Nuclear Forces and Spin Orbit Coupling

by

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Ph.D. Thesis, 1955.

Contents

CHAPTER I.

1. Introduction.

In section 2, a brief survey is given of the present position in the meson theory of nuclear forces. The principal aim of this is to provide the background to the main topic of this thesis, the existence of a strong spin orbit coupling in meson field theory.

In section 3, the Mayer nuclear shell model is briefly discussed in order to demonstrate that a strong spin orbit coupling in nuclear forces is required to explain the experimental results concerning the stability of certain nuclei.

The need for a strong spin orbit coupling is further emphasized in section 4, in which spin polarization of nucleons, elastically scattered by nuclei, is discussed. A possible explanation for this phenomenon is to introduce a spin orbit coupling, between the scattered nucleon and the nucleus, of the same magnitude as that required by the nuclear shell model.

Finally, in section 5, a survey is given of the most important contributions made so far to obtain a satisfactory spin orbit coupling in meson field theory.

2. Meson theory of nuclear forces.

The meson theory of nuclear forces began when Yukawa (1935) attempted to explain the strength and short range of nuclear forces by the assumption that nucleons interacted through the exchange of quanta, just as electrons exchanged photons. The strength of the nuclear forces could be explained by a large coupling constant, in contrast with the small value of $1/137$ of the fine structure constant $e^2/\hbar c$ in electrodynamics. The short range of nuclear forces could be obtained by assigning a mass of approximately 200-300 electron masses to the quanta or mesons.

The cosmic ray μ-meson, detected shortly after, was held responsible for nuclear forces, even although it was observed that it interacted weakly with nuclei. This belief was held until 1947 when the π-meson was discovered in Bristol. It was recognized, when it became fairly evident that the π-meson interacted strongly with nuclei, that this latter particle, and not the μ-meson, was in fact responsible for nuclear forces. It was thus with great interest that physicists attempted to establish the properties of the π-meson, both by photographic plate techniques and

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by means of high energy particle accelerators.

At the present time, the salient properties of the π - meson appear to be fairly well established, namely, that it exists in three charge states $+e, 0, -e$ and denoted by π^+, π^0, π^-; has zero in$trinsic spin; and has pseudo-scalar transformation properties. Further, due to the observation that, within certain limits, proton - proton and proton - neutron forces are alike, it seems likely, as well as desirable, that a charge symmetric π- meson theory should be used in any theoretical calculation.

The success of the covariant theory of quantum electrodynamics introduced by Schwinger, Tomonaga, Feynman and Dyson (1949), in which infinities were absorbed in terms of mass and charge renormalization, provided a further stimulus for efforts to obtain a satisfactory meson field theory of nuclear forces. However, since then, apart from a few notable advances, the successes of a meson field theory of nuclear forces have been very limited, a qualitative fit at most being obtained with experiment.

Matthews (1950) and Salam (1951) showed that the same mathematical techniques, as employed by Feynman and Dyson in quantum electrodynamics, could be applied to meson field theory with slight modifications, only provided that the

meson /

meson field were scalar, with scalar coupling to the nucleon field, denoted by S(S), or pseudoscalar with pseudoscalar coupling, denoted by PS(PS). As remarked above, it seems reasonably well established now that the π - meson is pseudoscalar. Thus the PS(PS) theory must be considered to be the only form of field theory which can be employed in a theoretical discussion of nuclear forces. The remainder of this section will contain a brief summary of the most successful attempts made so far to obtain a quantitative description of nuclear forces on the basis of charge symmetric PS(PS) meson field theory.

Firstly, it must be noted that the PS(PS) theory involves the Dirac operator γ_5 which, in the non-relativistic region, is (1) large (of order unity) for positive $\rightleftarrows$ negative energy transitions, and (2) small (of order v/c, the nucleon velocity) for energy transitions with no change of sign. It is quite clear that from the point of view of perturbation theory - some form of which is the only possible treatment at present - the lowest, second order, contribution, which must necessarily arise via an energy transition of type (2), need not be more, and is probably less, important than the fourth order contribution, which arises mainly from transitions of type (1). In

consequence /

consequence, two results follow, namely, that the coupling constant required is very large and that an exact calculation up to at least fourth order is required. These two consequences of the PS(PS) theory are in fact the major obstacles in the way of obtaining a potential with which one can compare the PS(PS) theory with experiment. A large coupling constant casts serious doubts on a perturbation theory. One saving factor is to invoke the argument of Wick (1953) that the range of nuclear forces to be expected from an interaction involving the simultaneous exchange of n mesons is $\hbar/n\mu c$ where μ is the meson mass. Thus, in the non-relativistic region, a reasonable approximation to the PS(PS) potential would still be obtained by including only the first few terms of a perturbation expansion. Further, an exact calculation of even a fourth order process is extremely tedious, and in fact has so far not been successfully carried out.

The various attempts made to obtain a potential up to fourth order in the coupling constant, which agreed with experimental data, were unsuccessful until Levy (1952) obtained a satisfactory potential by a new approach to the problem which was not based on the Feynman-Dyson techniques but on the non-covariant treatment of Tamm (1945) and

Dancoff /

Dancoff (1950) which Levy extended to include the creation and annihilation of virtual nucleon pairs and the exchange of an arbitrary number of mesons.

Levy's treatment consisted of expanding Ψ , the wave functional of the two-nucleon system, in terms of probability amplitudes $a^{(m,n)}(\lambda)$, where m and n denote the number of mesons and nucleon pairs respectively in the particular state considered, and λ specifies the momenta and spins of this set of mesons and nucleons. This expansion leads, in the continuum, to a set of coupled integral equations. Elimination of all the amplitudes except $a^{(0,0)}(\lambda)$ gives rise to an equation which has the form, in momentum space and centre of mass system:

$$(W - 2E_p)\, a^{(0,0)}(p, -p) = \frac{1}{(2\pi)^3}\int K(p, p'; W)\, a^{(0,0)}(p', -p')\, d^3p' \quad (1)$$

where W is the total energy of the system.

$E_p = (p^2 + M^2)^{1/2}$, using rational units $\hbar = c = 1$.
The kernel $K(p; p'; W)$ is expressed as a power series in the coupling constant g . Thus

$$K(p, p'; W) = \sum_n g^{2n} K_{2n}(p, p'; W) \quad (2)$$

Equation (1) can be transformed into co-ordinate space; after some manipulation it is:

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$$\left(\frac{\nabla^2}{M\rho} + \epsilon \right) \phi^{(0,0)}(\underline{r}) = \int U(\underline{r}, \underline{r}'; W) \, \phi^{(0,0)}(\underline{r}') \, d^3 r' \tag{3}$$

where $\epsilon = W - 2M =$ binding energy.

$$\rho = 1 + \epsilon / 4M$$

$$\phi^{(0,0)}(\underline{r}) = \frac{1}{(2\pi)^{3/2}} \int \left[\frac{4M\rho}{2E_p + W} \right]^{1/2} e^{i\underline{p}\cdot\underline{r}} \, a^{(0,0)}(\underline{p}, -p) \, d^3 p$$

$$U(\underline{r}, \underline{r}'; W) = \sum_n g^{2n} \, U_{2n}(\underline{r}, \underline{r}'; W)$$

where $U_{2n}(\underline{r}, \underline{r}'; W)$

$$= \frac{1}{(2\pi)^3} \int \frac{\left[(2E_p + W)(2E_{p'} + W) \right]^{1/2}}{4M\rho} \, K_{2n}(\underline{p}, \underline{p}'; W)$$

$$\times \exp\left(i\underline{p}\cdot\underline{r} - i\underline{p}'\cdot\underline{r}' \right) d^3 p \, d^3 p'$$

In the non-relativistic region, $U(\underline{r}, \underline{r}'; W)$, which is in general a non-local interaction operator, reduces to an interaction of the form

$$U(\underline{r}, \underline{r}'; W) = \delta(\underline{r} - \underline{r}') \, V\left(\frac{|\underline{r} + \underline{r}'|}{2} \right) \tag{4}$$

Thus equation (3) reduces in this case to a simple Schrodinger equation with $V(r)$ an ordinary potential.

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Various types of approximation may now be made in an attempt to obtain $V(r)$ analytically. Firstly, one may consider only the first two terms in the perturbation expansion of $K(p, p'; W)$ and thus obtain a kernel correct up to fourth order in g. Secondly, one may set $E_p = E_{p'} = \frac{1}{2}W = M$ to obtain the low energy properties of the two - nucleon system. This approximation is usually termed the adiabatic or perturbation - static approximation.

Making these two approximations, Levy erroneously obtained the potential

$$V(x) = \mu \left[V_c(x) + S_{12} V_t(x) \right] \tag{5}$$

where $x = \mu r$

$$V_c(x) = -g^2/4\pi \, (\mu/2M)^2 \, e^{-x}/x$$
$$-3\,(g^2/4\pi)^2 \, (\mu/2M)^2 \, \frac{1}{x^2} \left\{ \frac{2}{\pi} K_1(2x) + \frac{\mu}{2M} \left[\frac{2}{\pi} K_1(x) \right]^2 \right\}$$

where K_1 is the hankel function of imaginary argument as defined by Levy.

$$V_t(x) = -g^2/4\pi \, (\mu/2M)^2 \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) e^{-x}/x$$

S_{12} is the tensor operator $3(\sigma_1 \cdot r)(\sigma_2 \cdot r)/r^2 - \sigma_1 \cdot \sigma_2$

$V(x)$ is valid in the region $x \geqq x_c$, where x_c is a certain internucleon separation. For $x < x_c$, the wave function of the system is assumed to be zero. This is equivalent to

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introducing a repulsive core in the potential for $x < x_c$, which could be explained physically by the presence of heavy mesons as well as by interactions involving more than two mesons simultaneously. Thus, the value of x_c would be hoped to be approximately $1/3$, equivalent to the compton wave-length of a mass of three mesons.

x_c and g are taken to give the best fit for the binding energy of the deuteron and the singlet scattering length. It is hoped that the value of g thus obtained takes into account radiative corrections, which have so far not been considered. Levy finds good agreement with the other low-energy parameters, except the deuteron quadrupole moment in which there is a discrepancy of 20%, by taking $x_c = 0{\cdot}38 \pm 0{\cdot}03$ and $g^2/4\pi = 9{\cdot}7 \pm 1{\cdot}3$.

The radiative corrections, which cannot be treated unambiguously within the framework of the non-covariant form of the Tamm-Dancoff theory used by Levy, are obtained by a consideration of the covariant two-body integral equation, introduced by Salpeter and Bethe (1951), in which the kernel is obtained by Feynman-Dyson techniques. Renormalization can then be carried through unambiguously. Levy obtains the result that the main effect is to change the value of the coupling constant. The statement concerning the value of g in the previous paragraph is thus reasonably justified.

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The errors occurring in Levy's results outlined above arose in neglecting certain important terms involving the production of one nucleon pair in the intermediate state. The fourth order potential without radiative corrections was correctly evaluated in the same approximation$ by Klein (1953), whose starting point was the Bethe-Salpeter equation, which was reduced to a three-dimensional Schrodinger equation. A correspondence was thus obtained between the Tamm-Dancoff and Bethe-Salpeter methods. This method of obtaining the potential reduced considerably the risk of introducing errors. However, the result of this correction of Levy's potential was that the quantitative fit with experiment was lost.

Klein obtained the following fourth order potential:

$$V_4(x) = -3\mu\left(g^2/4\pi\right)^2\left(\mu/2M\right)^2 \frac{2}{\pi} K_1(2x)/x^2 \qquad (6)$$

The difference occurs in the correction to this term of order $\mu/2M$ (in the sense of an expansion in powers of $\mu/2M$) which Levy obtained, and which should not appear.

Since the perturbation - static potential up to fourth order is not in agreement with experiment, one can either attempt to obtain a fourth order potential in which nucleon recoil is taken into account, or calculate the potential to a higher order in the coupling constant while still

making /

making the same approximations. Klein adopted the second point of view to investigate whethor any change in the behaviour of the potential would occur, and in addition to determine of the series of potentials thus obtained did in fact convergo.

Klein shows that the potential $V_4(x)$ is the first term of a series

$$V(x) = \sum_{n=1}^{\infty} V_{4n}(x) \qquad (7)$$

where

$$V_{4n}(x) = -3\mu\,\alpha^{2n}\,\frac{2^{2(n-1)}}{n}\,\frac{2}{\pi}\,K_1(2nx)/x^{2n}$$

and $\alpha = g^2/4\pi \cdot \mu/2M$

$V_{4n}(x)$ is the potential arising from the totality of graphs of order $4n$ which represent an interaction between two nucleons in which $2n$ nucleon pairs are present in the intermediate state and $2n$ mesons are exchanged, (i.e. no radiative corrections are considered, and all processes are essentially convergent).

Klein assumes, by considering a canonical transformation of the interaction Hamiltonian:

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$$i g \bar{\Psi}(x) \tau_i \gamma_5 \Psi(x) \phi_i(x)$$

$$\longrightarrow \quad g^2/2M \; \bar{\Psi}(x) \Psi(x) \phi_i^2(x) \quad + \; g/2M \, \Psi^+(x) \tau_i \sigma_j \Psi(x) \partial_j \phi_i(x)$$

$$= \quad H'_{pair} \quad + \quad H'_{gradient} \tag{8}$$

that the maximum contribution to the potential is obtained from graphs with the maximum number of nucleon pairs in the intermediate state, since $g^2/2M$ is an order of magnitude greater than $g/2M$, if g is of the order of 10.

Subsequently, two other series are derived

$$V'(x) = \sum_{n=1}^{\infty} V'_{4n}(x) \tag{9}$$

where $V'_{4n}(x)$ arises from graphs with $2n-1$ pair interactions and 2 gradient interactions, and

$$V''(x) = \sum_{n=1}^{\infty} V''_{4n+2}(x) \tag{10}$$

where $V''_{4n+2}(x)$ is obtained from graphs with $2n$ pair interactions and 2 gradient interactions. The general terms of these two series are:

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$$V'_{4n}(x) = 3\mu\,\alpha^{2n}\,(\mu/2M)\,2^{2n-1}\left(1+\frac{1}{x}\right)^2 e^{-2nx}/x^{2n}$$

$$V''_{4n+2}(x) = \tfrac{1}{3}\mu\,\alpha^{2n+1}\,(\mu/2M)\,2^{2n}\;\underline{\tau}_1\cdot\underline{\tau}_2$$

$$\times\left[\underline{\sigma}_1\cdot\underline{\sigma}_2 + S_{12}\right]\left(1+\frac{1}{x}\right)^2 e^{-(2n+1)x}/x^{2n+1}$$

These three series have approximately the same radius of convergence x_c defined by the relation

$$x_c\, e^{x_c} > 2\alpha \tag{11}$$

For $g^2/4\pi = 10$, $x_c > 0.57$

For $g^2/4\pi = 15$, $x_c > 0.85$

The convergence of these series is very slow in the region of interest $x \doteq 1$, for the expected values of $g^2/4\pi$. Further, no better fit is obtained with experiment. However, it is suggested by Klein that nucleon pair production may be severely damped by radiative corrections and thus the effective coupling constant for H'_{pair} would be much smaller. This would have the effect of making convergence much more rapid. Unfortunately, the effect also invalidates the argument that V' and V'' are small corrections to V and,

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more important, that the series with more than 2 gradient interactions, not considered at all by Klein, are negligible. In fact, one must rewrite the transformed interaction Hamiltonian (8) as

$$H'(x) = \lambda H'_{pair} + H'_{gradient} \tag{12}$$

λ may be as small as $1/10$, in which case it is conceivable that a different approach should be made, where neither H'_{pair} nor $H'_{gradient}$ is considered more important than the other.

Brief mention may be made here of work performed by Brueckner and Watson (1953). Their starting point is the assumption that $H'_{gradient}$ is more important than H'_{pair}. The Schrodinger equation

$$(H_o + H')\,\Psi = E\,\Psi \tag{13}$$

is rewritten in terms of the Moller wave-matrix Ω

$$\Psi = \Omega\,\chi_a \tag{14}$$

where χ_a is an eigenstate of H_o.
The formalism of Lippman and Schwinger (1950) is used and extended, so that nucleon pair production and non linear interactions may be treated. A nuclear potential is derived up to fourth order in g, which gives a not unfavourable fit with experiment. It is also apparent that a different type of perturbation expansion results in this treatment,

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namely, that the interaction expansion has a general nth term proportional to g^{2^n} compared with the usual g^{2n}. The objections to this theory are that radiative corrections cannot be considerdd at all and that the damping required for H'_{pair} is rather critical and more severe than would be expected.

Recent work by Taylor (1954) and Kursunoglu (1954), in which the covariant new Tamm-Dancoff method introduced by Dyson (1953) is used, indicates that the fourth order potential obtained by Klein is reproduced in the same adiabatic approximation. In this new theory, the procedure of renormalisation is not so secure as in the Bethe-Salpeter equation, which was used by Levy, as mentioned above, in an attempt to find the effect of radiative corrections.

At the present time it seems likely that, until a thorough non-adiabatic treatment of the two-body nuclear potential up to fourth order is carried through, little can be said concerning the success or failure in obtaining a quantitative fit with experiment on the basis of the charge symmetric PS(PS) theory. In addition, calculation of the effect of higher order terms and of radiative corrections remains virtually impossible in any approximation other than the perturbation-static.

