HEAT TRANSFER IN A PIPE
WITH AN ABRUPT CHANGE
OF SECTION

The Effect of an Abrupt Change of Section on the Coefficient of Heat Transfer between a Pipe and Water flowing through it.

by

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CONTENTS

INTRODUCTION

Heat may be transferred by the three distinct mechanisms of conduction, convection and radiation.

The problem to be dealt with in this thesis is essentially a study in forced convection, since it is concerned with the transfer of heat between a pipe and a fluid flowing in it. Secondary considerations, such as heat transferred through the pipe wall, depend on conduction, but no consideration need be given to radiation effects which are negligible.

The flow of a fluid in a pipe can be either turbulent or laminar depending upon the value of the dimensionless Reynolds Number $\frac{\rho VD}{\mu}$, turbulent flow existing at the higher Reynolds Numbers.

In practice, most heat transfer equipment is operated under turbulent flow conditions and for this reason only turbulent flow will be considered in this thesis.

The quantity of heat transferred from a pipe to a fluid flowing in it depends to a large extent on the degree of turbulence of the fluid. If the pipe is long enough, a fully developed condition will be reached which will be such that the degree and nature of the turbulence will no longer vary, and the velocity distribution across the pipe will be the same at all cross-sections. The turbulence existing under such conditions will be called "Normal Turbulence". If, on the other hand, the fluid has just passed an entry section, a valve, a change of section or a bend, then a certain increase in turbulence will be noticed. Such turbulence will be called "Excess Turbulence". If no further irregularity in the pipe is encountered, then the excess turbulence will gradually die out and normal turbulence will be established at some distance downstream.

A great deal of work has been done on the transfer of heat between a pipe and a fluid flowing in it from both experimental and theoretical points of view. This work can be divided into three stages.

In the earliest work it was not appreciated that the entry conditions to an experimental pipe had any effect on the heat transfer between the pipe and the fluid. The experimenters did not, therefore, make any attempt to ensure normal turbulent conditions in the test sections of their pipes. They simply measured an average value of heat transfer coefficient for the whole pipe and thus missed any effect of excess turbulence at entry.

The second stage of the work began after it had been theoretically demonstrated, in 1921, that heat transfer coefficients depended on starting conditions in the pipe. "Calming sections" were now fitted to the experimental pipes to ensure normal turbulence in the test sections. The heat transfer coefficients measured were still average values, but were now slightly lower than before, since no effect of excess turbulence was included. It still remained to be shown, however, that this average value of heat transfer coefficient was equal to the local value at all points on the normal turbulent flow section.

The most recent stage deals with the measurement of local heat transfer coefficients. The great advantage of this method of measuring heat transfer is that the variation of the heat transfer coefficient along the pipe can be found. The effect of the entry conditions can be measured and the constancy of the local coefficients, after normal turbulence has been established, can be demonstrated.

In practice, a considerable proportion of the heat transfer surface in a condenser or heat exchanger may be in the form of pipes under excess turbulent conditions. Earlier research workers who thought in terms of average heat transfer coefficients for whole heat exchangers, were not equipped to deal with the problem of the variation of heat transfer throughout the exchanger.

With this problem in mind, an extensive research programme has been drawn up by the Heat Division of the Mechanical Engineering Research Laboratory, to study the effects of bends, elbows and changes of section on the heat transfer between a pipe and a fluid flowing in it.

The work described in this thesis, the greater part of which was carried out in a temporary engineering research annexe of the University of Glasgow, comprises the first part of this research programme. It is concerned with the study of the effect of an abrupt change of section on the heat transferred from a pipe to water flowing through it.

Before the experiments were completed, it became necessary for the University to evacuate the temporary research annexe. In order that the experiments might be carried on and completed, the apparatus was then moved to the Mechanical Engineering Research Laboratory at East Kilbride.

The author is indebted to Messrs. E. J. Le Fevre and A. J. Ede of M.E.R.L. for their invaluable suggestions, and to Professor James Small for his co-operation in providing laboratory space in the University for this project.

CHAPTER 1.

THE NUSSELT EQUATION.

A review of the literature on the measurement of heat transfer between pipes and fluids flowing in them has shown that the problem has two distinct aspects which can be considered separately. The first of these is dealt with in this chapter and concerns the development of an empirical equation from which heat transfer for normal turbulent flow can be evaluated. Chapter 2 deals with the effects of excess turbulence due to entry conditions on the heat transfer in pipes.

A theoretical equation proposed by Nusselt in 1910 has been the basis of nearly all correlations of results in this field. Many attempts have been made by research workers to find the "best" values of the constants in this equation.

1.1 Early Experiments.

The earliest work of importance was carried out by Stanton[28] in 1897. By modern standards his apparatus was very simple, consisting of two concentric vertical pipes. Cold water flowing down the inner pipe was heated by hot water flowing down the annulus. He assumed that the heat transmitted to the cold water could be given by

$$dH = K.\ 2\pi r\ \ dL\ \phi\ [T_o,\ t,\ (T_o - t),\ p,\ v,\ r]\ \ldots\ldots(1.1)$$

where

T_o = temperature at inner surface of pipe,

t = mean temperature of water in pipe at any cross section,

v = velocity of the water,

p = pressure of the water,

r = radius of the pipe,

L = length of the pipe,

K and ϕ were to be determined by experiment.

From his experiments Stanton showed that

(a) the heat transmitted for a given range of temperature was nearly proportional to the velocity of the water,

(b) the heat transmitted was proportional to $(T_o - t)$,

(c) the heat transmitted increased with increased water temperature.

In making these three statements, Stanton made the first contribution towards the present day theory of heat transfer in pipes.

This theory states that (comparing with a, b and c above):-

(a) the heat transmitted is proportional to velocity$^{0.8}$,

(b) the heat transmitted is proportional to $(T_0 - t)$,

(c) the heat transmitted is proportional to $(\frac{1}{\mu})^{0.4}$, where μ is the viscosity of the water which decreases with increase of water temperature.

Apparatus of a similar kind was used by Jordan[12] in 1909. In this case, air flowed through the inner pipe and water through the annulus, the water cooling the air. Jordan used Stanton's result that heat transmitted was proportional to the temperature difference between pipe wall and water $(T_0 - t)$, and went on to divide the quantity of heat transferred per unit surface area of pipe per unit time by $(T_0 - t)$. Thus, for the first time, was formed the now well known "heat transfer coefficient".

Jordan supported Stanton's assertion that the heat transmitted was nearly proportional to velocity.

These two sets of experiments, although giving a lead in the work, made no attempt at a quantitative measurement of the effects of velocity, viscosity, thermal conductivity, density, etc. on the heat transfer to a fluid flowing in a pipe. There now appeared to be a need for an equation which would attempt to correlate these variables.

1.2 Nusselt's Theoretical Contribution.

A very important advance in the theory of heat transmission was made by Nusselt[22] in 1910. He derived the equation:-

$$\frac{hD}{k} = \phi \left\{ \frac{\rho VD}{\mu} \; , \; \frac{C_p \mu}{k} \right\} \ldots\ldots\ldots\ldots (1.2)$$

wherein the functions relating the first group with each of the other two can be determined only by experiment.

This equation contains three dimensionless groups

1. $\frac{hD}{k}$ - The Nusselt Number - Nu
2. $\frac{\rho VD}{\mu}$ - The Reynolds Number - Re
3. $\frac{C_p\mu}{k}$ - The Prandtl Number - Pr

1.3 Derivation of the Nusselt Equation.

The heat transfer coefficient h for forced convection of heat inside a pipe depends on certain properties of the fluid and of the pipe. The heat is conducted through the fluid film, hence k should be a factor. Because the film thickness depends on the mass velocity which is the product of ρ and V, the pipe diameter D and the fluid viscosity these factors should affect h. Since for a given heat flow, the specific heat of the fluid affects the bulk temperature of the stream, C_p also should be considered.

Therefore let h be expressed by a series of terms of the form:

$$V^{a_1} D^{b1} \mu^{f1} k^{j1} \rho^{m1} C_p^{n1} + V^{a2} D^{b2} \mu^{f2} k^{j2} \rho^{m2} C_p^{n2} + \text{----}$$

The following symbols will be used for the basic units

H for heat energy

T for time

L for length

M for mass

Θ for temperature.

Then, equating the dimensions of h with those of the above expression

$$HT^{-1}L^{-2}\Theta^{-1} = (T^{-1}L)^{a} L^{b}(MT^{-1}L^{-1})^{f}(HT^{-1}L^{-1}\Theta^{-1})^{j}(ML^{-3})^{m}(HM^{-1}\Theta^{-1})^{n}$$

Identifying the sum of the exponents

$$
\begin{array}{lrl}
\text{For H} & 1 = & j + n \\
\text{T} & -1 = & -a - f - j \\
\text{L} & -2 = & a + b - f - j - 3m \\
\text{M} & 0 = & f + m - n \\
\Theta & -1 = & -j - n
\end{array}
$$

and therefore

$$a = m$$

$$b = m - 1$$

$$f = n - m$$

$$j = 1 - n$$

By substituting into the original expression:-
h may be expressed by a series of terms of the form:-

$$\frac{k}{D}\left(\frac{\rho VD}{\mu}\right)^{n_1}\left(\frac{C_p\mu}{k}\right)^{m_1} + \frac{k}{D}\left(\frac{\rho VD}{\mu}\right)^{n_2}\left(\frac{C_p\mu}{k}\right)^{m_2} + \ldots\ldots\ldots$$

Hence $\frac{hD}{k}$ is a function of $\frac{\rho VD}{\mu}$, $\frac{C_p\mu}{k}$ and the form of this function is not revealed.

Experimental data have shown that there is some justification for assuming that

$n_1 = n_2 = n_3$ --- $= n$ and $m_1 = m_2 = m_3$ ------- $= m$

This being the case, the equation can be written

$$\frac{hD}{k} = C\left(\frac{\rho VD}{\mu}\right)^{n}\left(\frac{C_p\mu}{k}\right)^{m}$$

or $\quad Nu = C(Re)^n (Pr)^m$ ....(1.3)

This equation will be known as the "Nusselt Equation".

The constants C, n and m can be determined only by experiment, as this theoretical treatment cannot attempt to define their values.

Nusselt first assumed n and m equal, since his early data on gases were not sufficient to indicate different values for these exponents. McAdams[16] analysed this data of Nusselt and plotted heat transfer coefficient against mass velocity (ρV) to logarithmic co-ordinates. The slope of the curve, and hence the value of n, was found to be 0.8.

1.4 Experimental Determinations of the Constants C, n and m.

In a paper written in 1928, Morris and Whitman[21] referred to the Nusselt Equation. They pointed out the limitations of dimensional analysis in forming such an equation: e.g. it throws no light on the validity of the assumption that the variables chosen are those important to the problem and it gives no hint of the functional relationship between the various dimensionless groups. They went on to state that the great importance of the analysis lies in the inferences it allows regarding variables which are constant for a particular set of experiments. For instance, in most tests the pipe diameter is constant throughout and the density of a liquid varies little. Thus no independent determination of their effects is possible, yet dimensional analysis can predict the effect of each separately.

In their experiments on oils and water it was shown that, for heating, the value of m was 0.37. For cooling, approximately the same value was obtained but the results were less consistent. All the properties of the fluid were taken at the mean temperature of the mass of the fluid.

Eagle and Ferguson,[11] in 1930, carried out a very important series of experiments to which reference will be made more fully in Chapter 2. They did not choose to correlate their data by an equation of the Nusselt form. In a written communication on this paper, Sherwood noted that one of the curves given might be closely expressed by the Nusselt Equation having a value of 0.85 for n. Nusselt[24] also analysed the data of Eagle and Ferguson and gave the values 0.819 and 0.365 for n and m respectively.

In 1930 Dittus and Boelter,[7] after studying the data of several investigators on air and water and the data of Morris and Whitman on heating and cooling oils, proposed the following forms of the Nusselt Equation.

For Heating $Nu = 0.0243\ (Re)^{0.8}(Pr)^{0.4}$ ....(1.4)

For Cooling $Nu = 0.0265\ (Re)^{0.8}(Pr)^{0.3}$ ....(1.5)

New data on heat transfer to five different liquids were presented in 1932 by Sherwood and Petrie[25] (Fig.1). The results were shown to be in excellent agreement with equation (1.4) of Dittus and Boelter.

Lawrence and Sherwood,[15] in 1931, obtained values of 0.056, 0.7 and 0.5 for C, n and m respectively in their experiments on heating water in horizontal pipes.

By correlating data of Morris and Whitman, Sherwood and Petrie and others, Colburn,[6] in 1933, proposed the following equation.

$$Nu = 0.023\ (Re)^{0.8}(Pr)^{0.33} \qquad \text{...............(1.6)}$$

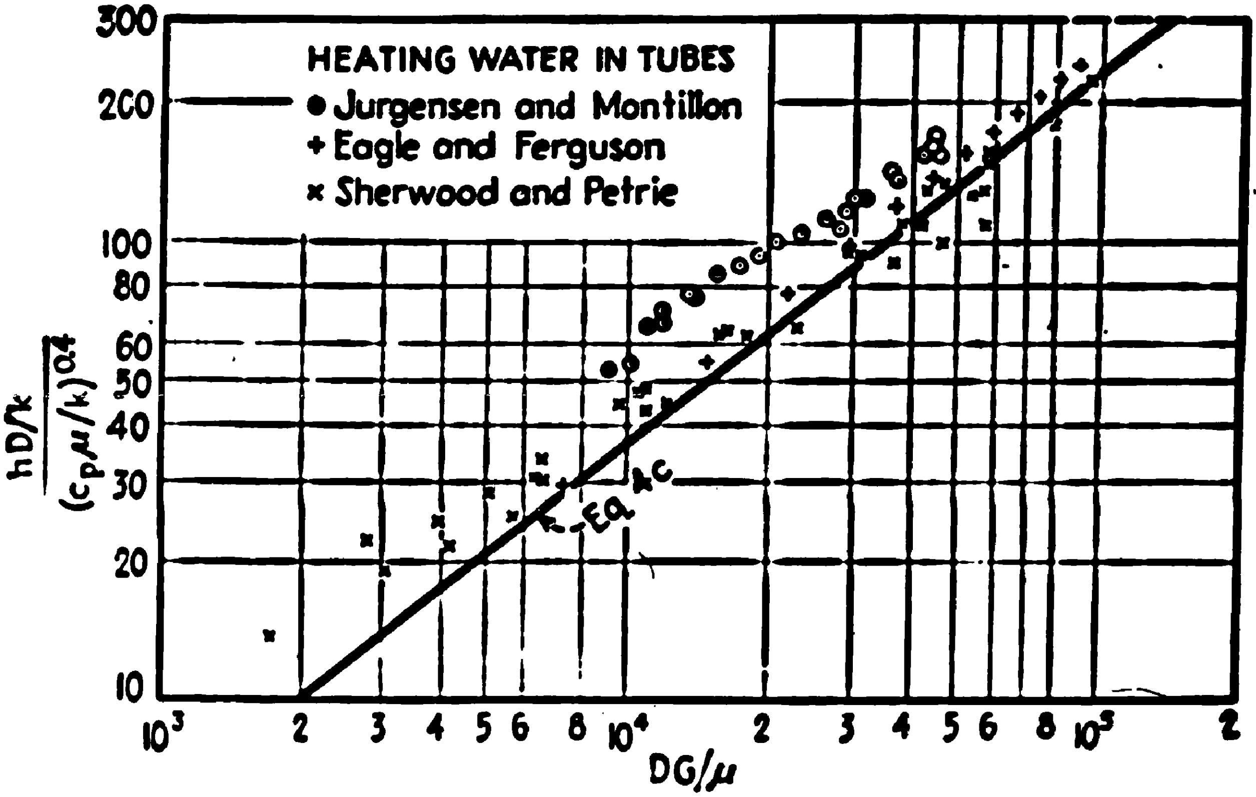


FIG 1 DATA BY M^c ADAMS

In a paper by Jurgensen and Montillon[13] (1935), the value of m was accepted as 0.4 and an experimental determination was made of n and C. The results obtained, which are shown in Fig. 1, were compared with the experimental results of Lawrence and Sherwood and with the correlations of Dittus and Boelter, and McAdams. The comparison was as follows:-

	C	n
Jurgensen and Montillon	0.0983	0.69
Lawrence and Sherwood	0.0583	0.71
Dittus and Boelter	0.0243	0.80
McAdams	0.023	0.80

It will be noticed that good agreement exists between the correlations of Dittus and Boelter, Colburn and McAdams but that results of individual experimenters vary widely. It can therefore be concluded that the experiments taken singly did not cover a wide enough range to give reliable values of C, n and m. Correlations of the results of many experiments, however, provided more consistent constants in the Nusselt Equation.

1.5 The Effect of Temperature Difference between Pipe Wall and Fluid.

In 1936 Sieder and Tate[26] pointed out that there was a need for a single heat transfer equation which would be applicable to both heating and cooling. Previous formulae had taken no account of radial temperature gradient, and hence radial viscosity gradient in the fluid.

The correlations available at that time fell into two classes, one using main stream properties and the other using film properties. The former resulted in two curves, one for heating and another for cooling (cf. Dittus and Boelter equations 1.4 and 1.5) while other effects of temperature difference between pipe wall and fluid were inadequately taken into account. The use of film properties resulted in a single curve for heating and cooling, only if the data under consideration were taken from fluids with nearly the same temperature coefficient of voscosity.

Sieder and Tate proposed an equation using main stream properties but introduced a new dimensionless group $\mu a/\mu w$, where μa is the viscosity of the fluid at its main stream temperature and μw the viscosity at the wall temperature.

The effect is not noticed for liquids having viscosities less than twice that of water. For liquids with viscosities greater than twice that of water they proposed:-

$$Nu = 0.027(Re)^{0.8}(Pr)^{1/3} \left(\frac{\mu a}{\mu w}\right)^{0.14} \quad \ldots\ldots\ldots(1.7)$$

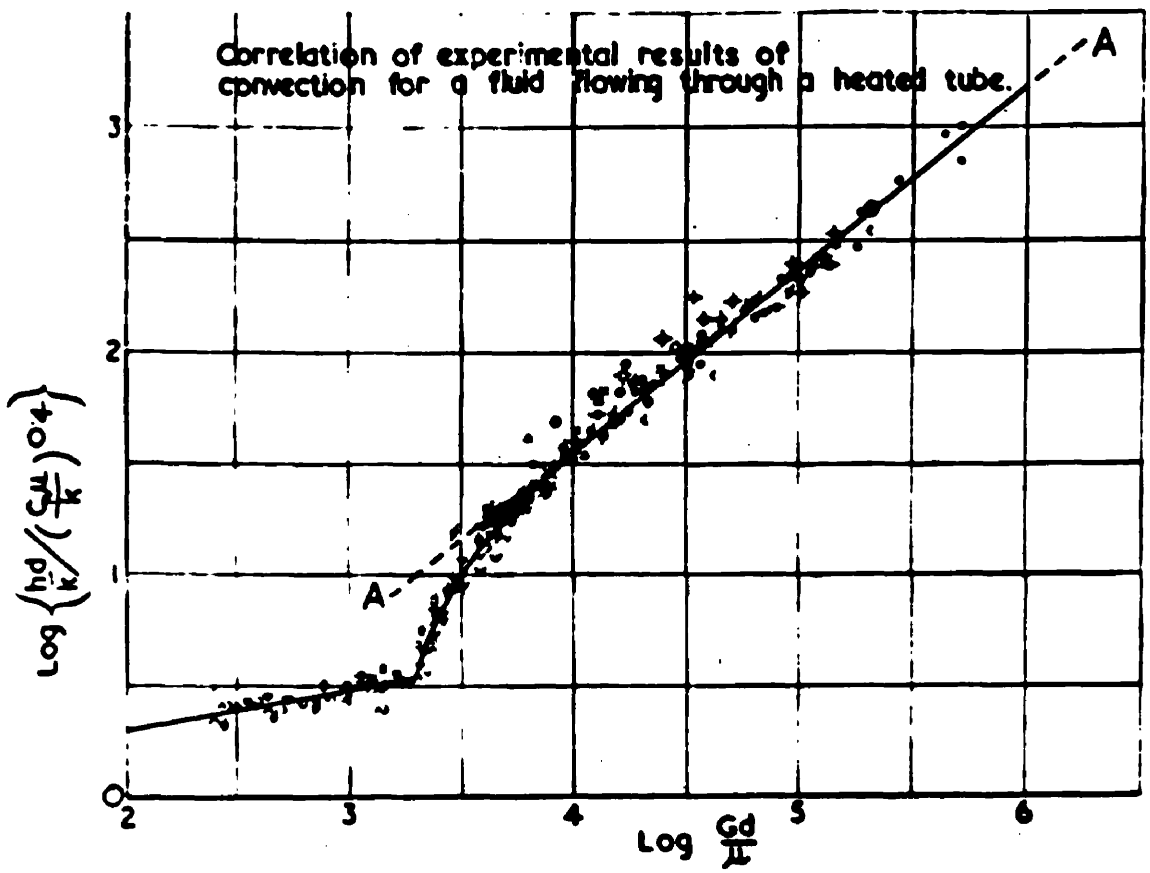


FIG 2 DATA BY BROWN,
FISHENDEN AND SAUNDERS

This equation only holds for Reynolds Numbers greater than 10,000.

1.6 Recent Correlations

The two most recent correlations given by McAdams[17] (1942) and Brown, Fishenden and Saunders[3] (1948) both arrive at the same equation

$$Nu = 0.023\ (Re)^{0.8}(Pr)^{0.4} \qquad \text{.................(1.8)}$$

McAdams states that this equation applies only for Reynolds Numbers greater than 2100 and for fluids with viscosities less than twice that of water. Physical properties of the fluid are evaluated at the convenient bulk fluid temperature.

Brown, Fishenden and Saunders maintain that the same equation applies only for Reynolds Numbers greater than 10,000 and for fluids with viscosities less than twice that of water. Their values of ${}^{Nu}/Pr^{0.4}$ plotted logarithmically against Reynolds Number are shown in Fig.2. It will be seen that for Reynolds Numbers greater than 10,000 the graph is a straight line whose equation is $Nu = 0.023(Re)^{0.8}(Pr)^{0.4}$. At lower Reynolds Numbers the graph lies below this straight line.

Brown, Fishenden and Saunders also state that, for gases and liquids with viscosities less than twice that of water, the heat transfer coefficient is the same whether the fluid is being heated or cooled; i.e. it is

Author	Date	C	n	m	Heating H or cooling C	Remarks
Nusselt	1910	-	0.80	-	H	Value given by McAdams
Morris & Whitman	1928	-	0.80	0.37	H and C	m less consistent for cooling
Eagle & Ferguson	1930	-	0.819	0.365	H	Values given by Nusselt
Dittus & Boelter	1930	0.0243	0.80	0.40	H	Correlation
Dittus & Boelter	1930	0.0265	0.80	0.30	C	Correlation
Lawrence & Sherwood	1931	0.056	0.70	0.50	H	Individual test
Sherwood & Petrie	1932	0.024	0.80	0.40	H	Test on 5 liquids
Colburn	1933	0.023	0.80	0.333	H and C	Correlation
Jurgensen & Montillon	1935	0.0983	0.69	0.40	H	Individual test
Sieder & Tate	1936	$0.027\left(\frac{\mu_a}{\mu_w}\right)^{0.14}$	0.80	0.333	H and C	Correlation
McAdams	1942	0.023	0.80	0.40	H and C	Correlation
Brown, Fishenden and Saunders	1948	0.023	0.80	0.40	H and C	Correlation

TABLE 1

Values of C, n and m in the equation $Nu = C\,(Re)^n\,(Pr)^m$

independent of heat input to the fluid. For liquids of high viscosity, the heat transfer coefficient is affected by variation of heat input. For this condition they support the Sieder and Tate form of the Nusselt equation (1.7).

Equation (1.7), when compared with equation (1.8) for the flow of gases and of liquids of low viscosity, shows good agreement only for values of Prandtl Number less than 40 when there is a small temperature difference between the fluid and the surface.

1.7 Summary.

A summary of the values found for the constants in the Nusselt equation is given in Table I.

CHAPTER 2

The Influence of Entry Conditions and Pipe Length on the Heat Transfer Coefficient

In general, any excess turbulence which exists in the entrance section of a pipe will cause an increase in heat transfer coefficient. The degree of excess turbulence will depend on the entry conditions, e.g. sharp-edged, right-angle bend, abrupt enlargement etc. and will diminish steadily with distance along the pipe until finally normal turbulent conditions will be established. In a similar manner, the heat transfer coefficient will drop steadily from a maximum and will attain a constant value beyond the point where normal turbulence is established. This constant value will be independent of the entry conditions.

The first mention of this effect was made by Nusselt[23] in 1917. He modified his equation by incorporating the term $(D/L)^p$ where D/L is the ratio of the diameter to the length of the pipe. His equation then read:-

$$Nu = C(Re)^n(Pr)^m(D/L)^p \qquad \text{..................(2.1)}$$

From his own data he gave p the value of 0.054.

2.1 Theoretical Investigation

In 1921 Latzko[14] devoted a thorough theoretical investigation to this subject. He divided the problem into three parts, each part referring to a particular set of entry conditions.

(a) Fully developed hydrodynamic and thermal fields

This condition is attained when the fluid has passed through a sufficient length of pipe for normal turbulence to be established, and has travelled sufficiently far past the commencement of heating for a fully developed or "normal" temperature distribution to exist across the section.

The heat transfer coefficient (h) is constant and is given by:-

$$h_{min} = 0.0384 \rho V C_p \left(\frac{1}{Re}\right)^{0.25}$$

or in the Nussel form

$$Nu_{min} = 0.0384 (Re)^{0.75} Pr \qquad \ldots\ldots\ldots\ldots (2.2)$$

(b) Fully developed hydrodynamic field, temperature uniform at entrance

This condition is realised by having an entrance section to the pipe which is maintained at the original fluid temperature by suitable heating. The heat transfer coefficient depends on distance along the pipe, falls very quickly from its maximum value and asymptotically approaches a minimum value.

It is given by:-

$$\left.\begin{array}{l} h = 0.0384\ \rho VC_p \left(\frac{1}{Re}\right)^{0.25} (\alpha) \\ \text{or}\quad Nu = 0.0384\ (Re)^{0.75}\ Pr\ (\alpha) \end{array}\right\} \quad \ldots\ldots\ldots\ldots (2.3)$$

$$\text{where}\ \alpha = 1 + 0.1e^{-2.7 \frac{x}{D} \left(\frac{1}{Re}\right)^{0.25}} + 0.9e^{-29.27 \frac{x}{D} \left(\frac{1}{Re}\right)^{0.25}} - 0.023e^{-31.96 \frac{x}{D} \left(\frac{1}{Re}\right)^{0.25}}$$

The ratio $\frac{x}{D}$ is the distance along the pipe measured in diameters. Equation (2.3) reduces to equation (2.2) for $x = \infty$.

(c) <u>Uniform velocity and temperature distributions across the entrance section</u>

This condition may be produced by having a bellmouth entrance immediately before the heating section. In this case two equations are necessary to define the heat transfer coefficient.

1. <u>For partly established turbulence</u>

$$\left.\begin{array}{l} h = \rho VC_p \left(\frac{1}{Re}\right)^{0.25} (\beta) \\ \text{or}\quad Nu = (Re)^{0.75}\ Pr\ (\beta) \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

where β is a very complicated function of χ

and $\chi = \left(\frac{1}{Re}\right)^{0.2} \left(\frac{x}{D}\right)^{0.8}$

2. <u>For fully established turbulence</u>

$$\left.\begin{aligned} h &= 0.0346\,\rho V\, C_p \left(\frac{1}{Re}\right)^{0.25} (\Upsilon) \\ \text{or}\quad Nu &= 0.0346\,(Re)^{0.75}\, Pr\ (\Upsilon) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2.5)$$

$$\text{where } \Upsilon = \frac{0.969e^{-m_1 x} + 0.038e^{-m_2 x}}{0.837e^{-m_1 x} + 0.0068e^{-m_2 x}}$$

$$m_1 = 0.151\,\frac{1}{D}\left(\frac{1}{Re}\right)^{0.25} \qquad m_2 = 2.844\,\frac{1}{D}\left(\frac{1}{Re}\right)^{0.25}$$

Equation (2.5) also reduces to equation (2.2) for $x = \infty$.

Latzko, in summing up, stated that "the great differences in results of the individual experimental works are now understandable".

2.2 Average Heat Transfer Coefficients

Definition

The average heat transfer coefficient h_{av} for a given length of pipe is defined to be the total amount of heat transferred between that pipe and a fluid flowing in it per unit time q divided by the product of the surface area of the pipe A and the average temperature difference between the pipe and fluid ΔT

$$\text{or}\quad h_{av} = \frac{q}{A\,\Delta T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

Until recently, experimental techniques were such that this was the only coefficient obtainable. It was noticed that, under exactly similar conditions, heat transfer coefficients were higher for short pipes than for longer ones; i.e. the influence of entry conditions was more marked for the short pipes, since the excess turbulent region extended for a larger proportion of the pipe length. Only for a pipe of infinite length could an average value of heat transfer coefficient be assumed to be equal to the local value for normal turbulent flow. This average value for a pipe of infinite length will be termed h_{∞}.

No effect of entry conditions or pipe length can be measured by the average heat transfer method using only one pipe, since for a given set of conditions only one value of h_{av} can be found, its value depending on the length of the pipe. It is therefore necessary to carry out a series of tests on different lengths of pipe to obtain a measure of this effect.

In describing their experiments, Morris and Whitman[21] made the following statement. "Another dimensionless group sometimes introduced into the Nusselt equation is the ratio of the clear length of the pipe to its diameter. There is no question that the excessive turbulence near a sudden contraction in cross section or an elbow, for instance, causes a local increase in coefficient. Hence a higher value of average coefficient would be obtained in a short pipe than in a long one".

In their experiments only one pipe was used, so that no attempt could be made to measure the effect of $L/_D$. They therefore decided to avoid end effects as far as possible by extending the pipe for 20 diameters before the heating section, i.e. by introducing a "calming section". It was therefore concluded that average heat transfer coefficients as calculated might be considered as applying to pipes of infinite length.

This conclusion was unjustified, since although they had produced a fully developed hydrodynamic flow, they had not produced a fully developed thermal field at entry. The conditions were in fact equivalent to Case b of Latzko, for which case h_{av}, for a finite length of pipe, is greater than h_{∞} .

In 1931 Lawrence and Sherwood[15] made a fresh investigation into the influence of pipe length on heat transfer coefficient. In their experiments, heat transfer was investigated during the heating of water flowing along horizontal pipes of four different lengths, these lengths varying from 59 to 224 diameters. To simulate conditions in a commercial condenser, no calming sections were used. No effect of pipe length on heat transfer was discovered. Thus, for the lengths of pipe tested, this method of measuring average coefficients did not show up any effect of pipe length. If, however, Lawrence and Sherwood had extended the range of pipe lengths to one of about 20 diameters, then a marked increase in the average heat transfer coefficient would have been found.

In 1931 Nusselt[24] analysed data of Burbach, and in his equation

$$Nu = C(Re)^n \, (Pr)^m \left(\frac{D}{L} \right)^p \quad \ldots\ldots\ldots\ldots\ldots(2.7)$$

gave values of 0.764, 0.355 and 0.0552 for n, m and p respectively. This value of p compares well with that given by him in 1917.

No limit was put on the length of pipe to which this equation was intended to apply. Obviously there should be some limit, since at $L = \infty$ $Nu = 0$, or $h = 0$.

2.3 Local Heat Transfer Coefficients

Definition

The local heat transfer coefficient may be defined as the average heat transfer coefficient for an infinitely short length of pipe. The ability to measure this type of coefficient immediately allows one to form a picture of the distribution of heat transfer coefficient along a pipe. The effect of entry conditions and pipe length can then be estimated from experiments carried out on one pipe.

Whether or not a local coefficient can be measured depends largely on the method of pipe heating. It is essential that a measurement can be made of heat flow between pipe and fluid per unit surface area of pipe for all positions on thepipe. The only method of heating so far used which allows for this measurement to be made accurately, is that which was used by Eagle and Ferguson[11]. They passed a high low-voltage alternating current through the pipe and measured the heat input per unit length by I^2R, where I was the current flowing and R the electrical resistance per unit length of pipe.

If a pipe is very carefully manufactured, then the cross-sectional area of material can be made constant throughout its length, so that the electrical resistance, and hence the heat input per unit length, will also be constant for the whole pipe. Under these conditions, the heat input per unit surface area of pipe and unit time is constant and is equal to the total heat input per unit time divided by the total surface area of the pipe. It then only remains to divide this quantity by the temperature difference between pipe and fluid at any point, to determine the local heat transfer coefficient at that point.

The more common method of pipe heating used in measuring local heat transfer coefficient is that of steam-jacketing. Here, in order to measure heat input per unit surface area of pipe, the steam jacket is divided into a large number of small compartments from each of which condensate can be collected. The quantities of condensate collected are then measures of the heat inputs to their corresponding sections of pipe.

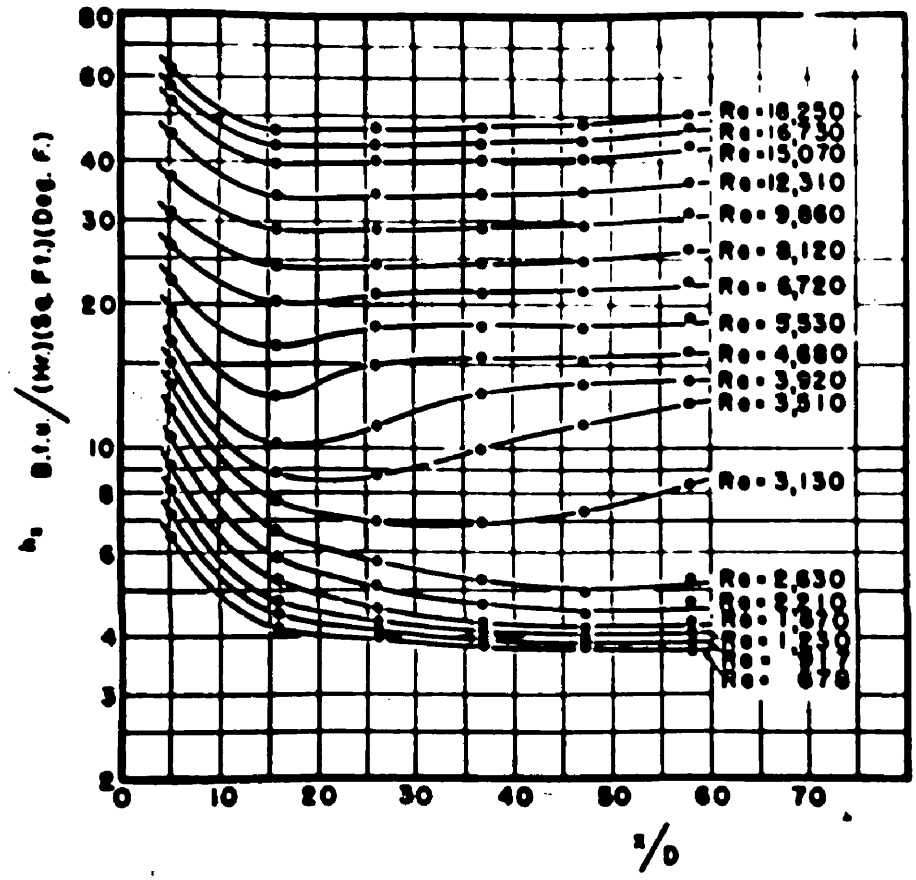
This method, at best, must be only approximate since no matter how short a steam compartment may be, it is of finite length. Hence heat transfer coefficients, measured one for each compartment, will be average values over the lengths of the compartments. The shorter the compartments the better will be the approximation to the true values of local heat transfer coefficients.

In their experiments, Eagle and Ferguson[11] used water as the working fluid flowing through a $\frac{3}{4}$ inch diameter pipe 15 feet long. Only the last 6 feet 3 inches of the pipe was heated, so that there was a calming section 140 diameters in length. Thus, as for the experiments of Morris and Whitman, the conditions corresponded to those for Case b of Latzko. A local heat transfer coefficient was measured at 13½ inches from each end of the heated section of the pipe. In all cases the coefficient measured near the inlet end of the pipe was slightly larger than that near the outlet end, showing that although the inlet coefficient was measured 18 diameters after commencement of heating, there was still some slight inlet thermal effect.

It is unfortunate that Eagle and Ferguson did not take advantage of their improved technique to measure the value of local heat transfer coefficient at more than two positions on the pipe. Their results would have been of greater value had they included the distribution of local heat transfer coefficient along the pipe under their own entry conditions (i.e. with a calming section) and had then repeated the tests without a calming section.

