STEAM TURBINES

A TEXT-BOOK FOR ENGINEERING STUDENTS

BY

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WITH 230 ILLUSTRATIONS AND NUMEROUS EXAMPLES

LONGMANS, GREEN AND CO.
39 PATERNOSTER ROW, LONDON
FOURTH AVENUE & 30TH STREET, NEW YORK
BOMBAY, CALCUTTA, AND MADRAS
1917

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PREFACE

THIS book has been written primarily to suit the requirements of engineering students; but the author hopes that the methods of calculation outlined in it will also be found useful by engineers, who have to deal with the design or operation of steam turbines.

The first portion of the text is devoted to detailed descriptions of commercial representatives of the various types now on the market, and gives the reader a fair idea of the variations in design. The second portion is devoted to what may be termed the "technical" part of the subject; and the method of treatment employed is that which experience has shown to be best suited to the requirements of the average student.

The steam turbine is largely the outcome of continuous experiment on the part of manufacturers, and the production of the commercially successful machine depends, to a considerable extent, on the use of design coefficients derived from practice. These have to be applied, with judgment and skill, to suit the conditions of each particular case.

Under these circumstances it is a somewhat difficult matter to treat the subject in a purely scientific way in a text-book, if the derived results are to be of practical value to the reader.

The author has tried, as far as possible, to reconcile conflicting conditions. While the fundamental principles underlying any scheme of design are clearly defined, the practical limitations to the application of "theory," which experience has shown to be necessary for the production of a commercially successful turbine, are also stated.

An endeavour has been made, by the introduction of practical coefficients and data collected from scientific and technical papers, the technical press, and occasionally from private sources, to place a practicable system of calculation before the reader.

In the development of the scheme care has been taken to arrange the matter progressively, so that the reader may work through the detail calculations relating to the various portions of the machine in connected order, and finally end with a general method of calculation for the provisional determination of the general proportions of any given type to fulfil specified conditions of operation.

The worked example is an educative factor of importance in a text-book. A large number of such examples has therefore been introduced throughout the text. The data, in most cases, are selected in conformity with practical requirements. Since, however, the majority of the calculations, relating to the subject of design, require the exercise of judgment in the selection of empirical coefficients, the numerical results obtained are to be regarded as only provisional. The main object of these examples is to show how the methods of calculation, developed in the text, can be applied to given cases under arbitrarily assumed conditions.

Only the turbine proper is discussed in the text. The allied subject of condensers and condensing plant has not been touched, as any attempt to deal with this in a chapter would be quite inadequate.

In compiling the subject-matter, the author has drawn largely from the valuable sources of information in the proceedings of the various technical societies, from the columns of the technical press, and also from the works of other writers on the steam turbine. Every care has been taken to acknowledge all sources from which the information thus quoted is drawn, either in the body of the text or in footnotes.

The author desires to express his thanks to the Councils of the following Societies who have kindly permitted him to reproduce illustrations that have appeared in their respective transactions :—

Institution of Mechanical Engineers, Institution of Engineers and Shipbuilders in Scotland, Institution of Electrical Engineers, Junior Institution of Engineers.

Also to thank the proprietors of *Engineering* for permission to reproduce illustrations of several turbines which have appeared in that journal, and the proprietors of the *Mechanical World* for permission to reproduce a set of curves.

The various engineering firms whose names are given in the text have supplied catalogue literature and other information, illustrations, drawings, and in some cases blocks for the descriptive section of the book; and the author also desires to thank them for the large amount of useful matter they have generously placed at his disposal.

Finally, he desires to express his thanks to Mr. B. J. Lloyd Evans, B.Sc., who has kindly read the proofs and undertaken the heavy task of checking all the numerical calculations throughout the book. As it is hardly possible, even with the most careful revision, to avoid mistakes, the author will be grateful for the notification of any numerical or clerical errors that may be found, and will also welcome suggestions for the improvement of the book.

W. J. GOUDIE.

UNIVERSITY OF LONDON,
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September, 1916.

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CHAPTER I

CLASSIFICATION OF STEAM TURBINES

1. Action of Steam in Reciprocating Engine and Steam Turbine.—In a steam engine the energy transformation is accomplished by the action of a statical pressure on the piston. At any instant during a stroke, the steam follows up the piston at approximately the same velocity, and its dynamical effect on the piston is negligible.

In a steam turbine the energy transformation is also accomplished by a pressure action on the blades of the mobile part or rotor, but the pressure is altogether dynamical. It is due solely to change of momentum of the steam in its passage through the blade channels.

In the reciprocator, the heat energy is directly transformed into work on the piston, as the expansion proceeds. In the turbine there is a double transformation of energy. The heat is first converted into kinetic energy by the expansion of the steam, and this kinetic energy is then transformed into mechanical work on the rotor blades.

2. Nozzle and Guide Passages.—The conversion of heat energy into kinetic energy is accomplished by expanding the steam in a suitably formed passage called a "nozzle."

Such a passage may be either stationary or moving. Moving nozzles are usually formed by the channels between consecutive blades on the rotor. The Parsons blading is typical of this class.

The term "nozzle" is usually applied, in its restricted sense, to the fixed type of passage. It is, however, generally applicable to any turbine passage, in which a drop in pressure takes place with expansion and generation of kinetic energy.

3. Classes of Steam Turbines.—Like reciprocating engines, steam turbines may be classified under the headings of "Simple" and "Compound." A more general classification is usually adopted. They are classed in accordance with the conditions of operation of the steam on the rotor blades, and may be conveniently divided into the four following groups :—

I. Impulse.
II. Pure Reaction.
III. Reaction.
IV. Combination.

In an impulse machine the whole of the expansion takes place in the fixed nozzle passages. As there is no expansion in the moving passages, between the rotor blades, the steam pressure is the same at the inlet and outlet edges of these blades.

In a pure reaction machine, the expansion takes place wholly in moving nozzle passages, formed at the circumference of a wheel or at the ends of a rotating arm. There are no stationary nozzle passages.

In the reaction type, expansion takes place in both fixed and moving nozzle passages, and the pressure at entrance to the rotor blades is therefore greater than at exit. The term "reaction" employed to denote this type of machine is not quite correct. Both impulse and reaction are impressed on the blades, and "impulse—reaction" is the more correct title. The term "reaction," however, has come into such general use among engineers that it will be used throughout to denote this class of machine.

The combination turbine is merely a combination in one machine of Classes I. and III. It consists of a high pressure impulse and a low pressure reaction section.

4. A secondary classification which has been adopted within recent years is based on the special conditions of industry in connection with which the turbines have to operate. Any machine of the foregoing classes may be employed as

(*a*) Back Pressure Turbine.
(*b*) Reducing ,,
(*c*) Exhaust ,,
(*d*) Mixed Pressure ,,

For the industrial processes carried out in chemical works, sugar refineries, breweries, paper mills, textile factories, laundries, etc., low pressure steam has to be used, in addition to the high pressure steam required for power generation.

This low pressure steam is obtained either from a low pressure boiler or by the reduction in pressure of the high pressure steam.

Back Pressure Turbines.—When a steam turbine is installed to generate the power for the establishment, both these expedients are unnecessary. By running the turbine at a high exhaust or "back" pressure, the exhaust can be utilised for the manufacturing processes.

A turbine designed to meet such conditions is called a "back pressure turbine."

Reducing Turbine.—The back pressure machine is suitable where there is a steady demand for the exhaust steam of the turbine running at its full or most economical load. In certain establishments the demand may be very variable, much less steam than the full output being required at one time than at another. During certain periods the demand may even cease altogether. Such variable conditions are met by the use of a "reducing" turbine.

In this case the steam is drawn off at some intermediate stage of the machine instead of being taken from the last or exhaust stage.

When the demand for steam ceases the turbine works as a pure condensing machine. When the maximum amount of steam is required, practically all the steam entering the turbine byepasses into the auxiliary steam pipe. Between the limits of full exhaust and no exhaust supply, the operative conditions can be economically adjusted for any demand; the steam not required for the reduced auxiliary supply being usefully employed in the L.P. section of the machine.

Exhaust Turbine.—One of the important advantages which the steam turbine has over the reciprocating engine is its capacity to economically use steam at low pressures which are quite impracticable in the steam engine. The recognition of this fact has led to the employment of exhaust steam turbines. An exhaust turbine is supplied with low pressure steam about atmospheric pressure, from a reciprocating engine or a series of such engines. It materially increases the power output of a plant, for the same output of the boilers.

Mixed Pressure Turbine.—Before the introduction of the steam turbine the exhaust steam from the numerous engines employed at collieries, rolling mills, forges, blast-furnaces, etc., was discharged into the atmosphere. There resulted an enormous waste of heat, which might have been usefully employed, had it been practically possible to expand the steam to much lower pressure.

The remedy for this uneconomical state of affairs was found in the exhaust turbine, but the very intermittent supply of steam in most cases created a serious difficulty.

The problem was successfully solved by Professor Rateau, by the introduction of the "heat accumulator," the principle and action of which is discussed in Chapter XII.

The accumulator is still extensively employed, but the more recent practice is to further supplement the intermittent supply by high pressure steam in what is termed a mixed pressure turbine. This is a high pressure turbine, so arranged that the auxiliary exhaust supply regenerated in the accumulator can be led into the low pressure section. When the supply of exhaust steam is sufficient to run the turbine at its full capacity, the high pressure supply is automatically cut out.

When the auxiliary exhaust supply ceases the low pressure inlet is automatically closed and the high pressure one opened, and the machine runs as a high pressure turbine. Between full exhaust and no exhaust the high pressure supply can be so adjusted that the deficiency of steam to maintain full output is made good from the high pressure supply.

The governing gears in these machines have been well developed, and they run very steadily under most variable conditions of exhaust supply. This fact and the obvious simplicity of the system has led to the extensive adoption of this type where an intermittent supply of exhaust steam has to be handled.

5. Classification of Impulse Turbines.—Returning to the first system of classification, turbines of Class I. may be subdivided into four groups as follows:—

(1) Simple or Single Stage Impulse.
(2) Velocity Compounded Impulse.
(3) Pressure-Velocity Compounded Impulse.
(4) Pressure Compounded Impulse.

The conditions of operation of the steam in each of these types of impulse machine, and also in the reaction and combination (Classes II., III., and IV.), are diagrammatically illustrated in Figs. 1 to 7.

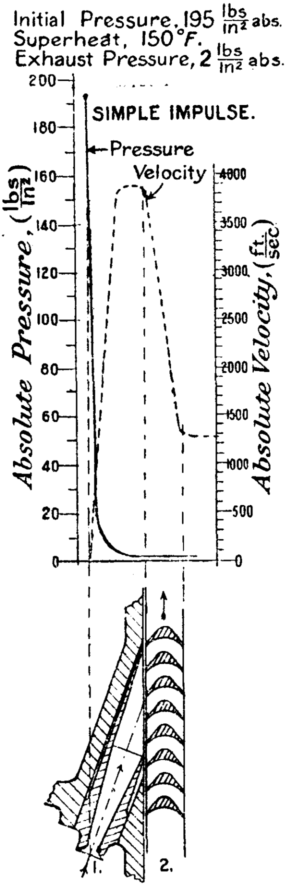


FIG. 1.

For the sake of comparison the diagrams have all been drawn for the same initial and final pressure conditions. These are 195 lbs./in.^{2} abs. 150° F. superheat and 2 lbs./in.^{2} abs. corresponding to a vacuum of 26 inches. Large-powered compound machines run on vacua varying from 27 inches to 29 inches; smaller machines from 24 inches to 27 inches. The exhaust pressure chosen here is an average for all the types.

6. Definition of a "Stage."—In the case of an impulse machine, a stage, which without qualification means a pressure stage, is any portion of the machine in which a drop of pressure takes place with generation of kinetic energy. It consists of a set of fixed nozzle passages and one or more adjacent sets of moving blade passages into which the nozzle set discharges.

In the case of a reaction machine there is some difference of opinion as to what represents a stage. Most engineers include in this a fixed ring of nozzle passages and its adjacent moving ring.

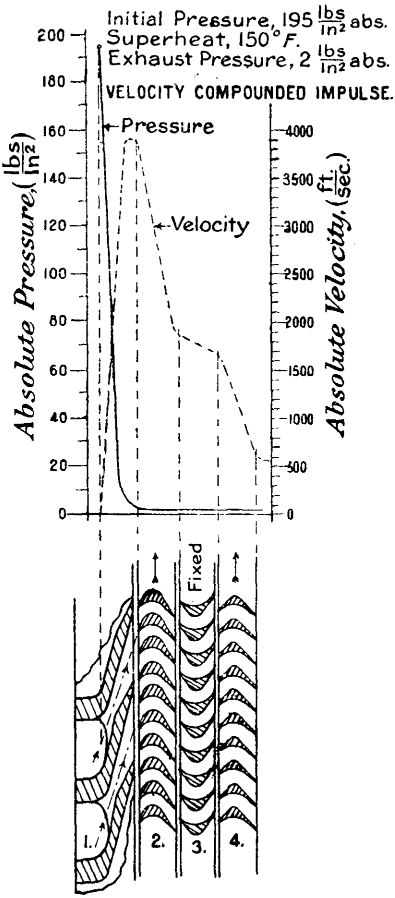
According to the definition just given for the impulse turbine, each of these rings represents a stage, since there is a drop in pressure in each. In dealing with the work done on the moving ring, it is, however, necessary to consider the total drop in the two rings, and to cover this condition the combination may be termed a "double-stage."

7. Simple or Single Stage Impulse.—This machine is diagrammatically represented in Fig. 1. It consists of one or more fixed nozzle passages, 1, and one ring of moving blades, 2, mounted on a rotor.

The steam, as indicated by the pressure curve on the diagram above, is expanded in the nozzle from the initial pressure of 195 lbs./in.^{2} abs. to the exhaust pressure 2 lbs./in.^{2} abs. The velocity, as shown by the dotted curve, rises from zero to a maximum of 3870 ft./sec. The steam is discharged on to the rotor blades with this velocity, and in passing through the blade channels produces a dynamical pressure on the blades and drives them forward. The steam is discharged into the casing with an absolute velocity of 1270 ft./sec., and then exhausted to the condenser.

The wheel carrying the blades is so arranged that the steam at the lower pressure has free access to each side of it. There is thus the same pressure at the inlet and outlet edges of the blades.

The form of the nozzle passage is important. The passage converges and then diverges, and the nozzle is in consequence called a "convergent-divergent" nozzle. The reason for the use of this form of passage is given in Chapter VII.



Two Velocity Stages.

FIG. 2.

8. Velocity Compounded Impulse.—This machine, as shown in Fig. 2, consists of a set of fixed nozzle passages, 1, also convergent-divergent, two rings of moving blades, 2 and 4, and a set of fixed guide blades or intermediates, 3, between them. Complete expansion of the steam between 195 and 2 lbs./in.^{2} takes place again in the nozzle passage, and the steam is discharged into the first moving ring, 2, with an absolute velocity of 3870 ft./sec. This ring runs at a much lower speed than that of the simple impulse. The absolute velocity of exit from it is 1900 ft./sec. The steam next passes into the ring of fixed blades, 3, in which its direction is changed, and, on account of frictional and other losses in the channel, its velocity is slightly diminished. It is discharged from the "guide" blades into the second ring of moving blades, 4, and finally leaves this ring with an absolute velocity of 550 ft./sec.

The kinetic energy of the steam at exit from the last ring is what remains of the original energy of the jet at entrance to the first ring, and is termed the residual energy or "carry over." It represents a loss and should be kept down to as low a value as possible. It will be noted that this carry-over loss is much less than that of the simple impulse, in which the exit velocity is 1270 ft./sec.

In this case again the steam pressure is kept constant in the wheel chamber, the object usually being accomplished by perforating the disc wheel, carrying the blades, with several holes.

There is, therefore, no drop in pressure in the fixed and moving rings.

The combination constitutes a pressure stage; but as the velocity and kinetic energy are reduced by passing the steam through two moving rings it is said to have two velocity stages.

In a machine of this type the number of velocity stages per pressure stage is the same as the number of moving rings in the stage.

There is, however, a variant of the type in which only one ring of moving blades is used. The steam, by means of specially arranged guide channels, is reversed several times, and passed at each reversal through the ring channels. The number of velocity stages in this case is given by the number of times the steam is passed through the moving ring.

9. Pressure-Velocity Compounded Impulse.—A machine of this type consists of two or more velocity compounded machines in series. Each of these constitutes a pressure stage.

A two-stage machine is diagrammatically shown in Fig. 3. It has two velocity stages per pressure stage.

The total pressure drop is so divided between the two stages that about the same amount of energy is absorbed in each.

In the first stage nozzles, 1, the pressure falls from 195 lbs./in.^{2} to 48 lbs./in.^{2} This drop produces an exit velocity of 2870 ft./sec. The steam, after discharge from the second moving ring, 4, has an absolute velocity of 350 ft./sec.

It is assumed that the steam does not flow directly into the nozzles of the second stage, but is brought nearly to rest by impingement on the wall, 5, of the nozzle diaphragm.

The second drop in pressure from 48 lbs./in.^{2} to 2 lbs./in.^{2} takes place in the second-stage nozzles, 1, and the velocity at exit again rises to 2870 ft./sec. The steam is again discharged from the last moving ring, 4, with an absolute velocity of 350 ft./sec.

It is shown, in Chapter VIII., that for the most efficient absorption of energy by the moving blades, the ratio of exit nozzle velocity and blade velocity must have a definite value for each type of machine. Approximately this ratio for a velocity-compounded machine having n velocity stages is $\frac{1}{n}$th of the value for a simple machine. This means that with the same nozzle velocity in each case the blade speed, and hence for a given diameter, the rotational speed of the velocity compounded machine is $\frac{1}{n}$th that of the simple machine.

In the pressure-velocity compounded type the reduction of the nozzle velocity, due to the subdivision of the total pressure drop, permits of a further reduction of rotational speed.

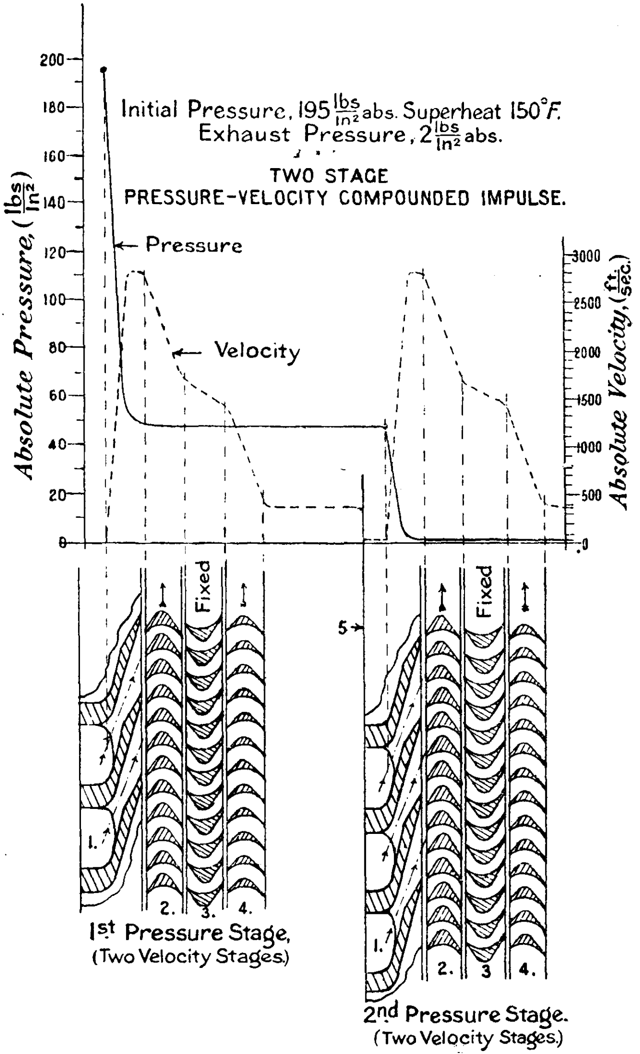


FIG. 3.

An outstanding feature of this machine is the difference of pressure drop in the first and second stages, required to produce the same jet energy. It requires a pressure drop from 195 to 48 lbs./in.^{2} to produce the same kinetic energy as a drop from 48 to 2 lbs./in.^{2} This case indicates in a general way the importance of low-pressure steam for power production.

The pressure-velocity compound turbine is made for land-work, with from two to six pressure stages, each stage usually having two velocity stages.

In directly coupled marine turbines a low rotational speed is required for reasonable propeller efficiency, and necessitates a low blade speed. This again requires the use of a large number of pressure stages.

For mechanical reasons a large proportion of the total energy has to be utilised in the first stage, usually about 30 per cent., and this stage is provided with four velocity stages.

Intermediate pressure stages usually have three and low pressure stages two velocity stages per pressure stage.

By the employment of gearing with this type of turbine the conditions of operation can be much improved, and a faster-running machine corresponding to the land type can be used.

The pressure-velocity compounded impulse is universally known as the Curtis turbine, after the American engineer Curtis, who first commercially developed it.

10. Pressure Compounded Impulse.—This machine consists of a number of simple turbines in series. It may also be regarded as the limiting case of the pressure-velocity compound, in which the number of velocity stages per pressure stage is reduced to one.

The first three stages of a twelve-stage machine are shown diagrammatically in Fig. 4, together with the pressure and velocity curves for the whole machine.

The total pressure drop between 195 and 2 lbs./in.^{2} abs. is subdivided into twelve steps. Each drop is arranged to give the same jet energy per stage, corresponding, as shown by the velocity curve, to a constant nozzle exit velocity of 1200 ft./sec. Steam from the nozzles, 1, enters the ring of moving blades, 2, with this velocity, and is discharged from the ring with an absolute velocity of 360 ft./sec. It is assumed that the steam flows without obstruction into each set of nozzles with this velocity, which also represents the final carry over velocity at exhaust.

In comparison with the simple impulse working between the same pressure limits, and having the same ratio of steam speed to blade speed, the number of revolutions of this compound machine is about one-third that of the simple one. A larger number of stages would give a proportionately greater reduction.

This machine is made with numbers of stages ranging from eight to twenty. It was commercially developed, about the same time, by Professor Rateau and Mr. Zoelly, although the type is usually spoken of as the Rateau turbine.

In the land Zoelly machines there are usually fewer stages for a given pressure drop than in the Rateau ones.

The conditions indicated above are approximately those occurring in the Zoelly machine.

The form of the nozzle passage is different from that shown for the other types. It is convergent. This form has to be used for the efficient production of the smaller jet energy. The nozzle groups at the first few stages, as a rule, cover only a small portion of the corresponding circumferences of the blade rings, so that there is "partial" admission at these stages.

A variant of the type consists of a machine in which a group of the single impulse high pressure stages is replaced by one velocity compounded stage, having usually two velocity stages.

This machine is sometimes called a Curtis-Rateau turbine.

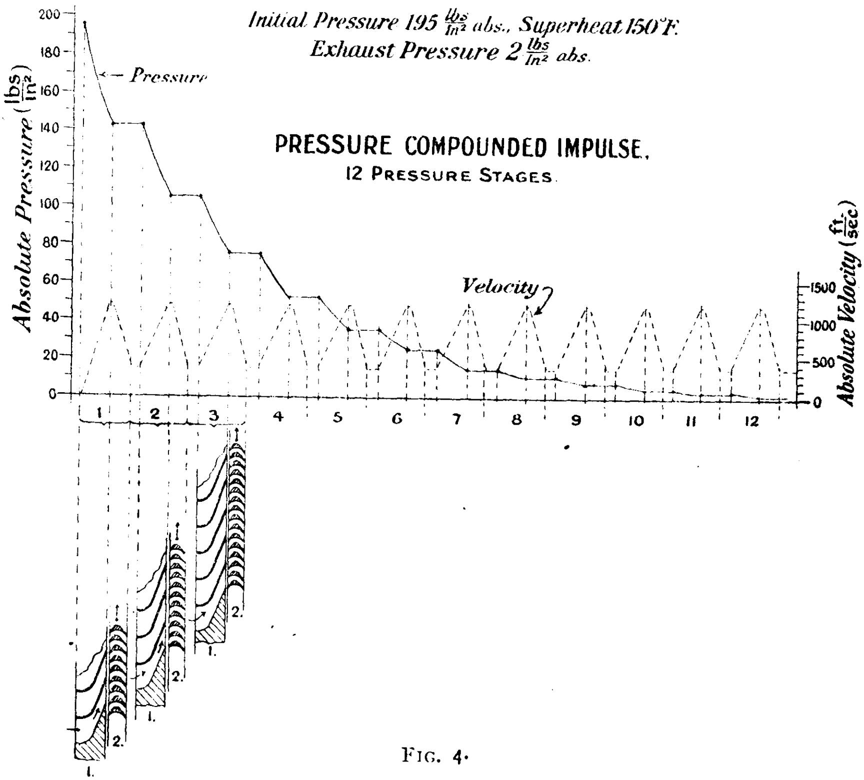


FIG. 4.

11. Pure Reaction Turbine.—This form of machine, which may be regarded as a steam Barker's mill, is shown diagrammatically in Fig. 5. It consists of two divergent nozzles, 1 and 2, mounted at the ends of a hollow rotating arm. This arm is attached to a hollow shaft, through which steam is supplied to the passage, 3, in the arm and thence to the nozzles. The reaction produced at each nozzle by the outflowing steam rotates the arm and shaft.

The pressure and velocity curves are shown on the left for nozzle, 1.

The steam is assumed to expand from 195 lbs./in.^{2} at entrance to 2 lbs./in.^{2} at exit from the nozzle, issuing with a velocity relative to the nozzle of 3870 ft./sec. The nozzle is here assumed to move with a velocity of 1770 ft./sec. in the opposite direction, so that the absolute velocity of exit is 2100 ft./sec.

As in the case of a Barker's mill, the pressure head at entrance to the nozzle, on account of centrifugal action, is greater than the pressure at the centre of the arm. The rise in pressure between the centre and the nozzle can only be roughly estimated by a doubtful and tedious method of calculation, and for this case its estimation is not worth the trouble involved. It is assumed, therefore, that this pressure is such that the final value at the nozzle entrance is 195 lbs./in.^{2}.

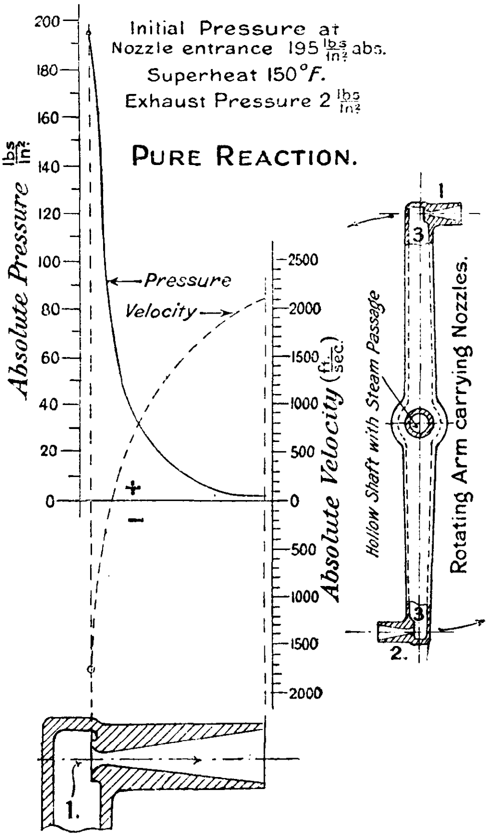


FIG. 5.

Since the steam at entrance has no velocity relatively to the nozzle —that is, in the forward direction—its absolute velocity is the same as that of the nozzle in the backward direction. The velocity curve thus starts at a negative velocity of 1770 ft./sec.

Small machines of this type were used by Dr. de Laval for driving cream separators before he developed the well-known simple impulse machine. Sir Charles Parsons also carried out experiments with the type, but did not adopt it.[1] Small machines have also had a restricted use in America.

There is, however, no present commercial representative of the type. The machine is of little practical importance, and need not be further considered.

12. Axial Flow Reaction. Parsons' Turbine.—This machine is

[1] Richardson's "Evolution of the Parsons Steam Turbine," p. 42.

shown in Fig. 6. On account of the large number of stages and the

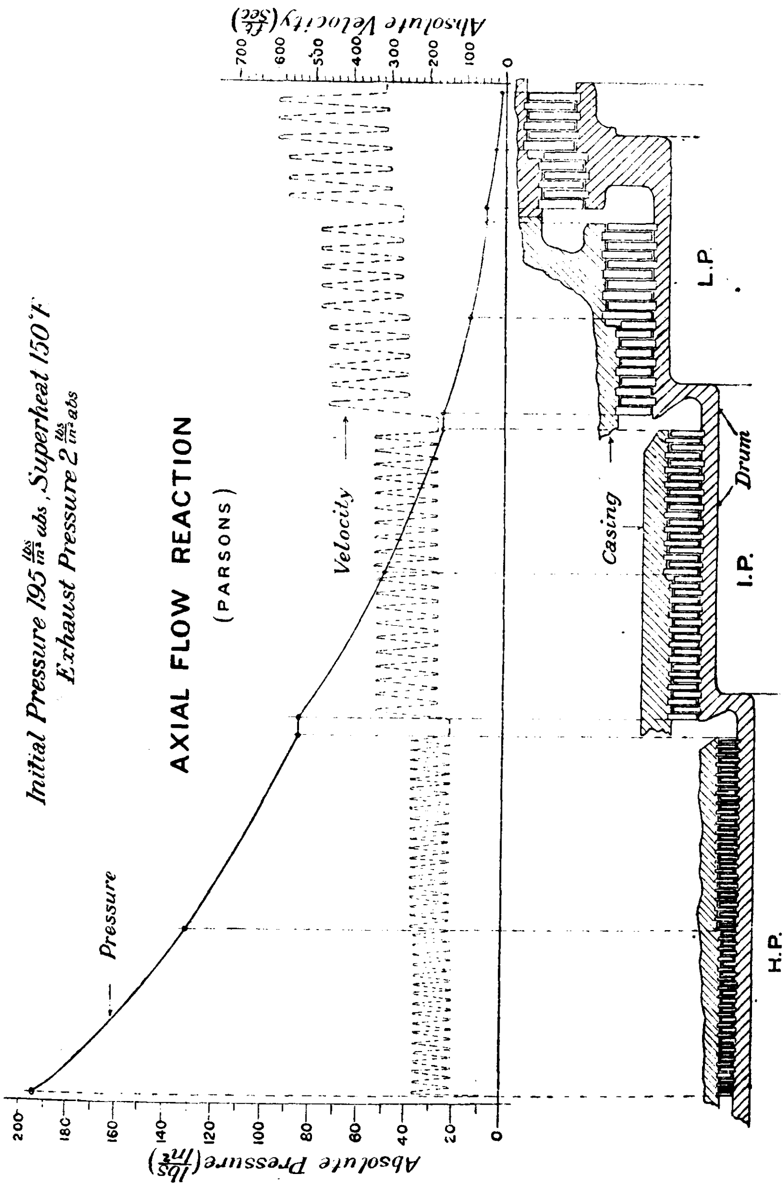


FIG. 6.

small blade sections, the developed plan of the blading as shown for the other machines is replaced by an elevation of the various groups or expansions. In impulse machines the blades are usually mounted on the circumferences of disc wheels. In the multistage reaction machine this construction is impracticable for several reasons, and they are mounted on the circumference of a drum, which is usually stepped into three or, as shown in this case, into four diameters. The channel sections of any ring of blades fixed to the casing are identical with those fixed on the drum.

The steam enters a ring of fixed nozzle passages with the absolute velocity of exit from the previous moving ring. It drops in pressure between inlet and outlet, as in the nozzles of the impulse machine, and is discharged with an increased velocity into the channels of the next moving ring. In these channels it still continues to expand and fall in pressure, and there is a further transformation from heat to kinetic energy between entrance and exit. During the passage through a moving ring work is being done on the blades, and the absolute velocity of exit is lower than that from the fixed ring.

In this case the ratio of blade velocity to exit nozzle velocity is taken as a constant for the whole machine.

Since the mean diameter of blade rings for each expansion increases on account of the stepping of the blade heights, and also the stepping of the drum, the absolute entrance and exit velocities of the steam progressively increase at the successive expansions.

The machine is divided into high, intermediate and low pressure sections. In the high pressure section, consisting of 37 pairs of fixed and moving rings (double-stages), the pressure falls continuously from 195 lbs./in.^{2} to 84 lbs./in.^{2}. For the first seventeen pairs the absolute velocities at entrance to and exit from the moving rings are 230 and 125 ft./sec. For the next twenty pairs the velocities are 242 and 135 ft./sec. The rise is due to the increased length of blade in the second expansion.

In the intermediate section, which consists of twenty pairs, the pressure drops continuously from 84 lbs./in.^{2} to 26 lbs./in.^{2}. The blades are again divided into two groups or expansions, each containing ten pairs. The height of the second is greater than that of the first. The corresponding absolute velocities are 337 and 170 ft./sec. for the first and 340 and 180 ft./sec. for the second.

In the low pressure section there are four expansions, the first two consisting of five pairs, and the last two of three pairs each. In passing through the first two expansions the steam falls in pressure from 26 to 8 lbs./in.^{2}. The absolute velocities are 470 and 260 ft./sec. for the first and 490 and 270 ft./sec. for the second.

In passing through the last two expansions the steam falls in pressure from 8 to 2 lb./in.^{2}. It is then exhausted to the condenser. The corresponding absolute velocities are 570 and 300 ft./sec. for the first, and 600 and 310 ft./sec. for the second expansion. The latter is the residual velocity of the machine.

The general direction of the flow of steam in this machine between admission and exhaust is parallel to the axis of the shaft, and the machine is in consequence called an axial flow turbine. The actual path of a particle is a spiral having its axis coincident with that of the shaft.

This type of machine, which was the commercial pioneer of all the others, is universally known as the Parsons turbine. It is the outcome of the genius and engineering skill of Sir Charles Parsons.

It will be noted that for the absorption of the same amount of energy the axial flow reaction type requires the employment of a much larger number of stages than the multistage pressure compounded impulse (Zoelly) machine. This is due to the fact that a much higher ratio of blade speed to steam speed is required for the best efficiency of a reaction stage than for an impulse one.

In the particular case illustrated there are seventy-three double stages or pairs of fixed and moving rings as against twelve in the pressure compounded impulse working between the same pressure limits. There are one hundred and forty-six pressure drops in the one case and twelve in the other.

13. Radial Flow Reaction. Ljungström Turbine.—There is another type of reaction turbine in which the steam flows radially outward from the shaft. This is called a Radial Flow Turbine. Sir Charles Parsons carried out extensive experiments with various designs of this type before finally adopting the axial flow machine as a standard.[1] The great success of the axial flow machine, especially in marine work, together with the development of the different classes of axial flow impulse turbines withdrew the attention of engineers for some time from the radial type.

In 1910, however, two Swedish engineers, Mr. Birger Ljungström and his brother Mr. Fredrik Ljungström, revived the interest in the radial flow turbine, by the production of a machine of a unique character.

The Ljungström radial flow turbine, while it embodies some devices well tried in the axial machine, has, on the whole, a design which is a radical departure from that of any other turbine on the market.

For a given output its size is small relatively to that of the corresponding Parsons axial flow machine. The ingenious methods employed to prevent unequal expansion enables high superheat to be used, while the employment of "double motion" discs ensures speed ratios of blading and steam-jet, much in advance of the maximum that, so far, has been found practicable in the Parsons machine.

The turbine is constructed either for "single" or "double" motion. For the purpose of comparison with the Parsons axial flow machine the single motion is selected here. A section of the blade channel is shown in Fig. 7. The fixed rings, 1, are mounted concentrically on the casing by special bulb shaped expansion rings, 3 (indicated diagrammatically). The moving rings, 2, are similarly fixed to the face of a steel disc, which is provided with "expansion" slots. This is the original arrangement. The "disc" in the later designs consists of a series of broad rings fastened by means of expansion bulb rings similar to those

[1] A full account of these and other experimental Parsons machines will be found in Richardson's "Evolution of the Parsons Steam Turbine."

used for the blade rings. These details are more fully illustrated in Chapter VIII.

The disc rotates about the shaft axis XX. Steam at 195 lbs./in.^{2} abs. and 150° F. superheat is admitted through suitable ports at the centre, I, and flows alternately through the fixed rings, 1, and the moving disc rings, 2, to the circumference, O, being discharged to the condenser at a pressure of 2 lbs./in.^{2} abs.

As in the case of the Parsons machine the absolute velocity rises and falls at entrance to and exit from the moving blades; but the magnitudes progressively increase from the first to the last pair of rings.

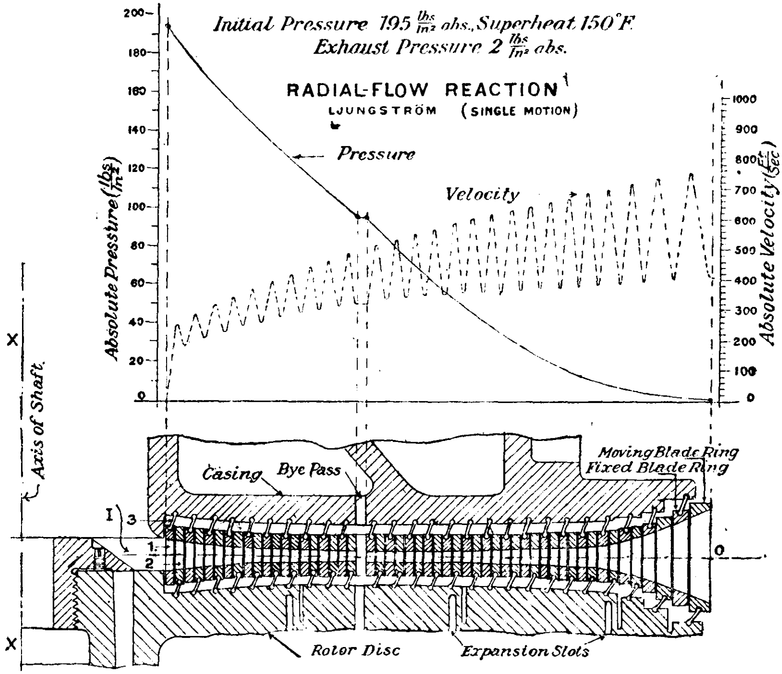


FIG. 7.

At the first ring the steam is assumed to issue with a velocity of 275 ft./sec. At the last fixed ring its value is taken as 740. The residual velocity is taken as 410, and the residual loss is somewhat greater than that of the Parsons.

The total energy in this case is absorbed by twenty-six pairs of rings as against seventy-three pairs in the Parsons. There are fifty-two pressure drops in the one and one hundred and fifty-six in the other. This is due to the fact that with the disc construction, the speed of rotation can be made much greater than that of the Parsons drum. It has been taken as twice that of the Parsons.

It is important to note the variation in the blade lengths, between inlet and exhaust. In the Parsons axial flow turbine these lengths progressively increase. In the Ljungström turbine they first decrease, then remain nearly constant for a considerable length of the blade channel, and only begin to increase rapidly at the last few L.P. stages. The reason for this difference in the two types is given in Chapters XIV. and XV. The arrangement of concentric blade rings is admirably suited for the production of a double motion turbine. If instead of a fixed casing, as shown in Fig. 7, a second disc with blade rings is substituted, the stationary element of the Parsons and other axial types disappears. Every blade becomes a moving and hence a "working" blade. The two discs under the propulsive action of the steam revolve in opposite directions. Kinematically the action is the same as though one disc and its blade rings were fixed and the other disc rotated at double the speed.

It is this "double motion" form of the Ljungström turbine that is likely to come into prominence in the future, as it realises the condition of a high velocity ratio between steam and blade speed, a condition which is essential for a high efficiency.

14. Combination Turbines.—There are certain mechanical drawbacks common to both the axial flow reaction and the impulse machines. One of these is the necessity for "running clearances" between the fixed and stationary parts in order to prevent "seizure." Through such clearance areas more or less leakage of steam takes place.

In the Parsons machine the weak spot is at the blade tips in the high pressure section. In the impulse machine it is at the stage diaphragm glands, through which the shaft passes.

Owing to the continuous nature of the expansion in the reaction blading full peripheral admission is necessary from the first stage onward.

For the passage of a given weight of steam the blade lengths at the H.P. end are much shorter than they would be if partial admission were possible.

In the majority of cases the blade tip clearance which is necessary for safe running, has, in consequence, to be made an appreciable fraction of the annular area between the drum and casing, and the leakage loss becomes appreciable. On the other hand, in the impulse machine where there are considerable pressure differences in the successive stages at the H.P. end, the reduction of the leakage area between the shaft and diaphragm gland is equally important, if the leakage loss is not to become appreciable.

As will be seen in a later discussion, it is only at the H.P. end of the Parsons' machine that the tip leakage is serious. By discarding the reaction blading of the H.P. section, and substituting a velocity compounded stage in which the steam is discharged from a set of nozzles, the H.P. tip leakage is entirely eliminated. Another advantage is the ability to use highly superheated steam in the first stage. This steam is in contact only with the small externally fixed nozzle box, and distortion of the turbine casing and rotor is avoided. Further, the absorption of a large proportion of the total energy in the impulse stage leaves the steam, at entrance to the reaction section, at a comparatively low pressure and temperature and simplifies the external packing glands. The replacement of the high pressure section of the drum by the velocity wheel very materially shortens the rotor, increasing its stiffness and at the same time shortening the length of the turbine.

The advantages enumerated above are purely mechanical. From the thermodynamic point of view the section of reaction blading would be more efficient than the impulse wheel, if the clearance defect did not exist.

The disadvantage on the thermodynamic side is outweighed by the advantages gained on the mechanical and commercial sides. This type of machine is usually called a "disc and drum" turbine, and is now being manufactured by most of the leading firms as a standard, in addition to the pure Parsons machine.

Pressure compounded high pressure impulse sections have been tried in place of the velocity compounded wheel. The arrangement necessitates the use of labyrinth packings to prevent "tip leakage" at the blade rings, which take the place of the diaphragm glands. The system has not found favour, and practically all the combination machines being put on the market are fitted with the velocity compounded wheel.

As regards the pressure and velocity curves for this type, it is not necessary to draw them. The diagram is simply a combination of the curves of Fig. 2, with the intermediate and low pressure curves of Fig. 6.

CHAPTER II

IMPULSE TURBINES

15. Simple Impulse. De Laval Turbine.—The chief commercial representative of this type is the de Laval turbine manufactured in this country by Messrs. Greenwood and Batley, Leeds, in standard sizes ranging from 5 to 500 B.H.P. On account of the very high rotational speed at which the rotor has to run, the machine is fitted with double helical reducing gear. The reduction in the smaller sizes is 10 : 1 ; in the larger 13 : 1.

A longitudinal section through a small machine is shown in Fig. 8, and a section through a large machine in Fig. 9. The parts are similarly numbered in each figure.

Steam from the stop valve is admitted at 1, and passes to the throttle, 2, through a strainer, 3. The throttle is controlled by a centrifugal governor, mounted on the second motion or driving shaft.

Steam flows from the throttle valve to a steam belt, 4, surrounding the wheel casing, which can be drained by the cock, 5. Several nozzles (not shown in Fig. 8) are fixed in the wall of the casing. Each nozzle is provided with a shut-off valve.

In large machines the practice is now to arrange the nozzles in groups of two or three in one nozzle plate, and to provide a shut-off valve for each group. One of these valves is shown at 6 in Fig. 9, and the complete set in Fig. 10, Plate I. Details of both types are given in Chapter VII.

The steam after expansion in the nozzles passes through the blading of the wheel, 7, into the exhaust belt, 8, and from thence through the exhaust pipe, 9, to the condenser.

The outstanding feature of this machine is its flexible shaft. This shaft, 10, is turned down to a small diameter, and supported in a spherically seated bearing, 12, and a fixed bearing, 14, placed widely apart. The shaft thus becomes flexible enough to bend when running at speed, and permits the centre of mass to coincide with the axis of rotation of the wheel. Without this provision it would not be possible to run these wheels at their extremely high speeds. These vary from 30,000 in the smallest to 10,600 R.P.M. in the largest machines. An inflexible shaft under these conditions would give rise to serious vibration troubles.

The shaft of the small machine passes through a hole in the boss of the wheel, but that of the large machine is in two parts which are bolted to the solid boss. The sections of these wheels are not the same. The reasons for this difference and the solid boss in the second case

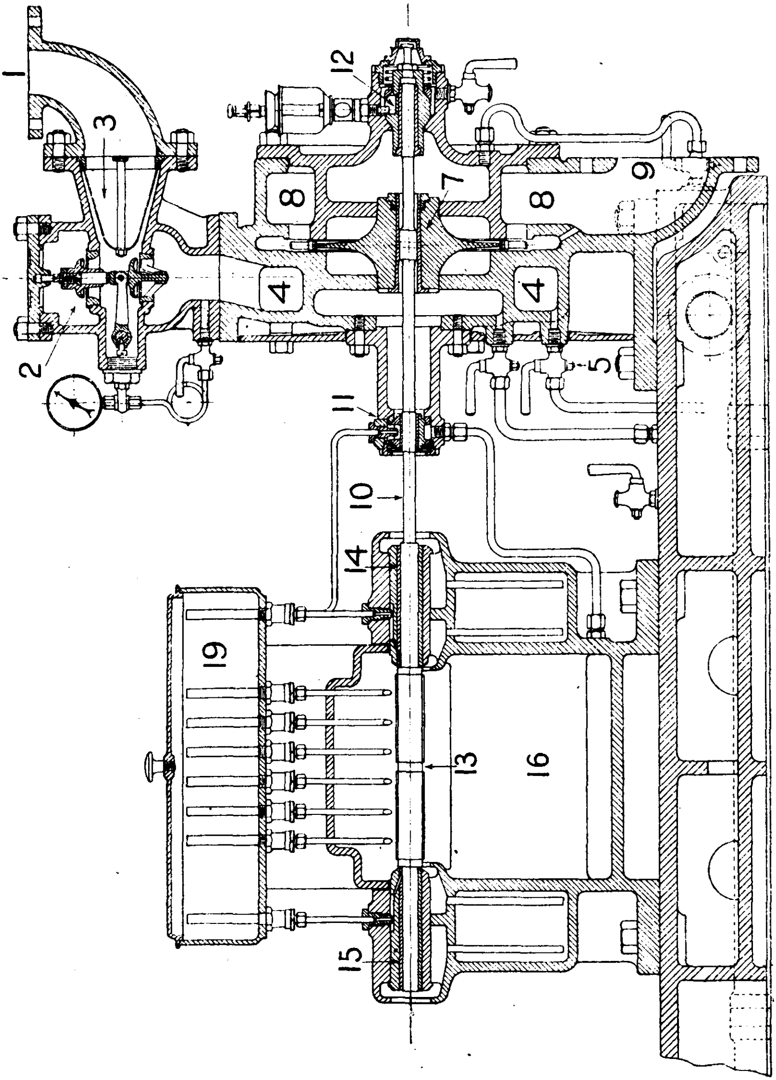


FIG. 8.—Small de Laval Turbine.

are stated in Chapter IX, where enlarged sections of the wheels, showing the details of construction, are given.

The driving pinion, 13, forms part of the turbine shaft, and is situated between the bearings, 14 and 15, at the ends of the gear case, 16, Fig. 8. In machines of 20 H.P. and upwards the pinion, 13, and gear

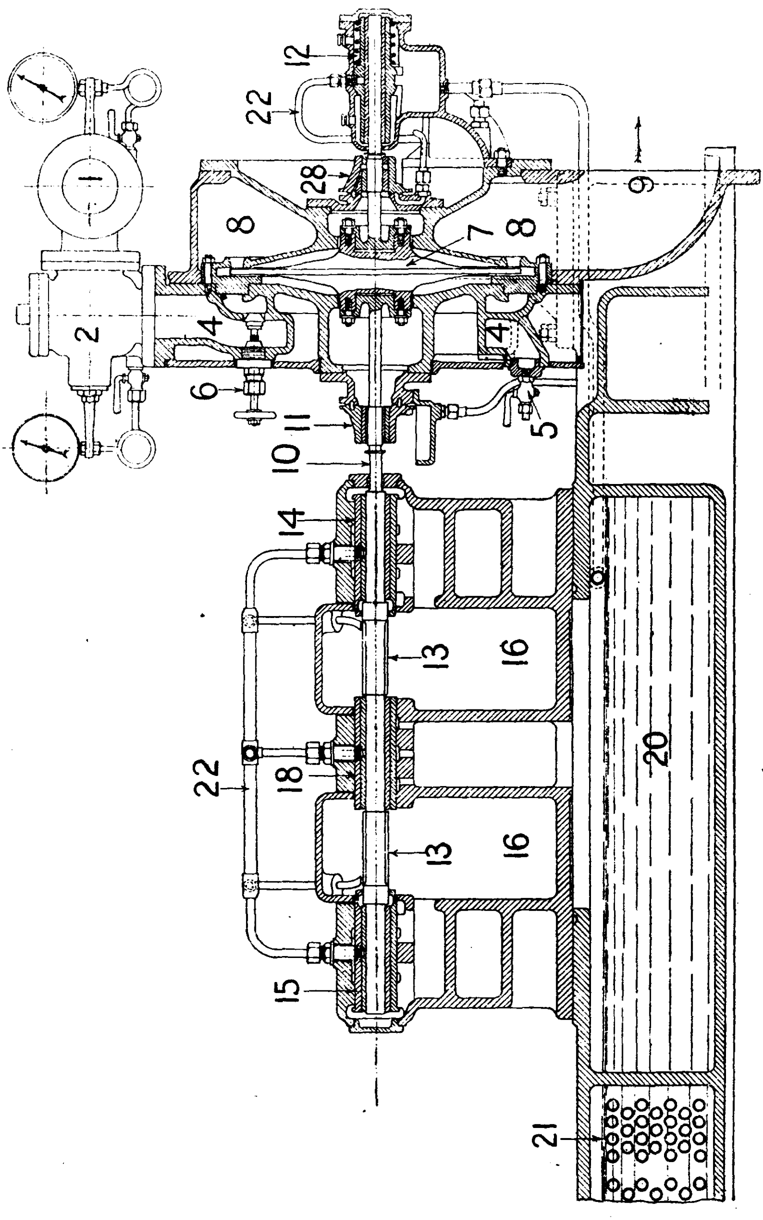


FIG. 9.—Large de Laval Single Gear Turbine.

wheel, 17 (Fig. 10) are divided, and intermediate bearings, 18, are provided in the gear case. Some machines are fitted with two driving shafts. In this arrangement the pinion is placed between the two gear wheels, and the driving shafts rotate in opposite directions.

The high speed bearings of the small machine, Fig. 8, are syphon lubricated from the oil box, 19, which is provided with "sight feeds." The low speed bearings of the driving shaft are "ring" lubricated. A sight feed lubricator is provided at bearing 12. Machines above 20 H.P. have forced lubrication. The hot oil from the bearings is returned to a tank, 20 (Fig. 9), cooled by the pipe coil, 21, through which water is circulated, and is then distributed to the bearings through the piping, 22, by a gear-wheel pump. This pump, placed in the oil tank, is driven by worm gear on the driving shaft of the machine. On the plan of the large machine shown in Fig. 10, with the gear case and top of dynamo removed, 23, 24, 25, and 30 are the main bearings of the driving shaft and dynamo. The worm gear driving the pump shaft is shown at 26.

Where the turbine shaft passes through the casing at 11, Fig. 8, and at 11 and 28, Fig. 9, tightening bushes are fitted to prevent leakage of air. An outside elevation of the large machine is shown in Fig. 11, Plate I. The centrifugal governor, 29, is fitted at the end of the driving shaft, and operates the lever, 31, which controls the throttle. If serious overspeeding occurs, it also operates an air valve, 32 (see Fig. 10). This valve admits air through the pipe, 33, to the cylinder of an auxiliary or emergency governor, 34. This governor closes a throttle flap in the exhaust pipe and cuts off the connection with the condenser, causing an immediate rise in the back pressure and reduction in speed. (See enlarged detail of this gear, Fig. 215, page 484.)

16. Velocity Compounded Impulse.—A longitudinal section and end elevation of a three velocity stage machine, by Messrs. Escher Wyss and Co., Zurich, is shown in Fig. 12. It is intended for use as an auxiliary turbine in driving fans, pumps, etc.

Steam is supplied through the pipe, 1, to the stop valve, 2, by which it is admitted to a double beat throttle valve, 3. This valve is directly controlled by a Hartung governor, 4, driven by worm gear, 5, from the shaft. The steam passes into the belt, 6, and from this to the nozzles, 7, which cover only a portion of the circumference of the wheel. The right-hand portion of the casing, 8, is connected to the box bed plate, 9. The left-hand portion carrying the nozzle plate forms an end cover, 10.

The right-hand portion and cover are divided horizontally so that the top half can be lifted for the insertion or removal of the rotor.

The packing glands, 11, are packed by sets of carbon rings. Water is also supplied through the pipes, 12, to the middle of each gland, to assist the lubrication.

The bearings, which are carried on pedestals bolted to the bed plate, are of the usual high speed ring lubricated type. At the inner end of each journal an oil-keep ring is provided to prevent the spreading

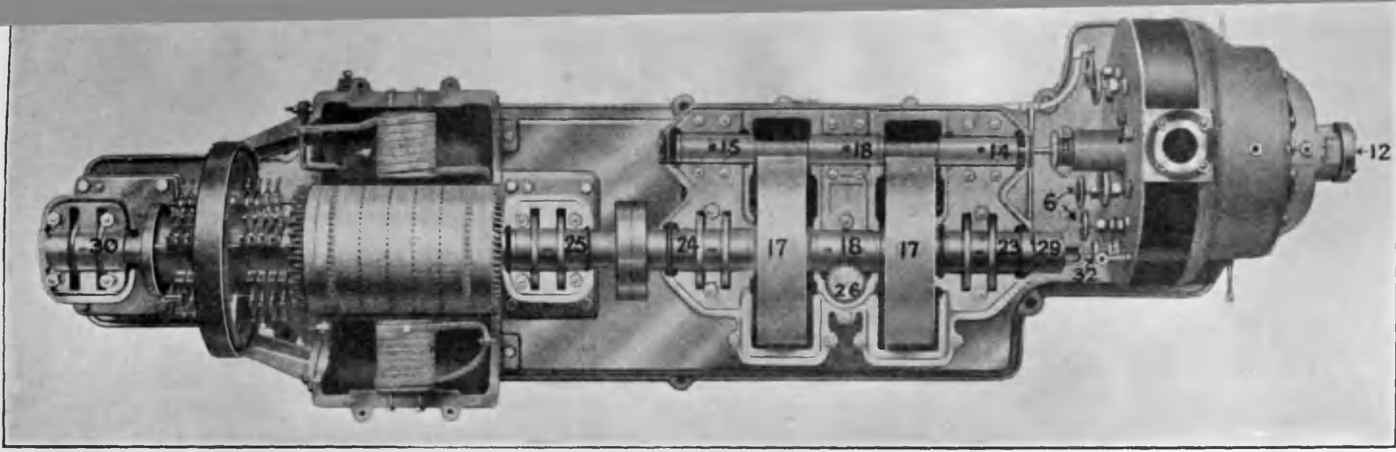


FIG. 10.—Plan of large de Laval Single Gear Turbine and Dynamo (with gear cover removed).

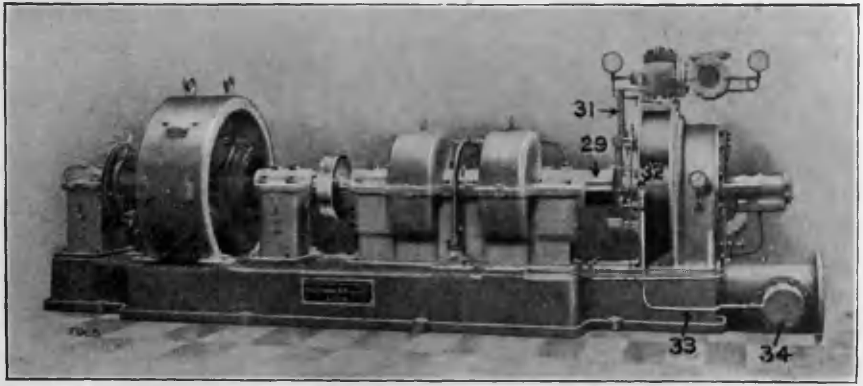


FIG. 11.—Elevation of large de Laval Turbine and Dynamo.

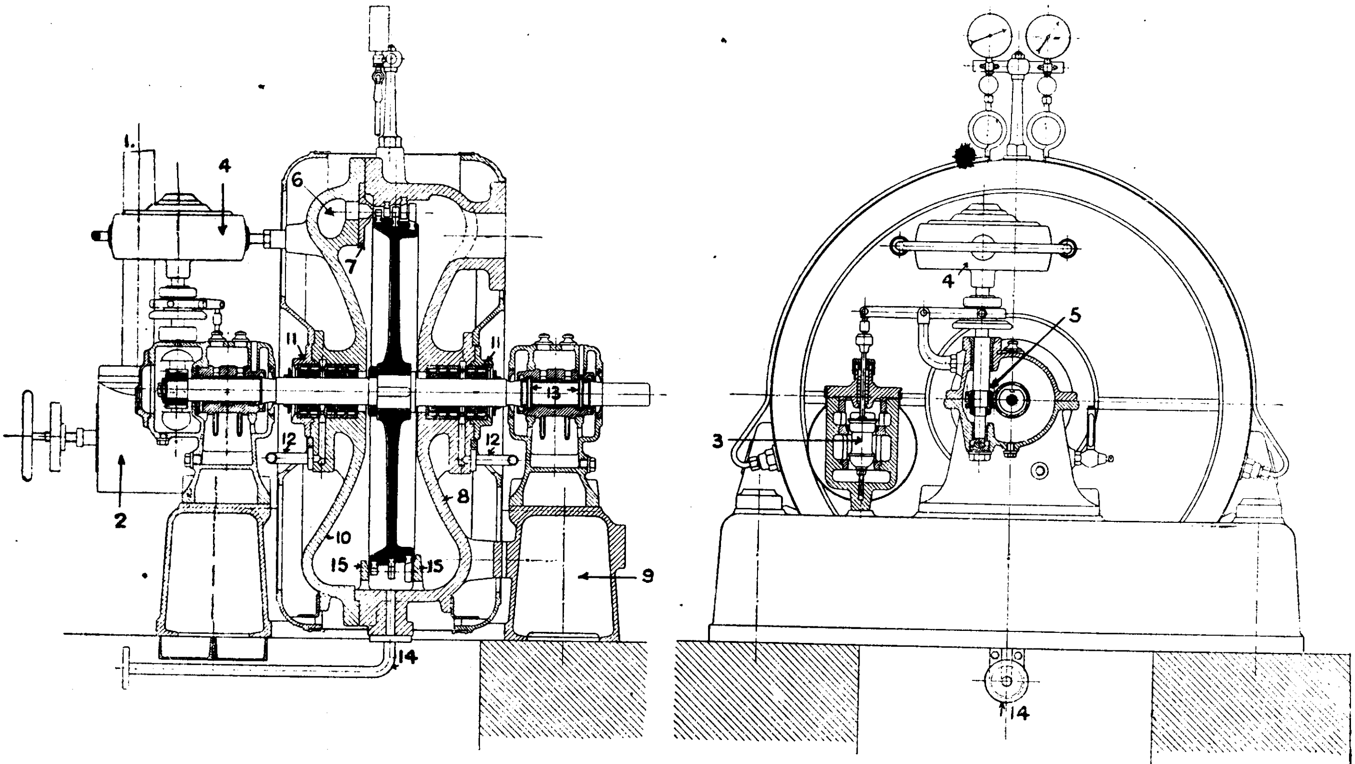


FIG. 12.—Escher Wyss Velocity Compounded Turbine.

of oil along the shaft. The right-hand journal has at each end a thrust ring, 13, to take any unbalanced axial force on the rotor. At the bottom of the casing a drain, 14, is provided. Here also are shown the sections of two ribs, 15, between which the blading is a running fit. These extend round that portion of the circumference of the wheel which is not covered by the nozzle arc, and help to reduce the "vane" action of the blading. The pressure gauges show the initial pressure in the steam belt, and the exhaust pressure in the casing.

17. Single Wheel Velocity Compound. Sturtevant Turbine.—A longitudinal section through a single ring velocity compounded machine

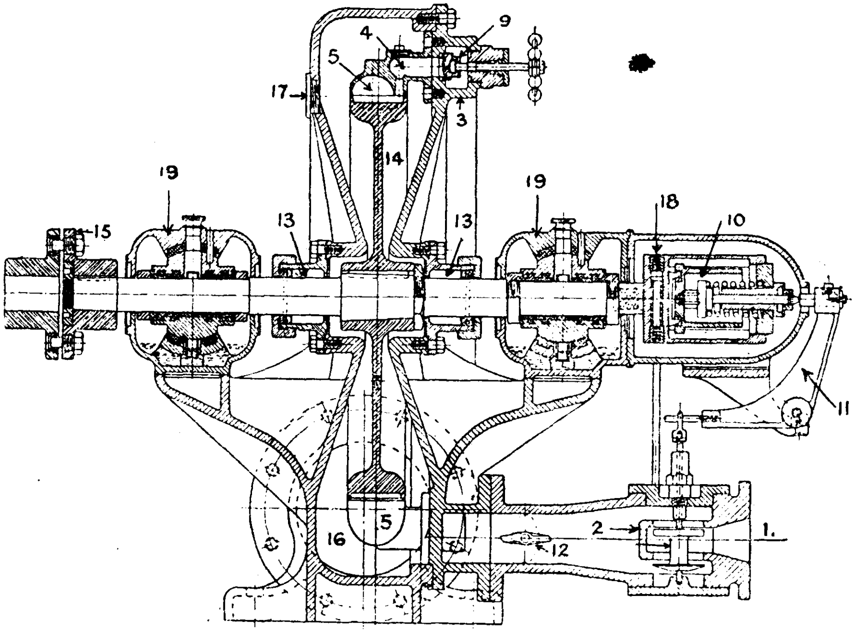


FIG. 13.—Sturtevant Velocity Compounded Turbine.

of the rim-bucket type, by the Sturtevant Engineering Company, is shown in Fig. 13.

A view of the machine when opened out and dismantled is shown in Fig. 14, Plate II. In this case the maker's object has been to produce a small moderate speed machine without complication of gearing, of low first cost and strong build, which can be used for auxiliary work, such as driving fan blowers, pumps, small lighting sets, etc. In such services high efficiency is of minor importance.

These machines are built in sizes ranging from 5 to 200 B.H.P., for speeds from 5000 to 1000 R.P.M.[1]

[1] For detail drawings of a 200 B.H.P. Sturtevant Turbine, see *Engineering*, July 11, 1913.

PLATE II.

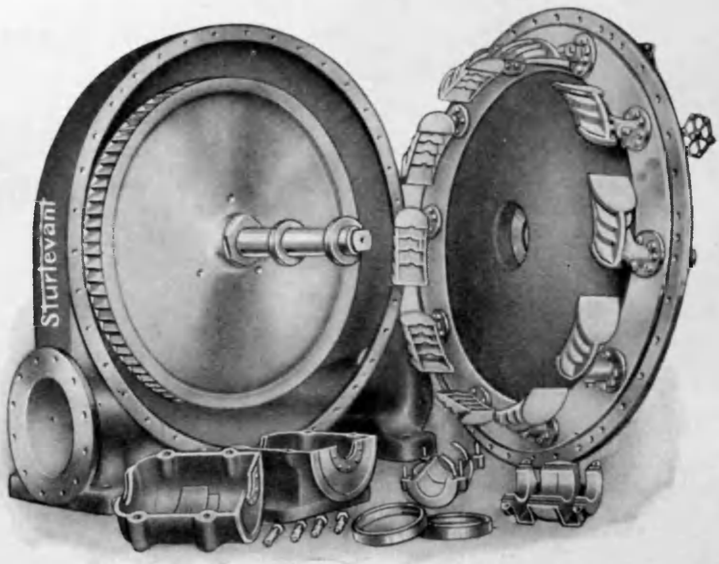


FIG. 14.—Parts of Sturtevant Velocity Compounded Turbine.

Steam entering at 1, Fig. 13, passes through the throttle valve, 2, into a steam belt, 3, from which it is distributed to nozzles, 4. Each nozzle and set of reversing passages, 5, are of bronze cast in one piece. Two sections of these are shown in Fig. 15. Instead of the wheel, 14, being "bladed," it is provided with "buckets," 6, milled in the face of the rim, as shown in the lower view of Fig. 15.[1] The flow of steam through the buckets is practically axial on account of the large inclination of the bucket walls to the radius. The directional arrows show the course of the steam through the buckets and guide passages, when the steam is turned on to start the machine.

The guide passage, 8, at the side of the nozzle, and the passage, 7, on the right (in which the arrow lines are dotted), act as starting

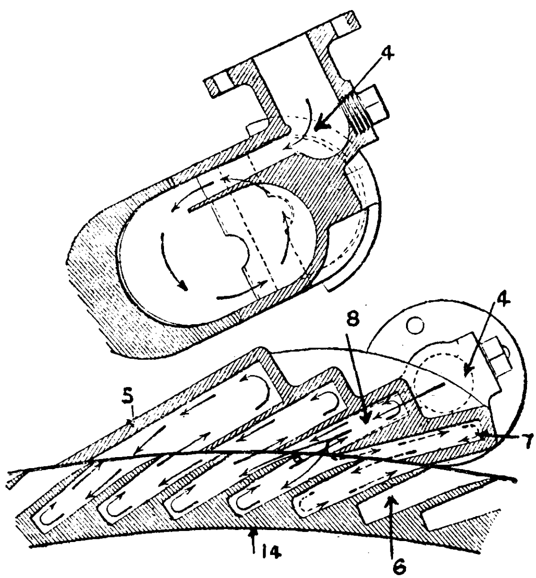


FIG. 15.

passages. They deflect the steam at the outset and help to increase the starting torque. They go out of action as soon as the machine runs up to speed.

The steam, discharged either from a nozzle or guide passage, enters a bucket at one side of the wheel, and is carried forward by the wheel, while it traverses the bucket to the other side. It is then discharged into the next guide to be reversed, and so on until it is finally exhausted into the casing, and discharged to the atmosphere or a condenser. The guide passages have to be skewed, and there is always the possibility of interference and spilling over the buckets. This type of machine

[1] The wheel of a similar machine, made by the Terry Steam Turbine Co., Hartford, Con., U.S.A., has pressed steel buckets clamped between two side plates.

has probably a lower efficiency than the one with three rings of moving blades, and intermediate guides.

The speed is controlled by a centrifugal shaft governor, 10, which operates the throttle through the lever, 11. An emergency governor is fitted at 18, and connected to an auxiliary butterfly valve, 12, which can shut off steam independently of the main throttle. The wheel is keyed on a nickel steel shaft, the end play of which is controlled by two steel collars, which can be adjusted to suit any change of position of the rotor. The bearings, 19, are spherically seated, lined with white metal, and provided with ring lubrication. The external glands, 13, are provided with a special type of metallic packing, and no lubrication is used. Sight holes are provided at 17, to enable the radial clearances between rotor and nozzles to be tested. Connection is made with the driven machine by a flexible coupling, 15. The steam is exhausted through the branch 16. A duplicate branch on the other side serves as an inspection door. Each nozzle is provided with a shut-off valve, 9, and the supply at full pressure can thus be adjusted for light loads by "cutting out" nozzles.

Another variant of the velocity compounded single ring machine has the ring of blades mounted on the side of the wheel rim, so as to overhang the rim. A set of nozzles discharges tangentially into the ring, and the steam is reversed and passed the requisite number of times by curved guide passages fitted at the outer and inner circumference of the ring. This is known as an "in-and-out" turbine. Although the guide passages are not in this case skewed, they have to be made with double curvature, and it is probable that considerable friction and disorderly flow occur in them. Some improvement is obtained by fitting guide blades at the outlet edges, to control the flow into the blades. From the mechanical point of view, the size of the guide or reversing passages, which are necessary with high output and high vacuum, precludes the construction of large units. A second variant of this class is made by the American Westinghouse Company. In this case the ring of blades is mounted in the usual way on the rim of the wheel, and the nozzles and reversing passages are fitted on either side, guide blades being provided at the outlet of each passage.

18. Pressure-Velocity Compound Impulse. Vertical Curtis Turbine.—This type of machine, introduced by the General Electric Company, U.S.A., was originally made vertical.[1]

An elevation, partly in section, of a G.E.C. vertical Curtis turbine and alternator, for an output of 15,000 K.W. at 720 R.P.M., is shown in Fig. 16.

The total pressure drop is divided into six stages. As an approximate estimate, for this output the limits would probably be an initial pressure from 170 to 180 lbs./in.^{2} gauge with 180° F. superheat, and a vacuum of 28 inches or 1 lb./in.^{2} abs.

Each pressure stage has two velocity stages. The steam is passed into two nozzle boxes, 1 and 2, containing a set of spring loaded valves

[1] An account of the earlier Curtis turbines is given in a paper by W. L. R. Emmett, *Proceedings Inst. Mech. Eng.*, June, 1904.

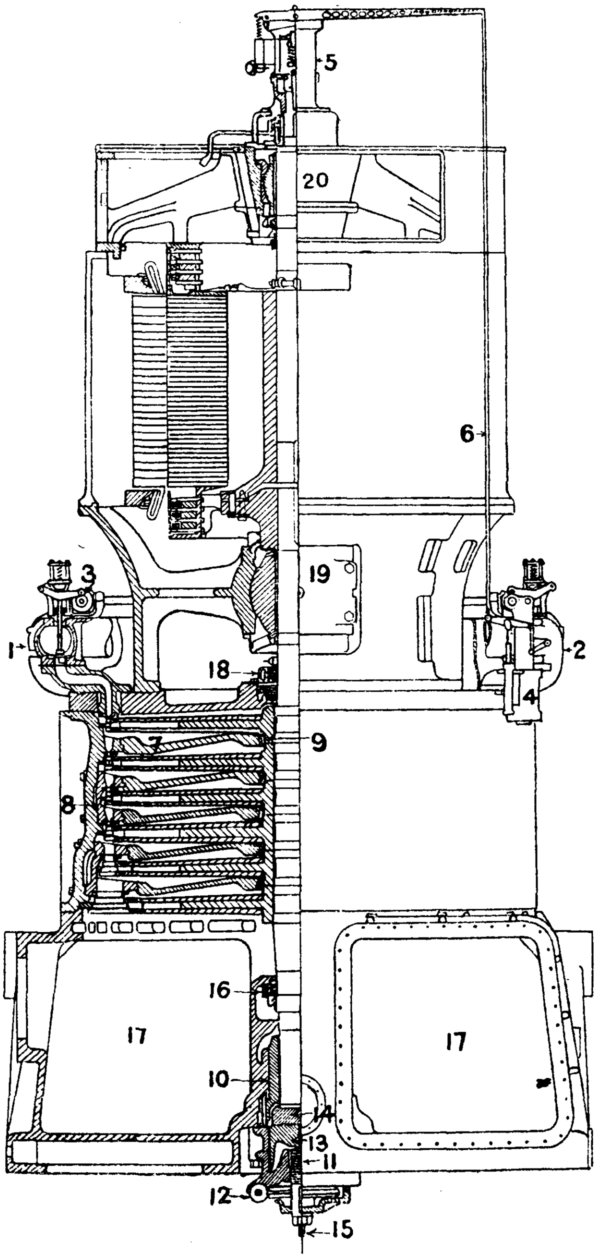


FIG. 16.—G.E.C. Vertical Curtis Turbine and Alternator.

which regulate the supply to groups of nozzles. These valves, as indicated on the left, are operated by cams, 3, on an auxiliary shaft. This shaft is rotated by the rod of an oil servo-motor, 4, the action of which is primarily controlled by the governor, 5, at the end of the generator shaft, through the medium of the rod, 6.

The casing space is divided horizontally into six compartments by dished diaphragms, 7. In a machine of this size the casing is made in six segments, with vertical flanges. To each of these segments the holders, 8, carrying the nozzles and intermediates are bolted.

The wheels and diaphragms are first assembled on the shaft, and the segments of the casing are then assembled round them. A projection on each holder, 8, fits into a corresponding groove in the circumference of the diaphragm and makes a joint. The boss of each wheel projects halfway into the adjoining diaphragm. Leakage of steam at the shaft is prevented by a special type of restrained packing ring, 9. This ring is so held between side plates that it has freedom of motion outwards, and can thus float in position in the diaphragm. Risk of seizure between the shaft and diaphragm is thus reduced to a minimum, and very fine clearances can be used at the high-pressure stages. This type of packing is illustrated in detail in Chapter X, Fig. 156.

The weight of the rotor is taken by a massive footstep bearing, 10. This bearing is provided at the lower end with an adjusting screw, 11, operated by a worm and wheel, 12.

The screw enables the shaft to be raised or lowered to give the necessary axial clearances between the stationary and running parts. It bears on the under side of a cast iron step, 13, which is kept from turning by two dowel pins. A corresponding step or block, 14, is placed below the shaft. It is keyed to the shaft by a cross key. The surface of each step is slightly recessed, leaving a narrow space in the centre and a broad bearing ring at the outside.

The screw and lower step are bored to take a central feed pipe, 15, through which oil under pressure is fed to the space between the steps, and thence between the bearing surfaces. The oil which escapes passes upward between the shaft and the sleeve bearing, and is drained back to the oil reservoir.

At the top of the bearing a gland, 16, usually water packed, is fitted to prevent leakage of air and oil into the condenser. The steam is exhausted through openings, 17. At the top of the casing a carbon-ring packing gland, 18, is fitted to prevent leakage of steam.

Alignment of the shaft is maintained by spherical seated sleeve bearings, 19, placed between the turbine and generator, and 20, at the top of the generator.

These bearings are provided with packing glands and oil catchers at the bottom, from which the leakage oil is drained back to the reservoir.

As the relay system of governing is employed, the governor at the top is simply a small spring loaded centrifugal one. Its function is to regulate the action of the servo-motor, which does all the work in operating the valve gear.

In a slow speed machine like this the strength of the disc wheel is of secondary importance in comparison with its lateral stiffness. A stiff and at the same time reasonably light and easily constructed wheel is obtained by building it up. Each wheel consists of a small disc provided with side plates. These are kept the proper distance apart near the outer circumference by distance pieces. Round the circumference of each of these plate discs is riveted the ring carrying the moving blades.

19. Horizontal Curtis Turbine.—With the exception of the Curtis machine, all the other types, from their inception, have been constructed to run with the shaft horizontal. This practice is now being generally adopted in the case of the Curtis turbine.

A longitudinal section of a six stage horizontal G.E.C. Curtis machine is shown in Fig. 17. An outside view, with the arrangement of governing gear, is shown in Fig. 18, Plate III. It has two velocity stages per pressure stage. With similar pressure limits as those of the previous machine, it is rated to develop 7500 K.W. at 1800 R.P.M.

Steam is admitted to the nozzle valve box, 1, from which its flow is controlled by a set of cam operated valves, 2. The cam shaft pinion is rotated by the rack, 3, at the end of the piston rod of the servo-motor, 4 (shown dotted in Fig. 17). The pilot valve, 16, of the motor (Fig. 18) is controlled by the lever and rod, 5, of the governor, 6. This governor is driven by a worm gear at the end of the shaft, and its spindle drives the oil pump, 7, which supplies oil under pressure to the bearings and servo-motor. The stop valve 8 (Fig. 18) is arranged to slide on its spindle. It is held open against the resistance of a strong spring by a trigger, 9. This is connected through the rod, 10, with an emergency ring governor on the main governor spindle. When the speed rises a predetermined amount above the normal, this rod trips the trigger, and the main valve instantly closes and automatically shuts down the machine. The corresponding arrangement in the vertical machine is not shown; but on this, as on all land turbines, such an emergency governor is fitted.

The end carrying the nozzle valve box, 1, the H.P. nozzle plate, 11, and external packing gland, 12 (Fig. 17), is bolted to the main casting. The turbine is divided along the centre, so that the top half forms a cover which can be lifted for the insertion or removal of the rotor and diaphragms. Each diaphragm, 13, is dished to strengthen it. Only one nozzle plate is used at the first stage. The nozzle passages in the succeeding stages are formed by steel plates cast in the diaphragm walls.

The external packing glands, 12 and 14, and also the diaphragm glands, 15, are of the same standard design as those used in the vertical machine.

The bearings are spherically seated and lined with white metal. The shaft runs under forced lubrication, the oil being supplied from the pump, 7.

The diameter of the rotor of this high-speed machine is less than half that of the larger slow-speed one, and solid forged steel disc wheels are used.

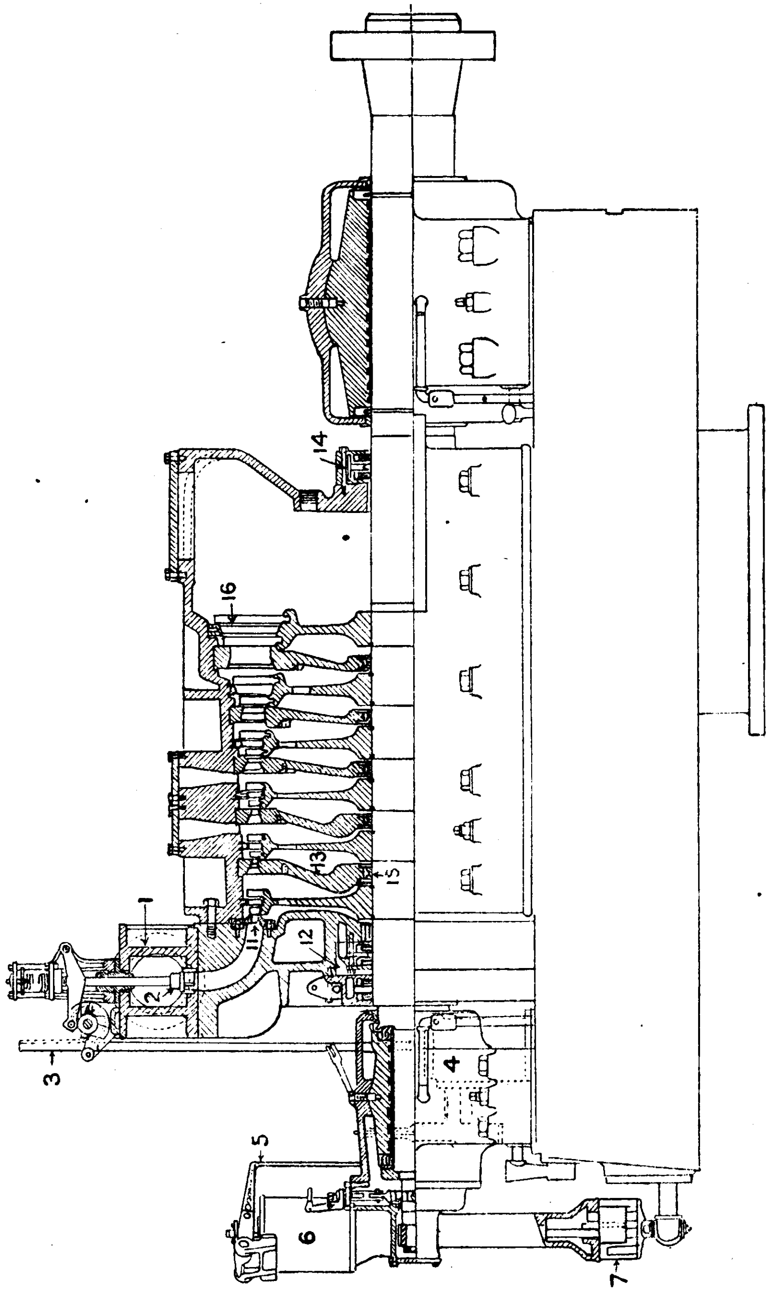


FIG. 17.—G.E.C. Horizontal Curtis Turbine.

PLATE III.

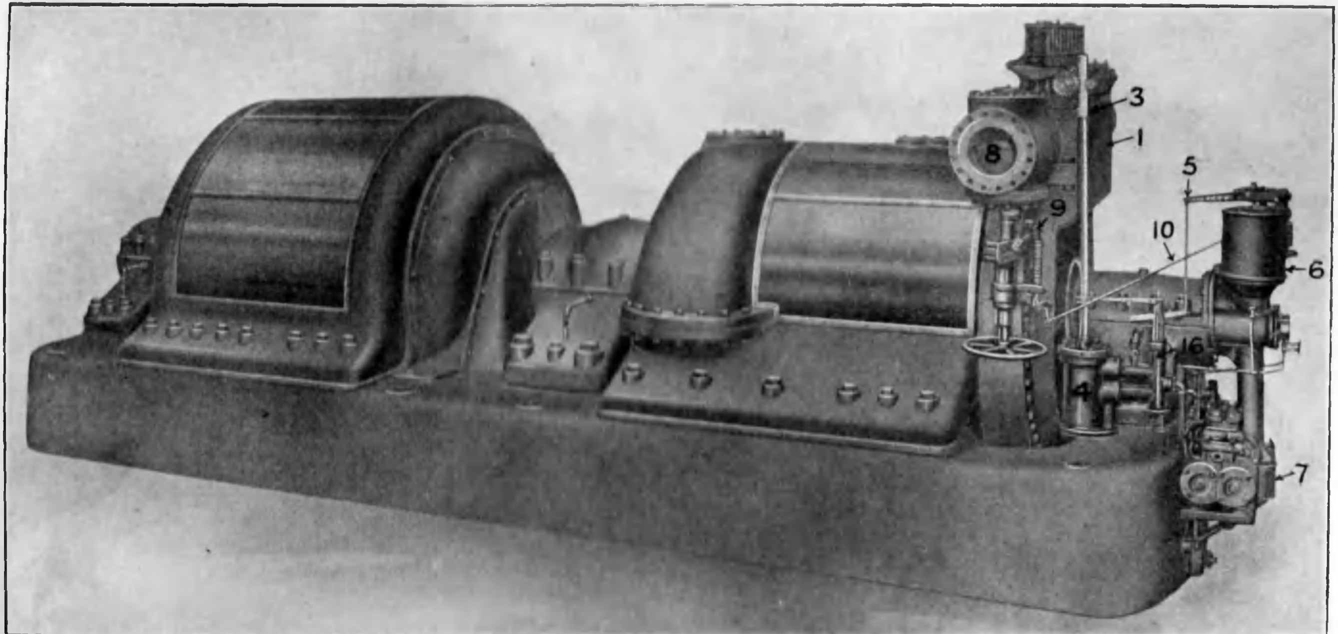


FIG. 18.—Elevation of G.E.C. Horizontal Curtis Turbine.

These wheels have broad rims with **T**-slot grooves, into which the blading is fitted. Each wheel is perforated with several large holes for equalisation of the pressure on each side.

The rings into which the intermediate guide blades, 16, are fitted are of channel section. The sides of the channel fit into grooves in the casing.

The horizontal Curtis turbine is made in this country by the British Thomson Houston Company. The machine is similar in general design to the American one. The details and method of operation of the relay governor gear are, however, somewhat different. These are discussed in Chapter XVI. (See Fig. 225, p. 495.)

In a recent development of the large five-stage B.T.H. horizontal Curtis, the third stage is made the high pressure one. The steam is successively expanded in the third, second, and first stage nozzles, the direction of flow being away from the exhaust end. It is then bye-passed from the first to the fourth stage, and flows toward the condenser expanding in the fourth and fifth stage nozzles.

This arrangement, it is claimed, has an advantage over the usual one, since the temperature and pressure of steam, at the end cover and external sealing glands, are low.

20. Marine Curtis Turbine.—The longitudinal section of a marine Curtis turbine by the Fore River Shipbuilding Company, U.S.A., is shown in Fig. 19. It is representative of the usual design of pure Curtis machine employed for the direct drive of a slow-running propeller.[1]

The machine consists of an "ahead" turbine A, and an astern turbine B, incorporated in one casing. It is designed for an output of 6000 S.H.P. at 600 R.P.M., the pressure limits being 270 lbs./in.^{2} abs. (dry steam) and 27 inches vacuum, or $1\frac{1}{2}$ lbs./in.^{2} abs.

The total pressure drop in the ahead portion, A, or turbine proper, is divided into fourteen stages. This large number of stages is due to the low rotational speed of the propeller and restricted diameter of the rotor. In the astern portion, B, there are only two stages, as economy is of secondary importance when going astern or manœuvring.

When the ship is going ahead the astern rotor revolves idly in the low-pressure exhaust steam of the ahead turbine. The disc and vane loss is thus reduced to a minimum. In the first six stages there are six wheels; in the next eight the wheels are replaced by a drum. The first pressure stage has four velocity stages, the others up to the seventh have three, and the last eight, on the drum, have two velocity stages. Each pressure stage of the astern turbine has four velocity stages.

The first stage nozzles are cast in a bronze nozzle plate, 1, which is bolted to the end of the casing. At the inlet side of the nozzle plate a steel face plate, 2, is fixed. This forms the bearing surface for a set of cut-off slide valves, 3. Each valve can be independently operated by means of the screwed spindle and handle, 4.

This arrangement for reducing the supply and maintaining a constant

[1] For dimensioned drawings of the principal details of a similar turbine, see *Engineering*, August 18, 1911.

initial pressure, combined with the large pressure drop used in the first stage, materially helps to improve the economy at "cruising" speeds.

The nozzle rings of the second and succeeding stages are of cast iron, the passages being formed by nickel steel plates, cast into the rings.

The nozzle segments of the first six stages are bolted to the faces of ribs, 5, cast on the inner circumference of the casing. The circumference of each rib is grooved to take a corresponding projection on the rim of the diaphragm, 6.

The aperture in the rib leading to the nozzle is bell-mouthed to give the steam free access to it.

In all these stages there is partial admission; but admission is complete at the first drum stage.

The astern half of the casing, which begins just behind the second drum stage, is made in a series of steps. Each nozzle ring, 7, is fixed in place with the flange carrying the intermediate guide, 8, between it and the inner circumference of the casing. The ring next the drum surface is specially formed with serrated edges to act as a packing gland. The diaphragms are built up. They have cast steel bosses, 9, and rims, 6, riveted to dished steel plates, 10. The combination is light, strong, and easily constructed. The wheels are built up in the same way, with cast steel rim, 12, and hub, 11, and two steel side plates, 13. The hubs are force fits on the shaft, and are spaced the correct distance apart axially, by distance pieces which project through the diaphragm glands.

The method of fixing the wheel blades is different from that usually employed in land turbines. The blades are first riveted at the proper angles into segments of channel-shaped foundation rings. These segments are fitted into grooves in the wheel rims and caulked in position. The sections of the intermediate blades are riveted into similar segments, 14, having dovetail grooves which fit corresponding projections in the casing. The construction is described in detail in Chapter VIII, Fig. 92.

The shaft is made of forged steel, and bored from end to end to lighten it.

Each diaphragm gland, 15, is formed by a serrated bush bolted by a flange to the face of the diaphragm. There is a space between the outer surface of the bush and the diaphragm, so that the bush can spring slightly outward if the shaft fouls it, and thus prevent seizure.

The drum portion of the rotor is mounted on a cast steel spider, 16, having eight arms. The drum is built in three stepped sections and riveted at the fore and aft ends to the spider. The after end is left open to the condenser; the forward end is completely closed. The result is that this end is subjected to a pressure forcing the whole rotor aft. In this way an axial thrust is induced, to partly or wholly balance the thrust of the propeller. In consequence, the heavy thrust block required for an unbalanced shaft is replaced by a smaller block, 17, placed at the ahead end of the machine. It is of the ordinary horseshoe adjustable type, and also serves as an adjusting block for

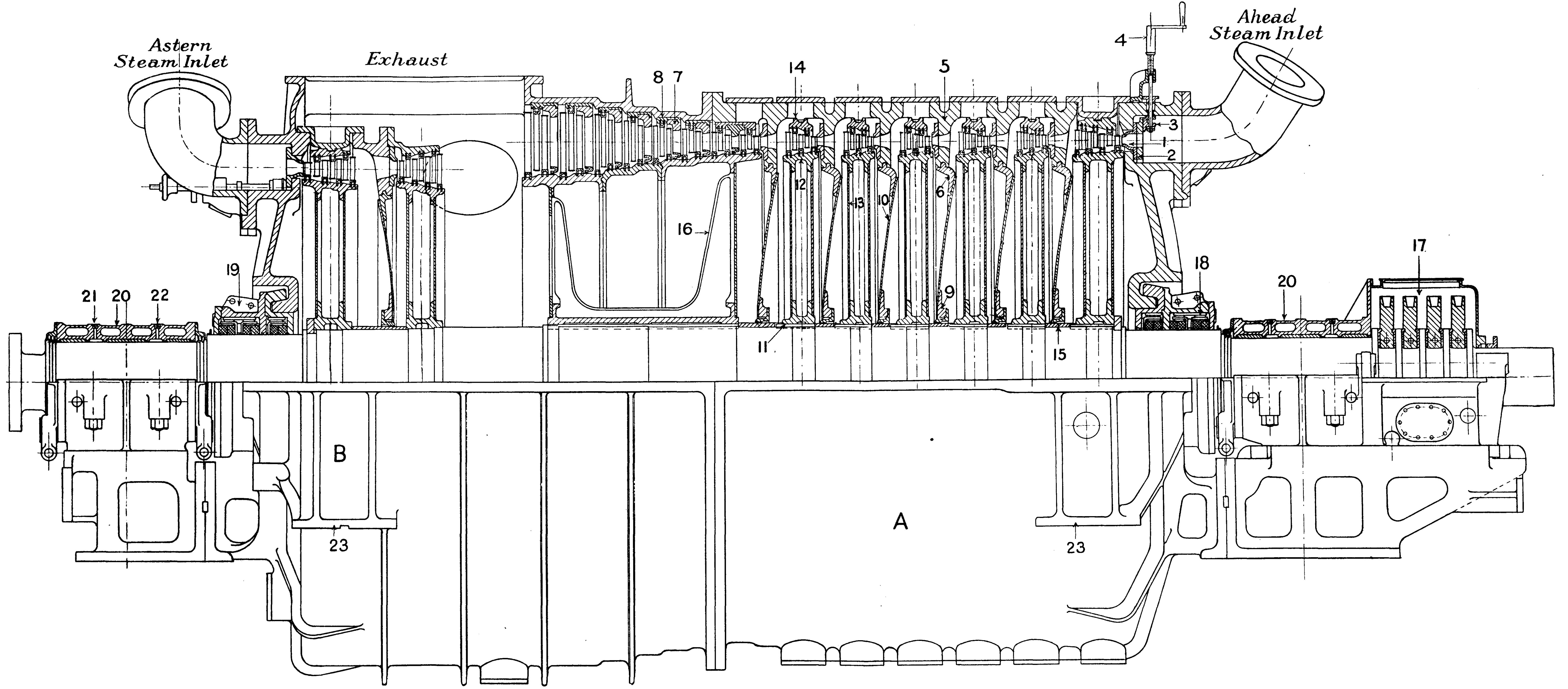


FIG. 19.—Fore River Marine Curtis Turbine.

the maintenance of the necessary clearances between the fixed and rotating parts.

Although the admission pressure to the turbine is very high, the large absorption of energy in the first stage (about quarter the total amount) reduces the pressure at the high pressure end of the casing to a moderate figure (80 to 100 lbs./in.^{2}). The H.P. shaft gland, 18, as in the previous cases, consists of segmental rings of carbon. Plate springs are employed to press the rings radially on to the shaft, and axially against the side surfaces of the holders. A similar gland, 19, is fitted at the after end.

The bearings are provided with shell casings, 20, through which cooling water is circulated.

The shaft runs under forced lubrication. The oil is fed in through the passages, 21 and 22, at the top of each bearing. It leaks to the ends and is caught by centrifugal oil throwers and drained back to the oil tank. The oil which escapes from the forward end of the forward bearing, flows directly into the oil well of the thrust bearing.

These bearings are formed in separate castings and bolted to the ends of the casing.

The casing is divided vertically into two parts at the end of the seventh stage, and horizontally along its length, so that the upper half can be lifted. Feet, 23, are cast on the lower half for the fixture of the turbine to the ship's frame. The details of the astern are similar to those of the ahead turbine, with the exception of the cut-out nozzle valves. These are not fitted.

21. Pressure Compounded Impulse. B.W. Rateau Turbine.—The longitudinal section of a large Rateau turbine by the British Westinghouse Company is shown in Fig. 20,[1] Plate IV. It is capable of developing from 5000 to 6000 K.W. at 750 R.P.M. The pressure limits are 180 lbs./in.^{2} gauge, 130° F. superheat and 28½-inch vacuum, or 0·75 lbs./in.^{2} There are twenty-four pressure drops or stages between these limits.

The steam is admitted to two nozzle boxes on either side of the H.P. casing end (not shown on the drawing). Each box contains three adjustable nozzle passages. Steam is discharged from these on to the blading of the first stage wheel, 1, from which it passes to the second stage nozzles, 2, covering a slightly greater arc than the first set. After expansion in these it is discharged on to the blading of the second wheel and so on until the ninth stage, 4, is reached. Up to this stage there is partial admission and the blades are kept a constant length

The nozzle passages are formed by carefully machined steel blades cast into segments. Each segment is bolted into a recess in the circumference of the diaphragm.

At the ninth stage there is full peripheral admission and the blade height is reduced, in consequence. At the third stage, 3, an additional set of nozzles (not shown in the drawing), is fitted to take steam from

[1] Reproduced by permission from *Engineering*. For dimensioned drawings of the details of this machine, see *Engineering*, January 13, 1911.

an automatic overload valve. A similar set for emergency overload is fitted at stage five, and supplied with steam from a hand-operated valve.

Each diaphragm is a steel casting, dished to strengthen it. It is divided horizontally, a check joint being provided to prevent leakage. At the circumference of the casing it fits into a groove. Each half is held in position by countersunk screws at the horizontal joint.

From the ninth stage onward, specially formed malleable cast blades are fitted into a dovetailed groove at the outer circumference.

The boss of each diaphragm is provided with a groove. Into this is fitted a split ring, 5, filled with white metal, which at the high-pressure diaphragms has a serrated surface, the serrations being knife edged. The diaphragms are fitted with these edges in contact with the shaft. They soon wear and accommodate themselves to the shaft without undue heating, and in this way very fine clearances are obtained. In the low-pressure section, from the ninth stage onward, where small clearance is not so essential, the rings are given an initial clearance of a few mils.

The high-pressure gland, 6, on account of the small drop in pressure, has to prevent the leakage of high pressure steam. It is a compound of a labyrinth and a centrifugal water gland.

The first portion consists of a "radial" packing, 7, which reduces the pressure, at exit to the chamber, 8, to about 20 lbs./in.^{2}. A leak-off pipe connects, 8, to an L.P. stage of the machine, in which a portion of the leakage steam is utilised. The second portion consists of a "face" labyrinth or dummy packing, which reduces the pressure to that of the condenser, the chamber, 9, being connected by the pipe, 10, with the condenser. The third portion of the gland is a paddle wheel, 11, which acts like the impeller of a centrifugal pump. Water is supplied to the wheel through a pipe, 12, and is thrown out by the wheel vanes to the circumference, effectively sealing the gland and preventing the access of air into the condenser.

The bearings, which are mounted on pedestals and quite separate from the body of the casing, are spherically seated, lined with white metal, and have forced lubrication.

The oil is supplied through the passages, 13, to the bearing and also the thrust block, 14. This block, as in the previous cases, is fitted for the adjustment of axial clearances. The axial thrust on the rotor is a negligible quantity.

The worm and wheel, 15, drive a horizontal governor which directly operates the throttle valve.

An emergency governor, 16, is fitted at the end of the shaft. It consists of a pin let into a hole in a casting, and held in place by a spring. When the turbine over-speeds, the pin is thrown out and trips the emergency gear that shuts down the main stop valve. (See also Fig. 219, page 487.)

The shaft is turned in a series of steps, each step taking the boss of one of the disc wheels. It tapers from the centre to each end, and has a hole bored the whole length. The discs are force fits on the shaft.

PLATE IV.

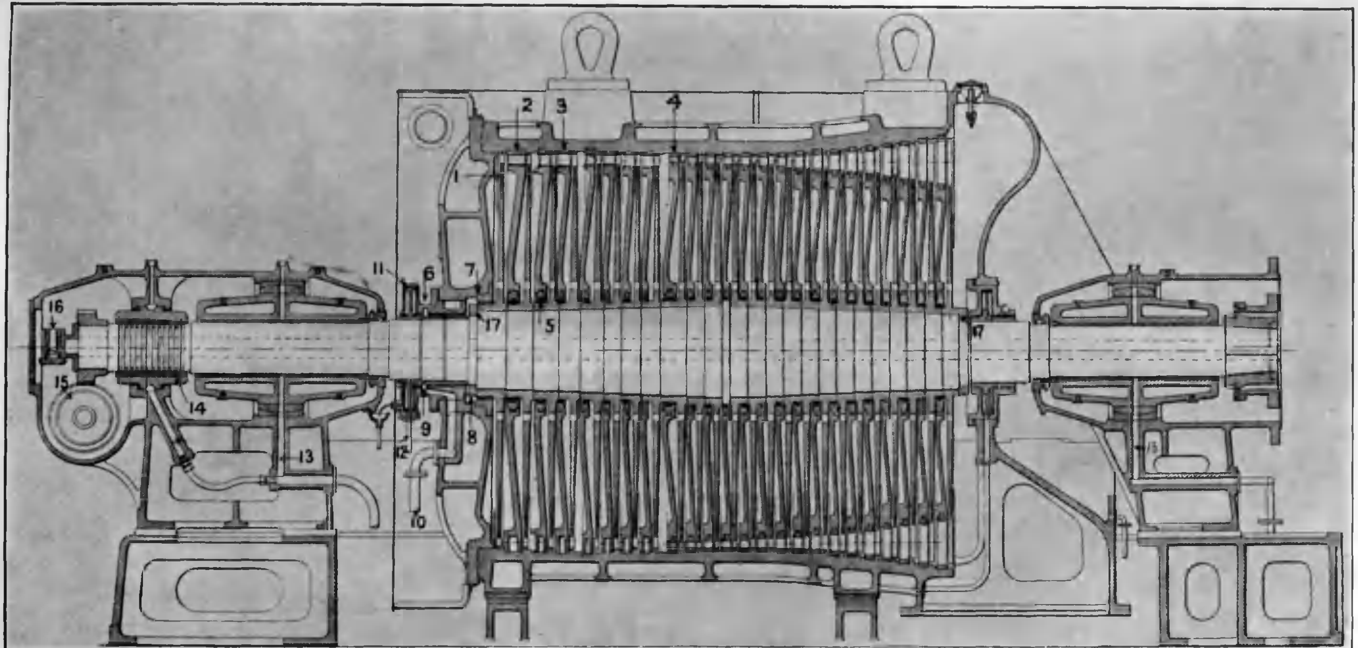


FIG. 20.—B.W. Rateau Turbine.

Each is provided with two keys. The set of wheels is "locked" by nuts, 17, at each end of the rotor.

The rims of the discs, unlike those of the Curtis machines, are made with a deep ridge, over which the forked ends of the blades are straddled. Holes are bored through the forks and the ridge, and the blades are permanently fixed to the wheel by countersunk rivets. The blades, like the discs, are made of steel and carefully milled to the required form. (See Fig. 88, page 151.)

The mean diameter of blade ring is practically constant throughout the machine, the discs being reduced at the L.P. end in order to get the requisite blade lengths.

This is the standard practice. Earlier machines were made with several sets of wheels of increasing diameter, and the casing was stepped in consequence.

The casing is divided along the centre so that the upper half and diaphragms can be lifted.

22. E.W. Zoelly Turbine.—The foregoing machine had to be built to comform to certain special conditions of operation, one of which was the low rotational speed, and hence the number of stages is larger than would normally be used if a higher blade speed could have been obtained.

The longitudinal section and end elevation of a similar pressure compounded impulse turbine designed for an output of 3000 K.W. at the usual speed of 1500 R.P.M. are shown in Figs. 21 and 22.

It is the standard design of Zoelly turbine, by Messrs. Escher, Wyss & Co., Zurich. The pressure limits are 180 lbs./in.^{2} gauge, 180° F. superheat, and 28-inch vacuum, or 1 lb./in.^{2} abs. For about the same limits of pressure as the previous machine, the number of stages is twelve.

Steam is supplied to a belt, 1, in the casing, Fig. 21. It passes into the first stage nozzles, 2. These are cast in a ring which is bolted to the end of the casing. The pressure in the casing, after expansion in the first stage nozzles, is in the neighbourhood of 100 lbs./in.^{2}, or about half the initial pressure. The steam is then expanded successively in the succeeding sets of nozzles. The diaphragms, 3, in this case are made of cast iron, and have nickel steel blades cast in them to form the nozzle passages.

To take an overload, live steam is bye-passed through the port, 4, to the third stage, Fig. 21, by means of the valve, 5, Fig. 22.

The external glands, 6, consist of a series of segmental carbon rings held on to the shaft by garter springs.

The diaphragm packing, 7, consists of segments of copper rings caulked into grooves and reduced to a fine edge next the shaft.

The bearings are cast iron lined with white metal, but are not spherically seated. Forced lubrication is employed. The oil is pumped by the gear wheel pump, 8, which is driven from the spindle of the governor, 9, Fig. 22. An auxiliary pump is also provided for starting purposes.

The governing is by oil relay, the arrangement being clearly shown in Fig. 22.

The governor, 9, by means of the lever, 10, operates a pilot valve in the cylinder, 11, which is connected by pipes to the top and bottom of the relay cylinder or servo-motor, 12. The oil is supplied to this motor at about 60 lbs./in.^{2} by a separate pump. The piston of the motor is connected directly to the balanced throttle, 13. The end of

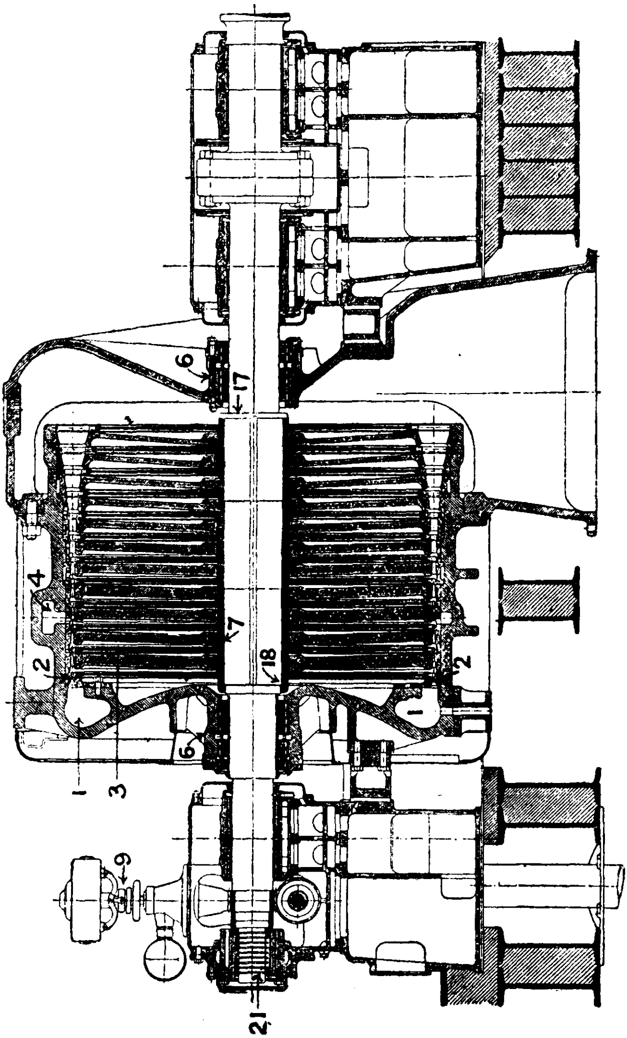


FIG. 21.—Sectional Elevation E.W. Zoelly Turbine.

lever, 10, is jointed to an adjustable block, 14. This block can be moved up and down by a wheel and pinion gear, 16, operated by a small electro-motor, 15. In this way the speed can be adjusted for paralleling alternators within ± 5 per cent. of the normal.

Steam is supplied to the throttle through the main stop valve, 19, operated through bevel and spur gearing by the hand wheel, 20.

The wheels are forged steel discs with thin hubs. They are not fixed directly on the shaft, but on expanding rings which are keyed to the shaft (see Fig. 153 *a*, page 292).

The blades and distance pieces are inserted like those of the Curtis machines in a T slot in the rim.

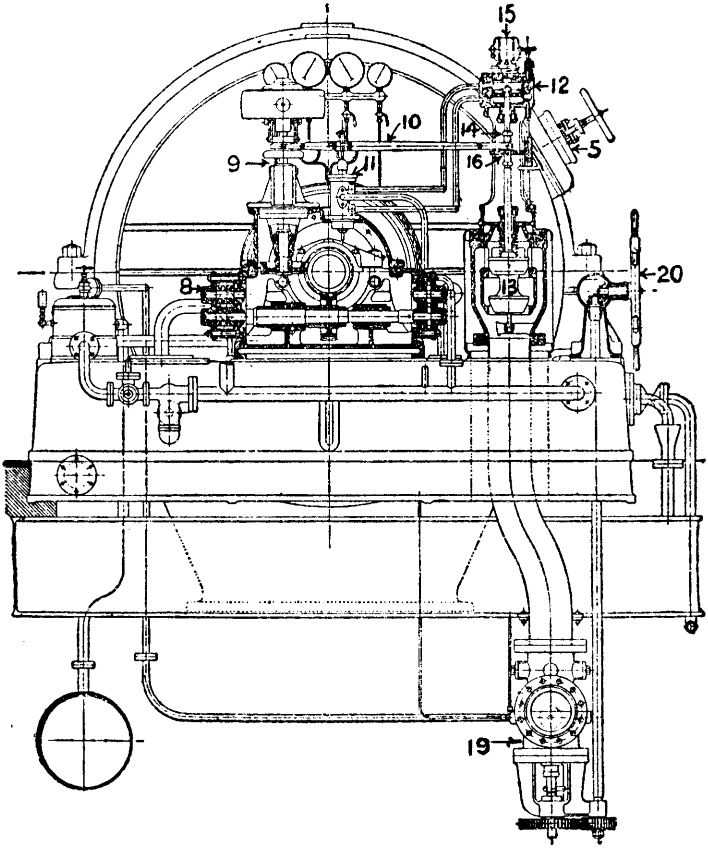


FIG. 22.—End Elevation E.W. Zoelly Turbine.

This is one of the differences in detail between the Rateau and Zoelly machines. The blades are also of uniform thickness, whereas those of the Rateau and Curtis machines are of the type with thickened back.

The set of wheels is locked on the shaft by the collar, 17, on the right and lock nut, 18, on the left side, Fig. 21. A thrust block, 21, is provided for axial adjustment.

As in the previous cases, the casing and diaphragms are in halves. The exhaust end is a separate casting and is bolted to the casing. The H.P. end is strengthened by dishing it.

The bedplate, which is of box section, in addition to carrying the bearings and casing, acts as an oil tank.

The casing is fixed to the bedplate only at the L.P. end. The feet at the H.P. end merely act as supports, and can slide as the casing expands. In this way expansion stresses and distortion are avoided.

23. Curtis-Rateau Turbine.—A standard type of impulse machine now being made by the British Westinghouse Company is the Curtis-Rateau turbine.

It has a high pressure velocity compounded stage followed by a series of simple impulse stages. A section through the machine is shown in Fig. 23.

Steam is supplied to the nozzle box, 1, expanded in the nozzles to the first stage pressure, and then passed through the blading of the velocity compounded wheel, 2. There are three nozzle boxes: the nozzles of one box are sufficient for half load, those of two for full load, and those of three for an overload or when the turbine is run non-condensing.

The steam is then passed into the Rateau section, and expanded in smaller drops to the exhaust pressure. The wheels, 3, are of the light pattern used in the Zoelly turbine, and are made of forged steel. The blades according to the standard Rateau practice, are straddled on the rim and riveted to it.

The velocity compounded wheel, however, has two rings of blades fitted in the customary way into T slots in the rim. The foundation rings carrying the intermediates or guide blades, as can be seen on Fig. 24, Plate V., which is an inverted plan of the upper half of the casing, are fitted into grooves in the cylinder.

The diaphragms, 4, are cast iron, the nozzle passages being formed by nickel steel plates, cast in. They are made in halves and held in position by countersunk screws. The arrangement is also clearly shown in Fig. 24.

In some cases the diaphragms, as in the large machine, Fig. 20, are made of cast steel with bolted-on nozzle plates.

The external glands, 5, and also the diaphragm glands are of the standard design described in connection with the large Rateau machine.

It will be noted that the bearings, 6, are separated from the glands by a considerable space. This arrangement conduces to coolness of running.

They are of cast iron lined with white metal, and adjusted by the insertion of steel pads, 7, one top, one bottom, and one at each side.

Forced lubrication is used. The oil returned from the bearings flows into the oil tank on the left side of the bed plate. It is pumped through a cooler to the bearings by a gear pump, 8, driven from the

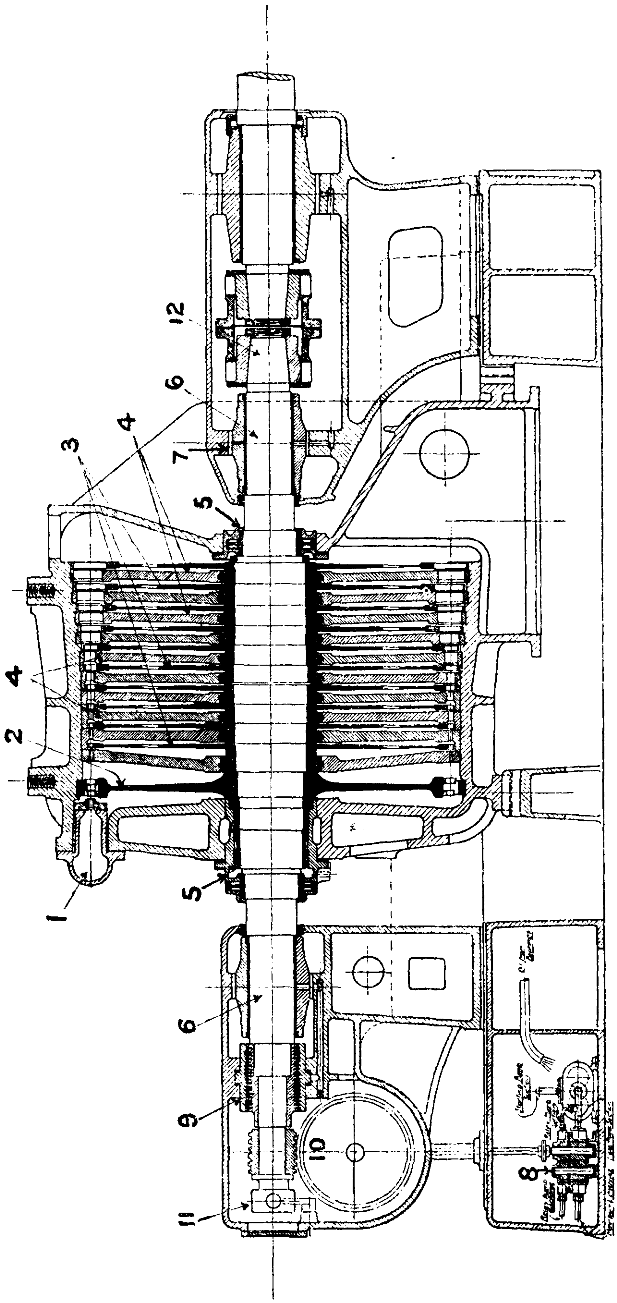


FIG. 23.—B.W. Curtis-Rateau Turbine.

governor spindle. An adjusting block is fitted at 9, for axial adjustment of the rotor.

The worm and wheel, 10, on the left drives a horizontal spring loaded governor, which in turn operates an oil relay servo-motor coupled to the throttle valve. An emergency governor, similar to that already described, is fitted at 11. A flexible coupling, 12, is placed between the turbine and generator shafts, to allow for any want of alignment. The casing, which is of cast iron, is stiffened with ribs. As in the previous cases, only one end is fixed rigidly to the bed plate, in order to avoid expansion trouble.

PLATE V.

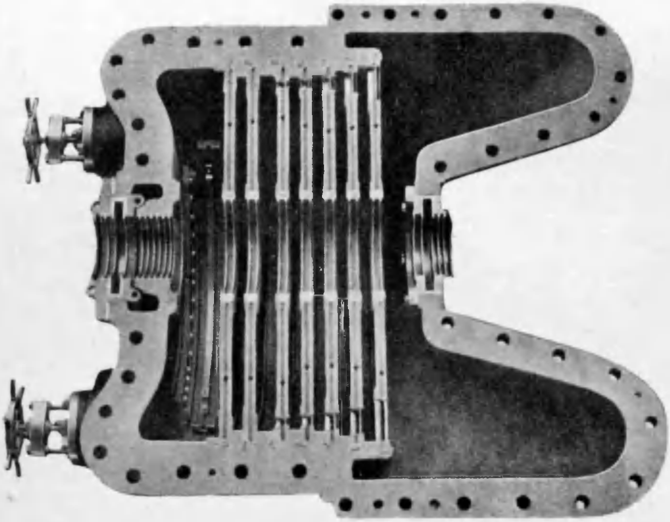


FIG. 24.—Inverted Plan of B.W. Curtis-Rateau Turbine.

CHAPTER III

REACTION TURBINES

24. Brush Parsons Turbine.—The longitudinal section of a standard design of pure Parsons turbine made by the Brush Electrical Engineering Co., Loughborough, is shown in Fig. 25. It is rated to develop 3000 K.W. at 1500 R.P.M., with initial pressure 175 lbs./in.^{2}, superheat 150° F., and vacuum 28 inches.[1]

Steam, after passing the throttle valve, which with the stop valve is not shown by the section, enters the machine by the pipe, 1, and is distributed to a steam belt, 2. It then enters the H.P. blading at 3, and expands continuously in the blading until it is exhausted to the condenser at 4.

When an overload has to be taken, high pressure steam is byepassed through the loaded valve, 6, to a second steam belt, 5, and admitted to the blading at the third expansion.

Normally the pressure in the belt, 2, is about 15 lbs./in.^{2} less than the pressure at the stop valve. When an overload comes on, the governor opens the throttle so as to increase the pressure below the valve, 6, which opens and bye-passes the steam. (See Art. 309, p. 501.)

The machine, according to the general practice, is divided into high, intermediate, and low-pressure sections. The drum and casing are stepped in three diameters.

The stepping of the drum and also the continuous fall in pressure in the blade channels causes an end thrust on the rotor, which has to be balanced. The balancing is accomplished by means of "dummy" balance pistons. The diameter of the drum at 12, is enlarged till it is approximately equal to the mean diameter of the blading on the H.P. drum. The steam at admission pressure acting on the annular area at 3, balances the thrust due to the H.P. blading. The spigot end, 11, of the H.P. shaft which is bolted to the drum is increased in diameter at 9, till this is equal to the mean diameter of the blading on the I.P. section. The annular surface thus formed is subjected to the initial pressure of the I.P. section, the connection being made by the external pipe, 10. The pressure on this piston balances the thrust due to the shoulder on the I.P. drum and the I.P. blading.

In the earlier designs of Parsons turbines a third and larger balance piston was fitted outside the I.P. one. This arrangement has now been

[1] For description and drawings of a larger machine, see *Engineering*, March 26 and April 9, 1915.

discarded, and what is known as the Fullager system of balancing is generally adopted. Holes are bored in the drum at 33 and in the spigot of the L.P. shaft flange, 8, at 34. Steam is thus admitted behind and to the inside of the drum at the initial pressure of the L.P. section. This pressure acting on the end of the drum balances the thrust due to the shoulder and the L.P. blading. The dummy piston, 7, is formed by the flange of the L.P. shaft. The effective pressure on each piston depends on the pressure at the outside surface of the drum at the H.P. end, that is, the area behind the I.P. dummy. This is fixed by the condenser pressure, the back of the dummy being connected to the eduction pipe by an external passage not shown in the section. All the dummy pistons are provided with a special form of labyrinth packing. The action of this packing is discussed in Chapter X.

The casing is made in three parts and is also divided horizontally along the centre line.

The H.P. section is made of steel, which is not liable to "growth," that attacks cast iron when it is subjected to the action of highly superheated steam. The I.P. and L.P. sections are made of cast iron. The circumferential joints are made with spigots to ensure correct alignment of the three sections.

The outstanding constructional feature of this machine is the method of supporting the turbine casing and the bearings in the frame. This is provided with bored seats, 13, and 14. The ends of the casing are turned to fit these seats, and the casing is held down in position, simply by its weight. The bearing shells, 15 and 16, are bolted to the casing ends and also rest on the cylindrical seats. The shell, 16, at the L.P. end, carrying the turbine and generator bearings, is recessed at 17, so that longitudinal motion of the whole casing is prevented.

The shell, 15, at the H.P. end has no recess, and can slide in the seating when expansion of the casing takes place under varying temperatures.

This arrangement admits of exact alignment of rotor and casing, and greatly facilitates the work of erection.

The bearings are made of cast iron lined with white metal and are supported in the bearing shells by four spherically seated pads. Forced lubrication is employed. The oil, at about 25 lbs./in.^{2} pressure, is pumped by the gear pump, 18 (shown dotted below the H.P. bearing). This is driven from the spindle of the governor, indicated at 19. The oil is fed to the bearings through the channels, 20 and 21. It is distributed from these to the bottom pad of each bearing and led through a channel cast in the body of the bearing shell to the horizontal diameter. It then passes to the surface of the journal through a hole in the side of the shell. The oil that leaks past the ends of the shell is drained back to the oil tank formed in the base of the frame, and cooled by a water coil (not shown in the drawing). The oil is prevented from creeping along the shaft by baffle discs, 22, fitted at the outside of each bearing.

The thrust or adjusting block is fitted at 23. The thrust rings are mounted on the same spindle as the gear wheel, 24, by which the

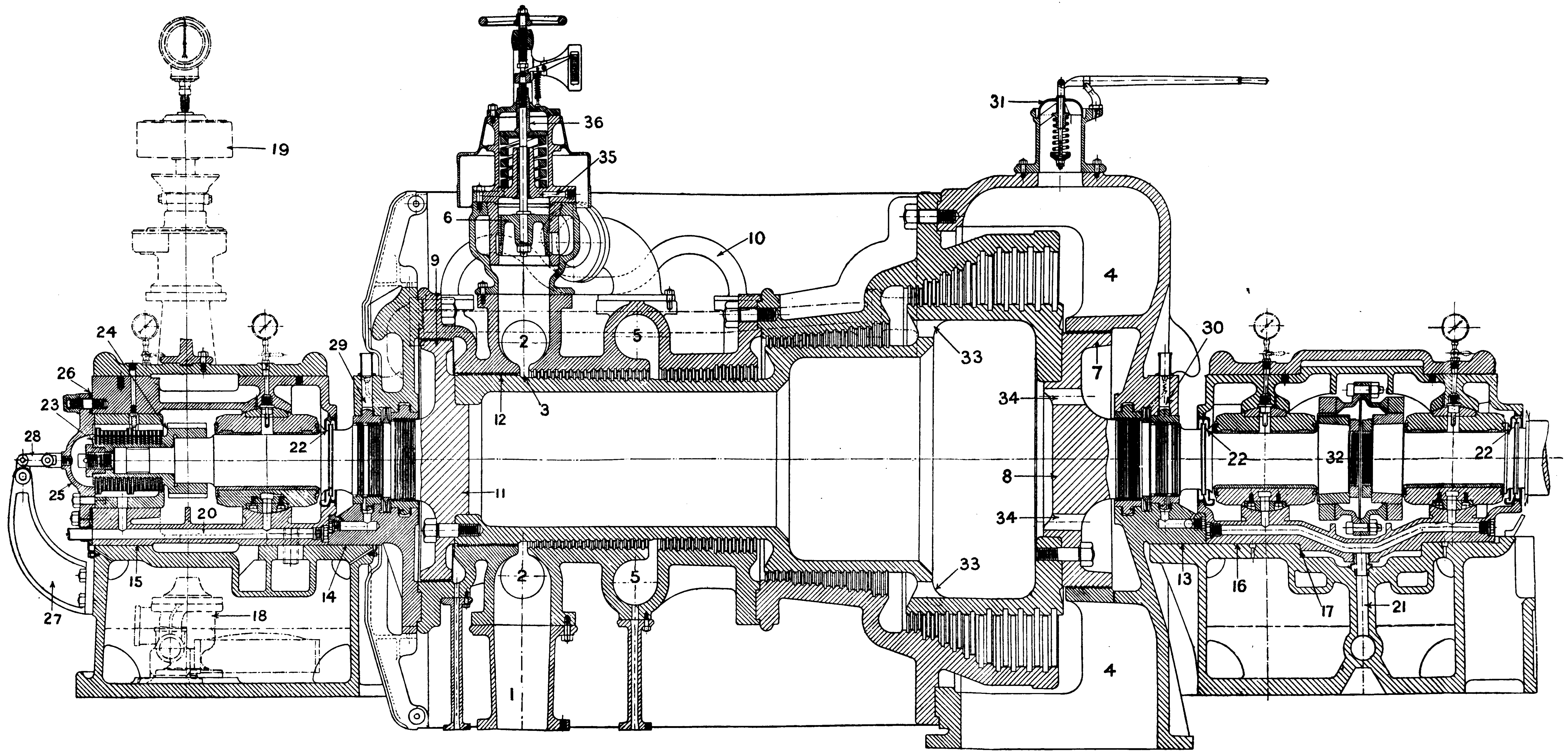


Fig. 25.—Brush-Parsons Turbine.

governor and oil pump are driven. The spindle and wheel are held in position by a cap nut. In this machine the emergency governor is driven from the governor spindle and is mounted on the governor standard. In larger machines, this governor, which consists of a weight held in position by a spiral spring, is fitted in the cap nut. The thrust body carrying the stationary rings is bolted to the casting, 25. The necessary adjustment for dummy clearance is obtained by packing strips inserted at 26. The bracket, 27, takes a bell crank lever, and a link, 28, attached to 25. This arrangement enables the thrust adjustment to be tested, and corrected when necessary. The details of this thrust block are given in Chapter X.

The external packing glands, 29 and 30, are of the combination type, which consists of a steam packed labyrinth gland in series with a water packed gland. The action of this type is discussed in Chapter X. At the outside portion of each gland a funnel is provided at the top, to carry away any vapour that may escape; and at the bottom there is a port for draining away any water or condensed vapour that may gather. In the most recent designs of this machine the water seal is discarded, and the ordinary type of steam packed labyrinth is employed.

A spring loaded relief valve, 31, which can also be operated by hand, is fitted on the top of the L.P. casing.

As in the previous case, a flexible claw coupling, 32, is fitted between the generator and turbine shafts.

The H.P. blading of this machine, which works with steam having considerable superheat, is made of copper. The rest of the blading is made of a special brass.

The governing gear, which is fitted outside the turbine proper, is worked by an oil relay and servo-motor. The oil is supplied by the gear pump which feeds the main bearings. In large machines an independent motor driven pump is used for starting purposes.

25. Brown-Boveri Parsons Turbine.—A skeleton section of the type of pure Parsons turbine made by Messrs. Brown, Boveri & Co., Baden (Switzerland), is shown in Fig. 26.

The steam which is under control of a double beat throttle valve operated by an oil relay (not shown on the drawing) is admitted to the steam belt, 1, and thence to the H.P. blading. The drum and casing, as in the previous machines, are stepped in three diameters. The drum construction in this case is slightly different. The I.P. dummy, 2, is forged in one piece with the H.P. shaft, and has a hollow spigot which fits the H.P. end of the drum, to which it is keyed. This spigot is closed by a plate, 3. A hole is bored through the drum and spigot at 4. Through this, steam at the initial temperature is admitted to the spigot chamber. The shaft end is thus kept at the same temperature as the end of the drum, and trouble due to unequal expansion and slackness of fit is avoided.

The L.P. shaft end is made with a spigot which fits the boss of a cylinder, 5. This cylinder takes the end of the drum and also carries the L.P. dummy packing at 6. The I.P. equalising pipe is not shown on the drawing. The connection between the eduction pipe, 7, and the back of the I.P. dummy is made by the external pipe, 8.

The overload arrangement is similar in principle to that of the Brush machine. As the speed falls under the increasing load, the governor, through an oil-relay system, opens the throttle, and the steam pressure in the belt, 1, increases. When the pressure reaches a predetermined value the overload valve, 9, is forced open against the resistance of a loaded piston in the cylinder, 10. The steam at initial pressure then bye-passes to the third expansion of the H.P. section at 11, and the first two expansions become inoperative. The larger area for the passage of steam at 11 enables a larger amount of steam to be passed, and additional power can thus be developed, at a somewhat reduced efficiency.

The special oil relay system used in these machines is described in Chapter XVI.

The two machines described serve to illustrate the main features

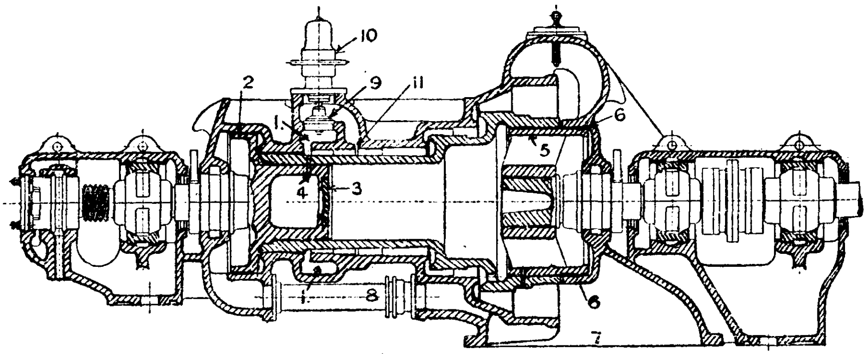


FIG. 26.—Brown-Boveri Parsons Turbine.

of the pure Parsons land type of axial flow turbine for small or fairly large outputs. They are of the "single flow" type.

When large outputs, 10,000 K.W. and over, have to be obtained, the size of single flow machine becomes impracticable. The general practice is now to employ a tandem type using two cylinders and a divided rotor, the power being about equally divided between the high- and low-pressure cylinders.

For smaller outputs this arrangement is sometimes employed, where a single cylinder would necessitate too wide a span between the bearings, and also cause difficulty in transport.

Another method of dealing with a large output, that is specially applicable to exhaust turbines, is to make the turbine of the double flow type. The trouble in such a case with single flow is the impracticable blade lengths required.

For very large output, 20,000 K.W. and over, a tandem machine with double flow L.P. section has to be used.

One of the best examples of this latest practice in pure Parsons turbine design is the 25,000 K.W. machine, supplied by Messrs. C. A. Parsons & Co. to the Commonwealth Edison Co., of Chicago, U.S.A. A complete description of this turbine with detail drawings and numerous photographs has been given in *Engineering*.[1] The reader is recommended to refer to them, as well as the descriptions of other machines mentioned in the footnotes, as it is not possible to adequately treat a large number of machines in the descriptive section of a general text book, without making the size of the book prohibitive.

This machine has a three-stepped drum in the H.P. cylinder, and a parallel drum with blading arranged for "double flow" in the L.P. cylinder. The drums are mounted on the same shaft. In addition to the end bearings, similar to those shown on the previous machines, there is an intermediate bearing between the H.P. and L.P. cylinders. The details in general do not differ materially from the usual Parsons' practice already illustrated. The H.P. end of the high-pressure shaft is forged solid with the drum. The L.P. end has a large spigot, and a flange by which it is bolted to the drum.

The low-pressure shaft is all in one piece, and coupled to the H.P. shaft by a flange coupling.

The L.P. drum, which is eleven feet long and seven feet diameter, is subject to variation of temperature under varying load, and special provision is made to counteract the relative axial expansion of the drum and the shaft.

The steam is admitted to the blading at the mid length of the drum and expands outwards from the centre. The shaft is keyed into the boss of a spider riveted inside the drum at mid length. At each end a flexible steel diaphragm is riveted to the drum. Where the shaft passes through a diaphragm it carries two collars with corrugated faces, one on each side of the diaphragm. These are riveted together. With this arrangement the ends of the drum can expand and contract independently of the shaft, while at the same time the drum is stiffly supported on the shaft.

The conditions of operation specified for this turbine have been chosen for an illustrative exercise, in proportioning a similar machine, in Chapter XIV.

26. Marine Parsons Turbines.—The directly connected Parsons Marine turbine has been standardised for a number of years. The high- and low-pressure turbines of a typical marine installation, made by the Parsons Marine Steam Turbine Company, are shown in half-section and elevation in Figs. 27 and 28.[2] They are fitted in an eighteen-knot steamer. The installation consists of one high-pressure and two low-pressure turbines, the combined output being about 20,000 S.H.P. at 290 revs. per minute. The two L.P. turbines run in parallel. Referring to the H.P. turbine (Fig. 27), steam is admitted at 1 to the wide steam belt, 2, and enters the blading round the whole circumference at 3. It expands continuously between, 3, and, 4, and is exhausted at 4, to the two L.P. turbines through two branches, one on each side.

[1] See *Engineering*, October 17, 1913.
[2] For dimensioned details, see *Engineering*, March 22, 1912.

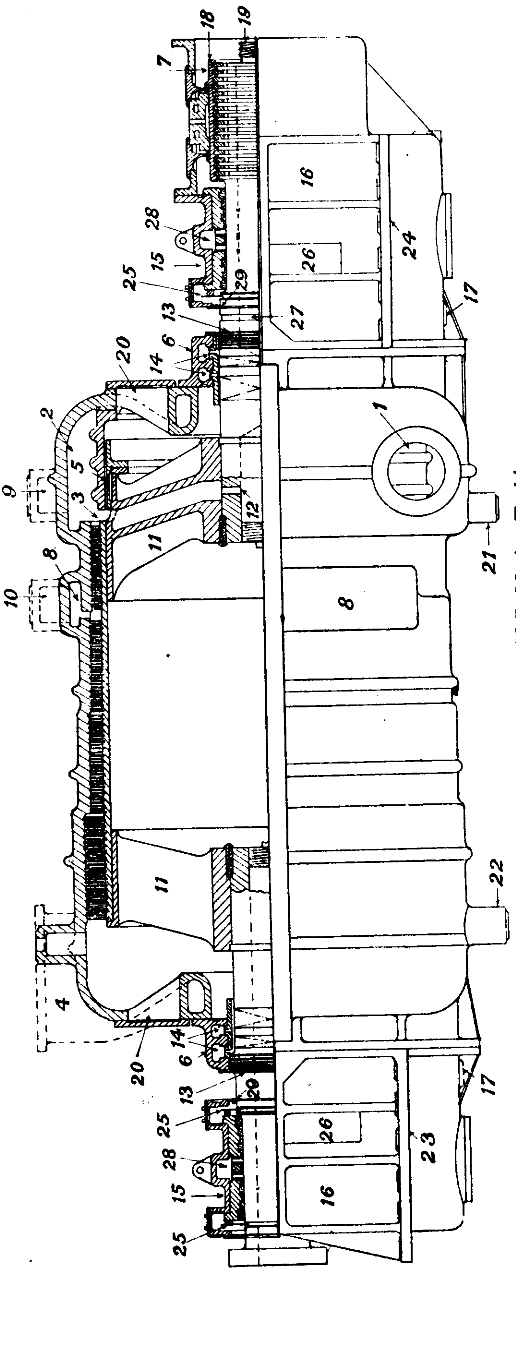


FIG. 27.—Parsons H.P. Marine Turbine.

One of these branches is indicated by dotted lines at, 4. In its correct position it is hidden by the casing. The drum, as is always the case in directly coupled marine turbines, is parallel. The dummy piston is fitted at 5. Its function is to prevent the steam at the high initial pressure from leaking to the interior of the drum. In the land machines the dummy diameter is made the same as the mean blade ring diameter, so as to counteract the thrust due to the blading. In this case it is made less, in order to obtain an annular area at 3. The pressure of steam on this surface increases the thrust aft on the rotor. The augmented steam thrust is utilised to balance the thrust of the propeller. The thrust block fitted at 7 has thus to take only the difference between the propeller and steam thrust. At full power, the rotor thrust is generally greater than the propeller thrust, and about half power it is slightly less.

As in the case of the land machine, additional power for increased speed can be obtained by bye-passing steam at the initial pressure from the belt, 2, to a second belt, 8. The branches, 9 and 10, on the belts, 2, and 8, are connected by an external pipe. Their correct position is at the side.

The drum is connected to the shaft at each end by means of spider wheels, 11, to the circumferences of which it is riveted. Each arm of the wheel at the H.P. end is hollow. The end of the shaft is bored and closed by a screwed plug. Live steam from the hollow arms is admitted through holes, 12, to the chamber in the shaft.

The temperature of the shaft is thus kept approximately the same as that of the casing, and both expand about the same amount. This precaution enables the fore and aft play of the thrust block to be reduced. The external packing glands, 6, are of the radial labyrinth type. Each is provided, at the outer end, with a set of Ramsbottom rings, to prevent leakage of steam into the engine room. Steam, slightly above atmospheric pressure, is supplied to the outer pocket at 14, and the leakage steam is carried away through the inner pocket. The steam passes out and into the pockets through a series of holes in the bushing carrying the labyrinth rings. Two of these holes are shown in each gland (see also Art. 143).

The bearings, 15, are carried on heavy brackets, 16, bolted to the fore and aft ends of the casing. They are lined with white metal. It is very important in case of seizure, if the white metal melts, that the rotor shall be prevented from dropping. To ensure this condition the lower half brass of each bearing is provided with a bottom bearing strip of brass. Oil is supplied to each bearing and the thrust block at the centre, 28, and escapes at the ends, falling into a well in the bottom of each bracket, from which it is drained, filtered, and passed to the bearings again. To prevent the oil from creeping along the shaft, serrations are turned on each side of grooves, into which oil wipers, 25, project. An additional plate, 29, is fitted into a groove in the shaft at the inner end of each bearing.

Water which escapes from the glands also falls to the bottom of the brackets, and is drained away at 17. At the H.P. end the oil which leaks past the end of the bearing falls into the well of the thrust block. This block consists of a cylindrical casting, 18, carrying the thrust rings. It is divided horizontally at the centre. The upper and lower halves are provided with lugs and eyes to take two longitudinal bolts. These bolts can be moved by means of worm gear, and the halves can be adjusted to fix the dummy clearance in either direction. The operating gear is omitted in this view. It is shown in the drawing of the L.P. turbine, Fig. 28. The worm, 19, shown at the forward end of the shaft is employed to drive an emergency governor. The casing is divided horizontally. The rotor is first placed in position, and the upper half is then lowered. In order that the top half may be raised and lowered without fouling the blading, it is always threaded on four guide pillars, two at each end. These pillars are fixed in the bosses, 26, cast on the sides of the brackets, 16. Manhole doors, 20, are provided at each end, so that the interior can be examined without lifting the top half of the casing. The casing is drained at 21 and 22. The turbine is fastened to the ship's frame by the feet, 23, and 24, cast on the brackets, 16.

The low-pressure turbine, shown in Fig. 28, has an ahead and an astern or reverse section. The latter is situated at the exhaust end. The casing is in two parts, and well ribbed to ensure stiffness. The casing of the reverse turbine is a separate casting bolted to the L.P. end of the main casing. The whole machine is divided along the horizontal centre line.

Steam exhausted from the H.P. turbine enters the inlet branch, 1, and flows into the wide steam belt, 2. The branch, indicated by dotted lines, is situated on the side near the top of the casing. The steam passes from 2, into the ahead blading at 3. It is exhausted to the condenser through the branch, 4.

For going astern high-pressure steam is admitted to the inlet branch, 5, and enters the astern blading from the steam belt, 6. For manœuvring purposes high-pressure steam can also be admitted to the belt, 2, through an auxiliary branch, 7, at the bottom of the main casing.

A dummy piston is fitted at 8, to prevent leakage of steam to the interior of the drum, and hence to the condenser. Like that of the H.P. turbine, it consists of a flanged cylinder registered and bolted on the end of the spider wheel. A similar dummy is fitted on the reverse turbine at 9.

The shoulder at 3 is greater than that of the H.P. turbine on account of the lower initial steam pressure.

The drum, which is connected to the shaft in the same way as the H.P. drum by spider wheels, 10, and 11, is in three parts. The ahead is connected to the astern section by a reducing cylinder, 12. The latter helps to stiffen the drum at the L.P. end. It is further stiffened at the middle of the ahead section by a channel ring, 13.

All other parts, such as bearings, thrust block, glands, brackets, etc., are identical with those of the H.P. turbine, and do not require any comment.

An additional ribbed bracket, 14, for securing the turbine to the

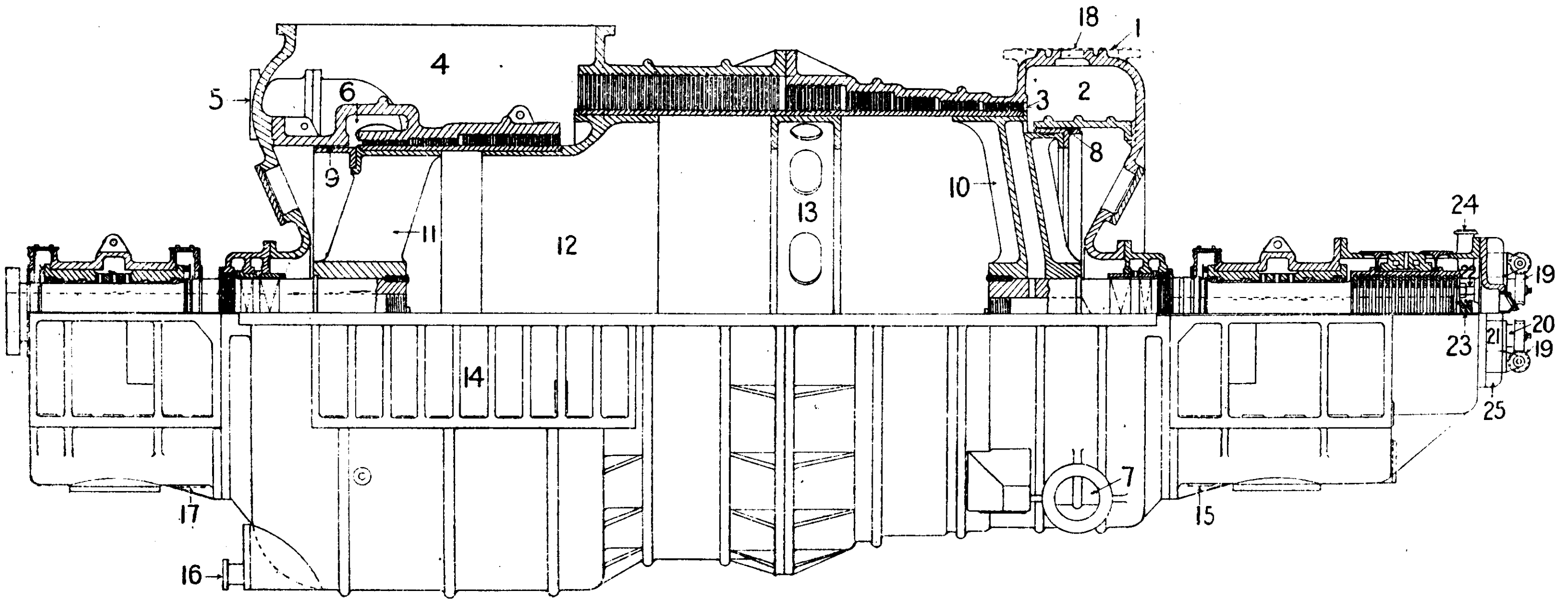


FIG. 28.—Parsons L.P. Marine Turbine.

ship's frame, is cast on each side at the exhaust end of the casing. The flat sides of the exhaust branch, 4, are stiffened by bar stays, which are not shown in the drawing.

Drains are fitted at 15, 16 and 17, and a relief valve on the top of the steam belt at 18.

The gearing for adjusting the thrust block is shown at the H.P. end. Each gear consists of a worm and wheel, 19. The wheel forms the outer portion of a nut, 20, which is recessed into the boss, 21, of a casting, 25, bolted on the end of the thrust block chamber. The nut is threaded on a bolt, 22, attached to one half of the thrust. There are two of these bolts in the upper and two in the lower half. The corresponding pairs of worm wheels are mounted on transverse spindles. When the worms are turned the bolts are moved either forward or aft, and the upper and lower halves of the thrust can thus be separately adjusted. The upper half is set to take any residual pull on the rotor due to steam pressure, and the lower half to take any residual thrust from the propeller. The block as a whole ensures the correct adjustment of the axial clearances at the dummy rings. A more detailed illustration of the thrust block and gear is given in Chapter X.

A similar gear is fitted on the high-pressure turbine.

The worm wheel, 23, fitted at the end of the shaft, drives an emergency governor, the driving spindle of which passes through the boss, 24, on the bearing cover.

27. Geared Marine Turbines.—Until within the last few years steam turbines have been fitted only in high-speed vessels, and even in these cases the limitation of the turbine speed by the propeller often militates considerably against efficiency.

In a slow-running cargo boat the revolutions of the propeller are necessarily low, and a direct coupled turbine requires a rotor of large diameter with inefficiently short blades. The result is that as regards size, weight, cost, and efficiency, the turbine cannot compete with the older reciprocating engine.

If a turbine is to be used in a slow-speed vessel some form of speed reduction becomes necessary.

In the case of the high-speed vessel, the substitution of the geared for the direct drive ensures a reduction in size and cost and an increase in efficiency.

The system of marine propulsion is at present in a state of rapid development; and it seems probable that, in a few years, the reciprocating engine will be, to a great extent, replaced in all classes of vessels by one or other of the geared systems of turbo drive now being advocated.

A full discussion of these systems is outside the scope of this text, and they are therefore dealt with briefly, only so far as their bearing on the operative conditions of the turbine is concerned. For further information the reader should consult the different papers referred to in the footnotes. The three systems advocated are: (1) Electrical; (2) Hydraulic; (3) Mechanical.

The electrical system requires a high-speed turbine and generator.

The current from the latter may be used to drive either slow-speed motors directly connected to the propeller shafts or high-speed motors which drive the propeller shafts at a lower speed through a mechanical gear of the helical type. The first practicable form of turbo-electric drive which has been put in operation in Europe has now been running successfully for more than a year in a Swedish vessel. It is of the latter class, and Ljungström turbines with superheated steam are used. Some particulars of this installation are given in Art. 31 in connection with the Ljungström turbine. For earlier discussions of the electrical system of reduction, a paper on "Marine Propulsion by Electric Motors," by H. A. Movor, is of considerable interest.[1] The most recent information on this subject is given in the discussion of a paper on "Some Alternative Types of Propelling Machinery," by J. Dornan.[2]

The hydraulic system, due to Dr. Fottinger, consists of a high-speed turbine and a directly driven centrifugal pump, which supplies water to a low-speed hydraulic turbine fitted on the propeller shaft.

In the Fottinger transmitter the pump and turbine are combined in one casing.[3]

In both the electrical and hydraulic systems there is a double transformation of energy between the prime mover and the propeller, and where the electrical reduction is further supplemented by a mechanical gear there is a third source of loss.

The efficiency of transmission in these cases does not rise above 90 per cent., and from this point of view they are inferior to a direct mechanical gear of the helical type, which experiment has shown to have an efficiency from 97 to 98 per cent.

The efficiency of the reduction process is, however, only one aspect of the general problem of the whole efficiency of the plant, which is the principal concern of the marine engineer and shipowner.

A directly coupled or mechanically geared turbine has to be reversed when the vessel "goes astern." When the reversal takes place high-pressure steam is admitted to the astern blading which has been running in steam of a much lower temperature, and a considerable and very sudden rise of temperature of the blading takes place. Further, before the vessel can "go astern," its forward velocity has to be reduced to zero, that is, its kinetic energy has to be absorbed. A large proportion is probably accounted for by the churning action of the propellers in the water, but there will also be a considerable amount of energy transmitted to the shafting, the result being a braking action on the turbine rotor, with the generation of friction heat on the

[1] *Proc. I.C.E.*, December 7, 1909. Also see Presidential address, by Sir A. J. Oram, Jun. Inst. Eng., 1909. Remarks by S. Z. Ferranti on Sir Charles Parsons' paper, "Mechanical Gearing for Ships" (*Trans. I.N.A.*, 1913, or reprint in *Engineering*, March 21, 1913).

[2] *Trans. Inst. of Engineers and Shipbuilders in Scotland*, November, 1915.

[3] See paper on "Recent Developments of the Hydraulic Transformer," by Dr. H. Fottinger, *Trans. I.N.A.*, 1914, or reprint in *Engineering*, September 25, 1914. See also Report by Sir J. H. Biles in *Engineering*, December 12, 1913, on the trials of the T.S.S. *Konigin Luise*.

steam. With saturated or wet steam, as generally used, the injurious effect on the blading is not great. When, however, superheated steam is used this temperature effect cannot be ignored.

The substitution of the initial velocity compounded stage at the astern turbine offers only a partial solution since, although the superheat may be removed by the expansion in the nozzles under the normal astern conditions, the braking action during stoppage may so check the flow and so reheat the steam that little reduction of the initial steam temperature may result. The detrimental action of variation of temperature on the blades, which are subjected to surface stressing, is not due so much to the high temperature as to the rapid change from low to high temperature, in the process of reversal. Even with the adoption of the velocity compounded stage, at the astern section of a reaction turbine, the use of a superheat range of more than 120° F. does not seem advisable.

Suggestions have been made to provide some means of bye-passing saturated steam from the boilers directly to the reverse turbine. This would meet the astern difficulty, if found feasible; but unless a high-pressure velocity compounded stage were used, there still would be the defect of admitting high-temperature steam to the high-pressure blading cooled down during the astern running to practically exhaust temperature.

It would appear from the above considerations that the use of highly superheated steam, which is conducive to higher efficiency, is precluded in the case of the directly coupled or mechanically geared turbine which requires to be reversed. This is one of the points claimed by the advocates of the hydraulic and electrical reduction systems. In either case, a high-speed turbine running continuously in one direction and thus capable of using steam of high superheat, is permissible; and it is claimed that the extra gain of efficiency of the turbine more than offsets the additional loss due to the double or treble transformation of energy.

The present trend of affairs in the mercantile marine is towards the reintroduction of superheaters and the use of geared turbine installations, using steam from 100° to 120° F. range of superheat, but it is questionable whether the double reduction hydraulic or electrical drive is likely to supersede the older, much simpler, and more efficient mechanical gear, despite the foregoing drawbacks attendant on the use of superheated steam.

The mechanical system, on account of its low cost, simplicity, reliability, and high efficiency, has of late years been extensively adopted for all classes of mercantile and naval vessels. It is simply an extension, to the large marine turbine, of the original helical reducing gear introduced by Dr. de Laval on the small de Laval turbines.

There is a variant of the usual arrangement in which the pinion is carried in a "floating frame," so as to ensure a more uniform distribution of pressure between the teeth. This is known as the Melville-Macalpine gear.[1]

[1] For description of this gear, see *Engineering*, September 17, 1909.

The development of the mechanical gear for marine turbines is due to the enterprise of the Parsons Marine Steam Turbine Company.

Experiments were first made with a small steam launch running at 9 knots, a 14 to 1 reduction gear being employed to transmit about 10 S.H.P. The success of this experiment led to the purchase of a small cargo steamer, S.S. *Vespasian*, 275 feet long and 4350 tons displacement. The reciprocating engines, developing about 1000 I.H.P., were replaced by a set of geared turbines having a reduction ratio of 19·9 to 1. The old propeller was retained. The results of the trials were conclusively in favour of gear transmission.[1] A comparison of the performance of the reciprocating engines and the geared turbines showed a reduction of 20 per cent. in the coal consumption. The reduction in weight was 25 per cent.

More recently, further comparative trials of geared turbines *versus* reciprocating engines have been made in two cargo boats of the Cairn Line.[2]

Each vessel is 370 feet long, 9900 tons displacement, the horse power at 10 knots being about 1800.

The turbine gear has a reduction of 26·2 to 1. The saving in coal consumption over that of the sister ship with reciprocating engines, taken on simultaneous 36-hour voyages, was 15 per cent.

In the cross-channel steamers *Normania* and *Hantonia*, the speed reduction ratio between H.P. shaft and gear shaft is 6·35, and between L.P. shaft and gear shaft 4·45. It has been estimated, in comparison with similar turbine steamers with direct drive, that the saving in coal consumption is 40 per cent. This high value is attributed to the higher efficiency of the plant and the substitution of twin for triple screws, with consequent reduction in weight of machinery.

The longitudinal sections of the H.P. and L.P. turbines and gears of the twin screw cross-channel steamer *Paris*, built by Messrs. Denny & Co., Dumbarton, are shown in Figs. 29 and 30.[3]

A diagrammatic plan of the port set of turbines and condenser is shown in Fig. 31.

The H.P. turbine, Fig. 29, is similar to the high-speed land turbines already illustrated, having a stepped drum and H.P. and I.P. balance pistons to counteract the steam thrust, since the interposition of the reduction gear requires the adoption of a thrust block to take the whole of the propeller thrust. The L.P. turbine, Fig. 30, is of the usual marine type with parallel drums for ahead and astern running. The ahead drum may be regarded as the third step of the equivalent single cylinder high-speed machine, with this difference, that the L.P. drum runs at 1849 and the H.P. and I.P. drums at 2610 revolutions per minute. Here, again, the land type of dummy piston is fitted at

[1] For a complete discussion of this installation, see a paper by Sir Charles Parsons, *Proc. I.N.A.*, 1910.

[2] See paper, "Geared Turbines in Cargo Steamers *Cairncross*," by C. W. Cairns, M.Sc., *Trans. Inst. N.E. Coast Engineers and Shipbuilders*, March, 1913; also *Engineering*, April 4, 1913.

[3] Reproduced by permission from *Engineering*. For arrangement of machinery and photographs, see *Engineering*, December 5, 1913.

FIG. 29.

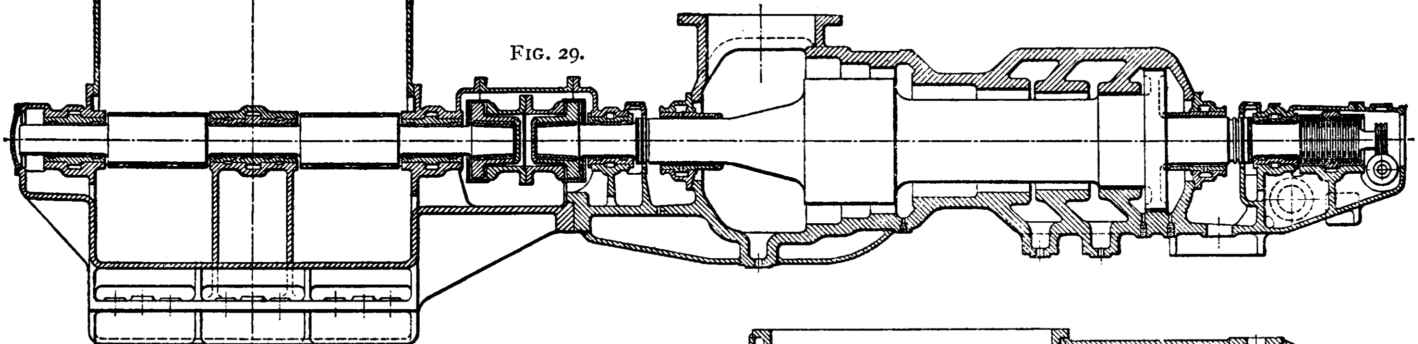
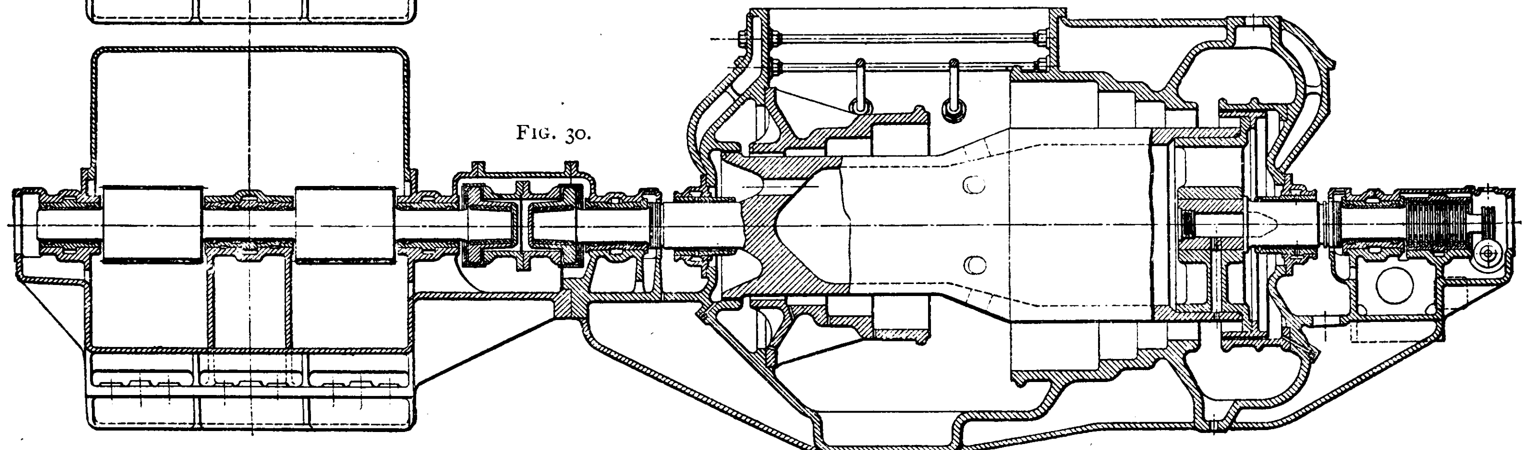


FIG. 30.



Parsons Geared Marine Turbines of T.S.S. *Paris*.

each end of the drum to counteract the steam thrust. There are six expansions on the H.P., six on the L.P. ahead, and five on the astern drum. The inlet branches supplying steam to the belts are not shown, as they are cast on the top sides of the halves of the casing, in front of the planes of section. The bearings, adjusting blocks, glands, etc., are of the usual Parsons' standard design and need not be further referred to. As in the case of the land machine, flexible claw couplings are fitted between the turbine shafts and the pinion shafts of the gear so that no residual thrust can be transmitted from the turbines to the gear wheel.

A section through the gear box, at the high-pressure pinion shaft, is shown to the left of Fig. 29. The pinion, as on the large de Laval machines, is divided and supported in three bearings. The corresponding section at the L.P. pinion shaft is shown to the left of Fig. 30. The subdivided pinion is larger in diameter than the H.P. one on

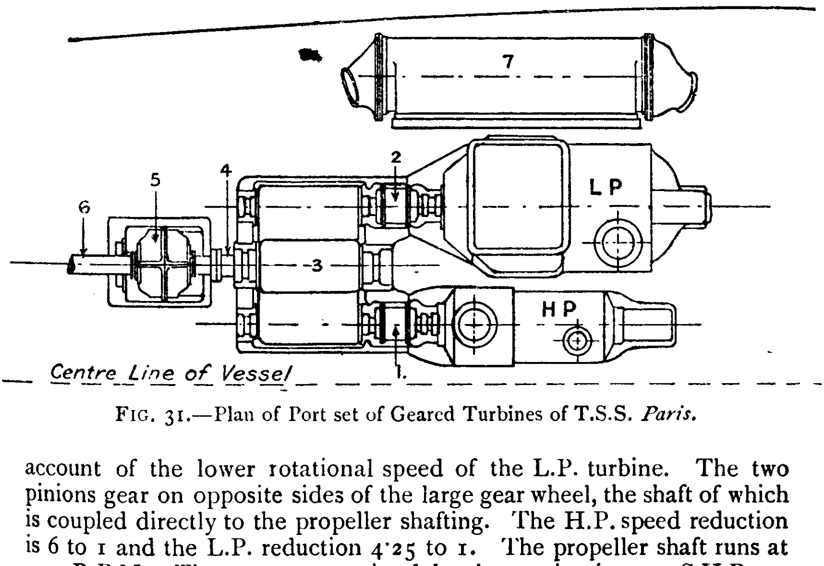


FIG. 31.—Plan of Port set of Geared Turbines of T.S.S. *Paris*.

account of the lower rotational speed of the L.P. turbine. The two pinions gear on opposite sides of the large gear wheel, the shaft of which is coupled directly to the propeller shafting. The H.P. speed reduction is 6 to 1 and the L.P. reduction 4·25 to 1. The propeller shaft runs at 435 R.P.M. The power transmitted by the gearing is 7000 S.H.P., or a total of 14,000 S.H.P. for the two sets of turbines.

The disposition of the turbine shafts gear box and propeller shafting is clearly shown in the sketch plan, Fig. 31. The condenser is shown at 7.

The flexible couplings are shown at 1 and 2 and the gear box at 3.

The gear wheel shaft, 4, is connected to the propeller shafting, 6, by a flange coupling; the thrust block, 5, is fitted on the latter shaft. It is a Michell segmental pivoted block, radically different in design from the ordinary marine thrust. It occupies much less space, can stand very high bearing pressures, while the friction loss is small compared with that of the ordinary type. It appears likely to replace the older design for this class of work. This type of block is discussed in Chapter X.

Longitudinal and transverse sections through the gear box are shown at Figs. 32 and 33.

The rim of the gear wheel, Fig. 32, has a channel section, and is connected to the boss by inclined and perforated webs. The wheel is centred on the shaft by a cone fit, keyed, and finally locked by the nut shown on the right. The gear teeth are cut in rings, shrunk on the wheel, and held in place by screws. The teeth on one half are inclined in the opposite direction to those on the other, to neutralise the end thrust due to obliquity.

Specially ductile steel is used for these rings. It is a carbon steel, having an ultimate strength of from 30 to 35 tons/in.^{2}, and an elongation of 26% on 2 inches. In the earlier arrangements, in order to reduce acoustic vibrations due to slight inaccuracies of the teeth, the pinion bearings were fixed on strong springs in the casings.

As the result of an improved method of cutting the gears,[1] absolute accuracy in the meshing of the teeth is now obtained, and there is no further need for the use of such springs.

Fig. 33 shows a transverse section through the centre of the gear box. The H.P. pinion bearing is on the right and the L.P. bearing on the left. The pinion shafts, like the wheel shaft, are bored. The pinions are made of nickel steel, having an ultimate strength of 40 tons/in.^{2} and 25% elongation on 2 inches.

All the bearings have forced lubrication, the oil being pumped to the pinion bearings through ports shown below them, and through similar ports to the gear-wheel bearings. The teeth are also oil lubricated. The oil is fed in at the top of the gear box by pipes, not shown on the drawing, and is sprayed on to the wheel. It enters the gear case on the top side, above the inspection doors indicated in Fig. 33.

With the proper quality of oil, which will form a film between the teeth, bearing friction and wear are practically eliminated. This condition contributes in no small degree to the high transmission efficiency obtained with these gears.

In order to give a better idea of a geared combination a photograph of the port set of turbines fitted in the twin-screw Anchor Liner *Tuscania*, by Messrs. Alex. Stephen and Son, Glasgow, is shown in Fig. 34, Plate VI.

The details of the reducing gear, with the oil-service pipes to the bearings, which are under forced lubrication, are very clearly shown.

In general design, these turbines are similar to those of T.S.S. *Paris*, with the exception of the astern turbine, shown at the end of the condenser on the left. This has a velocity compounded high-pressure impulse section.

The small centrifugal governor shown at the end of the L.P. turbine is an emergency one, which can come into operation in event of a shaft

[1] See paper by Sir C. A. Parsons, *Trans. Inst. Naval Architects*, March, 1913, or *Engineering*, March 14, 1913.

breakage, and operate the manœuvring valve. A similar governor is fitted on the H.P. turbine.

In this case the gear wheel is 120 inches diameter, and the speed reduction for both turbines is 12·5 to 1. On trial the maximum revolutions of the turbine shafts were 1707, and of the propeller 137 per min. The speed was 17·65 knots, and the total shaft horse power transmitted by the four turbines, 10,900.

A similar set of turbines is fitted in the Cunard liner T.S.S. *Transylvania*, built by Messrs. Scotts' Shipbuilding and Engineering Co., Greenock.[1]

In America the Westinghouse Co. have recently introduced a new system of geared turbine drive for warships. This has been described by J. W. McAlpine.[2] The outstanding feature of this installation,

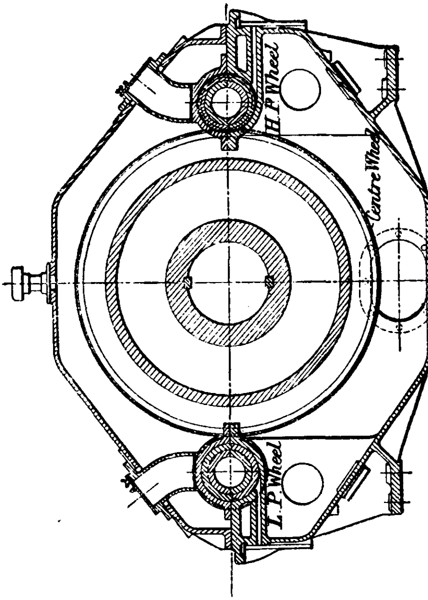


FIG. 33.

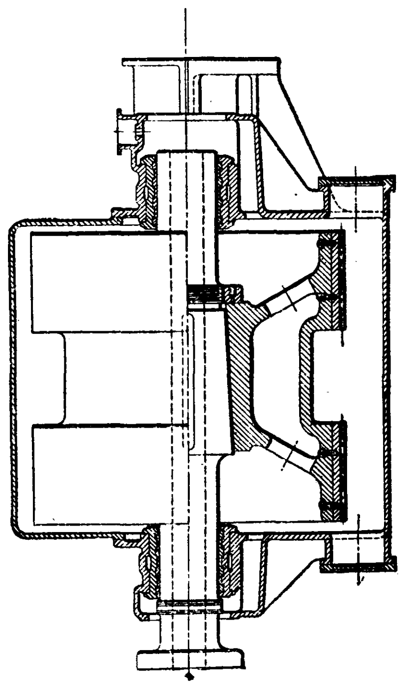


FIG. 32.

[1] For further particulars and descriptions of these two installations, see *Engineering*, January 29 and February 12, 1915. Also for particulars of turbines in T.S.S. *Orry*, see *Engineering*, June 27, 1913. For those of T.S.S. *Northumberland*, see *Engineering*, March 24, 1916.

[2] See *International Marine Engineering*, January, 1916. Also contribution to discussion on paper by J. Dornan on "Some Alternative Types of Propelling Machines for a 19½-Knot Steamer," by J. H. McAlpine, *Trans. Inst. Engineers and Shipbuilders in Scotland*, January, 1916. Also description of 11,000 S.H.P. divided turbine in *Engineering*, June 9, 1916, p. 547.

which is designed for an output of 22,000 shaft horse power at full speed, is the "divided flow."

The high-pressure turbine is a combination machine with high pressure velocity compounded impulse section and intermediate reaction section. At full power with all the H.P. nozzles open the total steam passes through the H.P. section. The reaction blading on the intermediate drum section is, however, designed to take only one-third of the total quantity. The remaining two-thirds passes through a suitable valve to the low-pressure turbine which is of the double flow type. Both turbines are connected directly to the condenser. The H.P. turbine drives the propeller shaft through a reducing gear. The double flow L.P. turbine is also geared to the shaft by a second pinion. The advantage of this arrangement is that at "cruising speeds," where a considerable reduction of power is required, the number of turbines in operation can be reduced by cutting out the L.P. turbine and using the H.P. turbine alone with the larger portion of the first stage nozzles "cut out." This turbine can then economically develop one-third of full power. The older system necessitated the use of additional cruising turbines.

It is said that this system gives as good steam consumption at cruising as at full speed.

The guarantee quoted is 10·8 lbs. per S.H.P. at full load, 10·5 from 80 per cent. to 85 per cent. of full power, and 11·5 at quarter full power.

For astern running, the reverse turbine is fitted at the L.P. end of the H.P. turbine casing. It is of the velocity compounded impulse type, and develops 60 per cent. of the full power.

In order to provide against emergency, the low-pressure turbine is fitted with an auxiliary connection for the supply of steam directly from the boiler.

28. Ljungström Radial Flow Turbine.[1]—A side and end elevation of the first experimental Ljungström turbine are shown in Fig. 35, and a section of one half of the turbine proper in Fig. 36.

The machine develops 1000 K.W. at 3000 R.P.M., the initial pressure being 170 lbs./in.^{2}, superheat 300° F., and vacuum 28·7 inch.

Referring to Fig. 35, the upper and lower halves of the turbine casing are shown at 1*a* and 1*b*. This casing, into which the steam is discharged on exhaust from the blade channel, is subjected only to the exhaust pressure and temperature. It is split horizontally so that the top half can be lifted for the insertion of the discs and shaft.

The generator casings, 2 and 3, and the bearing and field exciter casings are bolted to the ends of this main casing.

The rotating parts of the combination are supported by the bearings 5 and 6 on the left- and 7 and 8 on the right-hand half. The turbine proper has no bearings. Each of the two turbine wheels, 12 and 13, is bolted to the shaft, 11, of the corresponding generator, 9 or 10. The wheel chamber which fits into the main casing is kept in correct alignment by means of the flanges, 16.

The ventilation of the generators is accomplished by drawing air

[1] The Ljungström turbine is made by, Svenska Turbinfabriks Aktiebolaget Ljungström, Finspong, Sweden, and, Brush Electrical Engineering Co., Loughborough.

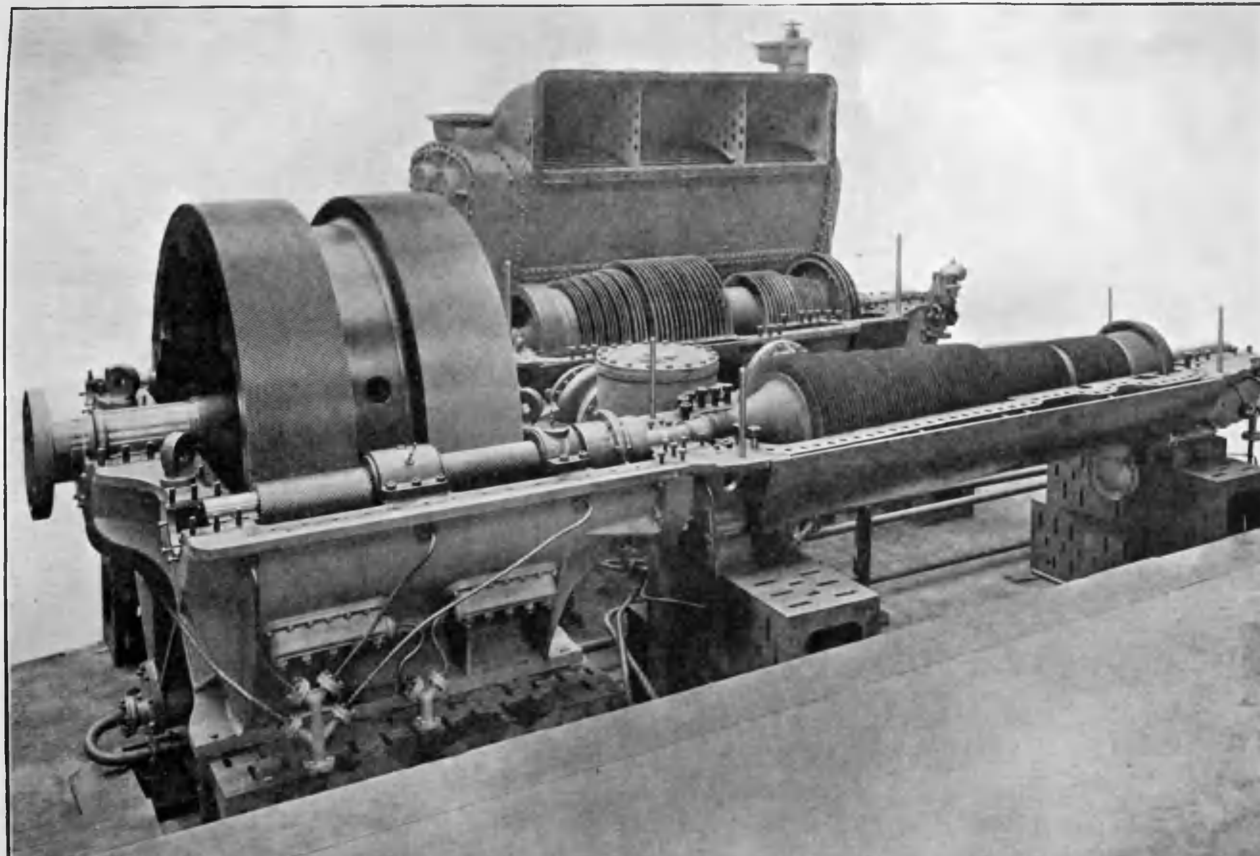


FIG. 34.—Geared Turbines of T.S.S. *Tuscania* (port set with top of casing removed).

passage, 23, and from thence through the orifices, 31, to the exhaust passage, 29.

The rotors of the two generators are connected in series and the stator windings in parallel. The rotors run in opposite directions, and on account of the arrangement of the turbine discs each is subjected to the same torque. The two generators thus work together as one machine.

The oil tank is shown at 20. It is provided with an oil straining and cooling arrangement. The oil is pumped by the usual form of gear-wheel pump, driven by worm gear situated in the cover casing at, 8.

The oil under pressure of about 34 lbs./in.^{2} is carried to the bearings by the pipe, 32. It is drained back by the pipe, 33. The method of supply and drainage is shown at bearing 6. The delivery pipe, 35, is enclosed in the larger draining pipe, 36. The oil under pressure is also used to operate the relay piston of the governor gear. It is reduced to a pressure of 15 lbs./in.^{2} at each bearing by a reducing valve. The rate of oil flow through the bearings is such that they are kept cool without the aid of a water circulation. Before the machine is started, the oil pressure has to be obtained by means of a hand pump, 21.

The bearings 5, 6, 7 and 8 are provided with special means of adjustment to bring the two discs into correct alignment.

The governor, 17, is driven by the same shaft as the oil pump. The governor lever, 37, and the vertical rod, 38, operate a pilot valve situated in the oil tank. This valve, in conjunction with levers, 44, and 42, and rod, 43, regulates the supply of oil to the piston of the relay cylinder, 40. The oil is passed through the pipe, 39. The relay piston and the throttle, 41, are connected by the spindle, 50.

An emergency governor is fitted on each shaft at the outer sides of the bearings, 6, and 7. It operates the valve, 45. This opens a communication passage between top and bottom of the relay piston. The top of the piston is loaded by a spring, and when the fluid equilibrium is established the spring forces down the piston and closes the throttle.

A stop valve is fitted on the same spindle as the throttle, and can be opened or closed by the hand wheel, 46. When starting up, this wheel is given a complete turn, to lift the valve off the seat, and allow a slight amount of steam to leak past the throttle and heat up the turbine. The hand wheel of an overload valve is shown at 47.

The steam after leaving the valve box, 41, passes into the main steam branch, 48; and from this by the supply pipes, 49, to the interior of the wheel chamber. Tachometers are fitted to each end of the machine at 51, and 52.

As the whole machine is self-contained and light, it is bolted directly to the condenser by the exhaust flange without any expansion joint. The ends of the generators are supported on springs, 14, fitted into supports, 15, on the top of the condenser.

Referring to the turbine proper, shown in half section in Fig. 36, 1 and 2 are the turbine discs with short hollow bosses bolted by the flanges, 3, to the corresponding flanges, 5, of the generator shafts. The

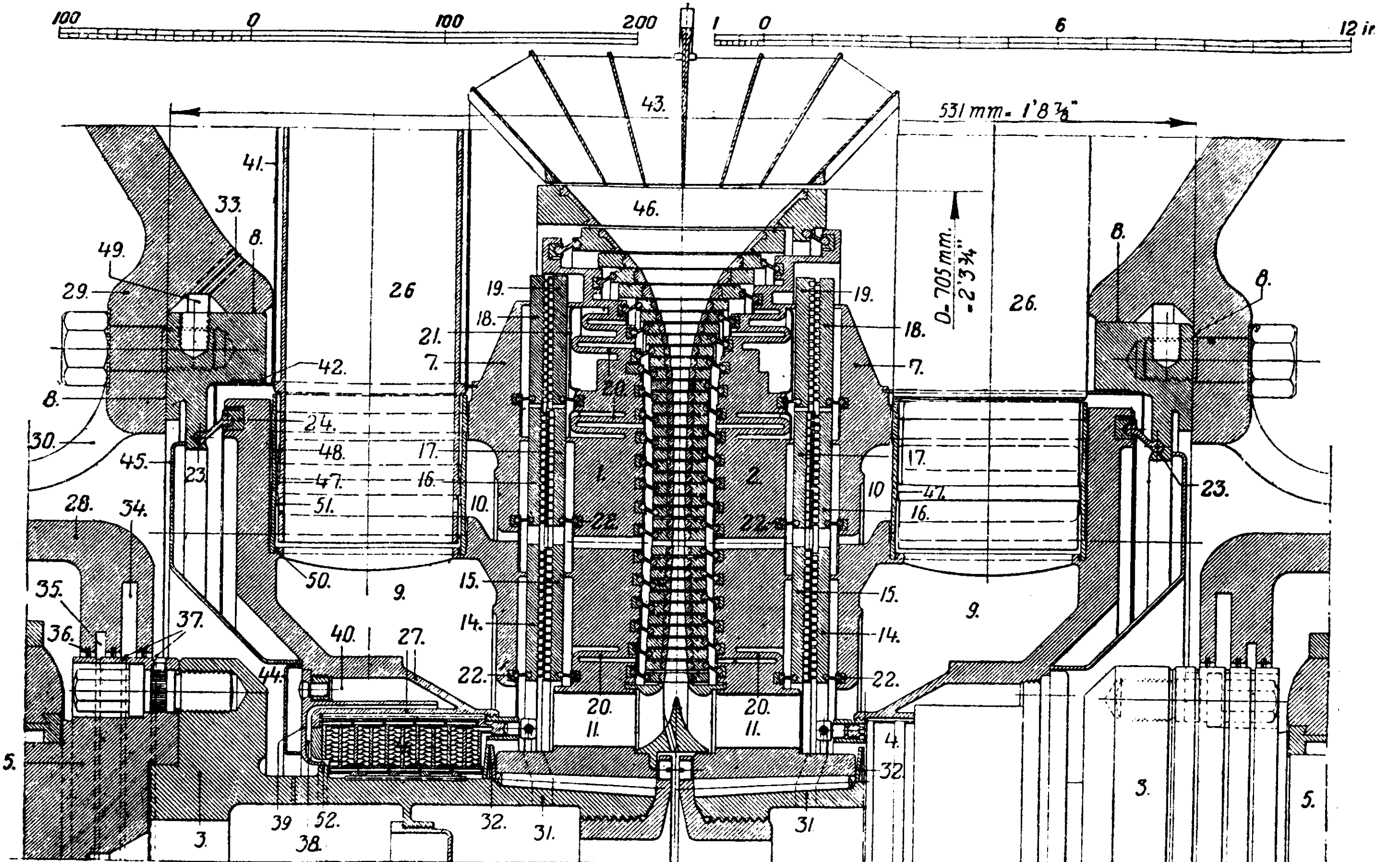


FIG. 36.—Original Section of 1000 K.W. Ljungström Turbine.

wheel chambers, 7, into which the steam supply pipes, 26, are fitted, are bolted to the main casing at 8. The flange at, 8, is not cast as a part of the chamber wall, but is a separate ring connected by a conical bulb ring, 23. The bulb ring acts as an expansion device when there is a large difference in temperature between the chamber walls and the casing metal.

Steam passes from the supply pipes, 26, to the annular space, 9, of the chamber, then through the ports at the bottom to the passages, 11. From thence it expands through the blading, and is discharged to the outer casing at the exhaust pressure. Steam for overload purposes is admitted to the annular space, 10, and flows into the blade channel, through the port shown, and through a ring of holes bored in the disc at about one-third of the radius from the centre.

A series of blade rings of increasing diameter is mounted on each disc. Each ring is fixed to its respective disc, 1 or 2, by the same type of conical expansion ring used at 23. This method of fixture is one of the important points of the machine. The inventors state that these joints have been tested with a difference of temperature between blade ring and disc of 570° F., and have been found to give a very uniform expansion of the blade ring. In this case each disc, which is exposed at the centre to the temperature of highly superheated steam, and at the periphery to the low temperature of the exhaust, is divided up into temperature zones by grooves, 20, which produce thin axial connections. These allow for a certain amount of expansion and prevent distortion. As already stated on page 13, this slotted form of disc is now replaced by a built-up "disc," which consists of several broad rings joined by bulb expansion rings. This latest construction is illustrated in Fig. 36A, also in Fig. 37, where the portions 1 and 2 of the discs of the large compound turbine are joined together. Three broad rings, 14, 16 and 18, are fixed to the side walls of the chamber by bulb rings, 22. Three corresponding rings, 15, 17, 19, which act as dummy pistons, are likewise fixed to the back of each disc (see Art. 143). The faces of these rings are provided with a special form of labyrinth packing. As the steam in this packing remains at nearly the initial superheat through the whole series, an insulating plate ring, 21, is fitted to prevent conduction of heat to the low temperature L.P. section of the disc.

Another ingenious labyrinth is fitted at 4, to prevent leakage of high-pressure steam between the shaft and the turbine chamber. The packing is carried in a thin-walled cover, 27, so as to avoid distortion trouble. It can expand independently of the turbine chamber. The shaft, 3, is made hollow to keep down the amount of heat conducted through the flanges to the bearings. In order to obtain uniformity of temperature in the shaft, steam is admitted to the ends by the ducts, 31. Spring washers are fitted at 32, to allow for any differential expansion between the shaft, 3, and labyrinth, 4.

The bearing is supported in the main casing by axial arms, 30.

When the turbine is heated up before starting, steam is admitted at 33, to the annular passages, 49. The turbine flange, 8, and casing flange, 29, are thus heated at the same rate as the other parts, having less thickness of metal. Unequal expansion and accompanying stress is thus avoided.

Steam which leaks past the labyrinth shaft packing, escapes at atmospheric pressure by the duct, 38. The action of the escaping steam in flowing through the passage is an aspirating one. It draws air from the engine room between the cover, 39, and the narrow flange, 52, on the disc boss. No steam leaks into the engine room. It is discharged into the annular chamber, 40, surrounding the packing, and is led from this by a pipe to the outside of the machine. It is finally condensed in the feed water, and the heat is thus returned to the boiler.

Heat insulating plates are provided at 42, 44 and 45 on the circumference and sides of the turbine chamber.

Each admission pipe, 26, has a ball-shaped projection, 47, which fits against the middle part of the lining sleeve, 48, which is caulked at 50, into the side of the turbine chamber. Holes are provided at 51, through which steam is admitted between the supply pipe and the sleeve. The middle part of the sleeve then takes the same temperature as the supply pipe, while the ends take the lower temperature of the chamber sides. In this way a satisfactory packing is obtained.

The fan-like arrangement, 43, fitted outside the discs is a diffuser. The original idea was to use this as an aspirator so as to reduce the pressure at the periphery of the wheel below the condenser pressure, and obtain a larger effective heat drop. Experience has shown that this is a needless refinement, and it has not been fitted in subsequent machines.

It is important to note that as the turbine proper is placed inside the main or exhaust casing, no external insulation is necessary.

At the bearings the casing has annular spaces, 34 and 35. Oil rings, 36, are fixed in the bearing cover, and fit closely on the circumference of the generator shaft flange. Oil grooves, 37 (shown more clearly at the right-hand bearing), are turned on the surface of the flange.

Oil thrown off from the bearing is prevented from leaking into the turbine by the rings. If any should so leak it is caught in the oil grooves, and thrown out by centrifugal force into the channels, 34, and 35. This oil is carried away from 35, by a draw pipe, which is connected to the oil return pipe.

One important feature of the machine is the small size for the power developed. The diameter of the turbine proper is only 2 feet $3\frac{3}{4}$ inches, and the axial length between generator flanges 1 foot $8\frac{7}{8}$ inches.

The original section of this turbine has been slightly modified. The latest arrangement is shown in Fig. 36A, reproduced from a paper by R. S. Portham,[1] on the "Ljungström Turbine and its Application to Marine Propulsion." The number of blade rings is increased from 38 to 42, and the slotted disc is replaced by the built-up type, having three sections joined by bulb expansion rings. The divergency of the

[1] *Trans. Inst. Engineers and Shipbuilders in Scotland*, January, 1916.

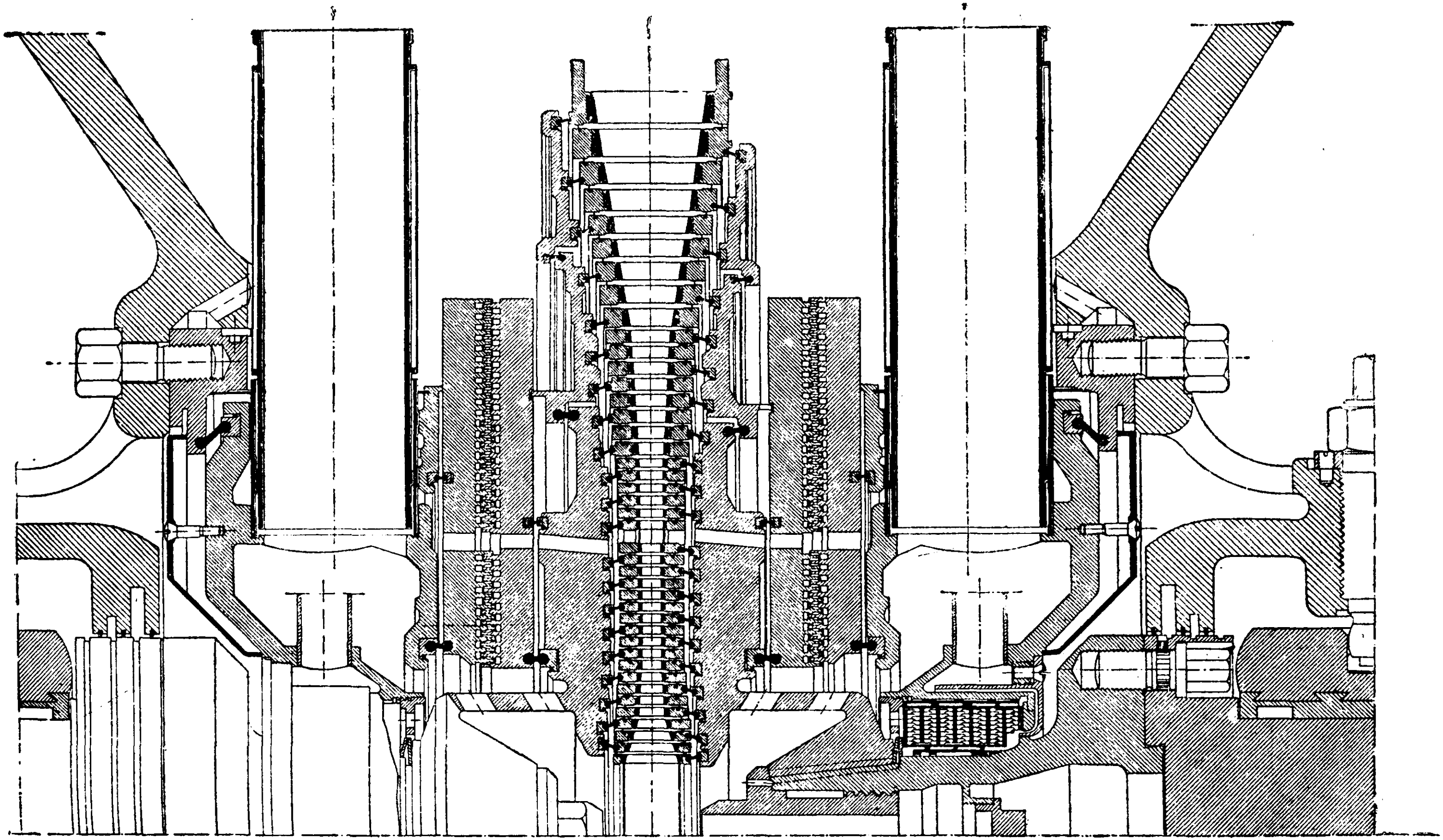


FIG. 36A.—Latest Section of 1000 K.W. Ljungström Turbine.

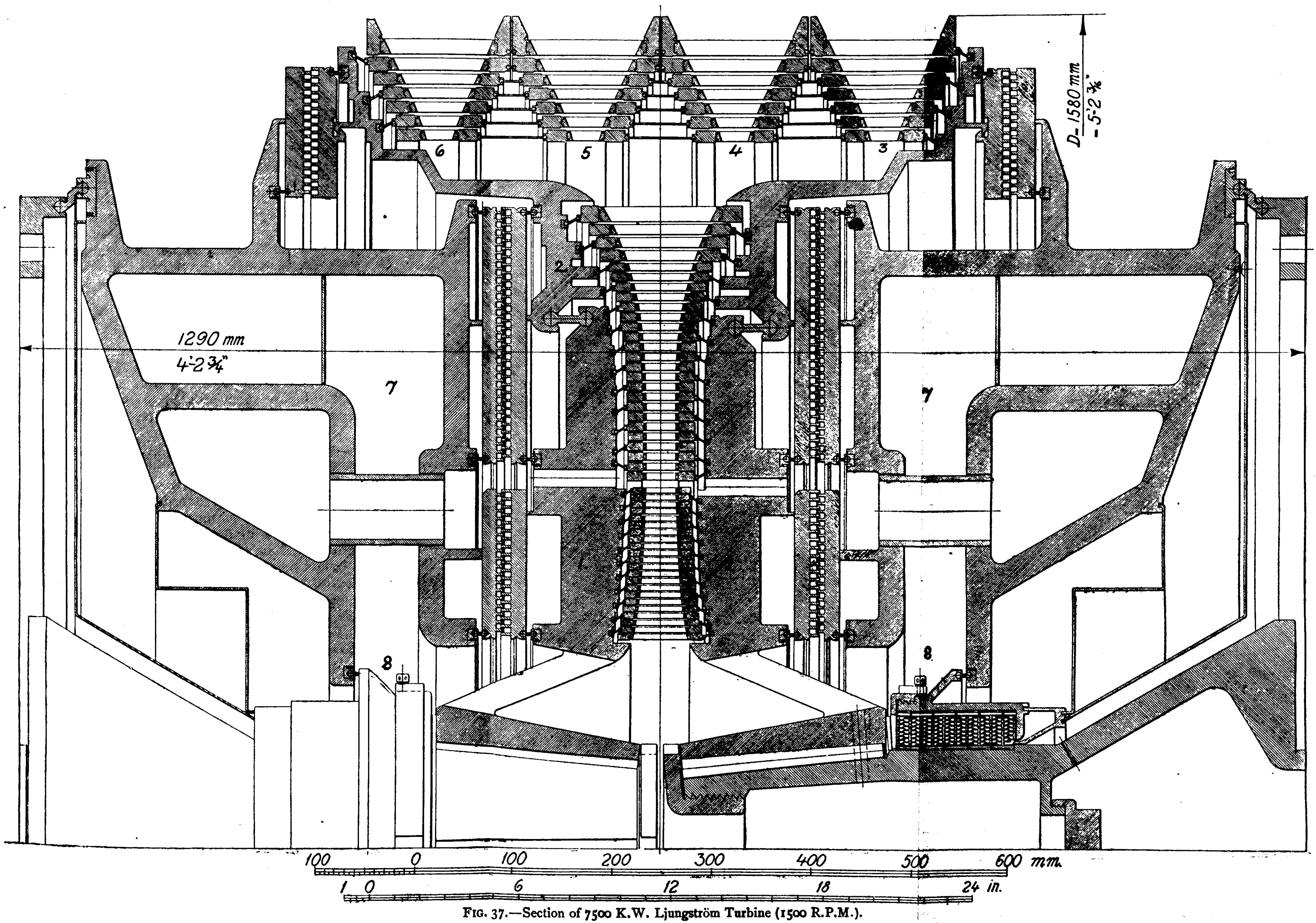


FIG. 37.—Section of 7500 K.W. Ljungström Turbine (1500 R.P.M.).

blade channel at the L.P. end is not as great as in the original design, and the diffuser is dispensed with. Other illustrations and photographs of the details are given in this recent paper, including a section of the largest unit, so far constructed, to develop 10,000 K.W. at 3000 R.P.M.

29. Where large power has to be developed in the case of an axial flow turbine a large L.P. diameter and long blading is required, and it may be necessary to adopt a double flow arrangement to keep the dimensions within practicable limits.

In the Ljungström type there is no difficulty in meeting the demand for very large outputs. The blade lengths at the L.P. end, otherwise quite impracticable in the axial type, can be subdivided into several sections. The result is, that at the L.P. section the single wide blade channel is subdivided into several narrower channels. This is equivalent, in fact, to replacing the L.P. turbine by several smaller turbines in parallel.

The resulting design is clearly shown in Fig. 37, which is a half section through a machine designed to develop 7500 K.W. at 1500 R.P.M.

Each "disc" wheel is built up in two parts, 1 and 2, representing the H.P. and L.P. sections respectively. The part 2 is increased axially, in order to obtain the requisite blade lengths 3, 4, 5 and 6, and area of passages to pass the large volume of steam at the lower pressures. Steam is admitted to the blading through the passages, 7 and 8.

Although at first sight the arrangement of L.P. sections may appear weak, it should be borne in mind that each overhanging ring is really the section of a squirrel cage well stiffened by the thick blade rings.

In this case, although the speed is half that of the 1000 K.W. machine, the diameter for an output seven and a half times as great is only 5 feet $2\frac{3}{4}$ inches, and the length 4 feet $2\frac{3}{4}$ inches. The 10,000 K.W. machine already mentioned has six L.P. blade divisions, and the maximum diameter of blade ring is 2 feet 10 inches.

Messrs. Ljungström state that machines of this design can easily be made for outputs up to 30,000 K.W.

30. While the "double-motion" turbine will undoubtedly come into general use for direct electrical driving, there are fields of operation in which the single-motion machine, despite its less favourable economy, may be found serviceable.

This machine is of simpler design, and requires only one generator. It is thus cheaper than the double motion and occupies less space. For mechanical drive the inventors claim that it can be advantageously used in place of the other types of machines. For example, it can be used as a geared marine turbine, as an exhaust turbine for rolling mills, etc.

As an illustration of the design of single-flow machine, the side and sectional elevations of a mixed-pressure turbine suitable for 2000 K.W. at 3000 R.P.M. are shown in Fig. 38, and Fig. 39, Plate VII.

The low-pressure inlet valve and relay cylinder are shown at 1, Fig. 38, and the high-pressure valve and relay at 4. The weight of the overhanging gear is taken by springs, 2, slung from the floor beams.

There are three admission passages to this turbine. The annular duct between 3 and 5 (see Fig. 39) is the passage for the L.P. steam. The inner pipe, 6, admits high-pressure steam to the first stage of the H.P. section.

The annular passage between 5 and 6 admits high-pressure steam to the "overload" passage, 17.

As in the case of the double-motion machine, 9 is a fixed turbine casing jointed to the exhaust casing, 10, and 11, is the single disc wheel.

The low-pressure steam from 3, can flow only through the L.P.

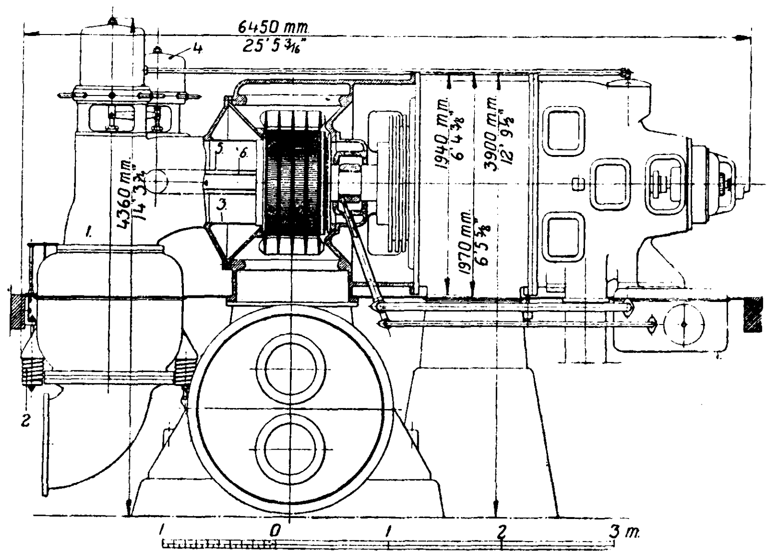


Fig. 38.—Side Elevation of Single Motion Mixed Pressure Ljungström Turbine.

section, the blading of which in this case is again subdivided into four sections, 12, 13, 14 and 15.

When the L.P. supply is in defect, the H.P. valve opens and admits steam by the pipe 6 to the high-pressure section at 7. This steam expands at 8, to the pressure of the exhaust steam entering at 3.

When the L.P. supply stops, the H.P. valve bye-passes steam to the middle of the blading through the passage, 17.

In this way the machine can be run as a high-pressure turbine, with the section of blading between, 7, and, 17, cut out.

31. One of the turbines employed for the turbo-electric marine installation referred to in Art. 27, is shown in Fig. 40, Plate VIII. This interesting installation is fitted in S.S. *Mjölner*, and, as in the case of the Cairn line vessels, for purposes of comparison, the sister ship, S.S. *Mimer*, is fitted with an ordinary set of triple expansion engines,

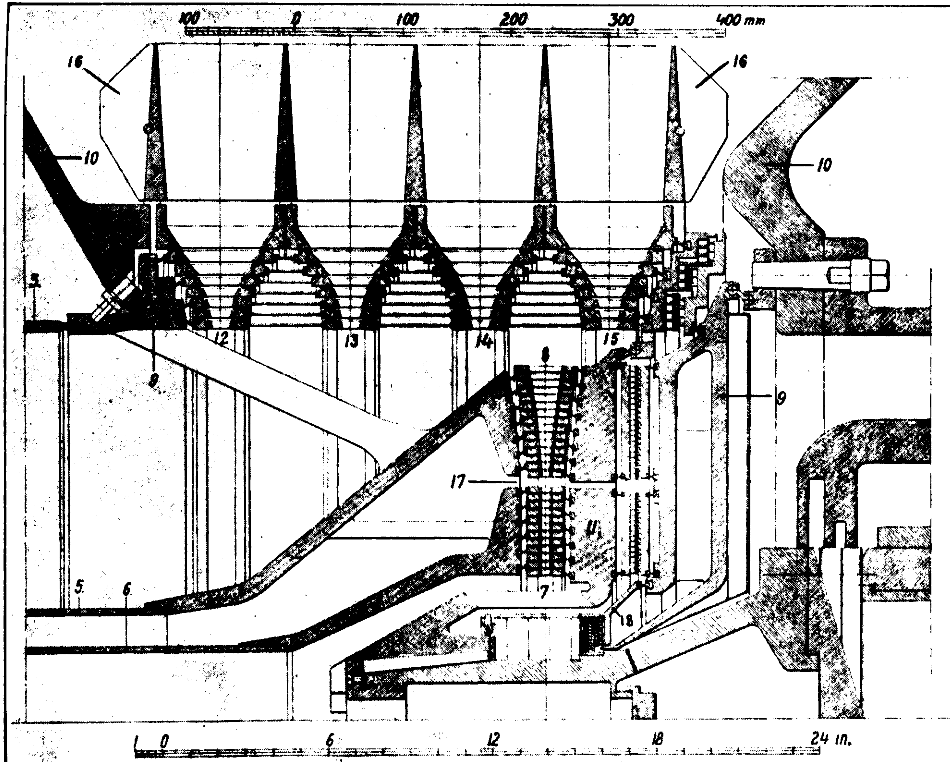


FIG. 39.—Sectional Elevation of Single Motion Mixed Pressure Ljungström Turbine.

running at 87 R.P.M., and developing about 900 I.H.P., with dry steam at 180 lbs./in.^{2} gauge.

