Transport Properties of Equal Mass Plasma

Thesis submitted to the University of Glasgow

by

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July 1991

To my Family

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Acknowledgements

I would like to thank my supervisor Professor E. W. Laing for his help and very useful suggestions during the period of this work .

I am also grateful to Dr. A. Garrett who made time for frequent discussion and made many helpful suggestions.

I am indebted to Dr. J. Cumming for the time he spent in checking this thesis .

Also I would like to express my gratitude to my friends in plasma physics group, especially Dr. A. S. Gill for his help in running the computer and G. Stewart for his helpful suggestions on all aspects of my research .

My parents have supported and encouraged me throughout my full time education from primary up to postgraduate studies, I really appreciate their help and unselfish consideration.

I am grateful to the people in the Department of Physics and Astronomy for their kindness during my difficulties at home .

Abstract

In plasma physics studies, transport coefficients such as electrical and thermal conductivities, diffusion and viscosity are very important . In this thesis we have studied transport in the case of equal mass plasma (e.g. electron - positron plasma) .

In chapter one we present a general review of plasma physics including plasma production, criteria for definition of plasma, natural occurrence of the plasma state. In this chapter we describe kinetic theory, in particular the equations of Boltzmann, Fokker - Planck and Liouville, then introduce the concepts of transport theory .

The use of irreversible thermodynamics to calculate the transport coefficients is presented in chapter two, in which we give an expression for the Gibbs equation as well as the equations for equal mass plasma in the centre of mass frame. The relaxation time method is presented , then we find equations for current , heat flux , particle flux and the stress tensor for equal mass plasma . From these equations we find the electrical and thermal conductivities , diffusion and viscosity coefficients

In chapter three , kinetic theory is used to calculate plasma transport coefficients . The kinetic equation used to describe the system is Boltzmann's . Temperatures and densities are assumed to be equal everywhere . We find the equations which describe the behaviour of the macroscopic variables such as density , mean velocity , pressure and temperature . Then frictional and thermal forces are found .

In this chapter departure from an equilibrium is caused by a temperature gradient , presence of electric field and relative velocity between the two species . Expressions for transport coefficients for equal mass plasma are found .

In chapter four , the results of chapter two and three are reviewed in various approximations .

Because I have done this work in case of electron - positron plasma , where the masses of the two species are equal , it is not possible to use the approximation of neglecting the terms consisting the ratio of (m_e / m_i) *(Braginskii 1965 and Kaneko 1960).*

We cannot assumed the ions are at rest or the velocity of the electrons is much bigger than the velocity of the ions, so in my work I assumed every species has its own independent velocity.

Hence this work is a repeat of existing work without its usual approximations.

CHAPTER ONE

Introduction

1.1. General properties of plasmas

A plasma is essentially a gas consisting of charged particles, rather than neutral atoms. In general the plasma is electrically neutral overall, but the existence of charged particles means that it can support electric current and react to electric and magnetic fields. It cannot, however, be treated simply as an ordinary gas which is electrically conducting. There is a very fundamental difference between a neutral gas and a plasma, resulting from a very different nature of the inter - particle forces in the two cases. In the former the forces are very strong, but of short range, so the dynamics of a gas is dominated by two - body billiard - ball - like collisions. In a plasma the forces are Coulomb forces, which are comparatively weak and of long range. This makes possible a variety of collective effects in plasma physics, involving the interaction of a large number of particles, and makes plasma physics a rich and complicated subject (Cairns 1985).

The universe is often classified in terms of four states solid, liquid, gaseous and plasma. The basic distinction between solids, liquids and gases lies in the difference between the strength of the bonds that hold their constituent particles together. These binding forces are relatively strong in solid, weak in liquids and essentially almost absent in the gaseous state. Whether a given substance is found in one of these states depends on the random kinetic energy of the particles, and on the average distance between particles, that is, on its temperature and density (Bittencourt 1986).

1.2. Plasma Production

A plasma can be produced by raising the temperature of a substance until a reasonably high fractional ionization is obtained. Under thermodynamic equilibrium conditions the degree of ionization and the electron temperature are closely related. This relation is given by the Saha equation

$$\frac{n_i}{n_j} = \frac{g_i}{g_j} \exp\left[-\frac{U_i - U_j}{k\,T}\right] \tag{1-1}$$

where g_i , g_j are statistical weights associated with energies U_i , U_j. Although plasmas in local thermodynamic equilibrium are found in many places in nature, as in the case of many astrophysical plasmas, they are not very common in the laboratory (Bittencourt 1986).

Plasma can also be generated by ionization processes that raise the degree of ionization much above its thermal equilibrium value. There are many methods of creating plasmas in laboratory, and depending on the method, the plasma may have high or low density, high or low temperature, stable or unstable and so on.

In the photoionization process, ionization occurs by absorption of incident photons whose energy is equal to or greater than the ionization potential of the absorbing atom. The excess energy of the photon is transformed into kinetic energy of the electron - ion pair formed. For example, the ionization potential energy for the outermost electron of atomic oxygen is 13.6 ev, which can be supplied by radiation of wavelength smaller than about $910\overset{\circ}{A}$ in the far ultraviolet. In the case of the electron - positron plasma, photons create electron - positron pairs; the minimum energy required is 1.02 Mev to create an electron - positron pair with zero kinetic energy.

In a gas discharge, an electric field is applied across the ionized gas, which accelerates the free electrons to energies sufficiently high to ionize other atoms by collisions. One characteristic of this process is that the applied electric field transfers energy much more efficiently to the light electrons than to the relatively heavy ions. But in the case of equal mass plasma this will not happen, since the rate of transfer of energy and momentum will be exactly the same for both species.

The electron temperature in gas discharges is usually higher than the ion temperature, since the transfer of thermal energy from the electrons to the heavier particles is very slow ; again this does not happen in electron - positron plasma. The temperatures for both species are equalised as rapidly as the approach to thermal equilibrium of each species.

1.3. Criteria for the definition of plasma :

1.3.1. Debye shielding

One of the related features of a plasma is that the charged particles tend to rearrange themselves in such away as to effectively shield any electrostatic fields due either to a surface at some nonzero potential or to a charge within the plasma. This rearrangement of the charged particles effectively cancels out any electrostatic fields within a distance on the order of a length λ_D which is known as the Debye length, and given by

$$\lambda_D = 69.0 \left(\frac{T}{n}\right)^{\frac{1}{2}} \quad meters \tag{1-2}$$

where T and n are, respectively, the temperature and density.

Figure (1-1) represents the plot of λ_D, and it can be seen from this figure that the Debye length is less than 1 cm for laboratory plasmas, although it becomes very large for gases with very low electron density or extremely high temperature (Tanenbaum 1967).

The Debye shielding effect, while characteristic of all plasma, does not occur in every mixture containing charged particles. For a mixture which can be treated classically there are two conditions for the shielding to occur. The first and most obvious requirement is that the physical dimensions of the system be large compared to λ_D, since otherwise there is simply not enough room for the shielding to occur. In addition, there must be enough electrons within a distance λ_D, hence the average distance between the electrons must be small compared with the Debye length. The number of electrons N_D in a sphere of radius λ_D must be much greater than one, from equation (1-2) we note that

$$N_D = \frac{4}{3} n_e \pi \lambda_D^3 = 1.37 \times 10^6 \frac{T^{3/2}}{n_e^{1/2}} \qquad (1\text{-}3)$$

A plot of N_D is shown in figure (1-2), where it can be seen that N_D is large for hot or rarefied gases and small for dense or cool material.

1.3.2. The plasma frequency

When a plasma is instantaneously disturbed from equilibrium, the resulting internal space charge fields give rise to collective particle motions which tend to restore the original charge neutrality. These collective motions are characterized by a natural frequency of oscillation known as the plasma frequency. In the case of ion - electron plasmas, because the mass of the ions is much larger than the electron mass, the ions are unable to follow the motion of the electrons. The electrons oscillate collectively about the heavy ions, the necessary collective restoring force being provided by the ion - electron attraction.

In the case of electron - positron plasma, because the masses are the same, the particles could follow the motion of the other particles. and as in the case of ion - electron plasmas, the positrons oscillate collectively as well as the electrons.

Consider a plasma initially uniform and at rest, and suppose that by some external means a small charge separation is produced inside it (figure (1-3)). The internal electric field resulting from charge separation collectively accelerates particles in an attempt to restore the charge neutrality. The angular frequency of this collective oscillation, called the plasma frequency ω_p is given by (Bittencourt 1986)

$$\omega_{pe^-} = [\frac{n\, e^2}{m\, \varepsilon_o}]^{\frac{1}{2}} \tag{1-4}$$

$$\omega_{pe^+} = [\frac{n\, e^2}{m\, \varepsilon_o}]^{\frac{1}{2}} \tag{1-5}$$

and for the plasma as a whole

$$\omega_p = [\frac{2n\, e^2}{m\, \varepsilon_o}]^{\frac{1}{2}} \tag{1-6}$$

where $m = m_{e^-} = m_{e^+}$,$n = n_{e^-} = n_{e^+}$

The collisions with other particles tend to damp these collective oscillations and gradually diminish their amplitude. If the oscillations are to be only slightly damped, it is necessary that the collision frequency, ν_c, be smaller than the plasma frequency

$$\frac{\omega_p}{2\pi} > \nu_c \qquad (1\text{-}7)$$

otherwise, the particles will not be able to behave in an independent way, but will be forced by collisions to be in equilibrium.

1.3.3. Cyclotron frequency

For a particle charge q and mass m the equation of motion in a uniform magnetic field **B** is

$$m \frac{d\mathbf{v}}{dt} = q\,(\mathbf{v}\times\mathbf{B}) \qquad (1\text{-}8)$$

v can be separated into two component, parallel $\mathbf{v}_{||}$ and perpendicular $\mathbf{v}_{\perp}$ to the magnetic field.

$$\mathbf{v} = \mathbf{v}_{||} + \mathbf{v}_{\perp} \qquad (1\text{-}9)$$

From equations (1-8) and (1-9) we obtain

$$\frac{d\,\mathbf{v}_{||}}{d\,t} + \frac{d\,\mathbf{v}_{\perp}}{d\,t} = \left(\frac{q}{m}\right)\left(\mathbf{v}_{\perp} \times \mathbf{B}\right) \qquad (1\text{-}10)$$

Since $\mathbf{v}_{\perp} \times \mathbf{B}$ is perpendicular to $\mathbf{B}$ then

$$\frac{d\,\mathbf{v}_{||}}{d\,t} = 0 \qquad (1\text{-}11)$$

$$\frac{d\,\mathbf{v}_{\perp}}{d\,t} = \left(\frac{q}{m}\right)\left(\mathbf{v}_{\perp} \times \mathbf{B}\right) \qquad (1\text{-}12)$$

From equation (1-11) we can see that the particle velocity along the magnetic field does not change. From equation (1-12) we can write

$$\frac{d\,\mathbf{v}_{\perp}}{d\,t} = -\,\mathbf{v}_{\perp} \times \Omega \qquad (1\text{-}13)$$

where Ω is a vector defined by

$$\Omega = -\,\frac{q\,\mathbf{B}}{m} \qquad (1\text{-}14)$$

Thus, Ω points in the direction of B for a negatively charged particle ($q<0$) and opposite for positively charged particle ($q>0$) as shown in figure (1-4). The radius of the circular orbit, given by

$$r_c = \frac{v_\perp}{|\Omega|} = m \frac{v_\perp}{|q| B} \tag{1-15}$$

is called the radius of gyration or gyroradius. It is important to note that Ω is directly proportional to B. Consequently, as B increases the gyrofrequency increases and the radius decreases. Also the smaller the particle mass the larger will be its gyrofrequency and the smaller its gyroradius.

1.4. The natural occurrence of the plasma state

Because air is normally nonconducting, potential differences of millions of volts can be generated between clouds and the earth and from one cloud to another, during the time when thunderstorm conditions prevail. A lightning discharge occurs in two phases with a leader stroke progressing in steps across the potential gap first. This establishes a low degree of ionization in the discharge path, thus providing the conditions for the second phase, the return stroke, to take place. The return stroke establishes a high conducting plasma path which permits a large current flow and the neutralization of the electrical charge which accumulated in the clouds.

At about 100 km above the surface of the earth we find that the non-conducting property of the atmosphere no longer applies. In the process of absorption of radiation a significant number of air molecules and atoms receive enough energy to become ionized. The resulting free electrons and positive ions are dense enough in the region called the ionosphere to satisfy the plasma criterion which has already been mentioned (Holt & Haskell 1965).

Many of the major features of the space within several earth radii of the earth have been establish by rocket and satellite studies. This region is called the magnetosphere because of the important role played by the earth's magnetic field. The charged particles continually streaming toward the earth from the sun are diverted, and sometimes even trapped, by the earth's magnetic field. The trapped particles are most dense in regions of high latitude and account for the Aurorae. The Van Allen radiation belts consist of electrons and protons with energies extending to several million electron volts which are trapped at equatorial latitudes within several earth radii of the earth. Those phenomena are illustrated in figure (1-5).

The plasma effects in the universe, which are of major interest, involve the interaction between plasma and magnetic fields. The widespread existence of magnetic fields in the galaxy has been demonstrated by independent measurements. A wide range of field magnitudes has been found, varying from 10^{-9} T in interstellar space to 1 T on the surface of magnetic variable stars, and up to 10^{8} T in pulsar atmospheres.

On the surface of the sun the general level of the field may be about one gauss, but in the region of sunspots it is known to rise to several thousand gauss. A highly conducting plasma and a magnetic field tend to remain frozen in whatever condition of intermixing occurs initially. This means that the plasma streaming from a solar flare not only follows the magnetic field, but also distorts the field and pulls it along in the stream (figure (1-6)). A dramatic example of mutual exclusion of charged particles and magnetic field is provided by the solar wind a continuous stream of charged particles coming from the sun. As it impinges upon the earth's magnetic field, the geomagnetic field is considerably compressed on the side of the magnetosphere which faces the sun and elongated on the remote side (figure (1-7)). Rocket and satellite measurements have shown that discontinuities occur in charged particle density and magnetic field in the region where the magnetosphere and the solar wind interact.

1.5. Plasma applications

A high particle energy, $\frac{1}{2} m \, v^2$, not only implies high speed v but also according to the kinetic picture of gas, a high temperature. Itappears that we will encounter matter in the plasma state in two most important diverse investigations. In fact it is even a little misleading to make a list of the present applications of plasma because this tends to narrow one's perspective. For example, it would be a frustrating task to attempt a brief discussion of the applications of the solid state of matter. Nevertheless we make the excuse that our subject is new, and in presenting a list of some applications we may at least indicate the present stage of its development.

1.5.1. Energy conversion

The most important application of man - made plasma is the control in of thermonuclear fusion reactions. Nuclear fusion is the process whereby two light nuclei combine to form a heavier one , the total final mass being slightly less than the total initial mass. The difference (Δm) appears as energy (E) according to Einstein's law $E = (\Delta m)c^2$ The nuclear fusion reaction is the source of energy in the stars , including the sun . The confinement of hot plasma in this case is provided by the self - gravity of the stars (Bittencourt 1986).

In the case of man - made plasma no conceivable material could confine it at thermonuclear temperatures Plasma at these temperatures coming into direct contact with the material wall would destroy it quickly . In order to achieve energy breakeven, we must contain the plasma and have a large enough density and temperature. For the first, we can use the fact that charged particles spiral about magnetic field lines which can confine the plasma in a magnetic field trap. For the second, we make the plasma sufficiently dense so that enough energy is released in 0.1 - 10 s. These conditions are satisfied in the Tokamak , which is a toroidal device in which a strong toroidal magnetic field is created by external currents - the toroidal field coils. A weaker poloidal magnetic field is created by current flowing through the plasma in the toroidal direction. This current is induced by means of a transformer which produces a large change magnetic flux through the hole in the torus exciting a current in the plasma which forms the secondary circuit of the transformer (Manheimer 1989).

In addition to the heating and confinement problems attention must be given to the energy loss by radation. These radiation losses constitute a serious factor in maintaining a self - sustaining fusion device. To generate more energy by fusion than is required to heat and confine the plasma, and supply the radiation losses, a condition is imposed on plasma density (n) and confinement time (t) , as well as on the temperature. It turns out that the product (nt) must be higher than minimum value which, for example, is estimated to be about 10^{20} m^{-3} s for deuterium - tritum (with $T > 10^7$) .

Since controlled nuclear fusion can provide an almost limitless source of energy, it is certainly one of the most important scientific challenges man faces today, and its achievement will cause an enormous impact on our civilization.

1.5.2. Electrical communications

The other applications in this area are long - distance radio propagation by refraction in the ionosphere and the communications with the space vehicle during the period of reentry into the earth's atmosphere, when the vehicle becomes covered by a plasma layer (Holt & Haskell 1965).

Long distance radio propagation is only possible over a fairly well - defined frequency band which ranges from about 3 - 25 MHz. At the low frequency end of the range the wavelength is long compared to the distance over which the electron density changes in the lower ionosphere, and the wave suffers considerable attenuation on being reflected. At the high frequency end of the range the wavelength is short compared to the characteristic length of the electron density gradient, and the wave can be pictured as a ray that is continuously refracted in the medium. Within the propagation frequency band the refraction causes the ray path to bend over and reemerge into the troposphere and return to the surface of the earth. However, as the frequency increases the amount of refraction decreases until the ray path is not bent over sufficiently to return to the earth. This effect is clearly dependent upon the angle of incidence of the radio wave upon the ionosphere. For example, a wave with vertical incidence may be reflected up to a frequency of 6 MHz for typical conditions of the ionosphere, while long distance communication under the same ionospheric conditions may be practical up to 22 MHz. The effect is shown in figure (1-8) which shows as well the structure of the ionosphere as a series of plateaux. These are commonly called the D, E and F layers. Attenuation of low - frequency waves occurs in the D - layer, and refraction of the upper frequencies occurs mainly in the F - layer.

1.6. Kinetic theory

The concept of a kinetic equation is introduced as a description of the statistical behaviour of a system of identical interacting particles as it evolves in time. First let us see what is the distribution function.

If $d^6 n_k\,(\mathbf{r},\mathrm{v},t)$ denotes the number of particles of type k within the volume element $d^3\mathbf{r}\, d^3\mathrm{v}$ around the position and velocity ($\mathbf{r}$,v) then the distribution function in phase space, $f_k\,(\mathbf{r},\mathrm{v},t)$, is defined as the density of representative points of type k particles in phase space (Bittencourt 1986)

$$f_k(\mathbf{r},\mathbf{v},t) = \frac{d^6 n_k(\mathbf{r},\mathbf{v},t)}{d^3\mathbf{r}\, d^3\mathbf{v}} \qquad (1\text{-}16)$$

It is assumed that the density of representative points in phase space does not vary rapidly from one volume element to the neighbouring element, so $f_k(\mathbf{r},\mathbf{v},t)$ can be considered as a continuous function of its argument. According to its definition f_k is positive and finite at any time, and f_k must tend to zero when the velocity becomes infinitely large.

We can formally set up an equation for f_k , the kinetic equation, as follows (Chapman & Cowling 1970)

$$\frac{\partial f_k}{\partial t} + \mathbf{v}\cdot\frac{\partial f_k}{\partial \mathbf{r}} + \mathbf{a}\cdot\frac{\partial f_k}{\partial \mathbf{v}} = \left(\frac{\delta f_k}{\delta t}\right)_{coll} \qquad (1\text{-}17)$$

The description of different types of plasma requires the use of inhomogeneous or homogeneous, as well as anisotropic or isotropic distribution function. A plasma in thermal equilibrium, for example, is characterized by a homogeneous, isotropic and time independent distribution function providing a complete description of the system under consideration. Knowing f_k we can deduce all the macroscopic variable of physical interest for the species k.

One of the primary problems of kinetic theory consists in determining the form of the distribution function for a given system. By integrating the distribution function we can obtain

The particle density = $n_k = \int f_k\,(\mathbf{r},\mathbf{v},t)\,d^3\mathbf{v}$

The mean velocity = $\mathbf{u}_k = \dfrac{1}{n_k} \int \mathbf{v} f_k\,(\mathbf{r},\mathbf{v},t)\,d^3\mathbf{v}$

The pressure tensor = $\mathbf{P}_k = m_k \int (\mathbf{v}-\mathbf{u}_k)(\mathbf{v}-\mathbf{u}_k) f_k\,(\mathbf{r},\mathbf{v},t)\,d^3\mathbf{v}$

The energy density = $\varepsilon_k = \dfrac{1}{2}\dfrac{m_k}{n_k} \int (\mathbf{v}-\mathbf{u}_k)^2 f_k\,(\mathbf{r},\mathbf{v},t)\,d^3\mathbf{v}$

The heat flux = $\mathbf{q}_k = \dfrac{1}{2} m_k \int (\mathbf{v}-\mathbf{u}_k)^2(\mathbf{v}-\mathbf{u}_k) f_k\,(\mathbf{r},\mathbf{v},\mathrm{t})\,d^3\mathbf{v}$

1.6.1. Boltzmann equation

In order to calculate the average values of the physical properties of the physical system, it is necessary to know the distribution function for the system under consideration. The dependence of the distribution function on the independent variables $\mathbf{r}$, $\mathbf{v}$ and t is governed by an equation known as the Boltzmann equation. Now if the plasma is collisionless, the particle of species k with coordinates $(\mathbf{r}, \mathbf{v})$ will be found after time dt in new coordinates $(\mathbf{r}', \mathbf{v}')$ such that

$$\mathbf{r}' = \mathbf{r} + \mathbf{v}\,dt$$

$$\mathbf{v}' = \mathbf{v} + \mathbf{a}\,dt$$

So all the particles in the volume $d^3v\, d^3r$ at time t will occupy a new volume $d^3\mathbf{r}'\, d^3\mathbf{v}'$ at time $(t+dt)$ (figure (1-9)). The Boltzmann equation in this case is written as

$$\frac{\partial f_k}{\partial t} + \mathbf{v}\cdot\frac{\partial f_k}{\partial \mathbf{r}} + \mathbf{a}\cdot\frac{\partial f_k}{\partial \mathbf{v}} = 0 \qquad (1\text{-}18)$$

Hance the collision term in the Boltzmann equation is set to zero we obtain the Vlasov equation (Chapman & Cowling 1970), (Laing 1976) and (Bittencourt 1986).

If the effect due to particle interactions is to be taken into account, equation (1-18) will not be correct. As a result of collisions during the time interval dt, some of the particles of type k which were initially within $d^3\mathbf{r}\, d^3\mathbf{v}$ may be removed from it, and particles of the other type initially outside may end up inside it as shown in figure (1-10). Generally the number of particles of type k inside $d^3\mathbf{r}\, d^3\mathbf{v}$ about the coordinates $(\mathbf{r}\ ,\ \mathbf{v})$ at time t, will be different from particles of type k inside the same volume element about the coordinates $(\mathbf{r}'\ ,\ \mathbf{v}')$ at the time $t+dt$. This net gain or loss of particles is as a result of collisions during the interval time dt in the volume $d^3\mathbf{r}\, d^3\mathbf{v}$.

1.6.2. Fokker - Planck equation

In this approach, if a single particle is chosen as a marker, or test particle, and observe its motion as it interacts with a large number of particles. These multi - particle collisions independently scatter the test particle in a random manner. We study the evolution of the system by obtaining a relation between the distribution function at time t and the distribution function at time $t + \Delta t$, thus

$$f(\mathbf{v}, t+\Delta t) = \int f(\mathbf{v}-\Delta\mathbf{v}, t)\, P(\mathbf{v}-\Delta\mathbf{v}, \Delta\mathbf{v})\, d^3\Delta\mathbf{v} \qquad (1\text{-}19)$$

where $P(\mathbf{v}, \Delta\mathbf{v})$ is the probability that a particle which has a velocity $\mathbf{v}$ at time t receives a velocity increment $\Delta\mathbf{v}$ at time interval between t and $t + \Delta t$. We define the averages (Laing 1976) and (Cairns 1985)

$$<\Delta\mathbf{v}> = \int d(\Delta v)\, \Delta\mathbf{v}\, P(\mathbf{v}, \Delta\mathbf{v}) \qquad (1\text{-}20)$$

$$<\Delta\mathbf{v}\,\Delta\mathbf{v}> = \int d(\Delta\mathbf{v})\, \Delta\mathbf{v}\,\Delta\mathbf{v}\, P(\mathbf{v}, \Delta\mathbf{v}) \qquad (1\text{-}21)$$

For sufficiently small Δt the collision term in equation (1-17) is given by

$$\left(\frac{\delta f}{\delta t}\right)_c = -\frac{\partial}{\partial \mathbf{v}}\cdot(\mathbf{D}_1 f) + \frac{1}{2}\frac{\partial^2}{\partial\mathbf{v}\,\partial\mathbf{v}} : (\mathbf{D}_2 f) \qquad (1\text{-}22)$$

where

$$\mathbf{D}_1 = \lim_{\Delta t \to 0} \frac{(<\Delta \mathbf{v}>)}{\delta t} \tag{1-23}$$

$$\mathbf{D}_2 = \lim_{\Delta t \to 0} \frac{(<\Delta \mathbf{v} \Delta \mathbf{v}>)}{\delta t} \tag{1-24}$$

$\mathbf{D}_1$ is known as the coefficient of dynamical friction, $\mathbf{D}_2$ the dispersion or velocity diffusion coefficient.