2.4 The Distribution of Local Heat Transfer Coefficient along a Pipe

It was not until quite recently that any attempt was made to measure this distribution of local heat transfer coefficient. In 1948 Cholette[5] presented his results on the heat transferred to air flowing in tubes. His apparatus consisted of a multi-tubular heat exchanger designed to obtain local, as well as average, heat transfer coefficients. The test section contained



VARIATION OF THE LOCAL COEFFICIENT h_x WITH POSITION x

FIG 3 DATA BY CHOLETTE

151 steam heated copper tubes $12\frac{1}{4}$ inches long and 0.19 inches internal diameter. The steam space was divided into six compartments from which condensate could be collected separately, by placing baffles at 2 inch intervals. Hence, what Cholette termed "local" coefficients were in fact average coefficients for pipe sections of 10.5 diameters in length.

From his experimental results he derived an equation indicating the effect of $^{L}/_{D}$.

$$\left.\begin{aligned} h_L &= 0.04\rho V\, C_p \left(\frac{1}{Re}\right)^{0.2} \left(\frac{L}{D}\right)^{-0.1} \\ \text{OR}\quad Nu_L &= 0.04(Re)^{0.8}\, Pr \left(\frac{L}{D}\right)^{-0.1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(2.8)$$

where h_L and Nu_L represent local values of h and Nu.

This equation is of the same form as equations (2.1) and (2.7) of Nusselt.

It will be noticed that the index of the Prandtl Number in equation (2.8) is unity. This is the case since the Prandtl Number for air is very nearly constant and is given by Brown, Fishenden and Saunders(4) as 0.72 between the temperatures of 0°C. and 500°C.

Fig. 3, taken from Cholette's paper, shows the variation of h_L with distance along the pipe and indicates that the entrance effect does not extend beyond $\frac{L}{D} = 10.5$.

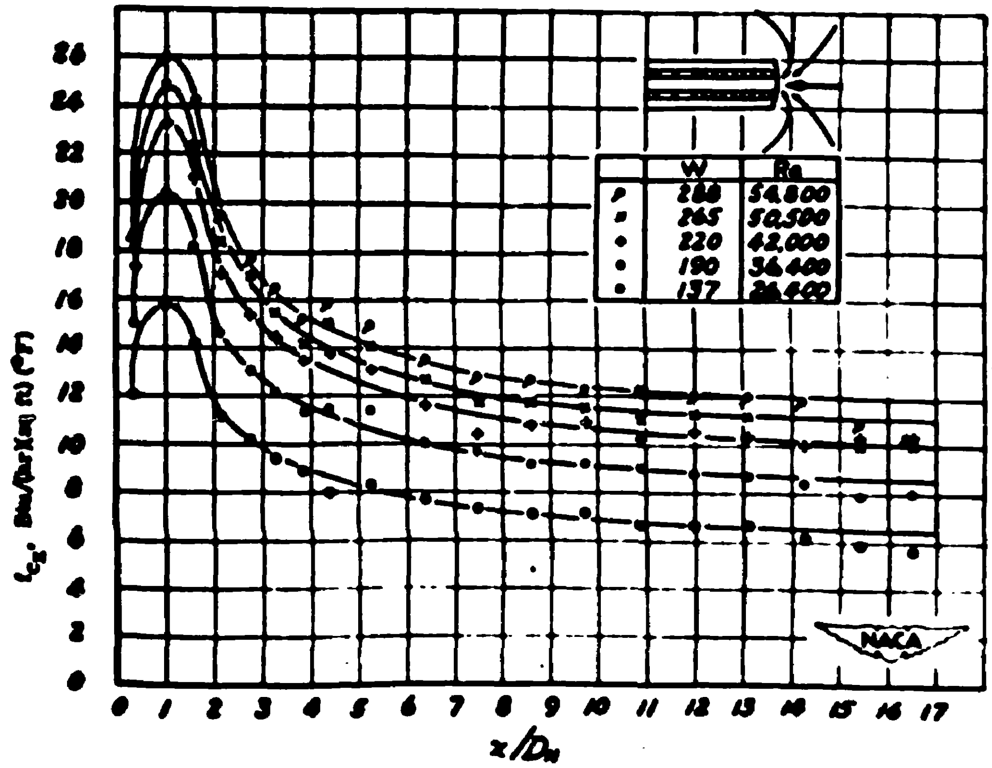
The method of plotting this curve is of interest. The average heat transfer coefficient measured over each 10.5 diameters is called the local coefficient for the mid-point of the section to which it refers. Such an approximation rules out the possibility of measuring any real effect in the first few diameters of the pipe length.

Cholette goes on to show that his value of h_{min} for Reynolds Number = 10,000 and $\frac{L}{D} > 10.5$ is in good agreement with that obtained from Latzko's equation (2.2).

This series of experiments would have been greatly improved had there been a large number of short steam compartments, especially near the entrance to the pipe. Considerably more information would then have been obtained about the variation of local heat transfer coefficient.

A somewhat similar series of experiments was carried out by Boelter, Young and Iversen[2] in 1948. Experimental data on the variation of local heat transfer coefficient in the entrance section of a pipe were presented for sixteen different flow conditions of entering air.

The test pipe was a highly polished seamless steel tube 32 inches long and 1.785 inches internal diameter. Again steam heating was employed, but in this case partitions were brazed to the pipe at approximately 1 inch intervals from the entrance section for the first 8 inches, and then at 2 inch intervals for the remaining length of

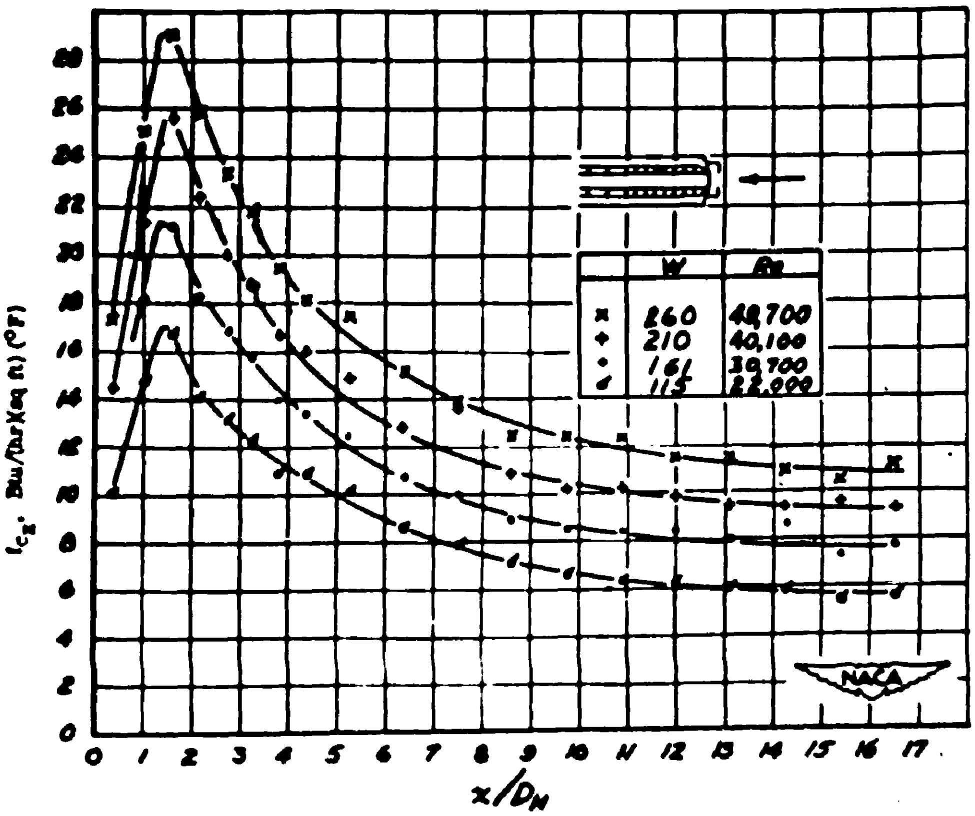


Variation of point unit thermal conductance in circular tube with bare sharp-edge entrance.

FIG 4 DATA BY BOELTER, YOUNG AND IVERSEN pipe. There was therefore reasonable justification for calling the coefficients measured in these experiments local heat transfer coefficients, since they were average values measured over pipe lengths of approximately one half diameter and one diameter.

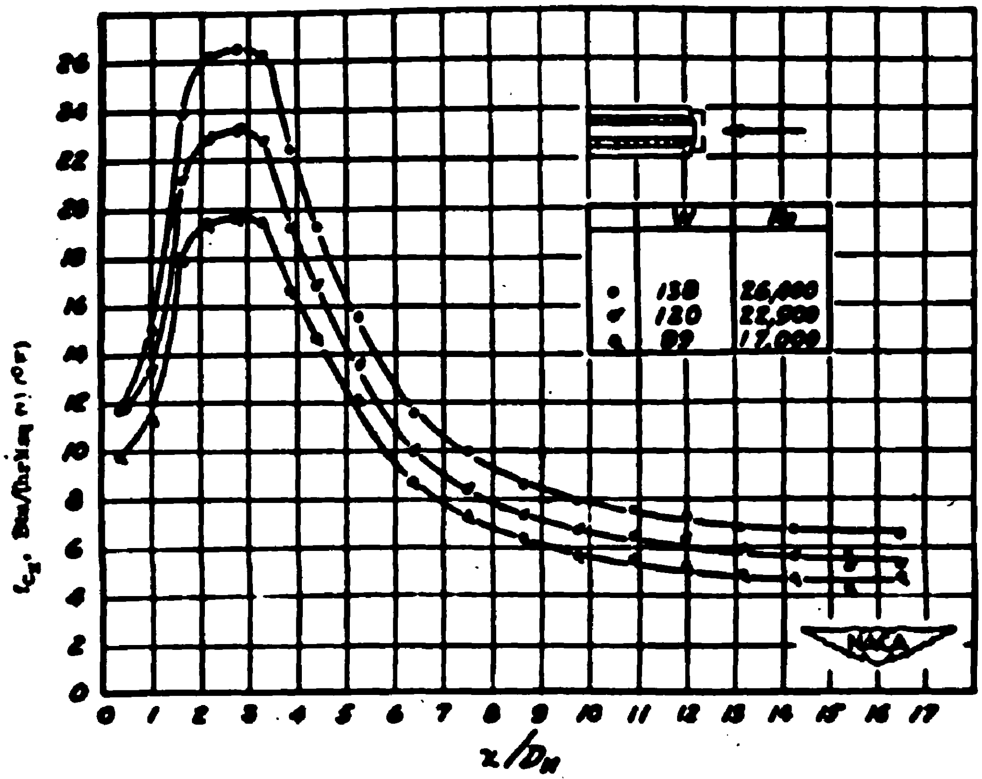
The results showed that the local heat transfer coefficient was large in the entrance section and fell away steadily with increasing L/D, and that its value depended on the type of fluid entrance. The effect in many cases appeared to die out after about 15 diameters, but since the test pipe was only 18 diameters long, insufficient length was provided to demonstrate the constancy of heat transfer coefficient for normal turbulent conditions. It is therefore felt that the test pipe should have been made at least 10 diameters longer.

A large number of extremely useful experimental results were given. In describing Fig. 4, which gives the distribution of local heat transfer coefficient along the pipe for the case of a sharp-edged entrance, Boelter, Young and Iversen said that a stagnant air pocket would be expected to develop at the leading edge for this entrance condition. This would cause an appreciable resistance to heat transfer, i.e. it would cause a reduction in the local value of heat transfer coefficient



Variation of point unit thermal conductance in circular tube with large orifice at entrance.

FIG 5 DATA BY BOELTER, YOUNG AND IVERSEN



Variation of point unit thermal conductance in circular tube with small orifice at entrance.

FIG 6 DATA OF BOELTER, YOUNG AND IVERSEN and the maximum value would occur at some small distance downstream. The shapes of the curves in Fig. 4 indicate that some such phenomenon did occur. A similar phenomenon might be expected to occur for the case of an abrupt contraction in a pipe.

The presence of an orifice at the entrance to the pipe moved the point of maximum local heat transfer coefficient further downstream, (Fig. 5 and 6). This might well be due to an enlarged air pocket. The corresponding case to this might be that of an abrupt enlargement in a pipe.

The authors did not derive an equation in the form of that given by Cholette to show the effect of entry conditions on local heat transfer coefficient. However, they gave the following equation, from which average coefficients may be calculated for $\frac{L}{D} > 5$

$$h_{av} = h_{\infty} \left(1 + K \frac{D}{L}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

where K is a different constant for each entrance condition.

Alad'ev[1] (1951) has carried out the most recent work in this field. In his experiments water flowed through a steam-jacketed pipe 10.2 mm. in internal diameter. The steam compartments were ½ pipe diameter in length near the entrance and the pipe itself was nearly 60 diameters long. It was therefore possible to measure

$\frac{L}{D}$	C	n	m
0.588	0.130	0.70	0.40
4.41	0.043	0.78	0.40
19.6	0.025	0.82	0.40
39.2	0.0156	0.86	0.40

TABLE 2

Values of C, n and m in the equation $Nu = CRe^{n}Pr^{m}$ given by Alad'ev.

reasonably exact local heat transfer coefficients and to demonstrate the constancy of local heat transfer coefficient for normal turbulent flow.

The experimental results were presented in a new form. In the Nusselt equation, the index of the Prandtl Number was assumed to be 0.4 and log $Nu/(Pr)^{0.4}$ was plotted against log Re for different values of L/D. The slopes of these curves then gave the indices of Re for different positions along the pipe. The index was found to increase with increasing $\frac{L}{D}$, while the value of C in the Nusselt equation decreased, as shown in Table 2.

Analysis of the experimental data showed that it was possible to assess the influence of L/D by using an exponential function with a negative index

$$Nu = 0.044\ Re^{0.8}\ Pr^{0.4}\ \left(\frac{L}{D}\right)^{-2.25\ Re^{-0.3}} \quad .....(2.10)$$

This equation holds for $\frac{L}{D} \leqslant 40$

For $\frac{L}{D} > 40$, the local heat transfer coefficient was found to be independent of $\frac{L}{D}$ and was given by

$$Nu = 0.0156\ Re^{0.086}\ Pr^{0.4} \quad .... (2.11)$$

2.5 Summary

The influence of entry conditions and pipe length on the heat transfer coefficient was first mentioned in a paper by Nusselt in 1917. It was probably this paper which stimulated Latzko to make his theoretical investigation of the problem, the results of which are given in his four important equations. As is often the case, theory was far in advance of experimental technique, since at the time of Latzko's investigation only average heat transfer coefficients were being measured and these were not capable of dealing with the problem. Most workers at the time preferred to avoid, rather than measure, the effect. Eagle and Ferguson, in 1930, were the first to measure a local heat transfer coefficient using their greatly improved technique, but they failed to take full advantage of it to measure the distribution of heat transfer coefficient along their pipe.

In recent years, three series of experiments have been carried out to find the effect of entry conditions and pipe length on the local heat transfer coefficients. In 1948 Cholette carried out a series of tests on a steam-heated pipe. His results would have been greatly improved had he used much shorter compartments in his steam jacket so that his assumption of local heat transfer coefficients might have been more accurate. In the same year Boelter, Young and Iversen described their experiments in which they measured the local heat transfer coefficient much more accurately by using very short steam compartments. Their test pipe, however, which was only 18 diameters long, is considered to have been too short to allow normal turbulent flow conditions to be established. Alad'ev, in 1951, avoided these two points of criticism by having short steam compartments and a long experimental pipe. His results are the most informative to date.

CHAPTER 3

Design and Description of Apparatus

3.1 Requirements

The problem, as stated in the title, is to determine the effect of an abrupt change of section on the coefficient of heat transfer between a pipe and a fluid flowing through it. The review of previous work on heat transfer in pipes has indicated that the apparatus should have the following requirements.

(a) It must allow for the accurate measurement of local heat transfer coefficient.

(b) There must be steady conditions of heating, fluid flow and temperature of the fluid.

(c) The experimental pipe must be of sufficient length to allow normal turbulent flow to be established.

(d) A complete picture of the distribution of local heat transfer coefficient along the pipe must be obtainable.

The following limitations were put on the scope of the experiments.

(a) Since the Prandtl Number of a gas, unlike that of a liquid, does not vary appreciably with temperature and since it was the intention to measure the effect of Prandtl Number on the heat transfer coefficient, it was decided to use a liquid as the working fluid. The most convenient liquid was water.

(b) The water flow measurement was to be made by weighing the discharge. This is essentially a positive method of measuring water flow which depends on the accuracy of the weighing machine and the stopwatch employed, both of which can be readily estimated. It avoids the complication of calibrating a flow-meter.

(c) The abrupt change of section in the pipe was to be confined to

1. An abrupt enlargement with diameter increasing in the ratio of 2:1
2. An abrupt contraction with diameter decreasing in the ratio of 2:1

This diameter ratio was chosen as the first of a range of ratios to be considered at some later date.

(d) For convenience, the enlargement and contraction experiments were both to be carried out using the same experimental pipe, the direction of water flow relative to the pipe to be adjusted accordingly.

(The term "experimental pipe" will be used to signify the combined pipe extending on both sides of the change of section).

(e) The maximum Reynolds Number in the smaller pipe was chosen to be 100,000 to conform with the maximum value used in previous experiments.

(f) The experiments were to be concerned with the heating, and not the cooling, of the water.

(g) It was proposed to measure temperatures to the nearest 0.01°C. No greater accuracy of temperature measurement could be expected from available thermocouples and potentiometers.

By developing these initial requirements and limitations a stage further, the following decisions could be made.

(a) A constant heat tank to be used to supply the steady flow of water, since water flow from the mains, or from a pump, would be liable to fluctuation. The available head of water was 15 feet.

(b) The length of pipe on each side of the change of section to be at least 100 diameters, in order to allow for the establishment of normal turbulent flow. It was realised in making this decision, that normal turbulent flow would probably be established well within the first 50 diameters of the pipe length. The remaining 50 diameters were to be used to demonstrate the constancy of the heat transfer coefficient for normal turbulent flow.

(c) Provision to be made for the experimental pipe to be turned round end to end, so that the flow of water, relative to the pipe, might be in either direction.

The design of the complete apparatus was based on the above considerations.

3.2 Method of Pipe Heating

It has been shown in Chapter 2 that the ability to measure local heat transfer coefficients depends on the method of pipe heating. Steam heating, which has been favoured by recent workers, allows only for average values of heat transfer coefficient to be measured over short lengths of pipe. Such values are therefore not true local values. Pipe surface temperature measurements are complicated by the fact that thermocouple leads must emerge through the steam jacket, the pipe itself being very inaccessible.

In what is commonly known as the "Eagle and Ferguson" method of pipe heating, either direct or alternating electric current is passed along the pipe, the heat being generated in the material of the pipe. This method lends itself very well to analytical treatment in the measurement of accurate local heat transfer coefficients, since the heat input per unit length of pipe can be calculated. With the possible exception of easily removable lagging, there is nothing to prevent access being obtained to thermocouples attached to the pipe. Since a calculation can be made of temperature drop through the pipe wall, (Appendix 3) only the outside surface temperatures of the pipe need be measured. A very precise control of the electrical heat input to the pipe can be maintained.

From the points of view of greater accessibility, flexibility of control and accuracy in measuring local heat transfer coefficients, the electrical method of pipe heating was adopted. Direct current was preferred to alternating current, since it provides a more uniform current distribution across the pipe wall and allows for the measurement of potential drops, and therefore heat inputs per unit length of pipe, on a D.C. Potentiometer. For the necessarily low voltage to be used in this method of heating, difficulties arise in measuring accurate heat inputs per unit length, if the heat is supplied by alternating current.

Inherent in this method of pipe heating is the condition that the heat input per unit length of pipe is proportional to the electrical resistance per unit length of pipe. Hence, for a pipe of uniform cross-sectional area throughout its length, the heat input per unit length will be constant. In the proposed experiments, the experimental pipe consists of two lengths of pipe of different diameters. The cross-sectional areas of material in the two pipes can be so arranged to give any desired ratio between the heat inputs per unit length in the two pipes.

Two conditions were considered:-

(a) That both pipes should have the same heat input per unit length,

(b) That both pipes should have the same heat input per unit inside surface area.

For condition (a), both pipes must have the same cross-sectional area of material, whereas for condition (b), the smaller pipe must have a cross-sectional area of material equal to twice that of the larger pipe.

Condition (a) conveniently provides for a linear water temperature rise throughout the whole experimental pipe. For this reason, it was decided to design the pipes to have the same cross-sectional area of material in them.

3.3 Pipe Diameters

The minimum size of experimental pipe consistent with the preliminary assumptions was estimated. The following factors governed this size.

(a) Available head of water = 15 feet

(b) Maximum Reynolds Number in smaller pipe = 100,000

(c) Ratio of pipe diameters = 2:1

(d) Each section of the pipe to be at least 100 diameters in length.

From an estimation of head lost in pipes of different diameters, allowance being made for the abrupt change of section, pipe connections, valves etc. it was found that the smallest suitable pipes were of 1 inch and 2 inch diameters.

The experimental pipe was therefore designed to consist of 9 feet of 1 inch diameter pipe joined with an abrupt change of section to 18 feet of 2 inch diameter pipe, 108 diameters of pipe length being provided on either side of the junction.

3.4 Pipe Material and D.C. Generator

The required maximum heat input to the pipe had to be sufficient to provide the minimum allowable temperature difference between the pipe and the water at the maximum Reynolds Number. This temperature difference, which was proportional to the heat input at a constant Reynolds Number, had to be not less than 1°C, in order that it might be measured to an accuracy of 1%.

The heat input to the pipe could be calculated from the temperature difference between pipe and water and the estimated heat transfer coefficient at the point in question. Unfortunately the minimum temperature difference between pipe and water occurred in the excess turbulent section of the pipe, for which no estimation of heat transfer coefficient could be made at this stage.

The required heat input was therefore based on an estimated heat transfer coefficient for a Reynolds Number of 100,000 in the normal turbulent section of the 1 inch diameter pipe and the somewhat larger assumed temperature difference of 3°C. between pipe and water.

The estimated power required was approximately 30 kW.

The choice of pipe material and D.C. generator to supply this power electrically had to be jointly considered, since the characteristics of the generator and the electrical resistance of the pipe were inter-related by Ohms Law. The choice had therefore to be made between a pipe resistance, and hence pipe material, to suit a given generator and a generator whose current and voltage were suited to the electrical resistance of a given pipe. This choice was finally governed by the availability of large D.C. generators.

A simultaneous decision was made to use brass as the pipe material and a generator with a rating of 5000 amps at 6 volts, since their electrical characteristics were appropriate to one another, and to the conditions of the experiment.

The composition of the pipe material used was

Copper	76%
Zinc	22%
Aluminium	2%

The resistivity of a short sample length of tube of this material was measured. It was then possible to calculate the required cross-sectional area of brass in the experimental pipe.

Cross-sectional area of brass = 1.12 square inches.

∴ Thickness of pipe wall for 1 inch diameter pipe = 0.279 inches

Thickness of pipe wall for 2 inch diameter pipe = 0.165 inches

The 1 inch diameter pipe was made accurately to the specified size, but the outside diameter of the 2 inch diameter pipe was slightly larger than specified, thus providing a cross-sectional area about 3% large. The pipes were tested for uniformity of cross-section by

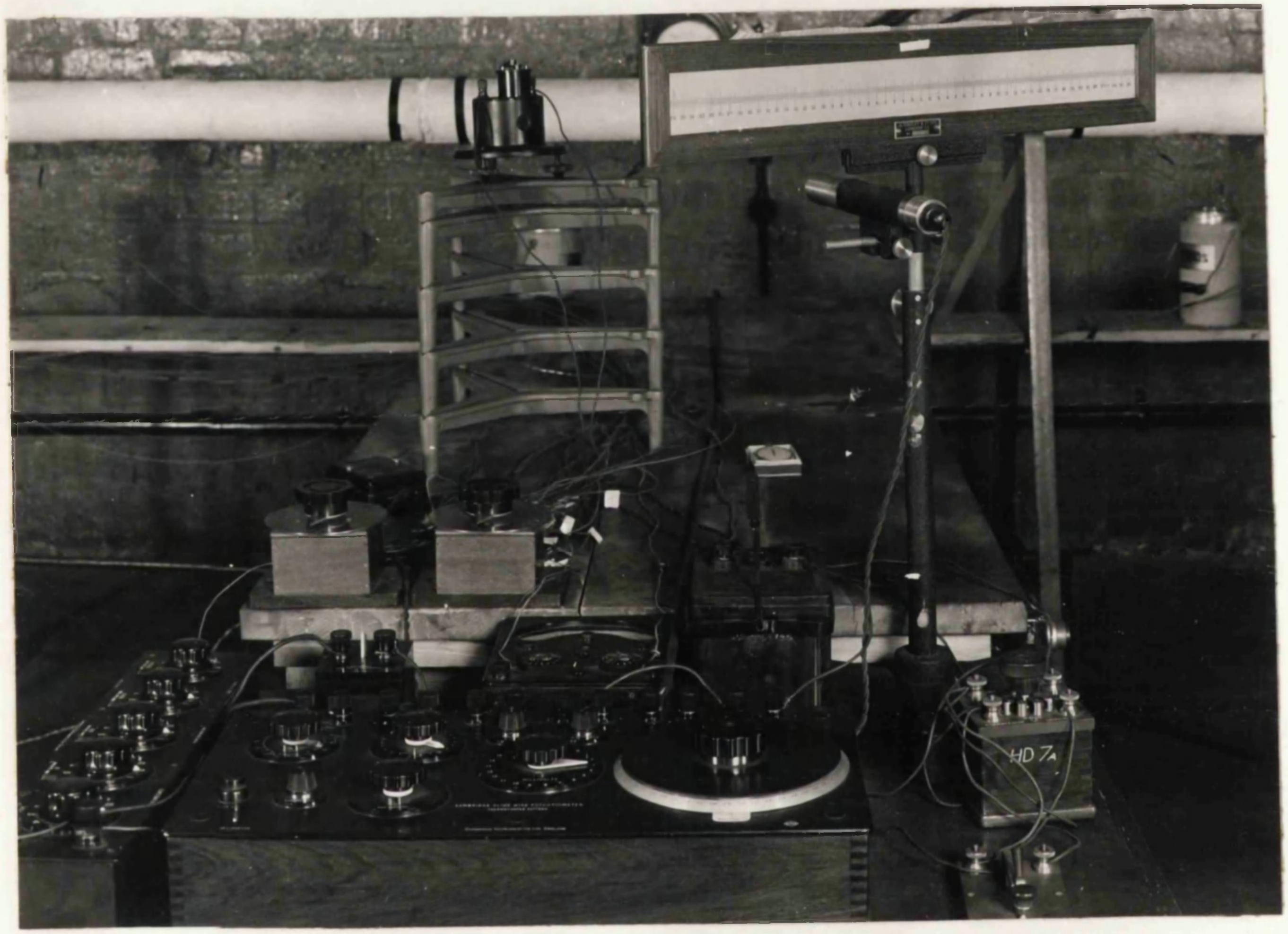


FIG 7 POTENTIOMETER

balancing them on a knife edge. In both cases the point of balance was within $^1/_{16}$th inch of their centre points. Reflections from a small electric light bulb pulled through the pipes showed the inner surfaces to be clean and highly polished. No ripples were noticed.

3.5 Thermocouples

The most convenient thermocouples for the range of temperatures to be measured in the proposed experiments (0°C. to 30°C.), are made from copper and constantan. The characteristics of copper-constantan thermocouples are such that a difference in temperature of 0.01°C. between hot and cold junctions produces an E.M.F. of approximately 0.0004 millivolts. The Cambridge potentiometer (Fig.7) used in the experiments had a range of from 0 to 180 millivolts, but could only be read to the nearest 0.002 millivolts.

In order, therefore, to allow for temperatures to be read to the nearest 0.01°C., a "Five-way thermocouple" was made by connecting up five single thermocouples in series. For a temperature difference of 0.01°C. between hot and cold junctions, such a thermocouple provides an E.M.F. of 0.002millivolts, this being the sum of the E.M.F.s of the five individual thermocouples. Thus, by using five-way thermocouples, it was possible to measure temperatures to the nearest 0.01°C. on the potentiometer.

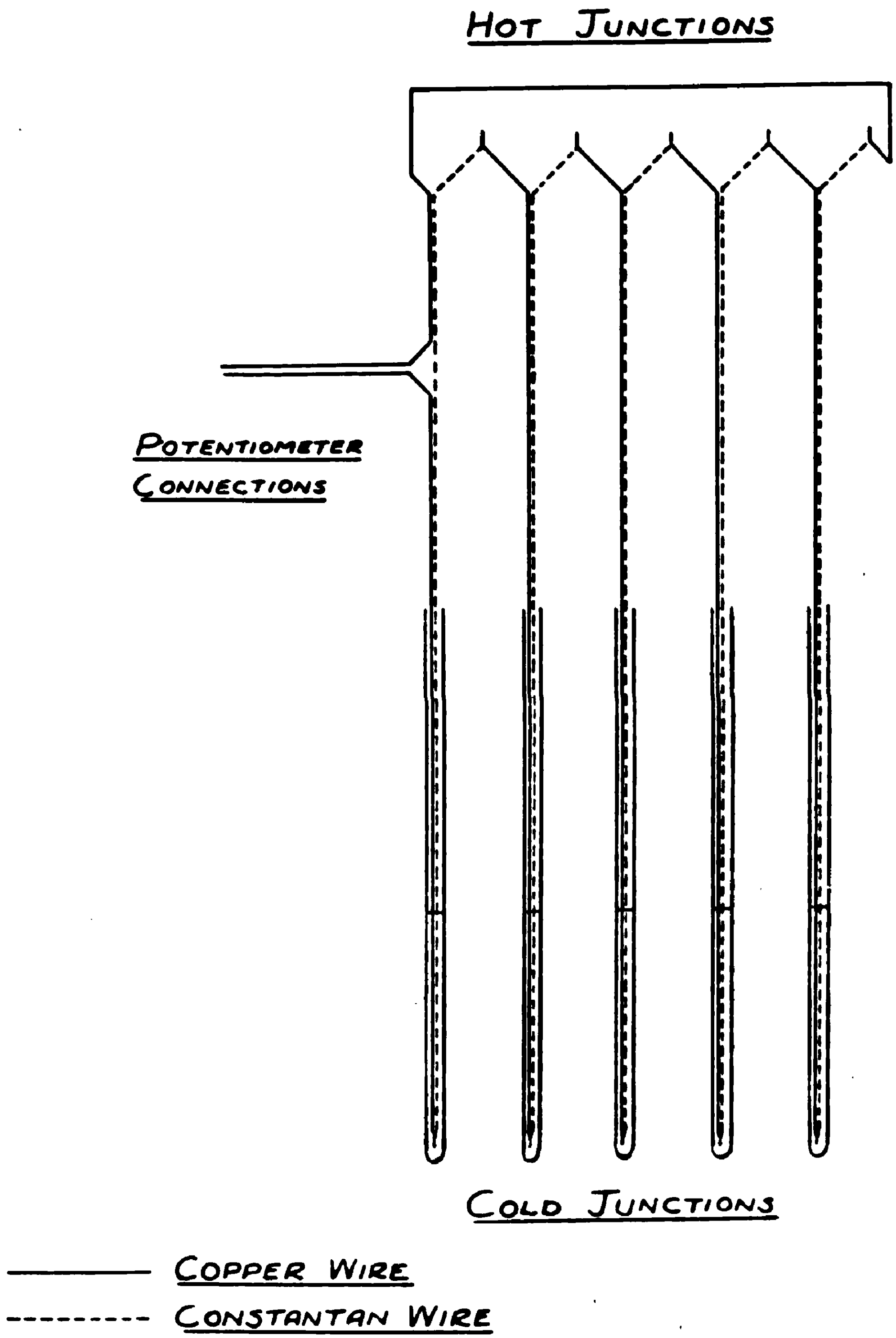


FIG 8 FIVE-WAY THERMOCOUPLE

The thermocouples were made from twin copper and constantan cotton-insulated wire 0.0092 inches in diameter. For each thermocouple, a cold junction was formed by soldering together the copper and constantan wires at one end of the double wire. This junction was then inserted into a 1/16th inch bore glass tube 8 inches long, sealed at its lower end. A few drops of paraffin wax held the junction in position.

Hot junctions were formed by soldering together copper and constantan wires from neighbouring lengths of double wire, the finished article being as shown in Fig. 8. Double copped wire connected the five-way thermocouple to the potentiometer via a multi-point switch.

The calibration of a five-way thermocouple is described in Appendix 1.

Local heat transfer coefficients can be calculated for each point at which the inside surface temperature of the pipe is known. Since a calculation can be made of temperature drop through the pipe wall, then in order to obtain a complete picture of the distribution of local heat transfer coefficients along the experimental pipe, the outside pipe surface temperatures must be measured at appropriate positions. The spacing of the thermocouples is closer for the interesting excess turbulent section of

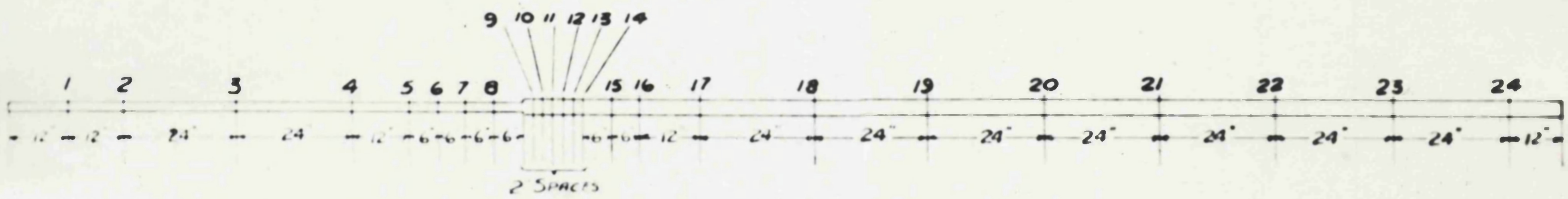


FIG 9 THERMOCOUPLE POSITIONS (ENLARGEMENT)

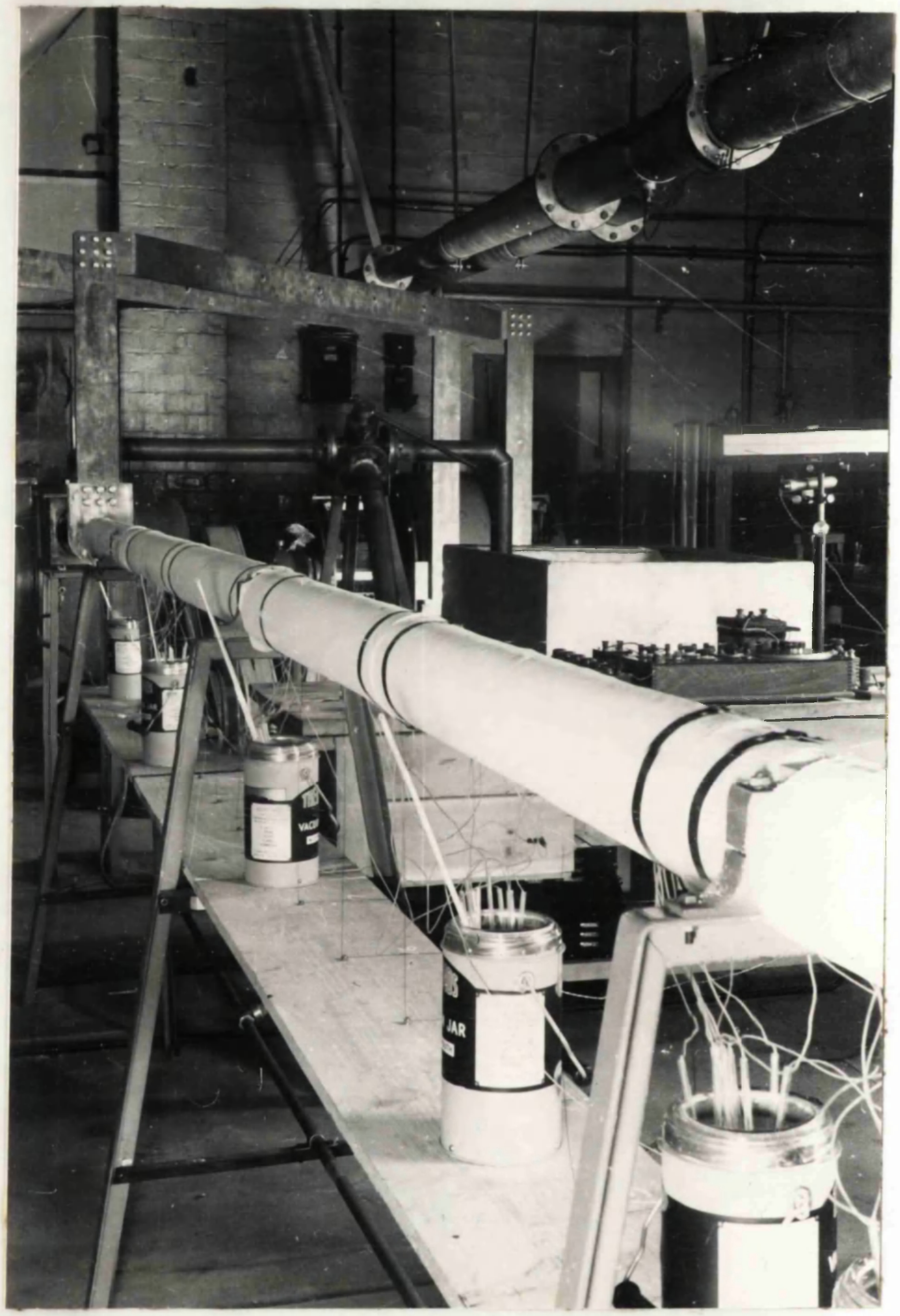


FIG 10 VACUUM JARS AND PIPE LAGGING

the pipe immediately downstream of the change of section. The positions, as chosen for the abrupt enlargement experiments, are shown in Fig. 9.

