INVESTIGATION OF TIDAL PHENOMENA

IN THE CLYDE ESTUARY

USING A SCALE MODEL,

by

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Introduction:-

Estuaries which have been studied in the past with the help of models, have had relatively shallow water at their mouths. The Clyde differs from most large British estuaries, in that the depth of the water at the mouth is considerable - off Greenock the low water depth increases from zero to 38 fathoms in a distance of 1·3 miles, and near Dunoon the depth is 46 fathoms. In comparison with the tidal ranges in some of the estuaries of which models have been made, the range at the mouth of the Clyde is very small. As this range is not inconvenient to shipping moored alongside the wharves, gates are not required in the docks at Glasgow. Two centuries ago the estuary above Port Glasgow could be navigated only at high water by ships of very shallow draught. At the present day large vessels can be docked in Glasgow at all stages of the tide. Vast quantities of material have been dredged from the channel, greatly altering the range of the tide in the upper reaches.

In the initial stages of work on the model it was decided to use a non-mobile bed, and the main dredged channel was kept unaltered throughout, at a depth corresponding to present day conditions. There are many problems to study in a tidal estuary, but any investigator would sooner or later become interested in the problems brought about by the reclamation of land on the banks, made by the possible spoiling of dredged material, which in the Clyde at present is transported a distance of 35 miles to sea. The model was used to study these and other problems on the Clyde, and the results are recorded in the following paper.

Section I

1. Summary:- The model was built in the James Watt Engineering Laboratories, Glasgow University.

The horizontal and vertical scales were 1 in 5280 and 1 in 115 respectively, giving a vertical exaggeration of 46. The time scale was 1 in 493, which gave the period of a 12 hours and 25 minutes tide as 90·3 seconds in the model.

The model was non-mobile, the bed being constructed of cement and sand mortar. In order to have conditions at the mouth of the estuary as representative as possible, the deep sea lochs were included. Details for these were obtained from the Admiralty Charts, and the Engineer of the Clyde Navigation Trust granted access to tide curves and recent cross sections of the estuary.

Tides, varying in the correct spring-neap cycle, were produced by a displacer, which was given the correct motion by mechanism designed on the principle used by Lord Kelvin in his tide-predicting machine. Tide curves were taken at various points on the estuary model by a gauge using reflected light, and the effects of spates and land reclamation were investigated. In addition, the following items were studied:-

(a) Increase of range of the tide as the tidal wave travels inland.

(b) The velocity of the tidal wave.

(c) The hump on the front of the advancing wave.

(d) Some current velocities.

(e) The natural period of oscillation of the estuary.

(f) Harmonic analysis of the tide curves.

(g) Frictional damping of the natural oscillation.

(h) The mean level of water in the estuary at various points.

(i) Profiles of the surface of the water in the estuary.

Wherever possible, comparisons were made throughout with the full scale estuary.

Section II

2. Choice of horizontal scale:- In order to have conditions as nearly representative as possible in the shallow part of the estuary, it was considered necessary to include all the sea area inside a line joining Dunoon to the Cloch. This meant that a large part of the area available had to be used for the construction of Loch Goil, Loch Long, Holy Loch, and the Gare Loch. The upper reaches of the tidal river were bent round in a curve of radius about 2 miles and Loch Goil and upper Loch Long were bent round as shown in Fig.1. The largest scale possible in the available space, and adopting these restrictions on the extremities, was 1/5280.

2.1. Choice of vertical scale:- The scale was taken as 1/115. As it depended to some extent on available gear wheels, this will be discussed later (Section 4.6.).

Section III

3. Construction of tank:- Preliminary work had commenced on the model in 1938-39. Reinforced concrete work on the tank had been completed, with emergency overflow, drainage plugs, and teak rail. The steel frame was in position to hold the tide displacer. The main gear wheels, to give the spring-neap variation in the tide, had been erected on their frame, and the constant speed motor and various chain wheels, lengths of chain etc. had been stored.

3.1. Construction of sea bed:- A scale plan, $\frac{1}{4}$ model size, was made. A grid with lines 1 mile apart was laid down on the plan and on the floor and teak rail of the model. This grid was transferred to the Admiralty charts for the areas in question, and the grid, the H.W., L.W., 5 fathom, 10 fathom, 20, 30, 40 and 50 fathom contours were enlarged by pantograph to model size. The charts used were:-

	No.	Scale
Loch Long and Loch Goil	3739	1/24,400
Lower Loch Long	3746	1/12,500
Gareloch	2000	1/12,500
River Clyde, Gourock to Dumbarton	2006	1/15,300
River Clyde, Dumbarton to Glasgow	2007	1/12,300

The model-size plan was laid on a flat floor - the various sheets (22" x 30") being correlated by the grid lines. Brass templates of the correct depths were now bent to the contour shapes and placed correctly in the model, again using the grid. The work was done in layers, each contour being completed before the next was begun. A level floor was constructed at each step to be used as a base for the next layer. The depth of a 10 fathom layer was $6\frac{1}{4}$ inches.

The stepped formation thus obtained was moulded in with mortar, using a trowel, and rubbed smooth. In the area between the L.W. and the 5 and 10 fathom contours, the moulding was done by taking spot depths from the chart. The land surface was finished off level at a height of 3 feet above H.W.

3.2. Rail datum:- Precision levels taken round the teak rail showed considerable variations and, to avoid the extensive planing that would be required to produce a level surface, it was decided to fix a steel strip to the inner edge, this strip being arranged in short lengths and carefully set level to within ± ·006 inches, using a surveyor's precision level. The bed levels etc. were referred to the rail which was taken as being at a level of O.D. + 48·00.

3.3. Construction of the river bed:- The river from Glasgow to a point near Port Glasgow, 18 miles down, is maintained and dredged by the Clyde Navigation Trust. Permission was obtained to make tracings of cross sections of the river as surveyed in 1944. These cross sections were re-plotted to the correct horizontal and vertical scales, and cardboard templates were cut and used to mould the river bed to its correct shape. The templates were spaced about 3 inches apart on the model. Below Port Glasgow the chart was again used, cross sections at 6 inch intervals being drawn, cut out in cardboard, and fixed as before. Salient features of the bed between cross sections were located by spot depths, using the grid.

3.4. Rivers:- Five rivers enter the estuary, namely the Clyde, Kelvin, Black Cart, White Cart, and the Leven. The Kelvin could be constructed to scale for its tidal stretch. As the other four rivers, however, are tidal for long distances, a labyrinth was made for each one, the complete tidal lengths for these rivers being determined from the Ordnance sheets. The Harbour Master of Paisley supplied information about the navigable part of the Cart. For the other three rivers, the depths were estimated, and the widths of the channels in the labyrinths made to correspond as closely as possible with the widths of the rivers.

The Clyde Weir was represented by a fixed weir at a level of O.D. + 5·0.

3.5. Run-off:- The catchment areas were estimated to be as follows:-

Clyde	...	840
Kelvin	...	127
White Cart		93
Black Cart		107
Leven	...	305
Total	...	1481 sq.miles.

The average rainfall over the whole catchment area may be estimated as 48 inches per annum, and taking 75% of this as run-off, the total annual run-off is estimated at $12{\cdot}4 \times 10^{10}$ cubic feet, or 3928 cub.ft./sec. The proportion which may be allocated to each river is, therefore, as follows:-

Clyde	...	2,253
Kelvin	...	337
White Cart		246
Black Cart		283
Leven	...	809
Total	...	3,928 cub.ft./sec.

3.6. Discharge scale:- The ratio of the discharge in the full size river to the discharge in the model is given by $\frac{Q}{q} = \frac{L}{l}\left(\frac{H}{h}\right)^{3/2}$ where $\frac{l}{L} = \frac{1}{5280}$, the horizontal scale, and $\frac{h}{H} = \frac{1}{115}$, the vertical scale. Thus $\frac{Q}{q} = 6 \cdot 5 \times 10^6$; and the required discharge in the model for each river is as follows:-

Clyde	$34 \cdot 53 \times 10^{-5}$
Kelvin	$5 \cdot 18 \times 10^{-5}$
White Cart	$3 \cdot 79 \times 10^{-5}$
Black Cart	$4 \cdot 36 \times 10^{-5}$
Leven	$12 \cdot 44 \times 10^{-5}$ cub.ft./sec.

3.7. Control of river inflow:- Water, at a constant level, was led from a tank into the lower end of a vertical glass tube, the amount entering the tube being controlled by a screw clip over a rubber section of the delivery pipe. The water escaped through a brass orifice at the base of the tube, the head of water in the glass tube controlling the flow out of the orifice. The size of the orifice for a given head was first calculated, and the gauge subsequently calibrated by direct measurement.

Five such orifices were assembled on a vertical board fixed on the rail of the model, and the discharge from each was led to the head of each river.

3.8. Overflow:- In order to keep the average water level in the model constant, water had to be removed at the same rate as the inflow. This removal was arranged to take place at the sea-ward end, firstly by a device similar to that used for the inflow, and secondly by a funnel set approximately at mean sea-level in the model. The orifice was designed to pass 80% of the total inflow from the rivers, and the conical funnel was set by trial to take 20% of the inflow.

This twin arrangement was adopted to keep the mean water level as constant as possible throughout a complete series of springs and neaps, covering a period, in the model, of about 43 minutes. The mean depth over the funnel varies considerably during this time with consequent variation in the overflow, thus affecting the mean level in the model.

3.9. Surface scum:- The dust etc. accumulating on the surface of the water, was removed by the funnel. The total quantity of surface dirt removed was increased considerably by tilting the funnel so that, as the water level rose, the meniscus broke more quickly at the rim of the funnel.

3.10. Measurement of water level:- In order to obtain an instantaneous measurement of the water level, use was made of the reflecting capacity of the surface. A beam of light from the carbon filament of a lamp was projected through a condenser on to the water surface, the reflection being observed on a scale fixed vertically as shown in Fig.2. As the water level varies, the filament remains in focus on the scale. The movement of the image of the filament is twice that of the water surface, and the scale was graduated accordingly. The scale could be read to ·02 of an inch, equivalent to ·01 of an inch in the model, or about $\frac{1}{10}$ of a foot on the full scale.

Owing to the narrowness of the model channel at the Broomielaw, Glasgow, the curvature of the meniscus prevented the formation of a good image and, therefore, the comparatively large area of Prince's Dock was used for the measurement of water levels in the upper reaches of the river.

Section IV

4. Tides:- The reproduction of the spring-neap cycle of the tides has been done in different ways. At Delft, Netherlands engineers have used electrically controlled pumping machinery to produce the model tides. At the National Physical Laboratory, tide displacers are actuated by electrically controlled hydraulic rams. Professor Gibson[1] used epicyclic gearing to move the tide-generating displacer in the Severn model, as did Allen[2] and Elsden[3] respectively in the Cheshire Dee and the Rangoon River models.

In a small model such as the Clyde model under investigation it was considered most suitable to use the principle employed by Lord Kelvin[4] in his tide-predicting machine to impart the spring-neap cycle of motions to a tide-producing displacer.

The periods of revolution of the sun and moon relative to the earth are in the ratio 1 : 1·0350501. The tide was assumed to be generated by the resultant of two forces, each having a simple harmonic motion of a different period and amplitude. The diurnal variation was neglected. Two chain wheels had to be found which would have the above ratio, 1 : 1·035051. Successive approximations to this ratio are:-

1	...	...	...	1·000000
$\frac{29}{28}$	...	...	...	1·035714
$\frac{30}{29}$	...	...	...	1·034483
$\frac{59}{57}$	...	...	...	1·0350877
$\frac{443}{428}$	...	...	...	1·035047

etc. etc.

The two wheels chosen have respectively 118 and 114 teeth, giving a ratio of $\frac{59}{57} = 1{\cdot}0350877$. These wheels were obtained from stock sizes.

4.1. Gearing layout:- Figs. 3 and 4 show the layout of the gearing of the tide-generating mechanism. The constant speed motor and gear box drive wheel A which drives wheel C through B. An endless chain, kept tight by the adjustable pulley P, drives the wheels E and D. A chain Q S_1 N M K is attached at K on the wheel D to a swivel pin fixed on a plate with a radial slot. The chain passes through a guide M, round a pulley N on wheel E, and over the wheel S_1 on the shaft lifting the displacer. The radii r and R of K and N respectively can be varied. The position of the guide M can be adjusted along a line joining the centres of the wheels D and E.

The motor drives wheel A at 22·3 r.p.m. and the following speeds are thus obtained:-

"Sun" wheel D ... ... 0·684 r.p.m.

"Moon" wheel E ... ... 0·661 r.p.m.

The time of one revolution of the sun wheel is thus $\frac{1}{0{\cdot}684}$ minutes and must correspond with the tide caused by the sun, rising to H.W. every 12 hours as indicated by the equilibrium theory.

Thus the time ratio $\frac{t}{T} = \frac{\text{model time}}{\text{actual time}} = \frac{1}{492{\cdot}6}$.

4.2. Layout of machinery:- The displacer is hung on chains from the shaft S_1 which is turned by the chain, the motion of which is determined by the moon and the sun wheels simultaneously. As the two wheels revolve, with a speed ratio of $\frac{114}{118}$, the length of the chain K M N S_1 varies and, in so doing, turns shaft S_1 and imparts a reciprocating motion to the displacer.

The sun tide, produced by the eccentricity r of the point K on the sun wheel, is combined with the moon tide, produced by the eccentricity R of the pulley N on the moon wheel. Were the wheel centres far apart, and no guide M in position, the tide would be the combination of the two periodic tides with amplitudes r and R and with periods in the ratio 1·035088. The moon wheel, with the larger amplitude R, determines the period, and the tide would take equal times to fall and rise. The actual mean spring tide at Greenock is unsymmetrical, and so the guide M had to be fixed to reproduce this lack of symmetry.

4.3. Unsymmetrical tide:- Reference to the Admiralty Tide Tables for 1945 (p.137) gave the following corrections for the times of H.W. and L.W. at Greenock:-

Correction to H.W. time	+ $00^h\ 24^m$
" " L.W. "	− $00^h\ 10^m$
The normal time of fall and rise is	$6^h\ 12^m$
therefore the correct (mean) time of fall is	$5^h\ 38^m$
and the correct (mean) time of rise is	$6^h\ 46^m$.

Since the complete tide of $12^h\ 24^m$ is represented by one revolution of the moon wheel, the time of fall of the tide will be represented by the rotation of the shaft through $163\frac{1}{2}^\circ$.

4.4. Position of chain guide:- The chain guide M consisted of two small sprocket wheels which could be moved in slots parallel to the line joining the centres of the moon wheel and the sun wheel. Referring to Fig.5, the circle with centre F represents the locus of the centre of the pulley N on the moon wheel. The radius R ($= FN_1 = FN_2$) can be adjusted. S_1 is the shaft which gives the periodic motion to the displacer.

$S_1\ N_2\ M$ is the position of the chain when $S_1\ N_2 + N_2\ M$ is a minimum length, while $S_1\ N_1\ M$ is the position when $S_1\ N_1 + N_1\ M$ is a maximum length. High water occurs when the pulley is at N_2.

From Section 4.3. the angle $N_1\ F\ N_2$ must be $(360^\circ - 163\frac{1}{2}^\circ)$

in a clockwise direction. The length F S_1 is fixed, and the range of tide is fixed. The variables are R– the radius of N– and the length of F M. The position of M and the value of R were found initially by trial on the drawing board, using the fact that for the length of chain S_1 N M to be a maximum or a minimum, the radius FN must bisect angle S_1 N M (Fig.6). Several trials gave R = 4·7 inches and F M = 8·48 inches, values which later were proved to be nearly correct when records of the tide were taken.

4.5. Displacer balancing cam:– When the displacer was at the upper limit of its range (Low water) the tension on its supporting chains was at its maximum. In order to reduce the tension in the driving chain and reduce the motor torque required, a balancing cam (Fig.4) was designed. It was fitted to the shaft S_2 which revolved only 276° as compared with shaft S_1 which revolved 630° for a normal spring tide. The displacer was loaded sufficiently to make it sink to the limit allowed by the gearing, and at this position the torque applied by the cam was at its maximum.

4.6. Scales:– The value of the time scale $\frac{T}{t}$ had been exactly determined (Section 4.1.) and the principle of dynamical similarity was used to calculate the values of the vertical scale, the discharge scale, and the horizontal velocity scale. The values of the model scales are given in the following table:–

Scale	Value
Horizontal	5280
Time	492·6
Vertical	115
Discharge	$6{\cdot}5 \times 10^6$
Horizontal Velocity	10·71

<u>4.7. Reynolds number:-</u> At high and low water in the model, the mean overall depth of the main channel is 0·32 and 0·23 feet respectively. The greatest velocity of the current is 0·32 ft./sec., and at 15° C the kinematic viscosity of water is 0·00001228 ft^2/sec. Consequently the Reynolds number is about 8,300 and 5,900 for high and low water conditions respectively. Reference will be made to the Reynolds number in Sections 7.2. and 14.4., which discuss the increase of frictional resistance in the model at low water.

Section V

<u>5. The choice of representative tides at Gourock:-</u> The range of tide to use in representative model experiments had to be determined. The following four methods were available:-

(a) Calculation of a theoretical tide from the Admiralty Tide Tables 1945.

(b) Estimation from data in Admiralty Tide Tables 1945.

(c) Estimation from data in Admiralty Tide Tables 1916.

(d) Estimation from data collected from the gauge records of the Clyde Navigation Trust.

For experimental work on the model, the tide range at Gourock was taken to be the same as at Greenock. Gourock was the most suitable position on the model at which to take readings and had the additional advantage that in the actual estuary a tide gauge was installed at this point.

<u>Method (a)</u> The moon coefficient M is given as 4·4 and the sun coefficient as 1·0. The mean spring range is thus 2 (M + S) = 10·8 feet and the mean neap range 2 (M – S) = 6·8 feet. There is a shallow water correction of –0·3 feet to H.W. and +0·3 feet to L.W. and, therefore, the mean spring and neap ranges are respectively 10·2 feet and 6·2 feet.

<u>Method (b)</u> The 1945 Admiralty Tide Tables give the following O.D. levels for tides at Greenock:-

Mean L.W. Springs	+ 0·29	
Mean L.W. Neaps	+ 2·32	
Mean H.W. Neaps	+ 8·40	
Mean H.W. Springs	+ 10·31	feet.

From these a mean spring range and a mean neap range of 10·02 feet and 6·08 feet respectively are deduced.

Method (c) The Admiralty Tide Tables 1916 give mean spring and neap ranges at Greenock as 10·04 feet and 6·02 feet respectively.

Method (d) Table 1 shows values of mean spring and neap tide ranges at Gourock as 10·5 and 6·5 feet respectively. The actual tide curves for Gourock for 1945 were consulted and the levels of H.W. and L.W. abstracted for each tide occurring exactly three tides after the moon's four phases, i.e. new moon and full moon for springs, and first and last quarters for neaps. Priming, lagging, and diurnal variation of the tides were neglected.

The two means of the results from (a), (b), (c) and (d) namely 10·19 feet (springs) and 6·20 feet (neaps) were taken as the representative tides. The moon and sun coefficients (Method (a)) are thus M = 4·1 feet and S = 1·0 feet, and these values were adopted for guidance in the first trials during adjustment of the tide machine.

5.1. Adjustment of the model tide:- The symbol n will be used to represent the ratio of the time of the fall of the tide to the time of the rise. From Section 4.3. the value of n for the mean tide is $\frac{5^h\ 38^m}{6^h\ 46^m} = 0{\cdot}831$. Trial curves were now taken on the model at Gourock and these agreed with a theoretical curve drawn by combining two curves of the correct period and amplitude and altering the resultant profile to make n = 0·831. These trial curves corresponded with the curves of the tides in the estuary; but it was evident that improvements could be made by altering the shape of the displacer.

An irregularity or low hump exists on the rising tide at Gourock, and this is evidently the beginning of the decided hump on the tide at Glasgow. In order to reproduce this part of the Gourock tide correctly in the model, a second displacer would be required, and it was decided to compromise by altering the shape of the displacer, which had been so constructed that alterations could easily be made. The total volume of the displacer remained the same, the box being "waisted", as shown in Fig.4, and the alteration obviously produced a similar irregularity on the falling tide. Trial curves of spring tides taken on the model at Gourock were found to be very similar to estuary tides at Gourock (Fig.8) allowing for the varying properties of any chosen natural tide.

Trial curves of neap tides were taken on the model at Gourock, and it was evident that the "waisting" of the displacer had diminished the range; the travel of the displacer was the same as formerly, but the amount of water displaced was diminished. Accordingly, the travel of the displacer was increased by altering the radius of point N on the moon wheel E, and the resultant spring and neap ranges at Gourock were 10·85 feet and 6·0 feet respectively. The shape of the displacer remained unaltered for the experiments carried out, and the travel of the displacer was kept the same except for the experiments performed in connection with the reclamation of large areas of sandbanks, described in Section 7.2.

Section VI

6. Tide recording:- Readings of water levels by the reflected beam gauge were made at intervals corresponding to one hour on the full scale and, to ensure that the timing of each reading was correct, the sun wheel was fitted with 12 brass screws spaced equally round a circle concentric with its centre. These screws made one contact per "model

hour" in a bell circuit, the bell thus giving a signal each "model hour". The water levels were taken at each signal, one of the brass contacts being made double to enable every 12th hour to be noted.

6.1. Velocity of wave crest:- The velocity of the tidal wave in the model was measured with the help of a drum recorder which gave a curve record of the motion of the displacer generating the wave (Fig.7). An ideal method of measuring the velocity would be to take tide curves simultaneously at five or six points on the model but, with one reflected beam gauge, this was impossible.

A wooden cylinder (Plate 3), with its axis vertical, was fixed near the displacer. A pencil, suitably guided, was fixed to the displacer, to trace a line on a sheet of paper pinned to the cylinder. The cylinder was geared to turn at half the speed of the sun wheel, and was of such a diameter that the circumference of the paper was exactly 12 inches. Using a horizontal scale of $\frac{1}{2}$" = 1 hour - the scale used to plot all the tide curves - the circumference of the cylinder thus represented to scale, 24 model hours. The curve traced on the paper on the drum gave the motion of the displacer producing the wave. The crests and troughs of the curve coincided with the troughs and crests of the actual wave produced. The signal given by the double bell was used to mark a vertical zero line on the cylinder, and the horizontal distance from this zero to the crest then gave the time interval to Low Water at the displacer. The tide curve taken at any point upstream had the zero time marked on it by using the signal from the double bell, and thus the time interval to Low Water at that point could be measured. The difference of the two times gave the time interval which had elapsed between the wave leaving the displacer and its arrival at the upstream point.