1.6.3. Liouville equation

Consider a gas of N particles in a volume V, and let $f_N(\mathbf{q}_1, \mathbf{p}_1, \mathbf{q}_2, \mathbf{p}_2, \ldots, \mathbf{q}_N, \mathbf{p}_N; t)$ be the distribution function of N particles. f_N is defined as probability that, at time t, particle 1 is in the state $\mathbf{q}_1, \mathbf{p}_1$, particle 2 in $\mathbf{q}_2, \mathbf{p}_2$ and so on. The Liouville equation can be expressed in the form

$$\frac{\partial f_N}{\partial t} + [f_N, H] = 0 \tag{1-25}$$

where H is the $N-$ particle hamiltonian and $[f_N, H]$ is a Poisson bracket. For the case of an external potential ψ and two - body potential Φ (Laing 1976)

$$H = \sum_{j=1}^{N} [\frac{1}{2m} \mathbf{p}_j^2 + \psi(\mathbf{q}_j)] + \sum_{i<j}^{N} \Phi(|\mathbf{q}_i - \mathbf{q}_j|) \tag{1-26}$$

and we can define a reduced distribution function

$$f_s(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_s ; t) = V^s \int f_N \, d\mathbf{x}_{s+1} \cdots d\mathbf{x}_N$$

where $\mathbf{x}_i = (\mathbf{q}_i, \mathbf{p}_i)$. Integrating the Liouville equation over $\mathbf{x}_{s+1}, ..., \mathbf{x}_N$ we obtain

$$\frac{\partial f_s}{\partial t} = [H_s, f_s] + \frac{N-s}{V} \int d\mathbf{x}_{s+1} [\sum_{i=1} \Phi_{i,s+1}, f_{s+1}] \tag{1-27}$$

$$\frac{\partial f_N}{\partial t} = [H, f_N] \tag{1-28}$$

In equation (1-28) H_s is the hamiltonian appropriate to a group of s identical interacting particles.

$$H_s = \sum_{i=1}^{s} [\frac{1}{2m} p_i^2 + \psi(\mathbf{q}_i)] + \sum_{i<j}^{s} \Phi(\mathbf{q}_{i,j}) \tag{1-29}$$

where $\Phi(\mathbf{q}_{i,j}) = \Phi(|\mathbf{q}_i - \mathbf{q}_j|)$. For $s=1$ and taking the thermodynamic limit

$$\frac{\partial f_1}{\partial t} + \frac{1}{m}\mathbf{p}_1 \cdot \frac{\partial f_1}{\partial \mathbf{q}_1} - \frac{\partial \psi}{\partial \mathbf{q}_1} \cdot \frac{\partial f_1}{\partial \mathbf{p}_1}$$

$$= n \int d\mathbf{q}_2 \, d\mathbf{p}_2 \frac{\partial \Phi_{1,2}}{\partial \mathbf{q}_1} \cdot \frac{\partial f_2}{\partial \mathbf{p}_1} \qquad (1\text{-}30)$$

f_1 is the probability of finding any particle in the state $(\mathbf{q}_1, \mathbf{p}_1)$ irrespective of the rest of the system. f_2 is the corresponding probability of finding a pair of particles in prescribed states. As we can see from equation (1-30), we cannot solve for f_1 unless we also solve the equation for f_2 which involves f_3, and so on up to f_N.

1.7. Transport coefficients

1.7.1. What is transport ?

In the previous section we described briefly the kinetic theory in the form required to describe plasma phenomena. This enables us to find the continuity, momentum and energy equations which describe the basic flow phenomena that arise in plasma. These phenomena are called *transport processes* (Kennard 1939) and (Present 1958)

The diffusion coefficient relates the flux of the particles to the density gradient. The electrical conductivity relates the electric current density to the electric field. The thermal conductivity relates the heat flux to the temperature gradient. Diffusion involves the transport of particles in the plasma, and the conductivities involve the transport of kinetic energy in the plasma.

Consider a single particle in a uniform magnetic field $B_\circ \hat{z}$. It spirals around a single field line, and is therefore confined in the $\hat{x}$, $\hat{y}$ directions.

Transport can occur at three levels.

1.7.1.1. Classical

In uniform magnetic field, the collision produces a jump in the position of a particle in the order of a Larmor radius, using transport of energy and particles (Bishop 1988).

1.7.1.2. Neoclassical

In a non-uniform field, the particle drifts ($\nabla \mathbf{B}$ and curvature) off field lines. For an axisymmetric system (e.g tokamaks) the orbits are closed and there is no net transport of a single particle. However, collisions cause interaction with the drifts and produce neoclassical transport which is generally much larger than classical, and a major study in its own right (Bishop 1988).

1.7.1.3. Anomalous

Many features of the experiments are not explained by collisional classical or even neoclassical transport. There are still unsolved problems, but in practice there is clear evidence for anomalous electron transport losses from ohmically heated tokamaks operating in a low - density regime. This would take us into theory of turbulence and is out of the scope of this review (Liewer 1985).

1.7.2. General transport equation

To derive the transport equation which we require to derive the transport coefficients, let $x \equiv x(\mathbf{v})$ represent any physical property of the particles in a plasma which depends on particle velocity. The average of x can be obtained by multiplying by the distribution function and integrating over the velocity. The equation governing the temporal and spatial variation of the average value of x can be obtained similarly by multiplying the Boltzmann equation by x and integrating over the velocity (Bittencourt 1985)

$$\int x \left(\frac{\partial f_k}{\partial t}\right) d^3\mathbf{v} + \int x\, \mathbf{v} \cdot \nabla f_k\, d^3\mathbf{v} + \int x\, \mathbf{a} \cdot \nabla_{\mathbf{v}} f_k\, d^3\mathbf{v}$$

$$= \int x \left(\frac{\delta f_k}{\delta t}\right)_{coll} d^3\mathbf{v} \qquad (1\text{-}31)$$

and after some manipulation we find

$$\frac{\partial}{\partial t}(n_k < x >_k) + \nabla \cdot (n_k < x \mathbf{v} >_k) - n_k < \mathbf{a} \cdot \nabla_{\mathrm{v}} x >_k$$

$$= [\frac{\delta}{\delta t}(n_k < x_k >_k)] \quad (1\text{-}32)$$

This is the general transport equation, the term on the right hand side denoting the change of quantity x per unit volume, for particles of type k due to collisions with other particles as well as those of type k

Let us set $x = 1$

$< x > = 1$

$< x\ \mathbf{v} >_k = < \mathbf{v} >_k = \mathbf{u}_k$

$\nabla_{\mathrm{v}}\, x = 0$

The substitution of these results into the general transport equation gives

$$\frac{\partial n_k}{\partial t} + \nabla \cdot (n_k\, \mathbf{u}_k) = S_k \quad (1\text{-}33)$$

where S_k represents the rate per unit volume at which particles of type k are produced or lost as a result of collisions. In the absence of interaction leading to particle creation or destruction, $S_k = 0$ and equation (1-33) will be

$$\frac{\partial n_k}{\partial t} + \nabla \cdot (n_k \, \mathbf{u}_k) = 0 \tag{1-34}$$

This equation is called the continuity equation. In the case of equal mass plasma (electron - positron) S is written as (Tajima & Taniuti 1990)

$$S = \nu_c \, n_\gamma - \nu \, n_- n_+$$

where n_- , n_+ are the electron and positron densities, respectively, n_γ the photon density, ν_c the pair creation density frequency and ν the pair annihilation rate. In this work to be presented in this thesis, we shall not consider these effects and instead assume constant electron and positron numbers, and thus set $S = 0$.

Now if $x = m_k \, \mathbf{v}$ and $\mathbf{v} = \mathbf{c}_k + \mathbf{u}_k$, where $\mathbf{c}_k$ is the random velocity $< \mathbf{c}_k > = 0$, the terms of the general transport equation will be

$$\frac{\partial}{\partial t} (m_k \, n_k < \mathbf{v} >_k) = m_k \, n_k \frac{\partial \mathbf{u}_k}{\partial t} + m_k \, \mathbf{u}_k \frac{\partial n_k}{\partial t}$$

$$\nabla\cdot(m_k\, n_k <\mathbf{v}\,\mathbf{v}>_k) = \nabla\cdot(m_k\, n_k\, \mathbf{u}_k\, \mathbf{u}_k + m_k\, n_k <\mathbf{c}_k\, \mathbf{c}_k>)$$

$$-\, n_k <\mathbf{F}\cdot\nabla_{\mathrm{v}}\mathbf{v}>_k = -n_k <\mathbf{F}>$$

where $\mathbf{F}$ is the force. Substituting the above result into the general transport equation gives

$$m_k\, n_k [\frac{\partial \mathbf{u}_k}{\partial t} + (\mathbf{u}_k\cdot\nabla)\mathbf{u}_k] + \nabla\cdot\mathbf{P}_k - n_k <\mathbf{F}> = \mathbf{R}_k \qquad (1\text{-}35)$$

where $\mathbf{P}_k$ = pressure tensor $= n\, m_k <\mathbf{c}_k\, \mathbf{c}_k>$.

$\mathbf{R}_k$ = rate of change of the momentum due to collisions . Equation (1-35) is called the momentum transport equation

To derive the energy transport equation, let $x = \frac{1}{2} m_k \mathrm{v}^2$, so the terms of the general transport equation will be

$$n_k <x> = \frac{m_k}{2} <\mathrm{v}^2>_k = \frac{n_k\, m_k}{2} <c_k^2> + \frac{n_k\, m_k}{2} u_k^2$$

$$= \frac{3}{2} p_k + \frac{n_k\, m_k}{2} u_k^2$$

$$\nabla_{\mathrm{v}} x = \frac{m_k}{2} \nabla_{\mathrm{v}} (\mathbf{v} \cdot \mathbf{v}) = m_k \, \mathbf{v}$$

where p_k = scalar pressure

Substituting into the general transport equation we find

$$\frac{3}{2}\frac{\partial p_k}{\partial t}\frac{\partial}{\partial t}(m_k n_k \frac{u_k^2}{2}) + \nabla \cdot [\frac{m_k n_k}{2} \langle (\mathbf{v}\cdot\mathbf{v})\mathbf{v} \rangle] - n_k \langle \mathbf{F}\cdot\mathbf{v} \rangle_k = Q_k \quad (1\text{-}36)$$

where Q_k represents the rate of change of the energy due to collisions. Now let us discuss some of transport coefficients individually

1.7.3. Viscosity

The phenomenon of viscosity occurs in a fluid when it is undergoing shearing motion. To represent a formula for the viscosity let us suppose that the plasma is in mass motion with a velocity everywhere the same in direction but varying in magnitude, and let the x-axis be in the direction of the assumed velocity gradient and y-axis be parallel to the direction of the velocity (figure (1-11)). If $\mathbf{v}_\circ$ the velocity, $\frac{d\mathbf{v}_\circ}{d\mathbf{x}}$ the velocity gradient and $\mathbf{P}_{xy}$ the shearing component of force in the y-direction, then we can write, in the absence of a magnetic field (Kennard 1938),

$$P_{xy} = \mu \frac{d\mathbf{v}_\circ}{d\mathbf{x}} \qquad (1\text{-}37)$$

The factor of proportionality μ in the equation above is called the coefficient of the viscosity of the fluid. In the presence of a magnetic field the viscosity becomes a tensor. When the magnetic field in the direction of the z-axis it is written as (Haines 1974) :

$$\mu = \begin{bmatrix} \mu_1 & \mu_2 & 0 \\ \mu_4 & \mu_3 & 0 \\ 0 & 0 & \mu_\circ \end{bmatrix} \qquad (1\text{-}38)$$

where $\mu_\circ$, μ_1 , μ_2 , μ_3 and μ_4 will be defined in the later chapters .

1.7.4. Thermal conductivity

When inequalities of temperature exist, heat will be transferred by the particle collisions from a hotter region to colder; this process is called heat or thermal conduction and is independent of any transfer of energy that may be going on simultaneously by means of radiation. The heat flows down the temperature gradient. The amount of heat conducted in a given time is found by experiment to be accurately proportional to the temperature gradient as long as the latter is approximately uniform over any distance equal in length to the particle mean free path; and the amount of heat transferred per second across unit area of any small plane divided by the temperature gradient is called the thermal conductivity. So if $\mathbf{q}$ is the heat flux, and λ the thermal conductivity then (Present 1958)

$$\mathbf{q} = -\,\lambda\,\nabla T \tag{1-39}$$

In a magnetic field in the direction of z-axis the thermal conductivity becomes a tensor (Clemmow and Dougherty 1969) :

$$\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & 0 \\ \lambda_3 & \lambda_4 & 0 \\ 0 & 0 & \lambda_\circ \end{bmatrix} \tag{1-40}$$

where $\lambda_\circ$, λ_1 , λ_2 , λ_3 and λ_4 will be defined later.

From equation (1-40) we can see that in the case of ion - electron plasmas and presence of a magnetic fieldheat flows both perpendicular and parallel to that field. In the case of equal mass plasma the situation will be different, as we shall see later.

1.7.5. Electrical conductivity

When an electric and magnetic field are present the force term in the Boltzmann equation will be $e_k\, n_k\, (\mathbf{E}+\mathbf{v}\times\mathbf{B})$. Here currents flow associated with the particle motion and in the absence of a magnetic field they are written (Bittencourt 1986)

$$\mathbf{J}_k = e_k\, n_k\, \mathbf{u}_k$$

$$= \sigma_k\, \mathbf{E}$$

where σ_k is the electrical conductivity. When a magnetic field is present there is a current flow perpendicular to both the electric and the magnetic fields, known as the Hall current, so when the magnetic field is in the direction of the z- axis the electrical conductivity becomes a tensor which is (Clemmow and Dougherty 1969):

$$\sigma = \begin{bmatrix} \sigma_{xx} & -\sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \tag{1-41}$$

To illustrate the physical meaning of the components of σ, it is convenient to separate the electric field in components parallel to $\mathbf{B}$, as shown in figure (1-12), the element $\sigma_{\perp}$ called σ perpendicular since it governs the electric current in the component of the electric field normal to the magnetic field, σ_H (known as Hall conductivity) governs the electric current in the direction perpendicular to both the electric and the magnetic fields. The element $\sigma_{||}$ is the longitudinal conductivity, since it governs the electric current in the direction of the electric field component along the magnetic field; this component of σ is not affected by the magnetic field.

1.7.6. Diffusion

When a gradient in the number density of charged particles exists, the particles will tend to move as a result of collisions with other particles toward the region of lower density. The particle flux is given by (Cairns 1985)

$$\Gamma_k = - D_k \nabla n_k \tag{1-42}$$

where Γ_k is the particle flux, D_k is the diffusion coefficient.

There is another kind of diffusion caused by the presence of the temperature gradient. By considering the thermal diffusion, the heat flow can be written as (Kennard 1938)

$$\mathrm{q}_k = n_k D_T \nabla T \tag{1-43}$$

where D_T is the thermal diffusion. In a magnetic field, the particle displacement across the field is restricted to one gyromagnetic radius between collisions, so that the diffusion and the heat conduction across the field are reduced by a factor $\frac{1}{\Omega_k \tau}$, where τ is the relaxation time to thermal equilibrium. Diffusion in a magnetic field along the z-axis is written as a tensor (Cairns 1985) and (Rose & Clark 1961) :

$$D = \begin{bmatrix} D_{xx} & D_{xy} & 0 \\ -D_{yx} & D_{yy} & 0 \\ 0 & 0 & D_{zz} \end{bmatrix} \tag{1-44}$$

1.8. How electron - positron plasma is created

Primary cosmic rays are for the most part high - energy protons; they also include nuclei of heavier elements as well as electrons and positrons. As they approach the earth, the trajectories of these particles are deflected by the earth's magnetic field. Upon entering the atmosphere, they collide with the atoms of the atmosphere and their nuclei. As we can see from figure (1-13), a primary particle (usually a proton) collides with a nucleus of oxygen or nitrogen in the atmosphere. The products are *neutrons* (n), *protons* (p), neutral $\pi^{\circ}-mesons$, charged $\pi^{-}-mesons$, $\pi^{+}-mesons$, *antiprotons* $(\bar{p})$, *antineutrons* $(\bar{n})$, heavy $\kappa-mesons$, and *hyperons* (Y).

Neutral $\pi-mesons$ decay into gamma rays (γ), which in turn materialize into *electrons* (e^{-}) and *positrons* (e^{+}). Charged mesons may strike other atmospheric nuclei or decay into $\mu-mesons$ and *neutrinos* (ν). The electrons and positrons radiate part of their energy in the form of gamma rays (Bremsstrahlung), which then decay into electrons and positrons. Broken lines in figure (1-13) indicate that further interactions may take place. Most charged particles end up as *electrons*, *positrons* and $\mu-$*mesons* (Rossi & Olbert 1970).

It was pointed out that $\gamma-$ rays produced in sufficiently luminous and compact astrophysical objects would create electron - positron pairs by collisions with lower energy photons $\gamma + \gamma \rightarrow e^{+} + e^{-}$. So we expect an electron - positron plasma to be found in the early universe, in active galactic cuclie (AGN) and in pulsar atmosphers. The other environment in which electron - positron plasma appears is the electron - positron collider. However good laboratory examples are found also in semiconductor plasma of holes and electrons (Tajima & Taniuti 1990).

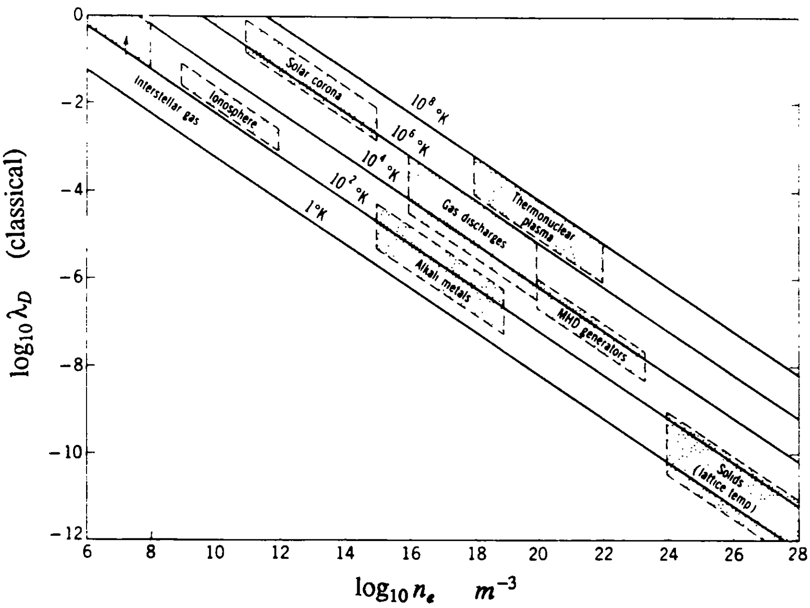

Figure (1-1). The Debye length (in meters) as a function of electron density and temperature .

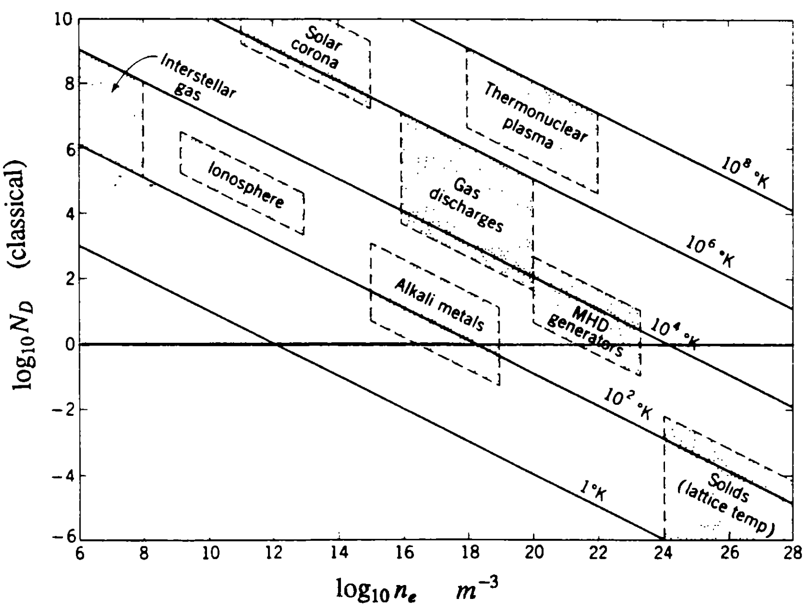

Figure (1-2). The number of particles in a Debye sphere as a function of electron density and temperature .

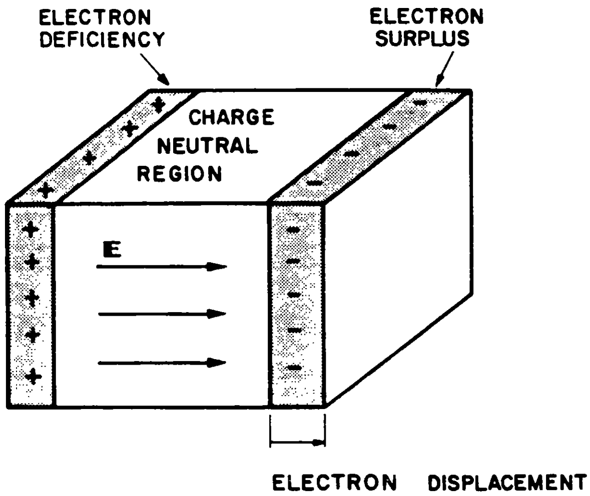

Figure (1-3). The electric field resulting from charge separation provide the force which generates the electron plasma oscillations.

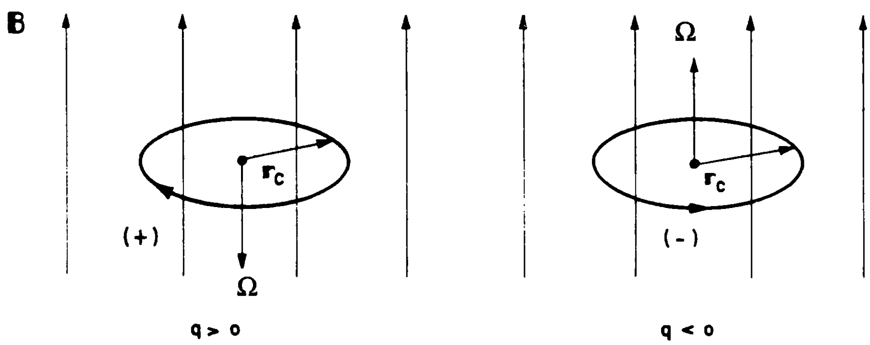

Figure (1-4). Circular motion of charged particle about the guiding centre G in a uniform magnetostatic field .

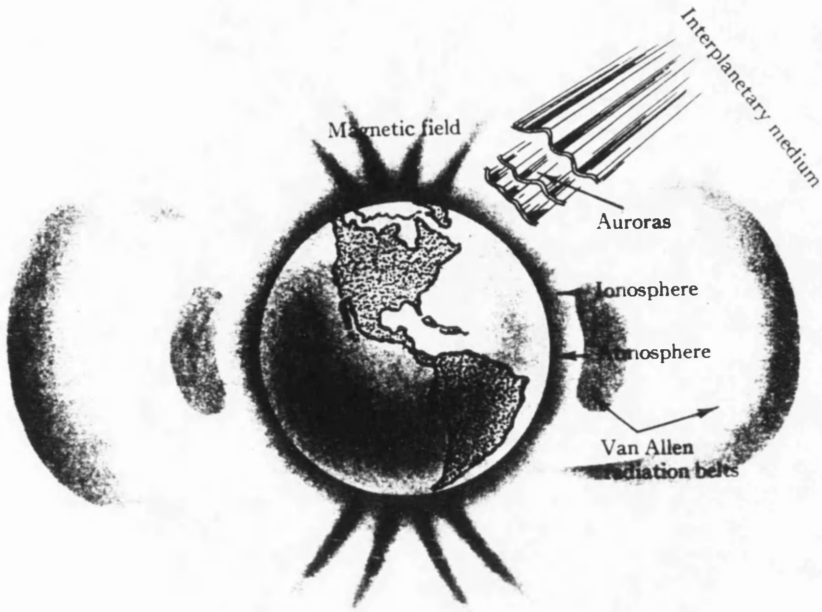

Figure (1-5). Plasma in the magnetosphere .