The turbo-electric installation consists of two double-motion Ljungström turbines. Each turbine drives two independent three-phase alternating current generators. One small direct current generator supplies current to the magnets. The turbine and generators are mounted in a cylindrical casing on the top of the condenser. In Fig. 40 the upper half of the centre of the casing is removed, showing the turbine, with its L.P. blading divided in three sections. The generators are shown at the left and right sides, and the starting gear in front.

This machine and alternators run at the high speed of 7200 R.P.M., and give a three-phase alternating current of 500 volts and 120 full periods per second, the maximum output being 400 K.W. The output for the whole installation is thus 800 K.W. Each machine, as in the previous cases, is throttle-governed through an oil relay. As precautions have to be taken to prevent trouble from salt and other dust in the stator and rotor, surface cooling only is depended upon for cooling the rotor. The air is circulated by the fan arrangement shown at the outer side of each turbine bearing.

The current from the generators is supplied to two three-phase induction motors, with slip rings, and designed for 120 periods. They run at about 900 R.P.M. The propeller revolutions at full speed are 90, so that the reduction is 10 to 1. Each motor is connected to the driving pinion of a double helical gear by means of a slipping coupling, introduced to prevent the fracture of the propeller, the shaft, or parts of the helical gear, if the propeller should strike any hard object. The control gear fitted enables the propeller to be reversed in 15 secs.

The driving pinions are made from chrome-nickel steel, and the gear wheel from forged carbon steel.

The two pinions and the gear wheel are all enclosed in a common casing which is fixed to the box-shaped bedplate supporting the two motors. The lubrication of the gearing is effected by a separate oil pump, which supplies oil under pressure to each bearing and to the wheel teeth.

The speed of the gear teeth is very low, being only 27 feet per second. The thrust is taken on a ball bearing; but a "stand-bye" horseshoe thrust block of the ordinary marine type is also provided, in case of a failure of the ball thrust. The efficiency of the gear is said to reach the high figure of 99·1. The turbines are supplied with steam of 220 lbs./in.^{2} gauge pressure, superheated through a range of about 200° F., the total temperature being about 594° F.

Comparative trials of the two vessels at an average speed of 11 knots, extending over several months, have shown that the average coal consumption of the *Mjölner* is fully 38 per cent. less than that of the *Mimer.*

The original guarantee of the Ljungström Company was a 30 per cent. decrease.

The S.S. *Mjölner* was put into commission in January, 1915, and has since been regularly employed in coasting trade between Stockholm and Gothenburg. There has been no trouble whatever either with the turbines or the electrical plant or gears.

The owners of these boats have since placed an order with the Ljungström Company for two similar sets for the turbo-electric propulsion of two 14-knot combined passenger and cargo steamers.

In addition to the reduction in coal consumption there is a substantial saving in running expenses and a gain in freight. The lower pressure of steam and lack of superheat in the case of the engine installation gives the turbine set a slight advantage; but if this is allowed for there still appears to be a substantial gain on the commercial side.

The author is indebted to the British Ljungström Marine Turbine Company, Ltd., for the illustration and particulars of this interesting departure in marine propulsion.

PLATE VIII.

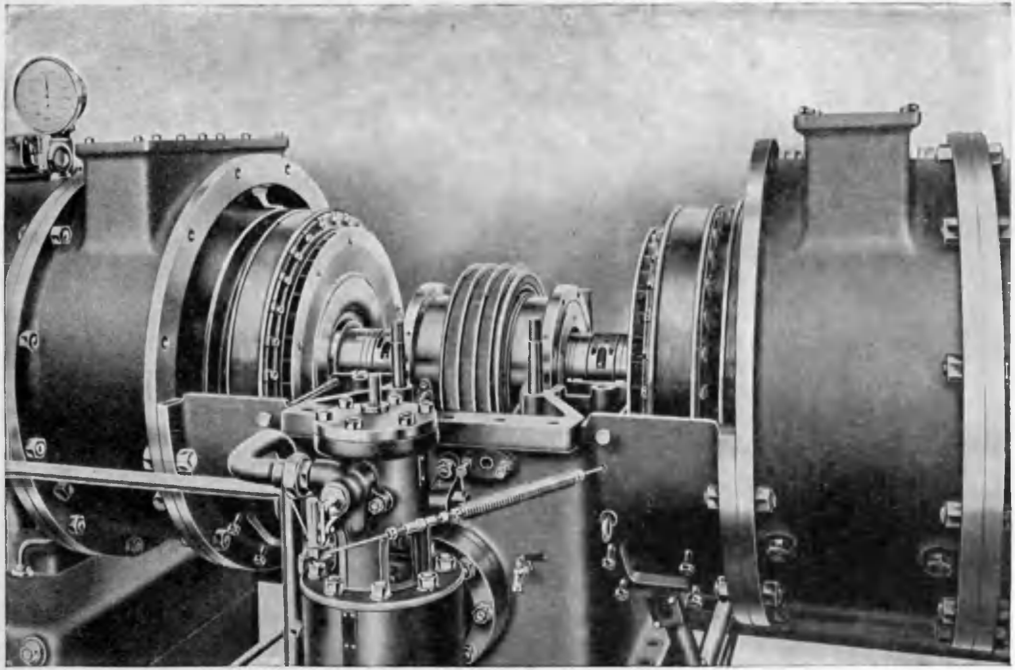


FIG. 40.—Ljungström Turbine and Generator, fitted in T.S.S. *Mjölner* (upper half of turbine casing removed).

CHAPTER IV

COMBINATION TURBINES

32. Westinghouse Double Flow Disc and Drum Turbine.—The Westinghouse Company was among the first to replace the H.P. reaction section by an impulse wheel. The machine was also made "double flow." A longitudinal section through a machine of this earlier design, made by the British Westinghouse Company, is shown in Fig. 41. It is rated to develop 10,000 K.W. at 1500 R.P.M.

The steam enters a nozzle box, 1, and expands in the nozzles, 2, of the two impulse stages. The wheels are merely a pair of deep ribs on the surface of the drum. It issues from each impulse stage at about atmospheric pressure, and enters the reaction blading at 3, one half flowing between 3 and 4 and the other between 3 and 5. It is exhausted to the condenser through the two exhaust branches 6 and 7. One feature, which somewhat simplifies the machine, is the absence of dummy pistons, as the two end thrusts on the reaction blading balance each other. The rotor is built up in three parts. The drum shell, 8, carrying the two impulse wheels is flanged internally at 9, and bolted to the rim of the conical casting 10. The spigot end of the shaft fits the boss of the casting, to which it is further bolted by the flange 11. A wheel, 12, threaded on the flange of the shaft and studded to the boss of casting, 10, carries the last three "expansions" at the L.P. end. The bearings, 13, like those of the B.W. machine previously described, are of cast iron, in halves, and lined with white metal. Adjustment is made by the pads top and bottom and at the sides, which are made with spherical seats. Liners are inserted between the pads and the seatings of the frame. Although not indicated in the section, the bearings are under forced lubrication. The oil is pumped by the gear pump, 14, in the oil tank on the left of the figure. This pump, normally, runs drowned, the oil being drawn through a strainer. It is delivered through a cooler to the bearings, and drained back from these by gravity. Centrifugal "oil catchers" are fitted on the shaft at 15. The pump is driven by a bevel gear on the worm-wheel spindle, 16, at the end of which a horizontal shaft governor is fitted. The worm wheel, 18, is driven by the worm, 17, fitted at the end of the shaft. To the left of the worm the emergency governor, 19, is fitted. The adjusting block is shown at 20. It is the same design as fitted in the Curtis-Rateau machine, Fig. 23. It consists of a cast-iron sleeve in halves. The "thrust" rings are lined with white metal. These bear against the faces of the steel rings on the

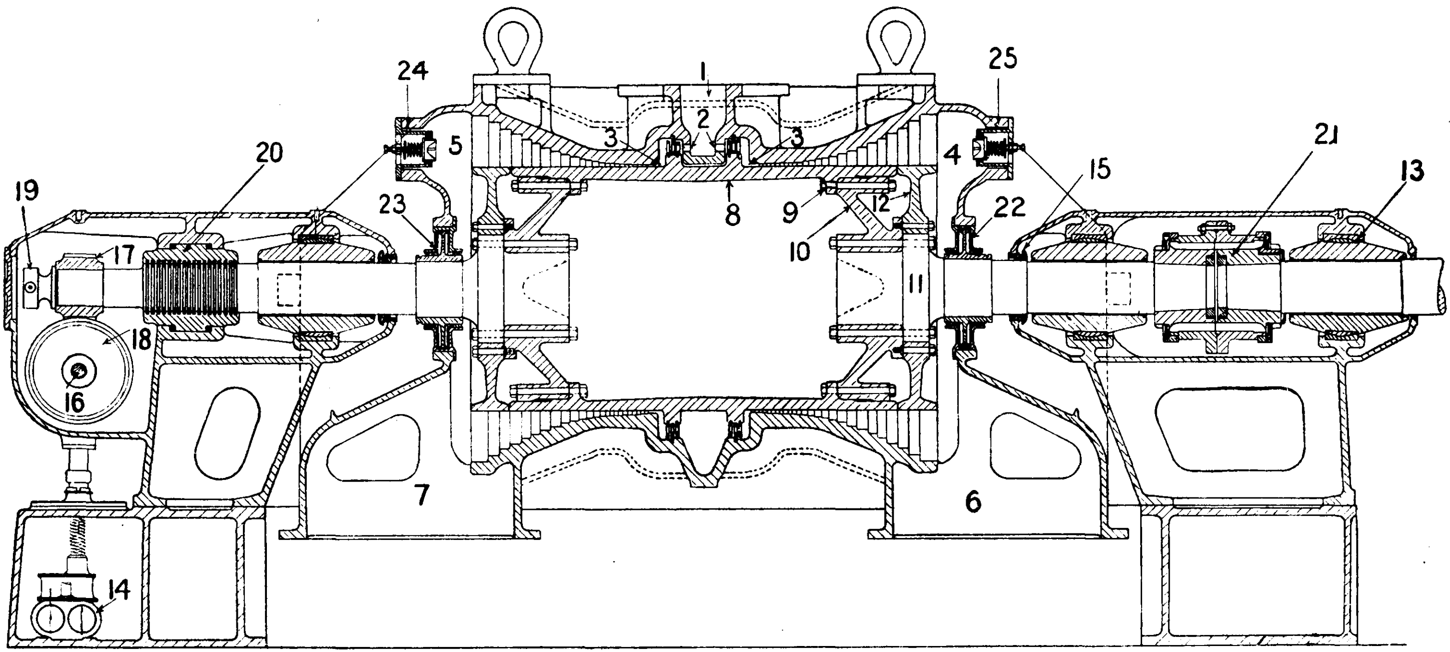


FIG. 41.—Westinghouse Double Flow Disc and Drum Turbine.

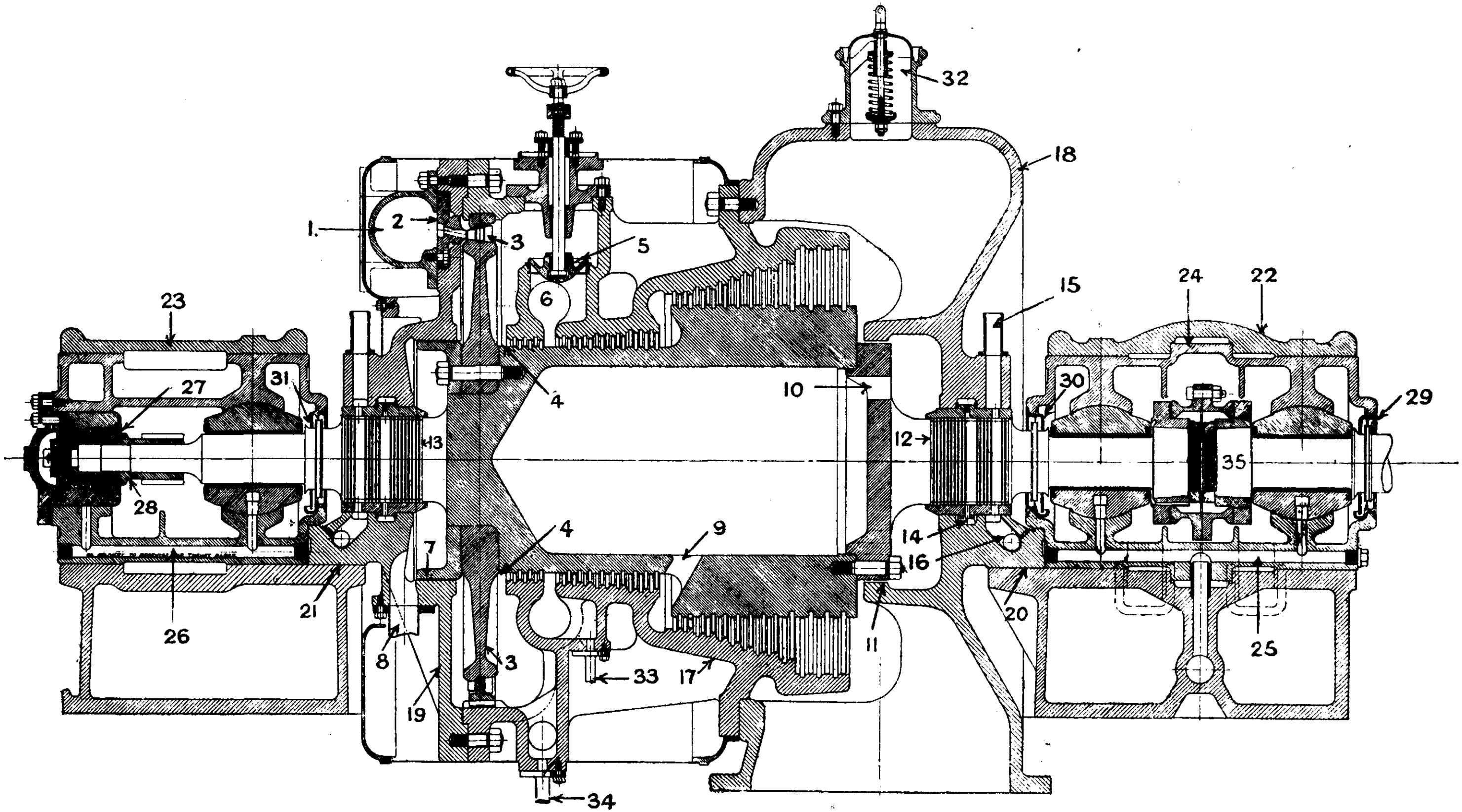


FIG. 42.—Brush Disc and Drum Turbine.

machine, Fig. 25, it is not fastened to the bedplate. The turned ends, 20 and 21, simply rest in the cylindrical seatings on the frame. The bearing shells, 22 and 23, likewise rest on these seatings, longitudinal movement again being prevented by the recessed collar, 24, on the shell at the L.P. end. The H.P. shell is free to move and allow for expansion.

The bearings are of cast iron, spherically seated, and lined with white metal. To guard against dropping the rotor in event of a seizure of the bearings, each lower half is provided with a gunmetal bearing strip.

Forced lubrication is used. The oil is fed through the passages, 25, and 26, in the lower halves of the bearing shells. It is distributed to a passage cast in the lower half of each bearing, and fed to the journal through a port near the centre line. The thrust or adjusting block, 27, is fitted at the H.P. end of the shaft. The rotating part is combined with the pinion sleeve 28. The arrangement is identical with that in Fig. 25 and need not be further considered. Oil catchers are fitted at 30, and 31, to prevent oil from spreading along the shaft to the external packing glands, and also at 29, to prevent it from getting into the generator.

An exhaust relief valve is fitted on the top of the casing at 32. The high-pressure casing is drained at 33 and 34. According to standard practice a flexible claw coupling is fitted between the turbine and generator shafts at 35.

34. Brush Mixed Pressure Disc and Drum Turbine.—A smaller machine by the same makers for an output of 500 K.W. at 2000 R.P.M. is shown in Fig. 43. It is designed to work as a mixed-pressure turbine.

The low-pressure steam, after passing an emergency and throttle valve in the exhaust supply main, enters the low-pressure section of the machine through the pipe 1, and flows into the Parsons blading at 2.

The high-pressure section has a two-velocity impulse stage. High-pressure steam is supplied to the steam chest, 3, in which are fitted several nozzle valves, 4. At the upper end of each valve spindle there is a spring-loaded piston, 5. The pressure in the chest is regulated by the main governor acting through an oil relay. The L.P. relay is also under control of the governor, and the two gears are interdependent.

When the L.P. supply is insufficient to maintain the load on the turbine, the automatic high-pressure nozzle valves open and admit steam to the nozzles of the impulse stage. It is expanded down to the L.P. pressure and passes with the L.P. steam through the reaction blading. The auto valves are progressively loaded so as to come into action one after the other as the demand for high-pressure steam increases.

When the turbine is run as a high-pressure one, an overload can be taken by opening the valve 6 and bye-passing steam to the second expansion of the drum.

The principal details of the design are the same as those of the other machines. The oil, however, is pumped into the top of the bearings through the pipes 7 and 8.

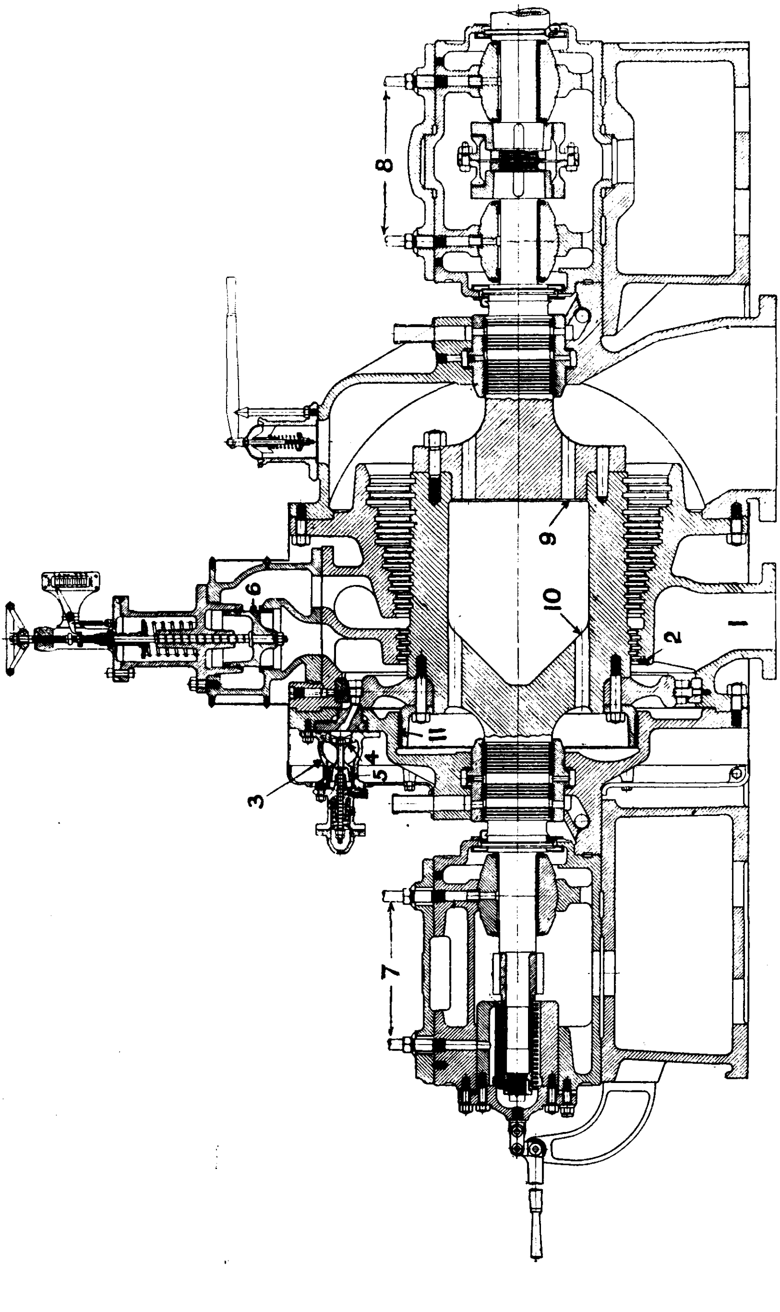


FIG. 43.—Brush Mixed Pressure Disc and Drum Turbine.

The drum in this case is made parallel, and both ends are opened to the condenser by means of the holes 9 and 10. The blade thrust is balanced by the dummy 11.

35. Willans Disc and Drum Turbine.—The standard design of combination turbine made by Messrs. Willans and Robinson, Rugby, in place of the pure reaction type, is shown in Fig. 44. Steam, after passing the throttle valve in the main inlet, enters the steam chest 1, and expands in the nozzles of the high-pressure impulse stage. The nozzle plate 2, having rectangular nozzle passages, is bolted to the inner face of the end cover. After passing through the blading, 3, of the impulse wheel, 4, the steam expands in the reaction blading, 5, and is discharged to the condenser through the exhaust branch, 6. When an overload has to be taken steam is admitted through the hand-operated valve, 7. In this case the drum, 8, is made parallel. For larger sizes a two-step drum is used.[1] The H.P. shaft is forged solid with the drum, and stepped in diameter from the main bearing journal, 9, to the drum, in order to take the external gland, 10, the dummy piston, 11, and the impulse wheel, 4. The latter are held in position by studs screwed into the drum. The circular rib, 12, which carries the sector of intermediate or guide blades is carried round the circumference of the casing, with a projecting rib which fits between the blade rings. In other cases angle rings are also fitted at the outside of the blade rings, over the portion of the circumference not covered by the nozzles. This arrangement tends to reduce the loss due to the "vane" action of the blades on the steam. The shaft at the low-pressure end is made with a spigoted flange and bolted to the drum. In the case of a large slow-speed machine the shafts are built into the drum ends by means of spider wheels riveted to the drum. The L.P. shaft is also stepped to take the external gland, 10. Oil baffle plates, 13, are fitted at the inner end of each journal, 9, to prevent creep of the oil towards the glands, 10, and a centrifugal oil thrower is fitted outside each gland (see Figs. 144, 146).

As the drum is parallel only one dummy piston, 11, is necessary. The space between this piston and the H.P. gland is connected with the eduction pipe by the external pipe, 15. Both the external glands at the inside and the back of the dummy piston are thus subjected to the condenser pressure. The glands are of the labyrinth type and are steam packed.

The bearings at 9 are cast-iron shells having white metal lining. The half shells are bolted together and have four pads, 16, secured to them top and bottom and at the sides by countersunk screws. Several steel packing pieces are fitted between the pads and the shell. The pads are then turned to fit a spherical surface in the frame of the machine. The packing pieces below the pads enable the bearings to be adjusted to get the shaft into correct alignment. The adjustment is made by interchanging some of the packing strips top and bottom or from side to side. The end bearing, 17, of the generator is placed at

[1] For description and drawings of a 7000 K.W. Willans Disc and Drum Turbine, see *Engineering*, November 14, 1913.

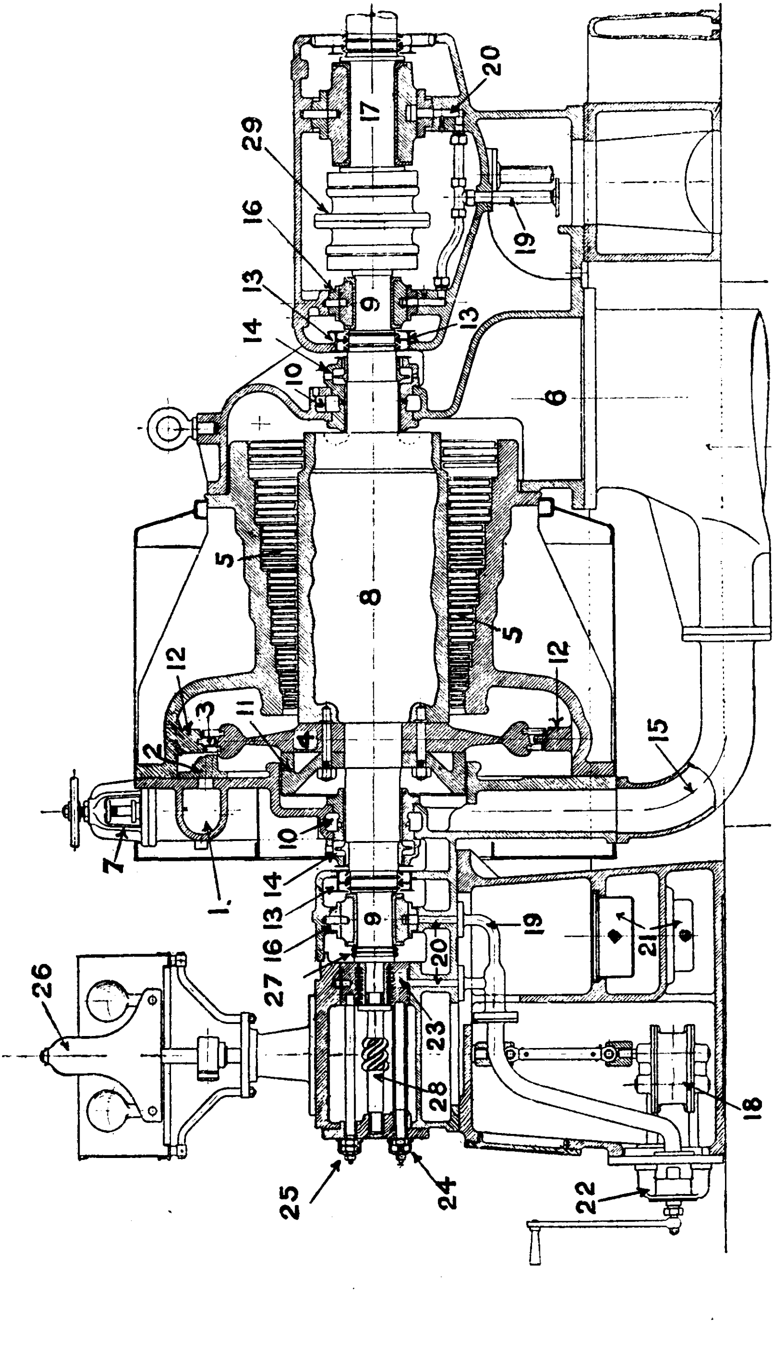


FIG. 44.—Willans Disc and Drum Turbine.

the end of the frame and fitted with similar pads. All the bearings and also the thrust block, 23, are under forced lubrication. The oil under pressure is pumped by a gear-wheel pump, 18, through a cooler to the distributing pipes, 19, and discharged from these through the ports, 20, in the frame. The oil passes through the bearing pads, and round a passage in the shell of each bearing to the horizontal diameter, where it is admitted to the journal through holes and gutters cut in the white metal. It passes axially along the journal, escaping at the ends into a receiver, from which it is drained back to the oil tank. Oil strainers are fitted at 21. A small oil pump, 22, is fitted to the side of the tank. This delivers oil to the main supply pipe, and can be used in an emergency or to flood the bearings before the machine is started. An adjusting block is fitted at 23. It is in halves, which can be separately adjusted by the bolts 24 and 25 (see Art. 166). The turbine is governed directly by a powerful centrifugal governor, 26, enclosed in a sheet metal casing. This operates a double beat throttle valve in the main steam inlet. This valve is combined with an emergency drop valve, under the control of an emergency governor trip gear, which is quite distinct from the main gear. The trip gear is operated by two spiral springs coiled round the shaft at 27. When the machine overspeeds (usually 10 per cent. above the normal) these springs expand under centrifugal force, and operate the trigger gear, holding the emergency valve open against the pull of a spring. The spring then closes the valve which shuts off the steam supply. The governor and oil-pump spindles are driven from the main shaft by the worm, 28.

The turbine and generator shafts are coupled between the bearings at 9 and 17 by a flexible claw coupling, 29.

36. Brown-Boveri Disc and Drum Reducing Turbine.—Another design of this type by Messrs. Brown, Boveri & Co., Baden (Switzerland), is shown partly in section in Fig. 45. It is arranged as a reducing turbine with an automatic exhaust valve on the reduced steam branch in order to maintain a constant pressure.

Steam enters the steam chest, 2, by the steam pipe, 1, and passes the stop valve, which is operated by the hand wheel, 3, through the medium of a worm gear. The spindle of this valve is held open by a trigger arrangement connected with an emergency gear, against the resistance of a spring in the end, 4, of the casing. The steam next passes a double beat throttle valve under control of the piston in the oil servo-motor, 5, and enters the steam belt through the branch 6. The casing, 7, on the left contains a very compact arrangement of centrifugal governor, oil pump and emergency governor. The pump supplies the oil under pressure to the servo-motor and the bearings. The governor shaft is driven by a worm gear from the turbine shaft. It also drives the tachometer, 8 (see Art. 300, p. 490).

There are four groups of nozzles. Steam is admitted directly to one set, and to two other sets through automatic lift valves. The casing of one of these valves is shown at 9.

Part of the steam, after passing through the velocity compounded impulse section, flows into the reduced steam main, 10, and part passes

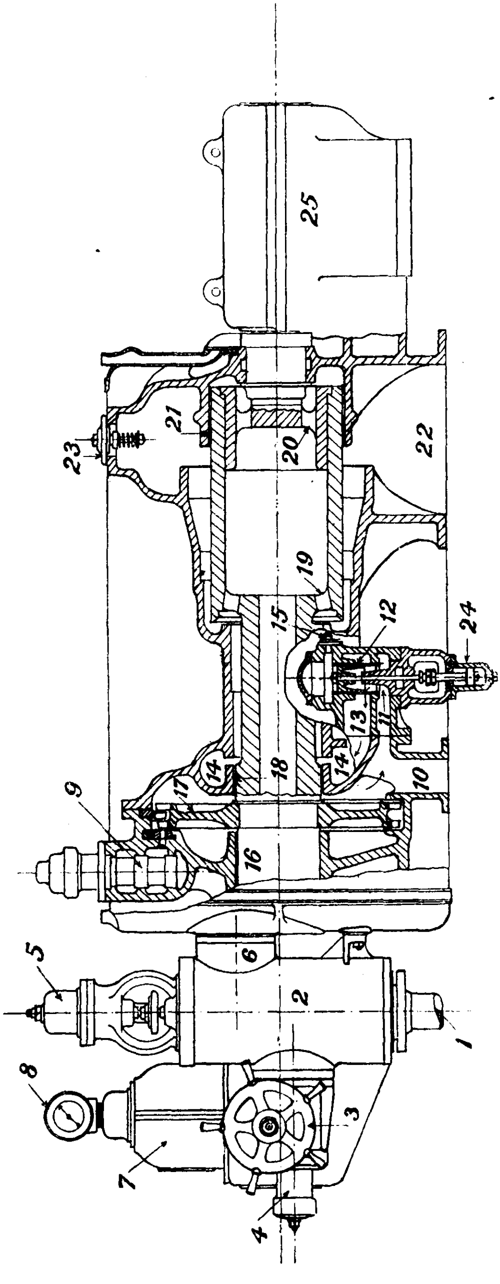


FIG. 45.—Brown-Boveri Disc and Drum Reducing Turbine.

along the passage, 11, to the under side of a piston valve, 12. It then flows through the partially opened port in the valve casing to the passage, 13, and from thence to the belt, 14, at the entrance to the Parsons section.

The piston valve, 12, is subjected to a constant steam or oil pressure on the top, the fluid being supplied through the pipe 15. If the demand for reduced steam falls off, the pressure below the valve "backs up," and raises the valve. The port area is thus increased, and a larger quantity of steam flows into the Parsons section, increasing the speed. The governor then comes into action, and reduces the supply to suit the conditions.

If the steam supply from the three sets of nozzles proves insufficient to meet the exhaust requirements and maintain full load, additional steam is obtained by admitting it to the fourth set of nozzles through a hand-operated valve.

The impulse and reaction sections are quite distinct. The high-pressure shaft is provided with a spigot end as in the case of the pure Parsons machine, Fig. 28, but no provision is made against temperature variation. The steam before it reaches the drum and shaft has fallen to a low pressure and temperature. A sleeve carrying labyrinth packings is fitted at 16, on the H.P. end of the drum. It is bolted by means of an internal flange to the drum end and shaft boss. The fastening studs are screwed partly into the drum and partly into the shaft boss. This sleeve holds the impulse wheel, 17, in position on the drum. The I.P. blade thrust is balanced by the dummy at 18. The thrust on the shoulder and L.P. blading is balanced, as in the previous cases, by admitting the steam to the interior and end of the drum through the ports 19 and 20. Labyrinth packings are fitted at 21 to prevent leakage to the condenser.

The steam, after passing through the Parsons blading, is exhausted through the branch 22. An exhaust relief valve, 23, is fitted at the L.P. end. A dash pot, 24, is fitted to the exhaust valve, 12, to steady the action.

The bearings, like those of the pure Parsons turbine, are of cast iron, lined with white metal, spherically seated in the frame with adjusting pads. The external glands are steam-packed labyrinths similar to those shown on the other machines.

The generator bearing is fitted at the right in the casing, 25; and the flexible claw coupling connecting the turbine and generator shafts is placed between the two bearings.

37. Franco Tosi Disc and Drum Turbine.—Two views, a longitudinal section and an end elevation of another continental machine by MM. Franco Tosi, Legnano, Italy, are shown in Figs. 46 and 47. The turbine is rated from 4500 to 5000 K.W. at 1500 R.P.M., with 180 lbs./in.^{2} pressure, 200° to 300° F. superheat, and 28½ inches vacuum. The special feature of this machine is the system of governing, which is entirely automatic.

Steam enters from the steam pipe 1, passes through a Ferranti stop valve, 2, into the pipe 3, Fig. 46. From this it passes to a steam

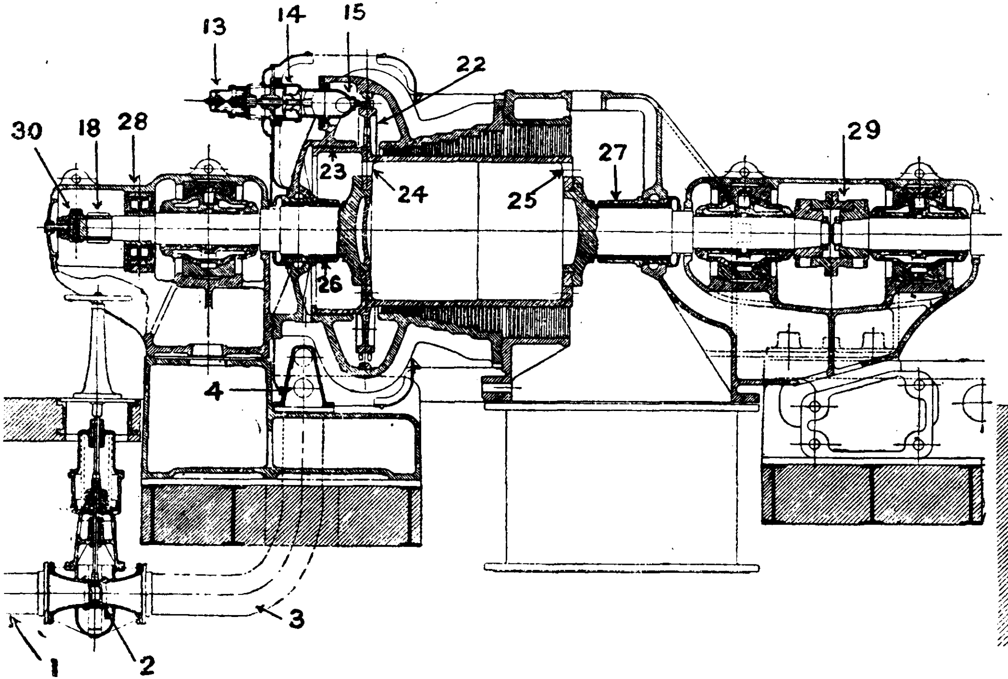


FIG. 46.—Franco Tosi Disc and Drum Turbine.

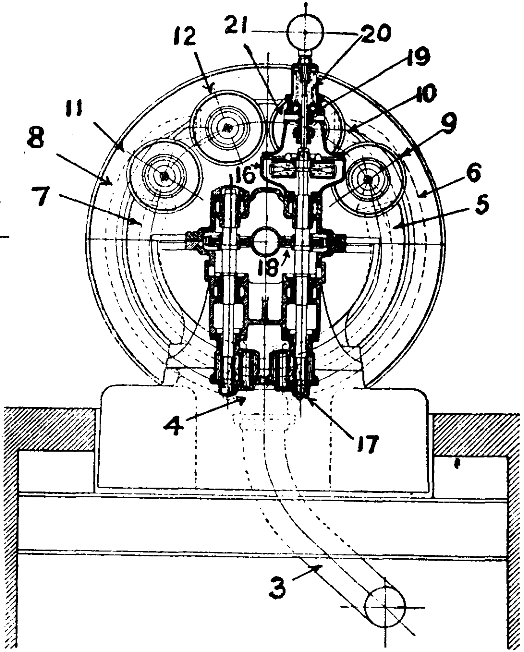


FIG. 47.

chest, 4, and is then distributed by pipes 5, 6, 7 and 8 to four nozzle boxes, 9, 10, 11 and 12, Fig. 47. Each nozzle box is provided with an automatic valve. A section of one of these boxes and valve is shown in Fig. 46. The valve, 14, which is of the double beat type, is connected to a spring-loaded piston in the cylinder, 13. Oil, under pressure from the pump, 17, Fig. 47, is admitted below the piston, which is an easy fit in the cylinder. It leaks past the piston and is drained back to the oil tank. If the rate of leakage is less than the feed, then the pressure below the piston increases, overcomes the resistance of the spring, and the piston moves out and opens the valve. The steam is thus admitted to the set of nozzles 15.

The springs are so adjusted that three of the nozzle valves are operated in succession, until full load is attained. This is the normal condition of running. The fourth set of nozzles is arranged to come into action when an overload has to be taken.

The supply of oil is controlled by the governor, 16, Fig. 47, driven from the shaft by the worm gear, 18. A cylinder, 20, with another spring-loaded piston, is fitted at the top of the governor casing. Oil from the pump is fed in at a port, 21, and passes through a sleeve valve fitted below the piston at 19, and operated by it.

At normal load the piston is in the lower position, and the ports are full open. After passing this valve the oil issues from a second port and passes to the nozzle valve pistons.

If the load falls off, the speed increases, and the pressure below the piston begins to increase. The piston rises, and throttles the flow through the sleeve valve, diminishing the supply to the pistons of the nozzle valves, which close down, progressively, on their seats as the load decreases. The reverse action takes place when the load increases. At the governor relay, a differential action, equivalent to the "overtaking" action produced by the pilot valve in the usual lever-operated steam relay, takes place (see Art. 296, p. 485).

By raising the piston, the oil supply to this is also throttled, and the pressure begins to fall. The piston then moves down under the action of the spring, and the sleeve valve again opens the ports and allows a larger quantity of oil to pass to the nozzle valve pistons.

An emergency gear is fitted to the governor bracket. It consists of a lever operated by the governor sleeve, which is connected to a valve on the oil supply pipe. If the speed increases 15 per cent. above the normal, the lever opens the valve and bye-passes the oil to the tank. The reduction in pressure at the nozzle valves causes these to close under the action of the springs, and the machine shuts down.

The drum, Fig. 46, is made of forged steel, and has the impulse wheel, 22, and the dummy piston, 23, solid with it. The shafts are registered and bolted to the ends of the drum, the spigot connection at the H.P. end being avoided to prevent trouble in case of slackness due to unequal expansion. The interior of the drum, dummy piston, and external packing gland are put in connection with the condenser by the holes, 24 and 25, in the ends of the drum. Both the glands, 26 and 27, are thus subjected to the same low pressure.

These glands are of an improved labyrinth type. Instead of the shaft acting as the inner surface of the rotating part, a thin sleeve is fitted, leaving a space between it and the shaft.

If the labyrinth rings should "touch," only this sleeve is heated up slightly, and the shaft remains unaffected. Very fine clearances can thus be used.

The bearings, as can be seen, are spherically seated, and, in addition to having forced lubrication, are also water cooled.

Any residual thrust, not taken by the dummy piston, is balanced by a special design of thrust block, 28.

This block, which is the same as that used in the next machine described, is illustrated in detail in Chapter X. As in the previous machines, a flexible coupling is inserted at 29, between the turbine and driven shaft. The casing of cast iron is in two parts, and divided horizontally along the centre line. An emergency governor is fitted on the end of the shaft at 30.

38. Franco Tosi Combination Marine Turbine.—An illustration of the type of combination marine turbine made by the same firm is shown in Fig. 48. In general design, it will be seen that it resembles the Curtis turbine, Fig. 19, with the exception that the drum section has reaction instead of impulse blading. It is designed for an output of 7500 S.H.P. at 600 R.P.M. The astern turbine develops about 3500 S.H.P. at 400 R.P.M. The initial pressure is 230 lbs./in.^{2}, and the vacuum 27 inches.[1]

In the ahead section there are six velocity compounded impulse stages. The blade rings of the first five are carried on the rims of disc wheels; those of the sixth on the H.P. end of the drum. The first stage has four rings of moving blades, each of the other stages has three. There are fourteen pairs of fixed and moving blades in the reaction section. The astern section has only one velocity compounded impulse stage, with four rings of moving blades. There are fourteen pairs of rings in the Parsons section.

Steam is admitted to the ahead section through the inlet, 1, and expands in the first stage nozzles, 2.

The nozzle blocks are bolted to the inside of the steam chest, and project through into the steam belt. Each nozzle is provided with a "cut-out" slide valve operated by hand through the medium of the screwed spindle, 4.

The second stage nozzles, 5, are similarly provided with cut-out valves, 6. In other designs the regulation of the supply at the second stage is effected by a sliding shutter, having a rack on the back gearing with a pinion at the end of the spindle. By means of this gear the shutter can be moved so as to cut out as many nozzles as are required. The nozzle passages for the other stages are formed by casting nickel-steel plates into the outer diaphragm rings, 7.

The diaphragm wall is made of forged steel, bolted to the outer ring, 7, and to the gland casting, 8. The latter is provided with a gunmetal sleeve, 9, serrated at the inner circumference.

[1] For detail drawings of a similar machine, see *Engineering*, April 26, 1912.

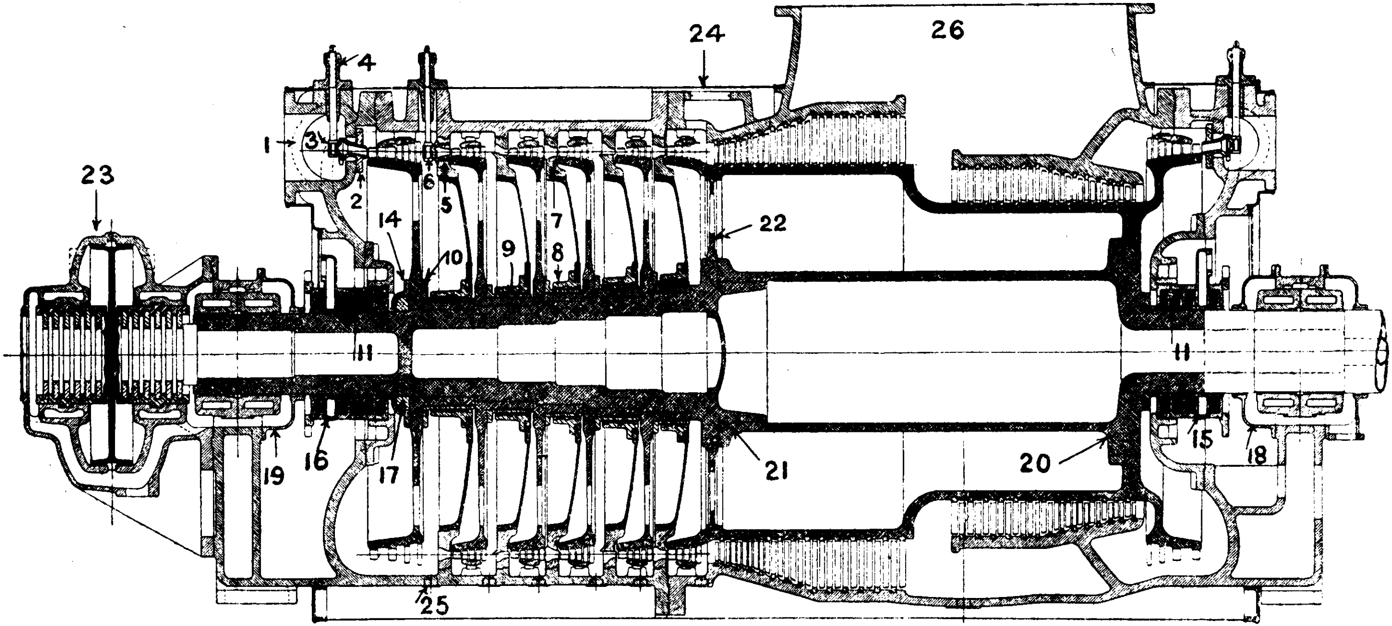


FIG. 48.—Franco Tosi Combination Marine Turbine.

A second thin sleeve, 10, bolted to the disc wheel, projects into this. The inner surface of this sleeve is quite clear of the surface of the boss. If the gland bush should "touch" the sleeve hard, it can spring slightly and prevent seizure. By this arrangement fine running clearances can be obtained at the diaphragm glands (see Fig. 155, page 293). The external packing glands, 11, are carried in castings bolted to the casing ends. They are packed by carbon rings. Each ring is in four segments, held up to the shaft surface by garter springs. It is also held up from below by a coach spring, fitted into the L-shaped ring carrier. Leak-off pockets are provided at 15 and 16, the steam being carried away by a pipe connection at the top of the gland. The carbons of the gland do not touch the surface of the shaft, but that of a thin sleeve bolted to the wheel by a flange, 14.

Unequal heating of the gland from any accidental cause does not produce distortion, and a damaged sleeve can be quickly replaced. The sleeve flange also acts as a cap to cover and lock the nut, 17, which holds the impulse wheels together on the shaft.

The main bearings are kept close to the glands, in order to keep down the span of the shaft and stiffen the rotor. They consist of cast-iron shells lined with white metal. They have forced lubrication, and are also water cooled. Oil wipers, which are not indicated on the drawing, are fitted at the gland side of the casting 18, which acts as an oil well. Similar wipers are provided at casting 19 at the high-pressure bearing. These prevent the spread of oil along the shaft to the glands.

Each disc wheel is a mild steel forging, perforated with large holes to lighten it and also equalise the steam pressure on each side. The wheel is a force fit on the shaft, which is progressively stepped. The shaft is hollow and in two parts. The L.P. section is really a small drum. It is registered and bolted to the drum end by the flange 20. In other designs the intermediate joint is dispensed with at 21, and the shaft up to the L.P. drum end made in one piece. At the L.P. end it is flanged and bolted to the corresponding flange on the L.P. shaft and the drum end. This arrangement gives a light and at the same time a stiff shaft. The forward end of the drum, like the wheels, is formed by a thin perforated plate web, 22, into which the shaft is registered.

The L.P. end of the drum being closed, it is acted on by the difference of pressure in the sixth stage and the condenser pressure. The thrust aft thus produced is utilised to balance the propeller thrust at cruising speed. At full speed, there is a residual thrust amounting to about 20 per cent. of the maximum propeller thrust which is unbalanced and is taken by the special thrust block 23 fitted at the forward end. This block is illustrated and described in Chapter X. The thrust is divided into two parts by a piston on the shaft carrying a face labyrinth at the rim. By means of this piston the block is automatically adjusted so that the ring surfaces do not touch. The thrust is thus practically free from wear and risk of seizure as long as the supply of oil under pressure is maintained. The block is water cooled.

Auxiliary exhaust steam can be admitted to the L.P. ahead section through the port 24.

Each H.P. wheel chamber is provided with a drain, 25, at the bottom. The flat sides of the exhaust branch, 26, are usually stayed with bar stays, not shown on the drawing.

CHAPTER V

PROPERTIES OF STEAM

39. Unit of Heat.—Throughout this text the British Thermal Unit is used. It is the unit of heat measurement used in the bulk of the technical literature, dealing with the steam turbine in this country and in America.

It may be defined, as the $\frac{1}{180}$th part of the quantity of heat required to raise 1 lb. of water from 32° F. to 212° F., and is denoted by the symbol B.Th.U.

In continental literature the major calorie is used. It is the $\frac{1}{100}$th part of the quantity of heat required to raise 1 Kg. of water from 0° C. to 100° C.

For purposes of comparison, the relation between the two units is given by

$$1 \text{ B.Th.U.} = 0{\cdot}252 \text{ Cal.}$$

Specific Heat of Water.—The heat capacity or heat "appetite" of water is not constant over a wide range of temperature. The specific heat K decreases slightly between 32° F. and 100° F., and then increases. It may be estimated approximately from the equation $K = 0{\cdot}5277\ T^{\frac{1}{10}}$, where T is the absolute temperature Fah. or $(t + 460)$.

The heat taken in by 1 lb. of water between temperatures T_1 and T_2 is given approximately by

$$h = \int_{T_1}^{T_2} K\,dT = 0{\cdot}5277 \int_{T_1}^{T_2} T\,dT^{\frac{1}{10}} = 0{\cdot}4797\left(T_2^{1{\cdot}1} - T_1^{1{\cdot}1}\right)$$

Taking the lower temperature as 32° F. or $T_1 = 492$

$$h = 0{\cdot}4797 T_2^{1{\cdot}1} - 438 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is the common practice for most calculations to assume $K = 1$, and with this approximation the heat taken in by 1 lb. of water between temperatures t_1 and t_2 is given by the temperature difference, $h = (t_2 - t_1)$. This value is always less than the real figure; and in dealing with steam calculations it is advisable to take the sensible heats directly from the steam tables.

EXAMPLE 1.—A boiler is supplied with feed water at 153° F. The gauge pressure is 180 lbs./in.², and the temperature of the dry steam is

380° F. Find the heat absorbed per lb. of water between these temperatures, (*a*) by direct calculation, (*b*) by difference of temperature, (*c*) by difference of the sensible heats from the steam tables.

(*a*) By equation (1), if T_2 and T_3 are the absolute temperatures

$$h = (h_3 - h_2) = 0{\cdot}4797(T_3^{1{\cdot}1} - T_2^{1{\cdot}1}) = 0{\cdot}4797(840^{1{\cdot}1} - 613^{1{\cdot}1})$$
$$= 0{\cdot}4797(1647-1165) = 0{\cdot}4797 \times 482 = 231{\cdot}2 \text{ B.Th.U.}$$

(*b*) Assuming $K = 1$, $h = (t_3 - t_2) = (380 - 153) = 227$ B.Th.U.

(*c*) From the steam tables at 153° F., $h_2 = 120{\cdot}9$

380° F., $h_3 = 352{\cdot}7$

and $h = (352{\cdot}7 - 120{\cdot}9) = 231{\cdot}8$ B.Th.U.

The value in (*a*) is 0·26 per cent. and in (*b*) 2·6 per cent. less than the tabular difference.

In all calculations that follow the sensible heats are taken directly from the steam tables.

40. Relation between Heat and Work.—This relation is given by the First Law of Thermodynamics, established by Dr. Joule.

If H heat units are converted into E units of mechanical work,

$$E = JH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

J is the factor of conversion known as Joule's equivalent.

The original value determined by Joule was 772, the heat being in B.Th.U. and the work in ft.-lbs. According to later determination 778 is a more correct figure, and this is now generally used.

Units of Power.—Power is a rate of performance of work or expenditure of energy. In mechanical problems the unit of power used is the Horse-power. This is 33,000 ft.-lbs./min., or 550 ft.-lbs./sec.

In electrical problems the unit is the Kilowatt. One Horse-power is equivalent to 746 Watts, so that 1 Kilowatt or (K.W.) = 1·34 Horse-power.

41. Generation of Steam.—During the process of "getting up steam" in a boiler with all valves closed, the steam is generated at "constant volume," as there is no sensible alteration of the volume of the steam space.

When the working pressure is reached and the stop valve opened to permit the steam to flow from the boiler at this pressure, as fast as it is formed, the generation takes place at "constant pressure."

Reckoned from a standard feed-water temperature of 32° F., one pound of steam generated under conditions of constant pressure, requires for its production a larger amount of heat than a pound generated at constant volume.

At constant pressure there is an increase in the volume of the pound of "stuff," which is resisted by the external pressure equal to the working pressure.

Work is done against the external resistance at the expense of some of the heat supplied. This work is called the "external work" of steam formation.

42. Sensible Heat (*h*).—In the production of a pound of steam at constant pressure, from a pound of water having an initial temperature 32° F., there is an absorption of heat producing a rise in temperature, until the boiling-point corresponding to the pressure is reached.

The temperature at this point is called the "saturation" temperature, and the heat absorbed, the "sensible" or liquid heat (h).

Latent Heat (L).—When the water boils there is a change of physical state, viz. conversion from liquid to vapour. The disintegration of the constituent particles of the stuff involves the performance of internal mechanical work, with corresponding conversion of heat energy. The absorption of heat for disintegration work is not indicated by a thermometric rise. The temperature remains constant at the saturation value until the whole pound of water is evaporated. This heat is said to become "latent." In addition, there is the heat required for the external work.

The sum of these two quantities is called the "latent" heat of the steam (L), at constant pressure.

Quality of Steam.—If the pound of stuff, in the form of steam, as it leaves the boiler is just at the boiling temperature corresponding to the pressure, the steam is said to be "saturated." If the whole pound of water is evaporated, the steam is saturated and "dry." If it contains a certain proportion of unevaporated water, it is said to be "wet." If by the addition of extra heat (or by throttling it) the steam reaches a higher temperature than that corresponding to the pressure, it is said to be "superheated."

The condition of wetness or superheat is spoken of generally as the "quality" of the steam.

Total Heat of Steam (H).—The "total" heat of a pound of wet, dry, or superheated steam is the amount of heat, reckoned from 32° F., that has to be supplied to the pound of stuff, initially in the form of water, for the production of the steam of given quality.

In the case of wet steam let the fraction q of the pound of water be converted to steam. The quality q is called the "dryness fraction," and $(1 - q)$ the "wetness fraction" of the steam. When the steam is dry, $q = 1$.

It will be evident that since the sensible heat (h) relates only to the liquid condition, and the proportion of latent heat required for q lbs. of steam is qL, that the heat required for the production of a pound of wet steam of q dryness is

$$\mathrm{H} = h + q\mathrm{L} = \text{Total heat of wet steam} \quad . \quad . \quad . \quad (3)$$

When the steam is dry $q = 1$, and

$$\mathrm{H} = h + \mathrm{L} = \text{Total heat of dry steam} \quad . \quad . \quad . \quad . \quad (4)$$

In the case of superheated steam, if t is the saturation temperature, and t_1 the superheat temperature, the range of superheat is $t_s = (t_1 - t)$.

When superheated this amount at constant pressure, the additional heat or "superheat" is $\mathrm{K}_p t_s$, where K_p is the mean specific heat at constant pressure for the range of superheat. Then,

$$\mathrm{H}_s = \mathrm{H} + \mathrm{K}_p t_s = \text{Total heat of superheated steam} \quad . \quad (5)$$

43. Specific Heat of Superheated Steam.—The specific heat of superheated steam at constant pressure is a variable quantity. It is some function of both pressure and temperature, increasing with increase of pressure and decreasing with increase of temperature. A great deal of experimental work has been done on this subject. The experimental results of Knoblauch and Jakob are regarded as being fairly reliable, and, for calculative purposes, the author has taken their values of mean specific heat from the original table, which is not suitable for direct application to turbine problems, and constructed the specific heat chart shown in Fig. 49.

It consists of a set of constant superheat curves plotted on a base of absolute pressure, the range of superheat varying from 40° to 350° F. and the pressure from 10 to 300 lbs./in.^{2} abs.

By following the pressure vertical to the given superheat-range curve, the specific heat is determined, and the superheat, $h_s = K_p t_s$, is calculable at once.

44. External Work.—If the volume of 1 lb. of water is b, and the final volume of the pound of dry saturated steam is v ft.^{3}, the total change of volume during formation is $(v - b)$. If the constant pressure of formation is p lbs./in.^{2} and the external work is E_e, then,

$$E_e = 144p(v - b) \text{ ft.-lbs. per lb.}$$

The volume b of the water is in all cases so small relatively to the final volume v, that it is quite negligible in practical calculations, and the external work, if P is the pressure in lbs./ft.^{2}, may be taken as

$$E_e = 144pv = Pv$$

Its heat equivalent is $h_e = \dfrac{144pv}{J} = \dfrac{Pv}{J}$ . . . (6)

Internal or Intrinsic Energy.—This quantity represents the difference between the total heat and the heat equivalent of the external work. Denoting it by I, then,

$$I = H - \frac{144pv}{J} = H - \frac{Pv}{J} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

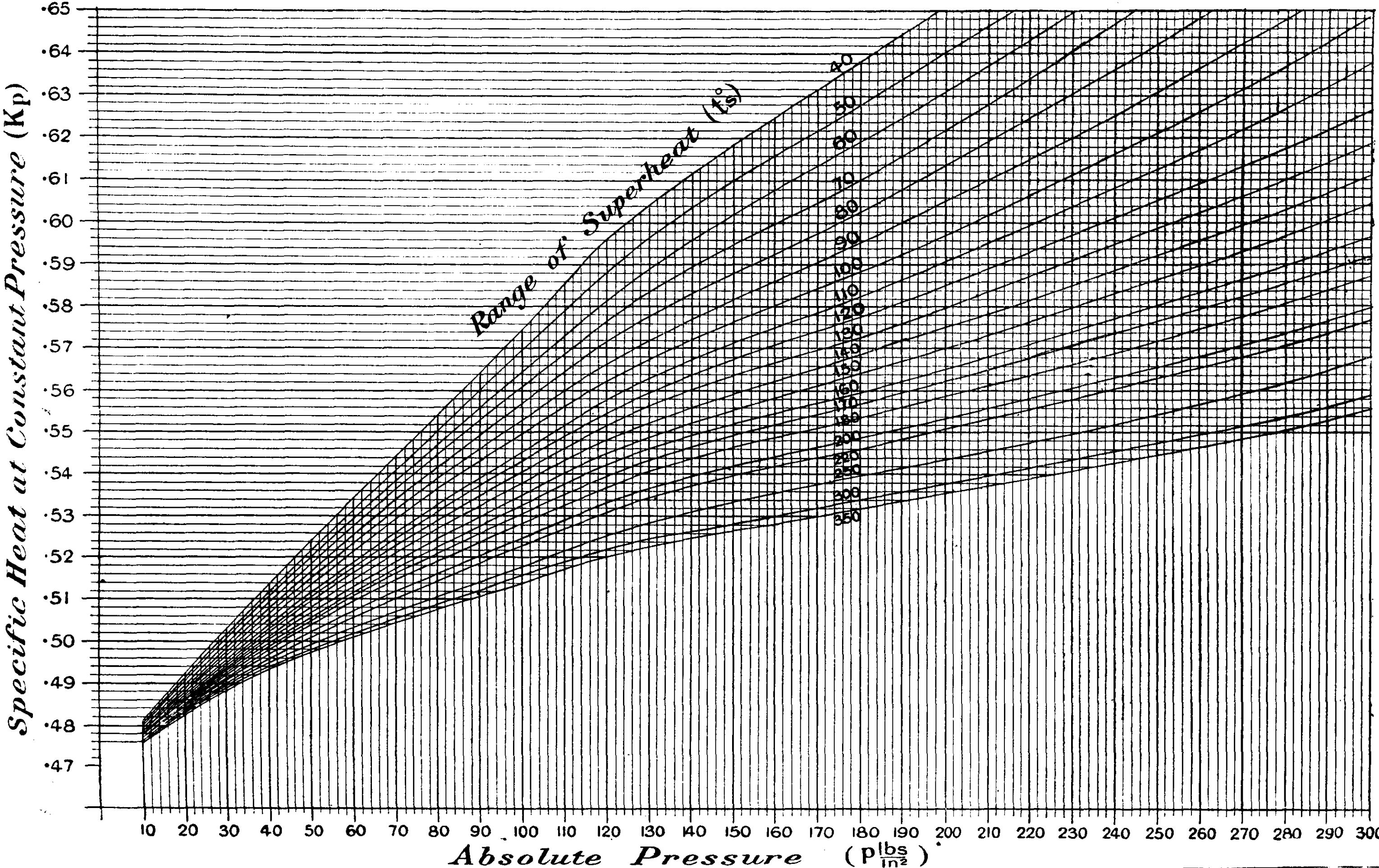
It may also be expressed as the sum of the sensible and disintegration heats. Thus, $I = (h + l)$, where l = disintegration heat or latent heat of the steam at constant volume.

45. Specific Volume of Steam.—In the case of dry steam the volume per lb. or specific volume, v, can be obtained directly from the steam tables.

When the steam is wet, if the small volume of water is neglected, the specific volume at any quality q is $v_w = qv$.

When the steam is superheated the specific volume v_s, at a given pressure, is dependent on the temperature of superheat.

A number of formulæ, based on experimental results, are in use


Specific Heat at Constant Pressure (Kp)
·65 ·64 ·63 ·62 ·61 ·60 ·59 ·58 ·57 ·56 ·55 ·54 ·53 ·52 ·51 ·50 ·49 ·48 ·47
Range of Superheat (t_s°)
40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 200 220 250 300 350
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300
Absolute Pressure ($P \frac{lbs}{in^2}$)

The equation due to Linde is considered reliable. It may be written in the form,

$$v_s = 0{\cdot}5962\frac{T}{p} - (1 + 0{\cdot}0014p)\left(\frac{150300000}{T^3} - 0{\cdot}0833\right)$$

where T is the temperature of superheat in degrees Fah. abs., and p the pressure in lbs./in.^{2} abs.

It is obviously far too clumsy for ordinary calculation, and does not take account of the range of superheat t_s.

Messrs. Marks and Davis have compiled a table of superheated steam volumes based on this formula.

From an examination of their figures the author finds that the volume at any given pressure for a given range of superheat t_s, can be determined by the following formula:

$$v_s = v(1 + 0{\cdot}0016t_s) \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where v is the specific volume of the dry steam at the given pressure, and t_s the superheat range in degrees Fah.

This formula lends itself readily to graphical representation, in the form of an alignment chart, which is shown in Fig. 50.[1] It consists of three vertical lines carrying scales of superheat range pressure and volume. By placing a straight-edge across the pressure and superheat values the corresponding volume is found where the edge cuts the volume scale. Additional scales of dryness and wet steam volume have been added, so that the chart can be used either for wet or dry or superheated steam volumes. In the case of dry steam, with $t_s = 0$ and $q = 1$, both volume scales give the same figure.

This chart on a large scale is incorporated with the heat-entropy diagram accompanying this book, and is used in subsequent calculations.

46. Saturated Steam Table.—The properties of saturated steam were originally made the subject of an extended investigation by Regnault, and were embodied in what is known as the Saturated Steam Table.

The experiments have since been repeated by various investigators, with the aid of more modern methods of physical research, and the results have, in consequence, undergone some modification.

Several revised steam tables have recently been published, and as far as most engineering calculations are concerned, it makes little difference what set of values is used, the variation in results is so slight.

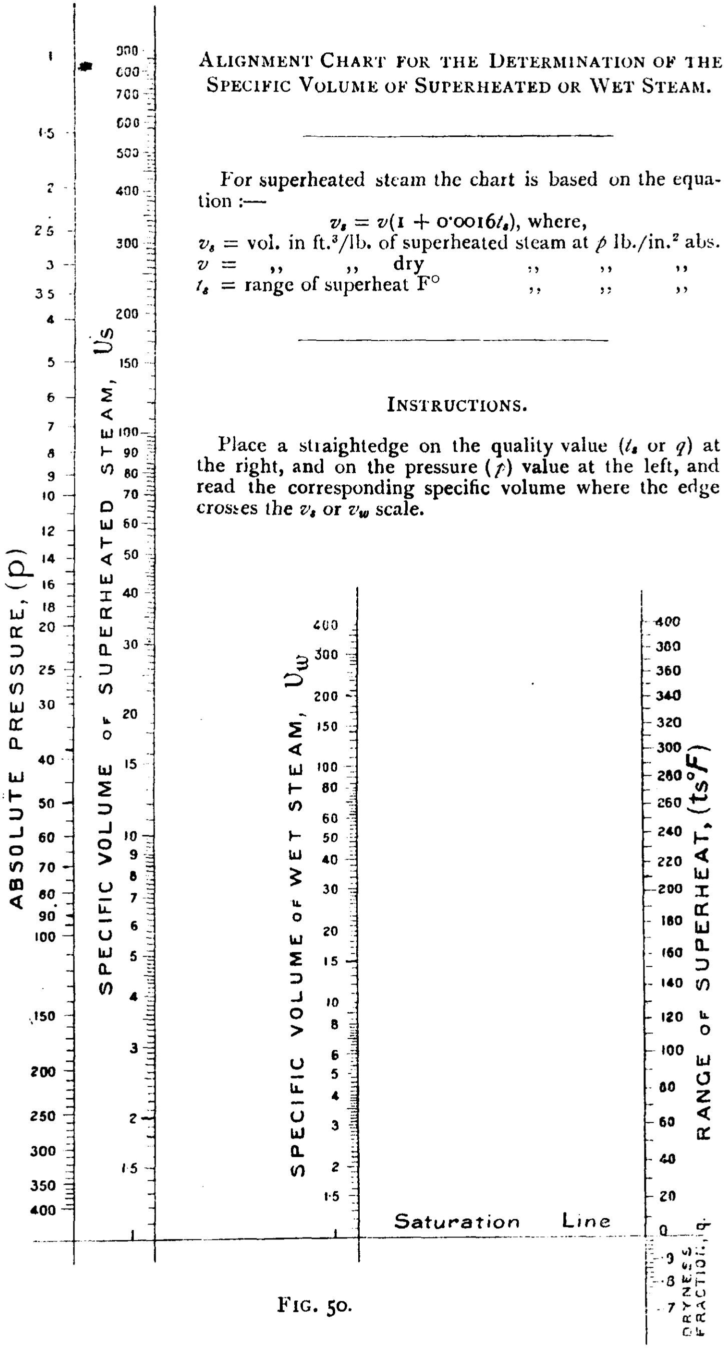
One of the most serviceable tables, which has in addition a very complete table of the properties of superheated steam, is that of Marks and Davis.

The Total-Heat-Entropy Diagram accompanying this book is plotted from the values given in these tables.

An abridgment of the saturated steam table for the series of pressures given in this diagram is added at the end of the book.

The following examples illustrate the practical use of the tables:—

[1] For a description of the method of constructing this chart, see an article by the author, *Engineering*, July 1, 1910.



ALIGNMENT CHART FOR THE DETERMINATION OF THE SPECIFIC VOLUME OF SUPERHEATED OR WET STEAM.

For superheated steam the chart is based on the equation:—

$$v_s = v(1 + 0{\cdot}0016t_s), \text{ where,}$$

v_s = vol. in ft.^{3}/lb. of superheated steam at p lb./in.^{2} abs.
v = ,, ,, dry ,, ,, ,,
t_s = range of superheat F° ,, ,, ,,

INSTRUCTIONS.

Place a straightedge on the quality value (t_s or q) at the right, and on the pressure (p) value at the left, and read the corresponding specific volume where the edge crosses the v_s or v_w scale.

FIG. 50.

EXAMPLE 2.—Feed water initially at 170° F. is converted, in a boiler, to steam at 200 lbs./in.^{2} abs., and superheated through a range of 200° F. Find the total amount of heat supplied by the gases per lb. of feed, and the specific volume of the steam.

From the steam tables at 170° F. $h_1 = 137{\cdot}9$
at 200 lbs./in.^{2} and 382° F. $h_2 = 354{\cdot}9$
" " " $L_2 = 843{\cdot}2$

From the chart, Fig. 49, at $p = 200$ and $t_s = 200$, $k_p = 0{\cdot}5538$, and the superheat is $h_s = k_p t_s = 0{\cdot}5538 \times 200 = 110{\cdot}76$.

Heat from the gases

$$= (h_2 - h_1) + L_2 + h_s$$
$$= (354{\cdot}9 - 137{\cdot}9) + \cancel{834{\cdot}2} + 110{\cdot}76 = \cancel{1161{\cdot}96} \text{ B.Th.U.}$$

From the tables the specific volume of dry steam at 200 lbs./in.^{2} is $v = 2{\cdot}29$ ft.^{3}, $t_s = 200$; hence by equation (8)

$$v_s = v(1 + 0{\cdot}0016 t_s) = 2{\cdot}29(1 + 0{\cdot}0016 \times 200)$$
$$= 2{\cdot}29 \times 1{\cdot}32 = 3{\cdot}023 \text{ ft.}^3$$

This figure is 0·5 per cent. less than that given in Marks and Davis' tables, which is 3·04.

EXAMPLE 3.—The difference of total heat at the admission and exhaust pressures of an ideal steam turbine is the heat for conversion into mechanical work. Determine in ft.-lbs. per lb. of steam the work done by such a machine, when the admission pressure is 180 lbs./in.^{2} gauge, steam temperature 500° F., exhaust pressure 1 lb./in.^{2} abs., and wetness 19 per cent.

If in an actual turbine 30 per cent. of this heat difference is not converted to mechanical energy, but remains in the steam at exhaust, what is the final quality of the steam?

From steam tables, saturation temp. at 195 lbs./in.^{2} is approx. 380° F., actual temp. 500° F. Hence the steam is superheated 120° F.

Total heat of dry steam at p_1 is $H_1 = 1197{\cdot}7$.

From the chart, Fig. 49, $k_p = 0{\cdot}583$; hence the superheat $h_s = 0{\cdot}583 \times 120 = 69{\cdot}96$.

Total heat at admission $H_s = H_1 + h_s = 1197{\cdot}7 + 69{\cdot}96 = 1267{\cdot}66$ B Th.U.

Dryness at 1 lb./in.^{2}, $q_0 = 0{\cdot}81$; $h_0 = 69{\cdot}8$; $L_0 = 1034{\cdot}6$.

Total heat at exhaust, $H_0 = h_0 + q_0 L_0 = 69{\cdot}8 + 0{\cdot}81 \times 1034{\cdot}6$
$= 69{\cdot}8 + 838{\cdot}026$
$= 907{\cdot}826$ B.Th.U.

Heat for conversion to work

$$= (H_s - H_0) = (1267{\cdot}66 - 907{\cdot}826) = 359{\cdot}834 \text{ B.Th.U.}$$

By equation (2),

$$\text{Work done} \doteq E = J(H_s - H_0) = 778 \times 359{\cdot}834$$
$$= 279{,}438 \text{ ft.-lbs.}$$

In the actual machine, heat retained $= h_f = 0{\cdot}3 \times 359{\cdot}834$
$= 107{\cdot}952$.

Actual total heat at exhaust, $H_0' = (H_0 + h_f) = 907{\cdot}826 + 107{\cdot}952$
$= 1015{\cdot}78$ B.Th.U.

Then, $H_0' = h_0 + q_0'L_0$, and actual quality

$$q_0' = \frac{H_0' - h_0}{L_0} = \frac{1015{\cdot}78 - 69{\cdot}8}{1034{\cdot}6} = \frac{945{\cdot}98}{1034{\cdot}6} = 0{\cdot}914$$

EXAMPLE 4.—Calculate the intrinsic energy of 1 lb. of steam at 150 lbs./in.^{2} abs. when (*a*) dry, (*b*) 20 per cent. wet, (*c*) superheated 200° F.

(*a*) From the tables, total heat at 150 lbs./in.^{2}, H $= 1193{\cdot}4$ B.Th.U., sp. vol. $v = 3{\cdot}012$ ft.^{3}

External heat by equation (6), $h_e = \frac{144pv}{J} = \frac{144 \times 150 \times 3{\cdot}012}{778}$
$= 83{\cdot}6.$

By equation (7), $I = H - \frac{144pv}{J}$
$= (1193{\cdot}4 - 83{\cdot}6) = 1109{\cdot}8$ B.Th.U.

(*b*) $h = 330{\cdot}2, \quad L = 863{\cdot}2, \quad q = 0{\cdot}8.$
By equation (3), $H = h + qL = 330{\cdot}2 + 0{\cdot}8 \times 863{\cdot}2$
$= 330{\cdot}2 + 690{\cdot}56 = 1020{\cdot}76$ B.Th.U.
Specific vol. (neglecting water) $= qv = 0{\cdot}8 \times 3{\cdot}012 = 2{\cdot}41$ ft.^{3}

External heat, $h_e = \frac{144 \times 150 \times 2{\cdot}41}{778} = \frac{83{\cdot}6 \times 2{\cdot}41}{3{\cdot}012} = 66{\cdot}9$

$\therefore I = 1020{\cdot}76 - 66{\cdot}9 = 953{\cdot}86$ B.Th.U.

(*c*) From the chart, Fig. 49, $k_p = 0{\cdot}542$; $t_s = 200$.

$\therefore h_s = 0{\cdot}542 \times 200 = 108{\cdot}4$

By equation (5), total heat, $H_s = H + k_p t_s = 1193{\cdot}4 + 108{\cdot}4$
$= 1301{\cdot}8$ B.Th.U.

Sp. vol. of steam $v_s = 3{\cdot}012(1 + 0{\cdot}0016 \times 200)$
$= 3{\cdot}012 \times 1{\cdot}32 = 3{\cdot}98$ ft.^{3}

External heat $h_e = \frac{144 \times 150 \times 3{\cdot}98}{778} = 110{\cdot}4$

$$\therefore I = H_s - \frac{144pv_s}{J} = 1301{\cdot}8 - 110{\cdot}4 = 1191{\cdot}4 \text{ B.Th.U.}$$

47. Graphical Representation of the Properties of Steam.—The relative importance of the sensible and disintegration heats and the heat equivalent of external work can be clearly shown by plotting the heat values on a base of absolute pressure.

The resulting diagram is shown in Fig. 51.

It will be noted that, except at the lower end of the pressure scale, the external work is nearly constant. This means that, as a first approximation, the law $pv =$ const. may be assumed to hold for saturated steam over nearly the whole pressure range employed in turbine practice.

Curves of specific volume, temperature, and entropy (dealt with in the next chapter) are also added to the diagram.

The very rapid increase of volume at the lower end of the pressure scale should be noted. It is this increase that makes the design of the last few stages of a compound condensing turbine so troublesome, and the utilization of high vacuum in a reciprocating steam engine impracticable.

This diagram is only one of several that can be plotted from the steam tables by varying the co-ordinates.

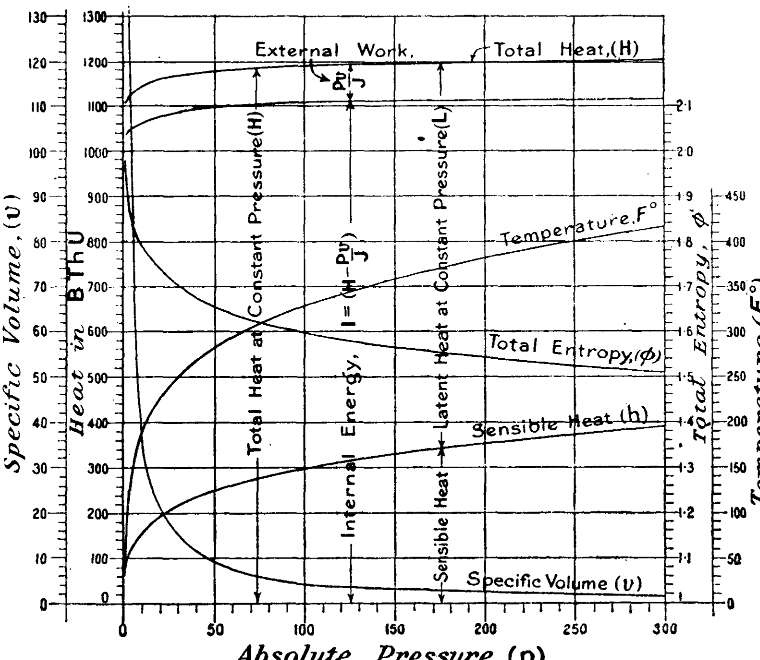


FIG. 51.

Two of these other diagrams are of the first importance for turbine calculations.

The first and earlier, employed in analytical investigations connected with the steam engine, has as co-ordinates absolute temperature and entropy. It is called the Temperature-Entropy Diagram.

The second and later, which is now used for direct and rapid estimation of heat drops and steam volumes in steam turbine calculations, has as co-ordinates total-heat and entropy. It is called the Total-Heat Entropy Diagram.

It was originally applied to turbine work by Prof. Mollier, and is also known as the Mollier Diagram.

CHAPTER VI

ENTROPY DIAGRAMS

48. Entropy.—When a substance gains heat from or loses heat to its surroundings, it is said to "change its entropy."

"Entropy" is not susceptible of direct definition. The term is used to denote a certain mathematical function of the absolute temperature.

If a unit mass of substance takes in H units of heat at an absolute temperature T, the change of entropy denoted by ϕ is expressed by the relation

$$\phi = \frac{H}{T}$$

The stock of energy of the substance is increased by

$$H = T\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the heat is taken in at varying temperature, then

$$\phi = \Sigma\frac{dh}{T}$$

where dh is an element of heat, and T the mean absolute temperature during its reception.

When the substance goes through a series of cyclic changes, and is brought back, at the end of the series, to the initial condition, then,

$$\Sigma\frac{dh}{T} = 0$$

If the internal energy of a substance is either increased by the performance of work on or decreased by the performance of work by the substance, and no heat is allowed to flow into or out of the substance through the containing envelope, then, at every instant of the process, $dh = 0$, and the entropy change is zero.

Such an operation is said to be "adiabatic." Further, since there is no change of entropy during the process, it is an operation at constant entropy, and hence "isentropic."

This is the condition which would exist during the expansion of steam behind a piston in a non-conducting cylinder, or in a turbine nozzle passage provided there were no disturbing influences, such as friction on the walls and eddies on the mass of the substance. The condition of "isentropic" expansion is never perfectly attained in either case, although in a very short nozzle passage it may be closely approached. Since in practice, friction effect at the walls of the containing envelope, and eddies in the fluid cause a reconversion of kinetic energy back to heat, there is a gradual growth of entropy as expansion proceeds. The expansion is adiabatic, since no heat flows through the walls of the envelope from the surroundings, but it is not isentropic. It is usual to refer to this expansion, accompanied by friction effect, as "adiabatic and resisted."

49. Temperature Entropy Diagram.—It will be apparent from

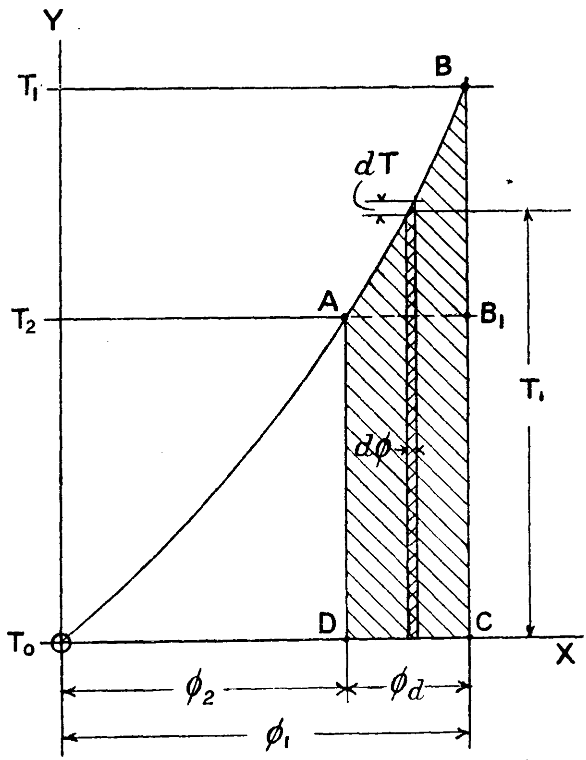


FIG. 52.

equation (1) that the area under any curve having absolute temperature and entropy as co-ordinates represents a quantity of heat.

Suppose that 1 lb. of a substance, whose volume keeps sensibly constant (say water) at a temperature T_2, has taken in H_2 units of heat reckoned from absolute zero of temperature, and that the absolute change of entropy is ϕ_2. The temperature entropy heating curve OA is shown in Fig. 52. The area OAD represents the heat H_2. Let a further quantity of heat, H, be added, with increase of temperature from T_2 to T_1, and entropy from ϕ_2 to ϕ_1.

The total amount of heat taken in by the substance is H_1, represented by the area OBC.

The added heat H is represented by the area DABC.

The change of entropy during this addition is $\phi_d = (\phi_1 - \phi_2)$. Consider the shaded element of area, representing the heat dh added during a change of temperature dT and entropy $d\phi$, the mean absolute temperature being T. If K is the specific heat of the substance, the energy equation, since $dh = KdT$, gives

$$KdT = Td\phi$$

and
$$d\phi = K\frac{dT}{T}$$

The whole change of entropy during the addition of H units is $\phi_d = (\phi_1 - \phi_2) = \int_{T_2}^{T_1} K\frac{dT}{T}$.

If the specific heat is constant, $(\phi_1 - \phi_2) = K \log_e \frac{T_1}{T_2}$.

It is assumed here that addition of heat between A and B causes a rise in temperature.

If there is no rise in temperature, the heating curve AB reduces to the isothermal AB_1, and the heat absorbed is represented by the area AB_1CD, $H = T_2\phi_d$, and change of entropy $(\phi_1 - \phi_2) = \frac{H}{T_2}$, where T_2 is the constant temperature of reception.

The same reasoning holds for abstraction of heat. In this case the entropy change ϕ_d has a negative value.

In steam turbine problems entropy changes have to be considered for (1) water during the heating process, (2) steam during evaporation, (3) steam during the process of superheat.

In all cases differences of entropy are dealt with, and an arbitrary zero of entropy can be selected to suit each particular problem.

The arbitrary zero chosen for steam is 32° F. or 492° F. abs. The entropy values of the liquid, given in the steam tables, are reckoned from this temperature.

Taking the case of steam, the temperature entropy curve for a pound of superheated steam at pressure p_1 is shown in Fig. 53. AB is the heating curve for the water starting with zero entropy at 32° F. or 492° F. abs.; BC is the isothermal during evaporation at the saturation temperature T_1; CF_1 is the heating curve during superheat between T_{s_1} and T_1.

The broken curve $ABCF_1$ is the curve of constant pressure p_1. In the illustration $p_1 = 300$ lbs./in.², $t_{s_1} = 300°$ F. The area under AB is the sensible heat h; under BC the latent heat L; under CF_1 the superheat h_s. The total heat is given by the area $OABCF_1G$.

The corresponding entropy changes are, for heating of water,

$\phi_w = \int_{492}^{T_1} K\frac{dT}{T}$; evaporation, $\phi_e = \frac{L_1}{T_1}$; superheat, $\phi_s = \int_{T_1}^{Ts_1} K_p\frac{dT}{T}$. The values of ϕ_ω and ϕ_e, the water and evaporation entropies, are given for each pressure in the steam tables. The superheat entropy for a given

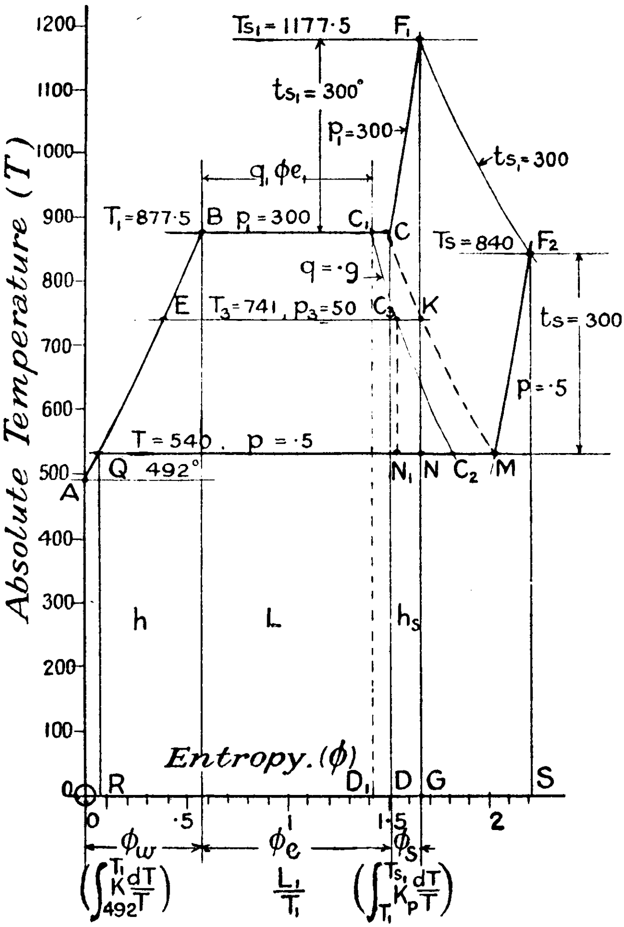


FIG. 53.

range of superheat can be fairly approximated by finding the corresponding value of the mean specific heat from the chart, Fig. 49, and calculating by the equation

$$\phi_s = K_p \log_e \frac{T_s}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where T_s and T are the absolute superheat and saturation temperatures.

50. Quality of Steam.—During the production of steam at constant pressure p_1, if the process of evaporation is stopped when the proportion q of water has been evaporated, the latent heat taken in will be qL. Hence in the Tϕ diagram, Fig. 53, the evaporation entropy at p_1 will be $BC_1 = \frac{q_1 L_1}{T_1} = q_1 \phi_{e_1}$, where $q_1 = \frac{BC_1}{BC}$.

The total heat of the wet steam is now given by the area $OABC_1D_1$. The point C_1 denoting the state or condition of the steam is called a "state point." In this case it represents the condition $q = 0{\cdot}9$.

The state point C is the saturation point, where $q = 1$, and the steam is dry. In this case the total heat of the dry steam is given by the area OABCD. For every pressure there is a curve similar to $ABCF_1$, having a corresponding saturation point. For example, at $p = 0{\cdot}5$, the curve is $OAQMF_2$, and the saturation point is M. The total heat is given by the area $OAQMF_2S$. The locus of the saturation points is called the "saturation curve," and is indicated by the dotted curve CM. It divides the co-ordinate field into two parts. That on the left is called the saturation field; that on the right the superheat field. If at a series of pressures the state points corresponding to a given quality (q or t_s) are located on the diagram, the corresponding dryness curve C_1C_2 or superheat curve F_1F_2 is obtained. When a set of these curves is drawn the quality at a given state point can be found by inspection. When the quality at a given pressure is known, then, as already seen from the examples in Chapter V, the total heats and volumes are readily calculable with the aid of the steam tables. The total heat in any case can, of course, be obtained by integrating the diagram under the pressure curve and multiplying the area by the heat scale. Calculation from the table, when once the quality is ascertained, is much the quicker and more accurate method.

51. Total Heat Entropy Diagram.—This serviceable diagram is obtained by replacing the temperature co-ordinate by total heat. A skeleton of this diagram with the same limiting pressures and quality curves as the Tϕ diagram, is shown in Fig. 54. This is only the upper portion of the diagram. At the lower end there is the water curve, as in the case of the Tϕ diagram, from which the straight constant pressure lines NM and C_1C branch out to the saturation curve CM. The saturation field is on the left, and the superheat field on the right of the saturation curve. In the superheat field, the pressure lines change to flat curves, CF_1, MF_2, but there is no abrupt change of direction, as in the case of the Tϕ diagram.

The curves of this diagram are plotted as follows: Take, for example, the curve for $p_1 = 300$. Referring to the tables the total entropy $(\phi_w + \phi_e) = 1{\cdot}5129$. The total heat is $H = 1204$. Projecting from the entropy and heat scales, the intersection C gives the saturation point at this pressure.

Find next the state point at a given superheat range, say $t_{s_1} = 300$. Calculate the superheat entropy change, ϕ_s. Then the total entropy is

$(\phi_\omega + \phi_e + \phi_s) = 1{\cdot}6765$. Calculate the total heat $H_s = H_1 + h_s = 1369$. The intersection of these two values gives the state point F_1. This is a common point in the constant pressure curve $p_1 = 300$ and the constant superheat curve F_1F_2 for $t_{s_1} = 300$.

Again, take some value of dryness, say $q = 0{\cdot}9$, and find the state

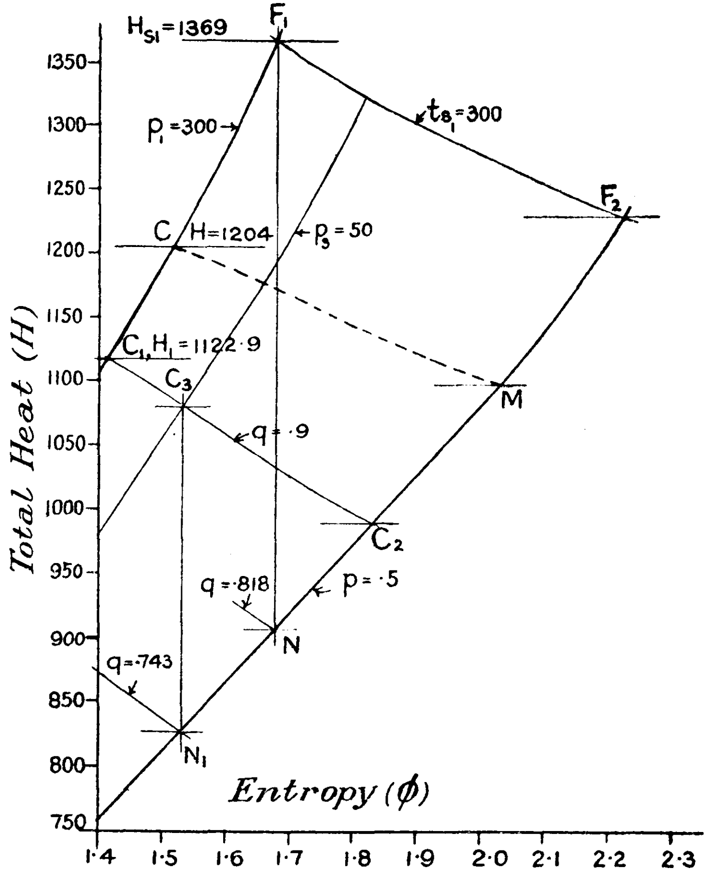


FIG. 54.

point. The total entropy is $\phi_\omega + q\phi_e = 1{\cdot}4204$, and total heat $H = h + qL = 1122{\cdot}9$.

The intersection C_1 of the projectors is a common point on the curve $p_1 = 300$, and the dryness curve C_1C_2 for $q = 0{\cdot}9$.

Any number of points can thus be located. In this way the combined set of pressure and quality curves, shown in the complete diagram, Fig. 55, is obtained. The heat and entropy values in the superheat field have been taken direct from Marks and Davis' table of superheated steam.

A copy of this diagram drawn to a scale that ensures results sufficiently accurate for practical purposes, is provided in the pocket at the end of the book. It is used in the majority of subsequent calculations relating to compound turbines.

52. Adiabatic Expansion.—When steam expands behind a piston in a non-conducting cylinder, and does work against an external resistance, or in a turbine nozzle (without friction or eddy disturbance), and does work in generating kinetic energy, the expansion is adiabatic. At all intermediate pressures between the initial and the final, the total entropy of the steam remains constant. Hence it follows that on either the $T\phi$ or $H\phi$ diagram the expansion curve is represented by a vertical line, drawn between the initial and final constant pressure curves.

If the state point at the initial pressure is known, the state point and corresponding quality of the steam at the final pressure are at once determined, by dropping a perpendicular to cut the final pressure curve.

The total heats of the steam before and after adiabatic expansion thus become determinate.

When the $T\phi$ diagram is used, these values have to be calculated for the given qualities. On the other hand, by using the $H\phi$ diagram they can be read directly on the heat scale of the diagram.

The figure of importance in all turbine calculations is the difference of these initial and final total heats. On the $H\phi$ diagram it is not necessary to read these, but simply to scale the vertical intercept between the curves in inches, and multiply it by the heat scale of the diagram.

This quantity representing the difference of total heats after adiabatic expansion is called the "heat drop." In subsequent discussions and calculations it is denoted either by h_r or H_r. The subscript r refers to the ideal turbine cycle or Rankine cycle, discussed in Art. 54.

The following examples show the practical application of these diagrams to the calculation of the qualities and heat drops :—

EXAMPLE 1.—Steam at $p_1 = 300$, and $t_{s_1} = 300°$ F. is expanded adiabatically to $p = 0{\cdot}5$. Find the quality at the lower pressure and the heat drop, (*a*) from the $T\phi$ diagram, (*b*) from the $H\phi$ diagram.

(*a*) Referring to Fig. 53, the state point at p_1 is F_1. Draw the vertical F_1N to cut the lower pressure curve p in N. The steam at p is wet, and the dryness is $q = \dfrac{QN}{QM} = 0{\cdot}818$.

From the chart, Fig. 49, $K_p = 0{\cdot}558$, $t^s = 300$, $h_s = 300 \times 0{\cdot}558 = 167{\cdot}4$.

$$\therefore\ H_{s_1} = H_1 + K_p t_s = 1204 + 167{\cdot}4 = 1371{\cdot}4 \text{ B.Th.U.}$$
$$H = h + qL,\quad h = 47{\cdot}7,\quad L = 1046{\cdot}8,\quad q = 0{\cdot}818$$
$$= 47{\cdot}7 + 0{\cdot}818 \times 1046{\cdot}8$$
$$= 47{\cdot}7 + 856{\cdot}280 = 903{\cdot}980 \text{ B.Th.U.}$$

Heat drop $H^r = (H_{s_1} - H) = (1371{\cdot}4 - 903{\cdot}980) = 467{\cdot}5$ B.Th.U.

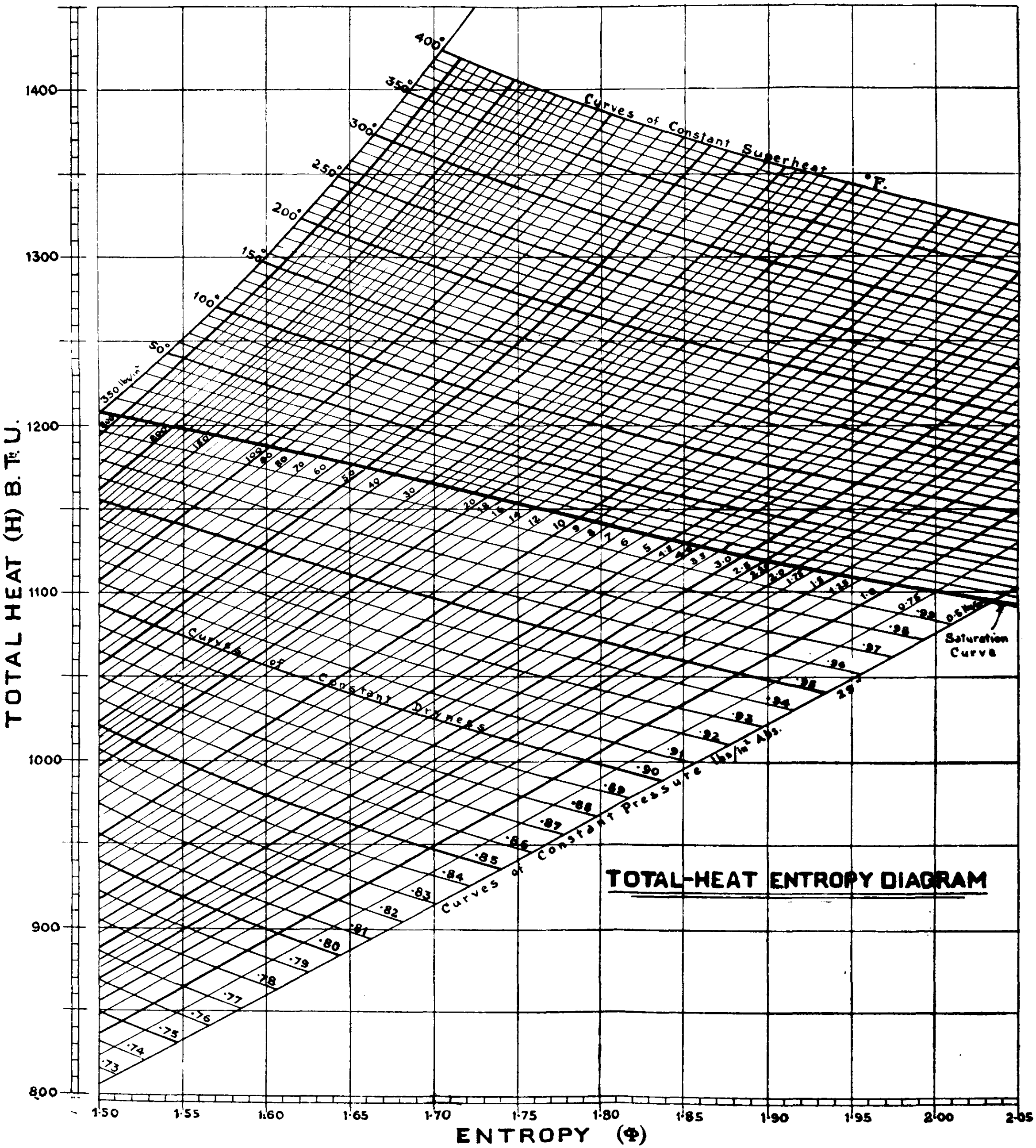

TOTAL HEAT (H) B. Th. U.
ENTROPY (Φ)
TOTAL-HEAT ENTROPY DIAGRAM
Curves of Constant Superheat °F.
Curves of Constant Dryness
Curves of Constant Pressure lbs/in² Abs.
Saturation Curve
1400
1300
1200
1100
1000
900
800
1·50
1·55
1·60
1·65
1·70
1·75
1·80
1·85
1·90
1·95
2·00
2·05

FIG. 55.

(*b*) Referring to Fig. 54, draw from the initial state point F_1 the vertical F_1N cutting p in N. It will be found that N falls at $q = 0{\cdot}818$. The intercept F_1N scales 9·34 inches.[1] The heat scale is 1 inch = 50 B.Th.U., hence the heat drop $H_r = 50 \times 9{\cdot}34 = 467$ B.Th.U., as against 467·5 by calculation from the tables. To guard against any error due to distorsion of the diagram in printing, the intercept F_1N should either be applied to the heat scale, or projectors should be drawn from F_1 and N to the scale and values of H_{s_1} and H determined. With the size of diagram employed here the distorsion error, however, is negligible, and the quicker method may be adopted.

The difference between the results obtained from the two diagrams is about 0·11 per cent., a negligible amount.

The upper limit of pressure chosen in this example is greater than is used in any present-day turbine, and the case also covers the extreme range of superheat.

EXAMPLE 2.—Steam at $p_3 = 50$ and $q_3 = 0{\cdot}9$ is expanded adiabatically to $p = 0{\cdot}5$. Find the quality after expansion and the heat drop, (*a*) from the $T\phi$, (*b*) from the $H\phi$ diagram.

The procedure is exactly the same as in the previous case.

(*a*) On Fig. 53 find the initial state point C_3 by scaling $EC_3 = 0{\cdot}9 \times EK$, and drop the perpendicular C_3N_1 to cut the pressure curve $p = 0{\cdot}5$ in N_1. Then $q = \dfrac{QN_1}{QM} = 0{\cdot}743$.

$$
\begin{aligned}
H_3 &= h_3 + q_3L_3, \quad h_3 = 250{\cdot}1, \quad L_3 = 923{\cdot}5, \quad q_3 = 0{\cdot}9 \\
&= 250{\cdot}1 + 0{\cdot}9 \times 923{\cdot}5 \\
&= 250{\cdot}1 + 831{\cdot}20 = 1081{\cdot}3 \text{ B.Th.U.} \\
H &= h + qL, \quad h = 47{\cdot}7, \quad L = 1046{\cdot}8, \quad q = 0{\cdot}743 \\
&= 47{\cdot}7 + 0{\cdot}743 \times 1046{\cdot}8 \\
&= 47{\cdot}7 + 777{\cdot}77 = 825{\cdot}4 \text{ B.Th.U.}
\end{aligned}
$$

$\therefore$ Heat drop $H_r = (H_3 - H) = 1081{\cdot}3 - 825{\cdot}4 = 255{\cdot}9$ B.Th.U.

(*b*) On the $H\phi$ diagram, Fig. 54, the initial state point at $p_3 = 50$ and $q_3 = 0{\cdot}9$ is C_3. Drop the perpendicular C_3N_1 to cut the curve $p = 0{\cdot}5$ in N_1. This point falls on the quality curve, $q = 0{\cdot}743$.

The intercept C_3N_1 scales 5·12 inches. $\therefore$ the heat drop $H_r = 256$ B.Th.U., which coincides with the previous value from the $T\phi$ diagram and the tables.

These two examples are sufficient to indicate the superiority of the $H\phi$ over the $T\phi$ diagram for this class of calculation.

53. For the majority of problems the qualities and heat drops can be determined directly from the $H\phi$ diagram in the manner just explained. In some cases where the pressure drops are small it is difficult to obtain reliable values, and it is advisable, under such circumstances, to check the results by direct calculation from the steam tables.

The method of determining the quality by calculation is as follows: Let the expansion take place from p_1 at t_{s_1} through a small range of

[1] This figure is obtained from the large diagram in the pocket of the book.

pressure. The quality at the lower pressure p depends on the initial superheat t_{s_1} and the drop in pressure. If the former is considerable the steam will still be superheated an amount t_s at the lower pressure. Taking this case and referring to the $T\phi$ diagram, Fig. 56, F_1 and F are the initial and final state points, T_{s_1} and T_s, the temperatures of superheat corresponding to the ranges t_{s_1} and t_s. With adiabatic expansion the total entropy remains constant; hence, denoting the entropy values, as shown, by the corresponding subscripts, and equating the sum of the values at p_1 to the sum at p,

$$\phi_{\omega_1} + \phi_{e_1} + \phi_{s_1} = \phi_\omega + \phi_e + \phi_s,$$

hence
$$\phi_s = (\phi_{\omega_1} + \phi_{e_1} + \phi_{s_1}) - (\phi_\omega + \phi_e),$$
$$= \phi_1 + \phi_{s_1} - \phi \quad \ldots \ldots \ldots \quad (3)$$

where ϕ_1 and ϕ are the total entropy values for dry steam, which can be read directly from the tables. The entropy of superheat at p_1 by equation (2) is $\phi_{s_1} = K_{p_1} \log_e \frac{T_{s_1}}{T_1}$. Similarly at p, $\phi_s = K_p \log_e \frac{T_s}{T}$.

The object of the calculation is to find T_s, and hence the lower superheat range t_s. As already shown the value of K_p is dependent on the range. The value of the pressure drop is, however, small, as an approximation it may be assumed that $K_{p_1} \doteqdot K_p$. T_s and t_s can then be calculated, and the heat drop determined. The latter is given by the equation

$$H_r = (H_{s_1} - H_s) = (H_1 + K_{p_1} t_{s_1}) - (H + K_p t_s) \quad . \quad . \quad (4)$$

It is represented by the area $DBCF_1FE$.

EXAMPLE 3.—Steam expands adiabatically from $p_1 = 180$ and $t_{s_1} = 150°$ F. to $p = 175$. Find the quality at the lower pressure.

Referring to the chart, Fig. 49, $K_p = 0·564$. By equation (2)

$$\phi_{s_1} = 0·564 \log_e \frac{T_{s_1}}{T_1}, \quad T_1 = 833, \quad T_{s_1} = 983$$
$$= 0·564 \log_e 1·18$$
$$= 0·564 \times 0·1655 = 0·09334. \quad \text{From the table } \begin{array}{l}\phi_1 = 1·5543 \\ \phi \;= 1·5567\end{array}$$

By equation (3), $\phi_s = (\phi_1 + \phi_{s_1} - \phi)$
$$= 1·5543 + 0·09334 - 1·5567$$
$$= 0·0909.$$

By equation (2), $\log_e \frac{T_s}{T} = \frac{\phi_s}{K_p} = \frac{0·0909}{0·564} = 0·1611$

$$\therefore \frac{T_s}{T} = 1·175, \quad \text{also } T = 830·9$$

$\therefore$ $T_s = 1·175 \times 830·9 = 976·2$ and $t_s = (976·2 - 830·9) = 145·3°$ F.

A reference to the chart, Fig. 49, shows that the value of K_p should be slightly greater than K_{p_1}, 0·566 instead of 0·564. This is a close enough approximation.

By equation (4) the heat drop is

$$\begin{aligned} H_r &= (H_1 + K_{p_1} t_{s_1}) - (H + K_p t_s) \\ &= (1196{\cdot}4 + 84{\cdot}6) - (1195{\cdot}9 + 81{\cdot}95) \\ &= 3{\cdot}15 \text{ B.Th.U.} \end{aligned}$$

Even with a drop as small as 5 lbs. in the high-pressure region of the diagram, the value obtained by this tedious calculation can be fairly estimated on the diagram. At the lower-pressure region, for such a drop, it can be read with greater accuracy since the spacing of the curves widens as the pressures decrease. Small drops like this are usually associated with multi-stage turbines, and, as a rule, do not require to be estimated in a step by step process.

When the steam expands to some pressure p_0, Fig. 56, and becomes wet, this condition is indicated by the negative sign of ϕ_{s_0} in equation (3). This equation reduces to

$$\phi_1 + \phi_{s_1} = \phi_{\omega_0} + q_0\phi_{e_0}$$

and the final quality is

$$q_0 = \frac{\phi_1 + \phi_{s_1} - \phi_{\omega_0}}{\phi_{e_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The heat drop is given by

$$H_r = (H_1 + K_{p_1} t_{s_1}) - (h_0 + q_0 L_0) \quad . \quad . \quad . \quad (6)$$

It is represented by the area $QBCF_1N$.

EXAMPLE 4.—Steam expands adiabatically from $p_1 = 150$ and $t_{s_1} = 200$ F. to $P_0 = 14$. Calculate the dryness at the lower pressure, and the heat drop.

From sp. ht. chart, $K_{p_1} = 0{\cdot}542$, $T_1 = 818{\cdot}5$, $T_{s_1} = 1018{\cdot}5$.

$$\begin{aligned} \phi_{s_1} &= 0{\cdot}542 \log_e \frac{1018{\cdot}5}{818{\cdot}5} \\ &= 0{\cdot}542 \log_e 1{\cdot}244 & \phi_1 &= 1{\cdot}5692 \\ &= 0{\cdot}542 \times 0{\cdot}2183 = 0{\cdot}1183 & \phi_{\omega_0} &= 0{\cdot}3081, \quad \phi_{e_0} = 1{\cdot}4523 \end{aligned}$$

$$q_0 = \frac{\phi_1 + \phi_{s_1} - \phi_{\omega_0}}{\phi_{e_0}} = \frac{1{\cdot}5692 + 0{\cdot}1183 - 0{\cdot}3081}{1{\cdot}4523} = \frac{1{\cdot}3794}{1{\cdot}4523} = 0{\cdot}95$$

This checks exactly with the value given by the $H\phi$ diagram.

$$h_0 = 177{\cdot}5, \quad L_0 = 971{\cdot}9, \quad q_0 = 0{\cdot}95, \quad H_1 = 1193{\cdot}4.$$

By equation (6),

$$\begin{aligned} H_r &= (H_1 + K_{p_1} t_{s_1}) - (h_0 + q_0 L_0) \\ &= (1193{\cdot}4 + 0{\cdot}542 \times 200) - (177{\cdot}5 + 0{\cdot}95 \times 971{\cdot}9) \\ &= 1301{\cdot}8 - 1100{\cdot}8 = 201 \text{ B.Th.U.} \end{aligned}$$

The value given by the $H\phi$ chart is 200 B.Th.U.

When the steam is initially wet with dryness fraction q_1 at p_1, equation (5) reduces to

$$q_0 = \frac{\phi_{\omega_1} + q_1\phi_{e_1} - \phi_{\omega_0}}{\phi_{e_0}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The heat drop is given by $H_r = (h_1 + q_1 L_1) - (h_0 + q_0 L_0)$ (8)

EXAMPLE 5.—Steam at $p_1 = 100$ and $q_1 = 0{\cdot}98$ is adiabatically expanded to $p_0 = 1$. Calculate the quality at the lower pressure, and the heat drop.

From the tables $h_1 = 298{\cdot}3,\ L_1 = 888,\ \phi_{\omega_1} = 0{\cdot}4743,\ \phi_{e_1} = 1{\cdot}1277$
$h_0 = 69{\cdot}8,\ L_0 = 1034{\cdot}6,\ \phi_{\omega_0} = 0{\cdot}1327,\ \phi_{e_0} = 1{\cdot}8427$

By equation (7)

$$q_0 = \frac{\phi_{\omega_1} + q_1\phi_{e_1} - \phi_{\omega_0}}{\phi_{e_0}} = \frac{0{\cdot}4743 + 0{\cdot}98 \times 1{\cdot}1277 - 0{\cdot}1327}{1{\cdot}8427}$$
$$= \frac{1{\cdot}4467}{1{\cdot}8427} = 0{\cdot}785$$

This checks with the value from the $H\phi$ diagram.

By equation (8) the heat drop is

$$H_r = (298{\cdot}3 + 0{\cdot}98 \times 888) - (69{\cdot}8 + 0{\cdot}785 \times 1034{\cdot}6)$$
$$= 1168{\cdot}54 - 881{\cdot}96 = 286{\cdot}58 \text{ B.Th.U.}$$

The value given by the $H\phi$ diagram is 286·5.

54. Cycle of the Steam Turbine.—A heat engine is a machine which converts heat into mechanical energy. The transformation is accomplished by means of a working substance, which is usually a fluid, either wholly or partly gaseous.

The working substance goes through a recurring series of changes called the "cycle." It starts with a given initial condition, and at the end of the cycle is brought back to this condition.

During the cycle, there is (1) heat supply from some source; (2) conversion of heat to work; (3) rejection of heat to a refrigerator or cooler.

In the case of the turbine engine, what is generally called the turbine, viz. the casing and rotor, is only the working chamber of the complete heat engine, in which transformation of heat energy to mechanical work takes place.

The complete turbine consists of the boiler, working chamber, and condenser.

The furnace of the boiler is the source of heat, and the condenser is the refrigerator.

The three elements, together with a feed pump, constitute a closed chain.

The working substance, in the form of water, at the exhaust or condenser temperature is pumped into the boiler, and converted to steam at constant pressure. The steam meantime is flowing steadily at this pressure through the connecting passage to the working chamber. In the working chamber a certain proportion of the heat supplied from the source is converted into mechanical work on the mobile part or rotor. The remainder is abstracted from the steam in the condenser, and the substance is brought back again to the condition of water at the original temperature. The water is again pumped to the boiler, and the operation is repeated.

The conversion of heat to work in the mobile part is accomplished by decrease of pressure and increase of volume of the steam. In the working chamber the pressure falls from some initial value p_1 to an exhaust value p_0, and the expansion is adiabatic.

Referring to the Tϕ diagram, Fig. 56, if superheated steam at p_1 is used, the heat supplied by the source per lb. of substance is the total heat, ${}_1H_s$, reckoned from the temperature T_0 at exhaust pressure p_0. This is obviously given by the area $RQBCF_1G$.

The heat rejected to the condenser, at constant pressure p_0 and temperature T_0, is the total heat ${}_0H$ at p_0, reckoned from T_0. This is given by the area RQNG.

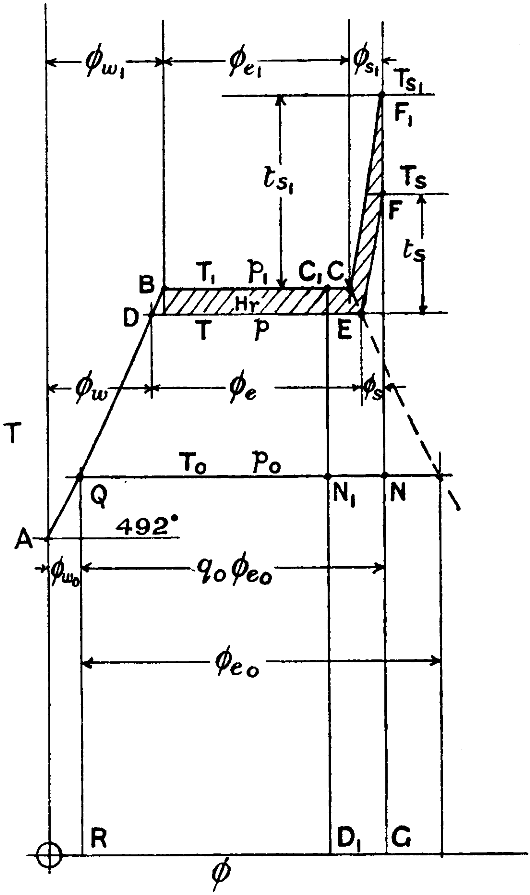


FIG. 56.

The difference $({}_1H_s - {}_0H)$ is given by the area $QBCF_1N$. This is also given by the difference of the total heats at p_1 and p_0 reckoned from 492° F, since the area OAQR is common to both quantities, and cancels out when the difference is taken.

That is, $({}_1H_s - {}_0H) = (H_{s_1} - H_0) = H_r =$ heat drop.

Similarly, if wet steam is supplied at p_1, and quality denoted by C_1, the corresponding heat converted to work is

$$(H_1 - H_0) = H_r = QBC_1N_1 = (RQBC_1D_1 - RQN_1D_1)$$

The "heat drop," calculated by any of the previous methods, between the initial and exhaust pressures, thus gives the amount of heat available for conversion into mechanical work in an ideal turbine. In an actual machine, only a portion of this heat drop can be used.

During the absorption of the sensible heat by the water its volume

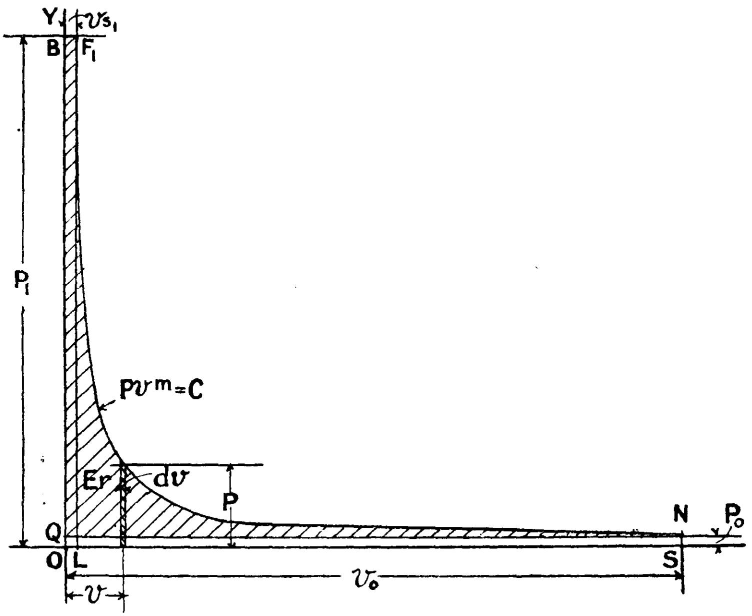


FIG. 57.

does not sensibly change, so that the $T\phi$ curve AB is a curve of heating at constant volume.

The diagram for the ideal turbine cycle is thus bounded by a constant volume, an adiabatic and two constant pressure curves.

The equivalent pressure-volume diagram is shown in Fig. 57. The pressures are expressed in lbs./ft.^{2}. QB is the curve of heating of the water at constant volume. This volume, as already stated in Art. 44, is so small that it may be neglected, and the heating curve is made to coincide with the axis OY. BF_1 is the curve of steam formation at constant pressure. F_1N is the curve of adiabatic expansion between P_1 and P_0, and NQ the constant-pressure cooling or heat-rejection curve at P_0. The area QBF_1N in mechanical units is equivalent to the heat drop area of the $T\phi$ diagram in heat units.

This cycle was taken by Rankine as a standard of comparison for steam-engine performances, and is always called the Rankine cycle.

The heat drop of a turbine is thus the heat available in the Rankine cycle engine, and may be termed the Rankine cycle heat. Denoting its mechanical equivalent by E_r, then $H_r = \frac{E_r}{J}$.

55. Taking the general case with superheated steam, the total heats H_{s_1} and H_0 by equation (7), Art. 44, are

$$H_{s_1} = I_1 + \frac{P_1 v_{s_1}}{J} \text{ and } H_0 = I_0 + \frac{P_0 v_0}{J}$$

where I_1 and I_0 are the corresponding intrinsic energies.

Hence the heat drop $$H_r = (I_1 - I_0) + \left(\frac{P_1 v_{s_1} - P_0 v_0}{J}\right) \quad . \quad . \quad (9)$$

or the work of the Rankine engine is

$$E_r = J(I_1 - I_0) + (P_1 v_{s_1} - P_0 v_0) \quad . \quad . \quad . \quad . \quad (10)$$

The adiabatic curve of expansion is assumed to follow the law Pv^m = constant, where m is a constant index. This latter assumption is not true since at some intermediate pressure the steam passes from the superheated to the wet condition, and the value of m decreases. Assuming this as an approximation to the actual condition, the value of the difference of intrinsic energy $I_1 - I_0$, in terms of the pressure and volume, can be easily deduced. Consider the area under the expansion curve F_1N. The initial condition is P_1 and v_{s_1}, and the final condition P_0 and v_0.

Area of an element dE at pressure P and volume v is Pdv. The whole area under the curve F_1N is

$$E = \int_{v_{s_1}}^{v_0} Pdv. \quad \text{But } P_1 v_{s_1}{}^m = P_0 v_0{}^m$$

hence $P = \frac{P_1 v_{s_1}{}^m}{v^m}$, and $E = P_1 v_{s_1}{}^m \int_{v_{s_1}}^{v} \frac{dv}{v^m} = \frac{P_1 v_{s_1}{}^m}{1 - m}\left(v_0^{1-m} - v_{s_1}^{1-m}\right)$

v_0

$P_0 v_0{}^m = P_1 v_{s_1}{}^m$. Substituting for $P_1 v_{s_1}{}^m$, $E = \frac{P_1 v_{s_1} - P_0 v_0}{m - 1}$

This is represented by the area LF_1NS.

The whole area OBF_1NS is given by $P_1 v_{s_1} + \frac{P_1 v_{s_1} - P_0 v_0}{m - 1}$

The area of the Rankine engine diagram is obtained by subtracting the area QNSO $= P_0 v_0$.

Hence $$QBF_1N = E_r = P_1 v_{s_1} + \frac{P_1 v_{s_1} - P_0 v_0}{m - 1} - P_0 v_0 \quad . \quad . \quad (11)$$

and $$J(I_1 - I_0) = \frac{P_1 v_{s_1} - P_0 v_0}{m - 1}$$

The work of the Rankine engine from (10) can be expressed either in the form

$$E_r = \frac{m}{m-1}(P_1 v_{s_1} - P_0 v_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

or

$$E_r = \frac{m}{m-1} P_1 v_{s_1} \left\{ 1 - \left(\frac{P_0}{P_1} \right)^{\frac{m-1}{m}} \right\} \quad . \quad . \quad . \quad (13)$$

the pressures being in lbs./ft.², volumes in ft.³, and work in ft.-lbs.

When saturated steam is used the dry steam volume v_1 replaces the superheat volume v_{s_1}.

In the Rankine engine cylinder this work is done directly on the piston. In the turbine it first appears as kinetic energy. In the simple turbine the whole heat drop is converted into kinetic energy at once. In the compound turbine the total heat drop is subdivided over a number of small drops. The work done in any stage of the compound turbine is the equivalent of the heat drop h_r between the pressure limits of the stage.

When expansion takes place in any stage between p_1 and p there is an increase V in velocity of the steam, and the Rankine engine work appears as the kinetic energy $\frac{V^2}{2g}$. This energy, in the ideal case, would be completely converted into work on the turbine blading.

Complete conversion is not possible in the actual machine. The reasons are discussed in subsequent chapters.

For any given stage, since

$$E_r = Jh_r = \frac{V^2}{2g}, \quad 2g = 64{\cdot}4, \text{ and } J = 778$$

$$h_r = \frac{V^2}{2gJ} = \frac{V^2}{(223{\cdot}7)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and the increase of velocity, due to the stage heat drop, h_r, is

$$V = 223{\cdot}7\sqrt{h_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

If the steam has an initial velocity V_1 at p_1, and final velocity V at p, the increase in kinetic energy is then $\frac{V^2 - V_1^2}{2g}$ and the stage heat drop

$$h_r = \frac{V^2 - V_1^2}{(223{\cdot}7)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

In the *ideal* compound turbine the sum of the stage heat drops Σh_r, which for the whole machine may be called the "cumulative" heat, is the same as the heat drop between the initial and exhaust pressures p_1 and p_0.

In an *actual* compound turbine the cumulative heat is always *greater* than the total heat drop.

The reason for this curious result, and the modifications in the turbine calculations which it causes, are discussed in Chapter XI.

EXAMPLE 6.—Steam expands from $p_1 = 200$ and $t_{s_1} = 200$ to $p_0 = 100$. Calculate the work done in the Rankine cycle, (*a*) from the pressures and volumes, (*b*) by direct measurement from the $H\phi$ diagram.

On the $H\phi$ diagram the quality at $p_0 = 100$ is $t_{s_0} = 97°$ F. The volume can be calculated by equation (7) or obtained from the alignment chart.

Its value is $v_{s_0} = 5{\cdot}12$. Similarly the volume at p_1 is $v_{s_1} = 3{\cdot}023$.
The law of expansion is $pv^m = c$, or $p_1 v_{s_1}{}^m = p_0 v_{s_0}{}^m$

$$\therefore\ m = \frac{\log p_1 - \log p_0}{\log v_{s_0} - \log v_{s_1}} = \frac{2{\cdot}3010 - 2}{0{\cdot}7093 - 0{\cdot}4804} = \frac{0{\cdot}3010}{0{\cdot}2289} = 1{\cdot}314$$

This figure is greater than the index usually taken for superheated steam, which is 1·3.

(*a*) $P_1 = 144 \times 200$, $P_0 = 144 \times 100$, hence by equation (12),

$$E_r = \frac{m}{m-1}(P_1 v_{s_1} - P_0 v_{s_0})$$

$$= \frac{1{\cdot}314 \times 144(200 \times 3{\cdot}023 - 100 \times 5{\cdot}12)}{0{\cdot}314}$$

$$= 55800 \text{ ft.-lbs.}$$

(*b*) From the $H\phi$ diagram the heat drop is 70 B.Th.U.

$$\therefore\ E_r = JH_r = 778 \times 70 = 54460 \text{ ft.-lbs.}$$

The first figure is 2·4 per cent. greater than the heat drop equivalent. A slight variation in the readings of the volumes has a considerable effect on the value of *m*.

EXAMPLE 7.—In the previous example if the steam is expanded in a nozzle passage, and friction effect is neglected, calculate the theoretical velocity of outflow from the nozzle, (*a*) when the initial velocity is zero, (*b*) when it is 300 ft./sec.

(*a*) Heat drop $H_r = 70$. $\therefore$ by equation (15),

$$V = 223{\cdot}7\sqrt{70} = 1872 \text{ ft./sec.}$$

(*b*) By equation (16), $h_r = \dfrac{V^2 - V_1^2}{(223{\cdot}7)^2}$, $V_1 = 300$

$$\therefore\ \frac{V^2}{(223{\cdot}7)^2} = h_r + \frac{V_1^2}{(223{\cdot}7)^2} = 70 + \left(\frac{300}{223{\cdot}7}\right)^2 = 70 + 1{\cdot}8 = 71{\cdot}8 \text{ B.Th.U.}$$

$$\therefore\ V = 223{\cdot}7\sqrt{71{\cdot}8} = 1895 \text{ ft./sec.}$$

The actual velocities would be less than these figures on account of friction effects in the nozzle passage.

CHAPTER VII

NOZZLES

56. Commercial Types.—The nozzle used on the simple impulse turbine is usually of the "straight" reamered convergent-divergent type. It is easily and cheaply made, as it can be machined to the required form without any hand filing. The arrangement of this type with shut-off valve, as fitted on the de Laval machine, is shown in Fig. 58. The convergent portion, 1, is simply a well-rounded entrance. Into this the coned end of the screwed spindle, 2, fits, when the steam is shut off. The

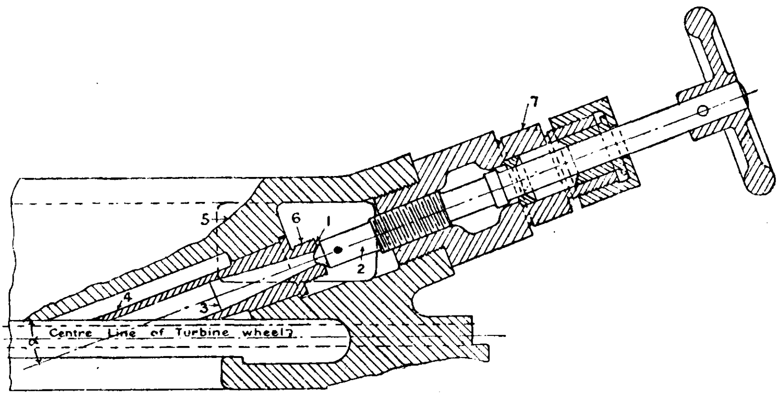


FIG. 58.

nozzle has a "straight" taper between the throat and the exit section, 3. The nozzle axis is inclined at the jet angle α to the plane of the wheel. On this account the part, 4, furthest from the wheel has to be carried forward to the wheel face. This portion is made cylindrical, with the same diameter as the exit section, 3. Some designers prefer to continue the taper right down to the wheel.

The nozzle is simply forced into the hole in the casing wall, 5. The upper part, 6, is reduced, and screwed to take a withdrawing tool. This can be inserted in the casing after the valve casting, 7, and spindle are removed.

As already mentioned in Art. 15, the group arrangement of nozzles is used for the larger sizes of de Laval turbines. A section through a valve box and nozzle plate is shown in Fig. 59. This is a conventional section showing the development of only one of the two or three nozzles supplied with steam from the belt, 1, by the valve, 2. The nozzle plate 3, is made of bronze.

The de Laval turbine is also used as an exhaust turbine, and a

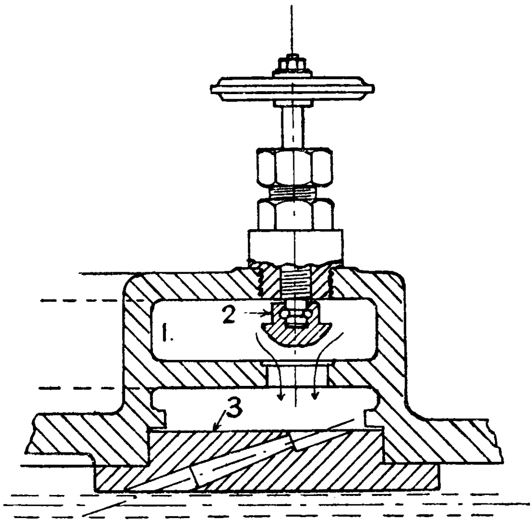


FIG. 59.

complete ring of these reamered nozzles, giving approximately full peripheral admission, is fitted.

Nozzle plates with straight reamered nozzles are also fitted on other types of turbines.

The inlet side of one of these plates used for the H.P. stage of a horizontal Curtis turbine is shown at Fig. 60, Plate IX. There are twelve nozzles arranged in pairs, the supply of steam to each pair being regulated by a drop valve under control of the governor. The exit side of the plate when bolted in position on the inside of the H.P. cover is shown in Fig. 61, Plate IX. The elliptical form of each nozzle exit is clearly indicated.

In the majority of cases the rectangular-shaped nozzle passage is used in compound turbines. A developed section through several of these passages is shown in Fig. 84. The inlet is bowl-shaped, as in the foregoing case, but the section changes to the rectangular form at the throat 2. The necessary divergence is obtained usually by tapering the passage in the circumferential direction between 2 and 3, the radial dimension being kept constant. Frequently the passage as a whole is given an inclination towards the axis of the shaft, so that the discharge of the jet is not axial. This is done to obtain a more effective tangential component as the steam flows through the wheel blade channels. An illustration of this arrangement is afforded by the Brush combination turbine shown in Fig. 43.

A somewhat special form of rectangular nozzle is fitted in the Franco Tosi combination marine turbine, Fig. 48. In this case the nozzle passage is formed of sheet metal expanded at the head into nozzle block 2. These projecting sheet-metal passages are packed close together at exit and can discharge a practically continuous band of steam on to the wheel blades. It will be noted that the passages have a considerable inclination towards the centre of the turbine.

An illustration of a cast nozzle plate for nearly complete peripheral admission is shown in Fig. 62, Plate IX. It is bolted to an intermediate stage diaphragm of a Curtis turbine. There are six groups of nozzles, with seven nozzles per group.

The exit side of the last diaphragm of this turbine is shown in Fig. 63, Plate IX. In this case the nozzle passages are formed by casting in steel plates in the walls of the diaphragm.

The type of nozzle plate fitted on the Brush disc and drum turbines is shown in Fig. 64, Plate IX. An arc of any length can be obtained by bolting on several of these plates in succession. Normally two are sufficient.

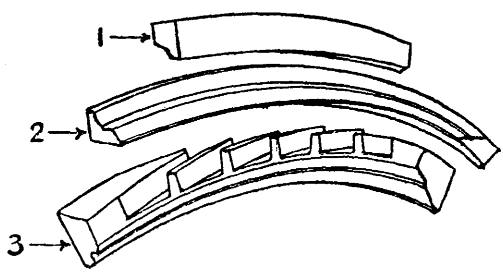


FIG. 65.

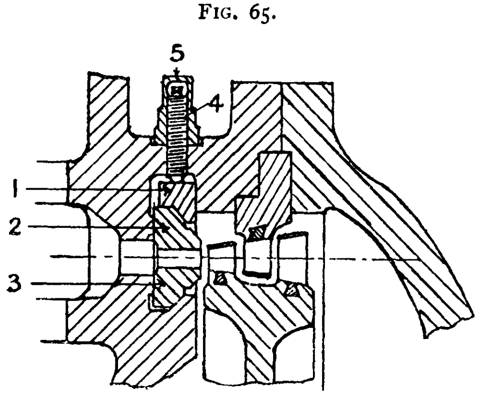


FIG. 66.

The type of nozzle passage used in the Brown Boveri disc and drum machines is also rectangular, but of a different construction to the foregoing. The solid cast nozzle plate is replaced by a built-up arrangement consisting of three machined strips. These are shown separately in Fig. 65, and in position in Fig. 66. The nozzle passages are cut in the strip 3, which forms the bottom half of the nozzle ring. The top half consists of a cover strip, 2. This is held down by a clamping strip, 1. The combination is held together by the set screw 4, locked and protected by the cap nut 5.

In the pressure compounded impulse turbine or the velocity com-

PLATE IX.

FIG. 60.

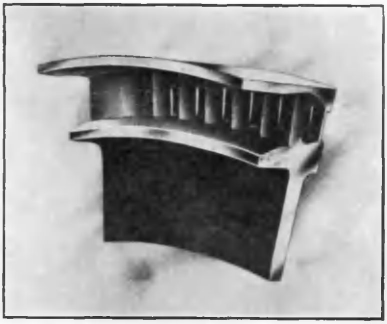


FIG. 68.

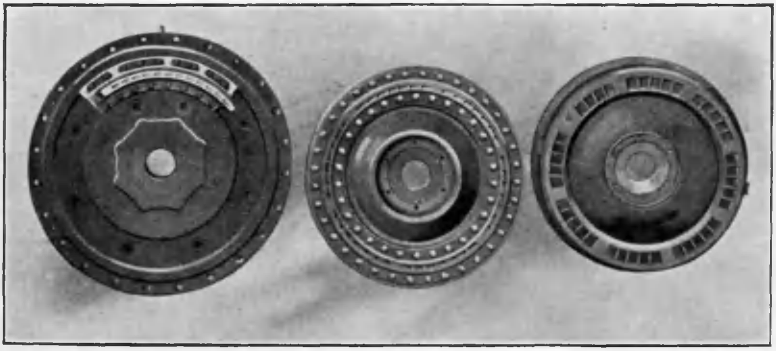


FIG. 61. FIG. 62. FIG. 63.

FIG. 64.

Nozzles and Diaphragms.

pounded one, where the stage heat drop is less than the critical value, the convergent nozzle passages are usually formed by casting steel plates in the rim of the diaphragm wall.

A sectional elevation and a plan of the arrangement used on the Zoelly turbine are shown in Fig. 67. The diaphragm at 1 is widened and provided with a bell-mouthed opening. This is subdivided along the arc into nozzle passages by the steel plates 2, which are parallel towards the exit end. Each diaphragm has a projecting flange, 3. The face of one diaphragm butts against the flange of the diaphragm in front of it. A perspective view of a portion of a nozzle arc is shown in Fig. 68, Plate IX.

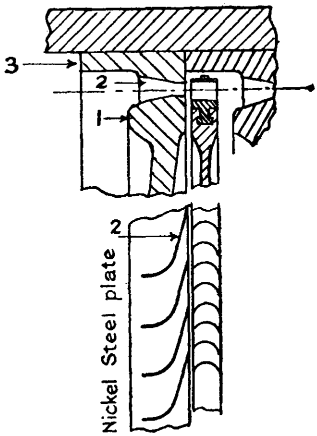


FIG. 67.

Two illustrations of the complete diaphragm with cast-in steel blades, as used in the Westinghouse Rateau machine, are shown in Figs. 69 and 70, Plate X. The diaphragms are made of cast iron and divided by a horizontal joint, so that the upper half can be lifted with the top half of the casing.

The diaphragm, Fig. 69, is for a stage with partial admission, the total nozzle arc being divided into two sectors, one top and one bottom.

The diaphragm, Fig. 70, is for an L.P. stage, where full peripheral admission is necessary. It will be seen in this case that the nozzles form a complete circle. In some impulse machines the horizontal

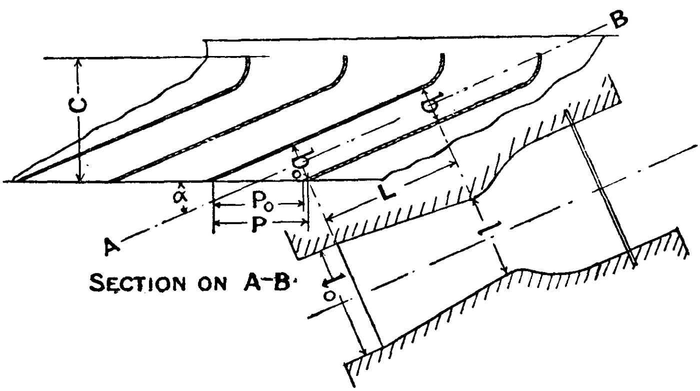


FIG. 71.

joint is carried across the whole diameter, necessitating several blank nozzle pitches at each side.

In some machines cast steel diaphragms are used, and in those cases nozzle blocks are fitted into segmental recesses in the rims of the diaphragms. In the Rateau machines the nozzle blades, cast in the diaphragms or segments, are of the humped or crescent shape, similar to those used for the wheels. These are usually set with an inlet angle to take the steam discharged from the previous wheel blades, with a minimum of shock, and thus utilise as much of the carry-over energy as possible. Usually in velocity compounded machines the sheet steel blade is cast into the intermediate and low-pressure diaphragms, the plates being kept parallel as shown in Fig. 71, and the dimensions at right angles to the axis being increased, as shown on the plan, between throat and exit when the passage has to be made divergent.

The special form of combined nozzle and guide passages used in the velocity compounded Sturtevant turbine is shown in Fig. 72, Plate X. The starting passage numbered 7, in Fig. 15 is the first on the right side. Next to it comes the passage 8, with the rectangular nozzle exit at the bottom.

57. Nozzle Design.—In the design of a nozzle passage for a given pressure drop, the principal factors which have to be considered are—the weight of steam that can pass the narrowest section or throat, and the velocity at exit. The actual exit velocity is calculable when the nett heat drop is determined. The weight of steam that can pass the throat of the nozzle, and conversely the throat area required for the passage of a given weight, can be calculated from the equation of continuity, when the volume and velocity at the throat are known. Thus if

W = weight of steam in lbs./sec.
V = velocity at the throat in ft./sec.
v = volume at the throat pressure in ft.^{3}/lb.
A = throat area in ft.^{2}

$$W = \frac{AV}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the conditions at exit are denoted by the subscript o, equation (1) becomes

$$W = \frac{A_0V_0}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the nozzle is convergent the throat and exit values are the same.

58. Critical Pressure.—In Fig. 73 two chambers, D and B, are connected by a convergent orifice, C, in the dividing wall. D is connected to a steam boiler which maintains a constant pressure, p_1, in the chamber. B is connected to some suitable exhausting arrangement by which a constant lower pressure, p, can be maintained. When the pressures in D and B are the same no steam flows through the orifice. As pressure in B is reduced, a gradually increasing discharge into B takes place. This continues until the pressure in B is between 50 and 60 per cent. of that in D. Further decrease of pressure in B has then no effect on the discharge, which remains constant at a maximum value.

PLATE X.

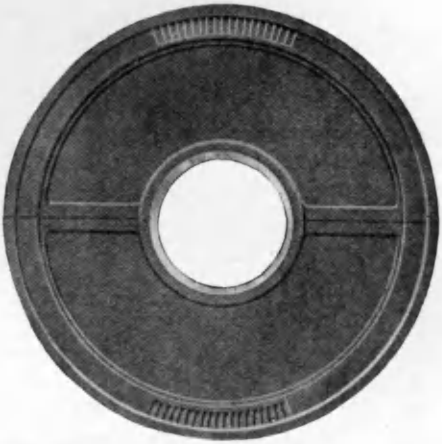


FIG. 69.

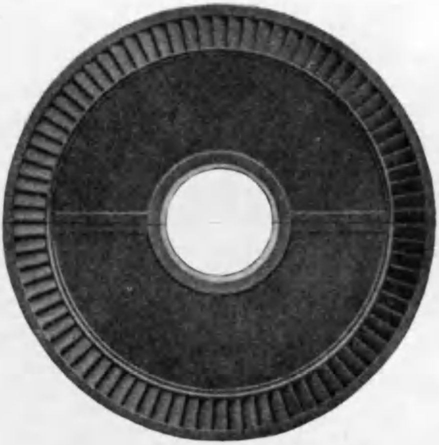


FIG. 70.

FIG. 72.

Nozzles and Diaphragms.

The lower exit pressure, at which the discharge becomes a maximum, is called the "critical pressure."

This, however, is not necessarily the pressure in the chamber outside the exit section. With certain formations of the passage at exit, it may be appreciably greater than the actual pressure at the "throat," or narrowest section of the converging passage.

This curious effect was first observed by Napier in 1867. He concluded that when the lower pressure p was less than half the initial pressure p_1, the pressure at the nozzle exit was never less than half the initial pressure. Under these circumstances the expansion in the nozzle was incomplete at exit and was completed after the steam had cleared the orifice.

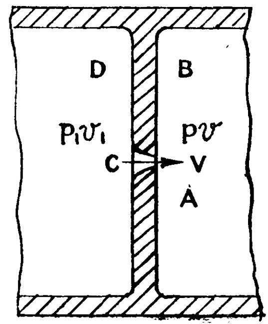


FIG. 73.

With a large pressure difference the residual expansion external to the orifice results in disorderly flow in the outer chamber, and reconversion of kinetic energy into heat. By adding a diverging channel the combination becomes a convergent-divergent nozzle, in which the expansion can be efficiently controlled. In this way the given weight of steam can be discharged with the maximum velocity corresponding to the nett heat drop between the initial pressure p_1 and final pressure p_0.

Napier demonstrated this fact; and subsequently Dr. de Laval successfully applied the converging-diverging nozzle to the simple impulse turbine.

59. The value of the critical pressure p can be found in terms of the initial pressure p_1, when it is assumed that, during adiabatic expansion between p_1 and p, the steam behaves like a gas.

In the first instance, suppose that the steam is superheated initially and finally, the initial pressure in lbs./ft.^{2} being P_1 and the throat or critical pressure P. Then by equation (13), p. 108, the kinetic energy at exit is given by

$$\frac{V^2}{2g} = \frac{m}{m-1} P_1 v_{s_1} \left\{ 1 - \left(\frac{P}{P_1}\right)^{\frac{m-1}{m}} \right\} \quad . \quad . \quad . \quad . \quad (3)$$

where V is the increase of velocity between inlet and throat, and v_{s_1} the volume of superheated steam at P_1.

Substituting V in equation (1), and squaring

$$\left(\frac{Wv}{A}\right)^2 = 2g \frac{m}{m-1} P v_{s_1} \left\{ 1 - \left(\frac{P}{P_1}\right)^{\frac{m-1}{m}} \right\}$$

but the volume at P is

$$v = v_{s_1} \left(\frac{P_1}{P}\right)^{\frac{1}{m}}$$

hence substituting and reducing

$$W = A\sqrt{2g\frac{m}{m-1}\frac{P_1}{v_{s_1}}\left\{\left(\frac{P}{P_1}\right)^{\frac{2}{m}} - \left(\frac{P}{P_1}\right)^{\frac{m+1}{m}}\right\}}. \quad \text{If} \left(\frac{P}{P_1}\right) = r$$

$$W = \text{constant} \times A\sqrt{r^{\frac{2}{m}} - r^{\frac{m+1}{m}}} = CAy \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The throat area A is a minimum, hence for W maximum the quantity under the root sign is a maximum. When y is a maximum

$$\frac{dy}{dr} = 0$$

$$\therefore \quad \frac{2}{m}r^{\left(\frac{2}{m}-1\right)} - \frac{m+1}{m}r^{\frac{1}{m}} = 0$$

and
$$r = \left(\frac{1+m}{2}\right)^{\frac{m}{1-m}} = \left(\frac{2}{m+1}\right)^{\frac{m}{m-1}} \quad . \quad . \quad . \quad . \quad (5)$$

The ratio of the throat pressure to the initial pressure is thus dependent on the value of the adiabatic exponent m.

In the case of superheated steam, $m = 1{\cdot}3$. When this is substituted in (5) the critical pressure ratio becomes $r = 0{\cdot}5457$.

60. Maximum Discharge of Superheated Steam.—Substituting this in equation (4), and reducing, the maximum discharge is,

$$W_s = 0{\cdot}3155A\sqrt{\frac{p_1}{v_{s_1}}} \text{ lbs./sec.} \quad . \quad . \quad . \quad . \quad (6)$$

where p_1 is in lbs./in.², A in sq. in., v_{s_1} in ft.³/lb.

Substituting in equation (3), and reducing, the corresponding throat velocity is

$$V = 72{\cdot}24\sqrt{p_1 v_{s_1}} \text{ ft./sec.} \quad . \quad (7)$$

where p_1 is in lbs./in.², and v_{s_1} in ft.³/lb.

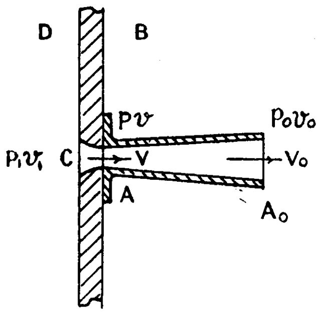


FIG. 74.

For a given throat area the discharge may be appreciably less than W_s, if the orifice is not provided with a suitably formed entrance. The velocity may also be less than V on account of friction and eddy disturbances which degrade some portion of the kinetic energy back to heat.

If expansion is carried below the critical pressure in a convergent-divergent nozzle, Fig. 74, to a pressure p_0, the final or exit velocity V_0 is calculated from the heat drop between p_1 and p_0, and volume v_0 by determining the quality at p_0. These figures can be got by calculation from the tables in the manner illustrated in Chapter V; but it is usually sufficient for practical purposes to obtain them directly from the Hϕ diagram.

61. Referring to Fig. 75, F_1 is the initial state point at p_1 and t_{s_1}, and N is the final state point at p_0, if the expansion is isentropic. The heat drop is given by F_1N. This is never the case in practice. There is always a loss of energy in the diverging part increasing with the range of the expansion; kinetic energy is converted back to heat, and the nett heat drop becomes some value F_1N_1. The amount NN_1, or friction heat loss h_f, remains in the steam as heat at the exhaust pressure

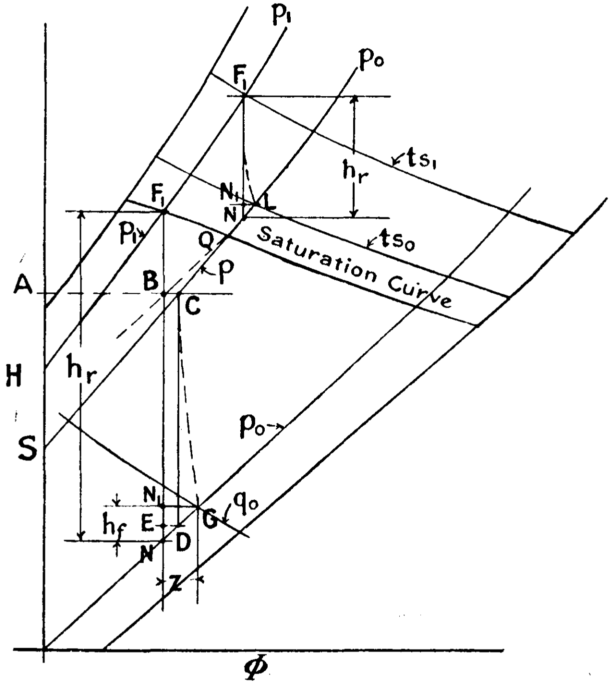


FIG. 75.

p_0, and "reheats" it. The result is an increase in quality and volume. The actual state point moves along the exhaust pressure curve from N to the point L, at which the projector from N_1 cuts the curve, and the actual quality becomes t_{s_0}.

The actual expansion curve departs to the right of the adiabatic vertical. As indicated by the dotted curve, its form is indeterminate, but as far as the design of the nozzle is concerned this is immaterial. The necessary data for the design are the nett heat drop and quality at exit.

The nett heat drop has to be estimated by the use of an empirical coefficient or efficiency factor derived from experiment. This subject is discussed in Arts. 69 and 70.

When the exit velocity V_0 is calculated from the nett heat drop, and the final volume v_{s_0} found from the quality t_{s_0}, the exit area A_0 is again obtained from the equation of continuity, thus

$$A_0 = \frac{144 W v_{s_0}}{V_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where A_0 is in inch², v_{s_0} in ft.³/lb., V_0 in ft./sec., and W in lbs./sec.

The ratio of exit to throat area or $\left(\frac{A_0}{A}\right)$ is called the "expansion ratio" of the nozzle.

EXAMPLE 1.—Steam is expanded in the first stage nozzles of a compound impulse turbine from $p_1 = 150$ and $t_{s_1} = 200°$ F. to $p = 50$. The weight of steam that has to pass the nozzles is 7 lbs./sec. Calculate the throat and exit areas on the assumption that the expansion from inlet to throat is adiabatic, and between throat and exit adiabatic and resisted, the friction loss representing 4 per cent. of the heat drop.

For the throat area by equation (6)

$$W = 0{\cdot}3155 A \sqrt{\frac{p_1}{v_{s_1}}} \qquad p_1 = 150$$

$$v_{s_1} = 3{\cdot}97$$

$$7 = 0{\cdot}3155 A \sqrt{\frac{150}{3{\cdot}97}} \qquad W = 7$$

$$\therefore\ A = \frac{7}{0{\cdot}3155 \times 6{\cdot}146} = 3{\cdot}61 \text{ in.}^2$$

Alternatively,

Throat pressure $p = 0{\cdot}5457 \times 150 = 82$ lbs./in.²

From Hϕ diagram quality $t_s = 110°$ F.

Specific volume at 82 lbs./in.² and 110° F. $= v^s = v(1 + 0{\cdot}0016 \times 110)$

$\therefore\ v_s = 5{\cdot}34 \times 1{\cdot}176 = 6{\cdot}28$ ft.³

Throat velocity by equation (7),

$$V = 72{\cdot}24\sqrt{p_1 v_{s_1}} = 72{\cdot}24\sqrt{150 \times 3{\cdot}97} = 1760 \text{ ft./sec.}$$

By equation (2), $A = \frac{144 W v_s}{V} = \frac{144 \times 7 \times 6{\cdot}28}{1760} = 3{\cdot}6$ in.²

To calculate the exit area, find the total heat drop between 150 and 50 lbs./in.² This, from the Hϕ diagram, or by calculation, is $h_r = 105$ B.Th.U.

The nett heat drop is $0{\cdot}96 h_r = 0{\cdot}96 \times 105 = 100{\cdot}8$ B.Th.U.

The reheat is $h_f = 4{\cdot}2$. Setting this off along the adiabatic and projecting to the 50 lbs./in.² curve the quality at 50 lbs./in.² is $t_{s_0} =$ 50° F. and volume $v_{s_0} = 9{\cdot}19$.

$$\therefore \text{ Exit velocity } V_0 = 223{\cdot}7\sqrt{100{\cdot}8} = 2246 \text{ ft./sec.}$$

Exit area $$A_0 = \frac{144 W v_{s_0}}{V_0} = \frac{144 \times 7 \times 9{\cdot}19}{2246} = 4{\cdot}12 \text{ in.}^2$$

The expansion ratio is $$\frac{A_0}{A} = 1{\cdot}142$$

62. Maximum Discharge of Saturated Steam.—Next, instead of superheated let dry saturated steam be supplied by the boiler to chamber D, Fig. 74. It is assumed that the steam again in expanding through the orifice follows the law $pv_n = c$, and the same calculation holds for the values of the critical pressure and the discharge. It has been the practice in making this assumption to take $m = 1{\cdot}135$ as the adiabatic exponent for saturated steam. For slightly wet steam having a dryness fraction q, the value of the exponent given by Zeuner is $m = 1{\cdot}035 + 0{\cdot}1q$. This gives $m = 1{\cdot}135$, when $q = 1$.

When $1{\cdot}135$ is substituted for m in equation (5), the critical pressure becomes $p = 0{\cdot}577p_1$.

Similarly substituting these values in equation (4), the maximum discharge becomes

$$W = 0{\cdot}304A\sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where v_1 is the specific volume of dry saturated steam at pressure p_1.

If the logs of pressures and volumes, given in the steam tables, are plotted, it will be found that the log. graph has a slope approximately $1{\cdot}066$. This can be more conveniently written as the fraction $\frac{16}{15}$, and the relation between the pressure and volume of saturated steam expressed by $pv^{\frac{16}{15}} = c$. Taking p in lbs./in.² and v in ft.³/lb., the value of c to the nearest round number is 490.

The law of the saturation curve may thus be stated as

$$pv^{\frac{16}{15}} = 490 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Substituting from (10) for v_1 in (9), the equation for the maximum discharge of saturated steam reduces to the simple form

$$W = 0{\cdot}01646p_1^{\frac{31}{32}}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This ought to be the value of the discharge from the convergent orifice if it is provided with a properly formed entrance, and there is no friction or eddy disturbance. Actually the conditions are never perfect, and the discharge may be expected to be slightly less than the calculated value.

It has been noted, however, by Rateau, Rosenhain, Parentry, and other experimenters, that instead of the discharge being less it is always greater than the value calculated on the foregoing hypothesis.

The excess may vary from 2 per cent. to 4 per cent. Allowing for a slight decrease from friction and contraction of the jet, the actual difference may reach a probable maximum of 5 per cent.

63. Supersaturated Steam.—There has been a great deal of speculation regarding the cause of this discrepancy. The most feasible explanation, which is now finding general acceptance, is that during the expansion between the entrance and throat the steam becomes supersaturated.

Under this condition no condensation takes place, and at successive pressures the temperatures fall below the corresponding saturation values. In other words, the steam becomes "undercooled" and gets into a state of thermal instability.

It is a recognised fact, when dry steam is rapidly cooled by expansion below the saturation temperature, that within a certain pressure limit, condensation cannot take place unless suitable nuclei are present. Even when they are present in sufficient quantity a certain time is required for the condensation. It is inferred that this interval, which in the case of the convergent-divergent nozzle is taken about $\frac{1}{10000}$ sec., is too small to permit of condensation before the throat is reached.

According to this hypothesis the steam thus continues to expand like a gas and the law of expansion is the same as that of superheated steam,

$$pv^{1 \cdot 3} = c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The volume at any pressure during the expansion between entrance and throat can be calculated from this relation. The heat drop, however, cannot be obtained from the $H\phi$ diagram without the use of supersaturation curves. These are continuations of the pressure curves from the superheat field, and depart to the left of the (straight) pressure curves in the saturation field, as indicated by the dotted curve BQ in Fig. 75.

The heat drop can, however, be calculated when the total heat of supersaturated steam at pressure p and volume v after adiabatic expansion is known.

64. Calculation of Heat Drop for Supersaturated Steam.—The equation for the total heat, given by Professor Callendar,[1] is

$$H = a(\tfrac{10}{3}m)p(v - b) + abp + B$$

a is a factor of conversion of work to heat units, having a value $\frac{144}{J}$, when p is in lbs./in.^{2}; b is the small initial volume of water, and B is a constant equal to 835·2 B.Th.U.; $m = 1 \cdot 3$.

The volume of water (b) is so small that it does not affect the heat value more than a fraction of a per cent. at high pressures. It may

[1] See paper on "Steady Flow of Steam through a Nozzle or Throttle." *Proc. I. Mech. E.*, February, 1915.

therefore be neglected for practical purposes. The total heat can thus be expressed as

$$H = \frac{144}{J} \times \tfrac{10}{3} mpv + 835$$

or

$$H = 0{\cdot}802pv + 835 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In this case again the critical pressure is $p = 0{\cdot}5457p_1$, and from equation (13) the corresponding volume is $v = 1{\cdot}5934v_1$, where p_1 and v_1 are the initial pressure and volume.

The heat drop between p_1 and any lower pressure p is

$$H_r = (H_1 - H) = H_1 - \{0{\cdot}802pv + 835\} \quad . \quad . \quad . \quad . \quad (14)$$

65. Maximum Discharge of Supersaturated Steam.—As in the case of the superheated steam the maximum discharge is given by

$$W = 0{\cdot}3155A\sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where A is the throat area in in.^{2}, p_1 the initial pressure in lbs./in.^{2}, and v_1 the initial volume in ft.^{3}/lb.

Substituting for v_1 from equation (10) the discharge, in terms of the initial pressure, is given by

$$W = 0{\cdot}0173p_1^{\frac{31}{32}}A \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

It is 5 per cent. greater than the value given by equation (11). This, as already indicated, is the amount by which the discharge, calculated on the assumption that the steam remains saturated, differs from the experimental value, when allowance is made for friction and contraction of the jet.

For the determination of the throat area, or the discharge when the pressure drop is greater than the critical, equation (16) should be used instead of (11).

66. The steam when it passes the throat section is in an unstable thermal condition. C. T. R. Wilson has shown [1] that the limit of this state of instability occurs when the expansion ratio is about 8 to 1. Whether nuclei are present or not the steam reverts to the stable condition, condensation takes place, and the steam becomes wet.

Experimental knowledge of the exact conditions which exist in a turbine nozzle, after the throat section is passed, is lacking.

Professor Callendar expresses the view that condensation probably begins with great rapidity, after the steam passes the throat.

The pressure at which this condensation begins is so far indeterminate. Suppose, however, as an approximation to the real condition, it is assumed that at some pressure slightly below the critical, condensation takes place instantaneously, then a simultaneous increase in temperature and entropy will take place.

The occurrence may be indicated on the Hϕ diagram, as shown in

[1] *Phil. Trans.*, 1898.

Fig. 75. First calculate the total heat of the supersaturated steam after expansion from the initial pressure p_1 to the chosen lower pressure p, using equations (12) and (14). The intersection of the total-heat horizontal AB and the adiabatic vertical F_1N, drawn from the initial state point F_1, gives the state point B of the supersaturated steam, just before condensation takes place and thermal equilibrium is established.

Instantaneous condensation causes the point to move to the right until it falls on the lower pressure curve at C. This point gives the actual state point of the now saturated steam at pressure p.

Constant Pressure Curves for Supersaturated Steam on Hϕ Diagram.—The point B lies on the supersaturation curve BQ, shown dotted. This is simply a continuation of the constant pressure p curve from the superheat field. It bends away from the straight saturation curve QS towards the left. By continuing the pressure curves in this way into the saturation field, the probable heat drops for supersaturated steam can be obtained directly from the chart. The state points do not, however, give the quality, and the volumes have to be separately calculated.

These curves can, if desired, be drawn by calculating a series of heat drops between chosen initial pressures and any given lower pressures and joining the state points (B) thus found. The curve BQ, shown in Fig. 75, is distorted for the sake of clearness.

67. Nett Heat Drop in a Nozzle.—If the steam is expanded further from p to some lower exhaust pressure p_0, the vertical through C to the lower pressure curve gives the additional heat drop CD and the quality at exhaust.

The projector from D cutting the adiabatic F_1N in E gives the nett heat drop F_1E, assuming no friction or eddy loss in the diverging part of the nozzle.

If the steam followed the usually accepted law the heat drop would be F_1N, and hence NE represents the loss of heat drop due to supersaturation.

Actually there is always friction and eddy loss in the diverging part, and, as in the previous case with superheated steam, the real expansion curve falls to the right of CD. It is indicated by the dotted curve CG. The final state point at p_0 moves along the curve from C to G, and the quality is increased.

The projector from G to F_1N now gives the nett heat drop F_1N_1 for the actual case of adiabatic and resisted expansion. The total reheat is NN_1, and covers the supersaturation and friction loss.

In the case of the convergent-divergent nozzle, the same method of procedure can therefore be followed for saturated and supersaturated steam when a certain percentage of the Rankine cycle heat or heat drop is allowed, to cover the energy loss in the passage, whether arising from friction, eddying, or supersaturation, or all these causes combined.

This allowance is a matter of individual judgment, and has to be based on the results of experiment.

In the majority of cases where the expansion is considerable the steam is initially superheated, and is still superheated at the throat pressure. There will be more or less supersaturation in the diverging part of the nozzle. Where this state ends and thermal equilibrium is restored, it is not possible to say in the present state of knowledge.

The ratio, between the pressure at which the adiabatic cuts the saturation curve and the exhaust pressure, will give some indication as to the probability of the steam being finally wet at exit. In the majority of cases it will be wet.

68. Quality and Volume of Steam at Exit from Nozzle.—In dealing with the practical problem of convergent-divergent nozzle design, for a large pressure drop, draw the adiabatic vertical from the initial to the exhaust pressure curve; scale off from the initial state point the value of the nett heat drop, calculated on the basis of a given "friction" loss. Project from the point thus obtained to the exhaust pressure curve and find the quality.

This quality can alternatively be found by calculation. The method can best be illustrated on the Tϕ diagram. Referring to Fig. 76, F_1 is the initial state point at p_1. With frictionless expansion, N is the final state point at p_0 giving the ideal quality q_{0_1}. If the friction heat loss is h_f, the total heat at p_0 is increased by this amount, which is represented by the cross-hatched area KNGL. The entropy is increased by an amount Z, and the state point is moved from N to G (see also Fig. 75).

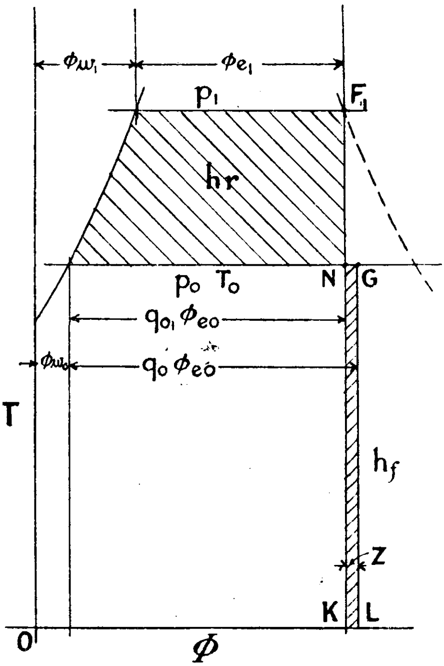


FIG. 76.

The actual quality determined from the entropy values is

$$q_0 = \frac{\phi_{w_1} + \phi_{e_1} - \phi_{w_0} + \frac{h_f}{T_0}}{\phi_{e_0}} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where T_0 is the absolute temperature at p_0.

This can also be expressed in terms of the ideal dryness and the latent heat L_0 at p_0, thus—

$$q_0 = \frac{q_{0_1}L_0 + h_f}{L_0} = q_{0_1} + \frac{h_f}{L_0} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In the case of a moderate pressure drop where it is estimated that the steam is supersaturated at exit, the volume may be approximately calculated from equation (12). This will be on the low side on account of friction.

The throat area is calculable from equation (6) when the steam is initially superheated, and either from equation (15) or (16) when initially dry or slightly wet.

When the nozzle is convergent and the steam initially dry, the exit area has to be calculated by applying successively equations (12), (13), and (8). When the pressure drop is just equal to the critical value, the exit area is calculable from equation (16).

Example 2.—A de Laval turbine is fitted with straight convergent-divergent nozzles. Each nozzle has a throat diameter of 0·25 inch. The initial pressure is 100 lbs./in.^{2} gauge, and the vacuum 26 inches. Calculate the discharge from each nozzle in lbs./hour; and find the exit diameter if the loss of energy is 12 per cent. of the heat drop.

Here $p_1 = 115$, $p_0 = 3$. The large pressure drop ensures wet steam at exit.

Throat area $A = 0{\cdot}049$ in.^{2}

By equation (16),
$$\begin{aligned} W &= 0{\cdot}0173\, A p_1^{\frac{31}{32}} \\ &= 0{\cdot}0173 \times 0{\cdot}049 \times 115^{\frac{31}{32}} \\ &= 0{\cdot}0840 \text{ lb./sec.} = 302 \text{ lbs./hour.} \end{aligned}$$

From the Hϕ diagram or by calculation, $h_r = 245$ B.Th.U. Nett heat with 12 per cent. loss is $0{\cdot}88 \times 245 = 215{\cdot}6$ B.Th.U. Scaling down 215·6 from the initial state point and projecting to the 3 lbs./in.^{2} curve, the actual quality at exit is found to be 0·854. The quality with no friction loss would be $0{\cdot}825 = q_{0_1}$.

Alternatively by calculation, since $q_{0_1} = 0{\cdot}825$, $h_f = 29{\cdot}4$, $L_0 = 1012{\cdot}3$, then by equation (18)

$$q_0 = q_{0_1} + \frac{h_f}{L_0} = 0{\cdot}825 + \frac{29{\cdot}4}{1012{\cdot}3} = 0{\cdot}854$$

The specific volume $v_0 = 0{\cdot}854 \times 118{\cdot}5 = 101{\cdot}2$ ft.^{3}

Exit velocity $V_0 = 223{\cdot}7\sqrt{215{\cdot}6} = 3285$ ft./sec.

Area at exit $A_0 = \dfrac{144 \times 0{\cdot}084 \times 101{\cdot}2}{3285} = 0{\cdot}373$ in.^{2}

Exit diameter $d_0 = 0{\cdot}69$ inch. Expansion ratio $= \dfrac{0{\cdot}373}{0{\cdot}049} = 7{\cdot}63$.

Example 3.—Steam expands in the stage nozzles of a compound impulse turbine from 50 lbs./in.^{2} abs. and 50° F. superheat to 12 lbs./in.^{2} abs.

Assuming that the condition of supersaturation exists at the end of the expansion, calculate the loss of energy due to this cause. If an additional loss of 3 per cent. is allowed for "friction," find the total throat and exit areas to pass 7 lbs. of steam per second.

Draw the adiabatic vertical in the Hϕ diagram between 50 and 12 lbs./in.^{2} This cuts the saturation curve at 35 lbs./in.^{2} The heat drop for this portion of the expansion is 30 B.Th.U.

Saturated steam is now assumed to expand adiabatically, and to become supersaturated, between 35 lbs. and 12 lbs./in.^{2}

Here $p_2 = 35$, $v_2 = 11{\cdot}89$, $p_0 = 12$.

By equation (12)

$$v_0 = \left(\frac{p_2}{p_0}\right)^{\frac{1}{m}} v_2 = \left(\frac{35}{12}\right)^{\frac{1}{1\cdot3}} 11{\cdot}89 = 2{\cdot}278 \times 11{\cdot}89 = 27{\cdot}1 \text{ ft.}^3$$

By equation (13) the total heat of supersaturated steam at $p_0 = 12$, after adiabatic expansion, is

$$\begin{aligned} H_0 &= 0{\cdot}802 p_0 v_0 + 835 \\ &= 0{\cdot}802 \times 12 \times 27{\cdot}1 + 835 \\ &= 261 + 835 = 1096 \text{ B.Th.U.} \end{aligned}$$

Total heat of dry steam at $p_2 = 35$, $H_2 = 1166{\cdot}8$.

Heat drop between 35 and 12 lbs./in.$^2 = (1166{\cdot}8 - 1096) = 70{\cdot}8$ B.Th.U.

Total heat drop from 50 lbs./in.^{2} to 12 lbs./in.$^2 = (30 + 70{\cdot}8) = 100{\cdot}8$.

The heat drop if there were no supersaturation loss would be 105 B.Th.U.

$\therefore$ Loss of energy due to supersaturation $= (105 - 100{\cdot}8) = 4{\cdot}2$ B.Th.U.

Probable total loss including 3 per cent. for friction = 7 per cent.

Hence the nett heat drop $= 0{\cdot}93 \times 105 = 97{\cdot}65$ B.Th.U.

Velocity at exit $V_0 = 223{\cdot}7\sqrt{97{\cdot}65} = 2211$ ft./sec.

Exit area $$A_0 = \frac{144 W v_0}{V_0} = \frac{144 \times 7 \times 27{\cdot}1}{2211} = 12{\cdot}35 \text{ in.}^2$$

This figure is on the low side, as the actual volume on account of the reheating effect of friction is slightly greater than 27·1 ft.^{3}

To find the throat area apply equation (6).

$$W = 0{\cdot}3155 A \sqrt{\frac{p_1}{v_{s_1}}} \qquad v_{s_1} = 9{\cdot}19$$

$$7 = 0{\cdot}3155 A \sqrt{\frac{50}{9{\cdot}19}}$$

$$\therefore A = \frac{7}{0{\cdot}3155 \times 2{\cdot}332} = 9{\cdot}513 \text{ in.}^2$$

The expansion ratio is $\dfrac{12{\cdot}35}{9{\cdot}513} = 1{\cdot}3$.

EXAMPLE 4.—At a certain stage of a pressure-compounded impulse turbine the steam enters the nozzles with an initial velocity of 300 ft./sec. It expands from $p_1 = 60$ to $p_0 = 40$ lbs./in.^{2}, and is supersaturated at exit. Find the percentage of the heat drop lost through supersaturation. If, in addition to this, there is a "friction" loss of 2 per cent. of the heat drop, find the exit area for the discharge of 7·5 lbs. steam per second, (*a*) when the initial kinetic energy appears at exit, (*b*) when 55 per cent. of this appears at exit.

Total heat at 60 lbs./in.^{2} = $H_1 = 1177$.
Volume of supersaturated steam at 40 lbs./in.^{2} by equation (12)

$$v_0 = v_1\left(\frac{p_1}{p_0}\right)^{\frac{1}{m}} = 7{\cdot}17\left(\frac{60}{40}\right)^{\frac{1}{1\cdot3}} = 7{\cdot}17 \times 1{\cdot}366 = 9{\cdot}8 \text{ ft.}^3$$

Total heat at 40 lbs./in.^{2} by equation (13)

$$\begin{aligned} H_0 &= 0{\cdot}802 p_0 v_0 + 835 \\ &= 0{\cdot}802 \times 40 \times 9{\cdot}8 + 835 \\ &= 1149{\cdot}4 \text{ B.Th.U.} \end{aligned}$$

Heat drop with supersaturated steam

$$H_r = (1177 - 1149{\cdot}4) = 27{\cdot}6$$

The heat drop with saturated steam would be 30 B.Th.U.

Hence loss due to supersaturation is 2·4 B.Th.U., or 8 per cent. of the heat drop. Loss from friction and supersaturation is probably 10 per cent.

Taking case (*a*), K.E. at entrance is equivalent to $\left(\frac{300}{223{\cdot}7}\right)^2 = (1{\cdot}34)^2$ $= 1{\cdot}8$ B.Th.U. Hence the nett equivalent heat drop is $0{\cdot}9 \times 30 + 1{\cdot}8$ $= 28{\cdot}8$ and $V_0 = 223{\cdot}7\sqrt{28{\cdot}8} = 1200$ ft./sec. With no reheat effect the volume is $v_0 = 9{\cdot}8$. To allow for slight reheat assume it is actually 10.

Thus $$A_0 = \frac{144 \times 7{\cdot}5 \times 10}{1200} = 9 \text{ in.}^2$$

In case (*b*) the effective heat equivalent of the carry over is $0{\cdot}55 \times 1{\cdot}8 = 0{\cdot}99$ B.Th.U., and the nett equivalent heat drop is $(27 + 0{\cdot}99)$.

Hence $V_0 = 223{\cdot}7\sqrt{27{\cdot}99} = 1184$ ft./sec. There will be a further slight increase in the volume due to the additional reheat. Assume a volume 10·1

$$A_0 = \frac{144 \times 7{\cdot}5 \times 10{\cdot}1}{1184} = 9{\cdot}2 \text{ in.}^2$$

The neglect of the initial K.E. at entrance leads to an increase of about 2 per cent. on the area at exit.

Considering the uncertainty of the assumptions made, probably an area of 9 ins.^{2} would be satisfactory.

These examples serve to indicate in a general way the method of approximating the supersaturation loss, and combining this with the probable friction loss. As a rule, separate estimation is not necessary; the whole energy loss can be conveniently covered by means of a correction factor, the nozzle efficiency.

69. Nozzle Efficiency (η_n).—The energy efficiency of a nozzle is expressed by the ratio between the actual and the theoretical kinetic energy at exit.

When the steam enters the nozzle with a negligibly small velocity, the efficiency is also the ratio of the nett heat drop h_r' to the total heat drop h_r.

If V_0' = theoretical velocity corresponding to h_r
V_0 = actual " " h_r'

$$\eta_n = \left(\frac{V_0}{V_0'}\right)^2 = \frac{h_r'}{h_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

When for a given nozzle the efficiency η_n is chosen, then

$$V_0 = 223 \cdot 7\sqrt{\eta\ h_r}$$

When the steam has an initial velocity V_1 of appreciable magnitude, the total energy available is

$$_1h_r = \left(\frac{V_1}{223 \cdot 7}\right)^2 + h_r$$

which may be regarded as an equivalent or virtual heat drop.

The actual kinetic energy at exit will depend on the proportion of the initial kinetic energy and the proportion of the heat drop lost through "friction" in the nozzle. The equivalent nett heat drop may be expressed as $h_r'' = m\left(\frac{V_1}{223 \cdot 7}\right)^2 + Mh_r$, where m and M are friction coefficients.

No experimental values of these are available. It is probable that m is less than M. For the ordinary run of cases in the pressure-compound impulse, the author would suggest the values of m as 0·8 and M from 0·90 to 0·93, the higher figure to be taken for stages where superheated steam is used.

Normally, however, it is sufficient to select a suitable nozzle efficiency, and base the calculation of the jet velocity on this arbitrary value and the equivalent heat drop.

In this case $$\eta_n = \frac{h_r''}{_1h_r}$$

and $$V_0 = 223 \cdot 7\sqrt{\eta_n \, _1h_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

In order to determine the quality at exit, when the steam is not supersaturated, the reheat $h_f = (1 - \eta_n)\,_1h_r$ has to be calculated, and scaled up from N on the exhaust pressure curve on the adiabatic vertical to obtain the point N_1, Fig. 75.

When it is assumed that the steam is supersaturated, the calculated volume should be increased from 1 to 2 per cent. to allow for the reheating effect. This calculation in any case is a tentative one, and an exercise of judgment is necessary.

70. Frictional Losses in Nozzles.—The quality of the steam, the velocity, the condition of the surfaces, and the form and length of the passage, etc., all contribute to the loss of kinetic energy, which, for convenience of expression, may be covered by the general term of "frictional loss."

Dr. Stodola has found that for short convergent-divergent nozzles of 2 inches length downwards, and having a small angle of divergence, the loss varies from 8 to 5 per cent., that is, the efficiency varies from 92 to 95 per cent. For longer nozzles 4 to 6 inches in length, the loss varies from 15 to 10 per cent., or the efficiency from 85 to 90 per cent.

Rosenhain's results [1] agree fairly well with Stodola's when complete expansion is attained, the efficiency for the larger types varying from 90 to 92 per cent.

Steinmetz [2] has given figures, derived from experiments on nozzles used in Curtis turbines, which indicate efficiencies from 94 to 96 per cent. The latter figures appear to be too large for large pressure drops. H. M. Martin has derived the following expression for the loss of kinetic energy in convergent-divergent nozzles. It is based on the results of various experimenters.

Percentage loss $= \frac{6}{100}(h_r - 45)$, where h_r is the heat drop. The corresponding efficiency is given by

$$\eta_n = 102{\cdot}7 - 0{\cdot}06h_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

It will be noted that when the heat drop is reduced to 45 B.Th.U. the loss is zero. With the usually accepted theory of the expansion, in the case of dry or slightly wet steam, 45 is the approximate value of the critical heat drop. Hence, for a convergent nozzle or orifice this makes the efficiency unity.

Even supposing there is no friction in a very short nozzle, there is, according to the later hypothesis of supersaturation, a loss when the steam at entry is in a dry or slightly wet condition. Further, in the pressure compounded impulse machine, when the critical pressure drop is used, the nozzle passages are not of the straight type, but rectangular passages, having considerable curvature and wall surface. The effect of centrifugal action and surface friction is to further accentuate the energy loss.

Equation (21) should be used to obtain the probable value of the nozzle efficiency only when the heat drop is well in excess of the critical value, say 100 B.Th.U. and over. The loss will be less with superheated than with wet steam. In the region in which the steam is dry or slightly wet the loss will be greater on account of supersaturation.

[1] *Proc. I.C.E.*, 1899.

[2] *Journal American Soc. Mech. Eng.*, May, 1908.

The value of the efficiency to be used in any case for a convergent-divergent nozzle is thus a matter for individual judgment. It may be taken between 85 and 96 per cent. When cast nozzles of rectangular section are employed the value should be less than that for straight nozzles.

As regards the convergent type of nozzle passage formed by curved blades cast in the diaphragms or nozzle blocks, the experimental results available are not sufficiently conclusive[1] to warrant any deduction of efficiency. As indicated in example (4) it is probable that the loss, with dry steam, is not less than 10 per cent.; and for dry steam a maximum value of 90 per cent. may be taken for the efficiency. For superheated steam a slightly higher value may be allowed, say 92 to 94 per cent.

71. Under-Expansion.—When the steam after expansion in a nozzle passage, issues at the exit section with a pressure equal to that of the external medium, it is completely expanded. The exit areas in the foregoing examples are worked out to ensure this condition, as closely as the uncertainty of the efficiency factor will admit.

When the pressure at exit is greater than the external pressure, the expansion is incomplete, and the nozzle is said to "under-expand" the steam. In this case the exit area is too small.

72. Over-Expansion.—When the area at exit is made too large the pressure in the nozzle falls below the external pressure, and then rises to the external value at discharge. The nozzle in this case "over-expands" the steam.

These phenomena have been experimentally investigated by Stodola, Jude, Thomas, and others, with the aid of a "searching tube," placed along the axis of the nozzle. Detailed information regarding these experiments and the results will be found in the steam turbine texts by these writers.

73. In order to show the peculiar effects which accompany the foregoing conditions, the author has replotted several of Stodola's curves, shown in Figs. 77, 78, 79, 80, the pressures being in lbs./in.^{2} and nozzle dimensions in inches.[2]

Three pressure curves for a short parallel orifice with rounded entrance are shown in Fig. 77. The pressure indicated by the curve at 2·5 inches from the exit side is the external pressure. In the case of curve B the pressure in the external medium approximates to the critical value. In each case, the pressure at the exit section falls below the external pressure, and compression occurs in the external medium till the pressure in the jet is equal to the external pressure. This parallel passage therefore over-expands the steam. The over-expansion is most marked in the case of B, where the pressure ratio approaches the critical value. In the case of A the over-expansion is relatively small, and there is none of the oscillatory disturbance shown by B. Curve C shows under-expansion and slight recompression outside the orifice. The orifice for A and B should be convergent, not parallel, if the

[1] See discussion of Christlein's results, *Engineering*, July 19, 1912.
[2] Reproduced from Stodola's "Steam Turbines," by permission of the author and publishers, Messrs. Constable and Co.

effects shown here are to be avoided. For C it should be divergent. Even if a very slight parallel portion is left at the exit end of such a converging passage, a recompression takes place.[1] This means a loss of effective energy or reduction of nozzle efficiency.

The three corresponding curves are shown in Fig. 78, for an orifice of the same dimensions with a sharp-edged entrance. The effect of the sharp entrance is considerable. Even with the small pressure ratio shown by A, the pressure falls inside the nozzle to nearly the critical

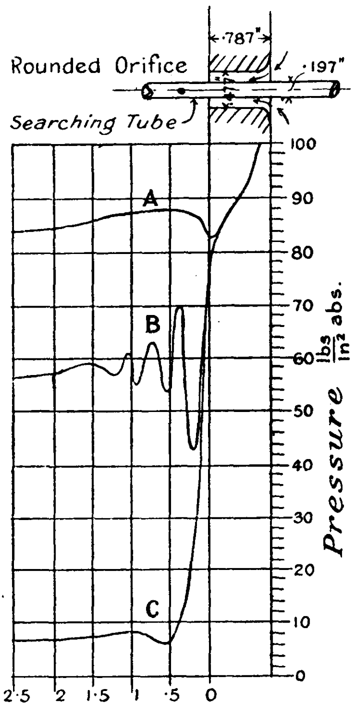


FIG. 77.

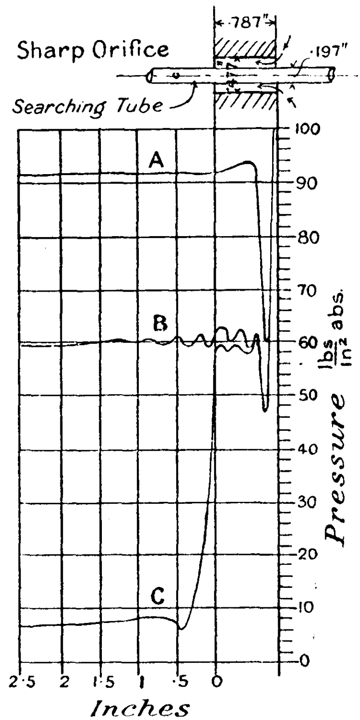


FIG. 78.

value, and there is nearly three times the amount of recompression as in the case of A, Fig. 77. For the larger ratio shown by curve B, the pressure again falls below the critical inside the nozzle, and the oscillatory disturbance takes place mostly in the nozzle.

The effect of the sharp entrance is similar to that produced by a sharp-edged orifice in the case of water. There is a "vena-contracta" inside, and the minimum section of the jet is less than the cross-section

[1] See paper on "Discharge of Steam through Nozzles," by Dr. W. G. Fisher, *Proc. Inst. Mech. Eng.*, December, 1914.

of the passage. In effect, the passage is equivalent to an inefficiently formed convergent-divergent nozzle.

In the case of curve C for high-pressure ratio there is recompression and oscillation in the passage near the critical pressure. The passage should be divergent to permit of the steady expansion to the lower pressure.

Beside the reduction of efficiency due to this spurious compression and expansion, the sharp edge, in producing a reduction of the minimum cross-section of the jet, reduces the discharge.

In the case of C, where the pressure drop is greater than the critical, a large proportion of the expansion takes place outside the passage. With discharge into a chamber of considerable dimensions the steam can expand in all directions producing disorderly flow. In the actual turbine case, however, it is not discharged into a wide chamber, but directly on to the wheel blades, across a very narrow clearance space, and hence the effect of a small amount of under-expansion is quite negligible. In fact, it is often considered advisable to design convergent-divergent nozzles in compounded machines for about 10 per cent. under-expansion at full load. At slightly lower loads, with reduced pressures, the nozzles then expand the steam correctly, and a slightly better efficiency is maintained.

The effects produced in the case of a convergent-divergent nozzle are shown in Figs. 79 and 80.

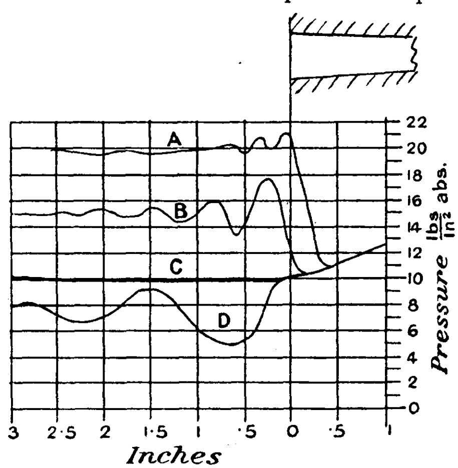


FIG. 79.

The curve C, Fig. 79, shows the condition of complete expansion. There is no oscillatory disturbance outside the nozzle. On the other hand, with a slight amount of over-expansion there is recompression just at exit, and considerable disturbance outside as shown by B.

With a larger amount of over-expansion, shown by A, the compression starts further in the nozzle. There is a quiescent compression, and somewhat less disturbance outside. For a small amount of under-expansion there is a large disturbance outside as shown by D.

As the back pressure is increased above the exhaust pressure for which the convergent-divergent nozzle is designed, the point of recompression recedes towards the throat.

The progressive effect is shown by the six curves of Fig. 80. The curve G is the proper curve of expansion between the upper and lower pressure limits. Curve F shows the recompression near exit for a slight over-expansion, curve E for somewhat greater amount. Curve D shows peculiar oscillations which Stodola attributed to separation of the steam from the nozzle walls. In the case of curve C, the external pressure is slightly greater than the critical value, but the fall in pressure at the throat is far below the critical. Even with the high exit pressure shown by B, the throat pressure is well below the critical value. Curve A shows the effect of a small difference of pressure at inlet and exit. There is an appreciable drop in pressure with corresponding increase in velocity at the throat, and then a quiescent compression to nearly the initial pressure. This effect has been employed with advan-

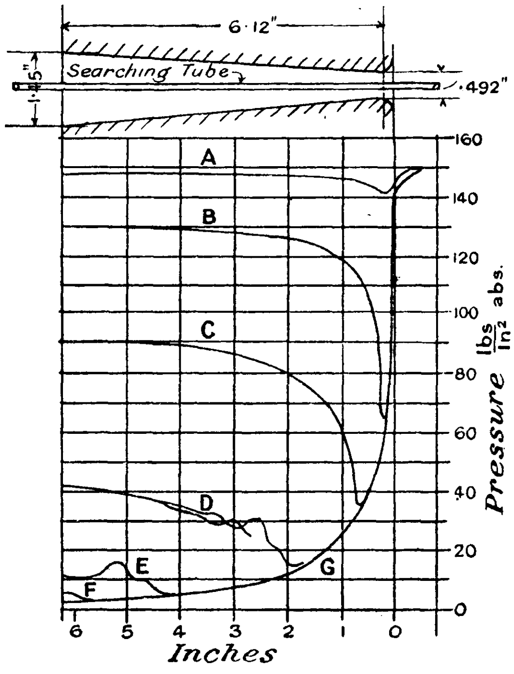


FIG. 80.

tage to the Hopkinson-Ferranti stop valve, which is simply a convergent-divergent nozzle, formed in the steam pipe, with a gate valve at the throat. In this way the size of the valve proper is much reduced, and there is comparatively small loss in pressure at the valve. A similar device is employed for the L.P. throttle of the Brush mixed pressure turbine.

While under-expansion within certain limits is not objectionable, over-expansion is. In designing a convergent-divergent nozzle it is better to have the exit area on the small rather than on the large side.

74. Effect of Under- and Over-Expansion on Nozzle Efficiency.—

The result of either under- or over-expansion is a reduction of the exit velocity of the steam.

From data published by Dr. Steinmetz [1] the author has derived the curves shown in Figs. 81 and 82.

If A_{0_1} is the actual exit area and A_0 the correct area for complete expansion, the condition of either under- or over-expansion can be

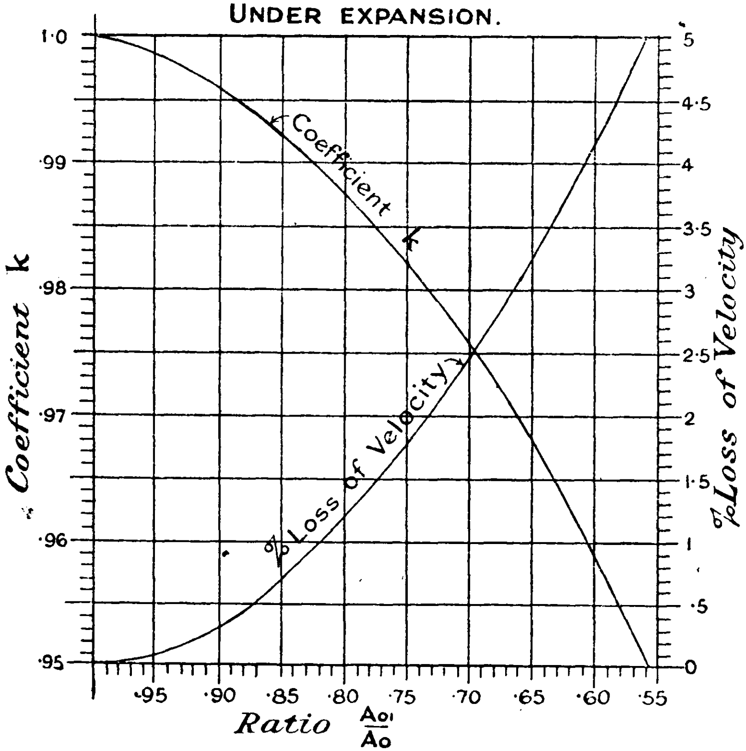


FIG. 81.

expressed by the ratio $\frac{A_{0_1}}{A_0}$. If $A_{0_1} > A_0$ there is over-expansion; if $A_{0_1} < A_0$ there is under-expansion.

If the correct velocity is V_0 and actual velocity V_{0_1}, then $V_{0_1} = kV_0$, where k is a velocity coefficient.

The values of k are plotted for a series of values of the ratio $\frac{A_{0_1}}{A_0}$, and to facilitate comparison the corresponding curves of percentage loss of velocity are also plotted.

If η_n is the nozzle efficiency for complete expansion, the reduced

[1] *Proc. American Soc. Mech. Eng.*, May, 1908.

efficiency with over- or under-expansion is given by $(k^2\eta_n) = \eta_n'$. The absolute values of the loss of velocity may be questionable; but the curves enable the relative losses from under- and over-expansion to be compared.

It will be noted that as much as 30 per cent. of under-expansion apparently reduces the exit velocity by about 2·5 per cent., while the same amount of over-expansion reduces it 9·4 per cent. A considerable amount of under-expansion may evidently be permitted without

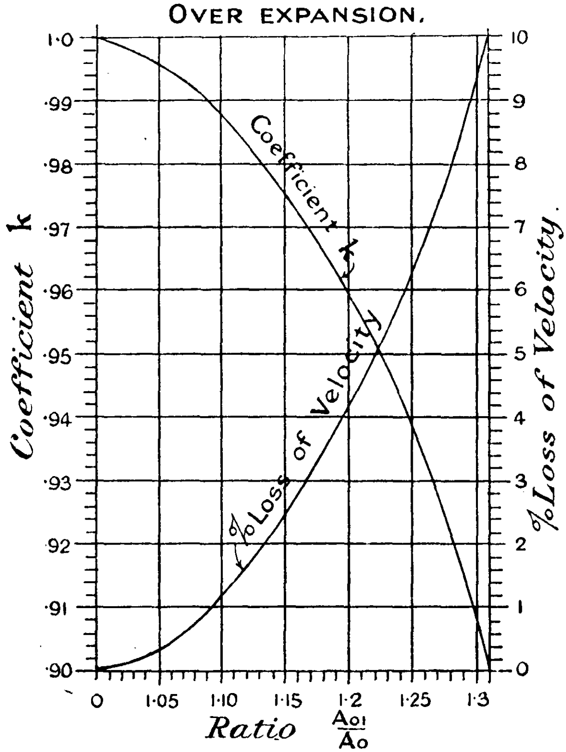


FIG. 82.

appreciably affecting the nozzle efficiency, and the 10 per cent. allowance previously quoted may quite well be exceeded. The over-expansion loss is an irrecoverable one, as far as the wheel blading is concerned, and ought to be avoided.

75. Proportions of Convergent-Divergent Nozzles.—In the straight reamered de Laval type of nozzle the convergent part is very short, usually about $\frac{1}{8}$ inch. It may be practically regarded as a well-rounded entrance. The diverging part is made with a "straight" taper. Referring to the section, Fig. 83, the length L between the throat and exit sections is controlled by mechanical considerations. In the same turbine casing, nozzles of various expansion ratio may have to be fitted to suit varying pressure ranges and output.

The axis of the nozzle may be inclined at an angle α, to the plane of the wheel, from 18° to 20°. The latter figure is practically the standard for the simple impulse type.

Generally the nozzle passage is continued parallel at the outlet diameter d_0, from the area of maximum section to the wheel blading, but some designers prefer to continue the taper on to the wheel.

The oblique section of the passage in the plane of the wheel is an ellipse, as shown in the plan, Fig. 83, and the nozzle only covers the full depth of blade at the centre. There is an aspirating action on the "dead" steam surrounding the nozzle, which tends to reduce the

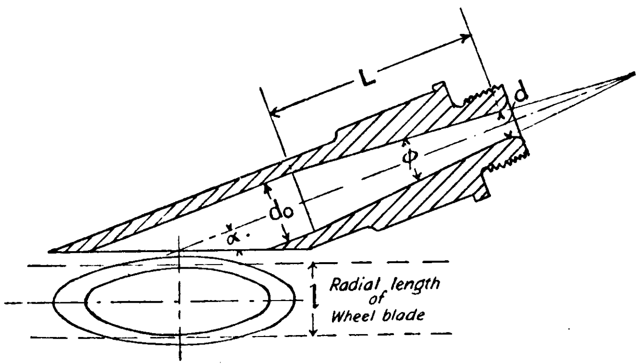


FIG. 83.

efficiency of the blading. This effect is not produced with a nozzle having a rectangular-shaped exit, the form always given to the cast type of nozzle. The conical reamered nozzle is, however, cheaper to manufacture than the rectangular one, which has to be finished by hand.

If the throat and exit diameters are d and d_0, and the cone angle ϕ, then from the geometry of the figure,

$$L \tan \frac{\phi}{2} = \frac{d_0 - d}{2}$$

$$\therefore L = \frac{d_0 - d}{2 \tan \frac{\phi}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The cone angle ϕ increases with the increase of pressure ratio $\frac{p_1}{p_0}$, and may reach a certain value at which the steam will separate from the nozzle walls, with resulting eddy disturbance and loss of energy. The maximum value of the cone angle used in the de Laval machines is 12°. It would appear from experimental records that ϕ may be increased to 20° without appreciable effect on the nozzle efficiency. Usually it varies according to the length and expansion ratio from 6° to 12°.

76. The general practice of makers of compound impulse turbines is to cast the high-pressure nozzle passages in nozzle blocks, the passages being made rectangular in section, and with a constant radial dimension from throat to exit.

A developed section through three passages of a nozzle block is shown in Fig. 84. The inlet is made bowl-shaped, and changes to rectangular section at the throat.

The circumferential distance P between each passage is the "pitch"

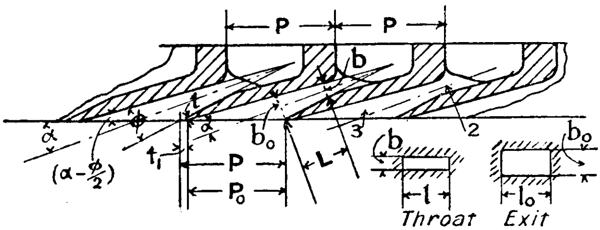


FIG. 84.

of the nozzles. The nett or effective pitch P_0 is this value less the circumferential thickness t_1 of the nozzle wall or plate at exit.

Let P $=$ pitch,
P_0 $=$ nett pitch,
l_0 $=$ radial dimension (constant),
b $=$ throat dimension perpendicular to nozzle axis,
b_0 $=$ exit " " " "
A_0 $=$ exit area,
n $=$ number of nozzles,
α $=$ jet angle,
ϕ $=$ angle between sides,
t $=$ thickness of nozzle wall at exit,

then $P_0 \sin\left(\alpha - \frac{\phi}{2}\right) = b_0$, as without sensible error it may also be assumed that this is the perpendicular distance to the forward side of the nozzle wall. The nozzle angle α is always specified, but $\frac{\phi}{2}$ has to be provisionally estimated. It may be taken from 2° to 4° for a first approximation to the number of nozzles. The nett pitch

$$P_0 = (P - t_1), \text{ where } t_1 = \frac{t}{\sin\left(\alpha - \frac{\phi}{2}\right)} \quad . \quad . \quad . \quad (23)$$

Instead of calculating t_1 it is sufficient to make the correction by means of a "thickness coefficient" c, which is the ratio of the nett to the gross pitch, or $c = \frac{P_0}{P}$. This varies from 0·94 to 0·98.

Since $l_0 b_0 = A_0$, then

$$A_0 = ncPl_0 \sin\left(a - \frac{\phi}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The nett arc is $x_0 = ncP$.
The gross arc is $x \ = nP$.
Where l_0 is kept constant between throat and exit the expansion ratio is

$$a = \frac{A_0}{A} = \frac{b_0}{b}$$

Hence the distance between throat and exit is

$$L = \frac{b(a-1)}{2 \tan \frac{\phi}{2}} = \frac{b_0(a-1)}{a \, . \, 2 \tan \frac{\phi}{2}} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

When the expansion ratio a is not much greater than unity, L should not have a value smaller than 1 inch.

EXAMPLE 5.—A simple impulse turbine is fitted with convergent-divergent nozzles of the type shown in Fig. 83. Each nozzle has a throat diameter of $\frac{1}{4}$ inch. Calculate the other dimensions if the initial pressure is 150 lbs./in.^{2} abs., superheat 180° Fahr., and vac. 26 inches. If the machine is to develop 200 B.H.P. on an estimated consumption of 16 lbs./B.H.P. hour, find the number of nozzles required.

