TRANSFER STRESSES IN THE END-ZONES OF PRETENSIONED PRESTRESSED CONCRETE I-BEAMS.

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C O N T E N T S

C H A P T E R 1

INTRODUCTION

CHAPTER 1. INTRODUCTION

One of the most important aspects of the design of prestressed concrete beams is the design of its ends. These are more susceptible to difficulties in manufacture, erection, and service than any other part. Anchorage stresses in post-tensioned beams, transfer stresses in pretensioned beams, and beam reactions all produce relatively high stress-concentration at the ends.

The distribution of stress in the ends of a prestressed concrete beam is a three-dimensional problem and is of considerable complexity. This is due to the localisation of prestressing forces which results in tensile stresses in planes perpendicular to the wires.

With a pre-tensioned member, the prestressing force is provided by a number of separate wires, and it is necessary to consider each of these as individual point loads. The conditions at the end of a pretensioned beam differ from those at the end of a post-tensioned one in that the prestressing force, instead of being applied abruptly on the end face, is developed over the transmission length, or prestress transfer length, of the wire.

The stress-distribution is affected by the arrangement and grouping of the cables or wires, the shape and size of the member, jacking procedures, releasing methods, form restraint at release, curing and handling methods, the diameter and transmission length of the wires.

The knowledge of these stresses is important, because several cases of cracking at the ends of prestressed concrete beams have been reported (39). The majority of these beams were plant-cast and were usually pretensioned, although some were entirely post-tensioned and others combined both pretensioning and post-tensioning. In most of the cases, these cracks appeared either near the centroidal axis or near one of the web-flange junctions in I-section and in inverted T-section beams.

It is known from the available information that these cracks extend only a few inches into the beam and are nothing but very fine hair-cracks. These do not open up with the application of dead and live loads on the member and have no effect on the ultimate load carried or the type of failure. Although apparently these may appear harmless, there is every possibility of moisture penetration through them, the result of which is the corrosion of the tendons. And, above all, one of the major claims of prestressed concrete is the elimination of all cracks from reinforced concrete; the presence of these cracks will, therefore, go against the principle of prestressed concrete and people will start losing faith in this outstanding structural development.

The possible remedies are: (a) the provision of end-blocks to assist the distribution of the end-forces, and (b) the provision of suitable end-stirrup reinforcement.

As regards the provision of end-blocks in the anchorage-zones of the prestressed concrete beams, although these are almost indispensable in the case of post-tensioned beams, they have not proved completely successful in pretensioned beams, as cracks have also been reported in beams with such end-blocks. The advantages and disadvantages of the use of end-blocks are discussed in detail in a later chapter.

A preferable remedy is the use of stirrups in the ends of prestressed concrete beams. The Codes of Practice on prestressed concrete in various countries make no suitable design recommendation for such end-zone stirrup reinforcement.

Many engineers have taken an interest in this problem, but most of the recent works deal with the design of the anchorage zone in post-tensioned prestressed concrete beams. Of the very few attempts made on the design of the transfer-zones in pretensioned prestressed concrete beams those of Guyon (10) and Marshall & Mattock (28, 35) require mention, and are discussed in detail in one of the following chapters.

This project deals with the investigation of end-zone stresses in pretensioned prestressed concrete beams at transfer. The worst condition of stresses in these end-zones occur at the time of transfer of the prestress. A knowledge of these stresses is, therefore, absolutely essential for designing the ends of such beams. Several theories and analyses about this stress set-up and empirical methods for design have been put forward for the anchorage zones of post-tensioned concrete beams. But very few of these are applicable to the pretensioned prestressed concrete beams.

Object of the present investigation

The object of the present investigation is the study of the nature of stresses that occur at transfer in the ends of pretensioned prestressed concrete beams under different conditions, and the comparison of the results obtained from the experiments with the theoretical results based on various theories available and thereafter recommendation of a simple, analytical method for finding out these stresses; the investigation of the various causes of cracking and its remedies. The design of the end-stirrup reinforcement has also been discussed.

Scope of the work

The investigation was carried out in the Concrete Laboratory of the Department of Civil Engineering at the University of Glasgow.

Twenty-three beams were tested, of which five were for the preliminary exploratory work. The rest were divided into two series, i.e. Series A, consisting of beams with the prestressing wires mainly concentrated in the bottom of the section and Series B consisting of beams with the prestressing wires equally divided between the top and the bottom of the section.

CHAPTER 2

DISCUSSION ON PREVIOUS WORKS AND INVESTIGATIONS

CHAPTER 2. PREVIOUS WORKS AND INVESTIGATIONS

Very little work has been carried out on the problem of stress-distribution in the ends of pretensioned prestressed concrete beams at the time of application of prestress. Some useful data are available from studies on relevant fields, the discussion of which will be of considerable importance here.

Photoelastic models were used to investigate the bond stresses and the stress along the faces of a pretensioned prestressed concrete beam due to the application of prestressing forces, by H. H. Racke (15, 16), and to find out the stresses in the end-anchorages of post-tensioned beams by S. P. Christodoulides (13, 17, 18, 20). Although very little information is available from Racke's experiments, he has pointed out that the zone of heaviest stress concentrations in the ends of a beam is that surrounding the wires. He has also noticed that the theoretical peak bond stresses are eliminated by a redistribution of stress.

Christodoulides carried out his tests on models of actual end-blocks of varied sections. The principal stresses in two-dimensional and three-dimensional models were calculated from values of the shears and the directions of the principal stresses observed photoelastically. This was achieved by using the equations of equilibrium to integrate along a line starting from a point where a stress was known. The concrete surface strains around the end-anchorages of a concrete beam, which was one of the precast members used in the erection of a railway overhead travelling gantry, were measured by a mechanical strain gauge, and by electrical resistance strain gauges stuck on the concrete. The three-dimensional stresses computed from the strains agreed reasonably well with those obtained photoelastically. His main conclusions from these experiments can be summarised as follows:-

The stress distribution obtained by using simplified two-dimensional photoelastic models representing end-blocks of prestressed concrete beams did not agree with previous experimental work and the theoretical solutions by Magnel and Guyon. In the case of two anchorages the photoelastic results are confirmed by a simplified mathematical solution.

The use of photoelastic models to represent prestressed concrete end-blocks was justified and the effect of Poisson's ratio was found to be negligible by comparing the results of the three-dimensional photoelastic analysis carried out by the "frozen stress" technique and the full scale tests on concrete beams where surface and internal strains were measured, and used to calculate the stresses.

The transverse tensile stresses around the end anchorages of the three-dimensional models were found considerably greater than the corresponding stresses obtained by Guyon and Magnel.

The influence of different types of cable ducts and embedded anchorages used in different post-tensioning systems on the anchorage zone stress-distribution was neglected by Christodoulides because of technical difficulties. But the importance of it was realised by Ban, Nuguruma and Ogaki (21), so they confined their work to Lee-McCall system only, and tried to study the influence of some specific factors upon the stress-distribution as well as upon cracking and ultimate load. In the tests carried out on small rectangular concrete end-blocks, the strain distribution showed fair agreement with that given by Bleich-Sievers' theory but deviated from those given by Guyon's and Magnel's theories. The agreement seemed very much influenced by the value of Poisson's ratio. Ban, Nuguruma and Ogaki are of opinion that the amount of transverse reinforcement has a considerable effect on the cracking and ultimate loads.

The Cement and Concrete Association, London, has carried out some full-scale investigations in order to throw some more light on the problem of the stress-distribution in the anchorage zones of post-tensioned prestressed concrete members and to prepare a simplified procedure for the design of the ends of such members. In this test programme, Zieliński and Rowe (29, 31, 33, 36) have dealt with (a) the stress-distribution in single, axially loaded end-blocks, (b) the behaviour of rectangular and I-section end-blocks subjected to up to five concentrated loads, and (c) the tensile stress-strain characteristics of concrete under complex stress conditions.

In (a) only three types of rectangular prism concrete specimens were used and all of the post-tensioning systems available in Great Britain at the time of tests were considered.

In (b), (i) eleven tests with symmetrical and eccentric loading were carried out on four I-section specimens; (ii) nine tests with symmetrical and eccentric loading were carried out on three rectangular section end-blocks.

The principal conclusions, drawn by Zieliński and Rowe, from these tests are as follows:

In individual end-blocks with single symmetrically applied loads -

(1) The distribution of the transverse stress and the ultimate load of an end-block are not significantly affected by the anchorage being either embedded or external, by the material of the anchorage, or by the method of anchoring the tendons.

(2) The dominant factor in the distribution of transverse stress and the ultimate load is the ratio of the loaded area to the gross cross-sectional area, but the latter does not affect the positions at which the maximum tensile stress and the zero transverse stress occur.

(3) The maximum transverse stresses, which always occur on the central axis of the prism, are considerably greater than those predicted by any existing theories, but the values of transverse tensile force are in agreement with those found experimentally by Christodoulides.

(4) The percentage of reinforcement has a significant effect on the bearing capacity of end-blocks. Of the types of reinforcement commonly recommended, helices were found to be more efficient than mats.

In end-blocks with groups of anchorages -

(5) The cross-section of the end-block and that of the beam adjoining the end-block have a significant effect on the stress-distribution by virtue of the deep beam behaviour.

(6) Tensile zones exist between the applied loads and near the loaded face of the end-block; these are a function of the distance apart of the applied loads.

(7) In a rectangular end-block with a depth-length ratio of unity, on an I-section beam loaded with a single eccentrically applied force, the tensile force in the tensile zone nearer the loaded face increases with decreasing values of the ratio of the loaded area to the cross-sectional area of the prism.

(8) In end-blocks, by virtue of the complex stress system that exists, the stress-strain relation of the concrete in tension is modified, the strain capacity prior to cracking being greater than in normal bending tension. In these circumstances, it would seem logical to increase the apparent permissible stresses in tension.

(9) A comparison between various deep beam theories and the experimental results indicates that there is reasonable agreement with regard to the position of the upper and lower tensile zones but not with regard to the magnitude of the tensile stresses.

Zieliński and Rowe have made recommendations for the design of the end-blocks and finally they have suggested some further experimental work (a) to clarify the problem of the most efficient type and arrangement of reinforcing steel to meet the wide variety of possible conditions; and (b) to investigate the effect of different lengths of end-block on the stress-distribution.

Douglas and Trahair (30), in an attempt to determine the effect of high concentrations of stress in the anchorage zone of a prestressed concrete beam, idealized their problem as that of a circular concentrated load acting at one end of a concrete cylinder. They have described a general method for evaluating the complete elastic stress distribution, and in order to verify the stress distribution thus obtained, they carried out test Series A and B on concrete cylinders, loaded by steel plungers and found that the failure loads were about four times greater than those predicted on the basis of the current theories of failure. Hence they investigated in a more direct way (series C) by measuring the surface strains with electrical resistance strain gauges. These observed strains agreed well with the theoretically predicted ones. The large discrepancies between the predicted ultimate loads and the test results suggest the inadequacy of the theories of failure used. The correctness of the developed theory is proved as there is good agreement between measured surface strains and those found by this analysis. No evidence either theoretical or experimental was found of the spalling stresses postulated by Guyon. The effect of the central prestressing hole was to reduce significantly the ultimate stress carried by the cylindrical test specimen.

Rydzewski and Whitbread (37) have suggested an analytical method for evaluating the tensile stresses present within a short end-block and in the web of the adjacent I-beam section. They have defined 'short end-blocks' as end-blocks with a length/depth (L/D) ratio somewhat less than one; to distinguish them from longer end-blocks, for which design methods are available. Their investigation dealt primarily with two shapes of end-block. The first was geometrically symmetrical, carrying a symmetrical arrangement of prestressing forces, and had an L/D ratio of 0.56. The second was an actual asymmetrical end-block used at Narrows Bridge, Perth. Photoelastic models of these two blocks were made and model tests were carried out. The results obtained from these were compared with the analytical results and were found to be in reasonable agreement with each other. They have also presented a simplified procedure for the design of the short end-blocks.

Evans (6, 14) and some of his colleagues at the University of Leeds, carried out investigations on the bond stress distribution in pretensioned prestressed concrete columns and beams. Evans has suggested a simple formula to give the transmission length for wires. One of his most important remarks is that bond resistance results from friction. He is of opinion that the slip, steel strains, concrete strains and bond stresses for a pretensioned member are distributed in the transmission length according to an exponential relation on release of the prestressing force. The transmission length has been observed to increase with time as a result of creep and shrinkage.

According to G. Marshall's (4) observations, the transmission length for prestressing wires depends also on the placing of concrete. He has recommended the use of some form of mechanical locking device when structures of short length are to be prestressed by 0.2 in. diam. wires.

Janney (11, 12) studied the nature of bond near the ends of a pretensioned prestressed concrete member just after the release of the wire tension, using prismatic specimens. The principal variables considered were diameter, surface condition, and the degree of pretension of the wire. He has noticed a variation in the anchorage length and the general shape of the stress transfer distribution for wires of different diameters, for different surface conditions ranging from rusted to lubricated, and for concretes of different strengths. He points out that the prestress transfer bond is largely a result of friction between concrete and steel. This has been confirmed by an elastic analysis of the deformations occurring when pretensioned steel is released. The effects of time, fatigue, impact and vibration may considerably alter the picture, as his conclusions are based on results obtained immediately after the prestress was transferred to the concrete.

As most of the previous investigations to assess the transmission length in pretensioned prestressed concrete units have been carried out under laboratory conditions, Base (22, 26, 32) made an attempt to determine the variation in transmission lengths that occurs in normal site or factory practice. His investigation was based on a large number of prestressed units produced in different factories and laboratories . The types of prestressing steel used in the members were high-tensile steel wires, strands, and Macalloy bars of different diameters and with various surface conditions. His main concluding remarks are as follows:-

The important factors in obtaining a rapid build-up of strain are the strength of concrete and the compaction of concrete at the ends of the actual unit. The use of additional poker vibrators , near the ends of the units has been suggested. An insufficient compaction due to too great aggregate size or too great a number of wires, may lead to honeycombing at the ends of units which results in greater transmission lengths.

Most of the bond is due to friction caused by the swelling wire exerting a radial force on the walls of a hole.

In terms of diameters, the transmission lengths of 0.08, 0.2 and 0.276 in diameter plain and indented wire in the factory produced units were the same with a range between 50 and 160 diameters and an average of 100 diameters.

Too great a concentration of the prestressing wires can lead to the formation of horizontal shear cracks immediately above the wires, particularly in I or inverted T-sections. In such sections, it may be advisable to distribute some of the wires in the lower part of the web.

While discussing the secondary effects associated with the build-up of stress at the ends of the beams, Base points out that the formation of peak stresses near the ends of the units having heavy concentrations of wires or short transmission lengths led to considerable cracking along the bottom of the web in some of his inverted T-beams even when heavy reinforcement was provided along some length in the end.

The wires near the top of a unit sometimes have greater transmission lengths than the wires near the bottom, and tension cracks may then occur near the end of the unit.

These were commonly found particularly in I and inverted T-beams with end-blocks. If there is zero prestress in the top fibres of an I or inverted T-section, there is likely to be tensile stress in the top of the end-block which combined with the effects of unequal transmission lengths of the top and bottom wires, may be sufficient to cause transverse cracks in the top of the end-blocks.

The sudden release of wires by flame-cutting or other means leads to a great increase in the transmission length in the units near the releasing end of the bed.

Base has recommended the use of his method of determining the build-up of strains at the ends of pretensioned units in factories and sites to provide a quick check for their products. This is suggested because the transmission or transfer length is of considerable importance to the structural engineer, since in short members the working bending moments and in some thin sections the shear resistance may depend upon the transmission length. It is important, too, when pretensioned units are joined to form continuous structures.

Marshall and Mattock (28, 34, 35) recently investigated the cause of horizontal cracks, which result due to the stresses occurring in the ends of pretensioned prestressed, concrete girders at the time of transfer of prestress. The tests carried out on I-section beams were divided into two groups:- Series A, for the measurement of the end-zone concrete stresses at transfer consisted of ten short I-girders of same cross-section and prestressed by the same-size strand; the variables being web thickness, arrangement of prestressing strand, and surface condition of the strand. Series B was for the measurement of the stresses set up at transfer in the vertical stirrup reinforcement provided near the ends of the girders; in this series twenty-five girders were tested having two basic cross-sections and containing two sizes of vertical stirrup reinforcement, the other variables being the size and location of the prestressing strands and the magnitude of the prestressing force.

Marshall used an adaptation of Sievers' theory for the distribution of stress in the end-zones of post-tensioned prestressed girders to calculate the maximum vertical tensile stress for each specimen at various stages of transfer, and these values were compared with those obtained from experimental results. In case of the girders in which the prestressing strands were equally distributed at top and bottom flanges, the estimated and the observed stresses were in reasonable agreement. But for the girders having more strands in the bottom flange, the estimated stresses were much less than the observed stresses. This may be due to the fact that the influence of a considerable horizontal shear which exists at the c.g. level of the section in the latter arrangement, was not taken into consideration when developing the expression for vertical tensile stresses.

The maximum vertical tensile stress existed near the mid-depth of the web and in all cases, in which no cracks appeared, became zero, at a distance from the end-face not greater than about one-third of the girder depth, indicating that the most effective position for the stirrups to control cracking should be as close to the end face of the girder as is practicable.

Marshall (28) also carried out a series of experiments on small reinforced concrete end blocks to provide some more information about the problem of stress distribution in the end zones of pretensioned concrete beams. All of his specimens were 20 inches high, reinforced with bars on one or both faces placed with varying eccentricities. The load was applied to each of these blocks through a system of beams in a testing machine. The load is transferred to the concrete by the deformations on the bar and the lateral expression due to the Poisson's Ratio effect is nearly the same manner as strands in a pretensioned prestressed beam. The information obtained from these tests confirmed the results of the test Series A.

The object of the test Series B was the measurement of the stresses in the vertical reinforcement provided near the ends of the pretensioned prestressed girders at the time of transfer. From the results obtained from this series, Marshall establishes an empirical formula for the design of the end-zone reinforcement.

His main conclusions and recommendations are:-

(1) Greater tensile forces are set up and there is a greater tendency of cracking when the strands are distributed equally in the top and bottom flanges.

The magnitude of the maximum tensile stress set up under these conditions is approximately $\frac{9M}{bd^2}$ where M = the moment taken about the neutral axis of the prestressing forces and the resulting prestress.

b = width of the rib.

d = depth or height of the unit.

(2) This maximum tensile stress is approximately $\frac{18M}{bd^2}$ when the tendons are largely concentrated in the bottom of the beam.

(3) Draped strands when used are not responsible for the formation of cracks, as the detensioning forces from inclined tendons are less than from the horizontal ones.

(4) End-blocks may be omitted as they do not serve at all in the prevention of the cracks and are at the same time considerably expensive.

(5) As the cracks under consideration are very fine and short and they will not affect the performance of the girder, it would be sufficient to provide suitable end reinforcement to stop cracking.

(6) As the distribution and arrangement of the prestressing wires play a very important role in the phenomenon of stress distribution of transfer, the layout of the wires should be such that the moment taken about the neutral axis of the prestressing forces and the resulting prestress will be as small as possible. Equal concentration of wires at top and bottom flanges should be avoided. They should be as uniformly distributed over the whole section as possible.

(7) Care should be taken in detensioning the wires. The wires should be cut beginning with the one nearest to the neutral axis, and working outwards above and below it.

A co-operative inspection (39) of 88 existing prestressed concrete highway bridges was carried out in different states of the U.S.A., to study the service performance of this relatively new construction technique of prestressing and cracks were found in many of the beams at their ends. The majority of these beams were plant-cast, and were usually pretensioned, although some were entirely post-tensioned and others combined both pretensioning and post-tensioning. Beams entirely pretensioned had straight steel only or had straight steel combined with deflected steel. 58 of these were I-beams and 12 were T-beams. In his concluding remarks, Fountain (39) says that horizontal cracks observed at the beam ends should be expected in beams with widely spaced concentrations of prestressing steel. Vertical reinforcement should be placed as close to the end of the beam as practical, to resist the tensile stresses. The vertical cracks which were found in the bottom flanges near the ends of a few beams, are attributed to resistance by the forms to elastic shortening of the concrete when prestressing was applied. They may be eliminated by use of a form construction and manufacturing procedures which allow movement of the beam during application of the stressing force. Cracks found along the junction of the webs and flanges are thought to be caused by volumetric changes in the concrete or forms during manufacture of the girders. They can be eliminated by controlled casting procedures and the use of low slump concretes with well-graded aggregates.

Ramaswamy and Goel (23) tried to solve the problem of the evaluation of stresses in end blocks of prestressed beams, by treating the end-block as a deep beam and using a 64-square lattice in case of a single concentrated load acting symmetrically on it. On comparing the results obtained from their study with Guyon's results, the authors remark that:

The "Bursting" Zone in this analysis is larger in extent than that of Guyon's.

The maximum tensile stress in the bursting zone is 10% higher than that of Guyon.

The maximum tensile stress in the spalling zone is much smaller than that according to Guyon's theory.

Ramaswamy and Goel indicate the possibility of extending the method to cover the case of a non-central load and hence drawing influence lines for stresses in all directions.

Various other deep-beam theories have been put forward by Chow, Conway and Winter (9,24), Geer (27), Kaar (25), etc.

In only one instance (28), measurements of the end-stresses at the time of transfer were taken in pretensioned prestressed concrete beams. But Marshall carried out the detensioning process by cutting the wires in predetermined groups which is not the practice in Britain. In this country, the prestress-transfer process is usually done by releasing the wires together slowly and gradually up to the stage of complete transfer to the unit. The latter method has been adopted in all the tests carried out under the present investigation.

In most of the cases concerned with the stress distribution in the anchorage zones of post-tensioned beams, none of the existing theories gave a satisfactory assessment of the stresses. Those due to Bleich and Sievers gave the closest approximation. It is not definitely known whether this is also true with pretensioned beams.

Referring to the papers 33 and 36, the comments made by the authors Zieliński and Rowe, that, in the ends of prestressed beams, due to the complex stress set-up, the stress-strain relation of the concrete in tension is modified, the strain capacity prior to cracking being greater than in normal bending tension, seem to be quite interesting.

Fountain's (3) comments on the end-zone cracking in prestressed concrete beams, seems to be reasonable and to be studied carefully in the present investigation.

CHAPTER 3

EXISTING THEORIES

CHAPTER 3. EXISTING THEORIES

Notation

f_y	=	vertical tensile splitting stress.
M	=	resultant moment of prestressing force and prestress produced, about neutral axis.
S	=	shearing force to be resisted at a horizontal plane.
s	=	shearing stress due to the shearing force "S".
P	=	concentrated force imposed on the beam-section by a single prestressing wire.
T	=	total force applied by all the prestressing wires.
e	=	eccentricity of the force P, measured from the neutral axis.
f_x	=	longitudinal stress.
t	=	bonding force between prestressing wire and concrete per unit length.
t_o	=	maximum value of bonding force.
x	=	abscissa measured along the line of prestressing wire.
λ	=	the characteristic length of the elastic anchorage zone.
l	=	length of the anchorage zone = 2λ
d	=	overall depth of beam.
b	=	breadth of beam at neutral axis.
s'	=	distance along the abscissa, between the origin and the point M' in "f_y" - diagram in Guyon's theory.
F	=	Airy stress function in Bleich's theory.
s''	=	half of the depth of the beam in Bleich's theory.
h	=	half of the length of the end block in Bleich's theory.
r	=	ordinate of the line of thrust in Siever's theory.
m	=	$e - \frac{d}{4}$
η	=	$\frac{x}{d/2}$
	=	constant in Sievers' formula.
K_1 & K_2	=	constants in Magnel's formula.

x' = distance from the end-section of beam to a section $X'-X'$ along the line of the prestressing wire in Marshall's theory.

K = constant in Marshall's formula.

= co-efficient of friction between prestressing wire and concrete.

= Poisson's ratio for wire.

= Poisson's ratio for concrete.

E_s & E_c = Young's modulus for wire and concrete respectively.

r_w = radius of prestressing wire used.

S' = total stirrup force.

l_t = transmission length of the wire used.

(1) Stresses

The problem of high stress concentration over a small area subjected to forces, has drawn the attention of mathematicians and technologists for a long time. The application of it to the comparatively newly developed structural technique of prestressed concrete has started only recently, with the work of the late Professor Magnel in 1949. From then onwards, up to the present day, several theories have been put forward, several investigations have been carried out. Of all the theories available on this subject those of Magnel, Guyon, Bleich-Sievers and Marshall are the major ones and have been recommended as a basis for the design of the ends of prestressed concrete beams. An attempt will be made in this chapter to present those theories in a simple and explanatory way.

Magnel's Theory (3, 5)

In his theory for the computation of the tensile principal stresses in the ends of a prestressed concrete beam, Magnel considered a plane AB [Fig. 1(a)] in the rectangular end block of a beam. The plane AB is parallel to the axis of the beam and normal to the direction of action of the load.

In developing this theory he assumed a bending moment "M" and a shearing force "S" to be resisted at this plane and that the tensile stress diagram due to "M" is of the shape of a cubic parabola [Fig. 1(b)]. The main two axes being Ox and Oy, the equation of the cubic parabola is

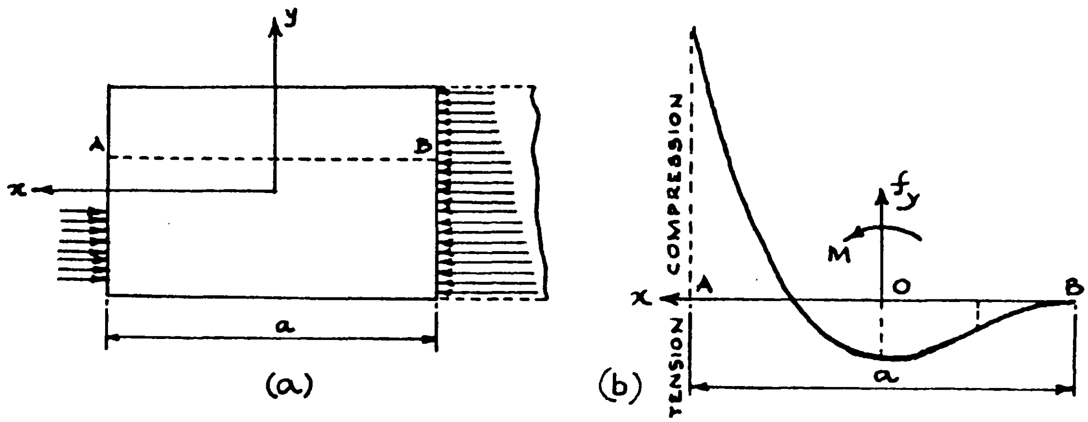

FIG. 1. END-BLOCK AND TRANSVERSE STRESS-DISTRIBUTION IN MAGNEL'S THEORY.

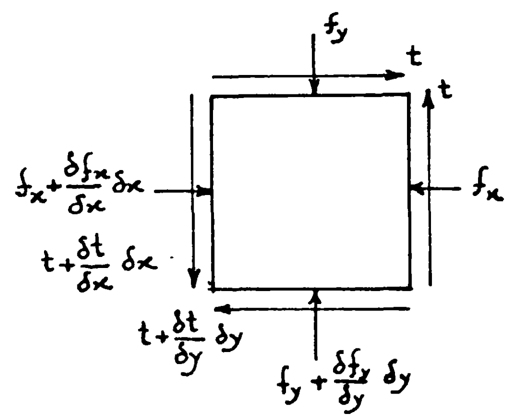

FIG. 2. EQUILIBRIUM OF INFINITESIMAL ELEMENT IN MAGNEL'S THEORY.

