solely by the utility expected value he would be acting in accordance with his true tastes. The proviso to this was that there existed an element of consistency in the person's tastes.

This consistency requirement was considered in some detail by Von Neumann and Morgenstern and they put forward a number of axioms, or postulates, of rational behaviour. Luce and Raiffa (1957), Ackoff (1962), MacCrimmon (1965), Krantz et.al. (1971) and others have also presented an axiomatic treatment of utility.

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Von Neumann and Morgenstern state their "excuse" for examining utilities to be, ".....the application of a classical preliminary device of scientific analysis : to divide the difficulties....."(p.16) This relates well to our earlier discussion on decomposition of decision problems.

The Axioms.

The principal axioms are listed below, with the phrases "is preferred to" and "is indifferent to" being represented by the symbols  $\succ$  and  $\sim$  respectively.

- a) Comparability. All consequences,  $C_1, C_2, \dots, C_n$ , can be ordered according to their attractiveness. For each pair  $(C_1, C_2)$ ;  
 $C_1 \succ C_2$ ,  $C_2 \succ C_1$  or  $C_1 \sim C_2$ .
- b) Transitivity. If  $C_1 \succ C_2$  and  $C_2 \succ C_3$  then  $C_1 \succ C_3$ . Similarly if  $C_1 \sim C_2$  and  $C_2 \sim C_3$  then  $C_1 \sim C_3$ .
- c) Probability Mixtures. A gamble with probability  $p$ ,  $[C_1, p, C_2]$ , can be formed by a mixture of consequences and when  $C_1 \succ C_2 \succ C_3$  a value of  $p$  can be found such that  $[C_1, p, C_3] \succ C_2$  and a further probability  $q$ , such that  $C_2 \succ [C_1, q, C_3]$ .
- d) Dominance. The fact that  $C_1 \succ C_2 \succ C_3$  implies that  $C_1 \succ [C_1, p, C_2]$  and that  $[C_2, p, C_3] \succ C_3$ . In other words a gamble is unattractive when it may worsen but not improve a person's position. On the other hand a gamble is always preferred when it may improve but not worsen the person's position.
- e) Sure-thing. When  $C_1 \succ C_2$  it implies that  $[C_1, p, C_3] \succ [C_2, p, C_3]$ . The preference relation among gambles is independent of the value of any fixed consequence, 'sure-thing'.

When a person is consistent, he obeys these axioms, then it can be said that there exists a preference ordering function, called a utility function, that assigns real numbers  $u(C)$  in such a way that;

$$C_1 \succ C_2 \text{ if and only if } u(C_1) > u(C_2).^3$$

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Notice that it would not be accurate to say that  $C_1 \succ C_2$  because  $C_1$  had the higher utility. Preferences existed prior to them being represented by utiles.  $C_1$  has a larger utility than  $C_2$  because it is more preferred, the opposite is not the case.

$$\text{and } u \left[ C_1, p, C_2 \right] = pu(C_1) + (1 - p) u(C_2).$$

This last result is the expected utility hypothesis stating that expected utility is calculated as the sum of the utiles of consequences with each consequence being multiplied by its probability of occurrence<sup>4</sup>.

Creating axioms will not, in itself, turn people into rational decision makers. Mosteller and Nogee (1951), Davidson et.al., (1957) Fellner (1961) and Ellsberg (1961) all showed that many individuals were irrational when making decisions. The inconsistencies were apparent despite the variety of research methodologies employed e.g., dice, lotteries and urns. After Ellsberg's publication Raiffa (1961) commented, inter alia, that the various experimental findings, particularly the findings of Ellsberg where subjects displayed behaviour that seemed contrary to "common sense" and persisted with it even when their inconsistencies were pointed out to them, did not negate the value of utility theory but rather, underlined the need for such a theory.

It is natural that most people will not always behave according to these axioms. Some will exhibit intransitiveness in preference orderings, many will enter gambling situations 'for the fun of it' even when it is highly unlikely they will gain. In certain cases all possible steps will be taken to avoid a gamble. Axiom (C), Probability Mixtures, is a good example of this last point. It is stated that a probability  $p$  can be found such that  $\left[ C_1, p, C_3 \right] \succ C_2$  substituting the consequences from

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This conclusion is often referred to by saying the utility function is linear. This can lead to confusion, in view of the many uses of the word 'linear' and as we will be using the word later for a distinct purpose its use at this point has been avoided.

the insurance purchasing decision at the end of chapter one of  $C_1 = 0$ ,  $C_2 = -£35$  and  $C_3 = -£10,000$  we can see that  $p$  would have to be 1 before most people would prefer not to purchase insurance.

Edwards et.al. (1965) Aumann (1962) and later MacCrimmon (1968) and Slovic and Tversky (1974) have looked at each axiom in detail. The general result has always been that decision makers do not abide by the basic postulates. In MacCrimmon's study most subjects accepted that they had violated reasonable postulates about consistent behaviour and were prepared to revise their choices when the inconsistencies were pointed out. From a practical point of view the axioms can be looked upon as no more than testable hypotheses about human behaviour.

The value of the utility function, in ordering preferences over consequences, is that it may minimise or eliminate breaches of the consistency axioms by prescribing the way in which a person should act in a manner consistent with his basic judgements and preferences.

Normative - Predictive - Descriptive.

In other words, Utility Theory suggests how a person should act, not how he does act or how he will act. It is normative as opposed to descriptive or predictive. The normative/descriptive dichotomy is easy to see in that Utility Theory can be used either to represent how men should act or it can be used, after a decision, to describe how they did act. This type of dichotomy is not uncommon in other real life situations with moral codes telling us how we should act and the news media telling

how we do act, with the law laying down rules as to how we should behave and court records describing how we do.

The normative/predictive dichotomy is not just as clear as the normative/descriptive. Some would say that to state how a man will act in a decision making situation is the same as suggesting how he should act. To many, to predict and prescribe are synonymous and there seems to be no consensus as to whether utility theory is normative or predictive.

Raiffa (1961) (p.690) writes "Savage's theory (the theory of maximising expected utilities) is not a descriptive or predictive theory of behaviour, it is a theory which purports to advise any one of its believers how he should behave in complicated situations". On the other hand Borch (1965) refers to the same concept of maximising expected utilities as being a method of predicting decision making behaviour.

In this research we take the view that Utility Theory is Normative. The value of a normative utility theory can be summarised by saying that it codifies preferences thus allowing the opportunity to alter any choices that deviate from "rational" preference postulates, it greatly eases the problems associated with determining preferences between alternatives made complex either by the multidimensional nature of the alternatives themselves or the fact that the preference is only one stage in a sequential choice process and finally it enables the decision maker to transform his preferences into a numerical scale with the consequent advantages of being able to reduce many alternatives to a common form prior to determining the optimum.

Later in this chapter we review the literature relating to the applications of Utility Theory, in particular prescriptive Utility Theory, but one important point that rarely appears in the literature is whether or not the use of normative Utility Theory does provide the "best" preference orderings. It certainly provides the rational decision maker with a set of preferences to which he should adhere, assuming he does not want to deviate from the axioms upon which the theory is based. Most would agree that this is a good thing as it introduces the opportunity to be consistent. The use of prescriptive Utility Theory may lead to the "best" decision as far as the individual is concerned, but it is still questionable whether such a decision will be best for the firm as a whole. If we assume that corporate decision makers do deviate from prescriptive Utility Theory axioms, as the literature would seem to suggest, one has to question if this degree of apparent irrationality is not the essence of entrepreneurial activity or at least a major factor in it.

What is rational behaviour will differ for each man and if Utility Theory is looked upon as a means of representing how a man should act then the prescriptive powers of the function have to be validated by the individual. If deviations from the prescribed course of action seem more acceptable than the prescribed behaviour then the model must be rejected as not being a good normative model. This assumes that the person has given considerable thought to what is acceptable and also that the function is looked upon as being without fault.

Neither of these two assumptions can readily be relied upon,

the former for reasons of human nature and the latter due to the "state of the art" of constructing utility functions. While human nature may inhibit attempts to ensure that people are completely aware of all the implications of rejecting the normative model's solution to choices, only the boundaries of our own knowledge of the measurement of utilities can inhibit our attempts at ensuring that the utility function is flawless.

#### THE MEASUREMENT OF UTILITIES.

Our knowledge and understanding of the measurement of utilities has been enhanced by many studies over the years. An examination of the literature relating to normative Utility Theory does reveal an almost even division between studies concerned with the measurement of utilities and those involved with the behavioural aspects of the theory.

If the literature is further classified then the behavioural studies can be divided into those concerned with the axioms and other doctrinal aspects of utility theory and a somewhat smaller number having to do with the applications of the theory either in the laboratory or in the field. Many of the studies referring to the axioms and other fundamentals of utility theory have been touched upon earlier and reports on the practical applications of the theory will be dealt with later in this chapter. In the meantime we will look at the literature bearing upon the measuring of utilities.

There would certainly be little point in formulating a normative utility theory if there was no satisfactory method open to us for the

measurement of utilities. Viewed this way the measurement problem assumes an importance that is not altogether reflected in the literature. The question of satisfaction with a method of measurement is inevitably impermanent as our knowledge expands with each experiment making use of a new method.

The measurement of utilities is a problem that continues to occupy the attention of those concerned with the application of utility theory. It is not possible to lay down a set of rules that, if followed, will lead eventually to the construction of a utility function, thus the measurement of utility differs significantly from the measurement of, for example, heat or sound. This does not imply that utility functions cannot be constructed for most problems and as we will see later in this chapter a wide range of problems have been approached using a utility function as an aid.

One result of the absence of a comprehensive system of measurement is that the literature on the creation of utility functions has tended to emphasize the value of a method of measurement in relation to the particular experiments or problems being tackled. Seldom, except perhaps in the field of investment analysis, are guidelines suggested to the reader which lead to a measurement technique capable of wide application. Keeney and Raiffa (1976) suggest a reason for this by saying, "this is mainly because the assessment of utility functions is as much of an art as it is a science and, therefore, no single set of rules can be laid down that invariably results in a utility function" (p.188.) We will look at five methods that have been employed to measure utility.

## A) Direct Rating Method.

The most straight forward method of measuring utilities is to ask the subject to rate the commodity in question either by selecting a number on a scale or by placing a mark on some straight line scale. This form of measurement is restricted to problems where the outcomes are discrete as it is difficult to see an application where continuous outcomes apply. Torgerson (1958) discusses this method of direct assessment and explains a number of variations on the original idea. Apart from the restriction to problems involving discrete outcomes, there is one other major drawback. The direct assessment technique could be used to construct a utility scale relating to a consumer and, say, his views of washing machines but if that same consumer was given different amounts of money and asked to rate them on a particular scale he would have the greatest difficulty in understanding what was being asked of him. The reason for his difficulty is not solely the direct assessment method, as he could use it quite well when thinking of the washing machines. What has caused the difficulty is that money is capable of being measured on other scales and the consumer would be confused concerning what scale to use. Would he rate them according to their absolute value, value to him or in any of a number of ways? Even if the consumer did produce some rating it would not be possible to state that the resultant utility scale represented his attitude to risk.

In cases where outcomes can be measured by some other scale another method must be employed. Three methods have been devised that are all based on the Von Neumann and Morgenstern (1947) concept of the "standard gamble".

The approach that each takes is slightly different but in essence they all involve establishing a relationship between a series of probabilities and a number of outcomes. The form of this standard gamble is shown in Fig.4. The subject is being asked to compare Z with  $pA + (1 - p) B$ , where  $p$  is the probability associated with achieving A, a given outcome, and  $(1 - p)$  is the probability of securing B, the other outcome. A questioner can elicit either the value of Z,  $p$ , A or B that result in the subject being indifferent between Z and  $pA + (1 - p) B$ .

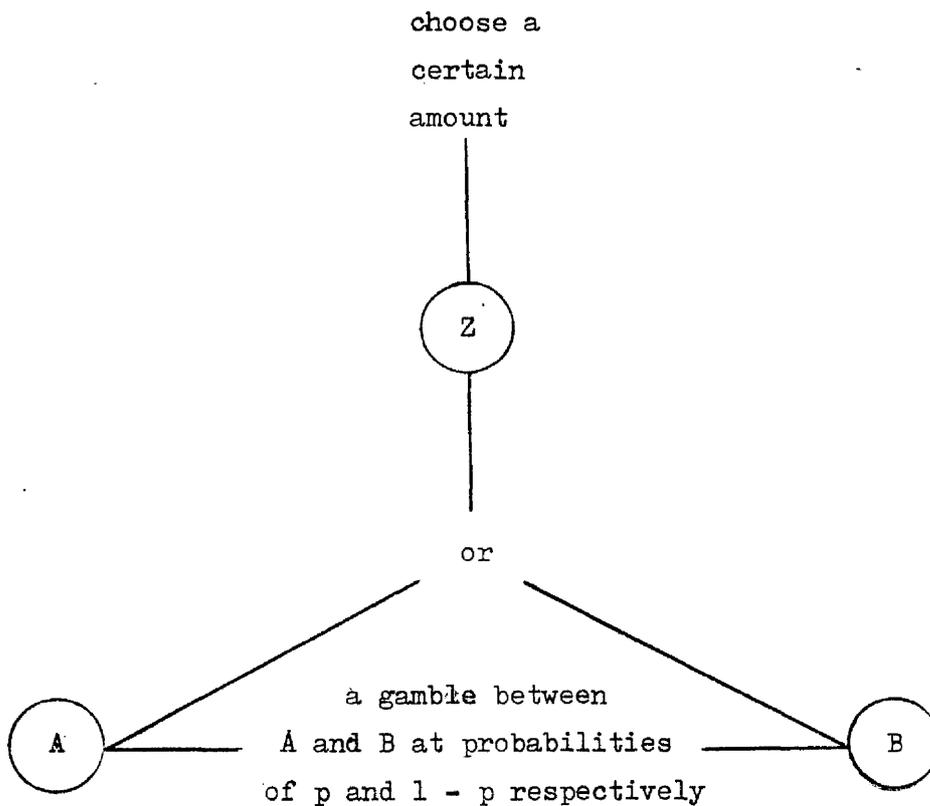


Fig.4

B) Sure Thing Method.

The first of the three methods based on the standard gamble principle can be referred to as the 'Sure Thing' method and involves the subject in selecting the particular value of Z where he is indifferent between

receiving it and entering a gamble where he will receive A, p number of times and B, (1 - p) number of times. In a simple example we could consider an investor who is trying to select the optimum investment alternative from a list of several options. He decides to substitute utilities for the absolute values of the interest rates offered on the various alternatives. He first selects the extreme outcomes, e.g., 15% and 6% and arbitrarily assigns the utility of one to be 1 and of the other to be 0. There is no significance in the scale chosen, he could equally well have assigned 47.35 and 97.82, for the two extremes, as no meaning attaches to the absolute utility value. Most experiments have used scales of 0 - 1 or 0 - 100. To find the utility value of the interest rates he must find an amount that will make him indifferent to the chance of securing 15% with a probability of p and 6% with a probability of (1 - p). Let us assume that the respective probabilities have been calculated as 0.75 and, consequently 0.25. In other words:

$$Z = p \cdot A + (1 - p) \cdot B$$

$$Z = 0.75 \cdot 15\% + 0.25 \cdot 6\%$$

At this stage the subject decided that he would be indifferent between receiving 11% as a sure thing and running the risk of getting either 15% or 6%. Substituting 1 and 0 for 15% and 6% we have:

$$U(Z) = p \cdot U(A) + (1 - p) \cdot U(B)$$

$$U(Z) = 0.75 \cdot 1 + 0.25 \cdot 0$$

$$U(Z) = 0.75$$

$$U(11\%) = 0.75$$

Knowing the utility value of 11% will then assist him in finding other utility values and in eventually constructing a utility function.

Some writers, such as Becker and McClintock (1967) refer to this method as the 'selling' method. This description has an appeal when considering outcomes that are positive, in the sense that they do not involve the subject in losing. In the previous example we could talk of the investor selling his chance of possibly getting a 15% or 6% return on his investment. If the interest rates were such that the investor was losing money no matter which alternative he selected it would be unusual to refer to him as selling his chance at losing either £X or £Y. It would be more common to say that he is trying to buy his way out of this situation by paying someone to take the chance for him rather than to say he sold the gamble for minus £W.

Describing the method as the 'sure thing' or 'certainty equivalent' method not only helps to explain the technique but resolves the problem of referring to buying and selling gambles. Krantz and Tversky (1965) raised the important point that it may not always be justifiable to relate the utility of a buying price with the utility of the wager being purchased and this caused many to reconsider findings based solely on buying prices, such as the work by Mosteller and Nogee (1951) and Edwards, Lichtenstein and Slovic (1962).

### C) The B R L T Method.

This method, developed by Raiffa (1968), reverses the problem facing the decision maker. In the sure thing method the decision maker selected his sure thing that made him indifferent to a gamble, Raiffa proposes a system that compels the subject to select a gamble that makes him indifferent to a given sure thing. The method has become

known as the B R L T (pronounced 'brilt') method. The letters B R L T stand for basic reference lottery ticket and refer to the hypothetical gamble that the decision maker is asked to imagine. Of all the measurement techniques B R L T may, at first, be the most difficult to grasp but it does in its own way highlight the very problem it attempts to solve.

The method relies on a person being able to equate each one of the outcomes in a decision problem with some imaginary lottery so that he can say he is indifferent between playing the imaginary lottery or getting the real payoff as represented by the outcome in the problem itself. In doing this the decision maker sets the probabilities associated with the lottery. He may think of the lottery as a perfect wheel of fortune, an urn filled with different coloured balls or any other game of chance involving events that are equally likely, mutually exclusive and exhaustive.

The basic reference point is the establishment of two reference payoffs, one which is at least as good as the best outcome in the real problem and another which is at least as bad as the worst. The lottery or reference gamble could involve anything that the decision maker really likes or would find attractive. He may decide that either life membership of the opera, an expensive new stereo set or a world cruise is at least as good as the most favourable outcome in the real problem and that a free subscription to a magazine he detests is at least as bad as the least favourable outcome. Working on a scale of 0 to 1 this means that one of the outcomes to the real problem, the most favourable one, is indifferent to a 1.0 B R L T and that one other of the outcomes, the

least favourable, is indifferent to a 0.0 B R L T. By probing his preferences for all outcomes between these two he can find what B R L T's will make him indifferent to the other outcomes in the real problem.

Fig.5 shows a decision problem represented by a simple decision tree.  $O_1$  and  $O_6$  have been assigned B R L T values, as already explained, and after considerable questioning the decision maker considers that he feels indifferent between;

$O_2$ and $a$	.4	B R L T
$O_3$ and $a$	.7	B R L T
$O_4$ and $a$	.6	B R L T
$O_5$ and $a$	.9	B R L T

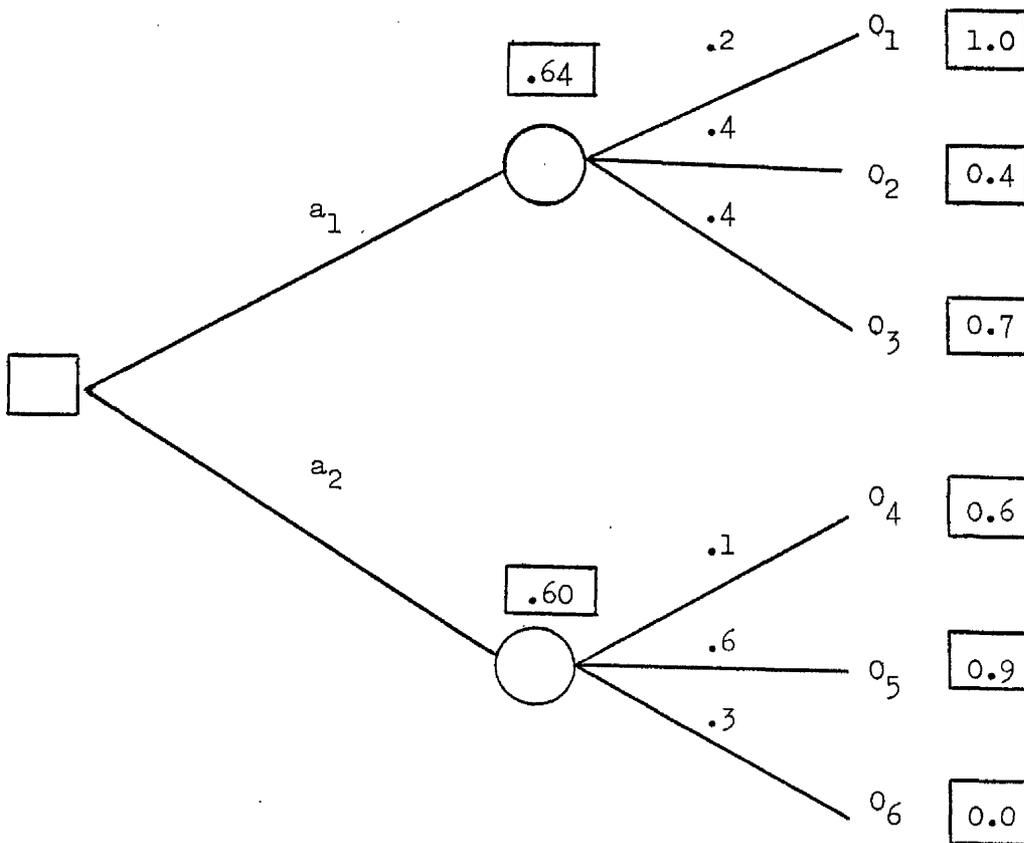


Fig. 5

The B R L T's are shown at the tip of each branch in place of the real payoffs, whatever they may have been. By averaging out and

rolling back to the chance nodes it appears that alternative  $a_1$  represents a .64 B R L T while  $a_2$  represents a .60 B R L T. The decision maker would obviously prefer the higher B R L T, the greater chance of securing his ideal prize, and would block off  $a_2$ .

This brief description of B R L T summarises more than a chapter of Raiffa's (1968) basic text. Descriptions of the B R L T method are also to be found in works by Becker and McClintock (1967) Hull, Moore and Thomas (1973) and Moore and Thomas (1973). We can envisage two problems with this method. Firstly the system is complicated and difficult to operate with people who are unfamiliar with the concept of utility. The explanations necessary may suppress a decision maker's true feelings and the method may end up becoming an end in itself as opposed to a means to an end. Secondly there is the problem of asking a person to select a gamble that makes him indifferent to a sure thing, rather than asking him to select a sure thing that makes him indifferent to a gamble. The latter, as was mentioned above, can be utilised in either a buying or selling situation but the B R L T approach does seem to be similar to the buying situation and consequently may encounter the difficulties, already mentioned, concerning the equating of the utility of a bid for a lottery with the utility of the lottery itself.

#### D) Mid-Point Method

The last of the three measurement techniques having their base in the Neumann - Morgenstern Standard gamble is the Mid Point Method. This technique is a combination of 'sure thing' and 'direct rating' in that it asks the decision maker to select the outcome that to him is

equivalent to a 50/50 gamble on two payoffs i.e. Given A and B as two outcomes in a problem the subject is asked to consider that each is equally likely and to decide what outcome is equivalent to that gamble. Having done that, each half can then be further sub-divided.

Despite certain problems that may arise due to a person's convictions about gambling and lotteries and the fear that he may not be describing his preferences accurately, these three standard gamble methods have found the most favour with researchers both in discrete and continuous variable situations.

#### E) Ordered Metric Method

Decision makers are not always able to look upon their problems as being a set of outcomes to rank or a set of problems equivalent to a lottery where the outcomes have a given chance of occurring. In many decisions, particularly business decisions, the decision maker knows the outcome that is least desirable to him. Having ordered the outcomes he can then very often, also order the differences between the outcomes. In problems where utility is a function of a discrete variable, a method relying on ordering, known as the 'Ordered Metric' method can be employed. Coombs (1950) is accorded the credit for this technique, which requires very little of the decision maker by way of understanding difficult concepts or in providing answers to complex questions.

The difficult part of the method is the ordering of differences between outcomes. Given three outcomes A, B and C the decision maker

may be sure that A is the worst outcome, that B is next to worse and that C is next to B, but when he is asked if the difference between A and B is less than the difference between B and C he may experience some difficulty. He can be helped by asking himself if a 50/50 gamble on A and B is preferred to a 50/50 gamble on B and C. This reintroduces the gamble and may not be advisable unless the subject is completely unable to order the differences. Let us assume that the difference between B and C is less than between A and B. If the utility of the worst outcome is ranked as 0 and subsequent differences are ranked 1, 2, 3...n, to the nth difference then this gives:

$$U(A) = 0, U(C) - U(B) = 1, U(B) - U(A) = 2 \text{ therefore}$$

$$U(A) = 0, U(B) = 2, U(C) = 3.$$

This review of measurement methods has not mentioned the work of Farrar (1961), Markowitz (1959), Arrow (1965), Meyer and Pratt (1968) and Hakansson (1971) who have concerned themselves primarily with the utility of an investor for money. Many assumptions have been made and challenged with the aim of avoiding bombarding the investor with a series of questions, of the form we have described in the above methods. The alternative they were seeking was to ask a few questions to elicit basic objectives and risk aversions and then, based on a number of assumptions, construct a theoretical utility function. This is very much like picking a function 'off the peg'. For this to work it demands a complete understanding, on the subjects part, of concepts such as compound returns, decreasing and increasing risk aversions. While such concepts may be known by many experienced investors it is unlikely that a business decision maker, in the normal course of business, would have reason to be aware of them.

## APPLICATIONS OF UTILITY THEORY

One point that was made earlier relating to the formal analysis of decisions was that a logical framework for decision analysis existed that was quite independent of any particular decision. This is evidenced by the application of Decision Theory Analysis to problems as wide ranging as the siting of nuclear power plants, Keeney and Nair (1975), family planning decisions Beach et.al. (1976), seeding hurricanes, Howard, Matheson and North (1972) and the choice of handgun ammunition for a police department, Hammond et.al. (1975). In comparison to reports of general decision analysis projects there are relatively few studies reporting the application of Utility Theory. The studies that have been reported, however, cover a wide range of problems and result in a similar conclusion to that in the paragraph above. Namely Utility Theory as a means to aid decision making stands distinct from the detail of any one problem. The only common feature of the decisions being that they require some form of preference ordering of particular attributes.

This study is concerned with management decision making and the application of Utility Theory but it is useful to remember that Utility Theory can and has been applied to many forms of problems. Krischer (1974) addressed the problems associated with the decision to operate on patients having a cleft lip. Brown and Peterson (1975) calculated utilities related to various political agreements drawn up to ensure the West's continued supply of oil.

## Business Applications.

The business applications of Utility Theory would seem to have been impeded by misunderstanding and lack of knowledge of the basic concept of utility. This is no different from attempts at applying the theory in other areas except that the business man places great value on his experience and intuition when confronted by a decision. This may explain, in part, his reluctance to consider the use of formal Decision Theory Analysis, and particularly the use of Utility Theory.

What must be impressed upon managers is that decision models are not intended to replace intuition or experience. Knowledge of the formal theory and methods of assessing utilities can be a valuable supplement to the usual intuitive approaches to decision making.

The work of Grayson (1960) marks one of the first attempts at assessing utility functions in an operational setting. Grayson confronted a number of oil men, engaged in the exploratory search for gas and oil, with hypothetical drilling opportunities. They were asked to accept or reject them on the basis of the investment required, potential payoff and probability of success in finding oil or gas. The majority of the oil men were 'wildcatters' so described as they were not averse to accepting unfair gambles.

The basic format of the experiments was that the 'wildcatter' was asked if he would invest, for example, \$20,000 in a drilling venture that had a potential gross payoff of \$100,000 if it were successful and had a .4 probability of success. Depending upon the answer the probability level was increased or decreased to the point where the 'wildcatter' was indifferent between accepting and rejecting the investment.

If indifference was found at probability  $p$ , then:

$$U(0) = p \cdot u(\$80,000) + (1 - p) \cdot u(-\$20,000)$$

This equation is founded upon the basic standard gamble and if two points on the utility scale are arbitrarily set then any number of points could be derived. Grayson obtained, on average, sixteen points for each 'wildcatter' and drew a line of 'best fit' through them. He had little success at introducing the utility concept to the 'wildcatters' who seemed to have a mistrust of using graphs and scales to represent their judgement. There were also technical problems with Grayson's study, many of which were due to the fact that he was one of the first to test Utility Theory in a practical business area. One such problem was that no attempt was made to mathematically describe the relationship between dollars and utiles. Later developments, if they had been known at the time, would have greatly enhanced Grayson's work. An example of this was highlighted by Kaufman (1963) when he found that a logarithmic utility function was a very good fit to the data produced for one of the 'wildcatters'.

Green (1963) produced work that underlined the feelings expressed by Grayson's 'wildcatters'. Green's subjects were sixteen executives from a large chemical company and he attempted to construct utility functions for them both as individuals and business executives. Scepticism was expressed by the subjects as to the validity of using utility functions.

The B R L T method of utility assessment was carried out by Cramer and Smith (1963) on a number of managers involved with research and development decisions. The method was found to be time consuming and

inadequate in determining the utilities at the upper and lower ends of the utility scale.

In contrast to the above mentioned studies, which indicated that managers did not place any great value on utilities, is the work of Marshak (1965). He reported the use of a prescriptive utility model by accountants in discussion of mergers and acquisitions. The accountants, mentioned by Marshak, had also progressed to the use of maximising expected utilities.

Immediately following these early studies came the, now very well known, work of Swalm (1966). Swalm's intention was to describe, rather than prescribe, but as a step forward in our understanding of Utility Theory it represents a valuable contribution. One hundred executives were involved and a utility function was created along similar lines to Grayson's 'wildcatters'. The questioning took the form of realistic business situations where the manager was being asked to act, as a business decision maker, in choosing a certain amount of money in preference to a 50 - 50 chance of gaining or losing a given amount. Each manager was individually questioned, which took some considerable time, and utility curves were drawn. It is important to note that, despite the findings of Kaufman, these curves were still drawn by the 'eyeball' process and no attempt was made at expressing a mathematical relationship between dollars and utilities.

Swalm concluded his study by asking if business managers really are the risk takers that everyone thinks they are. He found that his subjects did not attempt to maximise expected dollar income in situations involving risk and that Utility Theory was at least a step in the right

direction as far as describing their behaviour was concerned.

Thirty-six executives were used by Spetzler (1968) in his attempt to develop a corporate risk policy for capital investment decisions. Spetzler recognised the problems encountered by the previous researchers and set out to interview each manager and acquaint him with the idea of ordering and quantifying preferences.

Having described the role played by Utility Theory and having stressed the importance of assessing attitudes towards risk, Spetzler presented each manager with an investment opportunity involving two possible yields, one representing a high return and the other a low return. The probability of achieving a high return was given and then varied to find the point at which the subject was indifferent between accepting or rejecting the investment project. This method of assessment closely follows the earlier work and is again based on the standard gamble.

Later, Spetzler created logarithmic utility functions using a least squares error approach and returned to each subject for a further discussion. The conclusion was that each subject preferred the functions prescribed course of action to his original choice. This process of combining qualitative risk characteristics and quantitative risk assessment produced appealing results for almost all subjects, but it was long and involved. A great deal of time was spent with the managers and the practical value of the technique must be called into question.

## UNRESOLVED PROBLEMS AND RELEVANCE OF THIS RESEARCH.

Apart from the studies mentioned above there is little evidence of empirical work among managers, related to the application of normative Utility Theory. As was mentioned on page 26 much of the previous research, other than that related to the measurement of utilities, has concentrated on an axiomatic treatment of the theory. Some researchers moved on from looking exclusively at the axioms to try and classify basic principles and increase acceptability, Moskowitz (1974) being a good example of this, but their work was still centred on testing the validity of certain axioms. Even MacCrimmon and Larsson's work (1976) on a total re-evaluation of Utility Theory was spurred on by a diminishing belief in the justifiable nature of the axioms.

This present research is more in the style of the Grayson, Green, Swalm and Spetzler studies to the extent that it suggests a method of measuring utilities for business managers and then sets about using, or applying, the resultant utility functions.

### Unresolved Problems.

The comparison of actual behaviour with the axioms of Utility Theory present a number of unresolved problems, but in view of the direction of this study we will leave the discussion on basic postulates and concentrate on unresolved problems in the area of the measurement and application of utilities.

- a) In many of the experimental studies employing a utility function the fitting of the actual curve was left to free-hand drawing or as some have termed it, the eye-balling process. The loss of accuracy and the time necessary to use

such a curve, particularly where a large number of prescriptions have to be made, presents problems. It has to be said here that some very sophisticated mathematical devices have been employed by decision theorists to explain the relationship between utilities and different attributes. The various works of Robert Schlaifer, Kenneth Arrow, John Pratt, Ralph Keeney and Howard Raiffa are ample evidence of this. It does seem that many of these devices are applicable in particular circumstances and only when certain assumptions are made. These assumptions then have the effect of reducing the measurement task from one of obtaining a whole curve to one of determining a few parameters in a functional form. The utility of an investor for money represents one of the main areas where assumptions are made and a mathematical relationship consequently derived. The functions created in this way are theoretical functions, not based upon empirical observations, and as a result they are not specific to any one individual.

- b) Obtaining pound-utility co-ordinates is shown by the literature to have been an often long and complex business. The answer, as evidenced by previous research, has been to make assumptions that reduce the questioning necessitating only the determination of certain key parameters. A method is still sought that will quickly and efficiently enable co-ordinates to be derived for any given individual.
  
