SOME STUDIES OF THE INTERACTION OF FAST NEUTRONS WITH COMPLEX NUCLEI.

by

G. Brown
Department of Natural Philosophy,
University of Glasgow.

Presented as a thesis for the degree of Ph.D.,

in the University of Glasgow.

PREFACE

In this thesis an account is given of the research performed by the author, as a member of the Department of Natural Philosophy at the University of Glasgow, during the years 1953 to 1956.

In Chapter I a review is made of the theoretical and experimental investigations of the mode of interaction of nucleons with nuclei. It is concluded that many experimental results, on the nuclear reactions induced in nuclei by nucleons with intermediate energies, might be interpreted using a nuclear model basically similar to that which has been applied successfully to the reactions induced by nucleons of high energy.

The calculation of the cross-sections for the reactions induced in nuclei by nucleons with intermediate energies, is reported in Chapter II. It is shown that it is possible to calculate these cross-sections using a simple nuclear model which is basically similar to that used for nucleons of high energy. The derivation of the formulae for the total cross-section for the collision of an incident nucleon with an individual nucleon of a Fermi gas of nucleons, given in § 3 (a), was performed by Dr. H. Muirhead with assistance from the author. The responsibility for all other calculations, namely those of the angular distribution of the nucleons resulting from this collision process, of the level densities of excited nuclei and of the applications of this model, is equally that of Dr. H. Muirhead and the author.

An experimental investigation of the angular distribution of the protons emitted in reactions of the type ${}_{Z}N^{A}(n,p)_{Z-1}N^{A}$ induced by neutrons with an energy of 13.2 MeV, is reported in Chapter III. Nuclear emulsions were used to detect these protons. This work is original. Unfortunately, in these experiments it was not possible to employ professional scanners to advantage, and consequently the majority of the arduous scanning of these emulsions and the measurements involved has been performed by physicists. The apparatus which has been used in these investigations was designed in collaboration with Mr. G.C. Morrison. The measurements made for the protons emitted in the reaction ${}_{26}Fe^{56}(n,p)\,{}_{25}Mn^{56}$ and an initial analysis of this data were also the responsibility of Mr. G.C. Morrison and the author. A satisfactory analysis of this data was not possible until a method of correction for the loss in energy of the protons produced in a semi-thick target had been devised. A method of correction was suggested and applied by the author. The further analyses of (n,p) reactions, namely those of ${}_{13}Al^{27}(n,p)\,{}_{12}Mg^{27}$ and ${}_{45}Rh^{103}(n,p)\,{}_{44}Ru^{103}$ were performed solely by the author. The cross-sections for the emission of protons in these reactions have been determined, and these values have been compared with the predictions of the model discussed in Chapter II. In this work the author was assisted by Mr. W.T. Morton.

In Chapter IV the calculation of the cross-sections for the interaction of neutrons of low energies ($\leq$ 4 MeV) with nuclei is discussed. These cross-sections have been calculated using a potential of step form and the results have been compared with those obtained using a square well potential. These calculations were performed solely by the author.

Finally the information which has been derived from this research is discussed in Chapter V. Suggestions are made for its extension.

ACKNOWLEDGMENTS

The author wishes to express his gratitude to Professor P.I. Dee, F.R.S., for his interest, aid and encouragement during the course of this work. He is indebted to Dr. H. Muirhead for the advice and assistance given on many occasions. Finally he would like to thank Mr. G.C. Morrison and Mr. W.T. Morton for their co-operation.

CONTENTS PAGE

CHAPTER I.

A SURVEY OF THE EXPERIMENTAL AND THEORETICAL INVESTIGATIONS ON THE MODE OF INTERACTION OF NUCLEONS WITH NUCLEI.

§ 1. INTRODUCTION.

Many attempts have been made to interpret the results of experiments on the scattering of nucleons by nuclei in terms of the strength of the interaction between nucleons in nuclear matter. Mathematical treatments of the interaction of a small number of nucleons have been reasonably successful, but the extension of such treatments to the interaction of many nucleons in close proximity is too complex. However it is possible to obtain useful information on nucleon-nuclear interactions by the use of models employing certain simplifying assumptions.

The model used in the first theoretical attempts to interpret experimental results was based upon the assumption that the interaction between nucleons inside a nucleus was very weak. When it appeared that the predictions of this model were not substantiated by experimental results, it was replaced by its direct antithesis - that of the compound nucleus. In the model of the compound nucleus a strong interaction was assumed, so that the energy of the incident nucleon was shared rapidly among a large number of nucleons. It was shown however that at high energies ($\sim$ 100 MeV) this was not so; the experimental results indicated an interaction between the incident nucleon and individual nucleons of the nucleus. In recent years refined experiments have shown that the latter concept is also useful when the incident nucleon is of low or intermediate energy ($\lesssim$ 30 MeV). The discussion of these investigations will be divided into three sections:-

(a) the model of the compound nucleus,

(b) the study of the interactions of nucleons of high energy with nuclei, and

(c) the study of the interactions of nucleons of low and intermediate energies with nuclei.

§ 2. THE BOHR HYPOTHESIS.

(a) The potential well model.

The experimental results of the early investigations of nuclear reactions were explained using a simple potential well model of the nucleus. The effect of a nucleus upon an incident particle was assumed to be described by an attractive potential, V (r), such that

$$\begin{aligned} V(r) &= -V_0 \qquad\qquad r<R \\ &= 0 \qquad\qquad\quad r>R \end{aligned} \tag{I.1}$$

where R is the radius of the nucleus. The magnitude of V_0 was a few tens of MeV.

This model was used to explain the occurrence of resonances in the reactions initiated by slow neutrons and to predict that at these resonances both capture and scattering of the slow neutrons would be equally strong. The experiments of Bjerge and Westcott (1935), Moon and Tillman (1935), Szilard (1935), Fermi and Amaldi (1935) and others, showed, however, that the strong absorption which took place at resonance was not accompanied by strong scattering. This major weakness of the model led to its replacement in 1936.

(b) The model of a compound nucleus.

It was suggested by Bohr (1936) that a model which is the direct antithesis of the potential well model might be useful in the interpretation of experimental results. Bohr reasoned that the closely spaced resonances for the capture of slow neutrons by heavy nuclei were probably those produced in a strong interaction of the incident nucleon with a many particle system. He suggested that a neutron incident upon a nucleus would be subject to a very strong interaction and would be absorbed immediately on entering the nucleus, to produce a compound state in which all nucleons participated collectively. This is equivalent to assuming that the mean free path of a nucleon in nuclear matter is very short.

This new view suggested by Bohr was extremely successful in the interpretation of the experiments on the interaction of slow neutrons with nuclei; not only was it possible to explain the existence of closely spaced and narrow resonances, but more important to predict the actual predominance of the capture process over that of scattering.

It was suggested by Bohr that a nuclear reaction proceeds in two distinct stages; the formation of a compound nucleus and its subsequent decay. The mode of disintegration was assumed to be independent of the specific way in which the compound nucleus was formed. The conditions necessary for this hypothesis are that the energy of the incident nucleon is shared among all the nucleons contained in the nucleus, and that the average energy possessed by a nucleon is less than that required for escape from the compound nucleus. Therefore the time which elapses before a nucleon acquires sufficient energy to escape is long relative to the time it would take the incident nucleon to cross the nucleus. Under these conditions the independence hypothesis is probably justified.

According to the Bohr assumption the cross-section for a reaction of the type $X(a,b)Y$ may be written as

$$\sigma(a,b) = \sigma_c(a)\,\eta_b \qquad (I.2)$$

where $\sigma_c(a)$ = the cross-section for the formation of the compound system by particle a,

η_b = the probability that the compound system will decay by the emission of the particle b.

The calculation of the quantities involved in equation (I.2) will be considered in detail in the following two sections

(c) <u>The cross-sections for the formation of a compound nucleus.</u>

The calculation of these cross-sections on the continuum theory for nuclear reactions, described by Feshbach and Weisskopf (1949), did not involve the consideration of any detailed structure of the nucleus. In addition to the basic assumption that a nucleon was absorbed immediately on entering the nucleus, it was assumed that

(1) The nucleus had a well defined surface which was a sphere of radius R, and that the nuclear forces only acted on an incident particle when the distance between it and the centre of the nucleus was less than R.

(2) The average kinetic energy of the incident nucleon increased by an amount V on entering the nucleus.

(3) Many different modes of decay of the compound nucleus were possible, i.e. the emission of the incident nucleon with its original energy was assumed to be very improbable.

Because of this last assumption, the theory was only applicable for fairly heavy target nuclei ($A \gtrsim 50$) and for incident neutrons with an energy greater than 3 MeV. As a consequence of these assumptions, the radial wave function, u_ℓ, of a subwave of orbital angular momentum ℓ was written in the form of an ingoing wave only:-

$$u_\ell \sim \exp.-\left[iKr\right] \qquad (I.3)$$

where $$K = \frac{\left[2M(E+V)\right]^{1/2}}{\hbar}$$

= the wave number of the incident nucleon, of energy E, inside the nucleus.

The problem of determining cross-sections was merely one of calculating the phase and amplitude of the wave function for $r>R$ relative to the wave function for $r<R$, for each possible value of the angular momentum ℓ. This was achieved by matching the wave functions and their logarithmic derivatives, f_ℓ, at the nuclear surface.

$$f_\ell = R\left[\frac{du_\ell/dr}{u_\ell}\right]_{r=R.} \tag{I.4}$$

Integration over all possible values of the angular momentum was then performed.

Thus the only information about the nucleus which was used in this theory was contained in the form of the wave function inside the nucleus.

It was found by Feshbach and Weisskopf (loc.cit.) that when the energy of an incident neutron was high the cross-section, σ_{el}, for the elastic scattering of the incident nucleon and the cross-section, σ_c, for the capture of the incident nucleon were given by

$$\sigma_{el} \approx \sigma_c \approx \pi\left(R+\lambda\!\!\!^-\right)^2 \qquad \lambda\!\!\!^- < R \tag{I.5}$$

The total cross-section, σ_t, was given by

$$\sigma_t \approx 2\pi\left(R+\lambda\!\!\!^-\right)^2 \qquad \lambda\!\!\!^- < R \tag{I.6}$$

where $\lambda\!\!\!^-$ = the de Broglie wavelength of the relative motion divided by 2π.

To determine the behaviour of these cross-sections when the energy of the incident neutron was low the resonance theory of Feshbach, Peaslee and Weisskopf (1947) was used. In this theory assumption (3) was not made and thus the radial wave function inside the nucleus included a term representing an outgoing wave. The cross-sections were obtained from this theory in the same way as before except that the new form of the radial wave function was used. It was found that for incident neutrons of low energies the magnitude of the total cross-section when averaged over resonances was

$$\sigma_t \approx \frac{4\pi}{kK} \qquad \lambda \gg R \qquad (I.7)$$

where k = the wave number of the incident neutron outside the nucleus $= \frac{[2ME]^{1/2}}{\hbar}$

Feshbach and Weisskopf (loc.cit.) used the continuum theory to determine values for the radius of the nucleus from the measurements of Amaldi et. al. (1946) for the total cross-sections for neutrons with an energy of 14 MeV and of Sherr (1945) at 25 MeV. Using these values of the nuclear radius they computed the values of the cross-sections for the formation of a compound nucleus by neutrons with an energy of 14 MeV. These values were consistent with the cross-sections for the inelastic scattering of 14 MeV neutrons by nuclei, measured by Amaldi et. al. (loc.cit.) and Phillips et. al. (1952). This agreement suggested that neutrons with an energy of 14 MeV were immediately absorbed on entering a nucleus.

They also computed the values of the total cross-sections, when averaged over resonances, for neutrons with energies less than 1.5 MeV by using the resonance theory of Feshbach, Peaslee and Weisskopf (loc.cit.). They observed that there was agreement between these values and those measured by Barschall et. al. (1948) and by Bockelmann et. al. (1949).

(d) The statistical theory for the decay of the compound nucleus.

Weisskopf (1937) and Weisskopf and Ewing (1940) applied statistical methods to the decay of the compound nucleus. The probability, η_b (equation I.2), that the compound system decayed by the emission of the particle b was written as

$$\eta_b = \frac{\Gamma_b}{\sum_{b'} \Gamma_{b'}} \tag{I.8}$$

where Γ_b = the probability that the compound state decayed by the emission of the particle b.

The summation is over all possible modes of decay of the compound system.

The probability, $W_b(E)\,dE$, of decay of the compound system by the emission of the particle b with an energy between E and $E+dE$ was derived from the principle of detailed balancing to be

$$W_b(E)\,dE = \text{const.}\ M\,(2s+1)E\sigma_b(E)\,\omega_R\,(E_{max}-E)\,dE. \tag{I.9}$$

where E = the energy of the emitted particle b, of mass M and spin s,

$E_{max.}$ = the maximum energy of the emitted particle,

$\omega_R(E_{max}-E)dE$ = the number of levels of the residual nucleus with an excitation energy between $E_{max}-E$ and $E_{max}-E-dE$,

$\sigma_b(E)$ = the cross-section for the capture of the particle b, with an energy E, by the residual nucleus with an excitation energy of $E_{max}-E$.

This probability has no meaning unless the interval of energy dE is chosen sufficiently large that many levels of the residual nucleus are included. It is usual to assume under these conditions that the angular distribution of the particles emitted with energies between E and $E+dE$ is symmetrical about 90°.

The total probability, Γ_b, for the emission of particle b was obtained by the integration of equation (I.9) over all possible values for the energy of the emitted particle.

$$\Gamma_b = \text{const.}\, M(2s+1)\int_0^{E_{max}} E\sigma_b(E)\,\omega_R(E_{max}-E)\,dE \tag{I.10}$$

Thus the probability of emission of a particle is strongly dependent upon the magnitudes of both the cross-sections for the capture of that particle by an excited nucleus and the level density of the residual nucleus.

The magnitudes of $\sigma_b(E)$, as a function of the energy of the incident particle and the atomic weight of the target nucleus, have been calculated using the continuum theory by Feshbach and Weisskopf (loc.cit.) for neutrons, and by Shapiro (1953) for charged particles.

It was suggested by Weisskopf (loc.cit.) that the dependence of the level density, $\omega(U)$, of an excited nucleus upon its energy of excitation, U, might be given by the formula

$$\omega(U) = C \exp.\left[2\sqrt{aU}\right] \qquad (I.11)$$

where a and C are constants.

The values of a and C were determined by Weisskopf (loc.cit.) from the data available on the level densities of excited nuclei. He found that these constants varied with the atomic weight of the nucleus, and that C also depended on whether the number of neutrons and protons in the target nucleus was even or odd. The latter variation in C was found to be

$$\frac{C_{o-o}}{4} = \frac{C_{o-e}}{2} = C_{e-e} \qquad (I.12)$$

More recently Lang and Le Couteur (1954) have analysed experimental evidence related to the density of levels of excited nuclei. They have suggested a more complex formula which takes into account the possible variation of level density with the atomic weight of the nucleus.

§3. THE INFORMATION OBTAINED FROM EXPERIMENTS ON THE INTERACTION OF NUCLEONS OF HIGH ENERGY ($\gtrsim$ 100 MEV) WITH NUCLEI.

(a) The model of Goldberger.

The physical ideas for the description of the mode of interaction of nucleons of high energy with nuclei were proposed by Serber (1947), who noted several aspects of experimental results which differed from those at lower energies. He pointed out that for nucleons of high energy

(1) the de Broglie wavelength of the incident nucleon is small compared to nuclear dimensions,

(2) the collision time between the incident nucleon and a nucleon of the nucleus is less than that between the individual nucleons contained in the nucleus,

and (3) the cross-section for the interaction between free nucleons is small at high energies.

Thus at high energies the mean free path of a nucleon in nuclear matter might be of the order of nuclear dimensions, and nuclear reactions might take place as a result of collisions of the incident nucleon with single nucleons within the nucleus. This leads to the phenomenon of transparency of nuclei to incident nucleons of high energy. These considerations are just the opposite of those upon which the model of the compound nucleus was based.

Goldberger (1948), following these ideas of Serber (loc.cit), has made quantitative estimates of the mean free path of a nucleon inside nuclear matter. In addition he described a method of calculating the angular distributions and the energy spectra of the nucleons emitted from a nucleus after collisions of the type suggested by Serber (loc.cit.); this method has become known as the 'Monte Carlo' method. It was assumed by Goldberger (loc.cit.) that the neutrons and protons of the nucleus could be represented separately as non-interacting Fermi gases, and that the collision of the incident nucleon with one of these nucleons was subject to the restrictions of the Pauli Exclusion Principle. The calculation of the mean free path of a nucleon in nuclear matter consisted of determining the total cross-section for scattering of the incident nucleon by a nucleon inside the nucleus under these conditions.

The method of calculating the characteristics of nuclear disintegrations consisted of tracing the path of an incident nucleon and those of its collision partners, which arose in collisions for which the Pauli Exclusion Principle was not violated, until they either escaped from the nucleus or were captured to form an excited nucleus. This calculation was repeated a large number of times so that the average characteristics of such collisions were reproduced. The characteristics of the nuclear disintegrations produced by nucleons of high energy have been explained successfully in this way by several authors (Bernadini, 1952; Morrison, Muirhead and Rosser, 1953;

Combe, 1954, 1956). Since only a few nucleons are involved in this nucleon cascade, the forward momentum of the incident particle produces a forward peak in the angular distribution of the reaction products, and the energy spectra of these nucleons contain a large proportion of nucleons of high energy.

When the energy of a nucleon decreases its de Broglie wavelength and the cross-sections for the scattering of this nucleon by free nucleons increase. Thus the assumption that a particle of intermediate energies ($\lesssim$ 30 MeV outside the nucleus) interacts with a nucleus through single nucleon-nucleon collisions might be wrong. Indeed it was suggested by Peaslee (1952) that when the energy of an incident nucleon is less than 30 MeV outside the nucleus, that it will be captured on entering the nucleus to form a compound state. This hypothesis was tested by Morrison, Muirhead and Rosser (loc.cit.) They calculated the energy spectra and the angular distributions of the protons emitted from a heavy nucleus with energies less than 30 MeV as a result of the interaction of 140 MeV protons on the basis of two hypotheses. The first was that nucleons produced in the nucleon cascade with energies less than 60 MeV (V_0 = 30 MeV) were captured to form a compound nucleus, which was assumed to decay according to the statistical theory. The second was that only nucleons whose energy was below the height of the assumed potential well were captured to produce an excited nucleus. A comparison of the energy spectra and of the ratio of the number of protons emitted in the forward hemisphere to those emitted in the backward hemisphere, obtained on these hypotheses, was made with those observed experimentally by Lees, Morrison, Muirhead and Rosser (1953). It was shown that a comparison of both the energy spectra and the forward/backward ratio indicated that the nucleon cascade continued until the energy of the nucleons inside the nucleus was 50 MeV. For energies less than this the comparison of the energy spectra was insufficient to decide between the two hypotheses, but the forward/backward ratio was more consistent with the assumption that the cascade proceeded down to the lowest energies.

These authors calculated that if an incident nucleon with an energy of 1 MeV could be assumed to move as an independent particle on entering the nucleus, it would possess a mean free path of approximately 10^{-12} cm.

(b) The mathematical formulation of the 'Optical Model'.

Cook, McMillan, Peterson and Sewell (1947, 1949) have found that the total cross-sections of heavy elements for 83 MeV neutrons are approximately equal to the values given by equation (I.6), but those for light elements are considerably below this value. These results have been interpreted by Fernbach, Serber and Taylor (1949) in terms of the optical or transparent model of the nucleus. The basic assumptions made in this calculation are just those discussed in the previous section; namely, that the mean free path of a nucleon of high energy in nuclear matter is long, and that its interaction with the nucleus is by single nucleon-nucleon collisions with the nucleons contained in the nucleus.

The wave number of the incident nucleon inside the nucleus, k', was written as

$$k' = k + k_1 + \frac{iK}{2} \tag{I.13}$$

where k = the wave number of the incident nucleon, of energy E, outside the nucleus,

$$k_1 = k\left[\left(1 + \frac{V_0}{E}\right)^{\frac{1}{2}} - 1\right]$$

V_0 = the magnitude of the assumed potential well inside the nucleus (usually taken as $\sim$ 30 MeV),

and K = the absorption coefficient.

The absorption of the incident nucleon was assumed to take place by the mechanism suggested by Serber (loc.cit), and thus the absorption coefficient was related to the magnitudes of the cross-sections for the scattering of free nucleons by the formula

$$K = \frac{1}{\lambda} = \frac{A\sigma}{\frac{4}{3}\pi R^3} \tag{I.14}$$

where λ = the mean free path of the incident nucleon inside the nucleus of radius R.

$$\sigma = \frac{\left[Z\sigma_{Np} + (A-Z)\sigma_{Nn}\right]\rho}{A}$$

with Z = the charge of the target nucleus of mass A,

σ_{Nn} and σ_{Np} = the cross-sections for the scattering of the incident nucleon N by a free neutron and a free proton respectively,

ρ = a factor which reduces the magnitudes of these cross-sections inside nuclear matter, through the restrictions of the Pauli Exclusion Principle.

When the energy of the incident nucleon is high, the de Broglie wavelength is small compared to nuclear dimensions. It is sufficient then to determine the amplitude and phase of the emitted wave for one particular path through the nucleus. Thus the cross-sections for absorption and elastic (diffraction) scattering were obtained by integration of the appropriate quantities over all possible paths through the nucleus.

The previous considerations are equivalent to assuming (Francis and Watson 1953) that the effect of the nucleus upon the incident nucleon may be represented by an attractive potential, $V(r)$, such that

$$V(r) = -\left[V_0 + iV_1\right] \tag{I.15}$$

where

$$V_0 = E\left[\frac{k_1}{k} - \frac{1}{(4k^2\lambda^2)}\right]$$

and (I.16)

$$V_1 = \frac{\hbar}{\lambda\sqrt{2M}}\left(E + V_0\right)^{\frac{1}{2}}$$

A phase shift analysis can be performed using a potential of this form. The summation over all possible values of the angular momenta involved will give just the cross-sections derived by Fernbach, Serber and Taylor (loc.cit.).

§ 4. THE INTERACTION OF NUCLEONS OF LOW AND INTERMEDIATE ENERGIES WITH NUCLEI.

(a) Introduction.

The discussions contained in the previous sections indicate that the initial mode of interaction of nucleons of high energy with nuclei is completely different from that assumed in the model of the compound nucleus. The two models are generically related since absorption of the nucleons of low energy produced in the nucleon cascade, initiated by an incident nucleon, would lead to the formation of an excited nucleus. The evidence obtained by Morrison, Muirhead and Rosser (loc.cit.) suggests that such absorption may not occur until the energy of the nucleons is too low for escape from the nucleus to be possible. In recent years experimental results on the interaction of nucleons of low and intermediate energies have suggested also that immediate absorption of a nucleon does not occur at these energies. Indeed, theoretical investigations have shown that the optical model may be used successfully in this energy region which for many years had been thought to be the domain of the model of the compound nucleus.

(b) The use of the optical model at low and intermediate energies.

It has been shown by Barschall (1952) that the energy dependence of the total cross-sections for the scattering of neutrons with energies in the range 0 - 4 MeV does not show the simple behaviour predicted by equation (I.7). It is apparent (Friedman and Weisskopf 1956), when these cross-sections are plotted in three dimensions against the energy of the incident neutron and the atomic weight of the target nucleus, that there are large scale maxima and minima at positions predicted by the simple potential well model (§ 2 (a)), and under high resolution in energy the narrow and closely spaced resonances which led to the Bohr theory are present.

Measurements of the ratio of the neutron width to the level spacing for isolated resonances have shown that the concept of the immediate absorption of a neutron of low energy is not correct. It has been shown by Bollinger, Cotè and Le Blanc (1955) and by Carter, Harvey, Hughes and Pilcher (1954), that there are peak values of this ratio for nuclei of $A \sim 50$ and $A \sim 160$. The strong coupling theory does not explain such phenomena.

These experimental results have been explained successfully on the basis of an optical model by Feshbach, Porter and Weisskopf (1954). In using this model at low energies it is necessary to use a potential of the form given in equation (1.15) and to perform a phase shift analysis by fitting wave functions and their derivatives at the nuclear boundary.

The optical model has also been used by Culler, Fernbach and Sherman (1956) to calculate the total and inelastic cross-sections for the scattering of neutrons with an energy of 14 MeV by nuclei. The cross-sections calculated from this theory were found to agree with those measurements of Amaldi et. al. (loc. cit.), Phillips et. al. (loc.cit.) and Sherr (loc.cit.). This indicates that it is not necessary to assume the immediate formation of a compound nucleus to explain these results.

Finally it has been shown by Woods and Saxon (1954) that the experimental data on the elastic scattering of 20 MeV protons by nuclei may be predicted using the optical model.