6.2. Continuous recording of displacer motion:- The drum was allowed to run for 45 minutes and a series of waves was traced as shown in Fig.7. Any vertical line shows the position of the displacer at each succeeding 24 hour interval, model time. The drum revolved 15 times in 45 minutes, and as one revolution of the drum is equivalent to 24 hours on the full scale, the time scale is 480. A similar run on a different day gave the time scale as 492. The correct scale is 493 - the discrepancy being accounted for by variation in the voltage of the mains. Fifteen double tides are represented by the lines 0, 1, 2, 3, - - - 14 and 15 where they intersect the mean level line. The elapsed time, model scale, is 15·53 days, and the average time of one double tide is 24 hours 51 minutes, the correct value.

The cylinder was used to determine the mean position of the displacer and, before any series of readings was taken, the displacer was brought to its mean position and the still water level was adjusted to the required mean sea level.

Section VII

7. Investigation of tide ranges in the model under normal conditions:- In order to compare the shape of the tide wave as the wave advanced upstream, records were taken in the model at three stations, Gourock, Bowling, and Prince's Dock. Spring and neap tides were recorded at each place.

On starting the tide-generating mechanism, the tide level at Glasgow was at first irregular, showing a series of short period oscillations, which however rapidly died away and had disappeared by the end of the first tidal period. Before records were taken, therefore, the model was run for at least three tides to ensure that conditions were as nearly representative as possible.

The tide curves taken at the three stations on the model and tide curves taken from the actual estuary are shown in Fig.8. The spring and the neap tides on the estuary were taken on 15th January and 20th May 1945, dates when spring and neap tides were to be expected according to Table 1. The ranges of these actual river tides are shown in Table 2, while those observed on the model are given in Table 3 in the column headed "Normal". The range of the spring tide in the model increases as the wave advances upstream, and the total amplitude is plotted as a ratio, using the amplitude at Gourock as unity (Table 3, Fig.10). The ratio for the model at Prince's Dock is 1·22; for the estuary it is 1·30 as measured by tide gauge and 1·26 as recorded in the Admiralty Tide Tables. These values are shown in Table 5 and Fig.11.

Fig.8 shows that the model spring tide develops a hump on the rising tide at the same level as that on the estuary tide, becoming more pronounced as the wave advances upstream. Many curves from the gauges on the estuary were examined; in some the hump was very pronounced. Weather conditions, wind, barometric pressure, rainfall, seasons etc. have, however, a very marked effect and the tides detailed in Table 2 and Fig.8 were chosen because they occurred at springs and neaps, in a smooth series, showing settled weather conditions. It is considered that the model spring tide at Gourock, Bowling and Prince's Dock is sufficiently similar to the spring tide in the estuary to conclude that the model produces representative tidal conditions at springs.

Under normal conditions in the model, the variation in range of the neap tides, with distance from the estuary mouth is much less than that which occurs at springs.

It is of interest to note here that Allen and Matheson[14] give a table of the theoretical ratios of the rises in parallel and convergent model channels, for various values of $\frac{\sigma l}{c}$, where $\sigma = \frac{2\pi}{T}$, T = tidal period, l = length of inlet, h = depth of water, and $c = \sqrt{gh}$.

For the Clyde model, the value of $\frac{\sigma l}{c}$ is 0·52, and interpolation in the table gives the theoretical ratios, for parallel and convergent channels, as 1·16 and 1·08 respectively. At spring and neap tides, in the model, the ratios are 1·22 and 1·13 respectively (Table 4), and it appears that these ratios in the model, which has parallel sides for about 14 miles of the 23 miles of the estuary, correspond well with the theoretical ratios for parallel sided channels.

7.1. Investigation of the effect of increased river flow on the tide ranges:- Tide records were taken with no river water flowing into the model. Records were also taken with a spate in the Clyde only of ·00237 cusecs, corresponding to 15,300 cusecs full size, or 6·8 times the normal flow. The results are given in Table 3, Fig.9, and the tide curves shown in Figs. 12 and 13. The ratios of the ranges at spring tides when no river water is flowing, correspond closely with those recorded in the Admiralty Tide Tables. The ratios decrease with the increase of flow of river water for both springs and neaps.

7.2. Investigation of the effect of reclaiming the estuarine flats:- At L.W. springs, large areas of sandbanks are exposed on both sides of the dredged channel, mainly between Bowling and Greenock. Areas of shallow water were "reclaimed" in the model in four stages, I, II, III and IV, as detailed in Fig.1. The first stage was taken inside the Long Dyke, a low artificial barrier constructed in the early 19th century, which confines the river channel for a length of about 2 miles[5]. As these areas were raised above H.W. level, a corresponding decrease took place in the area of water in the model at H.W. The travel of the displacer was kept constant during trials I to IV, and the range of the spring tide at Gourock increased from 10·85 to 11·90 feet (Table 4). The travel of the displacer was reduced to give a tide range of 11·05 feet at Gourock -

a range nearer to the normal range at springs - and trial IVR was run. The tide curves for these five trials are shown in Figs. 14 to 18 inclusive.

The ratios of ranges, Table 4 and Fig.10, were compared, and it was seen that as the land was reclaimed, the range of spring and neap tides at first decreased with distance from the estuary mouth for condition I, and thereafter increased up to condition IV. The conclusion reached was that the reclamation of land IV would increase the spring and the neap range at Prince's Dock by 1% and 3% respectively.

In the estuary the increase of range of tide with distance from the mouth is 1·28 at Govan for neaps and 1·30 for springs (Fig.11). In the model tests, detailed in Figs. 9 and 10, the increase of range is greater at springs than at neaps, the range of the tide at Gourock having a direct effect on the ratio of ranges upstream. The model reproduces spring conditions better than neap conditions. The explanation may be that the depths at H.W. neaps are less than at H.W. springs - the Reynolds number (Section 4.7.) is less, and the model channel presents a greater frictional resistance to the advancing wave. Because the larger range at springs more readily overcomes the model friction the increase of range upstream is greater.

7.3. Tide range at the heads of the sea lochs:- Tide ranges were taken on the model at the heads of the sea lochs and the results are tabulated below.

Location	Range, feet		Range/Gourock Range	
	A	B	A	B
Gourock	10·9	10·2	1·00	1·00
Loch Long Head	11·1	10·5	1·02	1·03
Garelochhead	11·1	10·8	1·02	1·06
Loch Goil Head	11·0	--	1·01	--

These results may be compared with the value 1·05 given for each place, in the Admiralty Tide Tables.

Section VIII

8. Time lag of H.W. and L.W. after Gourock:- Table 6 and Figs. 19 and 20 show the time interval in hours between the instants of H.W. or L.W. at Gourock and those at Bowling and Prince's Dock. In constructing the table and the diagrams the information furnished in Figs. 8 and 12 to 18 (inclusive) was utilised.

Consideration of the spring tides, for normal conditions, shows that the times of L.W. correspond with those of the actual estuary but those of H.W. indicate some discrepancies at Glasgow. The lack of agreement occurs chiefly above Bowling. The effect of land reclamation is less noticeable on L.W. than on H.W. which at Bowling seems to be made earlier for all stages of reclamation. In all cases, Fig. 19, the H.W. and L.W. lines diverge, showing that the crest of the wave advances more quickly than the trough. Referring to the fact that high water in the model at Prince's Dock occurs slightly before H.W. at Gourock, a similar phenomenon has occasionally been observed on the actual river, when H.W. occurred at Glasgow before it occurred at Govan.

At neap tides, Fig.20, for normal conditions, the times of H.W. in the model and the estuary agree, but the times of L.W. differ. The spate makes both H.W. and L.W. earlier at Glasgow, and this is also generally the case with the schemes of reclamation examined.

In 1768, before a navigable channel was dredged in the estuary, the time interval elapsing between H.W. Gourock and H.W. Glasgow was 2 hours, as observed by Golbourne[6]. The deepening of the channel has advanced the time of H.W. at Glasgow about 1½ hours, by allowing the wave to travel up-stream more quickly. A comparison of the shape of the tide curve, condition IVR, with the normal tide curve shows that the time of H.W. is not brought forward in the same way in the model. The deepening of the estuary channel has a greater effect than the restriction of the model channel in the lower reaches.

Section IX

9. Comparison of times of fall and rise:- The tide curves as plotted show the level of the water at any instant after zero time, and thus give the shape of the advancing waves, considered as moving from right to left. The correct time of one tide should be about 12·43 hours, but in the records taken, both model and estuary, the sum of the times of fall and rise differ from this time, and in order to get a basis for the comparison of the data collected, the ratio of the time of fall to the time of rise (n) was calculated for each curve. The crests and troughs were marked on the tide curves and the horizontal distances apart gave the times of fall and rise. These times are given for springs and neaps in Table 7, and values of n are plotted in Fig.21. Each of the fall and rise times is the mean of 3 or 4 readings taken from observations at each station; one tide only is shown on the tide curve diagrams.

The normal value of n in the model for spring tides at Gourock is 0·86. The tide generating mechanism had been originally adjusted to make n equal to 0·83 (see Section 5). As seen in Section 6.2. and Fig.7, the mechanism corrects any discrepancy after 29 high tides, and any difference is small compared with the alteration in n as each wave advances upstream. The two spring tide curves taken in the estuary at Gourock on 15th January and 11th June, have values of n differing by 0·06 (see Table 7); and the period of the former tide at Gourock, Bowling and Glasgow is 12·34, 12·18 and 12·54 hours respectively. The period should be constant for the same wave as it advances. The discrepancy may be attributed to weather conditions.

9.1. Comparison of times of fall and rise, springs only:- A positive gradient of the graph shown in Fig. 21 indicates that n is increasing with distance from the estuary mouth, i.e. the front of the advancing wave is becoming steep, and the crest of the wave is moving faster than the trough.

The tide curves from the model, under normal conditions, show by comparison a large gradient in the graph, indicating an increase in the time of fall and a decrease in the time of rise. The crest of the wave is advancing faster than on the full scale, and the shallow water effect is intensified because of the greater retardation of the trough. The effect of friction is evidently greater in the model at L.W. or less at H.W. This agrees with the conclusion in 7.2.

It has been observed (Section 8) that since 1768 the duration of the rise at Glasgow has increased by 1½ hours while the level of H.W. has remained the same. Assuming that the tide at Gourock has not changed in character since 1768, the value of n at Glasgow in that year would be 1·62. The high value of n may be accounted for by the much greater friction in the undredged channel. The graph of n for the model under normal conditions (Fig.21), is steep, like the graph for the estuary in the year 1768, and a comparison would indicate that there is too much friction in the model at L.W.

The addition of river water in the model shortens the time of fall of the tide, the crest of the wave being retarded more than the trough.

Progressive reclamation of land in the model decreases the values of n at Glasgow until with condition IV R the shape of the wave is practically identical with the shape of the wave in the estuary. At H.W. the crest is retarded and the value of n is reduced because the long narrow channel presents more friction to the crest of the advancing wave. The time of fall decreases as the shallows are reclaimed, the velocity of the ebb current, therefore, increases and more silt will be moved on the ebb than on the flood, an indication that less dredging would be necessary to maintain the channel.

9.2. Comparison of times of fall and rise, neaps only:- The values of n for normal neap tides in the model, unlike those for the spring tides, decrease with distance from the mouth of the estuary (Fig.21). The same trend exists in the spate and land reclamation tests, and increasing the river flow does not appreciably alter the shape of the normal curve. A comparison of the graphs of the values of n for the neap and spring tides in the estuary shows that the value of n for neaps at Gourock is much greater than for springs, but that the gradients for both graphs are similar. The long narrow channel in the model (condition IV R) has possibly the shape required to reproduce the neap wave continually, the wave travelling forward with no change of profile[8].

Section X

10. Current velocities:- The velocity of the current was taken at Dumbarton and Clydebank on the model, by timing the passage of a drop of ink over a measured length. Observations were taken 3 or 4 times and the mean velocities are given in Table 8. The time of maximum velocity, which usually occurred about half tide, was estimated by eye. The following conclusions may be drawn from these observations:-

(a) The upstream and downstream velocities at spring tide diminish at Dumbarton and Clydebank as land is reclaimed. At neap tides, however, while the upward velocity diminishes at both places, the downward velocity at Clydebank increases slightly.

(b) The velocities for condition IV R are generally less than for condition IV, due to the smaller range at Gourock.

(c) Observations were made with no tide running (Table 9). Because of the narrower channel at Clydebank the velocity of the current there is greater than the velocity observed at Dumbarton.

(d) When the increase or decrease in the current velocities due to spate is found by subtracting the first from the second line of Table 8, figures are obtained which give a fair agreement with the spate velocities given in Table 9. The same trend is evident for average river water flow, except at Dumbarton for downstream velocities at springs.

(e) Near Dumbarton, in the model, the maximum observed velocities of the ebb and flood streams are (on the full scale) respectively 2·35 and 2·41 miles/hour, values which correspond well with the velocities of 2·5 and 3·0 miles/hour in the estuary. At Clydebank, in the model, maximum observed velocities, of both the ebb and the flood, are 2·26 miles/hour, the values in the estuary being respectively 1·73 and 2·15 miles/hour. It is apparent that the model tide ebbs more quickly than the tide in the upper reaches of the estuary.

Section XI

11. Comparison of profiles of the estuary and the model:- Surface profiles of the river were plotted at hourly intervals before and after high water springs at Gourock, by placing squared paper on the Gourock tide curve with an even hour at the crest of the curve, and marking the water level at each hour, along with the O.D. level and the zero time mark on the model curves (Section 6.1.), or the mid-day line on the estuary curves. The Bowling and Prince's Dock curves were now taken in turn, and use was made of the zero time mark to add two other points vertically above or below each first mark. From each of these three marks, a profile of the surface of the estuary and model was drawn as shown in Figs. 22 to 26. These profiles indicate the nature of the oscillation of the water level in the estuary.

A comparison of the profiles of the estuary (Fig.22) with the model, under normal conditions (Fig.24), indicates that they are similar to the extent that at 6 hours before and at the instant of H.W. Gourock, the level of the water at Glasgow is respectively lower and higher than at Gourock. In the model, by comparison, the level of the water does not fall quickly enough in the upper reaches. Increasing the flow of river water in the model raises the level very considerably at L.W. in the upper reaches, as shown in Figs. 23, 24 and 25. The profiles of the surface when land is reclaimed are not dissimilar to the profiles for the model under normal conditions.

Section XII

12. Natural period of the model estuary:- During early observations of the tide at Prince's Dock it was noted that the water level continued to rise and fall for several minutes after stopping the motion of the displacer. The range of the oscillations was, to scale, about 2 feet, a considerable proportion of the range of the tide. The range of the tide previous to stopping the motion of the displacer had a direct effect on the range of the transient oscillations. The curve of these oscillations in the model, normal conditions, was plotted as shown in Fig.27, for eight positions. In order to have conditions similar for each position, all the readings were taken at spring tides and the displacer was stopped each time at its mean position, the water finally settling to its mean level. An estimate was made of the period at each station and the results were plotted, as shown in Fig.28, on a base of distance downstream from Glasgow. The mean value of the period was 29·2 seconds or 4·0 hours full scale, a value which is approximately $\frac{1}{3}$ of the period of the lunar semi-diurnal tide.

12.1. Amplitude of natural oscillations:- The curves of the transient oscillations were used to determine the range of the oscillations at zero time, where zero time was determined by the point where the curve initially intersected the mean level line or the time axis, and the magnitude of the range (A) was taken as the length of the vertical line intercepted by the enveloping curves (Fig.27). The values of the ranges were plotted for each station on the model estuary on a base of distance from Glasgow, and as shown in Fig.29, the points lie on a line intersecting the axis at a point which is about 22 feet from the weir at Glasgow. The free oscillations of water in the model estuary are thus analogous to the vibrations in a closed organ pipe, and the wavelength will be four times the length of that part of the river above the node, the point where there ceases to be any appreciable oscillation. The wavelength of the natural oscillation in the model is therefore 88 feet.

The theoretical period of oscillation in a long channel of uniform section, closed at both ends, length $2\,l$ and depth h is [7]

$$T = \frac{2\,(2l)}{gh} \text{ seconds} \qquad \ldots \quad \ldots \quad \ldots \qquad (1).$$

In order to calculate l, values are required for T the period in seconds and h the depth in feet. The value of 29·2 seconds (Section 12) was used for T, and a depth was adopted equivalent to 31 feet full scale (26 feet low water depth, plus 5 feet). Substitution in (1) gives

$$l = 21{\cdot}5 \text{ feet} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

a value which agrees very closely with the value of 22 feet determined from Fig. 29. This calculation establishes the validity of the equation

$$V = \sqrt{gh} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

which can be deduced from (1), where V ft/sec. is the velocity of a wave advancing in shallow water of depth h.

When the areas of land were reclaimed as in condition <u>IV</u> R, the mouth of the restricted channel was 20·7 feet from Glasgow, a distance which nearly coincided with the length to the node (Fig.29). It is of interest to note that the natural period of oscillation for the restricted channel, taken at Prince's Dock, was 30·5 seconds (4·12 hours full scale), and that the shape of the spring tide curve had a closer resemblance to the estuary curve than the normal model curve. In the estuary, the alteration in character of the tide at Glasgow has been caused in the past two centuries by the dredging of the channel. The conclusion is that the reclamation of the shallow areas in the model did not have a great effect on the character of the tide because, both before and after the reclamation, the deeply dredged channel carried forward the tidal wave.

12.2. Velocity of tidal wave:- Assuming that the mean depth of the channel carrying the tidal wave in the estuary is 31 feet, the velocity of the main wave, from equation (3), is

$$V = 31{\cdot}6 \text{ ft/sec.} = 21{\cdot}52 \text{ miles/hour} \quad \ldots \quad \ldots \qquad (4).$$

This is also the velocity of the third harmonic component of the main wave, since a crest of the third harmonic will travel at the same velocity as a crest of the main wave. The velocity of the wave of natural oscillation in the model, using model units transposed to full size, is $V_1 = \frac{4l}{T} = \frac{88}{4}$ = 22 miles/hour, a value which corresponds with (4).

The wavelength of the main wave at the mouth of the estuary is, from equation (4),

$$21{\cdot}52 \times 12{\cdot}4 = 267 \text{ miles} \quad \ldots \quad \ldots \quad \ldots \qquad (5).$$

12.3. Increase of tidal range:- Doodson and Warburg[7] have suggested that the tendency towards resonance and the changes in the area of cross section of the channels cause the increase in the range of tides in estuaries. The profile of the surface of the water in a gulf depends on the length and cross section of the gulf, and the effect of resonance in a gulf of length less than $\frac{1}{4}$ of the wavelength is indicated in Fig.30, where it is seen that the tidal range is increased at the upstream end of the gulf. Maintenance of this increased tide depends on the equality of the periods of the oceanic tide and the free oscillation proper to the gulf. The free period of the Clyde estuary is $\frac{1}{3}$ of the period of the oceanic tide, and exaggeration of the third harmonic component of the applied tide would consequently be expected.

From the Admiralty Tide Tables, the crest of the incoming tidal wave travels the distance of 34 miles from Lamlash to Gourock in 20 minutes. The velocity of the wave is thus 102 miles per hour and its wavelength 1,270 miles, a length which exceeds four times the length of the Clyde estuary. An increase of range at Glasgow would thus be expected and is confirmed by observation.

12.4. Wavelength of the tidal wave:- From Table 6, the crest of the spring tide in the estuary on 15th January travelled the distance of 21·5 miles from Gourock to Glasgow in 0·66 of an hour. The velocity of the crest is thus 31·6 miles/hour and the wavelength, crest to crest, would be 392 miles ... ... (6). Similarly, the velocity of the trough is 19 miles/hour and the wavelength, trough to trough, would be 236 miles (7). The forward velocity of the wave is a function of the depth of water (Equation (3)), and the wave alters its shape because the crest travels faster than the trough. Profiles of the wave may be examined in Fig.8. The values (6) and (7) are of the same order as (5), the theoretical wavelength.

Examination of the data in Table 6 for the neap tide on 20th May indicates that the velocity of the trough and the crest is 41·4 miles/hour; the wavelength would then be 513 miles. The neap tide curves (Fig.8) are very rounded; at H.W. and L.W. the level remains nearly constant for a long period, and H.W. at Glasgow tends to occur simultaneously with H.W. at Gourock. The theoretical effect of resonance, indicated in Fig.30, makes H.W. in the gulf occur at the same time as in the ocean, and it would appear that this effect tends to occur in the estuary more at neap than at spring tides.

12.5 Standing oscillation:- A standing oscillation exists in a gulf when slack water occurs at both H.W. and L.W.; and the greatest rate of flow takes place across the nodal section at half tide[9], when the surface of the water is level throughout. Conditions in the Clyde estuary are not entirely suitable for this to happen, as the times of slack water, because of the downstream river current, do not occur at high and low water. The greatest known velocity, however, occurs in the channel at mile 15½ which is about 6 miles upstream from the nodal section indicated by the model. The profiles of the water surface shown in Fig.22 indicate that an oscillation similar to a standing oscillation exists in the Clyde estuary.

12.6. Initial wave:- The displacer was started from its mean position, and the initial disturbance advancing upstream was timed as it passed various points on the model estuary. The times were plotted on a base of distance upstream as shown in Fig.31, and the mean gradient of the graph gave the velocity of the disturbance as 2 ft/sec. in the model or 14·6 miles/hour full scale ... ... (8). Assuming that $v = \sqrt{gh}$ where v is the velocity of the disturbance and h is the mean depth of the channel, the value of h would be 0·125 feet, a value which does not agree with 0·27 feet, the actual mean depth of the model channel. A sketch of the shape of the actual disturbance when it reached Prince's Dock is given in Fig.32, which shows that the initial disturbance excites the oscillations natural to the model but that these are rapidly overcome by the major forced oscillation. The velocity (8) is much less than the velocity of the tidal wave calculated in equation (4), and the conclusion reached is that the initial wave may be the bore on the model, as its profile has a peak sharper than that of the normal wave.