$f_y = A + Bx + Cx^2 + Dx^3$ ..... (1). The values of the coefficients A, B, C and D may be obtained from the following conditions:-

$$\frac{dy}{dx} = o \quad \text{and} \quad f_y = o, \quad \text{when } x = -\frac{a}{2} \;;$$

$$\int_{-\frac{a}{2}}^{+\frac{a}{2}} f_y \cdot b \cdot dx = o; \quad \text{and} \qquad \int_{-\frac{a}{2}}^{+\frac{a}{2}} f_y \cdot x \cdot b \cdot d_x = M.$$

where "b" is the width of the beam.

Hence, $A = -\frac{5M}{ba^2}$, $B = o$, $C = \frac{60M}{ba^4}$ and $D = \frac{80M}{ba^5}$

Therefore, $f_y = \frac{5M}{ba^2}\left(-1 + \frac{12x^2}{a^2} = \frac{16x^3}{a^3}\right) = K_1 \frac{M}{ba^2}$ ..... (1a). *

This curve is horizontal at $x = o$ and $x = -\frac{a}{2}$ and the point of inflection is at $x = -\frac{a}{4}$.

When the law of variation of "f_y" is known, the shearing stress "s" can be evaluated from the equilibrium condition for an infinitesimal element, as shown in Fig. 2.

Hence, $s = \frac{5S}{ba}\left(\frac{1}{4} + \frac{x}{a} - \frac{4x^3}{a^3} - \frac{4x^4}{a^4}\right) = K_2 \frac{S}{ba}$ ..... (2).

The values of compressive stresses "f_x" on planes normal to Of_y must be determined to have a complete knowledge about the stress distribution, but the exact values for those cannot be determined.

In order to give a reasonable evaluation, Magnel assumed that the pressure under the anchorages of the cables disperses at an angle of 45° into the end of the beam and that at each vertical plane the ordinary laws of eccentric compression apply. In Fig. 3 the total force imposed by the cable is P and CDFE represents the diagram of the corresponding compression under the anchorages. If EG and FH are drawn at 45°, FHJIGE represents the end block. In a plane KL, "P" acts with an eccentricity "e" where "M" is the centroid of the plane KL.

All planes between NH and IJ have the same diagram of stresses "f_x", except those close to IJ, because the beam on the right-hand side of IJ is generally I-shaped, whereas the end-block is rectangular. Thus on IJ the stress-distribution is like that on the I-section and only at some distance to the left of IJ the stress has dispersed through the end-block and become the same as on the plane NH. According to Magnel the knowledge of the exact stress distribution is not so important as he assumed that the dispersion takes place in the right-hand quarter of the end-block and in this quarter "f_y" and "s" are not important.

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* See page 23 for author's extension of Magnel's Theory.

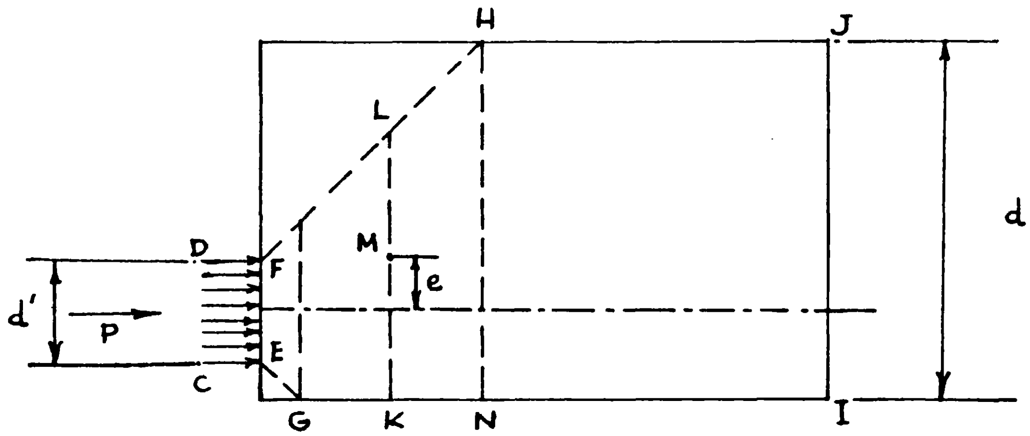


FIG. 3. END-BLOCK IN MAGNEL'S THEORY.

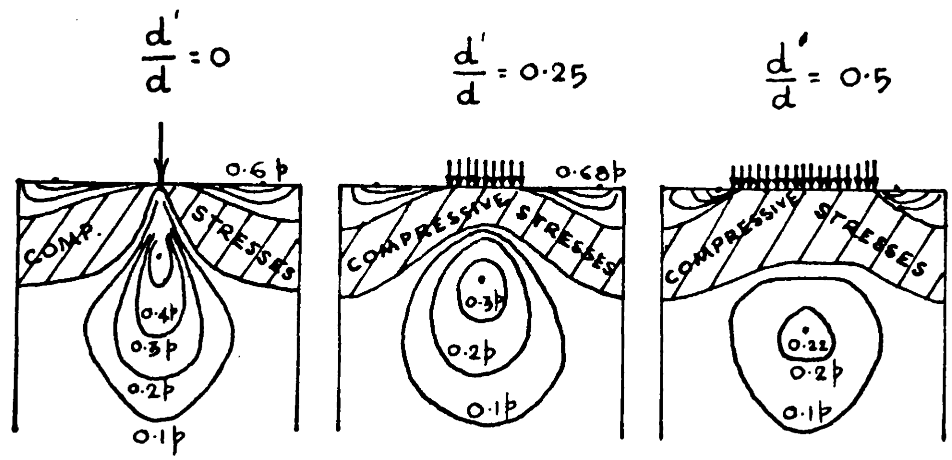


FIG.4. END-STRESSES DUE TO THE APPLICATION OF PRESTRESS WHEN $p = \frac{P}{2d}$, ACCORDING TO GUYON'S THEORY.

From the foregoing the stresses "f_y", "f_x" and "s" at any point in the end block can be determined and Mohr's circles for these points can be drawn to determine the tensile principal stresses, or the principal stresses can be calculated from the ordinary formulae.

A modification of the above theory can be done by assuming that the distribution of transverse stresses along the central axis is according to a parabola of a second degree and by expressing "f_x" not as distributed at 45° as Magnel did, but as a function of the "f_y" distribution. Chaikes (7) assumed

$$f_y = Ax^2 + Bx + C \quad \ldots\ldots (3).$$

From the boundary conditions,

$$\frac{dy}{dx} = o, \quad \text{when } x = -\frac{a}{2}$$

$$f_y = \frac{12M}{a^2}\left(-1 + \frac{4x}{a} - \frac{3x^2}{a^2}\right) \quad \ldots\ldots (3a).$$

Again the shear stress "s" from the equilibrium conditions for the element, shown in Fig. 2

$$s = \frac{12S}{a}\left(-\frac{x}{a} + \frac{2x^2}{a^2} - \frac{x^3}{a^3}\right) \quad \ldots\ldots (4).$$

The longitudinal stress $f_x = f_{x_o} - (f_{x_o} - f_{x_a})\left(-\frac{6x^2}{a^2} + \frac{8x^3}{a^3} - \frac{3x^4}{a^4}\right) \ldots\ldots (5)$,

where $f_{x_o} = \frac{P}{a_1 b_1}$ and $f_{x_a} = \frac{P}{ab}$, a_1 & b_1 being the anchorage plate dimensions.

<u>Guyon's Theory (10)</u>

Guyon's interpretation of the stress-distribution in the anchorage zones of post-tensioned beams is different from that in the ends of pretensioned beams at transfer. In this chapter, only the latter has been discussed.

According to Guyon, the stresses that come into play in the transfer-zones of pretensioned beams consist of the following:- (Ref. Fig.4)

(i) <u>Inter-wire tensions</u> which are made up of bursting tensions in the mass of the beam (maximum on the axis of each wire) and spalling tensions acting on sections very near the surface (maximum at the middle of the spaces between the wires)

(ii) <u>Inter-group tensions</u> which are due to the action of each group of wires, relative to the general equilibrium in the same manner, as the cone anchorage , except that the forces are distributed in depth along the anchorage length.

When the wires are distributed in the end-section in the same way as is required throughout the length of the beam, the only tensile forces that come into play are the inter-wire tensions. But when the wires are grouped in bundles the inter-group tensions also exist in addition to the inter-wire tensions.

There are also the effects of the swelling of the wires (Poisson's effect). Referring to Fig. 5. in order to discover the law according to which the transverse stresses "f_y" vary along the x-axis, Guyon assumes that "f_y" on a horizontal section through the wire is uniform in a given yz-plane, over the whole width of the beam.

As the distribution of "f_y" along Ox depends on the anchorage length in the case of elastic bond with plain wires,

$$t = t_o\, e^{-x/\pi} \quad \ldots\ldots (6)$$

where t = bonding force per unit length.
t_o = maximum value of bonding force.
x = distance along Ox-axis.
π = the characteristic length of the elastic anchorage zone.

Guyon replaces the above equation by the following simpler linear expression, giving approximately the same area under the curve,

$$t = \frac{2T}{l}\left(1 - \frac{x}{l}\right) \quad \ldots\ldots (7)$$

where T = total force applied by the wire.
l = length of the anchorage zone = 2π .

Now in determining the distribution of "f_y", Guyon considered the "f_y"-diagram for a concentrated force "P" acting along the axis of the prism of height "d" and width "b" (Ref. Fig. 6).

It can be seen in Fig. 7 that the tensile stress "f_y" at M' on the wire is given by the sum of elemental stresses set up by the elemental forces "t.dx" acting between the end and the point M'. Moreover, the total force, "f_y" at M' will be due to the forces t.dx in the length d. If a force "P" sets up a stress "f_y" at a distance "x" from the point of application of "P" and on its axis then $f_y = \frac{P}{db} f(x)$ where $f(x)$ is some function of 'x', "f_y" at M due to the force "t.dx" is $df_y = \frac{tdx}{db} f(x')$

The total "f_y" at M' is the sume of these elements for all positions of t.dx, between $x = s'-d$, and $x = s'$.

If, $d < s' < l$

$$f_y = \int_{s'-d}^{s'} \frac{tf(x')}{db}\,dx = \frac{2T}{dbl} \int_{s'-d}^{s'} \left(1 - \frac{x}{l}\right) f(x')\,dx \quad \ldots\ldots (9)$$

$$f_y = \frac{2T}{dbl}\left[\int_d^o \left(1 - \frac{s'}{l}\right) f(x')\,dx' + \frac{1}{l}\int_d^o x'f(x')\,dx\right]$$

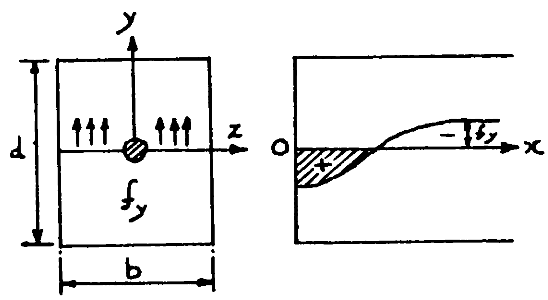


FIG. 5. BURSTING TENSIONS PERPENDICULAR TO THE WIRE.

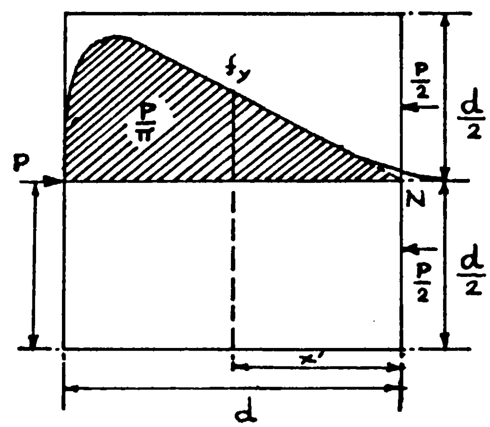


FIG. 6. "f_y" DIAGRAM FOR A CONCENTRATED FORCE "P" IN GUYON'S THEORY.

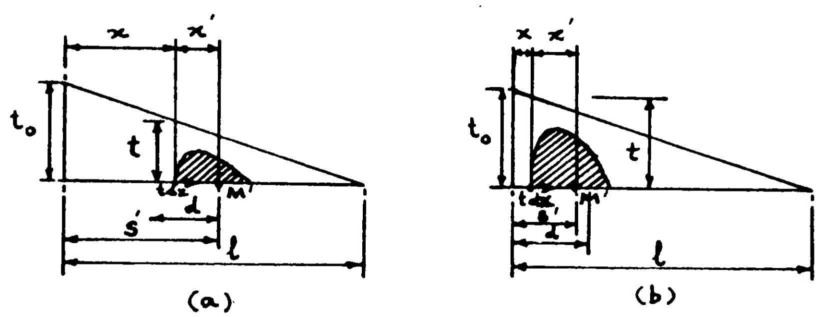


FIG. 7. "f_y" DIAGRAMS IN GUYON'S THEORY.

Now, $\frac{P}{dbl} \int_d^0 f(x')dx' = \frac{P}{db} \int_d^0 f_y dx' = 0$.

(taking into account the concentrated compressive stress at origin)

$$\int_d^0 f(x')\, dx' = 0 \;\ldots\ldots\; (a)$$

Also considering the general equilibrium of moments about N (Ref. Fig. 6)

$$\int_d^0 f_y \cdot dx' \cdot b \cdot x' = \frac{P}{2} \cdot \frac{d}{4}$$

or, $\frac{P}{d} \int_d^0 x' \cdot f(x') \cdot dx' = \frac{Pd}{8}$; or, $\int_d^0 x' f(x') dx' = \frac{d^2}{8}$ ..... (b)

From (a) and (b) $f_y = p \frac{d^2}{4l^2}$ .... (c)

where p = uniform compression, $\frac{"T"}{db}$ exerted by the wire on the area of concrete associated with it. "f_y" is thus a uniform tensile stress acting over the length from x=d to x=l-d.

When $s' < d$ [Fig. 7(b)] 'f_y' at origin is a compressive stress being expressed as,

$$f_y = \frac{2}{\pi} \; p \frac{d}{l} \;\ldots\ldots\; (11)$$

If $s' > l$, the stress decreases rapidly and becomes zero, when s = l+d.

The stesses due to spalling tensions as Guyon has pointed out are of less importance in bonded-wire prestressed units. The reason behind this is that the surface tension and end surface will be very small as the anchorage force has a longitudinal distribution.

The question of the swelling of the wire by Poisson's effect arises in case of plain smooth wires only. Although the swelling is very small, if the concrete is sufficiently hardened and the wires are closely arranged, the resulting stress is not negligible. However, in practice, in most of the cases, this swelling is automatically balanced by the decrease in cross-section which occurs due to initial tensioning of the wires.

As each group of wires acts, and interacts with others, in much the same way as the cone anchorage does in case of post-tensioned beams, a group effect is obtained and stresses result from it.

Bleich's and Sievers' Theories (1, 8, 19)

Bleich's (1) approach to the theory of stress analysis under concentrated loads is based on an Airy stress function 'f' such that (Fig.8)

$$f_y = \frac{\delta^2 F}{\delta x^2} \quad ; \quad f_x = \frac{\delta^2 F}{\delta y^2} \quad ; \quad t = \frac{\delta^2 F}{\delta x\, \delta y}$$

subject to the governing equation

$$\frac{\delta^4 F}{\delta x^2} + \frac{2\,\delta^4 F}{\delta x^2 \delta y^2} + \frac{\delta^4 F}{\delta y^4} = 0 \qquad \ldots\ldots\; (12)$$

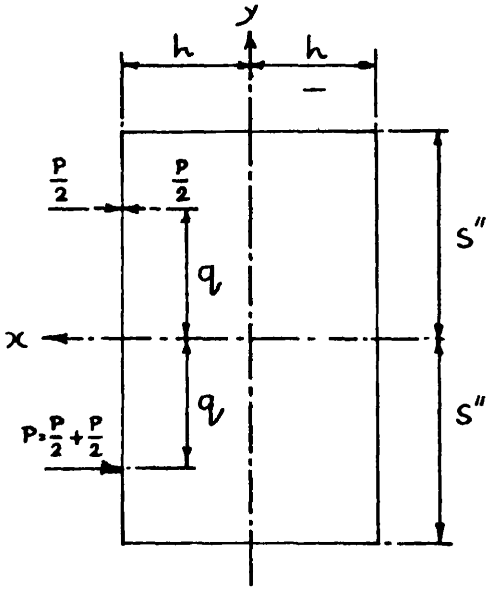


FIG. 8. SYMMETRICAL LOAD IN BLEICH'S THEORY.

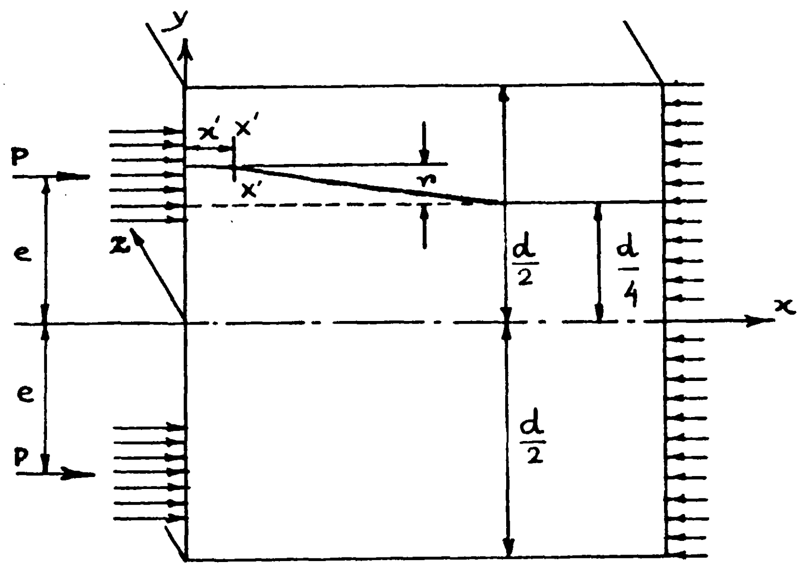


FIG. 9. NOTATION FOR SIEVERS'S FORMULAE.

He gives $f_y = 2 \frac{P}{s''} \sum_{n=1}^{\infty} \frac{(1-\beta_n h) \cosh \beta_n x + \beta_n x \sinh \beta_n x}{e^{\beta_n h}} \cos \beta_n \cdot y .$

..... (13)

For the case of symmetrical load, shown in Fig. 8, β_n being equal to $\frac{n\pi}{s''}$

The modified equation satisfying the boundary conditions, which should be used for plane y=0, is

$$f_y = 2 \sum_{n=1,3,5}^{\infty} \frac{P}{2s''b} \left[1 - \beta_n (h-x) \right] e^{-\beta_n (h-x)}$$

..... (14)

Using Bleich's results of the analysis of deep beams and following Mörsch's (2) assumptions and lines of thought Sievers (8, 19) approximated a formula for the transverse stress calculations which fulfilled the required boundary condition.

In his case a rectangular beam is prestressed uniformly by two equal forces "P", the eccentricity of the lines of action of each of these forces from the neutral axis, being 'e'. Although the resultant thrust is applied at the line of action of "P" at the end-section due to uniform prestress distribution the line of thrust must take such a shape that it finishes at a distance "$d/_4$" above the neutral axis at the end of the lead-in zone. (Ref. Fig. 9). The curvature of this resultant line of thrust must be governed by the fact that as the splitting tensions must be balanced by equal compressions, the total force along the neutral axis is zero.

This line of thrust can be presented by the following equation, one of the axes of reference being the horizontal line through "$d/_4$".

$$r = m (1 + \alpha\eta) e^{-\alpha\eta}$$

..... (15)

where r = ordinate of the line of thrust

$m = (e - d/_4)$

$\eta = \frac{x}{d/_2}$, x being abscissa.

α = a constant.

Now, $f_y = - \frac{P}{b} \frac{d^2r}{dx^2} = \frac{4mP}{bd^2} \alpha^2 (1 - \alpha\eta) e^{-\alpha\eta}$ ..... (16)

where f_y = the stress normal to the neutral axis

and b = the variable width at which the load acts in the direction of z.

Now, in order to make the value of "f_y" approximately equal to the more accurate determination of Bleich-Sievers takes $\alpha = 2.5$ and $\alpha^2 = 8$, and hence,

$$f_y = \frac{32mP}{bd^2} (1 - 2.5\eta) e^{-2.5\eta} \quad \ldots\ldots (17)$$

$$\text{or, } f_y = \frac{32M}{bd^2} (1 - 2.5\eta) e^{-2.5\eta} \quad \ldots\ldots (17a)$$

"M" being the resultant of the moments of the applied force "P" and the resulting prestress about the neutral axis and is equal to (P.m.) i.e. $[P(e - \frac{d}{4})]$

When x = o, i.e. "f_y" on the end face is given by

$$f_y = \frac{4x8M}{bd^2} = \frac{32M}{bd^2} \quad \ldots\ldots (17b)$$

Marshall's Theory (28)

Marshall (28) has used Sievers' theory for post-tensioned beams in determining the bursting stress in pretensioned beams and his modification of Sievers' theory is described as follows:-

x' is taken as the distance from the end-section to the section $X'-X'$, along the line of the prestressing wire (Ref. Fig. 9).

He expresses the equation of the line of thrust due to the force transmitted to the concrete by the element of wire at x' by

$$r = m \left[1 + \frac{2\alpha}{d}(x-x')\right] e^{-\frac{2\alpha}{d}(x-x')} \quad \text{where } x \geqslant x' \quad \ldots\ldots (18)$$

$$\text{Then } \frac{d^2r}{dx^2} = -\frac{4mP}{bd^2}\alpha^2 \cdot e^{-\frac{2\alpha}{d}(x-x')} \left[1 - \frac{2\alpha}{d}(x-x')\right] \quad \ldots\ldots (19)$$

Now, $P_{x'} = \frac{2P}{1}(1 - \frac{x'}{2}) \cdot dx'$ from Guyon's expression (7) where $P_{x'}$ = the force due to the element of wire at x'.

$$\therefore f_y = \frac{8Pm\alpha^2}{bd^2 1}(1 - \frac{x'}{2})\, dx' \left[1 - \frac{2\alpha}{d}(x-x')\right] e^{-\frac{2\alpha}{d} \cdot (x-x')} \quad \ldots\ldots (19a)$$

The above expression must be integrated between o to x' with $\alpha = 2.5$ and $\alpha^2 = 8$, as given by Sievers, in order to get the value of 'f_y' at any ordinate 'y' and proceeding in the above way finally, we get

$$f_y = \frac{64Pm}{bd^2 1} e^{-\frac{5x}{d}} \left[x + \frac{dx}{51} + \frac{d^2}{251} - \frac{d^2}{251} e^{\frac{5x}{d}}\right] \quad \ldots\ldots (19b)$$

This gives 'f_y' at any ordinate less than x'.

Marshall now assumes that the maximum value of 'f_y' will occur as with the post-tensioned beams, viz. when $x' - x = 0$, or $x = x'$.

Thus, $$f_y = \frac{64Pm}{bd^2 l} e^{\frac{-5x'}{d}} \left[x' + \frac{dx'}{5l} + \frac{d^2}{25l} - \frac{d^2}{25l} e^{\frac{5x'}{d}} \right] \qquad \text{..... (19c)}$$

Equating the first differential equation of the above expression (19c) to zero

$$x' = \frac{dl}{d+5l}$$ which is the condition for maximum 'f_y' .

Denoting a portion of the above expression (19c) by "K", it can be simply stated as:

$f_y = \frac{KmP}{bd^2}$, 'K' depending on 'l' and the breadth of the beam.

$$\therefore f_y = \frac{KM}{bd^2} \qquad \text{..... (19d)}$$

"l" can be obtained from the following expressions, as adapted by Marshall from Janney's (11) and Guyon's (10) theories:

$$\frac{1}{\lambda} = \frac{2\phi \mu_s}{r_w \left[1 + (1+\mu_c) \frac{E_s}{E_c} \right]} \qquad \text{..... (20)}$$

and $l = 2\lambda$ . ..... (21)

where λ = "characteristic length" of the elastic anchorage zone,

ϕ = coefficient of friction between steel and concrete,

μ_s = Poisson's ratio for the wire,

μ_c = Poisson's ratio for the concrete,

E_s & E_c = Young's modulus for the wire and the concrete, respectively,

r_w = radius of the wire.

Although Sievers' theory is based on symmetrical loads producing a uniform prestress, Marshall has suggested that this method may also be used in the case of asymmetrical loads producing a varying prestress, by calculating the correct value for M and then substituting it in expression (19d) where 'M' is the moment of prestressing forces and resultant prestress about the neutral axis.

In the equation, (19c), when $x = x' = 0$

$$f_y = \frac{64}{l} \cdot \frac{M}{bd^2} \qquad \text{..... (22)}$$

Author's Extension of Magnel's Theory

Magnel's theory, which is discussed earlier in this chapter is applicable only to post-tensioned prestressed concrete beams. The author suggests that it can be modified for use in pretensioned prestressed concrete beams, if "a" in Magnel's expression (1a) is taken as the transmission length of the prestressing tendons used in pretensioned beams.

Hence, from (1a)

$$f_y = \frac{5M}{ba^2}\left(-1 + \frac{12x^2}{a^2} + \frac{16x^3}{a^3}\right) = \frac{K_1M}{ba^2}$$

$$= \frac{K_1M}{bl_t^2} \quad \text{where } l_t = \text{transmission length of the prestressing tendons used} \qquad \text{........ (1b)}$$

From now onwards, the above expression will be referred to as the author's extension of Magnel's theory.

(2) Control of End-zone Cracking

The two remedies which have usually been applied to eliminate the cracks in the ends of pretensioned prestressed concrete beams are: firstly, the use of end blocks and secondly, the provision of vertical stirrup-reinforcement in the ends of the members.

It has been noticed in pretensioned concrete beams that, the end blocks, if provided, do not offer much help in minimising the high concentration of stresses that occur at the time of transfer. On the other hand, they add to the total cost of the structure.

A better remedy is to provide properly designed mild steel reinforcement in the ends of the beams than to provide end blocks. Two methods of determining the amount of the end-stirrup reinforcement is described as follows:

Method 1:

A simple rule for designing the end-stirrup reinforcement can be obtained based on the general expression for maximum tensile stress : $f_y = \frac{KM}{bd^2}$.

If it is assumed that the average tension equals half the maximum and that the tension only occurs over a distance $d/_4$ from the end, the total tension = $\frac{f_y \, db}{8}$ or $\frac{KM}{8d}$.

The area of web-reinforcement required in a distance $d/_4$, $A_w = \frac{KM}{8f_w d}$

..... (23)

where f_w = stress in the stirrups = 20,000p.s.i.

