OFF-DIAGONAL FERMION-HIGGS COUPLINGS

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To the Memory of my late FATHER who died with good wishes for my prosperous future.

To my Beloved MOTHER and my aunt TATA for their prayers, patience, forbearance, and encouragement.

To my brother MALEK for his moral support.

To my sisters.

I dedicate this thesis.

Salim.

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Contents

Abstract

The flavour changing neutral current (FCNC) decay induced by a Higgs boson in the standard model with three and four generations has been studied.

This process occurs first at one-loop order, where it has been calculated neglecting the mass of the outgoing quark. The possibility of producing the as yet undiscovered Higgs boson H by this decay has been considered. In particular we consider the decay of a fourth generation b' quark; the FCNC decay $b' \rightarrow bH$ may dominate the charged current (CC) decay modes if the b' mass is less than the top quark mass, for Higgs mass up to 60 Gev. The decay of the top quark, in the presence of a fourth generation, is also discussed.

INTRODUCTION

Rare decays due to flavour changing neutral currents have been a very useful tool to:

1- search for unknown particles. For example the existence of the charm quark c is based upon the GIM mechanism which ensures the absence of flavour changing processes at the tree level [1].

2- constrain physical quantities such as quark masses and the Kobayashi-Maskawa (K-M) weak mixing matrix elements V_{ij}[2]. For example the mass of the charm quark has been predicted from the strength of higher-order processes, see Gaillard [3]. The B decay which is a typical example of these decays determines five parameters of the standard model, namely the mass of the b and t- quarks, the relative sizes of two quark mixing angles (V_{bu} and V_{bc}) and the phase δ which could be responsible of the CP violation [4,5]. It can yield also limits on the mass of the Higgs particle H (a point which we want to investigate) which the theory does not specify.

3- ruling out and testing some models beyond the standard model, for example simple extended technicolour models are ruled out as they give a larger rate for strangeness changing neutral current processes than observed experimentally [6], to search for multi-Higgs models, the FCNC could be a test for supersymmetric theories [7].

4- Though the products of such decays are known fermions, the rate of such processes depends on the properties of the virtual particles present in the appropriate loop diagrams. Thus they will provide a window to particles which are too heavy to be produced experimentally (e.g. a fourth generation). These rare decays appear quite sensitive to the eventual existence of a new interaction beyond the standard model needed to understand:

i) the origin of masses, in other words to explain the spontaneous breakdown of symmetry .

ii) the problem of the fermion generations or the flavour problem.

These facts explain why the FCNC rare decays have been studied extensively in recent years. However decays involving the Z boson have received more attention [7-23] than the ones induced by the H boson [24-26].

The first calculation of the flavour changing coupling of the Higgs to quarks was made by Ellis et al. in 1976 [31]. From this date until the beginning of this year all efforts have been focused on decays involving experimentally observed particles [24-36]. This is due partly to the conflicting results of Ellis et al. [31] and the one obtained in 1982 by Willey and Yu [24,25] and independently in 1983 by Grzadkowski and Krawczyk [26] and secondly to try to find a very light Higgs particle in K and B decays [29, 30, 35]. As far as processes involving heavier as yet unobserved quarks are concerned there were no available results until the beginning of this year 1989, after the long controversy about the value of the one loop flavour changing Higgs coupling had been resolved in favour of Willey-Yu and Grzadkowski-Krawczyk result using a novel method [34]. These results on heavier quarks are due to Eilam et al [37] and more recently, May 1989, by Krawczyk [38].

Eilam et al have treated the case of the decay of a member of the fourth family b', however the details of their calculation have not been released. Moreover the case of the top quark decay which fulfils the same conditions, i.e. heavy external quarks, has not received any attention by them.

Krawczyk in a very recent preprint [38] gives more details about the calculation of the flavour changing coupling both in three and four generations scheme. However his investigation has been concerned with the estimation of the deviation from the low energy approximation in which the masses of the external particles are neglected compared to the mass of the W boson. Indeed there is no extension to the calculation of the branching ratio of the flavour changing process to a given semileptonic decay in order to have an idea whether the FCNC decay could be detected experimentally. To fill this gap, we adopt both the views of Eilam et al. and of Krawczyk, starting by recalculating the flavour changing couplings of the K and B decay and then extending our calculation to the T and B' decays. We try to speculate, as done for the K and B decays, whether the T and B' could be a probe for the detection of the Higgs particle.

In chapter 1 we give a brief review of the standard model, concentrating on the key points of charged and neutral currents, the GIM mechanism and the Higgs mass.

In chapter 2 there is a recalculation of the coupling $d_i \rightarrow d_j H$ using the Feynman gauge, considering the case where the external quarks d_i, d_j and the Higgs boson H are light compared to the mass of the W boson. There is no approximation made on the mass of the internal quark in the loop diagrams. We present the result of Ellis et al. and explain how their incorrect treatment of the trace anomaly led them to an over-suppressed flavour changing coupling.

At the end of the chapter we show that the ongoing controversy about the definition of the γ_5 matrix in the framework of the dimensional regularisation scheme does not represent , as far as our case is concerned, a real problem.

Chapter 3 deals with the same calculation but to the next order of approximation, in the masses of the Higgs particle H and the ingoing quark d_i. We thereby correct the result given in Grzadowski - Krawczyk [26]. The inconsistency of their results had already been noticed by Botella and Lim [27]: however no further details have been released. Next we extend our calculation and confirm our results using the unitary gauge. The chapter ends with a presentation of the calculation of the leading order result in an arbitrary gauge.

The flavour changing coupling in the case of the decay of a heavy quark is considered in chapter 4. Formulae are presented for the case in which the mass of the final state quark d_j is neglected but the masses of the initial quark d_i and the Higgs particle are treated exactly; these results are restricted to the kinematic regime in which the $d_i \rightarrow d_j H$ amplitude is real. the procedure for calculating the flavour changing decay branching ratio numerically is then explained.

In chapter 5 we present our numerical results for the flavour changing couplings and branching ratios. This is followed by a discussion of the results presented in this thesis, including a comparison with earlier published work.

The thesis ends with some appendices.

Chapter1

1.1 Review of the Standard Model

The standard model is a gauge theory based on SU(2) x U(1) invariance, where SU(2) is the group of weak isospin and U(1) is the group of weak hypercharge.

To give masses to particles, while preserving the renormalisability of the theory, a spontaneous breakdown of the above symmetry by the ground state is introduced (Higgs mechanism); the ground state gets a vacuum expectation value by means of one doublet Higgs field.

Three families of quarks and leptons are assumed. Left-handed fermions form doublets, however the right-handed ones form singlets. There are no right-handed neutrinos. CP violation is thereby naturally explained and the GIM (Glashow-Iliopolous-Maiani) mechanism suppresses flavour changing transitions.

This model predicts the existence of three bosons W^+, W^- and Z, their masses turn out to be in perfect concordance with the experimental data.

The other remnant of the theory still waiting for confirmation is the Higgs particle.

The standard model has three distinct sectors:

1- the gauge-Higgs sector,

2- the fermion-gauge sector,

3- the fermion-Higgs sector.

1.1.1 The gauge-Higgs sector:

It is represented by the following Lagrangian

$$L_{GH} = -\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu} - \frac{1}{4}F_Y^{\mu\nu}F_{Y\mu\nu} - (D_\mu\Phi)^+ (D^\mu\Phi) - V(\Phi) . \qquad (1.1.1)$$

Where Φ is the Higgs field, which leads to the breakdown of the SU(2) x U(1) gauge symmetry to $U(1)_{em}$.

$F_a^{\mu\nu}$, $F_Y^{\mu\nu}$ are the SU(2) and U(1) fields strengths respectively with

$$F_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu + ig\varepsilon_{abc} W_b^\mu W_c^\nu , \qquad (1.1.2)$$

and

$$F_Y^{\mu\nu} = \partial^\mu Y^\nu - \partial^\nu Y^\mu . \qquad (1.1.3)$$

$D_\mu \Phi$ is the SU(2) x U(1) covariant derivative given by

$$D_\mu \Phi = (\partial_\mu - ig \frac{\tau_a}{2} W_{a\mu} + ig' \frac{Y_\mu}{2}) \Phi . \qquad (1.1.4)$$

$W_{a\mu}$ and Y_μ are respectively the fields associated with the W boson and the photon. Also g and g' are the SU(2) and U(1) coupling constants respectively. $V(\Phi)$ is the Higgs potential.

1.1.2 The fermion-gauge fields sector.

The gauge-fermion Lagrangian is given by

$$L_{GF} = i \sum_i \bar{\Psi}_i \gamma^\mu D_\mu \Psi_i . \qquad (1.1.5)$$

The sum runs over all fermions (quarks and leptons) in the theory, with $D_\mu \Psi_i$ is the covariant derivative of the i^{th} fermion given by

$$D_\mu \Psi_i = [\partial_\mu - i g (t_a)_i W_{a\mu} - ig'(y)_i Y_\mu] \Psi_i . \qquad (1.1.6)$$

The left-handed components transform as an SU(2) doublet ($t_a = \tau_a/2$, where τ_a are Pauli matrices) while the right-handed components transform as SU(2) singlets (t_a=0).

1.1.3 The fermion-Higgs sector:

In general mass terms in the Lagrangian come in the following way

$$L_{Yukawa} = -\lambda^{u}_{ij}(\bar{u}_i\,\bar{d}_i)_L\,\Phi u_{jR} - \lambda^{d}_{ij}(\bar{u}_i\,\bar{d}_i)_L\,\tilde{\Phi}\,d_{jR} + \lambda^{l}_{ij}(\bar{\nu}_i\,\bar{l}_i)_L\,\tilde{\Phi} l_{jR} + \text{h.c.} \qquad (1.1.7)$$

Where i and j are family indices and λ_{ij} are the Yukawa couplings.

The charged conjugate Higgs field Φ is defined by

$$\tilde{\Phi} = i\tau_2\Phi^* \text{ with } \Phi = \frac{1}{\sqrt{2}}(v + H, 0)_T. \qquad (1.1.8)$$

The vacuum expectation value of Φ is given by

$$<\Phi> = \frac{v}{\sqrt{2}}(0, 1)_T. \qquad (1.1.9)$$

Replacing Φ and Φ by their values in (eq 1.1.7) we obtain

$$L_{mass} = -\bar{u}_{iL}M^{u}_{ij}u_{jR} - \bar{d}_{iL}M^{d}_{ij}d_{jR} - \bar{l}_{iL}M^{l}_{ij}l_{jR} + \text{h.c}. \qquad (1.1.10)$$

with

$$M^{f}_{ij} = \frac{1}{\sqrt{2}}\lambda^{f}_{ij}v, \; (f = \{u,d,l\}). \qquad (1.1.11)$$

However the mass matrices M are not necessarily diagonal or hermetian. To obtain physical masses we change to another basis called flavour basis where the fermion fields are replaced by

$$\Psi^{f}_{L,R} \to U^{f}_{L,R}\,\Psi^{f}_{L,R}, \qquad (1.1.12)$$

to diagonalise M_{ij}:

$$(U_L^f)^+ M^f (U_R^f) = M^f_{diag} . \qquad (1.1.13)$$

The Kobayashi-Maskawa matrix V is defined by

$$V = (U_L^u)^+ U_L^d . \qquad (1.1.14)$$

In addition to obtaining L_{mass} from L_{Yukawa} we have also

$$L_{HF} = -\frac{1}{\sqrt{2}} \bar{u}_{iL} \lambda^u_{ij} u_{jR} H + \ldots\ldots \qquad (1.1.15)$$

The lagrangian of the interaction Higgs-Fermions and its relation to the problem of the flavour changing transition will be discussed in paragraph 1. 5.

1.2 Charged current and the flavour changing transitions

In the standard model, flavour decays proceeds exclusively via charged-current interaction which are mediated by the $W^+$and W^- bosons.

These decays arise from the following part of the Lagrangian (if we consider just quarks fields).

$$L_{cc} = \frac{g}{2\sqrt{2}} (W^+_\mu J^\mu_- + W^-_\mu J^\mu_+) , \qquad (1.2.1)$$

where

$$J^\mu_- = (J^\mu_+)^+ = (\bar{u}, \bar{c}, \bar{t}) \gamma^\mu (1-\gamma_5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \qquad (1.2.2)$$

is the charged current associated to W bosons.

V is the Kobayashi - Maskawa matrix. Its appearance in the expression for the charged currents leads to the flavour changing transition.

1.3 The neutral current and the flavour changing transitions

One of the features of the standard model is the absence of flavour changing neutral currents at the tree level.

This observation is due to the fact that both the third weak isospin current J^3_μ and the electromagnetic current are diagonal when expressed in the mass eigenstate basis (or the flavour basis).

Indeed the weak isospin currents are given by:

$$J^i_\mu = \bar{Q}_{f_L} \gamma_\mu \frac{\tau_i}{2} Q_{f_L} . \qquad (1.3.1)$$

Here τ_i are SU(2) generators, and Q_{fL} are expressed in the weakeigenstate basis,

$$Q_{f_L} = \begin{pmatrix} u_f \\ d'_f \end{pmatrix} \qquad (f = 1, 2, 3). \qquad (1.3.2)$$

Also $u_1 = u$, $u_2 = c$ and $u_3 = t$, and $d_f' = V_{fg} d_g$ with $d_1 = d$, $d_2 = s$ and $d_3 = b$; V is the K-M matrix. Hence

$$J^3_\mu = \bar{Q}_{f_L} \gamma_\mu \frac{\tau_3}{2} Q_{f_i} \qquad (1.3.3)$$

$$= \frac{1}{2} (\bar{u}_{f_L} \gamma_\mu u_{f_L} - \bar{d}'_{f_L} \gamma_\mu d'_{f_L}) \qquad (1.3.4)$$

Using the unitarity of V, we have

$$J^3_\mu = \frac{1}{2}(\bar{u}_{f_L} \gamma_\mu u_{f_L} - \bar{d}_{f_L} \gamma_\mu d_{f_L}) . \qquad (1.3.5)$$

which clearly is flavour diagonal. The same procedure could be followed to show that the electromagnetic current satisfies this property.

Hence neither the electromagnetic field Y nor the Z field could induce a flavour changing transition.

1.4 The neutral flavour changing transitions and naturality

Glashow and Weinberg [39] postulate that the absence of flavour changing neutral current (FCNC) is natural, which means that it is due to choice of the group and of its representation rather than to the parameters of the theory, see Gatto [40,41].
They find that in order that, in SU(2)* U(1) group, neutral currents conserve flavour, all quarks of the same helicity and charge must:
i) have the same T_3 (the third component of the weak isospin),
ii) have the same T,
iii) receive mass from a single Higgs (i.e. same mass term in the Lagrangian).
The standard model, where the left handed quarks are in doublets and the right handed ones are in singlets, and the vector model of Glashow-Georgi where both left- and right-handed quarks are in doublets, satisfy the conditions i) and ii).
However the vector model does not respect the third one, indeed in this model the Higgs field could come in doublets and triplets, in other words in the vector model the Higgs particle mass comes from two terms in the Lagrangian.

1.5 The interaction Higgs-Fermion and the GIM mechanism

(Glashow - Iliopoulos - Maini)

The Lagrangian of interaction Higgs-quark is given by

$$L = -\frac{\sqrt{2}}{v}\,(\bar{q}_L M q_R + \bar{q}_R M^{+} q_L)\,H. \qquad (1.5.1)$$

where q and H are respectively the quark field and the Higgs field, v is the vacuum expectation value. The mass matrix M is arbitrary, not necessary diagonal or hermetian. We may diagonalise M the mass matrix by defining a new quark fields

$$q'_L = U_L q_L \;,\; q'_R = U_R q_R \;. \qquad (1.5.2)$$

Where U_L and U_R are unitary matrices acting on the flavour index of the quarks.

$$\bar{q}_L M q_R = \bar{q}'_L U_L M U_R^{-1} q'_R \,. \qquad (1.5.3)$$

We can choose U_L and U_R (unitary) such that $U_L M U_R^{-1}$ =M' is diagonal. So

$$L = -(\,\bar{q}'_L M'\, q'_R + \bar{q}'_R M'^{+}\, q'_L\,)\,H. \qquad (1.5.4)$$

which shows that there is no off diagonal transition. This is represented in the Feynman diagram [rules 4 and 5].

To obtain a flavour changing transition it is necessary to include the first order (g^3 where g is the SU(2) constant) loop diagrams where the charged bosons (W^+ and W^-) and the charged unphysical Goldstone bosons (Φ^+ and Φ^-) are exchanged. A flavour changing transition is then induced if the quark mixing angles are nonzero.

At this stage we encounter a further suppression due to the GIM mechanism, which occurs as follows:

If q_i and q_j are external quarks with flavour i and j, q_k is the internal quark with flavour k, then the contribution of q_k to the transition amplitude for $(q_i \rightarrow q_j H)$ is

$$\Gamma_k = V^*_{kj} V_{ki} \{ C_0 + \frac{m_k^2}{m_w^2} [C_1 + C_2 \log \frac{m_k^2}{m_w^2}] + \ldots\ldots \} . \quad (1.5.5)$$

Where C_0 , C_1 , ... are constants which are the object of our investigation.

V is the Kobayashi- Maskawa matrix which is unitary.

The amplitude for the transition is equal to

$$\Gamma = \sum_{k \text{ all flavours}} \Gamma_k . \quad (1.5.6)$$

So summing over all internal flavours k will eliminate C_0 since

$$\sum_k V^*_{kj} V_{ki} = \sum_k V^+_{jk} V_{ki} = (V^+ V)_{ji} = \delta_{ij} = 0 \text{ (because } i \neq j \text{).} \quad (1.5.7)$$

We will encounter this mechanism when calculating the contribution of diagrams E and H in the Feynman t' Hooft gauge and diagram E in the unitary gauge, see figure (1).

The most important feature of this mechanism is the elimination of divergences. Indeed we will encounter divergences in $1/\varepsilon$ ($\varepsilon \rightarrow 0$) in the dimensional regularisation procedure which are independent of the mass of the internal quark, and hence cancelled. This suppression is responsible for the rarity of the $s \rightarrow dH$ decay mode.

Another feature of the GIM mechanism is that the amplitude will depend on the quark mass squared differences. In other words the heaviest quark will be the one which contributes most.

Indeed from (1.5.5) we can write the amplitude as

$$\Gamma = \sum_k V^*_{kj} V_{ki} \, f(\text{masses})_k . \qquad (1.5.8)$$

$f(\text{masses})_k$ is some function of the masses which include the mass m_k .

If we suppose all the masses m_k for every k were equal, then f $(\text{masses})_k$ will be independent of k, so we may remove the subscript k and write

$$\Gamma = f(\text{masses}) \sum_k V^*_{kj} V_{ki} = f(\text{masses}) \delta_{ij} . \qquad (1.5.9)$$

For off-diagonal transitions Γ will vanish.Thus the amplitude will only be non-vanishing if there are non-zero quark mass differences. Naively the amplitude of the transition will depend on $\Delta(m_k^2)$.

The decay amplitude would be proportional to ($m_k/M_W)^2$ where k now is the flavour of the heaviest quark (top quark in the case of K and B decays). Thus the rate for this decay would be suppressed by the GIM mechanism unless m_k is comparable to m_W or exceeds it.

1.6 The Higgs mass

As the mass term in the Lagrangian violates the gauge symmetry, the electroweak theory begins as a theory with massless particles. To give masses to particles, a spontaneous symmetry breaking by the ground state is introduced. This mechanism is known as the Higgs mechanism.

Although this mechanism gives masses to all particles, in particular to the masses of the W and Z bosons which turn out to be in good agreement with the experimental data, such a mechanism does not fix the mass of the Higgs boson.

In the minimal (one Higgs doublet) standard model, the Higgs mass is given at tree level by

$$m_H = \sqrt{2\lambda}\, v , \tag{1.6.1}$$

where $v = (\sqrt{2} G_F)^{-1/2} = 246$ Gev (1.6.2)

is the vacuum expectation value of the Higgs field, and λ is the quartic self interacting coupling constant which is an arbitrary parameter. G_F is the Fermi constant.

There are a number of theoretical constraints on the mass of the Higgs:

1- Unitarity.

From the requirement that the lowest order contributions to amplitudes such as HH→ HH, HH→ WW etc are small enough to satisfy partial wave unitarity constraints we have

$$M_H < 0 (1 \text{ Tev}) . \tag{1.6.3}$$

For the deduction of this condition, see I. Hinchliffe [42].

2- Linde-Weinberg bound [43-44]:

A lower bound on the Higgs mass $\sim$ 7 Gev based on arguments on the stability of the vacuum could be obtained. This result depends on neglecting the fermion masses compared to the Z and W boson masses in the framework of the standard model with 3 generations.

If it turns out that the t-quark is heavy, which is quite likely (indeed a lower limit of 60 Gev is reported from the UA1 data, see Cline [45]) or a heavy fourth generation quark exists, this lower limit is no more valid.

If the mass of the top is around 80 Gev, the Higgs mass is not constrained at all by this limit and could be arbitrarily small.

To determine the mass of the Higgs we have to investigate the complete spectrum from zero mass to 1 Tev when the standard model ceases to be a perturbative theory.

For more details for a light Higgs < 5 Gev, see [46-49], for a Higgs mass up to 1 Tev, see [47].

Chapter 2

2.1 The study of the process $d_i \rightarrow d_j + H$ where d_i is light

Calculation of the coupling in the Feynman gauge.

In this chapter we recalculate the flavour changing $d_i \rightarrow d_j + H$ in the Feynman gauge. This calculation has been already done [24-26]. The reason of this is that the previous evaluation of the coupling contradicts the one due to [31]. Our purpose was to investigate the problem. However when this work was undertaken a preprint due to [31] appeared (April 1988) which confirmed the result of [24-26].

In this process five masses are involved in this process: the masses of the ingoing and outgoing quarks i and j, the mass of the internal quark k, the mass of the W boson and finally the mass of the Higgs particle.

Keeping all five masses non zero leads to an algebraic explosion. However in certain cases, for example $s \rightarrow d+H$ and $b \rightarrow s+H$, it is possible to neglect the mass of s, b, d and H compared to the mass of the W boson and the top quark see paragraphs 3.1 and 3.2, however we do not make any approximation of the mass of the internal quark vis-a-vis the mass of the W boson.

2.1.1Calculation scheme

The calculation proceeds as follows :

1) Using Feynman rules for propagators and vertices quoted in [appendix A] to find an analytical expression for the contribution of each diagram.

2) Integrals over the internal momentum are expressed in n dimensions [appendix B].

3) Using Dirac algebra in n dimension to simplify the expressions .

4) Applying Feynman parametrisation [appendix C].

5) Shifting the internal momentum to obtain formulae given in [appendix E] .

6)Using Dirac equation both to the left and to the right with considering quarks on-shell.

7)Applying GIM mechanism for terms in the individual contribution which does not depend on the mass of the internal quark.

8)Making the limit $n \to 4$.

The contribution of each diagram will be given in the following form

$$M_d = \bar{u}_j \Gamma_d R u_i \qquad (2.1.0)$$

where Γ_d denotes the flavour changing coupling for a given diagram d, $R = (1+\gamma_5)/2$ and u_j and u_i are the external quarks fields.

2.1.2 Flavour-changing quark self-energy.

Diagram A:

The unrenormalised self-energy and its renormalisation are represented respectively by (A1)+(A2) and (A3) see figure (1). The unrenormalised flavour-changing self energy receives contributions from loops with charged unphysical Goldstone bosons Φ^+, Φ^- (A1) and from loops with charged gauge bosons W (A2).

We denote the internal quark mass by m_k, the masses of the external quarks by m_i and m_j, using Feynman rules shown in [Appendix A], one obtains the contribution of diagram (A1) given by

$$M_{A1} = 4 \left(\frac{-ig}{2m_W}\right)\left(\frac{-ig}{2\sqrt{2}\, m_W}\right)^2 (-m_k m_i) V_{ki} V^*_{kj}$$

$$x\, \bar{u}_j(p)\, R\, iS_F(p-k)\, [m_i R - m_k L]\, iS_F(p)\, u_i(p-q)\, iD_F(k). \qquad (2.1.1)$$

The notation is given in [appendix A].

Both the summation over the internal flavour k and the integration over the internal momentum in n-dimensions (dimensional regularisation) are assumed implicitly in all our calculation.

By replacing the propagators by their expressions, using the properties of γ^μ matrices and taking the external quarks on shell with m_j=0, we obtain

$$M_{A1} = \alpha_{A1} \bar{u}_j (p) R I_{A1} (\not{p} + m_i) u_i (p-q) \qquad (2.1.2)$$

where

$$\alpha_{A1} = 4im_k^2 (\frac{-ig}{2m_W})(\frac{-ig}{2\sqrt{2}\, m_W})^2, \qquad (2.1.3)$$

and

$$I_{A1} = \int \frac{d^n k}{(2\pi)^n} \frac{m_i - (\not{p} - \not{k})}{[k^2 - m_W^2][(p-k)^2 - m_k^2]}. \qquad (2.1.4)$$

Using Feynman parametrisation, see [Appendix C], with $A=k^2-m_W{}^2$ and $B=(p-k)^2-m_k{}^2$, and neglecting the mass of the incoming quark compared to the mass of the W boson and the internal quark, we obtain

$$I_{A1} = \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \frac{m_i - (\not{p} - \not{k})}{[(k-l)^2 - m_W^2]^2} . \qquad (2.1.5)$$

Where

$$l = p(1-x), \qquad (2.1.6)$$

and

$$R^2 = m_w^2 [(1 - x_k) x + x_k], \qquad (2.1.7)$$

with

$$x_k = \frac{m_k^2}{m_W^2} . \qquad (2.1.8)$$

Next we shift k by l

$$k \rightarrow \tilde{k} = k\text{-}l. \qquad (2.1.9)$$

I_{A1} becomes (after dropping integrals with odd power in k)

$$I_{A1} = \int_0^1 dx \int \frac{d^n\tilde{k}}{(2\pi)^n} \frac{m_i - (\not{p} - \not{l})}{[\tilde{k}^2 - R^2]^2}. \qquad (2.1.10)$$

Using the formula (D10), we obtain

$$I_{A1} = \frac{i}{16\pi^2} \int_0^1 dx \left(\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{R^2}{\mu^2} \right) [m_i - (\not{p} - \not{l})] \qquad (2.1.11)$$

and using the Dirac equation to the right, we obtain

$$\Gamma_{A1} = -\sum_k C_{ij}^k \left[\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{m_W^2}{\mu^2} + 1 + \frac{x_k \log x_k}{1 - x_k} \right], \qquad (2.1.12)$$

where

$$C_{ij}^k = \frac{-i}{16\pi^2} \sqrt{2}\, m_i m_W g G_F x_k V_{kj}^* V_{ki}. \qquad (2.1.13)$$

Following the same steps as before, we find that

$$M_{A2} = \alpha_{A2}\, \bar{u}_j(p)\, R\, I_{A2} (\not{p} + m_i)\, u_i(p\text{-}q), \qquad (2.1.14)$$

with

$$\alpha_{A2} = \frac{-4i}{m_i} \left(\frac{-ig}{2\sqrt{2}} \right)^2 \left(\frac{-ig}{2m_W} \right) V_{kj}^* V_{ki}, \qquad (2.1.15)$$

and

$$I_{A2} = (2\text{-}n) \int \frac{d^n k}{(2\pi)^n} \frac{(\not{p} - \not{k})}{[(p-k)^2 - m_k^2][k^2 - m_W^2]}. \qquad (2.1.16)$$

It has the same Feynman parametrisation as I_{A1}. However, after making the shift in k, I_{A2} turns out to be proportional to p which acting to the left, gives m_j which has been taken equal to zero by hypothesis.

