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Essays in macroeconomics and fiscal policy

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Abstract

This thesis consists of three chapters. The first chapter contributes to the literature on the Laffer curve as a means of measuring the sustainability of public finances. In particular, it proposes to construct Laffer Curves via policy experiments where fiscal policy is set optimally and fiscal instruments are jointly varied along the transition to steady-state. This relation is labeled as an 'optimal Laffer curve'. It is shown that the tax revenue and welfare gains relative to the 'quasi-static' policy experiments examined by the previous literature are dramatic.

The second and the third chapters are instead dedicated to developing and estimating a DSGE model of the Scottish economy and the rest of the UK. The second chapter reviews the literature of DSGE models developed in academia and central banks in recent years, and outlines our model. In the third chapter, the model is then estimated and its quantitative implications are explored.

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Affidavit

I declare that, except where explicit reference is made to the contribution of others, this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution. The copyright of this thesis rests with the author. No quotation from it should be published in any format, including electronic and internet, without the authors prior written consent. All information derived from this thesis should be acknowledged appropriately.

Mattia Ricci
Edinburgh, April 22, 2019.

Chapter 1

Debt sustainability and welfare along an optimal Laffer curve

Debt Sustainability and Welfare along an Optimal Laffer Curve[1]

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Abstract

A recent literature on sovereign debt sustainability (see Trabandt and Uhlig (2011) and Mendoza et al. (2014)) has produced Laffer curve calculations for Eurozone countries. These calculations have been carried out mainly in a quasi-static fashion by considering policy experiments where individual tax rates are permanently set at a new value while keeping all others constant. However, such fiscal policy design disregards complementarities among tax instruments as well as the potential for altering tax rates during the transition to the steady-state in a manner which exploits expectations. Our paper addresses this issue by considering policy experiments where fiscal policy is set optimally and fiscal instruments are jointly varied along the transition to steady-state. Through the Ramsey problem we map the maximum amount of tax revenues a government can further raise to the welfare costs of the associated tax distortions. We label this relation as the 'optimal Laffer curve'. We show that tax revenue and welfare gains relative to the policy experiments examined by the previous literature are dramatic.

Keywords: Laffer Curve, Optimal Policy, Fiscal Sustainability, Fiscal Limit, Fiscal Consolidations

1.1 Introduction

Since Jude Wannisky's article in the 1970s, the Laffer curve has been the object of intense debate and the theoretical reference for a series of tax reforms. Much of this popularity is due to its simple interpretation and powerful implications. It implies that taxes and fiscal revenues are related by an inverted-U relationship. Progressive tax hikes are increasingly distortionary and eventually reduce tax revenues. In particular, the closer an economy is to the peak of the Laffer curve, the more self-defeating a tax hike will be and the lower the fiscal space for reducing any deficit.

More recently, the increase in debt-to-GDP ratios in a number of economies, particularly following the financial crisis and the associated increased risk of sovereign default, has triggered renewed interest in the Laffer curve as a means of assessing the sustainability of government finances. A large number of papers have investigated Laffer curves using a variety of economic models.[1] They seek to answer questions, such as: at which point on the Laffer curve is a particular country? To what extent might a tax increase be self-defeating, or a tax cut self-financing? And what is the sustainable level of government debt?

Despite this intense research effort, Laffer curve calculations have tended to be relatively mechanical. Typically, the conventional Laffer curve calculation for an individual tax is constructed by progressively varying its rate from 0% to 100%, while keeping all other fiscal instruments fixed. Moreover, a one-off permanent change in a single tax rate is often assumed in constructing the Laffer curve. However, these conventional approaches ignore two crucial issues. First, there is likely to be some degree of complementarity between fiscal instruments, such that appropriately designing the tax mix will generate more revenue than an instrument-by-instrument approach. Second, in a dynamic economy, the profile of fiscal instruments over time is likely to be important in assessing the discounted revenues generated. This is due to the fact that the tax elasticity of production factors varies over time, and tax revenues raised in the distant future matter less in present value terms.

[1] Those studies include Bruce and Turnovsky (1999), Agell and Persson (2001), Novales and Ruiz (2002),Mankiw and Weinzierl (2006), Leeper and Yang (2008), Trabandt and Uhlig (2011) and Mendoza et al. (2014).

Our paper attempts to address these issues by allowing fiscal policy to be conducted optimally. Therefore, the policy maker can vary both the level and the composition of fiscal instruments over time to achieve the maximum attainable tax revenues for a given welfare loss due to tax distortions. We then compare tax revenues under optimal policy with those implied by the conventional Laffer curve calculations considered by Trabandt and Uhlig (2011) and Mendoza et al. (2014). We show that the increase in tax revenues raised per unit of welfare loss is dramatic when the policy maker can vary multiple tax instruments over time. This result holds even when we allow for debt service costs to rise with debt levels and policy maker myopia. Our study implies that the previous literature significantly underestimates the sustainable level of government debt, or equivalently, overstates the welfare losses of achieving a given level of fiscal revenues.

Locating Our Contribution within the Related Literature

The European sovereign debt crisis of 2009-2011 has generated widespread interest in measuring the sustainability of government debt. In the new Handbook of Macroeconomics, D'Erasmo et al. (2015) identifies three main approaches to assessing fiscal sustainability. The first is empirical based on the estimation of fiscal reaction functions in the spirit of Bohn (2005). The latter two rely on calibrated theoretical models which either look at the government's optimal default decision (see Mendoza (2013) and Dovis et al. (2014)) or the fiscal limit (Mendoza et al. (2014) and Trabandt and Uhlig (2011)). The former approach asks what debt the government is prepared to support optimally, while the latter what debt it could potentially support. Whether or not the revenues implied by this final exercise can actually be attained then depends upon the credibility of the government's policies.

Our research builds on this latter literature which seeks to compute the fiscal limit, which is underpinned by the Laffer curve. The peak of the Laffer curve defines the maximum tax revenues which can be generated given the fiscal experiment considered. It serves as a measure of potential fiscal sustainability. The literature in this field has carried out fiscal limit calculations for groups of countries by constructing Laffer curves for appropriately calibrated model economies.

Trabandt and Uhlig (2011) have provided Laffer curve calculations by considering the steady-state economies calibrated to represent key European countries.[2] Their exercise is regarded as the conventional Laffer curve calculation where a curve for each tax instrument is constructed by letting its rate vary from 0% to 100%, while holding all the remaining fiscal instruments fixed. Similarly, Mendoza et al. (2014) have undertaken these calculations in the context of dynamic open economies to account for transitional dynamics and international spillovers. However, they also assume a one-off permanent change in a single tax rate as in Trabandt and Uhlig (2011). As a result, these calculations suggest that failing to account for transitional dynamics does not materially affect the construction of the Laffer curve.[3]

Our paper seeks to reconcile these conventional Laffer curve calculations with optimal policy results. In doing so, we produce an object we label the 'optimal Laffer curve', which plots a Laffer curve in welfare loss-sustainable debt space. Differently from the existing literature, we let the fiscal policy underpinning the Laffer curve be determined through an optimal policy problem. We employ the workhorse Neoclassical model allowing for variable capacity utilization of capital as in Mendoza et al. (2014).[4] In this environment, we study the maximum amount of tax revenues a government can raise for a given level of social welfare when policy is set optimally. In doing so, we extend the existing literature in a number of ways.

First, our approach takes full account of the dynamic path towards the eventual steady-state and allows tax rates to vary over time. We find that exploiting these transitional dynamics can often account for much of the tax revenue raising capability of the government under optimal policy. We show that both the steady-state or dynamic analyses with constant tax rates which

[2]Trabandt and Uhlig (2011) also conduct a robustness check on the significance of allowing for transitional dynamics. In the case of a labor income tax, transitional dynamics, following a permanent shift in the tax rate, make little difference. While the case of capital income tax is complicated by the fact the policy maker can exploit the initial holdings of capital in a non-distortionary way.

[3]Bi (2012) adopts a different approach to computing the fiscal limit which is closer in spirit to what we do. She considers a simple stochastic economy without capital and can analytically compute the peak of the Laffer curve in every period conditional on the realization of a technology shock. These conditional maximum revenues can then be combined to generate a distribution of the fiscal limit. Our model is richer, containing both capital and multiple fiscal instruments such that computing the Laffer curve is non-trivial.

[4]As shown in Mendoza et al. (2014) and Ferraro (2010), variable capacity utilization overturns the ability of the policy maker to tax initial holdings of capital in a lump-sum way. Effectively, the holders of capital choose to decrease their rate of utilization rather than allow the policy maker to tax them. The policymaker is, therefore, less able to exploit a predetermined tax base.

underpin conventional Laffer curve calculations significantly understate the potential fiscal limit as a result.

Second, we allow tax instruments to be varied simultaneously rather than, as in the conventional Laffer curve calculations, sequentially varying one instrument while holding all others fixed. The ability to vary multiple tax rates over time allows the policy maker to generate significantly higher revenues by committing to gradually eliminate capital income taxation in the long run, while, at the same time, slowly switching to labor income taxation.

Third, our optimal Laffer curve, plotted in welfare loss-sustainable debt space, combines the various tax distortions in a single welfare measure. We can map existing measures of the Laffer curve into the same space as our optimal Laffer curve and in doing so we can highlight how policy recommendations change both in steady-state and transition. Our findings suggest that these policy recommendations will be radically different.

Finally, we consider a number of extensions to our baseline model which include risk premia on government debt and policy maker myopia. Those extensions impact on the trade-offs which are so finely balanced in the benchmark optimal policy exercise.[5] Specifically, risk premia on government debt gives rise to incentives to stabilize debt quickly, while policy maker myopia captures the opposite tendency to delay fiscal adjustment. We show that the potential gains in terms of welfare and/or debt sustainability achieved when implementing fiscal policy optimally are robust to such extensions.

The remainder of the paper is structured as follows. Section 1.2 presents the main features of our model economy, while in Section 1.3 we describe the Ramsey problem solved by the government. Section 1.4 discusses the calibration strategy and introduces the optimal Laffer curve. In Section 1.5 we contrast our optimal Laffer curve with conventional analyses which examine one-off permanent changes in a single tax instrument. Robustness and extensions which include risk premia on debt and policy maker myopia are considered in Section 1.6. Section 1.7 concludes.

[5]Optimal policy in our baseline model will inherit one of the key features of tax smoothing whereby the policies sustaining steady-state debt will ensure that the discounted long-run benefits of reducing debt exactly match the short-run costs of doing so. This further implies that where this balancing point is found will define the steady-state level of debt which, in turn, depends on the initial debt level.

1.2 The Model Economy

Our baseline model follows the closed economy of Mendoza et al. (2014). The model economy features exogenous growth, at rate γ, which is driven by labor-augmenting technological change. Accordingly, all variables (except labor, leisure and the interest rate) are rendered stationary by dividing them by the level of technology.[6] This stationarity-inducing transformation of the model requires discounting the re-scaled utility flows at the rate $\widetilde{\beta} = \beta\,(1+\gamma)^{1-\sigma}$ where β is the standard subjective discount factor of time-separable preferences, and adjusting the laws of motion of physical and financial assets so that date $t+1$ stocks grow by the balanced-growth factor $1+\gamma$.

1.2.1 Households

The utility function of the representative household in our economy is

$$\sum_{t=0}^{\infty} \widetilde{\beta}^t U\left(c_t, 1-l_t\right), \tag{1.1}$$

where we assume the period utility function is a standard CRRA function in terms of a CES composite good made of consumption, c_t, and leisure, $1-l_t$ as follows:

$$U(c_t, 1-l_t) = \frac{\left[c_t(1-l_t)^a\right]^{1-\sigma}}{1-\sigma}, \sigma > 1, \text{ and}, \ a > 0.$$

The household's budget constraint is given by,

$$(1+\tau_t^c)c_t + x_t + (1+\gamma)q_t d_{t+1} = (1-\tau_t^l)w_t l_t + (1-\tau_t^k)r_t m_t k_t + \theta\tau_t^k\overline{\delta}k_t + d_t + e_t, \tag{1.2}$$

where τ_t^c, τ_t^l and τ_t^k are proportional tax rates on consumption, c_t, labor income, $w_t l_t$, and capital income, $r_t m_t k_t$, respectively. $\theta\tau_t^k\overline{\delta}$ is a capital tax depreciation allowance which is based on average rates of depreciation and only applies to a fraction of the capital stock since $\theta < 1$. Households also receive a lump-sum transfer from the government, e_t, which is treated as

[6]We could have presented the model in its non-stationary form and then undertaken the transformation of the equilibrium conditions at the end. This is equivalent to undertaking the scaling by technology when setting up the model, as we do.

being exogenous and sets to its steady-state value ($e_t = \bar{e}$). Finally, the household saves in the form of physical capital, k_{t+1}, as well as government bonds, d_{t+1}, which are priced at q_t.

Gross investment, x_t, is defined as,

$$x_t = (1+\gamma)k_{t+1} - [1-\delta(m_t)]\, k_t + \phi(k_{t+1}, k_t, m_t), \tag{1.3}$$

where the depreciation rate depends on the rate of capital utilization m_t as follows,

$$\delta(m_t) = \frac{\chi_0 m_t^{\chi_1}}{\chi_1},\ \chi_0 > 0 \text{ and } \chi_1 > 1, \tag{1.4}$$

and capital adjustment costs are defined as,

$$\phi(k_{t+1}, k_t, m_t) = \frac{\eta}{2}\left\{\frac{(1+\gamma)k_{t+1} - [1-\delta(m_t)]\, k_t}{k_t} - z\right\}^2 k_t,$$

where η determines the speed of adjustment of the capital stock and z is the long-run investment-capital ratio which removes adjustment costs from the steady-state.

The household chooses the path of consumption, leisure, government bonds, investment and the rate of capital utilization to maximize utility (1.1) subject to the budget constraint (1.2) and the law of motion for capital (1.3). Its optimization yields the following set of first order conditions.[7] The consumption Euler equation,

$$(1+\gamma)q_t = \widetilde{\beta}\frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{U'_{c_t}(c_t, 1-l_t)}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}, \tag{1.5}$$

consumption-leisure margin,

$$-\frac{U'_{l_t}(c_t, 1-l_t)}{U'_{c_t}(c_t, 1-l_t)} = \frac{1-\tau_t^l}{1+\tau_t^c} w_t, \tag{1.6}$$

gross investment,

[7] We use the notation $f'_{x_t}(.)$ to denote the partial derivative of function $f(.)$ with respect to argument x_t.

$$\frac{U'_{c_t}(c_t, 1-l_t)}{(1+\tau^c_t)}\left[1+\gamma+\phi'_{k_{t+1}}(k_{t+1}, k_t, m_t)\right] \tag{1.7}$$
$$= \widetilde{\beta}\frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{(1+\tau^c_{t+1})}\left[\begin{array}{c} 1-\delta(m_{t+1})-\phi'_{k_{t+1}}(k_{t+2}, k_{t+1}, m_{t+1}) \\ +(1-\tau^k_{t+1})r_{t+1}m_{t+1}+\theta\tau^k_{t+1}\overline{\delta} \end{array}\right],$$

and, finally, capital utilization condition,

$$(1-\tau^k_t)r_t k_t = \delta'_{m_t}(m_t)k_t + \phi'_{m_t}(k_{t+1}, k_t, m_t). \tag{1.8}$$

1.2.2 Firms

Firms rent labor, l_t, and capital services, s_t, from households at a given wage, w_t, and capital rental rate, r_t, to maximize profits,

$$\Pi_t = y_t - w_t l_t - r_t s_t,$$

subject to a production function which is assumed to be of the Cobb-Douglas form,

$$y_t = F(s_t, l_t) = s_t^{1-\alpha} l_t^{\alpha}.$$

The firms' maximization problem gives rise to standard first order conditions

$$F'_{s_t}(s_t, l_t) = r_t, \tag{1.9}$$

and

$$F'_{l_t}(s_t, l_t) = w_t, \tag{1.10}$$

while linear homogeneity implies $y_t = w_t l_t + s_t k_t$.

1.2.3 Public Sector

The government's budget constraint is given by,

$$d_t - (1+\gamma)q_t d_{t+1} = pb_t, \tag{1.11}$$

where the primary balance, pb_t, is defined as,

$$pb_t = \tau^c_t c_t + \tau^l_t w_t l_t + \tau^k_t (r_t m_t - \theta\overline{\delta})k_t - (g_t + e_t),$$

where government consumption, g_t, is set to its steady-state value $g_t = \bar{g}$.

1.2.4 Market Clearing

Market clearing in the goods market requires:

$$F(s_t, l_t) = c_t + g_t + (1+\gamma)k_{t+1} - [1 - \delta(m_t)]\, k_t + \phi(k_{t+1}, k_t, m_t), \quad (1.12)$$

while capital market clearing implies that

$$m_t k_t = s_t. \quad (1.13)$$

1.2.5 The Competitive Equilibrium

The equilibrium of our model consists of a sequence of prices $\{w_t,\ r_t,\ q_t\}_{t=0}^{\infty}$, government policy $\{\tau_t^c,\ \tau_t^l,\ \tau_t^k,\ d_{t+1}\}_{t=0}^{\infty}$ and allocations $\{c_t,\ l_t,\ s_t,\ x_t,\ m_t,\ k_{t+1}\}_{t=0}^{\infty}$ such that:

- $\{c_t, l_t, x_t, m_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}$ solves the households' problem given prices and government policy;
- $\{s_t, l_t\}_{t=0}^{\infty}$ solves firms' problem given prices;
- The government's budget constraint (1.11) holds for all $t \geq 0$;
- All markets clear as in (1.12) and (1.13).

The definition above implies that for any government policy $\{\tau_t^c,\ \tau_t^l,\ \tau_t^k,\ d_{t+1}\}_{t=0}^{\infty}$, satisfying the government budget constraint (1.11), we have a different competitive equilibrium. In section 1.3, we describe the optimal policy problem that selects the policy corresponding to the government's desired equilibrium. However, before considering such a problem, we need to put some structure on which instruments the government has access to.

The distortionary taxes in our model act on three margins. The first margin is the intratemporal consumption-leisure decision obtained by combining the first order conditions (1.6) and (1.10),

$$-\frac{U'_{l_t}(c_t, 1 - l_t)}{U'_{c_t}(c_t, 1 - l_t)} = \frac{1 - \tau_t^l}{1 + \tau_t^c} F'_{l_t}(m_t k_t, l_t). \quad (1.14)$$

The second margin is the intertemporal investment decision which is obtained by combining equations (1.7) and (1.9),

$$\begin{aligned} & \frac{U'_{c_t}(c_t, 1-l_t)}{(1+\tau_t^c)}\left[1+\gamma+\phi'_{k_{t+1}}(k_{t+1}, k_t, m_t)\right] \qquad (1.15) \\ = \; & \widetilde{\beta}\frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{(1+\tau_{t+1}^c)}\left[\begin{array}{c} (1-\tau_{t+1}^k)F'_{s_{t+1}}(m_{t+1}k_{t+1}, l_{t+1})m_{t+1}+1-\delta(m_{t+1}) \\ -\phi'_{k_{t+1}}(k_{t+2}, k_{t+1}, m_{t+1})+\theta\tau_{t+1}^k\overline{\delta} \end{array}\right]. \end{aligned}$$

Finally, combining equations (1.8) and (1.9) gives rise to the third margin, namely, the capital utilization condition,

$$(1-\tau_t^k)F'_{s_t}(m_t k_t, l_t)k_t = \delta'_{m_t}(m_t)k_t + \phi'_{m_t}(k_{t+1}, k_t, m_t). \qquad (1.16)$$

The labor tax, τ_t^l, can distort the first margin; the consumption tax, τ_t^c, distorts the first and second, while the capital tax, τ_t^k, affects the latter two. In the case of the intratemporal consumption-leisure and investment decision, if labor income is subsidized at the same constant rate as the policy maker taxes consumption (i.e. $-\tau_t^l = \tau_t^c = \tau$), it will eliminate these distortions. Given that those taxes and subsidies are then applied to different tax bases, this would enable the Ramsey policy maker to generate fiscal revenues without suffering any distortions.[8] It effectively gives them access to a lump-sum tax and renders the policy problem trivial. Since in the real world a lump-sum tax is typically not available, we rule out this possibility by fixing τ_t^c at a calibrated value consistent with the data, $\tau_t^c = \tau^c$.[9] Therefore, the capital and labor tax rates are the only fiscal instruments available to the Ramsey policy maker. Furthermore, in order to make the analytical solution of our Ramsey problem more tractable, we remove capital adjustment costs and capital depreciation allowances by setting $\eta = 0$ and $\theta = 0$ as in Debortoli and Nunes (2010). We will explore the implications of relaxing these assumptions in the robustness exercises in Section 1.6.

[8]Under such a tax policy the policy could also optimally set the capital tax rate to zero, $\tau_t^k = 0$.

[9]This is because, typically, $\tau^c \neq \tau$.

1.3 Ramsey Policy with Endogenous Capacity Utilization

In this section, we characterize the solution of the Ramsey model with endogenous capacity utilization. Under Ramsey policy, the policy maker chooses the sequences of labor and capital taxes and the implied path for debt, $\{\tau_t^l,\, \tau_t^k,\, d_{t+1}\}_{t=0}^{\infty}$, so as to maximize life-time utility. This problem is time inconsistent and we assume that government has access to a commitment technology.

We seek to make three main points which underpin the construction of our optimal Laffer curve in Section 1.4. First, the famous Chamley-Judd result (see Chamley (1986) and Judd (1985)) applies to our model. In the short-run the capital tax rate is positive as the Ramsey planner exploits the (quasi) lump-sum nature of the tax on the initial capital. However, in the long-run the capital tax approaches zero as the Ramsey planner attempts to raise revenues through the least distortative instrument which is the labor income tax. Second, with endogenous capacity utilization the tax on the initial stock of capital is bounded. In our model the presence of endogenous capacity utilization makes the capital base elastic in the short-run, limiting the extent to which the Ramsey planner can exploit this margin. This is in contrast to Chamley-Judd where capacity utilization is fixed. Third, the Ramsey policy features a unit root in steady-state debt. The steady-state level of debt the economy eventually achieves depends upon the initial level of debt the policy maker inherits.

To illustrate those three points, we follow Lucas and Stokey (1983) in writing the Ramsey policy problem in the primal form that solves for allocations only. Once allocations have been determined, prices and policy can be recovered from the competitive economy's equilibrium conditions.

1.3.1 The Primal Form

Our Ramsey problem in primal form consists of maximizing utility in (1.1) subject to four constraints. The first is the resource constraint implied by the market clearing conditions in the goods (1.12) and capital (1.13) markets, respectively,

$$F(m_t k_t, l_t) - c_t - g_t - (1+\gamma)k_{t+1} + [1-\delta(m_t)]\, k_t \geq 0. \tag{1.17}$$

The second constraint is the implementability constraint[10]

$$B - \sum_{t=0}^{\infty} \widetilde{\beta}^t \left[U'_{c_t}\left(c_t, 1-l_t\right) \left(c_t - \frac{\overline{e}}{1+\tau^c} \right) + U'_{l_t}\left(c_t, 1-l_t\right) l_t \right] \geq 0, \tag{1.18}$$

where B collects all period-0 terms such that

$$B \equiv \{d_0 + \left[\left(1-\tau_0^k\right) F'_{s_0}(m_0 k_0, l_0) m_0 \;+\; \left(1-\delta\left(m_0\right)\right) \right] k_0\} \frac{U'_{c_0}\left(c_0, 1-l_0\right)}{1+\tau^c}.$$

The resource and the implementability constraints are standard in the optimal policy literature, while the third constraint is due to the presence of endogenous capacity utilization. It is obtained by combining the intertemporal investment decision (1.15) and capital utilization condition (1.16) after leading the latter forward one period [11],

$$\frac{U'_{c_t}\left(c_t, 1-l_t\right)}{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)} = \frac{\widetilde{\beta}}{1+\gamma} \left[\delta'_{m_{t+1}}(m_{t+1}) m_{t+1} + 1 - \delta(m_{t+1}) \right]. \tag{1.19}$$

However, by leading the capital utilization condition (1.16) one-period forward, we omitted this condition at period-0 in the third constraint.

Therefore, we need to reintroduce the period-0 capital utilization condition,

$$\left(1-\tau_0^k\right) F'_{s_0}(m_0 k_0, l_0) = \delta'_{m_0}\left(m_0\right), \tag{1.20}$$

as a fourth constraint.

It is convenient to group all terms in the primal policy problem involving

[10] The derivation of the implementability constraint is shown in Appendix B.1.

[11] As discussed above, we temporarily remove capital adjustment costs and capital depreciation allowances to make the analytical solution of our Ramsey problem more tractable.

the utility function together as,

$$\begin{aligned} V(c_t, 1-l_t, \phi, \lambda_t^1) &= U\left(c_t, 1-l_t\right) \\ &+\phi\left[U'_{c_t}\left(c_t, 1-l_t\right)\left(c_t - \frac{\overline{e}}{1+\tau^c}\right) + U'_{l_t}\left(c_t, 1-l_t\right) l_t\right] \\ &+\lambda_t^1\left[\left(\frac{1+\gamma}{\widetilde{\beta}}\right)\frac{U'_{c_t}\left(c_t, 1-l_t\right)}{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)}\right], \end{aligned}$$

where ϕ and λ_t^1 are multipliers associated with the second constraint (1.18) and the third constraint (1.19), respectively. This expression can then be treated as the policy objective in a more compact representation of the Lagrangian describing the underlying policy problem, as follows,[12]

$$\underset{\{c_t, l_t, m_t, k_{t+1}, \tau_0^k\}_{t=0}^{\infty}}{max} E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \left\{ \begin{array}{c} V(c_t, 1-l_t, \phi, \lambda_t^1) \\ -\lambda_t^1\left[\delta'_{m_{t+1}}(m_{t+1})m_{t+1} + 1 - \delta(m_{t+1})\right] \\ +\lambda_t^2\left[F(m_t k_t, l_t) - c_t - \overline{g} - (1+\gamma)k_{t+1} + (1-\delta(m_t))\, k_t\right] \end{array} \right\} - \phi A,$$

where λ_t^2 is the multiplier attached to the resource constraint (1.17) and

$$A \equiv B - \frac{\varphi}{\phi}\left[\left(1-\tau_0^k\right) F'_{s_0}(m_0 k_0, l_0) - \delta'_{m_0}\left(m_0\right)\right].$$

Here, the term A captures all the period-0 constraints including B in the implementability constraint (1.18) and the period-0 capital utilization condition (1.20), where φ is the multiplier attached to this condition.

The first order conditions for $t \geq 0$ are:

$$\{c_t\} : V'_{c_t}(c_{t-1}, 1-l_{t-1}, \phi, \lambda_{t-1}^1) + \widetilde{\beta} V'_{c_t}(c_t, 1-l_t, \phi, \lambda_t^1) = \widetilde{\beta}\lambda_t^2, \tag{1.21}$$

$$\{l_t\} : V'_{l_t}(c_{t-1}, 1-l_{t-1}, \phi, \lambda_{t-1}^1) + \widetilde{\beta} V'_{l_t}(c_t, 1-l_t, \phi, \lambda_t^1) = -\widetilde{\beta}\lambda_t^2 F'_{l_t}(m_t k_t, l_t), \tag{1.22}$$

$$\{m_t\} : \lambda_{t-1}^1 \delta''_{m_t}(m_t) m_t = \widetilde{\beta}\lambda_t^2\left[F'_{m_t}(m_t k_t, l_t) - \delta'_{m_t}(m_t)\right] k_t, \tag{1.23}$$

$$\{k_{t+1}\} : \widetilde{\beta}\lambda_{t+1}^2\left[F'_{k_{t+1}}(m_{t+1}k_{t+1}, l_{t+1}) m_{t+1} + 1 - \delta(m_{t+1})\right] = \lambda_t^2(1+\gamma), \tag{1.24}$$

$$\{c_0\} : V'_{c_0}(c_0, 1-l_0, \phi, \lambda_0^1) = \lambda_0^2 + \phi A'_{c_0}, \tag{1.25}$$

[12]The details of the Lagrangian function are shown in Appendix B.2.

$$\{l_0\} : V'_{l_0}(c_0, 1-l_0, \phi, \lambda_0^1) = -\lambda_0^2 F'_{l_0}(m_0 k_0, l_0) + \phi A'_{l_0}, \tag{1.26}$$

$$\{m_0\} : \lambda_0^1 \delta''_{m_0}(m_0) m_0 = \lambda_0^2 \left[F'_{m_0}(m_0 k_0, l_0) - \delta'_{m_0}(m_0) \right] k_0 + \phi A'_{m_0}, \tag{1.27}$$

$$\left\{\tau_0^k\right\} : \phi \frac{U'_c(c_0, 1-l_0)}{1+\tau^c} F'_{k_0}(m_0 k_0, l_0) m_0 k_0 - \varphi F'_{k_0}(m_0 k_0, l_0) = 0. \tag{1.28}$$

The above set of first order conditions (1.21)-(1.28) and the four constraints characterize the solution of the Ramsey problem.

1.3.2 Long-run capital tax of zero

The famous long-run zero-capital tax applies to our model with endogenous capacity utilization. To illustrate this point, we compare the steady-state solution of the Ramsey first order condition for capital (1.24),

$$\widetilde{\beta} \left[F'_k(mk, l) m + 1 - \delta(m) \right] = 1 + \gamma, \tag{1.29}$$

to the intertemporal investment decision (1.15) implied by the competitive equilibrium[13]

$$\widetilde{\beta} \left[(1-\tau^k) F'_k(mk, l) m + 1 - \delta(m) \right] = 1 + \gamma. \tag{1.30}$$

Since the Ramsey allocation is a competitive equilibrium, equations (1.29) and (1.30) imply that the Ramsey capital tax, τ^k, is zero in the long-run.

1.3.3 Taxation of initial capital

In our model with endogenous capacity utilization, the first order condition with respect to the period-0 capital tax, τ_0^k, in (1.28),

$$\phi \frac{U'_c(c_0, 1-l_0)}{1+\tau^c} F'_{k_0}(m_0 k_0, l_0) m_0 k_0 - \varphi F'_{k_0}(m_0 k_0, l_0) = 0,$$

offers an insight of why the initial capital tax is bounded. In particular, the term, $\varphi F'_{k_0}(m_0 k_0, l_0)$, appears in the above first order condition because endogenous capital utilization introduces a distortionary component to the

[13] With $\eta = \theta = 0$, the terms associated with capital adjustment costs, $\phi'_{k_{t+1}}(k_{t+1}, k_t, m_t)$ and $\phi'_{k_{t+1}}(k_{t+2}, k_{t+1}, m_{t+1})$, and capital depreciation allowances, $\theta \tau_t^k \overline{\delta}$, disappear in equation (1.15). Therefore, the steady-states of those terms also disappear in equation (1.30).

period-0 capital tax. The multiplier, φ, measures the costs of adjusting capacity utilization, while ϕ represents the benefits associated with lower future distortions implicit in the present value of the budget constraint. As the government increases the capital tax, households will reduce capacity utilization. Therefore, when setting initial capital taxation, the Ramsey planner will need to balance the benefits associated with lower future distortions with the counteracting short-run costs associated with reduced capacity utilization.

This is in contrast to the corresponding condition in Chamley-Judd with an exogenous fixed utilization rate,

$$\phi \frac{U'_{c_0}\left(c_0, 1-l_0\right)}{1+\tau^c} F'_{k_0}\left(k_0, l_0\right) k_0 > 0,$$

where the period-0 stock of capital, k_0, is given and the capital tax rate is effectively a lump sum tax. Therefore, under Chamley-Judd without endogenous capital utilization, it is optimal to set the capital tax rate as high as needed to drive ϕ to zero.

1.3.4 The unit root in steady-state debt

While the steady-state rate of capital tax has been shown to be zero, the long-run value of the labor tax depends on the initial level of debt, d_0. This can be seen from the fact that the Lagrange multiplier of the implementability constraint, ϕ, enters the first order condition with respect to labor supply in (1.22) which pins down the labor tax rate. Since the value of this Lagrange multiplier captures the burden of initial debt, this links the Ramsey initial conditions to the steady-state rate of labor taxation. Therefore, a higher initial level of debt will result in a higher long-run labor tax to support a higher long-run debt level, *ceteris paribus*. Intuitively, our model features a form of tax smoothing which seeks to balance tax distortions over time, while at the same time satisfying the government's intertemporal budget constraint. In steady-state this means that the costs of transitory tax distortions which could reduce debt are exactly offset by the discounted value of the gains of that lower debt, such that the policy maker prefers to maintain debt at that higher level rather than act to return debt to a unique steady-state value.

Moreover, from the logic of the original Laffer curve, there are two steady-states associated with any initial debt level: one with a high and the other with a low value of the labor tax. Therefore, for a given initial level of government debt, there will be two potential steady-states which satisfy the Ramsey first order conditions. We can trace out the set of sustainable debt levels by varying the initial level of debt and assessing the two possible policy paths which both sustain that debt. Computing the welfare costs of each policy path then gives us two points on either side of our optimal Laffer curve which we will show in Section 1.4.

1.4 Optimal Laffer Curve

1.4.1 Calibration

Before constructing our optimal Laffer curve, we need to calibrate our baseline model presented in Section 1.2. Our model is calibrated at a quarterly frequency on the 15 largest countries in the Eurozone[14]. In general, our parametrization tracks closely Mendoza et al. (2014) and D'Erasmo et al. (2015), not only because we employ the same model for analyzing the fiscal position of the same group of countries, but also to keep our results directly comparable with theirs. Our calibration is reported in table 1.1.

Beginning with technology parameters, the labor share of production is set to 0.61, a value in line with Trabandt and Uhlig (2011) and Mendoza et al. (2014). The quarterly rate of labor augmenting technological change, γ, is set to 0.0022. This reflects a 0.9% annual average growth rate in real GDP per capita observed in Eurozone between 2000 and 2011. The depreciation function in (1.4) requires setting two parameters, χ_0 and χ_1. First, to calibrate χ_0, we use the steady-state of the capital utilization constraint (1.19) which implies that $\chi_0 m^{\chi_1} = \left(1 + \gamma - \widetilde{\beta}\right)/\widetilde{\beta} + \delta(m)$, and we normalize the long-run capacity utilization rate to $m = 1$. In order to match the long-run depreciation rate $\delta(m) = 0.0164$, a value in line with Mendoza et al. (2014) and D'Erasmo et al. (2015), χ_0 is set to 0.0266. Second, given the values of m, $\delta(m)$ and χ_0, $\chi_1 = 1.628$ which is derived from the depreciation function (1.4) in steady-state.

[14]Specifically, those countries include Austria, Belgium, Estonia, Finland, France, Germany, Luxembourg, the Netherlands, the Slovak Republic, Slovenia, Italy, Spain, Portugal, Greece and Ireland.

For preference parameters, σ is set to 2 to deliver the commonly used intertemporal elasticity of substitution of 0.5. The leisure utility parameter, a, is set to 2.675. This returns the average of 18.2 hours per week for a person aged between 15 to 64 in France, Germany and Italy, reported in Prescott (2004). The households' discount factor, β, is set to 0.9942 such that $\widetilde{\beta} = \beta\,(1+\gamma)^{1-\sigma} = 0.992$. This implies an annual real interest rate of 4.14% as the quarterly gross rate is $R \equiv \beta\,(1+\gamma)^{\sigma} = 1.0102$.

Fiscal variables include tax rates, government expenditures, transfers and debt. Although in our analysis labor and capital tax rates are solutions of the optimal policy problem, the initial equilibrium of our model is parametrized on the basis of the fiscal regime prior to 2008. In particular, tax rates are set to be consistent with Mendoza et al. (2014), where $\tau^c = 0.16$, $\tau^l = 0.35$ and $\tau^k = 0.20$. Government expenditures is set to be 21% of GDP in line with the OECD definition 'general government consumption expenditure as a percentage of GDP'. In addition, public debt to GDP ratio,$d/4y$, is calibrated to 66% to reflect the debt level in those countries at 2008. Finally, government transfers are determined as the residual of government's budget constraint in (1.11) such that[15]

$$\frac{e}{y} = \frac{Rev}{y} - \frac{g}{y} - \frac{d}{y}\left(1-\tilde{\beta}\right) = 0.152,$$

where $Rev \equiv \tau^c c + \tau^l wl + \tau^k (rm - \theta\overline{\delta})k$. In our baseline model we set both depreciation allowances, θ, and capital adjustment costs, η, to zero. When performing robustness we calibrate $\theta = 0.22$ and $\eta = 2$ as in Mendoza et al. (2014).

[15]Note that the consumption Euler equation in steady state implies that $\widetilde{\beta} = (1+\gamma)q$.

Parameter	Description	Value	Calibration strategy
	Technology		
α	labor income share	0.61	Mendoza et al. (2014)
γ	growth rate	0.0022	GDP p.c. growth EU-15
m	capacity utilization	1	steady-state normalization
$\delta(m)$	depreciation rate	0.0164	Mendoza et al. (2014)
χ_0	$\delta\left(m\right)$ coefficient	0.0266	set to yield $\delta(m) = 0.0164$
χ_1	$\delta\left(m\right)$ exponent	1.628	set to yield $m = 1$
	Preferences		
$\widetilde{\beta}$	discount factor	0.992	Mendoza et al. (2014)
σ	risk aversion	2.000	standard RBC value
a	labor supply elasticity	2.675	Mendoza et al. (2014)
	Fiscal Policy		
τ^c	consumption tax	0.16	Mendoza et al. (2014)
τ^l	labor tax	0.35	Mendoza et al. (2014)
τ^k	capital tax	0.20	Mendoza et al. (2014)
$\frac{d}{y}$	govt debt to GDP	0.66	Mendoza et al. (2014)
$\frac{g}{y}$	govt consumption to GDP	0.21	OECD
$\frac{e}{y}$	govt transfer to GDP	0.152	govt budget in SS

Table 1.1: Calibration

1.4.2 Constructing the Optimal Laffer Curve

To construct the optimal Laffer curve we employ the Ramsey policy discussed in Section 1.3. Compared to the conventional Laffer curve calculations in Trabandt and Uhlig (2011) and Mendoza et al. (2014), the optimal fiscal policy underneath our Laffer curve allows for variation of multiple tax instruments over time. Specifically, tax plans are constructed accounting for discounting, expectations and the dynamics of production factor elasticities.

We construct our optimal Laffer curve by iteratively solving the Ramsey problem conditional on different amounts of initial government debt. We then recover welfare costs associated with the optimal policy that sustains these debt levels.[16] These welfare costs capture the combined distortions implied by the fiscal mix optimally implemented by the Ramsey planner. We then plot each level of government debt (over GDP) against the implied welfare loss measured in consumption equivalent units.[17] This gives rise to our optimal Laffer curve in Figure 1.1. Such a curve represents welfare costs of sustaining any amount of government debt when fiscal policy is carried out optimally. It shows the key elements of a policy maker's problem: how much debt can be sustained and at what social cost?

The optimal Laffer curve inherits the bell shape of the conventional Laffer Curve: each amount of debt can be repaid in two ways, one of which is inefficient. This shape results from the properties of the optimal policy problem featuring exogenous government spending and endogenous distortionary taxation and debt. This problem is known to be non-ergodic as its steady-state depends on the initial level of government debt (and capital). However, such a steady state is not unique. In particular, there are two different steady-states satisfying the Ramsey first order conditions. One gives the positive sloping side of our optimal Laffer curve, while the other one features an inefficiently high level of tax distortions and welfare loss which is on the downward sloping side of the curve.

Finally, from Figure 1.1 some interesting insights can be appreciated. In particular, under zero welfare cost, our optimal Laffer curve implies a

[16] The initial government debt corresponds to the present value of tax revenues minus exogenous public spending. Since government spending is exogenous and fixed, the terms, such as the initial government debt, the sustainable government debt or discounted stream of tax revenues, are all equivalent. We can therefore use these terms interchangeably.

[17] See Appendix D.4 for the computation of consumption equivalent units of welfare.

sustainable debt to GDP ratio of 96% as opposed to 66% supported by the initial calibrated tax policies in Table 1.1. That means implementing an optimal tax policy can generate an additional 33% of GDP in discounted tax revenues at no welfare cost. In addition, moving along the optimal Laffer curve gives us a sense of the trade-offs facing a policy maker. The highest sustainable debt to GDP ratio is about 224%, with the associated tax distortions being equivalent to a welfare loss of 16.7% of steady-state consumption.

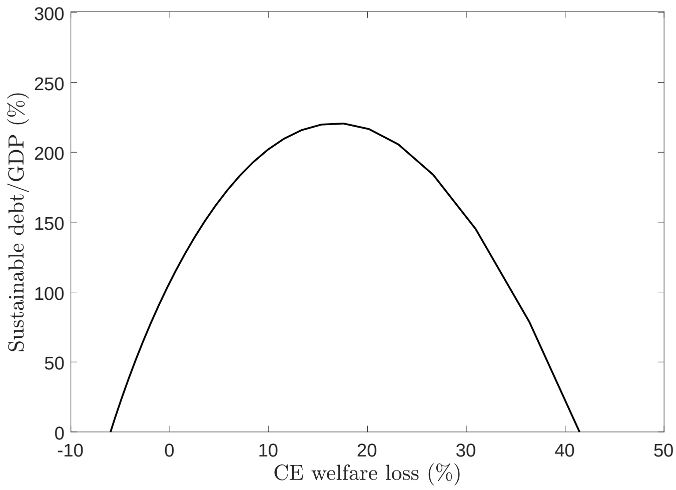

Figure 1.1: Optimal Laffer Curve

1.5 Optimal Laffer Curve versus Conventional Laffer Curve

We now turn to explore how our optimal Laffer curve compares to the conventional Laffer curves constructed following the methodologies of Trabandt and Uhlig (2011) and Mendoza et al. (2014). We aim to assess to what extent these latter calculations leave potential revenue or welfare gains unexploited by failing to conduct policy optimally. As Trabandt and Uhlig (2011) focus on a steady-state economy, whereas Mendoza et al. (2014) on a dynamic one, we consider these cases separately. Although in the conventional Laffer curve calculation, a dynamic analysis does not seem to be radically different from a steady-state one, we show that transitional dynamics can be hugely important when policy is conducted optimally. This is due to the fact that while the economy may be dynamic, the fiscal policies considered in conventional analyses are both static and rely on only varying one fiscal instrument at a time. Relaxing these assumptions can generate significant tax revenues and/or welfare gains.

1.5.1 Steady-State Laffer Curves

In this subsection, we first compare our optimal Laffer curve with the steady-state Laffer curve calculation carried out by Trabandt and Uhlig (2011), 'Trabandt-Uhlig' henceforth. Trabandt-Uhlig's calculation is produced by considering the economy at its steady-state and constructing two curves: the capital and the labor Laffer curve. Those curves are obtained by fixing all tax rates but one and observing how fiscal revenues change as the latter is varied from 0 to 100%. For most of their analysis transitional dynamics are disregarded, that is, following a policy change with respect to the initial equilibrium of the decentralized economy, all endogenous variables are analyzed after reaching their new long-run levels. We compare their Laffer curves calculations and fiscal policy experiment with a steady-state version of our optimal Laffer curve. We aim to show that there are significant differences between conventional Laffer curve calculation and the steady-state of our dynamic policy problem. Specifically, the latter does not generate as much tax revenues as the former in the long-run. The reasons why the Ramsey policy maker chooses to forgo revenues in the long-run underpins the gains from adjusting policy during the transition, which the steady-state approach ignores. We begin to explore these trade-offs in this subsection, and more fully in subsection 1.5.2. It is worth stressing here that what we will refer to in the following as 'Trabandt-Uhlig-like' Laffer Curve, is a Laffer curve constructed following Trabandt-Uhlig's approach. However, as we operate in the context of a slightly different model[18] and calibration, such a curve will not be immediately comparable with one presented in their paper. This will not impact our analysis which aims at comparing different approaches in the construction of the Laffer curves.