The value of the mean free path for a neutron of low energy in nuclear matter was estimated by Feshbach, Porter and Weisskopf (loc.cit.). This value was in good agreement with that obtained by Morrison, Muirhead and Rosser (loc.cit.) in their calculations on the development of a nucleon cascade in nuclear matter. In view of this, calculations were made by Morrison, Muirhead and Murdoch (1955) and by others (Clementel and Villi, 1955; Lane and Wandel, 1955;) of the magnitude of the imaginary potentials which have been used in these optical model analyses at low and intermediate energies. It was assumed by Morrison, Muirhead and Murdoch (loc.cit.) that since the real potential was deep ($\sim$ 40 MeV), even a slow neutron on entering the nucleus had a wavelength of approximately inter-nucleon dimensions, and could be treated as a free particle within the nucleus. It was assumed that the nucleons of a nucleus could be represented as two independent Fermi gases, and that reactions took place by single nucleon-nucleon collisions for which the Pauli Exclusion Principle was not violated. The value of the mean free path of nucleons in nuclear matter was calculated using this model, and the values obtained were inserted into equation (I.16) to obtain the magnitude of the imaginary potentials. The values of these potentials were in agreement with those which had been found necessary by Feshbach, Porter and Weisskopf (loc.cit.); by Culler, Fernbach and Sherman (loc.cit.); and by Woods and Saxon (loc.cit.).

The success of the optical model at these energies, and this interpretation in terms of semi-classical mechanics, leads to the possibility of calculating the yields of nuclear reactions at intermediate energies. Such calculations have not been performed. In the absence of such an application, a comparison of the experimental results with the predictions of the optical model can be made only in a qualitative manner.

(c) The experimental investigations of the nuclear reactions induced by nucleons of intermediate energies (10 - 30 MeV).

The optical and compound nucleus models predict different angular and energy distributions for the products of nuclear reactions. The angular distribution of the nucleons emitted from a compound nucleus is symmetrical about 90° provided the contributions from many levels of the residual nucleus are considered, while that of the nucleons emitted as a result of the type of collisions envisaged in the optical model is peaked in the direction of the incident nucleon. The average kinetic energy available for the emission of a nucleon from a compound nucleus is approximately 1 or 2 MeV, because of the large number of nucleons among which the energy of excitation is shared. Thus because of the coulomb barrier the emission of protons is seriously inhibited. However few nucleons are involved in the single nucleon-nucleon collisions envisaged in the optical model, and thus the majority of the protons produced in these collisions have energies approximately equal to the magnitude of the coulomb barrier. Consequently a higher ratio of the probability for emission of protons to that for neutrons is expected on the basis of an optical model than on the compound nucleus model.

Experimental investigations of the validity of the two models have proceeded along two lines:-

(1) measurements of the relative probabilities of emission of protons and neutrons, and

(2) measurements of the angular and energy distributions of the products of disintegration.

Measurements of the ratio of the cross-sections for the processes induced in Ni^{60} by alpha-particles and that for those induced in Cu^{63} by protons have been made by Ghoshal (1950). He has concluded that the alpha-particles and protons were captured to produce the compound nucleus Zn^{64}.

The first observation that the probability of emission of protons in a nuclear reaction was higher than that predicted by the compound nucleus model was made by Hirzel and Wäffler (1947). They found that the ratio of the cross-sections for the processes ${}_{Z}N^{A}(\gamma, p)_{Z-1}N^{A-1}$ and ${}_{Z}N^{A}(\gamma, n)_{Z}N^{A-1}$ induced in nuclei with $Z \approx 50$ was of the order of 10^2 times that predicted by the statistical model. Subsequently measurements were made by Wäffler (1950) of the cross-sections for the processes ${}_{Z}N^{A}(n, p)_{Z-1}N^{A}$ induced by neutrons produced by the bombardment of Lithium and Boron with deuterons. The experimental cross-sections were found to be higher than those predicted by statistical theory by factors between 3 and 14. Paul and Clarke (1953) observed that the ratio of the experimental cross-sections for these processes, induced by 14 MeV neutrons, to those predicted by the statistical model increases from a value of 1 for the lightest elements to a value of 10^4 for lead. The inadequacy of the statistical theory was demonstrated further by Cohen, Newman, Charpie and Handley (1954), in studies of the cross-sections for the processes ${}_{Z}N^{A}(p, pn)_{Z}N^{A-1}$ induced by 22 MeV protons.

The reactions induced in many nuclei of different charge by protons have been studied in detail by Gugelot (1951, 1954).

He has shown that the energy spectra of the neutrons emitted from nuclei bombarded with 16 MeV protons and of the protons produced in reactions with 18 MeV protons, do not show the simple behaviour predicted by the statistical model. It was found that too many nucleons of high energy were emitted. Moreover the angular distributions of the protons inelastically scattered by nuclei were found to be anisotropic. Similar observations were made by Eisberg and Igo (1954) in a study of the inelastic scattering of 31 MeV protons. It has been suggested by Eisberg (1954) that all these observations indicate that a direct interaction occurs between the incident nucleon and one of those forming the surface of the nucleus.

Recently measurements have been made by Rosen and Stewart (1956) of the angular distributions of the neutrons emitted from Bismuth as a result of the interaction of 14 MeV neutrons. It was found that the neutrons with energies less than 4 MeV were emitted in an isotropic manner; those with energies between 4 and 12 MeV were found to be emitted predominantly at angles less than 50°. These experimental results seem to suggest that the conditions necessary for both the optical and compound nucleus models might be present.

§5. CONCLUSION

It appears from the previous discussion that models based on the hypothesis of immediate absorption of an incident nucleon on entering a nucleus, are insufficient at low and intermediate energies. The applications of the optical model at these energies have not been considered fully. In particular no complete analysis of the experimental data on the yields of nuclear reactions has been made, and the analysis of the cross-sections for the interaction of neutrons of low energy with nuclei, performed by Feshbach, Porter and Weisskopf (loc.cit.), is somewhat unsatisfactory. To assist in a more complete assessment of the mode of interaction of nucleons with nuclei, it would be desirable to obtain further information on the angular distribution of the products of nuclear reactions at intermediate energies.

In Chapter II an analysis of the yields of nuclear reactions at intermediate energies will be reported. This analysis has been based on a very simple model of the nucleus, and has been confined to an analysis of the results obtained in experimental investigations performed by others.

In Chapter III some measurements of the angular distributions of the protons emitted in processes of the type ${}_{Z}N^{A}(n,p)_{Z-1}N^{A}$ will be reported. These results will be compared with the predictions of the model discussed in Chapter II.

Finally an account is given in Chapter IV of an attempt which has been made to resolve the discrepancies between the calculations of Feshbach, Porter and Weisskopf (loc.cit.) and experimental data.

CHAPTER II.

AN ANALYSIS OF THE REACTIONS INDUCED IN COMPLEX NUCLEI BY NUCLEONS WITH ENERGIES BETWEEN 10 AND 20 MEV.

§ 1. INTRODUCTION.

The discussion contained in the previous chapter indicates that the model of the compound nucleus is insufficient to explain the experimental results on the interaction of nucleons of intermediate energies with nuclei. In this chapter an account will be given of an extension of the approach suggested by Morrison, Muirhead and Murdoch (loc.cit.), to the calculation of the yields of nuclear reactions at intermediate energies. The calculations to be given here have shown for the first time that it is possible to calculate satisfactorily the cross-sections for most of the following processes:-

(1) ${}_{Z}N^{A}\,(n,p)\,{}_{Z-1}N^{A}$ for neutrons with an energy of 14.5 MeV.

(2) The excitation functions for the reactions: ${}_{12}Mg^{24}(n,p\gamma)\,{}_{11}Na^{24}$, ${}_{13}Al^{27}(n,p\gamma)\,{}_{12}Mg^{27}$, ${}_{14}Si^{28}(n,p\gamma)\,{}_{13}Al^{28}$, ${}_{17}Cl^{37}(n,p\gamma)\,{}_{16}S^{37}$ and ${}_{38}Sr^{88}(n,p\gamma)\,{}_{37}Rb^{88}$.

(3) The angular distribution of the neutrons emitted with energies between 4 and 12

MeV, following the interaction of 14 MeV neutrons with Bismuth.

(4) The energy spectrum of the protons emitted at 60° and 150° from Iron, Silver and Platinum when bombarded by 18 MeV protons.

§ 2. PROCEDURE.

It has been assumed, for simplicity of mathematical argument and physical description, that the nucleons contained in a nucleus may be represented as two independent Fermi gases of nucleons. This concept has been proved by many workers to be useful, and there appears to be some justification for considering it as an approximate physical description of the normal and excited states of a nucleus. Thus an assembly of non-interacting particles should approximate reasonably to the requirements of the shell model of the nucleus, in which definite physical states occur for considerable periods of nuclear time.

It has been assumed that a nuclear reaction proceeds in three stages:-

(1) an initial collision between the incident nucleon and an individual nucleon of the target nucleus (direct collision),

(2) the formation of an excited nucleus,

and (3) the decay of this excited nucleus.

It has been assumed that the excited nucleus is formed by the 'capture' of nucleons, produced in single nucleon-nucleon collisions, whose energy is too small to escape from the nucleus. The emission of nucleons by direct collisions takes place within a period of about 10^{-23} seconds after the nucleon enters the nucleus, whilst that by the decay of an excited nucleus may occur in times which are many orders of magnitude larger than this. Thus it is reasonable to consider the two processes separately.

§ 3. THE CROSS-SECTIONS FOR THE EMISSION OF NUCLEONS BY DIRECT COLLISIONS.

(a) The total cross-sections for the emission of nucleons.

The derivation of the formula for this cross-section will be given, to simplify the nomenclature, for a reaction of the type ${}_{Z}N^{A}(n,p)_{Z-1}N^{A}$. The formulae to be given are applicable, of course, to any other type of reaction.

It has been assumed that the incident neutron interacts with either neutrons or protons in the target nucleus. Assuming that the neutrons and protons in a nucleus may be represented as two independent Fermi gases, the cross-section, σ_{Ip} , for the collision of the incident neutron with a proton may be written as

$$\sigma_{IP} = \sigma_I \frac{\left[\alpha X(P_1)\right]_{np} \rho_p}{\left[\alpha X(P_1)\right]_{nn} \rho_n + \left[\alpha X(P_1)\right]_{np} \rho_p} \tag{II.1}$$

where σ_I = the cross-section for the interaction of the incident neutron with the target nucleus.

$X(P_1)$ = the cross-section for the collision of a neutron of momentum P_1 with a free nucleon.

α = a factor which reduces the magnitude of $X(P_1)$ inside the nucleus through the operation of the Pauli Exclusion Principle.

ρ = the density of the nucleons inside the nucleus.

The subscripts n and p refer to neutrons and protons respectively; an expression for α has been calculated by Goldberger (loc.cit.) for nucleons whose energies are twice the maximum Fermi energy. This calculation has been extended to cover the collisions of nucleons of lower energy and to include correction factors for the differences in binding energy of the last neutron and proton in a nucleus.

On crossing the nuclear surface the energy of the incident neutron has been assumed to be increased by an amount equal to the depth of the nuclear potential well, V_0, thus

$$P_1 = \sqrt{2M(V_0 + E_1)}$$

where M = the mass of the nucleon,

and E_1 = the energy of the incident nucleon.

The magnitude of the nuclear potential was assumed to be given by the sum of the maximum Fermi energy, ε_f , of the neutrons in the target nucleus and the binding energy of the last neutron in that nucleus. The magnitude of ε_f was calculated by assuming that the neutrons were contained within a sphere of radius $1{\cdot}37\,A^{1/3} \times 10^{-13}$ cm. (A is the atomic weight of the nucleus). It was assumed that V_0 for the protons was equal to that of the neutrons, and that the maximum Fermi energy of the protons, ε_F, was given by the difference between V_0 and the binding energy of the last proton. The values of the binding energies were obtained from the tables of Feather (1953).

A nucleon can escape from the nucleus only if its energy ε is greater than V_0. Thus it is necessary to calculate the probability for the proton to possess an energy between ε and $\varepsilon + d\varepsilon$ inside the nucleus after the collision, such that $\varepsilon > V_0$.

Consider the collision of a neutron of energy ε_1 with a proton of energy ε_2 and momentum P_2 moving in a direction ϕ with respect to that of the incident particle. It will be assumed that the cross-section for scattering in the centre of momentum system is isotropic. Physically this assumption is not correct but the error involved is small ($\alpha_{np} \sim 0{\cdot}8\,\alpha_{nn}$ see appendix 1a.). The value of the measured cross-section for the scattering of free neutrons and protons varies inversely as the energy available in the centre of momentum system. A similar assumption will

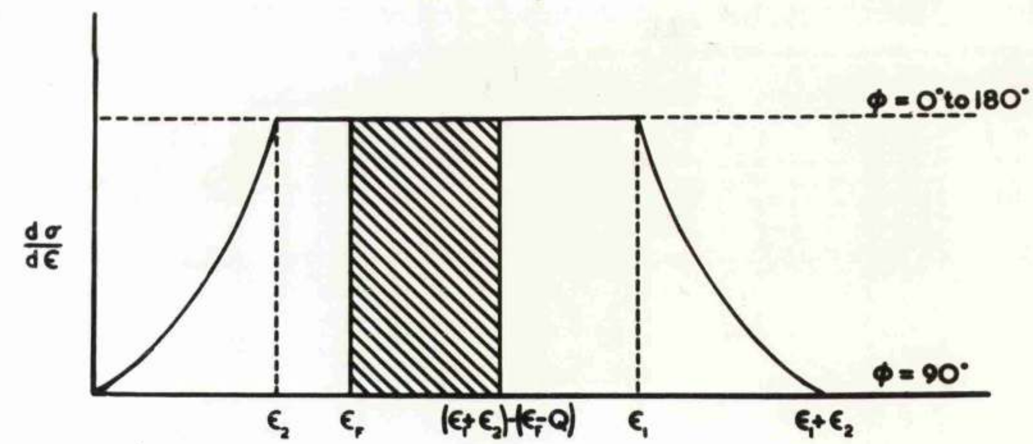


Figure II.1. The distribution in energy of the particles produced by the scattering of two nucleons of energies ϵ_1 and ϵ_2. The shaded area represents those collisions which are allowed by the Pauli Exclusion Principle.

be made here. The cross-section for scattering between the limits ε and $\varepsilon + d\varepsilon$ is given by

$$\left(\frac{d\sigma}{d\varepsilon}\right)_{\vec{P}_2} = \frac{P_1^2\, X(P_1)}{(|\vec{P}_1 - \vec{P}_2|)^2} \cdot \frac{|\vec{P}_1 - \vec{P}_2|}{P_1} \cdot \frac{2M}{|\vec{P}_1 - \vec{P}_2|\,|\vec{P}_1 + \vec{P}_2|} \qquad \text{(II.2)}$$

between the limits

$$\left\{|\vec{P}_1 + \vec{P}_2| + |\vec{P}_1 - \vec{P}_2|\right\}^2 \geq 8M\varepsilon \geq \left\{|\vec{P}_1 + \vec{P}_2| - |\vec{P}_1 - \vec{P}_2|\right\}^2$$

$$\left(\frac{d\sigma}{d\varepsilon}\right)_{\vec{P}_2} = 0$$

outside these limits.

The subscript in $d\sigma/d\varepsilon$ refers to fixed values of P_2 and ϕ. For convenience the operation of the Pauli Exclusion Principle will be ignored at present. Integration of equation (II.2) over all possible values of ϕ yields the following expression for the cross-section for the collision with a nucleon of fixed momentum P_2.

$$\left(\frac{d\sigma}{d\varepsilon}\right)_{P_2} = \frac{M\,X(P_1)}{2\sqrt{2}\,P_2\,(P_1^2 + P_2^2)^{\frac{1}{2}}} \left[\log_e \frac{(P_1^2 + P_2^2 + 2P_1P_2\cos\phi)^{\frac{1}{2}} + \sqrt{2}\,(P_1^2 + P_2^2)^{\frac{1}{2}}}{(P_1^2 + P_2^2 + 2P_1P_2\cos\phi)^{\frac{1}{2}} - \sqrt{2}\,(P_1^2 + P_2^2)^{\frac{1}{2}}}\right]_{\phi}^{\pi - \phi} \qquad \text{(II.3)}$$

The form of $\frac{d\sigma}{d\varepsilon}$, for protons, as a function of ε is displayed in figure II.1.

At this stage it is convenient to introduce the restrictions of the Pauli Exclusion Principle. For the collision of a neutron of energy ε_1 with a proton of energy ε_2 the Pauli Exclusion Principle is satisfied only in a reaction of the type ${}_{Z}N^{A}\,(n, p)\,{}_{Z-1}N^{A}$ if the energy of the proton is raised to a value such that

$$\varepsilon_F \leq \varepsilon \leq (\varepsilon_1 + \varepsilon_2) - (\varepsilon_F - Q)$$

where Q is the normal definition of the term representing the release of energy in a nuclear process* and ε_F is the maximum energy of the protons in the Fermi gas.

It follows that the protons which can participate in such collisions must possess energies which lie between the limits

$$\varepsilon + \varepsilon_F - Q - \varepsilon_1 \leq \varepsilon_2 \leq \varepsilon_F$$

The total cross-section for the production of a proton with an energy between ε and $\varepsilon + d\varepsilon$ is finally obtained by performing the integration

$$\frac{\int_x^y \left(\frac{d\sigma}{d\varepsilon}\right)_{P_2} P_2^2\, dP_2}{\int_0^y P_2^2\, dP_2} \tag{II.4}$$

where $y = \sqrt{2M\varepsilon_F}$

$x = \sqrt{2M(\varepsilon + \varepsilon_F - \varepsilon_1 - Q)}$

This integration yields the expression

$$\left(\frac{d\sigma}{d\varepsilon}\right) = \frac{3M\,X(P_1)}{\sqrt{2}\,P_F^3}\left[(P_1^2+P_2^2)^{\frac{1}{2}} \tanh^{-1}\frac{2P_2\sqrt{2(P_1^2+P_2^2)}}{P_1^2+3P_2^2} - 2\sqrt{2}\,P_1 \tanh^{-1}\frac{P_2}{P_1}\right]_x^y \tag{II.5}$$

---

* Footnote

In the nomenclature of this scattering problem the term $[(\varepsilon_1+\varepsilon_2)-(\varepsilon_F-Q)]$ implies that the neutron is left with sufficient energy that the exclusion principle is not violated for the neutron gas in the product nucleus $_{Z-1}N^A$. In the scattering of a neutron by a neutron in the target nucleus $Q = 0$.

It follows that the cross-section for producing a proton inside the target nucleus with kinetic energy between ε and $\varepsilon + d\varepsilon$ is given by

$$\left(\frac{d\sigma}{d\varepsilon}\right)_N = \sigma_{IP}\left(\frac{d\sigma}{d\varepsilon}\right) \Big/ \int_{\varepsilon_F}^{\varepsilon_1+Q} \left(\frac{d\sigma}{d\varepsilon}\right) d\varepsilon \qquad \text{(II.6)}$$

The subscript N refers to the target nucleus.

The entire term in the denominator of equation (II.6) is equal to the expression $\left[\alpha X(P_1)\right]_{np}$ of equation (II.1). The integrated expression is unwieldy but a numerically equivalent expression is

$$\alpha X(P_1) = \frac{X(P_1)}{2\sqrt{2}\,P_F^3}\left[\begin{array}{l}(P_1^2+P_2^2)^{\frac{1}{2}}(P_1^2+P_2^2-6P_F^2+6MQ)\,\tanh^{-1}\dfrac{2P_2\sqrt{2(P_1^2+P_2^2)}}{P_1^2+3P_2^2} \\ +4\sqrt{2}\,P_1(3P_F^2-3MQ-P_1^2)\tanh^{-1}\dfrac{P_2}{P_1} \quad + \; 2\sqrt{2}\,P_2 P_1^2\end{array}\right]_z^y \qquad \text{(II.7)}$$

where $z = \sqrt{2M(2\varepsilon_F - \varepsilon_1 - Q)}$

A proton produced in direct collisions can escape from the nucleus if its energy ε is greater than V_0. The cross-section, $\left(\frac{d\sigma}{dE}\right)_{np}$, for the emission of protons produced in direct collisions with kinetic energy between E and $E + dE\ (E = \varepsilon - V_0)$, is then

$$\left(\frac{d\sigma}{dE}\right)_{np} = \left(\frac{d\sigma}{d\varepsilon}\right)_N \phi(E) \qquad \text{(II.8)}$$

where $\phi(E)$ represents the probability of escape from the nucleus.

The total cross-section, $\sigma_p(\text{dir.})$, for the direct ejection of protons from the nucleus as a result of the first collision of a neutron with a proton can be obtained by numerical integration of equation (II.8).

$$\sigma_p(\text{dir.}) = \int_0^{E_1+Q} \left(\frac{d\sigma}{dE}\right)_{np} dE \qquad \text{(II.9)}$$

A rough numerical calculation showed that the probability for the direct emission of particles from second collisions is negligible, if the incident nucleon possesses an energy of less than 20 MeV.

The total cross-section, $\sigma_n(\text{dir.})$, for the emission of neutrons by direct collisions is given by

$$\sigma_n(\text{dir.}) = \int_0^{E_1} \left(\frac{d\sigma}{dE}\right)_{np} dE + 2\int_0^{E_1} \left(\frac{d\sigma}{dE}\right)_{nn} dE \qquad \text{(II.10)}$$

The factor two appears in the second term because it represents the collision of identical particles. The calculation of the individual terms of equation (II.10) is as already described.

(b) <u>The differential cross-section for the emission of nucleons produced in direct collisions.</u>

A calculation of the differential cross-section for the scattering of a nucleon by a Fermi gas of nucleons has been given by Hayakawa, Kawai and Kikuchi (1955). The method of calculation is basically correct, but there is an error in one of their equations. This mistake has been corrected and the calculation has been extended to include that for the scattering of non-identical particles. The details of their calculations will not be given here, instead the basic equations will be quoted and the error indicated. The nomenclature used will be as far as possible that of Hayakawa, Kawai and Kikuchi (loc.cit.). The equations given here differ occasionally by factors of

2 from those given by these authors.