Hence

$$\Gamma_{A2} = 0. \qquad (2.1.17)$$

The same conclusion has been obtained by Chia [51].

The unrenormalised contribution of diagram A is equal to the sum of Γ_{A1} and Γ_{A2}:

$$\Gamma_A = - \sum_k C^k_{ij} \left[\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{m_W^2}{\mu^2} + 1 + \frac{x_k \log x_k}{1 - x_k} \right]. \qquad (2.1.18)$$

A renormalisation is called for as a divergence appears in this self energy contribution. The couterterm is given by (A3).

All diagrams which we are going to calculate are unrenormalised. The study of the couterterms is relagated to a later discussion.

Diagram B.

Whereas diagram A is the contribution of the self-energy of the quark of mass m_i, diagram B is the contribution of the self-energy of quark of mass m_j. However due to the fact that we have considered this mass as vanishing, diagram B has no contribution because the vertex quark-Higgs-quark Yukawa coupling then vanishes for the quark j.

$$\Gamma_B = 0. \qquad (2.1.19)$$

However, if one had not made this assumption, we would obtain the same expression as diagram A with slight modification; the masses of the quarks have to be interchanged and the left-handed projection is replaced by the right-handed one. A divergence will appear and (B3) is the term which renormalise the amplitude for the diagram B.

2.1.3 Flavour-changing vertex.

All the contributions to the unrenormalised off-diagonal proper vertex and the counterterm needed to renormalise it are shown in fig (1) .

The Z boson and the photon, which we have already seen can not induce a flavour transition, are absent from these diagrams.

Diagram C

Following the same steps as before, we find that

$$M_C = \alpha_C \bar{u}_j\,(p+q)\,R\,I_C\,u_i(p), \tag{2.1.20}$$

where

$$\alpha_C = g^3 \frac{m_k^2}{m_W} V_{kj}^* V_{ki}, \tag{2.1.21}$$

and

$$I_C = \int \frac{d^n k}{(2\pi)^n} \frac{\not{q}+2\not{k}}{[(q+k)^2-m_k^2][k^2-m_k^2][(p+k)^2-m_W^2]}. \tag{2.1.22}$$

Using Feynman parametrisation and neglecting the momentum squared of the Higgs particle, we have

$$I_C = 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^n \tilde{k}}{(2\pi)^n} \frac{\not{q}+2\not{l}}{(\tilde{k}^2 - R^2)^3}, \tag{2.1.23}$$

where

$$l = p(1-x)-(p+q)y, \tag{2.1.24}$$

and

$$R^2 = m_W^2[(x_k-1)(x+y)+1]. \tag{2.1.25}$$

We have

$$\not{q}+2\not{l} = (1-2y)(\not{q}+\not{p})+(1-2x)\not{p}. \tag{2.1.26}$$

and using Dirac's equation both to the left and to the right

$$\bar{u}_j(p+q)\,(\not{p}+\not{q}) = 0,\quad \not{p}u_i(p) = m_i u_i(p), \tag{2.1.27}$$

we obtain for the coupling $\Gamma_C = \alpha_C I_C$

$$\Gamma_C = 2\sum_k C_{ij}^k \int_0^1 dx \int_0^{1-x} dy \frac{(1-2x)}{(x_k-1)(x+y)+1}. \qquad (2.1.28)$$

Performing both intgrations we obtain

$$\Gamma_C = -\sum_k C_{ij}^k [\frac{-x_k}{(1-x_k)^3} \log x_k + \frac{1}{2(1-x_k)} - \frac{1}{(1-x_k)^2}]. \qquad (2.1.29)$$

__Diagram D__

The amplitude for this diagram is given by

$$M_D = \alpha_D \bar{u}_j(p+q)\, R\, I_D u_i(p) \qquad (2.1.30)$$

where

$$\alpha_D = \frac{1}{4} g^3 (\frac{m_k^2}{m_W^2}) V_{kj}^* V_{ki}, \qquad (2.1.31)$$

and

$$I_D = \int \frac{d^n k}{(2\pi)^n} \frac{m_i[(\not{q}+\not{k})\not{k}+m_k^2]-m_k^2(\not{q}+2\not{k})}{[(q+k)^2-m_k^2][k^2-m_k^2][(p-k)^2-m_W^2]}. \qquad (2.1.32)$$

Following the same procedure as before we find that the vertex $\Gamma_D = \alpha_D I_D$ is equal to

$$\Gamma_D = 2\sum_k C_{ij}^k \int_0^1 dx \int_0^{1-x} dy \{ \frac{2}{\varepsilon} - \eta - \frac{1}{2} + \log 4\pi - \log\frac{m_W^2}{\mu^2} - \log((x_k-1)(x+y)+1)$$

$$- \frac{x x_k}{(x_k-1)(x+y)+1} \}. \qquad (2.1.33)$$

Terms in k^2 in the numerator of I_D give rise to divergences see appendix (D).

Performing both integrations gives

$$\Gamma_D = -\sum_k C_{ij}^k \{ -\frac{2}{\varepsilon} + \eta - \log 4\pi + \frac{1}{2} + \log \frac{m_W^2}{\mu^2} - \frac{x_k}{(1-x_k)^3} \log x_k - \frac{2x_k}{(1-x_k)^2} \log x_k$$

$$+ \frac{x_k^2}{(1-x_k)^2} \log x_k + \frac{x_k}{2(1-x_k)^2} - \frac{3}{2(1-x_k)^2} \}. \qquad (2.1.34)$$

A divergence appears in this amplitude, it has the same form as the one from diagram A but with an opposite sign.

Diagram E.

The amplitude for this diagram is

$$M_E = \alpha_E \bar{u}_j(p+q) R I_E u_i(p) \qquad (2.1.35)$$

where

$$\alpha_E = g^3 m_W V_{kj}^* V_{ki}, \qquad (2.1.36)$$

and

$$I_E = \int \frac{d^n k}{(2\pi)^n} \frac{\not{p} - \not{k}}{[(p-k)^2 - m_k^2][k^2 - m_W^2][(k+q)^2 - m_W^2]}. \qquad (2.1.37)$$

Due to fact that in this case we have to apply GIM mechanism since there is no x_k dependance in α_E, we will give more details about the procedure. Terms which do not depend on x_k are suppressed

After making the Feynman parametrisation and performing a shift on k, we obtain

$$I_E = 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^n \ell}{(2\pi)^n} \frac{\not{p} - \not{l}}{(\ell^2 - R^2)^3}, \qquad (2.1.38)$$

where l is defined in diagram C , and R^2 is given by

$$R^2 = m_W^2[(1-x_k)(x+y) + x_k]. \qquad (2.1.39)$$

Then we have

$$I_E = 2 \int_0^1 dx \int_0^{1-x} dy \left(\frac{-i}{16\pi^2}\right) \frac{(\not{p}-\not{l})}{2R^2}. \qquad (2.1.40)$$

After replacing l by its value in terms of x and y, we obtain

$$\Gamma_E = 4 \sum_k \frac{C_{ij}^k}{x_k} \int_0^1 dx \int_0^{1-x} dy \frac{x}{(1-x_k)(x+y)+x_k} \qquad (2.1.41)$$

where

$$\frac{C_{ij}^k}{x_k} = - \frac{i}{16\pi^2} \sqrt{2}\, g G_F m_i m_W V_{ki}^* V_{kj}. \qquad (2.1.42)$$

In general we obtain the following form for the double integrals:

$$h(x_k) = \frac{x_k f(x_k) \log x_k + g(x_k)}{(1-x_k)^m} \qquad (2.1.43)$$

Where $g(x_k)$ is a polynomial in x_k. The GIM mechanism is applied in the following way:

Let for simplicity 1 be the last term in $g(x_k)$ and let us calculate

$$\frac{1}{x_k(1-x_k)^m} = \frac{1}{x_k} + \frac{1}{1-x_k} + \dots\dots\dots + \frac{1}{(1-x_k)^m}. \qquad (2.1.44)$$

Where x_k in the denominator of the left-hand side of eq (2.1.44) comes from the coefficient of proportionality in eq (2.1.41). Since C_{ij}^k contains the term $x_k V_{ki}^* V_{kj}$, where V is the K- M matrix which is unitary , the first term in the right-hand side of eq (2.1.44) will be dropped. Applying the GIM mechanism in here and later in diagram H we obtain

$$\Gamma_E = - \sum_k C_{ij}^k \left\{ \frac{2x_k}{(1-x_k)^3} \log x_k + \frac{x_k}{(1-x_k)^2} + \frac{1}{(1-x_k)^2} \right\}. \qquad (2.1.45)$$

<u>**Diagram F**</u>

The contribution of diagram F is a next to leading order due to the factor $(m_H/m_W)^2$ from the Higgs - Goldstone bosons vertex. It will be considered in chapter 3, but here we take

$$\Gamma_F = 0. \tag{2.1.46}$$

Diagram G

The amplitude is

$$M_G = \alpha_G \bar{u}_j (p+q) R I_G u_i (p), \tag{2.1.47}$$

with

$$\alpha_G = -\frac{1}{4} g^3 \frac{m_k^2}{m_W} V_{kj}^* V_{ki}, \tag{2.1.48}$$

and

$$I_G = \int \frac{d^n k}{(2\pi)^n} \frac{2\not{q}+\not{k}}{[(p-k)^2 - m_k^2][k^2 - m_W^2][(k+q)^2 - m_W^2]}. \tag{2.1.49}$$

Following the same procedure as before we find

$$\Gamma_G = \sum_k C_{ij}^k \int_0^1 dx \int_0^{1-x} dy \frac{1+x}{(1-x_k)(x+y)+x_k} \tag{2.1.50}$$

which gives

$$\Gamma_G = -\sum_k C_{ij}^k \{ \frac{3x_k^2}{2(1-x_k)^3} \log x_k - \frac{x_k}{(1-x_k)^3} \log x_k + \frac{7x_k}{4(1-x_k)^2} - \frac{5}{4(1-x_k)^2} \}. \tag{2.1.51}$$

Diagram H

This diagram has the amplitude

$$M_H = \alpha_H \bar{u}_j (p+q) R I_H u_i (p), \tag{2.1.52}$$

with

$$\alpha_H = -\frac{1}{4}g^3\frac{1}{m_W}V^*_{kj}V_{ki}, \qquad (2.1.53)$$

and

$$I_H = \int\frac{d^nk}{(2\pi)^n}\frac{(\not{q}-\not{k})[m_i(\not{p}-\not{k})-m_k^2]}{[(p-k)^2-m_k^2][k^2-m_W^2][(k+q)^2-m_W^2]}. \qquad (2.1.54)$$

Here too we apply GIM exactly as in diagram E. A divergence occurs due to the presence of terms in k^2 in the numerator of I_H. However it is suppressed due to the GIM mechanism and we obtain

$$\Gamma_H = -2\sum_k\frac{C^k_{ij}}{x_k}\int_0^1 dx\int_0^{1-x}dy\left(\log((1-x_k)(x+y)+x_k)+\frac{(2-x)x_k}{2((1-x_k)(x+y)+x_k)}\right). \qquad (2.1.55)$$

Applying the GIM mechanism, we find

$$\Gamma_H = \sum_k C^k_{ij}\left\{\frac{-x_k^2}{2(1-x_k)^3}\log x_k - \frac{3x_k}{(1-x_k)^2}\log x_k + \frac{9x_k}{4(1-x_k)^2} - \frac{11}{4(1-x_k)^2}\right\}. \qquad (2.1.56)$$

We have now calculated all the contributions to the unrenormalised flavour changing vertex. The flavour changing vertex is renormalised by diagram (C3), which we discuss in the next section.

In fact the divergences cancel when the unrenormalised contributions from all the diagrams are added. The mass scale μ^2 is also cancelled.

The sum of all the unrenormalised contributions (i.e. diagrams A1, A2, B1, B2, C, D, E, F, G and H) gives

$$\Gamma = \sum_i \Gamma_i = \sum_k C^k_{ij}\left(-\frac{3}{2}\right). \qquad (2.1.57)$$

One point noticed here is the simple expression of the flavour changing coupling though the individual contribution is not as simple. Indeed we notice a cancellation between diagrams, for example there is an exact cancelation between diagram C and E.

2.2 Counterterms contribution in the coupling.

In the previous section we obtained a finite expression for the unrenormalised flavour changing vertex Γ. We now show that the combined contribution of the counterterms from diagrams A3, B3 and C3 is zero, thus leaving Γ unaltered as implicitly assumed by Willey and Yu [24,25] and Grzadkowski and Krawcyzk [26]. Our discussion follows that of Botella and Lim [28].

The relevant part of the Lagrangian is

$$L=\bar{\Psi}_B(i\partial\!\!\!/-M_B)\Psi_B-\frac{g_B}{2m_{W_B}}\bar{\Psi}_B M_B\Psi_B H. \qquad (2.2.1)$$

The first part of this Lagrangian is the free quarks Lagrangian and the second part is associated with the quark Yukawa interaction with the Higgs boson. Ψ_B and M_B are the column vector of down type quark fields and the mass matrix respectively. B denotes bare quantities.

At the one loop level (order g^2) only field and mass matrix renormalisation are responsible for the flavour changing transition (i.e. we do not include in our calculation the renormalisation of the mass of the W boson or of g the coupling constant of the SU(2) group).

The renormalisation of the left-handed and right-handed fields are different

$$\Psi_{BL}=Z_L\Psi_L \qquad (2.2.2a)$$

and

$$\Psi_{BR}=Z_R\Psi_R. \qquad (2.2.2b)$$

Where Z_L and Z_R are matrices. We have omitted the power 1/2 in the Z' s which has no effect on the following. The renormalisation on the mass is performed as follows, see Sirlin-Marciano [52]

$$M_B = M + \delta M L + \delta M^{+} R. \qquad (2.2.3)$$

The shift in the quark field is given by

$$\Psi_B - \Psi = [(Z_L - 1)L + (Z_R - 1)R]\Psi \qquad (2.2.4)$$

or

$$\delta\Psi = \Psi_B - \Psi = (\delta Z_L L + \delta Z_R R)\Psi. \qquad (2.2.5)$$

Where Z_L (resp Z_R) differs from the unity matrix by an infinitesimal matrix δZ_L (resp δZ_R).

The physical Lagrangian is given by (the same form as the bare one)

$$L = \bar{\Psi}(i\partial\!\!\!/ - M)\Psi - \frac{g}{2m_W}\bar{\Psi}M\Psi H. \qquad (2.2.6)$$

The counterterm Lagrangian is given by:

$$L_{count} = \delta L = L_B - L. \qquad (2.2.7)$$

Using the above renormalisation relations, and after some calculation, we find the expression for the counterterm Lagrangian (using the properties of γ_5 matrix and neglecting terms like $\delta M \delta Z$).

$$L_{count} = \bar{\Psi}\{ i\not\partial (AL+BR) + (CL+DR)\}\, \Psi + \frac{g}{2m_w} \bar{\Psi}(CL+DR)\, \Psi . \qquad (2.2.8)$$

The first term represents the counterterm due to the renormalisation of the quark propagator, the second due to the renormalisation of the proper vertex .

where

$$A = Z_L^+ Z_L - 1 \qquad (2.2.9)$$

$$B = Z_R^+ Z_R - 1, \qquad (2.2.10)$$

$$C = -(\delta M + Z_R^+ M Z_L - M), \qquad (2.2.11)$$

$$D = -(\delta M^+ + Z_L^+ M Z_R - M). \qquad (2.2.12)$$

2.2.1The structure of the counterterms:

At one loop order we have just considered the unrenormalised self energy Σ_{unren}which receives contribution from loops containing W boson (diagrams A1 and B1) and loops containing $\Phi^{+,-}$ Goldstone boson (diagrams A2 and B2). These contributions to diagrams A and B contain divergences. Here we do not neglect any quark masses. The counterterm diagrams (A3) and (B3) cancel the divergences in (A) and (B) and could contain other finite terms as the renormalisation scheme has not been specified.
The Yukawa coupling vertex which receives contribution from the loops shown in diagrams (C, D, E, F, G and H) is also unrenormalised and the counterterm is represented by diagram (C3).
The counterterm which we have to add to renormalise the self energy could be written as

follows,

$$\Sigma_{count} = A\not{p}L + B\not{p}R + CL + DR. \tag{2.2.13}$$

Where the form factors A,B,C and D do not depend on p^2. We work in a basis where the mass matrix is diagonal (the flavour basis). Hence

$$\Psi = (d , s , b)_T. \tag{2.2.14}$$

is a mass eigenstate .

The expressions for the amplitude of diagrams (1=A3) and (2=B3) are respectively (using Feynman rules) :

$$M^{(1)} = \bar{\Psi}(p)\, \Sigma_{count}(p)\, i\Delta(p)\, [M(D)]\, \Psi(p-q) \left(\frac{-ig}{2m_W} \right) \tag{2.2.15}$$

and

$$M^{(2)} = \bar{\Psi}(p+q)[M(D)]\, i\Delta(p)\, \Sigma_{count}(p)\, \Psi(p) \left(\frac{-ig}{2m_W} \right). \tag{2.2.16}$$

Where $i\Delta(p)$ is the quark propagator , and the term $(-ig/2m_W)$ [M(D)] is the contribution of Higgs-quark-quark vertex, see (Appendix A).

The formulae written above might be used to calculate any type of down quark type self energy counterterm transition . In particular for the transition s$\to$d , we have just to pick up the corresponding matrix element from [m(D)] $i\Delta(p)\, \Sigma_{Count}$.

So in the transition s$\to$d, we have to act on $u_d(p+q)$ to the right and on $u_s(p)$ to the left, obtaining the following forms:

$$M^{(1)} = \frac{gm_d}{2m_W} \frac{1}{(p^2 - m_d^2)} \bar{u}_s(p)\, \Sigma_{sd}(p)\, (\not{p}+m_d)\, u_d(p-q) \tag{2.2.17}$$

and

$$M^{(2)} = \frac{gm_s}{2m_W}\frac{1}{(p^2-m_s^2)}\bar{u}_s(p+q)\,(\not{p}+m_s)\Sigma_{sd}(p)\,u_d(p). \qquad (2.2.18)$$

Where $\Sigma_{sd}(p) = \alpha\not{p}L+ \beta\not{p}R+\gamma L+\delta R$, with $\alpha= A_{sd}$, $\beta= B_{sd}$, $\gamma = C_{sd}$ and $\delta= D_{sd}$ appropriate $s\rightarrow d$ matrix elements.

Applying the Dirac equation to the left in the first expression and to the right in the second one, we find after putting all external particles on the mass shell (in the following we omit the quark fields):

$$M^{(1)} = \frac{g}{2m_W(m_s^2-m_d^2)}\{ m_s m_d[m_d(\alpha L + \beta R) +m_s(\alpha R + \beta L)]+$$

$$m_d[m_d(\gamma L + \delta R) + m_s(\gamma R + \delta L)]\}, \qquad (2.2.19)$$

and

$$M^{(2)} = \frac{g}{2m_W(m_d^2-m_s^2)}\{ m_s m_d[m_d(\alpha L + \beta R) +m_s(\alpha R + \beta L)]+$$

$$m_s[m_d(\gamma R+ \delta L) + m_s(\gamma L + \delta R)]\}, \qquad (2.2.20)$$

There is a cancellation of terms in α , β and also of the term ($\gamma R+ \delta L$) when these two contributions are added :

$$M^{(1)} + M^{(2)} = \frac{-g}{2m_W}(\gamma L + \delta R) . \quad (2.2.21)$$

However the renormalisation of the proper vertex could be read off from L_{count}. It is given by

$$Y_c = \frac{g}{2m_W}(CL+DR) . \quad (2.2.22)$$

So if just the transition s→d is considered, we have

$$M^{(3)} = \frac{g}{2m_W}(\gamma L + \delta R). \quad (2.2.23)$$

The sum of these three diagrams (A3, B3 and C3) is equal to zero.

$$M^{(1)} + M^{(2)} + M^{(3)} = 0 . \quad (2.2.24)$$

Hence the counterterms has a vanishing contribution.

We should point out here the arbitrariness of A, B, C and D i.e the renormalisation scheme is not specified, accordingly this sum is renormalisation-scheme independent. This vanishing of counterterms is not peculiar to processes involving the H boson, but it is a special case of a more general theorem due to Feinberg, Kabir and Weinberg.

2.3 Feinberg-Kabir-Weinberg theorem.

The vanishing value of the sum of the counterterms in the process $d_i \to d_j H$ is not a peculiar to the decay of quark to the Higgs boson, but the same conclusion is found in several FCNC decays; for example the contribution of the counterterms in $d_i \to d_j Z$ (where Z is the neutral boson) is zero, see Clements et al.[15] and also there is the same conclusion in $d_i \to d_j \gamma$ (where γ is the photon), see Deshpande et al.[53]. All these results have a common origin. This is the Feinberg-Kabir-Weinberg theorem [54]. Although this work explains why muons do not decay electromagnetically into electrons the same procedure could be followed to explain the non-contribution of the counterterms in the above processes. Such procedure can be briefly stated in two equivalent ways:

1) The operators of dimension three and four such as $\bar{d}_i d_j$, $\bar{d}_i \gamma_5 d_j$, $\bar{d}_i \not{D} d_{jL}$ and $\bar{d}_{iR} \not{D} d_{jR}$, where D_μ is the covariant derivative, do not lead to any change in the flavour of the quark because they can be removed by a redefinition of the quark fields such that the corresponding mass matrix becomes diagonal.

2) If one uses the original basis called also gauge-eigenstate basis and deduces the Feynman rules for the relevant vertices and propagators from the effective Lagrangian, one obtains, when calculating the counterterms, a zero contribution as long as one deals with on-shell states. Indeed a cancellation takes place among several diagrams and such cancellation becomes exact when the particles are on-shell. Such a procedure was followed in the previous section.

2.4 Calculation of the amplitude b→sH using PCDC and the trace anomaly

Here we follow the discussion of Grinstein et al [34] and point out the error in the evaluation of the flavour changing b→sH amplitude by Ellis et al [31].

The starting point is the following observation:

The Higgs particle interacts with quarks via the trace of the improved energy-momentum tensor $\Theta^{\mu}{}_{\mu}$[55]. Indeed we have the following theorem; valid in the absence of the trace anomaly:

Theorem: if A and B are states which do not contain the Higgs boson, the amplitude for the process A→B+ H is given, to the lowest order in g the SU(2) group coupling, by :

$$<B, H \mid A> = \frac{-ig}{2m_w} < B \mid \Theta_{\mu}{}^{\mu} \mid A> . \qquad (2.4.1)$$

The above theorem can be proved easily in the unitary gauge; however as stated by [31], this theorem is still valid whatever the choice of the gauge.

Now let us consider the transition

$$q_{\alpha} \to q_{\beta} + H, \qquad (2.4.2)$$

where q stands for quark and α and β represent respectively the flavour of the ingoing and outgoing quarks.

There is a second theorem due also to Ellis et al [31] which states that the matrix element $<q_{\beta}| \Theta_{\mu}{}^{\mu} | q_{\alpha}>$ can be shown to be equal to [31,56]

$$<q_\beta \mid \Theta_\mu{}^\mu(k) \mid q_\alpha> = \bar{u}_\beta \{ m_\alpha^2 \delta_{\alpha\beta} + k^2[A(m_\alpha R + m_\beta L) + m_\alpha m_\beta A']$$

$$+ (m_\alpha^2 - m_\beta^2)[(m_\alpha R - m_\beta L)B + (m_\alpha - m_\beta) m_\alpha m_\beta C]\} u_\alpha = T(k^2) . \quad (2.4.3)$$

Where A, A', B and C are regular functions of k^2.

If we restrict ourselves to terms linear in quark masses, we obtain the simple form

$$T(k^2) = k^2 \bar{u}_\beta A(m_\alpha R + m_\beta L) u_\alpha . \quad (2.4.4)$$

Let us now consider the quantity

$$Q(k^2) = (k^2 - m_H^2) <q_\beta| H(k) | q_\alpha> . \quad (2.4.5)$$

There is a simple way to compute $Q(k^2)$ by using a similar method as PCAC (partially conserved axial current) which is PCDC (partially conserved dilatation (or scale) current).

The scale current s_μ is defined as

$$s^\mu = x^\nu \Theta^\mu{}_\nu . \quad (2.4.6)$$

Where the improved energy-momentum tensor satisfies the Noether theorem

$$\partial^\mu \Theta_\mu{}^\nu = 0. \quad (2.4.7)$$

Thus we have

$$\partial^\mu s_\mu = \Theta_\mu{}^\mu . \quad (2.4.8)$$

The fact that the Higgs particle interacts with quarks through the trace of the improved energy-momentum tensor and that this trace vanishes in massless theories (which are invariant under scale transformations) suggests that the Higgs particle could be considered (like the pion which is the Goldstone boson associated with the breakdown of the axial symmetry) as a dilaton or the Goldstone boson associated with the breakdown of scale invariance. Now

$$Q(k^2) = (k^2 - m_H^2) < q_\beta | H(k) | q_\alpha > \qquad (2.4.9)$$

and

$$T(k^2) = < q_\beta | \Theta_\mu{}^\mu(k) | q_\alpha >, \qquad (2.4.10)$$

could be related if we follow the same pattern as Jackiw in [57] in his discussion on $\pi \rightarrow 2\gamma$, where the pion field is interpolated by the divergence of the axial current, likewise in PCDC the divergence of the scale current is used as the Higgs interpolating field i.e.

$$H(k) = \frac{\partial^\mu s_\mu}{f m_H^2} . \qquad (2.4.11)$$

where fm_H^2 is introduced to normalise H(k) and

$$f = v = < 0 | H(k) | 0 >. \qquad (2.4.12)$$

Thus

$$Q(k^2) = (k^2 - m_H^2) < q_\beta | H(k) | q_\alpha > \qquad (2.4.13)$$

$$= (k^2 - m_H^2) < q_\beta | \frac{\partial_\mu s^\mu}{v m_H^2} | q_\alpha > \qquad (2.4.14)$$

$$= \frac{k^2 - m_H^2}{v m_H^2} < q_\beta | \Theta_\mu^\mu | q_\alpha > . \qquad (2.4.15)$$

Finally we have

$$Q(k^2) = \frac{k^2 - m_H^2}{v m_H^2} T(k^2). \qquad (2.4.16)$$

At this stage we introduce the hypothesis of PCDC and assume the smoothness of the function $Q(k^2)$, i.e, we assume

$$Q(m_H^2) \approx Q(0) . \qquad (2.4.17)$$

because at this point $k^2 = 0$ the computation of $T(k^2)$ is simpler as the explicit form of $A(k^2)$ in $T(k^2)$ is unknown. From eq (2.4.16)

$$Q(0) = -\frac{1}{v} T(0) . \qquad (2.4.18)$$

However $T(0)=0$ assuming the function $A(k^2)$ has no singularity at $k^2=0$, and we have the result

$$Q(0)=0. \qquad (2.4.19)$$

Ellis et al claimed that this result is unchanged by the trace anomaly and concluded that $Q(m_H^2)$, and thus the process $b \rightarrow sH$, is strongly suppressed.

Indeed [31,58] give an estimated value for the $q_\alpha \rightarrow q_\beta H$ vertex

$$\Gamma_{Hq_\alpha q_\beta} = O\left(\frac{g^3}{16\pi^2} \frac{m_{\alpha,\beta}^3}{m_W^3}\right), \qquad (2.4.20)$$

where $m_{\alpha,\beta}$ are the external quark masses. This is to be compared with the explicit one-loop calculation of eq (2.1.57) which gives

$$\Gamma_{Hq_\alpha q_\beta} = 0\left(\frac{g^3}{16\pi^2}\frac{m_{q_\alpha} m_Q^2}{m_W^3}\right). \tag{2.4.21}$$

where m_Q is the mass of the internal quark . However as noticed by Chanowitz [59] this discrepancy between the two results is due to the presence of the trace anomaly. The claim of Ellis and Franzini [58] that the coupling $\Gamma_{Hq\alpha q\beta}$ is unaffected by its presence is not confirmed by a careful analysis of the trace anomaly, see Grinstein et al [34].