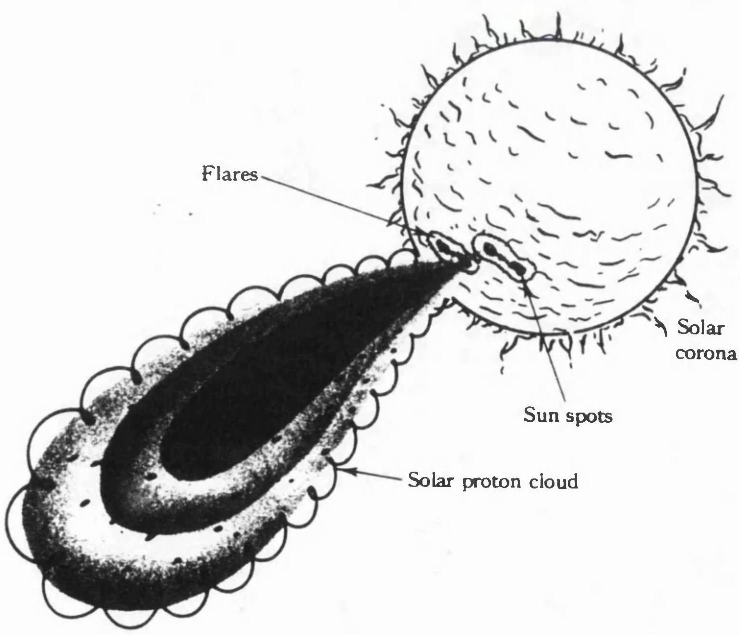

Figure (1-6). Plasma phenomena associated with the sun .

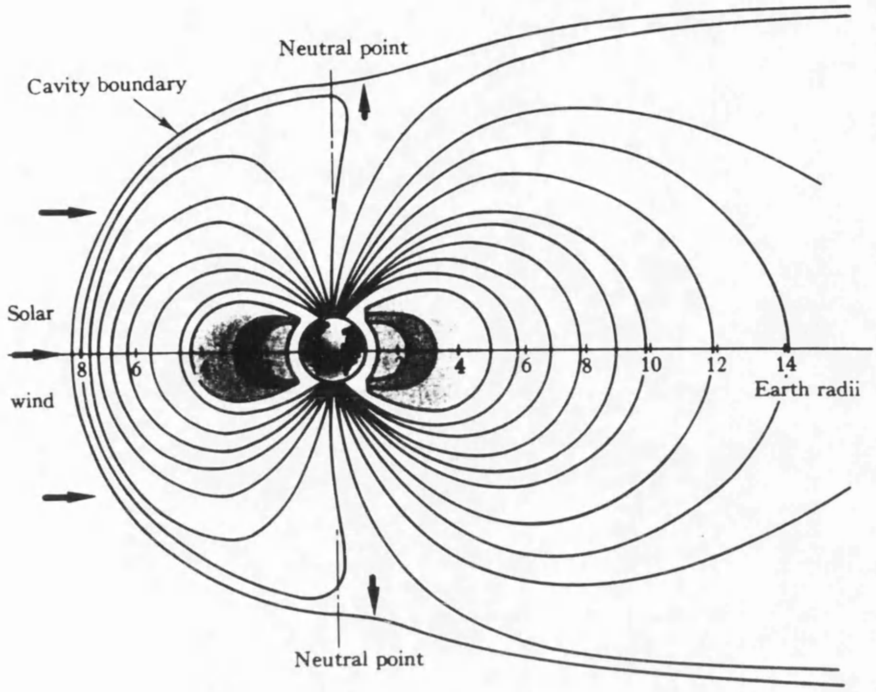

Figure (1-7). Distortion of the magnetosphere by the solar wind .

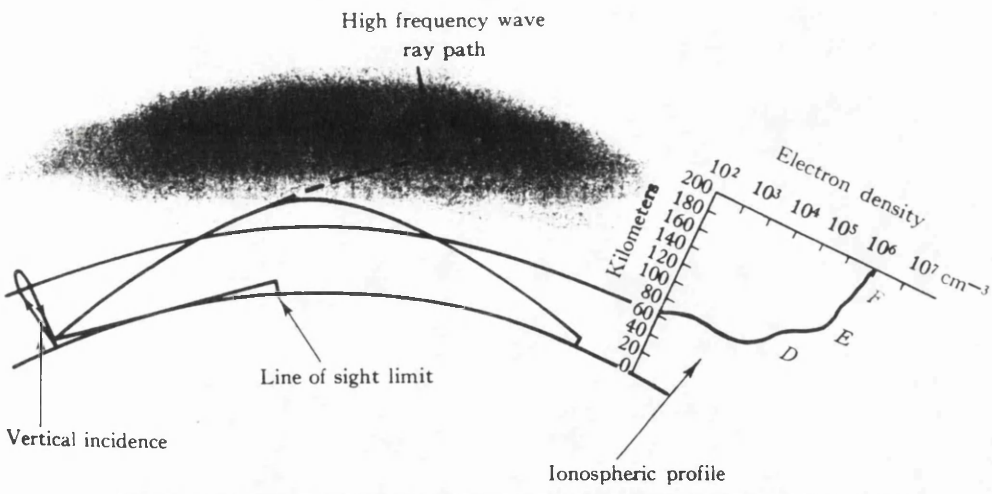

Figure (1-8). Ionospheric refraction of radio wave .

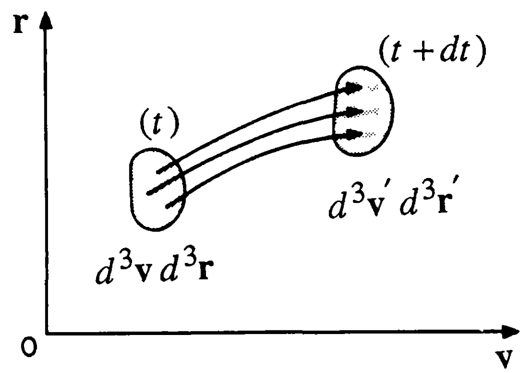

Figure (1-9). In the absence of collisions the particles within the volume $d^3\mathbf{r}\, d^3\mathbf{v}$ at time t , will occupy a new volume $d^3\mathbf{v}'\, d^3\mathbf{r}'$ at time $t+dt$.

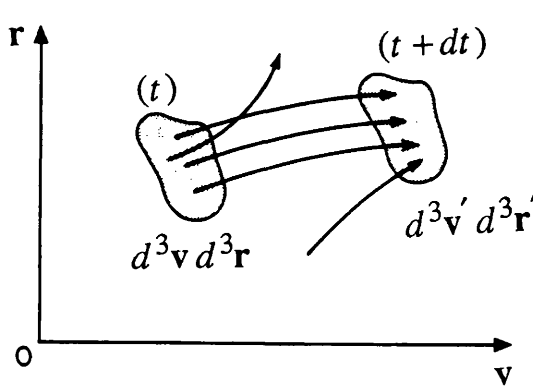

Figure (1-10). As a result of collisions during the time interval dt , the particles entering and leaving the volume element $d^3\mathbf{v}\, d^3\mathbf{r}$.

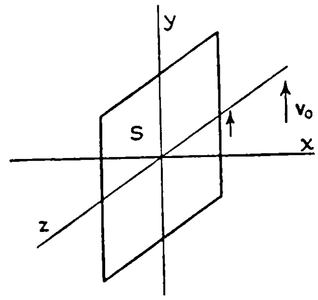

Figure (1-11). Illustrating viscosity.

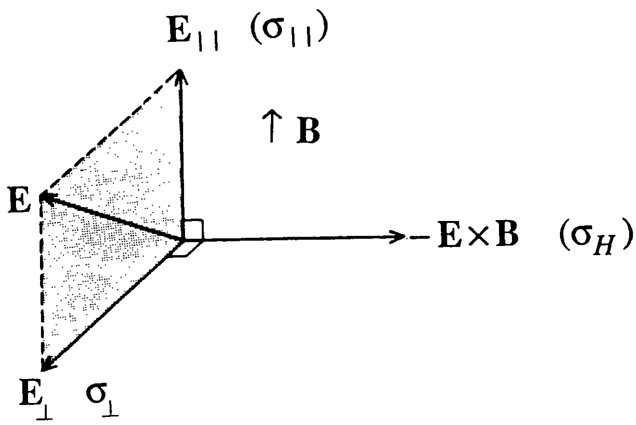

Figure (1-12). The conductivity components govern the magnitude of the current along the directions $\mathbf{E}_{||}$, $\mathbf{E}_{\perp}$ and $-\mathbf{E}\times\mathbf{B}$.

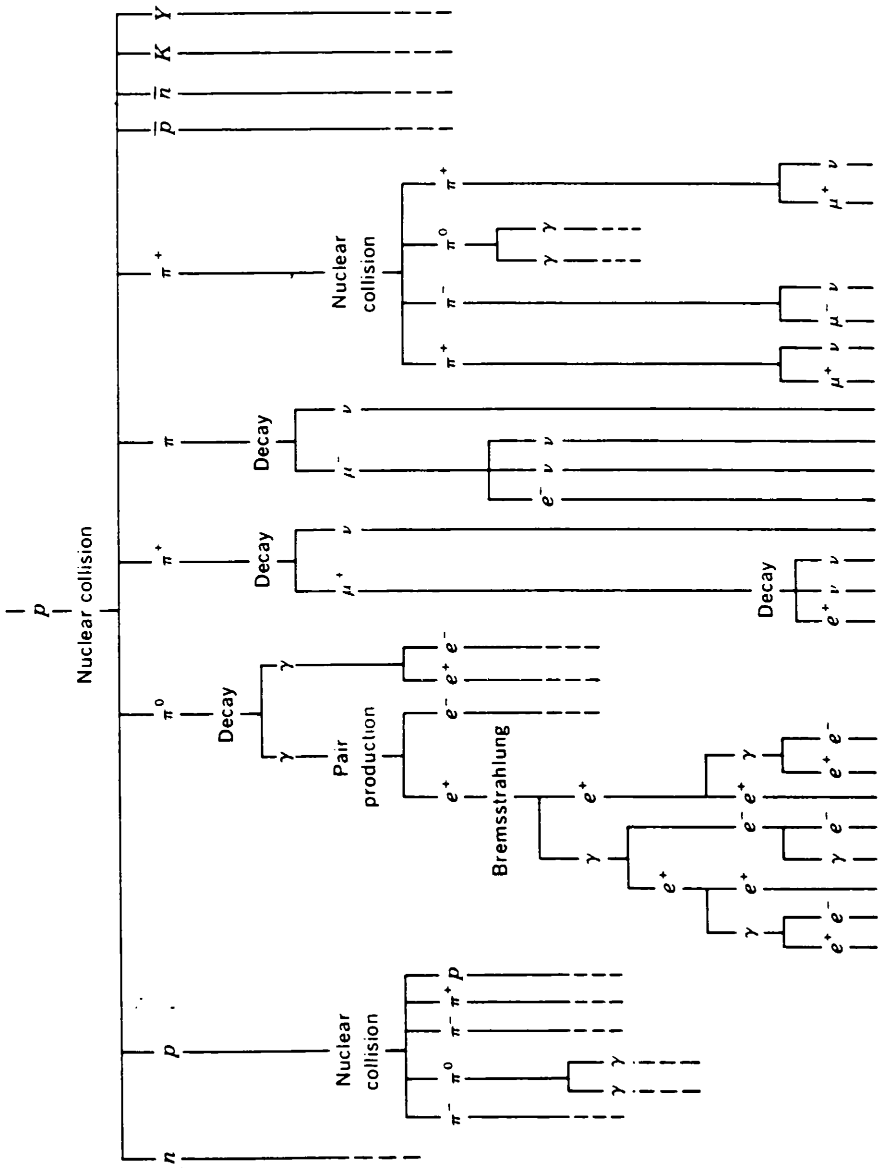

Figure (1-13). Progeny of a cosmic - ray particle .

CHAPTER TWO

Transport theory of equal mass plasma using the method of irreversible thermodynamics

2.1. Motivation

The usual method of deriving transport coefficients is through the expansion of the Boltzmann equation. This method is rather involved, as we shall see later, and the essential physics and qualitative picture can be obtained by the more direct macroscopic approach of phenomenological irreversible thermodynamics.

If we start with the momentum equation for each species, these equations simply describe in macroscopic terms the conservation of matter, momentum and energy. Next we combine these equations with the well known Gibbs equation for unit mass (Haines 1974).

$$T \ dS_m = dU_m + P \ dV_m \tag{2-1}$$

and the equation for local change of the entropy density is:

$$\frac{\partial S}{\partial t} + \nabla \cdot \mathbf{S} = \sigma \tag{2-2}$$

where T= temperature, U_m = energy per unit mass, S_m = entropy per unit mass, P = the pressure, $S = \rho\, S_m$, **S** = entropy flux σ = rate of production of the entropy, which by the second law of thermodynamics should be positive. Immediately we find that the thermodynamic fluxes, for example the heat flux or electric current, are related in a very definite and understandable way to the thermodynamic force, for example a temperature gradient or an electric field. The forces can be considered physically to be the direct cause of the fluxes. Cross phenomena such as the thermoelectric effect and the influence of a magnetic field arise naturally in this approach. The inventive set in non - equilibrium thermodynamics is the use of Gibbs equation for a non- but near - equilibrium system.

The Boltzmann equation for any species k and distribution function f_k (Chapman & Cowling 1970) is :

$$\frac{\partial f_k}{\partial t} + \mathbf{v}\cdot\nabla f_k + \frac{e_k}{m_k}(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c})\cdot\nabla_{\mathrm{v}} f_k = (\frac{\delta f_k}{\delta t})_{coll} \qquad (2\text{-}3)$$

From equation (2-3) we can get

$$\frac{\partial \rho_k}{\partial t} + \nabla\cdot(\rho_k\ \mathbf{u}_k) = 0 \qquad (2\text{-}4)$$

$$\frac{\partial}{\partial t}(\rho_k \ \mathbf{u}_k) + \nabla\cdot(\rho_k \ \mathbf{u}_k \ \mathbf{u}_k + \mathbf{P}_k) - n_k \ e_k(E + \frac{\mathbf{u}_k \times \mathbf{B}}{c}) = \sum_i \mathbf{R}_{ik} \quad (2\text{-}5)$$

$$\frac{\partial}{\partial t}(1/2 \ \rho_k \ u_k^2 + U_k) + \nabla\cdot(1/2 \ \rho_k \ u_k^2 \ \mathbf{u}_k +$$

$$U_k \ \mathbf{u}_k + \mathbf{P}_k\cdot\mathbf{u}_k + \mathbf{q}_k) - n_k \ e_k \ \mathbf{E}\cdot\mathbf{u}_k = \sum_i \varepsilon_{ik} \qquad (2\text{-}6)$$

where U_k = internal energy density = $1/2\int m_k \ (\mathbf{v}-\mathbf{u}_k)^2 \ f_k d\mathbf{v}=3/2p_k$ = 3/2 scalar pressure =3/2 $n_k \ k_B \ T_k$, T_k is the temperature of species k , R_{ik} the rate change of the momentum of species k due to the collisions with the species i , ε_{ik} the rate change in the energy of species k due to collisions with the species i and k_B Boltzmann constant (Chapman & Cowling 1970) and (Haines 1974).

By Newton's third law

$$\mathbf{R}_{ik} + \mathbf{R}_{ki} = 0$$

$$\varepsilon_{ik} + \varepsilon_{ki} = 0 \qquad (2\text{-}7)$$

If we take equation (2-6) and add it to equation (2-4) times $\frac{1}{2}u_k^2$ and subtract equation (2-5) times $\mathbf{u}_k$ we find

$$\frac{\partial U_k}{\partial t} + \nabla\cdot(U_k\ \mathbf{u}_k + \mathbf{q}_k) + (\mathbf{P}_k\cdot\nabla)\cdot\mathbf{u}_k = \sum_i (\varepsilon_{ik} + \mathbf{R}_{ik}\cdot\mathbf{u}_k) \quad (2\text{-}8)$$

In centre of mass frame and in general, the total pressure $\mathbf{P}$, the total internal energy U and the total heat flux $\mathbf{q}$ are not simply related to $\mathbf{P}_k$, U_k , $\mathbf{q}_k$, but should include correction terms, as shown

$$\mathbf{P} = \sum_k \mathbf{P}_k + \sum_k \rho_k(\mathbf{u}_k-\mathbf{V}_c)(\mathbf{u}_k-\mathbf{V}_c) \quad (2\text{-}9)$$

$$U = \sum_k U_k + \sum_k \frac{1}{2}\rho_k(\mathbf{u}_k-\mathbf{V}_c)^2 \quad (2\text{-}10)$$

$$\mathbf{q}=\sum_k \mathbf{q}_k+\sum_k [\,U_k(\mathbf{u}_k-\mathbf{V}_c)+\mathbf{P}_k\cdot(\mathbf{u}_k-\mathbf{V}_c)+\frac{1}{2}\rho_k(\mathbf{u}_k-\mathbf{V}_c)^2(\mathbf{u}_k-\mathbf{V}_c)\,] \quad (2\text{-}11)$$

where $\mathbf{V}_c$ is the center of mass velocity

2.2. Gibbs equation

If the quantities in equation (2-1) are per unit volume rather than per unit mass, Gibbs equation for species k will be (De Groot & Mazur 1962)

$$T_k\, d(\frac{S_k}{\rho_k}) = d(\frac{U_k}{\rho_k}) + p_k\, d(\frac{1}{\rho_k}) \tag{2-12}$$

or

$$T_k\, dS_k = dU_k - \mu_k\, d\rho_k \tag{2-13}$$

where μ_k is the chemical potential and

$$\rho_k\, \mu_k = U_k + p_k - T_k S_k = Gibbs\ potential \tag{2-14}$$

and from this we can find

$$dS = \sum_k dS_k = \sum_k \frac{1}{T_k}(dU_k - \mu_k\, d\rho_k) \tag{2-15}$$

Using equations (2-2) , (2-8) and (2-15) we find

$$\mathbf{S} = \sum_k \frac{1}{T_k}(U_k\, \mathbf{u}_k + p_k\, \mathbf{u}_k + \mathbf{q}_k - \rho_k\, \mu_k\, \mathbf{u}_k)$$

$$= \sum (S_k\, \mathbf{u}_k + \frac{\mathbf{q}_k}{T_k}) \tag{2-16}$$

The rate of generation of the entropy per unit volume is

$$\sigma = \sum_k \frac{1}{T_k}[-\tau_k : \nabla \mathbf{u}_k - \mathbf{q}_k \cdot \ln(T_k) + \sum_i (\varepsilon_{ki} - \mathbf{R}_{ki} \cdot \mathbf{u}_k)] \quad (2\text{-}17)$$

where $\mathbf{P}_k = p_k I + \tau_k$ = total pressure, p_k is the scalar pressure, τ_k the stress tensor and $\mathbf{q}_k = \int 1/2\ m_k\ (\mathbf{v}-\mathbf{u}_k)^2\ (\mathbf{v}-\mathbf{u}_k)\ f_k d\mathbf{v}$ = heat flux

2.3. Equal mass plasma

In the case of equal mass plasma the centre of mass velocity will be

$$\mathbf{V}_{c=}\ 1/2(\mathbf{u}_{e^-} + \mathbf{u}_{e^+})$$

so the velocities of the electrons and positrons in the centre of mass frame will be :

$$\mathbf{V}^- = \mathbf{u}_{e^-} - 1/2(\mathbf{u}_{e^-} + \mathbf{u}_{e^+}) = 1/2(\mathbf{u}_{e^-} - \mathbf{u}_{e^+}) \quad (2\text{-}18)$$

$$\mathbf{V}^+ = 1/2(\mathbf{u}_{e^+} - \mathbf{u}_{e^-}) \quad (2\text{-}19)$$

and equations (2-9) , (2-10) and (2-11) for electron - positron plasma will be

$$\mathbf{P} = \mathbf{P}_{e^-} + \mathbf{P}_{e^+} \quad (2\text{-}20)$$

$$U = U_{e^-} + U_{e^+} \tag{2-21}$$

$$\mathbf{q} = \mathbf{q}_{e^-} + \mathbf{q}_{e^+} \tag{2-22}$$

As we can see from equations (2-20) , (2-21) and (2-22) **P** , U and **q** in the case of equal mass plasma is not like the case of electron - ion plasmas.

From equation (2-5) and neglecting the inertial terms

$$\frac{\nabla p}{n\ e} \pm (\mathbf{E} \pm 1/2\ \frac{\mathbf{V}^- - \mathbf{V}^+}{c} \times \mathbf{B}) = \frac{R^\pm}{n\ e} = \mathbf{E}' \tag{2-23}$$

where $\mathbf{E}'$ = generalized electric field.

The rate of generation of entropy per unit volume for equal mass plasma can be obtained from equation (2-17)

$$\sigma = \frac{1}{T}[-(\tau_{e^-} - \tau_{e^+}):\nabla(\mathbf{u}_{e^-} - \mathbf{u}_{e^+}) - \mathbf{q}\cdot\nabla \ln T + \mathbf{j}\cdot\mathbf{E}'] \tag{2-24}$$

From equation (2-24) we can identify two thermodynamic forces, (Spitzer 1962)

$$-\nabla \ln T \quad \text{and} \quad \mathbf{E}'$$

and their conjugate fluxes, the heat flux and current density, can be written as a linear combination of the forces (Spitzer 1962) and (Haines 1974)

$$\mathbf{j} = \alpha_{11}\, \mathbf{E}' + \alpha_{12}(-\nabla \ln T)$$

$$\mathbf{q} = \alpha_{21}\, \mathbf{E}' + \alpha_{22}(-\nabla \ln T) \tag{2-25}$$

where α_{11} , α_{12} , α_{21} , α_{22} are unknown functions, to be determined.

2.4. Relaxation time method

In equilibrium, the thermal velocity of a particle is $(\frac{3\, k_B\, T}{m})^{1/2}$, and because the temperature and mass of the electron and positron are the same, they also have the same thermal velocity at this state, namely $(\frac{3\, k_B\, T}{m})^{1/2}$.

The relaxation time is (Laing 1981) and (Haines 1974) :

$$\tau^{\pm} = \frac{(3\, k_B\, T)^{3/2}\, (m)^{1/2}}{e^4\, 8\pi n\, \ln\Lambda} \tag{2-26}$$

where $\Lambda = \frac{3}{2e^3} \left[\frac{k_B^3\, T^3}{\pi n}\right]^{1/2}$

If we assume that collisions dominate, so that the lowest order distribution function is Maxwellian

$$f_k^{\circ} = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[\frac{-m\,(\mathbf{v}-\mathbf{u}_k)^2}{2 k_B T}\right] \qquad (2\text{-}27)$$

The actual distribution function will be assumed to differ only slightly from this, so that $f = f_k^{\circ} + f_k^{'}$, where $f_k^{'} \ll f_k^{\circ}$, and we have the following equation (Clemmow & Dougherty 1969) and (Haines 1974) :

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{e_k}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_{\mathrm{v}}\right] f_k^{\circ} = -\nu_c\, f_k^{'} \qquad (2\text{-}28)$$

ν_c is the collision frequency which is assumed to be velocity independent . We have made this approximation as a first attempt to work on electron - positron plasma.