At each position, the five thermocouple hot junctions were spaced evenly round the pipe circumference and were electrically insulated from the pipe. This electrical insulation was necessary for two reasons; firstly, to prevent the five thermocouples from shorting each other out and secondly, because the pipe itself was not at earth potential due to the current passing through it. The hot junctions were held in position by a layer of adhesive tape. It is explained in Appendix 3 that a negligible amount of heat flowed through the outside surface of the pipe, so that the radial temperature gradient at the surface could be assumed to be zero. There was therefore no temperature difference through the insulating varnish, and hence the temperatures as measured by the thermocouples were those of the pipe surface.

The cold junctions, in their individual glass tubes, were placed in vacuum jars filled with a mixture of crushed ice and water (Fig. 10).

3.6 <u>Description of Apparatus</u>.

The apparatus will be described as it was erected for the abrupt enlargement experiments (Fig.11). In the

FIG 11 EXPERIMENTAL APPARATUS

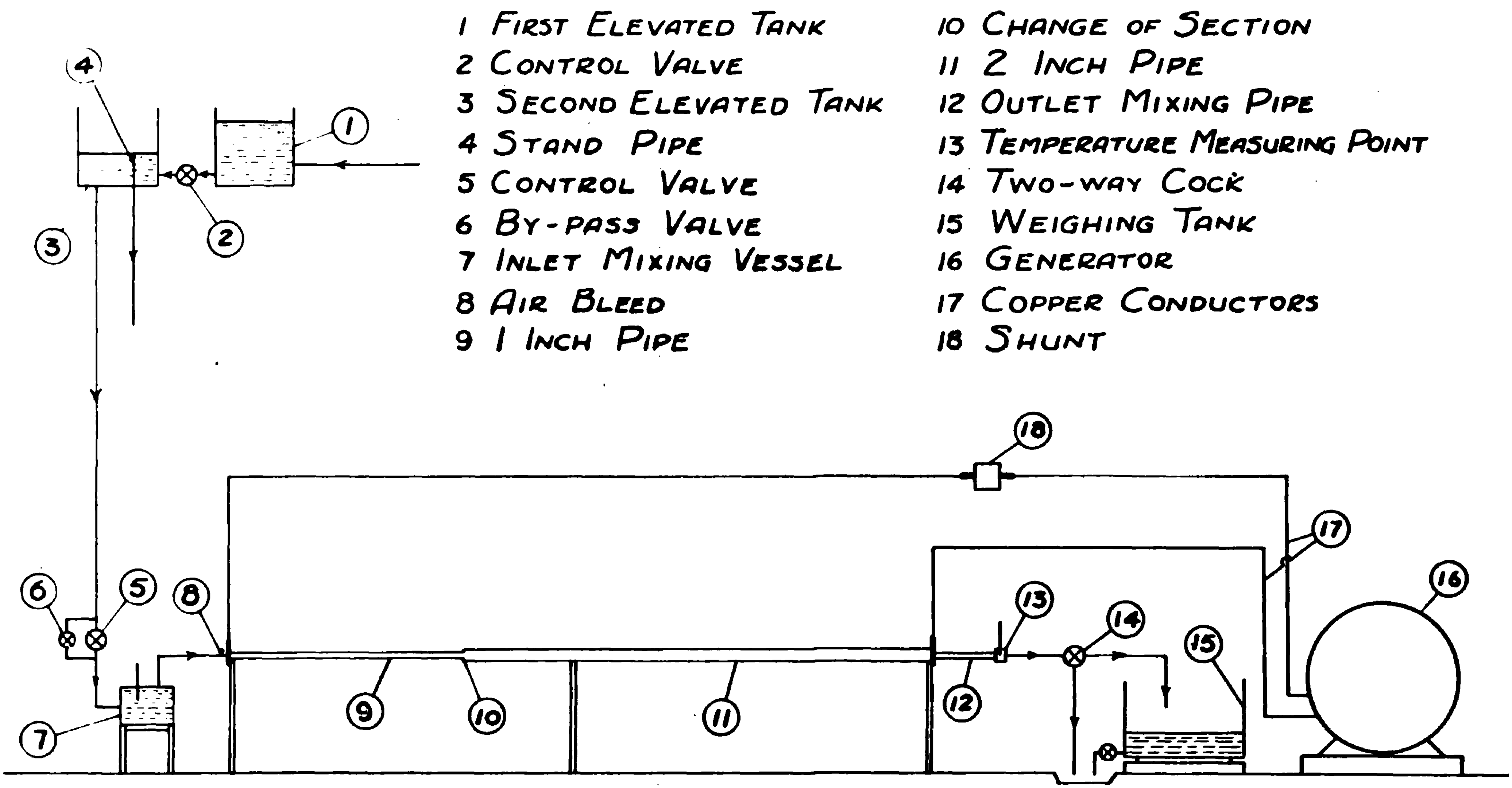


Fig 12 Experimental Apparatus

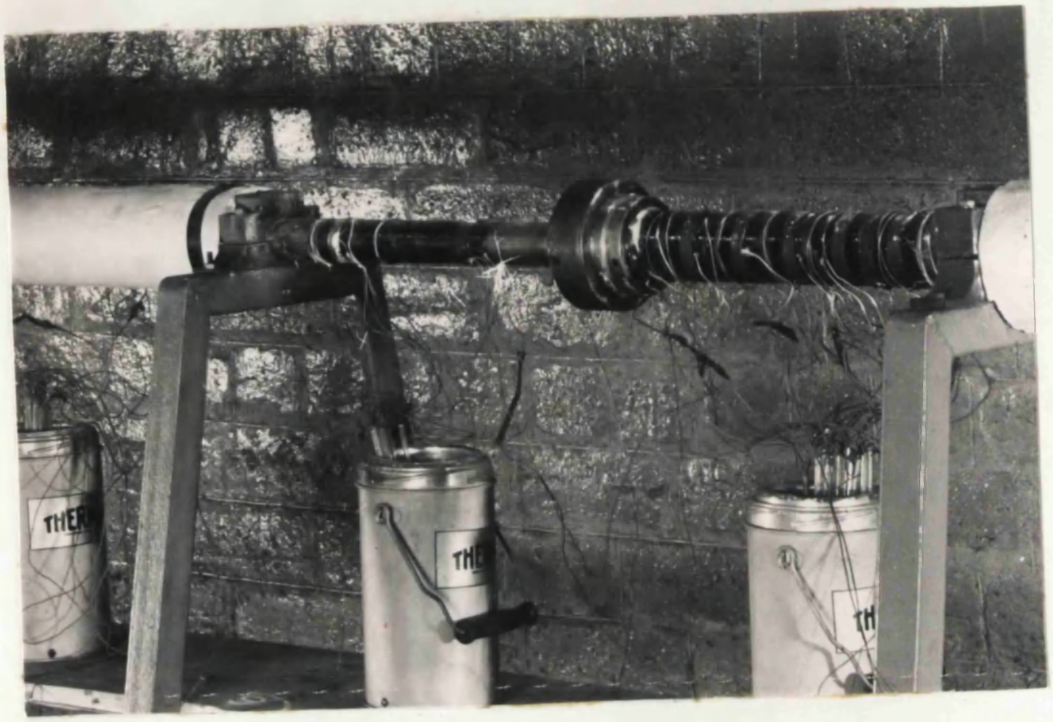
light of experience gained in this series of experiments, some minor modifications were made to the apparatus for the case of the abrupt contraction. These modifications will be referred to in Chapter 5.

The apparatus may be most conveniently described if the water flow and the electrical heating circuit are considered separately (Fig. 12).

(a) Water Flow

Referring to Fig. 12, mains water flows to the first tank (1), the flow being controlled by a ball-valve. From there it flows via a 4 inch breach pipe and control valve (2) to the second tank (3) which contains an overflow stand pipe (4). By suitably adjusting the control valve, the level of water in this tank can be maintained such that a small quantity of water flows to waste down the stand pipe. A constant head of water is thus assured.

Water from the second tank then flows down a 3 inch copper pipe to a valve (5), which controls the flow of water in the experimental pipe. A find adjustment is provided by a small by-pass valve (6). Water now passes to the inlet end mixing vessel (7), consisting of a brass box of about one cubic foot capacity lagged with 1 inch thick cork slabs. In this vessel are situated a thermometer and a five-way



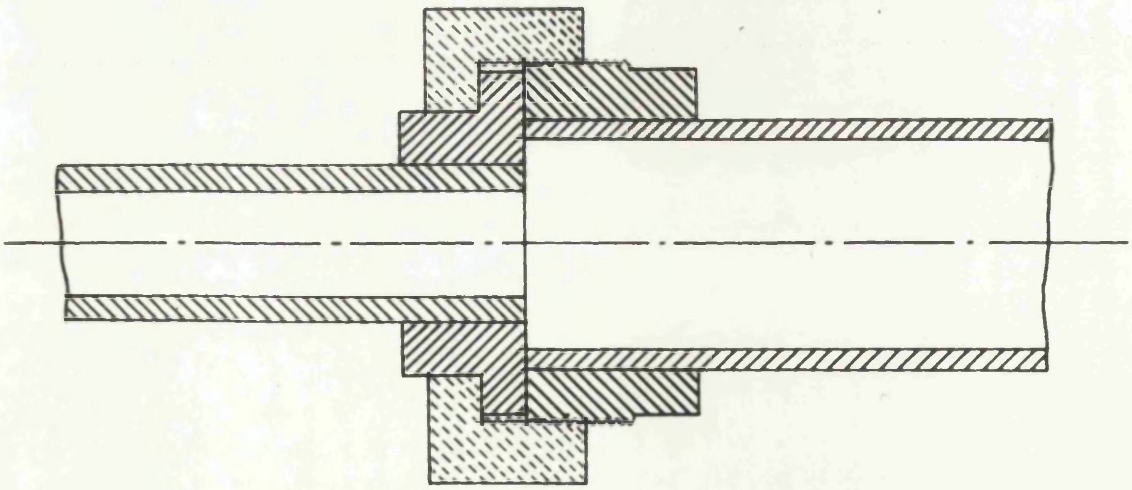


FIG 13 CHANGE OF SECTION

thermocouple in a glass tube, both of which measure inlet water temperature. A preliminary temperature traverse taken across this vessel indicated uniform temperature and hence adequate mixing. An air bleed (8) is situated immediately before the inlet to the experimental pipe to remove any air trapped there and thus prevent its being carried into the experimental pipe. The presence of air in theexperimental pipe can affect pipe surface temperatures and its progressive build-up might even affect the water flow.

Water flows through the 1 inch pipe (9), the change of section (10) (Fig. 13) and the 2 inch pipe (11) to the outlet **end** mixing device (12).

It is known that, due to the method of heating, the outlet water emerges from the experimental pipe with a non-uniform temperature distribution. It is therefore probable that, in order to measure the mean water temperature, more vigorous mixing will be required at outlet than at inlet. It was initially intended that the mixing of the outlet water should be brought about in a vessel similar to that at inlet, but a temperature traverse taken across such a vessel indicated a non-uniform temperature distribution, and hence inadequate mixing. In order to promote increased turbulence and at the same time to minimise heat conduction from the "hot" end of the

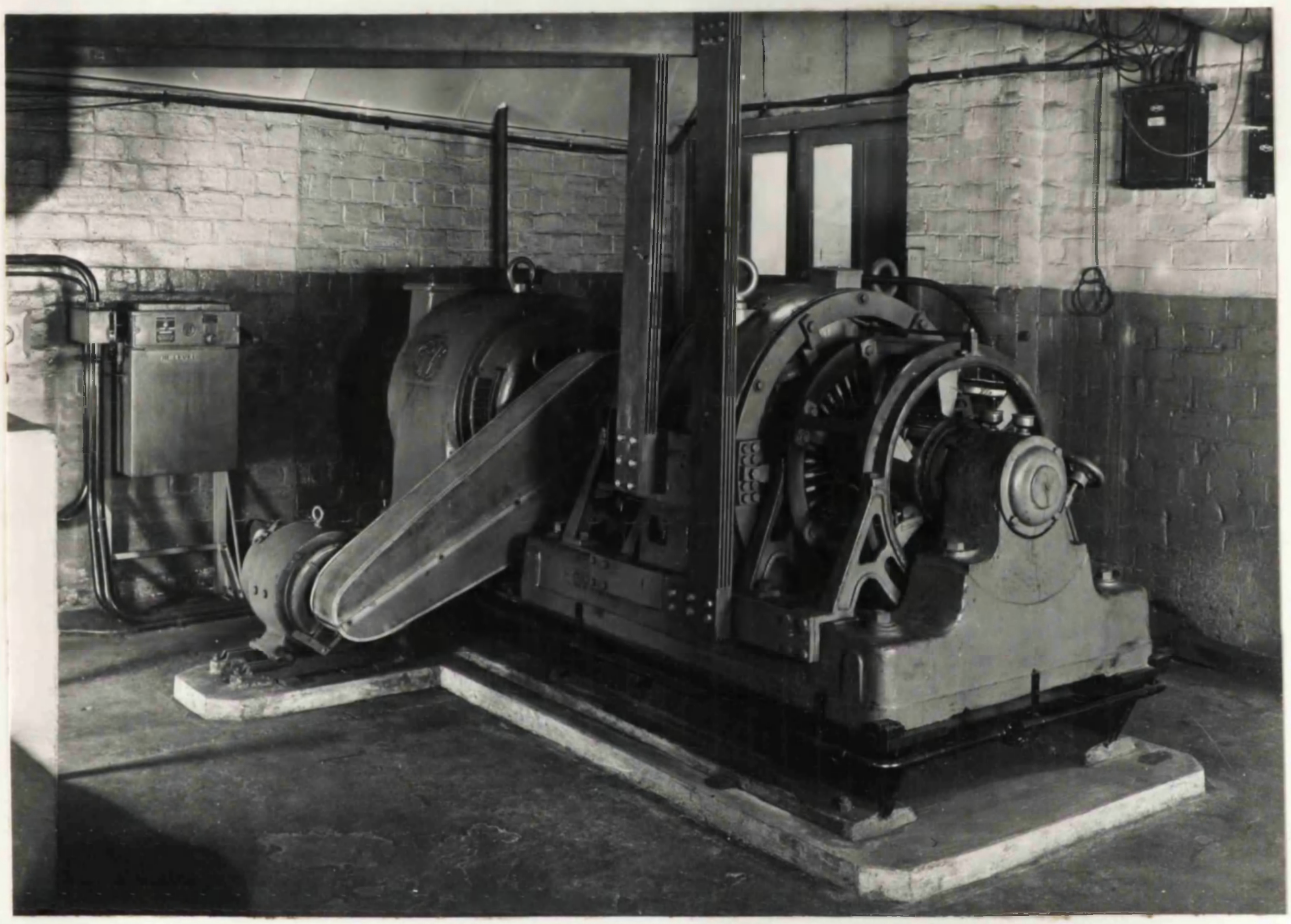


FIG 14 GENERATOR

experimental pipe to the temperature measuring point, a small bore "Tufnol" pipe of low thermal conductivity was introduced as the mixing device. The water temperature was measured in a larger diameter pipe at its outlet. Two alternative mixing pipes of 1 inch and ½ inch diameter were made, each being 15 inches long. The smaller pipe was necessary to provide sufficient mixing at the lower rates of water flow. A further reference to the adequacy of this mixing is made in Chapter 5.

Finally, from the outlet end temperature measuring point (13), containing a thermometer and a five-way thermocouple, water flows through a 3 inch copper pipe to a two-way cock (14) and thence to waste, either directly or via the weighing tank (15).

A determination was made of the effect on the water flow of redirection of the water from the waste pipe to the weighing tank. Any variation in the flow due to this redirection would cause a change of outlet water temperature. No such change occurred, and it was therefore assumed that the position of the two-way cock did not affect the water flow.

(b) Electrical Heating Circuit.

The generator (16) (Fig. 14) supplies the current which is conveyed via the copper conductors (17) to

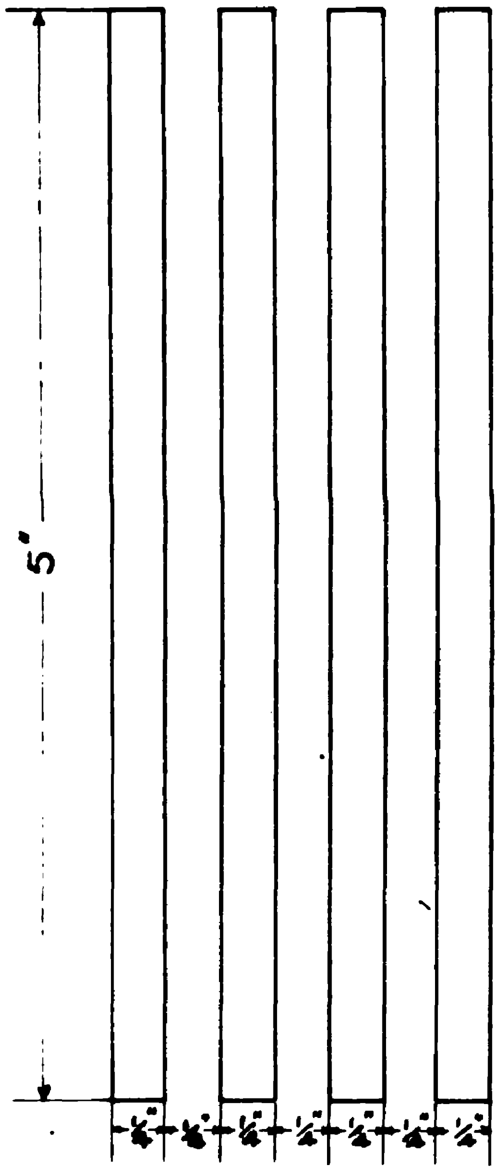
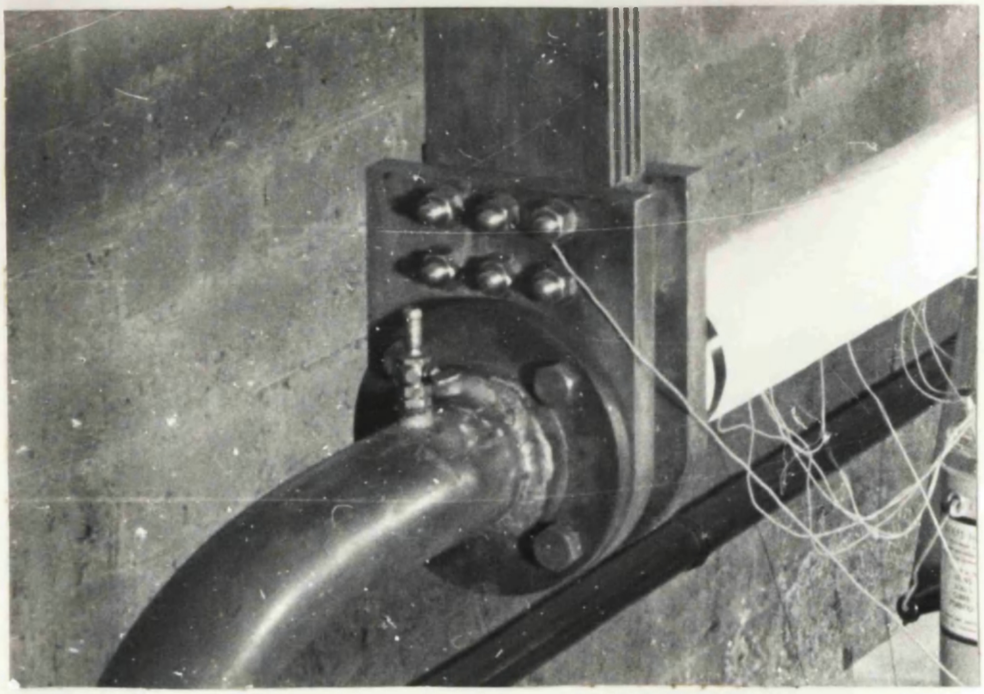


FIG 15 CROSS-SECTION OF COPPER CONDUCTORS



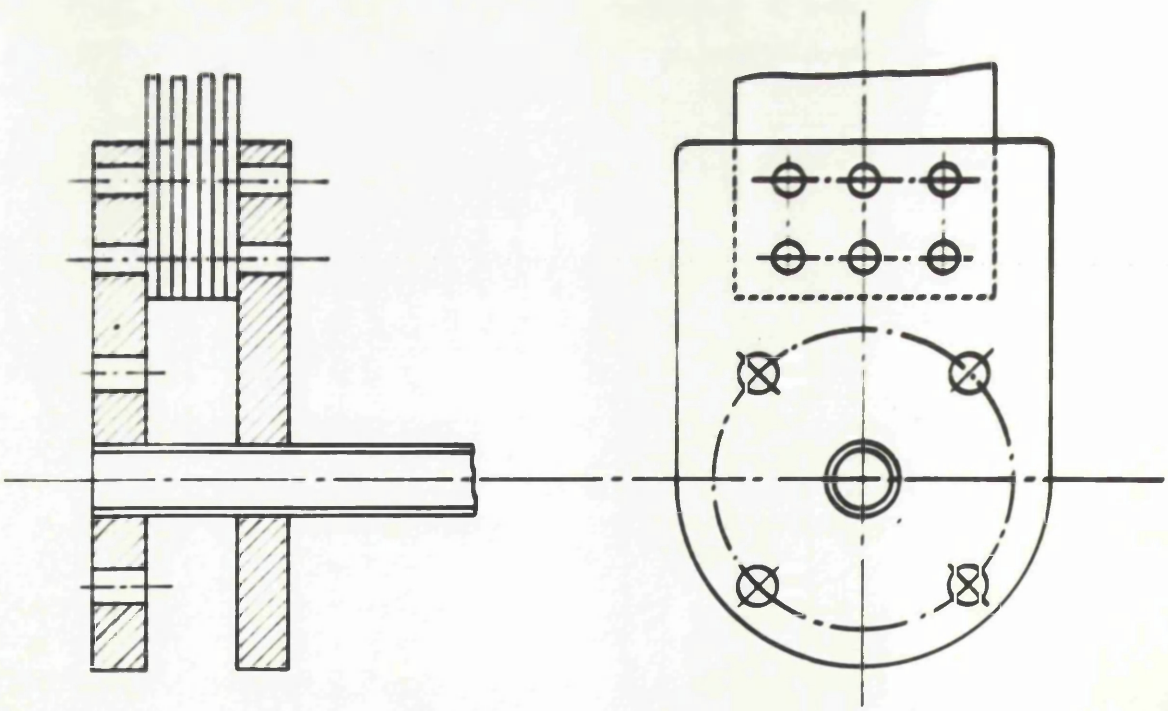


Fig 16 Double Brass Flanges

the ends of the experimental pipe. The conductors were made with a cross-sectional area of 5 square inches, to conform with the accepted standard of 1000 amps per square inch current density. The cross-section was made up as shown in Fig. 15. The electrical connection to the pipe were made through double brass flanges (Fig. 16).

For the purpose of measuring the heating current a 5000 amp shunt (18) was incorporated in the circuit. This shunt, which was designed to give a voltage drop of 60 millivolts at a current of 5000 amps, was calibrated at the National Physical Laboratory as follows:-

Duration of Test Current (minutes)	Current (amperes)	Voltage Drop (millivolts)	Thermo-E.M.F. (millivolts)
1	1000	11.99	0.00
30	3000	35.96	0.00
30	5000	59.85	0.00

The voltage drop across this shunt was measured on the Cambridge potentiometer.

The electrical insulation of the circuit was brought about by the placing of wooden blocks between the conductors and their supports, and by the use of wooden clamps to hold the experimental pipe in position.

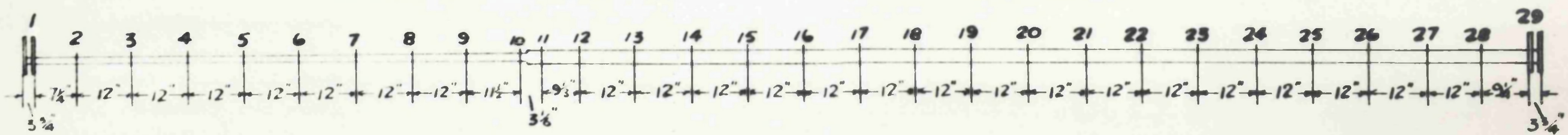


FIG 17 POTENTIAL POINTS (ENLARGEMENT)

A rubber washer placed between the connecting flanges insulated the experimental pipe from the inlet end pipe work, the "Tufnol" mixing pipe serving this purpose at the outlet end.

In order to be able to calculate water temperatures at all positions on the experimental pipe, it was necessary to measure the variation, if any, of electrical resistance per unit length of pipe. For this purpose, thin copper wires were soldered to the pipe at the positions shown in Fig. 17. These wires were connected, via a multi-point switch, to the potentiometer, this switch being so arranged that successive voltage drops could be measured throughout the length of the pipe. Since for a constant current, voltage drop is proportional to resistance, hence the variation of resistance per unit length of pipe could be found.

The electrical heat input to any section of the pipe was measured by the product of the sum of the voltage drops along that section of pipe and the current as measured on the shunt.

(c) Thermal Insulation of the Experimental Pipe

It had been initially assumed that no thermal insulation of the pipe would be necessary, since the pipe temperature would not be greatly different from that of the surrounding air and the heat transfer coefficient due to forced convection inside the pipe would be many times greater than the coefficient for natural convection outside the pipe (Appendix 3). However, preliminary test runs showed that air currents in the laboratory, due to the opening of doors, etc. produced forced convection effects at the outside of the pipe with resulting fluctuations of pipe temperature. It was therefore found necessary to lag the pipe.

The lagging used was in the form of easily removable 3 foot lengths of "Magnesia Section" of 1¼ inch wall thickness, split longitudinally into two halves, and held in position by steel bands. (Fig. 10). Thermocouple leads and potential wires emerged through the longitudinal division.

(d) <u>Factors to be considered in the Heat Balance.</u>

Measurements could be made of electrical heat input to the experimental pipe, and heat received by the water from its mass flow and temperature rise. In order to form a heat balance two further factors had to be considered.

(a) Heat transferred through the pipe lagging.

(b) Heat conduction between the experimental pipe and the copper conductors.

From a knowledge of air and pipe temperatures and of the thermal conductivity of the pipe lagging, an estimation could be made of the heat transferred through the pipe lagging (Appendix 2).

In order to estimate the amount of heat conducted between the copper conductors and the pipe, the temperature gradients of the copper conductors at their points of contact with the pipe had to be measured. For this purpose single thermocouples were placed near the end of each conductor (Appendix 2).

CHAPTER 4

Measurement of the Nusselt, Reynolds and Prandtl Numbers

It is the intention to present the experimental results in the form of the Nusselt equation so that certain comparisons with previous data may be made. The observations made must therefore be sufficient to provide the distribution of Nu, Re and Pr along the experimental pipe for all values of water flow and heat input.

The purpose of the present chapter is to show how the apparatus as described fulfills these requirements.

The measurement of Nu, Re and Pr, which may be written $\frac{hD}{k}$ $\frac{\rho VD}{\mu}$ and $\frac{C_p\mu}{k}$ respectively, depends on the determination of the following quantities:-

- D - The pipe diameter
- C_p - The specific heat of water
- ρ - The density of water
- μ - The viscosity of water
- k - The thermal conductivity of water
- h - The heat transfer coefficient
- V - The water velocity

These quantities may be divided into three groups

(1) Constants D

(2) Physical Properties of Water C_p ρ μ k which depend on water temperature.

Temperature °C	0	1	2	3	4	5	6	7	8	9
0	4.2174	2138	2104	2074	2045	2019	1996	1974	1954	1936
10	4.1919	1904	1890	1877	1866	1855	1846	1837	1829	1822
20	4.1816	1810	1805	1801	1797	1793	1790	1787	1785	1783
30	4.1782	1781	1780	1780	1779	1779	1780	1780	1781	1782

TABLE 3

Specific Heat of Water in joule/(gm)($^{\circ}$C)

Temperature °C	0	1	2	3	4	5	6	7	8	9
0	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
10	0.9997	0.9996	0.9995	0.9994	0.9993	0.9991	0.9990	0.9988	0.9986	0.9984
20	0.9982	0.9980	0.9978	0.9976	0.9973	0.9971	0.9968	0.9965	0.9963	0.9960
30	0.9957									

TABLE 4

Density of Water in gm/millilitre

1 gm/millilitre = 0.999973 gm/cm^3.

(3) Quantities which must be calculated from Experimental Observations h and V

4.1 The Pipe Diameter

The experimental pipe is composed of a 1 inch diameter pipe and a 2 inch diameter pipe. These diameters are constant throughout the pipe lengths and since the Metric System of Units is being used in the calculations:-

D is either 2.54 cm. or 5.08 cm.

4.2 The Physical Properties of Water

The variation of these properties with temperature will be considered over the temperature range of 0°C. to 30°C. All water temperatures will lie within these limits.

(a) Specific Heat

The variation of specific heat with temperature, as obtained from an unpublished correlation of data by the Heat Division of the Mechanical Engineering Research Laboratory, is shown in Table 3. This variation is sufficiently small to be neglected and the specific heat will therefore be considered constant and equal to 4.187 Joules/(gm.)(°C).

(b) Density

Table 4, taken from Dorsey,[9] shows the variation of density with temperature. Here again

Temperature °C	0	1	2	3	4	5	6	7	8	9
0	17.94	17.32	16.74	16.19	15.68	15.19	14.73	14.29	13.87	13.48
10	13.10	12.74	12.39	12.06	11.75	11.45	11.16	10.88	10.60	10.34
20	10.09	9.84	9.61	9.38	9.16	8.95	8.75	8.55	8.36	8.18
30	8.00									

TABLE 5

Viscosity of Water in millipoises

Viscosity in millipoises X 10^{-3} = Viscosity in gm/(sec)(cm)

Temperature ^{o}C	Thermal Conductivity
0	0.00571
5	0.00579
10	0.00586
15	0.00593
20	0.00601
25	0.00608
30	0.00616

TABLE 6

Thermal Conductivity of Water in joules/(sec)(cm)(^{o}C)

the variation is small and the density will therefore be taken as 1 gm/cm^3.

(c) Viscosity

The variation of viscosity with temperature is given by Dorsey[8], McAdams[19], and Brown Fishenden and Saunders[4]. Good agreement exists between the three sets of data, Table 5 being taken from Dorsey.

(d) Thermal Conductivity

Tables of the variation of thermal conductivity with temperature are given by Dorsey[10], McAdams[20] and Brown Fishenden and Saunders[4]. These three sets of data show a maximum deviation from one another of 2%. The most recent set, viz. that given by Brown, Fishenden and Saunders (Table 6), has been accepted.

4.3 The Heat Transfer Coefficient

The apparatus has been designed with a view to measuring the local values of heat transfer coefficient at the cross-sections of the pipe where outside pipe surface temperatures are measured. The heat transfer coefficient depends on the determination of two quantities:-

(a) The temperature difference between the inside surface of the pipe and the water.

(b) The heat transferred to the water per unit pipe surface area and unit time.

The coefficient is then the ratio $\frac{(b)}{(a)}$

(a) <u>Temperature Difference between Pipe and Water</u>

1. The outside surface temperature of the pipe is measured by a five-way thermocouple.
2. The temperature drop through the pipe wall can be calculated. This calculation is given in Appendix 3.
3. The inside surface temperature of the pipe can then be obtained by subtracting the temperature drop through the pipe wall from the outside pipe surface temperature.
4. Water temperature can be calculated at any position along the pipe but will only be calculated for the cross-sections at which pipe temperatures are measured.

   This calculation consists essentially of the addition to the measured inlet water temperature of a temperature rise calculated from the heat input to the water up to the point in question. Since there are two separate methods of estimating the heat input to the water, the equating of which forms the heat balance, it is therefore possible to use two alternative methods of calculating water temperatures in the pipe. These two methods will only give the same result if the heat balance is perfect.

On the assumption that the heat balances will not be perfect, the choice of method will depend on which side of the heat balance is considered to be the more accurate. This means, in effect, that if one experimental observation which contributes towards the heat balance can be shown to be the main cause of the discrepancy in the balance, then that observation can be neglected. The calculation of water temperatures will then be made by the method which does not depend on that observation.

The two alternative methods, to which fuller reference will be made later, will be called

(a) Water temperature calculation from electrical heat input, and

(b) Water temperature calculation from measured water temperature rise.

It may be emphasized at this stage that all measured and calculated water temperatures are bulk temperatures and that no attempt is made to measure temperature distribution across the water stream. Bulk water temperature at any cross-section of the pipe may be defined as that temperature which the water would attain if it were all brought to a constant temperature by mixing, having neither gained nor lost any heat.

From 3 and 4, the difference between inside pipe surface temperature and bulk water temperature can be found.

(b) Heat Transferred to Water per unit area and unit time

Of the two alternative methods of calculating heat transferred to the water, the one will be chosen which corresponds to the method used in calculating water temperatures in the pipe. Considering either the 1 inch or the 2 inch diameter pipe, since there is a constant generation of heat per unit length, the heat transferred to the water per unit area and unit time can be found accurately, being equal to all points on the pipe. This condition ensures that coefficients measured are "real" local heat transfer coefficients.

The calculation of heat transfer coefficient is now complete, being the ratio:-

$$\frac{\text{Heat Transferred to Water per unit area and unit time}}{\text{Temperature difference between Pipe and Water}}$$

4.4 <u>The Water Velocity</u>

The discharge of water is measured in the weighing tank. From this measurement, the water velocity in either pipe can be calculated directly. This velocity is constant throughout the length of each pipe because of the constant water density.

4.5 <u>The Nusselt, Reynolds and Prandtl Numbers</u>

Paragraphs 4.1 to 4.4 have shown how the quantities involved in Nu, Re and Pr are determined.

The Nusselt number thus obtained is a <u>local</u> Nusselt number, since it depends on a local heat transfer coefficient. For a normal turbulent length of pipe, although the heat transfer coefficient may remain constant, the Nusselt Number will vary slightly owing to the variation of thermal conductivity with temperature.

For a similar reason, the Reynolds Number of water flowing at a uniform speed in a heated pipe of uniform cross-sectional area, is not constant along the pipe. The water velocity remains constant, but the viscosity of the water varies with temperature.

The Prandtl Number of water depends only on water temperature and is therefore completely defined by Tables 1, 3 and 4.

4.6 Summary

The Nusselt, Reynolds and Prandtl Numbers depend on seven fundamental quantities D, C_p, ρ, μ, k, h and v. Of these, the first three can be considered to be constant, μ and k vary with water temperature which can be measured and the last two, h and V, can be determined experimentally.

Thus the Nusselt, Reynolds and Prandtl Numbers can themselves be determined by the apparatus as described in Chapter 3.

The variation of these quantities with position along the pipe can be found for any value of water flow and heat input.

CHAPTER 5

Experimental Tests.

The experimental tests were carried out in two parts, the first being concerned with the abrupt enlargement, and the second with the abrupt contraction, of the pipe.

Each of these two series of tests was carried out in descending order of Reynolds Numbers, three heat inputs in the ratio of about 3 : 2 : 1 being used at each Reynolds Number.

The maximum obtainable Reynolds Number for the enlargement tests, which were carried out in the University, was 90,000. The apparatus was re-erected in the Mechanical Engineering Research Laboratory for the contraction tests. Here the larger available head of water allowed for a maximum Reynolds Number of 105,000. These two maximum values were both of Reynolds Number measured in the 1 inch diameter pipe.