Weight of steam passing the nozzles per second by equation (6)

$$W = 0{\cdot}3155A\sqrt{\frac{p_1}{v_{s_1}}} \qquad p_1 = 150$$
$$v_{s_1} = 3{\cdot}01(1 + 0{\cdot}0016 \times 180) = 3{\cdot}877$$
$$A = 0{\cdot}049$$
$$= 0{\cdot}3155\sqrt{\frac{150}{3{\cdot}877}} \times 0{\cdot}049$$
$$= 0{\cdot}0962 \text{ lbs./sec.}$$

Heat drop between 150 and 2 lbs./in.^{2}, from Hϕ diagram or by calculation, $h_r = 317$ B.Th.U.
Quality after adiabatic expansion, $q_{0_1} = 0{\cdot}861$.
Probable nozzle efficiency by equation (21)

$$\eta_n = 102{\cdot}7 - 0{\cdot}06 \times 317 = 83{\cdot}5 \text{ per cent.}$$

Friction heat, $h_f = 0{\cdot}165 \times 317 = 52{\cdot}3$ B.Th.U. Nett heat drop $h_r' = 264{\cdot}7$ B.Th.U. Latent heat at 2 lbs./in.^{2}, $L_0 = 1021$.

$$\text{Actual quality at exit } q_0 = q_{0_1} + \frac{h_f}{L_0}$$
$$= 0{\cdot}861 + \frac{52{\cdot}3}{1021} = 0{\cdot}861 + 0{\cdot}0512 = 0{\cdot}9122$$

Velocity at exit, by equation (19)

$$V_0 = 223{\cdot}7\sqrt{\eta_n h_r} = 223{\cdot}7\sqrt{264{\cdot}7} = 3639 \text{ ft./sec.}$$

Specific volume at exit $= q_0 v_{0_1} = 0{\cdot}9122 \times 173{\cdot}5 = 158{\cdot}266 = v_0$

Exit area $\quad A_0 = \dfrac{144 \times W \times v_0}{V_0}$

$$= \frac{144 \times 0{\cdot}0962 \times 158{\cdot}266}{3639} = 0{\cdot}6014 \text{ in.}^2$$

$\therefore$ Exit diameter $d_0 = 0{\cdot}877$ inch, say $\frac{7}{8}$ inch.

Length between throat and exit, assuming a cone angle $\phi = 12°$, is by equation (22)

$$L = \frac{d_0 - d}{2 \tan \frac{\phi}{2}} = \frac{(0{\cdot}875 - 0{\cdot}25)}{2 \times 0{\cdot}105}. \quad \tan 6° = 0{\cdot}105$$

$$= \frac{0{\cdot}625}{0{\cdot}210} = 2{\cdot}97 \text{ inches.}$$

Total steam consumption $= \dfrac{w \text{ B.H.P.}}{3600} = \dfrac{16 \times 200}{3600} = 0{\cdot}888$ lbs./sec.

$\therefore$ Number of nozzles $= \dfrac{0{\cdot}888}{0{\cdot}0962} =$ say 9

EXAMPLE 6.—In a four-stage Curtis turbine the absolute admission pressures and qualities of the steam at entrance to each set of stage nozzles are as follows:—

Stage	1.	2.	3.	4.
Pressure. . . .	185	62	18	4
Quality	200° F.	118° F.	35° F.	$q = 0{\cdot}975$

The exhaust pressure is 0·75 lbs./in.^{2} abs. (28½ inches vac.). The steam to be passed through each set is 9·7 lbs./sec. Calculate the throat and exit areas for the four sets of nozzles.

Drawing the adiabatic verticals between the pressure limits specified for the successive stages, it will be found that the constant heat drop per stage is 103 B.Th.U.

In the first stage nozzles the expansion is altogether in the superheat field.

$$p_1 = 185, \; v_{s_1} = v_1(1 + 0{\cdot}0016 \times 200) = 2{\cdot}468 \times 1{\cdot}32 = 3{\cdot}26 \text{ ft.}^3$$

$$\sqrt{\frac{p}{v_{s_1}}} = \sqrt{\frac{185}{3{\cdot}26}} = 7{\cdot}534. \quad W = 9{\cdot}7.$$

Throat area by equation (6)

$$A = \frac{W}{0.3155\sqrt{\frac{p_1}{v_{s_1}}}} = \frac{9.7}{0.3155 \times 7.534} = 4.082 \text{ in.}^2$$

Nozzle efficiency by equation (21), $\eta_n = 102.7 - 0.06 \times 103 = 96.5$, say 96 per cent. Probable nett heat drop $= 0.96 \times 103 = 99$ B.Th.U. Scaling this from the initial state point and projecting to the 62 lbs./in.^{2} curve, $t_{s_0} = 57°$ F. Hence the volume at exit is $v_{s_0} = v_0(1 + 0.0016 \times 57) = 6.85 \times 1.0912 = 7.475$ ft.^{3}

Exit velocity by equation (19), $V_0 = 223.7\sqrt{99} = 2220$ ft./sec.

Exit area by equation (8)

$$A_0 = \frac{144 W v_{s_0}}{V_0} = \frac{144 \times 9.7 \times 7.475}{2220} = 4.7 \text{ in.}^2$$

Expansion ratio $\quad a = \dfrac{4.7}{4.082} = 1.15$

In the second stage nozzles the steam would be slightly supersaturated, if there were no friction, and the efficiency may be taken as 94 per cent. say.

Probable nett heat drop $= 0.94 \times 103 = 96.82$ B.Th.U.

As shown by the Hϕ chart, the steam will be supersaturated between 26 and 18 lbs./in.^{2}, and by equation (12) the volume at exit is $v_0 = 20.6$ ft.^{3} If the nett heat drop is scaled off and the quality at 18 lbs./in.^{2} taken without any regard to supersaturation, $q_0 = 0.985$ and $v_0 = 0.985 \times 22.16 = 21.8$, instead of 20.6. Allowing say 1 per cent. increase due to reheat, the volume may be taken $v_0 = 21$.

The exit velocity is $V_0 = 223.7\sqrt{96.82} = 2202$ ft./sec.

Here $p_1 = 62$, $\quad v_{s_1} = 8.24$, $\quad \sqrt{\dfrac{p_1}{v_{s_1}}} = \sqrt{\dfrac{62}{8.24}} = 2.74$

Throat area by equation (6)

$$A = \frac{9.7}{0.3155 \times 2.74} = 11.22 \text{ in.}^2$$

Exit area by equation (2)

$$A_0 = \frac{144 \times 9.7 \times 21}{2202} = 13.3 \text{ in.}^2$$

Expansion ratio $a = \dfrac{13.3}{11.22} = 1.182$

In the third stage nozzles, as can be seen from the Hϕ diagram, the steam is probably dry at 14 lbs./in.^{2}, and it may be assumed that it is still supersaturated at 4 lbs./in.^{2} Neglecting friction, the calculated volume of supersaturated steam, after adiabatic expansion from 14 to 4 lbs./in.^{2}, is by equation (12), $v_0 = 28 \times 3{\cdot}5^{\frac{1}{1\cdot3}} = 73{\cdot}5$ ft.^{3} This may be taken as 74 to allow for reheat.

On the assumption of complete supersaturation at exit, the loss from this cause may be taken about 6 per cent., and allowing, say, 3 per cent. for friction, the probable nozzle efficiency may be taken as, say, 91 per cent. Hence, nett heat drop $= 0{\cdot}91 \times 103 = 93{\cdot}73$ B.Th.U., and the exit velocity is $V_0 = 223{\cdot}7\sqrt{93{\cdot}73} = 2166$ ft./sec.

$$p_1 = 18, \quad v_{s_1} = 22{\cdot}16(1 + 0{\cdot}0016 \times 35) = 22{\cdot}16 \times 1{\cdot}056 = 23{\cdot}3$$

$$\sqrt{\frac{p_1}{v_{s_1}}} = \sqrt{\frac{18}{23{\cdot}3}} = \frac{1}{1{\cdot}136}$$

Throat area $\qquad A = \dfrac{9{\cdot}7 \times 1{\cdot}136}{0{\cdot}3155} = 35$ in.^{2}

Exit area $\qquad A_0 = \dfrac{144 \times 9{\cdot}7 \times 74}{2166} = 47{\cdot}7$ in.^{2}

Expansion ratio $\qquad a = \dfrac{47{\cdot}7}{35} = 1{\cdot}365$

In the fourth stage nozzles there will again be supersaturation for some part of the pressure drop. The steam, however, will probably be wet at exit. Assuming an efficiency of 92 per cent., the nett heat drop is $0{\cdot}92 \times 103 = 94{\cdot}76$. Scaling this from the initial state point and projecting to the 0·75 lbs./in.^{2} curve, the final quality is $q_0 = 0{\cdot}913$, and the volume at exit, $v_0 = 0{\cdot}913 \times 436 = 398$. The exit velocity $V_0 = 223{\cdot}7\sqrt{94{\cdot}76} = 2177$ ft./sec. Initial volume at 4 lbs./in.^{2}, $v_1 = 0{\cdot}975 \times 90{\cdot}5 = 89$.

$$\sqrt{\frac{p_1}{v_{s_1}}} = \sqrt{\frac{4}{89}} = \frac{1}{4{\cdot}75}$$

Throat area by equation (6)

$$A = \frac{9{\cdot}7 \times 4{\cdot}75}{0{\cdot}3155} = 146$$

Exit area $\qquad A_0 = \dfrac{144 \times 9{\cdot}7 \times 398}{2177} = 257$

Expansion ratio $\qquad a = \dfrac{257}{146} = 1{\cdot}76$

EXAMPLE 7.—Calculate the number and dimensions of the nozzles for the nozzle block of the first stage of the turbine (Example 6). The radial dimension of the nozzles at exit is to be $\frac{5}{8}$ inch, the pitch $1\frac{5}{8}$ inch, and the jet angle 20°. The thickness coefficient may be taken as 0·94. If the mean ring diameter is 69 inches, what percentage of the mean circumference is covered by the nozzle arc?

Since the expansion ratio is nearly unity, L may be taken as 1 inch and $\frac{\phi}{2}$ as $2°$.

By equation (24) the number of nozzles is

$$n = \frac{A_0}{cP \sin\left(\alpha - \frac{\phi}{2}\right) l_0} \qquad \begin{aligned} A_0 &= 4{\cdot}7 \\ P &= 1{\cdot}625 \\ c &= 0{\cdot}94 \\ l_0 &= 0{\cdot}625 \\ \alpha &= 20 \end{aligned}$$

$$= \frac{4{\cdot}7}{0{\cdot}94 \times 1{\cdot}625 \times 0{\cdot}309 \times 0{\cdot}625} \qquad \left(\alpha - \frac{\phi}{2}\right) = 18$$

$$\doteqdot 16 \qquad \sin 18 = 0{\cdot}309$$

Two blocks, each having eight nozzles, might be fitted.

Nozzle arc $x = nP = 16 \times 1{\cdot}625 = 26$ inches
Mean ring circumference $= 216{\cdot}74$

$$\text{proportion covered by nozzles} = \frac{26 \times 100}{216{\cdot}74} = 12 \text{ per cent.}$$

$$\text{with } l_0 = 0{\cdot}625, \quad b = \frac{A}{nl_0} = \frac{4{\cdot}082}{16 \times 0{\cdot}625} = 0{\cdot}408 \text{ inch}$$

Expansion ratio $a = 1{\cdot}15$

$$\therefore\ b_0 = 1{\cdot}15 \times 0{\cdot}408 = 0{\cdot}47 \text{ inch}$$

If the nozzle were made parallel the exit area would be 0·87 of the value calculated for complete expansion. This would give 13 per cent. of under-expansion, which would probably only have a very small effect on the efficiency (see Art. 74).

EXAMPLE 8.—The nozzle passages for the last stage of the turbine (Examples 6 and 7) are to be formed by steel plates cast in the rim of the diaphragm. The width perpendicular to the nozzle axis is to be kept constant, as shown in Fig. 71. The increase in area is to be obtained by increasing the radial dimension between throat and exit. The radial exit dimension is not to exceed $3\frac{1}{4}$ inches, and the nozzle arc, to allow for joints, etc., is not to cover more than 98 per cent. of the mean ring circumference. Mean ring diameter is 69 inches, nozzle pitch 2 inches, thickness coefficient 0·96. Calculate the jet angle and nozzle dimensions.

Mean circumference, 216·7 inches.
Nett nozzle arc

$$= x_0 = 0{\cdot}98 \times 216{\cdot}7 \times 0{\cdot}96 = 203{\cdot}8 \text{ inches.} \qquad A_0 = 263 \text{ in.}^2$$

Then $x_0 l_0 \sin \alpha = A_0$ $\qquad l_0 = 3{\cdot}25$

$$\therefore\ \sin \alpha = \frac{263}{203{\cdot}8 \times 3{\cdot}25} = 0{\cdot}398, \text{ and } \alpha = 23°\text{–}30'$$

Width of passage at exit perpendicular to the nozzle axis is

$$b_0 = P_0 \sin \alpha = cP \sin \alpha = 0{\cdot}96 \times 2 \times 0{\cdot}398 = 0{\cdot}764 \text{ inch}$$

This is also the throat width under the conditions specified. Expansion ratio $= 1{\cdot}76$; hence the radial dimension at the throat is

$$l = \frac{l_0}{a} = \frac{3{\cdot}25}{1{\cdot}76} = 1{\cdot}846 \text{ inches}$$

Assuming a slope of, say 10°, equivalent to a cone angle of $\phi = 20^\circ$, the length between throat and exit is $\mathrm{L} = \dfrac{l(a-1)}{2 \tan \frac{\phi}{2}}$

$$= \frac{1{\cdot}846 \times 0{\cdot}76}{0{\cdot}3526} = 3{\cdot}97 \text{ inches}$$

The axial dimension C, allowing for the curved portion of the plate at entrance, Fig. 71, should be about 2·5 inches.

The proportions worked out above are more or less arbitrary and depend, to a considerable extent, on judgment exercised in selecting the data which have to be assumed.

77. Proportions of Convergent Nozzles.—In the pressure compounded impulse machine the stage heat drop, which is kept sensibly constant throughout, is either equal to or less than the critical value, and the nozzles of the consecutive stages are convergent.

At any stage, for a given mean blade-ring diameter, the exit area A_0, as seen from equation (24), varies as the exit length l_0, the nett arc x_0, and the sine of the angle α which the nozzle axis makes with the plane of the wheel.

In fixing the exit dimensions there are thus three factors which can be arbitrarily varied.

It is the usual practice to maintain a constant value of l_0 and α, and to increase the nozzle arc until the gross value (np) approximates to the mean circumference of the blade ring. When this limit is reached there is full peripheral admission. To obtain greater areas either the exit length or the jet angle must be increased. Here again, as a rule, the length l_0 is varied and α kept constant, until a limiting value of l_0, reckoned as a percentage of the mean ring diameter, and determined by the practicable length of wheel blading, is reached. Any further increase of area has then to be obtained by an increase of angle.

In some instances, at the high-pressure end of the machine, where, with a minimum exit length, a small arc of admission results, the jet angle is decreased at the first few stages, in order to increase the arc. The standard angle α is 20°, but in those cases it may be varied from 15° to 20°.

The minimum exit length l_0, kept sensibly constant until full peripheral admission is approached, is dependent on the practicable minimum length of the wheel blading on to which the nozzles discharge. This minimum value of wheel-blade length may be taken from 1·5 to 2· per cent. of the mean ring diameter and is slightly greater than the radial exit length of the nozzle.

Except in the cases of machines having a very large output, the ratio of maximum blade length to wheel diameter does not exceed 20 per cent. at the last L.P. ring. Usually with moderate outputs it varies from 8 to 12 per cent.

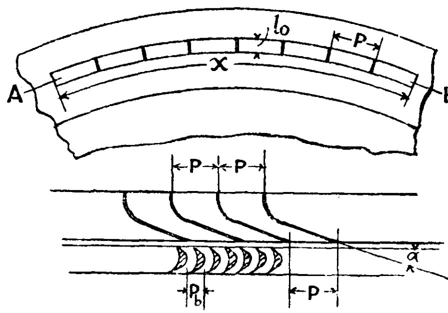
By means of a curve of cumulative heat, the construction of which is discussed in Chapter XI., the successive stage pressures for any arbitrary distribution of the total amount of heat can be approximated.

When the stage pressure limits are thus determined, the calculation of each set of nozzles can be carried out by the method just described.

The initial volumes are given by the condition curve, also discussed in Chapter XI.; but when the number of stages is small, the pressures can be readily found by trial on the Hϕ diagram, and the corresponding qualities and volumes obtained by allowing for a definite reheat in each stage.

The condition curve for the multistage impulse machine is drawn on the assumption that in the saturation region the steam reverts to the saturated condition in each wheel chamber, before entering the nozzles of the succeeding stage, and the approximate initial qualities are then given by the curve.

The assumption may not be quite true, but it enables the problem to be handled in a simple and expeditious manner without much material error. In any case the derived volume curve can, when desired, be modified for the range of pressure over which it may be assumed the condition of supersaturation exists.



DEVELOPED SECTION ON A-B.

FIG. 85.

When any stage heat drop h_r, exit velocity V_0, and exit volume v_0, are determined, the exit area is then calculable from equation (2) or (8).

Referring to Fig. 85, which shows a developed section of the usual type of convergent nozzle, for any arbitrary exit length l_0, thickness coefficient c, and jet angle α, the gross arc of admission is

$$x = n\mathrm{P} = \frac{\mathrm{A}_0}{cl_0 \sin \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The band of steam flowing through this arc has to be split into a series of elemental streams to prevent disorderly flow and loss of energy. This end is accomplished by the nozzle walls and plates, which transform the arc into a set of closely packed nozzle passages. The number of nozzles, on a given arc, varies inversely as the pitch P, which is an arbitrary quantity.

Usually it is found in practice that this pitch may be varied from three to four times the corresponding pitch of the wheel blades on to which the nozzles discharge.

If P_b is the pitch of the wheel blades, c_1 the corresponding thickness coefficient, and m the number of blade pitches per nozzle pitch, then

$$P = m\frac{c_1}{c}P_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If $c = c_1$, then $P = mP_b$.

When the exit areas are calculated and tabulated, the stage at which full peripheral admission begins is readily ascertained, by calculating the nett exit area, on an arc slightly less than the full circumference, so as to allow for joints.

A reference to the table shows at what stage the required area is in excess of this amount.

From this stage onward an arbitrary variation of height and angle can be adopted. This is a matter of judgment. The increase of exit height at the last few stages should not be too abrupt, as this entails a considerable radial spread of the steam between discharge from the wheel blades of the stage and entry to the nozzles of the next.

It is desirable also to make the inlet angle of the nozzles the same as that of the absolute velocity of the steam, as it leaves the wheel blades of the previous stage. By doing so a considerable proportion of the residual energy can be utilised, and a slight improvement of the over-all efficiency of the turbine obtained.

EXAMPLE 9.—A pressure compounded impulse turbine has eight stages. The absolute admission pressures and qualities at the successive stages are, approximately, those given below.[1]

Stage	1.	2.	3.	4.	5.	6.	7.	8.
Pressure	185	110	63	35	18	8·8	4	1·75
Quality	200° F.	152° F.	110° F.	65° F.	17° F.	$q = 0{\cdot}985$	0·963	0·943

The steam is dry at 15 lbs./in.^{2} abs. The exhaust pressure is 0·75 lb./in.^{2} abs. The effective carry-over velocity at entrance to the second and succeeding stage nozzles is 450 ft./sec. The steam consumption is 7·5 lbs./sec. Calculate the nozzle exit areas for the eight stages.

Scaling the heat drop for the first stage between 185 and 110 lbs./in.^{2}, this will be found equal to 55 B.Th.U. At the other stages it will be found equal approximately to 51 B.Th.U.

The effective carry-over velocity represents $\dfrac{V_1^2}{(223{\cdot}7)^2}$, or 4 B.Th.U., and the equivalent heat drop for each of these stages is ${}_1h_r = (51 + 4)$

[1] Refer to Example 4, Chapter XI. p. 339.

$= 55$ B.Th.U., the same figure as the actual heat drop in the first stage. The reheat for each of these stages is therefore $h_f = (1 - \eta_n)55$.

For the first four stages with superheated steam the higher efficiency of 92 per cent. may be assumed. At the fifth stage a slightly lower value, say 0·9, should be used. At stages 6, 7, and 8 the lower value of, say, 0·88 should be taken in order to allow for probable supersaturation loss.

The reheat for the eight stages, based on these assumed efficiencies, is then as follows:—

Stage	1.	2.	3.	4.	5.	6.	7.	8.
h_f	4·4	4·4	4·4	4·4	5·5	6·6	6·6	6·6

Scaling off $h_f = 4·4$ from the lower end of each adiabatic vertical on the $H\phi$ diagram for the first four stages, and projecting to the lower pressure curve, the probable qualities at exit are obtained, and from these the volumes. The exit velocity at each nozzle is $V_0 = 223·7\sqrt{55 \times 0·92} = 1590$ ft./sec. The results thus obtained are tabulated below:—

Stage	1.	2.	3.	4.
Effy. η_n	0·92	0·92	0·92	0·92
,, t_{s0}	130° F.	80° F.	35° F.	$q = 0·994$
,, v_{s0}	4·89	7·72	12·6	22·16

There may be slight supersaturation at the fourth stage at exit, but to allow for reheat the steam is assumed to be dry.

On the assumption that the steam is supersaturated in the nozzles of the other stages, the approximate volumes are calculated by equation (12), $v_0 = v_1\left(\frac{p_1}{p_0}\right)^{\frac{1}{1·3}}$.

For clearness both the initial and final values are tabulated below.

Stage	5.	6.	7.	8.
p_1	18	8·8	4	1·75
v_1	22·8	43	86	187
p_0	8·8	4	1·75	0·75
v_0	40 (40·5)	79 (80)	163 (166)	360 (364)

The calculated volumes are smaller than the actual, on account of the reheat in the nozzles, and are increased about 1 per cent. to take account of this. The figures are given in brackets in the last line.

The complete data for the calculation of the nozzle dimensions are collected below. The areas given in the last line are calculated by equation (8), $A_0 = \dfrac{144Wv_{s0}}{V_0}$.

Stage	1.	2.	3.	4.	5.	6.	7.	8.
Effy. η_n . .	0·92	0·92	0·92	0·92	0·9	0·88	0·88	0·88
Exit vel. V_0 .	1590	1590	1590	1590	1570	1552	1552	1552
,, vol. v_{s0} .	4·89	7·72	12·6	22·16	40·5	80	166	364
,, area A_0 .	3·33	5·26	8·58	15	27·8	55·6	115	253

EXAMPLE 10.—In the foregoing case calculate suitable nozzle dimensions at each stage, assuming that the passages are formed by steel plates cast in the diaphragm walls. (See Figs. 68, 85.)

The mean wheel diameter is 39 inches for each stage. The minimum exit length of the nozzles is to be 2 per cent. of the mean diameter, and the maximum length 12 per cent. The average jet angle at the H.P. end may be taken as 20°. The pitch of the nozzles may be varied from $1\frac{1}{2}$ to $1\frac{5}{8}$ inch, and the thickness coefficient may be taken as 0·96. It may also be assumed that from 2 to 3 per cent. of the mean circumference is taken up by the diaphragm joints at the full admission L.P. stages.

Taking the first stage, the minimum of 2 per cent. gives a nozzle exit length, $l_0 = 0{\cdot}02 \times 39 = 0{\cdot}78$, say $\frac{3}{4}$ inch. Taking the pitch, $P = 1{\cdot}5$, $c = 0{\cdot}96$, and $\alpha = 20°$, then, by equation (26), the gross arc of admission is

$$x_1 = \frac{A_0}{cl_0 \sin \alpha} = \frac{3{\cdot}33}{0{\cdot}96 \times 0{\cdot}75 \times 0{\cdot}342} = 13{\cdot}5 \text{ inches}$$

The mean ring circumference is 122·53, and the nozzle arc covers $\frac{13{\cdot}5 \times 100}{122{\cdot}53} = 11{\cdot}1$ per cent. of the circumference.

By equation (26) $\quad n_1 = \frac{x}{P} = \frac{13{\cdot}5}{1{\cdot}5} = 9$ nozzles

Exit width of nozzle passage

$$b_0 = \frac{A_0}{nl_0} = \frac{3{\cdot}33}{9 \times 0{\cdot}75} = 0{\cdot}495 \text{ inch}$$

If the length, blade angle, and pitch are kept the same in the succeeding stages till full admission is reached, then for stage 2

$$n_2 = \frac{9 \times 5{\cdot}26}{3{\cdot}33} = 14{\cdot}2, \text{ say } 14 \text{ nozzles}$$

and $\quad x_2 = 14 \times 1{\cdot}5 = 21$ inches or 17·15 per cent. of the circumference

For stage 3, $\quad n_3 = \frac{9 \times 8{\cdot}58}{3{\cdot}33} = 23{\cdot}2$, say 23 nozzles

$x_3 = 23 \times 1{\cdot}5 = 34{\cdot}8$ inches, or 28·4 per cent.

For stage 4, $\quad n_4 = \frac{9 \times 15}{3{\cdot}33} = 40{\cdot}6$, say 41

$x_4 = 41 \times 1{\cdot}5 = 61{\cdot}5$ inches or 50·2 per cent.

For stage 5, $\quad n_5 = \frac{9 \times 27{\cdot}8}{3{\cdot}33} = 75$

$x_5 = 75 \times 1{\cdot}5 = 112{\cdot}3$ or 92 per cent.

Full admission is necessary at the next three stages. Taking the gross arc as 98 per cent. of the circumference, then $x = 0{\cdot}98 \times 122{\cdot}53 = 120$ inches. The wider blade pitch, $1\frac{5}{8}$, may be taken here, as slightly broader blades would probably be used at these stages.

Taking stage 8, the maximum blade length is restricted to 12 per cent. of the diameter. Hence, $l_0 = 0{\cdot}12 \times 39 = 4{\cdot}7$, say $4\frac{3}{4}$ inches.

By equation (26)

$$\sin a = \frac{A_0}{cxl_0} = \frac{253}{0{\cdot}96 \times 120 \times 4{\cdot}75} = 0{\cdot}462$$

and $\quad a = 27^\circ\text{–}30'$

Also $\quad n_8 = \dfrac{120}{1{\cdot}625} = 74$

and $\quad b_0 = \dfrac{253}{74 \times 4{\cdot}75} = 0{\cdot}725$ inch

In order to obtain a reasonable progression of blade length, and to avoid excessive radial spread of the steam between each stage, the nozzle angle may be progressively increased from stage 5 to 8, say 22° at 6 and 24° at 7. The blade length at stage 6 is

$$l_0 = \frac{55{\cdot}6}{0{\cdot}96 \times 120 \times 0{\cdot}374} = 1{\cdot}29 \text{ inches, say } 1\tfrac{1}{4} \text{ inches}$$

The number of nozzles

$$n_6 = \frac{120}{1{\cdot}625} = 74, \text{ and } b_0 = \frac{55{\cdot}6}{74 \times 1{\cdot}25} = 0{\cdot}6 \text{ inch}$$

At stage 7,

$$l_0 = \frac{115}{0{\cdot}96 \times 120 \times 0{\cdot}4067} = 2{\cdot}46 \text{ inches, say } 2\tfrac{1}{2} \text{ inches}$$

$$n_7 = 74, \text{ and } b_0 = \frac{115}{74 \times 2{\cdot}5} = 0{\cdot}623 \text{ inch}$$

The complete nozzle data are collected in tabular form below :—

Stage	1.	2.	3.	4.	5.	6.	7.	8.
Area A_0. . . .	3·33	5·26	8·58	15	27·8	55·6	135	253
Exit length l_0 . .	$\frac{3}{4}''$	$\frac{3}{4}''$	$\frac{3}{4}''$	$\frac{3}{4}''$	$\frac{3}{4}''$	$1\frac{1}{4}''$	$2\frac{1}{2}''$	$4\frac{3}{4}''$
No. of nozzles n .	9	14	23	41	75	74	74	74
Nozzle arc per cent.	11·1	17·15	28·4	50·2	92	98	98	98
Pitch P	$1\frac{1}{2}''$	$1\frac{1}{2}''$	$1\frac{1}{2}''$	$1\frac{1}{2}''$	$1\frac{1}{2}''$	$1\frac{5}{8}''$	$1\frac{5}{8}''$	$1\frac{5}{8}''$
Exit width b_0 . .	0·495″	0·5″	0·496″	0·485″	0·495″	0·6″	0·623″	0·725″
Jet angle a . . .	20°	20°	20°	20°	20°	22°	24°	$27\frac{1}{2}°$

As in the case of the previous machine, the foregoing represents only one possible set of values that might be used. Considering the number of arbitrarily variable factors involved, and which have to be determined by the "personal equation," any pretence at extreme accuracy would be absurd. The figures are thus to be regarded as provisional and subject to alteration, in accordance with the individual judgment and experience of the designer.

78. Nozzle Lead.—As already stated in Art. 77, in order to utilise the residual energy at the successive stages of a compound impulse turbine, the inlet nozzle angles should be made the same as the jet exit angles at discharge from the wheel blades. Where there is partial admission the successive nozzle arcs should be so situated that the steam on discharge is directed on to them and not on to the blank wall of the diaphragm. Usually, in the case of the pressure compounded machine, in which the ratio of the relative steam velocity in the channel and the absolute blade velocity is small, all that is necessary is a fairly large axial clearance between the exit edges of the wheels and the walls of the successive diaphragms.

If the wheel at any stage were kept close up on the wall of the exit diaphragm the nozzle arc in this diaphragm would require to have an angular displacement or "lead" relatively to the arc of the previous diaphragm. Usually the increase in the length of arc, to pass the increased volume of steam at exit, is greater than the circumferential displacement required for "lead." Hence it is sufficient to provide only the axial clearance to permit of the circumferential spread of the band of steam after discharge from the wheel blades.

In the velocity compounded type the velocity ratios are lower than in the pressure compounded machine. As a rule no attempt is made, at a partial admission stage, to pass the steam directly into the succeeding stage nozzles, but it is necessary to ensure that the band of steam discharged from the nozzle arc, and which does not increase in circumferential length as it passes through the stage, is discharged properly into the sector of intermediate guide blades. Usually in a two-velocity stage "spilling" over the ends of the intermediate sector is prevented by extending the sector several pitches in the direction of rotation. In a three-velocity stage it may be necessary, in the case of the second sector of intermediates, to give this a certain amount of lead as well.

Although this question of nozzle lead is a minor one, it is desirable to indicate the method by which the "lead" for any given case may be calculated, if it is desired to ascertain the amount.

Referring to Fig. 86, suppose a moving blade, shown unsymmetrical, moves with an absolute velocity u, and at a given instant reaches the position (*a*).

It receives the steam discharged from the nozzle at the entrance edge I. As the blade moves forward the steam moves over the blade face between entrance I and exit O with some relative velocity V_r. The path traversed by the steam in the blade channel is y. In the time t taken by the steam to traverse the distance y in the channel, the blade moves from position (*a*) to position (*b*), a distance S.

Assuming uniform velocity in each case, $y = V_r t$ and $S = ut$, hence $S = \dfrac{uy}{V_r}$.

The blade for generality being assumed unsymmetrical and the inlet edge I being a distance d circumferentially behind the exit edge O, the nett displacement of the steam circumferentially is $(S-d)$.

This would be the amount of "lead" of the arc if the wheel were a close fit between the diaphragm walls and the steam were discharged from the wheel at O_1, into the next set of nozzles.

This lead is obviously modified by the amount of axial clearance Z_1 and Z_2, arbitrarily allowed, and the final inclination α_0 of the jet to the plane of the wheel at exit. The correct location is obtained by drawing the velocity diagram, shown dotted.

The total lead L may be calculated as follows, when the clearances Z_1 and Z_2 are provisionally fixed and the velocities and their directions determined by the methods discussed in Chap. VIII.

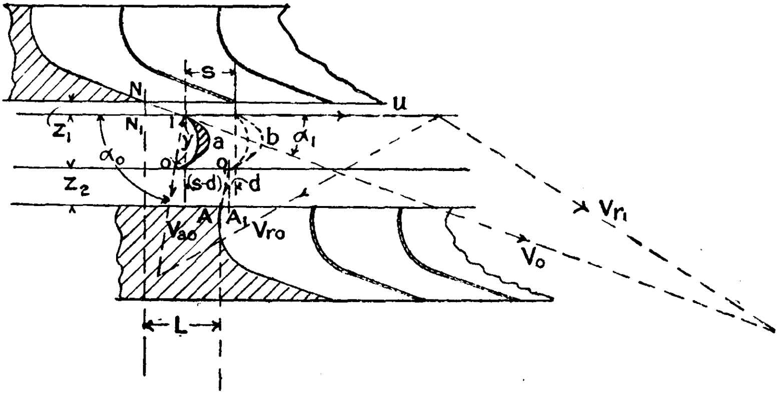


FIG. 86.

On account of the nozzle clearance Z_1 the steam after discharge from the nozzle at N has to traverse the diagonal distance NI $= Z_1/\sin\alpha_1$ before it reaches entrance edge I of the blade. Similarly after discharge from the wheel at O_1, it has to traverse the distance $O_1A = Z_2/\sin\alpha_0$, where α_0 is the jet angle at exit.

The circumferential components of these displacements are $N_1I = +Z_1 \cot\alpha_1$ and $AA_1 = -Z_2 \cot\alpha_0$.

The total lead is the sum of all the circumferential displacements

$$L = \left(Z_1 \cot\alpha_1 + \frac{uy}{V_r} - d - Z_2 \cot\alpha_0\right) \quad . \quad . \quad (28)$$

If the angle subtended by y is $\gamma = (180 - \theta_1 - \theta_0)$, where θ_1 and θ_0 are the entrance and exit blade angles in degrees and r the radius of the blade face, then

$$y = r\gamma = r\frac{(180-\theta_1-\theta_0)}{57{\cdot}3}$$

If the blade is symmetrical

$$\theta_1 = \theta_0,\ d = 0, \text{ and } y = r\frac{(180-2\theta)}{57{\cdot}3}$$

EXAMPLE 11.—In a pressure compounded impulse turbine the entrance jet angle $\alpha_1 = 18°$ and the exit jet angle $\alpha_0 = 60°$. The nozzle clearance $Z_1 = 0{\cdot}187$ inch and the wheel blade clearance $Z_2 = 1$ inch. The blade angles are $\theta_1 = 28°$ and $\theta_0 = 23°$, the radius of the blade face $r = 0{\cdot}6$ inch and the blade lap $d = 0{\cdot}05$ inch. The mean blade speed $u = 304$ ft./sec. and the average relative velocity $V_r = 516$ ft./sec. Calculate the nozzle lead required, and the angular advance of the nozzle arc if the mean ring diameter is 7 feet.

Here $$y = r\frac{(180 - \theta_1 - \theta_0)}{57{\cdot}3} = 0{\cdot}6\frac{(180 - 28 - 23)}{57{\cdot}3} = 1{\cdot}35$$

$$\alpha_1 = 18°, \quad \cot 18 = 3{\cdot}077, \quad \alpha_0 = 60°, \quad \cot 60° = 0{\cdot}5774$$

Hence by equation (28)

$$\begin{aligned} L &= \left(Z_1 \cot \alpha_1 + \frac{uy}{V_r} - d - Z_2 \cot \alpha_0\right) \\ &= 0{\cdot}187 \times 3{\cdot}077 + \frac{304 \times 1{\cdot}35}{516} - 0{\cdot}05 - 1 \times 0{\cdot}5774 \\ &= 0{\cdot}575 + 0{\cdot}795 - 0{\cdot}6274 \\ &= 1{\cdot}370 - 0{\cdot}6274 = 0{\cdot}742, \text{ say } \tfrac{3}{4} \text{ inch} \end{aligned}$$

This corresponds to an angular displacement of $\delta = \frac{0{\cdot}742}{42} = 0{\cdot}0177$ radian or $1{\cdot}0°$ on a circle of 7 feet diameter, a very small amount.

The circumferential lead required here is about half a nozzle pitch and much less than the increase in nozzle arc required to take the increased steam volume at the stage in question. Hence the increase added at the forward side of the arc is sufficient to prevent shock on the diaphragm wall at the forward edge of the jet, and nozzle lead need not be given.

CHAPTER VIII

BLADING

79. Impulse Blading.—The form of blade used in the de Laval turbine, and the method of fixing it to the wheel rim, is illustrated in Fig. 87.

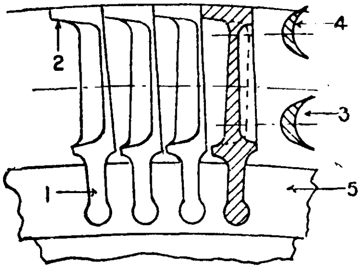


FIG. 87.

The blade is symmetrical, that is, the entrance and exit angles are equal, and it tapers in thickness between the root and tip. At the root it is provided with a thin tail-piece 1, enlarged and rounded at the bottom. This fits into a slot in the wheel rim 5, into which it is pressed. A rectangular projection 2 is provided at the tip towards the back. This butts on the face of the adjacent blade. When the wheel is bladed, these projections constitute an elastic shroud ring. This maintains the correct spacing of the blades, restricts the radial spread of the jet in the blade channels, and assists in the suppression of vibration. At the same time it admits of a slight amount of expansion of the blade ring under varying temperatures.

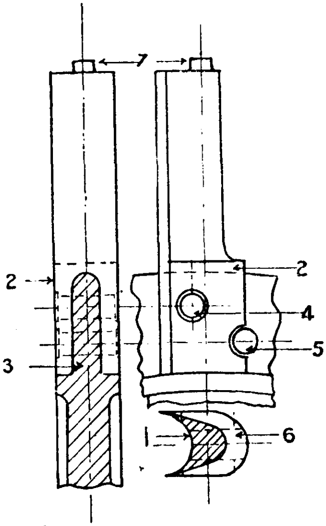


FIG. 88.

The blade has a crescent-shaped section, as shown at 3 and 4. In the small-powered machine it is stamped out of nickel steel; but for the larger machines it is milled out of solid steel bar. The illustration shown is drawn from the milled-out blading used in machines from 300 to 500 B.H.P. The outlet edge of the blade is ground to the required exit angle.

The standard blade used in the pressure compounded Rateau turbine is illustrated in Fig. 88. As shown at 1 it is crescent-shaped, and is usually unsymmetrical, the exit being less than the entrance angle. At the root 2 it is forked, and straddled over the reduced portion of the wheel rim 3. It is attached to this by two countersunk rivets, 4 and 5. The correct pitching of the blades is maintained by widening the blade circumferentially at the root. The projection 6, thus formed, is milled to fit the curve of the adjacent blade face. It should be noted that while one of the rivet holes, 4, is bored through the fork, the other, 5, is bored at the junction of the fork with the adjacent blade. This second rivet 5 serves to hold both forks. A tongue, 7, is formed at the tip. By means of this the sections of steel shroud ring, passed round the tips of the blades, are riveted in position. This ring is fastened on in sections to allow for a certain amount of expansion with variation of temperature.

It helps to stiffen the blades, keeps the pitching correct at the tips, and suppresses vibration. It is also of importance in preventing the radial spread of the jet in the blade channel.

The blade illustrated is that used in the Westinghouse turbines, Figs. 23 and 24. The arrangement of the blades in the rim of the wheel is shown in Fig. 89, Plate XI. These blades are milled from solid steel bar. A 5 per cent. nickel steel is used.

The type of blade used in velocity compounded and combination machines is shown in Fig. 90. The usual practice is to fit the blades into grooves, 1, turned in the rim, 2, of the wheel. The grooves shown in the illustration have a double dovetail. The corresponding groove in the stationary part or foundation ring 5, which carries the intermediate blades, has a single dovetail. This is the form of groove used by most makers, both for wheel and intermediate blading, but in some cases the **T** slot is also used. The section of the blade 3 is usually made unsymmetrical, the exit angle being less than the entrance angle. The section of the intermediate blades 4 is similar to that of the wheel blades. The diminishing thickness of the blades in the successive rows should be noted. This is due to the increasing angles at entrance and exit necessitated by the decreasing velocity of the steam.

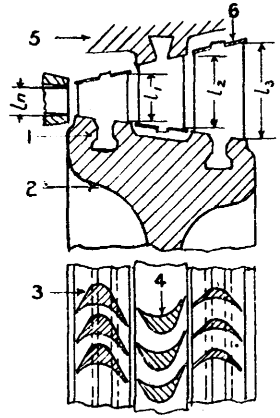


FIG. 90.

For the same reason, on account of the narrowing of the blade channels at exit, the radial depth or length of the blades progressively increases.

This increase is, however, of an arbitrary character, and subject to the judgment of the designer, as the actual conditions of flow through these blades is not known with any degree of certainty.

These blades are usually milled out of nickel steel, but sometimes drawn bronze blades are used, where the steam is not highly superheated.

PLATE XI.

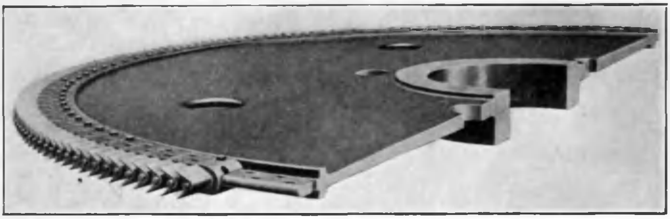


FIG. 89.—Method of Fixing Rateau Blading.

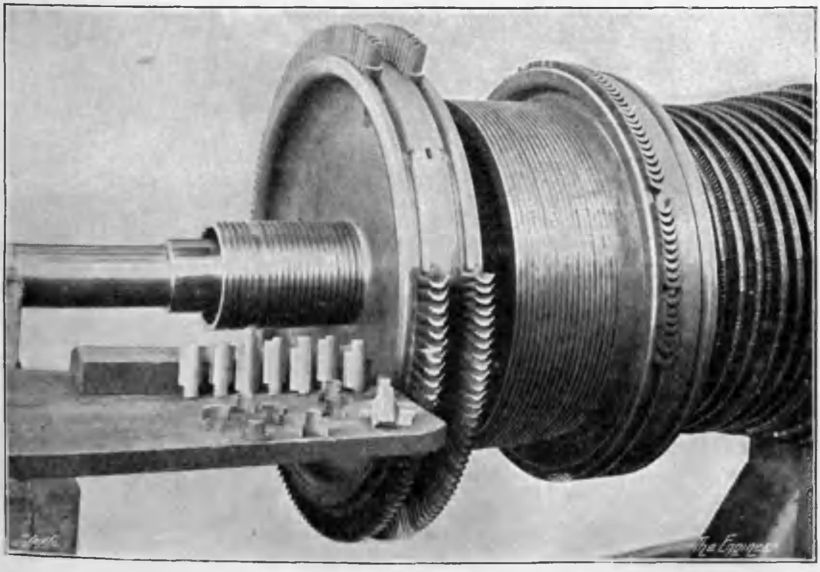


FIG. 91 —Method of Fixing Franco-Tosi Blading.

The pitching of the blades is determined by distance pieces fitted in the grooves. These and the blades are inserted through a wider gap on the circumference of the wheel rim. A special distance piece is fitted into this slot and riveted to the rim after the wheel is bladed.

Each ring of blades is provided with a shroud ring, 6, riveted round the circumference in sections.

The method of fixing the wheel and intermediate blades in the Franco Tosi combination turbines, Figs. 46 and 47, is illustrated in Fig. 91, Plate XI.

Each groove has parallel sides, and each side has several **V** grooves cut into its surface. The section thus resembles that of a **V**-threaded screw.

The tail end of each blade and distance piece, as shown by the

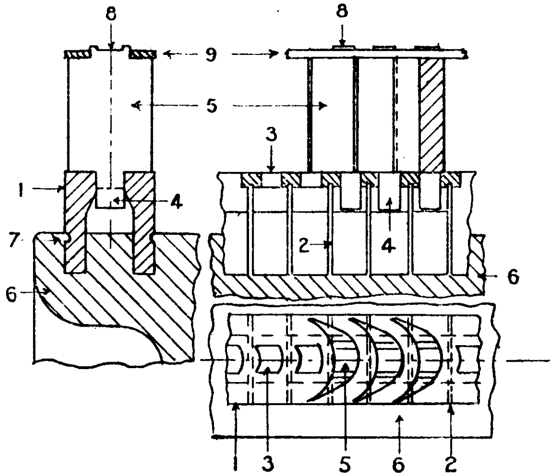


FIG. 92.

detached blading on the left, is cut in a similar manner. The blades and distance pieces are inserted through the two slots shown on the unbladed part of the rim. This method of fixture is also used in the marine turbine, Fig. 48.

The method of attachment employed by the Fore River Co. in the marine Curtis turbines is shown in Fig. 92. In this case a foundation ring is used to carry the wheel blades as well as the intermediates.

This steel ring 1 is channel-shaped in section, and divided into a number of sectors. The sides of each sector are provided with saw cuts, 2, so that it can easily be bent to any required radius.

Holes, 3, are punched in the ring to take the rectangular tails 4 of the blades 5. By means of the tails these blades, which are made of drawn bronze, are securely riveted to the foundation ring.

Two grooves are turned in the wheel rim 6, to take the sides of channel sectors. A small groove, 7, is turned on each side of the sector at the rim surface, and into this the rim metal is caulked.

In this way the sectors of the foundation ring are locked in place in the rim. A tongue, 8, is provided at the tip of each blade by which the shroud ring 9 is riveted.

In this design the radial depth of the blade is kept constant between entrance and exit. This blading arrangement can obviously be rapidly and cheaply put together. Also, the riveted-in blades obviate the use of the distance pieces used in the other constructions.

The special type of velocity compounded machine represented by the Sturtevant turbine has no attached blades. Their equivalents are "buckets" cut in the wheel rim (Fig. 103). They are milled out by a special cutter, and have walls $\frac{3}{32}$ inch thick. Each wall is filed to a sharp edge at entrance. The bucket has a slight taper, and the bottom has the form of a circular arc.

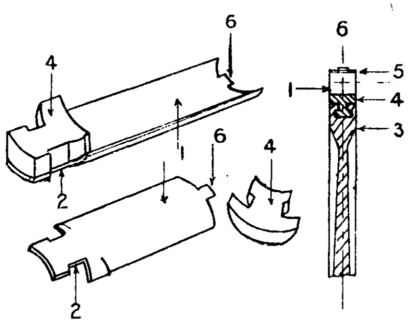


FIG. 93.

The blading, and method of fastening used on the Zoelly impulse machines are illustrated in Fig. 93.

The blade 1, in this case, is made of sheet steel in the land turbines. In marine turbines, where velocity compound stages are also employed, the usual crescent-shaped blade is used. The material is 5 per cent. nickel steel. It is bent to the required curvature, and notched at the ends 2, to fit the **T**-slot groove in the wheel rim 3. A steel distance piece, 4, is placed between each pair of blades to ensure the requisite pitch. A shroud ring, 5, is riveted round the tips by means of the tongue 6. The small-sized blades at the H.P. end are of uniform thickness, but at the L.P. stages the long blades taper slightly from root to tip. The inlet and outlet edges are ground to the requisite angles.

80. Reaction Blading.—The standard form of reaction or Parsons blading employed on the axial flow type of machine is shown in Fig. 94. This blading is drawn in lengths through special dies of various sizes standardised by the Parsons Company.

It is made of brass or copper or a special nickel copper alloy. From these drawn strips the requisite blade lengths are cut. Each blade is indented at 1, near the root, with one or two punch marks. The blades, with soft distance pieces 2, are packed into the groove 3, cut on the circumference of the drum 4.

Each side of the groove has several small grooves, 5, cut in it. By means of a special caulking tool the soft distance pieces are caulked down into the groove. These spread circumferentially into the notches on the blades and axially into the small grooves in the sides, and securely lock the blades in the wheel rim.

In order to maintain the requisite pitching at the blade tips, the blades are either cut or bored at the entrance side, and a thick wire, 6, is passed through the holes. At each blade this is further tied in by a fine copper wire, and the combination is made doubly secure by silver soldering at each blade. This arrangement further stiffens the blading and suppresses vibration, which may give considerable trouble in the case of long blades at the L.P. stages. In some instances a second binding wire is provided about the middle of the blade length.

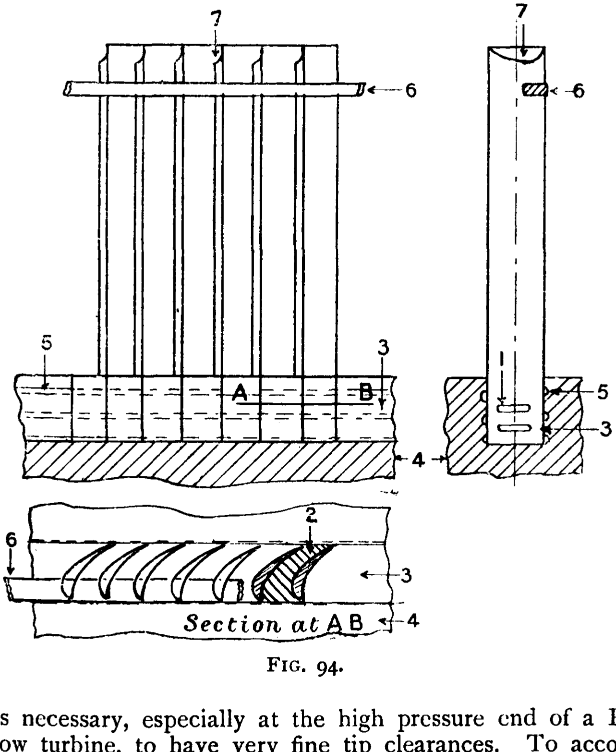


FIG. 94.

It is necessary, especially at the high pressure end of a Parsons axial flow turbine, to have very fine tip clearances. To accomplish this end each blade is thinned at the tip, 7, to a fine knife edge or "fin." If the blades should "touch" the casing or drum surface, this fine edge is rubbed down without serious generation of heat. Seizure and consequent "blade strip" is thus avoided. This simple device, which has been adopted for other running parts, where fine clearance is necessary, serves its purpose quite satisfactorily.

This method of blade fixture in the drum is the original one; and the blading of a drum is a tedious mechanical job when it is adopted.

It has now been practically superseded by a method analogous to that employed in the marine Curtis turbine. Sectors of blading are independently constructed and fitted successively into the groove in the drum.

These segments are constructed on the "rosary" system. The roots of the blades and distance pieces are provided with holes by means of which a series of blades and pieces can be strung on a wire. This set is assembled in a special jig or cramp, in which the parts are driven tightly together. The wire is then soldered at each end, and the binding wire fixed at the tips. The segment is then ready for fitting into the groove on the drum.

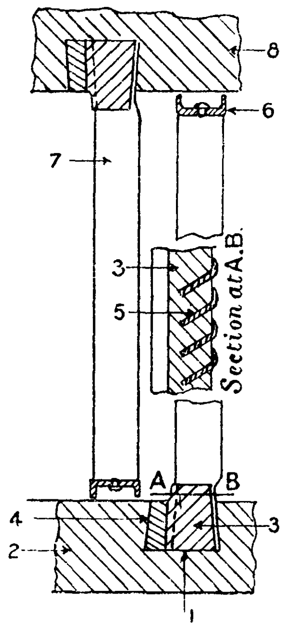


FIG. 95.

In some cases separate foundation rings are used and keyed into the drum grooves. The special fastening adopted in the Willans machines is illustrated in Fig. 95. A dovetail groove, 1, is cut in the drum, 2. The foundation ring, 3, is fitted into this. The ring is made of brass when saturated steam is used, and of steel when superheated steam is used. It is fixed in the groove by a caulking strip, 4, of soft metal, which is caulked in and wedges the ring into the groove. Each blade is cut with a dovetail projection or "tang," 5, at the root. This is driven into slits cut in the foundation ring 3. At the section AB, there is an abrupt change from the curved to the flat form of section at the exit edge, which is not desirable from the efficiency point of view. As the metal is probably severely punished in the clinching process, the chances of fracture under continuous vibration, due to incipient cracks thus set up, are greater than in the case of the rosary construction. Instead of the usual thinning at the tips, the blade thickness is maintained, and a tongue provided to fasten the channel-shaped shroud ring, 6. Each channel side is reduced to a knife edge to permit of the use of fine radial clearance. The stationary blade, 7, is fixed on the stator or casing, 8, in the same way as the wheel blade.

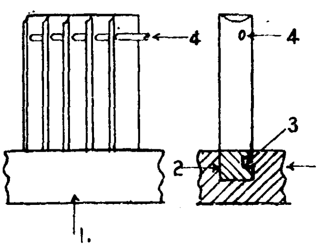


FIG. 96.

Another method of blading, used in the Brown-Boveri Parsons turbine, is illustrated in Fig. 96. In this instance, the groove, 2, in the drum, 1, is provided with a projecting ring, 3, turned on one side. The blades and distance pieces, inserted through a slot, are provided with corresponding slots into which the ring projects. The blades and distance pieces are simply slipped into position and pressed hard together. The distance pieces are not caulked, as the ring projection effectively prevents any loosening of the blades due to temperature change or other cause. The blades are bored near the tip to take the binding wire, 4, and the tips are thinned, as in the previous cases.

The most interesting type of blade construction is that employed in the radial flow Ljungström turbine. The section of the blade approximates to that of the Parsons. The main difference between this and other blade constructions is the autogenous welding of the blades to the blade rings, and the fixture in turn of these rings to the disc by special expansion rings. The whole construction is of such a nature that there is no possibility of the blades working loose when high ranges of superheat are employed. Another feature is the small widths

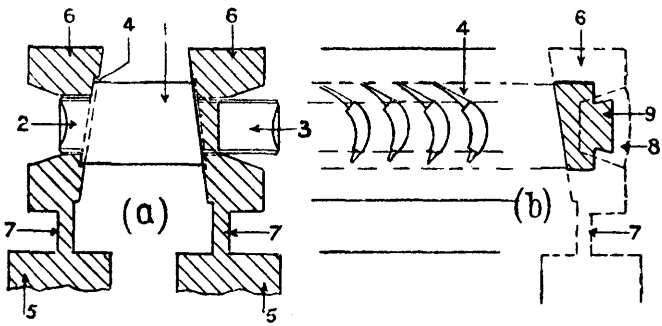


FIG. 97.

of the blades as compared with the widths used in the axial-flow Parsons turbine.

The blading, which is made of nickel steel, is milled from solid bar in strips 3 feet long. From these strips suitable blade lengths are cut. The blade surfaces are highly polished on face and back.

Referring to Fig. 97 (*a*), the ends of each blade, 1, are notched to form the projections 2 and 3. Each blade end is inserted in a check, 4, on the inner face of the ring, 6, which is formed with a tapering channel section. The projections, 2 and 3, pass through slots in the ring. Each ring is turned down from a disc of metal, 5, the connection being retained by the web, 7, until the blading operation is completed. The discs are threaded on a mandril, the blades are placed in position in the rings as shown at (*a*), and the combination is drawn tightly together. The ends, 2 and 3, are then welded into the rings. Metal is melted into each channel until it is filled as shown by 8 in (*b*). On account of the narrow connection, 7, the discs are not unduly heated during the welding process and distorsion is avoided. When the blades are welded in, the portion of the ring indicated by the dotted lines in (*b*) is turned off to give the final dovetail form, 9, shown in section. The blade ring is then parted from each disc, 5, and is ready for insertion in the main or strengthening ring, 10, shown in Fig. 98. This is the ring situated nearest the turbine disc. It is thickened at the edges, 11, as indicated by the dotted lines. A semicircular groove is turned to take the bulb expansion ring, 12, and a rectangular groove to take the dovetail, 9, of the blade ring. By rolling the internal and external circumferences of the ring, 10, the metal at 11 is forced into and grips the bulb at 12, and the dovetail at 9. The other end of the blade ring is likewise fastened to an outer stiffening ring. By this simple mechanical process the two strengthening rings, the blade ring and the expansion ring, are securely fastened together. The bulb, 15, of the expansion ring is similarly fitted in a clamping ring, by which it is attached to the disc face.

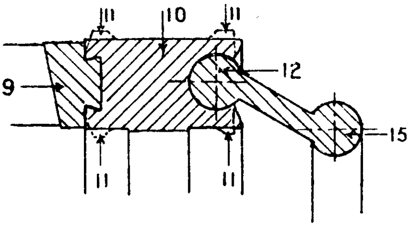


FIG. 98.

A section of the combination, fitted on the disc, is shown in Fig. 99. The numbers on the rings correspond to those given in Figs. 97 and

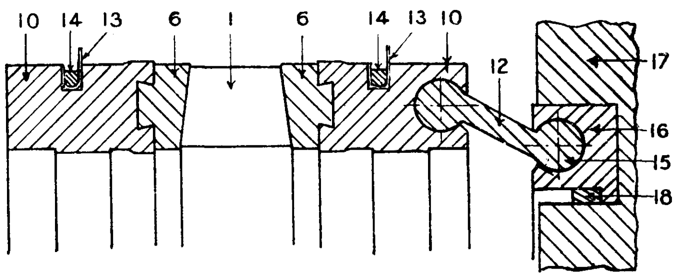


FIG. 99.

98. The circumference of each strengthening ring is grooved at 14. Into this groove is fitted and caulked a thin nickel strip, 13. The clearance between the circumference of each ring and the adjacent strengthening ring is very small, and leakage to either side is effectively prevented. The clamping ring, 16, fits into a groove cut in the face of the disc, 17, and is fixed in position by caulking strips, 18.

81. Work done on Turbine Blading.—The following notation is used in calculations relating to blading.

Subscripts a and r refer respectively to "absolute" and "relative," and 1 and 0 to "entrance" and "exit" values.

Blade Angles—

Angle of entrance to blade channel . . . . . θ_1

" exit from " " . . . . . . θ_0

Jet Angle—

Inclination of jet to plane of wheel, or to the tangent in a radial flow turbine $\left\{ \begin{array}{ll} \text{at entrance} & \alpha_1 \\ \text{at exit} & \alpha_0 \end{array} \right.$

Steam Velocities—

	Entrance.	Exit.
Absolute velocity	V_{a_1}	V_{a_0}
Relative ,, ...	V_{r_1}	V_{r_0}
Velocity of whirl	V_{w_1}	V_{w_0}

Angular velocity of blading . . .	ω
Peripheral velocity of blading . . .	u
Work done on blading	E_b

In its passage through the blade channels, at any stage, the steam impresses a dynamical force on the blade ring. The component of this force, in the direction of motion of the blades, is the effective turning

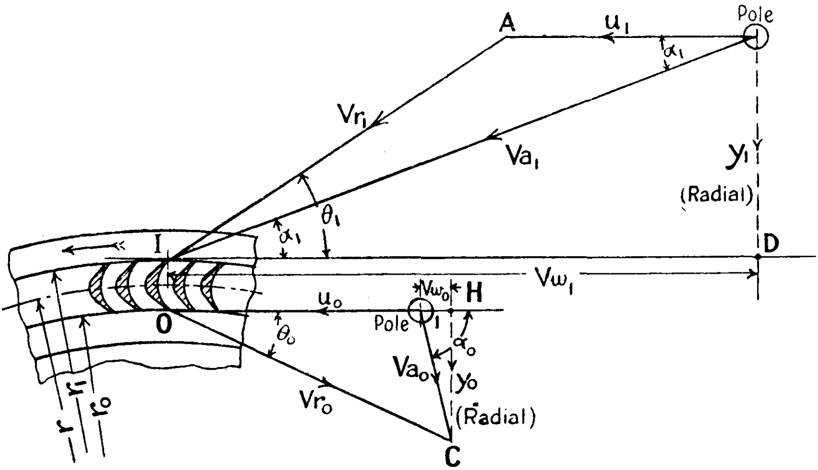


FIG. 100.

effort in the ring, and produces the torque on the shaft. In every case the torque is given by the "change of the moment of momentum" of the steam about the shaft axis.

A sectional elevation of a ring of blades, for a radial flow turbine, is shown in Fig. 100, and an elevation and sectional plan of a ring for an axial flow machine in Fig. 101. In each case the direction of motion of the blades is along DI. The steam is discharged from nozzle passages with an absolute velocity $OI = V_{a_1}$ at an angle α_1 to DI, and enters the blade channel at I. The component of this velocity in the direction of motion of the blade is $DI = V_{w_1}$, the velocity of whirl at entrance. The momentum per lb. of steam in the direction of motion is, mass $\times$ velocity, or $\frac{1}{g}V_{w_1} = \frac{1}{g}V_{a_1} \cos \alpha_1$. The moment of this momentum, in the case of the radial flow machine, Fig. 100, is $\frac{1}{g}V_{w_1}r_1$, where r_1 is the radius at the entrance edge I. In the axial machine, Fig. 101, it is $\frac{1}{g}V_{w_1}r$, where r is the mean blade radius at entrance.

In each case the steam is discharged from the channel at O with an absolute velocity V_{a_0} at an angle a_0 to the direction of motion of the blade, and similarly its momentum in the direction of motion of the blade is $\frac{1}{g}V_{w_0}$. Its sense, however, as shown here, is negative to that at entrance, since the direction of the jet is reversed.

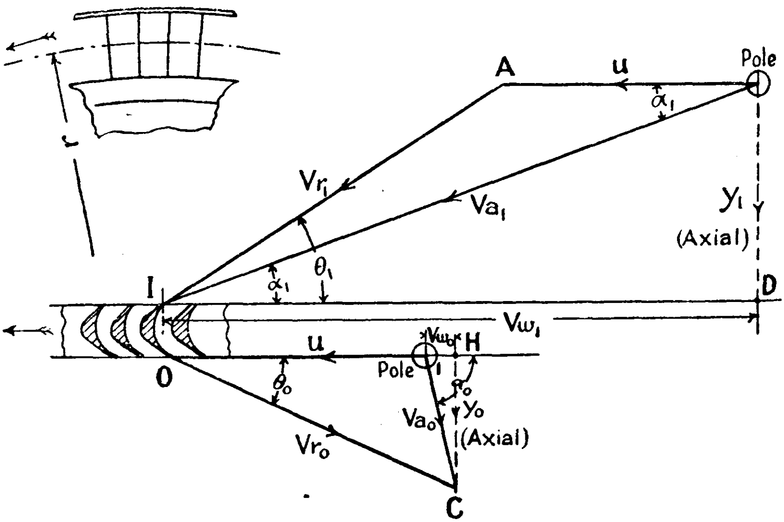


FIG. 101.

The corresponding moment of momentum at exit is thus, $\frac{1}{g}V_{w_0}r_0$ $= \frac{1}{g}V_{a_0}\cos a_0 r_0$ for the radial, and $\frac{1}{g}V_{w_0}r = \frac{1}{g}V_{a_0}\cos a_0 r$ for the axial machine, since the mean radius at entrance and exit is constant.

The change of moment of momentum between entrance and exit gives the torque on the shaft. Thus for the radial machine, Fig. 100,

$$\left.\begin{aligned} &\text{Torque} \qquad T = \frac{1}{g}(V_{w_1}r_1 - V_{w_0}r_0) \\ &\text{or} \qquad T = \frac{1}{g}(V_{a_1}\cos a_1 r_1 - V_{a_0}\cos a_0 r_0) \end{aligned}\right\} \quad . \quad . \quad (1)$$

where T is in ft.-lbs., when the velocities are in ft./sec. and the radii are in ft.

In the case of the axial machine, Fig. 101, since $r_1 = r_0 = r$

$$\left.\begin{aligned} T &= \frac{r}{g}(V_{w_1} - V_{w_0}) \\ &= \frac{r}{g}(V_{a_1} \cos \alpha_1 - V_{a_0} \cos \alpha_0) \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

These equations are fundamental, and apply equally to impulse and reaction machines.

In the case of the radial flow machine r_1 usually differs very little from r_0, and for most purposes it is sufficient to substitute the mean radius r, and calculate the torque by equation (2).

When, however, the summation of the work done on a series of rings of increasing diameter is required, as, for instance, in the Ljungström turbine, the exact value of the torque given by (1) should be used.

When the angular velocity ω and the torque T are known, the work done per lb. of steam flowing per second can be calculated from the fundamental equation,

$$E_b = T\omega$$

where ω is in radians per second.

Substitute for T from (1), then for the radial turbine

$$E_b = \frac{\omega}{g}(V_{w_1}r_1 - V_{w_0}r_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and for the axial turbine

$$E_b = \frac{\omega r}{g}(V_{w_1} - V_{w_0}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In the latter case, since $\omega r = u$, the mean peripheral speed of the blade, this can also be written

$$\left.\begin{aligned} E_b &= \frac{u}{g}(V_{w_1} - V_{w_0}) \\ &= \frac{u}{g}V_w \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where V_w represents the vector difference of velocities of whirl at entrance and exit.

Similarly for the radial turbine $\omega r_1 = u_1$ and $\omega r_0 = u_0$, the peripheral velocities at entrance and exit, and (3) may be written in the form

$$E_b = \frac{1}{g}(V_{w_1}u_1 - V_{w_0}u_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Here again, except in special cases, it will be sufficient to take the mean value u of u_1 and u_0, and apply either equation (4) or (5).

The bracketed term $(V_{w_1} - V_{w_0})$ represents the vector or geometrical difference of the velocities of whirl at entrance and exit. Normally, if V_{w_1} is positive in direction, V_{w_0} is negative, so that the sum of the numerical quantities has to be taken. This is the condition represented in Figs. 100 and 101, and $(V_{w_1} - V_{w_0}) = V_w = (DI + OH)$.

The value V_w is easily determined, for any case, whether the velocities of whirl have different signs or not, by the graphical methods of vector subtraction and addition.

82. Vector Subtraction.—The vector difference of two velocities is obtained by choosing a pole and drawing lines from it, parallel in direction and equal in magnitude to the two velocities, then joining the extremities of these lines. The closing line gives the vector difference. The essential point of the construction is that each velocity vector must be drawn with the "sense" arrow pointing away from the pole. For instance, in Fig. 101, $OI = V_{a_1}$ and $OA = u$ are the absolute velocities of the steam jet and the wheel blading, having the sense shown by the arrow heads. They are drawn from an arbitrarily chosen pole or point O with the sense reading from O. The closing line $AI = V_{r_1}$ is the vector difference $(\overline{OI} - \overline{OA}) = (\overline{V_{a_1} - u})$. The sense of the vector difference always reads from the extremity of the velocity that is subtracted, that is, the velocity having the negative sign in the bracket. In this case it is u, and hence the arrow head points from A to I.

Velocity Triangles at Entrance and Exit.—It can be easily demonstrated that the vector difference of the absolute velocities of two bodies also represents the velocity of one body relatively to the other. In this case the vector $AI = V_{r_1}$ is the velocity of the steam jet relatively to the wheel blade. It gives, in magnitude and direction, the equivalent absolute velocity with which the steam would enter the blade channel if the blade were at rest. Its inclination to the direction of motion of the blade obviously gives the necessary angle of entrance θ_1 for the blade.

The two absolute velocities and the relative entrance velocity determine the velocity triangle OIA at entrance.

Similarly, at exit, the blade has the mean velocity u and the steam an absolute velocity V_{a_0}. When the direction of V_{a_0} is known, then the same construction gives the velocity triangle O_1CO at exit; the vector difference of u and V_{a_0} is $OC = V_{r_0}$, the relative velocity of exit of the steam from the blades.

83. Vector Addition.—In the case of the inlet triangle the jet angle α_1 is always given. In the outlet triangle it is the exit blade angle that is arbitrarily fixed, and the exit jet angle α_0 is not known. The exit triangle has to be determined by the converse process of "vector addition."

In this construction the starting point is again the pole O, but the vector quantities to be summed are drawn consecutively with the sense arrows pointing forward. The closing line then gives the vector sum. Its sense reads away from the pole O.

Thus in Fig. 101 the vector $O_1O = u$, the mean blade velocity vector is drawn from the pole O_1, the sense reading from O_1 to O. From O the relative exit velocity vector OC is drawn, the sense reading from O to C. The closing line CO_1 gives the absolute velocity of exit, $O_1C = V_{a_0}$, the sense reading from O_1 to C.

84. Velocity Diagram for Simple Impulse Stage.—It will be evident that in the case of the axial machine where the blade velocity is the same for entrance and exit, and in that of the radial machine, where the mean velocity between entrance and exit is used, that the vector difference of the velocities of whirl can be automatically determined by superposing the entrance and exit triangles. The combined velocity diagram is shown in Fig. 102. O is the pole common to the two triangles of velocity. The perpendicular IG, from I to OA produced, gives the velocity of whirl V_{w_1} at entrance; and the perpendicular CH from C gives the velocity of whirl V_{w_0} at exit. The intercept HG is the "vector difference" V_w of these two quantities. Reckoned from the pole, V_{w_1} reads from O to G, while V_{w_0} reads in the

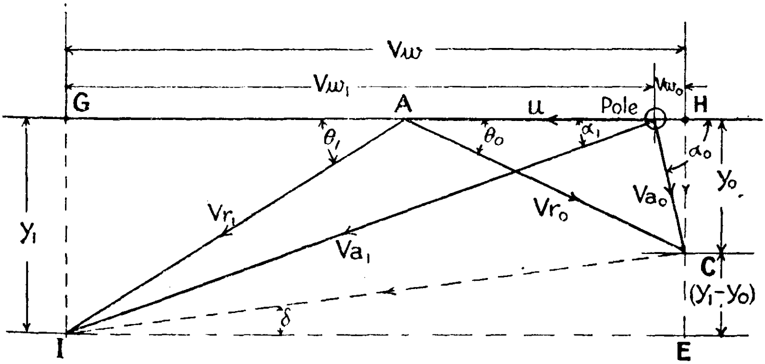


FIG. 102.

opposite direction from O to H. Hence the two quantities are added. If the exit jet angle α_0 were greater than 90°, H would obviously fall to left of the pole O, and OH would read in the same direction as OG. The two quantities would then be subtracted. In either case the addition or subtraction is automatically performed, when the complete velocity diagram is drawn.

For specified blade angles, jet velocity and angle, the principal object in drawing the velocity triangles is to determine the magnitude and direction of the absolute velocity of exit V_{a_0}.

The result of the construction in Fig. 102 is a pole O, from which are drawn the absolute velocity vectors OI and OC at entrance and exit. The change of velocity is the vector difference $\overline{(OI - OC)} = CI$. This has an inclination δ to the direction of motion of the blade. The effective change of velocity between entrance and exit is the component of this in the direction of motion of the blade, that is, the projection $IE = GH = V_w$.

For the practical purpose of calculating the work done on a ring of impulse blading the velocity diagram is constructed as follows :—

From any chosen point or pole O, Fig. 102, draw OA $= u$, the mean blade velocity, and OI $= V_{a_1} = V_0$, the velocity of the jet at exit from the nozzle, at the entrance jet angle α_1 to OA. Join AI. This gives the relative velocity of entrance V_{r_1}, and also fixes the correct entrance angle θ_1. If there were no losses in the blade channel the relative velocity between I and O would be constant. There is always a loss, and the relative velocity of exit V_{r_0} is less than the relative entrance velocity V_{r_1}, or $V_{r_0} = kV_{r_1}$, where k is a velocity coefficient arbitrarily chosen. Find V_{r_0} for the chosen value of k, and draw AC $= V_{r_0}$, at the given exit angle θ_0 to OA. Join OC. Then OC $= V_{a_0}$, the absolute velocity of exit, and α_0 is the jet angle at exit.

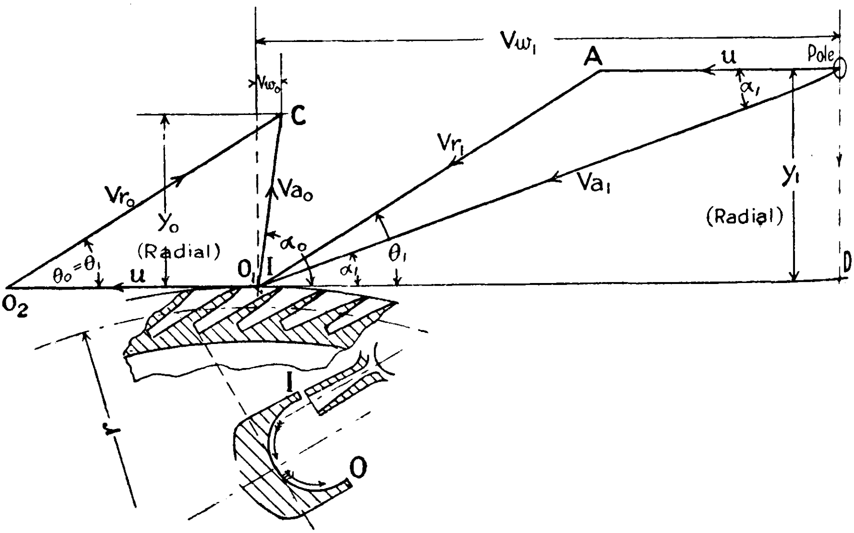


FIG. 103.

Project from C and I to cut AO produced in H and G. Then GH $= V_w$, the vector difference of the velocities of whirl to be used in equation (5). In the case of the impulse wheel with rim buckets, the foregoing method of constructing the velocity diagram also applies. This will be evident from an inspection of Fig. 103. In the sectional elevation the entrance and exit edges I and O coincide. Also since the blade channel is formed with the faces sensibly parallel the exit angle θ_0 is the same as the entrance angle θ_1, and the vector O_2C is parallel to AI. By superposing O_2CO_1 on AIO, the combination of Fig. 102 is again obtained. It is assumed that the blade velocity u is calculated at the mean radius r.

85. Dynamical Thrust on the Blades.—As already pointed out the actual " change of velocity " of the steam between entrance and exit is given in Fig. 102 by the vector CI inclined at an angle δ to the direction of motion of the blade. The effective change of motion along OA is $IE = GH$. There is, however, another component of the change, CE, perpendicular to the direction of motion, which serves no useful purpose, as far as the propulsion of the blades is concerned. It produces, per lb. of steam flowing, a dynamic force or thrust $F = \frac{1}{g}CE = \frac{1}{g}(y_1 - y_0)$, where y_1 and y_0 are the normal components of the jet velocities at entrance and exit. In the axial machine it is an axial thrust and tends to shift the rotor axially. In the radial machine it is radial and tends to press the shaft on to the journals. As a rule, however, the total thrust on the rotor is quite small in the impulse machine, and easily taken up by the adjusting block required for the maintenance of the necessary clearances.

86. Analytical Calculation of Work done on the Blading.— Referring to Fig. 102, it will be seen that

$$V_{a_1} \cos \alpha_1 = V_{r_1} \cos \theta_1 + u$$

hence
$$V_{r_1} = \frac{V_{a_1} \cos \alpha_1 - u}{\cos \theta_1} \quad \ldots \ldots \quad (7)$$

Also
$$V_{r_0} \cos \theta_0 + V_{r_1} \cos \theta_1 = V_w$$

or
$$V_{r_1}(\cos \theta_1 + k \cos \theta_0) = V_w$$

Hence
$$V_w = (V_{a_1} \cos \alpha_1 - u)\left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right) \quad \ldots \quad (8)$$

Substituting from (8) in equation (5), the expression for the work done per lb. on the blades is

$$E_b = \frac{u}{g}\left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right)(V_{a_1} \cos \alpha_1 - u) \text{ ft.-lbs./lb. sec.} \quad \ldots \quad (9)$$

If the ratio of the blade velocity to jet velocity at exit from the nozzle is, $\rho = \frac{u}{V_{a_1}} = \frac{u}{V_0}$, this becomes

$$E_b = \frac{u^2}{g}c\left(\frac{\cos \alpha_1}{\rho} - 1\right) \text{ ft.-lbs./lb. sec.} \quad \ldots \ldots \quad (10)$$

where
$$c = \left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right)$$

To find c for any given case θ_0 is arbitrarily assumed; but θ_1 is determined by V_{a_1}, u and α_1. Here

$$\left.\begin{aligned} \tan \theta_1 &= \frac{GI}{GA} = \frac{V_{a_1} \sin \alpha_1}{V_{a_1} \cos \alpha_1 - u} \\ \text{or} \quad \tan \theta_1 &= \frac{\sin \alpha_1}{\cos \alpha_1 - \rho} \end{aligned}\right\} \quad \ldots \quad (11)$$

and θ_1 is therefore calculable.

The horse-power expended on the blades by the steam is

$$\mathrm{HP} = \frac{E_b}{550} = \frac{u^2 c}{550 g}\left(\frac{\cos \alpha_1}{\rho} - 1\right) \quad . \quad . \quad . \quad . \quad (12)$$

and the heat converted to work on the blades is

$$\left.\begin{aligned} h_b &= \frac{u^2 c}{Jg}\left(\frac{\cos \alpha_1}{\rho} - 1\right) \\ \text{or} \qquad h_b &= \frac{E_b}{778} \text{ B.Th.U./lb. sec.} \end{aligned}\right\} \quad . \quad . \quad . \quad (13)$$

Axial components of the absolute velocities are

$$\left.\begin{aligned} y_1 &= (V_{a_1} \cos \alpha_1 - u) \tan \theta_1 \\ &= u\left(\frac{\cos \alpha_1}{\rho} - 1\right) \tan \theta_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

$y_0 = (u + V_w - V_{a_1} \cos \alpha_1) \tan \theta_0$. Substituting for V_w from equation (8) this, on reduction, becomes

$$\left.\begin{aligned} y_0 &= (c - 1)(V_{a_1} \cos \alpha_1 - u) \tan \theta_0 \\ &= (c - 1)u\left(\frac{\cos \alpha_1}{\rho} - 1\right) \tan \theta_0 \\ \text{or} \qquad y_0 &= \frac{k \sin \theta_0}{\cos \theta_1} u\left(\frac{\cos \alpha_1}{\rho} - 1\right) \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

Also $\quad V_{w_0} = (V_w - V_{a_1} \cos \alpha_1) = c(V_{a_1} \cos \alpha_1 - u) - V_{a_1} \cos \alpha_1$
$\qquad = V_{a_1} \cos \alpha_1 (c - 1) - cu$

Hence $\quad \tan \alpha_0 = \dfrac{y_0}{V_{w_0}} = \dfrac{(c - 1)(V_{a_1} \cos \alpha_1 - u) \tan \theta_0}{(c - 1)V_{a_1} \cos \alpha_1 - cu}$

$$\left.\begin{aligned} \text{or} \qquad \tan \alpha_0 &= \frac{(V_{a_1} \cos \alpha_1 - u) \tan \theta_0}{V_{a_1} \cos \alpha_1 - \dfrac{u}{1 - \dfrac{1}{c}}} \\ &= \frac{\left(\dfrac{\cos \alpha_1}{\rho} - 1\right) \tan \theta_0}{\dfrac{\cos \alpha_1}{\rho} - \dfrac{1}{1 - \dfrac{1}{c}}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Also $\quad V_{a_0} = \dfrac{y_0}{\sin \alpha_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$

and $\quad V_{a_0}^2 = V_{r_0}^2 + u^2 - 2uV_{r_0} \cos \theta_0$
$\qquad = (kV_{r_1})^2 + u^2 - 2ukV_{r_1} \cos \theta_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$

By means of this series of equations the work done on a ring of blading, the axial thrust, and residual energy can be calculated without the aid of the velocity diagram.

The graphical method is preferable in the majority of cases.

EXAMPLE I.—At a stage of a pressure compounded impulse turbine the exit velocity from the nozzles is $V_0 = V_{a_1} = 1590$ ft./sec. The mean ring diameter is 39 inches, and the speed 3000 R.P.M. The jet angle $\alpha_1 = 20°$, and the exit blade angle $\theta_0 = 25°$. Find the work done on the blades per lb. of steam, and its heat equivalent, (*a*) from the velocity diagram, (*b*) by direct calculation. Take $k = 0·78$.

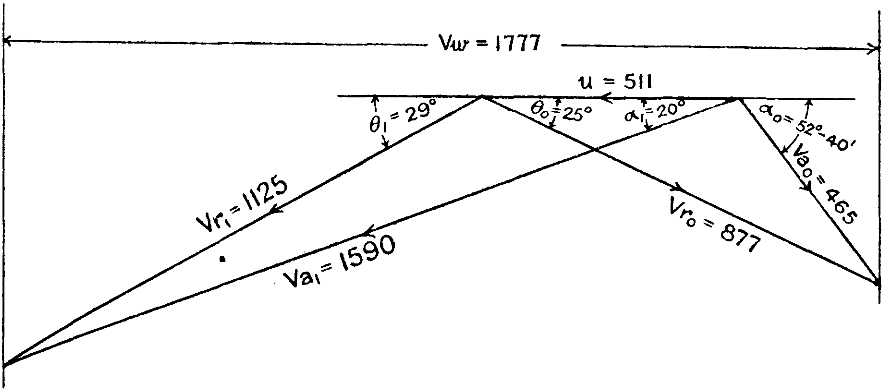


FIG. 104.

(*a*) The velocity diagram, drawn in accordance with the method of Art. 84, is shown in Fig 104.

$$V_0 = V_{a_1} = 1590$$

$$u = \frac{\pi DN}{60} = \frac{3·1416 \times 3·25 \times 3000}{60} = 511 \text{ ft./sec.}$$

$\alpha_1 = 20°$. From the diagram the entrance angle $\theta_1 = 29°$ and the relative entrance velocity $V_{r_1} = 1125$. With $k = 0·78$, the relative exit velocity $V_{r_0} = 0·78 \times 1125 = 877$. The absolute velocity of exit $V_{a_0} = 465$, and the vector difference of velocities of whirl $V_w = 1777$.

Hence by equation (5) the work done per lb. is—

$$E_b = \frac{u}{g} V_w = \frac{511 \times 1777}{32·2} = 28100 \text{ ft.-lbs.}$$

By equation (13) the heat expended in work on the blades is

$$h_b = \frac{E_b}{778} = \frac{28100}{778} = 36·2 \text{ B.Th.U./lb. sec.}$$

(*b*) $\alpha_1 = 20$, $\sin \alpha_1 = 0·342$, $\cos \alpha_1 = 0·94$, $\rho = \dfrac{u}{V_{a_1}} = \dfrac{511}{1590} = 0·322$

By equation (11)

$$\tan \theta_1 = \frac{\sin \alpha_1}{\cos \alpha_1 - \rho} = \frac{0{\cdot}342}{0{\cdot}94 - 0{\cdot}323} = 0{\cdot}553$$

and $\theta_1 = 29^\circ$, $\cos \theta_1 = 0{\cdot}8746$, $\theta_0 = 25$, $\cos \theta_0 = 0{\cdot}906$, $k = 0{\cdot}78$

$$\therefore\ c = 1 + \frac{k \cos \theta_0}{\cos \theta_1} = 1 + \frac{0{\cdot}78 \times 0{\cdot}906}{0{\cdot}8746} = 1{\cdot}81$$

By equation (10) $\mathrm{E}_b = \dfrac{u^2}{g} c\left(\dfrac{\cos \alpha_1}{\rho} - 1\right)$

$$= \frac{511^2 \times 1{\cdot}81}{32{\cdot}2}\left(\frac{0{\cdot}94}{0{\cdot}322} - 1\right)$$

$$= \frac{511^2 \times 1{\cdot}81 \times 1{\cdot}92}{32{\cdot}2} = 28100 \text{ ft.-lbs.}$$

This checks the value obtained from the velocity diagram. By equation (12) the horse-power expended on the blade per lb. is

$$\mathrm{HP} = \frac{\mathrm{E}_b}{550} = \frac{28100}{550} = 51{\cdot}2$$

EXAMPLE 2.—For the blading referred to in the previous example calculate the axial thrust and the residual energy carried over to the next stage.

Axial thrust per lb. of steam

$$\mathrm{F} = \frac{1}{g}(y_1 - y_0) \text{ lbs.}$$

By equation (14) $y_1 = u\left(\dfrac{\cos \alpha_1}{\rho} - 1\right) \tan \theta_1$

" " (15) $y_0 = u\left(\dfrac{\cos \alpha_1}{\rho} - 1\right) \dfrac{k \sin \theta_0}{\cos \theta_1}$

$$\therefore\ \mathrm{F} = \frac{u}{g}\left(\frac{\cos \alpha_1}{\rho} - 1\right)\left(\tan \theta_1 - k\frac{\sin \theta_0}{\cos \theta_1}\right)$$

$\tan \theta_1 = 0{\cdot}553$, $\cos \theta_1 = 0{\cdot}8746$, $\theta_0 = 25$, $\sin \theta_0 = 0{\cdot}4226$, $k = 0{\cdot}78$, $\cos \alpha_1 = 0{\cdot}94$, $\rho = 0{\cdot}322$

$$\therefore\ \mathrm{F} = \frac{511}{32{\cdot}2}\left(\frac{0{\cdot}94}{0{\cdot}322} - 1\right)\left(0{\cdot}553 - \frac{0{\cdot}78 \times 0{\cdot}4226}{0{\cdot}8746}\right)$$

$$= \frac{511 \times 1{\cdot}92 \times 0{\cdot}176}{32{\cdot}2} = 5{\cdot}36 \text{ lbs.}$$

By equation (16)
$$\tan \alpha_0 = \frac{\left(\dfrac{\cos \alpha_1}{\rho} - 1\right) \tan \theta_0}{\dfrac{\cos \alpha_1}{\rho} - \dfrac{1}{1 - \dfrac{1}{c}}}$$

$$c = 1\cdot81 \qquad \therefore \tan \alpha_0 = \frac{1\cdot92 \times 0\cdot4663}{\frac{0\cdot94}{0\cdot322} - \frac{1}{0\cdot4470}}$$

$$\left(1 - \frac{1}{c}\right) = 0\cdot447$$

$$\tan \theta_0 = \tan 25 = 0\cdot4663, \qquad = \frac{0\cdot894}{0\cdot68} = 1\cdot31$$

$$\therefore \alpha_0 = 52^\circ 40', \quad \sin \alpha_0 = 0\cdot795$$

By equation (17)

$$V_{a_0} = \frac{y_0}{\sin \alpha_0} = \frac{511 \times 1\cdot92 \times 0\cdot378}{0\cdot795} = 465 \text{ ft./sec.}$$

The residual energy is

$$h_e = \left(\frac{V_{a_0}}{223\cdot7}\right)^2 = \left(\frac{465}{223\cdot7}\right)^2 = 4\cdot33 \text{ B.Th.U./lb.}$$

The values scaled from the diagram will be found in close agreement with the above, if the diagram is drawn with reasonable care on a fairly large scale.

87. Diagram for Velocity Compounded Impulse Stage.—The extended velocity diagram for a two-velocity stage is shown in Fig. 105. In order to distinguish the velocities and angles at the successive rings, subscripts 1 and 2 are added to the left of the symbols, denoting first and second moving rings. The guide blade angles are denoted by dashes. The mean blade speed u is the same for both rings. It will be evident that the absolute velocity of exit ${}_1V_{a_0}$ from the first moving ring is the entrance velocity to the fixed or guide blade. The exit jet angle ${}_1\alpha_0$ is therefore the same as the entrance guide blade angle θ_1'.

The guide blade is given an exit angle θ_0', at which the steam is discharged into the second moving ring with the absolute entrance velocity ${}_2V_{a_1}$. This angle is the same as the second entrance jet angle ${}_2\alpha_1$.

The work done on each blade ring can, as before, be obtained by superposing the inlet and outlet triangles. As the mean blade speed is always the same for all the rings the whole series can be superposed to form one diagram, as shown in Fig. 106. For the sake of clearness, the triangles of the second moving rings are dotted.

The change of velocity of whirl for the first ring is ${}_1V_w$ and that for the second ${}_2V_w$, and the total work done on the two wheels is

$$E_b = \frac{u}{g}({}_1V_w + {}_2V_w) \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Any number of diagrams can be superposed and dealt with in this way, so that generally the total work done on a series of rings may be expressed by

$$E_b = \frac{u}{g}\Sigma V_w$$

At discharge from the nozzle the steam has an axial or normal component y_n. At discharge successively from the exit edges of the first moving, the fixed and the second moving rings, it has axial velocities y_1, y_2, and y_3. The corresponding radial lengths of the nozzle and blades at exit are also denoted by l_n, l_1, l_2, l_3 (see Fig. 90).

At the nozzle exit, for the passage of W lbs. having a volume v_0 through the exit area A_0, $W = \frac{A_0 V_{a_1}}{v_0}$ or $Wv_0 = A_0 V_{a_1}$. In the whole stage

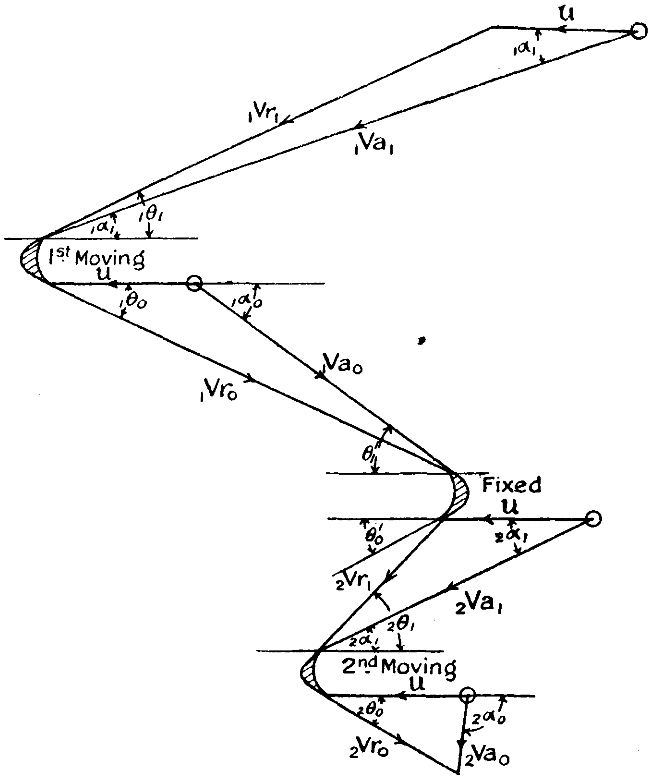


FIG. 105.

W and v_0 are constant, as there is no expansion in the blades. For a nozzle arc x_0, $A_0 = x_0 \pi D l_n \sin \alpha$, and since $x_0 \pi D$ is constant, $V_{a_1} l_n \sin \alpha_1$=constant. But $V_{a_1} \sin \alpha_1 = y_n$, hence $l_n = \frac{\text{constant}}{y_n}$, or the radial length varies inversely as the axial velocity. The same condition holds at the exit edge of each blade ring. Hence the ratio of the last blade length to the nozzle exit length is the reciprocal of the corresponding ratio of the axial velocities, that is, $\frac{l_3}{l_n} = \frac{y_n}{y_3}$. This is called the "height ratio" of the stage. When it and the exit blade angles are provisionally fixed, the complete velocity diagram can be easily drawn by the following method. The assumption is made that the progression in height at each blade exit is linear, that is, by scaling off the successive blade widths as abscissæ, setting up the nozzle exit length and last blade length as end ordinates, the line joining their

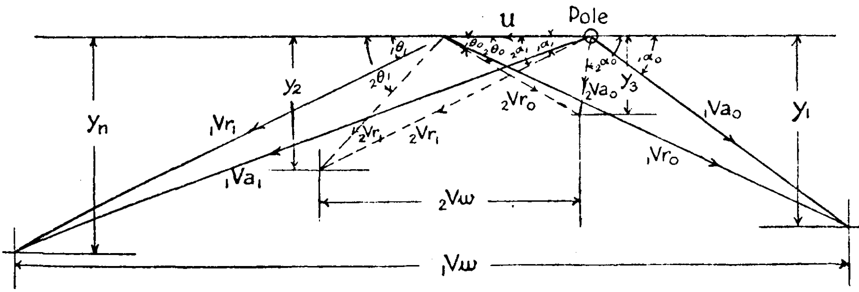


FIG. 106.

extremities gives the heights of the other blades. The general case is shown in Fig. 107, l_n and l_m being the nozzle exit and last blade length respectively. If there are m blades pitched b inches axially, the slope of the line NM is, $B = \dfrac{l_m - l_n}{mb}$. Any blade length l, corresponding to an axial distance Z, is given by $l = l_n + BZ$. If the number of blade pitches in distance Z is n, then $Z = nb$, and $BZ = \dfrac{(l_m - l_n)n}{m}$. The height ratio $K = \dfrac{l_m}{l_n}$ $\therefore$ $BZ = l_n\dfrac{(K - 1)n}{m}$, and $l = l_n\left\{1 + \dfrac{(K - 1)n}{m}\right\}$. But $y_n l_n = y_m l_m = yl$, hence

$$y = \frac{y_n}{1 + \dfrac{(K - 1)n}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The jet angle and velocity are always given, and either the blade speed or the speed ratio ρ is known.

An arbitrary choice of exit blade angles and height ratio is made. Assuming these figures to be fixed and referring to Fig. 107, draw $OA = u$ and $OB = V_{a_1}$ at the jet angle, α_1. Through B draw the horizontal BC. From the pole O drop the perpendicular OC on BC.

Find the points 1, 2, etc., on OC, so that $\dfrac{y_1}{y_n} = \dfrac{O1}{OC}$, $\dfrac{y_2}{y_n} = \dfrac{O2}{OC}$, etc., the values of the ratios being calculated by equation (20).

From A draw AD at the exit angle $_1\theta_0$ for the first moving ring and project horizontally from point 1, on OC, to cut this in D. B and D are the vertices of the two velocity triangles for the first moving ring.

Next from O draw OE, at the exit angle θ_0' of the fixed blade, and project horizontally from point 2 on OC to cut it in E. Finally, from A draw AG at the exit ${}_2\theta_0$ of the second moving blade, and project horizontally from point 3 (or m) to cut it in G. The points E and G are the vertices of the velocity triangles for the second moving ring. The triangles can now be completed, and the inlet angles and other data scaled from them.

For the most efficient performance of work, the final velocity vector OG should be perpendicular to the direction of motion of the blade, that is, OG and Om should be coincident. When with a given height ratio this condition is not closely approached, a slight readjustment of the exit angles may be made to ensure this. The value of the height ratio for the two-velocity stage varies in practice from 2 to 3 in high-

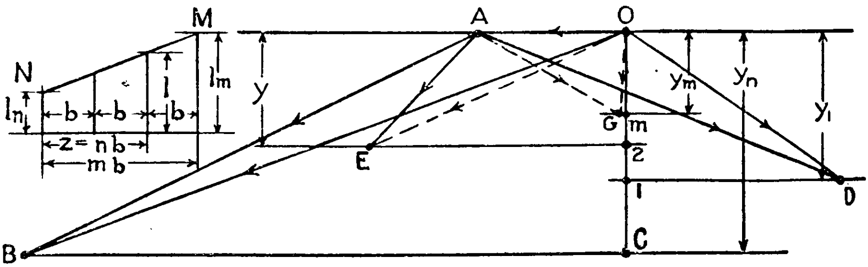


FIG. 107.

pressure land machines. In marine Curtis turbines it may vary from 1·2 to 2·5.

88. The diagram can also be drawn down by the method of Art. 84 if reliable values of the velocity coefficient k for the successive blade channels are known. For a given set of angles these coefficients determine the height ratio, which may be greater or less than the arbitrarily assumed value employed in the other construction. If the velocity coefficients are correct and a smaller height ratio than that derived by their use is given to the stage, then the steam will be partly "throttled" in its passage through the blades.

Unfortunately the knowledge regarding the coefficients for this type is scanty, and it is never possible to say definitely what value of k should be used for a particular blade. If the usual values employed for the simple impulse stage are applied to the various rings the height ratios are usually greater than those which, in practice, are found to give the best efficiency. This means generally that a certain amount of throttling is beneficial. The subject of the loss in blade channels is dealt with in Art. 94. For any chosen height ratio it can be easily ascertained if there is more or less throttling, by taking the ratio of relative exit to entrance velocity at each blade, and comparing with the probable velocity coefficient for the blade, as given by the method stated in Art. 94.

The analytical treatment of this case can, if desired, be carried out by applying the method of Art. 86 to each ring in succession.

89. In the case of the velocity-compounded turbine having a single wheel of rim bucket type, the velocity diagram differs from the foregoing case in having the inlet and outlet angle of the moving blades the same for each velocity stage. The diagram is shown in Fig. 108 for a four-

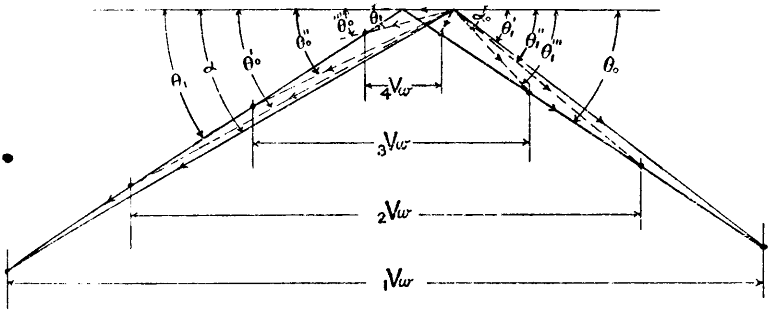


FIG. 108.

velocity stage. It will be seen that the guide inlet angles θ_1', θ_1'', θ_1''', etc., increase, while the exit angles θ_0', θ_0'', θ_0''', etc., decrease. On this account a large jet angle a is required. There are no data regarding the values of the velocity coefficient k to be used for the successive guide channels and wheel blades. They are probably lower than those which are applicable to the single-ring single-stage machine of this type, since the arrangement of guides must involve considerable friction and eddy loss.

EXAMPLE 3.—The first stage of a Curtis turbine has a wheel, with two moving rings. The jet angle is 20°, and the exit angles of the blades are, first moving, 22°; fixed, 24°; second moving, 35°. The blade speed is 455 ft./sec., height ratio 2·2, and the speed ratio 0·2.

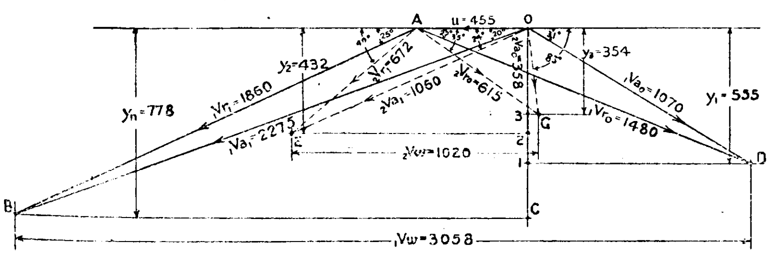


FIG. 109.

Draw the velocity diagram and find the work done on the blading per lb. of steam, and the horse-power expended. Also determine the entrance angle for each blade, the velocity coefficients, and the total axial thrust.

The velocity diagram, drawn in accordance with the method of Art. 87, is shown in Fig. 109.

$$u = 455, \quad \rho = 0{\cdot}2 \qquad \therefore \ V_{a_1} = \frac{455}{0{\cdot}2} = 2275, \quad \alpha = 20^\circ$$

For first ring $\ {}_1V_w = 3058$

„ second „ $\ {}_2V_w = 1020$

By equation (18) $\quad E_b = \frac{u}{g}({}_1V_w + {}_2V_w)$

$$= \frac{455 \times 4078}{32{\cdot}2} = 57600 \text{ ft.-lbs./lb.}$$

Heat equivalent $\quad h_b = \frac{57600}{778} = 74{\cdot}1$ B.Th.U./lb.

Horse power $\quad = \frac{E_b}{550} = \frac{57600}{550} = 105/\text{lb.}$

From the diagram, inlet angles, first moving 25°
fixed 31°
second moving 40°

Thrust on first ring $= \frac{1}{g}(y_n - y_1)$

„ second „ $= \frac{1}{g}(y_2 - y_3)$

Total thrust $= \frac{1}{g}(y_n + y_2 - y_1 - y_3)$

$$= \frac{1}{32{\cdot}2}(778 + 432 - 555 - 354)$$

$$= \frac{301}{32{\cdot}2} = 9{\cdot}35 \text{ lbs./lb. of steam}$$

Velocity coefficient for first blade $k_1 = \frac{{}_1V_{r_0}}{{}_1V_{r_1}} = \frac{1480}{1860} = 0{\cdot}796$

„ „ second „ $k_2 = \frac{{}_2V_{r_0}}{{}_1V_r} = \frac{615}{672} = 0{\cdot}915$

The probable coefficients treating each ring separately would be 0·8 and 0·76 respectively. It is therefore possible, that with the angles and height ratio adopted here, there is considerable throttling effect at the fixed and second moving rings. The reason for this assumption is given in Art. 94.

90. Velocity Diagram for Reaction Stage.—In a reaction stage there is an increase of relative velocity between entrance and exit, due to the pressure drop in the blade channel. The fixed and moving rings have identical blades, and the heat drop for the "double" stage is divided equally between the rings. Assuming the same friction losses in the identical rings, it follows that with the same exit angle $\theta = \alpha_1$ the exit and entrance triangles are identical, as shown in Fig. 110. Thus $V_{r_0} = V_{a_1}$, and $V_{a_0} = V_{r_1}$.

From the geometry of the figure it is evident that the difference of velocity of whirl is $V_w = (2V_{a_1} \cos\theta - u)$. Hence the work per lb. done on the moving ring of the "double-stage" is

$$E_b = \frac{u}{g}(2V_{a_1} \cos\theta - \mu)$$

$$E_b = \frac{u^2}{g}\left(\frac{2\cos\theta}{\rho} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where θ is the constant exit blade angle and $\rho = \frac{u}{V_{a_1}} = \frac{u}{V_{r_0}}$

The constant entrance angle for each ring is θ_1 and

$$\left.\begin{aligned} \tan\theta_1 &= \frac{V_{a_1}\sin\theta}{V_{a_1}\cos\theta - u} \\ \text{or} \qquad \tan\theta_1 &= \frac{\sin\theta}{\cos\theta - \rho} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

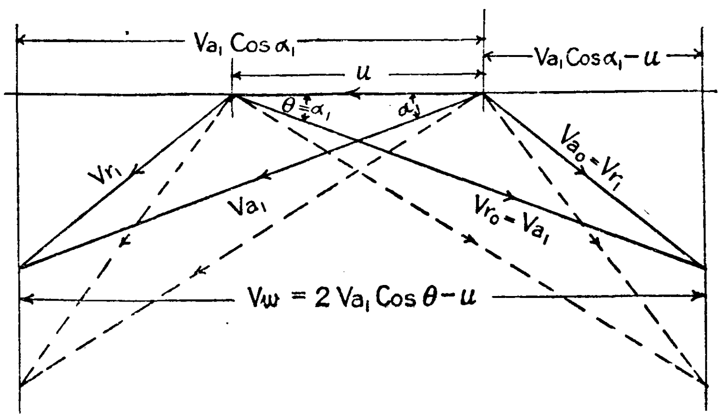


FIG. 110.

In this case, as a rule, the necessary particulars can be more easily obtained by direct calculation from equations (21) and (22).

It should be noted that if the exit blade angle θ is increased and the blade speed and change of velocity of whirl V_w remain constant, the speed ratio ρ is reduced, but the work done on the blades remains the same. This will be apparent from an examination of the dotted triangles, which show increased values of θ but the same value of V_w as the full-line diagram. In the first case the value of ρ is 0·5, in the second it is 0·455. This point is referred to later in Chap. XIV. in connection with the wide-angled blades at the L.P. stages of the Parsons axial flow turbine.

With the condition of identical velocity triangles for a double reaction stage, it will be evident that the axial components of the steam velocity are equal and the dynamical thrust is zero. With the usual construction, however, in which the blade heights are kept constant throughout a group or expansion, there is a progressive increase of velocity and consequently a slight thrust. Normally this is too small to be of any account, and it is neglected. There is, however, a statical thrust, which does not occur in the impulse machine. This is due to the difference of pressure between entrance and exit, at each moving ring. The cumulative effect of these small thrusts cannot be properly discussed at this stage. It is dealt with in Chap. IX., Art. 137.

EXAMPLE 4.—At the last two expansions of a Parsons turbine the blade rings have a common diameter of $54\frac{1}{2}$ inches. At the second last group the mean speed ratio $\rho = 0{\cdot}55$, the blade angle $\theta = 20^\circ$, and the speed of rotation is 1500 R.P.M. Calculate the heat equivalent of the work done on each blade ring and find the average value of the angle of entrance.

If the exit angle of the last group is increased to $\theta' = 31^\circ$, find the reduced speed ratio for the same amount of work per ring, as in the second last group.

Here $$u = \frac{\pi DN}{60} = \frac{3{\cdot}1416 \times 54{\cdot}5 \times 1500}{12 \times 60} = 356 \text{ ft./sec.}$$

$$\theta = 20^\circ, \quad \cos\theta = 0{\cdot}94, \quad \rho = 0{\cdot}55$$

By equation (21)

$$E_b = \frac{u^2}{g}\left(\frac{2\cos\theta}{\rho} - 1\right)$$

$$= \frac{356^2}{32{\cdot}2}\left(\frac{2 \times 0{\cdot}94}{0{\cdot}55} - 1\right)$$

$$= 9521 \text{ ft.-lbs./lb. of steam}$$

$$h_b = \frac{9521}{778} = 12{\cdot}24 \text{ B.Th.U./lb.}$$

Entrance angle by equation (22)

$$\tan\theta_1 = \frac{\sin\theta}{\cos\theta - \rho}$$

$$\sin 20^\circ = 0{\cdot}342$$

$$\therefore \tan\theta_1 = \frac{0{\cdot}342}{0{\cdot}94 - 0{\cdot}550} = \frac{0{\cdot}342}{0{\cdot}39} = 0{\cdot}88$$

$$\therefore \theta_1 = 41^\circ - 30'$$

Since the work done on the rings of the last group is the same as above, it follows from equation (21) that

$$\frac{\cos\theta}{\rho} = \frac{\cos\theta'}{\rho'}$$

where θ' and ρ' are the increased angle and decreased speed ratio.

$$\therefore \rho' = \rho\frac{\cos\theta'}{\cos\theta} = \frac{0{\cdot}55 \times 0{\cdot}8572}{0{\cdot}94} = 0{\cdot}502$$

91. Overspeeding and Underspeeding.—As already stated, the angle, which the relative entrance velocity vector makes with the blade velocity vector, is taken as the entrance angle of the blade. This means that the relative velocity is tangential to the blade surface at entrance, and hence the steam glides on to the blade without shock.

As it is necessary to thicken the blades from the entrance edge inwards, the entrance angle at the back is slightly less than that at the face. The angle of the back is usually taken as the value of θ_1 in the case of the impulse blade. In the case of the Parsons blade the exact angle of the back is indeterminate (see Fig. 122).

When the blading is designed for the above condition, the value

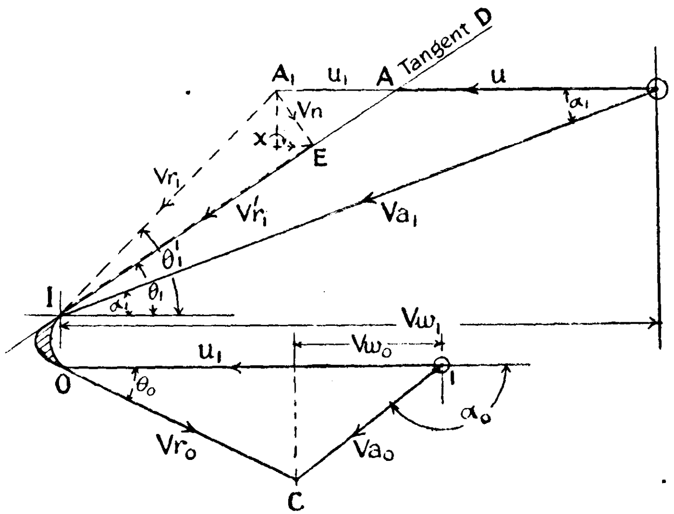


FIG. 111.

of u which, for a given jet velocity V_{a_1} fulfils this, is called the "synchronous" speed.

If the jet velocity V_{a_1} remains constant, and the blade speed u is increased, or if V_{a_1} is reduced while u remains constant, then, as can be seen from equations (11) and (22), the relative entrance angle is increased. The result is that the jet hits the back of the blade at entrance. The blade is said to be "overspeeded." This condition is illustrated for the impulse type in Fig. 111.

The blade velocity is increased from $OA = u$ to $OA_1 = u_1$, and the relative entrance angle from θ_1 to θ_1'.

The actual relative velocity at entrance is V_{r_1}', the component of V_{r_1} along the tangent DI, to the back of the blade at entrance.

The normal component is $V_n = A_1E$, and this has a relative component x in the direction of motion. Its sense is negative to that of the blade motion, and the corresponding force thus tends to retard the motion of the blades. There is also a second effect at exit. With the correct value of the ratio $\rho = \frac{u}{V_{a_1}}$, the absolute velocity vector at exit is nearly perpendicular to the direction of motion of the blade. With the increased value $\rho' = \frac{u_1}{V_{a_1}}$, this vector is inclined at the obtuse angle α_0, and the velocity of whirl at exit becomes V_{w_0}, with the same sense as the velocity of the blade. The result, as can be seen from equation (4), is a reduction of the work done on the blading.

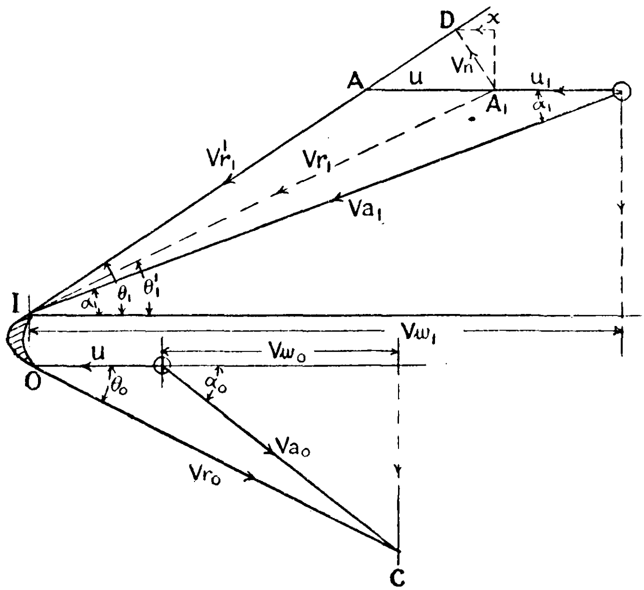


FIG. 112.

A high value of ρ has been chosen in order to exaggerate the angle at entrance, and make the overspeeding effect clearer.

Again, if the jet velocity is increased while the blade speed is kept constant, the speed ratio ρ' becomes less than the synchronous ratio ρ, and the relative entrance angle θ_1' is less than the blade angle θ_1.

The blade is now said to be "underspeeded." This condition for the impulse type is shown in Fig. 112.

As before, the actual relative velocity at entrance is V_{r_1}', the component of V_{r_1} along the tangent to the back of the blade. The normal component V_n has again the component x in the direction of motion of the blade, but its sense is the same as that of u. Hence the corresponding force tends to assist and not retard the motion. On this score it is evident that underspeeding is to be preferred to overspeeding. The secondary effect at exit is the reverse of that shown for overspeeding. The angle α_0 is acute, and the sense of V_{w_0} is negative to that of u, so that the change of velocity of whirl is increased. This value of u_1 is, however, too small for the best condition of efficiency, or, in other words, the value of ρ' is too small. Here again the choice has been made to exaggerate the decrease in angle.

It will be evident that while the best value of $\rho = \dfrac{u}{V_{a_1}}$ may be employed, the entrance angle may be so chosen that the jet may partly hit either the back or face of the blade producing the same effect at entrance as over- and underspeeding. It should obviously be the aim of the designer to avoid either condition, especially that of overspeeding. If the blading is designed for the maximum possible load on the machine, it is advisable to adjust the inlet angle so that a slight underspeeding effect may be produced. If the machine is run

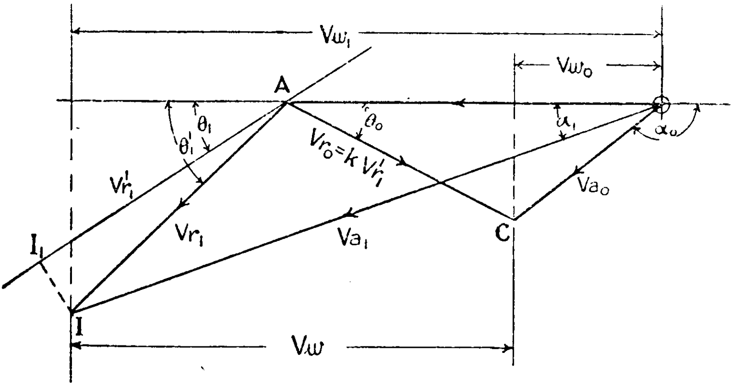


FIG. 113.

normally at a somewhat lower load, then the condition of synchronism will be closely approached.

There appears to be some doubt as to whether, in the case of an elastic fluid like steam, the foregoing assumptions, as to possible loss due to "shock," is warranted. In all probability, unless the underspeeding or overspeeding effect is excessive, this "shock loss" at entry is negligible. It should not be confused with the additional loss which results from an inefficient value of the speed ratio ρ. The shock condition affects only the values of the relative entrance and exit velocities.

When for a given blade, for which the angles of entrance and exit are known, it is desired to allow for any shock effect at entrance, the correction on the velocity diagram can be simply made. The method is illustrated in Fig. 113, which represents the diagram for the case of excessive overspeeding of the impulse blade shown in Fig. 111. Draw the velocity triangle OAI at entrance in the usual way. AI is inclined at the angle θ_1'. Through A draw AI_1 at the correct blade angle θ_1. From I draw II_1 perpendicular to AI_1, then AI_1 is the actual relative velocity, which is less than AI. Construct the exit triangle, making $V_{r_0} = kV_{r_1}'$.

For shock due to underspeeding AI_1 falls below AI, and the same construction applies.

92. Diagram Efficiency η_d.—The ratio between the work done on a blade ring and the kinetic energy of the jet at entrance to the ring is called the "diagram efficiency," since it can be derived directly from the velocity diagram.

$$\eta_d = \frac{E_b}{\frac{V_{a_1}^2}{2g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

For the simple impulse stage

$$\eta_d = \frac{2uV_w}{V_{a_1}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

For the velocity compounded stage

$$\eta_d = \frac{2u}{V_{a_1}^2}\Sigma V_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For the reaction stage

$$\eta_d = \frac{2u^2}{V_{a_1}^2}\left(\frac{2\cos\theta}{\rho} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (26)$$

For a given blade with constant angles, and a constant steam velocity (V_{a_1}), there is in each case some value of the speed ratio ρ, for which the work done on the ring, and hence the diagram efficiency, is a maximum.

By equation (10) the work done in the simple impulse stage is

$$E_b = \frac{u^2c}{g}\left(\frac{\cos\alpha_1}{\rho} - 1\right)$$

and since $\quad \dfrac{u}{V_{a_1}} = \rho, \quad E_b = \dfrac{V_{a_1}^2\rho^2c}{g}\left(\dfrac{\cos\alpha_1}{\rho} - 1\right)$

$$= \frac{cV_{a_1}^2\rho\cos\alpha_1}{g} - \frac{V_{a_1}^2\rho^2c}{g}$$

For E_b maximum $\dfrac{dE}{d\rho} = 0 \quad \therefore \cos\alpha_1 - 2\rho = 0 \quad$ and $\quad \rho = \frac{1}{2}\cos\alpha_1$

or $$u = \tfrac{1}{2}V_{a_1}\cos\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

For the normal angle $\alpha_1 = 20$, $\cos\alpha_1 = 0{\cdot}94$, and

$$u = \frac{0{\cdot}94}{2}V_{a_1} = 0{\cdot}47V_{a_1}$$

In the case of the velocity compounded stage, with the arbitrary variation of velocity coefficients and angles, the analytical expression for ΣV_w is extremely complicated, and no simple relation between u and V_{a_1} can be stated. When, however, the blades are made symmetrical, so that there is a definite progression of angle throughout the set, and the coefficient in each case is taken as unity, an approximation to the value of ρ can be derived from a consideration of the combined velocity diagram. In the ideal condition the work done in the stage would be given by the difference of the kinetic energies of the jet at entrance and exit, or $E_b = \frac{{}_1V^2_{a_1} - {}_nV^2_{a_0}}{2g}$, taking the number of moving rings as n, and ${}_nV_{a_0}$ as the absolute velocity of exit from the nth moving ring. From the combined diagram it follows that

$$nV^2_{a_0} = {}_1V^2_{a_1} + (2nu)^2 - 2(2nu)V_{a_1} \cos \alpha_1$$

$$\therefore\ E_b = \frac{1}{2g}(4n^2u^2 - 4nuV_{a_1} \cos \alpha_1)$$

$$= \frac{1}{2g}(4n^2V^2_{a_1}\rho^2 - 4nV^2_{a_1}\rho \cos \alpha_1)$$

$$\frac{dE}{d\rho} = 0 \quad \therefore\ 8n^2\rho - 4n \cos \alpha_1 = 0$$

and $$\rho = \frac{1}{2}\frac{\cos \alpha_1}{n}$$

or $$u = \frac{1}{n}\left(\frac{{}_1V_{a_1} \cos \alpha_1}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

That is, the blade speed for maximum efficiency of the velocity compound stage, in the actual case, will approximate to $\frac{1}{n}$th the speed for the single impulse stage. The exact value, for any given conditions, such as illustrated by example 3, can be determined, if desired, by trial and error adjustment of the speed ratio (see also Art. 95).

In the case of the reaction stage, from equation (21)

$$E_b = \frac{2V^2_{a_1}\rho \cos \theta}{g} - \frac{V^2_{a_1}\rho^2}{g}$$

$$\frac{dE}{d\rho} = 0 \quad \therefore\ 2 \cos \theta - 2\rho = 0$$

and $$\rho = \cos \theta$$

or $$u = V_{a_1} \cos \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

For normal angle $\theta = 20, \quad u = 0{\cdot}94V_{a_1}$

It will be evident from the energy equation (10) since the work varies as the square of the blade speed, that for any given blade angle the curve of diagram efficiency η_d on a base of speed ratio ρ is a parabola.

A series of efficiency curves can thus be plotted for varying entrance angle θ_1 and constant exit angle θ_0, and another series for varying exit and constant entrance angle. For symmetrical blades with $\theta_1 = \theta_0$ one series is sufficient. The calculation is most expeditiously done from the velocity diagram and equations (24) and (25) for the impulse stage, and from equation (26) for the reaction stage. It is not necessary to know the actual values of the velocities, as only their ratios are involved. The construction of the diagrams, which are similar to those shown in Figs. 114 to 118, is left as an exercise for the reader.

93. Practicable Values of Speed Ratio ρ.—In dealing with the blading design, the first item to be fixed is the value of the speed ratio ρ. From the thermodynamic standpoint the closer this is to the value corresponding to maximum diagram efficiency, the better will be the performance of the machine. On the other hand, a high value of efficiency may require a machine whose size, weight, and cost is, from the commercial standpoint, undesirable.

In turbine design, as in other branches of mechanical engineering, the commercial element is usually the dominating factor.

The question as to whether it is advisable, in any given case, to adopt a comparatively low value of speed ratio and obtain a smaller and cheaper unit of lower efficiency, or to adopt the maximum speed ratio and obtain a larger and more costly machine with high efficiency, is one that can only be settled from the consideration of the commercial aspects of the individual case.

As already pointed out, the approximate values of ρ for the three classes, to give maximum diagram efficiency, are—

		ρ
Simple impulse		0·47
Velocity compound	2 moving rings	0·235
	3 ,, ,,	0·156
	4 ,, ,,	0·117
Reaction		0·94

The values usually employed in practice are lower than the above. For the simple impulse ρ may vary from 0·3 to 0·35. For the velocity compound the values are about 0·21, 0·13, 0·11. For the land type of reaction turbine it varies from 0·5 to 0·6, and for the direct coupled marine turbine from 0·35 to 0·45. In some naval vessels where reduction of weight and space is a matter of importance, it falls as low as 0·3.

When it is considered that 0·94 is about the best value for the reaction stage, the handicap of the direct coupled marine turbine will be apparent. The adoption of the geared marine turbine is to a considerable extent remedying this defect.

The best value of the ratio, so far obtained in practice, is that of the double flow Ljungström turbine. In a large unit of this type it may vary from 0·6 to 0·8. It is in this respect that the Ljungström has a decided advantage over the axial flow turbine with "single motion."

94. Loss of Energy in Blade Channels.—In the blade channels of the impulse turbine there is a degradation of kinetic energy to heat, with the result that the relative exit is always less than the relative entrance velocity. A similar action takes place in the reaction machine, although the effect is not apparent on account of the increase of velocity due to pressure drop in the blade channels.

As in the case of the fixed nozzle passages, this energy loss may for convenience be included under the general heading of "frictional loss."

It may be due to various causes. In the impulse channel these may include—shock of the entering steam on the dead steam carried round in the channel, surface friction, spurious expansion and compression due to bad formation of the passage, centrifugal effect arising from blade curvature, disturbances set up at the nozzle exit and carried into the blade channel, etc.

Such experiments as have been made have been confined to the impulse type of blade, and these have been carried out on stationary rings. The principal results are those obtained by Rateau, who has experimented with a series of blades having angles varying from 25° to 34°. The general conclusion to be drawn from these is that within the limits of velocity employed the velocity coefficient k varies from 0·7 to 0·82. Briling[1] has also obtained values of the coefficients by a similar method. It would appear from these that sharp curvature of the blade face has a considerable effect in reducing the value of k. He gives the following formula for the coefficient:

$$k = k_0 - 0{\cdot}0004322\phi^{\frac{4}{3}}$$

where k_0 varies from 0·9 to 0·97 with blades having rounded edges, and from 0·7 to 0·99 with blades having sharp edges. The angle $\phi = 180 - (\theta_1 + \theta_0)$, or the "total angle of deviation" of the jet.

The values of k given by this formula are, however, lower than those warranted by the efficiency ratios obtained from modern pressure compounded impulse turbines.

One common feature of Rateau and Briling's results is the increase of the value of k with increase of the relative velocity.

From numerous cases he has tested, the author finds that the value of k, expressed as a linear function of the relative entrance velocity, gives efficiency ratios which approximate fairly to the actual values obtained on test from efficient impulse machines. The equation is

$$k = 0{\cdot}73 + \frac{V_{r_1}}{23000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

For the higher velocities used in the pressure compounded impulse turbine the values of k given by this equation agree fairly well with those of Rateau. For the lower velocities they are somewhat greater.

95. When these values are applied to the successive rings of the velocity compounded stage the height ratios, obtained with a given set of angles, are usually greater than those which give the best results in practice. On the hypothesis that the same general effect takes place

[1] See *Engineering*, April 29, 1910.

in the successive blade rings as in a single ring stage, the inference to be drawn, as already mentioned in Art. 88, is that the steam is progressively throttled in its passage through the rings, and that this throttling improves the stage efficiency.

Assuming that a reasonable value of k is fixed for a given impulse blade, the loss of energy in the blade channel is

$$E_f = \frac{V_{r_1}^2 - V_{r_0}^2}{2g}$$

and since
$$V_{r_0} = kV_{r_1}$$

$$E_f = \frac{(1 - k^2)V_{r_1}^2}{2g}$$

It is sometimes stated that for an impulse blade ring the work done on the blades is the difference between the kinetic energy of the jet at entrance and exit, that is

$$E_b = \frac{V_{a_1}^2 - V_{a_0}^2}{2g}$$

This could only be true if there were no "friction" losses in the blade channel, a condition that does not exist.

In order to obtain an approximation to the value of speed ratio ρ for maximum diagram efficiency in the case of the velocity compounded stage, this ideal condition was assumed. This, however, was done in order to avoid the very complicated expression which results when account is taken of the friction loss in the successive channels, and not because the expression is correct.

For the single ring the correct expression for the work done is

$$E_b = \frac{V_{a_1}^2 - (1 - k^2)V_{r_1}^2 - V_{a_0}^2}{2g}$$

It is merely quoted here to guard the reader against the common mistake mentioned above.

The work done on the blades should be calculated from the equations based on the change of moment of momentum of the steam.

96. While experimental results indicate the nature of the correction for friction in the impulse blade, and enable some estimate of the loss to be made, there is no means of directly ascertaining the corresponding loss in the blade channel of the reaction machine.

Some idea of this can, however, be obtained by an indirect method when certain assumptions are made.

If in each case the steam is regarded as a fluid like water flowing through the blade channel, and that the frictional loss of energy is proportional to the square of the velocity, then the calculation of the probable loss becomes determinate, and a comparison of the efficiencies of the two types can be made.

Taking the impulse case first, if f is the coefficient of frictional resistance, assumed constant, and V_r the relative velocity, then the loss of energy on any element dx of length of the channel, is $-f\dfrac{V_r^2}{2g}dx$.

This is also $d\left(\dfrac{V_r^2}{2g}\right)$

$$\therefore \; -f\frac{V_r dx}{2g} = \frac{d(V_r)^2}{2g}$$

and, $2V_r dV_r = -fV_r^2 dx$ — Integrating when $x = 0$, $V_r = V_{r_1}$

$\log_e V_r = -\frac{1}{2}fx + c$ — " $x = s =$ whole length of channel, $V_r = V_{r_0}$

$\therefore \; 2(\log_e V_r - \log_e V_{r_1}) = -fs$

and
$$k^2 = \left(\frac{V_{r_0}}{V_{r_1}}\right)^2 = \epsilon^{-fs}$$

hence
$$k = \epsilon^{\frac{-fs}{2}} = \frac{1}{\epsilon^{\frac{fs}{2}}} = \frac{1}{\epsilon^{\phi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where $\phi = \dfrac{fs}{2}$, a friction coefficient for the whole blade.

For $k = 0{\cdot}75$, $\phi = 0{\cdot}287$; for $k = 0{\cdot}82$, $\phi = 0{\cdot}2$

These are about the limits for the usual run of pressure compounded impulse stages.

Taking the reaction blade next and adopting the same hypothesis as to the variation of loss with velocity, and also assuming that the rate of increase of kinetic energy is constant between entrance and exit, it follows that the curve of energy or of relative velocity squares $(V_r)^2$, when plotted on a base of channel length, is a straight line, having a slope $\dfrac{V_{r_0}^2 - V_r^2}{s}$. Here s is the total channel length. $V_{r_1}^2$

The equation to the line is

$$V_r^2 = V_{r_1}^2 + \frac{(V_{r_0}^2 - V_{r_1}^2)}{s}x$$

As before, for any element of channel length dx the loss is $dE_f = \dfrac{f}{2g}V_r^2 dx$, and the total loss due to "friction" is

$$E_f = \frac{f}{2g}\int_0^s \left[V_{r_1}^2 + \left(\frac{V_{r_0}^2 - V_{r_1}^2}{s}\right)x\right]dx = \frac{fs}{2g}\{V_{r_1}^2 + \tfrac{1}{2}(V_{r_0}^2 - V_{r_1}^2\}$$
$$= \tfrac{1}{2}fs\left\{\frac{V_{r_0}^2 + V_{r_1}^2}{2g}\right\} = \phi\left\{\frac{V_{r_0}^2 + V_{r_1}^2}{2g}\right\}$$

Referring to Fig. 110, it can be seen from the geometry of this that

$$V_{r_1}^2 = V_{a_1}^2 - 2uV_{a_1}\cos\alpha_1 + u^2$$

so that
$$V_{r_0}^2 + V_{r_1}^2 = 2V_{a_1}^2 - 2uV_{a_1}\cos\alpha_1 + u^2$$

hence the frictional loss is

$$E_f = \frac{u^2\phi}{2g}\left\{\frac{2}{\rho^2} - 2\frac{\cos\alpha_1}{\rho} + 1\right\} \quad . \quad . \quad . \quad (32)$$

As the "carry over" is constant at each ring, the energy equivalent of the heat drop in the channel is the sum of the work done on the blade ring, and this friction loss, that is, $Jh_r = E_b + E_f$.

Substituting from equation (21) for E_b, the heat drop expressed in terms of the blade velocity, jet energy, and speed ratio becomes

$$h_r = \frac{u^2}{2gJ}\left\{\frac{\cos\alpha_1}{\rho}(4 - 2\phi) + \phi\left(\frac{2}{\rho^2} + 1\right) - 2\right\} \quad . \quad . \quad (33)$$

In the case of the impulse blade, with a constant carry-over velocity V_1, the heat drop as shown in Art. 69, Chap. VII., is given by

$$h_r = \frac{\frac{V_{a_1}^2}{(223 \cdot 7)^2} - \frac{mV_1^2}{(223 \cdot 7)^2}}{M}$$

If $V_1 = zV_{a_1}$, then $h_r = \left(\frac{V_{a_1}}{223 \cdot 7}\right)^2\left(\frac{1 - mz^2}{M}\right)$

$$= \frac{u^2}{2gJ\rho^2}\left(\frac{1 - mz^2}{M}\right) = \frac{u^2}{2gJ\rho^2} \times B \quad . \quad (34)$$

The doubtful factor in this equation is the coefficient B, since the values of m and M, the friction coefficients, are more or less indeterminate.

If as already suggested in Art. 69 the values are assumed as $m = 0 \cdot 8$ and $M = 0 \cdot 9$ to $0 \cdot 93$ for the type, the corresponding value of B may be ascertained. Normally the carry-over velocity for a stage of this kind is about 30 per cent. of the jet velocity at entrance. Taking this as an average $z \doteqdot 0 \cdot 3$. The corresponding value of B may thus vary from $1 \cdot 031$ to $1 \cdot 02$. The latter might be regarded as a minimum where superheated steam is used.

In the case of the few stage impulse turbines, where there is practically no effective carry over $z = 0$ and $M = \eta_n$, so that $B = \frac{1}{\eta_n}$.

97. Stage Efficiency.—The diagram efficiency, while it is important for the determination of the best ratio of jet and blade velocity, accounts only for part of the disturbing influences that affect the efficiency of the turbine as a whole, and each stage in particular.

At any stage of a compound turbine, there is a pressure drop, and the amount of heat that is available for conversion into kinetic energy, for the performance of work in the stage, is the Rankine cycle heat or heat drop h_r, between the stage pressure limits. Only a part of this energy is utilised in doing work on the rotor shaft. The remainder may all be reconverted to heat through friction in blade and nozzle channels, disc and vane friction, and in eddying and shock on the chamber walls. This is what occurs in the few stage impulse machines, where the steam is brought to rest in one stage before it passes into the next.

On the other hand, in the multistage impulse and the reaction machines, a proportion of the energy developed originally in the first stage is carried over from stage to stage, as the steam flows continuously, without sensible obstruction, into the successive nozzle passages.

The ratio between the nett work done on the rotor in the stage and the stage heat drop may be termed the stage efficiency, that is

$$\eta_s = \frac{E_b'}{Jh_r}$$

The nett work E_b' is the work per lb. of steam E_b corrected in the case of the impulse machine for the disc friction and leakage loss at the diaphragms, and in the case of the reaction machine, for tip leakage loss. The efficiency can be stated as

$$\eta_s = \frac{\gamma E_b}{Jh_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where γ is a coefficient. In the impulse machine towards the L.P. end, γ is practically unity. At the first few stages it may vary from 0·94 to 0·98. In the axial flow reaction turbine the drum and blade friction are negligible, but the tip leakage at the H.P. end may be considerable, especially in directly coupled marine turbines.

In this case γ represents the ratio between the "working steam" passing through the blade channels and the total steam passing through the stage, that is, through the channels and clearance spaces. The method of determining γ is discussed in Chap. X.

In the case of the impulse machine where there is no carry over, and the steam is brought to rest in each stage, with reconversion of kinetic energy to heat, it will be apparent that for the same amount of work per stage, a larger heat drop than that with carry over is required, and there is a reduction in the value of the stage efficiency. The effect, however, depends on the ratio of the residual to the jet velocity in each stage. If this is small, the effect on the stage efficiency is very slight.

If the nett work expended by the steam on the rotor is expressed as an equivalent amount of heat, that is, $\frac{\gamma E_b}{J} = h_s$, the stage efficiency is also given by

$$\eta = \frac{h_s}{h_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Since the relation between the values of k and ϕ for given values of coefficient of frictional resistance f, and common channel length s, is given by equation (31), it is possible to make a rough comparison between the stage efficiencies of impulse and reaction stages.

In the case of the impulse machine, the stage efficiency, on substitution for E_b and h_r, from equations (10) and (34), is given by

$$\eta_s = \frac{\gamma E_b}{J h_r} = \frac{2\gamma\rho}{B}\left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right)(\cos \alpha_1 - \rho) \quad . \quad . \quad (37)$$

When there is no carry over $B = \frac{1}{\eta_n}$, and hence

$$\frac{\eta_s}{\gamma\eta_n} = 2\rho\left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right)(\cos \alpha_1 - \rho)$$

The right-hand side of the equation is the diagram efficiency η_d, so that for this case

$$\eta_s = \gamma\eta_n\eta_d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The stage efficiency is therefore the product of the disc friction factor and the nozzle and diagram efficiencies.

In the case of the reaction stage, the stage efficiency, on substitution for E_b and E_f from equations (21) and (33), is given by

$$\eta_s = \frac{\gamma E_b}{E_b + E_f} = \frac{2\gamma\left\{2\frac{\cos \alpha_1}{\rho} - 1\right\}}{\frac{\cos \alpha_1}{\rho}(4 - 2\phi) + \phi\left(\frac{2}{\rho^2} + 1\right) - 2} \quad . \quad (39)$$

EXAMPLE 5.—Find the stage efficiency at the second stage of the impulse turbine (example 1) for which the following data are available: $\alpha = 20^\circ$, $\rho = 0{\cdot}322$, $k = 0{\cdot}78$, $\gamma = 0{\cdot}93$, $V_{a_1} = 1590$, $V_1 = 465$, $\theta_1 = 29^\circ$, $\theta_0 = 25^\circ$. Assume that $m = 0{\cdot}8$ and $M = 0{\cdot}92$.

Since

$$V_{a_1} = 1590, \quad \text{and } V_1 = 465, \quad z = \frac{465}{1590} = 0{\cdot}292, \quad \text{and } z^2 = 0{\cdot}0853$$

$$\text{Then} \quad B = \frac{1 - mz^2}{M} = \frac{1 - 0{\cdot}8 \times 0{\cdot}0853}{0{\cdot}92} = \frac{1 - 0{\cdot}0682}{0{\cdot}92} = 1{\cdot}013$$

By equation (37)

$$\eta_s = \frac{2\gamma\rho}{B}\left(1 + \frac{k \cos \theta_0}{\cos \theta_1}\right)(\cos \alpha_1 - \rho)$$

$$= \frac{2 \times 0{\cdot}93 \times 0{\cdot}322}{1{\cdot}013}\left(1 + \frac{0{\cdot}78 \times 0{\cdot}906}{0{\cdot}8746}\right)(0{\cdot}94 - 0{\cdot}322)$$

$$= \frac{0{\cdot}93 \times 0{\cdot}644 \times 1{\cdot}81 \times 0{\cdot}618}{1{\cdot}013} = 0{\cdot}66$$

The nozzle calculations for this stage are given in example 8, Chap. VII., and the heat drop, as checked in the $H\phi$ diagram, is approximately 51 B.Th.U. In example 1 the work done on the blading is 28,100 ft./lbs. or 36·2 B.Th.U. The nett heat is $0{\cdot}93 \times 36{\cdot}2 = 33{\cdot}7$. Hence by equation (36) the stage efficiency is

$$\eta_s = \frac{33{\cdot}7}{51} = 0{\cdot}66$$

which agrees with the value calculated by the other method.

At the first stage, where there is no carry over, the heat drop is 55 B.Th.U. and $\gamma = 0{\cdot}933$, so that for the same performance of work on the blades, the efficiency for the stage is

$$\eta_s = \frac{0{\cdot}933 \times 36{\cdot}2}{55} = 0{\cdot}615$$

These efficiencies are much lower at the first two wheels than at the others on account of disc friction and vane loss. The efficiency rises rapidly and reaches a figure over 70 per cent. at the L.P. stages. The average for the machine, which has eight stages, may be taken from 68 to 69 per cent.

EXAMPLE 6.—In the velocity compounded stage (example 3) calculate the stage efficiency if the nozzle efficiency is 0·96 and $\gamma = 0{\cdot}96$.

Here $\Sigma V_w = 4078$, $u = 455$, $V_{a_1} = 2275$, $\gamma = 0{\cdot}96$; hence by equation (25) the diagram efficiency is

$$\eta_d = \frac{2u\Sigma V_w}{V_{a_1}^2} = \frac{2 \times 455 \times 4078}{(2275)^2} \doteq 0{\cdot}72$$

By equation (38) stage efficiency $\eta_s = \gamma\eta_n\eta_d = 0{\cdot}96 \times 0{\cdot}96 \times 0{\cdot}72 = 0{\cdot}662$.

These two efficiencies are not directly comparable, since the impulse machine has $u = 511$, whereas the Curtis has $u = 455$, and the disc and vane friction, which principally determines γ, varies as the cube of the speed. Further, the pressure compounded wheel runs in steam of much higher pressure and density than the Curtis wheel, and the disc friction varies directly as the density.

The Curtis machine with four stages has a lower average value of stage efficiency than the pressure compounded one with eight, the value being 65 per cent. This figure is arrived at by combining the η_n values deduced in example 6, Chap. VII., with the γ values deduced in example 12, Chap. XI. (See page 301.)

EXAMPLE 7.—Calculate the probable stage efficiency of the reaction stage (example 4), using the value of the friction coefficient ϕ corresponding to the value of k used for the impulse stage (example 5), and compare the efficiencies. Take $\alpha = 20°$ and $\rho = 0{\cdot}55$, $\gamma = 0{\cdot}93$.

Since $k = 0{\cdot}78$, ϕ (by equation (31)) is 0·25.

By equation (39)

$$\eta_s = \frac{2\gamma\left\{2\frac{\cos\alpha_1}{\rho} - 1\right\}}{\frac{\cos\alpha_1}{\rho}(4 - 2\phi) + \phi\left(\frac{2}{\rho^2} + 1\right) - 2}$$

$$= \frac{2 \times 0{\cdot}93\left\{\frac{2 \times 0{\cdot}94}{0{\cdot}55} - 1\right\}}{\frac{0{\cdot}94}{0{\cdot}55}(4 - 2 \times 0{\cdot}25) + 0{\cdot}25\left(\frac{2}{0{\cdot}55^2} + 1\right) - 2}$$

$$= \frac{1{\cdot}86 \times 2{\cdot}42}{1{\cdot}71 \times 3{\cdot}5 + 0{\cdot}25 \times 7{\cdot}63 - 2}$$

$$= \frac{4{\cdot}5}{5{\cdot}877} = 0{\cdot}765$$

The value of γ in this case has been taken the same as that for the impulse stage, on the assumption that the tip leakage loss equals the disc and vane loss and leakage loss at the diaphragm gland. On this hypothesis, since the impulse stage has $\eta_s = 0{\cdot}66$, the ratio is

$$\frac{\text{Impulse stage efficiency}}{\text{Reaction stage efficiency}} = \frac{0{\cdot}66}{0{\cdot}765} = 0{\cdot}862$$

Taking $\rho = 0{\cdot}5$ as the maximum with the impulse, and $\rho = 0{\cdot}94$ maximum with the reaction stage, it is seen that the impulse stage with $\rho = 64{\cdot}4$ per cent. of the maximum has an efficiency 86·2 per cent. of the efficiency of the reaction stage, which has $\rho = 58{\cdot}5$ per cent. of its maximum value. At this comparatively low value of ρ, the reaction stage has still a decided superiority over the impulse one. The value of γ chosen is higher than that which usually occurs at the first H.P. expansion of a reaction machine. It may run from 0·88 to 0·9. With the lower figure the stage efficiency is reduced to 0·72. The value of γ rises to 0·98 at the L.P. end, and for the same value of $\rho = 0{\cdot}55$, with this figure, the efficiency becomes 81 per cent.

The speed ratio ρ in land reaction machines usually does not rise above 0·6, so that as a general inference from the above, it may be concluded that the stage efficiency for this type may vary from about 70 per cent. at the H.P. to 80 per cent. at the L.P. end of the machine.

98. Internal Efficiency.—If the stage efficiency η_s is assumed to be sensibly constant throughout a machine, the total nett heat is given by $\Sigma h_s = \eta_s \Sigma h_r$. In Chap. XI. it is shown that the quantity Σh_r, which may be termed the "cumulative heat," is greater than H_r the Rankine cycle heat between the pressure limits of the machine, that is

$$\Sigma h_r = \text{R}H_r$$

where R is a coefficient called the "reheat factor." From this it follows that

$$\Sigma h_s = \eta_s \text{R}H_r$$

Dividing each side by H_r

$$\frac{\Sigma h_s}{H_r} = R\eta_s = \eta_1$$

This ratio between the total nett heat or heat actually transformed into work at the shaft, and the total heat drop, is called the "internal efficiency" of the turbine. In a compound turbine it is greater than the stage efficiency.

99. Nett Efficiency or Efficiency Ratio.—The internal efficiency, which corresponds to the "hydraulic efficiency" of a water turbine, takes account of all the internal losses of friction and leakage which affect the quality of the steam at the successive stages.

In addition to these losses there are outside losses due to bearing friction at journals, friction at auxiliary gears as at governors and pumps. The effective work transmitted to the shaft throughout the turbine is $\Sigma h_s = \eta_1 H_r$. Part of this is absorbed in overcoming these external resistances and is lost at the driving end of the machine. In addition a certain proportion of the total heat drop H_r is wasted in external gland leakage, radiation, and carry over to the condenser.

All the losses internal and external are covered by a single factor known as the "efficiency ratio," or "turbine efficiency," or "nett efficiency." The correct term is "efficiency ratio." If the heat equivalent of the work transmitted by the shaft at the driving end of the machine, that is, the brake horse-power or shaft kilowatt output per lb., is denoted by H_B, and the heat drop between the initial and exhaust pressures by H_r, the efficiency ratio is

$$\eta = \frac{H_B}{H_r}$$

If the total nett heat $\Sigma h_s = H_I$, then

$$\frac{\eta}{\eta_1} = \frac{H_B}{H_I}$$

This latter ratio may be regarded as a "rotational efficiency." It is analogous to the mechanical efficiency of the steam engine, although it takes account of other than purely frictional losses.

The two efficiencies η and η_1 and the speed ratio ρ are the most important factors in the calculation of the proportions of any given type of turbine. They are discussed more fully in Chap. XI.

100. Experimental Values of Stage Efficiency.—In preliminary calculations relating to the general dimensions of impulse turbines the stage efficiency is one of the first factors to be ascertained. It is the practice, in the case of experimental values, to estimate these in terms, not of the actual but of the theoretical speed ratio. The latter is $\rho_t = \rho\sqrt{\eta_n} = \dfrac{u}{V_0'}$, where V_0' is the theoretical velocity of exit from the nozzle passage.

Very few experimental results have been published, but those obtained by Lasche, and quoted by Stodola,[1] give a fair idea of the limiting values of η_s to be obtained from the various types of impulse blading. In their original form the curves were plotted to a base of the reciprocal of the speed ratio. The author has plotted the values of η_s to a base of ρ_t. These are shown in Figs. 114, 115, 116, and 117.

For the pressure-compounded impulse stage the curves of Fig. 114 show a variation of maximum stage efficiency from 0·68 to 0·80 at a theoretical speed ratio of 0·45. The variation of value is probably due to the different blade angles used. It will be noted that between 0·4 and 0·45 the curves are flat. Taking, say, a maximum value of

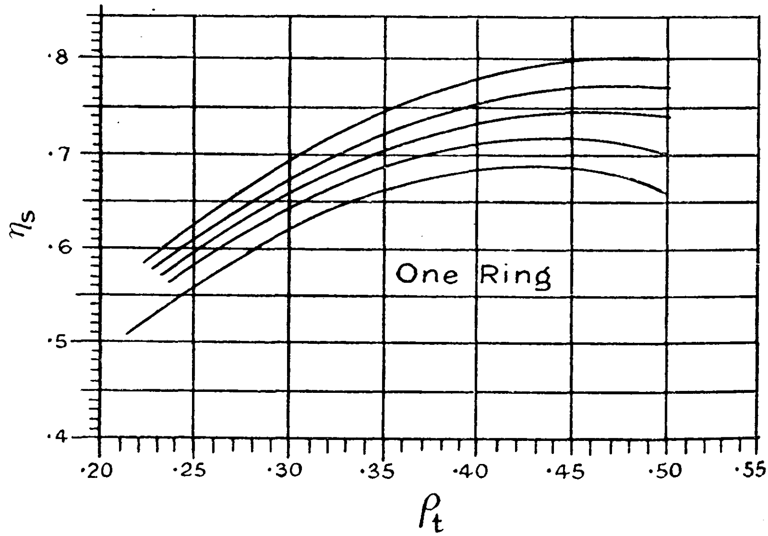


FIG. 114.

nozzle efficiency as 0·92 for the pressure-compounded machine the actual velocity ratio varies from 0·42 to 0·47. For commercial reasons principally it is kept between 0·3 and 0·35. With the latter figure $\rho_t = 0·96 \times 0·35 = 0·346$. This indicates a variation of η_s between 0·65 and 0·73.

Comparing the case considered in example 1, where $\rho = 0·322$ and $\eta_n = 0·92$, then $\rho_t = 0·96 \times 0·322 = 0·31$. At this speed ratio the value of η_s, as shown by the curves, may be between 0·63 and 0·71. The value of the stage efficiency as calculated for the second stage is 0·66, which is about an average. As the efficiency value rises towards the L.P. end of a multistage impulse turbine it would seem reasonable to use the fourth curve of Fig. 114 when deciding the average stage efficiency for the whole machine.

[1] See also *Engineering*, December 15, 1911, for reproduction of Lasche's curves.

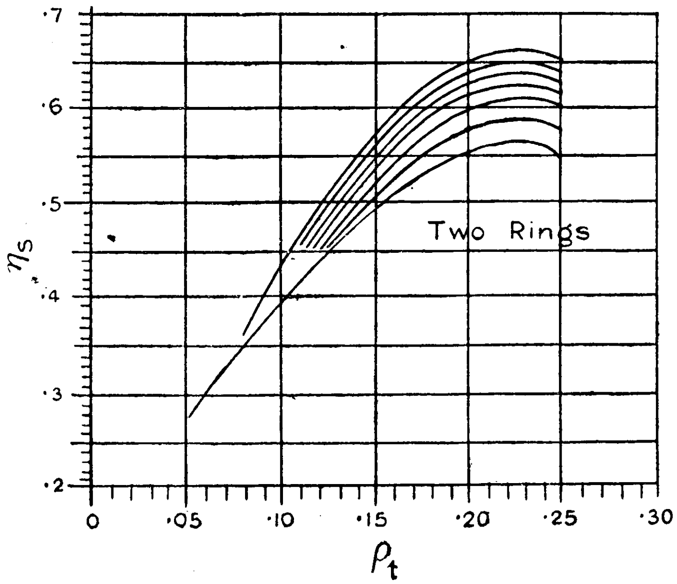


FIG. 115.

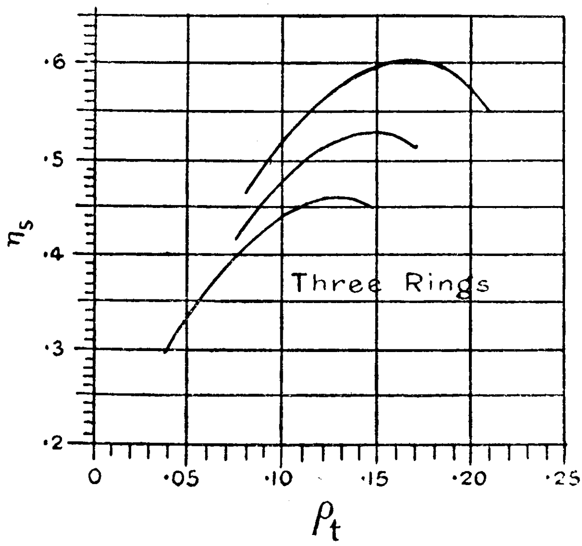


FIG. 116.

In Fig. 118 three curves for one, two, and three ring stages are shown. These are reproduced from Baumann's paper to the Institution of Electrical Engineers,[1] and are intended to represent average values of the stage efficiency for large machines of pressure compounded and pressure-velocity compounded types. They represent calculated values, while Lasche's values are experimental. A reference to the curve of one ring shows that for the above example, $\eta_s = 0{\cdot}71$, the same value as given by the top curve in Fig. 114. For the single ring stage it is probable that this curve gives too high values for the average stage efficiency.

101. Taking the two-velocity compounded stage next, the rate of variation of η_s with ρ_t is much more pronounced, although between 0·2 and 0·25, Fig. 115, the curves are fairly flat. The maximum value, both in Figs. 115 and 118, is shown at $\rho_t = 0{\cdot}225$. Normally the heat drop in the Curtis type with four stages is somewhat greater than

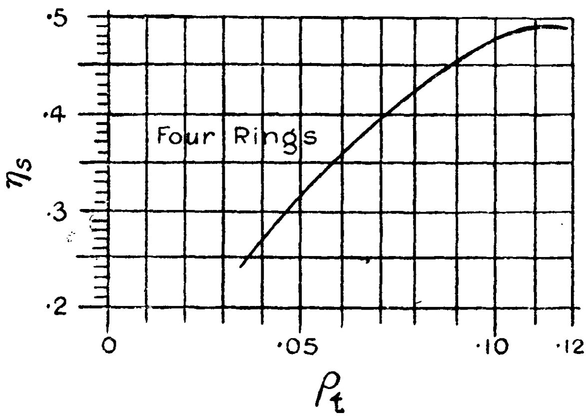


FIG. 117.

100 B.Th.U., and the maximum nozzle efficiency about 0·96. The corresponding value of the speed ratio is thus $\rho = \dfrac{0{\cdot}225}{0{\cdot}98} = 0{\cdot}23$. At $\rho_t = 0{\cdot}225$, the curves of Fig. 115 show a variation of η_s between 0·57 and 0·68. The curve of Fig. 118 shows the value 0·68. The upper curve of Fig. 115 or that of 118 may be used to ascertain the stage efficiency when there are two velocity stages per pressure stage. In the case of example 3, $\rho = 0{\cdot}2$ and $\eta_n = 0{\cdot}96$, so that $\rho_t = 0{\cdot}2 \times 0{\cdot}98 = 0{\cdot}196$. For this value the curve of Fig. 118 gives $\eta_s = 0{\cdot}655$, and the upper curve of Fig. 116 $\eta_s = 0{\cdot}65$. The efficiency calculated in example 6 is 0·662, and is probably on the high side.

102. For the three-velocity stage the curves of Fig. 116 do not give such definite information. The maximum value of the efficiency

[1] "Recent Developments of Steam Turbine Practice," by K. Baumann, *Proc. Inst. Elect. Engineers*, 1912, Vol. 48.

apparently falls anywhere between the speed ratio values 0·12 and 0·16. Taking an average $\rho_t = 0{\cdot}14$ and average value of nozzle efficiency $\eta_n = 0{\cdot}94$, the corresponding speed ratio is $\rho = 0{\cdot}144$. The stage efficiency may apparently vary from 0·47 to 0·6. The average

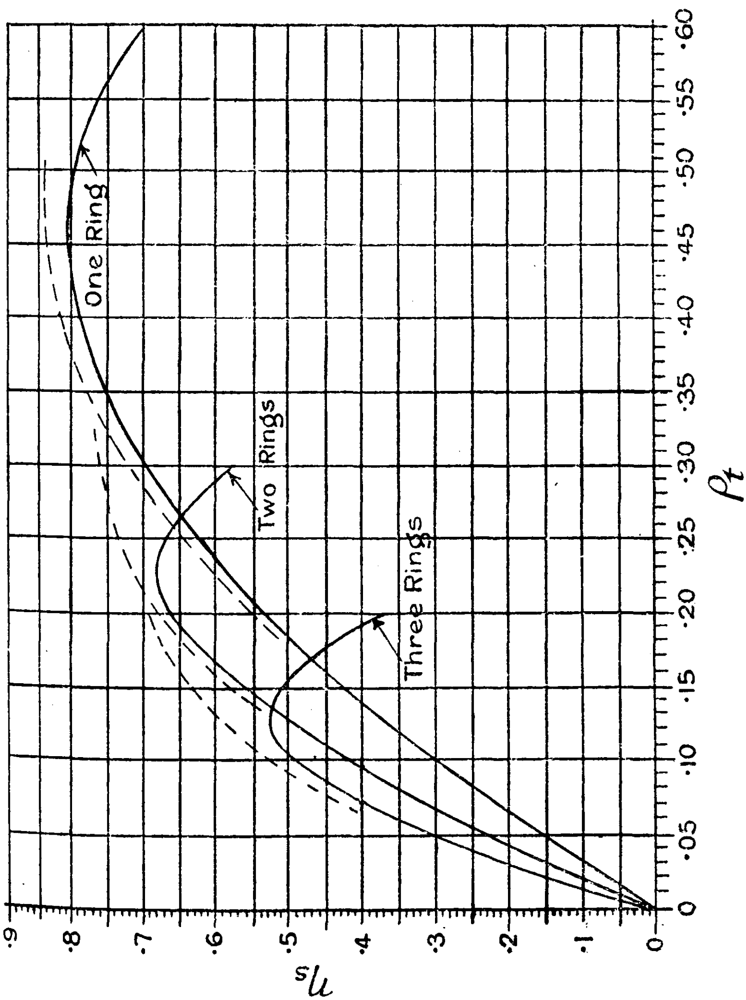


FIG. 118.

at $\rho_t = 0{\cdot}14$ is 0·53. This coincides with the maximum value given by the curve of three rings in Fig. 118, but the speed ratio in this case is $\rho_t = 0{\cdot}13$.

It will be noted that three dotted curves are shown in Fig. 118. These represent average values of the stage efficiency uncorrected for disc and vane friction loss. The effect of this friction at the high pressure end of a Curtis turbine having a three-velocity stage is considerable. It is not so pronounced in the case of the usual two-velocity stage. The subject of disc friction loss is considered in detail in Chap. X.

103. The four-velocity stage is only used in the marine type of Curtis machine, and in the marine combination turbine. In this case the blade speed is always much lower than that in the land turbine, and the disc friction effect is relatively smaller. The maximum efficiency is, however, necessarily lower than that of the three-velocity stage. An average curve for the four-velocity stage is shown in Fig. 117. The maximum value which approximates to 0·5 occurs at $\rho_t = 0{\cdot}11$. With a nozzle efficiency of 0·96 this gives the actual velocity ratio of 0·112. Usually the value employed in the first stage of the marine turbine is below 0·1, from 0·08 to 0·09, and the probable value of the stage efficiency will be about 0·45.

In the system of calculation for the determination of the general proportions of compound turbines, used by the author, these five sets of curves are taken as standards of reference for the stage efficiencies.

104. Exit Blade Angle θ_0.—The velocity diagram shows that the smaller this angle the greater the amount of work done on the blade ring. As the angle decreases the blade height at exit has to increase in order to obtain the necessary area to pass the steam. When θ_0 becomes very small the blade length becomes impracticable. Also the smaller the angle the greater the channel length and wall surface, a condition that increases the friction loss. Reduction of angle between entrance and exit conforms to the reduction in the circumferential width of the jet which takes place when a fluid flows over a curved surface. The jet tends to spread in the lateral or radial direction, and this condition involves some loss of energy. The steam acquires a velocity in the radial or non-effective direction. This spreading is checked by the shroud ring of the blading. The amount of this reduction of spread is a matter to be settled by experience, and the choice of the value of θ_0 is quite arbitrary. It is found in practice that it is not advisable to reduce the angle below 15°. It is usually varied from 16° to 30°. At low-pressure stages the practical limitation of blade length may require an increase to 35° or 40°.

In velocity-compounded machines at high-pressure stages the series of stage angles may vary from 22° to 30° or 40°. At low-pressure stages slightly higher values may be used, say 24° to 45°.

In Parsons axial flow turbine the angle may vary from 16° to 20°, except at the last few L.P. stages, where semiwing, wing, and double wing blades are used. In these cases it may increase from 25° to 50°.

In the Ljungström turbine the angle varies between 18° to 21° throughout the machine. At the last few rings somewhat larger angles are used to keep down the blade lengths and prevent too great divergence of the blade channel.

105. Maximum Blade Length.—When the length of the blade exceeds a certain proportion of the mean ring diameter, the circumferential pitching at the tips becomes too large for the proper control of the flow, and disorderly motion of the steam in the channel ensues. Further, as the blade length increases the centrifugal and bending stresses increase, and there is a greater tendency to vibration.

With regard to the strength of the blade it is desirable to ascertain what maximum value of the ratio of blade length to mean ring diameter may be allowed without unduly stressing the material.

Let A = sectional area of the blade in in.^2.
l = length of the blade in inches.
u = mean blade speed in ft./sec.
D = mean ring diameter in inches.
w = weight per cub. inch of the material of the blade.

Then the centrifugal force is

$$F_c = \frac{2wu^2Al \times 12}{gD}$$

In the case of the impulse blade the nett section, a, at the root, which takes this pull, is less than the full blade section, on account of the reduced width required to fit either a dovetail or a T slot in the rim.

If the blade were of uniform section A, the centrifugal stress would be $f_c = \dfrac{24wu^2}{g} \times \dfrac{l}{D}$ Taking $w = 0{\cdot}288$ for steel and $0{\cdot}3$ for brass blades, and denoting the ratio $\dfrac{l}{D}$ by z, the stress is given by

$$f_c = 0{\cdot}215u^2z, \text{ for steel,}$$
$$f_c = 0{\cdot}224u^2z, \text{ for brass.}$$

The last blade height at the L.P. end of an impulse machine may vary from 6 to 15 per cent. of the mean ring diameter according to the output and speed. In the reaction machine it varies from 8 to 15 per cent., and in marine turbines to 15 per cent. In some instances it may reach a value of 20 per cent. The latter figure may be taken as covering the general case, and the corresponding stresses for a blade of uniform section are as follow :—

u	100	200	300	400	500	600 ft./sec.	
f Steel . . .	430	1720	3870	6900	10,800	15,500	lbs./in.²
f Brass . . .	448	1790	4040	7170	11,200	16,100	lbs./in.²

In the case of the impulse blade, on account of the reduced section at the root, which takes the centrifugal pull, this stress is increased to $f_c' = f_c\left(\dfrac{A}{a}\right)$. The value of $\left(\dfrac{A}{a}\right)$ is variable, but assuming, say, $a = 0{\cdot}75A$, this gives $f_c' = 1{\cdot}33f_c$. It is seldom that a peripheral blade speed as high as 600 ft./sec. is used for an impulse wheel, but for this speed the probable stress, with a 20 per cent. ratio, is about 20,000 lbs./in.^2 for steel and 21,000 for brass.

If mild steel similar to that of the disc is used for the impulse blading an elastic strength of 20 tons/in.^{2} or 45,000 lbs./in.^{2} may be assumed. For the extreme limits of speed and blade height chosen here, there is thus a factor of safety of $2\frac{1}{4}$ on the elastic strength. Normally it is much greater, say $3\frac{1}{2}$ to 4, as, with the usual blade proportions and speeds, the working stresses are much less than the values given above.

In the case of drawn brass impulse blades an elastic strength of 20 tons/in.^{2} or again 45,000 lbs./in.^{2}, may be assumed, so that the brass blade may also be used up to the same limits as the mild steel. Where nickel steel is used for the blades the margin is much greater, as an elastic strength of 66,000 lbs./in.^{2} may be taken. The minimum value of the factor of safety in this case is 3·3.

The value, however, will be slightly reduced on account of the additional bending stress. This is a small quantity relatively to the centrifugal stress, and as its calculation, owing to the peculiar form of the section, is somewhat tedious it is seldom estimated. On the whole it would appear that a ratio of blade height to mean ring diameter in excess of 20 per cent. should not be used. It is advisable whenever possible to limit the maximum value to 15 per cent.

In the case of the reaction blade the full area of section is available. Further, the use of a drum instead of a disc restricts the maximum blade speed with mild steel to about 400 ft./sec., and with nickel steel 450 ft./sec.

With the usual drawn brass blading the probable stress will thus be slightly greater than 11,200 lbs./in.^{2}, giving a factor of safety of 4 on the elastic strength. With the usual blade proportions and speeds the working stress is always much less than this, about 6000 to 8000 lbs./in.^{2}.

In the Parsons turbine it is, normally, the provision of reasonable pitch at the tips and satisfactory fixture of the caulking pieces, not the question of strength that has to be considered.

For satisfactory conditions of blade pitch, both in impulse and reaction turbines, it is not advisable, except in special cases, to make the blade length much greater than 15 per cent. of the mean ring diameter. The lower this ratio can be kept the better the conditions for orderly flow through the blade channel.[1]

106. The large number of blades required in the Parsons axial flow turbine makes a system of standardisation a necessity.

The blading is usually made in four or five sizes. It ranges in axial width from $\frac{3}{8}$ inch to $1\frac{1}{8}$ inch. Blades from $\frac{3}{4}$ to $1\frac{1}{8}$ inch are usually semiwing, wing, and double wing. The large exit angles required by these blades reduce the cross-sections considerably, and

[1] The maximum blade length, 20 per cent. of the mean diameter, refers to machines of moderate output and speed. In recent practice, the single flow type of disc and drum turbine has been used for high output at high speed (5000 K.W. at 3000 R.P.M.), and in order to obtain the necessary area for flow at the L.P. end, the blade length has been increased to a maximum of 26 per cent. of the mean diameter. A special type of steel blade is used. The length of the blade is divided into three parts, of gradually diminishing thickness between root and tip, the radial section having a stepped appearance. The caulking piece at the root is made solid with the blade.

the strength and stiffness have to be maintained by increasing the axial widths. The widths of normal blades increase by $\frac{1}{8}$ inch.

The following may be taken as illustrations of the values usually employed, although the practice varies slightly with different makers:—

$\frac{3}{8}$ inch	blade	used	up to	4	inches	length
$\frac{1}{2}$ "	"	"	"	7	"	"
$\frac{5}{8}$ "	"	"	"	9	"	"
$\frac{3}{4}$ "	"	"	"	12	"	"
1 "	"	"	"	15	"	"
$1\frac{1}{8}$ "	"	"	"	20	"	"

The maximum value of the ratio of length to axial width given above is about 18. In the usual run of cases it is about 15. These long blades are stiffened against vibration by two and sometimes three rings of binding wire.

In the impulse machine the blade width may vary from $\frac{7}{8}$ inch at the H.P. end to $1\frac{1}{2}$ inches at the L.P. end. These blades are subjected to the action of steam at much higher velocities than are used with the reaction blades. At the L.P. stages the wider angles reduce the section, and it is advisable to keep the maximum limit of the ratio of length to width about 8 or 9, so as to ensure sufficient stiffness, and avoid undue vibration.

107. Minimum Blade Length.—In the impulse turbine this figure is settled by purely mechanical considerations. The smallest blade length adopted is $\frac{3}{8}$ inch. In large machines it runs from 1 to $1\frac{1}{2}$ inches, at the first high-pressure stage. As already pointed out in connection with the design of nozzles, the minimum value may be taken from $1\frac{1}{2}$ to 2 per cent. of the mean ring diameter. The progression of blade length, between the minimum and the maximum values in this type of machine, where there is partial admission, is an arbitrary one.

In the Parsons reaction turbine full peripheral admission is necessary from the first stage onward, and the minimum length is fixed with reference to the tip leakage. It should not be less than 3 per cent. of the mean ring diameter, if considerable loss of efficiency is to be avoided.

The increase in length in this case should be such that the curve of blade length approximates to the volume curve for the steam. Such an arrangement is commercially impracticable and the blades are arranged in groups or "expansions," the rings of a group all having the same blade length. The lengths for the successive groups are made to progress geometrically, the common ratio varying from 1·3 to 1·4. The determination of reaction blade lengths is fully dealt with in Chap. XIV.

108. Radial and Axial Blade Clearance in Impulse Turbines.—There is no thermodynamic reason for limiting the radial clearances in the impulse machine. In large pressure-compounded machines it may vary from 0·18 to 0·25 inch. In some Curtis machines the rings run quite clear of the casing. The clearance, when small, should be sufficient to prevent the blades from fouling the casing if the rotor should whip or otherwise get out of truth when running at speed. The real clearance difficulty does not arise at the blade tips but at the diaphragm glands.

Axial clearance between the nozzles and blades, on the other hand, should be kept as low as possible, especially at the high-pressure end where there is partial admission. Small clearances help to reduce the friction and eddy loss where the band of steam flowing between the nozzles and blades cuts across the edge of the zone of "dead" steam in the clearance space. This axial clearance varies from 0·1 to 0·2 inch, depending on the size of the machine.

In the case of the velocity-compounded impulse turbine, it is usual to give a radial increase at the entrance edge of each successive blade, of about 0·04 inch, in order to avoid "spilling" over the ends of the blades.

In the de Laval turbine the nozzle exit is kept 0·06 inch clear of the entrance side of the wheel blades. In the rim bucket type it is kept from 0·1 to 0·2 inch from the face of the rim.

109. Radial and Axial Blade Clearance in Parsons Turbines.—At the high-pressure end the radial clearance has to be reduced to the smallest possible value consistent with safe running. The practicable minimum is about 0·02 inch, when the turbine is cold. This fine clearance is rendered possible by the thinning of the blade tips.

E. M. Speakman[1] has derived a curve of radial clearance from values used in practice. It is fitted approximately by the straight line equation

$$c = 0{\cdot}01 + 0{\cdot}008D \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where c is the clearance in inches, and D the mean ring diameter in feet.

Axial clearance has to be fixed with reference to the differential expansion which occurs with the long rotor and casing having considerable temperature fall along the length.

J. M. Newton[2] has given the following empirical equation for the axial pitch of the blading which allows sufficient clearance, and also takes account of the possibility of bending of the blades in the axial direction :—

$$P_a = 2B + \frac{1}{4} + \frac{l}{16} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

where P_a = axial pitch in inches.
B = breadth of the blade in inches.
l = length " " "

Recently at the H.P. section a new method of reducing the leakage

[1] See paper on "The Determination of the Principal Dimensions of the Steam Turbine, with special reference to Marine Work," *Trans. Inst. of Engineers and Shipbuilders in Scotland*, 1905.

[2] See paper on "High Speed Turbine Rotor Design and Construction," *Trans. Junior Inst. of Engineers*, 1910.

loss at the blade tips has been introduced. Instead of thinning down the tips, the blades are provided with stout shroud rings. The shroud of the moving ring has an axial projection reduced to a fine edge, which runs almost in contact with the face of the corresponding shroud on the fixed ring. The question of fine tip clearance and its attendant difficulties is thus avoided, and extremely fine axial clearances are obtained at drum and casing with corresponding increase in efficiency at the H.P. end. The device is simply an extension of the well-known Parsons labyrinth packing used at glands and dummy pistons.

110. Circumferential Pitch of Blades.—The number of blades to be provided at any ring of a turbine disc or a drum should just be sufficient to efficiently direct the flow of the steam through them. An unnecessarily large number simply increases the friction loss through the addition of superfluous surface. The most suitable value of this pitch can be determined only by experiment.

It does not appear advisable to reduce it below $\frac{3}{8}$ inch or to increase it above 1 inch, when the ordinary form of crescent-shaped impulse blade is used.

For pitches between $\frac{3}{8}$ and $\frac{3}{4}$ inch the blade efficiency does not appear to vary to any appreciable extent.

In practice the pitch in impulse turbines runs from 0·3 to 0·7 inch, and in reaction turbines from 0·2 to 0·6 inch.

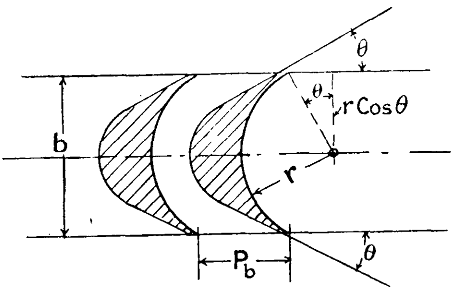


FIG. 119.

There is a rough practical rule used by some designers of impulse turbines by which the circumferential pitch P_b is made equal to the radius of curvature of the blade face r. Briling's experiments, already referred to, indicate that for the symmetrical impulse blade $(\theta_1 = \theta_0)$ the most efficient pitch is given by the relation

$$P_b = \frac{r}{2 \sin \theta} \quad \ldots \ldots \quad (42)$$

It will be seen from the geometry of Fig. 119, which shows the section of a symmetrical impulse blade, that

$$b = 2r \cos \theta$$

so that

$$b = 4P_b \sin \theta \cos \theta = 2P_b \sin 2\theta$$

or

$$P_b = \frac{b}{2 \sin 2\theta} \quad \ldots \ldots \quad (43)$$

This equation enables a suitable value of the pitch to be determined when the blade width and angle are known. For unsymmetrical blades the entrance angle may be taken.

The experiments also indicated that the impulse blade angle which gave the best efficiency was 30°, and it will be seen that if this is taken, the pitch by equation (42) becomes $P_b = r$. Thus experiment seems to justify the use of the working rule quoted above.

In the case of the velocity-compounded stage the pitch is fixed with reference to the last blade, which has the greatest entrance angle. As this may run from 40° to 45°, the denominator in (43) may vary from 1·96 to 2, so that the pitch may be taken from 0·52 to 0·5 of the axial breadth.

111. Method of Drawing Down the Section of Impulse Blading.— The following general method may be employed for "getting out"

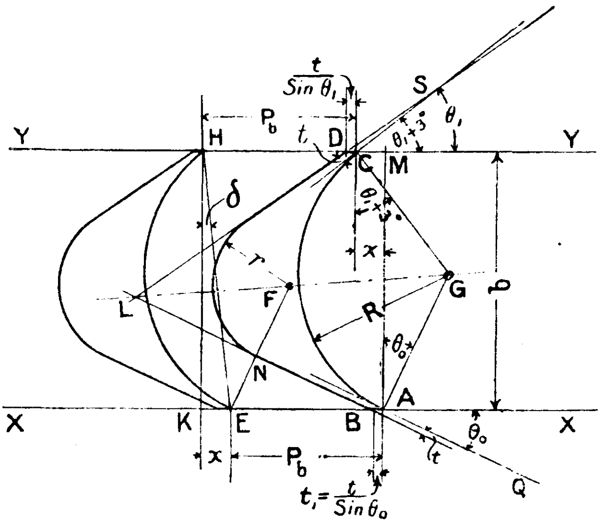


FIG. 120.

the section of an impulse blade whether it is symmetrical or unsymmetrical.

The section is usually drawn out several times full size.

Taking the most general case of the unsymmetrical blade ($\theta_1 > \theta_0$), and referring to Fig. 120, draw the parallel lines XX and YY at a distance b apart and equal to the axial blade width. From a chosen point A on XX set off AE $= P_b$, the circumferential pitch of the blades. Through A draw AQ inclined at the exit angle θ_0 to XX. Parallel to this line draw BL at a distance, equal to the blade thickness t, from it.

The length $AB = t_1 = \dfrac{t}{\sin\theta_0}$.

Since $\theta_0 < \theta_1$ the exit edge A overlaps the entrance edge C by an amount x. It is a common practice to make the entrance angle at the face about 3° greater than at the back to keep up the thickness at the inlet edge. If desired, however, the angle can be made the same as that of the back. Assuming the first condition, it can be seen from the geometry of the figure that the blade width is given by

$$b = \mathrm{R}\{\cos(\theta_1 + 3) + \cos\theta_0\} \quad . \quad . \quad . \quad . \quad (44)$$

and

$$x = \mathrm{R}\{\sin(\theta_1 + 3) - \sin\theta_0\} \quad . \quad . \quad . \quad . \quad (45)$$

Hence for given values of b, θ_1, and θ_0, calculate R and x from equations (44) and (45). Then draw AM perpendicular to YY and set off MC $= x$. Through C draw CS at $(\theta_1 + 3)$ to YY. Draw CG perpendicular to CS and AG perpendicular to AQ. The intersection G is the centre of curvature for the blade face and CG = AG = R. To obtain the profile of the back, set off CD $= \dfrac{t}{\sin\theta_1}$, and through D draw DL at θ_1 to YY, cutting BL in L. Join G and L, and through E draw EN perpendicular to BN to cut GL in F, the approximate centre of curvature for the back.[1] The radius of curvature is FN $= r$.

The preliminary calculation of x and R can be replaced by a graphical method which should be used in preference to the above. The construction for x is shown for clearness at the blade section on the left. In this case H is the entrance and E the exit edge of the blade, and EK $= x$.

By joining the edges A and C of the right-hand blade, it can be seen that the angle which the line AC makes with the perpendicular through C is $\delta = \frac{1}{2}\{(\theta_1 + 3) - \theta_0\}$.

Hence, to find x, draw HK perpendicular to XX, and HE inclined at the angle δ to HK, to cut XX in E. Then E is the required exit edge. The entrance and exit edges now being fixed the section can be drawn in by the method just stated.

When the blade is symmetrical $(\theta_1 = \theta_0)$ if the extra 3° are allowed at the entrance edge $\delta = \frac{1}{2} \times 3$ or $1\frac{1}{2}°$. This gives a negligible amount of lap x. Whether this refinement is omitted or not, the rest of the construction is exactly the same as that for the unsymmetrical blade.

EXAMPLE 8.—Make a dimensioned drawing of the impulse blading for the first stage of the Curtis turbine (example 3) showing an elevation and a developed sectional plan. Take the blade width as 1 inch, and thickness at entrance and exit is 0·02 inch. The nozzle exit is $\frac{5}{8}$ inch, and the exit length of the last blade $1\frac{3}{8}$ inch. The blades are to be let into grooves in the rim similar to that shown at Fig. 90. The intermediates are to be milled on the face of the foundation ring.

This example is given to show the practical application of the figures quoted in the previous paragraphs and to illustrate the general application of the geometrical construction for the blade sections.

[1] On account of the 3° increase at entrance, LG does not exactly bisect the angle DLB. The exact position of F is on the upper side of LG. The error, however, is so small that the additional bisection of the angle is not necessary.

The complete drawing is shown in Fig. 121. In order to draw the elevation and determine the successive lengths, a suitable value of axial clearance is chosen. This is taken as 0·11 (see Art. 108). The shrouding always projects beyond the blade edges, and it is assumed that this reduces the running axial clearance to 0·07 inch. The widths and clearance values are set off on the centre line BC, and verticals are

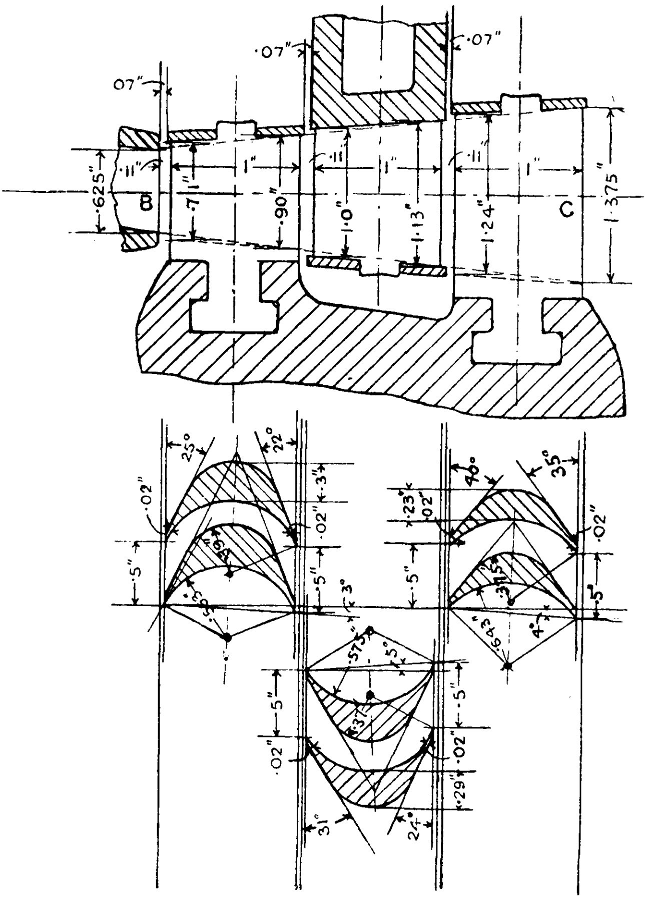


FIG. 121.

drawn. The nozzle exit length and last blade length are scaled off on the end verticals at B and C, and the points are joined by dotted lines. The entrance length at each blade as defined by these lines is increased by 0·08 inch (see Art. 108) to provide against radial spilling. The exact sizes of the dovetail and width of wheel rim are not given as these depend on the standard practice of the makers. In the case of the intermediate blade there is no slot, the condition being that the blades are milled out the solid.

In this case the pitch may be made $\frac{1}{2}$ inch (see Art. 110). The mean ring diameter is 69 inches, and the circumference 216·7 inches, so that there will be 433 blades in each moving ring. The intermediates, as shown in example 7, Chap. VII., cover an arc of 26 inches, and the minimum number of blades in the sector is 52. This should be increased to, say, 55, to allow for the "lead" effect at the forward side of the sector (see Art. 78).

The (developed) section of each blade in plan is drawn in accordance with the method of Art. 111. Two blades in each ring are shown, in order to distribute the dimensions, and avoid confusion of figures.

Although a uniform progression of blade length is taken here, this is not always given. In some designs, especially at the intermediate and low-pressure stages, the third blade length is increased at a greater rate between entrance and exit than the others. This slightly upsets the condition on which the diagram construction of Art. 87 is based. In other cases the entrance length is made the same as the exit length at each blade. This arrangement is used in the marine Curtis turbine shown in Fig. 19.

112. Section of Parsons Blading.—This type of blade is not susceptible of any exact geometrical treatment, like that of the impulse turbine. It is the result of a trial and error process of evolution. The blading is drawn through standard dies supplied by the Parsons Company to its liscensees.

An attempt has been made by Martin and Parsons[1] to obtain a geometrical construction to fit this type, from the examination of a photographically enlarged section of the blade.

The result is reproduced in Fig. 122. The suggested method of construction is as follows:—

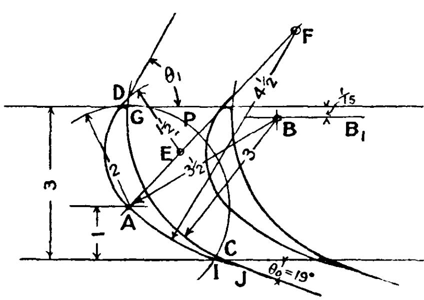


FIG. 122.

Draw two parallel lines distant 3 units apart. The unit for convenience may be taken as $\frac{1}{8}$ inch. From centre A, distant 1 unit from the bottom line, with radius 2 units, describe circle CPD. At $\frac{1}{15}$ unit below the top line, draw the parallel BB_1. From A with a radius $3\frac{1}{2}$ units describe an arc cutting BB_1 in B. With B as centre and radius 3 units draw the face curve CD cutting the circle CPD in D. With radius $1\frac{1}{2}$ units describe an arc from a centre E to pass through D and A. Make AF $= 4\frac{1}{2}$ units, and with F as a centre draw in the back curve AI cutting CD

[1] See paper on "The Theory of the Steam Turbine," by H. M. Martin and R. H. Parsons, *Trans. Jun. Inst. Engineers*, 1907.

in J. The bisectors of the angles between the tangents at D and G and I and C may be taken as the lines giving the entrance and exit angles θ_1 and θ_0. The latter value appears to be about 19°.

113. Section of Ljungström Blading.—The section of this blade is shown in Fig. 123. It is somewhat similar to the Parsons section, but,

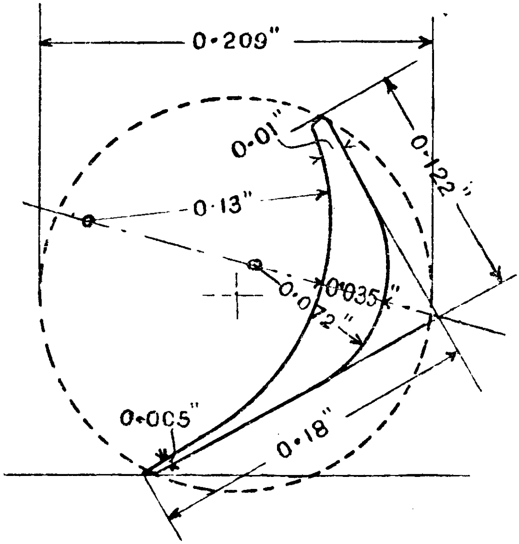


FIG. 123.

like the impulse blades, it has the profile formed by straight lines and circular arcs.

The blade illustrated is an enlarged section of the smallest size used at the H.P. section of the 1000 K.W. turbine. The actual dimensions are figured on it. It is enlarged ten times.

CHAPTER IX

ROTORS

114. Shaft and Disc Rotors.—With the exception of the disc rotor of the de Laval turbine, the rotors of the various types of impulse machine have already been sufficiently illustrated and described in Chap. II.

The type of wheel used on the small de Laval machine is shown

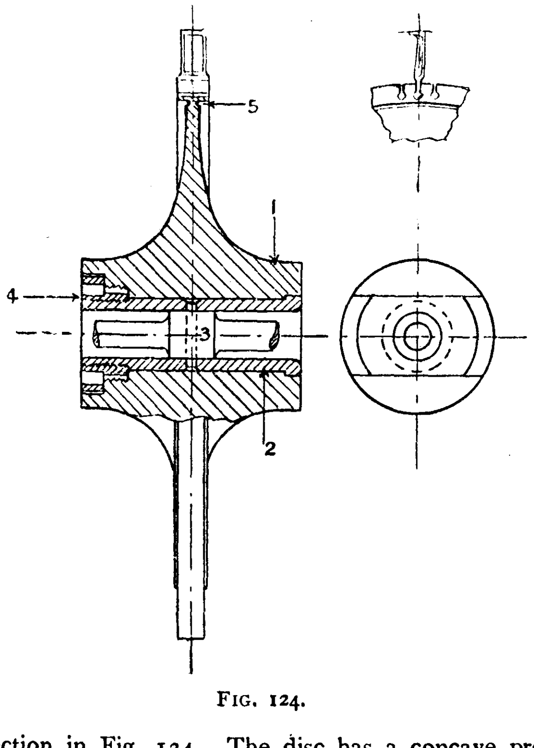


FIG. 124.

partly in section in Fig. 124. The disc has a concave profile which rounds off into a heavy boss, 1. This is bored to take a bush, 2. The shaft is enlarged at 3 to the same diameter as the bush, to which it is riveted. The bush and shaft are then inserted in the boss and locked in position by the nut, 4. A narrow safety groove, 5, is turned below the rim.

The wheel used on the large machine is shown in Fig. 125. In this case the disc is given a concavo-convex profile. It is not bored out at the boss or hub. The boss, 1, is recessed at each side to take the enlarged stud end, 2, of the shaft, 3. The shaft is bolted on by the flange, 4, and stud bolts, 6. This form of disc is said to be one of "constant strength," that is, the stress at any point between centre and

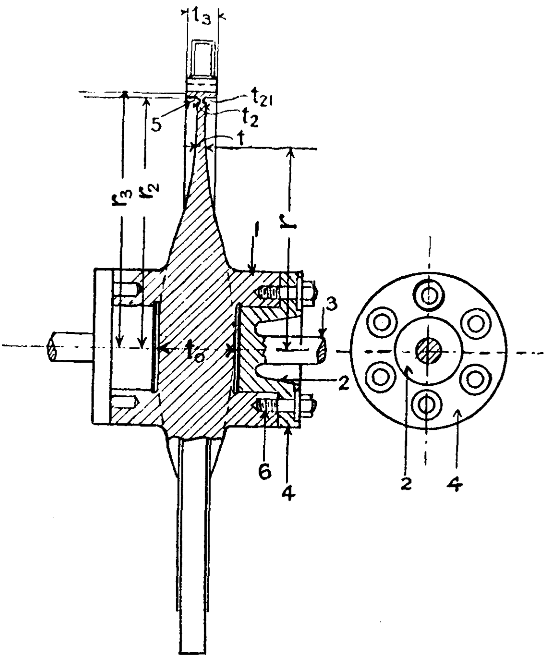


FIG. 125.

rim is supposed to be constant, when there are no hub projections at the centre.

It is very important to ensure that, in case of fracture due to overspeeding, the breakage shall take place at the rim and not at the hub. The disc is therefore reduced below the rim by cutting a safety groove, 5.

It has been experimentally proved that this type of wheel does fracture at the safety groove when excessively overspeeded. When such fracture occurs, the rim simply detaches from the disc, the driving effort on the wheel ceases, and the machine comes to rest without any material damage to the casing or other part.

If the wheel were to "burst" at the hub, a disastrous smash might result.

On account of the high stress and also the large energy loss by disc and vane friction, when the machine runs either non-condensing or under a poor vacuum the maximum wheel diameter of the de Laval turbine is restricted to the value of 30 inches. The standard speed for this wheel is 10,600 R.P.M., so that the mean peripheral velocity is 1378 ft./sec.

The wheels of pressure-compounded turbines which run at peripheral speeds of from 400 to 500 ft./sec., and which have relatively small rim dimensions, are made with flat tapering sections and light hubs. See Rateau and Zoelly turbines, Figs. 20, 21, 23.

In a machine of this type the larger number of stages required for a low rotational speed necessitates a wide span between the bearings. The turbine, Fig. 20, affords a good illustration of this. Increase in span for a given diameter of shaft reduces the stiffness of the rotor. Increase of diameter in order to maintain the stiffness increases the leakage area at the stage diaphragm glands. Thermodynamic considerations require a small shaft, dynamical ones a large shaft. There has to be a compromise between these two conflicting conditions.

If no special packing arrangement is adopted the leakage trouble may be acute at the high-pressure end. At the intermediate section it is of less account, and at the low-pressure end it is practically negligible. By tapering the shaft from the centre towards the ends its stiffness need not be much impaired, while the leakage area at the high-pressure diaphragms may be substantially reduced.

Improved types of gland packings are now employed in which, practically, running contact is ensured between the packing rings and the shaft; and the troublesome leakage defect has been to a large extent overcome.

Apart, however, from the provision against leakage, the shafts are tapered in a series of steps for the easy assembly and removal of the disc wheels.

The wheels are pitched closely together to keep the span as short as possible, and reduce deflection to a minimum. The smallest pitch of disc is controlled by mechanical conditions. The axial distance between each pair of discs must be sufficiently large to give access for cleaning up the shaft surface, if necessary, during an overhaul.

The wheel of the multistage-impulse turbine is lighter than that of the fewer stage Curtis turbine, which requires a heavier rim to carry double or treble rows of blades and runs at a higher peripheral speed, varying from 500 to 550, and in some cases to 600 ft./sec.

The disc is made with a more rapidly tapering section and a heavy hub (see Curtis turbines, Figs. 16, 17).

In the smaller types of velocity-compounded turbines the mean peripheral speed usually varies between 200 and 300 ft./sec., and the wheels are made with discs of approximately constant thickness having light hubs (see Figs. 12, 13).

In most cases the narrow span between the bearings enables a comparatively small size of shaft to be used. Although small, it is, at the same time, sufficiently stiff.

As already pointed out in Chap. II., where the diameter of the turbine is large and the rotational speed low, as in the marine Curtis and combination turbine, the question of strength is a minor one. Ease of production and cost of construction are more important items, and the wheels are "built up" of steel plates (see Figs. 16, 19).

115. Stresses in Disc Wheels.—The permissible diameter of a turbine disc to run at a stated speed of rotation is fixed by the safe limiting stress of the material used. Discs are made either of mild or nickel steel. It is possible to obtain cast steel discs, but there is always the risk of flaws which are revealed, only after time and labour have been expended in machining. On account of this commercial drawback, the forged disc is generally employed.

It is possible to reckon on an ultimate strength for mild steel of 36 ton/in.^{2}, and 20 per cent. elongation on 2 inches, and an elastic limit of 20 ton/in.^{2}. For 3 per cent. nickel steel the ultimate strength is 40 ton/in.^{2}, with 30 per cent. elongation on 2 inches, and an elastic limit of 29 ton/in.^{2}

These figures correspond to ultimate strengths of 80,000 lbs./in.^{2} for mild and 90,000 lbs./in.^{2} for nickel steel. The elastic strengths are 45,000 lbs./in.^{2} and 66,000 lbs./in.^{2}.

As a rule a safe stress of 16,000 lbs./in.^{2} for mild and 20,000 lbs./in.^{2} for nickel steel is not exceeded under normal conditions. The stresses thus employed ensure a factor of safety from 4 to 5 in the ultimate, and from 2·5 to 3 in the elastic strength. Designers of de Laval nickel steel discs of uniform strength, however, allow for higher stresses than the above. In estimating the probable maximum which may come on a given disc, it is necessary to take account of the maximum amount of overspeeding that may occur, before the emergency governor gear comes into action.

As a rule about 10 per cent. increase is allowed for an emergency shut down, but it is safer to reckon on a stress for a 20 per cent. increase of speed.

The stress varies as the square of the speed, so that 20 per cent. increase corresponds to 44 per cent. increase of stress. The factor of safety reckoned as the elastic strength is thus reduced slightly below 2.

116. Thickness of Wheel Disc.—Sections of the discs used in pressure-compounded impulse machines are shown in Fig. 126 (*a*) and (*b*), and the heavier type used in velocity-compounded machines in Fig. 127.

In each case the disc thickness t at any radius r is expressed by the equation

$$t = \frac{c}{r^n} \qquad (1)$$

where c is a constant. The value of the exponent n for the slightly tapering disc used for wheels with light rims and one ring of blades (Fig. 126), varies from 0·4 to 0·8. The average value 0·6 is most commonly used.

For the heavy type of rapidly tapering disc, Fig. 127, with heavy rim and several rings of blades, the exponent is about 1. In the special case of the small de Laval wheel, Fig. 124, it rises to 2.

As a rule, in fixing the dimensions of the disc for a given type of

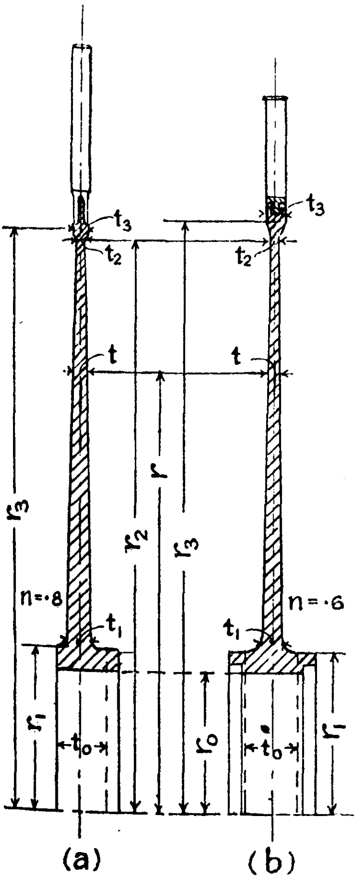


FIG. 126.

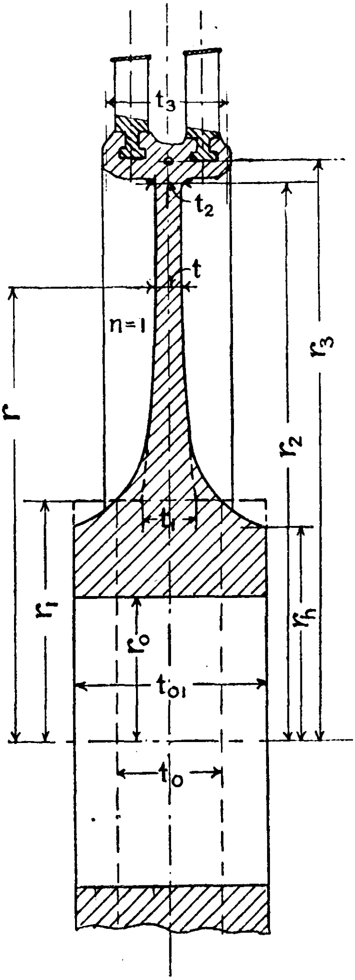


FIG. 127.

turbine, it is most convenient to start with the thickness of the disc below the rim.

When this value is fixed, and a provisional choice of the index n is made for equation (1), the disc profile can be drawn in. When a practicable hub has been added in accordance with the designer's judgment, the necessary dimensions are available for the calculation of the probable stresses in the wheel.

With regard to the thickness below the rim, it is necessary, apart from any question of strength, to provide sufficient metal to ensure that the disc shall be sufficiently stiff to resist buckling, during mechanical operations in handling and erection.

Except in the case of a very small wheel—as, for instance, that of the smallest size of de Laval turbine—the minimum thickness is $\frac{3}{8}$ inch.

As a rule, the maximum thickness below the rim of heavily loaded Curtis wheels does not exceed $1\frac{1}{2}$ inch. It usually runs from $\frac{5}{8}$ to $\frac{7}{8}$ inch for machines of moderate speed.

In the case of the lighter type of wheel used for pressure-compounded turbines this thickness runs from $\frac{3}{8}$ to $\frac{1}{2}$ inch.

For any given case it is desirable to ascertain the minimum thickness by calculation on the basis of the maximum permissible stress in the disc below the rim.

117. Approximate Calculation of Stresses in Disc Wheels.—Mathematicians have not as yet found any simple method for the exact calculation of the stresses, in such practicable forms of discs as are used in steam turbines. As far, however, as practical design is concerned, close refinement of calculation is not necessary.

Practicable hubs for the satisfactory fixture of the discs to the shaft have to be added according to judgment, and rims are necessary to carry the blading. Such modifications of the "theoretical" form of disc upset the results of exact calculation. The nearest approach to a solution is obtained by treating the wheel as a bored disc loaded by the rim and blading at the outer, and restrained by the hub at the inner circumference. When the approximate rim loading is ascertained the probable stresses at all points of the disc can be calculated.

In the mathematical investigation by means of which the stress equations are obtained, it is usual to consider only the two principal stresses, the radial stress and the hoop or tangential stress. The effect of axial and shear stress is neglected.

By assuming, as indicated above, the law of variation of thickness with radius, two sets of equations, one giving radial and tangential stresses and the other the corresponding strains, can be derived.

The investigation is outside the scope of this text; and the standard equations, given by Dr. Stodola, are taken and used as necessary tools for the attainment of a definite end.

The complete investigation will be found in Dr. Stodola's standard work on the steam turbine.[1]

Referring to the disc sections, Figs. 126 and 127, it is assumed that at any radius r the material of the disc is subjected to a radial stress f_r and a tangential or hoop stress f_t.

[1] See Stodola's "Steam Turbines," 2nd edition, chap. iii. Other forms of the stress and strain equations, and a somewhat different treatment to that given here, will be found in a paper on "The Calculation of Centrifugal Stresses in Turbine Rotors," by Wm. Kerr. *Trans. of Scientific Soc. of the Royal Tech. Coll., Glasgow*, March 1914.

When the form of the disc is such that $t=\frac{c}{r^n}$, the mathematical investigation referred to shows that these stresses are given by the following equations:—

Radial stress

$$f_r = 1\cdot 1\mathrm{E}\left[(a_1+\sigma)\mathrm{C}_1 r^{a_1-1}+(a_2+\sigma)\mathrm{C}_2 r^{a_2-1}-\frac{(3+\sigma)(1-\sigma^2)w\omega^2r^2}{\mathrm{E}g\{8-(3+\sigma)n\}}\right] \quad (2)$$

Tangential stress

$$f_t = 1\cdot 1\mathrm{E}\left[(1+a_1\sigma)\mathrm{C}_1 r^{a_1-1}+(1+a_2\sigma)\mathrm{C}_2 r^{a_2-1}-\frac{(1+3\sigma)(1-\sigma^2)w\omega^2r^2}{\mathrm{E}g\{8-(3+\sigma)n\}}\right]. \quad (3)$$

where f_r = radial stress in lbs./in.².
f_t = tangential stress in lbs./in.².
E = Young's modulus for steel $= 30 \times 10^6$ lbs./in.².
σ = Poisson's ratio = 0·3 for steel.
w = weight per cubic inch of steel = 0·288 lb.
r = radius in inches.
ω = angular velocity in radians/sec.
$g = 32\cdot 2 \times 12 = 386\cdot 4$ inches/sec.².

a_1 and a_2 are constants, and functions of n.
C_1 and C_2 are constants depending on the boundary conditions.
The values of a_1 and a_2 are given respectively by

$$a_1=\frac{n}{2}+\sqrt{\frac{n^2}{4}+\sigma n+1}\,; \quad a_2=\frac{n}{2}-\sqrt{\frac{n^2}{4}+\sigma n+1}$$

When the form of disc is provisionally fixed, and hence n is known, the constants a_1 and a_2 and the other factors depending on them are determinable, and equations (2) and (3) may be written in the simpler form—

$$f_r = 1\cdot 1(\mathrm{AC_1E} - \mathrm{BC_2E} - \mathrm{K}\omega^2) \quad . \quad . \quad . \quad . \quad (4)$$
$$f_t = 1\cdot 1(\mathrm{MC_1E} + \mathrm{NC_2E} - \mathrm{L}\omega^2) \quad . \quad . \quad . \quad . \quad (5)$$

where $\mathrm{A} = (a_1+\sigma)r^{a_1-1}$

$\mathrm{B} = -(a_2+\sigma)r^{a_2-1}$

$$\mathrm{K} = \frac{(3+\sigma)(1-\sigma^2)\omega r^2}{g\{8-(3+\sigma)n\}} = \frac{r^2}{3575-1474n}$$

The negative sign is used in (4), since B is always negative. In (5), however, N is positive.