Method 2 (35):

The analytical study discussed earlier in this chapter shows that for a given cross-section the magnitude of the vertical tensile stresses before cracking is a function of the transmission length of the steel used and the prestress force. Marshall and Mattock carried out a set of tests to determine whether these same parameters also influenced the tension in the stirrup reinforcement after cracking and it was observed by them that for the range of transmission lengths covered by those tests, the ratio of the total stirrup force to the prestressing force could be taken as inversely proportional to the transmission length of the strand for a given cross-section and distribution of prestressing strand. The way in which the strands were distributed across the section was seen to have

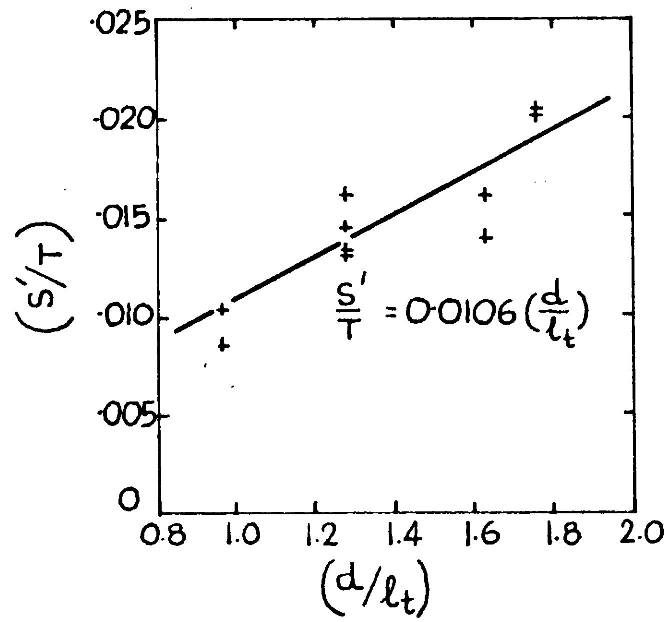


FIG. 10. RELATIONSHIP BETWEEN (s'/T) & (d/l_t) FROM MARSHALL & MATTOCK'S STUDY.

influence on the stirrup stresses. But ignoring the influence of the manner of distribution of the strand, the force in the stirrups is seen to be function of prestress force, strand transmission length, and depth of section. The ratio of the total stirrup force and prestress force was noted to be 0.0106 times the ratio between the depth of girder and transmission length, i.e. $\frac{S'}{P} = 0.0106 \frac{d}{l_t}$ where "S'", "P", "d" and "l_t" are the total stirrup force, prestress force, depth of girder, and transmission length respectively (Ref. Fig. 10).

Several checks were made by Marshall and Mattock on the possible influence on the stirrup forces of the manner of distribution of the strand and it appeared that the above equation can be used to calculate the total stirrup force regardless of the percentage of prestressing strand present at the top of the end-section of the beam.

The amount of stirrup reinforcement can thus be calculated by using the equation

$$A_w = \frac{S'}{f_{w/2}} = 0.021 \frac{P}{f_w} \frac{d}{l_t} \qquad \text{..... (24)}$$

where "A_w" is the total cross-sectional area of stirrups necessary, and "f_w" is the maximum allowable stress in the stirrups, the average stress being "$f_{w/2}$", since the stirrup stresses can be assumed to vary linearly from a maximum close to the end face of the girder, to zero near the end of the crack.

For design purposes "l_t" may be assumed to be 50 times the diameter of the prestressing steel.

While discussing the limitations of the applicability of the expression, it should be pointed out that it has been justified experimentally only for values of the ratio between depth of girder and transmission length of up to about '2'. As the ratio increases beyond '2' the expression will tend to become conservative, the degree of conservatism increasing as the ratio increases.

Further attention has to be given to the point that the design equation will tend to be conservative for beams in which the groups of prestressing steel are spaced more closely, or in which the prestressing steel is distributed uniformly over the end section of the beam.

This is simply because the equation had been developed from measurements made on beams in which the prestressing steel was divided into two groups placed in the top and bottom of the section. If the groups were placed over the end face then the stirrup forces would be less than those in the case considered in Marshall and Mattock's study.

C H A P T E R 4

EXPERIMENTAL WORK

CHAPTER 4. EXPERIMENTAL WORK

Test Beams

The tests were carried out in two series: Series A and Series B.

The cross-sections of the test beams are shown in Figs. 11 & 12. All test beams were 9 ft. 6 inches long.

In the Series A beams, the prestressing wires were mainly concentrated in the bottom of the section, the wire-distribution being two at the top and seven at the bottom. Each wire was stressed to 67 tons/sq.in. tension to give an average initial prestress on the cross-section of 975 lbs/sq.in.

In the Series B beams, the total prestressing force was kept the same as in the Series A beams but the prestressing wires were equally divided between the top and the bottom of the section, i.e., four wires at the top and four at bottom, the tension applied to each wire being 75 tons/sq.in.

The first beam which was cast to check the method of testing had rectangular end blocks, but the rest of the beams were of I-section without end blocks.

The prestressing steel used in all the test beams was 0.2in. diameter indented (Belgian pattern) high-tensile steel wire having a cross-sectional area of 0.0314 sq.in., an ultimate strength of 109 tons/sq.in. and an initial tangent modulus of elasticity of 28 x 10^6 lbs/sq.in. In all the beams, the wires were free of rust, and were cleaned of surface oil and grease before tensioning.

The stirrups, wherever used, were 0.2 in. diameter mild steel bars.

Fabrication and Test Procedure

The test beams were manufactured and tested one at a time in a short stretching bed set up on the concrete laboratory floor. The details of the pretensioning frame and the arrangement of the test set-up are shown in the Figs. 13 & 14.

Before the actual tensioning slacks were removed from the wires by stressing them by a Gifford-Udall-C.C.L. 1954 model single-wire jack, the stress range was 0-100 p.s.i. The wires were then tensioned all together by two Tangye 15-ton jacks (Ref. Fig. 15), built-in with the prestressing bed. The amount of extension applied was measured on two dial-gauges fitted to the two sides of the movable anchorage-block.

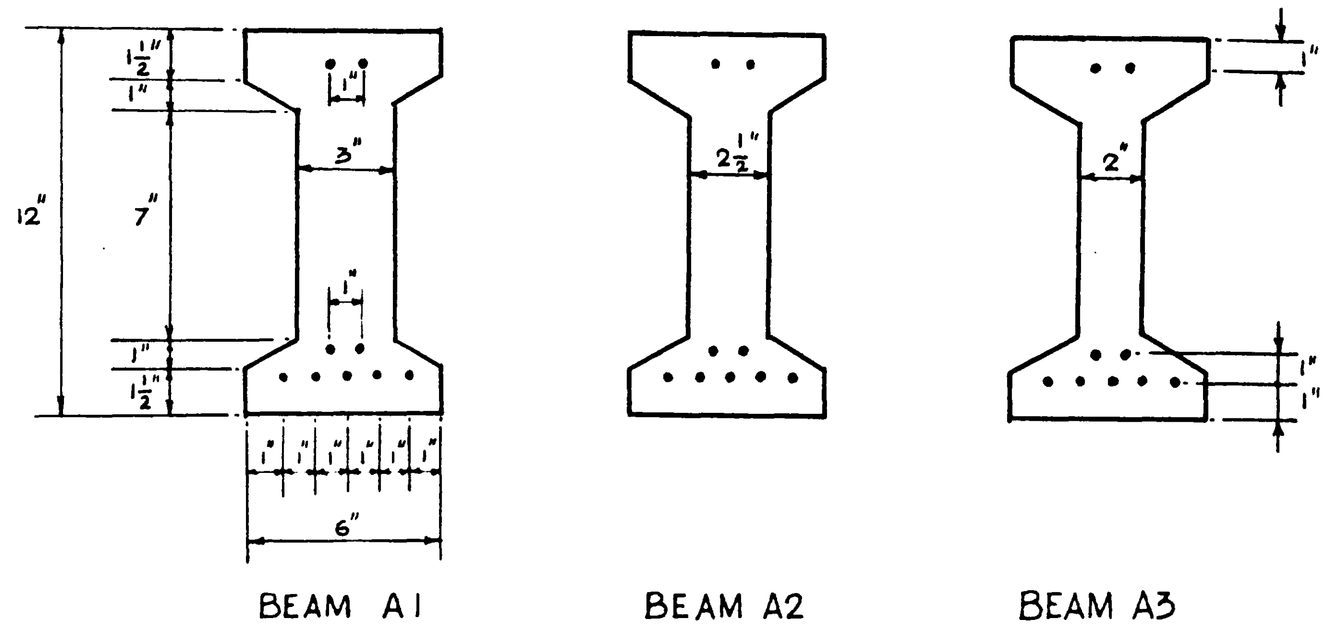


FIG. 11. SERIES A TEST BEAMS.

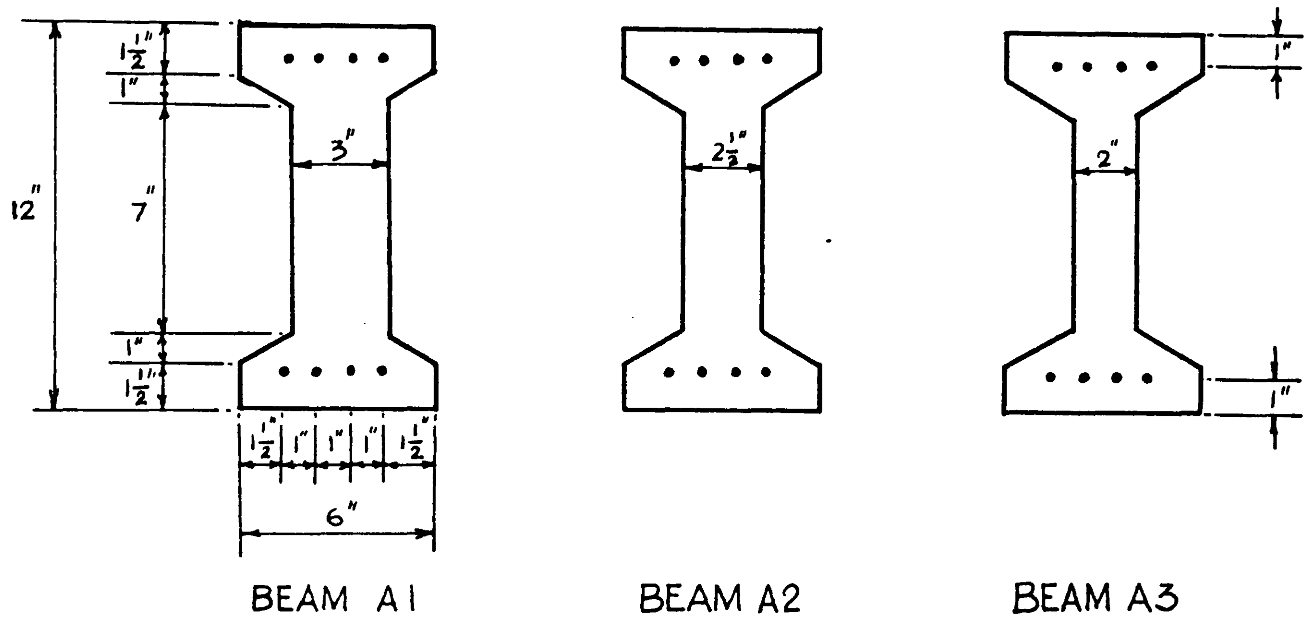


FIG. 12. SERIES B TEST BEAMS.

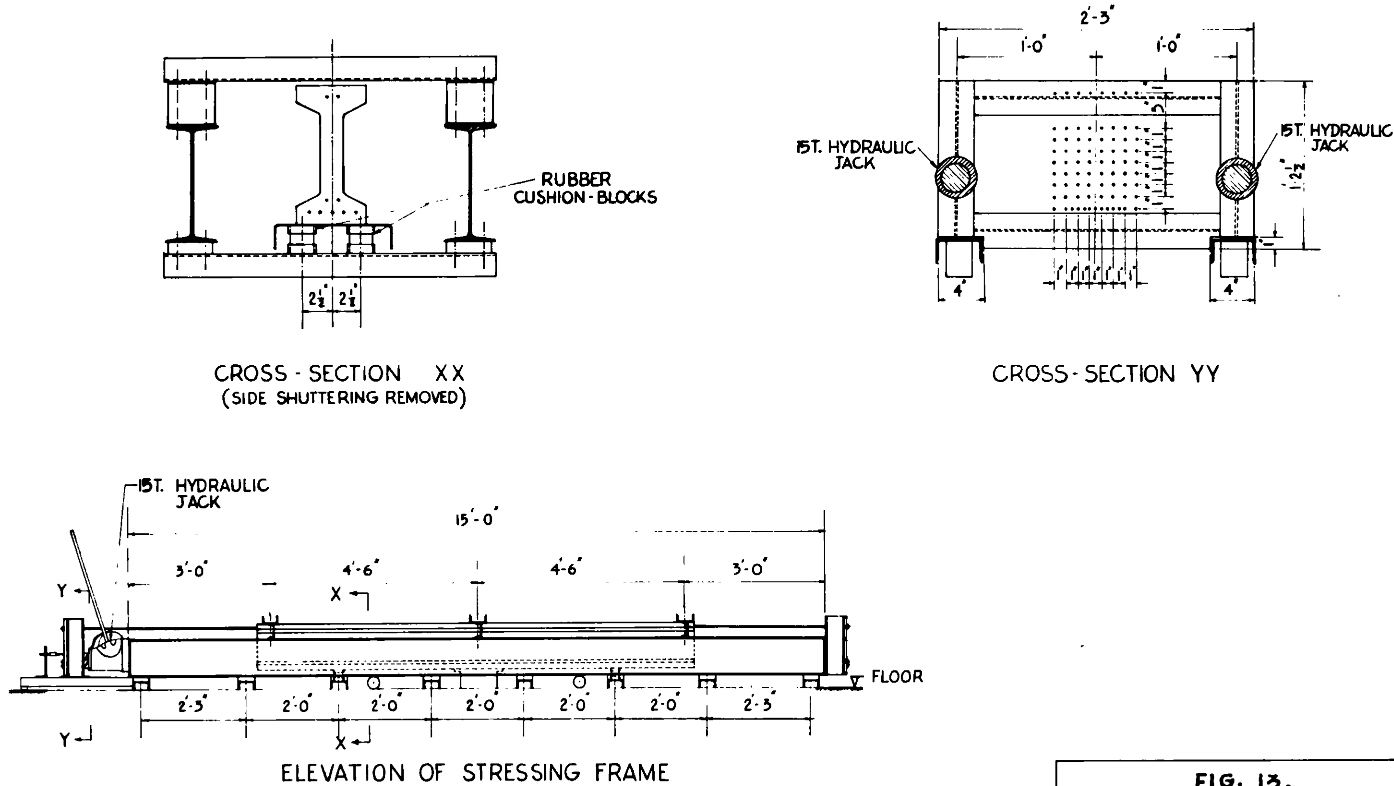


FIG. 13.
DETAILS OF PRETENSIONING FRAME

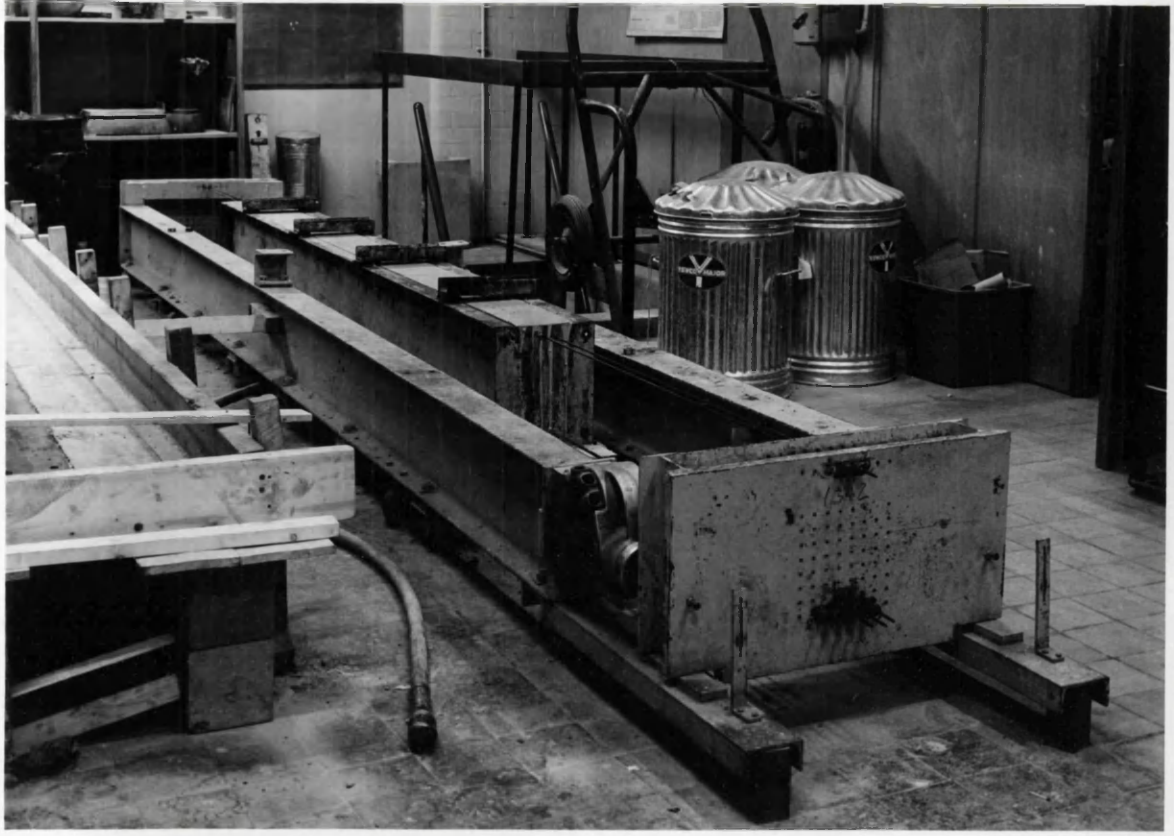


Fig. 14. A view of the test set-up.

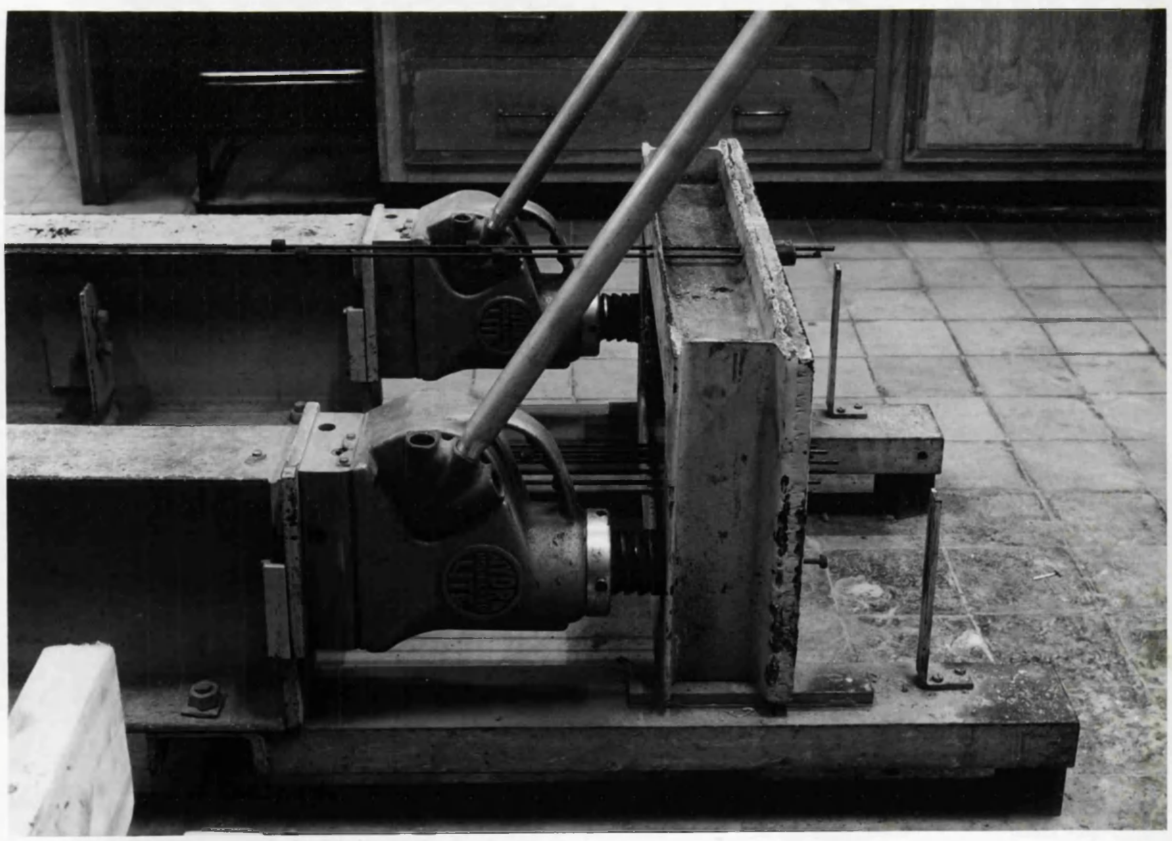


Fig. 15. Two Tangye 15-ton prestressing jacks.

The wires were overtensioned by approximately 5 percent, i.e. by 3 t.s.i. This overstress was maintained for two to three minutes and then reduced to the required initial prestress in order to minimise the loss in prestress due to the relaxation of the steel. The tension applied in the wires was checked by measuring the strain on an 8-inch gauge length on all accessible wires using a Demec gauge reading on collars attached to the wires (Fig.16).

The concrete used had an aggregate/cement ratio of 4.36 and a water/cement ratio of 0.47; the sand: coarse aggregate ratio was 1:3.8. The concrete was made with Ferrocrete rapid-hardening portland cement and $\frac{3}{8}$ in. coarse aggregate. Mid-Ross sand and gravel were used as aggregates. The concrete was cast the day after the stressing of the wires. The concrete in the beam was compacted by a Tremix vibrator bolted to the base of the mould and the average compacting factor was 0.86. A Kango hammer was used to compact the six 4-in. cubes which were cast along with each beam. The side-shutters were removed after 10 hours. Each beam was moist-cured under constantly watered jute bags for the first three days after casting. The cubes were cured under water at controlled temperature according to B.S. 1881.

On the fifth day after casting, the mechanical strain gauge points were mounted on the beam, the distribution of which is shown in the Figs. 16, 17 & 18.

When the beams were seven days' old, the prestress was transferred slowly in small equal steps. The amount of detension was measured by the rotation of the pointers of two dial-gauges fixed at the jacking end of the pretensioning frame. The concrete mix was designed so that the cube strength at this age was approximately 5500 p.s.i. The concrete cube strengths at transfer and at an age of twenty-eight days are listed in Table B. These concrete strengths are in each case the average of three 4-in. cubes.

The initial strain-gauge readings on the beams were taken before the transfer, readings were taken at every step of detensioning, and the final reading was taken when the transfer was over. In addition, in A2 and B2 beams, the vertical strains were measured twenty-four and forty-eight hours after the transfer to note the effect of time on those strain values. Readings were taken of the vertical strains on the end-faces and the side-faces and the horizontal strains along several lines on each of the side-faces.

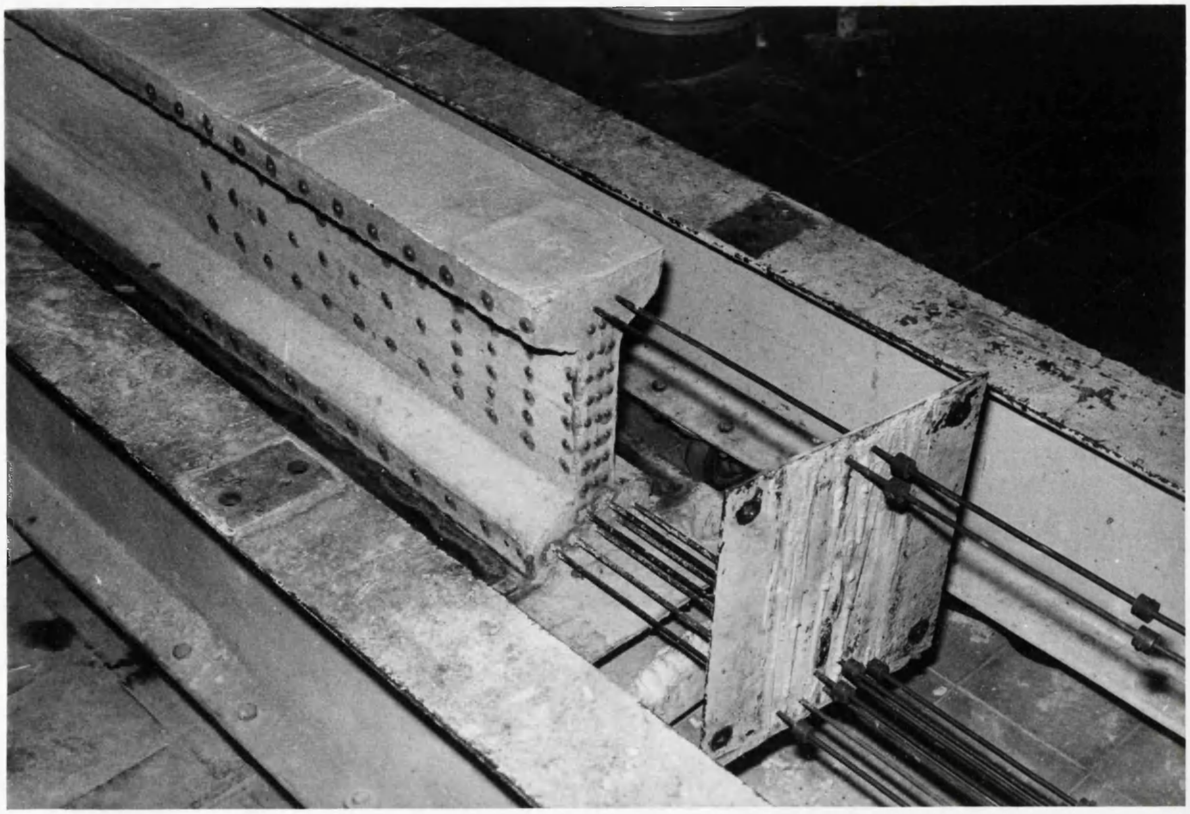


Fig. 16. A typical arrangement of the "Demec" gauge points in the ends of the beams.

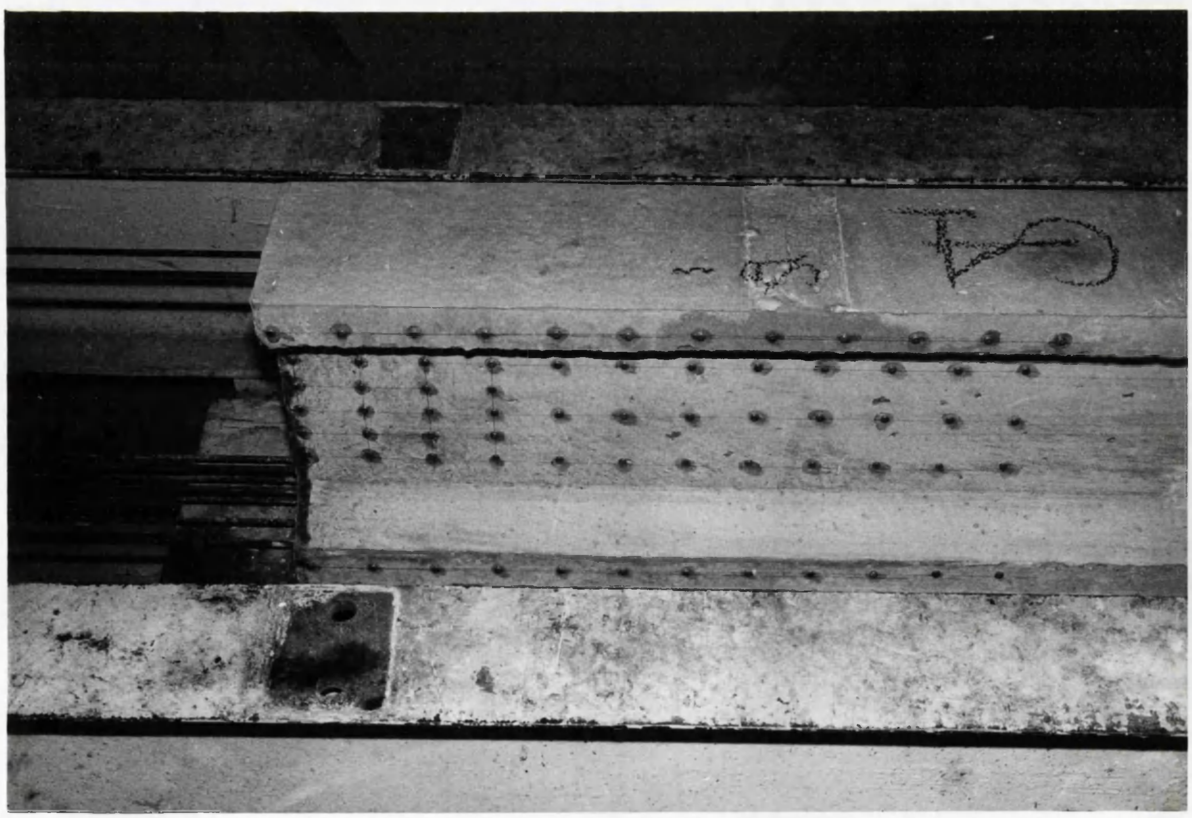


Fig. 17. Arrangement of the "Demec" gauge points on the side-faces of the beams.