For the collision of a nucleon of momentum P_1 with a nucleon of momentum P_2 the cross-section, $\sigma(\vec{P}_f)\, d\vec{P}_f$, for scattering such that the scattered nucleon has a final momentum between $\vec{P}_f$ and $\vec{P}_f + d\vec{P}_f$ is given by

$$\sigma(\vec{P}_f)\, d\vec{P}_f = \frac{1}{P_1\left(\frac{4\pi}{3}\right)P_F^3} \int 2P\, \sigma(\vec{P},\vec{P}')\, \frac{\delta(P-P')}{P^2}\, d\vec{P}_2\, d\vec{P}_f \qquad \text{(II.11)}$$

where $\vec{P}$ and $\vec{P}'$ = the momenta of the nucleons in the centre of momentum system before and after collision.

$$\sigma(\vec{P},\vec{P}') = \frac{1}{4\pi} \frac{P_1^2\, X(P_1)}{|\vec{P}_1 - \vec{P}_2|^2}$$

= the differential cross-section for scattering of the nucleons in the centre of momentum system.

and $$\delta(P-P') = 2P\, \delta(P^2 - P'^2)$$

A cylindrical co-ordinate system (z, ρ, ϕ) is introduced whose polar axis lies along the direction of the momentum transfer vector $\vec{q}$

$$\vec{q} = \vec{P}_f - \vec{P}_1 \qquad \text{(II.12)}$$

It then follows that

$$\delta(P^2 - P'^2) = \frac{1}{q}\, \delta(z - z_0) \qquad \text{(II.13)}$$

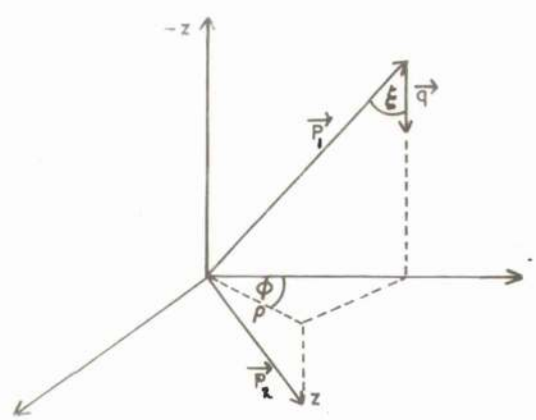


Figure II.2. The co-ordinate system used for the computation of $|\vec{P}_1 - \vec{P}_2|^2$.

where

$$z_0 = \frac{1}{2q}\left(P_f^2 + q^2 - P_1^2\right) \tag{II.14}$$

Equation (II.13) may now be written as

$$\sigma(\vec{P}_f)\, d\vec{P}_f = \frac{1}{P_1\left(\frac{4\pi}{3}\right)P_F^3}\cdot\frac{1}{\pi q}\int \sigma_t\, \delta(z-z_0)\, \rho\, d\rho\, dz\, d\phi\, d\vec{P}_f \tag{II.15}$$

where

$$\sigma(\vec{P},\vec{P}') = \frac{\sigma_t}{4\pi}$$

The energy dependence of this cross-section is introduced at this point

$$\sigma_t = \frac{2\sigma_L}{|\vec{P}_1 - \vec{P}_2|^2} \tag{II.16}$$

Thus in the nomenclature used in § 3(a), σ_L is given by

$$\sigma_L = \frac{P_1^2\, x(P_1)}{2} \tag{II.17}$$

The mistake made by Hayakawa, Kawai and Kikuchi (loc.cit.) was in the derivation of the magnitude of the quantity $|\vec{P}_1 - \vec{P}_2|^2$. The correct expression for this quantity can be seen from figure II.2 to be given by

$$|\vec{P}_1 - \vec{P}_2|^2 = P_1^2 + z^2 + \rho^2 + 2P_1\left(z\cos\xi - \rho\sin\xi\cos\phi\right) \tag{II.18}$$

When these quantities are inserted into equation (II.15) and the integration over z and ϕ performed, we have

$$\sigma(\vec{P}_f)\, d\vec{P}_f = \frac{4\sigma_L}{P_1\left(\frac{4\pi}{3}\right)P_F^3}\cdot\frac{1}{q}\int\frac{\rho\, d\rho}{\sqrt{\rho^4 + 2b\rho^2 + c}}\, d\vec{P}_f \tag{II.19}$$

where now the quantities b and c are given by

$$b = P_1^2 + z_0^2 + 2P_1 z_0 \cos\xi - 2P_1^2 \sin^2\xi \quad = z_0^2 + q^2 - P_f^2$$
$$c = \left(P_1^2 + z_0^2 + 2P_1 z_0 \cos\xi\right)^2 \quad = \left(P_f^2 + q^2 - z_0^2\right)^2 \tag{II.20}$$

The limits of integration are determined by the condition that before collision the target nucleon must have a momentum P_2 less than that of the Fermi maximum, after collision it must have a momentum such that the Pauli Exclusion Principle is not violated. Thus in the scattering of non-identical particles

$$P_2^2 \leq P_F^2 \qquad\qquad P_2'^2 \geq P_F^2 - 2Q \tag{II.21}$$

and the limits of integration are

$$\rho_2^2 = P_F^2 - z_0^2$$
$$\rho_1^2 = P_F^2 + P_f^2 - 2Q - P_1^2 - z_0^2 \tag{II.22}$$

The integration of equation (II.19) over ρ then yields

$$\sigma(\vec{P}_f)\, d\vec{P}_f = \frac{2\sigma_L}{P_1\left(\frac{4\pi}{3}\right)P_F^3} \cdot \frac{d\vec{P}_f}{q}\left[\log_e \frac{\sqrt{\rho_2^4 + 2b\rho_2^2 + c} + \rho_2^2 + b}{\sqrt{\rho_1^4 + 2b\rho_1^2 + c} + \rho_1^2 + b}\right] \tag{II.23}$$

Thus the differential cross-section, $\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)^\theta$, for the production of a nucleon of energy ε at an angle θ as a result of single nucleon-nucleon collisions may be written as

$$\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)^\theta = \frac{3M P_f P_1 X(P_1)}{4\pi q P_F^3}\left[\log_e \frac{\sqrt{\rho_2^4 + 2b\rho_2^2 + c} + \rho_2^2 + b}{\sqrt{\rho_1^4 + 2b\rho_1^2 + c} + \rho_1^2 + b}\right] \tag{II.24}$$

The differential cross-section, $\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)^\theta_N$, for the production of a proton of energy ε at an angle θ inside the nucleus, in the case of an (n,p) reaction, is then

$$\left(\frac{d^2\sigma}{d\varepsilon d\omega}\right)_N^\theta = \frac{\sigma_I}{[\alpha X(\rho_1)]_{np}\rho_p + [\alpha X(\rho_1)]_{nn}\rho_n} \cdot \left(\frac{d^2\sigma}{d\varepsilon d\omega}\right)_{np}^\theta \rho_p \qquad \text{(II.25)}$$

where $\left(\frac{d^2\sigma}{d\varepsilon d\omega}\right)_{np}^\theta$ is calculated from equation (II.24).

(c) The probability of emission of nucleons produced in direct collisions.

The magnitude of the mean free path of a nucleon in nuclear matter is at the energies under consideration quite large in comparison to the dimensions of the nucleus. This allows the possibility of the emission of nucleons produced in direct collisions. The probability of emission of a neutron is completely different from that for a proton, because of the inhibition to the emission of protons produced by the coulomb barrier.

It has been shown (Perkins 1949) that the average distance which a particle traverses, in a nucleus of radius R, before reaching the surface of the nucleus is $0{\cdot}75\,R$, when the direction of motion of the particles is random. In order to satisfy the principles of conservation of energy and momentum, the particles produced in the collision of two nucleons must move in the direction of the primary particle. A numerical computation showed that the average distance traversed by these nucleons is still about $0{\cdot}75\,R$..

The difficult problems associated with nucleons crossing the nuclear boundary have been avoided. It has been assumed that the nuclear boundary is sharp and that the probability, P , that a nucleon crosses this boundary is given by

$$P = 1 \quad \text{for neutrons}$$
$$P = \frac{\sigma_p}{\sigma_n} \quad \text{for protons} \tag{II.26}$$

where σ_p and σ_n represent the cross-sections for the interaction of protons and neutrons respectively with nuclei. The values of σ_p used were those given by Shapiro (loc.cit.) for a nuclear radius $R = 1.5 A^{1/3} \times 10^{-13}$ cm.; these cross-sections are in good agreement with the measurements of Tendam and Bradt (1947) and of Blaser and his co-workers (1951). The values of σ_n used were those measured by Beyster, Henkel and Nobles (1955) for neutrons with an energy of 4 MeV.

Thus the probability, p , that a nucleon of energy ε may reach the surface of the nucleus and escape in the first encounter with this surface is given by

$$p = P \exp. - \left[\frac{0.75 R}{\lambda(\varepsilon)}\right] \tag{II.27}$$

The value of the mean free path, $\lambda(\varepsilon)$, of a nucleon η of energy ε in nuclear matter was calculated from the formula

$$\frac{1}{\lambda(\varepsilon)} = \left[\alpha X(P_1)\right]_{\eta p} \rho_p + \left[\alpha X(P_1)\right]_{\eta n} \rho_n \tag{II.28}$$

Since P is less than unity for protons and the mean free path for a nucleon in nuclear matter is much larger than the diameter of the nucleus, it is possible for protons to be emitted after more than one traversal of the nucleus. The problem of calculating the relative probabilities of escape of protons from a nucleus, at specific angles with respect to the direction of the incident particle is difficult; the procedure which has been adopted can be regarded only as qualitative.

Since the protons move in the direction of the incident particle (forward) following the initial collision, it has been assumed that they move backward after the first, third, fifth etc., reflections and forward after the second, fourth etc., reflections. The average distance traversed by a proton in crossing the nucleus after such a reflection is $1{\cdot}33R$.

The probabilities, ϕ_1 and ϕ_2, for emission of reflected protons in the forward and backward hemispheres, respectively, are then

$$\phi_1 = px^2 + px^4 + \text{etc.} \qquad = \frac{px^2}{1-x^2} \qquad \text{(II.29)}$$

$$\phi_2 = px + px^3 + \text{etc.} \qquad = \frac{px}{1-x^2} \qquad \text{(II.30)}$$

where $$x = (1-P)\,\exp.-\left[\frac{1{\cdot}33R}{\lambda(\varepsilon)}\right]$$

The direction of emission of these protons emitted after multiple reflections will be assumed to be isotropic over forward and backward directions.

The total probability of escape, ϕ , as required in equation (II.8) is then obtained from equations (II.27), (II.29) and (II.30).

$$\phi = p + \frac{px}{1-x^2} + \frac{px^2}{1-x^2} = \frac{p}{1-x} \qquad \text{(II.31)}$$

The quantities ϕ_1 and ϕ_2 are required for the computation of the angular distributions of protons which are emitted as a result of direct collisions. When the angle of emission θ is less than 90° the differential cross-section, $\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^{\theta}$, for the emission of protons is composed of two terms; that for emission as a result of the first encounter with the nuclear surface and that for emission following multiple reflections

$$\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^{\theta} = p\left(\frac{d^2\sigma}{d\varepsilon\,d\omega}\right)_N^{\theta} + \frac{px^2}{1-x^2}\cdot\frac{1}{2\pi}\cdot\left(\frac{d\sigma}{d\varepsilon}\right)_N \qquad \theta < 90^{\circ} \text{ (II.32)}$$

When the angle θ is greater than 90° the protons emitted are only those which have been reflected at the surface of the nucleus. Thus

$$\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^{\theta} = \frac{px}{1-x^2}\cdot\frac{1}{2\pi}\left(\frac{d\sigma}{d\varepsilon}\right)_N \qquad \theta > 90^{\circ} \text{ (II.33)}$$

The magnitude of ϕ_1 is usually negligible,however the magnitude of ϕ_2 may be appreciable ($\phi_2 \sim 0.3\,\phi$ for protons with an energy of approximately 5 MeV, for a nucleus of $A \sim 50$).

§4. THE EMISSION OF NUCLEONS FROM AN EXCITED NUCLEUS.

(a) The formation and decay of an excited nucleus.

According to the model which has been used in these calculations, the nuclear excitation is initially concentrated in two nucleons. When these nucleons move through the nucleus their energy sometimes becomes degraded in further collisions until it is below the height of the nuclear potential well. These nucleons cannot escape from the nucleus and therefore as a result of the direct collision process an excited nucleus may be produced. This excited nucleus may exist in various states of excitation, but may easily be classified as being one of two types:-

(1) those nuclei which are left after the emission of nucleons by direct collisions does occur. These nuclei may have any value for the energy of excitation between 0 and E_1.

(2) those nuclei which are produced when the emission of nucleons by direct collisions does not occur. These nuclei have an energy of excitation, U, given by

$$U = E_1 + B$$

where B is the binding energy of the last nucleon in the excited nucleus. The cross-section, σ_c, for the formation of an excited nucleus with this energy of excitation is given by

$$\sigma_c = \sigma_I - \left[\sigma_p(\text{dir.}) + \sigma_n(\text{dir.}) \right] \qquad \text{(II.34)}$$

The value of this cross-section is approximately $\frac{2}{3}\sigma_I$ for nuclei of $A \sim 50$.

Detailed considerations of the mode of disintegration of the excited nuclei produced when the emission of nucleons by direct collisions occurs have not been made. It has been assumed at all times that if the energy of excitation is sufficiently high for the emission of a neutron to be energetically possible, then this process takes place; otherwise these excited nuclei have been assumed to decay by the emission of γ - radiation.

Those nuclei which are produced when the emission of nucleons by direct collisions does occur, have been considered in detail. Since the energy of excitation of these nuclei is high ($>$ 20 MeV), there will probably be several vacancies in the levels in the Fermi well. Thus the mean free path of the nucleons inside the nucleus may be reduced, and the conditions necessary for the maintenance of a statistical energy distribution with the gradual boiling off of particles might be established. Consequently it has been assumed that these nuclei decay according to the statistical theory of Weisskopf and Ewing (loc.cit.).

The cross-section, σ_b (comp.), for the emission of a particle, b , by the decay of this excited nucleus may be written then, I § 1.(d), as

$$\sigma_b(\text{comp.}) = \sigma_c \frac{\Gamma_b}{\sum_{b'} \Gamma_{b'}} \tag{II.35}$$

where the term Γ_b may be defined as

$$\Gamma_b = \text{const.}\, M(2s+1) \int_0^{E_1+Q} E\, \sigma_b(E)\, \omega_R(E_1+Q-E)\, dE \tag{II.36}$$

The terms in these expressions have the meanings given in I § 1(d) .

In all calculations it has been assumed that particles of all energies, emitted in the decay of this excited nucleus, are emitted isotropically. Thus the differential cross-section for the emission of the particle b by decay of the excited nucleus has been assumed to be given by

$$\left(\frac{d\sigma}{d\omega}\right) = \frac{\sigma_b(\text{comp.})}{4\pi} \tag{II.37}$$

For the value of $\sigma_b(E)$ for protons and alpha-particles the values given by Shapiro (loc.cit.), for $r_0 = 1.5 \times 10^{-13}$ cm., have been used. For neutrons this quantity has been assumed to be equal to the cross-section for the inelastic scattering of neutrons with an energy of 4 MeV; thus the values of this cross-section measured by Beyster, Henkel and Nobles (1955) have been used (appendix 1b.).

The probability of emission of a particle in the decay of an excited nucleus is strongly dependent on the level density of the residual nucleus. To compute the relative probabilities of the modes of decay of the excited nucleus satisfactorily, a semi-empirical determination of the possible variations in the level densities of excited nuclei has been made. This will be considered in the next section.

(b) <u>The determination of the density of levels of an excited nucleus.</u>

Lang and Le Couteur (loc.cit.) have made an extensive study of the spacings of nuclear levels, D , of excited nuclei. They have shown that the application of the thermodynamic properties of a Fermi gas to the nucleons of a nucleus yields the following equation for the spacing of nuclear levels of zero spin, D_0 , in a nucleus of atomic weight A and excitation energy U :-

$$D_0 = 0{\cdot}11\, A^2 (U+t)^2 \exp.-\left[2\left(\frac{AU}{11}\right)^{\frac{1}{2}} + \frac{3}{32}(11\,U)^{2/3}\right] \qquad \text{(II.38)}$$

where

$$t = \left(\frac{10{\cdot}5\,U}{A}\right)^{\frac{1}{2}} - \frac{7{\cdot}9}{A}$$

The spacing of nuclear levels of spin I , D_I , is assumed to be given by

$$(2I+1)\,D_I = D_0 \qquad \text{(II.39)}$$

Since the analysis of Lang and Le Couteur (loc.cit.) was made, a considerable amount of experimental data has been obtained concerning the density, ω (where $\omega D = 1$), of levels of many nuclei at excitation energies of approximately 8 MeV. This region of excitation energy is relevant to the present calculations since for most of the reactions considered the nuclei left after the emission of a charged particle have approximately this energy of excitation. Thus a comparison has been made of the predictions of the formula of Lang and Le Couteur (loc.cit.) with these experimental results.

The data on the level spacings in light nuclei have been taken from the tables of Endt and Kluyver (1954) and from recent publications by Buechner et.al. (1956), Paris et.al. (1955) and Paul et.al. (1956). Those on the scattering of slow neutrons from heavy nuclei have been taken from the work of Harvey, Hughes, Carter and Pilcher (1955). In the selection of these data on level spacings only those derived in the most reliable experiments have been used. Thus for the light nuclei the results of experiments on the scattering of neutrons by nuclei have not been considered.

An average quantum number of spin has been assigned to each group of levels, the assignments were made either from the information obtained in the individual experiments, or from reasonable assumptions concerning the magnitude of the initial and product nuclei involved in the individual reactions and the energy of the incident particles. These assignments are probably most reliable for the heavy nuclei with I greater than 1.

The comparisons between the predictions of the formula of Lang and Le Couteur (loc.cit.) are shown in tables 1,2 and 3.

Table 1.

Odd-odd nuclei.

Table 1.

Odd-odd nuclei.

Nucleus	Reaction	U in MeV	Measured average level spacing in ev	Average I	$(2I + 1)D_I$ in ev	$\frac{\text{Expt.}}{\text{Theory}}$
${}_{11}Na^{24}$	(d,p)	3.5 (2.5–4.5)	1.5×10^5	2	7.5×10^5	0.4
${}_{13}Al^{26}$	(p,γ)	7.3 (7.0–7.6)	4×10^4	$5/2$	2.4×10^5	1.0
${}_{13}Al^{28}$	(d,p)	4.0 (2.25–5.85)	7×10^4	2	3.5×10^5	0.3
	(d,p)	7.0 (6.3–7.7)	5×10^4	3	3.5×10^5	1.5
${}_{15}P^{32}$	(d,p)	3.2 (2.2–4.2)	2×10^5	2	1.0×10^6	0.7
${}_{49}In^{114}$	Neutron Scattering	7.2	14 ± 2	$9/2$	$(1.4 \pm 0.2)10^2$	0.3
${}_{49}In^{116}$	"	6.6	14 ± 2	$9/2$	$(1.4 \pm 0.2)10^2$	0.2

$_{55}Cs^{134}$	"	6.7	42 ± 5	$^7/_2$	$(3.4 + 0.4)10^2$	1.1
$_{63}Eu^{152,4}$	"	5.7	2.2 ± 0.3	$^5/_2$	13.2 ± 1.8	0.03
$_{65}Tb^{160}$	"	5.8	10 ± 1.0	$^3/_2$	40 ± 4	0.13
$_{67}Ho^{166}$	"	5.7	12 ± 1.3	$^7/_2$	96 ± 10	0.4
$_{69}Tm^{170}$	"	5.9	15 ± 2	$^1/_2$	30 ± 4	0.2
$_{71}Lu^{176}$	"	6.0	7 ± 2	$^7/_2$	56 ± 16	0.6
$_{73}Ta^{182}$	"	6.0	9 ± 1	$^7/_2$	72 ± 8	0.9

Table 2.

Even-odd, odd-even nuclei.

Table 2.

Even-odd, odd-even nuclei.

Nucleus	Reaction	U in MeV	Measured average level spacing in ev.	Average I	$(2I + 1)D_I$ in ev	$\frac{\text{Expt.}}{\text{Theory}}$
$_{11}Na^{21}$	(p,p')	5 (3.5-6.5)	2.5×10^5	3/2	1.0×10^6	1.0
$_{11}Na^{23}$	(p,γ)	9.8 (9.3-10.3)	5×10^4	1	1.5×10^5	1.4
$_{12}Mg^{25}$	(d,p)	3.3 (2.0-4.6)	3×10^5	2	1.5×10^6	0.8
$_{13}Al^{25}$	(p,p')	4.1 (2.9-5.3)	3×10^5	2	1.5×10^6	1.3
$_{13}Al^{27}$	(p,γ)	9.0 (8.5-9.5)	7×10^4	3/2	2.8×10^5	2.8
$_{14}Si^{29}$	(d,p)	4.0 (2.0-6.0)	4×10^5	2	2.0×10^6	1.9

$_{14}Si^{31}$	(d,p)	3.3 (2.3–4.3)	4×10^5	2	2.0×10^6	1.4
$_{15}P^{31}$	(p,γ)	9.3 (9.0–9.6)	2.5×10^4	2	1.2×10^5	2.0
$_{16}S^{33}$	(d,α)	3.1 (2.0–4.2)	2×10^5	3/2	8×10^5	0.6
$_{20}Ca^{41}$	(d,p)	3.0 (1.95–3.95)	2.5×10^5	3/2	1.0×10^6	0.8
$_{42}Mo^{93,5,7,9,101}$	Neutron Scattering	6.9	$(5 \pm 1.5)10^2$	1/2	$(10 \pm 3)10^2$	0.5
$_{50}Sn^{113}$	"	8.0	$(1.5 \pm 0.8)10^2$	1/2	$(3 \pm 1.6)10^2$	1.2
$_{50}Sn^{117}$	"	7.3	$(1.5 \pm 0.7)10^2$	1/2	$(3 \pm 1.4)10^2$	0.7
$_{50}Sn^{119}$	"	6.6	$(2 \pm 1)10^2$	1/2	$(4 \pm 2)10^2$	0.5
$_{50}Sn^{121,123,125}$	"	6.1	$(5 \pm 2)10^2$	1/2	$(10 \pm 4)10^2$	0.9
$_{72}Hf^{179,181}$	"	6.1	$(1 \pm 0.5)10^2$	1/2	$(2 \pm 1)10^2$	2.8
$_{92}U^{239}$	"	4.9	18 ± 2	1/2	(36 ± 4)	0.7

Table 3.

Even-even nuclei.

Table 3.

Even-even nuclei.

Nucleus	Reaction	U in MeV	Measured average level spacing in ev.	Average I	$(2I + 1)D_I$ in ev	$\frac{\text{Expt.}}{\text{Theory}}$
${}_{12}Mg^{24}$	(p,p')	4.8 (4.1-5.5)	4×10^5	3	2.8×10^6	3
	(p,γ)	12.8 (12.0-13.6)	7×10^4	2	3.5×10^5	12.
${}_{12}Mg^{26}$	(d,p)	4.3 (3.0-5.6)	3×10^5	5/2	1.8×10^6	2
${}_{14}Si^{28}$	(p,γ)(p,p')	12.0 (11.8-12.25)	5×10^4	5/2	3×10^5	13
${}_{16}S^{32}$	(p,p')	4.8 (3.8-5.8)	3×10^5	2	1.5×10^6	3
	(p,γ)	10.0 (9.2-10.9)	1.1×10^4	3/2	4.5×10^5	21
${}_{18}A^{38}$	(p,n)	12.3 (11.8-12.7)	5×10^3	5/2	3×10^4	5.5
${}_{42}Mo^{96}$	Neutron Scattering	9.2	$(3.7 \pm 1.2)10^2$	5/2	$(22 + 7)10^2$	8.5

$_{42}Mo^{98}$	"	8.3	$(3.7 \pm 1.2)10^2$	5/2	$(22 \pm 7)10^2$	4.5
$_{50}Sn^{118}$	"	9.3	$(1.2 \pm 0.3)10^2$	1/2	$(2.4 \pm 0.6)10^2$	5.3
$_{50}Sn^{120}$	"	8.6	$(3 \pm 1.4)10^2$	1/2	$(6 \pm 2.8)10^2$	7.7
$_{72}Hf^{178}$	"	7.6	5.6 ± 0.6	$\leq 3/2$	22 ± 2.4	2.3
$_{72}Hf^{180}$	"	7.4	8 ± 2	$\leq 3/2$	32 ± 8	2.8

The agreement between the experimental and theoretical data in table 2 is remarkable in view of the variation of the level spacing by a factor of 10^4. The tables show that there is a very pronounced difference between the spacings of levels in even-even nuclei and that of other nuclei; the difference between even-odd and odd-odd nuclei is not so certain. It is found, by taking an average of the final columns of the tables, that for nuclei of the same mass and excitation energy the average level densities are given by

$$\frac{\omega_{o-o}}{12} = \frac{\omega_{o-e}}{5} = \omega_{e-e} \qquad \text{(II.40)}$$

These variations in the level densities are much larger than those obtained by Weisskopf (loc.cit.) in the analysis of the data of Bardeen and Feenberg (1938). It is possible that, due to the lack of resolving power in the existing experimental investigations, some of the levels of excited nuclei have not been observed. If this is the case it is possible that the factors of 12 and 5 could be higher.

These factors of 12 and 5 have been used at all times when the emission of nucleons from an excited nucleus has been considered.

§ 5. RESULTS

(1) The cross-sections for the processes ${}_{z}N^{A}(n,p\gamma)_{z-1}N^{A}$ and ${}_{z}N^{A}(n,pn)_{z-1}N^{A-1}$ induced by neutrons with an energy of 14.5 MeV.

The cross-sections for the process ${}_{z}N^{A}(n,p\gamma)_{z-1}N^{A}$ * have been measured by Paul and Clarke (loc.cit.) and by Forbes (1952). To make comparisons with this data it has been assumed that it is not possible for an excited nucleus to emit γ- radiation if it possesses sufficient energy to emit a neutron. It follows that the permitted limits of energy for the emission of a proton such that a radioactive nucleus is left are ($E_1 + Q$) and ($E_1 + q$) where q represents the binding energy of the last proton in the target nucleus.

Thus the cross-sections $\sigma_p(\mathrm{dir.})$ for the emission of protons as a result of direct collisions, and $\sigma_p(\mathrm{comp.})$ for the emission of protons in the decay of the nuclei with excitation energy U , have been calculated by numerical integration of equations (II.9) and (II.35) between the appropriate limits. The results of these calculations are given in table 4.

---

* Footnote

The terminology $(n,p\gamma)$ and (n,pn) used here means that a proton is emitted first followed at a later stage either by the emission of γ- radiation or a neutron.

Table 4.

Element	Q MeV	$\sigma(n, p\gamma)$ mb.				
		σ_p(comp.)	σ_p(dir.)	σ_p(comp.) + σ_p(dir.)	Paul and Clarke	Forbes
O^{16}	-9.4	15	5	20	49±25	
F^{19}	-3.7	15	25	40	135±45	
Na^{23}	-3.4	15	30	45	34±15	
Mg^{24}	-4.7	160	45	205	190±20	
Mg^{25}	-2.9	60	60	120	45±18	
Al^{27}	-1.9	25	40	65	52±10	80±5
Si^{28}	-3.8	215	45	260	220±50	
Si^{29}	-3.0	45	55	100	100±30	
P^{31}	-0.7	30	35	65	64±10	90±10
S^{32}	-0.9	170	50	220	370±50	
S^{34}	-4.3	70	30	100	85±45	
Cl^{37}	-3.5	10	30	40	33±6	
K^{41}	-1.8	10	35	45	80±30	
Ti^{48}	-3.1	100	30	130	93±30	
V^{51}	-1.4	10	30	40	27±4	
Cr^{52}	-3.0	75	30	105	78±11	
Fe^{56}	-2.9	60	30	90	97±12	124±12
Ni^{61}	-0.5	30	45	75	182±30	

Table 4. (Contd.)