We proceed as follows. We use our decentralized economy in its steady-

[18] For example, our model allows for variable capacity utilization whereas Trabandt-Uhlig's does not.

state to produce Trabandt-Uhlig-like labor and capital Laffer curves. A labor Laffer curve is constructed by varying the labor tax rate and fixing consumption and capital tax rates at their calibrated values, while a capital Laffer curve is obtained by varying the capital tax rate while holding constant the other two tax rates at their calibrated values. We then compare these curves with a steady-state version of our optimal Laffer curve. The latter is constructed from the steady-state of our Ramsey model, iteratively solved by varying the initial level of debt and recovering the welfare cost implied by the associated steady-state. We plot all three curves in the welfare loss-sustainable debt space in Figure 1.2, where welfare costs are measured as losses of constant consumption equivalent units with respect to the decentralized economy and the sustainable debt as a percentage of GDP.[19]

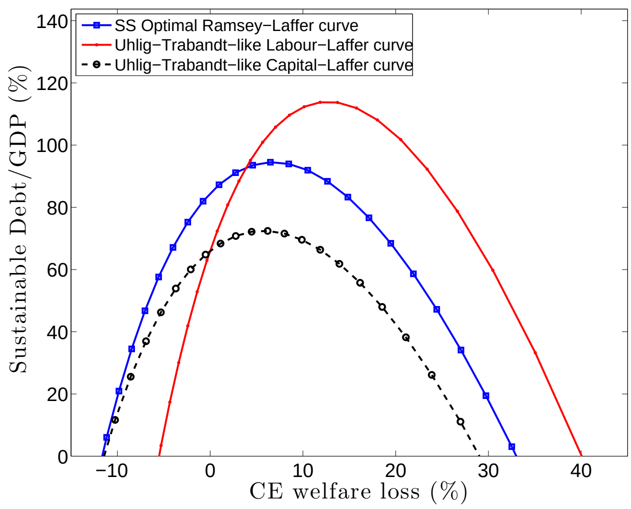

Figure 1.2: Comparing steady-state Laffer curves

The tallest and the lowest Laffer curves in Figure 1.2 are Trabandt-Uhlig-like labor and capital Laffer curves, respectively, while the intermediate curve represents the steady-state version of our optimal Laffer curve. The poor performance of the capital Laffer curve reflects the well known fact that the capital tax is the most distortive tax: a policy based on increasing

[19]Since the level of GDP varies across policies, for comparability we refer to its level in the calibrated decentralized economy.

capital taxation is therefore condemned to be relatively ineffective. For this reason, we focus on Trabandt-Uhlig-like labor Laffer curve for most of our comparisons. Figure 1.3 presents the fiscal policies underneath Trabandt-Uhlig-like labor Laffer curve and the steady-state version of our optimal Laffer curve. As discussed in Section 1.3, optimal policy in steady-state prescribes a capital tax of zero, with labor income bearing all the burden of taxation. In contrast, Trabandt-Uhlig-like policy implies a distortionary capital income tax of $\tau^k = 0.2$, which means that a given welfare loss is associated with a lower labor income tax as shown in the right panel of Figure 1.3.

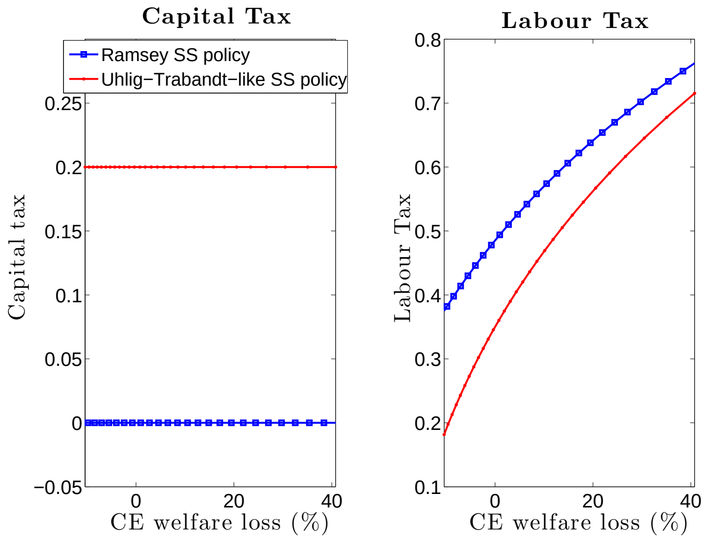

Figure 1.3: Comparing implied fiscal policies

The most striking implication of Figure 1.2 is that much of the Trabandt-Uhlig-like labor Laffer curve lies above the steady-state of our optimal Laffer curve. In other words, the optimizing policy maker is, in steady-state, sustaining a lower level of debt at a higher welfare cost. We now turn to explore why this is. The first point to make is that the optimal Laffer curve seeks to maximize welfare given the need to sustain a given level of debt in a dynamic economy. Therefore, the policy maker may not commit to achieving a steady-state that generates as much revenues as Trabandt-Uhlig in order to raise additional revenues during the transition at a lower welfare cost. To develop this intuition, we carry out the following analysis. Starting from the peak of the Trabandt-Uhlig-like Laffer curve, we move toward the optimal steady-state associated with such a point, as shown in Figure 1.4. In other words, we adopt a set of initial conditions implied by the peak of the Trabandt-Uhlig-like labor Laffer curve, and use our Ramsey problem to solve for the transitional dynamics toward the optimal steady-state these initial conditions imply. We find that such a steady-state is far from being appealing on the basis of static considerations. In particular, the optimal long-run equilibrium is associated with a large loss of tax revenues. The sustainable debt over GDP drops from 114% to 94% with only limited long-run welfare gains. Nevertheless, when transitional dynamics are accounted for, moving toward this optimal long-run steady-state is the right thing to do. As reported in Table 1.2, the overall tax revenues raised are such that sustainable debt is same as the peak of Trabandt-Uhlig-like Laffer curve, but welfare gains are dramatic, amounting to around 9.43% in consumption equivalent units.

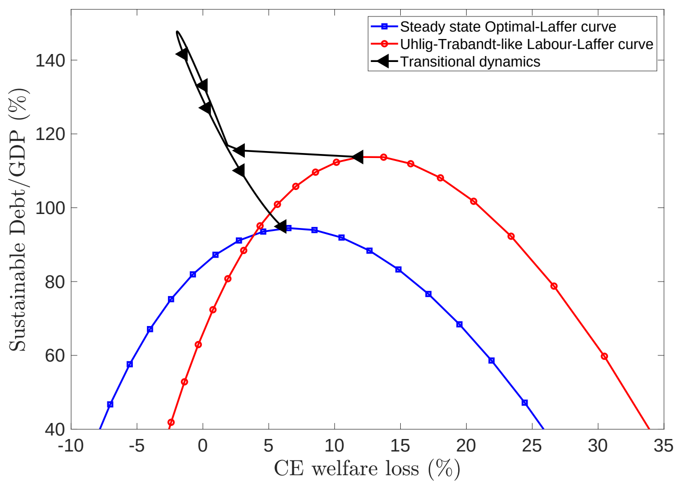

Figure 1.4: Transiting from Trabandt-Uhlig to Ramsey steady state

The above analysis can be applied to any point of Trabandt-Uhlig-like curve lying above the steady-state of our optimal Laffer curve. This implies that Trabandt-Uhlig-like long-run policies are superior in terms of tax revenue raised per unit of welfare loss but only when the policy maker's ability to exploit transitional dynamics are disregarded. Trabandt-Uhlig-like policies are dominated by those implied by the Ramsey model, when we include

	Capital tax	Labour tax	Debt/GDP
(1) Trabandt-Uhlig-like peak	20%	49%	114%
(2) Ramsey Laffer correspondent	0%	54%	94%
Transition from (1) to (2)	4%	51%	114%
By transiting from (1) to (2) we sustain same debt but gain 9.43% CE			

Table 1.2: Transition from Trabandt-Uhlig-like to optimal steady-state

the revenues generated during the transition to the steady-state. The main implication of this result is that, if a policy maker was to announce a long-run policy on the basis of steady-state calculations, she will most likely end up choosing a highly inefficient one. It follows that focusing on steady-state calculations alone is likely to be highly misleading. The evaluation of policy changes radically when the transition is taken into account and the policy maker is able to exploit that transition.

To appreciate this better, Figure 1.5 contrasts our optimal Laffer curve where transitional dynamics are accounted for as previously shown in Figure 1.1 with its steady-state version. We link three points on our optimal Laffer curve (one from the left side, the peak and one from the right side, respectively) to their associated steady-states which correspond to specific points on the steady-state curve.[20] It can be appreciated that the points on the efficient side of the optimal Laffer curve (e.g., the curve's peak) can imply long-run equilibria falling on the slippery side of the steady-state curve. Therefore, selecting a policy on the basis of steady-state analysis alone, a government would end up disregarding a number of policy options which are in fact optimal. How costly this could be can be seen by the distance between the optimal Laffer curve and its steady-state counterpart. The policy maker can more than double the discounted value of tax revenues generated when they actively exploit transitional dynamics.

[20]Recall that the steady state of our optimal policy model is dependent on the initial conditions. Each point in our optimal Laffer curve will have its own steady-state.

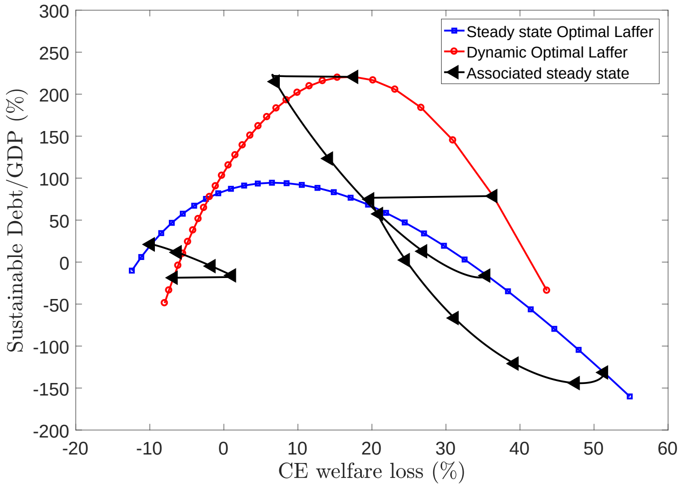

Figure 1.5: Optimal Laffer curve and associated steady states

1.5.2 Laffer Curve Comparison in Dynamic Economy

The main message of the previous section was that Laffer curve analyses should account for the potential gains from exploiting the transition to steady-state. We found that the Ramsey policy maker would not commit to the kind of static tax revenue maximizing policy implied by steady-state calculations when economic transition is accounted for. Instead they would commit to a policy mix with a less optimal steady-state since this maximizes discounted revenues generated in transition. In this subsection, we explore these transitional dynamics and the properties of the optimal tax mix. In particular, we address two issues. The first one is quantitative: how much additional fiscal revenues can be raised when the government is pursuing an optimal dynamic policy rather than adopting constant tax rates as assumed in conventional Laffer curve calculations? The second issue concerns policy design: how should an optimal fiscal policy be carried out during the economic transition?

To answer the first question, we need to account for the impact of transitional dynamics in conventional Laffer curve calculations and contrast this with the one implied by our optimal Laffer curve. Our benchmark will be the papers by D'Erasmo et al. (2015) and Mendoza et al. (2014), MTZ henceforth. MTZ construct capital and labor Laffer curves in dynamic economies which can be regarded as the dynamic counterpart of Trabandt and Uhlig (2011). Although the dynamic model enables them to account for transitional dynamics, the underlying fiscal policy assumes constant tax rates as in Trabandt and Uhlig (2011). Again, it is worth stressing that what we will refer to in the following as 'MTZ-like' Laffer Curve, is a Laffer curve constructed following MTZ's approach. However, as we operate in the context of the same model but of a slightly different calibration[21], such a curve will not be immediately comparable with one presented in their paper.

We plot these curves with our optimal Laffer curve in Figure 1.6. The capital Laffer curve, again, does not facilitate the efficient generation of tax revenues. Therefore, we focus on the comparison between our optimal Laffer curve and MTZ-like labor Laffer curve. The differences in these two curves are striking. MTZ-like labor Laffer peaks in correspondence of a welfare loss of 11.55%; at that point the difference between MTZ-like curve and the optimal Laffer curve in terms of sustainable debt is 82%. This means that the optimal policy could greatly enhance the amount of tax revenues raised at no additional welfare cost. In addition, differences in debt sustainability appear to be of broadly similar magnitude everywhere along the Laffer curve measuring about 63% of GDP. By inverting this argument, it can be noted that in sustaining the same level of debt optimal policy typically offers welfare gains of about 4% in constant consumption equivalent units. Therefore, we conclude that an optimal policy has strong quantitative implications for debt sustainability, tax revenues and welfare gains.

To illustrate the second issue as to how the optimal fiscal policy underlying our optimal Laffer curve is carried out during transition, we plot the transitional dynamics associated with a particular welfare loss of 1.53% under both the optimal policy and MTZ's constant tax rates in Figure 1.7. We note that with the constant tax rate policy adopted by MTZ there is very limited variation in the endogenous variables during the transition. In contrast, under the Ramsey policy, capital tax rates are front-loaded and coupled with a complementary cut in the labor tax rate. The capital tax rate then declines until converging to zero in the steady-state, while the labor income tax rate rises consistently until achieving a relatively high long-run

[21] I.e., in our baseline model we switched off the capital adjustment costs and the depreciation allowance

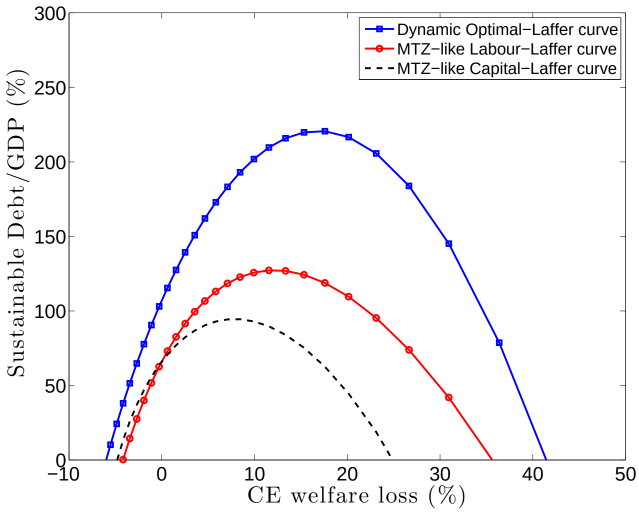

Figure 1.6: Comparing dynamic Laffer curves

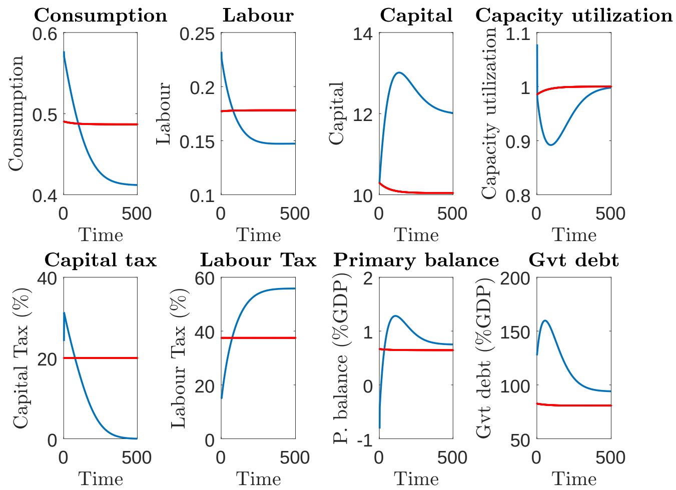

Figure 1.7: Transitional dynamics for CE=1.53%

value. Given the commitment to abolish capital tax in the long-run and the relatively low labor taxes during the initial stages of transition, capital keeps accumulating despite it being taxed at a positive rate. This underpins the core intuition behind our optimal steady-state: the low tax revenues and the zero capital tax chosen by the Ramsey planner in the long run imply large gains during the transitional dynamics. This can be further appreciated in Figure 1.8 where the highest revenues are raised when both capital and labor taxes are at 'intermediate rates' and the quantity of capital has reached its maximum. Therefore, we conclude that the striking gains in revenue generation arise from a combination of the gradual erosion of capital income taxation, while at the same time increasing labor income taxation. In other words, the gains are in part due to using one tax instrument to complement another, as well as allowing tax rates to vary over time. In the following analysis, we further explore to what extent the tax revenue generated by the Ramsey policy is due to complementarity of alternative tax instruments. To do so, we only allow the Ramsey policy maker to implement one tax instrument.

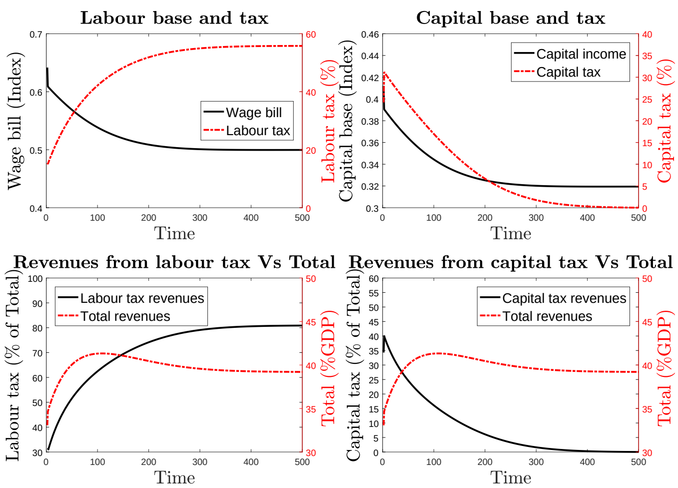

Figure 1.8: Optimal fiscal mix (CE=1.53%)

We construct optimal Laffer curves for both labor and capital taxes, respectively, in each case holding other tax rates fixed at their calibrated levels. In Figure 1.9, we compare the labor and capital optimal Laffer curves with the corresponding steady-state Trabandt-Uhlig-like and dynamic MTZ-like

Laffer curves. The left panel of Figure 1.9 features the labor Laffer curves. Here we see that the transitional dynamics contained within the MTZ calculation are negligible as the Trabandt-Uhlig-like and MTZ-like labor income Laffer curves are largely indistinguishable. In contrast, our optimal labor Laffer curve lies significantly above these solely as a result of the commitment to decrease and then gradually increase labor income tax. In contrast, when comparing capital tax Laffer curves plotted in the right panel, it is the MTZ-like and our optimal Laffer curves which are indistinguishable. This is partly because without the labor tax instrument being freely available to complement capital tax policy, capital taxes must sustain steady-state debt and cannot achieve the preferred policy of committing to reduce the capital tax rate to zero in the long-run. This brings the two forms of capital Laffer curve closer together. When comparing the labor and capital optimal Laffer curves with the optimal Laffer curve with both instruments as plotted in Figure 1.6 at their peak, it can be seen that two-thirds of the gains in terms of increased revenues are from gradually increasing labor tax during the transition. The other one-third comes from simultaneously eliminating capital income tax in the long-run. This finding sharply differs from the calculations of Chari et al. (1994) who find that, in a model with exogenous capacity utilization, most of the welfare gains of switching from the calibrated US fiscal policy to the Ramsey policy arise from the first period capital tax whereas labor tax plays a nearly irrelevant role. In our model with endogenous capacity utilization, the ability to exploit the capital tax of the predetermined capital stock is sharply reduced.

1.6 Robustness and Extensions

In describing the basic properties of the Ramsey policy and optimal Laffer curve, we removed frictions such as capital adjustment costs and capital depreciation allowances. We now consider whether reintroducing such factors significantly affects the results. Following that we consider a range of extensions to the basic model. These include introducing a risk premium on sovereign debt and assuming the policy maker is myopic.

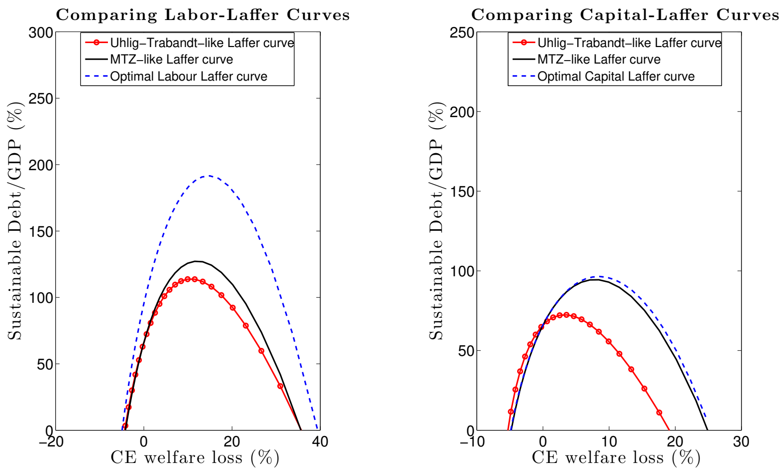

Figure 1.9: Comparing Trabandt-Uhlig and MTZ Laffer curves

1.6.1 On the Role of Adjustment Costs and Depreciation Allowances

In this subsection, we consider the implication of reinstating capital adjustment costs and depreciation allowances. These features of the benchmark model were temporarily removed for analytical convenience. Capital adjustment costs imply that the capital tax base is less elastic than it otherwise would be. This means that higher tax revenues can be generated across the MTZ-like and optimal Laffer curves which account for transitional dynamics. On the other hand, depreciation allowance reduces the capital income tax base and tax revenues.

Figure 1.10 plots the MTZ-like and optimal Laffer curves for our benchmark model with and without adjustment costs or depreciation allowances. It is clear from the figure that the relative movements in the Laffer curves across the three types of curve are similar, such that the presence of capital adjustment costs or depreciation allowances do not affect our main results. It remains the case that the gains from conducting fiscal policy optimally dramatically increases the revenues that can be generated at a given welfare cost.

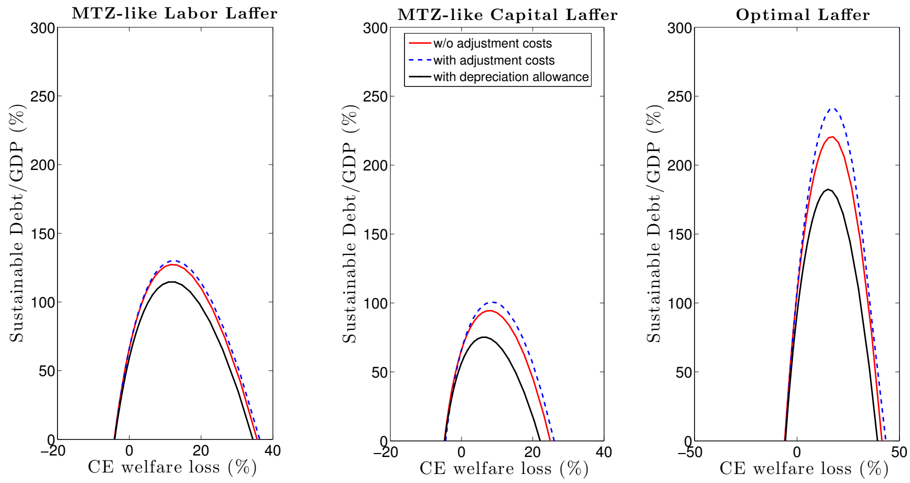

Figure 1.10: Comparing Laffer curves with and without adjust. costs

1.6.2 Risk Premia and Policy Maker Myopia

In previous sections, we have shown how the Ramsey policy can generate a significant degree of additional revenue relative to conventional approaches for assessing fiscal sustainability. We find that the gains are driven by a combination of being able to vary the labor tax over time alongside a commitment to eliminate capital taxation in the long-run. Implicitly, the prolonged transition to this time-inconsistent steady-state is entirely credible and the benevolent policy maker does not suffer any increased debt service costs when debt levels are high.

In this section, we explore the implications of relaxing these assumptions. We do so in two ways. First, we introduce bond holding costs as a tractable way of allowing debt service costs to rise with the level of debt. This overturns the unit root in steady-state debt ensuring that the economy returns to a unique steady-state with an associated debt to GDP ratio. However, it will remain the case that in the long-run the policy maker commits to eliminate capital taxation. Secondly, we shall relax the assumption that the policy maker is fully benevolent and introduce a myopia to policy making which means they wish to delay distortionary tax increases. On its own this extension would overcome the finely balanced trade-off implied by tax smoothing and governments would be tempted to allow debt levels to rise indefinitely. When it is combined with bond holding costs there will be a unique steady-state and a non-zero long-run capital tax.

Risk premium

Following Heaton and Lucas (1996), we assume that there are $\Psi_t = \frac{\psi}{2}\left(d_{t+1} - \bar{d}\right)^2$ insurance costs to be paid to a financial intermediary for the household to insure the unit gross return on government bond against repayment risk. This device introduces, in a reduced form way, a risk premium on government debt which is increasing in its level. Specifically, it produces a wedge between the interest rate the government pays and the return effectively realized by the household. This feature alters the bond-pricing condition such that:

$$(1+\gamma)q_t + \psi\left(d_{t+1} - \bar{d}\right) = \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)}{U'_{c_t}\left(c_t, 1-l_t\right)}. \tag{1.31}$$

We assume that the profits of these financial intermediaries are redistributed to the household in a lump-sum way.

A crucial consequence of this quadratic cost is to remove the unit root in government debt and therefore break the dependence of the model steady-state on initial conditions. In particular, in this version of the model, steady-state debt will be $d = \bar{d}/2$.[22] At the same time, the main features of the Ramsey policy (i.e. zero long-run capital income tax and bounded initial capital taxation) are preserved.[23] We calibrate $\bar{d}$ so that the long-run government debt over GDP is brought back to its pre-crisis level, i.e. $d/4y \times 100 = 66\%$. We consider this a useful reference as it implies that policy makers will seek to fully overturn increases in debt observed since 2008 and is roughly in line with the Maastricht criteria Euro-zone countries are required to meet. In addition, the parameter ψ in the function of insurance costs is related to the elasticity of interest rate to a 1% increase in debt over GDP with respect to its long-run level.[24] A recent empirical study by Laubach (2009) places this elasticity between 3 and 4 basis points while arguing that a standard

[22] Debt stationarity is shown in Appendix C.4.

[23] Properties of the Ramsey policy for the extended model are derived in Appendix C.

[24] The elasticity of interest rate to a 1% increase in debt over GDP with respect to its long-run level is defined as $\eta_{r,d/4y} = \partial r/\partial\frac{d}{4y} = 4y\partial r/\partial d$, where $r \approx log\left(R\right)$ denotes the net interest rate. To see how ψ is related to $\eta_{r,d/4y}$, we first note that equation (1.31) in steady-state implies the following relation that $(1+\gamma)/R = \widetilde{\beta} - \psi\left(d - \bar{d}\right)$. Second, we solve for R and take logs in both side of equation (1.31) in steady-state. When ψ is small, we obtain the following approximation that $\partial r/\partial d = \psi/\left[\beta - \psi\left(d - \bar{d}\right)\right] \approx \psi$. Substituting this approximation into the definition of $\eta_{r,d/4y}$, gives $\eta_{r,d/4y} \approx 4y\psi$.

RBC model tends to produce endogenously an elasticity of 2 basis points approximately. In the subsequent analysis, we then conservatively adopt a central value of 2 which corresponds to $\psi = 0.0057$.[25] Under this setting we produce an optimal Laffer curve, tracing out the relation between sustainable debt over GDP and welfare loss when risk premia are accounted for and the optimal fiscal policy is constrained to achieve a long-run equilibrium where the level of government debt is brought back to its pre-crisis level. Figure 1.11 plots both the MTZ-like labor Laffer curve and our optimal Laffer curve with bond holding costs. We can see this maintains the relative position of the two Laffer curves, and the substantial revenue gains of conducting fiscal policy optimally remains.

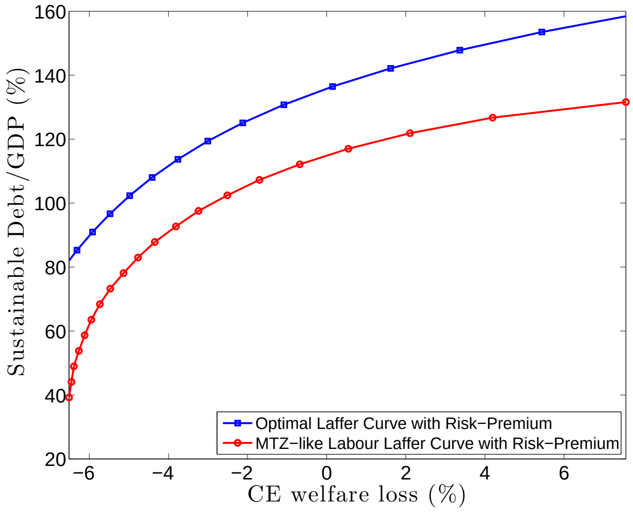

Figure 1.11: Comparing Laffer Curves with Risk-premium

Government myopia

We further augment the risk-premium model considered above to allow for an impatient policy maker featuring higher time-discounting than the private sector. This assumption will leave our model unchanged, but in

[25]We have also considered the cases where risk-premium elasticity is 1 or 3 basis points, and set ψ correspondingly to be 0.0028 and 0.0085. These additional results are available upon request.

the Ramsey problem the government will discount utility at $\mu\beta$ instead of β, with $\mu < 1$ representing the gap between household and government discount factors. We can think of this as capturing the shorter horizon governments typically face relative to the private sector. More specifically, we can interpret μ as the probability of a government being in charge in the next period, and therefore $1/(1-\mu)$ is the government's expected duration. In this framework, two important characteristics of the Ramsey steady-state, the level of debt and the capital tax rate, will depend on μ, such that, both the capital tax rate and the level of debt in steady state will be increasing in the gap between government and private discounting, and therefore decreasing in μ. The myopia on the part of the government would tend to support policies which lead to an unsustainable path for debt.[26] However, in the presence of bond holding costs, this would lead to increasing debt service costs. Eventually, these rising costs more than offset the myopia of the government and the policy acts to stabilize debt. This will be at a level above $d = \bar{d}/2$ and will also result in a positive rate of capital tax in the long-run.[27]

We set μ to 0.979 such that the Ramsey steady-state will feature a capital income tax rate matching the decentralized economy, i.e. $\tau^k = 0.20$. The calibrated value of $\mu = 0.979$ implies an expected government duration of 12 years. This is quite an extreme assumption of the degree of myopia experienced by the government given that many of the policies governments pursue have significantly longer periods of gestation before their full benefits are realized. In addition, we keep $\overline{d}$ at the value set above. The long-run level of debt will rise to about 98.50% of GDP. We construct the optimal Laffer curve for this model and compare it with the optimal Laffer curve with the risk-premium only in Figure 1.12. The optimal Laffer curve produced by the model with the additional assumption of government myopia always lies beneath the one for the risk-premium alone, meaning that for the same level of government debt to be sustained, the model with myopia implies larger welfare costs. However, the marginal increase in costs due to adding myopia is relatively small at high levels of debt but larger at low levels of debt.

[26]In essence, the myopia captures the various sources of deficit bias discussed in Alesina and Passalacqua (2016).

[27]The results of non-zero long-run capital tax rate and the level of steady-state debt are shown in Appendix C.3 and C.4, respectively.

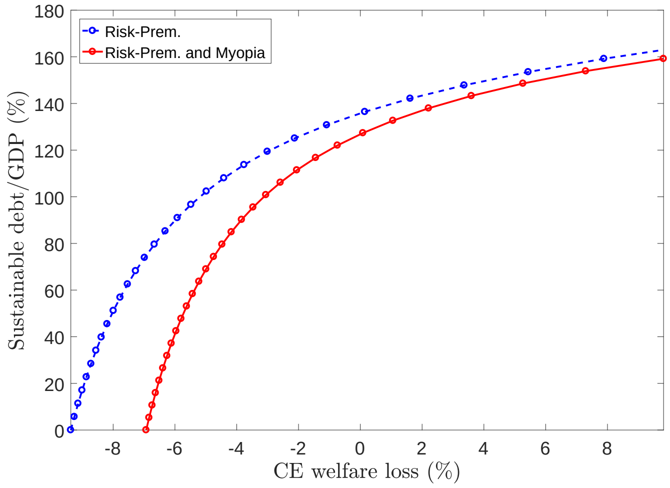

Figure 1.12: Comparing Laffer curves with Risk-premium and Myopia

To see why this is the case, Figure 1.13 plots the transitional dynamics for a number of endogenous variables at different initial debt to GDP ratios (e.g. 0%, 58% 122%). When the initial debt level is high (e.g. 122%), in the medium term, policy can still promise to reduce capital taxation to very low levels, thereby at least initially mimicking the promises inherent in the benchmark optimal Laffer curve over a more compressed time scale. In contrast, when initial debt levels are relatively low (e.g. 0% and 58%), we cannot obtain the combination of falling capital income tax rates and rising labor income tax rates, which was crucial in generating revenues by encouraging capital accumulation during the transition.

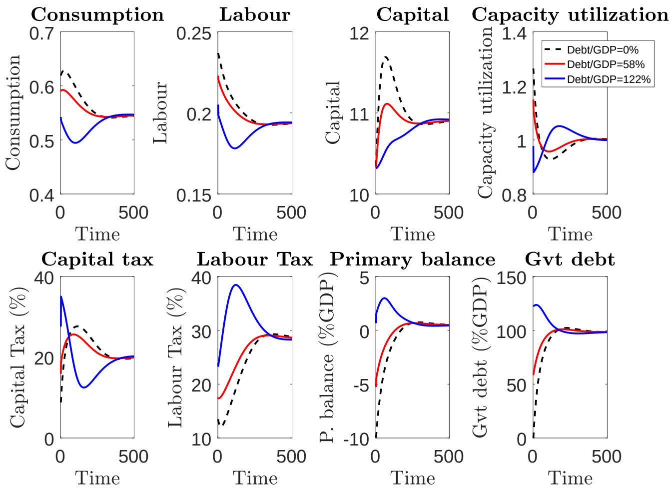

Figure 1.13: Myopic policies for different levels of initial debt

1.7 Conclusions

In the conventional Laffer curve calculations, discounted tax revenues are computed on the basis of varying individual tax instruments between 0% and 100%, while holding all other fiscal instruments constant. These studies are either carried out in a steady-state economy or a dynamic one but assume a one-off permanent change in a single tax rate.

Our paper is different from the conventional Laffer curve calculations. We plot the Laffer curve in welfare loss-sustainable debt space where the welfare loss captures the costs of the combined distortions implied by varying all tax rates optimally. As a result, each point on our Laffer curve reflects a full Ramsey problem, where the policy maker is optimally varying capital and labor income tax rates to maximize welfare given the need to satisfy the government's intertemporal budget constraint conditional on the initial level of debt. We find that, by committing to gradually eliminate capital tax and at the same time raising labor income tax, the optimal policy generates significant revenues during the economic transition. At the peak of the MTZ-like Laffer curve, the level of sustainable debt implied by the optimal policy (for the same welfare loss) is about 82% of GDP higher. However, this also assumed that the fiscal authority is fully credible and benevolent, and it does not suffer any increased debt service costs when debt levels are high.

Therefore, in a subsequent robustness analysis, we enrich the model by allowing debt service costs to rise with debt levels. This reduces the sustainable debt levels achievable by both the conventional and our optimal Laffer curves, but does not overturn the conclusion that there remain significant gains either in terms of revenues raised or welfare costs from conducting fiscal policy optimally. In addition, we further allow for a significant degree of policy maker myopia. Although the model with myopia implies larger welfare costs than the one with debt service costs only, the marginal increase in costs due to adding myopia is relatively small, especially at high levels of debt.

To sum up, our analysis suggests that conventional approaches to computing Laffer curves can significantly underestimate the amount of tax revenues that can be potentially generated or, equivalently, overstate the welfare costs of achieving a given level of fiscal revenues. In the future research, we will explore the degree to which time-inconsistency problem affects the revenue generating powers of a policy maker.

Appendix

A.1 The Decentralized Economy

The equilibrium conditions of the decentralized economy are summarized below. We remove capital adjustment costs and capital depreciation allowances (i.e. $\eta = 0$ and $\theta = 0$) and fix the consumption tax rate, τ_t^c, at a calibrated value consistent with the data, $\tau_t^c = \tau^c$. This is to facilitate the derivation of the primal form of our Ramsey problem.

$$(1+\gamma)q_t = \widetilde{\beta}\frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{U'_{c_t}(c_t, 1-l_t)}, \tag{A1}$$

$$-\frac{U'_{l_t}(c_t, 1-l_t)}{U'_{c_t}(c_t, 1-l_t)} = \frac{1-\tau_t^l}{1+\tau^c}w_t, \tag{A2}$$

$$U'_{c_t}(c_t, 1-l_t) = \frac{\widetilde{\beta}}{1+\gamma}U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})\left[\begin{array}{c} 1-\delta(m_{t+1}) \\ +(1-\tau_{t+1}^k)r_{t+1}m_{t+1} \end{array}\right], \tag{A3}$$

$$(1-\tau_t^k)r_t = \delta'_{m_t}(m_t), \tag{A4}$$

$$F'_{s_t}(s_t, l_t) = r_t, \tag{A5}$$

$$F'_{l_t}(s_t, l_t) = w_t, \tag{A6}$$

$$F(s_t, l_t) = c_t + g_t + (1+\gamma)k_{t+1} - [1-\delta(m_t)]\,k_t \tag{A7}$$

$$m_t k_t = s_t, \tag{A8}$$

$$g_t = \overline{g}, \tag{A9}$$

$$e_t = \overline{e}. \tag{A10}$$

B.2 The Primal Form of the Baseline Model

In this section, we present the primal form for the baseline model. Before showing how the Lagrangian function of the primal form is constructed, we first detail the derivation of the implementability constraint. The derivation of the other three constraints (i.e. the resource constraint, the capital utilization constraint, and the period-0 capital utilization condition) is illustrated in Section 1.3 of the maintext.

B.2.1 The Implementability Constraint

To derive the implementability constraint, we start with the household's budget constraint,

$$(1+\tau^c)c_t+(1+\gamma)q_t d_{t+1}+(1+\gamma)k_{t+1} = (1-\tau^l)w_t l_t+(1-\tau_t^k)r_t m_t k_t+[1-\delta(m_t)]\,k_t+d_t+\bar{e}. \tag{B1}$$

In addition, to simplify the notation below, we define

$$z_t \equiv (1+\tau^c)c_t - (1-\tau_t^l)w_t l_t, \tag{B2}$$

$$R_t^K \equiv \left(1-\tau_t^k\right) r_t m_t + 1 - \delta\left(m_t\right), \tag{B3}$$

and substitute the Euler equation (A1) into the above household budget constraint in (B1) to obtain

$$d_t = z_t + (1+\gamma)k_{t+1} - R_t^K k_t - \bar{e} + \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)}{U'_{c_t}\left(c_t, 1-l_t\right)} d_{t+1}, \tag{B4}$$

The corresponding expression in period-0 reads

$$d_0 = z_0 + (1+\gamma)\,k_1 - R_0^K k_0 - \bar{e} + \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)}{U'_{c_0}\left(c_0, 1-l_0\right)} d_1. \tag{B5}$$

We then substitute for all $d_{t\geqslant 1}$ recursively in equation (B5). The transversality conditions $\underset{t\to\infty}{lim}\widetilde{\beta}^{t+1}U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right) d_{t+1} = 0$ and $\underset{t\to\infty}{lim}\widetilde{\beta}^{t}U'_{c_t}\left(c_t, 1-l_t\right)$ $k_{t+1} = 0$, and the first order condition (A3) imply that the consolidated budget constraint at period-0 can be simplified to

$$d_0 + R_0^K k_0 = \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{U'_{c_t}\left(c_t, 1-l_t\right)}{U'_{c_0}\left(c_0, 1-l_0\right)}\left(z_t - \bar{e}\right) \tag{B6}$$

Finally, we substitute expression (B2) and (B3) back in (B6), and use the intratemporal consumption-leisure margin in (A2) to substitute out the labor tax. Therefore we obtain the implementability constraint,

$$B-\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\left[U_{c_t}'\left(c_t,1-l_t\right)\left(c_t-\frac{\overline{e}}{(1+\tau^c)}\right)+U_{l_t}'\left(c_t,1-l_t\right)l_t\right]=0,$$

where

$$B\equiv\left\{d_0+\left[\left(1-\tau_0^k\right)r_0m_0+1-\delta\left(m_0\right)\right]k_0\right\}\frac{U_{c_0}'\left(c_0,1-l_0\right)}{1+\tau^c}$$

collects the period-0 terms.

The implementability constraint, the resource constraint, the capital utilization constraint, and the period-0 capital utilization condition are the four constraints in the primal form of the Ramsey problem for our baseline model. In the next subsection we derive the Lagrangian function of the primal form presented in Section 1.3.

B.2.2 Ramsey Problem in the Primal Form

The Ramsey problem in primal form consists of maximizing the utility function,

$$\sum_{t=0}^{\infty}\widetilde{\beta}^{t}U\left(c_t,1-l_t\right),$$

subject to four contraints. These are the resource constraint

$$F(m_tk_t,l_t)-c_t-g_t-(1+\gamma)k_{t+1}+\left[1-\delta(m_t)\right]k_t\geq 0,$$

the implementability constraint

$$B-\sum_{t=0}^{\infty}\widetilde{\beta}^{t}\left[U_{c_t}'\left(c_t,1-l_t\right)\left(c_t-\frac{\overline{e}}{1+\tau^c}\right)+U_{l_t}'\left(c_t,1-l_t\right)l_t\right]\geq 0,$$

the capital utilization constraint for $t>0$

$$\frac{U_{c_t}'\left(c_t,1-l_t\right)}{U_{c_{t+1}}'\left(c_{t+1},1-l_{t+1}\right)}=\frac{\widetilde{\beta}}{1+\gamma}\left[\delta_{m_{t+1}}'(m_{t+1})m_{t+1}+1-\delta(m_{t+1})\right],$$

and the period-0 capital utilization condition,

$$\left(1-\tau_0^k\right)F_{s_0}'(m_0k_0,l_0)=\delta_{m_0}'\left(m_0\right).$$

The Lagrangian function of the primal form is then constructed as follows

$$\max_{\{c_t,l_t,m_t,k_{t+1},\tau_0^k\}_{t=0}^{\infty}} R =$$
$$\sum_{t=0}^{\infty}\widetilde{\beta}^t \left\{ \begin{array}{c} U\left(c_t,1-l_t\right) \\ +\phi\left\{B-\left[U'_{c_t}\left(c_t,1-l_t\right)\left(c_t-\frac{\overline{e}}{1+\tau^c}\right)+U'_{l_t}\left(c_t,1-l_t\right)l_t\right]\right\} \\ +\lambda_t^1\left[\left(\frac{1+\gamma}{\widetilde{\beta}}\right)\frac{U'_{c_t}(c_t,1-l_t)}{U'_{c_{t+1}}(c_{t+1},1-l_{t+1})}-\delta'_{m_{t+1}}(m_{t+1})m_{t+1}-1+\delta(m_{t+1})\right] \\ +\lambda_t^2\left[F(m_tk_t,l_t)-c_t-\overline{g}-(1+\gamma)k_{t+1}+(1-\delta(m_t))\,k_t\right] \\ +\varphi\left[\left(1-\tau_0^k\right)F'_{s_0}(m_0k_0,l_0)-\delta'_{m_0}\left(m_0\right)\right] \end{array} \right\}, \tag{B7}$$

where ϕ, λ_t^1, λ_t^2 and φ are the four multipliers attached to the resource constraint, the implementability constraint, the capital utilization constraint, and the period-0 capital utilization condition, respectively.