12.7. Bore:- The bore velocity is[10]

$$V_b = \left[\sqrt{\frac{2g\ (h + k)^2}{2h + k}} - v_c\right] , \quad \ldots \quad \ldots \qquad (9)$$

where h is the depth of the channel, k is the rise of the water level, and v_c is the current velocity. For a depth, rise, and velocity in the estuary of 31 feet, 6 feet and 0·2 ft./sec. respectively,

$$V_b = 35{\cdot}8 \text{ ft/sec. or } 24{\cdot}4 \text{ miles/hour} \quad \ldots \quad \ldots \qquad (10),$$

a value which exceeds the velocity of the tidal wave calculated in equation (4). In the normal model, the velocity of advance, between Gourock and Bowling, of the crest of the spring tide wave detailed in Table 6, is

$$21{\cdot}6 \text{ miles/hour} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (11).$$

The times given in the table for Prince's Dock cannot be used here, since in the model, H.W. occurs there before H.W. Gourock. The velocity of advance of the initial wave (8) is less than the velocity of the forced wave (11) and the bore (10), and the conclusion drawn is that the frictional resistance in the model is proportionally greater than it is in the estuary itself.

12.8. Alteration of the period of the tide:- The period of the forced tidal wave was altered to 29·2 seconds, equivalent to 4 hours full scale, and the time when the displacer was at the lower limit of its travel was noted, with the time when the wave reached Prince's Dock, 23 feet distant. Six trials were run, and the mean time gave the mean forward velocity of the crest of the wave to be 2·8 ft/sec. or 20·7 miles/hour full scale, a value which corresponds with (4), the theoretical velocity of advance of a wave in shallow water.

The amplitude of the forced wave with a period near to the natural period of the model increased about 2½ times as the wave advanced upstream. This fact will be discussed in the next section.

Section XIII

13. Frictional damping of tides in the model:- The long channel of water which has a natural period of T, is subjected at its open end to an oscillation of period T_1. Periodic frictional forces will act against the motion of the water, and it will be assumed that these resisting forces are proportional to the first power of the velocity. The following symbols will be used:-

p = angular velocity of oscillations natural to the model,

m = " " " forced oscillations,

r = coefficient of damping,

T = natural period of the system, in hours, full scale,

T_1 = period of the disturbing force, in hours, full scale,

$q = \frac{2r}{p}$, a quantity depending on the magnitude of the damping forces,

C = amplitude of forced oscillations at Prince's Dock,

and d = amplitude of forced oscillations at Gourock.

The magnification factor is [11].

$$\frac{C}{d} = \frac{1}{\sqrt{\left(1 - \frac{T^2}{T_1{}^2}\right)^2 + \frac{T^2\,q^2}{T_1{}^2}}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12).$$

The periods T_1 and T are equal for resonant conditions, and the magnification factor becomes

$$\frac{C}{d} = \frac{1}{q}. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13).$$

The mechanism was moved by hand to produce a tide having a range equivalent to 6·1 feet at Gourock, and a period equal to the natural period of the model (Section 12). Nine consecutive tides were observed, and the mean range at Prince's Dock may be taken as equivalent to 13·75 feet. The "static" range at Prince's Dock was 6·1 feet, therefore, in equation (13),

$$q = \frac{d}{C} = \frac{6\cdot1}{13\cdot75} = 0\cdot442 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14).$$

Substitution of this value for q, and of the values 4 and 12·4 hours for T and T_1 respectively, in equation (12), gives

$$\frac{C}{d} = 1{\cdot}107 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

a value approximately equal to the figure 1·13 given in Table 4 for the ratio between the neap range at Prince's Dock and Gourock in the model, under normal conditions.

13.1. Curve of damped oscillations:- The factor q may be found using the curve of the damped oscillations observed on the model at the River Cart, Fig.27. Assuming that the amplitude diminishes after each cycle in the ratio e^{-rT}, and taking the mean of four ratios of successive amplitudes from the curve to be 0·59, then $0{\cdot}59 = e^{-rT}$, and for T = 29·2 seconds, equivalent to 4 hours full scale,

$$r = 0{\cdot}131 \text{ radians/hour} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

∴ by definition,

$$q = \frac{2\,r}{p} = 0{\cdot}167 \quad \ldots \quad \ldots \quad \ldots \quad (17).$$

Substitution of this value for q in (12) makes $\frac{C}{d} = 1{\cdot}105$, which is again near the observed value. Frictional or damping forces become important only when $\frac{T}{T_1}$ approaches unity.

The ratio of successive amplitudes may be calculated for the curve of damped oscillations, using the value for q obtained by measurement of the resonant amplitude. Equation (14) is $q = 0{\cdot}442$, and by definition,

$$r = \frac{p\,q}{2} = 0{\cdot}347 \text{ radians/hour} \quad \ldots \quad \ldots \quad (18).$$

therefore $e^{-nT} = 0{\cdot}25$.

The value of e^{-nT} from the actual curve is 0·59; the difference is due to the low value of the magnification factor at Prince's Dock, where, in equation (14), $\frac{C}{d} = 2{\cdot}25$. The largest observed value of $\frac{C}{d}$ was 2·65. The theoretical magnification factor at resonance may be calculated using $r = 0{\cdot}131$ radians/hour obtained from the damped oscillation curve, when $\frac{C}{d} = \frac{1}{q} = \frac{p}{2r} = 5{\cdot}98$.

The conclusion is that the rate of dissipation of energy in the channel is much greater with the forced oscillation at the resonant period than with the simple damped oscillation, the inference being that the friction losses are proportional, not to the first power, but to the second power of the velocity of advance of the wave.

Section XIV

14. Harmonic Analysis:- From Section 12, and Figs. 27 and 28, it was discovered that the natural period of the estuary was 4 hours, about $\frac{1}{3}$ of the period of the semi-diurnal tide. With this in mind, a harmonic analysis of the tide in the estuary was commenced to search for an explanation of the hump developed about half-tide level on the rising tide in the upper reaches of the river. It might be anticipated that the large areas of shallow sand banks, covered by the rising water about half-tide level, retard the rate of rise, but as there is no equivalent variation in the rate of fall, another explanation must be sought. Doodson and Warburg[7] have discussed the phenomenon of double high water on the tide at Southampton and both Lamb[12] and McCowan[8] draw attention to the double high water in some estuaries. McCowan points out that the explanation given by Airy[13] was incorrect. Airy had made an approximation which led to the result that the wave would subdivide as it advanced up the river, so that at stations far enough up there would be two times of H.W. in each period, or even three or more at sufficiently distant stations. It was considered that the resonant properties of the Clyde estuary might be causing the hump by magnifying the third harmonic component of the wave, and a harmonic analysis of the tide was commenced.

14.1. A 24-Ordinate Analysis of tides in the Clyde:- A strip of tracing paper about 17 feet long was marked off in lengths of 12 inches, each length representing, to scale, one mean solar day. Tides for 16 consecutive days were traced from the gauge records at Gourock and Govan Wharf, a point situated 2·3 miles from Glasgow. The strip was then partitioned into 15 lengths, each representing one lunar day, and each lunar day was divided into 24 lunar hours by ordinates marked 0, 1, 2, 3, ... 23. A table was made of the lengths of these ordinates above the datum line, and the mean values of all the 15 sets of ordinates marked respectively 0, 1, 2, 3, ... 23, were then plotted, giving finally two mean curves, one for Gourock and the other for Govan Wharf (Fig.33). A harmonic analysis of the two mean curves thus obtained was carried out, using the 24 ordinates, and the amplitudes of the harmonic components are given in Table 10. If the series

$$U = a_0 + a_1 \sin B + a_2 \sin 2B + a_3 \sin 3B + \; .....$$
$$+ b_1 \cos B + b_2 \cos 2B + b_3 \cos 3B + \; .....$$

represents the shape of the wave profile, where the constants a_0, a_1, a_2, etc., and b_1, b_2, b_3, etc., are determined by the analysis, then the resultant amplitudes of the harmonic components are c_1, c_2, c_3, etc., where

$$c_1^2 = a_1^2 + b_1^2 ,$$
$$c_2^2 = a_2^2 + b_2^2 ,$$
etc.

Neglecting the presence of the diurnal tide, consideration need only be given to the even harmonics of the tide analysed by the 24-ordinate method. These harmonics are designated in Table 10 by c_2, c_4, c_6, c_8, etc., and have been called d_1, d_2, d_3, d_4, etc., for later comparison with the amplitudes of the harmonics of curves of period 12 lunar hours, analysed by the 12-ordinate system.

The values of the amplitudes of the first, second, third and fourth harmonics are plotted on a base of distance upstream, in Fig.34. It is evident that the slope of the graph of the third harmonic is greater than those of the second and fourth harmonics, and the reason suggested is that the period of the third harmonic coincides with the natural period of the estuary.

On further examination of the mean curves plotted in Fig. 33, it was noted that the hump was not in great evidence on the rising tide at Govan. A decided hump was observed on the spring tide of the 15th January, and an analysis of this tide was forthwith commenced.

14.2. An analysis of one tide from the estuary:- Runge's 12-ordinate method was used to analyse the spring tide on January 15th at Gourock, Bowling, and Glasgow. The results are plotted in Fig.35 where, in comparison with the results for the mean curve shown in Fig.34, it is seen that the presence of the hump has made a definite increase in the gradient of the curve representing the third harmonic.

14.3. Analyses of model tide curves:- The curves of spring tides in the model under normal conditions, with land reclaimed as in condition IVR, (Section 7.2.), and with no river water flowing, were analysed as above and the results plotted in Figs. 36, 37 and 38.

14.4. Harmonic analysis; conclusions:- On the estuary curves the amplitudes of all the harmonics increase with distance from the mouth. In comparison with the first harmonic, the increase of the third harmonic is more evident in the upper reaches. An examination of Figs. 34 and 35 shows that the amplitude of the third harmonic of the tide on 15th January at Gourock was already large (0·3 feet) compared with that of the average curve (0·1 feet). It would appear that the large amplitude at Gourock leads to a greater increase in amplitude with distance upstream. As the tidal wave advances from the ocean, it is forced by the shallow water to change shape (Section 9). The ranges of the higher harmonics therefore alter, and the development of the pronounced hump on the Glasgow tide can be traced to the magnification of the third harmonic component in the tide at Gourock, attributable to the fact that the natural period of the estuary is nearly one-third of the period of the oceanic tide.

On the model curves analysed, the amplitudes of all the harmonics, except the second, increased with distance upstream. The increase of amplitude of the first harmonic was similar to the increase on the estuary, and the most pronounced increase of the third harmonic was evident on the tide curve taken when the area of land IVR was reclaimed, and when the end of the narrow channel coincided with the nodal section under normal conditions (Section 12.1.).

The normal channel in the model allows the first harmonic to increase exactly as in nature; the increase of the third and fourth harmonics is insufficient for similarity, and the second harmonic is forced to decrease. It thus seems evident that due to the low Reynolds number, the model presents more friction to the higher harmonics of small amplitude than to the first harmonic of large amplitudes.

14.5. The mean level of the surface at points on the estuary:- Evaluation of the constant a_0, in the harmonic analysis of each tidal curve in the estuary, supplied the mean level of the surface at each point, and these levels are shown plotted in Fig.39 for five positions in the estuary, in conjunction with levels given on the Admiralty charts, and values obtained at spring and neap tides (Tables 11 and 12) by direct measurement of the heights of high and low water. The mean level evidently rises with distance upstream.

A comparison was attempted, with no tide in the model, by measuring the rises in level of the still water level at Prince's Dock caused by the normal river flow and by the spate; the still water level rose by 0·05 feet and 0·32 feet respectively (on the full scale). According to rainfall information supplied by the Meteorological Office, (a) the run-off to be expected during the 15-day period in May and June would approximate to the average flow, and (b) the rainfall on the 14th and 15th January was negligible. Examination of Fig.39 indicates that the mean water level at Glasgow was higher (0·85 feet above the Gourock level) on the 15th January than during the period in May and June (0·37 feet), the opposite to what would be expected if the river flow has any effect on the level. Before a definite conclusion could be reached, a much more exhaustive analysis of the estuary and model tides would be required.

Section XV

15. Conclusions:- Quantitative reproduction of tidal phenomena in a model built to such a small scale is difficult to achieve, and the possible experiments on the model are of necessity restricted to qualitative investigations. The bed of the channel was moulded in cement-sand mortar, and no attempt was made either to alter the roughness of the surface, or to insert wires, baffles, plates, etc. in order to change the shape of the tidal wave, because it was considered that the results obtained were sufficiently exact.

Conclusions arrived at from experimental work on the model may be summarised as follows:-

(a) Spring tides reproduced in the model corresponded, in the upper reaches of the river, with spring tides in the estuary. Magnification of the range of neap tides was insufficient in the model.

(b) Increase of flow of river water decreased the magnification of range of tide with distance upstream.

(c) Velocities of currents in the estuary were reproduced to scale in the model.

(d) Reclamation of shallows in the estuary altered the shape of the wave but had little effect on the range of tide at Glasgow. The velocity of the ebb current was increased, indicating that more silt would be transported downstream, and that less dredging would be necessary.

(e) The mean level of the surface of the water rises with distance upstream from the mouth of the estuary.

(f) The low Reynolds number causes the frictional resistance in the model to be relatively greater than the resistance in the estuary; the effect is more evident at L.W. than at H.W.

(g) Frictional losses are not proportional to the first power of the velocity of advance of the tidal wave.

(h) Dredging of the estuary in the past has altered the character of the tide at Glasgow by allowing the tidal wave to advance freely. Restriction of the broader parts of the waterway by any future reclamation of the shallows will have a minor effect on the tide at Glasgow, because the deep narrow channel will still allow the tidal wave to advance as before.

(i) A standing oscillation occurs in the model, similar to that in the estuary, and this leads to the conclusion that if the model has a natural period of oscillation of 4 hours to scale, the natural period of the estuary will also be 4 hours. This period corresponds with $\frac{1}{3}$ of the period of the tidal wave, resulting in the magnification, because of resonance, of the third harmonic component of the wave, which gives an explanation of the hump on the rising tide in the upper reaches of the estuary.

Acknowledgments

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Plates

Tables

Figures

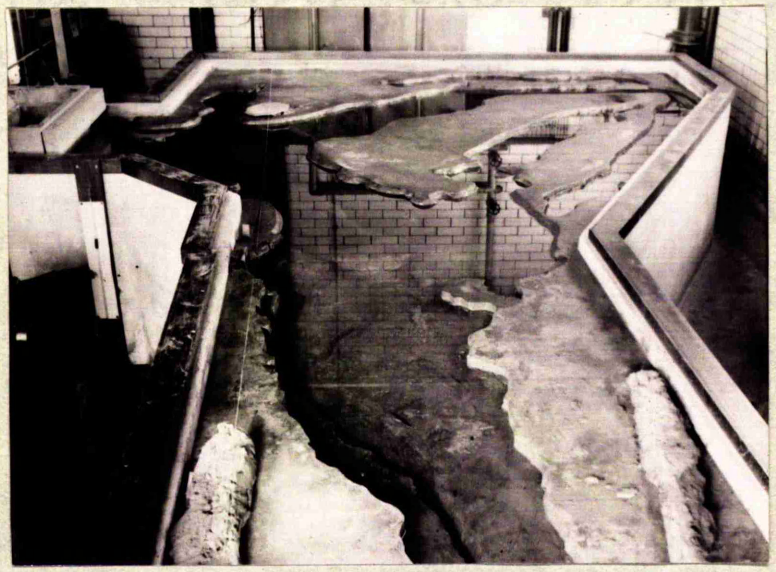


Plate 1

Model, normal conditions, high water.

Plate 2

Model, normal conditions, low water.

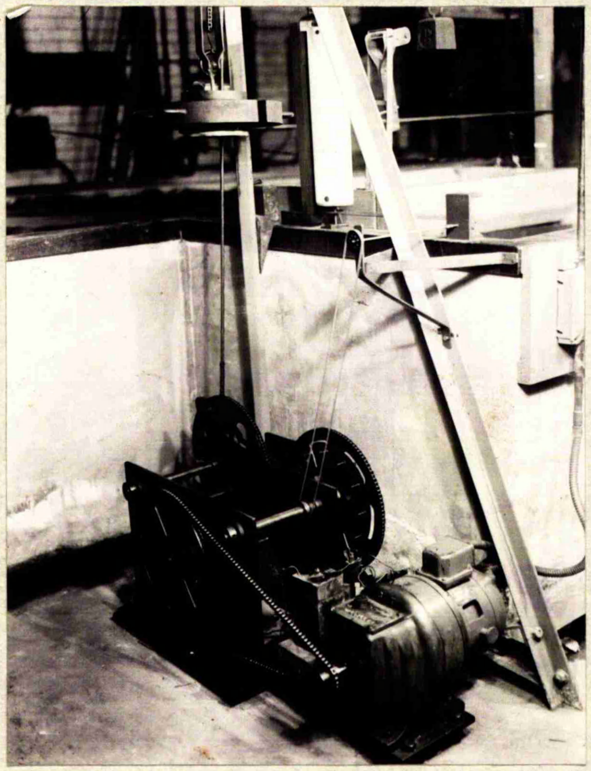


Plate 3

Tide Generating Mechanism.

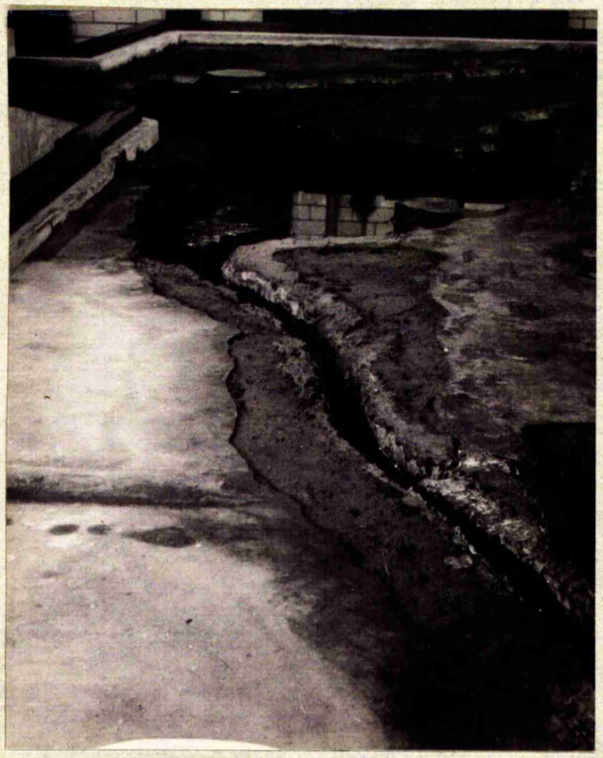


Plate 4

Model, land reclaimed, condition IV. Low water.

Plate 5

Model, land reclaimed, condition IV. Low water

Plate 6

Model, land reclaimed, condition IV. Part full of water.

1945

Phases of the Moon — Date of Full F	Last Quarter	Change C	First Quarter	Date of Full and Change + 1D 13h 15m (3 times 12h 25m): F + 1D 13h 15m = Time of Spring Tide	C + 1D 13h 15m = Time of Spring Tide	Tide Gauge at Gourock: H.W.	L.W.	Spring Range	Date of Last and First 1/4 + 1D 13h 15m (3 times 12h 25m): Last 1/4 + 1D 13h 15m = Time of Neap Tide	First 1/4 + 1D 13h 15m = Time of Neap Tide	Tide Gauge at Gourock: H.W.	L.W.	Neap Range
	D H M			D H		FT INS	FT INS	FT INS	D H		FT INS	FT INS	FT INS
	Jan 6 12 47			Jan 1 13		16 7	5 10	10 9	Jan 8 02		13 4	8 0	5 4
		Jan 14 05 06	Jan 20 23 48		Jan 15 18	15 7	5 4	10 3		Jan 22 13	16 2	7 9	8 5
Jan 28 06 41	Feb 5 09 55			Jan 29 20		17 7	8 8	8 11	Feb 6 23		16 0	9 8	6 4
		Feb 12 17 33	Feb 19 08 38		Feb 14 07	17 11	5 10	12 1		Feb 20 22	15 5	7 4	8 1
Feb 27 00 07	Mar 7 04 30			Feb 28 13		17 3	7 4	9 11	Mar 8 18		13 8	8 6	5 2
		Mar 14 03 51	Mar 20 19 11		Mar 15 17	16 8	5 2	11 6		Mar 22 08	14 8	8 10	5 10
Mar 28 17 44	Apr 5 19 18			Mar 30 07		17 7	8 1	9 6	Apr 7 09		13 7	7 6	6 1
		Apr 12 12 29	Apr 19 07 46		Apr 14 02	18 9	6 0	12 9		Apr 20 21	14 7	9 2	5 5
Apr 27 10 33	May 5 06 02			Apr 29 00		15 5	5 7	9 10	May 6 19		14 6	9 0	5 6
		May 11 20 21	May 18 22 12		May 13 10	18 4	6 10	11 6		May 20 11	15 6	9 10	5 8
May 27 01 49	Jun 3 13 15			May 28 15		16 9	7 4	9 5	Jun 5 03		15 9	7 3	8 6
		Jun 10 04 26	Jun 17 14 05		Jun 11 18	17 7	7 0	10 7		Jun 19 03	15 2	7 9	7 5
Jun 25 15 08	Jul 2 18 13			Jun 27 04		15 10	6 9	9 1	Jul 4 07		15 7	7 1	8 6
		Jul 9 13 35	Jul 17 07 01		Jul 11 03	15 11	6 4	9 7		Jul 18 20	16 3	9 9	6 6
Jul 25 02 25	Jul 31 22 30			Jul 26 16		16 0	6 6	9 6	Aug 2 12		14 11	6 7	8 4
		Aug 8 00 32	Aug 16 00 26		Aug 9 14	16 11	6 10	10 1		Aug 17 14	14 6	9 1	5 5
Aug 23 12 03	Aug 30 03 44			Aug 25 01		17 5	5 10	11 7	Aug 31 17		15 5	8 1	7 4
		Sep 6 13 43	Sep 14 17 38		Sep 8 03	15 3	5 10	9 5		Sep 16 07	15 0	10 5	4 7
Sep 21 20 46	Sep 28 11 24			Sep 23 10		16 10	4 0	12 10	Sep 30 01		13 10	8 5	5 5
		Oct 6 05 22	Oct 14 09 38		Oct 7 19	17 4	6 11	10 5		Oct 15 23	14 1	8 11	5 2
Oct 21 05 32	Oct 27 22 30			Oct 22 19		18 9	6 9	12 0	Oct 29 12		15 9	10 3	5 6
		Nov 4 23 11	Nov 12 23 34		Nov 6 12	16 6	6 10	9 8		Nov 14 13	15 0	7 6	7 6
Nov 19 15 13	Nov 26 13 28			Nov 20 18		17 3	5 10	11 5	Nov 28 03		14 10	9 6	5 4
		Dec 4 18 06	Dec 12 11 05		Dec 6 07	16 6	6 10	9 8		Dec 14 00	14 10	6 6	8 4
Dec 19 02 17	Dec 26 08 00			Dec 20 16		17 3	6 9	10 6	Dec 27 21		15 1	8 7	6 6
					Mean →	16 ft 11 ins	6 ft 5 ins	10 ft 6 ins		Mean →	14 ft 11 ins	8 ft 5 ins	6 ft 6 ins