Numerical calculations have been carried out by Blatt

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and Kalos (1954) using a potential

$$V = V_2 + \lambda V_4 \tag{15}$$

where V_2 and V_4 are the second and fourth order potentials respectively, originally obtained by Levy. These seemed to give the best fit, although their derivation was faulty. λ was regarded as an arbitrary parameter which, it was hoped, would represent radiative corrections. g, x_c and λ were allowed to vary arbitrarily and the low-energy properties of the two-nucleon system were calculated on an electronic computor. Results showed that there was no region of the parameter space (g, x_c, λ) in which all the low-energy properties were simultaneously fitted. Further, the previous fit obtained by Levy is not as good as it first appears, for exact fitting of the deuteron quadrupole moment is critical, a small variation outside the observed quadrupole moment producing a large fluctuation in the singlet and triplet effective ranges. According to Villars (1952), Deser (1953) and Sessler (1954), some variation of the quadrupole moment is possible if one takes into account the effect of meson exchange; thus there still remains the possibility that a potential of the Levy form will still fit experimental results. Even if this were so, it could still not be stated conclusively that the PS(PS) theory gives a good

account /

account of nuclear forces, since the Levy potential is certainly not obtained up to fourth order in a consistent perturbation - static (PS(PS) theory. The possibility still remains that higher order, radiative, and non-static corrections to the Klein fourth order static potential may change its character to such an extent that agreement with experiment would once more be obtained.

In conclusion, it can be said that, although a qualitative fit has not resulted as yet, the qualitative characteristics of PS(PS) theory, which certainly do not occur in S(S) theory, are encouraging. Thus, the long-range tensor force in the second order potential gives a large quadrupole moment, while the fourth and higher order potentials contribute to a strong short-range interaction which ensures that reasonable singlet and triplet effective ranges are obtained.

3. Nuclear shell structure.

It has been known since about 1930 that some kind of structure exists in nuclei, for experiments showed that nuclei composed of certain numbers of protons or neutrons were particularly stable.

Mayer (1948) presented a series of facts which showed conclusively that especially stable nuclei existed with 2, 8, 14, 20, 28, 50 and 82 protons or neutrons and with 126 neutrons. These numbers were called "magic numbers", no doubt due to the fact that no theory seemed capable of explaining their occurrence.

Briefly, the evidence for nuclear structure was obtained by observing that a greater number of isotopes and isotones existed at magic numbers. Further, nuclei with a magic number of nucleons are more abundant in nature; this occurs because of their higher binding energy.

More evidence for magic numbers was obtained by observing variation of energy of alpha-particles emitted by radio-active nuclei. Low energies were observed if the parent nucleus were magic; high energies if the daughter were magic. Similar trends were observed in beta-decay.

Finally, cross-sections for radiative capture of medium energy neutrons (Several MeV), and for thermal

neutron /

neutron capture, were seen to be very small in the case of magic nuclei, when compared with the average cross-sections for nuclei near these magic numbers.

These experimental results indicated that nucleons arranged themselves into shells, which contained a certain maximum number of either protons or neutrons. These shells would be very stable when completely filled. This is in complete analogy with atomic structure, in which electrons arrange themselves into shells of certain maximum numbers, leading to the explanation of the periodic table.

Pursuing further the analogy with atomic physics, a first attempt at explaining the occurrence of magic numbers is to assume that a nucleon in a nucleus moves under the influence of a constant central potential which is the effect of the presence of the other nucleons. This smoothing out of the variations in the interaction between a particular nucleon and the rest of the nucleus is usually termed the "independent particle model".

It appears at first sight that the criterion for the use of the independent particle model is not satisfied, namely, that the mean free path of a nucleon in nuclear matter is several times the diameter of the nucleus. This is due to the fact that nuclear forces are very strong, and the

nucleon /

nucleon density of nuclear matter is large. However, two effects contribute much towards lengthening the mean free path. Nuclear forces, although very strong, are extremely short-ranged; further, the Pauli exclusion principle is important in considering the interaction between nucleons in a nucleus, forbidding many collisions which would normally be thought possible in a less rigorous treatment.

A calculation based on the independent particle model, in which the common potential is a square well of reasonable depth and width, leads to a definite shell structure, each shell, as in the atom, having a definite angular momentum. However, the numbers appearing here are not the magic numbers, except for several at the beginning of the series. Variation of the square well within reason (such as rounding off the discontinuity) does not produce any significant changes.

It was suggested by Mayer that a simple explanation of the magic numbers could be obtained if one assumes that there is a spin orbit coupling between nucleons, as well as the static central force. The requirements on this spin orbit coupling are rather surprising. Firstly, inverted doublets are required; that is, an energy level, corresponding to a particular orbital angular momentum $l\hbar$, is split into two levels with total angular momentum $(l+\frac{1}{2})\hbar$ and

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$(l-\frac{1}{2})\hbar$, corresponding to the spin and orbital angular momentum of the nucleon being parallel or anti-parallel respectively, and the higher energy level must correspond to the anti-parallel state. Secondly, the level splitting must amount to at least 2 MeV, increasing with l. If these requirements are satisfied, and a potential intermediate between a square well and a harmonic oscillator is used, then the levels previously obtained cross each other in such a way that they now group themselves together quite naturally into shells containing the correct number of nucleons to produce the observed magic numbers.

It was stated above that the requirements on the spin orbit coupling were rather surprising. This is only so if one wishes to pursue the analogy with atomic physics to the end; for a spin orbit term occurs there, namely, the Thomas relativistic correction. If this were the source of the spin orbit coupling required for the nuclear shell model, then these requirements would not be satisfied. The opposite sign would be obtained, leading to ordinary doublets, and the magnitude would be approximately $1/15$ th too small.

However, the success of the nuclear shell model, in its subsequent application, for example, to intrinsic spins and magnetic moments of nuclei, is unquestionable. One is

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thus justified in believing that there must be some other source from which an adequate spin orbit force, both in magnitude and sign, can be obtained.

The purpose of the work to be discussed in detail later is in part an attempt to obtain a satisfactory field theoretical explanation for this large spin orbit coupling on the basis of the current pseudoscalar meson theory.

4. Polarization phenomena in double scattering.

In section 3, the existence of a spin orbit coupling in nuclear forces has been discussed with reference to the Mayer nuclear shell model. The possibility of a parallel effect in nucleon-nucleon scattering was reported by Case and Pais (1950), who showed that the high energy proton-proton scattering data could be explained by means of adding a spin orbit coupling to the static nuclear potential. The results they obtained were not reliable, as shown later by Goldfarb and Feldman (1952), due to the fact that a strongly singular potential had to be used to fit the observed variations in the scattering cross-sections. The importance of the work by Case and Pais, however, was that it indicated that the presence of a strong spin orbit coupling could considerably affect nucleon-nucleon scattering. These authors also suggested a possible connection with the Mayer shell model, although it was not expected that the model would still be valid at high energies.

The presence of a spin orbit coupling is very difficult to ascertain in single nucleon-nucleon scattering, where only scattering cross-sections and angular distributions can be measured. However, a spin orbit term produces a spin polarization of nucleons, due to a difference in scattering cross-sections for the two spin orientations in the incident beam. This cannot be observed directly, but by

measuring /

measuring the right - left asymmetry produced in a subsequent second scattering. A similar procedure can be followed in double scattering of high-energy protons by nuclei.

Recent experiments in double scattering of high energy protons by nuclei reveal considerably more polarization than is observed in proton - proton double scattering. Results also show that the largest asymmetry occurs in the range where elastic scattering by the nucleus is expected to predominate.

It was proposed independently by Fermi (1954), Malenka (1954) and others, that the asymmetry in nucleon-nucleus scattering could be explained by means of the spin orbit interaction used in the nuclear shell model. These authors assumed that essentially the same spin orbit interaction acted between the nucleus and the nucleon being elastically scattered. An 'optical model' was used to describe the static interaction. Thus, in Malenka's notation, the nucleus exerts a potential H' on the incident nucleon:

$$H' = (1 + i\epsilon) V_0(r) + \frac{1}{2} V_1(r) \underline{\sigma} \cdot \underline{L} \tag{16}$$

where $\underline{L} = \underline{r} \times \underline{p}$.

ϵ takes into account nuclear absorption. Its value at 100 MeV is approximately $0 \cdot 5$, and increases slowly with energy. For example, at 340 MeV, it has increased to $0 \cdot 6$.

$V_0(r)$ is a central potential well and $V_1(r)$ describes the radial dependence of the spin orbit coupling. The form of V_1 is not necessarily dependent on V_0. However, it is expected that V_0 will be best represented by a modified square well, and a possible form for V_1, in analogy with the Thomas coupling, is $V_1 \propto \frac{1}{r}\frac{dV_0}{dr}$. The spin orbit term is, according to Fermi, 15 times larger than the Thomas coupling.

Other forms of $V_1(r)$ envisaged are (1) a modified square well, assuming a uniform spin orbit effect, and (2) a generalization of the Thomas coupling, $\frac{1}{r}\frac{d\phi(r)}{dr}$, where $\phi(r)$ characterizes the collective action of the core.

Calculations were carried out using the first Born approximation, which is expected to give, in the high energy region, quantitatively correct results for the polarization. Results indicate that type (2) for $V_1(r)$ gives better results. Using the magnitude suggested by Fermi, these authors find that the approximate magnitude and angular location of the maximum polarization are 50% and 10° respectively, for 340 MeV nucleons on carbon. These results are in rough agreement with experiment.

Finally, the same model was employed in the low

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energy region by Adair et al. (1954), who investigated the scattering of 400keV polarized neutrons by heavy nuclei. The variation of the polarization with the atomic number is in agreement with that expected from an optical model modified by a spin orbit term $-\frac{3}{4}\,\mathrm{MeV} \times (\underline{\sigma} \cdot \underline{L})$. In these calculations, the absorption coefficient ϵ was taken to be 0·05 , in accordance with the results of Porter, Feshbach and Weisskopf (1954).

Thus, both high-energy and low-energy scattering data appear to be in agreement with calculations which depend on the introduction of a spin orbit coupling of the same magnitude and sign as that required by the nuclear shell model.

In the light of the discussion in this and the previous section on the nuclear shell model, the existence of a strong spin orbit coupling in nuclear forces, and the connection between polarization and the shell model, are reasonably beyond doubt. However, so far, there has been no attempt to justify its presence. In the next section, and in chapter II, a possible explanation will be given in detail.

5. Spin orbit coupling in meson field theory.

In sections 3 and 4 the desirability for a strong spin orbit coupling in nuclear forces has been stressed. An intimate connection is suspected between the strong spin orbit term which appears in the Mayer nuclear shell model and that required to explain polarization effects in double scattering experiments. However, the only justification given so far is that good agreement is obtained. The form which the coupling takes is suggested by the Thomas coupling, obtained as a relativistic correction to the energy of a fermion moving under the influence of a scalar potential, and which is one order of magnitude too small. This has already been discussed in the previous sections.

It was felt that there was need for some fundamental justification for the introduction of a spin orbit force of this strength. Although results obtained from meson field theory in other branches of physics have been somewhat disappointing, nevertheless, it may be argued that there is no better guide in theoretical physics at the present time, as far as obtaining a fundamental basis on which to build more exact theories; for field theory is applicable, in contrast with other theories, to a wide range of problems without alteration of parameters.

On the other hand, in a scattering problem in nuclear physics, for example, if field theory is not employed, one must specify the scattering potential, and perhaps obtain a quantitative fit with experiment only by varying certain arbitrary parameters in the potential.

Thus, a measure of success would be achieved if, out of some crude approximation, provided it were consistent, meson field theory yielded a strong spin orbit coupling, an order of magnitude larger than the Thomas effect, and opposite in sign, so that inverted doublets would result.

Immediately the need for such an investigation arose, a strong spin orbit force was obtained independently by Rosenfeld and Gaus (1949). Assuming vector meson theory, these authors found that a suitable choice of the vector and tensor constants g and f led to a spin orbit coupling proportional to gf. The spin orbit coupling thus obtained had the correct magnitude and sign required by the shell model. However, recent advances in meson experimental physics indicate that the π - meson is pseudoscalar (cf. section 2). Consequently, the spin orbit force of Rosenfeld and Gaus must be discarded.

It was not until 1953 that further progress was made.

Klein /

Klein (1953), in investigating the adiabatic nuclear potential on the basis of PS(PS) meson theory (cf. section 2), discovered that a more detailed treatment of the two-nucleon fourth order potential, without radiative corrections, revealed a velocity dependent correction in the form of a spin orbit force. The method used by Klein to obtain this correction is very similar to that which will be used in sections 3 and 4 of chapter II to extract spin orbit coupling from the total matrix element.

The spin orbit coupling obtained by Klein was not large enough, leading to a doublet splitting of approximately 1 MeV. In any case, the opposite sign was obtained to that required by the shell model.

Dresner (1953) has calculated spin orbit corrections, on the basis of PS(PS) theory, corresponding to some typical Feynman diagrams for two -, three -, and four - nucleon potentials. The resulting level splittings are too small, although here contributions of both signs are obtained. Little can be said further about these calculations, for it is not known whether in a fuller treatment, the total negative contribution will be larger or smaller than the positive. Again, these calculations were confined to the non-covariant Tamm - Dancoff method. Thus radiative corrections once more

were /

were not considered.

Klein's results, and in part Dresner's, can be applied to nuclei in a straightforward way, for Hughes and Le Couteur (1950) have shown that the resultant of the two-particle spin orbit forces acting between the nucleons of a saturated core and an external nucleon leads to a one-particle spin orbit force between the outer nucleon and the core. Thus, by following this procedure, one obtains a one-particle spin orbit coupling for the two-particle spin orbit forces obtained by Klein and Dresner.

Thus, so far, all treatments have shown that two-nucleon spin orbit forces are either of the wrong sign, or too small, for application to the shell model and similarly, as discussed in section 4, for high and low energy scattering.

A different approach to the problem was proposed by Chisholm and Touschek (1953) (referred to hereafter as CT). This consisted of representing the interaction between the saturated nuclear core and the external nucleon by a scalar central potential, and of calculating the change in the self-energy of the external nucleon due to the presence of the core. Physically, the spin orbit

coupling /

coupling arises in the same way as the anomalous magnetic moment of the electron in electrodynamics; it is considered to arise through the influence exerted on the mesonic cloud around the nucleon by the inner nuclear core. Due to the very much larger coupling constant in the meson - nucleon interaction, an important spin orbit force is likely to arise. The importance of this effect should certainly be investigated. The interesting aspect of this work is that it depends solely on radiative corrections to give rise to the required spin orbit effect.

Evaluation of the spin orbit coupling was carried out to order g^2 in the sense of a perturbation calculation and employing the Feynman - Dyson techniques. A spin orbit coupling was obtained strong enough to lead to a doublet splitting of several MeV for heavy nuclei. However, the wrong sign was again obtained.

This result is disappointing at first. However, CT neglected certain important terms in their evaluation of the spin orbit coupling. These terms contribute to the spin orbit force in a reduction of the matrix element to the non-relativistic limit. These corrections are in fact exactly twice the term originally obtained by CT, and are opposite in sign. The result os that this self energy effect now gives rise to a spin orbit coupling which is of the right order of magnitude and sign for application to the Mayer nuclear shell model. Details of these calculations will be given in Chapter II, section 3, and have recently been published by the author (1955). Finally, it should be noted that the magnitude obtained here is still too small to explain the He^5 splitting, which is much larger than the splitting in heavier nuclei.

On the strength of these calculations described above, it appears probable that the main contribution to spin orbit coupling arises from self-energy effects. This conclusion is supported to some extent by calculations recently carried out by Moorhouse (1954) in which the new Tamm - Dancoff method is modified to compare states of A and $A+1$ nucleons. The A nucleon state, rather than the vacuum, is taken to be the comparison state. In particular, the difference in the self-energies of the two states considered is obtained by considering a model in which bound virtual mesons are emitted and reabsorbed by the nucleus. The effect of the Pauli exclusion principle is taken into account. Results of these calculations indicate that in light - medium nuclei $(4 \leqq A \leqq 40)$ inverted doublets occur, with a large splitting in agreement with experiment. In the case of He^5, the position is not so secure, since the centre of mass motion is neglected, but there is a tendency for a

substantial /

substantial increase in the splitting, in qualitative agreement with experiment.

However, some doubt may be cast on the CT results, since no account has been taken of the possibility that the mesons exchanged between the nuclear core and the external nucleon, the effect of which is represented in this model by a scalar central potential $U(r)$, may in some way affect the strength of the coupling. To investigate this, it was thought worthwhile to estimate the spin orbit coupling obtained by particularising the potential $U(r)$ to be the potential between two nucleons which arises from the exchange of mesons and evaluating the spin orbit coupling directly from graphs analagous to that considered by CT. Since $U(r)$ is a scalar, one must assume that, since the mesons exchanged are pseudo-scalar, $U(r)$ arises from the exchange of an even number of mesons. Thus, the simplest case to consider is the exchange of two mesons. The evaluation of these graphs, which are of sixth order in g, comprises the main part of this thesis; a detailed account will be given in chapter II, section 4.