2.4.1 The trace anomaly.

The problem is that the trace of the tress tensor $\Theta^{\mu\nu}$ is in fact anomalous, in such a way that the interpolating field associated with the Higgs boson given by (2.4.11) has to be replaced by

$$H(k^2) = \frac{\partial_\mu s^\mu - \tilde{A}}{f m_H^2} \tag{2.4.22}$$

Where $\tilde{A}$ is the anomaly. Hence $Q(k^2)$ given by (2.4.16) is replaced by

$$Q(k^2) = \frac{k^2 - m_H^2}{v\, m_H^2}[T(k^2) - R(k^2)]\,, \tag{2.4.23}$$

with

$$R(k^2) = \langle q_\beta | \tilde{A} | q_\alpha \rangle \tag{2.4.24}$$

Extrapolating from $k^2 = m_H^2$ to zero and using the fact that $T(k^2)$ vanishes at the origin, we find that Q(0) is no more equal to zero and it is given by we find that Q(0) is no more equal to zero and it is given by

$$Q(0) = \left(\frac{1}{v}\right) R(0). \qquad (2.4.25)$$

Hence the problem of finding the amplitude for the process $q_\alpha \rightarrow q_\beta H$ is reduced to the calculation of the anomaly between these two states α and β.

The origin of this anomaly is due to the presence in the Lagrangian of terms of dimension four which depend on the choice of the mass scale .

Grinstein and Hill in [60] express the anomaly $\tilde{A}$ as a function of the renormalised Lagrangian density which depends on the choice of renormalisation mass scale μ. The anomalous divergence is then

$$\partial_\mu s^\mu = \Theta_\mu{}^\mu = \frac{\mu\,\partial}{\partial\mu} L . \qquad (2.4.26)$$

We pick up from the Lagrangian terms which could induce a flavour transition to the order of our investigation (g^3) and which depend on the mass scale μ; the only candidate is the Yukawa interaction. Indeed the Yukawa coupling matrix λ does depend on the mass scale.

Therefore we have

$$\tilde{A} = \mu \frac{\partial}{\partial\mu} \left(v\bar{\Psi}_L \lambda \Psi_R + \text{h.c} \right). \qquad (2.4.27)$$

As we are dealing with down type quark

$$\lambda = \lambda^D, \qquad (2.4.28)$$

so

$$\tilde{A} = - v \bar{\Psi} [\frac{1}{2}(1+ \gamma_5) \beta^D + \frac{1}{2}(1 - \gamma_5) \beta^{D^+}] \Psi, \qquad (2.4.29)$$

where

$$\beta^D = \mu \frac{\partial}{\partial\mu} \lambda^D. \qquad (2.4.30)$$

The expression for β^D is given by [61]:

$$\beta^D (\lambda^D)^{-1} = \frac{1}{16\pi^2} [8g_3^2 + \frac{9}{4} g_2^2 + \frac{1}{4} g_1^2 - \mathrm{Tr} (3\lambda^{U^+} \lambda^U + 3\lambda^{D^+} \lambda^D)$$

$$- \frac{3}{2} (\lambda^{D^+} \lambda^D - \lambda^{U^+} \lambda^U)]. \qquad (2.4.31)$$

where g_3, g_2, and g_1 are the SU(3) x SU(2) x U(1) gauge coupling constants of the standard model.

Let us go to a basis where the down quark masses are diagonal, the only term in β^D which gives off diagonal terms is

$$\beta^D = \frac{3/2}{16\pi^2} \lambda^{U^+} \lambda^U \lambda^D + \ldots\ldots \qquad (2.4.32)$$

Replacing the Yukawa coupling matrix by the mass matrix

$$\lambda = \frac{m}{v} \qquad (2.4.33)$$

in the previous equation and considering that

$$m_W^2 = \frac{g^2 v^2}{4}. \qquad (2.4.34)$$

we obtain

$$\beta^D = \frac{3}{64\pi^2}\frac{g^2}{vm_W^2} m^{U^+} m^U m^D \qquad (2.4.35)$$

where

$$m^D = \begin{bmatrix} m_d & & \\ & m_s & \\ & & m_b \end{bmatrix} \qquad (2.4.36)$$

is the diagonal mass matrix for the down-type quark and

$$m^U = V^+ \begin{bmatrix} m_u & & \\ & m_c & \\ & & m_t \end{bmatrix} V. \qquad (2.4.37)$$

is the mass matrix for the up-type quark SU(2) doublet partners of the down quark mass eigenstates.

Here V is the Kobayashi-Maskawa matrix,

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \qquad (2.4.38)$$

We can now evaluate eq (2.4.24), taking only the contribution of the transition b→sH, we find that the anomaly for the transition is given by

$$< s | \tilde{A} | b > = \frac{3}{64\pi^2}\frac{g^2}{vm_W^2} m_b m_t^2 V_{tb} V_{st}^+ \left(\frac{1+\gamma_5}{2}\right). \qquad (2.4.39)$$

where we have neglected m_s compared to m_b.

However since Q is related to the anomaly by (2.4.25) we have the flavour changing coupling given by

$$Q(0) = \frac{3}{128\pi^2} V_{tb} V^{+}_{st} g^3 \left(\frac{m_t}{m_w} \right)^2 m_b m_W \left(\frac{1+\gamma_5}{2} \right) . \qquad (2.4.40)$$

This result is now in agreement with the explicit one loop calculation of eq (2.1.57).

2.5 Some remarks on γ_5.

One of the motivations for this investigation is that there exists 3 conflicting results on the value of the coupling $d_i \rightarrow d_j H$. The first one is due to Ellis et. al (1976) which is exposed later, the second due to Wiley-Yu (1982) and independently Grzadkowski-Krawczyk (1983), the latest one due to Ruskov (1987) [33]. If efforts have been focused to explain why the first result is erroneous, there is no existing literature about the third one.

Faced with 3 different results for the same process, one has to be careful when dealing with the weakest points of the calculation.

One of these is the problem of γ_5 when using the dimensional regularisation scheme (see appendix B) for more example about the mistreatment of the γ_5 matrix see Korner [36].

Firstly let us indicate how the definition of γ_5 in n dimension represents a real risk to the evaluation of the coupling $d_i \rightarrow d_j H$.

The quark-W boson-quark vertex contributes in the coupling with a term proportional to $\gamma_\mu(1-\gamma_5)$ (see appendix A). So if we have 2 of these vertices, for example, we have the expression $\gamma_\mu(1-\gamma_5)\,\gamma^\mu(1-\gamma_5)$ which is in a careful analysis by Korner [36] is shown to be equal to (when letting $\varepsilon = n-4 \rightarrow 0$)

$$\gamma_\mu(1-\gamma_5)\gamma^\mu(1-\gamma_5) = -2\varepsilon(1-\gamma_5)\,. \qquad (2.5.1)$$

We can drop this contribution for convergent integrals over k; however we have encountered 2 types of divergent integral whose product with the previous expression leads to the appearance of additional finite terms in the coupling. Hence utmost care has to been taken when dealing with UV divergences and γ_5 interaction.

Let us return to our previous calculation but without assuming an anticommuting γ_5 matrix, and try to find the additional terms.

The supplementary contributions appear in diagrams A (precisely A2), C, and H.

Diagram C.

If we restrict ourselves just to the numerator of the amplitude we have

$$M_C = -\frac{g^3}{16}\left(\frac{m_k}{m_W}\right) V^+V\, \bar{u}_j(p+q)\,\gamma_\mu(1-\gamma_5)\,(\not{p}+\not{q}-\not{k}-m_k)(\not{p}-\not{k}-m_k)\gamma^\mu(1-\gamma_5)u_i(p)\,. \quad (2.5.2)$$

Where the integration over the internal momentum k and the sum over the internal flavour are understood.

From the product

$$(\not{p}+\not{q}-\not{k}-m_k)(\not{p}-\not{k}-m_k) = (\not{p}+\not{q})\not{p}-(\not{p}+\not{q})\not{k}-\not{k}\not{p}+k^2+\{-m_k\,[2\not{p}-2\not{k}+\not{q}]\} \quad (2.5.3)$$

We have taken just the term inside brackets, the remaining part does not contribute for an anticommuting γ_5. However k^2 in the previous expression leads to a divergent integral. Indeed after Feynman parametrisation we obtain an additional amplitude given by

$$\Delta M_C = -\frac{g^3}{16}\left(\frac{m_k}{m_W}\right) V^+V\, \bar{u}_j\,(p+q)\gamma_\mu(1-\gamma_5)\gamma^\mu(1-\gamma_5)\,u_i\,(p)\, I, \quad (2.5.4)$$

where (see appendix D)

$$I = 2\int_0^1 dx\int_0^{1-x} dy\int \frac{d^nk}{(2\pi)^n}\,\frac{k^2}{(k^2-R^2)^3} = \frac{i}{16\pi^2}\left(\frac{2}{\varepsilon} + \text{finite terms}\right). \quad (2.5.5)$$

R^2 is defined in eq (2.1.)

Using eq. (2.5.1) , we obtain

$$\Delta M_C = \frac{ig^3}{64\pi^2}\left(\frac{m_k}{m_W}\right) V^+V\bar{u}_j\,(p+q)(1-\gamma_5)u_i\,(p). \quad (2.5.6)$$

Diagram A.

Using the same procedure we find that

$$\Delta M_A = \Delta M_C\,. \quad (2.5.7)$$

Diagram H.

Similarly we have

$$M_H = \frac{g^3}{16m_W} V^+V \bar{u}_j (p+q) \gamma^\mu(1-\gamma_5) (\not{k}+m_k)(k-p+q)_\mu \{m_i (1+\gamma_5)-m_k (1-\gamma_5)\}$$

$$u_i(p). \qquad (2.5.8)$$

Taking the part which gives a divergent integral, we have

$$\Delta M_H = -\frac{g^3}{16}\frac{m_k}{m_W} V^+V \bar{u}_j (p+q) \gamma^\mu(1-\gamma_5) \not{k} k_\mu (1-\gamma_5) u_i (p). \qquad (2.5.9)$$

Using the property that (see appendix D eq 7)

$$\int \frac{d^nk}{(2\pi)^n} \frac{k^\mu k^\nu}{(k^2 - R^2)^3} = \frac{g^{\mu\nu}}{n} \int \frac{d^nk}{(2\pi)^n} \frac{k^2}{(k^2-R^2)^3} \qquad (2.5.10)$$

and eqs (2.5.1 and 2.5.9) , we have

$$\Delta M_H = -\frac{ig^3}{256\pi^2}\left(\frac{m_k}{m_W}\right) V^+V \bar{u}_j (p+q)(1-\gamma_5) u_i (p) . \qquad (2.5.11)$$

Finally there is an additional amplitude given by

$$\Delta M = \Delta M_A + \Delta M_C + \Delta M_H . \qquad (2.5.12)$$

There are two features which differs from the calculated amplitude first the nature of the helicity and second the coefficient of proportionality.

The cause of the appearance of this additional term ΔM is that we have applied the dimensional regularisation with Feynman rules obtained in four dimensional space-time i.e. using an anticommuting γ_5 matrix.

For consistency we have to apply to the original Lagrangian the method of dimensional regularisation and obtain within this scheme the different Feynman rules.

When this is done, the W-quark vertex is given by

$$\frac{-ig}{4\sqrt{2}}(1+\gamma_5)\gamma_\mu(1-\gamma_5), \qquad (2.5.13)$$

which reduces to the usual one for an anticommuting γ_5 matrix.

Using such a vertex, it is straightforward to show that

$$\Delta M_A = \Delta M_C = 0\,. \qquad (2.5.14)$$

However in diagram H we will have

$$(1+\gamma_5)\gamma_\mu(1-\gamma_5)\gamma^\mu(1-\gamma_5) \qquad (2.5.15)$$

which is equal to (using eq 2.5.1 and the properties of γ_5)

$$-2\varepsilon\,(1+\gamma_5)(1-\gamma_5) = 0. \qquad (2.5.16)$$

Finally

$$\Delta M = 0\,. \qquad (2.5.17)$$

Hence the problem of the definition of γ_5 in n dimensions does not affect the value of the coupling obtained from one loop flavour changing diagrams.

Chapter 3

3.1 Calculation of the next to leading term in the Feynman gauge

In [Grzadkowski, Krawczyk], the coupling is given in the following way:

$$\Gamma = - \sum_k C_{ij}^k \{ f_1(x_k) + \frac{m_H^2}{m_W^2} f_2(x_k) \} . \qquad (3.1.1)$$

Where $f_1(x_k)$ is the contribution of all diagrams except diagram F, the latter gives the contribution $f_2(x_k)$, where the coefficient $(m_H/m_W)^2$ stems from the Goldstone boson - Higgs particle vertex in diagram F.

However this procedure is not gauge invariant because, for example, in the unitary gauge the diagram F will disappear and hence $f_2(x_k)$ will vanish.

In others words the coupling in the unitary gauge will come exclusively from $f_1(x_k)$. In order to obtain the correct gauge invariant expression for $f_2(x_k)$, it is necessary to take the Higgs particle on mass shell, $q^2=m_H^2$, and include contributions of order $(m_H/m_W)^2$ from all diagrams. Botella and Lim point out that the result of Willey and Yu coincides with the one of Grzadkowski and Krawczyk when $m_H = 0$ as is required for a consistent leading order calculation. In this case $f_1(x_k) = 3/2$, as shown in chapter 2. If we consider the four momentum squared of the Higgs particle not to be negligible with respect to m_W^2 and the mass of the internal quark squared, the calculation will be very tedious, since m_i the mass of the incoming quark in the process $q_i \rightarrow q_j H$ will also not be negligible and Spence function will appear. To overcome this problem we make an approximation which turns out to be legitimate for the K decay and B decay.

After making Feynman parametrisation and the shift in the internal momentum we obtain the following expression

$$R^2 = R^2 - xyq^2 \qquad (3.1.2)$$

where q is the Higgs momentum, and

$$R^2 = (x_k - 1)(x+y) + 1 \quad \text{or} \quad R^2 = (1 - x_k)(x+y) + x_k \qquad (3.1.3)$$

depending on the diagram considered. There are also terms in q^2 from diagrams D and H due to the presence of terms in k^2 after making the shift by l.

The basic idea is to assume that xyq^2 is small compared to R^2 in order to make a Taylor expansion in (q^2/m_W^2) .

Let us take

$$f(x,y) = \frac{x_q xy}{(x_k - 1)(x+y) + 1} \qquad (3.1.4)$$

where $x_q = (q/m_W)^2$ which on shell is equal to $x_H = (m_H/m_W)^2$.

We want to find the condition on by x_q and x_k which ensures that, whatever the choice of x, y inside the triangle of integration, we have $f(x,y) \ll 1$.

To do that, let us find the maximum value of $f(x,y)/x_q$.

First we fix x and make y vary from 0 to 1-x, the maximum value of f is $x(1-x)/x_k$. Next x varies from 0 to 1 the maximum value is $1/4x_k$.

Hence the condition to make the approximation, i.e. xyx_q small compared to R^2, is $x_q \ll 4x_k$.

In terms of on-shell masses this condition is $m_H \ll 2m_k$. It is noticed that this constraint is a special case of processes below threshold.

Note that this condition is verified in the case of K and B decays because the Higgs particle must be light in order for the decay to be physical (less than the mass of the ingoing quark since we have neglected the mass of the outcoming one). So as far as we have $m_k = m_{top}$, which is expected to be bigger than 60 Gev, this condition is fulfilled.

We denote all contributions proportional to x_q or x_H by Γ^{Higgs} and calculate them following the same pattern as before.

Diagram A, B .

$$\Gamma^{Higgs}_{A,B} = 0 . \qquad (3.1.5)$$

Diagram C.

$$\Gamma^{Higgs}_{C} = 2 \sum_k C^k_{ij} x_q \int_0^1 dx \int_0^{1-x} dy \frac{(1-2x)xy}{[(x_k-1)(x+y)+1]^2} \qquad (3.1.6)$$

Hence

$$\Gamma^{Higgs}_{C} = 2 \sum_k C^k_{ij} x_q \{ \frac{18x_k \log x_k + 6\log x_k + x_k^3 - 9x_k^2 - 9x_k + 17}{36(x_k-1)^5} \}. \qquad (3.1.7)$$

Diagram D.

$$\Gamma^{Higgs}_{D} = 2 \sum_k C^k_{ij} x_q \int_0^1 dx \int_0^{1-x} dy \{ \frac{3xy}{2[(x_k-1)(x+y)+1]} - \frac{x^2 y x_k}{[(x_k-1)(x+y)+1]^2} \} \qquad (3.1.8)$$

Performing the integration, we obtain

$$\Gamma^{Higgs}_{D} = \sum_k C^k_{ij} x_q \{ \frac{6x_k \log x_k + 18\log x_k + 4x_k^4 - 21x_k^3 + 45x_k^2 - 67x_k + 39}{36(x_k-1)^5} \}. \qquad (3.1.9)$$

Diagram E.

$$\Gamma^{Higgs}_{E} = 4 \sum_k C^k_{ij} (\frac{x_q}{x_k}) \int_0^1 dx \int_0^{1-x} dy \frac{x^2 y}{[(1-x_k)(x+y) + x_k]^2}, \qquad (3.1.10)$$

After applying GIM mechanism we obtain (using the procedure described when discussing the diagram E in chapter 2)

$$\Gamma^{Higgs}_{E} = \sum_k C^k_{ij} x_q \{ \frac{-12x_k^2 \log x_k - x_k^4 + 8x_k^3 - 8x_k + 1}{9(x_k-1)^5} \}. \qquad (3.1.11)$$

Diagram F .

In this diagram we calculate the contribution as we have done for the other diagrams in chapter 2 when calculating the leading term in the Feynman gauge.

$$\Gamma_F^{Higgs} = -\sum_k C_{ij}^k x_H \int_0^1 dx \int_0^{1-x} dy \frac{(1-x)}{(1-x_k)(x+y)+x_k} \tag{3.1.12}$$

$$\Gamma_F^{Higgs} = \sum_k C_{ij}^k x_H \{ \frac{-2x_k^2 \log x_k + 4x_k \log x_k + x_k^2 - 4x_k + 3}{4(x_k-1)^3} \}. \tag{3.1.13}$$

Diagram G.

$$\Gamma_G^{Higgs} = \sum_k C_{ij}^k x_q \int_0^1 dx \int_0^{1-x} dy \frac{xy(1+x)}{[(1-x_k)(x+y)+x_k]^2} \tag{3.1.14}$$

$$\Gamma_G^{Higgs} = \sum_k C_{ij}^k x_q \{ \frac{-30x_k^3 \log x_k + 18x_k^2 \log x_k + 9x_k^4 + 13x_k^3 - 45x_k^2 + 27x_k - 4}{36(x_k-1)^5} \}. \tag{3.1.15}$$

Diagram H.

$$\Gamma_H^{Higgs} = \sum_k C_{ij}^k (\frac{x_q}{x_k}) \int_0^1 dx \int_0^{1-x} dy \{ \frac{y(3x-2)}{(1-x_k)(x+y)+x_k} + \frac{x_k xy(x-2)}{[(1-x_k)(x+y)+x_k]^2} \} \tag{3.1.16}$$

using GIM mechanism we obtain

$$\Gamma_H^{Higgs} = \sum_k C_{ij}^k x_q \{ \frac{6x_k^3 \log x_k + 18x_k^2 \log x_k - 36x_k \log x_k + 3x_k^4 - 35x_k^3 + 63x_k^2 - 21x_k - 10}{36(x_k-1)^5} \} \tag{3.1.17}$$

The sum of all diagrams.

$$\Gamma^{Higgs} = \sum_i \Gamma_i^{Higgs} = \sum_k C_{ij}^k \{ [(\frac{-2x_k^3 - x_k^2 - x_k + 2}{3(x_k-1)^5}) x_q + (\frac{-2x_k^2}{4(x_k-1)^3}) x_H] \log x + [(\frac{6x_k^3 + x_k^2 + 28x_k - 23}{18(x_k-1)^4}) x_q + (\frac{x_k - 3}{4(x_k-1)^2}) x_H] \} . \tag{3.1.18}$$

For on shell Higgs particle, we have $x_q = x_H$

and

$$\Gamma^{Higgs} = -\sum_k C_{ij}^k x_H f_2(x_k) \qquad (3.1.19)$$

where

$$f_2(x_k) = -\{(\frac{-3x_k^4+8x_k^3-17x_k^2+7x_k+5}{6(x_k-1)^5})\log x_k + (\frac{21x_k^3-43x_k^2+119x_k-73}{36(x_k-1)^4})\} \qquad (3.1.20)$$

3.2 Term quadratic in the mass of the ingoing quark.

Once $f_2(x_k)$ is calculated it is necessary to find also the amplitude quadratic in the mass of the ingoing quark because from the kinematics of the processes the mass of the ingoing quark is bigger than the mass of the Higgs particle.

We will apply the same type of approximation as in the previous one, we will notice that the constraint is no more than that the incoming quark mass must lie below the normal threshold for W boson plus top quark production.

Now we have after making the Feynman parametrisation

$$R'^2 = R^2 + x_i (x^2 + xy - x) , \qquad (3.2.1)$$

where R^2 is defined in eq (3.1.3) and $x_i = (m_i/m_W)^2$. Following the same pattern as before, see paragraph (3.1), to find the condition to make a Taylor expansion in x_i, we have

$$f(x, y) = \frac{-x_i(x^2+xy-x)}{(x_k-1)(x+y)+1} , \qquad (3.2.2)$$

We now consider the maximum value of $f(x,y)/x_i$. For a fixed x the maximum value of f(x,y) is

$$f_{max \backslash x \text{ fixed}} = \frac{-(x^2-x)}{(x_k-1)x+1} , \qquad (3.2.3)$$

next we let x varying in the domain [0 . 1], the maximum value obtained is

$$f_{max} = \left(\frac{1}{1+\sqrt{x}}\right)^2 \text{ for } x = \frac{1}{1+\sqrt{x}} . \qquad (3.2.4)$$

the condition of making the approximation is

$$x_i \ll (1 + \sqrt{x_k})^2 , \qquad (3.2.5)$$

which in terms of masses could be written as

$$m_i \ll m_k + m_W. \qquad (3.2.6)$$

In the Feynman gauge, these contributions are given by:

$$\Gamma_A^{d_i} = \sum_k C_{ij}^k x_i \{ \frac{x_k \log x_k}{(x_k-1)^3} - \frac{x_k+1}{2(x_k-1)^2} \} \qquad (3.2.7)$$

$$\Gamma_C^{d_i} = - \sum_k C_{ij}^k x_i \{ \frac{(36x_k^2+54x_k+6)\log x_k - 5x_k^3 - 63x_k^2 + 45x_k + 23}{18(x_k-1)^5} \} \qquad (3.2.8)$$

$$\Gamma_D^{d_i} = \sum_k C_{ij}^k x_i \{ \frac{(-18x_k^2-78x_k)\log x_k + 5x_k^4 - 27x_k^3 + 117x_k^2 - 77x_k - 18}{36(x_k-1)^5} \} \qquad (3.2.9)$$

$$\Gamma_E^{d_i} = -2 \sum_k C_{ij}^k x_i \{ \frac{(-6x_k^2-18x_k)\log x_k + x_k^4 - 5x_k^3 + 27x_k^2 - 19x_k - 4}{9(x_k-1)^5} \} \qquad (3.2.10)$$

$$\Gamma_G^{d_i} = \sum_k C_{ij}^k x_i \{ \frac{(30x_k^3+54x_k^2-36x_k)\log x_k - 79x_k^3 + 99x_k^2 - 9x_k - 11}{36(x_k-1)^5} \} \qquad (3.2.11)$$

$$\Gamma_H^{d_i} = - \sum_k C_{ij}^k x_i \{ \frac{(-30x_k^3+162x_k^2-180x_k)\log x_k - 36x_k^4 + 187x_k^3 - 495x_k^2 + 693x_k - 421}{36(x_k-1)^5} \} \qquad (3.2.12)$$

The sum of all contribution gives

$$\Gamma^{d_i} = \sum_i \Gamma_i^{d_i} = - \sum_k C_{ij}^k x_i f_3(x_k) \qquad (3.2.13)$$

where

$$f_3(x_k) = - \{ \frac{16x_k^2-21x_k+2}{6(x_k-1)^4} \log x_k + \frac{-19x_k^2+13x_k+12}{12(x_k-1)^3} \}. \qquad (3.2.14)$$

3.3 Calculation of the coupling in the unitary gauge.

In the unitary gauge there are less diagrams to calculate than in the Feynman gauge as the Goldstone bosons disappear in this gauge, indeed just diagrams C, E survive. At first sight it seems that the calculation will be rather simplified as the number of diagrams is less, however the boson W propagator acquires a new expression which increases the power of integration and the numbers of couplings or form factors. Divergences appear in each form factor and a careful calculation is required.

In the unitary gauge the W boson propagator is given by :

$$i D_F^{\mu\nu}(k) = \frac{-ig^{\mu\nu}}{k^2 - m_W^2} + \frac{ik^\mu k^\nu}{m_W^2(k^2-m_W^2)} . \tag{3.3.1}$$

The first term of the right-hand side is just the W propagator in the Feynman gauge, hence as noticed before in the unitary gauge we will expect more form factors.

In this gauge the contribution of diagrams A, B, D, G and H vanish (due to the fact that the unphysical Goldstone boson propagator vanishes in this gauge):

$$\Gamma_A = \Gamma_B = \Gamma_D = \Gamma_F = \Gamma_H = \Gamma_G = 0 . \tag{3.3.2}$$

We follow the same pattern as in the Feynman gauge, there is no change in the Feynman parametrisation and we obtain to leading order (where the superscripts ug and fg denote respectively the Feynman and the unitary gauges)

Diagram C

$$M_C^{ug} = M_C^{fg} + \alpha_C \bar{u}_j(p+q) R\, I_C u_i(p). \tag{3.3.3}$$

where

$$I_C = \int \frac{d^nk}{(2\pi)^n} \frac{\not k(2\not p+\not q-2\not k)\not k}{[(p+q-k)^2-m_k^2][(p-k)^2-m_k^2][k^2-m_W^2]} \tag{3.3.4}$$

Using the same parametrisation as before, shifting k by l obtained from it and using (appendix E eq 2) after dropping the integrand in odd power in k, and finally using Dirac equations to the left on $u_j(p+q)$ and to the right on $u_i(p)$, we obtain

$$\Gamma_C^{ug} = \Gamma_C^{fg} + \Gamma'_C \,, \qquad (3.3.5)$$

where

$$\Gamma'_C = \sum_k C_{ij}^k \{ \frac{3}{2}(\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{m_W^2}{\mu^2}) - \frac{1}{4} + \frac{1}{(1-x_k)} + \frac{1}{(1-x_k)^2}$$

$$+ \frac{3x_k^3}{2(1-x_k)^3} \log x_k - \frac{9x_k^2}{2(1-x_k)^3} \log x_k + \frac{4x_k}{(1-x_k)^3} \log x_k \} \qquad (3.3.6)$$

Diagram E.

