Spontaneous Creation of Quark-Antiquark Pairs in Few-Quark Systems

Thesis by

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To the Memory of Mebarak, Father and Teacher.

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Declaration

Except where specific reference is made to the work of others, this thesis has been composed by the author. It has not been accepted in any previous application for a degree. I further state that no part of this thesis has already been or is being submitted for any degree or qualification at any other university.

Keltoum Ayat

Contents

Abstract

There are growing hopes that QCD can explain the basic nucleon-nucleon interaction. Many studies have been made on the assumption that nucleons are three-quark systems and that N-N interaction arises from the exchange of gluons between quarks in different nucleons. It is now clear that these models can not account for the medium- and long-range attractive part of the interaction. These long range forces arise from the creation of a meson from a gluon emanating from a quark in one nucleon, which is then absorbed by the other nucleon.

A natural way to include these mesonic effects is to consider the One Gluon Exchange theory which includes $q\bar{q}$ pair creation terms. So the simple quark model extended to include these creation and annihilation terms is going to be very satisfactory for the study of the NN interaction.

As a first step to fully understand the NN interaction, the structure of the nucleon will be studied where these $q\bar{q}$ pairs are explicitly added to the model space. Antiquarks are distinguished from quarks by their intrinsic parity. We choose the shell model as the working framework. A significant innovation in this work is our solution to the problem that arises because the number of particles is not conserved. As these $q\bar{q}$ effects are introduced at an off-shell level, the set of parameters that produces the observed properties of the nucleon in the simple quark model is no longer valid. Therefore, we choose a new set of parameters appropriate to our model space.

Our model measures different configuration mixings. In the case of the nucleon, it predicts a dominance of the 3q component accounting 73% of the total wavefunction, 22% for the $4q\bar{q}$ component and only 5% for the $5q^2\bar{q}^2$ one. It predicts hardly any strangeness in the wavefunction of the nucleon. It also predicts that members of the baryon octet with S=-1 are less cloudy than those with S=0: a confirmation of the fact that the hyperon-(nucleon-) hyperon interaction is less attractive than the NN one. Encouraging results on the collective nature of the pion are obtained.

Chapter I

Introduction

The question "What are the actual constituents of the atomic nucleus?" has been continuously asked since the very beginning of nuclear research early in this century [1]. Ideas about nuclear constituents and consequently those of the structure of nuclear matter have been determined by the current understanding of elementary particles. In 1932 Chadwick [2] discovered the neutron and the outstanding puzzle of how to explain nuclear masses and atomic number was solved. Nuclei are made out of protons and neutrons. Protons and neutrons were considered as elementary particles and as such were pointlike. The interaction between the nucleons was mediated by the nuclear force noted for its strength and its short range.

Soon after the discovery of the neutron, Heisenberg [3] introduced the concept of isospin in nuclear physics. The proton and the neutron are just two projections of the nucleon in the isospin space, and the nuclear force is isospin independent. With this picture of the nucleus it was possible to attack many problems of nuclear structure.

The first systematic data on nuclei were obtained from hyperfine splitting of atom spectra. Magnetic and quadrupole moments of nuclear ground states were measured. Using these data on nuclei in 1937 Schmidt [4] proposed the independent-particle model to explain the systematics of magnetic moments. For states close to the ground state, the nuclear structure can be sufficiently well described with the assumption of pointlike nucleons interacting via a short range force which can be well approximated by a two-body potential. Detailed knowledge of the interaction is not essential to explain a great bulk of nuclear data.

In 1935, Yukawa [5] postulated a new particle, the pion, which mediated the nuclear force. In analogy to the photon and the electromagnetic interaction, the pion was the

quantum field responsible for the strong interaction. The question arose whether pions were additional constituents of the nucleus. The most important experiments influencing the picture of the nucleon were the determinations of the charge radii and form factors of the proton and the neutron. The root-mean-square charge radii of these two particles show that they are extended objects and therefore have structures [6].

It is not till 1964 that Gell-Mann and Zweig [7] independently proposed that these two particles and all other strongly interacting ones known as hadrons are composite particles with quarks as constituents. Quarks are fermions and have a new property called colour. They cannot be isolated. In an experiment one can see them as "free particles" only if one uses a resolution better than 1GeV/c. At separations of 1fm quarks hadronize. At small scales (1 GeV/c) quantum chromodynamics (QCD for short), the generally accepted theory of the strong interaction, seems to work well. The nuclear force is no longer the fundamental force of the strong interaction. The pion is a composite particle therefore it is no longer the field of the strong interaction which is the gluon field coupled to colour. Thus the nuclear force is just a remainder of the strong force of the colour neutral nucleon very much as the inter-molecular force is a remainder of the Coulomb force in the electric charge neutral atom.

As a consequence of these new ideas, many models have been proposed. They can be classified into two depending on their way of confining quarks. One is the bag model where the confinement is given by boundary conditions on the bag surface. The most popular bag model is the one which was first developed by the MIT and other groups [8]. The other is the potential model where the confinement is given by the interaction potentials between quarks. Both are fairly successful in reproducing observed masses, charge radii and magnetic moments of hadrons. These models were not only used to find properties of the ground states of baryons and their lowest excitations, which would limit their functions, but to study the nuclear force as well. The literature is very rich in the number of papers that deal with the nature of the short-range part of the nucleon-nucleon (NN for short) interaction using the quark model. The first attempt was made by

Liberman [9] to understand this repulsion using the Born-Oppenheimer approximation in which the distance between two nucleons is fixed. He calculated the energy of the six-quark system. The relative kinetic energy between the nucleons was subtracted and the remaining energy was identified with the nucleon-nucleon potential. Other groups [10] refined these early calculations. Their calculated phase shifts strongly indicate that a large part of the observed NN S-wave repulsion originates from the quark substructure of the nucleons. A similar mechanism appears in the $\alpha\alpha$ repulsion, which originates from the nucleon substructure of the α particle. One dominant effect in both cases is the requirement of antisymmetric wavefunctions, for the six quarks in the NN case [11] and for the eight nucleons in the $\alpha\alpha$ case. Myhrer and Wroldsen [12] give an excellent review of the current understanding of the NN force. They do not go through the details of these complex resonating group calculations, but they present the results and some physics arguments to understand these results. They also discuss the effects of space truncation to include only the NN channel. They point out the improvement of these results when Harvey *et al.* [13] and Storm [14] performed their calculations using the complete channel (NN, $\Delta\Delta$, B_8B_8).

One serious drawback of these models is that they do not provide the attractive part of the N-N interaction. This attractive part has been established for a very long time and so has the existence of the one pion exchange potential (OPEP), which is the most solid fact that can be gleaned from the extensive analyses of NN scattering [15]. Therefore, to describe the medium- and long-range parts of the NN interaction, meson-exchange potentials were added phenomenologically to the quark forces. Many groups [16] included meson-exchange potentials in resonant group method calculations. The calculated and measured NN $^3S_1$ and $^1S_0$ phase shifts are in remarkably good agreement with each other. The strengths of the effective long-range meson exchanges in these calculations have been adjusted to reproduce the scattering lengths. Similarly the energy dependence of the NN $^1P_1$ phase shift has been calculated and found to be close to the experimental one. It should be pointed out here that there were only two free parameters in these calculations. All the others were determined by reproducing static baryon

properties and by reproducing the long-range OPE potential. A microscopic description and how properly to join the long-range meson exchange forces with quark degrees of freedom are lacking in these partial-wave analysis. There is still much to be done on this question.

If we slightly change the direction of looking at things, this meson-exchange problem is not restricted to the two-nucleon system only. All nucleons are identical so that if a pion can be emitted by one nucleon and absorbed by another to give OPEP, a pion can be emitted and absorbed by the same nucleon giving rise to a pion cloud. As a consequence, the MIT cloudy bag model [17], in which the quark degrees of freedom of the nucleon interior are coupled to an external elementary meson field, has superseded the bag model. However, the finite extent of the pion was neglected. Therefore this model should only be used in low momentum transfer situations for which the size should be irrelevant. Despite its many nice features such as its relativistic nature, this model runs into serious difficulties in a number of areas, for example separating out the centre of mass motion. This problem was encountered previously in the simple MIT bag model and, of course, cannot be handled by including pionic effects which will make it worse.

A natural way to include the mesonic degrees of freedom is to consider the basic quark-gluon interaction Lagrangian which includes $q\bar{q}$ pair creation terms. This interaction leads to off-shell matrix elements coupling the 3q configurations of baryons to the $3q(q\bar{q})$ ones. These extra $q\bar{q}$ pairs may be absorbed on a different baryon to contribute to the one meson exchange potential or may change the physical properties of the baryon. This was the starting point for our motivation. In previous quark model studies, either the mesonic degrees of freedom are entirely neglected or the quarks and the mesonic fields are treated as separate entities, although diagrams with a strong resemblance to mesonic degrees of freedom have always been lying in the quark-gluon interaction Lagrangian (Fig. 1. 1). This is the term which gives creation to a quark-antiquark pair. So the simple quark model, extended to include these creation and annihilation pairs, is going to be much more satisfactory since the baryons and their surrounding mesonic cloud can be described

in terms of their constituents. There are no additional fields of mesons as mesons are now properly regarded as $q\bar{q}$ pairs and integrated into the framework of the quark model.

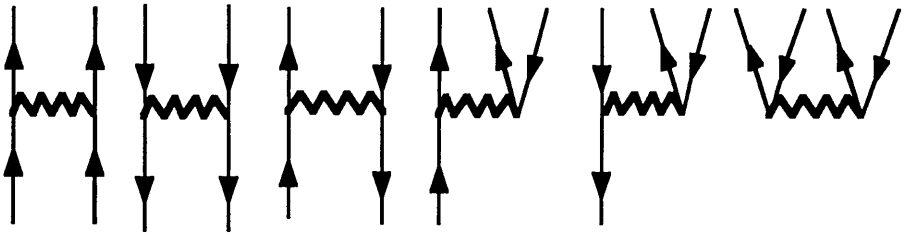


Fig. 1.1 quark- (antiquark-) gluon interaction

The annihilation diagram which contributes to the $q\bar{q}$ interaction is missing from Fig. 1. 1 mainly for reasons of simplicity. From a field theoretical study, in particular from the constructs of vertex functions and propagators in the non-relativistic limit, a transition potential [18] was derived to describe the quark-antiquark pair creation depicted by diagrams four and five of Fig. 1. 1.

It was Hecht and Fujiwara [19] who first studied the nucleon using this improved quark model as a first step towards the nucleon-nucleon interaction. They omitted the tensor and spin-orbit parts of the gluon exchange in their calculations. The contribution of the former was reported [20] to be negligible in calculations of the ground-state masses. The only way in which this force operates in the ground-state baryons is through the small admixture of higher oscillator functions in the ground-state wavefunctions. However, in calculations of masses of excited baryons the tensor part turns out to be of some importance [21], but we are not interested in this category of baryons. The latter is also ignored as it gives the wrong ordering of the energy levels of negative parity N^* and Δ^* on one hand and, on the other hand, it has been shown phenomenologically [22] that this **L.S** force is almost cancelled by the effective **L.S** force due to the confinement condition of the MIT model. They used the resonating group method for their calculations.

Hecht and Fujiwara included $q\bar{q}$ excitations in the model space to explain the medium-range attraction. The heights of the repulsive core given by the simple 3q-3q model were greatly reduced, but retained their strong linear energy dependence, with numerical values very similar to those given by the Paris potential. The effective potentials had acquired an attractive part of range 0.8 - 1.5 fm. However this attraction was too weak to fit the extreme low-energy S-wave phase shifts. One can expand on the idea of Hecht and Fujiwara and include $q\bar{q}$, $qq\bar{q}\bar{q}$, $q\bar{q}g$ ... in the model space to explain the medium-range attraction. However, beside the complex difficulties in the calculations that would result from these extra excitations, it is not clear that this expansion converges.

We have just mentioned the problem that occurs in the calculations if higher excitations are to be considered. In this type of calculation, the resonating-group or the generator-coordinate methods are usually used. There are other tools, perhaps less popular, which can describe and handle this kind of problem pretty well. One of them is the shell model which is known for the complex techniques and difficult concepts that it can handle .

We start off with a historical review of the shell model [23]. It emerged as a consequence of the inability of its competitors to provide one of the principal mechanisms for the understanding of the quantum many-body effects of the low-lying states of nuclei. A program of shell model research involves selecting the active single-particle orbits defining the model basis space, and then determining the effective two-body interaction between the particles considered in the model. With these two physics elements of the problem, the next step is to carry out the computational jobs of constructing the resultant Hamiltonian matrix for the chosen energy operator in the chosen space, of diagonalizing this matrix to obtain eigenvalues and eigenstates and, finally, of calculating expectation values for different observables such as J^2, T^2, the centre-of-mass excitation and other observables for characterizing a state. Other observable phenomena such as electro-magnetic transition rates may also be calculated and compared with experiment to assess one's confidence in the state.

The nuclear shell model has evolved enormously in the last forty years. In a modern

implementation [24], large basis calculations are progressing, better methods and models have been devised to predict which of the basis states are most important so that the reach of the active space can be extended without proportional expansion of dimensionalities. Also, improved mathematical and computational techniques have been developed for projecting angular momentum and setting-up and diagonalizing large matrices. These developments are proceeding in parallel with the use of computers that are faster and have large memories. The contribution of the Glasgow group to the shell model was a break-through. Whitehead [25] developed the well-known Glasgow code and the use of Lanczos method [26] for the eigenvalue problem.

The new phase of the Glasgow code is to handle quarks instead of protons and neutrons for the study of the quark effects in the conventional nuclear physics. The method was first used by Storm [27] to study QCD-dependent contributions to the NN interaction. Despite the criticism [28] that the nuclear shell model may break down by assuming free-moving quarks, we believe that it is a powerful tool in the absence of comprehensive lattice QCD calculations. The centre-of-mass motion is separated out exactly by using harmonic oscillator wavefunctions. The model was successfully used with the Breit-Fermi interaction to reproduce the spectra of baryons [29]. So there is no reason why it should not be used to investigate the quark-antiquark effects on the nucleon and more generally in other physical systems.

A first attempt at handling quark-antiquark effects was made by Malik [30] in parallel to the work by Hecht and Fujiwara discussed above. He studied the nucleon using a developed version of the shell model. He considered the nucleon as three valence quarks in the 0s shell and three filled shells $0\bar{s}_{1/2}$, $0\bar{p}_{1/2}$ and $0\bar{p}_{3/2}$ occupied by 48 ghost quarks below the lowest positive energy state with E=0 (the Fermi level) forming a sea of ghost quarks in analogy to the Dirac-Fermi vacuum of negative energy states all filled with fictitious electrons. The quark-antiquark excitations are then generated by exciting a ghost quark from the filled-shell sea into the real space. The hole created in the sea is treated as an antiquark as the ghost quarks are assigned negative intrinsic parity to distinguish them

from the valence quarks.

So for parity conservation reason, the model space must be expanded to include the real $0p_{1/2}$ and $0p_{3/2}$ shells, because the excitation of a quark from the $0\bar{s}$ sea shell can only take place to the 0p real shell and a similar excitation can happen from the $0\bar{p}$ sea shell to the real 0s shell. These excitations give rise to $(0s)^3(0p)(0\bar{s})$ and $(0s)^4(0\bar{p})$ configurations which introduce $3q(q\bar{q})$-component into the wavefunction of the nucleon consisting conventionally of the 3q-component only.

In Malik's model, to study the nucleon simple as it seems, 51 quarks (48 filling the sea shells + 3 valence quarks) must be handled in a 96 single-particle basis states. This is a difficult and cumbersome shell model calculation, adding to the problem of handling holes. That is without accounting for the multi-excitations in the sea shells which are likely to occur when a hole is there. The advantage of this approach is that the number of quarks, though large, is always constant.

TABLE I. Some properties of the three quarks(antiquarks) used in this work

Flavour	Symbol	Charge in units of e	z-projection of flavour	Strangeness	Mass(MeV.c^{-2})
Up (Upbar)	u ($\bar{u}$)	$\frac{2}{3}(-\frac{2}{3})$	$\frac{1}{2}(-\frac{1}{2})$	0	300-400
Down (Downbar)	d ($\bar{d}$)	$-\frac{1}{3}(\frac{1}{3})$	$-\frac{1}{2}(\frac{1}{2})$	0	300-400
Strange(Strangebar)	s ($\bar{s}$)	$-\frac{1}{3}(\frac{1}{3})$	0	-1(1)	≈500

A simpler way of integrating the discussed mesonic character into the wavefunction of the nucleon is to regard the antiquark as a new type of particle. It must be assigned an intrinsic parity to distinguish it from the quark. In this way, the number of particles is sharply cut from 51 particle to 3 and $5(3q+q\bar{q})$ particles only. The quarks and antiquarks come in three flavours u, d and s and are the fundamental (3 and $\bar{3}$) representations of flavour SU(3) [31]. Some of the properties of the light quarks and antiquarks are displayed in TABLE I.

Standard steps have to be followed for a shell model study. In our approach, the most sensitive one is the construction of basis states with a variable number of particles. For the case of the nucleon for example, three-quark as well as four-quark and an antiquark states must be constructed and are then allowed to mix by action of the transition potential. That is where the difficulty of the model lies.

The main feature of this work is that we have devised and implemented a shell model which

1) can handle a variable number of particles
2) treats antiquarks as well as quarks
3) handles particles of different mass
4) can be used with strange quarks in addition to up and down quarks, and
5) has been expressed in an ℓ-s coupling scheme.

These five points are treated in great detail in chapter II. As far as the author is aware all these features are new and many of them may be used in other applications. Chapter III deals with the computation of single and two-body matrix elements of the Hamiltonian and the operators discussed above. Those of the transition potential are also given. In chapter IV, the model is applied to the study of few light systems. A discussion of the parameters of the model and results are presented. Prompted by curiosity as to whether the nucleon contains "strange things" [32], we discovered that the model can be extended to investigate this too. The strangeness quantum number is then added. The parts of the model affected by this change are discussed. This has opened new prospects for the study of systems usually inaccessible to investigation by this kind of technique. These are the strange baryons and mesons. Revision of the parameters is vital. Relevant results are presented.

In chapter V, our work has been concluded with recommendations for future work. A preliminary presentation of the work was given at Strasbourg in March 1990 at the Annual Nuclear Physics Conference. A paper has been accepted for publication by the Nuclear Physics Journal, a copy of which is given in Appendix D.

Chapter II

Computational Techniques

II.1 INTRODUCTION

We attempt to construct a model that enables one to assess the importance of quark-antiquark effects on various nuclear properties. Therefore, the Hamiltonian must have two terms in addition to the kinetic energy operator: one which involves inter-particle interactions, and the other which describes ($q\bar{q}$) pair creation. So the main points to be discussed in this chapter are the standard shell model techniques stressing the innovations needed to cope with the new features of our approach such as the variable number of particles, and how we can handle the antiquarks as real particles by assigning to them an intrinsic-parity quantum-number. The language for this type of treatment is the second-quantization technique which is widely used in many-body theory and briefly reviewed in section 2. The nature of the problem requires separating the number-conserving part from the non-conserving one as we deal with particle excitations. So the two parts needed to calculate the Hamiltonian are given in section 3. Then the general sequences of a shell model program are tackled. We start by defining the working space and therefore defining the single particle basis in section 4. Choice of the representation and construction of the many-particle states is discussed in section 5 with emphasis on a colour constraint without which the calculations are not possible. Storage and manipulations of these many-particle basis states are considered in section 6. Construction of the energy matrix is developed in section 7. The general structure of this matrix is given first, then the many-body Hamiltonian and the use of symmetries are described. The Lanczos method for the solution of the eigenvalue problem is summarised. In sections 8 and 9, the forms of centre-of-mass and kinetic energy operators are given. They are inter-linked in a many-body problem since the centre of mass motion tends to smear the physics we get from

these models. The density matrix is reviewed in section 10. Finally, forms of operators representing physical observables vital for a better interpretation of states are given in section 11.

II.2 SECOND-QUANTIZATION TECHNIQUE

Basic concepts of this method can be found in any textbook on quantum mechanics [33]. We deal with the results here in a pragmatic manner without going into any detailed proof.

Let us introduce an operator a_α^+ which, operating on any state $|\Phi>$, produces an extra particle in the state α, in addition to what is already contained in $|\Phi>$. Thus, if $|\Phi>$ stands for a state with no particle (called the "vacuum state"), the $a_\alpha^+ |\Phi>$ represents the state $|\alpha>$ of a single particle. On the other hand, if $|\Phi>$ denotes a determinantal state of A particles, then $a_\alpha^+ |\Phi>$ is a state of (A+1) particles, the last state added to what is already contained in $|\Phi>$ being the state α. With these concepts, a_α^+ is the creation operator for the single particle state α. Similarly, the Hermitian conjugate operator a_α removes a single particle in the state α operating on any state $|\Phi>$. Thus, if $|\Phi>$ is an A-particle state, $a_\alpha |\Phi>$ is the state of (A-1) particles, which is obtained by eliminating the state α contained in $|\Phi>$. It therefore follows that, if the state α is not initially present in $|\Phi>$, then $a_\alpha |\Phi>$ must be zero. The operator is called the destruction or annihilation operator for the single-particle α state. If we keep the Pauli principle in mind, then $a_\alpha^+ |\Phi>$ must be zero if the state α is already present in $|\Phi>$. The two results

$$a_\alpha^+ |\Phi> = 0 \qquad \text{if } \alpha \text{ is an occupied state in } \Phi, \tag{II.1a}$$

$$a_\alpha |\Phi> = 0 \qquad \text{if } \alpha \text{ is not an occupied state in } \Phi \tag{II.1b}$$

are very useful in the algebraic manipulations of the creation and destructions operators. We now state the anticommutation properties of these operators. (AB+BA) which is called the anticommutator of any two operators A and B, is denoted by {A,B}. We then have

$$\{a_\alpha , a_\beta\} = \{a_\alpha^+ , a_\beta^+\} = 0 \qquad \text{(II.2a)}$$

$$\{a_\alpha , a_\beta^+\} = \delta_{\alpha\beta}. \qquad \text{(II.2b)}$$

(II.2a) states that whenever we interchange two destruction or two creation operators we get a minus sign. It also follows that $\{a_\alpha^+, a_\beta^+\} = 0$ guarantees the Pauli exclusion principle. We proceed to state the expressions of the operators in this new language.

Any one-body type operator is given by

$$F = \sum_{i=1}^{A} f_i = \sum_{\alpha,\beta} \langle \alpha | f | \beta \rangle \, a_\alpha^+ a_\beta \qquad \text{(II.3a)}$$

and a two-body type operator by

$$G = \sum_{i<j=1}^{A} g_{ij} = \frac{1}{4} \sum_{\alpha,\beta,\gamma,\delta} \langle \alpha\beta | g | \gamma\delta \rangle \, a_\alpha^+ a_\beta^+ a_\delta a_\gamma \qquad \text{(II.3b)}$$

It should be noted that the restriction $i < j$ in (II.3b) takes care of the fact that each pair (ij) has to be counted only once and the matrix element of g is the antisymmetric matrix element. The second point to note is that the labels γ and δ in $|\gamma\delta>$ appearing in the matrix element occur in the reverse order in the corresponding operator $a_\delta a_\gamma$.

II.3 HAMILTONIAN

We have chosen to use the shell model techniques as a working framework. In such a conventional treatment, this section would normally appear after defining the single particle states, then the many-particle states. However, we opt to consider the Hamiltonian first to inform the reader about the structure of the many-particle basis states as the number of particles is variable.

We want to write the Hamiltonian of a system of particles which could consist of quarks only or could[1] be heterogeneous to include antiquarks as well. It must contain the

single-particle kinetic energy operator, and the two-particle interaction potential, denoted by T and V respectively. That is where the application of (II.3) comes about, as the Hamiltonian of the system is the sum of (II.3a) and (II.3b) if we replace f by T and g by V. Is that the whole story?

The answer is no, since our main aim is to see how the $(q\bar{q})$ pair creation effects alter our old view of the nucleon as a three-quark system. The wavefunction [19] is now written as

$$\Psi_N = C_0\phi_0(3q) + \sum_\alpha C_\alpha \phi_\alpha ((3q)(q\bar{q})) + \sum_\beta C_\beta \phi_\beta ((3q)(q^2\bar{q}^2)) \qquad \text{(II.4)}$$

The coefficients C_α and C_β measure the importance of $q\bar{q}$ and $q^2\bar{q}^2$ components in the wavefunction. We want to calculate the magnitudes of these terms which we can do if we include all the interaction terms for a correct description of the Hamiltonian and hence a proper study of the system.

It is not worthwhile at this stage giving the interaction potential, but what is of importance to us is to devise a framework to handle antiquarks in a correct manner. The creation and destruction operator of an antiquark in the state α are denoted by b_α^+ and b_α respectively. So in the $3q(q\bar{q})$- or $3q(q^2\bar{q}^2)$-components of the nucleon wavefunction, we ought to add to the simplified form of the Hamiltonian the following terms:

$$\frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta}\langle\alpha\beta|V(1,2)|\gamma\delta\rangle\, a_\alpha^+ b_\beta^+ a_\delta b_\gamma \qquad \text{(II.5a)}$$

and also

$$\frac{1}{4}\sum_{\alpha,\beta,\gamma,\delta}\langle\alpha\beta|V(1,2)|\gamma\delta\rangle\, b_\alpha^+ b_\beta^+ b_\delta b_\gamma \qquad \text{(II.5b)}$$

The interesting feature of (II.5) is that the potential V(1,2) describing the quark-antiquark

[1] for the study of the meson for instance, the antiquark must be there, but not necessarily for the nucleon unless it has a mesonic cloud

or antiquark-antiquark interaction has mainly the same form as the quark-quark one which is going to be discussed later. The reason for this is that the minus sign in $H_{q\bar{q}} = -H_{qq}$ is balanced by another change in sign in the SU(3) reduced matrix element which follows from SU(3) conjugation under the $q \rightarrow \bar{q}$ transformation [19].

Another significant innovation in our work is the capability of handling a variable number of particles. In other words, we are faced with the situation where the operator for particle number does not commute with the Hamiltonian. This is going to be introduced in the model by considering, in the case of the nucleon for instance, the action that links 3q- to $3q(q\bar{q})$-components and $3q(q\bar{q})$- to $3q(q^2\bar{q}^2)$-components and so on. The terms described by diagrams 4 and 5 of Fig.1.1 are responsible for these two mixings. If the first mixing is to be considered, the contribution [34] which must be added to the Hamiltonian is

$$V_{q \rightarrow qq\bar{q}} = \sum_{\alpha,\beta,\delta,\gamma} <\beta\ \delta\ \gamma|\ V\ |\alpha>\ a^+_\gamma\ a^+_\delta\ b^+_\beta\ a_\alpha \qquad \text{(II.6a)}$$

where V is a transition potential describing the $(q\bar{q})$ pair creation. Whereas if we want to go further to consider the second mixing, an expression similar to (II.5a) ought to be added too as

$$V_{\bar{q} \rightarrow \bar{q}q\bar{q}} = \sum_{\alpha,\beta,\delta,\gamma} <\beta\ \delta\ \gamma|\ V\ |\alpha>\ b^+_\gamma\ b^+_\delta\ a^+_\beta\ b_\alpha \qquad \text{(II.6b)}$$

as the antiquark in the $3q(q\bar{q})$ configuration undergoes an excitation to create another $(q\bar{q})$ pair and by this process the $3q(q^2\bar{q}^2)$ configuration is obtained. Discussion and evaluation of matrix elements occuring in (II.6) will be given in the next chapter.

So from equation (II.6) it has become obvious, that we could construct basis states with a definite number of particles N fulfilling given conditions such as spin, flavour and parity quantum numbers and the action of the operators in (II.6), increments the number of

particles to N+2 to give a new state with conservation of quantum numbers because of the nature of the transition potential. In fact it is more convenient to proceed by handling the problem in another way as will become clear later.

II.4 SINGLE PARTICLE BASIS

Because of the form of the interaction involved, a single particle basis in which orbital and spin angular momenta are not coupled is preferred. A single particle state is then defined by a string of quantum numbers: n the principal quantum number, ℓ the orbital one, and m_ℓ, m_s and m_f are the z-projections of the orbital angular momentum, intrinsic spin and flavour respectively. Also as antiquarks are explicitly used, an intrinsic parity quantum number is assigned to the antiquark to distinguish it from the quark: 0 corresponds to a quark and 1 to an antiquark. The main feature of quark dynamics is confinement associated with the colour degree of freedom which means that a colour quantum number also must be added to the list of quantum numbers. The colour quantum number has three values: let them be red, green and blue (r, g, b) for a quark and ($\bar{r}$, $\bar{g}$, $\bar{b}$) for an antiquark. So a single particle state could be one of the following two forms[2]:

a quark state	$\lvert n\ \ell\ m_\ell\ m_s\ m_f\ c\ 0\rangle$	(II.7)
an antiquark state	$\lvert n\ \ell\ m_\ell\ m_s\ \bar{m}_f\ \bar{c}\ 1\rangle$	(II.8)

II.5 MANY-PARTICLE BASIS

II.5.1 CHOICE OF THE BASIS

We are trying to set up a model where the wavefunction of the nucleon has components with different numbers of particles and is presented by (II.4). In fact, it turns out that one can generate not only extra configurations of related interest in the nucleon, but others for other physical systems can be introduced by the same technique. More generally, a basis of the form nq(mq-m$\bar{q}$) where n=0, 3, 6... and m=0,1,2... is the right one to choose but

[2] quarks and antiquarks are fermions, have spin and flavour 1/2 which are omitted from the single particle wavefunction

we start off by fixing the value of n. For example, to study the pion n and m must be 0 and 1 respectively unless we want to investigate whether the pion has a mesonic cloud, in this case n is 0 but m is 1 and 2. The basis states would look like $q\bar{q}$ and $q^2\bar{q}^2$. Similarly, exotic states can also be studied by setting n and m to 0 and 2 respectively.

The problem of the method is that the number operator does not commute with the Hamiltonian. So if we want to investigate the quark-antiquark ($q\bar{q}$) pair creation effects in any physical system, we need to generate all its configurations which could be linked by the transition potential. To represent these configurations which are the basis states, the uncoupled antisymmetric product wavefunctions known as Slater determinants (or SD, for short) are chosen [25]. A typical determinant of a system of A quarks and B antiquarks in the occupation number representation is

$$b^+_{\alpha_{A+B}} b^+_{\alpha_{A+B-1}} \ldots b^+_{\alpha_{A+1}} a^+_{\alpha_A} a^+_{\alpha_{A-1}} \ldots a^+_{\alpha_1} |0\rangle \qquad \text{(II.9)}$$

where b^+ and a^+ are the creation operators of an antiquark and a quark respectively and α_i stands for a complete set of spin, flavour and colour as well as space coordinates as defined earlier.The corresponding wavefunction for the Slater determinant is

$$\frac{1}{\sqrt{A!}} \sum_{p \in S_A} \delta_p P \frac{1}{\sqrt{B!}} \sum_{q \in S_B} \delta_q Q \Psi_{AB}(\alpha_1 \alpha_2 \ldots \alpha_{A+B}) \qquad \text{(II.10)}$$

where the permutation operators P and Q act in separate spaces of the two families of fermions.

II.5.2 COMPUTER REPRESENTATION OF SDs

An SD is specified by n labels, representing the n occupied orbits. In a computer, a bit-mapped representation of SDs may be used. A single bit of the computer word is used to represent one single-particle orbital, the values 0 or 1 that the bit can have indicate whether

the orbit is empty or filled. However, such representation is inadequate in our case because of the large number of orbits: 96 orbits ($1\hbar\omega$ space) do not fit in a computer word of 32 bits. An alternative representation is to use a code for each colour: let them be Red, Green and Blue as each basis state has three colour components. Each code contains all the information about all the particles of that colour and is stored in a word. As the number of SDs may be enormous and we are only interested in colourless (physical) states, a colour hierarchy can be imposed [30]: the Red code must not be greater than the Green one which must not be greater than the Blue one ($R \leq G \leq B$). This reduces the number of basis states. As will be seen later, this colour condition cleverly used does not remove the colourless states which are exactly the same as in the full space, but removes the vast majority of coloured ones.