Element	Q MeV	$\sigma\,(n,p\gamma)$ mb.				
		σ_p(comp.)	σ_p(dir.)	σ_p(comp.) + σ_p(dir.)	Paul and Clarke	Forbes
Cu^{65}	-1.3	5	30	35		19 ± 4
Zn^{64}	+0.2	110	45	155	386 ± 60	
Zn^{66}	-1.9	45	30	75	100 ± 17	
Ga^{69}	-0.1	5	25	30	24 ± 18	
Ge^{70}	-1.0	60	30	90	130 ± 65	
Ge^{72}	-3.2	10	20	30	65 ± 30	
Ge^{73}	-0.6	6	30	36	137 ± 68	
As^{75}	-0.4	3	30	33	12 ± 2	
Se^{77}	0.0	5	20	25	45 ± 23	
Sr^{88}	-4.4	2	9	11	18 ± 3	
Zr^{90}	-1.4	20	20	40	250 ± 100	
Zr^{94}	-4.6	0.3	6	6	10.5 ± 5	
Mo^{97}	-1.3	2	20	22	108 ± 54	
Ru^{101}	-0.7	3	22	25	2 ± 1	
Pd^{104}	-1.5	6	17	23	132 ± 66	
Pd^{105}	+0.2	4	25	29	740 ± 520	
I^{127}	0.0	1	16	17	230 ± 140	
Ba^{138}	-4.1	0	3	3	6 ± 2	
La^{139}	-1.5	0	9	9	6 ± 2.5	
Tl^{205}	-1.0	0	3	3	3 ± 1.5	
Pb^{208}	-4.2	0	0.3	0.3	1 ± 1	

It can be seen that the calculations follow closely the trend of the experimental results. The large fluctuations in the calculated cross-sections are produced by the variations in the Q-values for the processes ${}_{Z}N^{A}(n,p\gamma)_{Z-1}N^{A}$ and ${}_{Z}N^{A}(n,pn)_{Z-1}N^{A-1}$ and also by the variations in the level densities of excited nuclei. It should be noted that in addition to the fluctuations resulting from the even-even, even-odd and odd-odd characteristics of nuclei, deviations from the formula of Lang and Le Couteur might arise from the detailed structure of an individual nucleus. These fluctuations should only affect the magnitude of $\sigma_p(\text{comp.})$.

It can be seen that the mechanism of direct collisions is responsible for the agreement obtained for the heaviest nuclei. It was found by Paul and Clarke (loc.cit.) that the cross-section obtained using merely statistical theory was too small by a factor of 10^4 for lead.

The total cross-sections for the emission of protons as a result of the interaction of neutrons with an energy of 14 MeV with some nuclei have been measured by Allan (1956) and by Armstrong and Rosen (1956). To make a comparison with these measurements the cross-sections for the processes ${}_{Z}N^{A}(n,p\gamma)_{Z-1}N^{A}$ and ${}_{Z}N^{A}(n,pn)_{Z-1}N^{A-1}$ have been calculated. Unfortunately for some of the nuclei considered the cross-section for the process ${}_{Z}N^{A}(n,np)_{Z-1}N^{A-1}$ is not negligible. This occurs when it is not energetically possible for the emission of two neutrons to take

Table 5.

Element	Q MeV	$\sigma(n,p\gamma) + \sigma(n,pn)$ mb.			Experimental results in mb.	
		σ_p(comp.)	σ_p(dir.)	σ_p(comp.) + σ_p(dir.)	Total cross-section	Corrected for (n,np)
Al^{27}	-1.9	75	100	175	140*	70
Fe^{54}	+0.3	540	55	595	580*	460
Fe^{56}	-2.9	80	30	110	190*	190
Ni^{58}	+0.6	470	60	530	530*	310
Ni^{60}	-2.0	115	30	145	300*	240
Cu^{63}	+0.7	80	65	145	250*	120
Cu^{65}	-1.3	15	45	60	< 80*	< 40
Zn^{64}	+0.2	360	55	415	590†	200
Pd^{105}	+0.2	6	24	30	10†	10

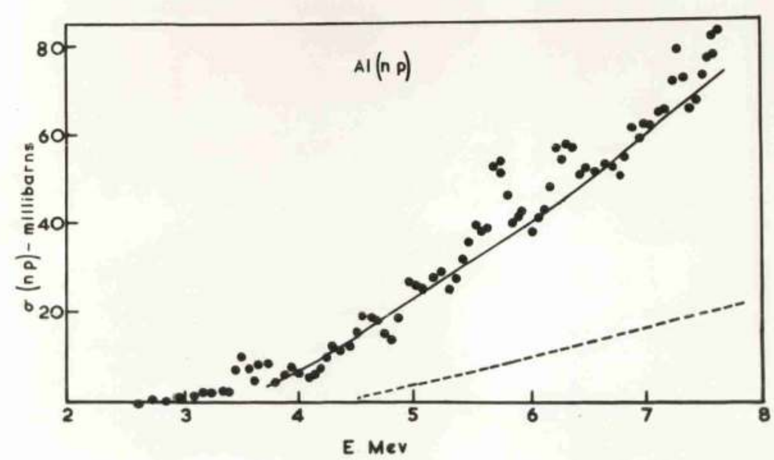
*Allan
†Armstrong and Rosen.

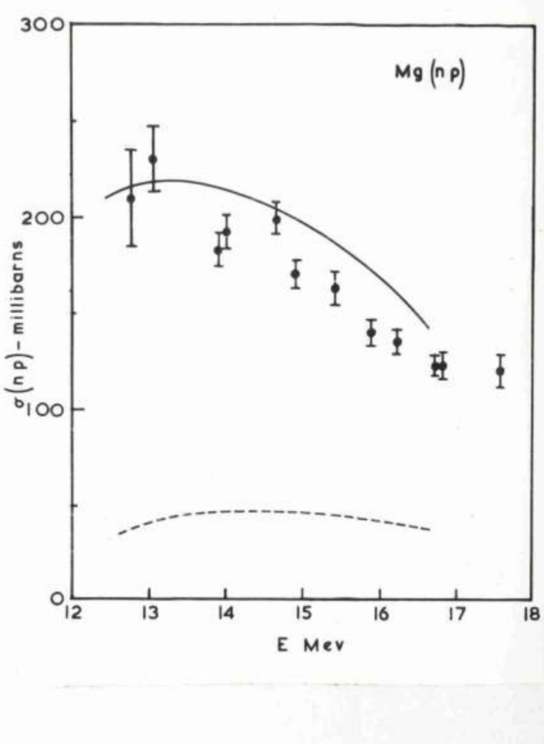
place. No attempt has been made to calculate the cross-section for this process since it involves the calculation of the relative probabilities of the emission of protons and γ-radiation; sufficient information concerning these relative probabilities is not available. However the protons arising in this process are readily distinguished from those arising in the processes ${}_{Z}N^{A}(n,p\gamma)_{Z-1}N^{A}$ and ${}_{Z}N^{A}(n,pn)_{Z-1}N^{A-1}$ (Allan, private communication), and corrections for this effect have been made by the experimental investigators.

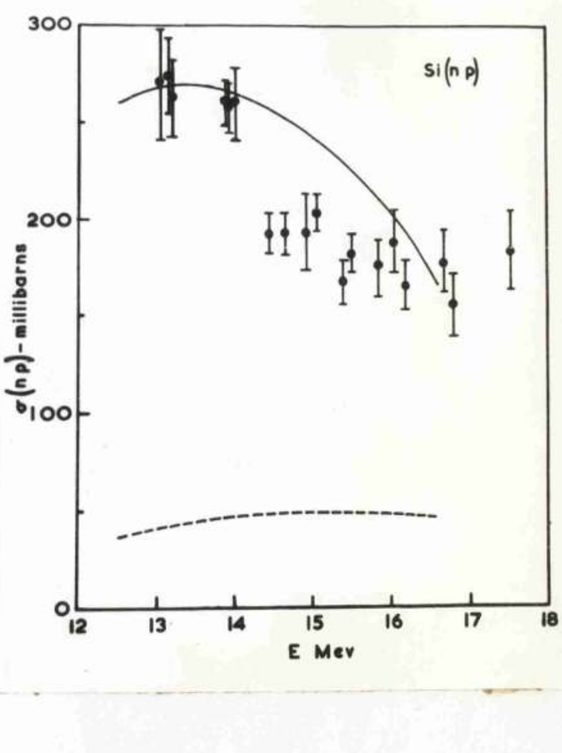
The comparison given in table 5 shows that the overall agreement between the calculated and experimental cross-sections is reasonably good, in view of the wide range of mass values. It should be pointed out that in the experiments performed by Allan (loc.cit.) only the protons emitted at forward angles were observed. The cross-sections were obtained by assuming that the angular distributions were isotropic; thus the cross-sections quoted by Allan (loc.cit.) might be overestimates when the cross-section for the emission of protons by direct collisions is the predominant one. A more satisfactory test of the model could be made by observations of the angular distributions of the protons emitted in these reactions.

(2) <u>The excitation functions for the reactions ${}_{12}Mg^{24}(n,p\gamma)_{11}Na^{24}$, ${}_{13}Al^{27}(n,p\gamma)_{12}Mg^{27}$, ${}_{14}Si^{28}(n,p\gamma)_{13}Al^{28}$, ${}_{17}Cl^{37}(n,p\gamma)_{16}S^{37}$ and ${}_{38}Sr^{88}(n,p\gamma)_{37}Rb^{88}$.</u>

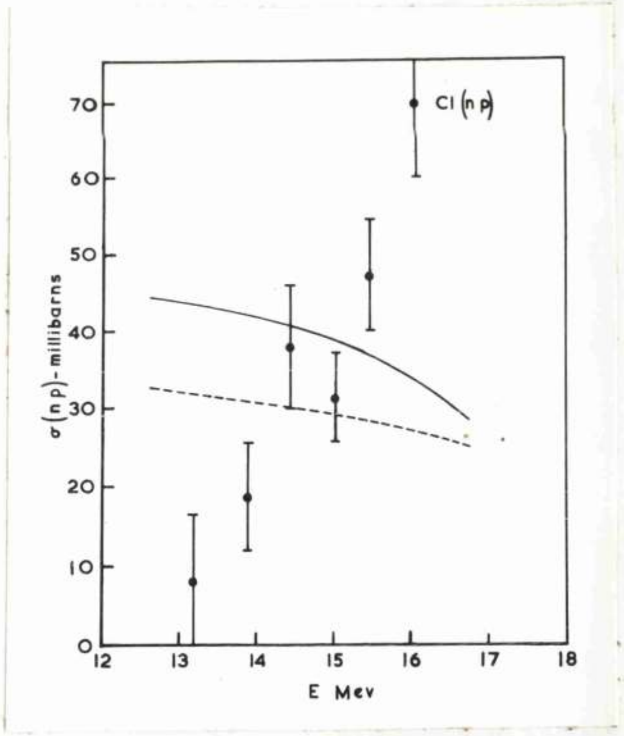
The excitation function for ${}_{13}Al^{27}$ has been measured by workers at Los Alamos (Hughes and Harvey 1955), while those for

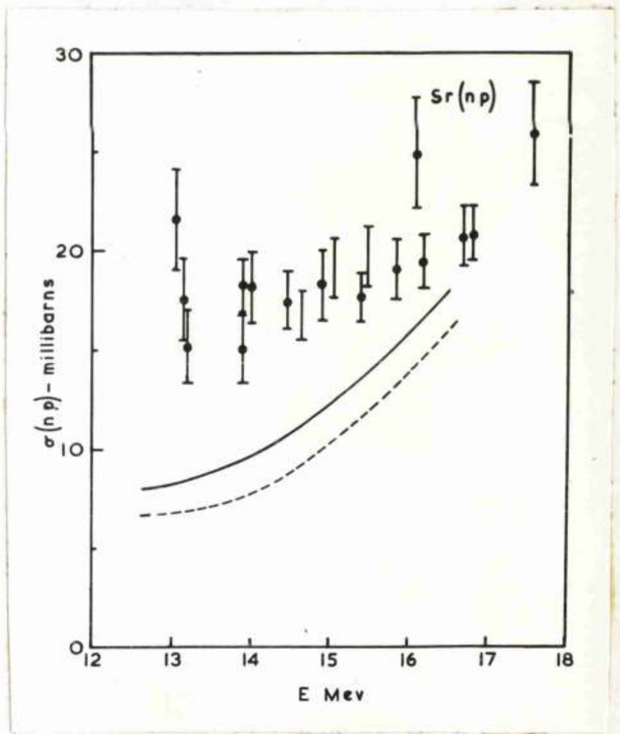






Figures II.3 - II.5. The excitation functions for the reactions $_{13}Al^{27}(n,p\gamma)_{12}Mg^{27}$, $_{12}Mg^{24}(n,p\gamma)_{11}Na^{24}$ and $_{14}Si^{28}(n,p\gamma)_{13}Al^{28}$.
- - - - $= \sigma_p$ (dir.), ——— $= \sigma_p$ (dir.) $+ \sigma_p$ (comp.)





Figures II.6 and II.7. The excitation functions for the reactions $_{17}Cl^{37}(n,p\gamma)_{16}S^{37}$ and $_{38}Sr^{88}(n,p\gamma)_{37}Rb^{88}$.
$----= \sigma_p(\text{dir.})$, $—— = \sigma_p(\text{dir.}) + \sigma_p(\text{comp.})$

${}_{12}Mg^{24}$, ${}_{14}Si^{28}$, ${}_{17}Cl^{37}$ and ${}_{38}Sr^{88}$ have been measured by Cohen and White (1956). The latter workers have measured all cross-sections for a particular nucleus relative to the cross-sections measured for neutrons of 14.5 MeV by Paul and Clarke (loc.cit.).

The calculations have been made in exactly the way discussed in (1). To calculate the excitation function for ${}_{13}Al^{27}$ a value of σ_I used was 0.8 barns; in calculating the other excitation functions the value of σ_I was assumed to be equal to the cross-sections for the inelastic scattering of a neutron with an energy of 14 MeV, measured by Amaldi et al (loc.cit.) and Phillips et al (loc.cit.). (The measurements of Taylor, Lönsjö and Bonner, 1955, show that this cross-section does not vary appreciably for neutrons with energies between 10 and 15 MeV).

The results of these calculations are compared in figures II.3 - II.7 with the experimental data.

The reactions ${}_{12}Mg^{24}(n, p\gamma){}_{11}Na^{24}$, ${}_{13}Al^{27}(n, p\gamma){}_{12}Mg^{27}$ and ${}_{14}Si^{28}(n, p\gamma){}_{13}Al^{28}$ are similar in that σ_p (comp.) is the dominating cross-section, while for ${}_{38}Sr^{88}(n, p\gamma){}_{37}Rb^{88}$, $\sigma_p(\text{dir.})$ is dominant. There appears to be no simple explanation why the excitation function for ${}_{17}Cl^{37}(n, p\gamma){}_{16}S^{37}$ should rise with increasing neutron energy.

(3) <u>The angular distribution of the neutrons emitted, with energies between 4 and 12 MeV, after the interaction of neutrons of 14 MeV with Bismuth.</u>

The experimental data with which a comparison has been made was obtained by Rosen and Stewart (loc.cit.).

The differential cross-section, $\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_N^\theta$, for the production of a neutron of energy ε and in a direction θ inside the target nucleus as a result of direct collisions may be written as

$$\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_N^\theta = \frac{\sigma_I}{[\alpha X(P_1)]_{nn}\rho_n + [\alpha X(P_1)]_{np}\rho_p}\left[\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_{np}^\theta \rho_p + 2\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_{nn}^\theta \rho_n\right] \qquad \text{(II.41)}$$

where the differential cross-section $\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_{n\{^n_p\}}^\theta$ is given by equation (II.24). For simplicity in calculating this latter cross-section it was assumed that $\varepsilon_f = \varepsilon_F = 30$ MeV and $V_0 = 36$ MeV; the errors introduced into this calculation by this simplification are negligible. The value of σ_I used was taken from the data of Amaldi et.al. (loc.cit.) and Phillips et.al. (loc.cit.).

The differential cross-section, $\left(\frac{d^2\sigma}{dE\, d\omega}\right)_n^\theta$, for the emission of a neutron of energy E at an angle θ to the direction of the incident neutron is given by

$$\left(\frac{d^2\sigma}{dE\, d\omega}\right)_n^\theta = \left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_N^\theta \exp.-\left[\frac{0.75R}{\lambda(\varepsilon)}\right] \qquad \text{(II.42)}$$

The angular distribution of the neutrons emitted with energies between 4 and 12 MeV was obtained by the numerical integration of equation (II.42). The cross-section for the emission of neutrons with an energy greater than 4 MeV by the

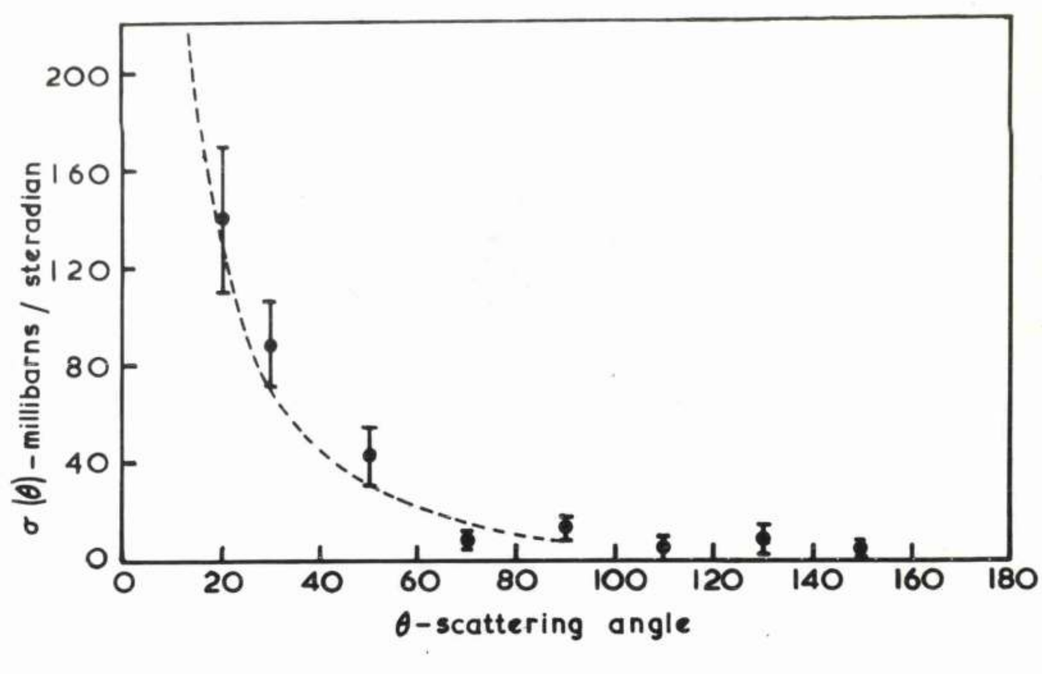


Figure II.8. The angular distribution of the neutrons emitted from Bismuth with energies between 4 and 12 MeV.

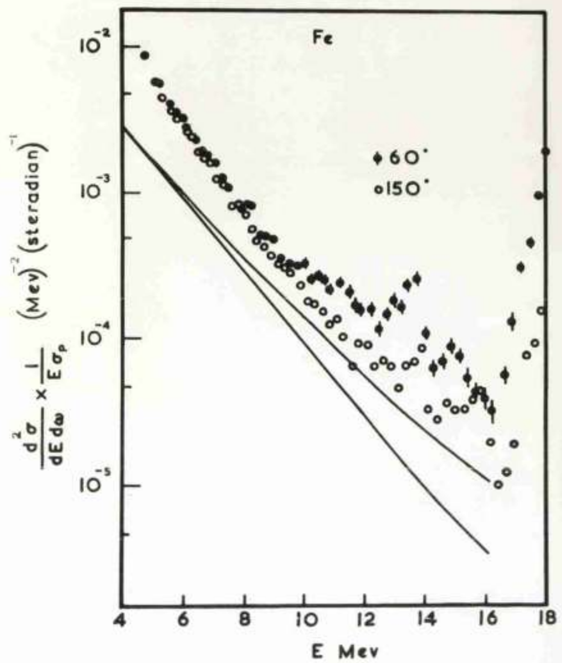
decay of a compound nucleus was found to be negligible. A comparison is made in figure II.8 of the results of the calculation with the experimental data.

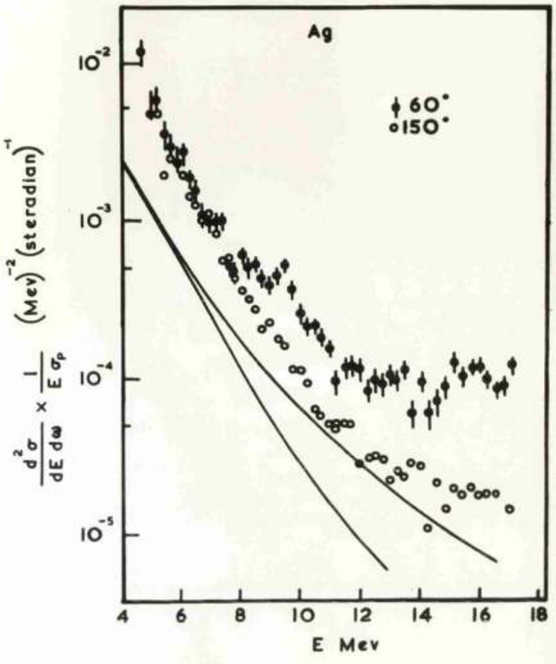
(4) The energy spectrum of the protons emitted at 60° and 150° from iron, silver and platinum when bombarded by 18 MeV protons.

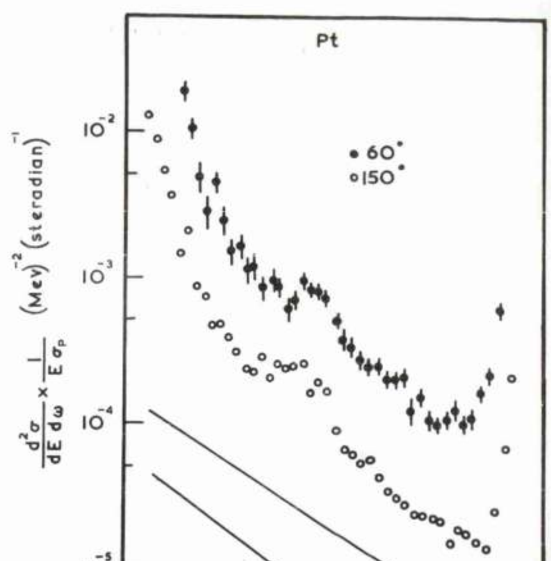
The expression for the differential cross-section, $\left(\frac{d^2\sigma}{d\varepsilon\, d\omega}\right)_N^\theta$, for the production of a proton of energy ε and in a direction θ is the same as that given in equation (II.41) with a suitable alteration in the subscripts. The differential cross-sections, $\left(\frac{d^2\sigma}{dE\, d\omega}\right)_p^{60^\circ}$ and $\left(\frac{d^2\sigma}{dE\, d\omega}\right)_p^{150^\circ}$, for the emission of protons at 60° and 150° were calculated using the equations (II.32) and (II.33) given in § 3 (c). In the calculation of these quantities it was assumed that $\varepsilon_f = \varepsilon_F =$ 30 MeV and that $V_0 =$ 38 MeV. The values of σ_I used were those given by Shapiro (loc.cit.) for the value $r_0 = 1.5 \times 10^{-13}$ cm.

The differential cross-section for the emission of a proton, of energy E, by the decay of an excited nucleus has also been calculated. The sum of this cross-section and that for emission as a result of direct collisions, appropriate to the angle considered, is compared in figures II.9, II.10 and II.11 to the data of Gugelot (loc.cit.).

The ratio of the cross-section observed by Gugelot (loc. cit.) to that obtained in these calculations is 2 for Iron, 3 for Silver and 20 for Platinum. It is difficult to observe any reasonable explanation for this serious discrepancy.







Figures II.9 - II.11. The energy spectra of the protons inelastically scattered at 60° and 150°, from iron, silver and platinum.

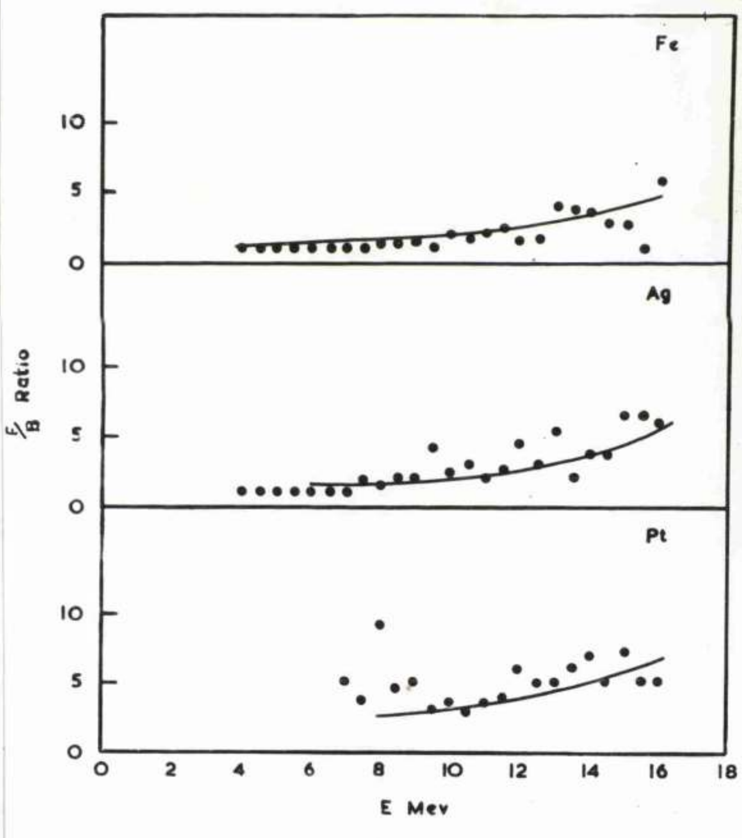


Figure II.12. The ratio of the cross-sections at 60^{0} and 150^{0} for protons inelastically scattered from iron, silver and platinum.