There are three more form factors arising from the product of the two W propagators, in addition to the one from the Feynman gauge

$$M_E^{ug} = M_E^{fg} + M_E^1 + M_E^2 + M_E^3 \qquad (3.3.7)$$

where

$$M_E^a = \alpha_E^\alpha \bar{u}_j(p+q)\, R\, I_E^a\, u_i(p) \qquad (3.3.8)$$

with a = 1 , 2 , 3 and

$$I_E^1 = \int \frac{d^nk}{(2\pi)^n} \frac{\not{k}(\not{p}+\not{k})\not{k}}{[(p+k)^2-m_k^2][k^2-m_W^2][(q-k)^2-m_W^2]} \qquad (3.3.9)$$

$$I_E^2 = \int \frac{d^nk}{(2\pi)^n} \frac{(\not{q}-\not{k})(\not{p}+\not{k})(\not{q}-\not{k})}{[(p+k)^2-m_k^2][k^2-m_W^2][(q-k)^2-m_W^2]} \qquad (3.3.10)$$

$$I_E^3 = \int \frac{d^nk}{(2\pi)^n} \frac{(\not{q}-\not{k})(\not{p}+\not{k})\not{k}(q-k)k}{[(p+k)^2-m_k^2][k^2-m_W^2][(q-k)^2-m_W^2]} \qquad (3.3.11)$$

After using GIM mechanism, we obtain (in I_E^3 we have to consider the term in k^4 which gives divergences see appendix D)

$$\Gamma_E^1+\Gamma_E^2=\sum_k C_{ij}^k \{ \frac{2x_k^2}{(1-x_k)^3}\log x_k-\frac{4x_k}{(1-x_k)^3}\log x_k-\frac{2}{(1-x_k)^2}-\frac{1}{(1-x_k)}\} \qquad (3.3.12)$$

and

$$\Gamma_E^3=\sum_k C_{ij}^k\{ -\frac{3}{2}(\frac{2}{\varepsilon}-\eta+\log 4\pi-\log\frac{m_W^2}{\mu^2})-\frac{5}{4}-\frac{3x_k^3}{2(1-x_k)^3}\log x_k+$$

$$\frac{5x_k^2}{2(1-x_k)^3}\log x_k+\frac{1}{(1-x_k)^2}\} \qquad (3.3.13)$$

Since $\Gamma_C{}^{fg}+\Gamma_E{}^{fg}=0$, we have

$$\Gamma^{ug}=\sum_k C_{ij}^k(-\frac{3}{2}). \qquad (3.3.14)$$

So the leading order expression is the same both in the Feynman and the unitary gauges.

It remains to show the same thing for the next to leading term in an expansion in $(m_H/m_W)^2$.

3.4 $F_2(x_k)$ in the unitary gauge.

Making the same approximation as in the Feynman gauge, we obtain

$$\Gamma_i^{Higgs}=0 \text{ for } i = A,B,D,F,G,\text{and } H. \qquad (3.4.1)$$

$$\Gamma_C^{Higgs,ug}=\Gamma_C^{Higgs,fg}+\sum_k C_{ij}^k\{\frac{(6x_k+1)\log x_k+4x_k^4-21x_k^3+45x_k^2-67x_k+39}{36(x_k-1)^5}\} \qquad (3.4.2)$$

$$\Gamma_E^{Higgs,ug}=\Gamma_E^{Higgs,fg}+\Gamma_E^1+\Gamma_E^2+\Gamma_E^3 \qquad (3.4.3)$$

with

$$\Gamma_E^1=\sum_k C_{ij}^k x_q\{\frac{(3x_k^3-9x_k)\log x_k-7x_k^3+18x_k^2-9x_k-2}{9(x_k-1)^5}\} \qquad (3.4.4)$$

$$\Gamma_E^2=\sum_k C_{ij}^k x_q\{\frac{-12x_k^3\log x_k+3x_k^4+10x_k^3-18x_k^2+6x_k-1}{18(x_k-1)^5}\} \qquad (3.4.5)$$

and

$$\Gamma_E^3 = \sum_k C_{ij}^k x_q \{ \frac{(-18x_k^4+60x_k^3-54x_k^2+36x_k)\log x_k+15x_k^4-68x_k^3+90x_k^2-60x_k+23}{36(x_k-1)^5} \} \tag{3.4.6}$$

In the unitary gauge, the next to leading term is given by

$$f_2(x_k) = -\{ (\frac{-3x_k^4+8x_k^3-17x_k^2+7x_k+5}{6(x_k-1)^5})\log x_k + (\frac{21x_k^3-43x_k^2+119x_k-73}{36(x_k-1)^4})\} \tag{3.4.7}$$

which shows that on-shell the next to leading term is the same in both gauges (Feynman and the unitary gauges).

3.5 The coupling in a general gauge.

In this case all diagrams contribute in the coupling, the calculation is very tedious although the procedure is the same as before. We give just the contribution of each diagram to leading order.

Diagram A.

$$\Gamma^{gg}_{A} = - \sum_{k} C^{k}_{ij} \{ \frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{m_W^2}{\mu^2} + 1 + \frac{x_k \log x_k}{\xi - x_k} - \frac{\xi \log \xi}{\xi - x_k} \} \qquad (3.5.1)$$

Diagram B.

$$\Gamma^{gg}_{B} = 0 . \qquad (3.5.2)$$

Diagram C.

$$\Gamma^{gg}_{C} = \Gamma^{fg}_{C} + C^{k}_{ij}[x_k \log x_k \{ \frac{-3x_k^2 + 9\xi x_k - 8\xi^2}{2(\xi - x_k)^3} - (\xi = 1) \} + \{ \frac{-7x_k^2 + 26\xi x_k - 31\xi^2}{12(\xi - x_k)^2} - (\xi = 1) \}$$
$$+ \frac{3\xi^3 - \xi^2 x_k}{2(\xi - x_k)^3} \log \xi] \qquad (3.5.3)$$

Diagram D.

$$\Gamma^{gg}_{D} = - \sum_{k} C^{k}_{ij} \{ - \frac{2}{\varepsilon} + \eta - \log 4\pi + \frac{1}{2} + \log \frac{m_W^2}{\mu^2} - x_k (\frac{3\xi^2 - 3\xi x_k + x_k^2}{(\xi - x_k)^3}) \log x_k$$
$$+ \frac{\xi^3}{(\xi - x_k)^3} \log \xi + \frac{\xi x_k}{2(\xi - x_k)^2} \} \qquad (3.5.4)$$

Diagram E.

After using GIM mechanism, which ensures cancellation of the divergences, we obtain

$$\Gamma_E^{gg} = \Gamma_E^{fg} + C_{ij}^k \{ \frac{(\xi-1)x_k}{2(1-x_k)(\xi-x_k)^3} [(\xi+1)(x_k^3-3x_k^2(\xi+1)+9\xi x_k)-8\xi^2] \log x_k$$

$$+ (\frac{\xi^2}{(\xi-x_k)^2} - \frac{\xi}{(\xi-x_k)} - (\xi=1)) + \frac{\xi^2(x_k-3\xi)}{2(\xi-x_k)^3} \log\xi \} \qquad (3.5.5)$$

Diagram F.

This diagram contributes in the next to leading term .

Diagram G.

$$\Gamma_G^{gg} = \sum_k C_{ij}^k \{ \frac{x_k(3x_k^2+\xi x_k(\xi-6)+2\xi^2}{2(1-x_k)(\xi-x_k)^3} \log x_k + \frac{\xi(-3x_k^2-\xi x_k(\xi-7)-3\xi^2)}{2(1-\xi)(\xi-x_k)^3} \log\xi$$

$$- \frac{\xi(\xi+x_k)}{4(\xi-x_k)^2} \} \qquad (3.5.6)$$

Diagram H.

$$\Gamma_H^{gg} = \sum_k C_{ij}^k \{ \frac{x_k}{2(1-x_k)(\xi-x_k)^3} (-x_k^2(3+2\xi)+ \xi x_k(3\xi+8)-6\xi^2) \log x_k$$

$$+ \frac{\xi}{2(\xi-1)(\xi-x_k)^3} (-(1+2\xi)x_k^2+3\xi x_k(1+\xi) -3\xi^2) \log\xi + \frac{\xi(3x_k-5\xi)}{4(\xi-x_k)^2} \} . \qquad (3.5.7)$$

We check the contribution of each diagram by taking two special cases, the Feynman and the unitary gauges.

The amplitude, as expected, is finite, there is a cancellation of divergences between diagram A and diagram D. When all diagrams are added we notice a cancellation of terms in $\log x_k$, $\log\xi$, fraction in $(1-x_k)$ and $(\xi-x_k)$.

The final result does not depend on ξ.

The flavour changing coupling Γ is equal to

$$\Gamma = \sum_k C^k_{ij} \left(\frac{-3}{2} \right). \tag{3.5.8}$$

This agrees with our results in the Feynman and unitary gauges, confirming the general gauge invariance of the amplitude.

The next to leading term, although we have calculated the analytical contributions, is more complicated.

Chapter 4

Study of the process $d_i \rightarrow d_j H$ where d_i is heavy.

In this chapter we consider the weak decay of a hadron which contains a heavy quark. The latter decays into a light quark. Since the energy released by the heavy quark is much bigger that the one which binds the quarks with each other, it is possible to suppose that the heavy quark decays as if it is independent of its surrounding quarks. In other words the spectator approximation becomes more valid than in the case of the light quark seen in chapter 2.

The study of the processes involving the decay of quarks of each generation within the standard model with 3 generations has given small rates.

The B-decay which has been intensively studied these recent years could not be suitable for the detection of the Higgs particle.

Our calculation shows that the FCNC processes are enhanced if heavy quarks are involved and particularly the internal ones. We will investigate the T-decay which contains the top quark which could be at least 10 times the mass of the bottom quark.

It is tempting to investigate the situation where the fourth family is introduced. The latter could provide us with massive quarks, either external or internal, which seems to be the optimum case for rare decays if one, of course, makes some speculations on the K-M matrix in 4 generations.

The idea of introducing a fourth family has no compelling arguments. However there are some speculation that the introduction of an additional family could lead to an explanation of the observed large mixing in B_d- $\bar{B}_d$ family, see Hamzaoui et al [62]. The large mixing observed by ARGUS Calloboration and confirmed later by CLEO

Calloboration is explained in the framework of the standard model with three generations if the top quark mass is bigger than 100 Gev or if certain elements of the K- M matrix are in the neighbourood of their upper limit. If it turns out that it is not the case, the resolution of this dilemma could be provided by assuming the existence of a fourth generation. However its inclusion leads to an enhancement of the CP violating parameter in the K^0-$\bar{K}^0$ mixing. Without entering in details we assume the existence of the fourth family and we will investigate the consequences. For more details about the effect of the fourth generation on the B^0-$\bar{B}^0$ mixing, see P.J. Franzini [63] and on the CP violation, see E. A. Paschos and U. Türke [64].

Contrary to H decay, the Z decay has received a great deal of attention . Three reasons could be recorded for that:

1) Less unkown paramaters are involved: the Higgs mass which is still unknown is replaced by the Z mass which has more or less a stringent value.

2) Rich literature dealing with special cases,

i) neglecting both the internal and external quarks, see Gaillard et al. [3]

ii) neglecting just the external quark masses compared to the mass of the W boson, see Ma - Pramudita [19]

iii) neglecting just the outgoing external quark mass and considering the others, see Clements et al.[15], Ganapathi et al. [16].

These special cases could be used as a test for the exact calculations, see for the exact calculations Hou- Stuart [8,9,12-14] and Busch [11] .

3) Possibility of using the Ward identities for the Z decay, see Hou-Stuart [9], for the neutral decay induced by an emission of a photon, see Deshpande [53].

The main target for the study of the FCNC Z decay is the search for the top quark. The first investigation of FCNC Z decay was made by Gaillard and Lee in 1974 [3]. Duncan

[17] consider the value 10^{-5} of the branching ratio as the benchmark for detecting the FCNC Z decay at LEP which corresponds to a sample of 100 events per year.
Unfortunately, within the standard model, it seems that this decay is not the mechanism which could produce the top quark, see Axelrod [22]. That is why more calculation have been undertaken introducing a fourth generation and/ or another Higgs doublet, see [8] and [11]. Others have been done using others models see Duncan [18] and Gronau [20]. For a review of all these calculations and its implication as far as the discovery of the top quark is concerned see Duncan [17].
In the contrary the investigations of the FCNC decays induced by H boson are less advanced. Indeed all efforts have been focused from 1976 until the beginning of this year on K and B decays within the minimal standard model. The only exception is the calculation of Hall and Wise [32] using 2 Higgs standard model in the case of the B decay.
These two decays, i.e. the K and B decays, provide a probe for the search of a light Higgs which explains why all investigations have been constrained only on these two processes. However according to the actual experimental data it is likely that the lower limit for the Higgs mass is 5 Gev. Hence the decay of more massive quarks yet to be observed could be a good candidate for the search of the Higgs above this limit.
In this chapter we will study the decay of a heavy quark to a Higgs boson and another light quark. The Higgs boson will decay in its turn to the final state with the maximum fermion masses allowed by kinematics.
There is no literature about the process $d_i \rightarrow d_j H$ for heavy d_i expect the work of Eilam et al.[37] and more recently Krawcyzk [38]. Eilam et al do not give any explicit details of their calculation and focus on the ratio of the b' to bH decay compared to the electronic decay mode, Krawcyzk concentrates his efforts on the deviation of the coupling in the

exact calculation from the one obtained if adopting the low-energy approximation. We will present in this chapter both views, i.e. investigate the effect of introducing a heavy quark compared to a light quark and second the consequences on the value of the branching ratio of $d_i \rightarrow d_j H$ over the semileptonic mode.

In this case the approximation made in the chapters 2 and 3 viz. requiring that

$m_i \ll m_k + m_W$, $m_H \ll 2 m_k$ and $m_H \ll 2m_W$ particularly the first condition is not valid. Therefore we should take into account all masses involved in the process.

We follow the same pattern described in chapter 2 but considering not negligible the mass of the incoming quark and the Higgs boson compared to the mass of the W boson. (To simplify the calculation we set the mass of the outgoing quark equal to zero straight from the beginning). The only difference is that when using Feynman parametrisation and the Dirac equation we will not drop the mass of the incoming quark and the momentum of the Higgs particle.

Using the Feynman gauge the individual contributions are given by below (where the summation over the internal flavour is understood).

We recall that the amplitude of the transition is given by

$$M = \bar{u}_j\,(p+q)\,\Gamma R\, u_i(p) \qquad (4.1)$$

Where Γ is the coupling of the Higgs boson to two quarks with different flavours i andj at one-loop order.

$$\Gamma_A = C_{ij}^k \{ \frac{-2}{\varepsilon} + \eta - \log 4\pi + \log \frac{m_W^2}{\mu^2} + \int_0^1 dx \log(R_0). \qquad (4.2)$$

$$\Gamma_B = 0. \qquad (4.3)$$

$$\Gamma_C = 2\, C_{ij}^k \{ \int_0^1 dx \int_0^{1-x} dy \frac{(1-2x)}{R_1} \}. \qquad (4.4)$$

$$\Gamma_D = C_{ij}^k \{ (\frac{2}{\varepsilon} - \eta + \log 4\pi - \frac{1}{2} - \log \frac{m_W^2}{\mu^2} - 2\int_0^1 dx \int_0^{1-x} dy \log (R_0)$$

$$- \int_0^1 dx \int_0^{1-x} dy \; \frac{x(x+y-1)\, x_i - xyx_H + 2xx_k}{R_1} \}. \qquad (4.5)$$

$$\Gamma_E = 4 \frac{C_{ij}^k}{x_k} \int_0^1 dx \int_0^{1-x} dy \frac{x}{R_2}. \qquad (4.6)$$

$$\Gamma_F = - C_{ij}^k x_H \int_0^1 dx \int_0^{1-x} dy \frac{1-x}{R_1}. \qquad (4.7)$$

$$\Gamma_G = C_{ij}^k \int_0^1 dx \int_0^{1-x} dy \frac{1+x}{R_2}. \qquad (4.8)$$

$$\Gamma_H = - \frac{2}{x_k} C_{ij}^k \int_0^1 dx \int_0^{1-x} dy \; \{ \frac{(x-2)((x+y)x_i - yx_H - x_k)}{2R_2} + \log R_2 \}. \qquad (4.9)$$

Where

$$R_0 = x_i x^2 - (x_i - x_k + 1)x + 1, \qquad (4.10)$$

$$R_1 = x_i x^2 + (x_i - x_H)xy - (x_i - x_k + 1)\, x + (x_k - 1\,)y + 1, \qquad (4.11)$$

$$R_2 = x_i x^2 + (x_i - x_H)xy - (x_i + x_k - 1)x - (x_k - 1)y + x_k. \qquad (4.12)$$

Notice that if we set m_i and m_H equal to zero we obtain the same contributions seen in chapter 2.

Because the calculation is quite tedious, we will give the analytic contribution of each diagram seperately.

Spence functions will appear in all diagrams expect diagram A. The contribution of diagram B is nil beause of the hypothesis of neglecting the mass of the outgoing quark. The method of the integration proceeds as follow:

First performing the first integration over y using the REDUCE programme, we will obtain an expression which depends on x; after extracting the part responsible for the appearance of the Spence functions see (appendix F), the remainder is integrated using REDUCE.

After performing the first integration four logarithms will appear and the sign of their arguments will matter giving different results, although one is analytic continuation of the second (see appendix F).

These arguments are:

$$P_1(x) = x_H x^2 - x_H x + 1, \qquad (4.13)$$

$$P_2(x) = x_H x^2 - x_H x + x_k. \qquad (4.14)$$

$$P_3(x) = x_i x^2 - (x_i - x_k + 1) x + 1, \qquad (4.15)$$

$$P_4 (x) = x_i x^2 - (x_i + x_k - 1) x + x_k. \qquad (4.16)$$

The signs of the discriminants gives the physical thresholds.

A real contribution comes if we choose this sign (of the discriminants) negative. In this case the sign of the argument will be the same as the sign of the coefficient of x^2 which is always positive in all arguments. However there are cases where the coupling is real although the descriminant is positive, it corresponds to the case where the roots are outside the domain of integartion [0 - 1].

For example for the first argument we have

$$\Delta_1 = x_H^2 - 4x_H, \qquad (4.17)$$

which is negative for $x_H < 4$, which in terms of masses means for $m_H < 2m_W$.

For the second we have

$$\Delta_2 = x_H^2 - 4\, x_k x_H, \qquad (4.18)$$

which is negative for $m_H < 2m_k$.

The last threshold is abtained from the third or the fourth arguments (in fact they have the same discriminant)

$$\Delta_{3,4} = (\, x_i - x_k + 1)^2 - 4x_i \;= x_i^2 - 2x_i\,(\, x_k+1) + (x_k - 1)^2. \qquad (4.19)$$

If we fix x_i , we have

$$\Delta_{\Delta_{3,4}} = 4\, x_k. \qquad (4.20)$$

Which is always positive and hence the sign of $\Delta_{3,4}$ depends on the relative position of x_i and the roots of it, which are

$$x_{1,2} = (\, x_k + 1\,) + 2\sqrt{x_k}\,. \qquad (4.21)$$

In terms of masses we have the physical threshold (we will not discuss the other "unphysical " one) condition

$$m_i < m_W + m_k. \qquad (4.22)$$

Our calculation is performed in the kinematic region where the below physical threshold conditions (i.e. $m_H < 2m_W$, $m_H < 2m_k$ and $m_i < m_W + m_k$) are satisfied and the amplitude is real.

4.1 The analytical calculation.

Diagram A.

$$\Gamma_A = C_{ij}^k \{ -(\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{m_W^2}{\mu^2}) + \frac{x_i + x_k - 1}{2x_i} \log x_k + \frac{d_1}{x_i} \text{atan} \frac{d_1}{x_k - x_i + 1}$$

$$-2 \}. \tag{4.23}$$

Diagram B.

$$\Gamma_B = 0. \tag{4.24}$$

Diagram C.

$$\Gamma_C = 2C_{ij}^k \frac{1}{(x_i - x_H)} \{ -2 [(\frac{x_i - x_k + 1}{2x_i}) \log x_k + \frac{d_2}{x_H} \text{atan} \frac{d_2}{(2x_k - x_H)} -$$

$$\frac{d_3}{x_i} \text{atan} \frac{d_3}{1 + x_k - x_i}] + (1 + 2\alpha) S_C \}. \tag{4.25}$$

Diagram E.

$$\Gamma_E = C_{ij}^k \frac{4}{(x_i - x_H)} (\frac{1}{x_k}) \{ \frac{(1 - x_i - x_k)}{2x_i} \log x_k - \frac{d_3}{x_i} \text{atan} (\frac{d_3}{x_k - x_i + 1}) + \alpha S_E \}. \tag{4.26}$$

In this diagram and also in diagram H we use the GIM mechanism to suppress any terms not depending on the mass of the internal quark. However, after performing both integrations, the arctangant and the Spence functions appear. Hence it is not possible to extract from them the part independent of the mass of the internal quark. That is why the term

($1/x_k$) appears in the expression for the coupling.

Diagram F.

In the case of the decay of a heavy quark, diagram F is no more negligible. Indeed the mass of the Higgs boson could be important. So the contribution of diagram F has to be included in the final coupling.

$$\Gamma_F = C_{ij}^k x_H \{ \frac{x_i - x_k + 1}{2x_i} \log x_k + \frac{d_2}{x_H} \operatorname{atan} \frac{d_2}{2x_k - x_H} - \frac{d_3}{x_i} \operatorname{atan} \frac{d_3}{1 + x_k - x_i}$$

$$+ (1 + \alpha) S_F \}. \qquad (4.27)$$

Diagram G.

$$\Gamma_G = C_{ij}^k \frac{1}{(x_i - x_H)} \{ \frac{(1 - x_i - x_k)}{2x_i} \log x_k + \frac{d_1}{x_H} \operatorname{atan} \frac{d_1}{(2 - x_H)} - \frac{d_3}{x_i} \operatorname{atan} \frac{d_3}{(x_k - x_i + 1)}$$

$$+ (1 + \alpha) S_G \}. \qquad (4.28)$$

Diagrams D and H.

The contribution of these two diagrams is too cumbersome to be presented.

The d_i's are defined by

$$d_i = \sqrt{-\Delta_i} \ . \qquad (4.29)$$

Δ_i are defined in eqs (4.17-19).

Similarly, we have

$$S_c = \int_0^1 dx \frac{\log P_1(x) - \log P_3(x)}{(x + \alpha)}, \qquad (4.30)$$

and

$$S_E = \int_0^1 dx \frac{\log P_1(x) - \log P_4(x)}{(x - \alpha)} . \qquad (4.31)$$

with

$$S_F = S_C \tag{4.32a}$$

and

$$S_G = S_E \tag{4.32b}$$

Where

$$\alpha = \frac{x_k - 1}{x_i - x_H}. \tag{4.33}$$

The S's could be expressed in terms of the Spence functions in addition to some terms in logarithms, see appendix (F).

4.2 The numerical calculation.

It is obvious from the expressions for the couplings that there is no practical advantage to consider them further and that one should resort immediately to a numerical calculation. In fact if we had expressed the S functions in terms of Spence functions we would have had an algebraic explosion. That is why in almost all studies about the neutral rare decays there is no tendency to give the explicit analytic expressions for the coupling. To have an idea about the form of the expressions in the case the FCNC Z decay see Busch [11] and Clements et al.[15].

A problem encountered when using the numerical calculation is implementing the GIM mechanism. Such a mechanism is applied after performing the integration; terms which do not depend on the mass of an internal quark are suppressed. However to use the numerical calculation we have to give to all particles in the process numerical masses including the mass of the internal quarks, in such a way that we do include in our results for each internal quark separately terms which are in fact absent due to GIM. These terms are independent of the mass of the internal quarks, however they depend on the masses of the external particles.

We have seen that diagrams E and H, due to the fact the coefficient of proportionality is C_{ij}^{k}/x_k instead of C_{ij}^{k}, contain some terms which are in fact suppressed due to GIM. The flavour changing coupling could be written as

$$\Gamma = \sum_k X_{ij}^{k} F_k(x_k, x_i, x_H) \qquad (4.34)$$

where

$$X_{ij}^{k} = \frac{C_{ij}^{k}}{x_k} = \frac{-i}{16\pi^2}\sqrt{2}\, g G_F m_i m_W V_{ki}^{*} V_{kj}. \qquad (4.35)$$

The function $F_k(x_k, x_i, x_H)$ contains a part which does not depend on the mass of the internal quark. We can evaluate this part for $x_k=0$, eq (4.34) becomes

$$\Gamma = \sum_k C_{ij}^{k} [\, F_k(x_k, x_i, x_H) - F_k(0, x_i, x_H)\,] \qquad (4.36)$$

We have seen in chapter 2 that in the low-energy approximation the coupling is given by

$$\Gamma_{\text{low-energy}} = \sum_k X_{ij}^{k} \left(\frac{-3}{2} x_k \right) \qquad (4.37)$$

Thus the deviation from this approximation is

$$z = \left| \frac{F_k(x_k, x_i, x_H) - F_k(0, x_i, x_H)}{\frac{3}{2} x_k} \right| \qquad (4.38)$$

Where k is now the heaviest internal quark.

We will use this expression when dealing with rare decays in the framework of the standard model with 3 generations. Indeed the value $x_k=0$ physically means that we have neglected the masses of the other internal quarks compared to the heaviest one.

For 4 generations it is preferable to use a slightly different formula for the coupling to take into account the mass splitting between the two heaviest internal quarks. Let us take the decay of the b'-quark as an example. If considering the K-M matrix in 3 generations, we have $V_{ub} \ll V_{cb} \ll V_{tb}$. If we assume the same scenario in 4 generations, we have $V_{ub'} \ll V_{cb'} \ll V_{tb'}$ and $V_{ub'} \ll V_{ub}$, $V_{cb'} \ll V_{cb}$ in other words we assume a decoupling of heavy and light quarks. However there is no information about the remaining K-M elements namely $V_{tb'}$, $V_{t'b}$, $V_{t'b'}$ and V_{tb}.

If ignoring CP violation we have, see Hou [12]:

$$\sum_i v_i = \sum_i V_{ib} V_{ib'} = 0, \qquad (4.39)$$

so

$$V_{tb} V_{tb'} + V_{t'b} V_{t'b'} + V_{ub} V_{ub'} + V_{cb} V_{cb'} = 0. \qquad (4.40)$$

due to the unitarity of V. Neglecting V_{ub} and V_{cb} compared to V_{tb} and $V_{ub'}$ and $V_{cb'}$ compared to $V_{tb'}$ we have

$$V_{tb} V_{tb'} = - V_{t'b} V_{t'b'}. \qquad (4.41)$$

Hence in FCNC b' decay since we can ignore the contribution ot the u- and c- quarks due to the suppression of the corresponding K-M matrix elements and also to the smallness of their masses, the form factors depend on the contribution of the t- and t'-quarks. The corresponding mixing angles differing only by a sign can be factored out. Let us assume more generally that these two quarks are k and k', the coupling is equal to:

$$\Gamma = X_{ij}^{k} F_k(x_k, x_i, x_H) + X_{ij}^{k'} F_{k'}(x_{k'}, x_i, x_H) \qquad (4.42)$$

As before, splitting the two couplings to terms dependent and independent on x_k and $x_{k'}$

we have

$$\Gamma = X_{ij}^{k}[F_k(x_k, x_i , x_H) - F_k(0, x_i, x_H)] + X_{ij}^{k'}[F_{k'}(x_k, x_i, x_H) - F_{k'}(0, x_i, x_H)] \qquad (4.43)$$

The term canceled by GIM is the same for both couplings because it does not depend on the mass of the internal quark.