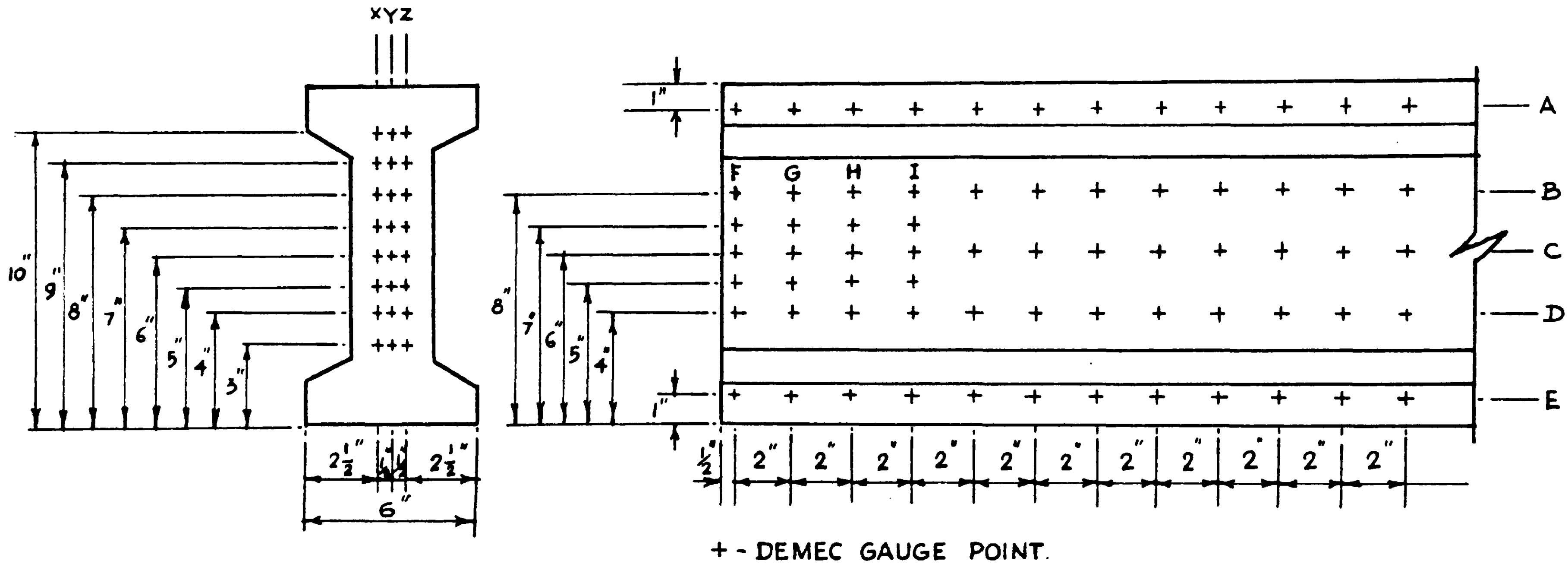


FIG. 18. TYPICAL ARRANGEMENT OF GAUGE POINTS.

CHAPTER 5

TEST RESULTS

CHAPTER 5. TEST RESULTS

Thirteen beams were tested in Series A, of which three were of the A1 type, another three of the A2 type and the rest of the A3 type (see Fig.11). Series B consisted of ten beams, three, each of the B1 and B2 types and four of the B3 type (see Fig.12).

A3(1), A3(2), A3(3), A3(4), and B3(1) were prelininary beams made to explore the test procedure. The beams A3(1), A3(2) and A3(4) were provided with vertical stirrups in the ends, while all the other beams tested in this project had no end-stirrup reinforcements. The spacing of the stirrups in the beams A3(1), A3(2) and A3(4) is shown in Fig.19.

The following factors were studied from the experiments:

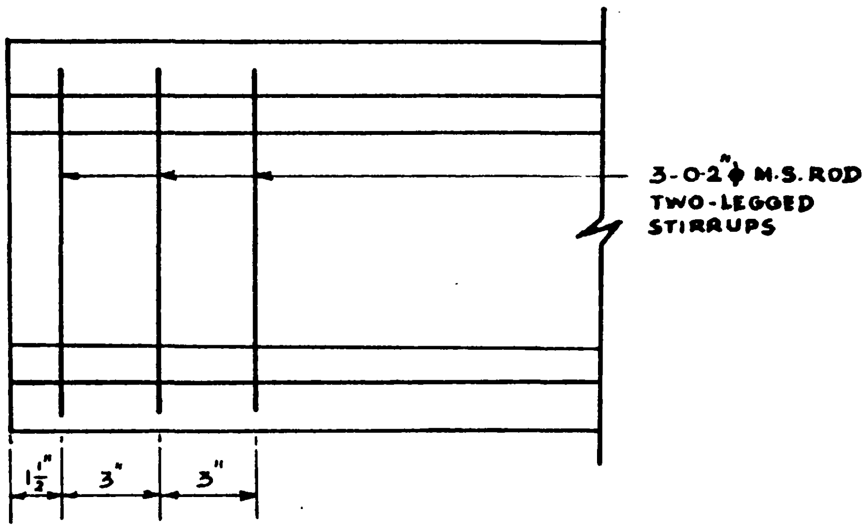
1. Build-up of horizontal strain along the sides of the beams and the transmission length of the wires used.
2. Vertical strain-distribution on the side and the end-faces of the beams and the effect of time on these vertical tensile strains.
3. The mechanism of web-cracking due to transfer-stresses, Various other causes of cracking and their respective remedies.

Figs. 20 to 25 are plots of the results obtained in cases 1, 2 and 3.

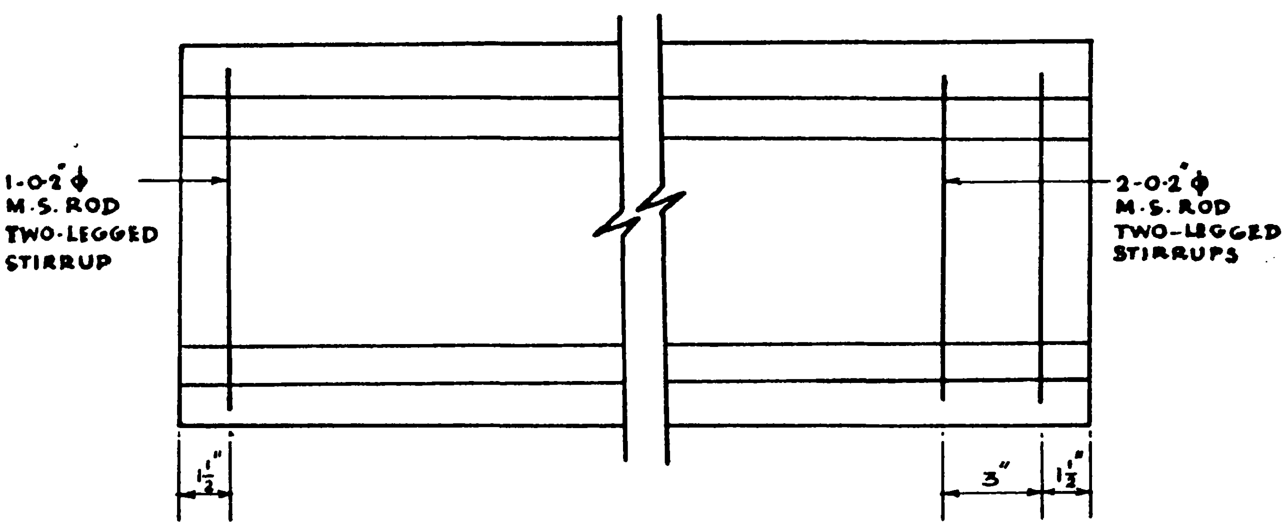
1. Horizontal strain build-up on the sides of the beams (Ref. Figs.20a-d)

A large number of gauge lengths were placed along horizontal lines, A, B, C, D and E, on each side of the beams (Ref. Figs. 17 & 18). Readings were taken on the gauge lengths immediately before and after the release of the stressing wires and the strains were then calculated. Curves were then plotted through the series of points obtained at 2-in. centres and these gave an indication of the build-up of stress in the concrete.

It is clear from Fig. 20a that the peak strain and the minimum strain are occurring at the level of the main group of prestressing wires and the top layer of steel respectively, in the Series A beams. While testing the first few beams, viz. A3(1) to A3(6), the build-up of horizontal strain was recorded for all the lines A, B, C, D & E, but as the strain values along the lines B & D were noticed to lie between those along A and C, and, C and E, respectively, it was decided later to cancel the lines B and D and to note the strain-distribution along the lines A, C and F only. The strain values on all the four sides of the beams were in agreement with each other.



BEAMS A3(1) & A3 (2)



BEAM A3 (4)

FIG. 19. SPACING OF VERTICAL END-STIRRUPS IN A3 BEAMS.

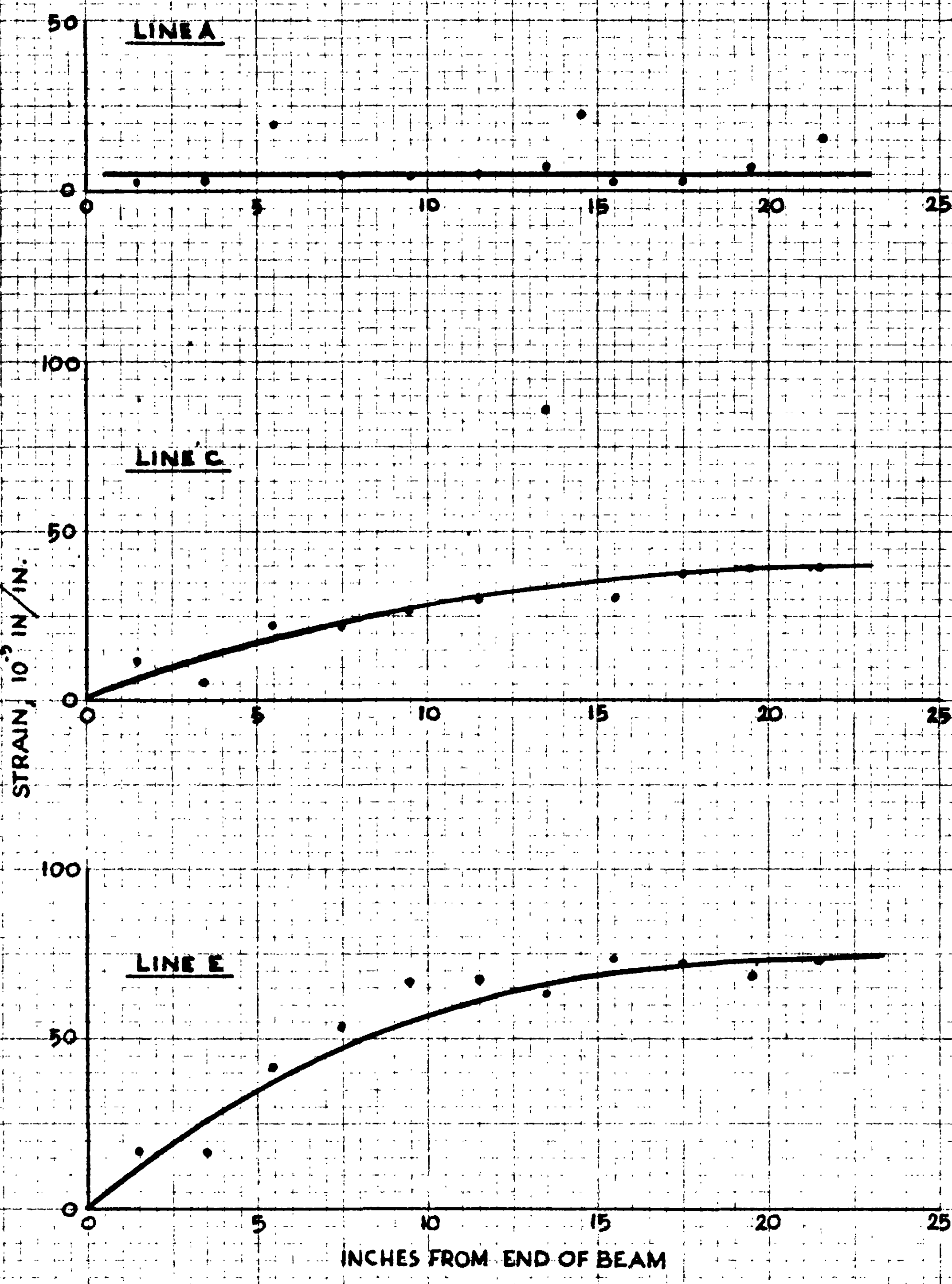


FIG. 20a. TYPICAL CURVES OF THE BUILD-UP OF HORIZONTAL STRAIN IN SERIES A BEAMS. [BEAM A2(2)]

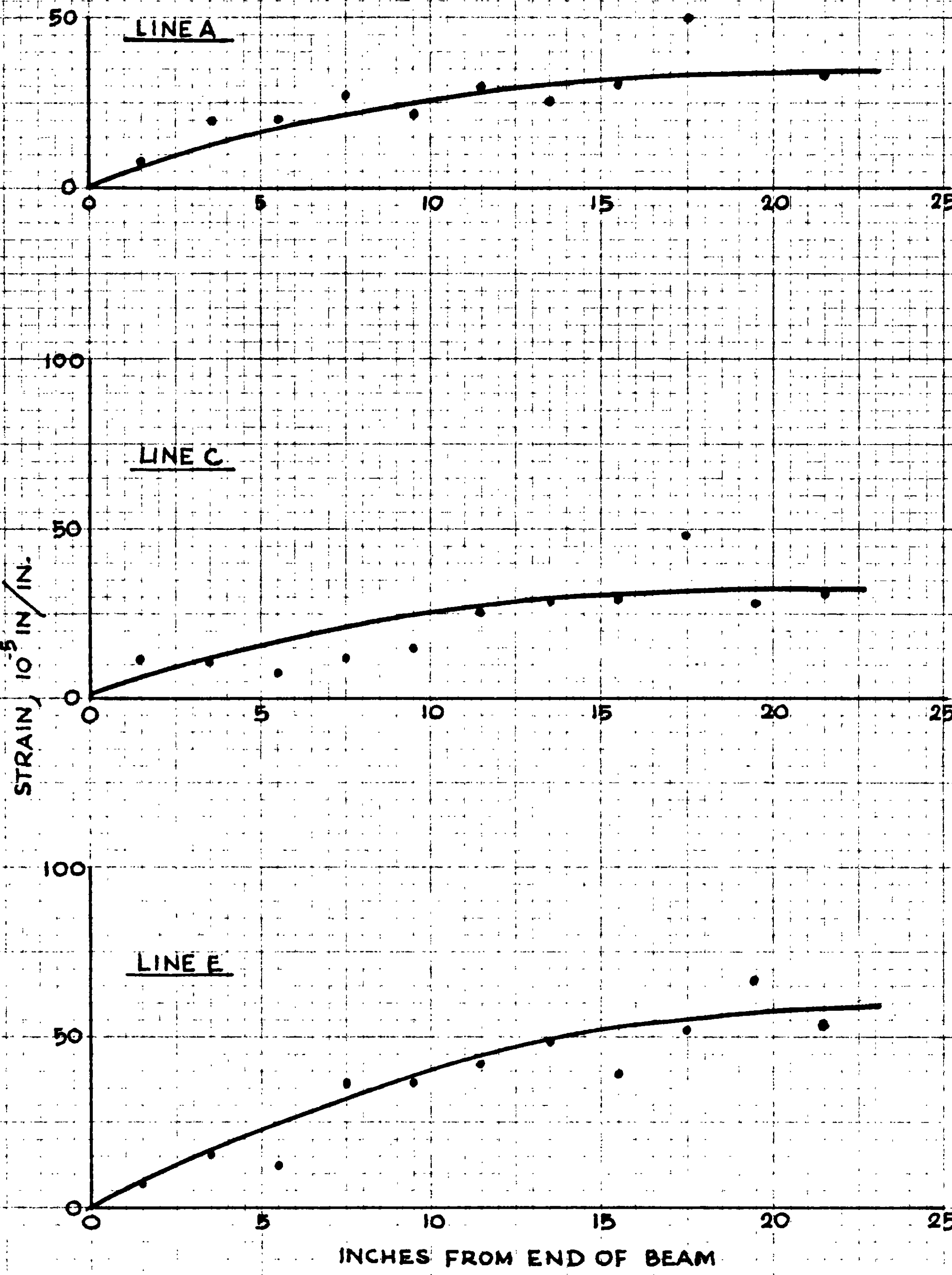


FIG. 20b. TYPICAL CURVES OF THE BUILD-UP OF HORIZONTAL STRAIN IN SERIES B BEAMS. [BEAM B1(2)]

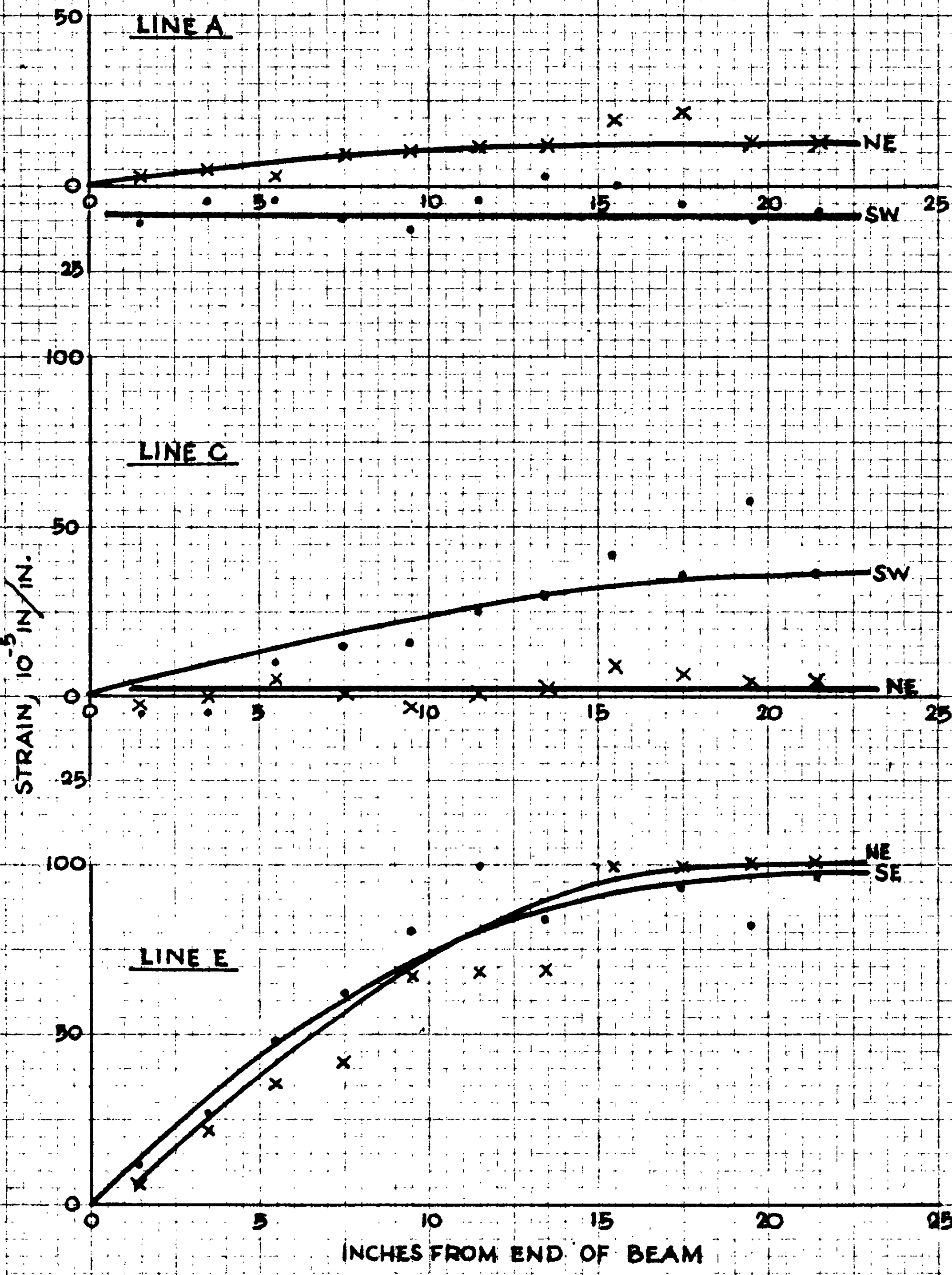


FIG. 20c. CURVES OF THE BUILD-UP OF ~~HORZ.~~ STRAIN IN BEAM A3(5). (CRACKED AT TOP FLANGE - WEB JUNCTION AT NORTH END)

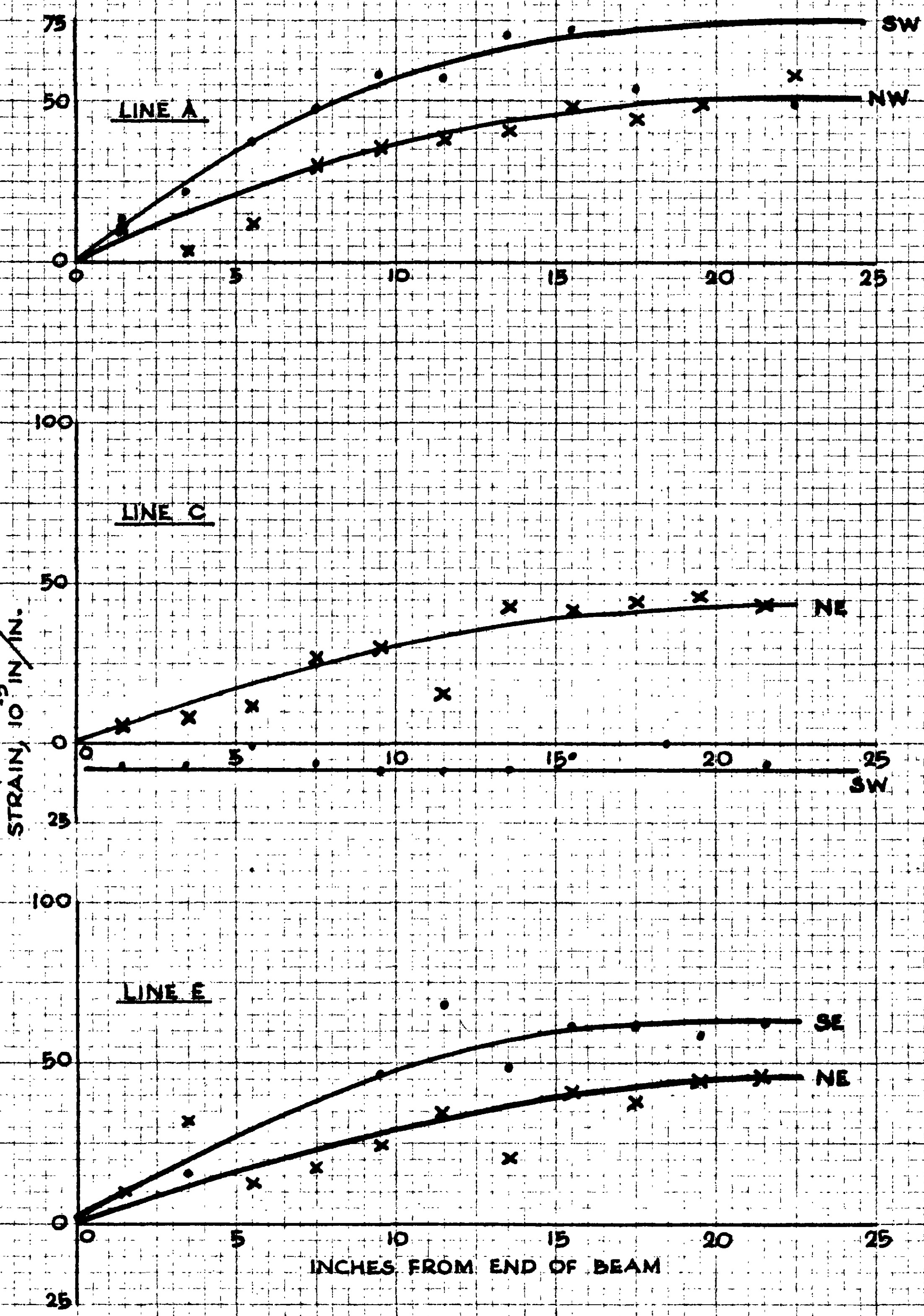


FIG. 20d. CURVES OF THE BUILD-UP OF ~~HORZ.~~ STRAIN IN BEAM B3(2). (CRACKED AT THE N.A. AT SOUTH END.)

The nature of the horizontal strain build-up in the Series B beams was different. The build-up of horizontal strain was found to be affected by the change in the distribution of the prestressing wires on the section of the beam. The strain values at one point along all the lines A, C and E were approximately equal due to the equal distribution of the prestressing wires at the top and the bottom of the section (Ref. Fig. 20b). The values of the horizontal strains measured on the sides of the Series B beams were quite different from the corresponding values in the Series A beams.

The average of the maximum values of all the strains in Series A beams was 85×10^{-5} in/in, the corresponding value in Series B beams being 42×10^{-5} in/in.

An interesting behaviour of the strains was noticed in two cases while making this study. The total prestress applied on the section, when severe cracking took place at the level of the neutral axis of the beam, was transmitted along the lines A and E only and the strains along the line C were nearly zero (Ref. Figs. 20c & 20 d). In the beams, where cracking was not so severe, the strains did not behave in a manner mentioned above.

The horizontal strains were found to be affected by the provision of stirrups in the ends of some of the beams, viz. A3(1), A3(2) and A3(3). The strains on the sides of the beams A3(1) and A3(2) had very much scattered values due to the fact that the elasticity of concrete was not constant all along the end-zone as the ends of those beams were reinforced with stirrups. The scatter was far less in the case of the beam A3(3) as lesser amount of stirrups were provided in it.

In almost all the beams, the first flattening of the curve of the build-up of strain was generally quite clearly defined and so this was taken as the limit of the transmission length. The transmission length of the 0.2" diam. indented, rustless, high-tensile steel wire used was observed to lie between 18 and 21.5 inches (i.e. 90 and 107 diameters), with an average of 19.5 inches in the Series A beams and between 18 and 19 inches (i.e. 90 and 95 diameters) with an average of 18.5 inches in the Series B beams (Ref. Table B at page 35). The influence of the width of the web of the section on the transmission length of the wire used was negligible.

