A STUDY OF MODE CHOICE
FOR THE JOURNEY TO WORK IN GLASGOW

by

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This thesis is dedicated to

my family, wife and children.

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SUMMARY

Selected data, obtained from a conventional household interview survey, conducted in 1978 and 1979 as a part of the Glasgow Rail Impact Study, were used to study mode choice for journeys to work in the city of Glasgow and to identify the most significant factors influencing that choice. A number of disaggregate multinomial logit mode choice models with five modes, viz. car driver, car passenger, bus, train and walk, were investigated initially. On the basis of validation tests and statistical evaluation, two of the models, one simple and one complex, were identified as being the best-specified and were then used for aggregate prediction and policy change analyses.

In general, the study has demonstrated the feasibility of using the multinomial logit approach to the development of multi-modal disaggregate travel demand models and that such models can be calibrated using data from a traditional household interview survey. More particularly, the major influencing factors on the mode choice decision were identified: travel time was found to be more significant than travel cost, which was also found to have the wrong (i.e. positive) sign; the central business district was found to affect significantly the choice of public transport modes; distance was found to have a significant effect on the choice of the walk mode; and car availability and the position in a household were found to be significant influences on the use of a car.

The aggregate prediction analysis revealed the feasibility and desirability of using disaggregate models for such analyses and confirmed the superiority of the simple model over the complex one. It was concluded from the policy change analysis

that changes in travel times would affect mode choice significantly but that changes in travel costs would not.

CHAPTER ONE

INTRODUCTION

1.1 General

1.2 Study Objectives

1.3 Outline of the Study

CHAPTER ONE

INTRODUCTION

1.1 GENERAL

Since the mid–1960s there has been increasing interest in mathematical models of urban transportation systems. Such models can assist transportation planners and decision–makers in understanding existing travel patterns and predicting future transportation needs. Within the sphere of travel demand modelling the specific problem of modal split or mode choice is of particular interest; mode choice modelling is of the utmost importance when deciding among alternative transportation proposals and, as yet, universally accepted mode choice models have not been developed. Throughout the transportation modelling process the need for understanding of travel behaviour at the level of the individual traveller is paramount.

1.2 STUDY OBJECTIVES

The city of Glasgow is the focus of industrial, commercial and retail activity for the associated conurbation with its population of around 1.7 million; this study is an examination of mode choice for journeys to work in Glasgow. The principal objective of the study is the identification of the most significant factors influencing the choice of transportation mode through the development of mode choice models. To achieve this objective the study employs recently developed disaggregate behavioural probabilistic choice models of the multinomial logit (MNL) type. The models have been calibrated using disaggregated data obtained from the Glasgow Rail Impact Study (GRIS) survey in 1978–1979. This survey was carried out independently of the present study which has consequently been constrained, to a degree, by the quality of the available data. Nevertheless, this study illustrates that disaggregate probabilistic choice models can be successfully developed from data obtained by traditional survey methods and can provide useful analyses of travel demand. This confirms the feasibility of using such models in the transportation planning process.

The present study also examines the issues involved in using disaggregate probabilistic choice models for the prediction of aggregate travel behaviour and the estimation of the sensitivity of transport mode choice to various changes in policy–controllable variables. The results may assist urban transportation planners and decision–makers to shape their pricing and investment policies, effecting more efficient utilization of transportation resources, and to anticipate future transportation needs.

1.3 STUDY OUTLINE

A general review, together with a discussion of the conventional urban transportation model system (UTMS) and the various alternative approaches to travel demand modelling are presented in Chapter 2. These provide an introduction to the understanding of urban travel demand modelling and emphasise the usefulness of analysing travel behaviour at the individual level. The theoretical framework for modelling individual travel behaviour with respect to the choice of transportation mode is outlined in Chapter 3. This involves the presentation of the deterministic and the probabilistic choice theories; the generation of two important choice models, the MNL and the multinomial probit (MNP) models; statistical techniques for the estimation of the unknown parameters of the various MNL model specifications; and statistical goodness-of-fit measures for assessing the validity of the various calibrated models. Finally, the specific issues of the specification of variables in the utility function and choice set generation are discussed.

Having provided the general form of the MNL model, the next stage of the study is concerned with the empirical analysis of the journey to work in the city of Glasgow. In Chapter 4 a brief description of the GRIS survey data and the study area are presented. Descriptions of the sample preparation and the investigation of the practical limitations of and problems inherent with the use of the GRIS data are also given. The chapter concludes with the selection of the most important explanatory variables for inclusion in the various model specifications and an explanation of how the variables are represented in the model formulations. In Chapter 5 various model specifications are calibrated and evaluated statistically and the final model forms are selected. The results obtained are then compared with previous analyses of journeys to work.

The following two chapters are concerned with the use of the selected models in the prediction of aggregate travel behaviour and with policy change analysis. Chapter 6 presents and discusses the aggregation problem; the available aggregation procedures; and the various sources and types of aggregation errors. Finally, the empirical results of using the naive, classification, and enumeration procedures are presented and a comparative assessment of their desirability in terms of their aggregation error values is made. Chapter 7 then deals with the prediction of the effects of a wide range of policy decisions on the choice of transport mode. The properties of a policy– sensitive model and the various techniques available for analysing different policy decisions are also presented. Lastly, the impacts of various policy changes on the aggregation error values of different aggregation procedures are examined. The final chapter presents the general conclusions of the study and the suggested directions for further research.

CHAPTER TWO

TRAVEL DEMAND MODELLING: AN OVERVIEW

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2.1 INTRODUCTION

The objective of this chapter is to put in perspective what has been done over the past three decades in the area of travel demand modelling. Without claiming to be exhaustive, the chapter reviews and discusses that work which is considered to contribute to the understanding of the specific problem of modal split or mode choice modelling.

The topics covered in this chapter are organised into three major areas: the conventional Urban Transportation Model System (UTMS) and the aggregate and disaggregate travel demand modelling approaches.

The first section presents a brief description of UTMS. The second section focuses on the earlier modelling approaches concerned with the development of UTMS and based on the prediction of travel demand at an aggregate level using zonal or city characteristics. These techniques are often called "aggregate modelling approaches". The last section is concerned with recent developments in modelling and analysing individual traveller behaviour. These procedures are usually called "disaggregate behavioural modelling approaches".

2.2 THE URBAN TRANSPORTATION MODEL SYSTEM

The general approach to forecasting travel demand in transportation studies has traditionally been through the well known sequence of the prediction of trip generation, trip distribution, modal split and traffic assignment. This set of four models, which is collectively called UTMS, has been the most widely used technique for the prediction of future travel demand.

Figure 2.1 presents the most typical UTMS structure. As may be seen from this figure, UTMS consist of a series of models which are executed sequentially, with the output of one model comprising the input for the next. Each model predicts one aspect of transportation demand viz. total trips leaving and entering each zone; the proportion of trips leaving a zone going to each possible destination; the proportion of these trips using each available mode of transport; and finally, the routes taken by these trips through the transport network. The serial model shown in Figure 2.1 is a simplification of a more complex recursive process. Outputs from later stages of the model are used as inputs to earlier stages (feedbacks) in the iterative process required in the solution of a more realistic model.

UTMS is discussed at length in virtually every transportation planning text[1] and is also well documented in the literature[2]. Nevertheless, a brief exposition of the constituent models is essential, since they form the conceptual framework within which transportation demand theory and practice have evolved.

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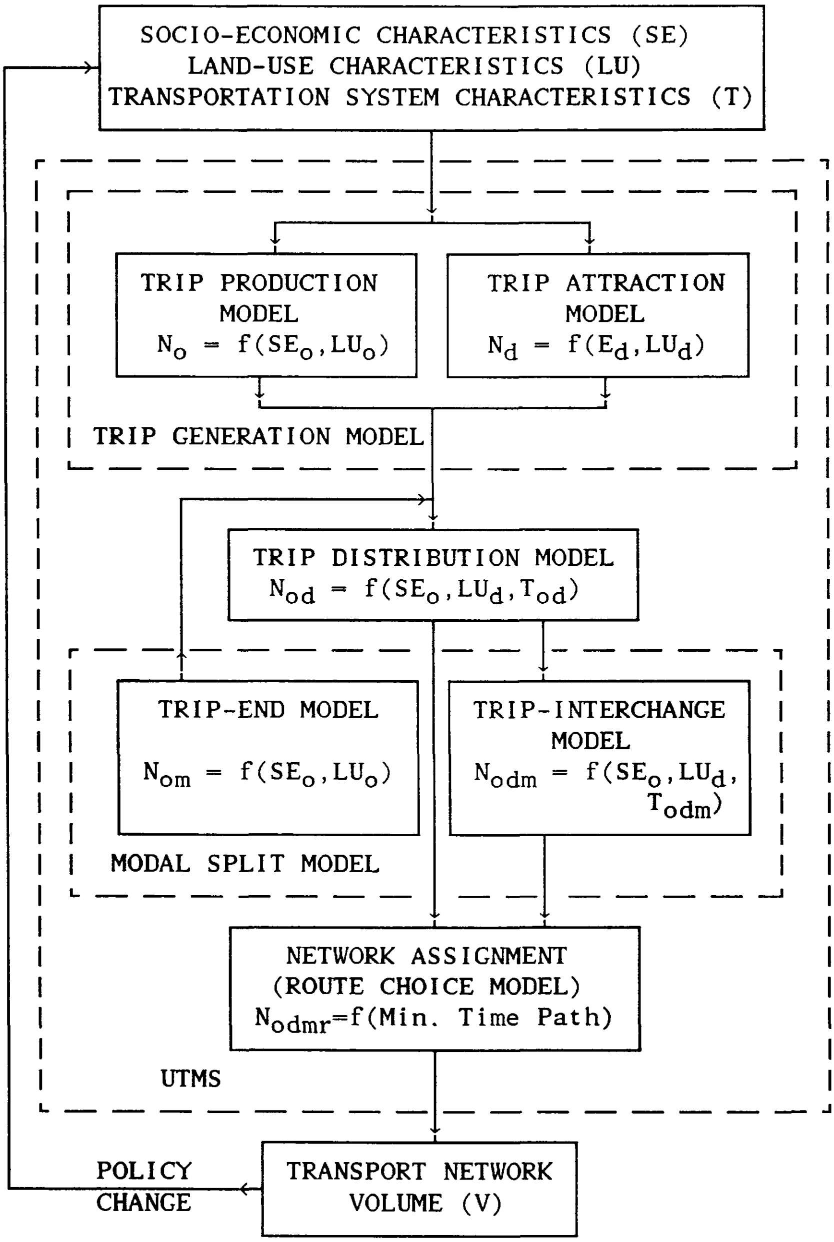
[1] See for example: Oi and Shuldiner (1962); Overgaard (1966); Hutchinson (1974); Burton (1975); Stopher and Meyburg (1975); Salter (1976); Morlok (1978).
[2] See for example: Stopher and Lisco (1970); Hartgen and Tanner (1971) Reichman and Stopher (1971); Charles River Associates (CRA) (1972, 1976); Ruiter (1973); Wilson (1974); Domencich and McFadden (1975).



where,

o	is the origin zone.
d	is the destination zone.
m	is the travel mode.
r	is the route chosen.
E_d	is the employment of the destination zone.
T_{od}	is the distance between the origin and destination zones, and
T_{odm}	is the travel time between the origin and destination zones.

FIGURE 2.1 The urban transportation planning process.

2.2.1 THE TRIP GENERATION MODEL

The trip generation model predicts the total number of trips starting and finishing in each zone in the study area; that is, for each zone, the total number of trip productions (i.e. trips originating in the zone, regardless of their destinations) and trip attractions (i.e. trips destined for the zone, regardless of their origins). All models of trip generation assume that the trip generation rate is a function only of the spatial and socio-economic characteristics of the generating zone and not the characteristics of the zone at the other end of the trip, or of the level of service provided by the transportation system connecting the two zones. The output from these models are zonal trip productions and attractions, typically disaggregated by trip purpose (e.g. work or shopping) and by trip type (e.g. home-based or non home-based). Regression and category analyses are the most commonly used techniques in the evaluation of trip generation[1].

2.2.2 THE TRIP DISTRIBUTION MODEL

The trip distribution model takes the zonal trip productions and attractions predicted by the trip generation model and links them together to predict the total flow between each production zone and each attraction zone. The distributed flow is a function of the socio-economic characteristics of the origin zone, the land-use characteristics of the destination zone and the time and distance between these zones. While many techniques exist for estimating trip distribution, the overwhelmingly dominant one is the gravity model[2].

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1 See for example: Shuldiner (1962); Federal Highway Administration (1967); Wotton and Pick (1967); Kassoff and Deutschman (1969); Douglas and Lewis (1970); Kannel and Heathington (1973); Tardiff (1977); Mohamad (1978).

2 See for example: Wilson (1967, 1969); Phibrick (1971); Cochrane (1975).

2.2.3 THE MODAL SPLIT MODEL

The modal split or mode choice model is concerned primarily with the allocation of the various trips which have been predicted among all the available modes of transport (e.g. car, public transport, walking). A modal split model is classified as being either a trip-end model (if it follows immediately after the trip generation model in the UTMS sequence) or a trip-interchange model (if it follows the trip distribution stage). As shown in Figure 2.1, trip-end models, since they precede the trip distribution phase (and hence destinations and possible routes are not known), cannot utilise the transportation system characteristics in their predictions and must depend upon the same set of socio-economic characteristics as are used in trip generation. This approach is clearly most reliable where a high proportion of public transport users are captive. The trip distribution model which follows the trip-end model involves the construction of separate distribution models for each mode of transport [Stopher and Meyburg(1975)].

Since the trip-interchange model follows the trip distribution phase, the origin-destination flows are already known (i.e. the distribution of total trips from all origins to all destinations is assumed completed). Then, based on transport service levels for each zonal interchange (origin-destination pair) as well as zonal socio-economic and land-use characteristics, the allocation of total travel is made among the available transport modes. The trip-interchange model permits the best possible reflection of the effects of relative service levels of the transport modes that exist between each pair of origin and destination zones.

2.2.4 THE TRAFFIC ASSIGNMENT MODEL

The traffic assignment model or, as it is sometimes called, the route choice model, takes the flows between each pair of origin and destination zones for a given mode and assigns them to one or more specific routes through the transport network. Conventionally, flows are assigned by route on the basis of minimum time paths and, therefore, the assignment process becomes that of attempting to allocate trips to a minimum time path through the network between the zone of production and the zone of attraction. The majority of traffic assignment models have focused on the car as the main mode of interest given its dominant role in causing traffic congestion. Public transport trip assignment is often a relatively straightforward task, primarily because there typically exists one dominant path between any given origin and destination pair.

2.3 THE AGGREGATE MODELLING APPROACH

Traditionally, travel demand models (i.e. UTMS) have been developed using aggregated data, mostly on the basis of traffic zones, and these models have generally attempted to predict aggregated traffic flows between pairs of zones. The explanatory variables included in these aggregate models represented in most cases the mean values of some characteristics that were somehow distributed across the zonal populations (e.g. average zonal income, average zonal car-ownership, average travel time between zones) and this made the use of the models conveniently simple. Prediction then involves the application of these zonal averages to each of the travellers in the zone. Thus, an implicit assumption made in using aggregated data is that the characteristics of the individual travellers and the attributes of the transportation system within each zone are relatively homogeneous, as compared with differences between the zones. Hence, each zonal group can be suitably represented by an average value of each explanatory variable.

In fact, zones are never homogeneous as the aggregate approach implies, because some degree of within-zone variance is inevitable. As has been shown by Fleet and Roberston (1968) and McCarthy (1969), the within-group variance that is neglected by averaging the data over traffic zones tends to be greater than the between-groups variance. This dispersal of the actual values around the mean can be great, and it is these actual values that are relevant in analysing and predicting travel behaviour. These intra-zonal variations are concealed in aggregation.

Thus, aggregation before the model construction phase of the analysis will cloud the underlying behavioural relationships and result in a significant loss of information. It may in some cases also create ecological fallacies in the statistical inferences, whereby factors that coincidentally dominate the behaviour of the arbitrary groups of an aggregate analysis are interpreted as affecting the behaviour of individuals[1].

An aggregate model that is largely based on an associated relationship in the data rather than on peoples' behaviour and preferences does not necessarily represent an individual traveller's behaviour, nor the average behaviour of the aggregated group under a variety of conditions. Therefore, there is no reason to expect that the same relationships would hold in other instances or in other locations [Richards and Ben-Akiva (1975)].

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1 See for example: Robinson (1950); De Neufville and Stafford (1971); DeDonnea (1971).

In addition to the above problems concerning the use of aggregated data, a number of other shortcomings in the aggregate modelling approach may be noted. Firstly, it is inflexible and static rather than dynamic. That is, it is based upon measurements and estimated relationships from a single point in time, with an assumption that these relationships and estimates will not change over time except in terms of extraneous changes in total population, wealth, etc. Secondly, an important failing has been the exclusion of the effects of transport system–controllable variables, so that the modelling processes do not respond precisely to transport policy changes. Finally, there is the separation of travel demand prediction into four stages (trip generation, trip distribution, modal split, and traffic assignment) which are assumed to interact in a logical fashion to represent a complex travel behavioural process. The individual models were for the most part developed and modified independently of each other and the well known problem of trip generation being assumed to be independent of the supply of transportation is a direct consequence of this separability assumption[1].

In response to some of the above shortcomings, a number of strategies have been adopted to improve the aggregate approach to travel demand forecasting. Firstly, attempts have been made to improve the internal efficiency and applicability of the aggregate models by giving them more rigorous theoretical bases, by considering more exactly the determination of important variables, and by analysing the interactions among demand, cost and pricing[2]. Secondly, a simultaneous travel modelling approach has been used in an attempt to make the

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1 See for example: Ben–Akiva (1974); Burnett (1974); Liou and Talvitie (1974); Domencich and McFadden (1975); Richards and Ben–Akiva (1975); Stopher and Meyburg (1974); Dalvi (1978).

2 See for example: Moses and Williamson (1963); Beesley (1965); Meyer, Kain, and Wohl (1966); Wilson (1967); Der Serpa (1971); Evans (1972); Cochrane (1975); Fairhurst (1975); Goodwin (1976); Heggie (1976); Zahavi (1977); Bruzelius (1979).

entire aggregate approach much more interactive and the models themselves more plausible and more responsive to policy changes. This development, which combines the stages of trip generation, modal split, and trip distribution, is called the "direct demand model". This model seems to be conceptually more valid than the conventional models since it takes into account the effect of changes in system characteristics on trip generation[1]. However, due to the large number of alternative trips which a traveller may face and the large number of attributes which describe an alternative trip, a simultaneous model can become more complex and computationally more difficult. Therefore, this raises some important issues concerned with the feasibility of a simultaneous model and the sensitivity of travel predictions to the simplifying assumption of a recursive structure. Thirdly, in order to avoid problems related to the use of the aggregate modelling approach, researchers have attempted to develop a completely new approach explaining travel behaviour at the level of the individual traveller. This approach is termed the "disaggregate behavioural modelling approach".

2.4 THE DISAGGREGATE MODELLING APPROACH

Disaggregate travel demand models represent a relatively new development in travel demand forecasting, the first models having appeared at the beginning of the 1960s, and having evolved slowly into the latter part of the decade[2]. Those models were initially developed as research tools, the main objective of the

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1 See for example: Kraft (1963); Quandt and Baumal (1966); Kraft and Wohl (1967); McLynn and Goldman (1967); Plourde (1968); Hartgen and Tanner (1970); Stopher and Lisco (1970); Reichman and Stopher (1971); CRA (1972); Shepherd (1974); Richards and Ben-Akiva (1975); Stopher and Meyburg (1975, 1980); Adler and Ben-Akiva (1976).

2 See for example: Warner (1962); Quarmby (1967); Lave (1967); Lisco (1967); Stopher (1969).

analyses being to improve the understanding of traveller decision-making behaviour. Since then, development of the disaggregate modelling technique has accelerated markedly as a result of growing disenchantment with the conventional aggregate approaches and in the hope that the newer approach has the potential to replace the conventional method[1].

Interest in disaggregate behavioural models can be justified on several counts. Firstly, disaggregate modelling provides a most natural setting for the development of causal relations among their components, based on simple assumptions about the behaviour of the decision-maker. Secondly, they usually allow a building block approach that can be extremely useful as a strategy for the development of urban models based on interrelated blocks describing the urban transportation system and the housing, educational, and other sectors. Thirdly, they provide useful guidance as to the appropriate way to aggregate data in the development of more efficient and operational aggregate prediction models [Koppelman (1974)]. Fourthly, using disaggregate data directly in disaggregate travel models can bring about large savings in the cost of data collection and processing. Since the data are used in the original disaggregate form, and are not aggregated to the zonal level, a large-scale home interview survey is not essential as is the case with the aggregate models [Ben-Akiva (1973)]. Finally, because travel decisions and factors that influence them are measured and analysed at the individual decision-maker level, using disaggregate data seems more plausible in the sense that actual behavioural relationships may be reflected in a more successful model rather than in simple exploitation of ecological correlations in the data. This provides increased confidence in the process of forecasting future travel demand.

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[1] See for example: Ben-Akiva (1973); Watson (1973, 1974); Domencich and McFadden (1975); Richards and Ben-Akiva (1975); Stopher and Meyburg (1975); Brand (1976); Bullen and Boekenkroeger (1979); Burnett and Thrift (1979).

The experience of previous work with the disaggregate travel modelling approach indicates that it is a feasible approach and the most promising avenue for improving future travel forecasting techniques [Stopher, Meyburg, and Brog (1981)].

CHAPTER THREE

MODE CHOICE MODELLING: THE THEORETICAL FRAMEWORK

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3.1 INTRODUCTION

The primary objective of this chapter is to outline the theoretical framework which is appropriate in modelling individual choice behaviour and, within this framework, to derive a tractable mathematical model of mode choice, namely, the MNL model.

The remainder of the chapter is divided into four sections. The first section is a survey of some principles of the deterministic and the probabilistic theories of individual choice behaviour. In the second section, the regression analysis technique and the maximum likelihood method are discussed. The latter has been chosen here as the most suitable technique for calibrating the MNL model.

To assess the validity of various calibrated models, different statistical goodness-of-fit measures are presented in the third section. These are the t-test for assessing the significance of each specified variable, the log likelihood ratio test, the log likelihood ratio index test, and the percentage of observations correctly predicted. The last three tests are used to assess the statistical significance of various models calibrated using the maximum likelihood method.

The last section presents and discusses some specific issues related to the development of various discrete choice models. These issues are concerned with the various ways of specifying different explanatory variables in the utility function of each available alternative and the definition of the set of available alternatives for each individual in the sample population.

3.2 THEORIES OF TRAVEL CHOICE BEHAVIOUR

In general, in the transportation planning process, planners and researchers are interested in the behaviour of aggregate groups of travellers. However, this aggregate behaviour is the result of individual or disaggregate behaviour. Thus, the modelling of individual behaviour is either explicitly or implicitly at the core of all prediction models of aggregate behaviour.

Although disaggregate behavioural travel demand models offer great promise for future travel demand analysis, a fully operational model has still to be developed. The reason for this is that there does not exist a single, universally accepted behavioural choice theory which adequately explains the observed choice behaviour of each individual and predicts their future travel demands[1]. Therefore this section is designed to present the various theories of individual travel choice behaviour.

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[1] See for example: Stoner and Milione (1975); Atiken (1977, 1986); Dalvi (1978); Kanafini (1983); Supernak (1983, 1984); Supernak and Stevens (1987); Ben-Akiva and Lerman (1985).

3.2.1 THE DETERMINISTIC CHOICE THEORY

Although the earlier disaggregate travel demand models[1] have achieved considerable success in their applications, their theoretical foundations have remained weak and, at times, even shaky. This is mainly the result of the inadequacy of the well-known conventional microeconomic consumer theory[2] in dealing with the problems inherent in transportation demand analysis[3]. In particular, the major difficulties encountered relate to the identification of an independent set of travel alternatives and the choice decision among them. It is widely agreed that travel alternatives are best defined in terms of trip characteristics rather than their name or the sort of physical equipment of which they are composed. Unfortunately, the conventional microeconomic theory was developed without any assumption as to the nature of the alternatives from which the consumer has to make a choice. However, the new approach to the microeconomic theory which has been suggested by Lancaster (1966) has paved the way for the development of more sound theoretical structures for analysing travel decisions behaviour. Lancaster postulated that utility or preference is derived not from the actual commodities themselves but from the characteristics which they possess. The most important advantage of this approach to travel demand analysis was that the difficulties of identifying independent sets of travel alternatives were overcome. Alternatives could now be defined by their attributes, such as travel time and travel cost. Hence, individuals could choose the alternative which maximised their derived utility; the corresponding vector of

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1 See for example: Warner (1962); Beesly (1965); Quarmby (1967); Lisco (1967); Lave (1968); Stopher (1969); Blackburn (1970); Golob and Beckmann (1971).

2 See for example: Lancaster (1966); McFadden and Winter (1970); Henderson and Quandt (1971); Green (1978); Layad and Walters (1978); Varian (1978); Kanafani (1983); Ben-Akiva and Lerman (1985).

3 See for example: Hanson (1974); Dalvi (1978); Manheim (1979, 1981); Kanafani (1983); Ben-Akiva and Lerman (1985).

characteristics then determined their observed travel choices.

However, travel demand analysis differs from traditional microeconomic theory in that the choices of concern in the former field usually are among qualitative and discrete sets of alternatives (e.g. destinations, modes, routes), whereas the latter field is concerned with choices among continuous sets of alternatives. Consequently, the standard mathematical techniques of microeconomics, which rely heavily on the assumption of choice among a continuum of alternatives, are no longer applicable to travel demand analysis [Horowitz (1985)]. Thus, a discrete representation of the alternatives necessitates a different analytical approach which is also based on the principle of utility maximization and the rational choice behaviour of the decision-maker. The only difference from the conventional microeconomic consumer theory is that, instead of deriving demand functions, this approach is concerned directly with the comparative utilities of the alternatives as the basis for specifying the resulting choice (i.e. a utility value is associated with each alternative in the choice set, and is used to compare the alternatives; the alternative with the highest utility is chosen). Therefore, modelling of the choice decision is formalised as follows:

Assume that an individual (n) faces a set (A_n) of mutually exclusive (discrete) alternatives, and that the utility of an alternative (i) to that individual is denoted by U_{in}. Following the approach of Lancaster, each alternative can be specified by a vector (Z_i) of characteristics which describes it. Then the utility of alternative i to individual n can be expressed in the form:

$$U_{in} = U_n (Z_i), \forall i, \ i \in A_n \tag{3.1}$$

However, $U_n(\ )$ is a specific utility function for each individual n. Therefore, to specify how tastes, and consequently utility functions, vary from one individual to another, an additional vector (S_n) of socio-economic variables describing individual n is introduced in the utility function of alternative i. Thus:

$$U_{in} = U\ (Z_i, S_n),\ \forall\ i,\ i \in A_n \tag{3.2}$$

Using the above notation, the deterministic choice theory postulates that an individual (n) will choose an alternative (i) out of all available alternatives in a choice set (A_n), if and only if,

$$U_{in} > U_{jn}\ ,\ \forall\ j \neq i,\ j \in A_n \tag{3.3}$$

where,

U_{in} is the utility of alternative i to individual n, and

U_{jn} is the utility of alternative j to individual n.

The above model of choice decision results in behaviour which is perfectly deterministic. However, to accept such a model requires the assumption that all individuals have perfect and complete information: they know all of the alternatives open to them, they know all of their characteristics, and they know their own preferences so that they behave as if they had well defined utility functions. Therefore they would always choose the alternative with the greatest utility. This is clearly an unrealistic assumption since empirical evidence shows that individuals do not select the same alternatives in repetitions of the same choice situations, under the same conditions. Moreover, by changing choice sets, violations of the transitive-preferences assumption are also observed. It has also been observed that individuals with identical choice sets, attributes, and socio-economic characteristics, select different alternatives.

Several factors may contribute to these inconsistencies. Firstly, it is usually not possible to include in the utility function (U_{in}) all the attributes that can possibly influence the choice decision. If such a function were possible it would no doubt be so complicated as to render it impractical. Secondly, a typical individual is not likely to have perfect information about the available alternatives. Thus, the set of alternatives (A_n) identified by the analyst may be larger or smaller than that encountered in fact by the individual, or the utility function (U_{in}) may contain variables about which information, as perceived by the individual, may be absent or incomplete. Finally, the individual may not always adopt the rational choice exactly and so the idiosyncrasies of individual behaviour cannot be anticipated in a deterministic model. Therefore, there may be essentially random elements in the behaviour of individuals, in that their preferences may vary from day to day or be influenced by external events (e.g. weather or availability of the household car).

One important way of partially overcoming these limitations of deterministic choice theory is to recognise that individuals do not make decisions with certainty. That is, there is a random or probabilistic element in the decision-making process. The probabilistic analysis of choice decision can be used to capture the effects of taste variations among individuals and unobserved characteristics of the alternatives. It can also take into account pure random behaviour as well as errors due to incorrect perceptions of the attributes and choices of suboptimal alternatives. Thus, probabilistic choice theory can be more readily adapted to formulate travel demand models. This is discussed next.

3.2.2 THE PROBABILISTIC CHOICE THEORY

As noted in the preceding section, the introduction of the probabilistic choice theory was the result of the inadequacy of the deterministic choice theory in explaining the individuals' behavioural inconsistencies that were observed. The earliest developments of probabilistic travel choice models[1] were founded on relatively simple postulates of human behaviour. These postulates stated firstly that individuals make travel choices on the basis of comparison of alternative levels of service provided by the travel alternatives, modified by attributes of the individual. Secondly, it was asserted that decision-making of individuals was to be modelled by the use of probabilities of choice, where these probabilities must satisfy the basic rules of probability as shown in the following equations:

$$0 < P(i : A_n) < 1, \forall i \tag{3.4}$$

$$\sum_i P(i : A_n) = 1, \forall i \tag{3.5}$$

where,

$P(i : A_n)$ is the probability of individual n choosing alternative i, and

A_n is the entire choice set of available alternatives for individual n.

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[1] A comprehensive review of these early developments is given in Luce and Suppes (1965); Reichman and Stopher (1971).

The probabilities are assigned to specified choice alternatives on the basis of consideration by the individual of the travel alternatives' characteristics, modified by the relevant attributes of the individual. This procedure is consistent with modern theories of human discrimination and choice. These theories state that every human decision is, in essence, probabilistic since there is a minimum variance in discrimination and there are dynamic changes in preference [Stopher and Meyburg (1974)]. This is an extremely important concept, since it leads to two conclusions of considerable importance in attempting to formulate choice theoretic models. These conclusions are:

1. Disaggregate probabilistic models can be formulated with a relatively small number of variables required to achieve good predictions.

2. Individuals do not have irrational or unquantifiable biases toward specific alternative choices.

This statement of hypotheses does not lead directly to any specific model formulation, but it does provide a broad framework within which choice models can be constructed. A more formal theoretical basis to travel choice modelling has been based upon two disciplines dealing with human behaviour; the psychological choice theory, through the strict utility approach, and the economic choice theory, through the random utility approach. In fact, both approaches, as will be seen, lead to similar forms of model [Ben-Akiva (1973)].

3.2.2.1 THE STRICT UTILITY APPROACH

This approach to the modelling of individual choice behaviour derives its theoretical underpinning from the psychological foundation of human behaviour. The view of the psychologist is that human decisions are probabilistic in nature, but are based upon an evaluation of utilities. These utilities, for each alternative, constitute a basis for estimating the probabilities of choice for each alternative. The psychological approach to the theory of disaggregate behavioural travel demand models is formalised through the application of Luce's Axiom of Independence of Irrelevant Alternatives (IIA) which states that, "If a set of alternative choices exists, then the relative probability of an individual choosing any two alternatives is unaffected by the removal (or introduction) of any set of other alternatives". Mathematically, this can be expressed as:

$$\frac{P_n(i : A_n)}{P_n(j : A_n)} = \frac{P_n(i : B_n)}{P_n(j : B_n)} \qquad (3.6)$$

where,

$P_n(i : A_n)$ is as defined previously,

A_n is the choice set of alternatives containing only i and j, and

B_n is the set of all alternatives including i, j, k, etc (i.e. A is a subset of B).

In other words, if some alternatives are introduced or removed from the set of alternative choices, the relative probabilities among the remaining alternatives are unchanged. The choice from the subset A_n is independent of what other alternatives exist in the main set B_n.

As mentioned in the previous section, individuals are assumed to associate a utility value with each alternative in the choice set available to them and subsequently to draw weighted lots to determine their choices. In other words, they know the exact utility of each alternative, but their choices are still probabilistic. It is further assumed that there is a direct correlation between the probabilities of choice and the levels of utility; the higher the level of utility of an alternative the higher the probability of its being chosen. Therefore, it seems reasonable to postulate that a ratio of probabilities can be expressed as a ratio of utilities. So,

$$\frac{P_n\ (i\ :\ A_n)}{P_n\ (j\ :\ A_n)} = \frac{P_n(i\ :\ B_n)}{P_n(j\ :\ B_n)} = \frac{U_{in}}{U_{jn}} \tag{3.7}$$

Thus, Equation 3.7 implies that the ratio of the probabilities is determined by the ratio of the utilities of the only two alternatives under consideration.

It is necessary to define a functional form for the utility. Without loss of generality, the functional form may be assumed to be exponential. This was found to be easy to use for computation and to provide a reasonable fit to real-world data [OECD (SEPT. 1980)]. Thus:

$$U_{in} = \exp\ (V_{in}) \tag{3.8}$$

where,

V_{in} is the linear function of the characteristics of both individual n and alternative i.

Thus Equation (3.7) can be written:

$$\frac{P_n\ (i : A_n)}{P_n\ (j : A_n)} = \frac{\exp\ (V_{in})}{\exp\ (V_{jn})} \tag{3.9}$$

Application of the probability rule (3.5) for only two alternatives in the choice set leads to the following equations:

$$P_n\ (i : A_n) = \frac{\exp\ (V_{in})}{\exp\ (V_{in}) + \exp\ (V_{jn})} \tag{3.10}$$

$$P_n\ (j : A_n) = \frac{\exp\ (V_{jn})}{\exp\ (V_{in}) + \exp\ (V_{jn})} \tag{3.11}$$

Given an assumption of linearity in V_{in}, these equations may be simplified by dividing throughout by either exp (V_{in}) or exp (V_{jn}). Thus,

$$P_n\ (i : A_n) = \frac{\exp\ (V_{in} - V_{jn})}{1 + \exp\ (V_{in} - V_{jn})} \tag{3.12}$$

$$P_n\ (j : A_n) = \frac{1}{1 + \exp\ (V_{in} - V_{jn})} \qquad (3.13)$$

The above two equations define the standard binary logit model [Berkson (1944)]. Where there are more than two alternatives, Equation 3.9 leads to the equation of the multinomial logit model [Rassam et al (1971); Ben-Akiva (1973); McFadden (1973)],

$$P_{in} = \frac{\exp\ (V_{in})}{\sum_{j \epsilon A_n} \exp\ (V_{jn})} \qquad (3.14)$$

Thus, the application of Luce's Axiom, with some reasonable assumptions about the form of the utility function, leads to the specification of a model structure for the analysis of travel choice behaviour. However, the Independence of Irrelevant Alternatives assumption of the strict utility approach is the principal strength on the one hand and principal weakness on the other. It is a strength in that, firstly, the parameters which determine the choice probabilities, conditioned on selection from a subset of alternatives, can be utilized in determining the probabilities for the full set. Thus, the dimension of the calibration data set can be reduced substantially, particularly with a large full set of alternatives. Further, data for the omitted alternatives need not be collected, leading to economy in data collection and the possibility of improving detail on the examined alternatives. Secondly, the strict utility approach allows quick analysis of the effects of introducing new alternatives using the predetermined parameters for models containing only generic variables (i.e. variables common to all alternatives in the choice set). Finally, sequential or recursive structures of travel demand decisions can be modelled based on the separability property of the IIA axiom[1].