By grouping all terms in (B7) containing the utility function, we define the objective function as

$$\begin{aligned} V(c_t,1-l_t,\phi,\lambda_t^1) &= U\left(c_t,1-l_t\right) \\ &\quad +\phi\left[U'_{c_t}\left(c_t,1-l_t\right)\left(c_t-\frac{\overline{e}}{1+\tau^c}\right)+U'_{l_t}\left(c_t,1-l_t\right)l_t\right] \\ &\quad +\lambda_t^1\left[\left(\frac{1+\gamma}{\widetilde{\beta}}\right)\frac{U'_{c_t}\left(c_t,1-l_t\right)}{U'_{c_{t+1}}\left(c_{t+1},1-l_{t+1}\right)}\right], \end{aligned}$$

the above Lagrangian function can be then rewritten as follows,

$$\max_{\{c_t,l_t,m_t,k_{t+1},\tau_0^k\}_{t=0}^{\infty}} R =$$
$$\sum_{t=0}^{\infty}\widetilde{\beta}^t \left\{ \begin{array}{c} V(c_t,1-l_t,\phi,\lambda_t^1) \\ -\lambda_t^1\left[\delta'_{m_{t+1}}(m_{t+1})m_{t+1}+1-\delta(m_{t+1})\right] \\ +\lambda_t^2\left[F(m_tk_t,l_t)-c_t-\overline{g}-(1+\gamma)k_{t+1}+(1-\delta(m_t))\,k_t\right] \end{array} \right\}$$
$$-\phi\left\{B-\frac{\varphi}{\phi}\left[\left(1-\tau_0^k\right)F'_{s_0}(m_0k_0,l_0)-\delta'_{m_0}\left(m_0\right)\right]\right\} \tag{B8}$$

Where

$$A\equiv B-\frac{\varphi}{\phi}\left[\left(1-\tau_0^k\right)F'_{s_0}(m_0k_0,l_0)-\delta'_{m_0}\left(m_0\right)\right].$$

captures all period-0 constraints including B in the implementability constraint and the period-0 capital utilization condition. In Section 1.3, we present the Lagrangian function in (B8).

C.3 The Primal Form of the Extended Model

This section derives the primal form for our extended model in Section 1.6. Due to the presence of risk premium on government bond, households pay an insurance, $\Psi_t = \frac{\psi}{2}\left(d_{t+1} - \bar{d}\right)^2$, to a financial intermediary to secure a unit return on government bond against repayment risk. In addition, the profits of such as a financial institution, ϵ_t, are redistributed to households in a lump-sum way. Therefore, the household budget constraint is modified as follows,

$$\begin{aligned} & (1+\tau^c)c_t + (1+\gamma)q_t d_{t+1} + (1+\gamma)k_{t+1} + \Psi_t \\ = \; & (1-\tau^l)w_t l_t + (1-\tau_t^k)r_t m_t k_t + \left[1-\delta(m_t)\right]k_t + d_t + \overline{e} + \epsilon_t. \end{aligned}$$

Since the financial intermediary has zero marginal and fixed costs, $\Psi_t = \epsilon_t$ holds in equilibrium. In addition, the Euler equation (A1) is modified to incorporate the insurance costs as follows,

$$(1+\gamma)q_t + \psi\left(d_{t+1} - \bar{d}\right) = \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1}, 1-l_{t+1}\right)}{U'_{c_t}\left(c_t, 1-l_t\right)}. \tag{C1}$$

Therefore, the presence of insurance costs alters the implementability constraint derived above, while the other constraints (i.e. the resource constraint and the capital utilization constraint) remain the same as in the baseline model.

In the following subsections, we derive the new implementability constraint for our model with risk premium, and then we will show the Lagrangian function of the primal form Ramsey problem with both risk premium and government myopia.

C.3.1 The Implementability Constraint with Risk Premia

By substituting the new Euler equation in (C1) into the household budget constraint, we get:

$$(1+\tau^c)c_t - \psi\left(d_{t+1}-\bar{d}\right)d_{t+1} + \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1},1-l_{t+1}\right)}{U'_{c_t}\left(c_t,1-l_t\right)}d_{t+1} + (1+\gamma)k_{t+1} =$$
$$(1-\tau^l)w_t l_t + (1-\tau^k_t)r_t m_t k_t + \left[1-\delta(m_t)\right]k_t + d_t + \bar{e}. \tag{C2}$$

We then rearrange equation (C2) as follows

$$\begin{aligned} d_t = \quad & z_t - \psi\left(d_{t+1}-\bar{d}\right)d_{t+1} + (1+\gamma)k_{t+1} - R^K_t k_t - \bar{e} \\ & + \widetilde{\beta}\frac{U'_{c_{t+1}}\left(c_{t+1},1-l_{t+1}\right)}{U'_{c_t}\left(c_t,1-l_t\right)}d_{t+1}, \end{aligned} \tag{C3}$$

where z_t and R^K_t are defined as in equations (B2) and (B3).

The corresponding consolidated budget constraint at period-0 is given by

$$d_0 + R^K_0 k_0 = \sum_{t=0}^{\infty}\widetilde{\beta}^t\frac{U'_{c_t}\left(c_t,1-l_t\right)}{U'_{c_0}\left(c_0,1-l_0\right)}\left[\begin{array}{c} z_t - \bar{e} \\ -\psi\left(d_{t+1}-\bar{d}\right)d_{t+1} \end{array}\right].$$

Differently from the baseline model, the government debt that describes the risk premium cannot be easily substituted away. This implies that we cannot discard the sequence of budget constraints for periods $t = 1, 2, 3, ...,$ after consolidating at period-0. Therefore, we have a sequence of implementability constraints for all periods $t \geq 0$ for the extended model.

$$d_t + R^K_t k_t = \sum_{j=0}^{\infty}\widetilde{\beta}^j\frac{U'_{c_{t+j}}\left(c_{t+j},1-l_{t+j}\right)}{U'_{c_t}\left(c_t,1-l_t\right)}\left[\begin{array}{c} z_{t+j} - \bar{e} \\ -\psi\left(d_{t+j+1}-\bar{d}\right)d_{t+j+1} \end{array}\right]. \tag{C4}$$

Finally, we substitute the intratemporal consumption-leisure margin in (A2), the capacity utilization condition (A4) and expression (B2) and (B3) into the sequence of consolidated budget constraints in (C4), we get the implementability constraint for period t as follows,

$$\left\{d_t + \left[\delta'_{m_t}(m_t)m_t + 1 - \delta\left(m_t\right)\right]k_t\right\}\frac{U'_{c_t}\left(c_t,1-l_t\right)}{1+\tau^c} = \sum_{j=0}^{\infty}\widetilde{\beta}^j\{U'_{c_{t+j}}\left(c_{t+j},1-l_{t+j}\right)\times$$
$$\left[c_{t+j} - \frac{\psi\left(d_{t+j+1}-\bar{d}\right)d_{t+j+1}}{1+\tau^c} - \frac{\bar{e}}{1+\tau^c}\right] + U'_{l_{t+j}}\left(c_{t+j},1-l_{t+j}\right)l_{t+j}\}. \tag{C5}$$

C.3.2 Ramsey Problem with Risk-premium and Myopia in the Primal Form

This subsection outlines the Lagrangian function for the primal form Ramsey problem with risk premium and myopia. In the latter case, the government will discount utility at $\mu\widetilde{\beta}$, instead of $\widetilde{\beta}$, with $\mu < 1$ representing the gap between household and government discount factors. In addition, the presence of a sequence of implementability constraints in (C5) for all $t \geqslant 0$ complicates the setup of the Lagrangian function of this problem. In order to write the Lagrangian function in a compact form, we follow Aiyagari et al. (2002) and Rieth (2017) in defining a recursive multiplier, $\lambda_t^3 = \frac{\lambda_{t-1}^3}{\mu} + \upsilon_t$, with $\lambda_{-1}^3 = 0$, to be attached to the implementability constraint in (C5).

Therefore, the Lagrangian function is constructed as follow,

$$\max_{\{c_t, l_t, m_t, k_{t+1}, d_{t+1}\}_{t=0}^{\infty}} R =$$

$$\sum_{t=0}^{\infty} \left(\mu\widetilde{\beta}\right)^t \left\{ \begin{array}{c} U\left(c_t, 1-l_t\right) \\ +\lambda_t^1 \left[\left(\frac{1+\gamma}{\widetilde{\beta}}\right) \frac{U'_{c_t}(c_t, 1-l_t)}{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})} - \delta'_{m_{t+1}}(m_{t+1}) m_{t+1} - 1 + \delta(m_{t+1}) \right] \\ +\lambda_t^2 \left[F(m_t k_t, l_t) - c_t - \overline{g} - (1+\gamma) k_{t+1} + (1-\delta(m_t)) k_t\right] \\ +\lambda_t^3 \left\{ U'_{c_t}\left(c_t, 1-l_t\right) \left[c_t - \frac{\psi\left(d_{t+1} - \bar{d}\right) d_{t+1}}{1+\tau^c} - \frac{\overline{e}}{1+\tau^c} \right] + U'_{l_t}\left(c_t, 1-l_t\right) l_t \right\} \\ -\upsilon_t \left\{ \left[d_t + \left[\delta'_{m_t}(m_t) m_t + 1 - \delta\left(m_t\right)\right] k_t \right] \frac{U'_{c_t}(c_t, 1-l_t)}{1+\tau^c} \right\} \end{array} \right\}, \tag{C6}$$

where λ_t^1 and λ_t^2 again, are multipliers associated with the resource constraint and the capital utilization constraint. It is important to clarify that the period-0 capital utilization condition now is embeded in the new the implementability constraint for the extended model.

The first order conditions for $t \geq 0$ are:

$$\{c_t\}: \quad \lambda_t^2 - U'_{c_t}(c_t, 1-l_t) - \lambda_t^3 U'_{c_t}(c_t, 1-l_t) - \lambda_t^3 U''_{l_t c_t}(c_t, 1-l_t)\, l_t =$$
$$\left\{ \begin{array}{c} -\lambda_{t-1}^1 \frac{(1+\gamma)U'_{c_{t-1}}(c_{t-1},1-l_{t-1})}{\mu\widetilde{\beta}^2 U'_{c_t}(c_t,1-l_t)^2} \\ +\lambda_t^1 \frac{1+\gamma}{\widetilde{\beta}U'_{c_{t+1}}(c_{t+1},1-l_{t+1})} \\ +\lambda_t^3\left[c_t - \frac{\psi\left(d_{t+1}-\bar{d}\right)d_{t+1}}{1+\tau^c} - \frac{\bar{e}}{1+\tau^c}\right] \\ -\frac{\upsilon_t}{1+\tau^c}\left[d_t + \left[\delta'_{m_t}(m_t)\, m_t + 1 - \delta(m_t)\right] k_t\right] \end{array} \right\} U''_{c_t}(c_t, 1-l_t), \tag{C7}$$

$$\{l_t\}: \quad -\lambda_t^2 F'_{l_t}(m_t k_t, l_t) - U'_{l_t}(c_t, 1-l_t) - \lambda_t^3 U'_{l_t}(c_t, 1-l_t) - \lambda_t^3 U''_{l_t}(c_t, 1-l_t)\, l_t =$$
$$\left\{ \begin{array}{c} \lambda_{t-1}^1 \frac{(1+\gamma)U'_{c_{t-1}}(c_{t-1},1-l_{t-1})}{\mu\widetilde{\beta}^2 U'_{c_t}(c_t,1-l_t)^2} \\ +\lambda_t^1 \frac{1+\gamma}{\widetilde{\beta}U'_{c_{t+1}}(c_{t+1},1-l_{t+1})} \\ +\lambda_t^3\left[c_t - \frac{\psi\left(d_{t+1}-\bar{d}\right)d_{t+1}}{1+\tau^c} - \frac{\bar{e}}{1+\tau^c}\right] \\ -\frac{\upsilon_t}{1+\tau^c}\left[d_t + \left[\delta'_{m_t}(m_t)\, m_t + 1 - \delta(m_t)\right] k_t\right] \end{array} \right\} U''_{c_t l_t}(c_t, 1-l_t), \tag{C8}$$

$$\begin{aligned} \{m_t\}: \quad & \lambda_{t-1}^1 \delta''_{m_t}(m_t) m_t = -\mu\widetilde{\beta}\upsilon_t \delta''_{m_t}(m_t)\, m_t k_t \frac{U'_{c_t}(c_t, 1-l_t)}{1+\tau^c} + \\ & \mu\widetilde{\beta}\lambda_t^2\left[F'_{m_t}(m_t k_t, l_t) - \delta'_{m_t}(m_t)\right] k_t, \end{aligned} \tag{C9}$$

$$\begin{aligned} \{k_{t+1}\}: \quad & \lambda_t^2(1+\gamma) = \mu\widetilde{\beta}\lambda_{t+1}^2\left[\; F'_{k_{t+1}}(m_{t+1}k_{t+1}, l_{t+1})m_{t+1} + 1 - \delta(m_{t+1}) \;\right] \\ & - \mu\widetilde{\beta}\upsilon_{t+1}\left[\delta'_{m_{t+1}}(m_{t+1})\, m_{t+1} + 1 - \delta(m_{t+1})\right] \frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{1+\tau^c}, \end{aligned} \tag{C10}$$

$$\{d_{t+1}\}: \quad \mu\widetilde{\beta}\upsilon_{t+1}\frac{U'_{c_{t+1}}(c_{t+1}, 1-l_{t+1})}{1+\tau^c} = \lambda_t^3 \frac{U'_{c_t}(c_t, 1-l_t)}{1+\tau^c}\left(\psi\bar{d} - 2\psi d_{t+1}\right), \tag{C11}$$

$$\{c_0\}: \quad \lambda_0^2 - U'_{c_0}(c_0, 1-l_0) - \lambda_t^3 U'_{c_0}(c_0, 1-l_0) - \lambda_t^3 U''_{l_0 c_0}(c_0, 1-l_0)\, l_0$$

$$= \left\{ \begin{array}{c} \lambda_0^1 \frac{1+\gamma}{\widetilde{\beta} U'_{c_1}(c_1, 1-l_1)} \\ +\lambda_t^3 \left[c_0 - \frac{\psi\left(d_1 - \bar{d}\right) d_1}{1+\tau^c} - \frac{\bar{e}}{1+\tau^c} \right] \\ -\frac{\upsilon_0}{1+\tau^c} \left[d_0 + \left[\delta'_{m_0}(m_0) m_0 + 1 - \delta(m_0) \right] k_0 \right] \end{array} \right\} U''_{c_0}(c_0, 1-l_0) \tag{C12}$$

$$\{l_0\}: \quad -\lambda_0^2 F'_{l_0}(m_0 k_0, l_0) - U'_{l_0}(c_0, 1-l_0) - \lambda_0^3 U'_{l_0}(c_0, 1-l_0) - \lambda_0^3 U''_{l_0}(c_0, 1-l_0)\, l_0$$

$$= \left\{ \begin{array}{c} \lambda_0^1 \frac{(1+\gamma)}{\widetilde{\beta} U'_{c_1}(c_1, 1-l_1)} \\ +\lambda_0^3 \left[c_0 - \frac{\psi\left(d_1 - \bar{d}\right) d_1}{1+\tau^c} - \frac{\bar{e}}{1+\tau^c} \right] \\ -\frac{\upsilon_0}{1+\tau^c} \left[d_0 + \left[\delta'_{m_0}(m_0) m_0 + 1 - \delta(m_0) \right] k_0 \right] \end{array} \right\} U''_{c_0 l_0}(c_0, 1-l_0) \tag{C13}$$

$$\{m_0\}: \quad \mu \widetilde{\beta} \upsilon_0 \delta''_{m_0}(m_0)\, m_0 \frac{U'_{c_t}(c_t, 1-l_t)}{1+\tau^c} = \mu \widetilde{\beta} \lambda_0^2 \left[F'_{m_0}(m_0 k_0, l_0) - \delta'_{m_0}(m_0) \right], \tag{C14}$$

The above set of first order conditions (C7)-(C14) and the three contraints characterize the solution of Ramsey problem for the extended model.

C.3.3 Non-zero Long-run capital tax

Introducing risk premium does not alter the zero long-run capital tax result in the baseline model. However, when allowing for policy myopia, the long-run capital tax becomes positive and increasing in the degree of myopia. In this subsection, we show how the two extensions of the model affect the long-run first order condition with respect to capital.

We first substitute υ and $\delta'_m(m)$ in the long-run Ramsey first order condition with respect to capital (C10), using the steady-state multiplier of the implementability constraint and capacity utilization condition to obtain,

$$\begin{aligned} &\lambda^2(1+\gamma) = \mu \widetilde{\beta} \lambda^2 \left[F'_k(mk, l) m + 1 - \delta(m) \right] \\ &+ (1-\mu)\, \widetilde{\beta} \lambda^3 \left[(1-\tau^k) F'_k(mk, l) m + 1 - \delta(m) \right] \frac{U'_c(c, 1-l)}{1+\tau^c} \end{aligned} \tag{C15}$$

Given that the Ramsey allocation is a competitive equilibrium, combining equation (C15) and the long-run intertemporal investment decision under the competitive equilibrium,

$$\widetilde{\beta}\left[(1-\tau^k)F_k'(mk,l)m+1-\delta(m)\right]=1+\gamma,$$

we obtain the following expression for the Ramsey capital tax in the long-run:

$$\tau^k=\frac{(1-\mu)\left[\lambda^2-\lambda^3\frac{U_c'(c,1-l)}{1+\tau^c}\right]\left[F_k'(mk,l)m+1-\delta\left(m\right)\right]}{\left[\lambda^2-(1-\mu)\,\lambda^3\frac{U_c'(c,1-l)}{1+\tau^c}\right]F_k'(mk,l)m}. \tag{C16}$$

In the absence of myopia, $\mu=1$, the Ramsey capital tax, τ^k, is zero in the long-run, which is consistent with the baseline model. However, under the assumption of policy myopia, $\mu<1$, to further show that the Ramsey capital tax is positive, we need to show that $\lambda^2>\lambda^3\frac{U_c'(c,1-l)}{1+\tau^c}$, which is difficult to show analytically, but does hold for our benchmark calibration and any other permutation of parameters we have tried.

C.3.4 Debt stationarity

In the extended model, government debt is no longer a unit root process; instead it will be mean-revering in the long-run. We show below this is the case.

The steady-state Ramsey first order condition with respect to debt in (C11) is

$$d=\frac{\widetilde{\beta}\,(1-\mu)}{2\psi}+\frac{\bar{d}}{2},$$

which implies that without myopia, $\mu=1$, Ramsey policy will prescribe $d=\frac{\bar{d}}{2}$. However, under the assumption of government myopia, $\mu<1$, $d>\frac{\bar{d}}{2}$ and increasing in the degree of Myopia. In both cases, the Ramsey solution implies a unique long-run level of debt independent from the initial conditions.

C.3.5 Bounded period-0 capital tax

In the extended model, the trade-off in the period-0 capital tax optimality condition remains qualitatively unchanged, when choosing the period-0 capital tax, the government will still trade-off the benefit of reducing debt burden versus the cost of household reducing capacity utilization.

D.4 Measuring Welfare Costs

In our Laffer curve calculations welfare associated with alternative fiscal policies (and revenues) are computed in equivalent constant consumption units as in Schmitt-Grohé and Uribe (2007). This procedure is a natural way for quantitatively comparing welfare across alternative policies when the utility function does not support a cardinal interpretation. We briefly outline the procedure below. Consider two alternative policy regimes A and B, we define life-time welfare as:

$$W^A = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t U\left(c_t^A, 1 - l_t^A\right) \tag{D1}$$

and

$$W^B = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t U\left(c_t^B, 1 - l_t^B\right) \tag{D2}$$

Let us denote λ^c the welfare cost of adopting the policy regimes B in place of the policy regimes A in terms of constant consumption units. Then λ^c would be implicitly defined as:

$$W^B = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t U\left(\left(1 - \lambda^c\right) c_t^A, 1 - l_t^A\right) \tag{D3}$$

For the utility function we employ the above expression can be re-written as:

$$W^B = E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \frac{\left[\left(1 - \lambda^c\right) c_t^A \left(1 - l_t^A\right)^a\right]^{1-\sigma}}{1-\sigma} = \frac{\left(1 - \lambda^c\right)^{1-\sigma}}{1-\sigma} W^A \tag{D4}$$

and solving for λ^c we obtain:

$$\lambda^c = 1 - \left[\frac{W^B}{W^A}\left(1 - \sigma\right)\right]^{\frac{1}{1-\sigma}} \tag{D5}$$

Chapter 2

A DSGE model for Scotland and the rUK

2.1 Introduction

The purpose of this chapter is to develop a dynamic stochastic general equilibrium model to carry out fiscal devolution analysis within a macroeconomic framework. Such a device will allow for the study of the macroeconomic and public finance implications of alternative fiscal devolution settings. This is a key aspect of the policy debate around Europe as many countries (among others: UK, Spain and Italy) are currently facing increasing requests of autonomy from their own regions. Such analysis requires to employ a two-country model (a region versus the rest of the country) capturing the key features of sub-national economies. These include: (i) unique monetary policy, (ii) multiple fiscal authorities (central and local governments); as well as (iii) significant price pass-through both in the good and in the labour market.

When designing the model, and when estimating it in the next chapter, we will consider the case of Scotland and the rUK. This case study is particularly interesting since the policy debate surrounding Scottish fiscal devolution and independence, as well as the availability of quarterly national account statistics, make the position of this country unique. To the best of our knowledge, no such a model is currently available for Scotland and, therefore, our DSGE model of the Scottish economy is innovative.

The design of the model draws from the DSGE literature, developed in policy institutions and academia in recent years (which is reviewed subsequently). This modeling approach reflects the new consensus in macroeconomics and accounts for the main features of countries business cycle. As such, it can offer insights on some of the key academic and policy questions surrounding fiscal devolution in Scotland, for example: what are the costs and benefits of the current devolution settlements? Can we decompose the welfare costs/benefits across individual policy measures e.g. with and without the control of income tax? Would Scotland have been better off without devolving a particular fiscal instrument? What are the costs associated with the lost of risk sharing? How the Scottish Laffer curve compares with the UK's one? How an independence scenario would affect all the above?

Within this chapter we will also discuss some of the limitations of our modeling approach. In particular, we do not explicitly model migration between Scotland and the rUK. This feature is typically not present in the

DSGE framework and it is not easily introduced; furthermore, migration statistics in and out of Scotland are limited. Such an assumption, nonetheless, will need to be re-assessed in future for its centrality in policy analysis and in light of the evidence from recent fiscal reforms in Scotland[1]. Similarly, the oil sector in the UK is modeled in a stylized manner which does not allow to account for the impact of oil price fluctuations over the private sector in Scotland. This aspect is likely to be quantitatively relevant. Such a simplifying assumption is made to limit the scale and the complexity of the model, which is, in its current formulation, already significant.

Our model will contribute to the economic literature and policy making along several lines. Firstly, it will provide an original framework able to capture the nature of the economic linkages between Scotland and the rUK as part of a fiscal and a monetary union. These include a unique central bank, strong trade linkages and significant price pass-through both in the goods and in the labour markets. Recent contributions (among others: Petrova et al. (2017); Bhattarai and Trzeciakiewicz (2017); Aminu (2018)) have provided DSGE models for the UK economy. However, none of these papers model Scotland as part of the UK; nor deals with the interrelations between sub-national economies within the UK. Secondly, our model will provide a useful device for running policy analysis and counter-factual experiments, to assess alternative devolution arrangements between Scotland and the rest of the UK. For the reasons just discussed, and to the best of our knowledge, none of the models currently available for the UK allows for this type of analysis. Thirdly, our model will allow future research to shed light on the optimal design of fiscal federalism.

We begin this chapter by reviewing the literature on DSGE models developed in academia and in central banks and discuss our model proposal in section 2.2. We then outline our model in section 2.3. In order to be solved and simulated in standard software the model needs, however, to be transformed in a stationary recursive form. In section 2.4, we therefore transform the model in stationary recursive terms and present the set of normalized equilibrium conditions. In the final section of this chapter we provide some concluding remarks.

[1]Recent income tax reform in Scotland might shed light on the extent of the migration response, if any. These are not captured by the data as yet, since the migration time-series ends in 2016-17.

2.2 The literature and our model

In the last twenty years, macroeconomic models have evolved towards a common framework known as Dynamic Stochastic General Equilibrium (DSGE henceforth). DSGE models combine the Neo-classical workhorse with a number of nominal rigidities (e.g. staggered prices and wages) and market imperfections (e.g. imperfectly competitive product and labour markets) of the New-Keynesian framework. Successive research efforts have brought DSGE models to reach the necessary level of sophistication to account for the properties of main macroeconomic time series and to be a reference tool for policy analysis. A cornerstone in this literature is the celebrated contribution of Smets and Wouters (2003, 2007) (SW, henceforth). Their medium-scale closed-economy model has proved a goodness of fit comparable to the best Bayesian SVAR and constitutes the new consensus in macroeconomics. More recently, some authors have extended this framework to open economy. Specifically, Adolfson et al. (2007) developed what today is the reference small open economy model. Similarly, Rabanal and Tuesta (2010) were among the first in successfully developing and estimating a medium-to-large scale two-country DSGE model.

We begin by reviewing small open-economy frameworks, describing in detail the model of Adolfson et al. (2007). This model is a key reference for it constitutes the common skeleton of most recent open-economy literature, including multi-country models. Accordingly, we find it useful to dig into the details of its structure, highlighting the role of its main assumptions. We then discuss some contributions extending and refining this framework. Finally, we review the literature on multi-country models and outline our model proposal.

2.2.1 The new open economy model (NOEM)

A baseline model

A central contribution in the new open economy literature has been provided by Adolfson et al. (2007) (henceforth, ALLV). They built on the SW's closed economy framework, adding a full set of open economy features. This framework is sometimes referred to as "The new open economy model" (NOEM, henceforth). Their economy is constituted by one endogenously modeled country (in their case Euro Area) trading with an exogenous rest-of-the-word (ROW, henceforth). In the following, we discuss the design of the model and the rationale behind its main assumptions; whereas in the next subsection, we present a number of related contributions refining and extending this framework.

Begin with the supply side of the the ALLV's economy. This features three types of firms - importers, intermediate good producers, and exporters - producing differentiated goods. Beside, there are goods packers whose purpose is to aggregate differentiated goods into consumption, investment and export baskets eventually consumed by households. All firms operate into imperfectly competitive markets featuring partially indexed calvo-regime[2], whereas packers operate under perfect competition and flexible price regime. Each intermediate good firm produces a differentiated good using capital and labour. These differentiated goods are then combined by good packers with imported goods to produce the final good baskets for consumption, investment and export. These imported goods are acquired abroad and sold at home by importing firms who price them in the domestic currency; exporting firms are just symmetrical. Importantly, good packers face a sector-specific and time-varying production elasticities with respect to the differentiated goods they aggregate; this gives rise to an equal number of time-varying, sector-specific mark-ups. This is a key feature for the model to replicate the properties of the exchange rate dynamics and inflation differential assuming a reasonable degree of price elasticities and without detrending the data[3]. Figure 2.1 summarizes the structure of the supply side we just described. Next to the good markets there is a non-competitive labour market, subject as well to the partially indexed calvo-regime, where differentiated labour is supplied by households to a labour packer who rents it, in turn, to the intermediate good producers. Before turning to the description of the demand side, it is worth discussing the role and the implications of the main assumptions so far. Firstly, the presence of nominal rigidities allows prices and wages to respond with some delay to changes in real variables, in line with the empirical evidence (and due, for example, to contractual rigidities, menu costs etc.). Secondly, the partial indexation mechanism allows non re-

[2] Under partially indexed calvo-regime, firms that are unable to re-optimize are allowed to index their prices at the past period inflation.

[3] Indeed, time-varying markups have a rather persistent effects on real exchange rate dynamics, while generating much less fluctuation in quantities and prices.

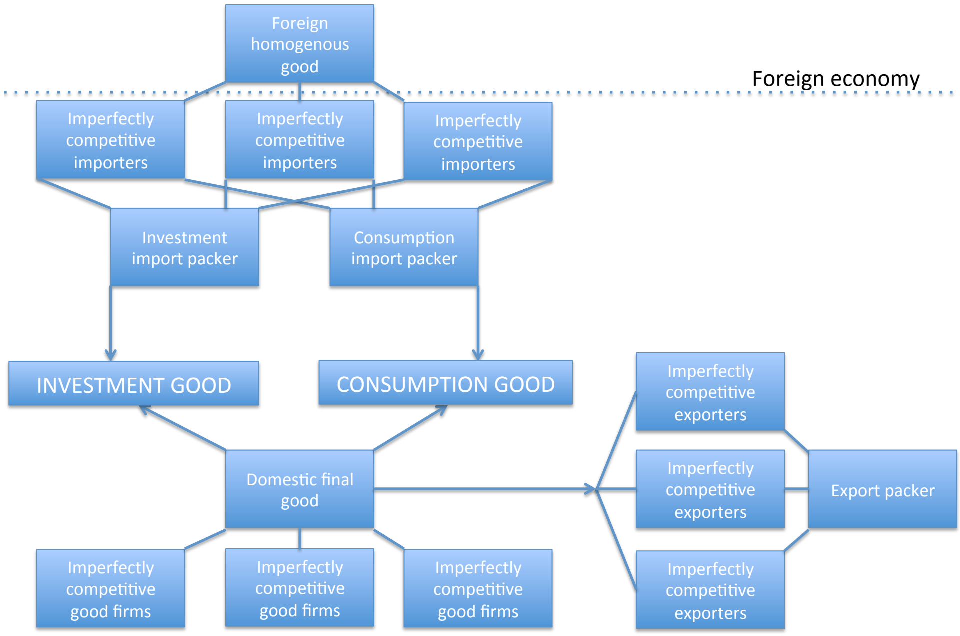

Figure 2.1: A summary of the supply side of ALLV's model

optimizing firms (workers) to partially index their prices (wages) to the last period inflation determining a stronger persistence in the level of price and wage inflation. That results in a more general specification of the Philips curve, where the inflation features both "forward" and "backward" looking components[4]. Thirdly, the combined presence of imperfect competition and calvo-pricing implies that the price and wage mark-ups change over time in a counter cyclical manner; since firms (workers) cannot re-optimize prices (wages) in every period, a positive demand shock lowers the markup and stimulates employment, investment and real output. Finally, local currency pricing operated by importing firms, together with calvo-regime, determines an imperfect price pass-through in response to changes in the exchange rate or in the supply conditions of the rest of the world.

The demand block of the economy is composed by the rest of world (which imports domestic goods) and by the domestic households who consume and

[4]However, Chari et al. (2008) noted that this specification, while allowing for a good empirical performance, has the counterfactual implication that firms update their prices every period.

invest. Specifically, domestic goods are demanded abroad depending on their relative prices, as well as on the dynamics of ROW GDP whose evolution is governed by an identified VAR model. On the other hand, the domestic household faces the problem of allocating his income between consumption, cash holding, investment in capital, as well as in the domestic and in the foreign bond. Consumption and investment are undertaken buying a composite basket of domestic and foreign produced goods. Both are subject to real rigidities, for consumption is subject to habits formation, while investment is subject to adjustment cots. Furthermore, as in SW, investment is not the only way for increasing capital services; capital utilization can, in fact, be increased at the cost of a faster depreciation. Again, we briefly discuss the role of the main assumptions of the demand side. Firstly, the set of real rigidities constituted by habits formation and investment adjustment costs play a central role in guaranteeing, respectively, greater persistence in consumption and in investment. Similarly, allowing for a variable utilization of capital, smooths the adjustment of its rental rate in response to changes in output. Secondly, imported goods enter both the consumption and the investment baskets; this is crucial for ensuring that imports variability can match its empirical counterpart[5]. Thirdly, the choice between domestic and foreign bonds balances to a no-arbitrage condition pinning down the expected real exchange rate appreciation, otherwise known as *uncovered interest rate parity condition* (UIP, henceforth). Fourthly, the absence of contingent bonds trading with the ROW determines an imperfect international risk sharing; accordingly, in presence of idiosyncratic shocks, wealth effects are not neutralized by international transfers. Finally, the model embodies no-optimizing monetary and fiscal authorities. The former adjusts the short-term interest rate in response to the CPI inflation rate, the output gap and the real exchange rate deviations. The latter, collects a set of distortionary taxes whose revenues are employed to finance government consumption under budget balance. In terms of performance, the model displays a quite satisfactory in-sample fit, with the exception of the real wages series which is predicted to grow too fast. The posterior predictive analysis shows that the realized volatility and auto-correlation of most variables are

[5] Import volatility is typically higher than consumption and closer to investment volatility (see Uribe (2014)). Including an imported component in the investment production allows to increase import volatility.

well captured by the model. Finally, looking at cross correlations, it is clear that the model has hard time replicating the joint behaviour of domestic inflation and imports; the model predicts a positive relation while in the data it appears to be negative. It is interesting to note that the marginal likelihood comparison favors a version of the model without working capital.

Model extensions and related contributions

Below we review a number of open economy models extending or amending ALLV's framework. We briefly discuss in each case the nature of such extensions and their rationale.

A slight refinement of ALLV's framework is proposed by Adolfson et al. (2008), in a model of the Swedish economy. Their contribution is to amend the UIP condition, which is a key component since it drives the real exchange rate dynamics. A large piece of empirical evidence is motivating this modification. Indeed, standard VAR analyses suggest that, after a monetary policy shock, the real exchange rate impulse response function is hump-shaped with a peak after 1 year. Standard UIP, instead, implies a peak within the quarter interested by the shock, followed by a relatively quick mean reversion. Moreover, DSGE models with standard UIP cannot account for the so-called 'forward premium puzzle', i.e. a currency whose interest rate is high tends to appreciate, implying a negative correlation between the risk premium and the expected exchange rate depreciation. Adolfson et al. (2008)'s approach is to include the expected exchange rate appreciation within the risk premium reduced-form formula. The empirical performance of such a "modified UIP" condition versus its standard version is assessed through marginal likelihood, impulse response function and out-of-sample forecast. In all cases the amended version of the model appears to perform better.

In a similar framework[6] designed for the Norway's economy, Bache et al. (2010) studied the performance of an instrument rule versus an optimal rule for monetary policy. There are both empirical and theoretical reasons motivating this exercise. From a theoretical point of view, the use of an instrument rule is somehow unsatisfactory since it implies an asymmetric treatment of the central bank with respect to the private sector. With the

[6]their model differs from Adolfson et al. (2007, 2008) because of the absence of money in the utility function and of the working capital channel

former being a less sophisticated, non-optimizing agent unable to exploit all the available information in the economy. Such assumption appears hard to justify and requires the introduction of an ad-hoc rule in an otherwise coherent framework. From an empirical point of view, there are two opposite mechanisms at play. On the one hand, the optimal policy produces a behavioural rule for monetary policy which is more comprehensive, since it contains a larger set of variables than the simple instrument rule. On the other, this comes at the cost of introducing a new set of restrictions on the reduced form solution of the model; restrictions that are potentially at odd with the data. Moved from these premises, these authors analyse which of these specifications provides a better account of the data. In-sample fit results superior under optimal policy, however, in terms of forecasting accuracy these models display a comparable performance.

Harrison and Oomen (2010) provide an analysis of the role of structural shocks in an open economy DSGE model of the UK. They begin by calibrating a baseline version of the model[7] (e.g. ALLV), containing only seven structural shocks[8]. Then they investigate which other shocks are to be introduced to address the most serious deficiencies of the model in fitting the data. This is a key question, since introducing shocks can potentially change the behaviour of the model and it affects the variance of core variables. For this purpose, authors employ spectral analysis[9], as well as the study of the coherence function[10], from which they derive three main insights. First, the variability of the model is generally low as its spectra mostly lies below the data spectra (this is especially true for investment, inflation and output). Moreover, the slope of model spectra differs from the data correspondent implying that the model does not capture the persistence in the data (mostly true for hours worked and real wages). Second, while the baseline model succeeds in capturing the lower coherence between the output and the main

[7]It should be noted that their model differs from ALLV in the design of the pricing regime: rotemberg price adjustment is used in place of Calvo pricing.

[8]Specifically, these are: productivity, government spending, monetary policy, ROW demand, ROW interest rate, ROW inflation and ROW export prices.

[9]The power spectrum of a time-series $x(t)$ describes how the variance of the data $x(t)$ is distributed over the frequency components into which $x(t)$ may be decomposed.

[10]The coherence between two series lies in the range (0,1) and gives a measure of the degree of correlation between the series at any frequency. A strong correlation is indicated by a coherence measure close to unity. In Harrison and Oomen (2010)'s article macro-variables coherence is always measured with respect to output.

macro-variables found in the data at high frequency, it does not match the strong increase in coherence at lower frequency. Third, shutting off nominal and real rigidities increases output coherence at any frequency, since main variables tend to co-move and to be driven by few common shocks. They conclude that further sources of variability should be incorporated in the model to increase low frequency coherence of main macro-variables with output. At the same time, lower coherence at high frequencies - generated by model rigidities - should be retained. They therefore seek to introduce shocks that affect a relatively small number of equilibrium conditions in the model and that are, consequently, likely to affect the spectra and coherence of a small number of variables in a predictable manner. The following shocks are inserted: preference shock (likely to increase the coherence of consumption and output at low frequencies), shock to capital adjustment costs (for similar reasons), mark-up shock (allowing the nominal interest rate and inflation to become less correlated at lower frequencies) and labour supply shock (increasing the variability of real wages and hours worked at low frequency). Overall this version of the model seems to capture the properties of the UK data better for most variables. Improvements are most visible at low frequencies reflecting the model increased ability to capture the coherence in the data. However, model performance at high frequency deteriorates, particularly for consumption and hours worked which become more volatile than in the data.

After Harrison and Oomen (2010)'s paper, a couple of recent contributions, ie. Petrova et al. (2017); Bhattarai and Trzeciakiewicz (2017); Aminu (2018), focused on modeling the UK economy within a DSGE framework. In particular, Bhattarai and Trzeciakiewicz (2017) adopted the same framework as Harrison and Oomen (2010), but with the difference of allowing for a share of households to be non-Ricardian as well as for a full-blown public sector balance sheet (including public consumption and investment, as well as a full set of distortative fiscal instruments). This extended version of the model is employed for fiscal policy analysis; particularly, to investigate the GDP multiplier implied by distortionary taxation, government consumption and investment in the UK. They find that government consumption and investment shocks are the most stimulating in the short-run. In the longer horizon the capital income tax and the public investment shock result in the highest multipliers. Furthermore, when the nominal interest rate is at the zero lower bound the effectiveness of consumption taxes and public expenditure increases, but decreases that of capital and labour income taxes. Finally, they show that non-Ricardian households make the fiscal policy more effective (i.e. generally enhancing the effects of a fiscal stimulus), and that nominal rigidities improve effectiveness of public spending and consumption taxes, whereas decrease effectiveness of income taxes.

Aminu (2018)'s paper built over the standard framework by allowing for a multi-sector model. This features a retail, a non-oil production, and a petrol production sector, where energy factors (gas and oil) constitute both an input and an output of the economic production. Such a model is then employed to examine the impact of energy price shock (oil prices shock and gas prices shock) on the economic activities in the UK. He finds that the fall in output during the financial crisis period was driven by domestic demand shock, energy prices shock and world demand shock. Furthermore, the effects of the energy prices shock on output in the UK are only temporary.

Finally, Petrova et al. (2017) estimated the Bank of England COMPASS model using Bayesian techniques with time-varying parameters. Such a model extends Harrison and Oomen (2010)'s framework by featuring hand-to-mouth agents, as well as a final good production sector with imperfect competition and price adjustment costs. The estimation is ran over a sample dating back to 1975-Q1 and running until 2014-Q4. Given the significant changes in policy regimes the UK economy has went through, structural parameters are not expected to be constant over these years. This motivates allowing for their time-variation. Their results seem to confirm such an insight: their estimation detects the transition to a monetary policy regime characterized by long-term inflation expectations anchored at the target, an increased responsiveness of policy rates to inflation and a reduction in the importance of the non-systematic component of monetary policy. Finally, they show that allowing for time-variation in parameters improves both point and density forecast performance for most variables and horizons[11].

[11] Petrova et al. (2017)'s paper has important implications for estimating DSGE models over the UK economy. Specifically, it highlights the importance of allowing time-variation in model parameters, when the estimation is ran over a long time-series. It should be noted, however, that this result is not relevant for the estimation of our model discussed in the next chapter. In fact, because of limitations in the Scottish statistics, our estimation sample will date back only until 1998-Q1.

2.2.2 Beyond small open economy: multi-country models

The NOEM allows to capture the key features of countries business cycle and to account for the main mechanisms through which shocks propagate within an economy. However, by leaving the rest of the world exogenous, this framework is unable to deal with the propagation of shocks, policy changes and business cycle across countries. Since we precisely aim to represent the interrelations between the Scottish economy and the rest of the UK, we need to move a step beyond NOEM toward multi-country models. It is worth stressing that the discussion above stays, nonetheless, central. Indeed, multi-country DSGE literature mostly built on NOEM, typically using the shortcut of representing each country in the model as a symmetric ALLV's economy. By the time we are writing, multi-country DSGE literature is not densely populated; there are only a couple of relevant contributions which we review in the following.

One of the earliest attempt to provide a framework for a multi-country analysis is the contribution of De Walque et al. (2005). Their model is meant to represent a two-country, open economy version of the original SW's closed economy framework. However, the structure of the supply side is much more sophisticated including oil among the production inputs and allowing imported goods to enter different phases of the production process. We do not enter the details of this model here for the main reason that, while adding more complexity than Rabanal and Tuesta (2010) (discussed next), it performs significantly worst, e.g. cross country GDP correlation is nearly zero. Subsequent literature did not build further on this framework.