- c) having derived a utility function the problem of changes in that function over time arises. Bell (1975) examined this

and tackled the problem of the devastation caused in forests by the spruce budworm. This particular problem is typical of many where attributes are of a long term nature. It would seem that little work has been directed at the time problem, with the work that has been done concentrating on modifying or updating functions, as opposed to making it simpler to derive them.

- d) The importance of attitudes towards risk in business problems, with purely monetary outcomes, has been mentioned. Psychologists in the utility area have examined risk taking behaviour and personality factors. Decision theorists have on occasion taken personality factors and related them to decision behaviour but there is little trace of emphasis being given to the important link between underlying attitudes towards risk and decision behaviour.

Assuming that the prescriptive utility model suggests how a person should behave in a decision making situation and assuming further that the utility model does itself in part reflect attitudes towards risk then there does seem to be some justification for examining the relationship between attitudes towards risk and what a model suggests a person's attitude towards risk should be.

- e) The willingness of a decision maker to use the information provided by a normative utility function is crucial. The general receptiveness of the individual can be as much of a deterrent

in the use of Utility Theory as the validity of the actual prescriptions themselves. Conrath (1973) touches on this topic and Wade and Longbottom (1972) report one user of Decision Analysis as saying (p214), " .... increasing their (management's) understanding of what we are trying to do, should make Decision Analysis more widely accepted and used in the future". In relation to the utility component of Decision Analysis the unresolved problems include when to reveal utility prescriptions, in what manner is the information to be provided, how long should a decision maker be allowed to reflect on differences between actual and prescribed behaviour.

#### Relevance Of This Research.

As with most forms of research the value of this work is not to be found as a study in itself but as a contributory link in the knowledge chain. It is neither possible nor practical to examine the full list of unresolved problems shown above, which is itself not exhaustive, but the relevance of this present study in relation to certain of those problems, and to previous research in general, is as follows:

- a) Business managers are used as subjects in the creation and application of utility functions. This avoids the problems involved in using students or other non-management subjects and averts some of the criticisms leveled at earlier work.
- b) A new test is devised to obtain pound - utile co-ordinates in a manner more appealing to the business manager.
- c) A method of describing the relationship between pounds and utiles is put forward which brings some rigor to the process and allows a comparatively quick calculation of prescriptions

in any situation.

- d) Emphasis is given to the attitude towards risk that is evidenced both by actual behaviour and prescribed behaviour and a formula is derived by which attitudes to risk can be ranked.
- e) The strength of belief in original decisions and willingness to change in the light of prescriptions is gauged by revealing utility prescriptions to different subjects at different times.

#### Multiattribute Utiles.

So far we have considered the assessment of a utility function as a function of a single variable. In many cases more than one attribute, often non-monetary attributes, will be involved. The purchase of new machinery may be measured by attributes including cost, design, capacity, durability, replacement times etc., etc.

In situations like this Multiattribute Utility Theory ( M A U T ) can be applied. No further mention of M A U T will be made as the nature of this present research involves a number of problems in the application of Normative Utility Theory that are common to both uni - and multi - dimensional utilities.

## CHAPTER 3

### THE PROBLEM

The area of decision making is so vast that almost any review would be incomplete in some respect. The preceding survey of past work has attempted to portray certain general theoretical points while concentrating on those topics that are important to this present research.

As was stated in the introduction this study is concerned with attitudes towards risk and the use of Utility Theory as a prescriptive decision making aid. Utility has been used in a number of disciplines for varying reasons but apart from decision theorists possibly the main use of Utility Theory has been made by economists and psychologists. Economists have generally relied upon utility as a means of explaining choice behaviour in situations involving two or more commodities where risk is absent. Psychologists have tended to look upon utility as being useful in the analysis and description of choice behaviour, whether under conditions of risk or not.

These very brief accounts of the role of utility in economics and psychology have been provided solely to highlight the different approach of the decision theorist in his use of utilities. In decision theory we are not so much concerned with analysis and

description as we are with prediction and prescription in situations involving varying amounts of a commodity and varying degrees of risk.

Many have suggested that if Von Neumann and Morgenstern had selected a term other than Utility, to describe their work, much of the criticism of the concept could have been avoided.

The semantic problem has certainly not assisted the growth and acceptance of Utility Theory in decision making and this fact has been acknowledged by, at least, one writer. Hammond (1967) entitled his paper on Utility Theory, "Better Decisions with Preference Theory" and explains his choice of the word "Preference" as being a deliberate attempt to avoid any ambiguity.

The term Utility Theory is now firmly established as an important part of decision theory and few text books on decision making omit reference to it.

We mentioned in the previous chapter that the need for Utility Theory arises as people assign different preference orderings to the same commodity and also display varying attitudes to degrees of risk. Utility Theory, if it is to be viable in a decision making situation must take account of both direct preferences and attitudes to risk.

Keeney and Raiffa (1976), Raiffa (1968) and Thomas (1972) all introduce the concept of unidimensional Utility Theory as being a mechanism that combines a person's preferences and attitudes allowing

Utilities to be substituted for real values in any decision problem. In situations involving non-monetary outcomes such as levels of pollution, social attitudes, health, tourist attractiveness, military efficiency and many more, it is clear that each decision maker will have his own preference ordering priorities. It is unlikely to have a situation where all people ranked differing levels of each outcome exactly alike.

When the problem of uncertainty is added to the decision then each decision maker's attitude towards the degree of risk involved will combine with, and possibly modify, his preference ordering.

When the outcome of a decision or the attribute used to measure the extent to which an objective has been attained is in monetary terms then the direct preference can almost be assumed. Most people will prefer progressively higher amounts of money. As we saw in the previous chapter, the effect that risk has in decisions involving monetary pay-offs is quite substantial. Although most people will exhibit very similar preference orderings for the real value of different levels of money they will display widely varying attitudes towards the risk involved in attaining or not attaining these particular amounts.

In business decisions involving monetary attributes the role of Utility Theory lies more in its ability to cope with different attitudes to risk than its ability to combine direct preferences and risk attitudes. This study, therefore, concentrates on attitudes towards risk and Utility Theory rather than on the axiomatic

intricacies of preference orderings. Having decided that utilities can be used the major task is how to arrive at a utility value for each level of each attribute.

Keeney and Raiffa (1976) sum up their introduction to unidimensional utility theory by saying (p.134) " If someone is sold on the merits of the above argument (Utility Theory) as we are, then the critical issue becomes: How can appropriate values (utilities) be assessed in a responsible manner? This is really the essence of our problem".

The study of Utility Theory and its business application is still in its early years, and this research endeavours to represent one more step along the path leading to an increased knowledge of the value of utility in decision making.

The actual location of this present study in relation to previous research was discussed on pages 42 to 46 of the last chapter.

What now follows is a statement of the exact problem to be tackled together with the various hypotheses that will be tested in an attempt to provide a solution to the problem.

#### THE RESEARCH PROBLEM.

The main problem, in question form, is as follows:

"To what extent can a workable, utility based, normative model be created for

decision making under risky conditions?"

This general problem lends itself to subdivision into two parts. Each of these two parts then gives rise to a number of sub-problems. It is only when these sub-problems are examined that the real direction of this research is made clear. The two divisions of the main problem and their various sub-problems are as follows:

- A) To what extent can a utility based normative model be created for decision making under risky conditions?
  - i) In what way can pound/utile co-ordinates be derived?
  - ii) Is a self-administered utility questionnaire viable?
  - iii) How is a curve to be fitted to the derived co-ordinates?
  
- B) To what extent will the model be workable?
  - i) How will prescribed choices compare with actual choices in a number of decision making problems?
  - ii) To what extent will decision making behaviour be influenced by knowledge of the models' prescriptions?
  - iii) How will the model reflect risk taking attitudes, as defined for the purposes of this study?

- iv) Will risk taking attitude be influenced by management background and the nature of the decisions posed?

A number of hypotheses have been created and these are detailed below. Prior to reciting the hypotheses it may be important to emphasize that although established statistical techniques will be employed in testing them there are hypotheses that are concerned with the methodological problems associated with the creation of the utility function.

Some would argue that methodological points do not require to be included in the list of hypotheses. This would be the case if established methodological procedures were being employed but in this study it has been thought advisable to prepare a comprehensive list of hypotheses including those that some may look upon as methodological considerations.

#### HYPOTHESES.

##### A) Creating a Utility Function.

- 1) A self administered test incorporating the choice of certainty equivalents, by subjects, can yield pound/utile co-ordinates.
- 2) A least squares regression programme will yield a non-linear monotonically increasing utility function from the derived pound/utile co-ordinates.

B) The Use of the Utility Function.

- 3) Any differences between model prescriptions and actual choices, in a number of decision problems, will be associated with the knowledge each subject had of how the model suggested he should act.
- 4) Differences between model prescriptions and final actual choices, in a number of decision making situations, will be smaller for those subjects who knew of their prescribed behaviour than for those subjects who had no knowledge of their prescribed behaviour.
- 5) Differences between actual and prescribed behaviour will be affected by whether a subject a) knew of his prescribed behaviour before being asked to make his choice or b) was given the opportunity to change his first choice in the light of his prescribed behaviour.
- 6) There will be no relationship between the risk taking attitudes of each subject as displayed by both the prescribed course of behaviour and actual behaviour, in a number of decision making situations.

C) Attitudes Towards Risk in Decision Making.

A particular method of sample selection was employed, for reasons

that will be discussed in the following chapter, page 58 , and this results in our being able to divide the total number of subjects into two sub-sets each representing a different management function. Having done this, certain further hypotheses, relating to sub-problem (B) (iv) can now be tested and these are:

- 7) There will be a relationship between risk taking attitude, as defined for the purposes of this study, and management type.
- 8) Risk managers will be more risk averse than non-risk managers in decisions involving potential losses.
- 9) There will be a relationship between risk taking attitudes and different probabilities of loss.

## CHAPTER 4.

### METHODOLOGY.

The way in which this study relates to previous research has been discussed at the end of chapter two and in the last chapter the research problem and hypotheses to be tested were stated. This present chapter looks at the methodology employed in carrying out the work. Under four main headings the chapter looks at sample selection, measurement of risk taking attitudes, creation of a utility function and using the utility function.

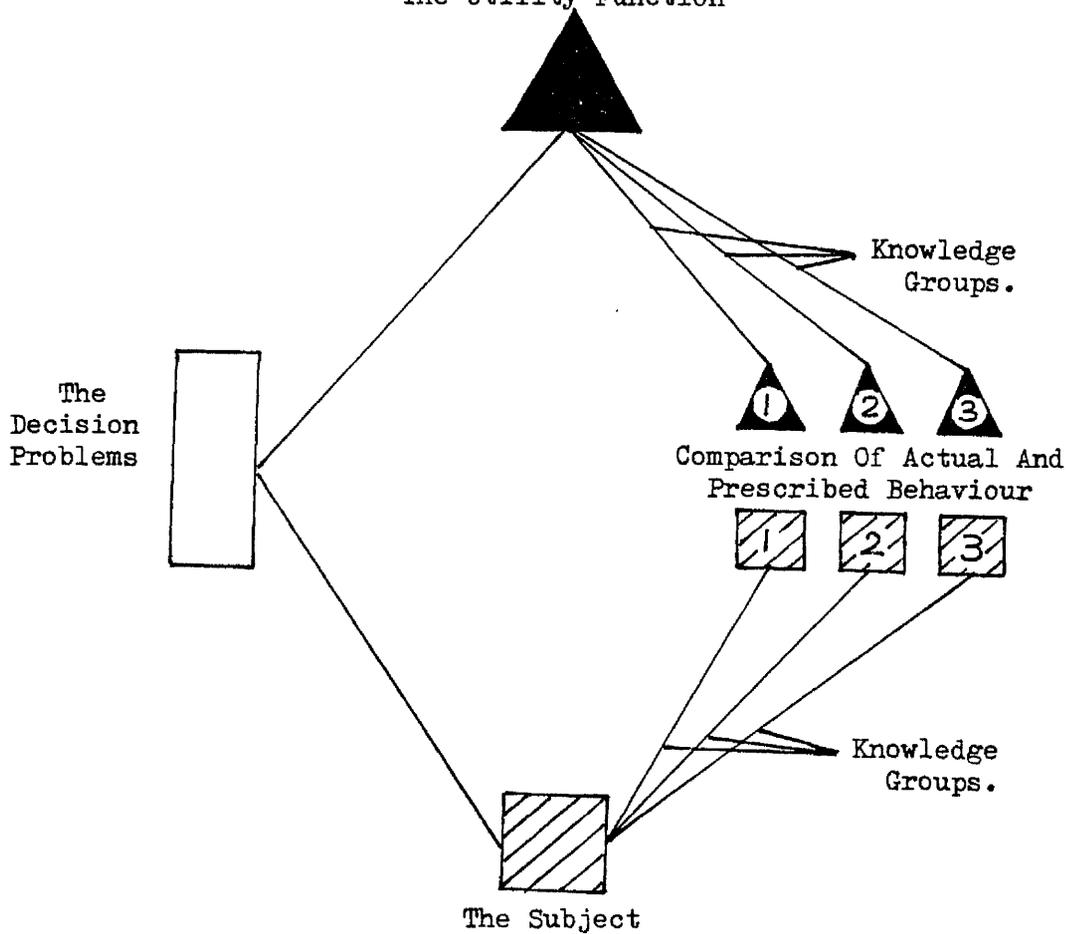
The central concept was the creation of a utility function and the comparison of actual decision making behaviour with the behaviour prescribed by the function, for a number of subjects. As is discussed at page 51 , of chapter 3, it was important to gauge differences in behaviour dependent upon when a subject was given knowledge of his utility prescriptions. The plan for carrying out the experiments is illustrated in Fig.6.

Each subject tackled a number of decision problems and his actual behaviour was then compared with what his personal utility function suggested he should have done. This comparison of actual with prescribed behaviour also takes account of the timing of knowledge, as indicated by the three knowledge groups.

Fig. 6.

EXPERIMENTAL PLAN

The Utility Function



## SAMPLE SELECTION

The previous chapter indicated that this study relied on experiments employing practising business managers. The desirability of using business managers in experiments involving business decision making is evident. A well known formula for the analysis of behaviour was given by Lewin (1936) as  $B = f(P,E)$ . Behaviour is a function of the interaction between P, all the person's inner determinants including attitudes, and E, all the environmental factors as perceived by the individual. In a management sense, the degree of perception of the environment will be dictated to a great extent by worldly experience, in particular experience of the business world. It is difficult to quantify what effect experience has on a manager's day to day work. Experience is certainly a word that crops up frequently as being an invaluable attribute and one much to be sought after. It may be justifiable to say that much of what a manager does is dictated by his experience.

When business decision making is being examined we must ensure that this valuable input of experience is not relegated to a minor role but that it is allowed to play its normal part. One way to ensure that this experience factor is not omitted is to use practising, mature managers. The use of practising managers has its problems. There are problems of selection, of time constraints, of location and of persuasion. The use of College or University students has been made by some experimenters and in these cases the above problems are far less important. Assuming that a sufficient number of

managers can be found, that they are willing to give of their time, that their geographical location presents no difficulties and that they can be persuaded to take part there remains one final problem. How can managers be motivated to tackle the various experiments conscientiously?. MacCrimmon (1965) appealed to a number of managers on academic grounds, Dickson (1978) provided an incentive by offering a bottle of whisky to three subjects drawn at random after the experiments in exchange for points gained in the experiments. Unlike students a manager will not be motivated by the possibility of course credits or the offer of a small fee, both of which are used with students. Information on the results of the experiments would be more likely to interest managers.

With these problems in mind it was nevertheless felt desirable to use business managers and to appeal to their interest in management education as a motivating factor.

The previous chapter, page 54 , did indicate that a particular method of sample selection made it possible to test a number of hypotheses related to the nature of work performed by the managers. It was decided to ensure that the total number of managers was equally divided between risk managers, sometimes known as, insurance managers, and managers performing some other management role.

The reason for selecting risk managers as one half of the total sample is threefold. First was the risk manager's familiarity with risk particularly in a loss making situation. The function of the risk manager in the identification, evaluation and economic control

of risk within a business is described by the author, Dickson (1978a) and others, as one dominated by decision making under conditions of risk. It was thought valuable to compare their responses with those of managers whose function is not solely related to loss making situations. Secondly a previous study carried out by the author Dickson (1978b) brought him in touch with a number of risk managers who appeared willing to help with any further projects and this seemed to provide a ready source of subjects. Thirdly a research grant was awarded by the Risk Studies Foundation<sup>1</sup> who were interested in the general problem of decision making under risky conditions but were also interested in the comparison of the responses of risk managers and non risk managers.

The first approach was made by telephone to a number of risk managers who were asked to participate in the study. This personal contact, although time consuming, was thought more advisable than a first approach by way of a letter. Those who agreed to take part were then asked if they could obtain the assistance of one other, non-risk manager, from within their company.

No attempt was, therefore, made to select the subjects on a random basis as those who were approached were managers who had previously

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<sup>1</sup>  
Risk Studies Foundation Inc. 205, East 42nd Street, Suite 1504,  
York, 10017, United States of America.

assisted the author or who had expressed a willingness to help in the future. No conclusion will be drawn about a wider population of managers and all interpretations of the results will be restricted to conclusions relating to those who participated.

#### MEASUREMENT OF RISK TAKING ATTITUDES

The utility function has already been described as a means of representing attitudes towards risk and direct preferences for different attributes. The importance of the utility function as a reflection of attitudes towards risk in decisions with monetary attributes has also been emphasized. The extent to which an individual's utility function reflects his own attitude towards risk can be measured if we have some method of quantifying a person's risk taking attitude and then of comparing this with what the utility function suggests his attitude is.

It is essential, therefore, to measure attitude towards risk, as distinct from creating a utility function.

As the whole study is concerned with decision making it was felt appropriate to measure attitude towards risk as evidenced by decisions taken as opposed to utilising attitudinal tests based on other forms of behaviour.

## Method Of Observation.

Decision making can be monitored by either observing actual decision behaviour or creating a number of hypothetical decision problems.

Previous studies have made use of both approaches.

The literature is divided on the subject of actual versus hypothetical decisions. To observe actual decision making behaviour would involve lengthy sessions with the researcher closely following each action taken by the subject. It is not impossible as Clarkson (1962) showed in his classic study of the investment manager. An alternative to observing actual behaviour in this way is to create a number of decision problems, where the decision maker still stands to win or lose, and monitor behaviour. Suppes and Walsh (1959) gave each of their subjects \$ 2 and allowed them to make decisions with pay-offs in the range of + \$ 1 to - \$ 1. Tversky (1967) used prisoners as subjects and their decisions had actual pay-offs of varying amounts of cigarettes. Dolbear (1963) introduced higher values of money when he offered his subjects pay-offs in the range + \$ 9.75 to - \$ 1.50.

It can be seen that where a decision making situation is created and behaviour is being monitored, the pay-offs are necessarily restricted by the finance available to the researcher. This does create a less than realistic situation when contemplating the decision making behaviour of managers and their attitudes towards risk. The amounts of money used as pay-offs would have to be extremely large to be of any meaning to the average business manager and even then they would be far less than the amounts he was concerned with as a corporate decision maker.

Recognizing this problem a number of researchers have used hypothetical decision problems where no actual gain or loss was involved. The work of Galanter (1962), Marshak (1964), Swalm (1966) and Spetzler (1968) is based on such a methodology.

The dilemma of actual versus hypothetical decision behaviour is that we do not know, for certain, that the decision maker would act as he said he would if money was at stake. Galanter (1962) supports his use of hypothetical decisions, and provides a valuable justification for their use in general, when he writes (p212) ".....we have asked people about hypothetical increments of money, and we do not have any information about what would happen if they actually get the money. This is often stated as an argument against techniques of this kind : but the fact of the matter is that prior to a decision one always considers the alternatives as hypotheticals and, presumably, bases his action on these considerations."

We have selected the use of hypothetical decision problems and have created a number of decision situations all of which should be familiar to a business manager. The value of having the decisions incorporated in a story line rather than as a set of bare facts is in the fact that we want to encourage the decision maker to act as a corporate decision maker and not as a participator in a lottery. Twenty-four decision problems were included in a questionnaire, see appendix 1, which was sent to each manager. The financial amounts or the probabilities may not appear realistic to every manager but the story itself should bring a certain realism that would be missing in

a straight forward bet or lottery.

The instructions that precede the decision problems did, inter alia, ask the manager to act as a business decision maker, doing what he considered he would do in the real world and not what he thinks he should do or what he thinks is expected of him.

#### Type Of Questioning.

It is possible to measure attitude towards risk by comparing a subject's preferences for certain outcomes over others. Coombs and Komorita (1958) provide a good example of this method. The success of this technique lies in there being a sufficient number of questions to enable averaging to be applied. This method has an appeal as business decisions often involve a choice among competing alternatives.

An equally frequent business choice is where the manager must decide at what stage he considers one alternative to be less attractive than another. He is deciding when he is indifferent between one alternative and another. A form of questioning based on this style of decision can be created which can reduce the total number of individual questions asked but still allow an interpretation of risk taking attitudes.

There are a number of variations on the basic method of having a manager decide his indifference point but they are all rooted in the standard gamble, already discussed.

The form of the problem is:

$$p \cdot \pounds x + (1 - p) \cdot \pounds y = \pounds z.$$

Where  $p$  is a probability resulting in a  $p$  chance of getting  $\pounds x$  or a  $(1 - p)$  chance of obtaining  $\pounds y$  and where  $p \cdot \pounds x$  plus  $(1 - p) \cdot \pounds y$  is equivalent to obtaining  $\pounds z$  with certainty.

The researcher can then omit either  $\pounds x, \pounds y, \pounds z$  or  $p$  leaving the subject to complete the equation.

The standard gamble method has been adopted in this study. All the questions involve the manager in providing the value  $\pounds z$  in problems where he has been given  $\pounds x, \pounds y$  and  $p$ . The questions were phrased in such a way that the  $z$  value should represent the highest amount he would be prepared to pay in order to avoid taking the chancy alternative with the risk of losing a given amount. In problems involving possible gains he was asked to provide the lowest amount he would accept below which he would prefer to take his chances. This particular form of questioning was considered to be more realistic, for business managers, than a question that asked for an indifference value.

Throughout this study the words 'certainty equivalent' and 'indifference value' will be construed as being the value derived in response to this line of questioning.

Compelling the manager to select the value of  $\pounds z$ , that makes him indifferent to the chance of  $\pounds x$  or  $\pounds y$  has been prompted by three considerations. The omission of the value of  $\pounds z$  can be achieved quite easily in a story situation, whereas some difficulty is encountered when a story is constructed that omits  $\pounds x, \pounds y$  or  $p$ . The selection of  $p$ , by subjects, i.e., the indifference probabilities could result in a set pattern which would not alter as the pound values changed.

Such a finding was described by Murray (1970) who also commented on the third reason for letting the subject select £z. This is, that when either £x or £y is omitted the subject finds great difficulty in understanding the form of the problem. Murray concludes that his subjects were more readily able to provide the indifference amount, £z.

Mention has been made of the need to simulate, as far as possible, the actual values that would be involved in a business decision. This necessitates the use of extremely large sums of money and was a sound reason for not using actual decision behaviour, with small betting pay-offs, as a measure of risk taking attitude. The criticism of using small amounts is that they may be meaningless to the manager. A similar criticism is often levied against the use of extremely high values.

The questions in this study are in the range - £55,000 to + £50,000 and are therefore considerably lower than those used by, for example, Swalm (1966) and Spetzler (1968). They are still large enough to be realistic. In each question one of the outcomes to the chance alternative will always be zero pounds, and the other will be stated. Half of the questions involve the chance of gaining and half involve the possibility of a loss. This has been done in a further attempt to add realism and to make the subject examine each question carefully. Zero is shown as one of the outcomes to ensure that depending upon the other outcome, the indifference value £z, will be negative in loss making problems and positive in profit making situations.

With the subject being left to select the indifference value, £z, and the values of £x and £y being decided upon, a priori, it was then necessary to consider what probabilities should be used. Apart from six questions, which will be discussed later, it was decided to use  $p = 0.5$ . The resultant form of each question is that the subject must decide the value at which he is indifferent between obtaining it for certain and a 50/50 chance of attaining £x or £y, whatever they may be.

What is being tested is a person's attitude towards risk, as measured by his choice of indifference value and to this extent it was not thought that any purpose would be served by varying the probabilities. It is not certain that subjects could be looked upon as being consistent, in their choice of certainty equivalent, over a range of different probabilities.

If a choice, other than a 50/50 choice, was selected it would mean that within each question there would be an unequal chance of winning or losing, i.e., there could be a 40/60 chance of winning, meaning a 40 percent chance of winning and a 60 percent chance of losing.

Slovic and Lichtenstein (1968) found that gambles involving unequal chances of winning or losing produced less consistent responses.

They ascribed this to the fact that with 50/50 choices a person only has two dimensions to integrate whereas with an unequal chance, say 40/60, he has four dimensions.

This difficulty of coping with varying probabilities could not be

ignored and rather than become involved with the problem we have limited our probabilities, with one exception, to .5. Swalm (1966) sums up the problem when he writes (p 127) "because of the possible confounding of utility and subjective probabilities, and because there was considerable evidence that few could sense fine distinctions between one course of action that had, say, a 90 per cent probability of success and another that had, say, a 95 per cent probability, we limited all our risks to those involving a 50/50 chance. These were easily understood as equivalent to a flip of a coin."

An exception to the use of 50/50 choices is made in the case of six questions where there is a 10 per cent chance of losing a high amount of money and a 90 per cent chance of breaking even. These questions are included as the low probability, high loss event is one with which the risk manager is particularly familiar. The list of hypotheses include one related to this particular point and it was therefore necessary to deviate from a strict 50/50 choice to allow this hypothesis to be tested.

#### The Measurement Device.

What is sought is a method of scoring each subject according to his risk taking propensities. The data produced by the hypothetical decision problems is in the form of positive or negative monetary values, each representing the subjects certainty equivalent from the twenty-four decision problems. Three options seemed to exist to use this data in obtaining the eventual measure of risk taking attitude.

The real value in monetary terms could be used, the deviation of actual certainty equivalents from the mean certainty equivalent of all subjects could be calculated or the deviation of actual certainty equivalents from some independent bench mark, static for all subjects, could be used.

The latter was selected. The use of the real money values was rejected as these values were absolute terms and, in themselves, did not reflect any underlying preferences. The subject with the highest certainty equivalent need not necessarily be the most risk seeking person. In addition to this was the problem of positive and negative values cancelling each other out and thus concealing risk taking attitude.

The use of the mean certainty equivalent was rejected as it is simply the mean and not necessarily reflective of risk taking attitudes. The majority of subjects may have exhibited risk taking tendencies in a question and the fact that one subject deviated from that pattern does not necessarily mean he was risk averse, he may simply have been less of a risk taker than the others, but nevertheless a taker and not an averter.

The third option, of finding a suitable bench mark, held out the most logical approach. If a base, the same for each subject, was selected which represented a particular risk taking attitude then deviations from such a base would give a measure of an individual's own attitude. The following describes a new approach to measuring attitude towards risk devised for this study.

Keeney and Raiffa (1976) look upon expected monetary value as indifference to risk when they define risk prone and risk averse behaviour. They are not alone in using the expected monetary value criterion to represent the actions of a person who is indifferent to risk. Using the expected monetary value ( E M V ) of each problem as the bench mark, any deviation from that could be used to describe risk taking attitude.

With an E M V of + £2,000 any certainty equivalent (C.E.) greater than that would indicate a risk taker and any C.E. less than £2,000 would show risk aversion. In other words faced with an E M V calculated to be £2,000 the risk averter would prefer a lesser amount for certain rather than run any risk while the risk taker would always take the chance until he was offered a certainty equivalent that was much higher than the expected value of the gamble.

Similarly if the E M V was - £2,000, risk aversion would be indicated by a C.E. that was greater, i.e., the person was willing to pay more than the E M V to avoid the chance of possibly losing £2,000. Risk taking propensities would be evidenced by someone selecting a C.E. that was not as great as the E M V. In such a case the subject would not be prepared to pay a great deal to avoid the possibility of losing, he would rather take his chances.

Having decided to use the E M V as the bench mark, two problems still remain. The first is that the signs will be different according to a positive or negative certainty equivalent. The solution is to alter

the signs to indicate risk aversion or taking attitudes rather than losses or gains of actual amounts of money. With an E M V of +£2,000 and a subject selects a C. E. of + £1,800 then this would be the action of a risk averter and his deviation from the E M V would be shown as - £200. If the C. E. had been + £2,500 the person would be classed a risk taker and his deviation shown as + £500.

In cases involving negative E M V's the same principle can be applied. If the E M V is - £2,000 and the C. E. is - £1,700 then this represents a risk taker and deviation will be shown as + £300. If the C. E. had been - £2,300, indicating an averter, the deviation would have been shown as - £300.

Large positive scores will indicate risk takers and large negative scores, risk averters.

The second problem concerns the use of the C. E.'s in absolute money terms. The following set of figures in Fig.7. illustrates the problem.  $S_1$  and  $S_2$  represent two subjects and  $Q_1$  and  $Q_2$  represent two different questions.

The deviations, in absolute terms, are displayed in brackets directly beneath the C. E. figures. When these are summed for each subject  $S_1$  has a score of - 170 and  $S_2$  a score of + 230. On this basis  $S_1$  is an averter and  $S_2$  a taker. A closer examination of the details in Fig.7 shows that this conclusion does not adequately reflect the behaviour of these two subjects. By using the deviation figure on its own we

are ignoring the important relationship between it and the benchmark of the E M V.

		S <sub>1</sub>	S <sub>2</sub>
Q <sub>1</sub>	E M V	+2200	+2200
	C E	+2000	+2500
		(-200)	(+300)
		{-.09}	{+.14}
Q <sub>2</sub>	E M V	-230	-230
	C E	-200	-300
		(+30)	(-70)
		{+.13}	

Fig. 7

When these deviations are transformed and shown as a fraction of the E M V, as indicated in the squared brackets, the position reverses to show S<sub>1</sub> as a taker and S<sub>2</sub> as an averter. This is a more appealing result when the extent to which each subject deviated from the E M V, in relation to the size of the E M V itself, is studied.

A formula can be presented to represent what has been done and it is;

$$\frac{C E - E M V}{|E M V|}$$

It is necessary to use modulus E M V,  $|EMV|$ , in order to preserve the sign of the numerator. It is this sign, positive or negative, that is important and must be preserved. Taking the modulus E M V allows this to be done.

A score has now been arrived at for each subject and it must now be used to rank the subjects according to their calculated attitudes towards risk. It is not being suggested that the high scores are definitely risk takers or vice versa but simply that among all the subjects tested there were differences in risky behaviour and that the subjects with high scores are more risk taking, in attitude, and those with low scores are more risk averse.

Each subject will have a score for each question as found by the formula. These question scores will be summed for the twenty-four questions and a mean taken as the eventual total score.

The reason for selecting the mean can be explained using the following figures:

	S <sub>1</sub>	S <sub>2</sub>
Q <sub>1</sub>	+ .30	- .30
Q <sub>2</sub>	- .30	+ .30
Q <sub>3</sub>	- .20	- .20
Q <sub>4</sub>	+ .10	- .10
Q <sub>5</sub>	<u>+ .10</u>	<u>+ .10</u>
	0	- .20

$S_2$  appears a lot closer to  $S_1$  than the sum of their scores seems to suggest and to reduce the possible effect that high and low numbers may have, the mean can be employed. In this example the mean score of  $S_2$  would be  $-.04$ .

What is then shown as the score for each subject is the reflection of his average attitude towards risk as described in the twenty-four questions.

Risk taking attitude can therefore be defined for the purposes of this research as the mean of the deviations of actual choices from the expected monetary values, expressed as a fraction of the expected monetary values, over a number of decision problems.

It is emphasized that, as yet, it is not suggested that one subject is a risk taker and another a risk averter but only that in relation to all other subjects each has shown himself to be different.

#### CREATING A UTILITY FUNCTION.

The problems involved in creating a utility function for an individual decision maker can be grouped under two main headings, obtaining the pound/utile co-ordinates and fitting a curve. Prior to examining the methods used in attempting to solve these problems let us state again that it is a utility function for an individual that is being created.

## Obtaining the Pound - Utile Co-ordinates.

The literature indicates the traditional method of deriving co-ordinates to be a question and answer session with each subject. This is very time consuming, and demands a degree of expertise on the part of the person asking the questions. The questioner probes the subject's answers until he is satisfied that the answers reflect the subject's 'true' feelings.

In an effort to avoid lengthy interview sessions with participating managers a self administered test has been devised. Should this test yield information suitable for the creation of utility functions then it will not only assist researchers by reducing time, but may also encourage more managers to give some consideration to utility theory in view of the relatively quick way in which a function can be derived.

A copy of the questionnaire used to derive the utility co-ordinates is shown in appendix 2. A decision problem was posed which involved each subject in the selection of his certainty equivalent. As before, the certainty equivalent was looked upon as being the highest amount the subject would be prepared to pay to avoid a chance of a possible loss or the lowest amount he would accept, for certain, rather than take a chance of gaining an amount of money.

The chances were all of the 50/50 type, the reasons for this and for having the subject select the certainty equivalent are the same as have been put forward in explanation of similar procedures when

measuring attitudes towards risk.