At any given radius with a given profile or value of n, the coefficients M and N are multiples of A and B, that is

$$\mathrm{M} = \zeta_1\mathrm{A}\,;\ \mathrm{N} = \zeta_2\mathrm{B}$$

An inspection of the last term in (3) will show that for all values of n at any given radius, $\mathrm{L} = 0\cdot 577\mathrm{K}$.

Hence the equation for the tangential stress may be written

$$f_t = 1\cdot 1(\zeta_1\mathrm{AC_1E} + \zeta_2\mathrm{BC_2E} - 0\cdot 577\mathrm{K}\omega^2) \quad . \quad . \quad (6)$$

The work of calculation is facilitated by the use of curves giving values of the coefficients A, B, K, ζ_1, and ζ_2.

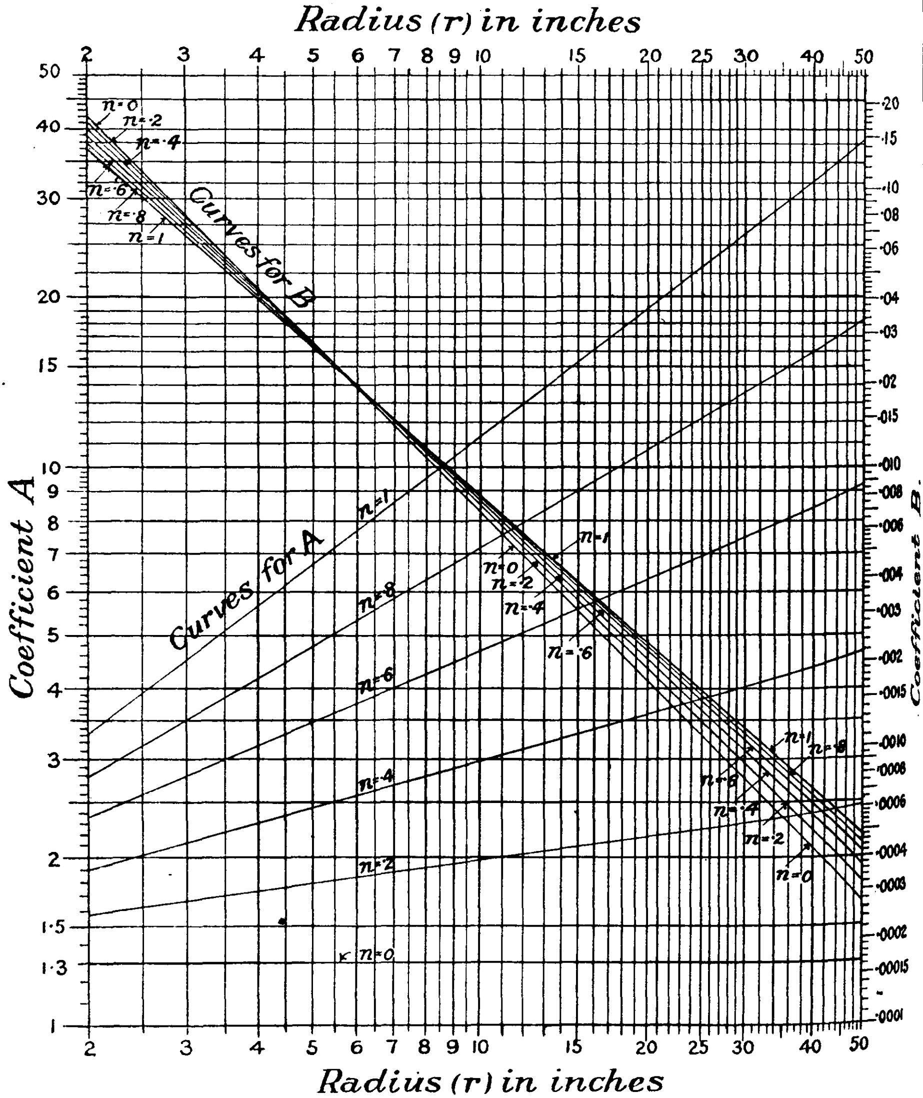

Radius (r) in inches
2 3 4 5 6 7 8 9 10 15 20 25 30 40 50
50 40 30 20 15 10 9 8 7 6 5 4 3 2 1·5 1·3 1
·20 ·15 ·10 ·08 ·06 ·04 ·03 ·02 ·015 ·010 ·008 ·006 ·004 ·003 ·002 ·0015 ·0010 ·0008 ·0006 ·0004 ·0003 ·0002 ·00015 ·0001
Coefficient A
Coefficient B.
Curves for B
Curves for A
n=0
n=·2
n=·4
n=·6
n=·8
n=1
Radius (r) in inches

FIG. 128.

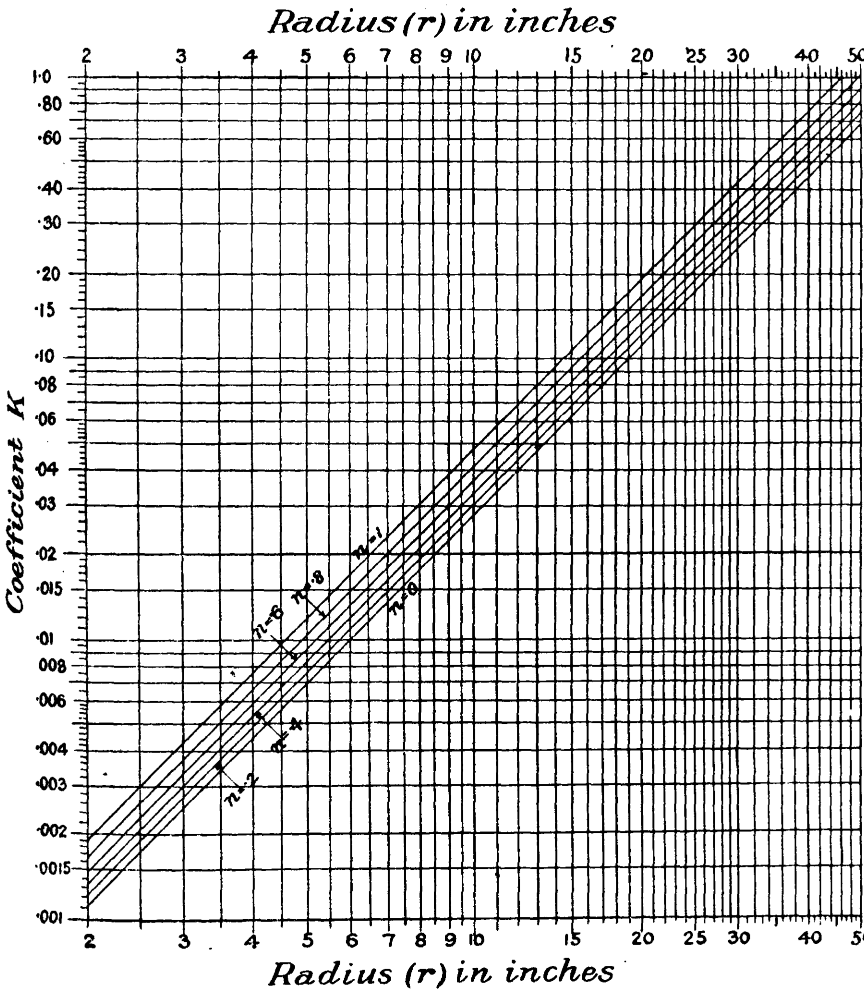

Radius (r) in inches
2
3
4
5
6
7
8
9
10
15
20
25
30
40
50
1·0
·80
·60
·40
·30
·20
·15
·10
·08
·06
·04
·03
·02
·015
·01
·008
·006
·004
·003
·002
·0015
·001
Coefficient K
n=1
n=·8
n=·6
n=·4
n=·2
n=0
Radius (r) in inches

FIG. 129.

Log graphs of A and B are plotted in the form of a chart in Fig. 128. A similar set of curves, giving values of K, is plotted in Fig. 129, while the values of ζ_1 and ζ_2 for a series of values of n are plotted in Fig. 130.

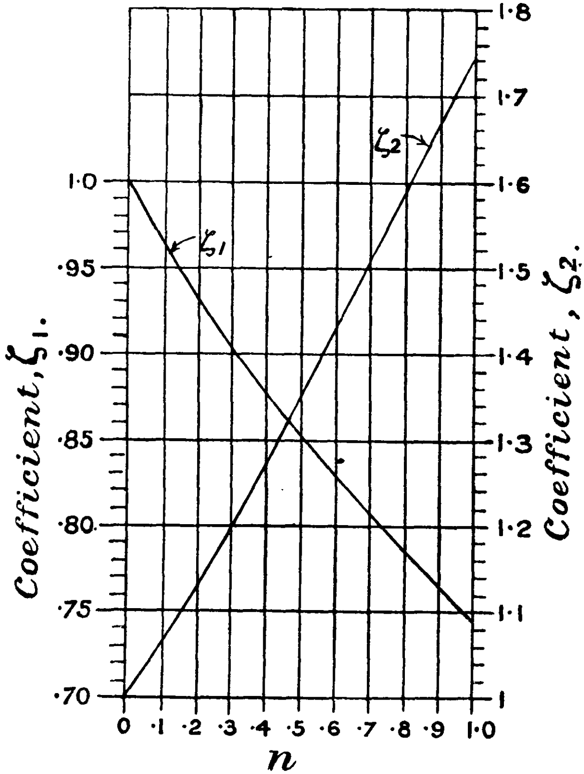


FIG. 130.

118. The coefficients C_1E and C_2E for each particular case have to be determined by the "boundary conditions." It is the estimation of these that somewhat complicates the calculation and makes the preliminary arithmetical work tedious. In this calculation it is necessary to use the equations for the common strain at rim and disc.

The conditions at the hub are much less definite than those at the rim, and it is preferable to consider only the rim and disc strain, and to use a method of successive approximation for the calculation of C_1E and C_2E.

In the analytical investigation already referred to it is also shown that the radial strain, that is, the increase of radius at any given radius r, is expressed by

$$S_d = C_1 r^{a_1} + C_2 r^{a_2} - \frac{b}{E} r^3 \omega^2$$

where C_1 and C_2 and a_1 and a_2 are the same constants as appear in equations (2) and (3), and b is another constant given by

$$b = \frac{(1 - \sigma^2)w}{g\{8 - (3 + \sigma)n\}}$$

It will be noted that this is $\dfrac{K}{(3 + \sigma)r^2} = \dfrac{0{\cdot}302K}{r^2}$.

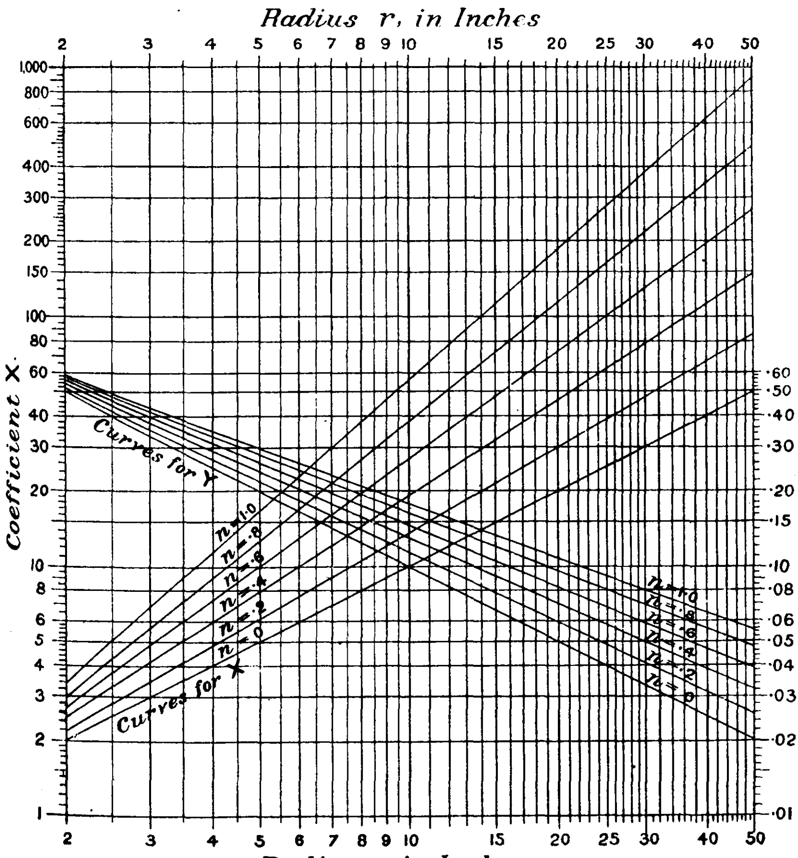


FIG. 131.

The disc strain at any radius r can therefore be expressed by

$$S_d = C_1X + C_2Y - \frac{0{\cdot}302Krw^2}{E} \quad . \quad . \quad . \quad . \quad (7)$$

where $X = r^{a_1}$, and $Y = r^{a_2}$.

In order to facilitate calculation, the log graphs of X and Y are plotted in the form of a chart, Fig. 131.

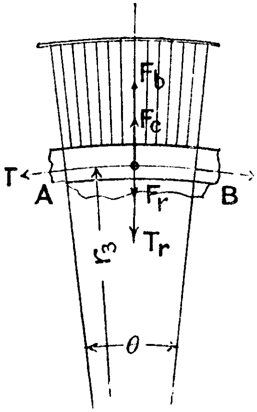


FIG. 132.

At the rim section of the disc, where $r = r_2$

$$S_{d_2}E = C_1EX_2 + C_2EY_2 - 0{\cdot}302K_2r_2\omega^2 \quad (8)$$

For the calculation of the corresponding strain of the rim, which must be the same as that of the disc, the rim is treated as a thin ring. This condition is fairly approximated to, as the rim depth or radial thickness is usually small relatively to the mean rim radius.

When running at speed the rim is in equilibrium under a system of four forces.

Referring to Fig. 132, consider the elementary sector AB of the rim, subtending the small angle θ at the centre. The forces in statical equilibrium acting on AB are

F_c = centrifugal force of the rim } radially outward.
F_b = " " " blades }
F_r = radial pull of the disc on the rim } radially inward.
T_r = radial component of the tangential force T on the rim }

Here $$F_c + F_b = F_r + T_r \quad . \quad . \quad . \quad . \quad (9)$$

Let a_3 = sectional area of the rim in square inches.
w = weight per cubic inch of the material of the rim.

Then the centrifugal force on the sector is

$$F_c = \frac{a_3r_3\theta w\omega^2r_3}{g} = \frac{a_3w\omega^2r_3^2\theta}{g}$$

Let f_b = radial force in lbs. per square inch of rim surface at r_3, due to the centrifugal blade load.
t_3 = width of the rim.

Then $$F_b = f_bt_3r_3\theta$$

Let ${}_rf_t$ = tangential stress produced in the rim, then the tangential load is

$$T = {}_rf_ta_3$$

Since θ is a small angle, the radial component of this force is

$$T_r = {}_rf_ta_3\theta$$

Finally, let f_{r_2} = radial stress on the disc below the rim at r_2.
t_2 = thickness of disc below the rim.

Then $$F_{r_2} = f_{r_2}r_2t_2\theta$$

This load referred to the mean rim radius r_3 is

$$F_r = f_{r_2} r_2 t_2 \theta \frac{r_2}{r_3} = f_{r_2} \frac{r_2^2}{r_3} t_2 \theta$$

Substituting these values in equation (9)

$$a_3 \frac{w}{g} \omega^2 r_3^2 + f_b t_3 r_3 = f_{r_2} \frac{r_2^2}{r_3} t_2 + {}_r f_t a_3 \quad . \quad . \quad . \quad (10)$$

Normally r_2 does not differ greatly from r_3, so that as a first approximation r_3 may be substituted for $\frac{r_2^2}{r_3}$ in the first term on the right, and the tangential stress in the rim may be expressed by the equation

$${}_r f_t = r_3 \left(\frac{w \omega^2 r_3}{g} + \frac{f_b t_3}{a_3} - \frac{f_{r_2} t_2}{a_3} \right) \quad . \quad . \quad . \quad . \quad (11)$$

Also the thickness t_2 of the disc below the rim is given by

$$t_2 = \frac{\frac{w}{g} \omega^2 r_3 a_3 + f_b t_3}{f_{r_2}} - \frac{{}_r f_t a_3}{f_{r_2} r_3} \quad . \quad . \quad . \quad . \quad (12)$$

The approximate disc thickness for a given case may be provisionally estimated from equation (12) by selecting some limiting value for the radial stress f_{r_2} and assuming $\frac{{}_r f}{f_{r_2}}$ to be unity or slightly greater.

If the thickness so calculated is less than the minimum value considered necessary to ensure sufficient stiffness, then this minimum value has to be adopted, and substituted in (12) to obtain the probable value of the disc stress f_{r_2}.

This provisional value of f_{r_2} is to be used in equation (14) for the first trial calculation of C_1E.

As regards the maximum value of f_{r_2}, this may be taken as 16,000 lbs./in.^{2} for mild steel and 18,000 lbs./in.^{2} for nickel steel.

These values, however, are obtained only on the high-speed wheels of Curtis turbines, which have heavy rim loads. In the light-rimmed disc wheels used in Rateau and Zoelly machines, running with a moderate peripheral velocity, the stress is quite low. In fact, with this type it is hardly necessary to trouble about the strength at all, as long as the conditions for stiffness are properly attended to. As shown in Fig. 126, the Zoelly wheel (*b*) is usually made with slightly tapering disc having $t = \frac{c}{r^{0 \cdot 6}}$, and the Rateau (*a*) with $t = \frac{c}{r^{0 \cdot 8}}$.

If the radial stress in the rim, regarded as a thin ring, is neglected, then the initial strain of the rim, S_r, or increase of radius per unit radius, is given by the fundamental elastic equation $S_r = \frac{{}_r f_t}{E}$. This value multiplied by the radius r_3 gives the total rim strain.

Substituting for ${}_{r}f_t$ from equation (11) the total strain is

$$S_r = \frac{r_3\,{}_rf_t}{E} = \frac{r_3{}^2}{E}\left(\frac{w\omega^2 r_3}{g} + \frac{f_b t_3}{a_3} - \frac{f_{r_2} t_2}{a_3}\right) \quad . \quad . \quad . \quad (13)$$

When for a given case the values of C_1E, C_2E, and f_{r_2} are correct, the equality $S_rE = S_{d_2}E$ will be satisfied.

Now it will be found, when the conditions at the rim section are taken, that for a given value of f_{r_2} the value of C_1E is not appreciably affected by neglecting the second term in the bracket on the right of equation (4), that is, by first neglecting C_2E. Similarly, in the strain equation the value of $S_{d_2}E$ is not greatly affected by the omission of the second term. Hence as a first approximation the provisional value of C_1E for a given value of f_{r_2} can be calculated from

$$f_{r_2} = 1{\cdot}1(C_1EA_2 - K_2\omega^2) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and the approximate value of the strain from

$$S_{d_2}E = C_1EX_2 - 0{\cdot}302\,K_2r_2\omega^2 \quad . \quad . \quad . \quad . \quad (15)$$

For the same value of f_{r_2} the corresponding value of the rim strain can be calculated from equation (13).

The value of S_rE should be in excess of the above value of $S_{d_2}E$ to allow for the influence of the term containing C_2E.

If there is a large discrepancy between $S_{d_2}E$ and S_rE the value of the product $(f_{r_2}t_2)$ in equation (13) has to be increased or decreased.

If, as is usual, the thickness t_2 is taken as the fixed value, then f_{r_2} requires to be altered. It will be found that a small increase or decrease in the value of f_{r_2} is sufficient to produce a considerable alteration in the value of S_rE. The actual value of f_{r_2} can therefore be fairly approximated in this way.

When this figure is settled it should be inserted on the left of the stress equation (4) and the value of $S_{d_2}E$ on the left of the complete strain equation (8).

These simultaneous equations, repeated here for convenience, can then be solved for C_1E and C_2E.[1]

$$f_{r_2} = 1{\cdot}1(C_1EA_2 - C_2EB_2 - K_2\omega^2) \quad . \quad . \quad . \quad (16)$$
$$S_rE = C_1EX_2 + C_2EY_2 - 0{\cdot}302K_2r_2\omega^2 \quad . \quad . \quad (17)$$

Alternatively, instead of using (17), a provisional value of f_r for some other radius may be used in the stress equation (4) and C_1E calculated. If this is approximately the same as the previous value, then the stress value assumed will be correct; if not, another value has to be tried. The solution by (16) and (17) is the preferable method. The values of C_1E and C_2E can now be inserted in equations (4) and (6), and the radial and tangential stresses at various radii can be calculated. Curves of stress can then be plotted and a fair idea obtained as to the

[1] The arithmetical work has to be done with care, and all values have to be taken out to the second decimal place.

probable distribution of stress in the disc, and the maximum value obtained.

119. In the case of the light type of wheel, Fig. 126, the hub may be treated as a thin ring, subjected to the external pull of the disc, to its own centrifugal force, and any residual "force fit" which may exist when the wheel runs at speed, all acting radially outward.

These are balanced by the radial component of the tangential force on the hub acting radially inward.

Let t_1 = disc thickness at hub.
t_0 = hub thickness.
r_1 = external hub radius.
r_{0_1} = radius to centroid of hub.
r_0 = radius of bore.
f_{r_1} = radial disc stress.
f_{r_0} = radial pressure at bore due to force fit.
f_{t_0} = tangential stress in hub.
a_0 = cross-sectional area of half hub.

Then, by a similar process of reasoning as used for the rim, the tangential force is given by

$$f_{t_0} = r_{0_1}\left(\frac{w\omega^2 r_{0_1}}{g} + \frac{f_{r_0} t_0}{a_0} + \frac{f_{r_1} t_1}{a_0}\right) \quad . \quad . \quad . \quad . \quad (18)$$

Neglecting the effect of radial stress, the strains of the hub may be expressed approximately by the fundamental elastic equation

$$S_h = \frac{f_{t_0} r_{0_1}}{E}$$

This will be the same as the disc strain at r_1, which may be calculated by equation (8), or more simply, since the stresses f_{r_1} and f_{t_1} are now known, again from the fundamental equation

$$S_{d_1} = \frac{r_1}{E}(f_{t_1} - \sigma f_{r_1})$$

Equating these, the tangential stress at the hub is given by

$$f_{t_0} = \frac{r_1}{r_{0_1}}(f_{t_1} - \sigma f_{r_1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

When f_{t_0} is determined from equation (19) and a suitable value of f_{r_0} is selected the hub area is calculable from equation (18), and the necessary width of the hub from

$$t_0 = \frac{a_0}{(r_1 - r_0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The exact conditions of the hub loading, however, on account of the change from the narrow disc to the wide hub are really indeterminate. The figures obtained by the above calculation for the hub should only be regarded as rough approximations.

In order to ensure as good a distribution as possible of the disc pull over the hub, liberal fillets should be provided at the junction.

120. Taking the case of the heavy type of wheel, Fig. 127, used for high-speed Curtis machines, the hub cannot be treated as a thin ring, as the difference between the radius of bore and radius of hub is appreciable.

It has to be regarded as a disc of constant thickness. The method of determinating the constants C_1E and C_2E, however, is less definite than in the case of the disc proper.

With a hub only slightly greater in axial width t_0 than the disc thickness t_1 at the junction, the form of the wheel may be taken as that shown by the dotted profile. In this case the strains of the whole hub and the disc will be practically the same. Again, the conditions of loading at the surface of the hole may be specified, and hence from the strain equation for the outside and the radial stress equation for the inside, the value of the constants can be calculated.

Normally the hub is made much wider than t_1, and to admit of an equalisation of the stress and strain, the actual hub is made smaller in diameter and rounded into the disc by large fillets.

In dealing with an actual case, then the radius of the virtual hub r_1 should be chosen to equalise the areas below the fillet curves. The calculation for the constants C_1E and C_2E can then be made for the virtual hub, and the stresses in the hub determined from the resulting equations. Here again the results can only be rough approximations, but with the assumption made it is probable the maximum stress obtained at the surface of the hole is on the high side. The probable form of the stress curve over the filleted portion may be approximated by drawing a fair curve to join up the disc and hub curves in the stress diagrams.

The stresses anywhere in the filleted portion will be less than the maximum on either disc or hub, and this part of the curve is not therefore of much importance. The strain curve can be dealt with similarly.

Adopting the foregoing method

Let r_1 = chosen virtual hub radius.
S_{d_1} = radial strain calculated for the disc at r_1.
r_0 = radius of the bore.
$-f_{r_0}$ = negative stress or radial pressure at the bore.

Then substituting in equations (7) and (4) the necessary coefficients for $n = 0$

$$S_{d_1}E = C_1EX_1 + C_2EY_1 - 0{\cdot}302K_1\omega^2 r_1 \quad . \quad . \quad (21)$$
$$-f_{r_0} = 1{\cdot}1(C_1EA_0 - C_2EB_0 - K_0\omega^2) \quad . \quad . \quad . \quad (22)$$

The constants C_1E and C_2E are determinable from (21) and (22).

Since for $n = 0$, $A = 1{\cdot}3$ for all radii, and $\zeta_1 = 1$, $\zeta_2 = -1$, the stress equations become

$$f_r = 1{\cdot}43C_1E - 1{\cdot}1BC_2E - 1{\cdot}1K\omega^2 \quad . \quad . \quad . \quad . \quad (23)$$
$$f_t = 1{\cdot}43C_1E + 1{\cdot}1BC_2E - 0{\cdot}6347K\omega^2 \quad . \quad . \quad . \quad (24)$$

121. With regard to the radial pressure $-f_{r_0}$ at the surface of the hole in the hub, to be inserted in equation (22), this will depend on the initial amount of force fit given to the hub when the wheel is at rest.

When running at speed the wheel should remain a fit on the shaft, and to ensure this, the distension of the bore or twice the radial strain, should not be greater than the distension produced by the force fit between bore and shaft when at rest.

Experiments carried out by S. A. Moss [1] on a G.E.C. Curtis wheel have shown that for the usual form of hub and a solid shaft the bore expands about 60 per cent. of the force fit, the remainder of the fit being taken up by compression of the shaft. If the shaft is hollow this proportion decreases with increase of the diameter of the hole in the shaft. For instance, when the hole is about half the diameter of the shaft the proportion is 50 per cent.

The initial force fit which will ensure tightness of the wheel on the shaft when running at speed can thus be estimated by taking $-f_{r_0} = 0$ in equation (22), and dividing the calculated strain S_{h_0} of the bore by a factor varying from 0·5 to 0·6.

Force fits of $1\frac{1}{2}$ mils. per inch diameter of bore are sometimes used with steel on steel, but these are on the high side and produce initial permanent set of the hub at the bore.

The maximum value for steel on steel should not exceed 1 mil. per inch diameter of bore.

In some cases of high speed wheels with heavy rims and high blade loading, a larger force fit is required to ensure tightness between the hub and shaft. The difficulty in such cases is overcome by the insertion of a bronze bush in the hub.[2]

The pressure is then distributed between steel and bronze instead of steel and steel, the bronze taking a larger proportion of the total force fit, and reducing the initial distension of the hub.

With this arrangement a force fit of $1\frac{1}{2}$ mils. per inch diameter of bore may be used.

The hub and bush can then be keyed to the shaft by a single key, whereas in the absence of the bush in order to prevent rocking of the wheel on the shaft, at least three keys would have to be fitted.

In some designs this bush is given a slight taper, and drawn into the hub of the wheel by means of a nut. In this way any desired pressure between shaft and hub and bush can be produced.

In other designs expansion rings are forced in between the shaft and hub at each end of the hub (see Fig. 153*a*).

The following examples illustrate the practical application of the foregoing methods to the light and heavy types of disc wheels.

EXAMPLE 1.—The section of a Zoelly wheel is shown at (*b*), Fig. 126. The dimensions are as follows:—

[1] See paper on "Increase of Bore of High Speed Wheels by Centrifugal Stresses," *Journal of American Society of Mech. Engineers*, September, 1912.

[2] See description of experiments by F. Samuelson to determine the "Deformation, by Centrifugal Stress, of Turbine Wheels," *Engineering*, May 9, 1913.

Radius of centroid of rim	r_3	= 28·5 inches
" of disc at rim	r_2	= 28 "
" of hub	r_1	= 7 "
" of centroid of hub	r_0	= 6 "
" of bore	r_0	= 5 "
Width of rim	t_3	= 1 "
Thickness of disc at rim	t_2	= 0·4 "
" " hub	t_1	= 0·93 "
Area of rim	a_3	= 1 in.^{2}

The thickness of the disc is given by $t = \frac{c}{r^{0·6}}$. The total weight of blading and packing pieces referred to, the radius r_3, is W = 70 lbs. The normal speed of the wheel is 1500 R.P.M.

Find the values of the coefficients C_1E and C_2E for the stress and strain equations, calculate the stresses and strains at several radii, and plot curves of radial and tangential stress and the curve of strain on a radius base.

Also calculate the tangential stress in the hub and the minimum hub width t_0.

The first step is the provisional determination of the probable stress f_{r_2} on the disc at the rim section.

Here $$\omega = \frac{2\pi N}{60} = \frac{6·28 \times 1500}{60} = 157 \text{ radians/sec.}$$

and $$\omega^2 = 24649 \frac{\text{rad.}^2}{\text{sec.}^2}$$

Total weight of blading at $r_3 = W = 70$ lbs.

Total centrifugal blade load $F_b = \frac{W\omega^2 r_3}{g}$.

Circumferential area of rim, $A = 2\pi r_3 t_3$.
Intensity of blade load

$$= f_b = \frac{F_b}{A} = \frac{W\omega^2}{2\pi g t_3} = \frac{70 \times 24649}{6·28 \times 32·2 \times 12 \times 1} = 711 \text{ lbs./in.}^2$$

Assuming for a trial $\frac{{}_rf_t}{f_{r_2}} = 1$, and using equation (12)

$$t_2 = \frac{\frac{w}{g}\omega^2 r_3 a_3 + f_b t_3}{f_{r_2}} - \frac{a_3}{r_3}$$

$$0·4 = \frac{0·288 \times 24649 \times 28·5}{32·2 \times 12 \times f_{r_2}} + \frac{711}{f_{r_2}} - \frac{1}{28·5}$$

$$0·4 + 0·0352 = \frac{523 + 711}{f_{r_2}}$$

$$\therefore f_{r_2} = \frac{1234}{0·4352} = 2860 \text{ lbs./in.}^2$$

The next step is to ascertain the more accurate value of f_{r_2} by substituting this approximate stress in equation (14).

The values of the coefficients required (obtained from the curves) are as follows :—

$$A_2 = 7{\cdot}294 \qquad B_2 = 0{\cdot}00119 \qquad K_2 = 0{\cdot}2904$$
$$X_2 = 117{\cdot}3 \qquad Y_2 = 0{\cdot}0629$$

The rim strain is given by equation (13)

$$\begin{aligned} S_rE &= r_3^2\left(\frac{w\omega^2 r_3}{g} + \frac{f_b t_3}{a_3} - \frac{f_{r_2} t_2}{a_3}\right) \\ &= 28{\cdot}5^2(523 + 711 - 2860 \times 0{\cdot}4) \\ &= 812{\cdot}2 \times 90 = 73103 \end{aligned}$$

Applying the approximate equation (14) for the radial stress

$$\begin{aligned} f_{r_2} &= 1{\cdot}1(C_1EA_2 - K_2\omega^2) \\ 2860 &= 1{\cdot}1(C_1E \times 7{\cdot}294 - 0{\cdot}2904 \times 24649) \\ \frac{2860}{1{\cdot}1} &= 7{\cdot}294C_1E - 7158 \end{aligned}$$

$$\therefore\ C_1E = \frac{7158 + 2600}{7{\cdot}294} = \frac{9758}{7{\cdot}294} = 1338$$

Substituting this value in the approximate strain equation (15)

$$\begin{aligned} S_{d_2}E &= C_1EX_2 - 0{\cdot}302K_2\omega^2 r_2 \\ &= 1338 \times 117{\cdot}3 - 0{\cdot}302 \times 0{\cdot}2904 \times 24649 \times 28 \\ &= 156947 - 60528 = 96419 \end{aligned}$$

This is fully 30 per cent. greater than the value calculated by equation (13), so that the value of f_{r_2} should be slightly decreased.

Reducing this to, say, 2780, the numerical value of the third term in equation (13) becomes 1112, and the quantity in the bracket is increased to 122, so that

$$S_rE = 812{\cdot}25 \times 122 = 99095$$

Substitute these values of f_{r_2} and S_rE in equations (16) and (17)

$$2780 = 1{\cdot}1(7{\cdot}294C_1E - 0{\cdot}00119C_2E - 0{\cdot}2904 \times 24649) \quad (a)$$
$$99095 = 117{\cdot}3C_1E + 0{\cdot}0629C_2E - 60528 \;.\;.\;.\;.\;.\;.\; \quad (b)$$

Dividing (*b*) throughout by 52·84 the ratio of the coefficients of C_2E, adding and solving for C_1E, the value obtained is $C_1E = 1335{\cdot}6$.

Substituting in (*a*), the value of $C_2E = 47180$ is obtained. These values substituted in (*b*) check the value of S_rE.

The final form of the radial stress equation (4) is

$$fr = 1469{\cdot}2A - 51898B - 27114K$$

For the tangential stress equation (6), since $n = 0{\cdot}6$, $\zeta_1 = 0{\cdot}83$, and $\zeta_2 = -1{\cdot}42$, the final form of the equation is

$$f_t = 1219{\cdot}4\text{A} + 73695\text{B} - 15645\text{K}$$

The strain equation (8) takes the final form

$$\text{S}_d\text{E} = 1335{\cdot}6\text{X} + 47180\text{Y} - 7444\text{K}$$

In order to obtain the stress and strain curves it is sufficient in addition to the calculated value at the rim to take five other values at, say, 25, 20, 15, 10, and 7 inches radii.

The various coefficients for the series, obtained from the log graphs, along with the calculated stresses and strains, are given in tabular form below:—

r . . .	7	10	15	20	25	28
A . . .	3·968	4·7	5·55	6·3	6·9	7·294
B . . .	0·0154	0·0078	0·0037	0·00219	0·00145	0·00119
K . . .	0·0178	0·036	0·082	0·147	0·23	0·2904
X . . .	16	27	48	72	100	117·3
Y . . .	0·195	0·145	0·105	0·08	0·067	0·0629
f_r . . .	4558	5524	5740	5155	3286	2780
f_t . . .	5695	5741	5761	5544	4923	4448
S_d . . .	0·0010	0·00136	0·0020	0·00266	0·00314	0·00337

The curves of stress and strain are shown in Fig. 133. It will be noted that the maximum radial stress has a value about 5800 lbs./in.^{2} at 13·5 inches radius. The maximum tangential stress is also 5800 lbs./in.^{2} at 13·5 inches radius.

The strains are very small, the maximum value occurring at the rim is 3·37 thousandths and the minimum occurring at the hub section is 1 thousandth of an inch or 1 mil. This represents 0·166 mil. per inch mean diameter of hub, and indicates roughly the amount of force fit to be given to the expansion rings fitted in the recesses at each end of the hub.

The stress on the rim by equation (11) is

$$\begin{aligned} {}_rf_t &= 28{\cdot}5(523 + 711 - 2780 \times 0{\cdot}4) \\ &= 28{\cdot}5 \times 122 = 3477 \end{aligned}$$

The ratio $\dfrac{{}_rf_t}{f_{r_2}} = 1{\cdot}25$ instead of the unit value assumed for the preliminary estimate of f_{r_2}.

The tangential stress in the hub by equation (19) is

$$\begin{aligned} f_{t_0} &= \frac{r_1}{r_{0_1}}(f_{t_1} - \sigma f_{r_1}) \\ &= \frac{7}{6}(5695 - 0{\cdot}3 \times 4558) \\ &= \frac{7 \times 4327}{6} \\ &= 5048 \end{aligned}$$

Assuming that at speed there is no residual force fit, viz. $-f_{r_0} = 0$, and substituting in equation (18) the hub area is obtained

$$f_{t_0} = r_{0_1}\left(\frac{w\omega^2 r_{0_1}}{g} + \frac{f_{r_1} t_1}{a_0}\right)$$

$$5048 = 6\left(110 + \frac{4558 \times 0{\cdot}93}{a_0}\right)$$

$$841{\cdot}3 - 110 = \frac{4239}{a_0}$$

$$\therefore\ a_0 = \frac{4239}{731{\cdot}5} = 5{\cdot}8 \text{ in.}^2$$

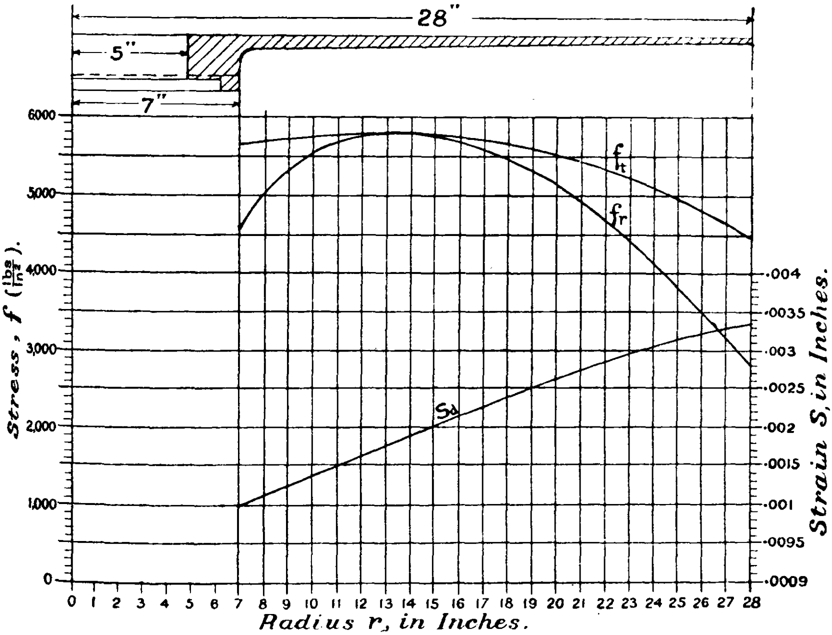


FIG. 133.

The minimum hub thickness by equation (20) is

$$t_0 = \frac{a_0}{r_1 - r_0} = \frac{5{\cdot}8}{2} = 2{\cdot}9, \text{ say } 3 \text{ inches}$$

The general inference which may be drawn from the whole set of results is that this form of disc wheel running at moderate speed is quite lightly stressed. It might be very considerably overspeeded without unduly stressing the material.

EXAMPLE 2.—The section of a heavy type of wheel used in a G.E.C. Curtis turbine is shown in Fig. 127, the leading dimensions being as follows :—

$r_3 = 18$ inches	$t_3 = 3{\cdot}75$
$r_2 = 17{\cdot}5$	$t_2 = 0{\cdot}8125$
$r_1 = 7{\cdot}5$ (assumed)	$t_1 = 1{\cdot}9$
$r_0 = 4{\cdot}5$	$t_{01} = 6$ inches
$r_h = 6\frac{3}{4}$ inches	$a_3 = 5$ in.^{2}

The thickness of the disc is given by $t = \frac{c}{r}$. The total weight of the two rings of blading and distance pieces, referred to the mean rim radius r_3, is 122 lbs. The speed of rotation is 3600 RP.M.

Calculate the probable stresses and strains in the disc and hub, and plot curves showing the distribution. Assume when running at speed that $-f_{r_0} = 0$.

Angular velocity

$$\omega = \frac{6{\cdot}28 \times 3600}{60} = 376{\cdot}8 \text{ radians/sec.}$$

$$\omega^2 = 141978$$

Centrifugal blade load

$$f_b = \frac{W\omega^2}{2\pi g t_3} = \frac{122 \times 141978}{6{\cdot}28 \times 32{\cdot}2 \times 12 \times 3{\cdot}75} = 1902 \text{ lbs./in.}^2$$

Taking for a trial $\frac{rf_t}{f_{r_2}} = 1$, then by equation (12)

$$t_2 = \frac{w\omega^2 r_3 a_3}{g f_{r_2}} + \frac{f_b t_3}{f_{r_2}} - \frac{a_3}{r_3}$$

$$0{\cdot}8125 = \frac{0{\cdot}288 \times 141978 \times 18 \times 5}{32{\cdot}2 \times 12 \times f_{r_2}} + \frac{1902 \times 3{\cdot}75}{f_{r_2}} - \frac{5}{18}$$

$$0{\cdot}8125 + 0{\cdot}278 = \frac{9519 + 7133}{f_{r_2}}$$

$$\therefore f_{r_2} = \frac{16652}{1{\cdot}09} = 15277 \text{ lbs./in.}^2$$

The rim strain for this stress value by equation (13) is

$$S_r = \frac{r_3^2}{E}\left(\frac{w\omega^2 r_3}{g} + \frac{f_b t_3}{a_3} - \frac{f_{r_2} t_2}{a_3}\right)$$

$$\therefore S_r E = 18^2\left(1904 + 1426 - \frac{15277 \times 0{\cdot}8125}{5}\right)$$

$$= 324(3330 - 2482)$$

$$= 324 \times 848 = 284752$$

For the stress and strain equations the coefficients are

$$A_2 = 17{\cdot}2 \quad B_2 = 0{\cdot}0029 \quad K_2 = 0{\cdot}146$$
$$X_2 = 145 \quad Y_2 = 0{\cdot}118$$

Substituting in equation (14)

$$f_{r_2} = 1{\cdot}1(C_1EA_2 - K_2\omega^2)$$
$$15277 = 1{\cdot}1(C_1E \times 17{\cdot}2 - 0{\cdot}146 \times 141978)$$
$$\therefore\ 17{\cdot}2C_1E = 13888 + 20729$$
$$\therefore\ C_1E = \frac{34617}{17{\cdot}2} = 2012{\cdot}6$$

Substituting this value in equation (15)

$$\begin{aligned} S_{d_2}E &= C_1EX_2 - 0{\cdot}302 \times K_2r_2\omega^2 \\ &= 2012{\cdot}6 \times 145 - 0{\cdot}302 \times 0{\cdot}14 \times 17{\cdot}5 \times 141978 \\ &= 219{,}827 - 109{,}552 = 182{,}275 \end{aligned}$$

The value is about 36 per cent. less than that of the rim strain obtained by assuming $f_{r_2} = 15{,}277$. This stress value has therefore to be increased to obtain closer coincidence.

It will be found that $f_{r_2} = 16800$, when substituted in equations (13), (14), and (15), gives the values

$$S_rE = 194{,}400 \qquad C_1E = 2093{\cdot}1 \qquad S_{d_2}E = 193{,}950$$

The disc strain value is about 6 per cent. less than the rim strain. This will be increased by the effect of C_2E in the complete strain equation (8), so that $f_{r_2} = 16800$ may be taken as a reasonably approximate value for the disc stress.

Substituting this and the rim strain in equations (16) and (17)

$$16800 = 1{\cdot}1(C_1E \times 17{\cdot}2 - 0{\cdot}0029C_2E - 0{\cdot}146 \times 141978)\ .\ (a)$$
$$194400 = 145C_1E + 0{\cdot}118C_2E - 0{\cdot}302 \times 20729 \times 17{\cdot}5\ \ .\ (b)$$

Eliminating C_2E between (*a*) and (*b*), the result is $C_1E = 2093{\cdot}66$.

Substituting this value in equation (*a*) and solving, then $C_2E = 3160{\cdot}7$.

As a check substitute these values in the complete strain equation (8)

$$\begin{aligned} S_{d_2}E &= 2093{\cdot}66 \times 145 + 0{\cdot}118 \times 3160{\cdot}7 - 0{\cdot}302 \times 20729 \times 17{\cdot}5 \\ &= 303580 + 373 - 109553 \\ &= 194400 \end{aligned}$$

This is the value calculated for the rim strain, so that the chosen radial stress is fairly correct.

Substituting the values of C_1E and C_2E in equation (4)

then $$f_r = 2303A - 3477B - 156176K$$

since $n = 1$, $\zeta_1 = 0{\cdot}745$ and $\zeta_2 = -1{\cdot}745$. Substitute in equation (6),

and $$f_t = 1716A + 6067B - 90113K$$

The strains can be calculated by equation (8), but it is simpler to calculate them from the stresses by the fundamental elastic equation, which obviates the use of the coefficients X and Y.

Then $$S_d E = r(f_t - 0{\cdot}3 f_r)$$

These equations are applied to determine the stresses and strains at the following radii: 17·5, 16, 14, 12, and 10 inches.

The coefficients A, B, and K, together with the resulting stresses and strains, are given below for the disc proper, to the radius 10 inches at which the fillets start.

The values for the disc at the virtual hub radius $r_1 = 7{\cdot}5$ are also given in the last line. These are used to determine the values of C_1E and C_2E for the hub, which is treated as a disc of constant thickness.

r.	A.	B.	K.	f_r.	f_t.	S_d.
17·5	17·2	0·0029	0·14	16800	16377	0·00661 inch
16	16·18	0·0035	0·122	18198	16792	0·00604 ,,
14	14·7	0·00445	0·0935	19237	16826	0·00516 ,,
12	13	0·0056	0·069	19144	16124	0·00415 ,,
10	11·2	0·0078	0·046	18589	15121	0·00318 ,,
7·5	9	0·0135	0·026	16619	13183	—

Assuming the actual form of disc at the hub junction replaced by the dotted form, and that the hub and disc strains are approximately the same, then

$$\begin{aligned} S_h E = S_{d_1} E = r_1(f_{t_1} - 0{\cdot}3 f_{r_1}) &= 7{\cdot}5(13183 - 0{\cdot}3 \times 16619) \\ &= 7{\cdot}5 \times 8197 = 61478 \end{aligned}$$

The values of the coefficients of the stress strain equations, for $n = 0$, are

$$\begin{array}{llll} r_0 = 4{\cdot}5 & A_0 = 1{\cdot}3 & B_0 = 0{\cdot}034 & K_0 = 0{\cdot}0055 \\ r_1 = 7{\cdot}5 & X_1 = 7\ 5 & Y_1 = 0{\cdot}132 & K_1 = 0{\cdot}0155 \end{array}$$

It is assumed that at speed the residual force fit is negligible or $f_{r_0} \doteq 0$. Substituting these values in equations (21) and (22)

$$61478 = 7{\cdot}5C_1E + 0{\cdot}132C_2E - 0{\cdot}302 \times 0{\cdot}0155 \times 141978 \times 7{\cdot}5 \quad (a)$$
$$0 = 1{\cdot}3C_1E - 0{\cdot}034C_2E - 0{\cdot}0055 \times 141978 \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Divide (a) by the ratio of the coefficients of C_2E, and add (a) and (b), and $15840{\cdot}8 = 3{\cdot}233C_1E - 2065{\cdot}1$

$$\therefore C_1E = \frac{17905{\cdot}9}{3{\cdot}233} = 5539{\cdot}3$$

Substitute this value in (b)

$$0 = 1{\cdot}3 \times 5539{\cdot}3 - 0{\cdot}034C_2E - 781$$
$$\therefore C_2E = \frac{6420}{0{\cdot}034} = 188831$$

The stress equations (23) and (24) thus become

$$f_r = 7921 - 207714B - 156176K$$
$$f_t = 7921 + 207714B - 90113K$$

It is sufficient to calculate the stress and strain values for, say, three points of the hub. Taking radii 4·5, 5, and 6·75, the coefficients and calculated stresses and strains are given below—

r.	B.	K.	f_r.	f_t.	S_h.
4·5	0·034	0·0055	0	14492	0·00217 inch
5	0·028	0·007	1012	13111	0·00213 ,,
6·75	0·016	0·0125	2646	10123	0·00210 ,,

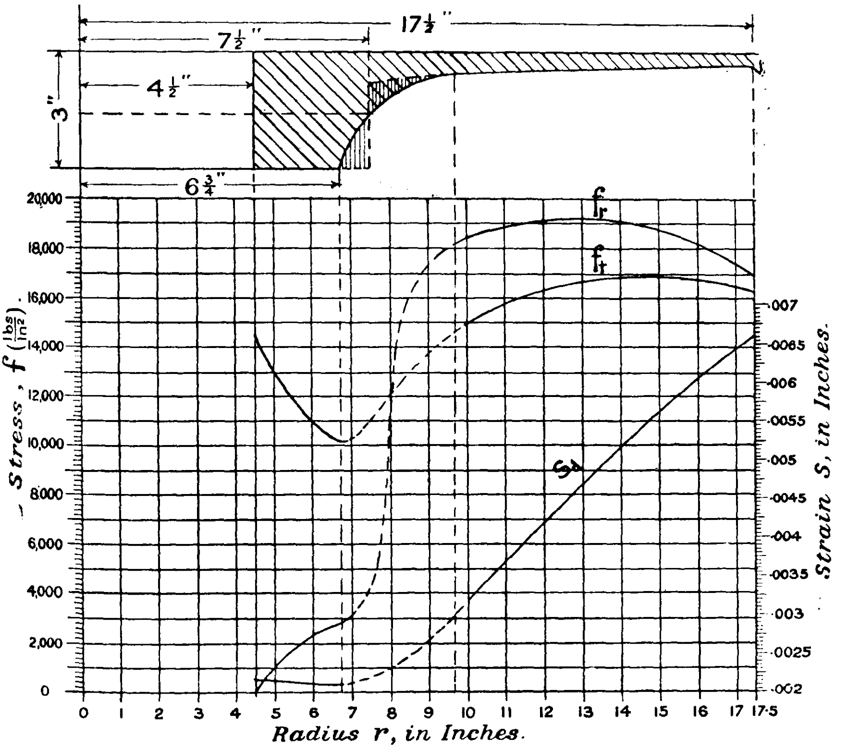


FIG. 134.

The curves of stress and strain are shown in Fig. 134. The full line portions show the values calculated as above for the disc and hub. The dotted portions are drawn to represent an assumed distribution of stress and strain over the filleted part of the disc. The conditions here are practically indeterminate.

In the case of the disc the maximum stress is radial and in excess of 19,000 lbs./in.^{2}. This is larger than the practicable limit permissible for mild steel, so this wheel, if made to the dimensions specified, requires to be of nickel steel.

The maximum stress at the bore of the hub is apparently in the neighbourhood of 15,000 lbs./in.^{2}. It may be less than this, as the calculation of the basis on the virtual hub and disc dimensions is very tentative, and the actual hub width t_0' is much greater than the virtual width t_0. The effect of the keyway has, however, been neglected. This may probably increase the effective hub radius from 4·5 to 5 inches, and the tangential stress to some value between 16,000 and 18,000 lbs./in.^{2}.

The probable total increase in diameter of the bore is 0·00434 or 4·34 mils., so that the increase per inch of bore is $\frac{4·34}{9} = 0·482$ mil. According to the experimental evidence quoted the expansion of the bore is usually about 60 per cent. of the actual force fit, the force being taken by the compression of the shaft. The actual initial force fit of this wheel should therefore be about 1 mil. per inch diameter of bore. This will ensure a tight fit of the wheel on the shaft when running at speed.

The foregoing methods of treatment leave a good deal to be desired, especially with regard to the real conditions existing at the hub; but they serve, in the absence of more definite means of calculation, a useful purpose, in giving the designer roughly approximate values for the maximum stresses produced in a given form of disc under given conditions of operation.

122. The types of disc wheel just considered are always perforated with several large holes about three-quarters of the radius out from the centre (see Fig. 89). These are required to ensure equality of steam pressure on each side of the wheel.

The value and distribution of stress in the immediate neighbourhood of these holes is very uncertain. It may rise to two or three times the value for the unpierced disc. To prevent such an undesirable increase, especially in the case of a highly stressed wheel, it is the custom either to thicken up the metal round the holes or to thicken up the whole disc at the radius where the balancing holes are provided.

The case of a built-up plate wheel can be treated by considering each plate as a disc of uniform thickness ($n = 0$) carrying half the rim and blade load. The radius r_0 is the radius of the hole in the plate where it fits over the hub. A similar proceeding might be applied to the end plates of certain drum rotors; but as these run at quite low speeds such stress calculations are unnecessary.

What is of more importance here is rigidity against end thrust.

123. Disc of Uniform Strength.—This is the type of wheel used in the larger sizes of the de Laval turbine, and is shown in Fig. 125. It is made of such a form that the stress at any point between the centre and the rim is constant, that is, $f_r = f_t = f =$ constant.

In the mathematical investigation by which equations (2) and (3) are deduced, if this condition is assumed, the following simple expression results :—

$$t = t_2 \epsilon^{\frac{w\omega^2}{2fg}(r_2^2 - r^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where f = (constant) stress in lbs./in.^{2}.

w = weight per cubic inch of material = 0·288 for steel.

ω = angular velocity in radians/sec.

$g = 32·2 \times 12$ inches/sec.^{2}.

r = radius of disc in inches.

t = thickness in inches at radius r.

t_2 = " " " r_2 below rim.

$\epsilon = 2·7183$.

The resulting profile is concavo-convex.

In this case the disc strain at the rim, since $f_{t_2} = f_{r_2} = f$, is given by $S_{d_2} = \frac{r_2}{E} f(1 - \sigma)$. Equating this against the rim strain given by equation (13)

$$r_3^2\left(\frac{w\omega^2 r_3}{g} + \frac{f_b t_3}{a_3} - \frac{f t_2}{a_3}\right) = r_2 f(1 - \sigma)$$

Taking as a first approximation $\frac{r_3^2}{r_2} = r_3$, then

$$t_2 = \frac{\frac{w}{g}\omega^2 r_3 a_3 + f_b t_3}{f} - 0·7\frac{a_3}{r_3} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

This is equation (12) with $\frac{r f_t}{f_{r_2}} = 0·7$ and $f_{r_2} = f$, and from it the thickness t_2 for any chosen value of the limiting stress f is calculable. This stress must be so chosen that t_2 has at least a practicable minimum of $\frac{3}{8}$ inch.

When suitable values of t_2 and f are fixed, then the necessary thicknesses at all radii are calculable by equation (25).

The strain, at any radius r, is given by the fundamental equation

$$S_d = rf(1 - \sigma) = 0·7rf \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

As the disc is made solid it varies from zero at the centre to a maximum at the rim section. The system of calculation, although in this case simple in comparison with that of the other forms of wheel, is regarded with considerable suspicion by many designers. If the ratio between the thickness under the rim and at the centre is considerable, then the rapidly bulging section towards the centre is subjected to shear stress, the effect of which is not taken into account by equation (25).

The provision of a section of weakness below the rim by the use of the safety groove appears, however, to satisfactorily guard against under estimation of stress near the centre.

The makers of de Laval turbines have found that such wheels, if not undercut at the rim, burst through the centre in two or three heavy pieces. In an experiment of this kind the pieces have been driven through an experimental cast steel casing 2 inches thick. When undercut with a safety groove so as to increase the radial stress at the reduced section about 50 per cent. above the presumably constant stress in the rest of the wheel, it is found that the fracture takes place at the rim, which is broken into small pieces that do no harm to the wheel case. As soon as the rim parts from the wheel there is an immediate reduction of the stress in the body. The wheel also gets out of balance, and the hub, which is made an easy fit in the end walls of the casing fouls the casing. This acts as a brake, and brings the wheel to rest.

Designers of these solid wheels use stresses as high as $11\frac{1}{2}$ tons/in.^{2} or 27,000 lbs./in.^{2} for the body of the wheel, and 16 tons/in.^{2} or 36,000 lbs./in.^{2} for the section at the safety groove, the material being nickel steel.

In determining the proportions of the wheel, the dimension t_2 of the disc below the rim can first be calculated. If t_{2_1} is the thickness at the safety groove, and f_1 the increased stress at the section, then

$$t_{2_1} = t_2 \frac{f}{f_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (28)$$

In some cases where the peripheral speed of the rim is not excessive this form of wheel is used in conjunction with a bored hub. In such a case the same tentative method of calculation outlined for the heavy type of Curtis wheel may be employed to find the approximate stress conditions at the hub.

EXAMPLE 3.—The solid disc wheel required for a turbine of 300 B.H.P. runs at 10,600 R.P.M. The radius below the rim is 13 inches, and the rim radius below the slot is 10·25 inches. The rim width may be taken as $\frac{3}{4}$ inch.

The total weight of blades and the slot projections of the rim may be taken as 20 lbs.; the rim area is 0·1 in.^{2}.

Allowing a stress of 27,000 lbs./in.^{2} in the body and 36,000 lbs./in.^{2} in the metal at the safety groove, find the thickness at the rim and centre of the disc (neglecting any hub effect).

Also compare the stress at the centre with the stress of a solid disc having a uniform thickness equal to the thickness at the rim, and carrying the same rim load.

$$\text{Angular velocity} \qquad \omega = \frac{6{\cdot}28 \times 10600}{60} = 1109 \text{ rad./sec.}$$

$$\omega^2 = 1229881$$

$$\text{Centrifugal blade load } f_b = \frac{W\omega^2}{2\pi g t_3} = \frac{20 \times 1229881}{6{\cdot}28 \times 32{\cdot}2 \times 12 \times 0{\cdot}75}$$

$$= 13500 \text{ lbs./in.}^2$$

Substitute in equation (26)

$$t_2 = \frac{0{\cdot}288 \times 1229881 \times 13{\cdot}25 \times 0{\cdot}1}{32{\cdot}2 \times 12 \times f} + \frac{13500 \times 0{\cdot}75}{f} - \frac{0{\cdot}7 \times 0{\cdot}1}{13{\cdot}25}$$
$$= \frac{1213 + 10100}{27000} - 0{\cdot}005$$
$$= 0{\cdot}42 - 0{\cdot}005 = 0{\cdot}415 \text{ inch}$$

By equation (28) the thickness at safety groove is

$$t_{2_1} = \frac{0{\cdot}415 \times 27}{36} = 0{\cdot}31 \text{ inch}$$

For the calculation of the various thicknesses the index of ϵ in equation (25) is

$$\frac{w\omega^2}{2gf}(r_2^2 - r^2) = \frac{0{\cdot}288 \times 1229881}{773 \times 27000}(13^2 - r^2)$$
$$= 0{\cdot}0169(169 - r^2)$$

At the centre $r = 0$, hence

$$\frac{t_0}{t_2} = \epsilon^{2{\cdot}85} = 2{\cdot}7183^{2{\cdot}85} = 17{\cdot}4$$
$$\therefore\ t_0 = 0{\cdot}415 \times 17{\cdot}4 = 7{\cdot}2 \text{ inches}$$

In the case of a solid disc of uniform thickness the second term in the radial and tangential stress equations (23) and (24) is zero.

Hence at the rim section of a disc of uniform thickness t_2 the radial stress is

$$f_{r_2} = 1{\cdot}43C_1E - 1{\cdot}1K_2\omega^2$$

f_{r_2} is taken as 27,000 lbs./in.^{2}, and K_2 corresponding to $n = 0$ and $r_2 = 13$, is 0·0475.

At the centre $r_0 = 0$, so that $K_0 = 0$.

Hence $\quad f_{r_0} = 1{\cdot}43C_1E$

or

$$f_{r_0} = f_{r_2} + 1{\cdot}1K_2\omega^2$$
$$= 27000 + 1{\cdot}1 \times 0{\cdot}0475 \times 1229881$$
$$= 27000 + 64000$$
$$= 91000 \text{ lbs./in.}^2$$

This is the maximum stress, and is equal to the ultimate strength of the nickel steel. The calculation serves to show how impracticable a thin disc, even if solid, would be, for a high-speed wheel like this.

124. Drum Rotors.—As already stated, the fine-running tip clearance in the case of the reaction machine necessitates a stiff construction of rotor, and the more elastic shaft and disc type is replaced by a drum.

Apart from this condition of stiffness, the large increase in the number of stages requires the use of an impracticable number of disc wheels.

In land turbines the complete drop in pressure is usually carried out in one cylinder, and the drum, partly to obtain reduction of tip clearance at the H.P. end, and partly to reduce the length and the number of blade rings, is stepped in three diameters.

In marine work, on account of the low rotational speed required for direct coupled turbines, the single cylinder machine would be excessively long. The question of the suitability of twin, treble, or quadruple screws, however, also affects the determination of the turbine equipment. The total drop may be divided between two or three turbines, according to whether a two-, three-, or four-shaft arrangement is adopted.

In each case the rotor of the marine turbine is formed with a parallel drum (see Figs. 27 and 28).

In the geared type of marine turbine the conditions on account of the higher peripheral speed, approximate to those of the land machine, and the stepped type of drum is employed (see Fig. 29).

The maximum diameter of the drum, whether of the parallel or stepped type, is usually determined by the maximum permissible stress in the drum shell. In the case of the land turbine another controlling factor, restricting the maximum diameter to about 8 feet 6 inches, is the facility for transport on the railways.

In the case of the marine rotor, the question of available space athwartship, and available head room, are also determining factors.

In the earlier designs of drum rotors the shaft ends or "spindles" were either forced into sockets on the ends of the drum, or these were shrunk on to the spindles.

With superheated steam the fluctuation of temperature due to variation in superheat at the H.P. end produced a differential expansion of the boss and shaft end since the boss metal heated quicker than that of the shaft. The result in many cases was slackness of fit between the shaft and drum; the rotor got out of truth, and a "blade-strip" ensued.

Several constructions, already described in connection with the turbines illustrated in Chaps. III. and IV., have been successfully applied to overcome this temperature defect.

In the case of the small drum the H.P. end may be forged solid with the drum (see Fig. 44).

Again, the end may be bolted to the drum, and provided with a boss projecting into the interior of the drum and surrounded by the L.P. steam, which is admitted to get the necessary axial balance at the L.P. section.

This low-pressure steam keeps the boss at a low and uniform temperature, and prevents the shaft end, which is forced into the boss, from getting slack.

In other cases the shaft is provided with a large hollow spigot, which fits directly into the socket on the drum end. The end of the shaft is closed, and high-pressure steam is admitted to the interior, through a radial hole (see Fig. 26).

At the exhaust end of the rotor no special precautions are necessary. The shafts are either flanged and bolted to the ends (see Figs. 42, 43) or spider wheels are riveted to the shell, and the shafts are pressed into the holes in the bosses (see Figs. 26, 27, 28, 30).

Land machines have drums forged either from mild or nickel steel, and turned all over inside and outside.

As in the case of the disc it is preferable to use a forged instead of a cast steel drum, on account of the risk of loss due to unsound casting.

In the case of the disc and drum type, the disc is either bolted on to or shrunk over the drum end.

Owing to the large pressure drop in the first stage nozzles, expansion trouble due to temperature variation at the H.P. end is eliminated.

The dummy piston may be forged with the disc end; but usually it is bolted either to the drum or the boss of the disc wheel.

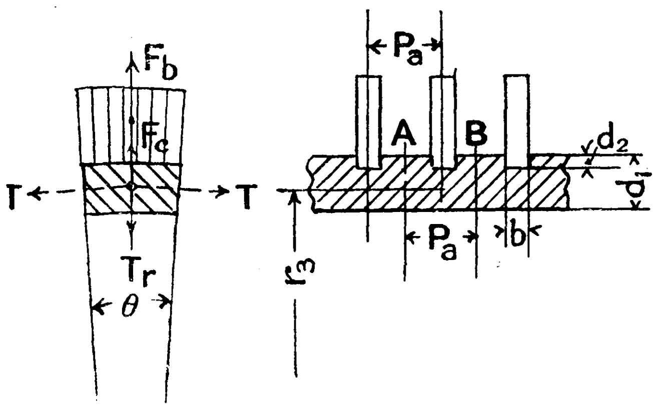


FIG. 135.

125. Stresses in Drums.—The calculation for the stress in the drum shell is similar to that for the rim of the disc wheel. At the ends or where there is an internal stiffening ring, the restraining radial pull is present as in the case of the disc; but in the calculation for strength it is the weakest portion of the drum, where such an effect is negligible, that has to be considered.

Referring to Fig. 135, consider a sector of the drum subtending a small angle θ, and having an axial length AB $= t_3 = P_a =$ axial pitch of the blades in inches.

When running at speed this sector is in equilibrium under the three forces, F_c, F_b, and T_r.

Let r_3 = mean radius of the drum in inches.

w = weight in lbs./in.^{3} of the steel.

a_3 = nett area in ins.^{2} allowing for groove.

ω = angular velocity in radians/sec.

Then the centrifugal force is

$$F_c = \frac{wr_3\theta a_3\omega^2 r_3}{g} = \frac{w}{g}a_3\omega^2 r_3{}^2\theta$$

Let f_b = radial force of the blades in lbs./in.^{2} of surface at r_3 due to the centrifugal load.
$t_3 = P_a$ = width of sector = axial pitch in inches.

Then the centrifugal blade load is

$$F_b = f_b t_3 r_3 \theta$$

Let f_t = tangential stress induced in the drum. Then the tangential load is

$$T = f_t a_3$$

With θ a small angle, the radial (inward) component of this is

$$T_r = f_t a_3 \theta$$

For equilibrium $$F_c + F_b = T_r$$

Hence $$\frac{w}{g}a_3\omega^2 r_3{}^2 + f_b t_3 r_3 = f_t a_3$$

and $$f_t = \frac{w}{g}\omega^2 r_3{}^2 + \frac{f_b t_3 r_3}{a_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Let d = equivalent thickness of rim allowing for the groove, then the nett area is $a_3 = t_3 d = P_a d$.

The first term on the right of equation (29) is the stress f_{t_1} in the drum when there are no blades. The second term is the additional stress f_{t_b} due to the blade load.

It is convenient, in the case of the Parsons blading, to express the weight of the blading per ring, in lbs. per inch circumference.

Let w_b = weight of blading per ring per inch circumference at r_3.

Then the total blade weight $W = w_b 2\pi r_3$.

The centrifugal blade load

$$F_b = \frac{W\omega^2 r_3}{g} = \frac{w_b}{g}2\pi\omega^2 r_3{}^2$$

The distributed blade pull over width t_3 is

$$f_b = \frac{F_b}{2\pi r_3 t_3} = \frac{w_b}{g}\frac{\omega^2 r_3}{t_3}$$

Hence $$f_{t_b} = \frac{w_b\omega^2 r_3}{gt_3} \times \frac{t_3 r_3}{P_a d} = \frac{w_b\omega^2 r_3{}^2}{gP_a d}$$

Equation (29) can thus be written in the form

$$f_t = f_{t_1} + \frac{w_b\omega^2 r_3{}^2}{gP_a d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and the equivalent thickness is

$$d = \frac{w_b \omega^2 r_3^2}{g P_a (f_t - f_{t_1})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Let d_1 = actual thickness of the drum.
d_2 = depth of groove.
b = width ,,

Then $$P_a d = P_a d_1 - b d_2$$

$$d_1 = d + \frac{b d_2}{P_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

126. The following empirical formula for the weight of blading has been given by J. M. Newton:—[1]

$$w_b = \alpha B^{1 \cdot 5} + \beta B l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

where w_b = weight in lbs./inch circumference.
B = axial width of blade in inches.
l = length of blade ,, ,,
α = coefficient, 0·15 to 0·3.
β = ,, 0·06 to 0·11.

If the additional stress f_{tb} due to the blade load is expressed as a fraction of the stress due to the drum alone, and an average value of the ratio of the blade height to mean ring diameter at the L.P. end of the turbine be assumed, the limiting blade velocity for mild and nickel steel drums can be approximated.

Let $f_{tb} = x f_{t_1}$.

Then $$f_t = (1 + x)\frac{w}{g}\omega^2 r_3^2 = (1 + x)\frac{w}{g}u_3^2 \quad . \quad . \quad . \quad (34)$$

where u_3 = mean velocity of the drum.

If the blade length at the L.P. end is given an average value of 15 per cent. of the mean ring diameter, then the outside drum diameter is 85 per cent. of the mean blade diameter, and the mean drum velocity, allowing for the thickness, is less than 85 per cent. of the mean blade velocity. For the present purpose, however, it is sufficient to take $u_3 = 0 \cdot 85 u$.

It may be assumed that, on an average, the additional stress due to the blading is about 30 per cent. of the stress due to the drum alone, $x = 0 \cdot 3$.

Substituting these assumed values in (34) and expressing the blade velocity in ft./sec.

$$f_t = \frac{1 \cdot 3 \times 0 \cdot 288}{32 \cdot 2 \times 12} \times (0 \cdot 85 \times 12 \times u)^2 = \frac{u^2}{9 \cdot 9}$$

$$\therefore u = 3 \cdot 14 \sqrt{f_t}$$

[1] *Junior Inst. of Engineers*, "High Speed Steam Turbine Rotor Design and Construction," April, 1910.

Taking the same maximum limits of stress as for the disc, $f_t = 16{,}000$ for mild and $f_t = 20{,}000$ for nickel steel.

$$u = 398 \text{ ft./sec. for mild steel}$$
$$u = 445 \text{ ft./sec. for nickel steel}$$

If instead of 15 per cent., a maximum value of 20 per cent. is taken for the blade length diameter ratio, these velocities are increased 7 per cent. For an extreme value of 26 per cent. (see footnote p. 198), the increase in blade velocity is 15 per cent. The drum stress due to the blading will, however, be greater than the value asumed above, and the increase may be taken from 10 to 12 per cent.

As a general rule, except in the case of a large output, the mean peripheral blade speed at the last ring L.P. does not exceed 400 ft./sec.

EXAMPLE 4.—The blading at the L.P. end of a drum rotor is $\frac{1}{2}$ inch wide and $6\frac{3}{4}$ inches long, and has an axial pitch of 1·67 inches. The groove may be taken as $\frac{1}{2}$ inch wide and $\frac{3}{8}$ inch deep. The approximate mean radius of the drum is $23\frac{1}{4}$ inches, and the speed 1500 R.P.M.

Calculate the minimum thickness of the drum so that the stress may not exceed 13,000 lbs./in.^{2}.

For the blade weight assume $\alpha = 0{\cdot}16$, $\beta = 0{\cdot}06$.

By equation (33) the blade weight is approximately

$$\begin{aligned} w_b &= 0{\cdot}16 \times 0{\cdot}5^{1\cdot5} + 0{\cdot}06 \times 0{\cdot}5 \times 6{\cdot}75 \\ &= 0{\cdot}057 + 0{\cdot}202 = 0{\cdot}26 \text{ lb./inch} \end{aligned}$$

$$\omega = \frac{6{\cdot}28 \times 1500}{60} = 157 \quad \text{and } \omega^2 = 24649$$

$$f_{t_1} = \frac{w\omega^2 r_3^2}{g} = \frac{0{\cdot}288 \times 24649 \times 23{\cdot}25^2}{32{\cdot}2 \times 12} = 10000 \text{ lbs./in.}^2$$

By equation (31) the equivalent drum thickness is

$$\begin{aligned} d &= \frac{w_b\omega^2 r_3^2}{g\mathrm{P}_a(f_t - f_{t_1})} \\ &= \frac{0{\cdot}26 \times 24649 \times 23{\cdot}25^2}{32{\cdot}2 \times 12 \times 1{\cdot}67(13000 - 10000)} \\ &= 1{\cdot}8 \text{ inches} \end{aligned}$$

By equation (32) the actual thickness is

$$\begin{aligned} d_1 &= d + \frac{bd_2}{\mathrm{P}_a} \\ &= 1{\cdot}8 + \frac{0{\cdot}5 \times 0{\cdot}375}{1{\cdot}67} \\ &= 1{\cdot}8 + 0{\cdot}112 \\ &= 1{\cdot}912, \text{ say } 1\tfrac{7}{8} \text{ inches} \end{aligned}$$

If the permissible stress, which is arbitrarily chosen according to the designer's judgment, is increased to 13,500, the thickness becomes $1\frac{5}{8}$ inches.

As there are several arbitrarily variable factors involved it will be evident that a considerable exercise of individual judgment is necessary in applying the foregoing method of calculation for drum thickness. It is not advisable at the H.P. end of a stepped drum to reduce the thickness below $1\frac{1}{2}$ inches, as with smaller thickness any inequality in the caulking of the distance pieces may lead to distorsion of the drum.

127. Balancing of Rotors.—In order to avoid injurious vibration a rotor has to be as accurately balanced as possible. Even with the most careful adjustment there is always some particular speed known as the "critical speed," at which, if the rotor is run continuously, excessive vibration will be produced.

In the design of a given type of rotor care must be taken that the running speed shall either be well above or well below the critical value.

Lack of balance may arise from various causes, such as non-homogeneity of the material of the discs or drum, or shaft, eccentricity of such masses as pins and keys fixing the shafts, discs, and drum, errors in workmanship, etc.

Statical balance is first obtained by placing the shaft journals on carefully levelled knife-edges, marking the circumference at six or more equidistant points, bringing these in succession to the top position, and finding the weight required, at either side, to prevent the rotor from turning.

Disc wheels are balanced separately before being put on the shaft. Each wheel is usually mounted on a short shaft whose axis is coincident with the axis of rotation of a weighing beam. The wheel is clamped in successive positions on this shaft, and the out-of-balance weights are registered by placing weights in the scale-pan of the weighing beam.

Drum rotors are balanced by bolting on weights usually at the ends. In marine rotors these weights are bolted to the "spider" wheels, on which the drum shells are mounted. In some instances balance strips are provided on the wheels, from which portions can be cut to obtain the necessary balance.

Although a rotor may be in good balance statically it may, owing to faulty distribution of the balance weights, be badly out of balance dynamically.

In order to test the dynamic balance the rotor is connected through a flexible coupling with a shaft which is driven at a speed usually 20 per cent. above the rated speed of the rotor. The rotor is run in spring-supported bearings.

If it is out of balance at speed, vibration, more or less pronounced, will result.

This effect is due to the incorrect disposition of the balance weights, which, under centrifugal action, set up a tilting couple that rocks the rotor.

A speed is reached at which the period of vibration of the rotor coincides with that of the spring supports, and the vibration is thus intensified.

A pencil is held in light contact with the drum surface, at several sections which are found to be true when the drum is turned slowly.

Owing to the tilting of the shaft the pencil marks only one side of the drum circumference. The marks produced along the length may not be in the same plane, they may even fall at opposite sides of the rotor at the ends. A redistribution of the balance weights at the ends has then to be tried.

The process is purely experimental. A weight is transferred from one end to the other, and the rotor is run again. If the dynamic balance is improved the marking will extend further round the circumference.

This adjustment is continued until the markings extend all round. It is, however, very difficult to ascertain when the final adjustment has been made, as the marking becomes very faint near the ends. It is to remedy this drawback that the elastic bearings supports are used.

The final accuracy of running balance is dependent solely on the experience and skill of the man who adjusts the balance weights. The rotor may be considered in good balance when the final adjustment of the weights reaches the limit of about $\frac{1}{2}$ oz. in a rotor weighing several tons.

The balance weights, sometimes in the form of screws or washers, are fixed to the rotor as far out as possible from the axis. They may either be left permanently in position or compensated for by drilling holes in the rotor diametrically opposite to remove a weight of material, having the same moment about the axis as the weights.

128. Critical or Whirling Speed of a Rotor.—It is never possible to obtain an exact coincidence between the mass centre of the rotor and the axis of rotation. In the usual case of the horizontal rotor the inevitable deflection under its own weight precludes the attainment of this condition, apart from any of the other causes of eccentricity.

As the rotor speeds up from rest the eccentricity of the mass centre causes the imposition of a gradually increasing centrifugal force tending to deflect the shaft.

This action goes on till a certain definite speed is reached. If the rotor is run steadily at this speed the shaft deflection progressively increases.

If the shaft were unrestrained this deflection and the accompanying vibration would increase till the shaft would fracture or become crippled.

Before any such extreme condition can arise in the actual case the rotor fouls the casing or the shaft fouls the diaphragm glands, or seizes on the bearings.

This speed, which for a small initial eccentricity of the mass centre of the rotor produces indefinitely large deflection of the rotor, is called the "critical speed" or "whirling speed" of the rotor.

A loaded shaft may have more than one critical speed. If the speed is increased beyond the first or fundamental critical speed, the shaft begins to straighten, and if the speed is further increased the shaft again bends and takes another configuration at a higher speed. If when run steadily at this new speed the deflection for this new configuration continues to increase, this is the second critical speed. A further increase may again change the form of the "elastic" curve into which the shaft bends, and a third speed may be reached, and so on.

The number of these critical speeds depends on the nature of the loading of the shaft. A uniform shaft with a uniformly distributed load has an infinite number of critical speeds.[1] The first is called the "fundamental" critical speed.

The analytical determination of critical speeds except in the most simple cases is exceedingly difficult. As far as turbine rotors are concerned it is as a rule not necessary to consider any speed except the fundamental one. For this again no great refinement of calculation is called for.

The designer has merely to ascertain whether at the rated speed the rotor will run either well below or well above the critical value, so that appreciable vibration may be avoided.

The running speed should be at least some 30 per cent. either above or below the critical speed.

129. The cases which usually arise are:—

(*a*) Small shaft loaded with a single wheel whose weight is large compared with that of the shaft.

(*b*) Approximately uniform shaft loaded by its own weight and a series of wheels.

(*c*) A shaft of varying section, constituted by a stepped drum and short journal ends, and loaded at varying rates along its length, by shaft and drum weight, blade rings, and flanges, dummies, and disc wheels.

In any of these cases the shaft ends may be regarded as either supported on spherical bearing blocks reducing the rotor to the condition of a supported beam, or held in "fixed" bearings which constrain the ends horizontally and reduce it to the condition of an encastre beam.

In some cases the shaft and disc type of rotor is run above the critical speed. In the majority of cases it is run below it. The drum rotor is almost invariably run well below the critical. On account of the fine blade clearance required in this type, the slight "whirl" which always takes place when the critical speed is passed through, tends to produce a "touch" between the blade rings and casing, especially at the H.P. end. This action may increase the tip clearance and reduce the efficiency.

As in the case of the stress estimation in the disc wheels, it is not proposed to enter into this question in detail. For an exhaustive discussion and analysis the reader should refer to Dr. Stodola's standard

[1] The successive critical speeds of a uniformly loaded shaft are in the proportion

$1 : 2^2 : 3^2 : 4^2 \ldots$ for supported shaft.
$3^2 : 5^2 : 7^2 : 9^2 \ldots$ for shaft fixed at the ends.

For an extended discussion of the subject of critical speeds, see Stodola's text, chap. III. See also article by Professor Morley on "Calculation of Vibration and Whirling Speeds," *Engineering*, July 30, 1909.

work.[1] The estimation of the fundamental or first critical speed, for the three cases outlined above, will only be considered.

130. Taking the simplest case (*a*), viz. the small shaft with a single heavy wheel, and referring to Fig. 136,

Let W = weight of wheel in lbs.

e = eccentricity of the mass centre.

L = span of bearings A and B in inches.

a = distance of wheel from A.

b = " " " B.

y = deflection from the central position AB in inches.

ω = angular velocity in radians/sec.

n = number of revolutions/sec.

$g = 32 \cdot 2 \times 12$ inches/sec.^{2}.

When the wheel rotates at speed ω, the displacement of the mass

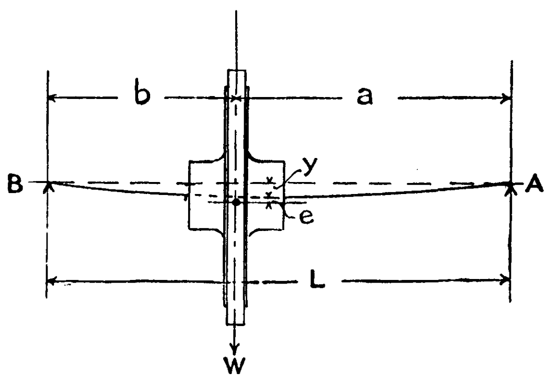


FIG. 136.

centre from the axis of rotation is $(y + e)$, hence the centrifugal force is

$$F_c = \frac{W}{g}\omega^2(y + e)$$

This must be equal to the elastic force of resistance of the shaft.

Let F = stiffness or force required to produce unit deflection at the wheel, then the elastic resistance is Fy.

Hence
$$\frac{W}{g}\omega^2(y + e) = Fy$$

and
$$y = \frac{\omega^2 e}{\frac{Fg}{W} - \omega^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

[1] See also a series of articles on "The Whirling Speeds of Loaded Shafts," by Wm. Kerr, in *Engineering*, February 18, 25, March 3, 10, 17, 1916; and correspondence relating to these in consecutive issues from March 24 to June 2, 1916.

When the speed is increased until $\omega^2 = \frac{Fg}{W}$, then the denominator on the right of (35) becomes zero and y takes an infinite value.

The condition for the critical speed in this case is therefore

$$\left.\begin{aligned} \omega_c &= \sqrt{\frac{Fg}{W}} \\ \text{or} \qquad n_c &= \frac{1}{2\pi}\sqrt{\frac{Fg}{W}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If ω is increased beyond the critical value, that is, if $\omega^2 > \frac{Fg}{W}$, then the denominator in (35) becomes negative, and y becomes negative. When ω^2 approaches such a value that $\frac{Fg}{W}$ is negligibly small compared with it, then in the limit the deflection approaches $-e$. In other words, the shaft begins to straighten again until the speed reaches some upper value, at which $y = -e$ and the mass centre coincides with the axis of rotation.

In order to attain this condition, however, ω would require to become indefinitely great, and this would mean an altogether impracticable speed of rotation.

Equation (35) may be written

$$y = \frac{+\omega^2 e}{\omega_c^2 - \omega^2} \quad \text{or } y = \frac{e}{\frac{\omega_c^2}{\omega^2} - 1}$$

where ω is the practicable running speed of the rotor.

In the de Laval machine the value arbitrarily adopted for the ratio $\frac{\omega_c}{\omega}$ is about $\frac{1}{7}$, so that the deflection at speed is

$$y = -\frac{e}{0{\cdot}98} = -1{\cdot}02e$$

The critical speed in a case like this is passed through so quickly in "speeding up" that the critical condition exists only for an instant and no harm is done to the shaft.

The slight whirl as the critical speed is reached is immaterial in the case of a single wheel machine with ample radial clearances; but in the other types, as already pointed out where there are fine gland or tip clearances, the whirl is objectionable, and is to be avoided wherever possible.

131. In order to calculate the whirling speed ω_c, it is necessary to evaluate the stiffness F of the shaft, for the given disposition of the wheel and the bearing supports.

In every case the deflection due to a load W (or force F) is given by

$$y = \frac{WL^3}{KIE} \quad \cdots\cdots\cdots \quad (37)$$

where y = deflection at point of application of the load.
L = span in inches.
I = 2nd moment of the uniform shaft about a diametrical axis.
E = Young's modulus = 30×10^6 lbs./in.^{2} for steel.
W (or F) = load in lbs.
K = coefficient depending on the values of a and b, and the nature of the end supports.

The values of K, which can be found in any text dealing with the strength of materials,[1] are for

Simply supported shaft

$$K = \frac{3L^4}{a^2b^2}$$

Shaft axially fixed

$$K = \frac{3L^6}{a^3b^3}$$

End A fixed and B supported

$$K = \frac{12L^6}{a^3b^2(3a + 4b)}$$

In the case of the stiffness or force to produce unit deflection $y = 1$, and hence by equation (37)

$$F = \frac{KIE}{L^3}$$

Inserting this value of F in equation (36) the critical speed is given by

$$\omega_c^2 = \frac{IEgK}{L^3W}$$

And the critical number of revolutions per second by

$$n_c = \frac{1}{2\pi}\sqrt{\frac{IEgK}{L^3W}} \quad \cdots\cdots \quad (38)$$

132. An alternative method of arriving at the value of the critical speed is to equate the strain energy of the shaft at the extreme position against the kinetic energy of the wheel at mid position, when the shaft is placed, as shown in Fig. 137, with its axis AB vertical.

When the shaft, thus placed, is slightly deflected and released it vibrates about the mean position with a simple harmonic motion. The mass centre has an amplitude of $2y$, which is the component along CD

[1] See Morley's "Strength of Materials," chap. vi.

of the displacement of a rotating point P, revolving in the circle of radius y with the angular velocity $\omega = 2\pi n$.

The linear velocity of P is $V = 2\pi y n$, and is obviously the maximum velocity of W as it passes through the mid position. The kinetic energy of the vibrating wheel is thus $\frac{1}{2}\frac{W}{g}V^2 = \frac{1}{2}\frac{W}{g}(2\pi y n)^2$.

In this position the strain energy is zero.

At the extreme end of the swing the K.E. of W is zero, as it has now been transformed into the strain energy of the shaft which has the value $\left(\frac{F}{2}y\right) \times y = \frac{1}{2}Fy^2$.

Hence $$\frac{1}{2}Fy^2 = \frac{1}{2}\frac{W}{g}(2\pi y n)^2$$

and $$n_c = \frac{1}{2\pi}\sqrt{\frac{Fg}{W}}$$

This is the expression deduced by the other method for the critical number of revolutions (equation (36)).

The above calculation thus shows that the critical number of revolutions is the same as the natural period of vibration of the shaft.

The importance of this method lies in the extended application of it to the case (c) of the non-uniform shaft with irregular loading.

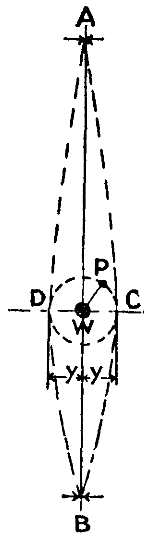


FIG. 137.

EXAMPLE 5.—The rotor of a small de Laval turbine has a wheel which weighs 1·65 lbs. By supporting the shaft in **V** blocks and loading the wheel, the stiffness obtained from the observed deflections was found to be 1108 lbs./inch deflection.

The apparent critical speed of the rotor was observed and found to be 5000 revolutions/minute. Calculate the critical speed from the above data and compare it with the observed value.

Here $W = 1·65$, $F = 1108$ lbs./in., $g = 12 \times 32·2$ in./sec.^{2}.

By equation (36) the critical speed is

$$n_c = \frac{1}{2\pi}\sqrt{\frac{Fg}{W}} = \frac{1}{6·28}\sqrt{\frac{1108 \times 386·4}{1·65}}$$
$$= 81·25 \text{ revs./sec.}$$
$$= 4875 \text{ revs./min.}$$

The calculated value is thus 97·5 per cent. of the observed value. The latter was obtained by connecting a tachometer to the driving shaft of the machine, and noting the reading when the pronounced "whirl" took place as the machine was speeded up. This whirl was easily detected by an observer placing his ear close to the wheel casing.

EXAMPLE 6.—The wheel of the Sturtevant turbine, Fig. 13, p. 22, weighs approximately 105 lbs., and the average value of the shaft diameter may be taken as $2\frac{1}{4}$ inches. The bearings are spherically seated, the span being 21 inches, and the wheel is placed centrally between them. Neglecting the effect of the shaft calculate the approximate whirling speed of the rotor.

Owing to the spherical seating it may be assumed that the shaft is simply supported at the ends.

For this case $\quad K = \dfrac{3L^4}{a^2b^2}$, and since $a = b$, $K = 48$.

Second moment of the shaft area

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64} \times 2{\cdot}25^4 = 1{\cdot}26 \text{ in.}^4$$

E for steel $= 30 \times 10^6$ lbs./in.^{2}, $\quad W = 105$ lbs., $\quad$ and $L = 21$ inches.

Hence by equation (38) the approximate value of the whirling speed is

$$\begin{aligned} n_c &= \frac{1}{2\pi}\sqrt{\frac{IEgK}{L^3W}} \\ &= \frac{1}{6{\cdot}28}\sqrt{\frac{1{\cdot}26 \times 30 \times 10^6 \times 32{\cdot}2 \times 12 \times 48}{21^3 \times 105}} \\ &= \frac{850}{6{\cdot}28} = 135 \text{ revs./sec.} = 8100 \text{ revs./min.} \end{aligned}$$

The running speed of this machine is 3000 R.P.M. or 37 per cent. of the critical speed.

The weight of the shaft is an appreciable fraction of the weight of wheel, and its effect is to decrease the above value by about 5 per cent. The method for taking account of the influence of the shaft is given in the next paragraph. If instead of supported ends, the condition of fixed ends is assumed, the value of K is quadrupled, and hence the value of the critical speed is doubled.

There is, however, an ample margin between the running and the lower critical value to ensure that there will be no vibration trouble if the wheel is properly balanced.

133. In case (*b*), where there is a uniform shaft carrying a number of wheels, a semi-empirical method due to Professor Dunkerley has to be applied.

This involves the preliminary calculation of the critical speed of the shaft without wheels, and the critical speed for each separate wheel on the assumption that it is mounted on a weightless shaft, as in the previous cases.

If n_1, n_2, n_3, etc., are the critical speeds for the successive wheels, and n_s the critical speed of the shaft alone, and N_c the critical speed of the whole rotor, then by Dunkerley's equation

$$\frac{1}{N_c^2}=\frac{1}{n_s^2}+\frac{1}{n_1^2}+\frac{1}{n_2^2}+\frac{1}{n_3^2}+\ \ldots$$

$$=\frac{1}{n_s^2}+\Sigma\frac{1}{n_c^2}\quad\ldots\ldots\ldots\quad(39)$$

It can be shown from first principles that the fundamental critical speed of the uniform shaft alone without wheels is given by

$$n_s=\frac{1}{2\pi}\sqrt{\frac{\mathrm{IE}g\mathrm{K}_s}{w\mathrm{L}^4}}$$

where w is the weight of the shaft per inch length in lbs.

The total weight of the shaft is $W_s = wL$, so that the critical number of revolutions is given by the same form of equation as that for the weightless shaft with a single wheel load W, that is

$$n_s=\frac{1}{2\pi}\sqrt{\frac{\mathrm{IE}g\mathrm{K}_s}{\mathrm{W}_s\mathrm{L}^3}}\quad\ldots\ldots\quad(40)$$

The coefficient K_s again depends on the nature of the end supports, and has the following values :—

End condition of shaft	K_s	a
Supported at each end	π^4	$0{\cdot}5\pi$
Fixed	$(\frac{3}{2}\pi)^4$	$1{\cdot}125\pi$
Fixed at one end and supported at the other	$(\frac{5}{4}\pi)^4$	$0{\cdot}78\pi$

This equation can also be written in the convenient form

$$n_s=a\sqrt{\frac{\mathrm{IE}g}{\mathrm{W}_s\mathrm{L}^3}}$$

The corresponding values of the coefficient a are given in the table above.

Equation (39) can be written in the form—

$$\frac{1}{N^2}=\frac{(2\pi)^2L^3}{\mathrm{IE}g\frac{K_s}{W_s}}+\frac{(2\pi)^2L^3}{\mathrm{IE}g\Sigma\frac{K}{W}}$$

or

$$\left.\begin{aligned}\frac{1}{N^2}&=\frac{(2\pi)^2L^3}{\mathrm{IE}g}\left(\frac{W_s}{K}+\Sigma\frac{W}{K}\right)\\&=C\left(\frac{W_s}{K}+\Sigma\frac{W}{K}\right)\end{aligned}\right\}\quad\ldots\ldots\quad(41)$$

The value of $\Sigma\frac{W}{K}$ can best be calculated by the tabular method shown on p. 251.

EXAMPLE 7.—A skeleton drawing of the shaft and disc rotor for a Curtis Rateau turbine is shown in Fig. 138.

The approximate values of the wheel weights and the distances a and b in inches, are as follows:—

Wheel.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
W (lbs.)	1371	1215	1155	1110	1070	1010	950	915	892	892	892
a (ins.)	70·25	62·25	58	53·75	49·5	45·25	41	36·75	32·5	28	23·5
b (ins.)	37·75	45·75	50	54·25	58·5	62·75	67	71·25	75·5	80	84·5

It is assumed that the stepped shaft is replaced by a uniform shaft of 10 inches diameter, which has a span of 108 inches between bearing centres, and that it is fixed in direction at the bearings.

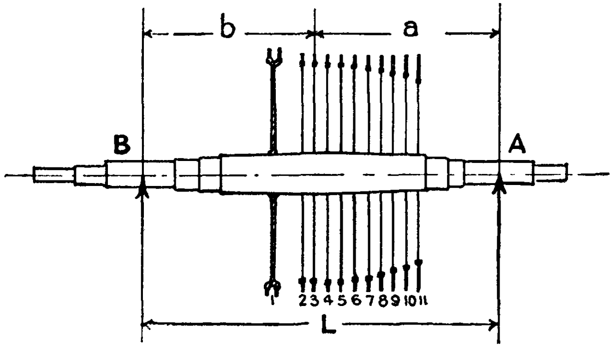


FIG. 138.

Apply Dunkerley's method to determine the critical speed of the rotor.

Here the second moment of the shaft section is

$$I = \frac{\pi}{64}d^4 = \frac{\pi \times 10^4}{64} = 490 \text{ in.}^4$$

$$L = 108'', \quad E = 30 \times 10^6 \text{ lbs./in.}^2, \quad g = 32{\cdot}2 \times 12 \text{ ins./sec.}^2$$

Hence for equation (41)

$$C = \frac{(2\pi)^2 L^3}{IEg} = \frac{(2\pi)^2 \times 108^3}{490 \times 30 \times 10^6 \times 32{\cdot}2 \times 12}$$

$$= \frac{1}{114400}$$

The values of K for the separate wheels are given by $K = \frac{3L^6}{a^3b^3}$. Thus for wheel 1

$$K = \frac{3 \times 108^6}{(70{\cdot}25)^3 \times (37{\cdot}75)^3} = 255$$

The necessary data and calculated values of $\frac{W}{K}$ are given in the accompanying table. The sum of the values in the last column gives the required value of $\Sigma\frac{W}{K}$.

No. of wheel.	*a* (inches).	*b* (inches).	K.	W (lbs.).	$\frac{W}{K}$.
1	70·25	37·75	255	1371	5·37
2	62·25	45·75	207	1215	5·87
3	58·00	50·00	195	1155	5·92
4	53·75	54·25	192	1110	5·82
5	49·50	58·50	195	1070	5·50
6	45·25	62·75	208	1010	4·85
7	41·00	67·00	230	950	4·12
8	36·75	71·25	265	915	3·46
9	32·50	75·50	324	892	2·76
10	28·00	80·00	425	892	2·10
11	23·5	84·50	608	892	1·46

$$\Sigma\frac{W}{K} = 47{\cdot}23.$$

The weight of the equivalent shaft is—

$$W_s = 0{\cdot}288 \times 108 \times 78{\cdot}54 = 2450 \text{ lbs.}$$

and
$$K_s = (\tfrac{3}{2}\pi)^4 = (4{\cdot}7)^4 = 488$$

Substituting the foregoing values in equation (41)

$$\frac{1}{N^2} = C\left(\frac{W_s}{K_s} + \Sigma\frac{W}{K}\right)$$

$$= \frac{1}{114400}\left(\frac{2450}{488} + 47{\cdot}23\right)$$

$$\therefore\ N^2 = \frac{114400}{52{\cdot}3} = 2187$$

and
$$\therefore\ N = 46{\cdot}7 \text{ revs./sec.}$$

$$= 2800 \text{ revs./min.}$$

This value is 86 per cent. in excess of the running speed of 1500 R.P.M., and leaves a liberal margin to take account of any lack of end fixture which may occur in the actual case.

The more exact value of the critical speed for the Sturtevant rotor (example 6) can also be calculated by equation (41). This is the limiting case where the number of wheels on the shaft is one.

134. There remains the case (*c*), of the drum type of rotor of irregular cross-section, with varying second moment of area and varying rate of loading.

The actual system of loading may be replaced by an equivalent series of concentrated loads; but it is out of the question to attempt to replace the drum and its attachments by an equivalent uniform shaft of equal stiffness.

The only practicable method of treating this case is to apply the principle already stated in Art. 132, and equate the total resilience of the rotor against the total kinetic energy in the mid position.

For this purpose it is necessary to know the form of the curve of deflection when the critical speed is reached. A laborious trial and error process might be adopted to find this, and at the end it would probably be found that this curve did not differ materially from that of the statical curve of deflection of the rotor under its own weight.

Generally this assumption is made, and the curve of statical deflection is taken as a first approximation to the actual curve when whirling commences.

If at any point in the length of the rotor, the deflection given by the curve is y, and W the substituted load at the point, the resilience is $\frac{1}{2}Wy$. For the whole rotor the total resilience, in the extreme position defined by the curve, is $\frac{1}{2}\Sigma Wy$.

Similarly the total kinetic energy at the mean (or straight) position is $\frac{1}{2}\Sigma\frac{W}{g}\omega^2 y^2$. Equating these energies

$$\Sigma Wy = \frac{\omega_c^2}{g}\Sigma Wy^2$$

and hence the critical number of revolutions is given approximately by

$$n_c = \frac{1}{2\pi}\sqrt{\frac{g\Sigma Wy}{\Sigma Wy^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

This method involves a good deal of graphical work in the determination of the curve of statical deflection. In determining this curve, the labour is somewhat reduced by the use of the force and funicular polygons.[1] This construction, although quite simple, sometimes proves troublesome on account of the difficulty in determining the scales of the derived diagrams. In order to systemise the procedure for the general case and show the relations between the scales explicitly, the disc and drum rotor of the turbine shown in Fig. 42 is chosen to illustrate this system of calculation.

EXAMPLE 8.—Draw the curve of statical deflection for the disc and drum rotor, Fig. 139 (*a*), by the force and funicular polygon method, and from this, by the method of Art. 134, calculate the approximate value of the critical speed. It may be assumed that the rotor is simply supported at the journals on a span of 7 feet 6 inches.

In a case of this kind the calculation may be carried out in the following order:—

I. Divide the span between the bearings into a number of sections, or lengths, over which the rotor has a uniform cross-section, calculate the load per inch run over each section and plot the load curve. In this case the stepped load curve obtained by the division of the rotor

[1] For the method of construction of these polygons, see any work on Graphical Statics.

shown by the verticals through (*a*) is shown at (*b*). The area of the cross-hatched portion between any two verticals gives the total load on the corresponding length of rotor. It is hardly necessary to find these areas as the loads are all calculable from the rates of loading. If, however, the area is used the load scale is obtained as follows :—

Let length scale be $1'' = n$ inches.
load „ $1'' = p$ lbs./inch.
Then area scale is $1 \text{ in.}^2 = np$ lbs.

II. Replace the distributed section loads by concentrated loads acting through the centres of the areas. Draw the verticals and letter the spaces in the usual way for the construction of the force and funicular polygons, as shown below diagram (*b*). Draw the load line AK as shown at (*f*). Fix a suitable polar distance S, locate the pole O arbitrarily, and draw the rays OA, OB, OC, etc., on the force polygon at (*f*) for the linkage of the funicular polygon at (*c*). Construct this polygon by drawing parallels to the rays through the correspondingly lettered spaces, that is, a parallel to OA through space A, a parallel to OB through space B, and so on. Close the polygon (*c*) by the line XX.[1]

This polygon, as can easily be proved, is the bending moment diagram of the rotor under the replaced system of concentrated loads.

It is at this point that trouble may arise with the scale of this bending moment diagram.

It can be easily avoided if the following simple relation is borne in mind:—

Let the length scale be $1'' = n$ inches (full size).
„ force „ $1'' = m$ lbs.
„ polar distance (chosen) = S inches.
Then $1'' = nm\text{S}$ inch-lb., the bending moment scale.

In the case illustrated the length scale chosen is

$$1'' = 15\cdot25 \text{ inches (on rotor).}$$

Force scale $1'' = 1000$ lbs.
Polar distance = 2 inches (on diagram).
Then moment scale is $1'' = 15\cdot25 \times 1000 \times 2 = 30{,}500$ in./lbs.

It is shown in works on the Strength of Materials that the area of the bending moment curve between any two points of a beam of uniform cross-section, when divided by EI, gives the change of slope between the two points. E is the Young's modulus for the material, and I is the second moment of the section area about an axis through the centroid of the section.

The areas of the diagram (C) can thus be reduced to equivalent "slope" areas.

These new areas may then be regarded as a new equivalent system of loads. Their values on the diagram are represented by ϕ radians.

[1] In this case XX is horizontal. For any other arbitrarily chosen position of the pole O in (*f*) it would be inclined to the horizontal.

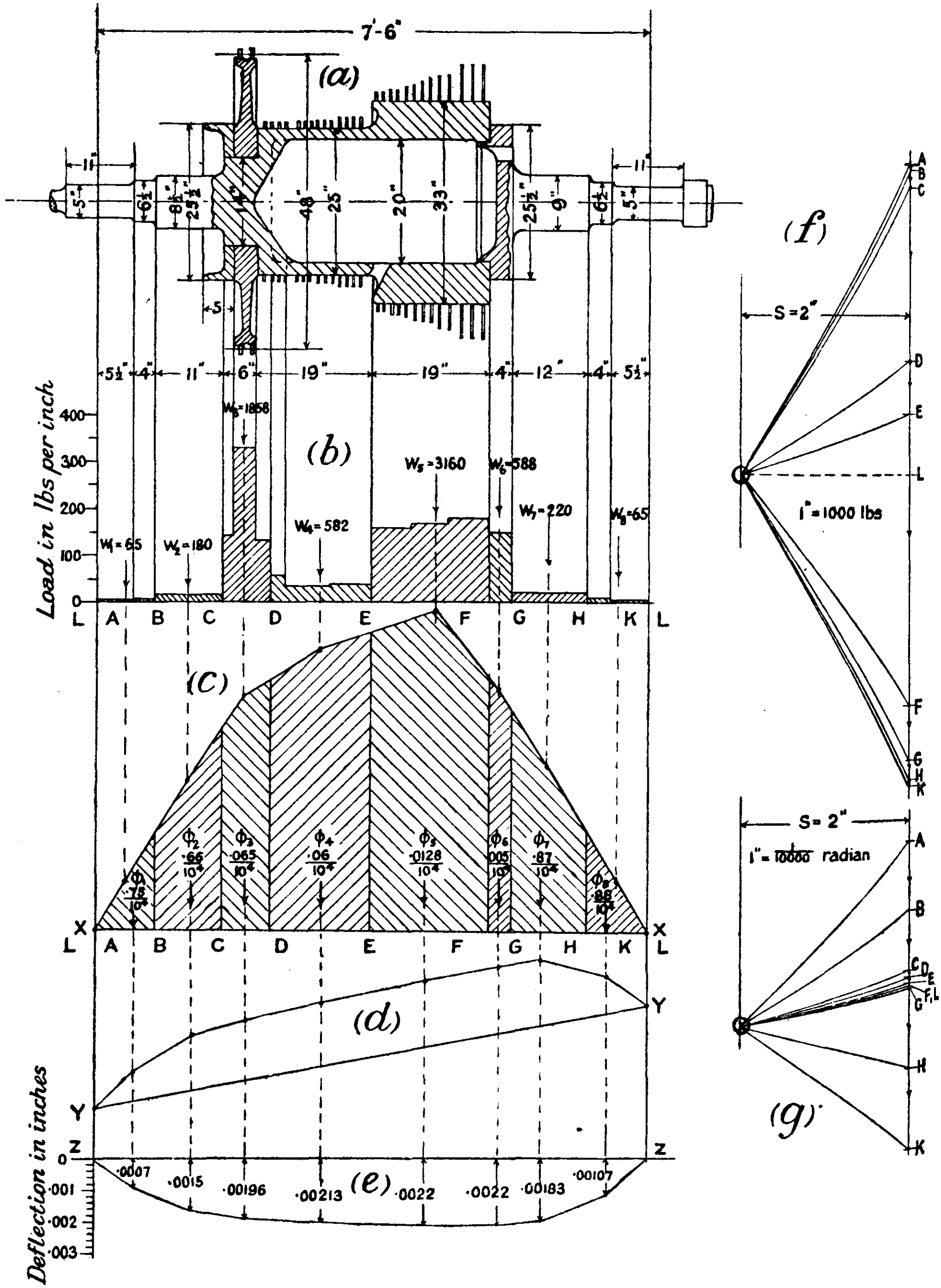


FIG. 139.
This diagram is reduced to about half the size of the original.

To find the slope or radian scale

Let the moment scale be $1'' = q$ inch-lb.

„ length „ $1'' = n$ inches.

Then 1 in.^{2} of B.M. scale $= qn$ in.^{2} lb. $= \dfrac{qn}{IE}$ radians

Since in this case $q = 30{,}500$, $n = 15{\cdot}25$, and $E = 30 \times 10^6$

$$1 \text{ in.}^2 \text{ B.M. diagram} = \frac{30500 \times 15{\cdot}25}{30 \times 10^6 I} = \frac{0{\cdot}0154}{I} \text{ radians}$$

In order to obtain the imaginary loads the value of I has to be found for each section of the rotor.

For solid sections $I = \dfrac{\pi}{64}D^4$; for hollow sections $I = \dfrac{\pi}{64}(D^4 - d^4)$, where D and d are external and internal diameters.

These have been figured for the above case, and are entered in the accompanying table of data.

Where there are attachments such as flanges, dummies, or wheels an exercise of judgment is necessary in calculating the value of I where these occur.

For instance, in the above rotor the shaft enlarges to 14 inches where the wheel and dummy are attached. It is joined to the drum by a slightly coned end. It is difficult to say what section should be taken here for the calculation of I.

The wheel also may help to stiffen the shaft.

To make allowance for the conditions existing at this part an equivalent diameter of 16 inches has been taken, and I calculated on this assumption.

III. When the I values are fixed find the areas of the B.M. curve, in inch^2, multiply each value by $\dfrac{0{\cdot}0154}{I}$, and tabulate the result. Through the centre of the areas draw the (imaginary) load verticals, and again letter the spaces below (c).

Draw the imaginary load line AK in diagram (g) to any convenient scale of radians, choose a polar distance S, and complete the vector polygon. From this derive the funicular polygon (d), closing it by the line YY. This is the deflection diagram.

In order to obtain the true form of the deflection curve plot the intercepts of the imaginary load verticals in (d) on a horizontal base (zz), and obtain the true deflection diagram (e). In order to determine the deflection scale, let $1'' = r$ radian.

$1'' = n$ inches.

Polar distance = S inches.

Then the deflection scale is $1'' = rns$ inches deflection.

In this case $1'' = \frac{1}{10000}$ radian.

$1'' = 15{\cdot}25$ inches (on rotor).

$S = 2$ inches (on diagram).

$$\therefore\ 1'' = \frac{2 \times 15{\cdot}25}{10000} = 0{\cdot}00305 \text{ inch deflection}$$

The deflections (y), obtained by scaling diagram (e) on the shaft-load verticals, are given in the fourth column of the data table.

No. of load.	I in.^{4}	$\phi \times 10^4$ radians.	Deflection y ins.	Load W lbs.	Products.	
					Wy.	Wy^2.
1	63·53	0·75	0·00065	65	0·0422	0·0000275
2	256	0·66	0·0015	180	0·2700	0·0004050
3	3216	0·065	0·00196	1858	3·640	0·0071200
4	8432	0·060	0·00213	582	1·240	0·0026400
5	50222	0·0128	0·0022	3160	6·952	0·0153000
6	19160	0·00505	0·0022	588	1·293	0·0028600
7	256	0·87	0·0018	220	0·396	0·0007120
8	63·53	0·88	0·0009	65	0·059	0·0000526

$\Sigma Wy = 13{\cdot}892, \quad \Sigma Wy^2 = 0{\cdot}0291.$

Substituting in equation (42) the critical number of revolutions

$$n_c = \frac{1}{2\pi}\sqrt{\frac{g\Sigma Wy}{\Sigma Wy^2}}$$

$$= \frac{1}{6{\cdot}28}\sqrt{\frac{13{\cdot}892 \times 32{\cdot}2 \times 12}{0{\cdot}0291}}$$

$$= \frac{430}{6{\cdot}28} = 68{\cdot}5 \text{ revs./sec.}$$

$$= 4100 \text{ revs./min.}$$

The rated speed of the turbine is 2000 R.P.M., so that the critical speed is at least double the running speed on the assumption of simply supported ends. If the ends are in any way constrained, the critical value will be higher still. There is thus an ample margin, and if the rotor is carefully balanced, there should be no trouble with vibration.

It will be noted that for this case the maximum value of the statical deflection obtained from the curve is 0·0022 inch. The deflections obtained with drum rotors vary from 0·001 to 0·005 inch.

In the disc type of rotor they are much greater, varying between 0·005 and 0·03 inch, reckoned on the condition of simply supported ends. For fixed ends these values may be halved.

In the foregoing discussion of critical speed it has been assumed that in the case of the shaft and disc rotor the various wheels move backward and forward in line as the shaft vibrates. This condition is not quite fulfilled in the case of wheels near the ends, which, owing to the bending of the shaft, also vibrate through a small angle. The "rotational inertia" of these wheels is thereby increased, and in consequence the critical speed is slightly reduced.

For practical purposes the correction is usually small, and as it involves a considerable amount of extra calculation, it has been neglected.

The mathematical treatment of this subject and the method of applying the approximate correction for rotational inertia is given by Professor Morley in the article on "Calculation of Vibration and Whirling Speeds," and by Wm. Kerr in the articles on the "Whirling Speeds of Loaded Shafts," already cited.

135. Rotor Shafts.—The shafting of rotors does not call for extended comment. It is usually made of mild steel. The diameter is fixed more with regard to stiffness than strength.

With the limits of deflection just quoted, the skin stress due to both torque and bending is usually comparatively low. In any case the maximum stress value can be calculated by using the Rankine form of the equation for combined bending and twisting moment. Thus if M = bending moment at the section considered, T = torque, the equivalent bending moment is

$$M_e = \frac{M}{2} + \tfrac{1}{2}\sqrt{M^2 + T^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

If Guest's Law is preferred to Rankine's, the equivalent twisting moment is given by

$$T_e = \sqrt{M^2 + T^2}$$

In either case the stress of tension or shear is given by

$$f_t = \frac{M}{z} \quad \text{or} \quad f_s = \frac{T}{z}$$

where z is the corresponding modulus of section.

The maximum value of f, which occurs at the journals in most impulse machines, should not exceed 5000 lbs./in.^{2} It has to be borne in mind that this stress is of a rapidly alternating kind. These may be slight out-of-balance loads, also accidental causes of shock, such as water carried over to the nozzles, to be provided against. A high factor of safety is therefore desirable.

A maximum value of 5000 lbs./in.^{2}, with mild steel, having an elastic strength of 40,000 lbs./in.^{2}, gives a factor of safety of about 8. Reckoned in the ultimate strength it is about 12.

The drum rotor is usually run on spherically seated bearings so that the ends are simply supported. The maximum bending moment thus occurs at the middle of the drum. The disc and shaft rotor, to keep the stiffness as great as possible, is run with "fixed ends," except in the case of the small machine, where a short span is required. The maximum bending moment thus occurs at the journals, and it is desirable to check this for any given case. The journal diameter, however, is also fixed with reference to the permissible surface velocity. It is advisable to keep the velocity below 60 ft./sec., although on some high-speed machines of small and medium power the speed rises to 70 or 80 ft./sec.

In marine turbines the surface velocity is uniformly lower than that of the land type. It varies from 15 to 30 ft./sec. In the land turbine the shaft is usually solid. In large marine turbines, as already seen from the illustrations in Chap. III., the shaft is made hollow, the internal diameter varying from 0·4 to 0·6 of the external.

EXAMPLE 9.—Assuming the condition of fixed ends, estimate a suitable journal diameter for the shaft of the Curtis Rateau turbine, for which the critical speed has been calculated in example 7.

For the purpose of this calculation assume that the minimum span between the journal necks is 88 inches, and that the shaft is subjected to a total load of 6 tons uniformly distributed, also that the maximum speed of the low-pressure journal which transmits the whole torque is not to exceed 60 ft./sec. The normal output at the generator is to be taken as 5000 K.W., the generator efficiency as 95 per cent., and the normal speed 1500 R.P.M.

Taking the surface velocity of the journal as V = 60, the corresponding diameter is

$$d = \frac{60\mathrm{V}}{\mathrm{N}\pi} = \frac{60 \times 60}{1500\pi} = 0{\cdot}765 \text{ ft.} \doteqdot 9 \text{ inches}$$

The maximum stress has now to be calculated to ascertain if it is within the limits specified above.

H.P. transmitted by the L.P. journal, is

$$\frac{5000 \times 1{\cdot}34}{0{\cdot}95} = 7000 \text{ say}$$

Then torque

$$\mathrm{T} = \frac{33000 \text{ H.P.}}{2\pi\mathrm{N}} = \frac{33000 \times 7000}{6{\cdot}28 \times 1500} = 24600 \text{ ft.-lbs.} = 11 \text{ ft.-tons}$$

For completely fixed ends the maximum B.M. occurs at the journal neck, and has the value

$$\mathrm{M} = \frac{\mathrm{WL}}{12} = \frac{6 \times 88}{12 \times 12} = 3{\cdot}66 \text{ ft.-tons}$$

By equation (43) the equivalent B.M.

$$\mathrm{M}_e = \frac{3{\cdot}66}{2} + \tfrac{1}{2}\sqrt{3{\cdot}66^2 + 11^2}$$
$$= 1{\cdot}83 + \tfrac{1}{2} \times 11{\cdot}6 = 7{\cdot}65 \text{ ft.-tons}$$

The second moment of the journal section about a diameter is

$$\mathrm{I} = \frac{\pi}{64} \times 9^4 = 322 \text{ in.}^4 \quad \therefore\ z = \frac{\mathrm{I}}{\frac{d}{2}} = \frac{322}{4{\cdot}5} = 71{\cdot}5$$

The equivalent stress on the journal is thus

$$f = \frac{7{\cdot}65 \times 12}{71{\cdot}5} = 1{\cdot}28 \text{ ton/in.}^2 = 2880 \text{ lbs./in.}^2$$

If the equivalent twisting moment is used instead of the bending moment (according to Guest)

$$\begin{aligned} T_e &= \sqrt{M^2 + T^2} \\ &= \sqrt{3{\cdot}66^2 + 11^2} = 11{\cdot}6 \text{ ft.-tons} \end{aligned}$$

and $$T_e = \frac{\pi}{16} f D^3$$

$$\therefore f = \frac{16 \times 11{\cdot}6 \times 12}{\pi \times 729} = 0{\cdot}978 \text{ ton/in.}^2 = 2200 \text{ lbs./in.}^2$$

In any case the size of the journal is ample for strength conditions. Actually the condition of fixed ends is never fulfilled, as there has necessarily to be a certain amount of slackness of the shaft in the bearings. If the other extreme is assumed, the condition of supported ends causes a maximum bending moment at the centre, which is $1\frac{1}{2}$ times the value at the journals with fixed ends.

The twist at the centre is much less than at the L.P. journal, as only about half the total torque is being transmitted by this section, because the steam has only imparted about half its energy to the rotor. The size of the shaft need not, therefore, be much greater than that at the journals. For the purpose, however, of the fixture and removal of the disc wheels, and also to allow for the external packing glands, the shaft is usually tapered from the journals to the centre, as shown in Figs. 19, 20, 23. The mid diameter in this case would be from 12 to 13 inches.

As regards the probable deflection, this may be roughly approximated by taking an equivalent uniform shaft, say 10 inches diameter. The value for fixed ends is

$$y = \frac{1}{384}\frac{WL^3}{EI} = \frac{6 \times 2240 \times 88^3}{384 \times 30 \times 10^6 \times 490} = 0{\cdot}00162 \text{ inch}$$

If the condition of supported ends is taken, the deflection at the centre is five times this amount, or $y_1 = 5y = 0{\cdot}0081$ inch. The actual deflection would be probably somewhere between these limits, say 0·002 or 0·003 inch.

136. Shaft Couplings.—For large and medium-sized machines driving electric generators directly, the Parsons type of flexible coupling is generally used to connect the turbine and generator shafts. This type is shown clearly, partly in section and partly in full elevation, on the Westinghouse machines (Fig. 23), and also on the Brush machines (Figs. 25, 42, and 43).

Each half of the coupling consists of two parts. The inner one or sleeve is keyed firmly to the shaft, and has the flange notched, so as to form a series of claws or teeth. The outer one, which has two flanges, is provided with a similar set of claws on one of the flanges. These fit into the spaces between the claws of the fixed sleeve. The two shafts are flexibly connected by bolting the outer sleeves together by means of the plain flanges.

Any want of alignment of the shafts is provided for by the freedom given by the claws. By giving these a certain amount of end play provision is made for the expansion of the rotor and adjustment of axial clearances.

Some machines have been fitted with a flexible coupling consisting of a series of small steel rods fixed on the coupling flanges parallel to the shaft axis, and near the outer circumferences of the flanges. The flexure of these rods provides the necessary play.

In a case of this kind stops are fitted on the flanges to provide against any sudden overload, which might overstrain and permanently damage the rod connections. This type is now practically superseded by the elastic claw coupling.

A simple arrangement is usually employed in the smaller sizes of machine.

The turbine shaft and the driven shaft are fitted each with one-half of an ordinary flange coupling.

The coupling bolt holes in the flange of the driven shaft are enlarged and fitted with rubber bushings.

The coupling bolts are screwed into the other flange on the driving shaft, and project through the rubber bushings in the driven shaft flange.

The bushing gives the necessary amount of elasticity to provide against want of alignment between the shafts, and also permits of any end play that may be necessary.

The connection between the rotor and tunnel shafting, in the case of a directly connected marine turbine, is usually made by a "ring coupling." The flange on the turbine shaft is recessed to take the end of the driven shaft. This shaft has a groove, cut near the end, and a ring, in halves, is placed in the groove. The recess in the turbine flange is enlarged to take this ring. The flanges are then bolted together in the ordinary way, but the faces are kept clear of each other, and the pressure comes on the ring, through which the thrust when "going ahead" and pull when "going astern" are transmitted.

A modification of the flexible or floating coupling is fitted between the turbine and pinion shafts in geared marine turbines, to keep the pinions in proper alignment with the gear wheel (see Figs. 29 and 30).

137. End Thrust on Rotor of Reaction Turbine.—The axial force in an impulse turbine due to the inequality of the axial component of the steam velocity at entrance to and exit from the blade channels is always small, and is fully provided for by the block which has to be fitted for the adjustment of axial clearance.

The dynamical force on the blading of a reaction drum is also a small quantity. There is, however, a thrust of considerable magnitude due to the cumulative effect of the difference of pressure at each moving ring, and the pressure on the annular surfaces where the drum is stepped up.

In order to balance the total thrust, either a thrust block or a series of balance pistons or "dummies" may be used. The general practice at present is to employ the balance pistons.

Consider a group of reaction blades having the same height throughout, and hence the same mean ring diameter. The statical axial force at any ring in the direction of flow is the difference between the entrance and exit pressures multiplied by the annular area between rotor and casing.

The total thrust in the whole group is the sum of these forces. The whole drop of pressure between entrance to and exit from the group is distributed between the fixed and moving rings, and it may be assumed without sensible error that the force on the moving rings is half the total amount.

Let p_1 = pressure at entrance to the group.
p_0 = " exit from the group.
a = annular area between drum and casing.

Then the positive statical blade thrust is

$$T_s = \tfrac{1}{2}(p_1 - p_0)a$$

The axial velocity at any ring is the steam volume divided by the annular area. This quantity, divided by g, gives the force per lb. of steam flowing. The sum of these forces gives the total force distributed over the group, and half of it again may be taken by the moving blades.

Hence, if W = lbs. of steam flowing per sec.,
v_1 = steam volume at entrance to the group,
v_0 = " " exit from the group,

the negative dynamic thrust is

$$T_d = \frac{1}{2}\frac{W}{g}\left(\frac{v_0 - v_1}{a}\right)$$

The total thrust on the group is thus

$$T = \tfrac{1}{2}(p_1 - p_0)a - \frac{W}{2g}\left(\frac{v_0 - v_1}{a}\right)$$

The dynamical thrust is usually less than 1 per cent., and does not exceed 2 per cent. of the total, and for practical purposes may be neglected.

If desired, in an exact calculation of dummy piston diameter, it can be allowed for by subtracting from 1 to 2 per cent. from the statical thrust.

In order to determine the dummy piston diameters, the equation of equilibrium for rotor and dummy thrusts has to be found. The method is to calculate the successive blade thrusts, and the thrust on the annular surface at the beginning of the drum section considered, and equate these against the dummy thrust under the action of the steam at the beginning of the section, the pressure being transmitted through the equalising passages already shown and described in Chap. III.

At any section of a stepped drum let a_1, a_2, a_3, etc., be the successive annular areas of the blade groups, and p_1, p_2, p_3, etc., the corresponding inlet pressures.

Let d = diameter of the drum at the previous section.
d_1 = " " at the section considered.
D_1 = " the dummy piston.
D = " dummy piston of previous section.

Then annular area at the step of the drum is

$$\frac{\pi}{4}(d_1^2 - d^2) = A_1$$

Annular area of the dummy piston face

$$\frac{\pi}{4}(D_1^2 - D^2) = a$$

Neglecting the small dynamical thrust, the approximate steam thrust on the blades and annular neck is

$$p_1A_1 + \tfrac{1}{2}\{(p_1 - p_2)a_1 + (p_2 - p_3)a_2 + (p_3 - p_4)a_3 \ldots + (p_n - p_{n+1})a_n\}$$

The dummy pull, which balances this, is

$$(p_1 - p_c)a$$

where p_c is the condenser pressure.

Hence

$$a = \frac{p_1A_1}{p_1 - p_c} + \frac{1}{2(p_1 - p_c)}\{(p_1 - p_2)a_1 + (p_2 - p_3)a_2 + (p_3 - p_4)a_3 \ldots + (p_n - p_{n+1})a_n\} \quad . \quad . \quad (44)$$

EXAMPLE 10.—The diameter of the high-pressure section of a three-stepped drum (similar to that of Fig. 25, Art. 24) is $23\frac{5}{8}$ inches, and the diameter of the I.P. section is $33\frac{3}{8}$ inches. The diameter of the high-pressure dummy is $25\frac{5}{8}$ inches. There are three groups of blades in the I.P. section, the heights being 1, $1\frac{3}{8}$, and $1\frac{7}{8}$ inches. The pressures are 68, 45, 30, and 19 lbs./in.^{2} abs., and the condenser pressure 1 lb./in.^{2} abs.

Calculate the necessary diameter of the I.P. dummy piston.

Here $$A_1 = \frac{\pi}{4}(33{\cdot}375^2 - 23{\cdot}625^2) = 436{\cdot}5 \text{ in.}^2$$

For the first expansion $$a_1 = \frac{\pi}{4}(35{\cdot}375^2 - 33{\cdot}375^2) = 108 \text{ in.}^2$$

" second " $$a_2 = \frac{\pi}{4}(36{\cdot}125^2 - 33{\cdot}375^2) = 150{\cdot}15 \text{ in.}^2$$

" third " $$a_3 = \frac{\pi}{4}(37{\cdot}125^2 - 33{\cdot}375^2) = 207{\cdot}65 \text{ in.}^2$$

By equation (44) the dummy area is

$$a = \frac{68 \times 436 \cdot 5}{67} + \frac{1}{2 \times 67}\{(68 - 45)108 + (45 - 30)150 \cdot 15 + (30 - 19)207 \cdot 65\}$$

$$= 443 \cdot 0 + (18 \cdot 5 + 16 \cdot 8 + 17)$$

$$= 495 \text{ in.}^2$$

Thus
$$D_1^2 = D^2 + \frac{4a}{\pi} = 25 \cdot 625^2 + \frac{495 \times 4}{3 \cdot 1416}$$

$$= 657 + 630 = 1287$$

$$\therefore\ D_1 \doteqdot 35\tfrac{7}{8} \text{ inches}$$

This is practically the same as the mean external diameter of the blading on the I.P. drum. Any slight residual thrust would be taken up by the clearance adjusting block. Usually this calculation may be omitted, and the mean external diameter taken for the dummy diameter.

138. In the case of the directly coupled marine turbine the shaft is subjected to the forward thrust of the propeller when the vessel "goes ahead."

This thrust, as in the case of the reciprocating engine, may be either completely taken up by a thrust block or partly by a block and partly by the rotor itself.

The standard practice, with directly coupled turbines, is to take the major part of the propeller thrust on the rotor. The total astern steam thrust is always less under full speed conditions than the forward propeller thrust, and in order to more nearly equalise the two the steam thrust on the blades is augmented by making the dummy piston diameter less than the drum diameter. The initial pressure of the steam, acting on the annular area so produced, increases the astern thrust. This area is so adjusted that the combined blade and dummy thrust balances the propeller thrust at full load.

The steam thrust at reduced load is less than the propeller thrust if it is just equal to it at full load.

To allow for this deficiency the dummy piston can, if desired, be proportioned to give, say, 10 per cent. excess thrust, the difference being taken by the small thrust block which has always to be fitted for the adjustment and maintenance of the axial dummy and blade clearances.

When the turbine is of the impulse type the rotor thrust is balanced by the steam pressure (usually about atmospheric) acting on the forward end of the drum section, which is made with a closed end. In some recent pressure compounded impulse machines, Rateau and Zoelly type,[1] the drum has been stepped, and the total astern thrust is due to the pressures on the annular areas thus produced.

In the case of the marine reaction drum, it is necessary first to determine the propeller thrust from the power and speed of the vessel.

[1] Illustrations will be found in Reed's "Steam Turbine."

Let T_p = effective thrust of the propeller in lbs.
E_s = shaft horse power.
S = speed of the vessel in knots (= 101·33S ft./min.).
η_p = efficiency of the propeller.

Then $$T_p = \frac{\eta_p E_s 33000}{S \times 101{\cdot}33} = \frac{325{\cdot}7\eta_p E_s}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

The efficiency η_p may vary from 0·5 to 0·6.

Referring to Fig. 140, consider the case of a drum with blading in four lengths, a_1, a_2, a_3, and a_4, being the annular areas in square inches between drum and casing, and p_1, p_2, p_3, p_4, the initial pressures, and p_5 the exhaust pressure in lbs./in.^{2}

The interior and ends of the drum are always open to the exhaust

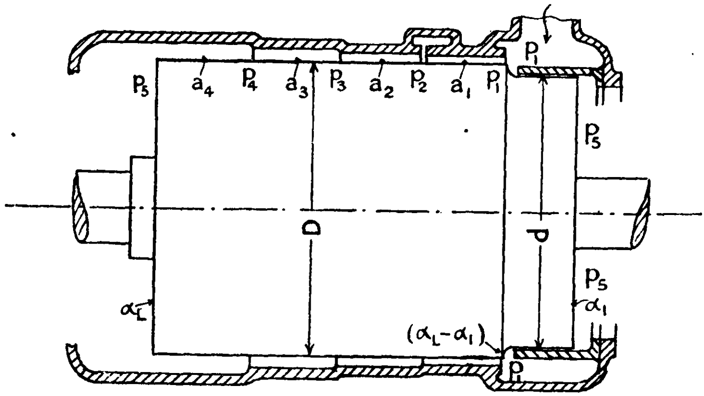


FIG. 140.

pressure p_5. If a_L is the total effective drum end area, and a_1 the end area of the dummy piston, the total forward thrust is

$$T_p + p_5 a_L$$

and the approximate total astern thrust where $(a_L - a_1)$ is the a nnular area of the dummy shoulder is

$$p_5 a_1 + p_1(a_L - a_1) + \tfrac{1}{2}(p_1 - p_2)a_1 + \tfrac{1}{2}(p_2 - p_3)a_2 + \tfrac{1}{2}(p_3 - p_4)a_3 + \tfrac{1}{2}(p_4 - p_5)a_4$$

Equating these and transposing terms

$$(a_L - a_1)(p_1 - p_5) = T_p - \tfrac{1}{2}\{(p_1 - p_2)a_1 + (p_2 - p_3)a_2 + (p_3 - p_4)a_3 + (p_4 - p_5)a_4\} . \quad (46)$$

Let D = diameter of the drum in inches.
d = „ „ dummy „

Then $$\frac{\pi}{4}(D^2 - d^2) = (a_L - a_1)$$

and $$d^2 = D^2 - \frac{4}{\pi}(a_L - a_1) \quad . \quad . \quad . \quad . \quad (47)$$

EXAMPLE 11.—The drum of a high-pressure marine turbine is $76\frac{3}{8}$ inches diameter, and the blading is in four lengths, $1\frac{5}{8}$, $2\frac{3}{8}$, $3\frac{1}{4}$, and $4\frac{5}{8}$ inches. The pressures are 170, 100, 55, 32, and the exhaust pressure 19 lbs./in.^{2}

The shaft output at a speed of 18 knots is 9000, and the propeller efficiency may be taken as 55 per cent. Calculate the diameter of the dummy piston, allowing for 10 per cent. excess thrust.

Calculating the annular blade areas as in the previous example, these are

a_1	a_2	a_3	a_4	
398·2	587·6	813	1177	in.^{2}

By equation (45) the propeller thrust is

$$T_p = \frac{325{\cdot}7 \times 0{\cdot}55 \times 9000}{18} = 89500 \text{ lbs.}$$

Substituting in equation (46) and allowing for 10 per cent. increase on thrust

$$(a_L - a_1)(170 - 19) = 1{\cdot}1 \times 89500 - \tfrac{1}{2}\{398{\cdot}2 \times 70 + 587{\cdot}6 \times 45 + 813 \times 23 + 1177 \times 13\}$$

Hence $$(a_L - a_1) = \frac{54292}{151} = 358 \text{ in.}^2$$

Substituting in equation (47)

$$\begin{aligned} d^2 &= 76{\cdot}375^2 - \frac{4 \times 358}{3{\cdot}1416} \\ &= 5833 - 456 \\ &= 5377 \\ \therefore\ d &= 73{\cdot}32, \text{ say } 73\tfrac{3}{8} \text{ inches.} \end{aligned}$$

139. Shaft Horse Power of Marine Turbine.—The brake horse power or power transmitted to the shaft by a land turbine of small output can be determined by some form of dynamometer, and that of a turbine of moderate or large output by coupling the machine to an electric generator of known efficiency.

The power transmitted from the turbine by marine shafting cannot be found in this direct manner. It can be, at the best, only approximated by observing the elastic deformation of the shaft due to the driving torque.

By the law of elasticity the twist of the shaft is proportional to the torque producing it, hence by measuring the angular displacement of two sections of the shaft at a given distance apart, the torque, and therefore the power transmitted at a given speed, can be estimated.

The angle of twist is measured by one or other of several forms of torsion meter which are at present on the market. It is not necessary to discuss the details of these apparatus here. They may be generally divided into three classes—

(*a*) Mechanical torsion meter which gives a magnified record of the arc of displacement at the shaft surface, and can be used to obtain an autographic record of the variation of torque in each revolution. The representative of this class is the Fottinger meter.[1]

(*b*) Optical meter, by which the angle of torque is measured by the deflection of a ray of light. The Bevis-Gibson and Hopkinson meters are representative of this class.[2]

(*c*) Electrical meter represented by the Denny Johnstone meter, which is probably the most extensively used in marine practice.[3]

In this instrument a rotating magnet attached to each end of the shaft length is made to traverse a series of inductor coils setting up a current in a circuit containing a telephone receiver. The two currents from the inductors are in opposition, and can be equalized by means of an adjustable resistance. When equal there is no sound from the receiver as the shaft rotates idly. When power is transmitted one of the magnets, owing to the twist, gets in advance of the other, and excitation of the coils takes place earlier than at the other inductor. This produces a "click" in the receiver. A further adjustment of the resistance is made till silence is restored in the receiver. The instrument is so calibrated that the equivalent of this in terms of the arc of twist at the inductor radius can be read off. This radius in the instructions issued with this instrument is called the "inductor constant" C.

140. Assuming that an efficient torsion meter is provided and the angle of twist θ on a length of shaft L is reasonably approximated to by this instrument, the expression of the shaft horse power transmitted is deduced as follows:—

Referring to Fig. 141, A and B are two discs, each of radius R

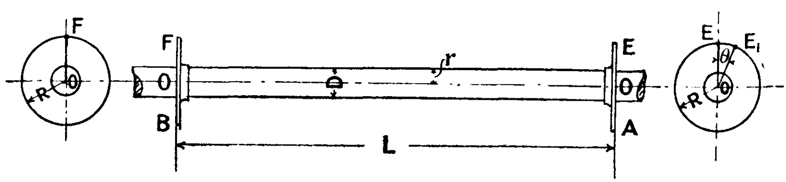


FIG. 141.

inches, fixed on the shaft L inches apart. The end points of the radii OE and OF, when the shaft rotates idly, remain in the same plane.

When the shaft transmits power let the deflection of OE relatively to OF be θ radians, the radius OE taking the position OE_1. The corresponding arc EE_1 at the radius R is x inches, and hence $\theta = \frac{x}{R}$. If f_s is the skin shear stress produced in the shaft of diameter

[1] See description in Paper on "Development and Present Status of Steam Turbines," by E. M. Speakman, *Trans. Inst. Engineers and Shipbuilders in Scotland*, 1907.

[2] See description of Bevis-Gibson Flash Light Torsion Meter, *N.E. Coast Inst. of Engineers and Shipbuilders*, 1908.

[3] An illustrated description was given in *Engineering*, April 7, 1905.

D inches, by a torque T, and J is the polar moment of the shaft section, then

$$\frac{T}{J} = \frac{f_s}{\frac{D}{2}}. \quad \text{Also the rigidity modulus is } G = \frac{f_s}{\frac{r\theta}{L}} = \frac{2f_sL}{\theta D}$$

where r = shaft radius $= \frac{D}{2}$.

Hence $G = \frac{TL}{J\theta}$

For a solid shaft, $J = \frac{\pi}{32}D^4$

" hollow " $J = \frac{\pi}{32}(D^4 - d^4)$

where d is the internal diameter.

$$\therefore\ G = \frac{32}{\pi}\frac{TL}{D^4\theta} \quad \text{or} \quad \frac{\frac{32}{\pi}TL}{(D^4 - d^4)\theta}$$

and the torque in ft.-lbs. is given by

Solid shaft. Hollow shaft.

$$T = \frac{\pi}{32} \times \frac{Gd^4\theta}{12L}\ \text{ft.-lbs.} \qquad T = \frac{\pi}{32} \times \frac{G(D^4 - d^4)\theta}{12L}\ \text{ft.-lbs.}$$

The modulus of rigidity varies slightly for different shafts, and it is desirable that the actual torque for any given angle of twist should be experimentally determined before a shaft is put in the vessel. A torque-angle diagram can thus be plotted and the torque read directly from it for any recorded value of the angle obtained under service conditions.

Failing any such preliminary adjustment the value of G for the calculation of the torque may be taken as 11,760,000 lbs./in.^{2}

If the shafting is run at N revolutions per minute the power transmitted is

$$\text{S.H.P.} = \frac{2\pi NT}{33000} \quad \ldots\ldots\ldots \quad (48)$$

The angle of twist is usually measured in degrees.

If D = external diameter in inches,
d = internal " "
L = length in feet,
θ = angle of twist in degrees,
$G = 11{\cdot}76 \times 10^6$ lbs./in.^{2},

the value of the torque in ft.-lbs. reduces to

Solid shaft. Hollow shaft.

$$T = \frac{140D^4\theta}{L} \quad \text{or} \quad T = 140\frac{(D^4 - d^4)\theta}{L} \quad \ldots \quad (49)$$

If the reading obtained from the particular meter used gives the value of the arc of twist x at radius R, then θ degrees $= \dfrac{57 \cdot 3x}{R}$.

Substituting this value in equation (48) the horse power is given by

$$\text{Solid shaft.} \qquad\qquad\qquad \text{Hollow shaft.}$$

$$\text{S.H.P.} = \frac{1 \cdot 53 D^4 N x}{RL} \quad \text{or} \quad \text{S.H.P.} = \frac{1 \cdot 53(D^4 - d^4)Nx}{RL} \quad . \quad (50)$$

In the case of the Denny Johnstone meter the inductor constant C, the value of which is marked on the instrument, is substituted for R.

EXAMPLE 12.—Each (solid) tunnel-shaft of a three-shaft compound turbine installation is $6\frac{3}{4}$ inches diameter. The recorded revolutions are H.P. (centre) 600, Port L.P. 655, Starboard L.P. 650. If a Denny Johnstone meter records for each shaft an arc of twist $x = 0 \cdot 68$ inch in a length of 36 feet, and the "inductor constant" $C = R = 12 \cdot 5$ inches, calculate the total horse power transmitted by the turbines to the propellers.

In this case the only variable quantity is N; hence equation (50) may be written

$$\begin{aligned}
\text{Total S.H.P.} &= \frac{1 \cdot 53 D^4 x}{RL}(N_H + N_P + N_S) \\
&= \frac{1 \cdot 53 \times 6 \cdot 75^4 \times 0 \cdot 68}{12 \cdot 5 \times 36}(600 + 655 + 650) \\
&= 4 \cdot 80 \times 1905 = 9144
\end{aligned}$$

CHAPTER X

MECHANICAL LOSSES AND THEIR PREVENTION

THE losses of heat energy which occur in a steam turbine may be classified into two groups: (1) internal losses which occur in nozzle and blade channels; (2) external losses due principally to mechanical causes.

The first class has already been considered in connection with nozzles and blading. The second class includes loss due to leakage at external glands, leakage at stage diaphragms, leakage past blade tips and dummy pistons in reaction turbines, disc and vane friction in impulse turbines, journal and gear friction, carry over at exhaust, and radiation.

141. External Glands.—In order to prevent leakage of steam out of or air into the turbine, where the shaft passes through the ends of the casing, either a ring-packed gland, or a steam-packed labyrinth, or a water-sealed gland may be used. The usual type of ring-packed gland consists of several carbon rings measuring about 1 inch × 1½ inches, which are held in casings inserted in the gland chambers at each end of the turbine. The design of the gland holders varies with different makers. Illustrations are given in the sections of the impulse turbines, Figs. 12, 16, 17, 19, and 21. Usually the rings are held up to the shaft or a steel liner shrunk on to it, by several garter springs, although, in some cases, for instance, the turbine, Fig. 19, plate springs are used to hold up the segments.

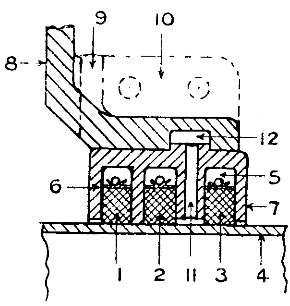


FIG. 142.

An enlarged section through the upper half of a gland suitable for the H.P. end of an impulse machine is shown in Fig. 142. Each of the rings, 1, 2, and 3, is divided into three segments. These are pressed on to the surface of a steel liner, 4, on the shaft by garter springs, 5, which bear on the back of a brass strip, 6, punched out at intervals to form seatings for the spring. A lug at one end of this strip engages a stop pin on the casing, and prevents the rotation of the ring. The casing, 7, is in halves. The lower half fits into the turbine casing. The upper half is held in place by a cover bolted to the casing end, 8, by a flange, 9, and by a horizontal flange, 10, at the shaft centre, as indicated by the dotted lines.

Radial holes, 11, are provided in the gland casing through which leakage steam can pass to a leak-off pocket, 12. From this pocket the steam can be led to some stage of the turbine where the pressure is above atmospheric, and a certain amount of the leakage heat can be usefully employed. In other designs the leak-off arrangement is omitted.

The rings used for this class of gland have to be made of special material, in order to withstand the temperature conditions to which they may be subjected. Ordinary carbon material, when used at the high-pressure glands, tends to disintegrate, under the action of superheated steam. The result is considerable wear both of rings and shaft, and to provide against this a steel liner, as shown in Fig. 142, is often fitted.

A packing ring of special composition called "morganite" is used for these rings by the Morgan Crucible Company of Battersea, who supply most of the leading turbine makers in this country and abroad.

This morganite gland ring is strong mechanically, has a low coefficient of friction, and does not disintegrate under the action of superheated steam.

The wear of shafts which have been running on morganite rings for years is a negligible quantity, and the makers state that with this material it is not necessary to fit the extra shaft liner.

To ensure this satisfactory result it is, however, necessary to have the glands properly fitted and quite free from any grit, or rust, if the shaft is allowed to stand and rust in the gland. If the faces of the ring segments get scored the highly polished surface necessary for satisfactory running cannot be obtained.

It is advisable after the turbine has passed its initial tests, to open out the glands, and if the rings have been scored to scrape them down to a smooth surface.

These rings are made in three or four segments, according to the size of the shaft. The joints, at the segments, are made either of butt or scarf type. Both types appear to give satisfaction, but the scarf joint is more convenient than the butt for following up wear, as a little clearance may be allowed for this purpose when the ring is made. The wear, however, as already indicated, is so slight that this is not a matter of any importance.

An illustration of one of those morganite gland rings for a large gland is shown in Fig. 143, Plate XII. It is in four segments and butt jointed. The front segment is recessed for the major part of its length. In the centre a small brass casting is fixed by two cheese-headed screws. Thin brass washer strips are placed on the face of the casting, and the screws are screwed down on these. After the screws are tightened up flats are filed on the heads, and the washer strips are bent up to them and lock the screws in position. They are shown quite clearly on the photograph. The head of a pin on the gland casing fits into the centre slot in this casting and prevents rotation of the ring. At each end of the recess brass plates are let into the ring. Each plate is secured by five "rivets." Each rivet consists of a thin brass split tube formed with countersunk head. The hole in the carbon segment into which the

PLATE XII.

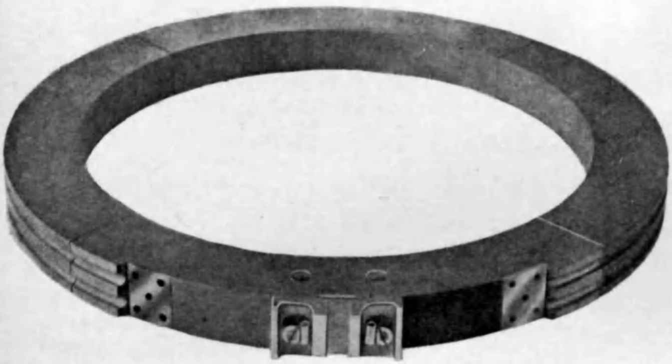


FIG. 143.—Morganite Packing Ring for Large Gland.

rivet is forced is undercut, and the prongs of the rivet are opened out into the carbon by caulking spongy copper powder into the tube. These brass plates take the bearing ends of a coach spring fixed on the gland casing. This spring carries the weight of the packing ring. The segments are held together and against the shaft surface by two garter springs, which fit into the grooves shown on the circumference of the ring.

Each lug, shown at the outer sides of the brass casting, is provided with two holes, through which the ends of the garter springs are passed and made fast.

142. Calculation of External Gland Loss.—Theoretically, in a gland of this type, the rings are in close contact with the shaft, and no leakage should take place. The condition is never completely fulfilled, and a certain amount of leakage occurs.

In order to estimate this approximately it is necessary to assume some value of clearance between rings and shaft under running conditions.

It may be taken between 0·003 inch and 0·01 inch (3 to 10 mils.). This small clearance space may be regarded as a nozzle passage, and the equations deduced in Chap. VII. can be applied to calculate the probable leakage under specified pressure conditions.

At the high-pressure gland the pressure drop, except in the case of a low-pressure turbine, is greater than the critical value.

The steam at the turbine side of the gland may either be superheated, dry, or slightly wet. In any of these conditions the approximate discharge is given by equations (6) and (16) for initially superheated and initially dry or slightly wet steam.

These equations for convenience of reference are repeated—

$$W_s = 0·3155A\sqrt{\frac{p_1}{v_{s_1}}} \text{ lbs./sec. for superheated steam} \quad . \quad (1)$$

$$W = 0·0173 p_1^{\frac{31}{32}} A \text{ lbs./sec. for dry or slightly wet steam} \quad (2)$$

where A = annular clearance area in square inches.

p_1 = initial pressure in lbs./in.^{2} abs. at turbine side of gland.

v_{s_1} = volume of superheated steam at p_1 in ft.^{3}/lbs.

As this equivalent nozzle passage is sharp-edged at entrance, a slightly lower discharge will result, and the above figure can, if desired, be corrected by a coefficient of discharge.

Owing to the uncertainty of the value of the area A the figure obtained can only be regarded as a rough approximation, and the use of a correction factor is hardly warranted.

At low-pressure glands it is usual to supply steam at a low pressure, slightly above atmospheric, to prevent leakage of air into the condenser. The general tendency of this steam leakage is towards the condenser, but a slight amount may leak outward.

This is hardly worth considering, but if desired it may be roughly estimated by an equation originally deduced by Rankine from Napier's results. Apart from the calculation of gland loss it is very convenient for the calculation of the approximate leakage at the dummy pistons of reaction turbines.

It can be deduced directly from equations (1) and (3), Art. 57, by making certain assumptions, which the author preferred to omit when dealing with the equations for the calculation of nozzle dimensions and discharge of steam.

For this and dummy piston leakage, however, where the value of the clearance at successive rings is very uncertain, and the result can only be considered as a rough approximation, this less precise equation is good enough.

It has the form

$$W = \frac{A}{K}\sqrt{p_0(p_1 - p_0)} \text{ lbs./sec.} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where A = clearance area in square inches.
p_1 = initial pressure inside the gland or dummy in lbs./in.^{2} abs.
p_0 = external pressure in lbs./in.^{2} abs.
K = coefficient.

The value of K is dependent on the assumption made regarding the law of the saturated steam curve.

The original assumption was $pv = c$.

This is not quite true. As already pointed out the more exact expression, as shown by recent steam tables, is $pv^{1.066} = c$. The value of K based on the average value of the product (pv) is usually taken as 34·3. If the more recent value of the index is used the constant reduces to 33·2.

One of the assumptions made in the deduction of equation (3) is that the pressure drop between p_1 and p_0 is small. If the drop is extended to the "critical" value, this assumption is no longer true. Here again, however, for the purpose of approximate calculation it is very convenient to use this value in equation (3), which reduces to the simple form

$$W = \frac{Ap_1}{K_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

With the older value of the critical pressure ratio

$$\frac{p_0}{p_1} = 0·58, \quad K_1 = 70$$

With the more recent value

$$\frac{p_0}{p_1} = 0·5457, \quad K_1 = 67$$

Equation (4), in the form $W = \frac{p_1 A}{70}$, is known as Napier's equation, and has generally been used to calculate the approximate discharge of saturated steam from the convergent-divergent type of nozzle.

In the light of the more recent knowledge of the flow of steam through nozzles, the more exact equations deduced in Chap. VII. should be used for the determination of nozzle proportions, and calculation of the discharge from nozzles.

EXAMPLE 1.—The pressure of steam in the first wheel chamber of a Curtis turbine is 63 lbs./in.² abs., and the superheat 130° F. If a gland of the carbon ring type is fitted at the H.P. end, calculate the probable value of the "leak off," the diameter of the shaft being taken as 10 inches and the clearance 5 mils. It may be assumed that the gland pocket is connected with a lower part of the turbine where the pressure is approximately atmospheric, and that any leakage to the atmosphere is negligible.

$$\text{Leakage area } A = 31{\cdot}416 \times 0{\cdot}005 = 0{\cdot}157 \text{ in.}^2$$
$$p_1 = 63, \quad t_{s_1} = 130, \quad v_1 = 6{\cdot}85$$
and
$$v_{s_1} = 6{\cdot}85(1 + 0{\cdot}0016 \times 130) = 8{\cdot}275$$

By equation (1), since the critical drop is exceeded

$$W_s = 0{\cdot}3155 A\sqrt{\frac{p_1}{v_{s_1}}}$$
$$= 0{\cdot}3155 \times 0{\cdot}157\sqrt{\frac{63}{8{\cdot}275}} = 0{\cdot}137 \text{ lb./sec.}$$

The estimated consumption of this machine is 9·7 lbs. per sec., so that for the assumed condition the H.P. gland leakage may be reckoned about 1·4 per cent. of the consumption. The heat of the leakage steam is not, however, a dead loss, as part of it can be usefully employed at a lower stage of the turbine.

EXAMPLE 2.—If the steam is admitted to the pocket of the low-pressure gland of the Curtis turbine (example 1) at 18 lbs./in.² abs., calculate the probable leakage to the condenser when the vacuum is 28 inches. Assume the same shaft diameter and clearance as at the H.P. gland.

In this case dry steam is supposed to be supplied to the gland pocket, and as the pressure drop is in excess of the critical value, equation (2) is applicable

$$P_1 = 18, \quad P_1^{\frac{31}{32}} = 16{\cdot}6, \quad A = 0{\cdot}157$$
$$\therefore\ W = 0{\cdot}0173 \times 16{\cdot}6 \times 0{\cdot}157 = 0{\cdot}045 \text{ lb./sec.}$$

If Napier's equation were used

$$W = \frac{18 \times 0{\cdot}157}{70} = 0{\cdot}0405 \text{ lb. per sec.}$$

There is probably a leakage of about 0·42 per cent. of the total consumption at the L.P. gland.

If the H.P. leakage is regarded as a total loss, the total for the two glands is in the neighbourhood of 2 per cent.

The effective heat loss is probably about half this value, say, 1 per cent.

143. Labyrinth Steam Gland and Dummy Piston Packing.—This type of packing was originated by Sir Charles Parsons, and is still used in most reaction turbines of the axial flow type, both for glands and dummy balance pistons.

The packing is a multiple throttling device. Two arrangements, the "face" and the "radial" packings, are employed for dummy pistons. For glands a combination of face and radial packing is often fitted. A section of the "face" type is shown at Fig. 150. The packing consists of a series of rings turned on the surface of the dummy piston, and corresponding series of brass rings let into the casing of the turbine.

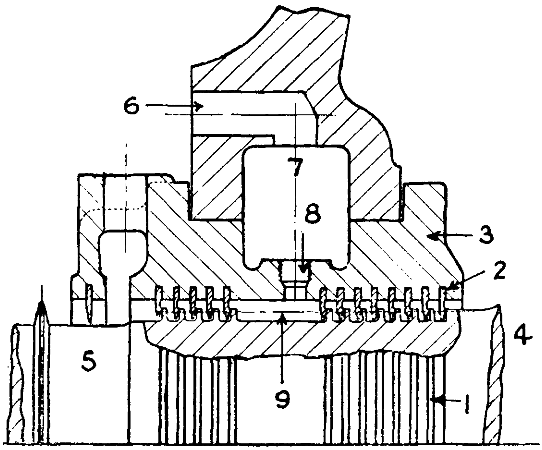


FIG. 144.

Each ring is formed, at the inner circumference, with a short angled projection which is thinned down to obtain a narrow face surface. The rotor is axially adjusted until there is a very small axial clearance between these thinned faces and the faces of the piston rings.

An example of the combined axial and radial type of packing for a high-pressure gland is shown in Fig. 144. This is the gland fitted on the Willans disc and drum turbine, Fig. 44. The gland rings, 1, are turned on the shaft, and the brass packing rings, 2, are let into the gland casing, 3. These rings have the thinned face projection, and are also carried down close to the surface of the shaft in each groove, the radial clearance being about 25 mils. (0·025 inch).

The rotor is adjusted axially till the axial clearance between the shaft and packing rings is from 8 to 10 mils. The back of the gland, as already stated in the descriptions of the machine (Art. 35), is subjected to the condenser pressure at 4, and the outside to the atmospheric pressure at 5. Low-pressure steam is supplied through the port, 6, to the pocket, 7, from which it passes through holes, 8, to the annular space, 9, between the two sets of shaft rings. The steam which tends to leak principally towards the condenser side is progressively throttled in passing through the axial and radial constrictions, and the amount of leakage to the condenser is reduced to a small percentage of the steam consumption.

The number of rings fitted may vary from 12 to 16. In this case there are 13. The amount of steam that leaks past to the atmosphere at 5 can be reduced to a negligible amount by regulating the steam pressure in the pocket, 7, by a hand valve, until just a whiff is visible at the outside. A slightly modified form of the combined axial and radial packing ring used in the Brush machines in shown in Fig. 145 (*a*). The shaft collars are shown at 1. Each packing ring is provided with a face projection, 2. The back of the ring is extended, as in the previous case, close to the surface of the shaft at 3; the face between the top and bottom of the groove is formed with a curved surface. The result is the formation of a smaller pocket, between the packing and shaft ring.

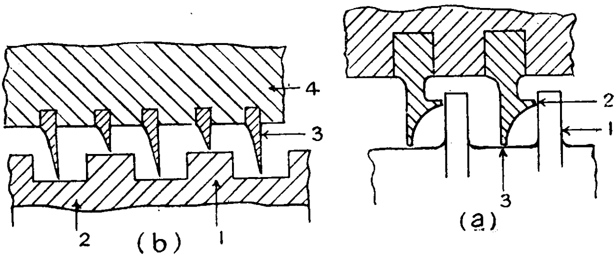


FIG. 145.

The steam is thus throttled twice in its passage from one shaft ring to the next instead of once, as in the case of the pure face type.

At the low-pressure end of a machine of moderate or large size, it is necessary to fit the radial type of gland and dummy packing. The modification is due to the uncertainty of the differential expansion which occurs between the rotor and casing, and precludes the use of fine axial clearances.

The form of packing used for the low-pressure gland of the Willans machine is shown in Fig. 146. The reference numbers correspond to those shown in Fig. 144. In this case the packing rings, 2, have a forked section, and are fixed in the gland casing by wedging rings. The circumferences of the forks run about 25 mils. clear of the shaft surface. The shaft rings are sufficiently spaced to enable the rotor to move relatively to the casing without risk of fouling the packing rings. In the previous cases it is probable that the steam undergoes two throttlings between each pair of shaft rings, or in each pocket of the shaft groove. In this case it is questionable whether a real throttling effect takes place at the second prong of the fork, as there is no definite baffling effect, and the steam tends to take the straight path and blow through. There is probably only a frictional effect which tends to diminish the flow.

Another form of the radial packing used in the Brown Bovrie machines is shown in Fig. 145 (*b*). A set of wide rings, 1, is turned on

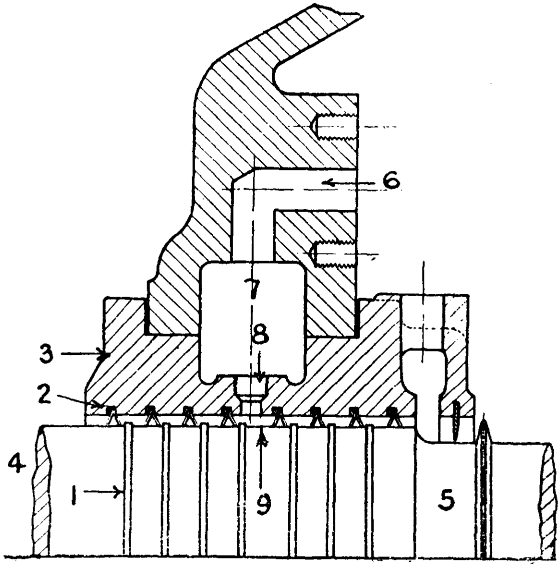


FIG. 146.

the shaft, 2. The packing rings, 3, inserted in the gland casing, 4, are tapered in section from outer to inner circumference, so as to resemble "fins." A packing ring is provided for each ring and groove. By this staggered arrangement there is a definite baffling effect, as the steam

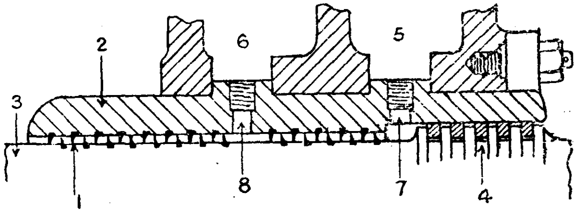


FIG. 147.

expands through the clearance between the packing rings and shaft surface; and the number of throttlings corresponds to the number of packing rings.

The usual type of labyrinth packing gland fitted in marine Parsons turbines is shown in Fig. 147. As in the previous case the brass packing rings, 1, are tapered to a fine edge at the inner circumference. The corresponding rings let into the shaft, 3, are similarly thinned at the outer circumference. At the external end of the gland several collars are turned on the shaft, and Ramsbottom rings, 4, are fitted between these. They constitute a mechanically packed auxiliary gland. Any residual leakage steam is carried away by a vapour pipe.

This gland has a steam pocket, 5, and a "leak off" pocket, 6. The latter is connected to the third or fourth expansion of the low-pressure turbine where the pressure is below the atmosphere. Steam slightly above atmospheric pressure is admitted to pocket 5, and prevents leakage of air into the turbine. A small amount of steam leaks to the outside, the bulk of the leakage passes into pocket 6, and then to the L.P. turbine. The remainder passes through the inner set of rings and passes out along with the H.P. exhaust steam. The pockets, 5 and 6, are connected with the interior of the gland by a series of radial holes, 7 and 8. The two holes shown, are tapped to take lifting screws, when the gland, which is in halves, has to be overhauled.

In other cases only a leak-off pocket is provided, and connected to an expansion of the turbine where the pressure is above atmospheric.

At the low-pressure end of the L.P. turbine, where the pressure is below atmospheric, low-pressure steam is likewise supplied to the pockets to prevent leakage of air into the condenser.

The high-pressure glands with one pocket are also usually fitted with an auxiliary steam connection, so that steam can be admitted to heat up the shaft before the turbine is started.

The brass packing rings used for this type of packing vary from $\frac{1}{8}$ to $\frac{3}{16}$ inch in thickness, and are pitched from $\frac{5}{8}$ to 1 inch. The radial clearance may vary from 10 to 20 mils.

The labyrinth packing fitted at the glands of the Ljungström turbine is different from any of the foregoing. It is illustrated in Fig. 148, and consists of a number of sections. Each section is formed by a narrow and deep cylinder or sleeve, 1 or 2, keyed to the shaft, 3, or casing, 4. Surrounding this are a series of thin concentric cylinders, 5, supported by the central web, 6. These are staggered, so that

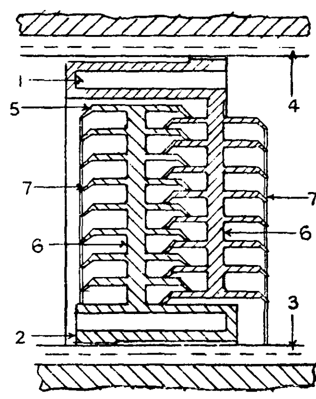


FIG. 148.

one set fits into the other. The edges, 7, of the cylinders are thinned down and turned over as shown, the thinned edge of one cylinder being brought close down to the surface of the cylinder into which it projects. The clearance is very fine, about 0·003 inch. It will be seen that steam entering either at the bottom or the top of a section has to pass sixteen constrictions between entrance and exit. There are no less than 158 of these constrictions on a 3-inch length of shaft. Since the labyrinths have all the same heating surface and thickness of metal, they expand equally under varying temperature. The leakage is small. The amount actually measured on the 1000 K.W. machine is 110 lbs./hour, or 0·03 lb./second. This is about 1 per cent. of the steam consumption. The packing is made of steel.

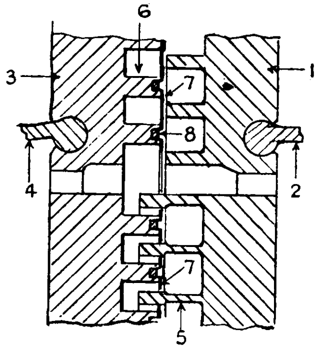


FIG. 149.

The special form of labyrinth ring used between the back of the disc and the casing is shown in Fig. 149.

The disc, 1, is connected to the casing by the expansion ring, 2. Similarly disc 3 is connected to the back of the disc wheel by the other expansion ring, 4 (see Fig. 36). Towards the high-pressure end projections, 5 on 1, fit into grooves in 3. The corresponding projections, 6 on 3, are fitted with sheet-nickel rings, 7, the edges of which fit closely to the surfaces of projections 5. These rings are caulked into the grooves in 6 by caulking rings, 8. The discs 1 and 3 have the same volume and heating surface, and the expansion due to the heating up of the turbine or to changes in steam temperature is simultaneous and uniform in each case.

This condition enables very fine clearances to be used and reduces the leakage of the labyrinth packings. The disc 3 fulfils the function of a dummy piston. The pressure in the blade channel of the disc tends to force the disc 3 to the right; the pressure in the labyrinth space tends to force it to the left. The disc is thus kept in a state of equilibrium. If the pressure, due to increase of load, rises in the blade channel, disc 3 is forced to the right till the nickel strips 7 pass the projections on 5. The high pressure steam, from the centre, then passes into the labyrinth space, forces back disc 3 and restores equilibrium.

The use of very thin nickel strips further obviates the risk of injurious heating and seizure, should hard contact occur between the running and stationary surfaces. The clearance may vary from 0·004 to 0·01 from the centre outward, even when a high superheat is used.

144. Calculation of Leakage at Labyrinth Glands and Dummy Packings.—The axial type of dummy packing for the external glands of a Parsons turbine is illustrated in Fig. 150. The H.P. and L.P. ahead dummy packings for the marine Parsons turbines, Figs. 27 and 28, are similar to this, but the astern dummy has a radial packing. The rings, like those of the glands, are usually $\frac{3}{16}$ inch thick and pitched $\frac{1}{2}$ to $\frac{5}{8}$ inch. In land machines the face clearance runs from 0·008 to 0·01 inch, and the radial clearance from 0·02 to 0·025 inch. In marine turbines the face clearance in H.P. machines is 0·015 to 0·02 inch, and in L.P. machines from 0·02 to 0·03 inch.

The calculation of the probable leakage from a labyrinth packing is more troublesome and tedious than that for the ring type of gland.

This is due to the fact that there is, throughout the series of pockets formed by the rings, a progressive drop in pressure. Four variables are involved. These are the initial pressure p_1 at the turbine end, the external pressure p_0 at the exhaust end, the number of throttlings, n, and the rate of leakage, w lbs. per second per in.^{2} of leakage area. By choosing the latter, the absolute value of the clearance area at any ring is eliminated. Referring to Fig. 150, steam enters the packing at A with pressure p_1, expands successively n times (in this case 15), and escapes at end B. With given values of p_1 and p_0, the number of rings may be sufficiently large to produce such a pressure p_a in the last pocket, at end B, that $\frac{p_0}{p_a}$ is greater than the critical ratio.

On the other hand, the number of rings may be sufficiently small to make this ratio equal to or less than the critical value.

Before a determination of the rate of leakage through the packing can be made, it is desirable to ascertain whether or not the critical drop occurs at the last pocket.

The problem can be solved by a step-by-step application of the

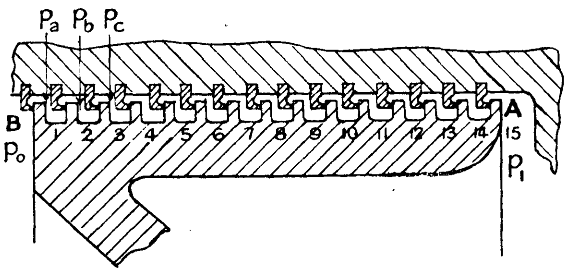


FIG. 150.

equations (3) and (4) for the approximate discharge of saturated steam through nozzle passages.

The action which takes place at each ring is supposed to be an adiabatic expansion through the clearance space (equivalent to a short nozzle passage).

Due to this, it is assumed the steam becomes slightly wet, but on being brought to rest by baffling action in the ring pocket, the kinetic energy generated by the expansion is assumed to be reconverted to heat, and the steam is dried and slightly superheated. Owing to the relatively large mass of metal in contact with the small ring of steam, a considerable conduction of heat takes place, and as a working hypothesis it is assumed that the steam in each pocket remains sensibly in a dry saturated state.

The assumption is open to question, in the light of the new supersaturation theory, but it gives a reasonable basis to work from without unduly complicating the calculation. For the reason that the leakage can only be a rough approximation, on account of the uncertain value of the absolute amount of clearance at any ring, it is sufficient for the purpose, to apply these approximate equations.

Referring to the packing, Fig. 150, the consecutive pockets, reading from the exhaust end B, are numbered 1, 2, 3, etc. Let the pressure in the last pocket (No. 1) be p_a, the pressure in pockets 2, 3, etc., be p_b, p_c, etc.

Assume, in the first instance, that $\frac{p_0}{p_a}$ is greater than the critical value, then by equation (3)

$$w = \frac{\sqrt{p_0(p_a - p_0)}}{K}$$

where w = rate of leakage in lbs./(in.^{2} clearance area) sec.
p_0 = external press in lbs./in.^{2} abs.
p_a = pressure in No. 1 pocket in lbs./in.^{2} abs.
K = coefficient.

Multiplying across and squaring each side

$$w^2K^2 = p_0p_a - p_0^2$$

Dividing by p_0^2

$$\frac{w^2K^2}{p_0^2} = \frac{p_a}{p_0} - 1$$

If

$$\frac{p_0}{w} = y$$

then

$$\left.\begin{aligned} p_a &= \left(1 + \frac{K^2}{y^2}\right)p_0 \\ \text{or} \quad p_a &= r_1p_0 \end{aligned}\right\} \quad \ldots \ldots \quad (5)$$

For any arbitrarily chosen value of the ratio y equation (5) gives the pressure p_a in No. 1 pocket.

Take No. 2 pocket, for which the pressure is p_b and the external pressure is p_a, then similarly

$$p_b = \left(1 + \frac{K^2}{y^2}\right)p_a$$

and substituting for p_a from equation (5), this again reduces to

$$p_b = r_2p_0$$

Similarly the pressure in No. 3 pocket is given by

$$p_c = r_3p_0$$

and so on, for the complete series of n pockets until $p_A = p_1 = r_np_0 = \rho p_0$ is reached.

In this way for any given number of rings or throttlings of the steam the values of the pressure ratio $\rho = \frac{p_1}{p_0}$ for a series of values of pressure-weight ratio $y = \frac{p_0}{w}$ can be calculated.

The values of y, when plotted on a p base, give a rectangular chart consisting of the series of n curves, as shown on the right of Fig. 151. For certain constructional reasons the scales used for this chart are logarithmic.

This rectangular chart is not applicable if the critical drop is reached at the last pocket.

If the critical ratio $\frac{p_0}{p_1} = \frac{1}{x}$, then substituting in equation (3)

$$w = \frac{\sqrt{p_0(p_0x - p_0)}}{K} \quad \text{or } w = p_0 \times \frac{\sqrt{x - 1}}{K} \quad \text{and } y = \frac{K}{\sqrt{x - 1}}$$

If the older values $K = 34{\cdot}3$ and $\frac{1}{x} = 0{\cdot}58$ are taken

$$y = \frac{34{\cdot}3}{\sqrt{0{\cdot}73}} = 40$$

If the later values $K = 33{\cdot}2$, $\frac{1}{x} = 0{\cdot}5475$ are taken

$$y = \frac{33{\cdot}2}{\sqrt{0{\cdot}83}} = 36{\cdot}6$$

There is thus some limiting value of y between 37 and 40, below which this rectangular chart will probably give too large a value for the rate of leakage.

It would seem preferable to take the higher value of 40; and in Fig. 151 each curve has been stopped at the base line X_1X_2, which may be called the "critical limit line," drawn through the value $y = 40$. The corresponding value of $K = 34{\cdot}3$ has been used in calculating the r values.

In order to deal with values of y less than this amount the ratio of the initial pressure to the pressure p_a in the end pocket No. 1 is taken instead of $\left(\frac{p_1}{p_0}\right)$.

Let the critical drop be exceeded at No. 1 pocket, then

$$w = \frac{p}{K_1} \quad \text{and } w^2 = \frac{p_a^2}{K_1^2}$$

To find the pressure p_b in pocket No. 2.

By equation (3)

$$w = \frac{\sqrt{p_a(p_b - p_a)}}{K}$$

squaring and substituting for w^2

$$\frac{p_a^2 K^2}{K_1^2} = p_b p_a - p_a^2$$

$$\frac{K^2}{K_1^2} p_a + p_a = p_b$$

or

$$p_b = p_a \left\{ 1 + \left(\frac{K}{K_1} \right)^2 \right\}$$
$$= \delta_1 p_a$$

To find the pressure p_c in the third pocket

$$w^2 K^2 = p_b p_c - p_b^2$$
$$\frac{p_a^2 K^2}{K_1^2} = p_b p_c - p_b^2$$

Substituting $\left(\frac{p_b}{\delta_1}\right)^2$ for p_a^2 and dividing out by p_b

$$p_c = \frac{p_b}{\delta_1^2} \frac{K^2}{K_1^2} + p_b$$
$$= p_b \left(1 + \frac{K^2}{\delta_1^2 K_1^2} \right)$$

Substitute for p_b

and

$$p_c = \delta_1 p_a \left(1 + \frac{K^2}{\delta_1^2 K_1^2} \right)$$
$$= p_a \left(\delta_1 + \frac{K^2}{\delta_1 K_1^2} \right) = \delta_2 p_a$$

In the same way it can be shown that the pressure on the fourth pocket is given by

$$p_d = p_c \left(1 + \frac{K^2}{\delta_2 K_1^2} \right)$$

or

$$p_d = p_a \left(\delta_2 + \frac{K^2}{\delta_2 K_1^2} \right) = \delta_3 p_a$$

and so on for any series of n pockets.

When the turbine pocket is reached the pressure is p_1

and

$$p_1 = p_a \left(\delta_{n-1} + \frac{K^2}{K_1^2 \delta_{n-2}} \right)$$

or

$$p_1 = C p_a$$

Substituting the original value of p_a in terms of w and K_1

$$p_1 = C K_1 w$$

and

$$w = \frac{p_1}{C K_1} \quad \ldots \ldots \ldots \quad (6)$$

The leakage rate is now calculable for any given value of the initial pressure p_1 when the value of the coefficient C is known for a given value of the number of pockets or throttlings n.

The values of δ depend on the value of the constant $\left(\frac{K}{K_1}\right)^2$.

If the older values of 34·3 and 70 are taken for K and K_1 this ratio is 0·24. If the more recent values 33·2 and 67 are taken it becomes 0·245.

The effect on the value of w is small, and the older values have again been taken in calculating the C values.

A curve can be drawn giving the values of C for given values n, and by means of this and equation (6) the leakage rate can be calculated when it is ascertained that the critical drop is exceeded at the last pocket. This condition is determined by the critical limit line on chart No. 1, Fig. 151.

The problem can, however, be much more rapidly solved, without any calculation, by graphing equation (6) in the form of an alignment chart, No. 2, and linking it up with the rectangular chart No. 1. This second chart is shown on the left of Fig. 151.

It is not necessary to go into the construction of this chart here. A detailed description of the method can be found in an article by the author.[1]

The complete combination can be used either to determine the leakage rate for given initial and final pressure conditions and a specified number of throttlings, or to calculate the required number of throttlings to limit the leakage to a specified amount.

The rules of operation for these two cases can be stated as follows :—

RULE I.

Given		To find
Initial pressure	p_1	
Exhaust pressure	p_0	Rate of leakage
Number of throttlings	n	w

(1) Calculate the value of $\rho = \frac{p_1}{p_0}$.

(2) Note if the projector from the ρ scale, chart No. 1, cuts the given n curve above X_1X_2.

(3) If it does so, project horizontally to the vertical MN, and mark the point of intersection. Lay a straight-edge across this point and the p_0 value, and read the leakage rate where the edge crosses the w scale.

(4) If the projector does not cut the n curve above X_1X_2, chart No. 2 is to be used.

Lay the edge across the n and p_1 values and read the leakage rate where it crosses the w scale.

[1] See article entitled "A Graphical Method of determining Dummy Piston Leakage in the Parsons Steam Turbine," *Engineering Review*, January 15, 1912.

RULE II.

Given		To find
Initial pressure	p_1	
Exhaust pressure	p_0	Number of throttlings
Rate of leakage	w	n

(1) Calculate the value of $\rho = \frac{p_1}{p_0}$.

(2) Lay the straight-edge across the w and p_0 values, and mark the intersection with MN.

(3) Project horizontally from this point, and vertically from the ρ value, and read the n value on the curve, where the two projectors intersect.

(4) If the edge cuts MN below X_1X_2, chart No. 2 is to be used. Lay the edge across the w and p_1 values, and read the number of rings where it cuts the N scale.

145. As already stated at the outset of this discussion, the rate of leakage in each case is probably on the high side as the equations used do not allow for reduction of discharge due to the sharp-edged passages. This error if it does occur is, however, on the right side. As far as the absolute amount of leakage is concerned this is affected by the area assumed in the subsequent calculation after the rate is provisionally fixed.

The indeterminate differential expansion which may occur between the rotor and casing, and in some machines the "play" of the adjusting block where the block is in halves and arranged for top and bottom adjustment in either direction, makes the real clearance under running conditions a doubtful figure.

In any case, taking the leakage, as determined by the foregoing method, and the dummy or gland clearance when the machine is cold, the designer will be well on the safe side in the estimation of the total leakage. The value can, if desired, be reduced by the use of a coefficient of discharge.

In the case of a high-pressure dummy where the steam has a high initial superheat, the leakage rate determined by the chart will be too large, and some judgment has to be exercised as to the reduction of this figure.

146. This problem of dummy leakage has been solved by H. M. Martin by a method somewhat different from that given above.[1]

His equation, with the constant modified to give the rate of leakage, is as follows:—

$$w = 0{\cdot}4722\sqrt{\frac{p_1\left(1 - \frac{1}{\rho^2}\right)}{v_1(n + \log_e \rho)}} \quad . \quad . \quad . \quad . \quad (7)$$

[1] See *Engineering*, January 10, 1908, or Martin's "Steam Turbine," chap. xviii.

Alignment Chart for the Determination of Gland and Dummy Piston Leakage.

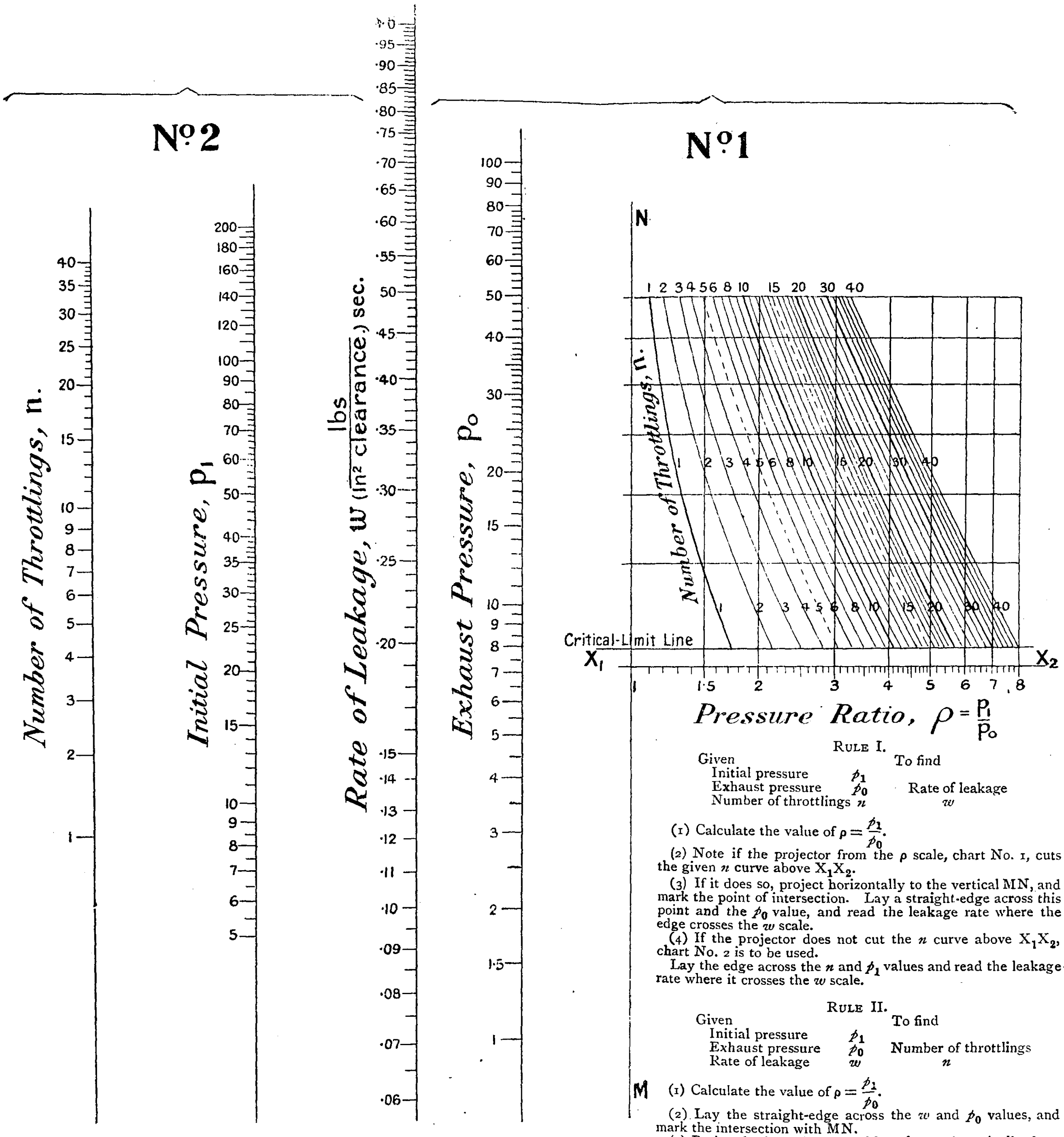


Fig. 151.

Rule I.

Given		To find
Initial pressure	p_1	
Exhaust pressure	p_0	Rate of leakage
Number of throttlings	n	w

(1) Calculate the value of $\rho = \frac{p_1}{p_0}$.

(2) Note if the projector from the ρ scale, chart No. 1, cuts the given n curve above X_1X_2.

(3) If it does so, project horizontally to the vertical MN, and mark the point of intersection. Lay a straight-edge across this point and the p_0 value, and read the leakage rate where the edge crosses the w scale.

(4) If the projector does not cut the n curve above X_1X_2, chart No. 2 is to be used.

Lay the edge across the n and p_1 values and read the leakage rate where it crosses the w scale.

Rule II.

Given		To find
Initial pressure	p_1	
Exhaust pressure	p_0	Number of throttlings
Rate of leakage	w	n

(1) Calculate the value of $\rho = \frac{p_1}{p_0}$.

(2) Lay the straight-edge across the w and p_0 values, and mark the intersection with MN.

(3) Project horizontally from this point, and vertically from the ρ value, and read the n value on the curve, where the two projectors intersect.

(4) If the edge cuts MN below X_1X_2, chart No. 2 is to be used. Lay the edge across the w and p_1 values, and read the number of rings where it cuts the N scale.

where w = leakage rate in lbs./(in.^{2} clearance area) sec.
p_1 = initial pressure lbs./in.^{2} abs.
p_0 = exhaust „ „ „
$\rho = \frac{p_1}{p_0}$.
n = number of throttlings.
v_1 = specific volume of steam at p_1 in ft.^{3}/lb.

This equation gives approximately the same value as obtained from the rectangular chart No. 1. It does not, however, give any definite information regarding the pressure conditions at the last pocket, and in order to ascertain if the critical value is reached a trial and error process has to be adopted.

As a rule, it will be found that the discrepancy resulting from the use of the equation instead of chart No. 2, when the critical drop is exceeded, is not sufficiently great to materially affect the practical design of the packing. The equation, although somewhat deficient in this respect, is suitable for most cases.

On the other hand, the graphical method, while it takes account of this element of the problem, also eliminates all calculation, and is superior to an equation for the quick and easy determination, either of the number of rings required or the probable leakage for a given number.

EXAMPLE 3.—A labyrinth packing gland having combined face and radial packing rings, has eight rings between the interior of the turbine and the steam pocket, and five rings between the pocket and the outside. Steam is supplied to the pocket at 16 lbs./in.^{2} abs. The mean diameter of the rings is 9 inches, and the average value of the axial and radial clearance may be taken as 0·008 inch. Calculate the probable steam leakage from the pocket to the condenser, the vacuum being 28 inches. Assume that there are two effective throttlings at each ring.

The pressure ratio is $\rho = 16$. This is outside the limits of chart No. 1. Laying a straight-edge across $p_1 = 16$ and $n = 16$ on chart No. 2, the edge cuts the leakage scale at $w = 0{\cdot}08$.

$$\text{Leakage area (mean of radial and axial)} = \pi \times 9 \times 0{\cdot}008 = 0{\cdot}226 \text{ in.}^2$$
$$\therefore \text{Leakage} = 0{\cdot}226 \times 0{\cdot}08 = 0{\cdot}0181 \text{ lb./sec.}$$

Owing to the small pressure difference of 1 lb. between the pocket and the outside of the gland, the external leakage will be negligibly small. The value of ρ is 1·06, a figure too small to enable the chart No. 1 to be used.

The consumption of the machine may, in this case, be taken as 8 lbs. per sec., so that with a corresponding leakage at the other gland, the probable total leak may amount to 0·45 per cent. of the steam consumption, a very small amount.

EXAMPLE 4.—The radial labyrinth gland, Fig. 147, is fitted at each end of a high-pressure marine turbine. The outer pocket, 5, is supplied with steam at 16 lbs./in.^{2} abs., and the inner pocket, 6, is connected to an "expansion" of the low-pressure turbine, where the vacuum is 10 inches. The initial pressure at the turbine side of the gland is 38 lbs./in. abs. The steam is throttled sixteen times at the inner and ten times at the outer group. Calculate the probable leakage from the H.P. turbine, and the total steam by-passed through the gland to the low-pressure turbine. The mean ring diameter of gland is 18 inches, and clearance 0·02.

Pressure in pocket 6 is 5 lbs./in.^{2} abs., hence pressure ratio of inner group is $\rho_1 = \frac{38}{5} = 7{\cdot}6$. For outer group, $\rho_0 = \frac{16}{5} = 3{\cdot}2$. Also $n_1 = 16$ and $n_0 = 10$.

Referring to the chart, it will be seen that at $\rho_1 = 7{\cdot}6$ the vertical does not cut $n_1 = 16$ above X_1X_2, and chart No. 2 is to be used. Apply Rule I., then for $p_1 = 38$ and $n_1 = 16$, $w_1 = 0{\cdot}19$. Mean leakage area $= 18 \times \pi \times 0{\cdot}02 = 1{\cdot}13$ in.^{2}; hence leakage $W_1 = 1{\cdot}13 \times 0{\cdot}19 = 0{\cdot}2147$ lb./sec.

For the outer group at $\rho_0 = 3{\cdot}2$, the vertical cuts the curve for $n = 10$ above X_1X_2. Projecting to MN and aligning the point with $p_0 = 5$, the value of w given on the leakage scale is 0·10.

Hence the leakage is $W_0 = 0{\cdot}10 \times 1{\cdot}13 = 0{\cdot}113$ lb./sec. The steam bye-passed to the L.P. turbine is approximately $W_1 + W_0 = 0{\cdot}3277$ lb. per sec. for each gland, or a total of 0·635 lb./sec. The slight leak through the Ramsbottom rings has been neglected.

The total consumption of this turbine is about 65 lbs./sec., so that the amount by-passed is about 1 per cent. of the consumption.

The heat of this steam is partly utilised in the L.P. turbine, so that the thermal loss from H.P. gland leakage is very small.

EXAMPLE 5.—The high-pressure dummy piston of a land turbine is $25\frac{5}{8}$ inches diameter. The pressure at the H.P. side is 190 lbs./in.^{2} abs., and at the side connected with the I.P. section 68 lbs./in.^{2} abs. The total consumption is 12 lbs. per sec.

Assuming a dummy clearance of 0·01 inch, calculate the number of rings to limit the H.P. dummy leakage to 5·5 per cent. of the consumption.

Leakage area $\quad A = 25{\cdot}625 \times \pi \times 0{\cdot}01 = 0{\cdot}805$ in.^{2}

Leakage specified $\quad W = 12 \times 0{\cdot}055 = 0{\cdot}66$ lb.

Leakage rate $\quad w = \dfrac{0{\cdot}66}{0{\cdot}805} = 0{\cdot}82$ lb./in.^{2} sec.

Pressure ratio $\quad \rho = \dfrac{190}{68} = 2{\cdot}8$

Applying Rule II., lay the edge across $w = 0{\cdot}82$ and $p_0 = 68$; project from the point on MN and from $\rho = 2{\cdot}8$. The projectors intersect on the curve for $n = 19$.

This is the minimum number of rings that should be fitted. In this case the steam is superheated 150° F., so that the probable leakage will be less, say 5 per cent. of the consumption.

EXAMPLE 6.—The I.P. dummy adjacent to the H.P. dummy (example 5) has a diameter of $35\frac{7}{8}$ inches. The initial pressure is 68 lbs./in.^{2} abs., and the pressure behind the piston is that of the condenser, or 1 lb./in.^{2} abs.

Assuming the same clearance as at the H.P. dummy, viz. 0·01 inch, calculate the number of rings which limits the leakage to the condenser to 4 per cent. of the consumption.

$$\text{Leakage area } A = \pi \times 35{\cdot}875 \times 0{\cdot}01 = 1{\cdot}13 \text{ in.}^2$$
$$\text{Leakage} \quad W = 12 \times 0{\cdot}04 = 0{\cdot}48$$
$$\text{Rate} \quad w = \frac{0{\cdot}48}{1{\cdot}13} = 0{\cdot}425$$

Pressure ratio $\rho = 68$, which is outside the limits of chart No. 1. Applying Rule II. to chart No. 2, the edge, when laid across $p_1 = 68$ and $w = 0{\cdot}425$, cuts the n scale at 9·6. The I.P. dummy should be fitted with a minimum of ten rings.

EXAMPLE 7.—The rotor carrying the H.P. and I.P. dummies of examples 5 and 6, has an L.P. dummy 37 inches in diameter. The initial pressure is 20 lbs./in.^{2}, and condenser pressure 1 lb./in.^{2} The packing is of the "radial fin" type, and the radial clearance is 0·025 inch. Calculate the number of rings to limit the leakage to 3 per cent. of the consumption of 12 lbs./sec.

$$\text{Mean leakage area } A = 37\pi \times 0{\cdot}025 = 2{\cdot}9 \text{ in.}^2$$
$$\text{leakage} \quad W = 12 \times 0{\cdot}03 = 0{\cdot}36 \text{ lb./sec.}$$
$$\text{and} \quad w = \frac{0{\cdot}36}{2{\cdot}9} = 0{\cdot}124$$

$\rho = 20$. Applying Rule II. to chart No. 1, the number of throttlings is found to be 10.

Hence there should be at least five rings on the casing and five on the dummy.

The last three examples refer to a 3000 K.W. turbine, for which the general proportions are worked out in Chap. XIV.

EXAMPLE 8.—The diameter of the dummy piston of a high-pressure marine turbine is $73\frac{3}{8}$ inches.

The initial and exhaust pressures are 170 and 19 lbs./in.^{2} abs. There are twenty-seven rings in the "face" dummy packing, and the clearance is 0·02. Calculate the probable percentage dummy leakage, (*a*) from the chart, (*b*) by equation (7), if the steam consumption is 65 lbs. per sec.

(*a*) Here $p_1 = 170$, $p_0 = 19$, $\rho = 8{\cdot}95$, and No. 1 chart is inapplicable.

Leakage area $A = 73{\cdot}375\pi \times 0{\cdot}02 = 4{\cdot}6$ in.^{2} and $n = 27$.

Applying Rule II. to chart No. 2, the line joining $p_1 = 170$ and $n = 27$, cuts the leakage scale at $w = 0{\cdot}66$.

$$\therefore \text{ Leakage } \quad W = 0{\cdot}66 \times 4{\cdot}6 = 3{\cdot}04 \text{ lbs./sec.}$$

$$\text{Percentage leak} = \frac{3{\cdot}04}{65} = 0{\cdot}047, \text{ or } 4{\cdot}7 \text{ per cent.}$$

(*b*) Applying equation (7)

$$w = 0{\cdot}4722\sqrt{\frac{p_1\left(1 - \frac{1}{\rho^2}\right)}{v_1(n + \log_e \rho)}}$$

From tables $\qquad v_1 = 2{\cdot}68\,;\ \log_e 8{\cdot}95 = 2{\cdot}1917$

squaring, $\qquad w^2 = \dfrac{0{\cdot}223 \times 170\left(1 - \frac{1}{80}\right)}{2{\cdot}68(27 + 2{\cdot}1917)} = 0{\cdot}48$

$$\therefore\ w = 0{\cdot}69, \text{ as against } 0{\cdot}66 \text{ obtained from the chart.}$$

This is a result to be expected, since the chart indicates that the critical drop is exceeded at the last pocket, and the equation does not give the true rate under this condition, but some value in excess of it. In this case the excess is 4·5 per cent.

147. Water-sealed Gland.—A gland of this type, if fitted at the high-pressure end of an impulse turbine, consists of a steam labyrinth which reduces the pressure below atmospheric, and an external water seal which prevents leakage of air into the turbine through the gland. If the gland is fitted at the low-pressure end, where the internal pressure is always below atmospheric, the water-seal only is provided.

In the usual arrangement the labyrinth has a leak-off pocket about the centre, which is connected to a lower stage of the turbine, in which the pressure is about atmospheric. Residual leakage, which passes the outer half of the labyrinth is discharged into a second leak-off pocket connected with the condenser. The standard type of this gland fitted on the Westinghouse impulse machines, is shown in plan at the H.P. and L.P. ends in Fig. 24, Plate V.

A slightly modified design of this gland is fitted on the Westinghouse-Rateau turbine, Fig. 20.

Rings are fitted on the casing to restrict the flow of high-pressure steam through the gland. There are no corresponding baffling rings on the shaft, and this arrangement is not quite a true labyrinth packing, since the steam tends to flow through the clearance between the rings and shaft, with impaired throttling effect. The leakage passes into a chamber, 8, at about 20 lbs./in.^{2} pressure, and is bye-passed to a stage of the turbine. A certain amount passes on through a face labyrinth, and is discharged into the pocket 9, connected with the condenser.

The shaft liner carrying the labyrinth rings also carries the impeller wheel, 7, which rotates in a cylindrical chamber, 8. At each side of this chamber two baffle rings are fitted. The chamber is supplied with water from a tank situated about 20 feet above the turbine.

The impeller wheel, which is similar to that of a centrifugal pump, has a set of curved vanes. The water, owing to centrifugal force, is thrown out to the circumference of the chamber, and forms a seal on each side of the wheel. The actual effect produced at speed is shown in Fig. 152, which represents a section through the seal at the low-pressure end of the turbine. If the pressures on each side of the chamber were the same, the water surface in compartment A would be on the same level as that of B. There is, however, atmospheric pressure at B and condenser pressure at A, and the centrifugal force on the column of water "h" is balanced by the difference of pressure between atmosphere and condenser. Any leakage of water outward between the baffles is drained off through a separate port.

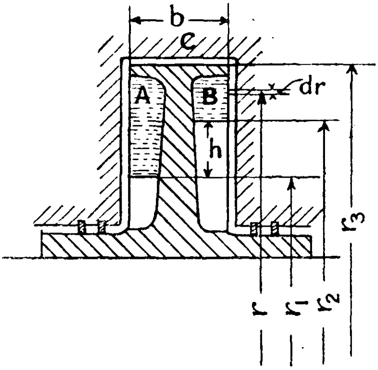


FIG. 152.

The advantages claimed for this type of gland are absolute air-tightness, no leakage of steam into the engine room, conditions of operation are not upset by change of vacuum, since if the vacuum changes h simply changes. Against these claims there has to be set the loss of energy in driving the impeller. The friction heat thus generated boils off the water at condenser pressure, and the steam thus formed is passed into the condenser. This energy is lost. The gland is also liable to silt up through deposit, if the water is hard, or contains solid matter in suspension.

148. Calculation of Friction Loss at Water-sealed Gland.—The loss can be approximated in the following manner:—

As shown in Fig. 152, the wheel when running at speed maintains the difference of head h by centrifugal action on the water. The value of h is readily calculated from the standard expression giving the centrifugal head in a pump, when there is no discharge of water.

Let ω = angular velocity of the wheel in radians/sec.
r_1 = smaller radius to water surface, in ft.
r_2 = larger radius to water surface, in ft.
r_3 = radius of wheel chamber surface, in ft.
b = width of the chamber surface, in ft.

Centrifugal pressure head h ft. is

$$\frac{\omega^2}{2g}(r_2^2 - r_1^2) = \frac{P}{w} \quad \ldots \ldots \quad (8)$$

where P = effective pressure head in lbs. per square foot, or difference between the atmospheric and condenser pressures, that is, $144(p_a - p_0)$, and w = weight per cubic foot of water.

As the wheel rotates the water is carried round in the chamber, and there is frictional resistance at the sides of A and B, and on the outer surface of the chamber at C.

Assuming that the friction varies as the area and the square of the velocity, then the friction force on an element of the side area of B at radius r and radial depth dr is

$$F = f(2\pi r dr V^2) \text{ lbs.}$$

where f = a coefficient.

$$V = \omega r = \text{velocity of the water in ft./sec.}$$

The friction work done on the elemental ring is

$$dE = FV = f 2\pi r^4 \omega^3 dr$$

Total friction work on side B is

$$E_b = 2\pi f\omega^3 \int_{r_1}^{r_3} r^4 dr$$

$$= \tfrac{2}{5}\pi f\omega^3(r_3^5 - r_1^5)$$

Similarly the friction work done on side A is

$$E_a = \tfrac{2}{5} f\pi\omega^3(r_3^5 - r_2^5)$$

The friction work done on the circumference of the chamber is

$$E_c = F_3 V_3 = f(2\pi r_3 b)\omega^2 r_3^2 \times \omega r_3 = 2\pi f\omega^3 b r_3^4$$

The total friction work is $E_a + E_b + E_c$,

or $$E_f = 2\pi f\omega^3\{\tfrac{1}{5}(2r_3^5 - r_1^5 - r_2^5) + br_3^4\} \quad . \quad . \quad . \quad (9)$$

Since ω is in radians per sec. and r in feet, the result is in ft.-lbs./sec.

The value of the coefficient f is rather uncertain. It may be taken between 0·004 and 0·008.

The horse power absorbed is

$$HP_f = \frac{E_f}{550} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The heat generated is

$$H_f = \frac{E_f}{778} \text{ B.Th.U./sec.}$$

or $$H_f = 0{\cdot}706 \text{H.P.}_f \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If L_0 = latent heat at the condenser pressure p_0 the water evaporated should be

$$\omega_0 = \frac{H_f}{L_0} \text{ lbs./sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This is the minimum supply required. A large quantity is necessary to allow for leakage.

EXAMPLE 9.—The water-seal chamber of an impulse turbine has a diameter of 18 inches and width of $2\frac{1}{4}$ inches. The speed of the turbine is 1500 R.P.M. The external pressure is 15 lbs./in.^{2} abs. and the internal pressure 1 lb./in.^{2} abs. If the diameter of the ring of water at the side of the impeller next the turbine is 14 inches, calculate the approximate horse power absorbed by the two gland seals, and the minimum amount of feed water required for the glands.