By using equation (2-27), (2-28) and replacing the time derivatives using the moment equations we get (Braginskii 1958), (Chapman & Cowling 1970) and (Haines 1974)

$$f_k^{\circ}\left\{-\frac{5}{2T}(\mathbf{v}-\mathbf{u}_k)\cdot\nabla T\,\left[1-\frac{m\,(\mathbf{v}-\mathbf{u}_{k)}^2}{5 k_B T}\right] + \frac{m}{k_B T} \times\right.$$

$$[(\mathbf{v}-\mathbf{u}_k)(\mathbf{v}-\mathbf{u}_k):\nabla\mathbf{u}_k - \frac{1}{3}(\mathbf{v}-\mathbf{u}_k)^2\,\nabla\cdot\mathbf{u}_k] + \frac{1}{n\,k_B\,T}[-(\varepsilon_k - \mathbf{R}_k\cdot\mathbf{u}_k)$$

$$\left.(1-\frac{m\,(\mathbf{v}-\mathbf{u}_k)^2}{3\,k_B\,T}) + \mathbf{R}_k\cdot(\mathbf{v}-\mathbf{u}_k)]\right\} = -\,\nu_c\, f_k' \qquad (2\text{-}29)$$

2.4.1. Heat flux

$$\mathbf{q}_k = \frac{1}{2}\int m\;(\mathbf{v}-\mathbf{u}_k)^2\;(\mathbf{v}-\mathbf{u}_k) f_k' d^3\mathbf{v} \qquad (2\text{-}30)$$

If we use equation (2-29) for the electron and positron separately in equation (2-30) we get

for the electron

$$\mathbf{q}_{e^-} = -\frac{5}{2}\,\frac{n\,k_B^2\,T}{\nu_c\,m}\,\nabla T\; - \;\frac{5}{2}\,\frac{k_B\,T}{\nu_c\,m}\,\mathbf{R}_{e^-} \qquad (2\text{-}31)$$

for the positron

$$\mathbf{q}_{e^+} = -\frac{5}{2}\,\frac{n\,k_B^2\,T}{\nu_c\,m}\,\nabla T\; + \;\frac{5}{2}\,\frac{k_B\,T}{\nu_c\,m}\,\mathbf{R}_{e^+} \qquad (2\text{-}32)$$

But we have $\mathbf{R}_{e^-} = -\mathbf{R}_{e^+} = \mathbf{R}$, so using equation (2-22) we get

$$\mathbf{q} = -5\,\frac{n\,k_B^2\,T}{\nu_c\,m}\nabla T \;-\; 5\,\frac{k_B\,T}{\nu_c\,m}\,\mathbf{R} \tag{2-33}$$

Now if we go back to equation (2-25) and compare it with equation (2-33) we get $\alpha_{22} = 5\,\dfrac{n\,k_B^2\,T^{2}}{\nu_c\,m}$ = thermal conductivity (λ) , and $\alpha_{21} = 5\,\dfrac{e\,n\,k_B\,T}{\nu_c\,m}$.

The thermal conductivity in a magnetic field can be written as a tensor namely (Clemmow & Dougherty 1969) :

$$\lambda_k = \begin{bmatrix} \lambda_{1k} & \lambda_{2k} & 0 \\ \lambda_{3k} & \lambda_{4k} & 0 \\ 0 & 0 & \lambda_{ok} \end{bmatrix} \tag{2-34}$$

where $\lambda_{ok} = \dfrac{5}{2}\,\dfrac{n_k\,k_B^2\,T_k}{\nu_c\,m_k}$, $\lambda_{1k} = \lambda_{4k} = \dfrac{\nu_c^2}{\nu_c^2+\Omega_k^2}\,\lambda_{ok}$, $\lambda_{3k} = -\lambda_{2k} = \dfrac{\nu_c\,\Omega_k}{\nu_c^2+\Omega_k^2}\,\lambda_{ok}$, and $\Omega_k = \dfrac{e_k\,\mathbf{B}}{m_k}$ = gyro frequency, $\mathbf{B}$ the magnetic field which in the direction of the z-axis.

In the case of equal mass plasma $\sum_k \Omega_k = 0$. So this means

$\sum_k \lambda_{3k} = \sum_k \lambda_{2k} = 0$, since also Ω_k^2 is independent of k . This leads to

$\sum_k \lambda_{1k} = \sum_k \lambda_{4k} = \dfrac{\nu_c^2}{\nu_c^2+\Omega^2}\,\lambda_0$, where $\Omega^2 = \Omega_{e^+}^2 = \Omega_{e^-}^2 = \dfrac{e^2 B^2}{m^2}$,

$\lambda_o = \lambda_o^- + \lambda_o^+ = 5\,\dfrac{n\,k_B^2 T}{\nu_c\, m}$.

$$\lambda = \lambda_+ + \lambda_- = \lambda_o \begin{bmatrix} \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 & 0 \\ 0 & \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-35}$$

$$i.e.\quad \lambda = 5\,\frac{n\,k_B^2 T}{\nu_c\, m} \begin{bmatrix} \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 & 0 \\ 0 & \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-36}$$

2.4.2. Current density

$$\mathbf{J} = -e\int(\mathbf{v}-\mathbf{u}_{e^-})f'_{e^-}\,d^3\mathbf{v} + e\int(\mathbf{v}-\mathbf{u}_{e^+})f'_{e^+}\,d^3\mathbf{v} \qquad (2\text{-}37)$$

Using equation (2-29) for equal mass plasma we get

$$\mathbf{j} = \frac{e}{m\nu_c}(\mathbf{R}_{e^-} - \mathbf{R}_{e^+})$$

$$e.\quad \mathbf{j} = \frac{2e}{m\nu_c}\mathbf{R} \qquad (2\text{-}38)$$

If we combine equation (2-38) with equation (2-25) we get $\alpha_{12} = 0$, and $\alpha_{11} = \dfrac{2ne^2}{m\,\nu_c}$ which represents the electrical conductivity.

Now to calculate the electrical conductivity, in general we have the equation of motion in the presence of the electric field

$$m_k\frac{d\mathbf{u}_k}{dt} = e_k\,\mathbf{E} - \frac{m_k\,\mathbf{u}_k}{\tau}$$

$$\mathbf{j} = e_k\,n_k\,\mathbf{u}_k = \frac{n_k\,e_k^2\,\tau}{m_k}\mathbf{E} = \frac{n_k\,e_k^2}{m_k\,\nu_c}\mathbf{E} = \sigma\mathbf{E} \qquad (2\text{-}39)$$

In a magnetic field, σ can be written as a tensor (Clemmow & Dougherty 1969) and (Bittencourt 1986)

$$\sigma_k = \begin{bmatrix} \sigma_{1k} & \sigma_{2k} & 0 \\ \sigma_{3k} & \sigma_{4k} & 0 \\ 0 & 0 & \sigma_{ok} \end{bmatrix} \qquad (2\text{-}40)$$

where $$\sigma_{1k} = \sigma_{4k} = \frac{n_k e_k^2}{m_k} \frac{\nu_c}{\Omega_k^2 + \nu_c^2} ,$$

$$\sigma_{2k} = -\sigma_{3k} = \frac{n_k e_k^2}{m_k} \frac{\Omega_k}{\Omega_k^2 + \nu_c^2} .$$

In the case of equal mass plasma $\sum \Omega_k = 0$. So the electrical conductivity tensor for equal mass plasma can be written as:

$$\sigma = \sigma_- + \sigma_+ = \sigma_o \begin{bmatrix} \frac{\nu_c^2}{\nu_c^2 + \Omega^2} & 0 & 0 \\ 0 & \frac{\nu_c^2}{\nu_c^2 + \Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}41)$$

where $\sigma_o = \sum_k \sigma_{ok} = \frac{2 e^2 n_e}{m \, \nu_c}$, since $\sigma_{ok} = \frac{n_k e_k^2}{m_k \nu_c}$.

$$\sigma = \frac{2e^2 n_e}{m\,\nu_c} \begin{bmatrix} \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 & 0 \\ 0 & \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2\text{-}42)$$

2.4.3. Diffusion coefficient:

The particle flux Γ is written (Cairns 1985) (Ferziger & Kaper 1972) and (Laing 1981)

$$\Gamma = \int \mathbf{v} f' \, d^3\mathbf{v} \qquad (2\text{-}43)$$

$$i.e. \quad \Gamma = -\frac{k_B\,T_k}{m_k\,\nu_c}\,\nabla n_k \qquad (2\text{-}44)$$

$$\Gamma = -\,D_k\,\nabla n_k \qquad (2\text{-}45)$$

where D_k is the diffusion coefficient, and from equations (2-44) and (2-45) $D_k = \dfrac{k_B\,T_k}{m_k\,\nu_c}$.

In the presence of a magnetic field we have instead the following equations

$$\frac{k_B T_k}{m_k} \frac{\partial n_k}{\partial x} = -\nu_c \Gamma_x - \Omega_k \Gamma_y$$

$$\frac{k_B T_k}{m_k} \frac{\partial n_k}{\partial y} = \nu_c \Gamma_y + \Omega_k \Gamma_x$$

$$\frac{k_B T_k}{m_k} \frac{\partial n_k}{\partial z} = -\nu_c \Gamma_z \tag{2-46}$$

So from this we can write the diffusion as a tensor given by (Rose & Clark 1961) and (Cairns 1985)

$$D_k = \frac{k_B T_k}{m_k} \begin{bmatrix} \dfrac{\nu_c}{\nu_c^2+\Omega_k^2} & \dfrac{-\Omega_k}{\nu_c^2+\Omega_k^2} & 0 \\ \dfrac{\Omega_k}{\nu_c^2+\Omega_k^2} & \dfrac{\nu_c}{\nu_c^2+\Omega_k^2} & 0 \\ 0 & 0 & \dfrac{1}{\nu_c} \end{bmatrix} \tag{2-47}$$

Again, in the case of equal mass plasma when we sum over the two species $\sum_k \Omega_k = 0$, and equation (3-32) becomes

$$D = \frac{2 k_B T}{m \nu_c} \begin{bmatrix} \frac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 & 0 \\ 0 & \frac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-48}$$

So this means there is symmetry in the diffusion coefficient tensor in the case of equal mass plasmas.

2.4.4. Stress tensor

$$\tau_k = \int m\,(\mathbf{v}-\mathbf{u}_k)(\mathbf{v}-\mathbf{u}_k) f_k' \, d^3\mathbf{v} - \frac{1}{3}\int m\,(\mathbf{v}-\mathbf{u}_k)^2 f_k' \, d^3\mathbf{v} \tag{2-49}$$

and we have :

$$\tau_k = -\mu_k \left[\nabla \mathbf{u}_k - \frac{2}{3} I \, \nabla \cdot \mathbf{u}_k \right] \tag{2-50}$$

μ_k is the viscosity tensor (Clemmow & Dougherty 1969) and (Haines 1974) :

$$\mu_k = \begin{bmatrix} \mu_{1k} & \mu_{2k} & 0 \\ \mu_{4k} & \mu_{3k} & 0 \\ 0 & 0 & \mu_{ok} \end{bmatrix} \tag{2-51}$$

where $\mu_{ok} = \dfrac{n_k\, k_B\, T_k}{\nu_c}$, $\mu_{1k} = \mu_{ok} \dfrac{\nu_c^2}{\nu_c^2 + \Omega_k^2}$, $\mu_{2k} = \mu_{ok} \dfrac{\nu_c\, \Omega_k}{\nu_c^2 + \Omega_k^2}$,

$\mu_{3k} = \mu_{ok} \dfrac{\nu_c^2}{\nu_c^2 + \Omega_k^2}$, and $\mu_{4k} = \mu_{ok} \dfrac{2\nu_c\, \Omega_k}{\nu_c^2 + 4\,\Omega_k^2}$.

Again, in the case of equal mass plasmas $\sum_k \Omega_k = 0$, and

$$\mu = \mu_o \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-52}$$

where $\mu = \mu_- + \mu_+$, $\mu_o = \sum_k \mu_{ok} = \dfrac{2n\, k_B\, T}{\nu_c}$,

$\mu_1 = \sum_k \mu_{1k} = \mu_o \dfrac{\nu_c^2}{\nu_c^2 + \Omega^2}$, $\mu_3 = \sum_k \mu_{3k} = \mu_o \dfrac{\nu_c^2}{\nu_c^2 + \Omega^2}$,

$\mu_2 = \sum_k \mu_{2k} = 0$, and $\mu_4 = \sum_k \mu_{4k} = 0$. So μ can be written as :

$$\mu = \mu_o \begin{bmatrix} \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 & 0 \\ 0 & \dfrac{\nu_c^2}{\nu_c^2+\Omega^2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2-53}$$

Let $S_{ij} = 1/2\,\nabla\mathbf{u} - 1/3\,\delta_{ij}\,\nabla\cdot\mathbf{u}$ which is :

$$S_{ij} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \tag{2-54}$$

where $S_{xy} = S_{yx}$, $S_{xz} = S_{zx}$, and $S_{yz} = S_{zy}$. We have

$$\tau_{xx} = -\mu_o\,(S_{xx} + S_{yy}) - \mu_3\,(S_{xx} - S_{yy}) - 2\,\mu_4\,S_{xy}$$

$$\tau_{yy} = -\mu_0\,(S_{xx} + S_{yy}) + \mu_3\,(S_{xx} - S_{yy}) + 2\,\mu_4\,S_{xy}$$

$$\tau_{zz} = -2\,\mu_o\,S_{zz}$$

$$\tau_{xy} = \tau_{yx} = -2\mu_3 S_{xy} + \mu_4 (S_{xx} - S_{yy})$$

$$\tau_{xz} = \tau_{zx} = -2\mu_1 S_{xz} - 2\mu_2 S_{yx}$$

$$\tau_{yz} = \tau_{zy} = -2\mu_1 S_{yz} + 2\mu_2 S_{xz} \qquad (2\text{-}55)$$

In the case of equal mass plasmas, equation (2-41) becomes

$$\tau_{xx} = -\mu_o (S_{xx} + S_{yy}) - \mu_3 (S_{xx} - S_{yy})$$

$$\tau_{yy} = -\mu_o (S_{xx} + S_{yy}) + \mu_3 (S_{xx} - S_{yy})$$

$$\tau_{zz} = -2\mu_o S_{zz}$$

$$\tau_{xy} = \tau_{yx} = -2\mu_3 S_{xy}$$

$$\tau_{xz} = \tau_{zx} = -2\mu_1 S_{xz}$$

$$\tau_{yz} = \tau_{zy} = -2\mu_1 S_{yz} \qquad (2\text{-}56)$$

From this it can be seen that the transport coefficients, perpendicular to the magnetic field are usually reduced by the factor $\dfrac{\nu_c^2}{\nu_c^2 + \Omega^2}$ compared to that in the absence of a field. Note that the component in the direction of the magnetic field, that is the zz component, is the same in the absence or presence of the this field.

In the case of equal mass plasmas we can see from the equations above that the off- diagonal coefficients of electrical and thermal conductivity, viscosity and diffusion all vanish. Also it can be seen that in the case of equal mass plasmas all the diagonal transport coefficients have twice the magnitude of corresponding coefficients for the ion - electron plasmas in the limit of the ion mass $\gg$ the electron mass.

CHAPTER THREE

Kinetic theory of transport

3.1. Motivation

The state of an ionized gas (plasma) is specified by the distribution function f_k for each particle species k . The function f_k(v,r,t) describes the phase space density of particles at time t at point **r** , **v** , so that$f_k\, d\mathbf{v}\, d\mathbf{r}$ represents the number of particles in the six - dimensional volume element *dvdr* . Our plasma consists solely of electrons and positrons, which are of equal mass.

Behaviour of the ionized gas is described by a system of kinetic equations (Boltzmann equations) which carry the distribution functions forward in time (Braginskii 1965) and (Chapman & Cowling 1970) :

$$\frac{\partial f_k}{\partial t} + \mathbf{v}\cdot\nabla f_k + \frac{e_k}{m_k}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathrm{v}} f_k = (\frac{\delta f_k}{\delta t})_{coll} \qquad (3\text{-}1)$$

where e_k = particle charge

E denotes the electric field, and **B** the magnetic field.

Particles of species k collide with particles of other species. Thus the collision term in equation (3-1) is written (Braginskii 1965)

$$C_k = \sum_l C_{kl}\,(f_k\,,f_l) \tag{3-2}$$

where C_{kl} is the change per unit time in the distribution function for particles of species k due to collisions with particles of species l. The term C_{kl} can describe either elastic or inelastic collisions.

For an equal mass plasma, such as electron-positron, we assume that

(1) it is supposed that the plasma is quasineutral, that is, to a good approximation, the density of the electrons n_- is equal to the density of the positrons n_+ which we denote by n

(2) the temperature $T_- = T_+ \equiv T$.

We can immediately state various conservation laws (mass, momentum, energy) that must be satisfied, whatever the collision terms. Thus

$$\int C_{e^-e^+}\,d\mathbf{v} = \int C_{e^+e^-}\,d\mathbf{v} = 0 \tag{3-3}$$

$$\int \mathbf{v}\,C_{e^-e^-}\,d\mathbf{v} = \int \mathbf{v}\,C_{e^+e^+}\,d\mathbf{v} = 0 \tag{3-4}$$

$$\int \mathbf{v}^2\,C_{e^-e^-}\,d\mathbf{v} = \int \mathbf{v}^2\,C_{e^+e^+}\,d\mathbf{v} = 0 \tag{3-5}$$

$$\int (C_{e^+e^-} + C_{e^-e^+})\,\mathbf{v}\,d\mathbf{v} = 0 \tag{3-6}$$

$$\int (C_{e^+e^-} + C_{e^-e^+}) v^2 d\mathbf{v} = 0 \tag{3-7}$$

The equations which describe the behaviour of a macroscopic variable such as density, mean velocity, pressure and temperature can be obtained by taking appropriate moments of the kinetic equations. Multiplying by 1, m_k $\mathbf{v}$ and m_k v^2 , then integrating over v leads to the following equations, after some manipulation :

(1) For the electron

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\, \mathbf{u}_{e^-}) = 0 \tag{3-8}$$

$$mn\left(\frac{\partial \mathbf{u}_{e^-}}{\partial t} + \mathbf{u}_{e^-} \cdot \nabla \mathbf{u}_{e^-}\right) = -\nabla p - \nabla \cdot \pi_{e^-} - e\, n\, (\mathbf{E} + \mathbf{u}_{e^-} \times \mathbf{B}) + \mathbf{R} \tag{3-9}$$

$$\frac{3}{2} n \left(\frac{\partial T}{\partial t} + \mathbf{u}_{e^-} \cdot \nabla T\right) + p\, \nabla \cdot \mathbf{u}_{e^-} = -\nabla \cdot \mathbf{q}_{e^-} - \pi_{e^-} : \nabla \mathbf{u}_{e^-} + Q_{e^-} \tag{3-10}$$

(2) For the positron

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\,\mathbf{u}_{e^+}) = 0 \qquad (3\text{-}11)$$

$$mn\left(\frac{\partial \mathbf{u}_{e^+}}{\partial t} + \mathbf{u}_{e^+}\cdot\nabla\mathbf{u}_{e^+}\right) = -\nabla p - \nabla\cdot\pi_{e^+} + e\,n\,(\mathbf{E} + \mathbf{u}_{e^+}\times\mathbf{B}) - \mathbf{R} \qquad (3\text{-}12)$$

$$\frac{3}{2}n\left(\frac{\partial T}{\partial t} + \mathbf{u}_{e^+}\cdot\nabla T\right) + p\,\nabla\cdot\mathbf{u}_{e^+} = -\nabla\cdot\mathbf{q}_{e^+} - \pi_{e^+}:\nabla\mathbf{u}_{e^+} + Q_{e^+} \qquad (3\text{-}13)$$

where

$$\mathbf{R}_{e^-} = \int m\,(\mathbf{v}-\mathbf{u}_{e^-})\,C_{e^-e^+}\,d\mathbf{v} = \mathbf{R}$$

$$\mathbf{R}_{e^+} = \int m\,(\mathbf{v}-\mathbf{u}_{e^+})\,C_{e^+e^-}\,d\mathbf{v} = -\,\mathbf{R} \qquad (3\text{-}14)$$

R = rate of change of the momentum of the electrons due to collisions with positrons.

$$\mathbf{q}_{e^-} = \int \frac{1}{2}\,m\,\mathbf{v}\,(\mathbf{v}-\mathbf{u}_{e^-})^2 f_{e^-}\,d\mathbf{v}\,,\quad \mathbf{q}_{e^+} = \int \frac{1}{2}\,m\,\mathbf{v}\,(\mathbf{v}-\mathbf{u}_{e^+})^2 f_{e^+}\,d\mathbf{v} \qquad (3\text{-}15)$$

$\mathbf{q}$ = heat flux vector

$$Q_{e^-} = \int \frac{1}{2} m \, (\mathbf{v} - \mathbf{u}_{e^-})^2 \, C_{e^-e^+} \, d\mathbf{v}$$

$$Q_{e^+} = \int \frac{1}{2} m \, (\mathbf{v} - \mathbf{u}_{e^+})^2 \, C_{e^+e^-} \, d\mathbf{v} \qquad (3\text{-}16)$$

Q = heat generation scalar;

p = scalar pressure = $n \, k_B \, T$

π_{e^-} , π_{e^+} = the stress tensor of the electrons and the positrons.

The transfer momentum $\mathbf{R}$ is made up of two parts.

$$\mathbf{R} = \mathbf{R}_u + \mathbf{R}_T \qquad (3\text{-}17)$$

where $\mathbf{R}_u$ is the frictional force which arises due to a non-zero relative velocity $\mathbf{U} = \mathbf{u}_{e^-} - \mathbf{u}_{e^+}$, and $\mathbf{R}_T$ is the thermal force which arises when there is a temperature gradient.

We can likewise consider the heat flux to be made up of two parts given by :

$$\mathbf{q} = \mathbf{q}_u + \mathbf{q}_T \cdot \qquad (3\text{-}18)$$

3.2. Qualitative Derivation of Transport Coefficients

The electron and the positron collision times are equal and can be expressed in the form :

$$\tau_{e^-} = \tau_{e^+} = \frac{3\, m^{\frac{1}{2}}\, \varepsilon_\circ^2 (2\pi k_B T)^{\frac{3}{2}}}{\ln\Lambda\, e^4 n} = \tau \qquad (3\text{-}19)$$

where $\ln\Lambda$ is the Coulomb scattering logarithmic term, which depends on the electron temperature. Thus (Laing 1981)

$$T < 50\,\mathrm{ev}. \quad \ln\Lambda = 30.3 - 1.15\log_{10} n + 3.45\log_{10} T\,/\mathrm{ev}$$

$$T > 50\,\mathrm{ev} \quad \ln\Lambda = 32.3 - 1.15\log_{10} n + 2.3\log_{10} T\,/\mathrm{ev}$$

3.2.1. Frictional Force $\mathbf{R}_u$

If we consider elastic collisions between the electrons and positrons, the main effect is that the ordered relative velocity between the two species

$$\mathbf{U} = \mathbf{u}_{e^-} - \mathbf{u}_{e^+} ,$$

is randomised in a time τ and consequently there is change in the momentum of the electron $m\,\mathbf{U}$. Thus, there is a frictional force exerted on the electrons by the positrons given by (Braginskii 1965) and (Laing 1981)

$$\mathbf{R}_u = \frac{m\,n}{\tau}\mathbf{U} \tag{3-20}$$

R_u is derived on page 69 *etseq.*

Also the frictional force exerted on the positrons by the electrons is equal and opposite to the frictional force exerted on the electrons by the positrons.

$$\mathbf{R}_{ue^-} = -\,\mathbf{R}_{ue^+} = \frac{m\,n}{\tau}\mathbf{U} \tag{3-21}$$

3.2.2. Thermal Force $\mathbf{R}_T$

Consider the case $\mathbf{u}_{e^-} = \mathbf{u}_{e^+} = 0$. The number of electrons or positrons moving in any particular direction will be exactly compensated by the number moving in the opposite sense . As a result of electron - positron collisions these fluxes experience frictional forces $\mathbf{R}^{\rightarrow}$ and $\mathbf{R}^{\leftarrow}$ of order $\dfrac{m\, n\, \mathrm{v}_{e^-}}{\tau}$, in a completely homogeneous plasma the frictional force balance exactly $\mathbf{R}^{\rightarrow} = -\,\mathbf{R}^{\leftarrow}$, which means there is no resultant force. We look at the simple case of a temperature gradient as shown in figure (3-1). For a mean free path λ for the electron - positron collisions, we have (Braginskii 1965) and (Laing 1981)

$$\mathbf{R}_{Te^-} = -\,\mathbf{R}_{e^-}^{\rightarrow} + \mathbf{R}_{e^-}^{\leftarrow} = \frac{m\, n\, \mathrm{v}_{e^-}}{\tau}\,\frac{\lambda_{e^-}}{T}\,\nabla T \tag{3-22}$$

$$\mathbf{R}_{Te^+} = -\,\mathbf{R}_{e^+}^{\rightarrow} + \mathbf{R}_{e^+}^{\leftarrow} = \frac{m\, n\, \mathrm{v}_{e^+}}{\tau}\,\frac{\lambda_{e^+}}{T}\,\nabla T \tag{3-23}$$

In the absence of a magnetic field, or for ∇T parallel to the field $\lambda = \mathrm{v}\,\tau$. So :

$$\mathbf{R}_{Te^-} = -\,\mathbf{R}_{Te^+} = -\,n\,\nabla T \tag{3-24}$$

We choose axes with a strong magnetic field along the z-axis, and the temperature gradient along x-axis, as shown in figure (3-2).