It is interesting to plot the ratio of the cross-section for the emission of protons at 60° to that for emission at 150°. This ratio which is less sensitive to the magnitudes of the penetrability of the coulomb barrier is shown in figure II.12. The agreement between the experimental ratios and the calculated ratios is satisfactory, but the discrepancy between the magnitudes of the cross-sections for Platinum is so large, that it cannot be attributed to the use of wrong values for the penetrability of the coulomb barrier.

§ 6. DISCUSSION

The model which has been used in these calculations is basically the same as that used in the interpretation of the nuclear reactions induced in heavy nuclei by nucleons with energies greater than 100 MeV (Bernadini et al. loc.cit., Goldberger, loc.cit. and Morrison et al. loc.cit.). It does not seem possible to achieve a satisfactory comparison with the experimental data by assuming the immediate formation of a compound nucleus. However the model suggested here does not abandon the concept of the compound nucleus, it merely defines the conditions which appear necessary for its formation. The calculations which have been performed using this model have shown that surprisingly satisfactory results may be obtained at an energy, for the primary nucleon, at which more formal treatments might be necessary.

During the course of the work reported here, a somewhat similar type of analysis was suggested by Hayakawa, Kawai and Kikuchi (loc.cit.). In the later stages of the investigations, their analysis of the angular distribution of the nucleons produced as a result of the collision of two nucleons inside a nucleus was considered (§ 3 (b)). These authors considered only the inelastic scattering of 31 MeV protons by Tin, and of 18 MeV protons by Iron. They observed that a satisfactory analysis of the scattering of 31 MeV protons by Tin is possible, but not for 18 MeV protons by Iron. The value of σ_p (dir.) quoted by these authors for the scattering of 18 MeV protons by Iron is not consistent with that found in the calculations described here.

CHAPTER III.

AN EXPERIMENTAL INVESTIGATION OF THE ANGULAR DISTRIBUTION OF THE PROTONS EMITTED FROM ALUMINIUM, IRON AND RHODIUM, FOLLOWING THE INTERACTION OF NEUTRONS WITH AN ENERGY OF 13.2 ± 0.2 MEV WITH THESE NUCLEI.

§ 1. INTRODUCTION.

The work which will be discussed in this Chapter was undertaken before the calculations presented in Chapter II were performed. Prior to this work the high cross-sections for reactions of the type ${}_{Z}N^{A}\,(n, p\,\gamma)\,{}_{Z-1}N^{A}$ induced by 14 MeV neutrons has been reported by Paul and Clarke (loc.cit.). It had been suggested by Austern, Butler and McManus (1953) that these high cross-sections are a consequence of some mode of interaction, of the incident neutron with the target nucleus, other than the formation of a compound nucleus. Moreover, it had been observed by Morrison, Muirhead and Rosser (loc.cit.) that it might be possible for nucleons with intermediate energies to move comparatively freely in nuclear matter. These considerations suggested that the model of the compound nucleus might not be applicable to the reactions induced by nucleons with such energies and thus these experiments were undertaken to provide more definite information concerning this possibility.

In these experiments on the angular distributions of the protons emitted in (n, p) reactions, nuclear emulsions have been used to detect the protons. These particular experiments were chosen rather than others because of the high cross-sections which were reported by Paul and Clarke (loc.cit.), and because it was possible to obtain monoenergetic neutrons from the reaction $T(d,n)He^4$. It was decided to study the angular distributions of the protons, because the predictions of the compound nucleus and optical models are vastly different for these distributions.

Since these experiments were commenced, other investigations of the angular and energy distributions of protons emitted in (n,p) reactions have been made. Thus Allan (1956, private communication) has investigated the energy spectra of the protons emitted in these reactions. Rosen and Armstrong (1956, private communication) have measured both the angular and energy distribution of the protons emitted as a result of the interaction of 14 MeV neutrons with Zn^{64}.

§2. EXPERIMENTAL PROCEDURE.

(a) Experimental arrangement.

There are two different methods which can be used to investigate the protons emitted in (n,p) reactions using nuclear emulsions as the detector of the protons. The element to be used as the target can either be placed external to the emulsion, or it might be introduced into the emulsion in the form of a wire.

An investigation was made first into the possibility of introducing metals in the form of wires into an emulsion. In the first attempts to do this it was found that it was not possible to use wires sufficiently thin that the loss in energy of the protons produced in (n,p) reactions was small. This was not possible because the wires had to be sufficiently strong for them to be rigidly attached to a metal frame while a nuclear emulsion was formed around them. Emulsions containing wires of nickel, 25 microns in diameter, were obtained; however on development of these emulsions an intolerable amount of distortion of the tracks originating inside the emulsion was produced. Attempts were made to reduce this distortion by treating the emulsions such that after development they possessed their original thickness. After development the nuclear emulsions were placed in several solutions of water and methyl alcohol of increasing concentrations of methyl alcohol. They were placed then in solutions of methyl alcohol and ether of increasing concentrations of ether. Finally they were placed in a solution of ether and resin. These attempts to introduce resin into the emulsion after development were unsuccessful. Thus since it was not found possible to eliminate this distortion, to use sufficiently thin wires and because of the limitations of the technique, produced by the corrosion of certain metals when placed inside an emulsion, these investigations were abandoned.

When the target is external to the nuclear emulsions, the ideal experimental arrangement is one which has been used by

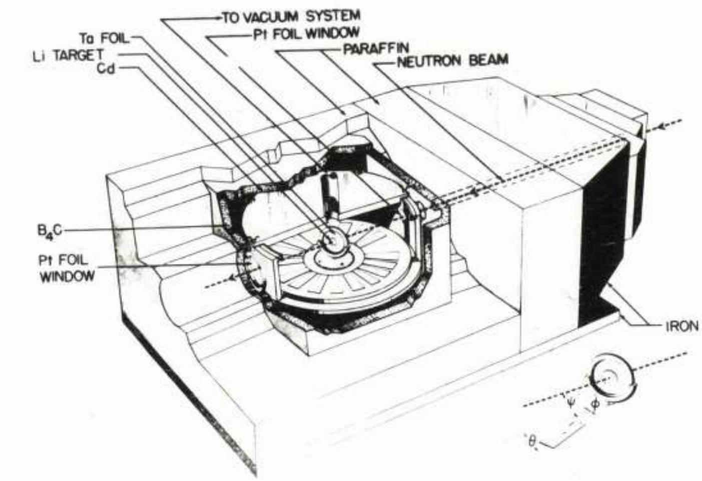


Figure III.1. The collimator and multiplate camera used by Frye (1953).

Allred, Armstrong and Rosen (1953). This arrangement was used by these authors to study the interaction of 14 MeV neutrons with protons and deuterons. A slightly modified form, shown in figure III.1, was used by Frye (1954) to investigate the interaction of 14 MeV neutrons with Li^6 and Li^7. In the investigation performed by Rosen and Armstrong (loc.cit.) this apparatus was used also. In this arrangement the neutrons are collimated by approximately eighteen inches of lead and eight inches of paraffin wax; the diameter of the hole used for collimation is approximately $\frac{3}{4}$ in. The nuclear emulsions, which are arranged around the target, are thus shielded from the incident beam of neutrons. The advantage of this arrangement is that there are few protons recorded by the emulsions which are not produced in the target. The disadvantage of introducing the collimator is that the flux of neutrons at the target is considerably reduced by the small solid angle subtended by the target to the source of neutrons. A further disadvantage of this arrangement is that few of the protons produced in the target are recorded by the nuclear emulsions.

The Tritium-Zirconium target obtained from Harwell for use in these experiments was 1 milligram per square cm. thick, and contained 0.054 cc. of tritium per square cm. It was calculated that a maximum yield of neutrons of 4×10^7 per microcoulomb of deuterons would be obtained when the incident

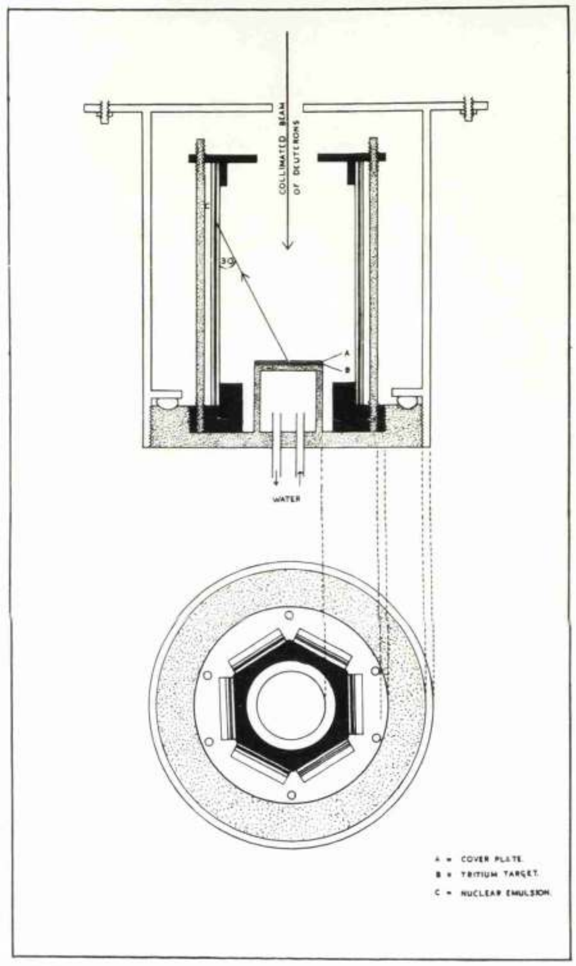


Figure III.2. The apparatus used in the experimental investigations reported here.

deuterons have an energy of 300 KeV. Unfortunately with the Glasgow H.T. set operating at this voltage it was possible to obtain a current of only $\sim$ 20 microamperes. Thus the maximum flux of neutrons which could be obtained was $\sim 10^9$ neutrons per second. No increase in this flux could be obtained with this target even though higher currents could be produced at higher accelerating voltages, because of the rapid decrease in the cross-section for the $T(d,n)He^4$ reaction at higher energies of the deuterons.

Using this flux of neutrons and the experimental arrangement of Allred, Armstrong and Rosen (loc.cit.), long exposure times ($\gtrsim$ 100 hours) would have been necessary, so that a sufficiently large number of protons would be recorded by the nuclear emulsions. In view of this consideration the plate holder shown in figure III.2 was designed. This apparatus was built so that it could be fitted on to the end of the accelerating column of the H.T. set. This necessitates the exposure of nuclear emulsions under vacuum conditions. Only a small amount of the tritium target, $^1/16"$ in diameter, is exposed to the incident beam of deuterons; thus the neutrons may be considered to arise from a point source. Collimation of the beam of deuterons above the plate holder ensures that the deuterons are incident upon the exposed portion of the tritium target. The scattering chamber is insulated from the rest of the accelerator, so that an estimate of the current of deuterons incident upon the target can be made. The plate holder itself consists of two small hexagonal pieces of brass separated by six supporting struts; this ensures that a minimum amount of material is near the nuclear emulsions. The advantages of this arrangement are

(1) a nuclear emulsion is placed close to the element (foil of metal) in which the protons originate. Thus a large fraction of the protons produced in the foil will be recorded by the nuclear emulsion.

(2) the angle of incidence of the neutrons at any point along a nuclear emulsion is well defined. Thus, by making measurements on the protons arising from a small area of the foil, it should be possible to determine the angular distribution of these protons over a large range of angle.

The disadvantage of this arrangement is that the nuclear emulsions are exposed to the source of neutrons. Consequently there is a limit to the time for which the nuclear emulsions may be exposed. This is reached when the number of recoil protons produced inside the emulsion is so large that the emulsion is blackened and measurements of individual tracks become impossible. The flux of neutrons which may be incident upon an emulsion before this occurs is approximately 10^9 per square cm.

As a result of this limitation in the time of exposure, there is a restriction imposed upon the minimum thickness of the foils which may be placed before the nuclear emulsions. It is necessary to use foils sufficiently thick that the number of protons recorded by the nuclear emulsion is not so low for a subsequent analysis to be too laborious. It was found that this number of protons was sufficiently high when for Aluminium the thickness of the foils used was 6 milligrams per square cm., and for both Iron and Rhodium the thickness was 13 milligrams per square cm.

(b) Exposure.

The method of exposure for the three elements was the same and thus the procedure will be discussed in a general manner.

Ilford C.2. emulsions, 4" x 1" and 400 microns thick, were used to detect the protons produced in the foils. A foil, attached to a gold strip 4" x 1" and 400 microns thick, was placed parallel to and in contact with the surface of an emulsion. Three foils of the same element were placed between a gold strip and a nuclear emulsion in this manner. Three similar pieces of gold strip were placed also directly in contact with an emulsion. These six nuclear emulsions were then attached symmetrically to the hexagonal plate holder so that the emulsions in contact with the foils were alternate plates. The gold, which has a negligible cross-section for the (n, p) process, was present to shield the emulsions from external sources of protons. The three emulsions covered only by gold were included so that allowance could be made for any protons arising in the gold strip to which the foils were attached. With the foils of Aluminium it was necessary to separate the foil and the surface of the emulsion to prevent any undesirable interaction of the foil with the surface of the emulsion. This was achieved by placing thin platinum wires at the top and bottom of the emulsion.

When handling these emulsions care was taken not to scratch the surface of the emulsions. This was necessary so that in the subsequent analysis of the emulsions the difficulties associated with deciding whether a proton entered the emulsion from outside would be minimised.

One exposure was made for each element and for each an unused portion of the tritium target was used. Foils of natural iron, aluminium and rhodium were used. The Cockroft-Walton accelerator was operated always at a voltage of 300 KeV. With the collimation used in the apparatus, it was found that a current of approximately one microampere was incident upon the exposed portion of the tritium target. This corresponds to a flux of neutrons of $\sim 4 \times 10^7$ per second at the tritium target. So that the emulsions would not be blackened, all exposures were restricted to two hours with this flux of neutrons.

The nuclear emulsions were exposed under conditions of vacuum and, for the subsequent analysis of the protons recorded by these emulsions, it was necessary to know the thickness of the emulsion at the time of exposure. Consequently measurements of this thickness were made immediately after all exposures, before the emulsions had time to absorb an appreciable amount of moisture.

Immediately after the exposure of the nuclear emulsions, each emulsion was covered by a thin piece of glass and the thickness of this assembly was measured in several places, along the edges of the emulsion, using a micrometer screw gauge. These measurements gave the total thickness of (1) the glass to which the emulsion was attached, (2) the emulsion - under conditions nearly identical to those of exposure - plus (3) the thickness of the glass cover. After development of the emulsions these measurements were repeated, in the same positions, using again the micrometer screw gauge and the same glass cover. Measurements were made then of the thickness of the emulsion at these positions using a microscope. From these measurements consistent values for the thickness of the emulsions immediately after exposure were obtained.

(c) The development of the emulsions.

Since the object of the analysis of the emulsions was to examine protons which entered an emulsion from outside, it was necessary to develop the nuclear emulsions such that a minimum amount of silver or other materials was deposited upon their surface. It was not possible to remove such deposits in the usual manner, i.e. by cleaning the surface of an emulsion during the fixing stage of development, since this removes a thickness of several microns of emulsion. The removal of such a layer of emulsion would have caused not only an error in subsequent measurements of the energy of the protons, but would have meant also that many protons which did not enter the emulsion from outside would appear to do so. The procedure given below was used successfully to prevent the deposition of silver or other materials upon the surface of the emulsion.

The two temperature method of development, suggested by Dilworth, Occhialini and Payne (1948), was used; amidol developer of normal strength was used but the solution was filtered to remove any amidol which had not dissolved. During the presoaking of the nuclear emulsions, successively in water and the developer for two hours, the temperature was kept constant at 2° Centigrade. The nuclear emulsions were then removed and excess developing solution was carefully dried from the surface of the emulsions with filter paper. The temperature of the emulsions was then raised to 22° Centigrade and kept constant for 30 minutes; it is at this stage that development takes place. The emulsions were placed then in a 1% solution of acetic acid for 30 minutes and finally in a fixing solution. The fixing solution was changed frequently during the time of fixation, to prevent the deposition of silver on the surface of the emulsions during this process.

§3. ANALYSIS.

(a) Selection of data.

For each exposure equal areas, 0·6 cm × 0·03cm*, in the centre of all six emulsions corresponding to an angle of incidence for the neutrons of $30° \pm 2°$ were examined. The energy of the neutrons incident in this region of the emulsions, as calculated from the dynamics of the reaction $T(d,n)He^4$, the energy of the incident deuterons and the thickness of the Tritium-Zirconium target used, was $13{\cdot}2 \pm 0{\cdot}2$ MeV. The emulsions were examined under x45 oil immersion objectives, using x10 eyepieces, for all tracks starting at the surface of the emulsion; the final decision of whether these tracks did start at the surface of the emulsion was made using x90 oil immersion objectives. Three measurements were made on each track which thus appeared to enter the emulsion from outside: (1) the projected length i.e. the length of the track in the plane of the emulsion, (2) the dip angle of the track with respect to the surface of the emulsion, and (3) the angle between the track and the long axis of the emulsion. The thickness of the emulsion was also measured with the microscope at the same time as these measurements were made.

---

* Footnote

For the exposure with Rhodium foils, it has been necessary to examine twice this area to obtain reasonably good statistical accuracy.

It was assumed that the tracks of unit charge starting at the surface of the emulsions were protons. It is possible that deuterons of low energies may be emitted as a result of the interaction of 13 MeV neutrons with the elements chosen. However, it is not possible to distinguish between protons and deuterons of low energies by grain counting in Ilford C.2. emulsions and thus no allowance for this could be made. The measurements made for each track, together with the knowledge of the thickness of the emulsion at the time of these measurements and its thickness at the time of exposure, were used to determine the energy of the proton and its spatial angle with respect to the direction of the incident neutron. The range-energy relationships which were used were those determined by Rotblat (1951) for protons recorded in nuclear emulsions exposed under vacuum conditions.

In the experiment with iron foils, protons with dip angles greater than 30° were not included in the analysis; in the other experiments this upper limit was extended to 45°. This limitation ensured that the majority ($\sim$ 95%) of all tracks selected for measurement stopped within the emulsion, and also simplified the problem of the determination of the direction of motion of the ionizing particle responsible for a given track.

Not all the protons, which were observed to start at the surface of an emulsion, originated in the elements in contact with the emulsion. Protons which start in the emulsion within a distance of $\sim$ 1 or 2 microns from the surface cannot be resolved optically from those entering the emulsion from outside. These protons may be easily divided into two groups:-

(1) recoil protons, which are produced in elastic collisions of the incident neutrons with hydrogen atoms present in the emulsion, and

(2) protons which might be produced as a result of the interaction of the incident neutrons with the nuclei of the emulsion, or as a result of the collisions of neutrons of lower energy with hydrogen atoms. These protons will be called background protons.

The recoil protons were distinguished by using the equation

$$E_p = E_n \cos^2\theta$$

where E_p and E_n = the energies of the recoil proton and the incident neutron respectively,

and θ = the spatial angle between the direction of the recoil proton and the incident neutron.

Protons for which the relationship between energy and angle was given by this formula, when all possible errors in the quantities involved were considered, were in the first instance assumed to be recoil protons. The angular distribution of these protons is shown in figure III.3; the full curve

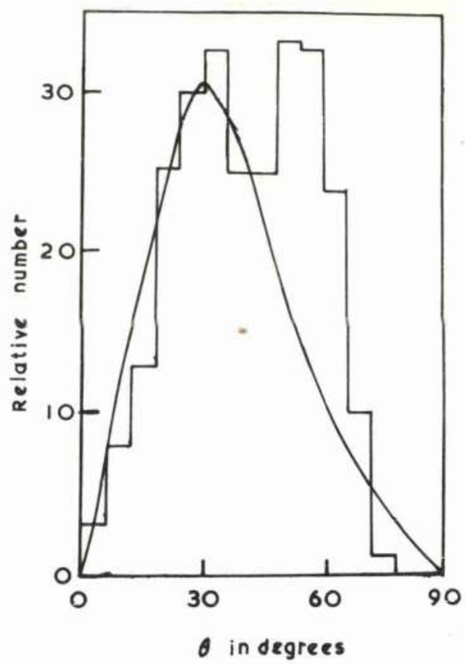


Figure III.3. The angular distribution of the recoil protons which start at the surface of the emulsions in contact with the foils and the gold strip.

indicates the angular distribution of the recoil protons calculated from considerations of the solid angle available at different angles in the geometrical considerations employed in these experiments. It was found that the angular distribution of the recoil protons observed to start at the surface of the emulsion in contact with gold was consistent with the full curve. That of those observed to start at the surface of the emulsion in contact with the foils indicated many more protons at angles greater than $\sim 40^{\circ}$. These protons were assumed to be those which originated in the foils, but which had an energy and spatial angle such that they appeared to be recoil protons. Thus the number of recoil protons were subtracted from the protons observed to start at the surface of the emulsions according to the full curve.

The correction for background protons was obtained from measurements of the protons occurring inside the emulsion. Measurements were made, for each exposure, of the protons originating between the surface and the half depth of the emulsion; a volume of 8×10^{-3} cc. of one emulsion being scanned for each exposure. Only protons which possessed an angle of dip of less than 45°, with respect to the surface of the emulsion, were measured. The angular distribution of the recoil protons which were observed in these measurements was that given by the full curve of figure III.3. This indicates that in the scanning for the protons starting at the surface of the emulsions in contact with the foils, there was no bias towards the observation of recoil protons with angles $> 40^{\circ}$. Of the background protons which were observed, 80% possessed energies below 3 MeV. Since the ratio of the background to recoil protons should be the same both at the surface and inside the emulsion, the number of protons discarded as being background protons was made proportional to that number removed as recoils. The subtractions were made in intervals of energy and angle, according to the relative numbers observed in these intervals inside the emulsion.

The remaining surface protons were then assumed to have arisen in the materials in contact with the photographic emulsion. The effect of the protons originating in the gold strip upon the protons starting at the surface of the emulsions in contact with a foil was found by considering the numbers of protons observed to originate, after these subtractions, in the gold strip. Allowance was made for the reduction in energy of these protons in passing through the foil and the protons were subtracted in intervals of energy and spatial angle.

The number of protons entering those various categories is given in table 1.

Table 1.

Element	Protons observed with dip angles $> 10^{\circ}$	Recoils	Back-ground protons	Gold	Protons from foil
Aluminium	1600	220	30	70	1280
Iron	1190	200	55	50	885
Rhodium	1140	490	60	90	500

The effect of these subtractions was different for all the elements studied. After the subtraction of the background for Rhodium there were few protons remaining with energies less than 4 MeV. For Iron the subtraction for energies less than 4 MeV was 25% of the observed data, and for Aluminium it was 10% of the observed data.

(b) The correction for the loss in energy of protons produced in the foils.

The thicknesses of the foils used in these investigations were 6, 13 and 13 milligrams per square cm. for Aluminium, Iron and Rhodium respectively. Since the protons which were accepted for analysis traversed the foil obliquely, the effective thicknesses of the foils were much larger than these values. The effect of these semi-thick foils can be seen more clearly by considering the loss in energy of a proton with an energy of 5 MeV which originates in the top layer of these foils. When the angle of this proton is 60° to the normal, i.e. it would have an angle of dip of 30° with respect to the surface of the emulsion, the loss in energy is 0.7, 1.2 and 1.0 MeV for Aluminium, Iron and Rhodium respectively. Thus the energy spectrum of protons produced in these foils may be modified considerably at low energies. The loss in energy for protons with angles of dip $< 10^\circ$ is exceptionally high; consequently these protons have been omitted from the analysis.

A simple method of correction for the loss in energy for protons with angles of dip $> 10^\circ$ has been devised. This correction was not applied to the observed energy spectrum for Rhodium, because after the subtraction of background few protons with energies less than 4 MeV were observed to originate in the Rhodium foil. It has been applied to the observed energy spectra for Aluminium and Iron. The effect of this correction to the observed energy spectrum for Aluminium was small because the foil was relatively thin, and because the majority of the protons observed had angles of dip $> 20^\circ$. The method of correction for the loss in energy will be discussed in the following paragraphs.

The protons observed to originate in a foil were separated into the regions of dip angles: $10^\circ - 20^\circ$,

$20^\circ - 30^\circ$ and $30^\circ - 45^\circ$. Measurements of the protons with angles of dip in this last region had not been made for Iron. The analysis was first performed in each case for the interval with the highest angle of dip.