II.5.3 JUSTIFICATION OF THE COLOUR CONDITION

The justification for this colour constraint lies in an illustration by Young tableaux [35]. A standard arrangement of a tableau is defined to be one in which positive integers are placed in a row, and increase in going from top to bottom in a column. Each number stands for a possible state in which a single particle can be. If n states are available to a single particle, these states are ordered in some manner, each being assigned a number between 1 and n. Then the number j in each box satisfies $1 \leq j \leq n$.

As an example, we consider two identical particles, each in a different state, with only two-single states available. The standard arrangements of the Young diagrams corresponding to this case are

$$\boxed{1}\boxed{2} \qquad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array} \tag{II.11}$$

There is only one symmetric and one antisymmetric combinations of two particles. The nonstandard arrangements correspond to the same states as the standard ones after properly symmetrising, and so must not be counted. For example, the nonstandard arrangement $\boxed{2}\boxed{1}$ is the same as the standard arrangement $\boxed{1}\boxed{2}$ after symmetrising with

respect to the two particles. If the two single states are denoted by u_1 and u_2, then the previously stated symmetric and antisymmetric normalised combinations corresponding to (II.11) are respectively

$$\psi_s = (u_1u_2 + u_2u_1)/2^{1/2}, \tag{II.12a}$$

and

$$\psi_{as} = (u_1u_2 - u_2u_1)/2^{1/2}. \tag{II.12b}$$

Then, a judicious choice of the Clebsh-Gordon coefficients of the SU(2) group of this example will decide the overall symmetry of the two-identical particle state when only two-single states are available. In other words, if we want to construct the symmetric state, we must choose the two coefficients of u_1u_2 and u_2u_1 to be the same and, of course take care of the fact that these states must be normalised.

We extend this simple two-particle state example with the availability of two-single states to a three-particle state with three-single states corresponding to the three colours. Only the antisymmetric state in colour is physical and is given by the following determinant

$$\frac{1}{\sqrt{3!}} \begin{vmatrix} r_1 & g_1 & b_1 \\ r_2 & g_2 & b_2 \\ r_3 & g_3 & b_3 \end{vmatrix} \tag{II.13}$$

where r, g, and b are the three particle states. We then write (II.13) as

$$(r\bar{r} + g\bar{g} + b\bar{b})/3^{1/2} \tag{II.14}$$

after having used the following notations

$$\bar{r} = \begin{vmatrix} g_2 & b_2 \\ g_3 & b_3 \end{vmatrix} /2^{1/2} \quad \bar{g} = - \begin{vmatrix} r_2 & b_2 \\ r_3 & b_3 \end{vmatrix} /2^{1/2} \quad \bar{b} = \begin{vmatrix} r_2 & g_2 \\ r_3 & g_3 \end{vmatrix} /2^{1/2} \tag{II.15}$$

We note from (II.14) that to construct the antisymmetric state in colour of a system of particles, if it exists, we only require the sign of the coefficients preceding the many-particle basis states corresponding to different arrangements of the same diagram to be the same. However, at this stage we should only deal with the colour condition. The nonstandard arrangements which we shall refer to as ghost states do not appear in the list of SDs and therefore are not counted, but that does not mean that they do not contribute to the expansion of the eigenstates. For that, we shall record the multiplicity of each generated state, obtained by all the possible permutations of the three colours. We note that this multiplicity is 6 or 3 if three or two codes are different, and 1 if they are all the same. Moreover, the ghost states, when used, will have the same coefficients as their corresponding real ones to ensure the antisymmetry in colour of the eigenstate as mentioned in the beginning of the current paragraph. The manipulation of the ghost states, despite their absence from the list of SDs, will be clarified when treating the many-body aspect of the approach later.

So far, we have only discussed the generation of basis states assuming that the particles involved are identical. However, we must point out that when it comes to generating states with the extra quark-antiquark pair, we must force the two extra particles to have a colour and its conjugate, for example r and $\bar{r}$. The reason for this is we only have $r \rightarrow r r \bar{r}$, $r \rightarrow r g \bar{g}$ or $r \rightarrow r b \bar{b}$ [3] to ensure the antisymmetry in colour of the state as it is ensured by (II.14).

II.5.4 GENERATION OF SDs

In the beginning of each calculation, we need to prescribe the quantum numbers of the complete set of SDs. They are: the z-projections of the total angular momentum, the flavour, and the overall parity of the state which is a very important entity as the particles involved are of different parities. This usually gives us an idea about what space we should use, although this was the subject of the previous section. We must also specify the variable numbers of quarks and antiquarks, and the restrictions on the occupancies of

[3] what is true for the colour r is also true for the two others g and b

the subshells of the considered space.

A program to handle the variable number of particles has to be set to generate the three codes constituting a SD. We proceed by generating all the codes for the one colour and then combining any three codes to yield the SD with the prescribed conditions on the quantum numbers and also $R \leq G \leq B$. For this purpose, a relation enabling one to find the particle distribution between the three colours has first to be worked out, bearing in mind that the quark and antiquark of the pair must be of the same colour, i.e. must be packed in the same code for the appropriate colour.

II.5.4.1 FIXING THE NUMBER OF RED QUARKS AND ANTIQUARKS

The process of finding the number of quarks and antiquarks in each colour for code generation is by far the most tricky part. If the minimum and maximum numbers of quarks and antiquarks of a given calculation are denoted by minq and maxq, and $\min\bar{q}$ and $\max\bar{q}$ respectively, then we need to generate all the codes with the appropriate number of red particles giving rise to different configurations. If minrq and maxrq are given by

$$(\text{minq} - \min\bar{q})/3 \qquad \text{and} \qquad (\text{maxq} + \max\bar{q}) - 2*|\text{minrq}| \tag{II.16}$$

then, for each number nrq varying between the absolute value of minrq and maxrq in steps of two, the corresponding number of red antiquarks, if there are any, is given by

$$nr\bar{q} = (\text{nrq} - \text{minrq})/2. \tag{II.17}$$

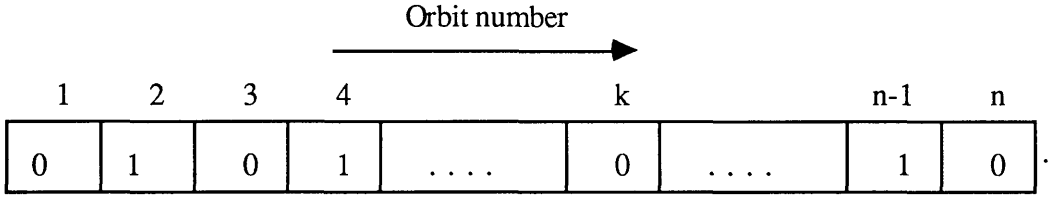
Then the current code has a quark-antiquark pair if $nr\bar{q}$ is not zero in which case it must be doubled and added to the minimum number of quarks to find the number of red particles

$$\text{nr} = \text{minrq} + 2*nr\bar{q}. \tag{II.18}$$

In this way, the number of particles is fixed to generate a set of codes.

II.5.4.2 CODE GENERATOR

For each number of red particles nr fixed by the previous scheme and the corresponding number of red antiquarks $nr\bar{q}$, we proceed to the generation of all the codes. We begin by filling the leftmost nr particle bits with 1's where each bit corresponds to a single-particle state. We illustrate this by an example. If for a given colour we want to represent three occupied orbits out of n available ones and let them be 2, 4 and n-1, then the computer representation of this code is pictured by



New codes are generated by moving the rightmost bit one place to the right. Each new code is then tested to ensure that it does not violate any of the conditions. The first one is if we are having too many antiquarks by checking the overall intrinsic parity of all the particles and the fixed number $nr\bar{q}$ for the current code. The second is if we are violating the shell occupancies set in the beginning of the calculation by putting more particles in a given subshell than is required. If the code does not fail these tests, then it is stored with all the related information such as its z-components of angular momentum and flavour, its spatial and intrinsic parities and so on. It is important to note at this stage that this operation generates the codes in an ascending order because the single particle states are ordered according to a scheme which is chosen to be the alphabetical order of the quantum numbers. That simplifies the procedure of combining the three codes with the colour condition $R \leq G \leq B$.

II.5.4.3 COMBINATION OF CODES

The list of codes already generated for one colour is the same for the two other colours:

we have then the three lists of codes for the three colours. We take each code from each list with the condition that the red code is not greater than the green which must not be greater than the blue one, then test whether these three codes combine to form a SD. That is, we check if the number of particles in the three codes does not exceed what we require. For example, if we are interested in the 3q and $3q(q\bar{q})$ configurations of the nucleon, then when we generate the list of codes for one colour, there will be codes with one quark and others with two quarks and an antiquark according to the previous scheme. Thus when the three codes are combined and around the end of each list of codes, we will face the situation where SDs with two quarks and an antiquark in each colour are being considered: a $3q(q\bar{q})^3$ configuration is formed but must be ruled out by the condition on the number of particles. Moreover, the additions of the z-components of angular momentum and flavour, and of the overall parities of the three codes must match the requirements of the calculation. Checks on the different occupancies of shells are also done. If the SD just formed fulfils all these conditions, its three codes and its multiplicity are then stored. As is clear from this process, the SDs are ordered in the three codes, but not in the number of particles. If we take the example of the nucleon just given, its list of SDs will consist of ones corresponding to the 3q configuration, then ones corresponding to the $3q(q\bar{q})$ configuration, and again others to the 3q one and so on. This is going to present a problem when setting up the Hamiltonian matrix as will be seen later.

II.6 MANIPULATION OF SDs

In this section we discuss a scheme to handle satisfactorily the action of the Hamiltonian on the SDs. There are two aspects which must be handled separately: one which deals with the part of the Hamiltonian that leaves the number of particles unchanged and the other which does not conserve it.

II.6.1 NUMBER-CONSERVING PART

This part involves the two-body operators occurring in (II.3) and (II.5) which act on a SD. This operation corresponds to the destruction of all the pairs of particles (δ,γ)

contained in the SDs taken one at a time, and the creation of all possible pairs (α,β) which are inserted into the original SD minus (δ,γ) without violating the Pauli principle. In this way new SDs are formed and a search operation must be performed to see whether they appear in the list. The created pairs (α,β) must have the same spin, flavour and overall parity with separate conservation of intrinsic and spatial parities as the destroyed pair (δ,γ). There is no need to explain the method here as its general framework is the same as the one for non-conserved number of particles which will be examined in detail in the next section. Whenever there are details peculiar to the number-conserving method, we shall point them out at the time.

II.6.2 NUMBER NON-CONSERVING PART

In this section we explain the method that deals with the part of the Hamiltonian given by (II.3) which creates three particles from one destroyed. When the operators (II.6) act on one of the basis SDs with given occupied orbits, we use the term in (II.6) that destroys particle i. From the index i we compute an address in a directory which contains the location of a block of storage containing all the matrix elements of the form $<jk\ell\,|\,V\,|\,i>$ which correspond to the destruction of the orbit i. Immediately above these matrix elements are stored two pointers, p_1 and p_2, which locate the beginning and end of a corresponding list of all the orbit triples that have the same z-components of total angular momentum and flavour as the particle destroyed i. Unlike the situation with the conserving number of particles, the intrinsic and spatial parities are separately non-conserved, which means that the intrinsic and spatial parities of the created triples must differ by one from that of the destroyed particle i. That allows the overall parity of the triples to be the same as that of the destroyed particle i.

The delicate part of this triple creation is to find the conservation of colour. We mentioned earlier when we dealt with the construction of SDs that the quark and the antiquark forming the pair must have a colour and its conjugate to ensure the overall antisymmetry in colour of the eigenstate. So in terms of the destroyed and created

particles, we have the following situation

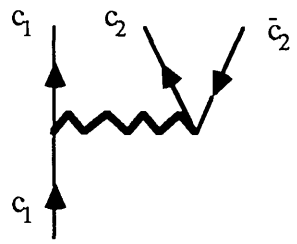


Fig. 2. 1 Colours of the quark and antiquark of the pair are conjugate

From the colour hierarchy ($R \leq G \leq B$), it follows that any permutation that takes $c_1c_2\bar{c}_2$ to $c_2\bar{c}_2c_1$ is even. We note this because it is by no means straightforward to see the order that the colours of the triples must be in to establish a logical correspondence. In other words, we need only to calculate the logical colours $0 \rightarrow 000$ when the quark-antiquark pair has the same colour as the destroyed particle and $0 \rightarrow 011$ otherwise. As an aside we mention that the logical colours that will be calculated in the case of the number-conserving case are in addition to $00 \rightarrow 00$, $01 \rightarrow 01$, $01 \rightarrow 10$, there is also $00 \rightarrow 11$ when a quark and an antiquark are destroyed, as the colour operator $\underset{\sim}{\lambda}_1 . \underset{\sim}{\lambda}_2$ can for example link $r\bar{r}$ and $g\bar{g}$ states. This will be treated in more detail when calculating the colour matrix elements in the next chapter.

We go back to the list of triples which have the same overall quantum numbers as the destroyed particle i. We run through this list until we find a triple which can be inserted into the original SD minus the particle i without violating the Pauli principle. It is not always possible to find such a triple. This is unlike the number-conserving case where there will always be at least one such pair since the pair itself will appear in the list. This method enables one to systematically work through all the occupied orbits one at a time (or pairs in the other case) in the initial SD and all initial SDs with almost no searching [25]. By virtue of this manipulation, an SD consisting of N particles is transformed into a new one with N+2 particles. So it remains to identify the new SD with one of those stored in the main list. A simple comparison with each SD in the list in turn will be far too-time

consuming. The most convenient searching technique for this purpose is the binary search, which makes use of the ascending ordering of the SDs already mentioned.

We compare the new SD with the middle one of the list and the comparison tells us in which half of the list it is to be found. This is continued, the area being searched diminishing by half at each step, until the SD is located. This method is used for both the number-conserving and non-conserving case. Only one detail has to be mentioned here concerning the new SD. Its three codes may no longer be ordered according to the colour condition $R \leq G \leq B$, so they must first be ordered before searching the list. This is true in both discussed cases.

Thus, the multiplication of the ith SD by different terms of H is equivalent to storing the contributions to SDs j, k, ... in the final state and their appropriate matrix elements which are either the two- or the one-three body ones connecting the SDs i and j, i and k and so on. We shall use this information in subsequent steps instead of going through the full process of manipulating the SDs again. However, in this process, we encountered a difficulty concerning the existence of ghost states, which we shall discuss later when we study the matrix elements of the Hamiltonian between two basis states.

II.6.3 OTHER ASPECTS AND ONE-BODY PART OF H

We should not leave this section of the program without mentioning that, at this stage, in addition to the evaluation of the one-body parts of different operators such as T^2 and C^2, the value of the mass number A is calculated. This is because A depends on the number of particles of the SD an operator is acting on. This is by no means feasible elsewhere and is of great importance for later stages because of the A dependence of all the operators. We shall see that the two-body matrix elements of these operators has to be evaluated without the A dependence, as A is a variable whose value is not known at that stage. This A dependence must be included later. Although the value of A is easy to find when a particular SD is taken out of the list, it is still a non-trivial operator.

II.7 SETTING-UP OF THE ENERGY MATRIX

II.7.1 PARTITION AND GENERAL FORM OF THE MATRIX

By construction, the basis states are ordered according to the constraint $R \leq G \leq B$ and not by the number of particles. The energy matrix is complex as the transition potential is a complex interaction, but because of the existence of its time reversal part, the entire Hamiltonian is Hermitian and therefore the eigenvalues are real. By exchanging rows and columns of the matrix we can end up with a matrix partitioned into block matrices of the form

$$H = \begin{bmatrix} A & iB \\ -iB' & C \end{bmatrix} \qquad \text{(II.19)}$$

where A, B, and C are real matrices and A and C are symmetric. B' is the transpose matrix of B. The basis states for A are $(nq)(q\bar{q})^{2n}$ and those for C are $nq(q\bar{q})^{2n+1}$. This operation is equivalent to reordering the basis states according to the number of particles. Although H is a complex matrix[4], its eigenvalues are real because it is Hermitian [36]. Even better, it can be decomposed into a product of three matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} A & B \\ B' & C \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \text{(II.20)}$$

The eigenvalues of H are found by finding the roots of the characteristic polynomial obtained by expanding

$$\det|H - \lambda\mathbf{1}| = 0 \qquad \text{(II.21)}$$

which is the same as

[4] complex matrices usually have a more complicated eigenvalue problem

$$\det|S - \lambda \mathbf{1}| = 0 \qquad \text{(II.22)}$$

where

$$S = \begin{bmatrix} A & B \\ B' & C \end{bmatrix} \qquad \text{(II.23)}$$

after having used the following property of the determinants of matrices

$$\det|A * B * C| = \det|A| * \det|B| * \det|C| \qquad \text{(II.24)}$$

and that the determinants of the leftmost and rightmost matrices in (II.24) are one. So we shall use the S matrix to find the eigenvalues of H.

The eigenstates of H are obtained from those of S quite simply. If (a,b) is an eigenstate of S corresponding to the eigenvalue E, then (a,-ib) is the eigenstate of H corresponding to the same eigenvalue E:

$$H(a,-ib) = E(a,-ib) \qquad \text{(II.25)}$$

We shall refer to the matrix S by H for the rest of this study.

II.7.2 MATRIX ELEMENTS OF H AND USE OF SYMMETRIES

We have seen earlier that not all the basis states are in the list and we did refer to absent ones as ghosts. As will become clear later, we shall have to use a new basis for the eigenvalue problem. If the SDs are denoted by $|1\rangle, |2\rangle, \ldots, |i\rangle, |j\rangle, \ldots |N\rangle$, then a new basis state can be written as

$$|\psi: \text{expanded}\rangle = \sum a_i (O_{exp}) |i\rangle \qquad \text{(II.26)}$$

where the operator (O_{exp}) takes care of all the permutations of the three colours of each SD

$|i>$. So the action of H on $|\psi$: expanded $>$ is given by

$$\sum_i a_i H (O_{exp}) |i> = \sum_{i\,\alpha} a_i H |\alpha\, i> \quad \text{(II.27a)}$$

where

$$(O_{exp}) |i> = \sum_{\alpha} |\alpha\, i> \quad \text{(II.27b)}$$

We insert the closure relation with index specification of the permuted SD of $|j>$in the colour space. (II.27a) then becomes

$$\sum_{\substack{i\,\alpha \\ j\,\beta}} a_i |\beta\, j><\beta\, j| H |\alpha\, i> = \sum_i a_i \sum_{\alpha,\,\beta\, j} H_{\beta\, j,\alpha\, i} |\beta\, j> \quad \text{(II.28)}$$

We make use of the multiplicity of $|i>$ denoted by M_i in (II.28)

$$\sum_i a_i \sum_j H_{j,i}\, M_i\, (O_{exp}) |j> \quad \text{(II.29)}$$

However, we are using the compressed form of ψ in which the ghost states do not appear in its expansion

$$|\psi\text{: compressed} > = \sum a_i |i> \quad \text{(II.30)}$$

Therefore the action of H on the compressed form of ψ yields

$$\sum_j [\sum_i [H_{j,i}\, M_i]\, a_i\,] |j> \quad \text{(II.31)}$$

which is obtained after rearranging (II.29). So the matrix element of H between two states ψ and ψ' in their compressed forms is

$$\sum_i \sum_j M_j \, b_j \, H_{j,i} \, a_i \, M_i \qquad \text{(II.32)}$$

where the b_j are the amplitudes of the state ψ' in the SDs basis. We have forced the ghost states of a given SD $|i\rangle$ to have the same amplitude as $|i\rangle$ in the expansions (II.26) and (II.30) to ensure the overall antisymmetry in the colour space of the eigenstates. By this process we have manipulated the ghost states without them being in the list and used for that the Hermicity of H.

Conventionally, we have a vector ψ expanded in terms of the SDs with amplitudes denoted by a(i). The Hamiltonian turns a given SD $|k\rangle$ into a new one $|\ell\rangle$ with a contribution of a(k)V to the ℓth amplitude of the new vector. However the situation is different here. Suppose that an SD $|i\rangle$ is of multiplicity 3, only [A, A, B] where A and B are the colour codes ([A, B, A] and [B, A, A] will be its two ghosts). So H is only going to act for example on [A, A, B] to yield [C, A, D], which is the ghost of [A, C, D] represented by the SD $|j\rangle$. Since H is Hermitian, there is an operator, part of H which turns [C, A, D] into [A, A, B], so we actually perform the contribution to the ith amplitude of the new vector which is

$$A(i) = A(i) + V * a(j) \qquad \text{(II.33)}$$

In this way, we find all the amplitudes A(i) of the new vector without accessing the ghost states and hence the contribution of [A, A, B] to the scalar product $\langle\psi|H|\psi\rangle$ is $A(i)*a(j)*M_i$.

By virtue of (II.32), the elements of the energy matrix linking either basis states of the same subspace or of different subspaces are found. The aesthetic side of the model is representing the matrix in a much smaller basis without loss of any physical property.

II.7.3 LANCZOS METHOD

The eigenvalues of matrices are calculated by their diagonalization. Usually, these matrices are large ones, and most procedures[5] around are for a complete diagonalization of the full matrix [37]. In most practical cases, however, we are only interested in the lowest eigenvalues as they usually are the ground-state energy and the energies of a few excited states. Therefore, since the dimensions of the configuration space may be large, it would be better not to have a full diagonalization. The Lanczos iterative method [25,26] generates a tri-diagonal form of the matrix and also allows us to obtain the lowest eigenvalues. The method is applicable for any sort of matrices, but we only consider real symmetric ones.

Let H be a NxN Hermitian matrix. One starts with an arbitrary normalised vector x_1 in a space of dimension N and then generates a set of orthonormal vectors x_i, for i = 2, 3, ... N by the following algorithm:

$$\alpha_i = x_i^\dagger H x_i \tag{II.34}$$

$$y_{i+1} = Hx_i - \alpha_i x_i - \beta_{i-1} x_{i-1} \tag{II.35}$$

$$\beta_i = (y_{i+1}^\dagger y_{i+1})^{1/2} \tag{II.36}$$

$$x_{i+1} = \beta_i^{-1} y_{i+1} \tag{II.37}$$

where

$$\beta_0 = 0 \tag{II.38}$$

Properly sequenced, these formulae define the Lanczos iteration and x_i are called the Lanczos vectors. The process terminates automatically since there can be no further vector orthogonal to $x_1, x_2, \ldots, x_N$. From (II.37) the Lanczos vector x_{i+1} is obtained by projecting out the component of Hx_i orthogonal to x_{i-1} and x_i and normalising to unity. The matrix elements of H in the x_i basis are then given by:

$$T_{ij} = x_i^\dagger H x_j = \beta_{i-1}\delta_{i\,j-1} + \alpha_i \delta_{i\,j} + \beta_i \delta_{i\,j+1} \tag{II.39}$$

[5] e.g the Householder method

As the Hermiticity of H ensures the reality of α_i, Eq. (II.39) indicates that T is a real symmetric tri-diagonal matrix in the x_i orthonormal basis. If we define

$$X = [x_1, x_2, \ldots, x_N] \tag{II.40}$$

a NxN matrix, then X is unitary and, the representation of H in the basis of Lanczos vectors is the tri-diagonal matrix T via the unitary transformation

$$X^{\dagger}HX = T \tag{II.41}$$

where

$$T = \begin{pmatrix} \alpha_1 & \beta_1 & & & & \\ \beta_1 & \alpha_2 & \beta_2 & & & \\ & \beta_2 & \alpha_3 & \beta_3 & & \\ & & \cdots & \cdots & \cdots & \\ & & & \cdots & \cdots & \cdots \\ & & & & \beta_{N-2} & \alpha_{N-1} & \beta_{N-1} \\ & & & & & \beta_{N-1} & \alpha_N \end{pmatrix} \tag{II.42}$$

The eigenvalues of T are those of H because of the unitary nature of X. Diagonalization of the tri-diagonal matrix T can be achieved rapidly by standard techniques [38]. The eigenvectors u of the original matrix, H, are related to those of T denoted by v by u=Xv.

A problem may arise here if the starting vector x_1 happens to be an eigenvector of H. The Lanczos process is therefore terminated at the first stage. So the process may be continued by choosing a new starting vector orthogonal to the one already taken. Despite a few numerical problems that could be encountered when using the Lanczos method, its important property is the fast rate of convergence to the lowest eigenvalues of the spectrum without completion of the process.

II.8 CENTRE OF MASS PROBLEM

The question of the centre of mass motion plagues all many-body wavefunctions based on individual states. This question was first discussed by Elliott and Skyrme [39] in the context of the harmonic oscillator shell model wavefunction.

The problem involved is as follows: the centre of mass of a system can be displaced through a distance a with the help of the operator $\exp(-i\hbar P.a)$, where P is the total momentum of the particles of the system. Under this displacement, the overall wavefunction should, at the most, get multiplied by a numerical factor $\exp(i\delta(a))$ where $\delta(a)$ is real. This implies that this wavefunction is an eigenfunction of the operator P, and the centre of mass part of the wavefunction is given by exp(iK.R), where R is the centre of mass coordinate vector and hK is the eigenvalue of P.

As it is not possible to ensure in a determinantal state that the centre of mass dependence of the wavefunction automatically becomes exp(iK.R), one aspect of dealing with the centre of mass is to ensure that the intrinsic energy is computed by eliminating the kinetic energy of the centre of mass from the total Hamiltonian. We consider this in the next section.

The single particle Hamiltonian for particles of mass m, moving in a harmonic oscillator potential can be written as the sum of a translationally invariant part for the relative motion and a purely CM part:

$$H_{HO} = \frac{1}{2m}\sum_{i=1}^{A} p_i^2 + \frac{m\omega^2}{2}\sum_{i=1}^{A} r_i^2$$

$$= H_{rel} + H_{CM} \qquad \text{(II.42)}$$

where

$$H_{rel} = \frac{1}{2m}\sum_{i=1}^{A}(p_i - \frac{P}{A})^2 + \frac{m\omega^2}{2}\sum_{i=1}^{A}(r_i - R)^2 \qquad \text{(II.43a)}$$

and

$$H_{CM} = \frac{P^2}{2mA} + \frac{Am\omega^2}{2} R^2. \qquad \text{(II.43b)}$$

The vectors R and P describing the coordinate and the momentum for the centre of mass are defined as

$$R = \frac{1}{A}\sum_{i=1}^{A} r_i \qquad P = \sum_{i=1}^{A} p_i \,. \qquad \text{(II.44)}$$

The eigenfunctions of the single particle Hamiltonian for an excitation energy of $N\hbar\omega$ can be written in the form of product wavefunctions:

$$\psi_N(1,2,\ldots A) = \Phi_\mu(R)\,\Phi_\nu(1,2,\ldots A\text{-}1). \qquad \text{(II.45)}$$

The $\Phi_\mu(R)$ and $\Phi_\nu(1,2,\ldots A\text{-}1)$ are the eigenfunctions of H_{CM} and H_{rel}, respectively, with oscillator energies $(\mu+3/2)\hbar\omega$ and $(\nu+(A-1)3/2)\hbar\omega$ and $\mu+\nu = N$

The shell-model states of a nuclear system, which are antisymmetrised products of one particle wavefunctions including spin, flavour and colour, should be a linear combination of the wavefunctions presented above. The states for which the CM quantum number μ is not equal to zero are referred to as spurious states. One way of proceeding to separate the non-spurious states from the spurious ones is to evaluate the expectation value of H_{CM}. It is expressed as a two-body operator since this is convenient in the shell model program and takes the following form:

$$\frac{1}{A(A-1)}\sum_{i<j}\left(\frac{p_i^2 + p_j^2}{2m} + \frac{m\omega^2}{2}(r_i^2 + r_j^2)\right) + \frac{1}{A}\sum_{i<j}\left(\frac{2p_i \cdot p_j}{2m} + \frac{\hbar\omega}{2}(2\rho_i \cdot \rho_j)\right) \qquad \text{(II.46)}$$

ρ is the dimensionless coordinate defined as: $\rho = r/b$, where b is the oscillator length

parameter of the particles.

Expression (II.46) is the sum of two one-body terms and two genuine two-body ones. Its great complexity lies in its dependence of the number A which is interpreted as the number operator and is variable as previously seen. In the previous versions of the shell model, A is constant and the A dependence is embodied in the two-body matrix element when being calculated. We evaluate them without the operator A but take care of it later by calculating two-body density matrices weighted by 1/A and 1/A(A-1) for the one- and the two-body density matrix respectively. They are used for the evaluation of the expectation value of the centre of mass operator. In the next chapter, we will calculate the two two-body matrix elements separately. The two one-body matrix elements are trivial.

II.9 KINETIC ENERGY OPERATOR

In the model, physical states (baryons and mesons) are constructed, upon which the transition potential acts to generate others differing from the first ones by a pair. In other words the number of particles involved is variable. As mentioned in the previous section, the kinetic energy of the centre of mass must be subtracted off the conventional kinetic energy operator. The total kinetic energy term is then expressed as a two-body operator

$$T = \sum_{i=1}^{A} \frac{p_i^2}{2m} - \frac{\left(\sum_{i=1}^{A} p_i\right)^2}{2mA} = \frac{1}{2mA} \sum_{i<j} (p_i - p_j)^2 \qquad \text{(II.47)}$$

In previous versions of the shell model with constant A, the two-body matrix elements of the kinetic energy have been added to the A-independent matrix elements of the interaction. However, as A is now variable, it is necessary to keep the two-body matrix elements of kinetic energy separate from those of the interaction. Because of the type of techniques developed to evaluate the matrix element of any central interaction, the two-body term of the kinetic energy is written as

$$\frac{(p_i - p_j)^2}{2mA} = \frac{1}{A}\left(\frac{p_i^2 + p_j^2}{2m} + \frac{m\omega^2}{2}(r_i^2 + r_j^2)\right) - \frac{m\omega^2}{2A}(r_i^2 + r_j^2) - 2\frac{p_i \cdot p_j}{2mA} \qquad \text{(II.48)}$$

We have chosen to give the kinetic energy operator the form in (II.48) to have less labour when calculating its matrix elements by having to evaluate only the central part $(r_i^2 + r_j^2)$. The evaluation of the momentum term $p_i.p_j/m$ is dealt with already as it also occurs in the centre of mass operator. By analogy with the centre of mass case, the matrix elements of the kinetic energy operator are evaluated without the A, with the difference that the A dependence must be taken care of when building the many-body energy matrix. Moreover, if we want the expectation value of T, there is only one weight of the density matrix, which is 1/A. Again we mention here the triviality of the one-body matrix elements and that the expressions of the two-body matrix elements are given separately in the next chapter.

II.10 DENSITY MATRIX

A very vital and convenient quantity in this kind of treatment is the statistical density matrix. The concept of a density matrix is broadbased and is applicable in any sort of space we could think of. We start by defining the density matrix [40] and move on to the applications relevant to our study.

An eigenstate $|\psi\rangle$ of the system under study is expressed in terms of the previously defined multiparticle-basis states denoted by $|I\rangle$, I = 1,2,... N as

$$|\psi\rangle = \sum_{I=1}^{N} A_I |I\rangle \qquad \text{(II.49)}$$

where A_I is the Ith amplitude of the eigenstate. So the expectation value of any two-body operator Ω is given by

$$\langle\psi|\Omega|\psi\rangle / \langle\psi|\psi\rangle. \qquad \text{(II.50a)}$$

If we use the normality of the states $|\psi>$, the definition of a two-body operator in the second quantization as in (II.3b) for Ω, the permutation of the two summations in (II.50a) and the fact that the amplitudes of the eigenstates are reals, the average value of Ω is

$$\sum_{i,j,k,\ell} \Omega_{ijk\ell} \sum_{I\,J} A_I A_J <I|\, a_i^+ a_j^+ a_k a_\ell \,|J> \qquad \text{(II.50b)}$$

where $\Omega_{ijk\ell}$ is its two-body matrix element. The element $<I|a_i^+ a_j^+ a_k a_\ell\,|J>$ is known as the two-body density-matrix element and is denoted by $\rho_{ijk\ell}$. Expression (II.50b) is in fact

$$\text{Tr}\ \rho\Omega \qquad \text{(II.50c)}$$

the sum of the diagonal elements of the matrix $\rho\Omega$ and denoted in (II.50c) by " Tr " referring to trace. The result (II.50) is very important in the respect that the density matrix can be computed and stored once and for all and then used in conjunction with any given operator Ω denoting a physical observable to calculate the corresponding expectation value. Some important properties of the density matrix can be used such as its Hermicity

$$<J|\,\rho\,|I> = <I|\,\rho\,|J>^* \qquad \text{(II.51)}$$

to cut down the number of density-matrix elements that require computation. However, these are not used in our case as we are interested only in the uncoupled representation of spin and isospin of the density-matrix elements.