*Mean W.L. Springs 11 ft 8 ins   *Mean W.L. Neaps 11 ft 8 ins

*Note. O. Datum = 11 ft $8\frac{1}{4}$ ins on gauge

TABLE 1

RIVER	SPRINGS 15 JAN 1945		SPRINGS 11 JUN 1945		NEAPS 20 MAY 1945	
	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO
GOUROCK	10·50	1·00	10·50	1·00	6·62	1·00
BOWLING	12·17	1·16	11·95	1·14	7·62	1·15
GOVAN WH.	13·65	1·30	13·50	1·29	8·45	1·28
GLASGOW	13·65	1·30	13·60	1·30	8·57	1·29

TABLE 2

MODEL		No RIVER WATER		NORMAL		SPATE	
		RANGE	RATIO	RANGE	RATIO	RANGE	RATIO
SPRINGS	GOUROCK	10·6	1·00	10·85	1·00	10·75	1·00
	BOWLING	12·05	1·14	12·25	1·13	12·05	1·12
	PR. DOCK	13·2	1·25	13·20	1·22	12·4	1·16
NEAPS	GOUROCK	5·85	1·00	6·0	1·00	6·0	1·00
	BOWLING	6·25	1·07	6·45	1·07	6·5	1·08
	PR. DOCK	6·75	1·15	6·80	1·13	6·55	1·09

TABLE 3

MODEL		NORMAL		I		II		III		IV		IVR	
		RANGE	RATIO	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO
SPRINGS	GOUROCK	10·85	1·00	11·10	1·00	11·20	1·00	11·40	1·00	11·90	1·00	11·05	1·00
	BOWLING	12·25	1·13	12·60	1·14	12·55	1·12	12·85	1·13	13·60	1·14	12·95	1·17
	PR. DOCK	13·20	1·22	13·10	1·18	13·35	1·19	13·50	1·19	14·60	1·23	13·6	1·23
NEAPS	GOUROCK	6·0	1·00	6·30	1·00	5·95	1·00	6·10	1·00	6·65	1·00	5·85	1·00
	BOWLING	6·45	1·07	6·60	1·05	6·65	1·10	6·75	1·11	7·40	1·11	6·50	1·11
	PR. DOCK	6·80	1·13	6·90	1·10	6·90	1·16	7·00	1·15	7·90	1·19	6·80	1·16

TABLE 4

	SPRINGS ADMY TABLES		SPRINGS MODEL NORMAL		NEAPS ADMY TABLES		NEAPS MODEL NORMAL	
	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO	RANGE	RATIO
GOUROCK	10·20	1·00	10·85	1·00	6·0	1·00	6·0	1·00
BOWLING	11·52	1·13	12·25	1·13	6·85	1·13	6·45	1·07
PR. DOCK	12·85	1·26	13·20	1·22	7·55	1·26	6·80	1·13

TABLE 5

TABLES TO SHOW RANGE IN FEET, AND RATIO OF RANGE TO THE GOUROCK RANGE.

		Time at Bowling after (+) Gourock of		Time at Pr. Dock after (+) Gourock of	
		H. W.	L. W.	H. W.	L. W.
Estuary	Admy Tables	+·4 hrs	+·92 hrs	+·68 hrs	+1·13 hrs
	15 Jan. 1945	+·25	+·66	+·66	+1·12
	11 Jan. 1945	+·25	+·68	+·45	+1·24
	No River Water	+·10	+·73	+·07	+1·18
	Normal	+·58	+1·04	−·05	+1·31
Springs	Spate	−·35	+·60	−·29	+1·02
	I	+·19	+·91	+·02	+1·22
	II	−·10	+·72	−·25	+1·10
	III	+·31	+·65	+·16	+1·17
	IV	+·19	+·70	−·06	+·91
	IV R	+·05	+·74	+·07	+·78
Estuary	Admy Tables	+·4	+·92	+·68	+1·13
	20 May 1945	+·37	+·60	+·52	+·52
	No River Water	+·38	+·67	+·57	+·48
	Normal	+·32	+·13	+·65	−·03
Neaps	Spate	+·04	+·26	+·24	+·48
	I	+·60	+·70	+·85	+·80
	II	+·31	+·27	+·08	+·32
	III	+·39	+·62	+·77	+·52
	IV	+·23	−·17	+·46	−·01
	IV R	+·42	+·03	+·43	+·21

TABLE 6

		Gourock			Bowling			Prince's Dock		
		Hrs. Fall	Hrs. Rise	Fall/Rise = m	Hrs. Fall	Hrs. Rise	Fall/Rise = m	Hrs. Fall	Hrs. Rise	Fall/Rise = m
Estuary	Admy Tables	5·63	6·77	·83	6·15	6·25	·98	6·25	6·15	1·02
	15 Jan 1945	5·29	7·05	·75	5·58	6·60	·85	6·07	6·47	·94
	11 Jun 1945	5·57	6·90	·81	5·93	6·16	·96	6·38	6·06	1·05
	No River Water	5·60	6·70	·84	6·55	5·77	·97	7·20	5·12	1·41
	Normal	5·68	6·64	·86	6·43	5·87	1·06	7·02	5·33	1·32
Springs	Spate	5·50	6·83	·81	6·55	5·67	1·16	7·03	5·43	1·30
	I	5·78	6·59	·88	6·25	6·04	1·04	6·91	5·43	1·27
	II	5·77	6·60	·87	6·28	6·00	1·05	6·83	5·45	1·25
	III	5·65	6·63	·85	6·26	6·04	1·04	6·87	5·60	1·23
	IV	5·55	6·63	·84	6·12	6·12	1·00	6·53	5·77	1·13
	IV R	5·72	6·54	·87	6·03	6·17	·98	6·61	5·74	1·13
	1768	5·43	6·97	·78				7·72	4·76	1·62
Estuary	Admy Tables	5·63	6·77	·83	6·15	6·25	·98	6·25	6·15	1·02
	20 May 1945	6·96	6·19	1·12	7·51	5·90	1·27	7·85	5·74	1·37
	No River Water	5·40	7·25	·76	5·55	7·05	·79	5·00	7·35	·68
	Normal	5·69	7·09	·80	5·36	7·35	·73	5·09	7·39	·68
	Spate	5·88	6·87	·86	5·70	6·96	·82	5·50	7·25	·76
Neaps	I	5·90	6·85	·86	5·80	6·85	·85	5·20	7·15	·73
	II	5·30	7·07	·75	5·50	7·06	·78	5·90	6·87	·86
	III	5·60	7·00	·80	5·65	6·85	·83	5·70	7·05	·81
	IV	5·65	6·97	·81	5·50	7·10	·77	5·35	7·27	·74
	IV R	5·45	7·00	·78	5·32	7·31	·73	5·25	7·36	·71

TABLE 7

	MODEL TIDES IN OPERATION							
	FT/SEC DOWNSTREAM CURRENT				FT/SEC UPSTREAM CURRENT			
	DUMBARTON		CLYDE BANK		DUMBARTON		CLYDE BANK	
NO RIVER WATER	·32 (·40)	·24 (·28)	·25 (·29)	·21 (·24)	·33 (·37)	·17 (·18)	·31 (·31)	·13 (·13)
SPATE	·35 (·40)	·34 (·35)	·31 (·40)	·33 (·35)	·24 (·28)	·10 (·11)	·19 (·20)	0 (0)
DIFFERENCE	·03 (0)	·10 (·07)	·06 (·11)	·12 (·11)	−·09 (−·09)	−·07 (−·07)	−·12 (−·10)	−·13 (−·13)
NORMAL	·26 (·32)	·26 (·28)	·29 (·31)	·24 (·24)	·33 (·33)	·15 (·16)	·30 (·31)	·14 (·17)
I	·31 (·33)	·26 (·30)	·27 (·30)	·22 (·25)	·26 (·27)	·13 (·16)	·31 (·34)	·12 (·16)
II	·31 (·32)	·24 (·25)	·30 (·31)	·19 (·24)	·24 (·33)	·12 (·14)	·31 (·33)	·09 (·09)
III	·30 (·36)	·22 (·24)	·27 (·29)	·23 (·25)	·29 (·36)	·13 (·15)	·29 (·33)	·11 (·13)
IV	·27 (·29)	·21 (·23)	·28 (·29)	·27 (·29)	·29 (·32)	·13 (·13)	·29 (·31)	·12 (·14)
IV R	·28 (·29)	·19 (·20)	·27 (·29)	·25 (·29)	·25 (·29)	·11 (·13)	·26 (·30)	·12 (·13)
	SPRINGS	NEAPS	SPRINGS	NEAPS	SPRINGS	NEAPS	SPRINGS	NEAPS

CURRENT VELOCITIES FT/SEC

TABLE 8

NOTE. BRACKETED FIGURES GIVE THE MAXIMA OBSERVED

NO TIDES IN OPERATION		
	DUMBARTON	CLYDEBANK
NORMAL	·02 (·02)	·03 (·03)
SPATE	·05 (·06)	·12 (·12)
I	·02 (·04)	·03 (·03)
II	·02 (·02)	·03 (·03)
IV & IV R	·02 (·02)	·03 (·03)

CURRENT VELOCITIES FT/SEC

TABLE 9

		GOUROCK	GOVAN
		AMPLITUDE	AMPLITUDE
	c_1	FEET ·28	FEET ·47
d_1	c_2	3·67	4·36
	c_3	·15	·17
d_2	c_4	·26	·44
	c_5	·03	·12
d_3	c_6	·10	·48
	c_7	·02	·17
d_4	c_8	·01	·14
	c_9	·03	·16
d_5	c_{10}	·01	·11
	c_{11}	·02	·16

c_1, c_2, c_3 ····· COEFFICIENTS IN 24 ORDINATE (24 LUNAR HOUR) SYSTEM.
d_1, d_2, d_3 ····· " " 12 " (12 " ") "

HARMONIC ANALYSIS OF MEAN TIDE CURVE TAKEN FROM 15 DAYS' TIDES

TABLE 10

		GOUROCK	BOWLING	ROTHESAY	GOVAN	GLASGOW
JUNE 26	HW	16' 6"	17' 8"	18' 2"	18' 5"	18' 3"
	LW	6 10	6' 3"	6' 0"	5' 8"	5' 4"
" 27	HW	16 6	17 8	18 1	18 3	18 1
"SPRING" DUE	LW	6 9	6 1	5 9	5 4	5 2
" 28	HW	16 9	17 9	18 2	18 4	18 3
	LW	6 9	6 1	5 9	5 4	5 1
" 29	HW	16 9	17 10	18 4	18 7	18 5
	LW	6 7	5 11	5 7	5 3	4 11
" 30	HW	16 11	17 11	18 6	18 8	18 6
	LW	6 10	6 3	5 11	5 7	5 4
JULY 1	HW	16 11	17 10	18 4	18 8	18 6
	LW	6 10	6 4	5 11	5 8	5 5
MEAN LEVEL		11 9	11 $11\frac{1}{2}$	12 $0\frac{1}{2}$	11 $11\frac{3}{4}$	11 $9\frac{1}{4}$
GAUGE O.D.		11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$
O.DATUM +		$\frac{3}{4}$	$3\frac{1}{4}$	$4\frac{1}{4}$	$3\frac{1}{2}$	1
LEVEL ABOVE GOUROCK		–	$2\frac{1}{2}$"	$3\frac{1}{2}$"	$2\frac{3}{4}$"	$\frac{1}{4}$"

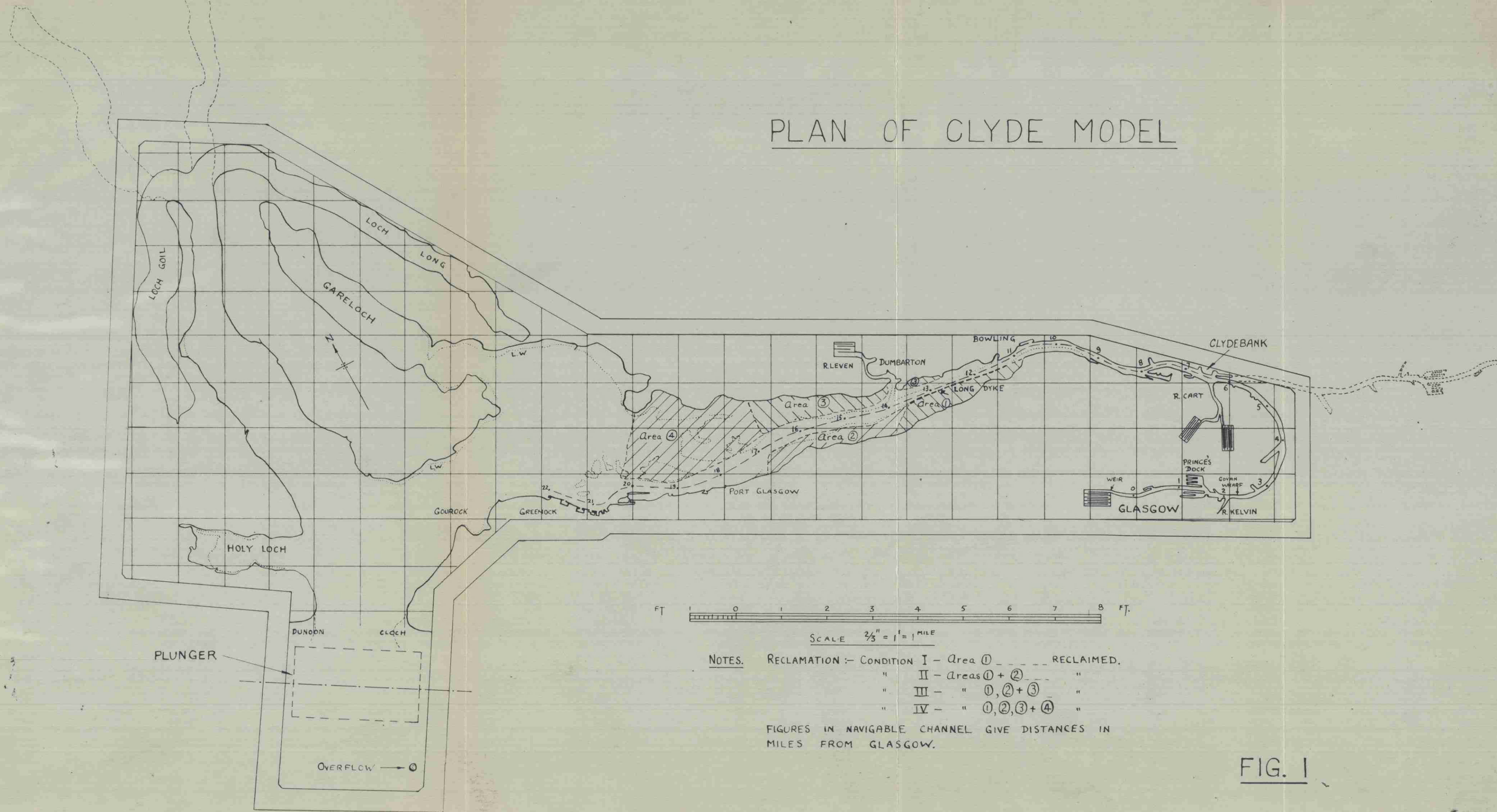
SPRINGS — TABLE 11

		GOUROCK	BOWLING	ROTHESAY	GOVAN	GLASGOW
MAY 15	HW	17' 7"	18' 9"	19' 3"	19' 4"	19' 4"
	LW	6' 9	6 4	5 10	5 5	5 3
" 16	HW	17 9	19 0	19 6	19 9	19 9
	LW	7 6	7 1	6 7	6 3	6 0
" 17	HW	17 6	19 0	19 5	19 6	19 6
	LW	7 7	7 8	7 4	7 2	7 0
" 18	HW	16 3	17 5	17 9	17 11	17 9
	LW	7 7	7 9	7 5	7 0	6 11
" 19	HW	15 11	16 11	17 2	17 3	17 0
	LW	8 0	7 11	7 8	7 4	7 2
" 20	HW	15 6	16 1	16 4	16 3	16 3
"NEAP" DUE	LW	8 2	8 0	7 8	7 5	7 3
MEAN LEVEL		12 2	12 8	12 8	12 $6\frac{1}{4}$	12 5
GAUGE O.D.		11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$	11 $8\frac{1}{4}$
O.DATUM +		$5\frac{3}{4}$"	$11\frac{3}{4}$"	$11\frac{3}{4}$"	$10\frac{1}{4}$"	$8\frac{3}{4}$"
LEVEL ABOVE GOUROCK			6"	6"	$4\frac{1}{2}$"	3"

NEAPS — TABLE 12

MEAN LEVEL OF WATER IN CLYDE

PLAN OF CLYDE MODEL

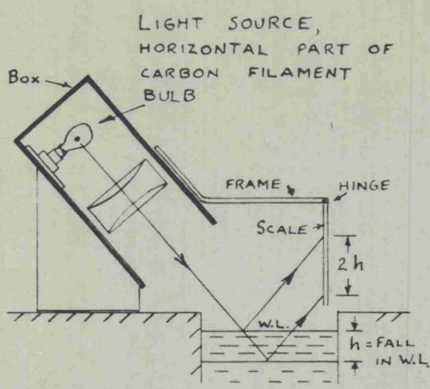


SCALE $2/3'' = 1' = 1$ MILE

NOTES. RECLAMATION :- CONDITION I - Area ① ..... RECLAIMED.
" II - Areas ① + ② ..... "
" III - " ①, ② + ③ ..... "
" IV - " ①, ②, ③ + ④ "

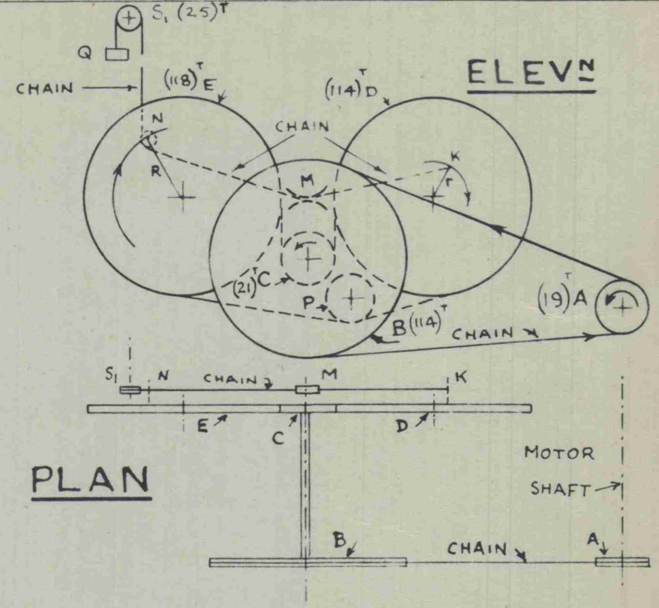
FIGURES IN NAVIGABLE CHANNEL GIVE DISTANCES IN MILES FROM GLASGOW.

FIG. I



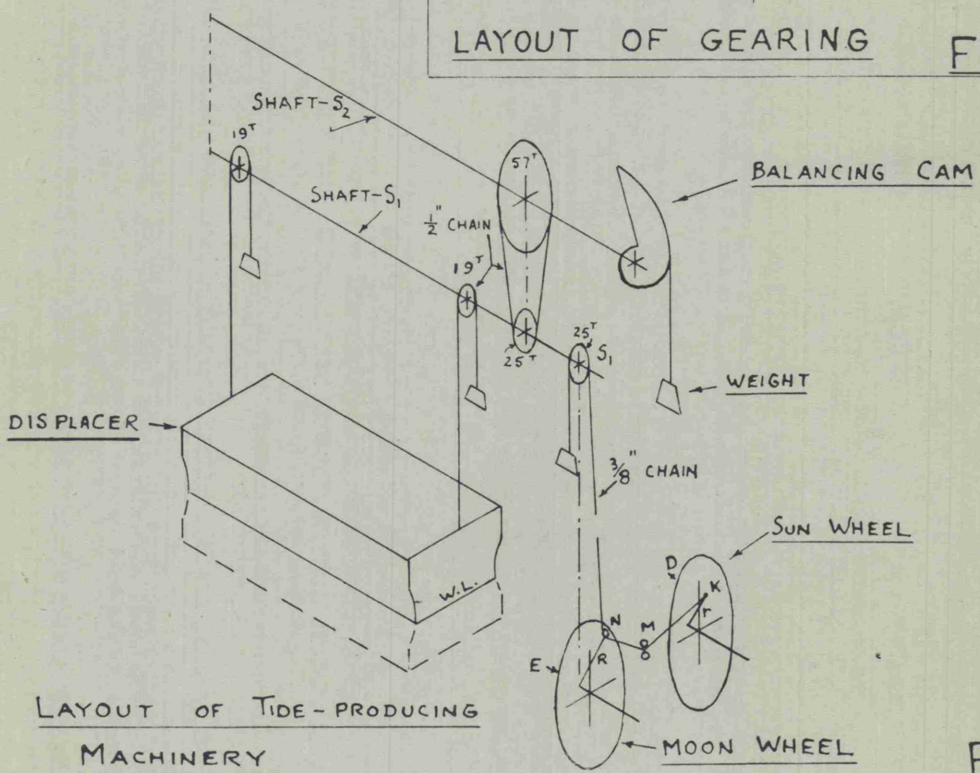
REFLECTED BEAM MEASURES CHANGE OF W. L.