The thermal force is written in this case as:

$$R_T = \frac{n}{\Omega \tau} \nabla T \cdot \tag{3-25}$$

R_T is derived on page 69 *etseq.*

3.2.3. Heat Generation

Since we assumed the temperature of the electrons and the positrons to be the same, and since (Braginskii 1965) and (Laing 1981)

$$Q_{ex} = \frac{n}{\tau} (T_{e^-} - T_{e^+}) \tag{3-26}$$

$$Q_{ex} = 0$$

where Q_{ex} is the energy exchange rate between the electrons and the positrons, which means from the equation above there is no overall energy transfer from the electrons to the positrons.

Another effect in heat generation is the rate of working of the frictional force when $\mathbf{U} = \mathbf{u}_{e^-} - \mathbf{u}_{e^+}$. This is $- \mathbf{R}\cdot\mathbf{U}$. The total heat generation rates in the case of ion - electron plasmas are :

$$Q_i = Q_{ex} \quad , \quad Q_e = - \mathbf{R}_u \cdot\mathbf{U} - \mathbf{R}_T \cdot\mathbf{U} - Q_{ex}$$

but in the case of equal mass plasma, $Q_{ex} = 0$, and

$$Q_{e^-} = - \mathbf{R}_{ue^-}\cdot\mathbf{U} - \mathbf{R}_{Te^-}\cdot\mathbf{U} \, , \qquad (3\text{-}27)$$

$$Q_{e^+} = \mathbf{R}_{ue^+}\cdot\mathbf{U} + \mathbf{R}_{Te^+}\cdot\mathbf{U} \cdot \qquad (3\text{-}28)$$

3.3. Derivation Of Transport Coefficients From Kinetic Theory

The distribution functions for electron and positron can be determined by a successive - approximation method that is described, for example, in the well known monograph of Chapman and Cowling. This approach can be described roughly as follows. The distribution functions are assumed to be Maxwellian with the parameters n , u , and T , which are slowly varying functions of the coordinates, and the time, and are expanded in the form (Braginskii 1965), (Clemmow & Dougherty 1969)

$$f_{e^-} = f_{e^-}^{\circ} + f_{e^-}^{1}$$

$$f_{e^+} = f_{e^+}^{\circ} + f_{e^+}^{1} \qquad (3\text{-}29)$$

where $f_{e^-}^{\circ}$, $f_{e^+}^{\circ}$ are Maxwellian distribution function.

The Boltzmann kinetic equation is :

$$\frac{\partial f_k}{\partial t} + \mathbf{v}\cdot\nabla f_k + \frac{e_k}{m_k}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{v}} f_k = (\frac{\delta f_k}{\delta t})_{coll} \qquad (3\text{-}30)$$

setting $\mathbf{v} = \mathbf{c} + \mathbf{u}$, where $\mathbf{c}$ is the random velocity

$$\frac{\partial f_k}{\partial t} + (\mathbf{c}_k + \mathbf{u}_k)\cdot \nabla f_k + \frac{e_k}{m}(\mathbf{E} + (\mathbf{c}_k + \mathbf{u}_k)\times\mathbf{B})\cdot\nabla_c f_k = C_{ab} \qquad (3\text{-}31)$$

We write the distribution function as $f = f^{\circ}(1 + \Phi)$, where Φ is a small correction. Substitute this in equation (3-31) , multiply by 1 , $m\mathbf{c}$ and $\frac{1}{2}mc^2$ and use the conditions (Chapman and Cowling 1970)

$$\int f^{\circ}\Phi d\mathbf{c} = 0 \ , \ \int m\mathbf{c} f^{\circ}\Phi d\mathbf{c} = 0 \ , \ \int 1/2 m\, c^2 f^{\circ}\Phi d\mathbf{c} = 0$$

(i) for the electrons :

$$f^{\circ}_{e^-}\left\{\left(\frac{m\,\mathbf{c}^2}{2k_B\,T}-\frac{5}{2}\right)\mathbf{c}\,\nabla\ln T+\frac{m}{2k_B\,T}\,\mathbf{c}\,\mathbf{c}:\pi_{e^-}+\frac{1}{n\,k_B\,T}\,\mathbf{R}^1_{e^-}\!\cdot\mathbf{c}\right.$$

$$\left.\frac{1}{n\,k_B\,T}\left[-(Q-\mathbf{R}^1_{e^-}\!\cdot\mathbf{c})\left(1-\frac{m\,c^2}{3\,k_B\,T}\right)\right]\right\}=$$

$$I_{e^-e^-}(\Phi)+I_{e^-e^+}-f^{\circ}_{e^-}\,[\,\mathbf{c}\times\Omega\,]\,\nabla_c\Phi \tag{3-32}$$

where $\mathbf{c}=\mathbf{v}-\mathbf{u}_{e^-}$

$$I_{e^-e^-}=C_{e^-e^-}(f^{\circ}_{e^-},f^{\circ}_{e^-}\Phi)+C_{e^-e^-}(f^{\circ}_{e^-}\Phi,f^{\circ}_{e^-})$$

$$I_{e^-e^+}=C_{e^-e^+}(f^{\circ}_{e^-},f^{\circ}_{e^+}\Phi)+C_{e^-e^+}(f^{\circ}_{e^-}\Phi,f^{\circ}_{e^+}) \tag{3-33}$$

Equation (3-33) is linear, so the solution can be written as a sum of terms

$$\Phi_T(c^2)=A_T\,\nabla_{||}\ln T+A_T^{'}\nabla_{\perp}\ln T+A_T^{''}\,\Omega\times\nabla\ln T$$

$$\Phi_u(c^2)=A_u\,\mathbf{U}_{||}+A_u^{'}\mathbf{U}_{\perp}+A_u^{''}\,\Omega\times\mathbf{U} \tag{3-34}$$

where $A(c^2)$, $A'(c^2)$, $A''(c^2)$ are scalar functions, and

$$A = A' + i\,(\Omega\cdot h\,)A'' \tag{3-35}$$

From equation (3-32) we obtain the equation that describes that part of the correction arising from $\nabla_\perp \ln T$ which is : (Braginskii 1958, 1965) and (Chapman & Cowling 1970)

$$f^{\circ}_{e^-}\left\{ (S^2 - \frac{5}{2})\,\mathbf{c}\,\nabla_\perp \ln T + \frac{\mathbf{R}_{Te^-}\cdot\mathbf{c}}{n\,k_B\,T}\left[2 - \frac{2}{3}\,S^2\right]\right\} =$$

$$I_{e^-e^-}(\Phi) + I_{e^-e^+}(\Phi) + f^{\circ}_{e^-}(\mathbf{c}\times\Omega_{e^-})\cdot\nabla\Phi \tag{3-36}$$

where $S^2 = \dfrac{m\,c^2}{2\,k_B\,T}$, and we write the thermal force as:

$$\mathbf{R}_{Te^-} = n\,k_B\,T\,(\,\kappa'\,\nabla_\perp \ln T \,+ \kappa''(\Omega_{e^-}\times\nabla\ln T)) \tag{3-37}$$

where $\kappa = \kappa' + i\,(\Omega\cdot h\,)\,\kappa''$. Also A can be written in terms of Sonine polynomials (see Appendix A). In fact there are two components, one thermal and the other frictional.

$$A_T^{'}(S^2) = \tau \sum_{k=0} a_k^{'} L_k^{(3/2)}(S^2)$$

$$A_T^{''}(S^2) = \tau \sum_{k=0} a_k^{''} L_k^{(3/2)}(S^2) \tag{3-38}$$

$$A_u^{'}(S^2) = -\frac{m}{k_B T} \sum_{k=0} b_k^{'} L_k^{(3/2)}(S^2)$$

$$A_u^{'}(S^2) = -\frac{m}{k_B T} \sum_{k=0} b_k^{''} L_k^{(3/2)}(S^2) \cdot \tag{3-39}$$

$k = 1, 2, 3, \ldots, n$

Multiplying equation (3-36) by

$$-\frac{4}{5}\frac{1}{n}\frac{m}{2k_B T}\mathbf{c} L_k^{(3/2)}(S^2) d\mathbf{c}$$

and integrating over the velocity we have (Landshoff 1949) and (Braginskii 1965)

$$\sum_k (\alpha_{kl} + \alpha_{kl}^{'}) a_l + i(\Omega_e \cdot h)\tau \frac{(k+\frac{3}{2})!}{k!\,(\frac{5}{2})!} a_k = \delta_{1k} \tag{3-40}$$

$$\sum_k (\alpha_{kl} + \alpha_{kl}^{'}) b_l + i(\Omega_e \cdot h)\tau \frac{(k+\frac{3}{2})!}{k!\,(\frac{5}{2})!} b_k = \delta_{1k} \tag{3-41}$$

where

$$\alpha_{kl} = -\frac{4\tau}{15n}\frac{m}{2k_B T}\int L_k^{3/2}(S^2)\,\mathbf{c}\,I_{e^-e^-}(L_l^{3/2}(S^2)\,\mathbf{c})\,d\mathbf{c}$$

$$\alpha'_{kl} = -\frac{4\tau}{15n}\frac{m}{2k_B T}\int L_k^{3/2}(S^2)\,\mathbf{c}\,I_{e^+e^-}(L_l^{3/2}(S^2)\,\mathbf{c})\,d\mathbf{c} \qquad (3\text{-}42)$$

$$\underset{\sim}{h} = \frac{\mathbf{B}}{B} = \text{unit vector in the direction of the magnetic field .}$$

Now to calculate the heat flow and the rate of change of momentum we have (Braginskii 1958)

$$\mathbf{R}_{e^-} = \mathbf{R}_{Te^-} + \mathbf{R}_{ue^-} = \int m\,\mathbf{c}\,\underset{\sim}{I}(\Phi_T)\,d\mathbf{c} + \int m\,\mathbf{c}\,\underset{\sim}{I}(\Phi_u)\,d\mathbf{c} \qquad (3\text{-}43)$$

where Φ_T , Φ_u are the thermal and frictional parts of the correction function. The integration in (3-43) gives :

$$\mathbf{R}_{e^-} = -\frac{5}{2}n\sum_{k=1}(\alpha_{0k} + \alpha'_{0k})\,[a'_k\nabla_\perp T + a''_k(\Omega_{e^-}\times\nabla T) -$$

$$-\frac{m}{\tau}(b'_k\mathbf{U}_\perp + b'_k(\Omega_{e^-}\times\mathbf{U})] \qquad (3\text{-}44)$$

$$\mathbf{q}_{e^-} = \mathbf{q}_{Te^-} + \mathbf{q}_{ue^-} = -\frac{5}{2}\, n\, k_B\, T\, [\frac{\tau}{m}\,(a_k'\, \nabla_\perp T \ + a_k'(\Omega_{e^-} \times \nabla T)) -$$

$$- (b_k' \mathbf{U}_\perp \ + b_k''(\Omega_{e^-} \times \mathbf{U}))] \qquad (3\text{-}45)$$

(ii) For the positrons the equations are :

$$\mathbf{R}_{e^+} = \mathbf{R}_{Te^+} + \mathbf{R}_{ue^+} = \int m\ \mathbf{c} I(\Phi_T)\, d\mathbf{c} + \int m\ \mathbf{c} I(\Phi_u)\, d\mathbf{c}$$

$$\mathbf{R}_{e^+} = +\frac{5}{2}\, n \sum_{k=1} (\beta_{0k} \ + \beta_{0k}')\, [a_k'\, \nabla_\perp T \ + a_k''(\Omega_{e^+} \times \nabla T) -$$

$$- \frac{m}{\tau}(b_k' \mathbf{U}_\perp \ + b_k''(\Omega_{e^+} \times \mathbf{U})] \qquad (3\text{-}46)$$

$$\mathbf{q}_{e^+} = \mathbf{q}_{Te^+} + \mathbf{q}_{ue^+} = -\frac{5}{2}\, n\, k_B\, T \times$$

$$[\frac{\tau}{m}(a_k'\, \nabla_\perp T \ + a_k''(\Omega_{e^+} \times \nabla T)) - (b_k' \mathbf{U}_\perp \ + b_k''(\Omega_{e^+} \times \mathbf{U})] \qquad (3\text{-}47)$$

where $\mathbf{c} = \mathbf{v} - \mathbf{u}_{e^+}$, and the coefficients β , β' are given by

$$\beta_{kl} = - \frac{4\tau}{15n} \frac{m}{2k_B T} \int L_k^{(3/2)}(S^2)\, \mathbf{c}\, I_{e^+e^+}(L_l^{(3/2)}(S^2)\, \mathbf{c}) d\mathbf{c}$$

$$\beta'_{kl} = - \frac{4\tau}{15n} \frac{m}{2k_B T} \int L_k^{(3/2)}(S^2)\, \mathbf{c}\, I_{e^+e^-}(L_l^{(3/2)}(S^2)\, \mathbf{c}) d\mathbf{c} \qquad (3\text{-}48)$$

Now we calculate the total heat flow $\mathbf{q}$:

$$\mathbf{q} = \mathbf{q}_{e^-} + \mathbf{q}_{e^+} = - 5n\, k_B\, T\, [\frac{\tau}{m} a'_k \nabla_\perp T - b'_k \mathbf{U}_\perp] \qquad (3\text{-}49)$$

In the appendix A we discuss the integrations which give the coefficients α_{kl} , α'_{kl} , β_{kl} , β'_{kl} , which are :

$$\alpha_{kl} = \beta_{kl} = \frac{4\sqrt{2}}{5} \begin{bmatrix} 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & \ldots\ldots \\ 0.00000 & 1.00000 & 0.75000 & 0.46875 & 0.27344 & 0.15381 & \ldots\ldots \\ 0.00000 & 0.75000 & 2.81250 & 2.41406 & 1.72851 & 1.14075 & \ldots\ldots \\ 0.00000 & 0.46875 & 2.41406 & 6.27441 & 5.90552 & 4.62410 & \ldots\ldots \\ 0.00000 & 0.27343 & 1.72850 & 5.90552 & 12.28058 & 12.13068 & \ldots\ldots \\ 0.00000 & 0.15381 & 1.14075 & 4.62410 & 12.13068 & 21.72864 & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{bmatrix} \qquad (3\text{-}50)$$

$$\alpha'_{lk} = \beta'_{lk} = \frac{4\sqrt{2}}{5} \begin{bmatrix} 0.50000 & 0.37500 & 0.23437 & 0.13672 & 0.07690 & 0.04229 & \ldots\ldots \\ 0.37500 & 0.71875 & -0.02734 & -0.10644 & 0.15875 & 0.49916 & \ldots\ldots \\ 0.23437 & -0.02734 & 2.38330 & 1.95227 & 1.61075 & 1.64798 & \ldots\ldots \\ 0.13672 & -0.10644 & 1.95227 & 6.01901 & 5.98933 & 5.33091 & \ldots\ldots \\ 0.07690 & 0.15875 & 1.61075 & 5.98933 & 11.57623 & 11.82884 & \ldots\ldots \\ 0.04229 & 0.49916 & 1.64798 & 5.33091 & 11.82884 & 18.78090 & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{bmatrix} \tag{3-51}$$

From these matrices we find that the desired coefficients α_{0k} , β_{0k} are all equal to zero, which means as always that there is no change in the momentum due to collisions between identical particles. but there is change due to electron-positron or positron-electron collisions. The changes are

$$\mathbf{R}_{e^-} = -\frac{5}{2} n \sum_{k=1} \alpha'_{0k} [a'_k \nabla_\perp T + a''_k (\Omega_{e^-} \times \nabla T)$$

$$- \frac{m}{\tau} (b'_k \mathbf{U}_\perp + b''_k (\Omega_{e^-} \times \mathbf{U}))] \tag{3-52}$$

$$\mathbf{R}_{e^+} = +\frac{5}{2} n \sum_{k=1} \beta'_{0k} [a'_k \nabla_\perp T + a''_k (\Omega_{e^+} \times \nabla T)$$

$$- \frac{m}{\tau} (b'_k \mathbf{U}_\perp + b''_k (\Omega_{e^+} \times \mathbf{U}))] \tag{3-53}$$

3.4. Electrical and thermal conductivities and thermal diffusion

To derive the electrical and the thermal conductivities and the thermal diffusion, the equation for the correction to the distribution functions can be written as (Kaneko 1960) :

$$\Phi_k = -\mathbf{D}_k \cdot \mathbf{E} - \mathbf{A}_k \cdot \nabla \ln T \tag{3-54}$$

where $\mathbf{D}_k$, $\mathbf{A}_k$ are vectors and written as :

$$\mathbf{A}_k = A_k^I \mathbf{S}_k + A_k^{II} \mathbf{S}_k \times h$$

$$\mathbf{D}_k = D_k^I \mathbf{S}_k + D_k^{II} \mathbf{S}_k \times h \tag{3-55}$$

and

$$A_k = \sum_m a^{(m)} L_p^{(m)}$$

$$D_k = \sum_m d^{(m)} L_p^{(m)} \tag{3-56}$$

where $L_k^{(m)}$ is a Sonine polynomial, and

$$a^{(m)} = a^I + i\, a^{II}$$

$$d^{(m)} = d^I + i\, d^{II} \qquad (3\text{-}57)$$

The correction function satisfies the following equation

$$f_k^{\circ}\,(\frac{5}{2} - S_k^2)\,(\mathbf{c}_k \cdot \nabla \ln T) - f_k^{\circ}\,\frac{e_k}{k_B\,T}\mathbf{c}_k \cdot \mathbf{E} =$$

$$- f_k^{\circ}\,\frac{e_k}{m_k c}\,\mathbf{c}_k \times \mathbf{B}\,\frac{\partial \Phi_k}{\partial \mathbf{c}_k} + \sum_l I_{kl}(\Phi_k) \qquad (3\text{-}58)$$

Now substituting equation (3-55) and (3-56) into equation (3-58) we find

$$-f_k^{\circ}\,\frac{e_k}{k_B\,T}\,\mathbf{c}_k = i\,\Omega_k\,D_k\,f_k^{\circ}\,\mathbf{S}_k - \sum_l I_{kl}(D_k\,\mathbf{S}_k) \qquad (3\text{-}59)$$

$$-f_k^{\circ}\left(\frac{5}{2}-S_k^2\right)\mathbf{c}_k = i\,\Omega_k\,A_k\,f_k^{\circ}\,\mathbf{S}_k - \sum_l I_{kl}(A_k\,\mathbf{S}_k) \tag{3-60}$$

By operating $\sum_k \int S_k\,L^r\,dc$ on equations (3-59) and (3-60) and using equations (3-42) , (3-48) and (3-56) we obtain for equal mass plasma (see appendix B)

$$0 = \sum_m \sum_r \frac{2\,i\,n}{\pi^{1/2}}\,\Omega\,(d_{e^+}^{(m)} + d_{e^-}^{(m)})\frac{\Gamma(\frac{5}{2}+r)}{r\,!}\delta_{rm}$$

$$+\frac{15}{4}\,\frac{1}{\tau}\sum_r \sum_m (\alpha_{rm} + \alpha'_{rm})\,(d_{e^-}^{(m)} + d_{e^+}^{(m)})\;, \tag{3-61}$$

$$\left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}}\delta_{r1} = \sum_m \sum_r \frac{2\,i\,n}{\pi^{1/2}}\,\Omega\,(a_{e^+}^{(m)} + a_{e^-}^{(m)})\frac{\Gamma(\frac{5}{2}+r)}{r\,!}\delta_{rm}$$

$$+\frac{1}{\tau}\sum_r \sum_m (\alpha_{rm} + \alpha'_{rm})\,(a_{e^-}^{(m)} + a_{e^+}^{(m)})\;\cdot \tag{3-62}$$

Here we have used the fact that $\alpha_{rm} = \beta_{rm}$, $\alpha'_{rm} = \beta'_{rm}$, and $\Omega = \Omega_{e^-} = -\Omega_{e^+}$.

The current density is

$$\mathbf{j} = \sum_k e_k \int f_k^{\circ} \Phi_k \, \mathbf{c}_k \, d\mathbf{c} = -\frac{n\,e}{2} \left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}}$$

$$\Big\{ [\,(d_{e^-}^{I(0)} - d_{e^+}^{I(0)})\,E + (d_{e^-}^{II(0)} - d_{e^+}^{II(0)})\,(h \times E)\,]$$

$$+ [\,(a_{e^-}^{I(0)} - a_{e^+}^{I(0)})\nabla \ln T + (a_{e^-}^{II(0)} - a_{e^+}^{II(0)})(h \times \nabla \ln T)\,] \Big\} \qquad (3\text{-}63)$$

and the heat flow is

$$\mathbf{q}^* = \frac{n\,e}{2} \left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}} [\,(a_{e^-}^{I(0)} - a_{e^+}^{I(0)})\,\mathbf{E} +$$

$$(a_{e^-}^{II(0)} - a_{e^+}^{II(0)})\,(h \times E)\,] + 5\,\frac{k_B\,T}{e}\,j + \frac{5}{4}\,n\,k_B\,T \left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}} \times$$

$$[\,(a_{e^-}^{I(1)} + a_{e^+}^{I(1)})\nabla \ln T + (a_{e^-}^{II(1)} + a_{e^+}^{II(1)})(h \times \nabla \ln T)\,] \qquad (3\text{-}64)$$

The reduced heat flow is

$$\mathbf{q} = \mathbf{q}^* - 5\frac{k_B T}{e}\mathbf{j} = \left(\frac{2 k_B T}{m}\right)^{\frac{1}{2}} \left\{ \frac{n e}{2} \left[(a_{e^-}^{I(0)} - a_{e^+}^{I(0)})\mathbf{E} \right.\right.$$

$$+ (a_{e^-}^{II(0)} - a_{e^+}^{II(0)})(h \times \mathrm{E}) + \frac{5}{4} n\, k_B\, T$$

$$\left.\left[(a_{e^-}^{I(1)} + a_{e^+}^{I(1)})\nabla \ln T + (a_{e^-}^{II(1)} + a_{e^+}^{II(1)})(h \times \nabla \ln T) \right] \right\} \quad (3\text{-}65)$$

In the magnetic field the electrical conductivity is written as :

$$\sigma = \begin{bmatrix} \sigma_1 & -\sigma_{11} \\ \sigma_{11} & \sigma_1 \end{bmatrix} \quad (3\text{-}66)$$

From equation (3-63) we can write

$$\sigma_1 + i\,\sigma_{11} = \frac{n e}{2}\left(\frac{2 k_B T}{m}\right)^{\frac{1}{2}} \left[(d_{e^-}^{I(0)} - d_{e^+}^{I(0)}) + i\,(d_{e^-}^{II(0)} - d_{e^+}^{II(0)}) \right] \quad (3\text{-}67)$$

The thermal conductivity is

$$\lambda = \begin{bmatrix} \lambda_1 & -\lambda_{11} \\ \lambda_{11} & \lambda_1 \end{bmatrix} \tag{3-68}$$

From equation (3-65)

$$\lambda_1 + i\,\lambda_{11} = -\frac{5}{4} n k_B \left(\frac{2k_B\,T}{m}\right)^{\frac{1}{2}} [(a_{e^-}^{I(1)} + a_{e^+}^{I(1)}) +$$

$$i\,(a_{e^-}^{II(1)} + a_{e^+}^{II(1)})] \tag{3-69}$$

And the thermal diffusion ϕ is written as :

$$\phi = \begin{bmatrix} \phi_1 & -\phi_{11} \\ \phi_{11} & \phi_1 \end{bmatrix} \tag{3-70}$$

From equation (3-65)

$$\phi_1 + i\,\phi_{11} = \frac{n\,e}{2T} \left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}} [(a_{e^-}^{I(0)} - a_{e^+}^{I(0)}) +$$

$$i\,(a_{e^-}^{II(0)} - a_{e^+}^{II(0)})\,] \tag{3-71}$$

To calculate the transport coefficients for a single particle

(i) for the electrons the equations above are written as :