5.1 Range of Reynolds Numbers and Heat Inputs.

In the experiments only two independent variables could be controlled, viz. water flow and heat input to the pipe.

(a) Water Flow.

The upper limit of the water flow was set by the available head of water. As stated above, this allowed for maximum Reynolds Numbers in the 1 inch pipe of 90,000 and 105,000 in the enlargement and contraction experiments respectively. The lower limit of the water flow was set, in both sets of experiments, by certain practical difficulties which will be described in Paragraph 5.3 of this Chapter.

The Reynolds Numbers used were chosen so that when plotted logarithmically, the points might be evenly spaced.

They were as follows:- (all values refer to the 1 inch pipe)

Englargement Experiments 90,000 47,500 25,000
12,600 7,400 and 3,700

Contraction Experiments 105,000 50,000 26,000
14,600 8,750 and 4,600

(b) Heat Input

The review of the literature has brought out the fact that very little had been discovered about the effect of variation of heat flow on the heat transfer coefficient. It was therefore decided to take advantage of the flexibility of the method of pipe heating to run three tests at each Reynolds Number, each test having a different value of heat input. The three heat flows were chosen to be evenly spaced, being in the ratio of approximately 3 : 2 : 1.

The absolute upper limit of heat input to the pipe was set by the available generator output. The general considerations which determined the maximum heat input to be used at any given Reynolds Numbers, were of maximum water temperature. Difficulties were encountered due to over-heating the water at the lower Reynolds Numbers. These will be described in Paragraph 5.3 of this chapter.

The lower limit of heat-input was theoretically zero, but in practice, the generator output became very unsteady when reduced to about $^{1}/100$th of its maximum.

A Abrupt Enlargement

5.2 Experimental Procedure

The water flow and generator output having been adjusted to their required values, a period of approximately two hours was necessary to allow inlet-water temperature and generator output to attain reasonably steady values.

Before any experimental observations were taken, the thermocouple cold junction temperatures were checked, and if necessary, the mixtures of crushed ice and water were stirred. The bleed off valve was opened to remove any trapped air, and the potentiometer was standardized. These three operations were repeated at intervals of approximately ten minutes throughout the experiment.

The successful operation of the experiment depended on steady conditions of water flow, heat input and inlet water temperature being maintained. Of these, water flow could be controlled, but preliminary tests had shown that, for a given setting of the regulating valve, the flow would remain constant to within 0.5% over a period of several hours. Heat input could also be controlled but did not remain constant for a given setting of the regulator. Small fluctuations of generator voltage necessitated fairly frequent adjustment of the regulator to maintain a constant heat input. The inlet water temperature, on the other hand, could not be controlled and although it often remained constant for periods of up to half an hour, it too varied slightly from time to time. The experimental observations had therefore to be taken during a period of constant inlet water temperature.

Measurements of water flow and air temperature were taken before and after the experiment. In no experiment was any variation of either of these quantities noticed.

Water Flow	cwt	5
	seconds	108.4
Shunt Voltage	millivolts	59.10
Voltage across pipe	volts	4.41
Inlet Water Temperature	Thermometer(°C)	9.61
	Thermocouple(mV)	1.861
Outlet Water Temperature	Thermometer(°C)	11.88
	Thermocouple(mV)	2.313
Air Temperature	°C	18.6
Copper Conductor Temperatures (mV)	1	2.391
	2	1.883
	3	1.733
	4	1.197
	5	1.118
	6	1.842
	7	1.570
	8	1.477
	9	1.082
	10	1.066

Thermocouple Position	Potentiometer Reading (mV)
1	2.589
2	2.635
3	2.664
4	2.692
5	2.711
6	2.722
7	2.730
8	2.738
9	2.442
10	2.385
11	2.410
12	2.460
13	2.540
14	2.648
15	2.818
16	2.886
17	2.941
18	3.004
19	3.033
20	3.068
21	3.096
22	3.130
23	3.160
24	3.190

Terminal Positions	Potential Drop (mV)
1- 2	110.6
2- 3	170.2
3- 4	170.4
4- 5	170.7
5- 6	169.6
6- 7	170.3
7- 8	170.2
8- 9	170.7
9-10	164.3
10-11	26.1
11-12	128.9
12-13	165.7
13-14	166.1
14-15	166.6
15-16	166.5
16-17	167.7
17-18	167.0
18-19	166.9
19-20	166.4
20-21	165.5
21-22	167.4
22-23	166.0
23-24	166.2
24-25	166.4
25-26	166.2
26-27	167.2
27-28	166.8
28-29	136.8

TABLE 7

Experimental Observations for

Abrupt Enlargement Test No. 1

The generator output was constantly regulated in order to maintain the voltage drop across the shunt at its required value. Measurements were taken of voltage drops per foot length of pipe and of copper conductor temperatures. Both of these quantities depended on heat input to the pipe and were therefore held constant.

The remaining measurements of water temperatures and outside pipe surface temperatures, all other quantities being constant, depended for their steadiness only on the constancy of the inlet water temperature. A complete set of pipe and water temperatures was measured at intervals of five minutes until two consistent sets were obtained.

A complete set of experimental observations is given in Table 7.

5.3 <u>Difficulties Encountered at Low Reynolds Numbers</u>.

The first nine tests, carried out at the top three Reynolds Numbers of 90,000 47,500 and 25,000 in a 1 inch diameter pipe, all gave smooth and consistent temperature distributions along the outside surface of the pipe. All of these distributions had the same general shape.

At the Reynolds Number of 12,600 in the 1 inch diameter pipe, however, some irregularities in pipe surface temperature were noticed in the downstream half of the

2 inch diameter pipe. Temperatures in this section were higher than the smooth curve would have predicted and were not repeatable.

The possibility of air being forced out of solution by the heating of the water, and being trapped in the 2 inch diameter pipe, was then considered. A series of air bubbles might then be formed towards the downstream end of the 2 inch diameter pipe causing bad heat transfer from pipe to water and hence local hot spots at the top of the pipe. These hot spots did, in fact, exist and were so marked that the temperature difference between the top and bottom of the pipe could be felt by touch. It was realised that air might also have come out of solution at the previous higher Reynolds Numbers, but it was thought, since no effect had been noticed, that any such air would have been carried away in the water stream because of the greater water speed existing at the higher Reynolds Numbers.

In order to test this hypothesis, an air bleed was introduced at the downstream end of the 2 inch diameter pipe, a glass inspection tube being incorporated in the rubber connection to it. The opening of this bleed off valve at the Reynolds Number of 12,600 in the 1 inch pipe (or 6,300 in the 2 inch pipe), released a series of air bubbles. Pipe surface temperatures measured after the removal of these bubbles, steadied down to give the "usual" distribution and were repeatable.

An attempt was made to discover the water temperature at which air began to be thrown out of solution. No conclusive results were obtained, but it could be demonstrated that as the water temperature increased, a stage was reached where the water flowing from the bleed off became aerated with tiny air bubbles. The temperature at which this occurred varied between 15°C. and 20°C.

No further trouble from this source was encountered during the remaining tests down to the lowest Reynolds Number, so long as air was bled off before a set of temperature readings was taken.

At the Reynolds Number of 3,700 in the 1 inch diameter pipe a new difficulty was encountered. Again it took the form of unsteadiness of pipe surface temperatures in the 2 inch diameter pipe. Air was bled off but the unsteadiness remained. The Reynolds Number of 1,850 in the 2 inch diameter pipe was in the transition region between laminar and turbulent flow, and this in itself might have accounted for the unsteadiness. Natural convection effects were also considered.

If water, flowing very slowly in a horizontal pipe, is heated, then natural convection currents will be formed transverse to the direction of the water flow. Hot water tends to rise to the top of the pipe causing an uneven distribution of temperature round the pipe circumference.

Dimensional analysis shows that, whereas for forced convection:-

$$Nu = \phi\,(Re,\ Pr)$$

for natural convection:-

$$Nu = \phi\,(Gr,\ Pr)$$

where Gr is the Grashof Number and may be written

$$Gr = \frac{D^3 \rho^2 \ g \beta \Delta t}{2}$$

g = Acceleration due to gravity

β = Coefficient of expansion

Δt = Temperature difference between surface and fluid.

Hence, for a given heat input, as the Reynolds Number is reduced, a stage will be reached where the Grashof Number comes into prominence. By reducing the heat input at a given Reynolds Number, the value of Δt will be reduced and so therefore will be the effect of the Grashof Number.

One experiment was carried out at the Reynolds Number of 3,700 in the 1 inch diameter pipe at a very low heat input. The pipe surface temperatures were now steadier than they had been at a higher value of heat input, but small fluctuations still existed. The generator output required to supply this low heat flow to the water, had practically reached its lower limit and was unsteady. For this reason, the results obtained in this test were not considered to be very reliable and no further abrupt enlargement tests were attempted.

5.4 <u>The Effect of Dirt in the Pipe.</u>

When this series of tests had been completed, the 1 inch and 2 inch diameter pipes were uncoupled and were examined by drawing a small electric light bulb through them. They both appeared to be in much the same condition as when originally inspected.

Both pipes were cleaned by pulling them through several times with a piece of cloth which had been dipped in a soap solution. A small amount of reddish-brown dirt was noticed on the cloth.

A typical test was then repeated under exactly similar conditions of heat input and water flow. Results calculated from this repeat test agreed almost exactly

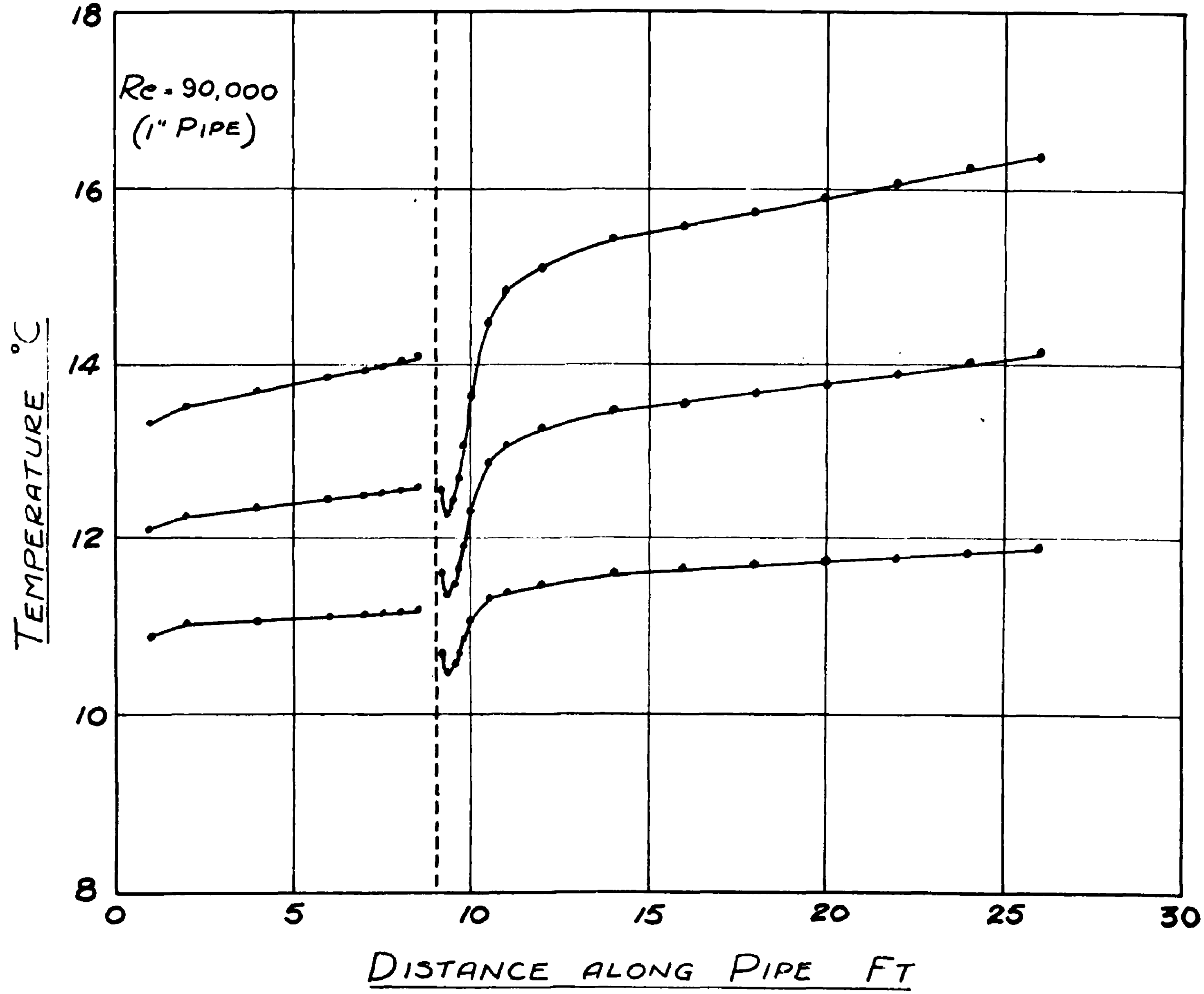


Fig 18a Pipe Surface Temperatures (Enlargement)

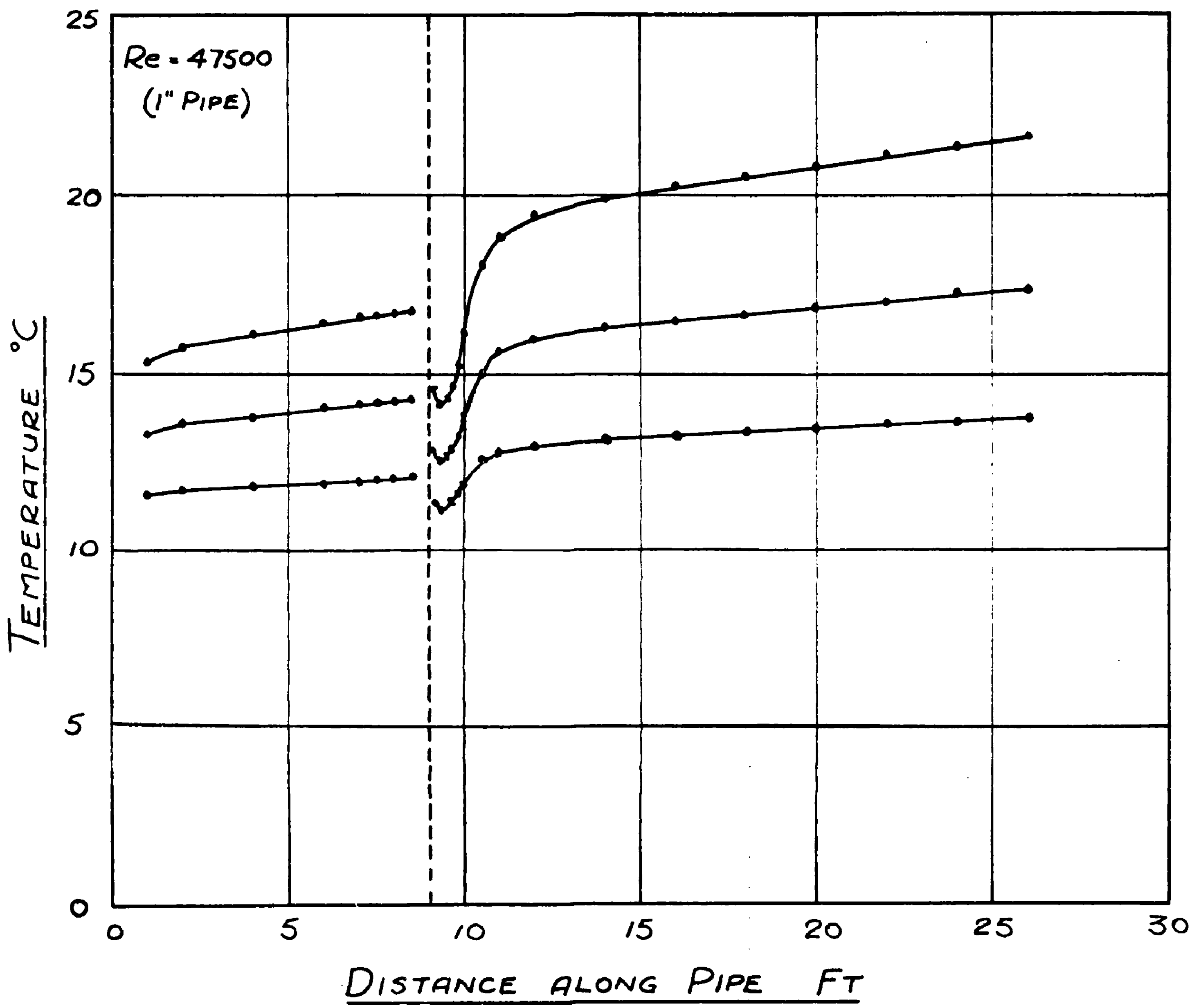


Fig 18b Pipe Surface Temperatures (Enlargement)

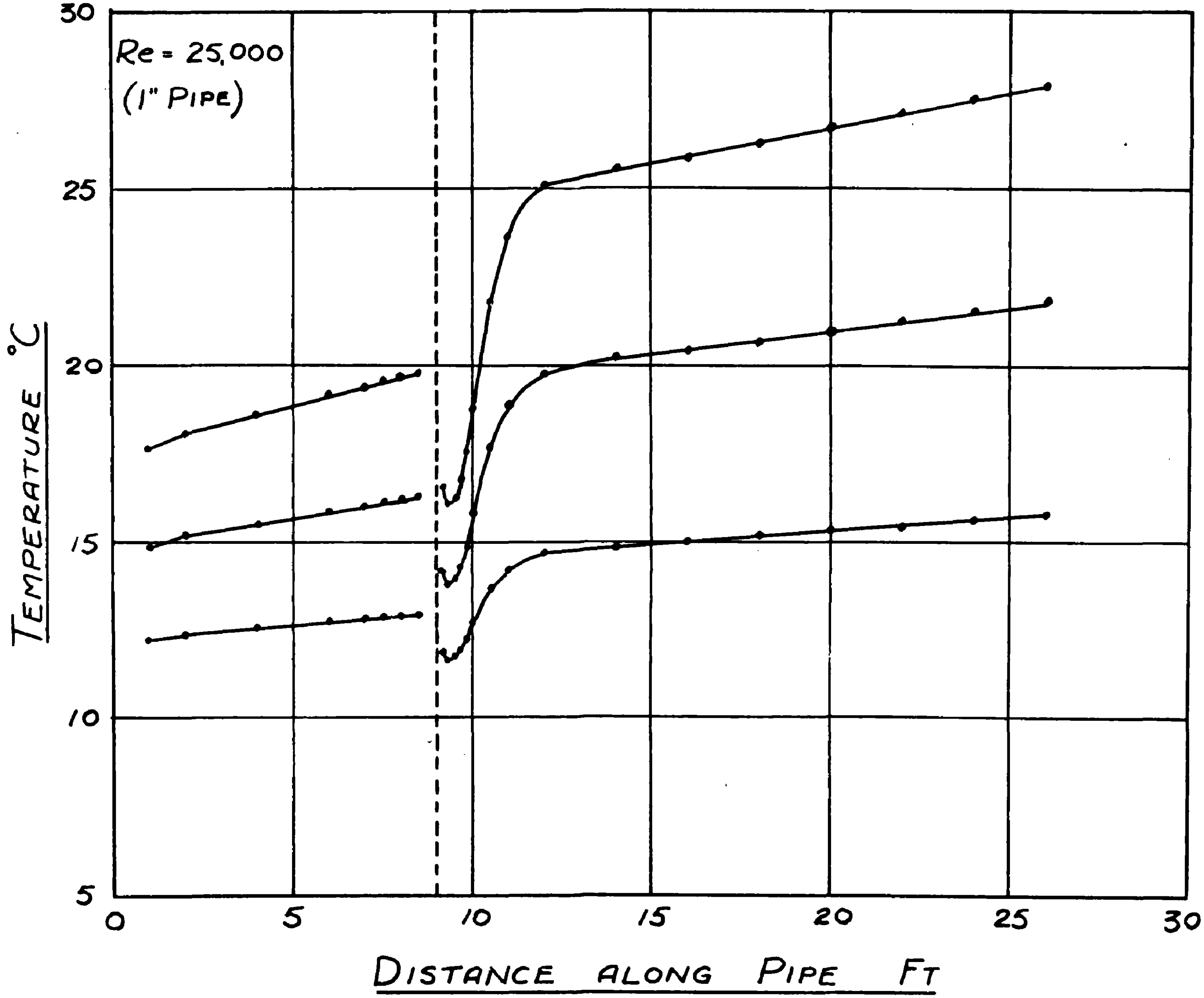


Fig 18c Pipe Surface Temperatures (Enlargement)

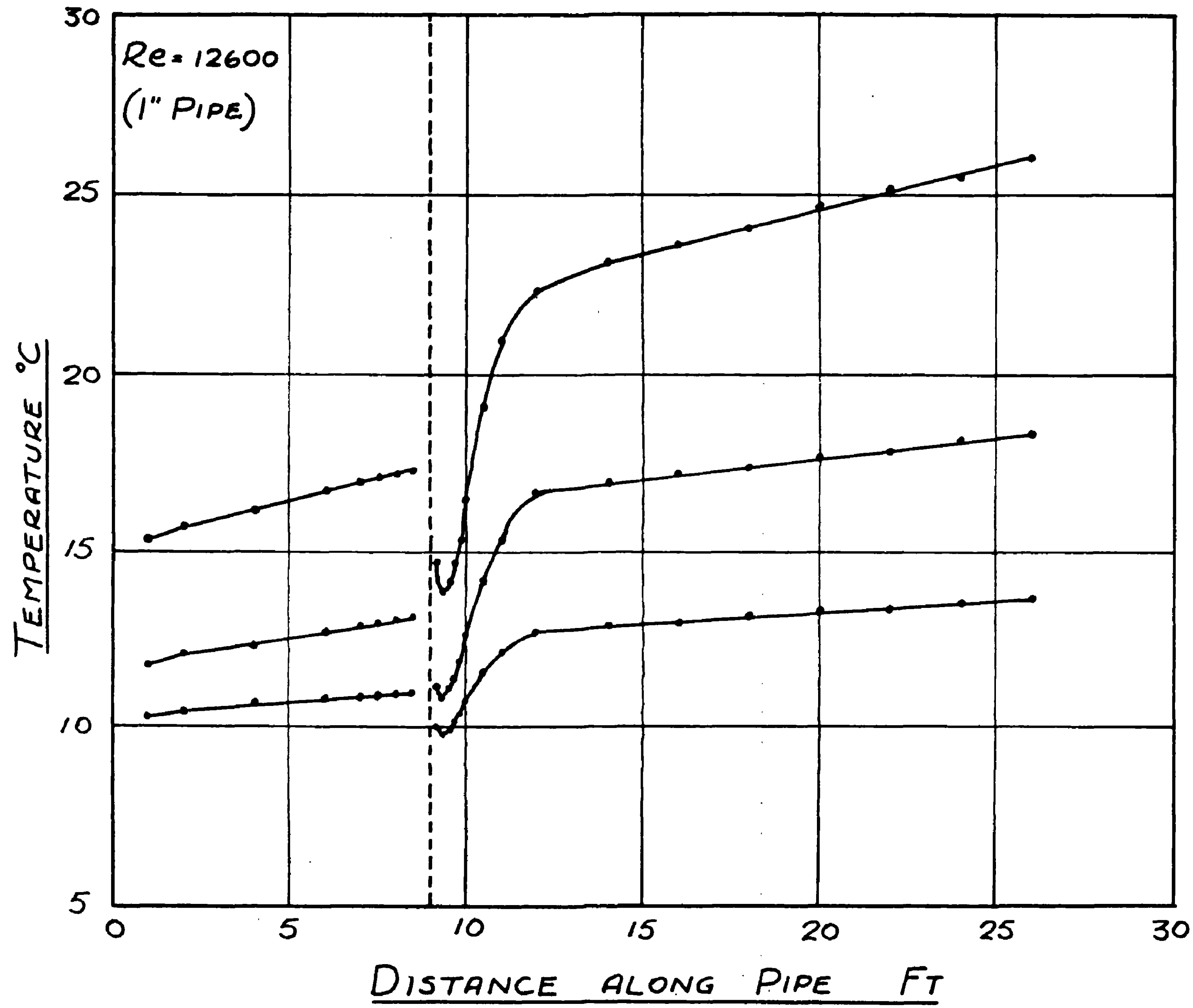


Fig 18d Pipe Surface Temperatures (Enlargement)

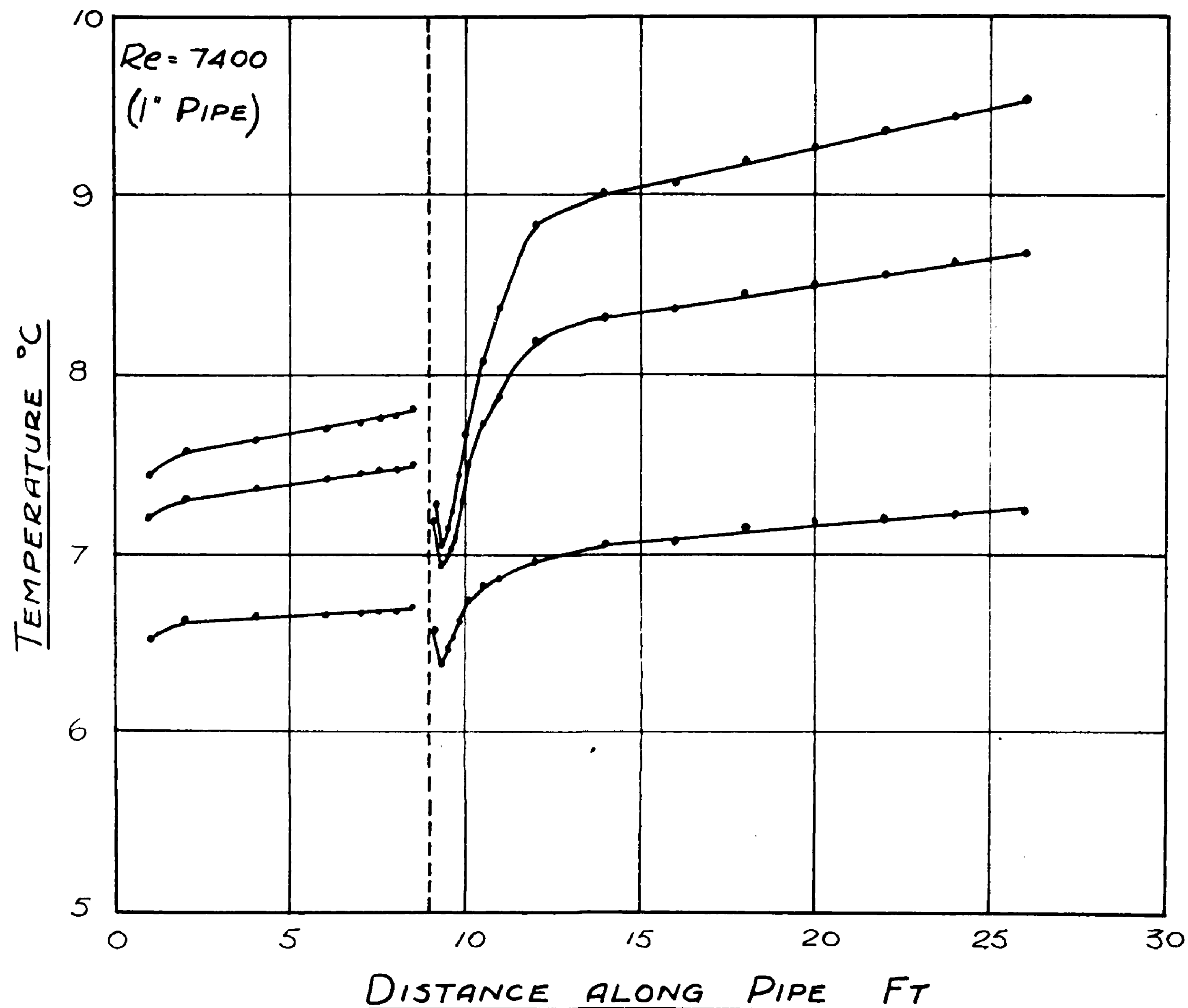


Fig 18e Pipe Surface Temperatures (Enlargement)

with those from the original test. The effect of dirt was therefore considered to be negligible.

5.5 Outside Surface Temperature Distributions along the Pipe

The temperature distributions along the outside of the pipe are shown in Fig. 18.

A fundamental point to be considered when studying these distributions is that, since the heat input to the water per unit surface area of pipe and unit time is a constant for either the 1 inch or 2 inch diameter pipe in a given test, then the heat transfer coefficient, by definition, must be inversely proportional to the temperature difference between the inside pipe surface and the water. Furthermore, since the water temperature rise is virtually linear throughout the whole experimental pipe, then for the normal turbulent sections of the pipe, where the heat transfer coefficient might be expected to be constant, there should be a constant temperature difference between pipe and water and therefore a linear rise of inside pipe surface temperature. It is shown in Appendix 3 that the temperature drop through the pipe wall is constant throughout the length of each section of the pipe. Hence, for normal turbulent sections of the pipe, the temperature distribution along the outside

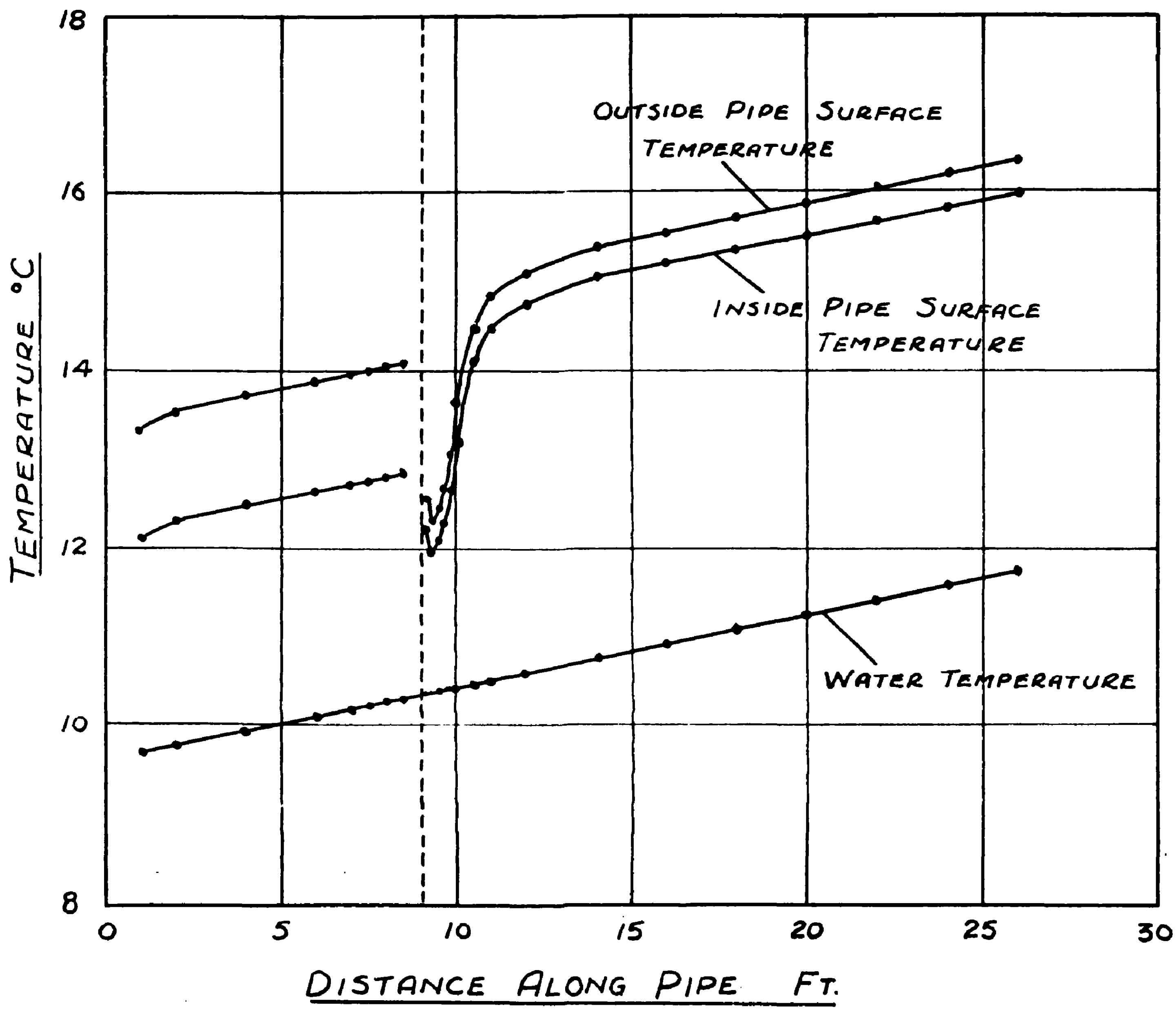


FIG. 19 PIPE AND WATER TEMPERATURES

surface of the pipe will be represented by a straight line parallel to that for the inside pipe surface temperature and both will be parallel to the water temperature distribution curve. These three curves, for a typical test, are shown in Fig. 19. This figure indicates that the above reasoning is correct, and that the assumption of constant values of local heat transfer coefficient for normal turbulent sections of the pipe is justified.

Since the heat transfer coefficient is inversely proportional to the temperature difference between pipe and water, then a "minimum" on the pipe surface temperature curve will correspond to a "maximum" heat transfer coefficient; i.e. for fixed conditions of heat input, water flow and inlet water temperature, the colder the pipe at a particular point the higher is the heat transfer coefficient at that point, or, in other words, the more efficient is the cooling of the pipe by the water.

All temperature distributions in Fig. 18 show a minimum at a position two diameters downstream of the abrupt enlargement, and therefore a maximum heat transfer coefficient for the 2 inch diameter pipe exists at that point.

Test No.	Reynolds Number in 1 inch Pipe	Electrical Heat Input (kW)	Heat from Conductors (kW)	Heat transferred through lagging (kW)	Corrected Electrical Heat Input (kW)	Heat Supplied to Water (kW)	Measured minus calculated Outlet Water Temperature (°C)
1	90,000	21.83	0.096	0.016	21.94	22.35	0.04
2	90,000	14.45	0.069	0.019	14.54	14.90	0.03
3	90,000	7.20	0.046	0.028	7.27	7.84	0.06
4	47,500	22.43	0.092	0.005	22.53	23.29	0.15
5	47,500	14.89	0.060	0.006	14.96	15.34	0.07
6	47,500	7.35	0.032	0.021	7.40	8.47	0.20
7	25,000	19.57	0.064	- 0.015	19.62	19.99	0.14
8	25,000	12.92	0.050	0.000	12.97	13.15	0.07
9	25,000	6.44	0.022	0.007	6.47	6.87	0.15
10	12,600	9.96	0.025	- 0.018	9.97	9.98	0.01
11	12,600	6.56	0.039	0.016	6.62	6.65	0.02
12	12,600	3.27	0.034	0.029	3.33	3.36	0.02
13	7,400	1.20	0.026	0.026	1.252	1.326	0.08
14	7,400	0.799	0.031	0.037	0.867	0.936	0.07
15	7,400	0.376	0.028	0.036	0.440	0.522	0.09

TABLE 8

Heat Balances for Abrupt Enlargement Experiments.

The temperature difference between pipe and water at the entrance to the 1 inch diameter pipe is slightly smaller than for the remainder of the pipe. This indicates that excess turbulence exists at entry to this pipe, with a resulting increase in heat transfer coefficient.

5.6 Heat Balance.

The corrected electrical heat input is balanced against the actual heat received by the water, where:-

Corrected electrical heat input is defined to be the electrical heat input to the pipe (volts x amps)
+ heat conducted from the copper conductors (Appendix 2)
+ heat transferred through the pipe lagging (Appendix 2).

Heat received by the water is calculated from the water flow and the water temperature rise.

The heat balances for all abrupt enlargement experiments are given in Table 8. The final column in this table indicates the difference between the measured outlet water temperature and that calculated from the corrected electrical heat input. For Tests Nos. 1-3, at a Reynolds Number of 90,000, this discrepancy is small, but increases with decreasing Reynolds Number as far as Test No. 9 at a Reynolds Number of 25,000. These first nine tests were carried out using the 1 inch diameter outlet mixing pipe. At this stage it was possible to change over to the $\frac{1}{2}$ inch diameter mixing pipe, since it would allow for a water flow having a Reynolds Number of 12,600 in the 1 inch section of the experimental pipe. For tests Nos. 10-12 at this Reynolds Number, a marked reduction in the outlet water temperature discrepancy is noticed. The indication is, therefore, that the accuracy of the outlet water temperature measurement, and hence of the heat balance depended on the degree of mixing of the water.