An energy spectrum of protons, which will be called the 'source spectrum', was chosen say of the form $N(E)\,dE$. The semi-thick target was then replaced by six thin targets and it was assumed that the source spectrum for the protons starting in each of these sections was the same. The number of protons, $N(E')\,dE'$, which would then emerge from the thick target is given by

$$N(E')dE' = \sum_{k=1}^{6} N_k\,(E + x_k)\,dE \qquad \text{(III.1)}$$

where x_k is the loss in energy of a proton which starts in section k , with an energy $E + x_k$, in traversing the remainder of the thick target.

The energy spectrum of the protons obtained from this choice of source spectrum was compared with that observed experimentally; a relaxation method was then applied to find the source spectrum which gave a spectrum consistent with that observed experimentally, after the allowance for the energy loss. This calculation was performed for all intervals; the spectra obtained in these intervals were in excellent agreement.

It was assumed that there was no scattering through very large angles for the protons originating in these thick foils.

Thus it is possible to relate the angular distribution of the protons observed experimentally with the energy of the protons in the 'source' spectrum.

It follows from the previous considerations that the number of protons, $N_s(E)\,dE$, in the source spectrum, with energies between E and $E+dE$ can be expressed in terms of the energy spectrum observed experimentally as

$$N_s(E)\,dE = X\,N(E_1')\,dE' + Y\,N(E_2')\,dE' + Z\,N(E_3')\,dE' + \text{etc.} \qquad \text{(III.2)}$$

where X is that fraction of the total observed number, $N(E_1')\,dE'$, of protons of energy E_1' which have an energy after correction for energy loss of E . The other terms have the same meanings.

It then follows that the angular distribution, $f(\theta)\,d\theta$, of the protons of energy E in the source spectrum is

$$f(\theta)\,d\theta = X f_1(\theta)\,d\theta + Y f_2(\theta)\,d\theta + Z f_3(\theta)\,d\theta + \text{etc.} \qquad \text{(III.3)}$$

where $f_1(\theta)\,d\theta$ etc., is the angular distribution of the protons of energy E_1' etc.

§ 4. THE DETERMINATION OF THE CROSS-SECTIONS FOR THE EMISSION OF PROTONS IN (n,p) REACTIONS.

(a) Experimental determination.

The flux of neutrons incident upon the nuclear emulsions was not measured at the time of exposure. Consequently it has been necessary to estimate this flux by measuring the number of recoil protons arising within a certain volume of a nuclear emulsion.

The differential cross-section, $\frac{d\sigma}{d\omega}$, for the elastic scattering, in the laboratory system of coordinates, of an incident neutron by a hydrogen atom is given by

$$\left(\frac{d\sigma}{d\omega}\right) = \frac{\sigma}{\pi} \cos\Theta \tag{III.4}$$

where σ = the total cross-section for elastic scattering,

and Θ = the angle between the direction of the incident neutron and the recoil proton.

The solid angle available for acceptance of a proton arising at an angle Θ was restricted in these experiments by the limitations imposed upon the protons selected for measurement. Thus the number, $N(\Theta_1)\,d\Theta_1$, of recoil protons observed between angles Θ_1 and $\Theta_1 + d\Theta_1$ with respect to the direction of the incident neutrons, is given by

$$N(\Theta_1)\,d\Theta_1 = \frac{\sigma}{\pi} \cos\Theta_1 . d\omega_{\Theta_1} . F . \rho_h N V \tag{III.5}$$

where $d\omega_{\Theta_1}$ = the solid angle of acceptance for the protons scattered at an angle Θ_1.

F = the flux of neutrons incident upon one square cm. of the emulsion,

ρ_h = the density of hydrogen atoms within the nuclear emulsion,

V = the volume of emulsion in which these protons were observed,

and N = Avogadro's number.

It follows that the flux of neutrons incident upon one square cm. of the emulsion is given by

$$F = \frac{N(\theta_1)\,d\theta_1}{\cos\theta_1\, d\omega_{\theta_1}} \cdot \frac{\pi}{\sigma} \cdot \frac{1}{\rho_h N V} \qquad \text{(III.6)}$$

i.e.

$$= R\ \frac{\pi}{\sigma} \cdot \frac{1}{\rho_h N V}$$

where R is a quantity easily obtained from the measurement of the angular distribution of the recoil protons produced inside an emulsion.

The number, $N_p(\theta_2)\,d\theta_2$, of protons observed to originate in the foil in contact with the emulsion may be written as

$$N_p(\theta_2)\,d\theta_2 = F.\left(\frac{d\sigma}{d\omega}\right)_{\theta_2} d\omega_{\theta_2} \cdot \frac{\rho_A t a N}{A} \qquad \text{(III.7)}$$

where $\left(\frac{d\sigma}{d\omega}\right)_{\theta_2}$ = the differential cross-section for the emission of protons, at an angle θ_2, from the nuclei of the foil,

$d\omega_{\theta_2}$ = the solid angle of acceptance for the protons emitted at an angle θ_2,

ρ_A = the density of the foil of thickness t cm. and atomic weight A.

and a = the area of the foil from which these protons have originated.

Thus the differential cross-section for the emission of protons is given by

$$\left(\frac{d\sigma}{d\omega}\right)_{\theta_2} = \frac{N_p(\theta_2)\,d\theta_2}{d\omega_{\theta_2}} \cdot \frac{\sigma}{\pi} \cdot \frac{\rho_h V}{R} \cdot \frac{A}{\rho_A t a} \qquad \text{(III.8)}$$

It is important, when using this method, that corrections are made for the efficiency of detection of the protons. When the emulsions were being scanned for protons starting at the surface, there were very few tracks crossing any one field of view and thus the efficiency for detection of these tracks was very high. However in the scanning performed for protons originating within the emulsion, there was a very large number of tracks crossing one field of view and there was thus a possibility that the beginning of some tracks starting in this field of view was obscured. Consequently the efficiency of detection is lower for these observations. The efficiency of the observer has been determined by examination of the same volume of emulsion made by two different observers to be $80 \pm 10\%$. The error in the determination of the flux of neutrons which was introduced in this manner was taken into account only in the calculation of the total cross-section for the emission of protons in (n, p) reactions. This error has not been considered in the determination of the differential cross-sections because it does not affect relative values.

The differential cross-sections have been determined only for angles 0° to 140°. This limitation is introduced because of the position in which the emulsions were examined, and because protons with angles of dip $< 10^{\circ}$ were discarded in the analysis.

(b) Theoretical calculations.

The cross-sections for the emission of protons in the reactions ${}_{Z}N^{A}(n, p\gamma)_{Z-1}N^{A}$ and ${}_{Z}N^{A}(n, pn)_{Z-1}N^{A-1}$ with the nuclei under consideration have been calculated on the basis of the simple model discussed in Chapter II. Thus the equations necessary for these calculations will be given here.

When the angle of emission of the protons is less than 90°, the differential cross-section for the emission of protons is composed of three terms:

(1) that for the emission of protons, produced in direct collisions, as a result of the first encounter with the surface of the nucleus.

(2) that for the emission of protons which have encountered the nuclear surface more than once, and

(3) that for the emission of protons in the decay of an excited nucleus.

When the angle of emission of the protons is less than 90°, the differential cross-section is composed of only the latter two terms.

Thus it is possible to write the differential cross-section, $\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^{\theta}$, for the emission of protons with an energy E at an angle θ as

$$\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^{\theta} = p\left(\frac{d^2\sigma}{dE\,d\omega}\right)_N^{\theta} + \frac{px^2}{1-x^2}\cdot\frac{1}{2\pi}\cdot\left(\frac{d\sigma}{dE}\right)_N + \frac{\sigma(\text{comp.})}{4\pi} \quad \theta < 90^{\circ} \qquad \text{(III.9)}$$

$$\left(\frac{d^2\sigma}{dE\,d\omega}\right)_p^\theta = \frac{px}{1-x^2}\cdot\frac{1}{2\pi}\cdot\left(\frac{d\sigma}{dE}\right)_N + \frac{\sigma(\text{comp.})}{4\pi} \qquad \theta > 90^\circ \qquad \text{(III.10)}$$

where

$$\left(\frac{d^2\sigma}{dE\,d\omega}\right)_N^\theta = \frac{\sigma_I}{[\alpha X(P_1)]_{nn}\rho_n + [\alpha X(P_1)]_{np}\rho_p}\cdot\left(\frac{d^2\sigma}{dE\,d\omega}\right)_{np}^\theta \rho_p$$

$\sigma(\text{comp.})$ = the total cross-section for the emission of a proton of energy E in the decay of an excited nucleus

and all other terms have the meanings given in Chapter II.

The method of calculation of the quantities involved was the same as those discussed in Chapter II.

§ 5. THE REACTION $_{13}Al^{27}(n,p)_{12}Mg^{27}$.

The energy spectra of the protons emitted in the reactions induced in Aluminium by the incident neutrons are shown, for various ranges of the spatial angle, in figure III.4. It can be seen from this figure that the maximum in the energy spectra occurs at an energy rather lower than that expected from considerations of the penetrabilities of the Coulomb barrier, for protons of low energies. This result is in agreement with the experimental investigations of Allan (private communication).

The angular distributions of the protons emitted in these reactions are shown in figure III.5. These distributions for the protons emitted with energies greater than 4 MeV show that there is preferential emission of protons in the forward direction. That for the emission of protons with energies less

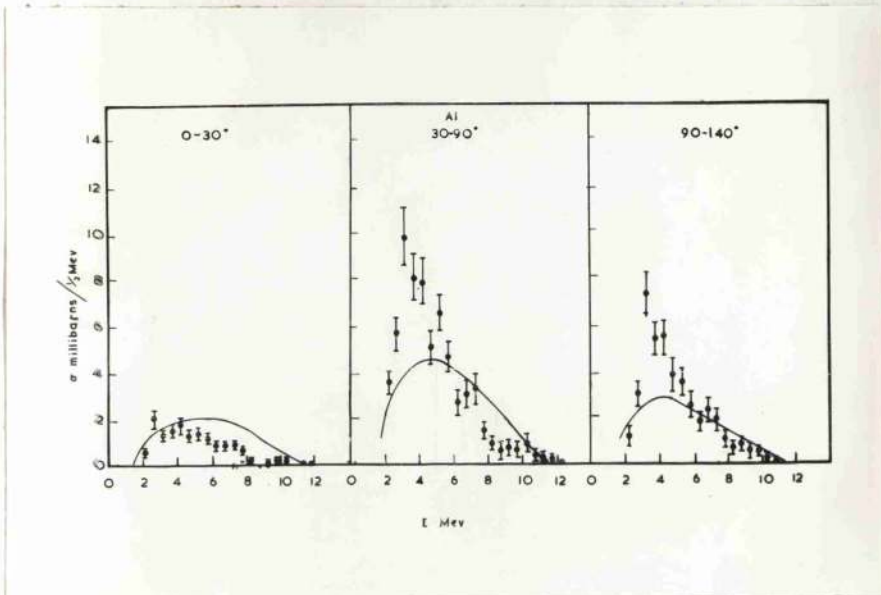


Figure III.4. The energy spectra of the protons emitted from Aluminium as a result of the interaction of 13.2 MeV neutrons with this nucleus.

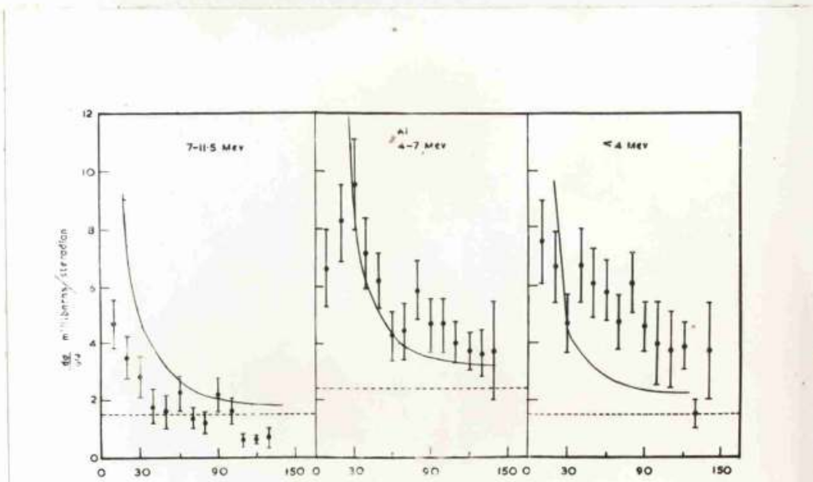


Figure III.5. The angular distributions of the protons emitted from Aluminium.

than 4 MeV does not show such a marked tendency. The anisotropy which is observed in the angular distributions of the protons emitted with energies greater than 4 MeV is not consistent with the assumption that a compound nucleus is formed when an incident neutron enters the nucleus. Thus it is of interest to compare these experimental results with the predictions of the simple model discussed in Chapter II.

The reactions which are energetically possible, and which lead to the emission of protons are

$$
\begin{aligned}
{}_{13}Al^{27} + n &= {}_{12}Mg^{27} + p + \gamma \quad - 1.8 \text{ Mev} \quad (a)\\
&= {}_{12}Mg^{26} + p + n \quad - 8.3 \text{ Mev} \quad (b)\\
&= {}_{12}Mg^{26} + n + p \quad - 8.3 \text{ Mev} \quad (c)
\end{aligned}
$$

The cross-sections for the emission of protons in the reactions (a) and (b) have been calculated on the basis of this model. It is found that σ_p(dir.) is 80 millibarns and σ_p(comp.) is 70 millibarns, thus giving a total cross-section for the emission of protons in these reactions of 150 millibarns. A comparison of these calculations and the experimental results is made also in figures III.4 and 5. The full curve in figures III.4 and 5 represents the sum of the cross-sections for the emission of protons as a result of the direct collisions and the decay of an excited nucleus. The dotted curve represents the cross-section for the emission of protons from the decay of an excited nucleus. The agreement between the calculated and experimental cross-sections is poor.

The high cross-section for the emission of protons with energies less than 5 MeV is not predicted by these calculations. The possible explanation for this discrepancy may be observed by a consideration of the possible modes of decay of the excited nuclei which are left after the emission of one neutron. The Q- values for the reactions ${}_{13}Al^{27}(n, 2n){}_{13}Al^{26}$ and ${}_{13}Al^{27}(n, np){}_{12}Mg^{26}$ are -13.0 MeV and -8.3 MeV respectively. Thus after the emission of one neutron, only the emission of a proton or γ- radiation is possible. The probability of emission of γ- radiation from excited nuclei is generally assumed to be small when the emission of nucleons is energetically possible. Consequently the cross-section for the emission of protons with energies less than 5 MeV might be expected to be high.

The angular distribution of the protons emitted with energies greater than 7 MeV is in qualitative agreement with the predictions of the model. Those for the protons emitted with energies less than 7 MeV do not have the shape predicted by the model. In fact that for the energy range 4 - 7 MeV appears to have a maximum at approximately 30°, and that for energies less than 4 MeV is nearly isotropic.

The experimental results of this investigation are in agreement with the results of other workers. The total cross-section for the emission of protons with angles between 0° and

180° may be obtained by assuming that the angular distribution of the protons emitted with angles $> 140^{\circ}$ is isotropic and equal in magnitude to that for angles between 90° and 140°. The error which may be introduced by this assumption is not large since the solid angle factors are reasonably small for angles greater than 140°. The value of the total cross-section of 140 ± 30 millibarns is in agreement with a similar value of 140 millibarns obtained by Allan (loc.cit.) from measurements of the energy spectra of the protons emitted, at angles $0^{\circ} - 34 \pm 20^{\circ}$, in these reactions. The cross-section for the emission of protons in the reaction ${}_{13}Al^{27}(n, p\gamma)_{12}Mg^{27}$ may also be obtained from the experimental results, if it is assumed that neutron emission always occurs when it is energetically possible. The cross-section for emission of protons in the $(n, p\gamma)$ reaction is then obtained by the addition of the cross-sections for the emission of protons with energies greater than 5 MeV. The value obtained is 55 ± 15 millibarns; the value of this cross-section measured for neutrons with an energy of 14 MeV was found by Paul and Clarke (loc.cit.) to be 52 ± 10 millibarns and by Forbes (loc.cit.) to be 82 ± 5 millibarns. The calculated value was found to be 85 millibarns.

§ 6. THE REACTION ${}_{26}Fe^{56}(n, p)_{25}Mn^{56}$.

The energy and angular distributions of the protons emitted in this reaction are shown in figures III.6 and 7.

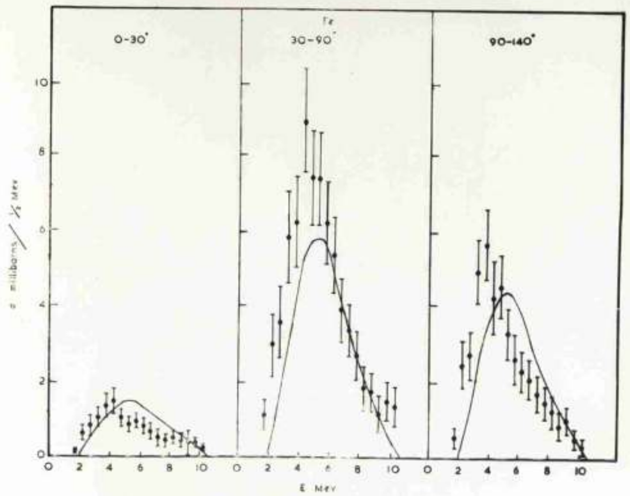


Figure III.6. The energy spectra of the protons emitted from Iron.

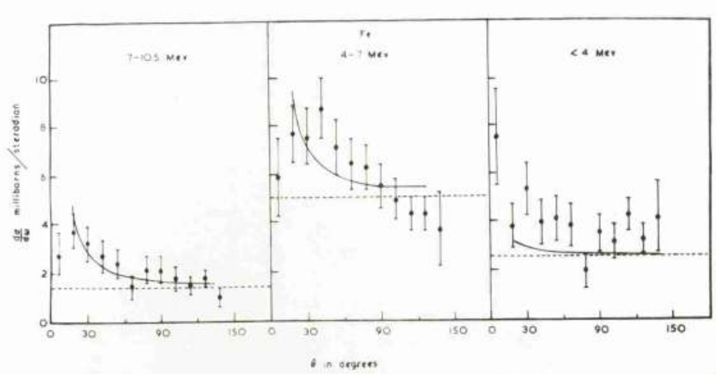


Figure III.7. The angular distributions of the protons emitted from Iron.

The position of the maximum in the energy spectra of the protons emitted in this reaction is in that position expected from coulomb barrier considerations. The experimental results for the angular distributions are similar to those observed in the experiment with Aluminium; i.e. that for the protons emitted with energies between 4 and 7 MeV appears to possess a maximum at approximately 30^{0}, while that for protons with energies greater than 7 MeV does not.

Since foils of natural iron were used in these investigations it is necessary to consider the emission of protons in the reactions ${}_{Z}N^{A}(n,p\gamma)_{Z-1}N^{A}$ and ${}_{Z}N^{A}(n,pn)_{Z-1}N^{A-1}$ for both Fe^{54} and Fe^{56}. For Fe^{54} it is found that $\sigma_p(\text{dir.})$ is 50 millibarns and $\sigma_p(\text{comp.})$ is 500 millibarns; for Fe^{56} the values are 25 and 70 millibarns respectively. To achieve a comparison with the experimental results, these cross-sections have been added according to the composition of the isotopes in natural iron. The cross-section for the emission of protons as a result of direct collisions for Fe^{54} has been neglected; the contribution from this process is only 3 millibarns, since this isotope is present to only 6% in natural iron. The results of these calculations are shown also in figures III.6 and 7.

It can be seen that the agreement between the calculated and experimental results is good for the protons emitted with energies greater than 7 MeV. The agreement for energies less than this is poor. The experimental results appear to indicate that there are slightly more protons emitted with energies less than 4 MeV than expected. This is not as certain as the results for Aluminium, because the background subtractions were of more importance for protons with energies less than 4 MeV in this experiment. However it is interesting to note that a high cross-section for the emission of protons of low energy in the reaction ${}_{26}Fe^{54}(n,np)_{25}Mn^{54}$ has been observed by Allan (loc.cit.); the emission of protons in this reaction is again enhanced because the emission of two neutrons is not energetically possible ($Q = -13.8$ MeV).

The total cross-section for the emission of protons in the reactions induced in the two isotopes of iron is found from this experimental data, to be 150 ± 30 millibarns. The subtraction of the cross-section for the emission of protons from Fe^{54} yields the value of 120 ± 30 millibarns for Fe^{56}. A small correction to this value is necessary to obtain the cross-section for the reaction ${}_{26}Fe^{56}(n,p\gamma)_{25}Mn^{56}$, since the maximum energy of the protons emitted in this reaction is 3 MeV. This value of 110 ± 30 millibarns compares favourably with the values for 14 MeV neutrons of 97 ± 12 millibarns obtained by Paul and Clarke (loc.cit.) and 124 ± 12 millibarns obtained by Forbes (loc.cit.).

The value of 190 millibarns quoted by Allan (loc.cit.) is higher than that found here presumably because only protons

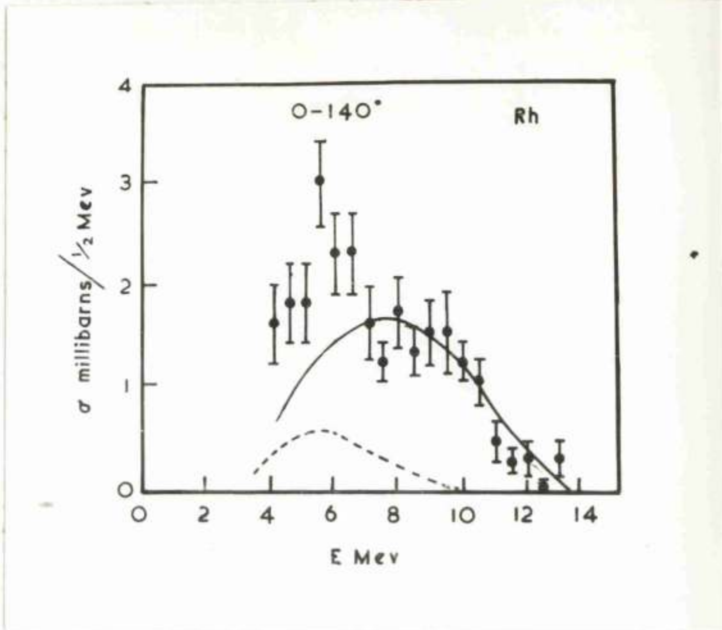


Figure III.8. The energy distribution of the protons emitted from Rhodium.

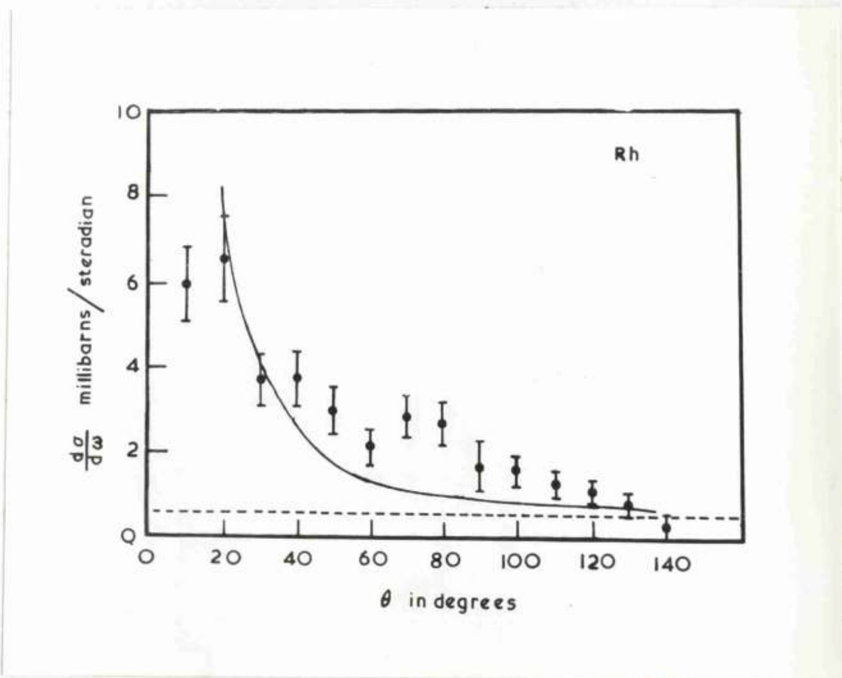


Figure III.9. The angular distribution of the protons emitted from Rhodium.

emitted at forward angles were observed in that experiment.

§7. THE REACTION ${}_{45}Rh^{103}(n,p){}_{44}Ru^{103}$.