FIG. 2



LAYOUT OF GEARING

FIG. 3



LAYOUT OF TIDE-PRODUCING MACHINERY

FIG. 4

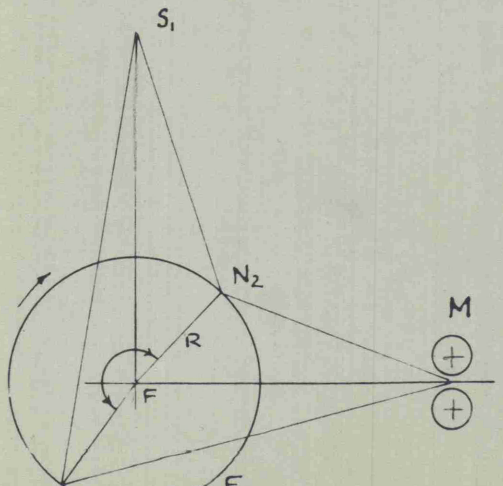


FIG. 5

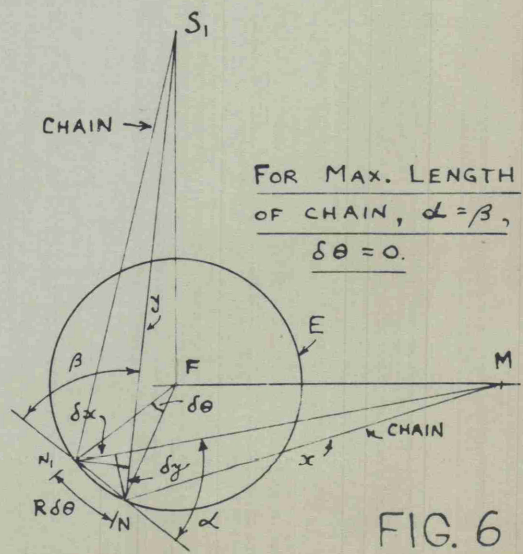
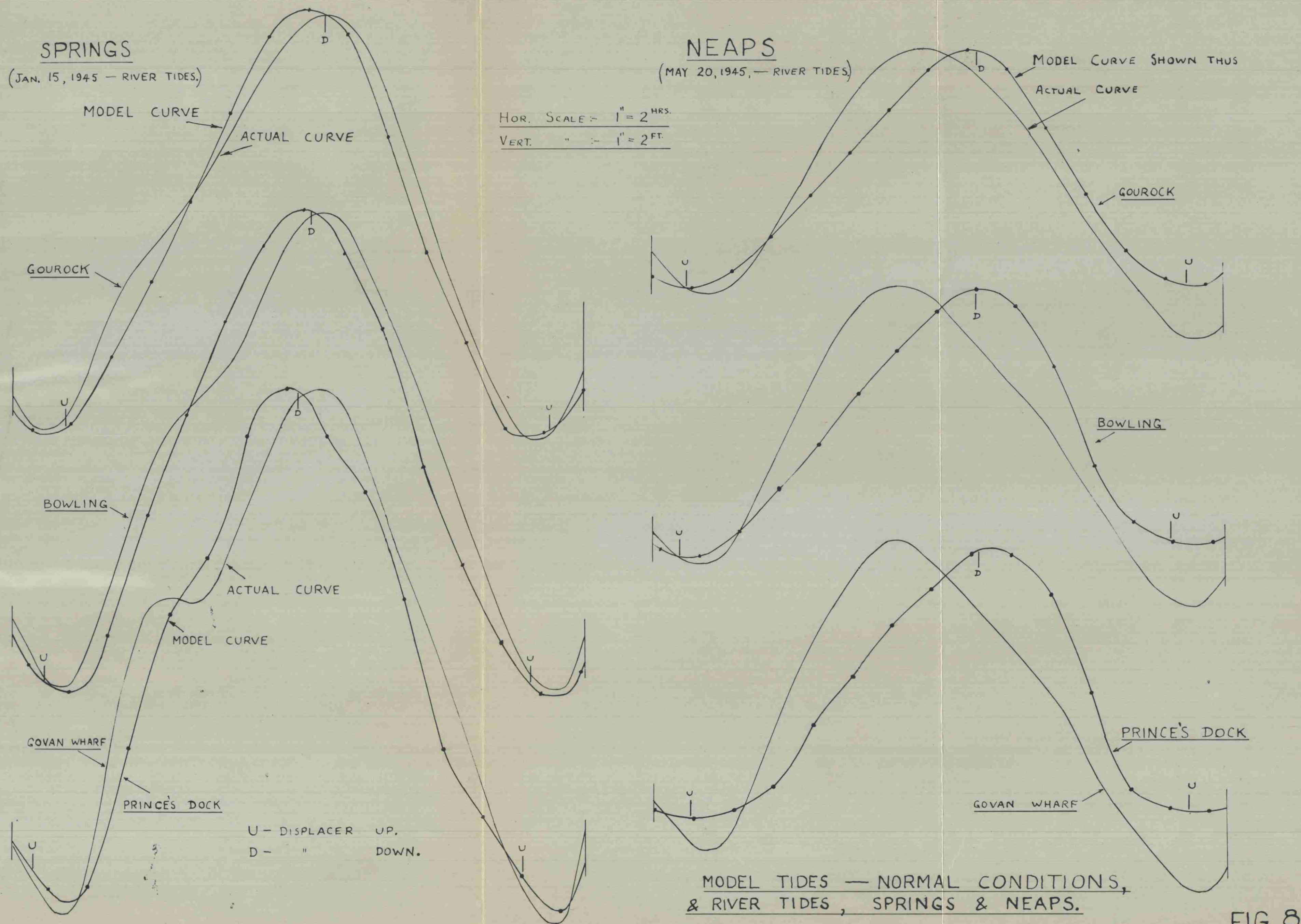


FIG. 6

DISPLACER MOTION FOR 15 CONSECUTIVE DAYS.

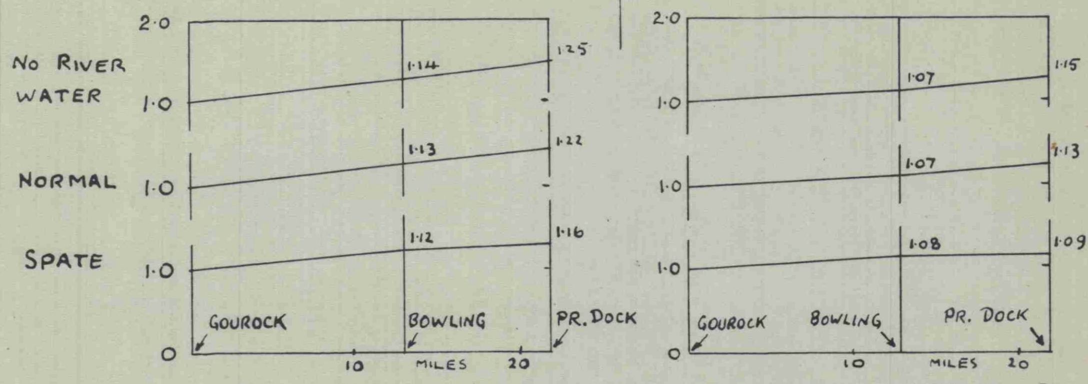
FIG. 7



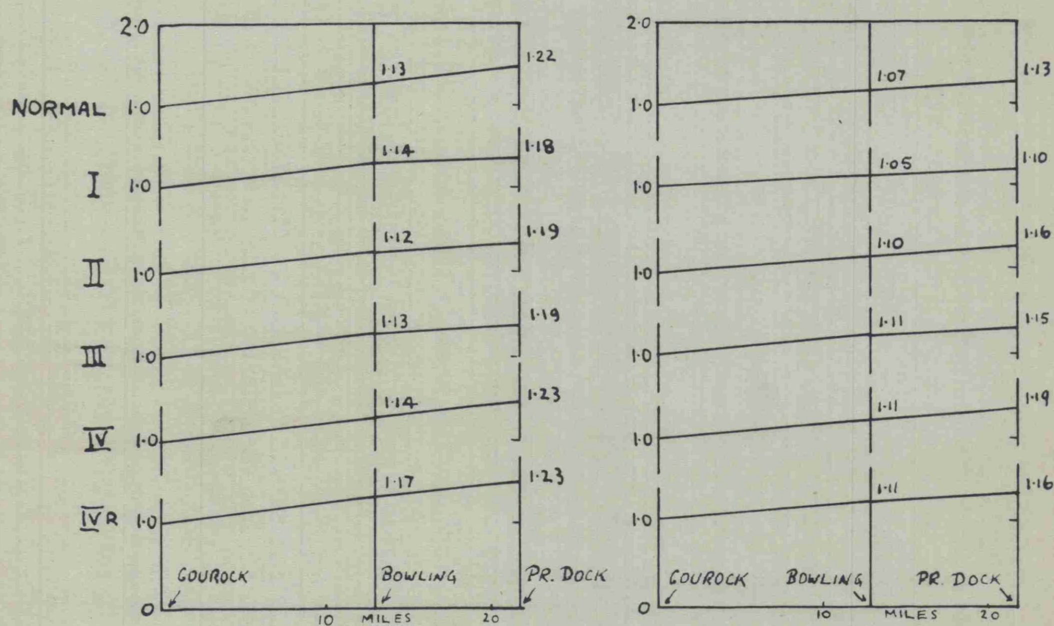
MODEL TIDES — NORMAL CONDITIONS, & RIVER TIDES, SPRINGS & NEAPS.

FIG. 8

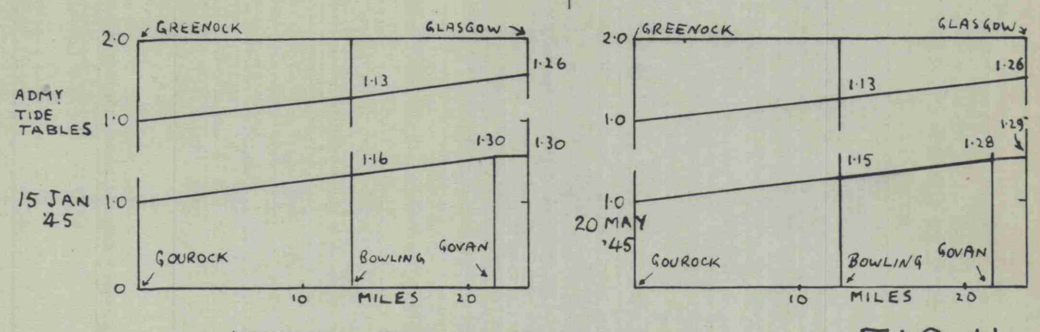
SPRINGS	NEAPS



EFFECT OF RIVER WATER. FIG. 9



EFFECT OF LAND RECLAMATION. FIG. 10



ACTUAL RIVER TIDES. FIG. 11

RATIO OF RANGES.

HOR. SCALE :- $1'' = 10^{M.}$

VER^{T} " :- $1'' = \text{UNIT RATIO.} = \dfrac{\text{RANGE}}{\text{GOUROCK RANGE}}$.

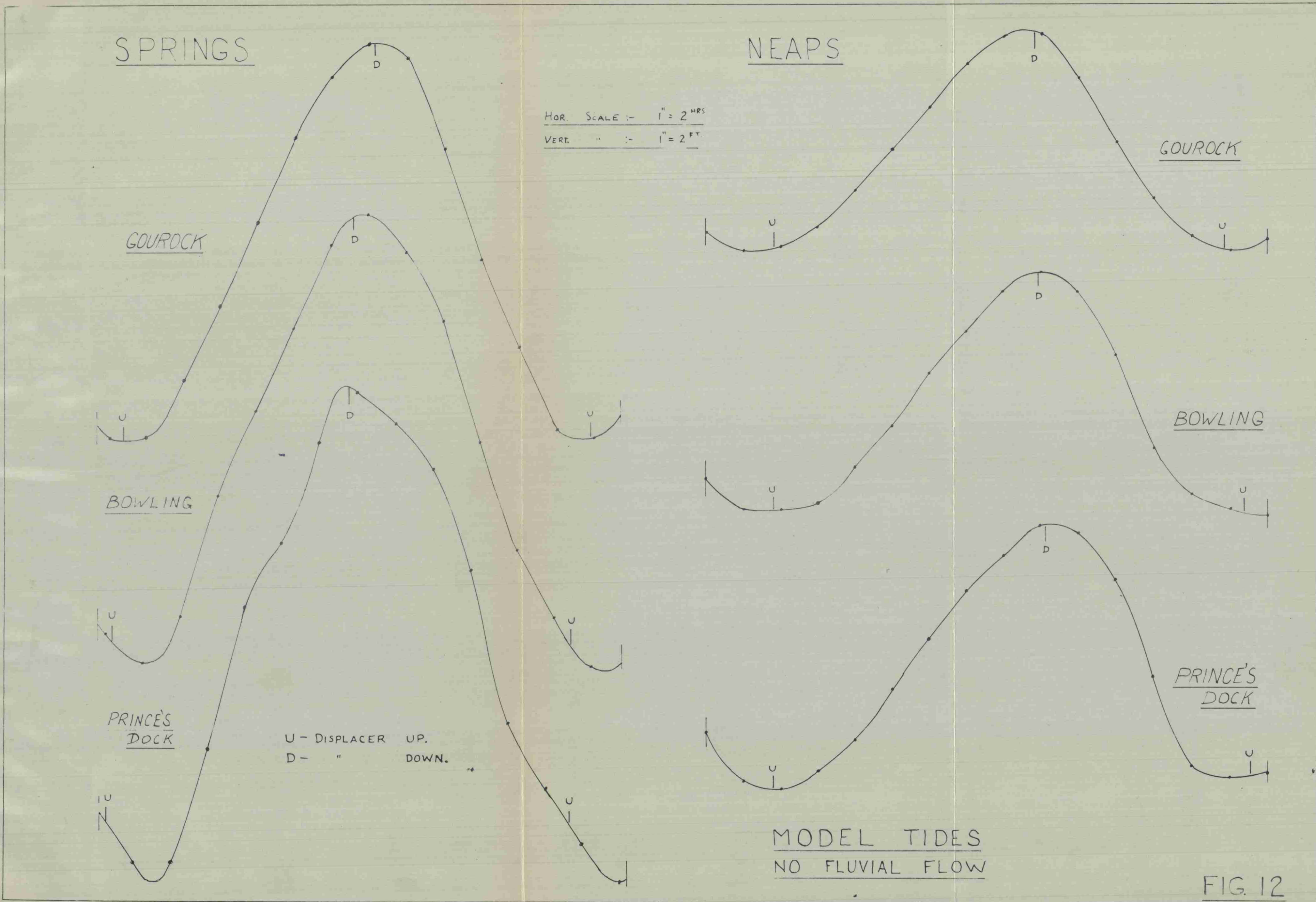

SPRINGS
NEAPS
HOR. SCALE :- 1" = 2 HRS
VERT. " :- 1" = 2 FT
GOUROCK
BOWLING
PRINCE'S DOCK
U - DISPLACER UP.
D - " DOWN.
U
D
MODEL TIDES
NO FLUVIAL FLOW

FIG. 12

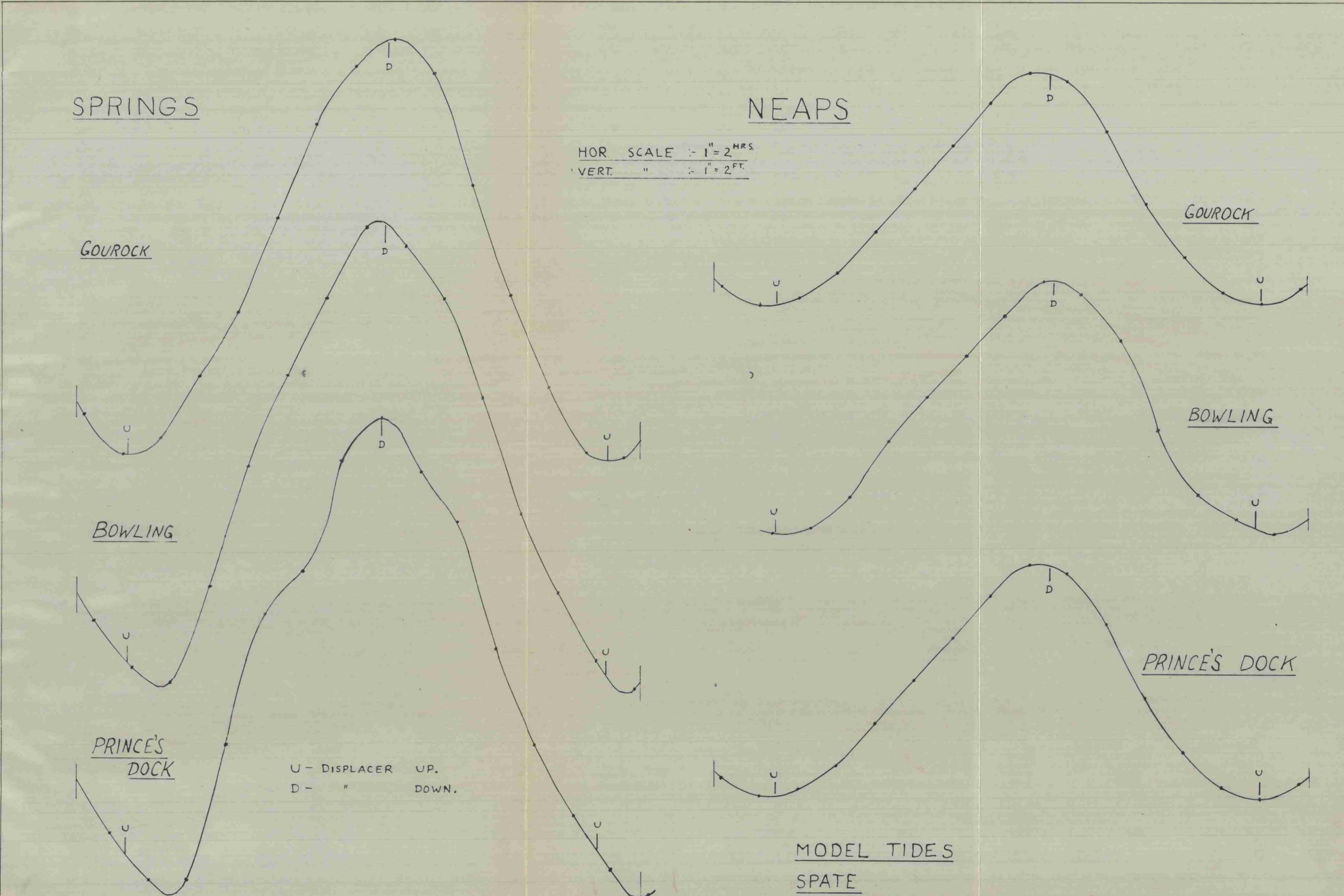
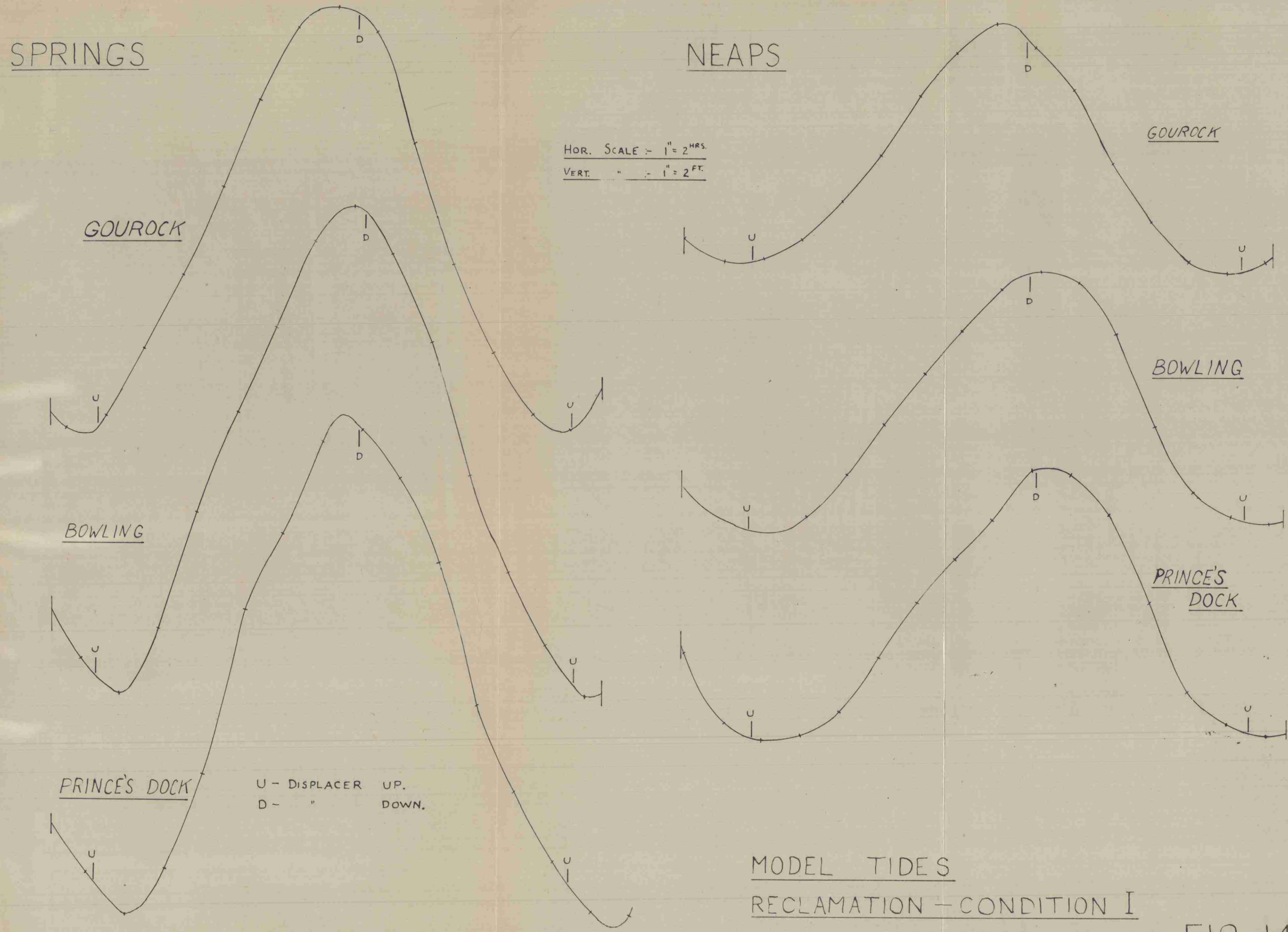