We have chosen to define the general concepts of a density matrix for a two-body operator. The one-body one can be introduced in a similar manner as will be seen when finding the expectation values of operators that have one- and two-body parts.

II.11 CALCULATION OF J , T AND C FOR EIGENSTATES' TEST

Earlier on, we proposed to calculate the expectation value of the Centre-of-mass operator to distinguish spurious from non-spurious states. Moreover and for a better identification of the states, their J, T and colour are found by computing the expectation values of J^2, T^2, and C^2 the colour Casimir operator. This is going to be an immediate test on the convergence of the eigenvalues: a complete converged state will yield sharp values for these Casimir operators. This relationship between convergence and expectation values of these operators is a consequence of the commutation of these operators with the Hamiltonian. Once these operators are written in the second quantization, it will remain to evaluate their matrix elements and with the help of the density matrix, it is straightforward to obtain their expectation values. For these operators, the following identity is used

$$(\sum_{i=1}^{A} a_i)^2 = \frac{A}{2(A-1)} \sum_{i<j} (a_i + a_j)^2 - \frac{(A-2)}{2(A-1)} \sum_{i<j} (a_i - a_j)^2 \qquad \text{(II.52)}$$

Hence we require the two-body matrix elements of $(a_1+a_2)^2$ and $(a_1-a_2)^2$, and the two-body density matrices which are obtained by multiplying the standard two-body density matrix already discussed by the numbers A/2(A-1) and (A-2)/2(A-1) respectively.

Although the evaluation of matrix elements is the subject of the coming chapter, it is worthwhile noting that if the single-particle basis states are denoted by $|i>$, then the two-body matrix elements of the operators in (II.52) are given by

$$<i\,j|(a_1 \pm a_2)^2|k\ell> = <i|a_1^2|k>\delta_{j\ell} + <j|a_2^2|\ell>\delta_{ik} \pm 2<i\,j|\underline{a}_1.\underline{a}_2|k\,\ell> \qquad \text{(II.53)}$$

The operators of the two first matrix elements on the right-hand side in (II.53) contain only one particle index, therefore can, at the most, change the state of the particle they act upon. Therefore, these matrix elements are either diagonal as in the case of T^2 and C^2 or non-diagonal and non-trivial ones at all as in the case of J^2. Thus, using

$$<i\,|a^2|\,k> = \text{constant} * \delta_{ik} \qquad \text{(II.54)}$$

where the value of the constant is 3/4 which is t(t+1) for t equals 1/2 as the particles are of flavour 1/2 in the case of T^2, and 16/3 which is the expectation value of the one-body colour operator λ^2, simpler expressions for T^2 and C^2 are derived:

$$2\frac{3}{4}\sum_{i<j} 1 + 2\sum_{i<j} \underline{t}_i \cdot \underline{t}_j \qquad \text{(II.55)}$$

$$2\frac{16}{3}\sum_{i<j} 1 + 2\sum_{i<j} \underline{\lambda}_i \cdot \underline{\lambda}_j \qquad \text{(II.56)}$$

where we require the matrix elements of the unit two-body operator. Hence (II.55) and (II.56) suggest that only the knowledge of the genuine two-body density matrix is required unlike what has been stated earlier on, and the two-body matrix elements of $\underline{t}_1 \cdot \underline{t}_2$ and of $\underline{\lambda}_1 \cdot \underline{\lambda}_2$ to find the expectation values of T^2 and C^2.

The evaluation of the expectation value of J^2 is slightly more complicated as the one-body part of the operator is non-diagonal. We would rather write (II.52) for J^2 as

$$\frac{1}{(A\text{-}1)}\sum_{i<j} (J_i^2 + J_j^2) + \sum_{i<j} \underline{J}_i \cdot \underline{J}_j \qquad \text{(II.57)}$$

to minimise the number of density matrices we need to calculate and therefore store, since the expression of J^2 will make use of the density matrix of weight 1/(A-1) previously discussed for the centre of mass case. We do worry about the number of density matrices as we do not have separate pieces of program to calculate the expectation values of these observables. Thus, a problem of computer storage arises if we do not take care to be economical.

Chapter III

Computation of Matrix Elements

PART A

In this first part, we confine our discussion to the number-conserving part of the Hamiltonian. The operators occurring there are symmetric in all the particle variables and are single particle and two-body type ones. The only operator of the latter type that concerns us is the two-particle interaction potential, the form of which will be given after discussion of the symmetry of the wavefunction.

III.1 TWO-PARTICLE WAVEFUNCTIONS

In the previous chapter we constructed determinantal states which are antisymmetric under the exchange of any two particles because of the symmetry of the operators. Thus any two-particle state contained in a determinant is also antisymmetric. As we have a cocktail of particles made up of quarks and antiquarks, the quark-antiquark wavefunction does not have to be antisymmetric under exchange of particle labels. In a fashion similar to the case of protons and neutrons where we do not have to distinguish between equivalent and inequivalent particles, an antisymmetric wavefunction specified by J M, T M_T is constructed [41] to cover the equivalent and inequivalent cases and whose expression in the j-scheme is

$$|(n\ell j),\,(n'\ell'j') : J\,M;\,T\,M_T> = [\,|(n\ell j),\,(n'\ell'j') : J\,M;\,T\,M_T> - (-1)^{j+j'-J}(-1)^{1-T}$$
$$|(n'\ell'j'),\,(n\ell j) : J\,M;\,T\,M_T>]/(2(1+\delta_{nn'}\delta_{\ell\ell'}\delta_{jj'})^{1/2} \qquad \text{(III.1)}$$

The second term on the right hand side of (III.1) is the result of the operation of the permutation operator $-P_{12}$ on the first term. The δ-term in the normalisation factor takes

care of the special case of equivalent particles n = n', $\ell = \ell'$ and j = j'. For such particles, J + T must be an odd integer to prevent the expression (III.1) from vanishing. In the case of inequivalent particles, however, there is no such restriction on the value of J. For each T, J can have any value between |j - j'| and j + j'.

We adopt the same method for quarks and antiquarks with the exception that the spin and angular momentum are not coupled to a j as explained earlier on and therefore the two-particle states are not specified by a J and a coupled flavour F replacing the isospin T. We first define an abbreviated notation for the rest of our shell model work. The sets of quantum numbers $(n\ \ell\ m_\ell\ m_s\ f\ c\ p)$, $(n'\ \ell'\ m_\ell'\ m_s'\ f'\ c'\ p')$ of a single particle as defined earlier on are, from now on, denoted by single letters α, α' for brevity. Whenever we need to show these quantum numbers explicitly, we shall do so. Therefore, the antisymmetric wavefunction in this uncoupled scheme equivalent to (III.1) is less cumbersome and has the following form

$$|\alpha_1\ \alpha_2>_{AS} = [\,|\alpha_1\ \alpha_2>_{NAS} - |\alpha_2\ \alpha_1>_{NAS}\,] / 2^{1/2} \qquad \text{(III.2)}$$

Therefore, it remains to sandwich any two-body potential V between two such states for matrix element evaluation, which will be the subject of the next section.

III.2 ANTISYMMETRISED TWO-BODY MATRIX ELEMENTS

Before proceeding with the actual evaluation, we shall derive a very important result which is applicable to any symmetric operator, i.e. any operator that does not change under an exchange of particle coordinates. Using the antisymmetric states $<\alpha_1\ \alpha_2|$ and $|\alpha_3\ \alpha_4>$ given by (III.2), in the expression of the matrix element of the two-body potential V, there will be four such terms and given by

$$<\alpha_1\ \alpha_2|V|\alpha_3\ \alpha_4>_{AS} = [<\alpha_1\ \alpha_2|V|\alpha_3\ \alpha_4>_{NAS} - <\alpha_1\ \alpha_2|V|\alpha_4\ \alpha_3>_{NAS} - <\alpha_2\ \alpha_1|V|\alpha_3\ \alpha_4>_{NAS} + <\alpha_2\ \alpha_1|V|\alpha_4\ \alpha_3>_{NAS}] / 2 \qquad \text{(III.3)}$$

Since the particle variables are dummy-integration variables in the matrix element, we can relabel them by exchanging $1 \leftrightarrow 2$ throughout, V is not affected by the exchange because it is a symmetric operator. Therefore, the first two terms in (III.3) are equal and add up to cancel the factor 1/2 , analogous are the last two ones in (III.3) which becomes

$$<\alpha_1\ \alpha_2|V|\alpha_3\ \alpha_4>_{AS} = <\alpha_1\ \alpha_2|V|\alpha_3\ \alpha_4>_{NAS} - <\alpha_1\ \alpha_2|V|\alpha_4\ \alpha_3>_{NAS} \quad \text{(III.4)}$$

The two terms in (III.4) are usually called the direct and exchange terms respectively. The matrix element of the two-body potential between antisymmetric states is therefore a combination of the direct and exchange terms which are evaluated separately. Next we give the general form of the interaction potential.

III.3 FORM OF THE TWO-BODY HAMILTONIAN

The Hamiltonian for a system of quarks and antiquarks commonly used in a non-relativistic quark model consists of the kinetic energy term, T, and the interaction term V

$$H = T + V \quad \text{(III.5)}$$

T is the sum of individual particle kinetic energies minus the kinetic energy for the centre-of-mass of the total system as discussed in the previous chapter. It also includes the rest masses, i.e.

$$T = \sum_i T_i - T_{CM},\ T_i = m_i + \mathbf{p}_i^2/2m_i,$$

$$T_{CM} = \mathbf{P}^2/2M,\ \mathbf{P} = \sum_i \mathbf{p}_i,\ M = \sum_i m_i, \quad \text{(III.6)}$$

where m_i and $\mathbf{p}_i$ are the mass and the momentum of the ith particle, respectively. V is assumed to consist of a confinement term, V^{conf}, and a two-body residual term V^{res}, i.e.

$$\sum_{i<j} V_{ij} = \sum_{i<j} V_{ij}^{res} + V_{ij}^{conf} . \qquad \text{(III.7)}$$

The confinement term is supposed to represent the non-perturbative effects of QCD and is responsible for confining quarks and antiquarks within colour singlet physical systems. A commonly used form for this term is a two-body quadratic confinement potential which will be added to the residual term due to the perturbative effects and is usually taken to be the momentum-independent part of the One-Gluon Exchange Fermi-Breit potential describing the first three diagrams of Fig. 1. 1. Therefore V_{ij} has the following form [19]

$$\lambda_i . \lambda_j \frac{\alpha_s}{4} \left[\frac{1}{r_{ij}} - \frac{\pi}{m^2} (1 + \frac{2}{3} \sigma_i . \sigma_j) \delta(r_{ij}) \right] - a_c \lambda_i . \lambda_j r_{ij}^2 \qquad \text{(III.8)}$$

where λ_i is the generator of the colour SU(3) group for the ith particle, α_s is the particle-gluon coupling constant, σ_i is the Pauli spin matrix for the ith particle, $\mathbf{r}_{ij}$ is the interparticle vector spacing, $\delta(\mathbf{r}_{ij})$ is the contact term which is going to be responsible for mass splittings and a_c is the strength parameter of the confining potential. The values of these parameters will be discussed below. Now that we have chosen the two-body interaction, we proceed to the evaluation of its matrix elements between the states previously defined, in other words finding the numerical values of the direct and exchange terms in (III.4). So the problem reduces to finding one, as the evaluation of the other is very similar.

III.4 ACTUAL EVALUATION OF TWO-BODY MATRIX ELEMENTS

A close look at the expressions (III.6) and (III.8) of the Hamiltonian and the structure of the antisymmetric two-body states (III.2) without the couplings of orbital angular momentum and spin and flavour will give us an idea of the simplicity of the calculations resulting from this choice. The non-antisymmetric matrix element which we referred to earlier as the direct term of the antisymmetric matrix element can be expressed as a sum of

terms which factorise as follows:

$$<\alpha_1\alpha_2|V|\alpha_3\alpha_4>_{NAS} = \sum <a_1a_2|O_{SPA}|a_3a_4> x <s_1s_2|O_S|s_3s_4>$$
$$x<f_1f_2|O_F|f_3f_4> x <p_1\, p_2|O_P|p_3\, p_4> <c_1\, c_2|O_C|c_3\, c_4> \qquad \text{(III.9)}$$

where "a" is a short notation of the spatial coordinates n ℓ m_ℓ, m_s has been replaced by s just to have an abbreviated expression for the non-antisymmetric matrix element and O_{SPA}, O_S, O_F, O_P, and O_C are the spatial, spin, flavour, intrinsic parity and colour operators respectively whose explicit expressions could be obtained from (III.8). We do see from (III.9) the advantage of the uncoupled scheme of spin, angular momentum, and flavour as there is no coupling angular momentum algebra involved and each operator acts in the appropriate space. Further, rules for conserved quantum numbers can be derived and hence we know in advance whether a matrix element vanishes or not.

III.5 CONSERVED QUANTUM NUMBERS

We mentioned in the previous chapter, when we were treating the action of the number-conserving part of the Hamiltonian on a SD, that to each pair of particles destroyed corresponds a list of pairs that can be created, some of whose quantum numbers must be the same as those of the pair destroyed. This becomes clearer now as we treat each term on the right hand side of (III.9) individually. It all stems from finding the expressions of O_{SPA}, O_S, O_F, O_P, and O_C occurring in (III.9) from (III.8).

We first deal with the flavour and intrinsic parity operators O_F and O_P as the Hamiltonian and all the other operators previously discussed are independent of these quantum numbers, except for the transition potential which will be treated separately in part B of the current chapter. Therefore, the third and fourth terms on the righthand side of (III.9) reduce to delta terms, i.e.

$$<f_1\, f_2|O_F|f_3\, f_4> = \delta(f_1, f_3)\, \delta(f_2, f_4)$$
$$<p_1\, p_2|O_P|p_3\, p_4> = \delta(p_1, p_3)\, \delta(p_2, p_4). \qquad \text{(III.10)}$$

So to have the non-vanishing matrix element (III.9), the flavour and nature of the particles involved must be both conserved. In other words, the matrix element must be evaluated between two ups, two downs or an up and down for two-quark states, two up bars, two down bars or a down bar and an up bar for two-antiquark states and, finally, an up bar and an up, an up bar and a down, a down and a down bar or a down bar and an up for quark-antiquark states.

The conservation of the colour quantum number emerging from $<c_1\ c_2|O_C|c_3\ c_4>$ is by no means straightforward to derive despite the scalar nature of the colour operator $\lambda_1.\lambda_2$. This is because of the occurrence of the three colours and also their three conjugate ones due to the presence of quarks as well as antiquarks. This will be treated in great detail in section III.9.2.

The two remaining matrix elements to be found are the spin and the spatial ones. The former does not involve much effort as the spin operators that we encounter in our treatment are either scalars like $\sigma_1.\sigma_2$ as in (III.8) or delta functions as the matrix elements for other operators are also evaluated in parallel. Therefore, the problem reduces to dealing with different spatial operators and there are few of them depending only on the norm denoted by r of the interspacing vector $\mathbf{r}_{12}$ between the two-particles. In the next section, we shall develop a general method for any two-body interaction depending on r.

III.6 MATRIX ELEMENT OF A CENTRAL INTERACTION

To avoid all the mathematical effort which consists of coupling the angular momentum and spin for a complete evaluation of the shell-model matrix elements of the two-body spatial potential, an alternative method entails using Slater integrals. These were originally used in atomic physics where the uncoupled scheme is retained and the shape function is denoted by f(r).

We can expand f(r) [42] in terms of Legendre polynomials of the angle θ_{12} between the vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ of the particle 1 and 2 respectively, and $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$. The expansion coefficients will obviously be functions of the magnitudes r_1, r_2 of the two vectors and

are denoted by $f_k(r_1,r_2)$. Thus,

$$f(r) = \sum_{k=0}^{\infty} f_k\,(r_1,r_2)\; P_k(\cos\theta_{12}) \qquad \text{(III.11)}$$

Multiplying (III.11) by $P_{k'}(\cos\theta_{12})$ and making use of the orthogonality of Legendre polynomials given by

$$\int_{-1}^{1} P_\ell(\alpha)\; P_{\ell'}(\alpha)\; d\alpha = \frac{2}{2\ell+1}\delta_{\ell\,\ell'} \qquad \text{(III.12)}$$

the unknown quantities $f_k(r_1,r_2)$ are given in terms of f(r) by the integral

$$f_k(r_1,r_2) = \frac{2k+1}{2}\int_{-1}^{1} f(r)\; P_k(\cos\theta_{12})\; d(\cos\theta_{12}) \qquad \text{(III.13)}$$

So the matrix element of f(r) between two-particle states in the uncoupled form of spin and angular momentum is denoted by

$$\langle n_1\;\ell_1\;m_1\;n_2\;\ell_2\;m_2\,|f(r)|\,n_3\;\ell_3\;m_3\;n_4\;\ell_4\;m_4\rangle \qquad \text{(III.14)}$$

where m is now the z-projection of the angular momentum previously denoted by m_ℓ to distinguish it from m_s. The expression (III.14) is an integral where the two particles coordinates are implicit. Its explicit form is given by

$$\sum_{k=0}^{\infty}\int_0^\infty\int_0^\infty R_{n_1\ell_1}(r_1)\,R_{n_2\ell_2}(r_2)\,f_k(r_1,r_2)\,R_{n_3\ell_3}(r_1)\,R_{n_4\ell_4}(r_2)\,dr_1\,dr_2\,\times$$
$$\iint Y^*_{\ell_1 m_1}(\theta_1\varphi_1)\;Y^*_{\ell_2 m_2}(\theta_2\varphi_2)\;P_k(\cos\theta_{12})\;Y_{\ell_3 m_3}(\theta_1\varphi_1)\;Y_{\ell_4 m_4}(\theta_2\varphi_2)d\Omega_1 d\Omega_2 \qquad \text{(III.15)}$$

where

$d\Omega = \sin\theta d\theta d\varphi$

$Y_{\ell m}$ are the spherical harmonics and $R_{n\ell}$ are the harmonic oscillator radial functions[6] given in terms of the dimensionless variable $\rho=r/b$, where b is the oscillator length parameter, by

$$b^{-\frac{1}{2}}\left\{\frac{2\Gamma(n+\ell+\frac{3}{2})}{n!}\right\}^{\frac{1}{2}}\rho^{\ell+1}\exp(-\tfrac{1}{2}\rho^2)\sum_{k=0}^{n}(-1)^k\Gamma^{-1}(k+\ell+\tfrac{3}{2})\binom{n}{k}\rho^{2k} \qquad \text{(III.16)}$$

The calculation of (III.15) is done in two steps. The first one deals with the angle part and is straightforward requiring only the orthogonality of the spherical harmonics after expanding the Legendre polynomials [43] in the angle part as

$$P_k(\cos\theta_{12}) = \frac{4\pi}{2k+1}\sum_{m=-k}^{k} Y^*_{km}(\theta_1\varphi_1)Y_{km}(\theta_2\varphi_2) \qquad \text{(III.17)}$$

In fact, because of this expansion, the angle part is only a product of Clebsch-Gordan coefficients (see Appendix A.3). This product will vanish unless

1. $|\ell_1 - \ell_3| \le k \le \ell_1 + \ell_3$ and $|\ell_2 - \ell_4| \le k \le \ell_2 + \ell_4$
2. $\ell_1 + \ell_3 + k$ and $\ell_2 + \ell_4 + k$ are equal to even integers. (III.18)

These two conditions will have a big impact in the second step of the process of the calculation of (III.5) dealing with the radial part this time. As the functions $f_k(r_1,r_2)$ are not at all simple functions of any of the familiar shape functions except the Coulomb one whose form[7] is given by

[6] see reference [41] on page 40

[7] see reference [40] on page 399

$$\frac{1}{r_{12}} = (r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_{12})^{-1/2} = \sum_{k=0}^{\infty} \frac{r_<^k}{r_>^{k+1}} P_k(\cos\theta_{12}), \qquad \text{(III.19)}$$

the definition (III.13) has to be inserted in (III.15) and then this radial part, known as the Slater integral, is a triple integral given by

$$\frac{2k+1}{2} \int_0^{\infty} dr_1 \int_0^{\infty} dr_2 \int_{-1}^{1} R_{n_1\ell_1}(r_1)\, R_{n_2\ell_2}(r_2)\, f(r)\, P_k(\cos\theta_{12}) R_{n_3\ell_3}(r_1)\, R_{n_4\ell_4}(r_2)\, d(\cos\theta_{12}) \qquad \text{(III.20)}$$

The product of the radial functions of r_1 occurring in (III.20) can be written as

$$R_{n_1\ell_1}(r_1) R_{n_3\ell_3}(r_1) = b^{-1} \sum_{i=0}^{N} C_i(n_1\ell_1\, n_3\ell_3)\, \rho_1^{2i+L+2} \exp(-\rho_1^2) \qquad \text{(III.21)}$$

where $N=n_1+n_3$, $L=\ell_1+\ell_3$, and $\rho_1=r_1/b$. The coefficients C_i are obtained in appendix (A.2). A similar expression can be written for the radial functions of r_2 too, i.e.

$$R_{n_2\ell_2}(r_2) R_{n_4\ell_4}(r_2) = b^{-1} \sum_{i'=0}^{N'} C_{i'}(n_2\ell_2 n_4\ell_4)\, \rho_2^{2i'+L'+2} \exp(-\rho_2^2) \qquad \text{(III.22)}$$

with $N'=n_2+n_4$, $L'=\ell_2+\ell_4$ and $\rho_2 = r_2/b$. The factors b^{-1} appearing in (III.8) convert $dr_1 dr_2$ into $d\rho_1 d\rho_2$. The factors ρ_1^2 and ρ_2^2 can also be extracted from (III.8) to yield the elementary integration volume $\rho_1^2 \rho_2^2\, d\rho_1 d\rho_2 d(\cos\theta_{12})$.

The dimensionless variables ρ_1, ρ_2, and $\cos\theta_{12}$ are transformed as follows

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{\rho}_1 + \vec{\rho}_2) \text{ and } \vec{\bar{\rho}} = \frac{1}{\sqrt{2}} (\vec{\rho}_1 - \vec{\rho}_2)$$

so that

$$\vec{\rho}_1 = \frac{1}{\sqrt{2}}(\vec{\rho} + \vec{\bar{\rho}}) \text{ and } \vec{\rho}_2 = \frac{1}{\sqrt{2}}(\vec{\rho} - \vec{\bar{\rho}}).$$

Therefore

$$\vec{\rho}_1 \cdot \vec{\rho}_2 = \rho_1 \rho_2 \cos\theta_{12} = \frac{1}{2}(\rho^2 - \bar{\rho}^2) \text{ and } \rho_1^2 + \rho_2^2 = \rho^2 + \bar{\rho}^2 \qquad \text{(III.23)}$$

If the angle between $\vec{\bar{\rho}}$ and $\vec{\rho}$ is denoted by α then ρ_1^2 and ρ_2^2 are expressed as

$$\rho_1^2 = \frac{1}{2}(\rho^2 + \bar{\rho}^2 + 2\rho\bar{\rho}\cos\alpha) \text{ and } \rho_2^2 = \frac{1}{2}(\rho^2 + \bar{\rho}^2 - 2\rho\bar{\rho}\cos\alpha) \qquad \text{(III.24)}$$

The Jacobian of the transformation from one set of coordinates to the other is 1. Therefore the element of integration is conserved and expressed as

$$\rho_1^2 \, d\rho_1 \, \rho_2^2 \, d\rho_2 \, d(\cos\theta_{12}) \rightarrow \rho^2 \bar{\rho}^2 \, d\rho \, d\bar{\rho} \, d(\cos\alpha)$$

The potential function f(r) becomes

$$f(r) = f(\sqrt{2}\, b\bar{\rho}) = \bar{f}(\bar{\rho})$$

We now write the Legendre polynomial $P_k(z)$, where $z = \cos\theta_{12}$ in powers of z [44], as

$$P_k(z) = \sum_{\substack{t=0 \\ \text{(parity of k)}}}^{k} C_t(k)\, z^t \qquad \text{(III.25)}$$

where the coefficients $C_t(k)$ have been calculated in Appendix A.1 and have been found

to be

$$C_t(k) = \frac{(-1)^{\frac{k-t}{2}}(k+t)!}{2^k t!\,(\frac{k-t}{2})!\,(\frac{k+t}{2})!} \qquad \text{(III.26)}$$

In this newly defined set of coordinates and denoting cosα by x, F^k takes the following form

$$\frac{2k+1}{2}\int_0^\infty \bar{\rho}^2 d\bar{\rho}\int_0^\infty \rho^2 d\rho\int_{-1}^{1} dx \sum_{i=0}^{N}\sum_{i'=0}^{N'}\sum_{\substack{t=0 \\ \text{(parity of k)}}}^{k} C_i C_{i'} C_t(k)\exp(-(\bar{\rho}^2+\rho^2))\bar{f}(\bar{\rho})\{\rho_1^{2i+L}\rho_2^{2i'+L'}z^t\} \qquad \text{(III.27)}$$

The quantity within braces should also be expressed in terms of the new coordinates. A manipulation is necessary to tidy up the expression of F^k, and to abbreviate the notation we have introduced μ = 2i+L and ν = 2i'+L'. A combination of the fact that t and k have the same parity and the two conditions from the angle part ensures that, for any t in the expansion, the quantities $\ell_1+\ell_3-t$ and $\ell_2+\ell_4-t$ are either a positive even integer or 0. Thus, δ=(μ+ν)/2 is an integer. The first two terms in the part within braces are written as

$$\rho_1^{2i+L} = \rho_1^{2i+L-t}\rho_1^t = \rho_1^{\mu-t}\rho_1^t \quad \text{and} \quad \rho_2^{2i'+L'} = \rho_2^{2i'+L'-t}\rho_2^t = \rho_2^{\nu-t}\rho_2^t \qquad \text{(III.28a)}$$

From what we have discussed, we know that L-t and L'-t are positive integers or zero, and hence both (L-t)/2 and (L'-t)/2 are integers greater than or equal to 0. Thus,

$$\rho_1^{2i+L-t} = (\rho_1^2)^{i+\frac{L-t}{2}} = \frac{1}{2^{\frac{\mu-t}{2}}}(\rho^2+\bar{\rho}^2+2\rho\bar{\rho}x)^{\frac{\mu-t}{2}}, \qquad \text{(III.28b)}$$

$$\rho_2^{2i'+L'-t} = (\rho_2^2)^{i'+\frac{L'-t}{2}} = \frac{1}{2^{\frac{\nu-t}{2}}}(\rho^2+\bar{\rho}^2-2\rho\bar{\rho}x)^{\frac{\nu-t}{2}}. \qquad \text{(III.28d)}$$

With the help of (III.23) and (III.24) the expression within braces becomes

$$\left\{\rho_1^{2i+L}\rho_2^{2i'+L'}z^t\right\}=\frac{1}{2^{\frac{\mu+\nu}{2}}}(\rho^2+\bar{\rho}^2+2\rho\bar{\rho}x)^{\frac{\mu-t}{2}}(\rho^2+\bar{\rho}^2-2\rho\bar{\rho}x)^{\frac{\nu-t}{2}}\left\{\rho^2-\bar{\rho}^2\right\}^t \qquad \text{(III.29)}$$

The resultant triple integral is denoted by G(μ,ν,t) and has the following form

$$G(\mu,\nu,t)=2^{-\frac{\mu+\nu}{2}}\int_0^\infty\bar{\rho}^2d\bar{\rho}\int_0^\infty\rho^2d\rho\int_{-1}^1dx\exp(-(\rho^2-\bar{\rho}^2))\bar{f}(\bar{\rho})\left\{\rho^2+\bar{\rho}^2+2\rho\bar{\rho}x\right\}^{\frac{\mu-t}{2}}\left\{\rho^2+\bar{\rho}^2-2\rho\bar{\rho}x\right\}^{\frac{\nu-t}{2}}\left\{\rho^2-\bar{\rho}^2\right\}^t$$

$$\frac{1}{2^{\frac{\mu+\nu}{2}}}\int_0^\infty\bar{\rho}^2d\bar{\rho}\int_0^\infty\rho^2d\rho\int_{-1}^1dx\exp(-(\rho^2-\bar{\rho}^2))\bar{f}(\bar{\rho})\left\{\rho^2+\bar{\rho}^2+2\rho\bar{\rho}x\right\}^{\frac{\mu-t}{2}}\left\{\rho^2+\bar{\rho}^2-2\rho\bar{\rho}x\right\}^{\frac{\nu-t}{2}}\left\{\rho^2-\bar{\rho}^2\right\}^t \qquad \text{(III.30)}$$

Its evaluation is possible only if the three expressions within braces are binomially expanded

$$\left\{(\rho^2+\bar{\rho}^2)+2\rho\bar{\rho}x\right\}^{\frac{\mu-t}{2}}=\sum_\xi\binom{\frac{\mu-t}{2}}{\xi}\left\{\rho^2+\bar{\rho}^2\right\}^{\frac{\mu-t}{2}-\xi}\left\{2\rho\bar{\rho}\right\}^\xi x^\xi \qquad \text{(III.31a)}$$

$$\left\{(\rho^2+\bar{\rho}^2)-2\rho\bar{\rho}x\right\}^{\frac{\nu-t}{2}}=\sum_{\xi'}\binom{\frac{\nu-t}{2}}{\xi'}\left\{\rho^2+\bar{\rho}^2\right\}^{\frac{\nu-t}{2}-\xi'}(-2)^{\xi'}\left\{\rho\,\bar{\rho}\right\}^{\xi'}x^{\xi'} \qquad \text{(III.31b)}$$

$$\left\{\rho^2-\bar{\rho}^2\right\}^t=\sum_{y=0}^t\binom{t}{y}\rho^{2y}\bar{\rho}^{2(t-y)}(-1)^{t-y} \qquad \text{(III.31c)}$$

A fourth expansion emerging from the two first expressions is

$$\left\{\rho^2+\bar{\rho}^2\right\}^\alpha=\sum_{x=0}^\alpha\binom{\alpha}{x}\rho^{2x}\bar{\rho}^{2(\alpha-x)} \qquad \text{(III.32)}$$

where $\alpha=\delta-t-\xi-\xi'$.

The first integration to be performed is that of x, which will give the nonvanishing result

for even powers of x only as

$$\int_{-1}^{1} x^s \, dx = \frac{2}{2s+1}. \tag{III.33}$$

The next integration to be done is that of ρ, which will have the general form

$$\int_0^{\infty} \rho^{2s} \exp(-\rho^2) d\rho = \frac{1}{2}\Gamma(s+\frac{1}{2}). \tag{III.34}$$

The last integration we have to do will be that of $\bar{\rho}$, and clearly the integrals will have the general form

$$I_s = \int_0^{\infty} \exp(-\bar{\rho}^2)\, \bar{f}(\bar{\rho})\, \bar{\rho}^{2(s+1)} d\bar{\rho} \tag{III.35}$$

which is known as the Talmi integral. Carrying out the binomial expansions and the integrations in the indicated order, the final expression of $G(\mu,\nu,t)$ is

$$\frac{1}{2^{\delta}} \sum_{\eta=0}^{\eta max} \frac{2^{2\eta}}{2\eta+1} \sum_x \sum_y \binom{\delta-t-2\eta}{x} \binom{t}{y} \Gamma(x+y+\eta+1.5)(-1)^{t-y} I_s \sum_{\xi,\xi'} (-1)^{\xi'} \binom{\frac{\mu-t}{2}}{\xi} \binom{\frac{\nu-t}{2}}{\xi'} \tag{III.36}$$

where $s = \delta - x - y$, $2\eta = \xi + \xi'$, and the maximum value of η, namely η_{max}, is given by the nonvanishing of the first binomial coefficient. The requirement for nonvanishing is that $\delta - t - 2\eta$ must be greater than or equal to 0, and hence, if $\delta - t$ is an even integer, then $2\eta_{max} = \delta - t$; in the contrary case $2\eta_{max} = \delta - t - 1$. The limits of the summation of x and y are restricted, once again, by the appropriate binomial coefficient; the limits on ξ and ξ' are determined by the binomial coefficients as well as the condition $\xi + \xi' = 2\eta$.