The additional problem, when trying to derive utility co-ordinates, is that there must be some relationship between the various certainty equivalents selected by a subject. We have already looked at the literature on the measurement of utility and from an examination of any one of the methods it can be seen that if the researcher is with the subject then he can question and re-question, using previous responses, until he has a sufficient number of certainty equivalents.

The self administered test must make the subject re-question himself, in such a way that previous answers are included in any new questions so that a relationship can be built up among the certainty equivalents. Depending upon the number of points to be derived, this procedure could involve the subject in reading through a large number of questions.

A new approach was devised based on a reduction in the number of questions to one, with that same question being used several times by each subject. The technique developed to obtain the co-ordinates is now described and an extract from the questionnaire is shown in Fig.8.

Each subject is instructed to read the decision story that precedes the columns of boxes wherein he will be given the financial outcomes to the 50/50 chance alternative. These outcomes are also shown in boxes 

1
---

 and 

2
---

 on line (a) and the subject is asked to place his certainty equivalent value in box 

3
---

 on

line (a). Having done this he is then instructed to move to line (b) and fill in box 3 on that line with the amount he filled in for it on the line above. In other words he carries down his answer from line (a) and fills in box 3 in line (b). By doing this he is presented with a new problem. He re-reads the story substituting for £58,000 whatever figure is in box 3 and he then selects his certainty equivalent to this problem and inserts it in box 4. This procedure is repeated down to line (g) by which time he has provided seven certainty equivalents using the same basic story line but with different chance outcomes each time.

	Outcomes		Certainty equivalents
a	1 + 8 0 0 0 0	2 - 5 8 0 0 0	3
b	1 + 8 0 0 0 0	3	4
c	1 + 8 0 0 0 0	4	5
d	3	4	6
e	3	2 - 5 8 0 0 0	7
f	7	2 - 5 8 0 0 0	8
g	7	3	9

Fig. 8

These seven certainty equivalents are now looked upon as the pound values of seven pound/utile co-ordinates. The corresponding utile values are obtained in the following manner.

We have decided on a utile scale of 0 to 1 and arbitrarily assign a utile value 0 to one payoff and utile value 1 to another in order that intermediate utilities can be derived<sup>2</sup>. The two initial outcomes to the chance alternative, + £80,000 and - £58,000 can be looked upon as having a utile value of 1 and 0 respectively. (These particular monetary outcomes were selected as they encompass all the values involved in the risk attitude measurement tests.) Another way of phrasing this is to say that box 1 has a utile value of 1 and box 2 a utile value of 0.

As the outcomes to the chance alternative are equally likely the value that is inserted as the certainty equivalent in box 3 will have a utile value of 0.5 according to the following calculations

---

<sup>2</sup>

Although we have said that the end points of 0 and 1 were arbitrarily assigned to - £58,000 and + £80,000 respectively this is not entirely accurate. When considering the fact that a curve was eventually to be fitted to the data it was thought desirable to have such a curve rise, monotonically, from left to right. Should 0 and 1 utilities be assigned to + £80,000 and - £58,000 this would not be the case and the result would be a line sloping down from left to right. 0 and 1 were therefore arbitrarily assigned to the monetary extremes but discretion was exercised in deciding which utile value to assign to each end point.

where £Z is the certainty equivalent;

$$0.5 ( + £80,000 ) + 0.5 ( - £58,000 ) = £Z$$

now substitute utiles for the two monetary outcomes,

$$0.5 \quad 1.0 + 0.5 \quad 0 = £Z$$

$$\therefore \quad 0.5 = £Z$$

Let us assume that a subject selects + £20,000 as his certainty equivalent, then + £20,000 has a utile value of 0.5.

The value inserted in box 3 in line (a) is carried down to line (b) and becomes one of the alternatives to a new problem, as the following calculations show.

$$0.5 ( + £80,000 ) + 0.5 ( + £20,000 ) = £Z$$

now substitute utiles for the two monetary outcomes,

$$0.5 \quad 1.0 + 0.5 \quad 0.5 = £Z$$

$$\therefore \quad £Z = 0.75$$

We can now say that whatever value is inserted in box 4 on line (b), as the certainty equivalent, has a utile equivalent of 0.75.

What can be concluded is that each certainty equivalent box assumes a utile value so that whatever monetary amount is shown in the box, and it will vary for each subject, it has the same utile equivalent.

Fig. 9 shows the complete relationship.

Box numbers	0	8	7	9	3	6	4	5	1
Utile values	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1.0

Fig. 9

This display shows that the subject is selecting his personal mid-point and that value is then equated with the corresponding mid-point on the utile scale. For example box 3 is the mid-point between boxes 0 and 1 and if the subject was indifferent to risk the value in box 3 would be the pure expected monetary value. It is unlikely that any subject will select the E M V hence the use of the phrase, personal mid-point. Whatever value is selected and placed in box 3 it still represents the mid-point on the utile scale i.e., 0.5. Similar comments apply to box 4 being the mid-point between 3 and 1 etc.

The certainty equivalent boxes were placed to the side of the outcome columns in an attempt to dissuade the subject simply from selecting the mid-point, as could have been the case if the "certainty equivalent" column had been placed between the two alternatives columns.

#### Fitting A Curve.

The above procedure yields seven pound-utile co-ordinates which in addition to the two end points of 0 and 1 results in nine co-ordinates in total. This number of co-ordinates is greater than some have obtained, for example the experiments carried out by Pratt, Raiffa and Schlaiffer (1965) with three and five points. Others have used many more points than this before fitting a curve.

In this study we were anxious that the subjects did not become

aware of the idea behind the questions and then provide us with figures that they considered were expected of them. In an attempt to eliminate, as far as possible, this contamination of data only nine point estimates were derived. Had more than nine points been asked for we could not have been sure that 'true' responses were being given. There is no guarantee that the nine points themselves are 'true' in the sense that the subject is revealing his actual preferences and not what he perceives as his expected course of action. In our estimation it is likely that he will work his way through these questions without realising what conclusions are to be drawn from his responses.

One major comment on previous work is the extent to which the fitting of the curve was left to an "eyeball" process, a free hand drawing of a smooth line through a number of point estimates.

Much of the previous literature would seem to suggest that an analyst either selects an "off the peg" utility function, or he relies on a freehand drawing of the curve. Keeney and Raiffa (1976), in what is one of the most advanced texts on utility theory in recent times, suggest that two steps are necessary before a utility function can be selected. First, certain tests must be carried out to decide on the qualitative characteristics of the function such as monotonicity, risk aversion or proneness and the decreasing or increasing nature of such aversion or proneness. Secondly the quantitative restrictions on the function must be decided upon which they suggest usually involves determining certainty equivalents

for a "few" lotteries. Having done this they continue by saying (p.196) "After we have determined some qualitative and quantitative characteristics of the utility function, we must determine whether these assessments are consistent, that is, does a utility function exist that simultaneously satisfies all of them? If there is such a utility function, how restrictive are these assessments, and how should an appropriate utility function be determined? If there is no such utility function, how should we obtain a consistent set of assessments?".

The implication may be that if the assessments do not satisfy the assumptions demanded by a number of pre-determined utility functions then the analyst is to consider how he can obtain assessments that will allow one of such a number of functions to be employed.

We have already discussed this question of using functions based on a number of assumptions and have decided not to rely on such a procedure. In its place we have devised a procedure that relies on minimum assumptions but still allows for a mathematical relationship between pound values and utiles.

Our raw data was in the form of nine point estimates, two of which had been arbitrarily set and the remainder provided by the subject. Nine points did seem to be sufficient in comparison with previous studies where assumptions were made as to the qualitative and quantitative characteristics of the eventual function. The question to be faced was whether or not nine points were sufficient when

little was to be done by way of ascertaining the basic characteristics of the function.

This question had to be confronted as, for reasons already discussed, we did not wish to ask each subject for more than nine points but at the same time wanted to describe the relationship between pounds and utiles mathematically.

One assumption we can make, regarding the shape of the eventual curve, is that it will monotonically increase. This assumption is possible due to the nature of the self administered test. The style of the questioning did not allow a subject to give the same pound value different utile equivalents nor did it allow for different pound values to be assigned the same utile equivalent. This has proved to be a problem in previous studies and it was decided in this case to force the subject to yield co-ordinates that monotonically increased<sup>3</sup>. What is required is a method of describing the relationship between any pound value and its corresponding

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<sup>3</sup>  
Monotonicity is a reasonable assumption when dealing with financial outcomes, but can still be broken when subjects are inconsistent in their responses, as indicated above. In non-monetary outcome problems monotonicity is less important. An example of this is the treatment of a diabetic and the measure of effectiveness of different treatments by the blood sugar count. There will be a normal blood sugar count that is most desired and have a certain utile equivalent. A lower count will be less desired and have a lower utile value and in the same way a higher count may be equally undesirable and have a lower utile equivalent than that assigned to the normal count.

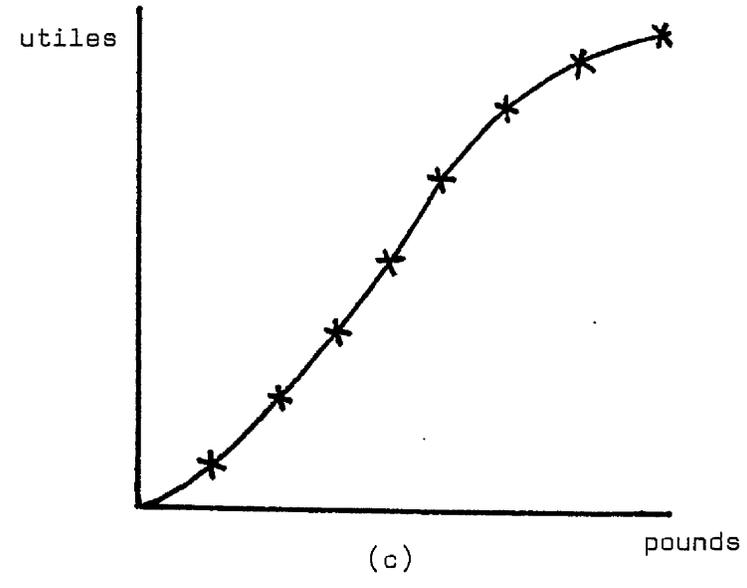
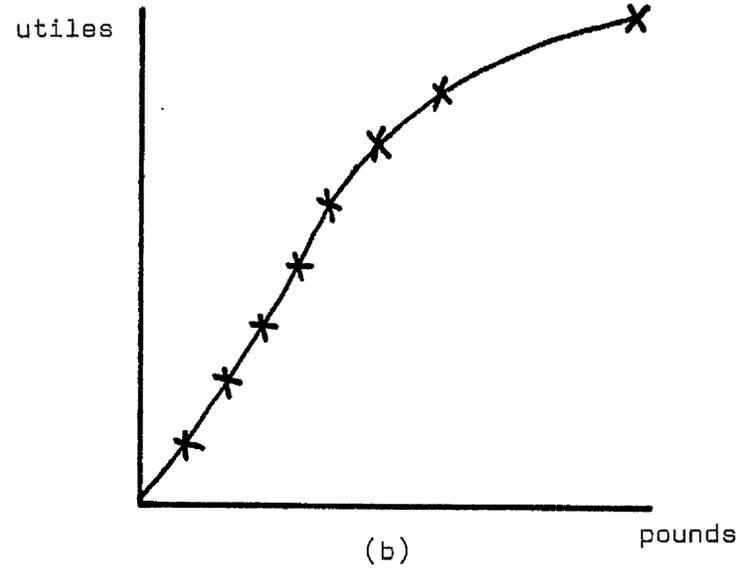
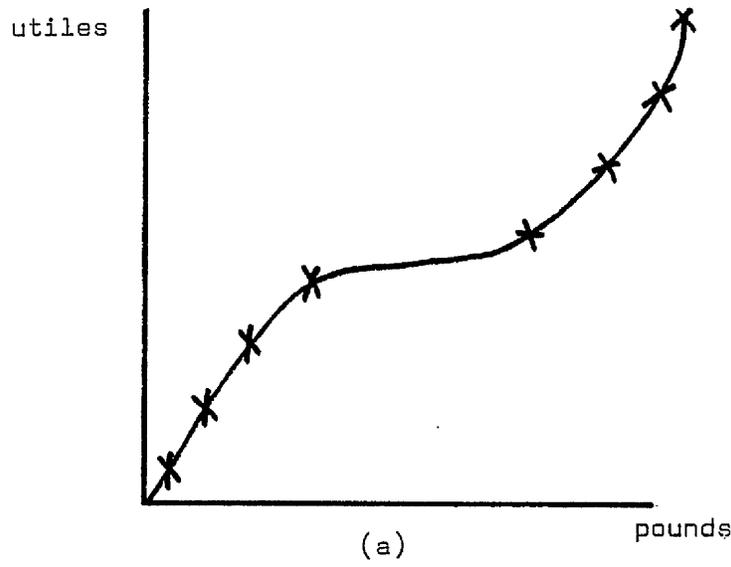
utile equivalent, in order that this function can then be used to provide prescriptions as to how each subject should act.

The relationship between pounds and utiles is unlikely to be linear, as this would represent the completely risk neutral person. Daniel Bernoulli utilised a logarithmic function, as explained by Pfeffer (1956) but such a function has been found in practice to yield too severe a curvilinear relationship. The logarithmic curve allows for only one consistent change in the rate of increase of utile values for increasingly larger pound amounts. This means that a subject will emerge as either a risk taker or risk averter over the whole range of monetary outcomes. Such a finding would not accord with much of the theoretical rationale for utility work, in particular the knowledge that a subject may exhibit risk taking propensities over one range of pound values while showing himself to be risk averse over another range.

We were eventually attracted to a polynomial function and two principal factors led us to its eventual selection. Firstly, the data produced by a pilot study, to be described later, showed us that the nine point estimates resulted in a curve that had approximately three points of inflection. When a free hand curve was drawn through the original nine points a shape for each person was visible quite clearly. Three examples are shown in Fig.10.

It is this basic shape that represents the picture of each subject's risk taking attitude and this shape must be captured by means of a mathematical relationship.

Fig. 10



Experiments were carried out using polynomials of various degrees but with the higher powered polynomials it was found that the coefficients of the high powers contributed very little to the description of that variance not already explained.

As these higher power coefficients did very little to reduce the residual variation it seemed reasonable to omit them. An examination of the pilot study data, confirmed by the actual data, showed that most subjects produced point estimates that, when a line was drawn freehand through them, revealed on average three points of inflection. For this reason a polynomial to the fourth power, a quartic, was decided upon as the most suitable method of describing the resultant relationship.

The second reason prompting the use of the polynomial was the support for its use provided by previous scholars. Borch (1963) comments on the fitting of a curve to utility - money co-ordinates that (p.700) ".....the utility function  $U(X)$  can be a polynomial

of the third degree, and this class of function agrees fairly well with our intuitive ideas of the utility of money". This quotation does serve to remind us that we have a basic shape and that the task of the function is only to describe the pound utile relationship, not dictate the actual shape or relationship. The polynomial is also proposed by Pashigan et.al.,(1966) as being a function that closely approximates an individual's own feelings. Finally Murray (1971) employed a quartic function and a logarithmic function with the same data and found the polynomial to be preferable partly for the reasons we have stated above, the polynomial being less severe than the logarithmic, and partly because the polynomial seemed to represent the shape indicated by the raw data more adequately.

One problem did arise while the data relating to the utility functions was being processed. A least squares regression technique was used to derive the regression coefficients for the quartic function. The computer software package which was used<sup>4</sup> also printed a scattergram. From the scattergram for each individual it was possible to draw, freehand, a smooth line through the point estimates. The graphs shown in fig.10 were produced by this technique.

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<sup>4</sup>

E.S.P. (Econometric Software Package) version 5/1/73 was employed for the derivation of the utility function. This was preferred over S.P.S.S. (Statistical Package for the Social Sciences) as it was possible to retrieve the coefficients when it was necessary to use the function to prescribe.

In order to visualise the shape of the curve described by the quartic function twelve pound values were chosen, arbitrarily, and the corresponding utilities calculated. These twelve together with the original nine point estimates produced twenty-one points.

Fig. 11 displays the result of this exercise for the three examples already described in Fig.10.

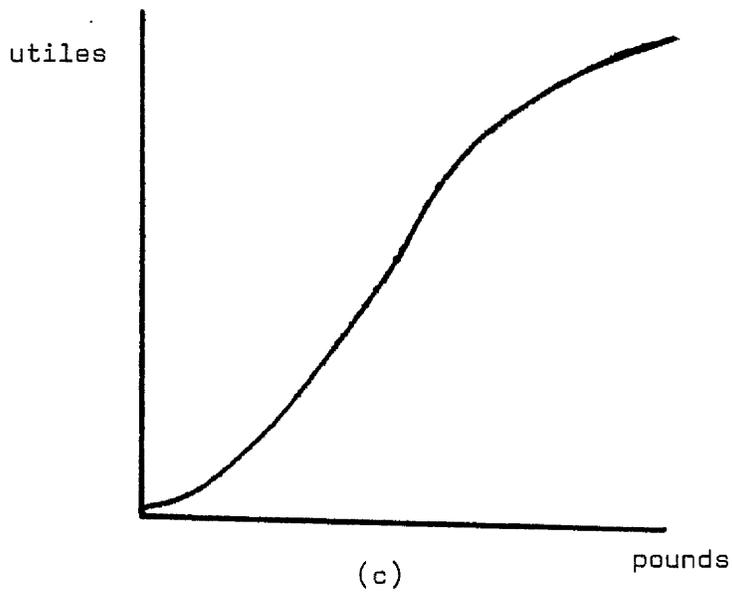
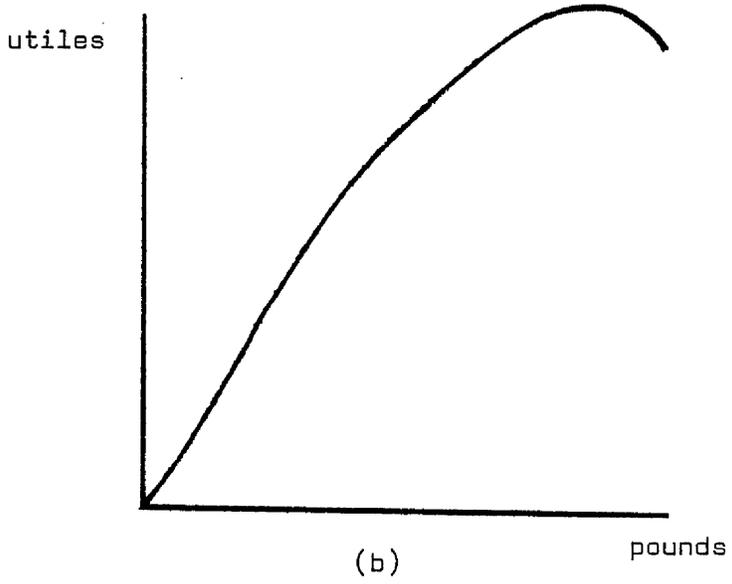
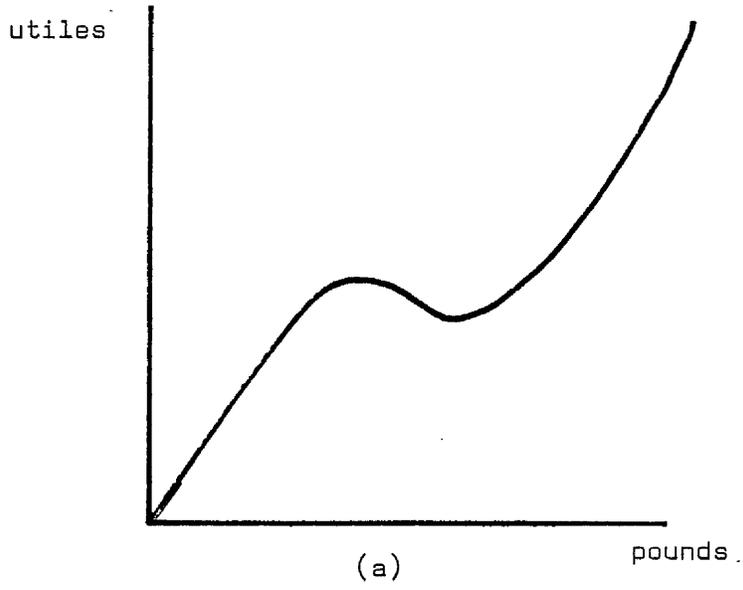
The result was that in some cases the same utility value was being given to different pound values. This was unsatisfactory as it produced a part of the curve that decreased thus contravening the monotonically increasing assumption.

From the nature of the questioning it was impossible to have a decreasing curve, therefore the reason was not a lack of consistency in the responses.

Further investigation revealed that the problem only arose in situations where a large gap existed between two of the original nine point estimates. Example (c) in Figs. 10 and 11 show that where the co-ordinates are evenly spaced the resultant shape described by the function was the same as the shape indicated by the nine points.

In the cases involving large gaps between two points the function was describing a relationship that was completely different from that indicated by the basic nine co-ordinates. A number of

Fig 11



solutions were tried: the power of the polynomial was increased, decreased, the end points were cut off but the problem remained that in certain cases, 5 in number, 17 per cent of the total number, the polynomial was describing a relationship that could not exist.

What has been seen as the solution is rooted in the basic notion that the shape of the curve, as displayed by the original nine points, is what is important and that the function is intended to describe the relationship between pounds and utiles along that curve. We have some knowledge of what that relationship should be and seek a method of describing it mathematically. This is unlike many problems where little is known of the relationship between variables.

What was happening was that the original line of the curve was being distorted by the sophistication of the function used. The least squares programme attempted to fit the best line through the nine points, but it had not been told that there was a constraint, namely the curve had to monotonically increase. The polynomial regression equation was the result of the analysis of the effect of all the point estimates and the best line was consequently derived. The problem was that when a gap appeared either mid-way on the curve or at an end the equation had to consider the influence of the points that bounded such a gap. In example (b) in Figs. 10 and 11, the result of including the influence of the last two points was that the line, which was monotonically increasing up to that stage was pulled down. It was as if the best line was proceeding in a

monotonically increasing fashion but was confronted with a large gap before it could reach the next point estimate. The influence of that point estimate had to be included and the consequence was that the line took what, for it, was the best course to reach it. In this way the original shape was being distorted by the function's sophistication.

What was required was some method of giving more direction to the equation. The most satisfactory way of achieving this would be to have more than nine points but we have already discussed the problems that could be associated with asking for more than nine point estimates.

As the solution to the problem a system of interpolation was devised to produce a number of additional points that could be looked upon as guiding points intended to assist the regression equation in describing the relationship that was indicated by the curve drawn through the original nine points.

This interpolation was carried out along the pound axis. The pound axis was divided to produce thirty-two equally spaced points. If the utile values could be derived for these points then they, together with the original nine points, would result in forty-one co-ordinates, which should assist the fitting of a curve.

When an upwards projection was made from these thirty-two points they landed somewhere between any two of the existing co-ordinates. A method of assuming at exactly what point each projection would

intersect the line joining the two co-ordinates was required.

Fig. 12 graphically represents what has been said so far.

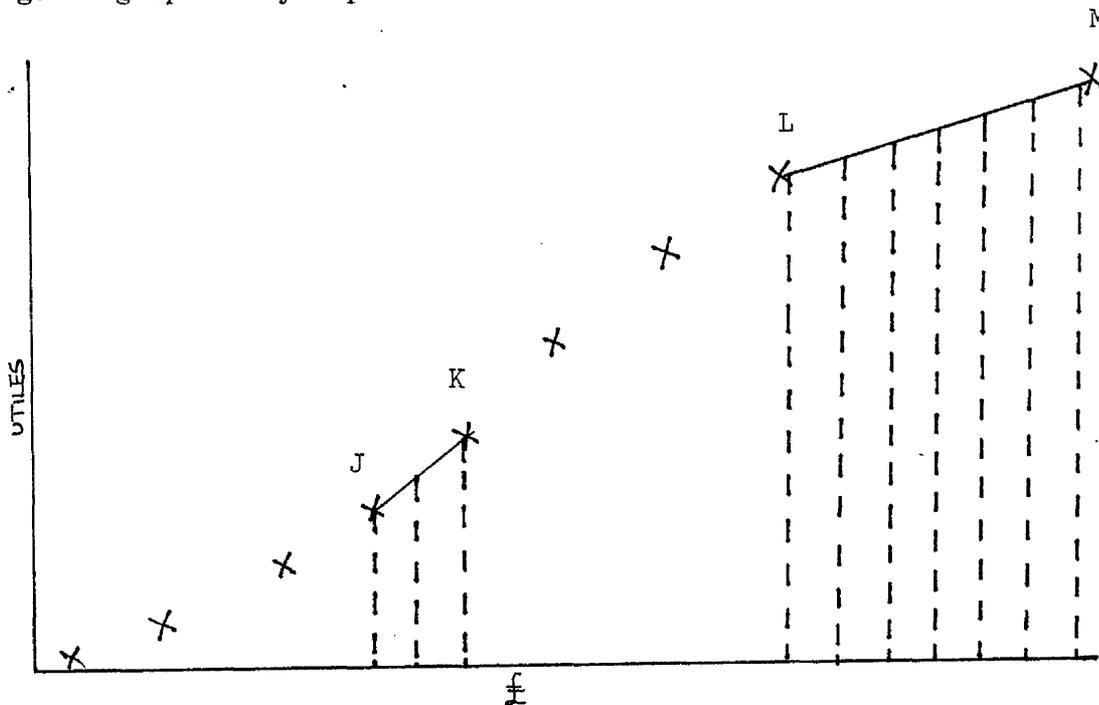


Fig.12

The exact line that the curve will take between any two points is not known. All that is known is that it is monotonically increasing. From the position of the two points we could make some assumptions as to whether the line would be concave or convex but this would be an estimate. In order to obtain the guiding co-ordinates a linear relationship was assumed for pound and utiles lying between any two original co-ordinates. Each set of two original co-ordinates was joined by an imaginary straight line and this assumption then allowed the derivation of the thirty-two guiding co-ordinates to be completed. The result was forty-one point estimates. The least squares programme was run again with this new data and the decreasing curves disappeared leaving all subjects with a monotonically increasing

curve that followed closely the shape indicated originally. The programme used to carry out this interpolation is shown in appendix 3.

An interpolation along the utile axis was considered but this only allowed for exactly the same number of guiding co-ordinates to be placed between any two original co-ordinates, regardless of the range of pound values covered by these two points. For example if the utile axis had been divided to produce thirty-two equally spaced utiles this would have meant four utiles in the spaces left by each of the nine basic utiles. As Fig.12 shows there is a considerable range of pound values covered by the space between points L and M. Despite this, the utile interpolation would only give four new points.

Interpolation on the pound axis allows for these large ranges to be 'filled' by more guiding co-ordinates. The space between points J and K is small and only three new co-ordinates would be placed in the gap.

The reason for interpolating is to provide guidance to the function in the large spaces between point estimates and this seems to be achieved more satisfactorily by interpolation on the pound axis.

In conclusion all that has been done is to 'eyeball' the shape of the curve and to give the programme enough information to compel it to fit a curve which corresponds to the image presented by the original nine points. The visual view of the shape is closer to reality

than the computed shape which is constrained by the type of function chosen.

The fact that the lack of monotonicity was apparent in five cases only, does not imply that the curves for the remaining subjects were satisfactory. There could well have been distortions in them but not so pronounced as to result in a downward sloping curve. The five subjects who contravened the monotonicity assumption could be looked upon as the tip of the iceberg and consequently the solution, as described above, had to be employed with all subjects.

#### Reliability.

The self administered utility test provided for the creation of two separate utility functions. These two separate functions can be used to assist in establishing the reliability of the function. The external validity will be considered when the functions are used to prescribe courses of action.

One method of testing reliability would have been to take two certainty equivalents, for example the values shown in boxes 6 and 9, and ask the subject to choose a certainty equivalent to a gamble involving a 50/50 chance of securing either of these two amounts.

We know that the value in box 6 equals a 0.375 utile and that the value in box 9 is equal to a 0.625 utile. Any certainty equivalent to a 50/50 gamble involving these values as outcomes should have a

utile equivalent of 0.5 in the following manner.

$$0.5 \cdot U(6) + 0.5 \cdot U(9) = U(Z)$$

$$\therefore 0.5 \cdot 0.375 + 0.5 \cdot 0.625 = U(Z)$$

$$\therefore 0.5 = U(Z)$$

Where  $U(6)$  and  $U(9)$  = the utile equivalent of whatever value is shown in box 6 and 9 respectively.

$U(Z)$  = the utile value of the certainty equivalent  $Z$ , to a gamble involving a 50/50 chance of securing either the amount in box 6 or the amount in box 9.

Whatever certainty equivalent is selected we can say it has a utile value of 0.5, but we know already that the value having a utile equivalent of 0.5 is shown in box 3, as per Fig.9.

It would then be necessary to examine the relationship between the value of  $Z$  and the value in box 3. It is unlikely that they would be identical as subjects select their certainty equivalent from within some subjectively assessed range. To comment on reliability would require some boundary to be placed on this range so that if the two values were outwith the range the function could be looked upon as unreliable.

This form of reliability test could not be achieved easily by means of a self administered test and if a question to this end was inserted it would be extremely difficult to decide, a priori, what the

boundaries of the acceptable range were.

Having accepted that a subject is unlikely to produce exactly the same answer a second time round, two separate functions were derived, each according to the method described above.

Each function resulted in a curve and both curves indicated the subject's risk profile. A different utile value for the same pound amount may be shown by each function. These utile equivalents are absolute amounts and in themselves did not reflect any particular attitude i.e., a utile equivalent on one curve could be 0.45 and for the same pound value the utile on the second curve could be 0.40. The second utile of 0.40 although less in real terms does not necessarily imply that the person is less risk averse or less risk prone. It is the shape of each curve that is important and a method was sought of combining the implications of these two curves.

The solution was to construct a new curve by taking the mean coefficients of the individual quartic functions:

$$y = \frac{C^1 + C^2}{2} + \frac{a^1 + a^2}{2} x + \frac{b^1 + b^2}{2} x^2 + \frac{c^1 + c^2}{2} x^3 + \frac{d^1 + d^2}{2} x^4$$

where; y is the utile

C is the constant

x is the pound value

a )  
 b )  
 c )  
 d )

are the regression coefficients.

$\frac{a^1 + a^2}{2} x$  represents the mean of the regression coefficients, as shown by the first and second quartic functions, for the value associated with x.

This procedure gave the mean coefficients that described a line midway between the two original curves, as shown in Fig.13.

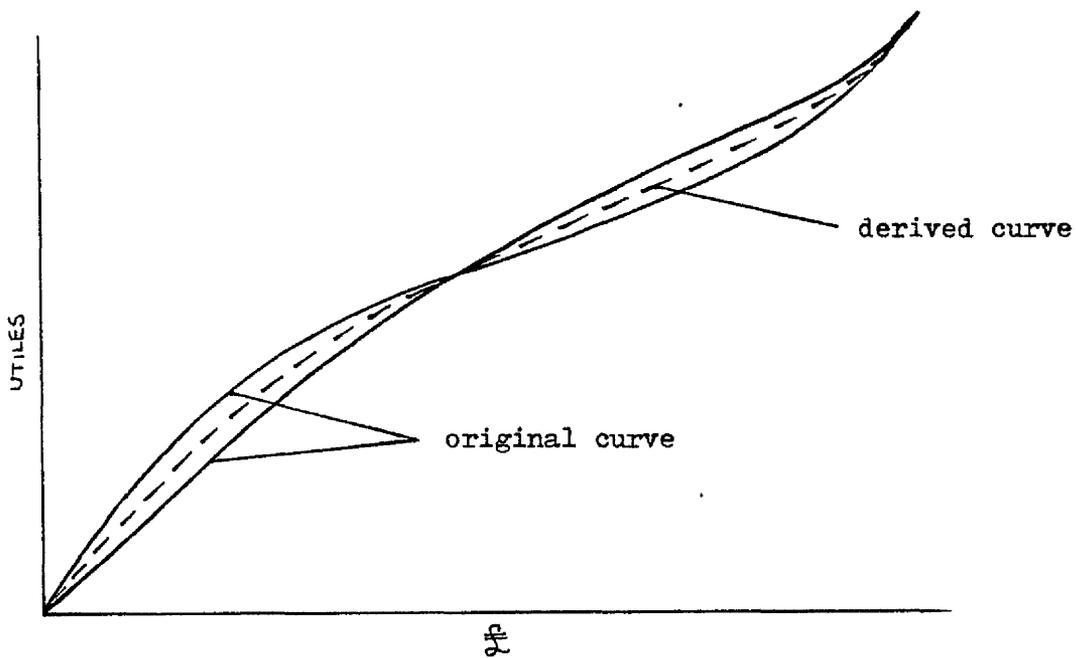


Fig.13

The two separate functions were tested to ascertain the extent to which each represented the same risk taking attitude of the subject.

## USING THE UTILITY FUNCTION.

One aspect of the research problem, stated in the previous chapter, was the comparison of risk taking attitude as displayed by the utility function and risk taking attitude as defined for the purposes of this study.

The reason for creating a utility function is the hope that it may be of some assistance as a decision making aid and the remainder of the research problem was concerned with testing the prescriptive qualities of the function.