The main weakness of the strict utility approach is the definition of an alternative. Throughout the theory, distinct alternatives are assumed, but classification of alternatives is not a part of the theory. Clearly, inappropriate definitions of the alternatives could lead to erroneous probability definition. The IIA axiom will not yield accurate forecasts in situations where a new alternative competes more heavily with similar alternatives than it does with dissimilar ones. This problem is illustrated by the classical example of the red bus/blue bus anomaly. Consider a situation in which a traveller who is making a choice decision between the car mode and a service of red buses is indifferent between the two modes. Hence, the choice probabilities are equal (i.e. $P_{car} = 1/2$ and $P_{red\ bus} = 1/2$). Now an additional service of blue buses, which is identical in all respects to the red bus service, is introduced. Since the axiom states that the ratio of choice probabilities remains unchanged, the new choice probabilities will be one-third (1/3) for each of the three modes. This is an unrealistic assumption since the individual traveller will treat the two bus services as one in spite of the different colours. This example suggests that application of the strict utility approach should be limited to multiple choice situations where the alternatives can plausibly be assumed by the individual traveller to be distinct and independent. Therefore, great care must be taken in choosing the alternatives in order that the choice axiom is not too strong for the application.

The final point which it is essential to make in the evaluation of the strict utility approach is that, since the IIA property is extremely useful for practical planning,

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[1] See for example: CRA (1972, 1977); Domencich and McFadden (1975).

its acceptance or rejection should be based on empirical grounds depending on the circumstances[1].

3.2.2.2 THE RANDOM UTILITY APPROACH

There is a major difference between the strict utility approach and the random utility approach. The former approach assumes that individuals have an exact and measurable utility associated with each alternative in their choice sets, but are uncertain of their choice decisions even after assessing the comparative utilities. Nevertheless, they must still make their choice decisions even when facing such uncertainty. In such situations an individual cannot always be expected to choose the alternative with the greatest utility. On the other hand, the random utility approach assumes that each individual is a deterministic utility maximiser, choosing from the available alternatives the one which yields the highest utility. The basic hypothesis of the random utility approach is that the individual's utility is represented as the sum of two components, a systematic component (V_{in}) and a random component (ϵ_{in}). The systematic component of the utility function accounts for the effects of the average tastes of the population and the observable characteristics of the alternative and the individual. The random component accounts for the effects of the unobservable characteristics of the individual and the alternative, individual idiosyncrasies and taste variations over the population. So,

$$U_{in} = V_{in} + \epsilon_{in} , \forall i, \ i \in A_n \qquad (3.15)$$

[1] See for example: CRA (1972); Brand (1974); Hensher and Johnson (1981).

As stated above, individuals are considered to be deterministic utility maximizers; that is, they will always choose the alternative which has the maximum utility. However, the analyst can only measure the deterministic part of the utility, and must therefore assign a probability to the outcome based on that observation. Thus, the random utility model of choice decision can be written:

$$P_{in} = \text{Prob} \{ U_{in} > U_{jn} , \forall j \neq i, j \in A_n \} \tag{3.16}$$

Substituting Equation 3.15 in Equation 3.16, Equation 3.16 becomes:

$$P_{in} = \text{Prob} \{ V_{in} + \epsilon_{in} > V_{jn} + \epsilon_{jn} , \forall j \neq i, j \in A_n \} \tag{3.17}$$

Equation 3.17, which is called the choice probability function, is the fundamental equation of the random utility models. Rearranging Equation 3.17 gives:

$$P_{in} = \text{Prob} \{ \epsilon_{jn} - \epsilon_{in} < V_{in} - V_{jn} , \forall j \neq i, j \in A_n \} \tag{3.18}$$

or,

$$P_{in} = \text{Prob} \{ \epsilon_{jn} < V_{in} - V_{jn} + \epsilon_{in} , \forall j \neq i, j \in A_n \} \tag{3.19}$$

Any particular choice model can be derived using Equation 3.18, or equivalently Equation 3.19, given specific assumptions on the joint distribution of the random components. Let $f(\epsilon_{1n}, \epsilon_{2n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{Jn})$ denote the joint density function of the random components of J alternatives. Then the choice probability of an alternative i is given by,

$$P_{in} = \int_{\epsilon_{1n}=-\infty}^{\epsilon_{in}+V_{in}-V_{1n}} \cdots\cdots\cdots\cdots \int_{\epsilon_{in}=-\infty}^{\epsilon_{in}=+\infty} \cdots\cdots \int_{\epsilon_{Jn}=-\infty}^{\epsilon_{in}+V_{in}-V_{Jn}} f\,(\epsilon_{1n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{Jn})\, d\epsilon_{1n} \ldots d\epsilon_{in} \ldots d\epsilon_{Jn} \qquad (3.20)$$

Although Equation 3.20 represents the most direct way of expressing the choice probability function, it involves a multiple integration computation which makes it an inconvenient way of deriving the choice probability for particular situations (e.g. for a large number of alternatives in the choice set or for more complicated choice functions such as the probit model function). Therefore, an alternative and simpler way is to denote $F\,(\epsilon_{1n}, \epsilon_{2n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{Jn})$ as the cumulative joint density function of the random components and $F_i\,(\epsilon_{1n}, \epsilon_{2n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{Jn})$ as its partial derivative with respect to the i^{th} random component. Then,

$$P_{in} = \int_{\epsilon_{in}=-\infty}^{\infty} F_i\,(\epsilon_{in}+V_{in}-V_{1n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{in}+V_{in}-V_{Jn})\, d\epsilon_{in} \qquad (3.21)$$

This equation can be interpreted as follows. Set the random component ϵ_{in} to some given value. The integral is then the probability that ϵ_{in} equals that specified value and that all the other random components satisfy the condition given by Equation 3.19. Hence, the probability of individual n choosing alternative i can be obtained by integrating F_i over all possible values of ϵ_{in}.

It is possible to obtain a specific operational model from this choice probability function by specifying, firstly, the functional form of the systematic component (V_{in}) of the utility function and, secondly, the joint distribution of the random components (ϵ_{in}) for all alternatives in the choice set (A_n).

The deterministic utility V_{in} is a function of the characteristics of alternative i (e.g. travel time, cost, convenience, comfort, and safety) and the socio-economic characteristics of individual n (e.g. income, sex, age, car-ownership, and occupation). Hence the function V_{in} can be expressed as,

$$V_{in} = f_i \; (Z_{in}, S_n) \tag{3.22}$$

where,

Z_{in} is a vector of characteristics of alternative i as perceived by individual n, and

S_n is a vector of socio-economic characteristics of individual n.

For mathematical convenience, linearity in the unknown parameters' specification of the deterministic utility function V_{in} is usually assumed. Thus,

$$V_{in} = \sum_{k=1}^{K} \beta_{ik} \, f_k \, (X_{ikn}), \; \forall \, k, \; i \epsilon A_n \qquad (3.23)$$

where,

f_k (X_{ikn}) is a vector of K functions of attributes of an individual (n) and characteristics of an alternative (i). These functions represent the way in which each explanatory variable can be introduced into the utility function (such as linearly or logarithmically or exponentially).

β_{ik} is a vector of K unknown parameters reflecting the estimated influence of variable k on the utility of alternative i. These are assumed to be constants across individuals.

K is the total number of explanatory variables entered in the utility function

Once the functional form of the deterministic utility function is specified, the next step is the specification of the joint distribution of the random components ϵ_{in}. Different assumptions on the joint distribution of the random components ϵ_{in} lead to different mathematical forms of probabilistic choice model. Clearly, a number of distributional assumptions are possible [See for example: Domencich and McFadden (1975); Ben–Akiva and Lerman (1985)] and among them two special cases are of particular interest.

In the first case, if the random components of the utility function are assumed to be Independently and Identically Distributed (IID) across individuals and all the alternatives in the choice set, then the appropriate statistical distribution is the Weibull distribution. The use of the Weibull distribution results in a MNL model of the form[1],

$$P_{in} = \frac{\exp(V_{in})}{\sum_{j \epsilon A_n} \exp(V_{jn})} \tag{3.24}$$

Thus, the same model form has resulted from this assumption as was derived from the strict utility approach (see Equation 3.14). As was pointed out by CRA (1972), the assumption that the random components follow the Weibull distribution is equivalent to the IIA axiom. This means that for an individual n, the odds ratio of the choice probabilities of any two alternatives (i.e. P_{in} / P_{kn}) is entirely independent of the presence or absence of any other alternatives in the choice set. This can be easily shown in the following way:

$$\frac{P_{in}}{P_{mn}} = \frac{\exp(V_{in}) \;/\; \sum_{j \epsilon A_n} \exp(V_{jn})}{\exp(V_{mn}) \;/\; \sum_{j \epsilon A_n} \exp(V_{jn})} = \frac{\exp(V_{in})}{\exp(V_{mn})} \tag{3.25}$$

The MNL model is both mathematically transparent and computationally tractable. It has been applied successfully in a wide variety of travel demand forecasting contexts[2]. However, the assumption that the random components of utilities are IID severely restricts the flexibility of the model and can be a source of

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1 For complete derivation of the MNL model, see Hensher and Johnson (1981); Kanafani (1983); Maddala (1983).

2 See for example: Manski (1973, 1977); Domencich and McFadden (1975); Richards and Ben-Akiva (1975); Adler and Ben-Akiva (1976); Ben-Akiva and Atherton (1977); Parody (1977); Small (1977); Spear (1977); Horowitz (1979); Ortuzar (1980); Dunne (1982).

substantial forecasting error [see for example: Horowitz (1980, 1981 a & b)]. In applications where the utilities of some alternatives are correlated, the logit model may overpredict or underpredict substantially the shifts in the choice probabilities of those alternatives when the characteristics of one or more alternatives are changed[1]. Therefore, several other exponential models derived from the MNL model have been suggested in the literature in order to overcome the problems associated with the IIA property when the alternatives concerned are correlated. These include the Generalised Extreme Value (GEV) model suggested by McFadden (1977, 1978), the Cross Correlated Logit model suggested by Williams (1977), and the Dogit model suggested by Gaudary and Dagenais (1979). However, their use in actual planning studies has been infrequent.

In the second case, a more general model, the Multinomial Probit (MNP) model, permits tastes to vary among individuals with identical observable characteristics, and allows effects of unobserved variables to be correlated across alternatives. The MNP model can be obtained by assuming that the random component (ϵ_{in}) of the utility of each alternative has the Multivariate Normal (MVN) distribution with zero mean vector and a finite variance–covariance matrix (Σ). Thus,

$$\epsilon \sim \text{MVN}\ (0, \Sigma) \tag{3.26}$$

where,

ϵ is the J vector of random components ($\epsilon_{1n}, \epsilon_{2n}, \ldots, \epsilon_{in}, \ldots, \epsilon_{Jn}$), and

J is the number of alternatives in the choice set.

[1] See for example: Mayberry (1970), Schneider (1973), Sheffi (1979).

Note that n is discarded from the expression for simplicity.

ϵ is MVN distributed if its density function $f(\epsilon)$ is given by,

$$f(\epsilon) = \text{MVN}(0, \Sigma) = [\{(2\pi)^J |\Sigma|\}^{-1/2}$$

$$\exp\{-\frac{1}{2}(\epsilon.\Sigma^{-1}.\epsilon^T)\}] \tag{3.27}$$

Thus for the MNP model, Equation 3.20 can be written as:

$$P_{in} = \int_{\epsilon_{1n}=-\infty}^{\epsilon_{in}+V_{in}-V_{1n}} \cdots\cdots\cdots\cdots \int_{\epsilon_{in}=-\infty}^{\epsilon_{in}=+\infty} \cdots\cdots\cdots\cdots \int_{\epsilon_{Jn}=-\infty}^{\epsilon_{in}+V_{in}-V_{Jn}} [\{(2\pi)^J |\Sigma|\}^{-1/2}$$

$$\exp\{-\frac{1}{2}(\epsilon.\Sigma^{-1}.\epsilon^T)\}]d\epsilon_{in}\ldots d\epsilon_{in}\ldots d\epsilon_{Jn} \tag{3.28}$$

Despite its generality, the MNP model has received little use in travel demand analyses because of its computational intractability. Algorithms for computing the choice probabilities and statistically estimating the parameters of this model have only recently become available[1]. At the present time, a program exists for the

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[1] See for example: Albright, Lerman, and Manski (1978); Daganzo and Schoenfeld (1978); Hausman and Wise (1978); Daganzo, Bouthelier, and Sheffi (1979); Daganzo (1979); Sheffi, Hall, and Daganzo (1982); Langdon (1984).

computation and estimation of the choice probabilities of the MNP model for up to 20 alternatives and 20 explanatory variables[1]. However, the MNP model computations with these programs are reported to cost from two to more than ten times as much as the equivalent MNL model computations. Moreover, there are preliminary indications that obtaining precise statistical estimates of the parameters of the MNP models may require samples much larger than those needed to estimate the parameters of the MNL models, which would further increase the cost of the MNP model computations. Finally, the MNP model has the disagreeable property that the functional form of the choice probabilities can not be written in closed form.

A MNL model, on the other hand, presents a more efficient tool for providing travel demand estimates when many alternatives are considered at the same time. In addition, the MNL model can be used to analyse the possible shifts in the choice probabilities of the competing alternatives when the characteristics of one of the alternatives are altered. Lastly, the MNL model can be used for quick analysis of travel demand in other locations[2].

As a result of the aforementioned problems in the application of the MNP model on the one hand and the tractability of the MNL model on the other hand, the logit formulation is more likely to be preferred in many travel demand modelling applications provided that the IIA behavioural assumption of logit can be accepted. Some limited tests of MNL against MNP in situations where IIA is violated have, nevertheless, failed to show distinct differences between the two models [Spear (1977)]. Moreover, in cases where the IIA assumption is valid, the

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[1] Personal communication (late 1988) with Prof. Daganzo of Berkeley University who generously supplied a MNP program (CHOMP) capable of handling up to 20 alternatives and 20 explanatory variables.

[2] See for example: Atherton and Ben-Akiva (1976); Train (1978, 1979); Koppelman and Wilmot (1982, 1986).

two models are generally statistically indistinguishable[1].

3.3 STATISTICAL ESTIMATION OF THE PROBABILISTIC CHOICE MODELS

Several statistical techniques can be used to calibrate discrete choice models. The most commonly used ones are regression analysis and the maximum likelihood estimation method. The form and applicability of the techniques depend on the structure of the choice model. The calibration process consists of estimating the values of the unknown parameters in the model formulation which will give the best fit to the data collected.

The data available for the calibration process will typically be a sample of N observations. Each observation consists of an observed choice and a vector of explanatory variables. The observed choice of each individual can be denoted by y_{in} such that:

$$y_{in} = \begin{cases} 1 & \text{if alternative } i \text{ is chosen by individual } n \\ 0 & \text{otherwise} \end{cases} \tag{3.29}$$

The following two subsections examine the regression analysis and the maximum likelihood calibration techniques.

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[1] See for example: Afriat (1972); Amemiya (1976, 1981); Bouthelier and Daganzo (1979); Horowitz (1980a, 1981); Maddala (1983).

3.3.1 REGRESSION ANALYSIS

The ordinary least squares estimation technique is normally used in the case of linear regression, that is

$$y_n = \sum_{k=1}^{K} \beta_k X_{kn} + \epsilon_n \tag{3.30}$$

where,

- y_n is the dependent variable for observation n,
- β_k is the kth unknown parameter,
- X_{kn} is the kth explanatory variable, and
- ϵ_n is the error term which is assumed to be normally distributed with zero mean and constant variance.

The least squares technique estimates the values of the unknown parameters β_k that minimise the sum of squared differences (Q) between the observed and the expected values of the observations. Thus,

$$\min_{\beta_k} Q = \min_{\beta_k} \sum_{n=1}^{N} (y_{in} - \sum_{k=1}^{K} \beta_k X_{ikn})^2 \tag{3.31}$$

In many cases the dependent variable y_n can take on a large number of possible values (i.e. continuous variables such as the number of individuals choosing a particular mode in a given area). For such dependent variables, the standard regression technique provides an appropriate statistical model.

Sometimes, however, the dependent variable is dichotomous, such as in mode choice where an individual chooses a particular mode or not. For several reasons, the standard regression is not an appropriate model for such types of variable. Firstly, the error terms are heteroscedastic. To prove this, using Equation 3.30, it can be shown that the error term can take only two values:

$$\epsilon_{in} = \begin{cases} 1 - \sum_{k=1}^{K} \beta_k X_{ikn} & \text{if } y_{in} = 1 \\ - \sum_{k=1}^{K} \beta_k X_{ikn} & \text{if } y_{in} = 0 \end{cases} \tag{3.32}$$

where, $y_{in} = 1$ indicates that alternative i has been chosen by individual n and $y_{in} = 0$ that it has not.

Therefore, the variance of ϵ_{in} can be calculated by [See Hensher and Johnson (1981) for the complete derivation of this variance]:

$$\text{Var } \epsilon_{in} = (1 - \sum_{k=1}^{K} \beta_k X_{ikn}) (\sum_{k=1}^{K} \beta_k X_{ikn}) \tag{3.33}$$

This is clearly not constant for all the observations since it depends upon the values of $\beta_k X_{ikn}$ which vary across the observations. This fact is contrary to the least squares' property that the error term has zero mean and constant variance. Secondly, the predicted values of the observed choice are not necessarily confined to the appropriate interval (0, 1) and may fall either below zero or above one in some cases (see Figure 3.1). Finally, hypothetical tests of the estimated parameters, such as the t-test, rely on the normality assumption of the error terms, which is equivalent to assuming that the y_{in} are normally distributed. This is not the case since y_{in} is a discrete variable, so the usual t-test is not valid[1].

However, an alternative least squares approach which was developed by Berkson (1953) has been applied to binary choice models (e.g. logit and probit) in which the utility is a linear function of the unknown parameters. Berkson's approach involves the transformation of the model to a straight line function. Specifically, for the binary logit model, the choice probability of alternative 1 is given by:

$$P_{1n} = \frac{\exp(V_{1n})}{\exp(V_{1n}) + \exp(V_{2n})} \tag{3.34}$$

or,

$$P_{1n} = \frac{1}{1 + \exp\{-(V_{1n} - V_{2n})\}} \tag{3.35}$$

Hence,

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[1] See for example: Neville and Kennedy (1964); Draper and Smith (1966); Domencich and McFadden (1975); Hensher and Johnson (1981); Ben-Akiva and Lerman (1985).

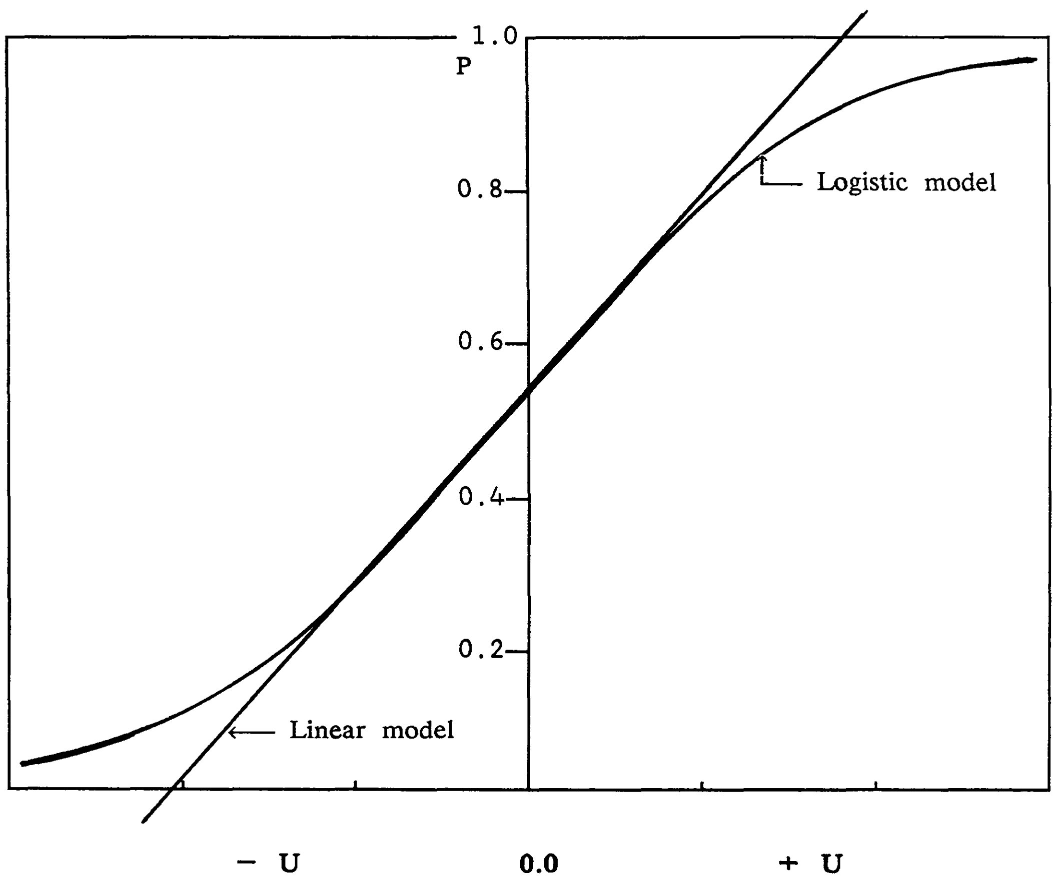

FIGURE 3.1 Comparison of linear and logistic models

$$P_{1n} / (1-P_{1n}) = \exp (V_{1n} - V_{2n}) \tag{3.36}$$

Taking the natural logarithm of both sides of Equation 3.36, results in:

$$\text{Log}[P_{1n} / (1-P_{1n})] = V_{1n} - V_{2n} \tag{3.37}$$

Since $\quad V_{1n} - V_{2n} = \beta_k X_{kn},$

and $\quad X_{1kn} - X_{2kn} = X_{kn}$

Equation 3.37 becomes:

$$\text{Log}[P_{1n}/(1-P_{1n})] = \beta_k X_{kn} \tag{3.38}$$

The problem with applying Equation 3.38 is that the choice probabilities are unknown. Therefore, one solution is to divide the observations into homogeneous groups with similar characteristics and use the choice share of each alternative for each group as an estimate of the choice probability of that alternative. So,

$$\text{Log}[R_{1g} / (1-R_{1g})] = \beta_k X_{kg} + \epsilon_g \tag{3.39}$$

where

R_{1g} is the share of the gth group choosing alternative 1,

X_{kg} is the kth explanatory variable for group g, and,

ϵ_g is the error term for group g attributed to the use of proportion as an estimate of the corresponding probabilities.

Since the right-hand side of Equation 3.39 is a simple linear function, the unknown parameters can be estimated using the ordinary least squares method. This will produce consistent estimates of β_k when the homogeneous groups have relatively large sizes [Domencich and McFadden (1975)].

Despite its obvious appeal, Berkson's approach is rarely applied in travel demand studies for a number of reasons. Firstly, a large sample of observations is required in order to divide it into homogeneous groups. As Domencich and McFadden (1976) pointed out, if each independent variable k in the model function takes only two values, then 2^k homogeneous groups are required (e.g. for k = 8, there would be 256 homogeneous groups). Secondly, by grouping the data some information will be lost and this makes the calibration less efficient. Finally, for continuous variables an arbitrary categorization is required and this can introduce biased estimates[1].

Theil (1969) extended Berkson's method to make it applicable to the calibration of the MNL model. However, the problems of finding homogeneous groups in the multinomial case are more difficult, especially when choice sets are varying across the observations. Therefore, application of the Berkson-Theil method to the multinomial model is extremely difficult.

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[1] See for example: Cox (1970); Domencich and McFadden (1975); Hensher and Johnson (1981); Ben-Akiva and Lerman (1985).

It would appear justified, from the above discussion of the applicability of the regression technique, to consider more appropriate techniques for calibrating discrete choice models. Fortunately, an appealing alternative is available, namely the maximum likelihood estimation method. This is discussed next.

3.3.2 MAXIMUM LIKELIHOOD

Maximum likelihood estimation is the most general and straightforward technique for calibrating discrete choice models. The idea behind this method is very simple. Given a sample of observations and a specified model, the estimated parameters are those values that are the most likely to generate the observed data.

The general likelihood function for the whole data sample is defined by:

$$L = \prod_{n=1}^{N} \prod_{i \varepsilon A_n} P_{in}^{y_{in}} \tag{3.40}$$

where,

- L is the likelihood function of the data sample,
- N is the total number of individuals in the sample,
- A_n is the choice set available to individual n,
- y_{in} is the observed choice indicator, and,
- P_{in} is the calculated choice probability of the n^{th} individual choosing alternative i. This probability is replaced by the specified model function.

Since the left-hand side of Equation 3.40 is the product of N probabilities, its value will usually be too small to be tractable. In addition, it is more convenient to work with the logarithm of the likelihood function which is a monotonically increasing function whose maximum occurs at the same value. Hence,

$$L^* = \sum_{n=1}^{N} \sum_{i \epsilon A_n} y_{in} \log P_{in} \tag{3.41}$$

where,

L^* is the log likelihood function.

For the MNL model, Equation 3.41 becomes,

$$L^* = \sum_{n=1}^{N} \sum_{i \epsilon A_n} y_{in} \log \frac{\exp (\beta_k X_{ikn})}{\sum_{j \epsilon A_n} \exp (\beta_k X_{jkn})} \tag{3.42}$$

or,

$$L^* = \sum_{n=1}^{N} \sum_{i \epsilon A_n} y_{in} [\beta_k X_{ikn} - \log \sum_{j \epsilon A_n} \exp (\beta_k X_{jkn})] \tag{3.43}$$

The maximum likelihood estimation method makes use of the fact that the calculated probability of observing the given sample should be highest when the unknown parameters k are near to their true values. Hence, the model calibration process involves finding the set of estimated values of β_k which maximises the log likelihood function. These values can be found in the ordinary way by differentiating Equation 3.43 with respect to each parameter β_k and setting each differential equal to zero. The first order condition for the maximization of the likelihood function is,

$$\frac{d\,L^*}{d\,\beta_k} = \sum_{n=1}^{N} \sum_{i \epsilon A_n} y_{in} \left[X_{ikn} - \frac{\sum_{j \epsilon A_n} \exp\,(\beta_k\, X_{jkn})\, X_{jkn}}{\sum_{j \epsilon A_n} \exp\,(\beta_k\, X_{jkn})} \right] = 0 \qquad (3.44)$$

Since y_{in} is a dichotomous variable, Equation 3.44 can be written in a more compact form,

$$\frac{d\,L^*}{d\,\beta_k} = \sum_{n=1}^{N} \sum_{i \epsilon A_n} [\, y_{in} - P_{in} \,]\, X_{ikn} = 0 \qquad (3.45)$$

The second order condition is,

$$\frac{d^2L^*}{d\beta_k\, d\beta_l} = -\sum_{n=1}^{N} \sum_{i \epsilon A_n} P_{in} \left[X_{ikn} - \sum_{j \epsilon A_n} X_{jkn}\, P_{jn} \right]$$

$$\left[X_{iln} - \sum_{j \epsilon A_n} X_{jln}\, P_{jn} \right] < 0 \qquad (3.46)$$

Equation 3.46 shows that the second partial derivative of L^* is negative definite. This implies that L is strictly concave and any estimator of β_k which satisfies Equation 3.45 is a unique maximizer of the likelihood function [McFadden (1973)]. This estimator will, for sufficiently large samples, have an asymptotic MVN distribution with the true parameters β_k as means, and variance–covariance matrix I given by the inverse of the matrix of the second derivative of L^* calculated at the true parameter vector multiplied by minus one [Theil (1971)]. Thus,

$$I_{kl} = \left| - \frac{d^2 L^*}{d\beta_k \; d\beta_l} \right|^{-1} \tag{3.47}$$

The maximization of the likelihood function which is equivalent to the solution of the K nonlinear equations in Equation 3.45 can be carried out by several numerical optimization techniques. The Newton–Raphson method which is often simple to implement and computationally efficient was used in this study[1].

3.4 GOODNESS–OF–FIT MEASURES

A calibrated discrete choice model provides calculated choice probabilities for any specified values of the explanatory variables. It is misleading to compare the estimated probabilities with the observed choices since the predicted choice is a probability, whereas the observed choice is either 0 or 1 [Hensher and Johnson (1981)]. Hence, a goodness–of–fit measure, such as the correlation coefficient

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[1] For more details of this method, see Broyden (1967); Ben–Akiva and Lerman (1985).

(R^2) of the ordinary least squares method, which is based on estimated residuals, does not make any sense. For the same reason, a comparison of the sum of the computed probabilities for a given alternative with the total number of individuals choosing that alternative is also statistically meaningless.

As a result, several alternative statistical goodness–of–fit measures based on the value of the log likelihood function calculated at the mean of the estimated parameters have been utilized to assess how well a calibrated model reproduces the observed data[1].

Statistical tests for assessing the validity of the MNL model which have been used in this study are described below.

3.4.1 THE t–TEST FOR SIGNIFICANCE OF EACH PARAMETER

The t–test is designed to indicate whether a particular variable in the model specification has a meaningful role in the utility function. The test seeks to determine if the coefficient associated with a particular variable is significantly different from zero (i.e. testing the null hypothesis that $\beta_k = 0$). If the hypothesis is accepted, then the conclusion is that the variable is not making a significant contribution in explaining part of the variation in the observed data. The rejection of the null hypothesis would indicate otherwise, namely that the contribution of the variable is significant. In other words, the greater the magnitude of the t–value (typically greater than 2 at the 5% level of significance or, equivalently, for the 95% confidence level), the more important is the contribution of that variable to the model.

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[1] See for example: Stopher (1975); Tardiff (1976); Hauser (1978); McFadden (1979).

The simplest form of this test entails the division of the estimated parameter value by its estimated standard deviation. Thus,

$$t = \beta_k / \sqrt{I_{kk}} \quad , \forall k \tag{3.48}$$

where t is the t–test value for parameter β_k and is normally distributed with zero mean and unit variance. I_{kk} is the estimated variance of parameter β_k and is obtained from the asymptotic variance–covariance matrix given by Equation 3.47.

Besides the t–value, the signs of the coefficients should also be reviewed for reasonableness.

3.4.2 THE LOG LIKELIHOOD RATIO TEST

The primary objective of this test is to assess the overall statistical significance of a particular model calibrated by the maximum likelihood estimation method. This can be done by comparing the tested model with another model resulting from imposing a linear restriction on some or all of the parameters of the tested model. To test the model as a whole involves testing the null hypothesis that the dependent variable is independent of the values of the explanatory variables. This implies that all the parameters are set equal to zero (i.e.the equal shares model). The rejection of the null hypothesis simply indicates that the tested model is considered better than the equal shares model, or, in other words, that the effects due to the parameters are to be regarded as significant [Hensher and Johnson (1981)].

Mathematically, let L_β^* denote the value of the log likelihood function of the tested model evaluated at the optimum values of the estimated parameters, and L_0^* denote the value of the log likelihood function of the model that assigns equal values of the choice probabilities of all alternatives, regardless of the values of the explanatory variables. Then, under the null hypothesis that all parameters are zero (i.e. $\beta_1 = \beta_2 = \ldots = \beta_k = 0$), the log likelihood ratio (LLR_0) is defined as,

$$LLR_0 = -2 \left(L_0^* - L_\beta^* \right) \tag{3.49}$$

and is X^2 distributed with K degrees of freedom, where K is the total number of parameters in the tested model [Wald (1943); Nerlove and Press (1973)].

However, this test is not very useful because almost always the null hypothesis can be rejected at very low levels of significance. Therefore, it is more informative to test the null hypothesis that all the parameters, except for the alternative-specific constants, are set to zero (i.e. the market share model). In this case, the log likelihood ratio (LLR_c) is given by,

$$LLR_c = -2 \left(L_c^* - L_\beta^* \right) \tag{3.50}$$

and is X^2 distributed with $K-J+1$ degrees of freedom, where J is the total number of alternatives in the choice set, and L_c is the log likelihood function value of a model with constants only. This value is given by [Sobel (1980)],

$$L_c^* = \sum_{j=1}^{J} M_j \ln (M_j / N_j) \tag{3.51}$$

where,

M_j is the number of individuals actually choosing alternative j, and

N_j is the total number of individuals having alternative j available (including those actually choosing alternative j).

The rejection of the above hypothesis would lead to the conclusion that the tested model is better than the market share model.

In general, however, the log likelihood ratio test is a relatively weak test for two reasons. Firstly, although the log likelihood ratio test can reject a null hypothesis model, it cannot give an indication of how well a calibrated model predicts, nor can it compare two models unless one model is a restriction of the other. Secondly, the log likelihood ratio test produces values of X^2 that are much larger than any tabulated values. Hence, comparison between alternative model formulations cannot be made based on log likelihood ratio values [Stopher (1975); Tardiff (1976)].

3.4.3 THE LOG LIKELIHOOD RATIO INDEX TEST

As a result of the aforementioned weaknesses in applying the log likelihood ratio test (specifically the unlimited values of LLR) and the fact that the observed dependent variable is discrete (i.e. 0 or 1), a more meaningful goodness-of-fit measure giving values between 0 and 1 is required. Consequently, the log likelihood ratio index ($\rho_0{}^2$), which is similar in many respects to the correlation coefficient (R^2) of the regression analysis, can be utilized in assessing the success of a particular choice model or in comparing models in terms of how well each model replicates the data from which it has been constructed.

The ratio index $\rho_0{}^2$ is most often defined when the null hypothesis model is one with all parameters equal to zero [Brownstone and Wills (1974)]. In this case, the log likelihood ratio index ($\rho_0{}^2$) is given by,

$$\rho_0{}^2 = 1 - L_\beta^* / L_0^* \tag{3.52}$$

The larger the value of $\rho_0{}^2$ for a given model, the better the model fits the data. It should be noted that values from 0.2 to 0.4 for $\rho_0{}^2$ are considered to indicate an excellent fit [McFadden (1976b); Hensher and Johnson (1981)].

Although this test is widely used in practice, it has been recognized that it is meaningless to compare $\rho_0{}^2$ for different data samples with different market shares. The reason is that the value of $\rho_0{}^2$ varies depending on the proportion of individuals choosing each alternative[1]. However, a more flexible log likelihood ratio index test which allows comparison between models estimated with different sample sets that have different market shares is given by McFadden (1973):

$$\rho_C{}^2 = 1 - L_\beta^* / L_C^* \tag{3.53}$$

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[1] For more details of this point see Tardiff (1976).

3.4.4 PERCENTAGE OF OBSERVATIONS CORRECTLY PREDICTED

This goodness-of-fit measure is based on the accuracy of a given model in reproducing the observed data. The simplest form of this measure is given by,

$$\% \text{ Right} = \frac{100}{N} \sum_{n=1}^{N} \hat{y}_n \tag{3.54}$$

where,

$\hat{y}_n$ is 1 if the highest predicted probability of choice corresponds to the actually chosen alternative, and 0 otherwise, and

N is the total number of observations in the data sample.

A higher value of % right indicates a better fit of the given model. However, this test is much less useful for comparing alternative models. The reason is that there are no readily available quantitative criteria for determining how large the differences between the values of % right for different models should be in order to justify a conclusion that the model with the higher value is more accurate.

3.5 SPECIFIC ISSUES IN THE APPLICATION OF DISCRETE CHOICE MODELS

This section presents, briefly, some important issues that are related to the application of probabilistic choice models. The first subsection discusses the ways in which various attributes enter the utility function of each alternative, in particular the distinction between generic and alternative specific variables. The next subsection is concerned with the identification of the feasible choice sets available to individuals in deciding which alternatives they will choose.