Rabanal and Tuesta (2010) estimate a two-country model of the US and the Euro Area. Both economies are modeled along the lines of Adolfson et al. (2007)[12]. Their aim is to develop a framework able to account for the dynamics of the exchange rate and cross country business cycle, while retaining the ability to match closed economy facts. For this purpose, two assumptions are proved to be key, these are: local currency pricing (henceforth, "LCP") and incomplete markets (e.g imperfect international risk sharing). Specifically, LCP breaks the law of one price[13] and, by ensuring that all the components

[12]The only substantial difference is the absence of money from the utility function and of the working capital channel.

[13]Since imported goods are priced in local currency and their price is sticky, short-term

of the price index are sticky, it is central for matching the real exchange rate dynamics. On other hand, market incompleteness averts the counter-factual prediction of (nearly) perfect correlation between the real exchange rate and the ratio of consumption across countries[14]. This framework is shown to do reasonably well in matching the volatility of the real exchange rate, the CPI inflation, the GDP deflator, the interest rate and the real wage growth. However, it produces a volatility of output, consumption and investment in both countries which is higher than in the data. In terms of cross-country dynamics, the model appears to under predict the correlation of output, consumption and investment. Furthermore, the correlation between (changes) in the real exchange rate and (changes) in relative consumption are found to be positive (though close to zero) in the model, as opposed to the negative correlation found in the data. Finally, authors assess the contribution of LCP and incomplete markets to the overall model fit. For this purpose, four versions of the model are compared; these are obtained by switching on and off these two features in turn. Marginal likelihood indicates a strong support for the model with LCP and incomplete markets. Moreover, the model with LCP performs better than any combination of LCP and producer currency pricing (e.g. where the LCP producers are only a fraction of the total).

A smaller two-country model was designed by Kolasa (2009) to study to what degree parameter estimates between Poland and the Euro area differ. These differences could account for divergence in economic institutions, market competitiveness and resilience to external shocks. Her framework combines the full set of closed economy features (i.e. standard real and nominal rigidities) with more a simplified open economy side with respect to Rabanal and Tuesta (2010). Specifically, in her model there are only two type of firms, producing either tradable or non-tradable goods, while producer currency pricing and perfect international risk sharing are assumed. Results do not allow the author to draw any firm conclusion on the difference in deep parameters. At the same time, they provide strong evidence of asymmetry in standard deviation of shocks and lack of business cycle syn-

fluctuations of the real exchange rates as well as of the price of foreign goods are not passed on consumers. This effectively means that the price of the same good might temporarily diverge in two the countries.

[14]For a general specification of the utility function, the real exchange rate is equal to the ratio of the marginal utility of consumption across country under complete markets

chronization between Poland and the Euro area. The in-sample fit of the model appears acceptable.

2.2.3 Our model proposal

Our model will belong to the current generation of DSGE models which have proved a good performance in accounting for in-sample dynamics and in forecasting exercises. Given the need of capturing the interconnections and feedback effects between Scotland and the rUK, our model will necessarily include an endogenous rest of the UK, and will therefore belong to the class of two-country models. Furthermore, each economy in our model will need to be comprehensively modeled for adequately representing within-country dynamics. In this respect, the model of Rabanal and Tuesta (2010) (henceforth, *RT*) appears to be the most suitable starting point. However, to serve our purposes, this framework needs to be amended and integrated along several lines. First, our model will represent the historical fiscal arrangements within the UK and include the different levels of government; i.e. the central government in Westminster as well as the devolved administration in Holyrood. Second, our framework should adequately represent the nature of the economic linkages between Scotland and the rUK as part of a fiscal and a monetary union. This means a unique central bank, strong trade linkages and significant price pass-through both in the goods and in the labour market. Third, our model will include an oil sector and account for the tax receipts from oil. Even though this sector does not weight much on the overall UK economy, it can be a key element in some scenarios for future research, such as Scottish independence. Indeed, in an independence scenario where oil is allocated between Scotland and the rUK according to population or geographical share, its relevance for Scottish public finances will be dramatic. Finally, to complete our open economy framework, the UK economy will need to be opened to the rest of world, which we will represent as exogenous and model it as a VAR. In the paragraphs below we discuss these extensions and amendments in greater detail.

The public sector

A major application of our model is dealing with public finance and fiscal devolution policy questions, such as: which fiscal instruments are most suitable to be devolved? By how much should borrowing powers be adjusted for managing the associated fiscal risk? How does the scope for a unique central bank change when devolving different sets of fiscal instruments and borrowing powers? In line with these objectives, our framework should necessarily include two different levels of government, the devolved administration in Scotland as well as the central government in Westminster. Differently from *RT*, we will allow for a full set of distortative fiscal instruments, together with fiscal rules for public spending. Furthermore, we will include a "Barnett formula" dictating the evolution of transfers from Westminster to Holyrood, which finance most of the Holyrood's spending. For the purpose of the estimation on historical data[15], in our baseline model no fiscal instrument will be devolved to Scotland. Once estimated the structural parameters, we will be able to vary those fiscal settings and run a full set of fiscal experiments and counter-factual exercises.

Price pass-through modeling

As part of a fiscal and a monetary union, Scotland and the rUK are characterized by strong trade linkages and significant price pass-through both in the goods and in the labour market. This a crucial aspect our model will need to account for. Trade linkages within the UK, together with the fact that Scotland is small relative to the rUK, imply that the baskets of consumption and investment goods in Scotland are largely composed by rUK produced goods. Importantly, over these goods a significant price pass-through take place. *RT*'s model is not suitable to reproduce this feature since it is meant to represent the trade between the US and the EU where price pass-through is limited[16]. Indeed, they assume segmented markets (i.e. LCP), implying that exporters price differently goods in the foreign markets with the respect to the domestic one. This assumption is unlikely to work well for the UK, where stores and chains selling UK-wide, e.g. Tesco, Boots etc, tend to charge the same prices across the country. We shall assume instead that markets are non-segmented, letting goods imported from the rUK to be acquired at the same price rUK consumers pay. This will

[15]Our sample will run from 1998:Q4 to 2007:Q4, as discussed later.

[16]Specifically, limited price pass-through is necessary for the model to match the dynamics of the real exchange rate in the data.

effectively imply that a significant share of the Scottish price index is lead by the rUK prices. A similar issue arises in the labour market. Again, much of the wage dynamics in Scotland are likely to be influenced by the ones in the rUK, for a number of jobs (e.g. public sector) see their pay growing uniformly within the country, often even paying the same wage rate. To deal with price pass-through in the labour market, we will assume that a share of workers in Scotland are "non-local". Non-local workers are such since they receive the same wage set in the rUK, as opposed to "local" workers whose wage is set in line with Scottish labour market idiosyncratic conditions.

The oil sector

As previously discussed, we are interested in modeling the tax receipts generated from oil. These are raised by the government in Westminster mainly by taxing the profits of companies operating in the North Sea. At the same time, we will not deal with oil price formation, optimal resources extraction or production technology. Accordingly, we will approach the modeling of oil through a few exogenous processes governing the dynamics of key oil variables as in Bodenstein et al. (2011), e.g. price, marginal costs etc. These processes will in turn govern the generation of tax receipt we are interested in. Since most of oil companies operating in the North Sea are foreign, we will assume that oil fields are drilled by foreign companies. Therefore, in our model oil tax receipts will enter the income component of the UK primary balance. As discussed in the introduction, such an approach to the design of the oil sector is admittedly stylized. Indeed, while it captures the rent generated by oil and its impact over public finances, it abstracts from the effect - quantitatively relevant - that oil price shocks are likely to have on the private sector in Scotland. Such a simplifying assumption is purely made to limit the scale and the complexity of the model, which is, in its current formulation, already significant.

Migration

Our model does not explicitly account for migration within the UK and with rest of the world. We acknowledge that fiscal policy changes, particularly those producing divergent fiscal regimes within the UK, might trigger a migration response. Especially considering the large degrees of social and economic integration within a country such as the United Kingdom. At the same time, there a number of reasons that motivate this choice. Firstly, migration is typically not a modeled feature in DSGE economies and its introduction is not straightforward. Especially given the need of limiting the size and the complexity of the model, which is already significant. Secondly, as observed by Mendoza et al. (2014), migration responses can still be partly captured by the response of labour supply to changes in fiscal policy. In more detail, an increase in income tax will generate incentives for households to substitute labour with leisure. Such a reduction in aggregate hours worked will, in turn, map - at the micro level- into phenomena such as a decrease in labour force participation or migration from one country to the other. However, since such mapping is not specified by the model, this results in a reduced form representation of the phenomena[17]. Thirdly, migration statistics currently available are very limited[18], for they do not cover any period involving income tax changes in Scotland with respect to the rUK. Our simplifying assumption over migration might be revised in future once newer data become available; meantime, it remains a limitation of the model.

[17]As such, it involves limitations such as the lack of model predictions on the exact extent of one response with respect to the other, e.g. migration response as opposed to, say, a reduction of labour force participation.

[18]Available series from ONS are provided on an annual basis and only cover the time-span 2001-2016.

2.3 The model

Below we outline the model we developed for the Scottish economy and the rest of the UK. As discussed, we built our framework using Rabanal and Tuesta (2010)'s model as a starting point. Such model is then amended to appropriately account for the economic linkages between Scotland and the rUK, as well as extended to allow for a number of features which are relevant for the policy exercises we aim to run in future.

2.3.1 The economic environment

The model includes three countries of different size: Scotland, the rest of the UK (rUK, henceforth) and the rest of the world (ROW, henceforth). The first two countries are endogenous, i.e. DSGE economies, whereas the ROW is modeled as an exogenous VAR. In particular, Scotland and the rUK economy are (quasi) symmetric blocks which together form the UK, a small open economy with international linkages with the ROW. We will refer to Scotland as the 'Home' country (or simply 'H') and to the rUK as the 'Foreign' country (or simply 'F'). All economies in our model are linked to each other by the means of trade in intermediate goods, indexed by $h \in [0, 1]$ in the home country, $f \in [0, 1]$ in the foreign and row in the rest of the world. In the DSGE economies, imported and domestically produced intermediate goods are then combined to produce a final good which is used for consumption, investment, and government spending. In what follows we present the problem for households, intermediate good producers, and final good producers in Scotland. The expressions for the rUK are analogous unless otherwise stated. We use the convention that variables and parameters with an asterisk denote the rUK counterparts, whereas a double asterisk will stand for the ROW. Also, variables for Scotland and the rUK are expressed in per capita terms and need to be weighted by the respective population shares, ϖ and $1 - \varpi$, when adding them up to produce UK aggregates.

2.3.2 Households

Households utility

In Scotland and in the rUK there are a continuum of infinitely lived households in the unit interval that obtain utility from consuming the final good and disutility from supplying hours of labour. In the home country households are indexed by $j \in [0,1]$ and their lifetime utility function is:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t D_{c,t} \left[log\left(C_t^j - bC_{t-1}\right) - D_{n,t} \frac{\left(N_t^j\right)^{1+\eta}}{1+\eta} \right] \tag{2.1}$$

the utility function displays external habit formation, $b \in [0,1]$ denotes the importance of the habit stock given by the last period aggregate consumption C_{t-1}. $\eta > 0$ is the inverse elasticity of labour supply with respect to the real wage; whereas, N_t^j, is the labour supply of the agent. $D_{c,t}$ and $D_{n,t}$ denote, respectively, inter-temporal and intra-temporal preference shocks. These shocks evolve as follows:

$$log\left(D_{c,t}\right) = \rho_c log\left(D_{c,t-1}\right) + \epsilon_t^{c,d} \tag{2.2}$$

$$log\left(D_{n,t}\right) = \rho_n log\left(D_{n,t-1}\right) + \epsilon_t^{n,d} \tag{2.3}$$

Households budget constraint

Markets are complete within each country and incomplete at the international (and UK) level. We allow for two type of assets: the first, B_t, which is issued by the UK government and costs the inverse of the rUK interest rate; the other, D_t, which is instead traded internationally and denominated in the ROW currency. We do not allow for a within-country asset as, given the complete market assumption, this asset would be redundant. The budget constraint of home-country household is given by:

$$\begin{aligned} &P_t\left(\left(1+\tau_t^c\right)C_t^j + I_t^j\right) + \frac{B_t^j}{R_t^*\Psi\left(\frac{B_t}{Y_tP_t}\right)} + \frac{S_tD_t^j}{R_t^{**}\Psi\left(\frac{S_tD_t}{Y_tP_t}\right)} + P_t\Upsilon_t = \\ &B_{t-1}^j + S_tD_{t-1}^j + \left(1-\tau_t^l\right)W_t^jN_t^j + \left(1-\tau_t^k\right)P_tR_t^km_t^jK_{t-1}^j + \Pi_t^j + T_t + \xi_t^j \end{aligned} \tag{2.4}$$

where B_t^j and D_t^j denote holdings of the UK and ROW bonds, whereas R_t^* and R_t^{**} are, respectively, the rUK and the ROW gross nominal interest rate. The households face an insurance cost to secure the unit return over the UK and ROW bond which is captured by the function $\Psi\left(\cdot\right)$ and it is increasing in the amount of asset held[19]. Since this function depends on the aggregate real holding of the asset, it is taken as given by individuals[20]. S_t is the exchange rate expressed in units of domestic currency needed to buy one unit of ROW currency[21]. P_t is the price of the final good and, ξ_t^j, denotes the pay-off from engaging in the trade of state-contingent securities. T_t are government transfers, whereas Υ_t represents the lump-sum tax used by the Westminster government to balance its budget in the long-run. Consumers obtain labour income from supplying labour to intermediate goods producers, for which they receive a nominal wage, W_t^j; furthermore they receive profits, Π_t^j, from intermediate and wholesale final goods producers as well as the financial intermediary which they own. The model includes sticky wages, and hence the wage received by each household is specific to that household and depends on the last time the wage was re-optimized. However, the assumption of complete markets within each country allows us to separate the consumption/saving decisions of the household from their labour supply decision. Furthermore, households rent capital, K_{t-1}^j, to the intermediate good firms at a real rental rate of R_t^k. Capital utilization, $0 \leq m_t^j \leq 1$, can be varied across periods; the more intense is the capacity utilization (i.e. the higher m_t^j) the higher is the rate of capital depreciation $\delta\left(m_t^j\right)$. Capital accumulation dynamics are given by the following expression:

$$K_t^j = \left(1 - \delta\left(m_t^j\right)\right) K_{t-1}^j + V_t \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right] I_t^j \tag{2.5}$$

where, I_t indicates investment, whereas $S(\cdot)$ represents the adjustment cost function which is increasing and convex (i.e S'(), S"()>0)[22]. The $S(\cdot)$ function summarizes the technology that transforms current and past investment into installed capital. This expression also includes an investment-specific technology shock, V_t, that evolves according to:

$$log\left(V_t\right) = \rho_v log\left(V_{t-1}\right) + \epsilon_t^v \tag{2.6}$$

Finally, following a standard convention, we define:

[19]This cost induces stationarity in the assets position of Scotland and rUK

[20]The financial intermediary faces no production costs and it is assumed to return its profits to the household.

[21]Under the UK monetary union, this is equal for Scotland and rUK.

[22]In steady state it will be the case that $\hat{S} = \hat{S'} = 0$ and $\hat{S''} > 0$.

$$\delta\left(m_t^j\right) = \chi_0 \frac{m_t^{j\chi_1}}{\chi_1} \tag{2.7}$$

Household consumption/saving decision

When allocating their income among consumption, investment and saving alternatives (i.e. domestic and foreign bonds), households face the following program:

$$\begin{aligned}
&\arg \max_{\{C_t, I_t, B_t, D_t, K_t, m_t\}_{t=0}^{\infty}} && \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t D_{c,t} \left[log\left(C_t^j - bC_{t-1}\right) - D_{n,t} \frac{\left(N_t^j\right)^{1+\eta}}{1+\eta} \right] \\
&\text{subject to} && (1)\ B_{t-1}^j + S_t D_{t-1}^j + \left(1-\tau_t^l\right) W_t^j N_t^j + \left(1-\tau_t^k\right) P_t R_t^k m_t^j K_{t-1}^j - P_t \Upsilon_t + \\
& && \Pi_t^j + \xi_t^j = P_t \left(\left(1+\tau_t^c\right) C_t^j + I_t^j\right) + \frac{B_t^j}{R_t^* \Psi\left(\frac{B_t}{Y_t P_t}\right)} + \frac{S_t {D_t}^j}{R_t^{**} \Psi\left(\frac{S_t D_t}{Y_t P_t}\right)} - T_t \\
& && (2)\ K_t^j = \left(1-\delta\left(m_t^j\right)\right) K_{t-1}^j + V_t \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right) I_t^j
\end{aligned} \tag{2.8}$$

Taking FOCs and substituting away Lagrangian multipliers we get the following set of conditions[23]:
the Euler equation representing the trade-off between today's and tomorrow's consumption,

$$1 = R_t^* \Psi\left(\frac{B_t}{Y_t P_t}\right) \beta \mathbb{E}_t \left(\frac{C_t - bC_{t-1}}{C_{t+1} - bC_t} \frac{D_{c,t+1}}{D_{c,t}} \frac{P_t}{P_{t+1}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) \tag{2.9}$$

the choice of capital determining the value of installed capital Q_t[24],

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_t - bC_{t-1}}{C_{t+1} - bC_t} \frac{D_{c,t+1}}{D_{c,t}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) \left[\left(1-\tau_{t+1}^k\right) R_{t+1}^k m_{t+1} + Q_{t+1}\left(1-\delta\left(m_{t+1}\right)\right)\right] \right\} \tag{2.10}$$

[23]note that, since perfect risk sharing is assumed, there is a representative agent and we can drop the j from the agent choice since all agents are identical and face the same marginal utility of consumption.

[24]Q_t can be interpreted as the shadow price of investment in terms of the consumption good.

and its capacity utilization,

$$\left(1-\tau_t^k\right) R_t^k = Q_t \delta'\left(m_t\right) \tag{2.11}$$

the investment decision,

$$\begin{aligned} &1 - Q_t V_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] = \\ &\beta \mathbb{E}_t\left(\frac{C_t - bC_{t-1}}{C_{t+1} - bC_t} \frac{D_{c,t+1}}{D_{c,t}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) Q_{t+1} V_{t+1}\left[\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \end{aligned} \tag{2.12}$$

the acquisition of the foreign bond,

$$1 = \beta R_t^{**} \Psi\left(\frac{S_t D_t}{Y_t P_t}\right) \mathbb{E}_t\left(\frac{C_t - bC_{t-1}}{C_{t+1} - bC_t} \frac{D_{c,t+1}}{D_{c,t}} \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right). \tag{2.13}$$

Furthermore, we define the real exchange rate with respect to the ROW, as the ratio between price indexes expressed in common currency:

$$RER_t^{H,row} = \frac{S_t P_t^{**}}{P_t} \tag{2.14}$$

and similarly, we define the real exchange rate with respect to the rUK as:

$$RER_t^{H,F} = \frac{P_t^*}{P_t} \tag{2.15}$$

The wage decision

By assumption any household is a monopoly supplier of a differentiated labour service, N_t^j. The household sells this service to a competitive firm that transforms it into an aggregate labour input that is used by the intermediate goods producers. Thus, one effective unit of labour that an intermediate goods producer firm, h, uses for production is given by:

$$N_t(h) = \left[\int_0^1 \left(N_t^j(h)\right)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{2.16}$$

Labour packer profit maximization, gives rise to the following demand schedule:

$$N_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\epsilon_w} N_t \tag{2.17}$$

and from the zero-profit condition, we obtain the following wage index:

$$W_t = \left[\int_0^1 \left(W_t^j\right)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}} \tag{2.18}$$

In the home country (i.e. Scotland), households belong to two categories: the local workers, in proportion θ^L, and the non-local workers, in proportion $1-\theta^L$. Local workers (whose wage index is W^L) have their wages set according to home labor market dynamics; we can imagine them to be workers of local SMEs. Non-local workers (whose wage index is W^{NL}) instead are those that, despite being employed in Scotland, have their salary set according to rUK wages; we can imagine them to be employed in public or private sector UK-wide organizations. Local workers set wages in a staggered way with Calvo pricing restriction: in each period, only a fraction of these households, $1 - \theta_W$, can re-optimize their nominal wage whereas the remaining, θ_W, will index their past wage to the previous period local-wage inflation (i.e, $\frac{W_t^L}{W_{t-1}^L}$); the extent of such an indexation is regulated by the parameter, λ_w[25]. Non-local workers, instead, will earn in any period the rUK wage index, i.e. $W^{NL} = W^*$. Since the foreign country (i.e. rUK) is large with respect to the home economy, we assume that its wages are not influenced by Scottish labour market dynamics and therefore we let its workers to be all locals.

Consider a local worker resetting its wage in period t, and let $\hat{W}_t$ the newly set wage. The household will choose $\hat{W}_t$ by solving:

[25]When $\lambda_w = 1$, indexation is perfect and non-re-optimizing workers see their salary growing at the same rate of wage inflation. When $\lambda_w = 0$, no indexation takes place.

$$\underset{\widehat{W}_t}{\text{argmax}} \quad \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k D_{c,t+k} \left[log\left(C^j_{t+k} - bC_{t+k-1}\right) - D_{n,t+k} \frac{\left(N^j_{t+k|t}\right)^{1+\eta}}{1+\eta} \right]$$

$$\begin{aligned} \text{s.t.} \quad & (1)\ P_{t+k}\left((1+\tau^c_t)\, C^j_{t+k} + I^j_{t+k}\right) + \frac{B^j_{t+k}}{R^*_{t+k}\Psi\left(\frac{B_{t+k}}{Y_{t+k}P_{t+k}}\right)} + \frac{S_{t+k}D^j_{t+k}}{R^{**}_{t+k}\Psi\left(\frac{S_{t+k}D_{t+k}}{Y_{t+k}P_{t+k}}\right)} - T_{t+k} + \varpi P_t \Upsilon_t \\ & = B^j_{t+k-1} + S_{t+k}D^j_{t+k-1} + \left(1-\tau^l_t\right)\widehat{W}_t N^j_{t+k|t} + \left(1-\tau^k_t\right) P_{t+k}R^k_{t+k}m^j_{t+k}K^j_{t+k-1} + \Pi^j_{t+k} + \xi^j_{t+k} \\ & (2)\ N^j_{t+k|t} = \left[\frac{\widehat{W}_t}{W_{t+k}}\left(\frac{W^L_{t+k-1}}{W^L_{t-1}}\right)^{\lambda_w}\right]^{-\epsilon_w} N_{t+k} \end{aligned} \tag{2.19}$$

where $N_{t+k|t}$ denotes labor supply in period $t+k$ of a household that last reset its wage in period t. Taking FOCs and re-arranging we obtain the following wage-setting equation:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{N_{t+k|t}D_{c,t+k}}{C_{t+k} - bC_{t+k-1}} \left\{ \left(\frac{1-\tau^l}{1+\tau^c}\right) \frac{\widehat{W}_t}{P_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} D_{n,t+k} N^{\eta}_{t+k|t} \left(C_{t+k} - bC_{t+k-1}\right) \right\} = 0 \tag{2.20}$$

where $N_{t+k|t} = \left(\frac{\widehat{W}_t}{W_{t+k}}\left(\frac{W^L_{t+k-1}}{W^L_{t-1}}\right)^{\lambda_w}\right)^{-\epsilon_w} N_{t+k}$. To be noted that we have removed the j subscript because, in presence of complete market (i.e. complete risk sharing), every agent is facing the same marginal utility of consumption in any period. Therefore, given the opportunity of re-optimizing, it is choosing the same wage (i.e. there is a representative worker).

Consistently with our discussion, we define local workers and non-local workers wage indices as follows:

$$W^L_t = (1-\theta^w_t)\,\widehat{W}_t + \theta^w_t W^L_{t-1}\left(\frac{W^L_{t-1}}{W^L_{t-2}}\right)^{\lambda_w} \tag{2.21}$$

$$W^{NL}_t = W^*_t \tag{2.22}$$

and derive the aggregate wage index, W_t, in Scotland:

$$
\begin{aligned}
W_t = &\left[\int_0^1 \left(W_t^j\right)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}} = \\
&\left[\int_0^{1-\theta^L} \left(W_t^j\right)^{1-\epsilon_w} dj + \int_{1-\theta^L}^1 \left(W_t^j\right)^{1-\epsilon_w} dj\right]^{\frac{1}{1-\epsilon_w}} = \\
&\left[\left(1-\theta^L\right)\left(W_t^{NL}\right)^{1-\epsilon_w} + \theta^L \left(W_t^L\right)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} = \\
&\left[\left(1-\theta^L\right)\left(W_t^*\right)^{1-\epsilon_w} + \theta^L \left(\int_0^{1-\theta^w} W_t^{L,j} dj + \int_{1-\theta^w}^1 W_t^{L,j} dj\right)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} = \\
&\left[\left(1-\theta^L\right)\left(W_t^*\right)^{1-\epsilon_w} + \theta^L \left(\left(1-\theta^w\right)\hat{W}_t + \theta^w W_{t-1}^L \left(\frac{W_{t-1}^L}{W_{t-2}^L}\right)^{\lambda_w}\right)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}
\end{aligned}
\tag{2.23}
$$

whereas in the rUK the wage index will simply read:

$$
W_t^* = \left[\int_0^1 \left(W^{j*}_t\right)^{1-\epsilon_w^*} dj\right]^{\frac{1}{1-\epsilon_w^*}} = \left[\left(1-\theta_w^*\right)\widehat{W}_t^{*1-\epsilon_w^*} + \theta_w^* \left(W_{t-1}^* \left(\frac{W_{t-1}^*}{W_{t-2}^*}\right)^{\lambda_w^*}\right)^{1-\epsilon_w^*}\right]^{\frac{1}{1-\epsilon_w^*}}
\tag{2.24}
$$

2.3.3 Firms

Final good producers

It is assumed that the production of the final good is performed in two stages. Firstly, a continuum of wholesale firms purchase a composite of intermediate home goods, $Y_{H,t}$, and a composite of intermediate foreign-produced goods, $Y_{F,t}$ and $Y_{row,t}$, to produce a differentiated final good product, $Y_t\left(i\right)$. Secondly, retail firms purchase the differentiated final goods from wholesale firms, and produce a homogeneous final good, Y_t, that is used for consumption, investment and government spending. Both firms operate under flexible prices, but the wholesale firms are assumed to be monopolistically competitive whereas the retail sector is perfectly competitive and inhabited by a representative firm. Furthermore, the price of the final good, P_t, can fluctuate over its real marginal cost due to the presence of exogenous mark-up shocks, $\mu_{f,t} = \frac{\epsilon_{f,t}}{\epsilon_{f,t}-1}$. In the following we illustrate retail and wholesale firms' problems.

Retail firm

The retail firm acts as a competitive good packer, with the following technology:

$$Y_t = \left[\int_0^1 Y_t\left(i\right)^{\frac{\epsilon_{f,t}-1}{\epsilon_{f,t}}} \right]^{\frac{\epsilon_{f,t}}{\epsilon_{f,t}-1}} \tag{2.25}$$

which implies the following demand function for the wholesale good i:

$$Y_t\left(i\right) = \left[\frac{P_t\left(i\right)}{P_t} \right]^{-\epsilon_{f,t}} Y_t \tag{2.26}$$

Wholesale firms

The continuum of wholesale firms purchase a composite of intermediate home goods, $Y_{H,t}$, and a composite of intermediate foreign-produced goods, $Y_{F,t}$ and $Y_{row,t}$, to produce a differentiated final good product, $Y_t\left(i\right)$, in the unit interval:

$$Y_t\left(i\right) = \left[\omega_1^{H\frac{1}{\theta}} Y_{H,t}^{\frac{(\theta-1)}{\theta}} + \omega_2^{H\frac{1}{\theta}} Y_{F,t}^{\frac{(\theta-1)}{\theta}} + \left(1 - \omega_1^H - \omega_2^H\right)^{\frac{1}{\theta}} Y_{row,t}^{\frac{(\theta-1)}{\theta}} \right]^{\frac{\theta}{(\theta-1)}} \tag{2.27}$$

Where ω_1 denotes the weight of Scottish-produced goods used in the production of the final good, ω_2 the weight of the rUK-produced good and $1 - \omega_1 - \omega_2$ the weight of ROW-produced goods; θ, instead, denotes the elasticity of substitution between domestic and imported goods. From firm's cost minimization we obtain the following schedules:

$$Y_{H,t} = \left(\frac{P_{H,t}}{MC_t^y} \right)^{-\theta} \omega_1^H Y_t\left(i\right) \tag{2.28}$$

$$Y_{F,t} = \left(\frac{P_{F,t}}{MC_t^y} \right)^{-\theta} \omega_2^H Y_t\left(i\right) \tag{2.29}$$

$$Y_{row,t} = \left(\frac{S_t P_{row,t}}{MC_t^y} \right)^{-\theta} \left(1 - \omega_1^H - \omega_2^H\right) Y_t\left(i\right) \tag{2.30}$$

where $MC_t^y = \left[\omega_1^H \left(P_{H,t}\right)^{1-\theta} + \omega_2^H \left(P_{F,t}\right)^{1-\theta} + \left(1-\omega_1^H-\omega_2^H\right)\left(S_t P_{row,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$ is the marginal cost. Note that foreign produced good prices are expressed in the producer currency; this is due to the Producer Currency Pricing assumption we make for the exporting firms, which is discussed below.

From the profit maximization problem, under optimized marginal costs, we can then derive the price-setting equation. In particular, any wholesale firm will face the following program:

$$\begin{aligned} &\underset{P_t(i)}{\operatorname{argmax}} \quad \left(P_t(i) - MC_t^y\right) Y_t(i) \\ &\text{subject to} \quad Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon_{f,t}} Y_t \end{aligned} \tag{2.31}$$

taking FOCs, we obtain the following price-setting equation:

$$P_t(i) = \frac{\epsilon_{f,t}}{\epsilon_{f,t}-1} MC_t^y = \mu_{f,t} MC_t^y \tag{2.32}$$

where $\mu_{f,t} = \frac{\epsilon_{f,t}}{\epsilon_{f,t}-1}$ is a time varying mark-up whose process takes the following form:

$$log\left(\mu_{f,t}\right) = log\left(\frac{\epsilon_{f,t}}{\epsilon_{f,t}-1}\right) = log\left(\frac{\epsilon_f}{\epsilon_f-1}\right) + \epsilon_t^{\mu^f} = \log\left(\mu_f\right) + \epsilon_t^{\mu^f} \tag{2.33}$$

where $\epsilon_t^{\mu^f}$ is the mark-up shock.

Finally, intermediate goods involved in wholesale firms' production are in turn an aggregate of heterogeneous intermediate goods produced by the monopolitistically competitive sector described below and aggregated by a competitive good packer according to the following technology:

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(h)^{\frac{\epsilon_p-1}{\epsilon_p}} dh\right]^{\frac{\epsilon_p}{\epsilon_p-1}} \tag{2.34}$$

$$Y_{F,t} = \left[\int_0^1 Y_{F,t}(f)^{\frac{\epsilon_p-1}{\epsilon_p}} df\right]^{\frac{\epsilon_p}{\epsilon_p-1}} \tag{2.35}$$

$$Y_{row,t} = \left[\int_0^1 Y_{row,t}(w)^{\frac{\epsilon_p-1}{\epsilon_p}} dw\right]^{\frac{\epsilon_p}{\epsilon_p-1}} \tag{2.36}$$

from which we can derive the following demand schedules:

$$Y_{H,t}(h) = \left[\frac{P_{H,t}(h)}{P_{H,t}}\right]^{-\epsilon_p} Y_{H,t} \tag{2.37}$$

$$Y_{F,t}(f) = \left[\frac{P_{F,t}(f)}{P_{F,t}}\right]^{-\epsilon_p} Y_{F,t} \tag{2.38}$$

$$Y_{row,t}(w) = \left[\frac{P_{row,t}(w)}{P_{row,t}}\right]^{-\epsilon_p} Y_{row,t} \tag{2.39}$$

and price indexes (obtained by imposing zero profit conditions):

$$P_{H,t} = \left[\int_0^1 P_{H,t}(h)^{1-\epsilon_p} dh\right]^{\frac{1}{1-\epsilon_p}} \tag{2.40}$$

$$P_{F,t} = \left[\int_0^1 P_{F,t}(f)^{1-\epsilon_p} df\right]^{\frac{1}{1-\epsilon_p}} \tag{2.41}$$

$$P_{row,t} = \left[\int_0^1 P_{row,t}(w)^{1-\epsilon_p} dw\right]^{\frac{1}{1-\epsilon_p}} \tag{2.42}$$

where $\epsilon_p > 1$ denotes the elasticity of substitution between types of intermediates goods. Note that $Y_{H,t}(h)$ denotes home intermediate goods whereas $Y_{F,t}(f)$ and $Y_{row,t}(w)$ denote imported intermediates goods used by a home wholesale firm.

Intermediate good firms

The continuum of intermediate firms operate in a monopolistically competitive market, producing differentiated goods for domestic consumption and export, pricing *à* la Calvo with partial indexation. They feature a Cobb-Douglas production technology over capital services (i.e. $m_t K_{t-1}$) and labour (N_t), which reads:

$$Y_{H,t}(h) + \frac{1-\varpi}{\varpi} Y^{*}_{H,t}(h) + Y^{**}_{H,t}(h) = \left[m_t K_{t-1}(h)\right]^{\alpha} \left[A_t N_t(h) X_t\right]^{1-\alpha} \tag{2.43}$$

where $Y_{H,t}$ represents the goods produced for the home market (Scotland) whereas, $Y^{*}_{H,t}$, represents those produced for the rUK and, $Y^{**}_{H,t}$, those produced for the ROW. The production function features two different types of labour-augmenting technical progress. These include a stationary process,

A_t, capturing the idiosyncratic technological progress in each country; as well as a UK-wide non-stationary technological process, X_t, which is common to Scotland and the rUK. They evolve according to the following laws of motion:

$$log\left(A_t\right) = \rho^a log\left(A_{t-1}\right) + \epsilon_t^a \tag{2.44}$$

and

$$log\left(X_t\right) = log\left(X_{t-1}\right) + \epsilon_t^x \tag{2.45}$$

We begin by considering the choice of inputs that minimize costs for firm h, having in mind that those conditions will be symmetrical for all firms in this sector:

$$\begin{aligned} &\min_{N_t(h),K_{t-1}(h)} && TC_t = R_t^k P_t m_t K_{t-1}\left(h\right) + W_t N_t\left(h\right) \\ &\text{s.t} && Y_{H,t}\left(h\right) + \frac{1-\varpi}{\varpi} Y_{H,t}^{*}\left(h\right) + Y_{H,t}^{**}\left(h\right) = \left[m_t K_{t-1}\left(h\right)\right]^{\alpha}\left[A_t N_t\left(h\right) X_t\right]^{1-\alpha} \end{aligned} \tag{2.46}$$

From the problem in (2.46), we can derive firm optimal demand for labour and capital, which underpins the following expression for the optimised real marginal cost MC_t:

$$MC_t = \frac{{R_t^k}^{\alpha}\left(\frac{W_t}{P_t}\right)^{1-\alpha} {A_t}^{\alpha-1}}{{X_t}^{1-\alpha}\left(1-\alpha\right)^{1-\alpha}\alpha^{\alpha}} \tag{2.47}$$

Having derived marginal costs, we can move to the second stage of the intermediate firm's problem which is choosing a price for its destination markets: the domestic and the foreign markets. We assume that intermediate good firms set prices in order to maximize discounted profits subject to a Calvo-type restriction with partial indexation. Since rUK and Scotland share the same currency, there is no difference between producer and local currency pricing regime[26]. Nevertheless, the literature typically further distinguishes between segmented and non-segmented markets: under the former, exporting firms are allowed to charge different prices in home and in foreign markets;

[26]This, clearly, will not hold true for the ROW. However, since ROW is modeled to be exogenous, this aspect will not play any relevant role (e.g the main difference is likely to show up in the correlation between exchange rate and UK export).

under the latter, they are instead forced to apply the same price at home and abroad. We assume non-segmented markets in our model, accordingly each firm sets a unique price for all the markets in which it operates (both domestic and foreign). The reason for this assumption is to allow for a part of Scottish prices not to adjust in response to asymmetric shocks to the Scottish economy. We can imagine this to be the result of companies selling and pricing UK-wide[27]. Similarly, we assume that UK companies selling to the ROW set prices in their home currency (PCP) and sell their goods at home and abroad at the same price. Admittedly, as far as the ROW is concerned, this assumption is partially counter-factual[28]. This is operated for the purpose of maintaining the pricing model symmetrical, at this stage. Moreover, we do not anticipate this assumption to significantly impact the analysis our model is meant to address, i.e. fiscal federalism within the UK, for the ROW mainly constitutes a source of exogenous demand shock in our model[29].

In each period a fraction $1-\theta_H$ of firms will change optimally their price, whereas the remaining will adjust theirs according to an indexation rule:

$$\frac{P_{H,t}\left(h\right)}{P_{H,t-1}\left(h\right)} = \left(\Pi_{H,t-1}\right)^{\lambda_H} \tag{2.48}$$

where $0 < \lambda_H < 1$. For each intermediate good firm, the demand schedule can be derived from equation 2.37, assuming symmetry with the rUK and the ROW. Specifically, the demand at time $t+k$ for an intermediate good firm h which last reset its price at t will be:

[27] This mechanism will also work in the opposite direction. A part of rUK prices will respond to Scottish dynamics rather than to rUK ones. However, the magnitude of this effect will be negligible given the small size of the Scottish economy relative to the rUK one.

[28] Note that PCP and non-segmented markets assumption will generate a form of 'law of one price', as individual good h will be sold at the same price in both countries. However, since the final good is produced using different combinations of intermediate goods in all countries (home bias), and because of idiosyncratic shocks to the mark-up rate of wholesale firms, the price of the consumption and investment basket will differ across countries.

[29] Furthermore, looking at Rabanal and Tuesta (2010), who assumes instead Local Currency Pricing in their two-country model of the EU and US, it can be appreciated how our framework can be easily extended in future to relax PCP assumption. Specifically, this can be done by a simple revision of the intermediate good firms price-setting problem (i.e. by allowing for a price-setting equation for each market in which domestic firms operate).

$$Y_{H,t+k|t}(h) = \left[\frac{P_{H,t+k|t}(h)}{P_{H,t+k}}\right]^{-\epsilon_p} Y_{H,t+k} = \left[\frac{P_{H,t}(h)}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right]^{-\epsilon_p} Y_{H,t+k} \tag{2.49}$$

With this at hand, we are ready to describe the intermediate firm's profit-maximization problem:

$$\begin{aligned}
\arg\max_{P_{H,t}(h)} E_t \quad & \sum_{k=0}^{\infty} (\beta\theta_h)^k \Lambda_{t,t+k}\left[\frac{P_{H,t}(h)}{P_{t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H} - MC_{t+k}\right] \times \\
& \left(Y_{H,t+k|t}(h) + \frac{1-\varpi}{\varpi} Y^*_{H,t+k|t} + Y^{**}_{H,t+k|k}\right) \\
\text{subject to} \quad & Y_{H,t+k|k}(h) = \left[\frac{P_{H,t}(h)}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right]^{-\epsilon_p} Y_{H,t+k} \\
& Y^*_{H,t+k|k}(h) = \left[\frac{P_{H,t}(h)}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right]^{-\epsilon_p} Y^*_{H,t+k} \\
& Y^{**}_{H,t+k|k}(h) = \left[\frac{P_{H,t}(h)}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right]^{-\epsilon_p} Y^{**}_{H,t+k}
\end{aligned} \tag{2.50}$$

Taking the FOC, we can derive the following price-setting equation:

$$\frac{\hat{P}_{H,t}(h)}{P_{H,t}} = \frac{\epsilon_p}{\epsilon_p - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \Lambda_{t,t+k} MC_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right)^{-\epsilon_p}\left(Y_{H,t+k} + \frac{1-\varpi}{\varpi} Y^*_{H,t+k} + Y^{**}_{H,t+k}\right)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \Lambda_{t,t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{1-\epsilon_p}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H(1-\epsilon_p)} \frac{P_{H,t+k}}{P_{t+k}}\left(Y_{H,t+k} + \frac{1-\varpi}{\varpi} Y^*_{H,t+k} + Y^{**}_{H,t+k}\right)} \tag{2.51}$$

Furthermore, using equations 2.40 and 2.41, we can derive the intermediate goods price index in Scotland:

$$P_{H,t} = \left[(1-\theta_H)\hat{P}_{H,t}^{1-\epsilon_p} + \theta_H\left(P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\lambda_H}\right)^{1-\epsilon_p}\right]^{\frac{1}{1-\epsilon_p}} \tag{2.52}$$

2.3.4 Oil sector

We allow for a stylized oil sector generating the revenues accruing to the UK government in the form of taxes on profits from oil production. In more detail, we imagine oil reserves being exploited by a number of ROW firms which then pay a tax on their profits to the government in Westminster. These profits depend on oil price, extracted quantity and marginal costs which are all modeled as an $AR(1)$ processes in logs:

$$\log \frac{P_t^o}{P_t^*} = \rho^{po} \log \frac{P_{t-1}^o}{P_{t-1}^*} + (1 - \rho^{po}) \log \overline{p}^o + \epsilon_t^{po} \tag{2.53}$$

$$\log \frac{Q_t^o}{X_t} = \rho^{qo} \log \frac{Q_{t-1}^o}{X_{t-1}} + (1 - \rho^{qo}) \log \overline{Q}^o + \epsilon_t^{qo} \tag{2.54}$$

and

$$\log \frac{MC_t^o}{P_t^*} = \rho^{mco} \log \frac{MC_{t-1}^o}{P_{t-1}^*} + (1 - \rho^{mco}) \log \overline{mc}^o + \epsilon_t^{mco} \tag{2.55}$$

This simple modeling strategy is close in spirit to the one of Bodenstein et al. (2011)[30]. This crucially means that, differently from other models explicitly dealing with the energy sector in the UK (e.g. Aminu (2018)), we abstract from the feedback effects that oil has on the private sector. Furthermore, we abstract from the resource (mis-)allocation problem following booms and windfalls from oil volumes and prices, which are studied by the New Dutch Disease literature (see among others: Benkhodja (2011) and Allegret and Benkhodja (2011)). In the context of our model, oil will simply be an exogenous rent in the hand of the Westminster government.

2.3.5 Fiscal sector

The design of the fiscal sector in Scotland and in the rUK is meant to represent the historical arrangements within the UK. Accordingly, we assume all tax revenues generated in the UK to accrue to the government in Westminster. Those revenues are then in turn employed to finance public expenditure in the rUK, whether devolved or not, together with non-devolved expenditure and transfers to Scotland. Scotland will in turn use transfers from Westminster to finance its devolved expenditure as well as transfers to households. When constructing the public sector balance sheet, we have to

[30] However, since we do not explicitly model an international market for oil, we let oil price to be exogenous too.

account for Scotland and the rUK populations differing in size.