The possibility exists of calculating heat transfer coefficient by two different methods which give the same result only if the heat balance is perfect. Heat balances are not perfect, and it is therefore thought reasonable to discard the method which is dependent on the least accurate observation, viz. the outlet water temperature.

The calculation for abrupt enlargement experiments will therefore be carried out by the method which estimates water temperatures in the pipe from the measured inlet water temperature plus temperature rises as calculated from the corrected electrical heat input.

B. Abrupt Contraction

5.7 Re-erection of Apparatus

The apparatus was re-erected in the Mechanical Engineering Research Laboratory with the experimental pipe reversed, so that water flowed from the 2 inch diameter pipe into the 1 inch diameter pipe.

New five-way thermocouples were made and were attached to the pipe at the positions indicated by the points on the temperature distribution curves of Fig. 20. A new calibration was made of a similar thermocouple cut from the same roll of thermocouple wire (Appendix 1).

5.7 Modifications to Apparatus

In the light of experience gained in the abrupt enlargement experiments, certain modifications were made to the apparatus.

With regard to the difficulties encountered due to the aeration of the water, it was thought that the violent disturbance caused by the water splashing into the first elevated tank from the ball valve would cause a large amount of air to become entrained, and subsequently dissolved, in the water. In order to avoid any such splashing, the ball valve was placed well below the level of the float and was therefore submerged in the water.

With the experimental pipe in its new position, the possibility existed of air being trapped immediately before the abrupt contraction and consequently an air bleed was placed at this position. A second air bleed was installed at the downstream end of the 1 inch diameter pipe. No aeration of the water emerging from these bleeds was noticed in any of the contraction experiments. This may have been due entirely to the modification mentioned, or to the fact that the water supply was obtained from a new source.

Five groups of five single thermocouples were attached to the pipe, two groups at the downstream end of the 2 inch diameter pipe and the remaining three in the interesting section after the abrupt contraction. Each group had its thermocouples evenly spaced round a circumference of the pipe. An estimation could therefore be made of temperature distribution round the pipe. In all experiments these five groups each gave five almost identical temperatures and thus demonstrated the absence of local heating due either to air bubbles or natural convection.

The outlet end mixing pipe was modified to be in the form of a $1\frac{1}{2}$ inch bore Tufnol pipe 15 inches long, into which could be fitted a 12 inch Tufnol pipe of $1\frac{1}{2}$ inch outside diameter with any required bore. Three such inner pipes were made having bores of 1 inch, $\frac{1}{2}$ inch and $\frac{1}{4}$ inch. In the last 3 inches of the downstream end of the outer pipe, provision was made to take a temperature traverse across the water stream by the introduction of ten single thermocouples. Five of these measured the distribution vertically, and five horizontally. A check could thus be kept on the degree of mixing of the water, and therefore on the accuracy of the outlet water temperature measurement.

5.9 Experimental Procedure

The experimental procedure was the same as that already described in Paragraph 5.2, except that the additional measurements mentioned in connection with the modifications were made.

No difficulties were encountered at the lower Reynolds Numbers due to air in the pipe. Again only one test was attempted at the lowest Reynolds Number, since the necessarily low generator output became unsteady. The results obtained in this last test were not considered to be very reliable.

On the completion of this set of tests the pipe was inspected and cleaned as before. A repeat test was again carried out, the results once more being in good agreement with the original test. The effect of dirt was therefore considered to be negligible as before.

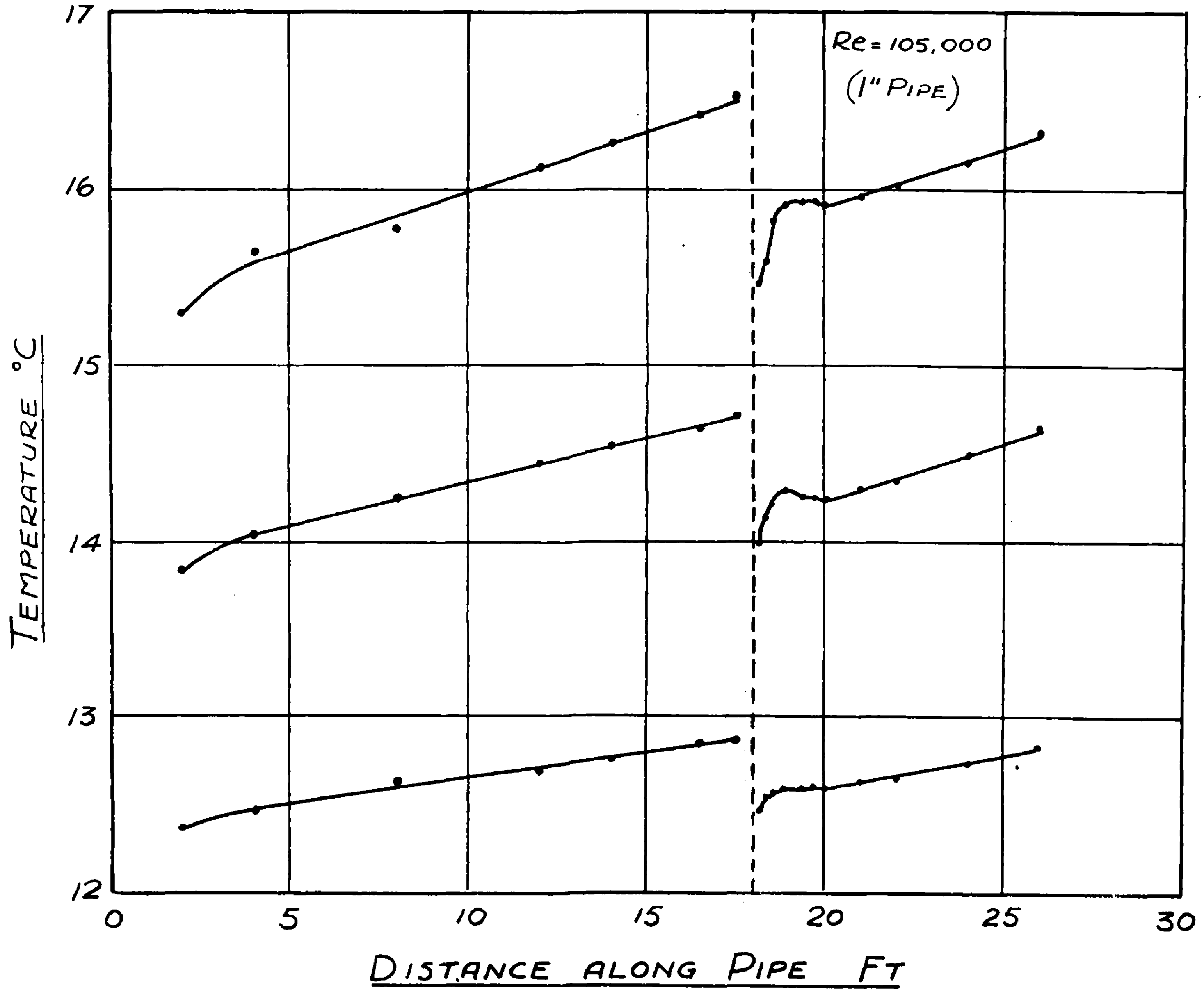


FIG 20a PIPE SURFACE TEMPERATURES (CONTRACTION)

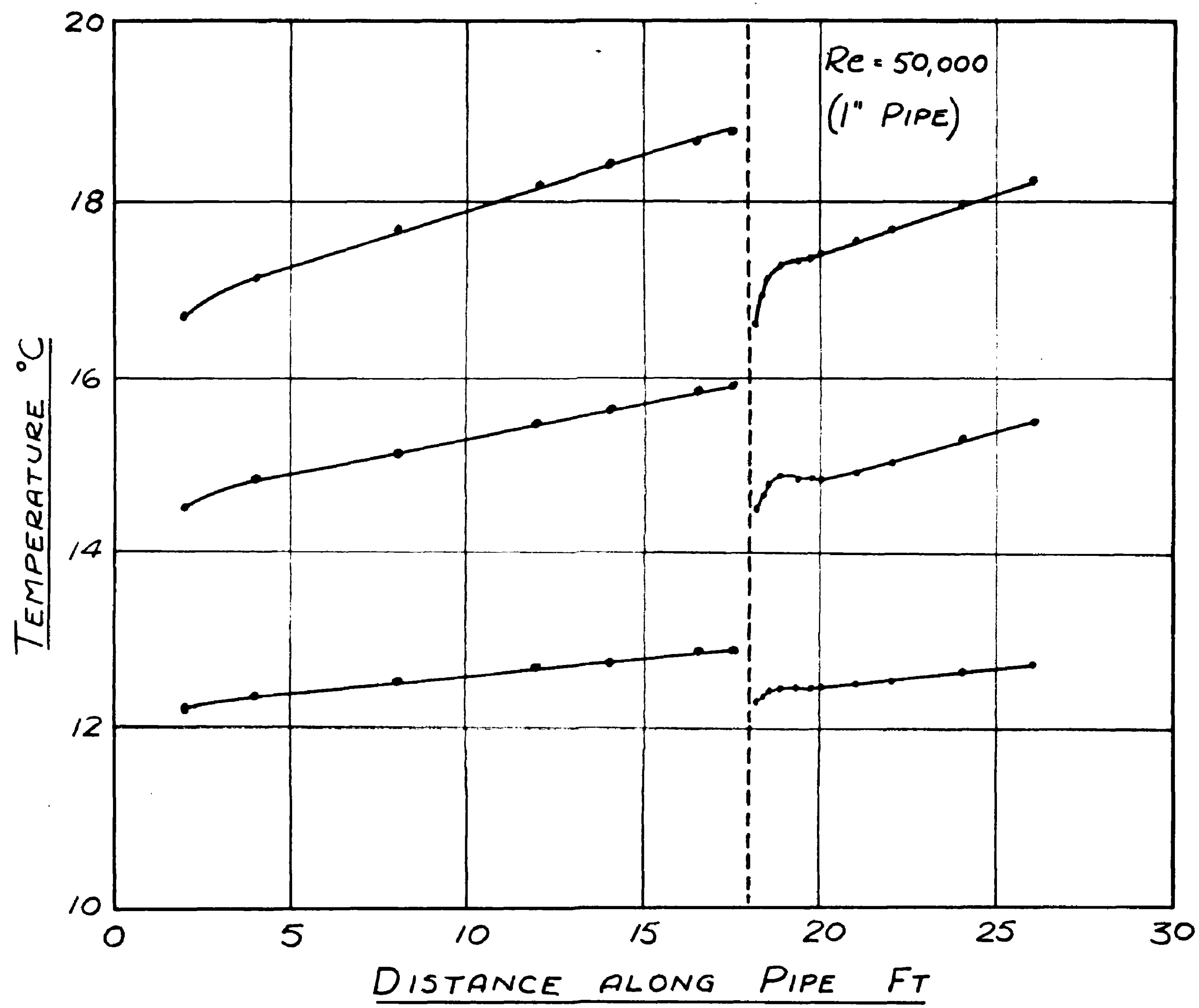


Fig 20b Pipe Surface Temperatures (Contraction)

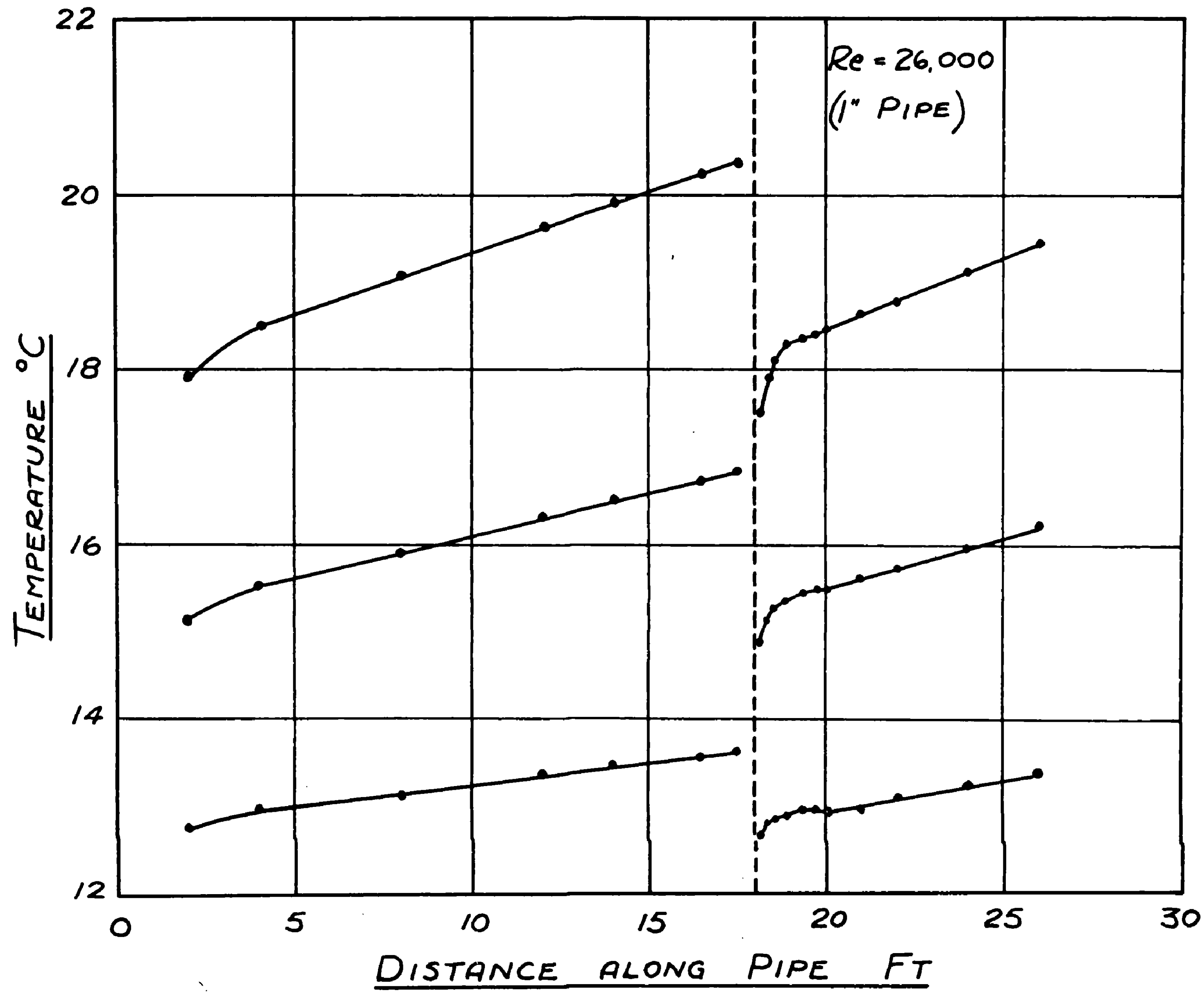


FIG 20c PIPE SURFACE TEMPERATURES (CONTRACTION)

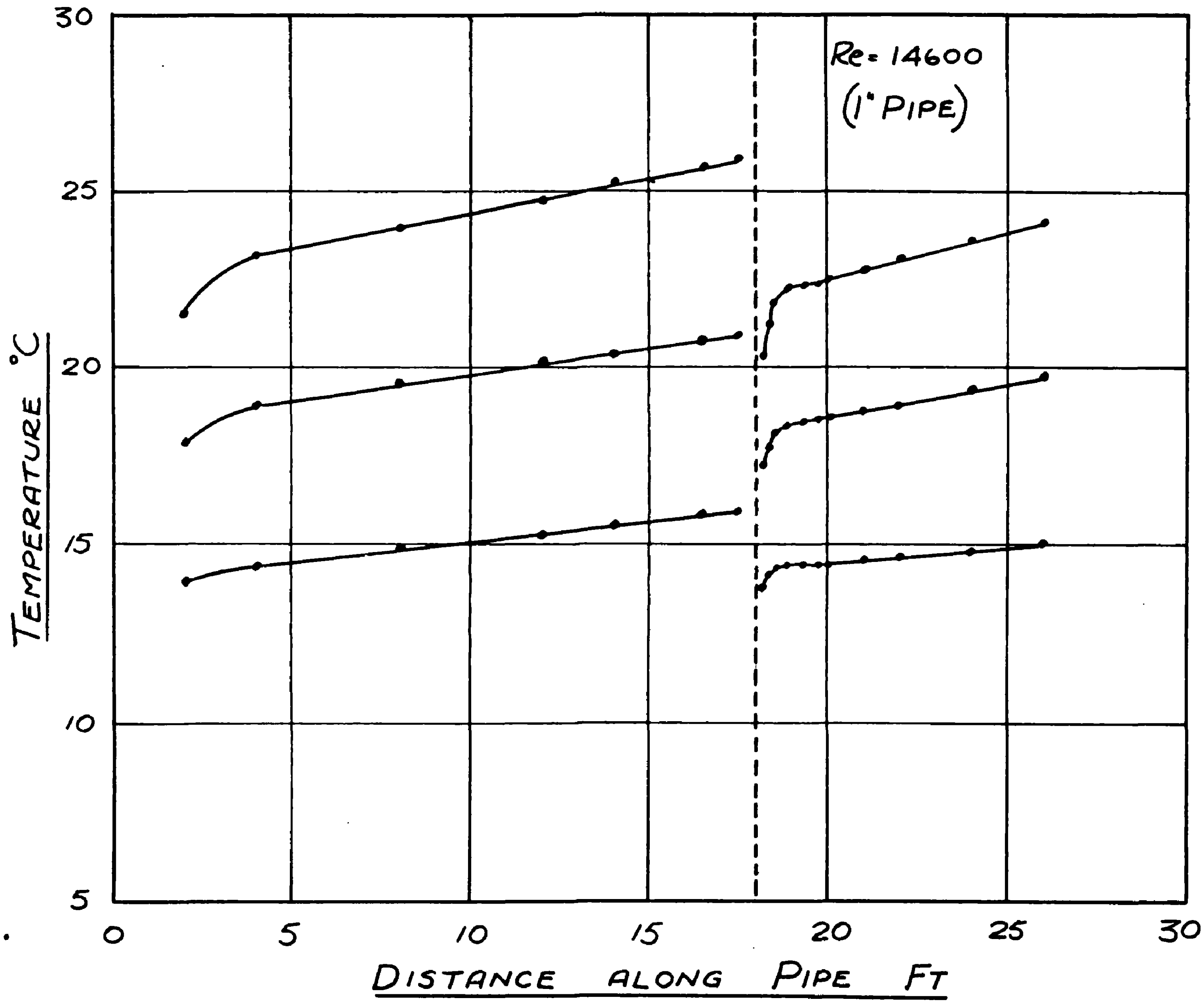


Fig 20d Pipe Surface Temperatures (Contraction)

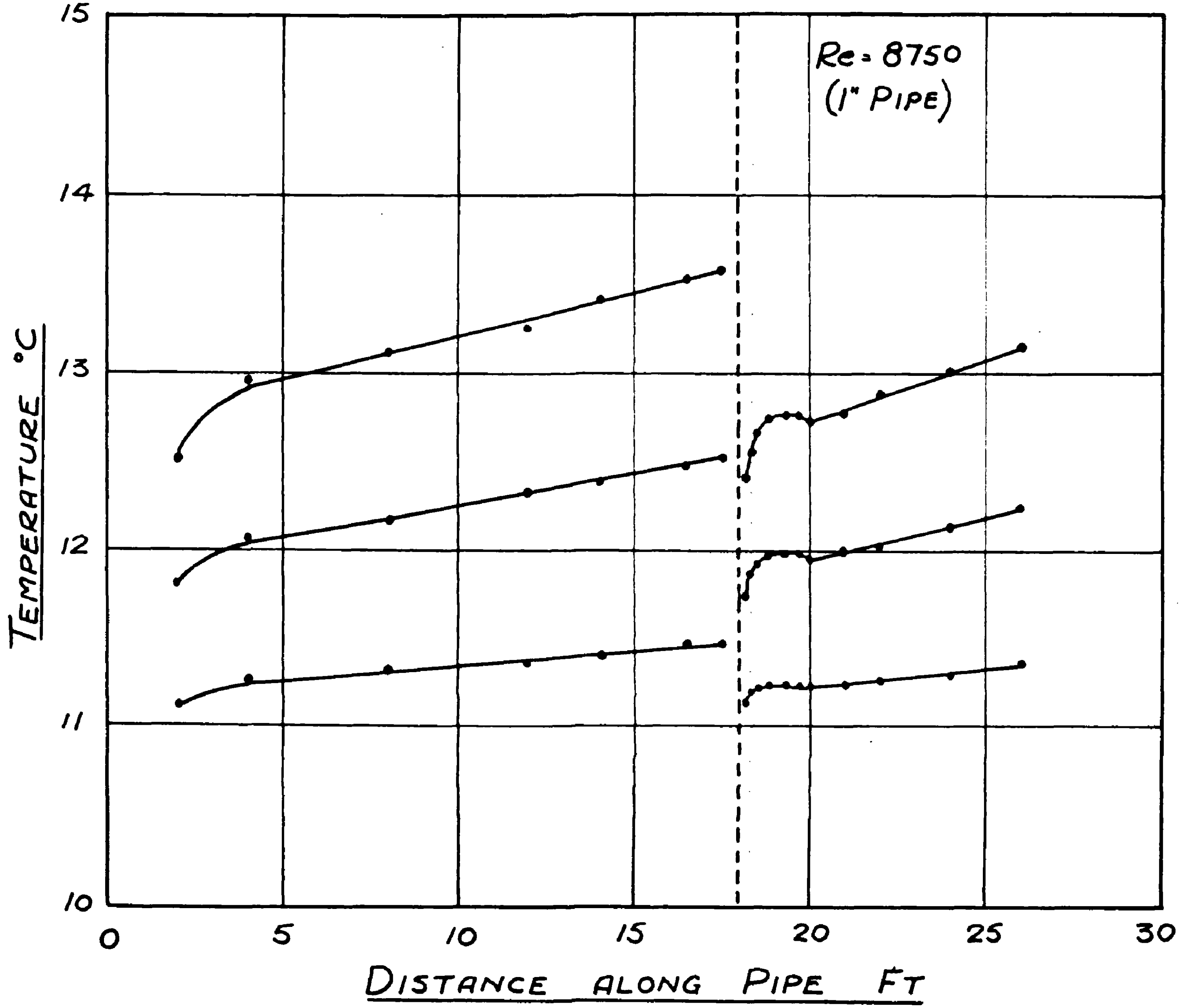


Fig 20e Pipe Surface Temperatures (Contraction)

Test No.	Reynolds Number in 1 inch Pipe	Electrical Heat Input (kW)	Heat from Conductors (kW)	Heat transferred through lagging (kW)	Corrected Electrical Heat Input (kW)	Heat Supplied to Water (kW)	Measured minus Calculated Outlet Water Temperature (°C)
1.	105,000	21.80	0.246	0.018	22.06	22.25	0.02
2.	105,000	14.61	0.206	0.023	14.84	14.98	0.01
3.	105,000	7.26	0.145	0.030	7.44	7.63	0.02
4.	50,000	16.24	0.202	0.013	16.47	16.68	0.04
5.	50,000	10.72	0.195	0.019	10.93	11.35	0.08
6.	50,000	5.39	0.175	0.024	5.59	5.54	- 0.01
7.	26,000	10.68	0.214	0.009	10.90	10.92	0.01
8.	26,000	7.18	0.130	0.015	7.33	7.43	0.04
9.	26,000	3.52	0.079	0.022	3.62	3.73	0.04
10.	14,600	10.15	0.135	- 0.008	10.28	10.27	0.01
11.	14,600	6.77	0.103	0.010	6.88	7.01	0.09
12.	14,600	3.36	0.081	0.014	3.46	3.52	0.04
13.	8,750	1.23	0.034	0.013	1.28	1.31	0.03
14.	8,750	0.823	0.024	0.017	0.864	0.911	0.05
15.	8,750	0.410	0.027	0.024	0.461	0.480	0.02

TABLE 9

Heat Balances for Abrupt Contraction Experiments.

5.10 Outside Surface Temperature Distributions along the Pipe

Fig. 20 shows the temperature distributions along the outside surface of the pipe.

Distributions for all tests are of the same general pattern. For the normal turbulent sections of the pipe they are, as might be expected, similar to those obtained for the enlargement experiments.

The effect of the abrupt contraction on the pipe surface temperatures is less marked than is the effect of the abrupt enlargement. There is, however, a consistently low temperature point at 2 diameters downstream of the abrupt contraction. The effect is therefore, once again, to cause an increase in the local heat transfer coefficient.

A slight inlet end effect is noticed, this time at entrance to the 2 inch diameter pipe.

5.11 Heat Balances

Heat balances similar to those for the abrupt enlargement experiments, were drawn up and are given in Table 9.

It was shown in Paragraph 5.6 that the accuracy of the heat balance depended on the degree of mixing of the outlet water. In this series of tests, the increased static head of water allowed for the introduction of the $\frac{1}{2}$ inch diameter mixing pipe at the higher Reynolds Number of 26,000, with a consequent improvement of both mixing and heat balance.

Also the reliability of the outlet water temperature measurement could be checked by the temperature traverse. In all tests, this traverse showed a negligible variation of temperature across the stream, and its individual temperatures agreed well with the outlet water temperature.

A general all-round improvement will be seen in the heat balances, as compared with those in Table 8.

Since the outlet water temperature has been proved to be reliable, the alternative and more straight-forward method of calculating water temperatures inside the pipe will be used, viz. that of dividing the water temperature difference between inlet and outlet in proportion to the heat inputs to the water.

CHAPTER 6

Calculation of Local Heat Transfer Coefficients

The calculation of the heat transfer coefficients from the experimental observations is a step by step process. The steps involved are as follows:-

1. By using the thermocouple calibration given in Appendix 1, the potentiometer readings are converted to outside pipe surface temperatures.
2. The temperature drop through the pipe wall ΔT is calculated by the method described in Appendix 3.
3. Hence, by subtraction, the inside pipe surface temperatures are obtained.
4. Water temperatures inside the experimental pipe can be calculated by either of two methods.
   (a) For the abrupt enlargement tests the method used is "Water temperature calculation from electrical heat input" (Paragraph 4.3a).
   (b) For the abrupt contraction tests the method used is "Water temperature calculation from measured water temperature rise" (Paragraph 4.3a)
5. Hence, by subtraction, the difference between the inside pipe surface temperature and the water temperature can be found.

Thermocouple Position	Potentiometer Reading (mV)	Outside Surface Temperature of Pipe (°C)	Temperature drop through Pipe Wall (°C)	Inside Surface Temperature of Pipe (°C)
1	2.589	13.34	1.23	12.11
2	2.635	13.57	1.23	12.34
3	2.664	13.72	1.23	12.49
4	2.692	13.86	1.23	12.63
5	2.711	13.95	1.23	12.72
6	2.722	14.00	1.23	12.77
7	2.730	14.05	1.23	12.82
8	2.738	14.09	1.23	12.86
9	2.442	12.58	0.37	12.21
10	2.385	12.30	0.37	11.93
11	2.410	12.42	0.37	12.05
12	2.460	12.68	0.37	12.31
13	2.540	13.08	0.37	12.71
14	2.648	13.63	0.37	13.26
15	2.818	14.49	0.37	14.12
16	2.886	14.84	0.37	14.47
17	2.941	15.12	0.37	14.75
18	3.004	15.43	0.37	15.06
19	3.033	15.58	0.37	15.21
20	3.068	15.76	0.37	15.39
21	3.096	15.90	0.37	15.53
22	3.130	16.07	0.37	15.70
23	3.160	16.22	0.37	15.81
24	3.190	16.37	0.37	16.00

TABLE 10

Calculation of Inside Pipe Surface Temperatures for Abrupt Enlargement Test No. 1

6. The heat input to the water per unit surface area and unit time is calculated by the method corresponding to that by which the water temperatures in the pipe are determined.

7. Finally, the heat input to the water per unit surface area and unit time is divided by the temperature difference between pipe and water, the result being the heat transfer coefficient.

The only differences between the methods of calculation for the enlargement and contraction experiments occur in steps Nos. 4 and 6.

A complete calculation of heat transfer coefficients for a typical abrupt enlargement experiment will be given. The calculations involved in steps 4 and 6 will be given for a typical abrupt contraction experiment.

A <u>Abrupt Enlargement</u>

6.1 <u>Calculation of Inside Pipe Surface Temperatures</u>

This calculation includes steps 1, 2 and 3.

Column 1 of Table 10 gives the potentiometer readings which are converted to temperatures, given in Column 2, using the thermocouple calibration (Appendix 1).

The temperature drops through the pipe walls were calculated as follows:-

Shunt voltage = 59.10 millivolts

$\therefore$ Current = 4940 amps

$\therefore$ Current^2 = 24.40×10^6 amps^2

(a) <u>1 inch diameter pipe</u>

Outside pipe surface temperatures range from 13.34°C. to 14.05°C.

$\therefore$ From Appendix 3 Table 20

$$\text{Approximate } \frac{\Delta T}{\text{current}^2} = 0.05022 \times 10^{-6} \ {}^{o}\text{C/amps}^2$$

$\therefore$ Approximate ΔT = <u>1.225°C</u>.

Then for thermocouple position 1

Temperature half way through the pipe wall

$$= 13.34 - \frac{1.225}{2} = 12.73^{o}\text{C.}$$

$\therefore$ More accurate $\dfrac{\Delta T}{\text{current}^2}$ at this position

$$= 0.050245 \times 10^{-6}$$

$\therefore$ More accurate ΔT = 1.226°C.

i.e. the second approximation makes a negligible difference in the value of ΔT.

By repeating this procedure for thermocouple positions 2-8, it is found that ΔT varies from 1.226°C. at position 1 to 1.225°C. at position 8 and can therefore be taken as 1.23°C. at all positions on the 1 inch diameter pipe.

(b) 2 inch diameter pipe

By using the same reasoning as above, the first approximation for ΔT at position 9 is $0.370^{\circ}C.$

The variation of the second approximation is from $0.369^{\circ}C.$ at position 9 to $0.368^{\circ}C.$ at position 24.

Hence ΔT can be taken as $0.37^{\circ}C.$ at all positions on the 2 inch diameter pipe.

These values of temperature drop through the pipe wall are given in Column 3 of Table 10 and the resulting inside pipe surface temperatures in Column 4.

6.2 Calculation of Water Temperatures.

The uniformity of electrical heat input per unit length of 1 inch or 2 inch diameter pipe, as measured by the voltage drops, was such that the maximum error in water temperature involved in assuming linear water temperature rise, was never greater than $0.005^{\circ}C.$ The effect of heat conduction along the pipe on the uniformity of heat input per unit length is considered in Appendix 2, and shown to be negligible. Hence, for all experiments, water temperature rise was considered to be linear, but of two slightly different gradients in the 1 inch and 2 inch diameter pipes.

The problem now resolves itself into finding the water

FIG 21 POSITIONS A, B, C, D, E, F (ENLARGEMENT)

temperatures at known positions near the ends of the 1 inch and 2 inch pipes. Any water temperatures between these known positions can then be easily calculated.

Referring to Fig. 21, positions A and F represent inlet and outlet temperature measuring points respectively. Positions B, C, D and E represent potential points near the ends of the pipes.

<u>Measured inlet water temperature at A</u>	= <u>9.62°C.</u>
Water flow	= 2341 gm/sec.
Specific heat of water (Table 3)	= 4.187 joules/(gm)(°C)
Heat from inlet conductors	= 44.9 watts
Heat generated in pipe between A and B	= 546.2 watts
Heat transferred through insulation between A and B	= 0.34 watts
∴ Total heat supplied to water between A and B	= 591.4 watts

∴ <u>Temperature at B</u> $= 9.62 + \dfrac{594.1}{2341 \times 4.187} = \underline{9.68^{\circ}C}$

Heat generated in pipe between B and C	= 6696 watts
Heat transferred through insulation between B and C	= 4.43 watts
∴ Total heat supplied to water between B and C	= 6700.4 watts

∴ <u>Temperature at C</u> $= 9.68 + \dfrac{6700.4}{2341 \times 4.187} = \underline{10.36^{\circ}C}$

Thermocouple Position	Distance from B (inches)	Proportion of Temperature Rise B to C	Water Temperature (°C)
1	1	0.0105	9.69
2	13	0.1361	9.77
3	37	0.3872	9.95
4	61	0.6384	10.12
5	73	0.7639	10.20
6	79	0.8267	10.24
7	85	0.8895	10.28
8	91	0.9523	10.32

Thermocouple Position	Distance from D (inches)	Proportion of Temperature Rise D to E	Water Temperature (°C)
9	0.3	0.0015	10.38
10	2.3	0.0114	10.40
11	4.3	0.0214	10.41
12	6.3	0.0313	10.42
13	8.3	0.0412	10.43
14	10.3	0.0512	10.45
15	16.3	0.0810	10.49
16	22.3	0.1108	10.53
17	34.3	0.1704	10.61
18	58.3	0.2896	10.78
19	82.3	0.4088	10.95
20	106.3	0.5281	11.12
21	130.3	0.6473	11.29
22	154.3	0.7665	11.46
23	178.3	0.8857	11.63
24	202.3	1.0050	11.79

TABLE 11

Calculation of Water Temperature for Abrupt Enlargement Test No. 1

By successive additions of corrected electrical heat inputs, the temperatures at D and E can be calculated in a similar manner.

Temperature at D $= 10.38^{\circ}C.$

Temperature at E $= 11.79^{\circ}C.$

Table 11 shows how the temperature differences B to C and D to E are divided so that water temperatures are obtainable at all thermocouple positions.