The final expression of the radial part of the Slater integral is given by

$$F^k(n_1\ell_1 n_2\ell_2\, n_3\ell_3\, n_4\ell_4) = \frac{2k+1}{2}\sum_{i=0}^{N}\sum_{i'=0}^{N'}\sum_{\substack{t=0 \\ \text{(parity of k)}}}^{K} C_i(n_1\ell_1\, n_3\ell_3)\; C_{i'}(n_2\ell_2\, n_4\ell_4)\; C_t(k)\; G(\mu,\nu,t) \qquad \text{(III.37)}$$

The expression of $G(\mu,\nu,t)$ is symmetric in μ and ν; furthermore, since L, L' and t have the same parity, we have to consider only the odd values of μ, ν for t odd, and the even values for t even.

III.7 APPLICATION OF THE METHOD

There are two major shape functions in the used Hamiltonian: one is the sum of Coulomb like and quadratic, the other one is a δ function. An application of the above described method will consist in finding the Talmi integral for the first one, and simplifying the obtained expression of F^k for the second one.

III.7.1 $f(r) = A/r + Br^2$

A quick glance at the expression of F^k suggests that the shape function must be expressed in terms of new coordinates. Thus, f(r) becomes

$$\bar{f}(\bar{\rho}) = \frac{A}{\bar{\rho}} + B\bar{\rho}^2 \qquad \text{(III.38)}$$

So the insertion of (III.38) in (III.35) yields a simple Talmi integral whose value is

$$\frac{A}{2\sqrt{2}\,b}\, s! + Bb^2\Gamma(s+2.5) \qquad \text{(III.39)}$$

where s is an integer defined earlier, and A and B are the strengths of the two potentials.

III.7.2 δ-FUNCTION POTENTIAL

The Slater integral method is straightforward to apply in the case of a zero-range potential because the radial functions $f_k(r_1,r_2)$ are extremely simple. Furthermore the summation over k (the order of the Legendre polynomial) which appears in the matrix element can be carried out in a close form, yielding very simple final expressions for the matrix elements.

In terms of the polar coordinates, the three-dimensional δ-function is actually a product of three one-dimensional δ-functions corresponding to the three coordinates

$$\delta(\vec{r}_{12}) = \frac{\delta(r_{12})}{r_1^2}\,\delta(\cos\theta_1 - \cos\theta_2)\delta(\varphi_1 - \varphi_2) \qquad \text{(III.40)}$$

where the subscripts 1 and 2 refer to the two interacting particles.

Incidentally, the occurrence of r_1^2 in the elementary integration volume in terms of polar coordinates demands that an r_1^2 be put in the denominator of (III.40). The two δ-functions in the angle space can be replaced by a closure relation for the complete set of spherical harmonics

$$\sum_{k=0}^{\infty}\sum_{q=-k}^{k}(-1)^q \mathbb{Y}_q^k(\theta_1\,\varphi_1)\mathbb{Y}_{-q}^k(\theta_2\varphi_2) \qquad \text{(III.41)}$$

We switch over to the renormalised spherical harmonics[8] defined by

$$Y_q^k(\theta\,\varphi) = \sqrt{\frac{4\pi}{2k+1}}\;\mathbb{Y}_q^k(\theta\,\varphi) \qquad \text{(III.42)}$$

and replace the q-summation by the scalar-product notation, the expression (III.40) becomes

[8] see reference [40] on page 597

$$\delta(\vec{r}_{12}) = \frac{\delta(r_{12})}{r_1^2} \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} Y^k(1).Y^k(2). \qquad \text{(III.43)}$$

Comparing the above expression to the definition of the radial function in (III.13), we derive

$$f_k(r_1,r_2) = \frac{2k+1}{4\pi} \frac{\delta(r_{12})}{r_1^2} \qquad \text{(III.44)}$$

Thus the k-dependence of the Slater integral of the δ-function potential is through the factor 2k+1 only. Following the same path as before, the spatial part F^k of the Slater integral of a δ-function potential is found having this following simplified form

$$\frac{2k+1}{\pi b^3 2^{\frac{\mu+\nu+9}{2}}} \sum_{i=0}^{N} \sum_{i'=0}^{N'} C_i C_{i'} \Gamma\left(\frac{\mu+\nu+3}{2}\right) \qquad \text{(III.45)}$$

If we examine closely the form of the interaction potential in (III.8), we notice that it is written as the product of a depth V_0 and the sum of a central term and the δ-function potential which is multiplied by a constant term for dimensionality reason. There is an extra division by b^3 for the δ term in (III.45), which means that the depth of the δ-function potential cannot be specified in MeV only; it has to be specified in MeV fm^3.

III.7.3 MISCELLANEOUS APPLICATIONS

By virtue of the method, diverse matrix elements for different types of central operators can be evaluated. As discussed in the previous chapter we are interested in calculating the matrix element of the kinetic energy and centre of mass operators. It was shown that they both had central terms. Their Slater integrals have been worked out and their expressions are presented in the foregoing sub-sections.

III.7.3.1 KINETIC ENERGY TERM

A brief look back at the expression of the two-body kinetic energy operator shows that the part which can be evaluated by the described method, is

$$-\frac{\hbar\omega}{2A}(\rho^2+\bar{\rho}^2) \qquad \text{(III.46)}$$

The expression of $G(\mu,\nu,t)$ for $(\rho^2+\bar{\rho}^2)$ is

$$\frac{1}{2}\sum_{x=0}^{\alpha}\sum_{y=0}^{t}(-1)^{t-y}\binom{t}{y}\binom{\alpha+1}{x}\Gamma(\delta-\eta-x-y+2.5)\,\Gamma(x+y+\eta+1.5)$$
$$+\frac{1}{2}\sum_{y=0}^{t}(-1)^{t-y}\binom{t}{y}\Gamma(\eta+t-y+1.5)\,\Gamma(\delta-\eta-t+y+2.5) \qquad \text{(III.47)}$$

III.7.3.2 CENTRE OF MASS TERM

Following the same philosophy as for the kinetic energy operator, the part of the centre of mass which requires analogous treatment is

$$\frac{\hbar\omega}{2A}(\rho^2-\bar{\rho}^2) \qquad \text{(III.48)}$$

Similarly, the expression of $G(\mu,\nu,t)$ for $(\rho^2-\bar{\rho}^2)$ is given by

$$\frac{1}{2}\sum_{x=0}^{\alpha}\sum_{y=0}^{t}(-1)^{t+1-y}\binom{\alpha}{x}\binom{t+1}{y}\Gamma(\delta-\eta-x-y+2.5)\,\Gamma(x+y+\eta+1.5)$$
$$+\frac{1}{2}\sum_{x=0}^{\alpha}\binom{\alpha}{x}\Gamma(\eta+x+t+2.5)\Gamma(\delta-\eta-t-x+1.5) \qquad \text{(III.49)}$$

III.8 NABLA-NABLA TERM

The last operator involving spatial coordinates that requires evaluation, is the nabla-nabla

term present in both the kinetic energy and centre-of-mass operators. It is expressed as follows

$$\frac{\hbar\omega}{A} b^2 \nabla(1).\nabla(2) \qquad \text{(III.50)}$$

where 1 and 2 refer to the two particles respectively. $\nabla(1).\nabla(2)$ is the scalar product of two tensors of rank 1 and conventionally[9] represented by

$$\sum_{\mu=-1}^{1} (-1)^{\mu} \nabla_{\mu}(1)\, \nabla_{-\mu}(2) \qquad \text{(III.51)}$$

which is going to be evaluated between states defined in (III.2), so $\nabla_{\mu}(1)$ and $\nabla_{-\mu}(2)$ are going to act on the spatial coordinates of particle 1 and particle 2 respectively

$$\sum_{\mu=-1}^{1} (-1)^{\mu} \langle n_1 \ell_1 m_1 | \nabla_{\mu}(1) | n_3 \ell_3 m_3 \rangle \langle n_2 \ell_2 m_2 | \nabla_{-\mu}(2) | n_4 \ell_4 m_4 \rangle \qquad \text{(III.52)}$$

So for each single particle, the general matrix element is given by

$$\langle \ell'\, m' | \nabla_{\mu} | \ell\ m \rangle = (-1)^{m'} \frac{\begin{pmatrix} \ell' & 1 & \ell \\ -m' & \mu & m \end{pmatrix}}{\begin{pmatrix} \ell' & 1 & \ell \\ 0 & 0 & 0 \end{pmatrix}} \langle \ell'\, 0 | \nabla_0 | \ell\ 0 \rangle \qquad \text{(III.53)}$$

n and n' have been omitted from (III.53) to make it less cumbersome but will be considered when required. To ensure that (III.53) does not vanish, the triangular condition of the 3-j symbols has to be fulfilled, thus, only $\ell'=\ell+1$ and $\ell'=\ell-1$ will remain, for which the matrix elements of type $\langle \ell'\, 0 | \nabla_0 | \ell\ 0 \rangle$ satisfy

[9] see reference [43] on page 79

$$<\ell+1\ 0|\nabla_0|\ell\ 0>=\frac{\ell+1}{((2\ell+1)(2\ell+3))^{1/2}}\left(\frac{d}{dr}-\frac{\ell}{r}\right) \qquad \text{(III.54a)}$$

$$<\ell-1\ 0|\nabla_0|\ell\ 0>=\frac{\ell}{((2\ell+1)(2\ell-1))^{1/2}}\left(\frac{d}{dr}-\frac{\ell+1}{r}\right) \qquad \text{(III.54b)}$$

If we make use [45] of

$$\left(\frac{d}{dr}+\frac{\ell+1}{r}\right)R_{n\ell}=b^{-1}\sqrt{\ell+n+0.5}\,R_{n\,\ell-1}+b^{-1}\sqrt{n+1}\,R_{n+1\,\ell-1} \qquad \text{(III.55a)}$$

$$\left(\frac{d}{dr}-\frac{\ell}{r}\right)R_{n\ell}=-b^{-1}\sqrt{n+\ell+1.5}\,R_{n\,\ell+1}-b^{-1}\sqrt{n}\,R_{n-1\,\ell+1}, \qquad \text{(III.55b)}$$

of the Wigner-Eckart theorem

$$<n'\ \ell'\ m'|\nabla_\mu|n\ \ell\ m>=(-1)^{\ell'-m'}\begin{pmatrix}\ell' & 1 & \ell\\ -m' & \mu & m\end{pmatrix}<n'\ \ell'||\nabla||n\ \ell> \qquad \text{(III.56)}$$

and of the fact that only a few reduced matrix elements of the nabla operator are not zero. They are the following

$$<n-1\ \ell+1||\nabla||n\ \ell>=-b^{-1}\sqrt{n(\ell+1)} \qquad \text{(III.57a)}$$

$$<n+1\ \ell-1||\nabla||n\ \ell>=-b^{-1}\sqrt{\ell(n+1)} \qquad \text{(III.57b)}$$

$$<n\quad \ell-1||\nabla||n\ \ell>=-b^{-1}\sqrt{\ell(n+\ell+.5)} \qquad \text{(III.57c)}$$

$$<n\quad \ell+1||\nabla||n\ \ell>=-b^{-1}\sqrt{(\ell+1)(n+\ell+1.5)} \qquad \text{(III.57d)}$$

and if A, B, C, D are

$$A=\sum_{\mu=-1}^{1}(-1)^{\mu}<\ell_3\,m_3\ 1\ \mu|\ell_1\,m_3+\mu><\ell_4\,m_4\ 1\ -\mu|\ell_2\,m_4-\mu> \qquad \text{(III.58a)}$$

$$B=b^{-2}[\ell_1\ell_2/(2\ell_1+1)(2\ell_2+1)]^{1/2},\ C=[n_3+\ell_1+0.5]^{1/2}\ \text{and}\ D=[n_4+\ell_2+0.5]^{1/2} \qquad \text{(III.58b)}$$

then, according to the values of ℓ_1 and ℓ_3, and ℓ_2 and ℓ_4, four cases have to be envisaged yielding four expressions of (III.53)

1. $\ell_1=\ell_3+1$ and $\ell_2=\ell_4+1$

$$AB\{C\delta(n_1,n_3)+(n_3)^{1/2}\delta(n_1,n_3-1)\}\{D\delta(n_2,n_4)+(n_4)^{1/2}\delta(n_2,n_4-1)\}$$

2. $\ell_1=\ell_3+1$ and $\ell_2=\ell_4-1$

$$AB\{C\delta(n_1,n_3)+(n_3)^{1/2}\delta(n_1,n_3-1)\}\{D\delta(n_2,n_4)+(n_4)^{1/2}\delta(n_2,n_4+1)\}$$

3. $\ell_1=\ell_3-1$ and $\ell_2=\ell_4+1$

$$AB\{C\delta(n_1,n_3)+(n_3)^{1/2}\delta(n_1,n_3+1)\}\{D\delta(n_2,n_4)+(n_4)^{1/2}\delta(n_2,n_4-1)\}$$

4. $\ell_1=\ell_3-1$ and $\ell_2=\ell_4-1$

$$AB\{C\delta(n_1,n_3)+(n_3)^{1/2}\delta(n_1,n_3+1)\}\{D\delta(n_2,n_4)+(n_4)^{1/2}\delta(n_2,n_4+1)\}$$

(III.59)

III.9 MATRIX ELEMENTS IN OTHER SPACES

III.9.1 SPIN PART

Most of the spin functions occurring in the Hamiltonian are delta functions between the two two-particle states, except for the $s_1.s_2$ term responsible for the hyperfine splitting. The term $s_1.s_2$ may be written as

$$[(s_{+1} + s_{-1})(s_{+2} + s_{-2})+(s_{+1} - s_{-1})(s_{-2} - s_{+2})+4s_{1z}s_{2z}]/4 \qquad \text{(III.60)}$$

where

$$s_+ = (s_x + is_y) \text{ and } s_- = (s_x - is_y)$$

known as the raising and lowering operators in the spin space. If the two two-particle spin states are denoted by

$$\left|\tfrac{1}{2}\, m_{s1}\, \tfrac{1}{2}\, m_{s2}\right\rangle \quad \text{and} \quad \left|\tfrac{1}{2}\, m_{s3}\, \tfrac{1}{2}\, m_{s4}\right\rangle \qquad \text{(III.61)}$$

then the matrix element of (III.60) between these two states is straightforward and reduces to

$$1/2 \text{ if } m_{s1}=m_{s3}\ -1 \text{ and } m_{s2}=m_{s4}+\ 1 \text{ or } m_{s1}=m_{s3}\ +\ 1 \text{ and } m_{s2}=m_{s4}-1,$$
$$m_{s1}m_{s2} \text{ if } m_{s1}=m_{s3} \text{ and } m_{s2}=m_{s4}. \qquad \text{(III.62)}$$

III.9.2 COLOUR PART

As previously discussed, the advantage of evaluating the two-body matrix elements in the uncoupled scheme is the flexibility in handling the operators involved. That is certainly true for the spatial and the spin ones as already seen but not for the colour one where "coupling of colour" is preferable.

In our study, both quarks and antiquarks are handled. This means that we will have to calculate matrix elements of $\underset{\sim}{\lambda}_1 \cdot \underset{\sim}{\lambda}_2$ between two-quark, quark-antiquark, or two anti-quark colour states. The difficulty is then in handling the two different representations of the colour group SU(3). In other words, the problem is confined in finding the Clebsch-Gordan coefficients of SU(3).

We need to digress here and switch over to group theory. Let us consider the Clebsch-Gordan coefficients that arise in reduction of the decomposition of the Kronecker product[10] of $3 \otimes 3$. The basis tensors of the Kronecker product representation are the nine two-particle states $u_i u_j$, i, j = 1, 2, 3.

We know that the product representation can be composed into a symmetric and an antisymmetric representations with $\square\square$ and $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ Young tableaux respectively.

By writing down the symmetric and antisymmetric combinations, we immediately get the relevant Clebsch-Gordan coefficients, except for possible sign ambiguities which must be decided by convention. In the symmetric case, the normalised irreducible functions are

[10] see for example reference [35]

$$\boxed{1}\boxed{1} = u_1u_1 = \psi_1$$

$$\boxed{1}\boxed{2} = 2^{-1/2}(u_1u_2+u_2u_1) = \psi_2$$

$$\boxed{2}\boxed{2} = u_2u_2 = \psi_3$$

$$\boxed{1}\boxed{3} = 2^{-1/2}(u_1u_3+u_3u_1) = \psi_4$$

$$\boxed{2}\boxed{3} = 2^{-1/2}(u_2u_3+u_3u_2) = \psi_5$$

$$\boxed{3}\boxed{3} = u_3u_3 = \psi_6 \qquad \text{(III.63)}$$

The Clebsch-Gordan coefficients are just the coefficients in these sums. In the antisymmetric case, the irreducible functions [46] are

$$\begin{matrix}\boxed{1}\\\boxed{2}\end{matrix} = 2^{-1/2}(u_1u_2-u_2u_1) = \bar{u}_3 = v_1$$

$$-\begin{matrix}\boxed{1}\\\boxed{3}\end{matrix} = 2^{-1/2}(u_3u_1-u_1u_3) = \bar{u}_2 = -v_2$$

$$\begin{matrix}\boxed{2}\\\boxed{3}\end{matrix} = 2^{-1/2}(u_2u_3-u_3u_2) = \bar{u}_1 = v_3 \qquad \text{(III.64)}$$

with a particular sign convention which we shall discuss shortly.

The notation $\bar{u}_1$, $\bar{u}_2$, and $\bar{u}_3$ has been introduced for the basis functions of the antisymmetric representation because these functions also serve as basis vectors for the second fundamental representation of SU(3). These states can be interpreted as the ones which denote the antiparticles of the states u_1, u_2, and u_3. A direct application of this is the construction of the Clebsch-Gordan coefficients arising in the decomposition $3 \otimes \bar{3}=8+1$ which is of great interest for us. By symmetry, the six states at the periphery of the octet weight diagram can be obtained and they are

$$\psi_1 = u_1v_1, \quad \psi_2 = u_2v_1, \quad \psi_3 = u_1v_2,$$
$$\psi_5 = u_2v_3, \quad \psi_7 = u_3v_2, \quad \psi_8 = u_3v_3. \qquad \text{(III.65)}$$

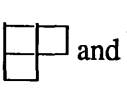
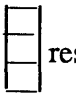
The two other states of the octet can be obtained by operating on ψ_3 and ψ_6 with lowering

operators, and then normalising. They are

$$\psi_4 = 2^{-1/2}(u_1v_3+u_2v_2) \text{ and } \psi_6 = 6^{-1/2}(2u_3v_1+u_2v_2-u_1v_3) \qquad \text{(III.66)}$$

and the singlet state is constructed orthogonal to the octet and it is found to be

$$\psi = 3^{-1/2}(u_1v_3-u_2v_2+u_3v_1) \qquad \text{(III.67)}$$

The Young tableaux of the octet and the singlet are  and  respectively.

From the two previous equations, the inverse transformation enabling us to go from the coupled basis states to the uncoupled ones (u_1v_3, u_2v_2, u_1v_3) is

$$\begin{pmatrix} u_1v_3 \\ u_2v_2 \\ u_3v_1 \end{pmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{pmatrix} \square\square_4 = \psi_4 \\ \square\square_6 = \psi_6 \\ \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array} = \psi \end{pmatrix} \qquad \text{(III.68)}$$

The reason for all these manipulations is that we want to evaluate the expectation value of the colour operator $\lambda_1.\lambda_2$, which can be written as

$$\frac{1}{2}\left[(\lambda_1+\lambda_2)^2 - \lambda_1^2 - \lambda_2^2\right] \qquad \text{(III.69)}$$

where $(\lambda_1+\lambda_2)^2$ is the colour Casimir operator whose value is 10/3, 4/3, 3, or 0 [47] if taken between two sextets (6), triplets($\bar{3}$), octet(8) or singlet states respectively of the colour SU(3) group. On the other hand, the expectation value of λ_1^2 is 4/3, thus the expectation value of (III.69) is

$C\bar{3}$=-8/3, C6=4/3, C8=2/3 and Csinglet=-16/3. (III.70)

So it becomes easy to apply this to colour. The six states of (III.65) correspond to $r\bar{b}$, $g\bar{b}$, $r\bar{g}$, $g\bar{r}$, $b\bar{g}$, and $b\bar{r}$, whereas the three in (III.66) and (III.67) correspond to $b\bar{b}$, $-g\bar{g}$, $r\bar{r}$ respectively. An important feature was unveiled at this stage concerning the fact that although the colour operator is scalar, in the case of a quark-antiquark state it could change the colour of the state. In other words, for the first case $<b\bar{r}|\underline{\lambda}_1.\underline{\lambda}_2|r\bar{b}>=0$ as $b\bar{r}$ and $r\bar{b}$ are orthogonal but for example $<r\bar{r}|\underline{\lambda}_1.\underline{\lambda}_2|b\bar{b}>$ is not zero. From (III.66) and (III.67), we have

$$r\bar{r}=(3^{1/2}\psi_4 - \psi_6 + 2^{1/2}\psi)/6^{-1/2} \text{ and } b\bar{b}=(\psi + 2^{1/2}\psi_6)/3^{-1/2} \quad \text{(III.71)}$$

then $<r\bar{r}|\underline{\lambda}_1.\underline{\lambda}_2|b\bar{b}>$=[-C8+Csinglet]/12 and $<r\bar{r}|\underline{\lambda}_1.\underline{\lambda}_2|r\bar{r}>$ = [2C8+Csinglet]/12.
To recapitulate, the expectation value between two quark-antiquark states of $\lambda_1.\lambda_2$ using the logical code for colour (00, 11), and (01,10), indicating that the two colours are the same or different respectively, is

$$<11|\underline{\lambda}_1.\underline{\lambda}_2|00>=-1/3,\ <00|\underline{\lambda}_1.\underline{\lambda}_2|00>=-1/2 \quad \text{(III.72a)}$$

$$<01|\underline{\lambda}_1.\underline{\lambda}_2|01>=1/6,\ <10|\underline{\lambda}_1.\underline{\lambda}_2|01>=0 \quad \text{(III.72b)}$$

Similarly, if the two particles are the same (two quarks or two antiquarks), and if the two colours are the same, then from (III.63) and (III.64) we are dealing with $<\square\square|\underline{\lambda}_1.\underline{\lambda}_2|\square\square>$=C6 but when the colours are different, there are two terms: the direct one and the exchange one, and are respectively

$$[<\square\square|\underline{\lambda}_1.\underline{\lambda}_2|\square\square>+<\begin{smallmatrix}\square\\\square\end{smallmatrix}|\underline{\lambda}_1.\underline{\lambda}_2|\begin{smallmatrix}\square\\\square\end{smallmatrix}>]/2 = [C6+C\bar{3}]/2 \quad \text{(III.73a)}$$

$$[<\square\square|\underline{\lambda}_1.\underline{\lambda}_2|\square\square>-<\begin{smallmatrix}\square\\\square\end{smallmatrix}|\underline{\lambda}_1.\underline{\lambda}_2|\begin{smallmatrix}\square\\\square\end{smallmatrix}>]/2 = [C6-C\bar{3}]/2 \quad \text{(III.73b)}$$

In terms of the logical colour we handle in the computer program,

$$<00|\underset{\sim}{\lambda}_1.\underset{\sim}{\lambda}_2|00>=1/3\ ,\ <01|\underset{\sim}{\lambda}_1.\underset{\sim}{\lambda}_2|01>=-1/6 \text{ and } <01|\underset{\sim}{\lambda}_1.\underset{\sim}{\lambda}_2|10>=1/3. \qquad \text{(III.74)}$$

A computer program is set to evaluate the two-body matrix elements for all the different operators in parallel, then combine different parts for matrix elements of the type (III.9).

PART B

In this second part, we shall deal with the unfamiliar aspects of the model such as the "symmetry" of the operators and the form of the matrix elements that occurred in the number non-conserving part of the Hamiltonian. The form of the transition potential and methods for matrix elements evaluation will be presented.

III.10 EXPRESSION OF THE TRANSITION POTENTIAL

As discussed earlier on, our model aims to investigate the effects of the sea-quark effects in the study of physical systems such as baryons and mesons and how this will be considered as a first fundamental step towards a deep understanding of the conventional problem of the nucleon-nucleon interaction (N-N for short). These sea-quark effects have always been present in the one gluon exchange theory and are presented by diagrams 4 and 5 of Fig. 1. 1, but it is only recently that they have been thought of seriously as a possible answer to many mysterious problems, such as that the attractive part of the N-N interaction is due to the exchange of mesons which are quark-antiquark pairs.

We discussed in the previous chapter in full detail the different terms appearing in the Hamiltonian: the ones that conserve the number of particles and the ones that do not. The former is in fact a classical problem, whereas the latter is unfamiliar as it requires handling a variable number of particles. We did overcome this problem by developing a general method that takes care of these obstacles by choosing the shell model as a framework with all the renovations needed due to the nature of the approach. However,

when treating the number non-conserving part, we did mention a transition potential which describes the quark-antiquark pair creation illustrated by diagram 4 of Fig.1.1. Its expression is derived from a non-relativistic field theoretical study by Yu and Zhang [34]:

$$v(1,2)=\frac{1}{8\pi}\alpha_s(\lambda_1\cdot\lambda_2)[\frac{\pi}{2m_c}\{\frac{1}{\rho^2}(\sigma_1\cdot\mathbf{n})+\frac{i}{2\rho^2}(\sigma_1\times\sigma_2)\cdot\mathbf{n}+\frac{i\sqrt{2}}{\rho}(\sigma_1\cdot\mathbf{p}_2)\}] \quad \text{(III.75)}$$

$$v(1,2)=\frac{1}{8\pi}\alpha_s(\lambda_1\cdot\lambda_2)[\frac{\pi}{2m_c}\{\frac{1}{\rho^2}(\sigma_1\cdot\mathbf{n})+\frac{i}{2\rho^2}(\sigma_1\times\sigma_2)\cdot\mathbf{n}+\frac{i\sqrt{2}}{\rho}(\sigma_1\cdot\mathbf{p}_2)\}] \quad \text{(III.75)}$$

where

$$\mathbf{n}=\frac{\mathbf{r}_1-\mathbf{r}_2}{|\mathbf{r}_1-\mathbf{r}_2|} \quad \text{and} \quad \rho=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) \qquad \text{(III.76)}$$

Here the labels 1 and 2 denote sea and valence quarks respectively, $\mathbf{p}_2$ is the momentum operator of the valence quark, λ are the generators of the SU(3) colour group, σ are the Pauli spin matrices, and α_s is the quark-gluon coupling constant. We must now proceed to the discussion of the evaluation of the matrix elements occurring in (II.75).

III.11 FORM OF THE TRANSITION MATRIX ELEMENT

Its form is not familiar in the many-body theory where the number of particles is always conserved and therefore by no means straightforward as they are one-three body ones

$$\langle\beta\delta\gamma|v|\alpha\rangle \qquad \text{(III.77)}$$

where α, β, δ, and γ are four single-particle states and v is some sort of potential.

However, as the transition potential in (III.75) is a two-body one, its matrix element must be evaluated between two-particle states. Therefore, if β is the odd particle in (III.77), then the matrix element we have to evaluate is

$$\langle\delta\gamma|v(1,2)|\alpha\bar{\beta}\rangle \qquad \text{(III.78)}$$

by converting the odd particle into its conjugate [19]. If the wavefunction of the state β is

$$\beta(1) = \omega_{\beta}(\mathbf{r})\chi^{s}_{ms_\beta}\chi^{f}_{mf_\beta}\chi^{c}_{(\mu\nu)\,c_\beta} \tag{III.79}$$

where ω, χ^s, χ^f, χ^c are the spatial, spin, flavour, and colour parts of the wavefunction and $(\mu\nu)$ is the SU(3) irreducible representation which is (10) if β is a quark state and (01) otherwise. Therefore under particle conversion, the new-particle state is

$$\bar{\beta}(1) = (-1)^{1+mf_\beta+ms_\beta+\eta(c_\beta)}\,\omega^{*}_{\beta}(\mathbf{r})\chi^{s}_{-ms_\beta}\chi^{f}_{-mf_\beta}\chi^{c}_{(\bar{\mu}\bar{\nu})\,c_\beta} \tag{III.80}$$

where a phase factor has emerged and most importantly $(-1)^{\eta(c)}$ due to colour conjugation and is given by the SU(3) Clebsch-Gordan coefficient depending on the nature of the converted particle. If it is a quark, then

$$(-1)^{\eta(c)} = \sqrt{3}\langle (10)\,c\,(01)\,\bar{c}\,|\,(00)\rangle \tag{III.81a}$$

otherwise, it is

$$(-1)^{\eta(c)} = \sqrt{3}\langle (01)\,\bar{c}\,(10)\,c\,|\,(00)\rangle. \tag{III.81b}$$

Hence, the equivalence between the one-three body matrix element, which does not mean much to us and the two-body one is established. The effects of the asymmetry of the transition potential in particle exchange on the requirements of the antisymmetry of the two-body states will be discussed next.

III.12 ASYMMETRY EFFECTS OF V ON THE TWO-BODY STATES

To discuss this point, we must consider the term of the Hamiltonian that does not conserve the number of particles given in (II.6) where the summation is on the four particles. In section II.6.2 we set up the many-body aspects of (II.6). In other words, to each particle destroyed corresponds a list of triples of particles that could be created. To

count these triples systematically, an ordering of the indices is chosen and therefore (II.6a) becomes

$$\sum_{\alpha;\beta<\delta<\gamma} a_\beta^+ a_\delta^+ a_\gamma^+ a_\alpha \left\{ v_{\alpha\beta\delta\gamma} - v_{\alpha\beta\gamma\delta} - v_{\alpha\delta\beta\gamma} + v_{\alpha\delta\gamma\beta} + v_{\alpha\gamma\beta\delta} - v_{\alpha\gamma\delta\beta} \right\} \qquad \text{(III.82)}$$

which is an identity which comes from relabelling the summation indices. Only the first two terms are non-zero if β is the odd particle, or the third and the fourth if it is δ, or the last two terms if the odd particle is γ. We want to stress this point because despite the asymmetry of the transition potential in particle exchange, the problem of its matrix element reduces to two terms: a "direct" one and an "exchange" one once the odd particle is identified. This could be seen directly from (III.77) that the antisymmetry must be imposed only on two particles of the three particle state $|\beta\delta\gamma>$ because of the Pauli principle as the third particle is different anyway.

To recapitulate, to evaluate a matrix element of the transition potential, we first search for the odd particle in the triple, and once it is found and it is always designated by β, the "antisymmetric" matrix element to be evaluated is

$$<\delta\gamma|v(1,2)|\alpha\bar{\beta}>_{NAS} - <\gamma\delta|v(1,2)|\alpha\bar{\beta}>_{NAS} \qquad \text{(III.83)}$$

which consists of the two non-antisymmetric terms which will be evaluated one at a time in a manner similar to that in which the two-body matrix elements of the number conserving part of the Hamiltonian was previously treated. The conservation of quantum numbers was discussed in section II.6.2, but whenever a situation occurs to highlight what was already stated, we shall not hesitate to do so.

In the next section, mathematical machinery will be developed for evaluating one term of (III.83), and the other one is treated similarly. There will be three such terms corresponding to the three parts of the transition potential.