Westminster

The UK government balance sheet will read:

$$\frac{B_t^{UK}}{R_t^*} + \varpi\tau_t^k m_t P_t R_t^k K_{t-1} + (1-\varpi)\,\tau_t^k m_t^* P_t^* R_t^{k*} K_{t-1}^* + \varpi\tau_t^l W_t N_t + \\ (1-\varpi)\,\tau_t^l W_t^* N_t^* + \varpi\tau_t^c P_t C_t + (1-\varpi)\,\tau_t^c P_t^* C_t^* + TO_t + P_t\Upsilon_t = B_{t-1}^{UK} + (1-\varpi)\,P_t^* Gdev_t^* + \\ \varpi P_t Gnodev_t + (1-\varpi)\,P_t^* Gnodev_t^* + (1-\varpi)\,P_t^* T_t^* + \varpi P_t Tsc_t \tag{2.56}$$

On the left hand side, B_t^{uk} represents the UK bond held by household in Scotland (B_t) and in the rUK (B_t^*), whereas ϖ and $(1-\varpi)$ represent Scotland and rUK population weights. TO_t, instead, are the tax revenues from oil. On the right hand side, $Gdev_t^*$ represents devolved expenditures in the rUK, while $Gnodev_t^*, Gnodev_t$ the non-devolved ones in, respectively, the rUK and Scotland, Tsc_t are transfers to Scotland and T_t^* transfers to the rUK households. Oil revenues are derived from a flat rate of tax, τ_t^o, over oil profits:

$$TO_t = (P_t^o - MC_t^o)\,Q_t^o \tau_t^o \tag{2.57}$$

Finally Υ_t represents a lump-sum tax evolving according to:

$$\Upsilon_t = \rho^{\Upsilon}\left(b_t^{UK} - \bar{b}\right) + \left(1-\rho^{\Upsilon}\right)\bar{\Upsilon} \tag{2.58}$$

This lump-sum tax ensures that the Debt-GDP ratio, b_t^{UK}[31], will eventually approach its long-run value $\bar{b}$. As we assume that Lump-sum taxes have zero mean, this condition simply goes down to:

$$\Upsilon_t = \rho^{\Upsilon}\left(b_t^{UK} - \bar{b}\right) \tag{2.59}$$

Holyrood

Scottish government's balance sheet will then simply read:

[31] We define $b_t^{UK} = \frac{B_t^{UK}}{P_t^* Y_t^*}$. Similarly, Scottish households bond holding with respect to (Scottish) GDP will read $b_t = \frac{B_t}{P_t Y_t}$; whereas for rUK households this will read $b_t^* = \frac{B_t^*}{P_t^* Y_t^*}$.

$$Tsc_t = Gdev_t + T_t \tag{2.60}$$

On the LHS, Tsc_t represents total transfers or grants from the UK central government to the Scottish devolved administration. These grants are used to finance Holyrood's devolved expenditure, $Gdev_t$, and transfers to Scottish households, T_t, on the RHS of the equation. As we discuss below, the devolved expenditure in Scotland will evolve endogenously according to the Barnett formula, whereas transfers to households will follow an exogenous process.

Barnett formula

The devolved administration in Scotland receives grants, amounting to Tsc_t, from the UK government which fund its spending. The Barnett formula determines how the largest of these grants -the block grants, which in our model finances the whole of the devolved expenditure $Gdev_t$ - changes from one year to the next[32]. It should be noted that the formula does not determine the total amount of the block grant, just the yearly change. The main idea behind the Barnett formula is to give the same pounds-per-person change in funding for comparable government services in the rUK: for instance, if the funding for education in the rUK increases of £200 per person, the Scottish administration block grant will increase by £200 per person, assuming education is a fully devolved subject. More specifically, the Barnett formula takes the change in the rUK devolved expenditure and applies two factors: the first accounts for the relative size of its population (*population proportion*), whereas the second accounts for the extent to which the specific service is devolved (*comparability percentange*). In our model, this will be represented through the following processes for $Gdev_t$:

$$\overline{Gdev}_t = \overline{Gdev}_{t-1} (1 + gg_t) \tag{2.61}$$

[32]More in detail, UK public expenditure is divided between Departmental Expenditure Limits (DEL) and Annually Managed Expenditure (AME). The former are departmental expenditures responding to strict budget limits (covering, for example, resources and services running costs), whereas the latter involves spent on demand lead areas whose budget is less strict as harder to be forecast (covering for example, tax credit smf public pensions). The Barnett formula determined block grant makes up the majority of the devolved administrations DEL, whereas the AME allocations are provided in separately negotiated grants.

$$gg_t = \frac{Gdev_t^* - Gdev_{t-1}^*}{Gdev_{t-1}^*} * DD_t \tag{2.62}$$

$$DD_t = \frac{Gdev_t^*}{Gdev_t} \tag{2.63}$$

and

$$Gdev_t = \overline{Gdev}_t \exp(\epsilon_t^{Gdev}) \tag{2.64}$$

Where equation 2.61 represents the process for the block grant to Scotland which grows according to a rate, gg_t, consistent with the Barnett formula. Equation 2.62 represents then the Barnett formula itself determining the per-period per-capita growth of the block grant received by Scotland. The latter depends upon the growth of the rUK devolved expenditure per capita[33], $\frac{Gdev_t^* - Gdev_{t-1}^*}{Gdev_{t-1}^*}$, as well as a comparability factor or devolution degree DD_t. Furthermore, equation 2.63 establishes how the comparability factor changes over time: this has been designed so that per-capita devolved expenditure in the rUK and in Scotland converge over time. Note that the rate of convergence is faster in presence of economic growth[34]. Finally, given that our Barnett formula is somehow stylized and aggregated, we allow for stochastic deviations from this rule, ϵ_t^{Gdev}, in equation 2.64. We expect allowing for such stochastic deviations to be useful for the matching data (i.e. when estimating the model).

Public expenditure composition

Total government consumption, both in Scotland and in the rUK, is given by the sum of the devolved and of the non-devolved government spending purchased within each country. Therefore, the definition of government consumption in Scotland reads:

$$G_t = Gdev_t + Gnodev_t \tag{2.65}$$

[33]It is worth noting that this is an approximation. In reality, Scottish devolved expenditure grows in line with the English one. That means, Wales and Northern Ireland spending should be excluded.

[34]To see why this is the case, just notice that $Gdev_t^*$ grows at the rate of technological change.

Furthermore, we define the non-devolved component of public consumption, respectively in Scotland and in the rUK, as follows[35]:

$$Gnodev_t = \varsigma Gnodev_t^{uk} \tag{2.66}$$

$$Gnodev_t^* = \varsigma^* Gnodev_t^{uk} \tag{2.67}$$

where ς and ς^* are the shares of non-devolved expenditure spent in each country (with respect to the UK average), and $Gnodev_t^{uk}$ is total non-devolved spending per-capita in the UK. As such, any time that $\varsigma \neq 1$ we will have that more/less no-devolved spending is purchased in Scotland with respect to rUK (in per-capita basis). Furthermore, aggregation consistency requires that $\varpi\varsigma + (1-\varpi)\varsigma^* = 1$. Finally, the non-devolved expenditure in the UK evolves according to the following $AR(1)$ process in logs:

$$\log\left(Gnodev_t^{uk}\right) = \rho_{gnodev}\log\left(Gnodev_{t-1}^{uk}\right) + (1-\rho_{gnodev})\log\left(\overline{Gnodev}^{uk}\right) + \epsilon_t^{gnodev} \tag{2.68}$$

2.3.6 Monetary policy

Finally, it is assumed that the Bank of England follows a monetary policy rule that targets deviations of domestic CPI inflation and real GDP growth from their steady state values, that are normalized to zero:

$$\frac{R_t^*}{R} = \left(\frac{R_{t-1}^*}{R}\right)^{\varphi_R}\left[\left(\varpi\frac{P_t}{P_{t-1}} + (1-\varpi)\frac{P_t^*}{P_{t-1}^*}\right)^{\varphi_\pi}\left(\varpi\frac{GDP_t}{GDP_{t-1}} + (1-\varpi)\frac{GDP_t^*}{GDP_{t-1}^*}\right)^{\varphi_y}\right]^{1-\varphi_R}\exp\left(\epsilon_t^{m*}\right) \tag{2.69}$$

2.3.7 Market clearing conditions

Final good (national) market:

Market clearing in the final good market requires:

[35] It is worth noting that, a controversial aspect in the accounting of transfers and subsidies within the UK, is the allocation of the non-devolved spending. Indeed, *while the amount spent on the behalf of each country reflects its population share, the amount spent within each country does not have to.*

$$Y_t = C_t + I_t + G_t \tag{2.70}$$

UK government bond market:

The bonds issued by the UK government are held by, both, the Scottish and the rUK households. Market clearing requires, therefore, bond holdings from households (weighted by the population share of each country) to equal the total supply from the UK government:

$$B_t^{uk} = (1 - \varpi)\, B_t^* + \varpi B_t \tag{2.71}$$

International markets:

The evolution of the UK external position is determined by aggregating the trade balance of Scotland (TB_t) and of the rUK (TB_t^*) and then adding the tax paid by ROW firms on oil profits. The latter is, in essence, the result of an 'oil export' from the UK government to the ROW. The UK government lets foreign firms exploiting national oil fields while taxing their profits in return.

$$\frac{S_t D_t^{uk}}{R_t^{**}} - S_t D_{t-1}^{uk} = \varpi TB_t + (1 - \varpi)\, TB_t^* + TO_t \tag{2.72}$$

where

$$TB_t = \frac{1 - \varpi}{\varpi} P_{H,t} Y_{H,t}^* + P_{H,t} Y_{H,t}^{**} - P_{F,t} Y_{F,t} - S_t P_{row,t} Y_{row,t} \tag{2.73}$$

$$TB_t^* = \frac{\varpi}{1 - \varpi} P_{F,t} Y_{F,t} + P_{F,t} Y_{F,t}^{**} - P_{H,t} Y_{H,t}^* - S_t P_{row,t} Y_{row,t}^* \tag{2.74}$$

finally we require the UK external position, e.g. D_t^{uk}, to be consistent with the sum of the Scottish and rUK individual positions. Namely:

$$D_t^{uk} = \varpi D_t + (1 - \varpi)\, D_t^* \tag{2.75}$$

2.3.8 Rest of the world

We let the rest of the world being exogenous and we model it as a VAR(1) in the following variables: output (Y_t^{**}), inflation (π_t^{**}) and interest rate (R_t^{**}),

$$F_{t+1} = AF_t + \epsilon_t^{row} \tag{2.76}$$

where $F_t = [Y_t^{**}, \Pi_t^{**}, R_t^{**}]$

We then link ROW output to Scottish and rUK export by imposing symmetric demand schedules to the ones of the DSGE economies:

$$\frac{Y_{row,t}^{**}}{Y_{F,t}^{**}} = \frac{\omega_1^{row}}{1 - \omega_1^{row} - \omega_2^{row}} \left(\frac{S_t P_{row,t}}{P_{F,t}} \right)^{-\theta} \tag{2.77}$$

$$\frac{Y_{row,t}^{**}}{Y_{H,t}^{**}} = \frac{\omega_1^{row}}{1 - \omega_1^{row} - \omega_2^{row}} \left(\frac{S_t P_{row,t}}{P_{H,t}} \right)^{-\theta} \tag{2.78}$$

$$Y_t^{**} = \left[\omega_1^{row \frac{1}{\theta}} Y_{row,t}^{** \frac{(\theta-1)}{\theta}} + (1 - \omega_1^{row} - \omega_2^{row})^{\frac{1}{\theta}} Y_{F,t}^{** \frac{(\theta-1)}{\theta}} + (1 - \omega_1^{row} - \omega_2^{row})^{\frac{1}{\theta}} Y_{H,t}^{** \frac{(\theta-1)}{\theta}} \right]^{\frac{\theta}{(\theta-1)}} \tag{2.79}$$

Finally (as it is standard in the literature) we assume that ROW exported good price equals ROW CPI

$$P_t^{row} = P_t^{**} \tag{2.80}$$

2.4 Recursive stationary equilibrium conditions

In this section we normalize the model and we re-express it in a form useful to be input in standard software for solution and simulation (e.g. Dynare). We begin by scaling growing variables for the level of technology and transforming the model in real terms. We then rewrite in recursive terms all equations involving infinite forward summations and integrals. Finally, we make explicit functional form assumptions for adjustment costs processes (e.g. investment adjustment costs, bold holding costs etc.). At the end of this section, we then present the system of stationarized, recursive equilibrium conditions.

2.4.1 Technology scaling

Our model assumes that the technological process in the UK, X_t, features a unit root. This implies that real output, consumption, capital, investment, real wages, and the level of government spending inherit the same property and, therefore, they are not stationary in levels. In order to obtain stationarity, we divide them by the level of technology. We denote with "tilde" variables which have been normalized, e.g $\tilde{C}_t = \frac{C_t}{X_t}$. We then define, $g_{x,t} = \frac{X_t}{X_{t-1}}$, as the gross growth rate of technology in the UK.

2.4.2 Real prices and inflation

In order to induce stationarity, we need to re-write the model in real terms. That requires re-expressing nominal prices in terms of relative prices or price growth. For this end, we define the following variables:

$$\frac{P_{H,t}}{P_{H,t-1}} = \Pi_t^H, \frac{\hat{P}_{H,t}}{P_{H,t}} = \hat{p}_{H,t}, \frac{P_{H,t}}{P_t} = p_{H,t}, \frac{P_t}{P_{t-1}} = \Pi_t, RER_t^{H,F} = \frac{P_t^*}{P_t} \quad (2.81)$$

we then replace the corresponding expressions everywhere in the model. Furthermore, when operating such substitutions, the following definitions are to be added:

$$p_{H,t} = \frac{\Pi_t^H}{\Pi_t} p_{H,t-1}, RER_t^{H,F} = \frac{\Pi_t^*}{\Pi_t} RER_{t-1}^{H,F} \quad (2.82)$$

2.4.3 Recursive transformation

For the purpose of inputting the model in solution software (e.g Dynare), equations involving infinite forward summations and integrals need to be transformed in recursive terms. Specifically, that requires to transform the following set of conditions: (i) the price setting equation, (ii) the wage setting equation and (iii) the price dispersion index. We deal with each of them in a separate subsection below.

Price setting

Begin recalling that the price setting equation reads:

$$\frac{\hat{P}_{H,t}}{P_{H,t}}=\frac{\epsilon_p}{\epsilon_p-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}\left(\beta\theta_H\right)^k\Lambda_{t,t+k}MC_{t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H}\right)^{-\epsilon_p}\frac{X_{t+k}}{X_t}\left(\tilde{Y}_{H,t+k}+\frac{1-\varpi}{\varpi}\tilde{Y}^*_{H,t+k}+\tilde{Y}^{**}_{H,t+k}\right)}{\mathbb{E}_t\sum_{k=0}^{\infty}\left(\beta\theta_H\right)^k\Lambda_{t,t+k}\left(\frac{P_{H,t}}{P_{H,t+k}}\right)^{1-\epsilon_p}\left(\frac{P_{H,t+k-1}}{P_{H,t-1}}\right)^{\lambda_H(1-\epsilon_p)}\frac{P_{H,t+k}}{P_{t+k}}\frac{X_{t+k}}{X_t}\left(\tilde{Y}_{H,t+k}+\frac{1-\varpi}{\varpi}\tilde{Y}^*_{H,t+k}+\tilde{Y}^{**}_{H,t+k}\right)} \tag{2.83}$$

Now, just note that the numerator can be expressed as:

$$F^1_t=MC_t\left(\tilde{Y}_{H,t}+\frac{1-\varpi}{\varpi}\tilde{Y}^*_{H,t}+\tilde{Y}^{**}_{H,t}\right)+\beta\theta_H\Lambda_{t,t+1}g_{x,t+1}\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{-\epsilon_p}\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{-\epsilon_p\lambda_H}F^1_{t+1} \tag{2.84}$$

Similarly, the denominator can be expressed as:

$$F^2_t=\left(\tilde{Y}_{H,t}+\frac{1-\varpi}{\varpi}\tilde{Y}^*_{H,t}+\tilde{Y}^{**}_{H,t}\right)\frac{P_{H,t}}{P_t}+\beta\theta_H\Lambda_{t,t+1}g_{x,t+1}\left(\frac{P_{H,t}}{P_{H,t+1}}\right)^{1-\epsilon_p}\left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\lambda_H(1-\epsilon_p)}F^2_{t+1} \tag{2.85}$$

We can therefore re-express equation 2.83 as follows:

$$\frac{\hat{P}_{H,t}}{P_{H,t}}=\frac{\epsilon_p}{(\epsilon_p-1)}\frac{F^1_t}{F^2_t} \tag{2.86}$$

Wage setting

Let us now derive the recursive wage-setting equation. Recall that wage-setting reads:

$$\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\frac{\left(\frac{\tilde{\widehat{W}}_{real,t}}{\tilde{W}_{real,t+k}}\frac{P_t}{P_{t+k}}\left(\frac{\tilde{W}^L_{real,t+k-1}}{\tilde{W}^L_{real,t-1}}\frac{P_{t+k-1}}{P_{t-1}}\right)^{\lambda_w}\right)^{-\epsilon_w}\left[\frac{X_t}{X_{t+k}}\left(\frac{X_{t+k-1}}{X_{t-1}}\right)^{\lambda_w}\right]^{-\epsilon_w}N_{t+k}D_{c,t+k}}{\tilde{C}_{t+k}-b\tilde{C}_{t+k-1}\frac{1}{g_{t+k}}}$$

$$\{\frac{1-\tau^l}{1+\tau^c}\tilde{\widehat{W}}_{real,t}\frac{X_t}{X_{t+k}}\frac{P_t}{P_{t+k}}-\frac{\epsilon_w}{\epsilon_w-1}D_{n,t+k}\left(\frac{\tilde{\widehat{W}}_{real,t}}{\tilde{W}_{real,t+k}}\frac{P_t}{P_{t+k}}\left(\frac{\tilde{W}^L_{real,t+k-1}}{\tilde{W}^L_{real,t-1}}\frac{P_{t+k-1}}{P_{t-1}}\right)^{\lambda_w}\right)^{-\epsilon_w\eta}\times$$

$$\left[\frac{X_t}{X_{t+k}}\left(\frac{X_{t+k-1}}{X_{t-1}}\right)^{\lambda_w}\right]^{-\epsilon_w\eta}N^{\eta}_{t+k}\left(\tilde{C}_{t+k}-b\tilde{C}_{t+k-1}\frac{1}{g_{t+k}}\right)\}=0 \tag{2.87}$$

first note that we can re-write the above as follows:

$$\frac{1-\tau^l}{1+\tau^c}\tilde{\widehat{W}}^{1+\epsilon\eta}_{real,t}=\frac{\epsilon_w}{\epsilon_w-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\tilde{A}^{1+\eta}_{t+k|t}\left(\frac{N_{t+k}}{\tilde{W}^{-\epsilon}_{real,t+k}}\right)^{1+\eta}D_{c,t+k}D_{n,t+k}}{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta_w)^k\frac{\tilde{A}_{t+k|t}\frac{N_{t+k}}{\tilde{W}^{-\epsilon}_{real,t+k}}D_{c,t+k}\frac{X_t}{X_{t+k}}\frac{P_t}{P_{t+k}}}{\left(\tilde{C}_{t+k}-b\tilde{C}_{t+k-1}\frac{1}{g_{t+k}}\right)}} \tag{2.88}$$

where: $\tilde{A}_{t+k|t}=\left(\frac{P_t}{P_{t+k}}\left(\frac{\tilde{W}^L_{real,t+k-1}}{\tilde{W}^L_{real,t-1}}\frac{P_{t+k-1}}{P_{t-1}}\right)^{\lambda_w}\right)^{-\epsilon_w}\left[\frac{X_t}{X_{t+k}}\left(\frac{X_{t+k-1}}{X_{t-1}}\right)^{\lambda_w}\right]^{-\epsilon_w}$

Now, just note that the numerator of equation 2.88 can be re-expressed in a recursive form as follows:

$$Z^1_t=\left(\frac{N_t}{\tilde{W}^{-\epsilon}_{real,t}}\right)^{1+\eta}D_{c,t}D_{n,t}+\beta\theta_w\tilde{A}^{1+\eta}_{t+1|t}Z^1_{t+1} \tag{2.89}$$

Similarly we can re-write the denominator of equation 2.88 as follows:

$$Z^2_t=\frac{\left(\frac{1}{\tilde{W}_{real,t}}\right)^{-\epsilon_w}N_tD_{c,t}}{\left(\tilde{C}_t-b\tilde{C}_{t-1}\frac{1}{g_{x,t}}\right)}+\beta\theta_w\tilde{A}_{t+1|t}\frac{1}{g_{x,t+1}}\frac{P_t}{P_{t+1}}Z^2_{t+1} \tag{2.90}$$

Therefore equation 2.88 can be re-written as:

$$\frac{1-\tau^l}{1+\tau^c}\tilde{\widehat{W}}^{1+\epsilon_w\eta}_{real,t}=\frac{\epsilon_w}{\epsilon_w-1}\frac{Z^1_t}{Z^2_t} \tag{2.91}$$

where $\tilde{A}_{t+1|t}=\left(\frac{P_t}{P_{t+1}}\left(\frac{\tilde{W}^L_{real,t}}{\tilde{W}^L_{real,t-1}}\frac{P_t}{P_{t-1}}\right)^{\lambda_w}\right)^{-\epsilon_w}\left[\frac{g^{\lambda_w}_{x,t}}{g_{x,t+1}}\right]^{-\epsilon_w}$.

Due to the absence of non-local workers in the rUK, the corresponding condition for $\tilde{A}_{t+1|t}$ will not be symmetrical. Instead, it will read:

$$\tilde{A}^*_{t+1|t} = \left(\frac{P^*_t}{P^*_{t+1}} \left(\frac{\tilde{W}^*_{real,t}}{\tilde{W}^*_{real,t-1}} \frac{P^*_t}{P^*_{t-1}} \right)^{\lambda^*_w} \right)^{-\epsilon^*_w} \left[\frac{g^{\lambda^*_w}_{x,t}}{g_{x,t+1}} \right]^{-\epsilon^*_w} \tag{2.92}$$

Price dispersion

Finally we deal with the infinitesimal summation in the left hand side of the (aggregate) production function, namely:

$$\int_0^1 Y_{H,t}(h) + \frac{1-\varpi}{\varpi} Y^*_{H,t}(h) + Y^{**}_{H,t}(h)\, dh = (m_t K_{t-1})^{\alpha} (A_t N_t X_t)^{1-\alpha} \tag{2.93}$$

We begin by summing up the demand schedules for the domestic and exported goods produced by firm h, from which we obtain:

$$\tilde{Y}_{H,t}(h) + \frac{1-\varpi}{\varpi} \tilde{Y}^*_{H,t}(h) + \tilde{Y}^{**}_{H,t}(h) = \left[\frac{P_{H,t}(h)}{P_{H,t}} \right]^{-\epsilon_p} \left(\tilde{Y}_{H,t} + \frac{1-\varpi}{\varpi} \tilde{Y}^*_{H,t} + \tilde{Y}^{**}_{H,t} \right) \tag{2.94}$$

At this point, we can define a price dispersion index $PD_{H,t}$ as follows:

$$\begin{aligned} PD_{H,t} = \int_0^1 \left[\frac{P_{H,t}(h)}{P_{H,t}} \right]^{-\epsilon_p} dh = \int_0^{1-\theta_H} \left(\frac{\hat{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon_p} dh + \\ \int_{1-\theta_H}^1 \left[\frac{P_{H,t-1}(h)}{P_{H,t}} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\lambda_H} \right]^{-\epsilon_p} dh = (1-\theta_H) \left(\frac{\hat{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon_p} + \\ \theta_H \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{-\epsilon_p \lambda_H} \left(\frac{P_{H,t-1}}{P_{H,t}} \right)^{-\epsilon_p} PD_{H,t-1} \end{aligned} \tag{2.95}$$

The infinitesimal summation in the equation 2.93 can then be substituted with the right hand side of the equation below:

$$\int_0^1 \tilde{Y}_{H,t}(h) + \frac{1-\varpi}{\varpi} \tilde{Y}^*_{H,t}(h) + \tilde{Y}^{**}_{H,t}(h)\, dh = PD_{H,t} \left(\tilde{Y}_{H,t} + \frac{1-\varpi}{\varpi} \tilde{Y}^*_{H,t} + \tilde{Y}^{**}_{H,t} \right) \tag{2.96}$$

2.4.4 Functional forms specification

In the following we make explicit functional form assumptions on the investment adjustment costs function and foreign debt holding costs. We start with the investment adjustment costs function, S. This is an increasing and convex function (i.e. $S'(\cdot)$, $S''(\cdot) > 0$) satisfying in the steady state $\bar{S} = \bar{S}' = 0$ and $\bar{S}'' > 0$. Consistently with these requirements, we impose the functional form below:

$$S\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{x,t}\right) = \frac{\phi}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{x,t} - 1\right)^2 = \frac{\phi}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{x,t} - 1\right)^2 \tag{2.97}$$

where $\phi > 0$. Then:

$$S'(\cdot) = \frac{\partial S(\cdot)}{\partial \tilde{I}_t} = \frac{1}{\tilde{I}_{t-1}} g_{x,t} \phi \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{x,t} - 1\right) \tag{2.98}$$

Let us now move to the risk premium function, $\Psi(\cdot)$. Recall that $\Psi(\cdot)$ has to be differentiable and decreasing in the neighborhood of zero, furthermore $\Psi(0) = 1$. Consistently with these requirements, we impose the functional form below:

$$\Psi(d_t) = 1 - \chi^d \left(d_t - \bar{d}\right) \tag{2.99}$$

$$\Psi(b_t) = 1 - \chi^b \left(b_t - \bar{b}\right) \tag{2.100}$$

where χ^d, χ^b are calibrated to be positive.

2.4.5 The system of stationary recursive equilibrium conditions

In the following we report the system of stationary recursive equilibrium conditions. Given the (quasi) symmetric structure of our model, most of the conditions describing the Scottish and the rUK economy are symmetrical. For convenience, we therefore report the conditions for Scotland only, listing those for the rUK where symmetry does not apply (e.g. 'wage-setting' equation).

Households saving and consumption decision

From the Scottish households problem of consumption and saving, we have the following set of conditions:

- the aggregate budget constraint,

$$\frac{b_t}{R_t^*} - \frac{b_{t-1}}{\Pi_t}\frac{\tilde{Y}_{t-1}}{\tilde{Y}_t}\frac{1}{g_{x,t}} - \frac{1-\varpi}{\varpi}RER_t^{H,F}\left(\frac{d_t^*}{R_t^{**}}\frac{\tilde{Y}_t^*}{\tilde{Y}_t} - \frac{d_{t-1}^*}{\Pi_t^{**}}\frac{RER_t^{F,row}}{RER_{t-1}^{F,row}}\frac{\tilde{Y}_{t-1}^*}{\tilde{Y}_t}\frac{1}{g_{x,t}} - \frac{\widetilde{TB}_{real,t}^*}{\tilde{Y}_t} - \frac{\widetilde{TO}_{real,t}}{(1-\varpi)\tilde{Y}_t}\right)$$
$$+\frac{\widetilde{TR}_{real,t} - \tilde{G}_t - \tilde{T}_t + \tilde{\Upsilon}_t}{\tilde{Y}_t} = 0 \tag{2.101}$$

together with definition of the total tax burden the households are subject to,

$$\widetilde{TR}_{real,t} = \tau^c\tilde{C}_t + \tau_t^l\tilde{W}_{real,t}N_t + \tau_t^kR_t^km_t\tilde{K}_{t-1}\frac{1}{g_{x,t}} \tag{2.102}$$

- the Euler equation representing the choice between today and tomorrow's consumption (and investment in the UK asset),

$$1 = R_t^*\left[1-\chi^b\left(b_t-\bar{b}\right)\right]\beta\mathbb{E}_t\left(\frac{g_{x,t}\tilde{C}_t - b\tilde{C}_{t-1}}{g_{x,t+1}\tilde{C}_{t+1} - b\tilde{C}_t}\frac{1}{g_{x,t}}\frac{D_{c,t+1}}{D_{c,t}}\frac{1}{\Pi_{t+1}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) \tag{2.103}$$

- the Tobin's q,

$$Q_t = \beta\mathbb{E}_t\{\left(\frac{g_{x,t}\tilde{C}_t - b\tilde{C}_{t-1}}{g_{x,t+1}\tilde{C}_{t+1} - b\tilde{C}_t}\frac{1}{g_{x,t}}\frac{D_{c,t+1}}{D_{c,t}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left[\left(1-\tau_{t+1}^k\right)R_{t+1}^km_{t+1} + Q_{t+1}\left(1-\delta\left(m_{t+1}\right)\right)\right]\} \tag{2.104}$$

- the investment decision in physical capital,

$$1 - Q_tV_t\left[1-\frac{\phi}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{x,t}-1\right)^2 - \frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{x,t}\phi\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{x,t}-1\right)\right] =$$
$$\beta\mathbb{E}_t\left(\frac{g_{x,t}\tilde{C}_t - b\tilde{C}_{t-1}}{g_{x,t+1}\tilde{C}_{t+1} - b\tilde{C}_t}\frac{1}{g_{x,t}}\frac{D_{c,t+1}}{D_{c,t}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)Q_{t+1}V_{t+1}\left[\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}g_{x,t+1}\right)^2\phi\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}g_{x,t+1}-1\right)\right] \tag{2.105}$$

- the investment decision in the foreign asset,

$$1 = \beta R_t^{**} \mathbb{E}_t \left(\frac{g_{x,t}\tilde{C}_t - b\tilde{C}_{t-1}}{g_{x,t+1}\tilde{C}_{t+1} - b\tilde{C}_t} \frac{1}{g_{x,t}} \frac{D_{c,t+1}}{D_{c,t}} \frac{RER_{t+1}^{H,row}}{RER_t^{H,row}} \frac{1}{\Pi_{t+1}^{**}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \left[1 - \chi^d \left(d_t - \bar{d}\right)\right] \tag{2.106}$$

- the choice of capacity utilization,

$$\left(1-\tau^k\right) R_t^k = Q_t \delta' \left(m_t\right) \tag{2.107}$$

- the law of motion of capital,

$$\tilde{K}_t = \left(1 - \delta\left(m_t\right)\right) \tilde{K}_{t-1} \frac{1}{g_{x,t}} + V_t \left[1 - \frac{\phi}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} g_{x,t} - 1\right)^2\right] \tilde{I}_t \tag{2.108}$$

Households labour supply decision

From the Scottish households labour supply problem we have the following set of conditions:

- the wage-setting equation,

$$\frac{1-\tau^l}{1+\tau^c} \widehat{\tilde{W}}_{real,t}^{1+\epsilon_w \eta} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{Z_t^1}{Z_t^2} \tag{2.109}$$

- the recursive definition of the numerator of the wage-setting equation,

$$Z_t^1 = \left(\frac{N_t}{\tilde{W}_{real,t}^{-\epsilon}}\right)^{1+\eta} D_{c,t} D_{n,t} + \beta\theta_w \tilde{A}_{t+1|t}^{1+\eta} Z_{t+1}^1 \tag{2.110}$$

- the recursive definition of the denominator of the wage-setting equation,

$$Z_t^2 = \frac{\left(\frac{1}{\tilde{W}_{real,t}}\right)^{-\epsilon_w} N_t D_{c,t}}{\left(\tilde{C}_t - b\tilde{C}_{t-1}\frac{1}{g_{x,t}}\right)} + \beta\theta_w \tilde{A}_{t+1|t} \frac{1}{g_{x,t+1}} \frac{1}{\Pi_{t+1}} Z_{t+1}^2 \tag{2.111}$$

- an auxiliary condition entering the wage-setting equation,

$$\tilde{A}_{t+1|t} = \left(\frac{1}{\Pi_{t+1}} \left(\frac{\tilde{W}^L_{real,t}}{\tilde{W}^L_{real,t-1}} \Pi_t \right)^{\lambda_w} \right)^{-\epsilon_w} \left[\frac{g^{\lambda_w}_{x,t}}{g_{x,t+1}} \right]^{-\epsilon_w} \tag{2.112}$$

- the aggregate wage index in Scotland,

$$\begin{aligned}\tilde{W}_{real,t} =& [(1-\theta^L) \left(\tilde{W}^*_{real,t} RER^{H,F}_t \right)^{1-\epsilon_w} + \theta^L ((1-\theta^w) \hat{\tilde{W}}_{real,t} + \\ & \theta^w \tilde{W^L}_{real,t-1} \left(\frac{{g_{x,t-1}}^{\lambda_w}}{g_{x,t}} \right) \frac{\Pi^{\lambda_w}_{t-1}}{\Pi_t} \left(\frac{\tilde{W^L}_{real,t-1}}{\tilde{W}^L_{real,t-2}} \right)^{\lambda_w})^{1-\epsilon_w}]^{\frac{1}{1-\epsilon_w}}\end{aligned} \tag{2.113}$$

- the local workers wage index in Scotland,

$$\tilde{W}^L_{real,t} = (1-\theta^w_t) \widehat{\tilde{W}}_{real,t} + \theta^w_t \tilde{W}^L_{real,t-1} \frac{\Pi^{\lambda^w}_{t-1}}{\Pi_t} \left(\frac{{g_{x,t-1}}^{\lambda_w}}{g_{x,t}} \right) \left(\frac{\tilde{W}^L_{real,t-1}}{\tilde{W}^L_{real,t-2}} \right)^{\lambda_w} \tag{2.114}$$

Intermediate good firm production and pricing

From the intermediate good firms problem of cost minimization and price-setting we have the following set of conditions:

- the optimal combination of production factors,

$$\frac{N_t}{m_t \tilde{K}_{t-1}} = \frac{1-\alpha}{\alpha} \left(\frac{R^k_t}{\tilde{W}_{real,t}} \right) \frac{1}{g_{x,t}} \tag{2.115}$$

- the marginal production costs,

$$\tilde{MC}_t = \frac{{R^k_t}^{\alpha} \left(\tilde{W}_{real,t} \right)^{1-\alpha} {A_t}^{\alpha-1}}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}} \tag{2.116}$$

- the production function,

$$PD_{H,t}\left(\tilde{Y}_{H,t}+\frac{1-\varpi}{\varpi}\tilde{Y}^{*}_{H,t}+\tilde{Y}^{**}_{H,t}\right)=\left(m_t\tilde{K}_{t-1}\right)^{\alpha}\left(g_{x,t}\right)^{-\alpha}\left(A_tN_t\right)^{1-\alpha} \tag{2.117}$$

- the price dispersion index,

$$PD_{H,t}=\left(1-\theta_H\right)\left(\hat{p}_{H,t}\right)^{-\epsilon_p}+\theta_H\left(\frac{{\Pi^H_{t-1}}^{\lambda_H}}{\Pi^H_t}\right)^{-\epsilon_p}PD_{H,t-1} \tag{2.118}$$

- the price-setting equation,

$$\hat{p}_{H,t}=\frac{\epsilon_p}{\left(\epsilon_p-1\right)}\frac{F^1_t}{F^2_t} \tag{2.119}$$

- the recursive definition of the numerator of the price-setting equation,

$$F^1_t=MC_t\left(\tilde{Y}_{H,t}+\frac{1-\varpi}{\varpi}\tilde{Y}^{*}_{H,t}+\tilde{Y}^{**}_{H,t}\right)+\beta\theta_H\Lambda_{t,t+1}g_{x,t+1}\left(\frac{{\Pi^H_t}^{\lambda_H}}{\Pi^H_{t+1}}\right)^{-\epsilon_p}F^1_{t+1} \tag{2.120}$$

- the recursive definition of the denominator of the price-setting equation,

$$F^2_t=\left(\tilde{Y}_{H,t}+\frac{1-\varpi}{\varpi}\tilde{Y}^{*}_{H,t}+\tilde{Y}^{**}_{H,t}\right)p_{H,t}+\beta\theta_H\Lambda_{t,t+1}g_{x,t+1}\left(\frac{{\Pi^H_t}^{\lambda_H}}{\Pi^H_{t+1}}\right)^{1-\epsilon_p}F^2_{t+1} \tag{2.121}$$

- the intermediate goods price index,

$$1=\left[\left(1-\theta_H\right)\left(\hat{p}_{H,t}\right)^{1-\epsilon_p}+\theta_H\left(\frac{{\Pi^H_{t-1}}^{\lambda_H}}{\Pi^H_t}\right)^{1-\epsilon_p}\right]^{\frac{1}{1-\epsilon_p}} \tag{2.122}$$

- the intermediate goods price definition,

$$p_{H,t}=\frac{\Pi^H_t}{\Pi_t}p_{H,t-1} \tag{2.123}$$

Wholesale good firm production and pricing

From the wholesale good firm problem of cost minimization and price-setting we have the following set of conditions:

- the production function,

$$\tilde{Y}_t = \left[{\omega_1^H}^{\frac{1}{\theta}} \tilde{Y}_{H,t}^{\frac{(\theta-1)}{\theta}} + {\omega_2^H}^{\frac{1}{\theta}} \tilde{Y}_{F,t}^{\frac{(\theta-1)}{\theta}} + \left(1 - \omega_1^H - \omega_2^H\right)^{\frac{1}{\theta}} \tilde{Y}_{row,t}^{\frac{(\theta-1)}{\theta}} \right]^{\frac{\theta}{(\theta-1)}} \tag{2.124}$$

- the optimal demand of rUK-produced intermediate goods,

$$\frac{\tilde{Y}_{H,t}}{\tilde{Y}_{F,t}} = \frac{\omega_1^H}{\omega_2^H} \left(\frac{p_{H,t}}{p_{F,t}} \frac{1}{RER_t^{H,F}} \right)^{-\theta} \tag{2.125}$$

- the optimal demand of ROW-produced intermediate goods,

$$\frac{\tilde{Y}_{H,t}}{\tilde{Y}_{row,t}} = \frac{\omega_1^H}{1 - \omega_1^H - \omega_2^H} \left(\frac{p_{H,t}}{RER_t^{H,row}} \right)^{-\theta} \tag{2.126}$$

- the price-setting equation,

$$1 = \mu_{f,t} \left[\omega_1^H \left(p_{H,t}\right)^{1-\theta} + \omega_2^H \left(RER_t^{H,F} p_{F,t} \right)^{1-\theta} + \left(1 - \omega_1^H - \omega_2^H\right) \left(RER_t^{H,row} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.127}$$

Public sector

From the public sector we have a number of conditions establishing the composition of the public finance aggregates as well as setting out the budget constraints faced by the different fiscal authorities, including:

- the definition of total public consumption in Scotland,

$$\widetilde{G}_t = \widetilde{Gdev}_t + \varsigma \widetilde{Gnodev}_t^{uk} \tag{2.128}$$

- the definition of total transfers to Scotland,

$$\widetilde{Tsc}_t = \widetilde{Gdev}_t + \widetilde{T}_t \tag{2.129}$$

- the growth of non-devolved expenditure in Scotland,

$$gg_t = \frac{\widetilde{Gdev}^*_t g_{x,t} - \widetilde{Gdev}^*_{t-1}}{\widetilde{Gdev}^*_{t-1}} * DD_t \tag{2.130}$$

- the evolution of the comparability factor for the Barnett formula,

$$DD_t = \frac{\widetilde{Gdev}^*_t}{\widetilde{Gdev}_t} \tag{2.131}$$

- the Barnett formula,

$$\overline{\widetilde{Gdev}}_t = \overline{\widetilde{Gdev}}_{t-1} \tag{2.132}$$

- the deviations from the Barnett formula,

$$\widetilde{Gdev}_t = \overline{\widetilde{Gdev}}_t \exp(\epsilon_t^{Gdev}) \tag{2.133}$$

- the Westminster budget constraint,

$$\begin{aligned}
&\frac{b_t^{UK}}{R_t^*} + \varpi\tau_t^k m_t R_t^k g_{x,t}\frac{\tilde{K}_{t-1}}{\tilde{Y}_t^*}RER_t^{F,H} + (1-\varpi)\,\tau_t^k m_t^* R_t^{k*} g_{x,t}\frac{\tilde{K}_{t-1}^*}{\tilde{Y}_t^*} + \\
&\varpi\tau_t^l\frac{\tilde{W}_{real,t}}{\tilde{Y}_t^*}N_t RER_t^{F,H} + (1-\varpi)\,\tau_t^l\frac{\tilde{W}_{real,t}^*}{\tilde{Y}_t^*}N_t^* + \varpi\tau_t^c\frac{\tilde{C}_t}{\tilde{Y}_t^*}RER_t^{F,H} + \\
&(1-\varpi)\,\tau_t^c\frac{\tilde{C}_t^*}{\tilde{Y}_t^*} + \frac{\widetilde{TO}_{real,t}}{\tilde{Y}_t^*} + \frac{\widetilde{\Upsilon}_t}{\tilde{Y}_t^*} = b_{t-1}^{UK} + (1-\varpi)\,\frac{\widetilde{Gdev}^*_t}{\tilde{Y}_t^*} + \\
&\varsigma\varpi RER_t^{F,H}\frac{\widetilde{Gnodev}^{uk}_t}{\tilde{Y}_t^*} + (1-\varpi)\,\varsigma^*\frac{\widetilde{Gnodev}^{uk}_t}{\tilde{Y}_t^*} + (1-\varpi)\,\frac{\tilde{T}_t^*}{\tilde{Y}_t^*} + \varpi RER_t^{F,H}\frac{\widetilde{Tsc}_t}{\tilde{Y}_t^*}
\end{aligned} \tag{2.134}$$

- the tax revenues from oil,

$$\widetilde{TO}_{real,t} = (p_t^o - mc_t^o)\,\tilde{Q}_t^o\tau_t^o \tag{2.135}$$

- the debt stabilizing, lump sum tax rule,

$$\widetilde{\Upsilon}_t = \rho^{\Upsilon}\left(b_t^{UK} - \bar{b}\right) \tag{2.136}$$

- the monetary rule,

$$\frac{R_t^*}{R} = \left(\frac{R_{t-1}^*}{R}\right)^{\varphi_R} \left[(\varpi\Pi_t + (1-\varpi)\Pi_t^*)^{\varphi_\pi} \left(\varpi\frac{GDP_t}{GDP_{t-1}} + (1-\varpi)\frac{GDP_t^*}{GDP_{t-1}^*}\right)^{\varphi_y} (g_{x,t})^{\varphi_y}\right]^{1-\varphi_R} \exp\left(\epsilon_t^{m*}\right) \tag{2.137}$$

Market clearing conditions

Equilibrium requires domestic and international markets to clear. This is ensured by the following conditions:

- the final good market clearing,

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t \tag{2.138}$$

- the UK's capital market clearing,

$$b_t^{uk} = (1-\varpi)\, b_t^* + \varpi b_t \frac{\tilde{Y}_t}{\tilde{Y}_t^*} RER_t^{F,H} \tag{2.139}$$