3.5.1 SPECIFICATION OF VARIABLES IN THE UTILITY FUNCTION

Travel demand models are concerned with the definition of the comparative utilities of alternatives, as the bases for specifying the resultant choice, and the utility is a function of all attributes that describe each alternative. It is, therefore, essential to present the ways in which these attributes enter the utility function.

In general, two main types of explanatory variable are used in specifying the utility function of each available alternative; these are the generic and the alternative—specific variables. Whether a particular variable is a generic or alternative—specific variable depends on the way that the variable enters the utility function. If the variable appears in the utility functions of all the alternatives with the same coefficient in each, then it is a generic variable. On the other hand, if the variable appears in the utility function of one alternative, then it is an alternative—specific variable.

To distinguish between generic and alternative—specific variables, consider, for example, the variable of travel time in a mode choice model. If the travel times by different modes are assumed to have a common valuation for all modes (i.e. a common weighting or coefficient), then travel time should be specified as a generic variable. However, if this is not considered correct, then the variables may be specified such that each one appears only in the utility function of one alternative and is zero otherwise. In this case the variables are specified as alternative– specific variables.

If all variables in a model are generic, then the model is termed an "abstract alternative" model [Quandt and Baumol (1966); CRA (1972)]. This type of model has variables relating only to characteristics common to all alternatives and is therefore highly suited, for forecasting purposes, to situations substantially different from those used for model estimation, and especially to systems not currently in use. Thus generic variables should be used whenever possible [Richards and Ben– Akiva (1975); Hensher and Johnson (1981)].

Generic variables are used only when there is little correspondence among the sets of alternatives available to different individuals; otherwise alternative– specific variables have to be used. For example, if the set of alternative shopping centres at one location is entirely different from the set of shopping centres at another location, there is no correspondence among sets of alternatives, and so these alternatives can be described only through the use of generic variables. But if only one of those shopping centres is common to every individual's set of alternatives, then alternative– specific variables can be used to describe that common shopping centre.

If a variable has the same value for all alternatives to all individuals, then it will have no impact on the model. This is because of the linear specification. In other words, the same term would appear in the numerator and in each member of the sum in the denominator, and thus it would cancel out as a common factor. In order to maintain the influence of such variables (e.g. socio– economic variables), they must be specified in one of the following ways:

1– The variable may be introduced as an alternative– specific variable (or alternative-specific dummy variable) which takes the specified value of that variable or has a value one for one alternative (or more), and is zero otherwise.

2- The variable may be combined or interacted with another variable (e.g. cost /income) so that it has an alternative-specific value and can be used to define either a generic or an alternative-specific variable.

An alternative-specific constant which has the value one for a particular alternative and zero for all other alternatives can also be included in the specification of the model to capture the impact of unobserved attributes affecting the choice of an alternative. Such constants cannot be included in the utility of all alternatives, since the result would be a condition analogous to perfect multicollinearity in regression analysis. Therefore, at least J - 1 constants can be identified, where J is the total number of alternatives available to each individual in the data sample. It is apparent that this also applies to alternative-specific dummy variables (such as sex or occupation) which can be regarded as additional alternative-specific constants [Richards and Ben-Akiva (1975)].

3.5.2 CHOICE SET DEFINITION

The most fundamental problem that the analyst has to solve is the definition of the set of available alternatives for each individual in the data sample. To define exactly a choice set for an individual is extremely difficult. However, two possibilities are available. These are: to treat all available alternatives as the set of relevant choices for all individuals, and let the coefficients and the model structure take care that the resulting choice probabilities of the infeasible alternatives are very low or even zero; or to select only the important modes, that is those modes used in the highest proportions. The former way requires additional data measurements and so results in an undesirable model and a high computational cost. Furthermore, the inclusion of unrealistic alternatives in the choice set may considerably reduce the comparative ability of the model, and the possibility exists that a model capable of dealing with unrealistic alternatives may not be able to describe sufficiently the choices between the realistic alternatives [Ruijgrok (1979); Ortuzar (1980)]. In the latter way, the reduction of the choice set by the exclusion of some alternatives with low choice frequencies will sometimes result in omission of some important alternatives that are not chosen due to the specific sample or sampling technique.

An appropriate alternative method, which neither considers all alternatives nor eliminates the low choice proportion alternatives, is heavily reliant on a priori logical arguments and observations of current behaviour in determining the feasible set of relevant alternatives. In this method, the definition of the relevant choice set for each individual is carried out by imposing some logical constraints on the availability of each alternative to each individual in the data sample. In other words, the feasibility of an alternative is defined by a variety of constraints such as physical availability (e.g. a bus service is an available alternative only when the bus stops are close to the home and place of work of a given individual); time availability (e.g. walking is an infeasible alternative for long distance travelling); monetary resources (e.g. a taxi is an infeasible alternative for low income workers); limited information (e.g. an individual's lack of knowledge about bus services, routeing, scheduling and the locations of stops may result in the unavailability of bus services to that individual); and so on[1]

[1] See for example: CRA (1972); Stopher (1980); Zahavi and Ryan (1980); Goodwin (1981); Gunn (1981); O'Neill and Nelson (1981); Richardson (1982); Kitamura and Lam (1984); Ben-Akiva and Swait (1984); Swait and Ben-Akiva (1987 a, b).

CHAPTER FOUR

GRIS SURVEY AND THE SELECTED DATA BASE

CHAPTER FOUR

GRIS SURVEY AND THE SELECTED DATA BASE

4.1 INTRODUCTION

This chapter briefly describes the data available for the calibration of the set of models presented in Chapter 5. Section 4.2 examines the GRIS survey: the study area; how the study data were collected; and details of the collected data. In Section 4.3 the sample used in this study is described. The division of the total sample into two subsamples, one for the calibration of the set of models and the other for the validation of the calibrated models, is also explained. Section 4.5 considers the problems inherent in the available data in terms of the requirements of this study. The last section deals with the selection of the most important level-of-service and socio-economic variables.

4.2 GRIS DATA SURVEY

The Central Clydeside Conurbation is centred on the City of Glasgow and incorporates a number of surrounding Districts; at the time of the GRIS survey the population was approximately 1.7 million. Glasgow itself is an important administrative, commercial and industrial centre and as such is an important attractor for people seeking employment. It is also the main focus for the conurbation, and much beyond, of shopping, leisure and educational facilities.

The conurbation has an extensive suburban railway network, much of it electrified during the 1960s. The Glasgow Underground, originally opened as a cable railway in 1896, comprises a loop located slightly to the west of the present city centre. Major road developments in the 1960s and 1970s resulted in the construction of the M8 motorway through the city from west to east and incorporating the west and north flanks of an intended ring motorway around the city centre. The conurbation has long been characterised by lower-than-average levels of car-ownership due to a combination of low income, housing density and good public transport.

GRIS was set up in 1978 to investigate the effects on part of the conurbation of two major rail investments in Glasgow, viz. the construction of the new Argyle Line which links the north and the south sides of the River Clyde and passes beneath the most important shopping centre in Scotland, and the modernisation of the Glasgow Underground (see Figure 4.1). GRIS was concerned only with that part of the conurbation (the suburban rail corridor between Dumbarton in the west and Hamilton in the east, via the city centre) likely to be affected by the investments.

The basic data source for this study is the household interview survey carried out by Martin and Voorhees Associates (MVA) in the autumn of 1978 and spring of 1979[1] as part of the GRIS study conducted by the Scottish Development Department (SDD), the Transport and Road Research Laboratory (TRRL) and MVA.

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[1] The data used in this study is from the "before" household interview survey, and was supplied from tapes held by TRRL together with data from the "after" household interview survey which was carried out in the late spring of 1980 after the opening of the Argyle Line and the modernised underground. The author is indebted in particular to Mr. H. Gentleman of SDD who supplied much helpful information on the organisation of the survey.

The household interview survey was conventional and provided details of one weekday's travel by all members of a sample of households along the rail corridor. The area surveyed was that within 1 km of the railway and underground stations. An additional sample was taken between 1 km and 2 km of two stations: Bearsden and Hamilton.

Within this study area addresses were selected randomly from the Regional Assessor's rating lists and grouped, for convenience of field work, into 55 clusters of about 60 addresses each. In total, 2598 households were surveyed. In each household all residents aged over 5 were asked to supply details of their travel during the previous day, including journeys on foot of more than 5 minutes. A total of 6944 persons were interviewed, and 17528 daily trips for different purposes reported. The overall response rate was 84%, although this varied between 80% and 90% in different parts of the study area [GRIS final report (1981)].

The questionnaire (see Appendix 1) was divided into three basic parts relating to different levels of data, viz. household data, person data and trip data. These data were stored on magnetic tape containing three files (i.e. household, person, and trip files). The data have, therefore, been arranged to allow analysis at three levels of detail, considering travel by household or by person, or considering travel in terms of the trip.

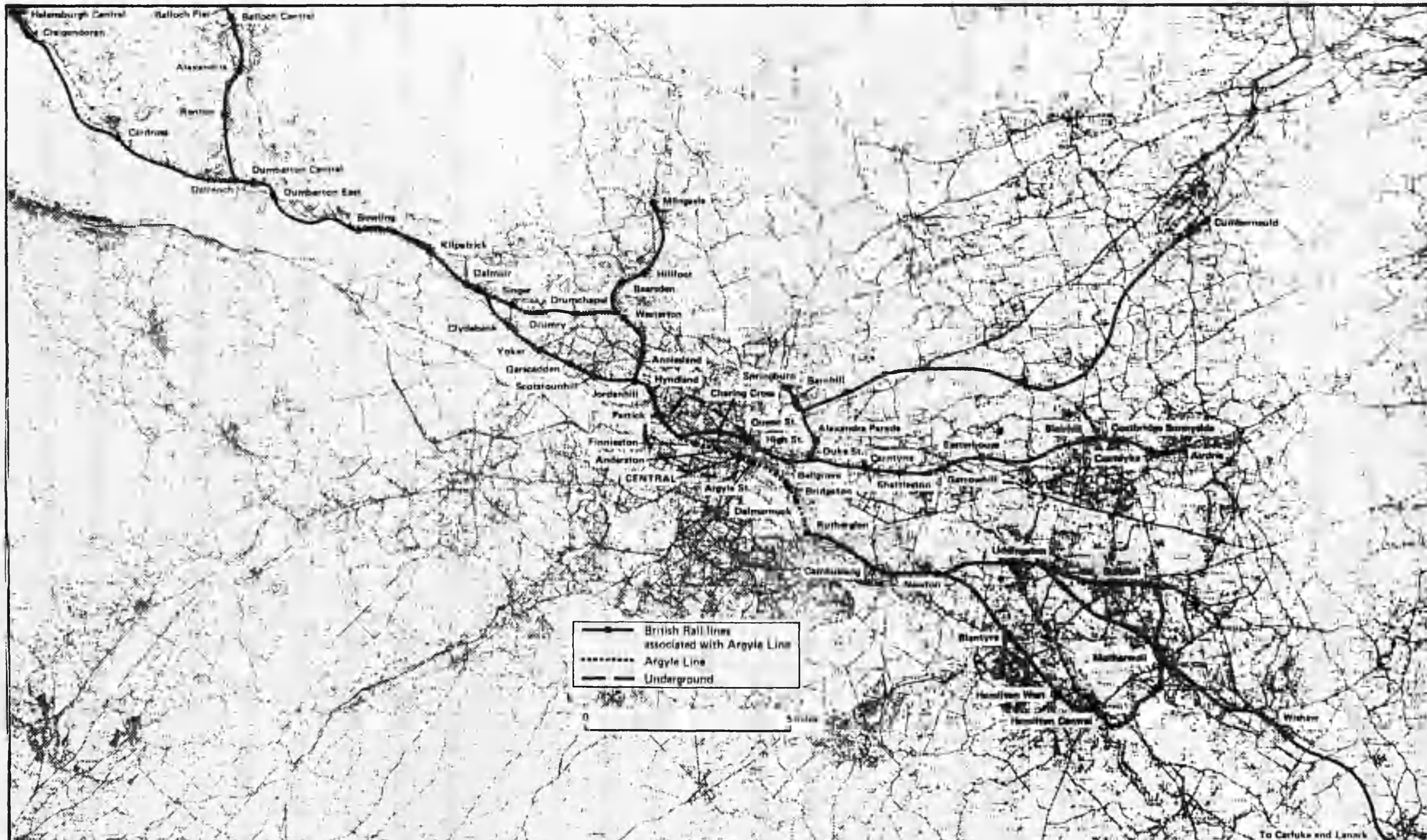

FIGURE 4.1 Glasgow suburban rail network and the locations of Glasgow Underground and the Argyle Line.

The household file contains information such as household size, structure, income and economic activity. The economic activity of a household is indicated by the number of persons employed and by characteristics of the head of the household. An indication of the household's theoretical mobility is given by variables such as car–ownership and the number of persons with driving licences and public transport seasonal tickets.

The person file includes personal characteristics of the trip–maker, such as age, sex, personal economic activity and type of occupation, and, in addition, factors affecting personal mobility, such as possession of a driving licence or some form of seasonal ticket.

Since the household and person files can consider trip–making only in terms of the numbers of trips involving particular purposes or modes, their usefulness in analysing travel behaviour is limited because they tell little about the characteristics of the trip itself. For this reason the trip file takes the trip itself, a whole journey made to achieve a specific purpose, and allows it to be used as the analysis base.

As shown in the questionnaire (see Appendix 1), the raw data were collected in terms of the individual stages of each trip and a simplification was made in linking them to form the whole trip. However, frequencies from the raw data indicated some trips of up to six stages. These were relatively few and have, therefore, been compressed to retain data on a maximum of three stages. Thus, a three–stage trip comprises access, main and egress stages.

The trip file summarises data for the whole trip in terms of the locations of its origin and destination, the start time and duration, the purpose at origin and destination, the main mode used in the case of multi-stage trips, the cost if using public transport modes, and the total walking time involved. For multi-stage trips, further details are given on the location of the destination of each stage and the mode used in each stage together with, in the case of car stages, more information on occupancy and parking and, for public transport stages, information on ticket types and costs. In addition, the mode name only is given for the best alternative mode which could have been used.

While these data on trip characteristics are rich in themselves, their effectiveness is increased by the ability to relate them to the characteristics of the trip-maker. Consequently, the trip file includes, for each trip, the same basic data on personal socio-economic characteristics as forms part of the person file, and also the general characteristics of the household from which the individual comes. Using combinations of these groups of data within the trip file, a number of approaches for analysing this data are possible. Thus, trips may be analysed by themselves in terms of purpose, mode, duration, origin and destination, peak and off-peak start time, etc. By any of these, they may be related to the socio-economic characteristics of trip makers, such as sex, age, and occupation, and may be further related to the characteristics of the household, such as car-ownership, income, and family size.

4.3 PREPARATION OF THE DATA BASE

Since the objective of this study is to build mode choice models for the journey to work in Glasgow, only work trips have been used. From the 17528 trips for various purposes, a total of 2498 work trips have been extracted. These work trips are distributed across 12 modes of transport (see Table 4.1).

Mode chosen	Chosen by	Proportion (%)
Household Car Driver	559	22.4
Other Car Driver	30	1.2
Car Passenger	275	11.0
M / C Driver	11	0.4
Taxi	19	0.8
Pedal Cycle	8	0.3
Walk	611	24.4
Goods Vehicle Driver	4	0.2
Train	150	6.0
Scheduled Bus	759	30.4
Unscheduled Bus	64	2.6
Other Passenger	8	0.3
TOTAL	2498	100.0

TABLE 4.1 Distribution of work trips across available modes

As shown in the above table, although twelve modes were used, there are few observations for seven of these (viz. other car driver, motor cycle driver, taxi, pedal cycle, goods vehicle driver, unscheduled bus, and other passenger). These modes have, therefore, been excluded primarily because of their low frequencies of use, but also, in some cases, because of their infeasibility or their lack of clear definition in the GRIS questionnaire. The motor cycle and pedal cycle modes have been excluded entirely due to their low frequencies of use. Trips by goods vehicle have been excluded also because such vehicles may be used at work so that their drivers are captive to this trip mode. Trips by unscheduled bus have been rejected also, since travellers may be captive to this mode where a company supplies a bus to collect its workers. The taxi mode has been excluded because of its infeasibility as a daily mode. The other car driver and other passenger modes have been excluded because there was no clear definition of either in the GRIS questionnaire. Generally, in the case of excluded modes, it is impossible to assume them as alternative modes since there are no logical reasons for their availability and because of the difficulties of measuring the level-of-service variables for them. Thus Table 4.2 shows the frequencies of the modes selected for this study. The work trips shown in Table 4.1 have thus been reduced to 2354, representing only the observed choice frequencies of the five modes shown in Table 4.2. Since journeys to work are normally assumed to be similar to journeys from work, it was decided that the analysis should be carried out only for morning peak journeys to work, so further reducing the number of work trips to 1524, as shown in Table 4.3.

After final checking of all information available on each observation in Table 4.3, it was decided to reject the following cases:

1. Trips with incomplete information.
2. Trips of individuals from the same household.

Mode chosen	Chosen by	Proportion (%)
Car Driver	559	23.7
Car Passenger	275	11.7
Bus	759	32.2
Train	150	6.4
Walk	611	26.0
TOTAL	2354	100.0

TABLE 4.2 Distribution of work trips across alternative modes (peak and off-peak).

Mode chosen	Chosen by	Proportion (%)
Car Driver	382	25.1
Car Passenger	199	13.1
Bus	483	31.7
Train	118	7.7
Walk	342	22.4
TOTAL	1524	100.0

TABLE 4.3 Distribution of work trips across alternative modes (morning peak).

3. Trips wrongly coded (e.g. wrongly reported mode or travel time).

4. Trips with origin and destination in the same zone.

5. Trips with more than one mode (e.g. mixed mode such as kiss–and–ride or park–and–ride).

Table 4.4 shows the amended number of morning peak work trips available from the GRIS data survey.

The alternative modes shown in Table 4.4 are defined as follows:

1. Car Driver: drove the household car all the way.

2. Car Passenger: driven all the way by car.

3. Bus: walked from home to the stop, waited, caught a scheduled bus, then walked to the work place.

4. Train: walked from home to the station, waited, caught the train, then walked to the work place.

5. Walk: walked all the way.

In order to reduce the amount of data preparation, the statistical package SPSSx (Statistical Package for the Social Sciences) was used to select a reasonably–sized, random sample of 650 trips from the 1141 available. The sample was subsequently divided randomly into two subsamples: a subsample of 530 trips for the calibration of the choice models, and a subsample of 120 trips for the validation of these calibrated models. The subsamples are shown in Table 4.5.

Mode chosen	Chosen by	Proportion (%)
Car Driver	298	26.1
Car Passenger	158	13.8
Bus	320	28.1
Train	70	6.1
Walk	295	25.9
TOTAL	1141	100.0

TABLE 4.4 Revised distribution of trips to work across alternative modes (morning peak).

Mode chosen	Calibration sample		Validation sample	
	Chosen by	Proportion (%)	Chosen by	Proportion (%)
Car Driver	144	27.2	35	29.2
Car Passenger	64	12.1	15	12.5
Bus	139	26.2	30	25.0
Train	45	8.5	10	8.3
Walk	138	26.0	30	25.0
TOTAL	530	100.0	120	100.0

TABLE 4.5 Distribution of trips across alternative modes for the two subsamples (calibration and validation).

In order to test whether the validation sample was correctly chosen, it had to be established that there were no significant differences between the characteristics of the total sample before division and the two subsamples (i.e. the calibration and validation samples). The following tables present some properties of the three samples. Tables 4.6 to 4.9 present, respectively, the distributions of: the number of cars in the household; the individual's position in the household; trips destined to the Central Business District (CBD); and the sex of the trip-maker. As can be seen from these tables, comparisons of the computed and corresponding tabulated X^2 values indicate that there is no sign of serious bias in the validation sample.

The set of alternative modes available to each individual in the data survey was not reported, the only information given being the name of the best alternative mode. Thus, the determination of the set of relevant alternatives for each individual is a difficult problem. As was mentioned in the previous chapter, if an alternative has zero or very close to zero choice probability, then its inclusion or exclusion from the set of alternatives has negligible effect on the estimation and prediction results of the calibrated model. However, from practical considerations, usually of cost and time saving, the set of alternatives must be reduced to include only the feasible alternatives. Unfortunately, there is, at present, no specified criterion for determining a priori which alternatives are considered available to a particular individual and which not (in essence, the analyst does not know exactly the choice sets available to individuals unless the individuals are asked about their sets of alternatives during the data collection). The only way to define the available choice set is to impose certain constraints or rules on the availability of each alternative mode. Then, from the observed trip-making pattern, the availability of each mode can be determined.

Number of cars in household	Proportion (%) in		
	Total	Calibration	Validation
0	54.9	54.5	56.6
1	35.4	35.1	35.2
2+	9.7	10.4	8.2
TOTAL	100.0	100.0	100.0

Computed $X^2 = 0.49$. Tabulated X^2 at the 95% level = 5.991

TABLE 4.6 Distribution of number of cars in the household for the subsamples.

Household Position	Coded Value	Proportion (%) in		
		Total	Calibration	Validation
Non-head	0	49.6	50.1	48.5
Head	1	50.4	49.9	51.5
TOTAL		100.0	100.0	100.0

Computed $X^2 = 0.132$. Tabulated X^2 at the 95% level = 3.841

TABLE 4.7 Distribution of household position for the subsamples.

Type of Destination	Coded Value	Proportion % in		
		Total	Calibration	Validation
CBD	1	67.5	68.8	70.0
Non-CBD	0	32.5	31.2	30.0
TOTAL		100.0	100.0	100.0

Computed $X^2 = 0.059$. Tabulated X^2 at the 95% level = 3.841

TABLE 4.8 Distribution of trips across types of destination for the subsamples.

Sex of Individual	Coded Value	Proportion % in		
		Total	Calibration	Validation
Female	0	40.5	40.0	37.3
Male	1	59.5	60.0	62.7
TOTAL		100.0	100.0	100.0

Computed $X^2 = 0.246$. Tabulated X^2 at the 95% level = 3.841

TABLE 4.9 Distribution of trip– maker sex for the subsamples.

In this study, the following rules have been used in the identification of the valid alternatives:

1. Car Driver is available if the individual is a member of a car-owning household and possesses a valid driving licence.

2. Car Passenger is assumed to be a universally available mode, in the sense that all individuals can be carried as passengers by their family drivers or friends.

3. Bus is almost universally available except for some short trips when the required walking distance at both ends of the bus trip is greater than the distance walking all the way.

4. Train is available if the access distance to the nearest station is less than 1 km[1] or the total distance at both ends of the trip is less than 3 km.

5. Walk is a valid alternative mode if the total distance of the trip is less than 2.75 km[2].

Based on the above considerations, the distribution of the choice set sizes for each subsample is given in Table 4.10. Table 4.11 shows in more detail the availability of each mode for each subsample.

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1 The value of 1 km was chosen on the basis that the GRIS survey was carried out on an area within 1 km of the rail stations, except for only two areas which were within 1 to 2 km.

2 It was found that, at the 95% confidence level, the farthest people were prepared to walk was 2.65 km.

Choice set size	Calibration sample		Validation sample	
	Frequency	Proportion (%)	Frequency	Proportion (%)
2	41	7.7	10	8.3
3	244	46.0	59	49.2
4	207	39.1	38	31.7
5	38	7.2	13	10.8
TOTAL	530	100.0	120	100.0

TABLE 4.10 Distribution of choice set sizes for both subsamples (calibration and validation).

Mode	Calibration subsample				Validation subsample			
Chosen	Chosen by	Proportion (%)	Available to	Proportion (%)	Chosen by	Proportion (%)	Available to	Proportion (%)
Car Driver	144	27.2	192	36.2	35	29.2	45	37.5
Car Passenger	64	12.1	530	100.0	15	12.5	120	100.0
Bus	139	26.2	499	94.2	30	25.0	113	94.2
Train	45	8.5	373	70.4	10	8.3	82	68.3
Walk	138	26.0	238	44.9	30	25.0	54	45.0
TOTAL	530	100.0			120	100.0		

TABLE 4.11 Distribution of the choice and availability of alternative modes for the two subsamples.

4.4 PRACTICAL LIMITATIONS OF THE GRIS DATA

It is clear from the review of the GRIS data survey (see Section 4.2 and Appendix 1) that there is a considerable amount of information available on household, individual, and trip characteristics. Nevertheless, there are a number of problems associated with the use of this data in this study.

The first problem is the lack of detailed information about the alternative modes available for each individual in the data; the only information available is the best alternative mode. Thus, it is difficult to identify the relevant set of alternatives. This was discussed in the previous section.

A second problem in using the GRIS data is the omission of all stages with a walking time of less than 5 minutes. This produces difficulties in determining walking times from home to bus stop or train station and vice versa.

The third problem is the lack of information on the costs of travel by the car driver and car passenger modes; the waiting times for public transport modes (bus and train); comfort, convenience, and safety of all of the modes; and the routes taken by all modes.

The last important problem is that there is no clear definition of the car passenger mode. A car passenger may be a passenger in the family or other car or may be part of a car-pooling scheme in which car owners travel together using the car of each one in turn. This results in difficulties in the allocation of costs of travel to car passengers.

All of the above problems are discussed in the next section.

4.5 SELECTION OF THE EXPLANATORY VARIABLES

Probably the most difficult task for the analyst is the selection of the variables to be used in the alternative model specifications. The reason, as was discussed in Chapter 3, is that the analyst does not know exactly what variables individuals considered in making their choice decisions. Individuals, in their choice decisions, must evaluate the characteristics of the competing modes. However, the perception of the characteristics of the alternatives modes varies from individual to individual and may depend on several socio-economic characteristics of the individuals and their households, as well as on the characteristics of the mode and trip. Therefore, the variables which affect the individual choice decision can be classified as:

1. Level-of-service variables (mode and trip characteristics).
2. Socio-economic variables (individual and family characteristics).

4.5.1 LEVEL-OF-SERVICE VARIABLES

To specify each alternative mode in the relevant set of alternatives available for each individual in the data sample, a set of level-of-service variables is required. For the chosen modes, values of some of these variables were reported. These values were used directly in the calibration of each specified model since, for journeys to work, the reported values are almost equal to the true values. For daily repeated trips, such as work trips, a learning process is involved which causes the reported values to converge towards their actual values as the journey is continually repeated. Hence, for such journeys there should be little difference between the reported and the measured values.

Since there was no information about alternative modes, a set of measured values of their variables was required. These measured values were derived by manually locating each pair of home and work addresses on large scales maps utilising the Ordnance Survey Grid Reference (OSGR) of six digits which was coded with the GRIS data. By this means location within an area of 0.01 km^2 can be defined. Thus, the values of the level-of-service variables can be more accurately measured than by using the ordinary centroid zonal system to represent the locations of the trip ends.

The following level-of-service variables were used in the specification of the alternative choice models:

Travel time

There is virtually no travel choice situation wherein the influence of travel time is absent. Travel time plays an important role in modelling travel choices within a transport system. It is a predominant explanatory variable of travel choice behaviour and, in addition, it often serves as an evaluation measure for transportation systems.

In considering the travel time taken for a particular trip by a particular mode, the in-vehicle travel time must be distinguished, where it is available, from the time spent outside the mode (walking and waiting times). It is necessary to split the travel time into its components and weight them differently.

For each chosen mode, the in-vehicle travel time used was the reported value for that mode. To measure the in-vehicle travel time for car driver and car passenger as alternative modes, an average speed of 19 kph was assumed[1]. Since the precise routes taken by car drivers were not known, in order to measure the corresponding journey distances, likely routes were selected on the basis of local knowledge and judgement. This procedure may result in the adoption of erroneous values. To avoid this problem, direct airline distances between home and work locations were measured and then multiplied by an average balancing factor to convert them to their route distances. In this study a random sample of 92 car trips was selected in order to establish an average value of balancing factor. For all trips, the lengths of possible alternative routes were measured on large scale maps and compared with the corresponding airline distances. This yielded an average value of balancing factor of 1.35[2]. The car driver and car passenger in-vehicle travel times were then derived from the distance between the ends of the trip using the assumed average speed of 19 kph. As regards the out-of-vehicle travel times for the two above modes, it was simply assumed that there would be no walking and waiting times. This assumption was based on the fact that the associated trips were reported as one-stage trips (i.e. the car was parked close to or garaged at the house or work place of each individual).

For bus and train trips, in-vehicle travel times were measured from the relevant bus and train time tables, the selected times being those of the fastest available services. If transfer was required, the total in-vehicle time for the trip was

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1 This value was determined statistically from the observations where car driver or car passenger were the chosen modes and was also proved empirically during the course of the study. The same figure was used by Sobieniak et al (1979).

2 Wilson (1967) used balancing factors of (0.38 + 1.15 d) and (0.51 + 1.18 d) for Coventry and the London area, respectively, where d is the direct distance. DeDonnea (1971) used a balancing factor of 1.4. See also Bock (1968).

equal to the sum of the in-vehicle times for all of the stages involved[1].

The out-of-vehicle time for bus and train trips was divided into walking time and waiting time. The walking times to and from bus stops or train stations were determined using distances obtained from large scale maps and an assumed average walking speed of 5.5 kph[2]. Locations of home and work places for each trip were defined to within 100 metres using the OSGR. Walking times were determined from the distance from the centroid of the grid square to the nearest stop or station. In the event of transfer between two bus or train services, the transfer walking time was also added.

Waiting times for bus or train were computed as half of the scheduled headway, up to a maximum of 7 minutes for small headways, or 10 minutes for large headways, plus the expected waiting time for transfer when required.

Travel cost

The GRIS survey did not gather any direct information on the travel costs of car drivers and car passengers, and so it was necessary to estimate them. To do this required knowledge of the costs to be attributed to the car drivers and car passengers and how they could be estimated.

It is not at all certain how car drivers perceive their travel costs; whether they

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[1] The author is indebted to Mr. B. Longworth of Strathclyde Buses, Mr. B. Bryson of Central SMT Buses and to Western SMT Buses for supplying bus timetables and details of bus fares; to Mr. T. Hart of the Department of Economic and Business History at the University of Glasgow for supplying train timetables; and to Mr. Birnie of British Rail for supplying details of train fares.
[2] This figure was derived from the observed data. The range of walking speeds used in most previous studies was from 5 to 6 kph.

consider a total cost comprising standing costs (i.e. car licence, insurance, depreciation and garage and parking costs) and running costs (i.e. the costs of petrol, oil, tyres, servicing and replacements), or take account only of out-of-pocket costs (i.e. petrol, oil and parking costs). If an individual buys a car specifically for journeys to and from work, then the total cost is probably the more appropriate one. However, in most cases a car is owned for a variety of reasons and so it seems more reasonable to consider only out-of-pocket travel costs. In this study, car travel costs were calculated, in pence, by multiplying the total distance between the origin and destination of a trip by 2.7 pence / mile, the estimated cost for an average family car. This estimated figure was determined from information kindly supplied by the Automobile Association and was based on an average family car of 1000 to 1500 cc engine capacity travelling 10000 miles per year, and a petrol cost (in 1978) of £ 0.78 per gallon.

In the GRIS survey there was no clear definition of the type of car passenger. It was decided, therefore, for a car carrying a passenger, to allocate half of the car driver's travel costs to the passenger on the assumption that both driver and passenger were car-owning individuals who shared their trips and used their cars alternately, or were members of the same family.

Travel costs for bus and train trips were reported where these modes were chosen. Travel costs for these modes as alternatives were determined from the relevant daily fare table or on the basis of a weekly ticket (Transcard), whichever was cheaper.

Zero cost was allocated to the walk mode.

Travel distance

Distances between the origins and destinations of all trips were measured in order to test the effects of distance on the choice of transport mode. It appeared, for example, that the car was the preferred mode, if available, for long trips, while walking was preferred for relatively short distances, though there were some notable examples of long-distance walks.

CBD

This dummy variable was introduced to test the effect on the mode choice decision of trips passing through, or destined for, the CBD. For example, such trips may not be undertaken by car, where possible, because of the associated traffic problems and delays.

4.5.2 SOCIO-ECONOMIC VARIABLES

In order to evaluate the effects of taste variations amongst travellers on their mode choice decisions, socio-economic variables were introduced into the analysis. The socio-economic variables used in the study were:

Household position

This dummy variable was constructed to differentiate between the head of a household and other members of the household, and to test the effect of that position on the car driver mode choice decision.

Number of workers in a household

This variable was introduced to test the effect of the number of workers in the household on the likelihood of their sharing the family car for work trips.

Number of cars per driving licence holder

This variable was used to reflect the competition for available household cars amongst household driving licence holders; it was not permitted to have a value in excess of one.

Household Income

In the GRIS survey, estimates of gross annual household income were reported. These were classified into twenty one ranges as shown and coded in Table 4.12. These codes were used in this study to test the association between household income and the choice of transport mode.

Sex

This dummy variable was introduced to test the effect of an individual's sex on the mode choice decision. It appeared, for example, that females preferred not to walk all the way to work even where the travel distances were short.

Household Annual Income (£)	Code used	Household Annual Income (£)	Code used
0 - 499	1	5500 - 5999	12
500 - 999	2	6000 - 6499	13
1000 - 1499	3	6500 - 6999	14
1500 - 1999	4	7000 - 7499	15
2000 - 2499	5	7500 - 7999	16
2500 - 2999	6	8000 - 8499	17
3000 - 3499	7	8500 - 8999	18
3500 - 3999	8	9000 - 9499	19
4000 - 4499	9	9500 - 9999	20
4500 - 4999	10	10000 +	21
5000 - 5499	11		

TABLE 4.12 Household annual income coding

Occupation

This dummy variable was used to test the effect of an individual's occupation on the choice of car mode and differentiated between professionals, managers and skilled foremen and others.

CHAPTER FIVE

MULTINOMIAL LOGIT MODELS FOR GLASGOW

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MULTINOMIAL LOGIT MODELS FOR GLASGOW

5.1 INTRODUCTION

This chapter presents details of the derivation of the best-specified MNL model with five transport modes: car driver, car passenger, bus, train, and walk.

The second section describes how a number of alternative specifications of the model were examined. Backward elimination and stepwise statistical techniques were used in the refinement process which involved the elimination of those variables with the wrong signs or whose significance effects on the choice context were minimal. This procedure was continued until a set of model specifications was left in which all the variables were significant.

In the third section, a statistical comparison is made in order to choose the best specified set of models. Almost all the models were found to be strongly significant and from them four were chosen for further analysis on the basis of the goodness-of-fit statistics and the values of the alternative-specific constants.

In the last section, further tests of model validation are carried out on the four selected models in order to select the final two models; one simple and one complex. These models are then used for the aggregate prediction analysis (see Chapter 6).

5.2 MODEL SPECIFICATION AND VARIABLE SELECTION

Having developed the theoretical model for this study, the MNL model (see Chapter 3), and completed the the data preparation (see Chapter 4), the next step was to build a MNL model that describes individuals' mode choice behaviour relative to their journeys to work in Glasgow.

One of the advantages of disaggregate behavioural travel demand models over conventional aggregate models is the ability of the former models to accommodate, in their specifications, a large number of explanatory variables (see Chapter 2). On the other hand, this is also a problem since the analyst does not know with certainty which variables have significant influences on the model performance. This means that the analyst does not know a priori the effects of these variables on the individual mode choice decision except in the case of some important variables such as travel time and cost which have different values for different alternatives. The problem arises in particular with socio-economic variables (see Chapter 3), since these variables have the same values for all modes and their effects (i.e. coefficient value and sign) vary from mode to mode. Therefore, the analyst must try various model specifications until a specification is obtained which is consistent with a priori beliefs and fits the data fairly well[1].