SPRINGS
NEAPS
HOR SCALE :- 1" = 2 HRS.
VERT. " :- 1" = 2 FT.
GOUROCK
BOWLING
PRINCE'S DOCK
U - DISPLACER UP.
D - " DOWN.
GOUROCK
BOWLING
PRINCE'S DOCK
MODEL TIDES
SPATE

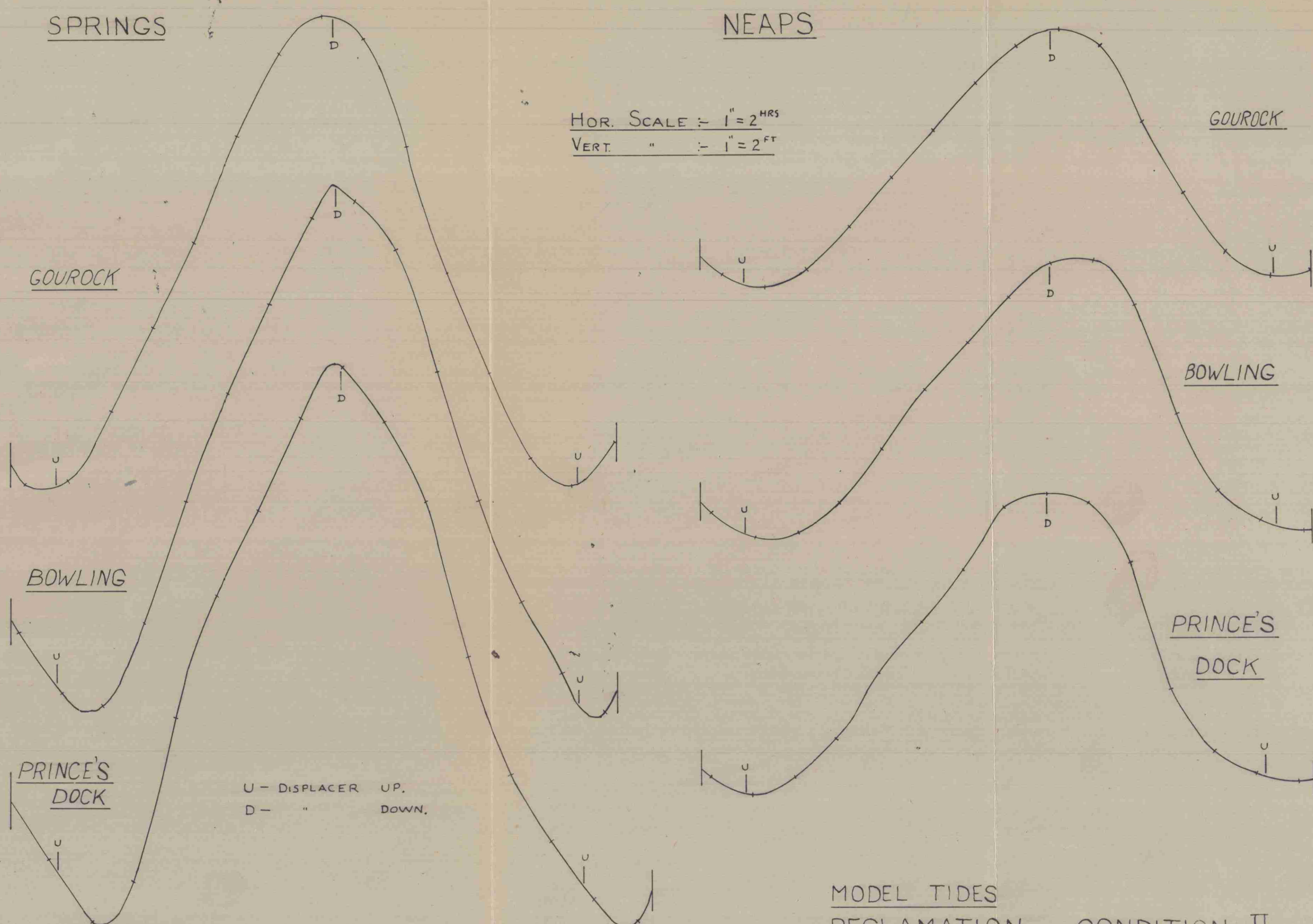
FIG. 13



MODEL TIDES

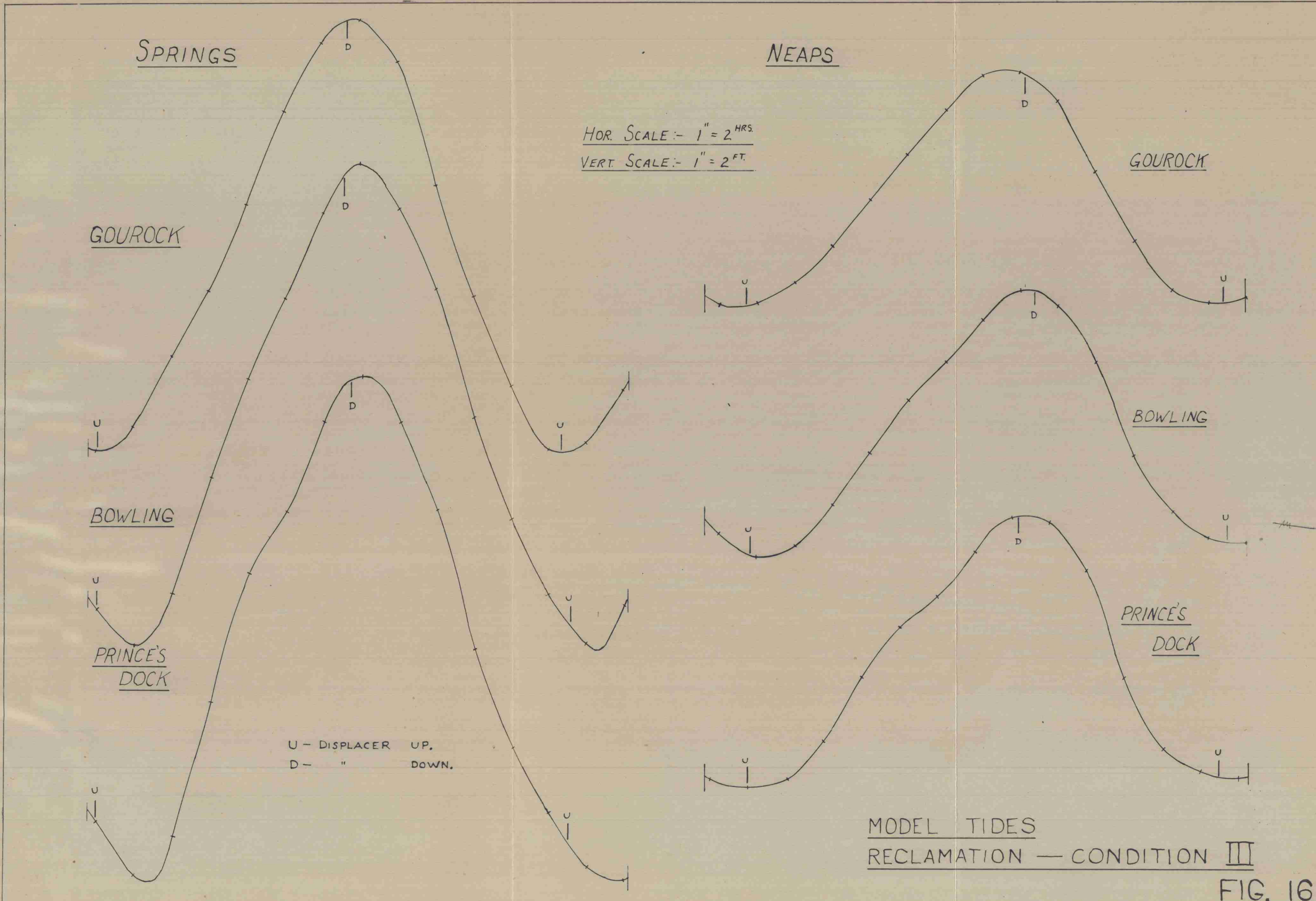
RECLAMATION - CONDITION I

FIG. 14



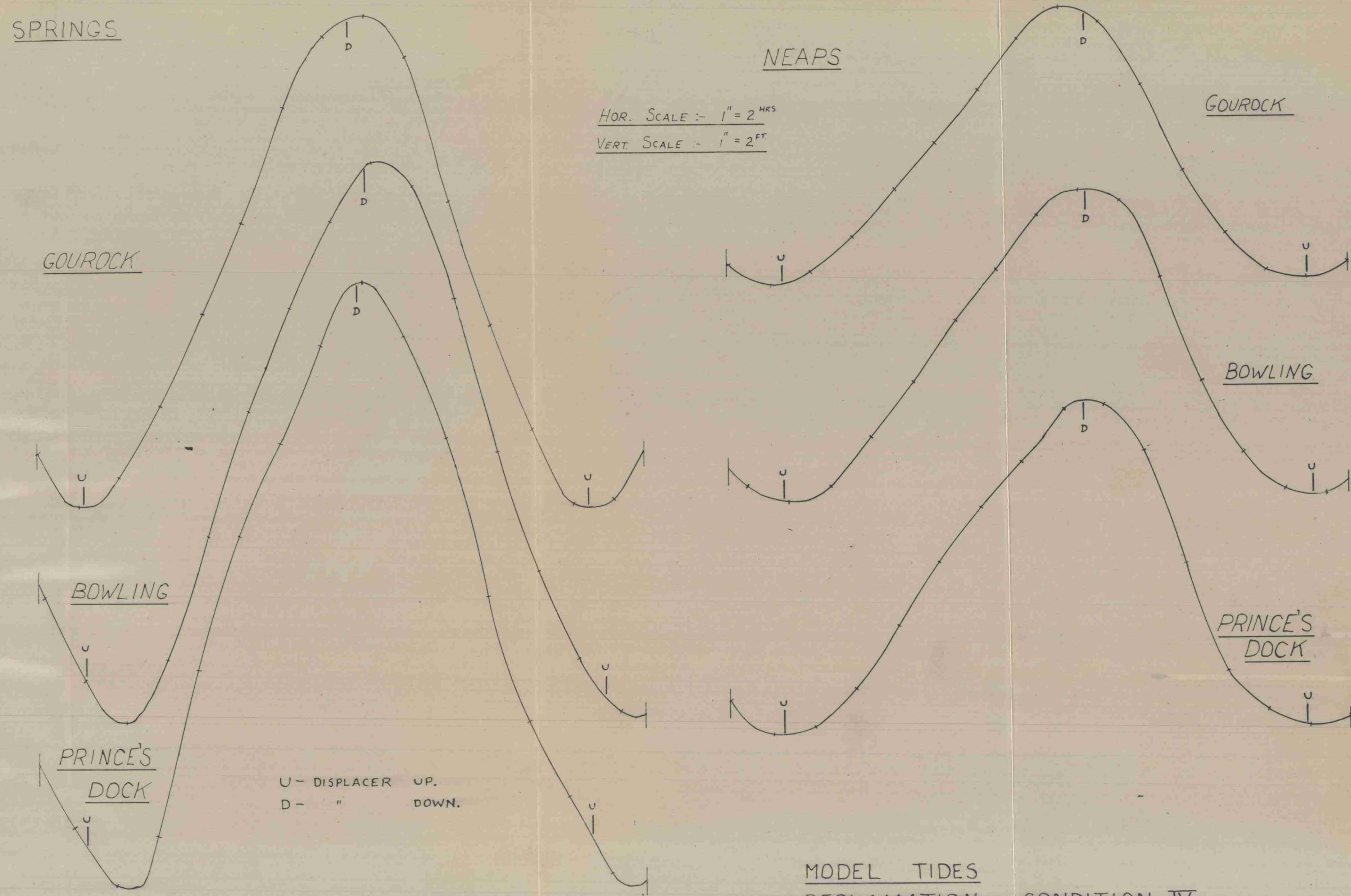
MODEL TIDES
RECLAMATION — CONDITION II

FIG. 15



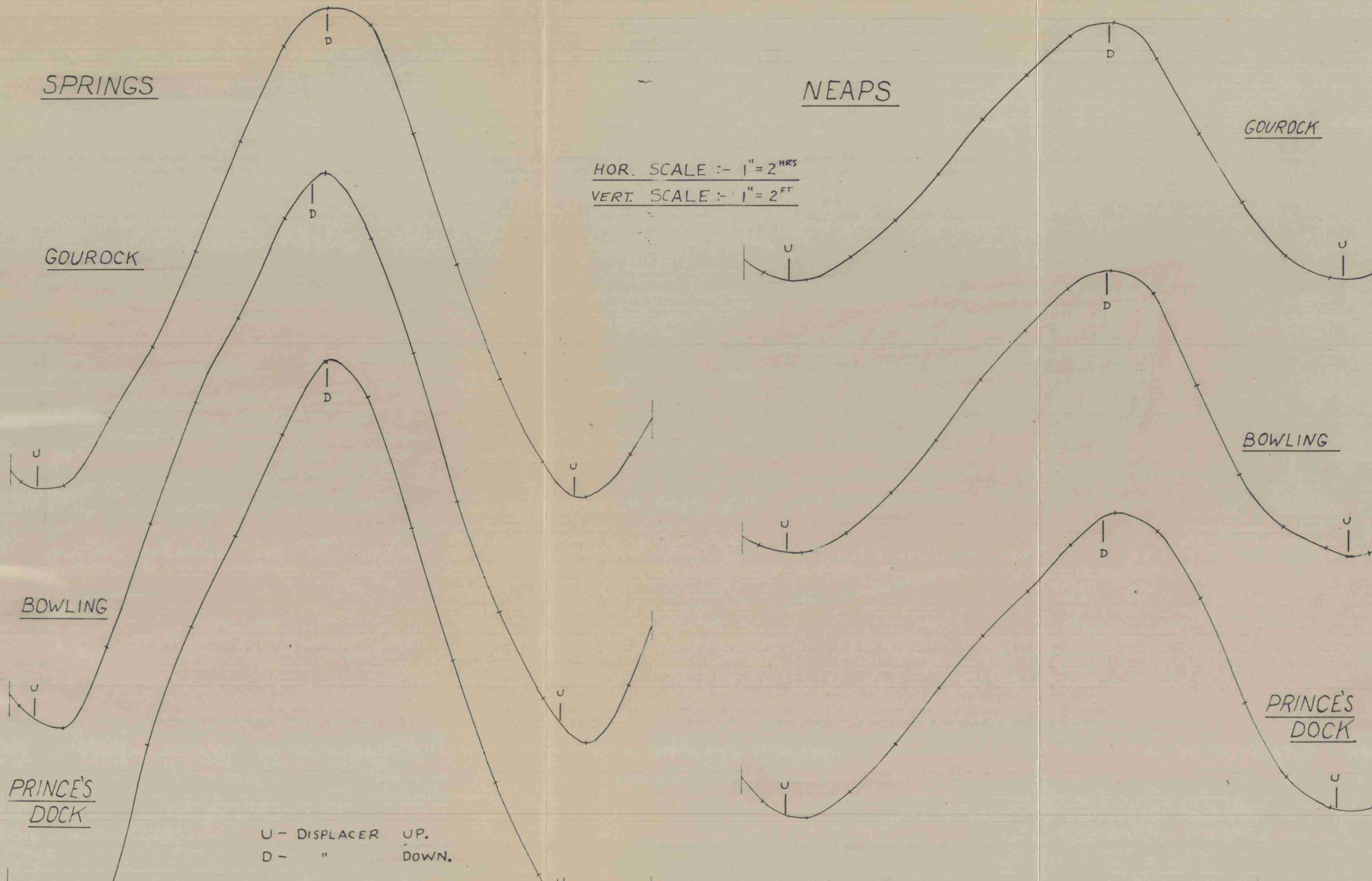
MODEL TIDES
RECLAMATION — CONDITION III

FIG. 16



MODEL TIDES
RECLAMATION — CONDITION IV.