Since (eq 4.41)

$$X_{ij}^{k} + X_{ij}^{k'} = 0, \qquad (4.44)$$

eq (4.43) becomes

$$\Gamma = X_{ij}^{k} [F_k(x_k, x_i, x_H) - F_{k'}(x_{k'}, x_i, x_H)] \qquad (4.45)$$

The deviation from the low-energy approximation becomes

$$z = \left| \frac{F_k(x_k, x_i, x_H) - F_{k'}(x_{k'}, x_i, x_H)}{(\frac{3}{2})(x_k - x_{k'})} \right| \qquad (4.46)$$

where we have also considered the two heavy quarks in the expression for the flavour changing couplings.

After extracting the divergences from diagrams A and D, it is possible now to engage into the numerical calculation. Also the terms containing the Euler number and the arbitrary mass scale are suppressed see eqs (4.2) and (4.5). Our programme performs the sum of the remaining parts of the individual contributions given by

$$\Gamma(x_k, x_i , x_H) = \int_0^1 dx\, [\log (R_0) + \int_0^{1-x} dy \{ \frac{2(1-2x)}{R_1} - 2 \log(R_1) - \frac{x(x+y-1)x_i - xyx_H + 2xx_k}{R_1} -$$

$$\frac{1}{2} + \frac{4}{x_k} \frac{x}{R_2} + \frac{1+x}{R_2} - \frac{2}{x_k} (\frac{(x-2)((x+y)x_i - yx_H - x_k)}{2R_2} + \log(R_2)) -$$

$$x_H \frac{1-x}{R_1} \}]. \qquad (4.47)$$

Using the numerical calculation we have checked the correctness of the analytical contribution of each diagram given above.

Moreover setting m_i and m_H equal to zero in our programme the results are identical to the ones obtained in the low energy approximation obtained in chapter 2.

4.3 The semileptonic decay.

Eilam et al. [37] express the branching ratio of the process b'→Hb to the process b'→ceν by:

$$\frac{\Gamma(b' \rightarrow Hb)}{\Gamma(b' \rightarrow ce\bar{\nu})} = \frac{27\sqrt{2}}{64\pi^2} G_F \frac{m_{b'}^2}{f\left(\frac{m_{b'}^2}{m_W^2}, \frac{m_c^2}{m_{b'}^2}\right)} \left[1 - \frac{m_H^2}{m_{b'}^2}\right]^2 \left|\frac{V_{bt'}V_{t'b'}}{V_{cb'}}\right|^2 \left(\frac{m_{t'}}{m_{b'}}\right)^4. \qquad (4.48)$$

This formula is none other than an approximate expression obtained by dividing the decay width of b' to H boson and b- quark in the low energy approximation by another approximate form of the decay of b' to the charm quark and an electron - neutrino pair, in which all masses are neglected compared to the mass of the W. However if this formula is valid for the K and B decays i.e. in decays involving light quarks from the first and the second generations, it is no more legitimate for processes involving heavy quarks from the third or fourth generations which could have the same mass if not exceed that of the W boson.

Hence, and similarly to the decay b'→Hb where we have calculated the exact expression in the limit where we neglect the mass of the b-quark, it is necessary to find the exact expression for Γ (b'→ce$\bar{\nu}$) where we include the contribution of the W propagator.

The nearly-diagonal character of the K-M matrix makes the most favoured channel for a heavy quark decay either to the same generation , and if it is kinematically impossible to the nearest generation.

The tree level width for $\Gamma(b'\to ce\bar{\nu})$ is given by Gilman et al.[65] .

$$\Gamma(b'\to ce\bar{\nu}) = \frac{G_F^2 m_{b'}^5}{192\,\pi^3}\,|V_{b'c}|^2 \int_0^{(m_{b'}-m_c)^2} dQ^2\, \frac{m_W^4 |\mathbf{Q}|}{(Q^2-m_W^2)^2+m_W^2\Gamma_W^2}\,[\,2|\mathbf{Q}|^2 +$$

$$3Q^2\,(1-\frac{Q_0}{m_{b'}})], \qquad (4.49)$$

where Q is the momentum of the W boson,

$$Q_0 = \frac{(m_{b'}^2+Q^2-m_c^2)}{2m_{b'}}. \qquad (4.50)$$

and

$$|\mathbf{Q}|^2 = Q_0^2 - Q^2. \qquad (4.51)$$

Γ_W is the total width of the W boson.

The above integral is not easy to perform, as it consists of elliptical functions, see Cortes et al. [66]; however it is possible to make a transformation on Q^2 to obtain a more simplified form on which we can make a numerical treatment.

Indeed we have, see Rosner [67]

$$\Gamma(b'\to ce\bar{\nu}) = \frac{G_F^2 m_{b'}^5}{192\pi^3}\,|V_{b'c}|^2 f(\frac{m_{b'}^2}{m_W^2},\frac{m_c^2}{m_{b'}^2}). \qquad (4.52)$$

where the phase space factor is given by

$$f(\rho,\mu) = 2\int_0^{(1-\sqrt{\mu})^2} dx\, \frac{[(1-\mu)^2+x(1+\mu)-2x^2]}{(1-x\rho)^2+(\Gamma_W/m_W)^2}\,[1+\mu^2+x^2-2(\mu x+\mu+x)]^{1/2}. \qquad (4.53)$$

With

$$\rho = m_{b'}^2/m_W^2 \text{ and } \mu = m_c^2/m_{b'}^2. \tag{4.54}$$

The function given by eq (4.55) can be evaluated numerically once the values of masses are given.

4.4 The leptonic decay $W \to e\nu$.

The partial decay width of the W boson to a lepton-antilepton pair is given by

$$\Gamma(W \to e\nu) = \frac{G_F}{\sqrt{2}} \frac{m_W^3}{6\pi} = \Gamma_W^0. \tag{4.55}$$

Decays to μ and τ leptons are similar.

The partial decay width to quark-antiquarks pairs is obtained similarly but with introducing the corresponding K-M matrix elements,

$$\Gamma(W \to q'q) = 3|V_{q'q}|^2\Gamma_W^0. \tag{4.56}$$

The factor 3 comes from summing over three colours for each flavour.

The total hadronic width is equal to

$$\Gamma(W \to \text{hadrons}) = \sum_{qq'} 3\,|V_{qq'}|^2\Gamma_W^0 = 3n_G\Gamma_W^0. \tag{4.57}$$

Where n_G is the number of generations. For 3 generations of quarks the total width is given by

$$\Gamma(W \to \text{all}) = 3\Gamma_W^0 + 3 \times 3\Gamma_W^0 = 12\,\Gamma_W^0 = \frac{\sqrt{2}\,G_F m_W^3}{\pi}. \tag{4.58}$$

Such procedure is valid just in case where we can neglect the quark masses in comparison with the mass of the W boson , however the decay mode $W \to tb$ does not have the same partial width as the other channels. A full treatment of this decay is needed if it is kinematically allowed.

Barger and Phillips [74] show that

$$\Gamma(W\rightarrow tb) = 3\,\Gamma^0_W |V_{tb}|^2 \lambda^{1/2}(1, r_t, r_b)\,[\,1 - \frac{1}{2}r_t - \frac{1}{2}r_b - \frac{1}{2}(r_t - r_b)^2]. \qquad (4.59)$$

with

$$r_t = \frac{m_t^2}{m_W^2},\quad r_b = \frac{m_b^2}{m_W^2} \quad \text{and} \quad \lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \qquad (4.60)$$

Considering $r_b \ll 1$ and setting $|V_{tb}| = 1$, we have

$$\Gamma(W\rightarrow tb) = 3\Gamma^0_W(1 - \frac{3}{2}r_t + \frac{1}{2}r_t^3), \qquad (4.61)$$

and

$$\Gamma(W\rightarrow \text{all}) = 12\,\Gamma^0_W(1 - \frac{3}{8}r_t + \frac{1}{8}r_t^3). \qquad (4.62)$$

To have an idea about the importance of the decay of a heavy quark to a light one plus a Higgs boson, we will compare it to the corresponding leptonic and semileptonic processes.

In the case of the B' decay we will consider as [37] 3 cases depending on the mass of the b'-quark vis-a-vis the masses of the t-quark and the W boson.

1) $m_{b'} < m_W$ and $m_{b'} < m_t$,

2) $m_{b'} < m_W$ and $m_{b'} > m_t$,

3) $m_{b'} > m_W$.

In the first case the only semileptonic decay which occurs is $b'\rightarrow c$ (pair quark-antiquark), however the decay $b'\rightarrow t$ (pair quark-antiquark) though K-M favoured is kinematically forbidden. In these processes there is an exchange of a virtual

W boson.

In the second case both these two processes could occur always with a virtual W boson . In the third case, in addition to the semileptonic decay b'→t (pair quark-antiquark), a leptonic decay b'→cW followed by the decay of the real W takes place.

In summary we have to compare for example :

1) $\Gamma(b' \to bH)$ to $\Gamma(b' \to ce\bar{\nu})$,

2) $\Gamma(b' \to bH)$ to $\Gamma(b' \to ce\bar{\nu}) + \Gamma(b' \to te\bar{\nu})$,

3) $\Gamma(b' \to bH)$ to $\Gamma(b' \to te\bar{\nu}) + \Gamma(b' \to c + W \to ce\bar{\nu})$.

The amplitude for the process $d_i \to d_j H$ is given by

$$M = \bar{u}_j \, \Gamma \, R \, u_i \tag{4.63}$$

The problem consists in finding the value of the flavour changing coupling Γ.

To obtain the decay rate of this process, we use the standard formula

$$d\Gamma = (2\pi)^{-2} \delta^{(4)}(p_H + p_j - p_i) \, (2m_i) \, (2m_j) \, |M|^2 \frac{d^3 p_H d^3 p_j}{2E_H 2E_i 2E_j} \, . \tag{4.66}$$

In the following a confusion could arise between the decay width and the flavour changing coupling. <u>Γ followed by the process</u> $d_i \to d_j H$ <u>represents the decay width and not the flavour changing coupling</u>. After using the standard techniques, we obtain

$$\Gamma(d_i \to d_j H) = \frac{\sqrt{2}}{512\pi^5} G_F^3 m_k^4 m_i^3 \, | V_{ik}^* V_{jk} |^2 [1 - \frac{m_H^2}{m_i^2}]^2 \, \Gamma^2 . \tag{4.65}$$

Hence using this formula and the expressions for the semileptonic decays corresponding to each case we have

<u>The first case.</u>

$$\frac{\Gamma(b'\to bH)}{\Gamma(b'\to ce\bar{\nu})}=\frac{3\sqrt{2}}{16\pi^2}G_F m_{b'}^2\,[\,1-\frac{m_H^2}{m_{b'}^2}]^2\,(\frac{m_{t'}}{m_{b'}})^4\left|\frac{V_{b't'}V_{t'b}}{V_{b'c}}\right|^2\frac{\Gamma^2}{f(\frac{m_{b'}^2}{m_W^2},\frac{m_c^2}{m_{b'}^2})}. \qquad (4.66)$$

where we have just considered the contribution of the t'- quark.

In low-energy approximation we have $\Gamma = 3/2$, we obtain the eq (4.48) which is given by Eilam et al [37].

Second case.

$$\frac{\Gamma(b'\to bH)}{\Gamma(b'\to ce\bar{\nu})+\Gamma(b'\to te\bar{\nu})}=\frac{3\sqrt{2}}{16\pi^2}G_F m_{b'}^2\,[\,1-\frac{m_H^2}{m_{b'}^2}]^2\,(\frac{m_{t'}}{m_{b'}})^4\,|V_{b't'}V_{tb}|^2$$

$$\times\frac{\Gamma^2}{|V_{b'c}|^2 f(\frac{m_{b'}^2}{m_W^2},\frac{m_c^2}{m_{b'}^2})+|V_{b't}|^2 f(\frac{m_{b'}^2}{m_W^2},\frac{m_t^2}{m_{b'}^2})}. \qquad (4.67)$$

Third case.

The branching ratio is given now by

$$\frac{\Gamma(b'\to bH)}{\Gamma(b'\to Wc)\,B(W\to e\nu)+\Gamma(b'\to te\bar{\nu})}$$

where the second term in the denominator has been already calculated. The branching ratio of the W boson to eν is equal to 1/12, neglecting the mass of the top quark compared to the mass of the W. However $\Gamma(b'\to Wc)$ is equal to [68, p102]

$$\Gamma(b'\to Wc)=\frac{G_F m_{b'}^3}{8\pi\sqrt{2}}|V_{b'c}|^2\lambda^{1/2}(1,\frac{m_W^2}{m_{b'}^2},\frac{m_c^2}{m_{b'}^2})$$

$$\times[(1-\frac{m_c^2}{m_{b'}^2})^2+\frac{m_W^2}{m_{b'}^2}(1+\frac{m_c^2}{m_{b'}^2})-2\frac{m_W^4}{m_{b'}^4}]. \qquad (4.68)$$

λ is defined in eq (4.62).

While the K-M matrix elements are relatively well known in the standard model with three generations of fermions, in the four generation model the matrix is poorly known. During our approach we try whenever possible to obviate the question of mixing angles by considering the contribution of one or two heavy internal quarks where we could factor out the corresponding mixing angles when calculating the decay width of a heavy quark. Such factor (K-M matrix elements) is suppressed when comparing the amplitude of the decay to its low-energy limit i.e. the deviation z.

However when estimating the branching ratio in the three cases seen before, we have to speculate on the values of $V_{b't'}$, $V_{b't}$, $V_{bt'}$ and $V_{b'c}$.

The decay width of the process $b' \rightarrow bH$ is seen to be proportional to $m_{t'}^4$, where t' is the heaviest internal quark. Hence the decay width is bounded only if the mass of t'-quark is bounded. However since the standard model is a perturbative theory, the expansion fails when the couplings in the theory get stronger. One of these couplings is the Yukawa coupling constant which is directly related to the masses of the fermions.

In addition to these constraints peculiar to the theory, there are others imposed by experimental data e.g.the radiative corrections to ρ-parameter restrict, when considering a fourth generation, the masses of b'- and t'-quarks to obey, see Marciano [69].

$$|m_{t'} - m_{b'}| \leq 180 \text{ Gev.} \qquad (4.69)$$

In other words if we fix $m_{b'}$, $m_{t'}$ is restricted to lie in a certain domain of masses. If as [37] we choose $m_{b'}$= 70 Gev, the maximum decay width of $b' \rightarrow bH$ is obtained for $m_{t'}$ = 250 Gev.

Chapter 5

5.1 Results.

The B decay in 3 generations standard model

Low-energy approximation

Taking the coupling Γ equal to 3/2 as seen in chapter 2 we deduce that the b decay width normalised by the semileptonic decay is given by

$$\frac{\Gamma(b\to sH)}{\Gamma(b\to ce\nu)} = \frac{27\sqrt{2}}{64\pi^2} G_F m_b^2 \frac{\left[1-\frac{m_H^2}{m_b^2}\right]^2}{f\left(\frac{m_b^2}{m_W^2},\frac{m_c^2}{m_b^2}\right)} \left(\frac{m_t}{m_b}\right)^4 \left|\frac{V_{bt}V_{st}}{V_{bc}}\right|^2. \qquad (5.1)$$

Taking the phase space factor f for the process $b\to ce\nu$ equal to 0.5 and $m_b = 4.9$ Gev with $|V_{bt}V_{st}/V_{bc}| = 1$, we have

$$\frac{\Gamma(b\to sH)}{\Gamma(b\to ce\nu)} = 5\ 10^{-8}\left[1-\frac{m_H^2}{m_b^2}\right]^2 m_t^4. \qquad (5.2)$$

where m_t is measured in Gev units.

If we take m_t= 80 Gev, and also the fact that the branching ratio of $b\to ce\nu$ equal to 0.123 [87], we have

$$Br\,(b\to sH) = 0.25\left[1-\frac{m_H^2}{m_b^2}\right]^2. \qquad (5.3)$$

The exact calculation.

The results obtained are not significantly affected if we take in account the mass of the b-quark and the mass of the Higgs boson. Indeed if we calculate the deviation from the low-energy approximation given by z from the previous chapter for m_b=4.9 Gev, m_H=200 Mev and m_t=80 Gev, the approximate coupling is accurate up to 0.2% see figure (2).

In figure (2) we show the dependence of the deviation z on the mass of the t-quark.

The T decay in 3 generations standard model.

Low-energy approximation.

$$\frac{\Gamma(t\to cH)}{\Gamma(t\to b e\nu)} = \frac{27\sqrt{2}}{64\pi^2} G_F m_t^2 \frac{\left[1-\frac{m_H^2}{m_t^2}\right]^2}{f\left(\frac{m_t^2}{m_W^2},\frac{m_b^2}{m_t^2}\right)} \left(\frac{m_b}{m_t}\right)^4 \left|\frac{V_{tb}V_{cb}}{V_{tb}}\right|^2 . \qquad (5.4)$$

Where as in the b decay we have taken the coupling Γ equal to 3/2.

For m_t= 80 Gev and for small masses of Higgs, we have (taking $V_{cb}\sim\theta_c^2$ see appendix G) a branching ratio equal to 6.6 10^{-10}.

The exact calculation.

Within the standard model with 3 generations it is not possible to give the values of the coupling Γ for all masses involved in the process. Our programme computes the coupling for masses below the physical thresholds. In other words the case where the coupling Γ is expected to be complex is not numerically available.

The thresholds for this case are

$$m_t \le m_b + m_W , \qquad m_H \le 2m_W , \qquad m_H \le 2m_b . \qquad (5.5)$$

The first and second conditions are satisfied for a mass of t-quark up to 80 Gev and for Higgs mass less than m_t, the last condition restricts the mass of the latter to be less than 10 Gev.

Let us study the process $t \to cH$ for small masses of the Higgs with m_t = 80 Gev and m_b= 4.9 Gev. We have the phase space function for the process $t\to bH$ equal to 3.28 for m_H=0. The coupling Γ_t is equal to 61.11, taking $V_{cb} \sim \theta_c^2$ (θ_c is Cabbibo angle) we have

$$\frac{\Gamma(t\rightarrow cH)}{\Gamma(t\rightarrow b e\nu)} \approx 3.7\ 10^{-8}\ . \qquad (5.6)$$

For m_H=5 Gev corresponds a coupling equal to 60.66 which gives the value 3.6 10^{-8} to the branching ratio.

Notice that with the exact calculation, the branching ratio is increased by 2 orders of magnitude compared to the one obtained using the approximate coupling. However the branching ratio remains uninterestingly small.

The T decay in 4 generations standard model.

The smallness of the branching ratio is primarily due to the factor $(m_b/m_t)^4 \sim 10^{-5}$. It is tempting to investigate the situation when an additional family is introduced. Indeed the term $(m_b/m_t)^4$ is replaced by $(m_{b'}/m_t)^4$ which could reach if $m_{b'}$= 500 Gev a factor 10^3. In other words 8 orders of magnitude bigger than in the 3 generations scheme i.e. the branching ratio will attain the value 1 which is quite interesting. However the mixing angles will also change and it is more likely that what we have gained in the factor $(m_{b'}/m_t)^4$ is lost in the magnitude of the mixing angles, if of course, we adopt the naive form of the K- M matrix seen in appendix G.

Let us calculate the ratio of the mixing angles using the ansatz G.6 (appendix G)

$$\left|\frac{V_{tb'}V_{cb'}}{V_{tb}V_{cb}}\right|^2 \approx \left(\frac{\theta_c^3\theta_c^4}{\theta_c^2}\right)^2 \approx 10^{-7} \qquad (5.7)$$

Taking this expression into consideration, the branching ratio in 4 generations is enhanced over the one in 3 generations by a factor 10. The branching ratio will be in the order of 10^{-7}. Indeed for m_t= 80 Gev and $m_{b'}$= 500 Gev we have

$$\frac{\Gamma(t\rightarrow cH)}{\Gamma(t\rightarrow b e\nu)} \approx 1.3\ 10^{-7}\ . \qquad (5.8)$$

The decay rate $b' \to bH$.

Using the expression (eq 4.68) of the previous chapter if considering the contribution of t- and t'-quarks having equal and opposite mixing elements (eq 4.43)

$$\frac{\Gamma(b' \to bH)}{|V_{b't'}V_{bt'}|^2} = \frac{1}{512\sqrt{2}}\frac{1}{\pi^5} G_F^3 m_{b'}^3 m_{t'}^2 [1 - \frac{m_H^2}{m_{b'}^2}]^2 (\Gamma_{t'} - (\frac{m_t}{m_{t'}})^2 \Gamma_t)^2. \qquad (5.9)$$

Where Γ_t and $\Gamma_{t'}$ are respectively the coupling corresponding to the t- and t'-quarks .

If we assume $m_{b'}$= 70 Gev, m_H= 50 Gev and m_t= 60 Gev and let $m_{t'}$ varying we have Γ_t = 2.52.

For each value of $m_{t'}$ in domain 70-250 Gev we find the corresponding coupling $\Gamma_{t'}$. The decay rate (K-M elements factor out) vs. the mass of t'-quark are shown in fig (5). Although this graph has no practical importance, we have included it just to have an idea about the behaviour of the GIM mechnism when the masses of the t and t'- quarks are comparable; also to conclude that the curve of the decay width is similar to the one obtained in the FCNC Z decay, see fig(5) and Clements et al.[15] and Hou and Stuart [8].

Study of the H branching ratio to the semileptonic one.

The study of the three cases seen in chapter 4 reduces to the calculation of the couplings Γ corresponding to t- and t'-quarks in the decay $b' \to bH$ and the phase space functions f of the semileptonic processes involved in each case. We remind that the couplings and the phase space functions have been calculated numerically.

1st case.

We study the case in which $m_{b'}$= 70 Gev, $m_{t'}$ = 250 Gev. We take the mass of the charm quark = 1.5 Gev and the total decay width of the W boson Γ_W= 2.25 Gev, see Gilman et al [65].

As far as the mass of the b' is several Gev from the mass of the W, we can ignore the effect of the non-vanishing W width see I. Bigi et al. [71]. The phase space factor computed is equal to 2.00.

Taking $V_{b't'} \sim 1$, $V_{t'b} \sim \theta_c^2$ and $V_{b'c} \sim \theta_c^3$ we have (the Fermi constant has been taken to be equal to 10^{-5} Gev $^{-2}$)

$$\frac{\Gamma(b' \to bH)}{\Gamma(b' \to ce\nu)} = 2.66 \left(\Gamma_{t'} - \left(\frac{m_t}{m_{t'}}\right)^2 \Gamma_t\right)^2 \left[1 - \frac{m_H^2}{m_{b'}^2}\right]^2 \qquad (5.10)$$

The graphs corresponding to m_t= 80 and 150 Gev are shown in figure (6).

2nd case.

We still take $m_{b'}$= 70 Gev and $m_{t'}$ = 250 Gev, but let us take now m_t= 50 Gev, hence

$$f\left(\frac{m_{b'}^2}{m_W^2}, \frac{m_c^2}{m_{b'}^2}\right) = 2.00 \;, \quad f\left(\frac{m_{b'}^2}{m_W^2}, \frac{m_t^2}{m_{b'}^2}\right) = 0.025 \qquad (5.11)$$

Taking $V_{b't} \sim \theta_c^3$, we have

$$\frac{\Gamma(b' \to bH)}{\Gamma(b' \to ce\nu) + \Gamma(b' \to te\nu)} = 2.01\left(\Gamma_{t'} - \left(\frac{m_t}{m_{t'}}\right)^2 \Gamma_t\right)^2 \left[1 - \frac{m_H^2}{m_{b'}^2}\right]^2 \qquad (5.12)$$

The graph corresponding to this case is shown in figure (7).

3rd case.

Now let us take $m_{b'}$=100 Gev, $m_{t'}$=250 Gev and m_t= 60 Gev.

The branching ratio takes a complicated form compared to the first two cases, because more factors are involved. However it is possible to simplify the expression by noticing that we can approximate the leptonic mode $\Gamma(b' \to Wc)$ $B(W \to e\nu)$ by the semileptonic decay $\Gamma(b' \to ce\bar{\nu})$. Indeed if we choose $m_{b'}$=100 Gev which is is several W widths above the threshold (83+1.5=84.5 Gev), such an approximation is valid, see Gilman et al [65].

Thus the branching reduced to the one of the second case. One has just to modify the values of the phase space functions f:

$$f(\frac{m_{b'}^2}{m_W^2}, \frac{m_c^2}{m_{b'}^2}) = 57.15 \ , \ f(\frac{m_{b'}^2}{m_W^2}, \frac{m_t^2}{m_{b'}^2}) = 0.10 \qquad (5.13)$$

which gives

$$\frac{\Gamma(b' \to bH)}{\Gamma(b' \to ce\bar{\nu}) + \Gamma(b' \to te\bar{\nu})} = 4.38 \times 10^{-2} (\Gamma_{t'} - (\frac{m_t}{m_{t'}})^2 \Gamma_t)^2 [1 - \frac{m_H^2}{m_{b'}^2}]^2 . \qquad (5.14)$$

The branching ratio vs. the mass of the Higgs boson is plotted in figure (8).

5.2 Discussion.

In chapter 2 we have verified, using the Feynman gauge, the expression for the coupling of the Higgs boson to two quarks with different flavour. At the tree level this coupling vanishes due to the fact that the Yukawa coupling is directly proportional to the mass matrix and once the latter is diagonal the former is diagonal too.

We have exposed the different steps leading to this result. We have also shown that the definition of γ_5 matrix in the dimensional scheme does not present any problem as far as our investigation is concerned see Körner [36]. We have also exposed the conflicting results, one due to Ellis et. al.[31], the other due to Ruskov [33]. We have showed why the Ellis' result is too suppressed due to the neglect of the trace anomaly which is responsible for this coupling to the leading order of the masses of the external quark [34]. This result also explains why the total coupling is simple although the seperate diagrams give complicated logarithmic terms. The Ruskov result appears to be wrong, but details of his calculation have still not appeared.

The next thing we have done is to calculate the next to leading order proportional to the mass squared of the Higgs particle. Indeed the result of Grzadkowski-Krawczyk [26] seems to be inconsistent because, to this order, they only include the calculation of diagram F, which is proportional to the mass squared of the Higgs, which is negligible in the case of the decay of light quark. Hence if we do consider this diagram, we will be forced to find from each diagram the part which has the same order magnitude as diagram F. To this end we let q be the momentum of the Higgs particle appearing in our calculation and at the end we make a Taylor expansion in $(q/m_W)^2$ which turns out to be legitimate in the kinematic region under study. The conditions on the Higgs mass for the validity of the expansion are

$$m_H \ll 2m_W \ , \ m_H \ll 2m_t \ . \qquad (5.15)$$

However, if we include the term proportional of $(q/m_W)^2$, it is necessary to include terms quadratic in the mass of the incoming quark due to the fact that the mass of the Higgs particle is expected (due to the kinematic of the process) to be smaller than the mass of the incoming external quark.

We proceed similarly to make an expansion in $(m_i/m_W)^2$, which is valid provided that

$$m_i \ll m_t + m_W. \qquad (5.16)$$

In chapter 3 we engage in the calculation of the coupling in a general gauge. This problem has been discussed by Botella and Lim in 1986 [27] who promised to publish the results; however nothing has been released since then. In the case of the flavour changing neutral current induced by the Z boson, the general case has already been done, see Voloshin [72]. We started by checking the coupling in the unitary gauge (both the leading and next to leading terms). Although the number of diagrams decreased because of the disappearance of the unphysical Goldstone boson in this gauge, the calculations got complicated. In this respect we have more form factors due to an additional term in the W propagator. Divergences will appear in each of the remaining diagrams, some of them are cancelled due to GIM, the remaining ones cancel when all diagrams are added. For on-shell external particles the coupling turns out to be the same (both the leading and next to leading terms) in the Feynman and the unitary gauge as expected.

The most complicated calculation is encountered when calculating the coupling in a general gauge. In this gauge the unphysical Goldstone bosons contribute. Due to the tediousness of the calculation we restrict ourselves just to the leading order and we verify that the latter is gauge invariant.