- the international market clearing,

$$\frac{d_t^{uk}}{R_t^{**}} - \frac{d_{t-1}^{uk}}{\Pi_t^{**}} \frac{RER_t^{F,row}}{RER_{t-1}^{F,row}} \frac{\tilde{Y}_{t-1}^*}{\tilde{Y}_t^*} \frac{1}{g_{x,t}} = \varpi \frac{\widetilde{TB}_{real,t}}{\tilde{Y}_t^*} RER_t^{F,H} + (1-\varpi)\frac{\widetilde{TB}^*_{real,t}}{\tilde{Y}_t^*} + \frac{\widetilde{TO}_{real,t}}{\tilde{Y}_t^*} \tag{2.140}$$

together with the definition of the Scottish balance of trade,

$$\widetilde{TB}_{real,t} = p_{H,t}\frac{1-\varpi}{\varpi}\tilde{Y}_{H,t}^* + p_{H,t}\tilde{Y}_{H,t}^{**} - RER_t^{H,F} p_{F,t}\tilde{Y}_{F,t} - RER_t^{H,row}\tilde{Y}_{row,t} \tag{2.141}$$

and the balance of trade in the rUK,

$$\widetilde{TB}^*_{real,t} = p_{F,t}\frac{\varpi}{1-\varpi}\tilde{Y}_{F,t} + p_{F,t}\tilde{Y}_{F,t}^{**} - RER_t^{F,H} p_{H,t}\tilde{Y}_{H,t}^* - RER_t^{F,row}\tilde{Y}_{row,t}^* \tag{2.142}$$

- the international capital market clearing,

$$d_t^{uk} = \varpi d_t RER_t^{F,H} \frac{\tilde{Y}_t}{\tilde{Y}_t^*} + (1-\varpi)\, d_t^* \tag{2.143}$$

RUK non-symmetrical conditions

In the rUK, equations 2.113 and 2.112 will not be symmetrical. In particular:

- the wage index will depend only on local workers wage,

$$\begin{aligned} \tilde{W}^*_{real,t} &= [(1-\theta_w^*)\,\widehat{\tilde{W}^*}_{real,t}^{\,1-\epsilon_w^*} + \\ \theta_w^* &\left(\tilde{W}^*_{real,t-1} \left(\frac{g_{x,t-1}^{\lambda_w^*}}{g_{x,t}}\right) \frac{\Pi_{t-1}^{*\,\lambda_w^*}}{\Pi_t^*} \left(\frac{\tilde{W}^*_{real,t-1}}{\tilde{W}^*_{real,t-2}}\right)^{\lambda_w^*} \right)^{1-\epsilon_w^*}]^{\frac{1}{1-\epsilon_w^*}} \end{aligned} \tag{2.144}$$

- the auxiliary condition will not include local workers wage,

$$\tilde{A}^*_{t+1|t} = \left(\frac{1}{\Pi^*_{t+1}} \left(\frac{\tilde{W}^*_{real,t}}{\tilde{W}^*_{real,t-1}} \Pi_t^* \right)^{\lambda_w^*} \right)^{-\epsilon_w^*} \left[\frac{g_{x,t}^{\lambda_w^*}}{g_{x,t+1}} \right]^{-\epsilon_w^*} \tag{2.145}$$

Rest of the world

The rest of world is exogenously modeled as a VAR(1), including ROW GDP, inflation and interest rate. Furthermore, symmetric demand schedules for imported goods to those of Scotland and the rUK are imposed. This is represented by the following set of conditions:

- the VAR,

$$F_{t+1} = AF_t + \epsilon_t^{row} \tag{2.146}$$

where $F_t = [Y_t^{**}, \pi_t^{**}, R_t^{**}]$

- the ROW demand for the rUK-produced intermediate goods,

$$\frac{Y^{**}_{row,t}}{Y^{**}_{F,t}} = \frac{\omega_1^{row}}{1-\omega_1^{row}-\omega_2^{row}} \left(\frac{RER_t^{F,row}}{p_{F,t}} \right)^{-\theta} \tag{2.147}$$

- the ROW demand for the Scottish-produced intermediate goods,

$$\frac{Y^{**}_{row,t}}{Y^{**}_{H,t}} = \frac{\omega_1^{row}}{1-\omega_1^{row}-\omega_2^{row}}\left(\frac{RER_t^{H,row}}{p_{H,t}}\right)^{-\theta} \tag{2.148}$$

- the ROW production function,

$$Y_t^{**} = \left[\omega_1^{row\,\frac{1}{\theta}} Y_{H,t}^{**\frac{(\theta-1)}{\theta}} + \omega_2^{row\,\frac{1}{\theta}} Y_{F,t}^{**\frac{(\theta-1)}{\theta}} + \left(1-\omega_1^{row}-\omega_2^{row}\right)^{\frac{1}{\theta}} Y_{row,t}^{**\frac{(\theta-1)}{\theta}}\right]^{\frac{\theta}{(\theta-1)}} \tag{2.149}$$

Remaining definitions

The model further contains a few definitions, including:

- the stochastic discount factor,

$$\Lambda_{t,t+1} = \mathbb{E}_t \frac{D_{c,t+1}}{D_{c,t}} \frac{1}{g_{x,t}} \frac{\left(g_{x,t}\tilde{C}_t - b\tilde{C}_{t-1}\right)}{\left(g_{x,t+1}\tilde{C}_{t+1} - b\tilde{C}_t\right)} \tag{2.150}$$

- the GDP,

$$GDP_t = \tilde{Y}_{H,t} + \frac{1-\varpi}{\varpi}\tilde{Y}^*_{H,t} + \tilde{Y}^{**}_{H,t} \tag{2.151}$$

- the real exchange rate between Scotland and the rUK,

$$RER_t^{H,F} = \frac{\Pi_t^*}{\Pi_t} RER_{t-1}^{H,F} \tag{2.152}$$

- the inverse of the real exchange rate between Scotland and the rUK,

$$RER_t^{F,H} = \frac{1}{RER_t^{H,F}} \tag{2.153}$$

Exogenous processes

Finally, the model contains a large set of exogenous processes whose law of motion is reported below. These includes:

- the discount factor shock,

$$\log\left(D_{c,t}\right)=\rho^{c}\log\left(D_{c,t-1}\right)+\epsilon_{t}^{c,d} \tag{2.154}$$

- the intra-temporal preference shock,

$$\log\left(D_{n,t}\right)=\rho^{n}\log\left(D_{n,t-1}\right)+\epsilon_{t}^{n,d} \tag{2.155}$$

- the investment technology shock,

$$\log\left(V_{t}\right)=\rho^{v}\log\left(V_{t-1}\right)+\epsilon_{t}^{v} \tag{2.156}$$

- the gross rate of growth of UK technology,

$$\log\left(g_{x,t}\right)=\epsilon_{t}^{x} \tag{2.157}$$

- the temporary productivity shock,

$$\log\left(A_{t}\right)=\rho^{a}\log\left(A_{t-1}\right)+\epsilon_{t}^{a} \tag{2.158}$$

- the mark-up shock,

$$\log\left(\mu_{f,t}\right)=\log\left(\mu_{f}\right)+\epsilon_{t}^{\mu^{f}} \tag{2.159}$$

- the price of oil,

$$\log\left(p_{t}^{o}\right)=\rho^{po}\log\left(p_{t-1}^{o}\right)+\left(1-\rho^{po}\right)\log\overline{p}^{o}+\epsilon_{t}^{po} \tag{2.160}$$

- the oil extracted quantity (in the UK),

$$\log\left(\tilde{Q}_{t}^{o}\right)=\rho^{qo}\log\left(\tilde{Q}_{t-1}^{o}\right)+\left(1-\rho^{qo}\right)\log\overline{Q}^{o}+\epsilon_{t}^{qo} \tag{2.161}$$

- the oil marginal production costs (in the UK),

$$\log\left(mc_{t}^{o}\right)=\rho^{mco}\log\left(mc_{t-1}^{o}\right)+\left(1-\rho^{mco}\right)\log\overline{mc}^{o}+\epsilon_{t}^{mco} \tag{2.162}$$

- the government transfers,

$$\log\widetilde{T}_{t}=\rho^{tr}\log\widetilde{T}_{t-1}+\left(1-\rho^{tr}\right)\overline{\widetilde{T}}+\epsilon_{t}^{tr} \tag{2.163}$$

- the devolved expenditure in the rUK,

$$\log \widetilde{Gdev}_t^{*} = \rho_{gdev} \log \widetilde{Gdev}_t^{*} + (1-\rho_{gdev}) \log \overline{\widetilde{Gdev}}^{*} + \epsilon_t^{gdev} \quad (2.164)$$

- the non-devolved expenditure in the rUK,

$$\log\left(\widetilde{Gnodev}_t^{uk}\right) = \rho_{gnodev} \log\left(\widetilde{Gnodev}_{t-1}^{uk}\right) + (1-\rho_{gnodev}) \log\left(\overline{\widetilde{Gnodev}}^{uk}\right) + \epsilon_t^{gnodev} \quad (2.165)$$

2.5 Concluding remarks

The objective of this chapter was to develop a dynamic stochastic general equilibrium model to be employed for carrying out fiscal devolution analysis within a macroeconomic framework. The model is designed considering the case of Scotland and the rUK economy. We began by reviewing the literature on DSGE models developed in recent years in academia and in central banks. The framework of Rabanal and Tuesta (2010) has been identified as a suitable starting point for developing our framework. We have then amended and extended this model to adequately represent the economic linkages between Scotland and the rUK. In particular, we have revisited key aspects such as the price pass-through and the structure of the public sector. Finally, we have transformed this model into stationary, recursive terms and presented the set of equilibrium conditions of our economy. This version of the model is suitable to be input into standard software for the purpose of solution and simulation.

The next chapter of this thesis will deal with the estimation of the model and its simulation. Together with shedding light on the dynamics of the economic adjustment within the UK, such an exercise will also provide an important source of model validation. Specifically, it will offer an insight on the model ability to replicate theory predictions, while accounting for the dynamics in the data.

Chapter 3

DSGE model for Scotland and the rUK: a Bayesian investigation

3.1 Introduction

In the previous chapter we developed a DSGE model of the Scottish and rUK economy for the analysis of fiscal devolution arrangements within the UK. However, for this model to be able to produce quantitative predictions we need to assign values to its parameters. Specifically, we need to parametrize our model for it to reproduce the main features of the Scottish and of the rUK economy. There are various possible approaches in parameter selection, offering alternative trade-off among the information which is conveyed from the data, the statistical treatment of the model and, not least, the overall robustness of the procedure. In the context of a large and densely parametrized model such as ours, this is a key aspect to be addressed. In particular, we aim to select a procedure able to satisfy a number of minimum requirements, namely: (i) to be able to reproduce most of the data features; (ii) to be able to suit alternative uses of the model (i.e. from policy analysis to academic research) and (iii) to be robust. We therefore begin this chapter by reviewing possible approaches in parameters selection and discuss their main features. For reasons to be discussed next, we will opt for estimating our model using Bayesian likelihood-based techniques. Bayesian estimation is based on a combination of statistical results, solution algorithms and powerful simulation techniques which have become a cornerstone in macroeconomics. Given the centrality and the complexity of this estimation technique, we provide a full description of its central aspects (e.g priors elicitation) as well as the derivation of the main statistical results (e.g. the Kalman filter) that underpin its foundation.

We will then proceed to estimate our model. This will require choosing a sample of macro-variables and appropriately transforming them to be linked to the their model counterparts. Estimation results will provide us with a first assessment of our framework. Specifically, parameter estimates which are economically meaningful, and broadly in line with the wider literature, will provide a first source of validation to our model.

Once estimated the model, we will explore its main quantitative predictions, for example: how a demand shock originating in the rUK propagates to Scotland? How does an increase in the interest rate from the Bank of England affect Scotland through its direct and indirect (e.g. feedback from rUK) effects? The study of the *Impulse Response Functions* will allow us to address these questions. This exercise, furthermore, will provide a second source of validation to our framework, whenever the dynamics produced by the model are largely consistent with the theory and with the literature.

We begin by reviewing the alternative approaches for parametrizing DSGE models, in section 3.2, where we will further discuss the theory behind the Bayesian estimation techniques. We will then move to estimate the model, documenting the passages involved in our estimation procedure and its results in section 3.3. Finally, in section 3.4, we will analyze the impulse response functions produced by our model and study the mechanic of the economic adjustment in Scotland and in the rUK. Few concluding remarks will then follow.

3.2 Parameters selection in DSGE models: estimation and other approaches

As discussed in the introduction, there are various possible approaches in parameter selection, offering alternative trade-off among the information which is conveyed from the data, the assumptions required on the model shocks and the statistical treatment of the model. In the context of a large and densely parametrized model such as ours, the choice a parameter selection procedure it is not an obvious one. We therefore review below the main approaches in parameters selection and discuss their central features. As we select Bayesian estimation techniques to estimate our model, we then discuss in detail the features and derive the main statistical results underpinning this approach.

3.2.1 Alternative approaches in parameters selection: Calibration Versus Estimation

A first characterization of the alternative approaches to parameter selection is the separation between the practice of calibration and estimation techniques. The main difference between these two consists in the statistical treatment of the model. In particular, calibration is substantially based on the denial of any probabilistic interpretation: since models constitute an abstraction of reality, they are necessarily false and any statistical test is expected to reject them. Moving from this premise, the calibration procedure aims at parametrizing models to ensure relevant stylized facts from the observed economy are replicated, without attempting to provide any probabilistic foundation. The procedure typically requires solving the model parameters as a function of endogenous variables in ratios at their steady state (e.g capital-output ratio, consumption-out ratio etc). Parameter values are then pinned down by setting these ratios to match the empirical ones (for a detailed description refer to Cooley and Prescott (1995)). Model evaluation is then carried out on the basis of an informal assessment of the model ability to match relevant characteristics of the data. The main critique researchers attach to calibration, is that it does not provide a framework for assessing the uncertainty surrounding model predictions; this is a serious impediment to the evaluation of the empirical performance of competing models. For this reason, calibration has been progressively overtaken by estimation techniques. Estimation, on the other hand, is a collection of procedures providing statistical foundation to the model and employing statistical inference tools, such as hypothesis testing and model likelihood. The set of procedures employed depend on the particular estimation technique selected. We survey below the main estimation techniques used in the DSGE literature.

3.2.2 Alternative approaches in model estimation: GMM, SMM, Classical ML, Bayesian approach

Standard textbooks (among others: DeJong and Dave (2011) and Canova (2007)) tend to characterize estimation techniques on the basis of two main criteria. The first involves the type of information which is used by the estimation procedure, distinguishing between *Full information* and *Partial information*. Partial information methods use only some of the information available in the sample for estimating the parameters of the model. The most widespread and well known technique in this category is the Generalized Method of Moments (GMM henceforth). Under GMM, model parameters are estimated to minimize the weighted distance between some model and sample moments. A slight variation over this approach is represented by the Simulated Method of Moments (SMM henceforth), which relies on the same moment matching procedure as GMM. However, SMM is applied in cases where the orthogonality conditions cannot be assessed analytically and relies on simulation instead[1]. On the contrary, full information methods exploit all the information available in the sample. In this category, maximum likelihood (ML) estimation represents a cornerstone. ML targets the whole distribution of empirical variables, not only some of their moments. This is reflected in the likelihood principle stating that, in parameter inference based on an observed sample, all the relevant information is contained in the likelihood function. When compared to partial information methods, this property constitutes at the same time the strength and the weakness of this methodology. Indeed, while not disregarding any of the information in the sample, likelihood-based methods require parametric assumptions over the distribution of the model's structural shocks carrying the risk of mis-

[1]More precisely, SMM exploits the fact that the asymptotic properties of the resultant testing procedure are retained when we replace the estimator derived analytically with the one obtained by simulation.

specification. The same does not occur with GMM and SMM which, not being parametric methods, do not require any parametric assumption. On the other hand, the problem with limited information methods is that inference may depend on the choice of the moments to be matched[2].

A second criterion used to characterize estimation procedures is by the type of statistical analysis employed, distinguishing between the *Classical* and the *Bayesian* approach. Following the treatment in main macroeconomics textbooks, we consider here the differences between the Classical and the Bayesian approach only in the context of full information methods. In classical analysis, model parameters are interpreted as fixed (but unknown), whereas the sample is interpreted as a random draw from the model likelihood. Parameter estimates are obtained through likelihood maximization, namely by selecting those values that maximize the probability of a given sample realization. Uncertainty surrounding point estimates derives from the sample at hand representing only a realization among all possible drawings, and it is conveyed through the use of standard errors. Inference is then pursued through classical hypothesis-testing, under the null hypothesis that a given parametrization of the model corresponds to the data generating process, we assess the probability of observing our sample. If this probability lies within a certain threshold, then the null hypothesis is not rejected, otherwise it is. In the Bayesian approach, instead, parameters are interpreted as random and the sample as fixed. The objective is to find the posterior distribution of parameters, in light of the observed data and of prior beliefs/knowledge about parameters values. Typically point estimates will be provided by the mean or mode of the posterior distribution, and uncertainty surrounding them will be measured by the standard deviation. In the Bayesian process of inference, competing models are assessed on the basis of their relative probability conditional on the observed sample, without maintaining any of them as the null hypothesis. This is an attractive feature; indeed, given that all models are necessarily false, the practice of Classical test of working under the imposition of one model as the true data generating process is somehow unsatisfactory. Moreover, another appealing property of the Bayesian techniques is that, being based on Bayes rule, it is coherent

[2]For a performance comparison between Full-information and Limited information procedures refer to Ruge-Murcia (2007).

with rational decision making and updating following new information accrual[3]. Furthermore, note that the use of priors in the Bayesian approach allows us to exploit pre-sample information that are often extraordinary rich. This extra source of information may become crucial when facing identification problems, since priors may induce curvatures along dimensions of the likelihood function that are, otherwise, nearly flat. Finally, it should be considered that, often, likelihood maximization could be an hard task and the results may be not entirely robust. For all these reasons, in recent times Bayesian estimation has become the prominent approach in the DSGE estimation. In the following we present the main concepts and the tools needed for its implementation.

3.2.3 Bayesian estimation

Basic ingredients

The core task in Bayesian estimation is to obtain and to evaluate the so called "posterior distribution" of model parameters given the series of observed data. The posterior distribution combines the information from the model likelihood, given data, and pre-sample knowledge (the so called "prior"). The theoretical result underpinning this approach is the Bayes theorem:

$$P\left(B|A\right) = \frac{P\left(A|B\right) P\left(B\right)}{P\left(A\right)} \tag{3.1}$$

To show how this results is transposed in DSGE models estimation, the following items are defined:

- The observed data series, Z$\equiv \left\{z_t\right\}_{t=1}^{T} \in R^{N\times T}$
- The parameter set, $\Theta \in R^K$
- The likelihood function, $f\left(Z|\theta\right)$: $R^{N\times T} \times \Theta \to R^+$
- The prior distribution, $\pi\left(\theta\right) : \Theta \to R^+$

[3]Note that Classical (or Frequentist) approach works ex-ante, defining a procedure that has certain properties on repeated sampling: e.g, in the 95% of samples the mean will fall within some interval. Instead, the Bayesian approach is entirely ex-post: we find the posterior distribution of our variable of interest, updating our prior knowledge on the basis of the observed sample (with no role for samples that have not been observed).

From items above, the following relations can then be derived:

- The joint distribution of data and parameters, $f(Z,\theta) = f(Z|\theta)\,\pi(\theta)$
- The marginal distribution of data, $P(Z) = \int f(Z,\theta)\,\mathrm{d}\theta = \int f(Z|\theta)\,\pi(\theta)\,\mathrm{d}\theta$
- The posterior distribution of parameters given data, $\pi(\theta|Z) = \frac{f(Z|\theta)\pi(\theta)}{\int f(Z|\theta)\pi(\theta)\,\mathrm{d}\theta}$

As anticipated, the goal of our estimation procedure will be to characterize the posterior distribution of parameters given data. For this purpose, the definition of the posterior distribution itself traces the road map of problems we need to deal with:

- how to specify the prior?
- how to evaluate the likelihood function?
- how to explore the likelihood function?

We address these questions below; before that, however, a remark is due. While dealing with the concepts of joint, marginal and posterior distribution, one should never forget that they are all conditional on the particular model under consideration. Accordingly we could rename our likelihood function as $f(Z|\theta,i)$ where $i \in M$ represents a particular model specification among the set of all possible model specifications M. Throughout this chapter such index is dropped for the sole purpose of easing notation.

Priors elicitation

Prior distributions are meant to reflect subjective beliefs, as well as information derived from other data sources (not included in the estimation sample) such as micro-level data and pre-sample information. As observed by An and Schorfheide (2007), priors play an important role in model estimation since: "they might downweigh regions of the parameter space that are at odds with observations not contained in the estimation sample. They may also add curvature to a likelihood function that is (nearly) flat in some dimensions of the parameter space and therefore strongly influence the shape of posterior distribution". In general, priors elicitation is a delicate task involving a number of issues which are briefly reviewed here. A first problem is how "loose" or "tight" priors should be, given that a loose prior will imply the posterior is largely dominated by the likelihood, and vice versa.

In this regard, Fernández-Villaverde (2010) suggests adopting different approaches depending on the model application. For policy making, it is better to adopt tighter priors which will guide the model toward more "reasonable" parametrizations whereas, for academic research, a looser prior will allow us to learn more about the model properties. A second issue is whether parameter priors should be chosen in isolation or on a joint basis and whether they should be model invariant. Typically, and for the sake of simplicity, it is assumed that model parameters are independent, this simplification however has important drawbacks. Indeed, the resulting joint distribution may end up assigning non-negligible probability mass to regions of the parameter space determining model predictions which are at odds with theory and empirical evidence. Furthermore, priors are often chosen to be invariant across competing model specifications. Del Negro and Schorfheide (2008), however, observe that identical parametrization of exogenous shocks may lead to very different dynamics through alternative models; therefore the use of the same set of priors may implicitly penalize some models while favouring others. To overcome these problems they propose a systematic approach for priors elicitation based on the division of parameters in three subsets: the steady states governing parameters, parameters governing the propagations of exogenous shocks (correlations, standard deviations etc..) and parameters governing endogenous propagation mechanisms (including taste, technology and policy parameters). They propose to calibrate the first group through "great ratios" (namely, long run averages for macro-economic variables in accordance with business cycle literature), while priors for the second ones are (jointly) selected through a quasi-likelihood function constructed on the basis of a set of endogenous variables and an approximated version of the model. For the last group, instead, no special procedure is necessary since typically researchers have beliefs originating from other sources of information (e.g. microeconomic studies). This procedure, while providing a systematic approach for prior elicitation, allows the attainment of two goals: (group of) priors distributions are jointly derived, and the priors of parameters governing the propagation of exogenous shocks are not model invariant. Finally, another issue to be aware of regards the use of other data sources for setting priors. Take, for example, micro-level data. As observed by Fernández-Villaverde (2010), translating micro-level data into macro models can often be less obvious than what one might think. Indeed, micro-data are estimated from a sub-population of individuals whereas the correspondent macro-model parameters are meant to represent the characteristics of an agent who is stand-in for the economy. Since the aggregation from micro to macro is far from trivial, one should be careful in operating such a translation as the classical example of the elasticity of labour supply suggests. At the same time, micro-estimates constitutes a natural reference point and departures from those, especially if big, should be appropriately justified. Similarly, the use of parameters estimates from other countries should be undertaken with care. This is a common practice and it is coherent with the idea, somewhat pervasive in economics, that individuals from all parts of the world are substantially the same, even though economical and cultural conditions may affect their attitudes. Taking the example of the discount factor, admittedly cultural differences, as well as differences in financial market development, may change its value from one country to another. Nonetheless, evidence from other countries constitute a source of information which should be taken into account, especially because those differences typically tend to be small.

Evaluating the likelihood function

Once priors have been elicited, the first step in exploring the posterior distribution is to evaluate the model likelihood at different points, θ, of the parameter space. In DSGE estimation this is done by employing the tools of state space representation and filtering theory. We discuss this below following closely the conceptual steps proposed by Fernández-Villaverde (2010). From the solution of the DSGE model we can get the law of motion of states in the state space representation consisting of:

- A transition equation, $X_t = f(X_{t-1}, W_t; \theta)$, where X_t is the vector of states, W_t is the vector of shocks and θ is the set of structural parameters;
- A measurement equation, $Z_t = g(X_t, V_t; \theta)$ where Z_t are the observed variables and V_t a vector of shocks defined over them (which typically have an interpretation in terms of measurements errors).

The transition equation results from the model solution and specifies the law of motion of states, X_t, whereas the measurement equation relates the set of observed data series, Z_t, to such states. These two equations allow us to link data and model variables whose joint distribution will in turn depend on the chosen parametrization, θ. This is the core mechanism at the basis of the model likelihood we derive below. Before that, a couple of aspects are worth noticing. The first one is that the number of variables in the measurement equation has to be lower or equal than the number of shocks (given by $dim\left(W_t + V_t\right)$). Whenever this condition is not satisfied the model suffers from stochastic singularity and its likelihood will be zero with probability one[4]. The second aspect involves the choice of observables. Since the condition over their number is the only restriction researchers face, this leaves many degrees of freedom. Such choice, nonetheless, should be treated carefully since variable selection may significantly affect inference, as shown by Guerron-Quintana (2010)[5]. In this regards, it is typically recommended to pick the series that are more relevant for the purpose of the model (e.g hours worked and wages should be included in a model of the labour market). Going back to model likelihood, note that given the knowledge of shocks distribution:

- from $X_t = f\left(X_{t-1}, W_t; \theta\right)$, we can get $p\left(X_t | X_{t-1}; \theta\right)$;
- from $Z_t = g\left(X_t, V_t; \theta\right)$, we can get $p\left(Z_t | X_t; \theta\right)$:
- by plugging $X_t = f\left(X_{t-1}, W_t; \theta\right)$ into $Z_t = g\left(X_t, V_t; \theta\right)$, we obtain $Z_t = g\left(f\left(X_{t-1}, W_t; \theta\right), V_t; \theta\right)$ from where we can get $p\left(Z_t | X_{t-1}; \theta\right)$

All these conditional probabilities are used in computing the likelihood, even though they enter in a kind of disguised way. In particular, given our state

[4] What would happen, in fact, is that extra observables would be a deterministic function of model endogenous variables. Since the probability of model variables to match exactly data series is zero, then the whole likelihood will be zero.

[5] In particular this author shows that the omission of some observables may dramatically influence the mode of certain parameters, affecting model prediction and forecasting performance. On the other hand, the introduction of observables for which the model has not be designed may complicate the estimation of structural parameters (although the usage of measurement errors may ameliorate this problem).

representation we can write our likelihood function $p\left(z^{T}|\theta\right)$ as follows[6]:

$$\begin{aligned} p\left(z^{T}|\theta\right) &= p\left(z_1|\theta\right)\prod_{t=2}^{T} p\left(z_t|z^{t-1};\theta\right) \\ &= \int p\left(z_1|X_1;\theta\right)\mathrm{d}X_1 \prod_{t=2}^{T}\int p\left(z_t|X_t;\theta\right)p\left(X_t|z^{t-1};\theta\right)\mathrm{d}X_t \end{aligned} \tag{3.2}$$

Note that we only need the knowledge of $\left\{p\left(X_t|z^{t-1};\theta\right)\right\}_{t=1}^{T}$ for assessing the likelihood of the model (since $p\left(z_t|X_t;\theta\right)$ is given by the measurement equation). For this task we rely on filtering theory, which is the branch of mathematics concerned with finding the sequence of states conditional distributions given noisy observations. We will take a recursive filtering approach where data are processed sequentially within a procedure consisting of two stages: *Prediction* and *Update*. The prediction stage uses the model to predict the state pdf one period ahead with respect to the measurement time. This stage relies on the Chapman-Kolmogorov equation:

$$p\left(X_{t+1}|z^{t};\theta\right) = \int p\left(X_{t+1}|X_t;\theta\right)p\left(X_t|z^{t};\theta\right)\mathrm{d}X_t \tag{3.3}$$

Whereas the update operation uses the latest measurement to modify the prediction pdf. This is achieved using Bayes theorem, which is the mechanism for updating knowledge about states in the light of extra information from new data:

$$p\left(X_t|z^{t};\theta\right) = \frac{p\left(z_t|X_t;\theta\right)p\left(X_t|z^{t-1};\theta\right)}{p\left(z_t|z^{t-1};\theta\right)} \tag{3.4}$$

where,

$$p\left(z_t|z^{t-1};\theta\right) = \int p\left(z_t|X_t;\theta\right)p\left(X_t|z^{t-1};\theta\right)\mathrm{d}X_t \tag{3.5}$$

is the conditional likelihood. Then, by recursive application of forecasting and updating we can generate the whole sequence $p\left(X_t|z^{t-1};\theta\right)_{t=1}^{T}$. However the problem of such a procedure is that involves the computation of a large number of integrals, making in many cases the whole process unfeasible. To

[6]In the following we use the standard Markov notation where z_t indicates the variable realization occurred at time t, whereas z^t is a vector of length t indicating the sequence of realizations occurred up to time t, namely $z^t = \{z_0, z_1......, z_t\}$

solve this problem we have basically two routes:

- The Kalman filter (based on restrictive assumptions that make an analytic solution available);
- The Particle filter (which relies on simulation).

In most cases, medium to large scale DSGE models are solved by log-linearizing around their deterministic steady-state and their shock distribution is assumed to be Gaussian. The resulting model is therefore linear and its likelihood function is known. Under these conditions, which apply in the context of our model, Kalman filter can be applied. Below, we will therefore focus on this approach.

The Kalman filter

The Kalman Filter is based on the assumptions that the measurement and transitional equations are linear and that shocks are normally distributed. Let us then define:

- Transition equation:

$$x_{t+1} = Fx_t + G\omega_{t+1},\ \omega_{t+1} \sim N\left(0, Q\right) \tag{3.6}$$

- Measurement equation:

$$z_t = Hx_t + v_t,\ v_t \sim N\left(0, R\right) \tag{3.7}$$

where x_t are the states and z_t are the observables. Our goal is to write and to evaluate the likelihood function for $z^t \equiv \{z_t\}_{t=1}^{T}$ at any point θ of the parameter space, namely:

$$L\left(z^T|\theta\right) = \prod_{t=1}^{T} L\left(z_t|z^{t-1}, \theta\right) \tag{3.8}$$

Given the Gaussian structure of the shocks, we will be able to rely on few properties:

- The likelihood of z^t is normal;
- the sequence of all conditional distributions $p\left(z_t|z^{t-1}, \theta\right)$ is normal;

- given normality, we will need to track only first and second central moments.

we can then develop the likelihood in equation 3.8 accordingly [7]:

$$\begin{aligned} logL\left(z^T|\theta\right) = & \sum_{t=1}^{T} logL\left(z_t|z^{t-1},\theta\right) = \\ & -\sum_{t=1}^{T}\left[\frac{n}{2}log2\pi + \frac{1}{2}log\left|\Omega_{t|t-1}\right| + \frac{1}{2}v_t{}'\Omega_{t|t-1}^{-1}v_t\right] \end{aligned} \tag{3.10}$$

where:

$$v_t = z_t - z_{t|t-1} = z_t - H'x_{t|t-1} \;\; and \;\; \Omega_{t|t-1} = H'\Sigma_{t|t-1}H + R \tag{3.11}$$

To understand the first group of equalities in equation 3.11, consider that:

$$z_{t|t-1} = E(z_t|z^{t-1}) = H'x_{t|t-1} = H'E\left(x_t|z^{t-1}\right) \tag{3.12}$$

where the first and last equality comes from arbitrary notational choice, whereas the second is derived from the measurement equation (see eq. 3.7). Finally note that:

$$\Omega_{t|t-1} = E\left[\left(z_t - z_{t|t-1}\right)\left(z_t - z_{t|t-1}\right)'|z^{t-1}\right] \tag{3.13}$$

and

$$\Sigma_{t|t-1} = E\left[\left(x_t - x_{t|t-1}\right)\left(x_t - x_{t|t-1}\right)'|z^{t-1}\right] \tag{3.14}$$

Then, from straightforward calculations, we can get the second group

[7] Note that this passage is straightforward once we note that the conditional distribution of a multivariate normal is normal itself. Then its density will be of the form:

$$f\left(z_t|z^{t-1},\theta\right) = \frac{1}{\sqrt{(2\pi)^n|\Omega_{t|t-1}|}}\exp\left(-\frac{1}{2}\left(z_t - z_{t|t-1}\right)'\Omega_{t|t-1}^{-1}\left(z_t - z_{t|t-1}\right)\right) \tag{3.9}$$

where $z_{t|t-1}$ and $\Omega_{t|t-1}$ are respectively the conditional mean and the conditional variance whose expressions, as mentioned later in the text, are known. By taking the logarithm of this density function we easily get into eq. 3.10.

of equalities in 3.11[8]. As stated above, given normality assumption (and linearity of our equations), in order to assess $L\left(z^T|\theta\right)$, we just need the sequence of all first and second central moments (v_t and $\Omega_{t|t-1}$,) from $t=0$ to T. Therefore the solution of our problem (evaluating the likelihood at a given θ) reduces to the following logical steps:

- start with $x_{t|t-1}$ and $\Sigma_{t|t-1}$ (forecast or initial condition[9]);
- observe a new z_t;
- obtain $x_{t|t}$ and $\Sigma_{t|t}$ (update);
- noting that $x_{t+1|t} = Fx_{t|t}$ and $\Sigma_{t+1|t} = F\Sigma_{t|t}F' + GQG'$, we will be able to come back to the first step and wait for z_{t+1} (new forecast).

Repeating this procedure from $t=0$ to T return us with the sequence of all first and second moments we are looking for; once we obtain them, evaluating the likelihood is just straightforward. Accordingly, our key question is how to get $x_{t|t}$ and $\Sigma_{t|t}$ from $x_{t|t-1}$, $\Sigma_{t|t-1}$ and z_t. For this task, we rely once more on the Gaussian structure of our processes. Recall that:

$$\begin{pmatrix} x_t & \\ & |z^{t-1} \\ z_t & \end{pmatrix} \sim N\left(\begin{bmatrix} x_{t|t-1} \\ H'x_{t|t-1} \end{bmatrix} \begin{bmatrix} \Sigma_{t|t-1} & \Sigma_{t|t-1}H \\ H'\Sigma_{t|t-1} & H'\Sigma_{t|t-1}H + R \end{bmatrix}\right) \tag{3.16}$$

then we can write:

$$x_t|z_t, z^{t-1} = x_t|z^t \sim N\left(x_{t|t}, \Sigma_{t|t}\right) \tag{3.17}$$

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[8] For this purpose, just note that

$$E\left[\left(z_t - z_{t|t-1}\right)\left(z_t - z_{t|t-1}\right)'|z^{t-1}\right] \equiv$$
$$E\left[H'\left(x_t - x_{t|t-1}\right)\left(x_t - x_{t|t-1}\right)'H + v_t\left(x_t - x_{t|t-1}\right)'H + H'\left(x_t - x_{t|t-1}\right)v_t' + v_tv_t'|z_{t-1}\right] =$$
$$H'\Sigma_{t|t-1}H + R \tag{3.15}$$

[9]The first iteration of the algorithm will run with x_0 and Σ_0 which are the (known) initial conditions.

where[10]:

$$x_{t|t} = x_{t|t-1} + \Sigma_{t|t-1} H \left(H' \Sigma_{t|t-1} H + R\right)^{-1} \left(z_t - H' x_{t|t-1}\right) \tag{3.18}$$

and

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1} H \left(H' \Sigma_{t|t-1} H + R\right)^{-1} H' \Sigma_{t|t-1} \tag{3.19}$$

Note that by defining the *Kalman gain* at time t, $K_t = \Sigma_{t|t-1} H \left(H' \Sigma_{t|t-1} H + R\right)^{-1}$, equations 3.18 and 3.19 can be re-written respectively as:

$$x_{t|t} = x_{t|t-1} + K_t \left(z_t - H' x_{t|t-1}\right) \tag{3.20}$$

and,

$$\Sigma_{t|t} = \Sigma_{t|t-1} - K_t H' \Sigma_{t|t-1} \tag{3.21}$$

This provides us with intuition about the updating mechanism of the state variable predictor $x_{t|t}$, whe new information z_t arrives. K_t acts as (an optimal[11]) weight defining the balance between past informations $x_{t|t-1}$ and the new information arriving a time t, z_t. In this regard, note that:

- Whenever the uncertainty associated with forecasting with past information, $x_{t|t-1}$, is large ($\Sigma_{t|t-1}$ is large) this procedure gives higher weight to the new information (K_t is large);

- If the new information tend to be noisy (R is large) we give more weight to the old prediction (K_t is small).

To conclude, notice the link between the procedure described here for the Kalman filter and the procedure described above for recursive filtering. Given the Gaussian assumption, we can determine the objects needed for computing the Chapman-Kolmogorov equation and the Bayes theorem, namely:

- $P\left(x_t | z^{t-1}; \theta\right) = N\left(x_{t|t-1}, \Sigma_{t|t-1}\right)$
- $P\left(x_t | z^{t}; \theta\right) = N\left(x_{t|t}, \Sigma_{t|t}\right)$
- $P\left(x_{t+1} | z^{t}; \theta\right) = N\left(x_{t+1|t}, \Sigma_{t+1|t}\right)$

[10] Note that eq. 3.18 and 3.19 come from the properties of conditional mean and conditional variance of a multivariate normal distribution.

[11] It can be shown that K_t is such that it minimizes the variance of $\Sigma_{t|t}$. A proof of this is provided by Fernández-Villaverde (2010).

Exploring the likelihood function

The instruments derived and described so far allow us to evaluate the model posterior distribution at any θ up to a constant, namely: $\pi\left(\theta|z^T\right) \propto f\left(z^t|\theta\right)\pi\left(\theta\right)$. Now, our issue is how to characterize this distribution in the parameter space Θ, given that we do not know its functional form. For this task we will rely on *Markov Chain Monte Carlo Simulation* (hereafter McMc) and on the *Metropolis-Hasting* algorithm which permits its implementation. To get into the core of this technique, note that typically a major concern in Markov chain theory is to determine the conditions under which there exists an invariant distribution λ and iterations of the transition probabilities $P\left(x,y\right)$ converges to it. The invariant distribution satisfies:

$$\lambda\left(y\right)=\int P\left(x,y\right)d\lambda\left(x\right) \tag{3.22}$$

McMc methods turn the theory around: the invariant distribution is known and it is the target density from which samples are desired (in our case the posterior distribution) but the transitional probabilities are not known (we don't know how to generate the chain). Accordingly to generate a sample from $\lambda\left(\cdot\right)$ the method finds and utilizes a transition function $P\left(x,y\right)$ whose n-th iterate converges to $\lambda\left(\cdot\right)$ for large n. After this large number, the distribution of the observations generated from the simulation is approximately the target distribution (namely our posterior distribution). The *Metropolis-Hastings* algorithm provides us with a constructive methods for specifying the chain[12]. Further technical details can be found in Chib and Greenberg (1995), here we provide the main intuition on the functioning of the algorithm: starting with an initial value for θ, we draw from a proposed density a new value and we evaluate the posterior at this point. If the posterior is greater at the new θ, we keep this value for the next iteration; if not, we reject it with a probability lower than one. This procedure allow us to travel towards the "higher probability region" of the likelihood function but also to explore, with some probability, lower regions. This avoid us to get trapped in local maxima. Algorithm 1 provides us a plain version of the pseudo-code for implementing Metropolis-Hastings algorithm as proposed by Fernández-

[12] In practice, what the Metropolis-Hasting does is to exploit a sufficient condition called "reversibility" for $p\left(x,y\right)$ to generate an invariant distribution

Villaverde (2010). The acceptance/rejection mechanism implied by step 3

Algorithm 1 Random Walk Metropolis-Hasting

1: **Step 0, Initalization:** Set $i = 0$ and an initial θ_i. Evaluate $\pi(\theta)$ and $p\left(z^t|\theta_i\right)$. Set $i = i + 1$.
2: **Step 1, Proposal draw:** Get a draw θ_i^* from a proposal density $q(\theta_{i-1}, \theta_i^*)$.
3: **Step 2, Proposal evaluation:** Evaluate $\pi(\theta_i^*)$ and $p\left(z^t|\theta_i^*\right)$.
4: **Step 3, Accept/Reject:** Draw $\chi_i \sim U(0,1)$. if $\chi_i \leq \frac{p\left(z^T|\theta_i^*\right)\pi\left(\theta_i^*\right)q\left(\theta_{i-1},\theta_i^*\right)}{p\left(z^T|\theta_{i-1}\right)\pi(\theta_{i-1})q\left(\theta_i^*,\theta_{i-1},\right)}$, set $\theta_i = \theta_i^*$; otherwise $\theta_i = \theta_{i-1}$.
5: **Step 4, Iteration:** if $i < M$, set $i = i + 1$ and go to step 1. Otherwise stop.

is the one discussed above: once noting that $max(\chi_i) = 1$ then, whenever $p\left(z^T|\theta_i^*\right)\pi(\theta_i^*)\, q(\theta_{i-1}, \theta_i^*) > p\left(z^T|\theta_{i-1}\right)\pi(\theta_{i-1})\, q(\theta_i^*, \theta_{i-1},)$, $\theta_i = \theta_i^*$ with probability one. However, when the opposite holds $\theta_i = \theta_{i-1}$ with probability less the one (which is higher, the lower is probability associated with $theta_i^*$). Finally, note that algorithm 1 requires us to specify a proposal density $q(\cdot,\cdot)$; the standard practice is to choose a random walk proposal (from where the name "Random Walk Metropolis Hasting"), $\theta_i^* = \theta_{i-1} + \kappa_i$, $\kappa_i \sim N(0, \Sigma_\kappa)$, where Σ_κ is a scaling matrix that researchers choose to obtain an appropriate acceptance ratio.

The next step: estimating our model

Having set out the main statistical results underpinning Bayesian estimation, as well as the computational techniques necessary for its implementation, we are now ready to move to the actual estimation of our model. We do this in the next section.

3.3 Model estimation

In this section we present the estimation of the model and discuss its results. We begin by introducing the main elements of our estimation procedure, i.e. from the summary of the structural shocks entering the model, to the observables variables employed and their treatment. Following the literature, we then calibrate a number of parameters which are typically poorly identified, and estimate the remaining. Finally, estimation results are discussed.

3.3.1 Structural shocks

The model contains 23 structural shocks. 7 of them are present in each of the two endogenous economies (i.e. Scotland and rUK), these are: (i) Temporary productivity shock; (ii) Inter-temporal preferences shock; (iii) Investment technology shock; (iv) Intra-temporal preference shock; (v) Mark-up shock; (vi) Government transfers shock and (vii) Devolved public spending shock. Each of these shocks enters twice the model, therefore they amount to 14 in total. We should then add 6 UK-wide structural shocks (including oil related shocks), namely: (i) Monetary policy shock; (ii) Non-devolved public spending shock; (iii) UK technology (unit root) shock; (iv) Oil price shock; (v) Oil production (in UK) shock;(vi) Oil marginal costs (in UK) shock. Finally there are 3 shocks associated with the ROW VAR block, these are: (i) ROW GDP shock; (ii) ROW inflation shock; (iii) ROW Interest rate shock. Table 3.1 summarizes the list above.