Here the angular velocity

$$\omega = \frac{3{\cdot}1416 \times 2 \times 1500}{60} = 157$$

$$\omega^2 = 24650$$

$$P = 144(15 - 1) = 144 \times 14, \quad r_1 = \tfrac{7}{12} \text{ feet}, \quad w = 62{\cdot}5.$$

Substituting in equation (8)

$$\frac{24650}{64{\cdot}4}\left(r_2^2 - \frac{49}{144}\right) = \frac{144 \times 14}{62{\cdot}5}$$

$$\therefore\ r_2^2 = 0{\cdot}0845 + 0{\cdot}34 = 0{\cdot}4245$$

$$\therefore\ r_2 = 0{\cdot}652 \text{ feet} = 7{\cdot}84 \text{ inches}$$

The difference of head h, Fig. 151, is thus less than 1 inch

$$r_3 = 9'' = 0{\cdot}75 \text{ ft.}, \quad r_1 = 7'' = 0{\cdot}583 \text{ ft.}, \quad r_2 = 7{\cdot}84'' = 0{\cdot}652 \text{ ft.},$$

$$r_3^5 = 0{\cdot}2375, \quad r_3^4 = 0{\cdot}316, \quad r_1^5 = 0{\cdot}0674, \quad r_2^5 = 0{\cdot}118,$$

$$b = \frac{2{\cdot}25}{12} = 0{\cdot}187, \quad \tfrac{1}{5}(2r_3^5 - r_1^5 - r_2^5) = \tfrac{1}{5} \times 0{\cdot}2896 = 0{\cdot}0579;$$

$$br_3^4 = 0{\cdot}187 \times 0{\cdot}316 = 0{\cdot}06.$$

Substituting in equation (9)

$$E_f = 2\pi f\omega^3 \times 0{\cdot}1179$$

Assuming $f = 0{\cdot}005$, then by equation (10)

$$\text{H.P.}_f = \frac{6{\cdot}28 \times 0{\cdot}005 \times 157^3 \times 0{\cdot}1179}{550} = 26 = 19{\cdot}4 \text{ K.W.}$$

The total friction loss for the two water-seals thus appears to amount to 52 horse power, or 39 kilowatts, and would represent about 2 per cent. loss for a 2000 K.W., and about 0·8 per cent. for a 5000 K.W. machine running at 1500 R.P.M.

By equation (11) the total friction heat is

$$H_f = 0{\cdot}706 \times 52 = 36{\cdot}7 \text{ B.Th.U./sec.}$$

Latent heat at condenser pressure of 1 lb./in.^{2}, $L_0 = 1035$.

By equation (12) the minimum water supply required is

$$w_0 = \frac{36{\cdot}7 \times 60}{1035} = 2{\cdot}12 \text{ lbs./min.}$$

It will be obvious, that since the friction horse power varies as the fifth power of the wheel diameter, this diameter should be kept as small as possible, to reduce the loss to a minimum.

149. Stage Diaphragm Glands of Impulse Turbines.—In a multi-stage impulse turbine, where a disc-and-shaft rotor is used, the gland at each stage diaphragm has to be fitted with some packing device which will as far as possible prevent leakage from stage to stage, and at the same time permit of a slight deflection of the shaft, without risk of seizure.

When the packing is of the fixed type a certain amount of clearance has to be left between the shaft and packing, or wheel hub and packing.

The form of diaphragm gland packing fitted on Zoelly machines is shown in Fig. 153 (*a*). The diaphragm, 1, has several grooves cut at the inner circumference, and into these brass packing rings, 2, are fitted. Each ring is reduced to a fine edge. These rings are a close fit on the wheel hubs, 3, which, with the press rings, 4, used to fix the wheels securely on the shaft, form a continuous shaft liner. The ring edges are so fine that they are simply worn down if a "touch" occurs, and no seizure ensues, although the leakage area is thereby increased. In a machine where there is considerable difference of pressure between the two sides of the gland, this packing is replaced by an inserted gland, fitted with carbon rings, similar to those used for the external glands.

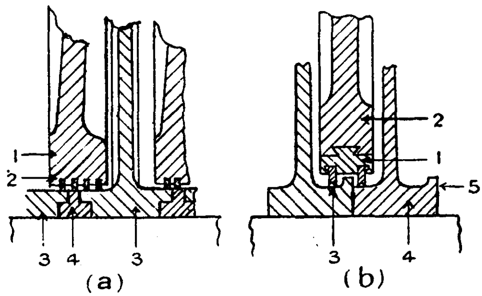


FIG. 153.

Another form of packing used in Westinghouse impulse turbines is shown at Fig. 153 (*b*). In this case a ring, 1, dovetailed into the diaphragm, 2, carries two copper rings, 3, which are grooved at the inner circumference to form two sharp edges. The wheel bosses, 4, are butted, and each boss is provided with a collar, 5, which fits between the two packing rings, 3. The combination thus forms a radial labyrinth having three throttlings.

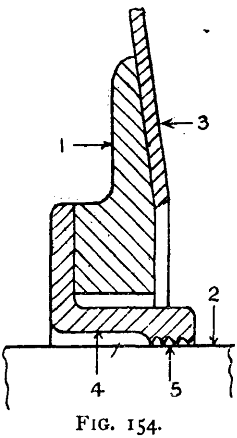


FIG. 154.

Another type of packing used in marine Curtis turbines is shown in Fig. 154. The boss, 1, with internal diameter greater than that of the shaft, 2, is bolted to the steel plate diaphragm, 3. A long bush, 4, is bolted to the casting and projects into it. This is quite clear of both boss and shaft except at the inner end, 5, where it is of reduced diameter, and provided with sharp serrations. If the shaft deflects and produces a "touch," the thin edges are simply worn away. If the touch is considerable, the long bush is sufficiently elastic to spring slightly, and accommodate itself to the shaft without seizure.

A modification of this form is shown in Fig. 155. It is the bushing fitted at the high-pressure end of the Franco Tosi marine turbine, Fig. 48.

The diaphragm, 1, which is made of forged steel for lightness, is provided with a boss sleeve, 2, into which are caulked the packing rings, 3. These are made of brass, and thinned down to a fine edge. The hub, 4, of the disc wheel, 5, is pressed on to the shaft, 6, but the packing rings do not come next this, as in the previous cases. A thin sleeve, 7, bolted to the wheel and quite clear of the hub, is interposed between hub and packing. If a serious "touch" should occur, pro-

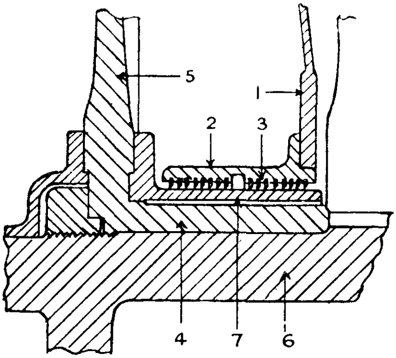


FIG. 155.

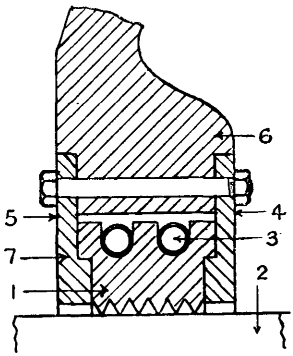


FIG. 156.

ducing considerable friction heat, no distortion of the shaft can take place.

The most effective form of packing, so far devised, is a type now used on Curtis turbines, where there is a considerable difference of pressure at each side of the gland. One design of this type is shown in Fig. 156. A serrated cast-iron bush, 1, cut in several segments, is held up to the surface of the shaft, 2, by garter springs, 3. It is made a good fit between the plates 4 and 5, fastened by bolts to each side of the diaphragm boss, 6. The ring and plates are shouldered at 7 to prevent too much pressure on the shaft. The ring is first made slightly less in diameter than the shaft, and worn down to a good fit by turning the shaft slowly. The segments can move out radially between the plates, and if the shaft should deflect slightly, they are simply pressed outward against the resistance of the springs. This type of gland has provided a satisfactory solution of the leakage problem at stage diaphragms.

150. Calculation of Leakage at Diaphragm Glands.—In the few-stage impulse machine the drop of pressure between stages is, as a rule, in excess of the critical, and at each diaphragm the approximate leakage can be calculated by equations (1) and (2), p. 271, as in the case of the external glands.

Some allowance may be made, if desired, for the frictional effect of the serrated bushings in restricting the flow, and also the fact that the contracted jet area is less than the clearance area. These bushings are usually referred to as labyrinths, but they do not baffle and change the direction of flow like a true labyrinth. The steam can take the "line of least resistance," and blow through the clear area between the ring edges and shaft or wheel boss, although the serrations probably produce some eddy effect which retards the flow.

The clearance area may be estimated for a clearance, in the case of the fixed type, of from 0·01 to 0·04 inch.

At any stage the heat of the leakage steam is not available for work on the blading, and this leakage is one of the minor causes of reduction in stage efficiency (see Art. 97).

If $W =$ total steam passing through any stage in lbs./sec.,
$w =$ steam leakage at the diaphragm gland in lbs./sec.,
$h_r =$ heat drop in the stage,
then total heat available for work $= Wh_r$.

Available heat bye-passed through the gland $= wh_r$.

$$\therefore \text{ Percentage loss of available heat in the stage} = \frac{w}{W} \times 100 \quad . \quad (13)$$

This percentage diminishes from the high- to the low-pressure end of the machine. In the application of the general method of determining the proportions of multistage turbines, developed later, a mean value of the stage efficiency for the whole machine is taken for the construction of the cumulative heat curve. For the estimation of the cumulative heat the mean value of the leakage loss, and also the disc and vane friction loss for the whole machine, is assumed.

EXAMPLE 10.—The absolute pressures and qualities of the steam at the high-pressure sides of the three diaphragms of a four-stage Curtis turbine are approximately—

Diaphragm	1	2	3
Pressure	62	18	4
Quality	118° F.	35° F.	q=0·975

The gland diameter at each diaphragm is 10 inches, and the clearances may be taken as 0·015, 0·03, and 0·04 inch respectively. The exhaust pressure is 0·75 lb./in.² abs.; and the weight of steam passing through the turbine 9·7 lbs./sec. Calculate the probable percentage loss of heat due to leakage at each stage.

There is no leakage in the first stage of an impulse turbine, since the steam passes directly to the nozzles from the supply pipe.

In the second stage the steam expands in the nozzles and diaphragm clearance, from 62 to 18 lbs./in.², and the critical drop is thus exceeded. The same condition holds for the second and third diaphragm glands.

At the first diaphragm, the leakage area $A_1 = 31{\cdot}41 \times 0{\cdot}015 = 0{\cdot}47$ in.^{2}. The steam volume at 62 lbs./in.^{2} and 118° F. superheat is $v_{s1} = 8{\cdot}24$ ft.

Hence, leak at first diaphragm

$$= w_1 = 0{\cdot}3155 \times 0{\cdot}47 \times \sqrt{\frac{62}{8{\cdot}24}} = 0{\cdot}405 \text{ lb./sec.}$$

By equation (13)

$$\text{Percentage loss of heat in second stage} = \frac{100 \times 0{\cdot}405}{9{\cdot}7} = 4{\cdot}17 \text{ per cent.}$$

This is probably on the high side, as no allowance is made for the effect of the serrations and the jet contraction at entrance to the clearance.

On the other hand, a larger clearance might exist, and balance out this difference. It seems reasonable to assume a leakage of 4 per cent. for the second stage.

For the second diaphragm the steam volume is 23·3 ft.^{3} and the area 0·942 in.^{2}

For the third diaphragm the values are 89 ft.^{3} and 1·256 in.^{2}. The corresponding leakages, calculated as above, are 0·26 and 0·085 lb./sec. The percentage leak at the third stage is thus 2·68 per cent., and at the fourth stage 0·875 per cent. The average for the whole machine is thus about 2·6 per cent.

151. The calculation of the approximate diaphragm leakage loss, as shown by the previous example, is simple enough for a few-stage impulse, where the critical drop is exceeded at each stage. In the multistage type, where this drop is not exceeded, the exact calculation for a considerable number of stages is tedious, as it is necessary to determinine the steam volumes at exit from the successive diaphragm glands.

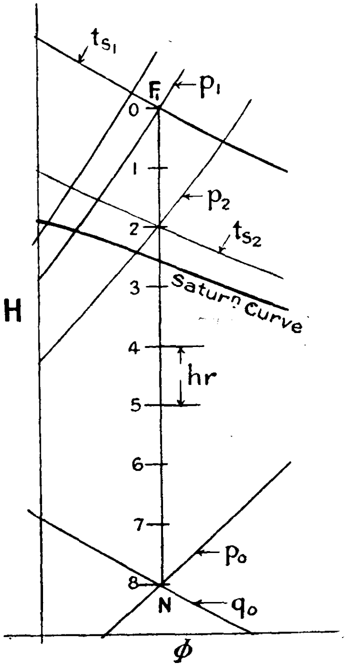


FIG. 157.

An approximation, sufficiently good for the purpose in view—that is, the determination of the average value of the leakage per diaphragm for the whole machine—can be obtained in the following manner. Draw the adiabatic vertical in the $H\phi$ chart between the initial and exhaust pressures p_1 and p_0 as indicated by F_1N, Fig. 157. Divide the vertical into the same number of equal parts as there are stages in the machine. Each division will give approximately the stage heat drop. The first division does not enter into the calculation. Beginning at point No. 2, read the pressures and qualities shown on the chart; then tabulate and find the corresponding volumes v_0.

If an average value of clearance is assumed for an average shaft diameter, the average clearance area A_0 for the diaphragm is determined. With the approximately constant heat drop h_r and a nozzle efficiency of, say, 85 per cent., to allow for serrations, the constant velocity of exit V_0 from each gland is calculable by the equation

$$V_0 = 223{\cdot}7\sqrt{0{\cdot}85 h_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

At any gland the leakage is given by

$$w = \frac{A_0 V_0}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and the percentage loss of heat $= 100 \times \dfrac{A_0 V_0}{W v_0} = \dfrac{\text{constant}}{v_0} \quad . \quad (16)$

Towards the L.P. end, when the number of stages is considerable, this loss becomes negligible.

EXAMPLE 11.—A pressure-compounded impulse turbine having eight stages works between 185 lbs./in.^{2} abs., superheat 200° F., and 0·75 lb./in.^{2} abs. The steam consumption is 7·5 lbs./sec. The average diameter of the shaft may be taken as 5 inches, and the average clearance, with the fin-edged packing (*a*), Fig. 153, as 0·01 inch.

Calculate the approximate leakage loss for the second and succeeding stages, and the mean value for the whole machine.

Applying the method of Art. 151, divide the adiabatic F_1N, Fig. 157, into eight parts. The total heat drop is 388 B.Th.U., so that each portion represents 48·5 B.Th.U.

The following values are read from the chart at the various points:

No.	2.	3.	4.	5.	6.	7.	8.
Pressure	70	38	20	10	4·6	2	0·75
Quality.	60° F.	q=0·99	0·955	0·92	0·887	0·855	0·825
Volume	6·8	11	19·8	35	70	148	360
Percentage loss of heat	3·03	1·89	1·05	0·595	0·298	0·141	0·058

Clearance area $A_0 = 5\pi \times 0{\cdot}01 = 0{\cdot}157$ in.^{2}. The value of the exit velocity will depend on what allowance is made for the serrations in the choice of the efficiency factor. Taking $\eta_n = 0{\cdot}85$ and $h_r = 48{\cdot}5$, $V_0 = 223{\cdot}7\sqrt{0{\cdot}85 \times 48{\cdot}5} = 1430$ ft./sec.

At any stage the approximate heat loss by equation (16) is

$$\text{Percentage heat loss} = \frac{0{\cdot}157 \times 1430 \times 100}{144 \times 7{\cdot}5 \times v_0} = \frac{20{\cdot}8}{v_0}$$

Substituting the volume values given in the table, the percentages in the last line are obtained.

The average value is just over 1 per cent.

It will be noted that the loss is most pronounced at the first diaphragm, that is, at the second stage of the machine, and that it rapidly decreases at the succeeding stages. The figures can only be regarded as rough approximations; but the calculation seems to indicate that the average value of the loss is probably below the order of 1·5 per cent.

The pressures obtained are higher than the initial pressures in the stage chambers, from 3 to 8 given in example 9, p. 144. When the more accurate pressure and quality values are known from a condition curve, this rougher method of calculation need not be used.

152. Disc and Vane Friction Loss.—When a disc wheel with one or more rings of blades is rotated in a fluid such as steam or air, there is a conversion of kinetic energy into heat by (1) disc friction, (2) vane action of the blades.

The first effect is due principally to the "pumping" action of the disc, which acts much in the same way as the impeller wheel of a centrifugal pump. When the disc is rotated in air or steam, in a large chamber, the weight of fluid taken on to the surface at the centre and thrown off at the circumference will vary as the area of the disc and the peripheral speed. Thus if u is the peripheral speed and D the diameter, the weight, $W \propto uD^2$.

The kinetic energy imparted to the fluid and dissipated in shock and eddies is proportional to the square of the speed, so that the work done $E \propto u^3D^2$ or $E \propto \omega^3D^5$, where ω is the angular velocity.

Experiment has shown that the work done increases directly as the density of the fluid, or that it is inversely proportional to the specific volume, viz. $E \propto \frac{1}{v}$.

As a first approximation it may thus be stated that the horse power absorbed in disc friction is

$$\text{H.P.}_d = \frac{cu^3D^2}{v}$$

where c is some constant.

Experiments by Odell,[1] Stodola and Lewecki,[2] have confirmed the above assumptions, as far as air is concerned.

Stodola's equation for the friction horse power has the form

$$\text{H.P.}_d = cD^n\left(\frac{u}{100}\right)^m \frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where c, n, and m are constants. The values given are $c = 0{\cdot}071$, $n = 2$, $m = 3$. The indices agree with those of the theoretical equation deduced above.

Moyer, from an investigation of numerous tests of Curtis turbines,[3] has derived the values $c = 0{\cdot}08$, $n = 2$, $m = 2{\cdot}8$.

As the effective pumping action of the wheel on the steam depends

[1] See *Engineering*, January, 1904.
[2] See Stodola, "Steam Turbine," p. 129.
[3] See "Steam Turbine," by J. A. Moyer, chap. v.

on the freedom of the steam to circulate in the containing chamber, it will be evident that a restriction of the chamber width will diminish this action, and reduce the power absorbed. The disc, especially at the high-pressure stages of multistage turbines, should be encased as well as possible by the adjacent diaphragms.

The second friction effect is due principally to the churning action of the blades on the steam at the portion of the circumference not covered by the nozzle arc. An eddying effect is produced at the blade edges. It is also assumed by some investigators that there is an axial pumping action through the blade passages. The friction loss due to the blade action is in consequence called the "ventilating" loss. This latter assumption is somewhat doubtful.

Experimental determinations of the loss from vane action show that the horse power absorbed is given by an equation of the form

$$\text{H.P.}_b = aDl^z\left(\frac{u}{100}\right)^n \frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $l =$ length of blade in inches, $u =$ peripheral speed in ft./sec., $D =$ diameter in feet, $v =$ specific volume in ft.^{3} a, z, and n are constants.

The values given by Stodola for discs rotating freely in air are $a = 0·6$, $z = 1·5$, $m = 3$, and by Moyer (from Curtis turbine tests), $a = 0·3$, $z = 1·5$, $m = 2·8$. There is considerable difference here between the values of the constant a.

Stodola's figures are obtained from wheels running "forward." When run backward and the wheel is encased so that the face edges encounter the fluid, the loss is increased from 20 to 30 per cent. When run in the open five to six times the power for forward running is necessary.

In order to reduce the vane action as much as possible, some makers now fit angle rings close to the blading round the circumference, where it is not covered by the nozzle arc. A ring is also fitted between the blades of velocity-compounded machines to further diminish the vane loss (see Fig. 12).

This vane loss does not occur over any arc where there is a continuous flow of steam from the nozzles through the blading.

It is a gradually diminishing quantity from the first stage to the stage at which full peripheral admission occurs. At this stage it is zero.

The effect of the variation of the number of rings on the vane loss is somewhat indeterminate. The only apparent experimental data available for guidance are results obtained by Lasche and quoted by Stodola. Unfortunately the figures given account for disc as well as vane loss, and the constants merely serve the purpose of giving relative values for the coefficients which may be introduced in Stodola's equation.

Lasche's equation is of the form

$$\text{H.P.} = 10^{-9} bDlN^3\frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where $D =$ mean ring diameter in ft.
$l =$ blade length in ft.
$N =$ revolutions per minute.
$v =$ specific volume of steam ft.^{3}/lbs.
$b =$ constant.

The values of b for different numbers of rings are given as

No. of rings	1	2	3	4
b	34·9	40·9	55·9	84·8

It is assumed here that the whole circumference of the blade ring is effective in producing "friction." This is never the case, even at the first high-pressure stage, where a small fraction of the circumference is necessarily covered by the nozzle arc.

Stodola quotes the result of an investigation on the effect of the arc of admission carried out by Jasinsky on a 50 H.P. de Laval turbine; but the resulting expression is too clumsy for practical use. Considering that the correction is required over only a limited section of the machine, and is very uncertain in any case, it seems sufficient, for practical purposes, to take the loss as directly proportional to the fraction of the circumference which is not covered by the nozzle arc.

153. Some important experimental work on this subject has been carried out by W. Kerr on a 250 K.W. impulse turbine at the Royal Technical College, Glasgow.[1] These experiments show that the horse power of the three-ringed disc wheel of the machine is given by

$$\text{H.P.} = 3{\cdot}016\left(\frac{u}{100}\right)^3\frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Lasche's equation (19), when reduced to the same form, is

$$\text{H.P.} = 4{\cdot}32\left(\frac{u}{100}\right)^3\frac{1}{v}$$

This gives too high a value of friction for the wheel tested, and is thus not reliable.

When Stodola's complete equation, $\text{H.P.} = \left(\frac{u}{100}\right)^3(cD^2 + aDl^{1\cdot5})\frac{1}{v}$ for air, is also expressed in the same form, this becomes for a single-ring wheel

$$\text{H.P.} = 1{\cdot}923\left(\frac{u}{100}\right)^3\frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The ratio of the constants of the two equations (20) and (21) is $\frac{3{\cdot}016}{1{\cdot}923} = 1{\cdot}58$. Now, taking Lasche's figures for one and three rings respectively, the ratio of these is $\frac{55{\cdot}9}{34{\cdot}9} = 1{\cdot}6$.

[1] See paper on "The Steam Friction of Turbine Wheels," by William Kerr, *Transactions of Scientific Society of Royal Technical College*, vol. ix. part 4, 1912–13, or *Engineering*, August 22, 1913.

These results appear to indicate that by inserting in Stodola's complete equation for air, the ratios of the constants of Lasche's equation, the coefficient for vane resistance may be modified to take account of the number of rings per wheel. When this is done, the equation for vane loss, taking account of both the number of rings and also arc of admission, can be written in the form

$$\text{H.P.}_b = a\text{B}s\text{D}l^{1\cdot5}\left(\frac{u}{100}\right)^3\frac{1}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where s = fraction of the mean circumference not covered by the arc.
D = mean ring diameter in ft.
l = blade length in inches.
u = mean blade speed in ft./sec.
v = specific volume of the steam in ft.^{3}.
$a = 0{\cdot}458$.

B is a correction factor for the number of rings. The values obtained by Kerr are:

No. of rings	1	2	3	4
B	1	1·23	1·8	2·9

The value of a given by Kerr lies between the values given by Stodola and Moyer. For the disc friction he gives $c = 0{\cdot}0607$, which is lower than Stodola's value for air.

154. For the whole turbine

$$\text{Total disc friction H.P.}_d = 0{\cdot}0607\text{D}^2\left(\frac{u}{100}\right)^3\Sigma\frac{1}{v} = \text{K}_d\Sigma\frac{1}{v}$$

$$\text{Total vane friction H.P.}_b = 0{\cdot}458\text{D}\left(\frac{u}{100}\right)^3\Sigma\frac{\text{B}sl^{1\cdot5}}{v} = \text{K}_b\Sigma\frac{\text{B}sl^{1\cdot5}}{v}$$

The corresponding heat values are given by

$$\text{H}_d = 0{\cdot}7066\text{K}_d\Sigma\frac{1}{v}\ \text{B.Th.U./sec.}$$

$$\text{H}_b = 0{\cdot}7066\text{K}_b\Sigma\frac{\text{B}sl^{1\cdot5}}{v} \quad ,, \quad ,,$$

If h_r = the average value of the heat drop per stage,
n = number of stages or wheels,
W = total steam consumption in lbs./sec.,
then the average loss per stage is given by

$$\left.\begin{array}{r}\%\ \text{loss from disc and}\\ \text{vane friction}\end{array}\right\} = \frac{70{\cdot}66}{\text{W}nh_r}\left(\text{K}_d\Sigma\frac{1}{v} + \text{K}_b\Sigma\frac{\text{B}sl^{1\cdot5}}{v}\right) \quad . \quad . \quad . \quad . \quad (23)$$

155. In the Parsons reaction turbine the friction loss due to drum and blading is very small and need not be considered. There is full peripheral admission at each ring, so that the vane action is eliminated. It has been suggested, that in the case of the astern marine turbine there is an appreciable loss from the vane action of the astern blading, when run in the ahead direction in the exhaust steam. Experiments carried out by the Parsons Company at the Turbania Works have disproved this contention. It has been shown that for an astern turbine, when run at the high speed of 1500 R.P.M., the drum and blade loss is only 0·3 of 1 per cent. at 28 inches vacuum, and 0·6 of 1 per cent. at 26 inches vacuum.

It may, therefore, be assumed that in general the loss at the astern turbine does not amount to more than $\frac{1}{2}$ of 1 per cent.

EXAMPLE 12.—In a four-stage Curtis turbine, the mean ring diameter of each wheel is 69 inches, and the blade velocity 455 ft./sec. There are two rings of blades on each wheel, and the approximate values of the successive mean blade lengths are 1, $1\frac{1}{2}$, 2, and 5 inches. The arcs of admission are approximately 12, 20, 60, and 97 per cent. of the mean circumference. The steam volumes are 8·24, 23·3, 89, and 408 ft.^{3} respectively. The heat drop per stage is 103 B.Th.U., and the steam consumption 9·7 lbs./sec. Calculate the loss due to disc and vane friction at each wheel, and the average loss per stage for the whole machine.

The solution is most expeditiously carried out in tabular form.

For each wheel, since $n = 1$, the general equation (23) becomes

$$\text{Percent. loss} = \frac{70·66}{Wh_r}\left(K_d\frac{1}{v} + K_b\frac{Bsl^{1·5}}{v}\right)$$

Here $$K_d = 0·0607D^2\left(\frac{u}{100}\right)^3 = 0·0607 \times (5·75)^2\left(\frac{455}{100}\right)^3 = 189$$

$$K_b = 0·458D\left(\frac{u}{100}\right)^3 = 0·458 \times 5·75 \times \left(\frac{455}{100}\right)^3 = 248$$

No. of wheel.	B.	s.	$l^{1·5}$.	$\frac{1}{v}$.	$\frac{Bsl^{1·5}}{v}$.	Percentage loss.
1	1·23	0·88	1	0·121	0·131	3·92
2	1·23	0·8	1·84	0·043	0·078	1·96
3	1·23	0·4	2·83	0·01125	0·0157	0·431
4	1·23	0·03	11·2	0·00245	0·00101	0·0505

$$\text{For wheel No. 1 percent. loss} = \frac{70·66}{9·7 \times 103}(189 \times 0·121 + 248 \times 0·131)$$
$$= 0·0708 \times 55·4 \doteqdot 3·92 \text{ per cent.}$$

The losses for the other wheels similarly calculated are entered in the last column of the table. The greatest loss occurs at the first wheel. The loss at the second is about half that at the first. The average value for the whole machine is 1·6 per cent., and the first wheel loss is about 2·5 times the average, and that of the second 1·22 times. The figures taken for this example refer to the turbine, for which the general proportions are calculated in example 3, Chap. XIII. The approximate diaphragm gland leakage has been calculated in example 10, and the values of the coefficient γ for insertion in the stage efficiency equation are now calculable.

At the first stage the percentage leak is zero. The loss is due to disc and vane friction only, hence $\gamma \doteqdot 0.96$. This is the value used in example 6, p. 189. At the second stage, leakage $= 4$ per cent., friction $= 1.96$ per cent., total loss $= 5.96$ per cent., and $\gamma \doteqdot 94$ per cent. At the third stage, leakage $= 2.68$ per cent., friction $= 0.431$ per cent., total loss $= 3.11$ per cent., and $\gamma \doteqdot 97$ per cent. At the fourth stage, leakage $= 0.875$ per cent., friction $= 0.050$ per cent., total loss $= 0.926$ per cent., and $\gamma \doteqdot 98$ per cent.

The average value of γ for the whole machine is 96.5.

In example 6, Chap. VII., p. 138, the nozzle efficiencies are given, and the average efficiency value is 93.3 per cent. In example 6, Chap. VIII., p. 189, the diagram efficiency, which may be assumed constant, is given as 0.72. The average stage efficiency for the machine by equation (38), p. 189.

$$\eta_s = \gamma\eta_n\eta_d = 0.965 \times 0.933 \times 0.72 \doteqdot 0.65$$

EXAMPLE 13.—In an eight-stage pressure compounded impulse turbine the mean ring diameter of each wheel is 39 inches, and the blade velocity 511 ft./sec. The successive blade lengths are $\frac{3}{4}$ inch from first to fifth wheel, and $1\frac{1}{4}$, $2\frac{1}{2}$, and $4\frac{3}{4}$ inches at the last three wheels. The arcs of admission are 11.1, 17.15, 28.4, 50.2, 92, 98, 98, 98 per cent. The steam volumes are 5.2, 8.3, 13.2, 23, 42, 85, 187, and 403 ft.^{3}. The steam consumption is 7.5 lbs./sec. The heat drop at the first stage is 55 B.Th.U., and at the other stages 51.

Calculate the loss due to disc and vane friction at each stage, and the average value for the whole machine.

Here
$$K_d = 0.0607(3.25)^2(\tfrac{511}{100})^3 = 85.5$$
$$K_b = 0.458 \times 3.25(\tfrac{511}{100})^3 = 199$$

Tabulate as before.

No. of wheel.	B.	s.	$l^{1.5}$.	$\frac{1}{v}$.	$\frac{Bsl^{1.5}}{v}$.	Percentage loss.
1	1	0.889	0.65	0.194	0.112	6.64
2	1	0.8285	0.65	0.121	0.066	4.32
3	1	0.716	0.65	0.0760	0.0354	2.49
4	1	0.498	0.65	0.0435	0.014	1.18
5	1	0.080	0.65	0.0238	0.00124	0.43
6	1	0.020	1.4	0.0118	0.00033	0.198
7	1	0.020	3.95	0.00535	0.000424	0.105
8	1	0.020	10.3	0.00248	0.000511	0.058

For No. 1 wheel since $h_r = 55$ and $W = 7{\cdot}5$

$$\text{Percentage loss of heat} = \frac{70{\cdot}66}{7{\cdot}5 \times 55}(85{\cdot}5 \times 0{\cdot}194 + 199 \times 0{\cdot}112)$$
$$= 0{\cdot}171 \times 38{\cdot}85 = 6{\cdot}64$$

For No. 2 wheel since $h_r = 51$

$$\text{Percentage loss} = \frac{70{\cdot}66}{7{\cdot}5 \times 51}(85{\cdot}5 \times 0{\cdot}121 + 199 \times 0{\cdot}066)$$
$$= 0{\cdot}184 \times 23{\cdot}5 = 4{\cdot}32$$

The other losses are similarly calculated and entered in the last column. The average for the whole machine is 1·9 per cent.

The loss at the first wheel is considerable, and is 3·3 times the average. That at the second is also appreciable, and 2·24 times the average. The high loss is due to the high pressures and steam densities, and also to the fairly high blade velocity of 511 ft./sec.

Taking the average value of the leakage loss as deduced in example 11, the average for leakage and friction is 3 per cent., and the average $\gamma = 0{\cdot}97$.

The heat expended in work on the blades from example 1, p. 167, is 36·2 B.Th.U. If the additional heat at the first stage is neglected, and the average for the whole machine taken as 51 B.Th.U., the average value of the stage efficiency becomes

$$\eta_s = \frac{36{\cdot}2 \times 0{\cdot}97}{51} = 0{\cdot}69$$

The actual average stage heat drop based on the cumulative heat of the turbine, as calculated in Chap. XI., is 51·5. This gives $\eta_s = 0{\cdot}683$.

For the purpose of drawing the condition curve in the $H\phi$ diagram 0·69 is quite a reasonable value to use. With the speed ratio $\rho = 0{\cdot}322$ given for the work calculation on p. 167, the fourth curve of Fig. 114, as already pointed out, indicates an efficiency from 70 to 71 per cent.

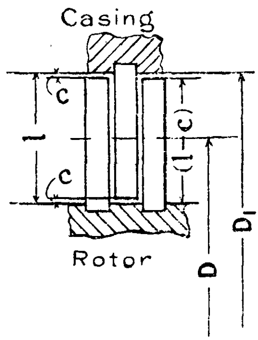


FIG. 158.

156. Leakage Loss at Blade Tips in Reaction Turbines.—Leakage in the Parsons machine occurs at the tips of the fixed and movable rings, between drum and casing. Referring to Fig. 158, this shows two moving blades on the drum, and the intermediate fixed blade on the casing. The tip clearance is denoted by c.

When the steam, continuously expanding, flows out of the moving blade at the left, a stream of radial depth, c, passes between the entrance and exit sides of the fixed blade. This stream may or may not flow across the blade width in the same direction as the main stream in the blade channel above, and finally leave the blade at the exit angle θ.

Recent experiments tend to confirm the assumption that this band of steam expands between entrance and exit in the axial direction. It may be inferred that it simply hits the edges of the next moving blade, and its energy is all practically reconverted to heat in shock and eddies. Again, at the top of the stream which leaves the fixed blade, there is another band of depth c which likewise flows across the width of the second moving blade, and again it may be assumed that the energy is practically wasted.

The steam passing the blade tips thus constitutes a leakage, similar to that at the diaphragm glands of the impulse machine.

157. Suppose, in the first instance, that there is no tip clearance, then the annular blade area between rotor and casing would be $\pi \mathrm{D}l$, where D is the mean ring diameter in inches, and l the distance between rotor and casing, or the theoretical blade length in inches. With a constant blade angle θ at exit, the weight of steam passing a given section of the turbine would be

$$\mathrm{W} = \frac{\pi \mathrm{D}l \sin \theta \mathrm{V}_0}{v_0 144} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

V_0 is the velocity due to the heat drop in the blade, and v_0 the corresponding specific volume at exit.

Suppose next that the clearance c is given. If the steam in the clearance space followed the main stream in the blade channel, the foregoing expression would still give the weight passing between the rotor and casing. Since, however, the leak is assumed to be axial in direction, the flow is increased. This may be regarded as the weight which would flow in the same direction as the main stream at angle θ, through an equivalent blade length $(l + ac)$, where a is some coefficient.

To find a, let $\mathrm{D}_1 =$ diameter at clearance space and $\mathrm{V}_0 =$ the steam velocity, which is the same irrespective of what direction the steam flows in.

With axial flow through the clearance c, the weight bye-passing the tip is

$$w = \frac{\pi \mathrm{D}_1 c \mathrm{V}_0}{144 v_0}$$

The equivalent clearance for the same leak is $(c + ac)$, when the flow is at θ and not axial, so that

$$w = \frac{\pi \mathrm{D}_1 (c + ac) \mathrm{V}_0 \sin \theta}{144 v_0}$$

Hence $$c = (c + ac) \sin \theta$$

and $$a = \left(\frac{1}{\sin \theta} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The values of a for given values of the blade angle θ are given by the curve, Fig. 159. For normal blades with $\theta = 20°$, $a = 1{\cdot}92$.

The weight of steam flowing between the rotor and the casing may thus be expressed by

$$W = \pi D \frac{(l + ac) \sin \theta V_0}{144 v_0} \text{ lbs./sec.} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

As the clearance steam cannot be reckoned as effective, the depth of the band of working steam which usefully expends energy

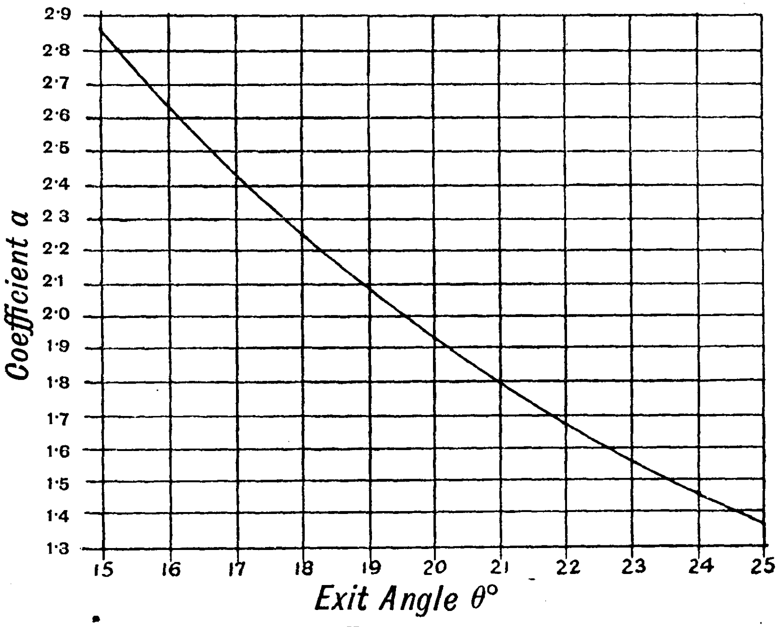


FIG. 159.

on the blades is $(l - c)$. Hence the weight of working steam is given by

$$W_1 = \frac{\pi D(l - c) \sin \theta V_0}{144 v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The leakage is $w = (W - W_1)$ and the percent. heat loss $= \dfrac{100 \times w}{W}$

or percentage loss $= 100(1 - \gamma)$

where $\gamma = \dfrac{W_1}{W}$. For a given group or expansion, γ is constant, as the clearance c remains constant. It increases from group since c does not increase proportionately with the blade length.

The loss consequently diminishes from the first to last expansion on any section of the drum.

Dividing (27) by (26) the ratio

$$\frac{W_1}{W} = \left(\frac{l - c}{l + ac}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

158. As indicated in Chap. VIII., Art. 109, the tip clearance may be expressed as a linear function of the mean blade ring diameter. Substituting the value of c from equation (40), p. 200, in equation (28), the ratio can be expressed in terms of the ring diameter for any constant values of the coefficient a and blade length l. For normal blades $\theta = 20^\circ$ and $a = 1{\cdot}92$. With this condition, the values of $\frac{W_1}{W}$ and D for a series of values of blade length have been calculated, and plotted to form the diagram shown in Fig. 160.

This diagram enables the value of γ for any given diameter and blade length to be at once ascertained by inspection. As numerous applications of this occur in connection with the calculations of blading proportions in Chap. XIV., special illustrations need not be given at this stage.

159. Bearings.—The bearings fitted to a steam turbine may be either of the fixed or spherically seated type. Except in the case of some disc and shaft rotors, where considerable stiffness is required, the majority of machines have spherical seated bearings.

The different designs in practical use are fairly well illustrated in the sections of the various machines given in Chaps. II., III., and IV.; and as the salient features of each have been pointed out in the descriptions, it is unnecessary to occupy space in further discussion of them.

One special bearing, however, is not shown in the illustrations, and deserves attention. This is the bearing of the Ljungström turbine, shown in Fig. 161. It is necessary, on account of the fine clearances between the blade rings, that the two shafts should be accurately centered, and maintained in alignment.

The centering arrangement consists of four screws, 1, screwed into the split bearing case, the halves of which, 2 and 3, are fastened together by the bolts, 4. These screws have spherical heads, 5, at the inner ends, which fit on to corresponding seatings in the blocks, 8. These blocks in turn fit on to the outer surface of each of the half bearings, 6 and 7. It will be evident that by adjusting the four screws the bearing centre can be raised or lowered, or set transversely as desired. A locking plate, 10, is fitted to each screw. The plate is provided with ten holes, 12, and the bearing case with eleven. The locking plate holes are tapped, so that any one can take a small pin, 9, the point of which projects into one of the holes in the bearing case. A fine adjustment is thus made possible, even with coarse pitch screws.

It is stated that this adjustment is kept with great precision, even after the turbine is dismantled and erected several times. It remains unaltered within limits of 0·0004 and 0·0008 inch. A fifth screw, 21, is used to provide for the axial adjustment of the bearing. It has an eccentric ball-shaped end, 22, which fits into a hole in the top of the

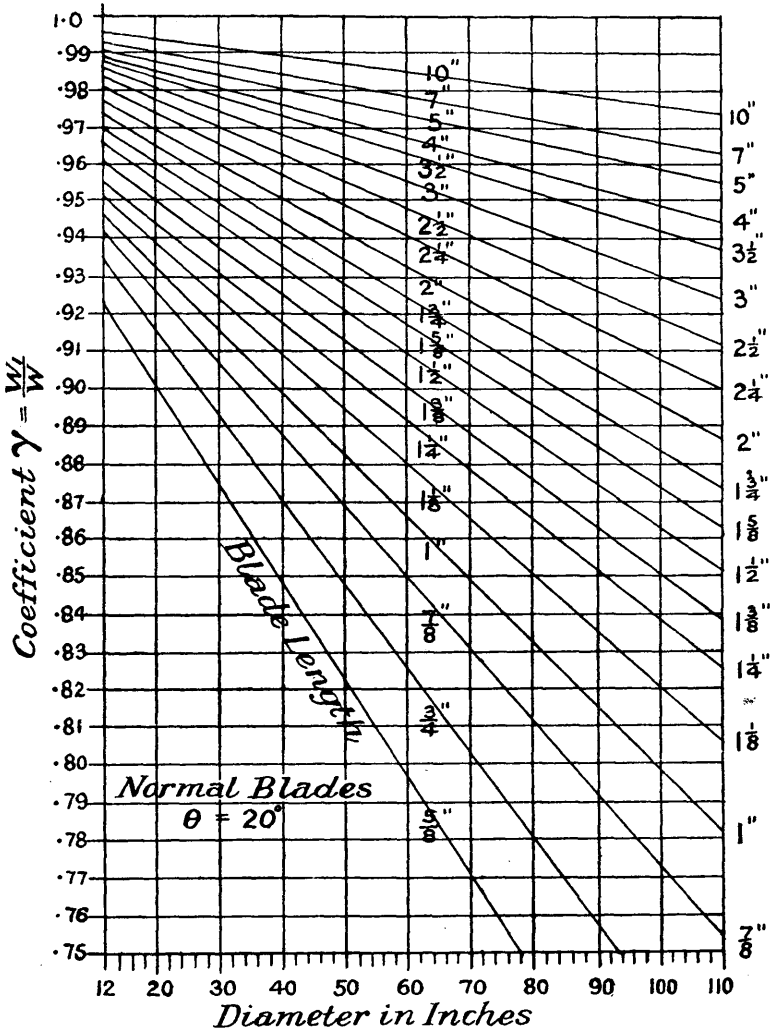


FIG. 160.

half bearing, 6. The two halves are bolted together by the recessed screws, 13, which are locked transversely and longitudinally by the pins, 14 and 15.

The lining, 16, of the bearing has oil channels, 17, supplied with oil through ports, 18, which are connected to the internal supply pipe, 19.

After passing along the bearing and out at the ends the oil falls to the bottom of the casing, and is drained back to the cooler, by the external pipe, 20. The oil grooves, 17, are also cut in the upper half of the shell, and are of sufficiently large area to permit of the efficient circulation of oil, about one pint per second, to cool the bearing. This forced oil circulation obviates the use of water cooling in the bearing.

The method of preventing leakage from the bearing is shown to

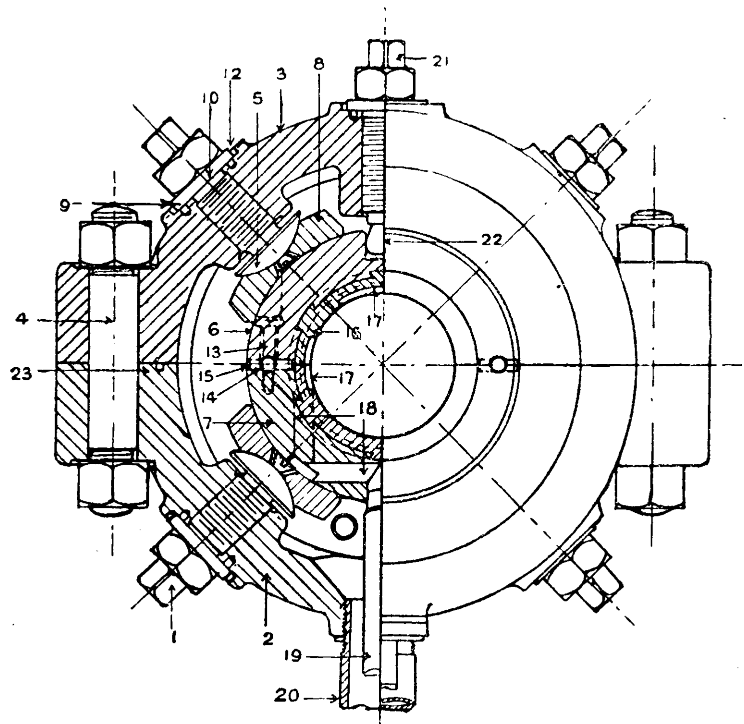


FIG. 161.

the left of Fig. 36, p. 59. The bearing casing, 28, has two grooves, 34 and 35, between which, and fitted close to the shaft 5, are thin packing rings, 36. A drain pipe is connected to 34, and groove 35 is connected to the drain from the bearing case, and the channel 23 cut along the lower half casing 2, Fig. 161. The latter prevents leakage of oil through the joint.

Another type of bearing, which is not illustrated in any of the machines described, is still used for machines running at from 3000 to 4000 R.P.M. This is the Parsons elastic sleeve bearing. The bearing case is fixed in the machine frame, but instead of the usual top and bottom brasses, a number of concentric sleeves are fitted. These fit loosely into one another with clearance varying from 0·002 to 0·006 inch, according to the size of the bearing. Oil is forced between them, and acts as an elastic cushion. The inner ring constitutes the bearing brass, and is flanged at one end, and provided with a ring nut at the other. By means of this, which acts as a flange when screwed in position, the inner sleeve is fixed loosely in the cushion-bearing block. The two outer sleeves are slipped over the inner sleeve, and kept in position by the flange and nut.

The inner sleeve is provided with oil grooves and a series of radial holes, to permit the oil to reach the outer sleeves. These are also perforated with a number of small holes for the same purpose. This bearing has to be slid endways on to the shaft. The hydraulic cushion, formed by the viscous oil film between the sleeves, damps the vibrations which are apt to arise at such high speeds.

This class of bearing is still fitted on some of the high-speed Zoelly turbines. The general practice, however, is to balance the rotor carefully and use the spherical seated bearing.

160. Calculation of Frictional Loss at Journal Bearings.—A turbine bearing is essentially a "high-speed" bearing. The journal diameters require to be kept large in order to obtain a stiff shaft. The skin stress, as already pointed out, is thus comparatively low, while the surface velocity as compared with that of the reciprocating engine shaft is high.

In land turbines the surface velocity of the journal may run from 30 to 60 ft./sec., and in marine turbines from 15 to 30 ft./sec.

As can be seen from the machines illustrated in Chaps. II., III., and IV., the length of these turbine bearings may vary from 2 to 3 times the diameter in land, and 1 to 2 in marine turbines.

In an ordinary "slow-speed" bearing, such as used for a reciprocating engine, increase of load at a low surface velocity tends to squeeze out the oil between the shaft and bearing, and a limit, depending on the viscosity of the oil, is reached at which contact between the metallic surfaces takes place. The law of solid friction for greasy surfaces then operates, the coefficient of friction increasing with the pressure. In the case of a high-speed bearing a different state of affairs exists. There is a certain limiting speed above which the friction is not affected by the pressure.

Beauchamp Tower's experiments [1] have shown that, in the case of a well-lubricated shaft running at high speed, the journal never comes in contact with the bearing. It is supported by a film of oil. Above the limiting speed the shaft produces a pumping action on the oil. The oil is dragged between the surface of shaft and bearing. The shaft has to continually shear through a film of oil, and friction work is done on the oil, with a resulting rise in temperature. This again has the secondary effect of reducing the viscosity, and a temperature may be reached

[1] *Proceedings Inst. Mech. Eng.*, 1885.

which causes so much reduction of viscosity that the oil may be squeezed out under the pressure, and metallic contact take place.

In a bearing of this type some means must be provided to carry off a sufficient portion of the friction heat and prevent undue loss of viscosity.

Water cooling of the bearing is sometimes tried; but the most satisfactory method is to pump sufficient oil through the bearing to carry off the heat. This oil is then cooled in an external cooler. The service piping and cooler arrangements have already been indicated in the descriptions of the machines in Chaps. II., III., IV.

In the case of very large bearings the oil is also caused to circulate in passages outside the bearings to assist in the cooling. At the high-pressure end the bearing may not only be heated by the journal friction, but also by conduction of heat from the steam along the shaft.

161. In practice, for land turbines, the temperature of the oil varies from 120° F. to 140° F. G. Stoney states[1] that it is not advisable to go above 160° F. owing to liability of the oil to oxidise. Also that 250° F. is the limit at which most oils cease to lubricate on account of the low viscosity produced.

The work done against friction at a journal is given by the general expression

$$E_J = \mu p l d V \text{ ft.-lbs./sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where μ = coefficient of friction.

p = pressure in lbs./in.^{2} of projected journal area.

l = length of journal in inches.

d = diameter " "

V = surface velocity of journal in ft./sec.

Experiments by Lasche have shown that with efficient lubrication and the maintenance of constant temperature and constant surface velocity, the increase of pressure does not increase the value of the friction work. This means that as p increases μ decreases, or in other words $\mu p = c$ is approximately constant. The value of c is not quite independent of the velocity V, but with values of V above 30 ft./sec. the increase is so slight as to be negligible. The curves obtained by Lasche are reproduced in the paper by Stoney, cited above.

With a constant temperature of about 122° F. and surface velocity 32 ft./sec., the mean curve of μ on a pressure base shows that $c = 0{\cdot}568$. Also the mean curve for μ on a temperature base shows that $\mu(t - 32) =$ constant, that is, for a given pressure, the value of c, varies inversely as the temperature, reckoned from the freezing-point.

For any temperature t the constant can thus be written in the form

$$c = \frac{0{\cdot}568(122 - 32)}{t - 32} = \frac{51{\cdot}12}{t - 32}$$

[1] See paper on "High Speed Bearings," *N.E.C. Inst. Engineers and Shipbuilders*, 1914, or *Engineering*, August 7, 1914.

Substituting in (29) for $c = up$, the friction work is given by

$$E_J = \frac{51 \cdot 12 ldV}{t - 32} \text{ ft.-lbs./sec.} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and the friction heat is

$$\left.\begin{aligned} h_J &= \frac{0 \cdot 0656 ldV}{t - 32} \text{ B.Th.U./sec.} \\ h_J &= \frac{236 ldV}{t - 32} \text{ B.Th.U./hour} \end{aligned}\right\} \quad . \quad . \quad . \quad (31)$$

162. In order that the maximum temperature at the bearing may not exceed t° F., this is the amount of heat that has to be carried away by the oil, if radiation is neglected.

Let K = specific heat of the oil.
ρ = specific gravity of the oil.
t_1 = temperature of oil supply.
t = final temperature of oil.
W = weight of oil in lbs. per sq. inch of projected surface, per min.

Then $$KWld(t - t_1) = \frac{236 ldV}{60(t - 32)}$$

and $$W = \frac{3 \cdot 936 V}{K(t - t_1)(t - 32)}$$

This quantity in gallons is

$$Q = \frac{W}{10\rho} \text{ gall./in.}^2 \text{ min.}$$

Assuming an average value of the final temperatuie, $t = 130^\circ$ F. and initial temperature $t_1 = 100^\circ$ F., $K = 0 \cdot 31$, and $\rho = 0 \cdot 88$.

$W = 0 \cdot 00431 V$

and $Q = 0 \cdot 00049 V$ gall./in.^{2} min.

With $V = 30$, $Q = 0 \cdot 0147$; and $V = 60$, $Q = 0 \cdot 0294$.

The practice of three turbine makers quoted by J. C. K. Balfry in a paper on "High Speed Bearings,"[1] is given as follows :—

Gall. per sq. inch per min.	Pressure in lbs./in.^{2}.
0·05	45 to 60
0·05	5 " 10
0·01	— —

In the first two cases the effect of the pressure, as might be expected, does not influence the quantity of oil circulated.

163. In fixing the proportions of a turbine bearing the diameter may either be settled for a low value of stress, and the surface velocity estimated, from the stated number of revolutions, or the velocity may

[1] See *Proc. Rugby Engineering Society*, vol. x., 1912–13.

first be arbitrarily fixed and the diameter calculated without reference to the stress. In any case when d is settled, it is the usual practice to determine the length l so that the bearing pressure per square inch of projected surface varies inversely as the surface velocity, that is, $pV =$ constant.

If W = weight on bearing in lbs., this is $\frac{WV}{ld} = C$.

Although, as stated above, the journal heat loss is independent of the pressure between the oil and shaft, it is necessary to ensure that an excessive pressure, which the viscosity of the oil could not withstand, is not produced. Such a condition would cause a breakdown of the oil film which is essential for efficient lubrication. Another element to be considered is the jarring action arising from vibration if the speed of rotation is very high. This action may materially assist in the breakdown if the pressure approaches its limiting value for the condition of temperature and viscosity of the oil.

G. Stoney, in the paper cited above, quotes the upper limits at $p = 75$ and $V = 75$, giving $C = 5600$. In American practice it appears that values as high as $p = 100$ and $V = 80$ are employed satisfactorily, when there is no risk of heavy vibration. These figures give $C = 8000$.

The usual limits of pressure in modern practice may be taken between 50 and 90 lbs./in.^{2}, and the values of the constant C from 2500 to 5500. These figures are for land turbines.

On the other hand, the journal velocities of marine turbines seldom exceed 30 ft./sec., and the pressures, so far, run from 80 to 100 lbs./in.^{2}. The usual oil temperature in marine work is from 100° F. to 110° F., and it is probable under this condition that pressures from 150 to 200 lbs./in.^{2} might be used without injurious effect.

EXAMPLE 14.—A pressure compounded impulse turbine running at 3000 R.P.M. has journals 4 inches diameter × 12 inches long. The steam consumption is 7·2 lbs./sec., and the heat drop for the machine 388 B.Th.U. Oil temperature at bearings 120° F. Calculate the percentage loss of the heat drop due to journal friction.

Here the journal velocity $V = 53$ ft./sec., $d = 4$, $l = 12$, $t = 120$

By equation (31)

$$h_J = \frac{0{\cdot}0656 \times 12 \times 4 \times 53}{120 - 32} = 1{\cdot}89$$

For the two bearings the total is 3·78 B.Th.U./sec. $= H_T$.
The total heat available per sec. $WH_r = 7{\cdot}2 \times 388$.

$$\text{Percentage loss at journals} = \frac{100H_T}{WH_r} = \frac{3{\cdot}76 \times 100}{7{\cdot}2 \times 388} = 0{\cdot}135 \text{ per cent.}$$

a very small proportion.

EXAMPLE 15.—Each bearing of a horizontal Curtis turbine is

9 inches diameter × 22·5 inches long. The speed is 1500 R.P.M. The heat drop is 388 B.Th.U., and the steam consumption 9·7 lbs./sec. Calculate the loss at the journals as a percentage of the whole heat drop, assuming an oil temperature of 120° F. at the bearings.

$$V = 59, \quad H_T = \frac{2 \times 0{\cdot}0656 \times 9 \times 22{\cdot}5 \times 59}{88} = 17{\cdot}8$$

$$WH_r = 9{\cdot}7 \times 388$$

$$\text{Percentage loss} = \frac{100 \times H_T}{WH_r} = \frac{100 \times 17{\cdot}8}{9{\cdot}7 \times 388} = 0{\cdot}474 \text{ per cent.}$$

164. These two illustrations indicate that the loss at the bearings of an efficiently lubricated machine of fairly large output is below the order of 1 per cent., reckoned on the total available heat. The heat drop per lb., taken in each case, is about the maximum limit used in practice. For smaller values the loss would be slightly greater.

In addition to the journal loss there is, however, the work of driving the governor and pump gears and the work done in circulating the oil at pressure through the piping and bearings.

This loss may run from 1 to 1·5 per cent., so that as an average estimate for machines of fairly large output, the journal, gear, and pump loss may be taken from 1·5 to 2·0 per cent.; and for smaller machines, say, 2 to 3 per cent.

165. While the high-speed journal bearing automatically lubricates itself the usual type of ring thrust bearing does not. Unless the latter is kept cool and well supplied with oil under pressure, the oil is squeezed out and metallic contact between the ring faces takes place. The same action would take place in the case of the journal bearing were it not for the curious fact that in such a bearing the journal axis gets slightly out of truth with the bearing axis. This want of concentricity produces a curved wedge-shaped film of oil. Points of no pressure occur at the "on" and "off" sides of the film dragged in by the shaft. The pressure increases from zero at the entering side to a maximum value at a point near the middle of the length and slightly past the centre of the brass surface in the direction of motion, and then decreases to zero at the exit side.

The conditions that such a distribution of oil pressure shall exist at a bearing are two—

(1) The surfaces have to be inclined to each other, with the angle of inclination facing the entering oil film.

(2) There have to be two points of zero pressure—one where the oil enters, and one where it leaves.

There is a viscous flow due to the rotation, and the wedge-shaped film enables the oil to bear considerable pressure. The theory of the pumping action and viscous flow which occurs at a well-lubricated high-speed journal bearing has been worked out mathematically by Professor Osborne Reynolds.[1] It has been recently applied by A. G. M. Michell to the case of the thrust bearing, with marked success.

[1] *Phil. Trans.*, 1886.

166. Thrust Bearings.—In the land type of impulse turbine, as can be seen from the machines illustrated, the adjusting blocks used are comparatively small in size, and are subjected to very small thrust pressures. In the case of the reaction turbine with correct proportions of dummy pistons, there should be a negligible thrust in the blocks when running at normal load. To provide against contingencies it is advisable to design the block to take the whole of the rotor thrust in a land, the whole of the propeller thrust in a marine turbine.

An illustration of a typical "thrust" block for a land turbine is

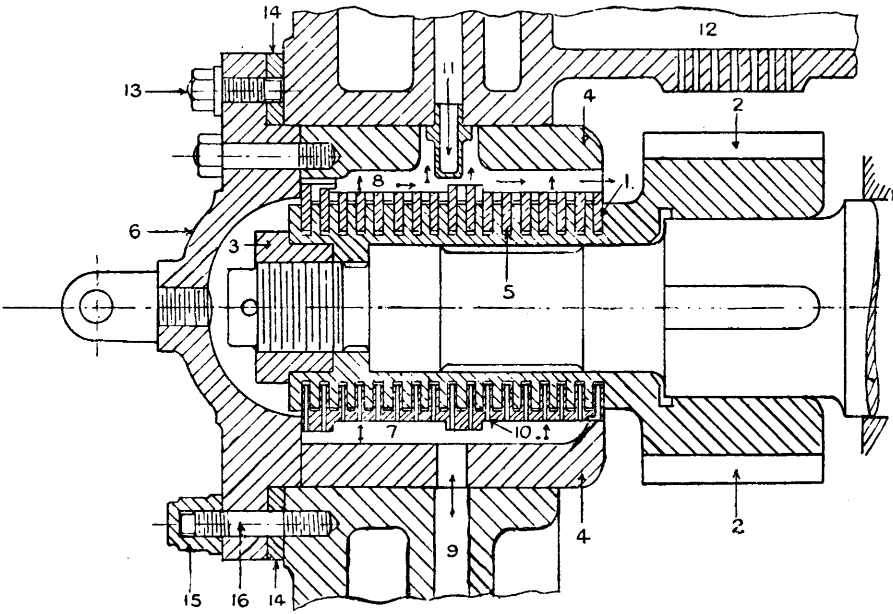


FIG. 162.

shown in Fig. 162. It is the standard type fitted in the Brush Parsons machines.

The thrust rings, 1, are turned on a sleeve carrying the worm, 2, which is keyed and held in place by the nut, 3. The sleeve, 4, carrying the thrust collars, 5, which are made of gun-metal, is bolted to the end cover, 6. The sleeve is in halves, but these are bolted rigidly together when in position and form a solid block, giving full ring circumference for the thrust. Passages, 7 and 8, are cut in the sleeve at bottom and top to permit of the supply and delivery of oil. The oil is pumped through the supply passage, 9, passes into 7, and is then forced through the holes 10 in the lower half of the collars to the inner circumference. Before it can escape it must pass over the thrust ring faces, and lubricate them. It flows outward partly under pressure and partly under centrifugal force, and is discharged at the top into the passage 8. From this it flows, as indicated by the directional arrows, towards the worm and downwards into the oil well, from which it is in turn drained back to the cooler in the base of the machine. A thermometer pocket is provided at 11, to ascertain the thermal condition of the bearing. The worm is lubricated from the oil chamber, 12, at the top. The clearance allowed between the rings and collars runs from 0·002 to 0·003 inch.

The adjustment of the block to give the correct dummy piston clearance in the case of the reaction or the disc and drum machine is made as follows. The sleeve, 4, being placed in position, the end casing, 6, is bolted on, and the block and rotor are pulled to the left by means of the bracket and lever arrangement, shown in Fig. 43, p. 72, until the dummy faces are in contact. The stop pins, 13, are then adjusted to give 10 mils. clearance at the face. The rotor is then set back until the required clearance at the dummy is obtained, and a spacing liner, 14, is fitted between the flange of the end cover, 6, and the face of the turbine frame. The fastening studs, 16, are provided with cap nuts, 15, which have an allowance of 3 mils. between the top and end of the stud to permit of slight further increase in clearance if necessary.

The dummy clearance can be tested at any time by slacking the nuts, drawing back the cover, 6, and block till the dummy rings are hard on the faces, and measuring the distance between the end of the stop pin, 13, and the face by feeler gauges. If there has been any wear of the ring faces this measurement will be greater than the initial value of 10 mils. to which the pin was originally set.

A new liner can then be fitted to give the clearance required. This block is different in design to the type ordinarily used on reaction turbines. It is usual to make the block in halves, which are relatively adjustable. A block of this kind is fitted in the Willans machine, Fig. 44, p. 74.

The block, 23, is fitted into a recess bored in the frame. The end cover corresponding to 6, Fig. 162, is provided at the bottom with a stop bolt, 24, having a fine thread and a micrometer adjustment, and at the top with a bolt, 25, fitting into the upper half of the block. This half, and with it the rotor, is pulled to the left by the bolt 25, until the dummy packing rings are hard against the collar faces. The micrometer bolt, 24, is then screwed in, until the lower half of the block is in contact with the thrust collars. When this preliminary adjustment is made, the upper half of the block is eased back, and the micrometer bolt, 24, further screwed in till the desired dummy clearance, about 10 mils., is obtained. The bolt is then locked in position by a jamb nut, and a cover, usually secured with a padlock, is placed over the end to prevent any tampering with the adjustment.

The bolt, 25, is then pulled up until the clearance between the thrust collars and upper half block is about 5 mils. It is then secured like the lower bolt.

In this class of thrust only half the collar surface is available, while in the other the whole surface is, and there is only one adjustment.

The adjusting blocks of impulse machines do not require to be set with such nicety as those of the reaction, and adjustment is made by the insertion of permanent liner rings between the body of the block and the bearing pedestal on which they are usually mounted. On the Westinghouse turbine, Fig. 20, packing pieces are fitted on each side of the central collar which keeps the block in position in the housing. On the Zoelly turbine, Fig. 21, a liner is fitted between the flange and the face at 21.

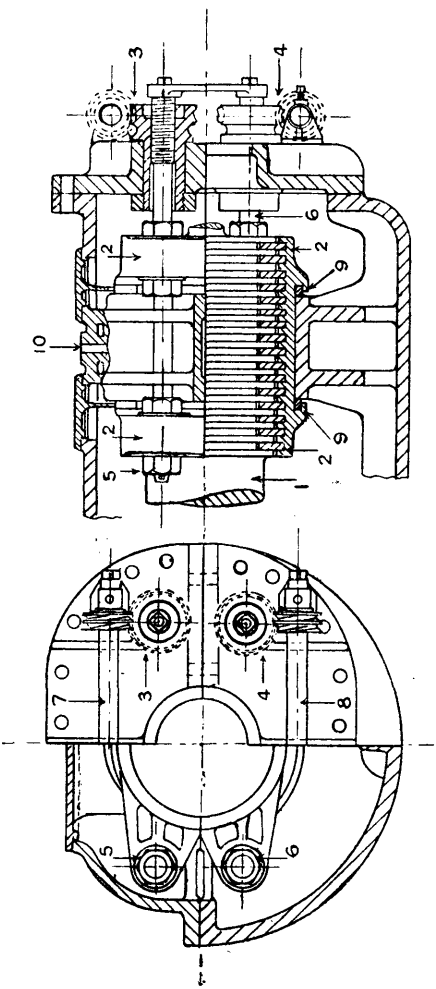


Fig. 163.

A typical marine thrust block is shown in end and side elevation in Fig. 163. It is fitted in the H.P. and L.P. Parsons turbines illustrated in Figs. 27 and 28. The thrust rings are turned on the shaft, 1, and the thrust collars, which are of brass, are fitted in a sleeve, 2. The top and bottom halves of this sleeve can be adjusted by means of the worm gears, 3 and 4, and the bolts, 5 and 6, which pass through the lugs at each side. By means of a handle fitted on the squared ends of the worm shafts, 7 and 8, the combined worm wheel and nuts are rotated and move the bolts fore or aft.

The lower half of the block is first drawn forward till the dummies are in contact with the rings. It is then eased back until the desired dummy clearance is obtained. The packing rings, 9, are then fitted to fix the lower block in position. After the lower block is so adjusted the upper block is moved forward till the collars and rings are in contact. It is then eased back 10 or 12 mils. to give space for the inflow of oil. By means of a micrometer gauge, not indicated on the drawing, the amount of clearance is adjusted. Corresponding packing rings are fitted at each end of the top half to permanently fix this in position.

In this case the lubricating oil is supplied to the top of the block through the passage, 10.

The method of measuring the dummy clearance in marine turbines varies slightly in detail. Usually a finger plate—that is, a narrow plate fitting into a groove in the shaft—is employed.

The clearance is ascertained by inserting a feeler between the plate and the side of the groove.

The groove, 27, shown in Fig. 27, p. 44, is provided to take one of these finger-plates.

The differential expansion of rotor and casing under steam, however, affects the dummy clearance, and the finger-plate reading is usually slightly different from the true value. It is now the practice to measure the actual clearance under running conditions. The particular arrangement used in the foregoing case is shown in elevation and plan in Fig. 164. A small angle stop piece, 1, is fixed in the casing. A

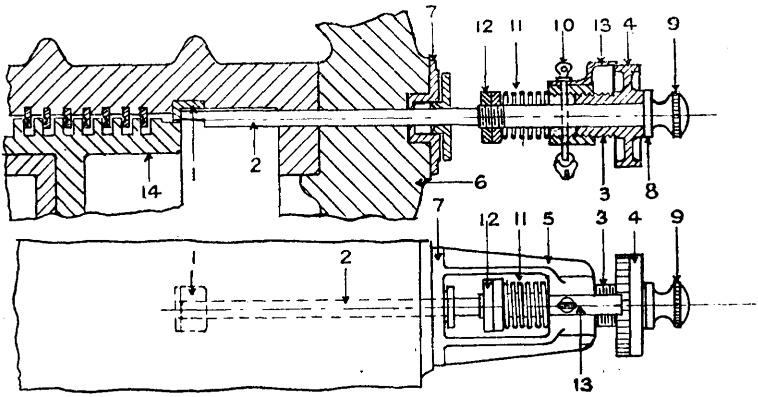


FIG. 164.

circular rod, 2, passes through a screwed bush, 3, provided with a micrometer wheel, 4. This bush is provided with a fine screw and screwed into the head of a bracket, 5. This is bolted to the casing at 6, and carries a stuffing box, 7. The rod is provided with a shoulder, 8, and an end knob, 9. It can be locked in position by a pin and padlock, 10. When this pin is removed, the shoulder, 8, is drawn hard against the face of the wheel, 4, by a spiral spring, 11, locked by the nuts, 12. An index plate, 13, is provided to determine the reading on the wheel scale, when adjustment is made. The end of the rod at 1 is notched, and when locked stands clear of the stop plate. When pin 10 is removed and the wheel bush screwed in slightly, the end of the rod presses against the stop. The wheel is then adjusted till it is in contact with collar, 8, and the reading is taken. The dummy rings are then brought hard against the collars by the adjusting gear, the knob, 9, is turned through 180°, the wheel bush screwed sufficiently far in to permit the end of the rod to bear on the end of the dummy piston, 14. The wheel is then screwed back till in contact with the collar, 8, and the reading is again taken. The difference gives the distance between the stop and piston when there is no clearance. The dummy is then set with clearance, and the wheel screwed well in till the rod presses on the piston again. It is then adjusted back to the collar and the reading is taken. The difference between the first and third reading gives the distance between the stop and piston, with clearance. The difference of these two readings gives the dummy clearance.

The advantage of this arrangement is the measurement of the clearance under running conditions.

An ingenious variant of the ordinary marine thrust block is used in the Franco Tosi turbine, Fig. 48, p. 81. It is shown on a larger scale in Fig. 165.

The block is divided into fore and aft sections. Between these there is fitted a piston, 1, with labyrinth packing at 2. Oil is pumped into the piston chamber to both the fore and aft sides through the ports, 3 and 4. The thrust rings and collars are given a certain amount of clearance, so that the oil can "leak off" at each side and drain away by the passages, 5 and 6, to the oil return branch, 7.

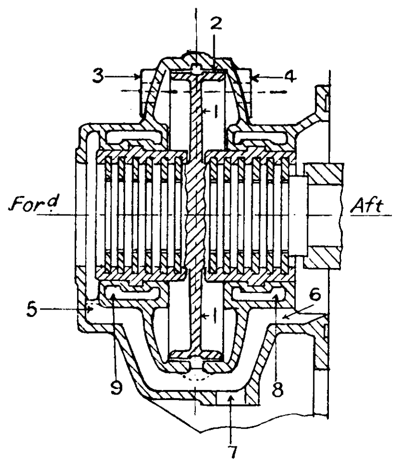


FIG. 165.

As long as the clearance at the fore and aft blocks is the same, the oil leaks equally through them and the fore and aft pressures on the piston balance. If now, through increase of propeller thrust, the rotor is forced slightly forward, the clearance at the forward rings is diminished, and increased at the after rings. The obvious result of this is an increase of pressure on the forward side, due to the throttling of the oil escape, and decrease on the aft side due to increased escape of oil. The unbalanced force from fore to aft then pushes back the rotor, equalises the leak, and hence the pressures.

This automatic adjustment keeps the thrust rings in a "floating" position, and practically no surface contact occurs. The bearing thus runs under the necessary condition for most efficient lubrication, viz. on a film of oil. The frictional heat generated is carried off by circulating water passed round the jackets, 8 and 9, on the housing. A similar design of adjusting block is employed in the combination type of land turbine, to take residual thrust.

The condition for efficient lubrication in this case, as in the other land thrust blocks cited, is artificially obtained by oil under considerable pressure.

167. When the thrust bearing is modified, so as to produce the condition which is automatically set up in the case of the high-speed journal bearing, no pressure lubrication is required; and further, the pressures which can be carried per square inch of thrust surface, under natural bath lubrication, are very much greater than can be taken by any ordinary thrust.

The result is a substantial reduction in the number of thrust rings and in the size of the bearing.

This modified type of thrust is due, as already indicated, to A. G. M. Michell, and is now coming into general use, especially in marine work.

There are various designs, but in all these the essential feature is the substitution of a bearing surface consisting of pivoted segmental blocks, in place of the fixed thrust rings. This modification establishes the condition which Professor Osborne Reynolds demonstrated was necessary for the perfect lubrication of two flat surfaces, viz. that they must not be kept parallel, but enabled to take a very small angle of inclination to each other.

In the Michell thrust, the fixed or standing collar is divided into several segments. Each segment, either by means of a small radial rib

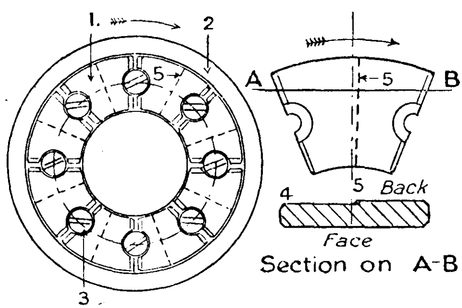


FIG. 166.

at the back or by resting on the point of a projecting stud in the housing of the bearing, is enabled to tilt, and automatically set itself at the requisite angle for efficient lubrication. In some instances one half of the back of the block is made to project slightly above the other half. An end elevation of the face of one form of the Michell thrust is shown in Fig. 166.

The thrust surface is covered by eight segmental blocks, 1. These are held in place on the thrust casing, 2, which is cylindrical. The back of the casing is formed with a spherical surface which fits on the stationary part of the machine or on a bearing head fixed to the frame. Each block is prevented from moving circumferentially by a recessed screw, 3. The block, as shown on the enlarged section to the right, is cut away at the back between 4 and 5, so that it can tilt about the edge, 5, as a fulcrum. This edge, as seen in the elevation, is not in the centre of the block, but beyond it, towards the rear edge, reckoned in the direction of motion shown by the arrow. It is found that centrally pivoted blocks work equally as well as eccentrically pivoted ones, but the friction loss is greater.

In an important paper on this subject,[1] H. T. Newbigin states that the results of an experiment to ascertain the effect of pivoting on the temperature were as follows:—

Position of Pivoting Point.	Temperature of Bearing.
$\frac{1}{16}$ inch in front of centre	135° F.
In centre	132° F.
$\frac{1}{16}$ inch behind centre	126° F.
$\frac{1}{8}$ " " "	124° F.

The increase in temperature gives a fair indication of the increased friction. The bearing collar ran at 1750 R.P.M., and the intensity of pressure on the blocks was 500 lbs./in.^{2}. The theoretical position of the pivoting axis was $\frac{1}{8}$ inch behind the centre. It will be noted that the bearing ran quite well with the pivoting axis $\frac{1}{16}$ inch in front of the centre. The explanation suggested for this result is that the increased temperature reduced the viscosity of the oil, and hence reduced its ability to support the trailing edge. The fact that the blocks may be centrally pivoted without injurious effect is of importance where, as in the case of a marine thrust shaft, the direction of rotation has to be reversed. The Michell blocks fitted on each thrust of the T.S.S. Paris (see Fig. 31) are centrally pivoted. Each block carries a load of 24 tons at a speed of 300 R.P.M.

A considerable number of pivoted thrust blocks are fitted to steam turbines of the land type. At the higher speeds employed in turbine practice, experience has shown that the best results are obtained when the pivoted blocks cover only a portion of the thrust surface. Usually three segmental blocks, set at 120° to each other, are fitted in the casing. They are kept the proper distance apart by fixed spacing blocks.

A bearing of this type is shown in Fig. 167, Plate XIII. The three spacing blocks, between which the segmental blocks fit, are fixed in the casing by recessed screws. One of the segmental blocks is shown on the right, beside the thrust ring. It will be noted that this is cut away on the back as in the previous illustration.

168. For the high-speed geared marine turbine, where the whole of the propeller thrust has to be taken by the block, the segmental thrust is of the first importance.

Newbigin states that, so far, it has not been necessary to use the multiple horseshoe arrangement of the ordinary fixed block, and that one collar for ahead and one for astern thrust only need be fitted. The loads that can be carried by these bearings without trouble are very high. Pressure up to 1500 lbs./in.^{2} can be safely carried at high velocities. An experiment carried out in America by the

[1] "The Problem of the Thrust Bearing," by H. T. Newbigin, *Proc. Inst. C.E.*, vol. cxcvi., February 3, 1914.

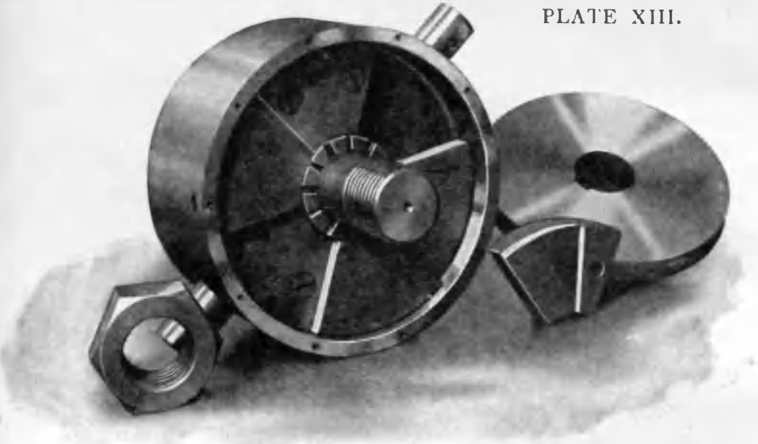


FIG. 167.

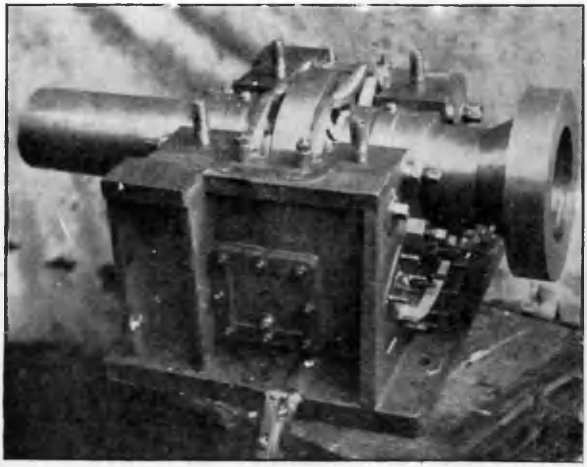


FIG. 169.

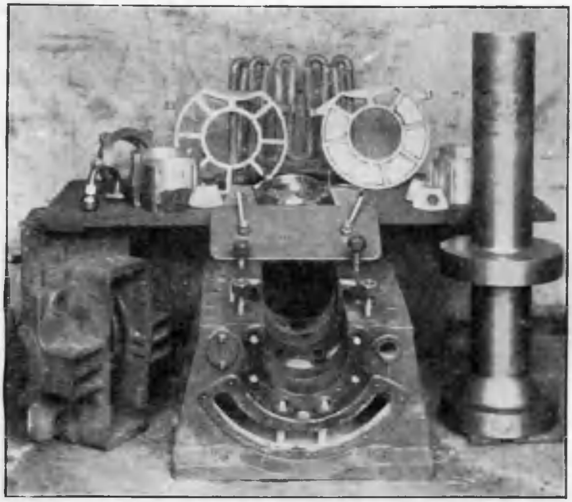


FIG. 170.

Michell Thrust Bearings.

Westinghouse Company on a segmental thrust block fitted to a steam turbine showed that with mean surface speed of 54 ft./sec. the pressure could be increased to 10,000 lbs./in.^{2} before failure took place. At this pressure the white-metal facing of the blocks flowed in all directions, but no heating occurred. This result indicates that the limit of pressure is fixed by the flow of the white metal and not by the breakdown of the oil film.

It is now becoming the practice to take the difference between propeller and steam thrust, in the case of the directly connected marine turbine, on a Michell thrust bearing. In some instances the auxiliary dummy thrust is dispensed with. Thrusts of 70,000 lbs. are being carried at mean rubbing speeds of 60 to 70 ft./sec.

In the case of the geared marine turbine the main blocks carry loads up to 130,000 lbs. at mean rubbing speeds of 40 to 50 ft./sec. In each instance only one thrust collar is used.

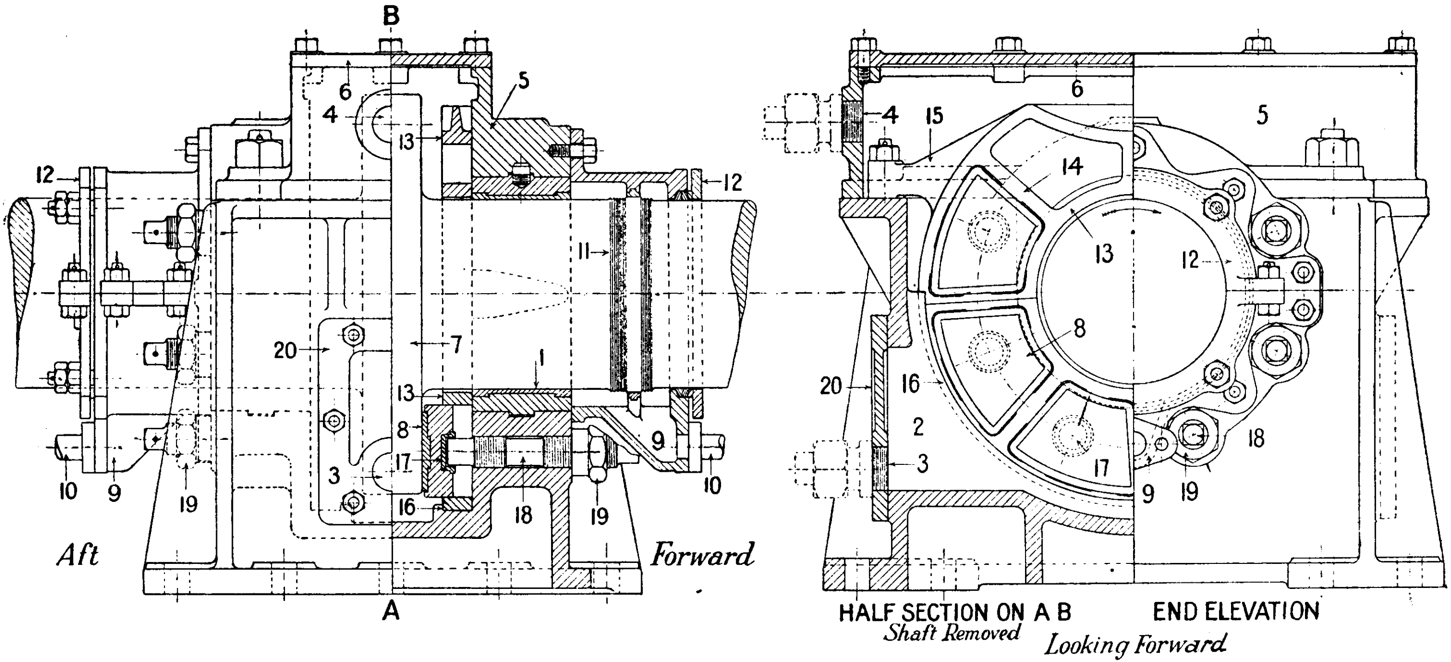
These thrust bearings are usually designed for pressures about 500 lbs./in.^{2} of bearing surface, for velocities from 60 to 70 ft./sec.

The "coefficient of friction," μ, or the ratio of the viscous resistance of the oil film to the load, may vary from 0·0008 to 0·003. Stoney suggests the average value of 0·002 as suitable for the usual case where the pressure is about 500 lbs./in.^{2}.

169. A design of single collar marine Michell thrust block, made by Messrs. Broom and Wade, High Wycombe, is shown, in side elevation, and end elevation, in Fig. 168. The shaft diameter is $7\frac{9}{16}$ inches and the thrust collar diameter 16 inches. The total thrust is 20,000 lbs., the nett area of the Michell blocks 69 in.^{2}, and the speed 300 R.P.M.

The shaft is supported on each side of the thrust by journal bearings, 1. The thrust chamber, 2, is flooded with cool oil from the turbine circulating system. The oil is admitted at the bottom through the port, 3, in the side cover, 20, and is discharged at the top through the port, 4, in the side of the upper casing, 5. The journal bearing caps and the upper half of the casing are formed in one piece, 5, the latter being closed at the top by an oil-tight cover, 6. The thrust collar, 7, and the Michell blocks, 8, are kept in a stream of oil which carries away the friction heat, as in the case of the main turbine bearings. Part of this oil is distributed from the thrust chamber to the journal bearings. This oil after passing through each bearing is caught in the oil well, 9, at the bottom, and drained away by the pipe, 10. Oil baffles are cut on the shaft at 11, and to further prevent external leakage, stuffing glands, 12, are fitted.

The Michell blocks, 8, as shown in the end elevation, do not cover the whole circumference of the thrust. The arrangement is similar to an inversion of the usual horseshoe distribution of surface. The blocks are carried in a ring frame, 13, which is divided into compartments by radial ribs, 14. The frame is in halves and secured in position by bolting down the top half to the housing by the arms, 15. This half carries the two top blocks. The lower half fits into a cylindrical groove, 16, which extends half round the circumference of



B
A
Aft
Forward
HALF SECTION ON A B
Shaft Removed
END ELEVATION
Looking Forward
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

FIG. 168.

the thrust chamber. When the upper half of the frame is removed, the lower half can be pushed round the groove and taken out.

Each block, which is of gunmetal, is recessed on the back to take a hardened steel bush, 17, and the bottom of this presses on the pivoting point of an adjustable hardened steel screw, 18, which is screwed into the housing.

When the thrust shaft is placed in position in the bearing, these screws are screwed in till they all take an equal bearing, and are then locked by the lock nuts, 19. When finally adjusted a wire is threaded through the pin-holes shown on the squared ends of the screws at the after side of the bearing, and the wire is then sealed. Each block is faced with white metal. It will be seen from the end elevation that the blocks are eccentrically pivoted in the direction of rotation as indicated by the arrow.

The sectional part of the side elevation on the right shows the forward bearing and thrust ring. This takes the thrust of the propeller when "going ahead." The corresponding set of blocks in the thrust chamber on the aft side of the thrust collar takes the pull of the propeller when "going astern."

This block was run on test for four hours without any oil circulation, and it was found that the radiation effect was quite sufficient to prevent undue increase in the oil temperature during that period. Under ordinary working conditions with proper circulation of oil no heating trouble need be anticipated.

This is the present practice as regards lubrication, but in some instances the journal bearings have been provided with syphon lubricators, and a cooling coil has been fitted in the bottom of the thrust chamber. Photographs of one of these earlier bearings by the same makers are shown in Figs. 169 and 170, Plate XIII.

In Fig. 169 the shaft and ring frames carrying the Michell blocks are shown in position. The lower part of the thrust chamber acts as an oil well, and the cooling pipe coil is inserted through the semicircular opening shown in front below the shaft. The coil and closing cover are removed. The oil can be drained from the chamber by the cock shown at the left side. The top half of the thrust chamber with the journal caps and lubricating boxes, and the various details, are clearly shown in Fig. 170.

The top of the casing and the journal caps, as in the previous case, are in one piece. Syphon boxes with the necessary oil channels are cast on the top of the caps, and the journals can be lubricated from them. These lubricators are fitted as a "stand-by" in case of heavy oil being used for the thrust, and trouble arising from the insufficient lubrication of the journals.

The pipe coil through which water is circulated is shown at the back of the photograph. The ring frames are shown one on the left without the blocks, and one on the right with all the blocks in place, while the face and back views of individual blocks are also shown on left and right sides. The brasses shown beside the blocks, as in bearing, Fig. 168, are semicircular, and are prevented from turning in the housing by stop pins screwed into the upper halves. As in the previous case, stuffing boxes and glands are fitted at each end of the thrust. One of these glands is shown at the back on the extreme left. The cover shown on the top of the main casting is used to close the open top of the upper half-casing, shown on the ground on the left. It is clamped down to form an oil-tight joint by means of the handle nuts shown.

For a test of this bearing at the maker's works, the "thrust" was produced by placing two thrust blocks in line and driving the common shaft by belt and pulley from an electromotor. One thrust block was fixed to the bedplate, the other was mounted on a movable frame, arranged to slide axially. The desired load was obtained by pressing on the movable part with a hydraulic jack. The test was run for four hours. During the first two and a half hours the total load was 15,000 lbs., giving a pressure of 300 lbs./in.^{2}, and the speed gradually increased from 425 to 450 R.P.M.

During the last hour and a half the load was 20,000 lbs., giving a pressure of 400 lbs./in.^{2}, and the speed increased from 450 to 460 R.P.M.

The temperature of the oil in one thrust chamber rose from 63° F. to 80° F. The temperature of the forward journal rose from 87° F. to 100° F., and of the aft journal from 84° F. to 98° F. The temperature of the collar rose from 63° F. to 112° F. The mean temperature rise of the circulating water of the cooling coil was 27° F., the water being circulated at the rate of 6·5 lbs./min.

When the combination was run at 460 R.P.M. without any thrust, the motor required 10 amps. at 420 volts. With a thrust of 20,000 lbs. on the two bearings the motor required 15 amps. Hence the friction work for the two thrusts, apart from any journal loss, is represented by 5 amps. at 420 volts, or 2·82 horse power. The friction heat generated is thus 120 B.Th.U. per min., or 60 B.Th.U. per min. for each block. The corresponding "coefficient of friction," μ, calculated from the above data, is 0·00155 (see example 17).

Another design of Michell thrust for a geared marine turbine, made by Messrs. Cammel Laird & Co., Birkenhead,[1] differs from the previous designs in having two thrust collars on the shaft, one for ahead and the other for astern thrust. Between them there is fitted a bridge piece divided in the centre, so that the two halves can be moved slightly forward and aft by the insertion of liners, to bring the Michell blocks up to the collar faces. The blocks, which are eight in number, are cut away at the back, and pivot on the faces of the bridge pieces. They extend round the whole circumference, and are held in position radially by half-hoops of steel. The bridge piece is strung on side rods on the housing, like the horseshoe collars of the ordinary thrust, and are locked in position by the usual lock nuts at each lug.

In this case also a cooling coil is provided in the oil well. On trial, however, the bearing ran quite satisfactorily without water cooling. The speed was 260 R.P.M., and the horse power transmitted was 2600.

[1] See *Engineering*, December 18, 1914.

The temperature of the air-cooled bearing did not rise above 148° F. The pressure was 300 lbs./in.^{2}.

170. Calculation of Frictional Loss at Thrust Bearings.—The energy absorbed in friction is calculable when the total thrust, the speed of rotation, and the coefficient of friction are known.

This coefficient is the ratio of the shearing force on the oil film to the total thrust, when the bearing is efficiently lubricated.

Let μ = coefficient of friction.
T = total thrust in lbs.
R = mean radius of collar in inches.
ω = angular velocity of shaft in radians/sec.

The total shearing force is $F = \mu T$ lb., and without sensible error in the case of the collar it may be assumed to act at the mean radius R. The friction torque is FR, and the friction heat generated is

$$h_t = \frac{\omega\mu TR}{12 \times 778} \text{ B.Th.U./sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In the case of a stationary ring surface it is necessary to limit the bearing pressures, and the object is attained by adding collars to the shaft until sufficient area is obtained to limit the bearing pressures from 50 to 60 lbs./in.^{2}. For the ordinary thrust an average value of $\mu = 0{\cdot}05$ may be taken. With this figure the friction heat is given by

$$h_t = \frac{\omega TR}{186720} \text{ B.Th.U./sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If W is the total consumption of the turbine and H_r the heat drop, the friction loss at the thrust expressed as a percentage of the heat drop is

$$\text{Percentage loss from thrust friction} = \frac{h_t \times 100}{WH_r} \quad . \quad . \quad (34)$$

In the case of the land turbine this loss is small. In the case of the marine turbine, when the whole thrust has to be taken by the block, it may become appreciable. The old problem of the large thrust bearing has been revived by the advent of the high-speed geared turbine, where the whole thrust has to be taken up. Fortunately the pivoted segmental thrust meets the case admirably. As already stated, the load that can be taken by such a bearing depends not on the breakdown of the oil film, but on the failure of the white metal facing, under the oil pressure.

For calculation of friction heat in the case where the pressures are in the neighbourhood of 500 lbs./in.^{2} the average value of 0·002 may be used for the coefficient.

EXAMPLE 16.—The thrust rings of the block of a H.P. marine turbine are 18 inches external and $13\frac{1}{2}$ inches internal diameter. The total propeller thrust is 89,500 lbs. If the thrust is overbalanced by the steam thrust at full load by 10 per cent., calculate the probable friction loss at the bearing. The speed is 290 R.P.M., the heat drop for the turbine is 112 B.Th.U./lb., and the consumption 62 lbs./sec.

The effective pressure on the block is T = 8950 lbs., the mean radius is

$$\frac{9 + 6\cdot75}{2} = 7\cdot875 \text{ inches}$$

$$\omega = \frac{6\cdot28 \times 290}{60} = 30\cdot4$$

By equation (33) friction heat is

$$h_t = \frac{30\cdot4 \times 8950 \times 7\cdot875}{186720}$$
$$= 11\cdot5 \text{ B.Th.U./sec.}$$

By equation (34)

$$\text{Percentage loss at thrust} = \frac{11\cdot5 \times 100}{62 \times 112} = 0\cdot16 \text{ per cent.}$$

If the whole thrust were to come on the block this loss would be 1·6 per cent.

The trouble with a block of this type is not the friction loss but the difficulty of maintaining equal pressure on all the rings and ensuring efficient lubrication to prevent seizure. In this case sixteen rings are employed. By using segmental blocks only one ring would be necessary, and the friction loss would be insignificant.

EXAMPLE 17.—A Michell thrust block was run during a test at 460 R.P.M., and the heat equivalent of the input from the driving motor was 1·0 B.Th.U./sec. The total thrust was 20,000 lbs. The radial distances of the edges of each block from the centre were 5 and $7\frac{1}{2}$ inches. Calculate the coefficient of friction for the block.

Here $T = 20{,}000$, $R = 6\cdot25$, $h_t = 1\cdot0$, $\omega = \dfrac{6\cdot28 \times 460}{60} = 48\cdot1$

By equation (32)

$$1\cdot0 = \frac{\mu \times 48\cdot1 \times 20000 \times 6\cdot25}{12 \times 778}$$

$$\therefore \mu = \frac{12 \times 778}{48\cdot1 \times 20000 \times 6\cdot25} = 0\cdot00155$$

EXAMPLE 18.—The Michell thrust block referred to in Art. 169 has blocks whose inner and outer radii are 5 and $8\frac{3}{8}$ inches. On trial the shaft horse power transmitted at 260 R.P.M. was 2600, and the total thrust taken by the block was 30,000 lbs. Calculate the friction loss as a percentage of the shaft horse power. Take $\mu = 0\cdot002$.

$\omega = 27\cdot3$ radians/sec., $T = 30{,}000$ lbs., $R = 6\cdot687$ inches.

By equation (32)

$$h_t = \frac{27{\cdot}3 \times 0{\cdot}002 \times 30000 \times 6{\cdot}687}{12 \times 778}$$

$$= 1{\cdot}17 \text{ B.Th.U./sec.}$$

$$2600 \text{ H.P.} = \frac{2600 \times 42{\cdot}4}{60} = 1840 \text{ B.Th.U./sec.}$$

$$\text{Percentage of H.P.} = \frac{1{\cdot}17 \times 100}{1840} = 0{\cdot}064 \text{ per cent.}$$

an insignificant amount.

171. Loss from Residual Energy at Exhaust.—For any case the exhaust loss is determined by the absolute velocity of the steam at exit from the last moving ring of the turbine. Thus

$$E_e = \frac{V_{a_0}{}^2}{2g} \text{ and } h_e = \left(\frac{V_{a_0}}{223{\cdot}7}\right)^2 \quad . \quad . \quad . \quad . \quad (35)$$

$$\text{The percentage loss} = \frac{h_e}{H_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where H_r is the heat drop between initial and exhaust pressures.

The heat drop may run from about 360 to 400. Basing the calculation on the lower value of 360 B.Th.U./lb., the velocities for percentage losses are as follow :—

Percentage loss at exhaust	1	2	3	4	5
Velocity in feet per sec.	424	598	733	846	948

As a rule, the exit velocity V_{a_0} in large impulse machines seldom exceeds 500 ft./sec., so that the loss on the average may be taken between 1 and 1·5 per cent. For moderate and large reaction machines it may be taken about 1 per cent.

For very large units, where it is necessary to open out the blading at the last stage, it may rise to 2 per cent.

For the simple impulse machine, where the residual velocity is usually about 1000 ft./sec., it may be taken from 4 to 5 per cent.

172. Radiation Loss.—There is very little information available on this subject. For small machines it may run about 1 per cent., and for large and medium machines from 0·5 to 0·7 per cent.

CHAPTER XI

CONDITION CURVE; REHEAT FACTOR; INTERNAL EFFICIENCY AND EFFICIENCY RATIO

173. Condition Curve of a Compound Turbine.—The quality of the steam on discharge from any stage of a compound turbine is always higher than that due to the adiabatic expansion in the stage. The steam is said to be "reheated." The reheat is caused by the friction losses in the nozzle and blade channels, the disc and vane loss, and leakage loss at diaphragm glands or blade tips. The cumulative effect of "reheating" is shown in the Tϕ diagram, Fig. 171, for a few-stage impulse machine. The pressures at entrance to the successive stages are p_1, p_2, p_3, etc., and the exhaust pressure is p_0.

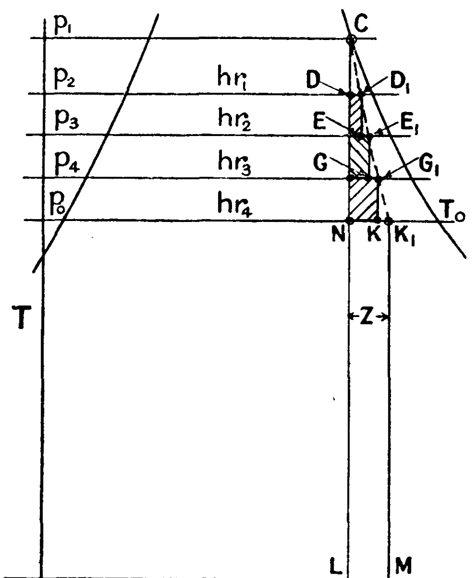


FIG. 171.

In order to simplify the explanation the steam is assumed to be initially dry at p_1.

In the first stage after adiabatic expansion to p_2 the state point would be D. On account of the friction losses and consequent reheat the quality is increased and the state point is moved to D_1. This point gives the initial condition of the steam at entrance to the second stage. After adiabatic expansion in this stage to p_3, E would be the state point, but again the reheat effect increases the quality, and the point is moved to E_1. Similarly, after expansion in the third stage, the state point is moved from G to G_1, and in the fourth stage from K to K_1.

The curve CK_1 drawn through these points, is called the "condition curve" of the turbine.

174. Reheat Factor.—There is a total increase of entropy, Z, due to reheat, and an amount of heat given by $T_0Z = NK_1ML$, in addition to the exhaust heat with adiabatic expansion, is thrown away.

It will be noted, however, that the heat drop in the successive stages is an increasing quantity. The sum of these heat drops is obviously greater than the heat drop H_r, between initial and exhaust pressures p_1 and p_0.

That is $$h_{r_1} + h_{r_2} + h_{r_3} + h_{r_4} > H_r.$$

This relation can be expressed by $\Sigma h_r = RH_r$, where R is a coefficient called the "Reheat Factor." It may vary from 1·02 to 1·08, according to the type and size of machine.

175. In what follows, the quantity Σh_r is called the "cumulative heat" of the compound turbine. It exceeds H_r by the amount of heat represented by the shaded areas on the right of CN.

As the number of stages increases the adiabatic drops become smaller and smaller, and in the limit, when the turbine has a large number of stages, the stepped curve coincides with the condition curve. The additional heat is then given by the triangular area CNK_1. The same effect is produced when the steam is initially superheated. This case is shown in Fig. 172. The initial state point at p_1 and superheat t_{s_1} is F_1, and the final state point at p_0 is K. The steam is still superheated at p_2 and p_3, whereas with adiabatic expansion it would be dry at p_2. The triangular area F_1NK represents the heat additional to H_r, for a large number of stages. For six stages and upward the value of this heat is not sensibly different from that obtained with an infinite number of stages, so that for the pressure compounded impulse and the Parsons reaction machines, the area below the curve F_1K, reckoned from the line QK, may be taken.

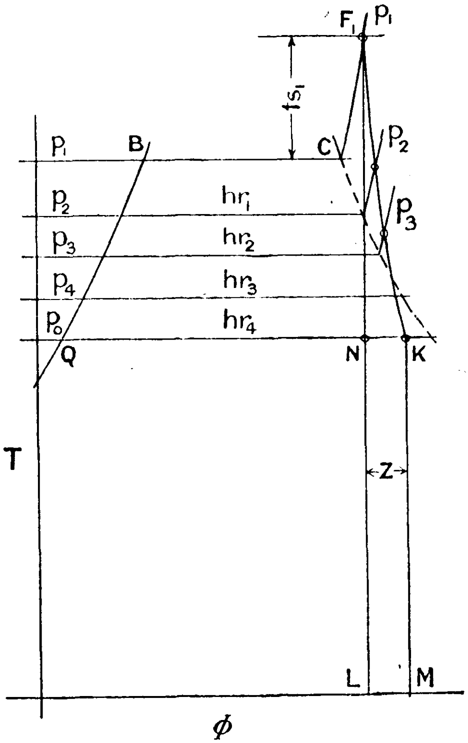


FIG. 172.

176. While the temperature entropy diagram serves to clearly demonstrate the meaning and effect of "reheat," it is inconvenient for practical use.

The Hϕ diagram is usually employed for the determination of heat drops and qualities.

The condition curve is readily located on the Hϕ diagram, when the stage friction loss or the stage efficiency is known.

Let h_r be the heat drop per stage, the stage pressures preferably being chosen to give the condition of equal heat drop, and η_s the stage efficiency, also assumed to be a constant throughout, then the nett heat converted to useful work at the shaft is

$$h_s = (1 - f)h_r = \eta_s h_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

f may be called the "stage friction factor."

Referring to Fig. 173, draw from the initial state point F_1 at p_1 and t_{s_1}, the vertical $F_1a = h_{r_1}$, the stage heat drop. The point a falls on the stage exhaust pressure p_2. From F_1 scale off $F_1b = \eta_s h^{r_1} = h_s$, and through b draw bF_2 horizontally to cut the p_2 curve in F_2. This is the initial state point for the second stage. Next draw $F_2c = h_{r_2}$, the heat drop in the second stage, and the point c gives the initial pressure p_3 for the third stage. Again set off from F_2, $F_2d = \eta_s h_{r_2}$, project horizontally from d to the p_3 curve, and F_3 gives the initial state point for the third stage.

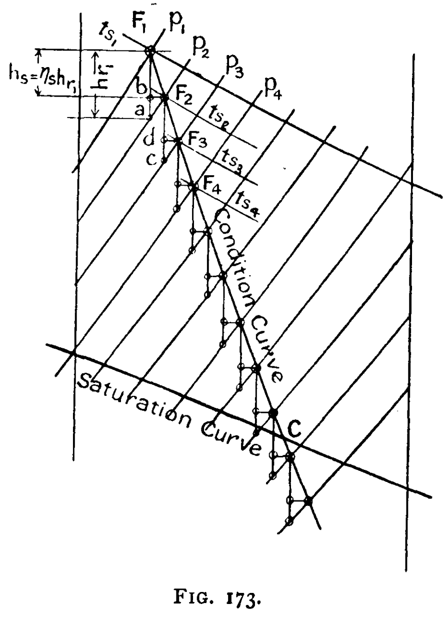


FIG. 173.

By repeating this projective operation throughout the whole machine, the series of state points F_1, F_2, F_3, F_4, etc., and the corresponding pressures p_1, p_2, p_3, etc., and qualities t_{s_1}, t_{s_2}, t_{s_3}, etc., are obtained.

The condition curve F_1C is the locus of these points.

Only the portion of the curve in the superheat field is shown in Fig. 173. A complete curve F_1CK is shown in both superheat and saturation fields in Fig. 174.

In the multistage machine the heat drop as already indicated is sensibly constant, and the condition curve can be drawn with sufficient accuracy for any given value of η_s by using a constant value of h_r.

The projective method outlined here, although practicable, is tedious. A sufficiently good approximation can be obtained when the point C, at which the curve crosses the saturation curve, and the final state point K are known.

When the steam is initially dry or slightly wet, and the curve falls wholly in the saturation field, it is quite sufficient to join the initial and final state points by a straight line.

177. Internal Efficiency of a Compound Turbine, η_1.—Referring to Fig. 174, the heat which would be converted to work in a frictionless turbine between p_1 and p_0 is $H_r = F_1N$. Owing to friction an amount of heat given by NN_1 is not converted to work. This is the "reheat," and the nett or useful heat which appears as work at the shaft is $F_1N_1 = H_I$.

The ratio of the nett heat transformed to work at the shaft, and the total heat drop, is the "internal efficiency," that is

$$\eta_1 = \frac{F_1N_1}{F_1N} = \frac{H_I}{H_r} = \frac{\Sigma h_s}{H_r} = \frac{\eta_s \Sigma h_r}{H_r}$$

Hence $$\eta_1 = \frac{\eta_s R H_r}{H_r}$$

and $$\eta_1 = R\eta_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the relation already deduced in Art. 98.

When the stage efficiency η_s and reheat factor R are known for a given machine, the final point on the condition curve can be found from the relation

$$\eta_1 = \frac{F_1N_1}{F_1N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Then determine η_1 from (2), scale off $F_1N_1 = \eta_1(F_1N)$ from the initial state point F_1, and project horizontally to cut the p_0 curve at K.

178. Approximate Method of Drawing the Condition Curve.—In every case the condition curve is flat, and can be drawn with sufficient accuracy for practical purposes, when, in addition to the initial and final points F_1 and K, the point C on the saturation curve is located.

From an examination of a series of condition curves, drawn in the high-pressure region of the superheat field, the author has deduced the following empirical equation, which gives approximately the slope of a straight line joining F_1 and C.

Let p_1 = initial pressure lbs./in.² abs.
t_{s_1} = „ superheat at p_1 in F°.
$f = (1 - \eta_s)$ = stage friction factor.
ϕ_f = entropy increase per 100 B.Th.U.

Then $$\phi_f = 0{\cdot}055 + 0{\cdot}8f^3 - 0{\cdot}00006(p_1 + t_{s_1}) \quad . \quad . \quad . \quad (4)$$

In order to determine the position of C in the $H\phi$ diagram, calculate ϕ_f. On the adiabatic vertical through F_1 scale off $F_1A = 100$ B.Th.U, Fig. 174. From A scale off horizontally, $AB = \phi_f$. Join F_1 and B and produce the line to cut the saturation curve at C. This point will

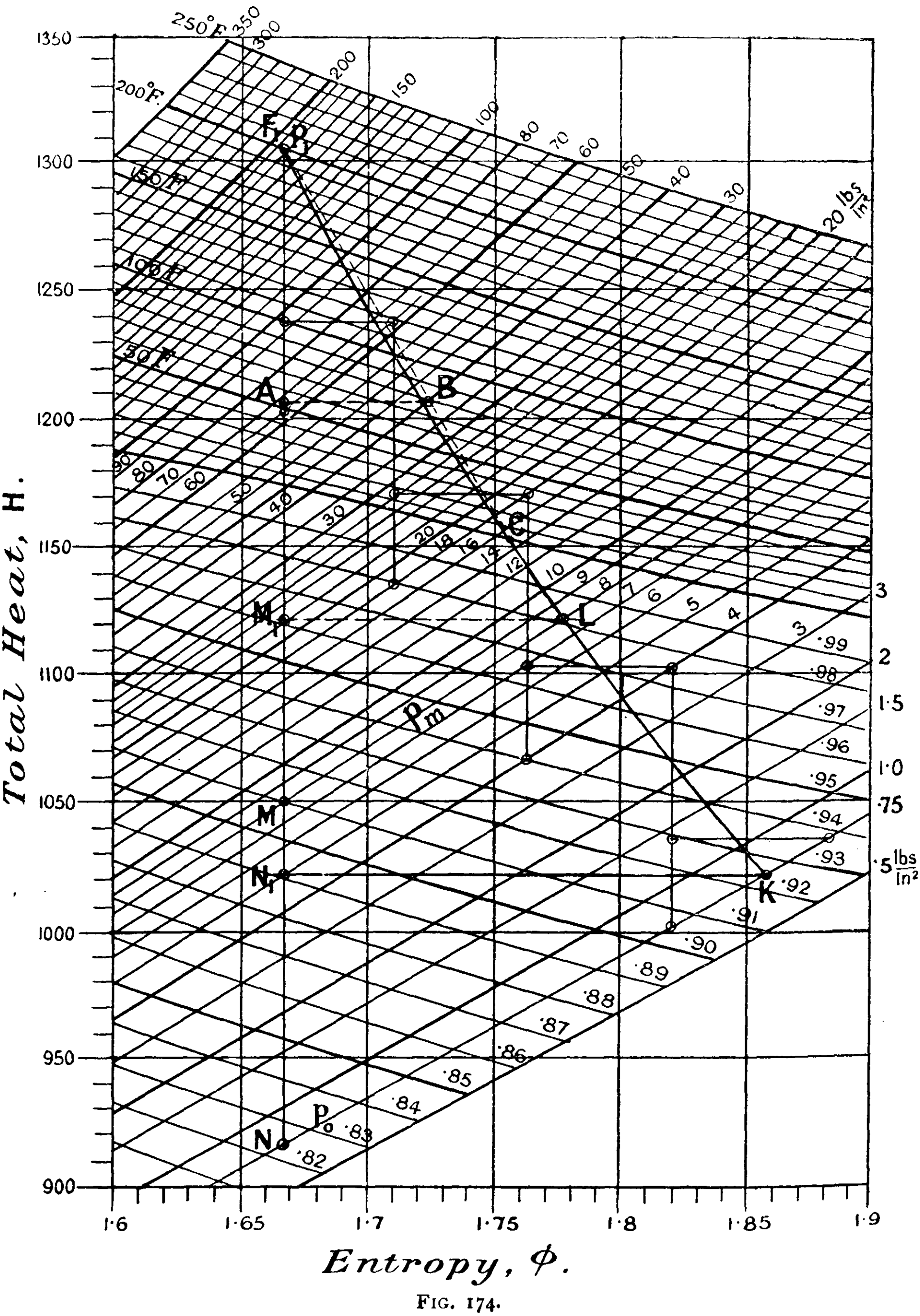

Total Heat, H.
Entropy, φ.
1350
1300
1250
1200
1150
1100
1050
1000
950
900
1·6
1·65
1·7
1·75
1·8
1·85
1·9
250°F
200°F.
150°F.
100°F.
50°F.
20 lbs/in²
3
2
1·5
1·0
·75
·5 lbs/in²
F
A
B
C
L
M
N
K
p_m
p_o
·99
·98
·97
·96
·95
·94
·93
·92
·91
·90
·89
·88
·87
·86
·85
·84
·83
·82

FIG. 174.

fall slightly to the left of a line joining F_1 and K. A fair curve can now be drawn through F_1, C, and K.

179. When expansion takes place wholly in the saturation field the condition curve for saturated steam could be obtained from the following equation, by plotting the entropy increases at several intermediate pressures between p_1 and p_0:—

$$z = \phi_w - \frac{1}{f} - (\phi_{w_1} + q_1\phi_{e_1}) + \left(q_1\phi_{e_1} + \frac{1}{f}\right)\frac{T_1}{T}f$$

where z = entropy increase between p_1 and p or T_1 and T.
ϕ_{w_1} = water entropy at p_1.
ϕ_{e_1} = evaporation entropy at p_1.
T_1 = abs. temperature at p_1.
ϕ_w = water entropy at p.
T = abs. temperature at p.
f = friction factor.

This method of obtaining the points on the condition curve is obviously so clumsy, that it need not be seriously considered. The projective method is much easier and quicker to apply. In any case, as already stated, no sensible error is involved if the initial and final state points are simply joined by a straight line. This method can also be adopted where the steam is slightly superheated.

It should not, however, be applied to the case of the few-stage impulse machine when there are only two, three, or four stages.

A provisional value of R should be assumed and RH_r determined. Then the approximate stage heat drop is

$$h_r = \frac{RH_r}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where n is the number of stages.

With this tentative value the stage pressures and qualities should be found by actual projection on the $H\phi$ diagram. If the final point at the last adiabatic falls either above or below the p_0 curve, the h_r value has to be slightly modified. Usually one trial is sufficient to get the proper adjustment.

The design of the stage nozzles can then be proceeded with, in the manner already illustrated in Chap. VII.

180. Determination of the Approximate Value of the Reheat Factor.—The value of the reheat factor for any given case can be determined when the initial and final state points are known on the $H\phi$ diagram. It will be found on trial, for values of the total pressure ratio $\left(\frac{p_1}{p_0}\right)$, ranging from 100 upwards, that for given values of the stage efficiency η_s, the value of R with initially dry steam is practically a constant. With the usual vacua employed and for pressures from 100 to 200 lbs./in.^{2}, the value of the reheat factor for dry steam may be approximated by the following equation:—

$$\left.\begin{aligned} R &= 1 + 0{\cdot}1f \\ \text{or} \quad R &= 1 + 0{\cdot}1(1 - \eta_s) \end{aligned}\right\} \quad \ldots \ldots \quad (6)$$

When, however, the steam has an initial superheat, the value of R increases with the superheat.

From the examination of a series of condition curves for steam at

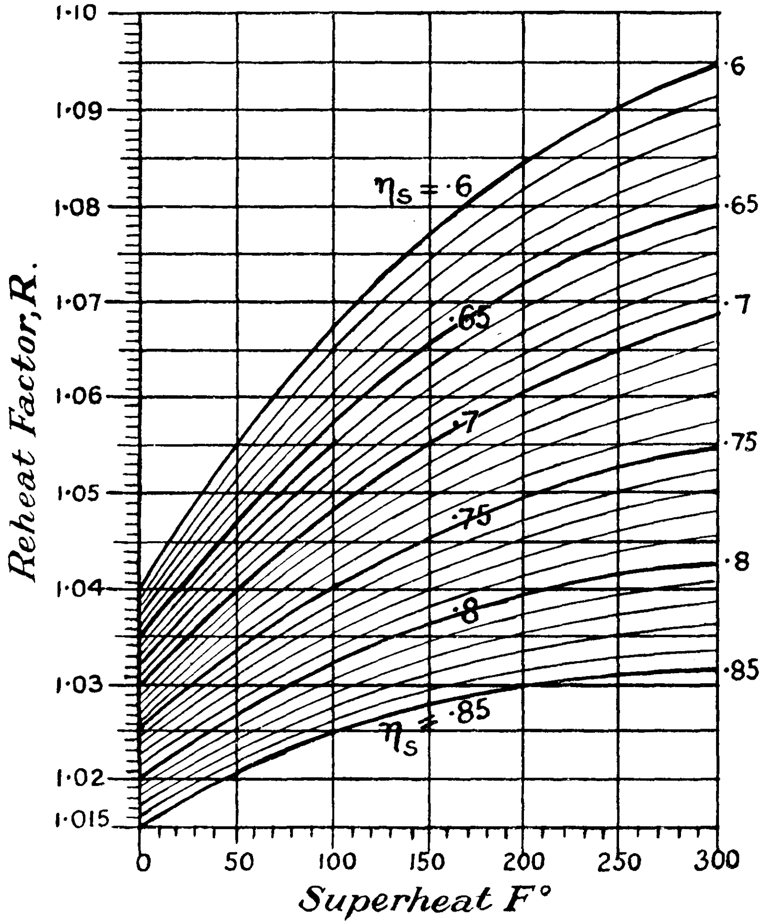


FIG. 175.

pressures between 100 and 200 lbs./in.^{2} and superheats from 0° F. to 300° F., the author has derived a set of mean values of R for multi-stage turbines, and embodied them in the diagram shown in Fig. 175. This diagram represents a series of constant stage efficiency curves plotted on a base of superheat range in F°.

By projecting from the specified superheat to the specified stage efficiency curve, the probable value of the reheat factor is obtained.

For the usual run of initial pressures, between 150 and 200 lbs./in.^{2}, the R values are about correct. Between 100 and 150 lbs./in.^{2} they are slightly on the high side.

EXAMPLE 1.—The initial pressure of a four-stage Curtis turbine is $p_1 = 185$ lbs./in.^{2} abs., superheat 200° F., and the exhaust pressure is 0·75 lb./in.^{2} abs. ($28\frac{1}{2}$ inches vac.).

Determine by projection on the Hϕ diagram, the successive initial stage pressures and qualities of the steam. Assume that the average value of the stage efficiency for the whole machine is 65 per cent.

An approximate value of the reheat factor has first to be chosen. The curves of Fig. 175 give too high values for few-stage machines, as they have been derived from condition curves of multistage machines. If this were a multistage turbine the value of R at 200° F. corresponding to $\eta_s = 65$ would be 1·072. For the four-stage machine a lower value is required. For trial take R = 1·062.

Drawing the adiabatic F_1N in Fig. 174 and scaling, the value of the heat drop is $H_r = 388$.

With R = 1·062, the cumulative heat $\Sigma h_r = RH_r = 1{\cdot}062 \times 388 = 412$.

By equation (5) the approximate stage heat drop is $h_r = \frac{RH_r}{n} = \frac{412}{4} = 103$ B.Th.U. The nett heat per stage is $h_s = 0{\cdot}65 \times 103 = 67$. Scaling $h_r = 103$ from F_1 in Fig. 174, the lower pressure is found to be $p_2 = 62$. Scaling again from F_1, $h_s = 67$, and projecting to the curve, the projector cuts it at $t_{s_2} = 118°$ F. Starting again with this point, scale off h_r and h_s and project. The next stage pressure is $p_3 = 18$ and quality $t_{s_3} = 35°$ F. Repeating the process for the third and fourth stages the corresponding values are found to be $p_4 = 4$, $q_4 = 0{\cdot}975$, and $p_0 = 0{\cdot}75$, $q_0 = 0{\cdot}938$. The last measurement between 4 and 0·75 lb./in.^{2} is exactly 103 B.Th.U., and shows that the original estimate of 103 for h_r is correct for the specified value of the stage efficiency. It follows from this that the internal efficiency by equation (2) is

$$\eta_1 = R\eta_s = 1{\cdot}062 \times 0{\cdot}65 = 0{\cdot}69$$

This checks with the value of the ratio $\frac{F_1N_1}{F_1N}$ obtained from the Hϕ diagram.

The required values are

Stage . . .	1	2	3	4	Exhaust
Pressure . .	185	62	18	4	0·75
Quality . . .	200° F.	118° F.	35° F.	$q = 0{\cdot}975$	0·938

These are the figures used for the calculation of the nozzle areas of this machine in example 6, p. 138.

EXAMPLE 2.—A multistage impulse turbine is supplied with steam at $p_1 = 185$ lbs./in.^{2} abs., $t_{s_1} = 200°$ F., and exhausts at $p_0 = 0{\cdot}75$ lb./in.^{2} abs. ($28\frac{1}{2}$ inches vac.). Assuming an average stage efficiency $\eta_s = 0{\cdot}69$, draw the condition curve on the Hϕ diagram.

Here at $t_{s_1} = 200$ and $\eta_s = 0{\cdot}69$, the curves of Fig. 175, which are applicable to this case, give $R = 1{\cdot}062$.

By equation (2) the internal efficiency is

$$\eta_1 = R\eta_s = 1{\cdot}062 \times 0{\cdot}69 = 0{\cdot}733$$

In order to avoid unnecessary duplication of diagrams the condition curve for this case is also drawn in Fig. 174. Here $F_1N = 338$; hence $F_1N_1 = 388 \times 0{\cdot}733 = 284{\cdot}4$. Projecting from N_1 to $p_0 = 0{\cdot}75$, the projector cuts the curve at $q_0 = 0{\cdot}925$. This is the final point K on the condition curve.

The point C where the condition curve cuts the saturation curve is found next.

Since $\eta_s = 0{\cdot}69$, $f = 0{\cdot}31$. Also $p_1 = 185$, $t_{s_1} = 200$; then by equation 4

$$\begin{aligned}\phi_f &= 0{\cdot}055 + 0{\cdot}8f^3 - 0{\cdot}00006(p_1 + t_{s_1})\\ &= 0{\cdot}055 + 0{\cdot}8 \times 0{\cdot}31^3 - 0{\cdot}00006(185 + 200)\\ &= 0{\cdot}055 + 0{\cdot}0238 - 0{\cdot}0231\\ &= 0{\cdot}0557\end{aligned}$$

Scale off $F_1A = 100$ B.Th.U. and $AB = 0{\cdot}0557$ entropy unit, and join F_1 and B. The line produced cuts the saturation curve at C on the pressure curve $p_c = 15$. The fair curve F_1CK can now be drawn.

181. Cumulative Heat Curve.—When the stage heat drop is sufficiently large to ensure a reasonably accurate measurement on the Hϕ diagram, the initial stage pressures can be found by successively measuring this vertically from the condition curve. The process, however, is tedious and troublesome when the number of stages is large. When the heat drop is small, the method is still more troublesome, and the result is liable to error through inaccuracy in scaling. In any case this method may be dispensed with, and the initial stage pressures easily and quickly determined when the curve of cumulative heat is plotted on a pressure base. With the aid of the condition curve the cumulative heat, Σh_r, between the initial pressure p_1 and any intermediate pressure p_m, can be calculated as follows. Referring to Fig. 174, the projector from the point of intersection L of the p_m and condition curves cuts the adiabatic vertical F_1M in M_1, and the internal efficiency of the section of the machine between p_1 and p_m is given by $\frac{F_1M_1}{F_1M} = \eta_1'$.

By equation (2) the reheat factor is

$$R' = \frac{\eta_1'}{\eta_s} = \frac{F_1M_1}{F_1M} \times \frac{1}{\eta_s}$$

The cumulative heat is

$$H_c = R'H_r' = R'(F_1M)$$

and

$$H_c = \frac{F_1M_1}{\eta_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Hence, to determine the cumulative heat at any arbitrarily chosen pressure, project horizontally from the condition curve at this pressure to cut the adiabatic vertical drawn through the initial state point, and divide the intercept on this vertical by the average value of the stage efficiency.

The cumulative heats thus calculated for several intermediate pressures can be plotted to obtain the H_c curve shown in Fig. 176.

The curve of Rankine cycle heat H_r is also plotted in this figure, and the ratio of the two ordinates at any pressure gives the corresponding value of the reheat factor.

182. Cumulative-Heat Volume Diagram.—As already shown by the examples of nozzle design given in Chap. VII. it is necessary for the complete nozzle calculation to know the stage heat drop, and the initial pressure, quality, and volume, at any given stage. Hence, for the purpose of rapid and easy calculation it is desirable also to plot the curve of quality MQ and specific volume EF on the same pressure base as the cumulative heat curve CB, as shown in Fig. 176.

These three curves are the "characteristic" curves of the turbine, and the combination may be called the "cumulative-heat volume diagram."

In the case of the axial flow reaction turbine it is not necessary to use the quality curve for the blading calculations, and only the cumulative heat and the volume curves need be drawn. In the case of the multistage impulse turbine, when the H_c curve is drawn, the end ordinate AB, which gives the total H_c value, can be arbitrarily divided into as many parts as there are stages. The divisions may either be equal or unequal. Each division represents the corresponding stage heat drop h_r.

When the "carry over" between stages is allowed for, the first stage heat drop may be made slightly greater than the others. Normally it is sufficient to make the remainder equal. By drawing horizontal projectors from the successive points on AB to the H_c curve, the corresponding initial stage pressures are found. The intersections of the verticals through these pressure values, with the t_s or q and v curves, then determine the initial values of the qualities and volumes.

The curves of the diagram are first obtained by choosing a suitable number of pressure values between p_1 and p_0, and deriving the H_c, t_s, and v values from the condition curve. The particular distribution of the stage pressures need not be considered.

If the diagram is plotted on a uniform scale of pressure the usual trouble will arise with the volume curve, on account of the rapid rate of increase between atmospheric and exhaust pressures. This defect can to a large extent be overcome by substituting a logarithmic scale of pressure for a uniform one, as shown in Fig. 176. With a reasonably open uniform scale of volumes a satisfactory volume curve EF is thus obtained. In order, however, to ensure greater accuracy at the higher pressures, when the volumes are small, an auxiliary curve GL, giving ten times the volume, should also be plotted.

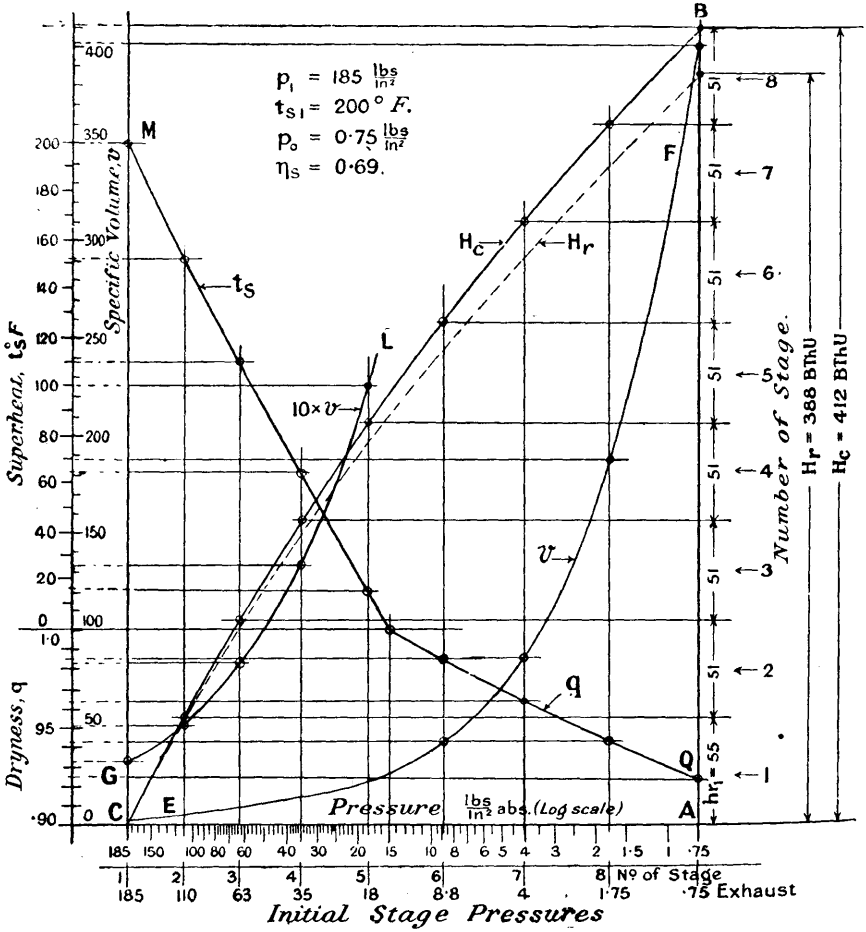


FIG. 176.

There need be no difficulty about the logarithmic pressure scale. It can be ticked off, on a strip of paper, from the upper scale of a 10-inch slide rule.

If it is desired to draw the diagram on a large scale, the lower scale of the slide rule may be used. It also simplifies the construction and operation of the chart to take the same unit length for the heat and volume scales.

EXAMPLE 3.—Draw the cumulative heat volume diagram for a compound multistage impulse turbine working with initial pressure $p_1 = 185$ lbs./in.^{2} abs., superheat 200° F., and exhaust pressure $p_0 = 0{\cdot}75$ lb./in.^{2} abs. Assume that the average value of the stage efficiency is 0·69.

The condition curve corresponding to the specified values is drawn as in the previous example in Fig. 174.

The following convenient set of pressures may be chosen: 185, 100, 60, 20, 10, 5, 2, 0·75.

From the points of intersection with the condition curve draw projectors to cut the adiabatic F_1N_1 and scale off the F_1M_1 values. Also read the qualities where the pressure and condition curves intersect. Tabulate the figures as shown below.

Pressure.	F_1M_1.	$\frac{F_1M_1}{\eta_s} = H_c$.	Quality.	Volume.
185	0	0	$t_s = 200°$ F.	3·27
100	45	65	143	5·5
60	75	109	106	8·5
20	138	200	23	22
10	173	251	$q = 0{\cdot}988$	38
5	204	297	0·968	71
2	245	352	0·946	164
0·75	284·4	412	0·925	402

The cumulative heat values are calculated by equation (7), using the value $\eta_s = 0{\cdot}69$. The first four volumes are obtained from the alignment chart, accompanying the large $H\phi$ diagram. The other four for the wet steam are obtained by calculation from the dry steam volumes, given in the steam tables.

These values when plotted in the logarithmic pressure base give the set of curves shown in Fig. 176.

EXAMPLE 4.—Apply the curves of the H_c—v diagram, obtained from the data of examples 2 and 3, to determine the initial stage pressures, qualities, and volumes for an eight-stage pressure compounded impulse turbine. Assume that the heat drop in the first stage is about 8 per cent. greater than the constant heat drop of the other seven stages.

The question is answered as soon as the heat-drop values are known.

Let h_r = heat drop in the second and succeeding stages, then

$$1{\cdot}08h_r + 7h_r = H_c$$

$$\therefore h_r = \frac{H_c}{8{\cdot}08} = \frac{412}{8{\cdot}08} = 51 \text{ B.Th.U.}$$

and

$$h_{r_1} = 1{\cdot}08 \times 51 = 55 \text{ B.Th.U.}$$

Scaling off these heat values in succession along the end ordinate AB, Fig. 176, and projecting horizontally to the H_c curve, the stage pressures, qualities, and volumes are found as follows :—

Stage . .	1	2	3	4	5	6	7	8
Pressure .	185	110	63	35	18	8·8	4	1·75
Quality . .	200° F.	152° F.	110° F.	65° F.	17° F.	q=0·985	0·963	0·943
Volume. .	3·27	5·1	8·3	13·3	22·8	43	86	187

The application, in detail, of these figures to the design of the stage nozzles for this turbine has already been given in example 9, Chap. VII. p. 144.

Practically the same results are obtained if the longer and more troublesome projection method is used in the determination of the condition curve.

This H_c—v diagram can be applied to any other turbine with a different number of stages, working between the same pressure limits, when the average value of the stage efficiency is taken as 0·69.

183. It will be noted that the dotted heat-drop or H_r curve is nearly coincident with the H_c curve for the first three stages, and that it only begins to diverge considerably near the L.P. end of the pressure range. A considerable variation in the value of η_s does not materially affect the relation of these two curves at the higher pressures, and the volume values on which the passage areas depend are likewise very little affected. It is obvious from this condition that, although the stage efficiency may be appreciably lower in the first two or three stages than in the remainder, it is legitimate to use the higher average value for the determination of the cumulative heats.

This condition applies equally to the case of the axial-flow reaction turbine, in which the stage efficiency runs about 0·7 at the H.P., 0·75 at the I.P., and 0·8 at the L.P. section.

184. Nett Efficiency or Efficiency Ratio.—The nett efficiency is meant when the efficiency of a steam turbine is stated without any qualification.

This term "efficiency" that has come into general use in connection with the steam turbine is not strictly correct. The figure referred to is an efficiency ratio, that is, the ratio between the thermodynamic efficiencies of the actual and ideal turbines, working between the same pressure and temperature limits.

Let H_r = heat drop or Rankine cycle heat between p_1 and p_0.
H_1 = total heat per lb. of steam at p_1 and t_{s_1} or q_1.
h_0 = heat in water at lower temperature t_0 at p_0.
H_B = heat equivalent of the nett work per lb. or B.H.P. heat of the actual turbine.

$$\text{Absolute efficiency of Rankine cycle } \eta_r = \frac{H_r}{H_1 - h_0}$$

$$\text{,, \quad ,, \quad actual turbine} = \eta_a = \frac{H_B}{H_1 - h_0}$$

$$\text{Efficiency ratio} = \eta = \frac{\eta_a}{\eta_r} = \frac{H_B}{H_r}, \text{ as already quoted in Art. 99.}$$

In the case of a turbo-generator the "over-all" efficiency of the turbine and generator η_0 is often stated. In order to distinguish between the two, the efficiency ratio is sometimes called the "Turbine efficiency." If $\eta_g =$ generator efficiency, then

$$\eta^g = \frac{\eta_0}{\eta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The nett efficiency covers all the losses of the machine. The internal efficiency does not cover the losses due to bearing friction, gear friction, pump friction, external gland and partial dummy leakage loss, radiation, and final carry over at exhaust. These are losses occurring outside the working chambers, and may be called "outside losses."

Reckoned as a percentage of the heat drop, which is taken here as the standard of comparison for all the others, the sum of these outside losses represents the difference between the internal and nett efficiencies. Thus if the outside loss is x per cent.

$$\eta = \eta_1 - x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For medium and large impulse machines the calculations, given in Chap. X., indicate that the outside loss may be taken from 2 to 4 per cent. For smaller units it may run from 4 to 6 per cent. of the heat drop.

In the case of the reaction turbine, where there are three or more dummy pistons, an additional allowance should be made. At each section, the leakage, as far as the blading of the section is concerned, is equivalent to an outside gland loss. Except at the L.P. dummy, where the leakage goes directly to the condenser, part of the leakage heat is available in the succeeding sections. It is better to overstate rather than understate this probable loss, and the addition may be taken from 3 to 5 per cent., according to the size of the turbine.

An allowance of from 6 to 9 per cent. for the total outside loss in this type may therefore be made, in order to obtain the approximate value of the efficiency ratio, when the internal efficiency is known. Figures within these limits give values of η, which are in fair agreement with the actual efficiency values calculated from test results.

185. Estimation of the Efficiency Ratio from the Steam Consumption.—When the value of η and the heat drop H_r for the total pressure range are determined, a fair estimation of the probable steam consumption for any proposed turbine can be made.

Conversely, when the actual consumption of a turbine is determined on test, the efficiency for the test conditions of operation can be calculated.

Let $w_r =$ Rankine cycle consumption in lbs./H.P. hour.
$w_a =$ actual turbine " " "
$H_r =$ heat drop per lb. in B.Th.U.
$\eta =$ nett efficiency or efficiency ratio of turbine.

For the ideal machine the heat available for the production of one horse power is $w_r H_r \dfrac{\text{B.Th.U.}}{\text{hour}}$.

Heat equivalent of (1) H.P. $= 42{\cdot}4 \times 60 = 2544 \dfrac{\text{B.Th.U.}}{\text{hour}}$.

Since the efficiency ratio of the Rankine cycle is unity these are equal, and

$$2544 = w_r H_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Again for the actual turbine the heat available for one H.P. is $w_a \eta H_r$, hence

$$w_a \eta H_r = 2544 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and $w_a \eta H_r = w_r H_r$, so that

$$\eta = \frac{w_r}{w_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

That is, the efficiency ratio is given by the ratio of the ideal and actual steam consumption.

Estimation of Steam Consumption for a given Efficiency Ratio.—From equation (11) the ideal consumption is given by

$$\left.\begin{aligned} w_r &= \frac{2544}{H_r} \text{ lbs./B.H.P. hour or lbs./S.H.P. hour} \\ \text{or} \quad w_r &= \frac{3414}{H_r} \text{ lbs./shaft K.W. hour} \end{aligned}\right\} \quad . \quad (13)$$

The actual consumption is given by

$$\left.\begin{aligned} w_a &= \frac{2544}{\eta H_r} \text{ lbs./B.H.P. hour or lbs./S.H.P. hour} \\ \text{or} \quad w_a &= \frac{3414}{\eta H_r} \text{ lbs./shaft K.W. hour} \end{aligned}\right\} \quad . \quad (14)$$

For any turbine, when the efficiency ratio is known from the heat conditions, the approximate internal efficiency can be estimated from equation (9) by substituting the probable value of the outside loss x, and *vice versâ*. Some idea can then be obtained of the average stage efficiency of the machine, from the curves of Fig. 175.

186. It should be noted that the value of w_a obtained from equation (14) depends on the initial pressure chosen in the calculation of the heat drop. This may either be the stop-valve pressure or the pressure below the governor valve. Sometimes the one, sometimes the other is used. Hence a statement of the efficiency ratio for any turbine, without specification of the particular initial pressure, is incomplete. The ratio is larger when the pressure below the throttle is used than when the stop-valve pressure is used.

It is desirable that a uniform system of calculation should be adopted for all tests. In the case of the steam engine the efficiency ratio, according to the recommendation of the Institution of Civil Engineers on the Method of Engine and Boiler Trials, is calculated for the initial pressure at the stop valve. For uniformity the same condition should be taken in the case of the steam turbine.

With certain types of automatic governing used on reaction turbines, the stop-valve pressure is sometimes about 15 lbs./in.^{2} higher than the pressure below the governor valve, when the machine runs under normal full load.

In dealing with such a case, the exact method for the location of the condition curve on the Hϕ diagram is to find the initial condition at the pressure below the governor valve by projection horizontally between the stop-valve and governor-valve pressures, and to take the intersection with the lower-pressure curve as the initial state point.

187. In other cases it may be assumed, without any sensible error, that the full stop-valve pressure exists at the throttle under the maximum load, and that the calculations may be made for the condition curve and curves of heat and volume, without further refinement of the pressure and superheat conditions and complication of the calculations.

In either case, the heat drop between the stop-valve pressure and the exhaust pressure should be used to calculate the internal efficiency for the location of the final state point on the condition curve.

EXAMPLE 5.—The conditions during the test of a pressure compounded impulse turbine, rated for a normal output of 5000 K.W. at 1000 R.P.M., were, pressure before the governor valve 162 lbs./in.^{2} abs., superheat 180° F., vacuum 27·7 inches. The output at the generator, which had an efficiency of 0·953, was 5118 K.W., and the steam consumption per hour was 77,640 lbs. Calculate the efficiency ratio and the over-all efficiency of the combination. Also, assuming a total outside loss of 3 per cent. of the heat drop, calculate the probable values of the reheat factor and the average stage efficiency.

Here the actual consumption

$$w_g = \frac{77640}{5118} = 15{\cdot}17 \text{ lbs./K.W. hour at generator}$$

or

$$w_a = 15{\cdot}17 \times 0{\cdot}953 = 14{\cdot}46 \text{ lbs./K.W. hour at the shaft}$$

Heat drop between 162 and 1·15 lbs./in.^{2} from the Hϕ diagram is $H_r = 355$ B.Th.U.

By equation (13) the Rankine engine consumption is

$$w_r = \frac{3414}{H_r} = \frac{3414}{355} = 9{\cdot}6$$

By equation (12)

$$\text{Efficiency ratio} = \eta = \frac{w_r}{w_a} = \frac{9{\cdot}6}{14{\cdot}46} = 0{\cdot}665$$

By equation (8)

$$\text{Over-all efficiency } \eta_0 = \eta\eta_g = 0{\cdot}665 \times 0{\cdot}953 = 0{\cdot}631$$

Since the percentage outside loss is $x = 0{\cdot}03$, by equation (9)

$$\text{Internal efficiency } \eta_1 = \eta + x = 0{\cdot}665 + 0{\cdot}03 = 0{\cdot}695$$

Referring to the curves, Fig. 175, assume for a trial an average stage efficiency $\eta = 0{\cdot}65$. At 180° F. superheat the corresponding reheat factor $R = 1{\cdot}07$. By equation (2)

$$\eta_1 = 1{\cdot}07 \times 0{\cdot}65 = 0{\cdot}6955, \text{ the value already deduced}$$

The probable values of the average stage efficiency and the reheat factor are thus 0·65 and 1·07 for this machine, under the conditions specified.

EXAMPLE 6.—A Curtis turbine designed for an output of 3000 K.W. at 1500 R.P.M. ran under test with initial pressure 155 lbs./in.^{2} abs., superheat 145° F., and vacuum 27 inches. The consumption was 15·96 lbs. per K.W. hour at a load of 2987 K.W. The efficiency of the generator was 94·5 per cent. Calculate the over-all efficiency, the efficiency ratio, and the probable internal efficiency, reheat factor, and mean stage efficiency.

Consumption $= w_a = 15{\cdot}96 \times 0{\cdot}945 = 15{\cdot}08$ lbs./shaft K.W. hour.
Heat drop from $H\phi$ diagram $H_r = 330$.
By equation (14)

$$w_a = \frac{3410}{\eta H_r}$$

$$\therefore \ \eta = \frac{3410}{15{\cdot}08 \times 330} = 0{\cdot}685$$

Over-all efficiency $\eta_0 = 0{\cdot}685 \times 0{\cdot}945 = 0{\cdot}647$
Internal efficiency $\eta_1 = 0{\cdot}685 + 0{\cdot}03 = 0{\cdot}713$
From the curves of Fig. 175 by trial, with $\eta_s = 0{\cdot}67$ at $t_s = 145°$ F., the reheat factor is $R = 1{\cdot}0615$.

And $$\eta_1 = R\eta_s = 0{\cdot}67 \times 1{\cdot}0615 = 0{\cdot}71.$$

By assuming a slightly lower value for the outside loss, the internal and stage efficiencies might be taken as 0·7 and 0·66, or 0·665 at the outside.

EXAMPLE 7.—An axial-flow reaction turbine is required to work between 205 lbs./in.^{2} abs., 175° F. superheat, and 29 inches vacuum. Find the probable steam consumption on the assumption of an average stage efficiency $\eta_s = 0{\cdot}75$ for the whole machine, and a generator efficiency $\eta_g = 0{\cdot}95$. The machine is to be capable of an output of 6250 KW at 1200 R.P.M., and the lower value of the total outside loss may be taken as 6 per cent. of the heat drop.

In the first place, the reheat factor corresponding to $t_s = 175$ and $\eta_s = 0{\cdot}75$ scaled from the curve, Fig. 175, is $R = 1{\cdot}048$. Hence

$$\eta_1 = R\eta_s = 1{\cdot}048 \times 0{\cdot}75 = 0{\cdot}786$$

Subtracting 6 per cent., outside loss,

$$\eta = 0{\cdot}786 - 0{\cdot}06 = 0{\cdot}726$$

The heat drop from Hϕ diagram between 205 and 0·5 lbs./in.^{2} abs. is $H_r = 410$.

By equation (14) the consumption is

$$w_a = \frac{3414}{0{\cdot}726 \times 410} = 11{\cdot}5 \text{ lbs./shaft K.W. hour}$$

$$w_g = \frac{11{\cdot}5}{0{\cdot}95} = 12{\cdot}1 \text{ lbs./K.W. hour at generator}$$

This consumption is $2\frac{1}{2}$ per cent. greater than was obtained from the actual machine running under the conditions specified here, so that it is probable that the average stage efficiency assumed is on the low side. A value of 75·5 gives about the correct experimental figure.

188. Average Values of Efficiency Ratio.—The maximum value of η that may be reached in the case of a very large machine is in the neighbourhood of 74 per cent. This so far may be taken as the extreme limit, where high pressure, high superheat, and high vacuum are employed. In recent practice, with large units developing from 10,000 to 20,000 K.W., the limits of 200 lbs./in.^{2} gauge, 200° F. superheat, and 29 inches vacuum, have been used. These conditions now apply to large Parsons turbines, where the superheat difficulty has been met by the use of special steel material for the casing at the H.P. end. It is claimed that at least 73 per cent. efficiency has been obtained from the multistage impulse machine working under somewhat similar conditions. In the case of a medium or fairly large machine working under less favourable pressure conditions, the value may run from 68 to 70 per cent. For small units and simple impulse machines it may range from 30 to 50 per cent.

In the case of the direct-coupled marine turbine of fairly large output using saturated steam the values may run between 58 and 62 per cent.

Low-pressure turbines, exhaust and mixed pressure, working between 16 lbs./in.^{2} abs. with dry steam and 27 to 28 inches vacuum, show a fairly high efficiency ratio. The figures appear to run from 68 to 70 per cent. at full-load conditions for machines of moderate output.

CHAPTER XII

STEAM CONSUMPTION

BEFORE the proportions of nozzles and blading throughout any proposed turbine can be calculated, the probable steam consumption at the maximum output has to be estimated.

This may be fixed either from a knowledge of the consumptions of similar machines working under approximately the same conditions of pressure, superheat, and vacuum, or deduced from the estimated efficiency ratio in the manner illustrated in example 6, p. 344.

For a land turbine the consumption is generally given in lbs./K.W. hour at the generator; for a turbine which does not drive a generator, in lbs./B.H.P. hour; and for a marine turbine, in lbs./S.H.P. hour.

189. Average Values of Steam Consumption. High Pressure Condensing Turbines.—For machines of moderate and large output (1000 to 6000 K.W.) working at pressures from 170 to 200 lbs./in.^{2} gauge (or 185 to 215 lbs./in.^{2} abs.), superheat about 200° F., and vacuum from 28 to 28½ inches, the consumption may run from 14 to 12 lbs./K.W. hour. The following figures are said to be obtained from the combination turbines manufactured by a leading firm :—

K.W.	500	1000	2000	3000	4000	5000	6000
lbs./K.W. hour . .	15·5	14·2	13	12·3	11·8	11·6	11·4

Similar results have been obtained from multistage impulse machines working under conditions approximating to those above; the value of the consumption ranges from 13·5 to 12 lbs./K.W. hour.

Published records of Curtis turbine tests are not numerous. From what information is available, it would appear that for machines from 1000 to 3000 K.W., working between limits 150 to 170 lbs./in.^{2} gauge (165 to 185 lbs./in.^{2} abs.), superheats from 100° to 150° F., and vacuum 28 to 28½ inches, the values run between 14 and 17 lbs./K.W. hour.

These figures would be proportionately reduced by the use of higher pressures and superheats (see example 7).

Parsons turbines, until recently, have been worked at lower superheats than impulse machines, the usual range being from 100° to 130° F.

With pressures from 180 to 200 lbs./in.^{2} gauge (195 to 215 lbs./in.^{2} abs.), and vacuum 28 to 29 inches, the figures run from 13 to 15 lbs./K.W. hour.

In recent practice, owing to the improvements in construction at the H.P. end of the casing, high superheats have been introduced, and a consumption of 11 lbs./K.W. hour obtained, with large units developing 20,000 K.W. and over.

There is not so much information available for the newer radial-flow Ljungström turbine, which is capable of standing higher superheats than have yet been employed in the axial reaction machine.

The earlier machines developing about 1000 K.W., with pressure about 170 lbs./in.^{2} gauge (185 lbs./in.^{2} abs.), superheat 300° F., and vacuum 29 inches, gave a consumption of 11·5 lbs./K.W. hour.

A substantial reduction in this figure may be expected at larger outputs, and the most recent tests carried out on a 5000 K.W. machine tend to confirm this estimate, as a consumption of 10·25 lbs./K.W. hour at full load has been obtained.

The foregoing figures are quoted to define, in a general way, the limits between which the consumption of a proposed high-pressure turbine may be expected to lie when run under conditions of pressure, superheat, and vacuum used in present-day practice.

190. Marine Turbines.—Marine turbines, until quite recently, have been worked with saturated or only slightly superheated steam at pressures ranging from 180 to 250 lbs./in.^{2} gauge (195 to 265 lbs./in.^{2} abs.), and vacuum 27 to 28 inches. The consumptions are inferior to those obtained with the high-pressure electrical type, using steam of high superheat, and running at more efficient speeds. They range from 15 to 12 lbs./S.H.P. hour. Assuming for comparison an average generator efficiency of 95 per cent., these figures represent from 21 to 17 lbs./K.W. hour for the electrical machine. The lower figure of 12 has been obtained with the mechanically geared turbine, and it is probable, after more experience with this type, that the consumption may be further reduced. The adoption of superheaters on shipboard will ensure an appreciable reduction and bring the consumption more in line with that of the land type.

191. Back Pressure Turbine.—This turbine is simply a high-pressure non-condensing one. The "back" pressure of the steam required for heating or industrial purposes may range from atmospheric to 70 lbs./in.^{2} above atmosphere. The consumption under a steady load increases with increase of back or exhaust pressure.

Figures for the type are scarce. In Fig. 177 a curve of consumption is given for a 1000 K.W. machine. It shows the probable consumption in lbs./K.W. hour for a series of back pressures ranging from atmospheric to 60 lbs./in.^{2} above atmosphere. The initial pressure is 170 lbs./in.^{2} gauge and the superheat 200° F. For any other initial pressure and superheat conditions the corresponding consumption, with a given back pressure, can be roughly estimated by the correction method discussed in Art. 204.

192. Reducing Turbine.—The field of application of the back-pressure turbine is limited, as in most industries the demand for exhaust steam is not constant. For increased efficiency of the combined power and heating plant a reducing turbine is sometimes fitted.

This is simply a high-pressure condensing machine, with provision for the abstraction of the required amount of steam for heating or industrial purposes, from an intermediate stage. With zero demand the machine runs as a high-pressure condensing one. Under this condition the consumption may vary from 16 to 19 lbs./K.W. hour. When the maximum demand for steam has to be met, the low-pressure section simply runs idle in a vacuum. The consumption is then practically that of a pure back-pressure turbine, and can be approximated from the figures given by the curve in Fig. 177.

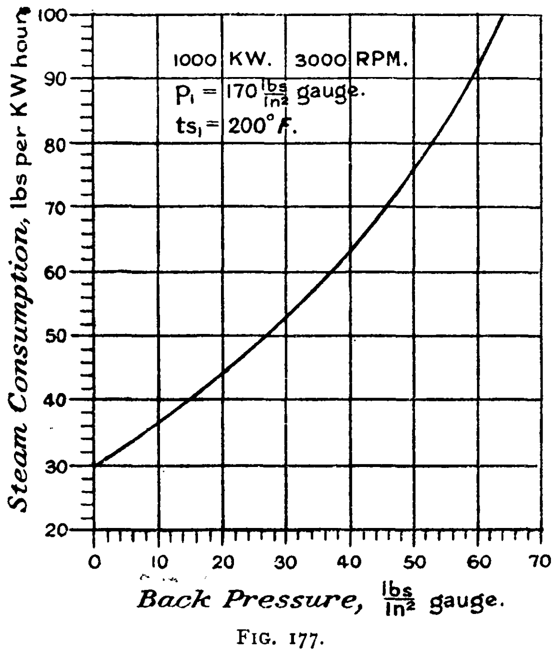


FIG. 177.

193. Exhaust Pressure Turbine.[1]—The low-pressure, like the back-pressure type has a limited field of operation. The supply of exhaust steam is in most cases of a fluctuating character. Before the advent of the geared marine turbine the combination of slow-running reciprocating engines with low-pressure turbines was adopted in a number of cases, with a view to the more economical propulsion of the vessels. The success of the geared turbine has to a large extent rendered this combination unnecessary. Some interesting particulars of a combination arrangement, fitted in the s.s. *Otaki*, are given in a paper by Engineer-Commander W. McK. Wisnom, R.N.[2] The consumption of the pure exhaust pressure turbine, which works with an initial pressure from 16

[1] For drawings and description of a pure exhaust pressure Brush turbine, see *Engineering*, July 1, 1910.
[2] *Trans. Inst. Engineers and Shipbuilders in Scotland*, vol. li., 1908–1909.

to 17 lbs./in.^{2} abs., steam saturated, and a vacuum from 27 to 28 inches, may vary from 30 to 35 lbs./K.W. hour.

194. Mixed Pressure Turbine.—The mixed pressure turbine is now generally fitted in cases where the exhaust steam from a number of engines giving more or less intermittent supply is to be utilised. In the majority of cases where it is used, for instance, in collieries and rolling mills, it is necessary to interpose a heat accumulator between the sources of exhaust supply and the turbine. This machine is usually constructed for an output of from 1000 to 2000 K.W. It may be regarded as a low-pressure turbine with a number of high-pressure stages added to enable additional high-pressure steam to be used, when the maximum available amount of low-pressure steam is insufficient to maintain full load.

The consumption of this type cannot be stated explicitly like that of the others.

It is usual to state the two extreme conditions, that is, the consumption when the machine is run on high-pressure steam during stoppage of exhaust supply, and the consumption when run as a low-pressure turbine on exhaust supply, with the high-pressure supply cut off.

Used as a high-pressure turbine with pressures from 100 to 150 lbs./in.^{2} gauge, steam dry or slightly superheated, and vacuum 27 to 28 inches, the consumption may vary from 17 to 22 lbs./K.W. hour. Used as a low-pressure turbine, with pressures from 16 to 17 lbs./in.^{2} abs., dry steam, and 27 to 28 inches vacuum, the consumption may vary from 33 to 38 lbs. per K.W. hour.

195. Graphical Record of Consumption Test.—This record, to be of value for the comparison of turbine performances, and the deduction of design coefficients, should contain the following data :—

(*a*) Curve of total consumption on a base of K.W. or B.H.P. or S.H.P.

(*b*) Either the curve of steam per K.W. hour at the generator or curve of steam per B.H.P. hour or shaft horse-power hour. If the first only is given the generator efficiency for each load should be stated.

(*c*) Curve of absolute pressure below the throttle.

Also at each fractional load ordinate the pressure above the throttle, the superheat, and the vacuum should be tabulated on the record sheet. The designed or guarantee conditions of pressure, superheat, and vacuum should be written at the head of the sheet.

A record executed on the above lines is shown in Fig. 178. The results are taken from a test of a ~~5~~000 K.W. Zoelly impulse turbine, made by Messrs. Escher Wyss & Co., Zurich. 2/

It will be seen that the curve of total steam consumption is a straight line (Willans Line). In some instances the curve tends to flatten out near the "no-load" end. As a rule, however, for all classes of multistage turbine with throttle governing, the curve is sensibly straight between no load and full load. In some instances, where a nozzle cut-out system is used at the first stage, the consumption line is stepped, the length at each step being straight (see Fig. 229). When an overload has to be taken by means of a bye-pass, the line begins to curve upward after the bye-pass is opened.

ZOELLY STEAM TURBINE.

2000 K.W. 3000 R.P.M.

Escher Wyss & Co., Zurich.

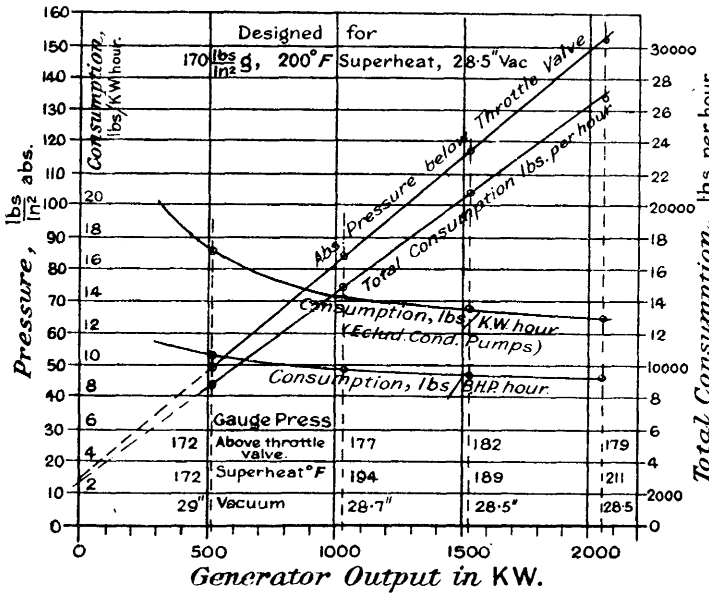


FIG. 178.

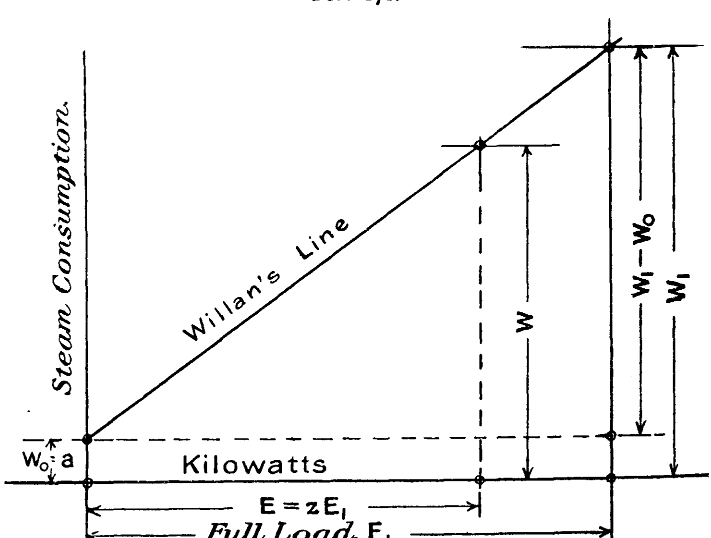


FIG. 179.

196. Calculation of Consumption at Fractional Loads.—When the full-load consumption of a proposed turbine is estimated, the probable consumptions at fractional loads may be approximated by assuming the law of the Willans Line.

Referring to Fig. 179, let

W_0 = no load consumption in lbs./hour.
W_1 = full " " "
E_1 = full load output at the generator.
$y = \dfrac{W_0}{W_1}$.
E = fractional load for which the consumption is required.
$x = \dfrac{E}{E_1}$ = fractional value of the load.
W = consumption at fractional load in lbs./hour.

The law of the Willans Line is

$$W = a + bE \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here $a = W_0 = yW_1$

$$b = \frac{W_1 - W_0}{E_1} = \frac{W_1}{E_1}(1 - y) = w_{g_1}(1 - y)$$

where w_{g_1} = rate of steam consumption at full load. Substituting in equation (1) for a and b, and dividing each side by the fractional load E

$$\frac{W}{E} = w_g = w_{g_1}\left\{y\frac{E_1}{E} + (1 - y)\right\}$$

The fractional load consumption in lbs. per K.W. hour at the generator is thus

$$w_g = w_{g_1}\left\{\frac{y}{x} + (1 - y)\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the case of a high-pressure turbine working between the usual limits of pressure, superheat, and vacuum, the value of y, the ratio of no-load to full-load consumption, varies with the size of the machine. From the test results of a considerable number of impulse and reaction machines, the author has calculated y, and derived the average curve shown in Fig. 180, for outputs ranging from 500 to 5000 K.W. The value usually lies between 0·1 and 0·15, and this curve may be used to obtain a rough approximation to the ratio for the range of output quoted.

In the case of the low-pressure turbine or the low-pressure section of the mixed pressure machine, the value may vary from 0·25 to 0·3. A fair average may be taken as 0·28.

For a proposed turbine the full-load rate of consumption w_{g_1} can first be calculated by the method given in Art. 185, and the approximate fractional load consumption can then be estimated by equation (2).

EXAMPLE 1.—Find the probable rates of consumption at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and full load of a multistage impulse turbine, for which the full-load conditions at 2500 K.W. are $p_1 = 185$ lbs./in.^{2} abs., $t_s = 200°$ F., vacuum $28\frac{1}{2}$ inches, $\eta_g = 0{\cdot}95$, and the efficiency ratio is estimated at $\eta = 0{\cdot}71$.

From the curve, Fig. 180, the probable value of y is 0·12.

Heat drop $H_r = 388$, $\eta = 0{\cdot}71$, $y = 0{\cdot}12$, $\eta_g = 0{\cdot}95$.

The full-load rate is

$$w_{g_1} = \frac{3414}{0{\cdot}71 \times 388 \times 0{\cdot}95} = 13{\cdot}03 \text{ lbs./K.W. hour}$$

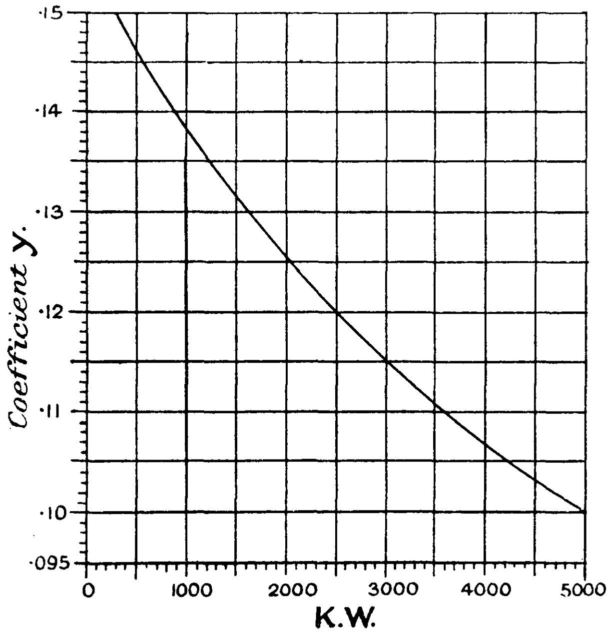


FIG. 180.

By equation (2) consumption at $\frac{3}{4}$ load ($z = 0{\cdot}75$) is

$$w_{\frac{3}{4}} = w_{g_1}\left\{\frac{0{\cdot}12}{0{\cdot}75} + (1 - 0{\cdot}12)\right\}$$
$$= 1{\cdot}04 w_{g_1}$$

The $\frac{3}{4}$-load consumption rate is probably 4 per cent. greater than the full-load value.

Calculating similarly, the consumption rates are at $\frac{1}{2}$ and $\frac{1}{4}$ load, 11·2 and 36 per cent. greater than the full-load value. At no load the rate is infinite. The probable values of the consumption rates are ∞, 17·71, 14·5, 13·55, and 13·03 lbs. per K.W. hour at the generator.

The corresponding rates in lbs./shaft K.W. hour depend on the decreasing value of the generator efficiency, and are calculable when the efficiency curve of the generator is available.

EXAMPLE 2.—An exhaust pressure turbine working with dry steam, 17 lbs./in.^{2} abs., and 26·4 inches vacuum, is required to develop a maximum output of 1000 K.W. at the generator. Assuming the efficiency ratio for the turbine as $\eta = 0·685$, and a generator efficiency $\eta_g = 0·95$, calculate the probable consumptions at full, $\frac{3}{4}$, and $\frac{1}{2}$ load. Assume that the no-load total consumption is 28 per cent. of the full-load value.

The heat drop is $H_r = 144$.

Actual consumption at full load

$$w_{g_1} = \frac{3414}{\eta H_r \eta_g} = \frac{3414}{0·685 \times 144 \times 0·95} = 36·5 \text{ lbs./K.W. hour}$$

$$\text{At } \tfrac{3}{4} \text{ load} \quad w_{\frac{3}{4}} = w_{g_1}\left\{\frac{0·28}{0·75} + (1 - 0·28)\right\} = 1·094 w_{g_1} = 40$$

$$\text{,, } \tfrac{1}{2} \text{ ,,} \quad w_{\frac{1}{2}} = w_{g_1}\left\{\frac{0·28}{0·5} + (1 - 0·28)\right\} = 1·28 w_{g_1} = 46·7$$

The consumptions are thus probably $9\frac{1}{2}$ and 28 per cent. greater than the full-load consumption.

197. Combined H.P. and L.P. Consumption of Mixed Pressure Turbine.—In order to approximate the probable consumption of high-pressure steam in a mixed pressure turbine, when it is run with L.P. supply insufficient to maintain full load, the following method may be used. The validity of the result depends on the rates of consumption chosen for the high- and low-pressure supply respectively.

When the L.P. steam is shut off, the machine runs as a high-pressure condensing turbine, having the minimum rate of consumption for H.P. steam. When the low-pressure supply is a maximum and the high-pressure steam is cut off, the rate of consumption for the low-pressure steam is a minimum.

When the low-pressure supply is some value between zero and maximum, the rate of consumption for both high- and low-pressure steam is increased. The difficulty is to determine the rate in each case for a stated total supply of exhaust steam.

If it is assumed for the reduced supply of low-pressure steam that the relation $pv = c$ is approximately true, then in consequence, the initial L.P. pressure is throttled down in the same proportion as the reduced to the full-load supply.

For instance, if the supply of low-pressure steam is reduced to 75 per cent. of the total at full load, the initial pressure is reduced to 75 per cent. of the full-load pressure. For a given reduction of pressure (assuming the vacuum is kept the same) a correction can be made for the low-pressure steam rate of consumption (see Art. 204). The increased high-pressure steam rate of consumption is not so easily ascertained. As the L.P. supply increases from zero to the maximum the supply of H.P. steam is decreased, and there is a progressive decrease of pressure at the H.P. throttle with some increase of superheat or dryness. This reduction is not determinable definitely. A rough approximation to the probable increased rate of high-pressure steam consumption can, however, be obtained by using the probable full-load consumption on H.P. steam in conjunction with equation (2), Art. 196.

198. Let E = total load to be maintained in K.W. at the generator.

W = total exhaust steam in lbs./hour, when the machine runs on exhaust steam only.

w = rate of consumption in lbs./K.W. hour corresponding to W.

W_e = reduced L.P. supply in lbs./hour when the machine is run as a mixed pressure turbine.

$r = \dfrac{W_e}{W}$ = ratio between full and reduced L.P. supply.

p_1 = initial pressure with full L.P. supply.

$p_e = rp_1$ „ reduced L.P. supply.

w_e = increased rate of consumption of L.P. steam corresponding to drop from p_1 to p_e (obtained from correction curve, Fig. 184).

Power developed in L.P. section under mixed pressure conditions is

$$E_1 = \frac{W_e}{w_e}$$

Power to be developed in H.P. and L.P. sections by the additional H.P. steam is

$$E_2 = (E - E_1) = E - \frac{W_e}{w_e}$$

Let $\dfrac{E_2}{E} = z$.

w_{g_1} = full-load consumption on H.P. steam only.

w_h = H.P. steam consumption under mixed pressure conditions.

Then by equation (2), assuming an average value of $y = 0{\cdot}12$

$$w_h = w_{g_1}\left(\frac{0{\cdot}12}{z} + 0{\cdot}88\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus the total supply of high-pressure steam under mixed-pressure conditions is

$$W_h = w_h E_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

EXAMPLE 3.—A mixed pressure turbine has to maintain a full load of 1000 K.W. at the generator.

When run on L.P. exhaust with 16 lbs./in.^{2} pressure the consumption rate is 33 lbs./K.W. hour. Find the probable consumption of high-pressure steam in lbs./hour when the total L.P. supply is reduced 25 per cent., and the balance of the full-load output is made up by the auxiliary high-pressure steam. Assume that the full-load high-pressure rate of consumption when the turbine is run as a high-pressure machine is 18 lbs./K.W. hour.

Here $r = 0{\cdot}75$. Reduced initial L.P. pressure $p_e = 0{\cdot}75 \times 16 = 12$.

By the correction method given in Art. 204 the rate of low-pressure consumption for a drop of pressure from 16 to 12 lbs./in.^{2} is increased by 13·4 per cent., so that $w_e = 1{\cdot}134 \times 33 = 37{\cdot}43$.

The reduced L.P. supply is $W_e = 0{\cdot}75 \times 33 \times 1000 = 24{,}750$ lbs. per hour, and approximate output of the L.P. section is

$$E_1 = \frac{24750}{37{\cdot}43} = 661 \text{ K.W.}$$

Power to be developed by H.P. steam in the H.P. and L.P. sections is $E_2 = (1000 - 661) = 339$ K.W.

$$\therefore\ z = \frac{339}{1000} = 0{\cdot}339$$

By equation (3) the probable increased H.P. rate is

$$\begin{aligned} w_h &= 18\left(\frac{0{\cdot}12}{0{\cdot}339} + 0{\cdot}88\right) \\ &= 18 \times 1{\cdot}234 = 22{\cdot}2 \text{ lbs./K.W. hour} \end{aligned}$$

By equation (4) the high pressure supply is

$$W_h = 22{\cdot}2 \times 339 = 7540 \text{ lbs./hour}$$

199. Comparison of Turbine Performances.—In order to make a fair comparison between the performances of a number of turbines, the initial pressures, superheat, and vacuum should be the same for all the cases.

This is due to the fact, which will be readily understood by a glance at the $H\phi$ diagram, that an increase either of pressure, superheat, or vacuum causes a reduction in steam consumption, or *vice versâ*.

In order to obtain a rational basis of comparison for turbines of all types, it is necessary to fix on some arbitrary standard turbine, working with fixed limits of pressure, superheat, and vacuum, and hence giving a standard value of Rankine cycle heat H_r and steam consumption w_r.

There does not appear to be any common agreement among turbine makers as to the most suitable standard of reference.

In the paper to the Institute of Electrical Engineers, Baumann has suggested the following figures as representing a fair average of those at present existing, for high- and low-pressure turbines :—

High-pressure turbines working between 100 and 200 lbs./in.^{2} gauge and 0° and 300° F.

Pressure (before governor valve) . .	180 lbs /in.^{2} g. = 195 lbs./ins.^{2} abs.
Superheat	150° F.
Vacuum	28 inches

Low-pressure turbines working at 14 to 16 lbs./in.^{2} abs. on dry steam.

Pressure (before governor valve)	16 lbs./ins.^{2} abs.
Superheat	0° F.
Vacuum	$27\frac{1}{2}$ inches

According to this proposed standard the heat drop as given by the Hϕ chart is

$$\left.\begin{aligned} H_r = 365 \text{ and } w_r &= 9{\cdot}35 \text{ lbs./shaft K.W. hour} \\ &= 6{\cdot}97 \text{ lbs/B.H.P. hour} \end{aligned}\right\} \text{ for high-pressure turbines}$$

$$\left.\begin{aligned} H_r = 164 \text{ and } w_r &= 20{\cdot}8 \text{ lbs./shaft K.W. hour} \\ &= 15{\cdot}5 \text{ lbs./B.H.P. hour} \end{aligned}\right\} \text{ for low-pressure turbines}$$

200. Reduction of Test Results to Standard Conditions.—When the pressure is increased, and the superheat and vacuum are kept constant, the heat drop, as can be seen from the Hϕ diagram, is increased, and the consumption proportionally decreased.

The theoretical reduction in consumption can be calculated from this diagram. The actual decrease, however, is less than the theoretical. Increase of pressure increases friction and leakage losses, and the efficiency is slightly reduced.

If the pressure and vacuum are kept constant and the superheat increased, the same result is obtained, viz. a reduction in the consumption. In this case, however, the actual reduction is greater than the theoretical. This is due to the fact that the friction and leakage losses are reduced by the increased superheat, and the efficiency is slightly improved.

When the pressure and superheat are kept constant and the vacuum is increased, the steam consumption is again decreased. The actual decrease, however, is less than the theoretical. With a given number of stages, the effect of the increased vacuum is to reduce the efficiency, by altering the velocity ratio and reducing the stage efficiency; also the residual loss at exhaust is increased.

On account of these effects the practicable values of the correction figures for pressure, superheat, and vacuum have to be obtained experimentally. The necessary data for any given type of turbine are obtained by running a series of tests on a machine at constant speed, and varying one factor at a time.

It would appear, from the experience of turbine builders, that the superheat correction is not affected by the initial pressure and the vacuum. Corrections for pressure and vacuum are, however, affected by the interdependence of these two factors when either is varied at a constant load.

201. Correction for Pressure.—A correction figure generally employed for high-pressure turbines working between 140 and 200 lbs./in.^{2} gauge is 0·1 per cent. decrease of consumption for every pound increase in pressure. This is equivalent for the range stated to a decrease of 1·4 per cent. for every 10 per cent. increase in pressure. Baumann, in the paper already quoted, shows, for impulse turbines, a correction curve which gives 1·21 per cent. decrease for 10 per cent. increase in pressure between 140 and 200 lbs./in.^{2} gauge, and 1·14 per cent. for 10 per cent. increase between 100 and 140 lbs./in.^{2}.

A similar curve, published by the British Thomson-Houston Company, shows 1·49 per cent. decrease between 140 and 200 lbs./in.^{2}, 1·62 per cent. between 100 and 140 lbs./in.^{2}, and 1·7 per cent. between 80 and 100 lbs./in.^{2} for every 10 per cent. increase in pressure.

According to Baumann's curve for low-pressure turbines there is a decrease of about 5 per cent. between 14 and 16 lbs./in.^{2} and 3·66 per cent. between 16 and 18 lbs./in.^{2} abs. for every 10 per cent. increase in pressure. The corresponding B.T.H. curve shows a decrease of 4·8 per cent. between 12 and 14 lbs./in.^{2}, 4·08 per cent. between 14 and 16 lbs./in.^{2}, and 3·36 per cent. between 16 and 18 lbs./in.^{2} abs., for every 10 per cent. increase in pressure. These values are in good agreement, and may be taken as representative of the average corrections, which may be applied both for impulse and reaction machines working within the range stated.

202. Correction for Superheat.—A rough rule often used is to allow 8 per cent. decrease in consumption for every 100° F. rise in superheat. This rule, however, gives too high a value at high, superheat ranges, and too low a value at low ranges.

Baumann's curve shows 9·3 per cent. decrease for every 100° F. rise between 0° and 100° F., 8 per cent. between 100° and 200° F., and 7·3 per cent. between 200° and 300° F.

The B.T.H. curve shows practically 7·7 per cent. decrease for every 100° F. between 0° and 200° F., which accords with the rough rule stated above.

The former figures are preferable, as it is probable that the frictional losses in impulse machines are reduced and the efficiency increased at a greater rate for the first 100° F. superheat, than for the subsequent increases.

For low-pressure turbines Baumann's curve shows a correction of 10 per cent. decrease in consumption per 100° F. rise between 0° and 100° F. The B.T.H. curve shows 8 per cent. per 100° F. rise between 0° and 100° F., and 6·9 per cent. per 100° F. between 100° and 200° F. superheat.

On the whole, since experience shows that the superheat correction is practically independent of the pressure, the general correction for both high- and low-pressure machines may be taken as, say, 10 per cent. between 0° to 100° F., 8 per cent. between 100° to 200° F., 7 per cent. between 200° to 300° F.

203. Correction for Vacuum.—The vacuum correction is considerably influenced by the initial pressure. This will be apparent if on the $H\phi$ diagram a case between 26 and 28 inches vacuum with 180 lbs./in.^{2} is compared with one between the same vacuum limits with 16 lbs./in.^{2} initial pressure. The decrease in consumption for the same increase in vacuum is much greater in the second than in the first case. In other words, the low-pressure turbine benefits most by the increase in vacuum.

It is, therefore, necessary, as in the pressure correction, to differentiate between the vacuum correction for high- and low-pressure turbines.

The correction is usually stated as a percentage decrease per inch increase in vacuum.

According to Baumann's curve for high-pressure turbines working at pressures from 100 to 200 lbs./in.^{2} gauge and superheat 0° to 300° F., the corrections with constant initial pressure are—

4 per cent.	decrease consumption	per 1 inch	increase of vac.	between	26–27	inches.
5 "	"	"	"		27–28	"
6 "	"	"	"		28–29	"

The values given by the B.T.H. curve are—

4·5 per cent.	decrease consumption	per 1 inch	increase between	24–25	inches.
5·1 "	"	"	"	25–26	"
6 "	"	"	"	26–27	"
6·5 "	"	"	"	27–28	"
8 "	"	"	"	28–29	"

It is not easy to obtain uniformity on account of the interdependence of pressure and vacuum. The latter figures, however, appear to be on the high side.

G. Stoney, in his Cantor lectures,[1] gives practically the same figures as Baumann. Also, in the discussion on Baumann's paper, it was stated that the corrections given for superheat and vacuum, in impulse turbines, were in substantial agreement with those used by the Parsons Company for their reaction machines.

As already indicated, the actual vacuum corrections for low-pressure turbines are greater than those for high-pressure machines.

The values given by Baumann's and the B.T.H. curves are—

Baumann.	B.T.H.				
11·5 per cent.	9·7 per cent.	decrease per 1 inch	increase vac. between	26–27	inches.
13 "	10·6 "	"	"	27–28	"
14·5 "	12 "	"	"	28–29	"

Considering the fact that the theoretical decrease between 28 and 29 inches, as shown by the Hϕ diagram, for steam initially at 16 lbs./in.^{2}, is about 18 per cent., the figures by the B.T.H. seem too low. The others are the more reasonable values to adopt.

As a rule, it is difficult to get the conditions of a turbine test exactly the same as those called for in a customer's specification, and it is necessary to correct the test results to conform to the specified conditions, and enable a more correct comparison to be made between the actual and the "guarantee" performance of the machine.

Also from the commercial standpoint it is of considerable importance for a manufacturer to be able to form a reasonable estimate of the expected performance of a machine, made from existing drawings, patterns, etc., when run under conditions different from those for which the original design was got out.

[1] *Journal Society of Arts*, October, 1909.

For both these purposes, and for the general comparison of all types with a common standard, the foregoing correction factors are of the first importance.

204. Correction Curves for Steam Consumption.—The corrections may be applied by using the average values first quoted; but the most expeditious way to obtain the results is to embody the corrections in a set of curves from which they can be obtained by inspection.

This is the usual practice with turbine builders. The method of setting out and using these curves varies slightly.

The following arrangement enables the total correction factor to be obtained with a minimum of trouble. On the basis of the figures quoted in the foregoing discussion, three average correction curves are plotted, for pressure, superheat, and vacuum respectively.

Primarily, the zero value of each correction scale is fixed to enable the correction, reducing the test to the standard condition, to be read as a positive or negative quantity. The corrections represent fractional increases or decreases in consumption. The curves for high-pressure turbines are shown in Figs. 181, 182, 183, and those for low-pressure turbines in Figs. 184, 185, 186. Each curve is obtained by plotting the set of straight lines given by the averages already quoted, and then drawing the fair curve coinciding most closely with the irregular one. The zero of the correction scale in each case has been fixed where the "standard" pressure, or superheat, or vacuum ordinate AB cuts the curve. The horizontal OX through this point O in each case thus becomes the datum line. When read above this line the correction is to be subtracted, when read below to be added to the test value, in order to obtain the equivalent standard consumption.

Positive values are thus read on the right and negative values on the left of the vertical AB.

Correction from Test to Standard Conditions.—The corrections being read from each curve, the algebraic sum, increased by unity, then gives a coefficient (x) by which the test consumption is to be multiplied in order to obtain the "standard" consumption.

Let w_g = test consumption in lbs./K.W. hour.

w_{g_s} = corrected or standard " "

and let c_p, c_s, and c_v be the fractional corrections for pressure, superheat, and vacuum.

Then $$w_{g_s} = w_g(1 + c_p + c_s + c_v) = xw_g \quad . \quad . \quad . \quad . \quad (5)$$

EXAMPLE 4.—A turbine when run under test with 190 lbs./in.^{2} gauge, 175° F. superheat, and 29 inches vacuum gave a consumption of 12 lbs./K.W. hour. Find the "standard" consumption referred to, 180 lbs./in.^{2} gauge, 150° F. superheat, and 28 inches vacuum.

For pressure correction, the vertical through pressure 190 cuts the curve, Fig. 181, to the right of AB, at $c_p = +0{\cdot}01$.

For superheat, the vertical through 175, Fig. 182, cuts the curve to the right of AB at $c^v = +0{\cdot}02$.

STEAM CONSUMPTION CORRECTION CURVES, FOR HIGH PRESSURE TURBINES.

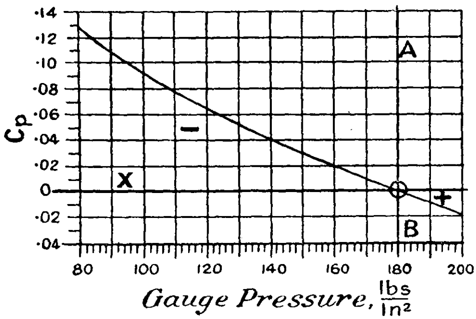


FIG. 181.

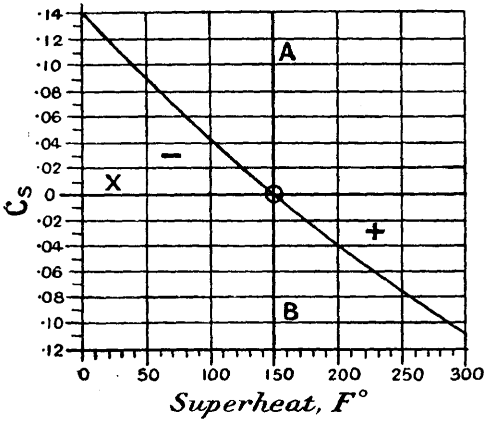


FIG. 182.

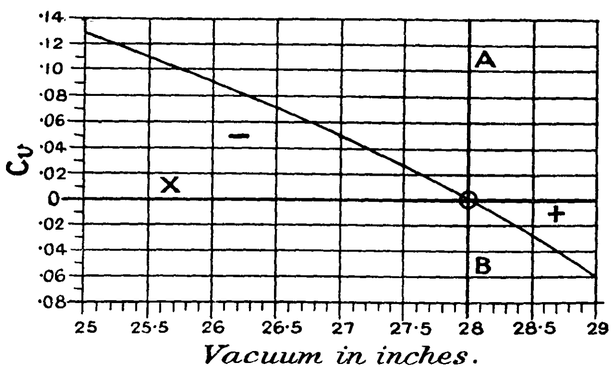


FIG. 183.

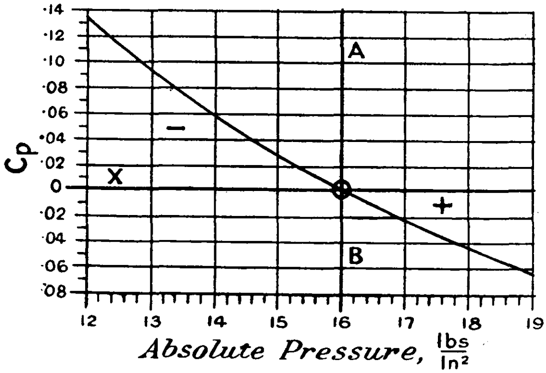


FIG. 184.

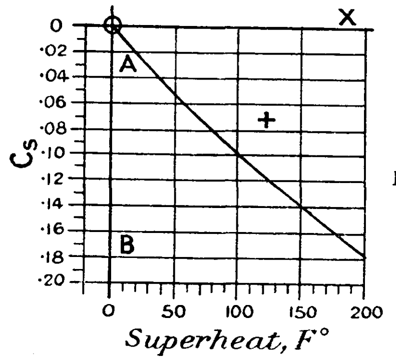


FIG. 185.

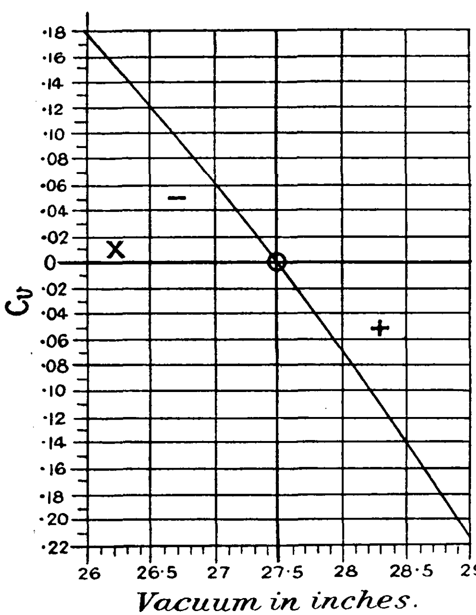


FIG. 186.

For vacuum, the vertical through 29 inches, Fig. 183, cuts the curve to the right of AB at $c_v = +0{\cdot}06$.

The correction coefficient $x = (1 + 0{\cdot}01 + 0{\cdot}02 + 0{\cdot}06) = 1{\cdot}09$.

By equation (5) the "standard" consumption is

$$w_{gs} = xw_g = 1{\cdot}09 \times 12 = 13{\cdot}08 \text{ lbs./K.W. hour}$$

These test figures refer to a 6000 K.W. Parsons turbine.

EXAMPLE 5.—Find the standard consumption for a turbine run with 160 lbs./in.^{2} gauge, 200° F. superheat, and $28\frac{1}{2}$ inches vacuum, if the steam consumption is 13 lbs./K.W. hour.

Proceeding, as in example 4, for the pressure correction the vertical at 160 cuts the curve to the left of AB at $c_p = -0{\cdot}02$. The corrections for superheat and vacuum are likewise found to be $c_s = +0{\cdot}04$, $c_v = +0{\cdot}03$.

$\therefore x (= 1 - 0{\cdot}02 + 0{\cdot}04 + 0{\cdot}03) = 1{\cdot}05$.

$\therefore$ standard consumption $W_{g_s} = 13 \times 1{\cdot}05 = 13{\cdot}65$ lbs./K.W. hour.

EXAMPLE 6.—A low-pressure turbine on test ran with initial pressure 17 lbs./in.^{2} abs., dry steam, and $26\frac{1}{2}$ inches vacuum. The steam consumption was $36{\cdot}5$ lbs./K.W. hour. Express this as an equivalent consumption at standard conditions of 16 lbs./in.^{2} abs. and $27\frac{1}{2}$ inches vacuum.

From Fig. 184 the pressure correction is $c_p = +0{\cdot}022$, and from Fig. 186 the vacuum correction is $c_v = -0{\cdot}12$.

Hence $x = (1 + 0{\cdot}022 - 0{\cdot}12) = 0{\cdot}902$.

Consumption at standard conditions $W_{g_s} = 0{\cdot}902 \times 36{\cdot}5 = 33{\cdot}92$ lbs./K.W. hour. The test figures refer to a 1000 K.W. impulse turbine.

205. Correction from Test to Design or Guarantee Conditions.—The correction scales are primarily arranged to facilitate the correction of test results to the arbitrary standard already specified. The curves can, however, be used for general purposes of correction from test to design or guarantee conditions, when these are not the same as the chosen standard. In such a case the correction is quickly ascertained by the use of a pair of compasses or dividers. Through the guarantee, and the test values of either pressure, superheat, or vacuum, draw verticals. Measure the smaller intercept between the base line and curve with dividers. Transfer this length to the other vertical scale from the point so obtained to the curve, and then find the value of the correction by applying the dividers to the correction scale. The process is illustrated for superheat correction in Fig. 187. A_1B_1 is drawn through the "test" superheat, and A_2B_2 through the guarantee value. B_2O_2 is measured by the dividers. These are transferred to B_1 and give the point C on the horizontal O_2C. The intercept CO_1 is measured by the dividers, which are then transferred to the correction scale, and the correction read from the zero of the scale. The rule for the sign of the correction is quite simple. If the correction is for an increase either of pressure, superheat, or vacuum, the sign is negative; if for a decrease, it is positive.

EXAMPLE 7.—The guarantee conditions for an impulse turbine were pressure 175 lbs./in.^{2} gauge, 200° F. superheat, and $28\frac{1}{2}$ inches vacuum.

The conditions on test at full load were 127 lbs./in.^{2} gauge, 68° F. superheat, and 28·4 inches vacuum. The consumption was 16 lbs./K.W. hour.

Calculate the consumption corrected to guarantee conditions.

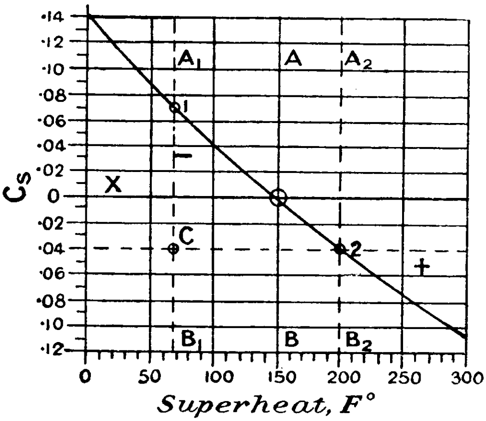


FIG. 187.

Applying the foregoing method, in Fig. 187 A_2B_2 is drawn through the guarantee value 200° F. superheat, and A_1B_1 through the test value 68° F. superheat. The intercept O_1C, when measured in the correction scale, gives $c_s = -0{\cdot}112$, the sign being negative, since the guarantee is larger than the test superheat. Dealing with the pressure and vacuum in the same way, the corrections are $c_p = -0{\cdot}05$ and $c_v = -0{\cdot}008$.

Correction factor, $x = 1-(0{\cdot}112 + 0{\cdot}05 + 0{\cdot}008) = (1 - 0{\cdot}17) = 0{\cdot}83$.

Consumption corrected to guarantee conditions

$$w_g' = xw_g = 0{\cdot}83 \times 16 = 13{\cdot}25 \text{ lbs./K.W. hour.}$$

These figures refer to a 2500 K.W. Curtis turbine.[1]

206. Correction of Efficiency Ratio to Standard or Guarantee Conditions.—In order to find the corresponding efficiency ratio, the heat drop of either standard or guarantee conditions has to be found from the $H\phi$ diagram, and from this the Rankine engine consumption w_r. Then the standard or guarantee efficiency ratio is

$$\eta' = \frac{w_r}{xw_g\eta_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

EXAMPLE 8.—Find the guarantee efficiency ratio for the turbine (example 7), and compare it with the value at test conditions, if in each case the generator efficiency $\eta_g = 0{\cdot}95$.

[1] For test results of this machine, see *Engineering*, July 23, 1909.

Guarantee heat drop $= 390$

$$\therefore\ w_r = \frac{3414}{390} = 8{\cdot}75$$

Since $x = 0{\cdot}83$, $w_g = 16$, $\eta_g = 0{\cdot}95$, by equation (6),

$$\eta' = \frac{8{\cdot}75}{0{\cdot}83 \times 16 \times 0{\cdot}95} = 0{\cdot}695$$

The heat drop at test conditions is 338. Rankine engine consumption $= w_r = \dfrac{3414}{338} = 10{\cdot}1$, also $w_g = 16$.

Test efficiency ratio

$$\eta_t' = \frac{10{\cdot}1}{16 \times 0{\cdot}95} = 0{\cdot}66$$

$$\therefore\ \eta' = \eta_t \frac{0{\cdot}695}{0{\cdot}66} = 1{\cdot}05\ \eta_t$$

The efficiency ratio at guarantee is therefore 5 per cent. better than at test conditions.

207. Corrections for Variation of Vacuum at Constant Load.—In the determination of the total correction factor in the foregoing cases, it is assumed that for the two separate sets of conditions compared, the turbine develops the full load possible with the constant maintenance of the specified pressure vacuum and superheat, at constant speed.

There is, however, another condition which has to be considered, for any given turbine, when it is run at a definite load at constant speed, and the vacuum is varied. In this case the result is more or less affected by the interdependence of the initial pressure with the vacuum.

As already pointed out, the relation $pv = c$ is approximately true for dry steam, and although it is not quite correct for superheated steam, the assumption gives a working basis for approximating the condition which arises under constant load, and varying vacuum in most compound turbines of the multistage type.

The power developed by any turbine varies as the amount of steam that passes the first-stage nozzles. The foregoing assumption regarding pressure and volume makes the density proportional to the absolute pressure. Hence to a first approximation a decrease in total consumption of steam arising from increased vacuum will proportionately decrease the pressure in front of the first-stage nozzles or ring passages. Decrease of initial pressure involves an increase in the rate of consumption, which offsets some of the decrease obtained by the improved vacuum.

In determining the total correction factor, in such a case for a turbine run under throttle governing on a steady load, the vacuum correction has to be decreased by the corresponding correction for the reduced pressure, carried with improved vacuum.

There is still another point. If the initial condition of vacuum at which the machine is run on "full" load is that for which the L.P. blading has been designed, then the efficiency of this blading will fall off more or less under the increased vacuum on account of the larger volume of steam to be dealt with.

The correction will thus be adversely affected by this second element, and the effect will depend on the amount of increase of vacuum above the designed vacuum. If the same pressure is maintained before the governor valve, the throttling to the reduced pressure below this valve will tend to increase the superheat slightly, and the slight reduction in consumption due to this cause may to some extent offset the increase due to insufficient area at exhaust. The actual conditions are obviously uncertain, but, on the whole, the decrease for a given range of vacuum will be dependent on the particular vacuum for which the turbine has been designed.

208. For the full-load condition in such a case, the correction factor c_v, for an increase of vacuum above the test value, may first be determined as in the previous cases, from the curve of Fig. 183. The reduced pressure in front of the nozzles is then given approximately by $p = p_1(1 - c_v)$. The corresponding increase c_p due to the change of pressure from p_1 to p can then be obtained from the curve of Fig. 181. Neglecting any superheat effect and restriction of area at exhaust, the total correction factor will be approximately $x = 1 - c_v + c_p$. Its value is less than unity.

209. If instead of an increase in vacuum, there is a decrease while the machine is run at constant load, then the conditions at the governor valve are reversed. The steam pressure has to be proportionately increased by opening up the valve so as to obtain the increased heat drop necessary to maintain the load. The rate of consumption is increased due to reduced vacuum, but correspondingly reduced by the increase of pressure. The total correction factor may thus be taken as approximating to $x = 1 + c_v - c_p$. Its value is greater than unity. In the case of an increase of vacuum, the value of x may lie on the low side if the test vacuum approximates closely to the designed value. In the case of a decrease in vacuum it may lie on the high side, since the blading designed for the higher vacuum can handle the smaller volume of steam at the lower vacuum more efficiently.

The corrective effect of an increase or decrease of vacuum is more marked in the case of the low- than the high-pressure turbine.

EXAMPLE 9.—A pressure compounded impulse turbine, designed for conditions of 160 lbs./in.^{2} gauge, and $27\frac{1}{2}$ inches vacuum, develops 1230 K.W. on full load, with 158 lbs./in.^{2} below the governor valve, and 28 inches vacuum. The consumption is 15·3 lbs./K.W. hour. Calculate the probable consumption if the load is maintained constant and the vacuum increased to $28\frac{1}{2}$ inches. Also if the vacuum were reduced to the designed value of $27\frac{1}{2}$ inches, and the main supply pressure maintained at 160 lbs./in.^{2}, calculate the value of the reduced load.

Referring to Fig. 183, the increase of vacuum from 28 to $28\frac{1}{2}$ inches gives $c_v = 0{\cdot}03$. The probable value of the reduced pressure below the governor valve or throttle is $p = p_1(1 - c_v) = 173 \times 0{\cdot}97 = 168$ lbs./in.^{2} abs. $= 153$ lbs./in.^{2} gauge.

Referring to Fig. 181 the decrease of pressure from 158 to 153 lbs./in.^{2} gives $c_p = 0{\cdot}005$, and the approximate total correction is $x = 1 - c_v + c_p = (1{\cdot}005 - 0{\cdot}03) = 0{\cdot}975$. As the test vacuum is already in excess of the designed vacuum, it is probable that this figure is on the small side. The probable corrected consumption with increased vacuum is $w' = 0{\cdot}975 \times 15{\cdot}3 = 14{\cdot}9$ lbs./K.W. hour. The influence of superheat increase, which is very small, is neglected.

Taking the second case, since the pressure before the first stage nozzles is kept constant, the same weight of steam W passes into the machine. Hence if E_1 and w_1 are the full load and consumption values in the first case with 28 inches vacuum, and E_2 and w_2 the corresponding values with $27\frac{1}{2}$ inches vacuum, then, $W = w_1E_1 = w_2E_2$ and $E_2 = \frac{w_1E_1}{w_2}$. The decrease of vacuum from 28 to $27\frac{1}{2}$ inches gives $c_v = 0{\cdot}028$ and $x = 1{\cdot}028$ approximately. Hence $w_2 = 1{\cdot}028w_1$, then the probable full load at $27\frac{1}{2}$ inches vacuum is

$$E_2 = \frac{1230 \times w_1}{1{\cdot}028w_1} = 1196$$

As x is probably on the high side, the full load at 160 lbs./in.^{2} gauge, 150° F., and $27\frac{1}{2}$ inches vacuum, the specified design conditions, is 1200 K.W. This corresponds to the rated output of the machine under these conditions at 3000 R.P.M.

EXAMPLE 10.—If it is required to run the turbine (example 9) at the steady load of 1230 K.W. and $27\frac{1}{2}$ inches vacuum, find the approximate value of the supply pressure required and the rate of consumption.