In chapter 4 we consider the situation where heavy quarks are involved, both internal and external, e.g the decay of the T decay in the framework of the standard model with 3 generations. The consequences of introducing a fourth generation is treated although there is no firm arguments for its existence. We have presented some of the analytic expressions for the case when the masses of the particles involved are below the physical thresholds where the amplitude of the process is expected to be real.

Krawczyk [38] finds also the analytic expression for the coupling in the general case i.e. below and above the physical thresholds. However the expression is given in terms of the two and three-points funtions of Veltman-Passarino [82]. In our case we express these functions too. The contribution of each diagram to the coupling turns out to be expressed in terms of the Spence function. To have an estimation of the decay width $\Gamma(d_i \to d_j H)$ we set up a programme which calculates the finite part of the coupling. The divergences and terms proportional to the Euler constant and the mass scale μ cancel when adding diagrams A and D. We mention that there are other divergences cancelled by the GIM mechanism.

We start with the K decay which is expected to be suppressed due to the smallness of the mixing angles. To have an idea about this process, let us compare it to the B decay: the loop effect in the B decay involves the element $V_{ts}^*V_{tb}$, however its counterpart in the K decay is $V_{td}^*V_{ts}$. The ratio of these mixing angles is

$$\left|\frac{V_{ts}^{*}V_{tb}}{V_{td}^{*}V_{ts}}\right|^{2} \sim \left(\frac{1}{\theta_c^{3}}\right)^{2} \sim 10^{5} \qquad (5.17)$$

Thus the B decay is more promising than the K decay. It is expected to be enhanced 10^5 times.

The branching ratio of the B decay for m_t=80 Gev is equal to

$$Br(b \to sH) = 0.25\left[1 - \frac{m_H^2}{m_b^2}\right]^2 \qquad (5.18)$$

Recall that the value m_t=80 Gev is the mass of the t-quark for which there is no theoretical lower limit of the Higgs particle (in the standard model with 3 generations). Indeed recent experimental results indicate that the top quark might be heavy; more than 60 Gev from the UA1 limit, see Cline [45] and more than 100 Gev inferred from the large B^0-$\bar{B}^0$ mixing measured by ARGUS Collaboration. These facts make the search for the light Higgs justified. For a very light Higgs we expect a branching ratio near 25% (8% for m_t=60 Gev, see Cahn [73]). In the standard model with three generations of quarks and leptons, the Higgs has to be light enough to appear in the B decay. The kinematics of the process forces the Higgs mass to be less than 4.9 Gev. However there are some experimental data which may exclude the possibility of finding the Higgs in this region but even then with a small window 100-200 Mev which remains a probe for further investigation, see Gunion [47] , Dawson et al. [49] and Spiro [50].

The value of the branching ratio previously published has been calculated using the low-energy approximation, which consists of neglecting the mass of the external quarks and the Higgs boson compared to the mass of the W boson; this is valid in the case of the B-decay. However if one nonetheless includes the mass of the b-quark and Higgs boson, there is no big deviation from the previous result. The approximate coupling is accurate up to 0.2%. In addition the inclusion of QCD corrections do not change too much the results, see Hall - Wise [32].

Let us turn now to another process, $t \to cH$, which has been treated, but only using the low-energy approximation, by Willey and Yu [24]. They find in the standard model with three generations an uninteresting small branching ratio (less than 10^{-8}). More recently Krawczyk claims that it is less than 10^{-10} .

However although the deviation z from the low energy approximation could attain 10 for m_t= 80 Gev and m_H= 50 Gev see figure(5) of [38] , it seems that it cannot counterbalance the small value of the mixing angles. Indeed adopting the low-energy approximation we obtain for m_t= 80 Gev and a very light Higgs ($m_H \approx 0$ Gev) (although the approximation is not valid for such a mass for the t-quark)

$$\frac{\Gamma(t \to cH)}{\Gamma(t \to be\nu)} \approx 6\ 10^{-10} . \tag{5.19}$$

Where the mixing angles factor is

$$\left|\frac{V^*_{tb} V_{sb}}{V_{tb}}\right|^2 \sim \theta_c^4 \approx 10^{-3}. \tag{5.20}$$

However performing the exact calculation, the branching ratio is replaced by

$$\frac{\Gamma(t \to cH)}{\Gamma(t \to be\nu)} \approx 4\ 10^{-8}. \tag{5.21}$$

Although the branching ratio has been enhanced by a factor 10^2, it remains uninterestingly small. Moreover we have

$$\frac{\Gamma(t \to cH)}{\Gamma(t \to be\nu)}\Big|_{exact} \Big/ \frac{\Gamma(t \to cH)}{\Gamma(t \to be\nu)}\Big|_{low\text{-}energ} = z^2 \tag{5.21a}$$

where z is the deviation of the exact calculation from the low-energy approximation defined in chapter 4.

We deduce that the deviation is approximatively equal to 10 which is the value obtained by [38] for m_t=80 Gev and m_H= 0 Gev, see fig (5) of [38].

We could search for more pronouced branching ratio by including a fourth generation. Indeed the smallness of the branching ratio is due primarily to the factor $(m_b/m_t)^4 \sim 10^{-5}$. So it is tempting to consider a fourth generation because the m_b will be replaced by $m_{b'}$ which could have a substantial mass. However if we adopt the form given in eq

(G.6) of the K-M matrix in 4 generations, it is likely that the factor which we have gained in $(m_b/m_t)^4$ is lost in the magnitude of the mixing angles. Indeed we have

$$\frac{Br\,(4\text{ gen})}{Br\,(3\text{ gen})} \approx \left|\frac{V_{tb'}V_{cb'}}{V_{cb}V_{tb}}\right|^2 \left(\frac{m_{b'}}{m_b}\right)^4 . \qquad (5.22)$$

and we adopt the naive form of the K-M matrix [see appendix G], we have

$$\left|\frac{V_{tb'}V_{cb'}}{V_{cb}V_{tb}}\right|^2 \sim \left(\frac{\theta_c^3\theta_c^4}{\theta_c^2}\right)^2 \approx 10^{-7} \qquad (5.23)$$

So taking $m_{b'} = 500$ Gev, we have $(m_{b'}/m_b)^4 \approx 10^8$ and

$$\frac{Br\,(4\text{ gen})}{Br\,(3\text{ gen})} \approx 10^{-7} \times 10^{8} = 10 \qquad (5.24)$$

In other words the branching ratio is enhanced just by a factor 10, we expect it in the order 10^{-7}. Nevertheless the exact calculation for $m_t = 80$ Gev and $m_{b'} = 500$ Gev gives

$$\frac{\Gamma(t\to cH)}{\Gamma(t\to be\nu)} \sim 10^{-10} \qquad (5.25)$$

This branching ratio gets smaller for an increasing Higgs mass, see figure (3).
Even with assuming the internal quark mass to be big, this mass could not compensate the effect of the mixing angles if, of course, we assume this naive form of the K-M matrix in four generations. The problem in the T decay is that the semileptonic decay is K-M favoured. The same pattern is exhibited in the case of the FCNC Z decay, see Busch [11].

$$Br(Z\to t\bar{c}) \sim 10^{-11}\ ,\ Br(Z\to b\bar{s}) \sim 10^{-8}. \qquad (5.26)$$

We do not need to restrict ourselves to the naive form of the K-M matrix given by (G.6). Indeed $V_{tb'} = \theta_c^3$ seems to be too small. For example the mass splitting between the t and b'- quarks could be small and hence a sizeable $V_{tb'}$ is expected.

Let us find the constaints on the K-M matrix elements to have a dominating FCNC T decay.

For a light Higgs below 5 Gev the condition is (for m_t=80 Gev and $m_{b'}$= 500 Gev)

$$\left|\frac{V_{tb}}{V_{tb'}V_{cb'}}\right| \leq 1.4 . \qquad (5.27)$$

and for a Higgs up to 50 Gev, the condition becomes

$$\left|\frac{V_{tb}}{V_{tb'}V_{cb'}}\right| \leq 0.8 . \qquad (5.28)$$

The question which arises is whether the values of the phenomenological K- M matrix elements seen in (G.7) do not exclude such conditions to be satisfied.

If for example, eq (5.28) is satisfied, the T decay could be interesting as far as the detection of the Higgs of mass less than 50 Gev is concerned. In figure (4) we show the allowed region for a dominant FCNC T decay. To satisfy such condition we require that the diagonal element V_{tb} to be small and the off-diagonal ones $V_{tb'}$ and $V_{cb'}$ to be big which seems to be unlikely.

The remaining decay is the B' decay. In figures 6, 7 and 8 we show the numerical results respectively for the cases 1, 2 and 3 of the decay width of the b' quark to the H boson divided by the semileptonic decay width denoted by Br as functions of the mass of the Higgs boson m_H. The results are given in tabular form below.

1st case.

$m_{b'}$=70 Gev, $m_{t'}$= 250 Gev

Table1 (m_t=100 Gev)

m_H	0	10	20	30	40	50	60
Br	4.2	4.1	3.6	2.9	2.0	1.0	0.3

Table 2 (m_t= 150 Gev)

m_H	0	10	20	30	40	50	60
Br	2.5	2.4	2.1	1.7	1.1	0.6	0.1

2nd case.

$m_{b'}$=70 Gev, $m_{t'}$=250 Gev

Table3. (m_t=50 Gev)

m_H	0	10	30	40	50	60
Br	4.6	4.4	3.1	2.1	1.1	0.6

3rd case.

$m_{b'}$=100 Gev, $m_{t'}$=250 Gev, m_t=60 Gev

Table 4.

m_H	0	10	20	30	40	50	60
Br	0.106	0.104	0.098	0.088	0.075	0.060	0.044

Notice that for a Higgs mass above 10 Gev, there is an exact comformity between the values of the branching ratio obtained here and the ones obtained in Eilam et. al [37]. For example in the first case with m_t=100 Gev, the branching ratio corresponding to m_H=0 and 10 Gev is equal respectively to 4.3 and 4.1, however its value in [37] is 4.0 and 3.9. For m_H=50 Gev the branching ratio is equal to 1.0 as obtained in [37].
The same remark could be made for m_t=150 Gev.
For m_t=50 Gev the branching ratio for m_H= 0 and 10 Gev is respectively 4.6 and 4.4 ; in [37] 4.7 and 4.5. However for m_H= 20Gev and above there is an exact concordance

between the two results. For example for m_H= 30, 40, 50 Gev, the branching ratio is respectively 3.1 , 2.1 and 1.1 .

Since there are some experimental data which may exclude the Higgs boson mass below 5 Gev, we will not consider this region any further. For m_H= 10 Gev our result for the branching ratio agree with [37] within 2%.

In [37] although there is no details of the calculation undertaken, it is obvious that they have included in their calculation at least the contributions of two quarks t and t' having equal and opposite mixing angles.

Indeed in the first case corresponding to $m_{b'} < m_t$, $m_{b'} < m_W$ it is clear from their figures that the curve which gives the branching ratio gets more flat and small once the mass of t-quark gets closer to the mass of t'-quark fixed to 250 Gev. This feature is due to the GIM mechanism which becomes more pronounced whenever the mass splitting gets small.

From the first case we see that the FCNC decay is dominant over the corresponding CC decay for $m_H \leq 40$ Gev if $m_t = 150$ Gev and this limit goes to 50 Gev if $m_t = 80$ Gev.

The idea of the dominating FCNC over a CC decay has been discussed for the first time by Barger et al. [74] and later by Hou and Stuart [8] both in the case of the Z boson.

The same behaviour (i.e. dominating FCNC) even more pronounced is noticed in the second case $m_t < m_{b'} < m_W$ for $m_H \leq 50$ Gev.

However for the last case corresponding to $m_t < m_w < m_{b'}$ this behaviour disappears once the leptonic mode $b' \rightarrow Wc$ becomes important. For illustration the Higgs-boson mode is only about 10 % of the electronic mode for $m_H \leq 15$ Gev. The percentage gets smaller with increasing Higgs mass e.g. 6% for $m_H = 50$ Gev, see table (4).

For the third case there is a difference between our results and [37], for example for m_H= 0 Gev the branching ratio is equal to 0.106 whereas in [37] 0.112. But in this case the difference persists for all Higgs masses at 6% level.

The main reason for the difference of our results from Eilam's et al for masses of the

Higgs below 10 Gev, both in the first and second case, is that our calculation is performed neglecting the mass of the outgoing quark (b-quark), however their result, as claimed, is obtained considering this mass. For masses of the Higgs above 10 Gev the mass of the b-quark is overshadowed by the mass of the Higgs which explains why the two results are identical. For small masses of the Higgs the effect of the b-quark appears.

However for the third case due to the fact that the branching ratio is small, the effect of the b-quark is more pronounced than in the previous two cases. In addition to that the fact of approximating the leptonic decay $\Gamma(b'\to cW)\ B(W\to e\nu)$ by the semileptonic decay $\Gamma(b'\to ce\nu)$ could have its effect on the branching ratio. The difference is anyhow small.

The direct computation supports the observation that for $m_{b'} < m_{t}$, a FCNC decay could be dominant, however it could be due to the simple form of the K-M matrix adopted. If we adopt a K-M matrix where the condition $V_{b't'}V_{bt'} = - V_{b't}V_{bt}$ is satisfied. The decay $b'\to b+H$ dominates $b'\to c+W$ (where W is virtual) for a Higgs mass up to 60 Gev provided that

$$\left|\frac{V_{cb'}}{V_{tb}V_{tb'}}\right| \leq 10^{-1} \text{ for } m_{b'} < m_{t} . \qquad (5.29)$$

which is the condition of the first case. For a Higgs mass equal to 50 Gev, the upper limit is replaced by 0.2 , however for $m_H=10$ Gev the limit is 0.4. Notice that the elements of the naive K-M matrix adopted before satisfies this condition for a Higgs mass less than 50 Gev. Indeed for this matrix the left-hand side of eq(5.29) is equal to 0.2. We can deduce similar constraints on the mixing angles for the second case and especialy the third one, however the situation gets complicated because in these two cases there is a competition of two electronic modes: ($b'\to ce\bar{\nu}$) and ($b'\to te\bar{\nu}$).

For the second case, assuming eq (4.41), the FCNC decay dominates the sum of the two semileptonic decays for $m_{b'}=70$ Gev, $m_t=50$ Gev, $m_{t'}=250$ Gev and $m_H=10$ Gev provided that

$$0.2 \leq V_{tb} \leq 1 \quad \text{and} \quad \left| \frac{V_{b'c}}{V_{b't}V_{t'b}} \right| \leq 0.5 \qquad (5.30)$$

Figure 9a gives the band of allowed $|V_{tb}|$ and $|V_{b'c}/V_{b't'}V_{bt'}|$.

For m_H=50 Gev, these conditions become

$$0.4 \leq V_{tb} \leq 1 \quad \text{and} \quad \left| \frac{V_{b'c}}{V_{b't'}V_{bt'}} \right| \leq 0.2 \qquad (5.31)$$

The allowed region is shown on figure 9b.

Notice that the naive form of the K-M matrix adopted previously satisfies these constraints.

Similarly we can deduce the same conditions for the third case:

For $m_{b'}$= 100 Gev , m_t=60 Gev , $m_{t'}$=250 Gev and m_H=10 Gev, these conditions are

$$0.6 \leq V_{tb} \leq 1 \quad \text{and} \quad \left| \frac{V_{cb'}}{V_{b't'}V_{bt'}} \right| \leq 0.05 \qquad (5.32)$$

For m_H=50 Gev, we have

$$0.8 \leq V_{tb} \leq 1 \quad \text{and} \quad \left| \frac{V_{cb'}}{V_{b't'}V_{bt'}} \right| \leq 0.03 \qquad (5.33)$$

Graphically these conditions are represented by figures 10a and 10b.

In the contrary to the first two cases, the naive K-M matrix of (G6) does not satisfy the second constraint on the ratio of $V_{cb'}$to $V_{b't'}V_{bt'}$.

Let us now turn to the possible signatures for the Higgs. The Higgs can decay to fermion - antifermion pair of mass m_f, to WW and ZZ final states depending on its mass. For $m_H < 2m_W$, the Higgs will decay dominantly to the heaviest fermion channel which is energetically allowed. If we assume also that $m_H < 2m_t$ (of course if the top is not too heavy), and above a mass of 10 Gev, the Higgs decays to a bb pair .

The branching ratio $\Gamma(H \rightarrow b\bar{b})$ is equal to [75]

$$\Gamma(H \rightarrow bb) = 3 \frac{G_F m_b^2 m_H}{4\pi\sqrt{2}} \left(1 - \frac{4m_b^2}{m_H^2} \right)^{3/2} . \qquad (5.34)$$

So with the Higgs of mass equal to 60 Gev, we obtain a decay width in the order of $2\ 10^{-3}$ Gev.

5.3 Conclusion.

We have verified in the first part of this work the gauge invariance of the flavour changing transition of the decay of a light quark. We have also verified that the next to leading term which is proportional to the mass of the Higgs boson squared is the same both in the unitary gauge and the Feynman gauge.

The other part of the thesis is reserved for the study of the decay of a heavy quark and the possibility of detecting the Higgs particle using such processes. We have the following deduction:

The B decay which could provide the detection of the Higgs below 5 Gev is ruled out by experimental data.

If there are just three generations of quarks and leptons and one Higgs doublet the T decay is not suitable for the search of the Higgs. The same conclusion is obtained if introducing a fourth family unless severe constraints are satisfied by the Kobayashi-Maskawa matrix elements.

If the b'-quark is not too heavy (compared to the t- quark), it can be a probe for detection of the Higgs particle with mass up to 60 Gev.

Figure captions

Figure 1: Diagrams contributing to the flavour changing transition induced by a Higgs boson. The blobs and the squares indicate respectively unrenormalised and renormalised quantities. d_i and d_j are external quarks with flavour i and j, u_k is the internal quark with flavour k, H is the Higgs boson and W and Φ are respectively the charged physical and charged unphysical bosons. The cross + indicates the counterterms.

Figure 2: The deviation z for the process $b \to sH$ vs. the mass of the t-quark with m_H= 0.2 Gev.

Figure 3: $\Gamma(t \to cH) / \Gamma(t \to bev)$ for m_t= 80, 100 Gev as function of m_H with $m_{b'}$=500 Gev.

Figure 4: Band of $|V_{tb'}V_{cb'}/V_{bt}|$ to have a dominating FCNC T decay for m_t= 80 Gev and $m_{b'}$=500 Gev vs. the mass of the Higgs.

Figure 5: The decay width for $b' \to bH$ in Mev, divided by the generation mixing parameter, as function of the mass of the t'-quark with m_t= 80 Gev and m_H = 50 Gev.

Figure 6: $\Gamma(b' \to bH) / \Gamma(b' \to cev)$ for m_t= 80, 100 and 150 Gev as function of m_H with $m_{b'}$=70 Gev and $m_{t'}$= 250 Gev.

Figure 7: $\Gamma(b' \to bH) / (\Gamma(b' \to cev) + \Gamma(b' \to tev))$ for m_t= 50 Gev, $m_{b'}$= 70 Gev and $m_{t'}$= 250 Gev and for m_t= 70 Gev, $m_{b'}$= 80 Gev and $m_{t'}$= 250 Gev as function of m_H.

Figure 8: $\Gamma(b'\to bH)$ / ($\Gamma(b'\to cW)$ $B(W\to e\nu)$ + $\Gamma(b'\to te\nu)$) for m_t= 60 Gev as function of m_H with $m_{b'}$= 100 Gev and $m_{t'}$= 250 Gev.

Figure 9

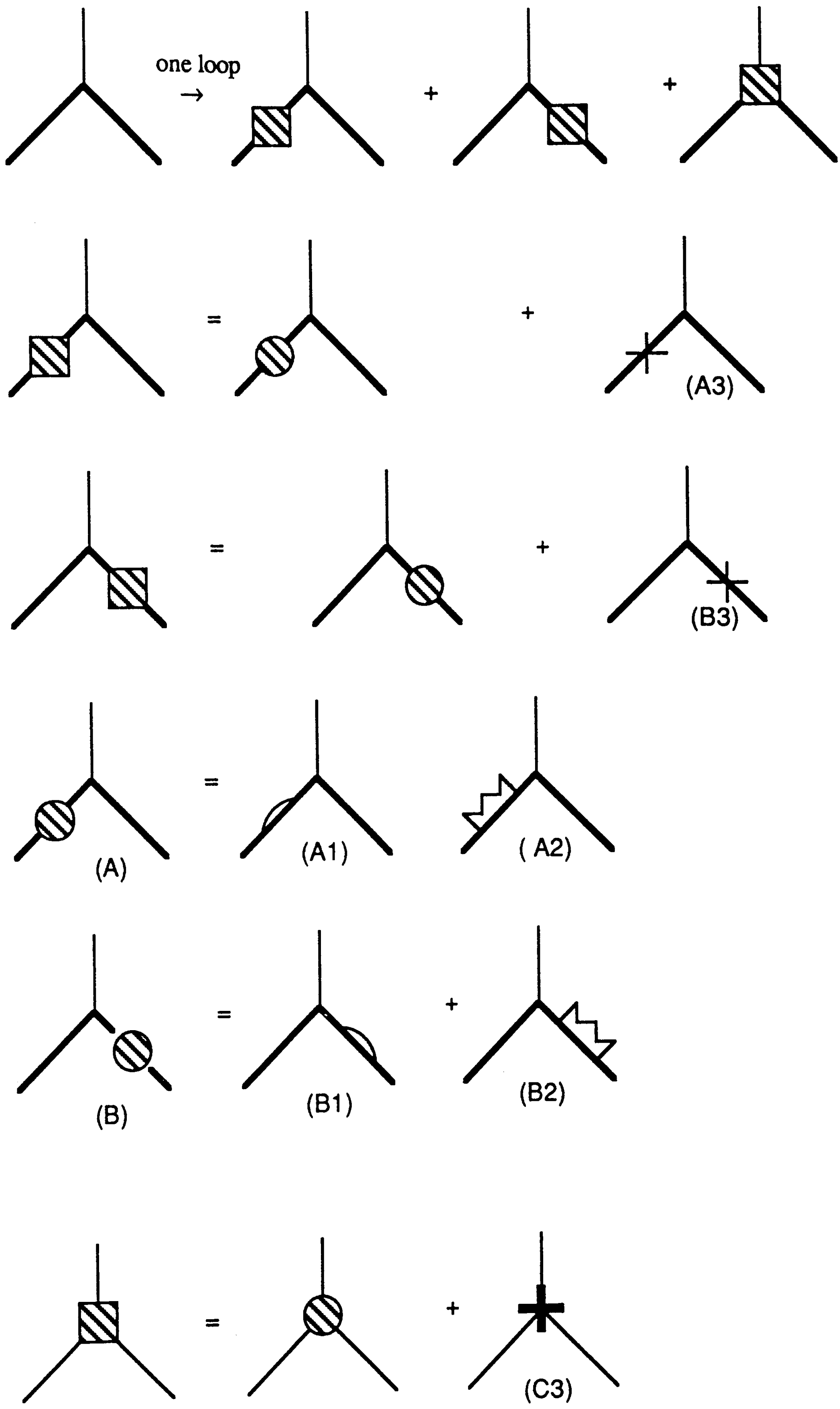
(a): Band of $|V_{tb}|$ and $|V_{cb'}/ V_{b't'}V_{bt'}|$ to have a dominating FCNC decay $\Gamma(b'\to bH)$ compared to ($\Gamma(b'\to ce\nu)$ + $\Gamma(b'\to te\nu)$) for m_t= 50 Gev, $m_{b'}$= 70 Gev, $m_{t'}$= 250 Gev and m_H= 10 Gev.

(b): same as (a) but m_H= 50 Gev.

Figure 10

(a): Band of $|V_{tb}|$ and $|V_{cb'}/ V_{b't'}V_{bt'}|$ to have a dominating FCNC decay $\Gamma(b'\to bH)$ compared to ($\Gamma(b'\to cW)$ $B(W\to e\nu)$ + $\Gamma(b'\to te\nu)$) for m_t= 60 Gev , $m_{b'}$= 100 Gev, $m_{t'}$= 250 Gev and m_H= 10 Gev.

(b): same as (a) but m_H= 50 Gev.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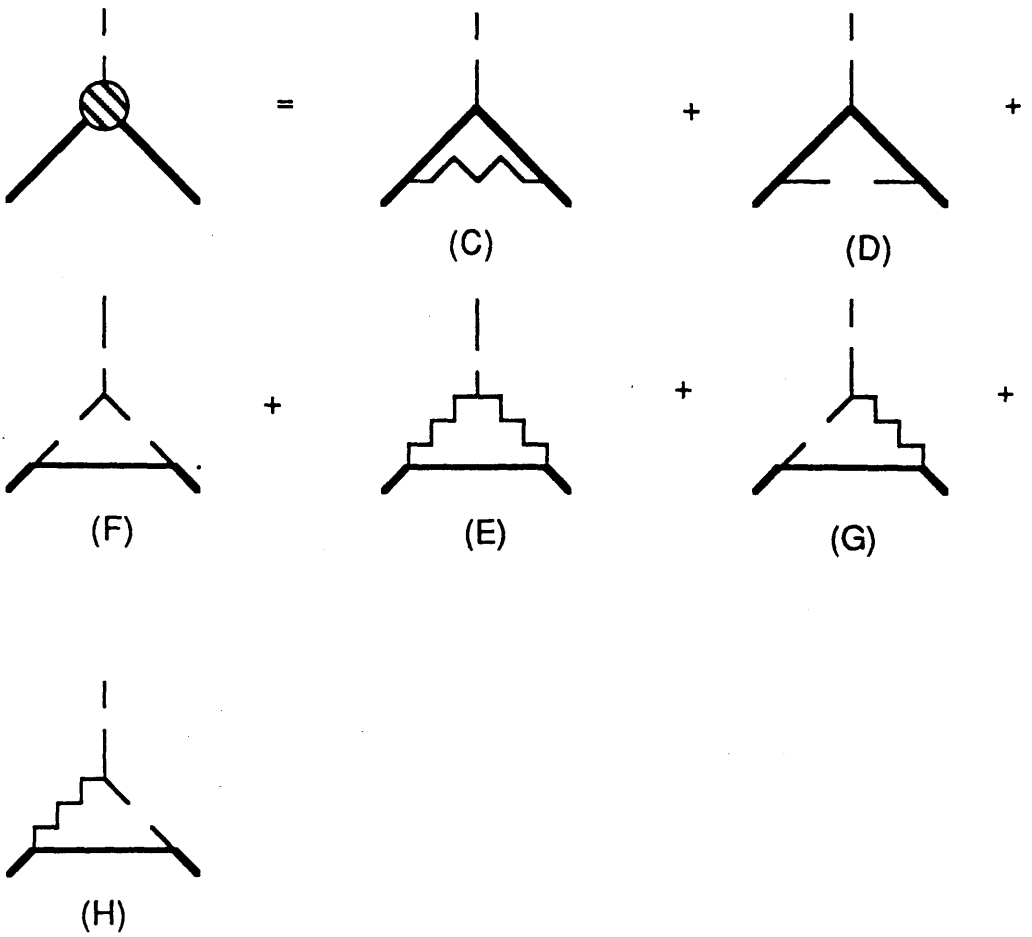
(B2)
(C3)

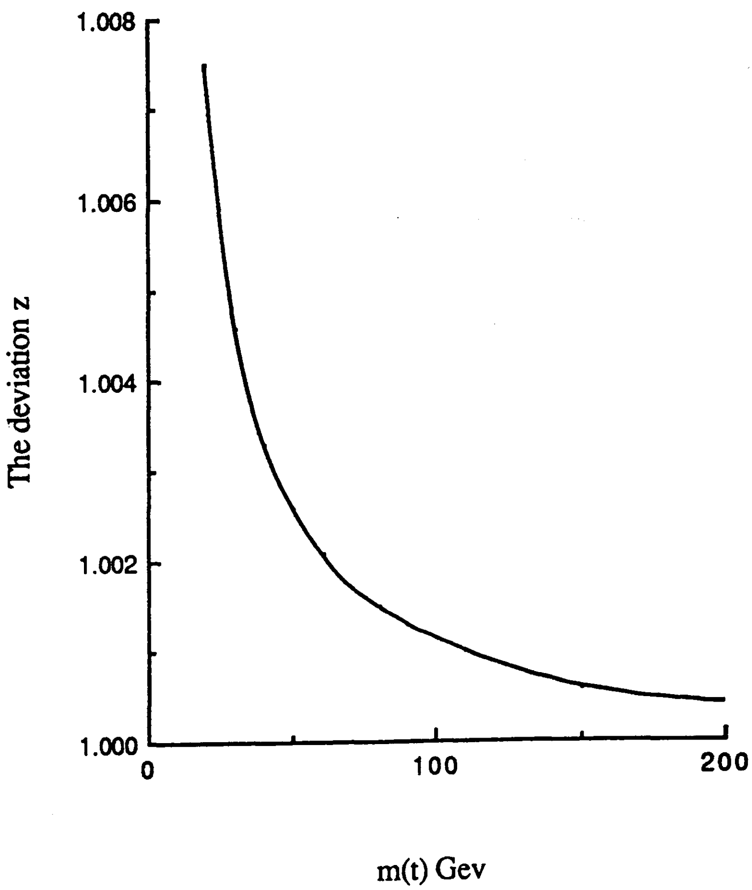


— for internal and external quark lines.