The energy and angular distributions of the protons emitted in this reaction are shown in figures III.8 and 9. Because of the small cross-section for this reaction, the statistical accuracy of the results is not sufficiently good to allow any comparison of the angular distributions in two intervals of energy. The values of the calculated cross-sections for the emission of protons in the reactions ${}_{45}Rh^{103}(n,p\gamma){}_{44}Ru^{103}$ and ${}_{45}Rh^{103}(n,pn){}_{44}Ru^{102}$ are shown in figures III.8 and 9. It can be seen from these comparisons that the cross-section for the emission of protons by the decay of an excited nucleus is very small and insufficient to explain the experimental results. The value of $\sigma_p(\text{comp.})$ is 5 millibarns, while that of $\sigma_p(\text{dir.})$ is 20 millibarns. The total cross-section obtained from the experimental results for the emission of protons with angles between 0° and 180° is 25 ± 5 millibarns. For this reaction there is no other experimental data with which a comparison may be made.

The agreement between the predictions of the model and the experimental results is very good. It appears that the cross-section for the emission of protons with energies less than 7 MeV is slightly higher than that predicted. This might be explained by considering the emission of protons in the reaction ${}_{45}Rh^{103}(n,np){}_{44}Ru^{102}$. The maximum energy of the protons which may be emitted in this reaction is 6.7 MeV. The nuclei which may emit protons with energies between 4 and 6.7 MeV must possess excitation energies between 10.5 and 13.2 MeV after the emission of the first neutron. This is equivalent to saying that the energy of the first neutron must lie between 0 and 2.7 MeV. The cross-section for the emission of neutrons with energies in this range is high, but when a neutron is emitted with such an energy it is possible also for a second neutron to be emitted (the Q-value for the reaction ${}_{45}Rh^{103}(n,2n)\,{}_{45}Rh^{102}$ is -9.4 MeV). Consequently the cross-section for the emission of protons in the reaction ${}_{45}Rh^{103}(n,np)\,{}_{44}Ru^{102}$ is not expected to be very high. The experimental results are in agreement with this.

§ 8. DISCUSSION

The total cross-sections for the emission of protons in the reactions which have been considered here, are in satisfactory agreement with the results of other investigations. The observation of preferential emission of protons in the forward direction is in agreement with a similar observation, for the protons emitted in (n,p) reactions induced in ${}_{30}Zn^{64}$ by 14 MeV neutrons, which has been reported recently by Rosen and Armstrong (loc.cit.).

The emission of deuterons has been neglected in these analyses. However the cross-section for the emission of deuterons is expected to be small, because of the large negative Q - values and the low coulomb barrier penetrabilities involved, and thus should not contribute significantly to these results.

CHAPTER IV

AN ANALYSIS OF THE CROSS-SECTIONS FOR THE INTERACTION OF NEUTRONS OF LOW ENERGY ($\leq$ 4 MEV) WITH NUCLEI.

§1. INTRODUCTION

For the calculations which have been reported in the previous chapters, a knowledge of the cross-sections for the interaction of nucleons with nuclei and their variation with the energy of the incident nucleon was required. Therefore an examination was made of the experimental data available on these cross-sections, and the extent to which this data could be reproduced by the various nuclear models which have been proposed.

The experimental data published for charged particles are reproduced very well by the calculations of Shapiro (loc.cit.). These calculations yield cross-sections which are in good agreement with the results of Tendam and Bradt (loc.cit.), who investigated the cross-sections for (α, n) and $(\alpha, 2n)$ reactions, and those of Blaser and his co-workers (loc.cit.) for (p, n) reactions. The model which was used was that of strong absorption. This model has recently been shown to be incorrect, but these calculations should not be in serious error since the cross-sections are for charged particles mainly determined by the probability that the particle traverses the coulomb barrier.

Calculations of these cross-sections for 4 MeV neutrons which were made with the model suggested by Feshbach, Porter and Weisskopf (loc.cit.), were not in agreement with the measurements of Beyster, Henkel and Nobles (loc.cit.). The observation of this discrepancy has led to an attempt to improve upon this by modifying slightly the model of Feshbach, Porter and Weisskopf (loc.cit.).

§2. THE NUCLEAR RADIUS AND NUCLEAR FORCES.

In recent years many experiments have been performed to determine the radius of the distribution of charged particles within nuclei. The experiments on the elastic scattering of electrons of high energy by nuclei (Hofstadter et.al. 1954, Pidd et.al. 1953, 1955) and those on the X-rays emitted from μ-mesonic atoms (Fitch and Rainwater, 1953) have shown that this radius is given approximately by $1.2 A^{1/3} \times 10^{-13}$ cm. The radius of the distribution of neutrons inside the nucleus is known with less certainty. It has been suggested by Johnson and Teller (1954), by Hess and Moyer (1954) and by Swiatecki (1955) that this radius is larger than that for protons. Estimates made of the value required in optical model analyses to obtain agreement with the total cross-sections for scattering of nucleons of high energy by nuclei suggest that this value is approximately $1.4 A^{1/3} \times 10^{-13}$ cm.

Thus the mass distribution of nucleons in nuclei appears to increase from zero at a distance of approximately $1.4 A^{1/3} \times 10^{-13}$ cm., from the centre of the nucleus, to a maximum at a distance of approximately $1 \cdot 2\, A^{1/3} \times 10^{-13}$ cm. Consequently the attractive forces between an incident nucleon and a target nucleus, which are represented by a potential well, might be expected to exhibit the same type of behaviour.

These considerations were not taken into account in the calculations of Feshbach, Porter and Weisskopf (loc.cit.). They found that the experimental measurements of the total cross-sections for neutrons with energies between 0 and 3 MeV (Barschall, 1952; Millar et.al. 1952; Walt et.al. 1953; and Okazaki et.al. 1954) could be reproduced excellently using a square well potential with the parameters

$$\begin{aligned} V_0 &= 42(1 + i\xi)\ \text{Mev} \\ 0 \cdot 03 \leqslant\ &\xi \leqslant 0 \cdot 05 \qquad\qquad r < R \qquad\qquad \text{(IV.1)} \\ V_0 &= 0 \qquad\qquad\qquad\qquad r > R \end{aligned}$$

with

$$R = 1 \cdot 45\, A^{1/3} \times 10^{-13}\ \text{cm.}$$

A more recent calculation by Emmerich (1955) has indicated that better agreement is obtained using this model with a radius given by

$$R = (1 \cdot 26\, A^{1/3} + 0 \cdot 7) \times 10^{-13}\ \text{cm.}$$

However, discrepancies other than those observed in the present investigation of the cross-sections for the interaction of neutrons with nuclei, § 1, were apparent. It was found that the predictions of this model for the reduced neutron width to level spacing, Γ_n^0/D, were not in good agreement with experimental results; the model predicts values of this ratio which are too high at the maxima and too low at the minima. This discrepancy might be removed by using a higher value of ξ , or it might be in part attributable to the fact that the shape of the nuclear potential well is badly approximated by a square well. Furthermore, it has been shown by Walt and Barschall (1954) that the angular distribution of 1 MeV neutrons which are elastically scattered by nuclei, is not consistent with the predictions of the model employing a square well potential.

Extensive calculations of the differential cross-section for the elastic scattering of protons by nuclei have been made. It has been shown by Dayton (1954) and by Burge, Fujimoto and Hossain (1955) that the experimental data on the elastic scattering of protons by light nuclei are consistent with the predictions of the square well potential model. The heaviest nucleus for which this appears to be true is Aluminium. However Chase and Rohrlich (1954) and Woods and Saxon (loc.cit.) have shown that this is not the case for heavy elements. In particular, Woods and Saxon (loc.cit.) have shown that by using a potential well which is rounded, it is possible to reproduce the experimental data for the elastic scattering of 20 MeV protons by Nickel and Platinum. The potential which was used was of the form

$$V(r) = \frac{V + iW}{1 + \exp.\left[\frac{r-R}{a}\right]} \qquad \text{(IV.2)}$$

with $$R = r_0 A^{\frac{1}{3}} \times 10^{-13} \text{ cm.}$$

$$a \approx 0.49 \times 10^{-13} \text{ cm.}$$

The parameter 'a' is interpreted as a measure of the diffuseness of the nuclear surface; it appears that this quantity may increase slightly with increasing atomic weight of the target nucleus.

Furthermore it has been shown by Culler, Fernbach and Sherman (loc.cit.) that it is also necessary to use potentials which are not square, to predict the differential cross-sections for the elastic scattering of 14 MeV neutrons by nuclei.

Thus it is of interest to examine the predictions of the optical model using non-square potentials, when the energy of the incident neutrons is low. The solution of the wave equation using a potential of the type suggested by Woods and Saxon (loc.cit.) is exceedingly difficult for neutrons with angular momentum greater than zero, and is usually performed using computing machines. However an analytical solution of the wave equation is possible if a potential of step form, as suggested by Culler, Fernbach and Serber (loc.cit), is used. A potential of step form should approximate reasonably to one which is rounded.

The calculations which are reported here have been performed using a step well potential. The form of this potential is shown in figure IV.1. For simplicity it was assumed that the imaginary potential may be located within

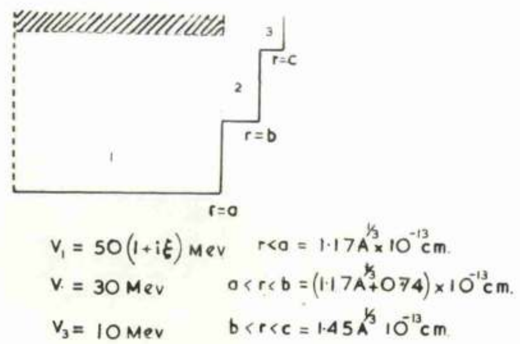


Figure IV.1. The form of the potential well which has been used in the calculations reported here.

a radius of approximately $1{\cdot}2\ A^{1/3} \times 10^{-13}$ cm., this does not take into account any possible absorption due to the presence of neutrons at distances greater than this from the centre of the nucleus. The third step of the potential was found necessary to reproduce the magnitudes of the total cross-sections for the heaviest nuclei.

§ 3. METHOD OF CALCULATION

(a) The theory of average cross-sections

The treatment of this problem, proposed by Feshbach, Porter and Weisskopf (loc.cit.) will be summarized briefly. When the energy of an incident nucleon is such that resonances are not observed the cross-sections for elastic and inelastic scattering of nucleons by nuclei are smooth functions of the energy of the incident particle. These cross-sections may be determined by considering the scattering of a plane wave by a nucleus. This scattering changes the amplitude of the outgoing wave, of the plane wave, by a complex reflection factor, η_ℓ , which is related to the complex phase shift, ϕ , by the equation

$$\eta_\ell = \exp.\left[2i\phi_\ell\right] \tag{IV.3}$$

The subscript ℓ refers to a wave of orbital angular momentum ℓ .

In terms of this complex reflection factor the cross-sections, σ_{el} , for elastic scattering by and , σ_I , for interaction of the incident nucleon with the target nucleus are given by

$$\sigma_{el} = \pi \lambda\!\!\!^{-2} \sum_{\ell=0}^{\infty} (2\ell+1) \left| 1 - \eta_\ell \right|^2$$

$$\sigma_I = \pi \lambda\!\!\!^{-2} \sum_{0}^{\infty} (2\ell+1) \left(1 - |\eta_\ell|^2 \right) \qquad \text{(IV.4)}$$

where $\lambda\!\!\!^-$ = the de Broglie wavelength of the incident nucleon divided by 2π .

When the energy of an incident nucleon is low, the reflection factor is a complicated function of this energy and exhibits fluctuations due to closely spaced resonances. It has been shown by Feshbach, Porter and Weisskopf (loc.cit.) that it is possible to define a new reflection factor, $\bar{\eta}_\ell$, which is averaged over resonances. In terms of this factor they have defined cross-sections which are similar in form to those given in equation IV.4.

$$\sigma_{se} = \pi \lambda\!\!\!^{-2} \sum_{0}^{\infty} (2\ell+1) \left| 1 - \bar{\eta}_\ell \right|^2$$

$$\sigma_c = \sigma_{ce} + \sigma_I = \pi \lambda\!\!\!^{-2} \sum_{0}^{\infty} (2\ell+1) \left(1 - |\bar{\eta}_\ell|^2 \right) \qquad \text{(IV.5)}$$

In this formulation the cross-section for elastic scattering is divided into two parts, that which appears as absorption from the incident beam with subsequent emission from the product nucleus, and that which represents elastic scattering as usually defined. The former is termed compound*-elastic, σ_{ce} , and the latter shape-elastic, σ_{se} . As a result of this definition, the cross-section, σ_c , for

---

* Footnote.

The term 'compound' does not imply the immediate absorption of a nucleon on entering a nucleus.

the formation of a compound nucleus should be greater than the experimentally measured cross-section, σ_I , for the interaction of an incident neutron with a nucleus, when compound-elastic scattering is of importance. This is expected to occur when the energy of the incident nucleon is approximately 1 or 2 MeV.

The replacement of η_ℓ by the average value $\bar{\eta}_\ell$ defines a new continuum problem. It is assumed that in the neighbourhood of resonances η_ℓ is given by a Breit-Wigner formula

$$\eta_\ell = \exp.-\left[2i\delta_\ell\right]\left(1 - \frac{i\Gamma_n}{\varepsilon - \varepsilon_s + i\Gamma/2}\right) \qquad \text{(IV.6)}$$

where δ_ℓ = the phase of the ingoing part of the radial wave function outside the nucleus with respect to the outgoing part, at $r = R$

Γ_n = the partial width of the resonance for emission of the incident particle. In this case only neutrons are considered.

Γ = the total width of the resonance.

and ε and ε_s = the energy of the incident neutron near and at a resonance.

If the distance between levels is D , then the average value $\bar{\eta}_\ell$ is given by

$$\bar{\eta}_\ell = \exp.-\left[2i\delta_\ell\right]\left(1 - \frac{\pi\Gamma_n}{D} + iG_\ell\right) \qquad \text{(IV.7)}$$

The magnitude of the term G_ℓ is Γ/D , all terms of magnitude Γ^2/D^2 are neglected.

The expression for $\bar{\eta}_\ell$ may be related to the logarithmic derivative, f_ℓ , of the wave function at the nuclear boundary

$$\bar{\eta}_\ell = \exp.-\left[2i\delta_\ell\right]\left(1 + \frac{2is_\ell}{f_\ell - \Delta_\ell - is_\ell}\right) \qquad \text{(IV.8)}$$

where the quantities Δ_ℓ and s_ℓ are determined by the conditions outside the nucleus.

$$\Delta_\ell + is_\ell = 1 + (kR)\,\frac{h_\ell'(kR)}{h_\ell(kR)} \qquad \text{(IV.9)}$$

where $h_\ell'(x)$ is the derivative of the Hankel function, of the first kind, $h_\ell(x)$.

Equation (IV.8) may be written in the form

$$\bar{\eta}_\ell = \exp.-\left[2i\delta_\ell\right]\left[1 - \frac{2s_\ell}{M_\ell + iN_\ell}\right] \qquad \text{(IV.10)}$$

where $\quad f_\ell = R_e(f_\ell) + \mathcal{I}_m(f_\ell)$

$$N_\ell = -\Delta_\ell + R_e(f_\ell)$$

and $\quad M_\ell = s_\ell - \mathcal{I}_m(f_\ell)$.

Using equations (IV.7) and (IV.9) the relationship between Γ_n/D and the logarithmic derivative may be obtained. The experimental quantity which has been plotted as a function of the atomic weight of the target nucleus is the reduced neutron width to level spacing Γ_n^0/D $\left(\Gamma_n^0 = \frac{\Gamma_n}{(E_n)^{\frac{1}{2}}}\right.$, where E_n

is the energy of the incident neutron). Thus it is only necessary to calculate the logarithmic derivative for $\ell = 0$. Since s_0 is small and $\Delta_0 = 0$, it is found that

$$\frac{\Gamma_n^0}{D} = \frac{-2s_0 \, \mathcal{I}m(f_0)}{\left[Re(f_0)\right]^2 + \left[\mathcal{I}m(f_0)\right]^2} \tag{IV.11}$$

The total cross-section for scattering and that for the formation of a compound nucleus may be written as

$$\sigma_t = \sum_{\ell=0}^{\infty} \sigma_t^\ell = \sum_0^\infty 4\pi \lambda\!\!\!^{-2}(2\ell+1)\left[\sin^2\delta_\ell + s_\ell \frac{M_\ell \cos 2\delta_\ell - N_\ell \sin 2\delta_\ell}{M_\ell^2 + N_\ell^2}\right]$$

$$\sigma_c = \sum_{\ell=0}^{\infty} \sigma_c^\ell = \sum_0^\infty 4\pi \lambda\!\!\!^{-2}(2\ell+1)s_\ell\left[\frac{-\mathcal{I}m(f_\ell)}{M_\ell^2 + N_\ell^2}\right] \tag{IV.12}$$

where δ_ℓ is given in terms of the spherical Bessel and Neumann functions, j_ℓ and n_ℓ respectively, as

$$\delta_\ell = \tan^{-1}\left[\frac{-j_\ell(kR)}{n_\ell(kR)}\right]$$

(b) <u>The calculation of the logarithmic derivative</u>.

The shape of the potential used is that shown in figure IV.1. The radial wave equation in these regions may be written in the form

$$\frac{d^2u_\ell}{dr^2} + \left[k^2 + \frac{2M}{\hbar^2}\left\{V_0(1+i\xi)\right\} - \frac{\ell(\ell+1)}{r^2}\right]u_\ell = 0 \qquad r < R$$

and (IV.13)

$$\frac{d^2u_\ell}{dr^2} + \left[k^2 - \frac{\ell(\ell+1)}{r^2}\right]u_\ell = 0 \qquad r > R$$

The most general solution of these equations is

$$u_\ell = r\left[A\, j_\ell(\alpha r) + i\, B\, n_\ell(\alpha r)\right] \qquad (IV.14)$$

where A and B are constants

$$\alpha = \sqrt{\frac{2M}{\hbar^2}}\left\{E + V_0(1+i\xi)\right\}^{1/2}$$

and E = the energy of the incident neutron, outside the nucleus.

The radial wave function in region 1 consists of only a Bessel function since the Neumann function has an infinite value for $r = 0$, and is thus physically unacceptable. In other regions the wave function is a combination of the Bessel and Neumann functions.

To derive the coefficients A and B for the wave functions in the different regions, it is necessary to match these functions and their logarithmic derivatives at the various boundaries. Using these boundary conditions, it is found that

$$u_\ell(r) = r.d\left\{c_\ell\, j_\ell(\alpha_2 r) + n_\ell(\alpha_2 r)\right\} \qquad a < r < b \qquad (IV.15)$$

where

$$d = \frac{j_\ell(\alpha_1 a)}{c_\ell\, j_\ell(\alpha_2 a) + n_\ell(\alpha_2 a)}$$

and

$$c_\ell = \frac{(\alpha_2 a)\, n_\ell'(\alpha_2 a)\, j_\ell(\alpha_1 a) - (\alpha_1 a)\, n_\ell(\alpha_2 a)\, j_\ell'(\alpha_1 a)}{(\alpha_1 a)\, j_\ell'(\alpha_1 a)\, j_\ell(\alpha_2 a) - (\alpha_2 a)\, j_\ell'(\alpha_2 a)\, j_\ell(\alpha_1 a)}$$

where $j_\ell'(\rho)$ represents the derivative with respect to ρ of $j_\ell(\rho)$

$$u_\ell(r) = r.g\left\{f\, j_\ell(\alpha_3 r) + n_\ell(\alpha_3 r)\right\} \qquad b < r < R \qquad (IV.16)$$

with

$$g = \frac{d\left[c_\ell j_\ell(\alpha_2 b) + n_\ell(\alpha_2 b)\right]}{\left[f j_\ell(\alpha_3 b) + n_\ell(\alpha_3 b)\right]}$$

and

$$f = \frac{(\alpha_2 b)\, n_\ell(\alpha_3 b)\left[c_\ell j_\ell'(\alpha_2 b) + n_\ell'(\alpha_2 b)\right] - (\alpha_3 b)\, n_\ell'(\alpha_3 b)\left[c_\ell j_\ell(\alpha_2 b) + n_\ell(\alpha_2 b)\right]}{(\alpha_3 b)\, j_\ell'(\alpha_3 b)\left[c_\ell j_\ell(\alpha_2 b) + n_\ell(\alpha_2 b)\right] - (\alpha_2 b)\, j_\ell(\alpha_3 b)\left[c_\ell j_\ell'(\alpha_2 b) + n_\ell'(\alpha_2 b)\right]}$$

the value of the logarithmic derivative, f_ℓ , at $r = R$ is then

$$f_\ell = 1 + (\alpha_3 R)\,\frac{\left[f j_\ell'(\alpha_3 R) + n_\ell'(\alpha_3 R)\right]}{\left[f j_\ell(\alpha_3 R) + n_\ell(\alpha_3 R)\right]} \qquad \text{(IV.17)}$$

These equations may be simplified by use of the relationships between the derivatives of the spherical Bessel and Neumann functions and these functions. Both the spherical Bessel and Neumann functions obey the relationships

$$j_{\ell-1}(\rho) = \frac{(\ell+1)}{\rho}\, j_\ell(\rho) + j_\ell'(\rho) \qquad \text{(IV.18)}$$

and

$$j_{\ell+1}(\rho) = \frac{\ell}{\rho}\, j_\ell(\rho) - j_\ell'(\rho)$$

Using these relationships it may be shown that

$$f_\ell = (\ell+1) - (\alpha_3 R)\,\frac{\left[f j_{\ell+1}(\alpha_3 R) + n_{\ell+1}(\alpha_3 R)\right]}{\left[f j_\ell(\alpha_3 R) + n_\ell(\alpha_3 R)\right]} \qquad \text{(IV.19)}$$

where

$$f = \frac{\alpha_3/\alpha_2\; n_{\ell+1}(\alpha_3 b) - n_\ell(\alpha_3 b)\left[\dfrac{c_\ell j_{\ell+1}(\alpha_2 b) + n_{\ell+1}(\alpha_2 b)}{c_\ell j_\ell(\alpha_2 b) + n_\ell(\alpha_2 b)}\right]}{j_\ell(\alpha_3 b)\left[\dfrac{c_\ell j_{\ell+1}(\alpha_2 b) + n_{\ell+1}(\alpha_2 b)}{c_\ell j_\ell(\alpha_2 b) + n_\ell(\alpha_2 b)}\right] - \alpha_3/\alpha_2\; j_{\ell+1}(\alpha_3 b)}$$

and

$$c_\ell = \frac{1}{(\alpha_2 a)^2\left[j_\ell(\alpha_2 a)\right]^2}\cdot\frac{1}{\left[\dfrac{j_{\ell+1}(\alpha_2 a)}{j_\ell(\alpha_2 a)} - \dfrac{\alpha_1}{\alpha_2}\cdot\dfrac{j_{\ell+1}(\alpha_1 a)}{j_\ell(\alpha_1 a)}\right]} - \frac{n_\ell(\alpha_2 a)}{j_\ell(\alpha_2 a)}$$

It can be seen that the calculation of the logarithmic derivative f_ℓ is not too complex when only α_1 is imaginary. The calculation of f_ℓ if other quantities were also assumed to be imaginary, would be very laborious and for this reason has not been considered.

§4. DISCUSSION

Calculations have been made of the ratio of the reduced width to level spacing, and of the total and interaction cross-sections for neutrons of 1 and 4 MeV. The results of these calculations and those performed using a square well potential are shown in figures IV.2 - 6. The experimental data for Γ_n°/D has been taken from the compilation of Carter, Harvey, Hughes and Pilcher (loc.cit.), those for the total cross-sections from the compilation of Hughes and Harvey (loc.cit.) and those cross-sections for the interaction of neutrons with nuclei from the data of Beyster, Henkel and Nobles (loc.cit.).

Preliminary calculations of Γ_n°/D indicated that with the potential used here it was necessary to use a value of V_1 of approximately 50 MeV, to ensure that the maximum was in that position determined experimentally. Consequently a value of 50 MeV was chosen and the value of 'a' found necessary was $1{\cdot}17\, A^{1/3} \times 10^{-13}$ cm. This depth of potential is in agreement with the similar value found necessary by Melkanoff, Maszkowski, Nodvik and Saxon (private communication), in an

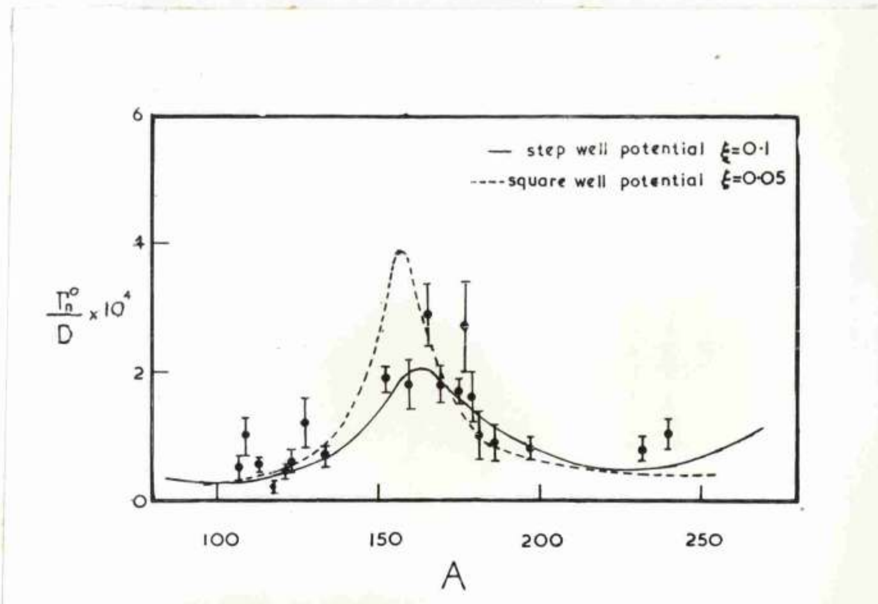


Figure IV.2. The ratio, Γ_n^0/D , of the neutron width to the level spacing.