Here the vacuum change is from 28 to $27\frac{1}{2}$ inches, giving $c_v = 0{\cdot}028$ as before. Without sensible error, it may be assumed that the increased pressure is obtained by the reversal of the process in the previous case, that is

$$p_1 = \frac{p}{(1 - c_v)} = \frac{173}{0{\cdot}97} = 178 \text{ lbs./in.}^2 \text{ abs.} = 163 \text{ lbs./in.}^2 \text{ gauge}$$

The correction for pressure rise from 158 to 163 lbs./in.^{2} from Fig. 181 is $c_p = 0{\cdot}003$.

Hence $$x = 1 + c_v - c_p = 1{\cdot}03 - 0{\cdot}003 = 1{\cdot}027$$

and the new rate of consumption is

$$w_1 = 1{\cdot}027w = 1{\cdot}027 \times 15{\cdot}3 = 15{\cdot}72 \text{ lbs./K.W. hour}$$

EXAMPLE 11.—A low-pressure turbine at full load runs with pressure of 17 lbs./in.^{2} abs. below the governor valve, and $26\frac{1}{2}$ inches vacuum.

Calculate the probable rate of consumption if the same load is maintained with an increased vacuum of 28 inches. The test consumption in the first instance is 36·5 lbs./K.W. hour.

The curve of Fig. 186, for the increase of vacuum from $26\frac{1}{2}$ to 28, gives $c_v = 0{\cdot}19$.

The reduced pressure at the throttle is

$$p = p_1(1 - c_v) = 17 \times 0{\cdot}81 = 13{\cdot}75 \text{ lbs./in.}^2 \text{ abs.}$$

The correction factor for drop of pressure from 17 to 13·75 lbs./in.^{2} from the curve of Fig. 184, is $c_p = 0{\cdot}09$.

The total correction factor, neglecting any wire-drawing effect at the throttle, is $x = (1 - 0{\cdot}19 + 0{\cdot}09) = 0{\cdot}90$.

Hence the probable reduced consumption rate is

$$w_1 = 0{\cdot}9w = 0{\cdot}9 \times 36{\cdot}5 = 32{\cdot}8 \text{ lbs./K.W. hour}$$

210. Correction for Increase or Decrease of Vacuum at Partial Loads.—The estimation of the correction coefficient at any partial load, for an increase or decrease of vacuum, is complicated by three factors.

In the first place, the pressure in front of the first-stage nozzles decreases with the decrease of load, so that the heat drop also decreases. The result, as can be seen from the Hϕ diagram, is a larger correction for a given vacuum increase at partial than at full load.

In the second place, the total consumption decreases with the load, as is clearly shown by the Willans Line. The result is that the L.P. blading can handle the increased steam volume at the higher vacuum more efficiently than at full load with the same vacuum increase.

In the third place, there is the effect of the progressive throttling at the governor valve. The superheat is gradually increased as the pressure below the valve is decreased.

The operation of all these conditions at partial loads makes this correction more or less uncertain.

211. Partial Load Correction Curves for H.P. Turbines.—A set of curves from which a rough estimate of the probable consumption corrections at partial loads may be made for high-pressure turbines working between 100 and 200 lbs./ins.^{2} abs., and superheat from 0° to 300° F., is given in Fig. 188.

Several sets of curves corresponding to a series of initial pressures at full load might be given; but considering the uncertainty of the correction, a refinement of this nature seems superfluous, and the probable average corrections for the above range have been used as in the previous cases, in plotting a single set of curves.

The top curve gives the correction at full load, which is practically the value that can be obtained from Figs. 181 and 183, when a slight allowance is made for the superheat effect due to the throttling. It is assumed, in the use of these curves, that the superheat at the supply pressure is maintained sensibly constant at all the loads. On this understanding the correction factor given by any of the curves is $c = (c_v - c_p)$. The sign is negative if the vacuum at the load is increased, and positive if it is decreased. The total correction factor is $x = (1 - c)$ for increase and $x = (1 + c)$ for decrease of vacuum. The correction between any two specified vacuum values is obtained by using dividers in the manner explained in Art. 205.

PARTIAL LOAD CORRECTION CURVES, FOR H.P. TURBINES.

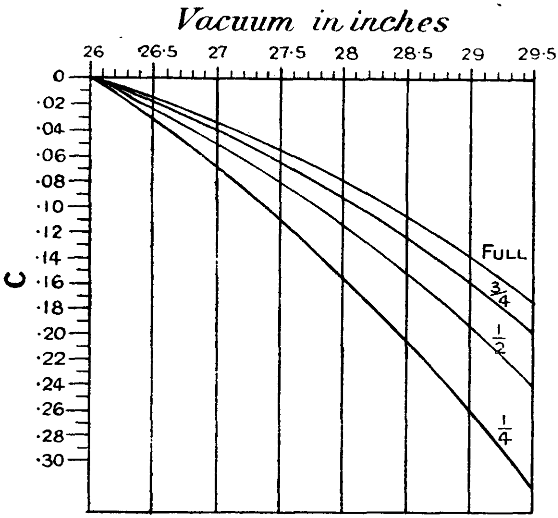


FIG. 188.

EXAMPLE 12.—The following test record was obtained from a pressure compounded impulse turbine :—

Load.	Full.	$\frac{3}{4}$.	$\frac{1}{2}$.
Pressure . . .	162	163	167 lbs./in.² gauge.
Vacuum . . .	28·2 inches	28·6 inches	28·9 inches.
Consumption . .	15·3	16	17·2 lbs./K.W. hour.

The superheat at each load only varied by a few degrees.

Calculate the probable rates of consumption if the vacuum at each load were maintained (*a*) at 29 inches, (*b*) at $27\frac{1}{2}$ inches.

(*a*) For full load, the intercept of the curve, Fig. 188, between the verticals at 28·2 and 29 inches vacuum scales, $c = 0{\cdot}047$; hence, $x = (1 - 0{\cdot}047) = 0{\cdot}953$.

At $\frac{3}{4}$ load the intercept at the verticals through 28·6 and 29 inches is $c = 0{\cdot}023$, and $x = (1 - 0{\cdot}023) = 0{\cdot}977$.

At $\frac{1}{2}$ load between the values 28·9 and 29 inches, $c = 0{\cdot}008$, and $x = (1 - 0{\cdot}008) = 0{\cdot}992$.

The probable consumption rates under the increased vacuum are—

Full load . .	$w_F = 0{\cdot}953 \times 15{\cdot}3 = 14{\cdot}6$	lbs./K.W. hour.
$\frac{3}{4}$,, . .	$w_{\frac{3}{4}} = 0{\cdot}977 \times 16 = 15{\cdot}65$	,, ,,
$\frac{1}{2}$,, . .	$w_{\frac{1}{2}} = 0{\cdot}992 \times 17{\cdot}2 = 17{\cdot}1$	,, ,,

(*b*) In the case of decreased vacuum, at full load, assuming the supply pressure is increased to maintain the same load value as at 28·2 inches vacuum, the intercept on the curve for 28·2 and 27·5 inches vacuum is $c = 0·03$, and $x = 1·03$. Similarly, at $\frac{3}{4}$ load between 28·6 and 27·5 inches, $c = 0·062$, and $x = 1·062$, and at $\frac{1}{2}$ load, between 28·9 and 27·5 inches, $c = 0·10$, and $x = 1·1$.

The probable consumption rates, under the decreased vacuum, are—

Full load . .	$w_F = 1·03 \times 15·3 = 15·76$	lbs./K.W. hour.
$\frac{3}{4}$,, . .	$w_{\frac{3}{4}} = 1·052 \times 16 = 16·85$	,, ,,
$\frac{1}{2}$,, . .	$w_{\frac{1}{2}} = 1·1 \times 17·1 = 18·8$	,, ,,

Other cases can be dealt with in a similar manner.

212. Partial Load Correction Curves for L.P. Turbines.—In the case of the low-pressure turbine working with dry steam at initial pressure from 14 to 16 lbs./in.^{2} abs., a reduction in vacuum, especially between 29 and 28 inches, gives a large positive correction. This, with the relatively smaller heat drop available at the larger vacuum, is not usually compensated by a reasonable rise of the supply pressure. Normally an increase does not occur. On the other hand, the corresponding increase in vacuum gives the same negative correction, which is then decreased owing to the drop in pressure below the governor valve in the same proportion as the pressure. It follows, therefore, that the correction for increase of vacuum in the case of the low-pressure turbine is less than that for the decrease of vacuum. The conditions hold generally for full or partial load.

To meet the two cases two sets of correction curves, derived from figures in Baumann's paper, are given in Figs. 189, 190. For the case of increased vacuum at any load, the total correction factor is $x = (1 - c)$, and for decreased vacuum, $x = (1 + c)$.

EXAMPLE 13.—The following test data were obtained from the L.P. section of a mixed-pressure turbine when run only with low-pressure steam :—

Load.	Full.	$\frac{3}{4}$.	$\frac{1}{2}$.
Pressure . . .	16·5	16·4	16·6 lbs./in.^{2} abs.
Vacuum . . .	27·56 inches	27·59 inches	27·54 inches.
Consumption .	29·6	31·3	37·3 lbs./K.W. hour.

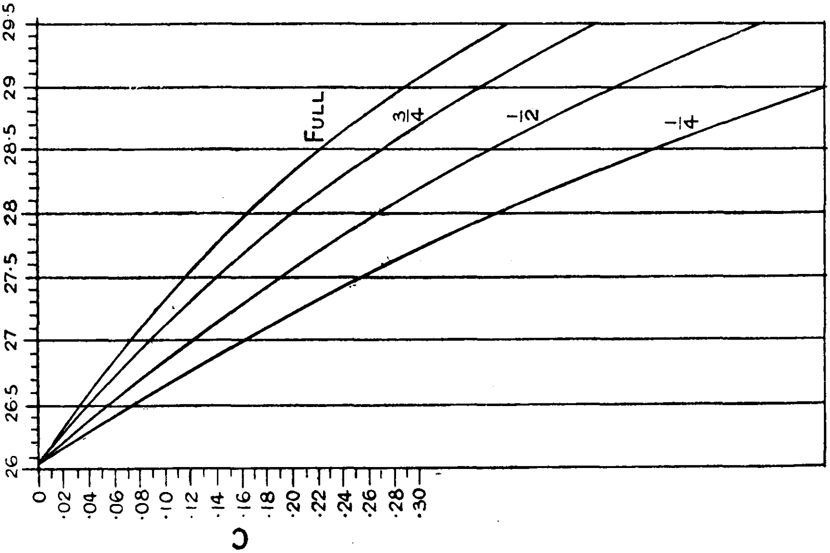
Calculate the probable values of the steam consumptions if the vacuum is (*a*) increased at each load to 28 inches, (*b*) decreased to 26·5 inches.

Proceeding as in the previous example, from the curves of Figs. 189, 190, the values obtained are—

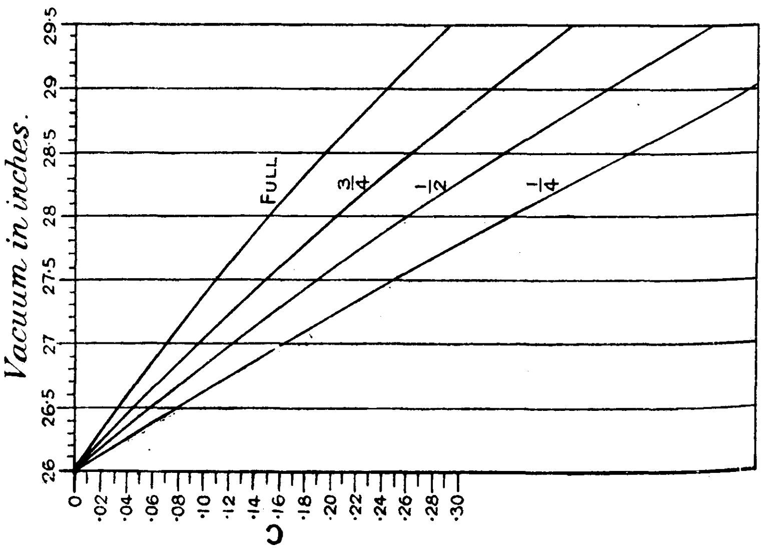
	Load.	Full.	$\frac{3}{4}$.	$\frac{1}{2}$.	
(*a*)	c	0·033	0·048	0·062	Increased vacuum.
	x	0·967	0·952	0·938	
	w'	28·6	29·8	35	
(*b*)	c	0·08	0·112	0·14	Decreased vacuum.
	x	1·08	1·112	1·14	
	w'	32	34·8	42·5	

213. Heat Accumulator.—As already stated in Chap. I., Art. 4, the introduction of the steam turbine provided the means for utilising the enormous amount of heat which for many years had been wasted

PARTIAL LOAD CORRECTION CURVES, FOR L.P. TURBINES.



Decrease of Vacuum.
Fig. 190.



Increase of Vacuum.
Fig. 189.

in the atmospheric exhaust of non-condensing engines in collieries, rolling mills, forges, blast furnaces, etc. The effective use in a low-pressure turbine, of this exhaust, which, in the majority of cases, is very intermittent in character, is made possible by the use of a heat accumulator or regenerator. The credit for its introduction is due to Professor Rateau, who applied the principle, practically, in 1904.[1]

The accumulator consists of a vessel containing some solid or liquid substance which acts as a "heat flywheel." The steam, on discharge into the vessel, is condensed. The heat is then "regenerated" by evaporation, when the exhaust discharge to the accumulator ceases or diminishes considerably. The temperature variations on which the condensation and re-evaporation depend are caused by fluctuation of pressure in the accumulator. The pressure rises during the condensation and falls during the evaporation, which goes on after the exhaust supply diminishes, and while the turbine continues to draw its supply.

In the earlier types iron, in the form of a set of old rails, was employed in a cylindrical shell. This costly material was soon superseded by water, which is the medium now universally employed.

Unfortunately water has a poor conductivity, and in order to ensure the rapid communication of large quantities of heat to the liquid mass in the accumulator, a large surface must be exposed to the action of the steam. Originally flat cast-iron trays containing thin layers of water were arranged in a vertical cylinder, but these soon gave place to an arrangement whereby the injection of the steam into the body of the liquid produced rapid circulation.

214. There are now several forms of accumulator in use. One of the most effective of the modern designs is the Rateau-Morrison accumulator,[2] shown in Figs. 191, 192, and 193. It consists of a cylindrical vessel, 1, Fig. 191, which is built up of cast-iron ribbed segments, 2, Fig. 192, having flanged joints, 3. This method of construction facilitates transport and erection. The exhaust from the engines enters the branch, 4. Part of this passes directly to the turbine through the opening of the flap valve, 5, the position of which is regulated by a spring-loaded piston in the cylinder, 6. The rest passes through the T pipe, 7, to the main distributing pipe, 8. From this it is distributed to the water in the vessel by means of the pipes, 9. Each pipe is provided with a special spraying nozzle, fitted in an outer tube, 10. This tube, which is open at the bottom, extends from the water level (WL) close down to the accumulator shell. The nozzle and pipe produce a very effective circulation of the accumulator water. While the engines continue to exhaust into the branch, 4, the surplus steam passes into the water and is condensed, and the temperature of the water and pressure rise. When the supply diminishes considerably or ceases altogether, the demand of the turbine causes the re-evaporation of some of the steam, and the pressure and temperature fall. The "regenerated" steam is drawn off through the passage, 11, into the receiver, 12, and thence

[1] For illustrations of the early types, see paper by Professor Rateau, on "Rateau Steam Turbines," *Trans. I. Mech. Eng.*, July, 1904. Also see paper on "Heat Accumulators and Use in Exhaust Steam Plants," by A. Alison, *Jour. Inst. Jun. Eng.*, June, 1913.

[2] See paper on "Steam Regenerative Accumulators," by D. B. Morrison, *Trans. Inst. Engineers and Shipbuilders in Scotland*, vol. lv., 1912.

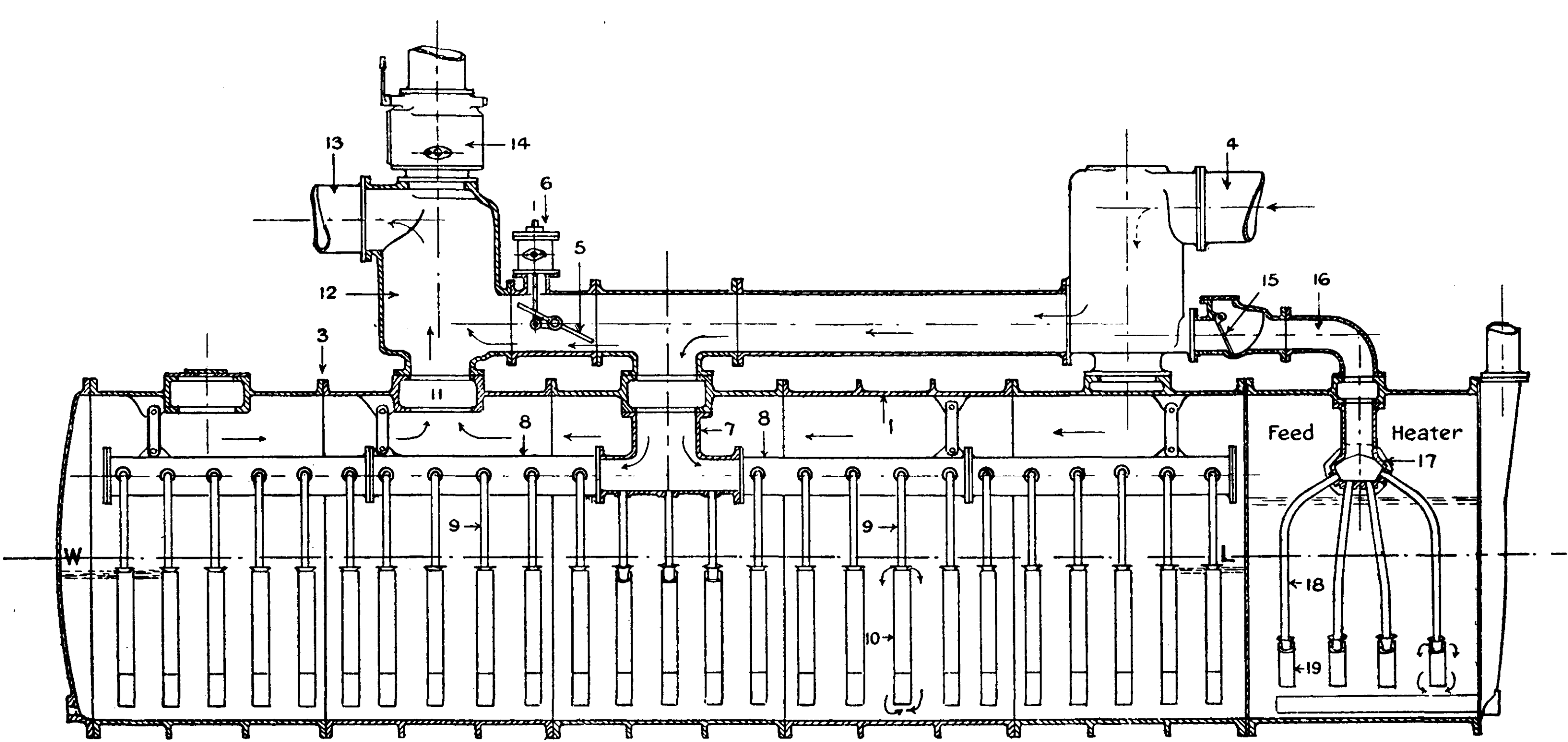
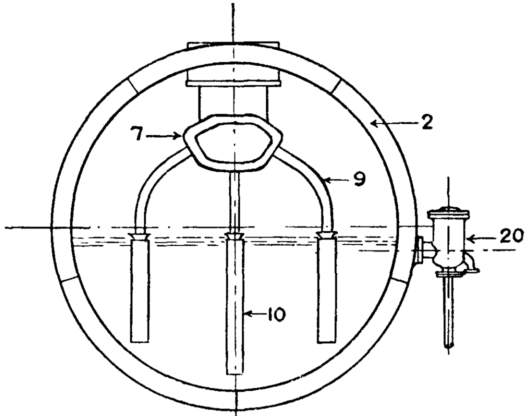


FIG. 191.

passes to the turbine by the branch, 13. A detail of the essential element, on which the successful operation of this accumulator depends,

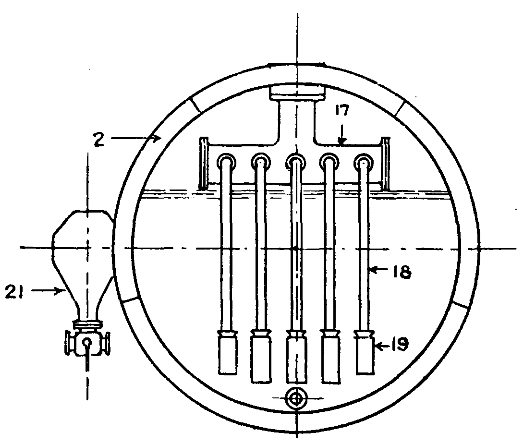


SECTION THRO. ACCUMULATOR.

FIG. 192.

is shown in Fig. 194. The distributing pipe, 1, is telescoped into the outer tube, 2, which is closed at the top by a cover, 3.

Several rectangular slots, 4, are cut in the side of the outer tube, near the top.



SECTION THRO. FEEDHEATER.

FIG. 193.

A star-shaped nozzle, 5, shown in plan at the bottom of the drawing, is fitted to the end of the distributing pipe, 1.

It consists of a series of radial arms, 6. Each arm has a narrow slit cut in its upper surface, through which the steam passes in a thin film, thus exposing a large surface to the upward stream of water which is produced between the arms.

The nozzle is placed centrally within the outer tube, and about 9 inches below the water level. This arrangement of nozzle is the outcome of considerable experimental work. It is designed to heat a given quantity of water with a minimum nozzle resistance, in as short a time as possible. Each unit is so proportioned that the kinetic energy of the jets is almost wholly absorbed in the production of the circulation of water through the outside tube, in the manner indicated by the directional arrows.

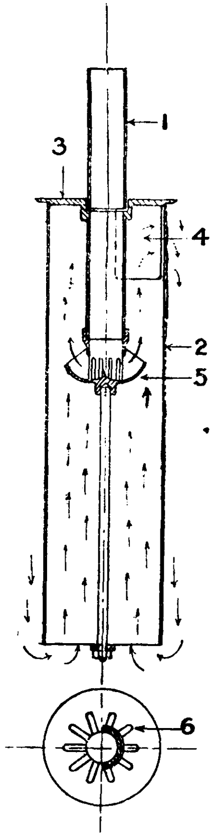


FIG. 194.

High circulating capacity is a necessity if the condensational efficiency is to be high, since the mean difference of temperature between the steam and the water is usually about 4° F.

In most exhaust systems there is a liability at times for the supply of exhaust steam to exceed the maximum capacity of the turbine and accumulator. The pressure in the latter is always regulated by a safety or escape valve, 14, Fig. 191, which is set for a few pounds above the exhaust pressure. If there is a frequent surplus, the continued blow-off either to the atmosphere or condenser involves a needless waste. This can be prevented by passing the extra steam into a feed heater. Such a heater arrangement is shown at the right-hand side of Fig. 191. An auxiliary flap valve, 15, is fitted in a branch pipe, 16. This is connected to a T pipe, 17 (Fig. 193). Distributing pipes, 18, and nozzles, 19, similar to those used in the accumulator, are fitted on this T pipe.

The nozzles are submerged to such a depth that the resistance to flow of the steam through them is slightly less than the load on the safety valve of the accumulator. The mass of feed water is considerable, and the intermittent exhaust supply is easily condensed, as there is no great variation of temperature of the water. The storage-water and feed-water connections are shown at 20, Fig. 192, and 21, Fig. 193.

215. In order to indicate the nature of the variation in exhaust supply from colliery winding and rolling-mill engines, Figs. 195, 196 have been reproduced from Morrison's paper, already cited.

In the case of the winding engine, Fig. 195, during about one-half of the total period, the rate of exhaust is in excess of the average of 21,600 lbs./hour. During the second half there is no exhaust supply or the rate is zero. If this exhaust is to be usefully employed in a low-pressure turbine, the accumulator must be of such capacity that it will store the surplus denoted by the area above the mean con-

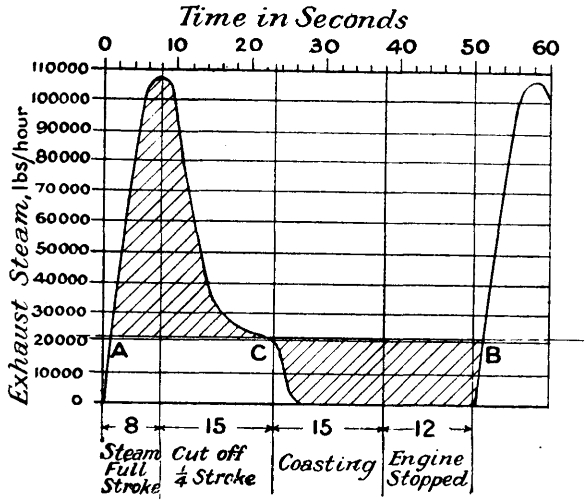


FIG. 195.

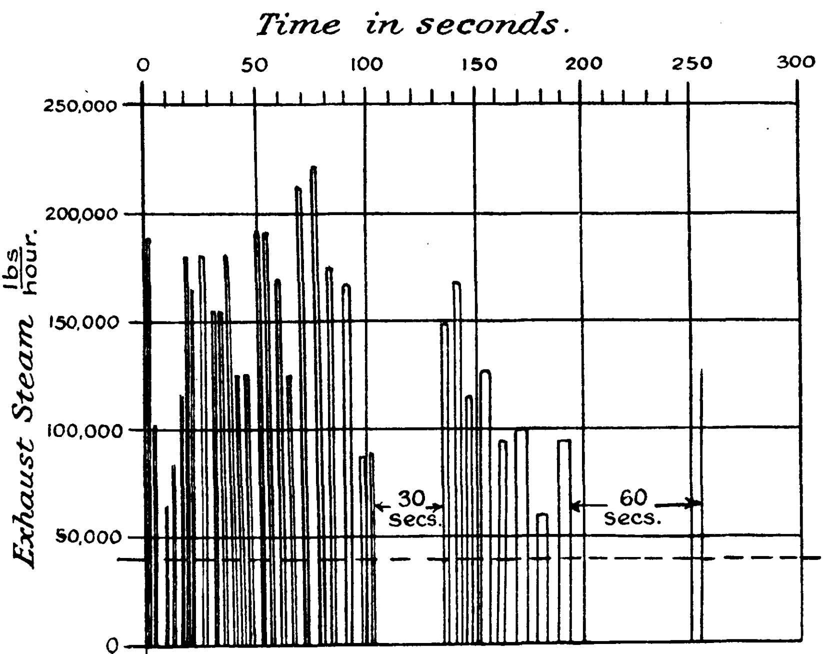


FIG. 196.

sumption line AB, while the engine is running. It must be able to restore this heat when the engine is stopped. In other words, it must make good the deficit of steam shown by the shaded area under the mean consumption line.

The same condition holds for the case of the rolling-mill engines, shown in Fig. 196. There is a rapid fluctuation of the exhaust supply due to the reversal of the engines during "rolling." The result is a stepped series of steam areas. About the middle and at the end of the "period" there is a much larger defect in the exhaust steam supply, the times being 30 and 60 seconds respectively.

The periods during which there is a reduction of direct exhaust supply to the turbine and the accumulator has to make good the deficit are called "bridge" periods. The duration of the time of the exhaust shortage varies with different services.

In rolling-mill work, where there may be several rolling engines exhausting into the main, it runs from 30 to 60 secs.

216. To illustrate the condition which arises in such a case, the combined exhaust steam curve for a set of four rolling-mill engines is shown in Fig. 197.[1]

This very irregular stepped curve shows for a complete rolling period of 300 secs. the simultaneous discharge from the engines of one blooming mill, one roughing mill, and two finishing mills, through which the steel has to pass successively before it can appear in the form of the finished rail.

The mean value of the rate is 80 lbs. per sec. By drawing the horizontal AB at this value, a new base line is obtained, relatively to which the supply curve shows the excess or defect at any instant above or below the mean value. This mean represents the steady rate at which the exhaust steam should be passed to, say, a set of mixed pressure turbines. Allowing, say, 10 per cent. loss in the system, the probable steam available per hour would be 260,000 lbs.

Taking the rate of consumption for a mixed pressure machine as 33 lbs./K.W. hour, the probable power available from this auxiliary steam is about 8000 K.W. as a minimum. If more efficient conditions exist it may reach 10,000 K.W.

The integral curve for the shaded areas of the adjusted consumption curve is shown in the lower part of the illustration. There is a continuous fluctuation, from instant to instant, in the total values. The curve in general rises from the beginning of the period and reaches a maximum value 90 secs. after the start. The ordinates show the amount of steam which has to be in storage in the accumulator water at each instant. The accumulator for the first 90 secs. is thus increasing its store of heat from the surplus steam. Between 90 and 150 secs. the curve falls, and during this period the quantity stored by the accumulator steadily decreases, as the accumulator makes good the deficit of the exhaust supply to the turbines. At 150 secs. the limit of the accumulator is reached, as no surplus is left. If this

[1] See article reprinted from "Power and the Engineer," in *Mech. World*, February 3, 1911, on "Exhaust Steam Regenerators," by C. H. Smoot.

condition were to persist, then the H.P. section of the mixed pressure turbine would require to come into action.

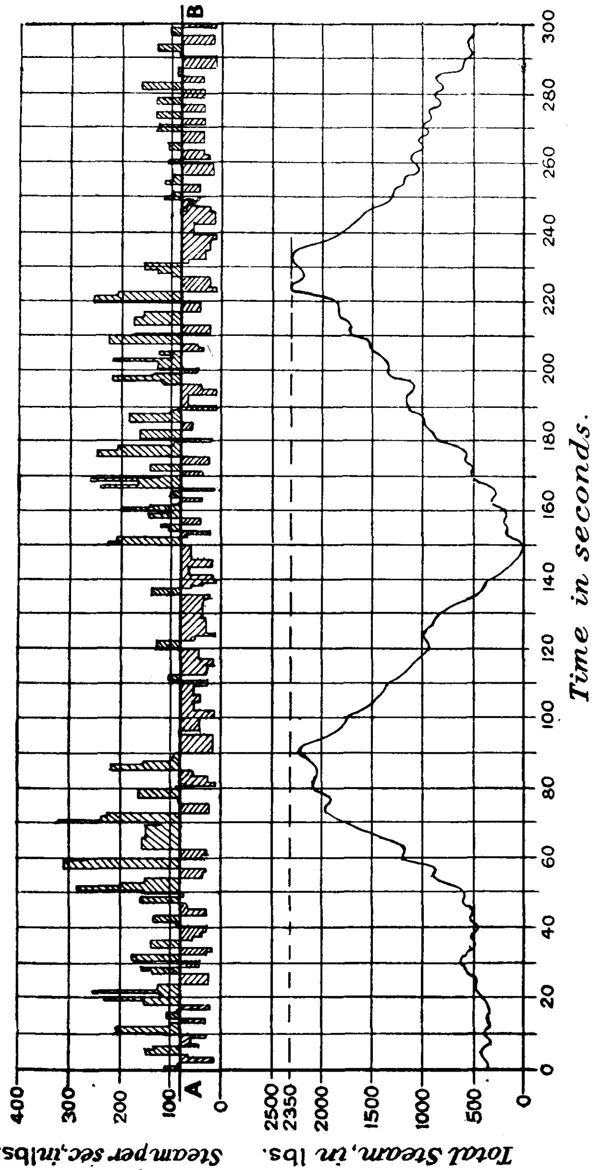


FIG. 197.

The curve, however, rises again between 150 and 233 secs., when the accumulator storage reaches its maximum value of 2350 lbs.

of steam. This is the quantity on which the calculation of the accumulator capacity should be based. Between 233 and 300 secs., when the operation ceases, the curve again falls till the storage value reaches 500 lbs.

This system thus shows two bridge periods, one between 90 and 150 secs., and one between 233 and 300 secs.; the values are 60 and 67 secs.

217. Accumulator Capacity.—As already indicated, exhaust and mixed pressure turbines work with steam from 16 to 17 lbs./in.^{2} abs., and in order to "regenerate" the steam at this pressure in the accumulator, the accumulator pressure range is usually from 15 to 18 lbs./in.^{2} abs. This gives a difference of temperature of about 10° F. A lower range might be employed. Some plants are running with exhaust pressure slightly above atmospheric, and the pressure in the accumulator has a lower limit of 12 lbs./in.^{2} abs.

The thermal conditions existing at any instant in an accumulator are too complex to admit of any exact calculation of capacity.

Approximate calculation is, however, quite sufficient for practical purposes.

Let w_s = steam to be regenerated per hour in lbs.

n = period of regeneration in secs.

Then total weight of steam regenerated in the period is

$$W_s = \frac{w_s n}{3600} \text{ lbs.}$$

Let t_2 and t_1 be the temperatures at the higher and lower pressures p_2 and p_1, and L the average latent heat, say, 970 B.Th.U.

Then the minimum amount of water required in the accumulator is

$$W = \frac{W_s L}{(t_2 - t_1)} \quad \ldots \ldots \ldots \quad (7)$$

If the limits of pressure are so chosen that $(t_2 - t_1) = 10°$ F., then

$$W = 97 W_s = 0{\cdot}027 w_s n \quad \ldots \ldots \quad (8)$$

218. From this equation it appears that the capacity of the accumulator is directly proportional to the period of regeneration, and independent of the time of absorption.

This could only be true if an extremely rapid absorption of heat could take place at the start of each exhaust period. This condition is not attained, even with the most excellent circulating arrangements.

An additional quantity of water is always required to avoid excess of pressure, and loss of steam through the relief valve at the commencement of each absorption period. 30 to 40 per cent. more water should be added to the calculated amount to take account of condensational inefficiency.

This can be allowed for generally, by using an efficiency factor, thus

$$W = \frac{W_s L}{\eta_a (t_2 - t_1)} \quad \ldots \ldots \ldots \quad (9)$$

where η_a = condensational efficiency of the accumulator.

For any given case the value of W_s is given by the area of the adjusted curve of the exhaust steam diagram during the "absorption period." It is the maximum value of the integral curve at the beginning of the bridge period. For example, in Fig. 195, AB, the mean exhaust supply line, is the base line for the accumulator curve. The integral of the curve, up to the beginning of the bridge period, CB, is the hatched area above AC. This gives the surplus steam to be stored in the accumulator water. The area of the curve below CB is equivalent to this, and is the amount to be regenerated during the bridge. In this case it is nearly equal to the product of the average rate and the bridge period.

In the more complex case illustrated in Fig. 197, the value of W is not so obvious owing to the frequent fluctuations of the exhaust values above and below the mean line AB; and it is necessary to derive the integral curve in order to ascertain the proper value, which is seen to be 2350 lbs.

219. In many works there are continuously working engines in addition to those that work intermittently. The exhausts from these engines can be more economically used by passing them through an accumulator system, than by the use of separate condensing plants.

With such a combination of exhausts the periods of several engines may synchronise at times, and the possibility of such a simultaneous discharge to the accumulator necessitates a much larger storage capacity than if these separate discharges took place in rotation.

The determination of the capacity under such circumstances is dependent on the conditions existing in the particular plant, and is more or less uncertain. As a rough estimate from 200 to 250 lbs. of water should be allowed per lb. of exhaust steam to be handled.

EXAMPLE 14.—The amount of steam to be absorbed and regenerated in the case of a winding engine is $W_s = 180$ lbs. The average value of the latent heat between the pressure limits is 970 B.Th.U. and the difference of temperature 10°. Calculate the size of accumulator required, assuming a condensational efficiency of 60 per cent. Make the length of the accumulator four times the diameter.

By equation (9) the capacity of the accumulator is

$$W = \frac{W_s L}{\eta_a(t_2 - t_1)} = \frac{180 \times 970}{0{\cdot}6 \times 10} = 29100 \text{ lbs. water}$$

The volume occupied by the water, with the water level at the centre of the shell may be taken as $\frac{3}{8}$ of the shell volume, to allow for displacement by pipes, etc.

$$\text{Volume of shell} = \frac{\pi}{4} D^2 \times 4D = \pi D^3$$

$$\text{,, water} = \tfrac{3}{8}\pi D^3 = \frac{29100}{62{\cdot}5}$$

$$\therefore D^3 = \frac{8 \times 29100}{3\pi \times 62{\cdot}5} = 396$$

$$\therefore D = \sqrt[3]{396} \doteqdot 7{\cdot}3 \text{ feet, and } L = \text{say } 30 \text{ feet}$$

Allowing, say, 5 per cent. loss between the engine and turbine, in this case, the rate of supply may be taken as 20,000 lbs./hour. At 33 lbs./K.W. hour for a mixed pressure turbine the output would be about 600 K.W., or about 80 per cent. of a 750 K.W. machine on exhaust steam alone. This is the capacity of the turbine, usually fitted, for an engine giving the exhaust specified in this example.

EXAMPLE 15.—Calculate the capacity of the accumulator system required to handle the total exhaust from the four rolling-mill engines, for which the exhaust curves are given in Fig. 195. Assume the same pressure and temperature conditions as in the previous example, and condensational efficiency of 60 per cent.

Here $W_s = 2350$ (from the integral curve)

$$\therefore\ W = \frac{2350 \times 970}{0{\cdot}6 \times 10} = 380000 \text{ lbs.}$$

$$\text{Total volume of water} = \frac{380000}{62{\cdot}5} = 6080 \text{ ft.}^3$$

$$\begin{aligned}\text{Taking total shell volume} &= 2 \times \text{volume of water}\\ &= 2 \times 6080 = 12160 \text{ ft.}^3\end{aligned}$$

The volume of a shell 9 feet 6 inches $\times$ 40 feet, a size commonly used, is 2800 ft.^{3}, so that, say, four accumulators of this size should be sufficient for the plant, as the inefficiency allowance is probably on the liberal side.

CHAPTER XIII

PROVISIONAL DETERMINATION OF GENERAL PROPORTIONS OF COMPOUND TURBINES

IMPULSE TURBINES

220. Speed of Rotation (R.P.M.).—The rotational speed of a turbine depends principally on the nature of the machinery it has to drive. In the case of a turbo-alternator the number of revolutions per min. is given by

$$N = \frac{60F}{\frac{1}{2}n}$$

where F = number of cycles per sec.
n = number of poles.

Recent improvements in alternator design have enabled turbine makers to design machines of given output for higher speeds than were formerly possible. The following table indicates generally the present practice of leading makers, in regard to output and speed :—

Output K.W.	Number of cycles.			
	25.	40.	50.	60.
	R.P.M.	R.P.M.	R.P.M.	R.P.M.
4,000	1500	2400	3000	3600
5,000	1500	2400	{3000, 1500}	{3600, 1800}
6,000	1500	2400	{3000, 1500}	{3600, 1800}
8,000	1500	1200	1500	1800
10,000	1500	1200	1500	1800
12,000	1500	1200	1500	{1800, 900}
15,000	{1500, 750}	1200	{1500, 1000}	900
20,000	750	800	{1000, 750}	900

The standard speeds of B.T.H. Curtis turbo-alternators are

K.W.	Cycles.	R.P.M.
3000–15,000	25	1500
500–10,000	40	2400
250–5000	50	3000
6000–15,000	50	1500
250–4000	60	3600
5000–15,000	60	1800

The speeds used for direct current machines may vary from 4000 R.P.M. with small to 1000 R.P.M. with fairly large units. The commutator difficulty for high output at high peripheral speed puts a restriction on the rotational speed of the D.C. machine.

The difficulty can, however, be overcome by the use of a reduction gear.[1]

The following are averages taken from a list of impulse machines:—

K.W.	300–500	500–1000	1000 1500
R.P.M. . . .	3000	2500	1500

Small-powered Curtis machines run about the following:—

K.W . .	25–35	35–75	75 150	150–300
R.P.M. . .	3600	2400	2000	1500 to 1800

The special class of velocity compounded turbine with single wheel made by the Sturtevant Company in five sizes, is run between the following speed limits:—

B.H.P. .	5–25	20–50	50–100	80–150	125–200
R.P.M. .	3000–5000	2000–4000	1600–3000	1300–2000	1000–2000

The standard speeds and mean wheel diameters of the de Laval machines are—

Output in B.H.P. .	5	15	30	50	100	300-500
R.P.M.	30,000	24,000	20,000	16,000	13,000	10,600
Mean dia. of wheel.	4 ins.	6 ins.	$8\frac{7}{8}$ ins.	$11\frac{3}{4}$ ins.	$19\frac{3}{4}$ ins.	30 ins.

These speeds are reduced at the driving shaft in the ratio of 10 to 1 for small and 13 to 1 for large machines.

221. In marine practice there has been considerable variation in the speeds employed for direct connected turbines. The speed in any case depends on the type of vessel and nature of service.

The following are averages:—

Mercantile Marine—

	R.P.M.
High-speed liners	180 to 250
Channel steamers	450 to 680

Naval vessels—

Battleships	300 to 350
Cruisers and destroyers	500 to 780
Torpedo boats	1000 to 1500

[1] See description of Westinghouse Reducing Gear for Direct Current Machines, *Engineering*, January 31, 1913; also description of 1500 K.W. Geared Turbo-generator at British Westinghouse Works, Manchester, *Engineering*, September 8, 1916.

The introduction of the geared turbine, as already indicated in Art. 27, Chap. III., is leading to the much-needed increase of the turbine speed. This has gone up to about 2000 R.P.M. with the mechanical gear, and may ultimately reach 3000 (see Art. 267).

222. General Considerations in a Preliminary Design.—In making a provisional estimate of the general dimensions of a given type of turbine, the output at a stated speed, and the pressure, superheat and vacuum conditions are usually specified.

The impulse machine may be constructed either with a large diameter and small number of stages, or with a small diameter and large number of stages. The axial flow reaction may likewise be constructed with small drum diameters and large number of expansions, or *vice versâ*.

In any case a satisfactory design depends primarily on the adjustment of the two factors, so that the losses inherent to the type and the resulting cost of the machine may be reduced to a minimum. The cost of production of such parts as governor gear, oil pumps, bearings, etc., is a much larger proportion of the total cost in the case of a small than a large turbine. Hence for equal efficiency the small machine is relatively more expensive than the large one.

For commercial reasons the small unit is usually designed for a lower efficiency than the large one. The question of efficiency primarily involves the speed ratio ρ, and the selection of this factor is one which has to be based on previous experience. For machines of moderate and large output the values already quoted in Art. 100 may be used.

When the leading particulars of output, speed, pressure, superheat, and vacuum are settled, the general proportions of the proposed machine can be provisionally estimated. Only one of various methods that may be adopted is given in what follows.

223. General Dimensions of Impulse Turbines—Pressure Compounded Impulse.—In this type the stage heat drop is usually in the neighbourhood of the critical value and the nozzle passages are convergent. The number of stages may run from 8 to 16, depending on the output and speed.

The value of the critical heat drop with superheated and supersaturated steam is in the neighbourhood of 50 B.Th.U. Hence for an equal distribution of the cumulative heat, the minimum number of stages is obtained by dividing the total cumulative heat by 50, or

$$n_{\text{min.}} = \frac{\text{RH}_r}{50} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let D = mean blade ring diameter in inches.
u = " " velocity in ft./sec.
N = revolutions/min.

Then $$u = \frac{\pi \text{DN}}{60 \times 12}$$

Let V'_0 = theoretical velocity due to stage heat drop h_r.

ρ_t = " " speed ratio $= \frac{u}{V'_0}$.

Then $$V'_0 = \frac{\pi DN}{\rho_t \times 720} = 223{\cdot}7\sqrt{h_r}$$

But $$h_r = \frac{RH_r}{n} \quad \therefore \; n = RH_r\left(\frac{\rho_t \times 51270}{DN}\right)^2 \quad . \quad (2)$$ [1]

For the limiting condition that $50n = RH_r$, the diameter is given by

$$D^2 = 50\left(\frac{\rho_t \times 51270}{N}\right)^2$$

or $$D = 362000\frac{\rho_t}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the value of D calculated from (3) gives a blade speed much in excess of 500 ft./sec., where the light type of disc wheel is used, or 600 ft./sec. with the heavier type, the figure should be reduced to keep the speed within these limits, and the number of stages increased if necessary.

Taking the four standard speeds usually employed with this type, the mean ring diameter for $u = 500$ are

N.	3000	1500	1000	750
D.	38	78	115	156

A solid disc 150 inches diameter is impracticable, since the cost would be prohibitive. When a large diameter is necessary the wheel is "built up," as in the case of a large Curtis turbine, Fig. 16, and in some marine Curtis turbines, where diameters as high as 12 feet are used.

In the case of the land turbine there is a practical limit of 13 feet imposed on the diameter, by the facilities for transport by rail in this country. As a rule, the diameter of the solid disc wheel does not exceed 7 feet.

224. The adjustment of mean wheel diameter and number of stages just considered takes no account of the weight of steam to be passed through the blading of the last stage. It is necessary to calculate the minimum diameter at the L.P. end to meet this second condition; and if the diameter is to be kept constant throughout to select the larger of the two. The minimum diameter of the last L.P. blade ring can be calculated as follows:—

Let w = consumption in lbs./K.W. hour or lbs./B.H.P. hour.
E = output in K.W. or B.H.P.
W = total steam passing through the machine, lbs./sec.

$$W = \frac{wE}{3600} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

[1] See footnote, p. 418.

The consumption w can be assumed on the basis of previous experience, but the most satisfactory method is to calculate it from the known heat drop H_r, the estimated value of efficiency ratio η and the generator efficiency η_g.

$$\left.\begin{aligned} \text{Thus} \qquad w &= \frac{3414}{\eta H_r \eta_g} \text{ lbs./KW hour} \\ \text{or} \qquad w &= \frac{2544}{\eta H_r \eta_g} \text{ lbs./B.H.P. hour} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

When the total weight W to be passed at the last stage is calculated, the mean ring diameter is easily obtained from the equation of continuity.

Let v_0 = specific volume of steam at exhaust pressure in ft.³/lb.
V_0 = theoretical velocity of steam from last stage nozzles in ft./sec.
α = jet angle of nozzle.
u = mean blade velocity $= \dfrac{\pi DN}{60}$ ft./sec.
D = mean ring diameter in ft.
N = revolutions/min.
ρ_t = theoretical speed ratio $= \dfrac{u}{V'_0}$.
z = ratio $\dfrac{\text{blade length}}{\text{mean diameter}} = \dfrac{l}{D}$.
A_0 = exit area of the last stage nozzles in ft.²

Then
$$W = \frac{A_0 V_0}{v_0} = \frac{\pi z D^2 V_0 \sin \alpha}{v_0} = \frac{\pi^2 z D^3 N \sin \alpha}{60 v_0 \rho_t}$$

and
$$D^3 = \frac{60 \rho_t W v_0}{\pi^2 z N \sin \alpha} \text{ ft.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This general equation can be used either to find the blade length-diameter ratio z, or to check the diameter for any specified conditions.

225. The specific volume v_0 depends on the vacuum and quality at exhaust. The latter figure is usually in the neighbourhood of 90 per cent. With this as a first approximation, the corresponding volumes for the usual range of vacuum may, for a preliminary estimate, be taken as follows :—

Vacuum . . .	27 ins.	27½ ins.	28 ins.	28½ ins.	29 ins.
Spec. vol. v_0 . .	210	250	300	390	575

The average angle may be taken as $\alpha = 20$ and sin $\alpha = 0{\cdot}342$.
Equation (6) may be written in the simpler form

$$D = C\sqrt[3]{\frac{\rho_t W}{N}} \text{ ft.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where the coefficient depending on the values of α, z, and v_0 is

$$C = \sqrt[3]{\frac{60 v_0}{\pi^2 z \sin \alpha}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For convenience of calculation, values of C corresponding to the vacua quoted above are plotted on a base of length ratio z, in Fig. 198. If α', a larger value than 20°, is taken, the C values must be multiplied by $\sqrt[3]{\frac{\sin \alpha}{\sin \alpha'}}$.

The arbitrary choice of speed ratio ρ_t and blade length-diameter ratio z, requires an exercise of judgment.

For large outputs at speeds between 750 and 1000 R.P.M., z may vary from 0·15 to 0·12.

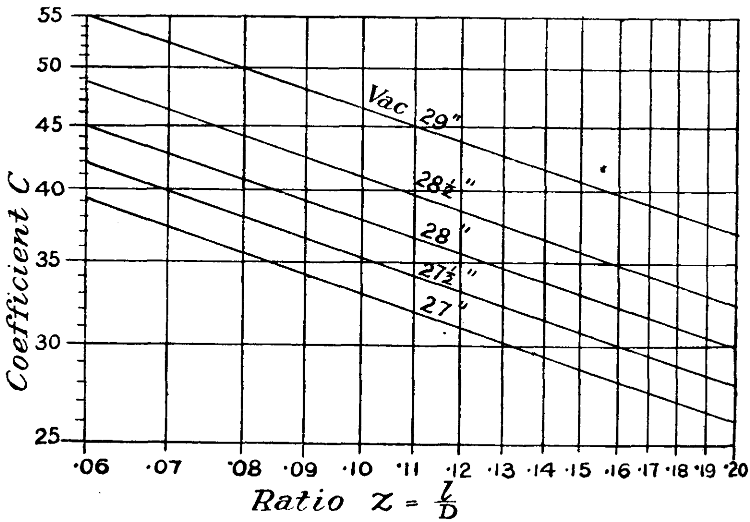


FIG. 198.

For moderate outputs at 1500 to 3000 R.P.M., it may run from 0·12 to 0·06.

226. It is the speed ratio that is the more important factor, as it has a direct bearing on the efficiency and cost.

As already indicated by the experimental curves of Figs. 114 and 118, an average value of stage efficiency $\eta_s = 0{\cdot}8$ at $\rho_t = 0{\cdot}45$ appears possible.

Making the necessary corrections for internal efficiency and outside loss, it would appear from this that an efficiency ratio 0·78 and 0·79 may be obtained. Such a high value, even with the improved conditions now common, has not been found practicable, the value of ρ^t being kept about 0·3 to 0·33. With the most favourable conditions the efficiency ratio of this type may be reckoned at about 0·72 to 0·73 ; but as a conservative estimate, for calculation, it is better to use a value $\eta = 0·7$. For smaller units with less favourable conditions it is preferable to take the efficiency ratio between 0·65 and 0·68. The corresponding range of internal efficiency may be taken from 0·68 to 0·72.

On this assumption the ρ_t values given by the curves of Fig. 114 vary between 0·3 and 0·32.

These limits also enable the approximate values of the reheat factor R to be determined from the curves of Fig. 175, p. 334.

227. The nozzles are assumed here to be convergent. As already seen from the examples worked out in Chap. X., the disc and vane friction loss in the impulse machine is greatest at the first wheel where the steam has the greatest density.

This can be reduced by allowing a much larger heat drop in the first-stage nozzles. For a given speed of rotation and constant mean ring diameter throughout, the velocity ratio at the first wheel, and hence the diagram efficiency, is reduced slightly, but the friction loss is considerably reduced owing to the lower density of the steam. Some makers adopt this practice of giving a large heat drop in the first-stage nozzles and using diverging passages.

EXAMPLE 1.—Determine provisionally the diameter and number of stages suitable for a pressure compounded impulse turbine, which is to develop 2000 K.W. at 3000 R.P.M., with initial pressure 170 lbs./in.^{2} gauge, superheat 200° F., vacuum $28\frac{1}{2}$ inches, and generator efficiency 0·95.

In this case an efficiency ratio $\eta = 0·7$ may be assumed. Allowing, say, 3 per cent. outside loss, the approximate value of the internal efficiency is $\eta_1 = 0·73$. From the curves of Fig. 175, a stage efficiency $\eta_s = 0·69$ at superheat $t_s = 200$ gives a reheat factor $R = 1·062$. Hence $\eta_1 = 1·062 \times 0·69 = 0·733$.

The mean-stage efficiency may be reckoned as 0·69, and from the curves of Fig. 114, the theoretical speed ratio may be taken as $\rho_t = 0·322$.

If the diameter is calculated for the minimum number of stages, then by equation (3)

$$D = \frac{362000 \times 0·322}{3000} = 39 \text{ inches}$$

$$u = \frac{\pi d N}{60} = \frac{\pi \times 39 \times 3000}{60 \times 12} = 511 \text{ ft./sec.}$$

a satisfactory velocity for the lighter type of disc wheel used in this class of machine.

The heat drop from the Hϕ diagram is $H_r = 388$, $R = 1·062$, $RH_r = 412$.

Minimum number of stages

$$n = \frac{RH_r}{50} = \frac{412}{50} = 8{\cdot}2, \text{ say } 8$$

The conditions of pressure, superheat, and vacuum are those specified for the standard Zoelly turbine running at 3000 R.P.M., and the machine for this output has eight stages.

A check has to be made for the limiting value of the last blade height.

By equation (5)

$$w = \frac{3414}{\eta H_r \eta_g} = \frac{3414}{0{\cdot}7 \times 388 \times 0{\cdot}95} = 13{\cdot}2 \text{ lbs./K.W. hour}$$

By equation (4) the weight per sec. is

$$W = \frac{wE}{3600} = \frac{13{\cdot}2 \times 2000}{3600} = 7{\cdot}35 \text{ lb.}$$

Substituting in equation (7)

$$D = C\sqrt[3]{\frac{\rho_t W}{N}}$$

$$\frac{39}{12} = C\sqrt[3]{\frac{0{\cdot}322 \times 7{\cdot}35}{3000}}$$

and

$$C = \frac{39 \times 10{\cdot}82}{12} = 35{\cdot}2$$

From Fig. 198 at $C = 35{\cdot}2$ and vacuum $= 28{\cdot}5$ inches, the value of the blade length ratio is $z = 0{\cdot}159$.

This value is for a jet angle of $20°$, and it can be reduced to any desired figure by increasing the angle.

If z' = decreased length ratio, α' the increased angle, then $z' = z\dfrac{\sin \alpha}{\sin \alpha'}$. It is reasonable to take $z' = 0{\cdot}12$, so that for this figure $\sin \alpha' = \dfrac{0{\cdot}159 \times 0{\cdot}342}{0{\cdot}12} = 0{\cdot}455$, hence $\alpha' = 27°$. In example 10, p. 147, one possible set of nozzles for this turbine has been worked out provisionally, and the more exact calculation taking account of the actual volume, thickness factor, etc., gives $\alpha' = 27\frac{1}{2}°$. A mean diameter of 39 inches and eight stages would be sufficient to fulfil the specified conditions of output, and ensure a reasonable efficiency ratio.

EXAMPLE 2.—Determine the mean ring diameter and number of stages of a pressure compounded impulse turbine to develop 500 K.W. at 750 R.P.M. Initial pressure 180 lbs./in.^{2} gauge, superheat $120°$ F., vacuum $28\frac{1}{2}$ inches, generator efficiency $0{\cdot}95$.

Find the probable output on a reduced vacuum of 28 inches when the first two stages are bye-passed.

The data chosen here are those specified for the Rateau turbine shown in Fig. 20, so that the results obtained by the foregoing method of calculation may be compared with an actual case.

It is probable in this case, considering the lower superheat, that an efficiency ratio of $\eta = 0{\cdot}68$ might be obtained. Allowing, say, 3 per cent. outside loss the internal efficiency is $\eta_1 = 71$.

Referring to Fig. 175 at $t_s = 120$, the corresponding values are approximately $\eta_s = 0{\cdot}68$ and $R = 1{\cdot}055$.

Referring to the curves of Fig. 114 and taking the second highest curve (as indicated in Art. 100, p. 192), the theoretical speed ratio for $\eta = 0{\cdot}68$ is $\rho_t = 0{\cdot}31$.

The heat drop is $H_r = 375$.

By equation (5)

$$w = \frac{3414}{0{\cdot}68 \times 375 \times 0{\cdot}95} = 14{\cdot}1 \text{ lbs./K.W. hour}$$

This is less than the maker's guarantee, which was 14·5. By equation (4)

$$\text{Weight of steam} = W = \frac{14{\cdot}1 \times 5000}{3600} = 19{\cdot}6 \text{ lbs./sec.}$$

It is obvious here that the condition for sufficient area of flow at the last stage controls the diameter. Suppose an outside limit of 0·15 is taken for z, then the value of C for $28\frac{1}{2}$ inches vacuum from Fig. 198 is 36.

By equation (7)

$$D = 36\sqrt[3]{\frac{0{\cdot}31 \times 19{\cdot}6}{750}}$$

$$= \frac{36}{4{\cdot}97} = 7{\cdot}25 \quad \text{or 7 feet 3 inches}$$

To find the number of stages, by equation (2)

$$n = \frac{RH_r\rho_t^2 \times 51270^2}{D^2N^2}$$

$$= \frac{1{\cdot}055 \times 375 \times 0{\cdot}31^2 \times 51270^2}{87^2 \times 750^2}$$

$$= 23{\cdot}5, \text{ say } 24$$

The mean ring diameter of the actual machine is 7 feet 3 inches and the number of stages twenty-four, so that the calculation is satisfactory.

The last blade length shown on the drawings is 13·5 per cent. of the mean diameter instead of 15 per cent. assumed here. This latter value corresponds to an angle of $\alpha = 20°$. If this were opened out to 22°–20′ the smaller blade height as used, would be obtained.

These dimensions have been calculated for a slightly lower consumption rate than the guarantee figure; but with this larger value of 14·5 instead of 14·1 there would be no material difference in the general proportions thus provisionally fixed.

At an overload, with the reduced vacuum of 28 inches, the heat drop is reduced from 375 to 360.

The weight which passes the last ring varies directly as the exit velocity from the nozzles, and inversely as the volume, assuming the other factors to remain constant.

The weight per second passing the last L.P. nozzles is thus, approximately

$$W_1 = W\left(\frac{v_0}{v_{0_1}}\right)\sqrt{\frac{H_{r_1}n}{H_r n_1}}$$

Since
$$\frac{V_{01}}{V_0} = \sqrt{\frac{\dfrac{H_{r_1}}{n_1}}{\dfrac{H_r}{n}}} = \sqrt{\frac{h_{r_1}}{h_r}}$$

The subscript 1 denotes the values at reduced vacuum. Assuming the quality about 0·9 as before

$$v_{0_1} = 0·9 \times 333 = 299·7, \quad n = 24, \quad n_1 = 22$$

Also
$$v_0 = 390, \quad H_{r_1} = 360, \quad H_r = 375, \quad W = 19·6$$

$$\therefore W_1 = 19·6 \times \frac{390}{299·7} \times \sqrt{\frac{360 \times 24}{375 \times 22}}$$

$$= \frac{19·6 \times 390}{299·7 \times 0·976} = 26·15 \text{ lbs./sec.}$$

There is an increase of the rate of consumption due to reduced vacuum. From Fig. 183, by the method of Art. 204, $c_v = +0·03$, hence $x = 1·03$, and the new rate without bye-pass would be $14·1 \times 1·03 = 14·5$. Due to cutting out two stages it is probably increased to 15. Hence the probable output at 28 inches vacuum is

$$E = \frac{3600 \times 26·15}{15} = 6260 \text{ K.W.}$$

228. Pressure-Velocity Compounded Turbine. Curtis Type.—This machine is made with four stages for outputs from 1000 to 5000 K.W., and with five or six for outputs between 5000 and 15,000.

The maximum solid disc diameter usually does not exceed 7 feet. Larger discs are "built up."

In the case of a machine of moderate output the ratio of maximum L.P. blade length to mean ring diameter does not usually exceed 0·16. For a large output at low rotational speed it may reach the value of 0·2, but this figure is exceptional.

In some designs, in order to keep the blade length within reasonable limits without excessive increase of blade angle, the last wheel is made from 5 to 8 per cent. larger in mean diameter than the others.

These L.P. blades, on account of the wide angles, have a smaller cross-sectional area than the blades of the other wheels, and to keep down the stress and obtain a reasonable amount of stiffness the maximum blade length should not be greater than 9 × width. As the largest blade width is about $1\frac{1}{2}$ inches it is not advisable to use a steel blade longer than 13 inches.

It is the standard practice to use a two-velocity stage per pressure stage. The three- and four-velocity stages are confined principally to the marine type, and the small sizes of land turbine, which are usually made with one-pressure stage.

229. Within the narrow limits of four and six stages, which are found to be commercially practicable, a trial value can, if desired, be chosen first, and the diameter suitable for the H.P. wheels can be calculated from the general equation (2). The last L.P. diameter can then be calculated for efficient flow at discharge, and adjustment made between the limits of diameter thus found. It is obviously not possible to state a hard-and-fast method where a simultaneous variation can be made in the L P. height ratio, the blade length-diameter ratio, the nozzle angle, the L.P. mean diameter, and the total number of stages. In applying equation 7, to calculate the L.P. mean diameter for efficient discharge, it should be remembered that the maximum blade length at the last wheel is $l_3 = Kl_n$, where K is the "height ratio" and l_n the nozzle exit length. For a chosen value of length-diameter ratio z, the coefficient C, for this case, becomes $C' = C\sqrt[3]{K}$, and the diameter is given by

$$D = C'\sqrt{\rho t \frac{W}{N}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

At the rated speed, with the heavy type of disc, as used in this turbine, the mean blade velocity should not exceed 600 ft./sec. With the dimensions used in current practice the velocity usually does not exceed 500 ft./sec., and the stresses in disc and blading are therefore reasonable.

230. In using the curves of Fig. 198 it should be noted that the C values are given for a standard nozzle angle, $\alpha = 20°$. If the angle is increased to α' to reduce the diameter, the value of the coefficient is

$$C' = C\sqrt[3]{K \times \frac{\sin \alpha}{\sin \alpha'}}$$

where $\sin \alpha = 0{\cdot}342$.

As regards the "height ratio" "K" at the L.P. end, it appears to vary between 2 and 3, but in some instances falls as low as 1·5. A low value ensures a smaller diameter, but causes a lower efficiency at this stage. As already indicated in Art. 88, p. 172, the relations between height ratio and nozzle and blade angles, in the velocity compounded stage, are, in the present state of knowledge, more or less indeterminate. It does not seem desirable, however, to go below a value of 2 for the last height ratio, unless there is considerable difficulty with a large output at a low rotational speed.

At the high and intermediate stages, as indicated by the example 3, p. 173, it may be taken between 2 and 2·5.

231. The frictional losses of this type of turbine are somewhat greater than those of the pressure compounded impulse, and the efficiency ratio for the same conditions of operation is usually slightly less.

The experimental curves, Fig. 115, for two rings show that under the most favourable operative and blading conditions, the stage efficiency may rise to 0·67, and under less favourable conditions may fall to 0·55. The curve of Fig. 118, which is said to represent average conditions for large units working under modern conditions of pressure, superheat, and vacuum, shows a maximum value of 0·68 for the stage efficiency. In each case the theoretical speed ratio corresponding to the maximum stage efficiency is about 0·23.

At the best, the efficiency ratio of this type for a moderate or large output, reckoned on the stop-valve pressure, does not usually exceed 0·69.

EXAMPLE 3.—Determine suitable proportions for a Curtis turbine having two velocity stages per pressure stage for an output of 2500 K.W. at 1500 R.P.M., with 170 lbs./in.^{2} gauge pressure, 200° F. superheat, and $28\frac{1}{2}$ inches vacuum. Assume a generator efficiency of 95 per cent.

The usual number of stages used for the output and speed specified is four. This number may be assumed here and the diameter calculated from equation (2). Alternatively the L.P. diameter can first be ascertained for chosen blade conditions, and the number calculated for this mean value throughout.

In either case the values of speed ratio, cumulative heat, and steam per second have first to be ascertained.

Taking a conservative value $\rho_t = 0·2$, and referring to Fig. 115, the upper curve gives $\eta_s = 0·65$.

The curves of Fig. 175, when used to determine R for number of stages below six, gives too large a value. In this case, as the number may be expected to be less than six, the R value should be reduced. At $t_s = 200$ and $\eta_s = 0·65$, $R = 1·072$, and the reduced value may be taken as $R = 1·062$. Hence the corresponding internal efficiency $\eta_1 = R\eta_s = 0·69$. These are the figures, which are checked on the Hϕ diagram in example 1, p. 335, for a four-stage machine working between the limits specified above.

Allowing 3 per cent. outside loss, the probable efficiency ratio is $\eta = 0·66$. The heat drop is $H_r = 388$ B.Th.U.; hence by equation (2)

$$n = \frac{RH_r\rho_t^2 \times 51270^2}{D^2N^2}$$

$$4 = \frac{1·062 \times 388 \times 0·2^2 \times 51270^2}{D^2 \times 1500^2}$$

$$\therefore D^2 = 4830$$

$$D = 69·3, \text{ say 69 inches, or 5 feet 9 inches}$$

The mean ring diameter of the first three wheels may be made this value. It has now to be ascertained if this diameter at the last wheel permits the use of a reasonable blade length, or whether an enlargement of diameter, increase of angle, or restriction of height ratio is required, for the efficient discharge of the steam.

For this moderate output choose for a trial, say, height ratio $K = 2{\cdot}5$ and blade length-diameter ratio $z = 0{\cdot}1$ to $0{\cdot}12$.

Referring to the curves, Fig. 198, at $28\frac{1}{2}$ inches vacuum, and $z = 0{\cdot}12, C = 38{\cdot}5$, hence $C' = C\sqrt[3]{K} = 38{\cdot}5\sqrt[3]{2{\cdot}5} = 38{\cdot}5 \times 1{\cdot}358 = 52{\cdot}3$.

By equation (9) the diameter is

$$D = C'\sqrt[3]{\rho_t \frac{W}{N}}$$

Here by equation (5)

$$w = \frac{3414}{0{\cdot}66 \times 388 \times 0{\cdot}95} = 14 \text{ lbs./K.W. hour}$$

and weight per sec.

$$W = \frac{14 \times 2500}{3600} = 9{\cdot}72 \text{ lbs.}$$

$$\therefore\ D = 52{\cdot}3\sqrt[3]{\frac{0{\cdot}2 \times 9{\cdot}72}{1500}} = \frac{52{\cdot}3}{9{\cdot}16} = 5{\cdot}7 \text{ ft.} = 68{\cdot}5, \text{ say } 69 \text{ inches}$$

It would appear from this that a uniform diameter of 69 inches may be used throughout.

The adoption of this diameter at the last wheel gives with $z = 0{\cdot}12$, the last blade length $8{\cdot}3$ inches, say $8\frac{1}{4}$, a reasonable value. With $m = 2{\cdot}5$ the nozzle exit length is $\frac{8{\cdot}3}{2{\cdot}5} = 3{\cdot}32$. The nozzle proportions have been calculated for this stage in example 8, p. 141, a provisional value of $l_n = 3{\cdot}25$ being assumed, and the exit angle, allowing for the correct steam volume and thickness factor, works out at $23\frac{1}{2}^\circ$. The result obtained may be modified to suit individual judgment. If, for instance, the height ratio is reduced, the diameter may be slightly increased, keeping other factors the same, or the angles may be increased. This latter modification is not advisable, as the general result is to lower the stage efficiency and increase the carry-over loss.

The general dimensions on which the detailed design of this proposed machine can be based may thus be taken as—

Mean ring diameter, 69 inches. Number of stages, 4.

232. Marine Curtis Turbine.—In this type the peripheral speed with directly connected propeller may vary from 150 to 200 ft./sec.

The efficiency ratio is much lower than that obtained by the higher speed land type working under favourable conditions of pressure, superheat, and vacuum. Usually dry steam is used, although in some naval installations a slight superheat has been given.

In order to reduce the pressure in the high-pressure casing to a reasonable figure and keep down the scantlings at this end, about one quarter of the total heat drop is apportioned to the first stage. With the pressure and vacuum limits customary, this heat drop is in the neighbourhood of 86 B.Th.U., and the pressure in the first stage (H.P.) casing varies from 90 to 100 lbs./in.^{2} abs.

233. The first step in the provisional design should be the location of the condition curve on the $H\phi$ diagram. On account of the proportionally larger heat drop in the first stage, this curve at the higher pressures is discontinuous.

The condition curve proper starts from the initial state point for the second stage, at the pressure p_2.

Let H_r = heat drop between the initial pressure p_1 and exhaust pressure p_0. For $\frac{1}{4}$ of the total heat drop, the drop for the H.P. stage is $h_{r_1} = \frac{H_r}{4}$.

From the initial state point on p_1 scale vertically the value of h_{r_1} and locate the p_2 curve. With this heat drop and the low peripheral speed imposed by the propeller conditions, a four velocity stage has usually to be employed. Its efficiency is thus low. It may run from 0·45 to 0·48. The efficiency ratio of the whole machine may not be much more than 0·56 at the full speed conditions, and an internal efficiency from 0·59 to 0·6 may be assumed for the location of the approximate state point at the exhaust pressure. This, as in the previous cases, is obtained by scaling the value of $\eta_1 H_r$ vertically from the initial state point on p_1, and projecting horizontally to cut the p_0 curve. The condition curve can then be drawn between the second and the final state points. Since with initially dry or slightly superheated steam the state point at p_2 falls in the saturation field, the condition curve is sufficiently approximated, by a straight line drawn between this and the final state point.

The qualities, volumes, and cumulative heat values can be obtained from the condition curve by the method already explained in Art. 181, and plotted to give the cumulative heat volume diagram shown in Fig. 199.

In the first stage the cumulative heat is simply h_{r_1}, the proportion of the total heat drop allotted. This value scaled up at the initial pressure p_2 of the I.P. section of the machine gives the starting point A of the cumulative heat curve for the I.P. and L.P. sections. The cumulative heats between A and C are reckoned from p_2 and plotted from the new base line AB. The total cumulative heat BC has to be apportioned suitably between the I.P. and L P. sections.

234. The usual practice is to use a drum construction for the L.P. section, and to close the H.P. end of the drum. The pressure at exhaust from the first stage drum nozzles, acting on the effective area of the closed drum end, is used to partly or wholly balance the propeller thrust at full load. As in the case of the reaction turbine, the steam thrust on the drum is less than the propeller thrust at light load or reduced speed. To compensate for this decrease at cruising speed a 10 per cent. increase of thrust may be allowed under full-load conditions.

Let p = pressure on the drum end in lbs./in.^{2} abs.
A = effective area of the end in inch^2.
T = propeller thrust in lbs.
p_0 = exhaust pressure in lbs./in.^{2} abs.

Then $T = (p - p_0)A$ . . . . (10)

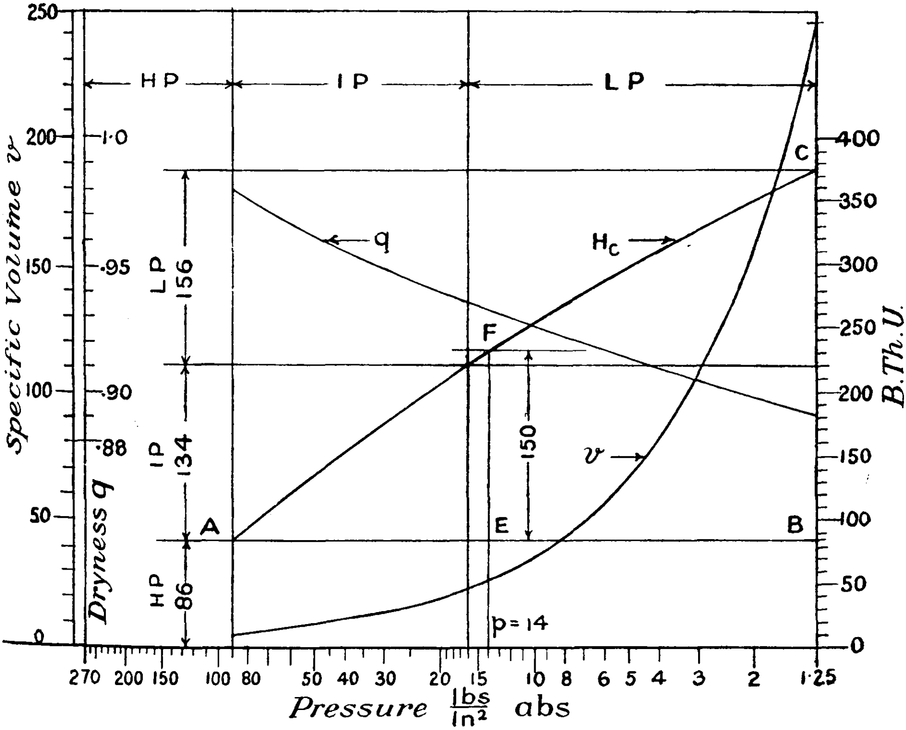


FIG. 199.

In this class of turbine the shaft diameter may vary from 20 to 24 per cent. of the mean diameter of the blading. The effective area may therefore be taken as $A = 0{\cdot}63D^2$, where D = mean ring diameter in inches.

By équation (45), p. 264, the thrust is given by

$$T = \frac{325{\cdot}7\eta_p E_s}{S}$$

hence, substituting for T from equation (10)

$$(p - p_0)0{\cdot}63D^2 = \frac{325{\cdot}7\eta_p E_s}{S} \quad . \quad . \quad . \quad . \quad (11)$$

If 10 per cent. increased thrust is allowed for, the right-hand side is multiplied by 1·1.

When the value of p is determined, the corresponding volume is given by the volume curve, Fig. 199. The length-diameter ratio z, calculated from the general equation when this volume is used, should run from 0·015 to 0·02. In any case it is not advisable, with full peripheral admission at the first drum stage, to use a shorter blade length than 1 inch. If this calculation is not satisfactory, the value of p may be arbitrarily varied to suit the condition desired.

When p is determined, the H_c curve gives the cumulative heat between p_2 and p. This is the amount of heat for the I.P. section and the first L.P. drum stage. By subtracting the first L.P. heat drop from this, the amount of heat for the I.P. section is known.

235. The first step towards the estimation of p is the calculation of the mean diameter D by equation (9). Here the speed ratio ρ_t is much lower than in the land machine. For the L.P. (two velocity stages) it may reach a maximum of 0·19 and for the I.P. (three velocity stages) about 0·11 to 0·13. Also the height ratio at the last one or two L.P. stages does not exceed 1·5, and the jet angle is greater than 20°, say about 25°.

The last blade length may run from 0·12 to 0·15 of the mean ring diameter. These figures are to be regarded as rough averages. As in the case of the land machine with so many factors that can be arbitrarily varied, it is out of the question to suggest any hard-and-fast method of adjustment at the L.P. end. The result, in any case, must depend on the personal equation.

236. Assuming that a provisional value of D is estimated, then from the chosen value of the theoretical speed ratio ρ_t and the mean blade velocity u, the constant L.P. stage heat drop is calculable from

$$h_{r_L}=\left(\frac{u}{223\cdot7\rho_t}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This value subtracted from the intercept between the base line AB, Fig. 199, and the curve AC gives the cumulative heat value for the I.P. section, or H_I, and the cumulative heat for the L.P. section is $H_L=(BC-H_I)$.

Substituting the corresponding value of ρ_t for each three-velocity stage of the I.P. section in equation 12, the corresponding I.P. stage heat drop h_{r_I} is obtained.

Then the number of stages in the I.P. section is

$$n_I=\frac{H_I}{h_{r_I}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The number in the L.P. section is

$$n_L=\frac{H_L}{h_{r_L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The pressure and qualities at entrance to each set of stage nozzles are now determinable from the cumulative-heat volume diagram, and the dimensions of the successive nozzles throughout the I.P. and L.P. sections can be calculated in detail by the methods given in Chap. VII. The blading designs can then be carried out by the methods given in Chap. VIII.

In order to keep down the dimensions, the astern or reverse turbine is usually made with two pressure stages, each having two velocity stages. The mean ring diameter is also made slightly less than that of the ahead turbine, say about 90 per cent. of it. The choice is, however, an arbitrary one. When the provisional diameter is fixed the probable blade velocity may be estimated by assuming that the revolutions when going "full speed astern" are the same as those when going "full speed ahead."

The total heat drop, when equally divided between the H.P. and L.P. stages, gives the approximate value of the theoretical jet velocity; and the theoretical speed ratio is calculable. The approximate value of the stage efficiency is then obtained from the curve of Fig. 117. It is necessarily low, probably about 0·3. The stage pressure limits are then ascertained on the $H\phi$ diagram by the trial method of Art. 175.

When these are satisfactorily adjusted, the necessary data for the calculation of the proportions of nozzles and blading are obtained from the diagram.

The internal efficiency, reduced by say 5 per cent. to allow for outside loss, gives the approximate efficiency ratio. By assuming the same weight of steam per second for the astern as for the ahead turbine, the probable full power astern is calculable from equations (4) and (5). (See also Art. 264.)

EXAMPLE 4.—Determine provisionally the mean diameter and number of stages for a marine Curtis turbine of the type shown in Fig. 19. The output is to be 7500 S.H.P. at 450 R.P.M. The initial pressure p_1 is 275 lbs./in.^{2} abs., steam dry, vacuum $27\frac{1}{2}$ inches. Allow $\frac{1}{4}$ of the power to the H.P. section, and take the steam consumption as 13·5 lbs./S.H.P. hour at full load.

The turbine is one of an installation of three to develop 22,500 S.H.P. when the full speed of the vessel is 28 knots.

From the $H\phi$ diagram the heat drop between $p_1 = 275$ and $27\frac{1}{2}$ inches vacuum is $H_r = 345$, hence the heat drop for the H.P. stage is

$$h_1 = \frac{345}{4} = 86 \text{ B.Th.U.}$$

Scaling down 86 units from the initial state point on 275 pressure curve, the lower pressure is found to be $p_2 = 90$ lbs./in.^{2} abs.

Assuming a value of $\eta_s = 0{\cdot}45$ for this stage, scaling down $0{\cdot}45 \times 86 = 39$ again from the initial state point on p_1, and projecting to p_2, the quality is found to be $q_2 = 0{\cdot}98$.

For a steam consumption of 13·5 lbs./S.H.P. hour, equation (5) gives

$$\eta = \frac{2544}{wH_r} = \frac{2544}{13{\cdot}5 \times 345} = 0{\cdot}55$$

Allowing for, say, 5 per cent. outside loss the probable value of the internal efficiency may be taken as $\eta_1 = 0{\cdot}6$, and used for the location of the final state point at $p_0 = 1{\cdot}25$ lbs./in.^{2}.

Scaling down $0{\cdot}6 \times 345 = 207$ from the initial state point on p_1 and projecting to $p_0 = 1{\cdot}25$, the final state point falls at $q_0 = 0{\cdot}89$. The second and final state points can now be joined by a straight line.

In order to derive the curves of the cumulative-heat volume diagram, Fig. 199, the following convenient pressures may be chosen. The values obtained by the method of Art. 181 are tabulated below.

Pressure, p . . .	90	50	20	10	5	3	2	1·2
Quality, q . . .	0·98	0·96	0·94	0·928	0·913	0·903	0·898	0·89
Volume, v_0 . . .	4·8	8·2	18·8	35·7	66·7	107	155	246
Cumulative heat, H_c	0	45	112	164	207	242	268	290

The cumulative heats are reckoned from $p_2 = 90$ lbs./in.^{2} abs., and scaled up from AB, Fig. 199. The total cumulative heat for the I.P. and L.P. sections is BC = 290 B.Th.U.

For the whole machine the total cumulative heat is $290 + 86 = 376$, so that the probable reheat factor is $R = \frac{376}{345} = 1{\cdot}09$. With the assumed internal efficiency of 0·6, the average stage efficiency for the whole machine is $\eta_s = \dfrac{0{\cdot}6}{1{\cdot}09} = 0{\cdot}55$. The first stage value is much less, and the value of each I.P. and L.P. stage greater in proportion.

In order to find the mean ring diameter the following conditions may be assumed for the last L.P. stage :—

$$K = 1{\cdot}2, \quad z = 0{\cdot}12, \quad \rho_t = 0{\cdot}185, \quad \alpha' = 25^\circ$$

$$W = \frac{13{\cdot}5 \times 7500}{3600} = 28{\cdot}2, \quad N = 450$$

with $z = 0{\cdot}12$, from curves at Fig. 198 at $27\frac{1}{2}$ inches vacuum $C = 33$.

Then
$$C' = C\sqrt[3]{K\frac{\sin\alpha}{\sin\alpha'}} = 33\sqrt[3]{1{\cdot}2 \times \frac{0{\cdot}342}{0{\cdot}4226}}$$
$$= 0{\cdot}99 \times 33 = 32{\cdot}6$$

By equation (9)

$$D = C'\sqrt[3]{\rho_t\frac{W}{N}}$$
$$= 32{\cdot}6\sqrt[3]{\frac{0{\cdot}185 \times 28{\cdot}2}{450}} = \frac{32{\cdot}6}{4{\cdot}42} = 7{\cdot}4 \text{ feet}$$

To the nearest round number the diameter might be fixed at 7 feet, or 84 inches.

This, with a blade length-diameter ratio of 0·12, gives a blade length of 10 inches, which is just about the limit that would be used with the drawn bronze blade fitted in these machines.

To find the balancing pressure p at the drum face, the propeller thrust is given by $T = \frac{325 \cdot 7 \times 0 \cdot 6 \times 7500}{28}$, taking the propeller efficiency as 60 per cent.

By equation (11)

$$(p - 1 \cdot 25) 0 \cdot 63 \times 84^2 = \frac{325 \cdot 7 \times 0 \cdot 6 \times 7500}{28}$$

Hence $$p = 11 \cdot 75 + 1 \cdot 25 = 13 \text{ lbs./in.}^2 \text{ abs.}$$

If an additional 10 per cent. thrust is allowed for

$$p = 11 \cdot 75 \times 1 \cdot 1 + 1 \cdot 25 = 14 \text{ lbs./in.}^2 \text{ abs.}$$

Taking the latter figure, the intercept EF between AB and the curve AC (Fig. 199), gives $h_c = 150$ B.Th.U.
The mean blade velocity is

$$u = \frac{\pi DN}{60} = \frac{3 \cdot 1416 \times 84 \times 450}{12 \times 60} = 165 \text{ ft./sec.}$$

By equation (12)

$$h_{r_L} = \left(\frac{165}{223 \cdot 7 \times 0 \cdot 185}\right)^2 = 4^2 = 16$$

Hence the cumulative heat for the I.P. section $H_I = (150 - 16) = 134$.

The heat for the L.P. section is $H_c - H_I = (290 - 134) = 156$.

From the curves of Fig. 116 a value of $\rho_t = 0 \cdot 13$ may be chosen for the three-velocity stages of the I.P. section. With this value by equation (12) the I.P. heat drop is

$$h_{r_I} = \left(\frac{165}{223 \cdot 7 \times 0 \cdot 13}\right)^2 = 5 \cdot 68^2 = 32 \cdot 2$$

By equation (13) the number of stages in I.P. section

$$n_I = \frac{H_I}{h_{r_I}} = \frac{134}{32 \cdot 2} = \text{say } 4$$

By equation (14) number of stages in L.P. section

$$n_L = \frac{H_L}{h_{r_L}} = \frac{156}{16} = \text{say } 10$$

The provisional make up of the turbine on the basis of the assumptions made is thus—

Diameter, 7 feet; 1 H.P. four-velocity stage, 4 I.P. three-velocity stages, 10 L.P. two-velocity stages.

The I.P. and L.P. stages might easily be increased one or two, by a different choice of the factors which have to be tentatively assumed.

For the astern turbine a mean ring diameter of 6·3 ft. or 90 per cent. of the ahead diameter may be assumed. With "full speed of 450 R.P.M. astern," the mean blade velocity is $u = 148$ ft./sec. The total heat drop between the assumed pressure limits of 275 lbs./in.^{2} abs. and 1·25 lbs./in.^{2} abs. is $H_r = 345$, so that the approximate stage heat drop, with the two-stage arrangement, is $h_r = 177$. The theoretical jet velocity, making some allowance for reheat, may thus be taken as 3000 ft./sec., and the theoretical speed ratio $\rho_t = \frac{148}{3000} = 0·0493$, or say 0·05.

The corresponding stage efficiency from Fig. 117 is $\eta_s = 0·32$. From the $H\phi$ diagram, with this value, the initial pressure at the second L.P. stage is 23 lbs./in.^{2} abs., and the quality $q = 0·984$. The internal efficiency is $\eta_1 = 0·35$. Deducting 0·05 for outside loss, the approximate efficiency ratio is $\eta = 0·30$.

When equations (4) and (5) are combined, the expression for the shaft horse power becomes—

$$E = 1·415 W \eta H_r$$

$$\text{Here } W = \frac{13·5 \times 7500}{3600} = 28·2 \text{ lbs./sec.}$$

and the full astern shaft horse power is

$$E = 1·415 \times 28·2 \times 0·3 \times 345 = 4130$$

This is a maximum value. Normally, it may be considerably less, as the pressure limits and speed chosen above are probably on the high side. The corresponding ratio of full astern to full ahead power is 55 per cent. Under normal conditions it would probably be from 45 to 50 per cent.

The provisional make up of the astern turbine on the basis of the assumptions made is thus—

Diameter, 6·3 ft.; 1 H.P. four-velocity stage; 1 L.P. four-velocity stage.

237. The conditions of the design are those at the full speed conditions of the vessel when the turbine is developing its full normal load.

In a naval vessel the difficult question of "cruising speed" has also to be considered. A compromise is made with this type of turbine, by designing the machine, as indicated here, for the full speed conditions, and adopting a nozzle cut-out system at the H.P. stage

Usually about 20 per cent. of the nozzles provided at this stage are used for cruising purposes. The others are shut off by means of special valves. In some instances a slide valve arrangement is also applied to the I.P. section. These cruising nozzles are of a different design from the ordinary full-speed nozzles, and are shut off when the others are brought into use. Of the full-load nozzles only about 75 per cent. are usually necessary to give the full-speed conditions. The other 25 per cent. are opened when a temporary overload for increased rate of speed is necessary.

238. Curtis-Rateau Impulse.—It is now the practice to make this machine with all the wheels the same diameter. The size of the first-stage velocity compounded wheel is thus fixed by the maximum limit for the pressure compounded wheels.

The machine may be treated in the first instance as a pure pressure compounded impulse, the diameter and number of stages being chosen according to judgment between the maximum and minimum values, fixed by the limiting stress in the disc, and the area for efficient discharge at the L.P. end.

When the mean diameter and number of stages are selected, the number can be reduced by the proportion corresponding to the single velocity compounded stage.

Thus, if x is the fraction of the heat drop H_r to be utilised in the first stage, and n the number of stages required for a pure pressure compounded machine, the number of stages for the modified arrangement is given approximately by

$$n' = (1 - x)n \quad \ldots \ldots \quad (15)$$

From the general equation (2), since $n = 1$ for the first stage, the heat drop in this stage is given by

$$h_{r_1} = \frac{D^2N^2}{(51270\rho_t)^2} \quad \ldots \ldots \quad (16)$$

and

$$x = \frac{h_{r_1}}{H_r}$$

For the preliminary adjustment of the diameter and number of stages, the maximum diameter for light wheels, assuming the limiting blade speed of 500 ft./sec., is given by

$$D = \frac{9550}{N} \text{ feet} \quad \ldots \ldots \quad (17)$$

The diameter for a minimum number of stages is given by equation (3), while the minimum diameter for efficient L.P. discharge is given by equation (7).

The adjustment of diameter between the limits given by the above equations is a matter of experience.

EXAMPLE 5.—Determine a suitable diameter and number of stages for a Curtis-Rateau turbine to develop 5000 K.W. at 1500 R.P.M. with initial pressure 190 lbs./in.^{2} gauge, superheat 180° F., vacuum $28\frac{1}{2}$ inches, and generator efficiency 0·95.

In this case the heat drop is $H_r = 391$, and the efficiency ratio may be taken as $\eta = 0{\cdot}69$. The probable reheat factor is $R = 1{\cdot}06$; $\rho_t = 0{\cdot}32$ may be taken for the single-ring wheels and $\rho_t = 0{\cdot}2$ for the two-ring wheel.

By equation (5)

$$w = \frac{3414}{391 \times 0{\cdot}69 \times 0{\cdot}95} = 13{\cdot}3 \text{ lbs./K.W. hour}$$

$$W = \frac{13{\cdot}3 \times 5000}{3600} = 18{\cdot}5 \text{ lbs./sec.}$$

For the limiting stress condition by equation (17)

$$D = \frac{9550}{1500} = 6{\cdot}3 \text{ feet}$$

For the minimum number of stages, by equation (3)

$$D = 362000 \frac{\rho t}{N}$$

$$= \frac{362000 \times 0{\cdot}32}{1500} = 77{\cdot}3 \text{ inches} = 6{\cdot}45 \text{ feet}$$

Take a blade-diameter ratio $z = 0{\cdot}15$, say, and allow for an increased angle, say $\alpha' = 25^\circ$, at the last L.P. stage. From Fig. 198 at $28\frac{1}{2}$ inches vacuum and $z = 0{\cdot}15$, $C = 35{\cdot}8$. This reduced for the larger angle is

$$C = 35{\cdot}8\sqrt[3]{\frac{0{\cdot}342}{0{\cdot}4226}} = 35{\cdot}8 \times 0{\cdot}93 = 33{\cdot}3$$

By equation (7)

$$D = 33{\cdot}3\sqrt[3]{\frac{0{\cdot}32 \times 18{\cdot}5}{1500}} = \frac{33{\cdot}3}{6{\cdot}35} = 5{\cdot}25 \text{ feet} = 63 \text{ inches}$$

This gives a maximum blade length of $9\frac{1}{2}$ inches at the last wheel, which is a reasonable value. There is some latitude of choice here between 6·3 and 5·25 feet. An increase would further reduce the blade length. Suppose 65 inches is chosen. Then the heat drop in the first stage by equation (16) is

$$h_{r_1} = \frac{D^2N^2}{(51270\rho t)^2} = \left(\frac{65 \times 1500}{51270 \times 0{\cdot}2}\right)^2 \doteqdot 90 \text{ B.Th.U.}$$

Hence $\quad x = \frac{90}{391} = 0{\cdot}23$ or 23 per cent. of the heat drop.

By equation (2) for a pure pressure compounded machine

$$n = RH_r\frac{(51270\rho t)^2}{D^2N^2} = 1{\cdot}06 \times 391\left(\frac{51270 \times 0{\cdot}32}{65 \times 1500}\right)^2$$

$$= 11{\cdot}8, \text{ say } 12$$

By equation (15) $n' = (1 - x)n = (1 - 0{\cdot}23) \times 12 = 0{\cdot}77 \times 12 = 9{\cdot}3$, say 10, 9 pressure compounded and 1 velocity compounded stage. Other values might be chosen; but the number of stages deduced corresponds closely to that used in large machines of this type.

CHAPTER XIV

PROVISIONAL DETERMINATION OF GENERAL PROPORTIONS OF COMPOUND TURBINES.

AXIAL FLOW REACTION (PARSONS TYPE) AND COMBINATION TURBINES

SEVERAL methods may be employed to calculate the general dimensions of a machine of this type.

Owing, however, to the nature of the problem, the process of adjustment is not so simple as that for an impulse turbine. In any case, more or less trial and error calculation is necessary, whilst a good deal of assumption has to be made regarding the various design coefficients to be used.

The method of calculation developed and illustrated in this chapter is based on the cumulative-heat volume diagram discussed in Chap. XI.

239. Distribution of Power in the Turbine.—In the land type of turbine there are usually three sections—high pressure, intermediate, and low pressure, and a three-stepped drum is used. As the result of experience the usual distribution of the total power is $\frac{1}{4}$ in the H.P., $\frac{1}{4}$ in the I.P., and $\frac{1}{2}$ in the L.P. section.

When a two-cylinder tandem arrangement is adopted, the power is divided equally between the H.P. and L.P. cylinders. A three-stepped drum is used for the H.P. and a parallel drum for the L.P. cylinder, usually arranged for "double flow."

The preliminary step towards the calculation of the drum and blading proportions is the construction of the cumulative-heat volume diagram.

240. For the purpose of locating the condition curve on the $H\phi$ diagram, the stage efficiency, as already indicated in Art. 97, may be assumed to vary from 0·7 at the H.P. to 0·8 at the L.P. end.

For the practical purpose of curve construction, an average value of $\eta_s = 0{\cdot}75$ may be used.

With this value the approximate reheat factor R and internal efficiency η_1 are calculable from the curves of Fig. 175. A complete $H_c - v$ diagram, drawn, by the methods given in Chap. XI., for $\eta_s = 0{\cdot}75, p_1 = 190$ lbs./in.^{2} abs., $t_{s_1} = 150°$ F., and vacuum $= 28$ inches, is shown in Fig. 200.

When this diagram is drawn, the total cumulative heat ordinate AB is divided in the proportions chosen for the powers. Projectors, drawn from the points of division to the H_c curve, then determine the limiting pressures and volumes at each section. Thus, in Fig. 200, a distribution of $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$ is shown. H.P. heat AC $= \frac{1}{4}$ AB; I.P. heat CD $= \frac{1}{4}$ AB; L.P. heat DB $= \frac{1}{2}$ AB. The corresponding pressure limits are shown on the horizontal scale.

When, in addition to the cumulative heat and initial and final

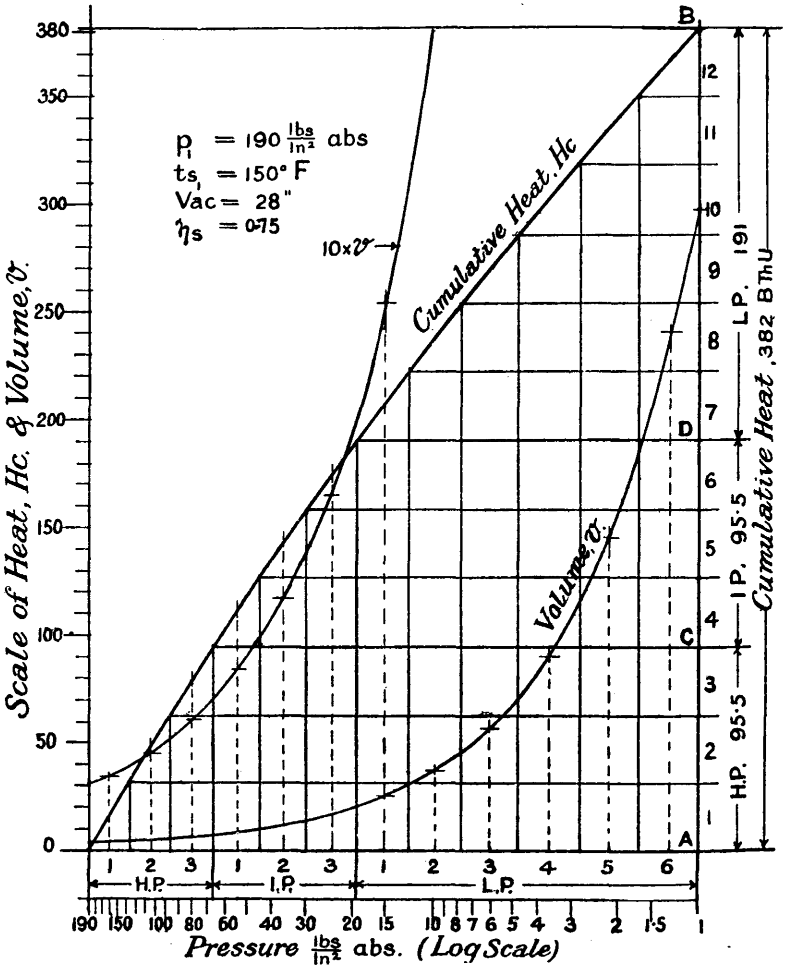


FIG. 200.

pressures and volumes, the weight W passing through the turbine is known, the drum and blading dimensions at any section are calculable.

With regard to W, this, as in the case of the impulse turbine, can be calculated from equations (4) and (5), p. 384. The value of the efficiency ratio η for (5) can be deduced for an average stage efficiency of 0·75 and a reasonable correction made for outside loss, in the manner indicated in Art. 184.

The external gland leakage may be neglected, since, as already indicated by the examples in Chap. X., with a properly designed labyrinth gland, this loss is a very small one. The dummy piston leakage at each section has, however, to be allowed for, in the calculation of the blading dimensions.

241. Number of Expansions.—On any section of the drum the curve of blade lengths should be progressive, and similar to that of the specific volume of the steam between the limiting pressures of the section. Such progressive blade length is mechanically impracticable, and the continuous curve is replaced by a stepped one. The cumulative heat of the section is arbitrarily divided into a number of parts, which may be equal or approximately so. Each part is utilised in a group of blades called an "expansion." The blades of the group are all made the same length, but the group length progressively increases from the first to the last group in the section. The choice of the number of expansions for any given section is quite arbitrary. As a rule, the number for the whole machine varies from 12 to 16. With the $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$ distribution of the power, when 12 expansions are used, the numbers for the three sections become 3 in the H.P., 3 in the I.P., and 6 in the L.P. section.

When 16 expansions are used, the section numbers become 4, 4, and 8. A 12-expansion arrangement is shown in Fig. 200.

Any other arbitrary number of expansions can be chosen, but the foregoing appear to meet the practical requirements satisfactorily.

242. When the number of expansions for any section is chosen, the cumulative heat for the section is usually divided equally among them. An unequal distribution can be adopted if desired. In any case, the limiting pressures and volumes for each expansion are obtained by dividing the heat ordinate in the selected proportions and projecting to the H_c curve. In Fig. 200 an equal division, which is quite good enough for the end in view, is adopted, AC and CD being divided into three and DB into six equal parts.

This is the system usually employed; but another division arranged to give equal increments of volume at each expansion is sometimes used. In this case the heat per expansion is a decreasing quantity.

As a rule, the blading proportions calculated by the second differ but slightly from those calculated by the first method. As the first is the simpler one to handle graphically, and involves less calculation, it is adopted and used throughout this chapter.

243. Calculation of H.P. Drum Diameter.—When the foregoing division of the $H_c - v$ diagram has been made, the necessary data are available for the calculation of the diameter of the drum at the H.P. end.

The blades of any group having constant length may all have the same discharge angle θ, and in this case, since the mean blade speed remains constant, the speed ratio ρ increases between the first and the last stages of the group. This is due to the fact that the steam velocity increases on account of increase of volume, while the outlet area at the successive blade exits remains constant.

If a definite value of ρ is to be maintained throughout an expansion, then the blade angles have to progressively increase from the first to the last stage of the group. When this condition is fulfilled, the blades are said to be "gauged." A few makers follow this practice, but the majority keep the blade angle constant throughout the group, and allow the velocity to increase and velocity ratio to diminish.

The result of gradually increasing the velocity throughout a group is an increase of the stage heat drop, so that the work done on the series of rings is not quite equally distributed.

For the estimation of the number of rings it is quite sufficient, for practical purposes, to calculate the average heat drop per stage, by using the mean value of ρ in the energy equation (see Art. 253).

When the mean speed ratio is thus chosen, the steam velocity used for the calculation of area for flow at the first expansion is that which exists at the middle of the expansion. Consequently the volume used in the calculation should be the value given by the volume curve of the $H_c - v$ diagram, at the mid-ordinate. These volume values for the different expansions in Fig. 200 are shown by dotted lines.

Denoting the mid volume value at the first expansion by v_1, the exit angle by θ, and using the notation already employed in Art. 224, the equation for flow at the first expansion gives the mean H.P. ring diameter.

Thus
$$D_1^3 = \frac{60\rho W v_1}{\pi^2 z N \sin\theta}$$

and
$$D_1 = B\sqrt[3]{\frac{Wv_1}{N}} \text{ feet} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here the coefficient
$$B = \sqrt[3]{\frac{60\rho}{z\pi^2 \sin\theta}}$$

In order to facilitate calculation, the values of B for a series of values of speed ratio ρ and length-diameter ratio z, with constant angle $\theta = 20$, are plotted in Fig. 201.

When a limiting value of the blade length-diameter ratio z is fixed and W and v_1 are known, then for the chosen mean ρ value, the mean ring diameter at the first expansion can be calculated by equation (1).

The maximum value of D_1 is fixed by a length-diameter ratio of 3 per cent. ($z = 0{\cdot}03$). Below this maximum diameter a considerable latitude of choice is usually possible. This is purely a matter of judgment, and no definite rule can be formulated.

244. Alternatively, as a first step, the mean blade speed may be chosen and the diameter provisonally calculated from the equation

$$D_1 = \frac{60u}{\pi N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In this case equation (1) can be used to find B. The corresponding value of z is then obtained from the curves of Fig. 201.

The H.P. drum diameter is given by $D_H = D_1(1 - z)$. (3)

The blade speed u, in small and medium machines, may vary from 100 to 130 ft./sec., and in large machines from 130 to 170 ft./sec. Between these limits, it will be apparent that considerable variation in ring diameter is possible, at the first expansion.

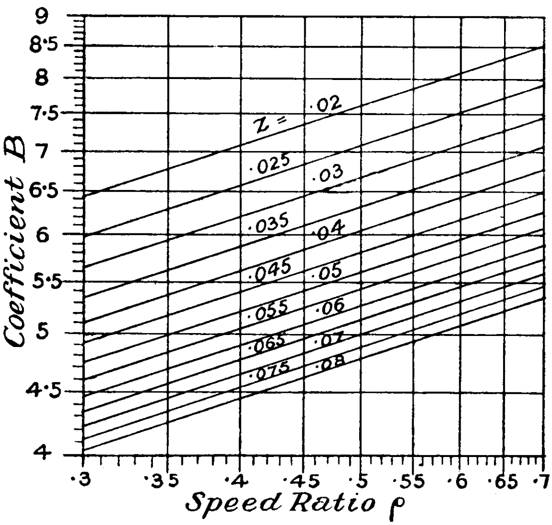


FIG. 201.

245. Calculation of L.P. Drum Diameter.—As in the case of the impulse turbine the maximum diameter of drum, at the L.P. end, is determined by the stress condition. For a given speed of revolution, as shown in Art. 126, the maximum diameter should have such a value that the mean speed of the blading is kept below 400 ft./sec. for mild, and 450 ft./sec. for nickel steel. In any case the maximum drum diameter should not exceed 8 feet 6 inches, as this is the practicable limit for transport by rail in this country.

On the other hand, the minimum limit of diameter is determined by the area for efficient discharge of the exhaust steam at the last blade ring. The length-diameter ratio z should not exceed 0·20. It should preferably be kept below 0·15.

In the majority of cases, with the relatively lower steam velocities used in the reaction turbine, the limitation of L.P. blade length involves the use of wide-angled blades, semiwing, wing, and, in some cases, double wing, at the last few expansions.

When the minimum L.P. drum diameter, determined by the condition for efficient discharge, is in excess of 8 feet 6 inches, a double flow arrangement has to be used, at the low pressure section. The drum has to be made of smaller diameter and greater length.

In order to calculate the minimum L.P. drum diameter, the equation already deduced for the impulse turbine can be used, that is

$$D_l = C\sqrt[3]{\frac{\rho W}{N}} \text{ feet}$$

In this case the actual speed ratio is used. In the case of the impulse machine the theoretical speed ratio was taken because the experimental curves were originally plotted on this base. There are no experimental records for reaction turbines.

The values of C given by the curve of Fig. 198 correspond to a standard nozzle angle of 20°. This is the normal value of the fixed (nozzle) and moving (nozzle) blade angle θ in the reaction machine, except at the low pressure end where the angle may vary from 30° to 50°.

Such increased angle, as already shown by the example 4 on p. 176, and in Art. 257, reduces the value of the speed ratio to $\rho' = \rho\frac{\cos\theta'}{\cos\theta}$, where θ is the normal angle and θ' the increased angle. Also, as in the case of the impulse machine, the value of the coefficient C is modified to

$$C' = C\sqrt[3]{\frac{\sin\theta}{\sin\theta'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By assuming a trial value of the blade angle θ at the least stage and taking C from the curves of Fig. 198, the values of ρ' and C' are found and the approximate minimum L.P. ring diameter is calculable from

$$D_l = C'\sqrt[3]{\frac{\rho' W}{N}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The corresponding L.P. drum diameter is given by

$$D_L = D_l(1 - z) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Between this and the maximum value any diameter may be arbitrarily chosen according to judgment. The drum is always stepped so that the diameters are in geometrical progression. Thus, if there are four steps, the common ratio is $r_1 = \sqrt[3]{\frac{D_L}{D_H}}$. If there are three, $r_1 = \sqrt[2]{\frac{D_L}{D_H}}$.

The common ratio thus being determined, the progressive diameters are calculable.

246. Instead of proceeding in the manner outlined above, a provisional value of the common drum ratio r_1 may first be chosen, and the L.P. diameter calculated from the relation

$$D_L = D_H r_1^{m-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where m = number of steps on the drum.

For the given weight of steam to be passed, the limiting blade length can be calculated by equation (5).

If this is not satisfactory the diameter may be modified.

It is apparent, whatever method is used, that, just as in the case of the impulse turbine, the determination of practicable rotor diameters to meet the conditions of any particular design is, to a great extent, a matter of individual judgment.

Further, it sometimes happens, that for the specified conditions of operation it may be possible to use the patterns of a previous job with a little alteration. Circumstances of this kind practically fix the leading dimensions of the drum, and confine the other adjustments principally to the blading design.

247. Calculation of Successive Blade Lengths.—When the drum diameters have been settled the successive series of expansion blade lengths on each section can be calculated.

It has been shown in Chap. X., Art. 157, that the equation for the flow of W lbs. through any blade ring of a group, when the clearance effect is taken into account, is

$$W = \frac{\pi D(l + ac) \sin \theta V_0}{144v}$$

so that the blade length is given by

$$l = \frac{144Wv}{\pi D \sin \theta V_0} - ac \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation does not allow for any reduction of effective area at exit, due to blade thickness. It is usually assumed that there is a slight reduction of area as in the case of the impulse turbine nozzle and wheel blades; but considering the special form of the Parsons blade surfaces, at the face and back, which tends to produce convergency of the stream lines at exit (see Fig. 122), it is doubtful if there is any such reduction.

With the usual assumption, the length given by equation (8) is on the small side. By neglecting the term ac, which accounts for tip leakage, a slight offset against diminution of area is obtained at the H.P. end. It is, in any case, a small quantity, and at the L.P. end is negligible. Also since the standard blade lengths, for commercial reasons, progress by $\frac{1}{8}$ inch, it becomes of minor importance in this equation, and may be neglected.

248. The exit velocity of the steam corresponding to the chosen blade velocity u and speed ratio ρ, is $V_0 = \frac{u}{\rho}$.

Since $u=\frac{\pi DN}{12 \times 60}$, where D is the mean blade ring diameter in inches, and N is the revolutions/min., then $V_0=\frac{\pi DN}{12 \times 60 \times \rho}$. Substituting in equation (8) the blade length is given by

$$l=\frac{Wv\rho \times 720 \times 144}{\pi^2 D^2 N \sin\theta}=\frac{10500 Wv\rho}{D^2 N \sin\theta} \quad . \quad . \quad . \quad (9)$$

The normal angle, except at the last one or two expansions, is $\theta \doteqdot 20^\circ$, and the value of $\sin\theta$ may be replaced by the factor $\frac{1}{3}$, hence the blade length may be taken as

$$l=\frac{31500 Wv\rho}{D^2 N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For any given section of the drum H.P., I.P., or L.P., this can be further reduced to

$$l=\frac{bv}{D^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where

$$b=\frac{31500 W\rho}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The value $\frac{1}{3}$ introduced in place of $\sin\theta$ in the equation (which is simply the equation of continuity in a modified form) is usually called the "annular area coefficient." It is the factor which reduces the annual area between the rotor and casing to the nett area for flow at any blade ring, making slight allowance for the clearance effect.

In applying equation (11) to determine the blade length at any expansion, v is the mean volume value given by the mid-ordinate of the volume curve between the limiting values for the group. The speed ratio ρ is the mean value for the expansion, and is arbitrarily chosen.

249. A direct solution cannot, however, be obtained until the correct value of the mean ring diameter D is known, and this is dependent on the value of l, the quantity required.

A trial and error process has to be employed.

An approximation to the probable blade length is chosen, and added to the drum diameter in order to obtain D for insertion in equation (10).

If the value of l, as calculated for this diameter, does not closely approach the assumed value, the calculation has to be repeated with an altered value of l. This process may appear troublesome, but a little practice will show that the trouble is more apparent than real. Further, it is not, as a rule, necessary to calculate the expansion blade lengths successively. When this process is employed it will be found that the lengths so obtained are in approximately geometrical progression. If the initial and final heights on the H.P. and I.P.

sections are calculated by the foregoing trial and error method to conform to the volume conditions, the intermediate heights can then be found with sufficient accuracy from the common ratio r, obtained from the first and last expansion lengths. Thus if l_1 is the length of the first and l_m of the last or mth expansion

$$r^{m-1} = \frac{l_m}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The consecutive expansion lengths are then given by

$$l_1, \quad rl_1, \quad r^2l_1, \quad r^3l_1, \text{ etc.}$$

In the case of the L.P. section, the last two expansions have usually the same blade length.

To determine the common ratio r for the L.P. section, the first and the second last lengths may be used.

In any case, in order to find an approximation to the length of the mth expansion on the section, the corresponding value of the total volume ratio $\frac{v_m}{v_1}$ can be used.

Thus $l_m \doteq \left(\frac{v_m}{v_1}\right) l_1$, where l_1 is the first expansion blade length. This value for the L.P. drum will be in excess of the true blade height, and a lower value has to be chosen, to obtain the ring diameter D for insertion in equation (11).

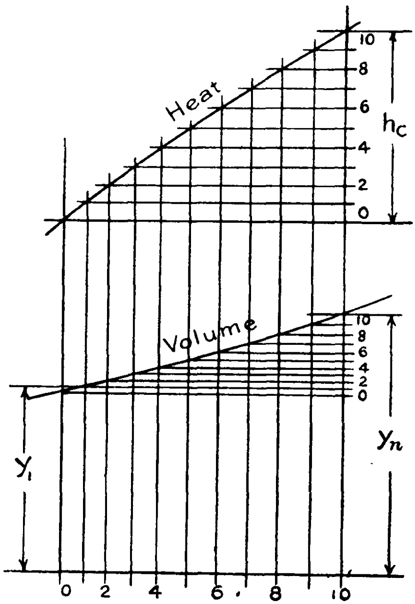
Frequently the blade lengths are increased in geometrical progression throughout the turbine, one arbitrarily chosen value of r being used. It varies from 1·3 to 1·4, or $\sqrt{2}$. This method is convenient where a rapid preliminary estimate of blade dimensions is required. It neglects the probable volumes at the various sections of the machine, which ought to be taken properly into account if a reasonable adjustment of areas, to suit the conditions of operation, is to be obtained. The final adjustment of the lengths should be made either by the above method or some equivalent method, based on the ascertained volume values at each expansion.

When the small amount of preliminary calculation and plotting for the H_c—v diagram is done, the necessary information regarding the volumes is ready to hand.

250. Gauging of Blade Angles throughout an Expansion.— When it is desired to maintain a constant value of speed ratio ρ from the first to the last stage of any expansion, and the constant length of this is calculated for a constant value of the angle, the blades have to be gauged, that is, the angle θ has to be progressively increased throughout the group.

Referring to Fig. 202, which shows the portion of the cumulative heat and volume curves lying between the pressure limits of a given expansion, the total cumulative heat h_c of the expansion can be divided into the same number of parts as there are rings in the group. In this case a ten-ring group of fixed and moving rings is shown.

Projectors to the heat curve determine the volume values at exit from the successive moving rings from 1 to 10. For constant speed ratio ρ, the steam velocity is constant, since the length and consequently the mean blade velocity is constant. With constant length, the width of passage at exit, which is proportional to the sine of the angle, has to increase directly as the volume. The sine of each increased blade angle has thus to be increased in approximately the same ratio as the successive volume ordinates. These may be scaled



10 Ring Expansion.

FIG. 202.

from the curve, but the preferable method is to take the total ratio $\left(\frac{y_n}{y_1}\right)$ and find what common ratio r corresponds to the number of fixed and moving rings in the group. Thus $\left(\frac{y_n}{y_1}\right) = r^{n-1}$, where n is the total number of rings. In the case illustrated, $n = 10$, and $\left(\frac{y_5}{y_1}\right) = 1{\cdot}42$. Hence, $r = \sqrt[9]{1{\cdot}42} = 1{\cdot}04$.

The middle ring of the group may be taken as that where the normal angle θ, used in the previous case, is retained. Denoting this generally by θ_n, the angle of the next blade ring towards the L.P. end is θ_{n+1}, and towards the H.P. end θ_{n-1}.

Hence, reckoned from the mid-ring towards the L.P. end the conditions for the calculation of the angles are, $\sin \theta_{n+1} = r \sin \theta_n$; $\sin \theta_{n+2} = r^2 \sin \theta_n$, and so on. Reckoned towards the H.P. end, these are, $\sin \theta_{n-1} = \frac{1}{r} \sin \theta_n$; $\sin \theta_{n-2} = \frac{1}{r^2} \sin \theta_n$, and so on.

Thus, in the case illustrated in Fig. 202, if $\theta_n = \theta_6 = 20^\circ$, then with $r = 1{\cdot}04$, the consecutive angles work out as follows :—

$\theta_1 = 16^\circ\ 20'$	$\theta_6 = 20^\circ\ 0'$
$\theta_2 = 17^\circ\ 0'$	$\theta_7 = 20^\circ\ 50'$
$\theta_3 = 17^\circ\ 42'$	$\theta_8 = 21^\circ\ 42'$
$\theta_4 = 18^\circ\ 26'$	$\theta_9 = 22^\circ\ 37'$
$\theta_5 = 19^\circ\ 12'$	$\theta_{10} = 23^\circ\ 34'$

Actually, in "gauging" the blades in the shop, the blader does not work to any angle values, but uses a gauging templet, which gives the opening between the blades at any ring, corresponding to the specified angle.

251. Calculation of First Blade Length at Enlarged Drum Diameter.—Where the drum is stepped up from H.P. to I.P. or I.P. to

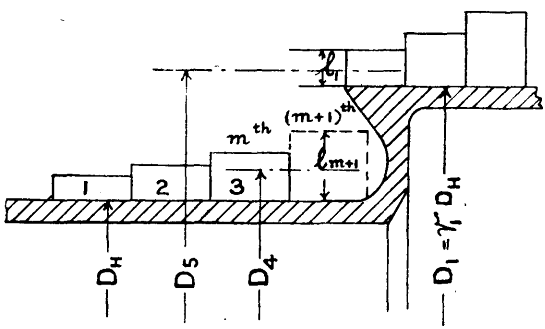


FIG. 203.

L.P. diameter, the first blade length on the enlarged drum is reduced below the value it would have if the diameter were kept constant.

Suppose, to fix ideas, that the first expansion length is required on the I.P. section. Let the number of expansions on the H.P. section be denoted by m. In the illustration, Fig. 203, $m = 3$.

If another group were added on the H.P. section, this would be the $(m+1)$ expansion. As a first approximation its height would be $l_{m+1} = rl_m$, where r is the common ratio for the H.P. expansion lengths, and l_m the mth expansion length.

Now, instead of placing this expansion on the drum of diameter D_H, let it be placed on the drum of larger diameter $D_I = r_1 D_H$, and given a length l_1 (the value to be determined).

By this change the mean ring diameter is increased from D_4 to D_5. Since the blade lengths are relatively small fractions of the diameters, then, as a first approximation, the ratio of the enlarged to the original ring diameter may be taken the same as that of the drum diameters, that is, $D_5 \doteqdot r_1 D_4$. For the maintenance of the same value of the speed ratio ρ, as in the previous section, the steam velocity will also be increased to approximately $r_1 V_0$, as the volume passing the ring of enlarged diameter is the same as that for the smaller ring. Hence, by the equation of continuity

$$\pi r D_4 l_m V_0 = \pi r_1 D_4 l_1 r_1 V_0$$

and

$$l_1 = \frac{r l_m}{r_1^2} = \frac{l_{m+1}}{r_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

The provisional value can then be used to fix the value of D_5, for a check calculation, by equation (11).

A similar process can be applied to the first expansion of the L.P. section.

252. Stepping of the Drum.—Before proceeding to the final portion of the blading calculation, that is, the determination of the number of stages or number of moving rings in the successive groups, it is desirable to indicate the reasons for the customary stepping of the drum in the land turbine.

A glance at the volume curve, Fig. 200, shows that if the drum were made the same diameter throughout, the blade lengths would become quite impracticable towards the L.P. end of the machine. By stepping the drum a reduction in the absolute values of the blade lengths is obtained, without any alteration of their relative values. By adding a second step to the drum a further reduction is obtained, giving reasonable proportions at the L.P. end.

There is, however, a second and equally important advantage gained by the stepping.

At any blade ring the work done on the ring is proportional to the square of the mean blade velocity u, that is

$$E_b \propto u^2$$

If the mean ring diameter D of a group is increased to a value r_1D, and E_{b1} is the work now done per ring, then

$$E_b \propto r_1^2 u^2$$

With the same number of blades, n, per group, the work done is increased r_1^2 times.

For the performance of the original amount of work by the group of enlarged diameter a smaller number of rings n_1 is required, and

$$n_1 E_{b_1} = n E_b$$

hence

$$n_1 = \frac{n}{r_1^2}$$

For example, suppose $r_1 = \sqrt{2}$, then the number of moving rings on the I.P. drum would be reduced to $\frac{1}{r_1^2}$, or 50 per cent. of the number required if the drum were kept the same diameter as the H.P. section. Similarly the number on the L.P. enlarged drum would be 50 per cent. of that required, if the drum were kept the same diameter as the I.P. one.

If the ratio were $r_1' = 1{\cdot}3$, the proportions would be 45 instead of 50 per cent.

The result of stepping the drum is thus twofold. There is a reduction in blade length, and a reduction in the number of stages (and hence of moving rings) by which the lengths of the cylinder and rotor, and hence the size and cost of the turbine, are reduced.

253. Calculation of Number of Rings in Each Expansion.—The number of moving rings (or double stages) in any expansion is readily calculated from the energy equation, when the cumulative heat allotted to the expansion and the mean speed ratio ρ are known.

Let h_c = cumulative heat of the expansion,

n = number of moving rings or double stages of the expansion, then the average heat drop for each "double stage" of the group is

$$h_r = \frac{h_c}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

From what has already been shown in Art. 97, if γ is the ratio of the actual working steam to total steam passing through the expansion, and η_s is the mean stage efficiency for the group, it follows that

$$n\gamma u^2\left(2\frac{\cos\theta}{\rho} - 1\right) = h_c J \eta_s g$$

Hence
$$n = \frac{J\eta_s h_c g}{\gamma u^2\left(2\dfrac{\cos\theta}{\rho} - 1\right)} \quad . \quad . \quad . \quad (16)$$

Taking the condition of equal heat drop throughout, then h_c is the same for all the expansions on a given section. On this hypothesis, equation (16) can be reduced to the simple form

$$n = \frac{\text{const. } \eta_s}{\gamma u^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The value of the weight ratio γ may have an appreciable effect at the H.P. section. It has less effect at the I.P., and towards the end of the L.P. section it is practically negligible. It can, if desired, be omitted for all the sections, the result being a slightly reduced value for the number of rings at the H.P. section. Since, however, its value for any case is obtained by inspection from the curves of Fig. 160, p. 307, no extra work is involved. It is, therefore, desirable, in each instance, to introduce it in the equation for all the sections, when a final adjustment of blade ring numbers is made.

The foregoing is a direct method of determining the number of blade rings, in which the values of the essential factors of the problem are known for any given section.

254. Coefficient Method of Calculating the Number of Rings.—There is another method by which a rapid estimate of the approximate number of rings on any section of the axial flow turbine can be made.

It involves the use of a certain coefficient, the precise significance of which does not appear to be quite understood by some designers, who usually "guess" its value for a given case.

The value is directly calculable when the heat drop, the reheat factor, and the speed ratio (assumed constant for the whole machine) are known.

It is obtained as follows. Consider the mid pair of rings in an ungauged group, one fixed and the other movable. The pair constitutes a "double stage."

Let V_1 = carry-over velocity from previous moving ring.
V_0 = exit velocity from the fixed ring.
ρ = speed ratio.
h_r = heat drop for the double stage, assumed equally divided between fixed and moving ring.

These represent the average values for the group.

Then the heat drop for the fixed ring is $\frac{h_r}{2}$. The actual amount of this heat drop converted into mechanical energy at exit is some value $M\frac{h_r}{2}$, where M is a friction coefficient. The corresponding proportion of the initial K.E. at entrance, which appears at exit, will be some value, $m\frac{V_1^2}{(223{\cdot}7^2)}$, where m is another friction coefficient (see also Art. 96). Then from the energy equation

$$\frac{V_0^2}{(223{\cdot}7)^2} = M\frac{h_r}{2} + m\frac{V_1^2}{(223{\cdot}7)^2} \quad . \quad . \quad . \quad . \quad (18)$$

There are no direct experimental values for M and m, but Martin states that from the results of an analysis of tests on a large marine turbine, it would appear that $M = 0{\cdot}9$ and $m = 0{\cdot}52$ may be taken as average values for this type of machine.

Expressing the carry-over velocity V_1 as a fraction of the velocity of exit V_0, that is, $V_1 = zV_0$, the energy equation (18) for the pair of rings may be written in the form

$$\frac{V_0}{(223{\cdot}7)^2}(1 - mz^2) = M\frac{h_r}{2} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This expression is similar to that deduced for the impulse stage in Art. 96.

But $$V_0 = \frac{u}{\rho} = \frac{\pi DN}{\rho \times 60 \times 12}$$

where D = mean ring diameter in inches,
N = revolutions/min.

Also the average value of the heat drop for the group is

$$h_r = \frac{h_c}{n}$$

Hence, substituting for V_0 and h_r in equation (19), and reducing, the result can be written in the form

$$\rho^2 = \frac{2nD^2N^2(1 - mz^2)}{(51270)^2 h_c M} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Here
$$\frac{2}{(51270)^2} = \frac{2}{2628} \times \frac{1}{10^6}$$

so that
$$\frac{2nD^2N^2}{(51270)^2} = \frac{nD^2N^2}{10^6} \times \frac{1}{1314} = \frac{\lambda_1}{1314}$$

Let
$$\left(\frac{M}{1 - mz^2}\right) = \delta$$

then substituting in equation (20)

$$\rho^2 = \frac{\lambda_1}{1314 h_c \delta} = \frac{1}{1314\delta} \times \frac{\lambda_1}{h_c}$$

$$1314\delta\rho^2 h_c = \lambda_1$$

Since ρ is constant, then for the whole machine

$$1314\delta\rho^2 \Sigma h_c = \Sigma\lambda_1$$
$$1314\delta\rho^2 H_c = \lambda$$

where λ is the corresponding coefficient for all the groups of blades, $H_c = RH_r$, where R is the reheat factor and H_r the heat drop between initial and exhaust pressures. Hence the value of λ is given by

$$\lambda = 1314\rho^2 RH_r\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

This is the design coefficient usually assumed for the calculation of the number of blade rings on the whole machine. It will be evident, from the explanation just given, that it is calculable for any given case, with a reasonable degree of accuracy, when the average value of the speed ratio ρ is decided.

The coefficient δ for a given value of angle and M and m can be calculated when the ratio z is obtained from the velocity diagram. The ratio can be read directly from the curves of Fig. 210, which are drawn for a series of ρ values and angles of 20°, 25°, and 30°.

The curve for 20° may be taken as the standard for this type of turbine, as this is the average normal angle. If $M = 0{\cdot}9$ and $m = 0{\cdot}52$ are also taken as the practicable averages, then the value of δ with $\theta = 20°$ for a series of ρ values can be plotted to give a standard coefficient curve as shown in Fig. 204.

The curve shows the effect of this factor on the value of the design coefficient λ for the customary range of ρ values at present used for the axial flow reaction machine. In the case of the land turbine, where ρ may vary from 0·55 to 0·6, δ appears to vary between 1·02 and 1·04. For marine turbines, where the speed ratio ρ may vary from 0·35 to 0·45, the variation appears to be between 1·1 and 1·18. Neglect of this factor may thus give a value of λ from 2 to 4 per cent. too low for the land and 10 to 18 per cent. for the direct-coupled marine turbine.

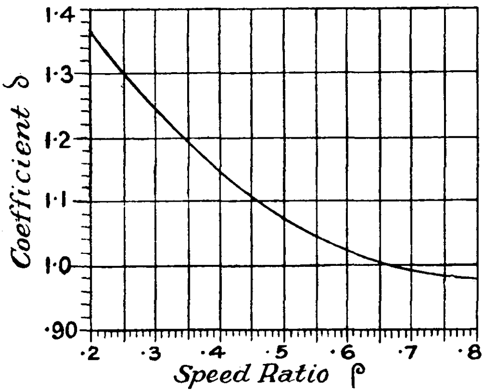


FIG. 204.

255. Application of λ to Calculate the Number of Moving Rings on any Section.—If the cumulative heat allotted to any section is $\frac{1}{x}$ of the total cumulative value RH_r for the whole machine, the coefficient for the particular section becomes $\frac{\lambda}{x}$. The number of moving rings on the section is given by

$$n = \frac{10^6\lambda}{xD^2N^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)^1$$

This number has to be divided up among the expansions on the section.

The I.P. and L.P. mean blade ring diameters can be expressed, approximately, in terms of the mean ring diameter D_1 of the H.P. section, and the common drum ratio r_1.

Thus, the average diameter of the I.P. expansions is, approximately, r_1D_1, and that of the L.P. expansions $r_1^2D_1$.

If the fractions of the total cumulative heat allotted to the H.P., I.P., and L.P. sections are respectively $\frac{1}{x_1}$, $\frac{1}{x_2}$, $\frac{1}{x_3}$, then from equation (22) the approximate number of moving rings on each section is

For H.P. $$n_1 = \frac{10^6\lambda}{x_1D_1^2N^2} = \frac{A}{x_1}$$

[1] A similar expression, taking account of carry-over between stages, may be used to calculate the number of stages, n, of the pressure compounded impulse machine.

$$n = \frac{10^6\lambda'}{D^2N^2}$$

where $\lambda' = \frac{2628RH_r\rho_t^2}{B}$ and $B = \frac{1 - mx^2}{M}$ (See Art. 96, p. 186.)

For I.P. $$n_2 = \frac{A}{x_2 r_1^2}$$

„ L.P. $$n_3 = \frac{A}{x_3 r_1^4}$$

Generally, at an mth drum, the value is given by

$$n_m = \frac{10^6 \lambda}{x_m D_1^2 N^2 r_1^{2(m-1)}} = \frac{A}{x_m r_1^{2(m-1)}} \quad . \quad . \quad . \quad . \quad (23)$$

where $$A = \frac{10^6 \lambda}{D_1^2 N^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

D_1 = mean ring diameter of the expansions on the H.P. section.
N = revolutions/min.

256. Calculation of Increased Blade Angle at the L.P. Stages.— At the last two or three expansions on the L.P. drum it is necessary, with the usual arrangement of ungauged blades, to progressively increase the group blade angle, in order to obtain sufficient area for flow with a reasonable blade length. The length is kept the same for each of these expansions.

Referring to Fig. 110, p. 175, which shows the velocity diagram for the mid pair of a group having a constant blade angle, since for reaction blading

$$V_{a_1} = V_0 \quad \text{and} \quad \alpha_1 = \theta$$

the velocity of whirl

$$V_{w_1} = V_0 \cos \theta \quad \text{or} \quad V_0 = \frac{V_{w_1}}{\cos \theta}$$

hence $$\rho = \frac{u}{V_0} = \frac{u \cos \theta}{V_{w_1}}$$

Substituting in equation (9), the blade length is given by

$$l = \frac{\text{const. } Wv}{D^2 N \sin \theta} \times \frac{u \cos \theta}{V_{w_1}}$$

where v is the volume of the steam at the middle of the group.

But l, W, u, D, and N are all constant for the L.P. groups having the same length, hence it follows that

$$\frac{v \cos \theta}{V_{w_1}} \propto \sin \theta$$

or $$v \propto V_{w_1} \tan \theta$$

Now, as can be seen from the dotted velocity triangles, there is an increase of the average exit velocity $V_{a_1} = V_0$, and a decrease of the average speed ratio ρ, if the velocity of whirl V_{w_1} is kept constant. The total change of velocity of whirl or V_w, and the blade velocity u remaining constant, the average work which is done per blade ring, with the increased group angle, is the same as the average work done per blade ring, with the normal group angle.

If then the blades of successive groups are to be arranged for this condition of equal work, that is with V_{w_1} constant, then

$$v \propto \tan \theta$$

If at the mth expansion on the L.P. section the normal group angle is denoted by θ_m and the mean volume from the $H_c - v$ diagram by v_m, and if for the $(m+1)$, $(m+2)$ expansions, having the same blade length as the mth, the corresponding subscripts are used

$$\left.\begin{array}{lll} \text{then at the } (m+1) \text{ expansion,} & & \tan \theta_{m+1} = \tan \theta_m \dfrac{v_{m+1}}{v_m} \\ \text{and} \quad ,, \quad (m+2) & ,, & \tan \theta_{m+2} = \tan \theta_m \dfrac{v_{m+2}}{v_m} \end{array}\right\} \quad . \quad . \quad (25)$$

and so on for any further extension of the number of expansions.

257. It should be noted that, although the average work done per ring is the same, the average heat drop per stage is increased, since V_0 is increased. For a given amount of cumulative heat in a group, the number of stages and number of moving rings, for ungauged blades, is less, with the increased than with the normal group angle.

This increase in velocity, as can be seen from the diagram, means an increased leaving velocity and residual loss or final carry-over to the condenser.

In making these adjustments of length and angle, care should be taken that the carry-over does not become too great a percentage of the heat drop. It should not exceed 3 per cent.

For the same average value of work per moving blade ring, in groups with normal and increased group angles, $V_w =$ constant, that is

$$2V_0 \cos \theta - u = \text{const.} \quad \text{But } V_0 = \frac{u}{\rho},$$

hence

$$2\frac{u}{\rho} \cos \theta - u = c, \quad \text{and} \quad \frac{\cos \theta}{\rho} = \text{const.}$$

Hence for any increased group angle θ', and reduced ratio ρ',

$$\frac{\cos \theta}{\rho} = \frac{\cos \theta'}{\rho'} \quad \text{or} \quad \rho' = \rho \frac{\cos \theta'}{\cos \theta} \quad . \quad . \quad . \quad (26)$$

258. When in the case of a machine of large output the maximum practicable diameter of 8 feet 6 inches is reached at the L.P. section of the drum, this portion has to be made for double flow.

For outputs from 15,000 to 25,000 K.W. requiring this modification it is the practice to use a tandem construction. All the power is transmitted by one shaft, but the cylinder and rotor are divided into H.P. and L.P. sections.

Several advantages are gained by this arrangement. A single cylinder would necessitate too great a span between the end bearings. The tandem arrangement enables a mid bearing to be placed between the cylinders, and a short and stiff rotor is obtained in each cylinder. Clearances can be made much finer than with the long rotor of wide span, and a better efficiency obtained from the H.P. section. Superheated steam can be used with less risk of distorsion trouble in the comparatively small H.P. cylinder. Cast steel can be used for the H.P. end, instead of cast iron, which at high temperatures is subject to "growth."

A further advantage is the increased facility for transport of the turbine, in sections, from the works to the place of installation. The adoption of the double flow L.P. section also decreases the residual loss to the condenser, as the blade angles, on account of the smaller steam way at each end, can be kept at a more reasonable figure, without unduly increasing the length of blade.

Against these advantages there have to be offset the increased bearing friction, gland losses, and the possibility of loss in the pipe connections between the cylinders.

In some cases the drum of the H.P. cylinder is made parallel, and the L.P. drum in two steps.

In this arrangement half the steam discharged from the first step of the L.P. rotor (the I.P. section) passes in at the I.P. end of the L.P. blading, while the other half passes through the interior of the L.P. drum and flows inwards again, through the L.P. blading, from the other end. The common exhaust of the two streams takes place at the centre of the L.P. blading.

In other cases the H.P. drum has two or three steps, and the L.P. drum is made parallel, constituting the third or fourth step of the complete rotor. This is the arrangement adopted in the 25,000 K.W. machines by the Parsons Company, the H.P. drum being made in three steps.

EXAMPLE 1.—Determine suitable diameters of drum and number and lengths of blade rings for a Parsons turbine to develop 3000 K.W. at 1500 R.P.M. Initial pressure 175 lbs./in.^{2} gauge, superheat 150° F., vacuum 28 inches, generator efficiency 0·95.

The turbine is to have a three-step drum, the mean value of the speed ratio is to be 0·55 throughout, and the power distribution $\frac{1}{4}$ in the H.P., $\frac{1}{4}$ in the I.P., and $\frac{1}{2}$ in the L.P. section.

Taking the mean value of the stage efficiency $\eta_s = 0{\cdot}75$, then from Fig. 175 at 150° F. superheat the reheat factor is $R = 1{\cdot}045$. Hence the probable internal efficiency is $\eta_1 = 0{\cdot}75 \times 1{\cdot}045 = 0{\cdot}785$. Using this value to determine the final state point on the Hϕ diagram, the quality at 1 lb./in.^{2} is found to be $q_0 = 0{\cdot}898$.

The condition curve being drawn, and the cumulative heats and volumes determined, the H_c—v diagram, Fig. 200, is then obtained.

The heat drop, from the Hϕ diagram, is $H_r = 365$, so that

$$H_c = 1{\cdot}045 \times 365 = 382$$

This heat is divided in the proportions specified for the design, that is, 95·5 in the H.P., 95·5 in the I.P., and 191 in the L.P. section.

Suppose an arbitrary choice of twelve expansions is made, then, with the power distribution specified, the numbers of expansions are, three on the H.P., three on the I.P., and six on the L.P. section (see Art. 241).

Taking an equal distribution of the cumulative heat of each section among the expansions of the section, the section portions of AB = H_c, are divided into as many equal parts as there are expansions. Projectors drawn from the points of division to the cumulative heat curve, divide the diagram into twelve parts, each corresponding to an expansion. The volumes scaled at the (dotted) mid-ordinates are as follows:—

	H.P.			I.P.			L. P.					
Expansion .	1.	2.	3.	1.	2.	3.	1.	2.	3.	4.	5.	6.
Volume . .	3·4	4·6	6·2	8·5	11·7	16·2	25·5	38	58	90	145	240

Allowing for total outside loss and partial dummy loss, say, 9 per cent. of the heat drop, the approximate value of the efficiency ratio is $\eta = (\eta_1 - 0{\cdot}09) = (0{\cdot}785 - 0{\cdot}09) = 0{\cdot}69$.

Steam consumption

$$w = \frac{3414}{\eta H_r \eta_g} = \frac{3414}{0{\cdot}69 \times 365 \times 0{\cdot}95} = 14{\cdot}25$$

Total consumption

$$W_1 = \frac{14{\cdot}25 \times 3000}{3600} = 11{\cdot}87 \text{ lbs./sec.}$$

Assuming the H.P. dummy leak is 5 per cent. (see example 5, p. 286), the probable weight of steam entering the first expansion is

$$W = 0{\cdot}95 \times 11{\cdot}87 = 11{\cdot}3 \text{ lbs./sec.}$$

To find the mean ring diameter of the first H.P. expansion, take the minimum blade-diameter ratio $z = 0{\cdot}03$.

With $\rho = 0{\cdot}55$, the value of B from Fig. 201 at 28 inches vacuum is 6·9. From the data table the mean volume for the first expansion is $v_1 = 3{\cdot}4$.

Mean diameter of first expansion, by equation (1), is

$$D_1 = B\sqrt[3]{\frac{Wv_1}{N}} = 6{\cdot}9\sqrt[3]{\frac{11{\cdot}3 \times 3{\cdot}4}{1500}} = \frac{6{\cdot}9}{3{\cdot}4} = 2{\cdot}03 \text{ feet}$$

Blade length $l = 24{\cdot}36 \times 0{\cdot}03 = 0{\cdot}73$, and the nearest standard of $\frac{3}{4}$ inch would be fitted.

The corresponding drum diameter by equation (3) is

$$D_H = 2{\cdot}03 \times 0{\cdot}97 = 1{\cdot}97 \text{ feet}$$

The L.P. diameter can be found next. It may be assumed that the blade lengths of the last two expansions at least will require to be kept the same, and the increased angle θ_1 at the last expansion may be, say, over 29°.

With this tentative value

$$C' = C\sqrt{\frac{\sin\theta}{\sin\theta'}} = C\sqrt[3]{\frac{0{\cdot}342}{0{\cdot}485}} = \frac{C}{1{\cdot}123}$$

and $$\rho' = \rho\frac{\cos\theta'}{\cos\theta} = 0{\cdot}55 \times \frac{0{\cdot}875}{0{\cdot}94} = 0{\cdot}512$$

The L.P. dummy leak may be taken as 3 per cent. (see example 7, p. 287), and the probable weight passing the L.P. section is

$$W = 0{\cdot}97 \times 11{\cdot}87 = 11{\cdot}5 \text{ lbs./sec.}$$

Assume for the practicable limit of blade length at the last ring, $z = 0{\cdot}15$, then from Fig. 198, at this value, $C = 33$. By equation (5) the mean ring diameter is

$$D_l = C'\sqrt[3]{\frac{\rho' W}{N}}$$

$$= \frac{33}{1{\cdot}123}\sqrt[3]{\frac{0{\cdot}512 \times 11{\cdot}5}{1500}} = \frac{33}{6{\cdot}34 \times 1{\cdot}123} = 4{\cdot}65 \text{ feet}$$

The L.P. drum diameter $D_L = 4{\cdot}65 \times 0{\cdot}85 = 3{\cdot}96$ feet.

The total ratio of the H.P. and L.P. drum diameter is thus

$$\frac{D_L}{D_H} = \frac{3{\cdot}96}{1{\cdot}97} \doteq 2$$

so that the common ratio for the three diameters in geometrical progression is

$$r_1 = \sqrt[2]{\frac{D_L}{D_H}} = \sqrt{2}$$

This is the value most commonly used, when an arbitrary choice of this factor is made first.

The drum diameters thus provisionally fixed are

$$D_H = 1{\cdot}97 \text{ ft.} = 23\tfrac{5}{8}'' : D_I = 2{\cdot}782 \text{ ft.} = 33\tfrac{3}{8}'' : D_L = 3{\cdot}96 \text{ ft.} = 47\tfrac{3}{4}''$$

The surface velocity at the L.P. drum is only 312 ft./sec., a value well below the permissible maximum for mild steel.

The progressive blade lengths of the expansions can now be calculated.

At the I.P. section a dummy leak of, say, 4 per cent. may be allowed for, and the values of W for the three sections may be thus taken as 11·3, 11·4, 11·5 lbs./sec.

The corresponding values of the coefficient b by equation (12) are 130, 132, 134.

At the H.P. drum the total volume ratio is $\dfrac{6{\cdot}2}{3{\cdot}4} = 1{\cdot}83$.

The approximate blade length for the third expansion is thus

$$l_3 = 1{\cdot}83 l_1 = 1{\cdot}83 \times \tfrac{3}{4} = 1{\cdot}37, \text{ say } 1\tfrac{3}{8} \text{ inches}$$

This gives a mean ring diameter $D_1 = 25$ inches.

Checking the value of the blade length by equation (11)

$$l_3 = \frac{bv_3}{D^2} = \frac{130 \times 6{\cdot}2}{25^2} = 1{\cdot}3$$

The nearest standard blade is $1\frac{3}{8}$, so that this is the suitable value to use.

Increasing the three lengths in geometrical progression the second expansion length is $l_2 = \sqrt{1{\cdot}375 \times 0{\cdot}75} \doteqdot 1$ inch, to the nearest $\frac{1}{8}$ inch. The common ratio for the H.P. expansions is thus $r = 1{\cdot}375$.

Taking the I.P. blading next, since the common ratio for the drums is $r_1 = \sqrt{2}$, the approximate blade length of the I.P. expansion, by equation (14), is

$$l_1 = \frac{l_m r}{v_1^2} = \frac{1{\cdot}375 \times 1{\cdot}375}{2} = 0{\cdot}95$$

The nearest standard is 1 inch. This makes the mean diameter $34\frac{3}{8}$ inches. Checking this length by equation (11)

$$l_1 = \frac{bv_1}{D_1} = \frac{132 \times 8{\cdot}5}{(34{\cdot}375)^2} = 0{\cdot}95$$

so that 1 inch is satisfactory. The total volume ratio for the I.P. section is $\frac{16{\cdot}2}{8{\cdot}5} = 1{\cdot}91$, hence the approximate blade length of the third I.P. expansion is $l_3 = 1{\cdot}91 \times 1 \doteqdot 2$ inches, giving a ring diameter of $35\frac{3}{8}$ inches.

Checking

$$l_3 = \frac{132 \times 16{\cdot}2}{(35{\cdot}375)^2} = 1{\cdot}72$$

instead of 2 inches. The value may be taken between $1\frac{3}{4}$ inches and 2 inches, say, the next standard of $1\frac{7}{8}$ inches.

The second expansion blade length is $l_2 = \sqrt{1{\cdot}875} \times 1 = 1{\cdot}37$, and the next standard of $1\frac{3}{8}$ inches may be taken. The common ratio is $r = 1{\cdot}37$.

Taking the L.P. blading, the blade length of the first expansion is $l_1 = \frac{1{\cdot}875 \times 1{\cdot}37}{2} = 1{\cdot}3$; and taking the nearest standard $1\frac{3}{8}$ inches the ring diameter is $49\frac{1}{8}$ inches.

Checking

$$l_1 = \frac{134 \times 25{\cdot}5}{(49{\cdot}125)^2} = 1{\cdot}42$$

The $1\frac{3}{8}$-inch blade is thus on the low side, and the next standard of $1\frac{1}{2}$ inches should be fitted.

Assuming the blade length of the fifth and sixth expansions to be the same, the volume ratio for a trial may be taken for the first and fifth expansions. This is $\frac{145}{25{\cdot}5} = 5{\cdot}7$. This ratio would give $l_5 = 5{\cdot}7 \times 1{\cdot}5 = 8{\cdot}5$ inches, an amount in excess of what the practicable length-diameter ratio ought to give. The ring diameter corresponding to it is $56\frac{1}{4}$ inches.

Inserting this in equation (11)

$$l_5 = \frac{134 \times 145}{(56{\cdot}25)^2} = 6{\cdot}15$$

The actual value should lie somewhere between $6\frac{1}{4}$ and 7 inches. Try $6\frac{3}{4}$ inches, which gives a ring diameter of $54\frac{1}{2}$ inches.

Then $$l_5 = \frac{134 \times 145}{(54\cdot5)^2} = 6\cdot6, \text{ say } 6\tfrac{5}{8} \text{ inches}$$

The common ratio for the geometrical progression of the lengths between expansions 1 and 5 is given by equation (13), $r^{m-1} = \dfrac{l_m}{l_1}$. Here $m = 5$,

$$r^4 = \frac{6\cdot625}{1\cdot5}$$

and $$\cdot r = \sqrt[4]{4\cdot41} = 1\cdot45$$

With this ratio the length for the sixth expansion would be $9\frac{5}{8}$ inches, a value in excess of the chosen limit of 15 per cent. of the mean diameter. The assumption that the last two lengths should be kept the same is thus justified.

The angle of the sixth expansion blading has to be opened out

Here $\theta_5 = 20°$ and $\tan\theta_5 = 0\cdot364$ also $\dfrac{v_6}{v_5} = \dfrac{240}{145}$

Hence by equation (25)

$$\tan\theta_6 = 0\cdot364 \times \frac{240}{145} = 0\cdot6 \quad \text{and} \quad \theta_6 = 31°$$

The blades are semiwing at the last expansion.

The set of L.P. blade lengths, having the common ratio $r = 1\cdot45$, to the nearest $\frac{1}{8}$ inch are

$1\frac{1}{2}''$, $2\frac{1}{4}''$, $3\frac{1}{4}''$, $4\frac{5}{8}''$, $6\frac{5}{8}''$, $6\frac{5}{8}''$.

A progressive application of equation (11) to each expansion may show some inequality of blade lengths with the above values. It is desirable, however, to adhere to a uniform increase of length as given above. Further, the amount of calculation is reduced. The final ratio of blade length to mean ring diameter is now 12·2 per cent., instead of 15 per cent. originally assumed.

It is the custom, in this type of machine, to take an overload by admitting steam directly to the second expansion. The additional steam thrust thus produced is balanced by swelling the drum at the first expansion, so that the mean ring diameter is the same as that of the second expansion.

In this case the common ring diameter would be $24\frac{5}{8}$ inches, and the enlarged drum diameter $23\frac{7}{8}$ inches.

The first blade length should be slightly reduced, but it is not worth doing so, as the effect on ρ, when $l_1 = \frac{3}{4}$ is retained, is inappreciable.

The necessary information is now available for the calculation of the numbers of moving rings in each expansion.

The rest of the calculation can be most expeditiously done in tabular form, as shown below.

In the calculation of the number of rings the values of the weight ratio γ are read from the curves of Fig. 160, p. 307, and entered on the table. The mean blade velocity is calculated from $u = \dfrac{\pi DN}{60}$.

	High Pressure. Diam. of Drum, $23\frac{5}{8}$ inches.			Intermediate Pressure. Diam. of Drum, $33\frac{3}{8}$ inches.		
	No. of Expansion.			No. of Expansion.		
	1.	2.	3.	1.	2.	3.
Ring diam., D ins.	$24\frac{5}{8}$	$24\frac{5}{8}$	25	$34\frac{3}{8}$	$34\frac{3}{4}$	$35\frac{1}{4}$
Blade length, l ins.	$\frac{3}{4}$	1	$1\frac{3}{8}$	1	$1\frac{3}{8}$	$1\frac{7}{8}$
Blade vel., u ft./sec.	161	161	164	224	227	231
Coefficient, γ	0·905	0·923	0·943	0·908	0·932	0·952
No. of moving rings, n.	(9·85)	(9·7)	(9·1)	(5·45)	(5·15)	(4·9)
	10	10	9	5	5	5

Low Pressure. Diam. of Drum, $47\frac{3}{4}$ inches.

	No. of Expansion.					
	1.	2.	3.	4.	5.	6.
Ring diam., D ins.	$49\frac{1}{4}$	50	51	$52\frac{3}{8}$	$54\frac{3}{8}$	$54\frac{3}{8}$
Blade length, l ins.	$1\frac{1}{2}$	$2\frac{1}{4}$	$3\frac{1}{4}$	$4\frac{5}{8}$	$6\frac{5}{8}$	$6\frac{5}{8}$
Blade vel., u ft./sec.	322	327	333	342	355	355
Coefficient, γ	0·922	0·948	0·966	0·973	0·98	0·98
No. of moving rings, n.	(2·76)	(2·62)	(2·46)	(2·32)	(2·14)	(2·14)
	3	3	3	2	2	2

In equation (17)

$$\text{constant} = \frac{Jh_c g}{2\dfrac{\cos\theta}{\rho} - 1}$$

Since the cumulative heat is $H_c = 382$, and an equal distribution is made throughout twelve expansions, the cumulative heat per expansion is $h_c = \frac{382}{12} = 31\cdot8$, B.Th.U. Also $\theta = 20$, and $\rho = 0\cdot55$. Hence

$$\text{constant} = \frac{778 \times 31\cdot8 \times 32\cdot2}{\dfrac{2 \times 0\cdot94}{0\cdot55} - 1} = 330000$$

A reasonable value of the mean stage efficiency η_s has to be chosen for each of the three sections.

It may be taken as 0·7 for the H.P., 0·75 for the I.P., and 0·8 for the L.P. When these values are substituted with γ and u in equation (17).

$$n = \frac{\text{constant} \times \eta_s}{\gamma u^2}$$

the number of rings on each section, given in the last line of the table, are obtained.

Thus, at the first H.P. expansion

$$n_1 = \frac{330000 \times 0{\cdot}7}{0{\cdot}905 \times 161^2} = 9{\cdot}85$$

The nearest whole number (10) is chosen and entered below this figure. The other numbers are treated in the same way. The numbers are so adjusted that the sum is approximately the same as that of the calculated values. For instance, at the H.P. section, the sum of the calculated numbers is 29, and the number for the third expansion is taken as 9 instead of 10. This makes the actual sum 29, the same as the calculated value.

The final result of the blading calculation is 29 rings on H.P., 15 on I.P., and 15 on L.P. sections of the drum, giving a total of 59 moving rings for the whole machine.

Application of the Coefficient Method.—Instead of tabulating values as in the foregoing calculation, the approximate number of moving rings on each section of the drum may be obtained by the coefficient method, as follows.

$RH_r = 382$, $\rho = 0{\cdot}55$, and the probable value of the factor δ, from the curve of Fig. 204, is 1·04, hence by equation (21)

$$\begin{aligned}\lambda &= 1314\rho^2 RH_r\delta \\ &= 1314 \times 0{\cdot}55^2 \times 382 \times 1{\cdot}04 \\ &= 158000\end{aligned}$$

Also $\frac{1}{x_1} = \frac{1}{4}$, $\frac{1}{x_2} = \frac{1}{4}$, $\frac{1}{x_3} = \frac{1}{2}$, and the common drum ratio $r_1 = \sqrt{2}$

The mean blade ring diameter of the three H.P. expansions may be taken as $D_1 = 24\frac{3}{4}$ inches, and $N = 1500$.

By equation (24)

$$A = \frac{10^6 \times 158000}{(24{\cdot}75 \times 1500)^2} \doteq 115$$

Hence by equation (23)

No. of rings on H.P. section, $n_1 = \frac{A}{x_1} = \frac{115}{4} = 28{\cdot}75$, say 29

" " I.P. " $n_2 = \frac{A}{x_2 r_1^2} = \frac{115}{4 \times 2} = 14{\cdot}4$, say 15

" " L.P. " $n_3 = \frac{A}{x_3 r_1^4} = \frac{115}{2 \times 4} = 14{\cdot}4$, say 15

As might be expected, the total number given by the sum of the calculated values is 58 approximately, slightly less than the previously calculated value.

Taking the nearest whole number above the calculated value in each case, the numbers become the same as those calculated directly from the energy equation, and the total again is 59.

With this method of calculation, the number of rings in each section has to be divided by the number of expansions. If the number per expansion, thus obtained, is not a whole number, the value for each expansion has to be adjusted to obtain this result. The smaller value should be added to the last expansion on the section.

Thus taking the H.P. section, the number per expansion is $\frac{29}{3} = 9{\cdot}666$, and obviously two 10's and one 9 should be used, the 9 ring group being the third expansion.

The convenience of the coefficient method is well illustrated by this example. For a final adjustment, however, it is desirable to estimate the blading proportions and numbers by the somewhat longer but more definite method, involving the direct application of the energy equation to each expansion.

EXAMPLE 2.—Calculate the proportions of the turbine (example 1) when sixteen expansions are chosen instead of twelve, and the three-step drum is retained.

The calculation for the drum diameter is the same as previously given, the diameters being

H.P. . $23\frac{5}{8}$ inches, I.P. . $33\frac{3}{8}$ inches, L.P. . $47\frac{3}{4}$ inches.

With an equal heat distribution there will be sixteen heat values—four for the H.P., four for the I.P., and eight for the L.P., corresponding to the numbers of the expansions.

Dividing the cumulative heat ordinate AB, Fig. 200, into sixteen parts, projecting to the heat curve, and scaling the volumes at the mid ordinates, the mean volume values at the first and last expansion of each section, and the volume ratios, are found to be

	H.P.		I.P.		L.P.	
Expansion . .	1 . . .	4	1 . . .	4	1 . . .	8
Vol.	3·3	6·4	8	17·1	23	252
Vol. ratio . .	1·94		2·14		11	

As before, the blade length for the first H.P. expansion is $\frac{3}{4}$ inch.

From the volume ratio the approximate length at the fourth expansion is, $l_4 = 0{\cdot}75 \times 1{\cdot}94 = 1{\cdot}45$, say $1\frac{1}{2}$. This gives a ring diameter $25\frac{1}{8}$ inches.

Since $b = 130$, by equation (11)

$$l_4 = \frac{130 \times 6{\cdot}4}{(25{\cdot}125)^2} = 1{\cdot}33$$

so that again the standard $1\frac{3}{8}$-inch blade should be fitted. The common ratio is thus $r = \sqrt[3]{\frac{1{\cdot}375}{0{\cdot}75}} = 1{\cdot}225$, and the H.P. expansion blade lengths are 0·92 and 1·125. To the nearest $\frac{1}{8}$ inch these may be taken as

$\frac{3}{4}$ inch, 1 inch, $1\frac{1}{8}$ inches, $1\frac{3}{8}$ inches

The first length of the I.P. section is

$$l_1 = \frac{l_m r}{r_1^2} = \frac{1{\cdot}375 \times 1{\cdot}225}{2} = 0{\cdot}85, \text{ say } \tfrac{7}{8} \text{ inch}$$

giving a ring diameter $34\frac{1}{4}$ inches.

Since $b = 132$, the length, by equation (11), is

$$l_1 = \frac{132 \times 8}{(34{\cdot}25)^2} = 0{\cdot}9$$

and the next standard of 1 inch should be fitted.

The total volume ratio is 2·14, and the approximate blade length of the fourth expansion is $l_4 = 2{\cdot}14$, say $2\frac{1}{8}$ inches, giving a ring diameter $35\frac{1}{2}$ inches.

Then
$$l_4 = \frac{132 \times 17{\cdot}1}{(35{\cdot}5)^2} = 1{\cdot}8$$

Here again the next standard of $1\frac{7}{8}$ inches should be fitted.

The common ratio for the I.P. blading is thus $r = \sqrt[3]{\frac{1{\cdot}875}{1}} = 1{\cdot}233$, and the blade lengths calculated from this one to the nearest $\frac{1}{8}$ inch

1 inch, $1\frac{1}{4}$ inches, $1\frac{1}{2}$ inches, $1\frac{7}{8}$ inches

The approximate blade length for the first L.P. expansion is

$$l_1 = \frac{l_m r}{r_1^2} = \frac{1{\cdot}875 \times 1{\cdot}233}{2} = 1{\cdot}16, \text{ say } 1\tfrac{1}{4} \text{ inches}$$

giving a ring diameter 49 inches. Since $b = 134$

$$l_1 = \frac{134 \times 23}{(49)^2} = 1{\cdot}29$$

and the next standard is $1\frac{3}{8}$ inches.

Taking the total volume ratio of 11, the approximate length at the eighth L.P. expansion would be $l_8 = 1{\cdot}375 \times 11 = 15$ inches. This is greatly in excess of the permissible value, and it may again be assumed that the last two expansions require to have the same blade length.

From the H_c—v diagram the mean volume at the seventh expansion is $v_7 = 172$, hence the approximate length

$$l_7 = 1{\cdot}375 \times \tfrac{172}{23} = 10{\cdot}30$$

and the ring diameter is 58 inches.

Checking by equation (11)

$$l_7 = \frac{134 \times 172}{(58)^2} = 6{\cdot}9$$

The length is somewhere between 7 and 10 inches.

Try $7\frac{5}{8}$ inches, giving a ring diameter $55\frac{3}{8}$ inches, and

$$l_7 = \frac{134 \times 172}{(55{\cdot}375)^2} = 7{\cdot}55, \text{ say } 7\tfrac{5}{8} \text{ inches as assumed}$$

The common ratio for the first seven L.P. expansions is thus

$$r = \sqrt[6]{\frac{7{\cdot}625}{1{\cdot}375}} = \sqrt[6]{5{\cdot}55} = 1{\cdot}33$$

The consecutive blade lengths calculated from this figure are, to the nearest $\frac{1}{8}$ inch

$1\frac{3}{8}''$, $1\frac{7}{8}''$, $2\frac{1}{2}''$, $3\frac{1}{4}''$, $4\frac{3}{8}''$, $5\frac{3}{4}''$, $7\frac{5}{8}''$, $7\frac{5}{8}''$.

The maximum blade length in this case is 13·8 per cent. of the mean ring diameter, as against 12·2 per cent. with the twelve expansion arrangement.

To find the increased angle for the eighth expansion, $v_8 = 252$, $v_7 = 172$.

$$\therefore \tan \theta_7 = 0{\cdot}364 \times \tfrac{252}{172} = 0{\cdot}532, \text{ and } \theta_7 = 28^\circ$$

The number of rings can be worked out, as in the previous case, by the energy equation, or approximated by the coefficient method. This is left as an exercise for the reader.

The number in each section will not materially differ from that obtained in the previous case.

Assuming the same values, 29 on H.P. may be divided among the four expansions, thus, 8, 7, 7, 7; 15 on the I.P. among four expansions, 4, 4, 4, 3; and 15 on the L.P. among eight expansions, 2, 2, 2, 2, 2, 2, 2, 1. Thus at the last expansion there will be one fixed and one moving ring. If desired in this and also in the previous case, the equation of continuity can be finally applied to the last two blades, and the angle modified slightly to prevent too great a reduction of ρ at the last ring.

These two cases should be sufficient to show the application of this method of provisional design, based on the heat drop, to a land turbine of moderate output.

It is equally applicable to the machine of large output with divided cylinders, or double flow arrangement.

In order, as in the case of the impulse machine (example 2, p. 388), to obtain a comparison between the proportions determined by this method and those of an actual turbine, the conditions chosen for the next example are those specified for the 20,000 K.W. Parsons turbine already referred to on p. 43.

EXAMPLE 3.—Determine provisionally suitable proportions for a

Parsons turbine to develop 20,000 K.W. at 750 R.P.M. The turbine is to be of the tandem type, with a three-step H.P. rotor, and a parallel L.P. drum arranged for double flow. Initial pressure 200 lbs./in.^{2} gauge, superheat 200° F., vacuum 29 inches. The total power is to be equally divided between the H.P. and L.P. cylinders. The generator efficiency may be taken as 0·97.

Taking as before a conservative value of $\eta_s = 0{\cdot}75$ for the stage efficiency, the curves of Fig. 175, at $t_s = 200$, give a reheat factor, $R = 1{\cdot}049$. Hence the probable internal efficiency $\eta_1 = 0{\cdot}79$.

Since this is a machine of large output, it is reasonable to assume the smaller outside loss, say 6 per cent. of the heat drop. The efficiency ratio is therefore assumed to be

$$\eta = (0{\cdot}79 - 0{\cdot}06) = 0{\cdot}73$$

The heat drop scaled from the Hϕ diagram, between $p_1 = 215$ and $p_0 = 0{\cdot}5$ lbs./in.^{2}, is $H_r = 418$. Hence the cumulative heat is

$$RH_r = H_c = 438 \text{ B.Th.U.}$$

The steam consumption is

$$w = \frac{3414}{0{\cdot}73 \times 418 \times 0{\cdot}97} = 11{\cdot}5 \text{ lbs./K.W. hour}$$

The maker's guarantee, at full load of 20,000 K.W., was 11·25 lbs./K.W. hour.

The weight of steam entering the H.P. section per minute is

$$W = \frac{11{\cdot}5 \times 20000}{3600} = 64 \text{ lb.}$$

Allowing a H.P. dummy leak of, say, 5 per cent., the probable weight flowing through the H.P. blading is $0{\cdot}95 \times 64 = 61$ lbs./sec.

At the L.P. cylinder, since there is no dummy piston loss, and assuming other leakage to be negligible, the weight flowing through each half of the L.P. blading is 32 lbs./sec.

The condition curve on the Hϕ diagram and the curves of the H_c—v diagram can now be drawn. For the former, by equation (4), Art. 178, the entropy change per 100 B.Th.U. is

$$\phi_f = 0{\cdot}055 + 0{\cdot}8f^3 - 0{\cdot}00006(p_1 + t_{s_1})$$

Here $f = 0{\cdot}25$, $p_1 = 215$, $t_{s_1} = 200$

$$\begin{aligned} \therefore\ \phi_f &= 0{\cdot}055 + 0{\cdot}8 \times 0{\cdot}25^3 - 0{\cdot}00006 \times 415 \\ &= 0{\cdot}055 + 0{\cdot}0125 - 0{\cdot}0249 \\ &= 0{\cdot}0426 \end{aligned}$$

The straight line thus obtained cuts the saturation curve at the pressure 22 lbs./in.^{2} abs.

Scaling down $\eta_1 H_r = 330$ from the initial state point and projecting to the 0·5 lbs./in.^{2} curve, the final state point on the condition curve falls on the dryness curve 0·89. The condition curve can now be drawn. From this curve, by the method of Art. 181, the cumulative heats and volumes at the following pressures are obtained, and from them the $H_c - v$ diagram can be plotted.

Pressure . .	215	100	50	20	10	5	2	1	0·5
H_c	0	74	137	208	253	306	362	402	438
v	2·84	5·3	9·45	20	37·2	69·6	160	302	568

For an equal division of work between the H.P. and L.P. cylinders the total cumulative heat ordinate may be divided in two. This gives the cumulative heat value of 219 for H.P. and 219 for L.P. cylinder. There may be a slight loss between the H.P. and L.P., but if there is it may be assumed that the effect is covered by the efficiency factors used in the L.P. section. For the above values the H_c curve shows the final H.P. and initial L.P. pressure to be 17 lbs./in.^{2} abs., and that the steam at entrance to the L.P. cylinder is nearly dry.

Dealing first with the H.P. cylinder, the two arbitrary quantities to be assumed are the mean blade velocity of the first H.P. expansion, and the number of expansions. If the usual 12-expansion arrangement is adopted, then there would be, say, six on the H.P. drum, giving, with the three steps specified, two expansions per step. This, according to the published description of the machine, is the number chosen for the H.P. cylinder.

The diameter of the first expansion blading is less easily fixed. For the limits already quoted in Art. 244 a considerable variation in this diameter is possible. It would appear that, in this case, a mean blade speed of about 130 feet per sec. has been chosen.

This is adopted for the calculation, in order to obtain a valid comparison between the actual blading dimensions and the dimensions calculated by this method.

For the assumed velocity of 130 ft./sec. the mean diameter of the first H.P. expansion is

$$D_1 = \frac{60 \times 130}{\pi \times 750} = 3{\cdot}32 \text{ feet} = 40 \text{ inches}$$

With the distribution of two expansions on each section of the H.P. drum, the volumes scaled at the mid-ordinates, from the H_c-v diagram are as follows :—

	1st Step.		2nd Step.		3rd Step.	
Expansion . .	1	2	3	4	5	6
Volume . . .	3·2	4·6	6·5	9	13	19·5

By equation (1)

$$B = D_1\sqrt[3]{\frac{N}{Wv_1}}$$

$$= 3{\cdot}32\sqrt[3]{\frac{750}{61 \times 3{\cdot}2}} = 3{\cdot}32 \times 1{\cdot}568$$

$$= 5{\cdot}2$$

It is somewhat difficult to decide on the most suitable value of the speed ratio ρ, which may lie between, say, 0·55 and 0·6. Suppose a value 0·58 is taken, then at $\rho = 0·58$ and $B = 5·2$ the approximate value of the blade length-diameter ratio z, as shown by the curves of Fig. 201, may be taken as 7 per cent.

Hence the first H.P. blade length $l_1 = 0·07 \times 40 = 2·8$ inches.

As in the previous design the drum is swelled at the first expansion to take end thrust due to bye-pass at overload, so that with the slightly greater blade ring diameter the next standard of $2\frac{3}{4}$ inches may be taken as satisfactory, with a 20° angle. This, according to the published information, is the length of blade fitted.

The corresponding drum diameter (not enlarged) is thus 3 feet $1\frac{1}{4}$ inches. The enlarged value determined later is $38\frac{1}{2}$ inches at the first expansion. Taking the L.P. cylinder, a trial calculation will show that the diameter for single flow is quite outside the practicable limit of 8 feet 6 inches, and that the double-flow arrangement, as specified, is necessary.

It is probable that the last two expansions may have to be made the same length, and even an angle in excess of 30° may be required. For a trial calculation, assume $\theta' = 35°$.

$\sin 35° = 0·573$, $\cos 35° = 0·82$, $\sin 20° = 0·342$, $\cos 20° = 0·94$

Then by equation (26)

$$\rho' = \rho \frac{\cos \theta'}{\cos \theta} = 0·58 \times \frac{0·82}{0·94} = 0·505$$

also
$$C' = C \sqrt[3]{\frac{\sin \theta}{\sin \theta'}} = \sqrt[3]{\frac{0·342}{0·573}} = \frac{C}{1·187}$$

To find possible limits for the choice of the L.P. diameter try, say, two values of blade-diameter ratio z, 15 and 20 per cent. From the curves, Fig. 198,

$$\left.\begin{array}{ll} \text{With } z = 0·15, & C = 41 \\ z = 0·2 & C = 37·5 \end{array}\right\} \text{for 29 inches vacuum}$$

By equation (5)

for $z = 0·15$,
$$D_l = \frac{41}{1·187} \sqrt[3]{\frac{0·505 \times 32}{750}}$$
$$= \frac{34·6}{3·6} = 9·6 \text{ feet}$$

for $z = 0·20$,
$$D_l = \frac{37·5}{1·187 \times 3·6} = 8·8 \text{ feet}$$

The corresponding drum diameters are

for 0·15, $D_L = 0·85 \times 9·6 = 8·15$

for 0·20, $D_L = 0·8 \times 8·8 = 7·05$

It would appear, on the above assumptions, that the L.P. drum might be made any diameter between 7 and 8 feet. The actual value according to the published drawings is 7 feet 2 inches.

This, as in the case of the H.P. drum, is taken for the subsequent calculations, to enable a proper comparison of the blading dimensions to be made.

If the four drum diameters, in accordance with the usual practice, are made to increase in geometrical progression, the common ratio is

$$r_1 = \sqrt[3]{\frac{D_L}{D_H}} = \sqrt[3]{\frac{86}{37 \cdot 25}} = 1 \cdot 32$$

The successive diameters are thus

High Pressure.			Low Pressure.
$37\frac{1}{4}$ inches,	49 inches,	65 inches.	86 inches.

The successive blading lengths can now be calculated.

The dummy leakage at each step may be assumed as a slightly diminishing quantity, say 5, 4·5, and 4 per cent. With these assumed values, the value of the coefficient b for equation (11) is 1490, 1495, and 1500 respectively.

The blade length for the first expansion on first step is $2\frac{3}{4}$ inches. In the second expansion $l_2 \doteqdot 2 \cdot 75 \times \frac{4 \cdot 6}{3 \cdot 2} = 3 \cdot 96$, say 4 inches. This gives a diameter 41 inches.

Checking by equation (11)

$$l_2 = \frac{1490 \times 4 \cdot 6}{(41)^2} = 4 \cdot 08$$

Thus 4 inches is a satisfactory length.

The common ratio is $r = \frac{4}{2 \cdot 75} = 1 \cdot 45$

For the second step

$$l_1 = \frac{l_m r}{r_1^2} = \frac{4 \times 1 \cdot 45}{(1 \cdot 32)^2} = 3 \cdot 32, \text{ say } 3\tfrac{3}{8} \text{ inches}$$

this gives a diameter $52\frac{3}{8}$ inches.

Checking, $$l_1 = \frac{1495 \times 6 \cdot 5}{(52 \cdot 375)^2} = 3 \cdot 4$$

and the $3\frac{3}{8}$-inch blade may be used.

For the second length

$$l_2 \doteqdot 3 \cdot 4 \times \frac{9}{6 \cdot 5} = 4 \cdot 7, \text{ say } 4\tfrac{3}{4} \text{ inches}$$

this gives a diameter $53\frac{3}{4}$ inches.

Checking, $l_2 = \dfrac{1495 \times 9}{(53 \cdot 75)^2} = 4 \cdot 68$

so that $4\frac{3}{4}$ inches should do.

The ratio is $r = \dfrac{4 \cdot 75}{3 \cdot 375} = 1 \cdot 4.$

For the third step

$$l_1 = \frac{l_m r}{r_1^2} = \frac{4 \cdot 75 \times 1 \cdot 4}{(1 \cdot 32)^2} = 3 \cdot 8, \text{ say } 3\tfrac{7}{8} \text{ inches}$$

this gives a diameter $68\frac{7}{8}$ inches.

Checking, $l_1 = \dfrac{1500 \times 13}{(68 \cdot 875)^2} = 4 \cdot 1$

so that it is advisable to use a 4-inch blade.

For the last length

$$l_2 = \frac{4 \times 19 \cdot 5}{13} = 6 \text{ inches}$$

this gives a diameter 71 inches.

Checking, $l_2 = \dfrac{1500 \times 19 \cdot 5}{(71)^2} = 5 \cdot 82$

so that a blade length of $5\frac{7}{8}$ inches may be used.

The ratio is $r = \dfrac{5 \cdot 875}{4} = 1 \cdot 47$

The complete set of expansion blade lengths thus provisionally calculated are

1st Step.	2nd Step.	3rd Step.
$2\frac{3}{4}$ inches, 4 inches.	$3\frac{3}{8}$ inches, $4\frac{3}{4}$ inches.	4 inches, $5\frac{7}{8}$ inches

It is stated in the published description of the machine that the blade lengths on the H.P. cylinder vary from $2\frac{3}{4}$ inches at the H.P. to $6\frac{1}{2}$ inches at the L.P. end of the rotor.

Taking the L.P. cylinder, with a twelve expansion distribution, there would be six in the L.P. drum. In the published drawings there appear to be seven, although only six blade lengths are shown.

For the sake of comparison again, suppose that seven expansions are taken.

Dividing up the cumulative heat value for the L.P. section on the $H_c - v$ diagram, projecting to the heat curve, and scaling the volume values at the mid-ordinates, the following volumes are obtained :—

Expansion . .	1	2	3	4	5	6	7
Volume . . .	28·5	45	67	100	160	265	445

Since $W = 32, \quad \rho = 0 \cdot 58, \quad b = \dfrac{31500 \times 32 \times 0 \cdot 58}{750} = 780$

The approximate blade length of the first L.P. expansion is $l_1 = \frac{5{\cdot}875 \times 1{\cdot}47}{(1{\cdot}32)^2} = 4{\cdot}96$ for a single-flow arrangement. For double flow this is about $2\frac{1}{2}$ inches, giving a blade ring diameter $88\frac{1}{2}$ inches. Checking by equation (11),

$$l_1 = \frac{780 \times 28{\cdot}5}{(88{\cdot}5)^2} = 2{\cdot}84$$

The value is thus between $2\frac{1}{2}$ and $2\frac{7}{8}$ inches, and the $2\frac{3}{4}$ inches standard may be chosen.

Assuming that the last two expansion lengths require to be the same, take the volume ratio for the first and sixth expansions. This is $\frac{265}{28{\cdot}5} = 9{\cdot}3$. If the first and last blade ratio is made this amount the practicable limit is exceeded, as the length is $25\frac{1}{2}$ inches.

Suppose a value of 19 is chosen for trial, then the ring diameter at sixth and seventh expansions is 105 inches,

and $$l_6 = \frac{780 \times 265}{(105)^2} \doteqdot 18\tfrac{3}{4} \text{ inches}$$

The sixth blade length may thus be made $18\frac{3}{4}$ inches. This is 17·9 per cent. of the mean ring diameter, which is close on the maximum limit that it is desirable to work to. The figures in this case are in better agreement with the published values for the L.P. than for the H.P. cylinder. The lengths are stated to range from $2\frac{3}{4}$ to 19 inches.

The common ratio for the six blade lengths is $r = \sqrt[5]{\frac{18{\cdot}75}{2{\cdot}75}} = 1{\cdot}465$. The corresponding blade lengths, calculated from this figure to the nearest $\frac{1}{8}$ inch, are

$$2\tfrac{3}{4}'', \quad 4'', \quad 6'', \quad 8\tfrac{5}{8}'', \quad 12\tfrac{5}{8}'', \quad 18\tfrac{3}{4}'', \quad 18\tfrac{3}{4}''.$$

At the seventh expansion $v_7 = 445$, and at the sixth $v_6 = 265$, and therefore by equation (25)

$$\tan \theta_7 = 0{\cdot}364 \times \tfrac{445}{265} = 0{\cdot}61, \text{ hence } \theta_7 \doteqdot 32^\circ$$

The number of rings on each expansion can now be determined. The stage efficiency will be a progressively increasing quantity at each step, say for the H.P. cylinder 0·7, 0·73, and 0·76, and for the L.P. cylinder 0·8. The heat drop per expansion for the H.P. is

$$h_r = \tfrac{219}{6} = 36{\cdot}5 \text{ B.Th.U.}$$

The constant for equation (17) is

$$c = \frac{36{\cdot}5 \times 778 \times 32{\cdot}2}{\frac{2 \times 0{\cdot}94}{0{\cdot}58} - 1} = 408300$$

Hence, for any expansion of the H.P. cylinder the number of moving rings is given by

$$n = \frac{408300\eta_s}{\gamma u^2}$$

The necessary data for this calculation are tabulated below. The resulting number of rings obtained by substituting the chosen values of η_s are given in the last line.

H.P. CYLINDER.

No. of Expansion.	1st Step.		2nd Step.		3rd Step.	
	1.	2.	1.	2.	1.	2.
Diam. of drum, ins. .	$38\frac{1}{2}$	$37\frac{1}{4}$	49	49	65	65
Ring diam., D ins. .	$41\frac{1}{4}$	$41\frac{1}{4}$	$52\frac{3}{8}$	$53\frac{3}{4}$	69	$70\frac{7}{8}$
Blade length, l ins. .	$2\frac{3}{4}$	4	$3\frac{3}{8}$	$4\frac{3}{4}$	4	$5\frac{7}{8}$
Blade vel., u ft./sec.	135	135	171	176	226	232
Coefficient, γ . . .	0·965	0·975	0·968	0·973	0·963	0·973
No. of moving rings, n.	(16·3)	(16)	(10·5)	(9·9)	(6·3)	(5·92)
	16	16	11	10	6	6

The total number of rings on the H.P. rotor is thus 65. The number stated in the published description is 64.

In the L.P. cylinder with seven expansions and equal division of work, the heat drop per expansion is $\frac{219}{7} = 31·3$, and the constant for equation (17) is

$$c = \frac{31·3 \times 778 \times 32·2}{\frac{2 \times 0·94}{0·58} - 1} = 350000$$

with $\eta_s = 0·8$, $$n = \frac{280000}{\gamma u^2}$$

The corresponding table of data for the L.P. cylinder with the calculated number of rings is given below.

L.P. CYLINDER. Dia. of Drum = 86 inches.

No. of Expansion.	1.	2.	3.	4.	5.	6.	7.
Ring diam., D ins. .	$88\frac{3}{4}$	90	92	$94\frac{5}{8}$	$98\frac{5}{8}$	$104\frac{1}{2}$	$104\frac{1}{2}$
Blade length, l ins. .	$2\frac{3}{4}$	4	6	$8\frac{5}{8}$	$12\frac{5}{8}$	$18\frac{3}{4}$	$18\frac{3}{4}$
Blade vel., u ft./sec.	289	294	300	309	322	336	336
Coefficient, γ . . .	0·933	0·952	0·965	0·972	0·98	0·99	0·99
No. of moving rings, n.	(3·6)	(3·4)	(3·25)	(3·02)	(2·76)	(2·5)	(2·5)
	4	4	4	3	3	2	2

The total number of rings on the L.P. rotor is thus 22. The number shown in the published drawings is 24. On each half-length of drum there are shown six groups having the numbers 4, 4, 4, 4, 3, and 5. This last group presumably represents two expansions of three and two rings respectively.

It will be seen that, taking into consideration the numerous assumptions that may be made, the figures obtained by the foregoing method of calculation agree fairly well with those from the actual machine. Considering the unequalled experience of the Parsons Company in the design of this type, it is to be inferred that the proportions of this turbine are about the best possible for the specified conditions of operation.

It is of interest to again apply the less precise coefficient method for the calculation of the number of rings.

Referring to Fig. 204, the value of δ at $\rho = 0{\cdot}58$ is $1{\cdot}022$. The cumulative heat $RH_r = H_c = 438$. Hence, by equation (21)

$$\lambda = (0{\cdot}58)^2 \times 1314 \times 438 \times 1{\cdot}022 = 197800$$

The values of $\frac{1}{x}$ for the four sections are $\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2}$. The mean ring diameter of the first drum step is $41\frac{1}{4}$ inches; hence by equation (24)

$$A = \frac{10^6 \times 197800}{(41{\cdot}25)^2 \times (750)^2} = 206$$

Thus for H.P. cylinder, by equation (23)

No. of rings on 1st step $n_1 = \frac{A}{x_1} = \frac{206}{6} = 34{\cdot}33$

" " 2nd " $n_2 = \frac{A}{x_2 r_1^2} = \frac{206}{6 \times (1{\cdot}32)^2} = 19{\cdot}8$

" " 3rd " $n_3 = \frac{A}{x_3 r_1^4} = \frac{206}{6 \times (1{\cdot}32)^4} = 11{\cdot}35$

The total number is 65·48, or say 66, as against 65 by the detailed calculation.

These might be distributed as follows:—

1st Step.		2nd Step.		3rd Step.	
17	16.	11	10.	6	6.

For the L.P. cylinder

$$n_4 = \frac{A}{x_4 r_1^6} = \frac{206}{2 \times 1{\cdot}32^6} = 19{\cdot}5, \text{ say } 20$$

This result is not so satisfactory as that for the H.P. cylinder, the number being two less than the calculated value, and four less than the value for the actual machine. According to this coefficient calculation the number of L.P. moving rings might be stated as 4, 3, 3, 3, 3, 2, 2. An application of the detailed method, to check the value, would, however, show that a larger total number of rings is required.

If the calculated numbers are accepted as fairly correct for the condition of equal power distribution, it follows, in the case of the actual machine, that the power developed in the H.P. cylinder is slightly less, and that in the L.P. cylinder slightly greater than half the total amount.

259. Parsons Marine Turbine.—In a marine installation where there are two or four shafts, the total power developed by the turbines is equally divided among the shafts.

This distribution is necessary for the maintenance of the vessel on a straight course, without unnecessary displacement of the rudder. Such a condition would involve loss of power and speed.

Where there are three shafts it is usual to place the H.P. turbine on the centre and an L.P. turbine on each wing shaft. The same power may be allotted to the H.P. and each L.P. turbine, or a larger proportion may be allotted to the H.P. provided each wing turbine is arranged to develop an equal amount of power. These L.P. wing turbines run "in parallel."

With the condition of equal power distribution, one-third of the heat drop takes place in the H.P. and the remaining two-thirds in the divided L.P. turbine.

The condition is analogous to that of the tandem machine with double-flow L.P. section, the two halves of the L.P. drum being placed in separate casings.

Another arrangement used in small vessels is a triple expansion one. A high-pressure turbine is placed on one wing shaft, an intermediate-pressure turbine on the other wing shaft, and a low-pressure turbine on the centre shaft. The turbines in this case run "in series."

Where either the two- or the four-shaft arrangement is adopted, the total heat drop is divided between a high- and a low-pressure turbine. With four shafts the port H.P. and L.P. turbines constitute one unit, and the starboard turbines another unit. The H.P. turbines are placed on the outboard and the L.P. turbines on the inboard shafts.

260. There has been a departure from this arrangement in recent practice, a four-shaft triple-expansion combination being substituted for the two independent compounded sets.

This arrangement is fitted in the large Cunard liner *Aquitania.* The H.P. turbine is placed on the outboard shaft on the port side, the intermediate (in series with the H.P.) on the starboard shaft, and two L.P. turbines, in series with the I.P., but in parallel with each other, on the two inboard shafts.

The power distribution in this case is one-fourth in the H.P., one-fourth in the I.P., and half divided equally between the two L.P. turbines.

This triple compounding is said to give better economy of power, and to effect a considerable saving in size and weight, since the high and intermediate pressure turbines are each half the size of the H.P. turbine required with a two-stage compound arrangement.

It is of interest to note the leading dimensions of the drums and blading used in the excellent example of up-to-date marine turbine practice.[1]

[1] For more detailed information, see description and drawings in *Engineering*, May 8, 1914.

The H.P. ahead turbine has a rotor 9 feet 2 inches diameter, carrying four groups of blades. The lengths range from $3\frac{3}{8}$ to 7 inches. On the same shaft, in an independent casing, is fitted a H.P. astern turbine having a rotor 7 feet 10 inches diameter, and carrying four groups of blades. There are, however, only three blade lengths ranging from $1\frac{1}{2}$ inches at the first to 3 inches at the third and fourth groups. Semi-wing and wing blades are used for the latter groups.

The intermediate turbine has a rotor 10 feet 4 inches diameter carrying four groups of blades. These range in length from $6\frac{3}{4}$ to 14 inches. As in the case of the H.P. turbine, there is a separate astern turbine fitted in the same shaft. It is a duplicate of the turbine on the H.P. turbine shaft.

Each L.P. turbine (on the inboard shaft) is of the ordinary marine design, with the ahead and astern rotors in one casing.

The diameter of the ahead drum is 12 feet, and there are nine groups of blades. The lengths range from 7 inches at the first to 20 inches at the seventh group. The last three lengths are 20 inches, and semi-wing and wing blades are used.

Each astern drum has a diameter of 10 feet, and carries four groups of blades. These range in length from 5 to 7 inches. The last two groups being the same length, semi-wing and wing blades are again used.

The ratios of the drum diameters for the ahead rotors are $\frac{\text{I.P.}}{\text{H.P.}} = 1{\cdot}13$. $\frac{\text{L.P.}}{\text{I.P.}} = 1{\cdot}16$. Also the maximum L.P. ahead blade is only 12·2 per cent. of the mean ring diameter.

The estimated shaft horse power of this combination at full speed of 24 knots is 60,000.

261. General Dimensions of Direct-Coupled Marine Turbines.—In settling the general dimensions of a marine turbine installation the method of procedure is less definite than in the case of a land installation. The interdependent factors of available space, weight and cost, and the conflicting conditions of propeller and turbine efficiency have to be taken into account, and each case considered on its merits.

For the estimation of the general dimensions of directly coupled turbines, the ring diameters of the H.P. and L.P. rotors at the high-pressure ends may first be provisionally calculated, by choosing an average value for the mean blade velocity. Alternatively, the ring diameter may be calculated from equation (1), for a chosen minimum limit of blade length, say 2 per cent. of the mean ring diameter.

As a rough guide in the selection of suitable blade velocities and velocity ratios the following table, published originally in a paper by E. M. Speakman,[1] will be found useful. The values are those that

[1] "The Determination of the Principal Dimensions of the Steam Turbine, with special reference to Marine Work." *Trans. Inst. Engineers and Shipbuilders in Scotland*, 1905–1906.

experience has shown to give a fairly satisfactory compromise between the conflicting elements of weight, cost, and efficiency.

Type of vessel.	Mean blade velocity, ft./sec.		Mean speed ratio, ρ.	No. of shafts.
	H.P.	L.P.		
High-speed mail steamers . .	70– 80	110–130	0·45–0·5	4
Intermediate ,, ,, . .	80– 90	110–135	0·47–0·5	3 or 4
Channel ,, ,, . .	90–105	120–150	0·37–0·47	3
Battleships and cruisers . .	85–100	115–135	0·48–0·52	4
Small cruisers	105–120	130–160	0·47–0·50	3 or 4
Torpedo craft	110–130	160–210	0·47–0·57	3 or 4

When the drum diameters are calculated, the blade lengths and numbers of moving rings in the various expansions are obtained by the method already detailed and illustrated in the preceding paragraphs for the land type.

262. Number of Expansions.—With regard to the number of expansions, experience indicates that for a three-shaft arrangement, with two stage compounding (one H.P. and two L.P. in parallel), a total of twelve expansions, four on the H.P. and eight on each L.P. drum, gives satisfactory results. The common ratio r of the blade lengths is usually in the neighbourhood of $\sqrt{2}$.

For the four-shaft arrangement with two units, each consisting of an H.P. and an L.P. turbine, 12 expansions may again be used, 5 on the H.P. and 7 on the L.P. drum.

In the four-shaft arrangement with triple compounding, 16 or 17 expansions may be employed, say 4 on the H.P., 4 on the I.P., and 8 or 9 on each L.P. drum.

Considerable elasticity of choice is, however, permissible, and these figures are merely quoted as averages.

At the exhaust end of each L.P. turbine, as a rule, the blade length is kept within 15 per cent. of the mean ring diameter.

With the usual blade velocities, it will be found, on calculation from the steam volumes, that the last three expansions require the same blade length to keep them within the limit specified. Semi-wing and wing and sometimes double-wing blades have therefore to be used. In dealing with the L.P. blading it may be taken for granted, in any case, that the last three expansion lengths are the same, and the common ratio for the blade lengths can be calculated on this assumption.

263. As already pointed out, the efficiency ratio of a directly coupled marine turbine is always much lower than that of the corresponding land machine, on account of the lower speed ratio ρ and the use, usually of dry, instead of superheated steam.

An approximation to the efficiency ratio can be got by assuming the consumption of saturated steam from 13 to 15 lbs. per S.H.P. hour.

As the value of the efficiency ratio for this type is about 60 per cent., and about 10 per cent. may be allowed for outside losses, an average value of internal efficiency, say 0·7, may be used for large units, to determine the condition curve on the $H\phi$ diagram, and to derive the curve of the $H_c - v$ diagram.

264. Proportions of Astern or Reverse Turbine.—The proportions of astern turbines vary so much that it is scarcely possible to state any definite method of procedure.

Obviously the size of the astern turbine should be kept as small as possible. This end can be attained by a sacrifice of efficiency, which, for astern running, is of secondary importance.

The power required for effective manœuvring astern is very variable, and depends on the type of vessel and the service.

For directly coupled turbines the astern power may vary from 30 to 45 per cent. of the ahead power, for the same total supply of steam.

In high-speed geared turbines the usual proportion is 60 per cent. of the ahead power, for the same total steam consumption.

Astern turbines may be placed in tandem with H.P. or L.P. turbines on the same shafts, and with this arrangement the astern drum is made about the same diameter as the H.P. or I.P. drum.

When, as is usual, the astern turbine is incorporated with the L.P. turbine in the same casing, the diameter of the astern drum is made from 75 to 80 per cent. of the diameter of the ahead drum.

265. For the estimation of the ring numbers and blade lengths a reduced speed ratio of from $0{\cdot}5\rho$ for merchant to $0{\cdot}58\rho$ for naval vessels may be used, where ρ is the speed ratio for the ahead turbine.

For approximately the same heat drop the corresponding values of the astern design coefficient λ_a may thus be taken from $0{\cdot}2\lambda$ to $0{\cdot}3\lambda$, where λ is the coefficient for the ahead turbine.

The coefficient method is quite good enough in this case, as any nicety of calculation is superfluous. The conditions, usually, are very indefinite, and a good deal of judgment has to be exercised.

266. The number of expansions used in the astern drum may vary from four to six.

Taking the same pressure limits as used for the ahead turbine, with the much smaller number of stages employed to keep down the length, the volume increase per expansion is much greater than in the ahead section. The common ratio of the blade lengths is thus much greater. usually about 2 instead of $\sqrt{2}$. As a rule the figure 2 is not exceeded. The heights of the last two or three expansions are kept the same, and semi-wing, wing, and double-wing blades are employed.

EXAMPLE 4.—Determine suitable dimensions for a set of direct-coupled marine turbines to develop 18,000 S.H.P. at 290 R.P.M. The set is to consist of one H.P. and two L.P. turbines (in parallel). A reverse turbine is to be incorporated with each L.P. turbine. Initial pressure 170 lbs./in.^{2} abs., vacuum 28 inches. The steam consumption may be assumed as 13 lbs./S.H.P. hour, and the speed ratio as 0·47. The power is to be equally divided between the three turbines.

Here $p_1 = 170, p_0 = 1$, and heat drop $H_r = 326$. For the assumed consumption the efficiency ratio is

$$\eta = \frac{2544}{326 \times 13} = 0{\cdot}6$$

and with, say, 10 per cent. outside loss the internal efficiency may be taken as $\eta_1 = 0{\cdot}7$.

In this case, for the calculation of the cumulative heat, it is necessary to assume an average value of the stage efficiency for the whole installation. This will be less than the figure permissible in the case of a land turbine, which would be taken as 0·75.

Assuming, say, 0·64 for the H.P. and 0·70 for the L.P. turbine the mean stage efficiency may be taken as 0·67.

The probable reheat factor (with dry steam), as shown by equation (6) or the curves of Fig. 175, is $R = 1{\cdot}033$. The probable value of the total cumulative heat is thus $H_c = 1{\cdot}033 \times 326 = 336{\cdot}75$ B.Th.U.

Take the usual number of expansions for this arrangement, say 4 on the H.P. and 8 on each L.P. drum, and an equal distribution of heat per expansion. Since $\frac{1}{3}H_c$ is taken by the H.P. turbine, the heat per expansion is

$$h_c = \tfrac{1}{4} \times \tfrac{1}{3} H_c = \frac{336{\cdot}75}{12} \doteqdot 28{\cdot}1 \text{ B.Th.U.}$$

The value is the same for each L.P. expansion.

The condition line being drawn on the $H\phi$ diagram for $\eta_1 = 0{\cdot}7$ and the $H_c - v$ diagram for $\eta_s = 0{\cdot}67$, the volumes scaled at the mid-ordinate of each expansion for the latter diagram are found to be as follows :—

	H.P. Turbine.				L.P. Turbine.							
Expansion	1	2	3	4	1	2	3	4	5	6	7	8
Volume	3	4·5	6·4	9·4	13	18·2	25	37	56	85	132	220

The total steam per second is

$$W = \frac{13 \times 18000}{3600} = 65 \text{ lb.}$$

Allowing an H.P. dummy leak of 4 per cent., the weight passing through the H.P. blading may be taken as 62 lbs.

In order to estimate the first H.P. ring diameter, assume for a trial $z = 0{\cdot}02$. Then from the curves of Fig. 201, at $\rho = 0{\cdot}47$, $B = 7{\cdot}5$. Also $N = 290$, and $v_1 = 3$.

Hence by equation (1)

$$D_1 = B\sqrt[3]{\frac{Wv_1}{N}}$$

$$= 7{\cdot}5\sqrt[3]{\frac{62 \times 3}{290}} = \frac{7{\cdot}5}{1{\cdot}152} = 6{\cdot}5 \text{ feet or 78 inches}$$

This gives a blade velocity $u = 99$, which as a reference to the foregoing table will show, is a reasonable figure. The corresponding blade length is $l_1 = 1{\cdot}56$, and the next standard of $1\frac{5}{8}$ inches may be used.

The H.P. drum diameter is $D_H = 0{\cdot}98 \times 78 = 76\frac{3}{8}$ inches, or 6 feet $4\frac{3}{8}$ inches.

For the calculation of the blade lengths, the value of b for equation (11) is

$$b = \frac{31500 \times Wp}{N} = \frac{31500 \times 62 \times 0{\cdot}47}{290} = 3160$$

The total mean volume ratio for the H.P. drum is $\frac{9{\cdot}4}{3} = 3{\cdot}133$. Hence the approximate blade length of the last expansion is

$$l_4 = 3{\cdot}133 \times 1{\cdot}625 \doteqdot 5 \text{ inches}$$

This gives a ring diameter $81\frac{3}{8}$ inches. Applying equation (11) as a check,

$$l_4 = \frac{3160 \times 9{\cdot}4}{(81{\cdot}375)^2} = 4\tfrac{1}{2} \text{ inches}$$

The blade length should lie between $4\frac{1}{2}$ and 5 inches, and the nearest standard of $4\frac{5}{8}$ inches can be used.

For a geometrical progression of blade length, the common ratio is

$$r = \sqrt[3]{\frac{4{\cdot}625}{1{\cdot}625}} = 1{\cdot}416 \text{ or nearly } \sqrt{2}$$

The four blade lengths to the nearest $\frac{1}{8}$ inch are

$1\frac{5}{8}$ inches, $2\frac{3}{8}$ inches, $3\frac{1}{4}$ inches, $4\frac{5}{8}$ inches.

Taking the L.P. turbine next, a reference to the table indicates that a reasonable value for the mean blade velocity of the first expansion in 135 ft./sec. With this value, the corresponding mean ring diameter is $D_1 = 8{\cdot}9$, say 9 feet.

The mean volume value at the first expansion is $v_1 = 13$. Since half the total steam passes into each L.P. turbine, the weight entering the L.P. is $32{\cdot}5$ lbs. Allowing for, say, 3 per cent. dummy piston leak, the probable weight passing through the L.P. blading is $31{\cdot}5$ lbs.

The value of b is

$$b = \frac{31500 \times 31{\cdot}5 \times 0{\cdot}47}{290} = 1610$$

Hence the first L.P. blade length is

$$l_1 = \frac{1610 \times 13}{(108)^2} = 1{\cdot}8 \text{ inches}$$

As this is nearer $1\frac{3}{4}$ than $1\frac{7}{8}$ inches, the standard $1\frac{3}{4}$ inches length may be used. This gives an L.P. drum diameter of $D_L = 106{\cdot}25$. The drum diameter ratio is thus

$$r_1 = \frac{106{\cdot}25}{76{\cdot}375} = 1{\cdot}4$$

which is approximately the value commonly used in marine work.

A trial calculation will show that for the total volume ratio, the last blade length would be too long, and the last three expansions should be kept the same length. The mean volume at the sixth expansion is $v_6 = 85$. Hence $\frac{v_6}{v_1} = \frac{85}{13} = 6{\cdot}52$, and the approximate length of blade for the sixth expansion is $l_6 = 6{\cdot}52 \times 1{\cdot}75 = 11\frac{1}{2}$ inches.

This gives a ring diameter 117·75. Checking by equation (11)

$$l_6 = \frac{1610 \times 85}{(117{\cdot}75)^2} = 9{\cdot}9, \text{ say } 10 \text{ inches}$$

The length of the last three expansions should thus be 10 inches or 8·6 per cent. of the ring diameter.

The common ratio for the lengths between expansions one and six is

$$r = \sqrt[5]{\frac{10}{1{\cdot}75}} = 1{\cdot}416, \text{ again nearly } \sqrt{2}$$

The lengths calculated from this figure, to the nearest $\frac{1}{8}$ inch, are

$1\frac{3}{4}''$, $2\frac{1}{2}''$, $3\frac{1}{2}''$, $5''$, $7''$, $10''$, $10''$, $10''$.

A detailed check calculation using the volume at each expansion will give practically the same result.

The enlarged blade angles at the seventh and eighth expansion have to be calculated.

Here

$$\tan\theta_7 = \tan\theta_6\frac{v_7}{v_6} = 0{\cdot}364 \times \frac{132}{85} = 0{\cdot}56 \text{ and } \theta_7 = 29^\circ \text{ (semi-wing)}$$

$$\tan\theta_8 = \tan\theta_6\frac{v_8}{v_6} = 0{\cdot}364 \times \frac{220}{85} = 0{\cdot}94 \text{ and } \theta_8 = 43^\circ\ 20' \text{ (wing)}$$

The number of moving rings in each expansion can now be calculated. For the H.P. drum, $h_c = 28{\cdot}1$, and the assumed value of $\eta_s = 0{\cdot}64$; then by equation (17)

$$n = \frac{28{\cdot}1 \times 778 \times 32{\cdot}2 \times 0{\cdot}64}{\gamma u^2\left(2 \times \frac{0{\cdot}94}{0{\cdot}47} - 1\right)} = \frac{150400}{\gamma u^2}$$

The necessary data, as in the previous examples, are tabulated below, and the calculated values of the ring numbers are given in the last line.

H.P. TURBINE. Drum Diam., $76\frac{3}{8}$ inches.

Expansion.	1.	2.	3.	4.
Ring diam., D ins.	78	$78\frac{3}{4}$	$79\frac{5}{8}$	81
Blade length, l in.	$1\frac{5}{8}$	$2\frac{3}{8}$	$3\frac{1}{4}$	$4\frac{5}{8}$
Blade vel., u ft./sec.	99	100	101	102·5
Coefficient, γ	0·898	0·93	0·947	0·962
No. of moving rings, n.	(17)	(16·2)	(15·6)	(14·9)
	17	16	16	15

The total number of moving rings on the H.P. drum is thus 64. Usually the total number would be equally divided among the groups, giving four groups of sixteen rings each.