Having derived a mathematical relationship between pounds and utiles it is possible to arrive at the prescribed course of action in any decision problem quite accurately. The polynomial regression equation is of the following form:

$$y = C + a x + b x^2 + c x^3 + d x^4$$

where y is the utile

C is the constant

x is the pound value. (As the pound values are high amounts and as they are taken to the fourth power they have been transformed so that £1 = £10,000).

a )  
b )  
c ) are the regression coefficients.  
d )

The decision problems in the questionnaire used to measure attitude towards risk, shown in appendix 1, were utilised again.

Each decision involved two alternatives. On the one hand there was a 50/50 chance of breaking even or gaining or losing a given amount. (As mentioned earlier, six questions involved a 10 per cent chance of losing). On the other hand there was a course of action leading to a definite monetary outcome. The problem was, what is the maximum the subject would be prepared to pay rather than take the risky alternative involving the chance of losing? In questions incorporating the possibility of gaining money or breaking even the problem changed to be, what is the minimum amount the subject would accept for certain below which he would rather take his chances on possibly gaining an amount of money?

The response given to each of the twenty-four decision problems included in the questionnaire represented that subject's actual certainty equivalent. To obtain what his utility function prescribed as his course of action in each problem it was necessary to calculate the utile equivalents of the monetary outcomes to the chance alternative. These were then combined with the chance involved, i.e., 50/50 or 90/10 in order to calculate the certainty equivalent in terms of utiles. Whatever utile value was found it could then be transformed back to pounds.

To simplify this procedure the pound and utile axes were adjusted to the point where zero pounds equalled zero utiles. This had the effect

of reducing the calculations involved in each question as one of the outcomes, to the chance alternative, was always 'break even' which would now have a utile value of zero. Had this not been done it would have been necessary to calculate, for each subject, what his utile equivalent of zero pounds was.

In a problem where there was a 50/50 chance of breaking even or gaining £5,000 it was only necessary to calculate the utile equivalent of £5,000 as zero pounds equalled zero utiles.

The certainty equivalent to this chance alternative is:

$$Z = 0.5 \cdot 0 + 0.5 \cdot £5,000$$

where Z is the certainty equivalent.

The utility of Z (U (Z) ) can be expressed as:

$$U(Z) = 0.5 \cdot U(0) + 0.5 \cdot U(5,000)$$

$$\text{but } U(0) = 0$$

$$\therefore U(Z) = 0.5 \cdot U(5,000)$$

Let us assume that the utile value of £5,000 is found to be 0.45:

$$\therefore U(Z) = 0.225$$

This utile value of 0.225 can be looked upon as the y value in the basic regression equation and when this equation is solved for z we are left with the corresponding pound amount. If this figure is found to be say £4,250 then it can be said that such a person is a risk taker, i.e., he would require a far higher amount than the straightforward E M V of the chancey alternative before he would forego his option to select the risky alternative.

A list of the functions prescriptions was printed for each of the twenty-four questions for each subject.

As we are not predicting how people will act but are prescribing how they should act, given that they want to be consistent, it is likely that actual choices, in the twenty-four decisions, will vary from the utility prescriptions. This is to be expected and tests will show to what extent the actual varied from the prescribed. The particular research problem that was raised earlier concerned the extent to which a subject would use the prescriptions provided by the utility function.

In order to carry out these tests the subjects were divided into three groups. The self administered utility test was sent to each subject first and as they were returned each respondent was assigned to one of three groups in turn. Group one were then sent the second questionnaire together with a letter explaining that the answers given to the first questionnaire had been analysed to enable prescriptions to be made as to how that person should act. A brief description of the role of utility theory was provided and the subject was asked to refer to a separate sheet of paper that showed how it was thought he should act in the twenty-four questions. It was made clear that he could make whatever use of the prescriptions he wanted.

Subjects in group two were sent the second questionnaire together with a sealed envelope marked "not to be opened until after the

questionnaire has been completed". On completion of the questionnaire and on opening the sealed envelope the subject received a similar letter to the subjects in group one and was asked to refer to a sheet showing how it was thought he should have acted. The instruction was to look back at the answers he had given and make any alterations he wanted, in the light of the functions prescriptions.

Group three subjects were given the second questionnaire and no other information at all.

Both questionnaires involved in the study were pilot tested by practising managers prior to the final form being arrived at. This testing of the whole questionnaires was preceded by separate tests relating to choice of wordings and the general phraseology of certain questions.

#### LIMITATIONS

There were certain limitations inherent in the form of experimentation described in this chapter. The first is that hypothetical decisions were used to create the utility function and in the measurement of attitudes towards risk. The decisions were framed as realistically as possible and involved monetary values that seemed appropriate but there is still a limitation in any interpretation of results. The use of hypothetical problems was made throughout and to this extent there is an element of consistency.

A second limitation is that the pressures to which a manager is exposed in the real world cannot be simulated when experiments of the form used in this study are performed. Each subject was instructed to act, as far as possible, as if he was making decisions on behalf of his company. He was asked to make the choice that he would make if faced with the problem in real life. It cannot be stated categorically that subjects responded to this instruction and it may be that some were influenced by what they thought was expected of them or by how they thought they should act.

A third limitation is the lack of motivation. When students are utilized in experiments it is often possible to stimulate motivation in a number of ways but with practising managers who are to face decisions with high monetary values the problem is more acute. All that could be done was to appeal to their interest in management education and their general sympathy with research and development. This was done by way of personal contact with each subject but nevertheless the lack of any more specific motivation is a limiting factor.

## CHAPTER 5

### THE RESULTS.

This chapter contains the results of testing each of the hypotheses stated in chapter 3, and also the interpretation of these results. This method of approach has been preferred to one that separates the display of results from the analysis.

At page 52 of chapter 3 the hypotheses were grouped into three categories, those concerned with creating a utility function, the use of the utility functions and attitudes towards risk in decision making. This present chapter maintains these categories and is split into three sections, each dealing with one of the groups of hypotheses.

This review of results is preceded by details of the numbers of managers who participated together with a general biographical description of respondents.

### RESPONSE.

The experiments that each subject was asked to assist with fell into two parts and it was necessary for both parts to be completed.

Fifty-nine subjects returned the first questionnaire, out of which fifty-one returns were able to be used for the creation of a utility

function. The reasons for discounting eight questionnaires are discussed on page 106 . Twenty-nine, of the remaining fifty-one , subjects completed the second questionnaire and it is on their responses to both forms, that the final testing of hypotheses is carried out.

As was stated when the method of sample selection was discussed, original contact was by telephone with a number of risk or insurance managers who were asked to obtain the support of one other manager from within their company. The response of twenty-nine managers represents returns from thirteen companies,<sup>1</sup> in certain cases the return was only that of the risk manager. Table 1 displays the age and working experience of the respondents.

Of the twenty-nine respondents, fifteen or 51.7 percent are risk or insurance managers and 48.3 are non-risk managers. The particular management function carried out by these non-risk managers was not asked for but from some returns it can be seen that they range through business development, corporate planning, finance and production.

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Some companies had a risk or insurance function of such size that it involved more than one person in a management capacity. These companies were therefore in a position to return forms from two risk/insurance managers and two non-risk managers.

TABLE 1.

Number of subjects related to  
type of manager, age and working  
experience.

	<u>Subjects</u>	
	<u>No.</u>	<u>Percentage</u>
Risk Managers	15	51.7%
Non Risk Managers	14	48.3%
TOTAL	29	100.0%

## Age.

less than 30	3	10.3%
30 - 39	11	37.9%
40 - 49	7	24.2%
50 - 59	6	20.7%
over 60	2	6.9%
	29	100.0%

## Working experience.

less than 6 years	1	3.4%
6 - 10 years	3	10.3%
11 - 15 years	4	13.8%
16 - 20 years	4	13.8%
over 20 years	17	58.7%
	29	100.0%

The ages of respondents showed that 51.8 percent were beyond the age of forty while 89.7 percent were at least thirty. Experience of working life is shown by the fact that 86.3 percent of respondents had been in full time employment for more than ten years and some 58.7 percent had worked for more than twenty years. The respondents brought age and considerable experience of business life to the experiments.

Table 2 shows the educational qualifications of respondents.

Table 2.

Type of manager related to  
educational qualifications.

	Risk <u>Manager</u>	Non-risk <u>Manager</u>	<u>TOTAL</u>
First degree - Yes	1 6.6%	5 35.7%	6 20.7%
- No	14 93.4%	9 64.3%	23 79.3%
	15 100.0%	14 100.0%	29 100.0%
Post graduate qualification ) Yes	1 6.6%	1 7.1%	2 6.9%
) No	14 93.4%	13 92.9%	27 93.1%
	15 100.0%	14 100.0%	29 100.0%
Professional qualification ) Yes	12 80.0%	6 42.9%	18 62.0%
) No	3 20.00	8 57.1%	11 38.0%
	15 100.0%	14 100.0%	29 100.0%

This comparison is drawn to add to the biographical data already mentioned, relating to the respondents. No further cross-tabulations will be done to compare this biographical data with risk taking attitudes and decision behaviour.

At the moment it is interesting to note the age and working experience of subjects compared with the low numbers of managers holding qualifications indicating higher education. Slightly more non-risk managers than risk managers have a university qualification while this is reversed when looking at professional qualifications. 80 percent of the risk managers hold a professional qualification compared to only 43 percent among the non-risk managers. This can perhaps be accounted for by the lack of university courses in the area of risk and insurance, when these respondents were starting their careers, and the comparative strength of professional education in this area. It may also be that certain non-risk managers operate in areas where no professional qualification exists.

## SECTION A - CREATING A UTILITY FUNCTION.

In chapter 3, page 51, the problems associated with creating a utility based normative model were described. The first two hypotheses are related to these problems.

Each will be discussed in turn and that will be followed by an examination of the prescribing function and the shape of the plotted utility curves.

### HYPOTHESIS 1.

"A self-administered test incorporating the choice of certainty equivalents, by subjects, can yield pound/utile co-ordinates".

Eight returns to the first questionnaire could not be used and the subjects involved were not asked to continue with the study.

From these eight questionnaires it was noted that the difficulties were of two types, the subject either did not select a certainty equivalent from within the range bounded by the outcomes to the chance alternative or a mistake was made in the carrying down of figures from one row to the next. An example of the former is the subject who, faced with a situation involving a 50/50 chance of losing either £10,000 or £75,000 chose £8,000 as his certainty equivalent. Subjects were asked to make their choice from within

the range bounded by the outcomes to the chance alternative. This instruction could have been left out as any choice of certainty equivalent outside that range, whether involving losses or gains, could not be correct. The amount of £8,000 selected by this subject, would not seem to be reasonable, as the best outcome to the chance situation is still a loss of £10,000.

The second problem, of carrying down the wrong figures, may have been ameliorated by changes in the design of the question but no difficulties were experienced either during the pilot study or by the majority of subjects. The mistakes in this category could have been due to working too quickly with signs being switched and figures transferred incorrectly. Once such a mistake had been made then the certainty equivalent to a chance alternative incorporating an incorrect alternative, was meaningless.

The evidence would seem to suggest that the self-administered test incorporating certainty equivalent selection can produce pound/utile co-ordinates.

## HYPOTHESIS 2.

"A least squares regression programme will yield a non-linear monotonically increasing utility function from the derived pound/utile co-ordinates".

The pound/utile co-ordinates were obtained and were found to be satisfactory but when the quartic function was derived it was observed that, in five cases, the same utile value was being assigned to different pound amounts thus contravening the monotonicity rule. The analysis of this finding and the solution to the problem are both contained in chapter 4, pages 86 to 93 . In summary all that was done was to provide the programme with enough information to compel it to fit a curve that corresponded to the image presented by the original co-ordinates. The visual view of the shape of the curve was closer to reality than the computed shape which is constrained by the dictates of the selected function.

Given a very large number of co-ordinates then it would be reasonable to say that a least squares regression programme will yield a non-linear, monotonically increasing function. In this study, hypothesis two has to be read in the context of the whole project and particularly in the light of hypothesis one.

It is being hypothesised that after having used a self-administered test to obtain pound/utile co-ordinates, a least squares regression programme will yield a monotonically increasing function. A constraint when using the self-administered test is the number of co-ordinates that it is reasonable to obtain. The arguments in support of the self-administered test itself and of the number of points obtained have been set out in the previous chapter.

When the original nine co-ordinates were employed the quartic

function proved suitable for twenty-four out of the twenty-nine subjects. After interpolating along the pound axis the downward sloping curves were eliminated for all five subjects.

$R^2$ , the coefficient of determination, is high for each subject, with four exceptions, the average being 0.9815. In excess of ninety-eight percent of the variance of each utile value is determined by the combination of the various powers in the regression equation.

Four exceptions were mentioned above, these relate to subjects who exhibited tendencies that revealed complete indifference to risk.

Each of the two utility functions created for these subjects and the mean function used for prescription purposes were linear. In these cases the  $R^2$  value was 1.0000 and the regression coefficients associated with the powers of each pound value were such that the null hypothesis could not be rejected i.e., there was insufficient evidence to suggest that these coefficients differed significantly from zero. This could have been expected and if a stepwise programme had been used it would almost certainly not have gone beyond the equation of a straight line,  $y = a + bx$ .

In conclusion, the hypothesis that a least squares regression programme will yield a non-linear monotonically increasing function, when the co-ordinates are derived from a self-administered test of the form used in this study, can be supported when it is implied that interpolation will take place.

## THE PRESCRIBING FUNCTION.

A utility function was created for each subject in order that it could be employed in prescribing decision making behaviour.

As was discussed in chapter four it was decided to plot a mean curve between two derived curves and use the function describing that derived line to prescribe. This attempt at testing internal reliability was made as it was not known which of the two independent curves was "correct", if either, and with this doubt the mean curve should represent, fairly, the mean risk taking characteristics of the subject.

It was anticipated that should one of the original curves show a person to have a certain pattern, as far as risk taking characteristics were concerned, then the second curve would reveal a similar pattern. It was not thought likely, for example, that one subject would produce one curve showing him to be a risk taker and the second suggesting him to be an averter.

This was confirmed when the original two curves were plotted and the shape of the two curves was seen to be extremely close. This added more strength to the mean curve as now it would represent the same risk taking characteristics as the two original curves but with the smoothing of minor variations. The use of a mean curve would have been more difficult to justify if the independent curves were quite different.

To gauge the actual degree of association between the two original functions a rank order correlation test was carried out. Each function in turn was used to calculate the utile equivalent of twelve pound values for each subject, these pound values being equally spaced over the range of money covered by both functions. The utile equivalents of the twelve pound amounts were summed for the first and second functions. The total of the utile equivalents was taken as the score for that subject on each of the two functions. As it was the ability of each curve to prescribe the same characteristics that was being tested, and not necessarily the accuracy of the prescriptions as absolute amounts, the subjects were ranked according to the magnitude of the sum of their utiles for each function.

Spearman's coefficient of rank correlation was calculated to ascertain the degree of association between the two sets of ranking. Unlike Pearson's product - moment correlation computed from ranks, Spearman's rho considers only the ordinal position of each score and not the weight given to the absolute magnitude of the scores. In cases, such as this, when the original scores are not ranks the Spearman's rho will not be identical to a Pearson r computed from the same data.

It was, however, the ability of each curve to prescribe actions that represented similar risk taking characteristics for each subject that was important and for this reason Spearman's rho seemed to be the most suitable test.

The Spearman's rho,  $r_s$ , was calculated as 0.8122 which would suggest a reasonably strong relationship between the prescriptions made by each of the two functions. The mean function that describes the line lying between the two original curves should, therefore, represent the same characteristics as was evident in the individual curves, subject to an element of smoothing.

The Pearson  $r$  computed on the same data was 0.8812 thus showing the difference as a result of utilising the absolute value of scores and not their ordinal position.

Having derived the prescribing function, a curve was plotted for each subject to reveal the basic shape. The pound values used to provide the co-ordinates were the twenty-four financial outcomes to the problems contained in the risk taking attitude questionnaire, shown in Appendix 1.

All subjects fell into five categories according to the basic shape of their utility curves. Fig. 14 illustrates the shapes involved.

Utility Curve Shapes.

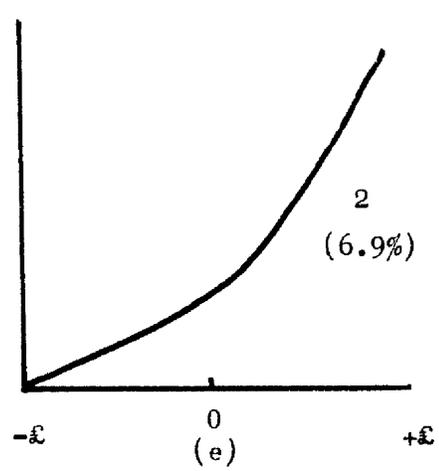
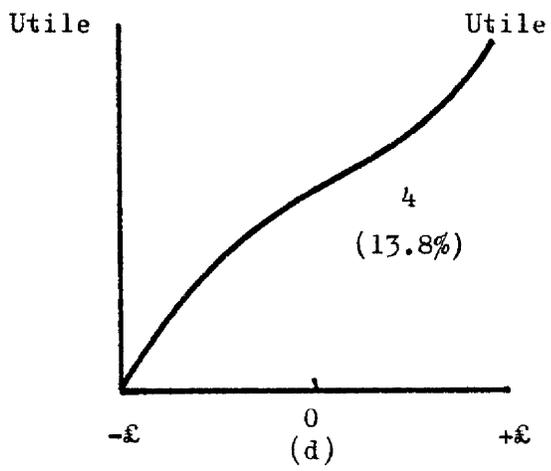
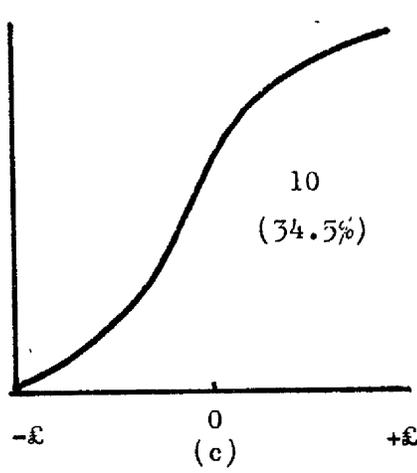
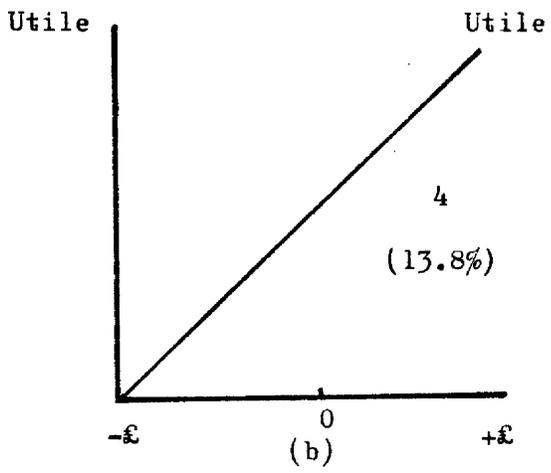
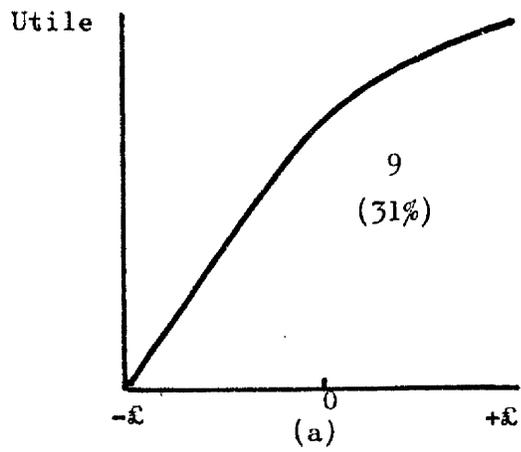


Fig.14

Shown with each graph is the number of subjects in that category, with the percentage indicated in brackets. The shapes are familiar utility curve patterns. The convex curve indicates risk aversion and the concave curve reveals risk taking tendencies. One half of the respondents indicated both aversion and taking characteristics which is revealed by a curve that is concave over certain pound values and changes to be convex over different pound amounts. Graphs (c) and (d) show this feature, with those subjects in the category represented by graph (c) being risk averse over positive values of money and risk takers over negative amounts. The opposite is the case for those subjects in category (d).

Risk aversion would appear to be the most prevalent attitude with the utility curves of 80 percent of respondents indicating aversion over some ranges of pound values. Constant risk aversion was indicated by the curves of 9 respondents whereas only 2 subjects revealed a consistent risk taking attitude.

One interesting group is represented by graph (b). These 4 subjects showed themselves to be linear, meaning that they are entirely indifferent towards risk. Howard Raiffa (1968) would refer to these subjects as E M V'ers, people who take decisions based on the expected monetary value criterion. In an experimental study such as this, it is not possible to state that such people are or will be guided by the E M V of alternatives in real world problems. A number of reasons can be suggested to explain their linearity; they may have chosen the simplest way to solve the experiments, they may

have assumed that they were expected to use some mathematical basis for decision making, they may have lacked interest and sought the quickest method, they may have thought that they should be consistent in their responses and regarded the E M V as a safe way of achieving this.

All the results, not just those where linearity is apparent, have to be interpreted with care as each subject was responding to hypothetical situations and there can be no guarantee that he would act as he said he would if faced with the same problems in real life. This problem of interpreting results is one that is not peculiar to this study or even to this area of work but is common throughout many areas of experimental research.

#### DIFFERENCES IN UTILITY CURVES

Examples of actual utility curves are displayed in appendix 4 and show how different these curves can be. Each subject responded to the same set of questions and yet quite different shapes are apparent for each person. These differences highlight the very personal nature of the utility curve.

To illustrate the different attitudes represented by these curves let us pose a decision problem and, from a visual examination of the curve, attempt to identify what action a particular subject should take.

Let us assume that the subject is managing director of a company

that is involved in litigation over damage to company property alleged to have been caused by the fault of some third party. A judgement is due to be made shortly and the best legal advice is that there is a 50 percent chance of success. Should the company succeed it will be awarded £48,000. The third party has made an offer of £24,000 to be accepted or rejected immediately.

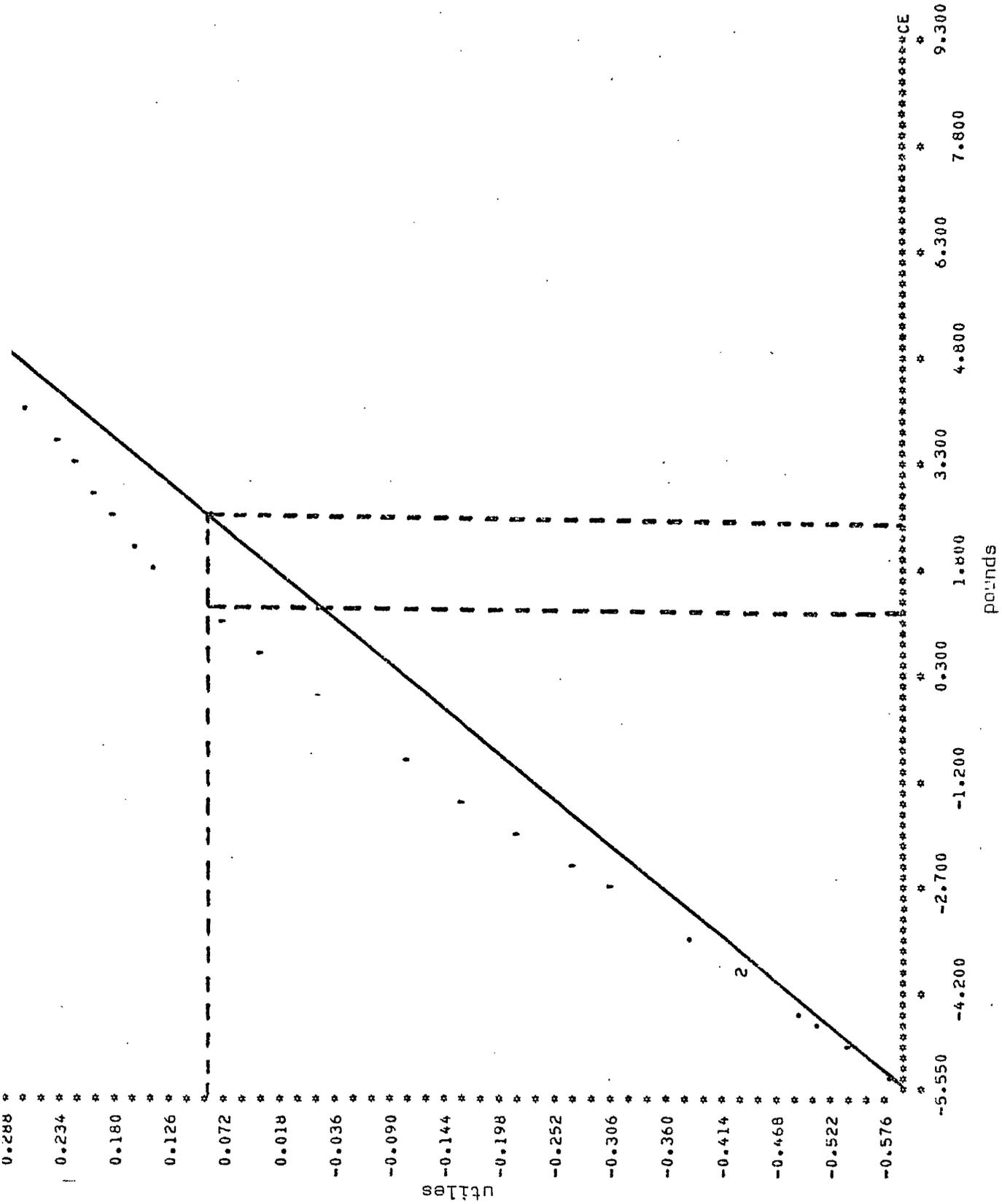
The decision facing the managing director is whether to recommend acceptance of the definite £24,000 or proceed with the action and take the chance of either being awarded £48,000 or losing the case and gaining nothing.

For subject 078 in appendix 4, and also shown in Fig.15, the utile equivalent of the expected value of the chance alternative, continue with court proceedings, is shown to be about 0.090. This is calculated by drawing the indifference line which is represented by a straight line drawn through the origin at 45 degrees. This line indicates complete indifference towards risk with increases in income being shown by proportional increases or decreases in utiles.

It can, however, be seen that subject 078 has exhibited risk aversion tendencies and is consequently not indifferent to risk. His personal utile equivalent of the expected value of £24,000 is far greater than 0.090, at about 0.180. As a result he should want to recommend acceptance of the definite amount rather than run the risks associated with continuing the action.

Fig. 15

Utility Curve For Subject 078



When a horizontal line is drawn from the point where the indifference line is cut by the value £24,000 it can be seen that this intersects the utility curve at a point equivalent to £12,000. In other words 0.090 utiles have a monetary equivalent of £12,000. This subject should then accept any definite amount down to £12,000 below which he would prefer to take his chances in court.

One interpretation of this person's risk averse behaviour is that he did not look upon the chance of winning or losing as a 50/50 possibility. His fear of losing was far greater than his hope of success and consequently his perception of the risks involved was affected. Even if the chance of success was 75 percent this subject would just be indifferent between taking the £24,000 for certain and going to court. This can be seen by projecting the vertical line through £24,000 beyond the indifference line until it cuts the utility curve. This yields a utile of 0.180 as already indicated. The expected value of a gamble having 0.180 as its utile equivalent is found by drawing a horizontal line to the indifference line and dropping down to the pound axis. The monetary amount found is about £36,000 which represents the expected value of a situation holding out a 75 percent chance of securing £48,000 and a 25 percent chance of gaining nothing.

Compare this to subject 031 who displays a predilection for risk. Faced with exactly the same decision this subject assigns a personal utile value of 0.270 to the value £24,000, which is far below the worth of the gamble measured by the E M V criterion. The result is that he should recommend continuing the action and should do so until

the out of court offer is increased to £37,500. When offered an amount in excess of £37,500 subject 031 should then prefer to accept the amount rather than take his chance as the utility value of any such higher amount would exceed the utility value of the chance alternative.

For this subject the hope of success is a far greater influence than the fear of failure. His liking or insensitivity towards risk is so marked that he would continue to recommend proceeding with the legal action even where the chance of success was forecast as being very low.

It is not only in comparisons between concave and convex curves that such differences in prescribed behaviour become apparent. Subject 091 is also risk averse but less so than subject 078. This can be judged by the convexity of the respective curves but can also be shown in their prescribed behaviour faced with the legal problem posed earlier. Subject 091 would accept any certain amount down to £1,500 rather than go on with the action. Even if offered £2,000 his utility curve suggests that he would accept rather than take his chance of possibly not being successful in his action. For this subject the chance of winning the action and being awarded the £48,000 would have to be extremely high before he would be indifferent between accepting the £24,000 for certain and taking his chance of winning.

Although each subject responded to the same set of questions when the utility curve was being derived, the resultant utility curve

for each person is personal to him. Should each subject want to be consistent among choices in terms of attitude towards risk then each should act differently. It may be necessary to state again that utility theory is being looked upon as a prescriptive theory in this study and consequently it is not being suggested that knowledge of a person's utility curve will enable us to predict how he will act. In the legal problem, quoted above, differing attitudes towards risk have been identified and guidance could then be given to the decision maker as to how he should act, assuming he wishes to be consistent.

#### SIMILARITIES IN UTILITY CURVES

Appendix 4 illustrated how different the shapes of individual utility curves could be. Appendix 5 on the other hand, shows some examples of curves that display similar characteristics. Subjects 047, 079 and 006 are all risk averters. There are differences in the degree of risk aversion exhibited by each one but the similarity in shape is quite marked. One feature of these three curves is the decreasing risk aversion over high negative amounts. From about -£27,000 to -£55,500 the utility curve becomes closer to the indifference line. This is quite apparent, in particular, for subject 006 where his own utility curve runs alongside the indifference line prior to breaking away.

The remaining subjects in Appendix 5, 106, 057, 063, 098 and 112 are also an example of similarity within category shape. These subjects

display characteristics of risk aversion over one range of pound values and risk taking over another. Almost 35% of all subjects displayed these characteristics. The degree of risk aversion and seeking is different for some as can be seen when 057 is compared to 106 but two interesting results can be identified.

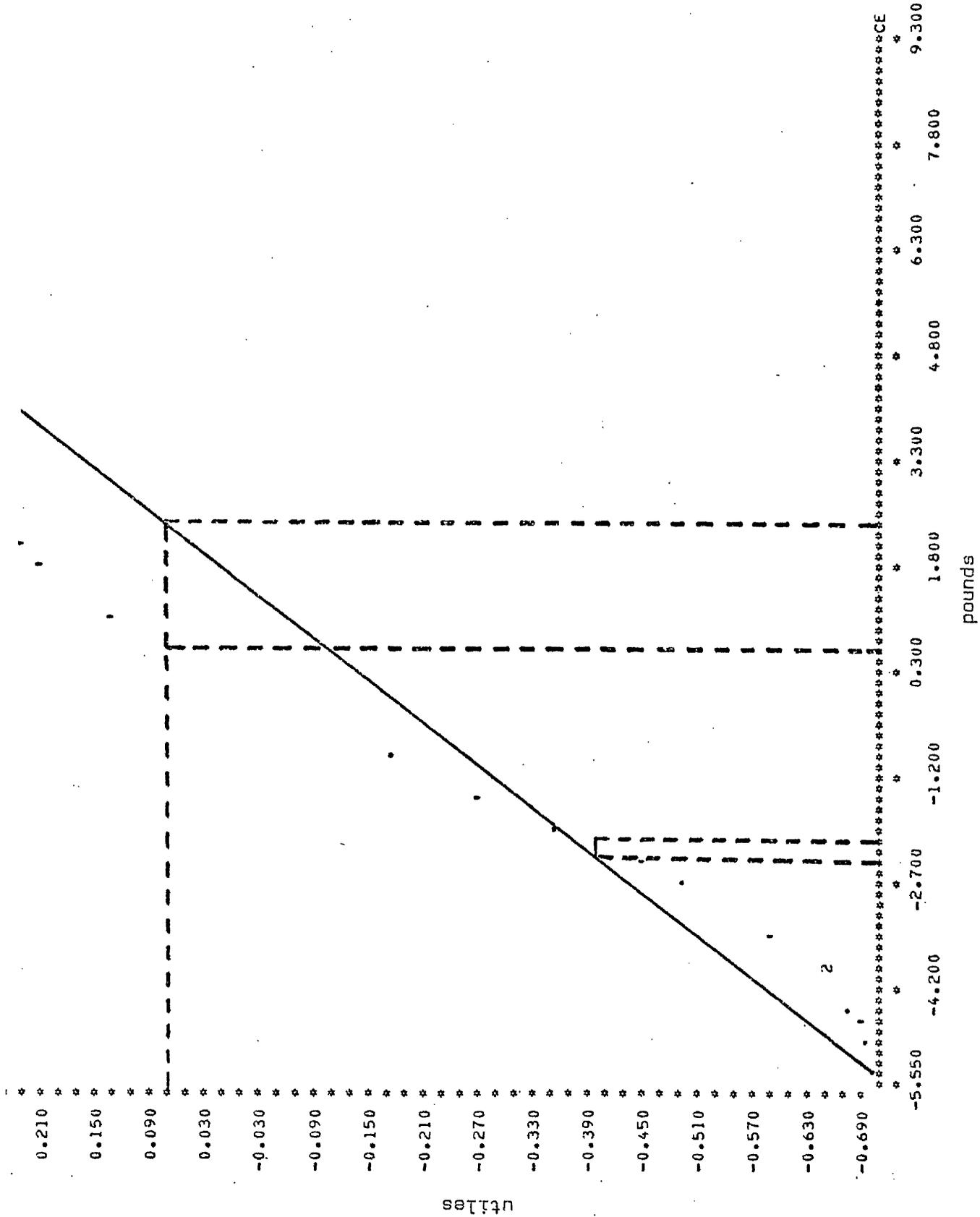
a) Risk Taking Over Negative Values.