With some units the curves did not readily indicate the transmission length because of the scatter which were presumably due to the variations in the elastic modulus of the concrete, and it was sometimes impossible to decide where transmission was complete.

The effect of friction between the beam and the formwork of its soffit was found to be negligible at transfer, little difference occurring in the stresses when the beam was lifted slightly and replaced.

The transmission length is a measure of the abruptness with which the prestress is transferred from the prestressing steel to the concrete. The shorter the transmission length, the more abruptly is the prestress transferred to the concrete and the higher are the vertical tensile stresses in the end zone. An analytical study by Marshall and Mattock (35) indicated that for the more general case of beams of different size, the relative abruptness of transfer of prestress, expressed as a function of both the transmission length and the depth of the beam is a more correct parameter to consider than the transmission length alone.

2. Vertical strain distribution (Ref. Figs. 21a-h & Table A)

The investigation of the vertical strain-distribution consisted of studying the strain-distribution along the lines (a) F, G, H and I, respectively $\frac{1}{2}$ in, $1\frac{1}{2}$ ins, $2\frac{1}{2}$ ins. and $3\frac{1}{2}$ ins. distant from the end on each side-face of the beam (Ref. Figs. 17 & 18) and (b) X, Y and Z on each end-face of the beam, Z being the vertical axis of the beam, and X and Y being placed 1 inch apart on either side of it (Ref. Figs. 16 and 18). Measurements of strain were taken at several different levels on each line at every step of release during the transfer.

The maximum vertical tension normally occurred at or near the level of the neutral axis decreasing on either side of it. The strain values showed a linear increase with the prestressing force applied on the section during the transfer. In the beams where cracking took place, this rate of increase was greater, from the very beginning than that in the uncracked ones.

On the side-faces the vertical strains were tensile up to a distance of about 4 inches from the end. Beyond this distance, they tended to become compressive. In the majority of the tests, strains were measured on all the lines F, G, H and I, but later on readings were taken on line F only, as the maximum vertical strain which was the critical feature was found to occur at the level of the neutral axis on line F.

In the Series A beams, sometimes the strains at levels below the neutral axis showed a considerable scatter in their values, as those were nearer to the main group of prestressing wires. But in most of the Series B beams the strains at levels below the neutral axis were of approximately the same values as those of the strains at corresponding levels above the neutral axis, as they were placed symmetrically with respect to the top and the bottom layers of steel.

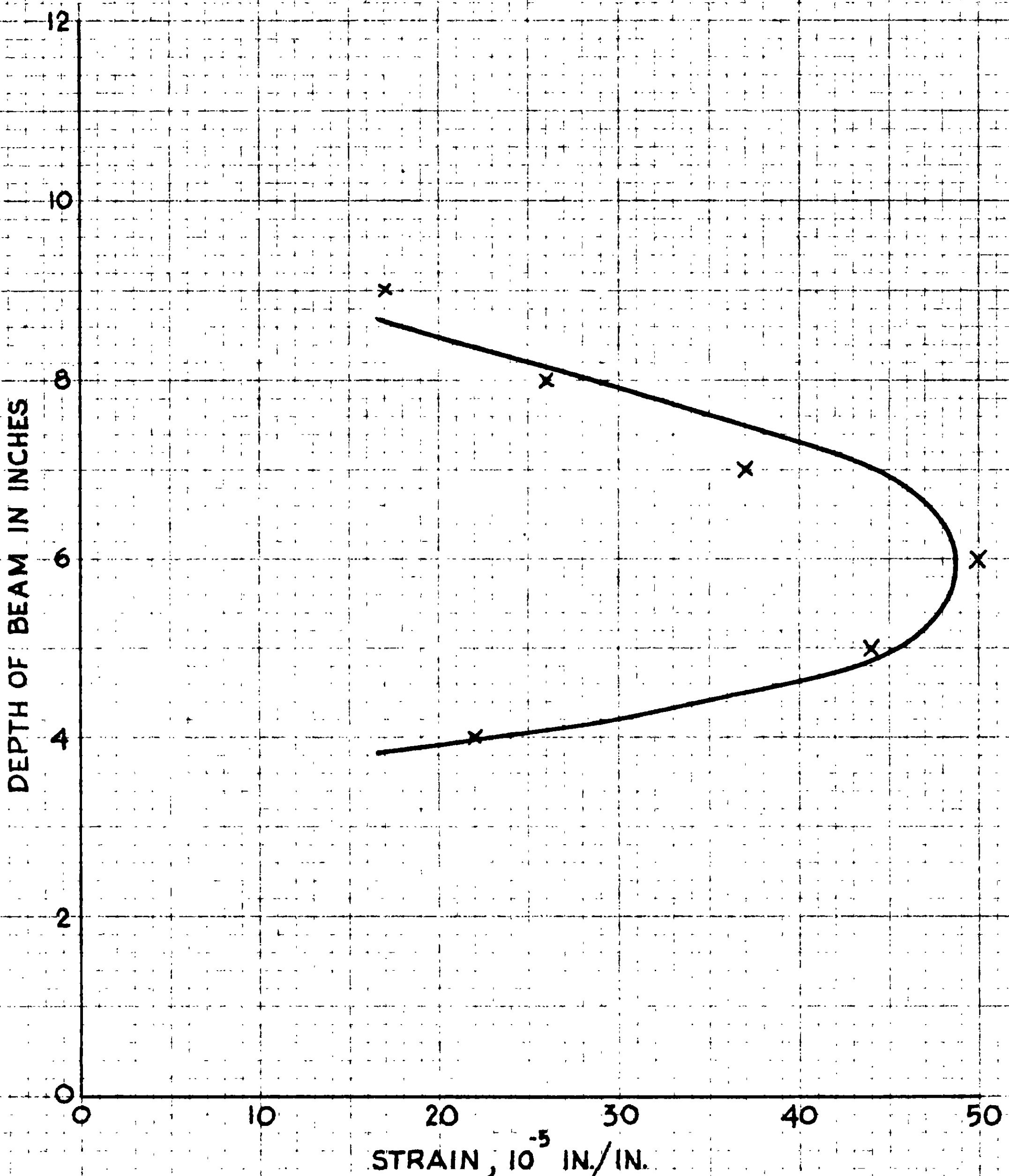


FIG. 21a. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF A1 BEAMS AT FULL RELEASE. [BEAM A1 (1)]

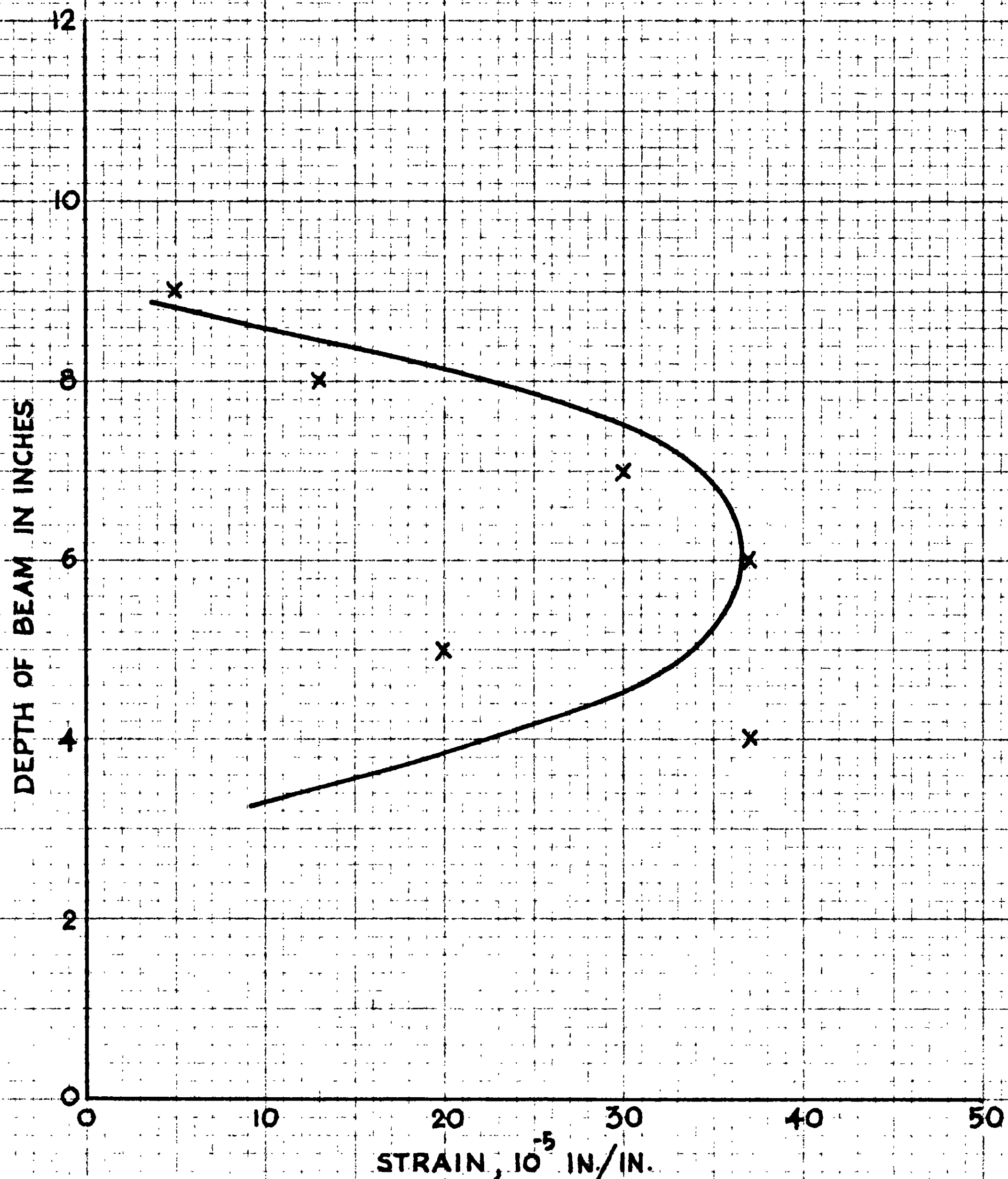


FIG.21b. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF A2 BEAMS AT FULL RELEASE. [BEAM A2(2)]

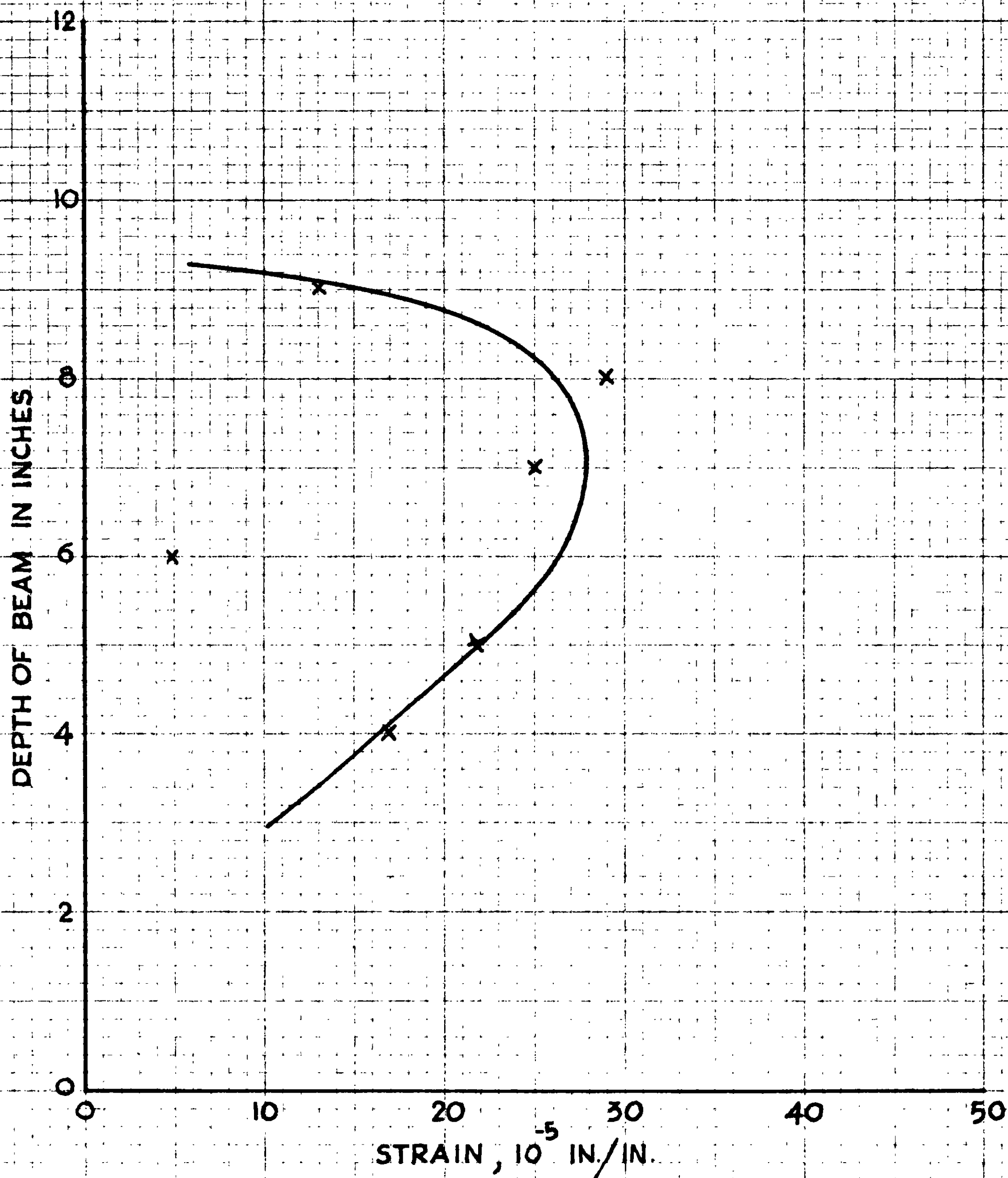


FIG. 21c. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF A3 BEAMS AT FULL RELEASE. [BEAM A3(6)]

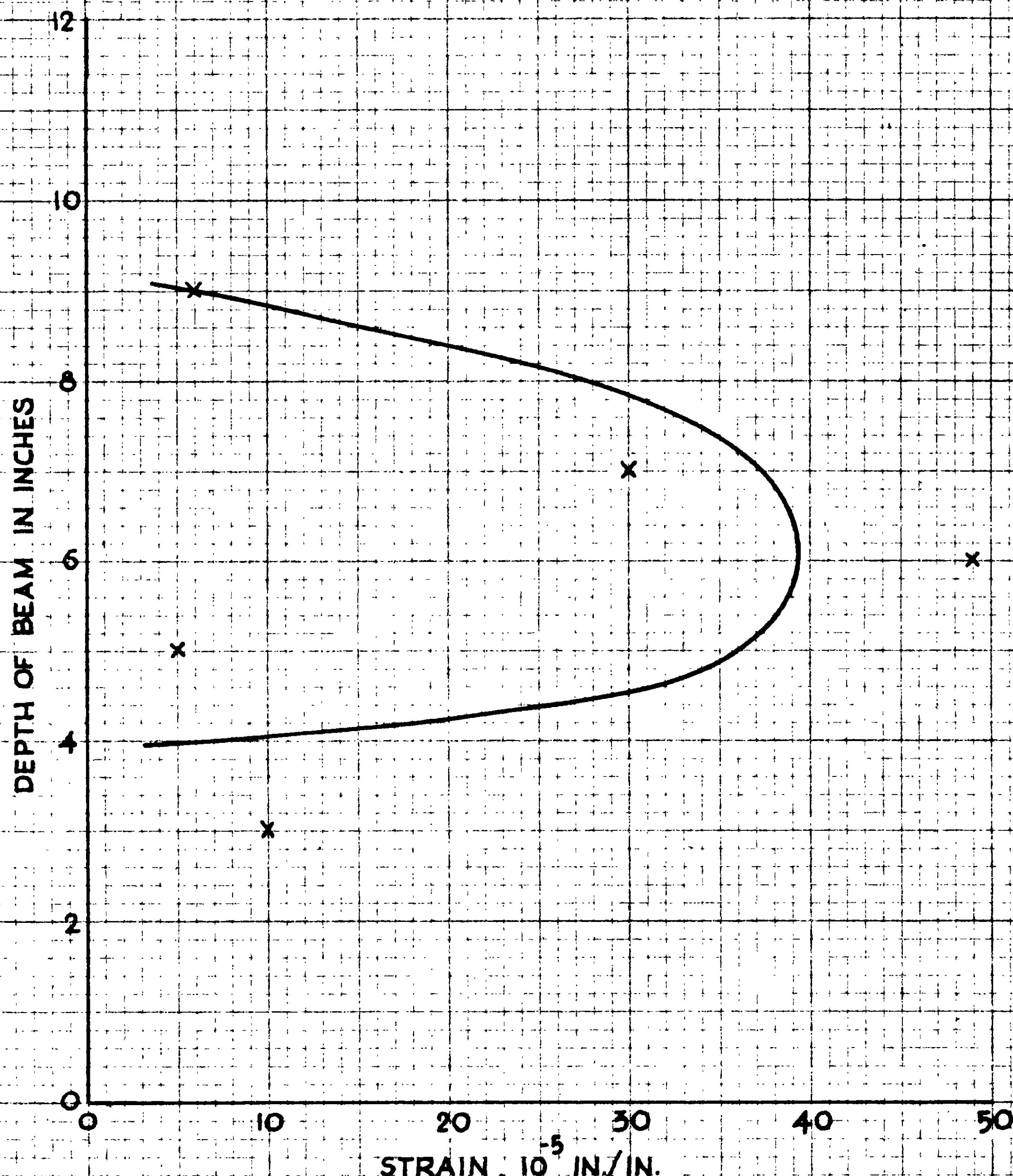


FIG. 21d. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF B1 BEAMS AT FULL RELEASE. [BEAM B1 (2)]

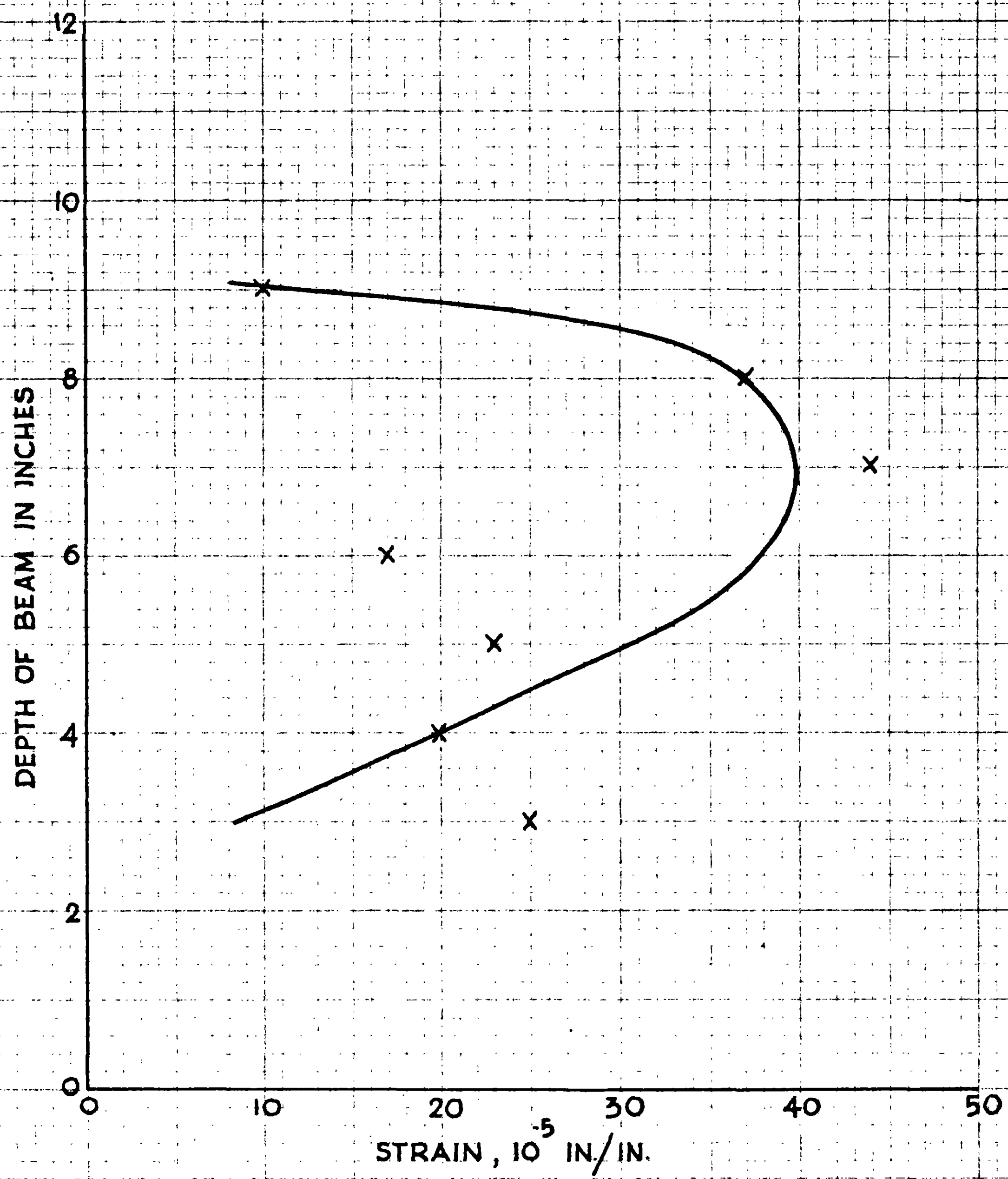


FIG.21e. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF B2 BEAMS AT FULL RELEASE. [BEAM B2(2)]

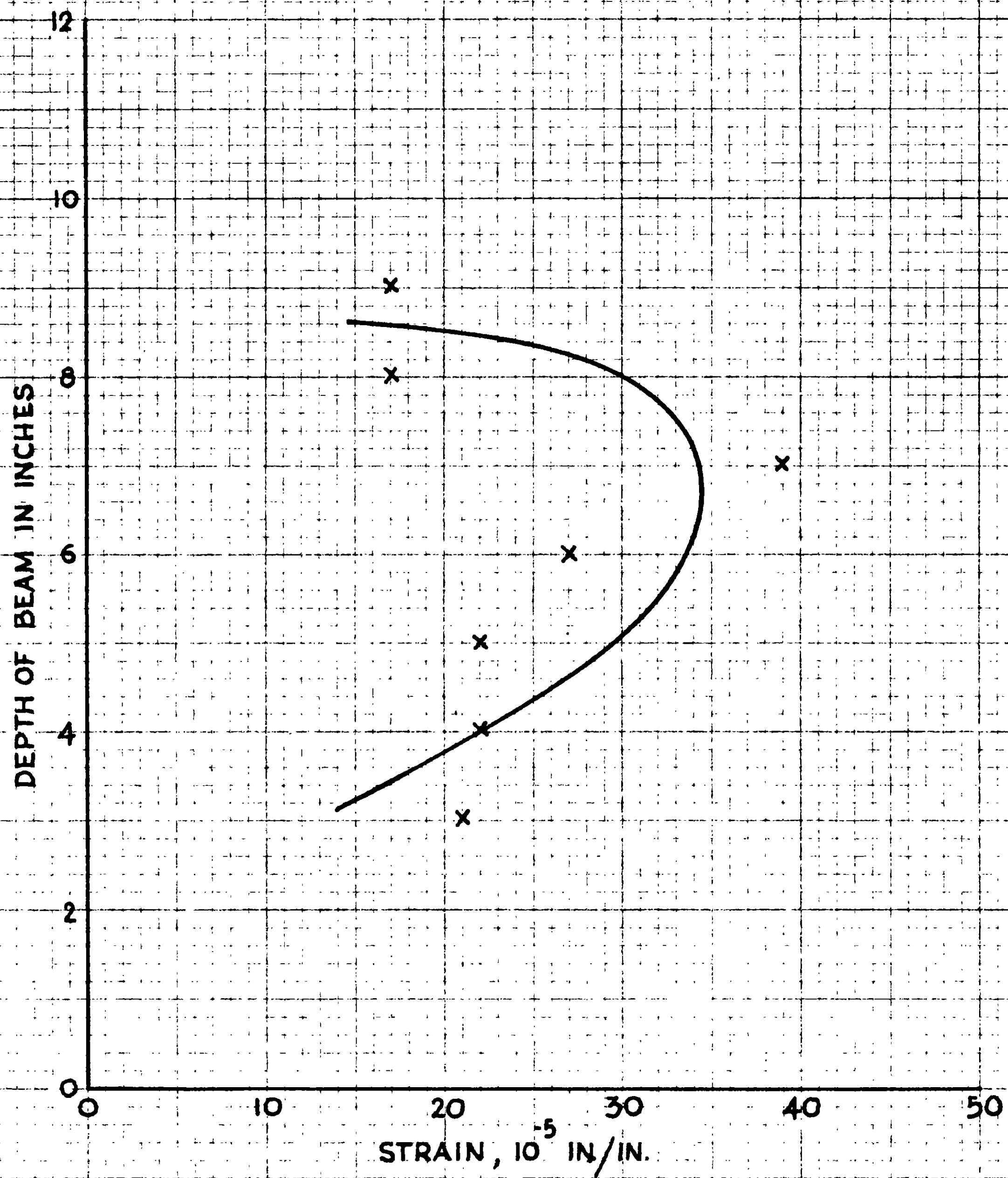


FIG. 21f. TYPICAL CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE END-FACES OF B3 BEAMS AT FULL RELEASE. [BEAM B3(3)]

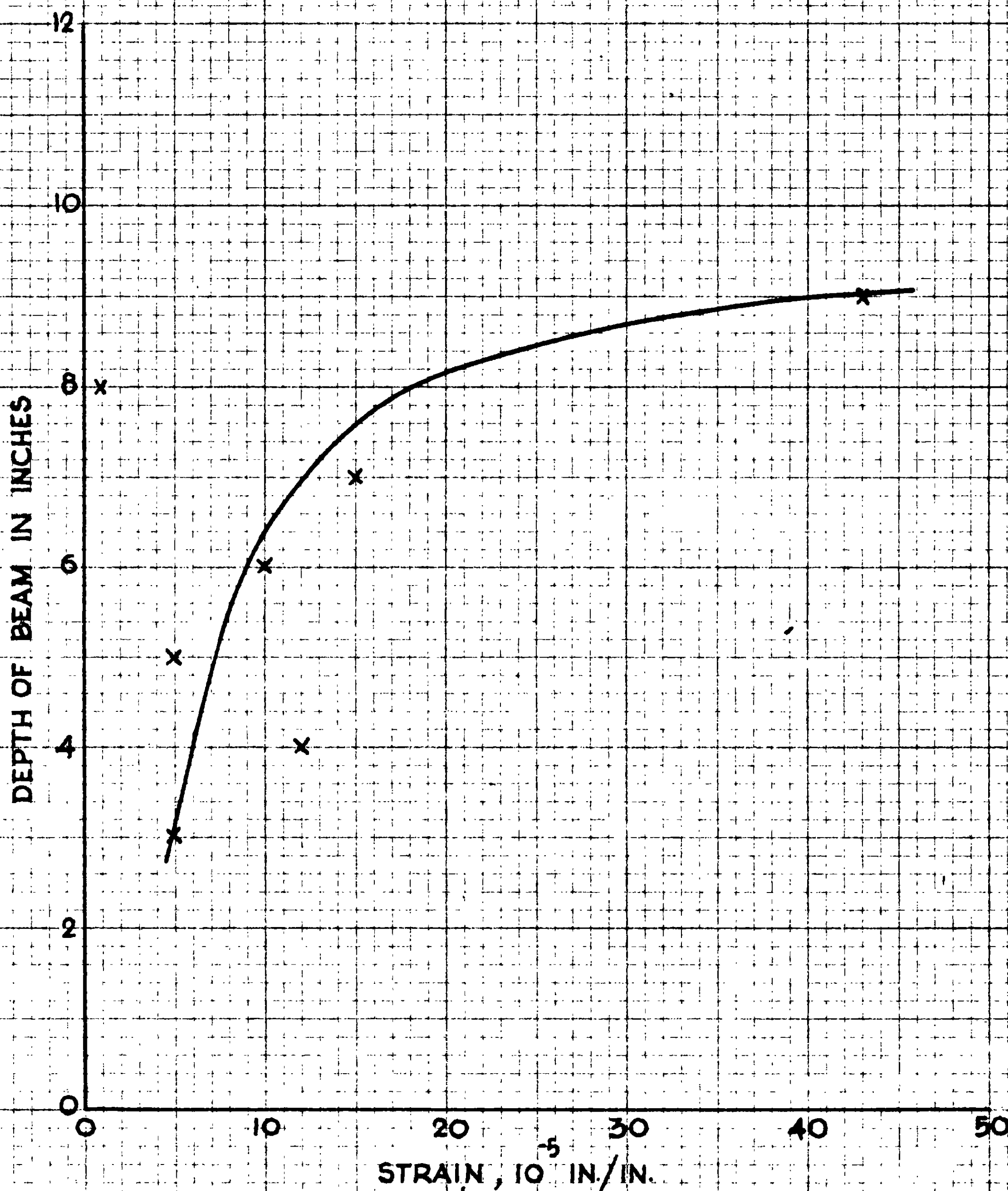


FIG. 219. CURVE OF THE VERTICAL STRAIN-DISTRIBUTION ON THE NORTH END-FACE OF BEAM B1(3) AT CRACKING.