For the L.P. turbine, since $h_c = 28{\cdot}1$ and the stage efficiency is assumed as $\eta_s = 0{\cdot}7$, the number of rings in any group is given by

$$n = \frac{28{\cdot}1 \times 778 \times 32{\cdot}2 \times 0{\cdot}7}{\gamma u^2 \times 3} = \frac{164000}{\gamma u^2}$$

L.P. TURBINE. Drum Diam., 106¼ inches.

Expansion.	1.	2.	3.	4.	5.	6.	7.	8.
Ring diam., D ins.	108	108¾	109¾	111¼	113¼	116¼	116¼	116¼
Blade length, l in.	1¾	2½	3½	5	7	10	10	10
Blade vel., u ft./sec.	136	137	138·5	140·5	143	147	147	147
Coefficient, γ	0·876	0·912	0·937	0·952	0·96	0·97	0·97	0·97
No. of moving rings, n.	(10·1) 10	(9·6) 9	(9·1) 9	(8·73) 9	(8·35) 8	(7·82) 8	(7·82) 8	(7·82) 8

The necessary data are again tabulated, and the number of rings are calculated from this equation.

The total number of moving rings on the L.P. drum is 69. Usually there are an equal number of rings per group, and instead of dividing up the number as above, eight groups of 8 rings, giving 64 total, might be fitted. This smaller value might be as satisfactory as the set calculated above, and the drum length would be slightly reduced.

The conditions of the design which are chosen for this example are approximately those under which the set of turbines, illustrated in Figs. 27 and 28, are operated. In this case there are four expansions of 16 rings each on the H.P. and eight expansions of 8 rings each on the L.P. drum. The assumption of a slightly lower value of mean stage efficiency for the L.P. cylinder would probably give the lower number of 64. As already pointed out, the value to be adopted in any given case must be subject to individual judgment on account of the uncertainty of the values of the design factors that have to be assumed throughout the calculation.

It is probably in marine practice that the coefficient method of calculation for the number of moving rings is most extensively used.

In the case with $\rho = 0{\cdot}47$ the coefficient δ from Fig. 204 is $1{\cdot}085$. Also $RH_r = 336{\cdot}75$. Hence

$$\lambda = (0{\cdot}47)^2 \times 1314 \times 336{\cdot}75 \times 1{\cdot}085 = 106000$$

The mean ring diameter of the H.P. turbine is $79{\cdot}35$ inches, $N = 290$, and

$$A = \frac{10^6 \times 106000}{(79{\cdot}35)^2 \times (290)^2} = 200$$

For the H.P. drum $\dfrac{1}{x_1} = \dfrac{1}{3}$; for the L.P. $\dfrac{1}{x_2} = \dfrac{2}{3}$, also $r_1 = 1{\cdot}4$

hence, No. of rings on H.P. $= n_1 = \dfrac{A}{x_1} = \dfrac{200}{3} = 66{\cdot}66$, say 66

" " L.P. $= n_2 = \dfrac{A}{x_2 r_1^2} = \dfrac{200 \times 2}{3 \times (1{\cdot}4)^2} = 68$

The total number for the whole turbine is 134 against 133 by the longer calculation, but the distribution in the two drums is slightly different.

Both these calculations therefore give a total number about 4 per cent. in excess of the number fitted in the corresponding set of turbines, Figs. 27 and 28, for which the value is 128.

The cause of the difference is probably due to the choice of higher efficiency values than have been allowed for in the other case.

Reverse Turbine.—Working on the general lines set out in Arts. 264, 265, 266, suppose, to fix ideas, that the L.P. astern drum diameter is made about 80 per cent. of the ahead drum diameter, say 84 inches, and that four expansions may be used.

If the same pressure conditions are assumed as for ahead running, and the heat drop taken to be the same, the volume for the four expansions as shown by the H_c—v diagram will be about 4·5, 13, 40, 130.

It is probable that in actual service the pressure limits would not be quite the same as those assumed, the initial pressure being lower than when going ahead and the vacuum probably less.

As an approximation at the first expansion the volume may be taken as 5. Allowing for, say, a reduction in speed ratio about 30 per cent., the value may be taken as $\rho = 0{\cdot}24$. With the same weight of steam passing as in the ahead turbine, that is 32 lbs. per second, and assuming the same speed of rotation, an approximation to the first blade length, which may be expected to be somewhere between the minimum value of $\frac{1}{2}$ and 1 inch, is given by equation

$$l_1 = \frac{31500 \times 32 \times 0{\cdot}24 \times 5}{(84)^2 \times 290} = 0{\cdot}6$$

so that a minimum length of $\frac{5}{8}$ inch may be taken for the first expansion.

With the volumes stated, the common ratio with normal blades would require to be 3. It is not usually made more than 2. Adopting this figure the four blade lengths may be taken as

$\frac{5}{8}$ inch, $1\frac{1}{4}$ inch, $2\frac{1}{2}$ inches, $2\frac{1}{2}$ inches.

The angles of the second, third, and fourth groups have to be opened out.

Taking a normal blade with $\theta_1 = 20$ at the first expansion,

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{3}{2}$$

$$\therefore \sin \theta_2 = 0{\cdot}342 \times \tfrac{3}{2} = 0{\cdot}51$$

so that the angle of the second expansion is $\theta_2 = 30°$.

Also $$\sin \theta_3 = \frac{0.51 \times 3}{2} = 0.765 \quad \text{and} \quad \theta_3 = 50^\circ$$

Since the last two lengths are kept the same,

$$\tan \theta_4 = \tan \theta_3 \frac{v_4}{v_3} \doteq 1.19 \times \frac{130}{40} = 1.19 \times 3.25 = 3.87$$

$$\therefore \theta_4 = 75^\circ$$

The first expansion has normal blades; the second, semi-wing; the third, wing; and the fourth, double-wing blades.

In order to estimate the number of moving rings by the coefficient method, the reduced value of speed ratio $\rho_a = 0.24$ is chosen, and since there would probably be a slight reduction of the heat drop due to lower pressure and vacuum, the value of the coefficient may be reasonably assumed as $\lambda_a = 0.20\lambda$ to 0.22λ, say 0.21λ. Hence, by equation (22) the probable number of rings is

$$n = \frac{10^6 \lambda_a}{D^2 N^2}$$

The mean ring diameter is 85·75 inches and N = 290,

$$\therefore n = \frac{10^6 \times 0.21 \times 10600}{(85.75)^2 \times (290)^2} = 36$$

Hence, four groups with 9 rings per group could be fitted in the astern drum.

The foregoing is only one of several sets of proportions that might be arbitrarily chosen for this case.

In the case of a naval vessel with directly connected turbines, special cruising turbines are usually fitted. A discussion of this aspect of marine propulsion, and also the allied subject of propeller design and efficiency, is outside the scope of this text. For these and other cognate matters relating to the general design and arrangement of marine turbine installations, the reader should consult the special text-books on the marine turbine, by Morrow and Reed, and also Swallow's translation of Bauer and Lasche.

A considerable amount of practical data, taken from turbines in service, together with information regarding constructural details and shop practice, is given in Southern's "Marine Steam Turbine."

For an advanced treatment of the "theory" of the reaction turbine, a mathematical analysis by H. M. Martin will be found in his text on the Steam Turbine.

267. Geared Marine Turbines.—In the case of the geared marine turbine, the design is identical with that of the high-speed land type. With a view to the preparation of normal designs, the following average values have been suggested by Messrs. McLaren and Welsh, in a paper on "Geared Turbines for Ship Propulsion."[1]

[1] *Trans. Inst. Eng. Shipbuilders in Scotland*, October 27, 1914.

Steam pressure, 180 lbs./in.^{2} gauge. Saturated steam.
Vacuum, 28 inches.
First expansion blade velocity H.P., 160 ft./sec.
Last " " " 260 "
Low pressure blading velocity exhaust end, 300 ft./sec.
Maximum limit of L.P. blade length, $\frac{1}{6}$ or L = 16·66 per cent.
Number of moving rings, H.P. turbine, with two-step drum:

H.P. section, $n = 33$. L.P. section, $n = 12$.

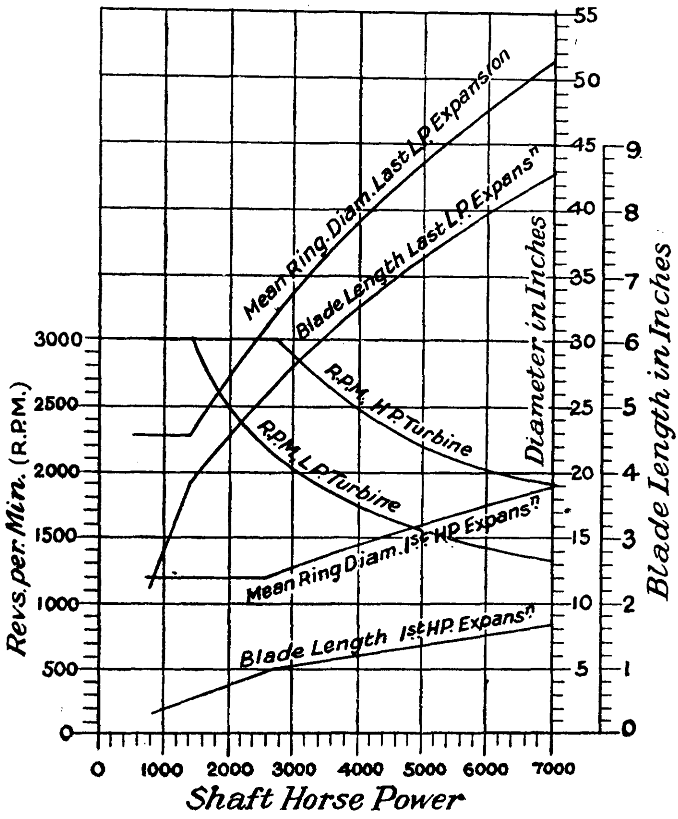


FIG. 205.

Number of moving rings L.P. turbine with parallel drum, $n = 21$.

It is suggested that in order to keep up the length of the high pressure turbine blades and reduce leakage loss, the H.P. turbine revolutions should be made greater than those of the L.P. A maximum speed of revolution of 3000 is proposed.

This is much higher than any of the speeds mentioned in Art. 27.

It is not however necessary, for any other reason, to run the H.P. at greater speed than the L.P. turbine, and if it is desired to use identical pinions in the H.P. and L.P. shafts, the dimensions of the H.P. rotor may be calculated for equal speed.

The provisional values of revolutions, blade length and ring diameters for H.P. and L.P. turbines, based on the foregoing assumptions, are given for shaft horse-powers, ranging from 1000 to 7000 in a set of curves, which is reproduced in Fig. 205.

It will be noted that the maximum ring diameter for this type is fixed slightly in excess of 4 ft., and the minimum about 1 ft., while the maximum blade length is about $8\frac{1}{2}$ inches.

As the calculations for a machine of this type are the same as those for the high-speed land turbine, it is not necessary to work out an illustrative example.

The reader, as an exercise, may take the necessary data at a specified shaft horse-power, from this diagram, and compare the proportions obtained by calculation as in the previous cases, with the averages quoted above.

268. Combination Turbines.—As in the case of the Curtis machines the maximum diameter of the impulse wheel for a stated speed of rotation is controlled by the maximum permissible stress in the wheel. A rim velocity of 600 ft./sec. should not be exceeded. As a rule, the mean blade speed does not exceed 500 ft./sec. It varies generally from 350 to 450 ft./sec.

As in the previous cases dealt with in Chap. XIII. a conservative value of the theoretical speed ratio for the velocity wheel may be taken as $\rho_t = 0{\cdot}2$.

When an arbitrary choice of blade speed u is made, then for the given ρ_t value the theoretical steam velocity and heat drop can be estimated.

Alternatively the proportion of the total heat drop allotted to the impulse stage can be assumed, in order to obtain the blade speed and the diameter corresponding to the stated revolutions.

This proportion may vary from 20 to 30 per cent. of the heat drop.

The heat drop for the impulse stage being settled, the value can be scaled off on the $H\phi$ diagram, and the initial pressure p_2 for the reaction section is thus determined.

269. To find the initial quality at p_2, the impulse stage efficiency can be taken from the curves of Fig. 115. Scaling the proportion from the initial state point and projecting to the p_2 curve, the initial point for the reaction section is found.

A vertical drawn from this point to the exhaust pressure curve gives the heat drop for the reaction section.

For this section it may be assumed that the mean stage efficiency varies from 0·76 to 0·78.

This value being provisionally selected, the reheat factor and internal efficiency are obtained from the curves of Fig. 175. Scaling down the corresponding proportion of the reaction section heat drop from the initial state point on p_2 curve, and projecting to the exhaust pressure curve, the final state point on the condition curve is obtained. The horizontal when carried across to cut the vertical drawn between the admission and exhaust pressure curves enables the internal efficiency of the whole machine to be calculated.

From this about 7 to 9 per cent. should be deducted to allow for outside loss. The result gives the approximate efficiency ratio for the combination.

This figure can be used for the calculation of the steam consumption.

The initial state point for the reaction section usually falls near the saturation curve in the superheat field, and it is sufficient to join this point and the final state point by a straight line, to obtain the condition curve of the reaction section.

If, however, the point should fall well above the saturation curve, then the condition curve should be drawn in by the method of Art. 178.

The H_c—v diagram for the reaction section can then be drawn, and the blading proportions calculated as in the previous cases.

270. With regard to the number of expansions, since the velocity wheel replaces the H.P. section of the reaction machine, the number may be taken from 9 to 12, say, 3 to 4 on the I.P. and 6 to 8 on the L.P. section of the drum. This is a matter of judgment.

In order to fix the drum diameters either a trial percentage blade length (not less than 3 per cent.) may be used to calculate the first I.P. ring diameter, or a blade velocity from 220 to 240 ft./sec. may be selected, and the corresponding diameter ascertained. The first I.P. blade length can then be calculated by equation (11), using the initial volume at the I.P. drum.

The minimum L.P. diameter may be provisionally fixed for a blade length preferably about 15 per cent. of the mean ring diameter, on the assumption that wide-angled blades are used at the last expansion.[1]

EXAMPLE 5.—Determine suitable disc and drum diameters, and number and lengths of blading rings for a combination turbine to develop 1200 K.W. at 2000 R.P.M. Initial pressure 150 lbs./ins.^{2} abs., superheat 110° F., vacuum 28 inches, generator efficiency 0·93. Assume for the reaction section a constant speed ratio $\rho = 0{\cdot}55$. The drum is to be made in two steps.

A considerable variation in the choice of the wheel diameter is possible. Suppose an average value of mean blade velocity $u = 400$ is selected for the two-velocity impulse stage, and $\rho_t = 0{\cdot}2$.

By equation (2) the ring diameter is

$$D = \frac{60 \times 400}{\pi \times 2000} = 3{\cdot}8$$

say, in round numbers, 4 feet.

[1] This refers to a machine of moderate output and speed. See footnote, p. 198.

This gives the slightly higher value, $u = 420$.

The theoretical velocity at the nozzle is

$$V_0 = \frac{420}{0{\cdot}2} = 2100$$

and the corresponding heat drop is

$$h_r = \left(\frac{2100}{223{\cdot}7}\right)^2 = 88 \text{ B.Th.U.}$$

From the Hϕ diagram the total heat drop betwen 150 lbs./in.^{2} at 110° F. and 1 lb./in.^{2}, is $H_r = 340$. Hence $\frac{88}{340} = 0{\cdot}26$ of the heat drop is utilised in the impulse stage.

By increasing the wheel diameter a larger percentage could, if desired, be utilised.

Retaining this value, and scaling it on the Hϕ diagram from the initial point on the 150 lbs./in.^{2} curve, the lower pressure p_2 is found to be 55 lbs./in.^{2}

The stage efficiency corresponding to $\rho_t = 0{\cdot}2$, from Fig. 115, is 0·66. Scaling down $0{\cdot}66 \times 88 = 58$ B.Th.U. from the initial state point and projecting to the 55 lbs./in.^{2} curve, the initial quality at the beginning of the reaction section is 40° F. superheat.

The heat drop for the reaction section scales 261 B.Th.U. Suppose an average value of stage efficiency is taken as $\eta^s = 0{\cdot}76$. Scaling down $0{\cdot}76 \times 261 = 199$ B.Th.U. from the initial point on the 55 lbs./in.^{2} curve, and projecting to the 1 lb./in.^{2} curve, the final state point falls on the quality curve for 0·894 dryness. Projecting horizontally across to cut the original vertical drawn from the initial state point on the 150 lbs./in.^{2} curve, and taking the ratio of the intercepts on this vertical, the internal efficiency of the combined machine is found to be $\eta_1 = 0{\cdot}758$. Allowing, say, 10 per cent. for outside loss, the probable efficiency ratio for the machine is $\eta = 0{\cdot}658$.

Hence the steam consumption is

$$w = \frac{3414}{0{\cdot}658 \times 340 \times 0{\cdot}93} = 16{\cdot}4 \text{ lbs./K.W. hour}$$

and weight passing is

$$W = \frac{16{\cdot}4 \times 1200}{3600} = 5{\cdot}46 \text{ lbs./sec.}$$

Next, the condition curve is obtained by joining the initial and final points, for the reaction section, on the Hϕ diagram.

In order to calculate the cumulative heats for the reaction section, it is found, by trial from the curves of Fig. 175, that $R = 1{\cdot}035$ and $\eta_s = 0{\cdot}73$ correspond to the assumed value of the internal efficiency $\eta_1 = 0{\cdot}76$. The heats calculated for this value of η^s, together with the volumes at a suitable set of pressures, are tabulated below.

	Impulse.	Reaction.									
Pressure	150–55	55	30	20	15	10	5	3	2	1·1	1
Cumulative heat	0–88	0	48	79·7	100	130	179	210	236	250	270
Quality	110°–40° F.	40°	1	0·983	0·972	0·96	0·94	0·925	0·913	0·908	0·894
Volume	—	8·3	13·74	19·8	25·6	36·8	68	110	158	206	300

The H_c—v diagram plotted from these values is shown in Fig. 206. DA is the heat for the impulse stage $h_r = 88$. AB is the heat for the reaction section $H_c = 270$. The latter quantity may be divided between the I.P. and L.P. sections, as in the case of the pure reaction machine, in the proportion of 1 to 2.

Thus $\frac{1}{3}$ is taken by the I.P. and $\frac{2}{3}$ by the L.P. section, I.P. heat AC = 90, and L.P. heat CB = 180.

For a pure reaction machine the number of expansions might be twelve, giving three on I.P. and six on L.P. section. Adopting these values and an equal distribution of the heat among them, the heat per expansion is $h_c = \frac{270}{9} = 30$ B.Th.U.

Dividing the H_c ordinate up into nine parts, projecting to the curve and scaling the volumes at the mid-ordinates, the following values are obtained :—

	I.P.			L. P.					
Expansion	1	2	3	1	2	3	4	5	6
Volume	9	13	18	26	38	58	87	140	236

Assuming a mean blade velocity of 220 ft./sec. for the first I.P. expansion, then by equation (2) the ring diameter is

$$D_1 = \frac{60 \times 220}{\pi \times 2000} = 2{\cdot}1 \text{ feet} = 25 \text{ inches}$$

The total steam is 5·46 lbs. per sec. Allowing for, say, 4 per cent. I.P. dummy leakage, this becomes 5·23.

The coefficient for equation (11) is

$$b = \frac{31500 \times 5{\cdot}23 \times 0{\cdot}55}{2000} = 45{\cdot}3$$

The first I.P. blade length is

$$l_1 = \frac{45{\cdot}3 \times 9}{(25)^2} = 0{\cdot}652$$

The next standard of $\frac{3}{4}$ inch should be fitted. This is 3 per cent. of the mean ring diameter, and it is not advisable to go below this figure.

The I.P. drum diameter is $D_I = 24\frac{1}{4}$ inches.

The volume ratio for the I.P. section is 2, so that the approximate blade length of the third expansion is $1\frac{1}{2}$ inches. This gives a ring diameter $25\frac{3}{4}$ inches.

Checking this length

$$l_3 = \frac{45.3 \times 18}{(25.75)^2} = 1.24$$

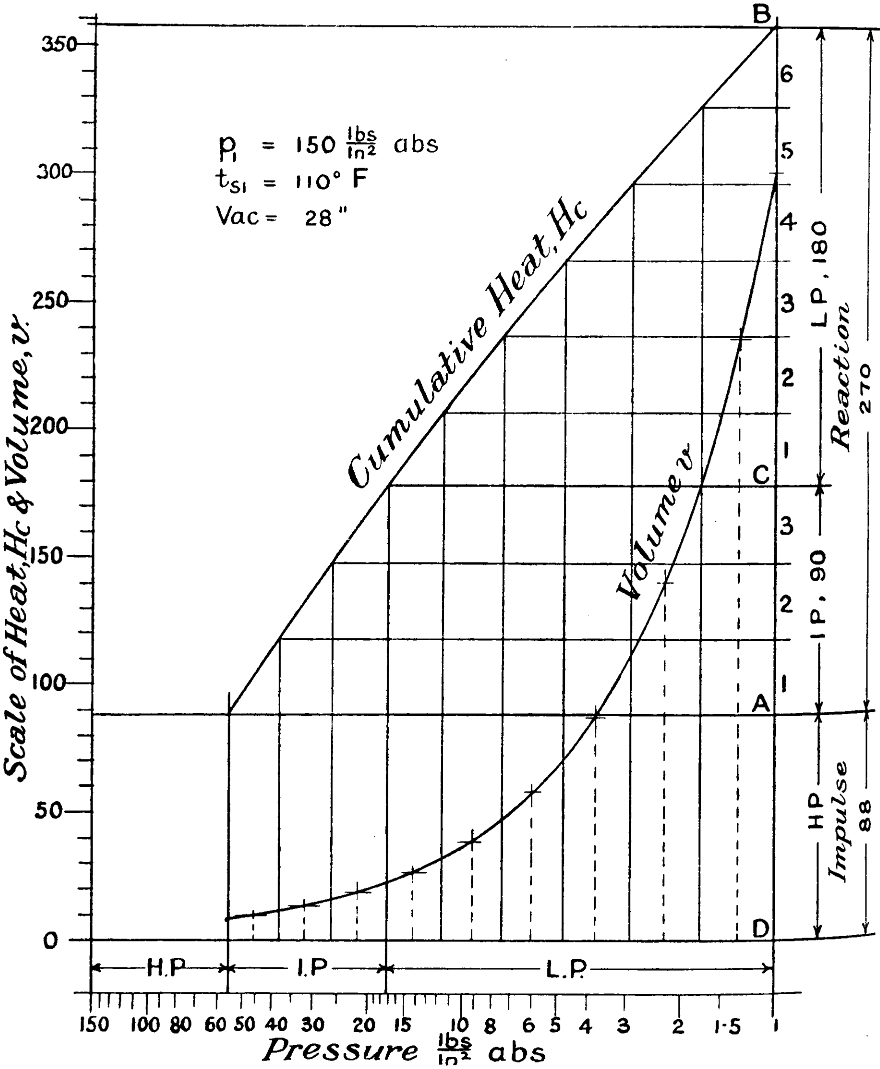


FIG. 206.

The value lies between $1\frac{1}{4}$ and $1\frac{1}{2}$ inches, so that the intermediate standard $1\frac{3}{8}$ inches may be taken. The nearest standard to the geometrical mean between first and third lengths is 1 inch.

The three lengths are thus

$$\tfrac{3}{4} \text{ inch}, \qquad 1 \text{ inch}, \qquad 1\tfrac{3}{8} \text{ inches},$$

and by equation (13) the common ratio is $r = 1{\cdot}355$.

At the L.P. section suppose an increased angle of 29° is used at the last ring, then as in example 1

$$\rho' = 0{\cdot}512 \quad \text{and} \quad C' = \frac{C}{1{\cdot}123}$$

By equation (5)

$$D_l = \frac{C}{1{\cdot}123}\sqrt[3]{\frac{\rho' W}{N}}$$

Allowing a 3 per cent. dummy leak, the value of W becomes 5·3, and taking for trial $z = 0{\cdot}15$, the coefficient from the curves of Fig. 198 is $C = 33$.

Hence $$D_l = \frac{33}{1{\cdot}123}\sqrt[3]{\frac{0{\cdot}512 \times 5{\cdot}3}{2000}} = \frac{33}{1{\cdot}123 \times 9{\cdot}03} \doteqdot 3{\cdot}2 \text{ feet}$$

The corresponding drum diameter is

$$D_L = 0{\cdot}85 \times 38{\cdot}4 = 32{\cdot}6 \text{ inches, say } 32\tfrac{5}{8} \text{ inches}$$

The common drum ratio is $r_1 = 1{\cdot}34$ or nearly the same as that of the I.P. blading.

By equation (14) the approximate length of the first L.P. blade is

$$l_1 = \frac{1{\cdot}375 \times 1{\cdot}355}{(1{\cdot}34)^2} = 1{\cdot}04, \text{ say } 1\tfrac{1}{8} \text{ inches to the nearest } \tfrac{1}{8} \text{ inch}$$

This gives a ring diameter $33\tfrac{3}{4}$ inches.

In this case

$$b = \frac{45{\cdot}3 \times 0{\cdot}97}{0{\cdot}96} = 45{\cdot}7$$

Checking the length

$$l_1 = \frac{45{\cdot}7 \times 26}{(33{\cdot}75)^2} = 1{\cdot}04$$

The volume ratio at first and fifth expansions is $\frac{140}{26} = 5{\cdot}4$, and the approximate length of the fifth expansion is $l_5 = 5\tfrac{3}{4}$ inches. This will be on the high side. For a trial, assume 5 inches. This gives a ring diameter $37\tfrac{5}{8}$ inches.

Then, $$l_5 = \frac{45{\cdot}7 \times 140}{(37{\cdot}625)^2} = 4{\cdot}53 \text{ inches}$$

With the slightly decreased diameter the length will be nearer $4\tfrac{5}{8}$ inches, so that this value may be taken. This gives a ratio value $z = 12{\cdot}5$ per cent., instead of 15 per cent. used in the preliminary calculation.

The common ratio for the lengths between first and fifth is

$$r = \sqrt[4]{\frac{4{\cdot}625}{1{\cdot}125}} \doteqdot \sqrt{2}$$

The standard blade lengths to the nearest $\frac{1}{8}$ inch are

$$1\tfrac{1}{8}'', \qquad 1\tfrac{5}{8}'', \qquad 2\tfrac{1}{4}'', \qquad 3\tfrac{1}{4}'', \qquad 4\tfrac{5}{8}'', \qquad 4\tfrac{5}{8}''.$$

For the increased angle, at the sixth expansion

$$\tan\theta_6 = \tan\theta_5 \frac{v_6}{v_5} = 0{\cdot}364 \times \frac{236}{140} = 0{\cdot}614, \text{ and } \theta_6 = 31^\circ\ 40', \text{ say } 32^\circ$$

For the calculation of the number of rings the constant of equation (17) is

$$c = \frac{30 \times 778 \times 32{\cdot}2}{\dfrac{2 \times 0{\cdot}94}{0{\cdot}55} - 1} = 310000$$

and
$$n = \frac{310000\eta_s}{\gamma u^2}$$

The mean value of the stage efficiency being assumed as 0·73, the values of 0·72 and 0·78 may be taken for the I.P. and L.P. sections. The necessary data with the derived values of n are given in the table below.

I.P. Drum Diam. $24\frac{1}{4}$ inches. L.P. Drum Diam. $32\frac{5}{8}$ inches.

Expansion.	1.	2.	3.	1.	2.	3.	4.	5.	6.
Ring diam., D ins.	25	$25\frac{1}{4}$	$25\frac{7}{8}$	$33\frac{3}{4}$	$34\frac{1}{4}$	$34\frac{7}{8}$	$35\frac{7}{8}$	$37\frac{1}{4}$	$37\frac{1}{4}$
Blade length, l ins.	$\frac{3}{4}$	1	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{5}{8}$	$2\frac{1}{4}$	$3\frac{1}{4}$	$4\frac{5}{8}$	$4\frac{5}{8}$
Blade vel., u ft./sec.	218	220	224	294	298	304	313	325	325
Coefficient, γ	0·905	0·924	0·943	0·92	0·943	0·961	0·972	0·98	0·98
No. of moving rings, n.	(5·2)	(5)	(4·73)	(3·04)	(2·9)	(2·72)	(2·54)	(2·34)	(2·34)
	5	5	5	3	3	3	3	2	2

The total adjusted number is 31 on the reaction section, 15 on the I.P. and 16 on the L.P.

Applying the coefficient method for a final comparison, from Fig. 204 at $\rho = 0{\cdot}55$, $\delta = 1{\cdot}04$. Also $RH_r = 270$.

Hence
$$\lambda = 0{\cdot}55^2 \times 1314 \times 270 \times 1{\cdot}04 = 112000$$

$$\frac{1}{x_2} = \frac{1}{3}, \qquad \frac{1}{x_3} = \frac{2}{3}$$

By equation (24)

$$A = \frac{10^6 \times 112000}{(25{\cdot}4)^2 \times (2000)^2} = 43{\cdot}5$$

No. of rings on H.P. section, $n_2 = \dfrac{43{\cdot}5}{3} = 14{\cdot}5$, say 15

" " L.P. " $n_3 = \dfrac{43{\cdot}5 \times 2}{3 \times (1{\cdot}34)^2} = 16{\cdot}1$, say 16

In this case the figures given by the coefficient method are the same as those by the longer calculation. If the higher average stage efficiencies of 0·75 and 0·80 for the I.P. and the L.P. sections had been taken, as in the previous cases, the calculated numbers would be greater.

It will be understood that in this, as in all the previous cases, the dimensions obtained represent only one of several alternatives that might be used, so much depends on the ideas of the man who designs, regarding speeds, efficiencies, etc.

A modified form of the general method given for the calculation of the axial flow reaction turbine might be applied to the case of the radial flow Ljungström type. It is, however, both tedious and clumsy. The method developed by the author is based on the use not of the cumulative heat, Σh_r, but on the nett heat, Σh_1. The nett heat is calculable, when the number of blade rings, that can be inserted between the limiting radii, is provisionally chosen. The general principles underlying the design of this type, and the particular method of design suggested, are discussed in the next chapter.

CHAPTER XV

PROVISIONAL DETERMINATION OF GENERAL PROPORTIONS OF COMPOUND TURBINES

RADIAL FLOW REACTION TURBINE (LJUNGSTRÖM TYPE)

271. Calculation of Work done on a Single-motion Group of Blade Rings.—In this type the steam expands successively from the centre outwards through a series of concentric blade rings.

A section of a group of these concentric rings is shown in Fig. 207.

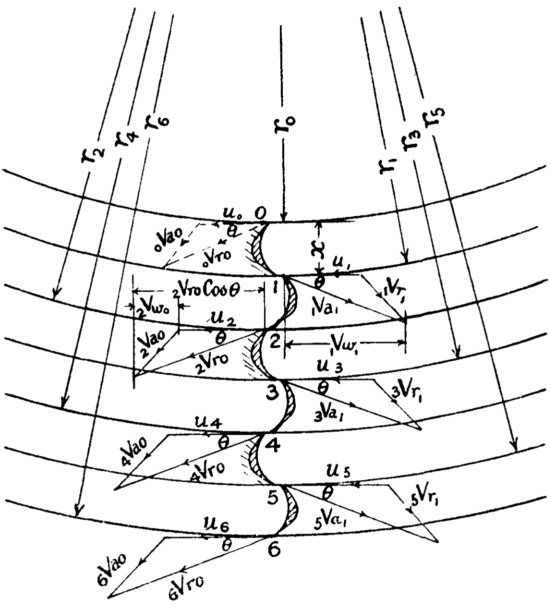


FIG. 207.

The first, third, and fifth are fixed, and the second, fourth, and sixth are movable.

In order to determine the work done on such a group, the fundamental equation (3), Art. 81, has to be applied to each ring.

Torque × angular velocity = work done

or $$\frac{\omega}{g}(r_1 V_{w_1} - r_0 V_{w_0}) = E_b \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where r_1 and r_0 are the radii at the entrance and exit edges of the blade ring, and V_{w_1} and V_{w_0} the corresponding entrance and exit velocities of whirl.

In this case the consecutive edges, beginning at the inlet edge of the first ring, are marked 0, 1, 2, 3, 4, 5, 6. The corresponding radii are denoted by these subscripts.

The entrance and exit triangles are drawn on the assumption that the ratio $\rho = \frac{\text{blade velocity}}{\text{steam velocity}}$ is constant, and that the exit blade angle θ is also constant.

Using the subscript notation, as shown in the figure, the velocities of whirl at entrance and exit edges of the first moving ring (the second of the group) are

$${}_1V_{w_1} = {}_1V_{a_1} \cos \theta, \quad \text{and} \quad {}_2V_{w_0} = {}_2V_{r_0} \cos \theta - u_2$$

Substituting in equation (1), the work done on this ring per lb. per second is

$${}_2E_b = \frac{\omega}{g}(r_1 \, {}_1V_{a_1} \cos \theta + r_2 \, {}_2V_{r_0} \cos \theta - r_2^2 \omega) \quad . \quad . \quad (2)$$

since $u_2 = r_2 \omega$.

The condition assumed for each blade is

$$\rho = \frac{u_1}{{}_1V_{a_1}} = \frac{u_2}{{}_2V_{r_0}} = \frac{u_3}{{}_3V_{a_1}} \quad . \quad . \quad . \quad \frac{u_6}{{}_6V_{r_0}} = \text{constant}$$

that is $$\rho = \frac{\omega r_1}{{}_1V_{a_1}} = \frac{\omega r_2}{{}_2V_{r_0}} = \frac{\omega r_3}{{}_3V_{a_1}} \quad . \quad . \quad . \quad \frac{\omega r_6}{{}_6V_{r_0}} = \text{constant}$$

Substituting for ${}_1V_{a_1}$, ${}_2V_{r_0}$, etc., in (2) the work done is

$${}_2E_b = \frac{\omega^2}{g}\left\{\frac{\cos \theta}{\rho}(r_1^2 + r_2^2) - r_2^2\right\}$$

This general form of the equation can be used to calculate the work done on any moving ring of the radial flow turbine

Hence the work done on the next moving ring, the fourth of the group, is

$${}_4E_b = \frac{\omega^2}{g}\left\{\frac{\cos \theta}{\rho}(r_3^2 + r_4^2) - r_4^2\right\}$$

The work done on the sixth ring is

$${}_6E_b = \frac{\omega^2}{g}\left\{\frac{\cos \theta}{\rho}(r_5^2 + r_6^2) - r_6^2\right\}$$

Expressed in general terms the work done on a group of n rings, having a constant blade angle θ and speed ratio ρ, is

$$_n\mathrm{E}_b = \frac{\omega^2}{g}\left(\frac{\cos\theta}{\rho}\Sigma r^2 - \Sigma r_e^2\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here Σr^2 denotes the sum of the squares of the exit radii of all the rings, the number of values being equal to the total number of rings.

Σr_e^2 denotes the sum of the squares of the exit radii of the moving rings (even numbers), and the number is half the total number of rings.

272. Work done on Double-motion Group of Blade Rings.—The combination constitutes a "single-motion" machine. If the initially fixed set of rings is also caused to rotate in the reverse direction, the combination becomes a "double-motion" machine. It is kinematically equivalent to the single-motion machine running at double the angular velocity.

The work done on a double-motion group can therefore be calculated, by treating the group as a single-motion combination, having *twice* the actual angular velocity. The calculation has to be made for an even number of rings.

If the radial pitch of the rings has a constant value x, the successive radii become

$$r_1, \; r_2 = (r_1 + x), \quad r_3 = (r_1 + 2x), \quad r_4 = (r_1 + 3x), \text{ etc.}$$

The summation of the squares of all the radii is given by

$$\Sigma r^2 = nr_1^2 + r_1 x(n^2 - n) + \frac{x^2}{6}(n^2 - n)(2n - 1)$$

The summation of the squares of the even numbers is

$$\Sigma r_e^2 = \frac{n}{2}r_1^2 + r_1 x\frac{n^2}{2} + \frac{x^2}{6}n(n^2 - 1)$$

Substituting these values in (3), the general equation for the work done on a group of n rings becomes

$$_n\mathrm{E}_b = \frac{\omega^2}{g}\left[\frac{\cos\theta}{\rho}\left\{nr_1^2 + r_1 x(n^2 - n) + \frac{x^2}{6}(n^2 - n)(2n - 1)\right\} - \left\{\frac{n}{2}r_1^2 + r_1 x\frac{n^2}{2} + \frac{x^2}{6}n(n^2 - 1)\right\}\right] \quad (4)$$

Since the pitch x is constant, it may be expressed as a fraction of the initial exit radius r_1, and equation (4) can be reduced to a more manageable form. Let $x = ar_1$, then

$$\Sigma r^2 = r_1^2\left\{n + a(n^2 - n) + \frac{a^2}{6}(n^2 - n)(2n - 1)\right\} = r_1^2\mathrm{A}$$

$$\Sigma r_e^2 = r_1^2\left\{\frac{n}{2} + a\frac{n^2}{2} + \frac{a^2}{6}n(n^2 - 1)\right\} = r_1^2\mathrm{B}$$

General Equation for Calculation of Work done on Single- or Double-motion Group.—When the initial radius is expressed in inches the work done on the group is given by

$$_{n}E_{b} = \frac{\omega^2 r_1^2}{144g}\left(\frac{\cos\theta}{\rho}A - B\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The values of the coefficients A and B corresponding to a series of n values can be calculated and plotted on a base of $\left(\frac{x}{r_1}\right)$. With the aid of the curves thus obtained, the work done on any group having a constant angle and speed ratio can be calculated with a minimum of labour.

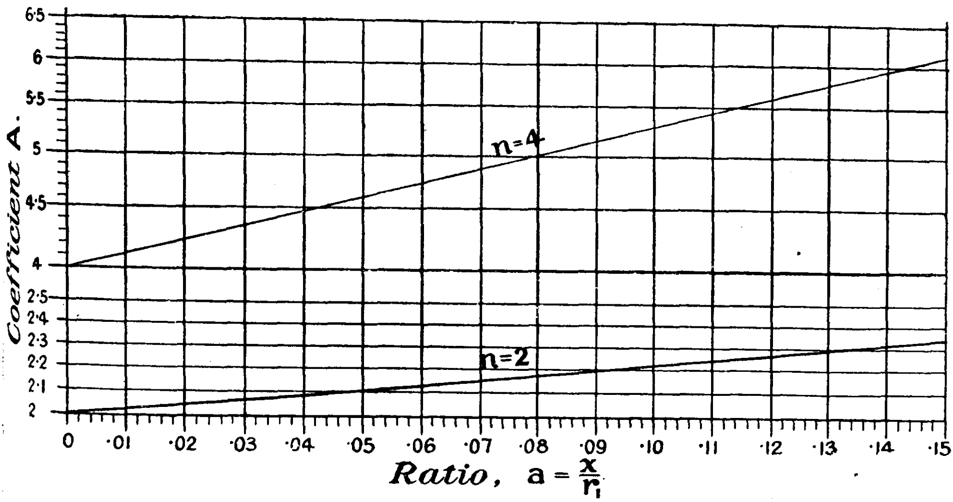


FIG. 208.

Subdivision of Total Number of Rings into Groups.—The present representative of the radial flow type, the Ljungström turbine, consists of a series of such groups, in which both ρ and x vary from the centre outwards.

It is advisable to confine the maximum number of rings per group to four, and for this condition only two A and two B curves, for two and four rings per group, are required. These four curves are given in Figs. 208 and 209. A uniform scale has been used for the $\left(\frac{x}{r_1}\right)$ values, and a logarithmic scale for the A and B values.

If desired, a chart consisting of a large number of curves may be constructed.

The total work done on the blading of a Ljungström turbine of specified blade dimensions can be calculated by the application of equation (5) to the successive groups.

Calculation of Nett Heat per Group.—For the determination of the blade lengths it is necessary to obtain the heat equivalent of the work done on any group. This is given by

$$h_1 = \frac{\omega^2 r_1^2}{144 \mathrm{J}g}\left(\frac{\cos\theta}{\rho}\mathrm{A} - \mathrm{B}\right) = cr_1^2\left(\frac{\cos\theta}{\rho}\mathrm{A} - \mathrm{B}\right) \text{B.Th.U./lb. sec.}\Big\} \quad (6)$$

273. "Carry-over" Increase at Successive Groups.—In the radial flow machine the blade speed is progressive throughout any group, and hence with constant speed ratio ρ, and blade angle θ, the "carry-over" velocity progressively increases from the first to the last ring of the group.

The heat drop in any blade ring is accounted for by the nett work

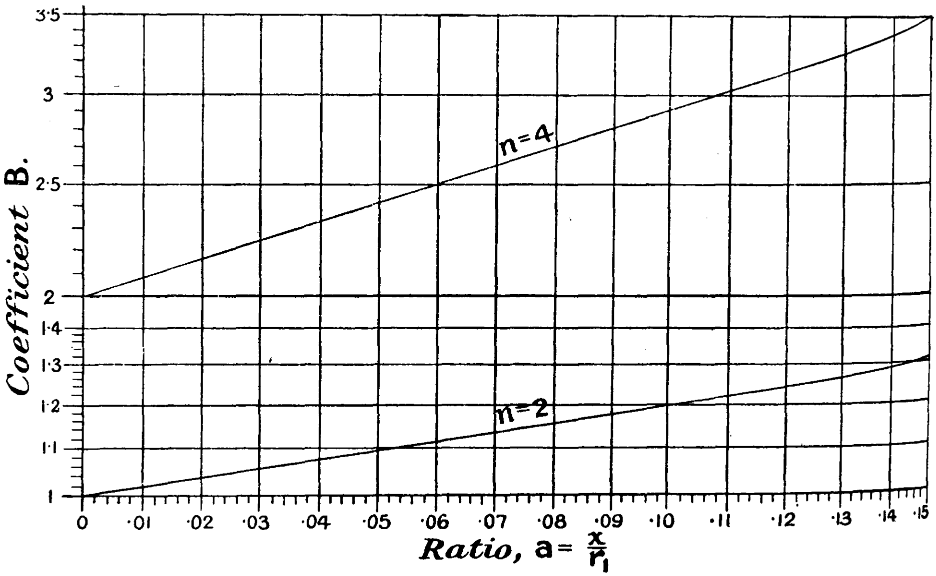


FIG. 209.

done on the blades, the friction loss, and the increase in the amount of carry-over.

The friction heat remains in the steam and affects the quality, but the increase in "carry-over" does not. The effect on the Hϕ diagram is the same as though this latter quantity represented additional work done.

For this case the energy equation may be written

$$\mathrm{J}\eta_s h_r = (\mathrm{E}_b + \mathrm{E}_e) = \mathrm{E}_b'$$

where h_r = heat drop in a stage.
E_b = work done on the blading.
E_e = increase in carry-over between entrance and exit.
η_s = stage efficiency.

Denoting $\frac{E_b'}{J}$ by h_s, $\eta_s = \frac{h_s}{h_r}$

With a constant value of η_s for the group, the total nett heat is

$$\Sigma h_s = \eta_s \Sigma h_r = h_I$$

If H_r is the Rankine cycle heat (or heat drop) between the pressure limits of the group, the internal efficiency is

$$\eta_1 = \frac{h_I}{H_r}$$

The value of $\frac{E_b}{J}$ is obtained from equation (6).

274. Calculation of Carry-Over Increase.—The carry-over increase $h_e = \frac{E_e}{J}$ can be calculated as follows:—

Referring to Fig. 207, consider the first ring of the group. The carry-over from the previous ring is the jet energy at entrance, or $\frac{{}_0V_{a_0}^2}{2g}$. The carry over to the next ring is $\frac{{}_1V_{r_1}^2}{2g}$.

Hence
$$E_e = \frac{{}_1V_{r_1}^2 - {}_0V_{a_0}^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

With a constant value of ρ, and constant angle θ, the ratio of the carry-over velocity at any ring and the exit velocity from the previous ring, is constant. Thus, in terms of the notation on the figure

$$z = \frac{{}_0V_{a_0}}{{}_0V_{r_0}} = \frac{{}_1V_{r_1}}{{}_1V_{a_1}} = \frac{{}_2V_{a_0}}{{}_2V_{r_0}}, \text{ etc.}$$

The constant z for several angles and speed ratios can be obtained from the curves of Fig. 210.

Now ${}_0V_{r_0} = \frac{u_0}{\rho}$ and ${}_1V_{a_1} = \frac{u_1}{\rho}$, hence ${}_1V_{r_1}^2 = \frac{z^2u_1^2}{\rho^2}$; ${}_0V_{a_0}^2 = \frac{z^2u_0^2}{\rho^2}$

Substituting in (7) the carry-over increase for the first moving ring is

$${}_1E_e = \frac{z^2}{2g\rho^2}(u_1^2 - u_0^2) = \frac{\omega^2z^2}{2g\rho^2}(r_1^2 - r_0^2)$$

and the equivalent heat value is

$${}_1h_e = c(r_1^2 - r_0^2)$$

Similarly for the rest of the rings, the heat values of carry-over increase are

$${}_2h_e = c(r_2^2 - r_1^2), \quad {}_3h_e = c(r_3^2 - r_2^2), \text{ and so on}$$

Summing all the values for the six rings the total is

$${}_6H_e = c(r_6^2 - r_0^2)$$

Expressed generally for a group of n rings, the total carry-over increase is

$$_{n}H_{e} = \frac{\omega^{2}}{144 J g} \frac{1}{2} \left(\frac{z}{\rho}\right)^{2} \{r_{n}^{2} - (r_{1} - x)^{2}\} \quad . \quad . \quad . \quad . \quad (8)$$

where r_1 and r_n are the exit radii of the first and nth rings, and x the common blade pitch.

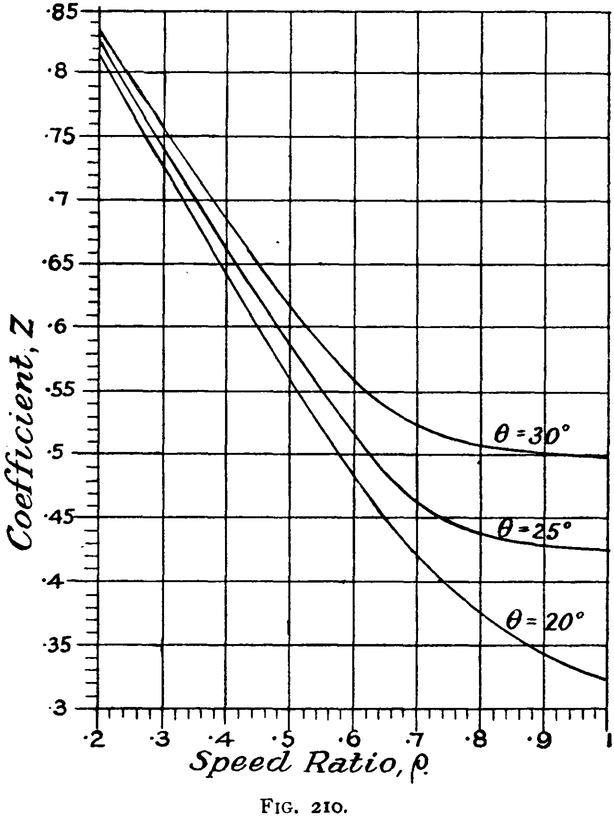


FIG. 210.

275. Influence of Carry-over Increase with Small Groups.—This quantity for small groups, as suggested here, is small in comparison with the heat expended in work on the blades, except in cases near the centre where the radii and the speed ratios are low.

For the practical purpose of determining the pressure and qualities of the steam from the $H\phi$ diagram for sets of four rings, this factor may be neglected for values of ρ above 0·5. For groups with ρ from 0·2 to 0·5 it is advisable to calculate the $_{n}H_{e}$ value, in order to ascertain if it has any appreciable effect on the nett heat value, or conversely for a given heat value the effect on the value of ρ.

In the latter case the combined equation for the equivalent nett heat becomes

$$h_1 = \frac{\omega^2 r_1^2}{144 Jg}\left[\left\{\frac{\cos\theta}{\rho}A - B\right\} + \frac{1}{2}\left(\frac{z}{\rho}\right)^2\left\{\left(\frac{r_n}{r_1}\right)^2 - \left(\frac{r_1 - x}{r_1}\right)^2\right\}\right] \quad (9)$$

The summation of the nett heat values for the whole machine or any given section of it should be approximately equal to $\eta_1 H_r$, where H_r is the total heat drop and η_1 the corresponding internal efficiency.

276. Variation of Speed Ratio throughout the Turbine.—The maximum value of the speed ratio ρ that may be reached will depend on the probable value of the mean stage efficiency η_s, to be expected with given conditions of output, pressure, superheat, and vacuum.

These ρ values are progressive, increasing towards the L.P. end. At the last two groups, consisting of two blade rings each, it has to be somewhat reduced, and the angle opened so as to prevent too great a divergence of the blade channel, and to keep down the length of blade. The values may run from 0·7 or 0·8 near this end, falling to, say, about 0·65 at the last group. By assuming trial values between 0·8 and 0·6 at the last four or five groups the nett heat values can be calculated, and a nett heat curve, sufficiently good for practical purposes, can then be drawn. From this curve the approximate heat values for the rest of the groups can be scaled, and the corresponding provisional values of ρ determined from equations (6) or (9).

277. Construction of Nett Heat Curve for Calculation of ρ Values.—In order to determine the nett heat curve, lay off on a base line EB, Fig. 211, a scale of ring numbers. At the end ordinate scale up the total nett heat value $BA = \eta_1 H_r$, and obtain the last point A on the curve. The point E at the zero of the scale is the first point on the curve. Calculate the h_1 values for the last four or five groups, using the assumed ρ values. Subtract these in succession from the total value AB, and obtain the corresponding ordinate values at the successive groups, giving the points *b*, *c*, *d*, *e*, etc. By using a good spline a satisfactory curve can be drawn through these points and the initial and final points E and A. The curve can now be used to ascertain approximately the rest of the group heat values, and also to determine the corresponding pressures and qualities on the $H\phi$ diagram.

In the calculation of the ρ values by equation (6), when the figure 0·5 is reached, the calculated value of ρ should be increased at the rest of the groups and used in equation (9), so as to obtain the desired nett heat values given by the curve.

As already indicated, it is quite sufficient for practical purposes to take four ring groups, unless where change of pitch or angle reduces the number to two. The arithmetical work is thus materially lessened.

278. Construction of Volume Curve from Nett Heat Curve.—When the nett heat curve is drawn on a base of ring numbers the volume curve on the same base is easily derived. The cumulative nett heat values are scaled down vertically in succession from the initial state point on the $H\phi$ diagram. Projectors drawn to the condition curve give the pressures and qualities at exit from the successive groups, and the volumes

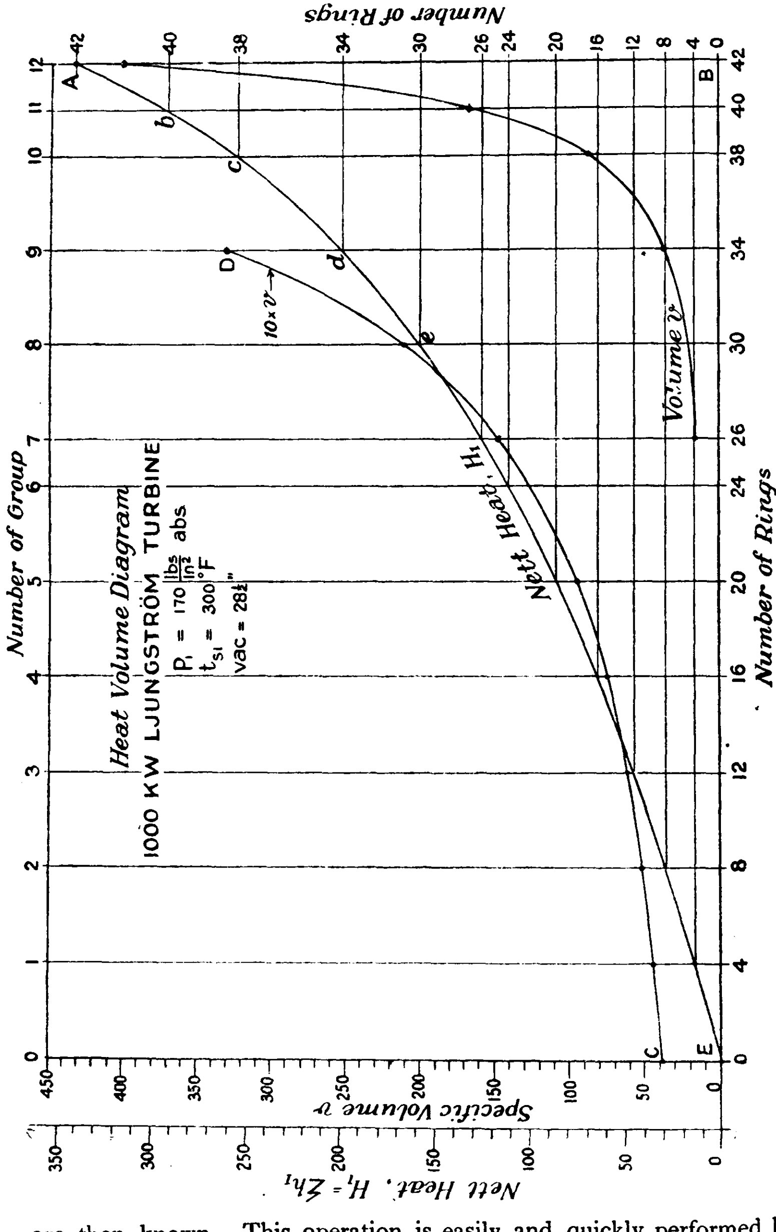


FIG. 211.

are then known. This operation is easily and quickly performed by transferring the values on a strip of paper. When the volume curve is plotted, all the data necessary for the calculation of the progressive blade lengths are available.

Calculation of Blade Lengths.—When the volume at exit from any group is ascertained from the volume curve, the corresponding blade length at the last ring can be calculated by the equation of continuity

$$W = \frac{2\pi r_0 l_0 V_0 \sin\theta_0}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where r_0 = exit radius of the ring.
l_0 = „ length.
V_0 = „ steam velocity.
v_0 = „ volume.
θ_0 = „ blade angle.
W = weight of steam in lbs./sec.

Since $V_0 = \frac{u_0}{\rho} = \frac{\omega r_0}{\rho}$, equation (10) can be written in the form

$$W = \frac{\omega}{\rho}\frac{r_0^2 2\pi l_0 \sin\theta_0}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For the normal angle $\theta_0 = 20°$, $\sin\theta_0 = 0{\cdot}342$. Expressing the angular velocity in terms of the revolutions per minute N_1, the blade length in inches, when r_0 is in inches, is given by

$$l_0 = \frac{7700 W v_0 \rho}{r_0^2 N_1} = \frac{C\rho v_0}{r_0^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

For single motion $N_1 = N$; for double motion $N_1 = 2N$, where N = revolutions/min. of the shaft.

In order to determine the exit height at the first ring of the turbine, the initial volume may be taken as that of the steam at the inlet pressure.

If the angle is increased at any group towards the L.P. end from θ_0 to θ_0' the coefficient C in (12) has to be multiplied by the ratio $\left(\frac{\sin\theta_0}{\sin\theta_0'}\right)$.

The blade lengths thus calculated can be scaled off on a previously drawn skeleton diagram of the blade pitching, and fair curves drawn through the points determine the form of the blade channel. When the blade widths are drawn with the requisite clearances, the exact values of inlet and outlet length of every blade can be scaled from this drawing.

279. Values of Speed Ratio ρ and Stage Efficiency η_s.—In order to fix the general dimensions of a machine of this type, to conform to given conditions of output, pressure, superheat, and vacuum, the condition curve has first to be drawn on the $H\phi$ diagram and the steam consumption estimated.

The special design of this turbine reduces the equivalent "tip leakage." In the case of double motion the average speed ratio is much more favourable than in the axial flow type. Higher values of stage and internal efficiencies are therefore attained.

The values of ρ for the double-motion machine may vary from 0·5 to 0·8, and the stage efficiency from 80 to 85 per cent. For single motion slightly higher values than those for Parsons machines, say, 75 to 80 per cent., may be attained.

In some cases where the inner ring is of small diameter the speed ratio may fall as low as 0·2, and rise to 0·75 or 0·8 at the low-pressure end. There may in such a case be a variation of stage efficiency between 0·68 and 0·85. A considerable variation of stage efficiency at the higher pressures, as already pointed out in Art. 183, does not, however, appreciably affect the volume values, and an average value of $\eta_s = 0{\cdot}8$ to $0{\cdot}82$ may be taken for the determination of reheat factor and internal efficiency, and the location of the condition curve on the $H\phi$ diagram.

280. Estimation of Efficiency Ratio η.—For the estimation of the steam consumption the efficiency ratio η may be calculated on the assumption of an additional loss from 6 to 4 per cent. of the heat drop, covering journal, pump, and gear friction, leakage, radiation, and carry-over. That is, the efficiency ratio for a small unit may be taken as $\eta = (\eta_1 - 0{\cdot}06)$, and for a large unit $\eta = (\eta_1 - 0{\cdot}04)$.

281. Limiting Diameters of the Turbine.—A provisional estimate of the number of blade rings that can be inserted between the limiting radii has to be made. The larger the number of rings with a given heat drop, the higher the average value of ρ, and the efficiency of the turbine.

At the rated speed the diameter of the outer blade ring is controlled by the centrifugal stress. To keep within the permissible stress limits, the outer ring should not be run at a speed greater than 450 ft./sec. The usual speed is somewhat less than this.

The maximum diameter with the above limit is restricted to 2·8 feet at 3000 R.P.M. and 5·6 feet at 1500 R.P.M. For lower speeds the diameter is proportionally increased.

The exit diameter of the first ring at the centre is controlled by the mechanical conditions of the design of disc hub, shaft, and the passages for the efficient supply of steam to the blade channel.

In the original 1000 K.W. machine running at 3000 R.P.M., this diameter was 6 inches. In recent designs by recessing the disc into the hollow hub it has been reduced to $3\frac{3}{4}$ inches, and a larger number of rings can be used.

In large units, 4000 to 10,000 K.W., this diameter may range from 10 to 18 inches. It is not possible to lay down any definite rule on this point.

When provisional values of inner and outer radii are chosen the approximate number of rings that can be inserted between the limits has to be estimated.

282. Standard Proportions of Ljungström Blades.—This number for a continuous blade channel depends on the blade proportions permissible at given radii.

The length of blade at any ring must be such that the blade will be sufficiently stiff to withstand the deflecting action of centrifugal force. The length and width are interdependent. Experience indicates that the maximum ratio of length to width may be taken from 8 to 10.

The inventors of this machine have endeavoured to reduce the width to as low a value as is consistent with safety and the limitations of manufacture.

The minimum width of blade adopted is $\frac{3}{16}$ inch. As can be seen from the drawing, Fig. 36A, the section of the blade channel is nearly parallel for the major portion of the length, and until the passage begins to diverge appreciably the smaller blade widths may be used.

Five standard widths are employed, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{5}{8}$ inch. The majority consists of $\frac{3}{16}$ and $\frac{5}{16}$ inch blades. The $\frac{1}{2}$ and $\frac{5}{8}$ inch blades are used at the last four rings, where the blade angle has to be opened out to keep down the divergence of the blade channel.

283. Radial Blade Clearance.—A radial clearance varying from $\frac{1}{16}$ inch at the H.P. to $\frac{1}{8}$ inch at the L.P. end of the channel is allowed between the blades. The corresponding approximate pitches are $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{4}$ inch. These may be used in heat calculations.

For large outputs the final blade length at the last ring would become prohibitive and the divergence of the channel sides excessive.

284. Subdivision into High and Low Pressure Sections for Large Output.—In such a case the turbine has to be divided into high and low pressure sections, as shown in Fig. 37.

The long L.P. blade length is subdivided into short lengths, each from five to eight times the width, by means of intermediate stiffening rings. This is equivalent to inserting several low-pressure turbines in parallel, each of which handles its proportion of the total steam that passes through the H.P. section.

285. Estimation of Total Number of Blade Rings.—For a continuous channel the approximate number of rings may be estimated by dividing the difference of the extreme radii by a mean pitch value, say, 0·33 inch. Fully half this number may be pitched $\frac{1}{4}$ inch, and the remainder divided in decreasing proportion among the other pitches, the second last pair being pitched $\frac{5}{8}$ inch and the last pair $\frac{3}{4}$ inch.

When a decision has been made regarding the set of rings a skeleton diagram of the blade pitching should be drawn down, preferably full size. In doing so provision has to be made for an overload bye-pass passage. This may be taken as equal to one of the smaller blade pitches in radial width. It is usually located at about half the number of rings out from the centre.

The set of rings should be divided into groups of four and two, and the exit radii of the first and last ring of each group should be marked on the drawing, as indicated in Figs. 212 and 213.

286. Calculation of Nett Heat or Speed Ratio Values for Successive Groups.—In order to calculate the nett heat values for the successive groups, or the ρ values when the heats are known, the data should be set out in tabular form as follows :—

Number of group; number of rings per group (n); pitch x; exit radius of first ring (r_1); ratio, $a = \frac{x}{r_1}$; coefficients A and B obtained from the curves, Figs. 208 and 209; speed ratio (ρ); nett heat per group h_I; total nett heat Σh_I; coefficient z.

287. Nett Heat-Volume Diagram ($H_I - v$).—The curve of nett heat values, Fig. 211, should be plotted on a base of ring numbers to the same heat scale as the $H\phi$ diagram. The ordinates at the end of each group should be ticked off on the edge of a strip of paper, and the points marked with the corresponding ring numbers.

This strip should be placed on the adiabatic vertical of the $H\phi$ diagram, with the zero value on the initial state point. Projectors from the points to the condition curve determine the corresponding pressures and qualities at exit from the successive groups. The volumes can then be obtained from the steam tables or preferably from the alignment chart, as a considerable number are volumes of superheated steam.

The volumes should be plotted on the same base as the nett heats, and a fair curve drawn. It is necessary to subdivide this curve, ten times the volume being plotted at the high-pressure end to give the curve CD, Fig. 211.

288. Tabular Method of Calculation for Blade Lengths.—From the volumes given by the faired curve, the exit blade heights at the different groups can be calculated. The data for this calculation should be tabulated below those of the nett heat and speed ratio calculation, as shown on p. 474. These are, exit radius (r_0), pressure p, quality t_s or q, volume v_0, blade length l_0, entrance angle θ_1, exit angle θ_0.

The calculated exit lengths are set off from the centre line of the skeleton diagram, as shown in Fig. 212; and fair curves, giving the contours of the blade channel, are drawn through the points. Any slight irregularity due to changes of ρ values from group to group can be easily smoothed out in this way.

289. Estimation of Blade Angles.—The exit angles are arbitrarily chosen, and except at the last two groups may be taken as sensibly constant at 20°. Actually, in these machines, the exit angle is varied slightly. Over groups of three or four rings it is alternately increased and decreased from 1° to 3°, the value ranging between 18° and 21°.

For a provisional estimate, it seems quite sufficient to take the average of 20°, except at the last few rings, where the influence of increasing volume at high vacuum is most marked.

The entrance angle θ_1 at each group, or each ring, if a subsequent detail adjustment is made, can be calculated from the equation (22) already deduced in Art. 90, and repeated for convenience

$$\tan \theta_1 = \frac{\sin \theta_0}{\cos \theta_0 - \rho} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where θ_0 is the exit angle of the previous blade ring.

290. Subdivision of Blade Rings into H.P. and L.P. Groups for Large Output.—In the case of the compounded type for large output, having a separate L.P. section, the most expeditious method of determining the total number of rings is first to assume a number for the L.P. section. Generally a set of six rings will suffice. The pitching, for machines up to, say, 10,000 K.W., may be taken as $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{4}$ inch. This provisional choice of blades and pitches enables the approximate value of the inlet diameter at the L.P. section to be fixed.

A radial distance varying from 1 to 3 inches may be allowed between the last H.P. blade ring and the first L.P. one, the value depending on the size of the machine. The space has to be sufficient to permit the efficient flow of steam from the central H.P. blade channel to the L.P. wing channels.

The external radius of the H.P. section thus being fixed, the number of H.P. blade rings can be estimated, as in the previous instance, by using a mean pitch about 0·33. The pitching may be $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ inch for this section. For larger outputs 15,000 to 30,000 K.W., where longer blades are required for the passage of the large quantities of steam, a modification of these figures would be necessary, so as to keep the ratio of length to breadth about the limit of 10 to 1.

So far, however, machines for such large output have not been constructed, and the guide of experience in this matter is lacking.

291. In settling the nett heat curve, the L.P. section should be dealt with first, taking, say, 0·65 to 0·68 for ρ at the last stage. With the subdivision of the L.P. channel into from four to six sections (see Art. 29), the divergency of the channel, producing the non-effective axial component of the velocity, is materially reduced, and a smaller discharge angle, ranging between 20° and 25°, may be taken, say, 20° at the second last, and 25° at the last two-ring stage. A considerable amount of judgment is, of course, necessary, but these limits give some idea of the values to choose for a trial. With the assumed values of θ_0 and ρ the nett heats of the L.P. blade groups can be calculated. By scaling these successively from the final state point on the condition curve and projecting to the curve, the pressures and volumes are found, and the blade lengths are then calculable. If the form of the subdivided L.P. blade channel, Fig. 213, is not considered satisfactory, some modification either of angle or speed ratio or both may be made, to give the required form. When this section is settled, the nett heat ordinates for the L.P. section can be used in conjunction with the initial and final points E and A, Figs. 211 and 214, to obtain the approximate heat curve, from which the h_I values can be scaled and the ρ values calculated. It may be necessary to adjust this curve slightly, to obtain a reasonable progression of ρ values and give the total, $H_I = \Sigma h_I = \eta_I H_r$. The rest of the calculation is the same as that given for the machine with continuous blade channel.

EXAMPLE 1.—Determine the blading proportions for a double-motion Ljungström turbine to develop 1000 K.W. at 3000 R.P.M. Initial pressure 170 lbs./in.^{2} abs., superheat 300° F., vacuum $28\frac{1}{2}$ inches, generator efficiency 0·95. Assume that the exit radius of the first ring may be reduced to $1\frac{3}{4}$ inches.

In accordance with the statement of Art. 279, an average stage efficiency $\eta_s = 0{\cdot}8$ may be assumed.

From the curves, Fig. 175, at 300° F. superheat, $R = 1{\cdot}043$ and $\eta_1 = 1{\cdot}043 \times 0{\cdot}8 = 0{\cdot}8344$.

Scaling the $H\phi$ diagram between 170 and 0·75 lbs./in.^{2} the heat drop is 408 B.Th.U.

The nett heat expended on the blading is $\eta_1 H_r = H_1 = 340{\cdot}5$.

Scaling this from the initial state point, and projecting to the 0·75 lbs./in.^{2} curve, the final state point for the condition curve falls on the quality curve $q = 0{\cdot}915$.

To find the intermediate point on the saturation curve

Since
$$
\begin{aligned}
f &= (1 - \eta_s) = 0{\cdot}2 \\
\phi_f &= 0{\cdot}055 + 0{\cdot}8f^3 - 0{\cdot}00006(p + t_s) \\
&= 0{\cdot}055 + 0{\cdot}8 \times 0{\cdot}2^3 - 0{\cdot}00006(170 + 300) = 0{\cdot}0332
\end{aligned}
$$

The corresponding line cuts the saturation curve at pressure 9·5.

It may be assumed in this case that external friction and leakage, etc., represent, say, 6 per cent. of the heat drop. The approximate efficiency ratio is thus

$$\eta = (\eta_1 - 0{\cdot}06) = (0{\cdot}8344 - 0{\cdot}06) = 0{\cdot}78$$

The rate of consumption is

$$w = \frac{3414}{\eta H_r \eta_g} = \frac{3414}{0{\cdot}78 \times 408 \times 0{\cdot}95} = 11{\cdot}25 \text{ lbs./K.W. hour at the generator.}$$

The total weight is

$$W = \frac{1000 \times 11{\cdot}25}{3600} = 3{\cdot}14 \text{ lbs./sec.}$$

Allowing, say, 2 per cent. leakage, the figure may be taken as

$$W = 3{\cdot}1$$

The external radius of the first ring is specified as $1\frac{3}{4}$ inches, and at the rated speed the external radius of the last ring should not be much in excess of 16 inches.

Into the difference $(16 - 1{\cdot}75) = 14{\cdot}25$ an arbitrary number of rings (n) has to be inserted. A blade pitch is to be allowed for an overload bye-pass.

With a mean pitch 0·33 the approximate number of pitches is 43, giving 42 rings.

The last pair of rings have a standard pitch of $\frac{3}{4}$ inch, and the second last pair $\frac{5}{8}$ inch.

The set may be distributed as follows :—

	26	at $\frac{1}{4}$ inch pitch	=	6·5 inches
	8	,, $\frac{3}{8}$,,	=	3 ,,
	4	,, $\frac{1}{2}$,,	=	2 ,,
	2	,, $\frac{5}{8}$,,	=	1·25 ,,
	2	,, $\frac{3}{4}$,,	=	1·50 ,,
Totals . .	42			$14\frac{1}{4}$,,

The blade pitch for the bye-pass passage may be fixed after the 20th ring.

The figures chosen are, of course, arbitrary, and subject to alteration according to individual judgment.

They represent approximately the values which have been adopted for the more recent designs of this size of the machine.

The skeleton arrangement of the blade pitching shown in Fig. 212

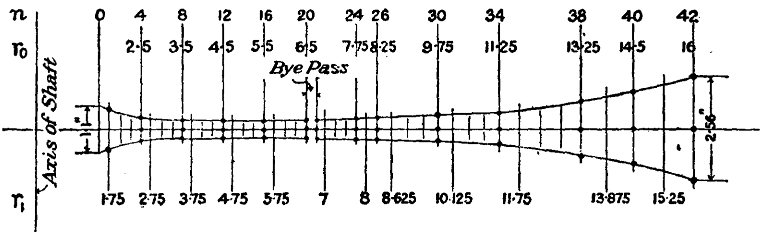


FIG. 212.

can now be drawn. As the $1\frac{3}{4}$-inch radius includes the width of the first blade ring, the total radius at the last ring is 16 inches, the limiting value fixed above.

The $\frac{1}{4}$-inch pitches divide into groups of four until the 24th ring is reached. The remainder of the rings pitched $\frac{1}{4}$ inch is two, and this pair is made a separate group. Between the 26th and 38th rings groups of four rings are taken. The 39th and 40th form one group, and the 41st and 42nd form another, the blade angle being progressively increased in each case.

The necessary data for the calculation of the nett heat and ρ values are given in the table below.

1000 KW. LJUNGSTRÖM TURBINE.

Group	1	2	3	4	5	6	7	8	9	10	11	12
No. of rings, n .	4	8	12	16	20	24	26	30	34	38	40	42
Rings per group .	4	4	4	4	4	4	2	4	4	4	2	2
Pitch, x inches .	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$
Radius, r_1 inches.	$1\frac{3}{4}$	$2\frac{3}{4}$	$3\frac{3}{4}$	$4\frac{3}{4}$	$5\frac{3}{4}$	7	8	$8\frac{5}{8}$	$10\frac{1}{8}$	$11\frac{3}{4}$	$13\frac{7}{8}$	$15\frac{1}{4}$
$a = \frac{x}{r_1}$	·143	·091	·067	·0526	·0435	·0358	·0313	·0434	·0370	·0425	·0450	·0491
Coefficients A .	6	5·2	4·85	4·65	4·55	4·42	2·05	4·54	4·43	4·5	2·1	2·11
Coefficients B .	3·45	2·83	2·58	2·45	2·35	2·28	1·05	2·35	2·32	2·3	1·075	1·09
Coefficient, z .	·813	·72	·68	·63	—	—	—	—	—	—	—	—
Speed ratio, ρ .	·2	·3	·35	·42	·51	·6	·63	·67	·7	·7	·68	·6
Nett heat, h_1 . .	12	14	17	20	22	25	14	33·6	40·7	56·5	36·2	49·5
$\Sigma h_1 = H_1$ . . .	12	26	43	63	85	110	124	157·6	198·3	254·8	291	340·5
Pressure, p lbs./ins.^{2} abs.	150	125	100	80	60	45	37	22	12	4·5	2	·75
Quality, t_s or q .	284	265	240	210	180	150	128	80	20	·97	·945	·915
Spec. vel., v_0 ft.^{3}.	4·4	5·1	6·1	7·3	9·3	12·6	14·8	21	33	87·6	164	400
Exit radius, r_0 ins.	2·5	3·5	4·5	5·5	6·5	7·75	8·25	9·75	11·25	13·25	14·5	16
Blade length, l_0 ins.	·56	·495	·42	·405	·445	·5	·545	·59	·73	1·39	1·72	2·56
Exit blade angle, θ_0°	20	20	20	20	20	20	20	20	20	20	25	30
Entrance blade angle, θ_1° * .	25	28	30	$33\frac{1}{2}$	$38\frac{1}{2}$	45	48	52	55	55	{53, 62	{55, 62

* The bracketed values in the last two columns give the entrance angles of rings 39, 40, 41, and 42.

At groups 12 and 11, the angles are opened out to 30° and 25° to keep down the divergence of the blade channel; and provisional values of ρ, at groups 12, 11, 10, and 9, are taken as 0·6, 0·68, 0·7, 0·7, for the trial calculation of the nett heat values. For a machine of this size, with the mean stage efficiency selected, it does not seem advisable to choose a greater value of ρ than 0·7.

Since the machine is a double-motion one, the effective angular velocity is $\omega = 2\pi \times 2\text{N} = 628 \frac{\text{rad.}}{\text{sec.}}$

Hence the value of the constant for equation (6) is

$$c = \frac{628^2}{144 \times 778 \times 32{\cdot}2} = 0{\cdot}109$$

Applying equation (6) for group 12,
Since cos 30 = 0·866

$$_{12}h_1 = 0{\cdot}109 \times 15{\cdot}25^2\left(\frac{0{\cdot}866}{0{\cdot}6} \times 2{\cdot}11 - 1{\cdot}09\right) = 49{\cdot}5$$

Similarly at the succeeding groups the calculated values are 36·2, 56·5, and 40·7.

The total nett heat is 340·5. Subtracting these values successively the corresponding group ordinates become 340·5, 291, 254·8, 198·3, 157·6.

These, when scaled from the base line EB, Fig. 211, give the points A, b, c, d, and e, and enable the heat curve AE to be drawn. The approximate values of the nett heats for the rest of the groups, obtained from this curve, are then entered in the table above, and equation (6) is used to calculate the corresponding values of ρ. The values so obtained for groups 1 to 4 are 0·155, 0·262, 0·333, and 0·43. These have to be increased to take account of the carry-over increase, since the actual heat expended in work on each group is less than the apparent value given in the table (see Art. 275).

Suppose at group 1 the ratio is taken as 0·2. From the curves, Fig. 210, $z = 0{\cdot}813$, also $x = 0{\cdot}25$, $r_1 = 1{\cdot}75$, and the exit radius of the last ring of the group is $r_n = 2{\cdot}5$. Hence by equation (9)

$$h_1 = 0{\cdot}109 \times 1{\cdot}75^2\left[\left\{\frac{0{\cdot}94}{0{\cdot}2} \times 6 - 3{\cdot}45\right\} + \frac{1}{2}\left(\frac{0{\cdot}813}{0{\cdot}2}\right)^2\left\{\left(\frac{2{\cdot}5}{1{\cdot}75}\right)^2 - \left(\frac{1{\cdot}75 - 0{\cdot}25}{1{\cdot}75}\right)^2\right\}\right] = 0{\cdot}109 \times 1{\cdot}75^2(24{\cdot}75 + 10{\cdot}7) = 11{\cdot}9$$

which is a close enough approximation to the value required, and 0·2 is a suitable figure to use for the calculation of the last blade length of the first group.

Selecting the trial value and calculating each of the others in the same way, the ratios for groups 2, 3, and 4 are found and entered in the table.

Actually the other ratios are slightly on the low side as the slight carry-over effect has been neglected. The complete set of values is a little uneven, but it should be understood that these are merely provisional, and any slight inequality in the blade heights resulting from this condition is easily remedied when the curves of the blade channel are drawn. When the final lengths are thus adjusted the actual value of the speed ratio ρ can, if desired, be calculated at each ring by means of the equation of continuity.

In order to determine the volume curve, horizontals are drawn from the successive points on the heat curve to cut the end ordinate BA, as shown in Fig. 211. The points are numbered to correspond with the ring numbers, the point B being the zero value of the derived scale.

Transferring this scale, on the edge of a strip of paper, to the adiabatic vertical on the Hϕ diagram with the zero value on the initial state point of the 170 lbs./in.^{2} curve, and projecting horizontally from the points to the condition curve, the pressures and qualities entered in the lower half of the table are found.

The corresponding volumes obtained from the alignment chart, Fig. 50, and the steam tables, are entered in the next line.

The blade lengths are now calculable. For groups with normal blades by equation (12)

$$l_0 = \frac{7700 \times 3{\cdot}1}{2 \times 3000}\frac{v_0\rho}{r_0^2} = 3{\cdot}98\frac{v_0\rho}{r_0^2}$$

The constant has to be modified at the last two groups.

At group 12, $\theta_0 = 30$, $\sin 30 = 0{\cdot}5$, also $\sin\theta = 0{\cdot}342$
" 11, $\theta_0 = 25$, $\sin 25 = 0{\cdot}422$

Then $$l_{12} = \frac{3{\cdot}98 \times 0{\cdot}342 \times 400 \times 0{\cdot}6}{0{\cdot}5 \times (16)^2} = 2{\cdot}56 \text{ inches}$$

$$l_{11} = \frac{3{\cdot}98 \times 0{\cdot}342 \times 164 \times 0{\cdot}68}{0{\cdot}422 \times (14{\cdot}5)^2} = 1{\cdot}72 \text{ "}$$

At the first normal group

$$l_{10} = \frac{3{\cdot}98 \times 87{\cdot}6 \times 0{\cdot}7}{(13{\cdot}25)^2} = 1{\cdot}39$$

All the other lengths are similarly calculated and entered in the table. At the first ring $r_0 = r_1 = 1{\cdot}75$, v_1 = values at 170 lbs./in.^{2} abs., and assuming the same speed ratio $\rho = 0{\cdot}2$

$$l \doteqdot \frac{3{\cdot}98 \times v_1 \times \rho}{r_1^2} \doteqdot \frac{3{\cdot}98 \times 3{\cdot}92 \times 0{\cdot}2}{1{\cdot}75^2} \doteqdot 1{\cdot}02 \text{ inches}$$

These values, set off equally on each side of the centre line of the skeleton blade diagram, Fig. 212, give a satisfactory form of blade channel. The angle of divergence of the channel at exit is about 30°. It is not advisable to have the angle much greater than this value. The entrance angles are calculated by equation (13), and the provisional data table is completed. With double motion the equivalent blade velocity at exit at the last ring is 840 ft./sec., and with a speed ratio $\rho = 0{\cdot}6$ as chosen, the velocity triangle with $\theta_0 = 30$ shows that the residual velocity of discharge is 800 ft./sec. The carry-over is thus $h_e = \left(\frac{800}{223{\cdot}7}\right)^2 = 12{\cdot}8$ B.Th.U. This represents $\frac{12{\cdot}8}{408} = 0{\cdot}0314$, or $3{\cdot}14$ per cent. of the heat drop, which is not an excessive figure.

EXAMPLE 2.—Determine suitable blading proportions for a compounded double-motion Ljungström turbine to develop 7500 K.W. at 1500 R.P.M. Initial pressure 185 lbs./in.^{2} abs., superheat 300° F., vacuum 29 inches, generator efficiency 95 per cent. Assume that the exit radius of the first ring may be made $8\frac{1}{2}$ inches, and the radial distance between the blades of the H.P. and L.P. sections is 3 inches.

In this case the conditions of pressure, superheat, and vacuum, combined with the larger size of unit, will ensure a higher stage efficiency. This may be taken as 0·83. The approximate value of the reheat factor from Fig. 175 is $R = 1{\cdot}037$, and the internal efficiency $\eta_i = 1{\cdot}037 \times 0{\cdot}83 = 0{\cdot}86$. The value of ϕ_f for the condition curve is 0·0298, while the final quality of the steam at exhaust is $q = 0{\cdot}895$.

The heat drop between 185 lbs./in.^{2} abs. and 0·5 lbs./in.^{2} abs. is 433 B.Th.U., and the nett heat $H_i = 0{\cdot}86 \times 433 = 372$..

The external friction loss will be smaller than that of the 1000 K.W. machine, say 4 per cent., and the efficiency ratio $\eta = (\eta_1 - 0{\cdot}04) = 82$ per cent.

Messrs. Ljungström, however, state that for a machine of from 7500 to 10,000 K.W., working under conditions similar to those assumed here, an efficiency ratio of 86 per cent. should be possible with a speed ratio of 0·75.

The steam consumption is

$$w = \frac{3414}{\eta H_r \eta_g} = \frac{3414}{0{\cdot}82 \times 433 \times 0{\cdot}95} \doteqdot 10 \text{ lbs./K.W. hour at the generator.}$$

Hence the weight per sec.

$$W = \frac{7500 \times 10}{3600} = 20{\cdot}8 \text{ lbs.}$$

Allowing about 2 per cent. for leakage the weight may be taken as $W = 20{\cdot}5$ lbs./sec.

The exit radius of the inner ring is $8\frac{1}{2}$ inches. Since the speed is reduced from 3000 to 1500 the outer diameter of the last ring may be made twice that of the 1000 K.W. turbine, that is 32 inches.

Taking the L.P. section first, assume that the last six blade rings are allocated to this, as follows, two at $\frac{3}{4}$-inch pitch, two at $\frac{5}{8}$-inch, and two at $\frac{1}{2}$-inch pitch. The inlet radius to the first L.P. ring is thus $(32 - 3\frac{3}{4}) = 28\frac{1}{4}$ inches. Deducting 3 inches from this, the exit radius of the last H.P. ring is $25\frac{1}{4}$ inches. The distance, $25\frac{1}{4} - 8\frac{1}{2} = 16\frac{3}{4}$, is available for the H.P. blading, which may be made up of blades pitched $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ inch. Taking an average pitch of 0·33, the approximate number of pitches is $\frac{16{\cdot}75}{0{\cdot}33} = 51$. Allowing a blank pitch for the bye-pass opening, the number of rings on the H.P. section may be taken as 50.

This gives a total of 56 rings for the whole machine. For the H.P. section they may be allocated as follows :—

	24 at $\frac{1}{4}$ inch pitch,		6	inches.
	20 ,, $\frac{3}{8}$,,	,,	7·5	,,
	6 ,, $\frac{1}{2}$,,	,,	3	,,
Total . .	50		16·5	,,

Allowing for a $\frac{3}{8}$-inch pitch at the bye-pass passage after the $\frac{1}{4}$-inch pitching, between rings 24 and 25 the exact value of the external radius of the last H.P. ring becomes $25\frac{1}{8}$ inches. With a 3-inch gap between H.P. and L.P. blading and the L.P. pitches just chosen, the external radius of the last L.P. ring becomes $31\frac{7}{8}$ inches.

The skeleton diagram of the blading is now drawn as shown in Fig. 213. The four L.P. sections are spaced sufficiently far apart to allow for a reasonable size of intermediate stiffening ring.

When this blade spacing is done and the requisite radii ascertained, the data for the calculation of the nett heats and speed ratios are entered in the table, as shown on opposite page.

Taking the L.P. section and proceeding by the method outlined in

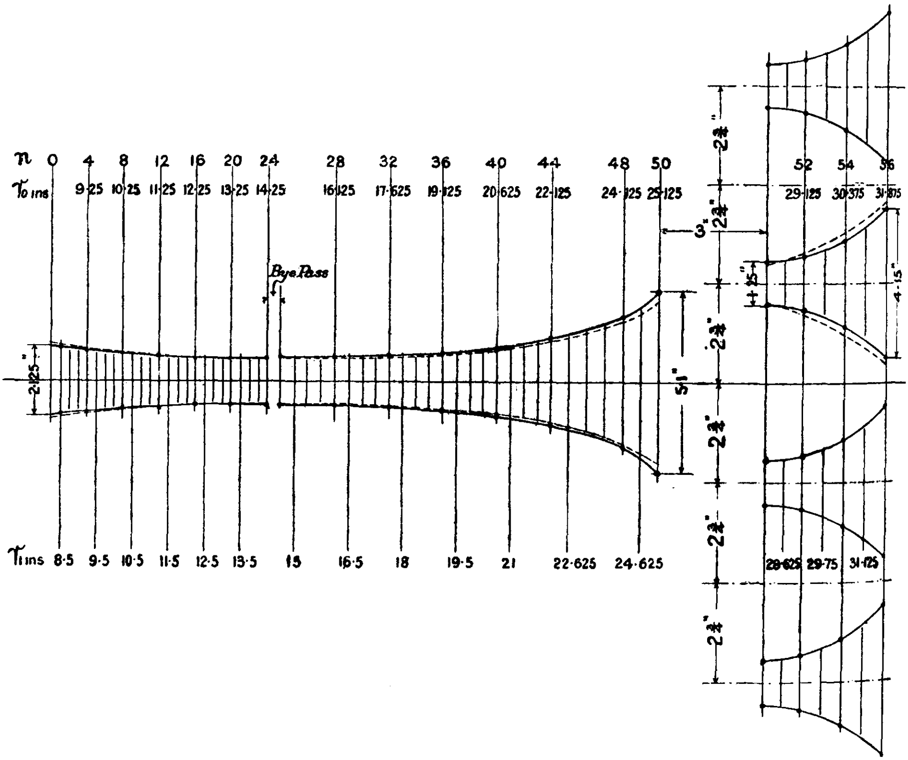


FIG. 213.

Art. 290, let the trial values of speed ratio for the three L.P. groups be 0·68, 0·72, 0·75, and the angles, say, 25°, 22°, and 20°.

Then at the last group (16) the nett heat is

$$_{16}h_{1} = c_{16}r_{1}^{2}\left(\frac{\cos\theta}{\rho}A - B\right)$$

Here the angular velocity with the double-motion condition is

$$\omega = \frac{2\pi \times 3000}{60} = 314. \quad \text{Hence the constant is}$$

$$c = \frac{314^2}{144 \times 778 \times 32{\cdot}2} = 0{\cdot}0272$$

Also $\rho = 0{\cdot}68$ (assumed), $\cos 25° = 0{\cdot}906$.

7500 K.W. Ljungström Turbine.

	H.P.													L. P.		
Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No. of rings, n	4	8	12	16	20	24	28	32	36	40	44	48	50	52	54	56
Rings per group	4	4	4	4	4	4	4	4	4	4	4	4	2	2	2	2
Pitch, x inches	¼	¼	¼	¼	¼	¼	⅜	⅜	⅜	⅜	⅜	½	½	½	⅝	¾
Radius, r_1 inches	8½	9½	10½	11½	12½	13½	15	16½	18	19½	21	22⅝	24⅝	28⅝	29¾	31⅛
$a = \frac{x}{r_1}$	·0295	·0264	·0238	·0218	·02	·0185	·025	·0227	·0208	·0192	·0178	·0221	·0203	·0174	·0209	·0242
Coefficients A	4·35	4·3	4·28	4·25	4·23	4·22	4·3	4·26	4·24	4·22	4·2	4·26	2·04	2·03	2·04	2·05
Coefficients B	2·25	2·2	2·19	2·17	2·15	2·14	2·2	2·18	2·16	2·14	2·13	2·18	1·04	1·03	1·04	1·045
Speed ratio, ρ	·7	·74	·75	·75	·75	·75	·79	·795	·8	·8	·81	·8	·76	·75	·72	·68
Nett heat, h_1	7	8	9·5	11·5	13·5	15·5	18	21	25	29	33	39	25	34	38·5	44·5
$\Sigma h_1 = H_1$	7	15	24·5	36	49·5	65	83	104	129	158	191	230	255	289	327·5	372
Pressure, p lbs./in.^{2} abs.	170	155	138	120	103	85	68	50	37	24	15	7·5	4·5	2·75	1·25	·5
Quality, t_s or q	290	280	270	250	230	210	182	150	120	80	35	·983	·98	·948	·922	·895
Spec. vol., v_0 ft.^{3}	3·9	4·25	4·7	5·25	5·9	6·8	8·26	10·5	13·4	19	28	49·5	80	126	250	580
Exit radius, r_0 in.	9¼	10¼	11¼	12¼	13¼	14¼	16⅛	17⅝	19⅛	20⅝	22⅛	24⅛	25⅛	29⅛	30⅜	31⅞
Blade length, l_0 in.	1·69	1·58	1·46	1·38	1·32	1·325	1·32	1·42	1·54	1·89	2·45	3·6	5·1	—	—	—
Divided ,, ,,	—	—	—	—	—	—	—	—	—	—	—	—	—	1·47	2·362	4·15
Exit blade angle, θ_0°	20	20	20	20	20	20	26	20	20	20	20	20	20	20	22	25
Entrance blade angle, θ_1°*	55	59½	61	61	61	61	66	67	68	68	69	68	62	61	{58 61	{57 62

* The bracketed values in the last two columns give the entrance angles of rings 53, 54, 55, and 56.

From table, $A = 2{\cdot}05$, $B = 1{\cdot}045$, $r_1 = 31{\cdot}125$; hence

$$_{16}h_1 = 0{\cdot}0272 \times (31{\cdot}125)^2\left(\frac{0{\cdot}906 \times 2{\cdot}05}{0{\cdot}68} - 1{\cdot}045\right) = 44{\cdot}5 \text{ B.Th.U.}$$

Similarly calculating the heat values for groups 15 and 14, the values are approximately 38·5 and 34. The nett heats, reckoned for this purpose from the final state point on the condition curve, are 44·5, 83, and 117, and these give the pressure and volume values shown in the lower part of the table. The resulting form of L.P. blade channel can now be determined.

For normal blades ($\theta_0 = 20^\circ$) the exit length by equation (12) is

$$l = \frac{7700 \times 20{\cdot}5 \times v_0\rho}{r_0^2 \times 3000} = \frac{52{\cdot}7 v_0\rho}{r_0^2}$$

Since $\sin 20 = 0{\cdot}342$, $\sin 22 = 0{\cdot}374$, $\sin 25 = 0{\cdot}422$, then

$$l_{16} = \frac{0{\cdot}342 \times 52{\cdot}7 \times 580 \times 0{\cdot}68}{0{\cdot}422 \times (31{\cdot}875)^2} = 16{\cdot}6 \text{ inches}$$

Hence the divided length is 4·15 inches.

$$l_{15} = \frac{0{\cdot}342 \times 52{\cdot}7 \times 250 \times 0{\cdot}72}{0{\cdot}374 \times (30{\cdot}375)^2} = 9{\cdot}45 \text{ inches}$$

and the divided length is 2·362 inches.

$$l_{14} = \frac{52{\cdot}7 \times 126 \times 0{\cdot}75}{(29{\cdot}125)^2} = 5{\cdot}87 \text{ inches}$$

and the divided length is 1·47 inches.

These values, when set off in Fig. 213, give a channel divergence which is not excessive, and they may therefore be taken as satisfactory for a preliminary adjustment, and the nett heats that can be used to obtain the approximate heat curve.

This is shown in Fig. 214. The corresponding group heat values and corresponding speed ratios (slightly adjusted from the approximate curve values), are given in the upper part of the table.

The derived heat scale, when applied to the $H\phi$ diagram, gives the pressure quality and volume values entered in the lower part of the table.

The H.P. blade lengths are then calculated, as in the case of l_{14}, using the tabular values of ρ, v_0, and r_0.

The last H.P. length is just at the desirable limit of 10 to 1.

The divergence at the last group is not objectionable, since the steam requires to have some lateral spread in order to flow freely into the L.P. passages.

The last H.P. blade length can easily be reduced by slightly increasing the angle, which is taken at 20°. Neglecting loss of velocity by the steam in traversing the gap between the H.P. and L.P. blading, the inlet length of each L.P. passage should run from 1 to $1\frac{1}{2}$ inches. In the skeleton drawing, Fig. 213, it has been made $1\frac{1}{4}$ inches.

At the first ring of the turbine the exit length is calculated on the

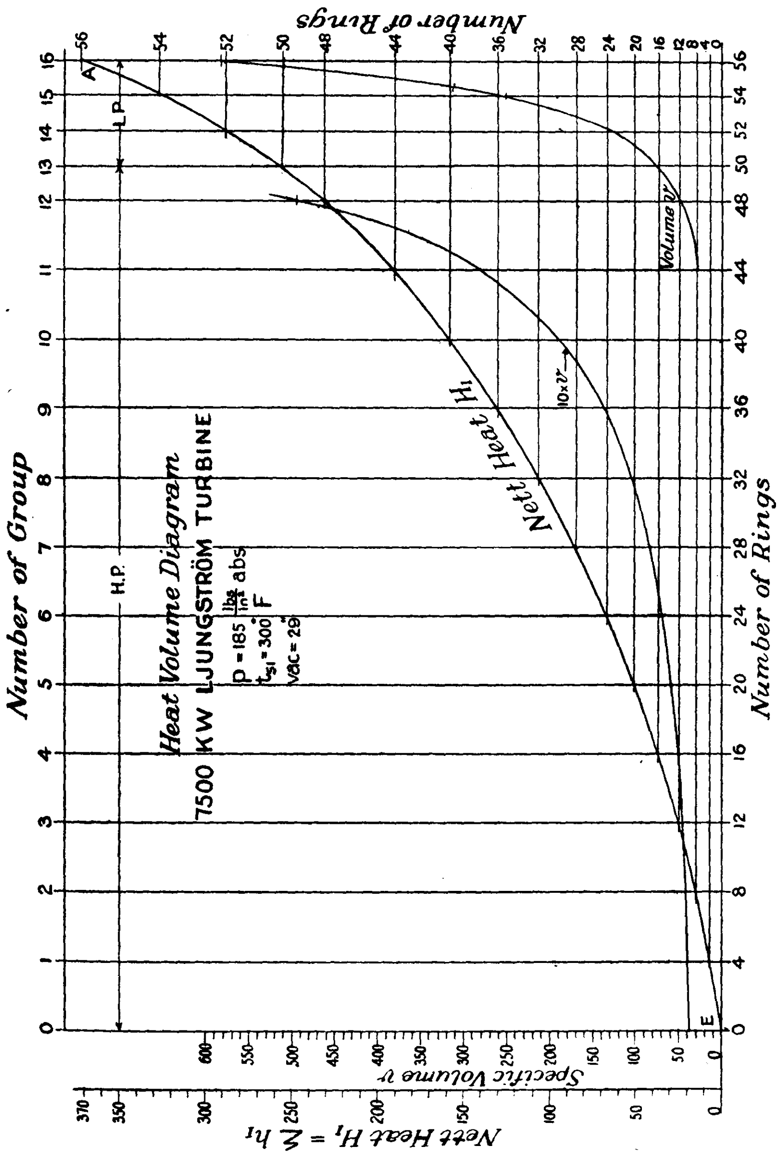


FIG. 214.

assumption that the speed ratio is 0·7, and the steam volume is that at the initial pressure of 185 lbs./in.². The latter is $v_1 = 3{\cdot}63$. Also $r_0 = r_1 = 8{\cdot}5$, hence

$$l = \frac{52{\cdot}7 \times 3{\cdot}63 \times 0{\cdot}7}{8{\cdot}5^2} = 1{\cdot}86 \text{ inches}$$

For a 10 to 1 ratio of length to breadth, the advisable limit of length is about 2 inches for the $\frac{3}{16}$-inch blade.

When the channel curve is drawn to the inlet edge, the figure obtained for the entrance length is $2\frac{1}{8}$ inches.

As can be seen from Fig. 213, these calculated lengths, when set out in the skeleton diagram, give a satisfactory form of channel.

This provisional drawing can now be used to find the actual entrance and exit lengths of the successive blades, when the necessary breadths, allowing for progressive radial clearances, are drawn in.

The conditions at each ring can be examined if desired, and slight alterations made in blade lengths or angles, as may be considered advisable.

As already mentioned, Messrs. Ljungström vary the angles in small groups by a few degrees.

Considering, however, the necessarily tentative nature of the design of a machine of this type, it is questionable if further refinement of the proportions of blading, obtained above, would ensure any better performance of the machine in service.

292. As in the case of the Parsons tandem machine, the data chosen for the last example are those specified by Messrs. Ljungström for a machine of this output, and the provisional design is worked out to enable a comparison to be made between the blading proportions obtained by the foregoing method, and the actual proportions of the design published by the makers.

For this purpose the blade channel outlines, taken from the corresponding Ljungström design, Fig. 37, are dotted in on Fig. 213.

It will be seen that the curves of the H.P. channel are in close coincidence. The actual inlet length, at the first ring, is slightly greater than that given by the calculated curve. At the last ring the actual value is about $4\frac{1}{2}$ inches, as compared with the calculated value of 5·1 inches. A slight alteration in the last blade angles would, however, bring the two curves into coincidence.

At the L.P. section the discrepancy is somewhat greater, although it is not excessive.

The actual lengths vary from 1 to $4\frac{1}{2}$ inches, as compared with the calculated values from $1\frac{1}{4}$ to 4·15 inches. Here again a slight alteration of blade angles would ensure coincidence. On the whole the comparison of the blading proportions is satisfactory. It will be noted that with the exception of the lower value, chosen to reduce the L.P. channel divergence, the speed ratio varies from 0·7 to 0·8. The average for the whole machine is 0·765.

Messrs. Ljungström state that their design is based on a value of 0·75, and anticipate an efficiency ratio of 0·86, as against the value 0·82 assumed for the foregoing calculation.

CHAPTER XVI

GOVERNING

293. The various methods adopted for the regulation of the supply of steam to suit the load on the turbines may be classified under four general headings—

(1) Throttling either directly or through "relay" gear.
(2) Nozzle cut-out.
(3) Blast governing.
(4) Bye-pass either by hand-controlled or automatic valves.

Direct Throttle Governing.—Small, simple, and compound impulse turbines are usually governed by direct throttling, a spring-loaded shaft governor being employed. Typical examples are afforded by the gears of the de Laval, Sturtevant, and Escher Wyss machines, which have already been described in connection with the illustrations in Chap. II.

294. **De Laval Emergency Governor.**—The emergency gear of the de Laval machine, which is of a special type, is illustrated in Fig. 215. On the sudden removal of the load, with accompanying increase of speed, the throttle comes into action and partly closes down, but when the machine discharges into a condenser the vacuum effect tends to counteract that of the throttle and to keep the speed up. In order to prevent a dangerous increase of speed the vacuum emergency valve, 1, is fitted in the exhaust branch. This is connected by a link, 2, with a spring-loaded piston, 3, in the cylinder, 4. The bottom of this piston is connected by the pipe, 5, with an air valve, 6, held closed by a spring. The spindle, 7, of this valve, when the turbine runs at speed, is just clear of the end of the lever, 8. The spindle, 9, of the spring-loaded shaft governor, enclosed in a sheet metal casing, 10, also runs clear of the lever end by a predetermined amount. The upper end of the lever is connected by a link, 11, to the bell crank of the double-beat throttle valve contained in the casing, 12. An increase of speed causes the governor spindle, 9, to move back and operate the throttle by pressing on the lower end of the lever, 8. If the throttle control is not sufficient to prevent the speed from rising above the normal by a predetermined amount, say 12 to 15 per cent., the lever end is pushed further back and opens the air-valve, 6. This puts the bottom of the piston, 3, into communication with the atmosphere, the piston is forced up against the resistance of the spring, and the valve, 1, closes and blocks the exhaust passage. The pressure in the casing immediately "backs up" and the speed is reduced.

295. This emergency system is different from that used on any other machine. The usual method is to cut off the steam supply by the automatic closure, either of the main stop valve or an auxiliary emergency valve fitted on the steam inlet. The gear for this purpose is always operated by an auxiliary or emergency governor, which acts independently of the main governor. In the majority of cases this

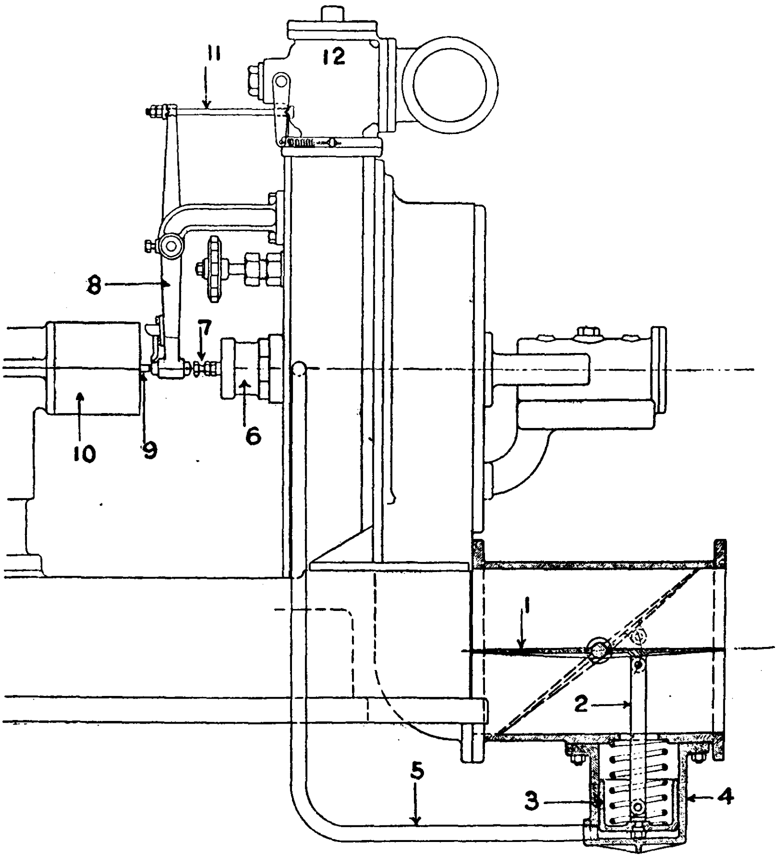


FIG. 215.

emergency governor is fitted and driven on the main shaft, and not on the auxiliary shaft driving the main governor or oil pump. This ensures certainty of action if the machine overspeeds. If driven by the secondary shaft any failure of this might render the emergency governor useless, and serious results might ensue before the main valve could be shut down.

It is not necessary to discuss at any length the other direct throttling arrangements used, as they are of the same nature as those applied to the reciprocating engine. An illustration of the direct throttle gear fitted on Willans turbines and adapted to govern a mixed pressure machine is shown in Fig. 222.

For large machines the general practice is to replace the large and powerful governor by a small high-speed one, and use this to operate the control gear of some form of relay.

296. Relay Throttle Governing.—In this system two methods of operating the servo motors which control the governor valve are employed. The fluid principally used to work the servo is the oil, supplied under pressure for the lubrication of the bearings. In some instances steam is used.

The method generally employed is to use the oil under equal pressure on each side of the piston of a servo-motor, when the turbine runs at normal speed, and to relieve the pressure on either side by means of a "pilot" valve, when the motion of the piston is necessary to counteract change of load and speed. The arrangement requires the use of a system of levers in addition to the pilot valve.

The method used by, at least, two prominent continental makers is somewhat different. The piston of the servo motor is loaded on one side by a spring and the supply of oil to the other side is throttled by the governor.

The principle of the first class of gear is clearly illustrated in Fig. 216. This represents the standard oil relay fitted in the Zoelly pressure compounded machines, by Messrs. Escher, Wyss & Co. The main governor, 1, is usually of the Hartung type and operates the pilot valve, 2, which controls the flow of oil to and from the cylinder of the servo motor, 3. The piston, 4, of this motor operates the double-beat throttle valve, 5.

When the turbine runs at normal speed the governor sleeve takes up a definite position, the throttle valve is opened the requisite amount and the pilot valve covers both the oil ports.

When the machine speeds up, the pin-joint, 6, of the control lever, 7, rises with the governor sleeve. This causes the lever to pivot about the pin, 9, of the other end at the collar on the valve spindle, and the pilot valve is raised by the pin at 8. Oil under pressure is supplied through the pipe, 10, to the centre of the valve at 11, so that when the valve rises it opens the top port to pressure supply and the bottom port to the exhaust passage, 14. The oil pressure being relieved below the piston, 4, through the escape of oil to the exhaust pipe, 13, the oil under pressure, supplied through pipe 12, causes the motor piston to descend. Pin 9 then ceases to be a fixed point. The pilot valve is thus simultaneously raised by the governor and lowered by the motor piston. Obviously this differential motion will continue until the pilot valve again covers the ports. The servo piston is then locked by the oil on each side and the throttle remains stationary in the new position. The reverse action takes place when the speed of the machine decreases, due to increase of load.

The differential leverage arrangement checks any tendency of the governor to "hunt."

In this particular gear a fine speed adjustment is provided. The collar, threaded on the valve rod, 15, is attached to a toothed wheel, that can be rotated by the pinion, 16, through the medium of the small electro motor, 17. This can be operated from the switchboard, and the position of the pin, 9, can be adjusted for "parallel running" to give a speed variation ± 5 per cent. of the normal. The general

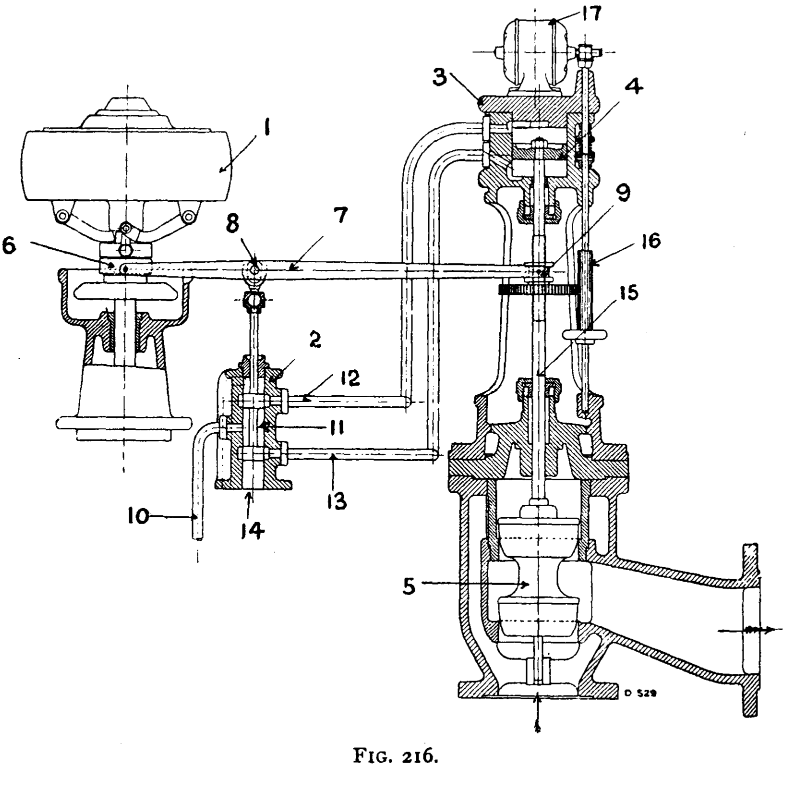


FIG. 216.

arrangement of the gear has already been described in Art. 22 in connection with the Zoelly machine.

297. The standard arrangement of relay throttling gear fitted on the British Westinghouse impulse turbines is shown in Fig. 217. A section through the governor case is also shown in Fig. 218. The governor, 1, Fig. 218, which is a horizontal spring-controlled one, is driven by the worm gear, 2, from the main shaft. This gear also drives the shaft of the oil pump, 3, and tachometer, 4, through the bevel wheels, 5. The governor lever is connected to an adjustable rod, 6, which is pin-jointed to one arm of a bell crank, 7. The other arm of the crank is coupled to the spindle of the pilot valve, 8, by a short link. This controls the piston of the servo motor, 9, in the manner already described.

The arrangement of the emergency gear is clearly shown on the full end elevation, Fig. 217. The main stop valve is actuated by the

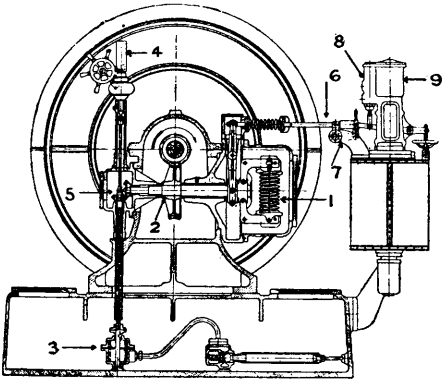


FIG. 218.

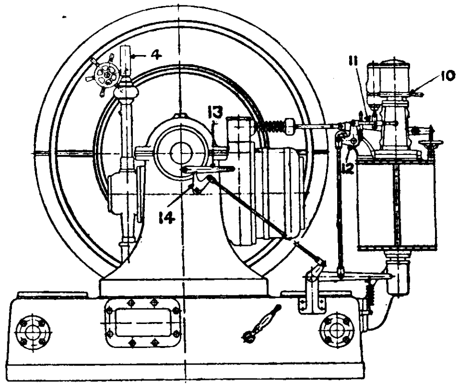


FIG. 217.

hand wheel, 10. It is held up against the resistance of a spring by a trigger arrangement, connected through the link, 11, with the vertical arm of a second bell crank, 12. This, in turn, through the system of links shown, is connected with the trigger gear operated by the emergency governor on the main shaft. When this throws up the lever, 13, the trigger, 14, is released, and the trip gear at the main spindle operates and closes down the valve (see also Fig. 221).

A section through the emergency governor, which is similar to that fitted on most large machines, is shown in Fig. 219. A hole is bored in the main shaft, and into this is fitted a weight, 1, with its centre of gravity slightly eccentric to the axis of the shaft. This is held in place by a spiral spring as long as the speed is not in excess of the normal. The rounded inner arm, 3, of the lever, 13, fits round the shaft.

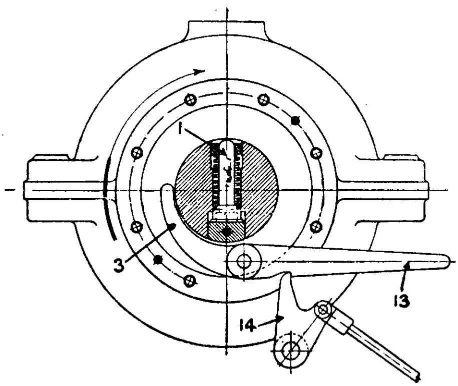


FIG. 219.

When the machine over-speeds, the unbalanced centrifugal force on the weight overcomes the resistance of the spring, the rounded end presses on the arm, 3, throws up the lever, 13, and releases the trigger, 14.

These two gears should serve to sufficiently exemplify typical arrangements of the form of relay throttle gear for high-pressure turbines.

298. Throttling Gear for Mixed Pressure Turbines.—The arrangement of relay required for the mixed pressure type is not quite so simple. One illustration of this class of gear is shown in Figs. 220 and 221. It is due to Professor Rateau, and is fitted on the British Westinghouse impulse machines.[1] Fig. 220 shows a section through the L.P. and H.P. valve chests. Fig. 221 is a section through the main H.P. valve chest and H.P. throttle chest on the plane at right angles to the paper. The servo motor, 1, is controlled by the main

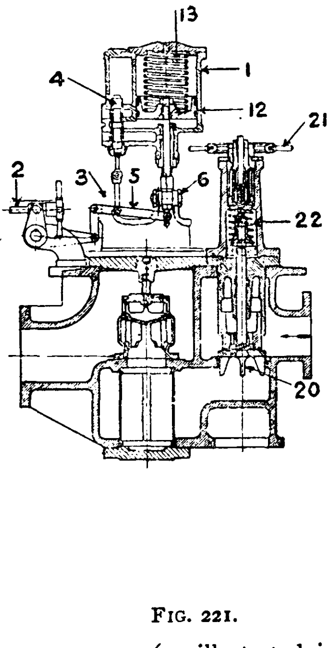


FIG. 221.

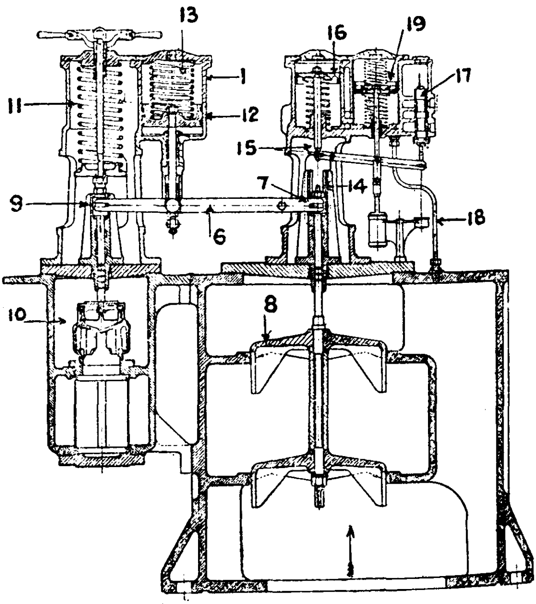


FIG. 220.

governor (as illustrated in the previous gear) through the rod, 2, the linkage, 3, and pilot valve, 4. The piston rod of the motor is connected to the lever, 5, and also to the long lever, 6 (Fig. 220), so that any movement of the piston raises or lowers these links simultaneously.

One end, 7, of the lever, 6, is connected to the spindle of the L.P. throttle, 8, and the other end, 9, to the spindle of the H.P. throttle, 10. This valve is held down on its seat by the strong spring, 11. It will be evident that when running on L.P. steam, if the speed decreases and the rod, 2, is moved to the right by the governor, the pilot valve

[1] Reproduced from paper on "Recent Developments in Steam Turbine Practice," by K. Baumann, *Proc. Inst. of Electrical Engineers*, January, 1912.

is lowered and admits oil under pressure below the piston, 12, lifting it against the resistance of spring, 13. End 9 being fixed, the motor then raises the lever, 6, and with it the L.P. throttle, and L.P. steam is admitted to take up the increased load. So far the gear performs the function of controlling the machine as a pure L.P. turbine by regulating the admission of L.P. steam to suit the load.

This action is only possible if there is sufficient head room between the end 14 of the L.P. spindle and the end 15 of the rod of the spring-loaded piston, 16. This piston is controlled by the pilot valve, 17. In the position shown it is at the top of the stroke. If it is lowered to some intermediate position the end 15 will force down the end 14 of the L.P. rod and partly close the valve.

This is what occurs when the L.P. supply is not sufficient to maintain full load. Under normal conditions with full L.P. supply the L.P. steam pressure transmitted through the pipe, 18, is sufficient to raise the piston, 19, to the top position. This operates the pilot valve, 17, which puts the top of piston, 16, into connection with the oil drain, and this piston is forced up to the top of the stroke by the spring, giving the necessary head room. When the L.P. supply falls off the pressure is reduced, piston 19 moves down, operates the pilot valve, and pressure oil is admitted above piston 16. This moves down and pushes down the L.P. throttle spindle. This action is independent of the main servo motor, 1, the piston of which remains stationary. The result is that lever 6 is turned about the end of the motor piston rod, and the end 9 rises, and with it the H.P. throttle, against the force of the spring, 11. If the main H.P. valve, 20, is open the additional H.P. steam, to maintain the full load, is admitted. Carried to the limit, if the L.P. supply ceases, piston 16 completely closes the L.P. throttle, and the main servo motor regulates the H.P. supply to the turbine as a high-pressure machine. The partial change of supply, it will be noted, is accomplished without alteration of speed of the machine, as the servo piston, 12, is stationary during the process.

The H.P. stop valve, 20, is opened by the hand-wheel, 21, against the spring, 22, and held open by the trigger gear already described in connection with Fig. 217.

299. The throttling gear used on the Willans mixed-pressure turbines is of a simpler construction, as no relay is used. The arrangement is shown in Fig. 222.

The sleeve of the centrifugal governor, 1, operates the main governor lever, 2, which is pivoted on a bracket at 3. The end, 4, of this lever is connected to the spindle of the low-pressure admission valve, 5, so that the governor operates this directly. The high-pressure admission valve, 6, is contained in the casing, 7, and its spindle, 8, is operated by a secondary rocking lever, 9, controlled by the main lever, 2. The end of this lever, to the left, is moved by tappets at 10, and the adjustment is so made that the L.P. admission valve is sufficiently opened to pass the full amount of L.P. steam before the auxiliary lever begins to open the H.P. admission valve. An adjusting screw is provided on the H.P. spindle at 11. A hand-regulation spring, 12, and a dashpot, 13, are fitted at the governor end of the main lever. Low-pressure steam enters at 14 and passes to the L.P. section through branch 15. High-pressure steam enters at 16 and passes to the H.P. section through branch 17. A steam strainer, 18, is fitted at the H.P. valve.

An emergency valve, 19, is fitted below the H.P. throttle and another emergency valve, 20, is fitted below the L.P. throttle. These are connected to a trip gear. This gear is operated by two spiral

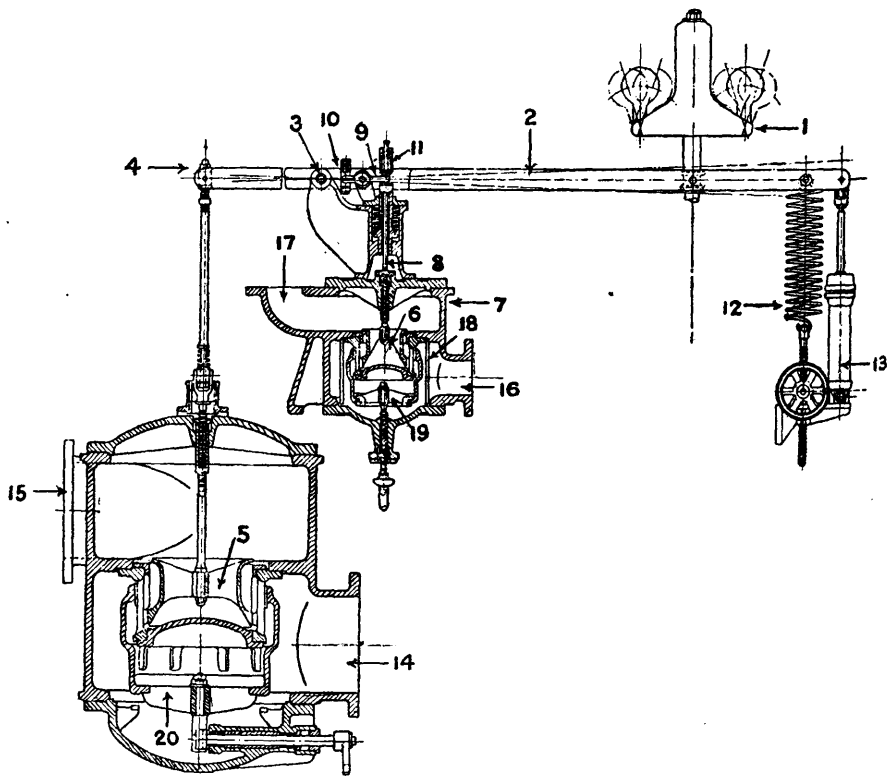


FIG. 222.

springs coiled round the shaft (see 27, Fig. 44). These distend under centrifugal force and release the gear.

300. The principle of the second class of oil relay gear, in which the rods and levers are replaced by a fluid link, is illustrated by the arrangement for combined throttle and cut-out governing, fitted on the Brown-Boveri combination turbines.

The general arrangement of the parts is shown on the elevation of the disc and drum machine, Fig. 45. The main stop-valve chest is shown at 4, the casing containing the main and emergency governors at 7, the throttle valve casing at 2, and the servo motor at 5.

The arrangement of the main and emergency governors, oil pump, etc., contained in casing 7, Fig. 45, is shown in detail in Fig. 223. The governor, 1, is driven by the auxiliary shaft, 2, from the main shaft, through the worm gear, 3. This shaft also drives the gear-wheel pump, 4, which supplies oil to the servo motor and bearings. The governor links operate a sleeve, 5, which works like a piston in an adjustable bush, 6. This is provided with ports opening to an annular chamber, 7, to which oil can flow from the servo motor through the passage, 8. A section through the main stop and throttle valve casings and the servo motor is shown in Fig. 224. Oil from the pump is discharged into the space below the servo motor piston, 9, through the pipe, 10, and passes to the passage, 8, and annular chamber, 7, Fig. 223, through the pipe, 11. The servo-motor piston is loaded by the spring, 12, which forces the throttle valve, 13, down on its seat, when the oil pressure, due to the action of the governor, is reduced.

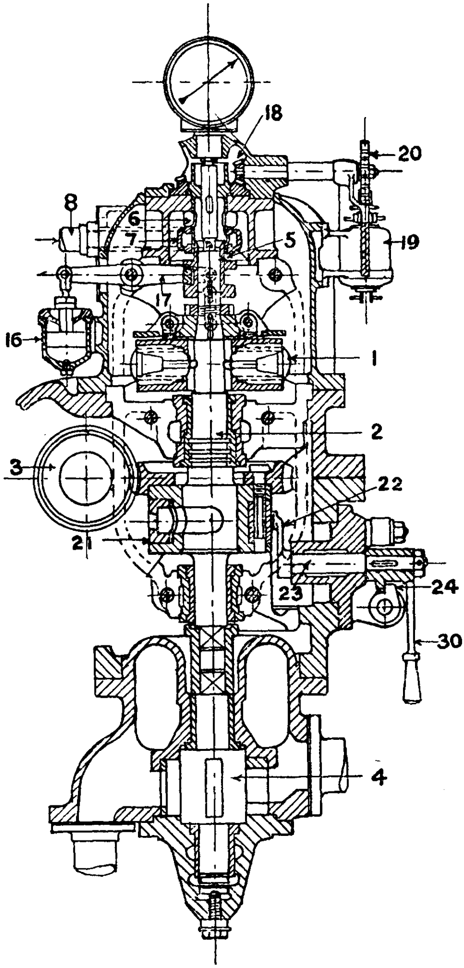


FIG. 223.

When running at normal speed, the position of the governor is such that the rotating sleeve, 5, partly closes the ports in the adjustable bush, 6, and throttles the oil at outlet. The oil pressure below the servo piston, 9, is thereby increased, and the throttle is raised to the position corresponding to normal load. If the speed increases due to decrease of load, the governor links pull down the sleeve, 5, and open the oil outlet ports in the bush, 6, and the pressure below the servo piston is reduced. The spring then forces down the piston and valve until equilibrium is restored at the lower load. This action is reversed when the speed is decreased due to increase of load. Since the same oil supply is used for the servo motor and lubrication of bearings, the servo motor shuts down the turbine in event of failure of the lubricating oil supply, before any seizure can take place at the bearings.

A common drawback with all such throttle gears is the statical friction that has to be overcome before the valve can begin to open or

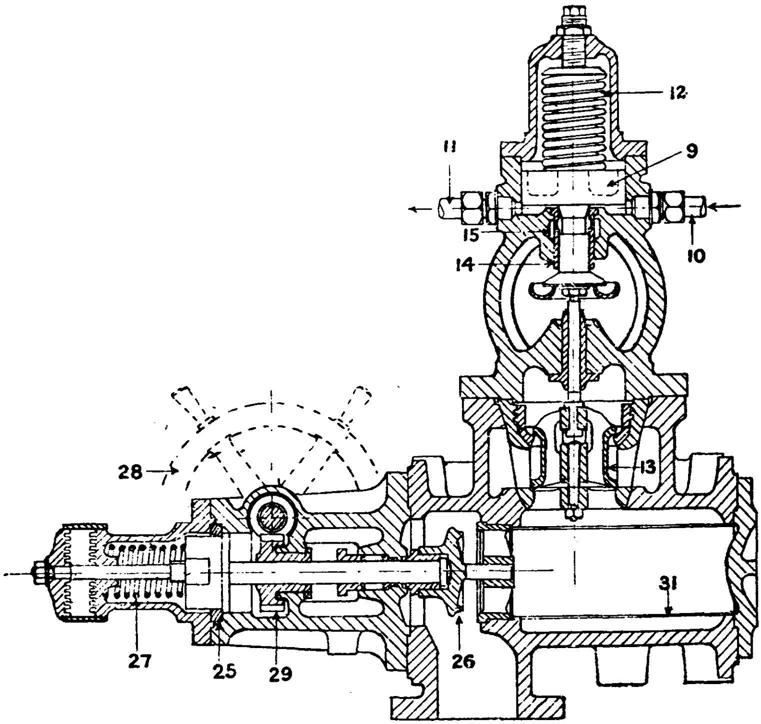


FIG. 224.

close under change of load. Where close governing is necessary, as in the case of the turbo-generator, the gear requires to have a continuous vibratory motion.

The end is attained in this arrangement by cutting a notch on the edge of the rotating sleeve, 5, Fig. 223. As the sleeve rotates, there is a momentary increase of outlet area in each revolution, when the notch passes the outlet ports. A continuous tremor of the servo piston and throttle valve is thus produced.

Where there is a sudden change of load, the inertia effect of the masses of the piston, rod, and valve comes into play, and the gear tends to "over-regulate." To counteract this tendency, the piston rod immediately below the piston is coned, and further down it is enlarged in diameter to a working fit in a bush, 14. This bush is provided with ports opening to the annular chamber, 15, from which escaping oil can be drained off.

If an abrupt change of load takes place, the sudden increase of pressure below the piston causes the mass of the gear to be set in rapid motion, and the tendency of the inertia is to lift the valve too high. The coned part of the spindle, acting as a valve, increases the outlet area from the cylinder, and additional oil is discharged through the ports into chamber, 15. The sudden decrease of pressure tends to check the upward motion of the valve.

When the turbine is run in parallel with any other prime mover less sensitively governed, a damping dashpot and piston, 16, Fig. 223, operated by a lever, 17, attached to the sleeve, 5, is fitted.

The speed of the turbine can be adjusted for a variation of 5 per cent. above or below the mean by adjusting the vertical position of the bush, 6. The adjustment is made by means of the bevel gear, 18, operated either by hand wheel 20, or an electromagnetic control, 19.

In this case the emergency governor is driven by the same spindle as the main governor. At the limiting overspeed, the weights, 21, fly outward and engage the arm, 22, of the emergency gear spindle, 23. This releases the pawl, 24, of the shaft which keeps a clutch, 25, Fig. 224, in position. This clutch holds the main stop valve, 26, open against the force of a spring, 27, and when released the spring forces this valve down on its seat. The valve is opened by the hand-wheel, 28, and worm gear, 29. The gear can also be tripped by means of the handle, 30, Fig. 223. A steam strainer, 31, Fig. 224, is fitted in the stop-valve casing.

301. This oil relay system, which obviates the use of links and levers, is also very suitable for mixed-pressure turbines. On a machine of this type, the H.P. relay cylinder is placed on the pipe line between the governor and the L.P. relay cylinder or servo motor. Between the H.P. and L.P. cylinders a shut-off valve is fitted in the pipe line. This valve is under the control of a motor piston in a third cylinder. The upper side of the piston is subjected to the steam pressure at the L.P. inlet, and the lower side to the upward force of a spring. When the oil pressure reaches the limit corresponding to the full opening of the L.P. throttle, the piston of the H.P. servo motor rises against the resistance of the spring and opens the H.P. throttle. When the pressure at the L.P. inlet falls below a minimum limit, the spring acting below the piston of the auxiliary valve forces this up and closes the valve. This cuts off the oil supply to the L.P. servo-motor piston which is then forced down by its spring and closes the L.P. throttle valve. A further decrease in the pressure of the exhaust steam is thus prevented.

302. Cut-out Governing.—In this system, which is applicable to the few-stage impulse—the Curtis-Rateau and combination turbines—the quantity of steam to maintain the speed at any given load is regulated by opening or closing groups of first-stage nozzles.

In the case of the simple impulse, and also in some combination turbines, the cut-out system is combined with throttling.

The regulation for small and temporary changes is effected by the throttle valve, and for greater changes of longer duration by the cut-out gear, or by hand-operated nozzle valves. The latter only are used on small machines.

Most of the large machines have automatic gears.

In principle the cut-out method of governing is superior to that of throttling, where only one or two stages are concerned.

When, however, it is applied to the multistage impulse turbine, it gives little better results than pure throttling of the steam on admission.

From its inception the Curtis turbine, which has a small number of stages, has been fitted with an automatic arrangement of nozzle cut-out valves at the first stage. These valves are, as a rule, operated by oil relays, but steam and electrical relays have also been used.

303. Nozzle Cut-out Gear for Curtis Turbine.—The arrangement fitted on the G.E.C. Curtis machine has already been described in Art. 19 in connection with the horizontal turbine, Fig. 17. It differs in detail from that fitted in this country by the British Thomson-Houston Co. The B.T.H. gear is shown in Fig. 225. The high-pressure nozzles are divided into groups, and each group is supplied with high-pressure steam through a separate valve, 1. This is raised from its seat against the force of a spring by a spindle, 2, which in turn is acted on at the lower end by a cam, 3, fixed on the shaft, 4. The series of cams is so arranged on the shaft that, as it rotates, the valves are either opened or closed in succession. The cam shaft is operated by the servo motor, 5, a sectional end-view of which is shown to the left. Instead of the usual cylinder and piston, a chamber, 6, containing a rotating vane, 7, is used, in order to get a direct turning effort on the cam shaft. The differential motion of the ends of the lever, 8, is obtained by means of a rack, 9, driven from a pinion, 10, at the end of the cam shaft. As in the previous cases, the oil, at a pressure of about 40 lbs./in.^{2}, is introduced to the centre of the pilot valve, 11, through the pipe 12, and exhaust takes place at the bottom through pipe 13.

The differential lever, 8, is connected to the main governor lever, 14, by the adjustable rod, 15. The governor, 16, which is encased, is of the ordinary spring-controlled high-speed type, and is driven from the main shaft through the worm gear, 17.

A hand-wheel, 18, is provided to adjust the compression of a balancing spring, 19, so that the speed may be varied 5 per cent. above or below the mean value, while the turbine is running. The oil pump is placed below the governor and driven from its spindle, 20.

An emergency governor is also fitted. The arrangement is shown in the next illustration, Fig. 226. This shows the corresponding system of cut-out and throttle control gear fitted on the B.T.H. type of mixed-pressure impulse turbine.

304. Cut-out and Throttle Gear for Curtis Mixed-Pressure Turbine.—In this case the L.P. supply is controlled by a butterfly throttle valve, 1, fitted on the L.P. steam inlet, 2. The series of H.P. nozzle valves (the spindles of which are omitted in the drawing) are contained in the casing 3, and operated by the cam shaft in casing 4 in the manner already stated, through the medium of the main governor.

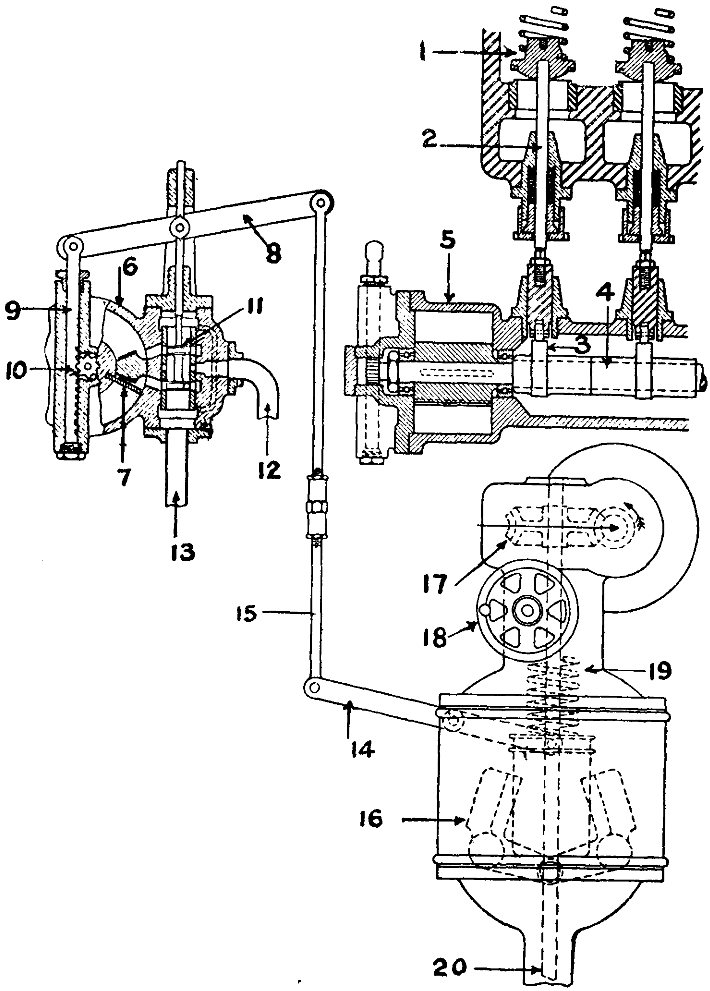


FIG. 225.

The cam shaft, 5, is continued into the end box and grooved as indicated. Into this groove the pin of a lever, 6, fits, and as the shaft rotates the lever turns about the pin, 7. This pin in turn carries a lever, 8, which is connected to the L.P. throttle lever, 9, by a link, 10. The latter is a tube containing a trip arrangement by which the connection between the levers 9 and 8 can be broken when the emergency gear acts.

The gear operates as follows. When the machine is at rest the governor weights are in the "in" position, and all the H.P. nozzle valves and the L.P. throttle are open.

When H.P. and L.P. steam is supplied the turbine speeds up, the main governor acts successively on the nozzle valves and "cuts out."

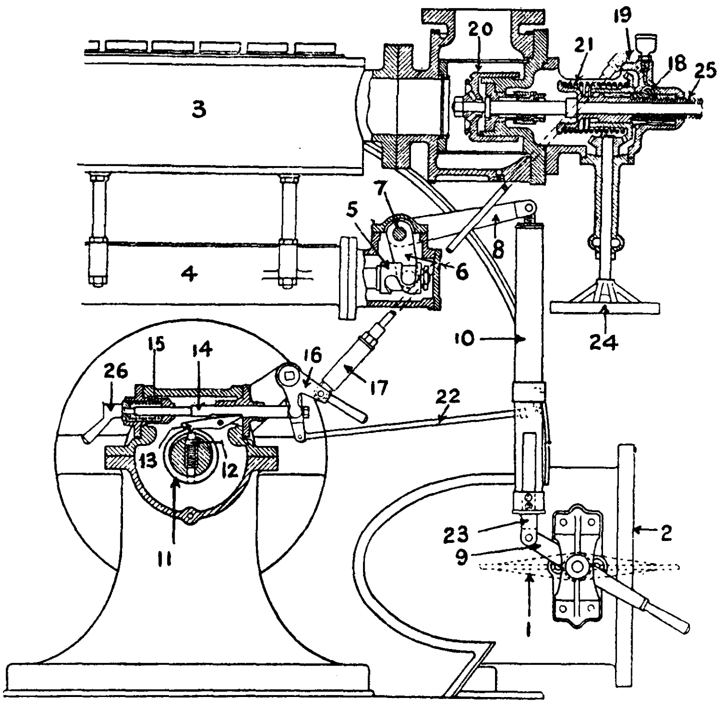


FIG. 226.

At normal speed with full-load pressure supply the last H.P. nozzle valve is just closing, and the cam sleeve through the medium of the linkage begins to close the L.P. throttle. About 2 per cent. above normal speed the L.P. valve is just sufficiently open to keep the turbine running at no load on L.P. steam. The governing between full and no load on L.P. steam is done through the medium of the grooved cam sleeve. When the L.P. supply falls off, the H.P. cams begin to operate the H.P. nozzle valves.

It is stated that the cam sleeve groove is so formed that the governor rises through half the range proportionally with the load, and this arrangement is superior to that of an ordinary throttle valve directly controlled by the governor.

The emergency gear is clearly shown. The governor proper consists of an eccentrically turned ring, 11, slipped over the end of the main shaft, and held concentrically by a pin, 12, passing through a hole in the shaft, and a spiral spring. The ring runs in contact with a trip lever, 13, which engages a notch on the rod, 14. This rod is under the pull of a spring, 15. One arm of a double bell crank, 16, is connected to a rod, 17, which can operate a trip gear, 18, through the medium of the lever, 19. This gear holds open the H.P. stop valve, 20, against the resistance of a spring, 21. The other arm of 16 is connected to another trip arrangement in the tube 10, by the rod 22. This second gear holds up the plunger, 23, in the tube against the resistance of a spring.

When the machine overspeeds the unbalanced centrifugal force on the ring, 11, overcomes the resistance of the spring, the ring runs out of truth, throws up the lever, 13, and releases the rod, 14, which pulls over the bell crank, 16. The two trip gears then operate, and the H.P. and L.P. valves are simultaneously shut down.

The emergency gear can also be tripped by hand through the medium of the handle, 26, at the end of the rod, 14.

305. Combined Throttling and Cut-out Gear.—The system of governing used in the Brown-Boveri combination machines is a combination of throttle and cut-out. The throttling gear, already described in Art. 300, is used to control the speed under small fluctuations of load. It regulates the supply of steam to one set of nozzles. For larger increases of load of longer duration, the steam supply to several sets of nozzles is automatically controlled by a series of servo-motor cylinders and nozzle valves. The arrangement employed is shown in Fig. 227.

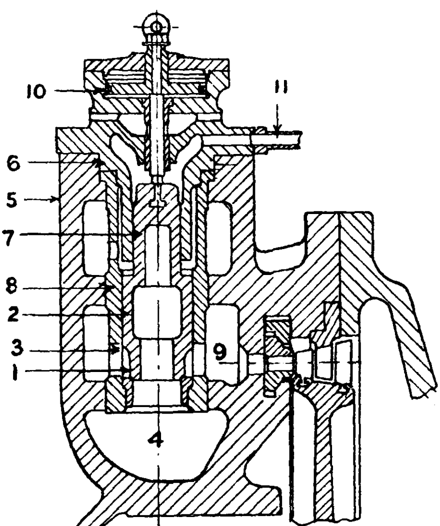


FIG. 227.

The principle of operation of the automatic valve is as follows. A valve, 1, of the poppet type has its diameter enlarged at 2, so as to form an annular area at 3. The space, 4, is in direct connection with the main throttle valve chest.

At the top of the casing, 5, a cylinder, 6, is fitted in which the upper part of the valve body, 7, reduced in diameter, can work as a piston, while the enlarged part, 2, works in the sleeve, 8. The cylinder at the top is connected to some source of constant pressure, either steam or oil, usually the latter. There is thus a constant force pressing the valve down on its seat. A dashpot is provided at 10. When the turbine runs well below its full load it is controlled by the main throttle gear, and the pressure below the valve, 1, is insufficient to overcome the downward force. The valve remains on its seat, and the corresponding set of nozzles is "cut out."

When the load increases the speed falls, the main gear opens the throttle further, and the pressure in the space, 4, increases. When it reaches a predetermined value, the upward force acting on the valve overcomes the constant downward force, and the valve begins to lift off its seat.

As soon as it gets clear, the high-pressure steam that passes it acts on the annular area at 3, and the valve opens fully.

The almost instantaneous opening of the valve thus prevents a secondary amount of throttling.

Several of these valves may be arranged to supply different sets of nozzles at the first stage of the turbine. These valves can be so loaded as to come into operation in succession as the load increases.

The system of governing in the Franco-Tosi combination turbine, Figs. 46 and 47, is also of the oil relay type. The auto-valves are, however, controlled not by the steam, but by oil servo motors.

306. With regard to "hand" cut-out arrangement, this is applied to the marine type of impulse turbine at the first stage, and in some cases also to the second stage nozzles. In a naval vessel this cut-out system gives an improved consumption at cruising speeds (see Art. 27).

In the mercantile marine, the turbine is always run at its full capacity, in normal weather, and there is no need for any special system of regulation beyond that afforded by the stop valve.

Owing to the considerable "flywheel" effect of the rotor, and also to the fact that the propellers are more deeply immersed than those of the reciprocating engine, the chance of dangerous racing in a heavy sea is practically eliminated. As far, then, as speed regulation is concerned the provision of a governor on the marine turbine is not necessary.

There is, however, as in the case of the land turbine, the possibility of the machine "running away" through the breakage of the shaft or propeller, and as a matter of safety some form of emergency governor gear is always fitted.

307. Blast Governing.—Various arrangements of gear are used to accomplish the end in view. The arrangement of levers used in Brown-Boveri machines seems best suited for the explanation of the principle of action of the gear, and it is therefore selected here for the purpose of illustration.

Referring to Fig. 228, the oil servo motor of the throttle gear, Fig. 224, is replaced by a cylinder, 1, having a piston loaded on the top by a spring, 2. The lower side of the piston is in communication through the small ports, 3, with the stop valve chest, 4. This piston, unlike the previous one, does not remain stationary during a steady load. It is caused to vibrate at a uniform rate under the alternate action of the spring acting downwards and the steam pressure upwards. This condition is produced by means of a pilot valve, 5, which is reciprocated so as to alternately open and close the exhaust port, 6. The motion of the valve is produced by a rod, 7, driven from an

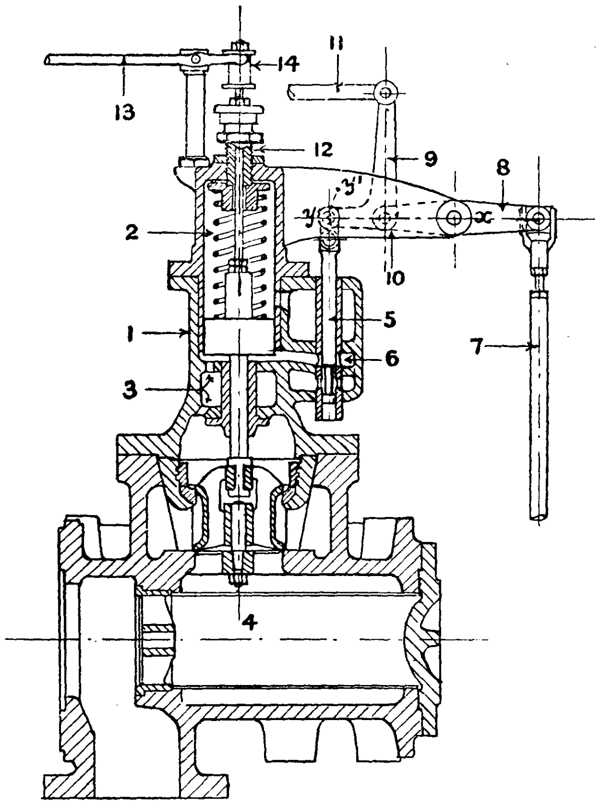


FIG. 228.

eccentric on an auxiliary shaft, and the lever, 8. The valve makes the same number of double strokes as the revolutions of the auxiliary shaft. This number may vary from 150 to 300, although in the majority of cases the usual limit is 250 reciprocations or "blasts" per minute.

The valve consists of a spindle reduced in diameter at the end. For the mean position shown, if the lever, 8, extended from x to y, then with the rod on the down stroke the edge of the pilot valve would just close the exhaust port. The area of this port is such that, when open, the steam is exhausted from it faster than it can be supplied through the steam ports, 3. When the valve closes the exhaust port the steam pressure on the bottom overcomes the resistance of the spring, and the piston is lifted, and with it the throttle valve, to the top position. The time during which the valve remains open obviously depends on the amount the valve overlaps the edge of the port when at the lowest position. If the lever were bent up at the end from y to the position y', then the amount of lap would be decreased by the approximate amount yy'. The portion of the period the port would remain open would be proportionally decreased, although the amplitude of the vibration and the periodicity would not be altered.

Conversely, if the end y were bent down so as to increase the lap of the valve, the conditions would be reversed.

The adjustment of the end y to ensure the above condition is attained by the automatic flexure of the end of lever 8. The end is made as one arm of a bell crank lever 9 pivoted on lever 8 at 10. The other arm of this lever is connected with the governor rod, 11.

If the speed increases due to decrease in load the governor rod pushes the bell crank to the right, raises the valve above the mean position and reduces the lap. The time during which the valve remains open during the "blast" is thereby reduced, and with it the weight of steam which passes the valve.

Conversely, with decrease of speed due to increase of load, the governor rod pulls the lever to the left depressing the valve and increasing the lap. The duration of the blast is thus increased and a greater weight of steam passes the valve.

The restoring spring, 2, is adjusted by a screw, 12. A hand lever, 13, fitting at an end on the deep collar, 14, at the end of the piston rod, can be used to lift the valve of its seat when necessary.

As the piston vibrates there is a continual discharge of steam from the exhaust port.

In order to prevent this from becoming a loss, it is usually led to the external packing glands.

308. Bye-pass Governing.—This system of governing is inherently detrimental to economy, and is suitable only for a condition of temporary overload, in which case economy is of minor importance.

When a turbine is loaded considerably in excess of its normal capacity the maximum inlet area at the first stage may be insufficient to pass the weight of steam required for the development of the "overload."

The high-pressure steam has then to be bye-passed to some lower stage where there is a larger admission area, and the section of the blading between this and the first stage is put out of action. A larger quantity of steam is passed through the machine, but the efficiency is reduced, and although the output is increased the rate of consumption is also increased.

As a rule, except in cases where turbines are used in traction work and the changes of load are very uncertain, the bye-pass for overload is accomplished by means of a hand-operated valve, and no additional gear is necessary.

An example of the arrangement is shown in Fig. 22.

309. The automatic type of bye-pass fitted in the Brush machines is shown in Fig. 25, p. 40. Steam at the stop valve pressure is supplied to the top side of the piston, 6, through a port, 35. The upper half of the valve spindle is threaded on a sleeve, 36, which takes the upward thrust of a spring, the compression of which can be regulated by the external gear. Under normal load the valve is held down in its seat by the steam pressure on the piston. As the load increases above normal, the pressure in the steam belt, 2, increases, and, finally, this, acting in conjunction with the upward pull of the spring, overcomes the downward force, the valve lifts off its seat and high-pressure steam is bye-passed to the second expansion at 5.

The same principle is applied to the case of the Brown-Boveri turbine, Fig. 26. The cylinder, 10, contains a piston loaded on top with a constant load, and in the bottom by the force of a spring and the variable pressure in the admission belt below the valve 9.

310. Comparison of Systems of Governing.—When pure throttling is applied to a simple or a few-stage impulse turbine, the consumption at fractional loads is greater than that which would be obtained by nozzle cut-out.

As already indicated, this is due to the reduction in heat drop per lb. with the decrease of initial pressure, and for the same out-put, the rate of consumption is therefore increased.

When, however, the cut-out system is applied to a multistage impulse turbine the resulting consumptions are little better than those given by pure throttling. The only way to ensure a more economical result is to apply the cut-out at every stage. This arrangement has been tried, but experience shows, from both the mechanical and commercial sides, that the complicated gears do not pay. The result is that the pressure compounded machines are simply controlled by pure throttling gears.

When a considerable proportion of the total energy is absorbed in the first stage, as in the case of the Curtis-Rateau and the various combination turbines, the cut-out system gives better results at partial loads than the pure throttling one.

The effect of nozzle cut-out at the first stage of an impulse or a combination machine is to transform the Willans line obtained by throttling into a stepped line, and the continuous consumption rate curve into a stepped curve.

The result is clearly shown by the curves of Fig. 229 taken from a Brown-Boveri combination turbine.[1]

The dotted total steam consumption line, A, is the usual Willans line obtained by governing on the throttle only. The corresponding curve of consumption rate, A, is shown dotted. The full line and curve, B, show the corresponding results with cut-out governing.

[1] See report of a lecture by Eric Brown, *Proc. Inst. Mech. Eng.*, July, 1911, Zürich meeting.

With nozzle cut-out (three to four nozzles per group) the throttling and cut-out curves with the full set of nozzles open, or throttle full open, coincide from 80 to 100 per cent. of the full load. From 60 to 80 per cent. of the full load, one set of nozzles is cut out, and the part of the Willans line between these limits falls below the throttling line. Between 40 and 60 per cent. of full load a further set of nozzles is cut out, the result being a second and much larger drop in the total consumption and rate curves below the throttling ones. At the low load of about 20 per cent. of full load where the whole of the automatically controlled nozzles are cut out and the rest of control is taken by the governor on the throttle, there is a further and much larger drop between the curves. This diagram indicates the advantage of the cut-out over the pure throttling gear between $\frac{1}{4}$ and $\frac{3}{4}$ load, for a combination turbine.

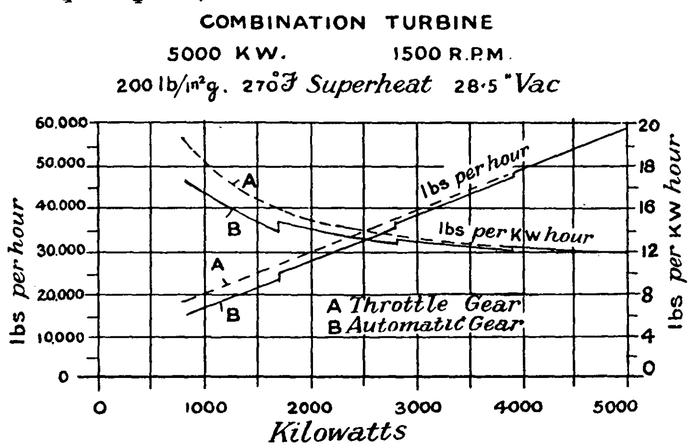


FIG. 229.

A cut-out system is only practicable where there is partial admission. Where, as in the case of the reaction machine, there has to be full admission from first stage onwards some other means has to be found to maintain the initial pressure at admission and at the same time to vary the quantity of steam to suit the load. The "blast" system is the outcome.

The gear does not, however, quite fulfil the desired conditions. This can be seen from Fig. 230, which is an autographic record of the initial steam pressure taken from a 2000 K.W. Westinghouse Parsons turbine.[1]

When the admission valve closes down, the steam in the machine continues to expand, and there is a fall in pressure at the admission end. At the normal load, about 2240, the dotted pressure curve shows a variation of pressure at each blast between 102 and 148 lbs./in.^{2} abs.

[1] Reproduced from paper on "Some Theoretical and Practical Considerations in Steam Turbine Work," by F. Hodgkinson, *Proc. Inst. Mech. Eng.*, June, 1904, Chicago meeting.

The lower full-line curve for about half-load shows a much greater variation, the limits being 63 and 133 lbs./in.^{2} abs. The upper curve for about 20 per cent. overload shows a smaller variation between 138 and 158 lbs./in.^{2} abs. The condition of constant initial pressure during each blast is thus not completely attained. The reduction of pressure is not, however, due to pure throttling but to the abstraction of steam. The condition that $pv \doteq c$ is approximately true, however,

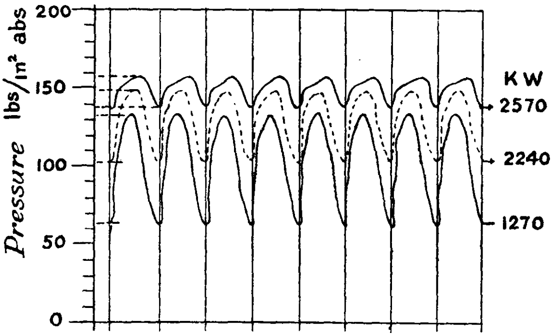


FIG. 230.

causes a disturbance of the velocity conditions with some reduction of speed ratio, and slight loss of efficiency results.

The primary condition is to keep the initial pressure constant at both full and partial load, and, as the diagram indicates, this is not completely accomplished. There is a distinct amount of pure throttling at the lower load. The maximum value of the initial pressure about $\frac{1}{2}$ load is 133 lbs./in.^{2} abs., while at full load it is 148 lbs./in.^{2} abs.

On the whole, this arrangement for partial loads gives slightly better results than the pure throttle.

PROPERTIES OF SATURATED STEAM

Compiled from Marks and Davis' "Tables and Diagrams of the Thermal Properties of Saturated and Superheated Steam," by permission of the authors and Messrs. Longmans, Green and Co.

Pressure lbs./in.² abs. p	Temperature ° Fah. t	Sp. vol. ft ³/lb. v	Heat in B.Th.U.			Entropy.		
			Sensible h	Latent L	Total H	Water ϕ_w	Evaporation ϕ_e	Total ϕ
0·5	79 5	643·0	49·7	1046·8	1094·5	0·0926	1·9413	2·0339
0·75	92·4	436·5	60·4	1039·8	1100·2	0·1157	1·8839	1·9996
1·0	101·83	333·0	69·8	1034·6	1104·4	0·1327	1·8427	1·9754
1·25	109·42	269·7	77·36	1030·35	1107·81	0·1461	1·8100	1·9561
1·5	115·79	227·0	83·7	1026·8	1110·5	0·1573	1·7845	1·9418
1·75	121·3	196·4	89·2	1023·7	1112·9	0·1670	1·7623	1·9293
2·0	126·15	173·5	94·0	1021·0	1115 0	0·1749	1·7431	1·9180
2·25	130·54	155·0	98·39	1018·5	1116·89	0·1824	1·7260	1·9084
2·5	134·5	140·4	102·36	1016·24	1118·6	0·1891	1·7106	1·8997
3·0	141·52	118·5	109·4	1012·3	1121·6	0·2008	1·6840	1·8848
3·5	147·62	102·4	115·48	1008·8	1124·28	0·2110	1·6612	1·8722
4·0	153·01	90·5	120·9	1005·7	1126·5	0·2198	1·6416	1·8614
4 5	157·84	81·0	125·7	1002·88	1128·58	0·2277	1·6241	1·8518
5	162·28	73·33	130·1	1000·3	1130·5	0·2348	1·6084	1·8432
6	170·06	61·89	137·9	995·8	1133·7	0·2471	1·5814	1·8285
7	176·85	53·56	144·7	991·8	1136·5	0·2579	1·5582	1·8161
8	182·86	47·27	150·8	988·2	1139·0	0·2673	1·5380	1·8053
9	188·27	42·36	156·2	985·0	1141·1	0·2756	1·5202	1·7958
10	193·22	38·38	161·1	982·0	1143·1	0·2832	1·5042	1·7874
12	201·96	32·36	169·9	976·6	1146·5	0·2967	1·4760	1·7727
14	209·55	28·02	177·5	971·9	1149·4	0·3081	1·4523	1·7604
16	216·3	24·79	184·4	967·6	1152·0	0·3183	1·4311	1·7494
18	222·4	22·16	190·5	963·7	1154·2	0·3273	1·4127	1·7400
20	228·0	20·08	196·1	960·0	1156·2	0·3355	1·3965	1·7320
22	233·1	18·37	201·3	956·7	1158·0	0·3430	1·3811	1·7241
24	237·8	16·93	206·1	953·5	1159·6	0·3499	1·3670	1·7169
26	242·2	15·72	210·6	950·6	1161·2	0·3564	1·3542	1·7106
28	246·4	14·67	214·8	947·8	1162·6	0·3623	1·3425	1·7048
30	250·3	13·74	218·8	945·1	1163·9	0·3680	1·3311	1·6991
32	254·1	12·93	222·6	942·5	1165·1	0·3733	1·3205	1·6938
34	257·6	12·22	226·2	940·1	1166·3	0·3784	1·3107	1·6891
36	261·0	11·58	229·6	937·7	1167·3	0·3832	1·3014	1·6846
38	264·2	11·01	232·9	935·5	1168·4	0·3877	1·2925	1·6802
40	267·3	10·49	236·1	933·3	1169·4	0·3920	1·2841	1·6761
42	270·2	10·02	239·1	931·2	1170·3	0·3962	1·2759	1·6721
44	273·1	9·59	242·0	929·2	1171·2	0·4002	1·2681	1·6683
46	275·8	9·20	244·8	927·2	1172·0	0·4040	1·2607	1·6647
48	278·5	8·84	247·5	925·3	1172·8	0·4077	1·2536	1·6613
50	281·0	8·51	250·1	923·5	1173·6	0·4113	1·2468	1·6581
52	283·5	8·20	252·6	921·7	1174·3	0·4147	1·2402	1·6549
54	285·9	7·91	255·1	919·9	1175·0	0·4180	1·2339	1·6519
56	288·2	7·65	257·5	918·2	1175·7	0·4212	1·2278	1·6490
58	290·5	7·40	259 8	916·5	1176·4	0·4242	1·2218	1·6460
60	292·7	7·17	262·1	914·9	1177·0	0·4272	1·2160	1·6432

PROPERTIES OF SATURATED STEAM—*continued.*

Pressure lbs./in.² abs. p	Temperature ° Fah. t	Sp. vol. ft.³/lb. v	Heat in B.Th.U.			Entropy		
			Sensible h	Latent L	Total H	Water ϕ_w	Evaporation ϕ_e	Total ϕ
62	294·9	6·95	264·3	913·3	1177·6	0·4302	1·2104	1·6406
64	297·0	6·75	266·4	911·8	1178·2	0·4330	1·2050	1·6380
66	299·0	6·56	268·5	910·2	1178·8	0·4358	1·1998	1·6355
68	301·0	6·38	270·6	908·7	1179·3	0·4385	1·1946	1·6331
70	302·9	6·20	272·6	907·2	1179·8	0·4411	1·1896	1·6307
72	304·8	6·04	274·5	905·8	1180·4	0·4437	1·1848	1·6285
74	306·7	5·89	276·5	904·4	1180·9	0·4462	1·1801	1·6263
76	308·5	5·74	278·3	903·0	1181·4	0 4487	1·1755	1·6242
78	310·3	5·60	280·2	901·7	1181·8	0·4511	1·1710	1·6221
80	312·0	5·47	282·0	900·3	1182·3	0·4535	1·1665	1·6200
82	313·8	5·34	283·8	899·0	1182·8	0·4557	1·1623	1·6180
84	315·4	5·22	285·5	897·7	1183·2	0·4579	1·1581	1·6160
86	317·1	5·10	287·2	896·4	1183·6	0·4601	1·1540	1·6141
88	318·7	5·00	288·9	895·2	1184·0	0·4623	1·1500	1·6123
90	320·3	4·89	290·5	893·9	1184·4	0·4644	1·1461	1·6105
92	321·8	4·79	292·1	892·7	1184·8	0·4664	1·1423	1·6087
94	323·4	4·69	293·7	891·5	1185·2	0·4684	1·1385	1·6069
96	324·9	4·60	295·3	890·3	1185·6	0·4704	1·1348	1·6052
98	326·4	4·51	296·8	889·2	1186·0	0·4724	1·1312	1·6036
100	327·8	4·429	298·3	888·0	1186·3	0·4743	1 1277	1·6020
105	331·4	4·23	302·0	885·2	1187·2	0·4789	1·1191	1·5980
110	334·8	4·047	305·5	882·5	1188·0	0·4834	1·1108	1·5942
115	338·1	3·880	309·0	879·8	1188·8	0·4877	1·1030	1·5907
120	341·3	3·726	312·3	877·2	1189·6	0·4919	1·0954	1·5873
125	344·4	3·583	315·5	874·7	1190·3	0·4959	1·0880	1·5839
130	347·4	3·452	318·6	872·3	1191·0	0·4998	1·0809	1·5807
135	350·3	3·331	321·7	869·9	1191·6	0·5035	1·0742	1·5777
140	353·1	3·219	324·6	867·6	1192·2	0·5072	1·0675	1·5747
145	355·8	3·112	327·4	865·4	1192·8	0·5107	1·0612	1·5719
150	358·5	3·012	330·2	863·2	1193·4	0·5142	1·0550	1·5692
155	361·0	2·92	332·9	861·0	1194·0	0·5175	1·0489	1·5664
160	363·6	2·834	335·6	858·8	1194·5	0·5208	1·0431	1·5639
165	366·0	2·753	338·2	856·8	1195·0	0·5239	1·0376	1·5615
170	368·5	2·675	340·7	854·7	1195·4	0·5269	1·0321	1·5590
175	370·8	2·602	343·2	852·7	1195·9	0·5299	1·0268	1·5567
180	373·1	2·533	345·6	850·8	1196·4	0·5328	1·0215	1·5543
185	375·4	2·468	348·0	848·8	1196·8	0·5356	1·0164	1·5520
190	377·6	2·406	350·4	846·9	1197·3	0·5384	1·0114	1·5498
195	379·8	2·346	352·7	845·0	1197·7	0·5410	1·0066	1·5476
200	381·9	2·29	354·9	843·2	1198·1	0·5437	1·0019	1·5456
210	386·0	2·187	359·2	839·6	1198·8	0·5488	0·9928	1·5416
220	389·9	2·091	363·4	836·2	1199·6	0·5538	0·9841	1·5379
230	393·8	2·004	367·5	832·8	1200·2	0·5586	0·9758	1·5344
240	397·4	1·924	371·4	829·5	1200·9	0·5633	0·9676	1·5309
250	401·1	1·85	375·2	826·3	1201·5	0·5676	0·9600	1·5276
260	404·5	1·782	378·9	823·1	1202·1	0·5719	0·9525	1·5244
270	407·9	1·718	382·5	820·1	1202·6	0·5760	0·9454	1·5214
280	411·2	1·658	386·0	817·1	1203·1	0·5800	0·9385	1·5185
290	414·4	1·602	389·4	814·2	1203·6	0·5840	0·9316	1·5156
300	417·5	1·551	392·7	811·3	1204·1	0·5878	0·9251	1·5129

Angle.									
Degrees.	Radians.	Chord.	Sine.	Tangent.	Co-tangent.	Cosine			
0°	0	000	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

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·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
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·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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PRINTED IN GREAT BRITAIN BY WILLIAM CLOWES AND SONS, LIMITED, BECCLES.

Thesis (old) 7.19

GOUDIE, WILLIAM J.

(b) Pub. journal article

(c) Pub. journal article

(d) MS. supplementary note

THESIS (OLD) 7.19

GOUDIE

TUBE OF CHARTS HERE

INSTITUTION OF ENGINEERS AND SHIPBUILDERS IN SCOTLAND.

SESSION, 1917-1918.

See Explanatory note on Page 1

SOME GRAPHICAL METHODS OF DETERMINING THE PROPERTIES OF SUPERHEATED, SUPERSATURATED, AND WET STEAM.

BY

MR. WILLIAM J. GOUDIE, B.Sc.
(MEMBER).

PUBLISHED BY THE INSTITUTION,
ELMBANK CRESCENT,
GLASGOW.
1918.

Matter for D.Sc. thesis

This paper is an extension of the Supplementary note to Chapter V of Steam Turbine Book, on "Properties of Steam"

(re-submitted 19th May 1919.) W. J. Goudie

SOME GRAPHICAL METHODS OF DETERMINING THE PROPERTIES OF SUPERHEATED, SUPERSATURATED, AND WET STEAM.

By Mr. WILLIAM J. GOUDIE, B.Sc. (Member).

23rd April, 1918.

IN dealing with calculations involving the properties of steam the engineer can, as a rule, find all the necessary data for saturated steam, ready to hand, in the saturated steam-table; but when the properties of superheated or supersaturated steam have to be handled, the conditions are somewhat different. In order to give sufficiently detailed information for direct use, without resort to interpolation, the superheated steam table has to be of considerable extent, and, even then, for reasonable accuracy in calculation, more or less interpolation may be necessary. The various tables available in the past have also been more or less thermodynamically inconsistent; and, although for most practical purposes, the discrepancies have not been serious, yet for close analyses or comparison they have usually been sufficient to vitiate conclusions.

Fortunately, the researches of Prof. Callendar have now put the subject on a definite basis. By the aid of his characteristic equation, all the properties of steam in a given state, superheated, dry, or supersaturated, can be calculated to a considerable degree of accuracy, over a wide range of temperature and pressure.

In using the Callendar tables, however, the author has experienced the limitation referred to above with respect to the total heat and specific volume of superheated and supersaturated steam. In order, therefore, to obtain a more extended range by direct inspection, he recently designed several charts and nomograms; and they are made the subject of this short note, in the hope that they may be found of some service in facilitating practical calculations connected with steam-turbine design.

The combinations described here are not, however, intended to replace the ordinary charts embodying the properties of steam, namely, the temperature-entropy, total heat temperature, and total heat-entropy diagrams; they are submitted simply as auxiliary aids to calculation in conjunction with the new steam tables.

TOTAL HEAT OF SUPERHEATED STEAM.

In commercial work the range of superheat is often specified. When the range, t_s°C or t_s°F, is stated for a given pressure, the superheat h_s is calculable, if the mean specific heat for the range is known. The total heat H of the dry steam can be obtained directly from the saturated steam table; and the total heat per lb. of superheated steam is then given by

$$H_s = H + h_s \qquad (1)$$

In order to facilitate calculation by this method, the specific heat chart, shown in Fig. 1, has been derived from the tabular values given in the superheated-steam table.

It gives the mean specific heat for pressures, increasing by 10 lbs. between 20 lbs./in.^{2} and 300 lbs./in.^{2} absolute, and superheat ranges, increasing from 0°C to 250°C, or 0°F to 450°F

The set of constant-pressure curves has been plotted, in preference to superheat-range curves, so that the chart can be used either for Fah. or Cent. units. The specific heat value can be read to the fourth decimal place, and the total heat values, obtained by means of the chart, check with the values given in the tables, to within a fraction of one per cent.

To illustrate its application, suppose that the total heat of

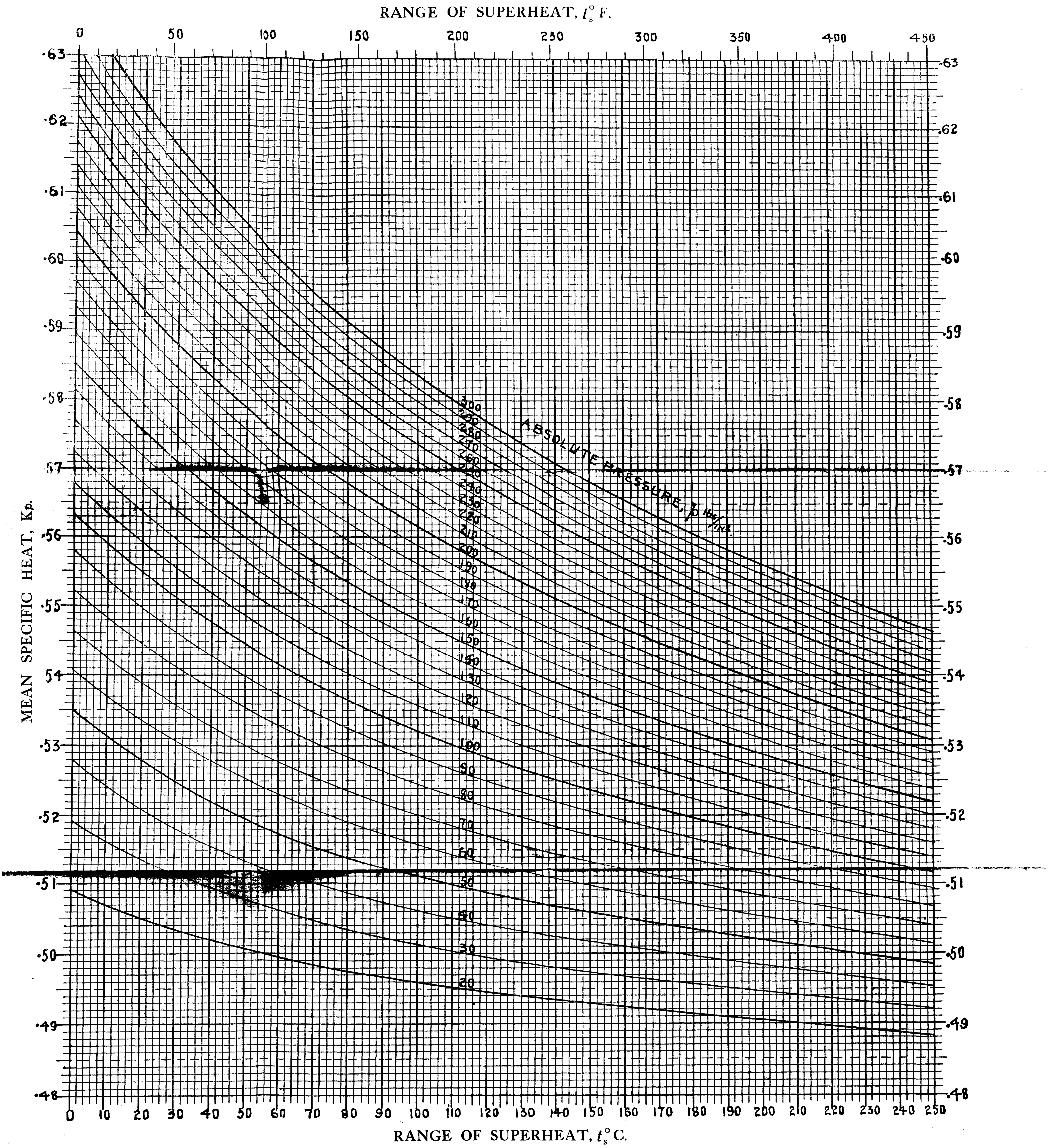


Fig. 1.—Specific Heat Chart.
(Based on Callendar's Tables).

steam at 180 lbs./in.^{2} absolute, and superheated through a range of 100·52°C, is required.

From the chart at $p=180$ and $t_s=100{\cdot}52$°C $Kp=0{\cdot}5560$, hence $h_s=K_p t_s=0{\cdot}5560\times100{\cdot}52=55{\cdot}78$ C.H.U./lb.

From the saturated-steam table $H=668{\cdot}53$, hence by (1)

$$H_s=668{\cdot}53+55{\cdot}78=724{\cdot}31 \quad \text{C.H.U./lb.}$$

The superheat range chosen here is such as to give one of the even total temperature figures of the superheated-steam table. The saturation temperature at 180 lb./in.^{2} is 189·48°C, so that the total superheat temperature is $t=189{\cdot}48+100{\cdot}52=290$°C. The value of H_s, corresponding to this total temperature in the table, is 724·43. The difference of ·13 C.H.U. on 724 C.H.U. is negligible for practical purposes.

If it is desired to avoid calculation, and to obtain a direct reading for a specified range of superheat, the end can be attained by plotting a set of constant-pressure curves on a base of superheat range. The drawback, in this case, is the size of diagram required to give a sufficiently large number of curves without undue crowding and confusion.

This combination is shown in Fig. 2. The constant-pressure curves have been plotted, with the aid of the specific heat chart, and their accuracy has been checked, at various points, by comparing the heat values against the values given in the superheated-steam table. The diagram is so arranged that either Fah. or Cent. temperature ranges, and the corresponding heat values in B.Th.U. or C.H.U., can be read with equal facility. The scales used for the original are, 1 inch = 10 C.H.U. and 1 inch = 20°C. It has been necessary, however, to increase the pressure differences between 100 and 300 lbs./in.^{2} from 10 to 20 lbs./in.^{2}. For practical use, in order to get smaller differences, it is advisable to replot this diagram to larger scales.

Applying this diagram to the previous case of steam at 180 lb./in.^{2} absolute and 100·52°C superheat range, it will be seen that it gives $H_s = 724 \cdot 4$ C.H.U./lb., which is practically the tabular value. The corresponding heat value on the left-hand scale is, $H_s = 1{,}304$ B.Th.U./lb.

This chart is combined with a nomogram, for the calculation of the specific volume, which will be considered later.

Total Heat of Superheated and Supersaturated Steam.

Under certain conditions it may be necessary to deal with the total heat of either superheated or supersaturated steam, in terms of the total temperature.

It is necessary here to consider the general equation given by Callendar* for this case.

The total heat is given as a function of the pressure and temperature as follows:—

$$H_s = 0{\cdot}4772\,T - 0{\cdot}10286\,p \left\{ (n+1)\,c - 0{\cdot}016 \right\} + 464$$

Where T = absolute temperature in °C = $(t + 273{\cdot}1)$

p = absolute pressure in lbs./in.^{2}

H_s = total heat of superheated, dry, or supersaturated steam in C.H.U./lb.

c = co-aggregation volume = $0{\cdot}4213 \left(\dfrac{373{\cdot}1}{T}\right)^n$.

n = constant = 10/3.

This, on substitution for n and c, becomes—

$$H_s = 0{\cdot}4772\,T - 0{\cdot}10286\,p \left\{ \frac{681{\cdot}3 \times 10^6}{T^{10/3}} - 0{\cdot}016 \right\} + 464 \frac{\text{C.H.U.}}{\text{lb.}} \quad (2)$$

$$\text{or } H_s = 0{\cdot}4772\,T - 0{\cdot}18514\,p \left\{ \frac{4842{\cdot}5 \times 10^6}{T^{10/3}} - 0{\cdot}016 \right\} + 835 \frac{\text{B.Th.U.}}{\text{lb.}}$$

Where T = absolute temperature in °F.

* The various equations, with notes, are given at the beginning of the "Callendar Steam Tables." Readers, who are not familiar with this recent work, will find a concise statement, and a derivation of the various expressions in Prof. Dalby's "Steam Power." A complete table of the properties of saturated steam is also given.

THESE NOMOGRAMS SOLVE CALLENDAR'S EQUATIONS.

$$H_s = 0{\cdot}4772T - 0{\cdot}10286\,p\,\{(n+1)\ c - 0{\cdot}016\} + 464$$

$$\text{or, } H_s = 0{\cdot}4772T - 0{\cdot}10286\,p\left\{\frac{681{\cdot}3 \times 10^6}{T^{10/3}} - 0{\cdot}016\right\} + 464$$

$$v_s = \frac{2{\cdot}243}{p}\,(H_s - 464) + 0{\cdot}0123$$

T = Absolute Temp. °C
p = „ Press. lbs./in.²
H_s = Total heat in C.H.U./lb.
v_s = Specific Volume in ft.³/lb.

OPERATIVE RULES

TOTAL HEAT.

Lay a straight-edge across the given pressure on the axis A B, and the given temperature on the axis C D, and read the total heat where it cuts the axis E F.

SPECIFIC VOLUME.

Lay the straight-edge across the total heat on the axis E F, and the given pressure on the axis G H, and read the specific volume where it crosses the axis K L.

When the given temperature falls above the given pressure on A B, the steam is superheated; when it falls below it the steam is supersaturated.

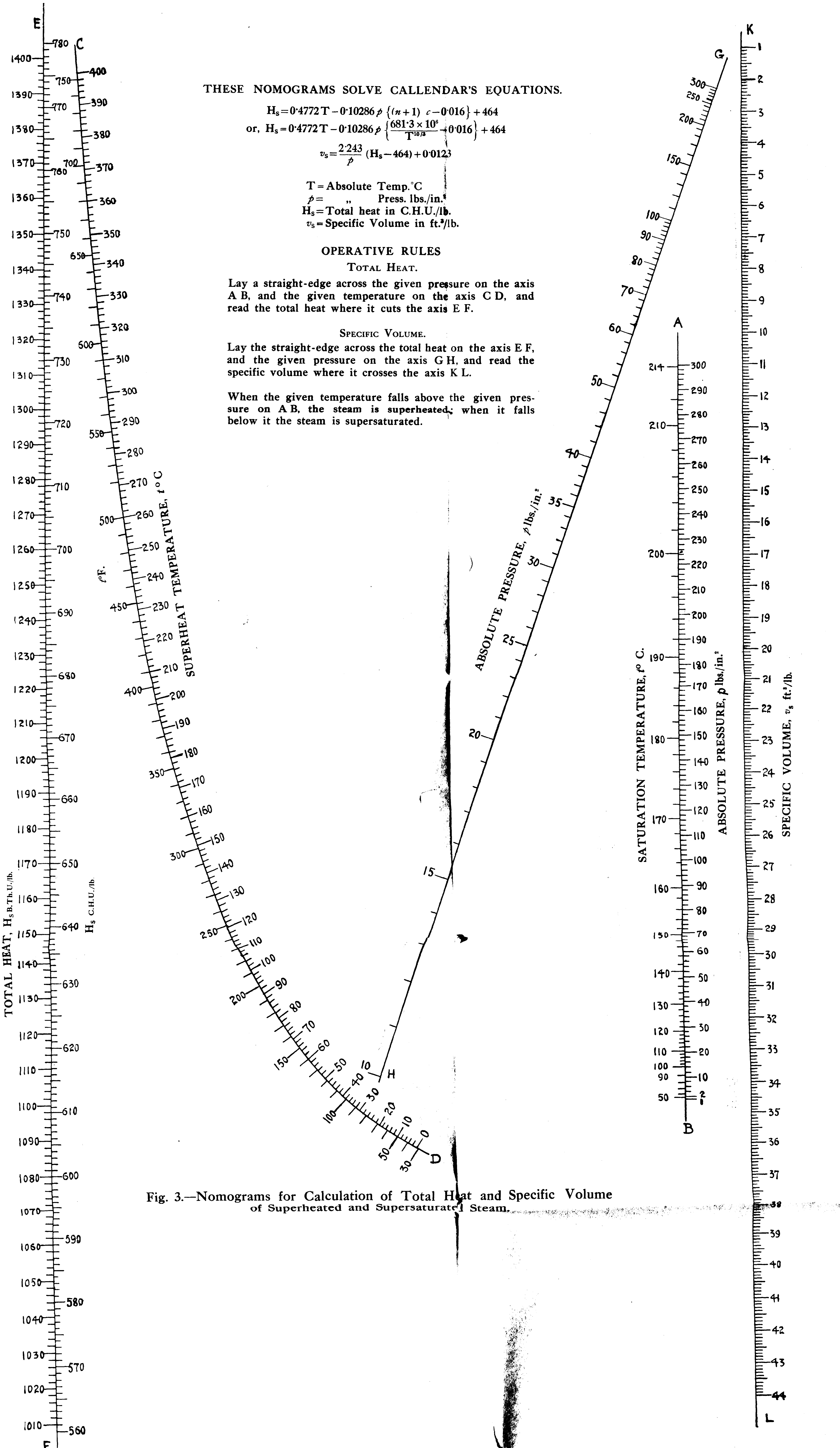


Fig. 3.—Nomograms for Calculation of Total Heat and Specific Volume of Superheated and Supersaturated Steam.

This equation can be represented graphically by a total heat-temperature chart, as in the "Callendar Steam Diagram." It can, however, be more simply represented by a three-scale nomogram, and this is the alternative form which the author has chosen.

According to the principles of nomography, an expression that can be put into the general form,

$$f(x)f(z)+f(y)\phi(z)+\mathrm{F}(z)=0,$$

can be nomographed on two parallel rectilinear axes and one curved axis.

Putting (2) in the form

$$p\times 0{\cdot}10286\left\{\frac{681{\cdot}3\times 10^{6}}{\mathrm{T}^{10/3}}-{\cdot}016\right\}+\mathrm{H}_s-(0{\cdot}4772\mathrm{T}+464)=0$$

it is seen that it is of this general type, where

$f(x)=p$; $f(z)=0{\cdot}10286\left\{\frac{681{\cdot}3\times 10^{6}}{\mathrm{T}^{10/3}}-{\cdot}016\right\}$; $f(y)=\mathrm{H}_s$

$\phi(z)=1$; and $\mathrm{F}(z)=(0{\cdot}4772\,\mathrm{T}+464)$

The nomogram, which is shown in Fig. 3, is very easily constructed. Two parallel axes, AB and EF, are drawn at any arbitrarily chosen distance apart. On the original drawing this distance is taken as 10 inches. A uniform scale of pressure is set off along AB, reading upward, and a uniform scale of heat along EF, also reading upward. These scales are also arbitrary, but their relative dimensions are controlled by the necessity for a sufficiently open temperature scale. The scales chosen by the author for the original drawing are, 1 inch = 10 C.H.U., and 1 cm. = 10 lb./in.^{2}.

When the p and H_s scales are set off on the parallel axes, the temperature-scale axis CD is easily obtained by taking a series of cross alignments.

If, for a chosen total temperature value, several corresponding pairs of pressure and total heat values are taken from the table, and these pairs of values are joined by straight lines on the chart, it will be found that all the lines intersect at a point. This is the point on the required temperature axis, corresponding to the chosen total temperature. In this way the points corresponding to 0, 50, 100, 150°C, etc., can be obtained, and the axis CD can then be drawn. The points for 10°C rise can then be located by taking a single pair of p and H_s values for each temperature; but it is advisable, for accuracy of spacing, to use at least two pairs. As the convergency of the scale is not pronounced, the subdivisions can be put in by dividers, without sensible error. A scale of saturation temperature in °C is drawn against the pressure scale on the axis AB. By means of this scale it can at once be seen whether, for the specified total temperature, the steam is superheated, dry, or supersaturated. When the total temperature at a given pressure is known, the total heat is found by laying a straight-edge across the pressure value on the axis AB, and the temperature value on the axis CD, and reading the total heat where it crosses the axis EF.

Applying this nomogram to the case of $p = 180$ and $t = 290$°C, it will be found to give $H_s = 724{\cdot}4$ C.H.U., which is again practically the value given at this temperature in the tables.

The condition of supersaturation or undercooling is one which arises during a rapid expansion of initially dry steam, or steam which expands from the superheated to the saturated condition, and continues to expand after the saturation temperature is reached.

There is some point at which the steam reverts to the wet saturated condition, but the author is not aware that this limit has yet been definitely fixed by experiments on steam alone.

The experiments of Mr. C. T. R. Wilson on air containing water vapour, indicated for that case a volume ratio from 1·24 to 1·38 for rapid expansion when a certain number of nuclei was present.*

* Phil. Trans. A., 1897.

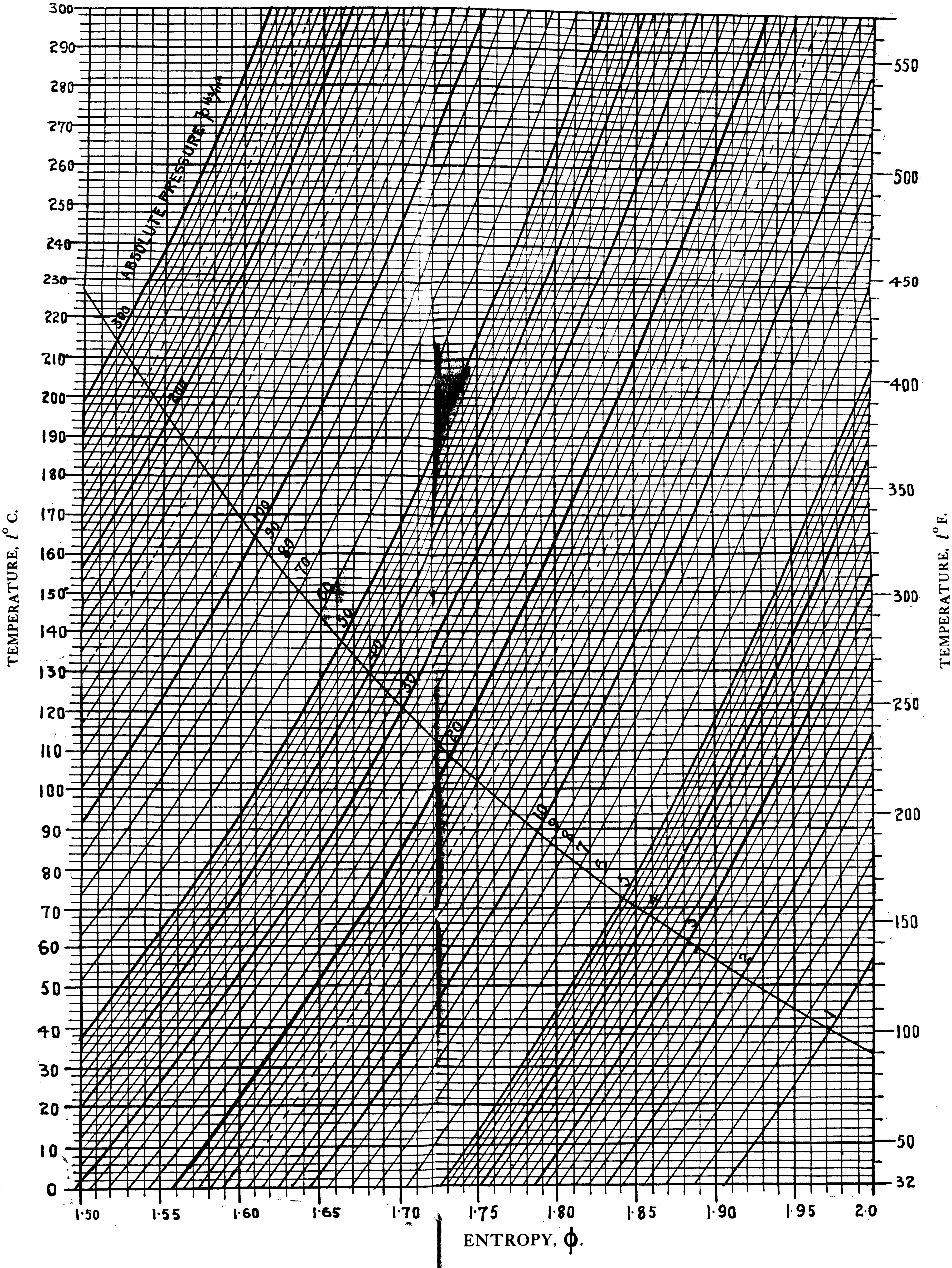


Fig. 4.—Temperature-Entropy Chart for Superheated and Supersaturated Steam.
(Based on Callendar's Tables).

The acceptance of the supersaturation theory, in the case of the discharge of steam from a convergent-divergent nozzle, settles the question regarding the apparent discrepancy between the calculated and actual discharge of initially dry steam, noted by various experimenters. How far this condition may exist in the multi-stage reaction type of turbine, where there is a continuous expansion between initial and exhaust pressures, is at present a matter for conjecture. There is scope here for experimental research, to discover whether this supersaturation condition exists in such types to an extent which justifies a modification of the usual methods of calculation based on the assumption of thermal stability throughout the whole range of expansion. For those who may wish to make tentative calculations, for arbitrarily assumed conditions, this nomogram is equally serviceable. Its application, however, is dependent on a knowledge of the supersaturation temperature at the lower pressure limit of the arbitrarily assumed isentropic expansion range. For the determination of this lower temperature it is possible to construct a nomogram from Callendar's equation for total entropy; but the author finds this method is too clumsy, and has, therefore, drawn the temperature-entropy chart for superheated and supersaturated steam shown in Fig. 4.

The zone above the saturation curve is the superheat field, and the zone below it the supersaturation field. The latter, of course, represents the saturation field, if the pressure curves in this region are neglected, and the temperature horizontals, which are constant-pressure lines for saturated steam, are taken instead. A set of dryness curves may, if desired, be drawn in this field. The author has omitted these, however, in order to avoid undue confusion.

For an isentropic expansion between arbitrarily chosen pressure limits, the final temperature for the condition of supersaturation at the lower pressure is obtained by following the vertical from the initial state point in the superheat field to the chosen lower pressure. The probable amount of undercooling at this pressure is given by the intercept on this vertical between the pressure curve and the corresponding pressure-line for saturated steam.

When the temperature at the lower pressure is determined, the total heat is obtained in the same way as that for superheated steam.

If the ratio of the volume at the lower pressure to the saturation volume, at the pressure where the vertical cuts the saturation curve, is greater than, say, 1·4, it is probable that the steam may have passed the critical stage, and become wet saturated. This process may be used to give a rough idea of the probable condition of steam during an approximately isentropic expansion in a nozzle passage.

This nomogram and the temperature-entropy chart, used in conjunction in this way, afford a ready means of ascertaining the heat drop or Rankine cycle heat, either in the superheat field, or partly in the superheat and partly in the supersaturation field.

By laying the straight-edge across the initial and final pressure and temperature values, the intercept on the axis EF gives the heat drop, which is the difference of the total heats.

To illustrate the application of the combination, suppose the initial pressure is 200 lb./in^2 absolute, and the total temperature of superheat is $t = 250°$C. What will be the probable state of the steam after isentropic expansion to 120 lb./in.^{2} absolute, and what is the heat drop or Rankine cycle heat H_r?

Following the vertical on the $T\phi$ diagram, Fig. 4, from $p_1 = 200$ and $t_1 = 250°$C to $p_2 = 120$, it is found that the point of intersection falls in the superheat field, and the temperature is, $t_2 = 192°$C. The saturation temperature at 120 lb./in.^{2} is 171·75°C, so that the steam is superheated 20·25°C.

Laying a straight-edge across $p_1 = 200$ and $t_1 = 250$, on the nomogram, Fig. 3, the total heat is $H_{s1} = 701·5$ C.H.U./lb.

Laying it across $p_2 = 120$ and $t_2 = 192$, $H_{s2} = 675{\cdot}3$.

Hence the heat drop is,

$$H_r = (H_{s1} - H_{s2}) = (701{\cdot}5 - 675{\cdot}3) = 26{\cdot}2 \text{ C.H.U./lb.}$$
$$= 47{\cdot}16 \text{ C.H.U./lb.}$$

Suppose in another case that the steam expands isentropically from $p_1 = 200$ and $t_1 = 250$ to $p_2 = 50$. Assuming that the wet saturated condition is not reached at the lower pressure, what is the probable temperature, and the heat drop?

The vertical from the initial state point, in this case, cuts the curve for $p_2 = 50$, at $t_2 = 107°$C. Using this value on the nomogram, Fig. 3, the total heat is $H_{s2} = 636{\cdot}5$ C.H.U./lb., and hence the heat drop is, $H_r = (701{\cdot}5 - 636{\cdot}5) = 65$ C.H.U./lb. or 117 B.Th.U./lb.

SPECIFIC VOLUME OF SUPERHEATED AND SUPERSATURATED STEAM.

The equation giving the volume of superheated or supersaturated steam, in terms of the pressure and temperature is.

$$v_s = 1{\cdot}07061\frac{T}{p} - 0{\cdot}4213\left(\frac{373{\cdot}1}{T}\right)^{10/3} + 0{\cdot}016 \qquad - \qquad (3)$$

where T = absolute temperature $°C = (t + 273{\cdot}1)$.

p = absolute pressure in lb./in².

v_s = specific volume in ft.³/lb.

This equation is of the same general type as that for the total heat, and can be represented by a three-scale nomogram, having a curved temperature-axis. The author has drawn this nomogram, but finds that it requires to be drawn to a large size in order to obtain a reasonably open temperature scale. As a much simpler method of calculating the volume is available, he has not included this nomogram.

If the temperature is eliminated between equations (2) and (3) a simple expression is obtained, giving the volume in terms of the pressure and total heat. This is,

$$v_s = \frac{2{\cdot}243}{p}(H_s - 464) + 0{\cdot}0123 \quad - \quad - \quad (4)$$

Writing it as,

$$\frac{p}{2{\cdot}243} \times (v_s - 0{\cdot}0123) - (H_s - 464) = 0$$

the type equation $f(x)\,f(y) + f(z) = 0$ is obtained, and this gives a three-scale nomogram with rectilinear axes, two parallel and one inclined.

$$\text{Here } f(x) = \frac{p}{2{\cdot}243}; \; f(y) = (v_s - 0{\cdot}0123) = l; \text{ and } f(z)$$
$$= (H_s - 464) = K$$

By setting off a uniform scale of volume v_s, reading downward on axis EF, Fig. 2, and a uniform scale of total heat reading upward on axis AB, the diagonal axis CD is obtained by joining the values of $l=0$ and $K=0$.

Here $l=0$ when $v_s=0{\cdot}0123$, and $K=0$ when $H=464$, so that CD is obtained by joining $H=464$ and $v_s=0{\cdot}0123$.

The position of the zero of the volume scale is arbitrarily fixed. On the full-sized diagram this scale is taken as 1 inch $= 2$ ft.^{3}. The pressure scale is derived from the volume and heat scales by a series of cross alignments, the H_s and v_s values for any given pressure being calculated from equation (4). It is advisable again, at the principal scale divisions, 10, 20, 30, etc., to take two pairs of values. One of the pairs can be taken, for the dry steam condition, directly from the saturated-steam table. Instead of laboriously constructing this scale, point by point, the intermediate divisions can be quickly filled in, by drawing the graph from the main divisions and projecting from this at the intermediate values to the scale axis.

On this diagram, find for the given range of superheat at the given pressure the point of intersection of the total heat horizontal with the axis AB Lay a straight-edge across this point and the pressure value on CD, and read the specific volume where it cuts the axis EF.

This nomogram is also combined with the total heat nomogram, Fig. 3, so that the total heats and volumes can be handled simultaneously. In order, however, to keep the combination within reasonable limits, the volume and pressure axes KL and GH are drawn on the right side of the total heat axis EF; and on account of the curvature of the temperature axis CD, it has been necessary to stop the pressure scale at $p=10$ lb/in². In order to get a sufficiently extended pressure scale, the axes KL and GH should be drawn on the left side of EF.

Applying this specific volume nomogram to the first case already considered, with $p=180$ and $t_s=100{\cdot}52°$C, the specific volume, obtained by laying the straight-edge across the point of intersection of the 724·4 heat horizontal on AB, and the 180 pressure value on CD, is $v_s=3{\cdot}26$ ft.³. This is approximately the value given in the steam table at $t=290°$C. The same result is given by the nomogram in Fig. 3.

A further illustration, to show the application of the combination of nomograms in Fig. 3 with the Tϕ diagram, Fig. 4, may be given. Suppose it is desired to ascertain if the assumed condition of supersaturation after expansion in the third case, from $p_1=200$ at $t_1=250°$C to $p_2=50$ lb./in.², is a reasonable one.

Following the vertical from the initial state point, Fig. 4, it will be found to cut the saturation curve at $p=87$ and $t=159°$C, and the curve for $p_2=50$ at $t_2=107°$C.

From the nomogram, Fig. 3, the total heat at 87 lb./in.² is 660·1 C.H.U./lb. and the volume is $v=5{\cdot}1$ ft.³/lb.

At 50 lb./in.² the total heat is 635·5 and the volume is $v_2=7{\cdot}65$. The volume ratio is thus

$$\frac{v_2}{v}=\frac{7{\cdot}65}{5{\cdot}1}=1{\cdot}5$$

It is, therefore, possible that the steam may have just reached the wet saturated condition at 50 lb./in.^{2}, if the tentative maximum ratio of 1·4 is taken as applicable to steam.

Although a result of this kind can not be regarded as conclusive, the example serves to indicate one method of procedure, which may be followed in the case of supersaturated steam, when more definite experimental data regarding this condition have become available.

The total heat and specific volume of wet steam are usually derived by calculation from the values given in the saturated steam table. Alternatively, they can be obtained without calculation directly from nomograms.

TOTAL HEAT OF WET STEAM.