Firstly, all the subjects exhibit their risk taking tendencies over large negative values. They appear to be conservative, to varying degrees, over positive values or small to medium sized negative amounts. They seem to prefer certainty when decisions involve a possible gain or only a small loss but when a large loss is at stake they begin to take on the characteristics of the risk taker.

An example of this can be illustrated by taking subject 063, Fig.16. By drawing the indifference line, as described earlier, it can be seen that for values between approximately - £19,500 and - £55,000 this subject is a risk taker. For all values between - £19,500 and + £50,000 he takes on the characteristics of a risk averter. Let us assume two problems, one involving a 50 percent chance of securing £48,000 and the other involving a 50 percent chance of losing £48,000.

Fig. 16

Utility Curve For Subject 063



In the first the E M V is + £24,000 and the utility equivalent of this gamble is approximately 0.070. The subject's personal utility for £24,000 is however much higher at 0.270. When a horizontal line is drawn from the point of intersection of the indifference line by the vertical line drawn through £24,000 it can be seen that this cuts the utility curve at a point equivalent to £6,000. The implication is that this subject should prefer to accept any amount down to about £6,000 in preference to the gamble which holds out the prospect of gaining £48,000.

In the second problem, if the same procedure is followed, it is found that the utility of the gamble having an E M V of - £24,000 is greater than the actual utility equivalent of the amount - £24,000. In fact this same subject would only pay up to £21,000 to get out of the gamble but no more than that. In other words while he was prepared to accept a definite amount less than the E M V when considering potential profits he is only prepared to pay a definite amount that is less than the E M V when contemplating a possible loss.

It is important to note here that while certain subjects exhibited this change in attitude, ten in total, the questions used in the measurement of attitudes towards risk contained in the questionnaire shown in appendix 1 did not involve high negative values. Where the subject was required to provide a negative amount as his certainty equivalent it has been found that they are all in sections of the respective utility curves that are convex i.e., in areas where the

subject is risk averse.

b) The Change From Aversion To Seeking.

The second point to note concerning those subjects who became risk takers over large negative amounts relates to the points at which they changed from being averters to seekers of risk. From the entire monetary range of £105,000 all subjects who exhibited these characteristics of aversion and seeking, began to assume the characteristics of the risk seeker within a comparatively narrow range of monetary amounts.

When a vertical line is drawn from the point on each subject's utility curve where it first becomes concave, to the horizontal axis, it is found that the range of money is approximately £4,500. All ten subjects who showed these tendencies changed from being averters to seekers within the range - £24,000 to - £19,500.

The finding that some subjects tended to be risk averters for positive and low negative amounts and risk takers over larger negative amounts does not seem to have an immediate, intuitive appeal. The popular image of the entrepreneurial risk taker does not compare well with these findings<sup>2</sup>. The vast majority of

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There is no implication that the results of this study will hold good given a far larger sample of managers. Reference to this point and the need for further research will be made later.

subjects in this study, 80 percent did not appear to be risk takers where a potential profit existed. The opposite was in fact the case as they were prepared to accept a small, but certain, profit rather than take any chance of not securing a profit at all. When confronted with possible large losses they changed their attitude and were prepared to assume much more risk. A number of possible reasons can be suggested as to why this should be.

i) The cautious decision taker who is prepared to accept the low but certain profit is rewarded by his company to a far greater extent than the person who is prepared to take chances. This reward may be sufficient motivation to encourage the manager to be cautious in his choices and to avoid the chance situation, even where the risks involved may be acceptable to the firm.

ii) Another factor to explain the behaviour of these subjects may be that they are unfamiliar with loss making situations. They may not deal with potential losses in the ordinary course of business. They certainly deal with degrees of profit and if a lower profit than expected is secured may refer to this as a loss, but this is significantly different from those situations where only losses are possible and decisions have to be made in such an environment. This unfamiliarity with losses could lead them to conclude that the loss will really not happen to them. In other words they are prepared to take the chance that they will escape free of loss as they believe that the loss is unlikely. This is a common enough attitude among the public at large and it may well be that it prevails in management to some extent.

iii) A further explanation may be that once a person knows that losses are a real possibility he could have a tendency to disregard common sense in an effort to retrieve lost ground. The gambler who has lost in a series of bets could be prone to increase his stake in any further round in order to recoup his outlays. In the business world stakes may also be increased in the sense that chances are taken that ordinarily would not be taken, in an effort to regain ground. This pattern is often evident in business failures where there is evidence of care being cast to one side once losses are being experienced.

The entrepreneur, like the "professional" gambler, is not a person who seeks risk. He prefers to play safe and take those decisions where the chancy element has been reduced to the minimum. The shapes of the utility curves in this study reinforce that view but also bring to the surface the problem of decision taking in loss making situations.

iv) A final reason may be that much of a manager's formal education is centred on profit maximisation and many decision making aids and investment appraisal techniques are concerned almost exclusively with profit or degrees of profit. Later hypotheses will make further reference to this question of aversion and taking tendencies in decisions involving potential losses and will, in particular, examine the behaviour of the risk managers whose background should lead to a different attitude in such problems.

## SECTION B - THE USE OF THE UTILITY FUNCTIONS.

Prior to examining the particular hypotheses relating to the use of the derived functions it is necessary to comment on the statistical tests employed in the analysis of the research data.

Whenever statistical tests are used certain assumptions are made. A number of conditions must be met if the use of particular tests is to be said to be valid. The existence of such conditions allows a researcher to be more specific in his conclusions and consequently the most powerful tests are often those that have the strongest or most extensive assumptions.

Parametric statistical tests are those that specify certain conditions relating to the parameters of the population from which the research sample has been drawn. These conditions are mainly that;

- a) the observations are independent.
- b) the observations are drawn from populations that are normally distributed.
- c) the variance of each population is the same.

With the possible exception of the homoscedasticity assumption it is not normal for tests to be carried out in an attempt to validate these conditions. It is assumed that they hold and the strength of any conclusions depends a great deal on the validity of such assumptions.

When there is reason to believe that the parametric assumptions are being satisfied in the research data then the parametric test will be the most powerful for rejecting a null hypothesis when it should be rejected.

The difficulty arises where there is no evidence to suggest that the parametric conditions are being met. Kerlinger (1973) indicates a number of studies where scholars have found the normal parametric tests to be robust against violations of basic assumptions. He concludes a discussion on the problems of parametric conditions by supporting the view that as parametric procedures are the standard tools of behavioural statistics they should be preferred. This view has not been accepted for this present study.

Much of the work on the robustness of parametric tests was carried out in the early to mid sixties and since then the use of alternatives to parametric procedures has become far more widespread. In addition the increasing sophistication of tests that do not make assumptions about the population from which the sample is drawn has led many more researchers to make use of them.

The decision has therefore been taken to utilize non-parametric statistical tests in the analysis of the remaining hypotheses. A non-parametric test is, as Siegel (1956) (p31) describes it ".... a test whose model does not specify conditions about the parameters of the population from which the sample was drawn".

The three principal reasons for making use of non-parametric

procedures were firstly that the research sample in this study is comparatively small at twenty-nine subjects. In addition, for certain hypotheses, this number is divided into three groups and in these cases the non-parametric tests seem to be more appropriate than the corresponding parametric procedures.

Secondly the probability statements obtained from the non-parametric tests are exact probabilities in that they do not rely on the shape of the population distribution from which the sample was drawn. The nature of the experiments in this study, the fact that certain subjects are given guidance in taking decisions, does cast some doubt on the equal variance assumption. With this in mind and no knowledge that normality could be relied upon, the non-parametric alternative seems more appropriate.

Thirdly and finally, the results from the non-parametric tests will be accurate for this data. Had parametric tests been employed with some doubt as to the validity of assumptions, or even with the presumption that the underlying conditions had been satisfied, then the results may have been less than accurate. To balance this argument it has to be said that if all the parametric assumptions are met then the non-parametric equivalent is wasteful of data. The level of this waste is not however high and was thought to be an acceptable price to pay when little evidence was available about the population parameters.

The non-parametric procedures that will be used are the Mann-Whitney U Test, Wilcoxon Matched Pairs Test and the Kruskal - Wallis One

Way Analysis of Variance. Spearman's Rank Correlation Coefficient has already been used and will be employed further.

The decision process in accepting or rejecting a research hypothesis is the same as for hypotheses to be tested by parametric procedures.

With the research hypothesis in mind a null hypothesis ( $H_0$ ) is framed. This  $H_0$  is tested against the alternative hypothesis ( $H_1$ ) which is the operational statement of the research hypothesis. The phrasing of  $H_1$  will dictate whether a one-tailed or two-tailed test is called for. The most suitable statistical test is selected, in this study one of those mentioned in the previous paragraph, and the level of significance is set. The test statistic is calculated and its associated probability of occurring under  $H_0$  is found. This probability is compared to the previously set level of significance and a decision is taken as to whether or not the  $H_0$  can be rejected.

Each of the non-parametric tests makes use of rankings and in one case the signs associated with these rankings. Whenever they are used in the testing of hypotheses the test statistic has been shown together with its associated probability of occurrence. In some cases the ranks, sums of ranks or mean ranks have also been indicated where this information is valuable in the interpretation of the data. A description of the method employed by each of the tests is also given whenever the test is used for the first time.

### HYPOTHESIS 3.

"Any differences between model prescriptions and actual choices in a number of decision problems, will be associated with the knowledge each subject had of how the model suggested he should act".

It is the normative property of the utility function, as opposed to its predictive ability, that is being tested in this study. Any strength a utility function may have as a normative decision aid will lie, largely, in the use to which it is put by managers. The series of hypotheses in this section of chapter 5 is aimed at gauging the use subjects made of knowledge they were given of their utility prescriptions.

Subjects were divided into three groups as follows, group one were given details of their utility prescriptions prior to embarking upon the questionnaire containing the decision problems, group two gave their answers to the problems in the questionnaire and were then informed of their prescriptions and group three were given no information at all concerning their utility prescriptions.

Risk taking attitude as defined by the formula  $\frac{CE - EMV}{|EMV|}$  was calculated for each of the twenty-four questions contained in the questionnaire shown in appendix 1 and the mean taken as the score for each subject. As a first test of the use made of the prescriptions, by

subjects, a significance level of 0.05 was set and the following null hypothesis was stated:

$H_0$ : There is no difference among the average risk taking attitude scores of subjects divided into three groups according to the knowledge they had of their utility function prescriptions.

$H_1$ : The three groups are not the same in their average risk taking attitude scores.

A Kruskal - Wallis one-way analysis of variance was carried out to provide evidence to accept or reject the hypothesis. The Kruskal - Wallis is a non-parametric alternative to the parametric analysis of variance to the extent that it is employed to test whether K independent samples are from different populations. Unlike the parametric analysis of variance the Kruskal - Wallis does not rely on the assumption that the observations are from normally distributed populations having equal variance. It does assume at least ordinal measurement of the variable being considered and that the variable has a continuous distribution, both of which are met by the data in this study. Differences will invariably arise in sample values and the problem is whether these differences signify actual population differences or whether they are chance variations. The null hypothesis that K independent samples come from identical populations is tested against the alternative that the means of these populations are not all equal.

The scores for each subject are ranked in one series, from lowest to highest. Each subject will then have a rank and it is possible to sum the ranks for subjects in the different treatment groups. In this study there are three groups and the sum of ranks for subjects in groups 1, 2 and 3 can be calculated. The test statistic H, which is proportional to the variance of these sums of ranks is then calculated by the following formula:

$$H = \frac{12}{N(N + 1)} \sum_{j=1}^k \frac{R_j^2}{n_j} - 3(N + 1)$$

where

k = number of samples

$n_j$  = number of cases in the  $j^{\text{th}}$  sample.

N = the number of cases in all samples combined.

$R_j$  = sum of ranks in the  $j^{\text{th}}$  sample.

Where the individual groups do not differ they should all have about the same proportion of high, medium and low ranks, in which case the sums of ranks will have low variance. Where the average rank for one group is higher than the others then the variance of the averages will be high. The test determines whether these sums of ranks are so disparate that they are not likely to have come from samples which were all drawn from the same population.

If the null hypothesis is true, that the samples are from identical populations, the sampling distribution of H can be approximated closely with a chi - square distribution with  $k - 1$  degrees of

freedom. The probability associated with the occurrence, under  $H_0$ , of a value as large as that of the computed  $H$  is ascertained by reference to a table of critical values of chi - square. The null hypothesis can be rejected and the alternative accepted, that the groups are not the same with respect to their mean ranks, when the value of  $H$  is such that the probability of its occurrence under  $H_0$  for degrees of freedom  $k - 1$  is equal to or less than the chosen significance level.

Table 3.

Kruskal - Wallis one-way analysis  
of variance on risk taking attitude  
scores related to knowledge groups.

<u>Knowledge</u> <u>group</u>	<u>Number</u> <u>of</u> <u>Subjects</u>	<u>Sum of</u> <u>ranks</u>	<u>Mean</u> <u>ranks</u>
1	11	176.00	16.00
2	9	162.00	18.00
3	9	97.00	10.78

Table 3 displays the individual groups, the sums of ranks and mean ranks. Having ranked all twenty-nine subjects in one series, the sums of ranks for individual groups are found and the mean ranks for groups calculated. The  $H$  statistic was 3.482 and the probability of having an  $H$  statistic of this size, under  $H_0$  is 0.175. It was not therefore possible to reject  $H_0$  at the 0.05 level of significance.

Insufficient evidence was found to suggest that risk taking attitude varied significantly with the knowledge subjects had of their utility prescriptions.

The meaning of the group rankings suggests that group three subjects, with a mean rank of 10.78 portrayed a higher degree of risk aversion than these in groups one or two. Ranking was carried out from lowest to highest and it could therefore be expected that group two subjects were least risk averse of all.

This is confirmed by Table 4 where it is shown that all group means indicated risk aversion, as might have been expected from our earlier interpretation of the utility curves, with group two revealing the least degree of aversion.

Table 4.

Risk Taking Attitude Scores  
related to knowledge groups.

<u>Knowledge</u> <u>Group</u>	<u>Number</u> <u>of</u> <u>Subjects</u>	<u>Sum of</u> <u>Scores</u>	<u>Mean</u> <u>Scores</u>	<u>Standard</u> <u>Deviations</u>
1	11	-1.614	-0.147	0.161
2	9	-0.710	-0.079	0.115
3	9	-2.865	-0.318	0.344

Those subjects who had knowledge of their utility prescriptions appear considerably less risk averse than those who were left to answer the twenty-four decision problems with no guidance. Although the null hypothesis could not be rejected there does seem to be some evidence to suggest differences in mean risk taking attitudes among knowledge groups. The differences were not large enough to produce a Kruskal - Wallis statistic that enabled the rejection of  $H_0$  but further work in this area would be of interest to examine the hypothesis that utility prescriptions tend to be less risk averse than initial choices. When risk taking attitudes were calculated using the prescribed choices in place of actual certainty equivalents in the formula  $\frac{CE - EMV}{|EMV|}$  the mean risk taking attitude over all subjects was -0.107. This compares with -0.179 when actual choices were used in the formula and does provide some evidence to suggest that the prescriptions tended to be less risk averse than original, actual choices.

Hypothesis three was, however, more concerned with the direct use to which subjects put their knowledge of utility prescriptions than with their attitudes to risk. The extent to which prescriptions were employed was measured by calculating the absolute differences between prescriptions and actual behaviour in the twenty-four decisions shown in appendix 1.

The difference was calculated in absolute terms, the direction of the difference not being important at this time, and these differences summed and the mean taken. This mean difference is looked upon as each subject's score and the scores for all subjects are shown in Table 5. The absolute difference was used as otherwise, negative

values would cancel out positive values and the magnitude of the discrepancies would be impossible to determine.

Table 5.

Mean absolute difference between actual and prescribed  
behaviour for all subjects in rank order.

<u>Rank</u>	<u>Knowledge Group</u>	<u>Subject</u>	<u>Mean absolute Difference</u>
1	2	006	492
2	2	096	825
3	2	078	1117
4	1	066	1162
5	2	070	1258
6	1	005	1471
7	3	095	1717
8	2	075	1821
9	2	091	2204
10	1	115	2479
11	1	097	2600
12	1	113	2721
13	1	047	2775
14	3	023	2835
15	1	057	3062
16	2	011	3383
17	3	055	3608
18	1	029	3683
19	3	106	3971
20	2	082	4317
21	1	094	4417
22	1	079	4579
23	3	112	4883
24	3	098	4921
25	1	031	5325
26	2	084	6006
27	3	058	6796
28	3	080	8094
29	3	012	9223

Table 5 displays subjects scores in rank order, from the lowest difference to the highest. There were no ties. The table also indicates each subject's knowledge group and a scan of these groups does show that groups one and two dominate the lower ranks. This does indicate that subjects who displayed the smallest mean difference between actual and prescribed behaviour were, predominantly, in groups that had some knowledge of their prescriptions.

Using the scores from table 5 a further Kruskal - Wallis one-way analysis of variance was carried out to see if evidence could be found to support the view that differences between actual and prescribed behaviour vary significantly with knowledge given.

Table 6 displays the sums and mean ranks used in the test.

Table 6.

Kruskal - Wallis one-way analysis of variance on differences in actual and prescribed behaviour related to knowledge groups.

<u>Knowledge group</u>	<u>Number of Subjects</u>	<u>Sum of ranks</u>	<u>Mean ranks</u>
1	11	157.00	14.27
2	9	90.00	10.00
3	9	188.00	20.89

What do the mean ranks used in the Kruskal - Wallis test signify?

The rank accorded to any subject describes that subject's score, in this case mean absolute difference between actual and prescribed

behaviour, in relation to all other scores. The sum of ranks and consequently the mean rank within groups is evidence of that group's performance as indicated by the score of each member of the group. Table 6 shows that those in group two had a very low average rank indicative of the fact that the scores for subjects in that group were also low and hence the mean absolute difference between actual and prescribed behaviour for these subjects was small.

The comparison of mean ranks between groups is evidence of the spread or dispersal of rankings, and consequently scores, across groups. The mean rank for group three, in table 6, shows that a large number of subjects with high scores were likely to be in that group.

The level of significance was set at 0.05 and the null hypothesis stated as:

$H_0$ : There is no difference in the average deviations of actual behaviour from prescribed behaviour among subjects divided into three groups according to the knowledge they had of their prescriptions.

$H_1$ : The three groups are not the same in their average deviations of actual from prescribed behaviour.

A considerable difference in group averages is indicated by table 6 and confirmed by an H statistic of 7.489 which has an associated probability of 0.024. The probability of obtaining an H Statistic

of this size under  $H_0$  is lower than the previously set level of significance and the alternative hypothesis can be accepted.

The mean ranks for groups can be different without the  $H_0$  being rejected, as was seen at page 136 but for this present  $H_0$  the probability associated with the occurrence of the H Statistic allowed its rejection and the alternative was accepted that the three knowledge groups are not the same in their average deviations of actual from prescribed behaviour.

What does rejection of the null hypothesis imply in this case?

Firstly, the variation in mean ranks between groups is due to more than just chance as is shown by the probability associated with the H statistic. Secondly, at least one group has a significantly higher mean rank than the others. Thirdly, the difference between actual and prescribed behaviour, reflected by the mean ranks, does vary significantly with the knowledge given to subjects of their utility prescriptions.

The Kruskal - Wallis test suggests that there are differences between groups but the test does not address itself to the question of variance. There is no underlying assumption that the group distributions have equal variance.

The dissimilarity between the mean ranks of groups may not be due to one group being comprised of predominantly one set of closely related scores. The means may have been similar apart from one or two extreme

scores that have influenced the average for the group and thus brought about the difference.

The sums of differences, the means and standard deviations for each group are shown in table 7 and there is no evidence, in these figures, of disproportionately high variance. If such a high variance had existed it should also have been reflected in the sums of differences and the standard deviations.

Table 7.

Mean difference between actual  
choices and prescribed choices  
related to knowledge group.

<u>Knowledge Group</u>	<u>Number of Subjects</u>	<u>Sum of differences</u>	<u>Mean differences</u>	<u>Standard deviations</u>
1	11	34275.01	3115.91	1284.68
2	9	21422.97	2380.33	1840.13
3	9	46047.96	5116.44	2474.59

The decision to accept the alternative hypothesis can be re-affirmed.

The mean differences between actual and prescribed behaviour vary with knowledge given of utility prescriptions and it is this difference in means and not a high within group variance that brought about the H statistic of 7.489.

The next question is to consider exactly where the identified differences between groups arises.

#### HYPOTHESIS 4.

" Differences between model prescriptions and final actual choices in a number of decision making situations will be smaller for those subjects who knew of their prescribed behaviour than for those subjects who had no knowledge of their prescribed behaviour ".

Table 5 displayed the mean absolute differences, for all subjects, between actual and prescribed behaviour in relation to the decisions posed in the questionnaire in appendix 1. The monetary range covered by the questions in appendix 1 was £105,000 and the mean difference for all subjects between actual and prescribed behaviour was £3,508. The range of differences was £492 to £9,223 with a standard deviation of £1,787.

Considering the range of money embraced by the decision problems a mean difference of £3,508 would seem to be reasonably small. Some minor element of error will be incorporated in this figure as the model prescriptions were made to two decimal places whereas no subject displayed an actual choice to a degree of accuracy greater than £50, that is, the vast majority of subjects gave answers rounded to the nearest hundred pounds with some rounding to the nearest fifty pounds but no one going below this.

As a predictor of behaviour the function could be said not to have performed too well but this study is concerned with it as a prescriber of behaviour. In this connection hypothesis 4 continues the investigation started in the previous hypothesis and attempts to show that the differences that have already been highlighted, between groups, is between those having knowledge and those not having knowledge of their utility prescriptions.

In order to identify where the differences, highlighted by the analysis of variance, arose it was decided to carry out a Mann - Whitney U Test. The hypothesis relates to whether those having knowledge of their prescriptions acted differently from those with no knowledge. A further hypothesis, dealt with as hypothesis five addresses itself to any differences that may exist between those groups who received knowledge of their prescriptions at different times.

All hypotheses, including this one and hypothesis five, were formulated prior to the experiments being carried out and consequently prior to the results being analysed. They were set up, a priori, based upon the theory of utilities and the nature of these particular experiments.

As the non parametric, Kruskal - Wallis analysis of variance had been carried out it was not possible to test these internal comparisons by means of Scheffés or Duncans Multiple Range tests, as would have been the case if the Parametric A.N.O.V.A. had been used, hence the need for the separate Mann - Whitney calculations.

The Mann - Whitney is described by Siegal (1956) p.116, as "..... one of the most powerful of the non-parametric tests, and it is a most useful alternative to the parametric t test when the researcher wishes to avoid the t tests' assumptions....". The test is one of the significance of differences in central tendency between independent groups when the research score has been replaced by a rank. The assumptions, like the Kruskal - Wallis test, are that at least an ordinal scale of measurement has been achieved and that the variable being considered, in this case, differences between actual and prescribed behaviour, has a continuous distribution.

The following  $H_0$  was set up to test against the alternative hypothesis that there was a larger difference between actual and prescribed behaviour for subjects who had no knowledge of their utility prescriptions.

$H_0$ : Any difference between actual and prescribed choices for those having knowledge of their utility prescriptions will be the same as those for subjects who had no knowledge of their utility prescriptions.

$H_1$ : Where subjects have no knowledge of their utility prescriptions the difference between actual and prescribed choices will be larger than for those with knowledge.

To carry out this test it was necessary to combine groups one and two as they both received knowledge of their prescriptions. The timing of their knowledge was different but this was not important for this present test.

Each subject's score was the mean of the absolute difference between actual and prescribed behaviour in the decision problems shown in appendix 1. These scores are ranked in increasing order of size regardless of group. The ranks for combined group one and two are then compared with the ranks of those in group three. If the  $H_0$  is true then it would be reasonable to assume that the means of the ranks assigned to the scores in both experimental groups would be very similar.

When the alternative hypothesis is true there should be a pronounced difference in the means of the ranks, thus indicating that most of the lower ranks were associated with one group while most of the higher ranks were linked with the other group.

A U statistic is calculated that relates to the number of times in the grand ranking that a subject from one group precedes a subject from the other. This U statistic can be found by counting, but when the number of scores are large, the following formula, which gives identical results, is used.

$$U = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - R_1$$

where  $n_1$  = number of subjects in the first group.  
 $n_2$  = number of subjects in the second group.  
 $R_1$  = sum of ranks assigned to the group having sample size  $n_1$ .

When  $n_1$  and  $n_2$  are both greater than 10 the sampling distribution of  $U$  can be approximated closely with a normal curve and the significance of an observed  $U$  can be determined by the formula;

$$Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{(n_1)(n_2)(n_1+n_2+1)}{12}}}$$

where  $Z$  = deviation of the observed value from the population mean under  $H_0$ .  
 $U$  = the test statistic calculated according to the formula on page 147.  
 $n_1$  = number of subjects in the first group.  
 $n_2$  = number of subjects in the second group.

The probability associated with the occurrence under  $H_0$  of a value as extreme as the calculated  $Z$  is found by consulting a table of probabilities associated with observed values of  $Z$  in the normal distribution.

Table 8.

Mann - Whitney test on differences  
 between actual and prescribed behaviour  
 for groups having and not having knowledge  
of their utility prescriptions.

Knowledge Group	Number of Subjects	Group means of actual differences	Mean rank	U	Z
1 & 2	20	2784.89	12.35	37.0	-2.498
3	9	5116.44	20.89		

Table 8 shows a substantial difference in the group means and this is reinforced by the mean rank of 12.35, accorded to those with knowledge, and 20.89 to those subjects with no knowledge of their utility prescriptions. From what has been said above, in the description of the Mann - Whitney test, it could be expected that sufficient evidence existed to enable rejection of the null hypothesis. The one tailed probability associated with the Z value of -2.498 is 0.006 and the  $H_0$  can therefore be rejected at that level of significance.

There is a significant difference in scores between those who had or had not knowledge and moreover the disparity between actual and prescribed choices was far larger where no knowledge was given of utility prescriptions.

The Research hypothesis can be accepted as there is a significantly smaller discrepancy between actual and prescribed behaviour for those having knowledge than for those without knowledge. The interpretation of this result however is less straight forward than the statistics may suggest.

Although evidence does exist to support the hypothesis it does not necessarily mean that it was knowledge of their prescriptions that resulted in each subject having a small difference between actual and prescribed behaviour. As in many forms of social science research it is not possible to control all the variables and in decision making problems of the form used in this series of experiments, other influencing factors may have existed. Having accepted this there does still exist a considerable difference between the two groups, those having knowledge and those not, which would suggest that a variable or variables did bring a large measure of influence to bear on the subjects. It does not therefore appear unreasonable to suggest that it was the knowledge held by those subjects in amalgamated group one and two that resulted in or contributed to the smaller difference, between actual and prescribed behaviour.

One other point that could be made is that there was no reason why managers in groups one or two should utilize the knowledge they were given of their prescriptions, unless they considered it to be useful information. It is certainly unlikely that they would have made use of their prescriptions in order to "please" the researcher, in fact, the opposite could possibly be the case in view of the

scepticism that often meets attempts to analyse business decision making.

HYPOTHESIS 5.

"Differences between actual and prescribed behaviour will be affected by whether a subject a) knew of his prescribed behaviour before being asked to make his choice or b) was given the opportunity to change his first choice in the light of his prescribed behaviour ".

No view was held, a priori, as to whether pre or post knowledge of prescriptions would result in the same or a different degree of useage by subjects. An intuitive thought was that those who were given their prescriptions prior to starting the decision questions would be more inclined to be guided by those prescriptions. This guidance may not have been sought consciously but having knowledge of how it was thought they should act, they may then fall into that pattern subjectively.

For those who answered the decision problems and were then asked to examine their initial answers in the light of the new knowledge of their prescriptions, it was felt that they might find it more difficult to consider that their first choice should be amended.

When the two groups were compared and the mean differences calculated, as shown in table 7, on page 143, the result was not as expected.

Group two subjects, who received knowledge of their prescriptions after they had made their initial choices had a smaller discrepancy between actual and prescribed behaviour than those in group one.

The two smallest mean difference scores, between actual and prescribed choices were found among group two subjects. Those were scores of £492 and £825. Out of the entire range of monetary values covered by the decision problems these mean differences reflect a very narrow gap between actual and prescribed choices. The subject who scored £492, subject 006, returned a questionnaire that showed a difference of only £50 between his choice and the prescription in one question and several questions where the difference was between £50 and £150 pounds. Bearing in mind, again, the values concerned in these decisions then there was a striking similarity between his own action and what had been prescribed.

A Mann - Whitney U test was used to test the significance of the difference between groups one and two and with the level of significance set at 0.05 the null hypothesis was stated as:

$H_0$ : Differences between actual and prescribed behaviour will be the same regardless of when subjects received knowledge of their prescriptions.

$H_1$ : Whether subjects received knowledge of their prescriptions before or after making their choices will affect the difference between actual and prescribed behaviour.

The mean rank of scores of those having prior knowledge was 12.18 and of those with post knowledge, 8.44.

The U statistic was calculated as 31 which has an associated two-tailed probability of 0.160. Insufficient evidence was found to enable rejection of the  $H_0$  and we must conclude that the time knowledge of prescriptions was given to the subjects did not affect the difference between actual and prescribed behaviour.

This is an area where more work is necessary both in the style of presentation of prescriptions and in the timing of delivery of this knowledge. There would seem to be reason to suggest that if initial choices are made, left for a while and then returned to, there would possibly be changes made. Whether such changes are as a result of the time lag alone or are in part due to the knowledge acquired in the interval, of prescriptions, is one that would require study. It may also be the case that too much thought is given to answers when a subject is presented with a note of his prescriptions prior to answering a problem. The initial first response, that may reveal his underlying attitudes, may be suppressed by some thought that he should consider the prescription and perhaps re-examine his own choice. This re-examination of his initial answer should, according

to the theory of utilities, result in his moving closer to the prescribed course of action. It could be, however, that an opposite effect may be caused where a subject reflects too long on both prescription and his initial choice resulting in him deciding that neither is representative of how he wants to act.

#### HYPOTHESIS 6.

" There will be no relationship between the risk taking attitudes of each subject as displayed by both the prescribed course of behaviour and actual behaviour in a number of decision making situations ".

This is the final hypothesis in section B dealing with the utility function. The remainder of this chapter, section C, concentrates on the attitudes towards risk of the various managers according to their backgrounds and the nature of the decisions posed.

To provide evidence for hypothesis six the risk taking attitude score was calculated for each subject according to the formula already mentioned in previous tests,  $\frac{CE - EMV}{|EMV|}$ . This formula is applied to each of the twenty-four decisions in the questionnaire shown in appendix 1 and the mean score for all questions is calculated for each subject. To provide the comparison between this score, that is looked upon the subject's attitude towards risk, and

the attitude revealed by the utility function, the same formula was utilized but this time the prescribed choice was substituted in place of C E (the subjects actual choice). In this way two scores were calculated for each subject, one showing his attitude to risk as revealed by his answers to the twenty-four decisions and the second showing his attitude to risk as indicated by his utility prescriptions of how he should act in the same decisions.

Subjects were ranked according to each of these scores and a rank order correlation test was carried out to calculate Spearman's rho. The  $r_s$  for all subjects was 0.0970 which suggests no relationship at all in rankings based on the two separate attitude to risk scores.

The research hypothesis can be supported in that the ranking of subjects by these two methods was not the same and as a result the risk taking attitude of the managers as determined from their actual behaviour was different from that revealed by their utility prescriptions. This was what was hypothesised and indeed there was no need for there to be any relationship between the two attitudes. The lack of any positive relationship between attitudes in different decisions is one of the main reasons why a prescriptive model is put forward. It was not suggested that the utility model would be a good predictor of behaviour and this view has been justified by the size of Spearman's rho. It was to be expected that a difference in attitude as described by the two scores would exist and the almost total lack of association between rankings has shown this to be so.

Table 9.

Rankings on attitude towards  
risk as displayed by actual  
choices and prescribed choices  
for those with knowledge of  
their prescriptions.

<u>Subjects</u>	<u>Rankings</u>		<u>Diff.</u>	<u>Diff.<sup>2</sup></u>
	Actual	Utile		
	<u>Choices</u>	<u>Prescriptions</u>		
005	14	14	0	0
006	8	8	0	0
011	18	18	0	0
029	1	1	0	0
031	2	20	18	324
047	3	3	0	0
057	11	11	0	0
066	13	10	3	9
070	16	16	0	0
075	12	12	0	0
078	7	9	2	4
079	19	6	13	169
082	20	13	7	49
084	5	2	3	9
091	4	5	1	1
094	6	4	2	4
096	10	15	5	25
097	15	19	4	16
113	9	7	2	4
115	17	17	0	0

Table 9 shows the rankings and differences in rankings for subjects in groups one and two. Table 10 shows similar figures for those having no knowledge at all of their utile prescriptions, i.e., those in group three.