In this study, a number of preliminary model specifications were tried, each restricted to a relatively small number of explanatory variables. The reason behind these trials was to test where the socio-economic variables had the highest significance. The results of these initial tests indicated that, for example,

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[1] **See for example: Talvitie (1972); Talvitie and Kirschner (1978); Talvitie and Dehghani (1979); Train (1979); Dehghani and Talvitie (1980, 1983); Ortuzar (1980).**

the household position variable had little effect when it was specified in the utility functions of the public transport modes, while it was more significant when specified in the utility function of the car driver mode. Conversely, the CBD variable had a relatively low effect on car driver and car passenger choice decisions and a high effect on public transport usage. Household income was also found to be more significant when specified in the utility functions of all modes except walk than when specified in the utility function of car driver only[1].

As was discussed in Chapter 4, socio-economic variables are included in order to explain the differences in individuals' choice behaviour across available alternative modes. Alternative-specific constants have totally different functions from those of the socio-economic variables; their inclusion in the model specification is to account for the effects of unobserved variables. If the variables used fully explain the individuals' choice behaviour then the alternative-specific constants should have zero values. Thus, with perfect model specification and perfect data, it can be argued that no alternative-specific constants are necessary. However, estimating a model without alternative-specific constants is not recommended in practice because the estimated values of the coefficients of the variables included are seriously affected if those variables do not fully explain the observed choice behaviour. Alternative-specific constants, therefore, represent the effect of those variables that influence individual choice behaviour but are not included explicitly in the model specification. The existence of significant and large values of the alternative-specific constants indicates the absence of a good model specification[2].

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1 Indeed this significance affects the choice of car driver, car passenger, bus, and train modes in preference to the walk mode only but cannot discriminate between the first four modes. This will be discussed later.

2 See for example: Domencich and McFadden (1975); Richards and Ben-Akiva (1975); Dehghani and Talvitie (1980, 1982); Talvitie and Dehghani (1979); Supernak (1984).

For a model with a maximum of five alternatives, only four alternative-specific constants can be specified (see Chapter 3). In this study the walk mode was, therefore, considered as the base mode and the value of the alternative-specific constant for this mode was set to zero. Other values of alternative-specific constants should be interpreted relative to that of the walk mode [see Richards and Ben-Akiva (1975)].

The variables selected for inclusion in the model specification are greatly restricted by the limitations of the available data and the possible existence of multi-collinearity between the variables.

Based on all of the above considerations, the specifications of the available variables which appeared to be the most suitable are shown in Table 5.1.

Backward elimination and stepwise statistical techniques were adopted in order to obtain the best specified model [see Draper and Smith (1966)]. Using this approach, a more general specified model containing all possible explanatory variables was first estimated. The variable that had the least influence on the model performance (lowest t-value, see Chapter 3) was then removed and a new model specification was tried. This refinement process was continued until all remaining variables were significant at the 95% significance level. These trial specifications of the model are shown in Table 5.2.

Variable Name	Designation	Car Driver	Car Passenger	Bus	Train	Walk
Mode	CODE	1	2	3	4	5
Household Position	HHPOS	x	0	0	0	0
No. of Persons Working	PERW	0	x	0	0	0
No. of Cars Owned	CAOD	x	x	0	0	0
No. of Cars per Driving Licence	CAPDL	x	0	0	0	0
Household Income	HINC	x	x	x	x	0
Sex	SEX	0	x	x	x	0
Occupation	OCC	x	0	0	0	0
Central Business District	CBD	0	0	x	x	0
Total Journey Time	TJT	x	x	x	x	x
In-Vehicle Time	IVT	x	x	x	x	0
Out-of-Vehicle Time	OVT	0	0	x	x	0
Walking Time	WK	0	0	x	x	0
Waiting Time	WT	0	0	x	x	0
Walk mode Walking[1] Time	WALK	0	0	0	0	x
Distance	DIST	0	0	0	0	x
Travel Cost	COST	x	x	x	x	0
Car Specific Constant	CCON	1	0	0	0	0
Car Passenger Specific Constant	PCON	0	1	0	0	0
Bus Specific Constant	BCON	0	0	1	0	0
Train Specific Constant	TCON	0	0	0	1	0

Note:

x equals the value taken by the specified variables.

[1] WALK variable was sometimes treated the same as WK variable in model MNL-10 [i.e. alternatives entered (3-5)].

TABLE 5.1 Variables used in the model specification.

Variable (Alternatives Entered)[1]	Coefficients (t-Values)					
	MNL-C	MNL-1	MNL-2	MNL-3	MNL-4	MNL-5
HHPOS (1)	---	1.4015 (3.34)	1.2236 (2.67)	1.2899 (2.71)	1.4868 (3.29)	1.4773 (3.27)
PERW (2)	---	0.0670* (0.45)	0.0677* (0.48)	---	---	---
CAOD (1-2)	---	0.7802 (2.92)	0.7826 (2.91)	0.7972 (2.95)	0.8086 (3.02)	0.7948 (2.98)
CAPDL (1)	---	4.7612 (7.64)	4.5564 (7.92)	4.5888 (7.68)	4.6154 (4.71)	4.6150 (4.70)
SEX (2-4)	---	-0.5320* (-1.93)	-0.4671* (-1.41)	-0.4699* (-1.42)	---	---
OCC (1)	---	-0.4943* (-1.02)	---	---	---	---
HINC (1-4)	---	0.0875 (2.22)	0.0854 (2.14)	0.0884 (2.25)	0.0919 (2.33)	0.0976 (2.49)
CBD (3-4)	---	1.3223 (4.28)	1.3274 (4.18)	1.3270 (4.16)	1.3440 (4.24)	1.4127 (4.47)
IVT (1-4)	---	-0.0512 (-2.84)	-0.0505 (-2.72)	-0.0508 (-2.72)	-0.0510 (-2.73)	-0.0439 (-2.43)
OVT (3-4)	---	-0.1856 (-8.21)	-0.1851 (-8.34)	-0.1851 (-8.01)	-0.1856 (-8.04)	-0.1829 (-8.08)
WALK (5)	---	-0.0585* (-1.54)	-0.0586* (-1.56)	-0.0585* (-1.54)	-0.0640* (-1.68)	-0.1327 (-5.56)

[1] For codes, see Table 5.1.

* Non-significant variable at the 95% level.

TABLE 5.2 The elimination of non-significant or wrongly-specified variables.

Continued:

Variable (Alternatives Entered)[1]	Coefficients (t-Values)					
	MNL-C	MNL-1	MNL-2	MNL-3	MNL-4	MNL-5
DIST (5)	---	-1.1539 (-2.57)	-1.1213 (-2.50)	-1.1232 (-2.50)	-1.0113 (-2.28)	---
COST (1-4)	---	0.0324 (2.36)	0.0325 (2.36)	0.0325 (2.35)	0.0326 (2.35)	0.0329 (2.39)
CCON (1)	0.9077 (4.62)	-7.4743 (-7.21)	-7.4834 (-6.94)	-7.5619 (-6.88)	-7.6156 (-6.97)	-7.7205 (-6.93)
PCON (2)	-1.9353 (-11.6)	-5.6296 (-6.88)	-5.5932 (-6.87)	-5.4851 (-6.87)	-5.7275 (-7.13)	-5.8362 (-7.27)
BCON (3)	-1.1019 (-7.11)	-2.2921 (-2.66)	-2.2699 (-2.62)	-2.3008 (-2.69)	-2.5389 (-2.98)	-2.7529 (-3.29)
TCON (1)	-1.9661 (-9.74)	-1.9848 (-2.01)	-1.9556* (-1.98)	-1.9900 (-2.03)	-2.2309 (-2.28)	-2.4001 (-2.51)
LLR_0	375.616	655.399	654.442	654.262	652.300	646.943
d.f.	4	17	16	15	14	13
% Right	67.74	77.55	77.36	77.36	77.74	77.93

[1] For codes, see Table 5.1.

* Non-significant variable at the 95% level.

TABLE 5.2 The elimination of non-significant or wrongly-specified variables.

Table 5.2 illustrates the set of models resulting from the above elimination procedure. The first model, MNL–C, represents the constant or market share model. This model is very important in the comparison of different alternative models (see Chapter 3). From models MNL–1, MNL–2, MNL–3, and MNL–4, the following observations can be made:

1. The PERW variable has no significant effect on the choice of car passenger mode. Due to the inconvenience of the relative locations of workplaces or different times of starting work, individuals within a household may use public transport modes or drive to their work places in their own cars rather than travel as car passengers.

2. The SEX variable is also found to have no significant effect on the choice of car passenger or public transport modes. This variable has a negative sign which is attributed to the fact that male individuals are more likely than females to prefer driving their cars if they are car–owners, or that they prefer to walk all the way if the distances are relatively short.

3. The OCC variable has no significant impact on the car driver mode choice decision and is also found not to have the expected positive sign. This is confirmed by the low number of professionals, managers, and skilled foremen in the data sample who drive their cars. It may be that those individuals leave their cars for household use whilst themselves using public transport or car–pooling arrangements.

4. The WALK variable is found not to be significant. This is, however, not the expected result since WALK was a priori expected to have an important effect on the choice of walk mode. It was found that this variable was specified with the

DIST variable in the utility function of the walk mode, and since these variables were, clearly, highly inter-correlated, their coefficients were wrongly predicted. Thus, their inclusion in the same mode utility function should be avoided[1].

5. The COST variable is found to be relatively significant, but has an unexpected positive sign. The reason for this is that, even in the presence of the car passenger alternative which has very low associated travel costs, individuals were found to choose other modes with higher travel costs[2].

Based on the above observations, variables PERW, SEX, and OCC were eliminated from the new model specifications. The WALK and DIST variables were used alternatively in different specifications and the COST variable was left for further tests due to its importance as a policy-controllable variable. These eliminations resulted in model MNL-5 (see Table 5.2). The comparison of model MNL-5 with the previous models (i.e. MNL-1 to MNL-4) indicates that the elimination process does not alter significantly the coefficient values of the remaining variables and, therefore, the null hypothesis that model MNL-5 is not significantly different from model MNL-1 is strongly accepted[3]. This indicates that the refinement procedure is empirically correct.

All other variables in model MNL-5 are significant at the 95% level and are consistent with a priori expectations. Therefore, model MNL-5 was adopted as the base model against which the new specifications could be compared.

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1 See for example: DeDonnea (1971); DeNeufville and Stafford (1971); Ossenbruggen and Li (1976); Lyles (1979); Stopher and Wilmot (1979); Khasnabis, Cynecki, and Flak (1983).

2 Several previous studies found the same positive sign for the cost variable. See for example: DeDonnia (1971); Richards and Ben-Akiva (1975); Lyles (1979).

3 The LLR_0 statistic between models MNL-5 and MNL-1 has the value of 8.456 with four degrees of freedom, whereas X^2 at the 95% level is equal to 9.488.

Since distance could be appraised more accurately than travel time, and since individuals may also be expected to attach more importance to distance travelled than to travel time, the DIST variable was substituted for the WALK variable in MNL-6 (see Table 5.3). This change resulted in a better specified model, as indicated by the decreases in the values of the alternative-specific constants[1]. This means that the utility function of the walk mode was better specified using the distance travelled rather than the travel time.

Although the effect of the out-of-vehicle (OVT) variable in models MNL-5 and MNL-6 is highly significant, it seemed logical from the policy viewpoint to explore the influence of dividing this variable into its two components, the WK and WT variables[2]. As can be seen from model MNL-7, this division has improved the model specification greatly through further reductions in the values and significance of the alternative-specific constants.

Since the travel cost (COST) variable still had a positive sign in all the previous models (see Tables 5.2 and 5.3), an attempt was made in model MNL-8 to introduce a combined variable describing the travel cost relative to the household income (i.e. COST/HINC). This variable, although resulting in a higher positive coefficient value (the increase in the coefficient value is attributed to the inclusion of household income), is not significant. Also, MNL-8 has lower values of alternative-specific constants than model MNL-7. This implies that the model specification has been improved. A further attempt was made (model MNL-9) excluding the COST/HINC variable due to its non-significance. This also resulted in a significantly better specified model than model MNL-5.

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[1] See for example: Talvitie and Kirschner (1978); Supernak (1984).

[2] See for example: Quarmby (1967); Talvitie (1972); Algers, Hansen, and Tenger (1975); Richards and Ben-Akiva (1975); Ortuzar (1980); Matzoros (1982).

Variable (Alternatives Entered)[1]	Coefficients (t-values)					
	MNL-5	MNL-6	MNL-7	MNL-8	MNL-9	MNL-10
HHPOS (1)	1.4773 (3.27)	1.4897 (3.32)	1.4830 (3.31)	1.4013 (3.15)	1.409 (3.15)	1.5095 (3.39)
CAOD (1-2)	0.7948 (2.99)	0.8320 (3.10)	0.8077 (3.00)	0.8289 (3.10)	0.8824 (3.33)	0.7165 (2.73)
CAPDL (1)	4.6150 (4.70)	4.6022 (4.70)	4.6054 (4.73)	4.5069 (4.66)	4.4631 (4.63)	4.6448 (4.80)
HINC (1-4)	0.0976 (2.49)	0.0808 (2.13)	0.0813 (2.13)	---	---	0.0847 (2.29)
CBD (3-4)	1.4127 (4.47)	1.3351 (4.22)	1.2611 (3.94)	1.3231 (4.18)	1.3562 (4.33)	1.3204 (4.13)
WK[2] (3-4)	---	---	-0.1728 (-7.20)	-0.1692 (-7.26)	-0.1702 (-7.34)	-0.1504 (-8.61)
WT (3-4)	---	---	-0.3208 (-4.19)	-0.3104 (-4.15)	-0.3100 (-4.16)	-0.3179 (-4.22)
IVT (1-4)	-0.0439 (-2.43)	-0.0509 (-2.75)	-0.0442 (-2.34)	-0.0321 (-2.01)	-0.0323 (-2.02)	-0.0290* (-1.68)
OVT (3-4)	-0.1829 (-8.08)	-0.1862 (-8.05)	---	---	---	---
WALK (5)	-0.1327 (-5.56)	---	---	---	---	---

1 For codes see Table 5.1.

2 The WK variable was treated the same as WALK variable in model MNL-10 [i.e. alternatives entered (3-5)].

* Non-significant variable at the 95% level.

TABLE 5.3 Model specifications and disaggregation of the variables.

Continued:

Variable (Alternatives Entered)[1]	Coefficients (t-values)					
	MNL-5	MNL-6	MNL-7	MNL-8	MNL-9	MNL-10
DIST (5)	---	-1.6240 (-6.08)	-1.5853 (-5.84)	-1.4697 (-5.68)	-1.5104 (-5.81)	---
COST (1-4)	0.0329 (2.39)	0.0328 (2.38)	0.0338 (2.39)	---	---	---
COST/HINC (1-4)	---	---	---	0.1324* (1.20)	---	---
COST/DIST (1-4)	---	---	---	---	---	-0.0960 (-2.70)
CCON (1)	-7.7205 (-6.97)	-7.0448 (-6.93)	-7.0264 (-6.84)	-5.8899 (-7.08)	-5.9582 (-7.12)	-8.0361 (-7.68)
PCON (1)	-5.8362 (-7.27)	-5.1536 (-7.49)	-5.1433 (-7.38)	-4.1908 (-9.34)	-4.3275 (-9.85)	-6.3169 (-8.79)
BCON (1)	-2.7529 (-3.29)	-1.9381 (-2.62)	-1.1948* (-1.39)	-0.0047* (-0.01)	0.1307* (0.02)	-1.4772* (-1.79)
TCON (1)	-2.4001 (-2.51)	-1.6233* (-1.85)	-0.9812* (-1.02)	0.4526* (0.58)	0.6727* (0.88)	-1.0007* (-1.25)
LLR_0	646.943	649.483	653.256	643.839	642.376	652.452
d.f.	13	13	14	13	12	13
% Right	77.93	77.36	76.98	77.36	77.74	76.98

1 For codes see Table 5.1.

* Non-significant variable at the 95% level.

TABLE 5.3 Model specifications and disaggregation of the variables.

An attempt was made, in model MNL–10, to include the COST/DIST variable. This again resulted in a better–specified model than model MNL–5. As can be seen from model MNL–10, the variable COST/DIST possesses a negative sign. This negative sign is attributed to the fact that the choice probability of any mode decreases as the unit cost per unit distance increases. The only worrying factor is the reduction in the coefficient value of the IVT variable and its impact on the individual mode choice decision. This decrease might be attributed to the interdependence of the IVT and DIST variables.

In order to obtain a relatively simple model with few variables included, all travel time components were aggregated into one single variable, namely the total journey time (TJT). In addition, the effect of excluding the COST variable from model MNL–5 was tested. As can be seen from model MNL–11 in Table 5.4, the resulting alternative–specific constants were slightly decreased in comparison with MNL–5 except for the train mode which has a slightly higher specific constant. Despite this, model MNL–11 seemed to have a better specification than model MNL–5. In model MNL–12, the HINC variable was excluded, and the model specification was considerably improved. Since travel cost is an important policy variable, and further to confirm that the exclusion of HINC would improve the model specification, the HINC and COST/DIST variables were then restored to model MNL–13. Although the COST/DIST variable clearly seemed to improve the specification, the resulting model was found to have a poorer specification than model MNL–12 as shown by slightly higher alternative–specific constants. Since the HINC variable was specified in the utility functions of all modes except walk (car driver, car passenger, bus, and train), it can discriminate between the choices of any one of these modes with respect to the walk mode only; it does not have the ability to discriminate between these modes. Hence, in model MNL–14, the HINC variable was excluded, and the model specification was further improved. It should be noted that the inclusion of the COST/DIST variable does not alter the coefficient value of the TJT variable (see Table 5.4). Thus the effect of correlation between the variables in this particular case was negligible.

In general, examination of Table 5.3 shows that the effects of some level-of-service variables confirmed a priori expectations. For example, one would expect the coefficient of the OVT variable to be greater than that of the IVT variable. In models MNL-5 and MNL-6 the OVT variable was, indeed, found to have a greater coefficient than IVT (by a factor of 3-4). In models MNL-5 and MNL-7 to MNL-10, the coefficients of the WALK and WK variables were approximately equal and 3-4 times the value of the IVT coefficient. The coefficient of the WT variable was found to have a value 7-10 times that of the IVT coefficient and twice that of the coefficient of the WK variable.

The differences between the coefficients of the WK and WT variables and that of IVT are greater than normally reported. Customary values of the WK and WT coefficients are, respectively, 2 and 2.5 times the value of the IVT coefficient[1]. The greater difference in the present study is attributed to the fact that individuals tend to be more conscious of the OVT variable components (i.e. WK and WT) due to the hilly nature of the topography of Glasgow, to the unpredictable weather and also to the effect of the River Clyde, the presence of which causes some of the trips to be split into more than one stage, resulting in more transfer (i.e. more walking and waiting times).

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[1] See for example: Quarmby (1967); Pratt and Deen (1967); McIntosh and Quarmby (1970). However, Algers, Hansen, and Tenger (1975) found that the waiting time coefficient was 7 to 12 times larger than the in-vehicle time coefficient.

Variable (Alternatives Entered)[1]	Coefficients (t-values)			
	MNL-11	MNL-12	MNL-13	MNL-14
HHPOS (1)	1.5851 (3.54)	1.5101 (3.50)	1.6322 (3.62)	1.5599 (3.54)
CAOD (1-2)	0.8673 (3.39)	0.9440 (3.71)	0.8396 (3.28)	0.9136 (3.63)
CAPDL (1)	4.6893 (4.82)	4.5286 (4.88)	4.6399 (4.65)	4.4863 (4.65)
HINC (1-4)	0.0819 (2.27)	---	0.0794 (2.15)	---
CBD (3-4)	1.4133 (4.73)	1.3817 (4.74)	1.2743 (4.10)	1.2453 (4.03)
TJT (1-5)	-0.1070 (-8.83)	-0.1029 (-8.68)	-0.1058 (-8.66)	-0.1018 (-8.45)
COST/DIST (1-4)	---	---	-0.0826 (-2.63)	-0.0849 (-2.68)
CCON (1)	-6.6817 (-7.16)	-5.5697 (-7.57)	-6.3065 (-6.68)	-5.2218 (-6.76)
PCON (1)	-4.8339 (-8.93)	-3.8513 (-13.14)	-4.5244 (-8.21)	-3.5656 (-11.49)
BCON (1)	-2.0153 (-4.42)	-1.0950 (-5.63)	-1.1536 (-2.10)	-0.2430* (-0.69)
TCON (1)	-2.8581 (-5.80)	-1.9236 (-7.53)	-1.9805 (-3.38)	-1.0539 (-2.66)
LLR_0	599.505	594.333	609.001	604.393
d.f.	10	9	11	10
% Right	74.53	73.77	74.72	74.72

[1] For codes see Table 5.1.

* Non- significant variable at the 95% level.

TABLE 5.4 Model specifications and aggregation of the variables.

5.3 COMPARISON OF ALTERNATIVE SPECIFICATIONS OF THE MODEL

Alternative model specifications were calibrated and presented in the previous section. To determine which models provide the most satisfactory specifications for use in further analyses, the alternative models should be compared statistically. The usual method of comparison is by defining the goodness-of-fit measures for the models in question (see Chapter 3) and comparing their values. The model with the greatest goodness-of-fit values among the models being compared is considered to provide the best explanation of the available data and to have the most satisfactory specification.

Goodness-of-fit measures such as LLR and ρ^2 are shown for all models in Table 5.5 and compared with the basic equally-likely and the market-share base models. The LLR measures have a X^2 distribution and can be compared with the critical 95 percent values shown in the table. Comparison of these values reveals the following:

1. All the models have very high values of LLR_0 and $\rho_0{}^2$. Thus the null hypothesis that the equally-likely model is not significantly different from these tested models at the 95% level is strongly rejected.

2. All the models have excellent values of LLR_C and $\rho_C{}^2$. Thus the null hypothesis that the market-share model is not significantly different from these models at the 95% level is also strongly rejected.

3. All the models have very high values of % right. This may also indicate that all these models were well specified.

Statistical Measures	Alternative Models					
	MNL-C	MNL-5	MNL-6	MNL-7	MNL-8	MNL-9
LLF_0	-644.60	-644.60	-644.60	-644.60	-644.60	-644.60
LLF_C	-456.79	-456.79	-456.79	-456.79	-456.79	-456.79
LLF_β	-456.79	-321.13	-319.86	-317.97	-322.68	-323.41
LLR_0 (d.f.) (χ^2, 0.95)	375.62 (4) (9.49)	646.94 (13) (22.36)	649.98 (13) (22.36)	653.26 (14) (23.69)	643.84 (13) (22.36)	642.38 (12) (21.03)
LLR_C (d.f.) (χ^2, 0.95)	n.a.	271.32 (9) (16.92)	273.86 (9) (16.92)	277.64 (10) (18.31)	268.22 (9) (16.92)	266.76 (8) (15.51)
$\rho_0{}^2$	0.291	0.502	0.504	0.507	0.499	0.500
$\rho_C{}^2$	n.a.	0.297	0.300	0.304	0.294	0.292
% Right	67.74	77.93	77.36	76.98	77.36	77.74

TABLE 5.5 Summary of goodness-of-fit measures for all model specifications.

Continued:

Statistical Measures	Alternative Models				
	MNL-10	MNL-11	MNL-12	MNL-13	MNL-14
LLF_0	-644.60	-644.60	-644.60	-644.60	-644.60
LLF_C	-456.79	-456.79	-456.79	-456.79	-456.79
LLF_β	-318.38	-344.85	-347.44	-340.10	-342.41
LLR_0 (d.f.) (χ^2, 0.95)	652.45 (13) (22.36)	599.51 (10) (18.31)	594.33 (9) (16.92)	609.00 (11) (19.68)	604.39 (10) (18.31)
LLR_C (d.f.) (χ^2, 0.95)	276.82 (9) (16.92)	223.88 (6) (12.59)	218.70 (5) (11.07)	233.38 (7) (14.07)	228.76 (6) (12.59)
ρ_0^2	0.506	0.465	0.461	0.472	0.469
ρ_C^2	0.303	0.245	0.239	0.256	0.250
% Right	76.98	74.53	73.77	74.72	74.72

TABLE 5.5 Summary of goodness–of–fit measures for all model specifications.

As can be seen from Table 5.5, models MNL-7 and MNL-9 have sightly higher values of all the measures. Therefore, these models could be considered as the best specified models. However, Horowtiz (1981, 1982) pointed out that: "Although it is generally recognised that these procedures (goodness-of-fit tests) can provide only rough indications of the quality of models, they often are the only diagnostic procedures that are carried out during the model estimation"; and, "It is easy to show that in comparisons of nested models[1], uncritical use of goodness-of-fit statistics can yield perverse results. For example, the well-known likelihood ratio index (McFadden, 1974) will nearly always lead to acceptance of the model with the largest number of parameters, even if many of these parameters are superfluous". Therefore, it is very difficult to discriminate between the tested models on the basis of statistical tests alone, especially when the statistical measures are nearly equal.

In this study, two basic criteria were employed to evaluate different model specifications: the statistical goodness-of-fit measures for the models under consideration; and the values of the alternative-specific constants which actually reflect the explanatory powers of these models. Reinvestigating Tables 5.3, 5.4, and 5.5, it can be concluded that models MNL-8, MNL-9, MNL-10, and MNL-14 have the best specifications. However, model MNL-8 has an unexpected positive COST / HINC coefficient and so was excluded from further analysis. Models MNL-9, MNL-10, and MNL-14 will, therefore, be discussed next.

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[1] Nested models are pairs of models based on the same mathematical theory, but where the one with the lower number of variables is considered as a linear restriction of the other (e.g. a logit model with different specifications). Non-nested models such as logit and probit are based on different theories.

5.4 VALIDATION TESTING OF THE SELECTED MODELS

To assess the predictive validity of the chosen models, the choice probabilities of each alternative mode were computed for each individual in the hold-back validation subsample. The predicted choice shares of the alternative modes were calculated in two different ways. Firstly, the choice probabilities of each mode for each individual were summed and averaged to give the expected choice shares of each mode. These are described as the expected share in Table 5.6. Secondly, the mode choice of each individual was predicted by the highest probability method and the percentage of individuals predicted to choose each mode then computed (i.e. each individual in the subsample was assigned to the highest-probability mode and the resulting proportion of individuals choosing each available mode computed). These are described as the predicted share in Table 5.6. These two shares are compared with the actual share of each mode in the validation subsample in Table 5.6. These results reflect the excellent fit of the tested data and may reflect the potential applicability of the chosen models to other locations similar to Glasgow.

As can be seen from Table 5.6, the models appear to provide an excellent match between the actual and the expected shares, and also a relatively good match between the actual and the predicted shares. Thus, none of these models appears to be much superior to the others.

Alter-native Model	Choice Share Distribution	Proportion (%)				
		Car Driver	Car Passenger	Bus	Train	Walk
MNL-9	Actual	29.2	12.5	25.0	8.3	25.0
	Expected	28.1	13.6	24.7	8.3	25.3
	Predicted	32.5	7.5	22.5	5.8	31.7
MNL-10	Actual	29.2	12.5	25.0	8.3	25.0
	Expected	27.8	13.1	24.9	8.2	26.0
	Predicted	31.6	5.2	26.5	7.5	29.2
MNL-14	Actual	29.2	12.5	25.0	8.3	25.0
	Expected	27.6	12.9	24.3	8.5	26.7
	Predicted	35.8	5.3	22.1	5.2	31.6

TABLE 5.6 Summary of the validation test results for the chosen models.

For further analyses (i.e. aggregate share prediction and policy change) two models, MNL–10 and MNL–14, were chosen as being the best–specified; MNL–9 was not considered further since it did not include the COST variable and so was less policy–responsive. The specifications of models MNL–10 and MNL–14 are similar and both include the time and cost variables, which are the most important policy–controlable variables, with the correct signs. The models MNL–10 and MNL–14, indeed, represent the complex and the simple model respectively.

CHAPTER SIX

AGGREGATE PREDICTION ANALYSIS

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AGGREGATE PREDICTION ANALYSIS

6.1 INTRODUCTION

In travel demand analysis and forecasting, the prediction of aggregate travel behaviour and of the performance of alternative transportation systems are always needed by transportation planners and decision–makers in order to determine the desirability of possible alternative transportation plans. The analysis of travel behaviour at the individual level is always preferred on theoretical grounds because of its correspondence with the actual behavioural choice process. This chapter considers aggregate prediction using disaggregate choice models.

The remainder of the chapter is divided into six sections. Section 6.2 presents and discusses the problems inherent in the application of disaggregate travel demand models to the prediction of aggregate behaviour. Section 6.3 describes the available alternative aggregation approaches. These are the naive, statistical differentials, classification, numerical integration, and enumeration procedures. Section 6.4 presents the different sources of prediction error in the application of these methods. In Section 6.5 the analytical measurement of aggregate prediction errors is considered. Section 6.6 presents the empirical results of the use of the naive, classification, and enumeration approaches. Finally, the last section compares the selected aggregation procedures.

6.2 THE APPLICATION OF DISAGGREGATE MODELS AT THE AGGREGATE LEVEL

The MNL models derived in Chapters 3 and 5 can be utilised to predict directly the behaviour of an individual selected randomly from the population. This is generally of little use to transportation planners and decision-makers since they are always interested in the prediction of aggregate travel behaviour (which is the accumulation of individuals' behaviour) in order to evaluate their alternative transportation plans and decisions.

Two alternative approaches to the prediction of aggregate travel behaviour are available. The first approach, the conventional one, uses an aggregate model calibrated with aggregate data to predict directly the aggregate travel behaviour. The second approach is to calibrate a disaggregate model using disaggregate data and to use this model for aggregate predictions.

There are a number of problems associated with the first approach. Firstly, a considerable number of observations is needed to calibrate an aggregate model and a direct consequence of this is that aggregate models are expensive and time-consuming to produce. Secondly, the loss of information experienced in aggregating values of the explanatory variables (i.e. no account is taken of the variation across the data observations for a zone) results in an aggregate model that has biased coefficients. Finally, the lack of policy-controllable variables in the specifications leads to models which are inflexible and less policy-oriented.

The only advantage of aggregate models is that they may be used directly to predict aggregate travel behaviour. However, considering the aforementioned problems, although these models may be able to simulate adequately the observed aggregate situation from which they were derived, their stability in other practical transportation situations is doubtful[1].

On the other hand, the advantages of using disaggregate models for the prediction of aggregate travel behaviour are just the converse of the disadvantages of using aggregate models. Firstly, disaggregate models may be calibrated using relatively few data points, and therefore, may be relatively quickly and inexpensively estimated. Secondly, there is no information loss due to aggregation because no aggregation is necessary to calibrate these models. Thirdly, their encompassing the policy-relevant variables provides disaggregate models with a potentially more useful role in prediction than the descriptive aggregate models. Finally, because they do not contain any aggregation scheme, disaggregate models can be used at different levels of aggregation, in different places and at different times.

Conceptually, therefore, disaggregate models are likely to be much more useful in the prediction of aggregate travel behaviour than the corresponding aggregate models. Past studies would seem to support this contention[2].

However, the problems of aggregation and possible loss of information, which are in fact equally applicable to the use of aggregate models, must be confronted when using disaggregate models in aggregate prediction.

While it is desirable to calibrate a model at the disaggregate level, it is not

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1 See for example: Watson (1973, 1974); Richards and Ben-Akiva (1975).

2 See for example: DeDonnea (1971); Kanafani (1972); Ben-Akiva (1973); Kannel and Heathington (1973); Tahir and Hovind (1973); Talvitie (1973); Koppelman (1974, 1975, 1976a); Miller (1974); Watson (1974, 1976); Westin (1974); Atherton (1975); Difilio and Reed (1975); Liou et al (1975); McFadden and Reid (1975); Watson and Westin (1975); Meyburg and Stopher (1975); Hensher and Johnson (1977); McFadden et al (1977); Bouthelier and Daganzo (1978, 1979); Hensher and Stopher (1978); Parody (1978); Ortuzar (1980) ;Dunne (1982).

always possible to use the same model directly for aggregate predictions, since the direct substitution of the average values of the relevant explanatory variables into the model formulation (the naive approach) may provide inaccurate predictions. For example, if the disaggregate model is non-linear, the disaggregate functional specification (with average values of the explanatory variables substituted for the individual values) will give a biased prediction of the average of the dependent variable, except in the special case when the population concerned is homogeneous with respect to those variables that influence the choice under study [Theil (1955); Green (1964)]. However, when the data are available at the disaggregate level, a more accurate aggregate prediction can be obtained directly. In this case, the expected choice behaviour can be estimated for each individual and then summed or averaged to obtain the aggregate travel predictions (the enumeration approach). This approach, however, requires voluminous data and exhaustive computation.

Between the extreme procedures for the prediction of aggregate travel behaviour, the naive and enumeration procedures, a number of alternative aggregation approaches have been proposed [Talvitie (1973); Westin (1974); Koppelman (1975)]. All of these methods are discussed next.

6.3 AGGREGATION APPROACHES

Koppelman (1975) defines five general types of aggregation procedure, according to the method by which the distribution of the explanatory variables is represented in the aggregate prediction models, though some of the five can be considered as special cases of the others. The purpose of these methods is to transform the disaggregate model and the distribution of the explanatory variables into a set of aggregate predictions. Each procedure reduces the problem of aggregation by imposing some simplifying assumptions about the choice model, the population or both. Each of these approaches is discussed in turn.

6.3.1 THE NAIVE APPROACH

The simplest, and possibly the most obvious, procedure for the prediction of aggregate travel behaviour involves the use of the sample average values of the explanatory variables together with the coefficients of the disaggregate model.

The general form of the disaggregate model is given by,

$$P_{in} = f_i \ (X_n) \tag{6.1}$$

where,

- P_{in} is the probability of individual n choosing alternative i,
- f_i is the choice function for alternative i, and
- X_n is the vector of the characteristics of available alternatives and individual attributes.

For the MNL model Equation 6.1 becomes,

$$P_{in} = \frac{\exp \ (V_{in})}{\sum\limits_{j \in A_n} \exp \ (V_{jn})} \tag{6.2}$$

where,

V_{in}, V_{jn} are the utility functions of alternatives i and j, respectively.

Thus, the expected aggregate choice share for alternative i in the sample of N observations is,

$$S_{iN} = f_i(\bar{X}) \tag{6.3}$$

or,

$$S_{iN} = \frac{\exp(\bar{V}_{iN})}{\sum_{j \epsilon A_N} \exp(\bar{V}_{jN})} \tag{6.4}$$

where,

S_{iN} is the aggregate choice share for alternative i, and

$\bar{X}$ is a vector of average values of explanatory variables for each alternative over all the prediction group.

Although this procedure uses the average values of the explanatory variables, it still has the advantage over the traditional aggregate prediction approach, which uses the aggregate model with the average values, since the coefficients are estimated at the disaggregate level. In addition, less data are required for the calibration of the disaggregate model. However, this approach implicitly assumes that each individual will behave as if represented by the average values of the explanatory variables, thus basing the analysis on the representative individual and taking no account of the distributions of the values of the variables across the prediction group. If this homogeneity of individuals does not hold, and if the functional form of the model is non-linear, then this approach will produce a biased prediction. However, the naive approach is the most likely to be used in the absence of recognition of the aggregation problem.

Prediction by the naive approach can be adjusted to account for differences in the choice set availability when such differences exist.

6.3.2 THE STATISTICAL DIFFERENTIALS APPROACH

In this approach, the expected aggregate shares are predicted on the basis of the moments of the distribution of the probabilities over the sample population. The method was first suggested as an approach to aggregate prediction by Talvitie (1973). He noted that choice probability could be expressed in terms of a Taylor series expansion of the disaggregate choice function about the mean variable values of an aggregate. However, the practical issues associated with estimating higher order moments and the instability of the series when the distribution is highly dispersed led the series to be terminated after the second moment or variance term [Johnson and Kotz (1969)]. Thus, for the binary choice case, the aggregate choice share of alternative i in a sample of N observations is given by,

$$S_{iN} = f_i(\bar{V}) + \frac{1}{2} \left. \frac{d^2 f_i(V)}{d V^2} \right|_{\bar{V}} \sigma_V^2 \tag{6.5}$$

where,

$f_i(\overline{V})$ is the choice function in terms of the net utility between the two alternatives, evaluated at the mean value of the net utility,

$\left.\frac{d^2 f_i(V)}{d V^2}\right|_{\bar{V}}$ is the second derivative of the choice function with respect to net utility, evaluated at the mean value of the net utility, and

σ_V^2 is the variance of the net utility distribution in the prediction group.