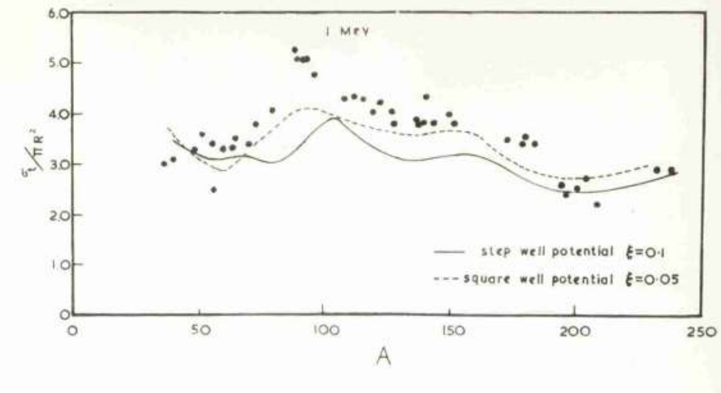


Figure IV.3. The total cross-sections for the scattering of 1 MeV neutrons by nuclei.

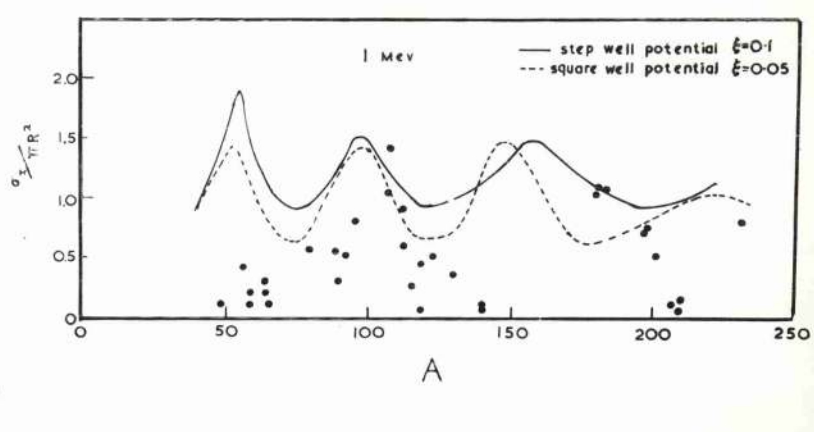


Figure IV.4. The cross-sections for the interaction of 1 MeV neutrons with nuclei.

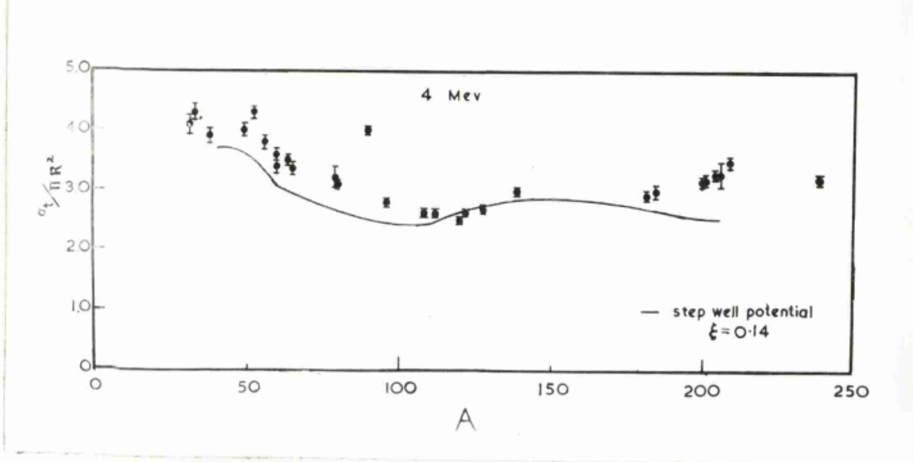


Figure IV.5. The total cross-sections for the scattering of 4 MeV neutrons by nuclei.

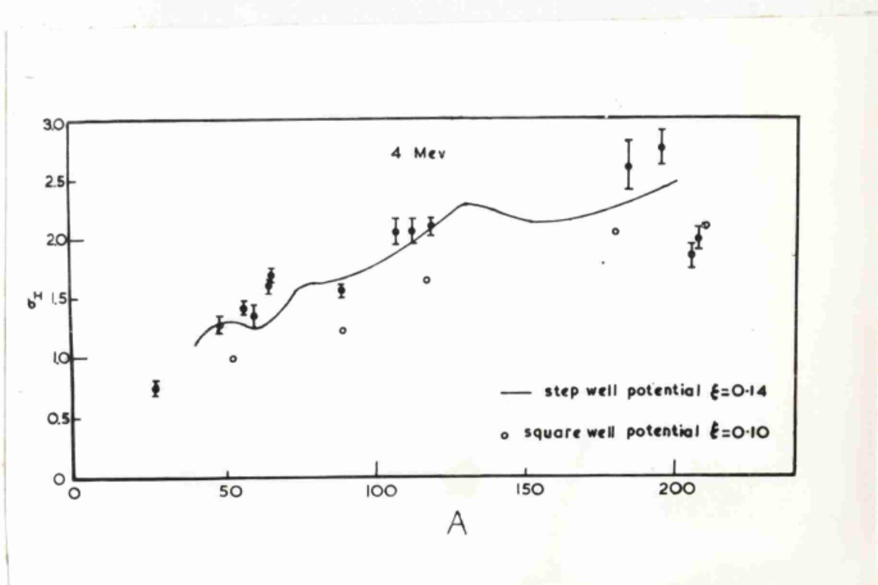


Figure IV.6. The cross-sections for the interaction of 4 MeV neutrons with nuclei.

optical model analysis of the elastic scattering of 5 MeV protons by nuclei. Preliminary calculations also indicated that it was necessary to use the third step of the potential to obtain agreement with the total cross-sections measured for nuclei of high atomic weight. The potentials $V_2 =$ 30 MeV and $V_3 =$ 10 MeV were chosen somewhat arbitrarily.

There is very little difference between the results of the calculations using the two different potentials when the energy of the incident neutrons is 1 MeV or less. The results of the calculations of Γ_n°/D depend very little on the shape of the potential well used but are strongly dependent upon the magnitude of ξ which is used. The results of calculations with a square well potential and $\xi =$ 0.10 are in as good agreement with the experimental data as those based on the step well potential. A good fit to the total cross-sections of nuclei for 1 MeV neutrons may be obtained using the square well potential with $\xi = 0.03$ - 0.05. Using such values for the step well potential produces rapid fluctuations in the variation of σ_T with atomic weight, which bear no resemblance to the experimental results. The results of these calculations are not shown in figure IV.3; instead the results of calculations using $\xi =$ 0.10 are shown, this value gives the best fit to the overall trend of the results. Furthermore this value of ξ gives cross-sections for the interaction of 1 MeV neutrons with nuclei which are consistent with those measured experimentally. The requirement that the calculated values should be greater than those measured experimentally is always fulfilled.

It can be seen from these results that improvement upon the model suggested by Feshbach, Porter and Weisskopf (loc.cit.) cannot be obtained when the energy of the incident neutron is 1 MeV or less. Thus the results of these calculations are unsatisfactory, since it is not possible to fit all the experimental data with one value of ξ using either shape of potential.

The importance of a rounding of the potential well can be seen from the results of calculations for the cross-sections for the interaction of 4 MeV neutrons with nuclei. Because of the high value of ξ which appeared necessary at lower energies, its value was taken to be 0.14 for neutrons with an energy of 4 MeV. The values of the total cross-sections obtained using a square well potential are in no better agreement with the experimental results than those obtained using the step well potential, and are therefore not shown. Agreement with the cross-sections for the interaction of neutrons with nuclei may be obtained using the step well potential. It is not possible to achieve this with the square well potential by merely increasing the value of ξ, since increasing this value produces small changes in the calculated values of the cross-sections.

These calculations show that it is necessary to use potential wells which are rounded in shape to obtain agreement with the high values of σ_I for neutrons with an energy of 4 MeV. This has recently been confirmed by Walt and Barschall (private communication). They have used a potential of the type suggested by Woods and Saxon (loc.cit.) and have found it necessary to use values of ξ similar to those used here.

CHAPTER V

A DISCUSSION OF THIS RESEARCH AND SUGGESTIONS FOR FURTHER RESEARCH.

It is apparent from the research reported in this thesis that the optical model is particularly suited to the description of the interaction of nucleons of low and intermediate energies with nuclei. However it has not been possible to obtain a completely satisfactory analysis of the experimental data available, and it is the purpose of this Chapter to discuss the experiments which may assist in further clarifications.

The model which has been used to determine the yields of nuclear reactions at intermediate energies is crude. It is an extension to lower energies of that model used for nucleons of high energy, and thus considers only the obvious quantities, such as Q values and coulomb barrier penetrabilities, which become of importance at these energies. Thus it has been assumed that only the conservation laws of momentum and energy are important, and that the momentum distribution of nucleons inside a nucleus is that of a Fermi gas of nucleons. This is an oversimplification, and it might be shown in future work that it is necessary to consider more detailed structure of nuclei. Moreover it has been assumed that the direct collision process takes place throughout the nuclear volume, and that there is no enhancement of interactions between an incident nucleon and those at the surface of nuclei. This type of interaction, suggested by Eisberg (loc.cit.) and by Austern, Butler and McManus (loc.cit.), might be of more importance in the case of proton emission, when the coulomb barrier severely inhibits the emission of protons produced throughout the nuclear volume. It might therefore be of interest to examine further the common characteristics of the angular distributions of neutrons and protons emitted in nuclear reactions. It is necessary to appeal to experiment for further information, since the experimental evidence is as yet still very scant.

The model used here seems adequate for the interpretation of the angular distribution of the nucleons emitted in the reactions ${}_{45}Rh^{103}(n,p)\,{}_{44}Ru^{103}$ and ${}_{83}Bi^{209}(n,n)\,{}_{83}Bi^{209}$ but not for that of the protons emitted, with energies between 4 and 7 MeV, in (n,p) reactions with lighter nuclei. It would be of interest to examine whether the features observed in the angular distributions of protons emitted in (n,p) reactions with lighter nuclei, are also present in the angular distributions of the neutrons emitted in (n,n) reactions with these nuclei. Such an investigation might show whether this effect is peculiar to (n,p) reactions and thus possibly associated with the coulomb barrier, or whether common to both types of reactions and thus probably dependent on detailed nuclear properties. Furthermore, investigations of the angular distributions of the protons emitted in reactions of the type ${}_{Z}N^{A}(p,p')\,{}_{Z}N^{A}$ would also be of assistance in this. Certainly the discrepancies observed for these reactions with nuclei of high charge, do not appear to be present for the reactions ${}_{Z}N^{A}(n,p)_{Z-1}N^{A}$. Thus further studies of the inelastic scattering of protons by nuclei are warranted to determine why the considerations used here are apparently inadequate.

The experimental data on the level density of excited nuclei is still very scant, but it does appear that there is a strong dependence upon the character of the nucleus. Thus the cross-section for the emission of nucleons in a nuclear reaction is expected to be strongly dependent on the particular target nucleus used. The experimental results on the cross-sections for the emission of nucleons in (n,p) reactions do tend to substantiate this, but it would be desirable to obtain more information. Many experiments could be chosen to establish this effect more conclusively; for example, further information may be obtained by studying the angular distribution of the protons emitted in the reaction ${}_{26}Fe^{54}(n,p)_{25}Mn^{54}$. It would be of considerable interest to determine whether the cross-section for the emission of protons from an excited nucleus is important for this reaction.

It should be noted that there is no experimental data on the angular distribution of nucleons emitted in (n,n) and (n,p) reactions which take place through discrete levels. Such information is of importance in the development of a theory for nuclear reactions.

It has been suggested by Paul and Clarke (loc.cit.) that the cross-sections for the processes ${}_{Z}N^{A}\,(n,\alpha)\,{}_{Z-2}N^{A-3}$ may be larger than those calculated on the basis of the statistical theory. During the course of the work discussed in Chapter II, calculations of these cross-sections were made by assuming that alpha-particles could be emitted only in the decay of an excited nucleus. The calculated values were found to vary rapidly with small changes in the Q- value for the process. Since these Q-values for nuclei with $Z \gtrsim 20$ are not sufficiently accurately known, it was not possible to draw any reliable conclusions from a comparison of these cross-sections and those measured by Paul and Clarke (loc.cit.) and by Blosser and his colleagues (private communication) for the heaviest nuclei. It would be of interest to obtain information on the angular distribution of the alpha-particles emitted in these reactions and the inverse reactions, particularly any variation with the character of the target nucleus.

The results of the calculations reported in Chapter IV are promising. It has been shown that it is possible to calculate satisfactorily the cross-sections for the interaction of neutrons with nuclei, i.e. the quantities which are necessary for the determination of the magnitudes of the cross-sections for particular nuclear reactions. The analysis is not entirely satisfactory, and might be considered further. It may be possible to achieve a more satisfactory analysis by using a smoother form of potential, and by smoothing and extending radially the imaginary part of the potential. To achieve further comparisons with experimental results, it would be desirable to calculate the differential cross-section for the elastic scattering of neutrons using a rounded potential well. Such extensive calculations would not be possible without the use of computing machines.

The further investigations suggested here, might yield additional information concerning the nuclear parameters which are important for the description of the interaction of nucleons of low and intermediate energies with nuclei.

APPENDIX 1.

(a) A CORRECTION FOR NON-ISOTROPIC SCATTERING IN THE CENTRE OF MOMENTUM SYSTEM.

The measured values of Hadley and his co-workers (1949), for the differential cross-section for the scattering of neutrons with an energy of 40 MeV from protons may be represented by the expression

$$\frac{d\sigma}{d\omega} \sim \left(13 + 8\cos^2\psi \right) \quad \text{mb. ster.}^{-1}$$

where ψ = the angle of scattering in the centre of momentum system.

This energy is of a similar magnitude to that considered in these calculations.

Numerical calculations show that collisions with a Fermi gas of nucleons are virtually forbidden for $\psi < 50^\circ$. Since the term involving $\cos^2\psi$ quoted above only makes a significant contribution at angles less than 50^0, it may be assumed that

$$\alpha_{np} \sim \frac{\int_0^{\pi} 13 \, d\omega}{\int_0^{\pi} (13 + 8\cos^2\psi)\, d\omega} \cdot \alpha_{nn}$$

$$\sim 0.8\, \alpha_{nn} = 0.8\, \alpha_{pp}$$

(b) THE CROSS-SECTION FOR THE CAPTURE OF A PARTICLE BY AN EXCITED NUCLEUS.

These cross-sections cannot be evaluated at present by experimental methods, and in all theoretical treatments it has been assumed that they are independent of the excitation energy of the nucleus. This assumption is probably correct for charged particles but not for neutrons.

At the energies considered in these calculations, the probability that a charged particle will traverse the coulomb barrier is small for heavy nuclei. Thus, despite a weak interaction between it and the nucleus, the probability of emission before it is incorporated into the compound system is small. Therefore the cross-section for capture should be independent of the excitation energy of the target nucleus. Consequently for charged particles, the cross-sections calculated by Shapiro (loc.cit.) have been used.

For neutrons the barrier penetrability is approximately unity, and as a result the cross-section for compound-elastic scattering is not negligible for neutrons of low energy. When a nucleus possesses excitation energy, this type of scattering should be reduced and thus the term $\sigma_b(E)$ in equation (II.36) should include the magnitude of this cross-section. It has been assumed that $\sigma_b(E)$ does not vary with the energy of the incident neutron, and that it is equal to the cross-section for interaction of a neutron with a nucleus in its ground state when compound-elastic scattering is not important. Thus the values of these cross-sections for 4 MeV neutrons measured by Beyster, Henkel and Nobles (loc.cit.) have been used.

REFERENCES.

Allan, D.L., 1955, Proc. Phys. Soc., A. 68, 925.

Allan, D.L., 1956, private communication.

Allred, J.C., Armstrong, A.H., and Rosen, L., 1953, Phys. Rev., 91, 90.

Amaldi, E., Bocciarelli, D., Cacciaputo, C., and Trabacchi, G., 1946, Nuovo Cimento, 3, 203.

Austern, N., Butler, S.T., and McManus, H., 1952, Phys. Rev., 87, 188.

Bardeen, J., and Feenberg, E., 1938, Phys. Rev., 54, 809.

Barschall, H.H., 1952, Phys. Rev., 86, 431.

Bernadini, G., Booth, E.T., and Lindenbaum, S.J., 1952, Phys. Rev., 88, 1017.

Beyster, J.R., Henkel, R.L., and Nobles, R.A., 1955, Phys. Rev., 97, 563.

Bjerge, T., and Westcott, C.H., 1935, Proc. Roy. Soc., 150, 709.

Blaser, J.P., Boehm, F., Marmier, P., and Peaslee, D.C., 1951, Helv. Phys. Acta., 24, 3.

Blaser, J.P., Boehm, F., Marmier, P., and Scherrer, P., 1951, Helv. Phys. Acta., 24, 441.

Bockelmann, C.K., Peterson, R.E., Adair, R.K., and Barschall, H.H., 1949, Phys. Rev., 76, 277.

Bohr, N., 1936, Nature, 137, 344.

Bollinger, L.M., Cotè, R., and Le Blanc, J., 1955, Bull. Am. Phys. Soc., 30 (1), 23.

Buechner, W.W., Mazuri, M., and Sperduto, A., 1956, Phys. Rev., 101, 188.

Burge, E.J., Fujimoto, Y., and Hossain, A., 1955, Phil. Mag., 1, 19.

Carter, R.S., Harvey, J.A., Hughes, D.J., and Pilcher, D.E., 1954, Phys. Rev., 96, 113.

Chase, D.M., and Rohrlich, F., 1954, Phys. Rev., 94, 81.

Cohen, B.L., Newman, E., Charpie, R.A., and Handley, T.H., 1954, Phys. Rev., 94, 620.

Cohen, A.V., and White, P.H., 1956, Nuclear Physics, 1, 73.

Clementel, E., and Villi, C., 1955, Nuovo Cimento, Vol., II, 176.

Combe, J., 1956, Suppl. Nuovo Cimento, III, 182.

Combe, J., and Cüer, P., 1954, Proc. of the 1954 Glasgow Conf., 89.

Cook, L.J., Mc Millan, E.M., Peterson, J.M., and Sewell, D.C., 1947, Phys. Rev., 72, 1264.

Cook, L.J., Mc Millan, E.M., Peterson, J.M., and Sewell, D.C., 1949, Phys. Rev., 75, 7.

Culler, G., Fernbach, S., and Sherman, N., 1956, Phys. Rev., 101, 1047.

Dayton, J.E., 1954, Phys. Rev., 95, 754.

Dilworth, C.C., Occhialini, G.P.S., and Payne, R.M., 1948, Nature, 162, 102.

Eisberg, R.M., 1954, Phys. Rev., 94, 739.

Eisberg, R.M., and Igo, G.J., 1954, Phys. Rev., 93, 1039.

Emmerich, W.S., 1955, Bull. Am. Phys. Soc., 30 (1), 9.

Endt, P.M., and Kluyver, J.C., 1954, Rev. Mod. Phys., 26, 95.

Feather, N., 1953, Adv. in Phys., 2, 141.

Fermi, E., and Amaldi, E., 1935, Ricerca Sci., 6 (2), 544.

Fernbach, S., Serber, R., and Taylor, T.B., 1949, Phys. Rev., 75, 1352.

Feshbach, H., Peaslee, D.C., and Weisskopf, V.F., 1947, Phys. Rev., 71, 1451.

Feshbach, H, Porter, C., and Weisskopf, V.F., 1954, Phys. Rev., 96, 448.

Feshbach, H., and Weisskopf, V.F., 1949, Phys. Rev., 76. 1550.

Fitch, V.L., and Rainwater, J., 1953, Phys. Rev., 92. 789.

Forbes, S.G., 1952, Phys. Rev., 88. 1309.

Francis, N.C., and Watson, K.M., 1953, Am. Journ. of Physics, 21, 659.

Friedman, F.L., and Weisskopf, V.F., 1956, Kgl. Danske Viderskab Selskab (anniv. vol. for Niels Bohr).

Frye, G.M., 1954, Phys. Rev., 93, 1086.

Ghoshal, S.N., 1950, Phys. Rev., 80, 939.

Goldberger, M.L., 1948, Phys., Rev., 74. 1269.

Gugelot, P.C., 1951, Phys. Rev., 81, 51.

Gugelot, P.C., 1954, Phys. Rev., 93, 425.

Hadley, J., Kelley, E., Segrè, E., Wiegand, C., and York, H., 1949, Phys. Rev., 75, 351.

Harvey, J.A., Hughes, D.J., Carter, R.S., and Pilcher, V.E., 1955, Phys. Rev., 99, 10.

Hayakawa, S., Kawai, M., and Kikuchi, K., 1955, Prog. of Theor. Phys., 13, 415.

Hess, W.N., and Moyer, B.J., 1954, Phys. Rev., 96. 859.

Hirzel, O., and Waffler, H., 1947, Helv. Phys. Acta., 20, 373.

Hofstadter, R., Hahn, B., Knudsen, A.W., and Mc Intyre, J.A., 1954, Phys. Rev., 95, 512.

Hughes, D.J., and Harvey, J.A., 1955, "Neutron Cross-Sections" Mc Graw-Hill.

Johnson, M.H., and Teller, E., 1954, Phys. Rev., 93, 357.

Lane, A.M., and Wandel, C.F., 1955, Phys. Rev., 98, 1524.

Lang, J.M.B., and Le Couteur, K.J., 1954, Proc. Phys. Soc., A., 67, 586.

Lees, C.F., Morrison, G.C., Muirhead, H., and Rosser, W.G.V., 1953, Phil. Mag., 44, 304.

Melkanoff, M.A., Moskowski, S.A., Nodvik, J., and Saxon, D.S., 1956, private communication.

Moon, P.B., and Tillman, J.R., 1935, Nature, 135, 904.

Millar, D.W., Adair, R.K., Bockelmann, C.K., and Darden, S.E., 1952, Phys. Rev., 88, 83.

Morrison, G.C., Muirhead, H., and Murdoch, P.A.B., 1955, Phil. Mag., 46, 795.

Morrison, G.C., Muirhead, H., and Rosser, W.G.V., 1953, Phil. Mag., 44, 1326.

Okazaki, A., Darden, S.E., and Walton, R.B., 1954, Phys. Rev., 93, 461.

Paris, C.H., Buechner, W.W., and Endt, P.M., 1955, Phys. Rev., 100, 1317.

Paul, E.B., and Clarke, R.L., 1953, Can. Journ. of Phys., 31. 267.

Paul, E.B., Bartholemew, G.A., Gove, H.E., and Litherland, A.E., 1956, Bull. Am. Phys. Soc., 1, 39.

Peaslee, D.C., 1952, Phys. Rev., 86, 269.

Perkins, D.H., 1949, Phil. Mag., 40, 601.

Phillips, D.D., Davis, R.W., and Graves, E.R., 1952, Phys. Rev., 88, 600.

Pidd, R.W., Hammer, C.L., and Raka, E.C., 1953, Phys. Rev., 92, 436.

Pidd, R.W., and Hammer, C.L., 1955, Phys. Rev., 99, 1396.

Rosen, L., and Armstrong, A.H., 1956, Bull. Am. Phys. Soc., 1 (4), 224.

Rosen, L., and Armstrong, A.H., 1956, private communication.

Rosen, L., and Stewart, L., 1955, Phys. Rev., 99, 1052.

Rotblat, J., 1951, Nature, 167, 550.

Serber, R., 1947, Phys. Rev., 72, 1114.

Sherr, R., 1945, Phys. Rev., 68, 240.

Shapiro, M.M., 1953, Phys. Rev., 90, 171.

Swiatecki, W.J., 1955, Phys. Rev., 98, 204.

Szilard, L., 1935, Nature, 136, 950.

Taylor, H.L., Lönsjö, O., and Bonner, T.W., 1955, Phys. Rev., 100, 174.

Tendam, D.J., and Bradt, H.L., 1947, Phys. Rev., 72, 1118.

Wäffler, H., 1950, Helv, Phys. Acta. 20 373.

Walt, M., and Barschall, H.H., 1954, Phys. Rev., 93, 1062.

Walt, M., and Barschall, H.H., 1956, private communication.

Walt, M., Becker, R.L., Okazaki, A., and Fields, R.E., 1953 Phys. Rev., 89, 1271.

Weisskopf, V.F., 1937, Phys. Rev., 52, 295.

Weisskopf, V.F., and Ewing, D.H., 1940, Phys. Rev., 57, 472.

Woods, R.D., and Saxon, D.S., 1954, Phys. Rev., 95, 577.