III.13 EVALUATION OF THE TWO-BODY MATRIX ELEMENTS

Once again, the aesthetic side of the uncoupled scheme is that each section of an operator is going to act on the relevant part of the wavefunction.We first notice the independence of the operators of the flavour of quantum number, and after particle conjugation of the odd particle, the two "two-particle" basis states of the matrix element become balanced in the nature of particles. Therefore an identical expression to the one given by (III.10) must be satisfied to have a non-vanishing matrix element. The colour part has already been treated as the operator $\underline{\lambda}_1 . \underline{\lambda}_2$ occurs in the number conserving part of the Hamiltonian too. So the remaining parts of the matrix element are three spin-spatial ones where each term is the scalar product of two tensors of rank one defined by (B.1), and are given by

$$1.\sum_{\mu=-1}^{1}(-1)^{\mu}\sigma_1^{\mu}\left(\frac{\vec{r}}{r^3}\right)^{-\mu} \qquad \text{(III.84a)}$$

$$2.\sum_{\mu=-1}^{1}(-1)^{\mu}(\vec{\sigma}_1\times\vec{\sigma}_2)^{\mu}\left(\frac{\vec{r}}{r^3}\right)^{-\mu} \qquad \text{(III.84b)}$$

$$3.\sum_{\mu=-1}^{1}(-1)^{\mu}\sigma_1^{\mu}\,\frac{\nabla_2^{-\mu}}{r} \qquad \text{(III.84c)}$$

where the subscripts 1 and 2 refer to particle 1 and 2 respectively. It is possible to use the Slater method developed to calculate the spatial integrals in part A to find the matrix element of $\nabla_2^{-\mu}/r$. Unfortunately it is not of great use to evaluate the spatial term common to both the first and the second operators where a totally different method is essential. We first start by giving the procedure to evaluate the matrix element of $(\vec{r}/r^3)^{-\mu}$ then proceed to the one of $\nabla_2^{-\mu}/r$. Finally, those of the spin parts will be treated, with more details in Appendix C.

III.14 MATRIX ELEMENT OF $(\vec{r}/r^3)^{\mu}$

Expressed in terms of the spatial quantum numbers, the matrix element we want to evaluate is denoted by

$$<n_1\ \ell_1\ m_1\ \ n_2\ \ell_2\ m_2|\frac{\vec{r}^{\mu}}{r^3}|n_3\ \ell_3\ m_3\ \ n_4\ \ell_4\ m_4>. \qquad \text{(III.85)}$$

A standard method to evaluate such matrix element is to switch over to the relative and the centre of mass coordinates as the operator depends on the relative coordinates only. The agent between the two sets of states is a Brody-Moshinsky transformation bracket. First, the two-body states must be written in the coupled form of angular momentum, so (III.85) becomes

$$\sum_{\Lambda,\Lambda'}<\ell_1 m_1\ \ell_2 m_2|\Lambda M><\ell_3 m_3\ \ell_4 m_4|\Lambda' M'>$$
$$<n_1\ell_1\ n_2\ell_2\ \ \Lambda M|\frac{\vec{r}^{\mu}}{r^3}|n_3\ell_3\ n_4\ell_4\ \ \Lambda' M'> \qquad \text{(III.86)}$$

where these two triangular conditions $|\ell_1-\ell_2| \leq \Lambda \leq \ell_1+\ell_2$ and $|\ell_3-\ell_4| \leq \Lambda' \leq \ell_3+\ell_4$ must be satisfied. We make use of the transformation, enabling us to go from one set of coordinates to the other, which is

$$|n_1\ell_1\ n_2\ell_2\ \Lambda M> = \sum_{n\ell,NL} <n\ell\ NL\ \Lambda|n_1\ell_1\ n_2\ell_2\ \Lambda>|n\ell\ NL\ \Lambda M> \qquad \text{(III.87)}$$

where $<n\ell\ NL\ \Lambda|n_1\ell_1\ n_2\ell_2\ \Lambda>$ is Baranger-Davies transformation brackets [48] and have been chosen, despite the popularity of the Brody-Moshinsky ones [49], because simpler formulae for their evaluation are available. The two brackets differ only by the order in the coupling of ℓ and L and are related by a phase factor

$$M_\lambda(n_1\ell_1\ n_2\ell_2\ ;\ NL\ n\ell\ ) = (-1)^{L+\ell+\lambda}<n\ell\ NL\ \Lambda|n_1\ell_1\ n_2\ell_2\ \Lambda>. \qquad \text{(III.88)}$$

The relation which is derived from such transformation bracket is

$$2n_1+\ell_1+2n_2+\ell_2 = 2n+\ell+2N+L \qquad \text{(III.89)}$$

as a result of the energy conservation and guarantees the conservation of parity by the condition $(-1)^{\ell_1+\ell_2} = (-1)^{\ell+L}$ too. Moreover, analogously to the triangular condition in the 3-j symbols, we must also have

$$\vec{\ell}_1 + \vec{\ell}_2 = \vec{\ell} + \vec{L} = \vec{\Lambda}\,. \qquad \text{(III.90)}$$

After substitution of (III.87) and use of the Wigner-Eckart theorem[11], (III.86) becomes

$$\sum_{\substack{\Lambda,\Lambda' \\ n\ell\ NL \\ n'\ell'N'L'}} AB\langle n\ell\ NL\ \Lambda\|\frac{\vec{r}}{r^3}\|n'\ell'\ N'L'\ \Lambda'\rangle\langle\Lambda'M'\ 1\mu|\Lambda M\rangle \qquad \text{(III.91)}$$

where A and B are products of two Clebsch-Gordan and two transformation brackets respectively given by

$$A = \langle \ell_1 m_1\ \ell_2 m_2\ |\Lambda M\rangle x \langle \ell_3 m_3\ \ell_4 m_4|\ \Lambda' M'\rangle \qquad \text{(III.92a)}$$

$$B = \langle n\ell\ \ NL\ \Lambda | n_1\ell_1\ n_2\ell_2\ \Lambda\rangle x \langle n'\ell'\ N'L'\ \Lambda' | n_3\ell_3\ n_4\ell_4\ \Lambda'\rangle\,. \qquad \text{(III.92b)}$$

Therefore the problem reduces to the evaluation of the reduced matrix element of $\vec{r}/r^3$.

III.15 EVALUATION OF $\langle n\ell\ NL\ \Lambda||\vec{r}/r^3||n'\ell'\ N'L'\ \Lambda'\rangle$

The operator, as it stands, acts only on the relative part of the $|n\ell\ NL\ \Lambda\rangle$, therefore we use the result in Appendix B.2. The reduced matrix element of $\vec{r}/r^3$ becomes[12]

$$(-1)^{\ell+L+1+\Lambda'}[(2\Lambda+1)(2\Lambda'+1)]^{1/2}\begin{Bmatrix} \ell & \Lambda & L \\ \Lambda' & \ell' & 1 \end{Bmatrix}\langle n\ell\,\|\frac{\vec{r}}{r^3}\|\,n'\ell'\rangle\delta_{NN'}\delta_{LL'} \qquad \text{(III.93)}$$

[11] see reference [43] on page 75

[12] see reference [43], expression 7.1.8 on page 111

Note that $\vec{r}$ is a tensor operator of rank 1 and negative parity, that forces ℓ and ℓ' differ by 1. Thus, there are only two reduced matrix elements worth considering: one is for $\ell'=\ell+1$ and the other is for $\ell'=\ell-1$. We only present their results here as the explicit detail of the calculations can be found in Appendix B. The two expressions are

$$\langle n'\ell+1\|\frac{\vec{r}}{r^3}\|n\ell\rangle=-b\sqrt{\ell+1}\{\sqrt{n+\ell+1.5}\langle n'\ell+1|r^{-3}|n\ell+1\rangle-\sqrt{n}\langle n'\ell+1|r^{-3}|n-1\ell+1\rangle\}$$

$$\langle n'\ell-1\|\frac{\vec{r}}{r^3}\|n\ell\rangle=-b\sqrt{\ell}\{\sqrt{n+\ell+.5}\langle n'\ell-1|r^{-3}|n\ell-1\rangle-\sqrt{n+1}\langle n'\ell-1|r^{-3}|n+1\ell-1\rangle\} \quad \text{(III.94)}$$

III.16 NOTE ON POLES IN THE RADIAL INTEGRALS

There is a problem in the evaluation of the matrix element $\langle n'\ell-1\|\frac{\vec{r}}{r^3}\|n\ell\rangle$ using III.94 as there is a pole at the origin in both the integrals on the right side when the value of ℓ is one. These poles are created only by the way in which we have written the equation and the left side is a well-behaved number. To lift up this divergence, rearrangement of the angular momenta in the matrix element is necessary, in other words we always choose to have the bigger angular momentum in the bra of the matrix element which will allow cancelling of poles. To clarify this, we shall make use of the following property

$$\langle n'\ \ell'\|\frac{\vec{r}}{r^3}\|n\ \ell\rangle=-\langle n\ \ell\|\frac{\vec{r}}{r^3}\|n'\ \ell'\rangle \quad \text{(III.95)}$$

which is proved in Appendix B. Therefore we have

$$\langle n'\ell-1\|\frac{\vec{r}}{r^3}\|n\ell\rangle=-\langle n\ \ell\|\frac{\vec{r}}{r^3}\|n'\ \ell-1\rangle. \quad \text{(III.96)}$$

We change the variable $\ell-1$ into λ and from the first expression in (III.94) to write the previous matrix element on the right side as

$$<n\lambda+1|\frac{\vec{r}}{r^3}|n'\lambda>=b\sqrt{\lambda+1}\{\sqrt{n'+\lambda+1.5}<n\,\lambda+1|r^{-3}|n'\,\lambda+1>-\sqrt{n'}<n\,\lambda+1|r^{-3}|n'-1\lambda+1>\}$$

(III.97a)

Now there are no poles on the right side. Expressing the last equation in terms of ℓ instead of λ, we have

$$<n'\ \ell-1\ |\frac{\vec{r}}{r^3}|\ n\ \ell> = -b\sqrt{\ell}\{\sqrt{n+\ell+.5}<n'\ \ell\,|r^{-3}|\,n\ \ell> - \sqrt{n}<n'\ \ell\,|r^{-3}|\,n-1\ \ell>\}$$

(III.97b)

Now we have the expression in terms of the biggest angular momentum in the expression of $<n'\ell-1\ |\frac{\vec{r}}{r^3}|\ n\ell>$ and therefore the poles are avoided.

III.17 MATRIX ELEMENT OF (∇_2^μ/r)

The last spatial matrix element we shall be calculating is

$$<n_1\ \ell_1\ m_1\ \ n_2\ \ell_2\ m_2|\nabla_2^\mu/r\,|n_3\ \ell_3\ m_3\ \ n_4\ \ell_4\ m_4>. \qquad \text{(III.98)}$$

The closure relation which is written as

$$\sum_{\substack{n'\ell'm'\\ n\ell m}} |n'\ \ell'\ m'\ \ n\ \ell\ m><n'\ \ell'\ m'\ \ n\ \ell\ m| \qquad \text{(III.99)}$$

is inserted in (III.98). As the operator nabla is only going to act on the coordinates of the second particle, the matrix element becomes rather simple

$$\sum_{n\ell m}<n_1\ell_1m_1\ \ n_2\ell_2m_2|r^{-1}|n_3\ell_3m_3\ \ n\ell m><n\ell m|\nabla^\mu|n_4\ell_4m_4> \qquad \text{(III.100)}$$

where the first matrix element is going to be evaluated according to the method given in part A once the values of the n, ℓ and m have been fixed by the second matrix element as

seen earlier on when treating the nabla-nabla term of the kinetic energy and of the centre-of-mass operators.

An interesting point concerning this round of spatial matrix elements evaluation is that as the operators are of odd parities, there always has been a condition that the angular momenta in the bra must be different by one from the one in the ket otherwise the matrix element vanishes. Therefore, as we are considering quarks and antiquarks in shells, we must provide the right space to study these quark-antiquark pair effects. In other words the space must consist of shells of different spatial parities. This will be highlighted in the next chapter when applications of the model will be considered.

III.18 SPIN MATRIX ELEMENTS

We close this part by mentioning that the last matrix elements we consider are those of spin. The uncoupled scheme of spin is again an advantage, i.e. each single particle operator is going to act on the relevant spin quantum numbers. The spin operators denoted by $(O_s)^{\mu}$ whose matrix elements are given by

$$<s_1\, s_2|(O_s)^{\mu}|s_3\, s_4>, \tag{III.101}$$

are the components of the two vectors $\underline{\sigma}_2$ and $i\underline{\sigma}_1 \times \underline{\sigma}_2$ respectively. These components will be transformed first into the spherical ones to ease the evaluation of their matrix elements which are given in Appendix C.

Chapter IV

Results and Discussions

IV.1 INTRODUCTION

Expectations of describing nuclei in terms of a fundamental theory are idealistic. However, there are growing hopes that QCD can explain the basic NN interaction which has recently become an active field of research. From the resonating group method calculation, when only one gluon exchange and confinement between two quarks are considered, a repulsive core of the NN interaction can be obtained [10]. This means that one can expect to get some convincing information about the short range part of the NN interaction from the quark potential model. On the other hand meson exchange theory successfully predicts the aspects of the medium and long range parts of the NN interactions. This effective meson exchange is always added phenomenologically to the quark potential model [16]. It is better if the pion could be incorporated in terms of quarks and gluons consistently in the study of the NN interactions. Naturally this is obtained by the process of creating a quark-antiquark pair as shown by diagram (4) of Fig. 1. 1. Therefore the NN interaction is pictured by the creation of a meson from a gluon emanating from a quark in one nucleon, which is then absorbed by the other nucleon [19]. This process can be regarded as the equivalent to the one-meson exchange mechanism, while the equivalent two-meson exchanges could be described by a double excitation in addition to the process given by diagram (6) of Fig.1.1.

It is not the subject of this work to study the NN interaction in which these described processes are incorporated, but a first step to achieve this aim is to perform one-centre calculations. In other words the structure of the nucleon is studied where these quark-antiquark pairs are added in the model space. This is equivalent to a cloudy nucleon where the meson(s) is(are) genuine quark-antiquark pair(s) unlike the cloudy bag model

in which the meson is added phenomenologically as an elementary field [17].

The shell model has the advantage that the method of approximation is well-defined. Therefore it is used to investigate these quark-antiquark effects. We restrict the space to include the 0s and 0p shells with both positive and negative intrinsic parities to accommodate quarks and antiquarks. This is because of the practical limitations of the shell model code, but the seriousness of this space limitation will be discussed later.

As a first part of this program, we shall mainly focus on only allowing the creation of light quark-antiquark pairs and investigate their effects on the wavefunction of the nucleon and the delta. We move on to considering the pion and the ρ meson as well. The results of the calculations will be presented and discussed. We make further use of the model to include the strangeness quantum number. It turned out that other members of the baryon octet are also accessible to investigation.

IV.2 CHOICE OF MODEL PARAMETERS

As in any model, the current one has a set of parameters which has to be chosen judiciously in order to reproduce physical properties. As there has been so much work done in this field, a primary set will not be too hard to find [50]. The coupling constant α_s is determined by the NΔ mass difference and the parameter a_c is the strength of the confining potential determined to fit the observed mass of the nucleon. Next it is necessary to find the oscillator length parameter b of the particles, determined by the stability condition, i.e. the value of b that yields the minimum mass for the nucleon. The particle mass is assumed to be the same for all the quarks and antiquarks of the up and down flavours. Thus the set that describes the interaction best is given in TABLE II. This set is found to give correctly the properties of the nucleon and the delta regarded as three quark systems, and is chosen just as a starting point for our model calculations. Readjustment of these parameters is inevitable when we extend the basis space to incorporate quark-antiquark pairs. This will become clear in later sections.

TABLE II. Parameters of the simple quark model

m_{ns} MeV/c^2	m_s MeV/c^2	a_c MeV/fm^2	b fm	α_s
313	513	23.67	0.6	1.517

IV.3 THE NUCLEON: ITS STRUCTURE AND ENERGY

Now that the choice of the model parameters has been made, there are various ways in which we can test that the computer programs work properly. The masses of the nucleon and the delta when regarded as three quark systems, must be reproduced correctly. To check that the antiquarks are handled properly, a calculation to find the mass of the anti-nucleon and the anti-delta was done. Considered as three-quark systems and in the $0\hbar\omega$ oscillator space, the nucleon and the antinucleon masses were found to be 938 MeV each as expected. Therefore a primary conclusion is that the program dealing with the number-conserving part of the Hamiltonian does work properly. Other tests on the number-nonconserving part are also vital and possible to ensure the correctness of handling the different particles. The main one is switching off the number conserving part of the interaction potential and checking that the trace of the energy matrix is nil as the quark-antiquark pair creation effects are off-shell ones. Also, the computation of the expectation values of the spin, isospin and colour Casimir operators ought to give good values as the transition potential commutes with these operators. In other words, the component with the extra quark-antiquark pair must have the same values of these operators as the one without the quark-antiquark pair to ensure the description of the physical system we are studying. On the contrary, the expectation value of the Centre-of-mass operator in this approach is not expected to be an exact value i.e. $3/2\hbar\omega$ or $5/2\hbar\omega$... because of the existence of the operator $\bar{\sigma}_1 \cdot \bar{p}_2$ in the transition potential that tends to excite the centre of mass by a very tiny amount to the extent that it can be neglected. This rather unsatisfactory aspect of the transition potential is a residual consequence of the

transformation from the fully relativistic form of the interaction to the non-relativistic approximation for the potential [34].

We confine ourselves to the smallest possible space, namely the 0s and the 0p oscillator shells. The reason for this choice is that, although the three quarks in the core of the nucleon are in the 0s shell, the 0p shell must be included to allow the overall parity of the $3q(q\bar{q})$-component to be even, as the antiquark has odd intrinsic parity.

We first look at the effects of creating light pairs ($u\bar{u}$ and $d\bar{d}$). The number of basis states for the nucleon's calculation was 312 including 3q and $3q(q\bar{q})$ states which are mixed by the transition potential. The importance of the mixing is presented in TABLE III where we see that the 3q component (84.8%) is dominant as, one might expect. However, the amplitude of the $3q(q\bar{q})$-component accounts for the remaining 15.2% and is a quite significant proportion of the wavefunction. We also note that it is much more probable that the added $\bar{q}$ is in the 0s shell than that it is in the 0p shell. This indicates that the internal forces between the antiquark $\bar{q}$ and a quark are stronger than between two quarks.

TABLE III. Composition of the N's and Δ's w/functions
Here the $3q(q\bar{q})$ components include $u\bar{u}$ and $d\bar{d}$ pairs only
a_c=23.67 MeV/fm^2

particle	3q-component	(3q)($q\bar{q}$)-component	
	$(0s)^3$ %	$(0s)^3(0\bar{s})(0p)$ %	$(0s)^4(0\bar{p})$ %
N	84.8	10.8	4.4
Δ^+	83.8	13.6	2.6

The $3q(q^2\bar{q}^2)$ states obtained by the action of the transition potential on $3q(q\bar{q})$ states should also be included. This increases the size of the basis to 24173 states. We have found that the amplitude of $3q(q\bar{q})$ has almost doubled from the previous results. The other interesting result is the considerable contribution of the $3q(q^2\bar{q}^2)$-component accounting for 8.5% of the nucleon's wavefunction which again would have been of

more importance if the effects of diagram (6) were also considered. The reason that we did not consider them is that they are vacuum-polarization effects which we have ignored. The spatial part of the $V_{\to q^2\bar{q}^2}$ potential is given by

$$V(\vec{r}) = -\frac{1}{\pi}\frac{1}{m}\frac{1}{r^2} + \pi\frac{1}{m}\delta(\vec{r}) \text{ where } \vec{r} = \vec{r}_1 - \vec{r}_2 \qquad \text{(IV.1)}$$

and there is no information on the dependence on other coordinates [51].

Since there is no explicit specification of the quantum numbers of the $q\bar{q}$ pairs created in our treatment (as we are only interested in $3q(q\bar{q})$- and $3q(q^2\bar{q}^2)$-components having the quantum numbers of the nucleon), the $3q(q^2\bar{q}^2)$ components include the possibility that there are two mesons around the nucleon and also the possibility that the seven particles form a collective state which cannot be decoupled into hadrons with the quantum numbers of physical particles. This will have an impact when it comes to study the medium and the long range attractive part of the N-N interaction believed to be due to the exchange of two and a single pions respectively. Our model generates the N-N interaction at long ranges by the following process: one cloudy baryon in the vicinity of another will exchange $q\bar{q}$ or $q^2\bar{q}^2$ clusters if the two baryons are sufficiently close.

When off-shell effects are considered, the lowest eigenvalues in the spectrum are lowered and the highest ones pushed higher up with no contribution to the trace of the matrix. For the calculation including $3q(q\bar{q})$ configurations the masses of the nucleon and the delta were respectively found equal to 680 and 954 MeV, a lowering in energy of about 250 MeV for a strength of the transition potential equal to 47.2 MeV.fm^2 obtained for α_s=1.517 (see expression III.75). For the space extension to $3q(q^2\bar{q}^2)$ terms, a further lowering was recorded with corresponding energies for the nucleon and the delta of 531 and 843 MeV respectively.

Although these results give us an idea about the proportions of different components constituting the wavefunction of the nucleon, they should not be trusted since the model

parameters are not set to give its observed mass. Therefore it is too early at this stage to ask which composition describes the more realistic physics of the nucleon. Also we cannot condemn the model because the composition of the nucleon has changed drastically as expected with the addition of the $5q\text{-}2\bar{q}$ configurations. Harmony will be restored and a clearer picture of the structure of the nucleon will be revealed after the right choice of parameters has been made.

IV.4 PRELIMINARY RESULTS ON THE PION

Chiral quark models with relativistic (massless, or almost massless) quarks confined in a bag have been introduced in order to unify the picture of the nucleon as a three-quark core surrounded by a pion cloud [52]. One of the drawbacks of these models is the choice of one of their parameters: the size of the model space used in performing perturbative calculations. Limitations and uncertainties are related to this latter point together with the centre of mass corrections. In these models, the pion is treated as a phenomenological structureless Goldstone boson (in the exact chiral limit the pion is massless). There is no relationship to an underlying quark-antiquark structure at this level. The only reminiscence of the pion intrinsic structure is the appearance of the pion decay constant and the mass m_π.

The simplest picture of the pion is that of a single quark-antiquark pair in the (0s) orbit of a confining potential. In the absence of the hyperfine interactions, the pion is degenerate with the ρ meson. If a hyperfine splitting is introduced consistent with the mass splitting between nucleon and the $\Delta(1232)$, the pion experiences a significant downward shift with respect to the ρ meson, but its mass is still too large, typically 300-400 MeV and these after introducing centre of mass corrections. The procedure for doing this is very ambiguous in most of the models but straightforward in our approach. In bag or potential models, there is nothing special about the pion. Its low mass, however, does not arise in any natural way.

A completely different picture is that of the pion as a collective quark-antiquark excitation of the non-pertubative vacuum. This makes the pion very special as compared

to all other mesons. This notion has a far-reaching analogy with low-energy collective modes in strongly interacting many-body systems, such as nuclei. In that sense, the pion is analogous to a low-lying, highly collective particle-hole state, whereas the heavier mesons correspond to non-collective particle excitations. The collectivity and other mechanisms, such as the admixture of quark-antiquark plus gluonic states, all have the tendency to lower the probability of finding the pion in a single quark-antiquark configuration. One question which might be worthwhile trying to answer in this section is: as expected for highly collective states, has the pion wavefunction strong admixtures of multiquark-multiantiquark components? These would be closely related to the ground state corrections in RPA (Random Phase Approximation) descriptions of low energy particle-hole states. We write the pion state [53] as

$$|\pi> = a|q\bar{q}> + b|qq\bar{q}\,\bar{q}> + \ldots, \qquad \text{(IV.2)}$$

then try to derive the values of the coefficients a and b and calculate its mass by using our newly obtained potential which works for the nucleon. Our results will be discussed and compared with experiment and with results from other models.

To begin with, we shall investigate the effects on the pion mass of creating light pairs only. We fix the quantum numbers of the calculation. They are the z-projections of the total angular momentum and the flavour which have values of 0, and 1 respectively, and an odd overall parity. We note here that the three pions corresponding to the three charge states which are the three isospin projections of T=1, are degenerate because the interaction potential is isospin scalar and does not include a Coulomb term. Therefore we shall not distinguish between π^-, π^0, and π^+. Although the $0\hbar\omega$ working space with both positive and negative shells can accommodate the $q\bar{q}$ states, the inclusion of the 0p shell is necessary for the $qq\overline{qq}$ states for parity reasons, therefore we choose to work in the 0s 0p space which allows a greater distribution of particles in different shells. Restrictions on the number of particles in each shell can also be imposed.

The first calculation we performed in an attempt to describe the pion by this improved quark potential model was done by extending the spatial space to include only the 0p shell for antiquarks for reasons of parity conservation. The number of basis states is 80. For consistency with the nucleon calculation, the set of parameters of TABLE II has been used. The off-shell effects have lowered the mass of the pion from 350 MeV to 260 MeV with 94% contribution of the $q\bar{q}$ component and the remaining 6% for the $q^2\bar{q}^2$ component. They are interesting results in the sense that there is an improvement of the mass of the pion given by the usual quark models and also the collective nature of the conventional description of the pion starts to appear.

The next step in this type of shell model approach is to extend the space even further to include the 0p shell for quarks. All the particles are allowed to occupy all the states permitted by the Pauli Principle. The number of such states was found to be 1296. However we note the dramatic decrease of the mass of the pion to only -76 MeV and the wavefunction composition of 77% and 13% for $q\bar{q}$- and $q^2\bar{q}^2$- components respectively. The probabilities of finding the $q\bar{q}$ state in $(0s)^1(0\bar{s})^1$ and $(0p)^1(0\bar{p})^1$ are 67.4% and 9.4% respectively as displayed in TABLE IV.

TABLE IV. Composition of the π's and ρ's w/functions
The $(q^2\bar{q}^2)$-components include $u\bar{u}$ and $d\bar{d}$ pairs only
a_c=23.67 MeV/fm^2

particle	$(q\bar{q})$-component (%)			$(q^2\text{-}\bar{q}^2)$-component (%)		
	$(0s)^1(0\bar{s})^1$	$(0p)^1(0\bar{p})^1$	$(0s)^1(0\bar{s})^2(0p)^1$	$(0s)^2(0\bar{s})^1(0\bar{p})^1$	$(0p)^2(0\bar{s})^1(0\bar{p})^1$	$(0s)^1(0p)^1(0\bar{p})^2$
π(-76)	67.4	9.6	5.4	5.4	6.1	6.1
ρ(99)	47.2	25.4	5.6	5.6	8.1	8.1

The results for the ρ are also of interest. The $q\bar{q}$ component is still fairly high at 72.6% but only 47.2% of this comes from the $(0s)(0\bar{s})$ configuration. The result for the mass

was expected because the set of parameters used was set for the nucleon's calculation without the off-shell effects due to the quark-antiquark pairs, therefore the parameters require readjustment for the case of the nucleon. This will be discussed in the second part of this chapter along with inclusion of the strangeness quantum number and, accordingly, the fitted parameters are used to find the mass of the pion and to investigate whether the $q^2\bar{q}^2$- component contains any strangeness.

IV.5 STRANGE FLAVOUR ADDED TO THE SPACE

In this second part, extension of the model to allow the creation of strange quark-antiquark pairs is considered. Incorporation of the strangeness quantum number is discussed. The question of whether the nucleon contains "strange things" [32] is examined and so is that of the pion being a collective state. Finally, the wavefunctions of baryons with the strangeness quantum-number different form zero are investigated.

IV.5.1 EFFECTS OF STRANGENESS ON THE BASIS STATES

Inclusion of the strangeness quantum number requires a formulation which incorporates flavour SU(3) instead of the isospin, the s and the $\bar{s}$ have I=0 in the SU(2) subgroup[13]. Therefore the affected parts by this strangeness implementation in the model space are discussed in the following subsections.

IV.5.1a SINGLE-PARTICLE STATES

The way the single-particle basis is generated remains the same with the exception that, for the same spatial space, it tends to get larger by a factor of 3/2 as we consider the three flavours instead of just two as previously seen. Some particle properties are summarised in TABLE I. As an example of this space extension, recall that to study the ground states of light systems requires a $1\hbar\omega$ excitation space for parity reason. In this space there are 96 single-particle orbits for the up and down flavours, and the inclusion of the strange flavour increases the number of single-particle states to 144. Consequently, the number

[13] see page 55 of reference [31]

of many particle-basis states will increase too. However, we shall expect additional complications in the process of generating these SDs.

IV.5.1b MANY-PARTICLE STATES

The two major additions brought about by an extension of the model are the specification of the charge of the system under investigation and the number of strange quarks we want it to have regardless of any strange quark-antiquark pairs that could be created by the action of the transition potential. The latter stems from the former as the evaluation of the charge of the SD is no longer given by the z-projection of flavour which is the difference in number of the up and down quarks as in part one, but has to be worked out separately. In other words, the strange particles do not account for z-projection of flavour, whereas they do contribute to the charge of the system which has then to be determined. Technically, for the generation of SDs, we must keep track of the number of strange and non-strange particles for each colour code in order to evaluate the overall charge and whether the number of strange particles is exceeded. Therefore, the flavour of the calculation and the charge of the system must be specified separately. The construction of SDs remains the same as previously discussed in Chapter II with the colour condition $R \leq G \leq B$. We note that for the case of SDs with the extra quark-antiquark pairs there are now the extra SDs with strange pairs in them.

It can also be seen that, when acting with the number non-conserving part of the Hamiltonian on the SDs, for each particle destroyed, there are more triples of particles to be created as now there is the possibility of the following diagram

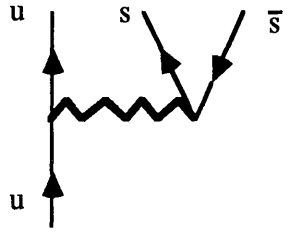


Fig. 4. 1 Strange-pair creation from the destruction of a light quark

and similarly with the down and strange quarks. We should expect a very large basis if we wanted to investigate the double excitation for the case of the nucleon, but we shall see later in the discussion of the results for the nucleon calculations that it is not worthwhile trying it at all.

IV.5.2 SYMMETRY BREAKINGS OF THE HAMILTONIAN

Three terms in the Hamiltonian actually break the SU(3) flavour symmetry:

1) the mass term i.e. the strange s quark is heavier than the u, d quarks which gives only a constant shift to the total energy
2) the kinetic energy term $p^2/2m$
3) the potential term i.e. the contact two-body term in (III.8) is mass dependent.

Among them the mass term (1) gives only a constant shift of the total energy for a given strangeness. The kinetic energy term (2) contributes mainly to the internal energy of the system, giving again a constant shift for a given strangeness. Although the introduction of different masses makes our procedure highly complicated, we shall not neglect in the following the symmetry breaking due to the kinetic energy and, most importantly, the symmetry breaking in the strength of the contact potential term.

IV.5.2a SYMMETRY BREAKING OF THE KINETIC ENERGY

This symmetry breaking of the kinetic energy has been neglected by all authors in this field because of its complexity, especially by those using the resonating group methods (RGM) to study the nucleon-nucleon interaction [54,55]. Therefore one would infer that the solution to this problem is by no means straightforward, but that does not mean that it is not feasible. The way our model is set to handle a variable number of particles for one specific calculation suggests that it will not be too hard to implement variable masses of particles as well. We propose, as we did earlier on in Chapter II, to treat all the two-body operators representing physical observables in a way that does not require a knowledge

of the total mass of the many-body system. Terms depending on the total mass will be taken care of when acting with the Hamiltonian on SDs at which point the mass number becomes known. This is going to be highlighted in the current section.

The general form of the kinetic energy operator was given by (II.47). This is no longer valid as the mass term is variable and cannot be taken out of the summation sign. Therefore we write the kinetic energy operator as

$$T=\sum_{i=1}^{A}\frac{p_i^2}{2m_i}-\frac{(\sum_{i=1}^{A}p_i)^2}{2\sum_j m_j} \tag{IV.3}$$

where the particle-mass dependence of the particles is explicit. Before going any further, we wish to point out that we do consider here that the symmetry of the isospin is exact and therefore we shall neglect the difference in mass of the up and down quarks (antiquarks) in the foregoing analysis. From now on, any difference in mass of the particles will mean between strange and non-strange particles (quarks and antiquarks). We write the kinetic energy operator in equation (IV.3) as follows

$$\sum_{i=1}^{A}\frac{p_i^2}{2m_i}-\frac{\sum_i p_i^2+2\sum_{i<j}p_i\cdot p_j}{2\sum_j m_j} \tag{IV.4}$$

which can be put as a one- and a genuine two-body terms as follows

$$\sum_{i=1}^{A}(1-\frac{m_i}{\sum_j m_j})\frac{p_i^2}{2m_i}-\sum_{i<j}\frac{p_i\cdot p_j}{\sum_j m_j}. \tag{IV.5}$$

We treat the two-body term as in Chapter II. In working out the many-body Hamiltonian, the occupied orbits of each SD are found, the mass of that particular term is calculated,

and used to complete the calculation of the two-body term. The single particle part of (IV.5) is slightly more complicated to evaluate. For each occupied single-particle state, the expectation value of the single-particle operator $p_i^2/2m_i$ is given by

$$\hbar\omega_i(2n_i+\ell_i+1.5)/2 \qquad \text{(IV.6)}$$

where n_i and ℓ_i are the principal and orbital quantum numbers respectively of the occupied orbit i, and $\hbar\omega_i$ is its oscillator energy. We also have to multiply each single-particle term by $(1 - m_i/\sum_j m_j)$ which is written as $m_i(1/m_i - 1/\sum_j m_j)$, in which case

$$\hbar\omega m_i = \hbar^2/b_i^2 \qquad \text{(IV.7)}$$

if we have made use of $m_i\omega = \hbar/b_i^2$. Therefore the only assumption we make in this process is that the strange particles have the same oscillator length parameter as the non-strange one while, on the other hand, the explicit mass-dependence of the one-body operator is considered.