Country specific (x2)	**UK-wide and oil**	**ROW**
Temporary productivity	Monetary policy	Inflation (VAR)
Intertemporal preference	Non-devolved public spending	GDP (VAR)
Investment technology	Technology unit root	Interest rate (VAR)
Intratem preference	Oil marginal costs	
Mark-up	Oil price	
Gvt Transfers	Oil production	
Gvt spending		

Table 3.1: Model structural shocks

3.3.2 Observables

We estimate the model using 22 quarterly time-series, for the period 1998:Q1 to 2007:Q4[13]. These can be grouped in a similar fashion as the structural shocks discussed above. In more detail, 7 observables are specific to each of the two endogenous economies, these are: (i) GDP; (ii) Consumption; (iii) Investment; (iv) Government transfers to household; (v) Price inflation; (vi) Wage rate and (vii) Government spending. As these observables are introduced for any of the two DSGE economies, they amount to a total of 14. To this, we should 5 UK-wide (and oil related) observables, namely: (i) Interest rate; (ii) Devolved public spending; (iii) Oil price; (iv) Oil production (in UK); (v) Oil marginal costs (in UK). Finally there are 3 series associated with the ROW VAR block, these are: (i) ROW GDP; (ii) ROW inflation; (iii) ROW Interest rate. Table 3.2 synthesizes the list above.

Country specific (x2)	**UK-wide and oil**	**ROW**
GDP	Interest rate	Inflation
Consumption	Devolved public spending	GDP
Investment	Oil marginal costs	Interest rate
Gvt Transfers	Oil production	
Gvt Spending	Oil price	
GDP deflator		
Wages		

Table 3.2: Observables

Variables construction and sources are discussed in greater detail below. Meantime, a few words on the choice of the observables are due. As Guerron-Quintana (2010) showed, this choice is indeed crucial as it might affect estimation results and the inference around model's parameters. We followed three core principles to inform our observables selection: consistency with the literature, coherency with model purposes and data availability. Our observables selection is largely consistent with the estimation of closed-economy (i.e. Smets and Wouters (2003, 2007)) and open-economy (Adolfson et al. (2007, 2008), Rabanal and Tuesta (2010), Bhattarai and Trzeciakiewicz (2017)) models for the UK and comparable countries. How-

[13]The choice of the sample period has been dictated, on the lower end, by the beginning of the Scottish national quarterly accounts (dating back only until to 1998:Q1) and, on the upper end, by the beginning of the financial crisis (which we exclude as our model is not equipped to account for it).

ever, few exceptions are to be noted. Firstly, differently from some of the open economy literature we do not use trade data series. That is because rUK-Scottish import flows are not reliably measured. Secondly, and for similar reasons, we use the GDP deflator for measuring inflation whereas much of the literature employs CPI. This is dictated by the absence of CPI series for Scotland. Thirdly, having a rich fiscal sector, we allow for a number of fiscal variables (e.g. Gvt devolved spending and Gvt transfers) and oil related variables (e.g. oil price prices, UK oil production etc) which are typically not present in other studies[14].

3.3.3 Data treatment and measurement equation

All time series[15] have been transformed in real per capita terms to be consistent with their model counterparts. In particular they have all been deflated using GDP deflator and divided by working age population (i.e. population aged 16-64). Resulting series have then been expressed in growth rates by first differencing their natural logarithms; growth rates series, finally, have been demeaned (as in Rabanal and Tuesta (2010)).

To understand why demeaning is applied, just recall the law of motion of our non-stationary technological process:

$$lnX_t = lnX_{t-1} + \epsilon_t^x \tag{3.23}$$

Under such a specification, UK-wide technological change follows a random-walk without drift in its natural logarithm. That means economic growth follows a stochastic process with zero mean. In what follows we present the measurement equation linking observables to model variables. Given the relative large size of our model, for representation purposes we find it convenient to break the set of measurement equations into three groups. Grouping is undertaken following the logic above i.e. Country specific variables, UK-wide (and oil related) variables and ROW variables. The following conventions apply: '$d\ln$' indicates the first differences of the natural logarithm, '$\tilde{}$' indi-

[14]One notable exception is given by Bhattarai and Trzeciakiewicz (2017)'s model for the UK, which features a full-blow public sector and, therefore, includes an unusual number of fiscal variables (i.e. government consumption, government investment, transfers, and effective tax revenue from consumption, labour and capital) in the estimation.

[15]With the exception of: hours worked, inflation, interest rate and oil related variables.

cates (consistently with our model notation) normalized model variables[16], 'μ_g' indicates the average rate of growth of a given time-series over the time period in our sample (i.e. 1998:Q1 to 2007:Q4). Before entering the actual description of the measurement equation(s), is worth remembering that the model is input in its non-linear form. Therefore each model variable should be interpreted as the level of its real per capita data counterpart, scaled by the innovations to the unit root technological process in equation 3.23.

Country specific observables

Country specific variables are introduced for, both, Scotland and the rUK. Given they enter symmetrically in the model; we only report here the Scottish ones. Equation 3.24 below presents the measurement equation and displays, on the LHS, actual data (whose definition is provided below) and, on the RHS, model variables. Note that model variables have been scaled by unit-root technology and therefore the associated shock enters in the RHS of the measurement equation.

$$\begin{bmatrix} d\ln\frac{GDP_t^{sc}}{Pop_t^{sc}*Def_t^{sc}}-\mu_{gy} \\ \\ d\ln\frac{PrivCons_t^{sc}}{Pop_t^{sc}*Def_t^{sc}}-\mu_{gc} \\ \\ d\ln\frac{PrivInvest_t^{sc}}{Pop_t^{sc}*Def_t^{sc}}-\mu_{gi} \\ \\ d\ln\frac{Wages_t^{sc}}{Hours_t^{sc}Def_t^{sc}}-\mu_{gw} \\ \\ \ln\frac{GvtTransf^{sc}}{Pop_t^{sc}*Def_t^{sc}}-\mu_{gt} \\ \\ d\ln Def_t^{sc}-\mu_{\pi} \\ \\ ln\frac{GvtSpend^{sc}}{Pop_t^{sc}*Def_t^{sc}}-\mu_{gg} \end{bmatrix} = \begin{bmatrix} d\ln\widetilde{GDP}_t+\epsilon_t^x \\ \\ d\ln\widetilde{C}_t+\epsilon_t^x \\ \\ d\ln\widetilde{I}_t+\epsilon_t^x \\ \\ d\ln\widetilde{W}_{real,t}+\epsilon_t^x \\ \\ d\ln\widetilde{T}_t+\epsilon_t^x \\ \\ \Pi_t-1 \\ \\ d\ln\widetilde{G}_t+\epsilon_t^x \end{bmatrix} \tag{3.24}$$

Time series descriptions, together with their sources, are reported below. All series are in current prices, seasonally adjusted and refer to the onshore economy.

[16]Normalization is done by dividing growing variables for the level of technology, X_t.

GDP^{sc}

Gross Domestic Product in Scotland (onshore): Expenditure Approach. Source: QNAS.

$PrivCons^{sc}$

Final Consumption Expenditure of Households and Non-Profit Institutions Serving Households in Scotland (onshore). Source: QNAS.

$PrivInvest^{sc}$

Gross fixed capital formation in Scotland (onshore), all sectors[17]. Source: QNAS.

$Wages^{sc}$

Compensation of employees in Scotland (onshore). Source: QNAS.

$Hours^{sc}$

Weekly average hours worked. Source: Labour Productivity Statistics.

$GvtTransf^{sc}$

Government transfers to household in Scotland (onshore). Source: NiGem.

$GvtSpend^{sc}$

Government consumption in Scotland (onshore). Source: QNAS.

Def^{sc}

Implied GDP deflator in Scotland (onshore). Source: QNAS.

Pop^{sc}

Population aged 16-64 in Scotland. Source: Regional labour market stastics.

UK-wide and oil related observables

UK-wide and oil related observables refer to the UK as a whole and, therefore, enter the model only once. Among those, devolved public spending is the series of public spending in Scotland which is managed by the local government; arguably, that series can be regard as more "Scottish specific" than "UK-wide". We, nonetheless, found it convenient to include it here and keep the country-specific block above fully symmetric between Scotland and the rUK. Finally it is worth noticing that, consistently with their model counterpart, total oil production costs are transformed in per unit basis by dividing them for oil revenues. Measurement equation for these variables is reported below in equation 3.25.

[17] Government investment is subtracted. That is, in fact, accounted for within public spending.

$$
\begin{bmatrix}
\frac{IntRate_t^{uk}}{400} - \mu_R \\
d\ln \frac{DevSpending_t^{sc}}{Pop_t^{sc} * Def_t^{sc}} - \mu_{Gdev} \\
d\ln \frac{BrentPrice_t}{Def_t^{uk}} - \mu_{po} \\
d\ln OilProd_t^{uk} - \mu_{po} \\
d\ln \frac{OilCost_t^{uk}}{UkOilProd_t^{uk} * BrentPrice_t} - \mu_{mco}
\end{bmatrix}
=
\begin{bmatrix}
R_t^* - R^* \\
d\ln \widetilde{Gdev_t} + \epsilon_t^x \\
d\ln p_t^o \\
d\ln q_t^o + \epsilon_t^x \\
d\ln mc_t^o
\end{bmatrix}
\tag{3.25}
$$

Time series descriptions, together with their sources, are reported below. All series are in current prices and refer to the onshore economy.

$IntRate^{uk}$
: Quarterly average rate of discount, 3 month treasury bills, sterling. Source: Bank of England.

$DevSpending^{sc}$
: Scottish Gvt devolved consumption expenditure. Source: NiGEM/OCEA.

Def^{uk}
: Implied GDP deflator in UK. Source: ONS.

Pop^{sc}
: Population aged 16-64 in Scotland. Source: Regional labour market stastics.

$BrentPrice$
: Europe Brent Spot Price. Source: US Energy Information Administration.

$OilProd^{uk}$
: Crude oil production in UK. Source: US Energy Information Administration.

$OilCost^{uk}$
: Oil and gas production operating expenditure plus Oil and gas production capital expenditure. Source: Scottish Government, Oil and production statistics.

ROW block

Rest of the world variables are constructed as weighted average of main trading partners of U.K., U.S. and Eurozone countries, using their (total) trade shares as weights[18]. We report in equation 3.26 below the measurement

[18]Total trade is given by the sum of import and export. Trade shares are $\varpi^{us} = 21\%$ for US and $1 - \varpi^{us} = 79\%$ for Eurozone countries and are calculated averaging over the

equation for these variables; all rest of the world variables are constructed to be consistent with the UK counterparts.

$$\begin{bmatrix} d\ln\frac{GDP^{us}_{real,t}*\varpi^{us}+GDP^{eu}_{real,t}*(1-\varpi^{us})}{(Pop^{us}_t*\varpi^{us}+Pop^{eu}_t*(1-\varpi^{us}))} - \mu_{gy} \\ \\ \frac{IntRate^{us}_t*\varpi^{us}+IntRate^{eu}_t*(1-\varpi^{us})}{400} - \mu_{R^{**}} \\ \\ \ln\left(Def^{us}_t * \varpi^{us} + Def^{eu}_t * (1-\varpi^{us})\right) - \mu_{\pi^{**}} \\ \end{bmatrix} = \begin{bmatrix} d\ln\widetilde{GDP^{**}_t} \\ \\ R^{**}_t - R^{**} \\ \\ \Pi^{**}_t - 1 \end{bmatrix} \tag{3.26}$$

GDP^{eu}_{real}
: Euro Area real GDP. Source: AWM database.

Def^{eu}
: Euro Area GDP deflator. Source: AWM database.

$IntRate^{eu}$
: Short term interest rate. Source: AWM database.

GDP^{us}_{real}
: US real GDP. Source: FRED.

Def^{us}
: US GDP deflator. Source: FRED.

$IntRate^{us}$
: Federal funds rate. Source: FRED.

ϖ^{us}
: Import and Export data (goods and services) for UK. Source: ONS.

3.3.4 Calibrated parameters

Following the literature, a number of parameters are calibrated. These parameters typically identified long-run ratios (e.g. income share of capital) and are not well identifiable by the estimation procedure. The list of parameters, their calibrated values, as well as the calibration strategy adopted in each case, are reported in table 3.3. Again, given the large size of our model, we find it convenient to split parameters in different groups according to the feature of the model they affect. The description of our approach to calibration for each parameter in these groups, is provided in the subsections below. It is worth noting here that the model has been parametrized such as

sample period 1999:2007 (we actually start from 1999 and not from 1998 because that's where ONS dataset starts)

Scotland, rUK and ROW steady states are symmetric[19] in aggregate/total terms[20].

Trade and oil

Scotland's share of UK population, ϖ, is set in line with its historical average over the sample period and equals 0.085. Inter and intra-UK trade are regulated by the trade shares, $\omega^2, \omega^1, \omega^{2*}, \omega^{1*}$; given Scotland's population share, these are pinned down by matching Scotland imports share of GDP from rUK, 36%, and UK exports share to ROW, 25%, and assuming balanced trade within UK and with the rest of the world. The oil block is associated with exogenous auto-regressive processes over the real price of oil (p_t^o), its real production cost (mc_t^o) and its total produced quantity ($\tilde{Q}_t^o$). Auto-regressive coefficients and standard deviations are estimated, whereas their steady state ($\overline{Q}^o, \overline{mc}^o, \overline{p}^o$) are calibrated to match long-run targets. In particular, we use the series of Europe Brent Spot prices[21] to calculate the long run real price of oil ($\overline{p}^o$). We then employ Scottish Government statistics on Oil and Gas Production in Scotland and rUK to calculate the production costs as a share of oil revenues[22]. Finally, by multiplying the so obtained series of cost shares by the real price oil, we obtain the series of real production costs in per unit terms of which we take the long-run mean ($\overline{mc}^o$). The long-run oil quantity ($\overline{Q}^o$) is instead set to match the empirical ratio 'oil revenues to UK GDP' (over the period in question this averages to 1.29%). Given oil price, quantity and production costs we then determine the rate of tax on oil profits such as tax receipts match the observed share of UK GDP (over the period in question this averages 0.43%). This implies a rate of tax of 52%. Finally, we let the foreign asset position be determined residually from the Balance of Payments: under our balanced trade assumption, this effectively means that foreign asset position is set to sustain the

[19]Steady state symmetry implies $Y = Y^* = GDP = GDP^*$, a property which we largely exploit while normalizing and calibrating the model.

[20]Whereas they are not necessarily symmetric in per-capita terms. The most prominent case being trade: for it to be balanced, given the small size of Scotland relative to the rUK, it has to be the case that the Scottish household imports/exports significantly more/less in per capita terms than its rUK counterpart.

[21]As for estimation, this series was converted in pounds and transformed it in real terms using the GDP deflator

[22]This is done dividing production costs (calculated as the sum of operational and capital costs)by total revenues from sales.

current account surplus induced by the oil tax receipts. This gives rise to a foreign asset position of -11% of UK GDP, versus an empirical counterpart of -17%.

Technology and preferences

From the depreciation function, $\delta_t = \chi_0 m_t^{\chi_1}$, we calibrate χ_0 and χ_1 to ensure full capital utilization in steady state, i.e. $m = 1$, as well as steady state depreciation, i.e. $\bar{\delta} = \chi_0$, to match the long run rate of investment. In more detail, using Trabandt and Uhlig (2011) data from AMECO, we calculate $\bar{\delta}$ for $\frac{I}{K}$ to match sample average, implying $\delta = 0.02$. Then, by normalizing $m = 1$, the above value above for $\bar{\delta}$ immediately implies $\chi_1 = 1.51$ and $\chi_0 = 0.03$. In the production function, we set $\alpha = 0.32$ to match the average labour share of income as provided by the OECD (Real Unit Cost of Labour). Finally, we set $\beta = 0.99$ to match a steady state quarterly interest rate of 1%, a value in line with the quarterly average rate of discount in UK for the period under consideration.

Public sector

The public sector block is composed by the Westminster and the Holyrood governments. We adopt the standard approach of calibrating all elements of their balance sheets according to historical averages, leaving government transfers to be determined residually to ensure long-run budget balance. Begin with government final consumption in Scotland and in the rUK (i.e. G_t, G_t^*). These represent the sum of devolved $(Gdev_t, Gdev_t^*)$ and non-devolved $(Gnodev_t, Gnodev_t^*)$ spending in Scotland and in the rUK, each driven by an associated exogenous process. As for oil, auto-regressive coefficients and standard deviations associated with these processes are estimated, whereas their steady states $(\overline{Gdev}, \overline{Gdev}^*, \overline{Gnodev}, \overline{Gnodev}^*)$ are set to match calibration targets. In particular, we match the government consumption share of GDP amounting to 18% over the sample period in UK (assuming this share is same in Scotland and in the rUK), as well as the share of devolved spending over total public spending in Scotland, of about 85%[23] (assuming this share is same in the rUK). It is worth noting here,

[23] To calculate this share, we employ the NiGEM series 'Scottish public consumption' (SCGC) and 'Holyrood consumption' (SCHGC) which are in turn based on OCEA cal-

this implies the non-devolved spending administered by the government in Westminster is allocated, in per-capita terms, equally among Scotland and the rUK, i.e. $\zeta = 1$. Furthermore, we set the debt/gdp ratio in UK to match the sample average amounting to 38% of GDP[24]. On the revenue side, we adopt the procedure proposed by Mendoza et al. (1994) to calculate aggregate tax rates on labour, capital and consumption. Using Trabandt and Uhlig (2011)'s dataset, we adjust the tax rates they originally calculated for the UK for them to cover our sampling period, 1998-2007. We find $\tau^c = 0.17$, $\tau^l = 0.28$ and $\tau^k = 0.47$. Finally, we set fiscal rule coefficient, $\rho^{up} = 0.09$, to ensure equilibrium stability.

Market structure and competitiveness

Most of the parameters in this group are estimated; we follow the literature in calibrating the ones which are typically poorly identified. In particular, ϵ_p and ϵ_w, regulating the long-run degree of mark-up in the intermediate good and in the labour market sectors, are set to $\epsilon_p = 11$ and $\epsilon_w = 6$, following a standard convention in the DSGE literature[25]. Similarly, we set $\bar{\mu} = \bar{\mu}^* = 1$, so as the mark-up in the final good sector - which comes on top of the one in the intermediate good sector - is normalized to be absent in steady state while fluctuating over the transitional dynamics in response to its own shock realizations. The elasticity of substitution between domestically produced and imported goods, θ, is set to 0.94 in line with Rabanal and Tuesta (2010). Finally, we set the coefficients regulating the risk premium, χ^b and χ^d, both equal to 0.005; this value was chosen to ensure equilibrium uniqueness/stability.

Table 3.3: Calibrated parameters

Parameter	Description	Value	Calibration strategy
	Trade & oil:		
ϖ	Scot population share	0.085	Population data

(Continued on next page)

culations. Taking the ratio between these two and averaging over our sample period, we find a devolved expenditure share of about 85%.

[24]This is calculated using the series of general government consolidated gross debt, which is consistent with Maastricht requirements.

[25]See, among others, Rabanal and Tuesta (2010).

Table 3.3: (continued)

Parameter	Description	Value	Calibration strategy
ω^2	Scot import from rUK	0.36	Export data & balanced trade
ω^1	Scot cons of domestic good	0.39	Export data & balanced trade
ω^{2*}	rUK import from Scot	0.03	Export data & balanced trade
ω^{1*}	rUK cons of domestic good	0.72	Export data & balanced trade
$\frac{\bar{D}^{uk}}{4Y^*}$	UK asset position	-0.11	Residually determined
$\frac{\bar{TO}}{\bar{Y}^*}$	Oil tax receipt	-0.0043	Gvt revenues data
τ^o	Tax on oil profits	0.52	Set to match TO
	Technology & Preferences:		
β	Discount factor	0.99	Set to match $R = 1.01$
α	Capital income share	0.32	Income share data
$\delta(\bar{m})$	Depreciation rate in ss	0.02	Set to match $\frac{I}{K}$
χ_0	$\delta(\cdot)$ coefficient	0.03	set to yield $\delta(\bar{m}) = 0.02$
χ_1	$\delta(\cdot)$ exponent	1.50	set to yield $\bar{m} = 1$
	Public sector:		
$\frac{B^{\bar{U}K}}{4Y^*}$	Debt/GDP in UK	0.38	Gvt debt data
$\frac{G}{Y}$	Gvt cons over GDP	0.18	Quarterly national account
$\frac{G^{dev}}{G}$	share of devolved gvt cons	0.85	NiGEM/OCEA calculations
ζ	Pc non-devolved gvt cons	1	rUK-Scot steady-state symmetry
$\frac{T}{Y}$	Gvt transfers over GDP	0.24	Budget balance
τ^c	Consumption tax	0.17	Trabandt and Uhlig (2011)
τ^l	Labour tax	0.28	Trabandt and Uhlig (2011)
τ^k	Labour tax	0.47	Trabandt and Uhlig (2011)
ρ^{up}	Fiscal rule	0.09	Equilibrium stability
	Markets & Competitiveness:		
ϵ_w	Elast of subst between types of labor	6	Rabanal and Tuesta (2010)
ϵ_p	Elast of subst between types of good	11	Rabanal and Tuesta (2010)
$\bar{\mu}$	Final good mark-up	1	Normalization
θ	Elast of subst between goods	0.94	Rabanal and Tuesta (2010)
χ^b	Uk bond holding cost	0.005	Equilibrium stability
χ^d	Uk bond holding cost	0.005	Equilibrium stability

3.3.5 Estimation results

We estimate the model using the Bayesian, likelihood-based, approach previously described. This procedure requires us, firstly, to assign a prior distribution to the parameters to be estimated and, then, to obtain and to evaluate their posterior. Such a posterior distribution combines the information from the model likelihood, given data, and the pre-sample knowledge represented by the priors. We begin by setting parameter priors to be largely consistent with the relevant literature, among others Smets and Wouters (2003, 2007), Justiniano and Preston (2010) and Rabanal and Tuesta (2010). Specifically, we employ diffuse priors over parameters governing model exogenous processes and shocks (i.e. standard deviations and persistence), given we own little information about those, i.e. they are largely model-specific as noted by Del Negro and Schorfheide (2008). On the other hand, we impose more informative priors over deep parameters such as the elasticity of labour supply and calvo lotteries for which we own significant evidence coming from related studies (e.g. DSGE models estimated for other countries), as well as from other pieces of literature (e.g. micro-level studies)[26]. We then approximate the mode of the posterior distribution by maximizing the log posterior function. Successively, we use the Metropolis-Hastings algorithm to explore the posterior distribution using a markov chain formed by 500,000 draws, the first 100,000 of which are dropped. Our acceptance rate of 23% is broadly in line with the optimal acceptance rate of 0.234 calculated by Roberts et al. (1997). Finally we test convergence of the Markov chain to its stationary distribution using the Brooks and Gelman (1998) diagnostic; this test suggests convergence is achieved.

Estimation results are presented in Table 3.4, for model parameters, and Table 3.5, for the standard deviations of structural shocks. Note that deep parameters are imposed to be symmetric between Scotland and the rUK. This is a quite common strategy in densely parametrized, medium-to-large scale, two-country models such as ours. In our model, this strategy appears even more appropriate once considering the short length of Scottish time-series[27]. On the other hand, shock processes (i.e. standard deviations and persistence) are country specific.

[26]In all cases, however, we maintain fairly diffuse priors in line with the positive purpose of this work.

[27]We have also considered experiments where deep parameters were allowed to be asymmetric between Scotland and the rUK. Results from these exercises suggested that the estimation of those parameters might, indeed, differ. This is an aspect worth of further future investigation. Differences in, say, the elasticity of labour supply could affect the economic adjustment within the UK and the policy implications of the model. At the same time, an even more densely parametrized structure can pose threats to mode optimization, as well as to parameters identification.

Stochastic processes

Begin by analyzing the estimation results for the shocks processes looking at their auto-regressive coefficients, reported in table 3.4, and their standard deviation, reported in table 3.5. It is typically the case that the size of shocks is larger in Scotland (with the exception of the consumption shock), whereas at the same time, those shocks tend to die out quicker in Scotland than in the rUK. Looking at the unconditional variances of the associated stochastic processes, in table 3.6, it appears that their volatility for Scotland and for the rUK are largely comparable. All in all, this seems to suggest that, being in a currency and fiscal union, those countries experience about the same volatility. However, smaller and less diversified parts of the union (i.e. Scotland) are more likely to be exposed to larger shocks which are quickly absorbed, thanks to the risk-sharing within the UK. On the contrary, shocks hitting larger parts of the union (i.e. the rUK) tend to be smaller in size but appear generally more persistent, likely because of the reduced extent of risk-sharing. Furthermore, it is worth noticing that the standard deviation of the labour supply shock in Scotland is of a larger magnitude compared to the rest. This could in principle occur for a number of reasons, from strong volatility in the Scottish labour market, to a mis-specification of the model along this dimension. It seems reasonable to believe that both factors are at play. Indeed, in the current policy debate, it is acknowledge that the labour market plays an important role in the economic adjustment of Scotland, thanks to phenomena such as labour force migration between Scotland and the rUK[28], underemployment (e.g. involuntary part-time working) and wage adjustment. These aspects are only partially captured by our model (e.g. migration and underemployment indistinctly map into leisure). Therefore, a large standard deviation of the Scottish labour supply shock may indicate that a significant part of the adjustment occurs through the Scottish labour market, as well as that our model is failing to capture in full the features and the mechanisms at work there. Finally, let us consider the shocks to

[28]Statistics on migration seem to indicate that this phenomenon is of modest size, with net migration typically involving less than 1% of the Scottish labour force on annual basis. However, these statistics have a number of limitations. For example, they do not provide information at quarterly frequency and they do not capture aspects such as people working in the rUK during the week while maintaining the Scottish residence. As it is not clear at this point how much these factors could weight, labour migration cannot be simply ruled out.

the exogenous block of the model, namely those associated with oil and the ROW. Not surprisingly oil processes are strongly volatile, reflecting the wide movements in this market over the last 30 years. As for the ROW, it is important to recall that the associated VAR(1) is estimated in a reduced-form fashion and, therefore, its coefficients do not have a structural interpretation. For this reason, parameters such as *ayr* and *apr* representing, respectively, the response of output and inflation to an increase in the interest rate, do not have the sign economic theory would suggest. VAR(1) shocks appear to be small.

Deep parameters

Let us now move to analyze the estimation of deep parameters in table 3.4. Estimated parameters appear to be broadly consistent with the DSGE literature considered so far. This is certainly the case for the Taylor rule coefficients, the inverse elasticity of labour supply with respect to wage and calvo lotteries for prices. On other hand, calvo lotteries for wages, as well habits, are somehow lower than in other studies[29]. Finally, consider the share of local workers in Scotland, θ_L. This parameter has no obvious counterpart in related studies and it was introduced to capture a feature of the interrelations between Scotland and the rUK. Our estimation indicates that this share is about 60%, meaning that 40% of workers in Scotland earn the wage prevailing in the rUK.

[29]When estimating the model letting deep parameters differing across Scotland rUK, calvo lottery parameters (both for prices and wages) are found to be lower in the Scotland than in the rUK. Typically they take, in the rUK, values which are very close to the ones from other estimated model for the UK, such as Harrison and Oomen (2010). On the other hand, when restricting them to be the same, they appear to be closer to the rUK ones (not surprisingly given the larger weight it has in the model) while, however, being lower than when estimating them for the rUK only (because of Scotland being factored in).

Table 3.4: Estimation results (parameters)

Parameter		Prior			Posterior	
Name	Description	Dist.	Mean	Stdev	Mode	Stdev
ρ_c	consumpt shock	beta	0.700	0.2000	0.8666	0.0912
ρ_n	labour supply shock	beta	0.700	0.2000	0.2443	0.1276
ρ_v	investment shock	beta	0.700	0.2000	0.4478	0.1915
ρ_a	technology shock	beta	0.700	0.2000	0.8940	0.0750
ρ_{tr}	gvt transfers shock	beta	0.700	0.2000	0.8917	0.1153
${\rho_c}^*$	consumpt shock	beta	0.700	0.2000	0.9839	0.0105
${\rho_n}^*$	labour supply shock	beta	0.700	0.2000	0.4082	0.1424
${\rho_v}^*$	investment shock	beta	0.700	0.2000	0.4937	0.2111
${\rho_a}^*$	technology shock	beta	0.700	0.2000	0.9659	0.0119
${\rho_{gdev}}^*$	gvt dev spend shock	beta	0.700	0.2000	0.9445	0.0825
ρ_{tr}^*	gvt transfers shock	beta	0.700	0.2000	0.8930	0.0531
ρ_{po}	oil price shock	beta	0.700	0.2000	0.9593	0.0249
ρ_{mco}	oil mg costs shock	beta	0.700	0.2000	0.9236	0.0583
ρ_{qo}	oil production shock	beta	0.700	0.2000	0.9967	0.0051
ρ_{Gnodev}	gvt non-dev spend shock	beta	0.700	0.2000	0.8444	0.0719
ayy	ROW VAR(1), y on y	norm	0.800	0.2000	0.9626	0.0502
ayp	ROW VAR(1), π on y	norm	0.000	0.3000	-0.3979	0.2120
ayr	ROW VAR(1), r on y	norm	0.000	0.3000	0.1735	0.1781
app	ROW VAR(1), π on π	norm	0.500	0.2000	0.2069	0.1471
apy	ROW VAR(1), y on π	norm	0.000	0.3000	0.0047	0.0312
apr	ROW VAR(1), r on π	norm	0.000	0.3000	0.0016	0.1645
arr	ROW VAR(1), r on r	norm	0.900	0.2000	0.8079	0.059
ary	ROW VAR(1), y on r	norm	0.000	0.3000	-0.0122	0.0137
arp	ROW VAR(1), π on r	norm	0.000	0.3000	0.1570	0.0912
θ_L	share of local workers	beta	0.660	0.2000	0.5959	0.1440
φ_R	Taylor rule, interest	beta	0.500	0.2500	0.7716	0.0398
φ_π	Taylor rule, inflation	norm	1.500	0.2500	1.8605	0.1455
φ_y	Taylor rule, output	norm	1.000	0.2000	1.1583	0.1405
b	Habit Sc	beta	0.500	0.2000	0.2215	0.0797
η	Inv. elast. of labour	gamm	1.000	0.2500	0.4856	0.1522

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Table 3.4: (continued)

Parameter		Prior			Posterior	
Name	Description	Dist.	Mean	Stdev	Mode	Stdev
θ_w	Calvo, wages	beta	0.600	0.2500	0.2927	0.0986
θ_H	Calvo, prices	beta	0.600	0.2500	0.6980	0.0544
λ_H	Price indexation	beta	0.500	0.2500	0.0229	0.0319
λ_W	Wage indexation	beta	0.500	0.2500	0.0832	0.1172
ϕ	Investment adj costs Sc	norm	1.000	0.5000	0.3912	0.1278

Table 3.5: Estimation results II (std structural shocks)

Parameter		Prior			Posterior	
Name	Description	Dist.	Mean	Stdev	Mode	Stdev
ϵ^c	std consumpt shock	invg	0.010	Inf	0.0123	0.0022
ϵ^n	std labour shock	invg	0.010	Inf	0.1501	0.0726
ϵ^v	std investment shock	invg	0.010	Inf	0.0175	0.0063
ϵ^a	std technology shock	invg	0.010	Inf	0.0460	0.0121
ϵ^{tr}	std transfers shock	invg	0.010	Inf	0.0120	0.0018
ϵ^{μ}	std mark-up shock	invg	0.010	Inf	0.0127	0.0015
ϵ^{gdev}	std dev speding shock	invg	0.010	Inf	0.0091	0.0028
ϵ^{c*}	std consumpt shock	invg	0.010	Inf	0.0300	0.0158
ϵ^{n*}	std labour shock	invg	0.010	Inf	0.0343	0.0142
ϵ^{v*}	std investment shock	invg	0.010	Inf	0.0127	0.0040
ϵ^{a*}	std technology shock	invg	0.010	Inf	0.0310	0.0071
ϵ^{tr*}	std transfers shock	invg	0.010	Inf	0.0193	0.0024
$\epsilon^{\mu*}$	std mark-up shock	invg	0.010	Inf	0.0041	0.0007
ϵ^{gdev*}	std dev speding shock	invg	0.010	Inf	0.0106	0.0016
ϵ^{x}	std unit root shock	invg	0.010	Inf	0.0125	0.0017
ϵ^{m*}	std taylor rule shock	invg	0.010	Inf	0.0026	0.0004
ϵ^{gnodev}	std non-dev speding shock	invg	0.010	Inf	0.0426	0.0049
ϵ^{po}	std oil price shock	invg	0.010	Inf	0.1323	0.0151

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Table 3.5: (continued)

Parameter		Prior			Posterior	
Name	Description	Dist.	Mean	Stdev	Mode	Stdev
ϵ^{mco}	std oil mg cost shock	invg	0.010	Inf	0.1436	0.0163
ϵ^{qo}	std oil production shock	invg	0.010	Inf	0.0396	0.0046
ϵ^{y**}	std ROW y shock	invg	0.010	Inf	0.0033	0.0004
$\epsilon^{\Pi**}$	std ROW π shock	invg	0.010	Inf	0.0021	0.0002
ϵ^{R**}	std ROW r shock	invg	0.010	Inf	0.0016	0.0002

Stochastic process	**Description**	**Unconditional variance**
Dc_t	Intertemporal shock (Scotland)	0.0006
Dc_t^*	Intertemporal shock (rUK)	0.0282
Dn_t	Intratemporal shock (Scotland)	0.0240
Dn_t^*	Intratemporal shock (rUK)	0.0014
V_t	Investment shock (Scotland)	0.0003
V_t^*	Investment shock (rUK)	0.0002
A_t	Productivity shock (Scotland)	0.0105
A_t^*	Productivity shock (rUK)	0.0143
μ_t	Mark-up shock (Scotland)	0.0001
μ_t^*	Mark-up shock (rUK)	0.0000
T_t	Gvt transfers shock (Scotland)	0.0007
T_t^*	Gvt transfers shock (rUK)	0.0018
$Gdev_t$	Gvt devolv. spending (Scotland)	0.0000
$Gdev_t^*$	Gvt devolv. spending (rUK)	0.0010

Table 3.6: Unconditional variances

3.4 Structural shocks in the UK economy

Having estimated the model, we now turn to analyse its quantitative properties in this section. In particular, we study the dynamics implied by a number of structural shocks to the Scottish and the rUK economies by assessing the associated IRFs produced by our model. In particular, we consider a shock to the following processes: (i) temporary productivity in Scotland, (ii) monetary policy in the UK, (iii) demand in the rUK, (iv) investment in the rUK, and (v) labour supply in Scotland. In each case we consider the response to one standard deviation innovation to the associated shock[30]. It should be recalled here that our model has been solved by linearizing around the non-stochastic steady state; the resulting IRFs should be interpreted accordingly. For any shock for which the dynamics are analysed, we present five different figures which can be broadly categorized as follows. First two figures describe the dynamics of Scottish specific variables, second two report their rUK correspondent, finally the last one looks at the movements of UK-wide aggregates.

3.4.1 Temporary productivity shock

We discuss below the effect of a temporary labour-augmenting technology shock affecting the Scottish Economy in an idiosyncratic manner.

Start with Figure 3.1 for Scotland. Consumption (C) increases whereas labour supply (N) falls, most likely as a results of a wealth effect and because of the increase in wages. As tobin-q is correctly anticipated to increase, investment increases and capital accumulates during the transition. Marginal costs of intermediate good firms (MC) fall and so does their price inflation (Pih). This dynamic is broadly in line with the one of Smets and Wouters (2003).

Figure 3.2 presents remaining set of Scotland-specific variables. Final good inflation (Pi) falls, as a result of falling intermediate good inflation. Similarly, because of the fall in the intermediate goods prices, real exchange rates with respect to the rUK (RER) and ROW (RERrow) both depreciate boosting exports toward the rUK (Yhstar) and the ROW (Yhstarstar);

[30]The standard deviations of these shocks, as well as the persistence of the associated forcing processes, are reported in tables 3.4 and 3.5.

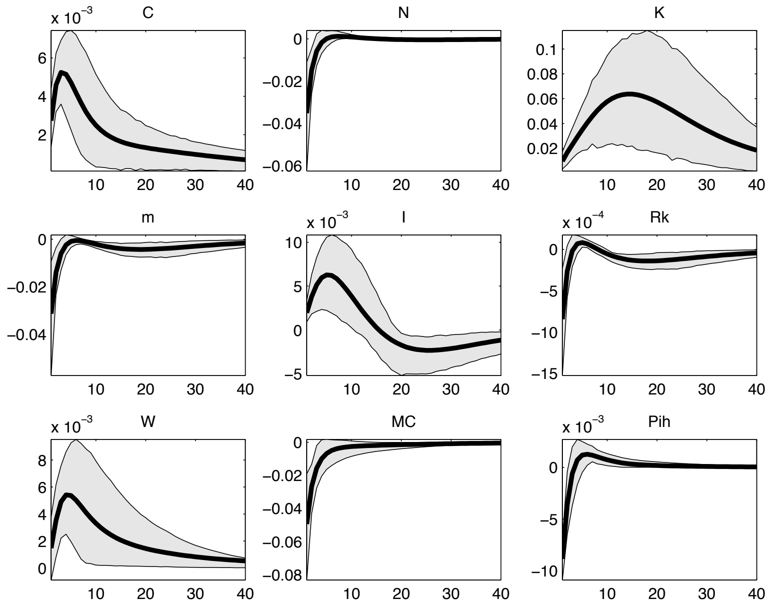

Figure 3.1: IRFs from temporary productivity shock - 1 (Scotland)

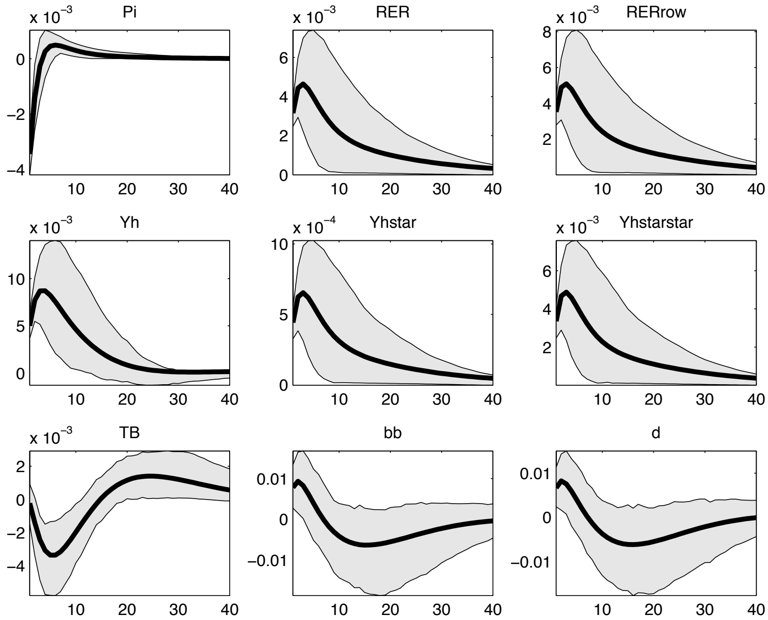

Figure 3.2: IRFs from temporary productivity shock - 2 (Scotland)

at the same time, domestic consumption of intermediate good increases as well. Since the productivity shock boosts both consumption (which includes imports) and exports, the effect on the trade balance is mixed resulting in fluctuating behaviour. A similar mixed behaviour is displayed by domestic and foreign denominated assets (i.e. 'bb' and 'd') for similar reasons.

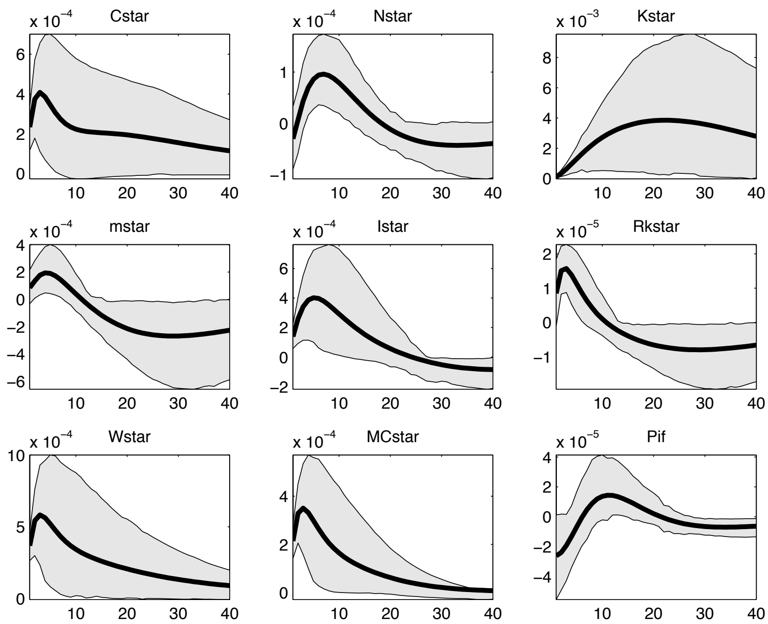

Figure 3.3: IRFs from temporary productivity shock - 3 (rUK)

Figure 3.3 presents Figure 3.1 counterparts for rUK variables. Since Scotland is relatively small with respect to the rUK, many feedback effects are small and their impact on rUK variables is negligible. Production of intermediate goods in the rUK increases, as a result of a greater demand from Scotland, boosting marginal costs, wages, investment and consumption in rUK. On the other hand, intermediate goods inflation appears mainly insensitive to these movements. This happens because lower prices in Scotland forces rUK firms to reduce their mark-up to maintain competitiveness.

Figure 3.4 presents the counterparts for rUK variables of figure 3.2 for Scotland. Inflation in rUK slightly reduces as a results of cheaper import from Scotland. However, as noticed above, the real exchange rate with respect to Scotland appreciates (RERstar) since the fall in Scottish CPI is of much larger magnitude. Imports into Scotland fluctuate, however this has a negligible impact on the rUK trade balance given the relative small size of

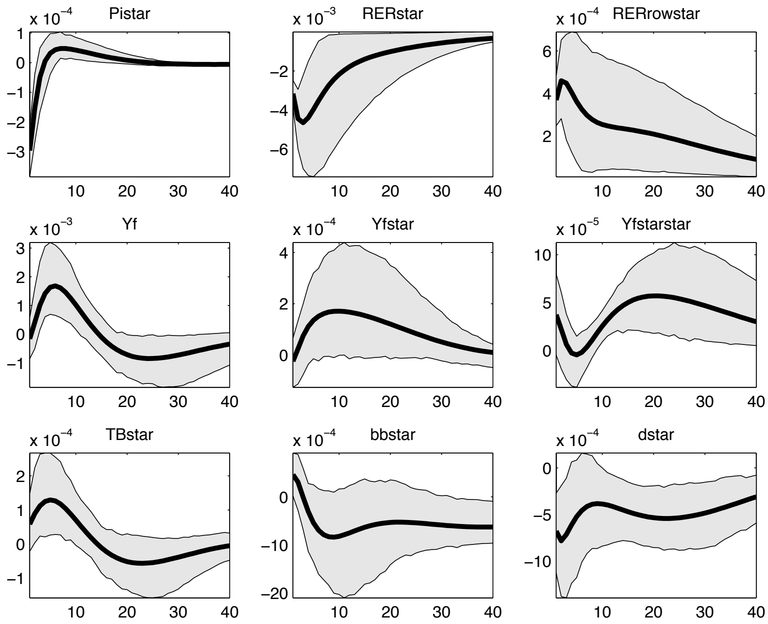

Figure 3.4: IRFs from temporary productivity shock - 4 (rUK)

Scotland.