Equation (3-61) for the electrons is

$$\frac{ne}{k_B T}\left(\frac{2k_B T}{m}\right)^{\frac{1}{2}}\delta_{r0} = \sum_r \sum_m \left[\frac{2}{3} h_{e^-}^{(m)}\, \delta_{rm} + \frac{5}{2}\,\frac{1}{\tau}\,(\alpha_{rm} + \alpha_{rm}^{'})\right] d_{e^-}^{(m)} \tag{3-72}$$

equation (3-62) for the electrons is

$$\frac{5}{2}\left(\frac{2k_B T}{m}\right)^{\frac{1}{2}}\delta_{r1} = \sum_r \sum_m \left[\frac{2}{3} h_{e^-}^{m}\, \delta_{rm} + \frac{5}{2}\,\frac{1}{\tau}\,(\alpha_{rm} + \alpha_{rm}^{'})\right] a_{e^-}^{(m)} \tag{3-73}$$

$$d_{e^-}^{(m)} = \frac{e}{k_B\, T}\left(\frac{2\,k_B\, T}{m}\right)^{\frac{1}{2}} \tau\, E_{e^-}^{(m)} \tag{3-74}$$

$$a_{e^-}^{(m)} = -\,\frac{5}{2}\left(\frac{2\,k_B\, T}{m}\right)^{\frac{1}{2}} \tau\, b_{e^-}^{(m)} \tag{3-75}$$

substitute equations (3-74) and (3-75) into equations (3-67) , (3-69) and (3-71) for the electrons, we obtain

$$\sigma_{e^-} = \frac{n\, e^2}{m}\, \tau\, E_{e^-}^{(0)} \tag{3-76}$$

$$\phi_{e^-} = -\frac{5}{2}\, \frac{k_B\, n\, e}{m}\, \tau\, b_{e^-}^{(0)} \tag{3-77}$$

$$\lambda_{e^-} = \frac{25}{4}\, \frac{n\, k_B^2\, T}{m}\, b_{e^-}^{(1)} \tag{3-78}$$

where $E_{e^-}^{(m)}$,$b_{e^-}^{(m)}$ are determined by :

$$\sum_r \sum_m [\frac{2}{3}\, h_{e^-}^{(r)}\, \delta_{rm}\, \frac{\tau}{n} + \frac{5}{2}\, (\alpha_{rm} + \alpha_{rm}^{'})\,]\, E_{e^-}^{(m)} = \delta_{r0} \tag{3-79}$$

$$\sum_r \sum_m [\frac{2}{3}\, h_{e^-}^{(r)}\, \delta_{rm}\, \frac{\tau}{n} + \frac{5}{2}\, (\alpha_{rm} + \alpha_{rm}^{'})\,]\, b_{e^-}^{(m)} = \delta_{r1} \tag{3-80}$$

and

$$h_{e^-}^{(r)} = i \, \frac{2n\,\Omega_{e^-}}{\sqrt{\pi}} \, \frac{\Gamma(r+5/2)}{r\,!} \tag{3-81}$$

From equations (3-63) and (3-65), we can see that the temperature gradient appears in the equation of the current density and the electric field appears in the equation of the heat flow. This is due to the influence of the electrical conductivity on the thermal conductivity and vice - versa . So from this we can write a coefficient (γ) to represent this interference between these conductivities.

$$\gamma_{e^-} = - \frac{T}{\sigma_{e^-}} \, \phi_{e^-}$$

$$\gamma_{e^-} = \frac{5}{2} \, \frac{k_B \, T}{e} \, C_{e^-}^{(0)} \tag{3-82}$$

where

$$C_{e^-}^{I(0)} = \frac{b_{e^-}^{I(0)} \, E_{e^-}^{I(0)} + b_{e^-}^{II(0)} \, E_{e^-}^{II(0)}}{(E_{e^-}^{I(0)})^2 + (E_{e^-}^{II(0)})^2}$$

$$C_{e^-}^{II(0)} = \frac{b_{e^-}^{II(0)} \, E_{e^-}^{I(0)} - b_{e^-}^{I(0)} \, E_{e^-}^{II(0)}}{(E_{e^-}^{I(0)})^2 + (E_{e^-}^{II(0)})^2} \tag{3-83}$$

(ii) For the positrons these equations above become

Equation (3-61) for the positrons is :

$$-\frac{ne}{k_B T}\left(\frac{2k_B T}{m}\right)^{\frac{1}{2}}\delta_{r0} = \sum_r \sum_m \left[\frac{2}{3} h_{e^+}^{(r)}\, \delta_{rm}\right.$$

$$\left. + \frac{5}{2}\frac{1}{\tau}(\beta_{rm} + \beta'_{rm})\right] d_{e^+}^{(m)} \qquad (3\text{-}84)$$

equation (3-62) for the positrons

$$\frac{5}{2}\left(\frac{2k_B T}{m}\right)^{\frac{1}{2}}\delta_{r1} = \sum_r \sum_m \left[\frac{2}{3} h_{e^+}^{(r)}\, \delta_{rm} \frac{5}{2}\frac{1}{\tau}(\beta_{rm} + \beta'_{rm})\right] a_{e^+}^{(m)} \qquad (3\text{-}85)$$

$$d_{e^+}^{(m)} = -\frac{e}{k_B T}\left(\frac{2 k_B T}{m}\right)^{\frac{1}{2}} \tau E_{e^+}^{(m)} \qquad (3\text{-}86)$$

$$a_{e^+}^{(m)} = -\frac{5}{2}\left(\frac{2 k_B T}{m}\right)^{\frac{1}{2}} \tau b_{e^+}^{(m)} \qquad (3\text{-}87)$$

On substituting equations (3-86) and (3-87) in to equations (3-67) , (3-69) and (3-71) we obtain

$$\sigma_{e^+} = \frac{n\, e^2}{m}\, \tau\, E_{e^+}^{(0)} \tag{3-88}$$

$$\phi_{e^+} = \frac{5}{2}\, \frac{k_B\, n\, e}{m}\, \tau\, b_{e^+}^{(0)} \tag{3-89}$$

$$\lambda_{e^+} = \frac{25}{4}\, \frac{n\, k_B^2\, T}{m}\, b_{e^+}^{(1)} \tag{3-90}$$

where $E_{e^+}^{(m)}$,$b_{e^+}^{(m)}$ are determined by :

$$\sum_r \sum_m [\frac{2}{3}\, h_{e^+}^{(r)}\, \delta_{rm}\, \frac{\tau}{n} + \frac{5}{2}\,(\beta_{rm} + \beta_{rm}^{'})\,]\, E_{e^+}^{(m)} = \delta_{r0} \tag{3-91}$$

$$\sum r \sum_m [\frac{2}{3}\, h_{e^+}^{(r)}\, \delta_{rm}\, \frac{\tau}{n} + \frac{5}{2}\,(\beta_{rm} + \beta_{rm}^{'})\,]\, b_{e^+}^{(m)} = \delta_{r1} \tag{3-92}$$

and

$$h_{e^+}^{(r)} = i\, \frac{2\, n\, \Omega_{e^+}}{\sqrt{\pi}}\, \frac{\Gamma(r+5/2)}{r\,!} \tag{3-93}$$

$$\gamma_{e^+} = -\, \frac{T}{\sigma_{e^+}}\, \phi_{e^+}$$

$$\gamma_{e^+} = -\frac{5}{2}\,\frac{k_B\,T}{e}\,C_{e^+}^{(0)} \tag{3-94}$$

where

$$C_{e^+}^{I(0)} = \frac{b_{e^+}^{I(0)}\,E_{e^+}^{I(0)} + b_{e^+}^{II(0)}\,E_{e^+}^{II(0)}}{(E_{e^+}^{I(0)})^2 + (E_{e^+}^{II(0)})^2}$$

$$C_{e^+}^{II(0)} = \frac{b_{e^+}^{II(0)}\,E_{e^+}^{I(0)} - b_{e^+}^{I(0)}\,E_{e^+}^{II(0)}}{(E_{e^+}^{I(0)})^2 + (E_{e^+}^{II(0)})^2} \tag{3-95}$$

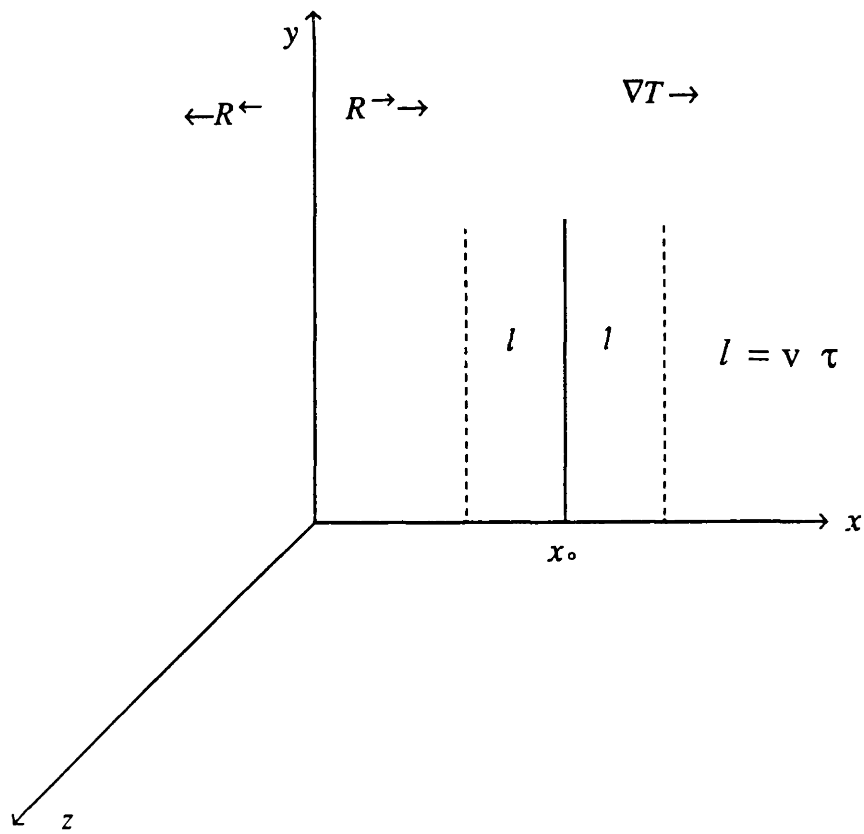

Figure (3-1). The thermal force in the absence of a magnetic field

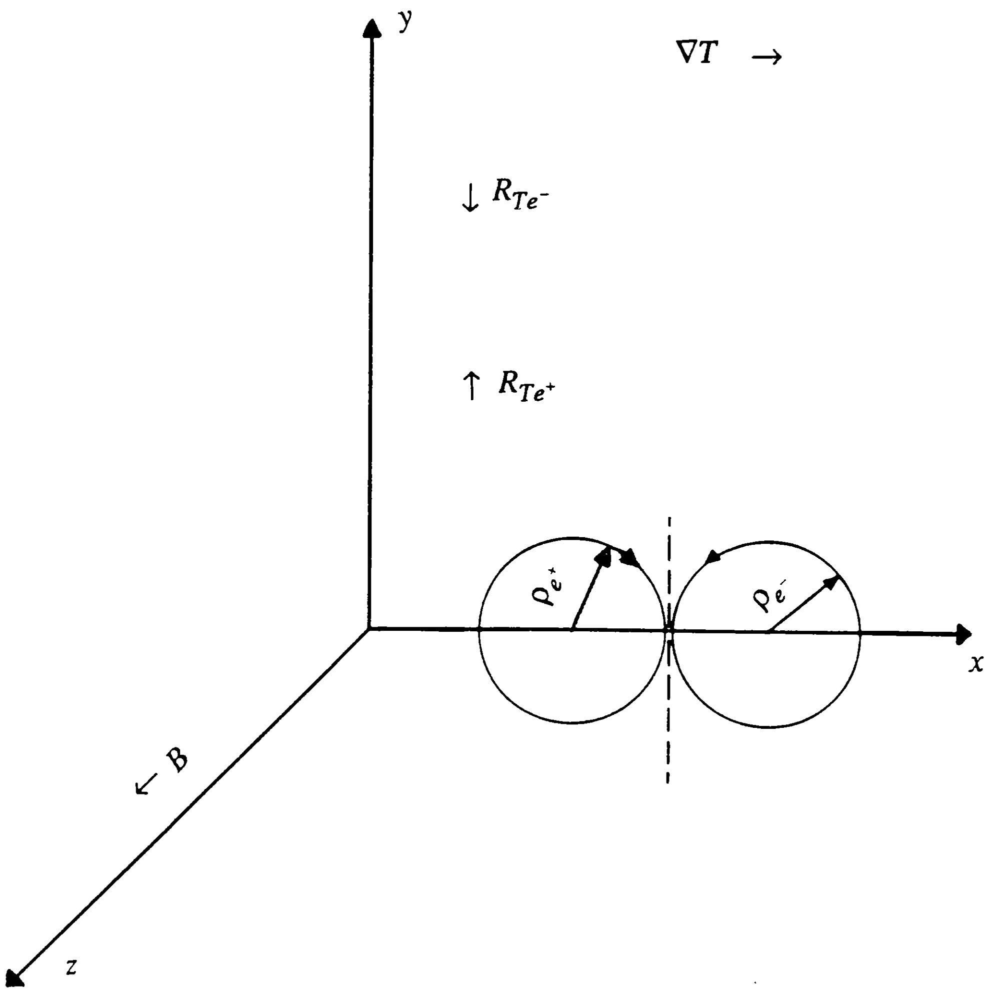

Figure (3-2). The thermal force in the presence of a magnetic field

CHAPTER FOUR

Results And Discussion

4.1. Motivation

Following the work of Chapman Enskog and Kaneko, we cut off the matrices in equations (3-79) , (3-80) , (3-91) and (3-92) at r=m= 0, 1, 2, 3, 4, and 5. We now solve the equations for the electrical conductivity, the thermal conductivity, the interference between these and the thermal diffusion and examine the convergence of this method.

For several values of $\Omega\tau$ of the electrons and the positrons $E^{(0)}$, $b^{(1)}$, $b^{(0)}$ and $C^{(0)}$ for the electrons and the positrons have been solved using the ORION 1/05 supermini at the University of Glasgow - Department of Physics and Astronomy and listed in tables 1 to 14.

4.2. The electrical conductivity

From figure (4-1) which represents the real part of $E^{(0)}$ for the electrical conductivity, it can be seen that the real part for both species are positive. So when we sum over the two species we obtain twice the magnitude of $E^{I(0)}$ for either the electrons or the positrons. But from figure (4-2) which represents the imaginary part of $E^{(0)}$, it can be seen when we sum over the two species we obtain zero for $E^{II(0)}$. Consequently we can rewrite the equations for the electrical conductivity. From equations (3-66) , (3-67) , (3-74) and (3-82) we find

$$\sigma_1 + i\,\sigma_{11} = n\,e\,(\frac{2\,k_B\,T}{m})^{\frac{1}{2}}\,d^{I\,(0)}\,, \tag{4-1}$$

where $d^{I\,(0)} = d^{I\,(0)}_{e^-} = -\,d^{I\,(0)}_{e^+}$;

and so

$$\sigma_{11} = 0 \tag{4-2}$$

$$\sigma_1 = n\,e\,(\frac{2\,k_B\,T}{m})^{\frac{1}{2}}\,d^{I\,(0)} \tag{4-3}$$

and

$$\sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{bmatrix} \tag{4-5}$$

This is the symmetrical form of the electrical conductivity in equal mass plasma.

If we go back to chapter two, equation (2-42) which represents the electrical conductivity for equal mass plasma, which was found by the method of irreversible thermodynamics compared with the relaxition time method. We can see that this equation agrees with equation (4-5) which represents the electrical conductivity as well but was found by the kinetic theory method.

From figures (4-1) and (4-2) we can see for a single particle the dependence of σ_{11} , σ_{22} , σ_{12} and σ_{21} on $\Omega\tau$. As $\Omega\tau$ increases σ_{11} and σ_{22} decrease, thus when $\Omega\tau$ is relatively large, very little current is produced across the magnetic field lines. But when $\Omega\tau$ increases σ_{12} and σ_{21} increase first till, $\Omega\tau=1$, then start decrease.

4.3. The thermal conductivity

From Figure (4-3) which represents the real part of $b^{(1)}$ for the thermal conductivity we also see when we sum over the two species the result is twice the magnitude of the result for either the electrons or the positrons. But from figure (4-4) which represents the imaginary part of $b^{(1)}$ we obtain zero. Consequently we can rewrite the equations for the thermal conductivity. From equations (3-68) , (3-69) , (3-75) and (3-84)

$$\lambda = \lambda_1 + i\,\lambda_{11} = -\frac{5}{2}\, n\, k_B \left(\frac{2\,k_B\, T}{m}\right)^{\frac{1}{2}} a^{I(1)} \tag{4-6}$$

where $a^{I(1)} = a_{e^-}^{I(1)} = a_{e^+}^{I(1)}$

and so

$$\lambda_{11} = 0 \tag{4-7}$$

$$\lambda_1 = -\frac{5}{2}\, n\, k_B \left(\frac{2 k_B T}{m}\right)^{\frac{1}{2}} a^{I(1)} \tag{4-8}$$

and

$$\lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix} \tag{4-9}$$

This is the symmetrical form of the thermal conductivity in equal mass plasma.

Equation (2-36) in chapter two which represents the thermal conductivity for equal mass plasma found by the irreversible thermodynamics method agrees with equation (4-9) which represents the same thing, but found by the kinetic theory.

From figures (4-3) and (4-4) we can see the dependence of the thermal conductivity of a single particles on $\Omega\,\tau$ is similar to the dependence of the electrical conductivity.

4.4. The thermal diffusion

Here we have different situation, figure (4-5) which represents the real part of $b^{(0)}$ for the thermal diffusion and figure (4-6) which represents the imaginary part. And because the equation of thermal diffusion contains the charge of the species which are equal and opposite we obtain when we sum over the two species, zero for the real part and twice the magnitude for the imaginary. Consequently the equations of thermal diffusion can be rewritten. From equations (3-70) , (3-71) , (3-75) and (3-84) we obtain :

$$\phi = \phi_1 + i\,\phi_{11} = 0 + i\,\frac{n\,e}{T}\left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}} a^{II\,(0)} \qquad (4\text{-}10)$$

where $a^{II\,(0)} = a^{II\,(0)}_{e^-} = -\,a^{II\,(0)}_{e^+}$

So

$$\phi_1 = 0 \qquad (4\text{-}11)$$

$$\phi_{11} = \frac{n\,e}{T}\left(\frac{2\,k_B\,T}{m}\right)^{\frac{1}{2}} a^{II\,(0)} \qquad (4\text{-}12)$$

and

$$\phi = \begin{bmatrix} 0 & -\phi_{11} \\ \phi_{11} & 0 \end{bmatrix} \qquad (4\text{-}13)$$

This is the antisymmetrical form of the thermal diffusion in equal mass plasma.

Here we have different situation than in chapter two, here we have investigated the diffusion caused by the presence of a temperature gradient, and we found that in the case of equal mass plasma, the particles diffuse in the *xy* and *yx* planes, and there is no diffusion in *xx* and *yy* planes. In chapter two we investigate the diffusion caused by the presence of a density gradient. We found that this kind of diffusion different than the previous one, there is diffusion in the *xx* and *yy* planes and non in the *xy* and *yx* planes.

4.5. The interference between the electrical and thermal conductivities

From figure (4-7) which represents the real part of $C^{(0)}$ for the interference we can see when we sum over the two species we get as well as for the other twice a magnitude of $C^{I(0)}$ of either the electrons or the positrons. But from figure (4-8) which represents the imaginary part of $C^{(0)}$ when we sum over the two species we get zero for this part. So from this we can rewrite the interference equations.

From equations (3-72) and (3-94) we get :

$$\gamma = \gamma_{e^-} + \gamma_{e^+} = 5\,\frac{k_B\,T}{e}\,C^{I(0)} \tag{4-14}$$

Which means that the imaginary part of γ for equal mass plasma is equal to zero.

Finally, we can see from the results exhibited graphically, the approximations made to $E^{(0)}$, $b^{(1)}$ and $b^{(0)}$ for the electrical conductivity, the thermal conductivity and the thermal diffusion have converged by the 5^{th} iteration.

4.6. Conclusion

By using the irreversible thermodynamics in chapter two, we find the thermodynamic forces which include the effect of the temperature gradient and presence of an electric field. After solving Boltzmann equation in non - equilibrium state with collision term $-\nu_c f'$, we find the current, heat flow and stress tensor. Comparing with the thermodynamic forces we have found earlier, the electrical and thermal conductivities and the viscosity can be found. For equal mass plasma and in the presence of magnetic field these coefficients are found to be symmetric (the off diagonal coefficients all vanish). This is the essential difference between electron - ion plasmas and equal mass plasma. In the case of equal mass plasma and a magnetic field along the z-axis there is no current and heat flow in the directions of xy and yx in contrast with the case of electron - ion plasmas. The diffusion caused by the presence of a density gradient is found as well to be symmetric as is the case of electrical and thermal conductivities.

By using the kinetic theory to find the transport coefficients in the presence of collisions and non - equilibrium state, we find the same symmetry we have found by using the irreversible thermodynamics; thus in the case of equal mass plasma there is no current or heat flow in the directions of xy and yx.

In this chapter we find the diffusion caused by the temperature gradient (not by the density gradient as in chapter two). The antisymmetry in the thermal diffusion coefficient means there is no current along the directions xx and yy but there are currents along xy and yx.

All the coefficients have been found in the presence of magnetic field and for equal mass plasma, all the diagonal components have twice the magnitude of corresponding coefficients for the ion - electron plasmas.

4.7. future work

In this thesis, transport coefficients for an electron - positron plasma have been derived on the assumption that the plasma departs only slightly from a Boltzmann distribution, and applying conventional linearisation techniques .

Future work would consist of applying the results have been obtained to various physical situations and comparing with results for an electron - ion plasma .

A further area of future work would be to consider non - linear effects. This would be require extensive computer simulation studies.

Finally, there are various problems of interest in astrophysics involving an electron - positron beam streaming through hydrogen plasma. A study of transport in such a three - component plasma (proton, electron and positron) would be a very interesting research topic for future work.