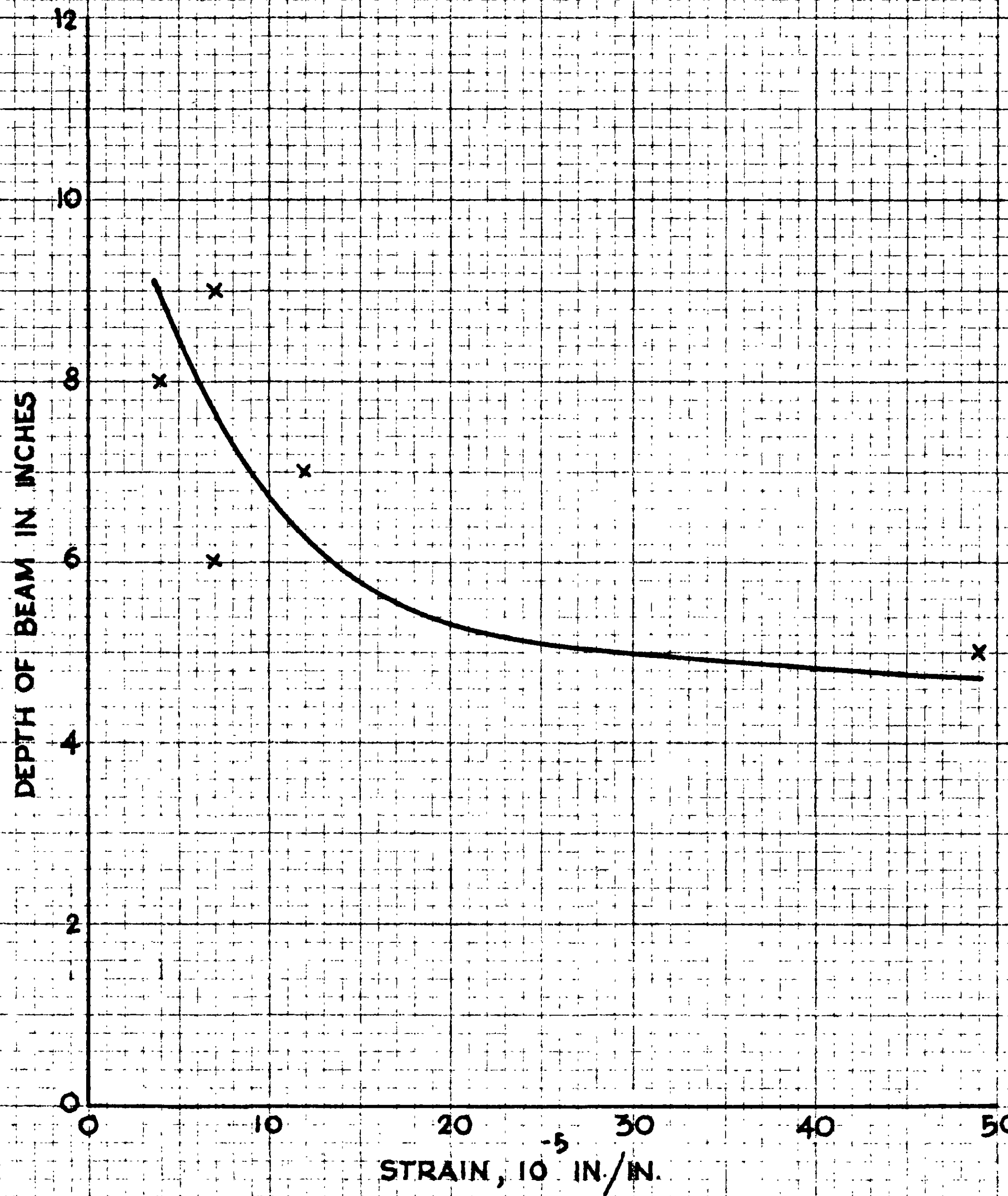


FIG.21h. CURVE OF THE VERTICAL STRAIN- DISTRIBUTION ON THE NORTH END-FACE OF BEAM B3(7) AT CRACKING.

TABLE A. VERTICAL STRAIN DISTRIBUTION ON END-FACES

Series A Beams

Beam No.	End Face	NO CRACKING Maxm. Vert. Strain at Full Release in/in.	CRACKING Maxm. Vert. Strain when Cracking took place in/in.	CRACKING % of full force at Cracking	End of Cracking	Remarks
A1(1)	N		32×10^{-5}	37.5	N	No visible crack at S.End in spite of high strains
	S	58×10^{-5}				
A1(2)	N		35×10^{-5}	43.75	N	-do-
	S	65×10^{-5}				
A1(3)	N		38×10^{-5}	37.5	N	S.End readings lost.
A2(1)	N	20×10^{-5}				No cracking - low strain readings.
	S	26×10^{-5}				
A2(2)	N	40×10^{-5}				-do-
	S	25×10^{-5}				
A2(3)	N	30×10^{-5}				Strain readings low.
	S		34×10^{-5}	43.75	S	
A3(5)	N		45×10^{-5}	31.25	Both ends	
	S		35×10^{-5}	43.75		
A3(6)	N	90×10^{-5}				No cracking at N.End in spite of high strains.
	S		35×10^{-5}	31.25	S	
A3(7)	N		35×10^{-5}	31.25	Both ends	
	S		35×10^{-5}	31.25		

TABLE A. VERTICAL STRAIN DISTRIBUTION ON END-FACES

(cont'd)

Series B Beams

Beam No.	End Face	NO CRACKING: Maxm. Vert. Strain at Full Release In/in.	CRACKING: Maxm. Vert. Strain when Cracking took place in/in.	CRACKING: % of full force at Cracking	End of Cracking	Remarks
B1(1)	N		42×10^{-5}	56.25	N	
	S	32×10^{-5}				
B1(2)	N	40×10^{-5}				No crack - low strains
	S	15×10^{-5}				
B1(3)	N		32×10^{-5}	31.25	Both ends	
	S		30×10^{-5}	68.75		
B2(1)	N	17×10^{-5}				No crack - low strains
	S	25×10^{-5}				
B2(2)	N	40×10^{-5}				
	S	40×10^{-5}				
B2(3)	N	40×10^{-5}				
	S		35×10^{-5}	43.75	S	
B3(1)	N		44×10^{-5}	56.25	Both ends	
	S		47×10^{-5}	31.25		
B3(2)	N		74×10^{-5}	100	Both ends	
	S		44×10^{-5}	56.25		
B3(3)	N	37×10^{-5}				No cracks - low strains
	S	27×10^{-5}				
B3(4)	N	32×10^{-5}				
	S		44×10^{-5}	37.5	S	

In the beams where cracking took place the average of the maximum strains was found to be 37×10^{-5} in/in. at the time of cracking. At the same time, in some cases, there was no sign of cracking, even when the strain had a much higher value.

The number of steps of release at the time of transfer did not seem to have any significant effect on the vertical strain-distribution. No conclusion could be drawn from the observations about whether (a) the change of web thickness and (b) the change of the prestressing wire-distribution have any direct influence on the stress-distribution.

On some occasions, the strains were scattered and the nature of the build-up of vertical strain curves were found to be not in agreement with the theoretical case, presumably because of variations in the concrete in the ends of the beams.

The values of vertical strain on the lines X, Y, Z were uniform and were greater than those on the line F.

Typical curves of vertical strain-distribution on end-faces have been shown in the Figs. 21a-h. Figs. 21g & 21h show curves of vertical strain on the end-faces of beams where cracking had occurred.

Effect of time on the vertical tensile strains

This study was made on six beams with $2\frac{1}{2}$" webs, three belonging to Series A and the remaining three being of Series B. Vertical tensile strains along the sides and the end-faces of the beams were recorded 24 and 48 hours after the full transfer. These were compared with those taken just after the transfer was over.

It was noticed that the tensile strains decrease and tend to become compressive with time and the maximum change takes place within 48 hours. The general nature of the curves of vertical strain distribution remained the same as before, the change in the values of strain being uniform.

The amounts of decrease in the strain values were about 2×10^{-5} in/in. and 10×10^{-5} in/in., recorded respectively at 24 and 48 hours after transfer.

At places, where web-cracking had taken place, the tensile strains, instead of decreasing in value, were noted to increase with time.

It was impossible to conclude whether the effect of time on the vertical tensile strain depends on factors such as the depth and size of the beam, the width of web, the amount of prestress and the elastic modulus of concrete.

3. <u>Study of the Mechanism of Web-cracking</u> (Ref. Table B)

Observation of the mechanism of web-cracking was made very carefully throughout the project.

One of the main objects of this project was to find out the value of the maximum vertical tensile stress occuring at the time of the transfer of prestress from the steel to the concrete. This was not possible in some of the preliminary test beams with end-stirrup reinforcement as the stirrups would control the value of the stress. So, afterwards, beams without any stirrup reinforcements in the ends were made and tested.

In testing beams without end-stirrups, it was noted that web-cracking was bound to occur during the transfer process, once the safe limit of concrete tensile stress is exceeded. And once cracks have opened up, strain readings to measure the limitations of such stress cannot be taken. The test procedure was thus modified by transferring the prestress from the steel to the concrete slowly in a number of equal small steps. This way, the exact point, when the crack becomes visible to the naked eye, could be easily detected, and the crack phenomenon could be studied more thoroughly.

The proposed final total force in the tendons was not varied during the experiments as evidently it had no influence over the concrete cracking because the wires were released in equal steps.

Average vertical strain at neutral axis level vs. percentage of initial prestressing force applied was plotted to make a thorough study of the mechanism of cracking. Typical curves are shown in the Figs. 22a-f.

In most of the cracked beams the web-cracking started somewhere between the steps 3 and 5 (the corresponding percentages of initial prestressing force applied to the beam being 31.25 and 56.25), although the cracks were visible to the naked eye only at a later step, when their thicknesses exceeded 0.001 inch.

In many of the cracked beams, cracks appeared near the level of the centroidal axis (Ref. Fig.23) and, in some cases, cracking took place near the junction of the web and either of the flanges (Ref. Fig.23). In beams A3(3), A3(5), A3(6), B3(1) and B3(4), cracking occurred near the junction of the web and the top flange, whereas in beam B1(3), it was near the junction of the web and the bottom flange.

TABLE B. WEB-CRACKING

Series A Beams

Beam No.	Concrete Str. at Transfer p.s.i.	Concrete Str. at 28 days p.s.i.	$E_c = 60,000 \sqrt{0.8 f_c}$ p.s.i.	Transm. Length of the wire used (in terms of wire diam.)	End of Crk. *	% of full force at Crk-ing	Position of crk on the End-face	Length of Crk visible at Full Release	Remarks
A1(1)	3950	5700	3.4×10^6	97d	N	37.5	$5\frac{3}{8}$" below top	1"	
A1(2)	4900	6400	3.75×10^6	97d	N	43.75	$5\frac{1}{2}$" below top	1'-4"	Honey-combing in N. End concrete
A1(3)	5400	6900	3.94×10^6	100d	N	37.5	$5\frac{1}{2}$" below top	1'-0"	
A2(1)	5300	6700	3.92×10^6	104d					No visible Crk - low Strains
A2(2)	4890	5950	3.8×10^6	97d					-do-
A2(3)	4650	6200	3.7×10^6	93d	S	43.75	$6\frac{1}{2}$" below top	Minute hair crk.	low strains
A3(5)	5650	6800	4.04×10^6	97d	Both ends	31.25 (N) 43.75 (S)	Juncn of top flange & web		severe crack
A3(6)	5350	7500	3.94×10^6	94d	S	31.25	$3\frac{3}{4}$" below top	1'-6"	-do-
A3(7)	2250	5700	2.54×10^6	90d	Both ends	31.25 (N) 31.25 (S)	8" below top (N) 6" below top (S)	2'-0" (N) None (S)	

* N - Fixed end. S - Jacking end.

Series B, Beams

Beam No.	Concrete Str. at Transfer P.s.i.	Concrete Str. at 28 days p.s.i.	$E_c = 60,000 \sqrt{0.8 f_c}$ p.s.i.	Transm. Length of the wires used (in terms of wire diam.)	End of Crk. *	% of full force at Crk- ing	Position of crk. on the End-face	Length of Crk. visible at Full Release	Remarks
B1(1)	5650	6550	4.04×10^6	92d	N	56.25	5" below top	None	Minute Cracking
B1(2)	5000	6250	3.8×10^6	95d					No Crk. - low strains
B1(3)	4000	5200	3.4×10^6	90d	Both ends	31.25 (N) 68.75 (S)	Top fla- ange Bot- tom fla- ange	None	Crk. visible before deten- sioning
B2(1)	4950	5750	3.77×10^6	90d					No Crk. - low strns.
B2(2)	5050	6550	3.85×10^6	90d					-do-
B2(3)	5050	6000	3.85×10^6	90d	S	43.75	9" below top	1'-6"	
B3(1)	6200	6720	4.23×10^6	95d	Both ends	56.25 (N) 31.25 (S)	Junc- tion of top fla- nge & web	4"(N) 2'-2" (S)	
B3(2)	5950	6750	4.15×10^6	95d	Both ends	100 (N) 56.25 (S)	-do- 5" above bottom	1"(N) 3"(S)	
B3(3)	5600	6950	4.15×10^6	95d					No Crk. - low strns.
B3(4)	3900	6550	3.36×10^6	95d	S	37.5	Junc- tion of top flange & web		Minute hair crack.

* N - Fixed end; S - Jacking end.

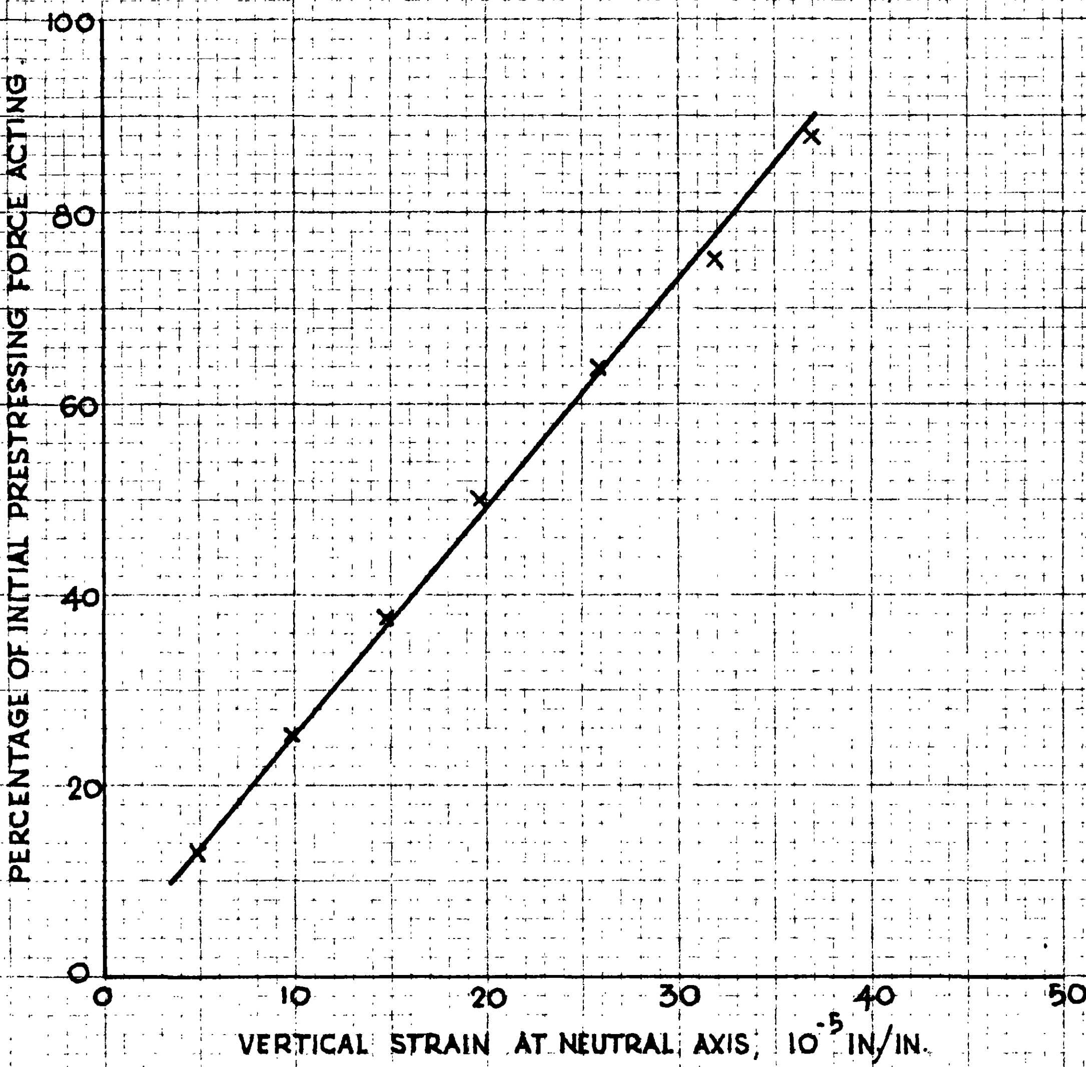


FIG. 22a. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN A1 BEAMS. [BEAM A1(1)]

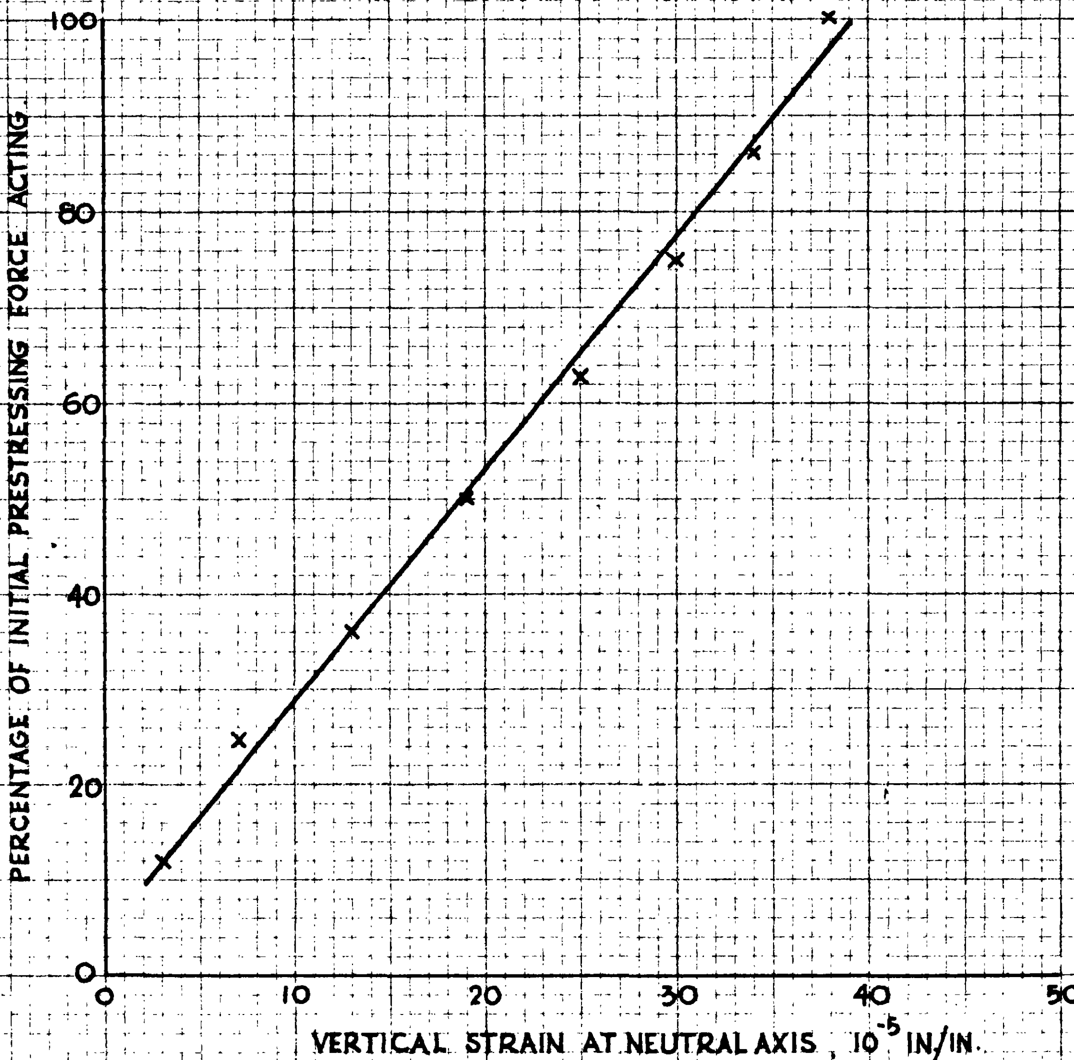


FIG.22b. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN A2 BEAMS. [BEAM A2(2)]

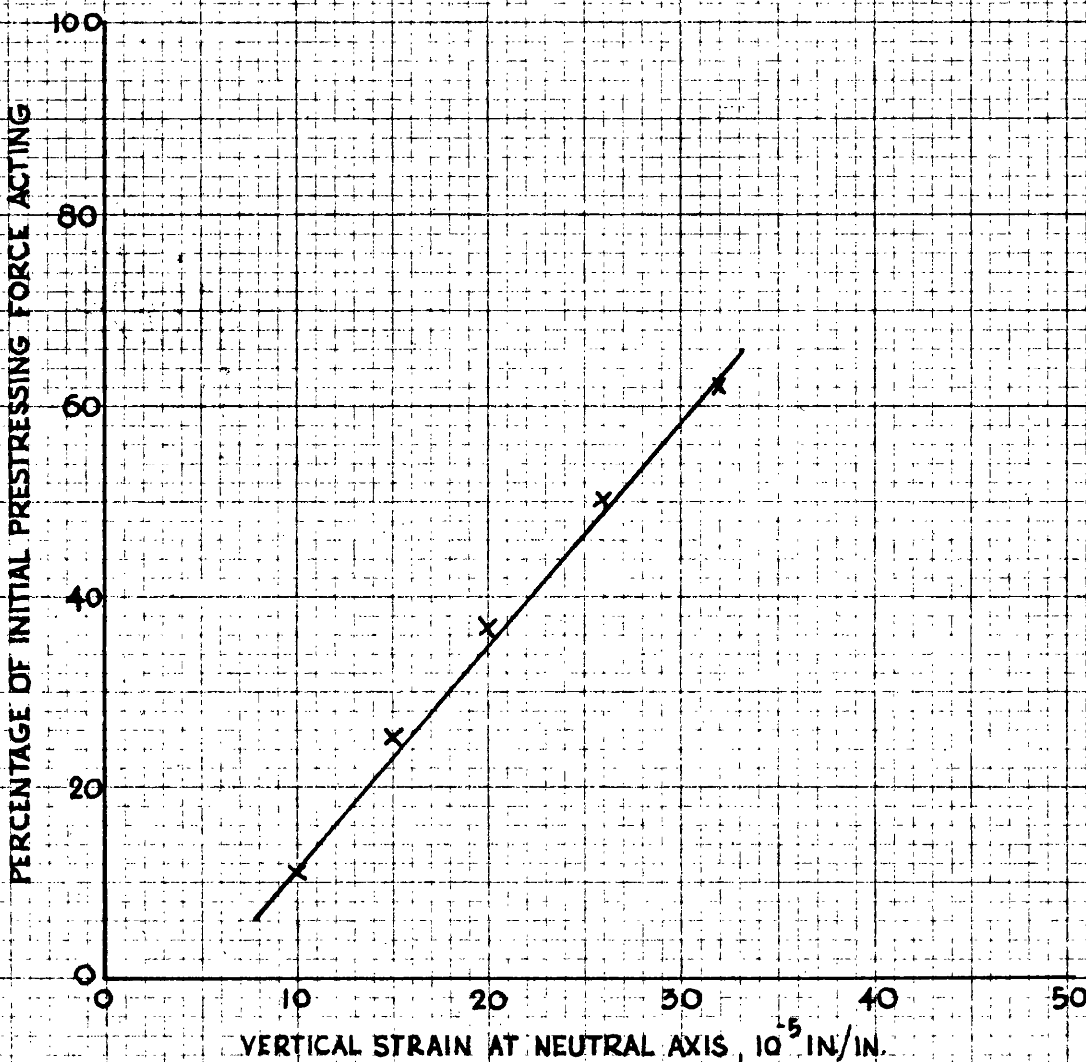


FIG 22c. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN A3 BEAMS. [BEAM A3(5)]