Table 10

Rankings on attitude towards risk as displayed by actual choices and prescribed choices for those with no knowledge of their prescriptions.

<u>Subjects</u>	<u>Rankings</u>		<u>Diff.</u>	<u>Diff<sup>2</sup></u>
	<u>Actual</u>	<u>Utile</u>		
	<u>Choices</u>	<u>Prescriptions</u>		
012	1	9	8	64
023	7	6.5	0.5	0.25
055	3	5	2	4
058	4	8	4	16
080	2	1	1	1
095	5	6.5	1.5	2.25
098	9	3	6	36
106	8	2	6	36
112	6	4	2	4

The similarity in rankings in table 9 is quite marked for those subjects who had knowledge of their prescriptions. This is as expected as earlier hypotheses had shown that where subjects had knowledge, the difference between actual and prescribed behaviour was narrower. As a result the underlying attitude towards risk displayed by actual choices would be very similar to the attitude revealed by the utility prescriptions. In nine cases the rankings were identical and in only two cases i.e., subjects 031 and 079 was any significant difference identified. Spearman's rho for the rankings in Table 9 was calculated as 0.538 which is less than might have been expected but is probably due to the two subjects mentioned above. Spearman's rho for Table 10 was - 0.364. The negative value of  $r_s$  indicates that the ranking in the two scores is diametrically opposed and this is certainly confirmed by the figures in Table 10.

SECTION C - ATTITUDE TOWARDS RISK, MANAGEMENT  
TYPE AND NATURE OF DECISIONS.

The remaining hypotheses are related to sub problem (B) (iv) as shown at page 52 in chapter three, that is, will risk taking attitude be influenced by management background and the nature of the decisions posed?

The method of sample selection, as already described, involved a number of risk managers in obtaining the assistance of non risk management colleagues for the experiments. The reasons for this method of selection are shown at page 58. Having subjects split into two categories, that is, risk and non risk managers it was possible to carry out a series of additional tests using their attitude to risk scores. These tests are concerned with differences in behaviour exhibited by the different management groups and also in their response to different forms of decision problems. No further tests were carried out on the utility functions other than to ascertain that there was no difference in the use made of the utility prescriptions by the two management groups. A Mann - Whitney U test produced a U statistic of 99 with an associated probability of 0.7948 when the differences between actual and prescribed behaviour was calculated and the two management groups compared. This did not allow rejection of the null hypothesis, 'there is no difference in the use made of the model between management groups'. There was no reason to believe that differences would arise between the two groups and the evidence is that there was very little difference. The mean difference between actual and prescribed behaviour for risk managers

was £3,532.46 and for non-risk managers was £3,482.64.

#### HYPOTHESIS 7.

" There will be a relationship  
between risk taking attitudes,  
as defined for the purposes of  
this study, and management type ".

The earlier examination of utility curves suggested that the majority of respondents should want to be risk averters when attempting the decision problems contained in the questionnaire shown in appendix 1.

The risk taking attitude scores for subjects supports this view with a mean score, for all subjects, of - 0.179. The range of scores was - 1.029 to + 0.292 with only three scores from the total number of subjects being positive that is only three people showed themselves to be risk takers, according to the formula  $\frac{CE - EMV}{|EMV|}$ .

Hypothesis seven was phrased as it was, because it was felt that risk managers may have a different attitude to risk in view of their constant dealings with problems that only hold out the prospect of a loss or no loss. The non-risk managers will not be so limited in their dealings with risky situations as they will be involved in many decisions where the outcomes may include a gain.

Table 11 displays the risk taking attitude scores for both management groups.

Table 11.

Risk taking attitude scores and rank by management type.

<u>Risk Managers</u>			<u>Non-Risk managers</u>		
<u>Rank</u>	<u>Score</u>	<u>Subject</u>	<u>Rank</u>	<u>Score</u>	<u>Subject</u>
2	-0.537	080	1	-1.029	012
3	-0.464	029	4.5	-0.451	055
4.5	-0.451	031	10	-0.209	112
6	-0.410	058	13	-0.181	078
7	-0.333	095	15	-0.147	006
8	-0.235	047	16	-0.102	113
9	-0.215	091	20	-0.053	066
11	-0.201	084	22.5	-0.040	097
12	-0.186	094	22.5	-0.040	106
14	-0.148	023	24	-0.014	070
17	-0.065	096	25	-0.013	115
18	-0.059	057	26	-0.006	011
19	-0.055	075	28	+0.174	082
21	-0.043	005	29	+0.292	098
27	+0.032	079			
<hr/>			<hr/>		
Sums :					
178.5	-3.370		256.5	-1.819	
Means :					
11.9	-0.225		18.3	-0.130	

The risk managers appear considerably more risk averse as a group than the non risk managers. This is evidenced both by the means of their respective scores and the mean rank of 11.9 accorded to the group of risk managers compared to a mean rank of 18.3 for the non risk managers.

The null hypothesis was stated as:

$H_0$ : Risk taking attitude will be the same  
for risk managers and non-risk managers.

$H_1$ : Risk managers will reveal a different risk  
taking attitude from non-risk managers.

The direction of any difference was not hypothesised, a priori, and a two tailed probability was therefore calculated by means of the Mann - Whitney U Test and the level of significance set at 0.05.

The U statistic was 58 which has an associated two tailed probability of 0.040.

The null hypothesis can therefore be rejected and the alternative accepted that risk taking attitude will not be the same for both management types.

The following hypothesis attempts, therefore, to identify where the differences in attitude lay.

HYPOTHESIS 8.

" Risk managers will be more  
risk averse than non-risk  
managers in decisions involving  
potential losses ".

A difference in decision making behaviour was identified in hypothesis 7. This was expected and it is now hypothesised that the difference arose in view of the risk manager's attitude towards risk where a potential loss is possible.

The decision problems in the questionnaire shown in appendix 1 had been designed so that one half of the problems held out the prospect of a loss only, while the remainder only held out the prospect of a gain. In each type of question the subject was being asked to state the certain amount, either to be paid or received, beyond which he would just prefer to accept the gamble entailing the prospect of a loss or gain respectively. As a first step towards testing hypothesis 8, decision making behaviour was compared for each of the two types of problems.

The scores for subjects were computed for each type of question and the results are shown in table 12 with those problems only holding at the prospect of a loss being termed "risk cost" and those involving a gain, "risk price".

Table 12.

Risk taking Attitude Score and Rankings for  
decisions involving gains and those involving losses.

Subject	Risk Cost Score	Risk Price Score	Difference	Rank of Difference
005	+ 0.073	- 0.158	+ 0.231	+ 16
006	- 0.078	- 0.216	+ 0.138	+ 12
011	+ 0.483	- 0.496	+ 0.979	+ 26
012	- 2.008	- 0.050	- 1.958	- 29
023	- 0.264	- 0.031	- 0.233	- 17
029	- 0.668	- 0.259	- 0.409	- 23
031	- 0.992	+ 0.091	- 1.083	- 27
047	- 0.274	- 0.196	- 0.078	- 7
055	- 0.829	- 0.072	- 0.757	- 25
057	+ 0.041	- 0.159	+ 0.200	+ 13
058	- 0.422	- 0.398	- 0.024	- 3
066	+ 0.110	- 0.217	+ 0.327	+ 21
070	0.000	- 0.029	+ 0.029	+ 4
075	- 0.090	- 0.020	- 0.070	- 6
078	- 0.187	- 0.175	- 0.012	- 1
079	- 0.086	+ 0.150	- 0.236	- 18
080	- 1.081	+ 0.007	- 1.088	- 28
082	+ 0.108	+ 0.240	- 0.132	- 11
084	- 0.315	- 0.086	- 0.229	- 15
091	- 0.168	- 0.261	+ 0.093	+ 10
094	- 0.305	- 0.068	- 0.237	- 19
095	- 0.667	0.000	- 0.667	- 24
096	- 0.055	- 0.075	+ 0.020	+ 2
097	- 0.150	+ 0.070	- 0.220	- 14
098	+ 0.467	+ 0.117	+ 0.350	+ 22
106	- 0.074	- 0.007	- 0.067	- 5
112	- 0.254	- 0.164	- 0.090	- 9
113	+ 0.051	- 0.256	+ 0.307	+ 20
115	<u>- 0.056</u>	<u>+ 0.030</u>	<u>- 0.086</u>	<u>- 8</u>
Mean:	- 0.266	- 0.093		
Standard			Sum of ' - ' ranks	289
Deviation:	0.497	0.162	Sum of ' + ' ranks	146

The null hypothesis was stated as:

$H_0$ : risk taking attitudes in decisions only involving the prospect of a loss will be no different from attitudes in decisions where the outcome will be a gain.

$H_1$ : there will be a difference in risk taking attitudes as revealed by questions only involving a loss and those only involving a gain.

The Wilcoxon matched pairs test was employed to test for any significant difference in attitude scores between the two types of decisions and the significance level was set at 0.05

The Wilcoxon test is used in the case of matched pairs or, as in this study, where each subject is exposed to both treatments. Each subject is given a score for both treatments, in this case his attitude towards risk in risk price and risk cost decisions. The difference between the scores is found and these differences are ranked according to the absolute value of the difference, that is without regard to sign. The sign of each difference is then assigned to the corresponding rank. This has been done for the scores in this hypothesis and the results are shown in table 12.

If there is no difference between the two treatments, i.e.,  $H_0$  is true, then the sum of the negative - sign ranks should be very

similar to the sum of the positive - sign ranks. This means that if either the sum of the negative - sign ranks or the sum of the positive - sign ranks is too small then  $H_0$  can be rejected. The smaller of these two sums of ranks is taken as the test statistic and termed T.

For sample sizes greater than twenty-five T is approximately normally distributed and the significance of any T value is then determined by the formula,

$$Z = \frac{T - \frac{N(N+1)}{4}}{\sqrt{\frac{N(N+1)(2N+1)}{24}}}$$

Where Z = Deviation of the observed value from the population mean under  $H_0$ .

T = The smaller of the two sums of ranks.

N = The total number in the sample.

A table of probabilities associated with observed values of Z in the normal distribution can then be consulted to ascertain the probability connected with the occurrence under  $H_0$  of a value as extreme as the calculated Z.

The differences in table 12 produce a Z value of - 1.546 with an associated two tailed probability of 0.122. There is insufficient evidence, therefore, to reject the  $H_0$ .

On referring back to table 12 it may appear strange that  $H_0$  cannot be rejected despite the difference between the mean scores but the test was between different treatments, i.e., risk cost and risk price decisions, for each subject. The Wilcoxon test gives more weight to

pairs that show a large difference than to pairs that only show a small difference and the discrepancy between the means in table 12 could be due to the three large differences displayed by subjects 012, 031 and 080.

The standard deviation of scores in those decisions involving a loss of 0.497, should be noted particularly. A standard deviation, some three times larger than the corresponding figure for decisions involving a gain does indicate a wide variety in attitudes in loss making situations. The mean score for these decisions involving losses indicates a far higher degree of risk aversion than for those involving gains but the deviation about this mean for gains was considerably less than for losses.

Although there was no significant difference, over all subjects, between scores on risk cost and risk price decisions there was some evidence to suggest that subjects were less homogeneous in attitude where losses were involved than where a gain was in prospect.

This line of enquiry was pursued with research hypothesis 8 in mind, risk managers will be more risk averse than non-risk managers in decisions involving potential losses.

The risk taking attitude scores in the loss making decisions were calculated for both management types and the result is shown in table 13.

Table 13.

Risk taking attitudes in loss making  
decisions related to management type.

<u>Management</u> <u>Type</u>	<u>Number of</u> <u>Subjects</u>	<u>Mean</u> <u>Scores</u>	<u>Standard</u> <u>Deviations</u>
Risk managers	15	- 0.3516	0.354
Non-risk managers	14	- 0.1727	0.615

A null hypothesis was formed and stated as:

$H_0$ : There will be no difference between the risk taking attitudes of risk managers and non-risk managers in decisions involving only the prospect of a loss.

$H_1$ : Risk managers will be more risk averse than non-risk managers in decisions involving only the prospect of a loss.

A Mann - Whitney U Test was utilized to test whether these two scores were drawn from the same population. The alternative hypothesis, against which  $H_0$  is being tested, is that the score for risk managers will be higher i.e., a larger negative value, than for non-risk

managers. This hypothesis can be accepted if the U statistic is so small that the probability associated with it under  $H_0$  is equal to or less than the significance level chosen.

With the level of significance set at 0.05 the U statistic was calculated as 57.  $H_1$  stated the direction of the predicted difference and the one-tailed probability associated with the U statistic was 0.018. The null hypothesis can be rejected and the alternative accepted that risk managers are more risk averse than non-risk managers in decisions that only involve the prospect of a loss.

Non - risk managers were much less risk averse than risk managers in the loss making problems and the high standard deviation of 0.615 would also suggest a lack of homogeneity among members of that group. Risk managers, as a group, were certainly more risk averse but in addition the standard deviation of their scores was much lower at 0.354.

These figures tend to suggest that where losses are concerned there was a high degree of variation, on the part of the non-risk managers, around a mean attitude score that was almost identical to the overall mean for all subjects in all questions of - 0.179. The fact that only losses were involved did not have any real effect on their mean risk taking attitude although the standard deviation of 0.615 was considerably higher than the overall standard deviation for all subjects in all questions of 0.250.

A discussion on attitudes to risk in loss making situations as

reflected in the utility curves for all subjects is contained on pages 123 to 128 .

The findings shown above, in relation to hypothesis 8, refer primarily to the group of risk managers and indicate that they may have acquired a respect for the potential loss that does not appear to be held by the non-risk managers. This respect could then have led them to be more risk averse than their colleagues.

As a comparison to what was being hypothesised above, a similar set of figures was collated for these decisions that held out the prospect of profit only. Table 14 displays the results of this collation.

Table 14.

Risk taking attitudes in profit  
making situations related to  
management type.

<u>Management</u> <u>type</u>	<u>Number</u> <u>of</u> <u>Subjects</u>	<u>Mean</u> <u>Scores</u>	<u>Standard</u> <u>Deviations.</u>
Risk managers	15	-0.0975	0.144
Non-risk managers	14	-0.0874	0.184

When dealing with decisions that only involved degrees of profit there was very little difference between the scores for each management group.

The standard deviations of these scores, for each group, were very similar at the same time as being less than the standard deviation of scores for all subjects in all questions.

No significant difference in risk taking attitudes between groups was detected in decisions involving profit only, suggesting that risk managers, as a group, reacted in a very similar manner to the group of non-risk managers. Although risk managers normally handle decisions holding out degrees of loss they were nevertheless capable of adjusting to the profit problem and of responding in a fashion almost identical to the non-risk managers.

This may imply that while the subjects are familiar with decisions involving levels of profit or gains either as a result of their business or private experience they are less familiar with loss making situations and are consequently more willing to take risks to avoid losses. The exception to this general rule would seem to be risk managers who reacted in much the same way as non-risk managers in profit decisions but differed significantly in attitude when deciding in loss making situations.

#### HYPOTHESIS 9.

" There will be a relationship between  
risk taking attitudes and different  
probabilities of loss ".

This final hypothesis was included to reflect the fact that many of the decision problems faced by risk managers involve high potential losses with low probabilities. This high loss, low probability event is found, for example, in the purchase of certain forms of insurance. The risk of a building being destroyed by earthquake in the United Kingdom is remote but should it happen the cost would be substantial. The earthquake risk is offered as a standard extension of cover on industrial property insurances and is effected by most insureds.

This hypothesis attempts to identify any difference in risk taking attitude between decisions involving a fifty percent chance of loss and those with a ten percent chance of loss. A Wilcoxon Matched Pairs test was carried out using the risk taking attitude scores in each of the two types of problem mentioned above.

The null hypothesis was:

$H_0$ : There will be no difference in the risk taking attitude of subjects between decisions having a fifty percent chance of loss and those having a ten percent chance of loss.

$H_1$ : Subjects will not reveal the same attitude towards risk in decisions with a ten percent chance of loss as they will in decisions with a fifty percent chance.

The significance level was set at 0.05 and the Wilcoxon test statistic calculated as 3.88. The associated two tailed probability was so low as not to be shown on most statistical tables and therefore enabled rejection of the null hypothesis and acceptance of the alternative. There was a significant difference in attitude towards risk between situations holding out a fifty percent chance of losing or breaking even and those with a low probability of a high loss.

The actual scores show a greater degree of aversion to risk in decisions with the ten percent chance of a high loss than in those with the fifty percent chance of losing.

When the scores were split according to management background it was found, as might have been expected from previous hypotheses, that the risk managers were again more averse to risk than their non-risk management colleagues.

## CHAPTER 6.

### CONCLUSIONS.

Decision making is more than just the final act of selecting an alternative course of action. It implies the construction of a decision with each component linking together to form the complete problem. The resultant examination of each part of the decision problem is then likely to be more beneficial than an attempt at trying to deal with all the various factors of the decision as one problem.

When the decision is broken down many areas of study become clear and the literature on formal decision analysis comprises several studies aimed at clarifying difficulties associated with individual parts of the decision structure. Such studies have been accompanied by attempts at applying decision aids in practical situations.

This study was confined to the business decision and to one aspect of the decision problem, namely the placing of some value on potential outcomes. When referring to outcomes it is necessary to distinguish between courses of action and their consequences. This study was concerned with the measurement of consequences. This aspect of the decision is as important as the identification of alternatives, evaluation of probabilities and the ascertainment of states of nature as if the outcomes, in real terms, influence the

decision taker and this fact is not taken into account, then much of the work that has been done in the building of the problem will have been in vain.

When outcomes are stated in absolute terms there is the possibility that different people will assign different preference orderings to them. In addition there is the chance that people will be influenced in some way by the likelihood that a particular outcome will come about.

When outcomes are restricted to monetary values the preference ordering becomes less of a problem but the decision taker's attitude towards risk is extremely important. Given simple decision problems the business manager may be able to perceive the risks and cope with them but when the risks are contained in a complex choice problem it is more likely that he will begin to be inconsistent among choices, despite the fact that he may be endeavouring to be true to some basic, underlying attitude towards risk.

Utility theory has been put forward as a means of meeting this problem. Actual values of money are replaced by their utility equivalents which then represent the decision taker's attitude towards risk combined with the absolute value of the monetary amount.

In this study utility theory was looked upon as a prescriptive decision aid. It was not suggested that utility equivalents of money would enable an accurate prediction of behaviour. What was

hypothesised was that utility theory can be employed to guide decision makers in making choices that they should make if they want to be consistent among choices and to some underlying attitude towards risk.

The research problem was to ascertain to what extent a workable, utility based, normative model could be created for decision making under risky conditions.

#### THE METHOD.

Business managers took part in the study in an effort to avoid the criticisms levelled at earlier work where subjects with little or no business experience were used. A utility function was derived for each manager and subjects were then assigned to one of three groups.

All subjects were subsequently presented with twenty-four business decision problems, each contained in short narrative form. Those managers in group one also received a note of how their utility function suggested they should act and were instructed to make whatever use they wanted of this additional information. Group two subjects responded to the decision problems and were then informed of their utility prescriptions and given the chance to make any alterations they wanted. Those in group three did not receive any knowledge of their prescriptions. In this way it was possible to note any variation in decision making behaviour among knowledge groups.

Utility functions were derived for fifty-one subjects but only twenty-nine of these subjects eventually completed the decision problems. As it was necessary for both parts of the study to be completed the results were based on the twenty-nine managers.

Certain techniques were devised specifically for use in this study.

- a) A Self Administered Test. Previous work relating to the creation of utility functions involved subjects in lengthy and often complex interview sessions. As business managers, who would only have a limited amount of time to offer, were being utilized in this study, a test was devised that could be completed by a subject on his own and in his own time. The test was designed to minimise the possibility of results becoming contaminated either by boredom or subject learning.
- b) A Mathematical Relationship Between Pounds and Utiles. One major weakness in previous research relating to the creation of personal utility functions is the lack of sophistication in the description of the resultant relationship between pounds and utiles. Very many studies involve a free hand drawing of a smooth line through a number of points. In this study a least squares regression programme was employed to fit a quartic function to the co-ordinates derived from the self administered test. The resultant polynomial equation was then used to prescribe behaviour as pounds could be transferred to utiles and utiles back to pounds as required.
- c) The Measurement of Risk Taking Attitudes.

As utility theory was being looked upon as prescriptive and representative of underlying risk taking attitudes it was also decided to measure such attitudes, as distinct from any measurement contained in the utility function. This was done and each subject was given a risk taking attitude score which was used to compare with what his utility function suggested his attitude should be. Further tests on risk taking attitude related to the nature of the decision problems and the background of the managers were also carried out using this risk taking attitude score.

#### THE RESULTS.

The results can be classified in three parts, the creation of the utility function, the use of the utility functions and attitudes towards risk in decision making.

##### A) The Creation of Utility Functions.

- i) The self administered utility test was completed satisfactorily and pound/utile co-ordinates were obtained. Eight out of the fifty-nine subjects who completed the utility questionnaires made errors that resulted in their response being rendered invalid. These were human errors but some further work on the form of the self administered test is referred to later and may reduce the possibility of mistakes.
- ii) A least squares regression programme performed satisfactorily in deriving a function that described the

mathematical relationship between pounds and utiles. For five subjects the curve described by the function included a downward sloping line. Such a curve meant that different pound amounts were being described by a single utile value. The line of questioning used in the self administered test had been designed to obviate this problem and produce co-ordinates that conformed to the basic assumption of monotonicity. The conclusion was that the dictates of the function resulted in a shape that was not representative of the image presented by the original co-ordinates. All that was sought was a method by which the relationship between any pound value and its corresponding utile equivalent could be described mathematically while still retaining the general shape of the curve as indicated by the initial point estimates. The existence of additional co-ordinates may have solved the problem but this would have resulted in a longer, more complex and time consuming questionnaire bringing with it all the difficulties that the test had been designed to avoid.

A system of interpolation along the pound axis was devised and additional, guiding co-ordinates were derived. The result was the elimination of the decreasing slope and creation of function, for each subject, that corresponded to the image presented by the original co-ordinates. The visual image of the shape dictated by the original co-ordinates was closer to reality than the computed

shape which had been constrained by the type of function chosen. What had been achieved by interpolation was to compel the least squares regression programme to bear in mind this original shape.

- iii) It was expected that the responses to the self administered test would represent, for each subject, the selection of one figure from within a range of possible figures. With this in mind it would not be possible to say how accurate the utility function was. Two curves were therefore derived and a mean curve plotted by using the mean of the coefficients of each of the two individual quartic functions. It was this mean function that was then used to provide the prescriptions. Some doubt would have been cast on this mean line if the two independent lines had been very dissimilar. In fact there was a marked similarity between the two initial curves which was confirmed when a rank order correlation test was carried out using each of the functions to prescribe behaviour. Subjects were ranked according to their eventual prescriptions and a high level of correlation in rankings was found.
- vi) The utility curves described by the prescribing functions revealed that the most prevalent attitude was one of risk aversion. Risk aversion of some degree was evidenced by the curves of twenty-three out of the twenty-nine participating subjects. The

largest single group of subjects, almost thirty-five percent displayed aversion to risk when potential profit was involved or a small loss was possible but where large losses were possible they began to assume the characteristics of the risk taker. This change from aversion to seeking took place within a very narrow range of pound values.

The concept of the entrepreneurial risk-taker is not borne out by this study. What was evidenced was a tendency to be conservative in decisions involving potential profit or small losses. The rewarding of "effective" decision making, the nature of management education, unfamiliarity with decisions only holding out the prospect of a loss and changes in attitude when losses are being experienced were discussed in an effort to explain the predominance of aversion and the shift to seeking for large losses.

B) The Use Of The Utility Functions.

- i) The main thrust of the experiments was to examine the use subjects were prepared to make of their utility prescriptions. Comparing actual with prescribed behaviour, in itself, would not address this problem as if utility theory is a normative theory it could be expected that differences between actual and prescribed behaviour would exist. It is because there are inconsistencies that a prescriptive decision

making aid has been developed.

Comparing actual with prescribed behaviour would be in order as long as subjects had knowledge of how they should act and their actual behaviour was observed after they had received such information. In this study subjects were in three groups some with prior knowledge, those with post knowledge and others with no knowledge of their prescriptions.

Using the mean of the absolute difference between actual and prescribed behaviour, in the twenty-four decision problems, as each subjects score, a significant difference was detected between the three knowledge groups.

- ii) The smallest difference was for those who had post knowledge of their prescriptions and by far the largest difference was displayed by subjects who had no knowledge of their prescriptions and had been left to tackle the decisions unaided.

Combining those with pre and post knowledge and comparing them with subjects having no knowledge of their prescriptions also produced a significant difference. The timing of knowledge of prescriptions did not, however, seem to result in any significant difference in the use made of the information.

With other variables controlled as far as possible, subjects subjected to identical decision problems and assigned to groups in a random fashion, knowledge would seem to have influenced their actual behaviour in as much as they behaved in a manner very similar to that prescribed for them.

There was certainly no reason to suggest that the subjects made use of their prescriptions because they thought this was expected of them, or as they wanted to "please" the experimenter. The opposite may be more appropriate as managers are often sceptical about attempts to analyse their behaviour in decisions. These subjects were mature managers with a considerable number of years experience behind them and were free to accept or reject the information they had been given.

- iii) The monetary range embraced by the decision questions that the managers were asked to answer was £105,000. The mean difference between actual behaviour and prescribed behaviour for all subjects was £3,508.48. For some subjects the difference was as low as a mean of £492.00 over the twenty-four decisions with the difference in individual questions being as low as £50.00. For other subjects the mean difference for the twenty-four decisions was as high as £9,222.91. In addition to the finding that knowledge groups

exhibited varying differences, the differences themselves, in real terms, were comparatively small.

- vi) There was no relationship between risk taking attitude as described by the model and as evidenced by actual behaviour. Such a finding, for all subjects, could have been expected as the function was not being looked upon as a predictor of behaviour.

The only relationship between these two methods of describing attitudes towards risk was found among subjects who had had knowledge of their prescriptions.

### C) Attitudes Towards Risk in Decision Making.

Of the twenty-nine subjects who participated in this study, fifteen were risk managers. The risk manager, or insurance manager, within a company is responsible for the management of firm's insurance portfolio, any self insurance fund, risk financing, investment in loss reduction or prevention equipment and the general ongoing evaluation and economic control of risk. The selection of risk managers was intentional as their familiarity with decision making in situations where there is only the prospect of a loss or status quo may have led them to have a different attitude to risk, in certain problems, than other managers.

The twenty-four decision problems presented to each subject were equally divided between these where only a loss or break even

existed and those where only a profit or break even was possible.

- i) There was a significant difference in decision making behaviour, as evidenced by risk taking attitude score, between risk and non-risk managers. Over all questions the risk managers showed themselves to be much more risk averse than their colleagues.
- ii) In analysing the decision making behaviour of risk managers where only a profit or break even position existed it was found that there was an insignificant difference between their behaviour and that of the non-risk managers.

The risk managers, however, were considerably more risk averse than others when confronted with the decisions that only held out the prospect of a loss. In addition they were, as a group, far more homogeneous in their behaviour. The non-risk managers by comparison, exhibited wide variation around a mean risk taking attitude that was much less risk averse.

Rather than their familiarity with potential losses resulting in them being less cautious, the risk managers were prepared to pay high amounts to be freed from risk. Their knowledge and experience of the loss producing event may have given them a respect for the potential loss that was not held by the other subjects.

iii) All subjects were more averse to risk where there was a low probability of losing a high amount than where the chance of loss was only fifty percent. This low probability, high loss phenomena is very common in industrial and commercial life.

While all subjects were risk averse when faced with such a problem the group of risk managers were far more so than their colleagues. This may again be attributable to the management role they perform as many of the perils against which they will purchase insurance protection will fall into the category of low probability high loss events. The purchase of insurance against material damage caused by earthquake in the United Kingdom is a good example of this situation in practice.

#### FURTHER RESEARCH.

A) The most important aspect of future research would be to expand the number of subjects selected and carry out the sample selection in such a way as to enable statistical inferences to be made concerning a larger population of managers. This could not be carried out without considerable effort and support from business managers. The benefits of such a large scale study do, however, merit further attention being given to the problem.

B) The self administered utility test questionnaire does cut down the amount of time required in obtaining pound - utile co-ordinates but further work is necessary on its design. One particular area that should be examined is the possibility of concealing answers once they have been made, in an attempt to minimise any learning effect that might exist at the moment.

Most managers would have access to a computer and it may be possible to write an "interactive" programme that would work in conjunction with a video display unit. Such a programme could ask the subject the questions that appear on the present questionnaire and possibly include many more while only showing one problem on the screen at any one time. An extension of this idea may be that such a programme could also fit the function to the co-ordinates that were derived. In this way an almost instantaneous utility function could be created and applied to an existing problem by feeding in the pound values of the problem and transferring them into utiles.

Should such a concept be possible then managers would have the ability to utilize utility theory on a day to day basis as and when required.

C) The problem of attitudes and preference orderings changing over time is still one that requires attention. In the

absence of a system as mentioned above, some delay will inevitably exist between creating a function and using it in problems. The longer it is between testings of a person's utility function the more likely it is to become out of date.

- D) The value of a normative utility theory is in its ability to point out what a manager should do if he wants to be consistent among decisions. More work is required into how knowledge of utility prescriptions should be disclosed to decision makers.

The choice would seem to be between giving them as much information as possible on the nature and theory of utilities or providing them with the basic facts they require, that is, the utility prescriptions. In this study each subject was given a little information on what the prescriptions represented.

This is possibly one of the crucial areas of future study. It is being suggested to a manager that he may have made a choice that he should not have made if he wants to be consistent. This fact can be difficult to convey to experienced business men and work is required to see whether more or less knowledge of what utility theory purports to be would be beneficial.

- E) It would also be interesting, in any future work, to return

to subjects sometime after the experiments had been completed and try to ascertain why they did or did not make use of the utility prescriptions that had been provided for them.

This could give an insight into the problems people may have in accepting guidance in decision making tasks.

- F) The form of management education and decision making education in particular has been mentioned during this study. An aspect of further research that could be beneficial would be into how best normative decision making aids can be taught.

Should it be possible to write an 'interactive' programme, as mentioned earlier, this could have substantial benefits to students in introducing them, in a personal way, to the problems of differing risk taking attitudes.

- G) The rationale behind this study is that management decision making may benefit. Decision making in the real world is far removed from a choice between alternatives in a hypothetical problem. Further research is required in the application of normative utility theory to actual business decisions.

This is difficult in view of the questioning of managers' initial choices that would be required when differences are observed between actual and prescribed behaviour.

This should not, however, deter further work being started.

- H) Decision making in loss producing situations did seem to be erratic on the part of those managers who had no great familiarity with the form of decision that only held out the prospect of a loss. More work is required in an effort to identify the causes of this change in attitude between profit making and loss producing problems.
  
- I) Finally a closer examination of the relationship between attitudes towards risk and certain biographical information is necessary. This would have some application in the personnel function but would also be of value in determining what factors contribute to various risk taking attitudes.

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# APPENDIX 1

QUESTIONNAIRE CONTAINING  
TWENTY FOUR DECISION  
PROBLEMS.

A STUDY OF  
MANAGEMENT DECISION MAKING

On the following pages you will find a series of decision making problems. Each problem involves you in a decision and what we want to know is how you would act if this was a decision you were facing, at work, today. Please approach these choices as a corporate decision maker not as a private individual and do not be influenced by what you feel you should do or what you think we expect you will do but rather make a choice based on what you consider you would do in real life.

All the decision making problems involve a choice between two mutually exclusive alternatives. The two alternatives will always be of the same basic form i.e., on the one hand there will be a chance situation involving a possibility of either breaking even or gaining a given amount of money (some of the chance situations will also involve a possibility of breaking even or losing a given amount). On the other hand, the second alternative will always be in the form of an option yielding a certain amount of money. You are being asked to decide what that certain amount of money would have to be so that you would prefer to take, or pay, it rather than take the alternative involving the chance situation. You must respond to each question. There is no correct answer to any of these questions, all that is being examined is how you, personally, respond.

All values in the problems can be taken as the present values of all cash flows, net of taxes, with payments and receipts taking place in the near future. The financial amounts involved and the probabilities may not always appear realistic but the basic choices have been incorporated in a story line to make it appear as a decision and not simply a lottery. Please try and place yourself in the position of the decision maker and answer the questions as you would if actually faced with these problems in real life. Please accept the facts given in each story and base your response on these facts alone, i.e., without giving consideration to facts not provided.

By looking at the following two examples you will see that your response, to each of the questions in this booklet will take the form of an amount of money which is to be inserted in the boxes provided. This amount of money could really be looked upon as a certainty equivalent i.e., it is the certain amount of money you would be prepared to lose, or gain, rather than choose the alternative with the chance outcomes of possibly breaking even or losing/gaining. In the problems with the chance of losing you are asked to indicate the maximum definite loss you would bear rather than take the risky alternative. In cases involving a chance of profit you are asked to indicate the minimum definite profit you would accept rather than take the risky alternative.

It is important that you show the maximum amount or the minimum amount you would bear or accept and before moving on to the next question you should spend a moment satisfying yourself that your answer adequately reflects your attitude.