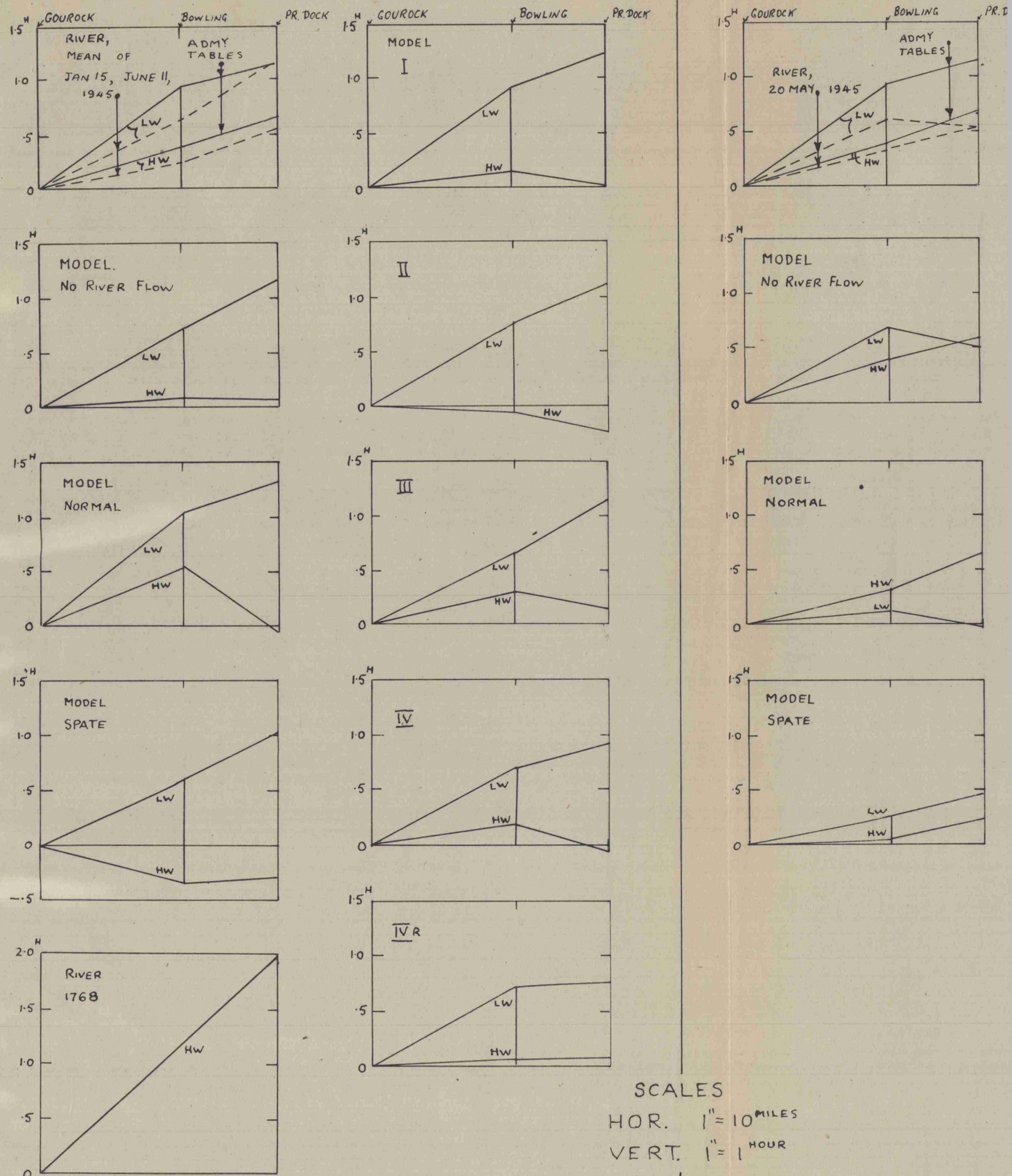
FIG. 17



MODEL TIDES

RECLAMATION — CONDITION IV R

FIG. 18

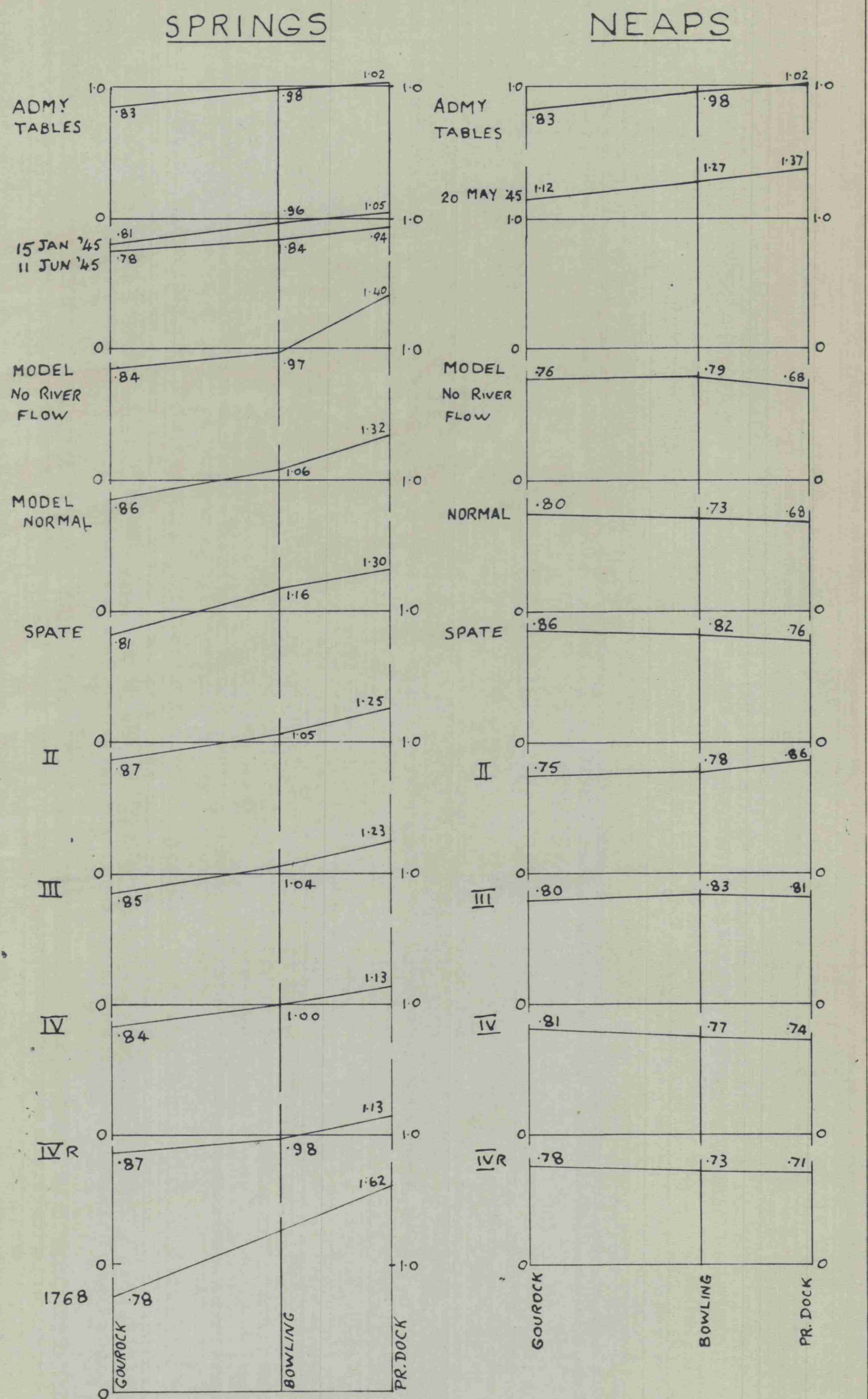


SPRINGS

FIG. 19

NE

TIMES OF L.W. & H.W. AFTER L.W. &

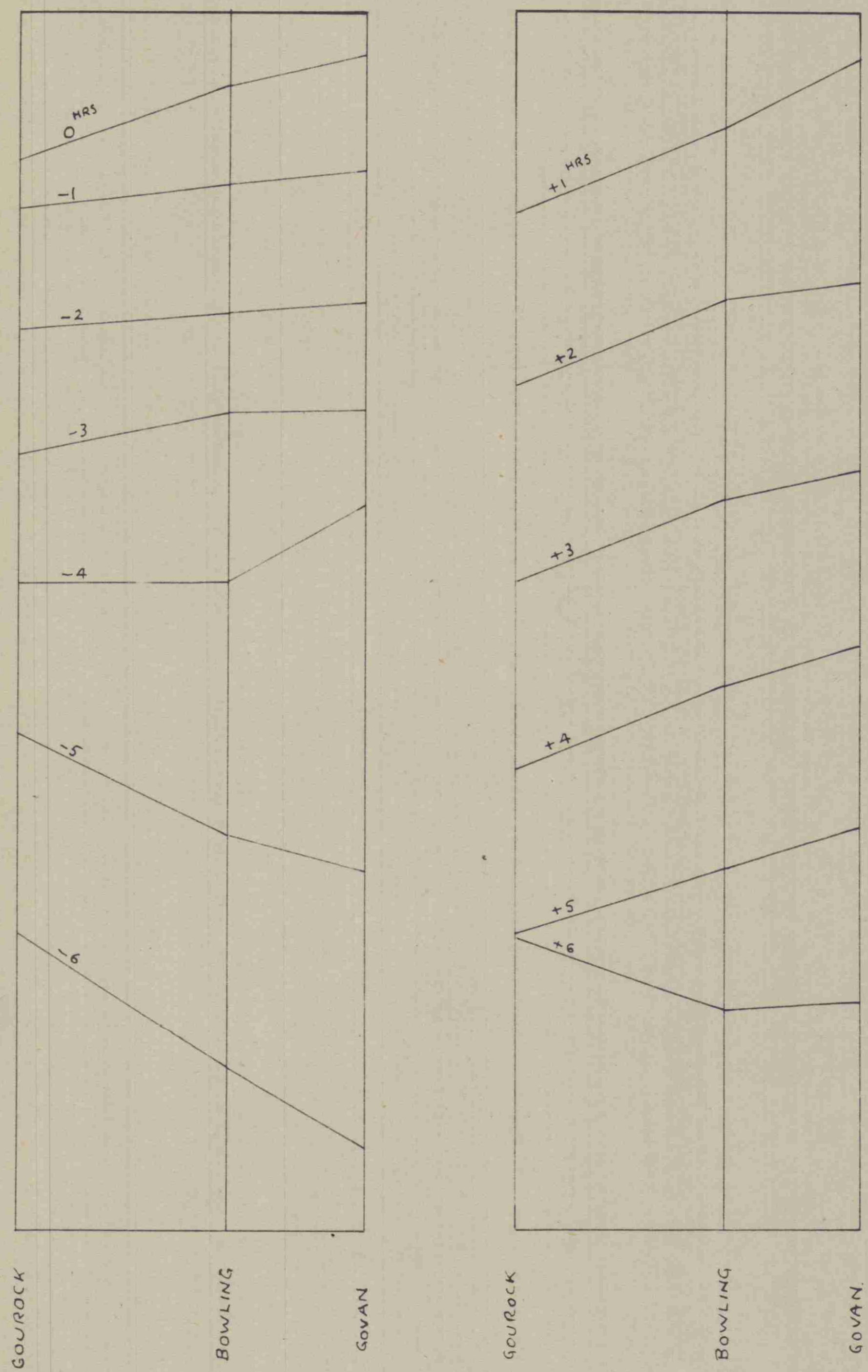


HOR. SCALE :- 1" = 10 MILES

VERT. SCALE :- 1" = UNIT RATIO $\frac{\text{FALL TIME}}{\text{RISE TIME}} = n$

COMPARISON, TIMES OF FALL & RISE.

FIG. 21



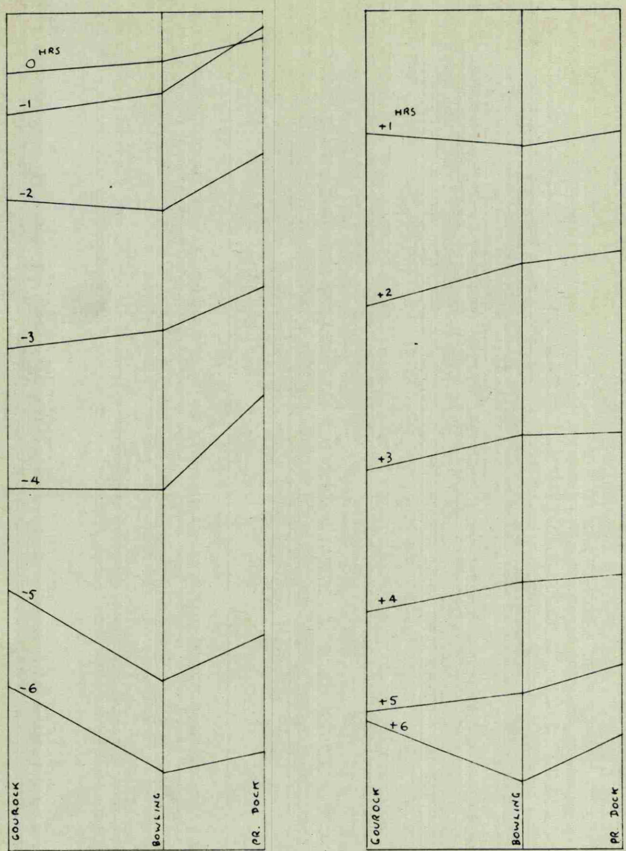
HOR. SCALE :- 1" = 10 MILES

VERT SCALE :- 1" = 2'

FIGURES SHOW TIMES BEFORE (–) AND AFTER (+) H.W. GOUROCK.

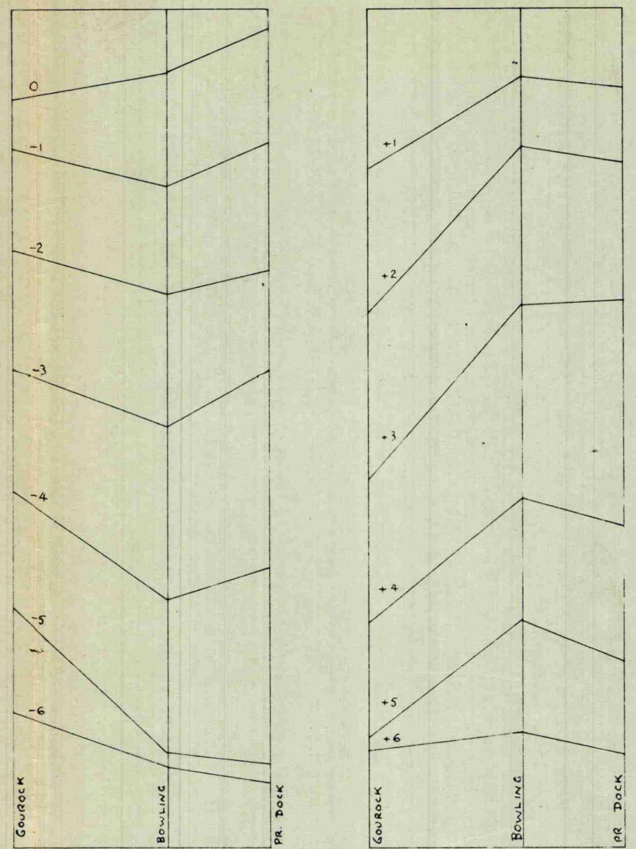
LONGITUDINAL SECTIONS OF THE RIVER ON 11 JUNE 1945.

FIG. 22



SPATE

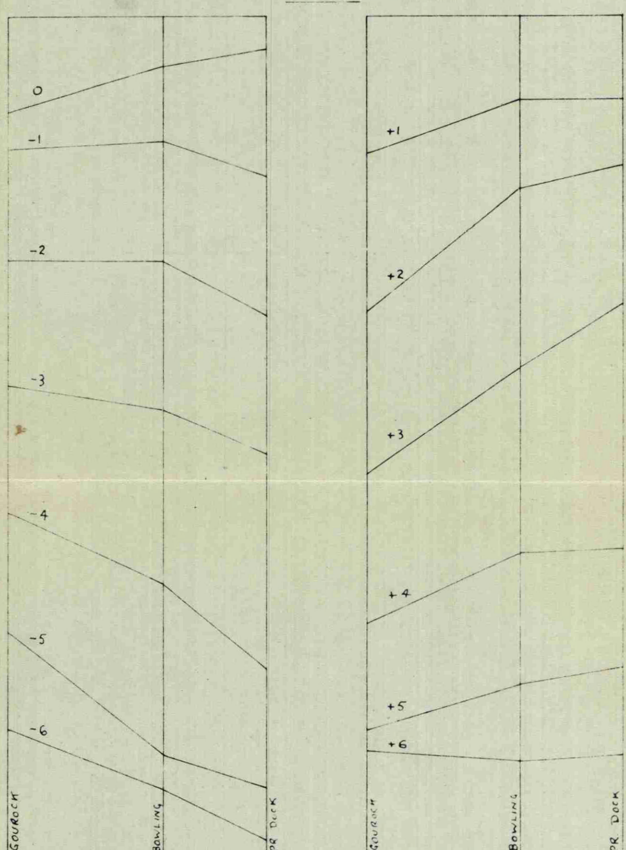
FIG. 23



NORMAL

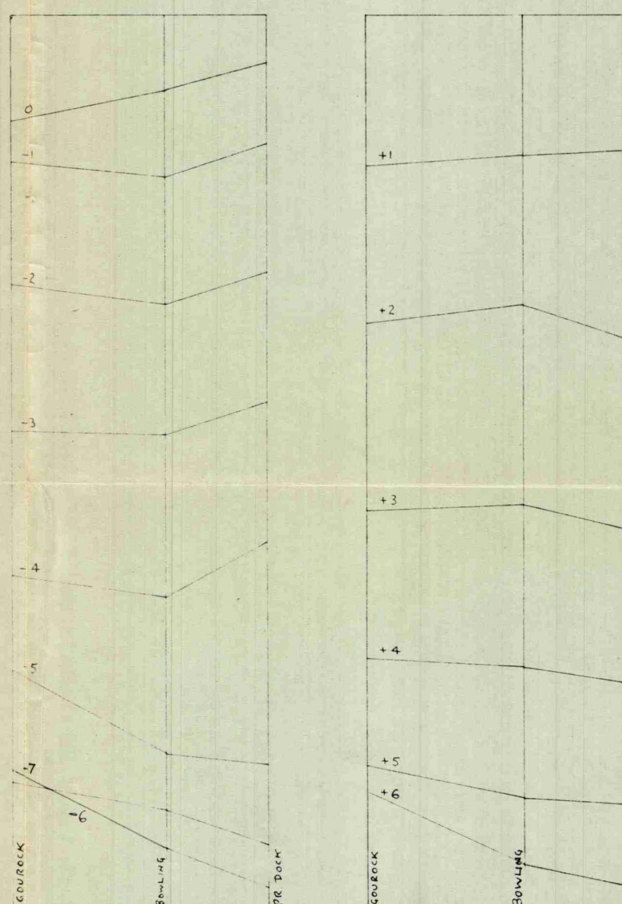
FIG. 24

NOTE:- FIGURES SHOW TIMES BEFORE (–) AND AFTER (+) H.W. GOUROCK.



NO RIVER WATER

FIG. 25

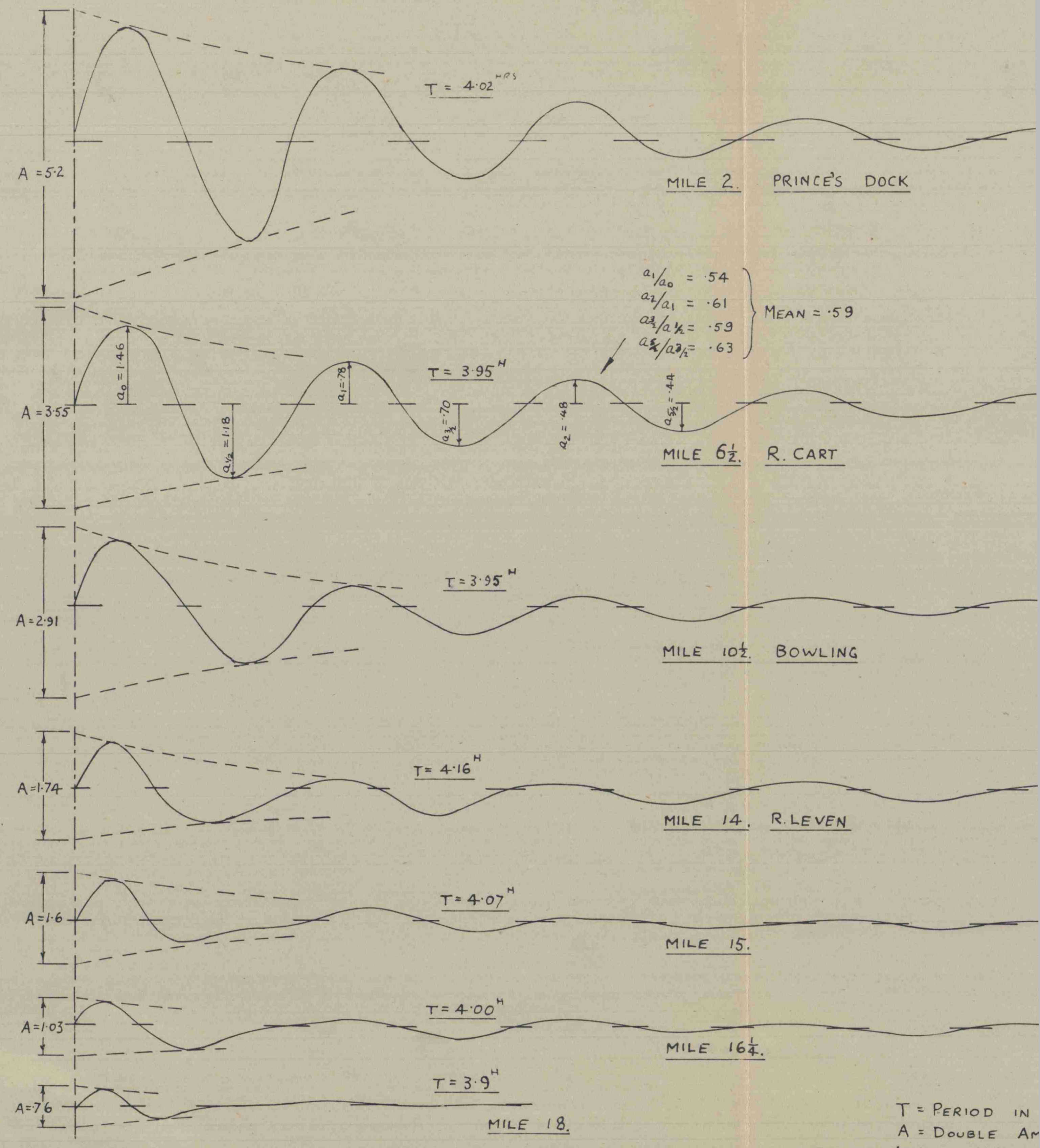


CONDITION IV R

FIG. 26

HOR SCALE - 1" = 10 MILES. VERT SCALE - 1" = 2'

LONGITUDINAL SECTIONS OF THE MODEL CHANNEL.



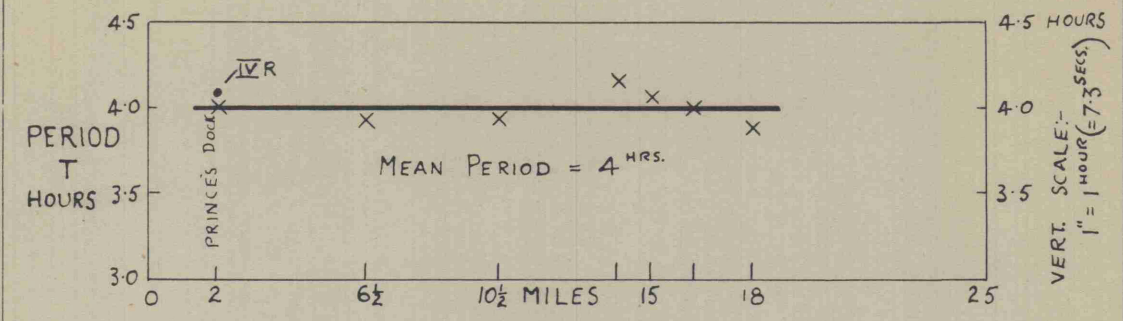
HOR. SCALE :- 1" = 2 HRS. (= 14·6 SECS)

VERT. SCALE :- 1" = 2 FT. (= 0·21 INS)

NOTE.

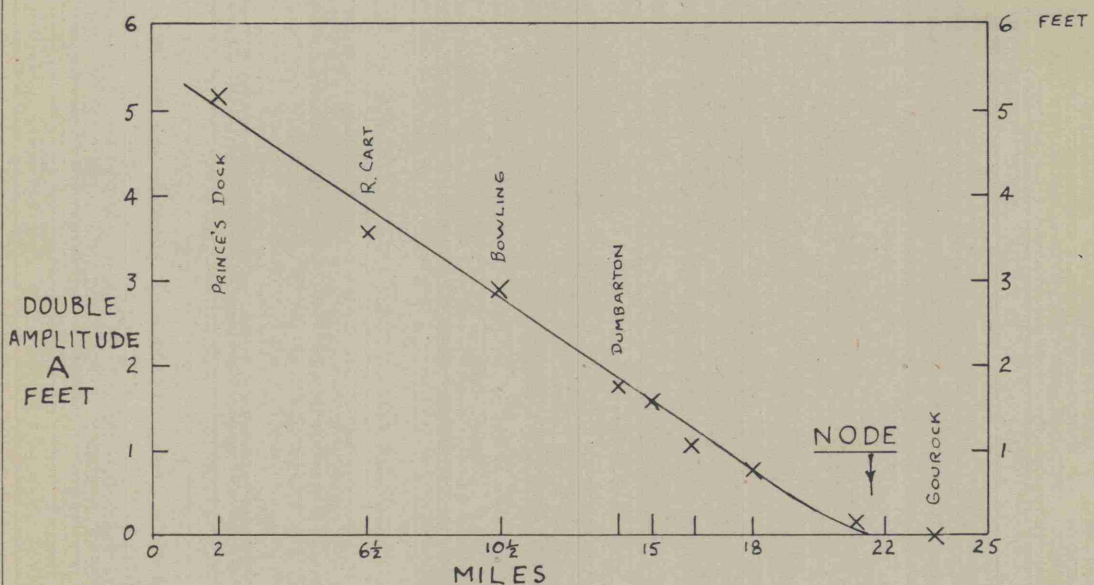
THE ABOVE OBSERVATIONS WERE TAKEN AT SPRINGS. ADDITIONAL OBSERVATIONS AT PRINCES DOCK GAVE THE FOLLOWING :- T = 4·1 H FOR CONDITION IVR SPRINGS, T = 3·85 H FOR SPATE, T = 4·0 H FOR NO RIVER WATER, T = 3·93 H NEAPS.

DAMPED OSCILLATION
ON THE MODEL ESTU



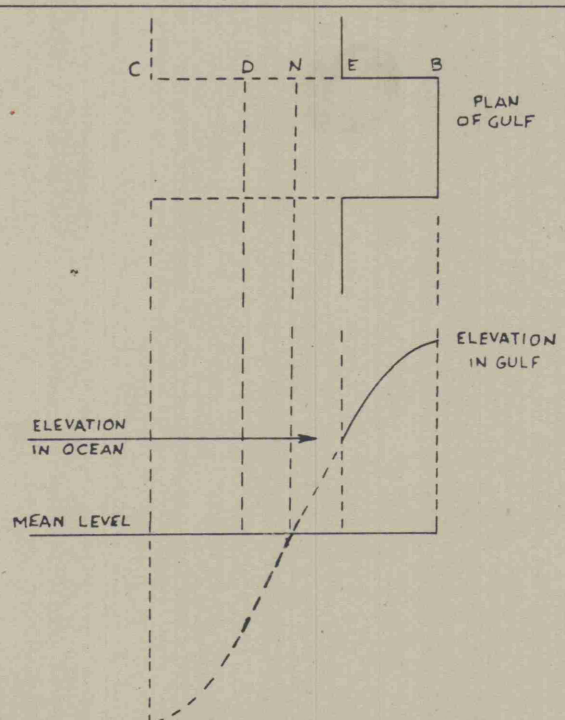
NATURAL PERIOD OF THE MODEL

FIG. 28



DOUBLE AMPLITUDE OF NATURAL OSCILLATIONS OF MODEL

FIG. 29



ADMIRALTY MANUAL OF TIDES, FIG. 19.3. "TIDES IN GULFS WHICH ARE SHORTER THAN A CERTAIN CRITICAL LENGTH."