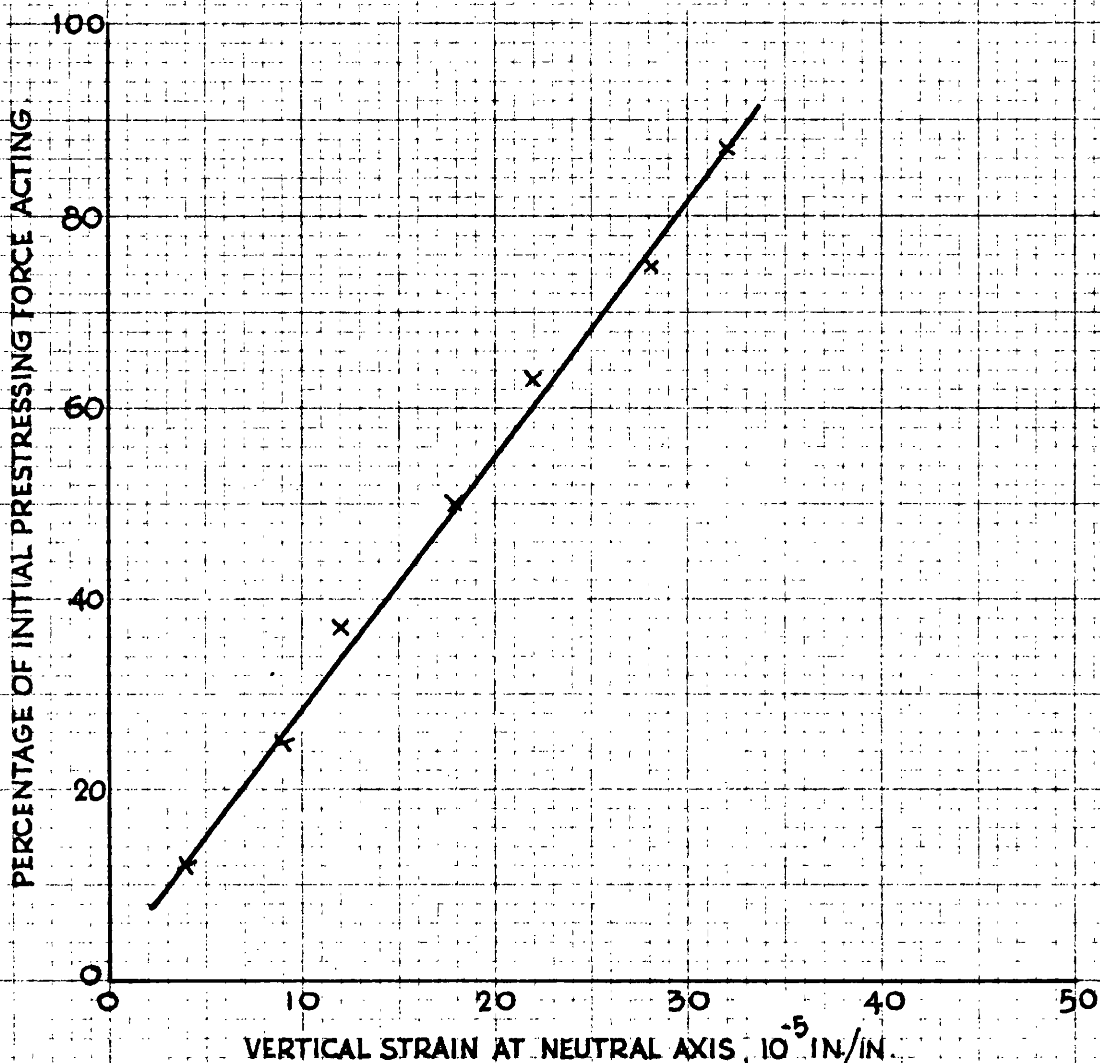


FIG. 22d. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN B1 BEAMS. [BEAM B1(2)]

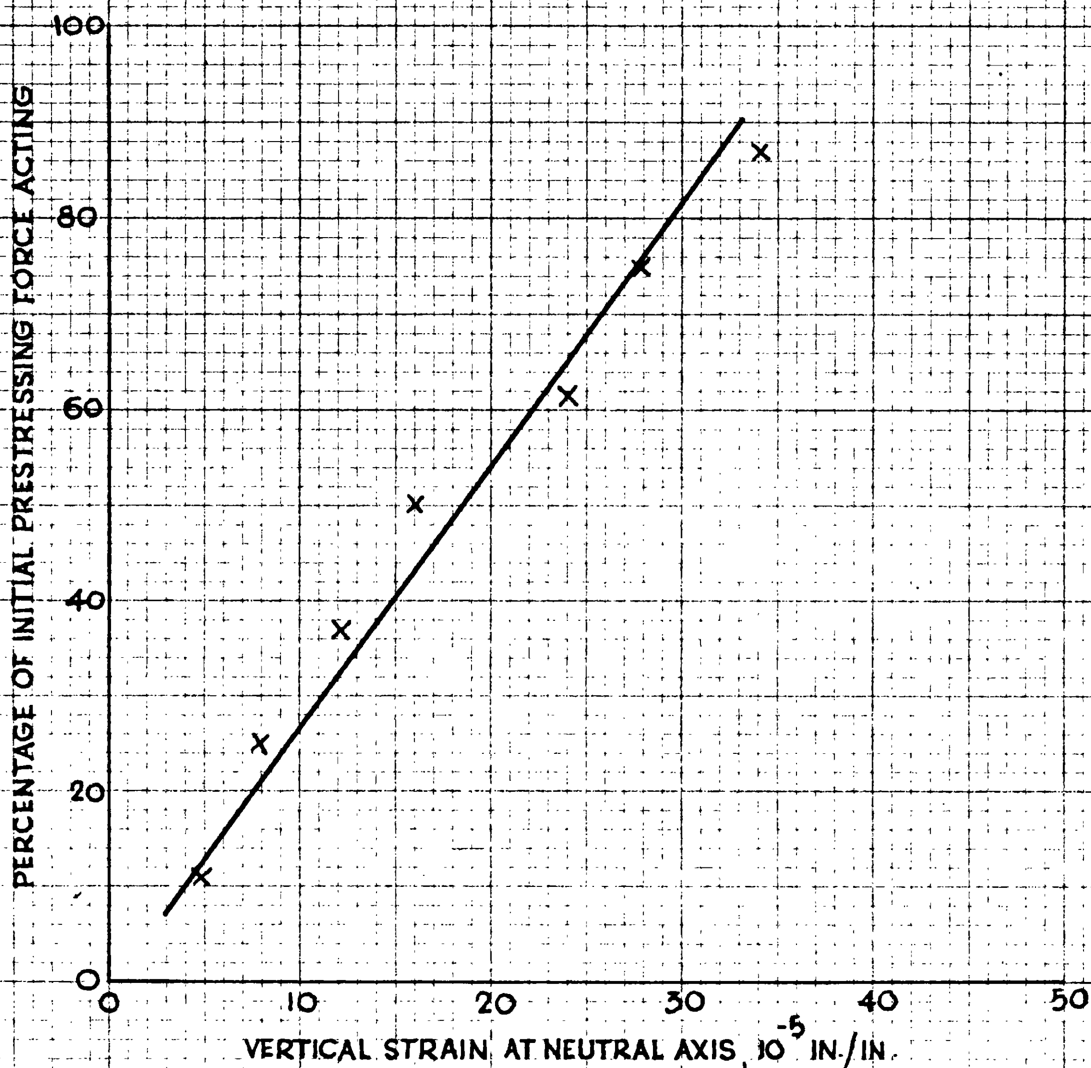


FIG. 22C. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN B2 BEAMS. [BEAM B2(2)]

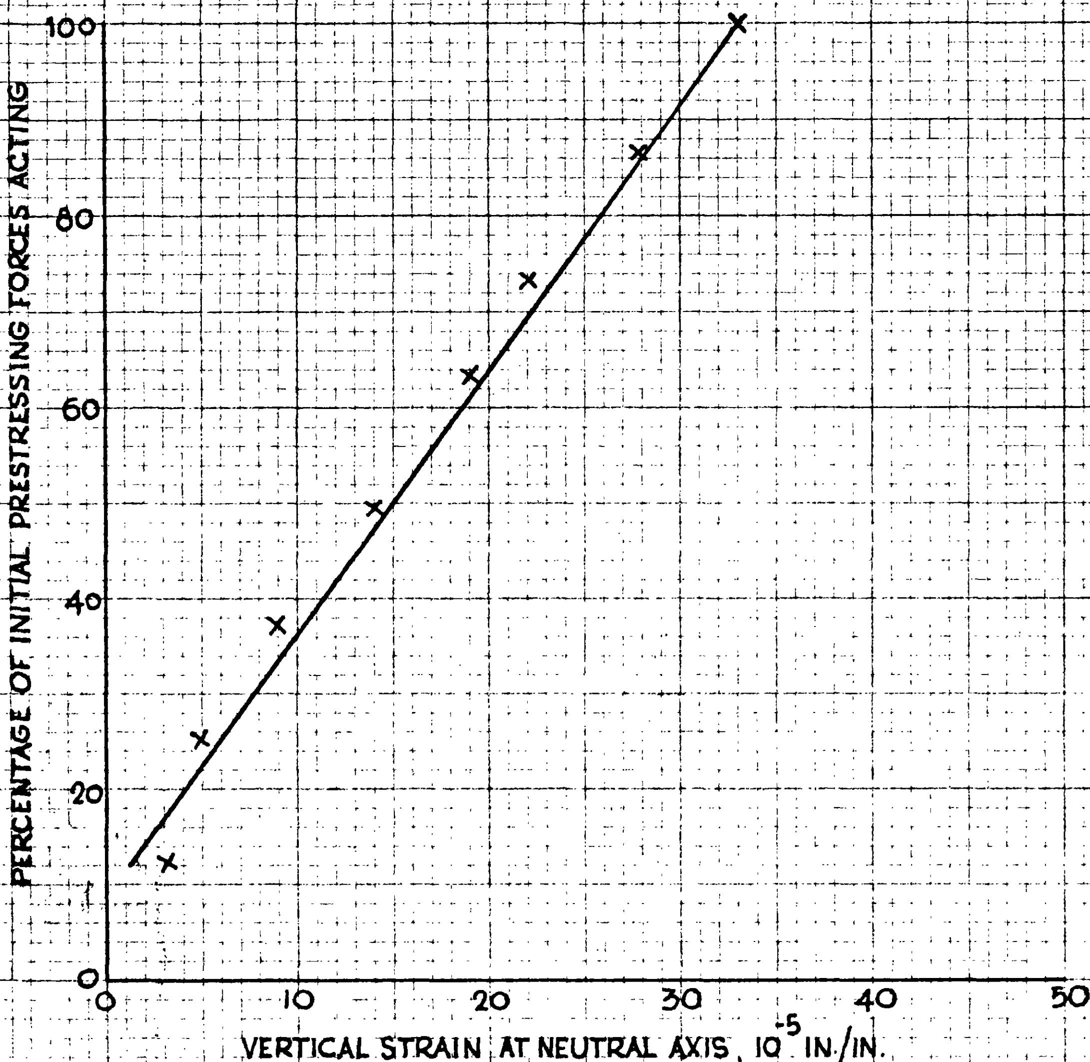


FIG. 22f. TYPICAL DIAGRAM OF AVERAGE VERTICAL STRAIN VS. PERCENTAGE OF PRESTRESSING FORCE APPLIED IN B3 BEAMS. [BEAM B3(3)]

Most of the time, cracking took place at one end of the beam only, viz. at the fixed end in beams A1(1), A1(2), A1(3), A3(1), A3(2), and B1(1) and at the jacking end in beams A2(3), A3(6), B3(4) and B2(3). In beams A3(5), A3(7), B1(3), B3(1) and B3(2), cracking occurred at both ends.

The end-crack, in average, travelled 1'-2" inside the beam, along the side-faces, but in some cases it was quite severe (Ref. Figs. 24a and 24b).

In beams A3(5) and A3(6), hair-cracks were found at the level of the top flange-web junction on both the end-faces before detensioning (Ref. Fig. 23a). In A3(5), these cracks started opening up in course of the detensioning, but in A3(6), little change occurred during the transfer process. The possible causes of such cracking are discussed in a later chapter.

In beams A3(1) and A3(2), which had stirrups at their ends, cracking did not take place at the time of transfer, but web-cracks were noticed at the north end of both the beams, $3\frac{1}{2}$" below the top surface, when examined three months after transfer. These were presumably due to the creep and shrinkage of concrete. Similar cases were reported from some of the precast prestressed concrete manufacturers.

In beams A2(1), A2(2) and B1(3), vertical cracks were observed in the bottom flange, 24 hours after transfer (Ref. Fig. 23a). These were obviously due to high localised stresses at those regions, and stresses resulting from form restraint.

The transmission length of the 0.2 inch diameter indented high-tensile steel wire used was fairly constant throughout the whole test series. That makes it clear that the phenomenon of cracking was not affected by the transmission length of the wires used.

It was felt while carrying out the detensioning that the amount of prestress applied on the end-section of the beam in each step had some influence on the web-cracking. In beams where the detensioning was carried out slowly in eight or sixteen equal steps, the cracking was not as severe as it was in beams A3(3) and B3(1) where the detensioning was carried out in three steps only. This suggests that the more abrupt is the application of forces on the end-section the more are the chances of cracking. It should, however, be confirmed from further research work by studying the effect of the process of detensioning on the vertical tensile stresses and the mechanism of cracking.

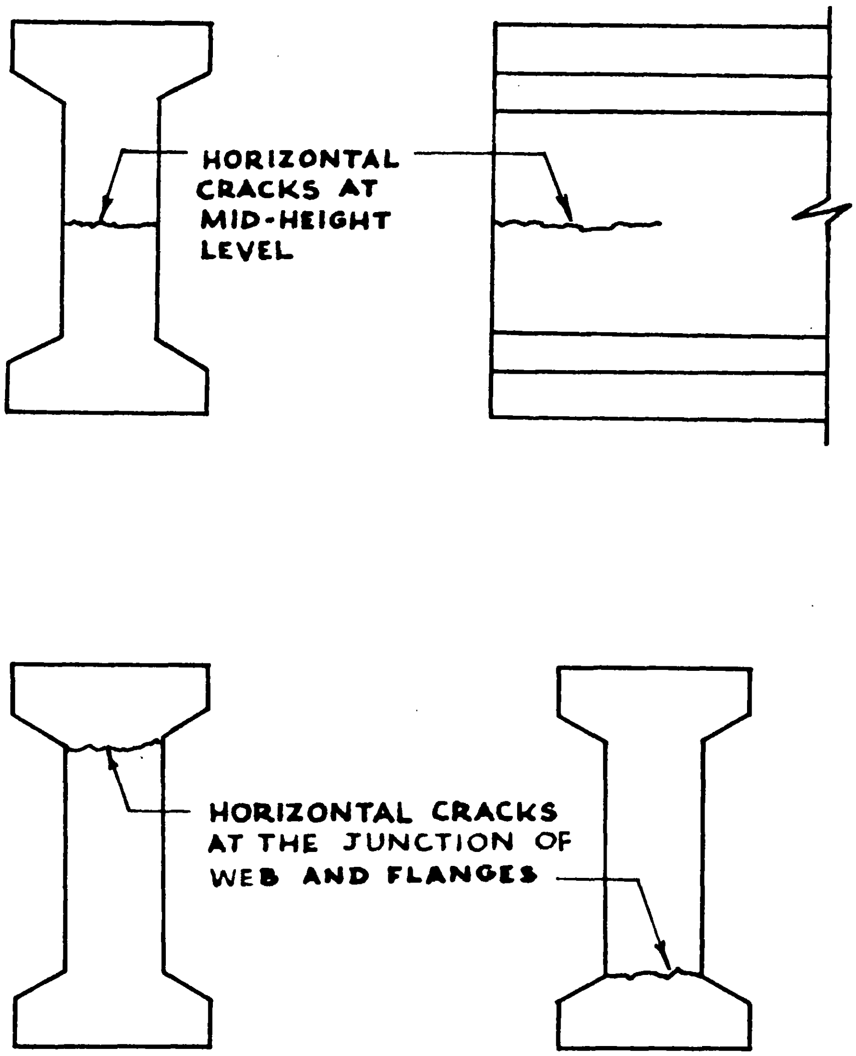


FIG. 23. VARIOUS TYPES OF WEB-CRACKING.

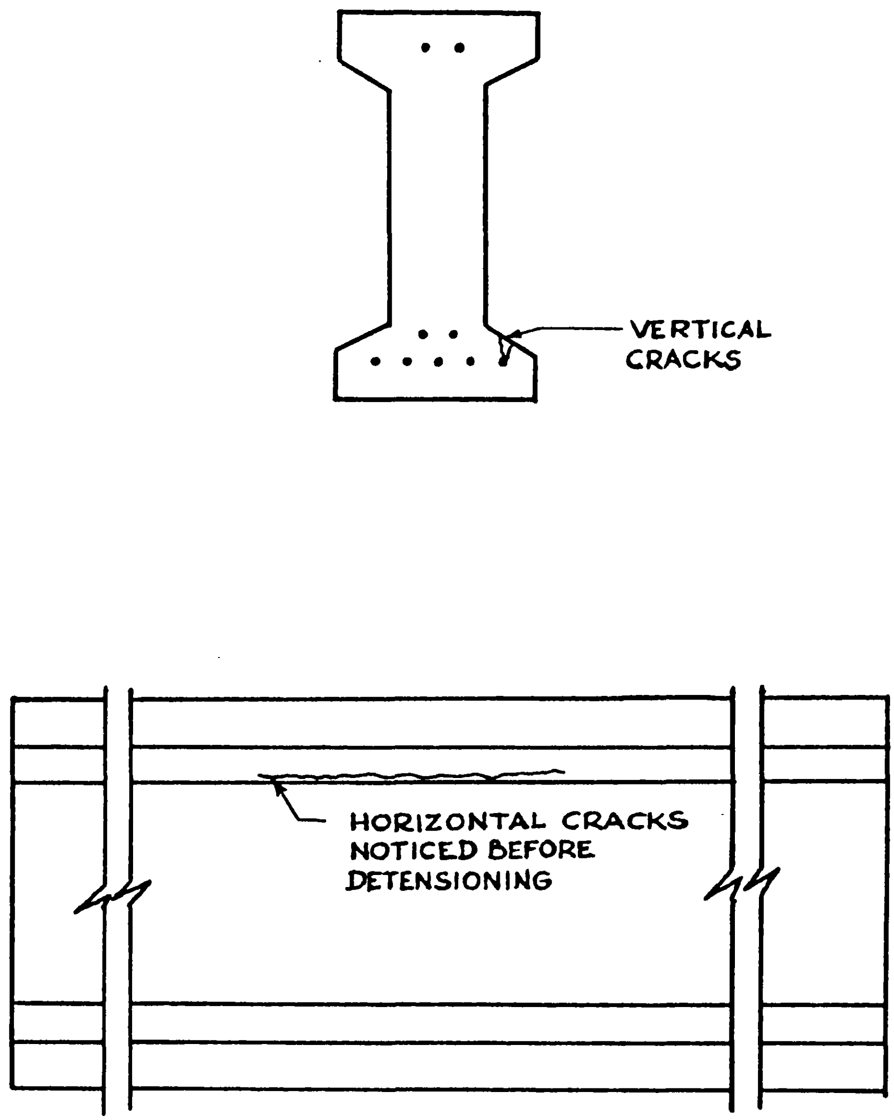


FIG. 23a. VARIOUS TYPES OF WEB-CRACKING.

Fig. 24a. Severe cracking in beam A3(3).

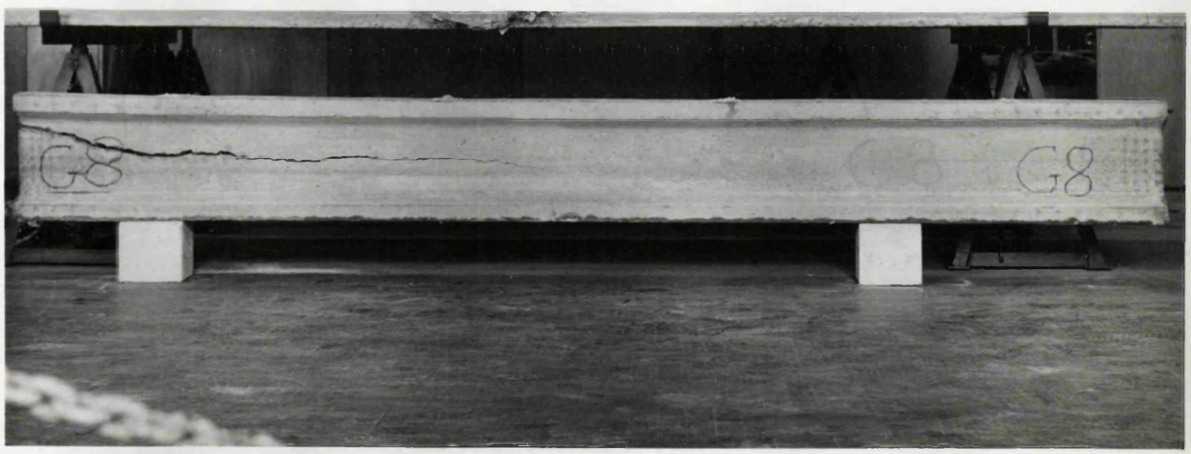


Fig. 24b. Severe cracking in beam B3(1).

No definite relationship between the 7-day cube strength of concrete and cracking could be made as all the concrete was of the same proportion. Further research, using different concrete qualities would perhaps show a relationship.

Cracking due to causes other than transfer stresses

A close inspection was made of each beam to see whether there was cracking due to any cause other than the transfer stresses, and the following cases of cracking were noted:

1) Horizontal cracking along the junction of the top flange and web near the centre of the span in beams A3(5) and A3(6), before the detensioning (Ref. Fig. 23a);

2) Vertical cracks in bottom flange in beams A2(1) and A2(2) after transfer (Ref. Fig. 23a);

3) Horizontal web-cracks noticed about three months after the transfer in beams A3(1) and A3(2).

1) Horizontal Cracking along the junction of the top flange and web:

These cracks were discovered in two of the beams while making a close inspection of them before detensioning. They were found near the centre of the span, on both sides of the web, and were ragged and discontinuous having lengths of about 2 to 6 inches. It is doubtful that they extended through the full web thickness. Relatively straight cracks, continuous over an appreciable length would have been observed, had the cracks been through the web. The probable causes of such cracking and their remedies are discussed as follows:-

(a) Settlement - the amount of stress build-up of the junction of the web and the top flange is a function of the settlement and the resistance to concrete flow caused by the form configuration.

The use of flange fillets reduces the resistance to flow of concrete by allowing the concrete to move toward the base of soffit form. The greater the fillet angle relative to the horizontal, the smaller will be the resistance and the smaller will be the build-up of stress.

The formation of such cracks can be avoided by proper aggregate gradation, controlling the water-content and reducing the slump thus minimising the concrete settlement. The placement of concrete in three separate stages; the bottom flange, web and top flanges has also been proved to be a successful method for the elimination of such cracks. This is applicable particularly to deep beams with narrow webs.

(b) Shrinkage - Shrinkage, through loss of moisture from concrete during the curing of a prestressed beam, if allowed to occur, may also affect stress build-up at the re-entrant corners of the web and the flanges. The tendency of the beam to shrink in a vertical direction is restrained by the side forms. This will obviously result in cracking at the junction of the webs and the flanges.

Shrinkage can be reduced for a given aggregate gradation by using minimum water per unit volume of concrete. In order to reduce the chance of cracks forming due to shrinkage the web forms were stripped as early as was practicable.

2) Vertical Cracks in bottom flange:

Vertical cracks in the bottom flange were detected at the time of detensioning in one beam, and, a day after stressing, in two beams. The only obvious reason of cracking is localisation of higher stresses in that region during the process of transfer.

Such cracks can be avoided if the detensioning is carried out slowly with much care and attention.

3) Horizontal cracks about three months after the transfer:

These horizontal web-cracks were discovered in beams A3(1) and A3(2) with end-stirrups while examining them about three months after the transfer of prestress from steel to the concrete. The cracks were very minute hair-cracks, visible to the naked eye, if inspected carefully, and they extended along both the sides for a distance of 6 inches.

These cracks were probably due to high tensile stresses resulting from the creep and the shrinkage of concrete.

All these cracks due to secondary causes changed very little during the process of detensioning and remained unaffected by time.

CHAPTER 6

DISCUSSION

CHAPTER 6. DISCUSSION

Comparison between theory and experiments (Ref. Table C)

(a) Values of "f_y"

The values of "f_y", i.e. the maximum vertical tensile stresses for each test beam at every step of detensioning were found from the measured vertical tensile strains using for the modulus of elasticity of concrete the expression ($60{,}000\sqrt{0.8f_c'}$) p.s.i. The mean values of these stresses were compared to their corresponding values found by the author's extension of Magnel's theory; and by Bleich-Sievers', Guyon's and Marshall's methods.

Guyon's method underestimated and Bleich-Sievers' method overestimated, while Marshall's method and the author's extension of Magnel's method gave the nearest values (Ref. Table C).

In using Sievers' theory, the expression he gives for the tension due to a single asymmetrical load has been used and not his equation for symmetrical loads which cannot apply in this case due to the lack of symmetry of the loads. Sievers' expression for an asymmetrical load is in error since it omits splitting forces which result from unbalanced horizontal shears along the neutral axis. This may account for the very large difference between theoretical and experimental values.

According to Magnel's theory for post-tensioned prestressed concrete beams, $f_y = \frac{5M}{ba^2}\left(-1 + \frac{12x^2}{a^2} + \frac{16x^3}{a^3}\right) = K_1 \frac{M}{ba^2}$. In this expression, "$f_y$" is inversely proportional to "a^2" where "a" is the length of the anchorage zone. "a" is generally taken as equal to the depth of the beam. In the present investigation, if "a" is taken as equal to the depth of the beams used, i.e. 12 inches, the values of "f_y" obtained by using the above expression are greater than the observed ones, while Chaikes' (7) modification of Magnel's theory gives values of "f_y" that are lesser than the observed ones. But when the author's extension of Magnel's theory, i.e. $f_y = \frac{K1M}{bl_t^2}$ (Ref. Chapter 3) is used, it gives the nearest values in both of the Series A and Series B beams (Ref. Table C). So the author's assumption that in pretensioned beams "a" of Magnel's theory is equal to the transmission length of the prestressing wires used is justified.

Marshall's method gave better results in the case of Series B beams only. In the present project Marshall's method did not give as satisfactory results as were expected because it was noted that Marshall's study was different from the present investigation in certain respects.

TABLE C. VALUES OF "f_y" AND "K"

Beam No.	$\frac{bd^2}{M}$ $\frac{in^2}{lb.}$	f_y obs. lb/in^2	Mean "f_y" for P/16 lb/in^2 Obs.	Ganguli's extension of Magnel's	Guyon's	Bleich-Siever's	Marshall's	Theoretical $K = \frac{64}{1}$ When = 0.2	When = 0.6	$K = \frac{f_y bd^2}{M}$ Obs. from Expts.	Mean "K"
A1(1)		106						3.4	10.3	10.0	
A1(2)	0.09	104	106	86	53	365	204	3.65	11.0	9.8	9.5
A1(3)		107						3.85	11.4	10.1	
A2(1)		72						3.85	11.4	5.4	
A2(2)	0.075	94	79	102	55	435	244	3.75	11.2	7.1	5.9
A2(3)		69						3.65	11.0	5.2	
A3(5)		125						3.95	11.9	8.0	
A3(6)	0.06	77	97	127	55	535	300	4	11.4	4.9	5.8
A3(7)		89						2.7	8.1	5.7	
B1(1)		57						3.95	11.9	9.7	
B1(2)	0.17	68	69	50	61	187	55	3.75	11.2	11.6	11.7
B1(3)		80						3.4	10.2	13.6	
B2(1)		83						3.70	11.0	12.7	
B2(2)	0.15	82	78	60	61	224	66	3.8	11.4	12.5	11.7
B2(3)		69						3.8	11.4	10.5	
B3(1)		85						4.15	12.1	10.4	
B3(2)		117						4.05	12.0	14.2	
	0.12		92	75	59	280	82				11.0
B3(3)		85						4.05	12.0	10.4	
B3(4)		80						3.05	9.2	9.8	

The basic points of difference noted between Marshall's study and the present investigation can be discussed as follows:

In the present investigation, the maximum vertical tensile stress always occurred on the end-face of the beam whereas in Marshall's project it occurred at a distance of not less than 1 inch from the edge of the beam. In other words, in his experiments "x" of the expression 19b always had a value, whereas in the present investigation "x" was always zero. This might be because he used strands instead of high-tensile steel wires as were used in the present project. Or, perhaps the maximum vertical tensile stress was occurring at a distance less than $1/2$ inch from the edge, and it could not be measured in the present investigation as the first line of gauge points was placed at a distance of $1/2$ inch from the edge. This might be possible as the ratio $\frac{"x"}{d}$ was $\frac{1}{22.5}$ in Marshall's experiments, which would require as "x" of less than $1/2$ inch in the present investigation as the "d" here is only 12 inches as compared to $22\frac{1}{2}$ inches of Marshall's beams.