As will be seen later, results of these calculations

are /

are encouraging, in that they support the treatment given by CT. Although both methods, that of CT and that to be described in the next chapter, involve perturbation theory and other approximations, and therefore must be suspect "ab initio", taken together they appear quite convincing. A relatively good fit between meson field theory and experiment is certainly obtained on the basis of regarding the spin orbit force as a self-energy effect. The presence of spin orbit forces of opposite sign must be taken into account, since they also appear in the same theory, but these effects seem to be smaller than the self-energy effects. It cannot, however, be said that the CT and Moorhouse results should be added. Rather, these should be viewed as two different models of the same basic effect, namely, the self-energy effect. That both methods agree with experiment should be considered most satisfactory.

CHAPTER II

1. Introduction.

In this chapter, details will be given of work carried out by the author on the possibility of the existence of a strong spin orbit coupling, arising from a self-energy effect, in PS(PS) meson field theory.

In section 2, an outline of Chisholm's method for evaluating S-matrix elements is given. This will serve two purposes. It will give the necessary mathematical background to the method of evaluation of the S-matrix elements in the latter part of the chapter and it will serve to introduce the notation to be used extensively later. The reason for using Chisholm's method is that it reduces all S-matrix elements to a standard form, thus reducing the risk of introducing errors. It allows one to evaluate graphs with two or more 'internal momenta' (that is, independent momentum integration variables), with the same facility as the simpler graphs. The complexity of the higher order graphs is instead absorbed into tedious algebraic analysis and, finally, a complicated integration over a set of parameters, which can in general be performed only numerically.

Section 3 contains details of the work carried out

by /

by Chisholm and Touschek, and subsequently revised by the author, to investigate whether the presence of a nuclear core would affect the self-field of a nucleon in such a way that a large spin orbit force, of the correct sign for the nuclear shell model, would arise. The interaction of the core and the outer nucleon is represented by a scalar central potential, $U(r)$.

In section 4, $U(r)$ is particularised to be the potential arising from double meson exchange between two nucleons. The resulting sixth order graphs, obtained in complete analogy with section 3, are evaluated by the methods developed in section 2.

The potential $U(r)$, of section 4, is estimated in section 5 in the same approximation as that made in section 4 for the spin orbit coupling. In this way, it is hoped that consistency is maintained.

Finally, in section 6, results of the calculation are given and compared with the results of section 3. A short discussion is then given of the significance of these results.

2. Method of calculating S-matrix elements.

Chisholm's (1952) method of calculating S-matrix elements consists of putting all line-factors in the form of scalar boson line-factors acted on by a product involving the usual Dirac matrices and a certain operator $\mathfrak{F}$, which depends only on the momenta of external lines and on particle masses. The integral over internal momenta now assumes a standard form. A set of parameters is introduced which permits this standard integral to be performed. The effect of the product of operators on the integral is determined and the S-matrix element is finally obtained as a multiple integral over the set of parameters. The method outlined above is essentially a generalization of Feynman's method of parameters. It is not in general applicable to divergent graphs, but only in the special case of graphs with one internal momentum. This is due to the fact that many graphs have a divergent part inserted, such as a vertex part, but appear to be convergent on first sight. A proof of the inapplicability of the method for those so-called 'primitively divergent graphs' can easily be constructed by considering the question of uniform convergence and the related question of changing the order of integrations in a multiple

integral /

integral.

Only the charge symmetric PS(PS) meson theory will be considered in the following more detailed account of Chisholm's method. Details will be restricted to those with direct application to the later discussion on spin orbit coupling.

Line and vertex factors are

Meson line: $\dfrac{-i}{(2\pi)^4}\,\dfrac{\delta_{ij}}{(p+k)^2+\mu^2}$

Nucleon line: $\dfrac{-i}{(2\pi)^4}\,\dfrac{i\gamma.(p+k)-M}{(p+k)^2+M^2}$ (1)

Meson-nucleon vertex: $g\,(2\pi)^4\,\tau_i\,\gamma_5$

where p is a linear combination of external momenta and k is a linear combination of internal momenta.

In the meson line factor, δ_{ij} refers to the isotopic spin matrices τ_i which occur at meson-nucleon vertices. $\gamma.(p+k)$ denotes the scalar product of the four-vectors γ_μ and $(p+k)_\mu$.

In order to reduce the integral over the internal momenta to a standard form, the meson and nucleon line factors are replaced by:

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Meson line:
$$\frac{-i}{(2\pi)^4} \frac{\delta_{ij}}{(p+k)^2 + \sigma}$$

Nucleon line:
$$\frac{-i}{(2\pi)^4} \left[-M - \frac{1}{2} i \gamma_\mu \frac{\partial}{\partial p_\mu} \int_\sigma^\infty d\sigma \right]$$

$$\times \frac{1}{(p+k)^2 + \sigma}$$

$$= \frac{-i}{(2\pi)^4} \mathcal{F}(p, \sigma) \frac{1}{(p+k)^2 + \sigma} \qquad (2)$$

Use of these new line factors reduces integrand to a product of factors of the form $\left[(p+k)^2 + \sigma \right]^{-1}$

The auxiliary parameters are now introduced, using the identity:

$$\frac{1}{\prod_{r=1}^{s} q_r} = (s-1)! \int_A \prod_{r=1}^{s} dc_r \frac{1}{\left[\sum_{r=1}^{s} c_r q_r \right]^s} \qquad (3)$$

where A is that portion of the hyperplane $\sum_{r=1}^{s} c_r = 1$ for which all c_r are positive or zero.

q_r is of the form $\left[(p_r+k)^2 + \sigma_r \right]$ where k is a linear combination of $k_1, k_2 \ldots\ldots k_l$, the internal momenta.

Thus, there occurs in the integral a quadratic

form /

form

$$Q(k_1, k_2, \ldots, k_l) \equiv \sum_{s,t=1}^{l} A_{st}\, k_s . k_t + 2 \sum_{s=1}^{l} A_s . k_s + C \tag{4}$$

where A_{st}, A_s, C are functions of the p_r, σ_r and c_r.

The integral over k_1, k_2, ..., k_l is of the form

$$\int d^4k_1 \int d^4k_2 \ldots \int d^4k_l \left[Q(k_1, k_2, \ldots, k_l) \right]^{-s}$$

The result is easily obtained and is

$$(s-1)! \int_A \prod_{r=1}^{s} dc_r \left[(i\pi^2)^l \frac{(s-2l-1)!\, \Lambda^{s-2l-2}}{(s-1)!\, \chi^{s-2l}} \right]$$

$$= (i\pi^2)^l (s-2l-1)! \int_A \prod_{r=1}^{s} dc_r \frac{\Lambda^{s-2l-2}}{\chi^{s-2l}} \tag{5}$$

where

$$\Lambda = \begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1l} \\ A_{21} & A_{22} & \ldots & A_{2l} \\ - & - & - & - \\ A_{l1} & A_{l2} & \ldots & A_{ll} \end{vmatrix} \tag{6}$$

and

$$\chi = \begin{vmatrix} A_{11} & A_{12} & \ldots & A_{1l} & A_1 \\ A_{21} & A_{22} & \ldots & A_{2l} & A_2 \\ - & - & - & - & - \\ A_{l1} & A_{l2} & \ldots & A_{ll} & A_l \\ A_1 & A_2 & \ldots & A_l & C \end{vmatrix} \tag{7}$$

Since Λ does not depend on the p_r and σ_r, only the effect of a product of operators $\mathcal{F}(p_r, \sigma_r)$ on χ^{-m} is required. That is,

$$\prod_{r=1}^{n} \mathcal{F}(p_r, \sigma_r)\, \chi^{-m}, \qquad m > \frac{n}{2}$$

where n is the number of nucleon lines in the graph. The assumption that m is not less than or $= \frac{n}{2}$ avoids divergent graphs, which cannot in any case be evaluated by Chisholm's method.

Usually, the product of $\mathcal{F}$ operators will be interspersed with γ_5 matrices, but this need not be considered here. Firstly, two quantities must be defined:

$$\Omega_{p_r} = -M + \frac{1}{\Lambda} \begin{vmatrix} A_{11} & A_{12} & \cdots & A_{1l} & \partial p_r k_1 \\ A_{21} & A_{22} & \cdots & A_{2l} & \partial p_r k_2 \\ - & - & - & - & - \\ A_{l1} & A_{l2} & \cdots & A_{ll} & \partial p_r k_l \\ i\gamma . A_1 & i\gamma . A_2 & \cdots & i\gamma . A_l & i\gamma . p_r \end{vmatrix} \tag{8}$$

and

$$\Omega_{p_r p_s} = \frac{1}{2\Lambda^2} \begin{vmatrix} A_{11} & A_{12} & \cdots & A_{1l} & \partial p_r k_1 \\ A_{21} & A_{22} & \cdots & A_{2l} & \partial p_r k_2 \\ - & - & - & - & - \\ A_{l1} & A_{l2} & \cdots & A_{ll} & \partial p_r k_l \\ \partial p_s k_1 & \partial p_s k_2 & \cdots & \partial p_s k_l & \frac{\delta_{rs}}{c_r} \end{vmatrix} \tag{9}$$

$\partial_{p_r k_t}$ is defined by the statement that $2\partial_{p_r k_t}$ is the coefficient of the scalar product $p_r \cdot k_t$ in $[(p_r+k)^2+\sigma_r]$. That is, $\partial_{p_r k_t} = 0, \pm 1$.

The result will now be stated. The proof may be obtained in the original publication.

Firstly, the 'basic term' is formed by replacing every $\mathcal{F}(p_r, \sigma_r)$ by Ω_{p_r}. The basic term is thus:

$$\prod_{r=1}^{n} \Omega_{p_r} \chi^{-m} \tag{10}$$

From this basic term, the 'derived terms' are obtained thus: a pair $(\Omega_{p_s}, \Omega_{p_t})$, not necessarily adjacent, is replaced by $(\gamma_\alpha, \gamma_\alpha)\, \Omega_{p_s p_t} \int_\chi^\infty d\chi$ with summation over α. This is repeated for all pairs, one at a time, then for two pairs simultaneously, and so on. The sum of the basic term and all the derived terms is the required result.

The general S-matrix element has therefore been evaluated in the terms of the quantities Λ, χ, Ω_{p_r}, $\Omega_{p_r p_s}$ and γ_s matrices. At this point, the p_r and σ_r are expressed in terms of the momenta of the external lines and particle masses, respectively. All

that /

that remains to be done now is a multiple integral over the set of parameters c_r .

Chisholm's method will be illustrated in sections 3, 4 and 5, where it will be extensively employed.

3. Spin orbit coupling in phenomenological treatment.

Reference has already been made in Chapter I, section 5, to the work performed by Chisholm, Touschek and the author in investigating the possibility that the self-energy corrections for a nucleon moving in a scalar potential well $U(r)$ lead to a strong spin orbit coupling in the charge symmetric PS(PS) meson theory. In this section, full details will be given of this work.

The potential $U(r)$ represents the interaction between the nuclear core and the external nucleon. The introduction of a scalar potential is justified if one assumes that the main contribution to the potential, from the point of view of field theory, would arise from the exchange of an even number of pseudoscalar mesons.

Following Dyson's prescriptions, the second order correction to the potential energy $U(r)$ of a nucleon at $\underline{r}$ is

$$\Delta_2 U(r) = -\frac{1}{2}\int d^4x_1\, d^4x_2\, P\left(H'(x_1), \bar{U}(\underline{r}), H'(x_2)\right) \qquad (1)$$

where $\bar{U}(\underline{r})$ is the potential density of the nucleon at $\underline{r}$

$$= \bar{\Psi}(\underline{r})\, U(r)\, \Psi(\underline{r}).$$

$$H'(x) = i g \bar{\Psi}(x) \gamma_5 \tau_i \phi_i(x) \Psi(x)$$

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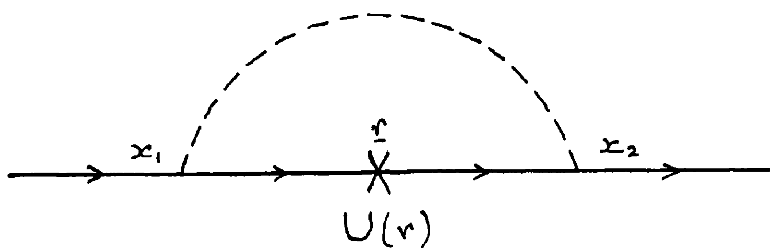
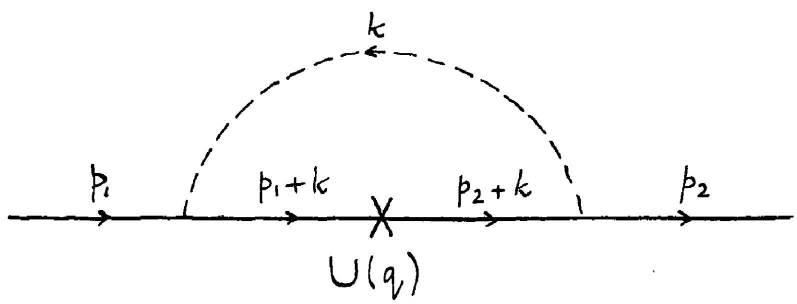
The only second order graph contributing to a spin orbit coupling corresponds to a Lambshift process, shown in Figure 1.



(Fig. 1)

This graph can be easily evaluated in momentum space. The equivalent graph in momentum space is shown in Figure 2.



(Fig. 2)

This leads to the following expression for the matrix element:

$$\Lambda(p_2, p_1)$$

$$= \frac{3ig^2}{(2\pi)^4} \int d^4k \, \frac{\bar{\Psi}(p_2)\,\gamma_5 \left[i\gamma.(p_2+k)-M\right] U(q) \left[i\gamma.(p_1+k)-M\right] \gamma_5\, \Psi(p_1)}{(k^2+\mu^2)\left[(p_2+k)^2+M^2\right]\left[(p_1+k)^2+M^2\right]} \tag{2}$$

where /

where

$$U(r) = \int d^3q \, e^{i\underline{q}\cdot\underline{r}} \, U(q)$$

$$\Psi(x) = \int d^4p \, e^{ip.x} \, \Psi(p)$$

The vector notation being used here is that three-vectors are underlined and four-vectors are not. Whether a quantity is a scalar or a four-vector must be inferred from the context. Thus, in the two definitions above, x and p are four-vectors, but r and q are scalars.

$\Lambda(p_2, p_1)$ is readily evaluated as a simple application of Chisholm's methods. However, it is more convenient to treat the divergent part differently and so the method will be applied only to those parts which are finite before subtraction. The reason for this procedure is that the additional finite terms after subtraction are very small; this is more easily seen by a direct treatment of the matrix element.

Now,

$$\Lambda(p_2, p_1) = \frac{3ig^2}{(2\pi)^4} \bar{\Psi}(p_2)\gamma_5 \mathcal{F}(p_2, \sigma_2) U(q) \mathcal{F}(p_1, \sigma_1)\gamma_5 \Psi(p_1) \, I \tag{3}$$

where finally $\sigma_1 = \sigma_2 = M^2$.

$$I = \int d^4k \frac{1}{(k^2+\mu^2)\left[(p_1+k)^2+\sigma_1\right]\left[(p_2+k)^2+\sigma_2\right]}$$

$$= 2\int_A dc_1\, dc_2\, dc_3 \int d^4k \; Q^{-3}$$

where $\quad Q = k^2 + 2p.k + C$

$\quad\quad p = c_1 p_1 + c_2 p_2$

$\quad\quad C = c_3 \mu^2$

Thus, $\quad I = i\pi^2/\chi$

where $\quad \chi = C - p^2$

$$\doteq (c_1+c_2)^2 M^2 + c_3\mu^2$$

$$= (c_1+c_2)^2 M^2 + (1-c_1-c_2)\mu^2$$

using the relation $\quad c_1 + c_2 + c_3 = 1.$

Thus $\quad \Lambda(p_2, p_1)$

$$= -\frac{3g^2}{16\pi^2}\int_A dc_1\, dc_2\, dc_3\; \bar{\Psi}(p_2)\,\frac{\bar{\Omega}_2\,\bar{\Omega}_1}{\chi}\,\Psi(p_1)\,U(q) \tag{4}$$

Here, the notation $\bar{\Omega}_i$ has been used for $\gamma_5\Omega_{p_i}\gamma_5$ This, and the symbol Ω_i for Ω_{p_i} will be used hereafter.