The corresponding equation for the multinomial choice situation is,

$$S_{iN} = f_i(\bar{V}) + \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} \left.\frac{d^2 f_i(V)}{dV_j dV_k}\right|_{\bar{V}} \mathrm{Cov}(V_j, V_k) \qquad (6.6)$$

where,

V, $\bar{V}$ are the vectors of utility and mean utility values for each alternative, respectively, and

$\mathrm{Cov}(V_j,V_k)$ is the covariance in the distribution of utilities for alternatives j and k; when j= k it is the variance of the utility distribution.

The advantage of this procedure is that it takes into account the within-group variance, through the use of the distribution moments of the explanatory variables, to achieve unbiased aggregate predictions, thus making it superior to the naive approach. However, Talvitie (1981) suggests that the use of the Taylor series approximation in multiple choice cases cannot be recommended due to its instability in the binary case [see also McFadden (1981)].

6.3.3 THE CLASSIFICATION APPROACH

The classification approach was initially developed to overcome the high prediction biases which result from the use of the naive approach [Koppelman (1975); Reid (1978)]. It involves, firstly, dividing the entire prediction group into relatively homogeneous groups, or market segments, so as to minimise within-group and maximise between-group variances [Ben-Akiva and Lerman (1985)]. The naive approach is then used to predict aggregate choice shares for each group or segment. Finally, the aggregate share of each alternative in the entire prediction group is computed from the weighted sum of all the naive shares of the groups. Thus,

$$S_{iN} = \sum_{g=1}^{G} \frac{N_g}{N} f_i(\bar{X}_g) \tag{6.7}$$

where,

$\bar{X}_g$ is the vector of the average values of the explanatory variables for N_g individuals in group g,

N_g is the total number of individuals in group g, and

N is the total number of individuals in all prediction groups.

The traditional geographical segmentation of a prediction group has been shown to be only a fair classifier of the level-of-service variables, and a poor classifier of the socio-economic variables, and results in groups which are insufficiently homogeneous. This leads to imperfect predictions. However, Koppelman (1975) suggests that further classification efforts to obtain more accurate aggregate predictions should concentrate on the distribution of the socio-economic variables which are not homogeneous within zones. The selection of the socio-economic variables for classification should be aimed at reducing the variance of the net utility distributions. This can be accomplished by the selection of those variables which exhibit the largest variances in the utility function. For example, in mode choice prediction, household income, car ownership, and the number of cars per driving licence holder are the most commonly used classifier variables.

However, classification by the value of a single explanatory variable sometimes gives unacceptable prediction results, especially in large aggregate prediction groups. Reid (1978) suggests classifying directly by the value of the utility function. The only problem with the use of Reid's approach is that utilities are not discrete, and intuition gives no guide as to how to divide the utility values into different utility classes [Ben-Akiva and Lerman (1985)].

6.3.4 THE NUMERICAL INTEGRATION APPROACH

This approach attempts to represent the variation of the explanatory variables across individuals in the prediction group in terms of their joint probability density function. The aggregate choice share of each alternative is then computed by integrating the disaggregate choice probability function weighted by the joint probability density function of the explanatory variables. Thus,

$$S_{iN} = \int_0^1 f_i(X)\ P_i(X)\ dX \qquad (6.8)$$

where,

X is the vector of the explanatory variables, and

$P_i(X)$ is their joint probability function.

However, $P_i(X)$ is generally unknown, so an approximate theoretical distribution is usually assumed. Westin (1974) has shown that, when individual choice can be represented by the binary logit function,

$$f_i(X) = \frac{1}{1 + \exp(\beta X_i)} \tag{6.9}$$

and the explanatory variables are normally distributed over the prediction group with mean $\bar{X}_i$ and variance-covariance matrix Σ, so that βX_i is normally distributed with mean $\beta \bar{X}_i$ and variance $\sigma^2 = \beta^T \Sigma \beta$, then the density function $P_i(X)$ is shown to have the beta distribution (S_B)[1] given by Equation 6.10.

$$P_i(X) = \frac{1}{\sqrt{2\Pi \sigma_i^2}} \frac{1}{P_i(1-P_i)}$$

$$\exp\left[- \frac{1}{2\sigma_i} \left\{ \ln\left(\frac{P_i}{1 - P_i} \right) - \bar{X}_i \right\}^2 \right] \tag{6.10}$$

No closed form for Equation 6.8 exists, but a table for the S_B distribution function can be utilised to reduce the computational burden of Equation 6.10.

[1] See for example: Johnson (1949); Johnson and Kotz (1972); Westin (1974).

McFadden and Reid (1975) used the same normal distributional assumption with the binary probit model. The aggregate share probit model which they obtained is,

$$S_{iN} = \Phi \left(\frac{\beta \bar{X}_i}{\sqrt{1 + \sigma_i^2}} \right) \qquad (6.11)$$

where,

Φ is the standard cumulative normal distribution.

This method was extended to the multinomial case by Bouthelier (1978).

In general, the numerical integration method would be quite cumbersome even for the binary cases. Its extension to the multiple cases would be difficult and the computational requirement of evaluating their integral may be prohibitive [Talvitie (1976); Ben–Akiva and Lerman (1985)].

6.3.5 THE ENUMERATION APPROACH

This approach represents the most explicit theoretical relationship between aggregate and disaggregate travel demand. The expected aggregate choice share for each alternative is obtained simply by averaging all of the estimated individual choice probabilities for that alternative. Thus,

$$S_{iN} = \frac{1}{N} \sum_{n=1}^{N} f_i\ (X_n) \qquad (6.12)$$

Although this method requires voluminous data and exhaustive computation due to the direct use of the values of the explanatory variables relevant to each individual in the prediction group, it has been shown, nevertheless, to give precise aggregate shares from the disaggregate models. For this reason it can be used as an ideal reference for evaluating the predictive performance of the alternative approaches [Koppelman (1975); Reid (1978)].

6.4 SOURCES OF PREDICTION ERRORS

Five aggregation approaches were identified and described in the previous section. These approaches are differentiated by their computational formulation and their input data requirements. All of the procedures are approximate and introduce errors into their aggregate predictions. It is necessary to consider the sources and types of these errors and how they are measured in order to evaluate the performance of the various aggregation procedures.

The major types of prediction errors associated with the use of disaggregate choice models are as follows:

1. Model specification errors: These are the result of applying the choice models to areas or situations different from the ones in which the models were calibrated (i.e. model transferability errors).

2. Data measurement errors: These comprise measurement errors associated with

the explanatory variables in both the calibration and prediction stages, and errors in estimating parameters[1].

Errors in model specification and data measurement may interact to produce errors in the prediction of the individual choice probabilities. These errors are propagated through the aggregation procedure to produce errors in aggregate prediction and may, therefore, be called collectively "propagation errors".

3. Aggregation errors: These result from the use of an approximate aggregate prediction approach to replace the most theoretically consistent aggregate prediction approach, the enumeration approach.

The above errors are determined in different ways. Errors due to model specification and data measurement (propagation errors) can be isolated by comparing the prediction by the enumeration method, which has no aggregation error, with the observed shares. The aggregation error from each aggregation approach can be obtained by comparing the prediction by each aggregation approach with the prediction by the enumeration approach. This comparison is the most commonly used since, firstly, the enumeration approach is consistent with relevant theories of individual travel behaviour; secondly, no aggregation error is involved; and finally, in most aggregate prediction applications the actual shares are unknown [Talvitie et al (1982)]. The total prediction error, which is the sum of all the above errors, is normally identified directly by comparing the aggregate predictions by each aggregate approach, except the enumeration approach, with the observed shares. However, this comparison is clouded due to the fact that the observed shares are not truly representative of the actual shares

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1 See for example: Manski (1975); Horowitz (1982, 1983); Ben–Akiva and Lerman (1985).

in the entire population because of sampling error [Koppelman (1975)].

6.5 MEASURES OF THE PREDICTION ERRORS

For comparative purposes, it is often desirable to express the error measure as a percentage of a reference value derived from an ideal procedure. In the case of the aggregate prediction error, two decisions must be made regarding the development of this error: how to express the prediction error of a single prediction unit (e.g. should it be an individual; a zone; a segment; a group of individuals; the entire sample; etc.?); and how to aggregate the error from the single prediction unit to some average aggregate prediction error.

In this study, the error measure chosen to describe the error in each single prediction unit is given by,

$$BEM = \frac{P_{pu} - P_{ru}}{P_{ru}} \tag{6.13}$$

where,

- BEM is the basic error measure in prediction per single unit of prediction,
- P_{pu} is the predicted value for the prediction unit estimated by the tested aggregation approach,
- P_{ru} is the reference value for the prediction unit estimated by the enumeration method, or the actual share (if available), and
- u is the unit of prediction.

In order to allow for an equitable summing of the total amount of prediction error for each prediction unit in the entire prediction group (i.e. to reflect the relative importance of each prediction unit), the error should be multiplied by a weighting value which is simply the reference value of that prediction unit or the size of the prediction unit or both. Thus, the average error measure for the entire prediction group is defined by,

$$AE = \sum_{u \epsilon U} \left(\frac{P_{pu} - P_{ru}}{P_{ru}}\right) P_{ru} \Big/ \sum_{u \epsilon U} P_{ru}$$

$$= \frac{\sum_{u \epsilon U} (P_{pu} - P_{ru})}{\sum_{u \epsilon U} P_{ru}} \tag{6.14}$$

where,

AE is the average prediction error for all prediction units, and

U is the total number of prediction units in the entire prediction group.

In order to treat the positive and negative errors alike (which is not the case with the average error), the entire prediction group error should be expressed as the average sum of squares of all prediction unit errors [i.e. the Root Mean Square Error (RMSE)], thus:

$$RMSE = \left[\sum_{u \epsilon U} \left(\frac{P_{pu} - P_{ru}}{P_{ru}}\right)^2 P_{ru} \Big/ \sum_{u \epsilon U} P_{ru} \right]^{1/2} \tag{6.15}$$

The Standard Deviation of this Error (SDE) is given by,

$$SDE = [\sum_{u \epsilon U} \{ (\frac{P_{pu} - P_{ru}}{P_{ru}}) - AE \}^2 \; P_{ru} / \sum_{u \epsilon U} P_{ru}]^{1/2} \qquad (6.16)$$

The relationship among the three error measures is,

$$RMSE^2 = AE^2 + SDE^2 \qquad (6.17)$$

In measuring the model specification and data measurement errors, P_{pu} and P_{ru} represent, respectively, the prediction values by the enumeration approach and the observed shares.

In estimating the aggregation error only, the values of P_{pu} and P_{ru} represent, respectively, the prediction values by each aggregation approach except the enumeration approach and the values by the enumeration approach.

In calculating the overall prediction error (which includes all of the types of error presented in the previous section), and provided that the actual shares are known, the values of P_{pu} and P_{ru} in Equations 6.13 and 6.14 represent, respectively, the prediction values by each aggregation approach except the enumeration approach and the actual or observed shares.

6.6 EMPIRICAL APPLICATION

The objectives of this empirical analysis are: firstly, to make a comparative evaluation of the performances of the different aggregation procedures through the identification of the magnitudes of their aggregation errors; and secondly, to test the predictive accuracies of the simple and complex models chosen in this study in terms of their aggregate prediction errors.

The choice of an aggregation approach for use in the prediction of aggregate travel behaviour depends mainly on the structure of the disaggregate model; the form of the available data; the accuracy required; and the economic considerations of the application of the chosen approach. In this study, aggregate prediction errors were examined for three different aggregation approaches; the naive, classification, and enumeration approaches. The naive and classification procedures were chosen for their conceptual simplicity and moderate data requirements (see Sections 6.3.1 and 6.3.2). The enumeration approach was chosen since it is conceptually simple to make aggregate predictions of travel behaviour when the data are available at the individual level. The statistical differentials and numerical integration procedures were excluded as their high computational requirements in multiple choice situations make them infeasible (see Sections 6.3.3 and 6.3.4)[1].

Unfortunately the data used in this empirical study are the data used also in the calibration of the disaggregate choice models. In fact, a true aggregate prediction

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[1] A personal communication (late 1988) from Prof. F. Koppelman of Northwestern University, USA, strongly advised against the use of the statistical differentials procedure. See for more details: Reid (1978); Hensher and Stopher (1979); McFadden (1981); Talvitie (1981); Supernak (1984, 1987); Ben-Akiva and Lerman (1985).

test should be carried out with another data set. However, the intention here is to show the ability of the tested models to reproduce the original data. While this is only the first step in the assessment of the models to be used in the aggregate prediction, if the models were to perform badly at this stage, it would certainly not be worthwhile using them with another data set or in another location.

The aggregate prediction errors for the three methods employed for the two choice models are presented and discussed in the following subsections.

6.6.1 AGGREGATION ERROR FOR THE NAIVE APPROACH

As was discussed earlier, the naive approach uses the average values of the explanatory variables for the entire study area in the disaggregate choice models and their computed probabilities as the expected choice shares for the entire study area.

Table 6.1 shows the aggregate prediction errors of the two models for the entire study area. As was expected, the error measures without choice set adjustment, shown in Table 6.1−a, have very high values. These are not surprising results since the higher error values for the entire study area as a single group are consistent with the increase in the average variance of the net utility distributions which results from aggregation over the wide range of individuals and level−of−service variables which are included in the entire study area. In addition, the use of one set of average values of all explanatory variables applied to all individuals in the study area implies that all alternative modes have effectively been available to all individuals for whom, in fact, some are not available. This will lead to an additional increase in the data variability which will in turn result in large aggregation errors.

It is also seen from Table 6.1–a that the complex model has lower error measures (8% lower) than the simple model. This indicates that the complex model is a better predictor when using the direct naive approach (i.e. the naive approach without choice set adjustment). However, since the error measures for the entire study area are based on a single representative observation, they are not reliable. On the other hand, when choice set variability is considered (see Table 6.1–b), the values of the error measures are reduced drastically, reflecting the importance of considering choice set variation across the individuals in the entire study area. In addition, the simple model has lower error values (3% lower) than the complex one. This indicates that the simple model predicts just as accurately as the complex model when considering choice set variation.

The naive error measures shown in Table 6.1–b are slightly larger than the values obtained in a previous study by Koppelman (1975) (e.g. naive RMSE = 10.2%) and substantially smaller than those in a study by Reid (1978) (e.g. naive RMSE = 40.0%). The errors obtained in this study are greater than might have been, since no account was taken of sample heterogeneity, and could have been further reduced by using a more consistent approach such as the classification approach.

Error Measure	Simple Model	Complex Model
AE	126.82	118.40
SDE	0	0
RMSE	126.82	118.40

a. Without choice set adjustment

Error Measure	Simple Model	Complex Model
AE	3.57	5.01
SDE	9.19	11.66
RMSE	9.86	12.69

b. With choice set adjustment

TABLE 6.1 Naive aggregation errors (percent) for the two models for the entire study area with and without choice set adjustment.

6.6.2 AGGREGATION ERROR FOR THE CLASSIFICATION APPROACH

In order to improve the predictive powers of the chosen disaggregate models (i.e. minimise their prediction error values), two classification methods have been used. These are:

6.6.2.1 GEOGRAPHICAL CLASSIFICATION

Aggregate predictions are traditionally made at the zonal level. However, in this study, aggregate predictions based on zonal average values of the explanatory variables seemed to be infeasible due to the small sample size (530 observations) and the large number of zones available (more than 600; see Figure 6.1). Thus, the aggregate predictions were carried out at three geographical levels. These were: the entire study area, six bands (Figure 6.2), and ten sectorgroups[1] (Figure 6.3). These bands and sectorgroups were originally defined by the GRIS study group in order to present the survey results in terms of simple statistical values at area levels. To be consistent with the GRIS study, the same divisions were used here.

Table 6.2 shows the variation in the aggregate prediction errors with the geographical scales of classification for the two disaggregate models.

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1 A trial was undertaken grouping the data into 42 sectors (see Figure 6.4), but it was found that most of these sectors were of small size, and so it was thought infeasible to consider them as aggregate groups.

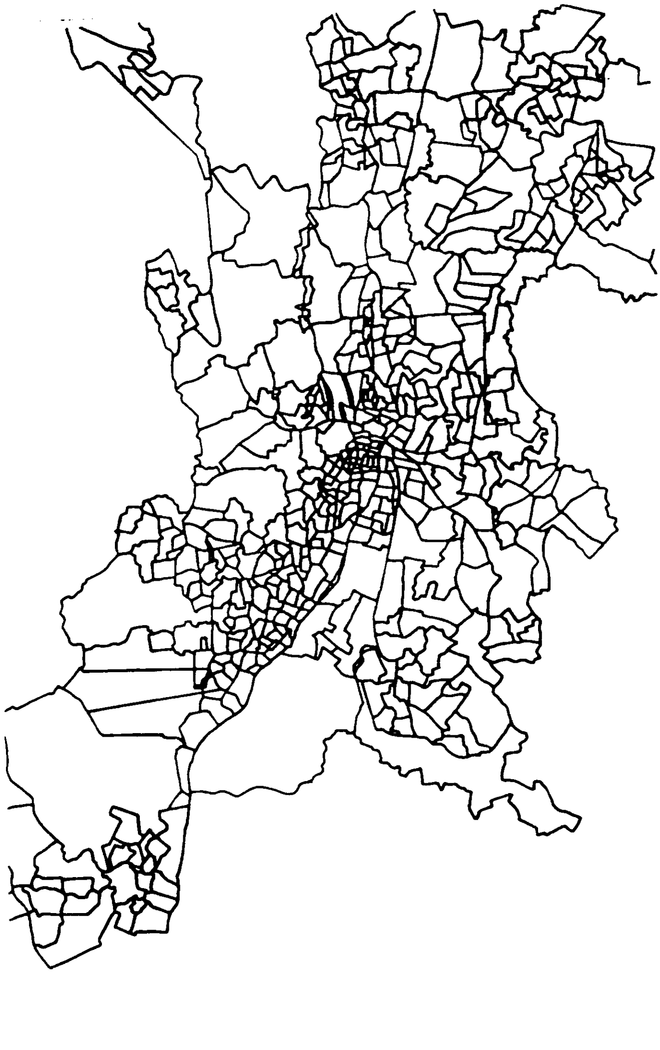
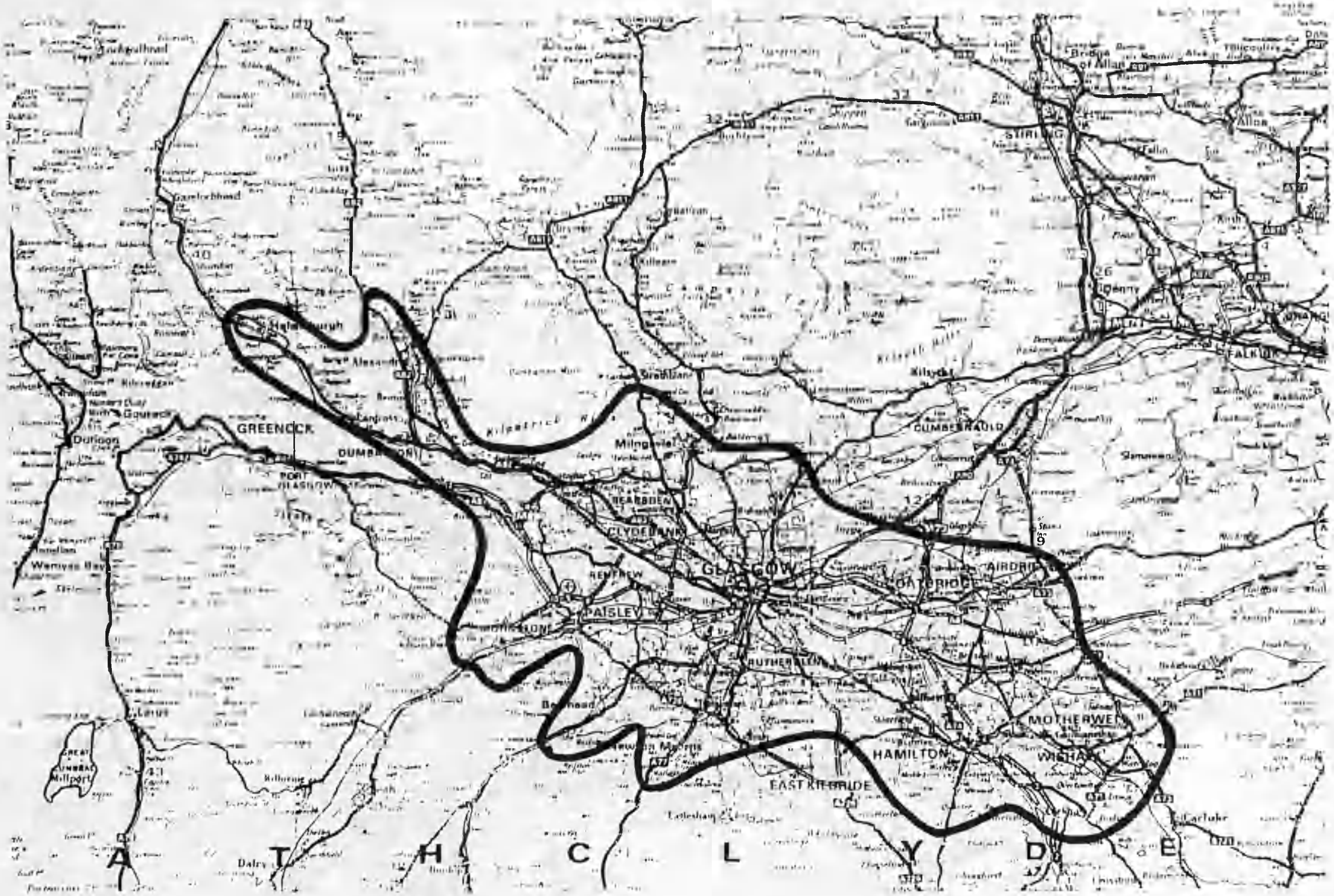


FIGURE 6.1 The GRIS study area (Zones).



GREENOCK
PORT GLASGOW
DUMBARTON
CLYDEBANK
BEARSDEN
Milngavie
GLASGOW
RENFREW
PAISLEY
RUTHERGLEN
COATBRIDGE
AIRDRIE
CUMBERNAULD
STIRLING
FALKIRK
MOTHERWELL
WISHAW
HAMILTON
EAST KILBRIDE
Gourock
Dunoon
Wemyss Bay
Largs
Millport
Kilsyth
Strathblane
Bridge of Allan
Alloa
Tillicoultry
Carluke
Eaglesham
Dalry
S T R A T H C L Y D E

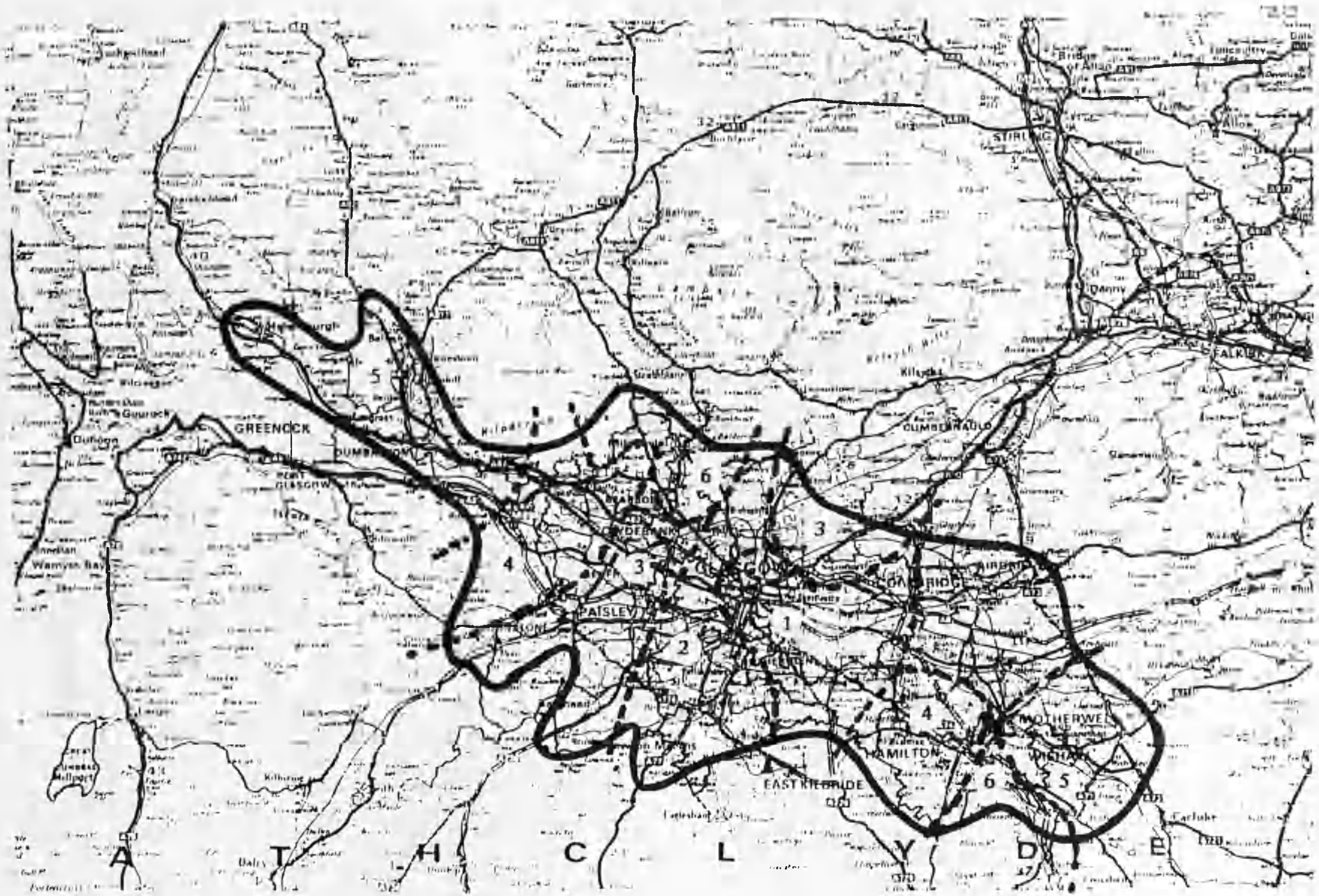


FIGURE 6.2 The GRIS study area (Bands).

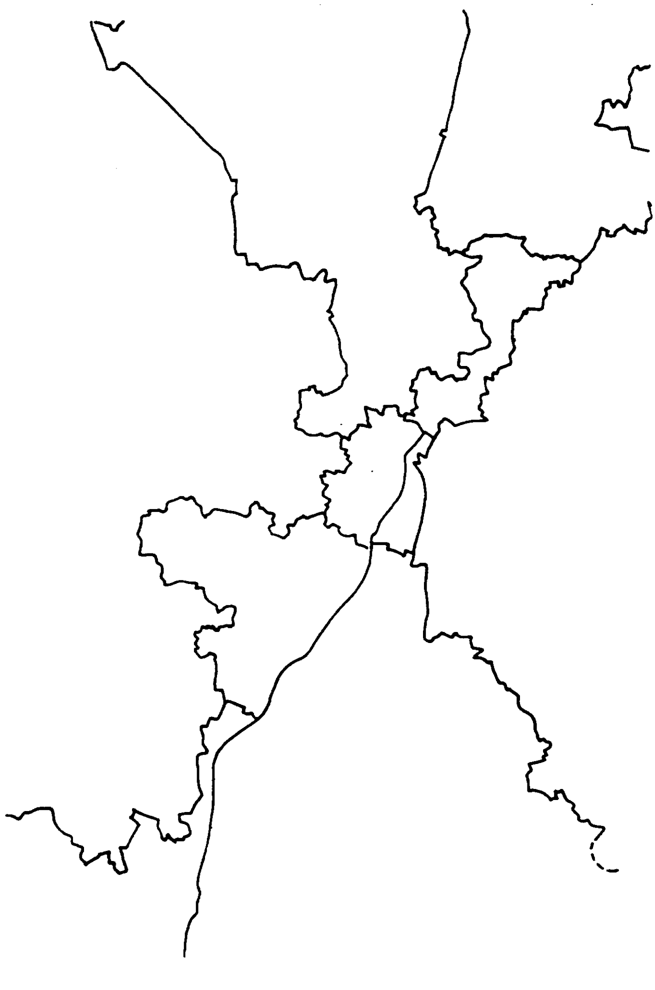
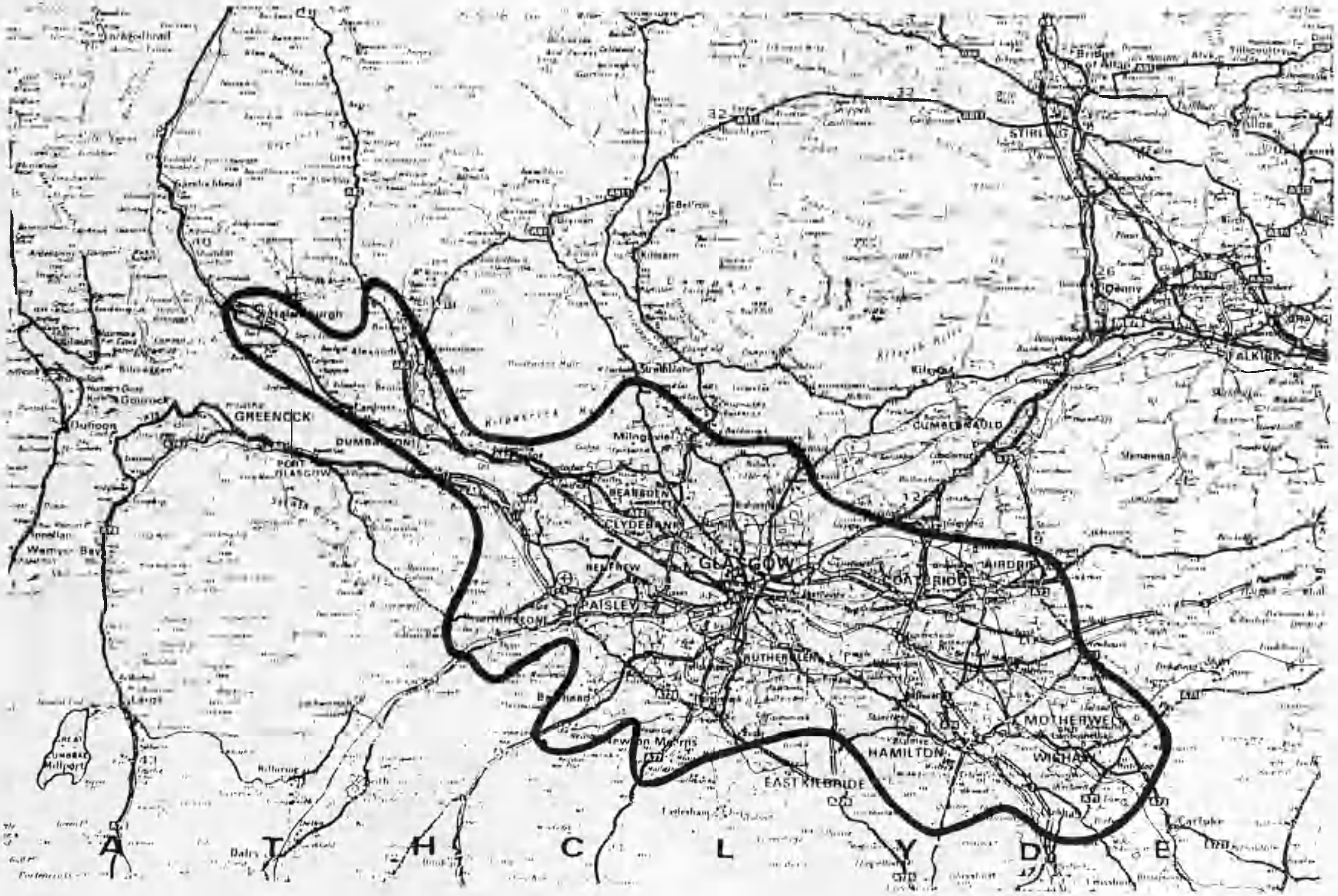


FIGURE 6.3 The GRIS study area (Sectorgroups).