TABLE (1)

Computed value of $(E^{II(0)})$ for the electrical conductivity of the electrons

Ω_{e^-}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.707108	0.845456	0.847894	0.847911	0.848477	0.850066
-0.2	0.693244	0.814742	0.817283	0.817303	0.817744	0.818193
-0.5	0.628540	0.687191	0.687464	0.690069	0.690164	0.690293
-1.0	0.471405	0.411303	0.456256	0.456184	0.456096	0.455968
-2.0	0.235702	0.210961	0.210685	0.210693	0.211740	0.210542
-4.0	0.078567	0.073603	0.072802	0.072872	0.072862	0.072842
-6.0	0.037216	0.035921	0.035586	0.035577	0.035605	0.035585

TABLE (2)

Computed value of $(E^{I(0)})$ for the electrical conductivity

of the electrons

Ω_e-	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-0.2	0.098039	0.153675	0.153771	0.153774	0.154064	0.154321
-0.5	0.222223	0.319051	0.314283	0.319695	0.320067	0.320463
-1.0	0.333334	0.293438	0.405430	0.405481	0.405657	0.405831
-2.0	0.333333	0.342862	0.345232	0.345098	0.344692	0.345142
-4.0	0.222222	0.219791	0.220079	0.220139	0.220098	0.220092
-6.0	0.157895	0.156561	0.156492	0.156540	0.156555	0.156521

TABLE (3)

Computed value of $(b^{I(1)})$ for the thermal conductivity of the electrons

Ω_{e^-}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.245950	0.256441	0.257332	0.258173	0.263791
-0.2	0.000000	0.239651	0.249560	0.250277	0.250936	0.251556
-0.5	0.000000	0.212953	0.219456	0.220452	0.220538	0.221051
-1.0	0.000000	0.160269	0.162815	0.162246	0.162015	0.162164
-2.0	0.000000	0.091895	0.089927	0.089899	0.090260	0.089714
-4.0	0.000000	0.037334	0.035994	0.036117	0.036124	0.036080
-6.0	0.000000	0.019005	0.018505	0.018504	0.018550	0.018486

TABLE (4)

Computed value of $(b^{II(1)})$ for the thermal conductivity of the electrons

Ω_{e^-}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-0.2	0.000000	0.035835	0.038655	0.039153	0.039600	0.039748
-0.5	0.000000	0.076892	0.081153	0.083543	0.084146	0.084511
-1.0	0.000000	0.106389	0.113459	0.113700	0.113859	0.114366
-2.0	0.000000	0.107466	0.111221	0.110862	0.110667	0.111029
-4.0	0.000000	0.080176	0.080361	0.080429	0.080436	0.080448
-6.0	0.000000	0.059851	0.059672	0.059735	0.059760	0.059762

TABLE (5)

Computed value of $(b^{I(0)})$ for the thermal diffusion

of the electrons

Ω_{e^-}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	-0.184463	-0.179406	-0.179283	-0.179974	-0.185143
-0.2	0.000000	-0.172489	-0.167435	-0.167309	-0.167848	-0.168382
-0.5	0.000000	-0.123845	-0.112702	-0.119026	-0.119120	-0.119423
-1.0	0.000000	-0.042520	-0.039743	-0.039946	-0.039792	-0.039733
-2.0	0.000000	0.015021	0.014083	0.014082	0.013158	0.014233
-4.0	0.000000	0.015786	0.014744	0.014838	0.014856	0.014887
-6.0	0.000000	0.009273	0.008862	0.008857	0.008901	0.008884

TABLE (6)

Computed value of $(b^{//(0)})$ for the thermal diffusion of the electrons

Ω_{e^-}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
-0.2	0.000000	-0.051270	-0.050467	-0.050415	-0.050774	-0.050985
-0.5	0.000000	-0.101455	-0.092797	-0.099050	-0.099525	-0.099937
-1.0	0.000000	-0.109858	-0.105784	-0.105670	-0.105847	-0.106180
-2.0	0.000000	-0.059356	-0.056317	-0.056538	-0.053759	-0.056590
-4.0	0.000000	-0.015481	-0.015225	-0.015160	-0.015170	-0.015169
-6.0	0.000000	-0.005545	-0.005566	-0.005605	-0.005577	-0.005592

TABLE (7)

Computed value of $(E^{I(0)})$ for the electrical conductivity of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.707108	0.845456	0.847894	0.847911	0.848477	0.850066
0.2	0.693244	0.814742	0.817283	0.817303	0.817744	0.818193
0.5	0.628540	0.687191	0.687464	0.690069	0.690164	0.690293
1.0	0.471405	0.411303	0.456256	0.456184	0.456096	0.455968
2.0	0.235702	0.210961	0.210685	0.210693	0.211740	0.210542
4.0	0.078567	0.073603	0.072802	0.072872	0.072862	0.072842
6.0	0.037216	0.035921	0.035586	0.035577	0.035605	0.035585

TABLE (8)

Computed value of $(E^{II(0)})$ for the electrical conductivity of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	-0.098039	-0.153675	-0.153771	-0.153774	-0.154064	-0.154321
0.5	-0.222223	-0.319051	-0.314283	-0.319695	-0.320067	-0.320463
1.0	-0.333334	-0.293438	-0.405430	-0.405481	-0.405657	-0.405831
2.0	-0.333333	-0.342862	-0.345232	-0.345098	-0.344692	-0.345142
4.0	-0.222222	-0.219791	-0.220079	-0.220139	-0.220098	-0.220092
6.0	-0.157895	-0.156561	-0.156492	-0.156540	-0.156555	-0.156521

TABLE (9)

Computed value of $(b^{I(1)})$ for the thermal conductivity

of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.245950	0.256441	0.257332	0.258173	0.263791
0.2	0.000000	0.239651	0.249560	0.250277	0.250936	0.251556
0.5	0.000000	0.212953	0.219456	0.220452	0.220538	0.221051
1.0	0.000000	0.160269	0.162815	0.162246	0.162015	0.162164
2.0	0.000000	0.091895	0.089927	0.089899	0.090260	0.089714
4.0	0.000000	0.037334	0.035994	0.036117	0.036124	0.0360800
6.0	0.000000	0.019005	0.018505	0.018504	0.018550	0.018486

TABLE (10)

Computed value of $(b^{II(1)})$ for the thermal conductivity of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000000	-0.035835	-0.038655	-0.039153	-0.039600	-0.039748
0.5	0.000000	-0.076892	-0.081153	-0.083543	-0.084146	-0.084511
1.0	0.000000	-0.106389	-0.113459	-0.113700	-0.113859	-0.114366
2.0	0.000000	-0.107466	-0.111221	-0.110862	-0.110667	-0.111029
4.0	0.000000	-0.080176	-0.080361	-0.080429	-0.080436	-0.080448
6.0	0.000000	-0.059851	-0.059672	-0.059735	-0.059760	-0.059762

TABLE (11)

Computed value of $(b^{I(0)})$ for the thermal diffusion

of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	-0.184463	-0.179406	-0.179283	-0.179974	-0.185143
0.2	0.000000	-0.172489	-0.167435	-0.167309	-0.167848	-0.168382
0.5	0.000000	-0.123845	-0.112702	-0.119026	-0.119120	-0.119423
1.0	0.000000	-0.042520	-0.039743	-0.039946	-0.039792	-0.039733
2.0	0.000000	0.015021	0.014083	0.014082	0.013158	0.014233
4.0	0.000000	0.015786	0.014744	0.014838	0.014856	0.014887
6.0	0.000000	0.009273	0.008862	0.008857	0.008901	0.008884

TABLE (12)

Computed value of ($b^{II(0)}$) for the thermal diffusion of the positrons

Ω_{e^+}	r=m=0	r=m=1	r=m=2	r=m=3	r=m=4	r=m=5
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.000000	0.051270	0.050467	0.050415	0.050774	0.050985
0.5	0.000000	0.101455	0.092797	0.099050	0.099525	0.099937
1.0	0.000000	0.109858	0.105784	0.105670	0.105847	0.106180
2.0	0.000000	0.059356	0.056317	0.056538	0.053759	0.056590
4.0	0.000000	0.015481	0.015225	0.015160	0.015170	0.015169
6.0	0.000000	0.005545	0.005566	0.005605	0.005577	0.005592

TABLE (13)

Computed value of ($E^{(0)}$, $b^{(0)}$ and $C^{(0)}$) in the 5th approximation of the electrons

Ω_{e^-}	$(E^{I(0)})$	$(E^{II(0)})$	$(b^{I(0)})$	$(b^{II(0)})$	$(C^{I(0)})$	$(C^{II(0)})$
0.0	0.850066	0.000000	-0.185143	0.000000	-0.217798	0.000000
-0.2	0.818193	0.154321	-0.168382	-0.050985	-0.210077	-0.022691
-0.5	0.690293	0.320463	-0.119423	-0.099937	-0.197622	-0.053030
-1.0	0.455968	0.405831	-0.039733	-0.106182	-0.164270	-0.086653
-2.0	0.210542	0.345142	0.014233	-0.056590	-0.101161	-0.102948
-4.0	0.072842	0.220092	0.014887	-0.015169	-0.041094	-0.081521
-6.0	0.035585	0.156521	0.008884	-0.005592	-0.021700	-0.061693

TABLE (14)

Computed value of ($E^{(0)}$, $b^{(0)}$ and $C^{(0)}$) in the 5th approximation of the positrons

Ω_{e^+}	$(E^{I(0)})$	$(E^{II(0)})$	$(b^{I(0)})$	$(b^{II(0)})$	$(C^{I(0)})$	$(C^{II(0)})$
0.0	0.850066	0.000000	-0.185143	0.000000	-0.217798	0.000000
-0.2	0.818193	-0.154321	-0.168382	0.050985	-0.210077	0.022691
-0.5	0.690293	-0.320463	-0.119423	0.099937	-0.197622	0.053030
-1.0	0.455968	-0.405831	-0.039733	0.106182	-0.164270	0.086653
-2.0	0.210542	-0.345142	0.014233	0.056590	-0.101161	0.102948
-4.0	0.072842	-0.220092	0.014887	0.015169	-0.041094	0.081521
-6.0	0.035585	-0.156521	0.008884	0.005592	-0.021700	0.061693