/

$\Lambda(p_2, p_1)$ as given in (4), now represents the part of the matrix element which is finite before subtraction.

$$\bar{\Omega}_i = -M - i\gamma.(p_i - p) , \qquad i = 1, 2.$$

$\bar{\Omega}_2 \bar{\Omega}_1$ contains four terms which all lead to a spin orbit coupling in the non-relativistic limit:

$$\begin{aligned} &(a) \quad -(1 - c_1 - c_2)(\underline{\gamma}.p_2)(\underline{\gamma}.p_1) \\ &(b) \quad iM(1 - c_1 - c_2)\gamma_4 \underline{\gamma}.(p_2 - p_1) \\ &(c) \quad iM\underline{\gamma}.[p_1(1 - 2c_1) + p_2(1 - 2c_2)] \\ &(d) \quad M^2(c_1 + c_2)^2 \end{aligned} \qquad (5)$$

That those terms all lead to a spin orbit coupling can be seen from the following considerations: consider the reduction to two-component spinors χ of the potential energy matrix element

$$\bar{\Psi}(p_2) V \Psi(p_1) \qquad (6)$$

where $V = A(\underline{\gamma}.p_2)(\underline{\gamma}.p_1) + B\gamma_4\underline{\gamma}.\underline{s} + C\underline{\gamma}.\underline{t} + D$

A, B, C, D are all functions of q, and thus all lead to scalar central potentials.

$$\underset{\sim}{s} = b_1 p_1 + b_2 p_2$$

$$\underset{\sim}{t} = c_1 p_1 + c_2 p_2$$

The reduction is effected by the substitution:

$$\bar{\Psi}(p_2) = \left(\chi^+ \quad \chi^+ \frac{\sigma . p_2}{2M} \right) \gamma_4$$

$$\Psi(p_1) = \begin{pmatrix} \chi \\ \frac{\sigma . p_1}{2M} \chi \end{pmatrix} \tag{7}$$

The matrix element then reduces to the form

$$\chi^+ \; V_{N.R.} \; \chi \tag{8}$$

where $V_{N.R.} \equiv V$ (non-relativistic)

$V_{N.R.}$ contains four spin orbit terms

(a) $A\,S$ (9)

where $S = \chi^+ (\underset{\sim}{\sigma} \cdot p_2)(\underset{\sim}{\sigma} \cdot p_1) \chi$

(b) $- i/_{2M} \, (b_2 - b_1) \; B\,S$

(c) $- i/_{2M} \, (c_1 + c_2) \; C\,S$

(d) $- 1/_{(2M)^2} \; D\,S$

$A S$, for example, can be written as

$$i \chi^{+} A(q) \; \underline{\sigma} \cdot (\underline{p}_2 \times \underline{p}_1) \; \chi$$

$$= i \chi^{+} A(q) \; \underline{\sigma} \cdot (\underline{q} \times \underline{p}) \; \chi \tag{10}$$

where $\underline{q} = \underline{p}_2 - \underline{p}_1$

$\underline{p} = \frac{1}{2}(\underline{p}_1 + \underline{p}_2)$

$\underline{q}$ is the momentum transferred by the core to the outer nucleon. $\underline{p}$ is the average momentum of the nucleon.

On taking the Fourier transform with respect to $\underline{q}$, one obtains

$$V_{\text{spin orbit}} = \underline{\sigma} \cdot \left(\nabla A(r) \times \underline{p} \right) \tag{11}$$

where $$A(r) = \int e^{i \underline{q} \cdot \underline{r}} A(q) \, d^3 q$$

Hence, the contributions to the spin orbit coupling are

$$(a) \quad \Lambda_1 = \frac{3g^2}{16\pi^2} \int_0^1 dc_1 \int_0^{1-c_1} dc_2 \, \frac{1 - c_1 - c_2}{(1-c_1-c_2)\mu^2 + (c_1+c_2)^2 M^2} \times \underline{\sigma} \cdot \left(\nabla U(r) \times \underline{p} \right)$$

$$= \frac{3g^2 I}{16\pi^2 M^2} \; \underline{\sigma} \cdot \left(\nabla U(r) \times \underline{p} \right) \tag{12}$$

where $$I = \int_0^1 \frac{x(1-x)\,dx}{(1-x)\lambda^2 + x^2}, \qquad \lambda = \mu/M .$$

(b) $$\Lambda_2 = -\frac{i}{2M}\, 2iM \left(\frac{-3g^2 I}{16\pi^2 M^2} \right) \sigma.(\nabla U \times p)$$

$$= -\Lambda_1$$

(c) $$\Lambda_3 = \frac{-i}{2M}\, iM \left(\frac{-3g^2}{16\pi^2 M^2} \right) 2I\, \sigma.(\nabla U \times p)$$

$$= -\Lambda_1$$

(d). This term will not be given here, since it is largely cancelled by the subtraction term. (d) can be regarded as the usual Thomas correction to the static radiative correction to $U(r)$.

The integral I has the approximate value $1{\cdot}2$ Thus the total spin orbit coupling arising from the self-energy effect is

$$\Lambda_1 + \Lambda_2 + \Lambda_3 = -3{\cdot}6\,(g/4\pi M)^2\, \sigma.(\nabla U \times p) \tag{13}$$

It may be noted in passing that the result obtained by

Chisholm /

Chisholm and Touschek had the opposite sign but the same magnitude. However, these authors neglected the importance of the terms Λ_2 and Λ_3, thus obtaining the total spin orbit coupling from Λ_1 alone. There is also a slight discrepancy in the magnitude, $3 \cdot 6$ being obtained instead of 3, but this is not of great importance.

The problem of renormalization of the graph will now be considered. The object of the discussion is to show that this can be carried through with negligible effect on the results given in (13). The matrix element (2) contains a term αU, where α is a scalar divergent quantity.

$$\alpha = \frac{3 i g^2}{(2\pi)^4} \int d^4k \frac{[i\gamma.(p_2+k)+M][i\gamma.(p_1+k)+M]}{(k^2+\mu^2)[(p_2+k)^2+M^2][(p_1+k)^2+M^2]} \tag{14}$$

α is logarithmically divergent. To remove the infinity unambiguously, a term α_0 must be subtracted. α_0 is obtained from α by setting $p_1 = p_2 = p_0$. p_0 is a four-momentum satisfying the Dirac equation $i\gamma.p_0 + M = 0$. Thus,

/

$$\alpha_0 = \frac{3ig^2}{(2\pi)^4} \int d^4k \frac{\left[i\gamma.(p_0+k)+M\right]^2}{(k^2+\mu^2)\left[(p_0+k)^2+M^2\right]^2}$$

(15)

In α, equation (14), p_1 and p_2 can be replaced by $(\underline{0}, iE)$ in the non-relativistic limit, and similarly $i\gamma.p_1 = i\gamma.p_2 = -E$.

$B = E - M =$ binding energy of nucleon and core

$= -$ 8MeV, on the average. (16)

Thus

$$\alpha \doteqdot \frac{3ig^2}{(2\pi)^4} \int d^4k \frac{\left[i\gamma.k + M - E\right]^2}{(k^2+\mu^2)\left[k^2 - 2k_0E + M^2 - E^2\right]^2}$$

$$\doteqdot \frac{3ig^2}{(2\pi)^4} \int d^4k \frac{\left[i\gamma.k - B\right]^2}{(k^2+\mu^2)\left[k^2 - 2k_0M - 2MB\right]^2}$$

(17)

Equation (17) is obtained by replacing k_0E by k_0M and $E^2 - M^2$ by $2MB$

α_0 is obtained from α by setting $B = 0$.

$$\alpha_0 = \frac{3ig^2}{(2\pi)^4} \int d^4k \frac{\left[-k^2\right]}{(k^2+\mu^2)(k^2 - 2k_0M)^2}$$

(18)

(17) can be written:

$$\alpha = \frac{3ig^2}{(2\pi)^4}\int_0^1 2x\,dx \int d^4k \, \frac{[i\gamma.k - B]^2}{[k^2 - 2x k_0 M - 2xMB + (1-x)\mu^2]^3}$$

Now let $k = q - M'$, $M' = (\underline{0}, iMx)$

Then

$$\alpha = \frac{3ig^2}{(2\pi)^4}\int_0^1 2x\,dx \int d^4q \, \frac{(i\gamma.q + Mx - B)^2}{(q^2 + L)^3}$$

where

$$L = M^2x^2 - (2MB + \mu^2)x + \mu^2$$

Thus,

$$\alpha = \frac{3ig^2}{(2\pi)^4}\int_0^1 2x\,dx \int d^4q \, \frac{(-q^2 + x^2M^2 - 2MBx)}{(q^2 + L)^3}$$

since denominator is a function of q^2.

$$\therefore \alpha = \frac{3ig^2}{(2\pi)^4}\int_0^1 2x\,dx \left\{ -\int \frac{d^4q}{(q^2+L)^2} + \left(2x^2M^2 - [4MB + \mu^2]x + \mu^2\right)\frac{i\pi^2}{2L} \right\}$$

(19)

Also, putting $B = 0$, one obtains:

$$\alpha_0 = \frac{3ig^2}{(2\pi)^4}\int_0^1 2x\,dx \left\{ -\int \frac{d^4q}{(q^2+L_0)} + \left(2x^2M^2 + \mu^2(1-x)\right)\frac{i\pi^2}{2L_0} \right\}$$

(20)

where $L_0 = M^2 x^2 + \mu^2 (1-x)$

Here approximation is introduced of neglecting μ^2, since at this stage it is seen that its neglect does not introduce any 'infra-red' divergences. The second terms in (19) and (20) are then e qual and thus cancel. (This approximation can be shown to be valid).

Thus,

$$\alpha_c = \alpha - \alpha_0$$

$$= \frac{3ig^2}{(2\pi)^4} \int_0^1 2x\,dx.\ i\pi^2 \log(L/L_0)$$

$$= -\frac{3g^2}{8\pi^2} \int_0^1 x \log\left(1 - 2B/Mx\right) dx \tag{21}$$

The integral in (21) can be easily evaluated and has the value $-2B/M$, neglecting higher powers in B/M.

$$\text{i.e.} \quad \alpha_c \doteqdot \frac{3g^2 B}{4\pi^2 M} \tag{22}$$

$$B/M \doteqdot -1/125 \qquad g \doteqdot 10.$$

$$\text{Thus} \quad \alpha_c \doteqdot -6\%. \tag{23}$$

The total effective static potential is thus

$$U_{renormalized} = \left(1 + \frac{3g^2 B}{4\pi^2 M}\right) U \qquad (24)$$

Substituting the value of α_c from (23) into (24), it is seen that the static radiative correction amounts to about 6% of U. Since the Thomas splitting is itself an order of magnitude, smaller than the spin orbit coupling obtained in (13), this additional contribution is negligible. It is important to go through with the renormalization, for it appears at first that term (d) of equations (12) leads to a sizeable spin orbit term. In any case, the result (24) is important for another reason. For it is $U_{renorm.}$ which is observed in nature and not U itself. The result (13) would thus be substantially altered for a completely different reason.

4. Spin orbit coupling in sixth order graphs.

4.(1). Introduction.

The programme adopted in section 3 of this chapter will be followed here as closely as possible. The general central potential $U(r)$ is now particularised to be the potential arising from double meson exchange between two nucleons. The assumptions involved here are (1) that $U(r)$ arises from a sum of two-nucleon interactions, and that many-body forces are not important; (2) that the two-nucleon interaction can itself be represented approximately by double meson exchange. Thus, in the calculation, $U(r)$ is replaced by $V_4(r)$, obtained from graphs 1 and 2. The spin orbit coupling, by analogy with the previous section, is to be obtained from graphs 1(a) and 2(a).

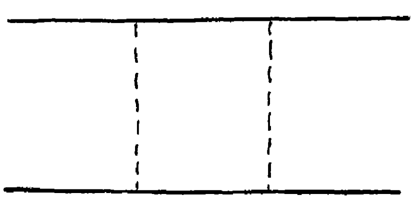


Fig. 1.

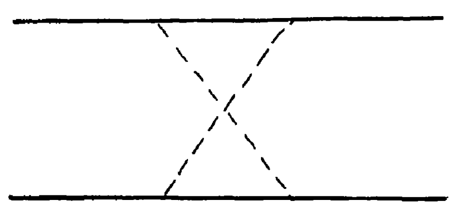


Fig. 2.

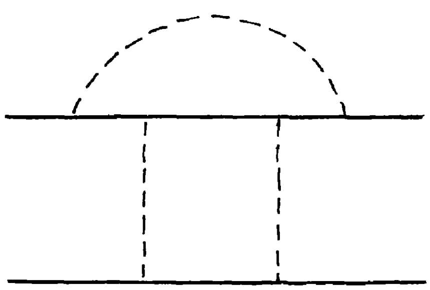


Fig. 1(a)

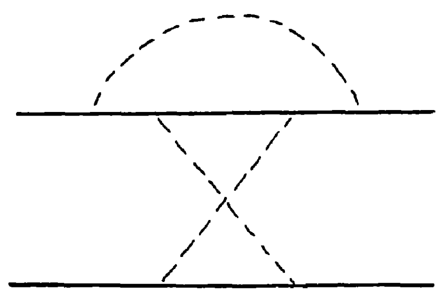


Fig. 2(a)

The final result $V_6^{s.o.}$ obtained from these graphs is put in the form

$$V_6^{s.o.} = \alpha \left(g/4\pi M \right)^2 \underset{\sim}{\sigma} \cdot \left(\nabla V_4 \times \underset{\sim}{p} \right) \tag{25}$$

The value of α, in magnitude and sign, must then be compared with the corresponding result in the phenomenological treatment of section 3.

It may be argued that there is a lack of consistency in omitting all the other sixth order graphs from the above programme. Corresponding to graph 1, for example, the graphs $1(b_1)$, $1(b_2)$, $1(c)$, $1(d_1)$ and $1(d_2)$ also yield a spin orbit coupling.

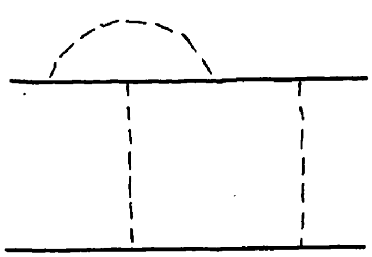


Fig. $1(b_1)$

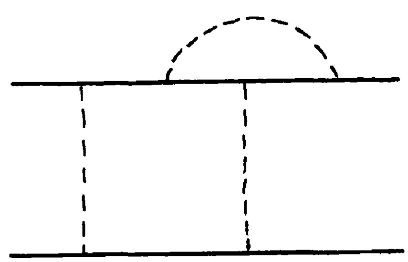


Fig. $1(b_2)$

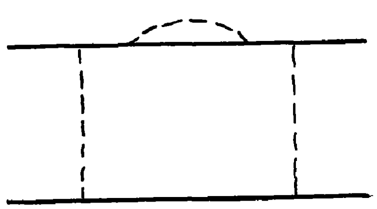


Fig. $1(c)$

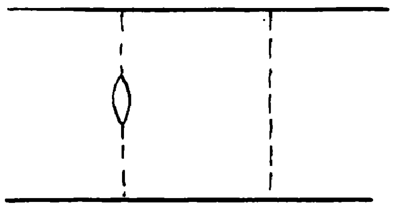


Fig. 1(d_1)

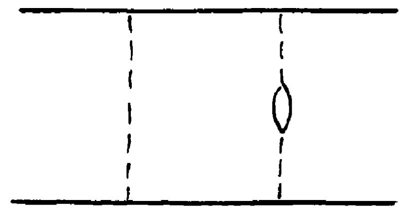
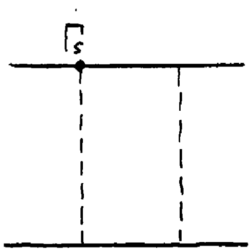
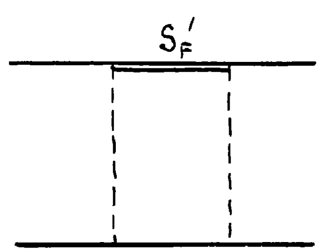


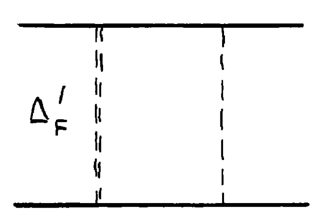
Fig. 1(d_2)

There is a similar set corresponding to graph 2. These graphs cannot be treated by Chisholm's methods, for the type (b) contains a vertex part, (c) a nucleon self-energy part, and (d) a meson self-energy part. These parts must all be renormalized before the total graph is evaluated. The matrix elements can thus not be reduced to a standard form.

However, the spin orbit coupling could be estimated by some other means, which would be much more laborious. The main reason for neglecting this latter set of graphs, of types (b), (c) and (d), is that they seem to belong more naturally to another programme. The graphs 1(b_1) . . 1(d_2) can all be obtained by modifying vertices or propagators in graph 1. Thus the set of graphs can be presented thus:







and similarly for $1(b_2)$ and $1(d_2)$.

Γ_5, S_F' and Δ_F' are the modified γ_5, S_F and Δ_F correct to second order in g.

Now, the two fourth order graphs 1 and 2 also contain a spin orbit coupling. Thus, it is more natural to consider graphs $1(b_1)$ - - $2(d_2)$ as supplying corrections to the spin orbit coupling in V_4 than to include them in the result which is to be directly compared with the results of section 3. Admittedly, a full treatment of the two-nucleon spin orbit coupling would lead one to consider both programmes, but this is not the object in view, namely, to obtain the spin orbit coupling as a self-energy effect.

Finally, the spin orbit coupling obtained by Klein, in a Tamm-Dancoff treatment of graphs 1 and 2, was much smaller than the self-energy effect. The hope to be expressed is that there will be the same tendency in the graphs $1(b_1)$ - - $2(d_2)$.