IV.5.2b SYMMETRY BREAKING OF THE CONTACT TERM

The symmetry breaking in the strength of the contact potential term [54] is the most important one. We introduce the SU(3) breaking as follows

$$\left\{\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{2}{m_i m_j}\frac{2}{3}\bar{\sigma}_i\cdot\bar{\sigma}_j\right\}\delta(\bar{r}_{ij}) \qquad \text{(IV.8)}$$

where the two indices i and j stand for two strange, two non-strange or one strange and the other non-strange interacting particles. By virtue of the extension of the model to include the strangeness quantum number, other members of the baryon octet are also accessible to investigation. In the next section, we shall see what effect the mass difference of particles has on the hyperfine mass splitting by taking the example of

baryons with S=0 and S=-1 respectively as three quark systems.

IV.5.2c COMPARISON OF THE ΔN AND Σ^*-Λ(Σ) SPLITTINGS

Let us forget for a while that these particles have mesonic clouds. If we replace one of the three quarks in the Δ by a quark of different flavour[14], i, where i = s,c,... then we obtain Σ_i^* (iqq). The nucleon has the quarks pairwise in either I = 1 or 0, and upon replacing the third quark by i yields respectively Σ_i (iqq) or Λ_i (iqq) states. For the choice of i ≡ s we have the familiar states

Σ_s^* (sqq) (1385) Σ_s (sqq) (1193) Λ_s (sqq) (1115)

We see that the act of substituting a strange quark for a u or a d quark in which m_s is 150 to 200 MeV/c^2 greater than m_u has:

1) increased the mass of the three quark system by around 150-200 MeV/c^2
2) decreased the $3/2^+$ - $1/2^+$ mass splittings
3) split the Λ(I=0) and Σ(I=1).

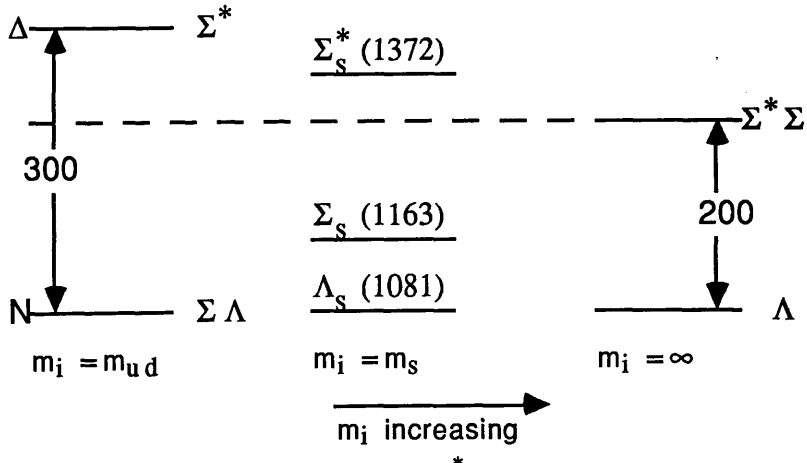


Fig. 4. 2 Hyperfine splittings of $(\Sigma^* \Sigma \Lambda)_i$ system as m_i increases from exact symmetry ($m_i = m_{ud}$) to infinity

[14] see page 388 of reference [31]

Since the hyperfine splittings are inversely proportional to the quark masses, then Σ^*-($\Sigma\Lambda$) will be in turn less split than the Δ-N (Fig. 4. 2). Finally, the overall spin-unitary spin symmetry of the wavefunction requires that the non-strange pair in Λ has spin 0 and in the Σ has spin 1. This will lead to different expectation values of the spin-spin operators for the states and hence to different masses.

IV.5.2d SYMMETRY BREAKING OF THE TRANSITION POTENTIAL

The last term of great relevance to the present study which is affected by the symmetry breaking is the form of the transition potential where the masses of the destroyed particle and created ones ought to be given explicitly. Yu and Zhang [51] give this required form as follows

$$\frac{i}{16c}\alpha_s(\lambda_1\cdot\lambda_2)[\frac{1}{2}(\frac{1}{m_1}+\frac{1}{m_2})\frac{1}{\rho^2}(\bar{\sigma}_1\cdot\bar{n})+\frac{i}{2m_2\rho^2}(\bar{\sigma}_1\times\bar{\sigma}_2)\cdot\bar{n}+\frac{i\sqrt{2}}{m_2\rho}(\bar{\sigma}_1\cdot\bar{p}_2)]\cdot \qquad \text{(IV.9)}$$

Therefore there will be extra complications in the computation of two-body matrix elements as it will be necessary to identify which type of particle is destroyed and which ones are being created. This can be done with an extra test on the flavours.

IV.6 APPLICATION TO S=-1 BARYONS

A simple test of this symmetry-breaking process is to study the baryons with the strangeness quantum number equal -1. In a first step towards a full investigation of these baryons by our model, only light pairs are allowed to be created. The quantum numbers of the calculations are set. They are the z-projections of isospin and total angular momentum whose values are 0 and 1/2 respectively. The charge and the strangeness quantum number are 1 (in units of e) and -1 respectively. Restrictions on shell occupancies by forcing three quarks in the 0s shell are also set as we are only interested

in ground states. The two-particles of the created pairs are free to settle in any orbits of the (0s 0p) space as long as the requirements of the Pauli Principle and overall parity are satisfied. In such conditions the number of basis states, which is expected to be larger than that of the case of the nucleon because the Pauli Principle is less effective due to the presence of the strange quark, is 1128. Using the set of parameters of TABLE II, the found masses of the Lambda(Λ) and the two Sigmas(Σ,Σ^*) are 1037, 1113 and 1292 MeV respectively with the transition potential switched off. In other words, these masses are those of the three particles considered as three-quark systems and are in good agreement with the observed ones [56]. We expect a lowering of these masses when we allow the three-quark, four-quark and antiquark states to mix by the transition potential. The recorded values of the three masses are 794, 916 and 1066 MeV respectively. Allowing the creation of strange pairs as well has lowered the masses to 736, 818 and 995 MeV respectively. In a later section, the adjusted parameters to fit the mass of the nucleon will be used to see their effects on these states.

IV.7 APPLICATION TO THE NUCLEON

The question whether the nucleon contains "strange things" [32] is an intriguing one. That could be a wide question, asking if the nucleon could consist of different particles besides the conventional up and down quarks. However, we take this question in its naive form to infer that the strange things mean inclusion of the strangeness quantum number and find out how much strangeness there is in the wavefunction of the nucleon if the creation of strange $q\bar{q}$ pairs could also be included in the model space (e.g. $u \rightarrow us\bar{s}$).

The calculation was limited to include 3q and 3q($q\bar{q}$) states only because the number of basis states was 687 and the extension to 3q($q^2\bar{q}^2$) configuration results in 124031 basis states which is too large for us to handle. The lowering in energy is an extra 90 MeV for the addition of the $s\bar{s}$ pair, summing up to the 250 MeV after the introduction of the $u\bar{u}$ and $d\bar{d}$ pairs: an overall lowering of 340 MeV. The masses of the nucleon and the delta were now 591 and 884 MeV respectively. The composition of the wavefunction of the

nucleon is displayed in TABLE V. If we compare the new partition to that in TABLE III, there is only an extra 3% into 3q($q\bar{q}$)-component due to the inclusion of the ($s\bar{s}$) pair creation: we conclude that the cloud surrounding the 3q core is mainly non-strange.This is a remarkable result indeed [57] as it agrees with the conclusion that MacGovern *et al.* [58] came to that the strange-quark content in the nucleon is less than 5%.

TABLE V. Composition of the N's and Δ's w/functions
Here the 3q($q\bar{q}$) components include $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs
a_c=23.67 MeV.fm^{-2}

particle	3q-component	(3q)($q\bar{q}$)-component	
	$(0s)^3$ %	$(0s)^3(0\bar{s})(0p)$ %	$(0s)^4(0\bar{p})$ %
N	81.3	13.6	5.1
Δ^+	80.9	16.1	3.0

A good picture of the effects of incorporating quark-antiquark pairs on the masses of S=0 and S=-1 baryon states is depicted by Fig. 4. 3. It is obtained by using the set of parameters given in TABLE I. Part (1) is a result of treating these particles as three quark systems. Part (2) is obtained by allowing these particles to have non-strange mesonic clouds. Part (3) is an outcome of implementing effectively the strangeness quantum number. The expected result, that the transition potential lowers the lowest eigenvalues of the spectrum as it has off-shell effects, is seen clearly. The other interesting result is that the downward shift in energy has not affected the hierarchy in which the particles usually come when treated as three-quark systems. However, the splittings between particles is affected by such a treatment. This was expected and is natural because of the contribution of the mesonic cloud to these splittings [50,59]. We would not worry too much about the values of these splittings as the approximation was made that the strange particles have the same oscillator length parameter as the non-strange ones.

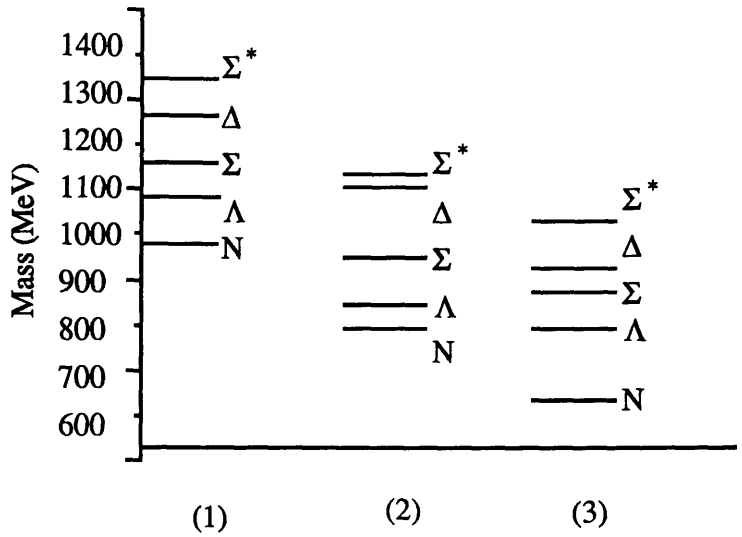


Fig. 4. 3 Downward shift of the masses of the S=0 and S=-1 baryons as quark-antiquark pairs are gradually allowed to be created for b=0.6 fm and a_c=23.67 MeV.fm^{-2}

IV.8 IMPORTANCE OF THE CONFIGURATION MIXING

The measurement of the configuration mixing is governed by the strength of the transition potential. Obviously the bigger this strength, the more important is the configuration mixing. However, the strength of the transition potential is fixed by the choice of the quark-gluon coupling constant (see equation IV.9) . This latter should take values around unity to fulfil the requirement of a perturbative treatment. Therefore we investigate the importance of the mixing in terms of fractions designated by λ of the strength of the transition potential. To clarify this, we propose to write the Hamiltonian as

$$H = H_{2\to2} + \lambda V_{1\to3} \qquad \text{(IV.10)}$$

where the immediate consequence on the previously given energy matrix (II.23) is

$$\begin{bmatrix} A & \lambda C \\ \lambda C & B \end{bmatrix} \qquad \text{(IV.11)}$$

whose eigenvalues are determined. We examine the effects of varying the values of λ on the mass of the nucleon. The results of the calculations are plotted in Fig. 4. 4.

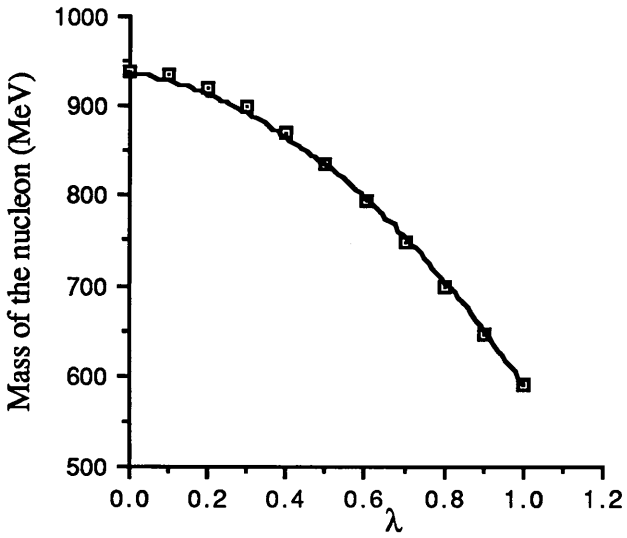


Fig. 4. 4 Mass of the nucleon as a mixture of 3q- and 3q($q\bar{q}$)-components including light and strange pairs versus the strength of the transition potential for b=0.6fm and a_c=23.67 MeV.fm^{-2}

Clearly for the value of λ equals zero, there is basically no mixing and the nucleon is a pure three-quark system and of mass the observed one. For the value of λ equals one, the mixing is maximum and the mass of the nucleon is minimum when the off-shell effects are at their peak. In between, the mass of the nucleon experiences a smooth parabolic shape between the two extreme values for λ equals zero and one respectively. This is exactly what is expected since the introduction of λ at an off-shell level has the effect of the term $-\lambda^2C^2$ in the characteristic equation instead of $-C^2$. Moreover this can be considered as a partial test for the correctness of our computer codes.

IV.9 RE-EVALUATION OF THE MODEL PARAMETERS

When we readjust the parameters to give the known mass of the dressed nucleon, an important consideration is the value of the strong interaction coupling constant. To maintain consistency with the idea of treating the one-gluon exchange as a perturbative

correction, values of α_s around unity are acceptable. Therefore the value of α_s fixed in TABLE II is satisfactory. The other parameter that needs consideration is the oscillator length parameter b. A search for the value of b that gives the stability condition, i.e. the value of b for which the mass of the nucleon regarded as a mixture of 3q and $3q(q\bar{q})$-components is minimum, was done. The recorded variations of the mass of the nucleon versus the variation of the oscillator length parameter b is plotted in Fig. 4. 5.

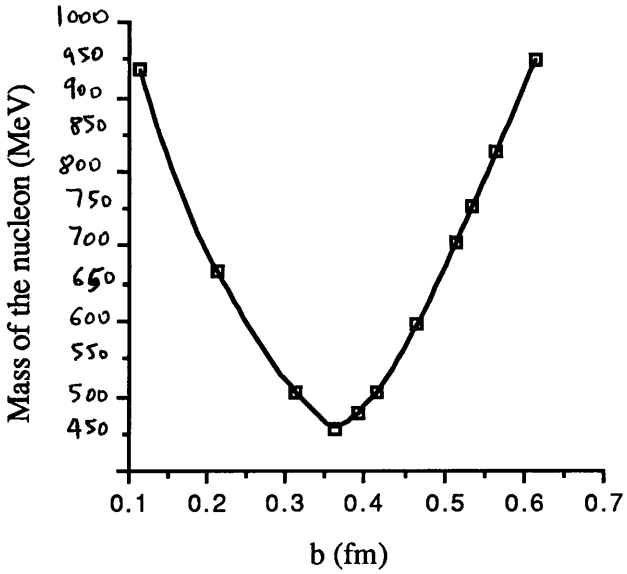


Fig. 4. 5 Mass of the nucleon as a mixture of 3q- and $3q(q\bar{q})$-components including light as well as strange pairs versus the oscillator length parameter b for a strength of the confining potential a_c=23.67 MeV.fm^{-2}.

We note that the mass of the nucleon is minimum for a value of b around 0.35 fm. This is by far the lowest value of b of all the ones given in the literature [30,54,55] when a similar treatment of the nucleon is done. However, this value of b did not surprise us much since this value of b is in some sort an average of the one of strange and non-strange particles. Moreover, as far as we are aware, no-one in the literature has included the strange pairs because of the complexities they will involve. Therefore, this value of b is not very reliable because we assumed the same value of b for both strange and non-strange particles. Consequently, in the rest of this approach, we keep the value of b of

0.6 fm.

Accordingly, the previous set of parameters which gives good results for the bare nucleon was retained, except that the strength of the confining potential was varied because of its dubious phenomenological origins and also because in previous studies [60], it was shown that a great many phenomena relevant to the baryons and their interaction are insensitive to this parameter. The variation of the mass of the nucleon with a_c is illustrated in Fig. 4. 6, showing that there is an almost linear increase of the mass of the nucleon when a_c increases. The value of a_c which gives the masses of the nucleon and the delta is 59.52 MeV.fm^{-2}. The other interesting result is that, while varying a_c, the mass splitting between the nucleon and the delta remained constant and equalled the observed value. In the simple quark model, the Δ and the nucleon are made up of three up and down quarks in overall S-wave with spins coupled to 3/2 or 1/2 respectively. The mass separation of about 300 MeV between these states is hypothesized to be a manifestation of QCD hyperfine splitting. The splitting is proportional to the product of the colour-magnetic moments of the quarks, defined in analogy to their electromagnetic moments. Even with the addition of the $q\bar{q}$ cloud, the structure of the nucleon and Δ are very similar and so the mass splitting is almost independent of the strength of the confining potential. We note that even for this value of the confining potential strength which gives the observed mass of the nucleon, the stability condition is not observed for b equals 0.6 fm but for 0.45 fm.

For the fitted parameters, the final composition of the wavefunction of the nucleon is given in TABLE VI. We see that the proportion of the 0p shell particles has been reduced. This can be understood because a stronger confining potential will raise the effective energy of a 0p shell particle.

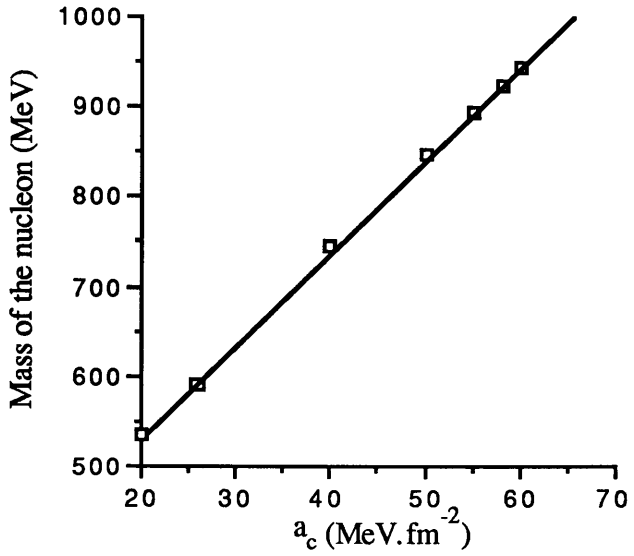


Fig. 4. 6 Mass of the nucleon as a mixture of 3q- and 3q($q\bar{q}$)-components including light and strange pairs versus the strength of the confining potential for b=0.6 fm.

One can estimate the seriousness of the restriction to the 0s and 0p shells only by considering results for three-quark calculations, without the additions of $q\bar{q}$ pairs with the final set of parameters. The results for the energy of the nucleon in the 0s space only is 1248.5 MeV. Adding the 0p and the (1s 0d) shells the energy becomes 1198.3 MeV, and adding the (1p 0f) and (2s 1d 0g) shells, the energy is 1129.7 MeV. Clearly these changes are smaller than the changes brought about by adding $q\bar{q}$ configurations, even with restrictions to the 0s and 0p shells.

We do not leave this section without mentioning the strongest test of all that has been made at this stage regarding the proper handling of antiquarks in our computer programs, which is to treat the antinucleon on an equal footing as the nucleon, i.e. find out the importance of its mesonic cloud and compare it to that of the nucleon. Therefore, with the exception of the overall negative parity and an overall charge of -e, the same quantum numbers as those of the previous nucleon calculation were set. The situation is just symmetrical in the nature of particles as the case of the nucleon. Nonetheless we would not have been too surprised if the outcome of such calculation does not give the observed

masses of the anti-nucleon and the anti-delta as different pair excitations might differ in the two cases. However, if we think of it, to each diagram in the one gluon exchange theory there corresponds a time reversal one. Therefore if the initial conditions are the same meaning by that the working spaces and the shell occupancies are provided, we expect the two cases to be identical. Indeed, the found masses of the anti-nucleon and the anti-delta are 938.5 and 1232.8 MeV respectively which match perfectly those of the nucleon and the delta and therefore so does their hyperfine splitting. Their wavefunction compositions are found to be exactly the same as those of the nucleon and the delta displayed in TABLE VI by reading each given shell by its negative one, for example, replacing $0\bar{s}$ by 0s and so on. Hence, we are confident that all codes in the programs do work correctly.

TABLE VI. Composition of the N's and Δ's w/functions
Here the 3q($q\bar{q}$) components include $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs
a_c=59.52 MeV/fm^2

particle	3q-component	(3q)($q\bar{q}$)-component	
	$(0s)^3$ %	$(0s)^3(0\bar{s})(0p)$ %	$(0s)^4(0\bar{p})$ %
N	85.0	10.8	4.2
Δ^+	84.6	12.9	2.5

Now that the parameters of the model have been adjusted, some predictions can be made concerning the second order excitations already discussed. We noted that we cannot allow the strange pairs in this order to be created because of the vast number of basis states and, therefore, the calculation would not be possible due to the many computing difficulties. However, the introduction of strange pairs as well as light ones in the first order excitations has roughly the same effects as the second excitations for light pairs only. In other words, the diagonal energies due to the strange ($s\bar{s}$) pairs is almost the same as two light pairs because we have:

$$s\bar{s} = \ell\bar{\ell} + 2*(513 - 313) = \ell\bar{\ell} + 400$$

and

$$\ell^2\bar{\ell}^2 = \ell\bar{\ell} + 2*313 = \ell\bar{\ell} + 626.$$

Subsequently we can guess the value of the strength of the confining potential that gives the observed mass of the nucleon being a mixture of 3q, $3q(q\bar{q})$ and $3q(q^2\bar{q}^2)$ configurations. Indeed it was found to have the value of 59.52 MeV.fm^{-2} for which the found probabilities for each component are displayed in TABLE VII.

TABLE VII. Composition of the N's and Δ's w/functions
The $3q(q\bar{q})$ and $3q(q^2\bar{q}^2)$ components include $u\bar{u}$ and $d\bar{d}$ pairs for a_c=59.52 MeV/fm^2.

particle	C_0 %	C_α %	C_β %
N (939)	73.2	22.3	4.5
Δ^+ (1232)	76.2	20.4	3.4

We can now discuss the results of TABLE VII. The pure three-quark configuration is dominant representing 73%, and a three-quark configuration with quark-antiquark pairs representing the rest of the wavefunction. We have constructed efficiently the nuclear wavefunction from different configurations. Although the order was up to two only because of the limitations due to computing difficulties, this picture of the nucleon is far better than the first order only, or what has been given qualitatively by Faessler [61]. Naturally the nucleon wavefunction as a single entity is in reality more complicated and is even more complicated if the nucleon is inserted into a nucleus where the different configurations will interact with the surrounding ones.

IV.10 EFFECTS OF STRANGE ($q\bar{q}$) PAIRS ON S=-1 BARYONS

In a similar fashion, we want to examine the effects of strange $q\bar{q}$ pair on the other members of the baryon ground state octet with S=-1. In other words, to see how cloudy these baryons are and also to find out if the information about their masses is still reproducible if the readjusted set of parameters for the nucleon and the delta is used. First of all, we found that this set gives very nearly the observed masses of the Λ, Σ $(1/2^+)$ and the $\Sigma^*(3/2^+)$. The calculated masses of the $\Lambda(\Sigma)(1/2^+)$ and the $\Sigma^*(3/2^+)$ are 1081.3 (1163.3) and 1339.6 MeV respectively compared with the ones given by the Particle Data Group [56] which are 1115.6 (1192.5) and 1383.7 MeV. The splittings of 176.3 MeV (258.3) are not in a very good agreement with the observed ones which are 191.2 MeV (268.4). That might be because we assumed that the non-strange and strange quarks (antiquarks) had the same oscillator length parameter b whereas the value of b for a strange quark should be less than that for a non-strange one as the oscillator parameter is inversely proportional to the mass. We could have considered it in the radial part of the single particle wavefunction but it leads to extra complications when it comes to the evaluation of all the transformation brackets and integrals of matrix elements.

TABLE VIII. Composition of the Λ's and the Σ's w/functions
Here the 3q($q\bar{q}$) components include $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs
a_c=59.52 MeV/fm^2

particle	3q-component	(3q)($q\bar{q}$)-component	
	$(0s)^3$	$(0s)^3(0\bar{s})(0p)$	$(0s)^4(0\bar{p})$
	%	%	%
$\Lambda(1081)$	86.2	9.4	4.4
$\Sigma(1163)$	86.3	10.2	3.5
$\Sigma^*(1340)$	86.1	11.4	2.5

The other interesting results are presented in TABLE VIII. The 3q($q\bar{q}$)-component represents 13.5% of the total strange baryons' wave functions, to be compared with

15.2% of the nonstrange one (TABLE VI). This means that the strange baryons are less cloudy than the nonstrange baryons due to the fact that the strange quark interacts less with the nonstrange ones: a confirmation of the well known fact that the hyperon-hyperon (-nucleon) interaction is less attractive than the nucleon-nucleon one.

IV.11 RESULTS ABOUT THE PION

If considered as a quark-antiquark system, the pion is by no means going to contain strange particles because of its low observed mass of 140 MeV. Therefore, as one might naively, expect inclusion of the strangeness quantum number is going to push higher the mass of the pion and which should not be there in the first place. However, this strangeness is going to be included at an off-shell level and hence a lowering in energy is justifiable and could lead to the observed mass of the pion which has not been predictable by any of the quark models around.

By virtue of our model, the strange quark-antiquark pair is going to appear in the wavefunction of the pion as a result of the excitation of the up or the down quark supposed to constitute the meson as depicted earlier on by Fig. 4. 1. We will then be able to assess the importance of the strangeness if there is any in the wavefunction of the meson in the same manner as we did with the nucleon.

The major idea behind this section is to calculate the mass of the pion from our model using consistently the parameters that reproduce the properties of the nucleon. We varied only the strength of the confining potential to find the mass of the nucleon, regarded as a mixture of three quark- and four quark and an antiquark system, and we found that this parameter has a value of 59.52 MeV/fm^2. This value of the strength potential gives the values of 39.2 and 293.4 MeV for the pion and the first excited state. This excited state has the quantum numbers of the ρ meson. We refer to it as the ρ meson even though the ρ meson is mainly a $q\bar{q}$ state. The results of the calculation are shown in TABLE IX. The result for the mass of the pion is not a bad one since assumptions once again about the oscillator length parameter for strange and non-strange quarks were considered to be the

same. Also a gain in the mass can be achieved through several channels, for example changing the masses of the constituent quarks and antiquarks. The probabilities of the two components $q\bar{q}$ and $q^2\bar{q}^2$ are 90.3% and 9.7% respectively. Our result does not agree with the one given in the literature [52] where the valence $q\bar{q}$ component in the total pion wavefunction is 20-25%. This is due to the fact that the parameters of the potential have not been chosen to give the observed mass of the pion on one hand, and that the $3q3\bar{q}$, $4q4\bar{q}$, ... configurations are not present in the wavefunction of the pion on the other hand.

TABLE IX. Composition of the π's and ρ's w/functions
The $(q^2\bar{q}^2)$-components include $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs
a_c=59.52 MeV/fm^2

particle	$(q\bar{q})$-component (%)			$(q^2\text{-}\bar{q}^2)$-component (%)		
	$(0s)^1(0\bar{s})^1$	$(0p)^1(o\bar{p})^1$	$(0s)^1(0\bar{s})^2(0p)^1$	$(0s)^2(0\bar{s})^1(0\bar{p})^1$	$(0p)^2(0\bar{s})^1(0\bar{p})^1$	$(0s)^1(0p)^1(0\bar{p})^2$
$\pi(39)$	73.2	6.6	5.1	5.1	5.0	5.0
$\rho(293)$	52.7	21.1	5.6	5.6	7.5	7.5

In order to extract how much strangeness there is in this composition, we propose to run the same calculation again with the same value of the strength potential but allowing the creation of light pairs only ($u\bar{u}$ and $d\bar{d}$ pairs). The results of the calculation are given in TABLE X.

Though the calculated masses for the pion have not been fitted to the observed one, they lie within an acceptable range of values similar to some extent to the ones obtained by other models. The model was set for the study of the nucleon as a cloudy system. Applied naively to the pion described in other models [53] as a collective excitation of the non-perturbative QCD vacuum, this collectivity was reproduced, but we did not see the strong reduction of the single $q\bar{q}$ component in the wavefunction of the pion as reported by Weise [52].

TABLE X. Composition of the π's and ρ's w/functions
The $(q^2\bar{q}^2)$-components include $u\bar{u}$ and $d\bar{d}$ pairs only
a_c=59.52 MeV/fm^2

particle	$(q\bar{q})$-component (%)		$(q^2\text{-}\bar{q}^2)$-component (%)			
	$(0s)^1(0\bar{s})^1$	$(0p)^1(o\bar{p})^1$	$(0s)^1(0\bar{s})^2(0p)^1$	$(0s)^2(0\bar{s})^1(0\bar{p})^1$	$(0p)^2(0\bar{s})^1(0\bar{p})^1$	$(0s)^1(0p)^1(0\bar{p})^2$
$\pi(176)$	77.5	11.5	2.6	2.6	2.9	2.9
$\rho(479)$	83.6	0.2	3.2	3.2	4.9	4.9

There is still a long way to go towards understanding the pion, but the known scenario for the pion is being challenged in an alternative picture where consistency can be reached with a bound state pion as a pure quark-antiquark state. Most importantly, we should mention that many details remain to be understood.

Chapter V

Conclusions and Future Work

As a first step towards the understanding of the NN interaction at medium and long ranges, the structure of the nucleon was studied by adding the quark-antiquark pairs to the model space. These quark-antiquark pairs are present in the one gluon exchange theory. Therefore as will be seen later, they will supersede the conventional meson exchange which is always added phenomenologically to the nuclear potential.

To carry out our study, we had to proceed systematically through a hierarchy of instructions. We started by choosing the shell model as the working framework. One reason for this choice is that the particles involved are fermions. As well as other quantum numbers, we had to incorporate the colour quantum number of the particles into consideration. Although there have been similar attempts to study the nucleon as a mixture of three-quark and four quark and an antiquark components, the awkwardness of these methods has left room for relatively easier ways to tackle the same problem.

The significant innovation in our work is the capability of handling a variable number of particles. In other words we solved the problem that arose because the particle-number operator does not commute with the Hamiltonian.

To study a physical system with given quantum numbers, we generated all the configurations which could be linked by the transition potential. In other words all those with particle number N and N+2 were generated without forgetting those with N+4 particles when we considered the second-order excitations. To represent these configurations, the uncoupled-antisymmetric wavefunctions known as Slater determinants (SD) were chosen. The other new feature in our work was the assignment of an "intrinsic parity" quantum number to antiquarks to distinguish them from quarks. This had made the computer representation of configurations more difficult because the

antiquarks have the colour denoted by the second fundamental representation of SU(3). In other words we were in a position to handle both the first and the second fundamental representations of the colour SU(3) group denoting particles and antiparticles respectively. After we solved this problem, the next main concern was to find an adequate representation for the SDs. We chose to assign a code for each of the three colours(anticolours) the SD is composed of. It turned out that there was a sort of repetition among the SDs and most of them were redundant for the construction of physical states which are colour singlets. Therefore a colour hierarchy was imposed: the Red code must not be greater than the Green one which must not be greater than the Blue one ($R \leq G \leq B$). This condition was used to remove the vast majority of coloured states without removing any of the colourless ones.