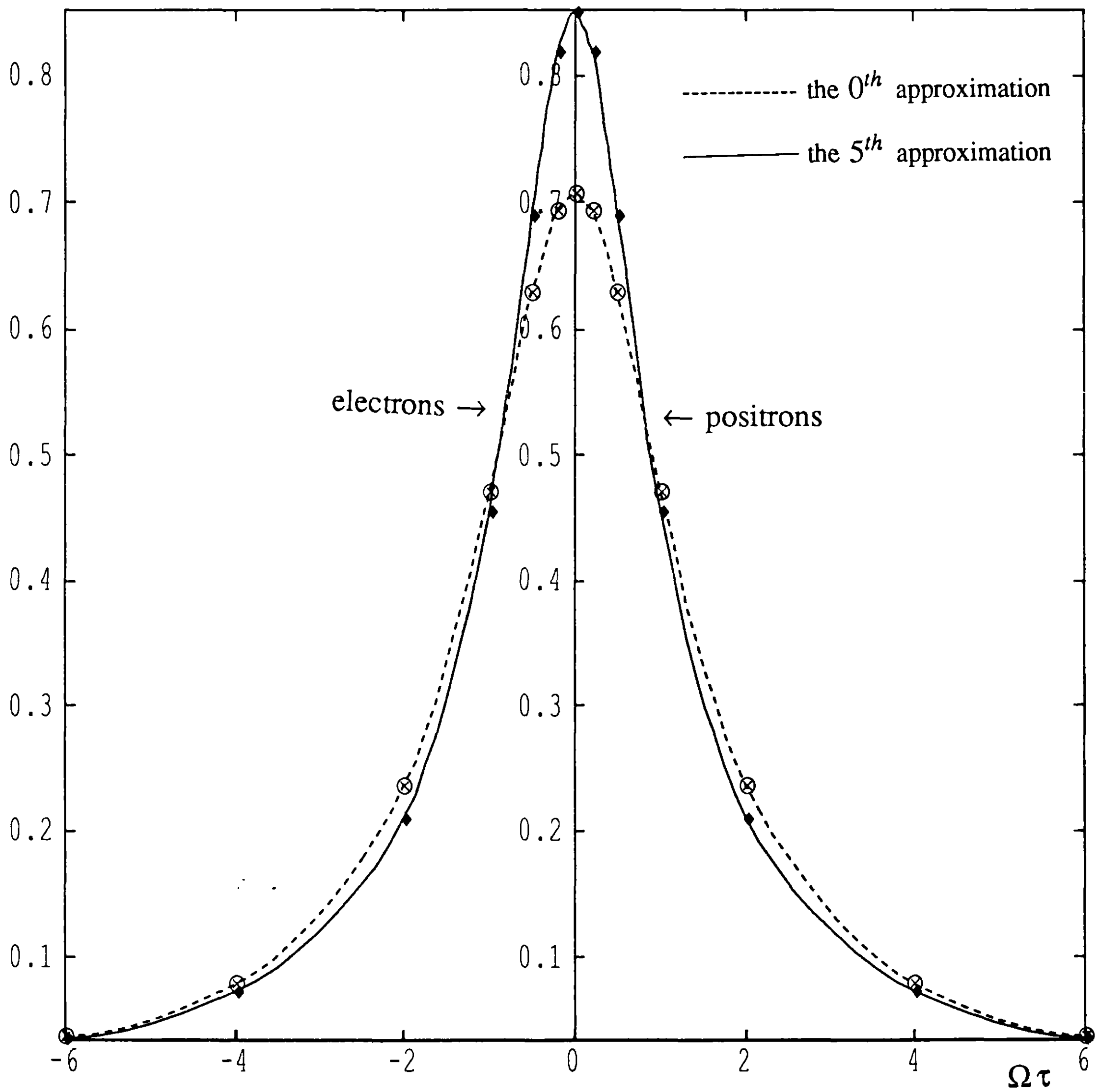

Figure (4-1). $E^{I(0)}$ for the electrical conductivity

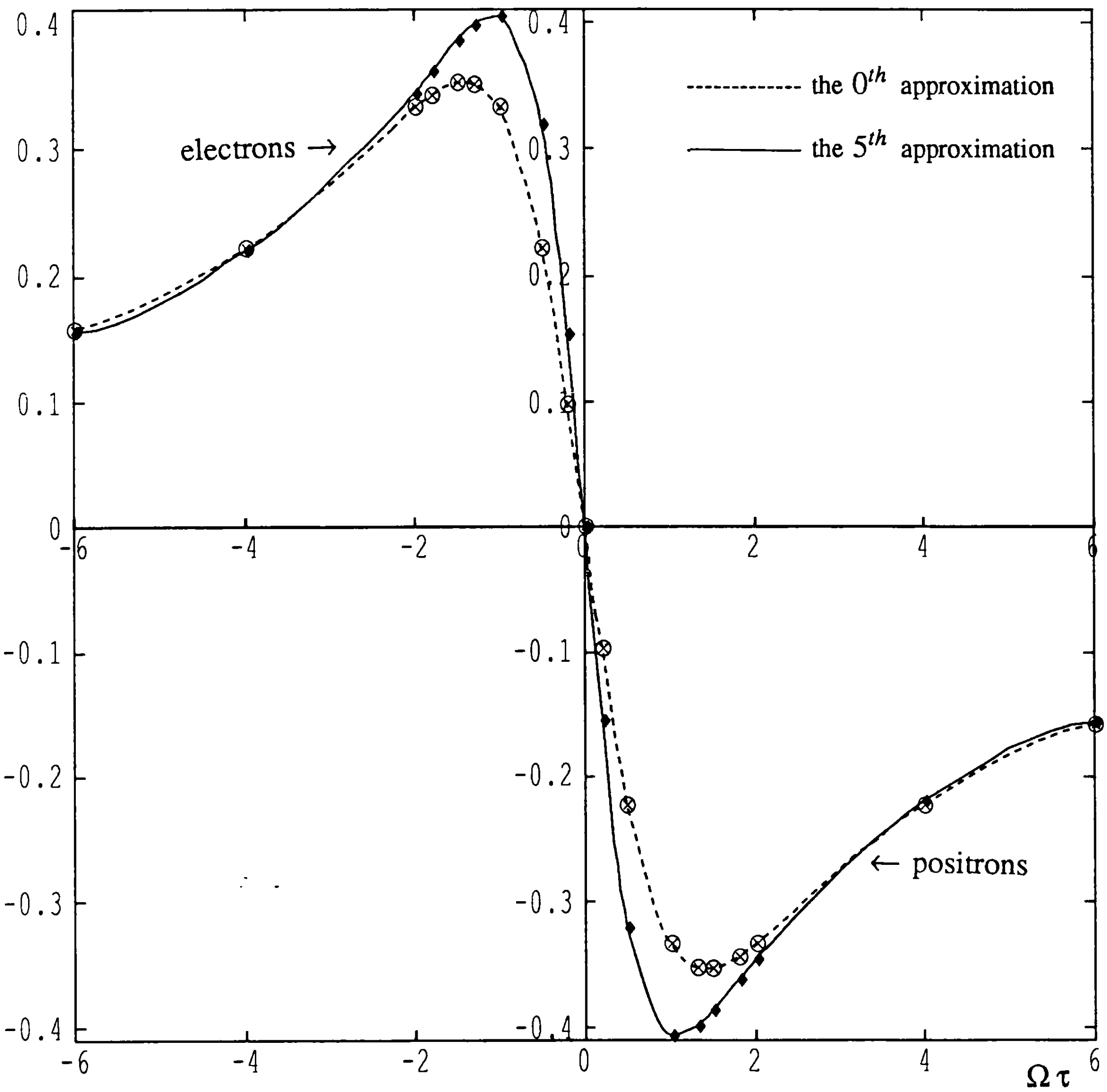

Figure (4-2). $E^{II(0)}$ for the electrical conductivity

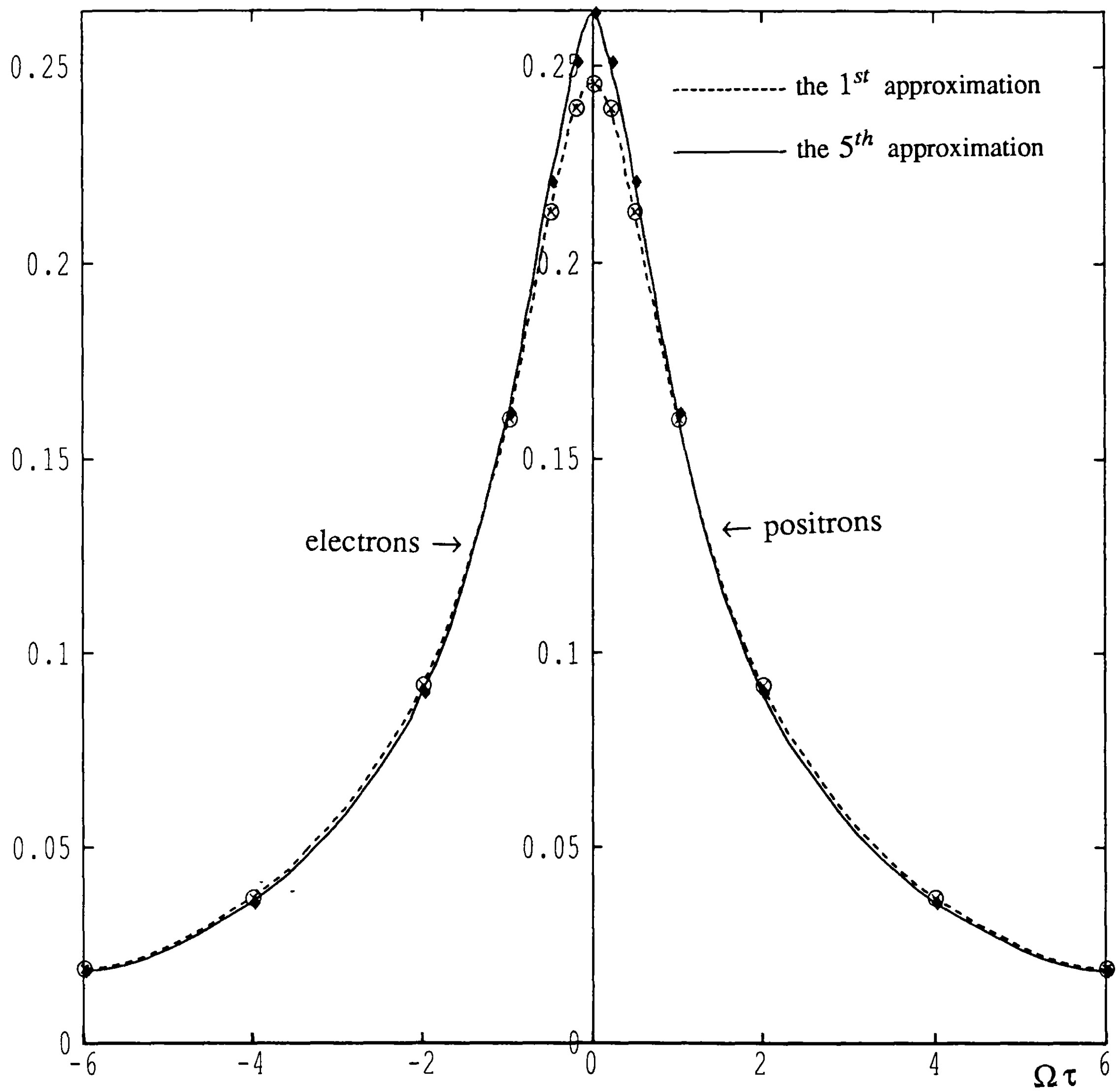

Figure (4-3). $b^{I(1)}$ for the thermal conductivity

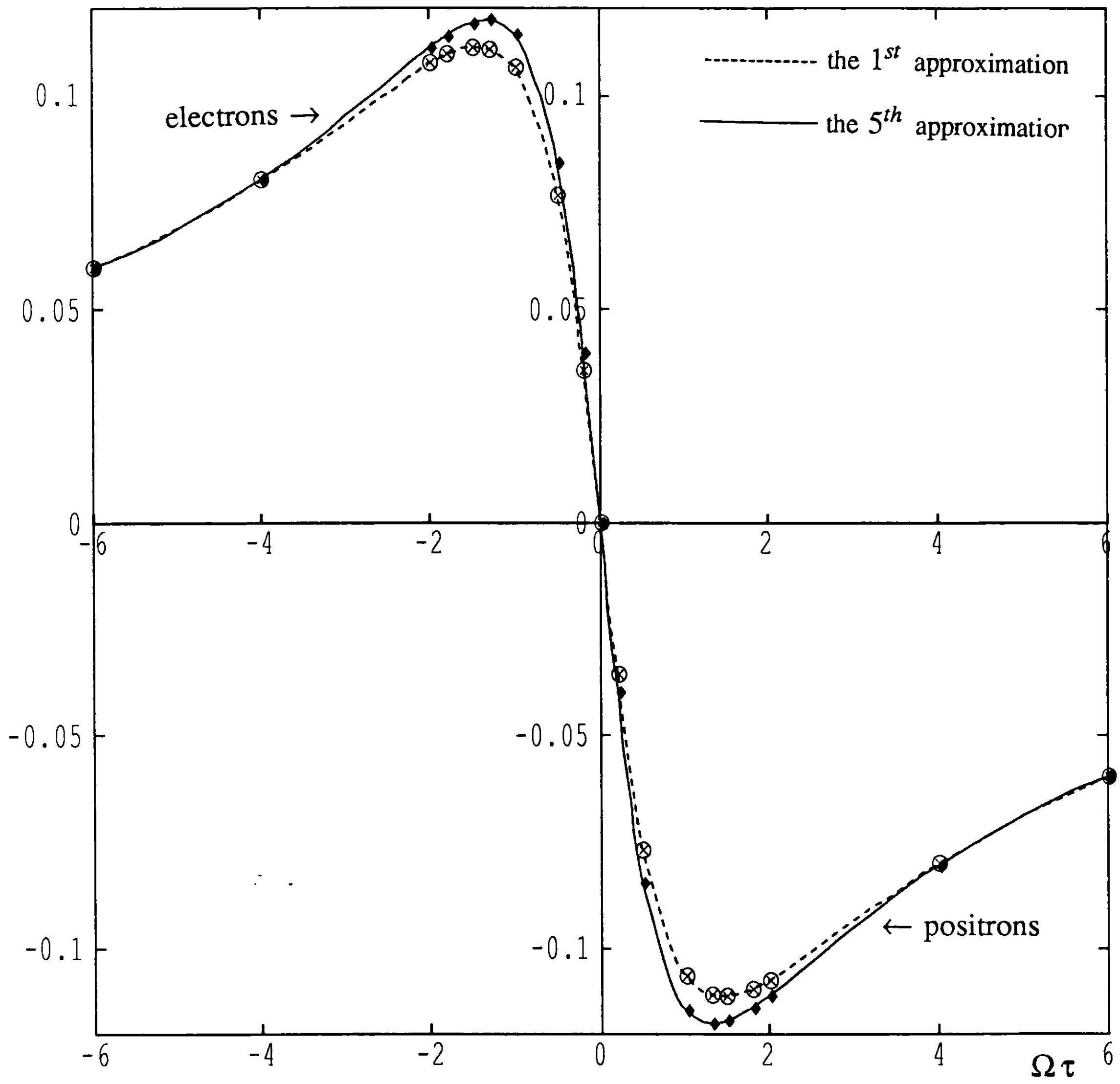

Figure (4-4). $b^{II(1)}$ for the thermal conductivity

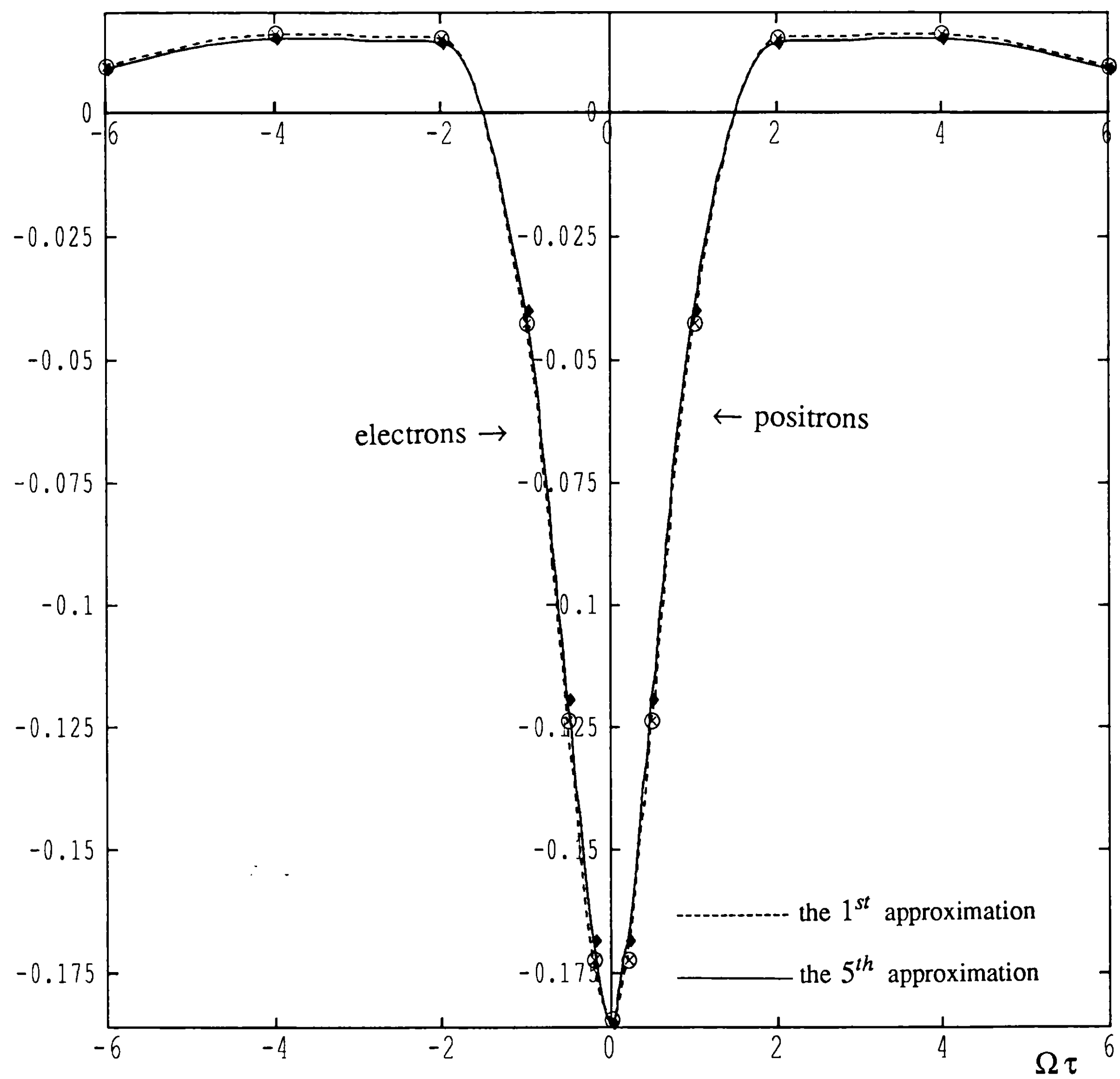

Figure (4-5). $b^{I(0)}$ for the thermal diffusion

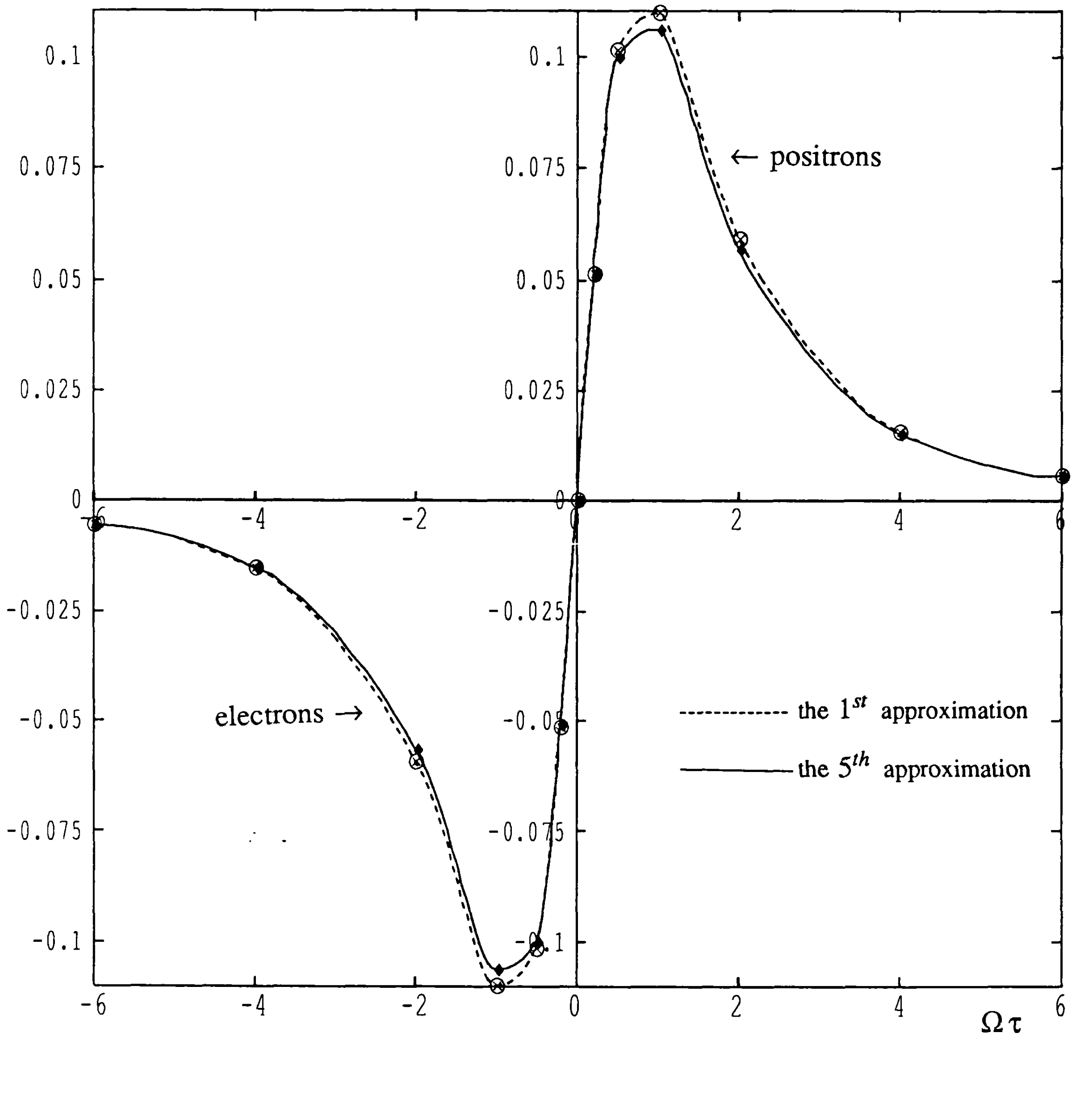

Figure (4-6). $b^{II(0)}$ for the thermal diffusion

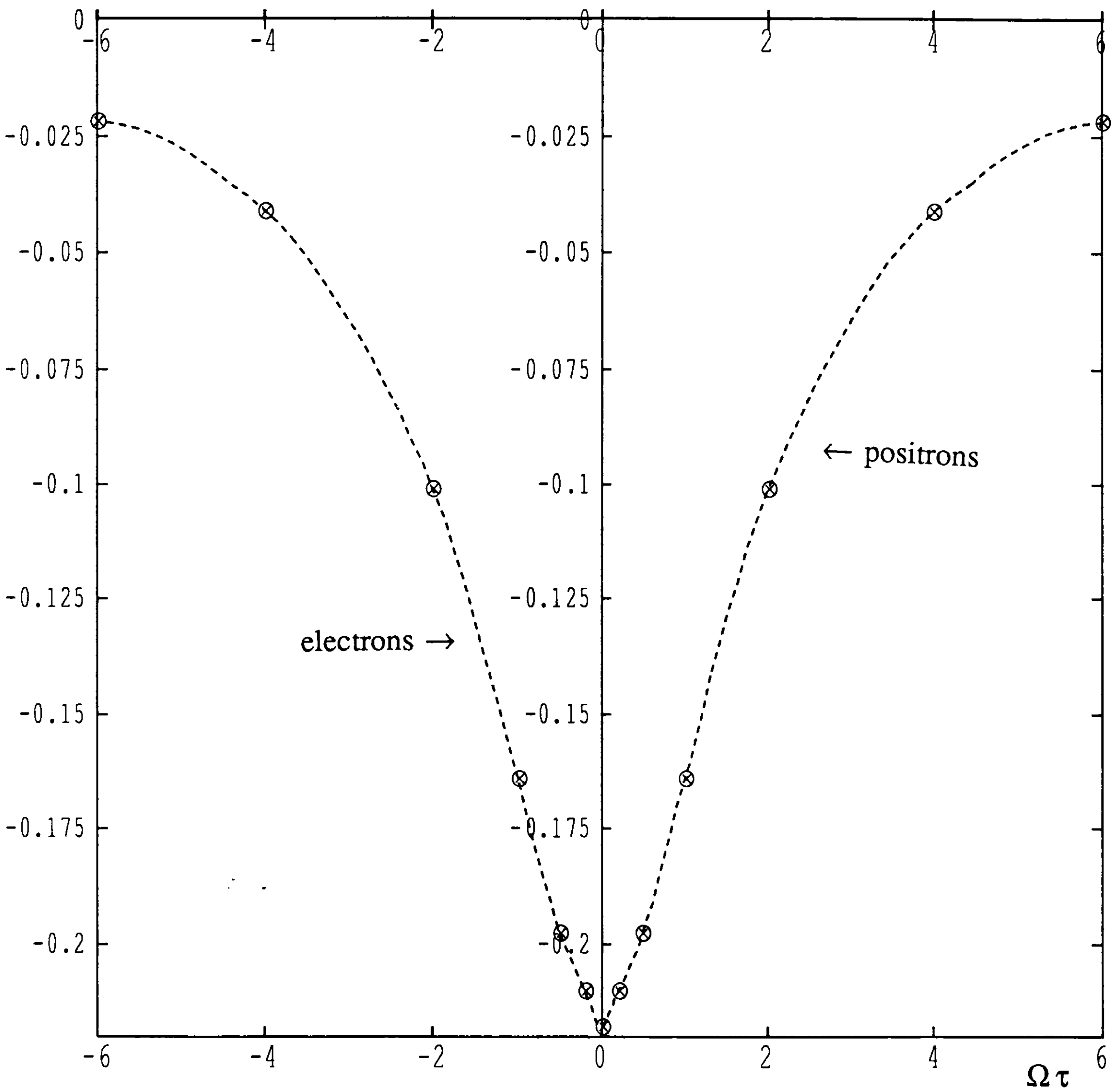

Figure (4-7). the fifth approximation of $C^{I(0)}$ for the interference between the electrical and thermal conductivities

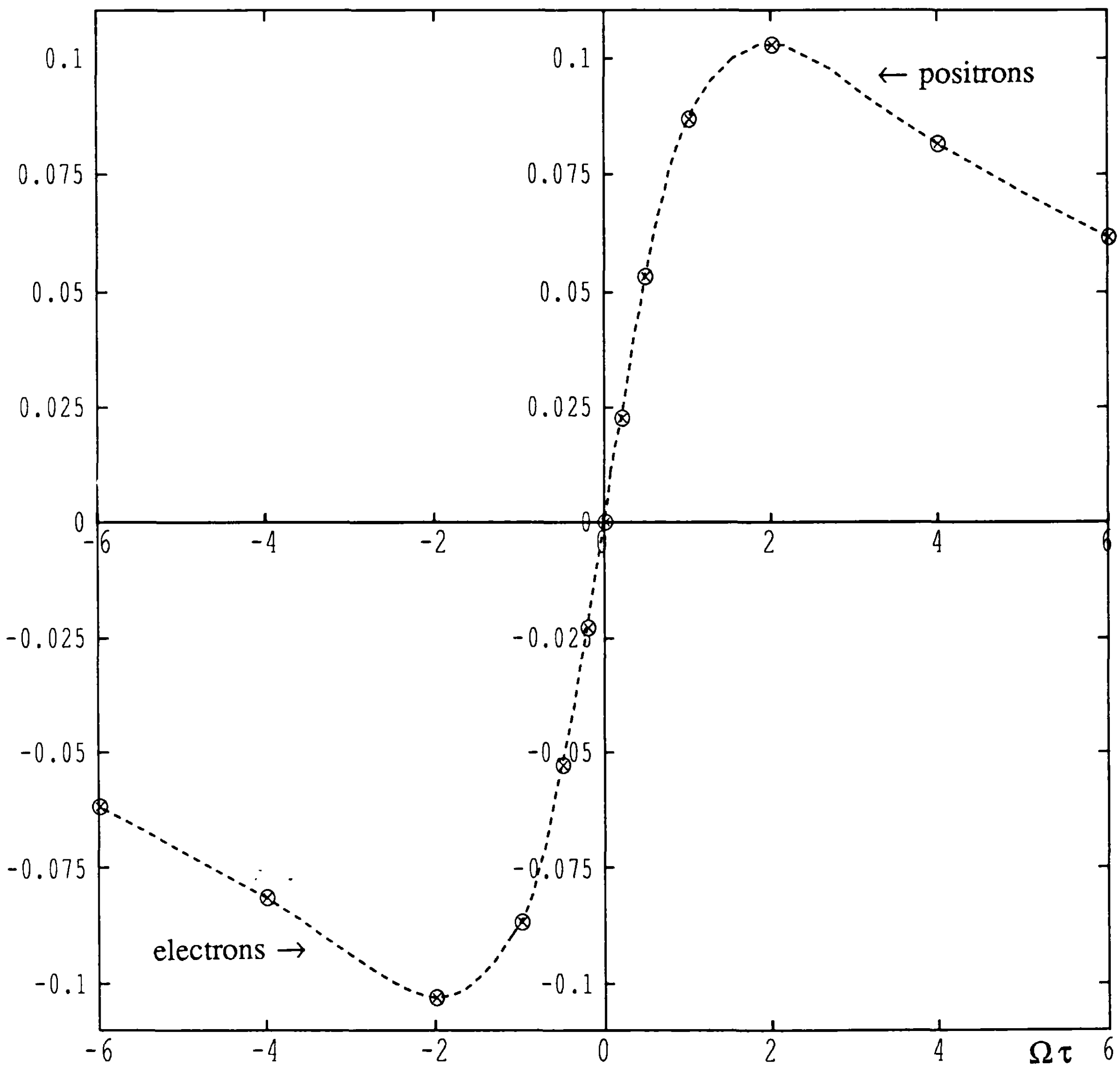

Figure (4-8). the fifth approximation of $C^{II(0)}$ for the interference between the electrical and thermal conductivities

APPENDIX -A-

To calculate the elements of the matrices (3-50) and (3-51), it is necessary to evaluate integrals of the form :

$$M_{pq}^{ij} = \int \psi_p \, C_{ij} \, (f_i \, \Phi_q \, , f_j) dc \tag{A-1}$$

$$N_{pq}^{ij} = \int \psi_p \, C_{ij} \, (f_i \, , f_j \, \Phi_q) dc \tag{A-2}$$

The role of ψ_p , Φ_q is played by the functions of the form

$$L_p^{(3/2)} \left[\frac{m \, c^2}{2 \, k_B \, T} \right] c$$

where $L_p^{(m)}$ are Sonine polynomials , defined by the generating function

$$(1-s) \exp \left(\frac{-s \chi}{1-s} \right) = \sum_p s^p \, L_p^{(3/2)} (\chi) \tag{A-3}$$

Instead of calculating these matrices individually for each pair of values of p , q , it is better to calculate the matrix of the generating functions (A-3). Then by expanding it in powers of s, t . for polynomial of order 3/2 we have (Landshoff 1949) , (Braginskii 1958) and (Kaneko 1960) :

$$M^{ij}(\frac{3}{2}) = \sum_{p,q} s^p\, t^q\, M^{ij}_{pq}(\frac{3}{2}) \tag{A-4}$$

Following the characteristics of Sonine polynomials and following Braginskii , without neglecting the terms of order $\frac{m_e}{m_i}$ as in electron - ion plasmas we obtain for the collisions between $e^- \rightarrow e^+$ and $e^+ \rightarrow e^-$:

$$M^{e^-e^+}(\frac{3}{2}) = M^{e^+e^-}(\frac{3}{2}) = \frac{4\sqrt{(2\pi)}\ln\Lambda\, e^4 n^2}{m^2}\,(\frac{m}{2\,k_B\,T})^{\frac{1}{2}}$$

$$(1-st)^{-1}(1-t)^{-1}(1-s)^{-1}(1-\frac{s+t}{2})^{-\frac{1}{2}}\left[1-\frac{1}{2}(s+t-2st)(1-\frac{s+t}{2})^{-1}+\right.$$

$$\frac{5}{4}st\,(1-s)(1-t)(1-\frac{s+t}{2})^{-2}$$

$$\left.+\frac{1}{2}st\,(1-\frac{s+t}{2})^{-2}(1-s)^2(1-t)^2\right] \tag{A-5}$$

$$N^{e^-e^+}(\frac{3}{2}) = N^{e^+e^-}(\frac{3}{2}) = -\frac{4\sqrt{(2\pi)}\ln\Lambda\, e^4 n^2}{m^2}(\frac{m}{2k_B T})^{\frac{1}{2}}$$

$$(1-t)^{-1}(1-s)^{-1}(1-\frac{s+t}{2})^{-\frac{1}{2}}\left[1-\frac{1}{2}(1-\frac{s+t}{2})^{-1}\right.$$

$$\left. + 3st\,(1-s)(1-t)(1-\frac{s+t}{2})^{-2}\right] \quad \text{(A-6)}$$

And for identical particles we have

$$M^{e^-e^-}(\frac{3}{2}) + N^{e^-e^-}(\frac{3}{2}) = M^{e^+e^+}(\frac{3}{2}) + N^{e^+e^+}(\frac{3}{2})$$

$$= \frac{4\sqrt{(2\pi)}\ln\Lambda\, e^4 n^2}{m^2}(\frac{m}{2k_B T})^{\frac{1}{2}}\, st\,(1-st)^{-2}(1-\frac{s+t}{2})^{-\frac{5}{2}}$$

$$\left[1-\frac{s+t}{2}-\frac{st}{8}+\frac{st(s+t)}{4}+\frac{3}{8}s^2t^2\right] \quad \text{(A-7)}$$

Then we expand equation (A.7) and neglect powers more than 8 , because ($1>s>0$ $1>t>0$) . Using equations (3-33) , (3-42) , (3-48) , (A-1) and (A-2) we obtain (equation (A-7) is also obtained by (Landshoff 1949) , (Kaneko 1960) and (Braginskii 1957) :

$$\alpha_{kl} = \beta_{kl} = \frac{4\sqrt{2}}{5} \begin{bmatrix} 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & \ldots\ldots \\ 0.00000 & 1.00000 & 0.75000 & 0.46875 & 0.27344 & 0.15381 & \ldots\ldots \\ 0.00000 & 0.75000 & 2.81250 & 2.41406 & 1.72851 & 1.14075 & \ldots\ldots \\ 0.00000 & 0.46875 & 2.41406 & 6.27441 & 5.90552 & 4.62410 & \ldots\ldots \\ 0.00000 & 0.27343 & 1.72850 & 5.90552 & 12.28058 & 12.13068 & \ldots\ldots \\ 0.00000 & 0.15381 & 1.14075 & 4.62410 & 12.13068 & 21.72864 & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{bmatrix} \quad \text{(A-8)}$$

From equations (A-5) , (A-6) and using equations (A-1) , (A-2) , (3-33) , (3-42) and (3-48) we obtain :

$$\alpha'_{lk} = \beta'_{lk} = \frac{4\sqrt{2}}{5} \begin{bmatrix} 0.50000 & 0.37500 & 0.23437 & 0.13672 & 0.07690 & 0.04229 & \ldots\ldots \\ 0.37500 & 0.71875 & -0.02734 & -0.10644 & 0.15875 & 0.49916 & \ldots\ldots \\ 0.23437 & -0.02734 & 2.38330 & 1.95227 & 1.61075 & 1.64798 & \ldots\ldots \\ 0.13672 & -0.10644 & 1.95227 & 6.01901 & 5.98933 & 5.33091 & \ldots\ldots \\ 0.07690 & 0.15875 & 1.61075 & 5.98933 & 11.57623 & 11.82884 & \ldots\ldots \\ 0.04229 & 0.49916 & 1.64798 & 5.33091 & 11.82884 & 18.78090 & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \end{bmatrix} \quad \text{(A-9)}$$

APPENDIX -B-

To find equations (3-61) and (3-62) in chapter 3 . equation (3-59) operated by $\sum \int S_k L_{(r)}^{3/2} \, dc_k$ becomes :

$$-\sum_k \int \frac{e_k}{k_B T} c_k f_k^{\circ} S_k L_r^{3/2} dc_k = i \sum_k \int \Omega_k D_k f_k^{\circ} S_k^2 L_r^{3/2} dc_k$$

$$-\sum_k \sum_l \int I_{kl} (D_k S_k) S_k L_r^{3/2} dc_k \qquad \text{(B-1)}$$

The L.H.S of equation (B-1) for the electrons

$$\frac{n}{\pi^{3/2}} \frac{e}{k_B T} \int c \, S \, e^{-S^2} L_r^{3/2} dS =$$

$$\left(\frac{2k_B T}{m}\right)^{\frac{1}{2}} \frac{n}{\pi^{3/2}} \frac{e}{k_B T} \int S^2 e^{-S^2} L_r^{3/2} dS \qquad \text{(B-2)}$$

where $f_{\circ} dc = \frac{n}{\pi^{3/2}} e^{-S^2} dS$ has been used , and the subscript been dropped for clarity . But Sonine polynomial has the following characteristic

$$\int_0^\infty e^{-x} L_p^m L_q^m x^m \, dx = \delta_{pq} \frac{\Gamma(m+p+1)}{p!} \qquad \text{(B-3)}$$

So equation (B-2) will be

$$= \frac{2n}{\pi^{1/2}} \left(\frac{2k_B T}{m}\right)^{\frac{1}{2}} \frac{e}{k_B T} \int_0^\infty S^4 e^{-S^2} L_r^{3/2} \, dS$$

$$= \frac{2n}{\pi^{1/2}} \frac{2k_B T}{m} \frac{e}{k_B T} \int_0^\infty (S^2)^{3/2} e^{-S^2} L_0^{3/2} L_r^{3/2} \, d(S^2)$$

$$= \left(\frac{8n^2 e^2}{\pi m k_B T}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{5}{2}+r)}{r!} \delta_{r0}$$

For the positrons we find the L.H.S of equation (B-1) is

$$= -\left(\frac{8n^2 e^2}{\pi m k_B T}\right)^{\frac{1}{2}} \frac{\Gamma(\frac{5}{2}+r)}{r!} \delta_{r0}$$

So when we sum over the two species we get zero .

The first term on the R.H.S of equation (B-1) for the electrons

$$\Omega_{e^-}\int d_{e^-}^{(m)}\, L_m^{3/2} L_r^{3/2} S^2 f^\circ\, dc$$

$$= \frac{n}{\pi^{3/2}}\, \Omega_{e^-} d_{e^-}^{(m)} \int L_m^{3/2} L_r^{3/2}\, S^2 e^{-S^2}\, dS$$

$$= \frac{2n}{\pi^{1/2}}\, \Omega_{e^-} d_{e^-}^{(m)} \int (S^2)^{\frac{3}{2}} L_m^{3/2} L_r^{3/2}\, e^{-S^2}\, d(S^2)$$

$$= \frac{2n}{\pi^{1/2}} \Omega_{e^-} d_{e^-}^{(m)}\, \frac{\Gamma(\frac{5}{2}+r)}{r\,!}\, \delta_{rm}$$

for the positrons

$$+ \frac{2n}{\pi^{1/2}} \Omega_{e^+} d_{e^+}^{(m)}\, \frac{\Gamma(\frac{5}{2}+r)}{r\,!}\, \delta_{rm}$$

The second term

$$d_{e^-}^{(m)} \left[\int I_{e^-e^+} (L_m^{3/2} S_{e^-}^2) S_{e^-} L_r^{3/2}\, dc_{e^-} + \int I_{e^-e^-} (L_m^{3/2} S_{e^-}^2) S_{e^-} L_r^{3/2}\, dc_{e^-}\right]$$

$$+ d_{e^+}^{(m)} \left[\int I_{e^+e^-} (L_m^{3/2} S_{e^+}^2) S_{e^+} L_r^{3/2} dc_{e^+} + \right.$$

$$\left. \int I_{e^+e^+} (L_m^{3/2} S_{e^+}^2) S_{e^+} L_r^{3/2} dc_{e^+} \right] \tag{B-4}$$

from equations (3-42) , (3-48) and (B-4) will be

$$- \frac{15}{4} \frac{1}{\tau} \sum_r \sum_m (\alpha_{rm} + \alpha'_{rm}) (d_{e^-}^{(m)} + d_{e^+}^{(m)})$$

So equation (B-1) will be

$$0 = \sum_r \sum_m \left(\frac{2in}{\pi^{1/2}} \Omega (d_{e^-}^{(m)} + d_{e^+}^{(m)}) \frac{\Gamma(\frac{5}{2} + r)}{r!} \delta_{rm} \right.$$

$$+ \frac{15}{4} \frac{1}{\tau} \sum_r \sum_m (\alpha_{rm} + \alpha'_{rm}) (d_{e^-}^{(m)} + d_{e^+}^{(m)}) \tag{B-5}$$

By using the same method equation (3-62) can be found .

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