STIRLING
FALKIRK
Denny
CUMBERNAULD
Milngavie
BEARSDEN
CLYDEBANK
GLASGOW
RENFREW
PAISLEY
AIRDRIE
COATBRIDGE
RUTHERGLEN
MOTHERWELL
WISHAW
HAMILTON
EAST KILBRIDE
Newton Mearns
Barrhead
Helensburgh
DUMBARTON
GREENOCK
PORT GLASGOW
Dunoon
Gourock
Wemyss Bay
Largs
Millport
Kilsyth
Killearn
Bridge of Allan
Carluke
A T H C L Y D E

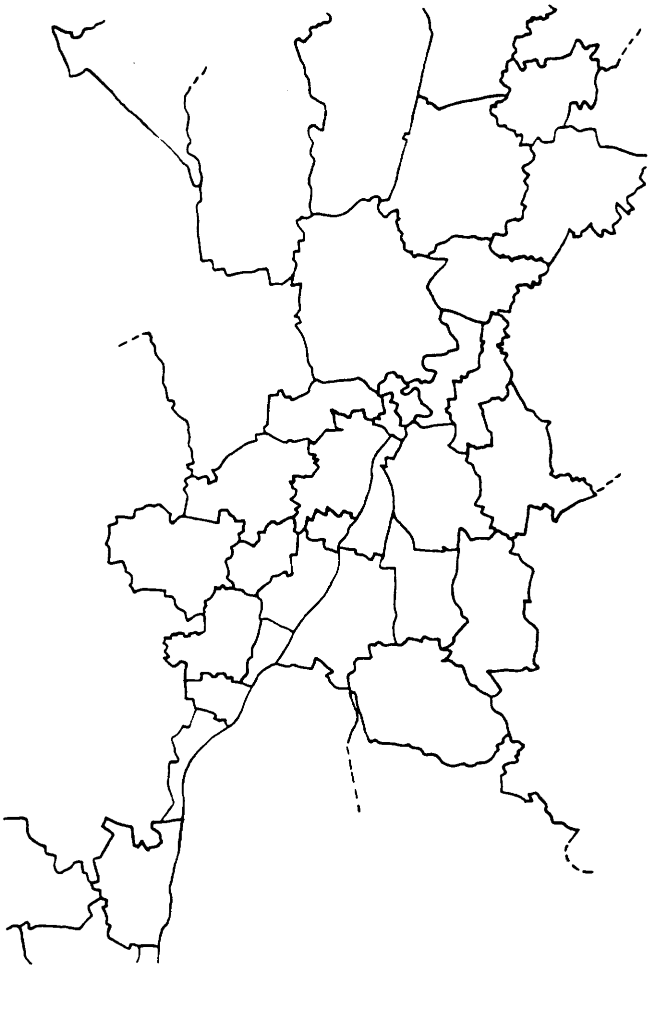
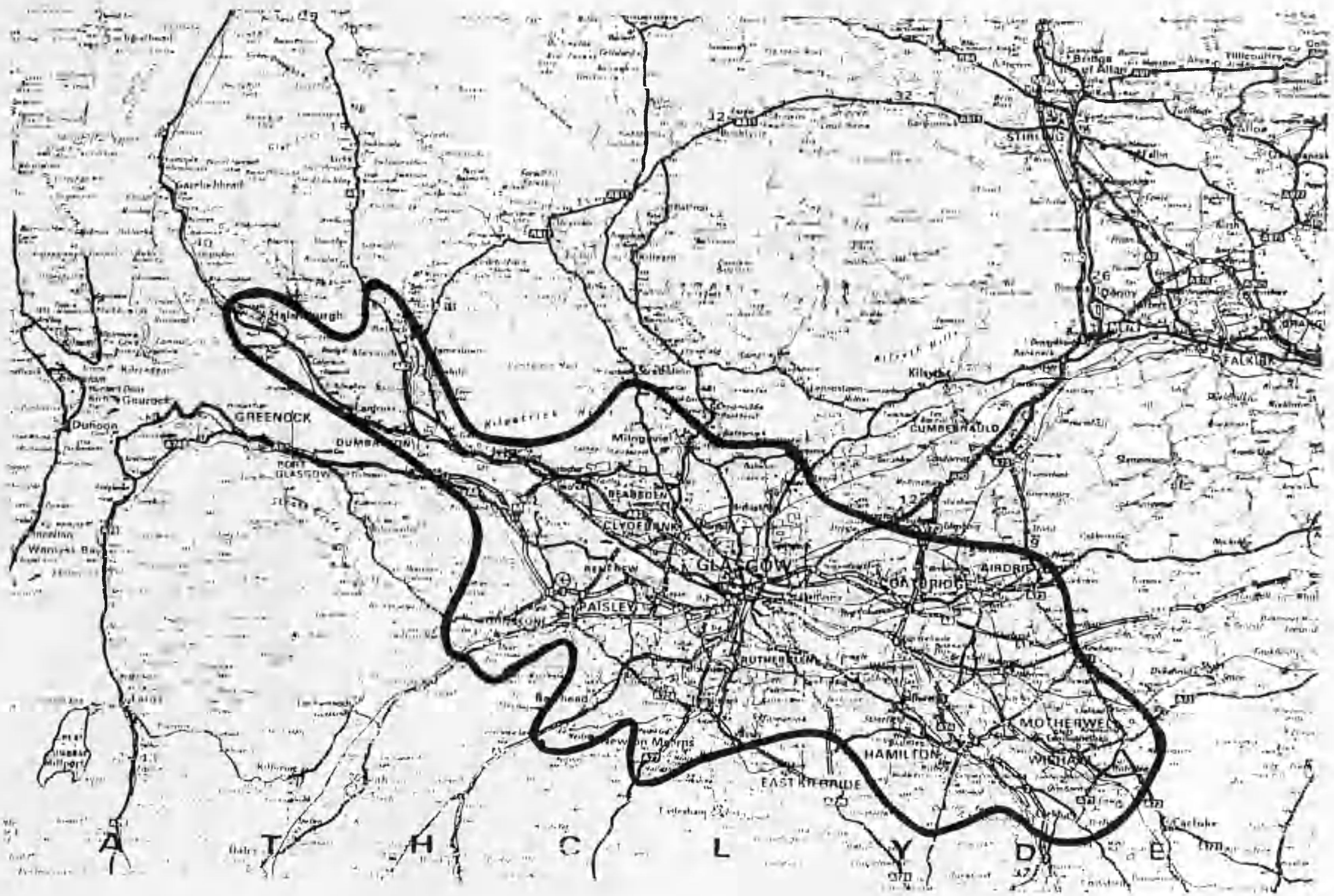


FIGURE 6.4 The GRIS study area (Sectors).



GREENOCK
PORT GLASGOW
DUMBARTON
Helensburgh
Gourock
Dunoon
Wemyss Bay
Largs
Millport
Garelochhead
Milngavie
BEARSDEN
CLYDEBANK
RENFREW
GLASGOW
PAISLEY
RUTHERGLEN
Barrhead
Newton Mearns
EAST KILBRIDE
HAMILTON
MOTHERWELL
WISHAW
COATBRIDGE
AIRDRIE
CUMBERNAULD
Kilsyth
Denny
FALKIRK
STIRLING
Bridge of Allan
Tillicoultry
Alloa
Carluke
S T R A T H C L Y D E

Types of Model	Types of Error	Classification by		
		Study area * (1)	Bands (6)	Sectorgroups (10)
Simple Model	AE	3.57	0.77	0.71
	SDE	9.19	4.94	4.88
	RMSE	9.86	5.00	4.93
Complex Model	AE	5.01	1.15	0.97
	SDE	11.66	6.28	5.81
	RMSE	12.69	6.38	5.89

* Note: the numbers in parentheses represent the numbers of groups at the different geographical levels.

TABLE 6.2 Percentage aggregation error for the three geographical levels for the two models.

It is clear from the table that the error measures increase with increasing geographical level. In other words, the whole study area as a single group has the highest error values, while the bands have smaller errors than the entire study area but larger errors than the sectorgroups. This is, indeed, consistent with the statistical notion that the aggregation error increases with increasing within-group variance [Fleet and Robertson (1968); DeNeufville and Stafford (1971)]. It is also apparent from Table 6.2 that the simple model is a slightly better predictor than the complex model at all three levels of geographical classification.

6.6.2.2 BY-VARIABLES CLASSIFICATION

An alternative to geographical classification is to divide the data sample into relatively homogeneous groups of individuals according to their attributes so as to minimise the within-group and maximise the between-group variances. In this approach the classification should be based on the important explanatory variables, that is, as Koppelman and Ben-Akiva (1977) point out, the variables with the highest variances.

In this study the classification was done in three ways. The data were classified: firstly, into three groups according to the number of cars per household (CAOD variable; CAOD = 0, CAOD = 1, and CAOD = 2+); secondly, into three groups based on the number of cars per driving licence (CAPDL variable; (CAPDL = 0, 0 < CAPDL < 1, and CAPDL = 1); and finally, into eight groups by different combinations of the variables HHPOS, CAOD, and CBD. The results of this classification approach are shown in Table 6.3.

Types of Model	Types of Error	Classification by		
		CAOD * (3)	CAPDL (3)	HHPOS, CAOD, and CBD (8)
Simple Model	AE	2.99	2.45	0.71
	SDE	8.59	6.87	4.20
	RMSE	9.10	7.29	4.26
Complex Model	AE	4.29	3.65	1.34
	SDE	11.05	9.61	6.32
	RMSE	11.85	10.28	6.46

* Note: the numbers in parentheses represent the number of classes for each variable.

TABLE 6.3 Percentage aggregation error for the three by–variables classifications for the two models.

As can be seen from Table 6.3, the error measures for the classification by combination of the three variables have the lowest values. This indicates that as the number of classifying variables increases the predictive powers of the disaggregate models also increase. This is consistent with the results obtained by Koppelman (1975) and Reid (1978). The results in Table 6.3 also show that the simple model has slightly lower errors than the complex one.

6.6.3 PREDICTION ERROR FOR THE ENUMERATION APPROACH

There are two basic objectives of using the enumeration approach in aggregate prediction. The first is to define a reference value for assessing the predictive performance of different aggregation approaches, since in most situations the actual shares are not known. The second, since the enumeration approach does not include any form of data aggregation errors, is to define other sources of prediction error such as model specification, transferability, or data measurement errors (i.e. errors in variable and parameter estimation).

In this study the same data were used for the calibration of the models and so there is no specification error and the only errors left are the data measurement errors. The identification of these errors is carried out by comparing the aggregate shares predicted by the enumeration approach with the observed or actual shares. The errors are shown in Table 6.4.

As was expected, the results in Table 6.4 show that error measures SDE and RMSE increase with decreasing geographical level [Koppelman (1975)].

Types of Model	Types of Error	Classification by		
		Study area * (1)	Bands (6)	Sectorgroups (10)
Simple Model	AE	0.02	0.00	0.46
	SDE	0	7.23	8.52
	RMSE	0.02	7.23	8.53
Complex Model	AE	0.03	0.01	0.32
	SDE	0	9.59	11.20
	RMSE	0.03	9.59	11.20

* Note: the numbers in parentheses represent the numbers of zones at the different geographical levels.

TABLE 6.4 Percentage prediction errors for the two models for the three geographical levels by the enumeration approach.

That is, the errors in the share prediction for the entire study area are much less than the errors for the bands which are also less than the errors for the sectorgroups. It is also clear from Table 6.4 that the values of AE are small for the three classification groups.

Once again, the predictive power of the simple model is slightly greater than that of the complex one.

6.7 COMPARISON OF THE PREDICTION ERRORS OF THE DIFFERENT AGGREGATION APPROACHES

In order to evaluate the desirability of the different aggregation methods to be used in the prediction of aggregate travel behaviour, the values of their prediction errors should be compared. Table 6.5 shows the prediction errors for the three aggregation procedures.

The enumeration approach leads to the smallest prediction errors. This is attributed to the exclusion of specification (or transferability) errors due to the use of the same calibration data set and also because the chosen disaggregate models appear to be well-specified.

The naive approach has substantially higher aggregation errors than those of the classification approaches for both models; the naive aggregation errors are, in fact, approximately twice those of the classification methods.

Types of Model	Types of Error	Naive * (1)	Enumeration (1)	Classification	
				Geographical (Sectorgroups) (10)	By-variables (HHPOS, CAOD, and CBD) (8)
Simple Model	AE	3.57	0.02	0.71	0.71
	SDE	9.19	0	4.88	4.20
	RMSE	9.86	0.02	4.93	4.26
Complex Model	AE	5.01	0.03	0.97	1.34
	SDE	11.66	0	5.81	6.32
	RMSE	12.69	0.03	5.89	6.46

* Note: the numbers in parentheses represent the numbers of zones and variable groups.

TABLE 6.5 Percentage prediction errors for the two models by the three aggregation approaches.

The error measures by the geographical classification method are slightly higher than those obtained by the by-variables classification method for the simple model, while the opposite is true for the complex model. This variation may be attributed to the different specifications of the two disaggregate models. Since the complex model includes a slightly larger number of variables (i.e. more level-of-service variables) the by-variables classification method, using the combination of some of these variables, may not improve the homogeneity of the remaining variables in each group (i.e. may result in imperfectly homogeneous groups).

Additional insight into the structure of the prediction errors for different aggregation procedures is obtained by disaggregating the error measures by alternative mode. The resulting errors are shown in Tables 6.6 and 6.7 for the simple and the complex models, respectively.

The prediction error measure for all alternative modes is simply equal to the square root of the average sum of the squared values of the corresponding prediction errors for each mode [Koppelman (1975)].

As can be seen from Tables 6.6 and 6.7, both models slightly overpredict the car driver and bus mode choice shares, while they underpredict the car passenger (slightly) and train mode choice shares. The error in the prediction of train travel may result from the small proportion of train travellers in the data sample.

Alternative Mode	Types of Error	Naive * (1)	Enumeration (1)	Classification	
				Geographical (Sectorgroups) (10)	By-variables (HHPOS, CAOD, and CBD) (8)
Car Driver	AE	1.35	0.01	0.44	0.46
	SDE	4.16	0	2.55	1.98
	RMSE	4.37	0.01	2.59	2.03
Car Passenger	AE	-2.03	-0.05	-0.64	-0.29
	SDE	6.87	0	5.04	3.03
	RMSE	7.16	0.05	5.08	3.04
Bus	AE	0.86	0.02	0.14	0.14
	SDE	10.31	0	5.68	5.41
	RMSE	10.35	0.02	5.68	5.41

* Note: the numbers in parentheses represent the numbers of zones and variable groups.

TABLE 6.6 Percentage prediction error by the three aggregation approaches for each mode for the simple model.

Continued:

Alternative Mode	Types of Error	Naive * (1)	Enumeration (1)	Classification	
				Geographical (Sectorgroups) (10)	By-variables (HHPOS, CAOD, and CBD) (8)
Train	AE	-7.56	-0.01	-1.36	-1.51
	SDE	15.71	0	7.31	6.58
	RMSE	17.43	0.01	7.44	6.75
Walk	AE	-0.15	-0.01	-0.05	-0.06
	SDE	2.14	0	1.16	1.55
	RMSE	2.15	0.01	1.16	1.55
TOTAL	AE	3.57	0.02	0.71	0.71
	SDE	9.19	0	4.88	4.20
	RMSE	9.86	0.02	4.93	4.26

* Note: the numbers in parentheses represent the numbers of zones and variable groups.

TABLE 6.6 Percentage prediction error by the three aggregation approaches for each mode for the simple model.

Alterna- tive Mode	Types of Error	Naive * (1)	Enumera- tion (1)	Classification	
				Geographical (Sectorgroups) (10)	By-variables (HHPOS, CAOD, and CBD) (8)
Car Driver	AE	1.45	0.01	0.46	0.54
	SDE	4.48	0	2.76	2.43
	RMSE	4.71	0.01	2.80	2.49
Car Passenger	AE	-1.69	-0.04	-0.41	-0.05
	SDE	7.22	0	3.73	3.81
	RMSE	7.41	0.04	3.75	3.81
Bus	AE	1.00	0.01	0.30	0.00
	SDE	11.54	0	5.97	6.28
	RMSE	11.58	0.01	5.98	6.28

* Note: the numbers in parentheses represent the numbers of zones and variable groups.

TABLE 6.7 Percentage prediction error by the three aggregation approaches for each mode for the complex model.

Continued:

Alterna-tive Mode	Types of Error	Naive (1)*	Enumera-tion (1)	Classification	
				Geographical (Sectorgroups) (10)	By-variables (HHPOS, CAOD, and CBD) (8)
Train	AE	-10.94	-0.05	-2.05	-2.95
	SDE	21.60	0	10.38	11.65
	RMSE	24.21	0.05	10.58	12.02
Walk	AE	0.00	0.02	0.02	0.24
	SDE	2.82	0	1.92	2.05
	RMSE	2.82	0.02	1.92	2.06
TOTAL	AE	5.01	0.03	0.97	1.34
	SDE	11.66	0	5.81	6.32
	RMSE	12.69	0.03	5.89	6.46

* Note: the numbers in parentheses represent the numbers of zones and variable groups.

TABLE 6.7 Percentage prediction error by the three aggregation approaches for each mode for the complex model.

The choice shares for the walk mode are predicted differently by the two models; the simple model underpredicts while the complex one overpredicts. This may result from differences in the representation of the WALK variable in the utility function of the walk mode, as well as the difference in the specifications of the two models.

In general, both models have relatively low prediction error values. However, the simple model appears to be more desirable for use in the prediction of aggregate travel behaviour, since it has lower error values than the complex model and is much cheaper to use, both in terms of computational requirements and data collection. Therefore, the simple model was chosen for the analysis of policy changes. This will be discussed in the next chapter.

CHAPTER SEVEN

POLICY CHANGE ANALYSIS

CHAPTER SEVEN

POLICY CHANGE ANALYSIS

7.1 INTRODUCTION

The development over the past three decades of disaggregate travel demand models has increased considerably the range and power of the tools available to the transportation analyst concerned with the prediction of future travel demand. It has been widely stated in the literature that disaggregate travel demand modelling techniques appear to hold the greater potential for providing the basis for accurate methods of estimating and predicting travel demand[1].

In addition to the use of these models for predicting aggregate travel behaviour (see Chapter 6), they can be used for assessing the effects of a wide range of policy decisions. This assessment is clearly an important aspect of travel demand prediction since it allows transportation planners and decision-makers to evaluate the effect of different proposed policy changes in the transportation system. This application of the models is examined in this chapter.

In Section 7.2, the general properties of policy-sensitive models are presented. Section 7.3 considers the various methods available for analysing different policy change decisions. Section 7.4 presents the various elasticity measures appropriate to the individual traveller and the aggregate group; these involve the analysis of

[1] See for example: Watson (1973, 1974); DeDonnea (1971); Richards and Ben-Akiva (1975); Domencich and McFadden (1975); Koppelman (1974, 1975); Supernak (1983, 1984); Ben-Akiva and Lerman (1985).

small and large changes in the various policy-relevant variables. The last section examines the use of the MNL model in predicting aggregate travel behaviour under different policy changes. It presents the impact of these policy changes on the aggregation error for various methods of aggregation.

7.2 POLICY-SENSITIVE MODELS

One of the most important aspects of any travel demand model is its sensitivity to changes in transportation system characteristics. It is essential to develop a model which can accurately reflect the possible effects of changes in the transportation system associated with a new alternative. The model must be able to test new transportation strategies that are of concern to the transportation planners and decision-makers.

In recent years the range of the policy alternatives analysed and policy questions considered has greatly expanded. Emphasis has shifted from long-term transportation planning to short-term planning. These shifts have placed a considerable strain on conventional aggregate prediction tools, which were originally developed to address problems of highway network design [McFadden (1976b)]. Thus, demand prediction methods have been sought which are especially capable of incorporating the behavioural forces linking individual transportation decisions and the relationships between individual travel choice and aggregate flow. The resulting behavioural disaggregate methods expand the policy sensitivity of travel prediction. Tests and practical experience with these approaches indicate that they are superior to the conventional aggregate prediction techniques in terms of data collection and computational requirements (see Chapters 2 and 6).

Some typical policy issues that the transportation planners would like to be able to address with disaggregate models include the following[1]:

1. What effects will changes in travel times and costs have on total travel demand and on the demands for alternative modes?

2. How can public transport modes be made more attractive alternatives, in the peak periods, for those who are currently travelling by car?

3. What are the effects of introducing new or substantially redesigned alternative transport modes on the distribution of trips across available modes?

In fact, disaggregate behavioural choice models are particularly well-suited for analysing such short-term transportation policy questions. They translate the questions into quantitative descriptions of their effects on the predetermined models in order to predict their consequences for future travel demands. This is discussed next.

7.3 POLICY CHANGE ANALYSIS TECHNIQUES

The basic concern of transportation planners and decision-makers is to be able to anticipate the consequences of any proposed changes in the transportation system. This can be done by using an estimated model for the analysis of these proposed changes. In general, demand models can only reflect the effects of

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[1] See for example : Domencich and McFadden (1975); Gwilliam and Mackie (1975); McFadden (1976); Nash (1976); Ssherret (1979); Hottler (1981); Richards (1981); Spear (1981).

changes in some policy-relevant variables that are of interest to the transportation planners and decision-makers if such changes are expressed as changes in relevant explanatory variables in the model.

In recent years a number of simplified techniques have been developed for analysing policy changes. Most of these techniques rely either on transferring or borrowing a model developed in one area to another area, or on simple methods which relate proportional changes in policy-dependent variables (e.g. travel time and travel cost) directly to proportional changes in a particular transport mode choice. These techniques are:

1. Development of a simplified model from locally-available data.

2. The use of borrowed or base year models with adjustments to the local data.

3. The use of borrowed or base year models without adjustments to the local data.

4. The use of elasticity models (i.e. simple models which relate policy-relevant variables directly to a transport mode choice probability or choice share).

The first technique requires that the transportation planners and decision-makers understand the econometric techniques involved in specifying and calibrating the demand model. In addition it requires an appropriate set of data for use in the model development. However, the development of the required model is an expensive task in terms of data collection and computational requirements.

Often, particularly for small scale studies involving minor policy decisions, there is neither the time nor the money to develop a new travel demand model. Consequently it seems more desirable to borrow a predetermined model for use in analysing such policy decisions.

Two alternative techniques which use borrowed or base year models in policy change analysis are available (listed as 2 and 3 above). The first technique updates a model using data available in the borrowing area to adjust the model parameters so that the model better replicates the current situation. The extent to which a borrowed model can be updated depends largely on the structure of the model together with the type of data available in the borrowing area [OECD (Sept. 1980); Supernak (1984)]. The other technique requires that the transportation planners and decision–makers assume that both the structure and the parameters of the borrowed model are representative of the borrowing area. This is clearly a considerable assumption, although it may sometimes be correct. If the policy alternatives are substantially different from the base year conditions, the use of the base year parameter values may be equivalent to extrapolation outside the range of the data. In this case, the use of the borrowed model will produce biased results. Nevertheless, in the absence of major policy changes (such as the introduction of important new transportation modes) this technique seems to be more desirable than the updating one due to its simplicity and straightforwardness of use, all of which make it a more economical approach to policy change analysis.

The use of elasticity models, widely applied in the United Kingdom for policy change analyses [OECD (Sept. 1980)], requires a good knowledge of both the modelling technique and the transport system being studied. Such models can be used to provide quick estimates of the effects of small scale policy changes in the transportation system[1].

7.4 ELASTICITY MEASURES

Travel demand elasticities can be considered in disaggregate or aggregate terms as defined below.

7.4.1 DISAGGREGATE ELASTICITIES

Since disaggregate choice models are concerned with the individual traveller and with the fact that the impact of any proposed changes in the transportation system varies across individuals, the disaggregate elasticities are of great importance since they reflect the true behaviour of each individual in response to policy changes.

The various types of disaggregate elasticities are:

7.4.1.1 POINT ELASTICITIES

These measures are often used to assess the responsiveness of the individual choice probability of a particular alternative with respect to changes in some explanatory variables relevant to that alternative or to other competing alternatives. Thus, direct and cross (indirect) point elasticities can be defined. Direct point elasticity is the percentage change in the individual choice probability

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[1] For more details of these methods see OECD (Sept. 1980).

of a particular alternative with respect to a given percentage change in an explanatory variable which relates directly to that alternative. Cross point elasticity, on the other hand, is defined as the percentage change in the individual choice probability of a particular alternative with respect to a given percentage change in an explanatory variable which is related directly to some other competing alternative. Thus the mathematical definition of these elasticities can be written as:

$$E^{P_{in}}_{X_{ikn}} = \frac{dP_{in}}{dX_{ikn}} \cdot \frac{X_{ikn}}{P_{in}} \qquad \text{(Direct point elasticity)} \qquad (7.1)$$

and,

$$E^{P_{in}}_{X_{jkn}} = \frac{dP_{in}}{dX_{jkn}} \cdot \frac{X_{jkn}}{P_{in}} \qquad \text{(Cross point elasticity)} \qquad (7.2)$$

where,

P_{in} is the probability of individual n choosing alternative i, and

X_{ikn} and X_{jkn} are the explanatory variables relating to alternatives i and j respectively.

For the logit model given by Equation 3.24, it is possible to derive the above point elasticities as follows[1]:

[1] For complete derivation of the elasticities see Hensher and Johnson (1981).

$$E^{P_{in}}_{X_{ikn}} = (1 - P_{in})\ \beta_k\ X_{ikn} \qquad \text{(Direct point elasticity)} \qquad (7.3)$$

and,

$$E^{P_{in}}_{X_{jkn}} = -\ P_{jn}\ \beta_k\ X_{jkn} \qquad \text{(Cross point elasticity)} \qquad (7.4)$$

Equation 7.4 shows that the cross point elasticity depends only on the variables associated with alternative j. Thus, the cross elasticities with respect to change in a variable related to alternative j are equal for all alternatives $i \neq j$. However, this constraint of equal elasticity (i.e. equal substitutability) can be considered as a limitation of the logit model since it is not necessarily logical in all cases and is therefore considered as another aspect of the IIA property limitation [Richards and Ben–Akiva (1975)].

In general, Equations 7.3 and 7.4 can be combined to yield a single point elasticity formula for the logit model,

$$E^{P_{in}}_{X_{jkn}} = (\delta_{ij} - P_{jn})\ \beta_k\ X_{jkn} \qquad (7.5)$$

where,

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \quad \text{(Direct point elasticity)} \\ 0 & \text{if } i \neq j \quad \text{(Cross point elasticity)} \end{cases}$$

As can be seen from Equation 7.5, the direct point elasticity approaches zero as the choice probability P_{jn} approaches one, and approaches $\beta_k X_{jkn}$ as P_{jn} approaches zero. This clearly implies that the direct point elasticity is greatest when the choice probability is lowest and vice versa. On the other hand, cross point elasticity behaves in precisely the converse manner (i.e. the cross point elasticity is a minimum when P_{jn} is a minimum).

Theoretically, it is clear from Equations 7.1 and 7.2 that point elasticities are relations between differentials and that they are relevant only for small changes in the values of the explanatory variables and indicate only a trend at a particular point [Richards and Ben-Akiva (1975)].

7.4.1.2 ARC ELASTICITIES

Arc elasticities are similar to point elasticities except that they are well suited for measuring the sensitivity of individual travellers to large changes in the policy-relevant variables. These elasticities represent the effect of moving from one situation to another (for example, before and after a travel cost increase or travel time decrease for a particular transport mode). To assess the effect of these changes, the before and after choice probabilities of any particular mode must be recalculated, and so arc elasticities must be determined using differences rather than differentials. Thus:

$$E^{P_{in}}_{X_{ikn}} = \frac{\Delta P_{in}}{\Delta X_{ikn}} \cdot \frac{X_{ikn}}{P_{in}} \qquad \text{(Direct arc elasticity)} \qquad (7.6)$$

$$E^{P_{in}}_{X_{jkn}} = \frac{\Delta P_{in}}{\Delta X_{jkn}} \cdot \frac{X_{jkn}}{P_{in}} \qquad \text{(Cross arc elasticity)} \qquad (7.7)$$

where,

ΔP_{in} is the difference between the after and before choice probabilities of mode i, and

ΔX_{ikn}, ΔX_{jkn} are the differences in the values of the explanatory variables X_{ikn} and X_{jkn}, respectively.

The problem inherent in the above definitions (i.e. Equations 7.6 and 7.7) is that inconsistent results can be obtained when a change in a given explanatory variable is reversed [Kanafani (1983)].

A number of alternative forms can be used to calculate arc elasticity measures (considering only the direct arc elasticity; for cross arc elasticity the subscript of the explanatory variable is simply changed to another competing mode index). A very simple way is to define arc elasticity as the ratio of the change in the choice probability to the change in the value of the explanatory variable in question. Thus:

$$E^{P_{in}}_{X_{ikn}} = \frac{\Delta P_{in}}{\Delta X_{ikn}} \qquad (7.8)$$

This can, alternatively, be expressed in a logarithmic form [Kemp (1973)]:

$$E^{P_{in}}_{X_{ikn}} = \frac{\Delta \log P_{in}}{\Delta \log X_{ikn}} \tag{7.9}$$

The only problem with the use of Equations 7.8 and 7.9 is that the resulting elasticities are not dimensionless measures, and so are of little use in comparing the effects of different explanatory variables.

Another simple method of determining arc elasticity is to assume a linear relationship between the choice probability and the explanatory variables. This is often done in conventional travel demand studies. In this case the arc elasticity is defined in terms of the average values of the parameters. Thus:

$$E^{P_{in}}_{X_{ikn}} = \frac{\Delta P_{in}}{\Delta X_{ikn}} \cdot \frac{\bar{X}_{ikn}}{\bar{P}_{in}} \tag{7.10}$$

where the bar sign on P_{in} and X_{ikn} is used to represent the average values for the before and after situations [OECD (Sept. 1980)].

Using the above definitions (i.e. Equations 7.8, 7.9 and 7.10), the problem of inconsistency does not occur when reversing changes in any of the explanatory variables [Kanafani (1983)].

To calculate the arc elasticities for the logit model, the probabilities for the before and after situations are computed and then substituted, together with the variable values for both situations, in any of the above elasticity forms.

7.4.2 AGGREGATE ELASTICITIES

Although disaggregate elasticities are more appropriate in reflecting the effect of any policy decision, in practice they are of little use since transportation planners and decision-makers are always interested in the responsiveness of the demand at an aggregate level to any proposed policy changes. Thus, some form of aggregation is required. The simplest way to derive the aggregate elasticities is to substitute the average probability and explanatory variable values into the disaggregate elasticity measures. Thus, for small changes in the explanatory variable, the aggregate point elasticity is simply expressed as:

$$AE^{\bar{P}_i}_{\bar{X}_{ik}} = (1 - \bar{P}_i)\ \beta_k\ \bar{X}_{ik} \tag{7.11}$$

where,

$$\bar{P}_i = \frac{1}{N} \sum_{n=1}^{N} P_{in} \tag{7.12}$$

is the average choice probability or the expected choice share of alternative i in a sample of N observations, and,

$$\bar{X}_{ik} = \frac{1}{N} \sum_{n=1}^{N} X_{ikn} \tag{7.13}$$

is the average value of the relevant explanatory variable X_{ikn} in the data sample.

This approach was used by Richards and Ben–Akiva (1975) for evaluating point elasticities based on the observed average probability (choice share) and the average values of the relevant explanatory variables. The results of using the same approach in this study are given in Table 7.1.

Table 7.1 shows the aggregate direct point elasticities for a specific group of individuals. This group was chosen on the basis that each individual belonged to a car–owning household and had all modes available. By this means, a relatively homogeneous group of individuals was produced.

The most important results shown in the table are the elasticities of public transport mode choice probabilities with respect to TJT (total journey time). These values were calculated to be −3.004 and −3.810 for bus and train, respectively. These large values indicate that any reductions in total journey time, which rely heavily on decreases in out–of–vehicle time (through increasing numbers of stops or stations or increasing public transport frequencies), would be highly effective ways of making these public transport modes more desirable to individuals not at present using them.

In general, Table 7.1 also shows that, for each mode, the elasticities with respect to travel time are higher than the elasticities with respect to the COST/DIST variable. This indicates that the travel time variable has more influence on the mode choice decision than has the cost variable.

Alternative Mode	Variable	Average Probability	Coefficient Value	Variable Value	Point Elasticity
Car	TJT	0.6536	-0.1018	9.89	-0.349
Driver	COST/DIST	0.6536	-0.0849	1.40	-0.041
Car	TJT	0.0594	-0.1018	9.89	-0.947
Passenger	COST/DIST	0.0594	-0.0849	0.70	-0.056
Bus	TJT	0.0472	-0.1018	30.79	-3.004
	COST/DIST	0.0472	-0.0849	11.82	-0.956
Train	TJT	0.0089	-0.1018	37.76	-3.810
	COST/DIST	0.0089	-0.849	13.88	-1.168
Walk	TJT	0.2309	-0.1018	22.11	-1.731

TABLE 7.1 Aggregate direct point elasticities.

On the other hand, for large changes in the relevant explanatory variable, the average choice probabilities need to be recalculated, and the aggregate arc elasticity is defined as:

$$AE^{\bar{P}_i}_{\bar{X}_{ik}} = \frac{\Delta \bar{P}_i}{\Delta \bar{X}_{in}} \cdot \frac{\bar{\bar{X}}_{ik}}{\bar{\bar{P}}_i} \qquad (7.14)$$

where,

$\Delta\bar{P}_i$ and $\Delta\bar{X}_{ik}$ are the differences between the after and before average choice probabilities and explanatory variable values, respectively, and

$\bar{\bar{P}}_i$ and $\bar{\bar{X}}_{ik}$ are the average values of the after and before average choice probabilities and explanatory variable values, respectively.

Table 7.2 shows the aggregate arc elasticities calculated for different percentage changes in the relevant explanatory variables. These elasticities apply to the same group of travellers as those given in Table 7.1.

As can be seen from Table 7.2, a twenty percent increase in total travel time for the car driver and car passenger modes has lower associated elasticities than the same percentage decrease in total travel time for the bus and train modes. This indicates that travel time for the public transport modes is more important than for the private modes.

Alternative Mode	Variable	Variable Value	Percent Change (%)	Choice Shares (%)		Arc Elasticity
				Before	After	
Car Driver	TJT	9.89	+20	0.6536	0.6293	-0.208
	COST/DIST	1.40	+25	0.6536	0.6499	-0.026
Car Passenger	TJT	9.89	+20	0.0594	0.0541	-0.514
	COST/DIST	0.70	+25	0.0594	0.0594	0.000
Bus	TJT	30.97	-20	0.0472	0.0745	-2.019
	COST/DIST	11.83	-25	0.0472	0.0572	-0.671
Train	TJT	37.76	-20	0.0089	0.0235	-2.553
	COST/DIST	13.88	-25	0.0089	0.0117	-0.951

TABLE 7.2 Aggregate direct arc elasticities.

It is also clear from the table that changes in travel costs have a higher effect for public transport modes, but that such changes have a smaller effect than have changes in travel time. This implies that travellers are significantly less sensitive to travel cost changes than to travel time changes.

In general, the above approach is based on the use of average values of the choice probabilities and explanatory variables for evaluating the aggregate point and arc elasticities. This will produce biased results if, firstly, the sample is not a homogeneous group of individuals and, secondly, the average values of the choice probabilities and explanatory variables lie beyond the ranges of the corresponding values for which the model was estimated [Richards and Ben–Akiva (1975); Hensher and Johnson (1981)]. A more appropriate procedure is to calculate the relevant elasticity of each individual and then sum the elasticities over the sample to obtain the required aggregate elasticity[1].

7.5 ANALYSIS OF THE AGGREGATION ERRORS FOR POLICY CHANGES

The purpose of this section is to examine the use of the MNL model for the prediction of aggregate travel behaviour under various policy changes. This can be done by comparing the aggregation errors for these policy changes with the aggregation error for the base case. For example, three different policy changes are considered. These are:

1. A fifty percent increase in the cost of travel for the car driver mode.

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[1] For more details of these approaches see McFadden (1979); Hensher and Johnson (1981).

2. Zero cost of travel for the car passenger mode.

3. A fifty percent decrease in the out-of-vehicle time for the bus mode.

The objective of these policy decisions was to examine the relative effects of different ways of reducing car and increasing bus usage. The expected choice shares for the base case and for the three policy changes for the entire study area using the complete enumeration method are given in Table 7.3.

As can be seen from Table 7.3, policy changes one and two do not have any significant effects on the choice shares of the various modes. Policy change three, however, has the effect of increasing the choice share of the bus mode by ten percent (i.e. from 26.35% to 36.30%). These policy changes indicate that the cost variable has less impact on the mode choice decision for the car driver and car passenger modes, whereas out-of-vehicle travel time has more effect on the bus mode choice decision.

For three methods of aggregation, the impacts of the above policy changes on the aggregation error have been examined and the results, together with the base case aggregation errors, are shown in Table 7.4. It is clear from Table 7.4 that the aggregation errors for the three policy changes by the three methods of aggregation are consistent with the aggregation errors for the base case. The by-variables classification method has the least error measure, whereas the naive method has the highest.

Alterna- tive Mode	Prediction Situation			
	Base Case	Change One	Change Two	Change Three
Car Driver	27.12	26.69	27.08	25.93
Car Passenger	12.08	12.26	12.59	8.84
Bus	26.22	26.35	25.99	36.30
Train	8.49	8.55	8.44	5.92
Walk	26.04	26.15	25.90	23.01

TABLE 7.3 Expected choice shares (percent) for the various modes for the base case and the three policy change proposals using the complete enumeration approach.

Prediction Situation	Naive Approach * (1)	Classification Approach	
		Geographical Classification (Sectorgroups) (10)	By-Variables Classification (HHPOS, CAOD, and CBD) (8)
Base Case	9.86	4.43	4.26
Change One	9.47	4.36	4.21
Change Two	9.84	4.60	4.25
Change Three	11.46	5.20	4.94

* Note: the numbers in parentheses represent the number of zones and variable groups.

TABLE 7.4 Percent aggregation error (RMSE) for the base case and the three policy change situations by three methods of aggregation.

Although policy change three has the highest aggregation errors for the three aggregation methods, the errors can still be considered small. This suggests that the MNL model developed here could be used to analyse other policy changes and policy changes in areas similar to the study area, provided that the changes in the relevant policy variables were within the range of their values for which the model was estimated.

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

CHAPTER EIGHT

CONCLUSIONS AND RECOMMENDATIONS

8.1 INTRODUCTION

The objectives of this final chapter are to present the main conclusions of the study and to identify possible areas for further research. General conclusions regarding the desirability of the approach used in the study are considered first. Specific conclusions relating to the model development and applications in aggregate prediction and policy change analyses are then drawn. The last section considers how the present study might be extended.

8.2 GENERAL CONCLUSIONS

This study has contributed empirical results to the development and application of disaggregate behavioural travel demand models in urban transportation planning studies in the U.K. context. A better understanding of travel behaviour with respect to mode choice for journeys to work in Glasgow has been obtained and the most important factors influencing the mode choice decision have been identified. The study has also demonstrated the feasibility of using the MNL approach to the development of multi-modal disaggregate travel demand models.