In section (4.2), the total matrix element, corresponding to graph 1(a), is obtained as a multiple integral over the auxiliary parameters c_r. This matrix element

contains /

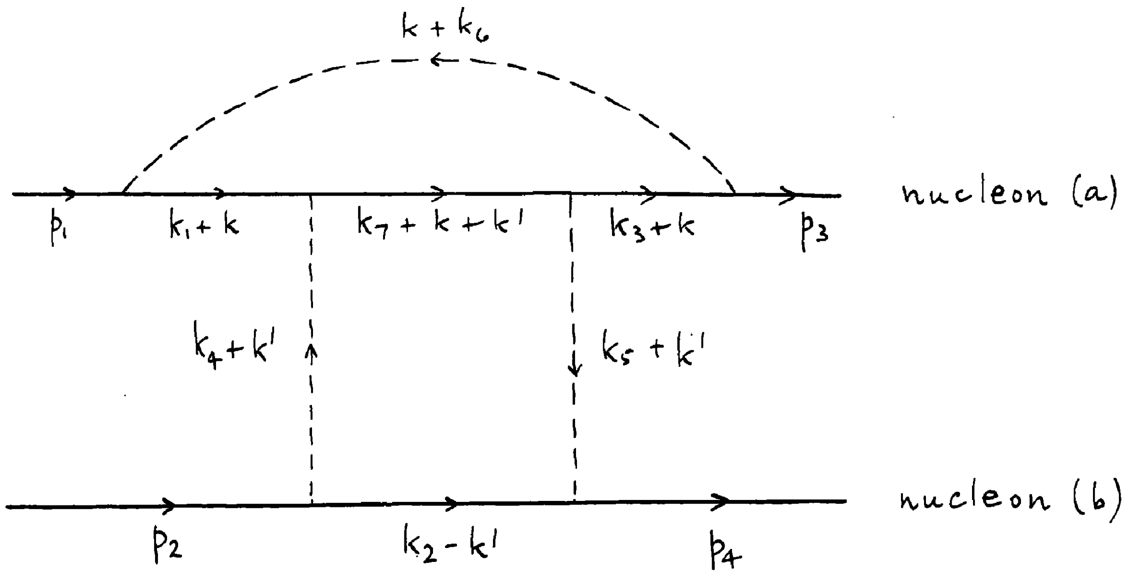
contains a spin orbit coupling, the extraction of which is carried out in section (4.3).

In section (4.4) the graph 2(a) is considered. A correspondence is established between 1(a) and 2(a), which enables one to obtain the spin orbit coupling directly in terms of quantities defined in section (4.3).

Section (4.5) deals with types of approximation which must be made in order to put the contribution to the spin orbit coupling in a manageable form.

Finally, in section (4.6), a short discussion is given on the dependence of the results on the isotopic spin state of the two nucleons.

4.2. Evaluation of graph 1(a).



Finally,

$$k_1 = p_1 \qquad k_4 = k_6 = 0 .$$
$$k_2 = p_2 \qquad k_5 = p_1 - p_3$$
$$k_3 = p_3 \qquad k_7 = p_1$$

Constant of graph

$$g^6 (2\pi)^{24} \frac{(-i)^4}{(2\pi)^{16}} \frac{(-i)^3}{(2\pi)^{12}} = \frac{i g^6}{(2\pi)^4} \qquad (26)$$

Operator

$$P = \gamma_5 \mathcal{F}_3 \gamma_5 \mathcal{F}_7 \gamma_5 \mathcal{F}_1 \gamma_5 \,\big|\, \gamma_5 \mathcal{F}_2 \gamma_5 \qquad (27)$$

In (27), et seq., the following notation is employed:

$$\Omega_{ij} = \Omega_{k_i k_j}$$

$$\mathcal{Y}_i = \mathcal{Y}(k_i, \sigma_i)$$

$$M|N = [M]_{\text{nucleon }(a)} \times [N]_{\text{nucleon }(b)}.$$

<u>k, k' - integration</u>

$$\iint d^4k\, d^4k' \Big[\{(k+k_1)^2+\sigma_1\}\{(k'-k_2)^2+\sigma_2\}\{(k+k_3)^2+\sigma_3\}$$
$$\times\{(k'+k_4)^2+\sigma_4\}\{(k'+k_5)^2+\sigma_5\}\{(k+k_6)^2+\sigma_6\}\{(k+k'+k_7)^2+\sigma_7\}\Big]^{-1}$$

$$= 6! \int_A \prod_{r=1}^{7} dc_r \iint d^4k\, d^4k'\; Q^{-7}$$

finally,
$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_7 = M^2.$
$\sigma_4 = \sigma_5 = \sigma_6 = \mu^2.$

$$= 6! \int_A \prod_{r=1}^{7} dc_r\; (i\pi^2)^2\; \frac{2!}{6!}\; \frac{\Lambda}{\chi_1^3}$$

$$= -2\pi^4 \int_A \prod_{r=1}^{7} dc_r\; \Lambda / \chi_1^3$$

(28)

$$Q = c_1\{(k+k_1)^2+\sigma_1\} + c_2\{(k'-k_2)^2+\sigma_2\}$$
$$+ - - - + c_7\{(k+k'+k_7)^2+\sigma_7\}$$

$$\equiv A k^2 + 2 H k.k' + B k'^2 + 2 G.k + 2 F.k' + C$$

where

$$A = c_1 + c_3 + c_6 + c_7$$

$$B = c_2 + c_4 + c_5 + c_7$$

$$H = c_7$$

$$G = c_1 k_1 + c_3 k_3 + c_6 k_6 + c_7 k_7$$

$$= (c_1 + c_7) p_1 + c_3 p_3 , \quad \text{finally.}$$

$$F = - c_2 k_2 + c_4 k_4 + c_5 k_5 + c_7 k_7$$

$$= (c_5 + c_7) p_1 - c_2 p_2 - c_5 p_3 , \quad \text{finally}$$

$$C = \sum_{r=1}^{7} c_r \left(k_r^2 + \sigma_r \right)$$

$$= (c_4 + c_5 + c_6) \mu^2 + c_5 (p_1 - p_3)^2$$

treating nucleons as free initially and finally.

$$\Lambda = \begin{vmatrix} A & H \\ H & B \end{vmatrix}$$

$$= AB - H^2 \qquad (29)$$

$$\chi = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix} = \Lambda C - AF^2 - BG^2 + 2H(G.F) \qquad (30)$$

$P\chi_1^{-3}$ must now be evaluated. The basic term is

$$\gamma_5 \Omega_3 \gamma_5 \Omega_7 \gamma_5 \Omega_1 \gamma_5 \mid \gamma_5 \Omega_2 \gamma_5 \; \chi_1^{-3}$$

$$= \bar{\Omega}_3 \; \Omega_7 \; \bar{\Omega}_1 \mid \bar{\Omega}_2 \; \chi_1^{-3}$$

Thus, following the prescription given in section 2,

$$P\chi_1^{-3}$$

$$= \bar{\Omega}_3 \; \Omega_7 \; \bar{\Omega}_1 \mid \bar{\Omega}_2 \; \chi_1^{-3}$$

$$+ \Big\{ -4\bar{\Omega}_1 \mid \bar{\Omega}_2 \Omega_{37} + \gamma_\alpha \Omega_7 \gamma_\alpha \mid \bar{\Omega}_2 \Omega_{13}$$

$$+ \gamma_\alpha \Omega_7 \bar{\Omega}_1 \mid \gamma_\alpha \Omega_{23} - 4\bar{\Omega}_3 \mid \bar{\Omega}_2 \Omega_{17}$$

$$- \bar{\Omega}_3 \gamma_\alpha \bar{\Omega}_1 \mid \gamma_\alpha \Omega_{27} + \bar{\Omega}_3 \Omega_7 \gamma_\alpha \mid \gamma_\alpha \Omega_{12} \Big\} \tfrac{1}{2} \chi_1^{-2}$$

$$+ \Big\{ \Omega_{13} \Omega_{27} - 4\Omega_{12} \Omega_{37} \Big\} \; 2\gamma_\alpha \mid \gamma_\alpha \; \chi_1^{-1} \qquad (31)$$

The term in χ_1^{-1} contains no spin orbit coupling. In the χ_1^{-2} term, only the $\gamma_4 \mid \gamma_4$ part of $\gamma_\alpha \mid \gamma_\alpha$ contributes. Thus, the remaining part of $P\chi_1^{-3}$, which contains all the contributions to the spin orbit coupling, is

$$\bar{\Omega}_3 \; \Omega_7 \; \bar{\Omega}_1 \mid \bar{\Omega}_2 \; \chi_1^{-3} \qquad (32)$$

$$+\Big\{-4\bar{\Omega}_1|\bar{\Omega}_2\Omega_{37} + \gamma_\alpha\Omega_7\gamma_\alpha|\bar{\Omega}_2\Omega_{13}$$
$$+\gamma_4\Omega_7\bar{\Omega}_1|\gamma_4\Omega_{23} - 4\bar{\Omega}_3|\bar{\Omega}_2\Omega_{17}$$
$$-\bar{\Omega}_3\gamma_4\bar{\Omega}_1|\gamma_4\Omega_{27} + \bar{\Omega}_3\Omega_7\gamma_4|\gamma_4\Omega_{12}\Big\}\frac{1}{2}\chi_1^{-2}$$

Evaluation of Ω_{st}.

$$\Omega_{st} = \frac{1}{2\Lambda^2}\begin{vmatrix} A & H & \partial_{sk} \\ H & B & \partial_{sk'} \\ \partial_{tk} & \partial_{tk'} & \frac{\delta_{st}}{c_s} \end{vmatrix} \qquad \text{by (9), §2.}$$

Results of this are grouped together in the form of a matrix (Ω_{st}) with s, t taking the values 1, 2, 3, 7 and 1, 2, 3, 5, 7 respectively.

$$[\Omega_{st}] \tag{33}$$

$$= \frac{1}{2\Lambda^2}\begin{bmatrix} \frac{\Lambda}{c_1} - B & -H & -B & H & H-B \\ -H & \frac{\Lambda}{c_2} - A & -H & A & -H+A \\ -B & -H & \frac{\Lambda}{c_3} - B & H & H-B \\ H-B & -H+A & H-B & H-A & \frac{\Lambda}{c_7} + 2H-A-B \end{bmatrix}$$

Evaluation of Ω_s.

Ω_s is an element of the column matrix

$$[\Omega_s] = -M + \frac{1}{\Lambda}[2\Lambda^2\Omega_{st}][C_t]$$

where $[C_t]$ is the column matrix

$$i\gamma.\{c_1k_1 \quad c_2k_2 \quad c_3k_3 \quad c_5k_5 \quad c_7k_7\}$$
$$= i\gamma.\{c_1p_1 \quad c_2p_2 \quad c_3p_3 \quad c_5(p_1-p_3) \quad c_7p_1\}$$

This can be shown to be equivalent to the Ω_s defined in section 2, (8). Ω_s can then be put into the form

$$\Omega_s = -M + i\gamma.\sum_{t=1}^{3} R_s^t\, p_t \quad , \quad s = 1, 2, 3, 7. \tag{34}$$

where the R_s^t are:

$$\Lambda R_1^1 = \Lambda - Bc_1 + Hc_5 + (H-B)c_7$$

$$\Lambda R_1^2 = -Hc_2$$

$$\Lambda R_1^3 = -Bc_3 - Hc_5$$

$$\Lambda R_2^1 = -Hc_1 + Ac_5 + (A-H)c_7$$

$$\Lambda R_2^2 = \Lambda - Ac_2$$

$$\Lambda R_2^3 = -Hc_3 - Ac_5$$

$$\Lambda R_3^1 = -Bc_1 + Hc_5 + (H-B)c_7$$

/

$$\Lambda R_3^2 = -Hc_2$$

$$\Lambda R_3^3 = \Lambda - Bc_3 - Hc_5$$

$$\Lambda R_7^1 = \Lambda + (H-B)c_1 + (H-A)c_5 + (2H-A-B)c_7$$

$$\Lambda R_7^2 = (A-H)c_2$$

$$\Lambda R_7^3 = (H-B)c_3 + (A-H)c_5$$

The following relations hold between the R_s^t which greatly simplify the extraction of the spin orbit coupling:

$$R_1^1 - R_3^1 = 1$$

$$R_1^2 - R_3^2 = 0$$

$$R_1^3 - R_3^3 = -1$$

These relations can be summarised by

$$R_3^r = R_1^r + \delta_{3r} - \delta_{1r} \qquad (35)$$

4.3. Extraction of spin orbit coupling.

All the material is now at hand for extracting the spin orbit coupling from the graph. All momenta will now be referred to the centre of mass of the two-nucleon system. Thus, $\underline{p}_1 = -\underline{p}_2$, $\underline{p}_3 = -\underline{p}_4$ and $p_s^{(4)} = iE$ for all s . As an approximation for low energy, E is set equal to the nucleon mass M .

The following notation will be adopted for simplicity

$$E_s = \sum_{t=1}^{3} R_s^t \quad , \quad s = 1, 2, 3, 7 \qquad (36)$$

In particular, $E_1 = E_3$

$$F_s = R_s^1 - R_s^2 + R_s^3 \quad , \quad s = 1, 2, 3, 7 \qquad (37)$$

$$= \sum_{t=1}^{3} \left(1 - 2\delta_{2t}\right) R_s^t$$

In particular, $F_1 = F_3$

S_1^b , $S_1^{b'}$, $S_1^{b''}$ = coefficients of spin orbit coupling from $\underline{\gamma}\,\underline{\gamma}$, $\underline{\gamma}$, $\gamma_4\underline{\gamma}$ terms, respectively, in basic term.

S_1^d , $S_1^{d'}$, $S_1^{d''}$ = coefficients of spin orbit coupling from $\underline{\gamma}\,\underline{\gamma}$, $\underline{\gamma}$, $\gamma_4\underline{\gamma}$ terms, respectively, in derived term.

These coefficients will now be evaluated in terms of

quantities /

quantities E_s , F_s and Ω_{st}. The results derived in section 3 will be used, concerning contributions from the small components of the initial and final spinor wave functions.

<u>Basic terms.</u>

Consider the expression

$$\left(M + i\gamma.R_3^r p_r\right)\left(-M + i\gamma.R_7^s p_s\right)\left(M + i\gamma.R_1^t p_t\right)$$

$$= \left\{M(1-E_3\gamma_4) + i\underline{\gamma}.R_3^r p_r\right\}\left\{-M(1+E_7\gamma_4) + i\underline{\gamma}.R_7^s p_s\right\}$$

$$\times\left\{M(1-E_1\gamma_4) + i\underline{\gamma}.R_1^t p_t\right\}$$

using (36)

This expression, multiplied by $-M(1-E_2)$, contains all the basic term contributions to the spin orbit coupling.

<u>(a) S_1^b :</u>

$$M^2(1-E_2)\Big\{(1-E_1\gamma_4)(\underline{\gamma}.R_7^s p_s)(\underline{\gamma}.R_1^t p_t)$$
$$- (\underline{\gamma}.R_3^r p_r)(1+E_7\gamma_4)(\underline{\gamma}.R_1^t p_t)$$
$$+ (\underline{\gamma}.R_3^r p_r)(\underline{\gamma}.R_7^s p_s)(1-E_1\gamma_4)\Big\}$$

on putting $E_3 = E_1$.

At this stage, γ_4 may be replaced by 1 , after it has been pulled through to the left (or right) of all other γ -matrices, since in S_1^b , only the large components of the initial and final spinor wave functions contribute. Terms which do not contribute to the spin orbit coupling will be discarded at each step. Thus,

$$S_1^b = M^2(1-E_2)\Big\{(1-E_1)(R_7^s R_1^t + R_3^s R_7^t) - (1-E_7)R_3^s R_1^t\Big\}(\underline{\gamma}\cdot p_s)(\underline{\gamma}\cdot p_t)$$

$$= M^2(1-E_2)\Big\{(1-E_1)R_7^t - (1-E_7)R_1^t\Big\}(\delta_{3s}-\delta_{1s})(\underline{\gamma}\cdot p_s)(\underline{\gamma}\cdot p_t)$$

using (35)

$$= M^2(1-E_2)\Big\{(1-E_1)(R_7^1 - R_7^2 + R_7^3) - (1-E_7)(R_1^1 - R_1^2 + R_1^3)\Big\}(\underline{\gamma}\cdot p_3)(\underline{\gamma}\cdot p_1)$$

On reducing S_1^b to the non-relativistic limit$\neq$, using (9a), this gives

$$S_1^b = M^2(1-E_2)\Big\{F_7(1-E_1) - F_1(1-E_7)\Big\}(\underline{\sigma}\cdot p_3)(\underline{\sigma}\cdot p_1) \tag{38}$$

where use has been made of the definition (37).

/

(b) $S_1^{b'}$:

$$i M^2 \underline{\gamma} \cdot p_r \left\{ (R_1^r + R_3^r)(E_1 E_7 - 1) + R_7^r (1 - E_1^2) \right\}$$
$$\times M (1 - E_2)$$
$$= i M^3 (1 - E_2) \underline{\gamma} \cdot p_r \left\{ 2 R_1^r (E_1 E_7 - 1) + R_7^r (1 - E_1^2) \right\}$$

using (35)

Thus,

$$S_1^{b'} = \tfrac{1}{2} M^2 (1 - E_2) \left\{ 2 (E_1 E_7 - 1)(R_1^1 - R_1^2 + R_1^3) + (1 - E_1^2)(R_7^1 - R_7^2 + R_7^3) \right\} (\underline{\sigma} \cdot p_3)(\underline{\sigma} \cdot p_1)$$

in the non-relativistic limit, using (9b). Thus,

$$S_1^{b'} = \tfrac{1}{2} M^2 (1 - E_2) \left\{ 2 (E_1 E_7 - 1) F_1 + (1 - E_1^2) F_7 \right\} \times (\underline{\sigma} \cdot p_3)(\underline{\sigma} \cdot p_1) \tag{39}$$

(c) $S_1^{b''}$:

$$i M^3 (1 - E_2) \gamma_4 \underline{\gamma} \cdot p_r \left\{ R_3^r (E_7 - E_1) + R_1^r (E_1 - E_7) \right\}$$
$$= i M^3 (1 - E_2) \gamma_4 \underline{\gamma} \cdot (p_3 - p_1)(E_7 - E_1)$$

using (35)

Thus, by (9b),

$$S_1^{b''} = M^2 (1 - E_2)(E_7 - E_1)(\underline{\sigma} \cdot p_3)(\underline{\sigma} \cdot p_1) \tag{40}$$

Derived terms.