Once these computing difficulties were overcome and tests were done to ensure that the programs work properly, it was time to start doing real calculations. We started by looking at the effects of implementing these quark-antiquark excitations in the wavefunction of the nucleon. Naturally, the first excited state known as the delta particle emerged in the calculations. This was an important state since the choice of the key parameter of the model which is the quark-gluon coupling constant, was determined from the difference in energy of the two states as discussed in section IV.2. The outcome of this first calculation was the dominance of the three-quark component for the two states representing 85% of the total wavefunctions and a three-quark and an antiquark components representing the rest. We noted also that as the quark-antiquark pairs were introduced at an off-shell level, their effects would be to lower the lowest eigenvalues and to push the highest ones higher up. So we recorded a lowering of the energies of the nucleon and the delta by 250 MeV.

The second calculation we did was to go to the second order in the expansion given by equation II.4. We found that the amplitude of the $3q(q\bar{q})$ component had doubled from the previous results and a contribution from the $3q(q^2\bar{q}^2)$ component of 8.5%. A confusion arose at this at stage about which results of the two calculations describe best

the structures of the nucleon and the delta. The answer was although these two pictures gave us an idea about the proportions of different components constituting the wavefunctions of the nucleon and the delta, they should not be trusted since the model parameters were not set to give the observed masses of the particles. This led us to choose our own set of parameters efficiently to bring the lowered masses back up to the observed ones.

Before choosing the final set of parameters, we aimed at including strange quarks and antiquarks into the game as so far only the two light flavours were considered. This limitation was mainly due to our incapability of handling the three flavours and their three conjugate ones. We judged it worth including the strangeness quantum number because we wanted to answer the intriguing questions whether the nucleon was only made of light-quark flavours (i.e. u, d, $\bar{u}$ and $\bar{d}$). This strangeness could come from the excitation of a light quark or an antiquark to another light quark or antiquark and a strange quark-antiquark pair (e.g. $u \rightarrow us\bar{s}$). This was perfectly allowed by the conservation rules of different quantum numbers. The problem then was to tackle the radical change in the programs brought about by this flavour addition. We discussed in the second part of Chapter IV the terms of the Hamiltonian that break under this flavour implementation and which is mainly due to the mass difference of about 150 MeV between the strange and nonstrange particles. Once we solved the technical side of this problem, new prospects for further investigations were open.

We performed the calculation for the nucleon allowing strange pairs as well as light pairs to be created and compared the results to the case when only light pairs were considered. The outcome of the comparison was that the cloud surrounding the three-quark core was mainly nonstrange accounting only 3% of the total wavefunction. This was in good agreement with the conclusion of other authors [58] using a different approach to ours.

We could not allow strange pairs to be created in the second order for two reasons. The first one was that the number of basis states was enormous and beyond the capacity of our computer. The second one was that it was not worthwhile doing as we noted that in

the first order excitation the "strange" contribution to the wavefunctions was small. An explanation for this is although the strange pairs were going to experience other strange excitations, their overall contribution would be small since the $3q(q^2\bar{q}^2)$-component in the wavefunction was also small anyway. Therefore we limited ourselves to the first order excitation including strangeness. Accordingly, we chose a set of parameters to fit the masses of the nucleon and the delta and their final wavefunction partitions were found.

We next performed calculations for other octet members with the strangeness quantum number equals minus one. These particles were the two sigmas and the lambda. We found two interesting results. The first one was that the set of parameters that was chosen to give the masses of the nucleon and the delta reproduced amazingly well the observed masses of these particles given by the Particle Data Group [56]. The second result was these particles like the nucleon and the delta had also mesonic clouds. However, what was of more importance than this was that the amplitude of these clouds measured by the $3q(q\bar{q})$ component was less than that in the case of the nucleon and the delta as discussed in section IV.10. We interpreted this as a confirmation of the well known fact that the hyperon-hyperon (-nucleon) interaction is less attractive than the nucleon-nucleon one. A paper has been written and a copy is attached as Appendix D.

The second major application of the model was in the field of mesons. The special one was the pion. It had always been known as the mysterious particle as its low mass had never been predicted by any of the successful models around. We had some hope that our model could well be the right one to explore the "collectivity" of the wavefunction of the pion. To keep consistency within the model, we had to use the parameters that were chosen for the nucleon. We recorded a low mass for the pion but it was not 140 MeV. What was interesting about this application was that the collectivity was seen by noting again the dominance of the $q\bar{q}$ component and the non-small contribution from the $q^2\bar{q}^2$ one. If we believed that the model is a good one for the pion then we concluded that a gain of 100 MeV could be achieved through several channels, for example, changing the masses of the constituent quarks and antiquarks.

Many more light physical systems can undergo the same kind of scanning. The conclusion is that these quark-antiquark excitations describe very well the usually known mesonic clouds. The only difference is that we had implemented them explicitly in the model when they were usually introduced as single entities as fields and that involved so many approximations. We consider the case of the cloudy baryons as a first step towards a full investigation of the medium range baryon-baryon interaction. What will be required there is to put a second centre in the vicinity of the cloudy baryon and then derive the potential between the two centres in terms of the distance separating them. In this picture the quark-antiquark pair will have the role of the meson exchange.

In this whole study we restricted ourselves to the 0s and 0p shells with both positive and negative parities only. This space cut was not very serious since the changes from a full space to this restricted one are smaller than the ones brought about by adding $q\bar{q}$ configurations as discussed in section IV.9. Therefore in future work to do the two-centre calculations, the space restriction will not have that much effect on the results. However, it is recommended that calculations should be performed in a larger space as this certainly will lead to more exact results despite the predictions of enormous computational difficulties which will certainly increase exponentially with larger basis.

In extending this work, the limitation of the non-relativistic quark model should be borne in mind. One should also include the spin-orbit and tensor forces, which are absent in our model. More detailed studies in considering these points are needed.

One further point for the use of the model and the associated and developed techniques is in conventional nuclear physics. The interacting boson approximation has enjoyed a considerable success in recent times [62]. One view is that the bosons are pairs of nucleons coupled to a definite angular momentum and isospin. In this picture the number of bosons in a nucleus is clearly constant and is half the number of nucleons. Another view is that a boson is merely a wave travelling in the nuclear medium. In this picture there is clearly no need for the number of bosons to be constant. If one wished to pursue this line of research, our method could be used to handle the variable number of particles.

Appendix A

A.1 LEGENDRE POLYNOMIALS

Any textbook of mathematics where special functions [44] are treated would contain a chapter or a section about the Legendre polynomials because of their wide-range use. They will be defined as

$$P_k(z) = \sum_{t=0}^{[\frac{k}{2}]} (-1)^t \frac{(2k - 2t)!\, z^{k-2t}}{2^k t!\, (k - t)!\, (k - 2t)!} \tag{A.1.1}$$

where [ ] is the greatest integer symbol, which would mean

$$[\tfrac{k}{2}] = \tfrac{k}{2} \text{ if k is even} \tag{A.1.2}$$

$$[\tfrac{k}{2}] = \tfrac{k-1}{2} \text{ if k is odd} \tag{A.1.3}$$

A change of variable is required. We put k - 2t = t'. The new limits of the summation are: for k even then $0 \le t' \le k$ but for k odd, $1 \le t' \le k$. There is also the fact that the factorials must be integers therefore $(k - t)! = (\frac{k + t'}{2})!$ and $t! = (\frac{k - t'}{2})!$ are allowed in terms of k and t' only and only if k and t' have the same parity. As k is fixed, t' starts from 0 or 1 depending on whether k is even or odd respectively. We thus obtain

$$P_k(z) = \sum_{\substack{t'=0 \\ (t',k \text{ same parity})}}^{k} \frac{(-1)^{\frac{k-t'}{2}}}{2^k} \frac{(k+t')!}{(\frac{k-t'}{2})!\, (\frac{k+t'}{2})!\, t'!} z^{t'} \tag{A.1.4}$$

A simplified form of (A.1.4) is

$$P_k(z) = \sum_{\substack{t'=0 \\ (t',k \text{ same parity})}}^{k} C_{t'}(k)\, z^{t'} \qquad \text{(A.1.5)}$$

where

$$C_{t'}(k) = \frac{(-1)^{\frac{k-t'}{2}}(k+t')!}{2^k t'!\,(\frac{k-t'}{2})!\,(\frac{k+t'}{2})!} \qquad \text{(A.1.6)}$$

A.2 EVALUATION OF THE COEFFICIENTS C_i

Here we evaluate the coefficients C_i which appeared in equation III.21 when two radial wavefunctions were expanded. These coefficients C_i can be obtained from

$$R_{n_1\ell_1}(\rho_1) = b^{-\frac{1}{2}} \sum_{k=0}^{n_1} A(k\, n_1 \ell_1)\, \rho_1^{2k+\ell_1+1} \exp(-\frac{\rho_1^2}{2}) \qquad \text{(A.2.1)}$$

where

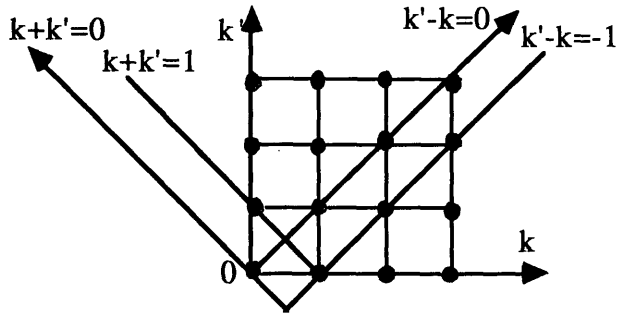
$$A(k\, n_1 \ell_1) = \left\{ \frac{2\Gamma(n_1+\ell_1+1.5}{n_1!} \right\}^{\frac{1}{2}} (-1)^k\, \Gamma^{-1}(k+\ell_1+1.5) \binom{n_1}{k} \qquad \text{(A.2.2)}$$

So the product of two radial wavefunctions of the same variable can be written as

$$b^{-1} \sum_{k=0}^{n1} \sum_{k'=0}^{n3} A(k\, n_1 \ell_1)\, A(k' n_3 \ell_3) \rho_1^{2(k+k')+L+2} \exp(-\rho_1^2) \qquad \text{(A.2.3)}$$

where $L=\ell_1+\ell_3$

The points of the two summations of (A.2.3) are described by the following diagram



If an axis rotation of 45^0 is performed, we end up with a new set of axes designated by k'+k and k'-k. The matrix $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ enables us to go from one set of axes to the other.

We express the old indices k and k' in terms of the new ones denoted by i and j satisfying $i = k+k'$ and $j = k'- k \Rightarrow k' = (i+j)/2$ and $k = (i-j)/2$

Since $0 \le k \le n_1$ and $0 \le k' \le n_3$ therefore $0 \le i \le N$ where $N = n_1+n_3$ But the limits on j are much harder to determine. A simplified form of (A.2.3) can be written as

$$b^{-1}\sum_{i=0}^{N} C_i(n_1\ell_1\, n_3\ell_3)\, \rho_1^{2i+L+2} \exp(-\rho_1^2) \qquad (A.2.4)$$

where the coefficients C_i are

$$\sum_j A(\tfrac{i-j}{2}\, n_1\ell_1)\, A(\tfrac{i+j}{2}\, n_3\ell_3) \qquad (A.2.5)$$

We put $n = \min(n_1,n_3)$ and $m = \max(n_1,n_3)$, the limits on j depend on i and are determined by the following scheme:

1.for $i \le n$ $\Rightarrow$ j goes from -i to i in steps of 2

2.for $i \ge m$ $\Rightarrow$ j goes from k to ℓ in steps of 2 where

if a= i-2m, b=2n-i $\Rightarrow$ { for $n_1 > n_3$, k=min(a,b) and ℓ=max(a,b)
for $n_1 < n_3$, k=min(-a,-b) and ℓ=max(-a,-b)

3. for $n < i < m \quad \Rightarrow \begin{cases} \text{for } n_1 < n_3 \Rightarrow j \text{ goes from } i-m+1 \text{ to } i \text{ in steps of } 2 \\ \text{for } n_1 < n_3 \Rightarrow j \text{ goes from } -i \text{ to } m-i-1 \text{ in steps of } 2 \end{cases}$

A.3 THE ANGLE PART

To evaluate the angle part of the Slater integral, which is given by

$$\iint_{\Omega_1}\iint_{\Omega_2} Y^*_{\ell_1 m_1}(\theta_1\varphi_1)\, Y^*_{\ell_2 m_2}(\theta_2\varphi_2) P_k(\cos\theta_{12})\, Y_{\ell_3 m_3}(\theta_1\varphi_1)\, Y_{\ell_4 m_4}(\theta_2\varphi_2)\, d\Omega_1\, d\Omega_2 \qquad (A.3.1)$$

where $d\Omega=\sin\theta\, d\theta\, d\varphi$.

We write the Legendre polynomials occuring in (A.3.1) in terms of the spherical harmonics [43], i.e.

$$P_k(\cos\theta_{12}) = \frac{4\pi}{2k+1}\sum_{m=-k}^{k} Y^*_{km}(\theta_1\varphi_1) Y_{km}(\theta_2\varphi_2). \qquad (A.3.2)$$

We make use of the following symmetry relation satisfied by the spherical harmonics

$$Y^*_{\ell m}(\theta\varphi) = (-1)^m Y_{\ell -m}(\theta\varphi) \qquad (A.3.3)$$

the expression of the angle part becomes

$$\frac{4\pi}{2k+1}\sum_{m=-k}^{k}(-1)^{m_1+m+m_2}\iint_{\Omega_1} Y_{\ell_1 -m_1} Y_{\ell_3 m_3} Y_{k -m} d\Omega_1 \iint_{\Omega 2} Y_{\ell_2 -m_2} Y_{\ell_4 m_4} Y_{k -m} d\Omega_2 \qquad (A.3.4)$$

The consultancy of any textbook which treats the angular momentum [43] would provide us with

$$\int_0^{2\pi}\int_0^{\pi} Y_{\ell_1 m_1} Y_{\ell_2 m_2} Y_{\ell_3 m_3} d\Omega = \left[\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}\right]^{\frac{1}{2}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.3.5}$$

replaced in (A.3.4) would give

$$\sum_{m=-k}^{k} (-1)^{m_1+m_2+m} C \begin{pmatrix} \ell_1 & \ell_3 & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_2 & \ell_4 & k \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_3 & k \\ m_1 & m_3-m & \end{pmatrix} \begin{pmatrix} \ell_2 & \ell_4 & k \\ m_2 & m_4 & m \end{pmatrix} \tag{A.3.6}$$

where C has the following value

$$C = \sqrt{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)(2\ell_4+1)} \tag{A.3.7}$$

To ensure that (A.3.6) does not vanish, the four 3-j symbols must be different from zero. In the case of the two first ones $\ell_1+\ell_3+k$ and $\ell_2+\ell_4+k$ must be even. Whereas for the last two ones ℓ_1,ℓ_3,k and ℓ_2,ℓ_4,k must satisfy the triangular condition. In this way, k does not take all the values from 0 to ∞ as in the expansion but is determined by the two restrictive conditions expressed as follows:

1) $\ell_1+\ell_3+k$ and $\ell_2+\ell_4+k$ must be even (A.3.8)

2) $|\ell_1-\ell_3| \leq k \leq \ell_1+\ell_3$ and $|\ell_2-\ell_4| \leq k \leq \ell_2+\ell_4$ (A.3.9)

Appendix B

Radial Matrix Elements

B.1 SCALAR PRODUCT OF TWO TENSOR OPERATORS

The scalar product of two tensor operators T and U of the same rank is represented conventionally by

$$(T.U) = \sum_{q} (-1)^{q} T(k\,q)\, U(k\,-q) \qquad \text{(B.1.1)}$$

B.2 MATRIX ELEMENT OF T(k) AS A SINGLE OPERATOR

We obtain the reduced matrix element of a tensor operator **T** of rank k working only on part 1 [43] in the coupled scheme of $(\gamma\, j_1\, j_2\, J\, M)$, i.e.

$$(-1)^{j'1+j_2+J+k}[(2J+1)(2J'+1)]^{1/2} \begin{Bmatrix} j'_1 & J' & j_2 \\ J & j_1 & k \end{Bmatrix} \langle \gamma' j_1 \| T(k) \| \gamma j_3 \rangle \qquad \text{(B.2.1)}$$

B.3 RADIAL WAVEFUNCTION

We give the expression of the radial wavefunction of a particle in the harmonic oscillator potential [41], i.e.

$$|n\,\ell\rangle = R_{n\ell}(r) = \sqrt{\frac{2n!}{b^{2\ell+3}\Gamma(n+\ell+1.5)}}\; r^{\ell} L_n^{\ell+0.5}\left(\frac{r^2}{b^2}\right) \exp\left(-\frac{r^2}{2b^2}\right) \qquad \text{(B.3.1)}$$

where b is the oscillator length parameter and L are Laguerre polynomials.

B.3 EVALUATION OF $\langle n\ell \| \frac{\vec{r}}{r^3} \| n'\ell' \rangle$

Applying the Wigner-Eckart theorem backwards to the matrix element, we get

$$\langle n\ell \left| \frac{\vec{r}}{r^3} \right| n'\ell' \rangle = \frac{1}{(-1)^{\ell}\begin{pmatrix} \ell & 1 & \ell' \\ 0 & 0 & 0 \end{pmatrix}} \langle n\ell\, 0 \left| \frac{z}{r^3} \right| n'\ell'\, 0 \rangle. \qquad \text{(B.3.1)}$$

Thus we have to evaluate the expression of the non-reduced matrix element first. For that,

$$z\,|\,n\ell\,0\rangle = rR_{n\ell}(r)\cos\theta\, Y_{\ell 0}(\theta,\phi) \qquad \text{(B.3.2)}$$

where the radial part is given by

$$rR_{n\ell}(r) = b[\sqrt{n+\ell+0.5}\, R_{n\ell-1}(r) - \sqrt{n+1}\, R_{n+1\ell-1}(r)] \qquad \text{(B.3.3a)}$$

and the angle part is obtained by using the properties of Laguerre polynomials which could be found in any textbook on polynomials, i.e.

$$\cos\theta Y_{\ell\,0}(\theta,\phi) = \frac{(\ell+1)}{[(2\ell+1)(2\ell+3)]^{1/2}} Y_{\ell+1\,0} + \frac{\ell}{[(2\ell+1)(2\ell-1)]^{1/2}} Y_{\ell-1\,0} \qquad \text{(B.3.3b)}$$

therefore the expression (B.3.2) becomes

$$z\,|\,n\ell\,0\rangle = b\frac{(\ell+1)}{[(2\ell+3)(2\ell+1)]^{1/2}}[\sqrt{n+\ell+1.5}\,|\,n\ell+1\,0\rangle - \sqrt{n}\,|\,n-1\,\ell+1\,0\rangle]$$

$$+ b\frac{\ell}{[(2\ell-1)(2\ell+1)]^{1/2}}[\sqrt{n+\ell+.5}\,|\,n\ell-1\,0\rangle - \sqrt{n+1}\,|\,n+1\,\ell-1\,0\rangle]. \qquad \text{(B.3.4)}$$

Hence if we multiply (B.3.4) by $\langle n'\ell+1\,0\,|/r$ and $\langle n'\ell-1\,0\,|/r$ respectively, we obtain

$$\langle n'\ \ell+1\ 0\left|\frac{z}{r^3}\right|n\ \ell\ 0\rangle = b\frac{(\ell+1)}{[(2\ell+3)(2\ell+1)]^{1/2}}\{\sqrt{n+\ell+1.5}\langle n'\ \ell+1\ 0\left|r^{-3}\right|n\ \ell+1\ 0\rangle$$
$$-\sqrt{n}\langle n'\ \ell+1\ 0\left|r^{-3}\right|n-1\ \ell+1\ 0\rangle\} \qquad \text{(B.3.5a)}$$

and

$$\langle n'\ \ell-1\ 0\left|\frac{z}{r^3}\right|n\ \ell\ 0\rangle = b\frac{\ell}{[(2\ell-1)(2\ell+1)]^{1/2}}\{\sqrt{n+\ell+.5}\langle n'\ \ell-1\ 0\left|r^{-3}\right|n\ \ell-1\ 0\rangle$$
$$-\sqrt{n+1}\langle n'\ \ell-1\ 0\left|r^{-3}\right|n+1\ \ell-1\ 0\rangle\}. \qquad \text{(B.3.5b)}$$

We make use of (B.3.1) to derive the reduced matrix element for ℓ' equals to $\ell+1$ expressed as

$$\langle n'\ell+1\left\|\frac{\vec{r}}{r^3}\right\|n\ \ell\rangle = -b\sqrt{\ell+1}\{\sqrt{n+\ell+1.5}\langle n'\ell+1\left|r^{-3}\right|n\ell+1\rangle$$
$$-\sqrt{n}\langle n'\ell+1\left|r^{-3}\right|n-1\ell+1\rangle\} \qquad \text{(B.3.6a)}$$

similarly we obtain the one in the second case, i.e. ℓ' equals to $\ell-1$

$$\langle n'\ell-1\left\|\frac{\vec{r}}{r^3}\right\|n\ell\rangle = -b\sqrt{\ell}\{\sqrt{n+\ell+.5}\langle n'\ell-1\left|r^{-3}\right|n\ell-1\rangle$$
$$-\sqrt{n+1}\langle n'\ell-1\left|r^{-3}\right|n+1\ell-1\rangle\} \qquad \text{(B.3.6b)}$$

where the values of the matrix elements of $1/r^{\lambda}$ are determined later on.

B.4 PROPERTY OF $\langle n'\ell-1\left\|\frac{\vec{r}}{r^3}\right\|n\ell\rangle$

We shall derive a useful symmetrical property of $\langle n'\ell-1\left\|\frac{\vec{r}}{r^3}\right\|n\ \ell\rangle$. We use the Wigner-Eckart theorm to write

$$\langle n'\ \ell-1\ 0\left|\frac{z}{r^3}\right|n\ \ell\ 0\rangle = \langle \ell\ 0\ 1\ 0\left|\ \ell-1\ 0\right\rangle\langle n'\ell-1\left\|\frac{\vec{r}}{r^3}\right\|n\ \ell\rangle/(2\ell-1)^{-1/2}. \qquad \text{(B.4.1)}$$

On the other hand, we have

$$\langle n\ \ell\ 0|\frac{z}{r^3}|n'\ell\text{-}1\ 0\rangle=\langle \ell\text{-}1\ 0\ 1\ 0|\ell\ 0\rangle\langle n\ \ell\ |\frac{\vec{r}}{r^3}|\ n'\ell\text{-}1\rangle/(2\ell+1)^{-1/2}. \quad \text{(B.4.2)}$$

If we replace the Clebsch-Gordan coefficient in (B.4.2) by the following relation

$$\langle \ell\text{-}1\ 0\ 1\ 0|\ell\ 0\rangle=-[\frac{2\ell+1}{2\ell-1}]^{1/2}\langle \ell\ 0\ 1\ 0|\ell\text{-}1\ 0\rangle \quad \text{(B.4.3)}$$

and as the operator $\frac{z}{r^3}$ is real

$$\langle n'\ \ell\text{-}1\ 0|\frac{z}{r^3}|n\ \ell\ 0\rangle=\langle n\ \ell\ 0|\frac{z}{r^3}|n'\ \ell\text{-}1\ 0\rangle \quad \text{(B.4.4)}$$

thus we must have

$$\langle n'\ \ell\ -1\ |\frac{\vec{r}}{r^3}|\ n\ \ell\ \rangle=-\langle n\ \ell\ |\frac{\vec{r}}{r^3}|\ n'\ell\text{-}1\rangle. \quad \text{(B.4.5)}$$

B.5 EVALUATION OF $\langle n'\ell\,|\,r^{-\lambda}\,|\,n\ell\rangle$

By using the expression of the wavefunction given in (B.3.1), $\langle n'\ell\,|\,r^{-\lambda}\,|\,n\ell\rangle$ is expressed as

$$\sqrt{\frac{4n!\,n'!\,\Gamma^{-1}(n'+\ell+1.5)}{b^{2(2\ell+3)}\,\Gamma(n+\ell+1.5)}}\int r^{2\ell+2-\lambda}L_{n'}^{\ell+0.5}(\frac{r^2}{b^2})\,L_n^{\ell+0.5}(\frac{r^2}{b^2})\exp(-\frac{r^2}{b^2})\,dr. \quad \text{(B.5.1)}$$

We shall use the generating function of Laguerre polynomials given by

$$\frac{\exp(-xt/(1-t)}{(1-t)^{k+1}}=\sum_{n=0}^{\infty}L_n^k(x)\,t^n \quad \text{(B.5.2)}$$

then the matrix element equals the coefficient of $u^{n'}v^{n}$ in the result of the evaluation of

$$\int r^{2\ell+2-\lambda} \exp(-\frac{r^2}{b^2}) \exp(-\frac{r^2}{b^2}(\frac{u}{1-u}+\frac{v}{1-v}))\frac{1}{((1-u)(1-v))^{\ell+1.5}}dr \qquad \text{(B.5.3)}$$

multiplied by the constant in (B.5.1).A change of variable is done,

$$x = \frac{r^2}{b^2} \qquad \text{(B.5.4)}$$

the integral (B.5.3) becomes

$$b^{2\ell+3-\lambda}\frac{1}{2}\int_0^\infty x^{\ell-(\lambda-1)/2} \exp(-x(\frac{1-uv}{(1-u)(1-v)}))\frac{1}{((1-u)(1-v))^{\ell+1.5}}dx \qquad \text{(B.5.5)}$$

λ equals to 3 in our case but the method is valid for any odd integer. The result of the integral (B.5.5) is

$$b^{2\ell+3-\lambda}\frac{1}{2}(\ell-(\lambda-1)/2)!\frac{1}{(1-uv)^{\ell+1.5-\lambda/2}}\frac{1}{((1-u)(1-v))^{\lambda/2}}. \qquad \text{(B.5.6)}$$

If we use the following expansion

$$(1+x)^{\alpha} = \sum_{r=0}^{\infty}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha-r+1)\,r!}x^r \qquad \text{(B.5.7)}$$

for the three expressions in u and v, then the coefficients of the term $u^{n'}v^{n}$ occurring in

$$b^{2\ell+3-\lambda}\frac{1}{2}(\ell-(\lambda-1)/2)!\sum_{ijk=0}^{\infty}\frac{\Gamma(\lambda/2+i)}{\Gamma(\lambda/2)\,i!}u^i\frac{\Gamma(\lambda/2+j)}{\Gamma(\lambda/2)\,j!}v^j\frac{\Gamma(\ell+1.5-\lambda/2+k)}{\Gamma(\ell+1.5-\lambda/2)\,k!}(uv)^k \qquad \text{(B.5.8)}$$

which arise from terms with i = n' - k and j = n - k and are given by

$$\text{Const}\sum_{k=0}^{\infty}\frac{\Gamma(\lambda/2+n'-k)\,\Gamma(\lambda/2+n-k)\,\Gamma(\ell+1.5+k-\lambda/2)}{(n'-k)!\,(n-k)!\;k!} \tag{B.5.9}$$

where the value of Const is

$$\frac{b^{2\ell+3-\lambda}}{2}(\ell+0.5-\lambda/2)!\,\frac{1}{(\Gamma(\lambda/2))^2\Gamma(\ell+1.5-\lambda/2)}. \tag{B.5.10}$$

Thus, the final answer for the the matrix element is

$$\sqrt{\frac{n!\,n'!\,\Gamma^{-1}(n'+\ell'+1.5)}{b^{2\lambda}\;\Gamma(n+\ell+1.5)}}\,\frac{1}{(\Gamma(\lambda/2))^2}\sum_{k=0}\frac{\Gamma(n'-k+\lambda/2)\,\Gamma(n-k+\lambda/2)\,(\ell+k+0.5-\lambda/2)!}{(n'-k)!\;(n-k)!\;k!} \tag{B.5.11}$$

Appendix C

Spin Matrix Elements

C.1 MATRIX ELEMENT OF σ_μ

The spherical components of a vector σ are defined by

$$\tilde{\sigma}_+ = -\frac{1}{\sqrt{2}}(\sigma_x + i\sigma_y) \qquad \text{(C.1.a)}$$

$$\tilde{\sigma}_- = \frac{1}{\sqrt{2}}(\sigma_x - i\sigma_y) \qquad \text{(C.1.b)}$$

$$\tilde{\sigma}_z = \sigma_z \qquad \text{(C.1.c)}$$

where σ_x, σ_y, and σ_z are its three components along three axes Ox, Oy, and Oz respectively. Therefore in terms of these spherical components and those of the vector $\underline{r}$, the scalar product of $\underline{\sigma}$ and $\underline{r}$ is defined as

$$\underline{\sigma}_1 \cdot \underline{r} = -\tilde{\sigma}_{1+} r_- - \tilde{\sigma}_{1-} r_+ + \tilde{\sigma}_{10} r_0 . \qquad \text{(C.2)}$$

We want to evaluate the matrix element of the operator spin occurring in (C.2). As we are working in the uncoupled scheme of spin and angular momentum, the σ is going to act on the spin coordinates of the first particle only

$$\langle s_1\, s_2 | (\tilde{\sigma}_1)_\mu | s_3\, s_4 \rangle = \delta(s_2, s_4) \langle s_1 | \tilde{\sigma}_\mu | s_3 \rangle \qquad \text{(C.3)}$$

where μ=+,0, -, hence we only require to give the three expressions of $\langle s_1 | \tilde{\sigma}_\mu | s_3 \rangle$ for the three components $\tilde{\sigma}_\mu$, they are

$$-\sqrt{2}\delta(s_1, s_3-1) \quad \sqrt{2}\delta(s_1-1, s_3) \quad s_1\delta(s_1, s_3) \tag{C.4}$$

for $\tilde{\sigma}_+$, $\tilde{\sigma}_-$, and $\tilde{\sigma}_0$ respectively. The result (C.4) will be used in the coming section.

C.2 EVALUATION OF THE MATRIX ELEMENT OF $[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]$

The other operator of spin occurring in the transition matrix element is

$$\underline{\Sigma} = (\underline{\sigma}_1 \times \underline{\sigma}_2) \tag{C.5}$$

which is a vector as it is the cross-product of two vectors. It may be expressed as a tensor of rank one. Therefore, in terms of the spherical components of $\underline{\sigma}_1$ and $\underline{\sigma}_2$ defined in (C.1) and after developing the cross-product, the three spherical components of the tensor $\underline{\Sigma}$ are

$$\Sigma_+ = -i(\tilde{\sigma}_{1+}\tilde{\sigma}_{20} - \tilde{\sigma}_{10}\tilde{\sigma}_{2+}) \tag{C.6.a}$$

$$\Sigma_- = i(\tilde{\sigma}_{1-}\tilde{\sigma}_{20} - \tilde{\sigma}_{10}\tilde{\sigma}_{2-}) \tag{C.6.b}$$

$$\Sigma_0 = -i(\tilde{\sigma}_{1+}\tilde{\sigma}_{2-} - \tilde{\sigma}_{1-}\tilde{\sigma}_{2+}) \tag{C.6.c}$$

Therefore the components of $i(\underline{\sigma}_1 \times \underline{\sigma}_2)$ are obtained by multiplying (C.6) by i, i.e.

$$[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_+ = (\tilde{\sigma}_{1+}\tilde{\sigma}_{20} - \tilde{\sigma}_{10}\tilde{\sigma}_{2+}) \tag{C.7.a}$$

$$[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_0 = (\tilde{\sigma}_{1+}\tilde{\sigma}_{2-} - \tilde{\sigma}_{1-}\tilde{\sigma}_{2+}) \tag{C.7.b}$$

$$[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_- = (\tilde{\sigma}_{10}\tilde{\sigma}_{2-} - \tilde{\sigma}_{1-}\tilde{\sigma}_{20}) \tag{C.7.c}$$

So from the righthand sides of (C.7) and using the results in (C.4), it is straightforward to evaluate the matrix elements between two two-spin states in the uncoupled sheme of spin. We have to take one matrix element of these operators at a time.