In each question you will be given the profit/loss for the risky alternative and also what the chances are of you making that profit/loss. The non-risky alternative will always lead to certain profit/loss and it is the minimum profit or maximum loss you are asked to arrive at so that you would just still prefer the non-risky alternative to the risky one.

The following two examples should clear up any remaining problems you may have.

Prior to starting the questions would you please place a tick in one of the boxes at the top of page 6. Place a tick in box  RIM  if you are the risk or insurance manager or tick box  NRM  if you perform some other management function.

## EXAMPLE I

Certain long term securities held by your company are about to mature and you must decide how to use the money. After consideration it is decided that the money is not required for any immediate purpose and can be reinvested. You have the opportunity of purchasing some shares with a high possible net profit of £20,000, during the next accounting period. There is, however, the possibility that the shares will not rise but will stay at the same value. The expert advice you have received indicates there is a 50/50 chance of the shares rising or staying the same. If they rise it is assumed they will give a profit of £20,000. An alternative open to you is to reinvest the money in Government Securities where the profit will be smaller but certain.

What would be the minimum amount you would be prepared to accept as a definite profit on the Government Securities rather than take your chance of possibly gaining £20,000 or breaking even?

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Please satisfy yourself that if the profit from the Government Securities was any lower than the figure you have selected, you would prefer to take the risky alternative.

In this example, which involves a possible gain, the respondent has inserted £19,500 as the minimum definite profit he would look for instead of taking the risky alternative. This would be an unusually high amount to insist upon and most people would select a figure somewhat less than that.

By inserting £19,500 this person indicates that should the profit from the Government Securities be, say £19,400 he would prefer to take the chance alternative of gaining £20,000 even although by doing so he also takes the equally likely chance of gaining nothing.

On the other hand someone else may have inserted £150 as the minimum definite amount he would look for meaning that should the profit from the securities be, say £200 he would buy the securities rather than take the chance of gaining nothing by buying the shares, despite the fact that he also would have the chance of gaining £20,000.

These are exaggerated extremes and most people would select a definite amount that lay somewhere between them.

## EXAMPLE II

Your company has been sued for libel as a result of an article published in a trade journal and written by your technical manager. You have taken expert legal advice and find that there is only a 10 per cent chance that you will be held liable. Should the other side win their case they will be awarded £17,500, should they fail, you will break even as they will pay costs.

You have heard that they will consider a reasonable "out of court" settlement figure. What is the maximum offer you would make to avoid becoming involved in litigation with the chance of breaking even or losing £17,500? Your firm is not insured for this risk.

-	1	7	0	0	0
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Are you satisfied that if the other side said they would accept a little more than the figure you have shown, you would then prefer to take your chances in litigation?

This is an example involving a possible loss situation and again the figure inserted of £17,000 is an unrealistically high amount. It means in fact that if the other side said they would take £16,900 he would pay this rather than take the chance of losing £17,500, despite the fact that there is only a 10 per cent chance of losing the case.

On the other hand if a person had said that the maximum amount they would pay "out of court" was £100 then this implied that if the other side said they would accept £120 he would not pay it but prefer to take his chance of paying nothing, despite the fact that there is still a 10 per cent chance of having to pay £17,500.

These two definite amounts £17,000 and £100 are obviously quite extreme and most people would probably select a figure somewhere between them. However, there will be considerable variation in the amount actually selected by people.

RIM	
NRM	

1 Your company presently trades in one foreign market in addition to the home market. The experience in this foreign market has not been good and consistent losses have been made. Your evaluation is that if you continue to trade there, you will lose £28,000 in total. You consider there to be a 50/50 chance of losing this amount or breaking even.

You could leave the foreign market immediately, and rely on the domestic market only, but by doing so would incur a definite loss in view of certain fixed costs that have already been paid.

What is the maximum loss level would you accept, by leaving the foreign market immediately, rather than take the chance of breaking even or losing £28,000?

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Were you to find that the definite loss from leaving the foreign market was slightly higher than the figure you have shown above, are you satisfied that you would then prefer to take your chance, by continuing to trade, of breaking even?

2 You are engaged in the analysis of a decision as to whether you should expand your business by entering a foreign market or should concentrate on the less uncertain alternative of expanding domestically.

Should you enter the foreign market, the best advice you have is that there is a 50/50 chance of achieving a net profit level of £40,000 otherwise the enterprise will fail and only provide sufficient funds to break even.

Should you expand domestically there is no doubt that the increased profit level could be exactly calculated.

What is the minimum increased profit level you would require, from domestic expansion, lower than which you would choose to enter the foreign market?

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If the definite profit from domestic expansion was just a little lower than the figure you have shown above, are you convinced you would then prefer to take your chances in the foreign market of possibly gaining £40,000?

3 Your company has purchased an amount of iron ore from a foreign producer and now find that you cannot obtain an import licence for it.

You have decided that you could attempt to sell the ore in some other foreign market. The chance of being able to sell the quantity of ore in question is put at 50/50 and if you succeed then you would break even. If you fail in selling the ore then the loss will be £25,000 being the purchase price and lost revenue.

The producer of the ore has also offered to buy it back at a figure obviously lower than the purchase price you paid, thus resulting in a definite loss.

What is the maximum certain loss you would accept before you would prefer to take your chances on selling the ore?

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If the producer of the ore offered you a price that resulted in a loss which was just a little more than what you have inserted above, are you satisfied that this would persuade you to take your chances on selling the ore and possibly breaking even?

4 You have been given the chance to make a bulk purchase of iron ore from abroad but you are not sure that an import licence would be granted. You must, unfortunately, decide whether or not to buy the ore before you know the outcome of the Government's deliberations over your import licence. A friend in the relevant government department has told you that there is only a 50/50 chance of getting the licence. If you get the licence the profit from selling in this country would be £20,000. If you fail to get the licence you would, of course, break even.

An alternative course of action is to spend the money on buying finished metal sheets in Britain which would be more expensive but will yield a definite profit.

What is the lowest definite profit you would accept rather than make the bulk purchase with the chances involved in doing that?

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Should the profit from buying finished metal sheets be lower than what you have inserted above are you sure that you would then prefer to take your chance by buying the ore, of possibly gaining £20,000?

5 After a recent take-over you have found that a large quantity of foreign products, all of the same kind, are lying unsold in a warehouse owned by the firm you have just acquired. The goods have only been partly paid for. There is, without doubt, a 90 per cent chance of you being able to sell the goods and if you do sell, you will break even, bearing in mind the outstanding purchase price and that the price will be so low in view of the age of the goods and the recent introduction of a more attractive modern version of the same item. You, therefore, have a 10 per cent chance of losing £55,000, the amount still owed. A foreign importer has offered to buy the goods from you but the amount he has offered will not allow you to make a profit, in fact you will definitely make a loss and you can calculate this loss exactly.

What is the highest definite loss you would bear rather than attempt to sell the foreign products?

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Should the price offered by the importer result in a loss just a little higher than you have indicated, are you sure you would then prefer to try and sell the products with the chance of breaking even?

6 Although you do not normally import goods you have received the opportunity of obtaining, by bulk purchase, a foreign product, very similar to one manufactured in this country which you presently market successfully. You have had little time to examine the market's potential response to this imported product but on the strength of all available evidence you conclude that there is a 50/50 chance of the item selling as well as the British version. The foreign product is much cheaper to make and if it does sell as well as the British equivalent the increased profit, in the coming accounting year, will be £28,000. The alternative to buying this bulk order is to acquire a local retail outlet and expand the marketing of your existing product. The shop to be purchased has produced a steady rate of profit over recent years and your accountants have forecast a target profit for the forthcoming accounting year, which they are certain will be attained.

What is the lowest certain profit you would accept from the shop beyond which you would prefer to take your chances by importing the foreign goods?

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On looking back at the figure you have shown above, are you satisfied that if the profit from the shop was lower than it, then you would prefer to take your chances, by importing the foreign products, of gaining £28,000?

7 After a recent take-over you have found that a large quantity of foreign products, all of the same kind, are lying unsold in a warehouse owned by the firm you have just acquired. The goods have only been partly paid for. There is, without doubt, only a 50/50 chance of you being able to sell the goods and even if you do sell, you will only break even, bearing in mind the outstanding purchase price and that the price will be so low in view of the age of the goods and the introduction of a more attractive modern version of the same item. You, therefore, have a 50/50 chance of breaking even or losing £15,000, the amount still owed.

A foreign importer has offered to buy the goods from you but the amount he has offered will not allow you to make a profit, in fact you will definitely make a loss and you can calculate this loss exactly.

What is the highest definite loss you would bear, rather than attempt to sell the foreign products?

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If the loss from selling to the foreign importer was more than you have shown above, are you sure you would then prefer to attempt to sell the goods with the chance of possibly breaking even?

In the remainder of the decisions you are not asked to look back at the figure you insert, to check that you are satisfied with it. The principle does however remain that you should make sure you are completely happy with the amount you choose before moving on to the next decision.

8 Your company is considering purchasing a new warehouse. The new premises will enable you to handle a wider variety of goods more speedily than before. You have carefully analysed all the issues involved and consider there to be a 50/50 chance that the new improved warehouse, with its wider range of goods, will yield additional profits, per year, of £25,000.

You are also considering an alternative and that is to redesign your existing warehouse so that the goods you presently handle can be processed more quickly and efficiently. This option will, without doubt, lead to additional profits, per year.

What is the minimum definite additional annual profit from redesigning you would accept rather than take the chance of the £25,000 profits increase by moving?

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9 Your Firm's premises are now too small for the level of production presently being achieved and anticipated in the future. You have the opportunity of purchasing a new custom built ready to enter factory unit in a neighbouring town. You fear that you may lose some of your market, by making this change, and have carefully evaluated there to be only a 10 per cent chance that you will lose some of your market. The amount you consider you would lose would be £47,500 over the period under discussion. The alternative is to completely refurbish your existing premises which will ameliorate the problem but will result in a definite loss, as the factory will be closed for these alterations, during the period under discussion.

What is the greatest definite loss you would be prepared to accept rather than take the chance of possibly losing £47,500 or breaking even?

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10 A new product had been launched by your company sometime ago. Unfortunately it has not done well and a decision must be made whether or not to continue to market the product. Your marketing departments research shows a 90 per cent chance of you regaining lost ground and breaking even during the period under consideration. If you should fail to break even then the net loss would be £45,000.

You could decide to immediately withdraw the product as a result of which there would be a definite loss in view of initial expenses etc.

What is the greatest loss you would accept, from withdrawing the product, rather than take the chances involved in continuing to market it with the possibility of breaking even or losing £45,000?

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11 You are engaged in the analysis of a decision as to whether you should expand your business by entering a foreign market or should concentrate on the less uncertain alternative of expanding domestically.

Should you enter the foreign market, the best advice you have is that there is a 50/50 chance of achieving a net profit level of £47,500 otherwise the enterprise will fail and only provide sufficient funds to break even.

Should you expand domestically there is no doubt that the increased profit level could be exactly calculated.

What is the minimum increased profit level you would require, from domestic expansion, lower than which you would choose to enter the foreign market?

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12 Your company presently trades in one foreign market in addition to the home market. The experience in this foreign market has been variable and losses have been made. Your evaluation is that if you continue to trade there, there is a 10 per cent chance that you will lose £40,000 in total. There is, therefore, a 90 percent chance of breaking even.

You could leave the market immediately, cut your losses, and rely on the domestic market only.

What is the maximum loss level you would accept, by leaving the foreign market immediately, rather than take the chance of breaking even or losing £40,000?

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13 Certain Government Securities, held by your company, are about to mature and you are deciding what is now to be done with the available funds. After consideration it is decided that the money is not required for any immediate capital investment project, it can therefore be re-invested. You have been given the opportunity of purchasing some shares with a high possible net profit of £10 000 during the next accounting period. There is the possibility that the shares will not rise but will stay at the same value. The experts in the field suggest that there is a 50/50 chance of the shares giving a profit of £10,000. The alternative is to re-invest the available funds in Government Securities where the profit will be smaller but guaranteed.

What is the lowest profit from the Government Securities you would accept below which you would buy the shares with the chance of gaining £10,000 or breaking even?

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14 Certain Government Securities, held by your company, are about to mature and you are deciding what is now to be done with the available funds. After consideration it is decided that the money is not required for any immediate capital investment project, it can therefore be re-invested.

You have been given the opportunity of purchasing some shares with a high possible net profit of £17 500 during the next accounting period. There is the possibility that the shares will not rise but will stay at the same value. The experts in the field suggest that there is a 50/50 chance of the shares giving a profit of £17,500. The alternative is to re-invest the available funds in Government Securities where the profit will be smaller but guaranteed.

What is the lowest profit from the Government Securities you would accept below which you would buy the shares with the chance of gaining £17,500 or breaking even?

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15 Your Firm's premises are now too small for the level of production presently being achieved and anticipated in the future. You have the opportunity of purchasing a new custom built ready to enter factory unit in a neighbouring town. You fear that you may lose some of your market, by making this change, and have carefully evaluated the chance to be 50/50 that you will lose or not lose any market. The amount you consider you would lose would be £40,000 over the period under discussion. The alternative is to completely refurbish your existing premises which will ameliorate the problem but will result in a definite loss, as the factory will be closed for these repairs, during the period under discussion.

What is the greatest definite loss you would be prepared to accept rather than take the chance of possibly losing £40,000 or breaking even?

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16 You have been given the chance to make a bulk purchase of iron ore from abroad but you are not sure that an import licence would be granted. You must, unfortunately, decide whether or not to buy the ore before you know the outcome of the Government's deliberations over your import licence.

A friend in the relevant government department has told you that there is only a 50/50 chance of getting the licence. If you get the licence the profit from selling in this country would be £50,000. If you fail to get the licence you would, of course, break even.

An alternative course of action is to spend the money on buying finished metal sheets in Britain which would be more expensive but will yield a definite profit.

What is the lowest definite profit you would accept rather than make the bulk purchase with the chances involved in doing that?

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17 Certain shares your company holds have been falling steadily in value. An investment analyst, employed by your firm, has examined the position carefully and has reported on the prospects during the next year. The analyst's opinion is that there is a 50/50 chance that the shares will regain their purchase value and if they don't, then the total loss to your company will be £20,000. A client of the company with which the analyst works has expressed an interest in the shares, from a speculative viewpoint, and is prepared to make an offer for them. The offer, in view of the shares uncertain future, will be less than the price you paid for them and this will mean a loss, but a loss that you could calculate exactly.

What is the greatest certain loss you would be prepared to suffer, as a result of selling to the analyst's client, rather than retain the shares with a 50 per cent chance of breaking even or losing £20,000?

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18 Your company is considering purchasing a new warehouse. The new premises will enable you to handle a wider variety of goods more speedily than before. You have carefully analysed all the issues involved and consider there to be a 50/50 chance that the new improved warehouse, with its wider range of goods, will yield additional profits, per year of £45,000.

You are also considering an alternative and that is to redesign your existing warehouse so that the goods you presently handle can be processed more quickly and efficiently. This option will without doubt lead to additional profits per year.

What is the minimum definite additional annual profit from redesigning you would accept rather than take the chance of the £45,000 profits increase by moving?

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19 A major competitor has recently left the market and you are considering production of one of their lines. The expected profit is £35 000 but due to the uncertainty as to whether you would be successful in cornering a share of the market or not the chance of you achieving this profit is put at 50/50. If you fail in your bid to capture the market you know that you will only gain enough to cover costs and thus break even.

There is an alternative and that is to extend the range of your own similar products. Extensive market research has indicated a certain profit level if this was done.

What is the lowest certain profit you would accept below which you would take the alternative of producing the competitor's product with the possibility of gaining £35,000 or breaking even?

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20 Your company has purchased an amount of iron ore from a foreign producer and now find that you cannot obtain an import licence for it.

You have decided that you could attempt to sell the ore in some other foreign market. The chance of being able to sell the quantity of ore in question is put at 90 per cent and if you succeed then you would break even. If you fail in selling the ore then the loss will be £35,000 being the purchase price and lost revenue.

The producer of the ore has also offered to buy it back at a figure obviously lower than the purchase price you paid, thus resulting in a definite loss.

What is the maximum certain loss you would accept before you would prefer to take your chances on selling the ore?

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21 Although you do not normally import goods you have received the opportunity of obtaining by bulk purchase a foreign product, very similar to one manufactured in this country which you presently market successfully. You have had little time to examine the market's potential response to this imported product but on the strength of all available evidence you conclude that there is a 50/50 chance of the item selling as well as the British version. The foreign product is much cheaper to make and if it does sell as well as the British equivalent the increased profit, in the coming accounting year, will be £5,000. The alternative to buying this bulk order is to acquire a local retail outlet and expand the marketing of your existing product. The shop to be purchased has produced a steady rate of profit over recent years and your accountants have forecast a target profit for the forthcoming accounting year, which they are certain will be attained.

What is the lowest certain profit you would accept from the shop beyond which you would prefer to take your chances by importing the foreign product?

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22 Certain shares your company holds have been fluctuating in value, always below the original purchase price. An investment analyst, employed by your firm, has examined the position carefully and has made a report to you. The analyst's opinion is that there is a 90 per cent chance that the shares will regain their purchase value, at which time you could sell and break even, and a 10 per cent chance they don't when the total loss to your company would be £50,000.

A client of the company with which the analyst works has expressed an interest in the shares, from a speculative viewpoint, and is prepared to make an offer for them. The offer, whatever it is, will still result in your company making a loss on these investments.

What is the greatest certain loss would you be prepared to suffer, as a result of selling to the analyst's client, rather than retain the shares with a 10 per cent chance of losing £50,000 ?

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23 A new product had been launched by your company sometime ago. Unfortunately it has not done well and a decision must be made whether or not to continue to market the product. Your marketing departments research shows a 50/50 chance of you regaining lost ground and breaking even. If you should fail to break even then the net loss would be £10,000.

You could decide to immediately withdraw the product as a result of which there would be a definite loss in view of initial expenses etc.

What is the greatest loss you would accept, from withdrawing the product, rather than take the chances involved in continuing to market it with the possibility of breaking even or losing £10,000?

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24 A major competitor has recently left the market and you are considering production of one of their lines. The expected profit is £31,500 but due to the uncertainty as to whether you would be successful in cornering a share of the market or not, the chance of you achieving this profit is put at 50/50. If you fail in your bid to capture the market you know that you will only gain enough to cover costs and thus break even.

There is an alternative and that is to extend the range of your own, similar, products. Extensive market research has indicated a certain profit level if this was done.

What is the lowest certain profit you would accept below which you would take the alternative of producing the competitor's product with the possibility of gaining £31,500 or breaking even?

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## INSTRUCTIONS.

The following questions are designed to provide some biographical information that will eventually be compared with your responses to the decision situations, which form the main part of the study. The information you provide here is, of course, entirely confidential.

As you work your way through the questions please place a tick in the box corresponding to the answer you select.

1. Are you the risk/insurance manager or do you perform some other management function?

Risk/insurance  
manager  
non-risk/  
insurance manager


2. What age will you be on the 1st. January 78?

Less than 30  
30 - 39  
40 - 49  
50 - 59  
over 60


3. Do you hold a university first degree? If the answer is yes please indicate the faculty.

No  
Yes


4. Do you hold a university post-graduate degree?

No  
Yes


5. Do you hold a professional qualification appropriate to your present position?

Yes  
No


6. Do you hold any other form of post-experience, post-qualifying diploma or certificate?

Yes \*

No

\* please indicate type.


7. How long have you been in full-time employment?

Less than 6 years

6 - 10 years

11 - 15 years

16 - 20 years

Over 20 years


## APPENDIX 2

SELF ADMINISTERED UTILITY  
QUESTIONNAIRE.

A STUDY OF  
MANAGEMENT DECISION MAKING

INSTRUCTIONS

The two decision problems shown on pages 5 and 6 of this booklet represent the first part of a two part programme designed to give an insight into certain aspects of management decision making.

Both of the questions you are asked to answer are of the same basic style. You will be faced with a decision situation where you are forced to act in choosing between two mutually exclusive alternatives. One alternative will always involve a chance situation where there will be two possible financial outcomes with a 50/50 chance of either occurring. The other alternative will result in a definite amount of money.

You will be given the two chancy financial outcomes and will be asked to decide what level the definite amount of money would have to be, at its limit, so that you would prefer to take the alternative leading to it rather than the one leading to the chance situation.

Depending upon the outcomes, to the chancy alternatives, you may be willing to accept a minus amount rather than take the chancy alternative. i.e. you would be willing to pay up to a certain amount of money rather than choose the risky alternative and risk losing a lot more. If this is the case then your answer should represent the maximum negative amount you would consider i.e., the maximum you would be prepared to pay. In other cases the alternatives will be such that you would require a positive amount before you would forego the risky alternative, i.e., you would accept an amount of money, down to some minimum figure, rather than take the risky alternative. In these cases your answer will be the minimum amount you would require, i.e., the minimum amount you would accept, below which you would just prefer to take your chances with the risky alternative.

The values in the chance outcome will be high and you are asked to look upon these decisions as a business decision maker and not as a private individual. It is important to remember that there are no correct answers to these questions, you must try not to be influenced by what you think you should do or think is expected of you but, rather, make a choice based on what you think you would do in real life.

An example of the form of decision problem follows together with some explanatory notes on the responses shown.

You will see that beneath the problem itself there are seven lines, (A) to (G) , with 3 boxes in each line. The first two boxes in each line are under the heading of "OUTCOMES" and the last box in each line is under the heading of "CERTAINTY EQUIVALENTS". Certainty equivalent is the name given to the definite amount of money that you would choose rather than take the alternative leading to the chance of securing one of the two outcomes you will have been given.

The boxes are numbered 1 to 9 and you will see that in line (A) boxes 

1	
---	--

 and 

2	
---	--

 show the outcomes from the problem itself. Once you have selected your certainty equivalent and put this in box 

3	
---	--

 you are ready to move on to line (B).

All you need do now is fill in box 

3	
---	--

 in line (B) by carrying down the value for box 

3	
---	--

 in line (A) , and by using the same basic story you now have a new decision with outcomes of 

1	
---	--

 and 

3	
---	--

 . There are two important restrictions, the first is that your certainty equivalent is to lie between the two outcomes , whatever they may be , and the second is that you cannot select one of the outcomes as your certainty equivalent.

By following this pattern you will see that the person ended up

by giving seven certainty equivalents in seven different decision situations by using the one original story and changing the outcomes as he worked from line to line.

Prior to starting the questions would you please place a tick in one of the boxes at the top right hand corner of page 5. Tick box  RIM  if you are the risk or insurance manager or tick box  NRM  if you perform some other management function.

#### EXAMPLE

You are faced with the following decision where you are compelled to act in selecting between two mutually exclusive alternatives.

The first alternative represents a chance situation which could result in one of two possible financial outcomes, either + £10,000 or - £1,000, there is a 50/50 chance of either occurring.

The second alternative is a course of action leading to a definite financial outcome about which there is no risk.

What you are asked to do is to decide at what level that definite amount of money would have to be, at its limit, so that you would prefer to take that alternative rather than the chancy one with the possibility of + £10,000 or - £1,000. This definite amount of money will be called a certainty equivalent.

Depending upon the outcomes, to the chancy alternatives, you may be willing to accept a minus amount rather than take the chancy alternative. If this is so then your certainty equivalent should represent the maximum negative amount you would accept. In other cases, the alternatives will be such that you would require a positive amount before you would forego the risky alternative. In these cases your certainty equivalent will be the minimum amount you require.

OUTCOMES

CERTAINTY EQUIVALENTS

0	0	0	0	2	-		1	0	0	0	3	-		5	0	0
0	0	0	0	3	-		5	0	0	4	-		2	0	0	
0	0	0	0	4	-		2	0	0	5	+		1	0	0	
5	0	0		4	-		2	0	0	6	-		4	0	0	
5	0	0		2	-		1	0	0	0	7	-		7	0	0
7	0	0		2	-		1	0	0	0	8	-		9	0	0
7	0	0		3	-		5	0	0	9	-		6	0	0	

You will notice, from this example, that as the person moved from line to line he filled in the blank boxes, in the outcomes column, with the figure he had already used for that number of box in the certainty equivalent column of lines above. eg in line (D) he filled in boxes  and  with the values he had inserted for these boxes in the certainty equivalent column of lines (A) and (B).

You will also see that, for example, in line (B) he would pay up to £200 rather than take the chancy alternative and stand to lose £500. This person obviously lays more importance on the amount to be lost than on the amount to be possibly gained. Another person may have inserted + £9 000 in box  on line (B) meaning that even if he found that the definite amount turned out to be £8,500 he would refuse it and prefer to take his chance of gaining £10,000. These are extremes, as you will have realised and most people would select a certainty equivalent between these figures.

If you are not entirely sure of what is to be done then a further explanation is given on the last page. If you feel quite happy with the explanation so far then you should proceed to the questions themselves.

1 You are faced with the following decision where you are compelled to act in choosing between two mutually exclusive alternatives.

On the one hand there is an alternative that leads to a 50/50 chance of the outcome being + £80,000 or - £58,000.

On the other hand you can follow a course of action that you know will result in a definite financial amount that you can calculate. What is the limit you would allow that definite amount to reach at which you would just prefer to take it rather than the alternative leading to the 50/50 chance of + £80,000 or - £58,000?

Depending upon the outcomes in the chancy alternatives, you may be willing to accept a minus amount rather than take the chancy alternative. If this is so then your answer (your certainty equivalent) should represent the maximum negative amount you would accept. In other cases, the alternatives will be such that you would require a positive amount before you would forego the risky alternative. In these instances, your answer will be the minimum amount you would require.

OUTCOMES

CERTAINTY EQUIVALENTS

8	0	0	0	0	2	-	5	8	0	0	0	3					
8	0	0	0	0	3							4					
8	0	0	0	0	4							5					
					4							6					
					2	-	5	8	0	0	0	7					
					2	-	5	8	0	0	0	8					
					3							9					

2 You are faced with a decision where you must act. There are two mutually exclusive alternatives. On the one hand there is an alternative leading to a 50/50 chance of the outcome being either + £60,000 or - £75,000.

On the other hand there is an alternative which will result in a definite financial outcome about which there is no risk. What is the limit you would allow that definite amount to reach, at which you would just prefer it rather than the alternative leading to the 50/50 chance of + £60,000 or - £75,000? Depending upon the outcomes in the chancy alternatives, you may be willing to accept a minus amount rather than take the chancy alternative. If this is so then your answer (your certainty equivalent) should represent the maximum negative amount you would accept. In other cases, the alternatives will be such that you would require a positive amount before you would forego the risky alternative. In these instances your answer will be the minimum amount you would require.

OUTCOMES

CERTAINTY EQUIVALENTS

0	0	0	0
0	0	0	0
0	0	0	0

2	-	7	5	0	0	0
3						
4						
4						
2	-	7	5	0	0	0
2	-	7	5	0	0	0
3						

3					
4					
5					
6					
7					
8					
9					

Each of the decision problems is rather like a situation where a person has presented you with two alternatives, on the one hand he has offered you £10 if a coin, he is about to flip, lands heads up and nothing if it lands tails up. On the other hand he is prepared to give you a definite amount of money, now, regardless of the outcome of the coin tossing. What you must decide upon is the minimum amount you would be prepared to accept, now, rather than take your chances on possibly winning £10 or breaking even.

Once you decide on this amount it could be looked upon as a certainty equivalent, in that it is the minimum amount you would prefer to have, for certain, as an alternative to choosing the risky alternative.

You might also have found a situation where you would have to pay £10 if a coin landed heads up and nothing if it landed tails up. An alternative is that you can pay a fixed amount of money now in preference to taking the risky alternative. In such a case the certainty equivalent would be the maximum amount you would be prepared to pay, now, rather than run the risk of possibly having to pay £10 or break even.

## APPENDIX 3

COMPUTER PROGRAMME FOR  
INTERPOLATION ON THE POUND  
AXIS.

```

C INTERPOLATING EXTRA POINTS AT EQUAL INTERVALS OF CE
  REAL UTILE1(41),UTILE2(41)
  REAL CE1(9),CE2(9),C1(41),C2(41)
  READ(5,111) CE1(5),CE1(3),CE1(2),CE1(4),CE1(7),CE1(8),CE1(6)
  READ(5,111) CE1(1),CE1(9)
  READ(5,111) CE2(5),CE2(3),CE2(2),CE2(4),CE2(7),CE2(8),CE2(6)
  READ(5,111) CE2(1),CE2(9)
111 FORMAT(6X,7F10.0)
112 FORMAT(6X,7F10.4)
  NINT=33
  CEINT=135000.0/NINT
  NK=NINT+8
  J1=1
  J2=1
  DO 10 K=10,NK
  CEINC=(K-9)*CEINT
  C1(K)=80000-CEINC
  C2(K)=60000-CEINC
 30 IF(C1(K).GE.CE1(J1+1)) GOTO 31
  J1=J1+1
  GOTO 30
 31 CONTINUE
 40 IF(C2(K).GE.CE2(J2+1)) GOTO 41
  J2=J2+1
  GOTO 40
 41 CONTINUE
  UTILE1(K)=(C1(K)-CE1(J1))/(CE1(J1+1)-CE1(J1)) + J1-1
  UTILE2(K)=(C2(K)-CE2(J2))/(CE2(J2+1)-CE2(J2)) + J2-1
  UTILE1(K)=UTILE1(K)*.125
  UTILE2(K)=UTILE2(K)*.125
 10 CONTINUE
  DO 20 K=1,9
  C1(K)=CE1(K)
  C2(K)=CE2(K)
  UTILE1(K)=(K-1)*.125
  UTILE2(K)=(K-1)*.125
 20 CONTINUE
  WRITE(10,112) UTILE1
  WRITE(10,112) UTILE2
  WRITE(10,111) C1
  WRITE(10,111) C2
  STOP
  END

```

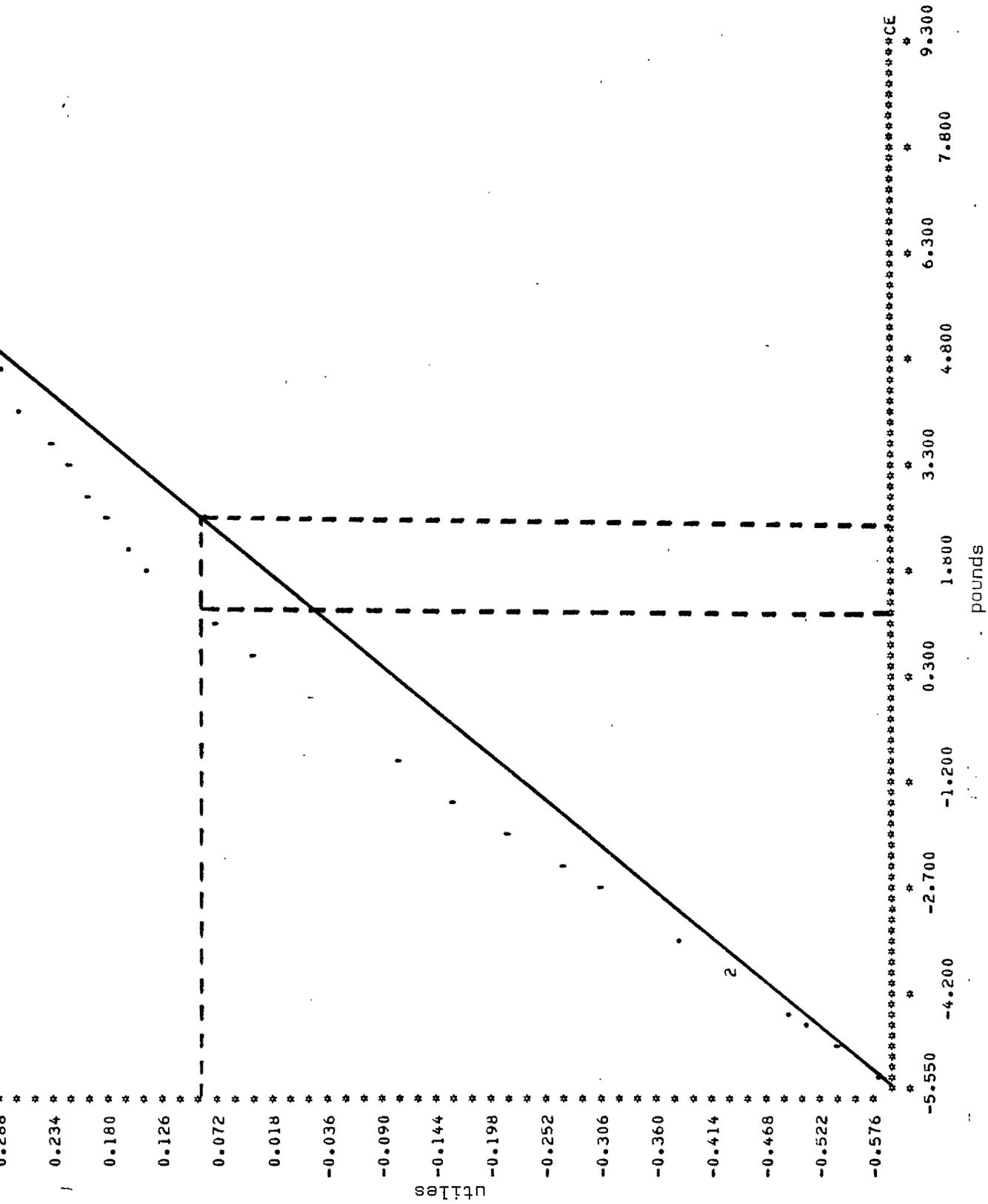
TILE HAS BEEN CALCULATED FOR VALUES OF EUV FROM EMIN IN STEPS OF ESTEP  
SSUME LINEAR INTERPOLATION BETWEEN VALUES OF UTILE

```
DIMENSION UPRED(24),UTILE(601),EUV(24)
INTEGER IEUV(24)
ESTEP=100.0
EMIN=-25000.0
READ(5,555) IDENT1,IDENT2
5 FORMAT(A3,1X,A2)
WRITE(6,666) IDENT1,IDENT2
6 FORMAT('1SUBJECT NUMBER ',A3,1X,A2///)
READ(9,111) UPRED
READ(9,111) POUND
READ(9,111) UTILE
1 FORMAT(4(1X,G19.6))
WRITE(6,222)
2 FORMAT(' QUESTION      PRED.UTILE      EUV')
DO 10 J=1,24
DO 20 K=1,601
UT2=UTILE(K)
IF(UPRED(J).LT.UTILE(K)) GOTO 21
UT1=UT2
0 CONTINUE
1 CONTINUE
PART=(UT2-UPRED(J))/(UT2-UT1)
EUV(J)=ESTEP*(K-1-PART)+EMIN
IEUV(J)=IFIX(EUV(J))
WRITE(6,333) J,UPRED(J),IEUV(J)
3 FORMAT(I9,F13.5,I11)
0 CONTINUE
DO 5 K=1,2
K1=K+2
J2=K*12
J1=J2-11
5 WRITE(10,444) IDENT1,K1,IDENT2,(IEUV(J),J=J1,J2)
4 FORMAT(A3,I1,A2,2X,12I6)
STOP
END
```