6.3 Calculation of Heat Transfer Coefficients

(a) For the 1 inch diameter pipe

Heat input to the water between B and C = 6700 watts

Inside surface area of pipe between B and C = 1936.3 sq.cm.

$$\therefore \text{Heat Input to Water} = \frac{6700}{1936.3}$$

$$= \underline{3.460 \text{ watts/sq.cm.}}$$

(b) For the 2 inch diameter pipe

Heat input to the water between D and E = 13801 watts

Inside surface area of pipe between D and E = 8155.9 sq.cm.

$$\therefore \text{Heat Input to Water} = \frac{13801}{8155.9}$$

$$= 1.692 \text{ watts/sq.cm.}$$

The local heat transfer coefficients can now be obtained by dividing the appropriate value of heat input

Thermocouple Position	Inside Surface Temperature of Pipe (^{o}C)	Water Temperature (^{o}C)	Temperature Difference Pipe-Water (^{o}C)	Heat Transfer Coefficient Joules/(sq.cm.)(sec)(^{o}C)
1	12.11	9.69	2.42	1.428
2	12.34	9.77	2.57	1.345
3	12.49	9.95	2.54	1.361
4	12.63	10.12	2.51	1.378
5	12.72	10.20	2.52	1.372
6	12.77	10.24	2.53	1.367
7	12.82	10.28	2.54	1.361
8	12.86	10.32	2.54	1.361
9	12.21	10.38	1.83	0.925
10	11.93	10.40	1.53	1.106
11	12.05	10.41	1.64	1.032
12	12.31	10.42	1.89	0.895
13	12.71	10.43	2.28	0.742
14	13.26	10.45	2.81	0.602
15	14.12	10.49	3.63	0.466
16	14.47	10.53	3.94	0.430
17	14.75	10.61	4.14	0.409
18	15.06	10.78	4.28	0.395
19	15.21	10.95	4.26	0.397
20	15.39	11.12	4.27	0.396
21	15.53	11.29	4.24	0.399
22	15.70	11.46	4.24	0.399
23	15.81	11.63	4.18	0.404
24	16.00	11.79	4.21	0.402

TABLE 12

Calculation of Heat Transfer Coefficients for Abrupt Enlargement Test No. 1

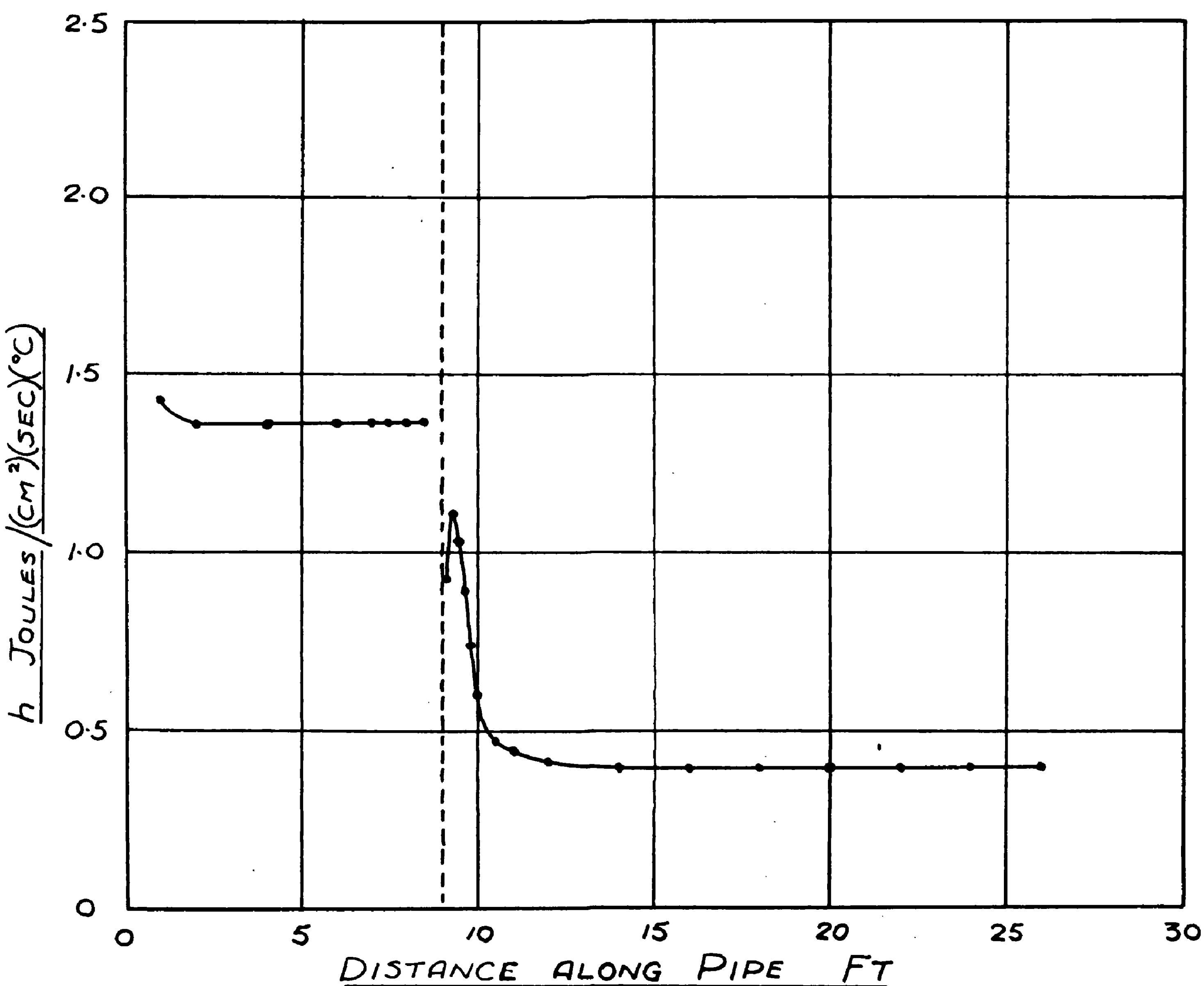


FIG. 22 HEAT TRANSFER COEFFICIENTS

Thermocouple Position	Potentiometer Reading (mV)	Outside Surface Temperature of Pipe (°C)	Temperature drop through Pipe Wall (°C)	Inside Surface Temperature of Pipe (°C)
1	2.985	15.30	0.36	14.94
2	3.055	15.66	0.36	15.30
3	3.075	15.76	0.36	15.40
4	3.145	16.11	0.36	15.75
5	3.174	16.26	0.36	15.90
6	3.204	16.41	0.36	16.05
7	3.231	16.54	0.36	16.18
8	3.015	15.46	1.21	14.25
9	3.042	15.59	1.21	14.38
10	3.085	15.81	1.21	14.60
11	3.106	15.92	1.21	14.71
12	3.107	15.92	1.21	14.71
13	3.107	15.92	1.21	14.71
14	3.113	15.95	1.21	14.74
15	3.098	15.88	1.21	14.67
16	3.118	15.97	1.21	14.76
17	3.125	16.01	1.21	14.80
18	3.149	16.13	1.21	14.92
19	3.186	16.32	1.21	15.11

TABLE 13

Calculation of Inside Pipe Surface Temperatures

for Abrupt Contraction Test No. 1

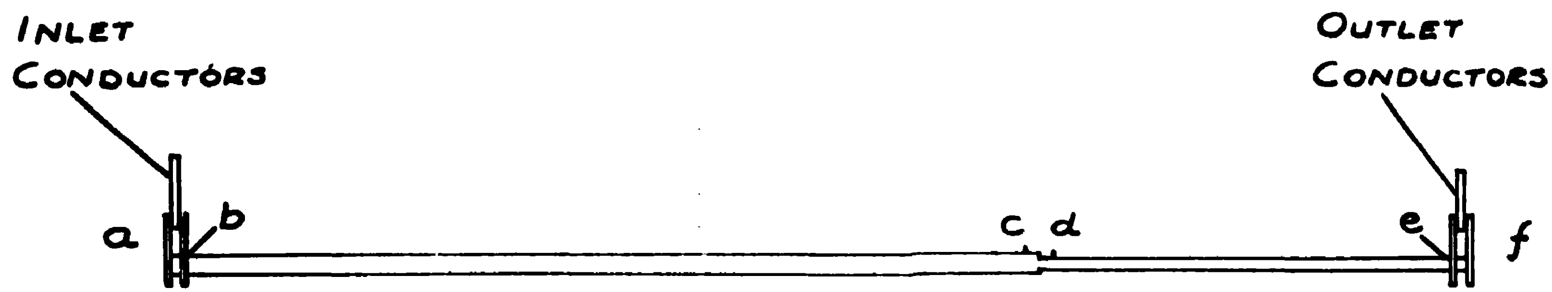


FIG 23 POSITIONS a, b, c, d, e, f (CONTRACTION)

per unit surface area and unit time by the temperature difference between the pipe and the water.

The last few stages of the calculation are indicated in Table 12. Fig. 22 shows the variation of heat transfer coefficient along the pipe for this experiment.

B Abrupt Contraction

6.4 Calculation of Inside Pipe Surface Temperatures

This part of the calculation is exactly similar in method to that given in paragraph 6.1. The results of this calculation are given in Table 13.

6.5 Calculation of Water Temperatures

In Fig. 23, a and f represent inlet and outlet temperature measuring points respectively. Positions b and e represent potential points immediately after inlet, and immediately before outlet copper conductors respectively. Positions c and d represent potential points on either side of the abrupt contraction.

T_a and T_f are measured and hence from an estimation of the heat conducted from inlet and outlet conductors, the values of $T_b - T_a$ and $T_f - T_e$ can be found. Hence T_b and T_e can be found. Therefore ΔT_{be} is known.

In this case, the problem is to divide the temperature difference $T_e - T_b$ in the proportion of the heat inputs, and hence find all water temperatures in the 1 inch and 2 inch diameter pipes. In order to do this, T_c and T_d

must be found.

The temperature rise ΔT_{cd} can be found from the potential difference between c and d.

Then $\Delta T_{bc} + \Delta T_{de} = \Delta T_{be} - \Delta T_{cd}$ ....(1)

The ratio of the electrical heat input b to c to the electrical heat input d to e is assumed to be equal to the ratio of the heat transferred through the pipe lagging b to c to the heat transferred through the pipe lagging d to e.

This was found to be the case to within 5%, from estimations of heat transferred through the pipe lagging, and since the quantity of heat transferred through the lagging is small compared with the heat generated in the pipe, the above assumption can be justified.

Then $$\frac{\Delta T_{bc}}{\Delta T_{de}} = \frac{\text{Voltage drop b-c}}{\text{Voltage drop d-e}} \quad \text{..........(2)}$$

From equations (1) and (2) can be found ΔT_{bc} and ΔT_{de} and hence T_c and T_d.

For the test under consideration

$\underline{T_a}$	= $\underline{11.00^\circ C}$
$\underline{T_f}$	= $\underline{13.04^\circ C}$
Water flow	= 2597 gm/sec.
Heat conducted from inlet conductors	= 41.87 watts

$\therefore$ Water temperature rise from a to b $= \dfrac{41.87}{2597 \times 4.187} = 0.004^{\circ}C$

$\therefore$ $\underline{T_b}$ $= \underline{11.00^{\circ}C}$

Heat conducted from outlet conductors = 204.4 watts

$\therefore$ Water temperature rise from e to f $= \dfrac{204.4}{2597 \times 4.187} = 0.019^{\circ}C$

$\therefore$ $\underline{T_e}$ $= \underline{13.02^{\circ}C}$

$\therefore$ $\underline{\Delta T_{be}}$ $= 13.02 - 11.00 = \underline{2.02^{\circ}C}$

Electrical heat input from c to d = 231.9 watts

$\therefore$ Water temperature rise ΔT_{cd} $= \dfrac{231.9}{2597 \times 4.187} = \underline{0.021^{\circ}C}$

$\therefore$ $\underline{\Delta T_{bc} + \Delta T_{de} = 2.02 - 0.02 = 2.00}$ ....(3)

Ratio of $\dfrac{\text{Voltage drop b-c}}{\text{Voltage drop d-e}} = 1.982$

$$\therefore \quad \frac{\Delta T_{bc}}{\Delta T_{de}} = 1.982 \qquad \text{...............(4)}$$

Solving equations (3) and (4)

$\Delta T_{bc} = 1.33^{\circ}C$

$\Delta T_{de} = 0.67^{\circ}C$

Hence $T_a = 11.00^{\circ}C$

$T_b = 11.00^{\circ}C$

$T_c = 12.33^{\circ}C$

$T_d = 12.35^{\circ}C$

Thermocouple Position	Distance from b (inches)	Proportion of Temperature Rise b to c	Water Temperature (°C)
1	20.25	0.0961	11.13
2	44.25	0.2101	11.28
3	92.25	0.4380	11.58
4	140.25	0.6659	11.89
5	164.25	0.7798	12.04
6	194.25	0.9223	12.23
7	206.25	0.9792	12.30

Thermocouple Position	Distance from d (inches)	Proportion of Temperature Rise d to e	Water Temperature (°C)
8	0.625	0.0061	12.35
9	2.625	0.0255	12.37
10	4.63	0.0450	12.38
11	8.63	0.0838	12.41
12	9.63	0.0936	12.41
13	14.63	0.1422	12.45
14	17.63	0.1713	12.46
15	22.63	0.2199	12.50
16	34.63	0.3366	12.58
17	46.63	0.4532	12.65
18	70.63	0.6865	12.81
19	94.63	0.9198	12.97

TABLE 14

Calculation of Water Temperatures for Abrupt Contraction Test No. 1

T_e = 13.02°C

T_f = 13.04°C

Table 14 shows how the water temperature rises in the 1 inch and 2 inch diameter pipes are divided to obtain the required values of water temperature.

6.6 Calculation of Heat Transfer Coefficients

(a) For the 2 inch diameter pipe

Heat input to the water between b and c

= Heat required to raise the water temperature by 1.33°C

= 14400 watts

Inside surface area of pipe between b and c

= 8533 sq.cm.

$\therefore$ Heat Input to Water $= \dfrac{14400}{8533}$

= 1.688 watts/sq.cm.

(b) For the 1 inch diameter pipe

Heat Input d - e = Heat required to raise water temperature by 0.67°C.

= 7260 watts

Inside surface area of pipe between d and e

= 2085 sq.cm.

$\therefore$ Heat Input to Water $= \dfrac{7260}{2085}$ = 3.480 watts/sq.cm.

Thermocouple Position	Inside Surface Temperature of Pipe (°C)	Water Temperature (°C)	Temperature Difference Pipe-Water (°C)	Heat Transfer Coefficient Joules/sq cm)(sec)(°C)
1	14.94	11.13	3.81	0.441
2	15.30	11.28	4.02	0.418
3	15.40	11.58	3.82	0.440
4	15.75	11.89	3.86	0.436
5	15.90	12.04	3.86	0.436
6	16.05	12.23	3.82	0.440
7	16.18	12.30	3.88	0.434
8	14.25	12.35	1.90	1.829
9	14.38	12.37	2.01	1.729
10	14.60	12.38	2.22	1.565
11	14.71	12.41	2.30	1.511
12	14.71	12.41	2.30	1.511
13	14.71	12.45	2.26	1.538
14	14.74	12.46	2.28	1.524
15	14.67	12.50	2.17	1.601
16	14.76	12.58	2.18	1.594
17	14.80	12.65	2.15	1.616
18	14.92	12.81	2.11	1.647
19	15.11	12.97	2.14	1.624

TABLE 15

Calculation of Heat Transfer Coefficients for Abrupt

Contraction Test No. 1

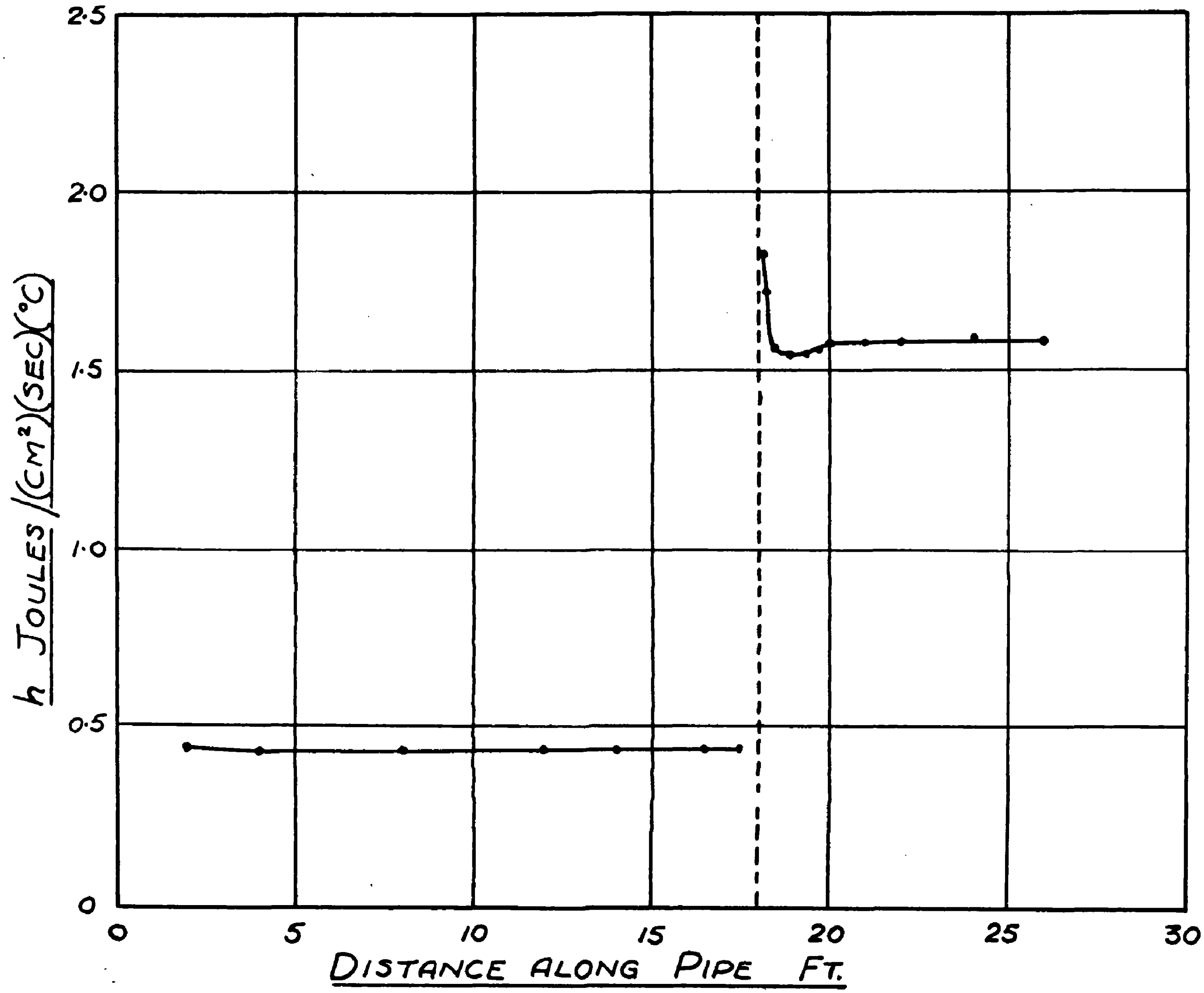


Fig 24 Heat Transfer Coefficients

The local heat transfer coefficients are now calculated as described in paragraph 6.3. Table 15 gives the last stages of the calculation. The distribution of heat transfer coefficient along the pipe for this experiment is given in Fig. 24.

CHAPTER 7

The Effect of the Change of Section

The review of the previous work in this field has shown that, for normal turbulent flow in pipes, the heat transfer coefficient can be expressed by some form of the Nusselt equation. Sieder and Tate[26] have indicated that the temperature difference between pipe and fluid can have an effect on the value of the heat transfer coefficient.

In order to assess the effect of the change of section on the heat transfer coeffidient in the present experiments, consideration will first be given to the normal turbulent flow sections of the pipe. A Nusselt equation will be derived for these sections neglecting, in the first instance, any effect of temperature difference between pipe and water. This Nusselt equation will be compared with those derived by previous workers.

The effect of temperature difference between pipe and water at constant Reynolds Number, which is equivalent to the effect of variation of heat input at constant Reynolds Number, will then be considered for normal turbulent flow sections. The values of heat transfer coefficient for normal turbulent flow will then be expressed accurately. The reliability of these values can then be estimated by comparison with previous data on the subject.

The effect of the change of section on the heat transfer coefficient will be based on a comparison of local heat transfer coefficients in the excess turbulent sections of the pipe with these well established coefficients for normal turbulent flow. Modified Nusselt equations will be derived which will apply at fixed positions in the excess turbulent flow sections of the pipe. The total effect of the change of section will be estimated in terms of an equivalent number of extra diameters of pipe length, the whole pipe being considered to be under normal turbulent flow conditions.

7.1 Nusselt Equation for Normal Turbulent Flow

The Nusselt, Reynolds and Prandtl Numbers were calculated for all pipe thermocouple positions in all enlargement and contraction experiments, as described in Chapter 4.

The Nusselt Number was found to have a constant value for the normal turbulent section of the 1 inch pipe, and a different constant value for the normal turbulent section of the 2 inch pipe, in each experiment. Hence two values of what may be termed the "normal turbulent"

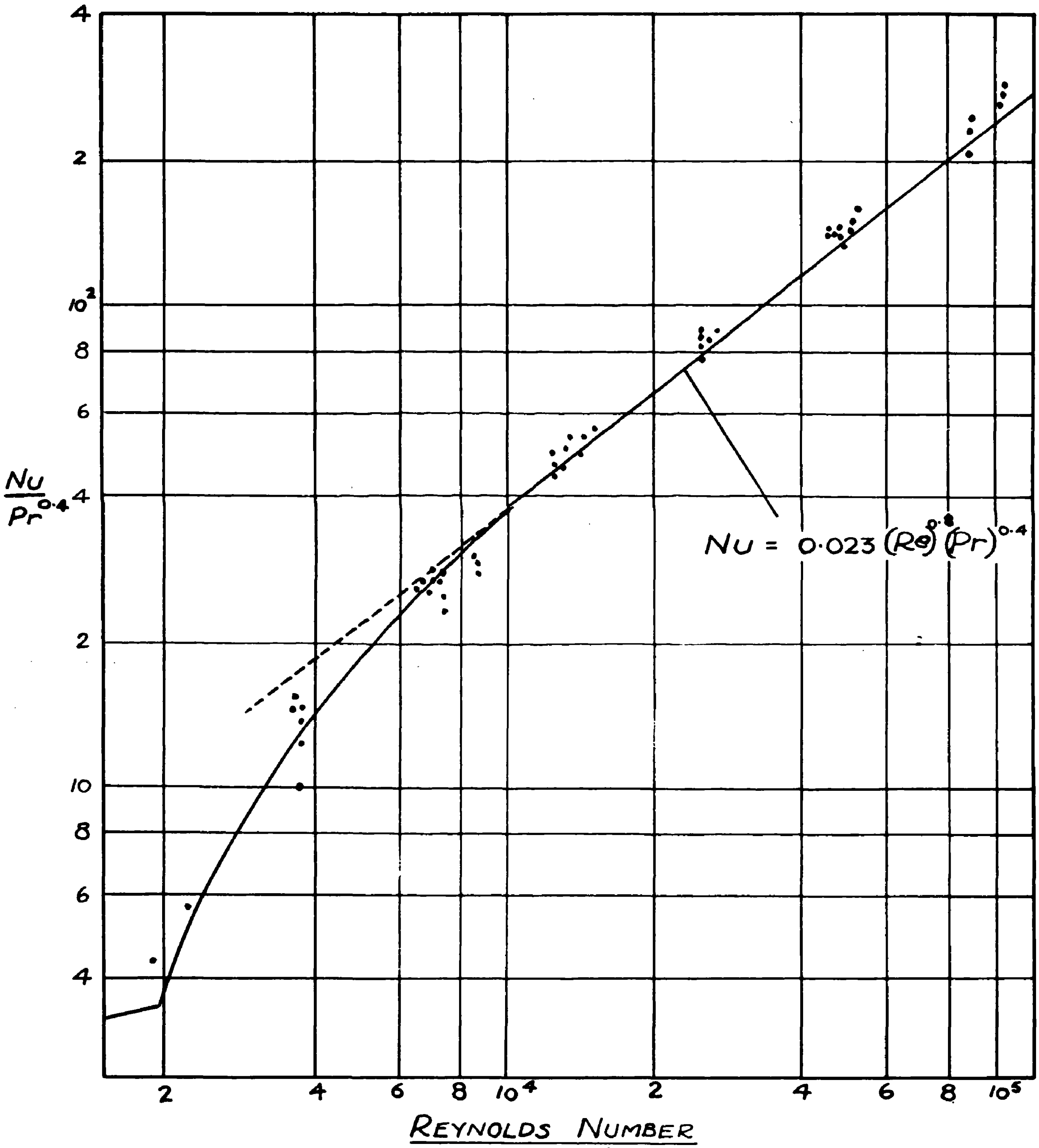


FIG 25 GRAPH OF $Nu/Pr^{0.4}$ v. Re FOR NORMAL TURBULENT FLOW

Nusselt Number were obtained for each experiment, one referring to the 1 inch pipe and the other to the 2 inch pipe.

Considering only these Nusselt Numbers, the ratio $\frac{Nu}{Pr^{0.4}}$ was plotted logarithmically against Re. This was done in order to compare the present data with that of Brown, Fishenden and Saunders[3], given in Fig. 2. The plot of $\frac{Nu}{Pr^{0.4}}$ v Re is shown in Fig. 25 and superimposed upon it is the curve from Fig. 2 whose equation is

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$$

for Reynolds Numbers greater than 10,000.

It will be seen that the experimental points lie very close to this line and that their scatter is considerably less than that of the points in Fig. 2.

Hence, for normal turbulent flow, the experimental data are closely represented by the equation

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$$

for Reynolds Numbers greater than 10,000.

7.2 <u>Effect of Heat Input on the Heat Transfer Coefficient</u>

The experimental points in Fig. 25 were plotted in groups of three at each Reynolds Number. The highest value of $\frac{Nu}{Pr^{0.4}}$ at each Reynolds Number corresponded to

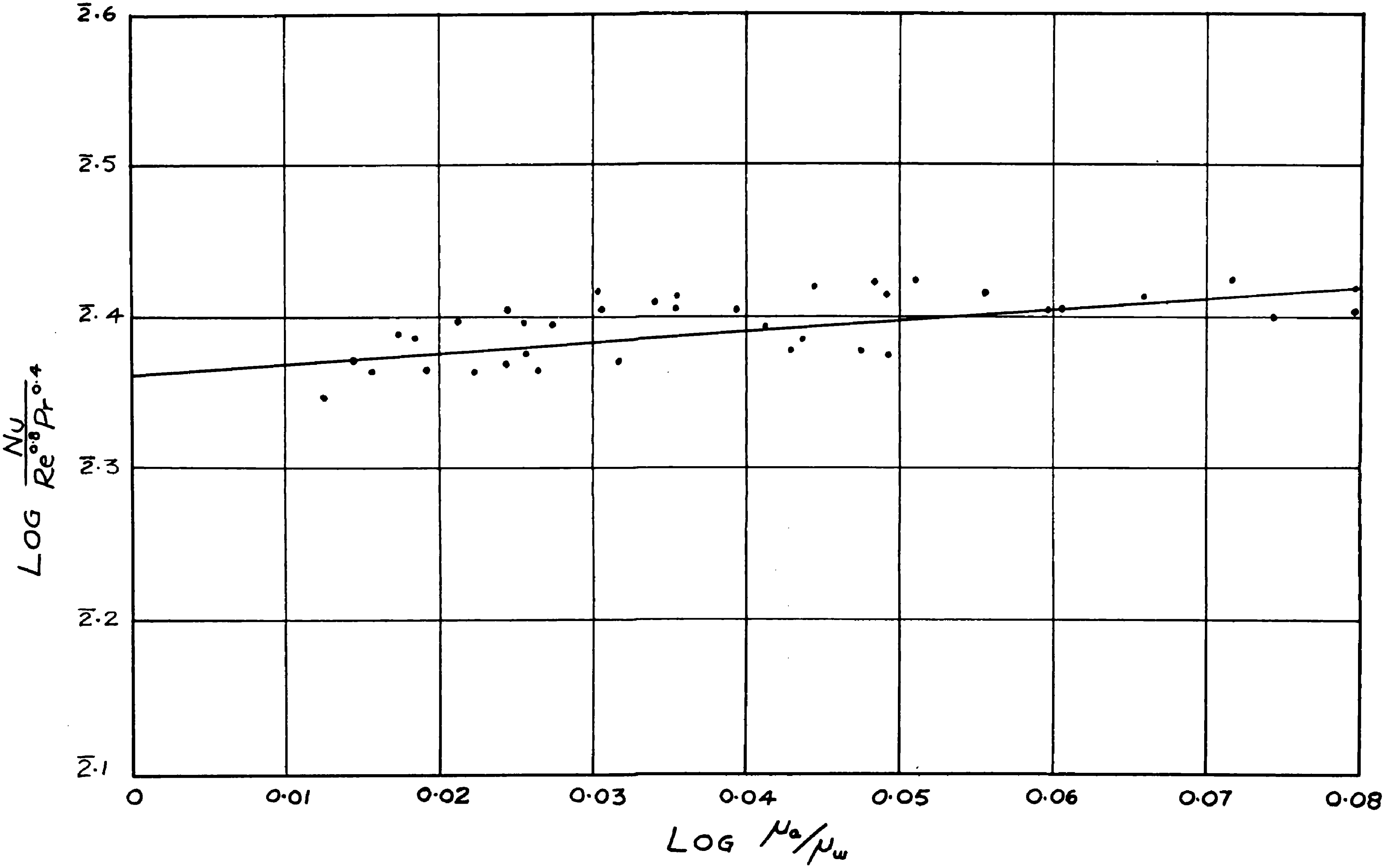


FIG 26 EFFECT OF HEAT INPUT ON HEAT TRANSFER COEFFICIENT

the highest heat input at that Reynolds Number, and the lowest value to the lowest heat input.

The effect of heat input on the heat transfer coefficient is therefore noticeable.

In their analysis of this effect, Sieder and Tate[26] pointed out that the experimental data at that time fell into two groups, one for heating and the other for cooling. Their method of eliminating the effect of heat input was to introduce the ratio $(\mu_a/\mu_w)^{0.14}$ into their form of the Nusselt equation, where μ_a was the viscosity of the liquid at its bulk temperature and μ_w the viscosity of the liquid at the wall temperature. They maintained that the effect was not noticed in heat transfer to liquids of viscosity lower than twice that of water.

Proceeding along the same lines as Sieder and Tate, the group $\dfrac{Nu}{Re^{0.8}Pr^{0.4}}$ was plotted logarithmically against the ratio (μ_a/μ_w) for normal turbulent flow, at all Reynolds Numbers above 10,000. The points are shown in Fig. 26 and are seen to lie close to the straight line whose slope is 0.07. This line makes an intercept of $\bar{2}.362$ with the y-axis.

Hence when $\mu_w = \mu_a$ i.e. at zero heat flow

$$\log_{10} \frac{Nu}{Re^{0.8}\ Pr^{0.4}} = \bar{2}.362$$

or $$\frac{Nu}{Re^{0.8}Pr^{0.4}} = 0.023$$

The experimental results, for normal turbulent flow, may therefore be expressed by the equation

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4} \left[\frac{(\mu_a)}{(\mu_w)}\right]^{0.07}$$

for Reynolds Numbers greater than 10,000.

The index of the $\left(\frac{\mu_a}{\mu_w}\right)$ group is half of that found by Sieder and Tate for liquids of higher viscosity, and hence the effect is smaller. In fact, it is small enough to have been unnoticed by Sieder and Tate and has been neglected by most workers.

7.3 <u>Extension of Analysis to cover Excess Turbulent Flow</u>

Reverting to the Nusselt equation for normal turbulent flow, neglecting the effect of heat input

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$$

This equation may be written

$$\frac{Nu}{Re^{0.8}Pr^{0.4}} = 0.023$$

In order to extend the analysis to cover the excess turbulent sections of the pipe, the group $\frac{Nu}{Re^{0.8}Pr^{0.4}}$ was calculated for all positions and all experiments.

This group is of importance, since variation of heat transfer coefficient due to different pipe diameters, water velocities and temperatures are absorbed by it, leaving in prominence the variations due simply to the

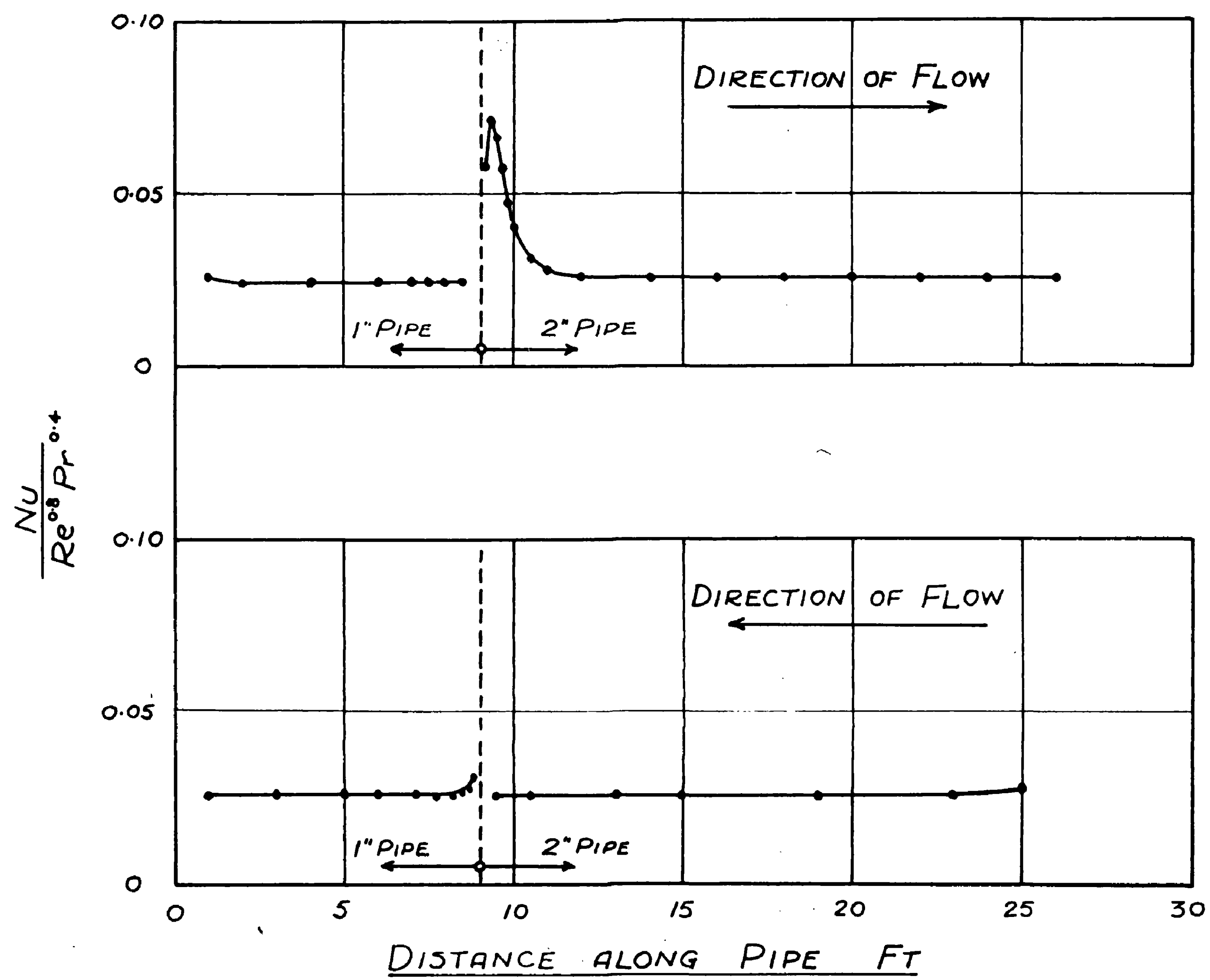


FIG 27 DISTRIBUTION OF $\frac{Nu}{Re^{0.8} Pr^{0.4}}$ ALONG THE PIPE

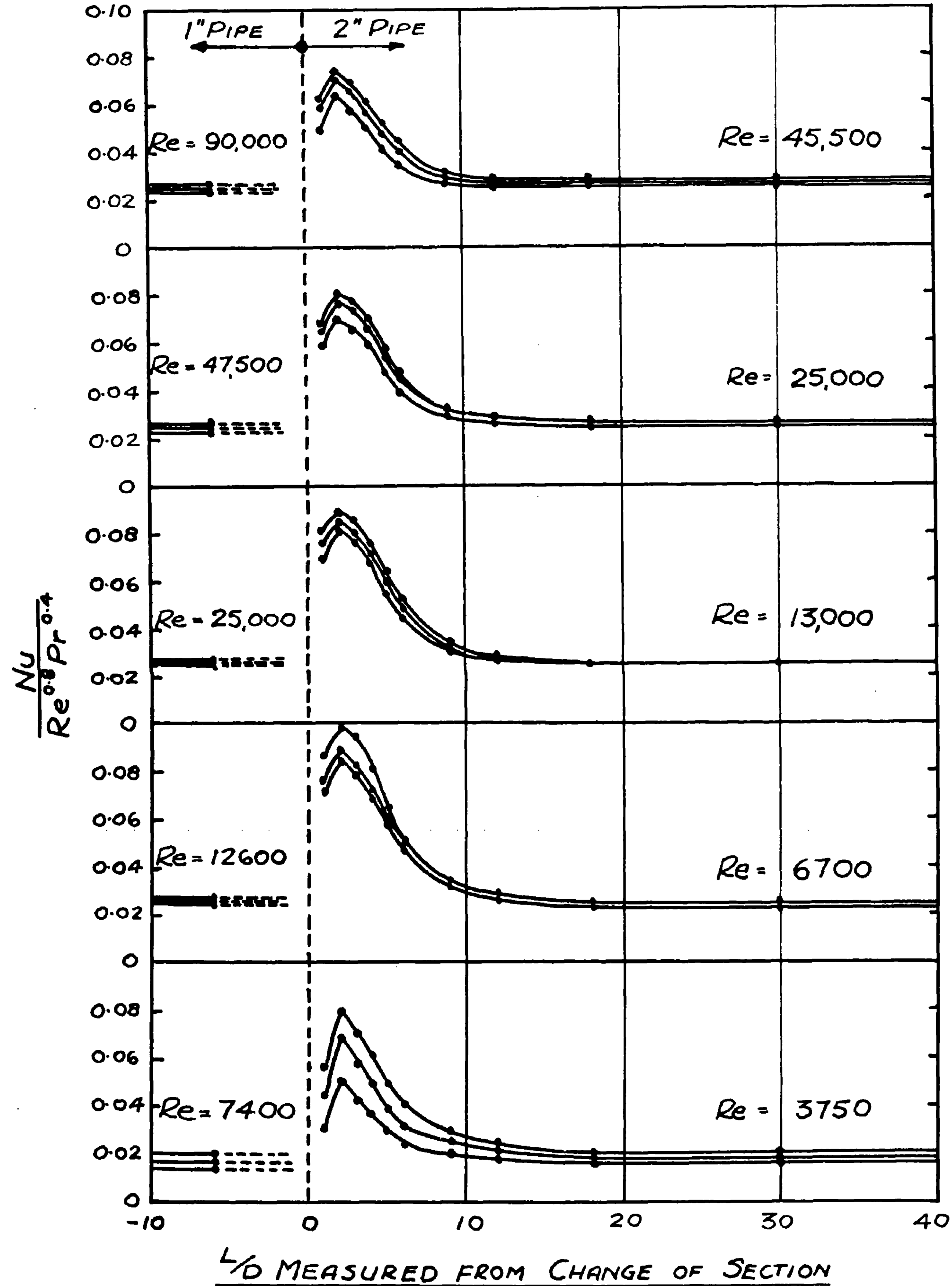


FIG 28 EFFECT OF CHANGE OF SECTION (ABRUPT ENLARGEMENT)

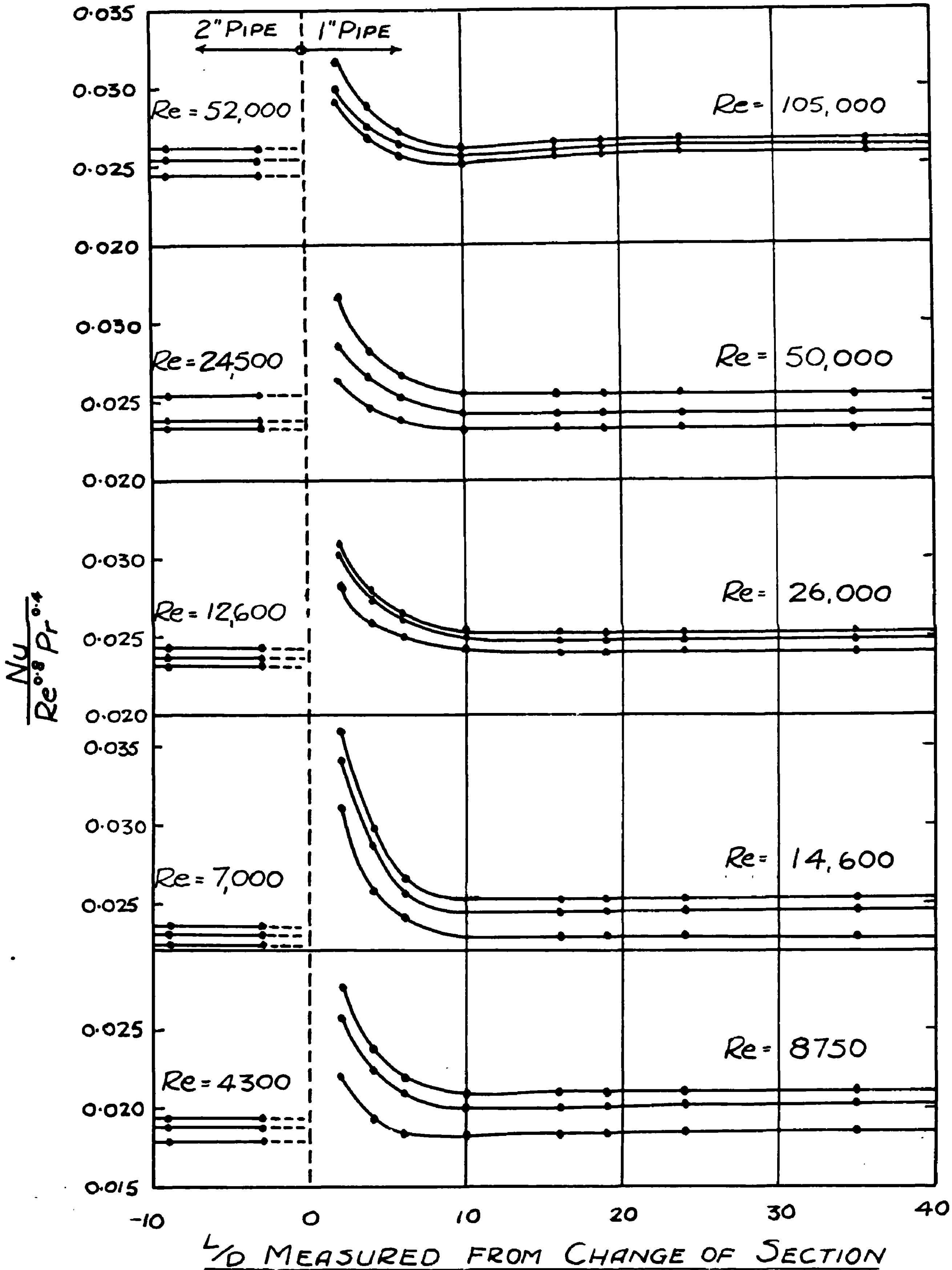


FIG 29 EFFECT OF CHANGE OF SECTION

(ABRUPT CONTRACTION)

change in section.