(a) S_1^d :

S_1^d, arises from those terms in (32) which contain two Ω - factors belonging to the same nucleon, namely:

$$\gamma_4 \Omega_7 \bar{\Omega}_1 \Omega_{23} - \bar{\Omega}_3 \gamma_4 \bar{\Omega}_1 \Omega_{27} + \bar{\Omega}_3 \Omega_7 \gamma_4 \Omega_{12}$$

Since, by (33), $\Omega_{12} = \Omega_{23}$, this becomes

$$\left[(\underline{\gamma}.R_7^s p_s)(\underline{\gamma}.R_1^t p_t) + (\underline{\gamma}.R_3^s p_s)(\underline{\gamma}.R_7^t p_t) \right] \Omega_{12}$$

$$- (\underline{\gamma}.R_3^s p_s)(\underline{\gamma}.R_1^t p_t) \Omega_{27}$$

The above terms have already been evaluated in obtaining S_1^b , (38). Thus,

$$S_1^d = \left\{ F_7 \Omega_{12} - F_1 \Omega_{27} \right\} (\underline{\sigma}.p_3)(\underline{\sigma}.p_1) \tag{41}$$

(b) $S_1^{d'}$:

$$i M \underline{\gamma}.p_r \left\{ (R_1^r + R_3^r)\left[4(1-E_2)\Omega_{17} + E_1\Omega_{27} + E_7 \Omega_{12} \right] - 2 R_7^r \left[(1-E_2)\Omega_{13} + E_1 \Omega_{12} \right] \right\}$$

using (36)

Thus,

$$S_1^{d'} = \left\{ F_1 \left[4(1-E_2)\Omega_{17} + E_1\Omega_{27} + E_7 \Omega_{12} \right] - F_7 \left[(1-E_2)\Omega_{13} + E_1\Omega_{12} \right] \right\} \times (\underline{\sigma}.p_3)(\underline{\sigma}.p_1) \tag{42}$$

using (35), (37) and (9b).

(c) $S_1^{d''}$:

$$i M \gamma_4 \underline{\gamma} \cdot p_r \left\{ R_3^r - R_1^r \right\} \left(\Omega_{27} - \Omega_{12} \right)$$

Thus,

$$S_1^{d''} = \left(\Omega_{27} - \Omega_{12} \right) (\underline{\sigma} \cdot p_3)(\underline{\sigma} \cdot p_1) \tag{43}$$

using (35) and (9b)

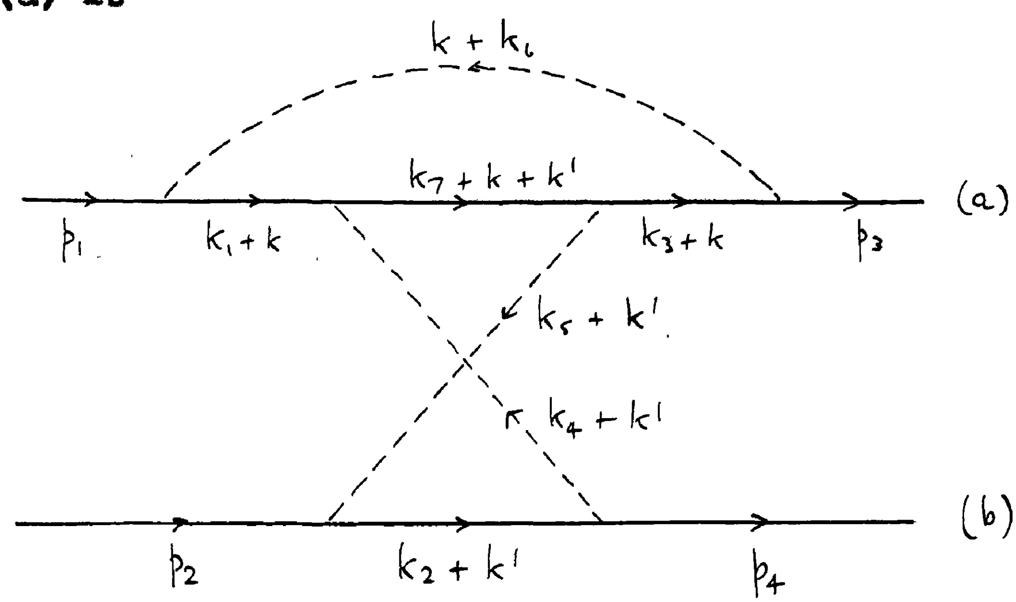
The six coefficients of the spin orbit coupling have thus been obtained in terms of quantities E_s , F_s and Ω_{st}, which in turn are simple functions of the parameters c_r . The above coefficients can thus easily be expressed in terms of the c_r . However, since a change of variable will be made later, this will not be done here.

4.4. Evaluation of graph 2(a).

The graphs 2(a) can be evaluated in exactly the same way as graph 1(a). Instead, however, simple relations between the two graphs will be derived, by which the coefficients of the spin orbit coupling can simply be inferred from the results obtained in section (4.3).

Let quantities in 2(a) which correspond to similar quantities in 1(a) be denoted by primes. Thus, for example, corresponding to Ω_{st} there now occurs Ω'_{st} .

The graph 2(a) is to be labelled similarly to 1(a) to obtain the correspondence as simply as possible. Thus, graph 2(a) is



Finally,

$$\begin{aligned} k_1 &= p_1 & k_4 &= k_6 = 0 \\ k_2 &= p_4 & k_5 &= p_1 - p_3 \\ k_3 &= p_3 & k_7 &= p_1 \end{aligned}$$

It is obvious that the constant, (26), of the graph and the operator P , (27), are unchanged.

Due to labelling the graph as in 1(a), the k, k'-integration is similar, involving only a change of sign in one factor of the integrand, namely,

$$(k' - k_2)^2 + \sigma_2 \longrightarrow (k' + k_2)^2 + \sigma_2 \qquad (44)$$

However, this change of sign is significant and accounts for most of the differences between the two graphs. Thus, A, B, C, G and H and hence Λ, (29), are unchanged, but F is altered.

$$F = c_2 k_2 + c_4 k_4 + c_5 k_5 + c_7 k_7$$

$$= (c_5 + c_7) p_1 + c_2 p_4 - c_5 p_3$$

χ_1 , is accordingly changed to χ_2 , say.

It is easily seen that the following relations hold:

$$\Omega'_{st} = (2\delta_{2s} - 1)(2\delta_{2t} - 1)\,\Omega_{st} \qquad (45)$$

$$\Omega'_s = -M + i\,\gamma . R'^{t}_{s}\, p'_t \qquad (46)$$

where

$$p_1' = p_1$$

$$p_2' = p_4$$

$$p_3' = p_3$$

and $$R'^t_s = (2\delta_{2s} - 1)(2\delta_{2t} - 1) R^t_s \qquad (47)$$

$$E_s' = \sum_{t=1}^{3} R'^t_s$$

$$= \sum_{t=1}^{3} (2\delta_{2s} - 1)(2\delta_{2t} - 1) R^t_s$$

$$= (1 - 2\delta_{2s}) F_s$$

(48)

$$F_s' = \sum_{t=1}^{3} (1 - 2\delta_{2t}) R'^t_s$$

$$= \sum_{t=1}^{3} (1 - 2\delta_{2t})^2 (1 - 2\delta_{2s}) R^t_s$$

$$= (1 - 2\delta_{2s}) E_s \qquad (49)$$

These primed quantities must now be substituted in the coefficient of the spin orbit coupling obtained in section (4.3). The results are as follows:

/

Basic terms.

$$S_2^{b} = M^2(1+F_2)\left\{E_7(1-F_1) - E_1(1-F_7)\right\}(\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{50}$$

$$S_2^{b'} = \tfrac{1}{2}M^2(1+F_2)\left\{2(F_1F_7-1)E_1 + (1-F_1^2)E_7\right\} \times (\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{51}$$

$$S_2^{b''} = M^2(1+F_2)(F_7-F_1)(\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{52}$$

Derived terms.

$$S_2^{d} = -\left\{E_7\Omega_{12} - E_1\Omega_{27}\right\}(\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{53}$$

$$S_2^{d'} = \Big\{E_1\left[4(1+F_2)\Omega_{17} - F_1\Omega_{27} - F_7\Omega_{12}\right] - E_7\left[(1+F_2)\Omega_{13} - F_1\Omega_{12}\right]\Big\}(\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{54}$$

$$S_2^{d''} = -\left(\Omega_{27} - \Omega_{12}\right)(\underline{\sigma}\cdot\underline{p}_3)(\underline{\sigma}\cdot\underline{p}_1) \tag{55}$$

$$= -S_1^{d''}$$

This completes the evaluation of the spin orbit coefficients of the two graphs in terms of the quantities E_s, F_s and Ω_{st}. The procedure developed in (10) and (11) is followed again here to effect the transformation from momentum

space /

space to co-ordinate space. Nucleon (b) in the sixth order graphs replaces the core of section 3, and p_2 is replaced by p_3 .

4.5. Discussion of approximations.

It is impossible to evaluate exactly the integral over the auxiliary parameters c_r. Some method of numerical integration must be employed, together with a certain amount of approximation. As the calculations to be carried out are not so much to stand by themselves, but rather to be normalised, as it were, to the fourth order potential, and then directly compared to the results obtained in section 3, it was felt that approximations which appeared rather drastic at first, but which simplified the amount of computation tenfold, are in fact justifiable.

Thus, the approximation of putting all external nucleon four-momenta as $(\underline{0}, iM)$ was made. This is in effect neglect of nucleon recoil and of the total momentum exchanged by the two nucleons. This is tantamount to having a δ - function (contact) interaction between the two nucleons, an interaction with no physical meaning. However, the justification for this lies in the assumption that if the same approximation is made in the fourth order potential, then the ratio of the spin orbit coupling to the fourth order potential is not much altered.

More precisely, if $V_6^{S.O.}$ and V_4 are the spin orbit coupling and the static potential, respectively,

/

$$V_6^{S.O.} = A\, \underset{\sim}{\sigma} \cdot \left(\nabla \delta(\underset{\sim}{r}) \times \underset{\sim}{p} \right) \quad \text{, say}$$

$$V_4 = B\, \delta(\underset{\sim}{r}) \quad \text{say} \tag{56}$$

$V_6^{S.O.}$ is to be written in the form

$$V_6^{S.O.} = \frac{A}{B}\, \underset{\sim}{\sigma} \cdot \left(\nabla V_4 \times \underset{\sim}{p} \right) \tag{57}$$

A/B is a constant here. The assumption outlined above is that A/B would not vary much if more realistic potentials were calculated. No proof can of course be given for this drastic assumption.

However, be fore the main computations were carried out, the results of which are given in section 6, another programme of numerical work was undertaken, which will be discussed briefly in this section. The coefficients of spin orbit coupling $S_{1,2}^{b}$ and $S_{1,2}^{d}$ alone were considered. The potentials obtained were central, static, but not contact, potentials. The sixth order potential, had not the same shape as the fourth order potential, and was of shorter range. The radial dependence was approximately $e^{-3\mu r}$ and $e^{-2\mu r}$ respectively. Thus, by setting

$$V_6^{S.O.} = \alpha\, \underset{\sim}{\sigma} \cdot \left[\nabla V_4 \times \underset{\sim}{p} \right]$$

it /

it is apparent that α is now a function of the internucleon separation. In the region of interest, α is of the same order of magnitude and sign as the result originally obtained by Chisholm and Touschek. This work was reported at the Nuclear Physics Conference, Glasgow (1954). This result is favourable to the assumption made above.

With the approximation of obtainign contact potentials, it is found that in the sixth order graphs a certain transformation to a new set of paramoters reduces the integrand to a function of four variables, and that the volume of integration is transformed from the hyperplane $\sum_{r=1}^{7} c_r = 1$ to the unit cube.

In the fourth order graphs, a similar transformation reduces the potential to a standard integral. This latter potential will be evaluated in section 5.

Firstly, χ_1 and χ_2 assume the following forms:

$$\chi_1 = \mu^2 \Lambda (c_4 + c_5 + c_6) + M^2 \left[\Lambda (c_7 - c_2)^2 + B (c_1 + c_3 + c_7)^2 - 2H (c_7 - c_2)(c_1 + c_3 + c_7) \right] \tag{58}$$

$$\chi_2 = \mu^2 \Lambda (c_4 + c_5 + c_6) + M^2 \left[\Lambda (c_7 + c_2)^2 + B (c_1 + c_3 + c_7)^2 - 2H (c_7 + c_2)(c_1 + c_3 + c_7) \right] \tag{59}$$

The spin orbit coupling coefficients are unchanged, being independent of the momenta apart from the term $(\sigma \cdot p_3)(\sigma \cdot p_1)$

It is to be noted that there is a certain degree of symmetry in the graphs; thus, c_4 and c_5 and likewise c_1 and c_3 always occur in the combinations $(c_4 + c_5)$ and $(c_1 + c_3)$.

A transformation is now made to a new set of parameters $u, v, w, x, y, z,$ by the relations:

$$
\begin{aligned}
c_4 &= u(1-v)(1-w)(1-x) \\
c_5 &= u(1-v)(1-w)x \\
c_2 &= u(1-v)w \\
c_7 &= uv \\
c_1 &= (1-u)(1-y)(1-z) \\
c_3 &= (1-u)(1-y)z \\
c_6 &= (1-u)y
\end{aligned}
\qquad (60)
$$

The Jacobian of the transformation (60) is

$$|J| = u^3 (1-u)^2 (1-v)^2 (1-w)\, y\, (1-y) \qquad (61)$$

$$\sum_{r=1}^{7} c_r = 1, \qquad \text{as required.}$$

The range of integration of each variable is $(0, 1)$.

Since /

Since $|J|$ does not contain x or z and $(c_4 + c_5)$ and $(c_1 + c_3)$ are also independent of x and z, the promised reduction to four variables is effected.

The method of numerical integration employed was the Gauss method. Three points were taken in each interval, making a total of $3^4 = 81$ points for the integral.

The integral

$$I = \int_0^1 du\, dv\, dw\, dy \quad f(u, v, w, y)$$

has, according to the Gauss formula, the approximate value

$$I = \sum_i W_i\, f(u_i, v_i, w_i, y_i) \tag{62}$$

where (u_i, v_i, w_i, y_i) is one of the 81 sets formed from the numbers

$$\tfrac{1}{2}\left(1 - \sqrt{\tfrac{3}{5}}\right) \quad ; \quad \tfrac{1}{2} \quad ; \quad \tfrac{1}{2}\left(1 + \sqrt{\tfrac{3}{5}}\right)$$

$$W_i = \left(5/18\right)^{4-n_i} \left(4/9\right)^{n_i}$$

where n_i is the number of times the 'mid point' occurs in the set (u_i, v_i, w_i, y_i).

The Gauss 3-point numerical integration formula, when applied to a one-dimensional integral, is exact for an integrand which is a polynomial of degree 5 in the integration variable. When applied to a four-dimensional

integral /

integral, the method of integration is thus exact for an integrand which is a polynomial of degree 20 in the four variables of integration, provided that when any three are held constant a polynomial of degree 5 results in the remaining variable.

It is hoped that the integrands obtained for the spin orbit coupling approximate roughly to these conditions. Tests carried out with integrands, which imitated the general behaviour of the ones actually obtained, but which could also be performed exactly, indicated that it was not unreasonable to hope that the estimate so obtained would at least give the correct order of magnitude. It is believed that, although some doubt may be cast on the magnitude, the sign obtained is definitely correct. It would indeed require a great fluctuation in the magnitude of the results to effect a reversal of sign; this was not noticeable when the computation was being carried through.

It will be assumed hereafter that the magnitude and sign of the results would be reproduced if computations were carried out with greater accuracy. This conclusion is supported to a great extent by the satisfactory results obtained.

All that remains to be done in this section is to give Λ, χ_1, χ_2, S_1^b, . . . , $S_2^{d''}$ in terms of the new parameters.