In Marshall's experiments, the wires were detensioned in several stages by cutting them in small groups at the time of transfer. In the beams with equal prestressing wire-distribution at top and bottom, the detensioning was carried out by cutting the wires concentrated in the top of the section from the topmost row downwards. When all the top-flange wires had been released, the bottom flange wires were released by cutting them starting from the bottom-most row and moving upwards. This resulted in values of "M" at each stage, which were greater than the corresponding values in beams with unequal prestressing wire-distribution at top and bottom where the wires were cut in groups of two in a different order, i.e. two outer wires from the bottom-most row, then the two outers from the one above it and so on.

In the present investigation the method of detensioning was different from the one used by Marshall all the wires being released together slowly in a number of small equal steps. The "M" at each step was thus calculated for all the forces acting together on the section and the "M" for beams with equal wire-distribution at top and bottom were greater than those in the beams with unequal wire-distribution at top and bottom.

Marshall's statement that "the tendency to crack is greatest when the tendons are divided between the top and the bottom of the beam" was not confirmed by the present investigation, as cracking occurred in the series A beams as well as in the series B beams. And, hence, the proposed theory does not differentiate between beams where splitting is likely to occur and those where it is not.

In the Series B beams, higher stresses were obtained in the sections with thinner webs which is predicted by Guyon's and Bleich-Sievers' theories. In Series A, there was a slight but not definite trend in the opposite direction.

(b) Values of "K" in Marshall's Method

While discussing the advantages of Marshall's theory, it must be pointed out that it is very dependent on the values taken for "K". If the fundamental approach combining Janney's and Guyon's theories is sued to find out the value of "K", "K" varies from 2.7 to 11.9 depending on the values of "O" and "Ec". Again, Marshall suggested that the value K = 18 should be used for beams with tendons largely concentrated in the bottom of the beam, and K = 9, for beams with tendons equally divided between the top and the bottom of the beam.

The values of "K" found out from the experiments using the expression $K = \frac{f_y bd^2}{M}$ have been recorded in Table C together with the observed and the theoretical values of "f_y" which gives a full comparison between the different values (Refer to Figs. 25a & 25b).

Theoretically, lower values of "K" are expected for the thinner web-sections which is also confirmed from the experiments. The observed values of "K" have been plotted for the Series A and the Series B beams in the Figs. 25a & b. The average "K" for the Series A beams is 7.0 and that for the Series B beams is 11.4.

The experimental values of "K" seem to agree with those found from the expression $K = \frac{64}{1}$, based on Janney's and Guyon's theories, but differ from those suggested by Marshall. (For the basic points of difference between Marshall's study and the present investigation, please refer to the discussion under the sub-title "Values of "f_y" on page 40.)

Design of the End-Stirrup Reinforcement

Two simple methods of designing the end-stirrup reinforcement have already been discussed in Chapter 3.

Those methods can be applied to estimate the amount of reinforcement required to be provided at the ends of the beams tested in the present investigation. The values to be taken for "K" are 7.0 and 11.4 for the Series A and the Series B beams respectively, which are the average of the values obtained from the experiments for each series.

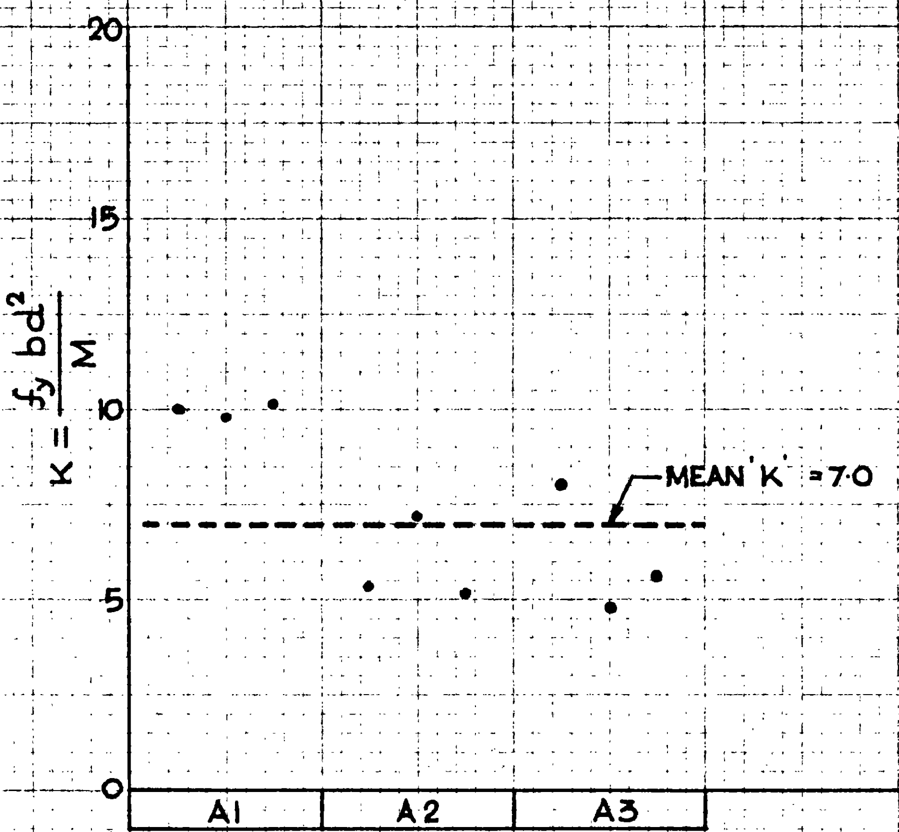


FIG. 25a. VALUES OF 'K' IN SERIES A BEAMS

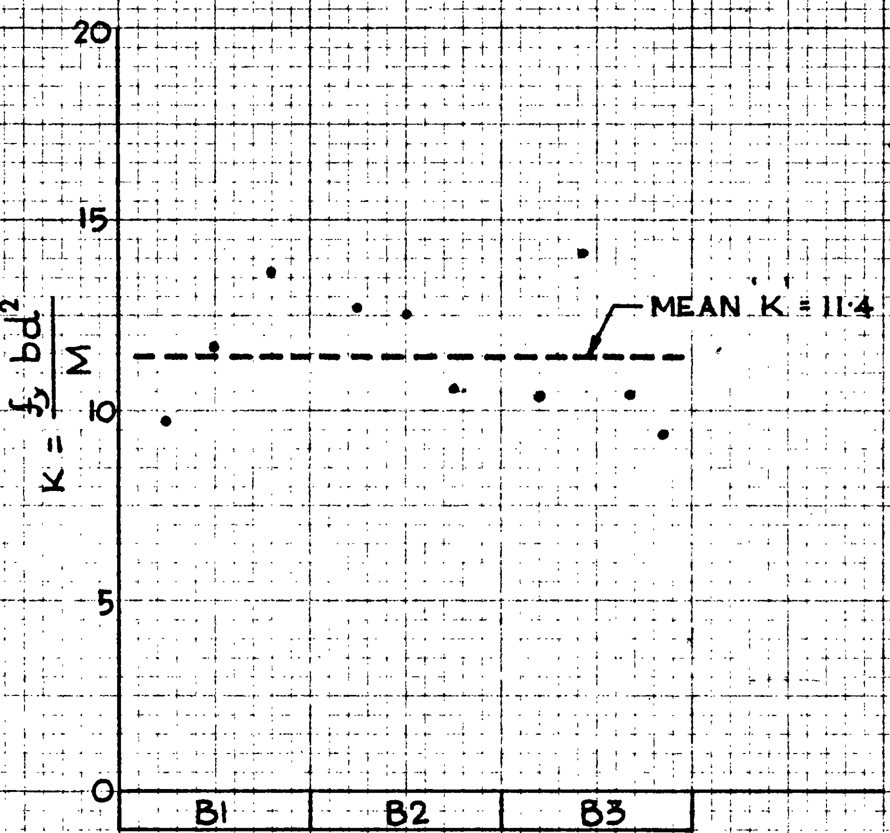


FIG. 25b. VALUES OF 'K' IN SERIES B BEAMS

i) Series A Beams

To apply method 1 in Series A beams we have $K = 7.0$; M (average of the three) = 4900 lb.inches; $d = 12$ inches and $f_w = 20,000$ p.s.i.

Hence from (23) $A_w = \frac{7.0 \times 4900}{8 \times 20,000 \times 12} = 0.018$ sq.in.

Now, to apply method 2 in Series A beams, we have $P = 9 \times 4194$ lbs; $d = 12$ inches; $l_t = 50 \times 0.2$ inches and $f_w = 20,000$ p.s.i.

Therefore, from eqn (24) $A_w = \frac{0.021 \times 9 \times 4194}{20,000} \times \frac{12}{50 \times 0.2}$ sq.in.

$= 0.045$ sq.in.

ii) Series B Beams

Proceeding in a similar way for the Series B Beams , we get $A_w = 0.015$ sq.in. and 0.05 sq.ins by method 1 and method 2 respectively.

So it is quite obvious that if we provide one 0.2 inch diam. m.s. stirrup at a distance of 1 inch from the edge of the beam, which gives an "A_w" of 0.063 sq.ins., cracking, if any, will be prevented. This is applicable to the Series A as well as the Series B beams. This reinforcement was provided in Beam A3(4) and there was no cracking.

Control of Cracking

The provision of end-stirrup reinforcement is the best solution for the cracking, as even a small amount of stirrups would restrict the growth of such cracks under all conditions and thus increase the serviceability and performance of the girders.

The amount of reinforcement should be distributed uniformly over a distance of one-fifth of the depth of the beam, measured from the end, the first stirrup should be placed as close to the end-face of the beam as practicable.

The possibility of web-cracking may be minimised in another way by distributing the prestressing wires as uniformly over the end-section as is practicable. Too great a concentration of the wires can lead to the formation of cracks immediately about them.

Chances of web-cracking can be reduced by keeping the end-plates in their usual positions, at the time of transfer.

Concluding Remarks

The values of the vertical tensile stresses near the web-cracks, measured at a step when the crack became visible, were about 1200 lbs/sq.in. But in some of the beams where cracks were not visible to the naked eye, stresses as high as 3000 lbs/sq.in. were recorded suggesting the formation of some invisible cracking. So using a factor of safety of 1.5 we can say that, if the value of the maximum vertical tensile stress found from the empirical formula by Marshall, $f_y = \frac{KM}{bd^2}$, exceeds 800 lbs/sq.in. for the quality of concrete which was used in the present investigation, it is a warning that cracking is likely to occur and the section should be reinforced. Hence, this formula can be used to predict where there are chances of cracking. The amount of end-stirrup reinforcement can also be determined by a simple method based on the same formula as has already been discussed.

The applicability of the values of "K" of the expression $f_y = \frac{KM}{bd^2}$ found from the present study should, however, be verified and confirmed by more experiments by varying the concrete mix and the water/cement ratio, the type and diameter of the wire, and the method of concrete curing, as the only variables in the present investigation were the width of the web and the distribution of the prestressing wires on the section.

The author's extension of Magnel's method of the determination of the values of vertical tensile stresses gave very satisfactory results. For its use in the design office as a check against the possibility of end-zone cracking it is recommended that the value of "l_t" should be taken as 80 times the diameter of the prestressing wires where plain and indented wires are used. It should, however, be modified where strands are used as prestressing tendons.

A P P E N D I C E S

A P P E N D I X I

DETERMINATION OF "M"

APPENDIX I. DETERMINATION OF 'M'

'M' is the resultant of the moments of all the prestressing forces and the resulting prestress distribution at the end of the transmission length about the neutral axis. It can be calculated as follows:-

Case 1. Series A Beams (Ref. Fig. 26a)

If "P_1" and "P_1'" are the prestressing forces applied at the top and the bottom of the section respectively, "e" and "e'" being their respective eccentricities, and "y" being the distance between the neutral axis and the extreme fibre of the section, then to plot the diagram for the prestress distribution at the end of the transmission length, the fibre stresses at top and bottom of the section have to be known.

Fibre stress at top: $$\frac{P_1}{A} + \frac{P_1.e.y}{I} + \frac{P_1'}{A} - \frac{P_1'.e'.y}{I}$$

Fibre stress at bottom: $$\frac{P_1}{A} - \frac{P_1.e.y}{I} + \frac{P_1'}{A} + \frac{P_1'.e.'.y}{I}$$

These being known, the moment of the prestress distribution diagram, about the neutral axis can be determined.

The moment of the prestressing forces P_1 and P_1' about the neutral axis is $(P_1'.e' - P_1.e)$.

Hence, "M", the resultant of these two moments can be found out.

Case 2. Series B Beams (Ref. Fig. 26b)

In case of the Series B beams, the prestressing force (P_1) applied at the top and the bottom of the section being of the same value, the prestress distribution at the end of the transmission length is uniform.

Hence referring to Fig. 26b and considering the forces above the neutral axis only,

$$M = P_1.e - P_1.\frac{d}{4} = P_1\left(e - \frac{d}{4}\right).$$

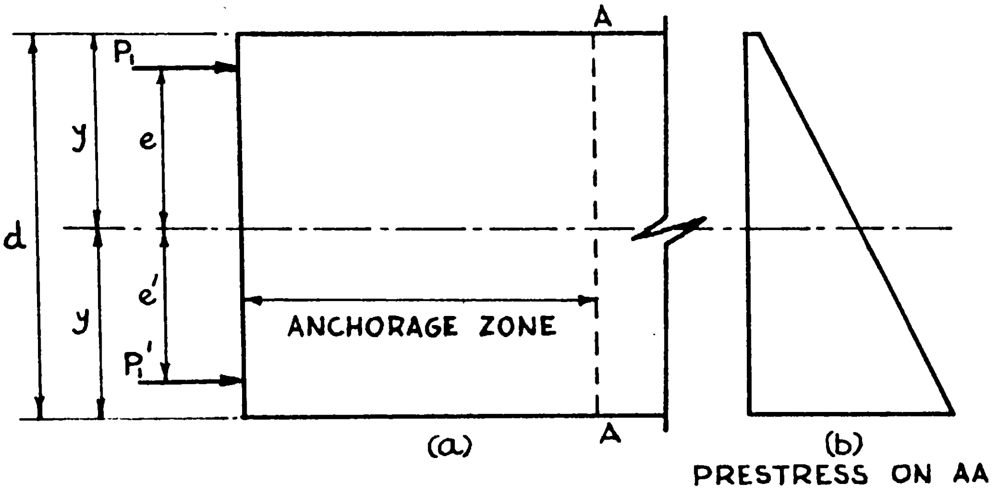


FIG. 26a. CALCULATION OF "M" AT TRANSFER FOR SERIES A BEAMS.

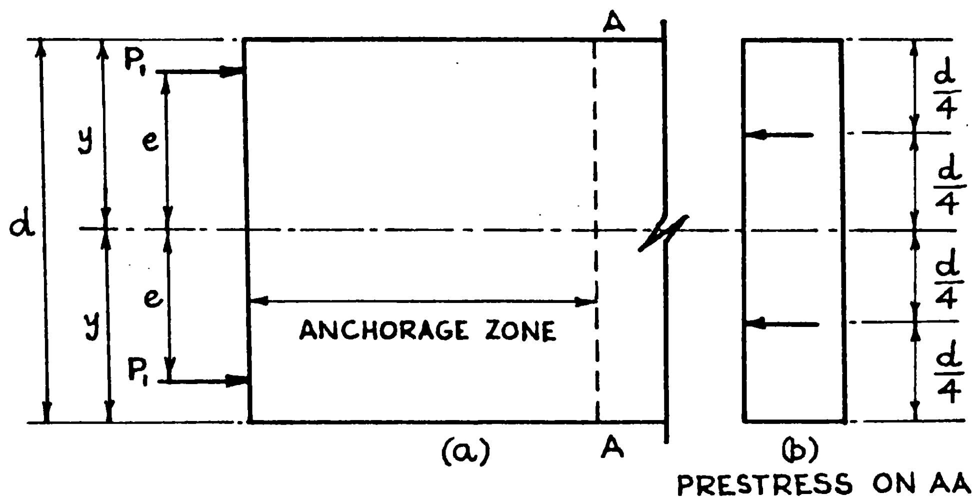


FIG. 26b. CALCULATION OF "M" AT TRANSFER FOR SERIES B BEAMS.

N.B. FIGS. 26a(b) & 26b(b) ARE THE DIAGRAMS OF PRESTRESS DISTRIBUTION AT THE END OF THE TRANSMISSION LENGTH i.e., ON SECTION AA

A P P E N D I X   II

SAMPLE CALCULATIONS

APPENDIX II. SAMPLE CALCULATIONS

Series A Beams

a) Determination of the extension required in each wire to produce the design prestress.

Breaking load for the 0.2" diam. h.t.s. indented wire used = 3.4 tons, and the ultimate stress for it = 109 tons/in^2.

Hence, working load = 0.7 x 3.4 tons, i.e. 2.4 tons, and the working stress = 75.6 tons/in^2.

$\therefore$ the total extension required in each wire to produce a design prestress of 2.1 tons = $\dfrac{2.1 \times 2240 \times 16 \times 12 \times 4}{\pi \times 0.04 \times 28 \times 10^6}$ inches or 1.02 inches.

b) Calculations of Elastic Loss of Prestress.

If, A_{st} = area of steel wires in the cable,

e = the eccentricity with which the prestressing force has been applied,

P_o = the force in wires before transfer,

P_1 = the force in wires after transfer,

m = modular ratio,

A = cross-sectional area of the beam,

I = moment of inertia of the section,

then $\left[1 + mA_{st} \left(\frac{e^2}{I} + \frac{1}{A} \right) \right]$ for A1 beams

$$= \left[1 + 7 \times 9 \times 0.0314 \left(\frac{25}{684} + \frac{1}{48} \right) \right]$$

$$= 1.12 \text{ (say)}$$

[For A1 beams $A_{st} = 9 \times 0.0314$ sq.in; $m = 7$; $e = 4.8$ inches; $P_o = 294$ lbs; $A = 48\text{in}^2$; $I = 684\ \text{in}^4$.]

Calculating in a similar way,

$\left[1 + mA_{st} \left(\frac{e^2}{I} + \frac{1}{A} \right) \right]$ for A2 and A3 beams are 1.12. *

$$\therefore P_1 \text{ for Series A beams} = \frac{P_o}{\left[1 + mA_{st} \left(\frac{e^2}{I} + \frac{1}{A} \right) \right]}$$

$$= \frac{294}{1.12} \text{ lbs or } 262 \text{ lbs.}$$

---

* NOTE: The variation in the values of $\left[1 + mA_{st} \left(\frac{e^2}{I} + \frac{1}{A} \right) \right]$ for A1, A2 & A3 beams is negligible, although they have different values of I and A.

c) Determination of "M"

To calculate "M" for each step for the A1 beams, we know $A = 48 in^2$; $I = 684 in^4$; $P_1 = 2 \times 262$ lbs; $P_1' = 7 \times 262$ lbs; $e = 5$ inches; $e' = 4.7$ inches and $y = 6$ inches.

$\therefore$ fibre stress at the top of the prestress-distribution diagram =

$$\left[\frac{2 \times 262}{48} + \frac{2 \times 262 \times 5 \times 6}{684} + \frac{7 \times 262}{48} - \frac{7 \times 262 \times 4.7 \times 6}{684} \right] \text{ lbs/in}^2.$$

$= 0$ lb/in^2

and, fibre stress at the bottom of the prestress-distribution diagram =

$$\left[\frac{2 \times 262}{48} - \frac{2 \times 262 \times 5 \times 6}{684} + \frac{7 \times 262}{48} - \frac{7 \times 262 \times 4.7 \times 6}{684} \right] \text{ lbs/in}^2$$

$= 98$ lbs/in^2.

The moment of this diagram, about the neutral axis of the section is calculated to be 1068 lb.inches.

$\therefore M = 7 \times 262 \times 4.7 - 2 \times 262 \times 5 - 1068$ lb.ins. or 4912 lb.ins.

In a similar way, "M" for the A2 and A3 beams are found to be 4873 lb.ins. and 4834 lb.ins., respectively.

SERIES A BEAMS				
P_o lbs.	P_1 lbs.	"M" at each step		lb.inches
		A1 Beams	A 2 Beams	A3 Beams
294	262	4912	4873	4834

d) Determination of "K" in Marshall's Method.

"K" is found from Marshall's expression $f_y = \frac{KM}{bd^2}$ by substituting the values of "f_y", "M", "b" and "d" in the same.

For beams A1(1), $f_y = 106$ lbs/in^2. (from expt.); $M = 4912$ lb.ins.; $b = 3$ inches; $d = 12$ inches.

$$\therefore K = \frac{f_y bd^2}{M} = \frac{106 \times 3 \times 12^2}{4912} = 10.0.$$

The values of"K" for other beams belonging to the Series A beams found out in a similar manner are listed in Table C in chapter 6.

e) Determination of "f_y" by the different existing methods.

i) The author's extension of Magnel's method.

In the present investigation $x = + \frac{a}{2}$

$$\therefore f_y = \frac{5M}{ba^2} \left(-1 + \frac{12a^2}{4a^2} + \frac{16a^3}{8a^3}\right) = \frac{20M}{ba^2} = \frac{20M}{bl_t^2} .$$

In Series A beams, l_t = 19.5 inches (average of the observed values). In A1 beams, M = 4192 lb.inches and b = 3 inches.

$$\therefore f_y = \frac{20 \times 4192}{3 \times (19.5)^2} \text{ lb/in}^2 = 86 \text{ lb/in}^2.$$

Similarly the values of f_y for A2 and A3 beams are calculated to be 102 lb/in^2 and 127 lb/in^2, respectively.

ii) Guyon's Method.

The values of "f_y" for the A1, A2 and A3 beams when calculated by Guyon's method are found to be 53 lb/in^2, 55 lb/in^2 and 55 lb/in^2, respectively.

NOTE: Guyon's method of determination of the values of "f_y" for a beam where prestressing forces are applied on the section in groups is too elaborate to be described here. Full details about it can be found in Guyon's book "Prestressed Concrete" published by the Contractors Record and Municipal Engineering, London.

iii) Bleich-Sievers' Method.

From Bleich-Sievers' theory, "f_y" on the end-face is given by the expression,

$$f_y = \frac{32M}{bd^2}$$

In A1 beams, M = 4912 lb.inches, b = 3 inches and d = 12 inches,

Hence, $f_y = \frac{32 \times 4912}{3 \times (12)^2}$ lb/in^2 = 365 lb/in^2.

Similarly, the values of "f_y" for A2 and A3 beams are 435 lb/in^2 and 535 lb/in^2, respectively.

iv) Marshall's Method.

According to Marshall, "f_y" on the end-face for beams with prestressing wires concentrated mainly at the bottom of the section, is given by the following expression,

$$f_y = \frac{18M}{bd^2}$$

In A1 beams, M = 4912 lb.inches, b = 3 inches and d = 12 inches.

Therefore, $f_y = \frac{18 \times 4912}{3 \times (12)^2}$ lb/in^2 = 204 lb/in^2.

Similarly, the values of "f_y" for A2 and A3 beams, are found to be 244 lb/in^2 and 300 lb/in^2, respectively.

Series B Beams.

a) Determination of the extension required in each wire to produce the design prestress.

If the design prestress in Series A beams is 2.1 tons, i.e. 69 tons/sq.in. for each wire, and if the total prestress is 9 x 2.1 tons or 18.9 tons in the Series B beams, the design prestress for each wire will be $\frac{18.9}{8}$ tons, i.e. 2.36 tons, the total number of wires being 8.

The total extension required in each wire to produce a design prestressing force of 2.36 tons in it

$$= \frac{2.36 \times 2240 \times 16 \times 12 \times 4}{\times 0.04 \times 28 \times 10^6} = 1.155 \text{ inches.}$$

b) Calculations of elastic loss of prestress.

$[1 + mA_{st} (\frac{e^2}{I} + \frac{1}{A})]$ for B1, B2 and B3 beams is 1.04. *

$$\text{Hence, } P_1 \text{ for Series B beams} = \frac{P_o}{1 + mA_{st} (\frac{e^2}{I} + \frac{1}{A})}$$

$$= \frac{327}{1.04} \text{ lbs or 315 lbs.}$$

* NOTE: The variation in the values of $[1 + mA_{st} (\frac{e^2}{I} + \frac{1}{A})]$ for B1, B2 and B3 beams is negligible, although they have different values of I and A.

c) Determination of "M".

For the Series B beams, P_1 = 4 x 315 lbs; e = 5 inches and d = 12 inches.

Therefore, "M" at each step = $4 \times 315 (5 - \frac{12}{4})$ lb.inches.
or 2515 lb.inches.

SERIES B BEAMS		
P_o lbs.	P_1 lbs.	"M" at each step lb.inches.
327	315	2515

d) Determination of "K" in Marshall's Method.

For beam B1(1), $f_y = 57$ lbs/in^2 (from expt.); M = 2515 lb.inches; b = 3 inches; d = 12 inches.

$$\therefore K = \frac{f_y bd^2}{M} = \frac{57 \times 3 \times 144}{2515} = 10.0.$$

The values of "K" for the other Series B beams are listed in Table in chapter 6.

e) Determination of "f_y" by the different existing methods.

i) Author's extension of Magnel's method.

In Series B beams l_t = 18.5 inches (average of the observed values).

In B1 beams, M = 2515 lb.inches and b = 3 inches.

$$\therefore f_y = \frac{20M}{bl_t^2} = \frac{20 \times 2515}{3 \times (18.5)^2} \text{ lb/in}^2 = 50 \text{ lb/in}^2.$$

Similarly the values of "f_y" for B2 and B3 beams are calculated to be 60 lb/in^2 and 75 lb/in^2, respectively.

ii) Guyon's Method.

See Note on page 49.

The values of "f_y" for the B1, B2 and B3 beams, when calculated by Guyon's method are found to be 61 lb/in^2, 61 lb/in^2 and 59 lb/in^2, respectively.

iii) Bleich-Sievers' Method.

In B1 beams, M = 2515 lb.inches, b = 3 inches and d = 12 inches.

$$\therefore f_y = \frac{32M}{bd^2} = \frac{32 \times 2515}{3 \times (12)^2} \text{ lb/in}^2 = 187 \text{ lb/in}^2.$$

Similarly, the values of "f_y" for B2 and B3 beams are calculated to be 224 lb/in^2 and 280 lb/in^2, respectively.

iv) Marshall's Method.

Marshall's "f_y" for beams with prestressing wires equally distributed at the top and bottom of the section is given by the following expression:-

$$f_y = \frac{9M}{bd^2}$$

In B1 beams, M = 2515 lb.inches, b = 3 inches and d = 12 inches.

$$\therefore f_y = \frac{9 \times 2515}{3 \times (12)^2} \text{ lb/in}^2 = 55 \text{ lb/in}^2.$$

Similarly, the values of "f_y" for B2 and B3 beams are calculated to be 66 lb/in^2 and 82 lb/in^2, respectively.

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