8.3 SPECIFIC CONCLUSIONS

The empirical findings of the study and their implications with respect to model development and applications in aggregate prediction and policy change analyses have been presented and discussed in the relevant chapters. In this section, summaries of the most important conclusions relating to the above three aspects of the study are outlined in order to show the extent to which the results obtained may be utilised to improve existing, or develop more advanced, mode choice models.

8.3.1 MODEL DEVELOPMENT

The model calibration stage of the study yields the following conclusions.

1. Travel time as a single variable (or its components: walking, waiting, and in-vehicle times) is statistically significant. The results confirm general assumptions about the relative weights of out-of-vehicle time (or its components) and in-vehicle time, and are reasonably consistent with those obtained from other studies.

2. Travel cost is found to have the wrong sign. This may be attributable to the way in which travel costs for the car driver and car passenger modes were calculated. Unfortunately, this has precluded the determination of any meaningful estimate of the value of travel time from the study.

3. The CBD was a dummy variable based upon whether or not a trip was

destined for, or passed through, the central business district and is found to have a significant effect on the choice of public transport modes. This is not surprising. The problems of driving and parking within the central business district strongly encourage the use of public transport modes and strongly discourage the use of the car driver and car passenger modes.

4. The effect of distance on the walk mode choice is found to be significant, as would be expected.

5. Car availability was included twice in all models via the CAOD and CAPDL variables which reflect, respectively, the effects of the number of cars in a household and the number of cars per driving licence holder. The latter is a measure of the competition within the household for the use of the car mode for the journey to work. Both variables add significant power to the models developed and are worthy of inclusion in them.

6. An individual's position in a household is found to be a highly significant influence on the choice of the car mode. Car use is much greater among heads of households than among other members of the household.

The overall conclusion of this stage of the study is that disaggregate behavioural travel demand models can be calibrated successfully using data obtained from a traditional home interview survey. Although it may be advantageous to have specially-designed data for this type of study, the results confirm the wide applicability of data from conventional home interview surveys.

8.3.2 AGGREGATE PREDICTION

A number of conclusions may be drawn from the aggregate prediction aspect of the study.

1. The aggregate prediction performances of the two models were compared. Aggregate prediction errors for various aggregation procedures for the simple model are found to be slightly lower than those for the complex model. This implies that the simple model is superior to the complex one, confirming results obtained by other investigators.

2. Significant reductions in the aggregation errors of the naive approach are obtained when the prediction is adjusted for choice set variation. This suggests strongly that if differences in choice set availability exist, these differences should be used as a basis for adjusting predictions for various methods of aggregation in order to improve their prediction performances.

3. The performance of the enumeration procedure for aggregate share prediction is found to improve with increasing size of prediction group. This implies that the enumeration procedure is preferable whenever an adequate data sample is available, although the associated data and computational requirements may be costly.

4. The prediction accuracy of the classification procedure increases with decreasing geographical dispersion of the prediction group or with increasing numbers of classifying variables, provided that adequate sample sizes are available within the classes.

In summary, this phase of the study shows the feasibility and desirability of using disaggregate models to provide aggregate predictions; their flexibility provides more appropriate means of data aggregation, which in turn provide more accurate aggregate predictions.

8.3.3 POLICY CHANGE ANALYSIS

The application of the model to policy change analysis leads to the following conclusions.

1. Although travel cost is one of the most important current policy issues in urban transportation planning, the study shows that the sensitivity of mode choice to changes in associated travel costs is very low in Glasgow. This may have been the case in 1978–79 when the GRIS was carried out, but need not necessarily be the case at present.

2. Changes in travel times were found to have a significant effect on mode choice, especially in relation to public transport modes. This indicates that travel time may play an important part in policy decisions, and that by increasing the frequencies of buses or trains, or the number of train stations, public transport may be made more accessible and attractive.

3. The aggregate prediction errors for various policy changes for different aggregation procedures are consistent with the aggregation errors for the base case. This suggests that the tested model may be used for analysing other policy changes provided that the changes in the variables concerned are within the range of their values in the data from which the model was developed.

These conclusions indicate that the model can be used for testing various policy changes, although it is not sensitive to travel cost policy changes.

The overall conclusion of the study is that the empirical results obtained can be considered satisfactory and the approach used both sound and flexible.

8.4 RECOMMENDATIONS

The limited scope of the current study together with the practical limitations of the available data mean that the analyses presented here could be expanded in numerous directions. The major areas in which the study could be extended are suggested below:

1. The specification of the developed models could be improved significantly if more information on level-of-service measures such as comfort, convenience, and safety were available. The need for more detailed data could have implications for the method of data collection.

2. More information about travel costs by the car driver and car passenger modes is essential for the improvement of the sensitivity of the developed models to changes in the travel costs of various transport modes.

3. The study could be extended to include the development of an aggregate MNL model using the GRIS data. This would allow comparison of the aggregate share predictions using aggregate and disaggregate models.

4. The prediction performance of the developed models could be checked using

the GRIS "after" survey data, which are readily available. These data would also allow testing of:

1. The temporal stability of the developed models.

2. The effect on the mode choice decisions of individuals of introducing new alternatives such as the Glasgow Underground.

5. The developed models could be applied to other areas similar to Glasgow in order to test their spatial transferability.

6. The study could be extended to analyse trips for purposes other than working.

7. A further extension of the study could be the development of more general models, such as nested logit or MNP models, which avoid the difficulties of the IIA property of the logit model.

APPENDICES

1. GRIS QUESTIONNAIRE
2. MULTINOMIAL LOGIT PROGRAM
3. AGGREGATE PREDICTION ERROR PROGRAM

APPENDIX 1

GRIS QUESTIONNAIRE

GREATER GLASGOW PASSENGER TRANSPORT EXECUTIVE

Your Ref: 48 ST. VINCENT STREET, GLASGOW G2 5TR

Our Ref:

November 1978.

Dear Householder,

GLASGOW RAIL IMPACT STUDY

Next year will see the completion of Glasgow's two major railway schemes. In May the Argyle Line, which will link Rutherglen and Partick via the former Central Low Level Line, will open to passenger traffic. Then towards the end of the year the Glasgow Underground will reopen after complete modernisation.

The purpose of the Glasgow Rail Impact Study is to discover what effects these new transport systems have on the Glasgow area. The results will help to decide how investment in public transport can best meet the needs of people living and working in the area. They will also help to show, in detail, how the new Clyderail and Underground services themselves can be developed to give maximum benefit to the general public.

For the study to be successful, we need to know where, how and why people travel. This means conducting a brief interview with people in their homes, and a random sample of households has been selected to help us. Yours is one of those chosen. It would help us a lot if you and the members of your household would agree to co-operate in this survey, as everyone we miss, for whatever reason, means that the sample is just that bit less representative.

A firm of experts, Martin and Voorhees Associates, have been contracted by the Scottish Development Department to carry out this work. One of their interviewers will call during the next few weeks. The interviewer will first ask for a few facts about you and your household and will then ask for some information regarding trips to work and shopping trips made by members of the household. All the information collected will remain absolutely confidential. Your identity is not required and the results of the Study will contain no reference to individual persons or households.

Each interviewer will carry an identity card. Please ask to see it before being interviewed. If you wish any further information please contact the Glasgow Rail Impact Study at 16 Princes Square, 48 Buchanan Street, Glasgow, G1 3JP (Telephone 041-226-4532).

I hope I can count on your full co-operation in this important survey. It is vital to its success that everyone takes part, including people who rarely go out as well as those who travel by car, bus or train. Its success will help to improve travel facilities both in Glasgow and elsewhere in the country.

Yours sincerely,

Andrew McKay

A.F. McKay,
Director General

Director General; A. F. McKAY ; Directors: J. COYLE, W. N. STIRLING, H. M. TAYLOR, N. TOWNEND ; Secretary: E. S. PAYNE

GLASGOW RAIL IMPACT STUDY

SCOTTISH DEVELOPMENT DEPARTMENT
TRANSPORT AND ROAD RESEARCH LABORATORY:
DEPARTMENT OF TRANSPORT
DEPARTMENT OF THE ENVIRONMENT
MARTIN AND VOORHEES ASSOCIATES

Card Type 0 1 (1 2)   Address Serial No. (3 4 5 6 7)   Household No. (8)

Post Code (9 10 11 12 13 14 15)

HOUSEHOLD SURVEY 1978

A. Date of Travel: _ _ _ _ _ _ _ _ _ _ _ _   D (16 17) M (18 19) Y (20 21)
[Interviewer: Enter date here]

B. Day of Travel: _ _ _ _ _ _ _ _ _ _ _ _ _   (22)
[Interviewer: Enter day here in full]

M = 1
T = 2
W = 3
Th = 4
F = 5

C. CONTACT RECORD

CALL	TIME	DATE	RESULT	APPOINTMENT DETAILS
1				
2				
3				
4				

D. DATA RECORD

PART	COMPLETE	PARTIALLY COMPLETE	NO DATA	
1				23
2				24
3				25
4				26
5				27

E. INTERVIEWER DETAILS

Name _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ No _ _ _ _ _ _ _ _

F. ADDITIONAL INTERVIEWER COMMENTS:

PART I: HOUSEHOLD DATA

1. (a) How many people of all ages, including yourself are there in this household (28 29)

(b) How many of these people were staying here on _ _ _ _ _ _ _ _ (30 31)
[Interviewer ask day of travel]

(c) How many of these people were staying away from here on _ _ _ _ _ _ _ (32 33)
[Interviewer ask day of travel]
[Interviewer check that b + c = a]

(d) How many visitors, of all ages, were staying here on _ _ _ _ _ _ _ _ _ (34 35)
[Interviewer ask day of travel]
[Interviewer check "that means that in all_____people (b + d) were staying here on _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ (travel day)]

2. (a) How many of each of the following types of vehicle are owned by members of the household. [complete column 'a' below].

(b) How many of each of these vehicles are provided with some assistance from an employer (eg car provided with job; free of or low interest loan; essential user car allowance etc) [complete column 'b' below]

(c) How many other vehicles of each of these types were also available to members of the household for their personal use on the travel day [complete column 'c' below]

	a	b	c	
Cars/Private Vans (including 3 wheelers and Motor caravans)	____	____	____	36 37 38
Motor cycles and motor scooters	____	____	____	39 40 41
Light Goods vehicles	____	____	____	42 43 44
Heavy Goods vehicles	____	____	____	45 46 47

3. I'd like you to look at this card and tell me which of the groups shows the total combined income of all members of the household, before deductions. (48 49)
[Interviewer Show CARD 'I']   Group ________

GRIS-HH PART 2: PERSON DATA

Card Type (1 2) 0 2 Address Serial No. (3 4 5 6 7) H/Hold No. (8)

4 Person No	5 Sex	6 Age	7 Status	8 Health	9 Driving Licence	10 Trip Maker	11 Ticket Status	12 Occupation	13 Industry/Profession	14 Work Journey
Start at 1	M = 1 F = 2		Enter Description Head of Household Wife Child Related Adult Unrelated Adult resident Visitor.	Is there anything about your health which makes it difficult for you to use public transport If 'YES' specify.	Car = 1 Motor Cycle = 2 Provisional = 3 None = 4	On the travel day were you: Present and making trips = 1 Present and not making trips = 2 Absent = 3	Do you usually hold any of these season tickets or travel cards. [Show Card B]	Enter detailed description of main occupation and indicate if: [Interviewer show CARD C] Full Time employed 1 Employed 10-29 hours per week 2 Employed less than 10 hours per week 3 Housewife 4 Full time student 5 Long term sick 6 Retired 7 Unemployed 8 [If 6 or 7 give previous occupation If 'manager' indicate no of employees]	Enter detailed description of the type of business or activity carried on at the respondent's main place of employment.	On the travel day did you make a work journey to: Usual Place 1 Other Place 2 Work at Home 3 Not working on travel day 4 Not employed 5
9 10	11	12 13	14	15	16	17	18 19	20 21 22	23 24	25
26 27	28	29 30	31	32	33	34	35 36	37 38 39	40 41	42
43 44	45	46 47	48	49	50	51	52 53	54 55 56	57 58	59
60 61	62	63 64	65	66	67	68	69 70	71 72 73	74 75	76
9 10	11	12 13	14	15	16	17	18 19	20 21 22	23 24	25
26 27	28	29 30	31	32	33	34	35 36	37 38 39	40 41	42

SHEET 1 SIDE 2

RE 65810/1 4700 10/78 TCL

GRIS-HH PART 3; TRIP DATA

Card Type (1–2): 0 3   Address Serial No (3–7)   Hh No. (8)   Person No (9)

15. Trip No Start from 1 for each person	10	10	10	10
16. Stage No Start from 1 for each trip	11	11	11	11
17. Start Time Enter time using 24-hour clock	12 13 14 15	12 13 14 15	12 13 14 15	12 13 14 15
18. Finish Time Enter time using 24-hour clock	16 17 18 19	16 17 18 19	16 17 18 19	16 17 18 19
19. Destination Address Interviewer write address in full for each stage	20 21 22 23 24 25 26 27	20 21 22 23 24 25 26 27	20 21 22 23 24 25 26 27	20 21 22 23 24 25 26 27
20. Travel Method H/hold car/van dr 01 Other car/van dr 02 H/hold car/van pass 03 M/cycle driver 04 Taxi 05 Pedal cycle 06 Walk 07 Goods veh dr 08 Train 09 Sched bus 10 Non-Sch bus 11 Other pass 12 Other spec 13	28 29	28 29	28 29	28 29
21. Stage purpose Home 01 Work 02 Education 03 Shopping 04 Pers Bus 05 Empl bus 06 Social/Recreation 07 Escort 08 Tow 09 Change mode 99	30 31	30 31	30 31	30 31
22. Lift If driver in 20: How many people incl yourself travelled in vehicle	32	32	32	32
23. Parking place On street 1 Stn car park 2 Other public CP 3 Private CP 4 At home 5 Did not park 6	33	33	33	33
24. Parking duration How long did you park? (If not at home enter mins up to 60 btn hrs)	34	34	34	34
25. Parking cost Enter actual cost (if free, private CP or home, enter FREE)	35 36 37	35 36 37	35 36 37	35 36 37
26. Ticket Type: RAIL / BUS Single 1 5 Day Ret 2 - Return 3 6 Season 4 7	38	38	38	38
27. Cost (if cash - ie not season) Enter actual cost	39 40 41	39 40 41	39 40 41	39 40 41
28. Other Mode: If on travel day you could not have made this stage of trip by ______ what would have been your first alternative. If no alternative enter X.	42	42	42	42
29. Frequency: Ask only if Q21 is Not "Change Mode" ie code 99. How often did you make this trip in the 7 days before travel day?	43 44	43 44	43 44	43 44

SHEET 2 SIDE 1/2

RS 65810/2 21,000 10/78 TCL

GRIS-HH PART 3: TRIP DATA

Card Type (1–2): 0 3 Address Serial No: (3–7) Hh No. (8) Person No (9)

Question	Stage 1	Stage 2	Stage 3	Stage 4
15. Trip No Start from 1 for each person	10	10	10	10
16. Stage No Start from 1 for each trip	11	11	11	11
17. Start Time Enter time using 24-hour clock	12 13 14 15	12 13 14 15	12 13 14 15	12 13 14 15
18. Finish Time Enter time using 24-hour clock	16 17 18 19	16 17 18 19	16 17 18 19	16 17 18 19
19. Destination Address Interviewer write address in full for each stage				
20. Travel Method H/hold car/van dr 01 Other car/van dr 02 H/hold car/van pass 03 M/cycle driver 04 Taxi 05 Pedal cycle 06 Walk 07 Goods veh dr 08 Train 09 Sched bus 10 Non-Sch bus 11 Other pass 12 Other spec 13	20 21 22 23 24 25 26 27 28 29	20 21 22 23 24 25 26 27 28 29	20 21 22 23 24 25 26 27 28 29	20 21 22 23 24 25 26 27 28 29
21. Stage purpose Home 01 Work 02 Education 03 Shopping 04 Pers Bus 05 Empl bus 06 Social/Recreation 07 Escort 08 Tov 09 Change mode 99	30 31	30 31	30 31	30 31
22. Lift If driver in 20: How many people incl yourself travelled in vehicle	32	32	32	32
23. Parking place On street 1 Stn car park 2 Other public CP 3 Private CP 4 At home 5 Did not park 6	33	33	33	33
24. Parking duration How long did you park? (If not at home enter mins up to 60 btn hrs)	34	34	34	34
25. Parking cost Enter actual cost (if free, private CP or home, enter FREE)	35 36 37	35 36 37	35 36 37	35 36 37
26. Ticket Type: RAIL BUS Single 1 5 Day Ret 2 6 Return 3 7 Season 4 8	38	38	38	38
27. Cost (if cash = is not season) Enter actual cost	39 40 41	39 40 41	39 40 41	39 40 41
28. Other Modes: If on travel day you could not have made this stage of trip by ______ what would have been your first alternative. If no alternative enter X.	42	42	42	42
29. Frequency: Ask only if Q21 is Not "Change Mode" is code 99. How often did you make this trip in the 7 days before travel day?	43 44	43 44	43 44	43 44

SHEET 2 SIDE 1/2

GRIS-HH PART 4 ATTITUDE DATA

Card Type 1 2 [0 4] Address Serial No. 3 4 5 6 7 H/hold No. 8 Person No. 9

10 11 12

30. (a) Where is your usual place of work? ________

(b) By what method do you usually travel to work? 13
[Interviewer show CARD D] ________

1 Walk or cycle 2 Motor cycle 3 Car 4 Bus 5 Train 6 Bus then train 7 Train then bus 8 Car then train 9 Other (specify) ________
10 Not applicable.

(c) How easy or difficult is it to get to work by this method? 14
[Interviewer show CARD E] ________

1 Very easy 2 Fairly easy 3 Neither easy nor difficult
4 Fairly difficult 5 Very Difficult 6 Not Applicable.
[If coded 4 or 5 Interviewer probe 'why do you say that?'] 15 16

____

____

(d) Which other methods could you use to travel to work? 17
[Interviewer show CARD 'D' again]
(i) ________
(ii) ________
(iii) ________

(e) How easy or difficult would it be for you to travel to work by each of these methods?
[Interviewer show CARD 'E' again and list response for each of (i) - (iii) in (d)] 20
(i) ________
(ii) ________
(iii) ________

31. (a) How often do you visit the centre of Glasgow for shopping? 23
[Interviewer show CARD F] ________

1 Daily 2 Twice or more a week 3 Once a week 4 Twice a month
5 Once a month 6 About every 2-3 months 7 Less often
8 Not applicable.

(b) By what method do you usually travel there for shopping? 24
[Interviewer show CARD 'D' again] ________

(c) How easy or difficult is to to get there for shopping? 25
[Interviewer show CARD E again] ________
[Interviewer if coded 4 or 5 'Fairly or Very Difficult' probe 'Why do you say that?'] 26 27

____

____

31. (Contd.)

(d) Which other methods could you use to travel to the centre of Glasgow for shopping? 28
[Interviewer show CARD D again list up to 3 methods]
(i) ________
(ii) ________
(iii) ________

(e) How easy or difficult would it be for you to travel to the centre of Glasgow for shopping by each of these methods?
[Interviewer show CARD E again and list response for each of (i) - (iii) in (d)] 31
(i) ________
(ii) ________
(iii) ________

32. (a) How often do you visit the centre of Glasgow for leisure or entertainment purposes? 34
[Interviewer show CARD F again] ________

(b) By what method do you usually travel there for leisure or entertainment? 35
[Interviewer show CARD D again] ________

(c) How easy or difficult is it to get there for leisure or entertainment? 36
[Interviewer show CARD E again] ________
[Interviewer if coded 4 or 5 'Fairly or Very Difficult' probe 'Why do you say that?'] 37 38

____

____

(d) Which other methods could you use to travel to the centre of Glasgow for entertainment? 39
[Interviewer show CARD D again and list up to 3 methods]
(i) ________
(ii) ________
(iii) ________

(e) How easy or difficult would it be for you to travel to the centre of Glasgow for entertainment by each of these methods? 42
[Interviewer show CARD E again and list response for each of (i) - (iii) in (d)]
(i) ________
(ii) ________
(iii) ________

SHEET 3 SIDE 1

33. How easy or difficult is it to move about from place to place in the centre of Glasgow?
[Interviewer show CARD E again]
[Interviewer if coded 4 or 5 'Fairly or Very Difficult' probe 'Why do you say that?']

____

____

45

46 47

34. (a) There are a number of places which people visit which are just outside the centre of Glasgow
[Interviewer show CARD G]
Which of these places have you visited within the past year?
i ________ ii ________ iii ________

48

(b) [For each of those mentioned]
How do you usuakly travel there?
[Interviewer show CARD D again]
i ________ ii ________ iii ________

51

(c) How easy or difficult is to to get to each of these places?
[Interviewer show CARD E again]
________ ii ________ iii ________

54

[Interviewer if coded 4 or 5. Probe 'Why do you say that]
i ______________________________
ii ______________________________
iii ______________________________

57 58

59 60

61 62

35. You have probably heard that the 'subway' is being modernised and will soon link in with the 'blue trains' at Partick Hill (Merkland St) and Buchanan St. Also there will be a new rail line through the city centre underneath Argyle Street from Partick to Rutherglen.

(a) Which of the following journeys will be made easier for you by these new services?
[Interviewer show CARD H] i ________ ii ________ iii ________

63

1 Centre/work 2 Other parts/work 3 Centre/other purposes
4 Around centre 5 Just outside city centre 6 Visiting friends or relatives elsewhere in Glasgow area 7 Don't know

35. (Contd.)

(b) Which of these journeys are you likely to use the new 'subway' for?
[Interviewer show CARD H again]
i ________
ii ________
iii ________

66

(c) Which of these journeys are ou likely to use the new Argyle St rail line for?
[Interviewer show CARD H again]
i. ________
ii ________
iii ________

69

(d) At some stations it will be made easier for people to change between trains and 'subway' for part of their journey. How likely are you to do this for any of these journeys?
[Interviewer show CARD H again]
i ________
ii ________
iii ________

72

36. What improvements, if any, do you think should be made to public transport in your area?

____

____

____

____

75 76

77 78

INVERVIEWER: - "I would like to thank you and the other members of your household for all your co-operation in this survey".

SHEET 3 SIDE 2

RE 65810/3 3750 10/78 TCL

GLASGOW RAIL IMPACT STUDY SCOTTISH DEVELOPMENT DEPARTMENT TRANSPORT AND ROAD RESEARCH LABORATORY: DEPARTMENT OF TRANSPORT DEPARTMENT OF THE ENVIRONMENT MARTIN AND VOORHEES ASSOCIATES	SATURDAY TRAVEL SURVEY CONFIDENTIAL	INTERVIEWER CODE Card Type 1 2: 0 5; Serial 3 4 5 6 7; HH 8; P 9	FOR OFFICE USE ONLY 10 11 12

Thank you for helping with this survey. First of all would you please complete the following details: Your sex ______ Your age ______

Please give the following details about each trip you made during the 24 hours from 4.00 a.m. last Saturday to 3.59 a.m. last Sunday. A trip is a one-way journey for a particular purpose. Use a separate row for each trip.

1. TRIP NUMBER	2. PLACE STARTED Please give the full address of where you began the trip (If at home write "HOME").	3. TIME STARTED When did you begin the trip. Please note whether AM or PM	4. PLACE FINISHED Please give the full address of where you ended the trip. (If at home write "HOME").	5. TIME FINISHED When did you end the trip. Please note whether AM or PM.	6. PURPOSE OF TRIP Please give the main purpose of your trip. e.g. to or from work. Employer's business School/College etc. Shopping Personal business Social/Recreational Carrying a passenger	7. MEANS OF TRAVEL If more than one means was used please list them in the order you used them. If you travelled by CAR or VAN please state whether it was as "CAR DRIVER" or "CAR PASSENGER" If you travelled by bus please state whether it was "SCHEDULED BUS" or "NON-SCHEDULED BUS/COACH" Do not forget "WALK" trips.
1 13 14 0 1	15	22	26	33	37	39 44
2 45 46 0 2	47	54	58	65	69	71 76
3 13 14 0 3	15	22	26	33	37	39 44
4 45 46 0 4	47	54	58	65	69	71 76

RE 65810/4 TCL

Continued overleaf

CONTINUE to record each of your trips on a separate row

FOR OFFICIAL USE ONLY

1	2	3	4	5	6	7	8	9	10	11	12
0	5										

1. TRIP NUMBER	2. PLACE STARTED Please give the full address of where you began the trip (If at home write "HOME").	3. TIME STARTED When did you begin the trip. Please note whether AM or PM	4. PLACE FINISHED Please give the full address of where you ended the trip. (If at home write "HOME").	5. TIME FINISHED When did you end the trip. Please note whether AM or PM.	6. PURPOSE OF TRIP Please give the main purpose of your trip. e.g. to or from work. Employer's business School/College etc. Shopping Personal business Social/Recreational Carrying a passenger	7. MEANS OF TRAVEL If more than one means was used please list them in the order you used them. If you travelled by CAR or VAN please state whether it was as "CAR DRIVER or "CAR PASSENGER". If you travelled by bus please state whether it was "SCHEDULED BUS" or "NON?SCHEDULED BUS/COACH". Do not forget "WALK" trips.
5 13 14 0 5	15	22	26	33	37	39 44
6 45 46 0 6	47	54	58	65	69	71 76
7 13 14 0 7	15	22	26	33	37	39 44
8 45 46 0 8	47	54	58	65	69	71 76
9 13 14 0 9	15	22	26	33	37	39 44
10 45 46 1 0	47	54	58	65	69	71 76

When you have recorded all your trips please place this form in the envelope provided and post it back to us. Thank you very much for your help.

RE 65810/4 3750 10/78 TCL

GLASGOW RAIL IMPACT STUDY

SCOTTISH DEVELOPMENT DEPARTMENT
TRANSPORT AND ROAD RESEARCH LABORATORY:
DEPARTMENT OF TRANSPORT
DEPARTMENT OF THE ENVIRONMENT
MARTIN AND VOORHEES ASSOCIATES

HOUSEHOLD SURVEY: control 1978

A. Date of Travel: - - - - - - - - - - - - D (16 17) M (18 19) Y (20 21)
[Interviewer: Enter date here]

B. Day of Travel: - - - - - - - - - - - - 22
[Interviewer: Enter day here in full]

[M = 1, T = 2, W = 3, Th = 4, F = 5]

C. CONTACT RECORD

CALL	TIME	DATE	RESULT	APPOINTMENT DETAILS
1				
2				
3				
4				

D. DATA RECORD

PART	COMPLETE	PARTIALLY COMPLETE	NO DATA	
1				23
2				24

E. INTERVIEWER DETAILS

Name - - - - - - - - - - - - - - - - - No - - - - - - - -

F. ADDITIONAL INTERVIEWER COMMENTS:

Card Type (1 2) 0 6 Address Serial No. (3 4 5 6 7) Household No. (8)

Post Code (9 10 11 12 13 14 15)

PART I: HOUSEHOLD DATA

1 How many people of all ages, including yourself are there in this household (25 26)

2 (a) How many of each of the following types of vehicle are owned by members of the household. [complete column 'a' below]. (27 28)

(b) How many of each of these vehicles are provided with some assistance from an employer (eg car provided with job; free of or low interest loan; essential user car allowance etc) [complete column 'b' below] (29 30)

(c) How many other vehicles of each of these types were also available to members of the household for their personal use on the travel day [complete column 'c' below] (31 32)

	a	b	c	
Cars/Private Vans (including 3 wheelers and Motor caravans)	____	____	____	33 34 35
Motor cycles and motor scooters	____	____	____	36 37 38
Light Goods vehicles	____	____	____	39 40 41
Heavy Goods vehicles	____	____	____	42 43 44

Go to Q3

14 How easy or difficult is it to get to work/shopping by your usual method? [Interviewer show CARD E] ________ (45)

1 Very easy 2 Fairly easy 3 Neither easy nor difficult
4 Fairly difficult 5 Very Difficult 6 Not Applicable.
[If coded 4 or 5 Interviewer probe 'why do you say that?']

____ (46 47)

____

15 I'd like you to look at this card and tell me which of the groups shows the total combined income of all members of the household, before deductions.
[Interviewer Show CARD 'I'] Group ________ (48 49)

SIDE 1

GRIS - HHc part 2: person data

Card type 0 7 (1–2) Address serial Nº (3–7) H/hold Nº (8)

Person Nº 3	Sex 4	Age 5	Status 6	Driving Licence 7	Occupation 8	Work journey 9	Work trip destination 10	Travel method 11	Shopping trip destination 12	Travel method → go to Q 14 13
Start at 1	M=1 F=2		Enter description Head of household Wife Child Related adult Unrelated adult resident Visitor	Car 1 M/cycle 2 Prov 3 None 4	Enter detailed description of main occupation and indicate if (interviewer show card C) Full time employed 1 Employed 10-29 hrs per week 2 Employed less than 10hrs per week 3 Housewife 4 Full time student 5 Long term sick 6 Retired 7 Unemployed 8 (if 6 or 7 give previous occupation) (if 'manager' indicate Nº of employees)	On the travel day did you make a work journey to: Usual place 1 Other place * 2 Work at home * 3 Not working on travel day * 4 Not employed 5	Interviewers write in full address of the work place	H/hold car/van dr 01 Other car/van dr 02 H/hold car/van pass 03 M/cycle dr 04 Taxi 05 Pedal cycle 06 Walk 07 Goods veh dr 08 Train 09 Sched bus 10 Non sch bus 11 Other pass 12 Other (specify) 13	Did you travel from home to the shops yesterday? Interviewer write in full address of shop or shopping centre visited If no shopping trip write NONE	H/hold car/van dr 01 Other car/van dr 02 H/hold car/van pass 03 M/cycle dr 04 Taxi 05 Pedal cycle 06 Walk 07 Goods veh dr 08 Train 09 Sched bus 10 Non sch bus 11 Other pass 12 Other (specify) 13
9 10	11	12 13	14	15	16 18	19	20 26	27 28	29 35	36 37
38 39	40	41 42	43	44	45 47	48	49 55	56 57	58 64	65 66
9 10	11	12 13	14	15	16 18	19	20 26	27 28	29 35	36 37
38 39	40	41 42	43	44	45 47	48	49 55	56 57	58 64	65 66
9 10	11	12 13	14	15	16 18	19	20 26	27 28	29 35	36 37
38 39	40	41 42	43	44	45 47	48	49 55	56 57	58 64	65 66