The /

The spin orbit coupling coefficients will be given throughout without the factor $(\sigma . p_3)(\sigma . p_1)$.

$$\Lambda = u\left(1 - u + uv - uv^2\right) \tag{63}$$

$$\begin{aligned} \chi_1 = {} & \mu^2 u\left(1 - u + uv - uv^2\right)\left\{u(1-v)(1-w) + y(1-u)\right\} \\ & + M^2 u\Big\{u(1-u+uv)(v - w + vw)^2 \\ & \quad + \left[uv + (1-u)(1-y)\right]^2 \\ & \quad - 2uv(v-w+vw)\left[uv + (1-u)(1-y)\right]\Big\} \end{aligned} \tag{64}$$

$$\begin{aligned} \chi_2 = {} & \mu^2 u\left(1 - u + uv - uv^2\right)\left\{u(1-v)(1-w) + y(1-u)\right\} \\ & + M^2 u\Big\{u(1-u+uv)(v + w - vw)^2 \\ & \quad + \left[uv + (1-u)(1-y)\right]^2 \\ & \quad - 2uv(v+w-vw)\left[uv + (1-u)(1-y)\right]\Big\} \end{aligned} \tag{65}$$

$$\begin{aligned} S_1^b = {} & \frac{M^2 u^3}{\Lambda^3}\Big[\left\{uv(v-w+vw) - uv - (1-u)(1-y)\right\} \\ & \quad \times (1-u)(1-v)(y-w) \\ & \quad + \left\{(1-v)\left[uv + (1-u)(1-y)\right] + (1-u)(v-w+vw)\right\} \\ & \quad \times \left\{y(1-u) + uvw(1-v)\right\}\Big] \\ & \quad \times \left[vy(1-u) - w(1-v)(1-u+uv)\right] \end{aligned} \tag{66}$$

$$S_1^{b'} = \frac{M^2 u^4}{2\Lambda^4}\Big[vy(1-u) - w(1-v)(1-u+uv)\Big]$$

$$\times \Big[\Big[(1-u)(1-v)(y-w)\big[1-u+uv-uv^2+y(1-u)-uvw(1-v)\big]$$

$$\times\big[(1-u)(1-y)+uv-uv(v-w+vw)\big]$$

$$-2\big[y(1-u)+uvw(1-v)\big]$$

$$\times\Big[\{1-u+uv-uv^2+y(1-u)-uvw(1-v)\}$$

$$\times\{(1-v)\,[uv+(1-u)(1-y)] + (1-u)(v-w+vw)\}$$

$$+\{1-u+uv-uv^2\}$$

$$\times\{w(1-v)(1-u+uv)-vy(1-u)\}\Big]\Big]$$

(67)

$$S_1^{b''} = -\frac{M^2 u^2}{\Lambda^2}\Big[w(1-v)(1-u+uv) - vy(1-u)\Big]^2 \qquad \textbf{(68)}$$

$$S_1^{d} = -\frac{u}{4\Lambda^3}\Big[uv(1-u)(1-v)(y-w)$$
$$+(1-u)\{y(1-u)+uvw(1-v)\}\Big] \qquad \textbf{(69)}$$

$$S_1^{d'} = \frac{u^2}{4\Lambda^4} \Big[\Big[\left\{ y(1-u) + uvw(1-v) \right\} \tag{70}$$
$$\times \left\{ 4u(1-v)\left[w(1-v)(1-u+uv) - vy(1-u) \right] \right.$$
$$\left. + (1-u)\left[y(1-u) - uvw(1-v) \right] - uv(y+w)(1-u)(1-v) \right\}$$
$$- u(1-u)(1-v)(y-w)$$
$$\times \left\{ w(1-v)(1-u+uv) - 2vy(1-u) + uv^2 w(1-v) \right\} \Big]\Big]$$

$$S_1^{d''} = \frac{1}{4\Lambda^2} \left(1 - u + uv \right) \tag{71}$$

The corresponding spin orbit coupling coefficients for graphs 2(a) are very similar. However, they will be given below for completeness.

$$S_2^{b} = - \frac{M^2 u^3}{\Lambda^3} \Big[\left\{ uv(v+w-vw) - uv - (1-u)(1-y) \right\}$$
$$\times (1-u)(1-v)(y+w)$$
$$+ \left\{ (1-v)\left[uv + (1-u)(1-y) \right] + (1-u)(v+w-vw) \right\}$$
$$\times \left\{ y(1-u) - uvw(1-v) \right\} \Big]$$

$$\times \left[vy(1-u) + w(1-v)(1-u+uv) \right] \tag{72}$$

$$
\begin{aligned}
S_2^{b'} = & -\frac{M^2 u^4}{2\Lambda^4}\Big[vy(1-u) + w(1-v)(1-u+uv)\Big] \\
& \times\Big[\Big[(1-u)(1-v)(y+w)\big[1-u+uv-uv^2+y(1-u)+uvw(1-v)\big] \\
& \qquad \times\big[(1-u)(1-y)+uv-uv(v+w-vw)\big] \\
& \quad -2\big[y(1-u)-uvw(1-v)\big] \\
& \quad \times\Big[\{1-u+uv-uv^2+y(1-u)+uvw(1-v)\} \\
& \qquad \times\{(1-v)[uv+(1-u)(1-y)]+(1-u)(v+w-vw)\} \\
& \quad -\{1-u+uv-uv^2\} \\
& \quad \times\{w(1-v)(1-u+uv)+vy(1-u)\}\Big]\Big]
\end{aligned} \tag{73}
$$

$$
S_2^{b''} = \frac{M^2 u^2}{\Lambda^2}\Big[w(1-v)(1-u+uv)+vy(1-u)\Big]^2 \tag{74}
$$

$$
\begin{aligned}
S_2^{d} = \frac{u}{4\Lambda^3}\Big[& uv(1-u)(1-v)(y+w) \\
& +(1-u)\{y(1-u)-uvw(1-v)\}\Big]
\end{aligned} \tag{75}
$$

$$S_2^{d'} = -\frac{u^2}{4\Lambda^4}\Big[\Big[\left\{ y(1-u) - uvw(1-v) \right\} \tag{76}$$

$$\times\Big\{-4u(1-v)\left[w(1-v)(1-u+uv) + vy(1-u)\right]$$

$$+(1-u)\left[y(1-u) + uvw(1-v)\right] - uv(y-w)(1-u)(1-v)\Big\}$$

$$+\; u(1-u)(1-v)(y+w)$$

$$\times\left\{w(1-v)(1-u+uv) + 2vy(1-u) + uv^2 w(1-v)\right\}\Big]\Big]$$

$$S_2^{d''} = -\frac{1}{4\Lambda^2}\left(1 - u + uv\right) \tag{77}$$

The total spin orbit matrix element in momentum space for graph 1(a) is, apart from integration over the parameters u, v, w, y,

$$-\frac{i g^6 \Lambda T_{1(a)}}{8}\left\{\frac{S_1^b + S_1^{b'} + S_1^{b''}}{\chi_1^3} + \frac{S_1^d + S_1^{d'} + S_1^{d''}}{\chi_1^2}\right\} \tag{78}$$

There is a similar term for graph 2(a).

$T_{1(a)}$ in (78) is an isotopic spin factor. This will be discussed in the next section.

4.6. Isotopic Spin Factors.

Since the charge symmetric PS(PS) meson theory is being employed, an operator τ_i appears at each meson-nucleon vertex of a graph. The terminology 'isotopic spin' has of course no connection with mechanical spin; 'spin' refers to the close resemblance that τ_i has with the spin matrix σ_i. Just as a spin $1/2$ particle has two spin states, so a nucleon can exist in two states, namely, as a neutron or a proton.

The properties of τ_i are the same as those of σ_i :

$$\tau_i \tau_j = \delta_{ij} + i \epsilon_{ijk} \tau_k$$

$$\begin{aligned} T_{1(a)} &= \tau_i \tau_j \tau_k \tau_i \,|\, \tau_j \tau_k \\ &= 9 + 2 \underline{\tau}_a \cdot \underline{\tau}_b \end{aligned} \qquad (79)$$

$$\begin{aligned} T_{2(a)} &= \tau_i \tau_j \tau_k \tau_i \,|\, \tau_k \tau_j \\ &= 9 - 2 \underline{\tau}_a \cdot \underline{\tau}_b \end{aligned} \qquad (80)$$

It should be noted that only if the two graphs 1(a) and 2(a) give equal contributions, and also, as will be seen in section 5, if the two fourth order graphs are equal, in the approximations made, will the spin orbit coupling be independent of whether the two nucleons are in the isotopic singlet or triplet state. As far as application to two-

nucleon /

nucleon phenomena is concerned, this dependence on isotopic spin is important. The main purpose of this work, however, is to supplement the phenomenological calculations of section 3, and thus, in the spirit of these calculations, one is led to assume that if any such isotopic spin dependence does arise in two-nucleon forces, the effect will be smoothed out in the interaction between the nuclear core and the external nucleon.

Let the strengths of the spin orbit coupling be α_T and α_S in the triplet and singlet states respectively. It is to be expected that an estimate of the spin orbit coupling α between the saturated nuclear core and the external nucleon will be given by

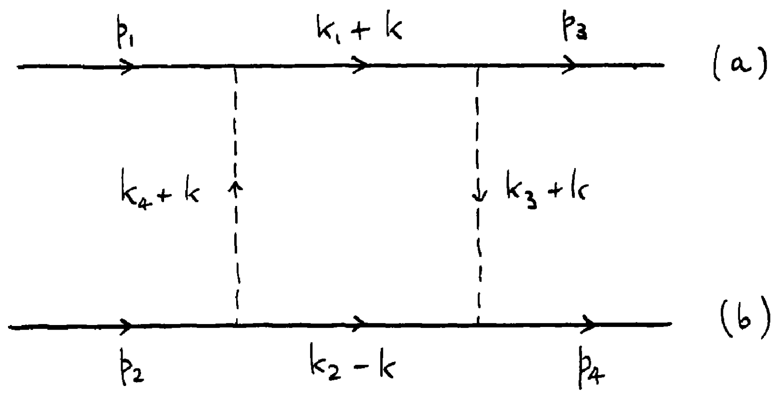
$$\alpha = \frac{1}{4}\left(3\alpha_T + \alpha_S\right) \tag{81}$$

This assumes an equal number of protons and neutrons in the core, which is not necessarily fulfilled, especially in heavy nuclei, and a statistical distribution of singlet and triplet states.

5. Potential from fourth order graphs.

In this section, the potential, arising from the two fourth order graphs 1 and 2 of section 4, is evaluated in the approximation made in the sixth order graphs. In this approximation, it will be seen that the potential can be evaluated exactly. The methods of section 2 are once again employed.

Evaluation of graph 1.



Finally, $k_1 = p_1 \qquad k_3 = p_1 - p_3$

$k_2 = p_2 \qquad k_4 = 0$

Constant of graph

$$g^4 \frac{(-i)^4}{(2\pi)^{16}} (2\pi)^{16} = g^4 \tag{82}$$

Operator

$$P = \gamma_5 \mathcal{F}_1 \gamma_5 \mid \gamma_5 \mathcal{F}_2 \gamma_5 \tag{83}$$

k-integration

k-integration

$$\int d^4k \left[\left\{ (k+k_1)^2 + \sigma_1 \right\} \left\{ (k-k_2)^2 + \sigma_2 \right\} \left\{ (k+k_3)^2 + \sigma_3 \right\} \left\{ (k+k_4)^2 + \sigma_4 \right\} \right]^{-1}$$

finally,

$$= 3! \int_A \prod_{r=1}^{4} dc_r \int d^4k \; Q^{-4} \qquad \sigma_1 = \sigma_2 = M^2, \quad \sigma_3 = \sigma_4 = \mu^2$$

$$= i\pi^2 \int_A \prod_{r=1}^{4} dc_r \; \frac{1}{\chi_1^2} \tag{84}$$

where

$$Q = c_1 \left\{ (k_1+k)^2 + \sigma_1 \right\} + c_2 \left\{ (k-k_2)^2 + \sigma_2 \right\} + c_3 \left\{ (k+k_3)^2 + \sigma_3 \right\} + c_4 \left\{ (k+k_4)^2 + \sigma_4 \right\}$$

$$\equiv k^2 + 2p.k + C$$

using the fact that $\sum_{r=1}^{4} c_r = 1$.

where

$$p = c_1 p_1 - c_2 p_2 + c_3 (p_1 - p_3)$$

$$C = (c_3 + c_4)\mu^2 + c_3 (p_1 - p_3)^2$$

treating nucleons as free initially and finally.

$$\chi_1 = C - p^2$$

$$= (1 - c_1 - c_2)\mu^2 + (c_1 - c_2)^2 M^2 \qquad (85)$$

χ_1 is given in (85) after the same approximations as in section 4 have been made; that is, p_i, $i = 1, 2, 3, 4$, has been set equal to $(\underline{0}, iM)$.

$P\chi_1^{-2}$ must now be evaluated. The basic term is

$$\gamma_5 \Omega_1 \gamma_5 \,|\, \gamma_5 \Omega_2 \gamma_5 \; \chi_1^{-2} .$$

Thus, again following the prescription given in section 2, and writing $\gamma_5 \Omega \gamma_5$ as $\bar{\Omega}$,

$$P\chi_1^{-2} = \bar{\Omega}_1 | \bar{\Omega}_2 \chi_1^{-2} + \gamma_\alpha | \gamma_\alpha \Omega_{12} \chi_1^{-1} \qquad (86)$$

Since the fourth order potential required must be central, static, and spin-independent, only the fourth components in $\bar{\Omega}_1$, $\bar{\Omega}_2$ and γ_α need be retained.

Evaluation of Ω_{st}

$$\Omega_{st} = \frac{1}{2} \begin{vmatrix} 1 & \partial_{sk} \\ \partial_{tk} & \frac{\delta_{st}}{c_s} \end{vmatrix} \qquad \text{by (9), §2.}$$

Results are grouped together in the form of a matrix $[\Omega_{st}]$ with s, t taking the values 1, 2, 3, and 1, 2, respectively /

respectively.

$$[\Omega_{st}] = \frac{1}{2}\begin{bmatrix} \frac{1}{c_1}-1 & 1 & -1 \\ 1 & \frac{1}{c_2}-1 & 1 \end{bmatrix} \tag{87}$$

Evaluation of Ω_s

Again, Ω_s is an element of the column matrix

$$[\Omega_s] = -M + 2\,[\Omega_{st}]\,[C_t]$$

since, here, $\Lambda = 1$.

$[C_t]$ is the column matrix

$$i\gamma.\left\{ c_1 k_1 \quad c_2 k_2 \quad c_3 k_3 \right\}$$

$$= i\gamma.\left\{ c_1 p_1 \quad c_2 p_2 \quad c_3 (p_1 - p_3) \right\}$$

Thus,

$$\Omega_1 = -M + i\gamma.(p_1 - p) \tag{88}$$

$$\Omega_2 = -M + i\gamma.(p_2 + p)$$

where p is the linear combination of p_1, p_2 and p_3 defined in (84).

Approximate forms to be used for $\bar{\Omega}_1$ and $\bar{\Omega}_2$ are accordingly

$$\bar{\Omega}_1 \doteq -M(c_1 - c_2)$$

$$\bar{\Omega}_2 \doteq M(c_1 - c_2) \qquad (89)$$

Thus, by (82), (84), (87) and (89), the approximate forms for the matrix element of graph 1 is

$$i g^4 \pi^2 \int_A \prod_{r=1}^{4} dc_r \left[\frac{-M^2 (c_1 - c_2)^2}{\chi_1^2} + \frac{1}{2\chi_1} \right]$$

$$= i g^4 \pi^2 \int_0^1 dc_1 \int_0^{1-c_1} dc_2 \int_0^{1-c_1-c_2} dc_3$$

$$\times \left[\frac{-M^2 (c_1 - c_2)^2}{[(1-c_1-c_2)\mu^2 + (c_1-c_2)^2 M^2]^2} + \frac{1/2}{(1-c_1-c_2)\mu^2 + (c_1-c_2)^2 M^2} \right]$$

$$= \frac{1}{2} i g^4 \pi^2 \int_0^1 dx \int_{-x}^{x} dy \, (1-x) \qquad (90)$$

$$\times \left[\frac{-M^2 y^2}{[(1-x)\mu^2 + y^2 M^2]^2} + \frac{1/2}{(1-x)\mu^2 + y^2 M^2} \right]$$

The form (90) is obtained by the transformation

$$x = c_1 + c_2$$

$$y = c_1 - c_2$$

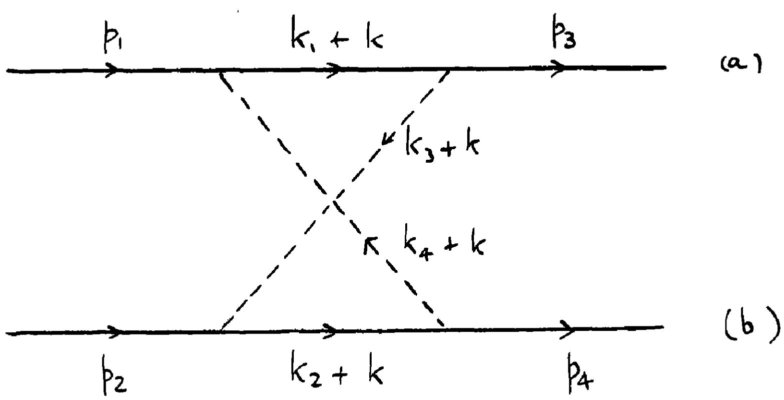
The integration over y is elementary and yields

$$\frac{1}{2}\, i g^4 \pi^2 \int_0^1 dx \; \frac{x(1-x)}{(1-x)\mu^2 + x^2 M^2} \tag{91}$$

This integral has already been obtained in (12).