– – for the Higgs and the Goldstone bosons.

for W bosons .

Figure 1.



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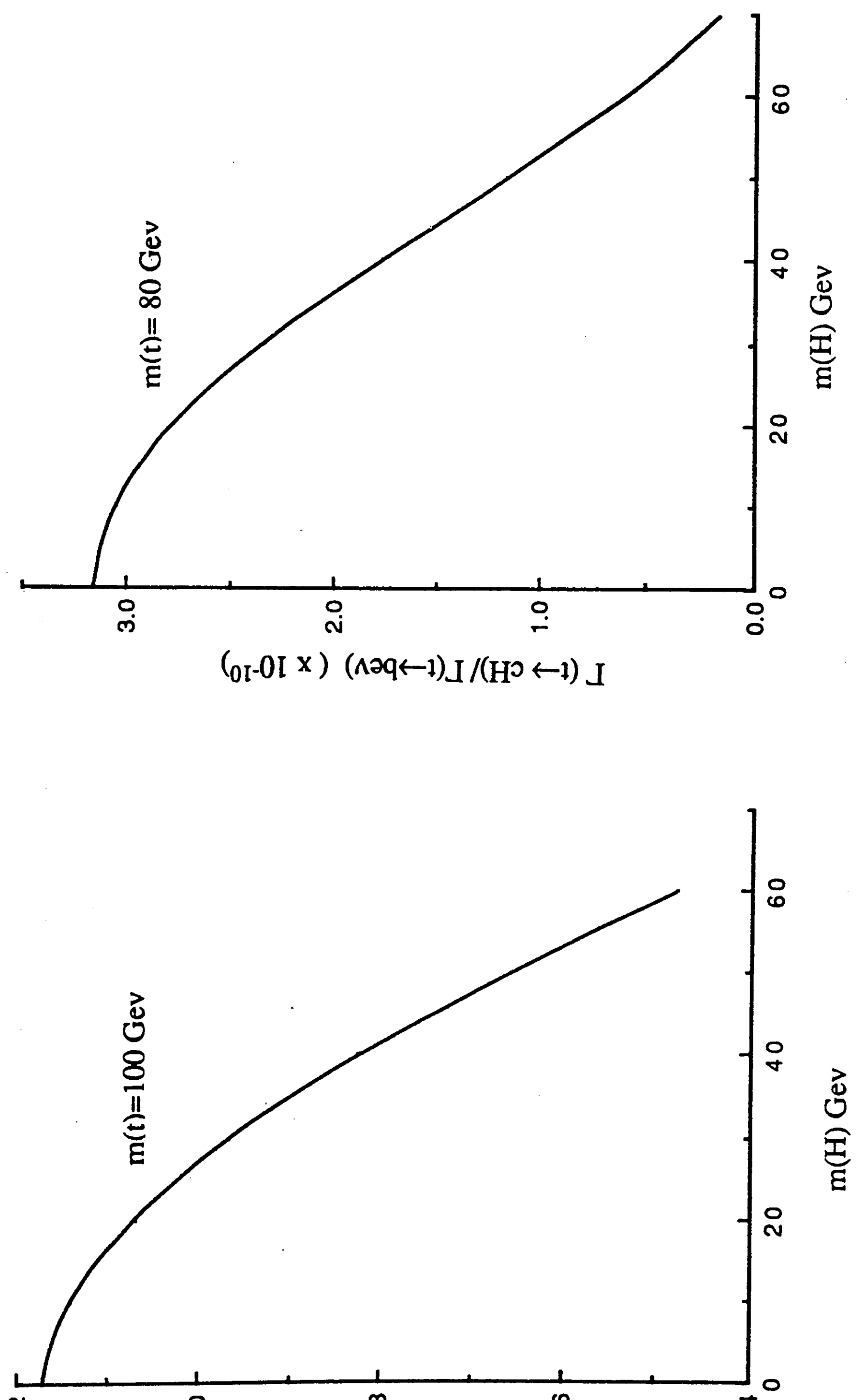


m(t)= 80 Gev
Γ (t→ cH)/ Γ(t→bev) (x 10-10)
3.0
2.0
1.0
0.0
0
20
40
60
m(H) Gev
m(t)=100 Gev
Γ (t→ cH)/ Γ(t→bev) (x 10-11)
1.2
1.0
0.8
0.6
0.4
0
20
40
60
m(H) Gev

Figure 3.

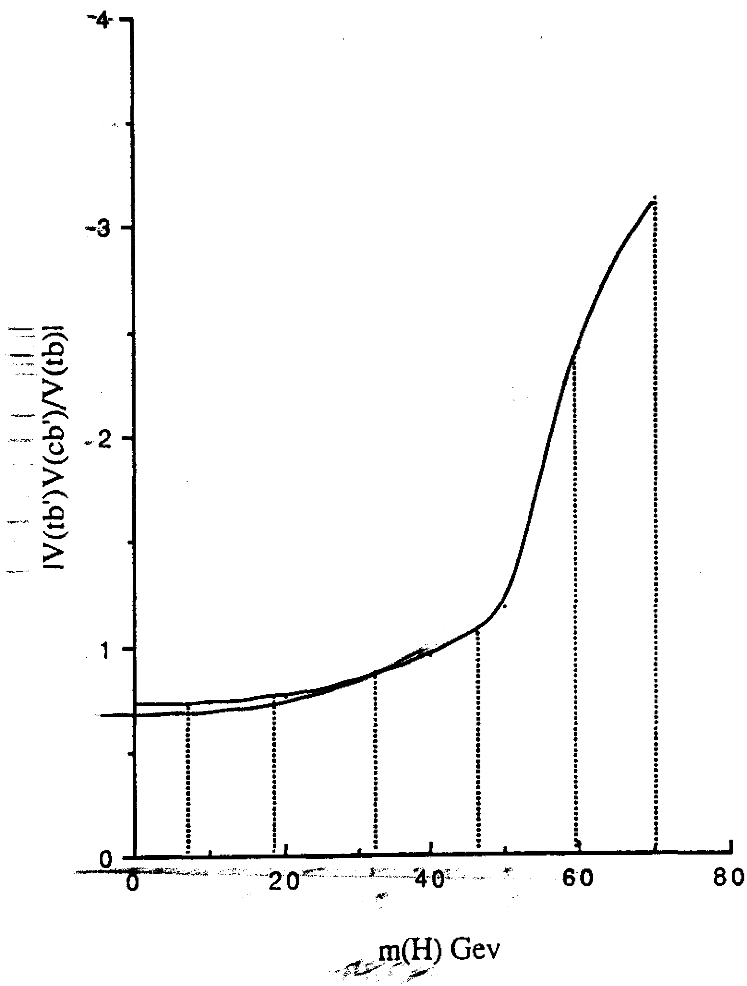
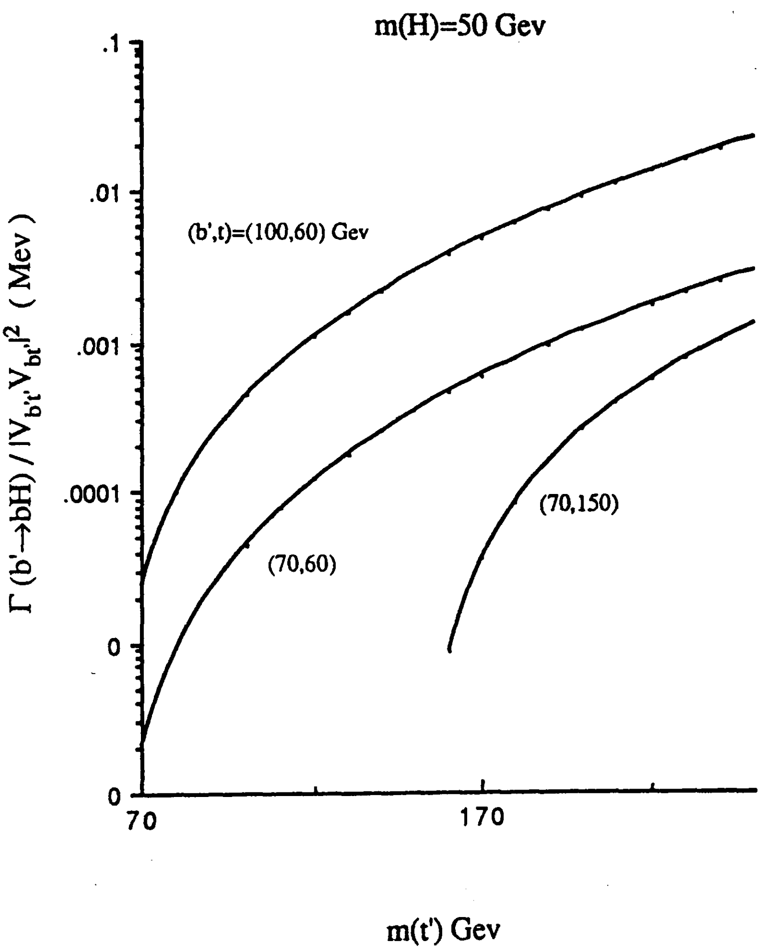


Figure 4.



m(H)=50 Gev
.1
.01
.001
.0001
0
0
70
170
(b',t)=(100,60) Gev
(70,60)
(70,150)
Γ (b'→bH) / $|V_{b't'}V_{bt'}|^2$ (Mev)
m(t') Gev

Figure 5.

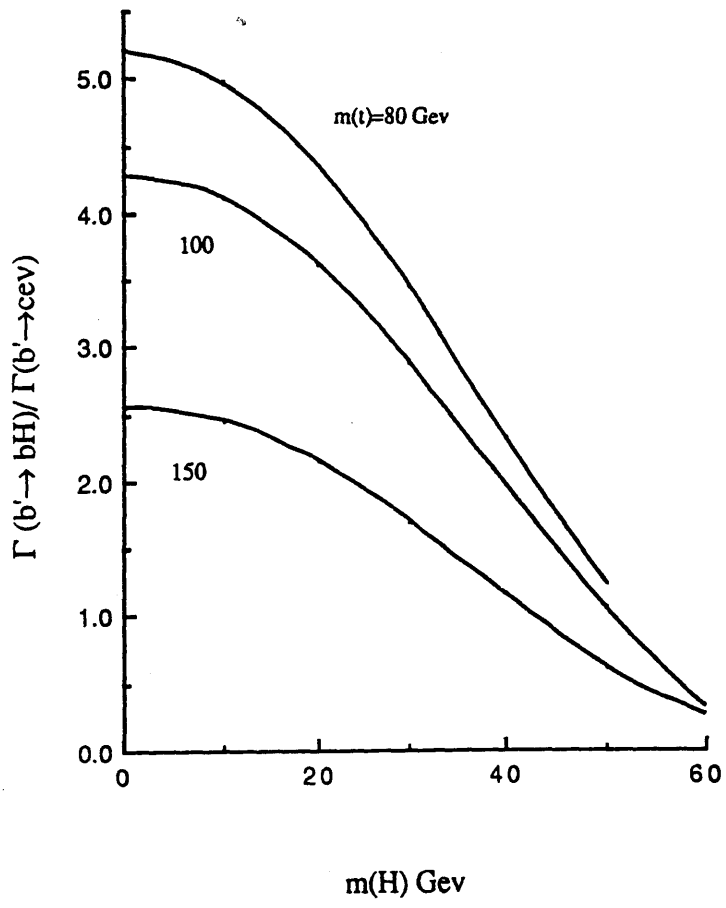


Figure 6.

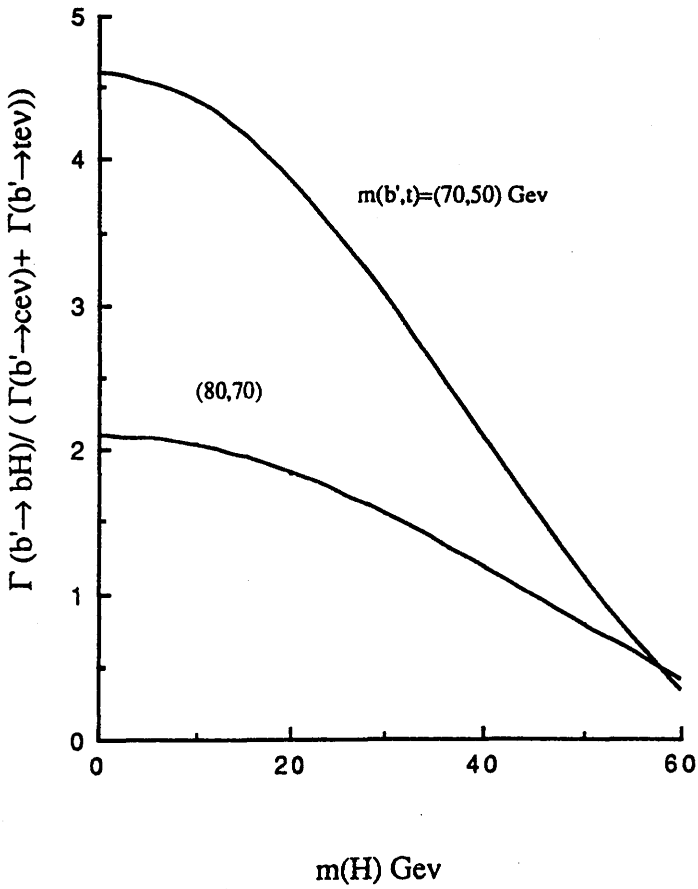


Figure 7.

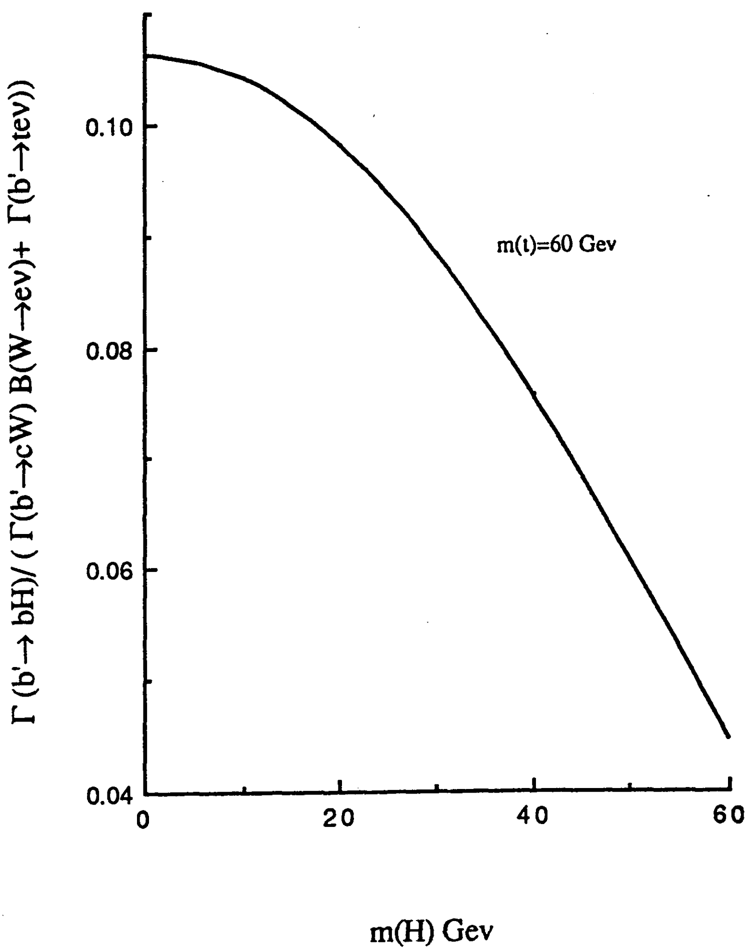


Figure 8.

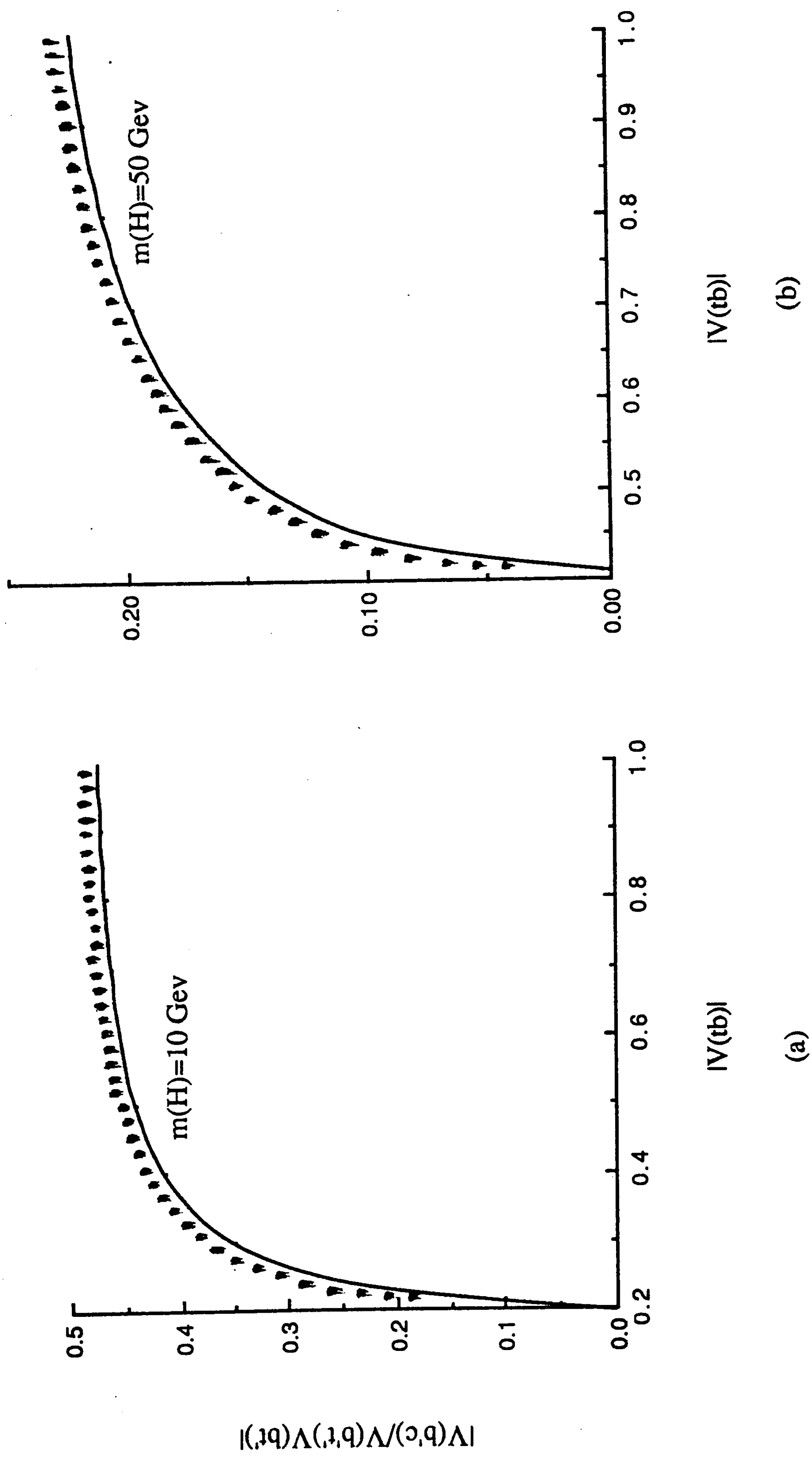


Figure 9

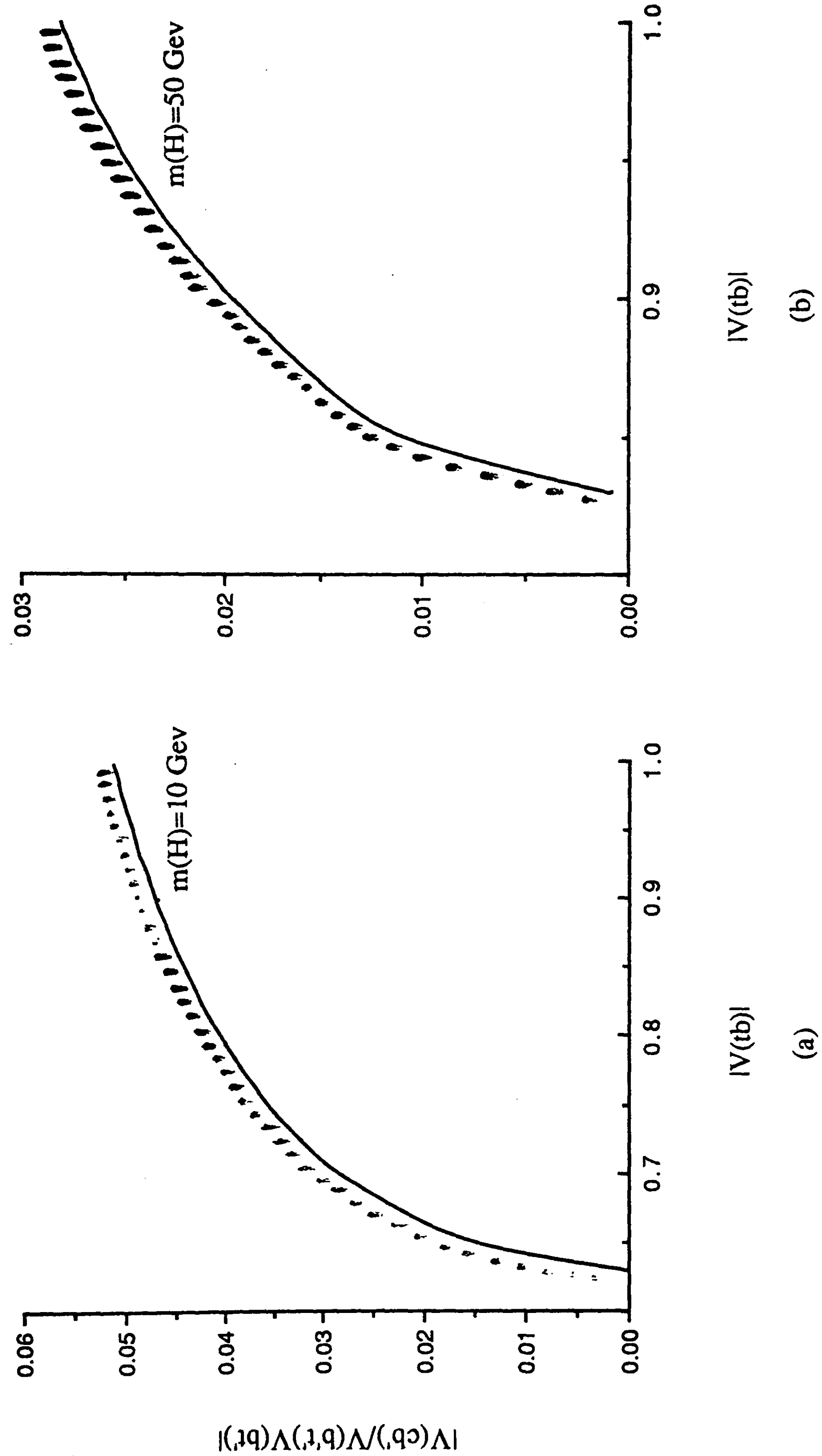

|V(cb')/V(b't')V(bt')|
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.7
0.8
0.9
1.0
m(H)=10 Gev
|V(tb)|
(a)
0.00
0.01
0.02
0.03
0.9
1.0
m(H)=50 Gev
|V(tb)|
(b)

Figure 10

Appendix A.

Feynman rules.

The relevant vertices and propagators are given in figure (1). Where i and j are the external quarks, k is the internal quark. The quarks are labelled by their flavours. W , $\Phi^{+,-}$ and H denote respectively the charged boson, charged unphysical scalar bosons and the neutral Higgs boson.

The Feynman rules for vertices and propagators are (in the general gauge, where ξ is the gauge fixing parameter) given, see Bailin and Love [76].

In our calulation we use the following convention:

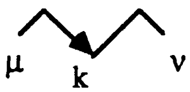
V is the Kobayashi - Maskawa matrix, m_i, m_j and m_k are respectively the mass of the incoming, outgoing and the internal quarks.

M(U) and M(D) are the mass matrices of the up-type and down type quarks respectively.

L , R are the left and right hand projection of the helicity.

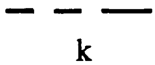
$$L,R = \left(\frac{1 -,+ \gamma_5}{2} \right).$$

Rule 1.



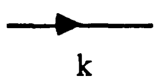
$$i\Delta_{\mu\nu}(k) = \frac{-i}{k^2 - m_W^2 + i\varepsilon}\left[g_{\mu\nu} - (1 - \xi)\frac{k_\mu k_\nu}{k^2 - \xi m_W^2} \right].$$

Rule 2.



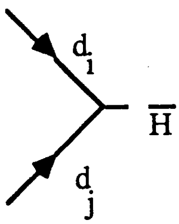
$$iD_\Phi(k) = \frac{i}{k^2 - \xi m_W^2 + i\varepsilon}.$$

Rule 3.



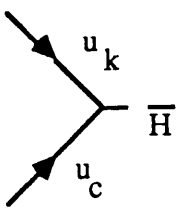
$$iS_F(k) = \frac{i(k + m_k - i\varepsilon)}{k^2 - m_k^2}.$$

Rule 4.



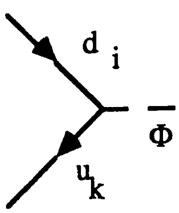
$$\frac{-ig}{2m_W}[M(D)]_{ij} = \frac{-ig}{2m_W} m_i \delta_{ij}.$$

Rule 5.



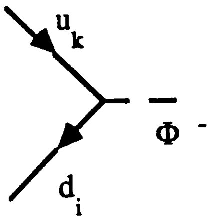
$$\frac{-ig}{2m_W}[m(U)]_{kc} = \frac{-ig}{2m_W} m_k \delta_{kc}.$$

Rule 6.