FIG. 30

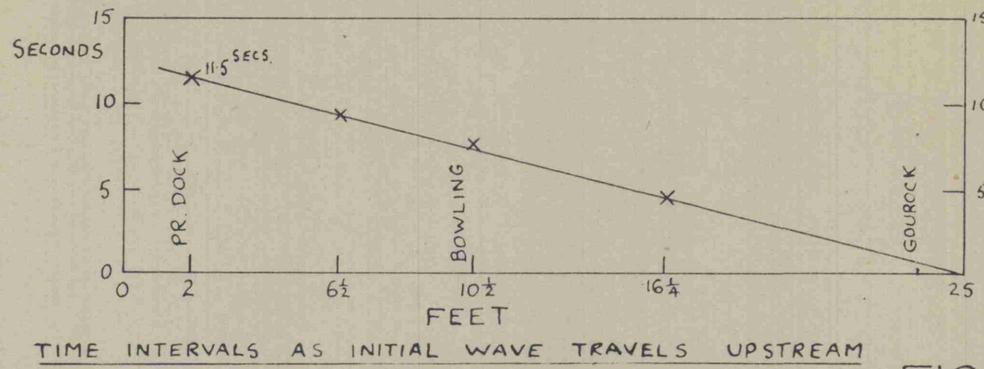
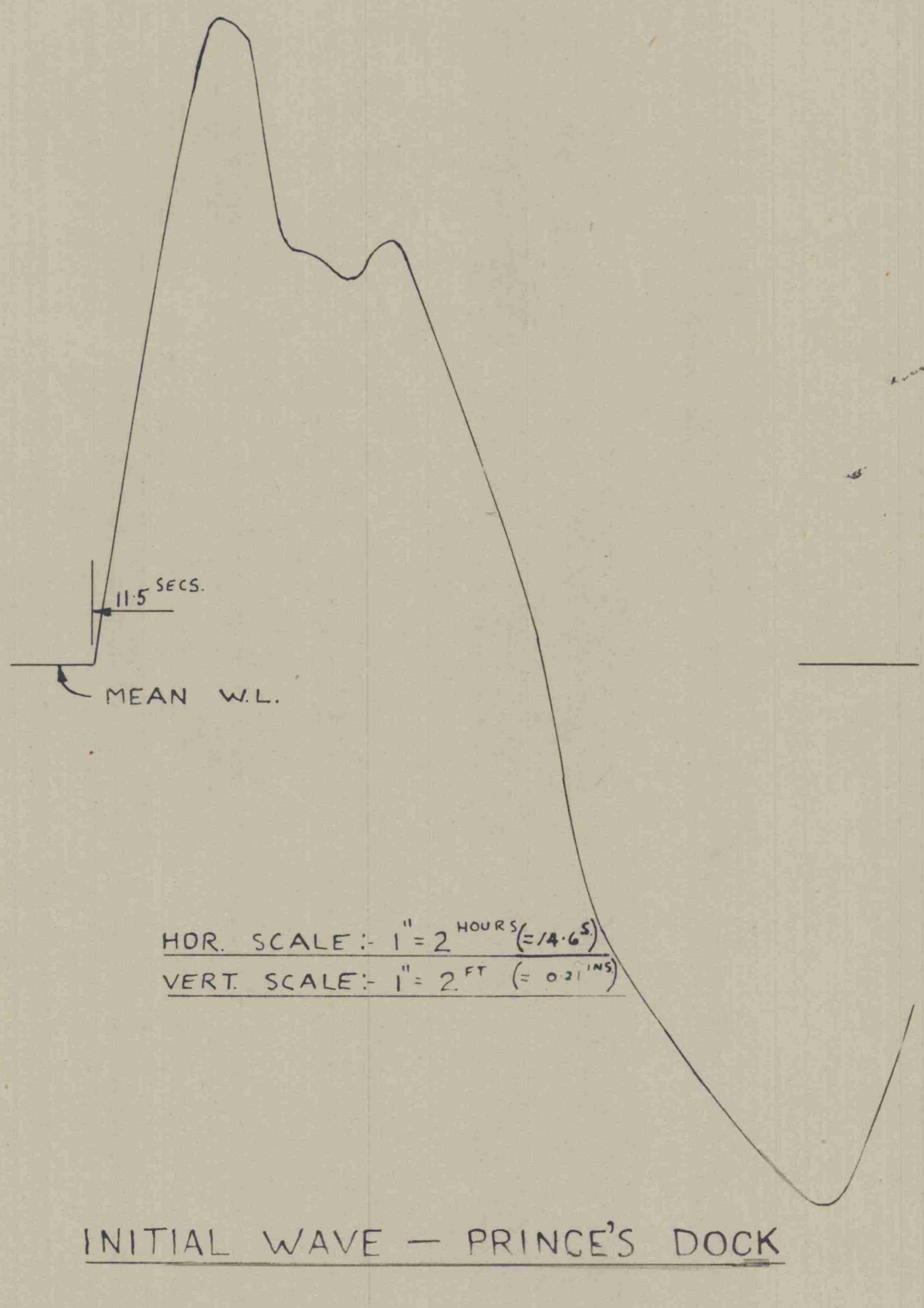
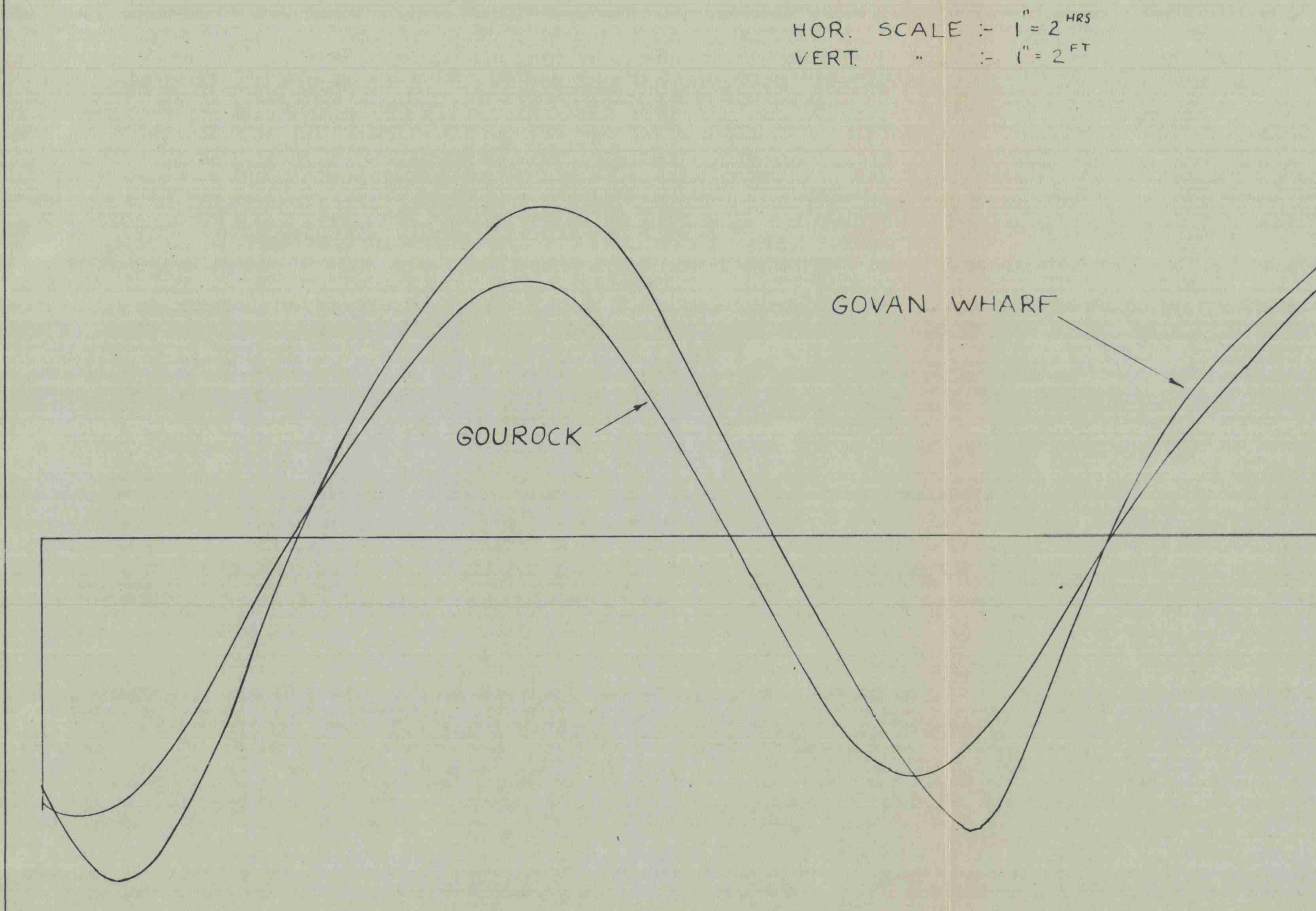


FIG. 31



INITIAL WAVE — PRINCE'S DOCK

FIG. 32



MEAN TIDE CURVES

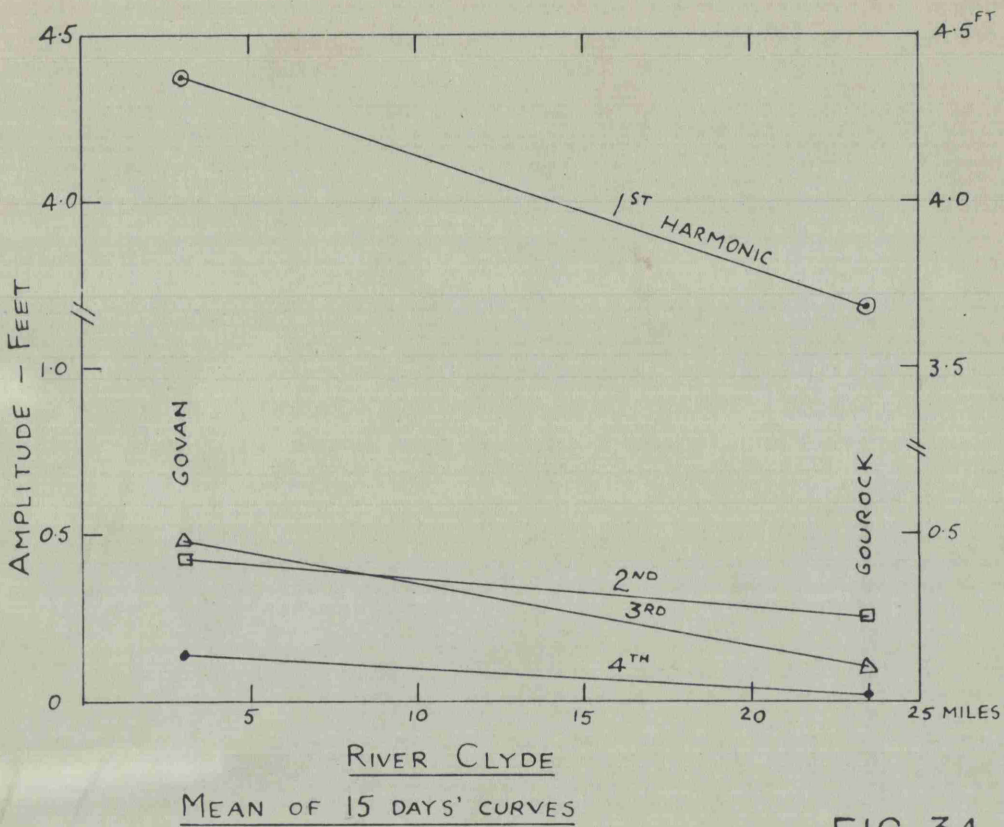
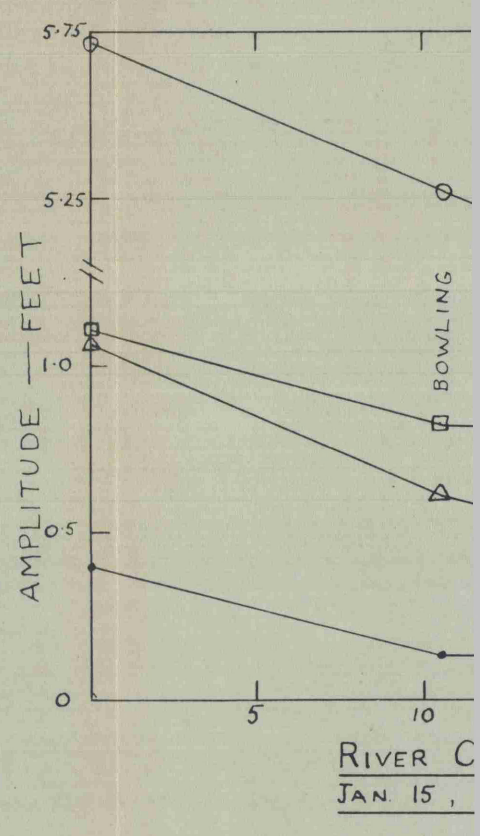


FIG. 34



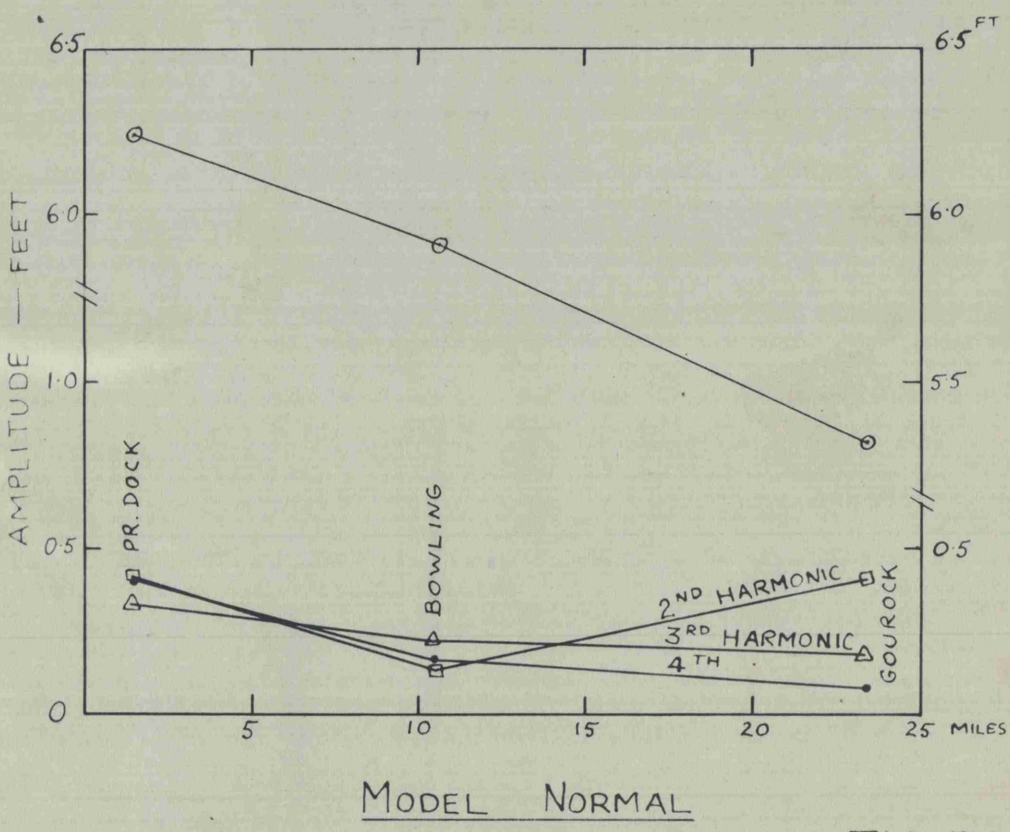
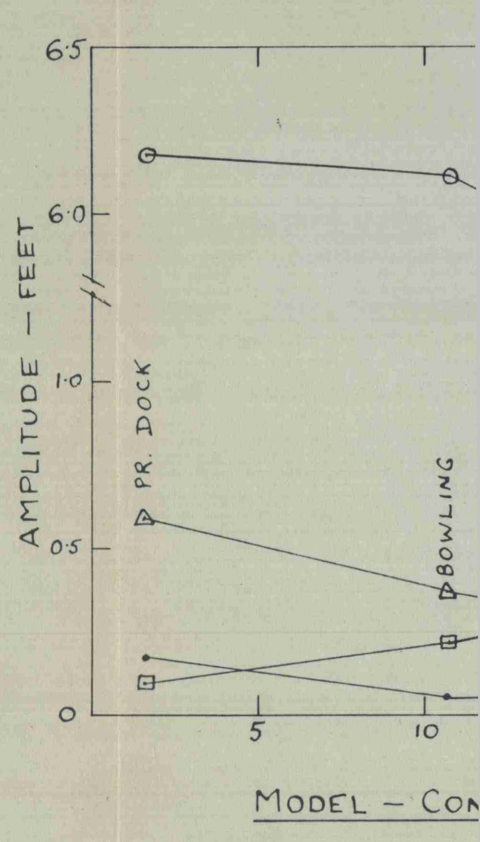
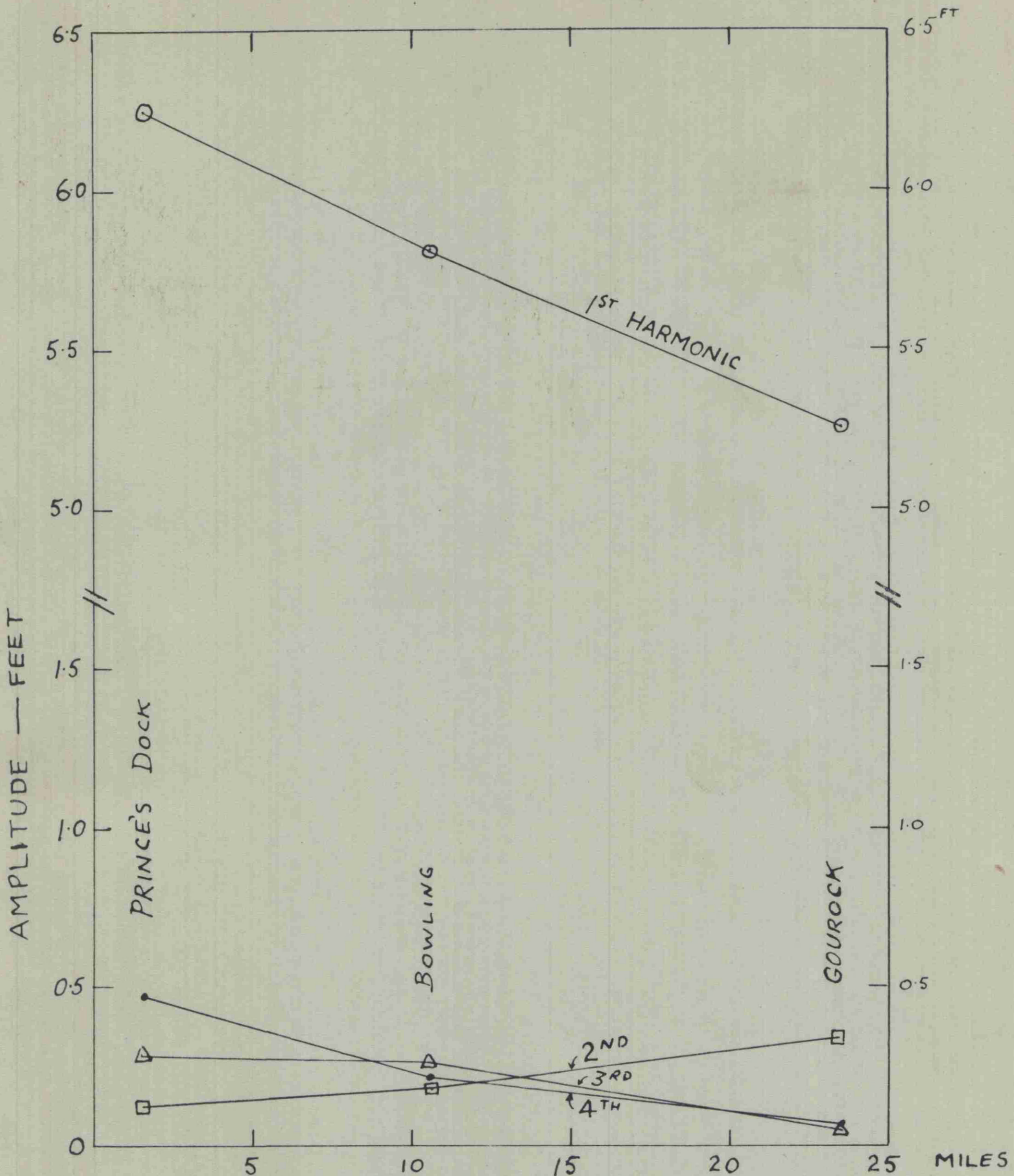


FIG. 36

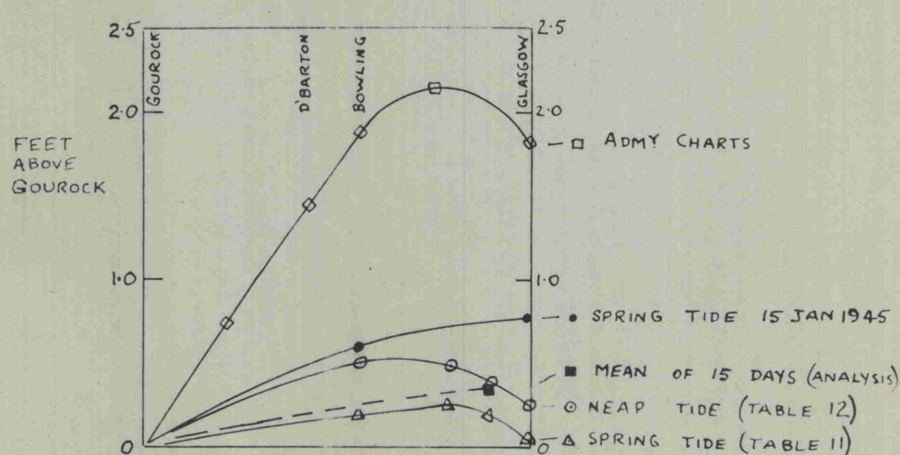


HARMONIC ANALYSIS OF TIDE CUR



HARMONIC ANALYSIS – MODEL TIDE, NO RIVER WATER

FIG. 38



MEAN LEVELS OF RIVER SURFACE

FIG. 39