Evaluation of graph 2.

As indicated in section (4.4), one can obtain results in graph 2 corresponding to similar results in graph 1 by very simple relationships. To obtain these results most simply, both graphs must be labelled similarly. Thus, graph 2 is



finally, $k_1 = p_1 \qquad k_3 = p_1 - p_3$

$k_2 = p_4 \qquad k_4 = 0$

The matrix element for graph 2 can be written down immediately, and is:

$$i g^4 \pi^2 \int_0^1 dc_1 \int_0^{1-c_1} dc_2 \int_0^{1-c_1-c_2} dc_3 \tag{92}$$

$$\times \left[\frac{M^2 (c_1 + c_2)^2}{\left[(1-c_1-c_2)\mu^2 + (c_1+c_2)^2 M^2\right]^2} - \frac{1/2}{(1-c_1-c_2)\mu^2 + (c_1+c_2)^2 M^2} \right]$$

Using the transformation

$$c_1 = x\,y$$

$$c_2 = x\,(1-y)$$

the matrix element is simplified, and is

$$i g^4 \pi^2 \int_0^1 dx \quad x(1-x)\left[\frac{M^2 x^2}{\left[(1-x)\mu^2 + x^2 M^2\right]^2} - \frac{1/2}{(1-x)\mu^2 + x^2 M^2}\right] \tag{93}$$

The integral in (93) is a standard form, and is readily evaluated.

Isotopic spin factors.

As in section (4.6) for the sixth order graphs, there are two isotopic spin factors to be obtained for graphs 1 and 2, denoted by T_1 and T_2, respectively.

$$\begin{aligned} T_1 &= \tau_i \tau_j \mid \tau_i \tau_j \\ &= 3 - 2\, \underline{\tau}_a \cdot \underline{\tau}_b \end{aligned} \tag{94}$$

$$\begin{aligned} T_2 &= \tau_i \tau_j \mid \tau_j \tau_i \\ &= 3 + 2\, \underline{\tau}_a \cdot \underline{\tau}_b \end{aligned} \tag{95}$$

Inserting the value $1 \cdot 2/M^2$ for the integral in (91) and $0 \cdot 14/M^2$ for the integral in (93), one obtains

$$\underline{\text{Graph 1}} \qquad \cdot 60\, i \pi^2 g^4\, T_1 / M^2 \tag{96}$$

$$\underline{\text{Graph 2}} \qquad \cdot 14\, i \pi^2 g^4\, T_2 / M^2 \tag{97}$$

As remarked in section (4.6), the spin orbit coupling finally obtained would be independent of the isotopic spin state of the two-nucleon system only provided graphs 1 and 2, at least, gave equal contributions. Reference to results (96) and (97) shows that this is not the case.

As a final note of more general applicability, it is seen that (96) and (97), the sum of which is proportional to the fourth order potential in the approximation made, contain the imaginary factor i. This factor i also appears in the sixth order matrix elements. This arises from the correspondence between the S-matrix for the interaction of two nucleons not influenced by an external potential,* whose source is a second nucleon. Thus, in the notation of Dyson (1949), there is a correspondence between $S(\infty)$ for two nucleons and U_1 for one nucleon acted upon by an external field.

* and the first Born approximation of the S-matrix for the scattering of a single nucleon by an external potential /

$$S(\infty) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dx_1 \ldots dx_n \, P\left(H'(x_1) \ldots H'(x_n) \right)$$

(98)

where

$$H'(x) = i g \bar{\Psi}(x) \gamma_5 \tau_i \phi_i(x) \Psi(x)$$

$$U_1 = -i \int_{-\infty}^{\infty} H_F(x) \, dx \qquad (99)$$

where

$$H_F(x) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dx_1 \ldots dx_n \, P\left(H^e(x) H'(x_1) \ldots H'(x_n) \right)$$

and

$$H^e(x) = \bar{\Psi}(x) \, U(x) \, \Psi(x)$$

These results were applied to obtain (1) of section 3. An S-matrix element, such as are obtained in sections 4 and 5, must be multiplied by i to obtain a real potential. It is important that this fact, though elementary, should be considered, for otherwise, although the factors i cancel out in the treatment of the spin orbit coupling given in this thesis, doubt might possibly be cast on the sign of the result.

Lastly, in connection with the cancellation referred to in the previous paragraph, normalization factors, which usually appear when transformation is effected from momentum to co-ordinate space, need not be considered here, as they cancel out in the 'normalization' of the sixth order spin orbit coupling to the fourth order potential.

6. <u>Discussion of results.</u>

Results will be given in the following form. The total spin orbit coupling will be given, as in (25), as

$$V_c^{s.o.} = \alpha \left(g/4\pi M \right)^2 \sigma \cdot \left(\nabla V_4 \times p \right)$$

where V_4 is the sum of the two fourth order graphs, (96) and (97) (multiplied by a factor i). Thus,

$$V_4 = \frac{-\pi^2 g^4}{M^2} \left[\cdot 60 \left(3 - 2 \tau_a \cdot \tau_b \right) + \cdot 14 \left(3 + 2 \tau_a \cdot \tau_b \right) \right]$$

$$= - 5 \cdot 0 \pi^2 g^4 / M^2 \quad \text{in the singlet state.}$$

$$= - 1 \cdot 3 \pi^2 g^4 / M^2 \quad \text{in the triplet state}$$

noting that $\tau_a \cdot \tau_b$ assumes the values -3 and 1 in the singlet and triplet states respectively. α is the sum of two contributions to the spin orbit coupling, α_1 from graph 1(a) and α_2 from graph 2(a). $V_c^{s.o.}$ is put into the form (25) with the factor $(g/4\pi M)^2$, so that direct comparison may be made with the results obtained in section 3.

Due to the fact that the result depends on the isotopic

spin /

spin state of the two-nucleon system, α_1 and α_2 and thus also α, are correspondingly two-valued. The values corresponding to the singlet and triplet states will be denoted, as in section (4.6), by the suffixes S and T respectively. The factors $T_{1(a)}$ and $T_{2(a)}$ are incorporated in the results and are

$$T_{1(a)} = 9 + 2\,\underline{\tau}_a \cdot \underline{\tau}_b = \begin{pmatrix} 3 \\ 11 \end{pmatrix} \begin{matrix} S \\ T \end{matrix}$$

$$T_{2(a)} = 9 - 2\,\underline{\tau}_a \cdot \underline{\tau}_b = \begin{pmatrix} 15 \\ 7 \end{pmatrix} \begin{matrix} S \\ T \end{matrix}$$

Results

$$\alpha_{1S} = -0.74$$

$$\alpha_{1T} = -10.4$$

$$\alpha_{2S} = -0.32$$

$$\alpha_{2T} = -0.57$$

Thus,

$$\alpha_S = -1.1$$

$$\alpha_T = -11.0$$

When applied to the case of a spin orbit coupling between a saturated nuclear core and an external nucleon, an estimate of the strength $\bar{\alpha}$ of the spin orbit coupling is obtained by taking a statistical average over the singlet and

triplet /

triplet states, as shown in (81). Thus,

$$\bar{\alpha} = \frac{1}{4}\left(3\,\alpha_T + \alpha_S\right)$$

$$= -8.5$$

on substituting the values of α_S and α_T given above. This result is to be compared with that obtained in section 3, namely,

$$\alpha = -3.6$$

It is encouraging that the results obtained agree so well in order of magnitude and, especially, in sign. It was stated in chapter I that the spin orbit coupling required had to be opposite in sign and 15 times larger than the Thomas coupling, which is

$$V_T^{S.O.} = \frac{1}{(2M)^2}\ \sigma \cdot \left(\nabla V_4 \times p\right)$$

The results given above can be put into the form

$$\frac{\alpha}{\pi} \cdot \left(g^2/4\pi\right) V_T^{S.O.}$$

The expected value of $g^2/4\pi$, according to Levy (cf. Chapter I, section 2) is approximately 10. Thus, the spin orbit coupling required for the nuclear shell model, and for polarization effects in double scattering, is

/

$$\alpha = -\frac{15\pi}{g^2/4\pi}$$

$$= -5.7$$

The values obtained in both treatments given in this thesis are thus close to the desired magnitude.

Both treatments are suspect in that they depend on perturbation theory, the validity of which is questionable with such a large coupling constant. Moreover, each treatment can perhaps be criticized on different points.

The phenomenological treatment of section 3 can be criticized on the point that a scalar potential has been used in a pseudoscalar meson theory. This fact can, however, be met by stating that the potential $U(r)$ arises from the exchange of an even number of mesons. Further, the calculation assumes the persistence of $U(r)$ in the intermediate state. It is doubtful whether this assumption would be valid if the negative energy components of the spinor field were more important, in the intermediate state. However, the substitution of a potential $\gamma_4 U(r)$ for $U(r)$ leaves the sign unchanged, whilst a corresponding change in the potential for the Thomas Coupling causes a change of sign. This fact makes the results of section 3 quite trustworthy.

The /

The second treatment can be criticized on many different points. The assumptions have been somewhat drastic. Firstly, the two-nucleon spin orbit coupling is intended to represent the interaction of the nuclear core with an external nucleon. This neglects the effect of many-body forces, the effect of which has been estimated to some extent by Dresner (1953). Secondly, the assumption has been made, and only partly justified, in section (4.5), that the strength α of the spin orbit coupling would not be substantially altered if more realistic potentials were calculated. Thus, the approximation is made of setting all four-momenta equal to $(\underline{0}, iM)$ after the spin orbit coupling has been extracted, and a similar approximation is made in the fourth order potential. Lastly, the method of numerical integration employed in the calculation of the sixth order graphs can be expected to give only a rough estimate of the actual value of the integral.

Nevertheless, the calculations described in this thesis were carried out in the hope that the assumptions referred to above would actually not be far removed from reality. The results, considered separately, are thus rather suspect in different aspects, although arguments can be given in answer to most of the criticisms. However, taken together, the results are much more convinc-

ing /

convincing; each result supports conclusions drawn from the other.

One final remark is that no more mention has been made of the existence of a spin orbit coupling in the fourth order potential and the sixth order terms which can be regarded as corrections to the fourth order. From the results of Klein (1953) the fourth order spin orbit coupling appears to be appreciably smaller than the self-energy effect, and it must be assumed that the sixth order corrections will not alter this conclusion. In fact, the effect of the Klein spin orbit coupling and the corrections must be considered in a consistent theory, although, as has been stated in section (4.1), this constitutes a separate programme. Their effect, however, is not expected to be detrimental. Rather, the spin orbit coupling obtained here would be reduced towards the value required by the nuclear shell model.

In conclusion, one may say that charge symmetric PS(PS) meson theory leads to a strong spin orbit coupling arising from a self-energy effect, of the right sign and order of magnitude for application to the Mayer nuclear shell model and associated phenomena.

Acknowledgements

The author wishes to express his gratitude to Dr. J.S.R. Chisholm for suggesting this problem and for helpful criticism in the initial stages of the work, and to Professor J.C. Gunn under whose guidance this work was performed.

The author also wishes to thank the University of Glasgow for a fellowship and the Department of Scientific and Industrial Research for a maintenance allowance.

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From the PHILOSOPHICAL MAGAZINE, Ser. 7. vol. 46, p. 106,
January 1955.

Spin Orbit Coupling and the Mesonic Lamb Shift

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[Received November 18, 1954]

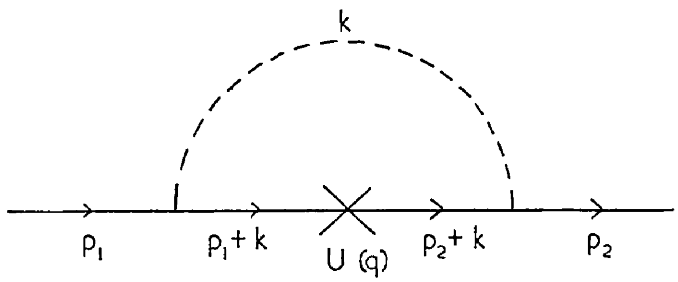
IT has recently been shown by Chisholm and Touschek (1953) that the self-energy corrections for a nucleon moving in a scalar potential well $U(r)$ lead to a strong spin orbit coupling for pseudoscalar mesons with pseudoscalar coupling. Assuming charge symmetric meson theory, these authors obtain a spin orbit coupling

$$\Delta U^{\text{S.O.}}=3(g/4\pi M)^2 \frac{1}{r}\frac{dU}{dr}\, \boldsymbol{\sigma}.\, \mathbf{L},\ \mathbf{L}=\mathbf{r}\times\mathbf{p}. \quad . \quad . \quad . \quad (1)$$

This is of the right order of magnitude, but of the wrong sign, for application to the nuclear shell model.

It is the purpose of this note to show that a more detailed investigation of the total matrix element reveals contributions to the spin orbit coupling from the small components of the initial and final spinor wave functions. These contributions are such that they reverse the sign of the spin orbit coupling, while leaving its magnitude unchanged.

We shall employ the usual Feynman–Dyson prescription for evaluating the S-matrix element $\Lambda(p_2, p_1)$ in momentum space. The only second order graph which contributes is shown in the figure.



This leads to the following expression for the matrix element

$$\Lambda(p_2, p_1)=\frac{3ig^2}{(2\pi)^4}\int d^4k\, \frac{\bar{\psi}(p_2)\gamma_5[i\gamma.(p_2+k)-M]U(q)[i\gamma.(p_1+k)-M]\gamma_5\psi(p_1)}{(k^2+\mu^2)[(p_2+k)^2+M^2][(p_1+k)^2+M^2]} \quad . \quad . \quad . \quad (2)$$

where $$U(r)=\int d^3q\, e^{i\mathbf{q}.\mathbf{r}}\, U(q).$$

$\Lambda(p_2, p_1)$ is logarithmically divergent. By the usual renormalization procedure, $\Lambda_0=\Lambda(p_0, p_0)$ is subtracted from $\Lambda(p_2, p_1)$. (p_0 in Λ_0 is a free particle momentum.)

The finite part $\Lambda_c=\Lambda-\Lambda_0$ contains four terms which contribute to the spin orbit coupling, namely :

$$\left.\begin{aligned}\Lambda_1&=A_1\,\overline{\psi}(p_2)\,\gamma_i\gamma_j p_{2i}p_{1j}\,\psi(p_1)\,U(q),\\ \Lambda_2&=A_2\,\overline{\psi}(p_2)\,\gamma_4\gamma_i P_{2i}\,\psi(p_1)\,U(q),\\ \Lambda_3&=A_3\,\overline{\psi}(p_2)\,\gamma_i P_{3i}\,\psi(p_1)\,U(q),\\ \Lambda_4&=A_4\,\overline{\psi}(p_2)\,\psi(p_1)\,U(q),\end{aligned}\right\}, \quad . \quad . \quad . \quad . \quad (3)$$

where $i, j=1, 2, 3$ and summation is implied. Here P_{ki} is a linear combination of p_{1i} and p_{2i}, say $a_k p_{1i}+b_k p_{2i}$, $k=2, 3$. The constants A_1, A_2, A_3, A_4, a_k, b_k are easily obtained by standard methods.

The $\Lambda_\mu(\mu=1, 2, 3, 4)$ contain the following terms, Λ_μ', on reduction to two-component spinors χ in the non-relativistic limit,

$$\left.\begin{aligned}\Lambda_1'&=iA_1\chi^+\boldsymbol{\sigma}\,.\,(\mathbf{p}_2\times\mathbf{p}_1)\,U(q)\chi\\ &=iA_1S,\ \text{say}\\ \Lambda_2'&=(b_2-a_2)\,A_2S/2M,\\ \Lambda_3'&=(a_3+b_3)\,A_3S/2M,\\ \Lambda_4'&=-i/(2M)^2\,A_4S,\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$iS=\frac{1}{r}\frac{dU}{dr}\,\boldsymbol{\sigma}\,.\,\mathbf{L},\ \text{in configuration space.}$$

The results obtained are as follows :—

$$\left.\begin{aligned}\Lambda_1'&=3(g/4\pi M)^2\,\frac{1}{r}\frac{dU}{dr}\,\boldsymbol{\sigma}\,.\,\mathbf{L},\\ \Lambda_2'&=\Lambda_3'=-\Lambda_1'.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Λ_4' is negligible, since, to the order considered, calculations show that the static radiative correction amounts to only 6% of $U(r)$.

Thus, the total contribution to the spin orbit coupling is

$$-3(g/4\pi M)^2\,\frac{1}{r}\frac{dU}{dr}\,\boldsymbol{\sigma}.\,\mathbf{L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This is just a change of sign from the previous work by Chisholm and Touschek, referred to above, which reported the contribution of the term Λ_1' alone.

The results obtained are encouraging insofar as they afford a possible field theoretical picture for the nuclear shell model. One must bear in mind, however, that a systematic search for spin orbit coupling from all possible sources should be made and that only the total contribution should be compared with the experimentally observed spin orbit coupling. For example, Klein (1953) obtains a spin orbit coupling of the wrong sign for the shell model as a velocity dependent correction to fourth order two-body forces. Further work is being done by the author along the lines indicated above.

In conclusion the author wishes to extend grateful thanks to Professor J. C. Gunn and Dr. J. S. R. Chisholm, under whose guidance this work was performed.

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