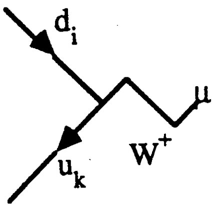
$$\frac{-ig}{\sqrt{2}\, m_W} V_{ki}(m_i R - m_k L).$$

Rule 7.



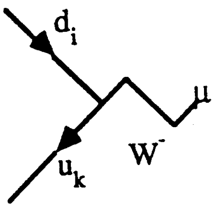
$$\frac{-ig}{\sqrt{2}\, m_W} V^{+}_{ki} \left(m_i L - m_k R \right).$$

Rule 8.



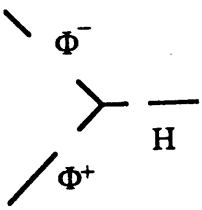
$$\frac{ig}{\sqrt{2}} V_{ki}\, \gamma_\mu L.$$

Rule 9.



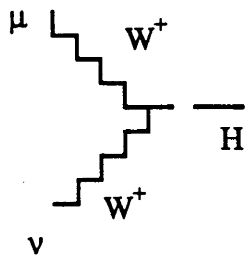
$$\frac{ig}{\sqrt{2}} V^{+}_{ki}\, \gamma_\mu L\, .$$

Rule 10.



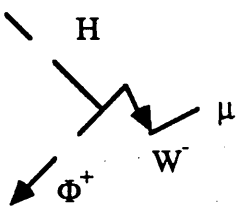
$$\frac{-ig}{2} \frac{m_H^2}{m_W}.$$

Rule 11.



$$i\, g\, m_W g_{\mu\nu}.$$

Rule 12.



$$\frac{ig}{2}(p+q)_\mu.$$

Appendix B.

Dimensional regularisation.

In general, in the loop calculation, the Green's functions develop ultra-violet divergences. The technique which enables us to handle these divergences is called regularisation. We will enumerate the well known procedures with a brief explanation and we choose the method suitable to our case.

1) Cut-off regularisation:

Since in our case the source of the divergence is the contribution of the high-momentum region, it is possible to regularise the theory by introducing a cut -off on the momenta. However such a method breaks the translation invariance and hence a shift in the momenta will change the value of the integral. Also it breaks in general the gauge invariance which makes it not suitable for our purpose.

2) Pauli-Villars regularisation:

The divergent propagator $1 / (k^2 - m^2)$ will be replaced by

$$\frac{1}{k^2 - m^2} - \frac{1}{k^2 - M^2} = \frac{m^2 - M^2}{(k^2 - M^2)(k^2 - m^2)} . \qquad (B.1)$$

The new propagator reduces to the original one if we set $M \rightarrow \infty$. The idea behind it is to increase the power of the internal momenta in the denominator and hence make the integral less divergent. This power increases with introducing more regulators until obtaining a convergent integral.

However such regularisation, although respects translation and Lorentz invariance, does not maintain in general the gauge invariance.

3) Lattice regularisation:

In this method the space-time is discretised accordingly the short distance contribution to the space-time and hence the high-momentum behaviour is eliminated. However it breaks the translation symmetry and Lorentz invariance.

4) Dimensional regularisation:

We keep the space-time dimension n lower than 4 and replace divergent 4-dimensional integral by convergent n-dimensional one. After making the integration we obtain an analytical expression as a function of n. At the end we set up $n \rightarrow 4$.

The most interesting feature of this regularisation is that the divergence are extracted as poles in (n-4).

This method respects all symmetries (which make it more suitable for our calculation). For example the translation property is respected, so we are free to perform shifts in integration variable and some results are obtained by making a repeated use of such property.

When replacing the 4-dimensional integral by the the n-dimensional one, some care has to be given to the Dirac algebra in n dimension.

First of all the space time index μ now runs from 0 to n-1, the components of k^{μ} are

$$k^{\mu} = (k^{0}, \ldots\ldots, k^{n-1}) \qquad (B.2)$$

and the contracted metric tensor is

$$g^{\mu}_{\mu} = g_{\mu\nu} g^{\mu\nu} = n \qquad (B.3)$$

The Dirac algebra in n-dimensions remains unchanged in others words the anti-commutation relation still valid

$$\{ \gamma^{\mu}, \gamma^{\nu} \} = 2g^{\mu\nu}. \qquad (B.4)$$

However, when making the contraction of indices we must take into account the equation (B.3), i.e.

$$\gamma^{\mu}\gamma_{\mu} = n1 \qquad (B.5)$$

$$\gamma_{\mu}\gamma^{\nu}\gamma_{\mu} = (2-n)\gamma^{\nu} \qquad (B.6)$$

However the immediate obstacle in the dimensional regularisation is the choice of γ^{5} in n dimensions . In 4 dimension γ^5 is defined by

$$\gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3. \qquad (B.7)$$

It is possible to define γ^5 in even dimension n by

$$\gamma_5 = i\gamma^0 \ldots\ldots\ldots\ldots \gamma^{n-1}. \qquad (B.8)$$

However if we define it in the same way when n is odd , γ_5 turns out to be a multiple of the identity, see Muta [77, p 118].

We require the existence of a quantity called γ_5 which verifies

$$\gamma_5^2 = 1 , \quad \{ \gamma_5 , \gamma_\mu \} = 0 , (\mu = 0 , 1 , \ldots\ldots\ldots\ldots n-1). \qquad (B.9)$$

APPENDIX C.

Feynman parametrisation.

Throughout our calculation we encounter terms of the following form 1/ABC........ where A, B.....are the denominators of the quark, W boson etc. It is useful to change this form to an equivalent one, suitable for performing more easily the integration over the internal loop-momentum. This transformation is called Feynman parametrisation which is widely employed in loop integration.

The general form is, see Muta [77 p 123].

$$\prod_{i=1}^{n} \frac{1}{A_i^{\alpha_i}} = \frac{\Gamma(\alpha)}{\prod_{i=1}^{n}\Gamma(\alpha_i)} \int_0^1 \left(\prod_{i=1}^{n} dx_i\, x_i^{\alpha_i - 1}\right) \frac{\delta(1-x)}{\left(\sum_{i=1}^{n} x_i A_i\right)^{\alpha}} . \quad (C.1)$$

Where

$$\alpha_i \ (i=1, \ldots\ldots\ldots, n) \quad (C.2)$$

are arbitrary complex numbers, and

$$\alpha = \sum_{i=1}^{n} \alpha_i, \quad (C.3)$$

$$x = \sum_{i=1}^{n} x_i. \quad (C.4)$$

However we will need just the Feynmann parametrisation of 1/AB and 1/ABC [78]:

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[(A-B)x+B]^2}, \quad (C.5)$$

and

$$\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{[A+(B-A)x+(C-A)y]^3}. \quad (C.6)$$

Appendix D.

Mathematical methods.

In the calculation of one loop integral we follow the steps below:

1) Feynman parametrisation.

2) shifting the internal momentum over which we perform the integration.

3) the reduction of the integral to a standard formula, which we evaluate by the dimensional regularisation.

The cited formula is

$$I(m,r) = \int \frac{d^n k}{(2\pi)^n} \frac{(k^2)^r}{(k^2 - R^2)^m} \qquad (D.1)$$

which is equal to [79]

$$I(m,r) = \frac{i}{(16\pi^2)^{n/4}} (-1)^{r-m} (R^2)^{r-m+n/2} \frac{\Gamma(r+n/2)\,\Gamma(m-r-n/2)}{\Gamma(n/2)\,\Gamma(m)}. \qquad (D.2)$$

In our case we have m=3 in all diagrams except diagram A where m=2, and n=4-ε. So the only parameter which varies with diagrams is r. It turns out for certain values of r the argument (m-r-n/2) of the Gamma function in I(m,r) is negative; this gives rise to divergences. These infinities show up as poles in ε.

Some properties of Γ function.

$$\Gamma(x+1) = x\,\Gamma(x) \qquad (D.3)$$

$$\Gamma(n) = (n-1)!, \qquad (D.4)$$

where n is in integer.

We have also

$$\lim_{\varepsilon \to 0} \Gamma(\varepsilon) = \frac{2}{\varepsilon} - \eta + \ldots\ldots \qquad (D.5)$$

For ε→0 we have the following development:

$$A^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2}\log(A) + O(\varepsilon^2). \qquad (D.6)$$

In addition to I(m,r) the following integrals are encountered in our calculation.

$$\int \frac{d^n k}{(2\pi)^n} \frac{k_\mu k_\nu}{(k^2 - R^2)^m} = \frac{1}{n} g_{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{k^2}{(k^2 - R^2)^m} \qquad (D.7)$$

and

$$\int \frac{d^n k}{(2\pi)^n} \frac{k_{\mu 1} \cdots k_{\mu r}}{(k^2 - R^2)^m} = 0 \qquad (D.8)$$

if r is an odd integer.

Moreover we have

$$\int \frac{d^n k}{(2\pi)^n} \frac{k^2 k_\mu k_\nu}{(k^2 - R^2)^m} = \frac{1}{n} g_{\mu\nu} \int \frac{d^n k}{(2\pi)^n} \frac{k^4}{(k^2 - R^2)^m}. \qquad (D.9)$$

Using the above formulae we have the following expressions:

$$\lim_{\varepsilon\to 0} \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - R^2)^2} = \lim_{\varepsilon\to 0} \frac{i}{16\pi^2}\left(\frac{2}{\varepsilon} - \eta + \log 4\pi - \log \frac{R^2}{\mu^2}\right) \qquad (D.10)$$

$$\lim_{\varepsilon\to 0} \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - R^2)^3} = -\frac{i}{16\pi^2} \frac{1}{2R^2} \qquad (D.11)$$

$$\lim_{\varepsilon\to 0} \int \frac{d^n k}{(2\pi)^n} \frac{k^2}{(k^2 - R^2)^3} = \lim_{\varepsilon\to 0} \frac{i}{16\pi^2}\left(\frac{2}{\varepsilon} - \eta - \frac{1}{2} + \log 4\pi - \log \frac{R^2}{\mu^2}\right) \qquad (D.12)$$

$$\lim_{\varepsilon\to 0} \int \frac{d^n k}{(2\pi)^n} \frac{k^4}{(k^2 - R^2)^3} = \lim_{\varepsilon\to 0} \frac{3iR^2}{(16\pi^2)}\left(\frac{2}{\varepsilon} - \eta + \frac{1}{6} + \log 4\pi - \log \frac{R^2}{\mu^2}\right). \qquad (D.13)$$

where we have introduced the parameter μ to make the argument of the logarithm dimensionless, however the physical result is, of course, independent of μ.

Appendix E.

The shift.

In order to obtain the same form as the expressions in the previous paragraph we make a shift in the internal momentum k. However such procedure has to be legitimated. Indeed in some types of integrands a shift could create a supplementary contribution. As far as our calculation is concerned there is no problem because the integrand is in the worst case quadratically divergent, hence its derivative vanishes on the surfaces at infinity [80].

The shift is

$$k \to \tilde{k} = k - l. \qquad (E.1)$$

The following development is needed to eliminate terms in odd powers of $k^{\sim}$ in the integral.

$$\not k(\not k + \not\alpha)\not k \to \not k\not k\not k + \not k\not k\not l + \not k(\not l + \not\alpha)\not k + \not l\not k\not k + \not k(\not l + \not\alpha)\not l + \not l\not k\not l$$
$$+ \not l(\not l + \not\alpha)\not k + \not l(\not l + \not\alpha)\not l. \qquad (E.2)$$

Appendix F.

The Spence function.

All one-loop, integrals could be expressed in terms of elementary functions which could be integrated analytically over x except for special one called Spence function which in its turn it is a special case of the generalised polylogarithms of Nielson [81]. Such functions appear just for massive theories at one loop calculation (for example in the case $b' \to Hb$ process where the mass of the Higgs particle and the b' quark are considered to be not negligible with respect to the mass of the W boson). However for massless theories (for example in the process $s \to dH$ where we neglect both the mass of the external quarks and the Higgs particle in respect to the mass of the W boson) such a function will reduce to elementary functions or numerical constants. In this appendix we will present some properties of the Spence function.

The generalised polylogarithms of Nielson are defined by [81]

$$S_{n,p}(x) = \frac{(-1)^{n+p-1}}{(n-1)!\,p!}\int_0^1 \frac{\log^{n-1} t \log^p(1-xt)}{t}dt, \quad n, p \geq 1. \qquad (F.1)$$

For n = p = 1 we have

$$Sp\,(x) = S_{1,1}(x) = -\int_0^1 dx \frac{\log(1-xt)}{t} \qquad (F.2)$$

Sp stands for Spence function, x is a complex variable in general.

Some properties of the Spence function.

$$Sp\,(0) = 0, \qquad (F.3)$$

$$Sp(1) = \frac{\pi^2}{6}, \quad (F.4)$$

$$Sp(x) + Sp(1-x) = \frac{\pi^2}{6} - \log x \log(1-x) \quad (F.5)$$

The following integral could be expresses in terms of the Spence function [81,82]:

$$I = \int_0^1 dx \frac{\log(x-x_0)}{x-x_0} = Sp(\frac{x_1-1}{x_1-x_0}) - Sp(\frac{x_1}{x_1-x_0}) + \log(\frac{1-x_0}{x_1-x_0})\log(1-x_1)$$
$$- \log(\frac{-x_0}{x_1-x_0})\log(-x_1). \quad (F.6)$$

After performing the first integration of the equations (4.2-4.9) see chapter 4, the more general form obtained is the following:

$$Q(x) = \frac{ax^3+bx^2+cx+d}{(x+\alpha)^2}\log P(x). \quad (F.7)$$

The P(x) 's are defined in eqs (4.13-4.16) in chapter (4). Q(x) could be taken as:

$$Q(x) = (\beta x + \gamma + \frac{\delta}{(x+\alpha)^2} + \frac{\lambda}{(x+\alpha)})\log P(x). \quad (F.8)$$

where β, γ, δ and λ could be expressed in terms of a,b,c,d and α.

The last term proportional to λ gives the Spence function.

During our calculation, we need the following formulae:

$$I_1 = \int_0^1 dx \log(ax^2 + bx + c) = \frac{1}{2a}\{ (2a+b)\log(a+b+c) - b\log(c)$$
$$+ 2d \operatorname{atan}(\frac{d}{2c+b}) - 4a \} \text{ for } d>0. \quad (F.9)$$

$$I_2 = \int_0^1 dx\, x\log(ax^2 + bx + c) = \frac{1}{4a^2}\{(2a^2+2ac - b^2)\log(a+b+c)$$

$$+ (b^2-2ac)\log(c) - 2bd\,\text{atan}\left(\frac{d}{2c+b}\right) - 2a^2+2ab\} \quad \text{for } d>0. \qquad (F.10)$$

$$I_3 = \int_0^1 dx\, \frac{\log(ax^2+bx+c)}{(x+\alpha)^2} = \left[\frac{(2a\alpha+b\alpha-b-2c)}{2(\alpha+1)}\log(a+b+c) + \frac{(-b\alpha+2c)}{2\alpha}\log(c)\right.$$

$$\left. + (-2a\alpha+b)\log\left(1+\frac{1}{\alpha}\right) + d\,\text{atan}\left(\frac{d}{2c+b}\right)\right]\frac{1}{(a\alpha^2-b\alpha+c)}, \quad \text{for } d>0. \qquad (F.11)$$

where

$$d = 4ac - b^2. \qquad (F.12)$$

For $d < 0$ we have another expressions for the I's which are the analytical continuation of previous ones which could be obtained by making the following transformation:

$$\text{atan}\, z = \frac{1}{2i}\log\frac{1+iz}{1-iz}. \qquad (F.13)$$

Appendix G.

The Kobayashi - Maskawa Matrix.

In the standard model left handed quarks come in doublets and right handed in singlets. These doublets are arranged in families as

$$\begin{bmatrix} u \\ d' \end{bmatrix} \quad \begin{bmatrix} c \\ s' \end{bmatrix} \quad \begin{bmatrix} t \\ b' \end{bmatrix} \qquad (G.1)$$

where the weak eigenstates, d' , s' , b' are mixture of the mass eigenstates, d, s, b. The transformation matrix which relates the two basis is called the Kobayashi- Maskawa (K-M) matrix.

The u , c , t are assumed (by convention) unmixed.

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \qquad (G.2)$$

Each of the elements of this matrix represents the coupling of a charge 2/3 quark to a charge -1/3 quark. There is no mixing angles between quarks having the same charge. The K-M matrix can be expressed in terms of three angles and one phase. The latter is responsible of the CP violation.

There are various representations of the K- M matrix, the one used in our calculation is due to Wolfenstein.

The Wolfenstein's parametrisation [83]:

The matrix elements are expanded in terms of small parameter $\lambda = \sin \theta_c$, where θ is the Cabbibo angle. Using the unitarity of the matrix, we obtain

$$V : \begin{bmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} \qquad (G.3)$$

The coefficients A, ρ and η are of order one or smaller:

$$0.218 \le \lambda \le 0.222, \qquad 0.80 \le A \le 1.2, \qquad 0 \le \sqrt{\rho^2+\eta^2} \le 1.0. \qquad (G.4)$$

In our calculation we adopt the following pattern (θ_c is the Cabbibo angle taken to be equal to 0.2) where we ignore the CP phase. For more details see Barger and Phillips [86 p 62]:

$$|V| \approx \begin{bmatrix} 1 & \theta_c & \theta_c^3 \\ \theta_c & 1 & \theta_c^2 \\ \theta_c^3 & \theta_c^2 & 1 \end{bmatrix} \qquad (G.5)$$

For the four generation scheme, an extrapolation of this 3 x 3 mixing matrix is often assumed see Barger and Phillips [74] and Bager [84].

$$|V| \approx \begin{array}{c|cccc|} & d & s & b & b' \\ u & 1 & \theta & \theta_c^3 & \theta_c^5 \\ c & \theta_c & 1 & \theta_c^2 & \theta_c^4 \\ t & \theta_c^3 & \theta_c^2 & 1 & \theta_c^3 \\ t' & \theta_c^5 & \theta_c^4 & \theta_c^3 & 1 \end{array} \qquad (G.6)$$

However there is a K- M matrix where the elements are obtained from a phenomological point of view, i.e. they are calculated basically from measurements of weak decays of light and heavy quarks and from neutrino production of charm quarks. For more details see Kleinknecht [85,86]

In four generation this matrix is see Kleinknecht [85,86]

	d	s	b	b'
u	0.9730-0.9758	0.218-0.224	0.0-0.008	0.0-0.07
c	0.180-0.229	0.84-0.98	0.039-0.050	0.0-0.50
t	0.0-0.13	0.0-0.47	0.0-0.999	0.0-0.999
t'	0.0-0.14	0.0-0.49	0.0-0.999	0.0-1.0

(G.7)

REFERENCE.

[1] S. Glashow, J. Iliopoulos and L. Miani, Phys. Rev. D **2**, 1285 (1970).

[2] M. Kobayashi and K. Maskawa, Prog. Theo. Phys. **49**, 652 (1973).

[3] M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).

[4] R. S. Galik, Proceedings of the Second Lake Louise Winter Institute on New Properties in Particles, February 1987, Canada, ed World scientific.

[5] H. Schroder, Rep. Prog. Phys. **52**, 767 (1989).

[6] C. S. Lim, Proceedings of the Second Meeting on Physics at Tev Energy Scale, KEK, Tsubuka, Ibaraki, Japan, May 1988.

[7] B. Mukhopadhyaya and A. Raychaudhuri, Phys. Rev. D. **39**, 280 (1989).

[8] W. S. Hou and R. G. Stuart, MPI - PAE / PTh 24/89, April 1989.

[9] W. S. Hou and R. G. Stuart, MPI - PAE / PTh 55/88, October 1988. Published in Nucl. Phys. B 320, 277 (1989).

[10] T. Inami, RIPF. 800 April 1989.

[11] C. Busch, DESY 88-148 , UTP-UH 9/88, September 1988. published in Nucl. Phys. B 319, 15, 1989.

[12] W. S. Hou and R. G. Stuart, Phys. Rev. Lett. **62**, 617 (1989).

[13] W. S. Hou, MPI - PAE / PTh 72/88, December 1988.

[14] W. S. Hou and R. G. Stuart, MPI - PAE / PTh 54/88, December 1988.

[15] M. Clements, C. Footman, S. Narasimhan, and D. Photiadis, Phys. Rev. D. **27**, 570 (1983).

[16] V. Ganaphati, E. Laermann, I. Schmitt, T. Weiller, and P. Zerwas, Phys. Rev. D. **27**, 579 (1983).

[17] M. J. Duncan, CERN-TH. 5429/89.

[18] M. J. Duncan, Phy. Rev. D. **31**, 1139 (1985).

[19] E. Ma and A. Pramudita, Phys. Rev. D. **22**, 214 (1980).

[20] W. Buchmüller and M. Groneau, DESY 88-171 (1988).

[21] F. J. Gilman and S. H. Rhie, Phys. Rev. D. **26,** 1898 (1989).

[22] A. Axelrod, Nucl. Phys. B **209** , 349 (1982).

[23] M. J. Savage, CALT-68-1496 (1988).

[24] R. S. Willey and H. L. Yu, Phys. Rev. D. **26** , 3086 (1982) .

[25] R. S. Willey and H. L. Yu. Phys. Rev. D. **26** , 3287 (1982).

[26] B. Grzadkowski and P. Krawczyk, Z. Phys. C **18** , 43 (1983).

[27] F. J. Botella and C. S. Lim, Phys. Rev. Lett. **56,** 1651 (1986).

[28] F. J. Botella and C. S. Lim, Phys. Rev. D. **34,** 301 (1986).

[29] R. M. Godbole, U. Türke and M. Wirbel. Phys. Lett. B **194** , 302 (1987).

[30] R. S. Chivukula, A. N. Manohar, Phys. Lett. B **206,** 86 (1988).

[31] J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **106,** 292 (1976).

[32] L. Hall and M. B. Wise, Nucl. Phys. B **187** , 397 (1981).

[33] R. Ruskov, Phys. Lett. B **187,** 165 (1987).

[34] B. Grinstein, L. Hall and L. Randall, Phys. Lett. B **211** , 363 (1988).

[35] R. S. Willey, Phys. Rev. D. **39,** 2784 (1989).

[36] J. G. Körner, N. Nasrallah and K. Schilcher, MZ-TH/ 89-05, February 1989.

[37] B. Haeri, G. Eilam and A. Soni, Phys. Rev. Lett **62** , 719 (1989).

[38] P. Krawczyk, DESY. 89-058.

[39] S. Glashow and S. Weinberg, Phys. Rev. D **15,** 1958 (1977).

[40] R. Gatto, Jerusalem Einstein Centenial Symposium on Gauge Theories and Unification of Physical Forces, edited by Y. Ne'eman, Addison 1981.

[41] R. Gatto, Proceedings of the International Shool of Physics " Enrico Fermi " Course LXXI, edited by M. Baldo Ceolin, 1979.

[42] I. Hinchliffe, Proceedings of the Second Lake Louise Winter Institute on New Properties in Particles, February 1987, Canada, ed World scientific, p 266.

[43] S. Coleman and E. Weinberg, Phys. Rev. D. **7**, 1888 (1973).

[44] M. Sher, Phys. Rep. **179**, 273 (1989).

[45] D. B. Cline, Proceedings of the Second International Symposium on the Fourth Family of Quarks and Leptons, edited by D. B. Cline and A. Soni to be published by the end of this year.

[46] S. Raby, G. W. West and C. M. Hoffman, Phys. Rev. D **39** , 828 (1989).

[47] J. F. Gunion, Proceedings of the XXIV international conference on High Energy Physics, Munich West Germany, 1988.

[48] J. Lee-Franzini, Proceedings of the XXIV International Conference On High Energy Physics, Munich West Germany, 1988.

[49] S. Dawson, J. F. Gunion and H. E. Haber, UCD-89-12, SCIPP-89-22.

[50] M. Spiro, DPhDE 89-12.

[51] S. P. Chia, Phys. Lett. B **147**, 361 (1984).

[52] W. J. Marciano and A. Sirlin, Nucl. Phys. B **93**, 303 (1975).

[53] N. G. Deshpande and G. Eilam, Phys. Rev. D. **26**, 2463 (1982).

[54] G. Feinberg, P. Kabir and S. Weinberg. Phys. Rev. **3** , 527 (1959).

[55] C. G. Callan, Jr, S. Coleman and R. Jackiw, Ann. Phy. (NY). **59**, 42, (1970).

[56] M. K. Gaillard, private communication.

[57] R. Jackiw et al, Lectures on Current Algebra and its applications. Princeton University Press.

[58] J. Ellis and P. J . Franzini, CERN-TH. 4952/88.

[59] M. S. Chanowitz, Ann. Rev. Nucl. Part. Sci. **38**, 323 (1988).

[60] B. Grinstein and C. T. Hill, Fermi-Pub-88/190-T. Submitted to Phys. Lett. B.

[61] M. E. Machacek and M. T. Vaughn, Nucl. Phys. B **236**, 221 (1984).

[62] C. Hamzaoui, A. I. Sanda and A. Soni. Phys. Rev. Lett. **63**, 128 (1989).

[63] P. J. Franzini, Phys. Rep. **173**, 1 (1989).

[64] E. A. Paschos and U. Türke, Phys. Rep. **178**, 4 (1989).

[65] F. J. Gilman and R. Kauffman, Phys. Rev. D. **37** , 2676 (1988).

[66] J. L. Cortes, X. Y. Pham and A.Tounsi, Phys. Rev. D. **25**, 188 (1982).

[67] J. L. Rosner, Lectures presented at the Banff Summer Institute, Banff, Alberta, Canada, August 1988. To be published in the Proceedings (World scientific, Singapore, 1989).

[68] V. Barger and R. Phillips in Collider Physics, Addison-Wesly, 1987.

[69] W. J. Marciano, Proceedings of the First International Symposium on the Fourth Family of quarks and Leptons , edited by D. B. Cline and A. Soni [Ann. N. Y. Acad. Sci. **518**, 1987]

[70] V. Barger, J. L. Hewett and T. G. Rizzo, MAD/PH/ 481, March 1989.

[71] I. Bigi, Y. Dodshitzer, V. Khoze, J. Kühn and P. Zerwas, Phys. Lett. B **181**, 157, (1986).

[72] M. B. Voloshin, Sov. J. Nucl. Phys. **24**, 422 (1976).

[73] R. N. Cahn, LBL-27134.

[74] V. Barger, R. Phillips and A. Soni, Phys. Rev. Lett. **57**, 1518 (1986).

[75] V. Barger and R. Phillips in Collider Physics, Addison-Wesly, 1987, p 432.

[76] D. Bailin and A. Love, Introduction to Gauge Theories, Adam Hilger, 1986, chapter 14.

[77] C. Muta, Foundations of Quantum Chromodynamics, World scientific, 1987.

[78] F. Mandl and G. Shaw, Quantum Field Theory, John Wiley, 1986, p. 224.

[79] S. Narisson, Phys. Rep. **84**C, 263 (1982).

[80] T. P. Cheng and L. F. Li, Gauge Theories of Elementary Particle Physics, Oxford University Press, 1984. p 176.

[81] R. Barbieri, Nuovo. Cim. **11** A, 824 (1972).

[82] G. Passarino and M. Veltman, Nucl. Phys. B **160**, 151 (1979).

[83] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[84] V.Barger, Proceedings of the First International Symposium on the Fourth Family of quarks and Leptons, edited by D. B. Cline and A. Soni [Ann. N. Y. Acad. Sci. **518**, 1987]

[85] K. Kleinknecht, B. Renk. Z. Phys. C **34** , 209 (1987).

[86] K. Kleinknecht, Proceedings of the XXIV International Conference On High Energy Physics, Munich West Germany, 1988.

[87] Particle Data Group, Phys. Lett. B **204**, 1, (1988).