Fig. 27 shows the distribution of $\frac{Nu}{Re^{0.8}Pr^{0.4}}$ along the pipe for a typical enlargement and a typical contraction experiment. The effect of the change of section is quite clearly marked. Again it will be noticed that the effect of the abrupt enlargement is greater than that of the abrupt contraction.

Figs. 28 and 29 show the same group plotted against numbers of pipe diameters in the interesting regions after the change of section, for all experiments.

The highest curve at each Reynolds Number corresponds to the highest heat input, and the lowest to the lowest heat input. The effect of heat input is therefore again apparent.

These curves have similar shapes to those given by Boelter, Young and Inversen[2] in Figs. 4, 5 and 6.

7.4 <u>The Effect of the Change of Section</u>.

Having established fairly conclusively the values of local heat transfer coefficients for normal turbulent flow in a pipe, a comparison will now be made of the local coefficients in the excess turbulent sections of the pipe.

Two separate methods have been used, the first of which determines an expression for the local values of

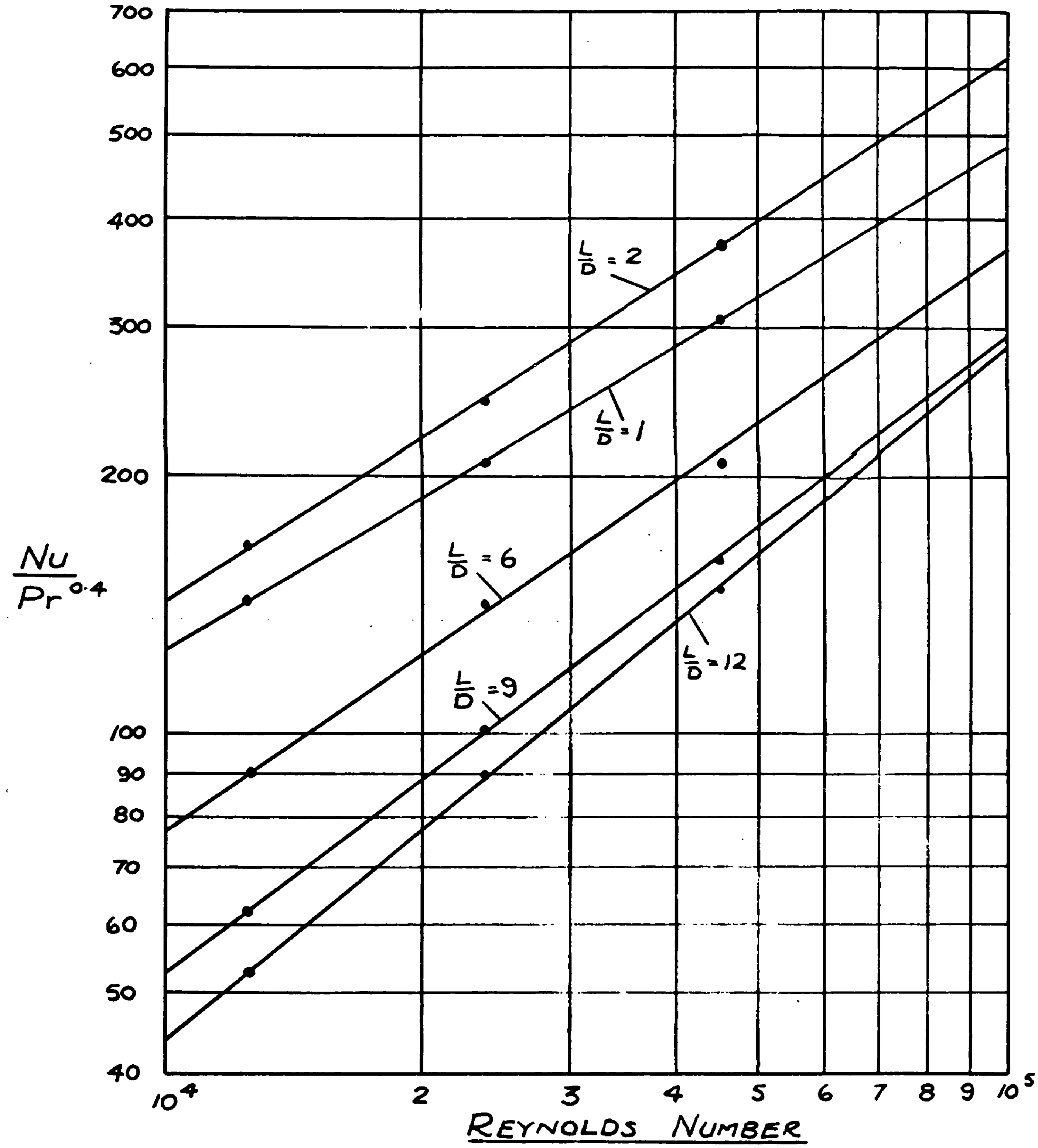


FIG 30 GRAPH OF $Nu/Pr^{0.4}$ V. Re
(ENLARGEMENT EFFECT)

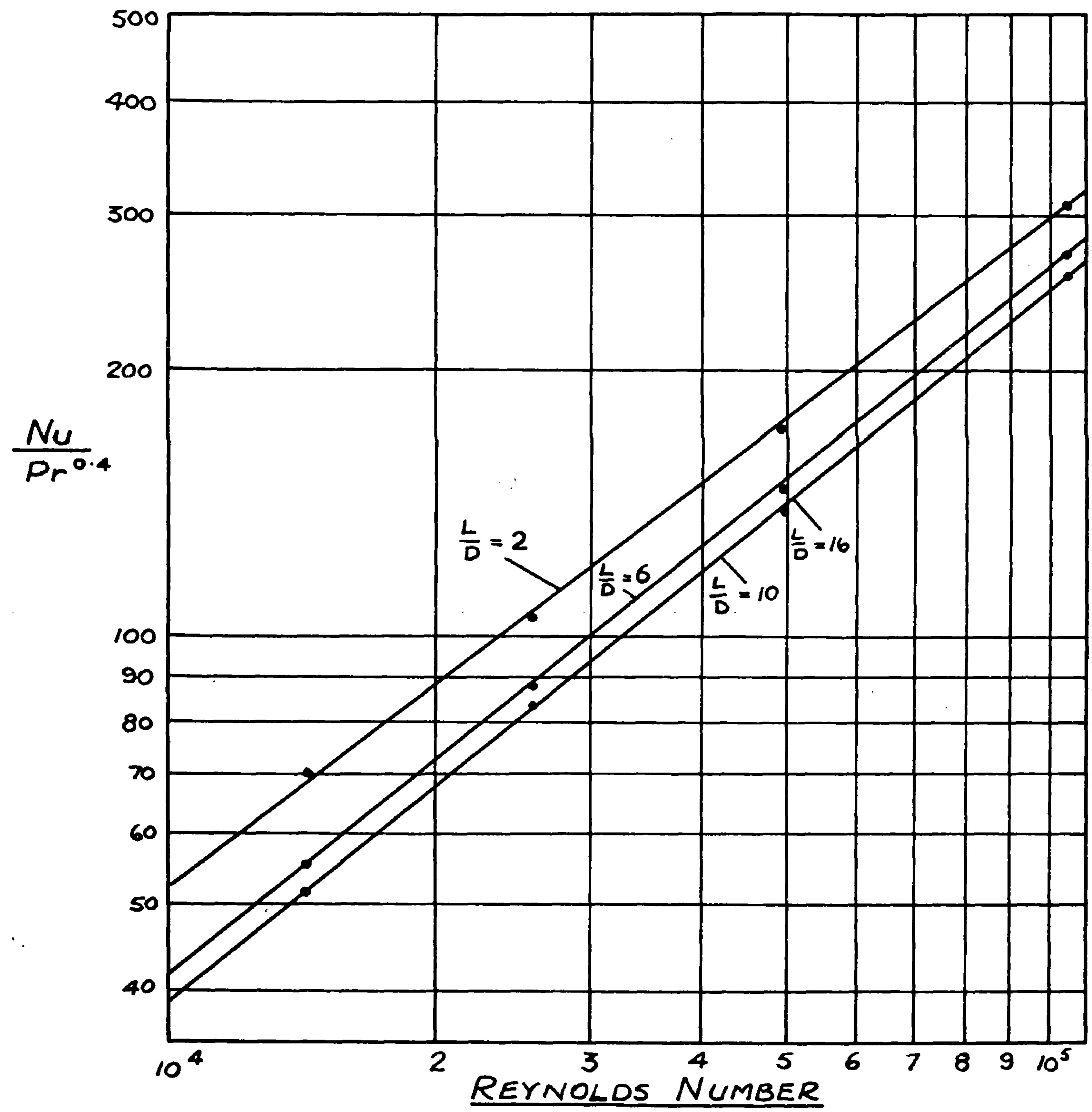


FIG 31 GRAPH OF $Nu/Pr^{0.4}$ v. Re
(CONTRACTION EFFECT)

$\frac{L}{D}$	C	n	m
1	0.497	0.60	0.40
2	0.396	0.64	0.40
6	0.122	0.70	0.40
9	0.0478	0.76	0.40
12	0.0227	0.82	0.40

Enlargement Tests.

$\frac{L}{D}$	C	n	m
2	0.0500	0.76	0.40
6	0.0257	0.80	0.40
10	0.0245	0.80	0.40
16	0.0245	0.80	0.40

Contraction Tests.

TABLE 16

Values of C, n and m in the equation $Nu = C\,(Re)^n(Pr)^m$ for fixed positions in the excess turbulent pipe sections.

heat transfer coefficient at various fixed positions in the excess turbulent sections, by the use of modified Nusselt equations. The second method expresses the total effect of the change of section in terms of an equivalent number of extra diameters of pipe length.

The complication of attempting to estimate the effect heat input in conjunction with the effect of change of section was considered to be too great. The effect of change of section is therefore considered for the tests carried out at the mean value of the three heat inputs at each Reynolds Number.

(a) Modification of the Nusselt Equation for Excess Turbulent Sections of Pipe

If the group $Nu/Pr^{0.4}$ is plotted logarithmically against Reynolds Number, for fixed positions in the excess turbulent sections of the pipe, values of C and n in the equation

$$Nu = C\ Re^{n}\ Pr^{0.4}$$

can be obtained for those positions. Figs. 30 and 31 show $Nu/Pr^{0.4}$ plotted against Reynolds Numbers greater than 10,000 for fixed positions in the enlargement and contraction experiments respectively. Table 16 gives the values of C and n obtained from these graphs.

Figs. 32 and 33 show C and n plotted against $\frac{L}{D}$

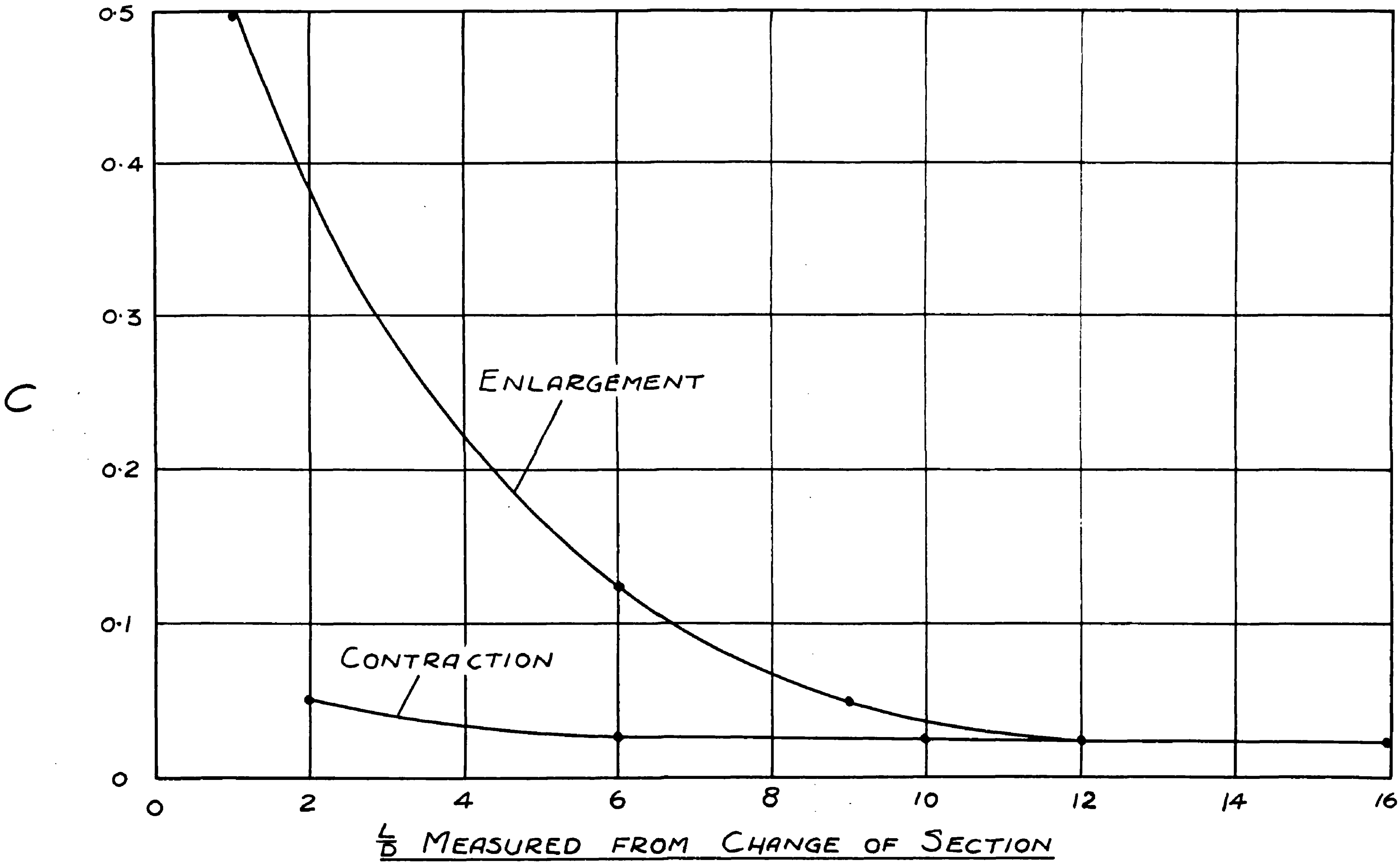


FIG 32 GRAPH OF C v. L/D

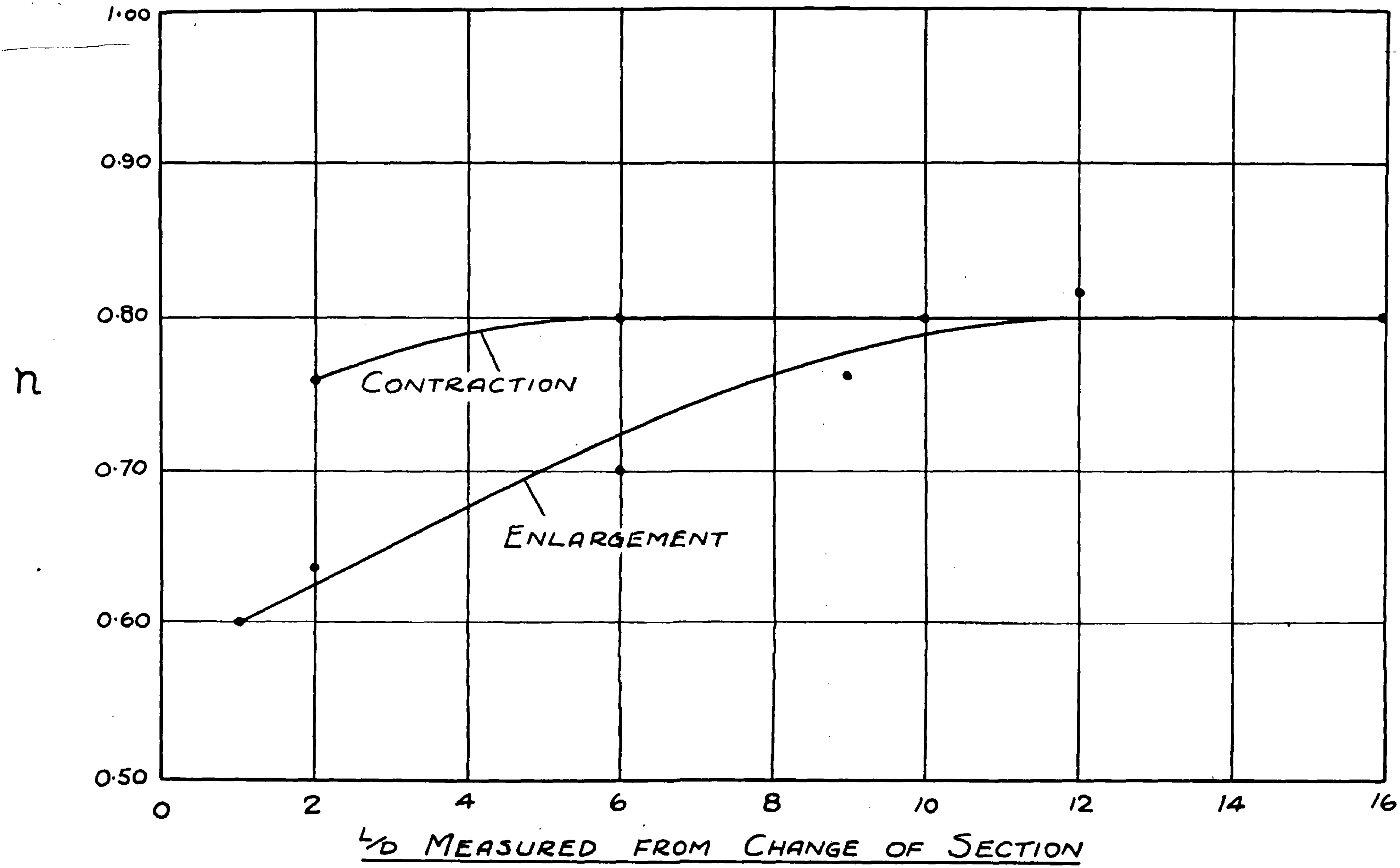


FIG 33 GRAPH OF η v. L/D

From these figures can be obtained one form of the Nusselt equation corresponding to any position in the excess turbulent sections of the pipe.

A comparison of these values of C and n can be made with those given by Alad'ev[1] in Table 2. In both cases C decreases and n increases with increasing $\frac{L}{D}$.

(b) The Total Effect of the Change of Section estimated in terms of Equivalent Extra Diameters of Pipe Length

Let $\emptyset$ represent the group $\frac{Nu}{Re^{0.8}Pr^{0.4}}$,

$\emptyset_L$ be the local value of $\emptyset$ at any point,

$\emptyset_\infty$ be the local value of $\emptyset$ in the normal turbulent section of the pipe.

In order to compare the effects of change of section in different tests, the ratio $\emptyset_L/\emptyset_\infty$ is considered. This ratio has a value of unity in normal turbulent sections of the pipe for all experiments. A comparison of effects of change of section in different experiments can therefore be made on this bases.

A graph of $\emptyset_L/\emptyset_\infty$ v $\frac{L}{D}$, measured from the change of section for a typical enlargement experiment, is shown in Fig. 34.

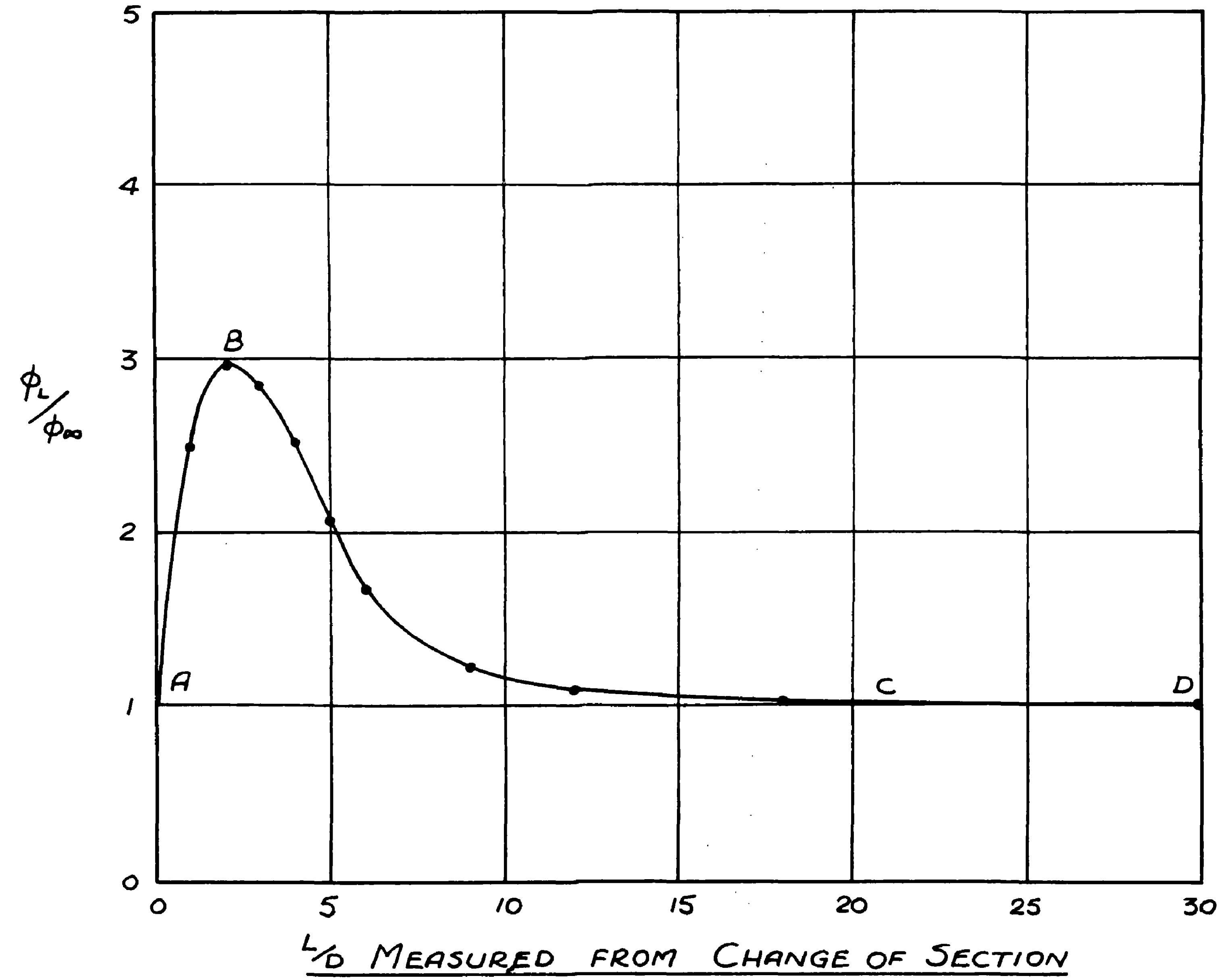


FIG. 34 GRAPH OF ϕ_L/ϕ_∞ v. L/D (ENLARGEMENT)

The area ABC represents the effect of the abrupt enlargement. If this area is divided by the area below the line CD for a length of one pipe diameter, then the ratio will represent an equivalent number N of "normal" pipe diameters. Hence, the total area under the curve ABCD, where D is at X diameters from the change of section, is equal to the area under a curve for X + N "normal" diameters of pipe length. The effect of the abrupt enlargement can therefore be considered to be equivalent to that of N extra diameters of pipe length, the whole pipe being considered to be under normal turbulent flow conditions.

It will be noticed in Fig. 34, that the first point on the curve is at $\frac{L}{D} = 1$, and that the curve is produced back to $\frac{L}{D} = 0$ in order that an area may be measured from this point. This is thought to be justified for all enlargement experiments, since there is always a maximum of ϕ_L/ϕ_∞ at $\frac{L}{D} = 2$ and the shape of the curve is such that, if produced, it gives a value of $\phi_L/\phi_\infty = 1$ at $\frac{L}{D} = 0$.

However, in the case of the contraction experiments, no such maximum was found. It was not possible to measure a pipe temperature at a position less than $\frac{L}{D} = 2$ from the change of section. The method of producing

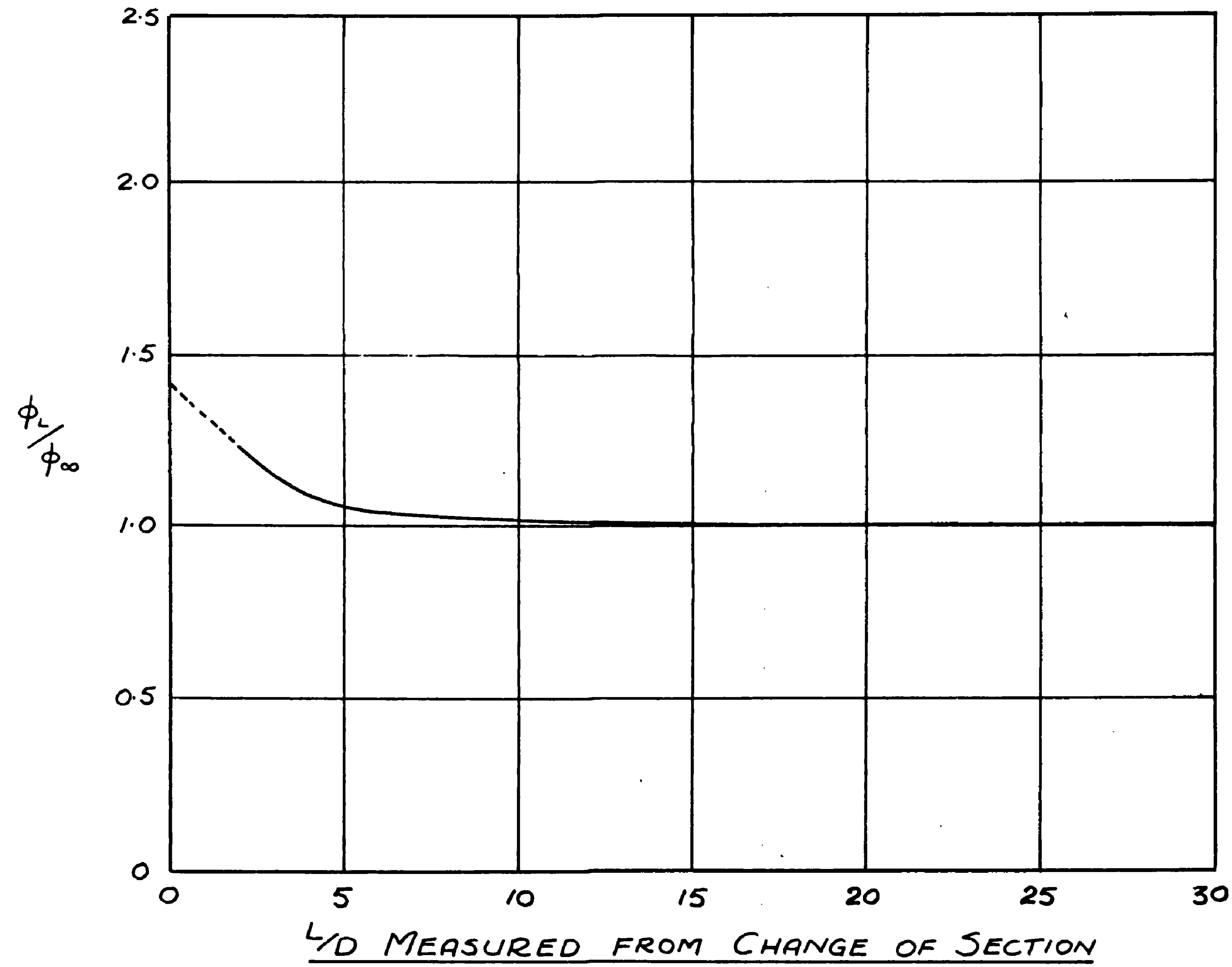


FIG 35 GRAPH OF ϕ_L/ϕ_∞ V. L/D (CONTRACTION)

Reynolds Number in the 2 inch Pipe	Extra diameters of Pipe Length N
45,500	8.45
25,000	10.19
13,000	13.04
6,700	15.79
3,750	11.49

Enlargement Tests

Reynolds Number in the 1 inch Pipe	Extra diameters of Pipe Length N
105,000	0.290
50,000	0.769
26,000	1.364
14,600	1.757
8,750	1.020

Contraction Tests

TABLE 17

Values of equivalent number of extra diameters of pipe length, N.

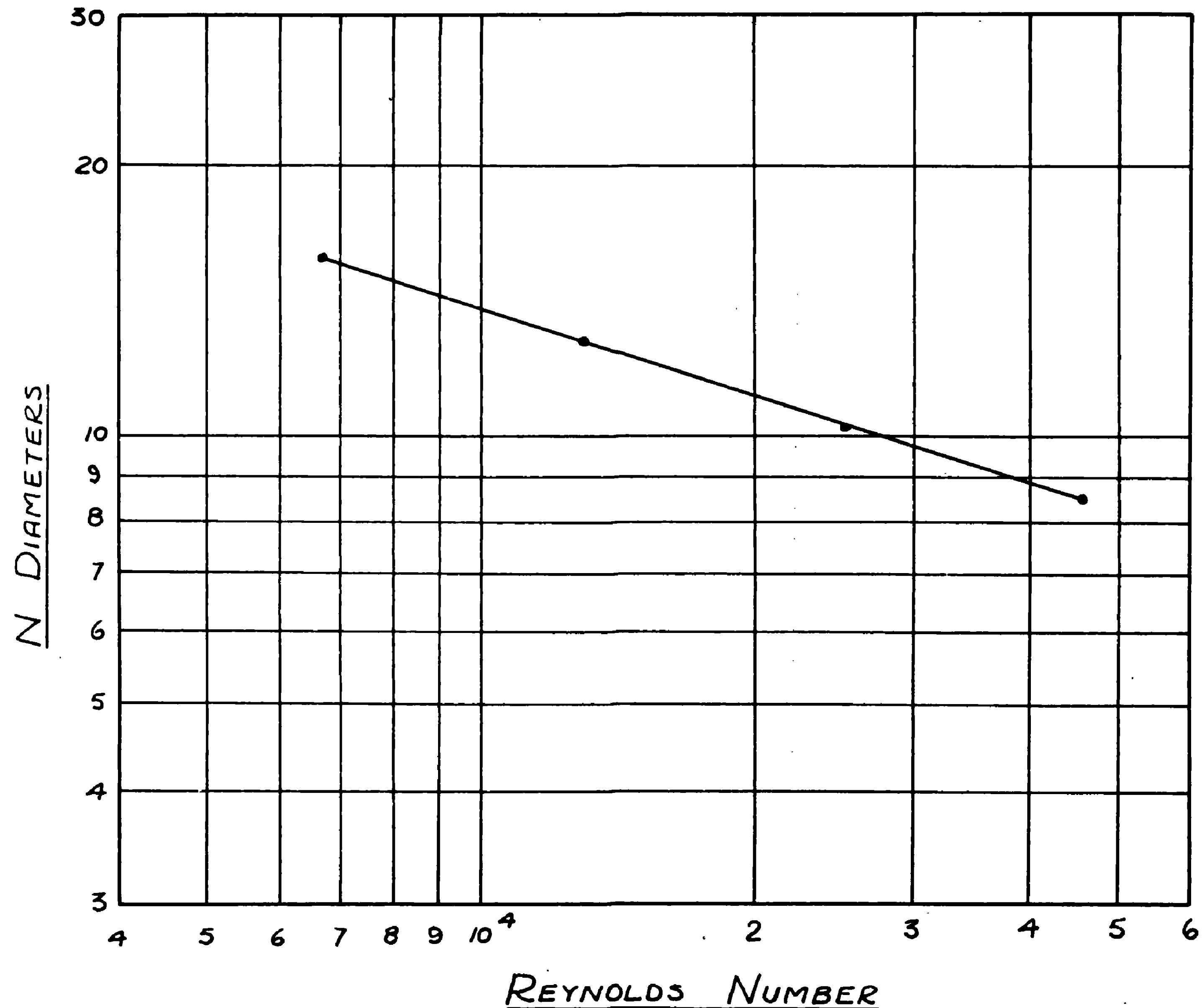


FIG 36 GRAPH OF N v. Re

this curve back to $\frac{L}{D} = 0$ must therefore be open to doubt. A maximum might exist anywhere between $\frac{L}{D} = 0$ and $\frac{L}{D} = 2$ but under the circumstances, for the purpose of measuring the area below it, the curve has been produced as shown in Fig. 35. Values of N obtained from such curves will probably be high, and will not be as reliable as those obtained for the enlargement experiments.