C.3 MATRIX ELEMENT OF $[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_+$

Its form is given just below

$$\langle s_1\, s_2 | [i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_+ | s_3\, s_4 \rangle = \langle s_1 | \tilde{\sigma}_{1+} | s_3 \rangle \langle s_2 | \tilde{\sigma}_{20} | s_4 \rangle - \langle s_1 | \tilde{\sigma}_{10} | s_3 \rangle \langle s_2 | \tilde{\sigma}_{2+} | s_4 \rangle \qquad \text{(C.8)}$$

where the spin 1/2 of the particles has been omitted from the states for simpler expressions of the matrix element whose value depends on the z-projections of the particles. There are four two-particle states the operator could act on, therefore there are 16 matrix elements to be evaluated. The matrix has been worked out in this space using the results in (C.4) and is given below. The sign (+) in there stands for the first spherical component of the operator and the z-projections of the spin have been doubled for the sake of simplicty of the form of the matrix.

(+)	s_3	1	1	-1	-1
s_1	s_4	1	-1	1	-1
1	1	0	$\sqrt{2}$	$-\sqrt{2}$	0
1	-1	0	0	0	$\sqrt{2}$
-1	1	0	0	0	$-\sqrt{2}$
-1	-1	0	0	0	0

C.4 MATRIX ELEMENT OF $[i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_0$

Similarly, the form of the matrix element of the operator in (C.7.b) is developed and is given by

$$\langle s_1\, s_2 | [i(\underline{\sigma}_1 \times \underline{\sigma}_2)]_0 | s_3\, s_4 \rangle = \langle s_1 | \tilde{\sigma}_{1+} | s_3 \rangle \langle s_2 | \tilde{\sigma}_{2-} | s_4 \rangle - \langle s_1 | \tilde{\sigma}_{1-} | s_3 \rangle \langle s_2 | \tilde{\sigma}_{2+} | s_4 \rangle \qquad \text{(C.9)}$$

The same remarks stated for the previous case hold here too. Thus any desired matrix element can be obtained from the following matrix

(0)	s_3	1	1	-1	-1
	s_4	1	-1	1	-1
s_1 s_2					
1 1		0	0	0	0
1 -1		0	0	-2	0
-1 1		0	2	0	0
-1 -1		0	0	0	0

C.5 MATRIX ELEMENT OF $[i(\underline{\sigma}_1 x\ \underline{\sigma}_2)]_-$

Finally, the last matrix element of the third component of the tensor in (C.7.c) is given by

$$\langle s_1\, s_2 | [i(\underline{\sigma}_1 x\, \underline{\sigma}_2)]_- | s_3\, s_4\rangle = \langle s_1 | \tilde{\sigma}_{10} | s_3\rangle\langle s_2 | \tilde{\sigma}_{2-} | s_4\rangle$$
$$-\langle s_1 | \tilde{\sigma}_{1-} | s_3\rangle\langle s_2 | \tilde{\sigma}_{20} | s_4\rangle \qquad \text{(C.10)}$$

which matrix is given by

(-)	s_3	1	1	-1	-1
	s_4	1	-1	1	-1
s_1 s_2					
1 1		0	0	0	0
1 -1		$\sqrt{2}$	0	0	0
-1 1		$-\sqrt{2}$	0	0	0
-1 -1		0	$\sqrt{2}$	$-\sqrt{2}$	0

Therefore any matrix element of any component of the tensor operator $i(\underline{s}_1 x\, \underline{s}_2)$ in (C.7) can be obtained from the relevant matrix of the three given above.

Appendix D

Quark-antiquark pair creation effects in the study of S=0 and S=-1 baryons

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Abstract

Improved wavefunctions of nonstrange and strange baryons are obtained by incorporating quark-antiquark excitations inherent in the quark-gluon interaction into the quark model. The working framework is the shell model where the antiquark is assigned an intrinsic parity quantum number to distinguish it from the quark. The parameters of the interaction are chosen to be consistent with the experimental data. It is found that the amplitudes of the $3q(q\bar{q})$ and $3q(q^2\bar{q}^2)$ components are not negligible and should be included in the calculation of the properties of the baryons. It is also found that the amplitude of the $3q(q\bar{q})$ component for strange baryons is less than for nonstrange ones, indicating that the hyperon-nucleon(-hyperon) interaction is less attractive than the N-N one.

1. Introduction

Much information has been gained about the internal structure of baryons from deep inelastic scattering of leptons and baryons. They consist of quarks and gluons. Also the existence of the one pion exchange potential (OPEP) is the most definite piece of information that has be gleaned from the extensive analyses of nucleon-nucleon scattering. Conventional quark models fail to explain the attractive part of N-N interaction, putting them in direct conflict with experiment[1,2]). Furthermore, this problem is not restricted to the two nucleon system. All nucleons are identical so that if a pion can be emitted by one nucleon and absorbed by another to give OPEP, a pion can be emitted and absorbed by the same nucleon giving rise to a pion cloud. As a consequence, the MIT cloudy bag model, in which the quark degrees of freedom of the nucleon interior are coupled to an external elementary meson field, has superseded the bag model. Despite its many nice features

such as its relativistic nature, this model runs into serious difficulties in a number of areas, for example separating out the centre of mass motion. A natural way to include the mesonic degrees of freedom is to consider the basic quark-gluon interaction lagrangian which includes $q\bar{q}$ pair creation terms. This interaction leads to off-shell matrix elements coupling the 3q configurations of the baryon to $3q(q\bar{q})$ components. These extra $q\bar{q}$ pairs may be absorbed on a different baryon to contribute to the one meson exchange potential or may change the physical properties of the baryon. Our main purpose in this paper is twofold. One is to explicitly incorporate the $q\bar{q}$ excitations described by a transition potential directly into the model space, and the other is to study their effects on the wavefunctions of baryons with S=0 and S=-1.

In section 2, we will explain the improved quark model we use with emphasis on the $q\bar{q}$ pair creation terms. In this study, we adopt the same starting point as Hecht and Fujiwara, but perform the calculations within the framework of the nuclear shell model extended to include colour, and quarks and antiquarks. In section 3, improved wavefunctions for the nucleon and baryons with S=-1 are presented. An extension of the model to study the baryon-baryon interaction at medium range is discussed along with a summary and conclusion in section 4.

2. Non-relativistic improved quark model

In this section we explain the non-relativistic improved quark model which we use to investigate the S=0 and S=-1 baryons' wavefunctions. The Hamiltonian for the study of a quark and antiquark system is the sum of the normal kinetic energy terms and an interaction derived from quantum chromodynamics (QCD). The kinetic energy of the centre of mass is subtracted from the kinetic energy which is expressed as

$$KE = \sum_i \frac{p_i^2}{2m_i} - \frac{P^2}{2M} \qquad (1)$$

where m_i and p_i are the mass and the momentum of particle i, while $P=\Sigma p_i$ and $M=\Sigma m_i$ are the total momentum and mass of the system. In this work, the m_i cannot be taken to be independent of i, as the mass of the strange quark is very different from the non-strange one. Also, M depends on the number of particles in the part of the wavefunction of which

T is acting, so that M itself is a non-trivial operator.

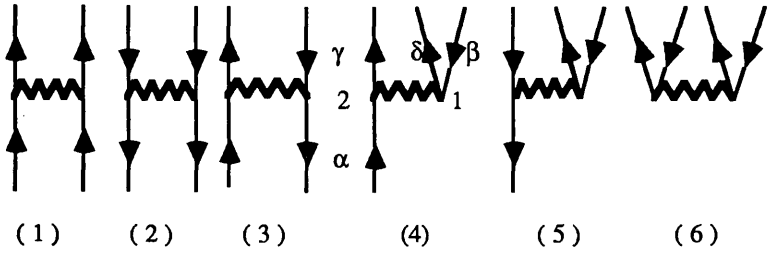

Fig.1 quark- (antiquark-) gluon interaction

The interaction potential is derived from a field theoretical study, in particular from the constructs of vertex functions and propagators in the non relativistic limit. Its theoretical framework supposes that the quarksand antiquarks interact (Fig.1.(1,2,3)) by exchanging a gluon and are confined to simulate the infrared slavery of QCD by a phenomenological confining potential of harmonic oscillator type. Their potential [3]) is written as:

$$v_{ij} = \alpha_s \frac{\hbar}{8\pi} c\,(\lambda_i.\lambda_j)\left\{\frac{1}{r_{ij}} - \frac{h^2}{4\pi m c^2}\,\delta(\bar{r}_{ij})\left[1+\frac{2}{3}(\bar{\sigma}_i.\bar{\sigma}_j)\right]\right\} - a_c(\lambda_i.\lambda_j)\,r_{ij}^2 \qquad (2)$$

where

$\delta(\bar{r}_{ij})$ is the contact term

$\bar{r}_{ij}$ is the interquark vector spacing

α_s is the quark-gluon coupling constant

a_c the strength parameter of the confining potential

λ_i is the set λ^α ($\alpha = 1, 2, \ldots 8$): the generators of the SU(3) colour group

Mechanisms represented by diagram (4) of Fig.1 give rise to the sea-quark effects: a quark-antiquark pair is created and a meson cloud can then be formed. Yu and Zhang [4]) derived a transition potential from OGE theory for diagram (4):

$$V_{q\to qq\bar{q}} = \sum_{\alpha,\beta,\delta,\gamma} \langle \psi_\delta(1)\,\psi_\gamma(2)\,|\,v(1,2)\,|\,\psi_{\bar{\beta}}(1)\psi_\alpha(2)\rangle\, a_\gamma^+ a_\delta^+ b_\beta^+ a_\alpha \qquad (3)$$

where

$$v(1,2) = \frac{i}{8\pi}\alpha_s\,(\lambda_1\cdot\lambda_2)\left[\frac{\pi}{2mc}\left\{\frac{1}{\rho^2}(\bar{\sigma}_1.\bar{n}) + \frac{i}{2\rho^2}(\bar{\sigma}_1\times\bar{\sigma}_2).\bar{n} + \frac{i\sqrt{2}}{\rho}(\bar{\sigma}_1.\bar{p}_2)\right\}\right] \qquad (4)$$

$$\bar{n} = \frac{\bar{r}}{r} = \frac{\bar{r}_1 - \bar{r}_2}{|r_1 - r_2|} \quad \text{and} \quad \bar{p} = \frac{1}{\sqrt{2}}(\bar{r}_1 - \bar{r}_2) \qquad (5)$$

Here the labels 1 and 2 denote sea and valence quarks respectively, and p2 is the

momentum operator of the valence quark.

Also a_α^+ (a_α) is the quark creation(destruction) operator, which creates(destroys) a quark in an α state with wavefunction

$$\Psi_\alpha(2) = \omega_\alpha(\mathbf{r})\, \chi^{\mathbf{s}}_{\frac{1}{2}\, s_\alpha}\, \chi^{\mathbf{f}}_{\frac{1}{2}\, f_\alpha}\, \chi^{\mathbf{c}}_{(10)\, c_\alpha} \qquad (6)$$

Here χ^s, χ^f and χ^c are the spin, flavour and colour parts of the wavefunction. The spatial part is a 0s or 0p shell oscillator wavefunction. For example, the expression of the 0s wavefunction is given by

$$\omega_0(r) = \frac{1}{\sqrt{\eta}} \exp(-r^2/2b^2) \qquad (7)$$

with $\eta = (\pi b^2)^{3/2}$ (8)

Similarly, b_β^+ (b_β) is the antiquark creation (destruction) operator which creates (destroys) an antiquark in a β state with wavefunction

$$\Psi_{\bar\beta}(1) = \omega_\beta(\mathbf{r})\, \chi^{\mathbf{s}}_{\frac{1}{2}\, s_\beta}\, \chi^{\mathbf{f}}_{\frac{1}{2}\, f_\beta}\, \chi^{\mathbf{c}}_{(01)\, c_\beta} \qquad (9)$$

In practical terms, the matrix element of diagram (4) is a one-three body one which can be evaluated if and only if it is transformed into a two-body one. So the odd particle (the antiquark β in our case)* in the diagram is converted into its conjugate

$$\Psi_{\bar\beta}(1) = \omega^*_\beta(\mathbf{r})\, (-1)^{\frac{1}{2}+s_\beta}\, \chi^{\mathbf{s}}_{\frac{1}{2}\, -s_\beta}\, (-1)^{\frac{1}{2}+f_\beta}\, \chi^{\mathbf{f}}_{\frac{1}{2}\, -f_\beta}\, (-1)^{\eta(c_\beta)}\, \chi^{\mathbf{c}}_{(10)\, c_\beta} \qquad (10)$$

the colour phase factor $(-1)^{\eta(c)}$ is given by the SU(3) Clebsch-Gordan coefficient

$$(-1)^{\eta(c)} = \sqrt{3}\langle (10)\, c\, (01)\, \bar c \,|\, (00)\rangle \qquad (11)$$

Although the transition potential (4) is pure imaginary, because of the existence of the adjoint operator $V_{qq\bar q \to q}$, the overall Hamiltonian is hermitian.

As a first step to explore this unconventional picture of the meson exchange, we first perform one centre calculations in which the wavefunctions of nonstrange and strange baryons are investigated. We also go to higher order to include $q^2\bar q^2$ excitation (Fig.1.(6)). In this way, the baryon wavefunction is extended to include these excitations and the solution of the Schrödinger equation will give improved values for the

* the quark undergoes similar transformation if the destroyed particle is an antiquark

eigenfunctions and the eigenvalues of the system . The wavefunction is expressed as

$$\Psi_B = C_0\phi_0(3q) + \sum_\alpha C_\alpha \phi_\alpha((3q)(q\bar{q})) + \sum_\beta C_\beta\phi_\beta((3q)(q^2\bar{q}^2)) \qquad (12)$$

For the nucleon, ϕ_0 has the three quarks in the 0s shell. The components ϕ_α but not necessarily ϕ_β require the 0s shell and also the 0p shell in order that parity is conserved. The mixing coefficients $C\alpha$ are determined from the transition potential $V_{q\to qq\bar{q}}$ already discussed. The same treatment is applicable to determine the coefficients C_β but there are actually two ways in which the $q^2\,\bar{q}^2$ could arise. The one we do not consider here comes from the $V_{\to q^2\bar{q}^2}$ creation potential describing the two-($q\bar{q}$) pair creation present in the OGE theory (Fig.1.(6)). The other one is a second order $q\bar{q}$ excitation which is incorporated into the model. The states that undergo a $q\bar{q}$-pair excitation can undergo another excitation. In terms of states and number of particles, we start off with 3 quarks, transformed into a 5 particle state (4 quarks +1 antiquark) which in turn undergoes another pair excitation to a 7 particle state (5 quarks + 2 antiquarks). In addition to the transition potential in eq. (2), the antiquark present in the 3q($q\bar{q}$) state may cause an excitation (Fig1.(5)) written by analogy to eq. (3) as

$$V_{\bar{q}\to\bar{q}q\bar{q}} = \sum_{\alpha,\beta,\delta,\gamma} \langle \psi_\delta(1)\,\psi_\gamma(2)\,|\,v(1,2)\,|\,\psi_{\bar{\beta}}(1)\psi_\alpha(2)\rangle\, b^+_\gamma b^+_\delta a^+_\beta b_\alpha \qquad (13)$$

More generally, the basis states consist of configurations with (3q) and (3q)(mq-m$\bar{q}$) where m= 1, 2.... The transition potential will mix states of the form 3q[mq-m$\bar{q}$] and 3q[(m+1)q-(m+1)$\bar{q}$] differing by a $q\bar{q}$ pair, giving rise to significant off-diagonal matrix elements.

In our treatment, the most awkward part is the setting up of the basis states, where non-constant number of particles has to be handled. We choose to work in the m-scheme, and the many-body basis states are Slater determinants. Even though the basis states do not have good J, the eigenfunctions will have good J values since the Hamiltonian is rotationally invariant. This method has the advantage that the Hamiltonian matrix elements can be calculated relatively quickly. In the J-scheme method, the matrix elements of the Hamiltonian between states with good J are constructed by specifying a complete set of vector-coupled states and the associated Clebsch-Gordan coefficients. The advantage of

the J-scheme method is that the number of basis states considered is typically an order of magnitude less than in the m-scheme. The disadvantage is that each matrix element takes longer to calculate since there is much vector recoupling algebra involved.

The energy eigenvalues of the Hamiltonian were calculated by setting up and diagonalising the Hamiltonian matrix using the Lanczos algorithm [5]) in the space of 3q, $3q(q\bar{q})$ and $3q(q^2\bar{q}^2)$ basis states which are constructed in such a way as to be totally antisymmetric in space, spin, isospin and colour. The improved single baryon wave function can also be obtained. The results presented below show that the effects of the $q\bar{q}$ excitations and also those of the $q^2\bar{q}^2$ excitations to the structures of baryons are by no means negligible. The calculations presented here involve starting with light quarks and then extending the calculations to include the strangeness quantum number explicitly.

3. Results and discussions

We confine ourselves to the smallest possible space, namely the 0s and the 0p oscillator shells. There are two resons for this choice. One is that, although the three quarks in the core of the nucleon are in the 0s shell, the 0p shell must be included to allow the overall parity of the $3q(q\bar{q})$-component to be even as the antiquark has odd intrinsic parity. The other one is the practical limitations of the shell-model code but the seriousness of this space limitation will be discussed later.

The interaction is determined by the parameters listed in Table I which have been fixed [6]) to yield the masses of the nucleon and the delta (and therefore their observed mass splitting of 293 MeV) when regarded as 3q systems. Readjustment of the these parameters will be required when we extend the basis space.

We first look at the effects of creating light pairs ($u\bar{u}$ and $d\bar{d}$). The number of basis states for the nucleon's calculation was 312 including 3q and $3q(q\bar{q})$ states which will be mixed by the transition potential. The importance of the mixing is presented in Table II where we see that the 3q component (84.8%) is dominant as one might expect. However the amplitude of the $3q(q\bar{q})$-component accounts for the remaining 15.2% and is a quite significant proportion of the wavefunction.

The $3q(q^2\bar{q}^2)$ states obtained by the action of the transition potential on $3q(q\bar{q})$ states

should also be included. This increases the size of the basis to 24173. We have found that the amplitude of $3q(q\bar{q})$ has almost doubled from the previous results. The other interesting result is the considerable contribution of the $3q(q^2\bar{q}^2)$-component accounting for 8.5% of the wavefunction of the nucleon which again would have been of more importance if the effects of diagram(6) were also considered.

Because there is no explicit specification of the quantum numbers of the $q\bar{q}$ pairs created in our treatment (as we are only interested in $3q(q\bar{q})$- and $3q(q^2\bar{q}^2)$-components having the quantum numbers of the nucleon), the $3q(q^2\bar{q}^2)$ component include the possibility that there are two mesons around the nucleon and also the possibility that the seven particles form a collective state which cannot be decoupled into hadrons with the quantum numbers of physical particles. This will have an impact when it comes to study the medium and the long range attractive part of the N-N interaction believed to be due to the exchange of two and a single pions respectively. Our model generates the N-N interaction at long ranges by the following process: one cloudy baryon in the vicinity of another will exchange $q\bar{q}$ or $q^2\bar{q}^2$ clusters if the two baryons are sufficiently close.

When off-shell effects are considered, the lowest eigenvalues in the spectrum are lowered and the highest ones pushed higher up with no contribution to the trace of the matrix. For the calculation including $3q(q\bar{q})$ configurations the masses of the nucleon and the delta were respectively found equal to 680 and 954 MeV, a lowering in energy of about 250 MeV for a strength of the transition potential equal to 47.2 MeV.fm^2. For the space extension to $3q(q^2\bar{q}^2)$ terms, a further lowering was recorded with corresponding energies for the nucleon and the delta of 530.8 and 843.2 MeV respectively.

Although these results give us an idea about the proportions of different components constituting the wavefunction of the nucleon, they should not be trusted since the model parameters are not set yet to give its observed mass. Therefore it is too early at this stage to ask which composition describes the more realistic physics of the nucleon. Also we cannot condemn the model because the composition of the nucleon has changed drastically as expected with the addition of the $5q\text{-}2\bar{q}$ configurations. Harmony will be restored and clearer picture of the structure of the nucleon will be revealed after the right choice of parameters has been made.

In this section, we extend the previous calculations based on light quarks and antiquarks to systems with strangeness. For the case of the nucleon (S=0), a question which one might ask is, how much strangeness is there in its wavefunction if the creation of strange $q\bar{q}$ pairs is also possible (e.g. $u \rightarrow us\bar{s}$). The calculation was limited to include 3q and $3q(q\bar{q})$ states only because the number of basis states was 687 and the extension to $3q(q^2\bar{q}^2)$ configuration results in 124031 basis states which is too large to us to handle. Inclusion of the strangeness quantum number requires a formulation which incorporates flavour SU(3) instead of the isospin, the s and the $\bar{s}$ have I=0 in the SU(2) subgroup. Three terms in the Hamiltonian actually break the SU(3) flavour symmetry:

1) the mass term i.e. the strange s quark is heavier than the u, d quarks which gives only a constant shift to the total energy
2) the kinetic energy term $p^2/2m$
3) the potential term i.e. the contact term $\left\{\frac{1}{m_i^2}+\frac{1}{m_j^2}+\frac{2}{m_i m_j}\frac{2}{3}\bar{\sigma}_i\cdot\bar{\sigma}_j\right\}\delta(\bar{r}_{ij})$

The introduction of different masses makes our procedure highly complicated but still feasible. We will not neglect in the following the symmetry breaking due to the kinetic energy and most importantly the symmetry breaking in the strength of the contact potential term. The possibility of the nucleon having kaons in the surrounding was investigated by introducing the strange quark and antiquark pair ($s\bar{s}$). We took care of the explicit mass dependence in both the interaction and the transition potential which is now expressed [7] as

$$v=\frac{i}{16c}\alpha_s(\lambda_1\cdot\lambda_2)\left[\frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\frac{1}{\rho^2}(\bar{\sigma}_1\cdot\bar{n})+\frac{i}{2m_2\rho^2}(\bar{\sigma}_1\times\bar{\sigma}_2)\cdot\bar{n}+\frac{i\sqrt{2}}{m_2\rho}(\bar{\sigma}_1\cdot\bar{p}_2)\right] \quad (14)$$

The lowering in energy is an extra 90 MeV for the addition of the $s\bar{s}$ pair, summing up to the 250 MeV after the introduction of the $u\bar{u}$ and $d\bar{d}$ pairs: an overall lowering of 340 MeV. The masses of the nucleon and the delta were now 591 and 884 MeV respectively. The composition of the wavefunction of the nucleon is displayed in the second half of Table II. If we compare the new partition to the one in the first half, there is only an extra 3% into $3q(q\bar{q})$-component due to the inclusion of the $(s\bar{s})$ pair creation: we conclude that the cloud surrounding the 3q core is mainly non-strange. This is a remarkable result

indeed as it agrees with the conclusion that MacGovern *et al.* [8]) came to that the strange content in the nucleon is less than 5%.

When we readjust the parameters to give the known mass of the dressed nucleon, an important consideration is the value of the strong-interaction coupling constant. To maintain consistency with the idea of treating the one-gluon exchange as a perturbative correction, values of α_s around unity are acceptable.

Accordingly, the previous set of parameters which gives good results for the bare nucleon was retained except that the strength of the confining potential was varied because of its dubious phenomenological origins and also because in previous studies [9]), it was shown that a great many phenomena relevant to the baryons and their interaction are insensitive to this parameter. The variation of the mass of the nucleon with a_c is illustrated in Fig. 2, showing that there is an almost linear increase of the energy of the nucleon when a_c increases. The value of a_c which gives the masses of the nucleon and the delta is 59.5 MeV.fm^{-2}. The other interesting result is while varying a_c, the mass splitting between the nucleon and the delta remained constant and equalled the observed value. In the simple quark model, the Δ and the nucleon are made up of three up and down quarks in overall S-wave with spins coupled to 3/2 or 1/2 respectively. The mass separation of about 300 MeV between these states is hypothesised to be a manifestation of QCD hyperfine splitting. The splitting is proportional to the product of the colour-magnetic moments of quarks defined in analogy to their electromagnetic moments [10]). Even with the addition of the $q\bar{q}$ cloud, the structure of the nucleon and Δ are very similar and so the mass splitting is independent of the strength of the confining potential.

For the adjusted parameters, the final composition of the wavefunction of the nucleon is given in Table III. We see that the proportion of the 0p shell particles has been reduced. This can be understood because a stronger confining potential will raise the effective energy of a 0p shell particle.

One can estimate the seriousness of the restriction to the 0s and 0p shells only by considering results for three-quark calculations, without the additions of $q\bar{q}$ pairs with the final set of parameters. The results for the energy of the nucleon in the 0s space only is 1248 MeV. Adding the 0p and the (1s 0d) shells the energy becomes 1198 MeV, and

adding the (1p 0f) and (2s 1d 0g) shells the energy is 1129 MeV. Clearly these changes are smaller than the changes brought about by adding $q\bar{q}$ configurations, even with restrictions to the 0s and 0p shells.

If we replace one of the three quarks in the Δ by a quark of different flavour, i, where i = s,c,... then we obtain Σ_i^* (iqq). The nucleon has the quarks pairwise in either I = 1 or 0, and upon replacing the third quark by i yields respectively Σ_i (iqq) or Λ_i (iqq) states. For the choice of i ≡ s we have the familiar states

Σ_s^* (sqq) (1385) Σ_s (sqq) (1193) Λ_s (sqq) (1115)

We see that the act of substituting a strange quark for a u or a d quark in which m_s is 150 to 200 MeV/c^2 greater than m_u has:

1) increased the mass of the three quark system by around 150-200 MeV/c^2
2) decreased the $3/2^+$ - $1/2^+$ mass splittings
3) split the Λ(I=0) and Σ(I=1)

Since the hyperfine splittings are inversely proportional to the quark masses, then Σ^*-($\Sigma\Lambda$) will be in turn less split than the Δ-N. Finally, the overall spin-unitary spin symmetry of the wavefunction requires that the non-strange pair in Λ has spin 0 and in the Σ has spin 1. This will lead to different spin-spin expectation values for the states and hence to different masses.

We now want to examine the effects of the $q\bar{q}$ pair on the other members of the baryon ground state octet, in other words to see how cloudy these baryons are and also to find out if the above information is still reproducible if strange baryons are allowed to be cloudy. Only the structures of those with S=-1 are studied. First of all, we found that the readjusted set of parameters used previously to obtain the observed masses of the nucleon and the delta gives very nearly the observed masses of the Λ, $\Sigma(1/2^+)$ and the $\Sigma^*(3/2^+)$. The calculated masses of the $\Lambda(\Sigma)(1/2^+)$ and the $\Sigma^*(3/2^+)$ are 1081.3 (1163.3) and 1339.6 MeV respectively compared with the ones given by the Particle Data Group [11]) which are 1115.6 (1192.5) and 1383.7 MeV. The splitting of 176.3 MeV (which should have been 258.4 MeV) was not perfectly reproduced. That might be because we assumed that the non-strange and strange quarks (antiquarks) had the same oscillator length parameter b whereas the value of b for a strange quark should be less than that for a non-

strange one as the oscillator parameter is inversely proportional to the mass. We could have considered it in the radial part of the single particle wavefunction but it leads to extra complications when it comes to the evaluation of all the transformation brackets and integrals of matrix elements.

The results are presented in the first half of Table III. The $3q(q\bar{q})$-component represents 13.5% of the total wavefunctions of the strange baryons, to be compared with 15.2% of that of the nonstrange ones. This means that the strange baryons are less cloudy than the nonstrange baryons due to the fact that the strange quark interacts less with the nonstrange ones: a confirmation of the well known fact that the hyperon-hyperon (-nucleon) interaction is less attractive than the nucleon-nucleon one.

Now that the parameters of the model have been adjusted, some predictions can be made concerning the second order excitations already discussed. We noted that we cannot allow strange pairs in this order to be created because of the vast number of basis states and, therefore, the calculations would not be possible due to the many computing difficulties. However, the introduction of strange pairs as well as light ones in the first order excitations has roughly the same effects as the second order excitations for light pairs only. In other words, the diagonal energies due to the $(s\bar{s})$ pairs is almost the same as two light pairs. Subsequently we can guess the value of the confing potential strength that gives the observed mass of the nucleon being a mixture of 3q, $3q(q\bar{q})$ and $3q(q^2\bar{q}^2)$ configurations.Indeed it was found to have the value of 59.5 $MeV.fm^{-2}$ for which the found probabilities for each component are dispayed in the second part of Table III. We now discuss these results. The pure three-quark configuration is dominant representing 73%, and a three-quark configuration with quark-antiquark pairs representing the rest of the wavefunction.

Although the order is set to two only because of the limitations due to the computing difficulties, this picture of the nucleon is far better that of the first order only.

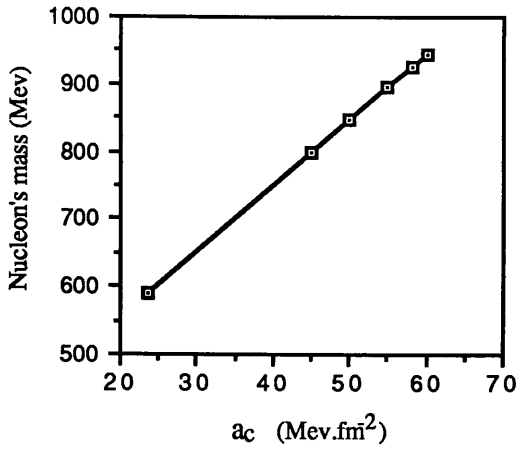

Fig. 2 Mass of the nucleon versus the confining potential strength ac

4. Summary

In this paper, we have investigated the effects of the $q\bar{q}$ pair creation into the wave function of non-strange and strange baryons using the non-relativistic quark model. The central and colour hyperfine terms of the One Gluon Exchange (OGE) potential as well as an effective quadratic confining potential are considered. We also included the transition potential describing the $q\bar{q}$ pair creation up to a second order. Strange quarks and antiquarks are explicitly considered with the only approximation that the oscillator length parameter b is flavour independent. By fitting the observed mass of the nucleon, it was found first that the masses of other members of the octet are also reproduced in good agreement with the experimental ones and the effects of the $q\bar{q}$ pair creation are smaller for strange baryons.

The shell model has been used in this analysis because it has the advantage that the method of approximation is well defined. It is certainly true that an increase in the model space would lead to more exact results and more work should be done to investigate this further. In extending this work, the limitation of the non-relativistic quark model should be borne in mind, and the computational difficulties of large basis calculations will become more serious. Larger model spaces may be truncated by comparing oscillator excitation energies with the extra of additional particles pairs.

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Table I. Parameters of the simple quark model

m_{ns} MeV/c^2	m_s MeV/c^2	a_c MeV/fm^2	b fm	α_s
313	513	23.67	0.6	1.517

Table II. Composition of N and Δ^+ wavefunctions for a_c=23.67 MeV.fm^{-2}

particle	3q-component	(3q)(q$\bar{q}$)-component	
	$(0s)^3$ %	$(0s)^3(0\bar{s})(0p)$ %	$(0s)^4(0\bar{p})$ %
The 3q(q$\bar{q}$) components include u$\bar{u}$ and d$\bar{d}$ pairs only			
N	84.8	10.8	4.4
Δ^+	83.8	13.7	2.5
The 3q(q$\bar{q}$) components include u$\bar{u}$, d$\bar{d}$ and s$\bar{s}$ pairs			
N	81.2	13.6	5.2
Δ^+	80.9	16.0	3.1

Table III. Composition of few baryons' wavefunctions for a_c=59.52 MeV.fm^{-2}

particle	3q-component	(3q)(q$\bar{q}$)-component	
	$(0s)^3$ %	$(0s)^3(0\bar{s})(0p)$ %	$(0s)^4(0\bar{p})$ %
The 3q(q$\bar{q}$) components include u$\bar{u}$, d$\bar{d}$ and s$\bar{s}$ pairs			
N(938)	85.0	10.9	4.1
Λ(1081)	86.2	9.5	4.4
Σ(1163)	86.3	10.2	3.5
Δ^+(1232)	84.6	12.9	2.5
Σ^*(1340)	86.1	11.4	2.5
Second-order excitation including u$\bar{u}$ and d$\bar{d}$ pairs only			
	C_0 %	C_α %	C_β %
N(938)	83.2	22.3	4.5
Δ^+(1232)	76.2	20.4	3.4

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