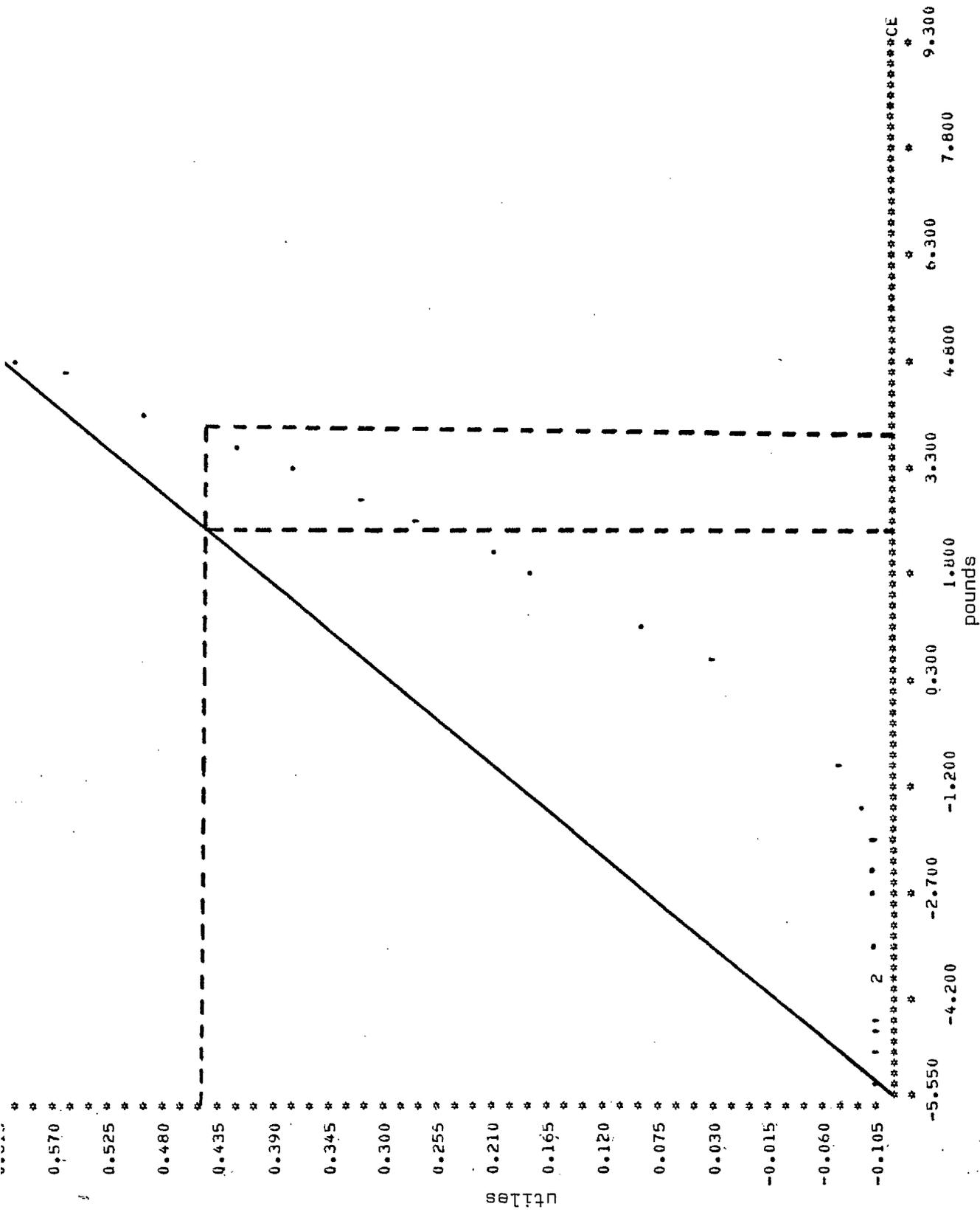
## APPENDIX 4

EXAMPLES OF DIFFERENCES IN  
UTILITY CURVES.

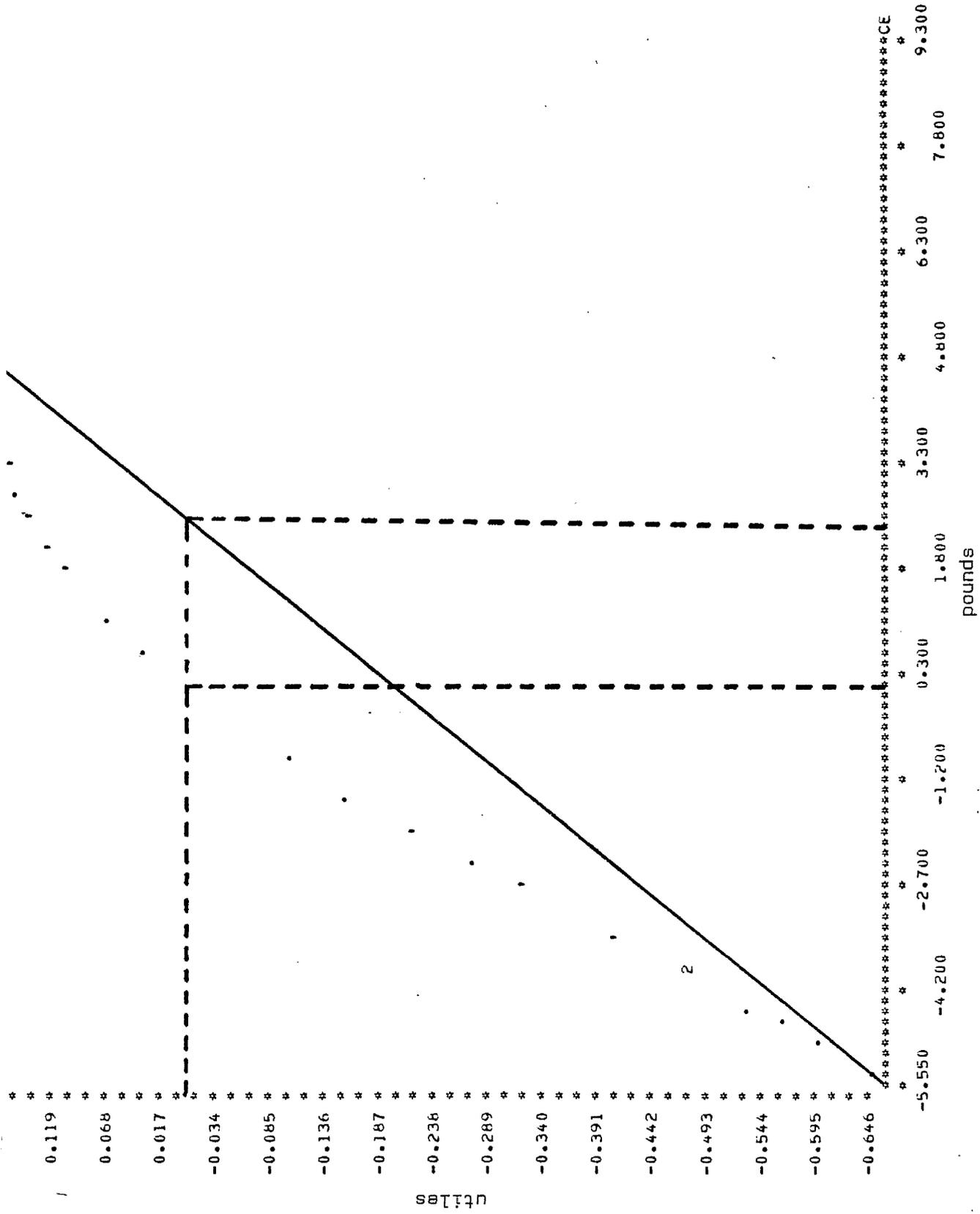
Subject 078



Subject 031



Subject 091



Subject 070

0.252  
 0.210  
 0.168  
 0.126  
 0.084  
 0.042  
 -0.000  
 -0.042  
 -0.084  
 -0.126  
 -0.168  
 -0.210  
 -0.252  
 -0.294  
 -0.336  
 -0.378

utlbs

238

2

\*\*\*\*\*  
 \* -5.650 \*  
 \* -4.200 \*  
 \* -2.700 \*  
 \* -1.200 \*  
 \* 0.300 \*  
 \* 1.800 \*  
 \* 3.300 \*  
 \* 4.800 \*  
 \* 6.300 \*  
 \* 7.800 \*  
 \* 9.300 \*  
 \*\*\*\*\*CE

pounds

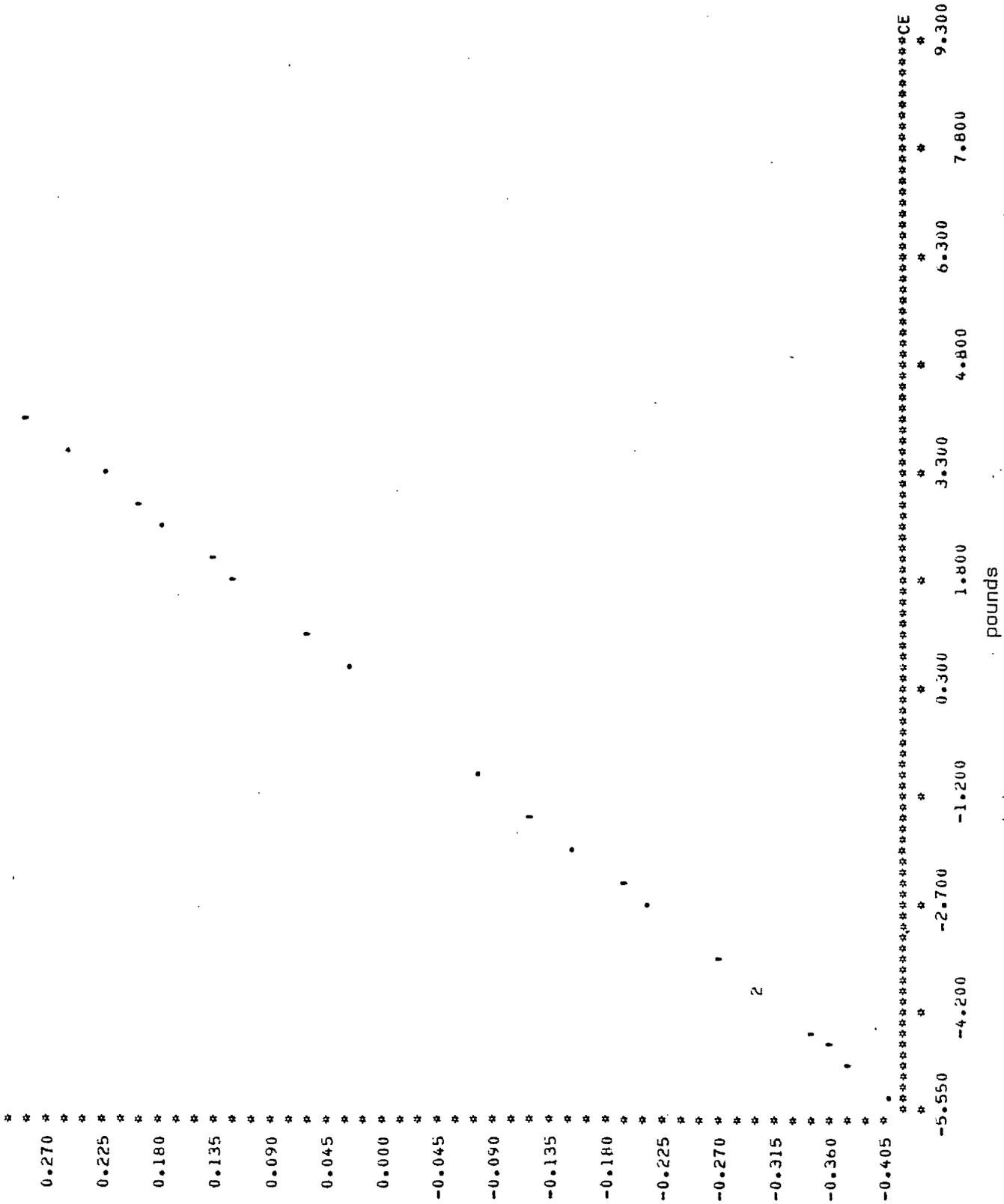
\* \* \*  
 \* 0.285 \*  
 \* \* \*  
 \* 0.228 \*  
 \* \* \*  
 \* 0.171 \*  
 \* \* \*  
 \* 0.114 \*  
 \* \* \*  
 \* 0.057 \*  
 \* \* \*  
 \* 0.000 \*  
 \* \* \*  
 \* -0.057 \*  
 \* \* \*  
 \* -0.114 \*  
 \* \* \*  
 \* -0.171 \*  
 \* \* \*  
 \* -0.228 \*  
 \* \* \*  
 \* -0.285 \*  
 \* \* \*  
 \* -0.342 \*  
 \* \* \*  
 \* -0.399 \*  
 \* \* \*  
 \* -0.456 \*  
 \* \* \*  
 \* -0.513 \*  
 \* \* \*  
 \* -0.570 \*  
 \* \* \*

utilies

\*\*\*\*\*  
 \* -5.550 \*  
 \* \* \*  
 \* -4.200 \*  
 \* \* \*  
 \* -2.700 \*  
 \* \* \*  
 \* -1.200 \*  
 \* \* \*  
 \* 0.300 \*  
 \* \* \*  
 \* 1.800 \*  
 \* \* \*  
 \* 3.300 \*  
 \* \* \*  
 \* 4.800 \*  
 \* \* \*  
 \* 6.300 \*  
 \* \* \*  
 \* 7.800 \*  
 \* \* \*  
 \* 9.300 \*  
 \* \* \*

pounds

Subject 023



## APPENDIX 5

EXAMPLES OF UTILITY CURVES  
DISPLAYING SIMILAR  
CHARACTERISTICS.

\* \* \*  
0.032 \*  
\* \* \*  
-0.016 \*  
\* \* \*  
-0.064 \*  
\* \* \*  
-0.112 \*  
\* \* \*  
-0.160 \*  
\* \* \*  
-0.208 \*  
\* \* \*  
-0.256 \*  
\* \* \*  
-0.304 \*  
\* \* \*  
-0.352 \*  
\* \* \*  
-0.400 \*  
\* \* \*  
-0.448 \*  
\* \* \*  
-0.496 \*  
\* \* \*  
-0.544 \*  
\* \* \*  
-0.592 \*  
\* \* \*  
-0.640 \*  
\* \* \*  
-0.688 \*

utils

2

\* \* \* \* \*  
\*\*\*\*\*  
\* \* \* \* \*  
-5.550 \* -4.200  
\* \* \* \* \*  
-2.700 \*  
\* \* \* \* \*  
-1.200 \*  
\* \* \* \* \*  
0.300 \*  
\* \* \* \* \*  
1.800 \*  
\* \* \* \* \*  
3.300 \*  
\* \* \* \* \*  
4.800 \*  
\* \* \* \* \*  
6.300 \*  
\* \* \* \* \*  
7.800 \*  
\* \* \* \* \*  
9.300 \*  
\*\*\*\*\*CE

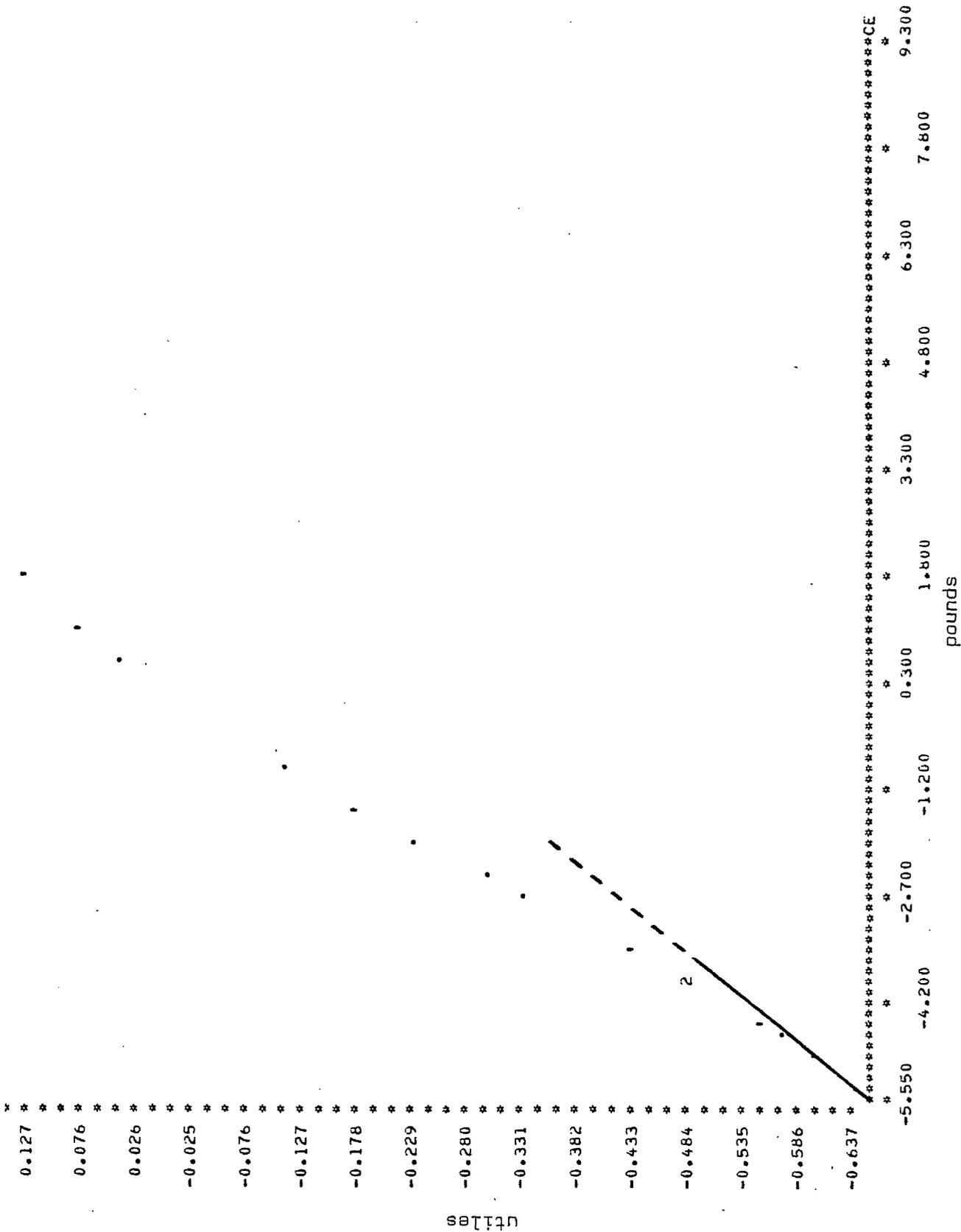
pounds

Subject 079

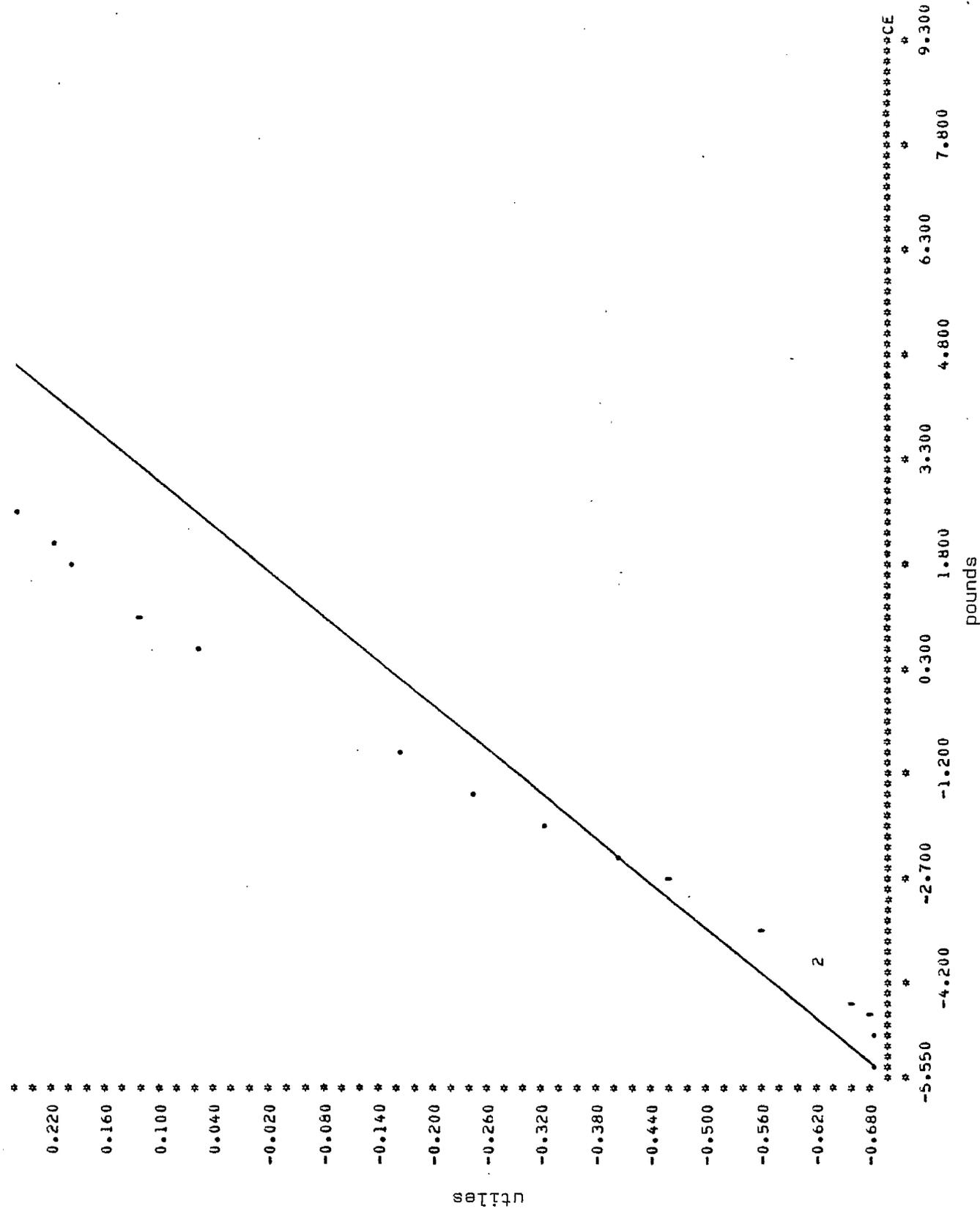
```
*
* 0.133 *
*
* 0.076 *
*
* 0.019 *
*
* -0.038 *
*
* -0.095 *
*
* -0.152 *
*
* -0.209 *
*
* -0.266 *
*
* -0.323 *
*
* -0.380 *
*
* -0.437 *
*
* -0.494 *
*
* -0.551 *
*
* -0.608 *
*
* -0.665 *
*
* -0.722 *
*
*****
* -5.550 *
* -4.200 *
* -2.700 *
* -1.200 *
* 0.300 *
* 1.800 *
* 3.300 *
* 4.800 *
* 6.300 *
* 7.800 *
* 9.300 *
*****
pounds
```

2

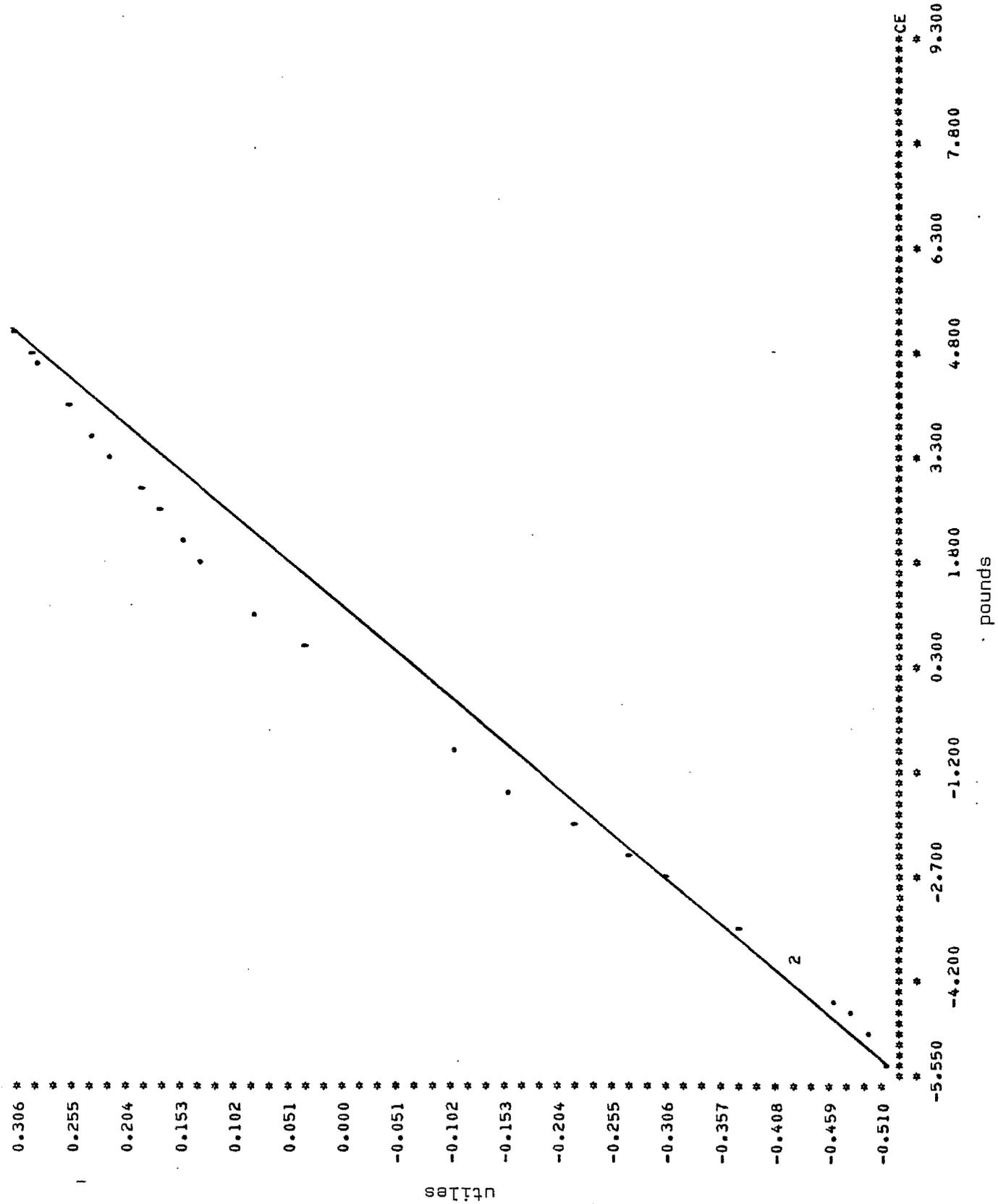
utils



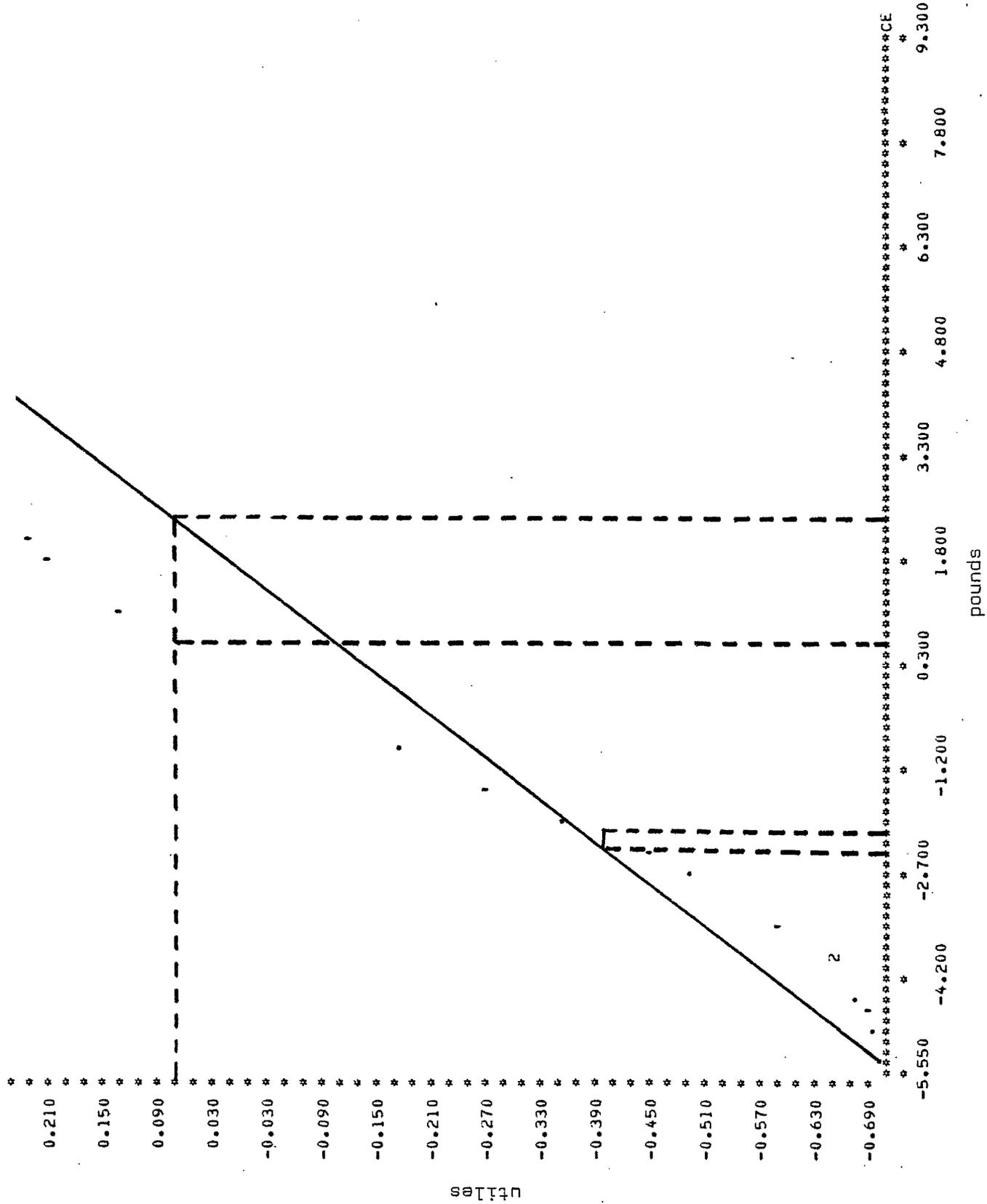
Subject 106



Subject 057



Subject 063



```
*
* 0.252 *
* *
* *
* 0.198 *
* *
* *
* 0.144 *
* *
* *
* 0.090 *
* *
* *
* 0.036 *
* *
* *
* -0.018 *
* *
* *
* -0.072 *
* *
* *
* -0.126 *
* *
* *
* -0.180 *
* *
* *
* -0.234 *
* *
* *
* -0.288 *
* *
* *
* -0.342 *
* *
* *
* -0.396 *
* *
* *
* -0.450 *
* *
* *
* -0.504 *
* *
* *
* -0.558 *
* *
* *
* -5.550 *
* *
* *
* -4.200 *
* *
* *
* -2.700 *
* *
* *
* -1.200 *
* *
* *
* 0.300 *
* *
* *
* 1.800 *
* *
* *
* 3.300 *
* *
* *
* 4.800 *
* *
* *
* 6.300 *
* *
* *
* 7.800 *
* *
* *
* 9.300 *
* *
* *
* *****CE
```

pounds

titles