APPENDIX 2

MULTINOMIAL LOGIT PROGRAM

```
C     THIS PROGRAM WAS WRITTEN BY THANOS MATZOROS
C     AT THE INSTITUTE FOR TRANSPORT STUDIES IN THE
C     UNIVERSITY OF LEEDS IN SUMMER 1982 AND GENEROUSLY
C     MADE AVAILABLE TO THE AUTHOR IN 1987.  IT WAS
C     AMENDED SLIGHTLY TO COPE WITH THE REQUIREMENTS
C     OF THE PRESENT STUDY.

C     AT PRESENT IT CAN HANDLE (EASILY EXTENDABLE THOUGH)
C     UP TO 20 ATTRIBUTES, CHARACTERISING BOTH INDIVIDUALS
C     AND ALTERNATIVES, AND 7 ALTERNATIVES.

C     IT USES A QUASI NEWTON-RAPHSON OPTIMISATION
C     TECHNIQUE, OBTAINED FROM NAG LIBRARY AS ROUTINE
C     E04JBF, AND CAN DETERMINE EITHER AN UNCONSTRAINED
C     MINIMUM/MAXIMUM (I.E. WHEN THE PARAMETERS CAN
C     TAKE ANY REAL VALUE) OR A CONSTRAINED MINIMUM/
C     MAXIMUM WHEN THE PARAMETERS ARE SUBJECT TO FIXED
C     UPPER AND/OR LOWER BOUNDS.  THIS FEATURE IS USEFUL
C     WHEN IT IS NEEDED TO CONSTRAIN THE VALUE OF
C     A PARAMETER TO BE WITHIN A PARTICULAR INTERVAL.

C     THE OPTIMISATION PROCESS INVOLVES THE EVALUATION
C     OF FIRST AND SECOND DERIVATIVES IN ORDER TO FIND
C     THE TURNING POINT.  THE FUNCTION TO BE OPTIMISED
C     MUST HAVE CONTINUOUS FIRST AND SECOND DERIVATIVES
C     (ALTHOUGH THE ALGORITHM WILL WORK EVEN IF THE
C     DERIVATIVES HAVE OCCASIONAL DISCONTINUITES).
```

```
C     NO DERIVATIVES NEED TO BE SUPPLIED ANALYTICALLY.
C     THE USER HAS ONLY TO PROVIDE A SUBROUTINE (FUNCT),
C     WHICH MUST EVALUATE THE LLF AT ANY POINT OVER THE
C     PARAMETER SPACE (SEE ALSO E04JBF MANNUAL), AND
C     A SUBROUTINE (MONIT) WHICH MONITORS THE PROGRESSION
C     OF THE OPTIMISATION PROCEDURE (SEE SUBROUTINES FUNCT
C     AND MONIT AS WELL).

C     NLOGIT IS THE MAIN PROGRAM, WHICH CALLS SUBROUTINES
C     ATTRB, THIS IS FOR DATA PREPARATION, E04JBF, AND
C     E04HBF.  AFTER SUCCESSFUL EXIT FROM THE E04JBF ROUTINE
C     IT CALCULATES THE PREDICTED PROBABILITIES OF THE
C     MODEL, THE LLF AT ZERO FROM WHICH THE RHO SQUARED
C     INDEX IS OBTAINED

      PROGRAM NLOGIT
      COMMON/DERIV/HESL,HESD
      COMMON/NUMB/N,NOBS,MXNLT
      COMMON/ACCUR/ETA,XTOL
      COMMON/HBFEVL/J
      CHARACTER ALTR*7,MODE*7
      REAL*8 ETA,F,FEST,STEPMX,XTOL,FIINV,FLLO,FLLR,
     + CCAR1,CCAR2,CCAR3,CPAS1,CPAS2,CPAS3,CBUS1,CBUS2,
     + CBUS3,CTRN1,CTRN2,CTRN3,CWLK1,CWLK2,CWLK3
      REAL*8 DELTA(20),HESD(20),HESL(190),W(180),
     + X(20),U(7),PA(7),A(20,7),P(7),XC(20),G(20),
     + BU(20),BL(20),PSUM(7),APSUM(7),WH(180)
      INTEGER ISTATE(20),IW(2),NL0(7),IX(7),IWH(2)
```

```
      LOGICAL LOCSCH
      EXTERNAL E04JBQ,FUNCT,MONIT
      CALL ATTRB
      IFAIL=1
      IFAILH=1
      LIW=2
      LW=180
      LIWH=2
      LWH=180
C     LLH=N*(N-1)/2
C     LH=MAX(LLH,1)
      LH=190
      IFLAG=0
C     INITIALISE AT ZERO OR SUPPLY INITIAL GUESSES FOR
C     THE UNKNOWN PARAMETERS
      DO 99 I=1,N
         X(I)=0.
 99   CONTINUE

C     THIS SUBROUTINE PROVIDES SUITABLE DIFFERENCING
C     INTERVALS TO E04JBF

      CALL E04HBF(N,FUNCT,X,J,DELTA,HESL,LH,HESD,F,G,
     + IWH,LIWH,WH,LWH,IFAILH)
      IF(IFAILH.NE.0)THEN
      WRITE(6,*)'IFAIL FOR E04HBF= ',IFAILH
      STOP
      ENDIF
```

```
      LOCSCH= .TRUE.
      IPRINT= 1
      INTYPE= 1
      MAXCAL= 40*N*(N+ 5)
      STEPMX= 100000.
      FEST= 0.
      IBOUND= 1

C     THIS SUBROUTINE DOES THE OPTIMISATION AND CALLS
C     FUNCT AND MONIT FOR THAT PURPOSE

      CALL E04JBF(N,FUNCT,MONIT,IPRINT,LOCSCH,INTYPE,
     + E04JBQ,MAXCAL,ETA,XTOL,STEPMX,FEST,DELTA,IBOUND,
     + BL,BU,X,HESL,LH,HESD,ISTATE,F,G,IW,LIW,W,LW,IFAIL)
      IF(IFAIL.NE.0)THEN
         WRITE(6,998)IFAIL
         STOP
      ENDIF

C     AFTER SUCCESFUL EXIT PROCEED TO THE CALCULATION
C     OF THE PREDICTED PROBABILITIES

      IPRD= 0
      PCAR1= 0.
      PCAR2= 0.
      PCAR3= 0.
      PPAS1= 0.
      PPAS2= 0.
```

```
      PPAS3= 0.
      PBUS1= 0.
      PBUS2= 0.
      PBUS3= 0.
      PTRN1= 0.
      PTRN2= 0.
      PTRN3= 0.
      PWLK1= 0.
      PWLK2= 0.
      PWLK3= 0.
      DO 92 I= 1,MXNLT
         NL0(I)= 0
         PSUM(I)= 0.
         APSUM(I)= 0.
 92   CONTINUE
      WRITE(6,990)
      REWIND 1
      REWIND 4
      DO 98 IOBS= 1,NOBS

C     READ DATA FROM SUBROUTINE ATTRB

         READ(1,ERR= 43,END= 98)ICH,NSEL,NALT,(IX(I),I= 1,MXNLT)
         READ(1,ERR= 63)((A(J,I),J= 1,N),I= 1,NALT)
         PD= 0.
         DO 96 I= 1,NALT
            U(I)= 0.
              DO 95 J= 1,N
```

```
              U(I)= U(I)+ X(J)*A(J,I)
95          CONTINUE
          PA(I)= DEXP(- U(I))
          PD= PD+ PA(I)
96      CONTINUE
        PMAX= - 1.
        LM= 0
        DO 94 I= 1,MXNLT
          IF(IX(I).EQ.0)THEN
            P(I)= 100.
            GO TO 94
          ENDIF
          LM= LM+ 1
          P(I)= PA(LM)/PD
          PSUM(I)= PSUM(I)+ P(I)
          IF(P(I).GT.PMAX)THEN
            PMAX= P(I)
            INDMX= I
          ENDIF
94      CONTINUE
        ALTR= '          '
        MODE= '          '
        IF(ICH.EQ.1)ALTR= 'CAR- 1    '
        IF(ICH.EQ.2)ALTR= 'PASS- 2  '
        IF(ICH.EQ.3)ALTR= 'BUS- 3    '
        IF(ICH.EQ.4)ALTR= 'TRAIN- 4'
        IF(ICH.EQ.5)ALTR= 'WALK- 5  '
        IF(IOBS.EQ.1)WRITE(6,995)
```

```
IF(INDMX.EQ.ICH)THEN
  IPRD= IPRD+ 1
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,ALTR,INDMX,ICH
ELSE IF (INDMX.EQ.1.AND.INDMX.NE.ICH) THEN
  MODE= 'CAR- 1   '
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,MODE,INDMX,ICH
ELSE IF (INDMX.EQ.2.AND.INDMX.NE.ICH) THEN
  MODE= 'PASS- 2  '
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,MODE,INDMX,ICH
ELSE IF (INDMX.EQ.3.AND.INDMX.NE.ICH) THEN
  MODE= 'BUS- 3   '
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,MODE,INDMX,ICH
ELSE IF (INDMX.EQ.4.AND.INDMX.NE.ICH) THEN
  MODE= 'TRAIN- 4'
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,MODE,INDMX,ICH
ELSE IF (INDMX.EQ.5.AND.INDMX.NE.ICH) THEN
  MODE= 'WALK- 5  '
  WRITE(6,994)IOBS,(P(I),I= 1,MXNLT),ALTR,MODE,INDMX,ICH
END IF
NL0(NALT)= NL0(NALT)+ 1
IF (ICH.EQ.1) THEN
  PCAR1= PCAR1+ 1
END IF
IF (ICH.EQ.1.AND.INDMX.EQ.1) THEN
  PCAR2= PCAR2+ 1
END IF
IF (INDMX.EQ.1) THEN
  PCAR3= PCAR3+ 1
```

```
END IF
IF (ICH.EQ.2) THEN
  PPAS1=PPAS1+1
END IF
IF (ICH.EQ.2.AND.INDMX.EQ.2) THEN
  PPAS2=PPAS2+1
END IF
IF (INDMX.EQ.2) THEN
  PPAS3=PPAS3+1
END IF
IF (ICH.EQ.3) THEN
  PBUS1=PBUS1+1
END IF
IF (ICH.EQ.3.AND.INDMX.EQ.3) THEN
  PBUS2=PBUS2+1
END IF
IF (INDMX.EQ.3) THEN
  PBUS3=PBUS3+1
END IF
IF (ICH.EQ.4) THEN
  PTRN1=PTRN1+1
END IF
IF (ICH.EQ.4.AND.INDMX.EQ.4) THEN
  PTRN2=PTRN2+1
END IF
IF (INDMX.EQ.4) THEN
  PTRN3=PTRN3+1
END IF
```

```
      IF (ICH.EQ.5) THEN
        PWLK1= PWLK1+ 1
      END IF
      IF (ICH.EQ.5.AND.INDMX.EQ.5) THEN
        PWLK2= PWLK2+ 1
      END IF
      IF (INDMX.EQ.5) THEN
        PWLK3= PWLK3+ 1
      END IF
98    CONTINUE
               CCAR1= 100*((PCAR1)/FLOAT(NOBS))
               CCAR2= 100*((PCAR2)/FLOAT(IPRD))
               CCAR3= 100*((PCAR3)/FLOAT(NOBS))
               CPAS1= 100*((PPAS1)/FLOAT(NOBS))
               CPAS2= 100*((PPAS2)/FLOAT(IPRD))
               CPAS3= 100*((PPAS3)/FLOAT(NOBS))
               CBUS1= 100*((PBUS1)/FLOAT(NOBS))
               CBUS2= 100*((PBUS2)/FLOAT(IPRD))
               CBUS3= 100*((PBUS3)/FLOAT(NOBS))
               CTRN1= 100*((PTRN1)/FLOAT(NOBS))
               CTRN2= 100*((PTRN2)/FLOAT(IPRD))
               CTRN3= 100*((PTRN3)/FLOAT(NOBS))
               CWLK1= 100*((PWLK1)/FLOAT(NOBS))
               CWLK2= 100*((PWLK2)/FLOAT(IPRD))
               CWLK3= 100*((PWLK3)/FLOAT(NOBS))
      DO 93 I= 1,MXNLT
          APSUM(I)= 100*(PSUM(I)/FLOAT(NOBS))
 93   CONTINUE
```

```
      FLL0= 0.
      DO 91  I= 2,MXNLT
         FIINV= 1./FLOAT(I)
         FLL0= FLL0+ NL0(I)*DLOG(FIINV)
91    CONTINUE
      FLLR= - 2*(FLL0+ F)
      WRITE(6,993)FLL0,- F
      WRITE(6,992)FLLR,N
      WRITE(6,989)(- X(J),J= 1,N)
      WRITE(6,777)(APSUM(I),I= 1,MXNLT)
      WRITE(6,555)
      WRITE(6,666)CCAR1,CPAS1,CBUS1,CTRN1,CWLK1
      WRITE(6,333)
      WRITE(6,666)CCAR2,CPAS2,CBUS2,CTRN2,CWLK2
      WRITE(6,111)
      WRITE(6,666)CCAR3,CPAS3,CBUS3,CTRN3,CWLK3
      CPRD= (FLOAT(IPRD)/FLOAT(NOBS))*100.
      WRITE(6,991)CPRD
 43   WRITE(6,*)'ERR IN READ DATA IN NLOGIT:ICH,NSEL,ETC...'
 63   WRITE(6,*)'ERR IN DATA READ.NLOGIT:A'
      STOP
990   FORMAT(///1X,'CONVERGENCE HAS BEEN COMPLETED')
998   FORMAT(///1X,'CONVERGENCE MAY NOT BE SUCEEDED.
     + IFAIL= ',I3,'  SEE E04JBF MANUAL')
666   FORMAT(/15X,5F10.5)
777   FORMAT(///15X,'EXPECTED SHARE (PERCENT) FOR EACH
     + MODE',//15X,5F10.5)
555   FORMAT(///15X,'OBSERVED SHARES (PERCENT) FOR EACH MODE')
```

```
333  FORMAT(///15X,'PER. OF OBSERVATIONS CORRECTLY PREDICTED
    + FOR EACH MODE')
111  FORMAT(///15X,'PREDICTED SHARE (PERCENT) FOR EACH MODE')
995  FORMAT(1H1///5X,'OBSERVATION',10X,'CHOICE PROBABILITIES'
    + ,9X,'CHOSEN MODE',4X,'ALTERNATIVE MODE',4X,'INDMX',5X,'ICH')
994  FORMAT(7X,I5,5X,5(2X,F5.3),5X,A,10X,A,10X,I2,7X,I2)
993  FORMAT(1H1/////' LOG LIKELIHOOD AT ZERO ',F9.3,5X,
    + 'FINAL LOG LIKELIHOOD ',F9.3)
992  FORMAT(////' L-LIKELIHOOD RATIO ',F9.3,5X,
    + 'DEGREES OF FREEDOM ',I3)
991  FORMAT(////' PERCENTAGE OF OBSERVATIONS CORRECTLY
    + PREDICTED',F9.3)
989  FORMAT(/////' FINAL VALUES OF COEFFICIENTS'/(13(2X,D11.5)))
     END

     SUBROUTINE FUNCT(IFLAG,N,XC,FC,GC,IW,LIW,W,LW)
     REAL*8 XC,FC,GC,W,U,PA,PD,FFC,DA
     INTEGER IFLAG,LIW,LW,IW,N
C    DIMENSION XC(20),GC(20),IW(2),W(180),DA(120)
     COMMON/NUMB/NQ,NOBS,MXNLT
     FFC=0.
     REWIND 4
     DO 99,IOBS=1,NOBS
        PD=0.
        READ(4,ERR=33)NX,(DA(I),I=1,NX)
     DO 97,I=1,NX/N
        U=0.
        DO 96 J=1,N
```

```
          U= U+ XC(J)*DA(J+ N*(I- 1))
 96     CONTINUE
      IF (U.LT.-170.)GO TO 97
      IF (U.GT.170.)GO TO 99
      PD= PD+ DEXP(U)
 97   CONTINUE
      PD= PD+ 1.
      FFC= FFC+ DLOG(1./PD)
 99   CONTINUE
      FC= - FFC
      RETURN
33    WRITE(6,*)'ERR IN DATA READ.FUNCT:NALT,DA...'
      STOP
      END

      SUBROUTINE MONIT(N,XC,FC,GC,ISTATE,GPJNRM,COND,
     + POSDEF,NITER,NF,IW,LIW,W,LW)
      COMMON/NUMB/NQ,NOBS,MXNLT
      COMMON/DERIV/HESL,HESD
      COMMON/HBFEVL/NFH
      REAL*8 COND,FC,GPJNRM,GC(20),W(180),XC(20),HESD(20),
     + HESL(190),A(21,20),B(20,20),Z(20),X02AAF,STD(20),
     + TRAT(20),RL(13,13)
      INTEGER ISTATE(20),IW(2)
      LOGICAL POSDEF,FREE,POSIT
      NFUN= NF+ NFH
      WRITE(6,999)NITER,NFUN
      FREE= .TRUE.
```

```
      DO  99,J=1,N
         IF(ISTATE(J).LE.0)THEN
            FREE= .FALSE.
            WRITE(6,998)J,ISTATE(J)
         ENDIF
99    CONTINUE
      IF(.NOT.FREE)STOP
      WRITE(6,997)(-XC(J),J=1,N)
      IA=21
      IB=20
      IFAIL=1
      DO  13  I=1,N
      DO  13  J=1,N
      RL(I,J)=0.
13    CONTINUE
      DO  23  J=1,N
      RL(J,J)=1.
23    CONTINUE
      K=0
      DO  34  I=2,N
      DO  34  J=1,I-1
       K=K+1
      RL(I,J)=HESL(K)
34    CONTINUE
      DO  44  I=1,N
      DO  44  J=1,N
       SUM=0
      DO  55  K=1,N
```

```
      SUM = SUM+ RL(I,K)*RL(J,K)*HESD(K)
 55   CONTINUE
      A(I,J)= SUM
 44   CONTINUE
      IF(.NOT.POSDEF)THEN
      WRITE(6,996)
      STOP
      ENDIF
      CALL F01ACF(N,X02AAF(IT),A,IA,B,IB,Z,L,IFAIL)
      IF(IFAIL.NE.0)THEN
         WRITE(6,995)IFAIL
         STOP
      ENDIF
      DO 93,I=1,N
         STD(I)= DSQRT(A(I+1,I))
         TRAT(I)= -(XC(I)/STD(I))
 93   CONTINUE
      WRITE(6,994)(STD(I),I=1,N)
      WRITE(6,993)(TRAT(I),I=1,N)
      WRITE(6,992)(GC(I),I=1,N)
      WRITE(6,990)
      DO 94,I=1,N
         WRITE(6,991)(A(I+1,J),J=1,I)
 94   CONTINUE
      WRITE(6,989)-FC
      WRITE(6,988)GPJNRM,COND
999   FORMAT(1H1//1X,'ITERATION NR ',I3,10X,'NR OF L-LIKELIHOOD
     + FUNCTION EVALS SO FAR ',I8)
```

```
998  FORMAT(//1X,'COEFFICIENT NR',I3,1X,'HAS REACHED -+10**6',
    + /1X,'ISTATE VALUE IS ',I3,'      PROCESS TERMINATED')
997  FORMAT(/1X,'COEFFICIENTS IN THIS ITERATION ',/,
    + (13(3X,D11.5)))
996  FORMAT(/1X,'MATRIX FOR INVERSION NOT POSITIVE DEFINITE')
995  FORMAT(/1X,'MATRIX OF SECOND DERIV CANNOT BE
    + INVERTED.IFAIL= ',I2)
994  FORMAT(/1X,'STANDARD DEVIATION ESTIMATES',/
    + ,(13(3X,D11.5)))
993  FORMAT(/1X,'T-RATIOS(ON ZERO)=COEFF/STD DEV',/,
    + (13(3X,D11.5)))
992  FORMAT(/1X,'FIRST DERIVATIVES ESTIMATES'/(13(3X,D11.5)))
990  FORMAT(/1X,'ESTIMATED VAR-COVAR MATRIX')
991  FORMAT(13(3X,D11.5))
989  FORMAT(/1X,'L-LIKELIHOOD FUNCTION VALUE',F10.4)
988  FORMAT(///1X,'GRAD.PROJ.NORM ',F10.3,6X,'COND NR OF
    + PROJ HESSIAN MATRIX ',F10.3)
      RETURN
      END

      SUBROUTINE ATTRB
      REAL*8 AVC,AVB,AVT,AVW,A(20,7),DA(120),ETA,XTOL,
    + PER,PERW,PERDL,HHPOS,SEX,OCC,HINC,HINCP,CBD,
    + CAOD,CAPDL,CAPW,WKB,WKT,WTB,WTT,IVTB,IVTT,
    + IVTCP,WALK,OVTB,OVTT,TTB,TTT,DIST,CSTB,CSTT,
    + CSCP1,CSCP2,CSCP3,CSC1,CSC2,CSC3,CPIPB,CPIPT,
    + CPICP1,CPICP2,CPICP3,CPIC1,CPIC2,CPIC3
      INTEGER IX(7)
```

```
      COMMON/ACCUR/ETA,XTOL
      COMMON/NUMB/N,NOBS,MXNLT
      REWIND 4
      REWIND 1
      NXS=0
      READ(5,999)N,NOBS,MXNLT,ETA,XTOL
      IF(N.EQ.1)ETA=0.
         DO 99 IOBS=1,NOBS
      READ(5,998)ICH
      READ(5,997)PER,PERW,PERDL,HHPOS,SEX,OCC,HINC,HINCP,CBD,
     + CAOD,CAPDL,CAPW
      READ(5,997)WKB,WKT,WTB,WTT,IVTB,IVTT,IVTCP,WALK,OVTB,
     + OVTT,TTB,TTT,DIST
      READ(5,997)CSTB,CSTT,CSCP1,CSCP2,CSCP3,CSC1,CSC2,CSC3
      READ(5,997)CPIPB,CPIPT,CPICP1,CPICP2,CPICP3,CPIC1,CPIC2,CPIC3
      READ(5,997)AVC,AVB,AVT,AVW

      II=0
      KK=0
      KK=KK+1
      IF(AVC.EQ.1.)THEN
      II=II+1
      IX(KK)=1
      IF(ICH.EQ.KK)NSEL=II
C*****------------------ C A R -------------------
         A(1,II)=HHPOS
         A(2,II)=CAPDL
         A(3,II)=CAOD
```

```
      A(4,II)= 0
      A(5,II)= IVTCP
      A(6,II)= CSC1/DIST
      A(7,II)= 1
      A(8,II)= 0
      A(9,II)= 0
      A(10,II)= 0
C        A(11,II)=
C        A(12,II)=
C        A(13,II)=
C        A(14,II)=


      ELSE
      IX(KK)= 0
      ENDIF
      KK= KK+ 1
      IX(KK)= 1
      II= II+ 1
      IF(ICH.EQ.KK)NSEL= II
C******--------------- P A S S -----------------
      A(1,II)= 0
      A(2,II)= 0
      A(3,II)= CAOD
      A(4,II)= 0
      A(5,II)= IVTCP
      A(6,II)= CSCP1/DIST
      A(7,II)= 0
```

```
      A(8,II)= 1
      A(9,II)= 0
      A(10,II)= 0
C     A(11,II)=
C     A(12,II)=
C     A(13,II)=
C     A(14,II)=


   KK= KK+ 1
   IF(AVB.EQ.1.)THEN
   II= II+ 1
   IX(KK)= 1
   IF(ICH.EQ.KK)NSEL= II
C******---------------- B U S ------------------
      A(1,II)= 0
      A(2,II)= 0
      A(3,II)= 0
      A(4,II)= CBD
      A(5,II)= TTB
      A(6,II)= CSTB/DIST
      A(7,II)= 0
      A(8,II)= 0
      A(9,II)= 1
      A(10,II)= 0
C     A(11,II)=
C     A(12,II)=
C     A(13,II)=
```

```
C        A(14,II)=

      ELSE
      IX(KK)= 0
      ENDIF
      KK= KK+ 1
      IF(AVT.EQ.1.)THEN
      II= II+ 1
      IX(KK)= 1
      IF(ICH.EQ.KK)NSEL= II
C******--------------- T R A I N ---------------
         A(1,II)= 0
         A(2,II)= 0
         A(3,II)= 0
         A(4,II)= CBD
         A(5,II)= TTT
         A(6,II)= CSTT/DIST
         A(7,II)= 0
         A(8,II)= 0
         A(9,II)= 0
         A(10,II)= 1
C        A(11,II)=
C        A(12,II)=
C        A(13,II)=
C        A(14,II)=
```

```
ELSE
IX(KK)= 0
ENDIF
KK= KK+ 1
IF(AVW.EQ.1.)THEN
II= II+ 1
IX(KK)= 1
IF(ICH.EQ.KK)NSEL= II
C******--------------- W A L K -----------------
      A(1,II)= 0
      A(2,II)= 0
      A(3,II)= 0
      A(4,II)= 0
      A(5,II)= WALK
      A(6,II)= 0
      A(7,II)= 0
      A(8,II)= 0
      A(9,II)= 0
      A(10,II)= 0
C     A(11,II)=
C     A(12,II)=
C     A(13,II)=
C     A(14,II)=


ELSE
IX(KK)= 0
```

```
      ENDIF
      NALT=II
      WRITE(1)ICH,NSEL,NALT,(IX(I),I=1,MXNLT)
      WRITE(1)((A(J,I),J=1,N),I=1,NALT)
      KKK=0
      DO 397,I=1,NALT
      IF(I.EQ.NSEL)GO TO 397
      KKK=KKK+1
      DO 97 J=1,N
      DA(J+N*(KKK-1))=A(J,NSEL)-A(J,I)
   97 CONTINUE
  397 CONTINUE
      NX=N*(NALT-1)
      NXS=NXS+NX
      WRITE(4)NX,(DA(I),I=1,NX)
   99 CONTINUE
      WRITE(6,*)'*****************SUM OF NX= ',NXS
      RETURN
  999 FORMAT(3I4,2F10.6)
  998 FORMAT(I1)
  997 FORMAT(13F8.4)
  222 FORMAT(9I4)
  223 FORMAT(7F11.5)
  224 FORMAT(I3,(7F11.5))
      END
```

APPENDIX 3

AGGREGATE PREDICTION ERROR PROGRAM

```
C      THIS SIMPLE PROGRAM WAS WRITTEN BY A.K. MOHAMAD AT THE
C      DEPARTMENT OF CIVIL ENGINEERING IN THE UNIVERSITY OF
C      GLASGOW IN 1988.

C      IT IS DESIGNED TO COMPUTE AGGREGATE PREDICTION ERRORS
C      IN TERMS OF THE AVERAGE, ROOT MEAN SQUARE AND STANDARD
C      DEVIATION ERRORS.

C      THE ERRORS CALCULATED IN THIS PROGRAM ARE WEIGHTED ERRORS
C      AND THE WEIGHTING IS GIVEN BY THE PROPORTION OF INDIVIDUALS
C      CHOOSING EACH MODE.


       PROGRAM PDTERR
       REAL*8 GS,TGS,SSAVE,SSAVGE,SSRMSE,SSRMSGE,SSSDE,SSSDGE,
      + GAVE,GAVGE,GRMSE,GRMSGE,SSAVET,SSAVGET,SSRMST,SSGRMST,
      + SSAVST,SSAVGST,TAVE,TAVGE,TRMSE,TRMSGE,TSDE,TSDGE,CS(100),
      + AVP(5,100),ENP(5,100),ACP(5,100),SENP(5),SACP(5),TENP(5),
      + TACP(5),E(5),GE(5),DFE(5),DFGE(5),SDFE(5),SDFGE(5),TDE(5),
      + TDGE(5),SE(5),SGE(5),WSE(5),WSGE(5),SWSE(5),SWSGE(5),TWSE(5),
      + TWSGE(5),AVE(5),AVGE(5),SAVE(5),SAVGE(5),RMSE(5),RMSGE(5),
      + SRMSE(5),SRMSGE(5),D(5),GD(5),WD(5),WGD(5),SWD(5),SWGD(5),
      + SDE(5),SDGE(5),SSDE(5),SSDGE(5),AVET(5),AVGET(5),SAVET(5),
      + SAVGET(5),RMST(5),GRMST(5),TD(5),SRMST(5),SGRMST(5),TGD(5),
      + WTD(5),WTGD(5),SWTD(5),SWTGD(5),AVST(5),AVGST(5),SAVST(5),
      + SAVGST(5),EI(5,100),GEI(5,100)
       INTEGER NM,NGRP,NGOBS(100)
       READ(5,*)TGS,NM,NGRP,(NGOBS(I),I=1,NGRP)
C      INATIALISE TO ZERO
```

```
      SSAVET= 0.
      SSAVGET= 0.
      SSRMST= 0.
      SSGRMST= 0.
      SSAVST= 0.
      SSAVGST= 0.
     DO 101 I=1,NM
      TDE(I)= 0.
      TDGE(I)= 0.
      TWSE(I)= 0.
      TWSGE(I)= 0.
      TENP(I)= 0.
      TACP(I)= 0.
      SWTD(I)= 0.
      SWTGD(I)= 0.
101     CONTINUE
    DO 100 IGRP=1,NGRP
      SSAVE= 0.
      SSAVGE= 0.
      SSRMSE= 0.
      SSRMSGE= 0.
      SSSDE= 0.
      SSSDGE= 0.
     DO 99 I=1,NM
      SDFE(I)= 0.
      SDFGE(I)= 0.
      SWSE(I)= 0.
      SWSGE(I)= 0.
      SWD(I)= 0.
```

```
        SWGD(I)= 0.
        SACP(I)= 0.
        SENP(I)= 0.
 99     CONTINUE
        GS= 0.
      ISTRT= 1
      IFNSH= NGOBS(IGRP)
C     READ THE AV. PROB. VALUES FOR EACH MODE FOR EACH GROUP
        DO 98 IOBS= ISTRT,IFNSH
          READ(5,998)CS(IOBS)
          READ(5,997)(AVP(I,IOBS),I= 1,NM)
          READ(5,997)(ENP(I,IOBS),I= 1,NM)
C         READ(5,997)(ACP(I,IOBS),I= 1,NM)
          GS= GS+ CS(IOBS)
C     SET TO ZERO IF THE ENUMERATION PROB. EQUAL ZERO
C     OTHERWISE CALCULATE THE VALUES OF ERRORS
            DO 97 I=1,NM
C             IF(ACP(I,IOBS).EQ.0.)THEN
C               E(I)= 0.
C             ELSE
C               E(I)= (ENP(I,IOBS)- ACP(I,IOBS))/ACP(I,IOBS)
C             ENDIF
              IF(ENP(I,IOBS).EQ.0.)THEN
                GE(I)= 0.
              ELSE
                GE(I)= (AVP(I,IOBS)- ENP(I,IOBS))/ENP(I,IOBS)
              ENDIF
C               DFE(I)= E(I)*ACP(I,IOBS)
                DFGE(I)= GE(I)*ENP(I,IOBS)
```

```
C                SDFE(I)= SDFE(I)+ DFE(I)*CS(IOBS)
                 SDFGE(I)= SDFGE(I)+ DFGE(I)*CS(IOBS)
C                TDE(I)= TDE(I)+ DFE(I)*CS(IOBS)
                 TDGE(I)= TDGE(I)+ DFGE(I)*CS(IOBS)
                 SENP(I)= SENP(I)+ ENP(I,IOBS)
C                SACP(I)= SACP(I)+ ACP(I,IOBS)
                 TENP(I)= TENP(I)+ ENP(I,IOBS)
C                TACP(I)= TACP(I)+ ACP(I,IOBS)
C                SE(I)= E(I)**2
                 SGE(I)= GE(I)**2
C                WSE(I)= SE(I)*ACP(I,IOBS)*CS(IOBS)
                 WSGE(I)= SGE(I)*ENP(I,IOBS)*CS(IOBS)
C                SWSE(I)= SWSE(I)+ WSE(I)
                 SWSGE(I)= SWSGE(I)+ WSGE(I)
C                TWSE(I)= TWSE(I)+ WSE(I)
                 TWSGE(I)= TWSGE(I)+ WSGE(I)
C                EI(I,IOBS)= E(I)
                 GEI(I,IOBS)= GE(I)
 97          CONTINUE
C      WRITE THE CALCULATED VALUES OF ERRORS
C          WRITE(6,996)
C          WRITE(6,995)IOBS,(E(I),I=1,NM)
C          WRITE(6,995)IOBS,(GE(I),I=1,NM)
C          WRITE(1)IGRP,CS(IOBS),(E(I),I=1,NM)
           WRITE(4)IGRP,CS(IOBS),(GE(I),I=1,NM)
 98      CONTINUE
C          WRITE(6,777)
C      FIND THE AV.VALUES OF:ERRORS,RMSE FOR EACH MODE FOR EACH
C      GROUP AND THEN WRITE THEM
```

```
      DO 96 I=1,NM
C           IF(SACP(I).EQ.0.)THEN
C             AVE(I)=0.
C             RMSE(I)=0.
C           ELSE
C       AVE(I)=SDFE(I)/(SACP(I)*GS)
C       RMSE(I)=SQRT(SWSE(I)/(SACP(I)*GS))
C           END IF
            IF(SENP(I).EQ.0.)THEN
              AVGE(I)=0.
              RMSGE(I)=0.
            ELSE
        AVGE(I)=SDFGE(I)/(SENP(I)*GS)
        RMSGE(I)=SQRT(SWSGE(I)/(SENP(I)*GS))
            ENDIF
C             SAVE(I)=AVE(I)**2
              SAVGE(I)=AVGE(I)**2
C             SSAVE=SSAVE+SAVE(I)
              SSAVGE=SSAVGE+SAVGE(I)
C             SRMSE(I)=RMSE(I)**2
              SRMSGE(I)=RMSGE(I)**2
C             SSRMSE=SSRMSE+SRMSE(I)
              SSRMSGE=SSRMSGE+SRMSGE(I)
 96   CONTINUE
C    FIND THE SDE FOR EACH MODE FOR EACH GROUP THEN WRITE THEM
      DO 95 IOBS=ISTRT,IFNSH
        DO 95 I=1,NM
C         D(I)=(EI(I,IOBS)-AVE(I))**2
          GD(I)=(GEI(I,IOBS)-AVGE(I))**2
```

```
C              WD(I)= D(I)*ACP(I,IOBS)*CS(IOBS)
               WGD(I)= GD(I)*ENP(I,IOBS)*CS(IOBS)
C              SWD(I)= SWD(I)+ WD(I)
               SWGD(I)= SWGD(I)+ WGD(I)
 95      CONTINUE
        DO 94 I=1,NM
C                 IF(SACP(I).EQ.0.)THEN
C                    SDE(I)= 0.
C                 ELSE
C            SDE(I)= SQRT(SWD(I)/(SACP(I)*GS))
C            END IF
                 IF(SENP(I).EQ.0.)THEN
                    SDGE(I)= 0.
                 ELSE
            SDGE(I)= SQRT(SWGD(I)/(SENP(I)*GS))
            END IF
C           SSDE(I)= SDE(I)**2
            SSDGE(I)= SDGE(I)**2
C           SSSDE= SSSDE+ SSDE(I)
            SSSDGE= SSSDGE+ SSDGE(I)
 94     CONTINUE
C           GAVE= SQRT(SSAVE/NM)
            GAVGE= SQRT(SSAVGE/NM)
C           GRMSE= SQRT(SSRMSE/NM)
            GRMSGE= SQRT(SSRMSGE/NM)
C           GSDE= SQRT(SSSDE/NM)
            GSDGE= SQRT(SSSDGE/NM)
C           WRITE(6,666)
CC          WRITE(6,994)(AVE(I),I= 1,NM),GAVE
```

```
C           WRITE(6,994)(AVGE(I),I=1,NM),GAVGE
C           WRITE(6,555)
C           WRITE(6,994)(RMSE(I),I=1,NM),GRMSE
C           WRITE(6,994)(RMSGE(I),I=1,NM),GRMSGE
C           WRITE(6,444)
C           WRITE(6,994)(SDE(I),I=1,NM),GSDE
C           WRITE(6,994)(SDGE(I),I=1,NM),GSDGE
 100    CONTINUE
        DO 93 I=1,NM
C               IF(TACP(I).EQ.0.)THEN
C                 AVET(I)=0.
C                 RMST(I)=0.
C               ELSE
C           AVET(I)=TDE(I)/(TACP(I)*TGS)
C           RMST(I)=SQRT(TWSE(I)/(TACP(I)*TGS))
C               ENDIF
                IF(TENP(I).EQ.0.)THEN
                  AVGET(I)=0.
                  GRMST(I)=0.
                ELSE
            AVGET(I)=TDGE(I)/(TENP(I)*TGS)
            GRMST(I)=SQRT(TWSGE(I)/(TENP(I)*TGS))
                ENDIF
C                 SAVET(I)=AVET(I)**2
                  SAVGET(I)=AVGET(I)**2
C                 SSAVET=SSAVET+SAVET(I)
                  SSAVGET=SSAVGET+SAVGET(I)
C                 SRMST(I)=RMST(I)**2
                  SGRMST(I)=GRMST(I)**2
```

```
C                 SSRMST= SSRMST+ SRMST(I)
                  SSGRMST= SSGRMST+ SGRMST(I)
 93     CONTINUE
C      REWIND 1
       REWIND 4
       DO 92 IGRP= 1,NGRP
         ISTRT= 1
         IFNSH= NGOBS(IGRP)
        DO 92 IOBS= ISTRT,IFNSH
C          READ(1)IGRP,CS(IOBS),(E(I),I= 1,NM)
           READ(4)IGRP,CS(IOBS),(GE(I),I= 1,NM)
         DO 92 I= 1,NM
C          TD(I)= (E(I)- AVET(I))**2
           TGD(I)= (GE(I)- AVGET(I))**2
C          WTD(I)= TD(I)*ACP(I,IOBS)*CS(IOBS)
           WTGD(I)= TGD(I)*ENP(I,IOBS)*CS(IOBS)
C          SWTD(I)= SWTD(I)+ WTD(I)
           SWTGD(I)= SWTGD(I)+ WTGD(I)
 92     CONTINUE
        DO 91 I= 1,NM
C               IF(TACP(I).EQ.0.)THEN
C                 AVST(I)= 0.
C               ELSE
C          AVST(I)= SQRT(SWTD(I)/(TACP(I)*TGS))
C               ENDIF
               IF(TENP(I).EQ.0.)THEN
                 AVGST(I)= 0.
               ELSE
           AVGST(I)= SQRT(SWTGD(I)/(TENP(I)*TGS))
```

```
      ENDIF
C         SAVST(I)= AVST(I)**2
          SAVGST(I)= AVGST(I)**2
C         SSAVST= SSAVST+ SAVST(I)
          SSAVGST= SSAVGST+ SAVGST(I)
 91   CONTINUE
C         TAVE= SQRT(SSAVET/NM)
          TAVGE= SQRT(SSAVGET/NM)
C         TRMSE= SQRT(SSRMST/NM)
          TRMSGE= SQRT(SSGRMST/NM)
C         TSDE= SQRT(SSAVST/NM)
          TSDGE= SQRT(SSAVGST/NM)
          WRITE(6,333)
C         WRITE(6,994)(AVET(I),I= 1,NM),TAVE
          WRITE(6,994)(AVGET(I),I= 1,NM),TAVGE
          WRITE(6,222)
C         WRITE(6,994)(RMST(I),I= 1,NM),TRMSE
          WRITE(6,994)(GRMST(I),I= 1,NM),TRMSGE
          WRITE(6,111)
C         WRITE(6,994)(AVST(I),I= 1,NM),TSDE
          WRITE(6,994)(AVGST(I),I= 1,NM),TSDGE
      STOP
 999  FORMAT(F4.0,3I4)
 998  FORMAT(6X,F4.0)
 997  FORMAT(14X,5F11.5)
 996  FORMAT(////3X,'              ')
 995  FORMAT(4X,I10,5F11.5)
 994  FORMAT(14X,6F11.5)
 777  FORMAT(/10X,'THE ABOVE VALUES ARE :FOR EACH OBS. THERE ARE
```

```
     + TWO VALUES. THESE ARE: 1-ERRORS MEASURE.  2-AGGREGATE
     + ERROR.',///)
666  FORMAT(///10X,'THE BELOW VALUES ARE : 1-AV. ERROR. 2-AGG. AV.
     + ERROR',//)
555  FORMAT(///10X,'THE BELOW  VALUES ARE: 1-RMSE. 2-AGG. RMSE',//)
444  FORMAT(///10X,'THE BELOW  VALUES ARE: 1-STD ERROR. 2-AGG.STD
     + ERROR',//)
333  FORMAT(///10X,'THE BELOW  VALUES ARE: 1-TOTAL AV. ERRORS.
     + 2-TOTAL AGG. ERROR',//)
222  FORMAT(///10X,'THESE ERRORS ARE: 1-TOTAL RMSE. 2-TOTAL AGG.
     + RMSE.',//)
111  FORMAT(///10X,'THESE ERRORS ARE: 1-TOTAL SDE. 2-TOTAL AGG.
     +  SDE.',//)
      END
```

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