Curves of ϕ_L/ϕ_∞ v $\frac{L}{D}$ were plotted, one for each Reynolds Number, for all enlargement and contraction experiments. The values of N obtained from these curves are shown in Table 17.

It will be seen from this table that the effect of change of section, as measured by this method, increases with decreasing Reynolds Numbers, reaching a maximum for the enlargement experiments at a Reynolds Number of about 6,700, and for the contraction experiments at a Reynolds Number of about 14,600.

Taking the analysis one step further, Fig. 36 shows N plotted logarithmically against Reynolds Number for the enlargement experiments. For the range of Reynolds Numbers from 7,000 to 45,000, the curve is in the form of a straight line having a slope of -1/3.

For this range of Reynolds Numbers in the enlargement experiments:-

$$N = 300 \, Re^{-1/3}$$

where N = equivalent number of extra diameters of pipe length.

For the reason stated above, the values of N obtained for the contraction experiments are not thought to be very reliable. These values will not, therefore, be analysed further.

CHAPTER 8

Conclusions

8.1 Summary of Results

(a) Normal Turbulent Flow

The heat transfer coefficient for normal turbulent flow in a pipe, neglecting the effect of heat input, is well expressed by the equation:-

$$Nu = 0.023\ Re^{0.8} Pr^{0.4}$$

for the range of Reynolds Numbers between 10,000 and 100,000.

The results are therefore in good agreement with those of McAdams[17] and Brown, Fishenden and Saunders,[3] given in Chapter 1.

(b) Effect of Heat Input

The effect of heat input on heat transfer coefficient for normal turbulent flow, although small, is quite noticeable. The above equation requires to be modified as follows, in order that this effect may be taken into account

$$Nu = 0.023\ Re^{0.8} Pr^{0.4} \left[\left(\frac{\mu_a}{\mu_w}\right)\right]^{0.07}$$

This effect has previously been noticed only for liquids having viscosities greater than twice that of water. (Sieder and Tate[26]).

Brown, Fishenden and Saunders[3] have stated that no effect of heat input on heat transfer coefficient is noticeable for liquids having viscosities less than twice that of water.

No more recent reference to this effect has been found, and it therefore seems probable that this is the first measurement to be made of the effect as regards heat transfer to water.

(c) Effect of Change of Section

The effect of the change of section on the heat transfer coefficient is expressed in two ways.

The first expression enables the local heat transfer coefficient to be found at any position in the excess turbulent section of the pipe for both abrupt enlargement and abrupt contraction experiments. Values of C and n, in the Nusselt equation $Nu = CRe^{n}Pr^{0.4}$, are determined, and are plotted against $\frac{L}{D}$ in the excess turbulent sections of the pipe. Thus one particular form of the Nusselt equation applies for each position in the excess turbulent regions.

The second expression enables an estimation to be made of the total effect of the change of section in terms of an equivalent number N of extra "normal" diameters of pipe length. For the abrupt enlargement tests, N can be expressed in terms of the Reynolds Number as follows:-

$$N = 300 \, Re^{-1/3}$$

for the range of Reynolds Numbers from 7000 to 45000 in the 2 inch diameter pipe.

For the abrupt contraction experiments, the method used in plotting the curves from which N is determined, is open to doubt. The values of N thus obtained are, if anything, higher than their true values. They are, however, of the order of 1/10th of the corresponding values for the enlargement experiments. Hence the effect of the change of section, when measured in terms of extra "normal" diameters of downstream pipe length, is at least ten times as great for the abrupt enlargement experiments as it is for the abrupt contraction experiments.

8.2 Future Work

It is the intention to modify the present apparatus so that measurements can be made of static pressures in the pipe at the positions where the local heat transfer coefficients were measured. An attempt will then be made to relate the loss of head due to change of section, to the increase of heat transfer coefficient caused by the same change of section.

With regard to new apparatus, two ways may be suggested for widening the scope of the work.

Firstly, the experiments might be repeated with the same ratio of pipe diameters, but using different working fluids ranging from air to oils of varying viscosity. Consideration might also be given to mercury as a working fluid.

Secondly, using only water as a convenient working fluid, a wide range of pipe diameter ratios might be studied. The extent to which the effect of change of section depended on the enlargement or contraction ratio could then be determined.

APPENDIX 1

Calibration of Thermocouples

Although the characteristics of copper-constantan thermocouples are fairly well established, in order to attain the accurary required in these experiments, a calibration is necessary.

The thermocouples used in the experiments being of the "five-way" type, it was decided to calibrate such a thermocouple. The thermocouple was constructed as described in Chapter 3 paragraph 5, but was arranged to have its hot junctions, as well as its cold junctions, in individual glass tubes. The cold junctions were placed in a vacuum jar containing a mixture of crushed ice and water and the hot junctions were placed in a similar vacuum jar containing water at a temperature of about 50°C. and a thermometer graduated in $\frac{1}{20}$ °C.

The water in the "hot" vacuum jar was found to cool at the rate of about 0.1°C. per minute and it was therefore necessary to instal a small electric heater in the water to maintain a constant temperature. The heater was designed in the form of a length of constantan wire wound on a formapex stirrer. The heating current came from a 12 volt transformer, a variable resistance being connected in series with the heater. The water was thus stirred and maintained at a constant temperature by the same instrument.

The calibration procedure was then as follows:- The mixture of crushed ice and water was stirred and its temperature seen to be 0°C. Alternate readings of thermometer and thermocouple in the "hot" vacuum jar, were taken at one minute intervals. By suitably regulating the variable resistance and constantly stirring, these readings could be maintained constant. The process was repeated at 5°C. intervals down to about 5°C., the water in the "hot" vacuum jar being cooled each time by the addition of cold water. To attain the lowest temperature of 5°C. it was necessary to completely melt a piece of ice in the jar.

A typical set of calibration readings are as follows:-

Thermometer Reading $t^{\circ}C$	Potentiometer Reading e millivolts
49.90	10.077
44.78	9.020
39.09	7.797
34.91	6.933
29.53	5.829
24.80	4.873
19.73	3.850
14.98	2.908
10.14	1.967
5.50	1.064

The relationship between e and t is known to be nearly linear. It is sufficient to include a term in t^2 as a first approximation.

The following equation is therefore assumed:-

$$e = at + bt^2$$

where a and b are constants to be determined.

The equation may be written

$$\frac{e}{t} = a + bt \quad \text{which is linear in } \frac{e}{t} \text{ and } t.$$

The "Method of Least Squares" can then be employed to find the best straight line through the experimental points.

<u>The Method of Least Squares</u>

Consider the straight line $y = a + bx$. The deviation of a point $x_1\ y_1$ from this line is $y_1 - (a + bx_1)$

The line is chosen for which the sum of the squares of the deviations from it, measured in a vertical direction, is a minimum.

The sum of the squares of the deviation is

$$Q = \sum \left[y - (a + bx)\right]^2$$

a and b are then chosen to make Q a minimum.

$$\frac{\partial Q}{\partial a} = -2 \sum \left[y - (a + bx)\right] = 0$$

$$\frac{\partial Q}{\partial b} = -2 \sum x \left[y - (a + bx)\right] = 0$$

$$\therefore \quad \Sigma y = na + b\Sigma x$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

where n is the number of observations.

The equation $y = a + bx$ corresponds to the assumed equation $\frac{e}{t} = a + bt$

Hence x corresponds to t

xy corresponds to e

y corresponds to $\frac{e}{t}$

x^2 corresponds to t^2

The equations to be solved are therefore:-

$$\Sigma \frac{e}{t} = na + b\Sigma t$$
$$\Sigma e = a\Sigma t + b\Sigma t^2$$

t	e	$\frac{e}{t}$	t^2
49.90	10.077	0.20194	2490.01
44.78	9.020	0.20142	2005.25
39.09	7.797	0.19946	1528.03
34.91	6.933	0.19859	1218.71
29.53	5.829	0.19739	872.02
24.80	4.873	0.19649	615.04
19.73	3.850	0.19513	389.27
14.98	2.908	0.19412	224.40
10.14	1.967	0.19398	102.82
5.50	1.064	0.19345	30.25
273.36	54.318	1.97197	9475.80

$$\therefore \quad 1.97197 = 10a + 273.36b$$

$$54.318 = 273.36a + 9475.8b$$

from which $a = 0.19158$

$b = 0.0002057$

$$\therefore \quad \underline{e = 0.19158t + 0.0002057t^2}$$

The figures given above refer to an early calibration.

A calibration, using the above method, was made of a thermocouple cut from the same length of wire as used for the thermocouples in the enlargement experiments.

The calibration curve had the following equation:-

$$\underline{e = 0.1913t + 0.000219t^2}$$

This was plotted to a large scale, the resulting chart being used for estimating all temperatures in the abrupt enlargement experiments.

A further calibration was made of a five-way thermocouple cut from the reel of wire used for the abrupt contraction test thermocouples.

The calibration was as follows:-

$$\underline{e = 0.1914t + 0.000238t^2}$$

A curve plotted from this equation was used to determine all temperatures in the abrupt contraction experiments.

APPENDIX 2

A Heat Conducted from Copper Conductors

A certain amount of heat was generated in the heavy copper electrical conductors. In all experiments, the conductor temperatures, at some distance from their ends, were higher than the temperatures at the ends of the experimental pipe to which they were attached. Hence, in all cases, heat was conducted from the conductors to the pipe.

An estimation of the quantity of heat conducted was made as follows:-

Let the heat generated in unit length of conductor $= H$

Then heat generated in length $\delta x = H\delta x$

Heat lost by convection in Length $\delta x = Sh\delta x(t - t_a)$

where S = surface area of conductor per unit length

h = heat transfer coefficient

t = temperature of conductor at any point

t_a = atmospheric temperature

Heat conducted through section at distance x from the end $= -kA\dfrac{dt}{dx}$

where A = cross-sectional area of conductor

k = thermal conductivity of copper

Heat conducted through section at distance $(x + \delta x)$

from end $= -kA \frac{dt}{dx} - kA \frac{d^2t}{dx^2} \delta x$

$\therefore$ Net heat lost from length δx by conduction

$$= -kA \frac{d^2t}{dx^2} \delta x$$

But heat generated in length δx = heat lost by convection + heat lost by conduction

$$\therefore \quad H \delta x = Sh \, \delta x(t - ta) - kA \frac{d^2t}{dx^2} \delta x$$

$$\therefore \quad \underline{H = Sh\,(t - ta) - kA \frac{d^2t}{dx^2}} \quad \ldots\ldots\ldots(1)$$

The solution of this equation is

$$\underline{t = \alpha e^{\sqrt{\frac{Sh}{ka}}\,x} + \beta e^{-\sqrt{\frac{Sh}{ka}}\,x} + ta + \frac{H}{Sh}} \quad \ldots\ldots(2)$$

where α and β are constants to be determined.

<u>At $x = \infty$</u>

the maximum temperature is attained and therefpre

$$\frac{dt}{dx} = 0 \quad \text{and} \quad \frac{d^2t}{dx^2} = 0$$

$\therefore$ From (1) $\quad H = Sh\,(t - ta)$

$$\therefore \quad \underline{t = ta + \frac{H}{Sh}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From (2)

$$t = \alpha e^{\infty} + \beta e^{-\infty} + ta + \frac{H}{Sh}$$

$$\therefore \text{ From (3)} \quad \alpha e^{\infty} + \beta e^{-\infty} = 0$$

$$\therefore \quad e = 0$$

$$\therefore \quad \underline{\alpha = 0}$$

<u>At x = 0</u>

$t_o = \alpha + \beta + t_a + \frac{H}{Sh}$ where t_o is the temperature at $x = 0$

$$\therefore \beta = t_o - (t_a + \frac{H}{Sh})$$

$$\therefore \quad \underline{t = \left[t_o - (t_a + \frac{H}{Sh})\right] e^{-\sqrt{\frac{Sh}{ka}}x} + t_a + \frac{H}{Sh}} \quad \ldots\ldots(4)$$

$$\therefore \quad \frac{dt}{dx} = \sqrt{\frac{Sh}{kA}}\,(t_a + \frac{H}{Sh} - t_o) \quad e^{-\sqrt{\frac{Sh}{kA}}\,x}$$

<u>At x = 0</u>

$$\underline{\frac{dt}{dx} = \sqrt{\frac{Sh}{kA}}\,(t_a + \frac{H}{Sh} - t_o)} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

The heat conducted from the copper conductors to the experimental pipe is $kA\,\frac{dt}{dx}$, where $\frac{dt}{dx}$ is measured at the end of the conductor, i.e. at $x = 0$

$\therefore$ Heat Conducted from Copper Conductor

$$= kA\,\frac{dt}{dx}_{x = o} \quad = \underline{\sqrt{kASh}\,(t_a + \frac{H}{Sh} - t_o)} \quad \ldots\ldots\ldots(6)$$

If t_o, the temperature at the end of the conductor, is measured, then the only unknown in equation (6) is the heat transfer coefficient h. The value of h can be determined by measuring the conductor temperature at some position, say 1 foot from the end, and substituting this value, along with t_o, into equation (4).

The heat conducted from thecopper conductor can then be calculated from equation (6).

In the enlargement experiments, conductor temperatures were measured one at two feet, and two at one foot, from the end and two at the end of each conductor. The temperature which was measured at two feet from the end of the conductor, could be used as a check by substituting it into equation (4). The temperature one foot from the end, and the temperature at the end of the conductor were taken as the mean of the two values measured at each of these positions.

The copper conductor temperatures are given, numbered 1 to 10, in Table 7. Numbers 1 to 5 refer to the inlet and conductor and 6 to 10 to the outlet end conductor.

The positions of the thermocouples were as follows:-

1 and 6 at two feet from the end.

2, 3, 7 and 8 at one foot from the end.

4, 5, 9 and 10 at the end.

The geometry of the copper conductors, as erected for the abrupt contraction experiments, was such that owing to bends and junctions only a few inches from their ends, the above method of determining $\frac{dt}{dx}$ for a straight length of conductor did not apply. In this case temperatures were measured at two points separated by 2 inches at the ends of the conductors. The mean temperature gradients measured over these 2 inch lengths were taken to be the values of $\frac{dt}{dx}$ at $x = 0$.

Then, heat conducted from the conductor was directly $kA \frac{dt}{dx}$.

B Heat Transferred through Pipe Lagging.

In order to complete the heat balance, it is necessary to make an estimate of the heat transferred through the pipe lagging. The quantity of heat transferred from the air to the lagging depends on the velocity of air currents passing over the pipe. These are variables, so that any estimation of heat transfer based on their assumed velocity cannot be very accurate. As will be seen from Tables 8 and 9, for the majority of the experiments the quantity of heat transferred through the lagging, as estimated by the following method, is a very small proportion of the heat generated in the pipe. A high degree of accuracy in the estimation need not, therefore, be achieved.

The calculation is based on an assumed air velocity of 4 feet per second transverse to the axis of the pipe. In order to simplify the calculation, it is assumed that the inside and outside surface temperatures of the lagging are constant along their length.

The method of calculation depends on equating the quantity of heat transferred between the air and the lagging to the quantity of heat conducted through the lagging.

If q is this quantity, then:-

$$q = 2\pi k L \frac{t - t_p}{\log_e \frac{R}{r}}$$

$$q = 2\pi RLh\,(t_a - t)$$

where

k = thermal conductivity of pipe lagging

L = length of either 1 inch or 2 inch pipe

t = outside surface temperature of lagging

t_p = mean outside surface temperature of pipe (assumed equal to the inside surface temperature of the lagging)

R = outside radius of lagging

r = inside radius of lagging

h = heat transfer coefficient at outside of lagging

t_a = atmospheric temperature.

An estimation can be made of h using the equation

$\frac{hD}{k} = 0.24\, Re^{0.6}$ given by McAdams[18]

All other quantities, except the outside surface temperature t, can be measured.

The following equations can be formed (subscripts 1 and 2 refer to 1 inch and 2 inch pipes respectively).

For the 1 inch pipe

$$q_1 = 2.06\ (t_1 - t_{p_1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$q_1 = 22.8\ (t_a - t_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For the 2 inch pipe

$$q_2 = 5.88\ (t_2 - t_{p_2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$q_2 = 48.8\ (t_a - t_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From (1) and (2) $t_1 = \dfrac{22.8t_a + 2.06t_{p_1}}{24.86}$

From (3) and (4) $t_2 = \dfrac{48.8t_a + 5.88t_{p_2}}{54.68}$

Thus the outside temperatures of the lagging t_1 and t_2 can be found.

Hence the heat transferred through the lagging q_1 and q_2 can be found by substituting t_1 and t_2 into equations (1) and (3) respectively.

The values of the total quantity of heat transferred through the lagging $q_1 + q_2$ are shown in Tables 8 and 9.

C Heat Conducted along the pipe

Although conduction of heat along the pipe has no effect on the overall heat balance, it might affect the uniformity of heat input to the water per unit length.

Since a temperature gradient exists along the pipe, there must be a certain amount of heat conducted along it. So long as there is a uniform temperature gradient over a given length of the pipe, as there is for normal turbulent flow, the quantity of heat conducted into any section of that length will be equal to the quantity of heat conducted away from it. Hence the net effect, with regard to heat transferred to the water from that section, will be zero.

In other words, the conditions under which heat conduction along the pipe affects the uniformity of heat input per unit length to the water, only occur where there is a change of temperature gradient along the pipe, i.e where the net quantity of heat conducted to a section of pipe is not zero.

The worst condition occurs in the enlargement experiments in the region of the minimum temperature position on the 2 inch pipe. At this position heat is being conducted inwards from both sides.

The effect will be calculated for this position.

Let A be the cross-sectional area of pipe material and let Q be the heat generated in the pipe per unit volume.

Then:-

Heat generated in length $\delta L = QA\delta L$

Heat conducted into length $\delta L = -kA\dfrac{dt}{dL}$

Heat conducted from Length $\delta L = -kA\dfrac{dt}{dL} - kA\dfrac{d^2t}{dL^2}\delta L$

$\therefore$ Heat transferred to water from length δL

$$= QA\delta L + kA\frac{d^2t}{dL^2}\delta L$$

$$= \underline{A\delta L\left(Q + k\frac{d^2t}{dL^2}\right)}$$

A measure of the effect can how be obtained by finding how $k\dfrac{d^2t}{dL^2}$ (the conduction term) compares with Q (the heat generated).

For a typical enlargement experiment, the rate of change of $\dfrac{dt}{dL}$ was measured across the minimum temperature position on the 2 inch pipe.

$k\dfrac{d^2t}{dL^2}$ was found to be equal to 0.19% of Q

The effect of heat conduction along the pipe can therefore be neglected.

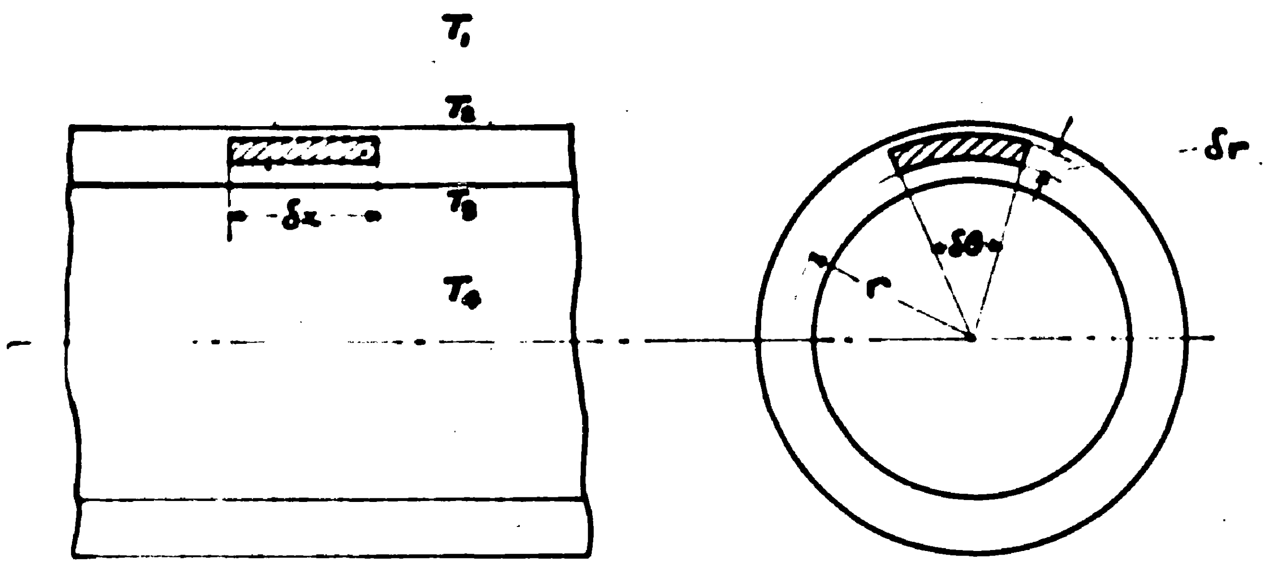


FIG 37 ΔT THROUGH PIPE WALL

APPENDIX 3

Estimation of Temperature Drop through Pipe Wall

Let J = Current density in the pipe (amps/cm.^{2})

σ = Specific resistance of pipe material (ohms.cm.)

Then $J^2\sigma$ = Electrical heat input per unit volume (watts/cm.^{2})

Let $J^2\sigma = Q$

In Fig. 37

If r = radial distance

θ = angle included by two radii

x = distance parallel to the axis of the pipe

Then $r\,\delta\theta\,\delta r\,\delta x$ is an element of volume of pipe material.

Conduction of heat in the pipe will be considered only in the radial direction, since heat conduction along the pipe has been shown to be negligible (Appendix 2).

Heat generated in element $r\,\delta\theta\,\delta r\,\delta x \quad = Q\,r\,\delta\theta\,\delta r\,\delta x$

Heat conducted to element at radius $r \quad = -\,kr\,\delta\theta\,\delta x\,\frac{dT}{dr}$

(where k = thermal conductivity of pipe material)

Heat conducted from element at radius $r + \delta r$

$$= -\,kr\,\delta\theta\,\delta x\,\frac{dT}{dr} - \frac{d}{dr}\left[kr\,\delta\theta\,\delta x\,\frac{dT}{dr}\right]\delta r$$

$$\therefore\; Qr\,\delta\theta\,\delta r\,\delta x = kr\,\delta\theta\,\delta x\,\frac{dT}{dr} - kr\,\delta\theta\,\delta x\,\frac{dT}{dr} - \frac{d}{dr}\left[kr\delta\theta\delta x\,\frac{dT}{dr}\right]\delta r$$

$$\therefore\; Q = -\,k\,\frac{d^2T}{dr^2} - \frac{k}{r}\,\frac{dT}{dr}$$

$$\therefore \quad \frac{d^2T}{dr^2} + \frac{1}{r} \cdot \frac{dT}{dr} = -\frac{Q}{k} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

The solution of this equation is:-

$$T = -\frac{Qr^2}{4k} + A \log_e r + B \quad \ldots\ldots\ldots\ldots\ldots(2)$$

and $$\frac{dT}{dr} = -\frac{Qr}{2k} + \frac{A}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Let R_1 = inside radius of pipe

R_2 = outside radius of pipe

T_1 = air temperature

T_2 = outside pipe surface temperature

T_3 = inside pipe surface temperature

T_4 = water temperature

<u>At $r = R_1$</u>

$$-k\frac{dT}{dr} = h_1 (T_4 - T_3)$$

where h is the heat transfer coefficient between pipe and water

$$\therefore \quad \left[\frac{QR_1}{2k} - \frac{A}{R_1}\right] k = h_1 (T_4 - T_3) \quad \ldots\ldots\ldots\ldots(4)$$

<u>Similarly at $r = R_2$</u>

$$\left[\frac{QR_2}{2k} - \frac{A}{R_2}\right] k = h_2 (T_2 - T_1) \quad \ldots\ldots\ldots\ldots(5)$$

where h_2 is the heat transfer coefficient between pipe and air.

At $r = R_1$

$$T_3 = -\frac{QR_1^{\,2}}{4k} + A \log_e R_1 + B \quad \ldots\ldots\ldots\ldots\ldots(6)$$

At $r = R_2$

$$T_2 = -\frac{QR_2^{\,2}}{4k} + A \log_e R_2 + B \quad \ldots\ldots\ldots\ldots(7)$$

From (4) and (5)

$$T_2 - T_3 = T_1 - T_4 + \frac{Q}{2}\left[\frac{R_1}{h_1} + \frac{R_2}{h_2}\right] - Ak\left[\frac{1}{R_1h_1} + \frac{1}{R_2h_2}\right] \ldots(8)$$

From (6) and (7)

$$T_2 - T_3 = \frac{-Q(R_2^{\,2} - R_1^{\,2})}{4k} + A \log_e \frac{R_2}{R_1} \quad \ldots\ldots\ldots\ldots(9)$$

From (8) and (9)

$$A \log_e \frac{R_2}{R_1} - \frac{Q(R_2^{\,2} - R_1^{\,2})}{4k} = T_1 - T_4 + \frac{Q}{2}\left[\frac{R_1}{h_1} + \frac{R_2}{h_2}\right] - Ak\left[\frac{1}{R_1h_1} + \frac{1}{R_2h_2}\right]$$

$$\therefore \quad A = \frac{\frac{Q}{2}\left[\frac{R_1^{\,2} - R_1^{\,2}}{2k} + \frac{R_1}{h_1} + \frac{R_2}{h_2}\right] + T_1 - T_4}{\log_e \frac{R_2}{R_1} + k\left[\frac{1}{R_1h_1} + \frac{1}{R_2h_2}\right]}$$

For the purposes of this calculation, it was assumed that the pipe was unlagged and that the air surrounding the pipe was still. Hence in this expression for A, h_1 is the heat transfer coefficient for forced convection of water inside the pipe and h_2 is the heat transfer coefficient for natural convection of air outside the pipe. An estimation was made of the two coefficients for typical working conditions, and h_1 was found to be greater than $1{,}000h_2$. This means that for still air conditions, virtually all the heat generated in the pipe is transferred to the water and that there is therefore no need to lag the pipe. In practice, however, there were air currents in the laboratory and pipe lagging became necessary as explained in Chapter 3.

Proceeding on the assumption that h_2 is negligible compared with h_1 and substituting their appropriate values, the expression for A reduces to:-

$$A = \frac{QR_2^{\,2}}{2k}$$

Hence Temperature Drop through Pipe Wall

$$\Delta T = \frac{QR_2^{\,2}}{2k} \log_e \frac{R_2}{R_1} - \frac{Q}{4k} (R_2^{\,2} - R_1^{\,2})$$

Temperature °C	Resistivity ohms.cm
2	8.082×10^{-6}
4	8.099×10^{-6}
6	8.116×10^{-6}
8	8.133×10^{-6}
10	8.150×10^{-6}
12	8.167×10^{-6}
14	8.184×10^{-6}
16	8.200×10^{-6}
18	8.216×10^{-6}
20	8.233×10^{-6}
22	8.250×10^{-6}
24	8.267×10^{-6}
26	8.283×10^{-6}
28	8.300×10^{-6}
30	8.315×10^{-6}

TABLE 18

Electrical Resistivity of Pipe Material (σ)

$$\therefore \Delta T = \frac{Q}{4k}\left[2R_2^{\,2}\log_e\frac{R_2}{R_1} - (R_2^{\,2} - R_1^{\,2})\right] \quad \ldots\ldots(10)$$

where all terms inside the square brackets are constant. The temperature drop is therefore proportional to the heat input per unit volume of metal $J^2\sigma$, and inversely proportional to the thermal conductivity of the pipe material k.

Electrical resistivity σ and thermal conductivity k both vary with temperature.

Electrical and Thermal Conductivities

A study was made by Smith and Palmer[27] of the relationship between electrical and thermal conductivities of 50 popper alloys including one of composition similar to that of the experimental pipe.

It was found that for all 50 copper alloys:-

$$k = 5.71 \times 10^{-9}\lambda T + 0.018$$

where k is thermal conductivity in cals/(cm)(sec)(^oC) and λ is electrical conductivity in $ohms^{-1}\ cm^{-1}$

Electrical conductivity is therefore the reciprocal of the electrical resistivity.

T is the absolute temperature ^oK

An estimation of the variation of electrical resistivity with temperature was made using values of resistance and temperature obtained from actual abrupt enlargement tests. Table 18 shows this variation.

Temperature °C	Thermal Conductivity cals/(cm)(sec)(°C)	Thermal Conductivity joules/(cm)(sec)(°C)
2	0.2123	0.8889
4	0.2133	0.8930
6	0.2143	0.8972
8	0.2153	0.9014
10	0.2163	0.9056
12	0.2172	0.9094
14	0.2182	0.9136
16	0.2192	0.9177
18	0.2203	0.9223
20	0.2212	0.9261
22	0.2221	0.9299
24	0.2231	0.9341
26	0.2240	0.9378
28	0.2251	0.9424
30	0.2260	0.9462

TABLE 19

Thermal Conductivity of Pipe Material (k)

The corresponding values of electrical conductivity were found, and on substituting into the equation of Smith and Palmer, values of k were obtained as given in Table 19. The pipe manufacturers gave the value $k = 0.22$ cals/(cm)(sec)(^{o}C) which is in good agreement.

Reverting to equation (10), and substituting in values for R_1 and R_2

(a) <u>For the 1 inch Pipe</u>

$$\Delta T = \frac{0.2924Q}{k} = \frac{0.2924J^2\sigma}{k}$$

(b) <u>For the 2 inch Pipe</u>

$$\Delta T = \frac{0.0928J^2\sigma}{k}$$

<u>Hence for the 1 inch Pipe</u>

$$J = \text{current density}$$

$$J^2 = \text{current}^2 \times 0.01915$$

$$\therefore \Delta T = \text{current}^2 \times 0.01915 \times 0.2924 \frac{\sigma}{k}$$

$$\therefore \frac{\Delta T}{\text{current}^2} = 0.005600 \frac{\sigma}{k}$$

<u>For the 2 inch Pipe</u>

$$J^2 = \text{current}^2 \times 0.01817$$

$$\therefore \frac{\Delta T}{\text{current}^2} = 0.001686 \frac{\sigma}{k}$$

Temperature °C	$\frac{\sigma}{k}$	$0.005600 \frac{\sigma}{k} = \frac{\Delta T}{\text{current}^2}$ °C/amps^2. 1 inch Pipe	$0.001686 \frac{\sigma}{k} = \frac{\Delta T}{\text{current}^2}$ °C/amps^2. 2 inch Pipe
2	9.092×10^{-6}	0.05092×10^{-6}	0.01533×10^{-6}
4	9.069×10^{-6}	0.05079×10^{-6}	0.01529×10^{-6}
6	9.046×10^{-6}	0.05066×10^{-6}	0.01525×10^{-6}
8	9.023×10^{-6}	0.05053×10^{-6}	0.01521×10^{-6}
10	9.000×10^{-6}	0.05040×10^{-6}	0.01517×10^{-6}
12	8.981×10^{-6}	0.05029×10^{-6}	0.01514×10^{-6}
14	8.958×10^{-6}	0.05017×10^{-6}	0.01510×10^{-6}
16	8.935×10^{-6}	0.05004×10^{-6}	0.01507×10^{-6}
18	8.908×10^{-6}	0.04989×10^{-6}	0.01502×10^{-6}
20	8.890×10^{-6}	0.04978×10^{-6}	0.01499×10^{-6}
22	8.872×10^{-6}	0.04968×10^{-6}	0.01496×10^{-6}
24	8.850×10^{-6}	0.04956×10^{-6}	0.01492×10^{-6}
26	8.832×10^{-6}	0.04946×10^{-6}	0.01489×10^{-6}
28	8.805×10^{-6}	0.04931×10^{-6}	0.01484×10^{-6}
30	8.788×10^{-6}	0.04921×10^{-6}	0.01482×10^{-6}

TABLE 20

Variation of $\frac{\Delta T}{\text{current}^2}$ with temperature

From Tables 18 and 19 the values of 0.005600 $\frac{\sigma}{k}$ and 0.001686 $\frac{\sigma}{k}$ can be found. The variation of these quantities with temperature is shown in Table 20.

Thus, if the electric current passing through the pipe and the temperature of the pipe at any point are known, then the temperature drop through the pipe wall ΔT can be found from Table 20.

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LIST OF SYMBOLS

A	Cross-sectional area
C	Constant in the Nusselt equation
C_p	Specific heat
D	Inside diameter of pipe
e	Electromotive force
Gr	Grashof Number
g	Acceleration due to gravity
H	Heat generated per unit length of conductor
h	Heat transfer coefficient
h_L	Local heat transfer coefficient
h_{av}	Average heat transfer coefficient
h_∞	Average heat transfer coefficient for a pipe of infinite length
I	Electric current
J	Electric current density
k	Thermal conductivity
L	Length measured along the pipe
m	Power to which the Prandtl Number is raised in the Nusselt equation
N	Number of pipe diameters
Nu	Nusselt Number
Nu_L	Local value of Nusselt Number

n	Power to which the Reynolds Number is raised in the Nusselt equation
Pr	Prandtl Number
p	Pressure
Q	Quantity of heat per unit time and unit volume
q	Quantity of heat per unit time
R	Electrical Resistance
Re	Reynolds Number
r	Radius of pipe
S	Surface area
T, t	Temperature
ΔT	Temperature difference
t_a	Air temperature
V, v	Velocity
x	Distance
β	Coefficient of expansion
ρ	Density
μ	Viscosity
σ	Specific resistance or resistivity
λ	Electrical conductivity
ϕ	The group $\frac{Nu}{Re^{0.8}Pr^{0.4}}$
ϕ_L	Local value of $\frac{Nu}{Re^{0.8}Pr^{0.4}}$
ϕ_∞	Value of $\frac{Nu}{Re^{0.8}Pr^{0.4}}$ for normal turbulent flow

SUMMARY

HEAT TRANSFER IN A PIPE WITH AN ABRUPT CHANGE OF SECTION

The Effect of an Abrupt Change of Section on the Coefficient of Heat Transfer between a Pipe and Water flowing through it.

by

Charles I. Hislop, B.Sc., D.I.C.

When a fluid is heated while flowing through a pipe the phenomenon of heat transfer between the pipe and the fluid may be broadly classified into two types. If the pipe is straight, of uniform diameter and of sufficient length, a fully developed condition will be reached such that the nature and degree of turbulence will no longer vary. Such turbulence is called "normal turbulence". Under this condition, heat transfer coefficients do not vary with distance along the pipe. If, on the other hand, the fluid has just passed an abrupt change of section in the pipe, "excess turbulence" will exist with a resulting increase in heat transfer coefficient.

The author has carried out a series of experiments, from which a measurement has been obtained of the variation of local heat transfer coefficient due to both an abrupt

enlargement, and an abrupt contraction, in a pipe.

The experimental pipe, which was of brass, consisted of 9 feet of 1 inch diameter pipe coupled to 18 feet of 2 inch diameter pipe, the coupling being in the form of an abrupt change of section. Water flowed through the pipe which was heated by the passage of a high, low-voltage current through it. The same experimental pipe was used for both abrupt enlargement and abrupt contraction experiments the direction of water flow relative to the pipe being adjusted accordingly.

Local heat transfer coefficients were measured at positions sufficient to give a complete picture of their distribution along the pipe. Tests were carried out for Reynolds Numbers ranging from 4,000 to 100,000 in the 1 inch diameter pipe. Three values of heat input were used at each Reynolds Number.

For the normal turbulent flow sections of the pipe, neglecting the effect of heat input, the experimental results were well expressed by the equation

$$Nu = 0.023\, Re^{0.8} Pr^{0.4}$$

This equation is in good agreement with the most recent data on heat transfer in normal turbulent flow in pipes.

The effect of heat input on the heat transfer coefficient for normal turbulent flow was taken into account by introducing an extra term into the above equation. Thus

$$Nu = 0.023\, Re^{0.8} Pr^{0.4} \left(\frac{\mu a}{\mu w}\right)^{0.07}$$

where μa is the viscosity of water at its bulk temperature and μw the viscosity of water at the pipe wall temperature. This effect had not previously been noticed for fluids with viscosities less than twice that of water.

The effect of the abrupt change of section on the heat transfer coefficient was expressed in two ways. Firstly, Nusselt equations were derived from which the local heat transfer coefficient could be calculated at any position in the excess turbulent flow sections of the pipe. Secondly, the total effect of the change of section was estimated in terms of an equivalent number of extra "normal" diameters of pipe length.