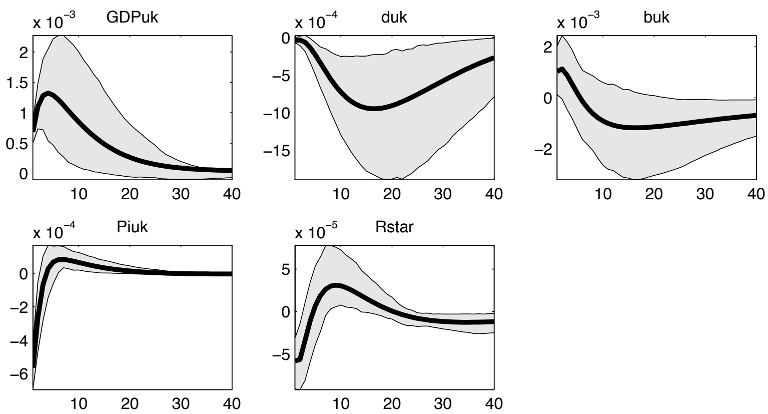

Figure 3.5: IRFs from temporary productivity shock - 5 (UK)

Finally, figure 3.5 displays the behaviour of UK aggregates. Since both Scottish and rUK GDP increase, so does UK GDP (GDPuk). However, the GDP response is much stronger in Scotland which weighs little in the Taylor rule and is accompanied by a decrease in price inflation; as a result, the interest rate response is negligible. The dynamics of the UK foreign asset position (duk) suggests that the the UK overall trade balance is somewhat countercyclical, though fluctuating. Finally, the productivity shock boosts tax revenues during the transition, temporarily reducing government debt.

3.4.2 Monetary policy shock

We now turn to analyze the effect of a monetary policy shock hitting the UK Economy.

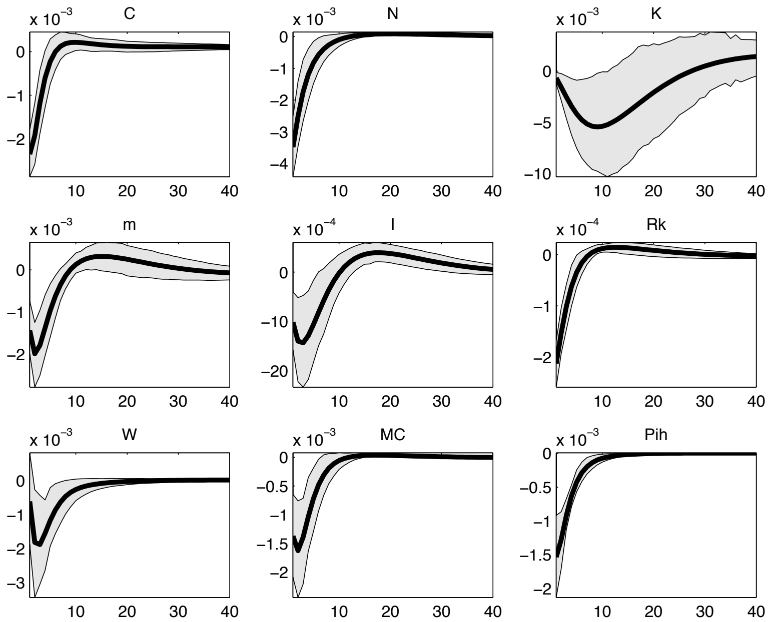

Figure 3.6: IRFs from monetary policy shock - 1 (Scotland)

Start with Figure 3.6 for Scotland. Consistently with the literature (again see Smets and Wouters (2003) as well as Christiano et al. (2005)) our model predicts that, following an increase in the interest rate, consumption, investment and employment all fall. A higher interest rate induces households to save (i.e. postpone consumption), and increases the user-cost of capital, reducing therefore investment and, consequently, capital during the transition. The real rental rate of capital initially decreases due to the fall of employment to subsequently recover as capital decumulates (and its productivity increases); for similar reasons, wages initially increases to then decrease. Altogether, this implies a reduction in marginal costs and consequently of intermediate goods price inflation.

Figure 3.7 presents the remaining set of Scottish variables. CPI inflation falls as a result of falling intermediate goods inflation, however since

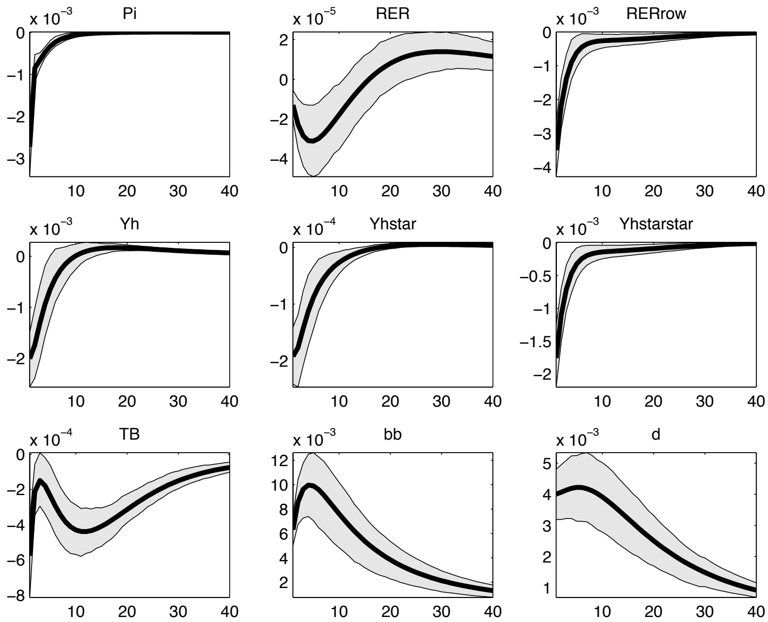

Figure 3.7: IRFs from monetary policy shock - 2 (Scotland)

Scotland and rUK are broadly symmetrically affected by this shock, the real exchange rate within UK remains largely unaffected. On the other hand, the real exchange rate with respect to the ROW appreciates on impact, coherently with the Uncover Interest Rate Parity (UIP) condition. UIP, indeed, requires an expected depreciation to balance the interest rate differential to ensure international financial markets equilibrium. The fall in consumption in Scotland and rUK, as well as the exchange rate appreciation with respect to ROW, causes intermediate goods production both for export and domestic market to fall. The trade balance initially improves then deteriorates, while capital is attracted from abroad (i.e. 'd' rises) and Scottish households save more (i.e. 'bb' increases).

Figures 3.8 and 3.9 present rUK counterparts of 3.6 and 3.7. As expected, responses in rUK are qualitatively similar to those in Scotland though somehow more pronounced. The main difference occurs in the dynamics of the trade balance, which in the rUK diplays a more significant improvement on impact and, subsequently, a much milder deterioration.

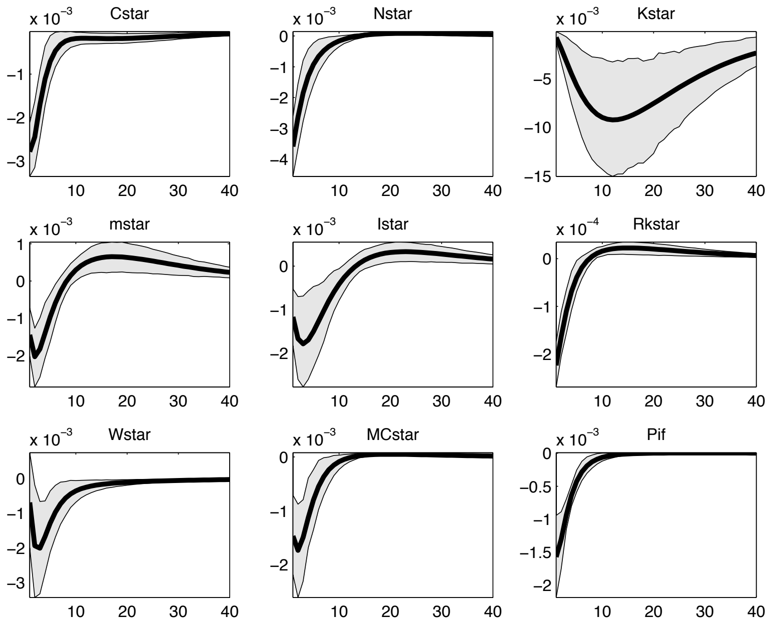

Figure 3.8: IRFs from monetary policy shock - 3 (rUK)

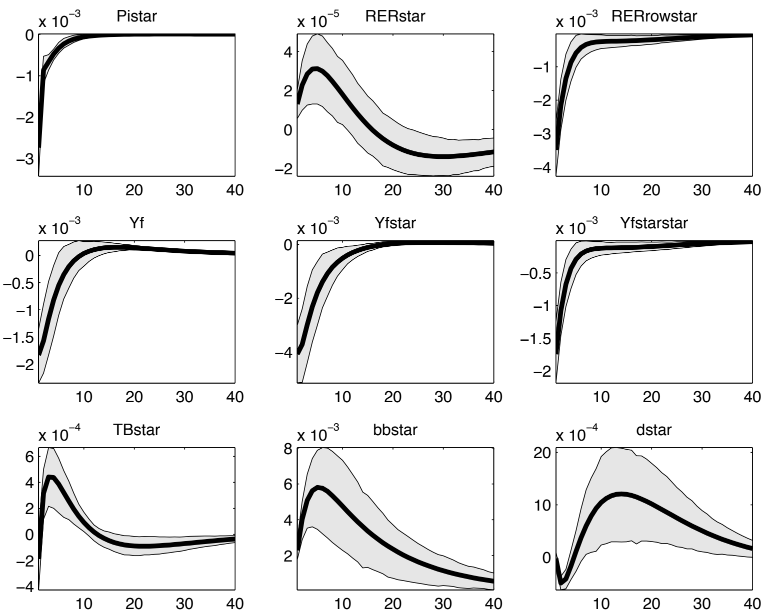

Figure 3.9: IRFs from monetary policy shock - 4 (rUK)

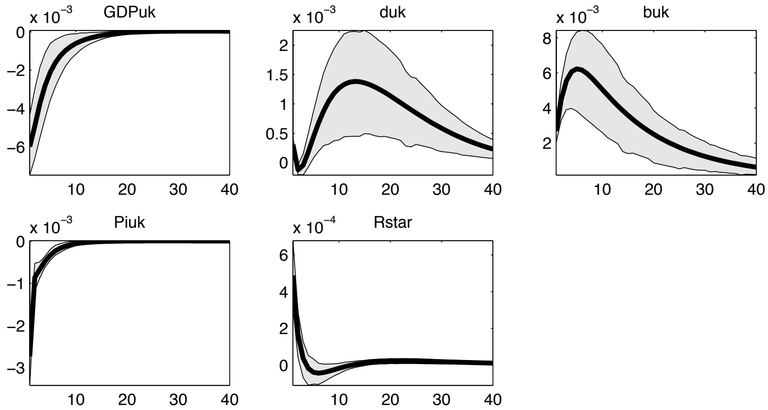

Figure 3.10: IRFs from monetary policy shock - 5 (UK)

Finally figure 3.10 presents UK-wide variables. UK GDP falls in response to the monetary policy shock, causing tax revenues to follow suit. As a result, government debt accumulates in the short run and so does foreign debt.

3.4.3 Demand (discount factor) shock

The next shock we consider is a demand shock hitting the rUK economy in an idiosyncratic manner.

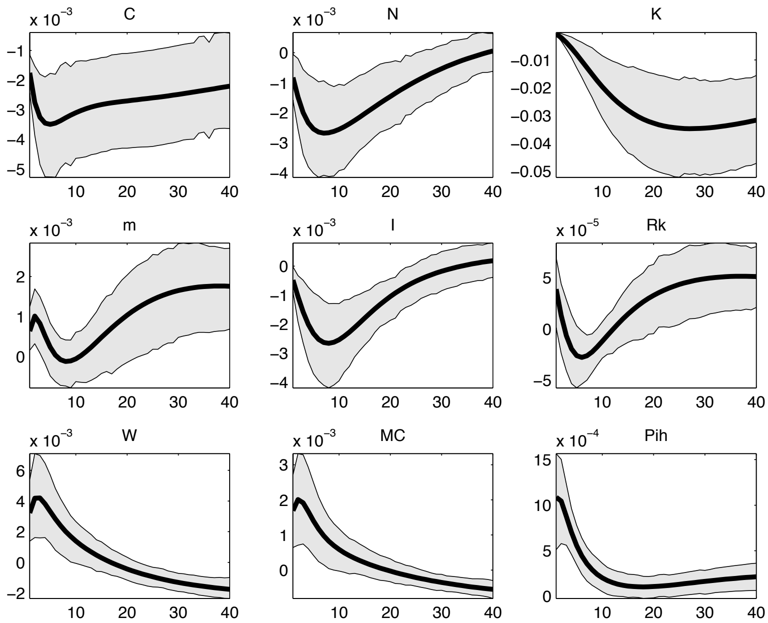

Figure 3.11: IRFs from demand shock in rUK- 1 (Scotland)

Start with figure 3.11 for Scotland. A demand shock in rUK feeds back to the Scottish economy via exported goods. As rUK is large relative to Scotland, an increase in rUK demand for Scottish exports substantially increases Scottish production causing an increase in marginal costs and therefore in prices. Contemporaneously, consumption falls as monetary policy responds (see figure 3.15) raising interest rates. On the other hand, investment and labour display an fluctuating behaviour which tracks the one in Scottish GDP (again, see figure 3.15). Such oscillating behaviour is in turn the result of contrasting forces: on the one hand an expansionary effect due to increased demand for Scottish exports from the rUK, on the other a contractionary one caused by the Bank of England's response (which depresses consumption etc.).

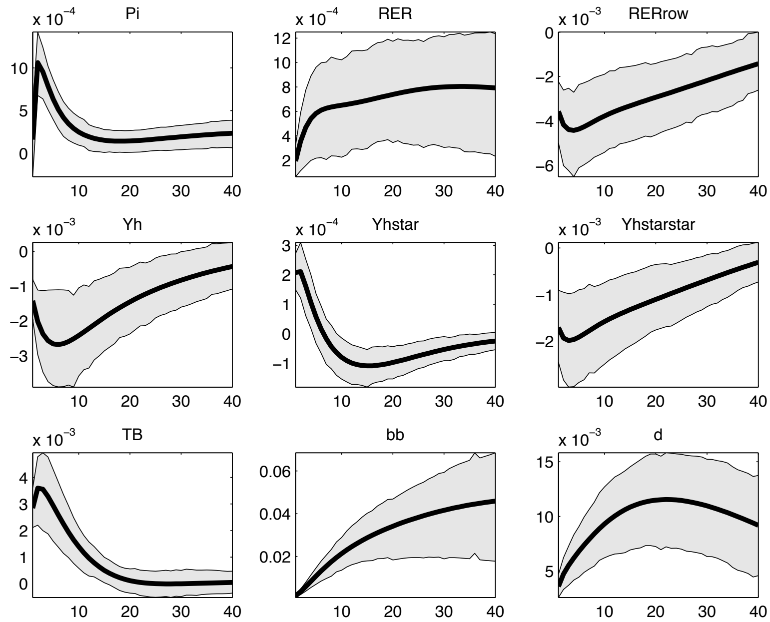

Figure 3.12: IRFs from demand shock in rUK- 2 (Scotland)

Figure 3.12 presents the remaining set of Scottish variables. Raising intermediate goods inflation causes their demand to fall at home and in the ROW; on the other hand, their export toward the rUK increases. Overall, the Scottish balance of trade exhibits a prolonged surplus. Meantime, Scottish households accumulate UK and ROW denominated assets as the increase in the interest rate positively affects their propensity to save. Finally, the Scottish real exchange rate with respect to the rUK depreciates indicating that inflation is raising faster in the rUK, where the demand shock originates.

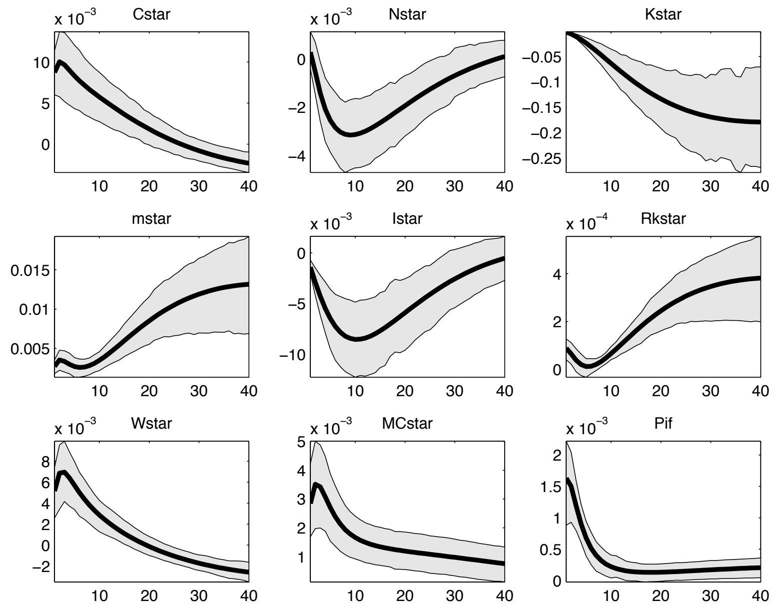

Figure 3.13: IRFs from demand shock in rUK- 3 (rUK)

In figure 3.13, rUK variables displays a similar pattern to their Scottish counterpart in figure 3.11, though the magnitude of their deviations from steady state appears stronger. The main exception is consumption which, contrary to the Scottish case, increases on impact. This is the result of the demand shock itself, i.e. a shock to the discount factor, affecting the rUK in an idiosyncratic manner. The latter increases the relative importance of utility today (adjusted for habits), pushing rUK households to consume more in the near term while sacrificing future consumption.

Because of the rise in rUK inflation (rising faster than in Scotland) and because of the BoE raising interest rate, the real exchange rate in the rUK appreciates both with respect to Scotland and with respect to the ROW. As a result, exports towards both regions fall and trade balance displays an initial, sustained deficit which is financed by borrowing from abroad. This is then subsequently offset by modest but prolonged surpluses which lead to debt repayment (see figure 3.14). This dynamic is consistent with the intertemporal approach to the trade balance discussed by Uribe (2014). Namely, households in the rUK use borrowing from abroad for optimally adjusting their inter-temporal consumption profile in response preference

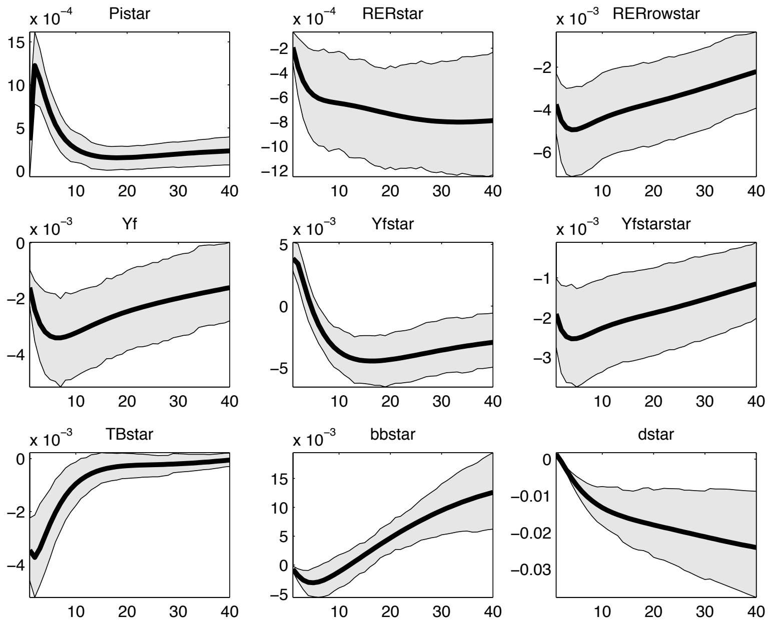

Figure 3.14: IRFs from demand shock in rUK- 4 (rUK)

shocks affecting their economy in an idiosyncratic manner.

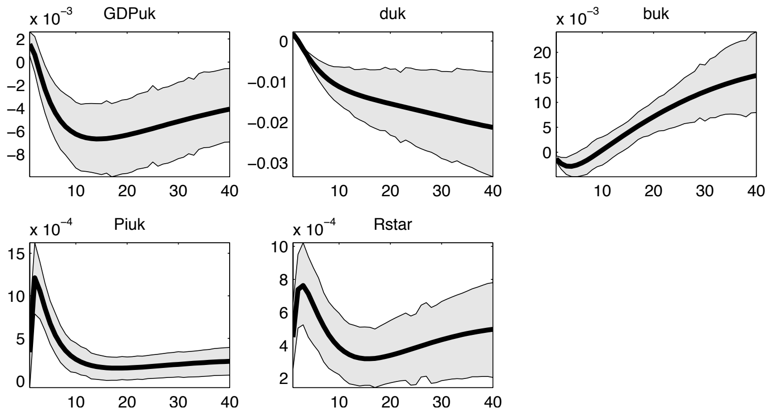

Figure 3.15: IRFs from demand shock in rUK- 5 (UK)

Finally, figure 3.15 shows the dynamics of main UK aggregates. There we can appreciate that the overall international asset position of UK deteriorates during the transition, for the rUK asset responses dominate the Scottish ones. Similarly government debt increases during the transition in response to public finance deficits due to falling tax revenues.

3.4.4 Investment technology shock

We now consider the effect of an investment technology shock hitting the rUK economy (household) in an idiosyncratic manner. Such a shock has the consequence of making the production of investment goods (capital) more efficient for a prolonged time.

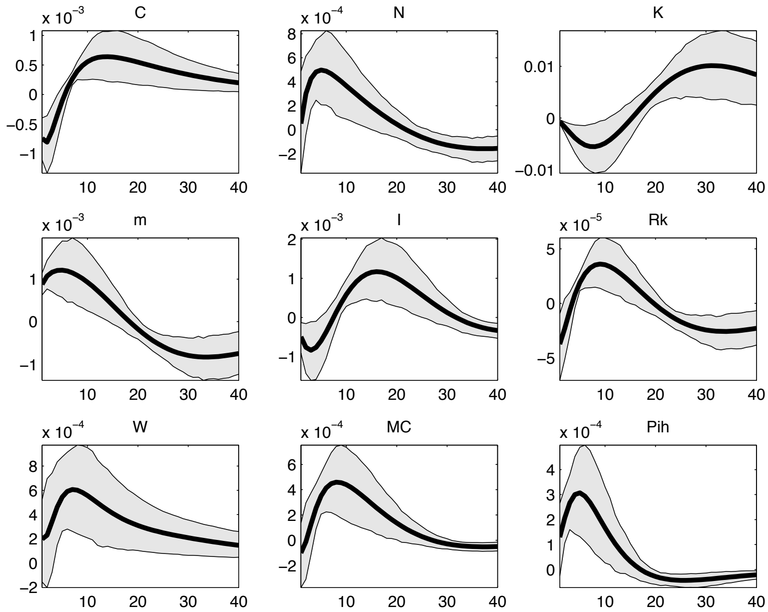

Figure 3.16: IRFs from investment shock in rUK- 1(Scotland)

Start with figure 3.16 for Scotland. An investment shock in rUK feeds-back to the Scottish economy mainly via exported goods. As in the case of a demand shock, an increase in the rUK demand for Scottish exports substantially increases Scottish production causing an increase in marginal costs and prices. However, differently from a demand shock, the increase in price inflation reverses since technological improvements in rUK makes imported goods cheaper over time, reducing the cost of final goods in Scotland too. Furthermore, consumption increases since the monetary policy response is far milder than for the case of a demand shock (i.e. compare figure 3.15 with figure 3.20). Therefore, the positive effect over consumption of greater economic activity dominates the depressing one of an increased interest rate. On the other hand, investment falls on impact due to the raised user cost of capital, whereas labour supply and wages increase following an increase in labour demand from firms.

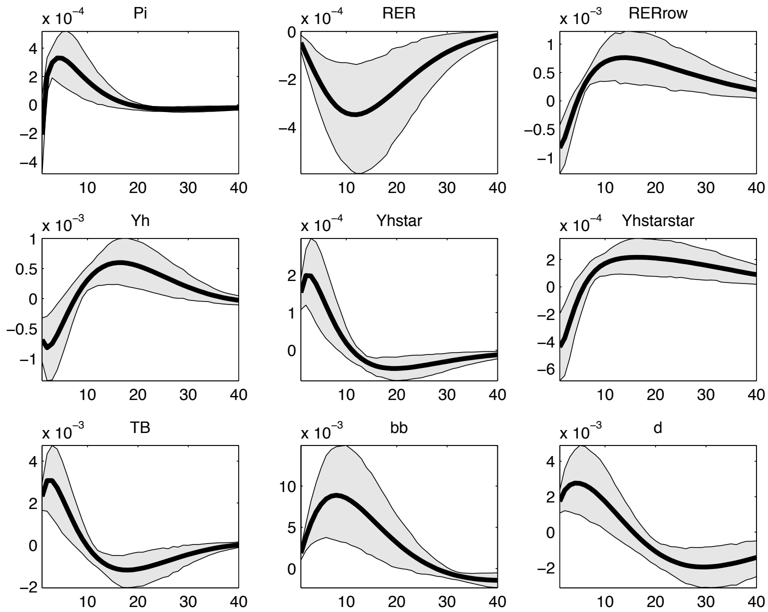

Figure 3.17: IRFs from investment shock in rUK- 2 (Scotland)

Figure 3.17 presents the remaining set of Scottish variables. The initial rise in intermediate goods inflation causes their demand to fall at home and in the ROW while their export towards the rUK increases. As inflation starts to fall, so does trade in intermediate goods; the asset position follows suit. Finally, thw Scottish real exchange rate with respect to the rUK appreciates whereas the one with respect to the ROW depreciates.

In figure 3.18, rUK variables display somehow a similar pattern to their Scottish counterpart in figure 3.16. However, in the rUK households initially substitute away from consumption to investment causing the former to fall and the latter to increase (i.e. in Scotland the opposite occurs). Meanwhile, the technological innovation in investment production allows capital to increase faster and marginal costs to fall. Despite the fall of marginal costs at home, intermediate goods inflation initially rises driven by an initial increase in final good inflation which is in turn driven by more expensive imports from Scotland.

Figure 3.19 shows the remaining set of rUK variables. The real exchange rate with respect to Scotland and the ROW both depreciate. In the first case, this is due to inflation rising faster in Scotland than in the rUK; in the latter, this is driven by the nominal exchange rate more than compensating

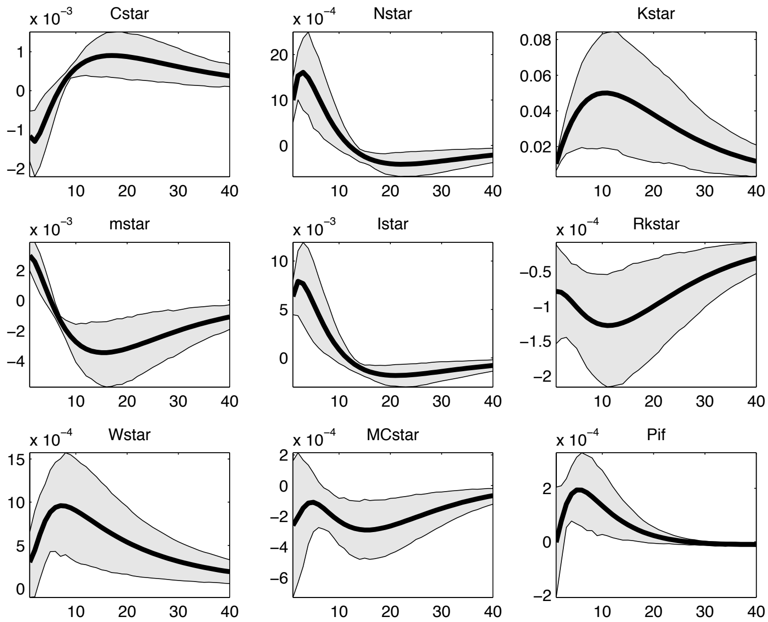

Figure 3.18: IRFs from investment shock in rUK- 3 (rUK)

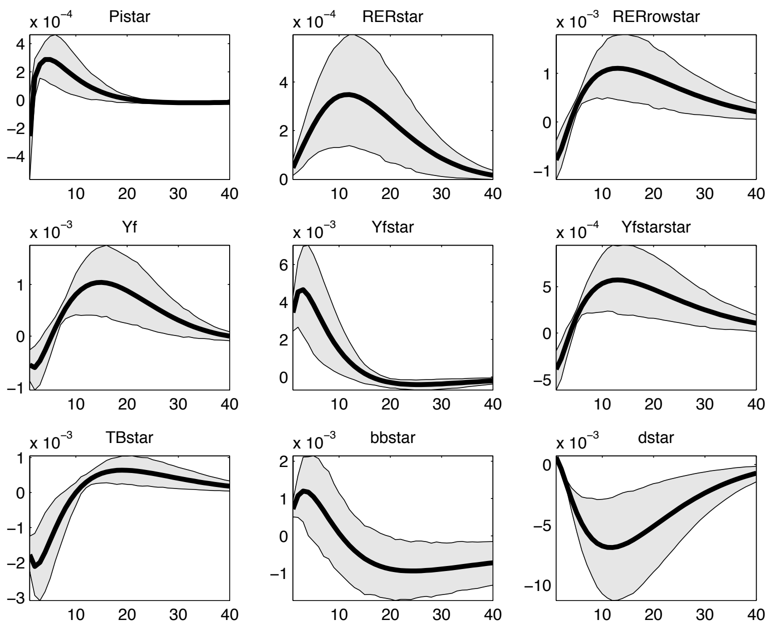

Figure 3.19: IRFs from investment shock in rUK- 4 (rUK)

the opposite movement in prices. Following the resultant real exchange rate depreciation and increased internal absorption (investment increases more than compensate initial fall in consumption) both rUK imports and exports increase. The balance of trade indicates that imports increase initially prevails until, subsequently, the opposite occurs.

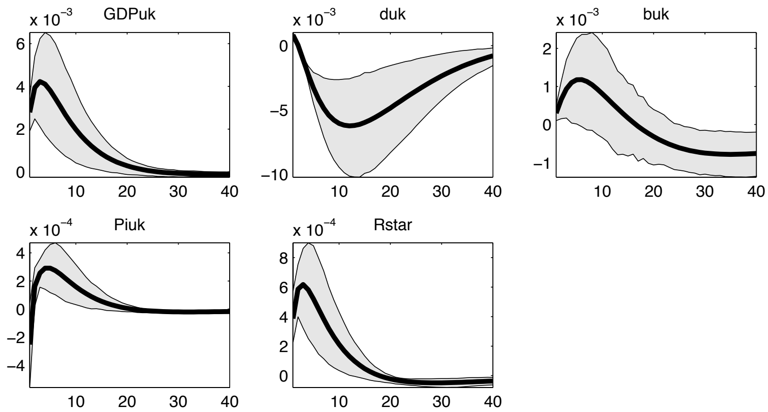

Figure 3.20: IRFs from investment shock in rUK- 5 (UK)

Finally, figure 3.20 shows UK GDP increases as a result of a GDP increase in both Scotland and in the rUK. Consequently, tax revenues rise and government debt is temporarily reduced. Meanwhile, the UK's international position temporarily worsens because of the initial increase in internal absorption. the monetary policy response is, as previously mentioned, relatively mild.

3.4.5 Labour supply shock

We comment below on the effect of an intra-temporal preference shock hitting the Scottish economy in an idiosyncratic manner. Such a shock has the consequence of temporarily increasing the disutility of labour, and therefore the cost for the household of supplying it.

As shown in figure 3.21, following such a shock, labour supply in Scotland falls while wage rate increases. Consumption and investment fall as production and disposable income are reduced. Meantime, higher wages drive up marginal costs and intermediate goods inflation in Scotland.

From figure 3.22, we can appreciate how intermediate goods inflation drives CPI inflation up, which in turn induces the real exchange with rUK

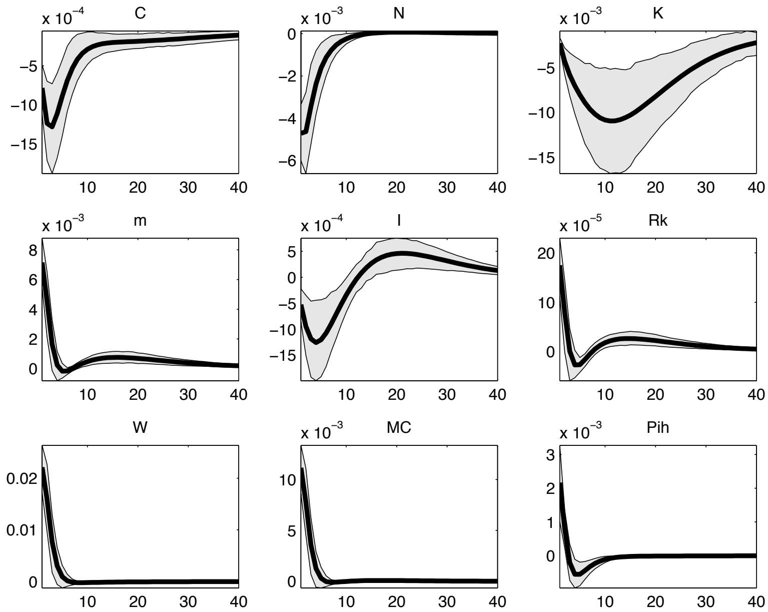

Figure 3.21: IRFs from labour supply shock in Scotland- 1 (Scotland)

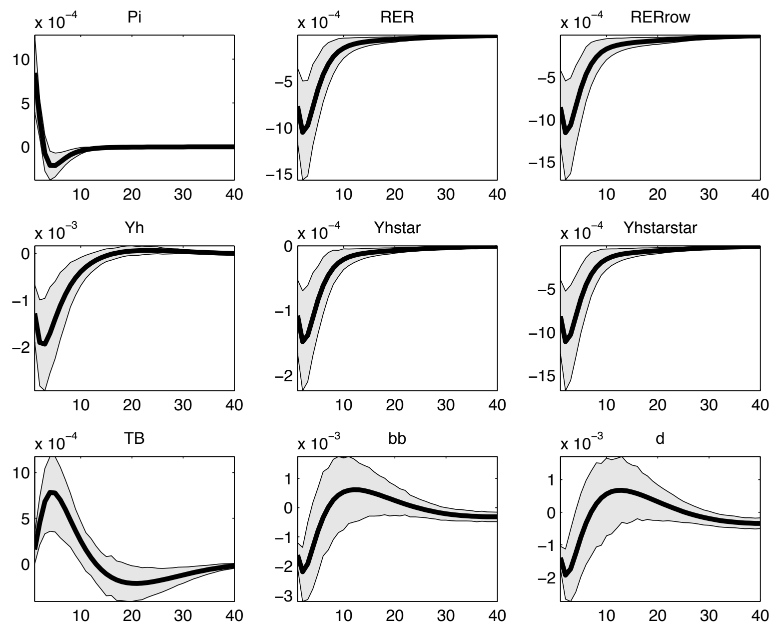

Figure 3.22: IRFs from labour supply shock in Scotland- 2 (Scotland)

and ROW to appreciate. Altogether this drives down the demand and production of Scottish intermediate goods for internal consumption and export. As Scottish imports fall as well, the final effect over the balance of trade is mixed: trade balance fluctuates along the transient dynamics and assets accumulation follows suit.

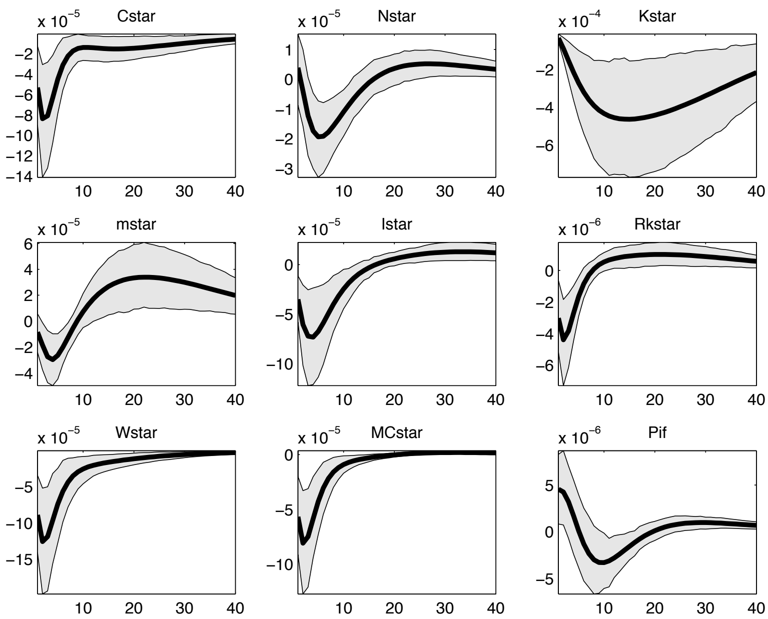

Figure 3.23: IRFs from labour supply shock in Scotland- 3 (rUK)

Figures 3.23 and 3.24 show that feedback to the rUK is, as expected, marginal. Looking at the scale of rUK endogenous variables responses, it is immediately apparent how the only variables displaying a significant impact are the real exchange rate with respect to Scotland, which depreciates, and the exports of intermediate goods, again toward Scotland, which increase. Given the relative small size of Scotland, however, the latter have a very small propagation over the rUK economy.

Finally, figure 3.25 shows that the fall in Scottish GDP does not translate into a UK-wide GDP fall as expected.

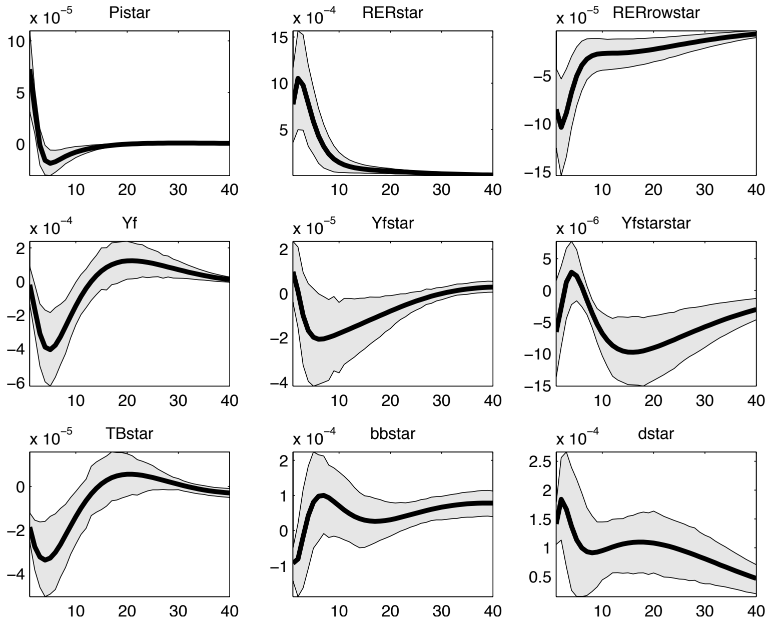

Figure 3.24: IRFs from labour supply shock in Scotland- 4 (rUK)

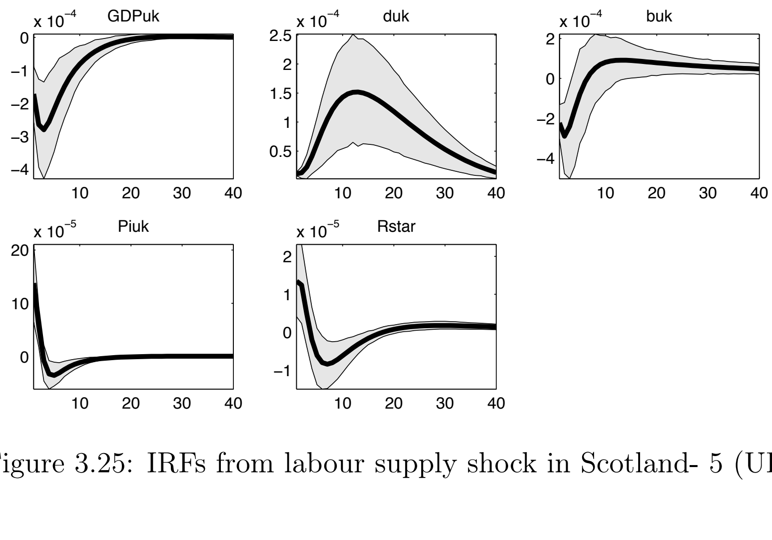

Figure 3.25: IRFs from labour supply shock in Scotland- 5 (UK)

3.5 Concluding remarks

In this chapter we have reviewed the main approaches for parametrizing the DSGE models, and have selected the Bayesian likelihood-based techniques for estimating our model. This procedure has several advantages. Firstly, being a full-information method, it exploits all of the relevant information available in the sample and not only some of its moments. Secondly, the Bayesian approach allows us to inform estimation with the pre-sample information coming from related studies and economic theory, which is often very rich. Prior distributions assigned to parameters, furthermore, tend to induce curvature to regions of the likelihood function which are otherwise flat, helping the identification and allowing for a more conservative stance for policy making. Thirdly, the Bayesian techniques rely on the "exploration" of parameters' posterior distribution, instead of attempting to maximize the likelihood function as in the classical ML. As discussed, the former approach is significantly more robust.

We then moved to estimate the model. We found that the modes of the estimated parameters are broadly in line with those found by most of the DSGE literature, providing a first validation to our framework. At the same time, we noted that the Calvo lottery for wage is relatively small, while the labour supply shock in Scotland tends to be quite large. There are a few reasons which could account for this. Scottish series tend to be more volatile than the ones from larger advanced economies over which these models are typically estimated. This, in turn, can be due to a larger measurement error in Scottish statistics as well as to the greater volatility smaller countries, such as Scotland, are likely to experience. Furthermore, it is generally believed that with a reduced scope for monetary and fiscal policy responses to Scottish idiosyncratic shocks, much of the adjustment in Scotland may occur through the labour market.

Finally, we analyzed the impulse response functions produced by our estimated model and studied the propagation of shocks and the dynamics of the economic adjustment in Scotland and in the rUK. We found that the dynamics produced by our model are well in line with what suggested by economic theory, and consistent with the classic predictions from DSGE models. This provides a further validation to the design of our model and to its estimation.

Future research will employ this framework to address a large set of policy relevant questions on alternative devolution settings for Scotland, as well as on the assessment of its current fiscal framework. For example: what are welfare costs/benefits of the current devolution settlements? Can we decompose them across individual policy measures e.g. with and without the control of income tax? By how much should the borrowing powers be extended to manage the fiscal risk in Scotland? What are the costs/benefits of Scottish independence? Etc. Together with informing the policy debate in Scotland, this line of research could more generally contribute to the literature on the optimal design of fiscal federalism, a branch of economics increasingly relevant to policy makers and surprisingly subdued.

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