The total heat of wet steam is given by the equation

$$H_w = H - (1-q)L \quad \text{- - - - -} \quad (5)$$

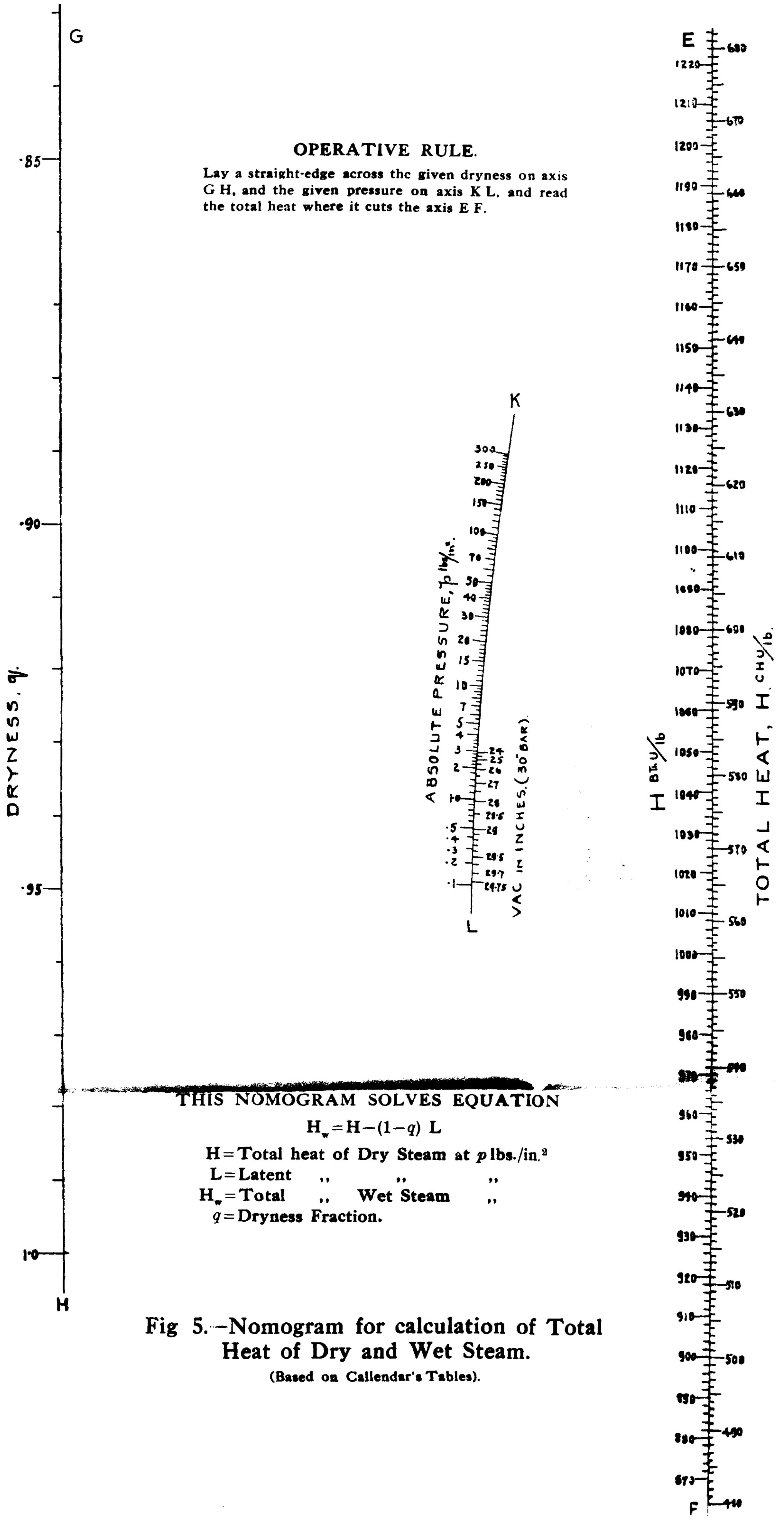
where H = total heat of dry steam in C.H.U./lb. or B.Th.U./lb.
L = latent ,, ,, ,, ,, ,,
q = dryness fraction.

Since H and L are functions of the pressure, equation (5) can be put in the form,

$$H_w + f(q)f(p) + F(p) = 0.$$

This is the general equation giving a three-scale nomogram, with two rectilinear and one curved axis. The nomogram is shown in Fig. 5.

The total heat (H_w), to a scale of 1 inch = 10 C.H.U., is set off on an axis EF, and a function scale of the dryness (q), that is, a scale of $(1-q)$, to 1 inch = 0·01 unit, and figured with the q values, is set off on a parallel axis GH. The curved pressure-scale axis KL and scale divisions are obtained from these scales, by calculating pairs of q and H_w values and taking cross alignments, as in the case of the nomogram, Fig. 3. The



OPERATIVE RULE.

Lay a straight-edge across the given dryness on axis G H, and the given pressure on axis K L, and read the total heat where it cuts the axis E F.

THIS NOMOGRAM SOLVES EQUATION

$$H_w = H-(1-q)\ L$$

H = Total heat of Dry Steam at p lbs./in.^{2}
L = Latent ,, ,, ,,
H_w = Total ,, Wet Steam ,,
q = Dryness Fraction.

Fig 5.—Nomogram for calculation of Total Heat of Dry and Wet Steam.

(Based on Callendar's Tables).

arbitrarily chosen distance between EF and GH is 9 inches. A straight-edge, laid across given q and p values, registers the corresponding H_w value on the heat scale. This is the simplest form of the nomogram for equation (5), and involves only one setting of the straight-edge. There is, however, an alternative arrangement which requires two settings; and as this may be preferred by some operators, since it gets rid of the curved pressure-axis, the author has also included it. It is shown in Fig. 6, and is really a combination of two nomograms embodying the equations,

$$z = (1 - q)\,L \qquad (6)$$

$$H_w = H - z \qquad (7)$$

The nomogram for equation (6) is constituted by a function pressure-scale on an axis NP, a function scale of dryness q, on a diagonal axis QF, and a z scale on a parallel axis EF. The latter is the " link " axis coupling the two nomograms together. The nomogram for equation (7) is constitued by this z scale on EF, a function pressure-scale on a parallel axis RS, and a total-heat scale on a parallel axis UV. The $f(p)$ scale on NP is obtained by first setting off a uniform scale of latent heat (L), reading downwards, to 1 inch = 10 C.H.U., and marking the pressure values against the corresponding latent-heat values given in the saturated-steam table.

Alternatively, the pressure scale can be set out by the general method described in the author's previous paper on the " Use of Nomograms, etc." * The " module " here is $l = 0{\cdot}10$ inch, and the latent-heat value corresponding to any given pressure, when multiplied by l, gives the fundamental y distance, in inches, from the pressure division to the arbitrarily chosen zero of the uniform-heat scale. The z scale set off on EF, to 1 inch = 10 units, has its zero value at the point of intersection with the axis QF, that is,

* Trans. Inst. Engineers and Shipbuilders in Scotland, vol. lx., p. 288.

when $q=1$ or $(1-q)=0$. This scale is omitted in the finished nomogram. The diagonal scale and axis are obtained by taking a number of cross alignments for pairs of q and L values, calculated from equation (6). The arbitrarily chosen distances between the axes NP and EF is 6 inches. The second function pressure-scale, on the axis RS, is obtained in the same way as that on NP, the total-heat values being used instead of the latent heats. The uniform base scale of total heat is again taken as 1 inch = 10 C.H.U., and the pressure scale module is $l=0{\cdot}10$ inch. Since the z and H scales are the same, it follows by a rule of nomography, that the axis UV, for total heat of wet steam, falls midway between EF and RS, and the unit of the uniform H_w scale is half that of the total-heat scale on RS. The scale is, therefore, 1 inch = 20 C.H.U. One or two points being obtained on UV, by cross alignment from the z and H values, the rest of the scale can be set off from an engine-divided scale.

The operative rule for this nomogram is as follows:—Given the pressure and the dryness fraction, lay the straight-edge across the p value on NP and the q value on QF, and note the point of intersection on EF. Pivot the edge about this point, until it lies across the p value on RS, and read the total heat of the wet steam where it crosses the axis UV.

In order to illustrate the application of these two nomograms, suppose that the steam at 10 lb./in.^{2} absolute is 10 per cent. wet, and find the total heat.

When the straight-edge is laid across $q=0{\cdot}9$ and $p=10$ in Fig. 5, it will be found to cut the total-heat scale at $H_w=580{\cdot}1$.

Using the nomogram, Fig. 6, lay the edge across $p=10$ on NP, and $q=0{\cdot}9$ on QF, and note the intersection on EF. Lay the edge across this point and $p=10$ on RS, and it will be found to cut the total-heat scale on UV at $H_w=580{\cdot}5$.

Checking by calculation from the steam tables,

OPERATIVE RULE.

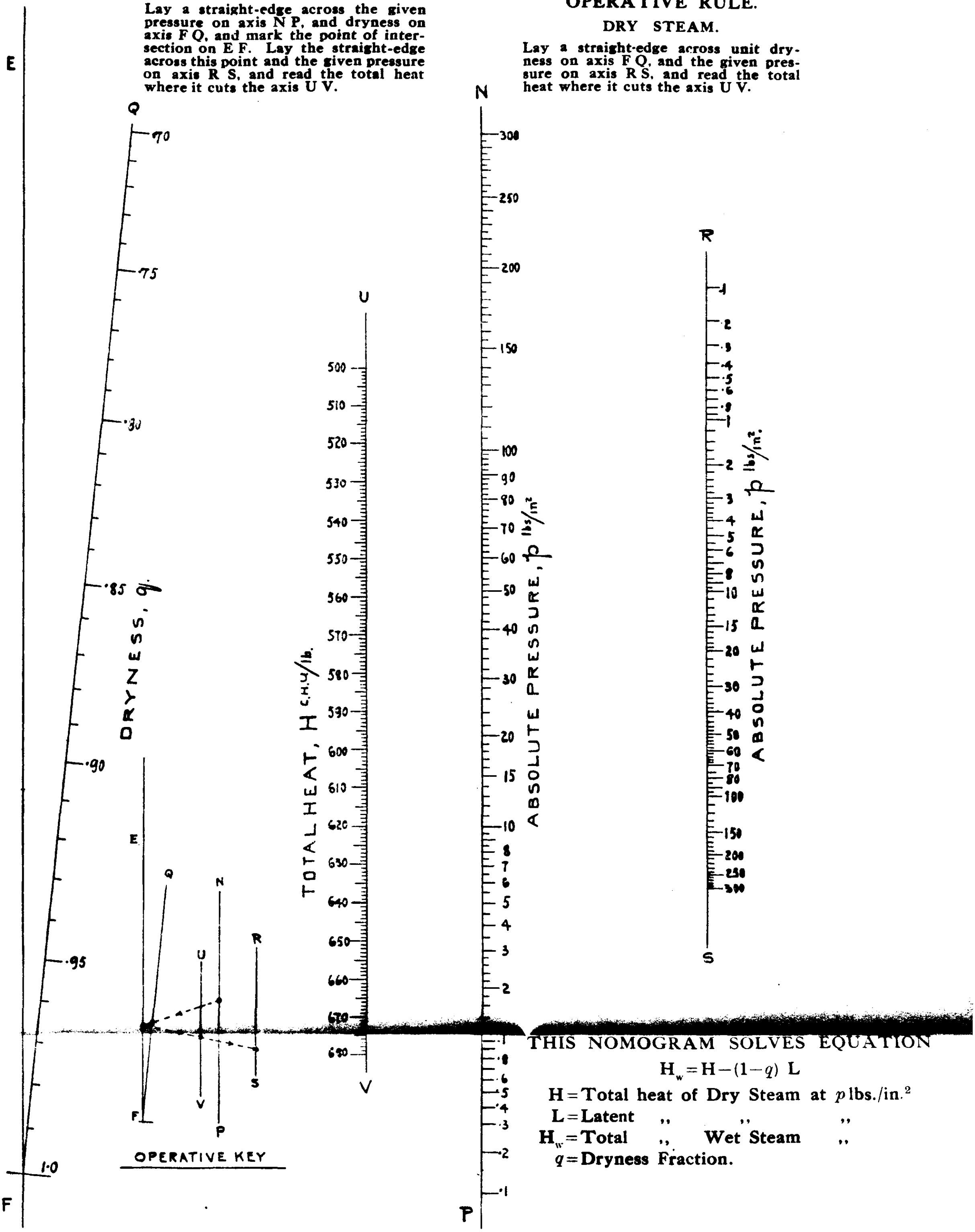
WET STEAM.

Lay a straight-edge across the given pressure on axis N P, and dryness on axis F Q, and mark the point of intersection on E F. Lay the straight-edge across this point and the given pressure on axis R S, and read the total heat where it cuts the axis U V.

OPERATIVE RULE.

DRY STEAM.

Lay a straight-edge across unit dryness on axis F Q, and the given pressure on axis R S, and read the total heat where it cuts the axis U V.



THIS NOMOGRAM SOLVES EQUATION

$$H_w = H - (1-q)\ L$$

H = Total heat of Dry Steam at p lbs./in.^{2}

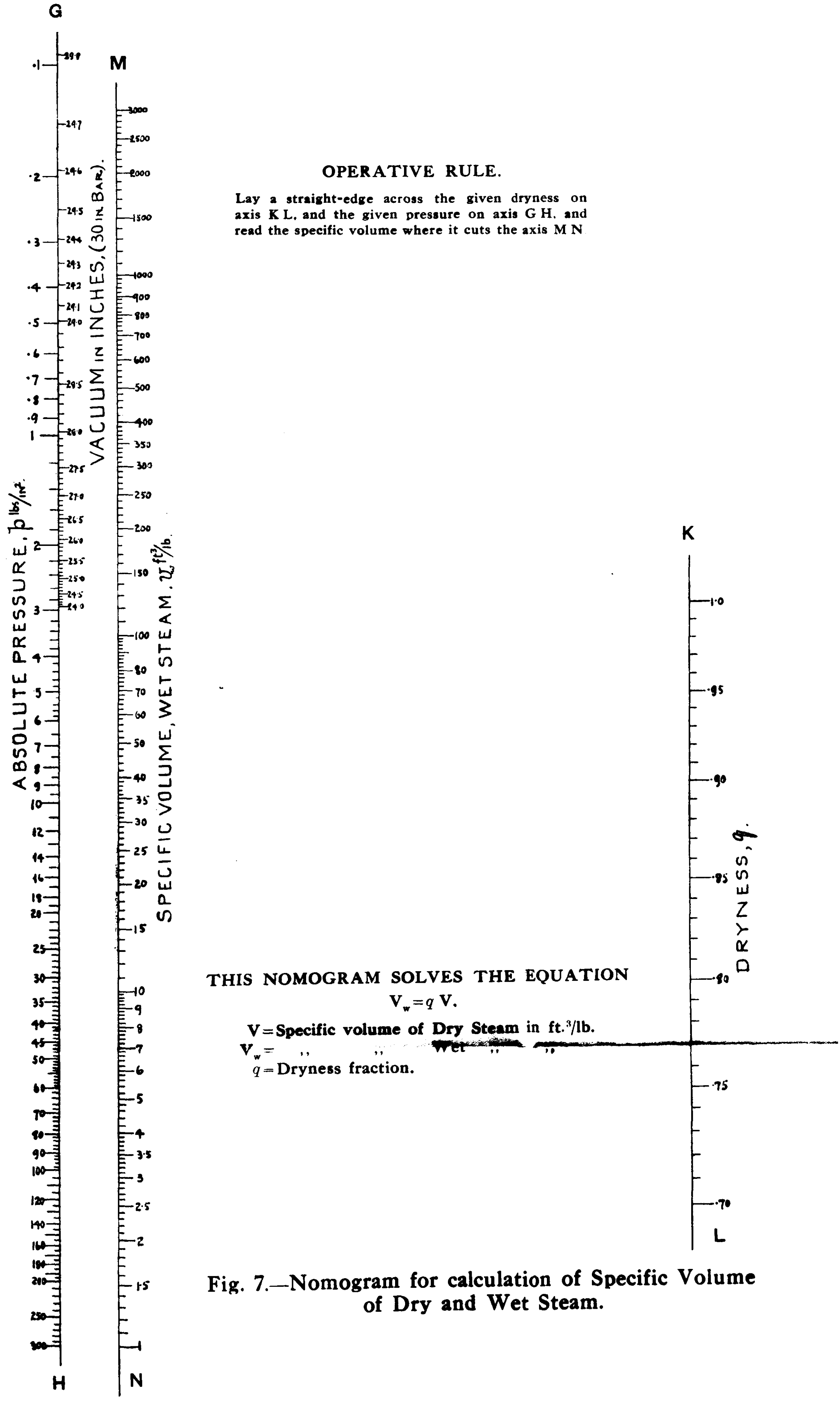
L = Latent ,, ,, ,,

H_w = Total ,, Wet Steam ,,

q = Dryness Fraction.

Fig. 6.—Nomogram for calculation of Total Heat of Dry and Wet Steam.

(Based on Callendar's Tables.)



OPERATIVE RULE.

Lay a straight-edge across the given dryness on axis K L, and the given pressure on axis G H, and read the specific volume where it cuts the axis M N

THIS NOMOGRAM SOLVES THE EQUATION

$$V_w = q\ V.$$

V = Specific volume of Dry Steam in ft.³/lb.

V_w = ,, ,, Wet ,, ,,

q = Dryness fraction.

Fig. 7.—Nomogram for calculation of Specific Volume of Dry and Wet Steam.

At $p=10$, $H=635{\cdot}01$ and $L=545{\cdot}5$, $q=0{\cdot}9$, so that,

$$\begin{aligned} H_w &= H-(1-q)L \\ &= 635{\cdot}01-0{\cdot}1\times 545{\cdot}5 \\ &= (635{\cdot}01-54{\cdot}55) \\ &= 580{\cdot}46. \end{aligned}$$

It will be seen that the check is fairly satisfactory considering the size of the nomograms used. The nomogram, Fig. 5, on account of the curved pressure axis and the crowded scale divisions, is not so reliable as that of Fig. 6, which the author prefers, from the point of view of accuracy. The first nomogram, however, can, if desired, be drawn on a much larger scale, say, twice the original size.

Specific Volume of Wet Steam.

The volume of wet steam is given by the equation

$$v_w = qv \qquad \text{- - - - - - -} \qquad (8)$$

Where v = volume of dry steam in ft.^{3}/lb.
q = dryness fraction.

Equation (8) is the simple form, which can be nomographed on three rectilinear axes, two being parallel and one inclined, or on three parallel axes, when logarithmic scales are used.

When it is desired to deal with the volumes at low pressures, such as occur in turbine practice, the wide range of volumes to be handled precludes the use of uniform scales, on a nomogram of reasonable dimensions. In the construction of the nomogram shown in Fig. 7, the author has, therefore, used logarithmic scales. This nomogram is constructed as follows:—Two parallel axes GH and KL are drawn at an arbitrarily chosen distance of 9 inches apart. A logarithmic scale of dry-steam volume is set off in GH reading upward. The particular machine-cut scale used has a unit length of 5·5 inches—that is, the distance on the scale from 1 to 10 is 5·5 inches. The pressure scale is obtained by marking the pressures against the corresponding dry-steam values given in the saturated-steam table. In order to obtain fairly accurate readings of the wet-steam volumes, the scale axis MN should be kept close up to the axis GH. The relative distances between the axes depends on the relative values of the v and q log-scale units. In the arrangement shown, the author has chosen the q log-scale unit ten times that of the v log-scale, so that the length between $q=0{\cdot}7$ and $q=1{\cdot}0$ is ten times the length, on GH, between $v=7$ and $v=10$. This q scale should be set out by the "module" method. Since the arbitrarily chosen unit of the q log-scale is 55 inches, the module is $l=55$ inches. Since the log of unity is 0, the arbitrarily chosen point on KL, for $q=1$, is the zero of the uniform base scale, from which the fundamental y lengths are to be measured. To make the procedure clear, take the case of $q=0{\cdot}7$. Log $0{\cdot}7=\bar{1}{\cdot}8451=-0{\cdot}1549$. Module $l=55$. Hence the fundamental distance of the scale division for $q={\cdot}7$ is $y_{\cdot 7}=l\times\log{\cdot}7=55\times-{\cdot}1549=-8{\cdot}519$ inches. This distance set off downwards from the chosen point for $q=1$, gives the point for $q={\cdot}7$. All other scale divisions are obtained in the same way.

The wet-steam volume axis MN is easily located by taking two pairs of v and q values and finding the points of intersection by cross alignment. When it is drawn in through these points, the v_w scale is completed by joining each v value on GH with the unit dryness value, $q=1$ on KL, and marking the points of intersection on MN with the volume values.

The steam volume at any pressure for a given dryness is obtained by laying the straight-edge across the p and q values on GH and KL, and reading the v_w value, where it crosses the axis MN. For convenience in turbine calculations, the vacuum values, in inches of mercury reckoned on a 30-inch barometer, are drawn against the pressure scale.

To show the application of this nomogram find the volume of steam at 29-inch vacuum or $0{\cdot}49$ lb./in.^{2} absolute, when the dryness is $q = 0{\cdot}90$. When the straight-edge is laid across $q = 0{\cdot}9$ and vacuum $= 29$ inches, it will be found to cut the volume scale at $v_w = 590$, reading to the nearest even figure.

Checking from the steam table, since the specific volume of dry steam at $p = 0{\cdot}49$ is $v = 655$ and $q = 0{\cdot}90$ then, $v_w = 0{\cdot}9 \times 655 = 589{\cdot}5$ ft.^{3}/lb. This check is sufficiently close for practical purposes.

In this note the discussion of these graphical methods of calculation has been confined to the two most important properties of steam which have to be handled in turbine design.

The other properties can be similarly treated; and, in this connection, the author hopes that the hints on construction given may be found useful by designers.

Discussion.

Mr. WILLIAM KERR (Associate Member) said that all those whose work involved calculations with the properties of steam must give a ready consideration to any graphical systems which promised evasion of the difficulties of arithmetical computation. The form of Callendar's equations almost forbade direct calculation therefrom in practical work, and the tabular arrangement of values in the steam tables was not always agreeable, so that graphical methods must usually be resorted to. In practical design the valuable heat-entropy diagram was of outstanding importance, and it was probable that in most cases that diagram, with direct reference to tables for specific volumes, covered the requirements. But when it came to close comparative analysis of test performances, or to problems involving accurate consideration of small pressure changes, it was necessary to deal entirely with the tables, or, if interpolation therein were too laborious, with the equations direct. Therein lay the applicability of Mr. Goudie's charts, in which he gave easy graphical methods for the determination of steam properties, as defined by Callendar's equations. These charts were very elegant and useful applications of the principles of nomography, and the simplicity of their final form no doubt hid to some extent the labour expended by Mr. Goudie on their compilation. The Callendar equations could hardly have been graphically resolved by the ordinary methods of co-ordinate axes, and it was instructive to note how readily they surrendered to the less known but more powerful processes so ably used by Mr. Goudie. Graphical forms to replace calculations from equations must possess the merit of close accuracy, and give representation of the full useful range of values. The first of these requirements was well satisfied by these charts, and a variety of tests of their accuracy did not give the slightest cause for complaint. The total heat values were in all cases satisfactory. The pressure scale in Fig. 5 was weak, and Mr. Goudie had been rightly inspired to offer the preferable form of Fig. 6. The specific volume alignment charts of Figs. 2 and 3 were excellent for the ranges given, but these required extension for practical work, since it was not uncommon to have to deal with superheat conditions at low pressures—for instance, in fractional power analyses of marine turbines. The wet steam volumes on logarithmic scales in Fig. 7 were most complete, and rendered that chart one of distinct utility. In defining specific volumes by pressure and total heat only (by the elimination of the temperature factor) the author just included those properties that were required, since the heat and pressure values were those usually known when the volume was wanted. The reverse operation on these volume charts was also valuable, as in rapid provisional designing with the use of the PV^{γ} law, the total heat was required when the pressure and volume were known. As an aid to the accurate and rapid calculation of steam properties, these charts fulfilled their purpose most completely; and, even where re-drawing or extension might be needed for practical use, the instructions given by the author would allow that to be done without serious trouble. But while the charts, as calculating agencies, called only for commendation, the prominence given to the determination of supersaturated values, and the facilities provided for finding the undercooling effect, Fig. 4, raised a controversial point of some importance, but also, unfortunately, of some obscurity. On that question one got the impression that Mr. Goudie had confined himself to giving methods, and had carefully refrained from giving opinions. It seemed undeniable that the supercooling of expanding steam might occur on occasion, and the explanation thereby obtained of excessive discharge through simple nozzles was very satisfactory. But much more would be required before designers would be willing to discard a reasonable procedure—which, after all, gave good results—in favour of a hypothesis of meagre substantiation. The conditions in actual turbines were vastly different from those in special nozzle experiments, and the causes of postponed condensation would seem to be to some extent accidental. Apparently the time period and the nuclei conditions governed the occurrence. Now, from the established cases of excessive discharge from nozzles, it would seem that the anomaly was confined to simple short convergent nozzles, and did not occur with long tailed nozzles or with the long guide passages common in impulse turbines. In simple convergent nozzles the period of expansion was very brief. In Loschge's nozzle experiments the discharge coefficient was noticeably affected by cylindrical tail lengths of a few millimetres beyond the throat; and he (Mr. Kerr), in a series of nozzle experiments, only observed excessive discharge with convergent nozzles having the plane of discharge at the throat section. If the existence of a tail on the nozzle had such an effect, the critical time period must lie within a very narrow limit, or the additional friction and confinement of the steam in the tail passage had a paramount influence. How much more then would the elaborate nozzle and bucket arrangements of actual turbines militate against the occurrence? Again, while in simple nozzles the nuclei conditions, in combination with the brief time interval, might easily be supposed to allow of supercooling, neither of these conditions were at all similarly advantageous in complete turbines. A large steam plant containing a turbine was widely different from a miniature apparatus containing a nozzle, and the complex and changing steam action in the former quite distinct from the simple and definite action in the latter. Relative to the time of action in a short nozzle, the period of development in a multi-stage turbine was quite prolonged. Mr. Goudie remarked on Wilson's experiments with air containing water vapour, but, apparently, in these experiments the air was either dust-free, or carried a certain number of nuclei. In actual turbines the water vapour might contain a little air, but it was most likely to be quite generously endowed with nuclei of foreign matter. The erosion of the blades on which it impinged, and the corrosion of the paths along which it flowed, might provide the necessary nuclei, even assuming insuperable difficulties to the entrance of extraneous dust into the cycle of events. The turbulence of the steam flow in the turbine, the eddying of the high-speed jets into stagnant steam spaces, and the intermixture of main and leakage lines of flow might all tell against the occurrence of undercooling. And it was noteworthy, if that effect were at all general in turbines, that discrepancies between pressure and temperature readings had not been more noticed. That was evidence which either discredited the application of the theory to turbines or postulated the existence of fundamental faults in the ordinary methods of thermometric measurement. Everything considered, therefore, there were a multitude of points which required elucidation before the supersaturation effect must be envisaged in methods of actual turbine design. It must be shown that it did occur in turbines, and not merely argued that it might occur because it was possible, and because it was seemingly present in certain nozzles. It must be discovered to what extent it was general, and to what extent adventitious; and whether it was possible over a wide range, or only within narrow limits. For instance, it was conceivable that supercooling might occur when superheated steam passed over the dry condition, and yet not be possible, or not equally so, when it expanded from a wet condition. It must be submitted, then, that Fig. 4 of Mr. Goudie's set of charts, while of considerable interest, had at present no very useful field of application. It might, of course, come to have such in the future, when more was known on the subject than the quoted experiments on saturated air could tell us; but somehow one got the impression that the occurrence was so fortuitous as to make calculation thereof ineffectual, and to take it into consideration was like admitting a diffidence in forecasting the boiling point of water, because it was known that with great care water might be heated beyond that point without ebullition.

Dr. G. H. Gulliver remarked that he had followed with much interest Mr. Goudie's ingenious graphical methods of obtaining the properties of steam. His accuracy left little room for criticism on the part of the casual reader, and the real test of the value of his work was the degree of usefulness which it would have in the daily problems of the designer of engines and turbines. The original diagrams must have been drawn to a considerably larger scale than the printed ones, as in Figs. 1 and 2 it was quite impossible to read, with accuracy, the temperature 100·52°C of the examples given on page 3. The expression, "total temperature," used several times on pages 5 and 6, was peculiar, and open to an obvious objection, though, of course, the meaning was clear.

Lieutenant W. J. Duncan observed that the charts which Mr. Goudie had devised would be of great service in calculations dealing with steam turbines and other heat engines. The nomogram, Fig. 3, for the determination of the total heat of superheated steam was particularly useful, as the calculation by formula was long and troublesome. The value of Mr. Goudie's work did not lie merely in the usefulness of the diagrams themselves, but also in that it tended to popularise the use of nomograms, which could be of great service in all branches of engineering. As an instance of their usefulness in another sphere, he might mention that he had found them of great assistance in the rapid calculation of the results of mechanical tests on metals. The only criticisms which he had to offer were of a very minor nature. With reference to Fig. 3, it appeared rather misleading to label the curved scale " superheat temperature," since that was very liable to be mistaken for " superheat range." It would be an improvement to substitute the word " actual " for " superheat." The formula for H_s given on page 4 was almost certainly correct, because it agreed with that given in Dalby's " Steam Power," and the derivation there given was easily verified; but it might be well to point out that it did not agree exactly with that given on page 7 of Callendar's Steam Tables. Callendar's equations were:—

$$H = S_0 T - S C P + B$$
$$\text{and } S C = a\,(n+1)\,c - b$$

where a was the reciprocal of J, the mechanical equivalent of heat. Combining these equations, the result was:—

$$H = S_0 T - Pa\left\{(n+1)\,c - \frac{b}{a}\right\} + B$$

The value of b was 0·016, so that this formula differed from that given in the paper inasmuch as $\frac{b}{a}$ was written instead of b inside the bracket. It appeared probable that there was a mistake in Callendar's Tables, and that the expression for SC should be:—

$$S C = a\left\{(n+1)c - b\right\}$$

Fig. 4. would be improved if the saturation curve were labelled.

Mr. FREDERICK SAMUELSON stated that Mr. Goudie's graphical charts on the properties of steam would be very useful, especially for approximate work. He regretted that Mr. Goudie had not carried the steam pressures up to at least

500 lbs. per square inch, as steam plants using pressures in that neighbourhood were being installed. A steam alignment diagram or graphical chart developed on similar lines to those adopted by Mr. Goudie had been devised by Mr. D. Halton Thomson, M.A., who included in one chart nearly all the information covered by Mr. Goudie's several charts. At the bottom of page 4 the author gave the formulæ for the total heat of superheated and supersaturated steam, and made a slip in converting the centigrade figures into Fahrenheit or B.Th.U. The formula, as given by Mr. Goudie,

$$H_s = 0{\cdot}859\,T - 0{\cdot}10286\,p\left\{\frac{1226{\cdot}5 \times 10^6}{T^{10/3}} - 0{\cdot}016\right\} + 835\ \frac{\text{B.Th.U.}}{\text{lb.}}$$

should be in rounded figures:—

$$H_S = 0{\cdot}4772\,T - 0{\cdot}18514\,p\left\{\frac{4842{\cdot}5 \times 10^6}{T^{10/3}} - 0{\cdot}016\right\} + 835\ \frac{\text{B.Th.U}}{\text{lb.}}$$

Mr. H. L. GUY observed that the paper was of great interest to engineers as an exposition of the art of constructing nomograms, and after studying it they would no doubt utilise that method of preparing charts for many other purposes than those covered by the paper. In choosing the range covered by the charts, Mr. Goudie had wisely chosen a wide range—from 0° Fah. to 450° Fah.—for superheat, a range which would cover the requirements of engineers for several years to come. On the other hand, the range of pressure did not go above 300 lbs. per square inch, which was not sufficiently extended to meet present-day work, as turbines were already being constructed for steam pressures of 450 lbs. per square inch gauge, and the designers of these turbines would have been in considerable difficulty had not Prof. Callendar's tables been prepared to cover these higher pressures. Unfortunately, these tables were not sufficiently elaborated to meet the requirements of engineers, steps between consecutive figures being so wide as to necessitate a resort to plotting in nearly all cases when it was desired to use them. The charts prepared by Mr. Goudie would dispense with the necessity for this plotting, although, personally, he would have preferred to see Callendar's tables published with pressure steps of not more than 10 lbs. per square inch, and with temperature steps of not more than 5° Fah. While the usefulness of the nomograms for the purpose stated would be considerable, he regretted that Mr. Goudie, or someone else interested in the subject, had not prepared a Mollier diagram, that was, one with entropy as abscissæ and total heats as ordinates. If such a chart were prepared showing temperature, pressure, wetness, and specific volume lines, together with saturated and supersaturated lines, a great boon would be conferred on engineers who had to handle turbine or reciprocating-engine problems. Such a curve would be more useful than a series of nomograms, because a single point on the chart would give all the information which would be obtained from reading several nomograms. Further, in working out, say, a complete turbine, the Mollier diagram gave a pictorial representation of the expansion through the turbine, which was of itself very instructive. Unfortunately, no such chart had been published in Great Britain, and it was regrettable that the only chart of this character, but without supersaturation lines, and one which was very extensively used, was published in Germany, and was not based on Callendar's steam tables. Another point in the preparation of such a chart which should be observed was to choose a scale which could be readily applied. Thus, if a scale for ordinates of one millimetre per centigrade heat unit was chosen the chart could be easily interpreted, because a centimetre scale was always available. Further, if a scale of one centimetre for each 0·1 of entropy were chosen a chart of very convenient working proportions was obtained. The Mollier diagram had proved its utility, whereas, in his opinion, none of the diagrams which had been more recently published with other co-ordinates possessed any advantages, from which it appeared regrettable that they should be introduced. Mr. Goudie's notes on the supersaturated condition which might exist during expansion in the turbine were of great interest, and it was now generally accepted that therein lay the explanation of the well-known anomaly in nozzle discharge. It was, however, at the present time very uncertain as to whether the steam remained in a supersaturated condition after leaving one set of the nozzles until it entered the next set. In the opinion of Prof. Stodola, the condition of thermal equilibrium was established with such rapidity that the steam was not in a supersaturated condition on leaving the moving row of blades of an impulse turbine. On the other hand, Mr. Martin had presented an argument in favour of the suggestion that the steam remained in a condition approximating to supersaturation during the whole of its expansion through the wet area of the diaphragm. Mr. Goudie rightly stated that "There is scope here for experimental research, to discover whether this supersaturated condition exists in such types to an extent which justifies the modification of the usual methods of calculation based on the assumption of thermal equilibrium throughout the whole range of expansion." Even if such research finally established that the steam did remain in a supersaturated condition, it was not probable that any substantial change in the construction of steam turbines would result from an adoption of methods of design based on that assumption.

Mr. H. M. Martin said it was gratifying to note that British engineers were at length beginning to appreciate the value of Callendar's work on the properties of steam, which, it might be observed, had been made the basis of the "Beama" heat-drop tables. It must, however, be admitted that Germany was the first to recognise the extreme importance of the advance made by Callendar. It had, unfortunately, long been the habit of British scientific men and technologists to ignore original work done in their own country, and to recognise it only when returned to them with a foreign cachet.

The attitude of British surgeons towards Lister was a case in point; whilst British engineers quite commonly attributed to Bauschinger the discovery of the importance of the elastic range of a material, and totally ignored the fact that James Thomson had demonstrated that so far back as 1848. Again, the theories as to the critical speeds of shafting, and as to the strength of rotating discs, were first worked out here, but Chree's researches were ignored by British engineers until his results were returned to them from overseas. The importance of Callendar's work was first impressed on him (Mr. Martin) by an unfortunate experience. Mr. Eric Brown had been good enough to give him, for his personal information, full particulars of certain tests of a steam turbine, in which superheats, pressures, and speeds were varied through a considerable range. The tests were made under practically laboratory conditions, and every precaution was taken to secure the utmost accuracy in all observations. In attempting to analyse the results with the aid of some very elaborate and convenient American tables, astonishing discrepancies were found in the turbine efficiencies as deduced from pressure and volume data and as deduced from heat entropy data. Some weeks of work were thus rendered nugatory, when it occurred to him to remake the calculations, using the much less convenient Callendar tables. When that was done the former discrepancies entirely disappeared, and a further examination of the American tables showed that the area (expressed in heat units) of a theoretical indicator diagram, as deduced from these tables did not agree with the area of a precisely corresponding temperature-entropy diagram. Further studies disclosed discrepancies of 15 to 20 per cent. in the specific heats of superheated steam, as deduced from the pressure-volume data and from the total-heat data. The tables in question were supposed to embody, and did, in fact, embody, the most recent experimental results on specific heats. The experiments were, however, "made in Germany," and were wholly unreliable. The great drawback to Callendar's tables was the need for interpolation, and that drawback was greatly reduced by Mr. Goudie's charts, since graphic interpolation was was much less tiresome than arithmetical. Charts I. and II. would be found extremely convenient, since in so many cases the data given were the superheats and not the temperature of the steam. Regarding the accuracy of the readings, it was, of course, quite true, as Mr. Goudie observed on page 3, that an error of 0·13 C.H.U. was negligible in comparison with 724 units of total heat. It had, however, to be observed that quite commonly they had to deal with differences of total heats, and when these differed little, as happened when the pressure drop was small, tables or formulæ must be employed if accuracy were essential. Chart III., which gave both total heats and specific volumes, would be particularly useful, as in most charts to which volume lines were added the ensuing complication was somewhat bewildering. With respect to undercooling, Mr. Goudie remarked on page 464 that "There is some point at which the steam reverts to the wet condition," but that that limit was not definitely fixed. That statement seemed to him (Mr. Martin) to underestimate the importance of the work done by Lord Kelvin and Mr. C. T. R. Wilson. If the size of the nuclei on which the condensation came down were known, Lord Kelvin's theory defined perfectly the limit at which that condensation occurred. Wilson's experiments showed that these nuclei were equivalent to spheres of about 5×10^{-8} cm. radius, and with that value of the radius Kelvin's theory led to the result that dust-free undercooled steam began to condense when

$$\log. \frac{p}{p_s} = \frac{3{\cdot}75\sigma}{\mathrm{T}}$$

Here p denoted the actual pressure of the undercooled steam, T its absolute temperature on the centigrade scale, σ the surface tension of water at the same temperature, and p_s the pressure of saturated steam at the temperature T. Probably further experiment might slightly modify the above coefficient, but it was no doubt substantially correct. The formula showed further that the limit of expansion possible without condensation was underrated by Mr. Goudie, being, as an average value, nearer 1·8 than 1·4. The latter figure seemed to have been suggested by the volume ratios in Wilson's experiments quoted on page 8. It should, however, be borne in mind that in these experiments the reduction of temperature due to an expansion of moist air was not the same as when pure undercooled steam was expanded. He (Mr. Martin) had recently* given reasons for concluding that, in actual steam turbines, wet steam never did attain a condition of thermal equilibrium, but was undercooled up to the very condenser. That view afforded a rational explanation of the observed values of the superheat and vacuum corrections, and would necessitate, if accepted, an abandonment of the "efficiency ratio" as the criterion of merit of a turbine. That some change of that kind was needed had long been recognised by turbine engineers. Fancy figures had on various occasions been obtained on the continent with steam at temperatures of 350°C. or so. These figures made the results obtained in this country with moderate superheats look poor. He (Mr. Martin) had long held the view that the "hydraulic efficiency" of a turbine best represented its figure of merit, and if Mr. Goudie would devise a nomograph by which that "hydraulic efficiency" could be determined directly from the trial data, he for one would be sincerely grateful, though he hesitated to urge the task on so extremely busy a man as Mr. Goudie. Few but those who had been actually engaged in similar work were in a position to appreciate the great amount of time and ingenuity which must have been expended in preparing the charts described in the paper, and for that engineers were deeply in Mr. Goudie's debt.

* "Engineering," July 5, *et seq.*

Prof. H. L. CALLENDAR, F.R.S., stated that, as Mr. Goudie remarked, it was impossible to avoid the necessity for interpolation in the use of steam tables for accurate work, especially when the desired result depended on small differences, as was often the case. With superheated or supersaturated steam, a double interpolation was commonly required, which aggravated the inconvenience considerably. The graphic methods which Mr. Goudie proposed would often prove useful and at the same time sufficiently accurate for most practical purposes. The diagrams illustrating the paper appeared to have been selected with great judgment, and drawn with all necessary care and attention to suitability of scale. They should prove well adapted for the purpose for which they were designed. With regard to the state of supersaturation, there seemed to be very little doubt that this state existed in the turbine, nearly if not quite up to the limit indicated by Wilson's experiments with water vapour in air. Further experiments were doubtless required to elucidate this point, but it appeared that the simple assumption that supersaturated steam followed the same equation as dry steam, and that the state of supersaturation might persist up to the limit of eight times the normal saturation pressure, gave a fair explanation of many anomalies. In his forthcoming book on the "Properties of Steam," he (Prof. Callendar) had already worked out various cases with satisfactory results, but publication had, unfortunately, been delayed by the war.

Prof. A. L. MELLANBY, D.Sc., Vice-President (Member), said that he had long felt that the equations representing the properties of steam as brought forward by Prof. Callendar were not sufficiently well known, and he considered that members of the Institution were indebted to Mr. Goudie, not only for bringing Callendar's work before them, but for introducing to them these ingenious methods of solving the rather complicated expressions necessary for obtaining values so often required in turbine design. It was interesting to note that the paper in which Callendar first published his equations was read before the Royal Society in 1900, and he believed that about 10 years elapsed before his results were published in any engineering text-book in this country, although their value had been recognised previously on the Continent. The objection to the various other tables in general use had been stated by Mr. Goudie, who had pointed out that they were somewhat thermodynamically inconsistent—a fact which anyone working on problems connected with the flow of steam through nozzles soon found out. He would suggest that arrangements might be made so that the diagrams proposed by Mr. Goudie could be issued separately, as it was somewhat difficult to obtain accurate results when the diagrams were folded up and bound in the Proceedings of the Institution. He had no actual criticism to make upon the paper, but would refer briefly to the remarks made regarding the apparent discrepancies observed by different experimenters between the calculated and actual discharges of steam through nozzles. Many experiments had been made upon the flow of steam through nozzles, and it had often been pointed out that the actual flow was greater than that obtained from calculation on the ordinary theory. It would be found, however, that many of the experiments published would not stand a critical examination. Often sufficient precautions had not been taken to ensure that the initial steam was perfectly dry, and in other cases when the steam was initially superheated it would be found that the superheat fluctuated considerably during the experiments. Again, in calculating the theoretical flow, the steam table values used in some cases were open to suspicion, and he felt that this fact alone was a sufficient reason for further investigation of this important point. In experiments that had been conducted by students at the Royal Technical College, when the initial steam was given one or two degrees of superheat—just sufficient to ensure that it was dry—and the Callendar steam values used, it had been found that the difference between the calculated and actual discharges were much less than those mentioned by other experimenters. He did not wish to imply that he disbelieved in the supersaturation theory; indeed, it appeared to him to afford a rational explanation of many of the phenomena of steam flow, but he thought that this theory would have to be combined with careful experiment and calculation before accurate values of the frictional resistance in nozzles could be obtained.

Mr. Goudie, in reply, stated that Mr. Kerr had endorsed the claim he made regarding the advantage of a nomographic estimation of total heat values of superheated steam over the method of interpolation from the steam tables, or the still more laborious method of calculation from the equations. It was satisfactory to note that the tests which he had made of the accuracy of the charts revealed no cause for complaint. In connection with the degree of accuracy obtainable, Mr. Martin had pointed out that the discrepancy between the calculated and graphically determined values of the total heat, which he had stated as being negligible for practical purposes, might not be so, in a case where a very small drop in pressure and corresponding small decrease of total heat was involved. In designing the charts and nomograms, he had not tried to adapt them for such a fine degree of accuracy; and if such was desired, in any case, then direct calculation from the equation became necessary. Leaving such extreme cases on one side, however, he thought that the specific heat and total heat charts and nomograms would serve for most practical purposes. He was pleased to note that this opinion was also endorsed by Prof. Callendar. Dr. Gulliver, while commending the accuracy of the diagrams, had questioned the possibility of reading a temperature range of 100·52°C on the scale of the specific heat chart. It was quite true that such a degree of accuracy was not obtainable, but the closest approximation to this was obviously the common sense interpretation of the figure, which was quoted, to the second decimal place, in order to obtain, for comparison, the exact temperature of the superheated steam given in the tables. Dr. Gulliver had commented on the use of the term "total temperature," although he agreed that its meaning was clear. He had used the term "superheat temperature" for the scale title on the nomogram, Fig. 3, in place of this, and Mr. Duncan had taken exception to it, as liable to be confused with superheat range. He had found it difficult to decide what was the most suitable expression. He was not sure that the term "actual," suggested by Mr. Duncan, was any better than the others. Probably the best way to avoid ambiguity would be to use the definite, though longer term, "temperature of superheated steam." The discrepancy between the total heat equation given in the paper and that given in the Callendar steam tables, pointed out by Mr. Duncan, was due simply to a typographical error in the tables. The brackets had been omitted. He had not labelled the saturation curve in Fig. 4, as the table, if printed near the curve, would have given rise to confusion in reading values in the vicinity of the curve. The curve might have been given distinguishing letters at each end; and he regretted he had been unable to add these, as all the diagrams for the journal had been printed off before the criticisms had reached him. Mr. Samuelson and Mr. Guy pointed out that the pressure range had not been carried high enough. He regretted he had not extended this to 500 lbs./inches^2. At the time the charts were designed, he did not anticipate the rapid progress that had since been made towards the use of higher pressures and temperatures in turbine practice. Mr. Kerr had also pointed out the desirability of extending the lower pressure limit in the case of the nomograms for calculation of specific volume at low pressures. Owing to lack of time he had not been able to look into this suggestion; but possibly it would be found that an auxiliary nomogram, having logarithmic scales, would be more suitable than an extension of the scales on the axes of Figs. 2 and 3. He had to thank Mr. Samuelson for pointing out the numerical error in the total-heat equation for superheated steam, which occurred in the advanced proof of the paper. In converting from the Fahrenheit to the centigrade system of units, he had overlooked the fact that the coefficient of the first term on the right side of the equation was the specific heat of steam at zero pressure ($S_0=0{\cdot}47719$), and remained constant, whatever scale of temperature was used. He had reconverted the figures and found that his values checked with those of Mr. Samuelson. The correct form of the equation for total heat in B.Th.U. was now given in the paper. As stated in the opening paragraphs, his object in writing the paper was to submit one or two auxiliary graphical aids to calculation for use in conjunction with the steam tables; and he had confined the discussion to the total heats and volumes. He had intended, when time permitted, to attempt the design of a combined nomogram, analogous to the well-known Mollier diagram; but pressure of work had so far prevented him. He was much interested, therefore, to learn from Mr. Samuelson's remarks, that Mr. D. Halton Thomson had, unknown to him, been working along the same lines on this subject. He had since had a conversation with Mr. Thomson regarding his new "steam alignment diagram," and found that this was the nomographic equivalent of the complete Mollier diagram which he had in his mind, taking account of all the properties of the steam in either saturated, supersaturated, or wet condition, and enabling calculations to be carried out, either on the ordinary hypothesis of thermal equilibrium throughout the range, in a multi-stage turbine, or on the new hypothesis advanced by Mr. Martin of complete or partial supersaturation. Mr. Thomson had kindly presented him with a provisional copy of the diagram; and after an examination and a test of it he was sure it would meet with the hearty commendation of engineers engaged in turbine design, when Mr. Thomson published it in its final form. Mr.

Martin had kindly suggested that he should try to devise a nomogram which would deal with the hydraulic efficiency of the steam turbine. He thought this new diagram of Mr Thomson's, although it might not satisfy the experimental conditions, would go a considerable distance towards the solution Mr. Martin was seeking for, as it made provision for the introduction of hydraulic efficiencies in provisional calculations dealing either with saturated or supersaturated steam. Obviously, in this reply, it would be out of place for him to enter into any details regarding the new diagram. He understood that Mr. Thomson intended to publish it, with full explanations, at an early date. He agreed with Mr. Guy in his desire to see the Callendar tables issued in a much more extended form, in order to obtain closer readings of pressure and temperature, etc.; and he hoped that in the forthcoming book, which he was pleased to learn Prof. Callendar had in preparation, the author would make this improvement, and also give values of sensible, evaporation, and total heats and entropies. It was also very desirable, since the bulk of the engineering profession still used the Fahrenheit system, that a complete table should be given in Fahrenheit as well as centigrade units. Mr. Guy had touched on the Mollier or total heat entropy diagram. He agreed with him that this diagram was the most useful member of the group of rectangular energy charts available for use by the turbine designer. He did not agree, however, that it should be loaded up with such a complete set of curves as Mr. Guy suggested. The introduction of supersaturation and saturation pressure curves and specific-volume curves, differing only slightly in slope, below the saturation curve, in addition to the usual dryness curves, produced a bewildering maze difficult to read from. Also, more or less interpolation between volume curves was necessary. In the long run, he considered that the volume for a given condition, shown by a state point on the diagram, could be obtained more expeditiously and accurately from a nomographic chart, than from a set of curves in the Hϕ field. His reference to supersaturation had elicited some interesting and instructive comments. Mr. Kerr remarked that he had confined himself to giving methods and carefully refrained from giving opinions. He had to admit that his mental attitude towards this subject was that of an "anxious inquirer." He accepted the supersaturation theory, in so far as it gave a satisfactory explanation of the discrepancies between actual and calculated discharge from nozzles. He had considerable doubt, however, in subscribing to the hypothesis formulated by Mr. Martin, that the supersaturation condition might persist right through the whole range of expansion in a multi-stage turbine. He gathered from Prof. Callendar's remarks that he considered the condition might persist in the turbine till the Wilson line, defined by Mr. Martin on his Hϕ diagram, had been reached. He was not sure if Prof. Callendar agreed with Mr. Martin as to its persistence for the remainder of the pressure-drop down to the condenser pressure. On the other hand, as Mr. Guy pointed out, Prof. Stodola, who was certainly one of the first authorities on the steam turbine in Europe, had expressed his opinion that the supersaturation condition produced at the nozzles in any stage did not even persist throughout the stage. In suggesting the probable volume ratio, in nozzle experiments, at which the condition of thermal equilibrium might be restored, he had stated 1·4 as a purely tentative value suggested by Wilson's experiments on air. He accepted Mr. Martin's higher estimate of 1·8 deduced on Kelvin's theory, provided the conditions in the case of the moist air in Wilson's experiments were the same as those in steam pure and simple. His difficulty lay in accepting deductions from experiments on a mixture of air and water vapour, as being applicable for the determination of conditions for water vapour alone. As he had already stated in the paper, he was not aware of any experiments on steam by which the limit of the thermal instability for steam had been definitely fixed.

The whole theory of the existence of supersaturation in a turbine, as far as he could judge, was, as yet, in a hypothetical state; and he thought that the first step towards a clearer understanding of the subject should be a direct experimental investigation on nozzles. Since the discussion, he had talked the matter over with Dr. Alfred W. Porter, and arranged, as soon as they could find time, to make a series of preliminary experiments on this subject. He thought the arguments Mr. Kerr had advanced against the occurrence of supersaturation from stage to stage of a multi-stage turbine were worth careful consideration. Although the complete conditions in the interior of a turbine could not be reproduced experimentally, a considerable number of conditions similar to them might easily be produced in a simple experimental nozzle apparatus, and much valuable information might thereby be obtained. Prof. Mellanby's statement regarding the first publication of Callendar's equations would probably come as a surprise to many members. He hoped that the state of apathy in such matters, which existed before the war among engineers, would now become a thing of the past. Prof. Mellanby had also commented on the supersaturation theory, and had drawn attention to the possibilities of experimental error and lack of proper precautions which rendered many published results on nozzle experiments open to question. There was no doubt that a good deal of such work had to be accepted with caution. It was much better to set it aside altogether and start a completely new line of investigation in which every precaution could be taken to ensure accurate determination of initial and final states of the steam and constancy in the conditions. Many of the older experiments were carried out by means of apparatus and recording instruments which were inferior to the more modern types now used for purposes of scientific research. Under the present improved conditions it should be possible to carry out experimental work on steam nozzles and obtain results free from the suspicion of inaccuracy that was attached to some of the earlier experiments.

A Spare copy of the Advance Proof of this paper is submitted, as, on account of delay in printing, the "Author's Copies", with the discussion, have not yet come ~~to~~ hand.

9th Jan 1918

W. J. Goudie.

This paper returned on 2nd May 1919 is resubmitted in accordance with the request contained in Clerk of Senate's letter dated 15th May 1919.

19 May 1919

W. J. Goudie

ADVANCE PROOF—PRIVATE. *SUBJECT TO REVISION.*

This proof is sent for your perusal in order that you may prepare such remarks as you may desire to make during the discussion.

Institution of Engineers and Shipbuilders in Scotland.

THE USE OF NOMOGRAMS IN THE DETERMINATION OF THERMAL EFFICIENCIES OF INTERNAL COMBUSTION ENGINES, AND COEFFICIENTS OF PERFORMANCE OF VAPOUR COMPRESSION REFRIGERATORS.

By Mr. WILLIAM J. GOUDIE, B.Sc.

24th April, 1917.

IN dealing with the thermal analysis of any heat-engine cycle, it has now become the custom to use either a graphical or semi-graphical method of calculation in preference to the often involved and tedious analytical method. The instrument used is commonly termed an "energy chart," and its character varies with the particular nature of the problem to be handled. The older and well-known temperature-entropy diagram and the newer total-heat entropy diagram for steam are familiar examples. These, however, represent only two of a possible series of fifteen charts which can be constructed to show the interdependent relations between the six characteristic properties of the working substance. These characteristics are—pressure, volume, temperature, entropy, internal energy, and total heat.

Five rectangular energy charts appear to be in use at present. These have respectively as co-ordinates—pressure and volume; temperature and entropy; total heat and entropy; temperature and pressure; total heat and temperature. Until recently, the use of such charts has been confined principally to the analyses of the cycles of engines and refrigerating machines, using saturated vapours as working substances. The $T\phi$ diagram has been used to a limited extent to study the heat changes in the gaseous working substance of the internal combustion engine; but the usual methods for conversion from *pv* to Tϕ co-ordinate fields have militated against its general adoption.

The calculation of the ideal thermal efficiency of an internal combustion engine is a very simple process, when it is assumed that the specific heat of the products of combustion is constant at all temperatures. On such a hypothesis, the efficiency is expressed simply as a function of the compression ratio. Modern research, however, has shown that this old hypothesis is not correct, and that at the very high temperatures attained in internal combustion engines the specific heat increases with the temperature. The precise law of variation is still undecided, some authorities considering that the specific heat may be taken as a linear, and some as a quadratic function of the absolute temperature. Whatever may be the true condition, the expression for the thermal efficiency deduced from this new hypothesis is a complicated one, and as a rule it can only be reduced to manageable proportions by taking a series of approximations.

The case for the gas engine has been worked out by H. E. Wimperis*; that for the Diesel engine by S. Lees†; and that for the *quasi* Diesel engine, with constant temperature reception of heat, by K. Takemura‡; and their results can be referred to by those who are interested in the analytical method.

This complexity of the calculation in the case of the internal combustion engine has naturally drawn attention to the need for some suitable form of energy chart, by which the thermodynamic problem can be handled with the same facility as that of the steam engine and turbine. In order to determine the ideal thermal efficiency, the heat supplied to the working substance, and the heat rejected by it, have to be found. When

* "Internal Combustion Engine," by H. E. Wimperis, M.A., p. 81.

† *Engineering*, 1st January, 1915, p. 1.

‡ *Engineering*, 26th November, 1915, p. 540.

these are known, the heat available for conversion into work is given by their difference. This quantity, divided by the heat supplied, gives the ideal efficiency of the particular cycle under consideration, which can then be compared with the actual thermal efficiency of the engine. For any given case, in practice, the heat supplied is known from the conditions of the test, and the problem resolves itself into one of finding the heat rejected. This, again, involves the determination of total heat or internal energy, at different points of the cycle. These two quantities are simply functions of the absolute temperature, so that, if the corresponding temperature values are found, the necessary data for the calculation of the efficiency are available.

Any form of energy chart for the complete graphical calculation of the heat values, should give the interdependent relations between, at least, the four characteristics—pressure, volume, temperature, and entropy. By this means the characteristic equation, and the equation for entropy change under varying conditions of pressure, volume, and temperature can then be solved.

At present the two principal diagrams in use for such analysis are the temperature-entropy ($T\phi$) and temperature-pressure (Tp) energy charts. In the case of the $T\phi$ diagram the four characteristics p, v, T, and ϕ are graphically defined by the horizontal and vertical temperature and entropy lines, and by sets of constant-volume and constant-pressure curves. Two of the latter are shown on the field on the right side of Fig. 7. When the pressure and volume values at any point of the cycle are known, the point of intersection of the corresponding pair of curves in the $T\phi$ field determines the temperature and entropy values.

It is not necessary, in this case, to provide a large number of curves. Only one or two reference or index curves need be drawn. Any other curves can be obtained from them by means of portable or operative scales of pressure and volume.

On this diagram adiabatic curves are vertical straight lines; and the rise or fall in temperature, during an adiabatic compression or expansion between given limits of pressure or volume, is readily ascertained by drawing the vertical between the two limiting curves. For the efficiency calculation this latter point is of importance.

The other and more recent form of energy chart has the entropy co-ordinate replaced by pressure. The relations between the four characteristics are defined by the horizontal and vertical temperature and pressure lines and sets of constant-volume, and constant-entropy or adiabatic curves. Two of the latter are shown in the Tp field, on the right side of Fig. 6. In this case, where the pressure and volume at any point of the cycle are given, the corresponding temperature and entropy values are again determined by the point of intersection of the pressure vertical and the constant-volume curve. Here, in order to determine the temperature rise or fall during an adiabatic compression or expansion, it is necessary to locate the corresponding adiabatic curve in the Tp field. This form of diagram, to be of sufficiently practical value, requires to be provided with a large number of adiabatic curves; and, even then, more or less interpolation is necessary. The constant-volume curves, however, are straight lines passing through the origin, so that any one can be very easily drawn in as required, to suit the conditions of the problem.

In some instances the temperature co-ordinate has been replaced by its function, the internal energy. In such a case the constant-volume lines are changed into flat curves, and a set requires to be plotted. When complete sets of constant-volume and adiabatic curves are drawn in the field the result is a confusing network. Further, the calculations of the p and T co-ordinate values, especially for the adiabatics, is a very laborious business. The crowding difficulty can be partly dodged by using logarithmic scales of pressure and temperature. In this case the adiabatics are flattened out almost into straight lines, and are nearly parallel. A smaller number can be used, and interpolation becomes easier. The volume curves still remain straight, but become parallel, and are inclined at 45 degrees to the pressure axis. Any curve can thus be drawn from a logarithmic scale, placed parallel to the pressure axis. In any case, a permanent set of adiabatics has to be provided, and the solution of the equation for temperature rise or fall, during an adiabatic operation, has to be made by following the nearest adiabatic till it intersects the given pressure or volume curve, as the case may be.

With a view to eliminating, as far as possible, the undesirable conditions indicated above, the author recently applied the principle of that somewhat neglected branch of applied mathematics, called "Nomography," to this particular problem. One result is the nomogram shown in Fig. 1, which has been designed for the expeditious determination of the heat quantities necessary for the calculation of the thermal efficiency of an internal combustion engine. It consists essentially of three axes—AB, EF, and CD—carrying scales of pressure, volume, and temperature, a reference volume curve GH, and a portable or operative volume scale. Total heat and internal energy scales are placed beside the temperature scale, so that, when a temperature value is determined for any point of a cycle, the corresponding total heat or internal energy can be read off. The necessary values for any given case can be obtained from the nomogram, by means of a straight-edge, and a strip of paper, having the necessary limiting volume values ticked off on it from the operative scale of volume. This is not the only form in which the nomogram can be drawn. Logarithmic scales can be used instead of the uniform ones shown here. The volume axis, owing to this change, also takes a vertical position. A slight advantage is obtained by this alteration of slope of the axis. The acute angle of intersection of the straight-edge with the axis, at lower pressure and higher volume values, as occurs at the beginning of compression periods, is partly avoided. The author finds, however, that this advantage is more than balanced by the disadvantage arising from the rapid convergence of the temperature and heat scales at the upper limits, which makes close reading of the values difficult. In addition, the work entailed in the construction is much greater than when uniform scales of pressure and temperature are employed.

There is a variant of one portion of this nomogram in which the reference volume curve GH and the operative volume scale can be replaced by a function scale of temperature and a fixed function scale of volume. This is shown in Fig 2; and, to avoid confusion, the axis carrying the pressure scale and the inclined axis carrying the volume scale, Fig. 1, are omitted. A function scale of temperature is provided on an axis AB, a function scale of volume on a parallel axis EF, and a scale of total entropy (the values have been omitted, as they are not required in making a calculation) is placed on a parallel axis KL. These two scales and reference axis, on the one hand, or the operative volume scale and reference curve on the other, enable the temperature rise or fall during adiabatic compression or expansion to be obtained in a simple manner, and without any calculation. This second form, however, is shown only for purposes of comparison with the first nomogram. For several reasons, the author prefers to use the reference volume curve in conjunction with the operative volume scale, and recommends the adoption of the nomogram, shown in Fig. 1, for practical use. When it is drawn on a reasonably large scale it will give results for stated conditions, with a degree of accuracy quite sufficient for practical purposes.

When it is desired to investigate the thermal changes taking place throughout an engine cycle, or, in other words; to transfer the indicator diagram of the actual engine from the pv to $T\phi$ or Tp co-ordinate field, this nomogram is

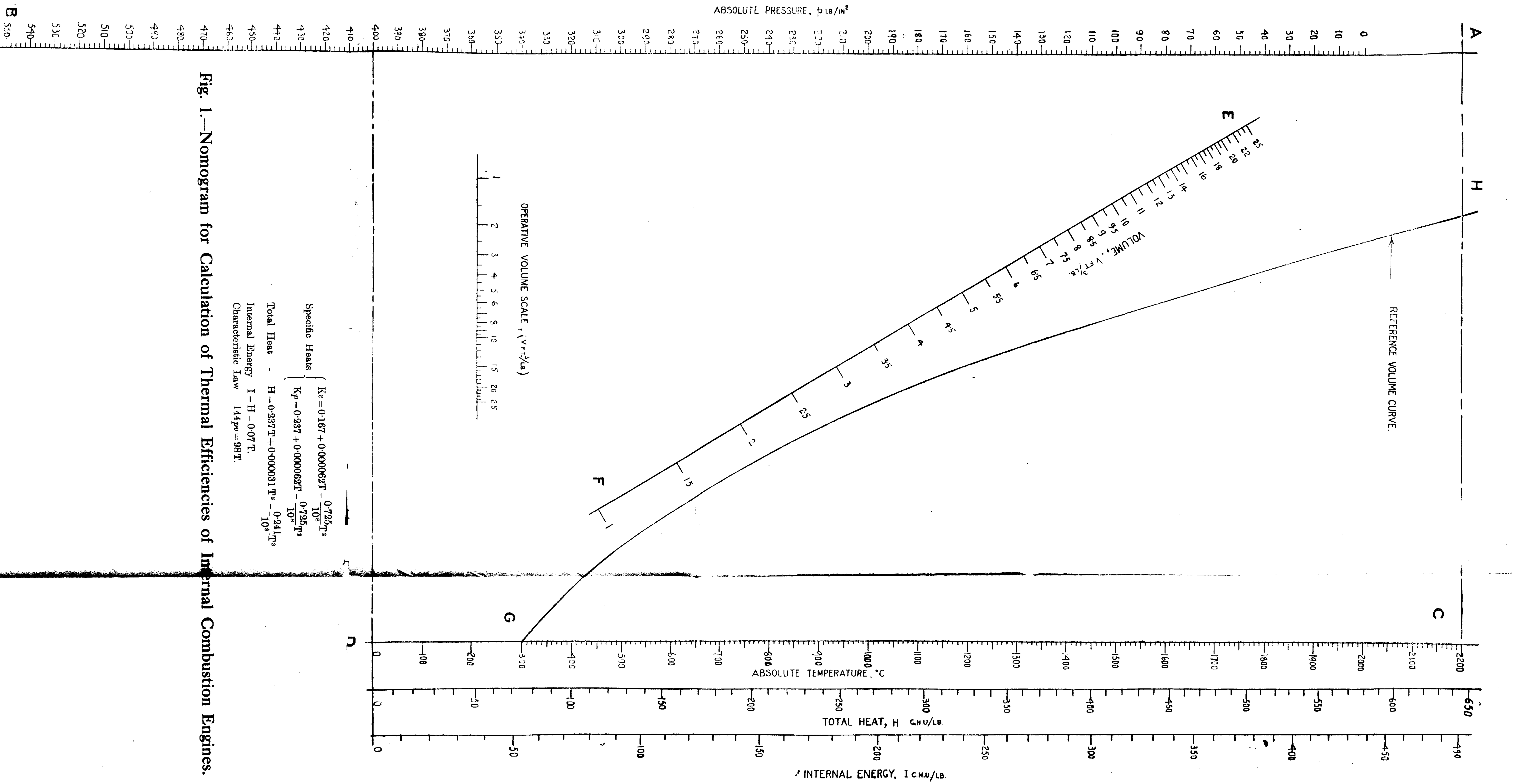


Fig. 1.—Nomogram for Calculation of Thermal Efficiencies of Internal Combustion Engines.

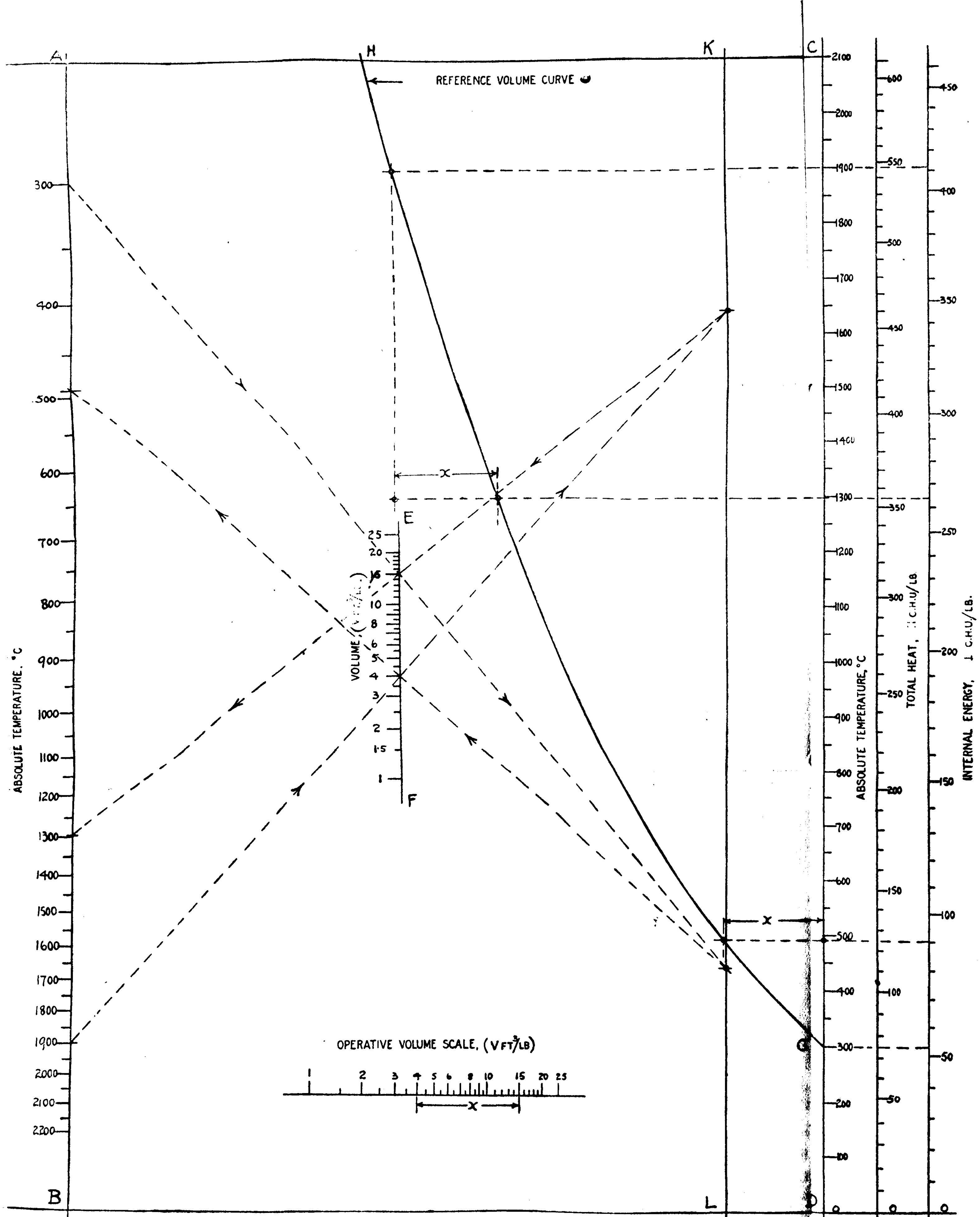


Fig. 2—Alternative Method of Calculating Temperatures.

equally serviceable. The modified arrangement for this purpose is shown in Fig. 3. The axis CD carrying the uniform

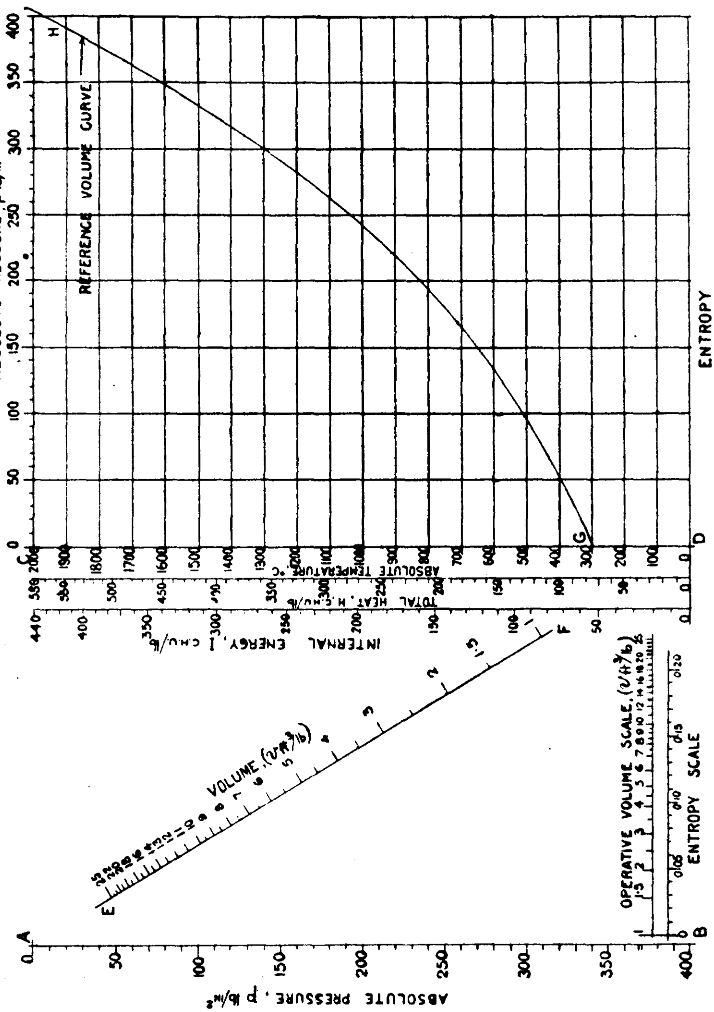


Fig. 3.—Alignment-Energy Chart for the Analysis of Gas and Oil Engine Indicator Diagrams

scale of temperature is used as the temperature axis of a common field of Tp and $T\phi$ co-ordinates, and the reference volume curve GH is drawn on the right instead of on the left of this axis.

This combination enables any constant-volume or constant-pressure curve to be located on the Tϕ field, or constant-volume or constant-entropy (adiabatic) curve to be located on the Tp field. The operator can draw either the transferred Tϕ or Tp indicator diagram just as he chooses, and the instruments required are a straight-edge, and a strip of paper for the transference of operative volume values. It will be noted that the network of curves, referred to above, is replaced by the single reference volume curve.

The equations used for the design of this nomogram, and simplified "alignment chart" are as follows:—

1. Characteristic equation of the gas.
2. Equation of entropy change under varying conditions of pressure volume and temperature.
3. Equations for total heat and internal energy of the gas.

As there appears to be a general opinion in favour of the specific heat as a quadratic function of the absolute temperature, the author has adopted this hypothesis, and used the resulting equations for 2 and 3. The specific heat equations are:—

$$Kv = a + sT - uT^2\,;\quad Kp = b + sT - uT^2$$

$$\text{when } a = 0{\cdot}167\,;\ b = 0{\cdot}237\,;\ s = 0{\cdot}000062\,;\ \text{and } u = \frac{0{\cdot}725}{10^8}.$$

The values of the constants s and u given here are due, the author understands, to Dr. Stodola. Those who prefer to use the linear form of the specific heat equation, or other values of the constants, can easily modify the resulting equations. The principles of the construction and methods of operation are in no way affected by such a change

The characteristic equation of a permanent gas is, as usual, assumed to hold for the products of combustion. This is—

$$144\ pv = RT \qquad (1)$$

when p = pressure in lbs./in.^{2} absolute.
v = volume ft.^{3}/lb.
T = absolute temperature °C.
R = gas constant.

In all calculations for values of entropy, volume, total heat, and internal energy, these values are reckoned for unit mass of the stuff, taken here as 1 lb. Also the temperatures, in such calculations, are now, as a rule, expressed in centigrade degrees, instead of Fahrenheit. The advantage of this change of temperature scale is that the numerical values of the heat quantities in lb. degree centigrade units, C.H.U., are the same as when the calorie or kilogramme degree centigrade is used for the statement of heat units per unit of mass.

The relation holds, that the gas constant is given, in mechanical units, by the difference of the specific heats. Thus,

$$R = (Kp - Kv)\ J = (b - a)\ J \qquad (2)$$

where Joule's equivalent J = 1,400 for the centigrade system of heat units. With the values of a and b taken here, the gas constant from equation (2) becomes, R = 98, and equation (1) reduces to,

$$T = 1{\cdot}47\ pv \qquad (3)$$

This equation is nomographed in Figs. 1 and 3 by the scales of pressure and temperature on the parallel axes AB, and CD, and the scale of volume on the diagonal axis EF. Equation (3) is of the general form

$$f(T) + f(p)\ f(v) = 0.$$

where functions T, p, and v are quantities depending for their values on values of the variables. By the principles of nomography, any algebraical expression which can be thrown into this form can be nomographed on three straight-line axes, two of which are parallel and the third inclined. In this case the function values of temperature and pressure are simply the temperature and pressure values, and function v is 1·47 v.

A consideration of nomographic principles, and rules of construction would occupy too much space here; and only one or two salient points relating to the charts and diagrams described in this paper are dealt with. The explanations given should be sufficient to enable those who desire to use the combinations to draw them down to a more practicable scale than the pages of the Transactions permit. For further information on this very interesting subject, reference should be made to the classical treatises in French, by Prof. M. d'Ocagne, also that of Dr. J. Clark, and the English texts by Col. R. K. Hezlet, R.A., and Prof. J. B. Peddle.*

In setting out the nomogram, shown in Figs. 1 and 3, and also shown in skeleton in Fig. 4, the scale axis AB is drawn at a distance $x = 12$ inches from the axis CD. A uniform scale of pressure, 1 inch = 20lbs./in.^{2} is set off on AB, and a uniform scale of temperature 1 inch = 100° C on CD. The two scales read in opposite directions. When these are set out, a pair of p and v values is arbitrarily chosen, and the corresponding T value calculated. A line is then drawn between the p and T values. With the same value of v and another value of p the process is repeated and the p and T values joined by a line. The point of intersection of these "operative" lines gives the corresponding v point on the diagonal axis. This process is repeated for some other value of v, and the point

* **"Traité de Nomographie," and "Calcul. Graphique en Nomographie," by M. d'Ocagne.**

"Théorie général des Abaques d' Alignment de tout Ordré," by Dr. J. Clark.

"Nomography or Graphical Representation of Formulæ," by Col. R. K. Hezlet, R.A.

"The Construction of Graphical Charts," by John B. Peddle.

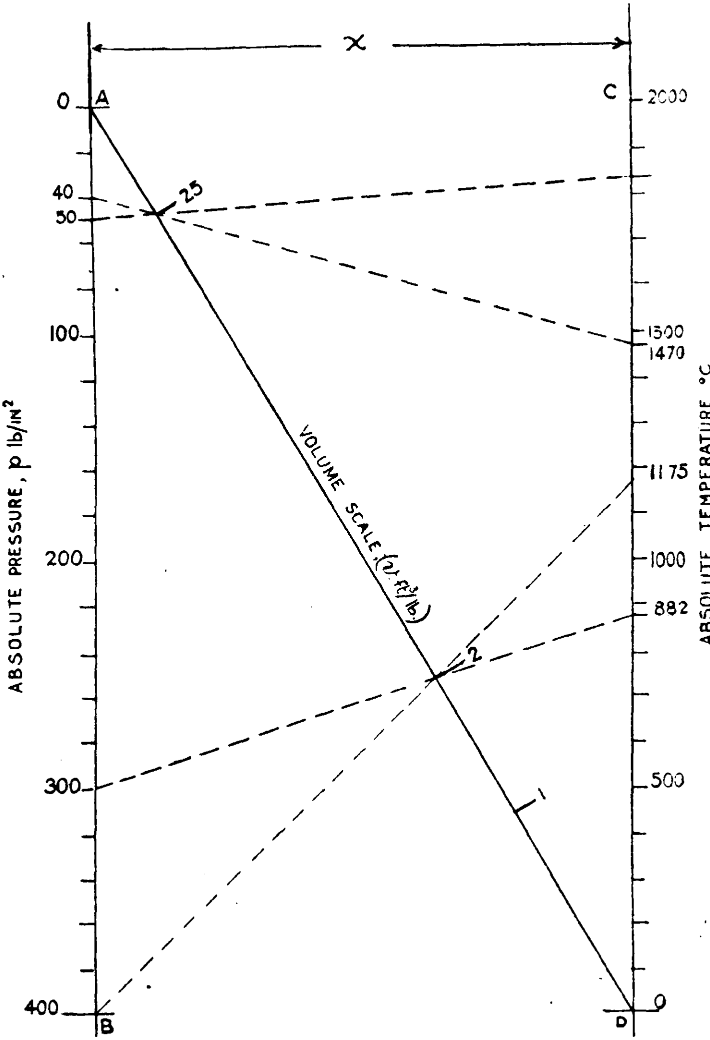


Fig. 4.—Method of constructing the Alignment Chart.

determined. A line drawn through the two v points then gives the diagonal scale axis. The process is shown in Fig. 4, for $v=2$, with $p=400$ and 300, and for $v=25$, with $p=50$ and 40. All other points on the v scale can be derived from the p and T scales in the same way; but when the v axis is drawn in, only one construction line is necessary at each v value. As can be seen, the diagonal axis obtained by the above method passes through the zero values of the p and T scales.

In general, for a case of this type, the diagonal axis always passes through the zero values of the function scales set out on the two parallel axes. There is, of course, a corresponding algebraical expression for determining the proportions of the scale on the diagonal; but in such a simple case as this the most suitable method is to graduate it from the scales on the other axes. The quickest plan is to choose a single value of one of the factors on a parallel axis, calculate the values of the factor on the other parallel axis for a series of values of that in the diagonal one, then to draw a set of "rays" from the single value. The process is clearly indicated on the nomogram on the left of Fig. 7, where for v values between 1 and 4 the value of $p=300$ is chosen.

Instead of deriving the complete set of scale divisions on the diagonal axis in this way, only one or two of these may be determined. With these lengths a graph of the function may be plotted, and the other scale divisions can then be derived by projecting from the curve to the scale axis. In this particular case, however, the T values can be so quickly run out on a slide rule, that the intersection method is the most expeditious one to use.

The characteristic equation can also be written in the logarithmic form—

$$\log \mathrm{T} = \log p + \log (cv).$$

This is of the general type—

$$f(\mathrm{T}) + f(p) + f(v) = 0.$$

Again, according to the rules of nomography, this equation can be nomographed (as in the previous case) on three straight line axes, the mid axis, however, being parallel to the other two.

As several function scales are used in the combinations dealt with, a word or two of explanation on the method of construction may be added.

If $f(z, v)=0$, and if v is the independent variable or argument, the dependent variable is given explicitly by

$$z=f(v)$$

where $f(v)$ may be any algebraical expression containing v and constants.

Referring to 5 (a), let O X be a scale axis having the origin

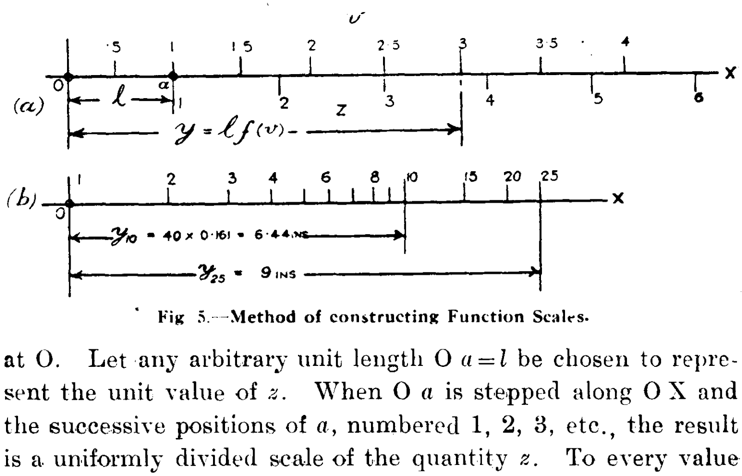


Fig 5.—Method of constructing Function Scales.

at O. Let any arbitrary unit length O $a=l$ be chosen to represent the unit value of z. When O a is stepped along O X and the successive positions of a, numbered 1, 2, 3, etc., the result is a uniformly divided scale of the quantity z. To every value of this scale there is a corresponding value of the variable v. If the v values are marked on the scale axis opposite the z values, a second scale is produced, which may either be uniformly or unequally divided, according to the nature of the function. The second scale is a "function scale" of the variable v. The combination of two scales on one axis constitutes a "simple" nomogram, and gives by inspection the solution of the equation $z=f(v)$.

When a function scale, which is a "partial aspect" of a simple nomogram, is used in a compound nomogram, the proportional length values handled are not those marked with the values of the variable (v), but the function value z implicitly expressed by the scale.

In a particular design, where a series of corresponding values of the variables z and v are known, such a scale is constructed as follows:—If the unit length in which l is measured be called the fundamental unit, the distance y, measured from the origin to any z value, will not be numerically equal to the z value figured on the scale, if l is either a multiple or a submultiple of the fundamental unit. In general, this is the case. The actual value of y in fundamental units is, therefore—

$$y = lz = l \ f(v).$$

Suppose, for instance, l is expressed in terms of the inch as the fundamental, then $y = l \ f(v)$ inches. The coefficient l is called the "module" of the function scale. In practical design this factor has to be so chosen that the scale length between any lower limit z_1 and higher limit z_n corresponding to values v_1 and v_n of the variable, may not exceed the limit of length permissible on the paper. If the maximum length is to be restricted to L inches, then the module is given by—

$$l = \frac{\mathrm{L}}{f(v_n) - f(v_1)}.$$

The case illustrated in Fig. 5 (b) is that of the function scale, $z = 0{\cdot}16118 \log v$. This is the scale marked "operative volume scale" in Figs. 1, 2, and 3. Suppose it is desired to limit the length of the v scale to a maximum of 9 inches between the values $v = 1$ and $v = 25$. For $v = 1$, $z = 0$; for $v = 25$, $z = 0{\cdot}255$. Hence the module required is—

$$l = \frac{\mathrm{L}}{f(\cdot 225) - f(1)} = \frac{9}{\cdot 225} = 40 \text{ inches}.$$

The values of z or $(\phi_2 - \phi_1)$ are given on page 18, and the corresponding fundamental (y) distances, which are to be scaled from the scale origin, are obtained by multiplying the $(\phi_2 - \phi_1)$

tabular values by 40. In this way any other function scale can be constructed.

In Fig. 2 a scale of $f(\mathrm{T})$ is thus laid off as the axis A B. A function scale $f(v)$ is also set off on the central axis E F, although it has been derived here directly from the fundamental scale on A B and an entropy scale (not shown) on the parallel axis K L. This $f(v)$ scale can, however, be constructed in exactly the same way when its module, derived from the modules of the scales on AB and KL, is determined. The three base scales on AB, EF, and KL are scales of entropy; and this nomogram is used either to perform the operation of addition, $f(\mathrm{T})+f(v)=z$, or of subtraction $z-(fv)=f(\mathrm{T})$. A knowledge of the z value is not required for the final temperature result, and the z scale is therefore omitted on KL. The function temperature scales shown on the nomograms of the total heat alignment charts for the NH_3 and CO_2 refrigerators, Figs. 14 and 15, are derived in the same way, tables of temperature and entropy and temperature and heat being used.

The nomogram constituted by the scales on the three axes AB, CD, and EF, Figs. 1 and 3, is sufficient for the determination of T, when p and v are known; or p, when T and v are known; or v, when T and p are known, at any point of a cycle. The required value of the third quantity is found by laying the straight-edge across the other two given values and noting the third value where the scale is cut by the edge.

The nomogram, however, is not sufficient for the complete determination of the data for the efficiency calculations, as has already been indicated, and a suitable means for the solution of the equation of entropy change has also to be found.

If the temperature of the gas changes from T_1 to T_2 under varying conditions of pressure and volume, the general equation for the corresponding entropy change is—

$$(\phi_2-\phi_1)=a\,log_e\frac{\mathrm{T}_2}{\mathrm{T}_1}+s\,(\mathrm{T}_2-\mathrm{T}_1)-\frac{u}{2}(\mathrm{T}_2{}^2-\mathrm{T}_1{}^2)+\frac{\mathrm{R}}{\mathrm{J}}log_e\frac{v_2}{v_1}\quad (4)$$

If the pressure ratio $\frac{p_2}{p_1}$ is substituted for the volume ratio $\frac{v_2}{v_1}$, the last term becomes $-\frac{R}{J}\,log_e \frac{p_2}{p_1}$ and the first term, $b\ log_e \frac{T_2}{T_1}$. For the purpose in view, however, it is not necessary to consider this alternative form of the entropy equation. Equation (4) may be put in the form—

$$z = \left\{ a\ log_e \frac{T_2}{T_1} + s\ (T_2 - T_1) - \frac{n}{2}(T_2{}^2 - T_1{}^2) \right\} + \frac{R}{J}\ log_e \frac{v_2}{v_1}$$

that is,

$$z = f\,(T) + f\,(v).$$

As already indicated the nomographic interpretation of this equation is given by the function scales of T and v on the parallel axes AB and EF, and the z scale on KL, Fig. 2. During an adiabatic operation the entropy value z remains constant.

If the initial and final volumes are given, together with the initial temperature, and the straight-edge is laid across the initial T and v values on AB and EF, it will register the entropy value z on the axis KL. If the straight-edge is now pivoted about this point on KL till it passes through the final volume value in EF, it will then register the final temperature, where it crosses the scale on AB.

To make this point clear, and also for comparison with the other method of calculation, the case for a gas engine is worked out on Fig. 2. The initial temperature at the beginning of adiabatic expansion is taken as $T_1 = 1{,}900°$ C, and the volume $v = 4$ ft.^{3}/lb.; the final volume is $v_2 = 15$. The dotted arrow lines show the positions of the straight-edge, and the direction of the operation. The straight-edge finally cuts the temperature scale at $T_2 = 1{,}295°$ C, and this would be the ideal temperature after adiabatic expansion, between the limiting volumes specified.

The other pair of operative lines show the corresponding calculation for adiabatic compression. The temperature at the end of suction is taken as $T_1 = 300°$ C, and the volume $v_1 = 15$, while the final volume is $v_2 = 4$. The ideal temperature after adiabatic compression would be $T_2 = 490$.

An inspection of equation (4) shows that $f(T)$ is the entropy change at constant volume, for when v is constant the fourth term on the right or $f(v)$ becomes zero, that is,

$$(\phi_2 - \phi_1)_v = a \log_e \frac{T_2}{T_1} + \frac{s}{2}(T_2^2 - T_1^2) - \frac{u}{2}(T_2^2 - T_1^2) \quad \cdot \quad (5)$$

This entropy change at constant volume being a function only of the absolute temperature, the graph of the function, drawn through the temperature value T_1, may therefore be taken to represent any arbitrarily chosen volume. A convenient value of T_1 is 300° C.. The entropy values reckoned from this temperature have been calculated from equation (5), and are given below—

T °C	300	500	700	800	1,000	1,200	1,400	1,600	1,800	2 000	2,200
$(\phi_2 - \phi_1)_v$	0	0·097	0·1645	0·1928	0·2431	0·2826	0·3186	·0·3516	0·3776	0·4081	0·4327

These values are used to obtain the reference-volume curve GH shown in Figs. 1, 2, and 3. In Figs. 1 and 2 the entropy scale is taken as 1 inch = 0·05 entropy unit, in order to keep the curve clear of the diagonal scale axis; but a larger scale can be used if desired. On the alignment energy-chart, Fig. 3, the scale is taken as 1 inch = 0·025 entropy unit. In the construction of the $f(T)$ scale on AB, and the entropy scale on KL, Fig. 2, the entropy scale is also taken as 1 inch = 0·025 units. The module for the $f(T)$ scale is thus $l = 40$ inches, and the y distances for this scale are obtained by multiplying the tabular values of the entropy by 40.

Again, the fourth term on the right of equation (4) is the entropy change at constant temperature, or during an isothermal operation. When T remains constant the ratio in each of the first three terms is unity, and each reduces to zero. Thus,

$$(\phi_2 - \phi_1)_T = \frac{R}{J} \log_e \frac{v_2}{v_1} = (b - a) \log_e \frac{v_2}{v_1} \qquad - \quad - \quad - \quad (6)$$

Equation (6) enables a function scale of volume to be constructed when some arbitrary value of the initial volume v_1 is chosen. A convenient value is $v_1 = 1$. With this value equation (6) reduces to—

$$(\phi_2 - \phi_1)_T = 2{\cdot}3026\ (b - a) \log \frac{v_2}{v_1} = 0{\cdot}16118 \log v \qquad - \quad (7)$$

The values of the entropy change calculated from equation (7) are given in Table I.

TABLE I.

v	$(\phi_2 - \phi_1)_T$	v	$(\phi_2 - \phi_1)_T$	v	$(\phi_2 - \phi_1)_T$
1	·0000	8·5	·1506	16	·1945
1·5	·0284	9	·1538	16·5	·1960
2	·0485	9·5	·1571	17	·1985
2·5	·0641	10	·1610	17·5	·2001
3	·0770	10·5	·1648	18	·2022
3·5	·0876	11	·1680	18·5	·2038
4	·0971	11·5	·1710	19	·2060
4·5	·1051	12	·1740	19·5	·2080
5	·1128	12·5	·1765	20	·2100
5·5	·1190	13	·1800	21	·2130
6	·1251	13·5	·1821	22	·2160
6·5	·1310	14	·1848	23	·2195
7	·1360	14·5	·1870	24	·2212
7·5	·1411	15	·1898	25	·2250
8	·1455	15·5	·1920		

The function scale of volumes or "operative volume scale" is derived from these tabular values in the manner already explained. In the nomogram, Figs. 1 and 2, the module is $l=20$ inches; but for the alignment energy chart, Fig. 3, since the base scale is taken as 1 inch $=0{\cdot}025$ entropy unit, the module is $l=40$ inches. According to a nomographic rule, since the same base scale is used on AB and KL, Fig. 2, the module of the scale on EF is half that on AB, or $l=20$ inches.

As already indicated, the curve GH plotted on a field of $T\phi$ co-ordinates, may be taken to represent any constant volume. It may, therefore, be chosen as the initial volume for an adiabatic expansion or compression. Referring to Fig. 6, let the horizontal AB drawn through the initial temperature corresponding to the initial volume cut the curve GH in B. Draw the adiabatic vertical YY through B. Let the operative volume scale be placed horizontally on the field and moved down vertically with the initial or " index volume " on the reference curve. The temperature after adiabatic expansion to any larger volume is obtained, when the final volume value on the scale coincides with the vertical YY. Thus, if the initial volume at the beginning of the expansion is $v=6$ and the corresponding temperature $T=1{,}060°$ C, the temperature after adiabatic expansion to $v=10$ will be $T=903°$ C. Conversely, if compression takes place from $v=6$ and $T=1{,}060°$ C to $v=4$, the final temperature will be $T=1{,}200°$ C.

The practical rule for the calculation of the temperature after adiabatic compression or expansion may be stated as follows:—

Given the initial or index volume and the corresponding temperature, draw a horizontal through the T value to cut GH, then through the point of intersection draw a vertical.

Mark on the edge of a strip of paper from the operative scale, the length x between the initial and final volume values.

Move the paper with the edge horizontal till the points on it are coincident with the vertical and the reference volume curve, and then note the corresponding temperature level.

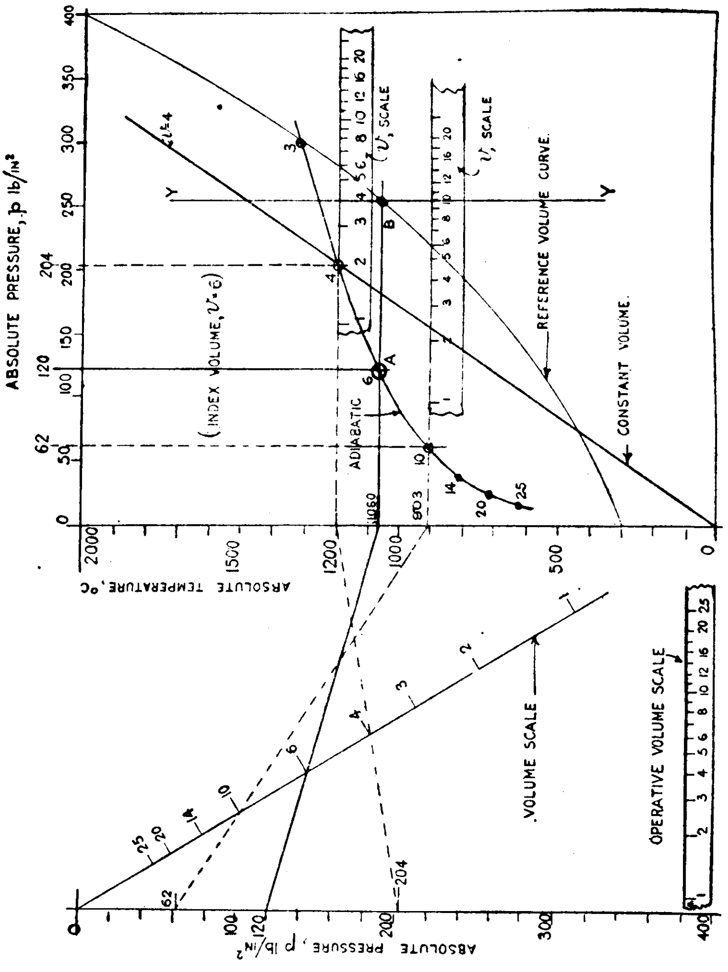


Fig. 6.—Method of drawing an Adiabatic Curve on the (Tp) Diagram.

The case for the gas engine has been worked out in this way in Fig. 2, as indicated by the dotted construction lines;

and it will be noted that the results obtained are exactly the same as those given by the previous method.

An extension of this method enables any adiabatic curve to be quickly drawn in the Tp field of co-ordinates. In this case the point is defined by the initial temperature and pressure values.

Referring again to Fig. 6, let A be any point on the Tp field through which an adiabatic is to be drawn.

Find the index volume corresponding to the T and p values, from the nomogram on the left, by means of the straight-edge.

In the case illustrated $p=120$, $T=1{,}060°$ C, and the nomogram gives $v=6$.

Draw the horizontal HB and then the vertical YY.

Suppose it is required to find the T and p values of a point on this adiabatic where $v=10$—

Move the operative volume scale down with $v=6$ on the reference volume curve, until $v=10$ coincides with YY, and note the temperature level.

Lay the straight-edge across this temperature $T=903°$ C, and the chosen volume $v=4$, and note the corresponding pressure $p=62$.

The point marked 10 is then found by dropping the vertical from $p=62$ on the upper pressure scale.

In this simple manner any number of points can be located, and the curve can be drawn.

This construction is necessary when it is desired to ascertain the relations between an actual transferred Tp expansion curve with the adiabatic curve through any point on it. It is also necessary when the ideal Tp diagram of an internal combustion engine is to be drawn in the field, for comparison with the actual diagram obtained from the engine.

For this purpose constant volume curves have also to be drawn. As these are straight lines through the origin, the process is quite simple. Choose any convenient pressure and calculate the temperature corresponding to the given volume.

Find the point of intersection of the temperature horizontal, and the pressure vertical from the upper scale, and draw a line through this and the zero value of the temperature scale.

The case for $v=4$ is shown in Fig. 6, where the arbitrarily chosen pressure is taken as $p=204$.

For the construction of the ideal Tϕ diagram of an internal combustion engine, and also for transference of actual indicator diagrams, it is necessary to draw constant volume and constant pressure curves in the Tϕ field. Here, again, the nomogram is extremely handy. Its application for this purpose is shown in Fig. 7.

A constant volume curve is obtained by the application of the operative volume scale to the reference volume curve, when an arbitrary choice of an "index volume" has been made. A similar choice is necessary for a constant pressure curve. For the transference of an indicator diagram, care must be taken that the same index volume is used for constant volume and constant pressure curves. This freedom of choice enables the operator to place the Tϕ diagram in any position he chooses in the co-ordinate field.

Assuming a choice of "index volume" is made, the operative scale is placed at several temperature levels with the index volume value on the reference curve, and the points for the required volume curve are marked from the scale.

The construction for $v=2$ is shown in the figure, the index volume chosen being $v=5$. The result is the same as though the reference volume curve were moved to the left through an entropy change of

$$(\phi_5 - \phi_2) = 0{\cdot}16118 \log \frac{v_5}{v_2}.$$

A constant pressure curve is drawn by finding the temperature levels, for several volumes, with one pressure value, from the nomogram, then by scaling the corresponding volume values at these levels from the operative volume scale.

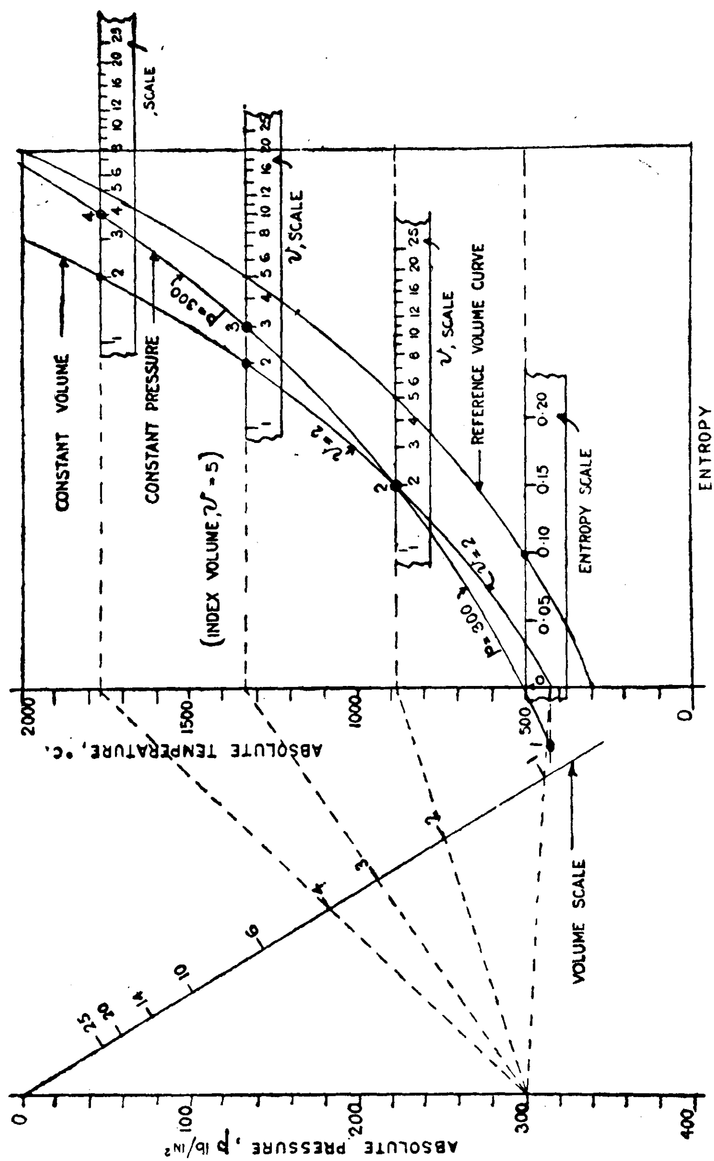


Fig. 7.—Method of drawing Constant Pressure and Constant Volume Curves on the Tϕ Diagram.

The construction is illustrated for the constant pressure curve $p=300$, the index volume again being taken as $v=5$.

When the gas is heated at constant pressure, the total heat energy, reckoned from absolute zero of temperature, is given by the equation—

$$H = bT + \frac{s}{2}T^2 - \frac{u}{3}T^3.$$

When it is heated at constant volume the corresponding internal energy is given by—

$$I = a\,T + \frac{s}{2}T^2 - \frac{u}{3}T^3,$$

or $$I = H - (b-a)\,T.$$

Substituting the values of the constants a, b, s, and u, the energy equations become—

$$H = 0{\cdot}237\ T + 0{\cdot}000031\ T^2 - \frac{0{\cdot}241\ T^3}{10^8} \qquad (8)$$

$$I = H - 0{\cdot}07\ T \qquad (9)$$

The heat values calculated for a series of temperatures from equations (8) and (9) are given in Table II.

TABLE II.

T	200	500	800	1,000	1,200	1,500	1,700	2,000	2,050	2,100	2,150	2,200
H	48·6	126	210	266	325	416	478	578	596	612	629	646
I	34·6	91	154	196	241	311	359	438	453	434	479	493

The scales shown in Figs. 1, 2, and 3 are most readily obtained by the method illustrated in Fig. 8. A scale of absolute temperature 1 inch = 100° C is laid off on the axis LM, and a scale of heat energy 1 inch = 50 heat units is laid off on the axis LN. The values of H and I given above are plotted on the temperature base and the graphs drawn. The heat scale divisions are then got by projecting vertically from the heat values, and horizontally to the two axes drawn parallel to the temperature axis. These scales for accuracy of projection should be constructed in sections, dealing in each section with the part of the graph covering a range of 100° C. The scales

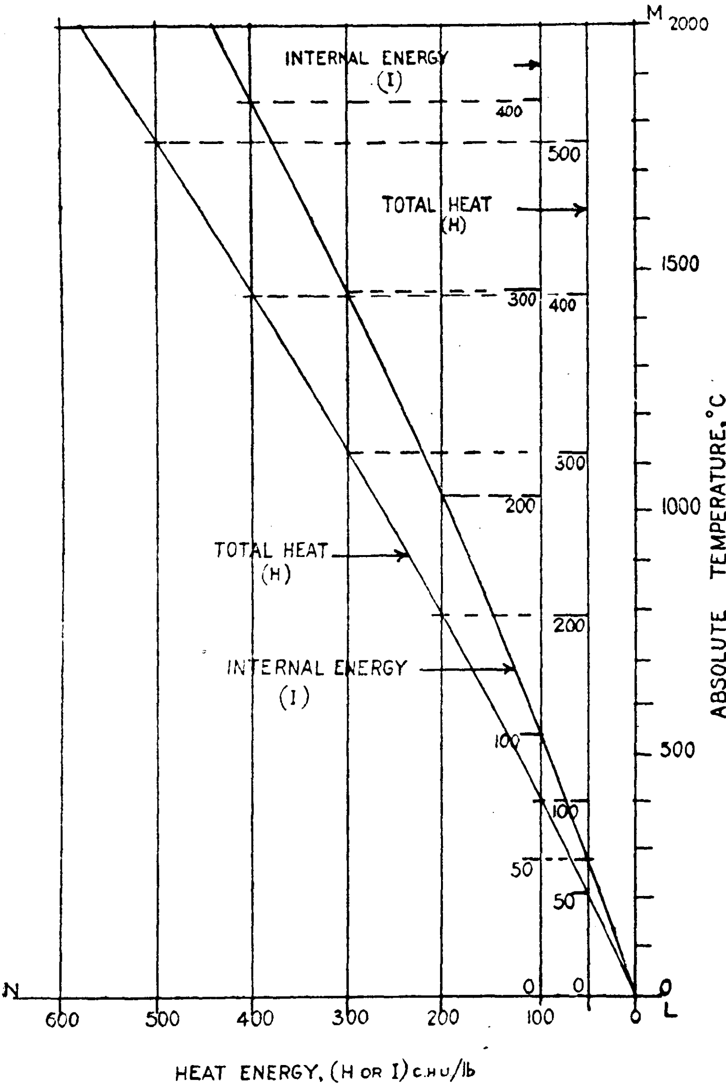


Fig. 8.—Method of constructing the Energy Scales.

can either be drawn in this way in position or separately constructed, and transferred to the nomogram or chart on a strip of paper.

In order to transfer any indicator diagram by means of the foregoing combination shown in Fig. 3, it is necessary, since the chart is designed for one pound of gas, to reduce the actual cylinder volumes of the engine, as shown by the indicator card, to equivalent volumes per pound of stuff in the cylinder. When the absolute temperature at any point of an expansion or compression curve is known, the volume per pound is calculable from the characteristic equation. The most convenient point for starting the transference is that at the end of the suction stroke, for which the pressure can usually be obtained from a light spring card. Usually the corresponding temperature is not known, and has either to be calculated from other known data, or assumed.

In the case of a gas engine the charge weight, assuming there is no leakage, is constant throughout the cycle. If the suction temperature and pressure are known, this weight can be calculated with a reasonable degree of accuracy.

Let V_1 = (stroke volume + clearance volume) in ft.^{3}.
p_1 = initial pressure in lbs./in.^{2}.
T_1 = initial temperature °C.
w_c = weight of stuff in cylinder in lbs.

then $$w_c = \frac{144\, p_1 V_1}{R\, T_1}. \qquad (10)$$

Conversely, if the weight of air and gas supplied is measured, and allowance is made for the additional products left in the clearance space, the value of the charge weight can again be approximated, and used to calculate the initial temperature T_1.

In the case of an oil engine of the Diesel type, the charge weight increases slightly during the combustion period, on account of the oil and injection air. The volume per pound from the point of cut-off to the end of the stroke is consequently slightly less than the value during the compression stroke and the admission part of the expansion stroke. If desired the expansion and compression strokes can be dealt with separately, but no great error is involved, and the work is simplified if the same final volume per pound is taken for the compression as for the expansion stroke.

As shown in Fig. 9, the diagram is prepared for transference by drawing the lines of zero volume O Y and zero pressure O X. The total base length OX is then divided into the same number of equal parts as the volume per pound at the end of the suction stroke. The corresponding diagram pressures are then scaled, and either tabulated or marked on the diagram, as shown. Where a light spring card is available the lower compression pressures should be taken from it.

Any point on the diagram is transferred to the T*p* field by noting the pressure and volume values, laying the straight-edge across these on the alignment chart, and drawing the temperature horizontal to cut the pressure vertical from the upper scale.

When the point is transferred to the Tϕ field, the temperature level is found, an index-volume value is arbitrarily chosen, and the corresponding volume value is set off from the reference curve by means of the operative scale.

In order to show the practical application of the alignment energy-chart, the transferred T*p* and Tϕ diagrams for a gas engine and a Diesel oil engine have been drawn, and are shown in Figs. 9 and 10.*

The gas engine diagram, Fig. 9, is from an old type of engine having hot-tube ignition, and a low compression ratio of 3·72. It has a cylinder 7 inches in diameter, and a stroke of 15 inches. The clearance volume is 0·123 ft.^{3}, and the stroke volume 0·334 ft.^{3}, so that the total volume represented by the base line O X is $V_1=0{\cdot}457$ ft.^{3}. The tem-

* These illustrations are reduced to about ¼th original size.

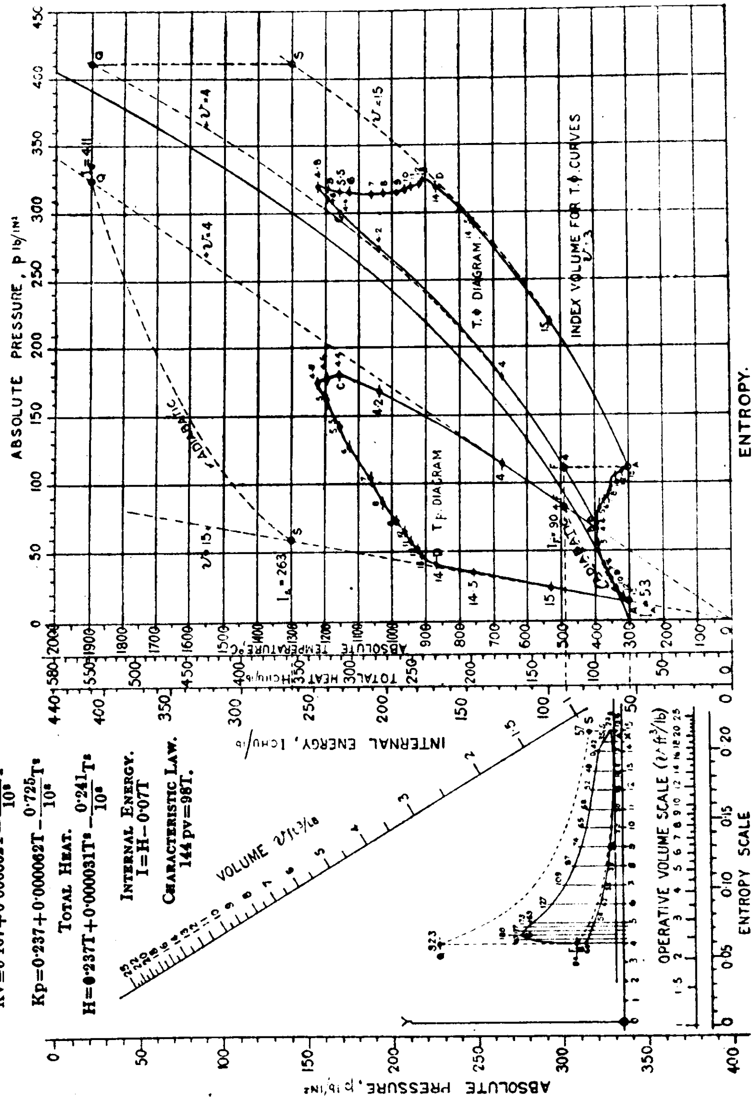


Fig. 9.—Tp and Tφ Diagrams of a Gas Engine.

perature at the end of suction is not definitely known, and it is assumed to be 300° C absolute. From a light spring card the suction pressure $p_1 = 13{\cdot}6$ lbs./in.^{2} absolute. For these conditions the volume per pound at the end of the suction stroke is $v = 15$. Also, from equation (10), the probable charge weight is $w_c = 0{\cdot}0305$ lb.

The base line OX is divided into fifteen equal parts to obtain the scale of volumes per lb. The transferred points on the Tp and Tϕ diagrams are numbered with the corresponding volume values on the pv diagram.

Each diagram starts at A, the initial point on the compression curve. The final point of compression is B on the $v = 4$ curve. The point of maximum pressure is C, and the point of release is D. In order to keep the Tϕ diagram clear of the Tp one, the index volume chosen is $v = 3$.

The "ideal" diagram for the cycle is shown by the dotted curves. It consists in each case of the constant volume curves for $v = 15$ and $v = 4$, and the adiabatics AF and QS drawn through the points A and Q.

It will be noted that the temperature level at F, the end of adiabatic compression, can be found either from the location of the final point of the Tp adiabatic, or by drawing in a part of the constant volume $v = 4$, Tϕ curve, and projecting vertically from A to cut this in F. The horizontal through F to the internal-energy scale shows that the absolute amount of internal energy at the end of adiabatic compression would be $I_F = 90$ C.H.U. The heat of combustion of the gas in the charge, when added to this, would give the total internal energy after heating at constant volume.

The gas supplied per cycle reduced to standard conditions of pressure and temperature, 14·7 lbs./in.^{2} absolute and 0° C, is 0·0348 ft.^{3}. It has a higher calorific value of 280 centigrade heat units per ft.^{3} at S.T.P., and a lower value of 262. The heat added per cycle is thus 321 C.H.U. per lb. of charge, hence the total internal energy at Q is $I_Q = 411$ C.H.U.

The horizontal drawn through this value to cut the constant volume $v=4$ line of the Tp and the curve of the Tϕ diagram, determines Q, the point of maximum pressure and temperature on the ideal diagram. The adiabatic expansion curve drawn through Q cuts the final volume curve $v=15$, at the horizontal passing through $I_s=263$. The ideal efficiency is given by—

$$\eta=\frac{(I_Q-I_F)-(I_S-I_A)}{(I_Q-I_F)}.$$

$$=\quad\frac{321-210}{321}=0{\cdot}346.$$

The brake horse power is 4·6, and the gas supplied per minute (reduced to S.T.P.) is 2·74 ft.³. Hence the thermal efficiency on the brake, reckoned on the higher calorific value of the gas, is—

$$\eta_B=\frac{4{\cdot}6\times23{\cdot}6}{2{\cdot}74\times280}=0{\cdot}142.$$

The brake efficiency ratio is—

$$\eta_r=\frac{\eta_B}{\eta}=\frac{0{\cdot}142}{0{\cdot}346}=0{\cdot}41.$$

On the older assumption, that the specific heats are constant, the ideal efficiency of the standard air engine is—

$$\eta=1-\left(\frac{1}{r}\right)^{\gamma-1}$$

where r is the compression ratio, and $\gamma=\frac{K_p}{K_v}=1{\cdot}4$.

Since $r=3{\cdot}72$, $\eta=1-\frac{1}{(3{\cdot}72)^{\cdot4}}=0{\cdot}41.$

The ideal efficiency, when variation of specific heat is allowed for, thus appears to be about 15 per cent. less than that of the standard air engine of comparison. The brake efficiency ratio is thereby increased from 0·347 to 0·41.

The Diesel engine diagram, shown in Fig. 10, together with trial and other data, has kindly been supplied to the author

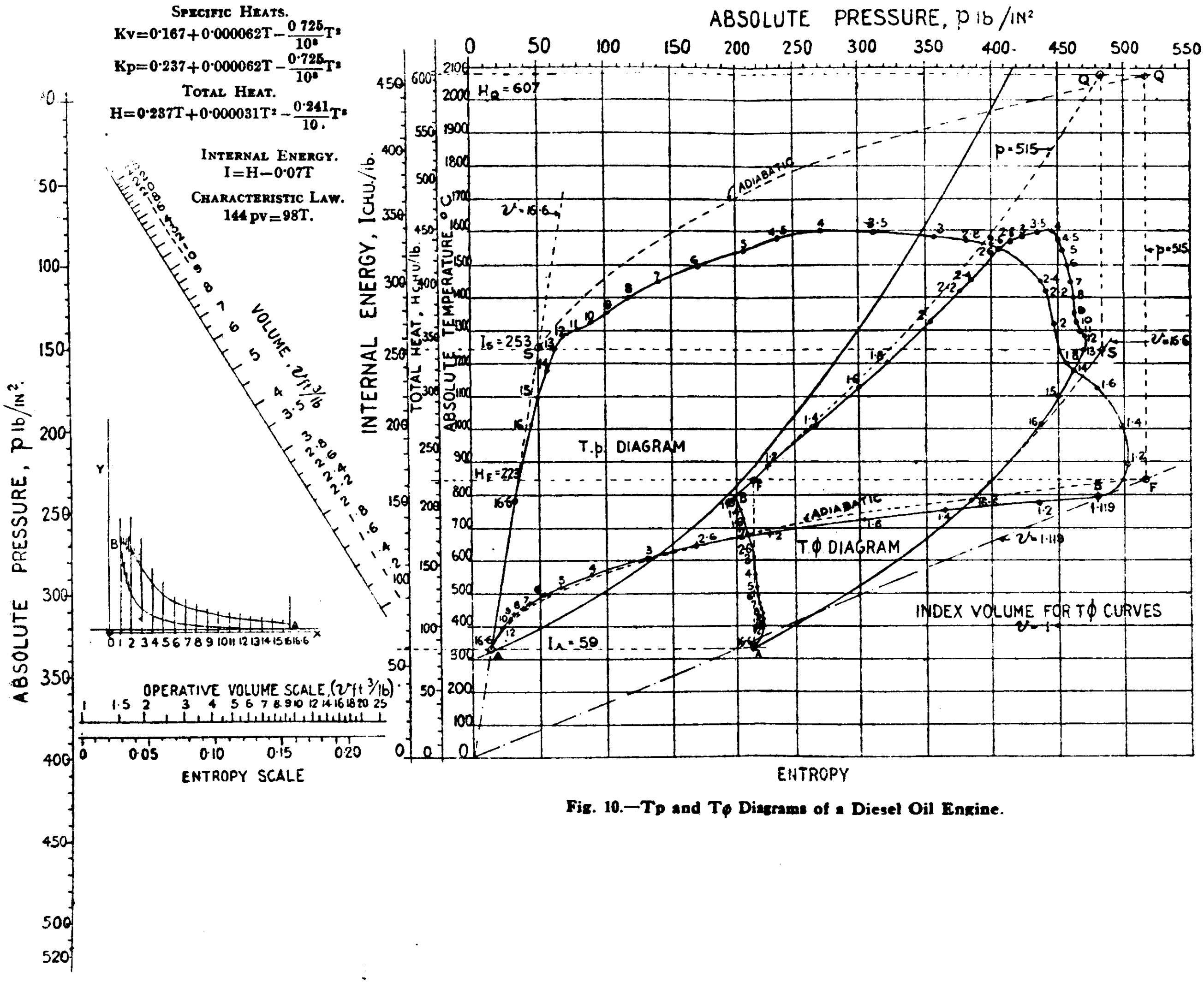


Fig. 10.—Tp and Tφ Diagrams of a Diesel Oil Engine.

by Prof. Mellanby. It is taken from the experimental engine in the mechanical laboratory of The Royal Technical College, Glasgow. The engine has a cylinder diameter of 12 inches and a stroke of 18¼ inches. The stroke volume is 2,062 in.^{3}, and the clearance volume is 150 in.^{3}. The total volume at the end of the suction stroke is thus $v_1 = 2{,}212$ in.^{3} or 1·28 ft.^{3}. For the trial under consideration the supply of oil per cycle works out at 0·00274 lb. The oil has a higher calorific value of 10,810 C.H.U., or 19,460 B.Th.U. per lb., and a lower value of 10,112 C.H.U., or 18,214 B.Th.U.

In a case of this kind, if the weight analysis of the fuel and the volume analysis of the exhaust gases are known, the average charge weight for the expansion stroke can be calculated with a fair degree of accuracy. The analyses for the trial show that the weight of products in the cylinder, at the end of the expansion stroke, if release did not take place till the end of the stroke, would be 28 lbs. per lb. of oil supplied. The actual weight of the stuff would, therefore, be

$$w_c = 28 \times 0{\cdot}00274 = 0{\cdot}077 \text{ lb., and the volume per lb.}$$

$$v = \frac{1{\cdot}28}{0{\cdot}077} = 16{\cdot}6 \text{ ft.}^3$$

By dividing up the total base length of the diagram into 16·6 parts, the approximate volume scale for the expansion stroke is obtained. At the end of the suction stroke the volume per lb. will be slightly greater, probably 17 ft.^{3}. The difference, however, of the scale divisions for 16·6 and 17 is small, and the effect in the pressure values is so slight that the 16·6 value is also taken for the compression stroke.

The full-sized indicator diagram, taken with a spring calibrated to metric units, has an equivalent pressure scale of 1 inch = 361 lbs./in.^{2}. With such a small card it is difficult to determine the pressures accurately, and although these have been carefully measured, they may not be quite correct. It has been necessary to assume a value of the suction pressure, and this is taken as 13·6 lbs./in.^{2} absolute. Taking the charge weight as approximately 0·077 lb., the probable suction temperature, calculated from equation (10), is $T_1 = 330°$ C absolute. The starting point A of the transferred diagrams is, therefore, taken at $p_1 = 13·6$ and $T_1 = 330$. With $v_1 = 16·6$ the corresponding volume at the end of compression works out at 1·119 ft.^{3} per lb. The maximum pressure of compression at this volume appears to be 488 lbs./in.^{2} absolute. The pressure rises to a maximum of 502 when combustion begins. There is then a fall in pressure with increasing temperature until about quarter stroke. The slight "constant pressure" period near the begining of the stroke, between $v = 1·8$ and $v = 2·4$, is clearly shown by the Tp diagram. This effect is not apparent on the heating curve of the Tϕ diagram. The Tϕ expansion curve shows that the gases receive heat throughout nearly the whole stroke. This fact can only be discovered on the Tp diagram by drawing an adiabatic curve through the point $v = 4$.

The convenience of having the two diagrams side by side for comparison will be obvious, as information afforded by one is supplemented by information given by the other. The heat exchanges between the cylinder and gases can also be more conveniently dealt with on the Tϕ than on the Tp diagram.

As in the previous case the ideal Tp and Tϕ diagrams are shown by the dotted lines and curves. Starting at A the adiabatic compression curve AF is drawn between $v = 16·6$ and $v = 1·119$. The final compression point F lies above the actual point B at pressure $p_F = 515$. Since, in the ideal cycle, heating is supposed to take place at constant pressure, the total heat at this point is taken. Its value is $H_F = 223$ C.H.U. The heat added to the charge, reckoned on the higher calorific value of the oil is 384 C.H.U. per lb. of charge, so that the maximum total heat for the ideal cycle is $H_Q = (223 + 384) = 607$ C.H.U./lb. The horizontal drawn through this value cuts the $p=515$, constant-pressure Tp vertical, and the Tϕ constant-pressure curve in Q. The Tp adiabatic curve and the Tϕ vertical through Q, cut the constant volume $v=16{\cdot}6$ line and curve at S. As the cooling is supposed to take place at constant volume $v=16{\cdot}6$, from T_s to T_A, the corresponding internal energies are taken to find the heat rejected. These are $I_s=253$ and $I_A=59$. The ideal efficiency is given by—

$$\eta=\frac{(H_Q-H_F)-(I_S-I_A)}{(H_Q-H_F)}$$

$$=\frac{384-19}{384}=0{\cdot}495.$$

The brake horse power is 34·9, and the oil supplied per min. is 0·2775 lb., so that the brake thermal efficiency reckoned on the higher calorific value is—

$$\eta_B=\frac{34{\cdot}9\times 23{\cdot}6}{0{\cdot}2775\times 10810}=0{\cdot}274.$$

The brake efficiency ratio is—

$$\eta_r=\frac{\eta_B}{\eta}=\frac{0{\cdot}274}{0{\cdot}495}=0{\cdot}552.$$

It can be shown that for the standard engine of comparison, when the specific heats are assumed to be constant, the thermal efficiency is given by—

$$\eta=1-\frac{\frac{1}{\gamma}\left(\rho^{\gamma}-1\right)}{r_c^{\,\gamma-1}\left(\rho-1\right)},$$

where r_c = compression ratio.

r_e = expansion ratio.

$$\rho=\frac{r_c}{r_e}.$$

$$\gamma=\frac{K_p}{K_v}=1{\cdot}4.$$

In this case the compression ratio $r_c=14{\cdot}85$. The point of closure of the fuel valve, ascertained by turning the engine round, is at 14 per cent. of the stroke, so that the expansion ratio may be taken as—

$$r_e = \frac{2212}{0{\cdot}14 \times 2062 + 150} = 5{\cdot}04.$$

$$\text{and } \rho = \frac{14{\cdot}85}{5{\cdot}04} = 2{\cdot}95.$$

Hence the ideal efficiency is—

$$\eta = 1 - \frac{\frac{1}{1{\cdot}4}\,(2{\cdot}95^{1{\cdot}4} - 1)}{{\cdot}1488^{1{\cdot}4}\,(2{\cdot}95 - 1)} = (1 - 0{\cdot}442) = 0{\cdot}558.$$

The ideal thermal efficiency thus appears to be about 11 per cent. less than that of the standard engine of comparison, with constant specific heat, and the brake efficiency ratio is increased from 0·49 to 0·552.

A discussion of the thermodynamic aspects of the two sets of Tp and Tϕ diagrams, shown in Figs. 9 and 10, is outside the scope of this paper. The diagrams are drawn merely to illustrate how easily the transformation, for analytical purposes, can be effected by means of this alignment form of energy chart.

The thermal efficiency in each case is calculable from the nomogram, Fig. 1, in a few minutes.

In using this nomogram for the estimation of the ideal thermal efficiency of any standard internal combustion cycle, the calculation of the efficiency ratio of the engine is placed on the same basis as that of the steam engine and steam turbine, by taking the heat actually supplied to the engine during a test.

The efficiency ratio of these steam motors, especially that of the turbine, is a factor of commercial importance.

The efficiency ratio for an internal combustion engine, of given type, might be made equally important, if a standard equation for the specific heat were agreed upon, and a nomographic method of calculation, such as suggested here, were adopted.

For stated conditions of test, the heat available for work, or what may be stated shortly as the "cycle heat," can be as easily estimated for the internal combustion engine as the "Rankine cycle heat" for the steam engine or turbine.

In the case of a refrigerator the ratio between the theoretical performance of the ideal cycle and the actual performance of the machine is as equally important as the efficiency ratio of the direct-heat engine.

The function of a refrigerator is to pump heat from a zone of lower to a zone of higher temperature; and in conformity with the second law of thermodynamics this can only be done through expenditure of work on the substance by some external agent. The end to be aimed at in the design and operation of the machine is the extraction of as much heat as possible from the substance refrigerated, for the expenditure of a given amount of mechanical energy.

The ideal performance of the machine is expressed by the refrigerating effect it can produce under specified cyclic conditions; and the ratio of the heat taken in to work done, analogous to the efficiency of the direct heat engine, is called the "co-efficient of performance." It is usually denoted by the symbol Ċ.

If H_2 = heat abstracted from the zone of lower temperature,
H_1 = heat discharged to the zone of higher temperature,
then the heat equivalent of the work done by the external agent, during the pumping process, is—

$$H_e = (H_1 - H_2) \quad \text{- - - -} \quad (11)$$

and the heat rejected is—

$$H_1 = H_2 + H_e \quad \text{- - - -} \quad (12)$$

The coefficient of performance for the ideal cycle is—

$$C = \frac{H_2}{H_1 - H_2} = \frac{H_2}{H_e} \quad \text{- - - -} \quad (13)$$

The actual coefficient of performance of the machine is always much less than the ideal, and is given by—

$$C_a = \frac{H_2}{H_a} \quad \text{- - - - - -} \quad (14)$$

where H_a is the heat equivalent of the actual amount of mechanical energy expended by the driving agent. While C covers the purely thermodynamic losses of the particular cycle employed, C_a covers all thermal and mechanical losses of the actual machine.

For a given type of refrigerator both C and C_a are calculable when certain test data are known, and their ratio is therefore calculable. This is—

$$C_r = \frac{C_a}{C} = \frac{H_e}{H_a} \quad \text{- - - - -} \quad (15)$$

In contradistinction to the " efficiency ratio " of the direct-heat engine, the author would suggest the use of the term " co-ratio " for this factor. This co-ratio is a figure of importance, as by means of it a standard system of comparison of the performances of refrigerating machines, working under varying test conditions, can be instituted; and, like the efficiency ratio of the steam turbine, it can be usefully employed in design.

For the vapour-compression type of refrigerator, the ideal coefficient of performance C is usually calculable by means either of a temperature-entropy or a heat-entropy diagram. The first method by the $T\phi$ diagram, illustrated in Figs. 11 and 12, is rather tedious; the second by the $H\phi$ diagram is simpler, since the heat quantities can be read directly.

Owing, however, to the fact that the liquid and vapour heats have to be ascertained at the lower limit of temperature and pressure, as well as the heat of the dry or superheated vapour at the upper limit, the ordinary form of the rectangular $H\phi$ diagram is not suitable. A diagram plotted to skew co-ordinates, originally introduced by Prof. Mollier, is used instead.

It occurred to the author that the skew co-ordinate construction could be avoided, and a serviceable chart produced by employing a nomogram to calculate the heat values at the lower limit, and the upper portion of the ordinary $H\phi$ diagram to calculate the heat at the upper limit. The modified type of $H\phi$ alignment chart evolved is shown in Fig. 14. for an NH_3 machine, and in Fig. 15 for a CO_2 machine.

The rectangular chart sections in those illustrations can, however, be replaced by nomograms, if certain approximations are taken, and the whole calculation can thus be performed by purely nomographic means.

The results given are not, however, so exact as those by the combined $H\phi$ alignment chart, and, in consequence, the author has omitted this alternative arrangement.

It is sufficient for the purpose in view simply to state what heat quantities have to be estimated for the calculation of C, as a complete statement of the thermodynamic reasoning, on which the conclusions are based, would occupy far too much space.

Those who are not familiar with the theory of the vapour-compression refrigerator will find a clear statement of the subject, by Sir A. J. Ewing, in the Report of the Refrigeration Research Committee of the Institution of Mechanical Engineers.*

Put in a few words, the estimation of the heat taken in (H_2), and the heat rejected (H_1), involves the determination of the total heat of the stuff (reckoned from a standard temperature level, usually 0° C) at three points of the cycle.

The most perfect type of refrigerator cycle is a reversed Carnot; but this is not possible in the case of a vapour-compression machine, because the use of an expansion cylinder is impracticable. The cylinder is replaced by an expansion

* Proceedings of Institute of Mech. Eng., Oct., 1914, p. 659.

valve, and instead of a reversible adiabatic expansion from the upper condenser pressure to the lower evaporator pressure, a throttling irreversible operation takes place. The liquid before the expansion valve may either be at the condensation temperature corresponding to the condenser pressure, or it may be completely "undercooled" in the condenser, to the lower temperature, corresponding to the evaporator pressure. Under the latter conditions the cycle becomes a reversed Rankine engine cycle.

Complete undercooling, however, never occurs, and the liquid, on admission to the expansion valve, is cooled down to some temperature between the upper and lower saturation limits.

During the throttling process the temperature drops to the lower temperature of the evaporator, but as no external work is done, the heat content or "total heat" of the stuff is the same at the end as at the beginning of the operation. This fact has to be borne in mind when the coefficient of performance is calculated from the heat areas of the Tϕ diagram, since the work area of the Rankine cycle does not give the work done on the substance.

The Tϕ diagrams showing the ideal cycle for both NH_3 and CO_2 machines are illustrated in Figs. 11 and 12. They are similarly lettered, and the following statements apply to both:—

The vapour at the upper pressure of the condenser p_c is condensed at the condensation or saturation temperature t_3, until it is all converted to liquid at C (on the liquid limit curve).

The liquid is then "undercooled" at pressure p_c from t_3 to some temperature t_4, the state point on the p_c curve being at B. The total heat of the liquid at B is H_B. It then passes through the expansion valve to the evaporator, and the pressure is reduced from p_c to p_e, the temperature falling from t_4 to t_1, the saturation temperature at p_e.

The heat content of the stuff, however, remains constant, and in consequence a portion of the liquid is vaporised. The stuff after passing the valve is then partly in a state of vapour, and its state point is K. The total heat of the stuff at the beginning of the absorption period is $H_K = H_B$, and the wet vapour has the quality given by NK/NM.

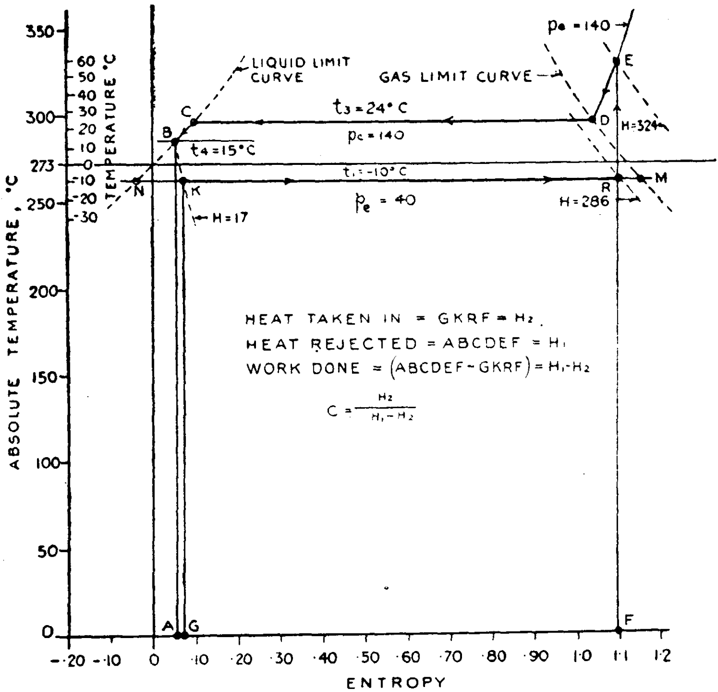


Fig. 11.—Tϕ Diagram for NH_3.

The liquid portion of the stuff absorbs the heat (H_2) from the substance refrigerated, at constant temperature t_1 and pressure p_e.

At the end of this operation the state point is at R and the total heat is H_R.

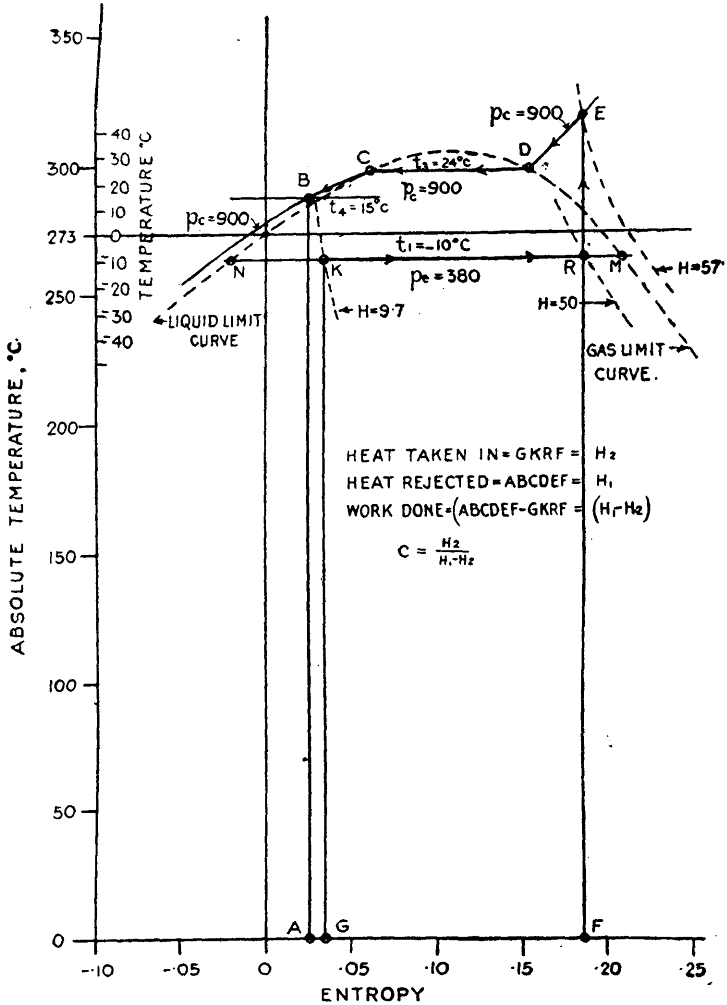


Fig. 12.—Tϕ Diagram for CO_2.

The heat taken in, given by the area under KR, is obviously the difference of the total heats at K and R, that is—

$$H_2=(H_R-H_K)=(H_R-H_B) \quad - \quad - \quad - \quad (16)$$

and the quality at R is, $q_1=NR/NM$.

The wet vapour is then taken into the compressor and compressed adiabatically from p_e to p_c.

The final condition of the vapour at the end of compression depends on the value of the dryness q_1 before compression. In each case illustrated, Figs. 11 and 12, the final state point E, on the condenser pressure curve p_c, falls in the superheat field—that is, the vapour is superheated at the end of compression. The total heat at E is H_E.

It can be shown that, without sensible error, the work done on the substance in the compressor is the difference of the total heats at R and E, hence equation (11) can be written—

$$H_e=(H_E-H_R) \quad - \quad - \quad - \quad (17)$$

The heat rejected is also given by the area below BCDE, for by equation (12)

$$H_1=(H_2+H_e)=(H_R-H_B)+(H_E-H_R)=(H_E-H_B).$$

Substituting in equation (13) from (16) and (17), the equation for the coefficient of performance becomes—

$$C=\frac{H_2}{H_1-H_2}=\frac{H_R-H_B}{H_e-H_R} \quad - \quad - \quad - \quad (18)$$

and the heat quantities which have to be determined are, therefore—

1. Total heat of the under-cooled liquid at t_4.
2. Total heat of the wet vapour at p_c and t_1.
3. Total heat of the wet, dry, or superheated vapour at p_c.

When a set of constant heat or throttling curves (H) is drawn on the $T\phi$ diagram, as indicated by the dotted curves in Figs.

11 and 12, these heat values can be read directly or obtained by interpolation. Evaluation from the diagram areas or by an analytical method is thus avoided.

At the best, however, the Tϕ diagram is inferior to the Hϕ diagram for this heat calculation.

The type of modified skew co-ordinate Hϕ diagram (for CO_2), now used, is shown in Fig. 13. The diagram for NH_3 or SO_2 is similar, but the pressure curves do not extend beyond the liquid limit curve at the lower end of the diagram.

The heat quantities, reckoned from 0° C, are read on the vertical heat scale on the left. When the state point on any pressure curve is defined, the corresponding total heat is obtained by projecting horizontally from it to the heat scale.

The diagonal lines show the slope of the entropy axis, relatively to the heat axis; and the adiabatic expansion or compression curve is a line drawn parallel to these entropy lines between the specified pressure limits.

The cycle of the CO_2 machine, for the same limits of pressure and temperature as shown on the Tϕ diagram, Fig. 12, is shown on this chart. The same letters are used, and the sequence of the operations is denoted by the directional arrows.

During the throttling operation at the valve, between $p_c = 900$ lbs./in.^{2} and $p_e = 380$ lbs./in^2, the state point moves from B to K on the constant heat curve BK. The total heat indicated on the scale is $H_B = H_K = 9{\cdot}7$ C.H.U./lb. During the evaporation period at $p_e = 380$ lbs./in.^{2} and $t_1 = -10°$ C, the point moves along the constant pressure curve, in the saturation field, from K to R. At R the quality is $q_1 = 0{\cdot}9$, that is the distance of the point R, from the liquid limit curve, is 90 per cent. of the distance between the liquid and gas limit curves.

The total heat at R, shown by the scale, is $H_2 = 50$ C.H.U. By equation (16) the heat taken in is—

$$H_2 = (H_K - H_B) = (50 - 9{\cdot}7) = 40{\cdot}3 \text{ C.H.U.}$$

During adiabatic compression from $p_e = 380$ to $p_c = 900$lbs./in.^{2}, the point moves along RE, crossing the gas limit curve, and taking its final position at E on the condenser pressure curve $p_c = 900$. The total heat at E is, $H_E = 57$ C.H.U.

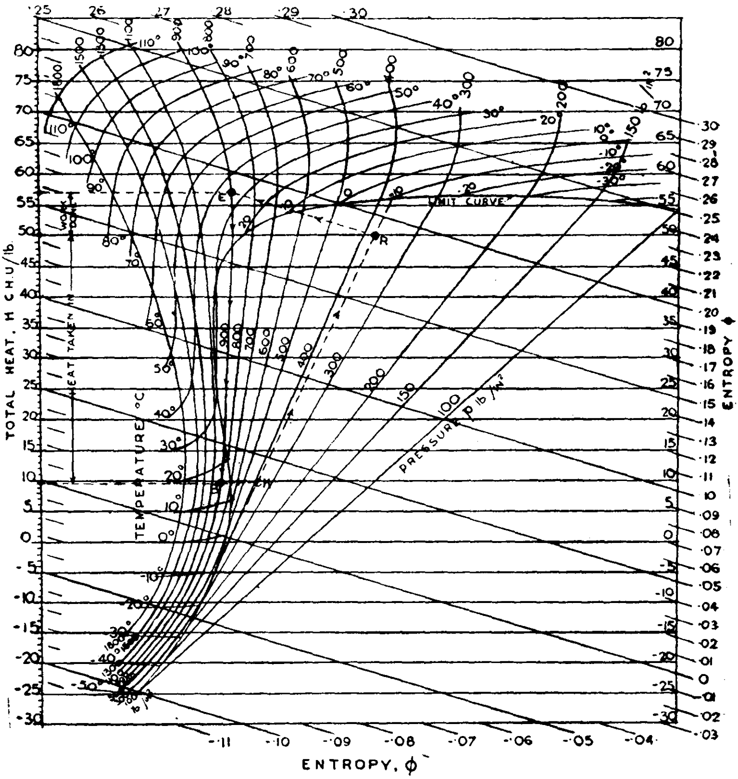


Fig. 13.—Total Heat Entropy Diagram for Carbonic Acid.

By equation (17) the heat equivalent of the work done is—

$$H_e = (H_E - H_R) = (57 - 50) = 7 \text{ C.H.U.}$$

and by equation (18) the coefficient of performance is—

$$C = \frac{H_R - H_B}{H_E - H_R} = \frac{40 \cdot 3}{7} = 5 \cdot 757.$$

During the condensation period, the point moves along the constant pressure curve $p_c = 900$ lbs./in.^{2}, first crossing the gas limit curve, and then the liquid limit curve, until the initial state at B on the temperature curve $t_3 = 15°$ C, is reached.

By equation (12) the heat rejected is—

$$H_1 = (H_2 + H_e) = (40{\cdot}3 + 7) = 47{\cdot}3 \text{ C.H.U.}$$

This is given by the vertical distance between the points B and E.

The curves shown in this skew diagram are those derived from the most recent research on the properties of CO_2, by Prof. C. F. Jenkin and Mr. R. D. Pye, the results of which were communicated to the Royal Society in 1915.*

The author, for the construction of the $H\phi$ alignment chart shown in Fig. 15, has taken the necessary pressure, temperature, heat, and entropy values from these curves, and from the $T\phi$ diagram also given in the paper.

Referring to the modified $H\phi$ diagram, shown in Fig. 14 for the NH_3 refrigerator, it will be seen that this consists of three sections, marked respectively—No. 1, No. 2, and No. 3. The sections are linked together by means of the common axis OC.

Section No. 1 consists merely of a function scale of temperature set out on an axis EF, which is drawn parallel to the principal axis OC.

It represents the partial aspect of the simple nomogram for, $h = f(t)$, the base scale being a scale of liquid heat (h).

This axis is drawn below OC, merely to avoid confusion, and makes the diagram clearer for purposes of explanation. The temperature scale can be marked directly on the under side of the main axis, if desired. This axis is also the scale axis for total heat, the origin being at 0.

When the temperature of the undercooled liquid is known,

* Phil. *Trans.*, 215 A, 1915; "Thermal Properties of Carbonic Acid at Low Temperatures."

a projector from its value, on EF to OC, registers the heat of the liquid on the heat scale. This scale, also to avoid confusion, is not drawn on the axis, but is transferred to a parallel axis at the top of the diagram. On the full-sized chart it has been taken as 1 inch = 25 C.H.U. In the case illustrated, the temperature of the undercooled liquid is taken as $t_4 = 15°$ C, and the total heat of the liquid is OA = 17 C.H.U.

This gives the total heat H_B at B, Fig. 11, and is also the heat at K, when absorption begins at $p_e = 40$ lbs./in.² and $t_1 = -10°$ C.

When the quality q_1 at R, Fig. 11, is known, the total heat at R is given by—

$$H_R = h_1 + q_1 L_1 \quad - \quad - \quad - \quad - \quad (19)$$

where h_1 and L_1 are the liquid and latent heats at t_1.

Equation (19) can be written thus—

$$H_R = h_1 + z \quad - \quad - \quad - \quad - \quad (20)$$

$$\text{Here } z = q_1 \ L_1 \quad - \quad - \quad - \quad - \quad (21)$$

Equation (21) is of the general form,

$$f(z) + f(q) \ f(t) = 0,$$

and can be nomographed on three straight-line axes two being parallel and one inclined. The nomogram is shown in section No. 2, and is constituted by the $f(t)$ scale on the axis GH, the z scale on the parallel axis IK, and the dryness (q) scale on the inclined axis LM.

The base scales on GH and IK are scales of heat; the module chosen for each is $l = 0{\cdot}04$ inch.

As the z value does not require to be read in the calculation, the scale divisions are omitted on the finished chart. The diagonal axis LM, and the q scale, are obtained from the scales on GH and IK by the projective method, already explained in connection with the other nomograms.

Equation (19) is of the form,

$$f(H) + \phi(t) + f(z) = 0,$$

and can, therefore, be nomographed on three parallel axes. The z value being common to both nomograms, the scale axis IK forms a connecting link between them. The second nomogram is constituted by the z scale on IK, the H scale on the main axis OC, and the $\phi(t)$ scale on the parallel axis NP.

The scale on IK is made the same as that on OC. This on the full-size chart is, 1 inch $=25$ C.H.U., so that the common module is, $l=0{\cdot}04$ inch.

By a rule of nomography, when this condition holds, the third axis is midway between the other two, and its scale module is half that of the others. Hence, for the $\phi(t)$ scale on NP, $l=0{\cdot}02$ inch.

The fundamental y lengths for the temperature scale on GH for the full size chart are obtained by multiplying the latent heat values given in Table III by 0·04, while the lengths for the scale on NP are obtained by multiplying the liquid heat (h) values by 0·02.

Table III.

t °C ..	−30	−25	−20	−15	−10	−5	0	5	10	15	20	25	30	35	40
L ..	327·75	324	320	316·5	312·72	308·5	303·5	299·5	294·9	289·75	284·47	278·99	273·25	267·0	261·5
h ..	−32·71	−27·41	−22·05	−16·6	−11·14	−5·566	0	5·64	11·36	17·11	22·95	28·82	34·76	40·79	46·21

The zero value of the scale on NP is fixed by joining a scale value on IK with the same value on OC, and noting the point of intersection of the line with NP.

Alternatively, instead of using a module, the scale on NP can be easily and quickly derived by projection, from the scales on IK and OC.

The positon of the temperature scale on GH is quite arbitrary. It should be so placed that, for the most usual temperature conditions, the operative lines may not cut the axes at too acute an angle, and cause inaccuracy in reading the results.

The base scales of heat on OC and GH read in the same direction; the scale on NP reads in the opposite direction. On the full-size chart the arbitrarily chosen distance between GH and IK and OC and IK is 3 inches.

When the straight-edge is laid across the given temperature value on GH and the dryness q_1 on LM, the latent heat of the wet vapour $q_1 L_1$ is registered by the scale on IK. When the edge is laid across this point and the t_1 value on NP, the liquid heat h_1 is added, and the sum $H_R = h_1 + q_1 L_1$ is registered at B, where the edge cuts the main axis OC.

In the case illustrated, with $p_e = 40$, $t_1 = -10°$ C and $q_1 = 0{\cdot}95$ OB $= 286$. The distance AB $= (\text{OB} - \text{OA}) = (H_R - H_K)$ $= H_2$, hence $H_2 = (286 - 17) = 269$ C.H.U.

Section No. 3 determines the total heat of the vapour after compression. It consists of a compound nomogram, similar to that of section No. 2, and the upper portion of an ordinary $H\phi$ diagram, plotted to rectangular co-ordinates. The main axis OC is drawn parallel to the total-heat axis of the chart, while the axis YZ is drawn parallel to the entropy axis.

The preliminary calculation involves the estimation of the total entropy of the vapour at the beginning of the compression—that is, at R, Fig. 11, since the entropy remains constant during adiabatic compression. Here,

$$\phi_R = \phi_{w_1} + q_1 \phi_{e_1} \quad - \quad - \quad - \quad - \quad (22)$$

Where ϕ_{w1} and ϕ_{e1} are the liquid and evaporation entropies at t_1, and ϕ_R is the total entropy.

Equation (22) is of the same form as (19), with entropy substituted for heat, and as the same process of reasoning applies, it is not necessary to go over the ground again.

The equation $z = q_1 \phi_{e1}$ is nomographed on the axes QR, ST, and UV. The function temperature scale on QR is the partial aspect of the simple nomogram for $\phi = f(t)$. The scale of evaporator pressure drawn against it is the partial aspect of the simple nomogram for $\phi = f(p)$, the two superposed nomograms having the common base scale of entropy. The module for each scale on the full-size chart has been taken as $l = 10$ inches, or the entropy base scale is 1 inch = 0·1 entropy unit.

The fundamental (y) distances for the temperature and pressure scales (measured upwards) are obtained by multiplying the corresponding evaporation entropies ϕ_e, given in the subjoined table, by 10.

In order to get a fairly open set of constant pressure curves, the entropy scale of the full size Hϕ diagram has been taken as 1 inch = 0·05 entropy unit.

The base scale on UV has been taken the same as that on QR, or 1 inch = 0·1 entropy unit. The module for the scale on UV is thus $l = 10$ inches, and the module for the scale on YZ, $l = 20$ inches.

According to nomographic rules, with this arbitrary choice of scales, the distance between the parallel axis WX, and the link axis UV is one-third of the distance between UV and YZ; while the module of the temperature scale on WX is—

$$l = \frac{10 \times 20}{10 + 20} = \frac{290}{30} = 6{\cdot}666 \text{ inches.}$$

The fundamental (y) distances for the temperature scale on WX are obtained by multiplying the liquid entropies (ϕ_w), given in Table IV by 6·666. The zero value of this scale is obtained by joining a scale value on YZ with the same value on UV, and finding the intersection of the line with WX.

The base scales on UV and YZ read in opposite directions, and the arbitrarily chosen distance between UV and QR, and UV and YZ is 4 inches.

The $H\phi$ chart on the right does not require any special comment.

The pressure curves are plotted in the usual way, the total heat and entropy values being taken from Table IV.

Table IV.

$t°$	p	h	L	H	ϕ_w	ϕ_e	ϕ	$t°$	p	h	L	H	ϕ_w	ϕ_e	ϕ
−23	20	−29·8	325·3	295·5	−0·104	1·32	1·216	24	140	27·2	280·7	307·9	0·0955	0·943	1·0385
−16	30	−19·0	318·5	299·5	− ·072	1·26	1·168	26	150	29·8	278·2	308·0	·104	·930	1·034
−10	40	−11·5	313·2	301·7	− ·045	1·20	1·156	28	160	32·2	275·8	308·0	·112	·915	1·027
−5	50	− 5·5	308·9	303·4	− ·022	1·155	1·133	30	170	34·4	273·6	308·05	·120	·90	1·02
−1	60	− 0·6	304·9	304·3	− ·002	1·12	1·118	32·3	180	36·8	271·3	308·1	·127	·890	1·017
3	70	+ 4	301·2	305·2	+0·014	1·088	1·102	33·8	190	39·0	269·2	308·2	·1342	·877	1·0112
6·3	80	8·1	297·8	305·9	·03	1·06	1·09	36	200	41·0	267·2	308·2	·141	·862	1·003
10	90	11·8	294·6	306·4	·042	1·037	1·08	37·2	210	43·0	265·2	308·2	·148	·852	1·000
13·5	100	15·3	291·3	306·6	·054	1·015	1·069	39	220	45·0	263·2	308·2	·154	·842	0·996
16·2	110	18·5	288·8	307·3	·065	0·996	1·061	41	230	46·8	261·4	308·2	·160	·830	·990
19	120	21·6	285·9	307·5	·076	·978	1·054	41·5	240	48·5	259·8	308·3	·166	·822	·988
21·3	130	24·5	283·2	307·7	·086	·960	1·046								

Any pressure curve is continued from the saturation to the superheat field by calculating entropy changes for several superheat ranges, from the usual equation—

$$\phi_s = Kp \log_e \frac{Ts}{T},$$

where ϕ_s = increase of entropy for the range $(Ts - T)$.

T = saturation temperature at the pressure considered.

Ts = temperature of superheated vapour.

Kp = sp. ht., taken as constant at 0·508.

When the straight-edge is laid across the evaporator temperature t_1 on QR, and the quality q_1 on ST, it registers the evaporation entropy $q_1 \ \phi_{e1}$ of the wet vapour on the axis UV. When placed across this point and the temperature t_1 value on WX, the liquid entropy ϕ_{w_1} is added, and the sum

$$\phi_K = \phi_{w_1} + q_1 \ \phi_{e_1}$$

is registered where the edge cuts the axis YZ. This on the entropy scale of the Hϕ diagram gives the total entropy at the end of the compression. By projecting horizontally from YZ to the condenser pressure curve p_c, then vertically to the main axis OC, the total heat at the end of compression is registered at C.

In the case illustrated the condenser pressure chosen is, $p_c = 140$ lbs./in.^{2}, and it will be noted that the state point falls in the superheat region. This condition is also shown in Fig. 11.

The total heat is given by the distance OC = 324 C.H.U. The heat rejected is (OC − OA), so that $H_1 = (324 - 17) = 307$ C.H.U.

The work done is thus, BC = (AC − AB) = $(H_1 - H_2)$, so that $H_e = (307 - 269) = 38$ C.H.U.

By equation (13) the coefficient of performance is—

$$C = \frac{H_2}{H_1 - H_2} = \frac{269}{3 \cdot 8} = 7 \cdot 1.$$

By varying the initial quality q_1, the dryness, which gives the maximum value of C, can be readily found. This will be in the neighbourhood of 0·9, and the vapour will be nearly dry at the end of compression.

The foregoing detailed explanation of the construction of the NH_3 chart has been given to enable those who may desire it to draw the chart out, either to larger scales, or from some other set of data; also to assist them in the construction of the SO_2 and CO_2 charts of the same type.

At present no really reliable table of the properties of NH_3 is available. The values used here have been derived from a somewhat old table, and can only be regarded as tentative.

When once a chart of this type is drawn down, to a good scale, it will be found useful for the investigation of the effect of varying the different factors involved in the refrigerator calculation.

The details of the alignment chart, Fig. 15, for the CO_2 refrigerator, are worked out in the same way as those for the NH_3 one, except those for section No. 1.

In the case of H_2O, NH_3, and SO_2, the liquid heat at any given temperature is not sensibly affected by the pressure, and all the constant pressure curves in the liquid region coincide with the liquid limit curve. CO_2 is an exceptional case, the heat value being affected both by temperature and pressure. The pressure curves branch out from the limit curve to the left, as indicated for $p=900$ lbs./in.^{2} in Fig. 12. Above the critical limit of pressure, about 1,000 lbs./in.^{2}, they fall outside the limit curve altogether, as shown in Fig. 13.

To determine the heat of the undercooled liquid for this case, both pressure and temperature have, therefore, to be taken into account. In consequence, the author has substituted. in Fig. 15, for the simple nomogram (Fig. 14) a heat temperature chart for the liquid in Section No. 1.

It consists of a set of constant pressure curves, ranging between $p_e = 600$ and 1,800 lbs./in.², plotted on a temperature base.

Projecting horizontally from the temperature scale to the given p_c curve, then vertically to the main axis gives the point A; and the liquid heat before expansion through the valve is again given by OA. It will be noted that between 0° C and 20° C these liquid pressure curves are practically straight lines. When calculations are confined within these limits any curve may be expressed approximately by the equation $h = a + b\,t$, where a and b are functions of the pressure. The two equations,

$$z = f(p)\ f(t) \text{ and } h = f(p) + z,$$

can be nomographed in exactly the same way as the previous sets, giving a compound nomogram similar to that of section No. 2, with p scales in place of the q_1 and t scales on the diagonal and parallel axes. This compound nomogram can, if desired, be substituted for the rectangular chart. The author has constructed this nomogram; but as a readable combination occupies just as much space as the rectangular one, and as it does not satisfactorily cover the wider range of temperature, he has preferred to retain the set of curves as shown.

Again, at the Hϕ section, it will be seen that the pressure curves lying outside the limit curve are very flat, and each as a first approximation might be taken as a straight line expressed by the equation $\phi = a + b\mathrm{H}$, where, again, b and a are functions of the pressure.

Another nomogram, which will give approximately the values obtained from the Hϕ diagram, can, if desired, be substituted for it. In this case the main axis OC is drawn in the vertical position.

In the case of the NH_3 diagram, if it is not intended to consider the pressure curves in the superheat field much beyond the gas limit curve, a similar substitution may be made. In each instance the substituted nomogram can be so disposed that the two link axes UV and YZ, Fig. 14, to the left of the Hϕ

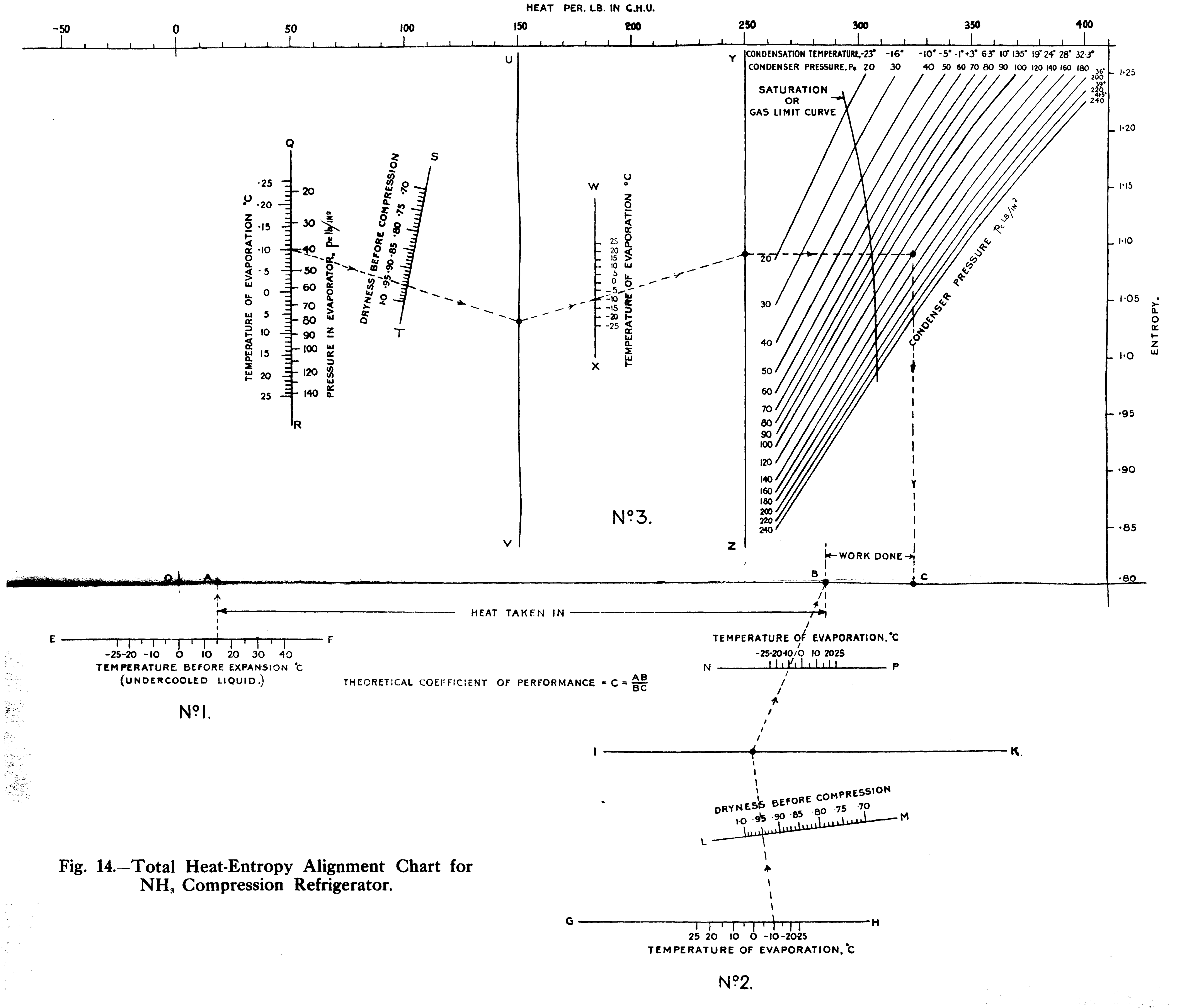


Fig. 14.—Total Heat-Entropy Alignment Chart for NH_3 Compression Refrigerator.

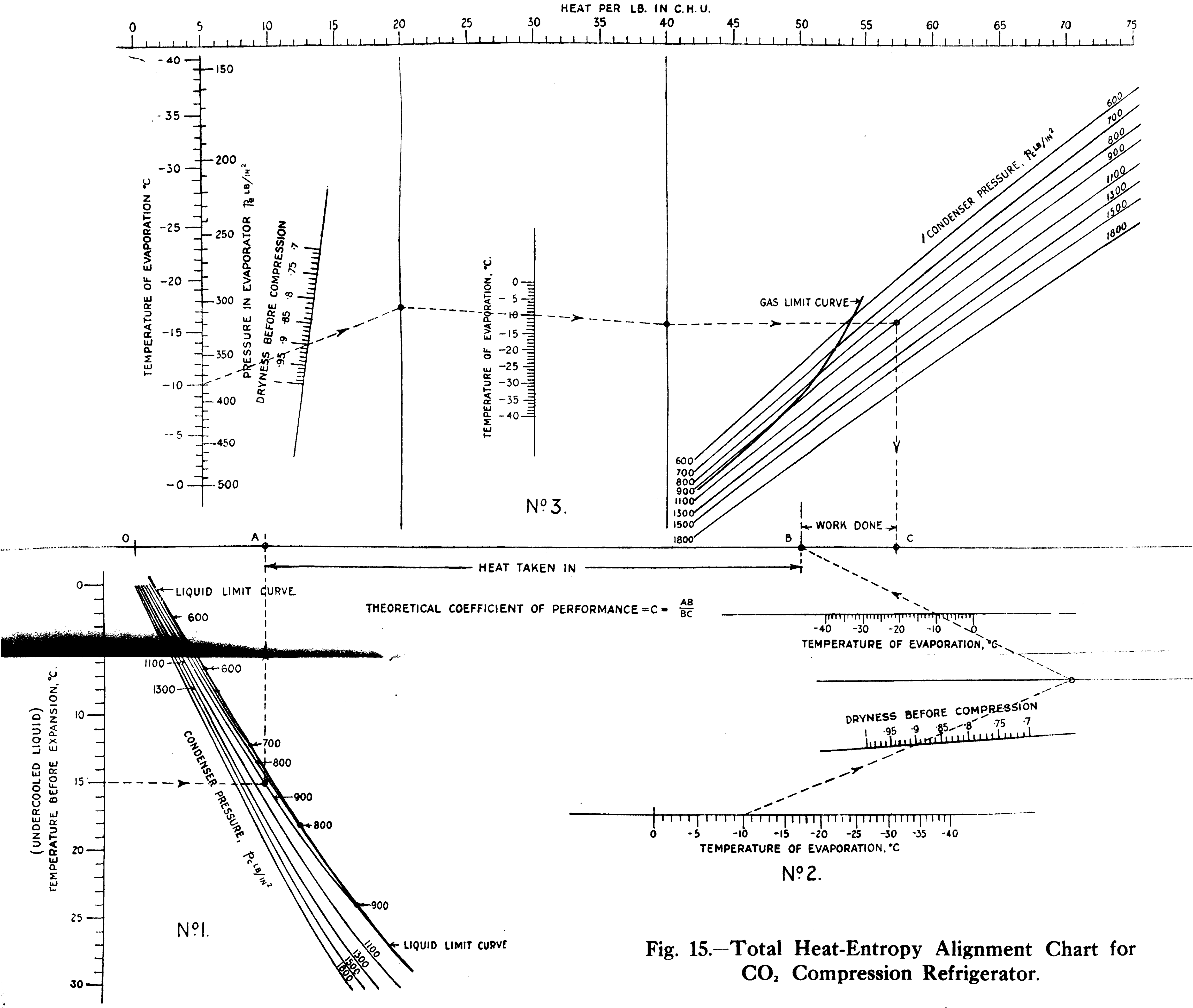


Fig. 15.—Total Heat-Entropy Alignment Chart for CO_2 Compression Refrigerator.

diagram, can be made also to serve as the link axes of the substituted one.

While the alternative combination is of considerable interest, from the point of view of chart design, it is not so satisfactory as the combination, shown in Fig. 14, and recommended for practical use.

The case worked out on the CO_2 alignment chart is that already dealt with in the skew $H\phi$ diagram, Fig. 13. It will be seen that the same heat values are obtained from the total heat alignment chart by the simpler projective process. As this chart embodies the most recent data on CO_2, the results obtained from it may be regarded as reliable.

In conclusion the author hopes that these total heat alignment charts may prove both interesting and useful to engineers engaged on refrigeration work; also that the few notes on nomographic methods given throughout the paper may help to stimulate the interest in this useful branch of graphical calculation, which can be employed to greatly facilitate the solution of many standard formulæ handled in the daily work of the civil, mechanical, and electrical engineer.

Work Submitted for D Sc Examination
by
Wm J. Goudie

7th Jan 1918

This Envelope Contains

(2) M.SS. Supplementary Note to Chapt X
of "Steam Turbine" Book

With the supplementary paper, since published in Trans Inst Engineers & Shipbuilders in Scotland, 23 April 1918.

(3) Proof Copy of Paper :-

"The Use of Nomograms in the Determination of thermal Efficiencies of Internal Combustion Engines & Coefficients of Performance of Vapour Compression Refrigerators" (now in Trans Inst Engineers & Shipbuilders in Scotland Vol LX. 1916-1917. page [illegible])

with paper & [illegible] to Clerk of Senate
[illegible] 15 Nov 1919 W.D.

Steam Turbines

Supplementary Note to Chapter V. on "Properties of Steam"

This note deals with:-
The Nomographic Representation of Callendar's Characteristic Equations for Steam

by

William J. Goudie.

7th Jan 1918.

This sketch is illustrated by photo prints of four nomograms, (on two sheets)

This manuscript, returned on 2nd May is resubmitted in accordance with the request contained in the Clerk of Senate's letter dated 15 May 1919

Since it was written ~~the~~ a paper, dealing more fully with the subject, has been published in the ~~the~~ "Trans. of Institution of Engineers & Shipbuilders in Scotland" (23rd April 1918)
A copy of this paper (with discussion) is sent herewith as a supplement.

W. J Goudie

19th May 1919

Steam Turbine

Chapter V

The Nomographic Representation of Callendar's Characteristic Equations for Steam.

At the time this chapter on the Properties of Steam was written the ~~author~~ tables of Messrs Marks and Davis appeared to be the most comprehensive and best suited for the work in hand, and the author therefore adopted and used their tabular values, for all the steam calculations throughout the book.
Since then, however, Callendar's work has been published; and there is no doubt that the adoption of his Characteristic equation and it's derivatives obviates any slight thermodynamic inconsistencies, to which former tables may have been liable.
The Callendar tables, where superheated steam is concerned, may however necessitate a considerable amount of interpolation, where practical calculations relating to turbine design, have to expeditiously handled.
In view of this fact, the author recently gave a nomographic interpretation to the Callendar equations, and four of the nomograms are submitted, along with this note.

Total Heat Nomograms

Callendar, in his published work, has shown, that the total heat of steam can be deduced from the general equation

$$H = 0.4772T - 0.10286\,p\{(n+1)c - 0.016\} + 464$$

Where:-

where:- H = Total heat in cent. heat units per lb. (c.4U/lb)
T = Absolute temp. °C
p = " " press lb/in²
n = const = $10/3$

& c is the "coaggregation volume" $= 0{\cdot}4213\left(\frac{T_1}{T}\right)^{10/3}$
where $T_1 = 373{\cdot}1$ °C abs.

When T is greater than the saturation temperature at p., the steam is Superheated; when it is less, the steam is undercooled, & hence Supersaturated.

This equation can be expressed in the general form

$$f(H) + F(T) + f(p)\,f(T) = 0 \text{ ------- } (1)$$

and this can be readily nomographed.

Let $$Z = f(p)\,f(T) \text{ ----- } (2)$$

Equation (2) is the type equation giving a three scale nomogram, having two parallel and one inclined axis, as shown in Fig (1). Applying the rules of nomographic construction to this case, a scale of the product (Z) is set off on an axis EF; a function scale of temperature (T) is set off on a parallel axis AB., and a function scale of pressure (p) is set off on a diagonal axis CD.

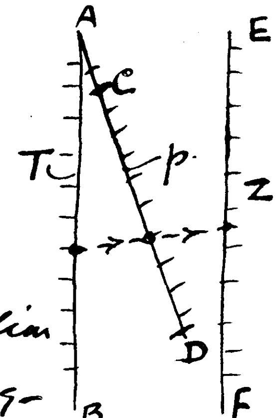


Fig (1)

The solution of equation (2) is obtained by inspection, when a straightedge is laid across the given p and T values.

Putting (Z) in equation (1), it becomes,

$$f(H) + F(T) + Z = 0 \text{ -- } (3)$$

and this again is the type equation for a three scale nomogram, having all the axes parallel, as shown in Fig (2). Again applying the rules of nomography, a function scale of temperature (T) is set off on an axis GK parallel to EF., and a

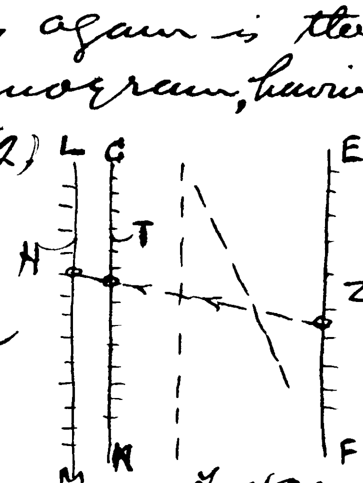


Fig (2)

function scale of ~~the~~ total heat (H) on another parallel axis LM. Equation ③ is solved by inspection, when a straightedge is laid across the given Z and T values. These two nomograms, when linked together by the common scale axis EF, can be used to determine the total heat of either superheated or supersaturated steam.
The complete combination, drawn to a practicable size, is shown in Fig (3), or the blue print (Sheet [illegible] † It has been designed to cover the practicable range of pressure, from 20 to 300 lbs/in², and temperature from 380 to 800° C. abs.
The author has used the C.H.V. & °C notation given by Callendar; but the B.T.H.U and °F notation can be substituted, if desired.
A pressure scale has also been provided on the axis AB, in order to show, at a glance, the saturation temperature at any given pressure, and to indicate, whether the steam at a specified temperature and pressure is, superheated, dry or supersaturated. These two scales of pressure and temperature are partial aspects of two simple nomograms, superposed on the axis AB, & having a common base scale.
If this nomogram is tested against the range of values given in Callendar's table of Superheated Steam†, it will be found to give results ~~correct~~ to a small fraction of a percent.

The Author is of the opinion that this graphical combination should be of material use in a turbine design department. It enables a far more comprehensive range of pressure and superheat values to be

† See, "The Callendar Steam Tables", Arnold 1915. Table IV.

† Equation ① can also be nomographed on three scales (one of them being curved) See pages 4 5 & 6 [illegible]

expeditiously handled, than is possible with a necessarily limited set of tabular values. The operation is easily and rapidly performed and all risk of arithmetical mistakes is eliminated.

It may, if desired be used as an alternative to the method of calculation involving the application of the superheat range (t_s) and the mean value of specific heat (K_p), given in Art. 43. p. 84.

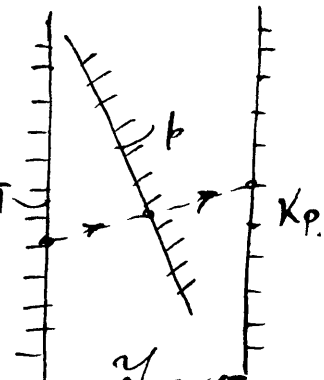
Some doubt has recently been expressed regarding the accuracy of Knoblauch & Jacob's specific heat results, on which the author based the specific heat Chart, shown on page 88.

It seems desirable, therefore, to substitute, for this Chart, a nomogram embodying Callendar's equation for K_p., which is,

$$K_p = 0.4772 + n(n+1)\frac{pc}{JT}$$

In general form this is,

$$f(K_p) + f(p) f(T) = 0$$



Fig(5)

And, as in the case of equation (2), it is represented by the type of three scale nomogram shown in Fig(5.)

Referring again to Sheet I. it will be seen that a second total heat nomogram Fig(4) is linked to the first nomogram Fig(3) by the common axis EF. By means of this nomogram, the total heat of either dry or wet steam can be quickly obtained, by inspection, in a few minutes, without calculation or reference to tables.

This nomogram solves equation (3), page 86, for the total heat of wet steam, but, for graphical purposes, in the more convenient form,

$$H_w = H - (1-q)L$$

where H_w = Total heat of wet steam at press^{re}
H = " " " dry " "
L Latent " " " " "
q = dryness fraction.

The general form for this nomogram is

$$f(H) + F(p) + f(q)\, f(p) = 0$$

The function of p scales ~~basic~~ give, implicitly, on the axis R.S., the total heat of dry steam (H) and on the axis N.P., the latent heat of dry steam (L). In other words, the base scales are scales of ~~total~~ heat.

As the methods of operation for these combinations are clearly indicated, by the accompanying rules and operating keys, it is not necessary to discuss them further.

Total Entropy Nomograms

The general equation for the total entropy of super heated or supersaturated steam can be handled in the same way as the equation for total heat. It has the form,

$$\phi = 1.09876 \log \frac{T}{T_1} - 0.25356 \log \frac{p}{p_1} - a\,n\,c\, \frac{p}{T} + 1.7630$$

Expressed generally this is

$$f(\phi) + F(T) + F(p) + f(p)\, f(T) = 0$$

Here two functions of pressure have to be graphed, instead of one in the case of the total heat nomogram. The complete nomogram, shown in fig 6, thus consists of three nomograms, linked by the two axes KL and EF. There are two pressure and

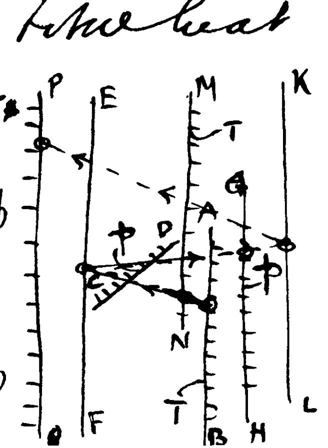


Fig (6)

two temperature scales, and the total entropy scale; and the solution of the equation involves three adjustments of the straightedge across the scales & link axes.

This combination, drawn to a practicable scale, is shown in Fig 7, on the blue print (Sheet II). It has been designed to cover the same pressure and temperature ranges as the total heat nomogram.

Here again, a second nomogram, for calculation of the total entropy of wet steam, has been linked to that for superheated steam, and is shown in Fig 8.

It solves the equation, analogous to that for the total heat of wet steam

$$\phi_q = \phi - (1-q)\phi_e$$

where ϕ = Total entropy of dry steam at p lb/in²
ϕ_e = Evaporation " " " " " "
ϕ_q = Total entropy of wet steam " " "
q = Dryness fraction

The only difference between this nomogram and its analogue, is the use of a base scale of total entropy for the function pressure scale on axis, WX and evaporation entropy for the function pressure scale on axis RS. †

The complete combination (figs 7 & 8), enables the final quality of steam after isentropic expansion from initial pressure p_1 to final pressure p_2 to be found, without troublesome reference to steam tables, and arithmetical calculation. The presumption here is that, in dropping from p_1 to p_2, the steam passes from the superheated to the wet state.

In the case of a comparatively small pressure

† A simpler form corresponding to Fig 5, accompanying ... could ... construct ...

drop where there is considerable initial superheat the steam is still superheated at p_2, and the wet steam nomogram does not apply.†
The author has not yet been able to test the trial method, set out under operative rule III, sufficiently, to ascertain if this graphical method has any advantage over the trial method given on page. 102.
In any case, however, the nomogram (fig 8) at once defines the final state at p_2, as either superheated or wet. If the former condition holds, the operator can choose which ever of the two methods he prefers.

It follows, at once, that when the qualities at p_1 & p_2 are known, the Rankine cycle heat is calculable from the corresponding values of the total heats H_1 & H_2, determined from the total heat nomogram.

It should be understood, that the author does not submit these new nomographic combinations as substitutes for the older $H\phi$ (Mollier) diagram, discussed in the book, and on which he has based and developed the general system of design calculations for compound turbines, discussed in Chapters XI XIII XIV & XV. In all round application to the class of problems, involved in such design, the Total heat ($H\phi$) diagram is probably unique.

These nomograms are rather to be regarded as useful supplements to the table of saturated steam, which enable superheated steam values to be easily and quickly ascertained & handled, & incidentally serve to facilitate, or check the num-

† In this case the point of intersection on T V falls above unit dryness.

erical calculations relating to wet steam.
Referring to the approximate equation (8) for the volume of superheated steam, given on page 89, while this gives values of V_s, sufficiently close for most practical purposes, it might together with the nomogram on page 90, now be replaced, with advantage, by a nomogram embodying Callendar's exact equation.

This equation is

$$V_s = 1.07061\frac{T}{p} - 0.4213\left(\frac{T_1}{T}\right)^{10/3} + 0.016$$

As in the previous cases the general form is

$$f(v) + F(T)\,f(p) + f(T) = 0.$$

The type of nomogram embodying this is shown in Fig 9.

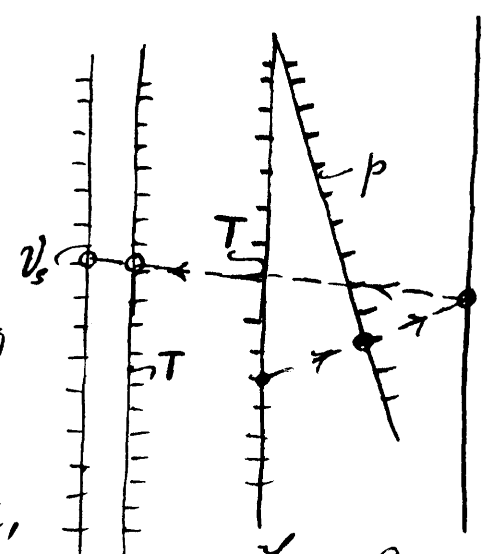


Fig 9.

Owing to pressure of other work, the Author has not yet found time to construct this† and the specific heat nomogram indicated in Fig 5.; but from what has been said above, it will be seen that both are perfectly practicable, and their inclusion in the set would thus give a complete account of all the independent characteristics of pressure (p), volume (v), temperature (T), ~~entropy (ϕ)~~ Total heat (H), and by inference, from p. V & H, the internal energy (I) of the steam in any given state.

9th Jan. 1918:

W. J. Goudie.

† This nomogram was drawn down later, but as it did not prove quite suitable, another equation was used, which gives a simpler easily handled nomogram. See pages 9 & 10 of accompanying paper and Figs 2 & 3. W. J. G.

SHEET II.

Fig. 8.

NOMOGRAM

FOR CALCULATION OF

TOTAL ENTROPY OF DRY & WET STEAM

THIS NOMOGRAM SOLVES THE EQUATION

$$\phi_q = \phi - (1-q)\,\phi_e$$

ϕ = TOTAL ENTROPY OF DRY STEAM AT p lbs/in².
ϕ_e = EVAPORATION " " " " " " "
ϕ_q = TOTAL ENTROPY OF WET " " " "
q = DRYNESS FRACTION

OPERATIVE RULE. II.

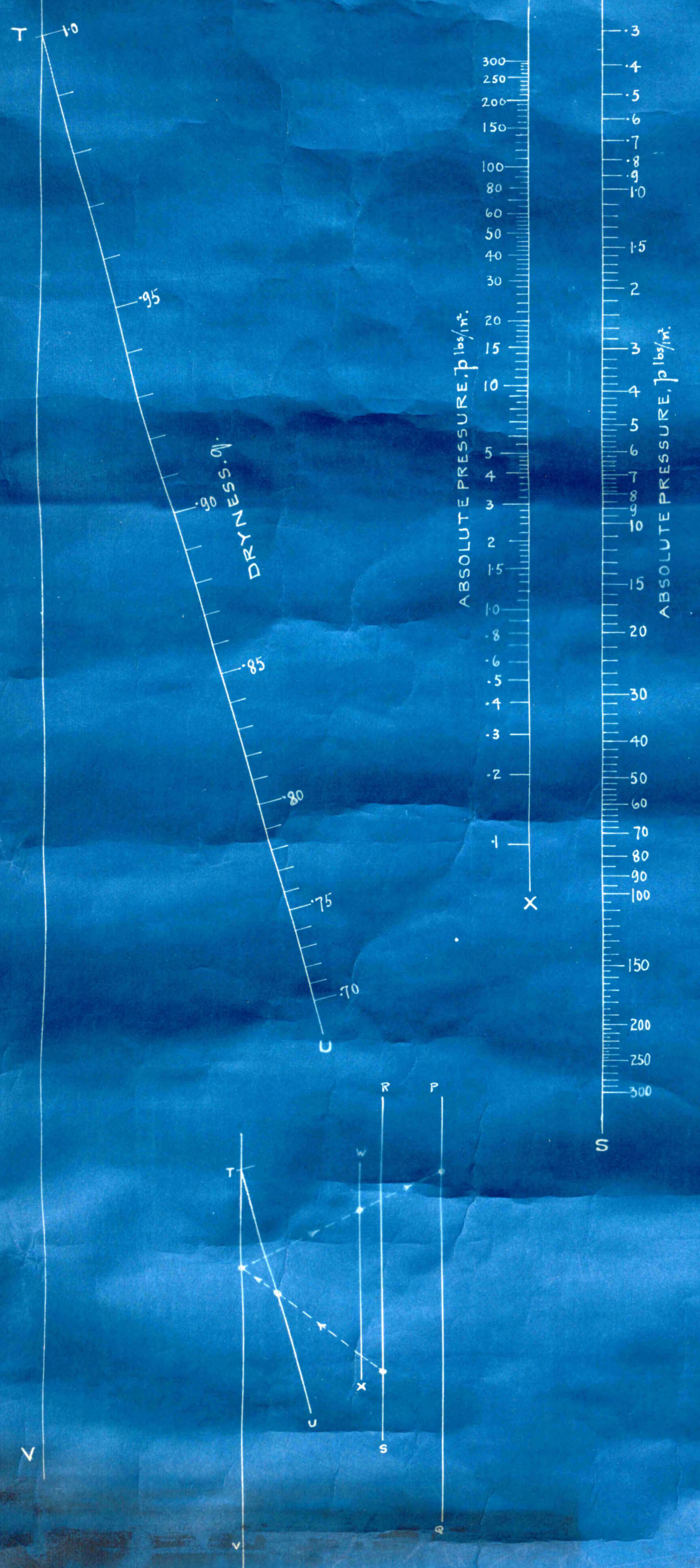
DRY STEAM

LAY A STRAIGHT EDGE ACROSS UNIT DRYNESS ON AXIS T.V. AND THE GIVEN PRESSURE ON AXIS W.X. AND READ THE TOTAL ENTROPY WHERE IT CUTS THE AXIS P.Q.

WET STEAM

LAY A STRAIGHT EDGE ACROSS THE GIVEN PRESSURE ON AXIS R.S. AND THE GIVEN DRYNESS ON AXIS T.U. AND MARK THE POINT OF INTERSECTION ON AXIS T.V. LAY THE EDGE ACROSS THIS POINT AND THE GIVEN PRESSURE ON AXIS W.X AND READ THE TOTAL ENTROPY WHERE IT CUTS THE AXIS P.Q.

DESIGNED BY
W. J. GOUDIE
JAN. 1918.



OPERATIVE KEY.

Fig. 7.

NOMOGRAM

FOR CALCULATION OF

TOTAL ENTROPY OF SUPERHEATED STEAM

THIS NOMOGRAM SOLVES CALLENDAR'S EQUATION

$$\phi = 1{\cdot}09876 \log \frac{T}{T_1} - 0{\cdot}25356 \log \frac{p}{p_1} - anc\frac{p}{T} + 1{\cdot}7630$$

$$\text{OR, } \phi = 1{\cdot}09876 \log \frac{T}{373{\cdot}1} - 0{\cdot}23536 \log \frac{p}{14{\cdot}896} - 54{\cdot}01 \times 10^{6} \frac{p}{T^{10/3}} + 1{\cdot}7630$$

T = ABSOLUTE TEMP °C
p = " " " PRESS lbs/in²
ϕ = TOTAL ENTROPY CHU/°C

OPERATIVE RULE. I.

LAY A STRAIGHT EDGE ACROSS THE GIVEN SUPERHEAT TEMPERATURE ON AXIS A.B. AND PRESSURE ON AXIS C.D. AND MARK THE POINT OF INTERSECTION ON AXIS E.F. LAY THE EDGE ACROSS THIS POINT AND THE GIVEN PRESSURE ON AXIS G.H. AND MARK THE POINT OF INTERSECTION ON AXIS K.L. LAY THE EDGE ACROSS THIS POINT AND THE GIVEN TEMPERATURE ON AXIS M.N. AND READ THE TOTAL ENTROPY WHERE IT CUTS THE AXIS. P.Q..

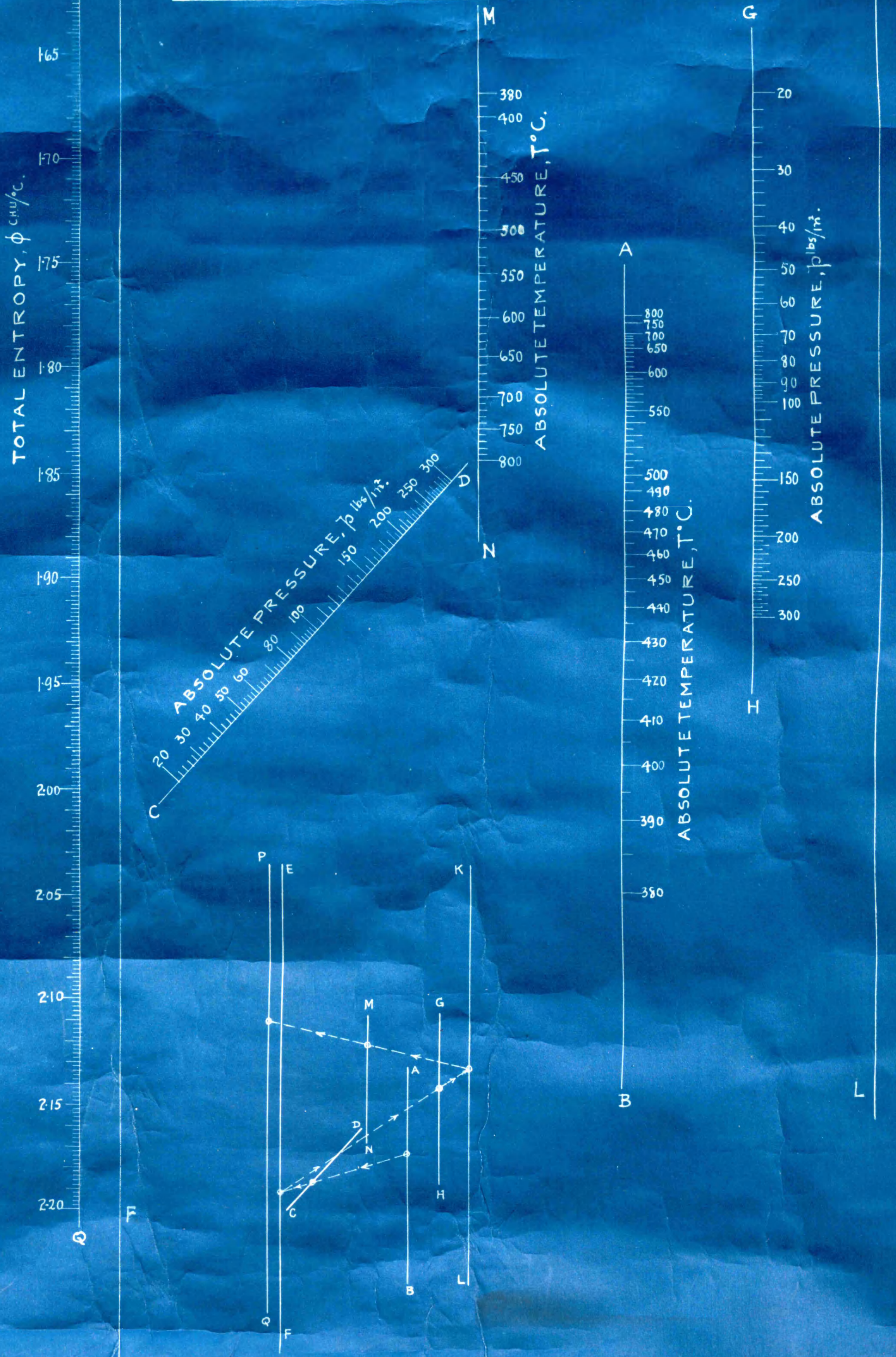
OPERATIVE RULE. III.

TO FIND THE DRYNESS AFTER ISENTROPIC EXPANSION FROM PRESSURE p_1 TO PRESSURE p_2

DETERMINE BY RULE I OR II THE TOTAL ENTROPY ϕ_1 AT p_1. LAY A STRAIGHT EDGE ACROSS THE ENTROPY ϕ_1 ON AXIS P.Q. AND PRESSURE p_2 ON AXIS W.X AND MARK THE POINT OF INTERSECTION ON AXIS T.V. LAY THE EDGE ACROSS THIS POINT AND PRESSURE p_2 ON AXIS R.S. AND READ THE DRYNESS q_2 WHERE IT CUTS THE AXIS T.U.

IF THE POINT OF INTERSECTION ON THE AXIS T.V. FALLS ABOVE UNIT DRYNESS THE STEAM IS STILL SUPERHEATED AT p_2. THE SUPERHEAT TEMPERATURE IS FOUND BY TRIAL. FIND, FROM THE SCALES ON AXIS A.B. OF THE TOTAL HEAT NOMOGRAM, THE SATURATION TEMPERATURE T_2 AT p_2. CHOOSE, FOR A TRIAL, SOME HIGHER VALUE T_{S_2} AND APPLY RULE I. IF THE RESULT IS LESS THAN ϕ_1, INCREASE T_{S_2}, IF GREATER DECREASE IT, AND RECALCULATE. REPEAT THE OPERATION UNTIL ϕ_1 IS OBTAINED.

TO FIND THE RANKINE CYCLE HEAT H_r, CALCULATE FROM THE TOTAL HEAT NOMOGRAM, THE TOTAL HEATS, H_1 AND H_2 CORRESPONDING TO THE QUALITIES AT PRESSURES p_1 AND p_2. THEN, $H_r = (H_1 - N_2)$.



OPERATIVE KEY.

SHEET. I.

Fig. 3.

NOMOGRAM

FOR CALCULATION OF

TOTAL HEAT

OF

SUPERHEATED & SUPERSATURATED STEAM

THIS NOMOGRAM SOLVES CALLENDAR'S EQUATION

$$H = 0{\cdot}4772T - 0{\cdot}10286\,p\left\{(n+1)c - 0{\cdot}016\right\} + 464$$

OR, $$H = 0{\cdot}4772T - 0{\cdot}10286\,p\left\{\frac{691{\cdot}3\times10^{6}}{T^{10/3}} - 0{\cdot}016\right\} + 464$$

T = ABSOLUTE TEMP. °C.
p = " " " PRESS. lbs/in².
H = TOTAL HEAT IN CHU/lb.

OPERATIVE RULE.

LAY A STRAIGHTEDGE ACROSS THE GIVEN TEMPERATURE ON AXIS AB. AND PRESSURE ON AXIS C.D., AND MARK THE POINT OF INTERSECTION ON AXIS E.F. LAY THE EDGE ACROSS THIS POINT AND THE GIVEN TEMPERATURE ON AXIS GK, AND READ THE TOTAL HEAT WHERE IT CUTS THE AXIS L.M.

WHEN THE GIVEN TEMPERATURE FALLS ABOVE THE GIVEN PRESSURE ON AXIS AB, THE STEAM IS SUPERHEATED; WHEN IT FALLS BELOW IT, THE STEAM IS SUPERSATURATED

DESIGNED BY
W. J. GOUDIE.
DEC. 1917.

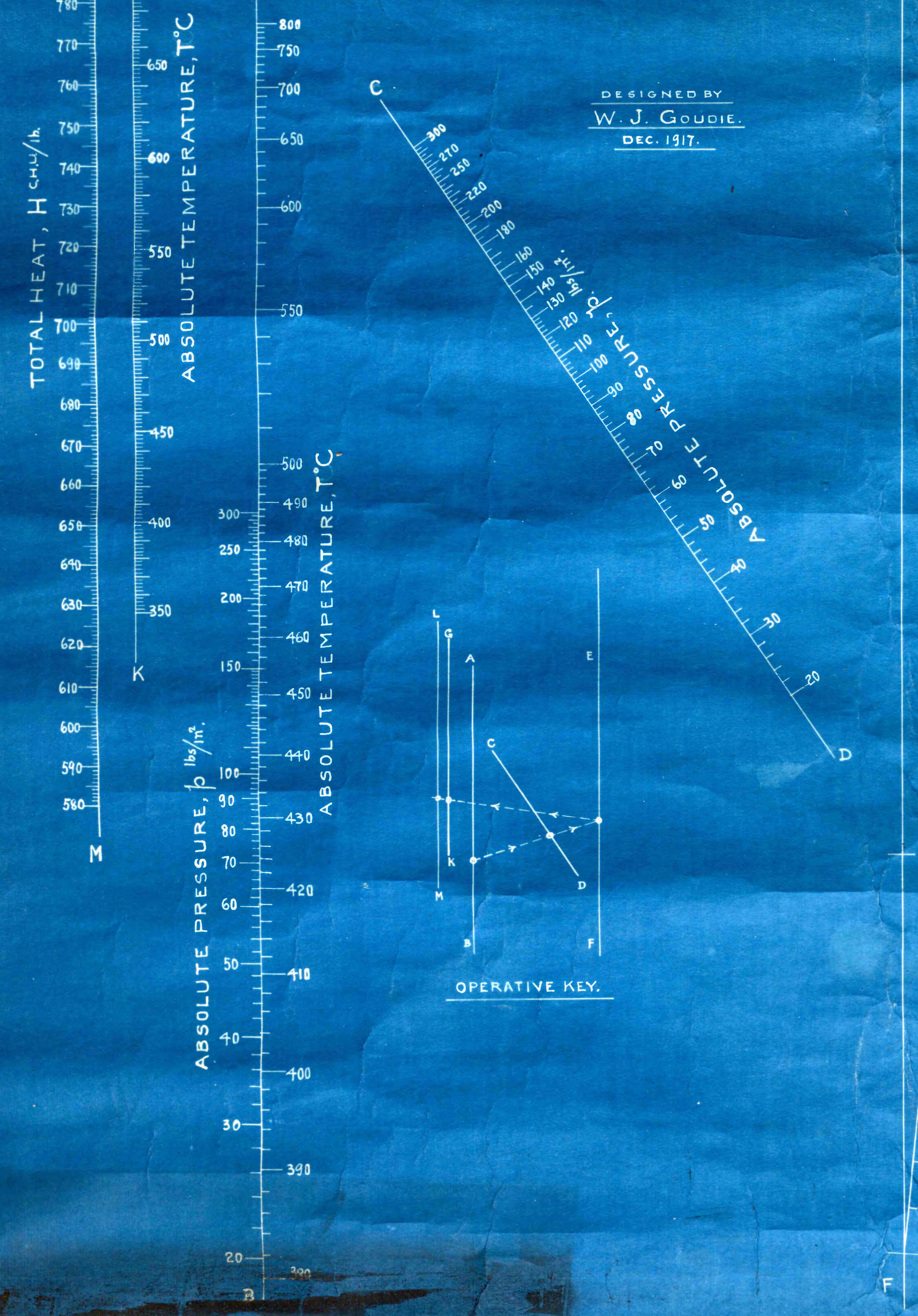


Fig. 4.

NOMOGRAM

FOR CALCULATION OF

TOTAL HEAT

OF

DRY & WET STEAM

THIS NOMOGRAM SOLVES THE EQUATION

$$H_w = H - (1-q)L$$

H = TOTAL HEAT OF DRY STEAM AT p lbs/in²
L = LATENT " " " " " " " "
H_w = TOTAL " " " WET " " " " "
q = DRYNESS FRACTION

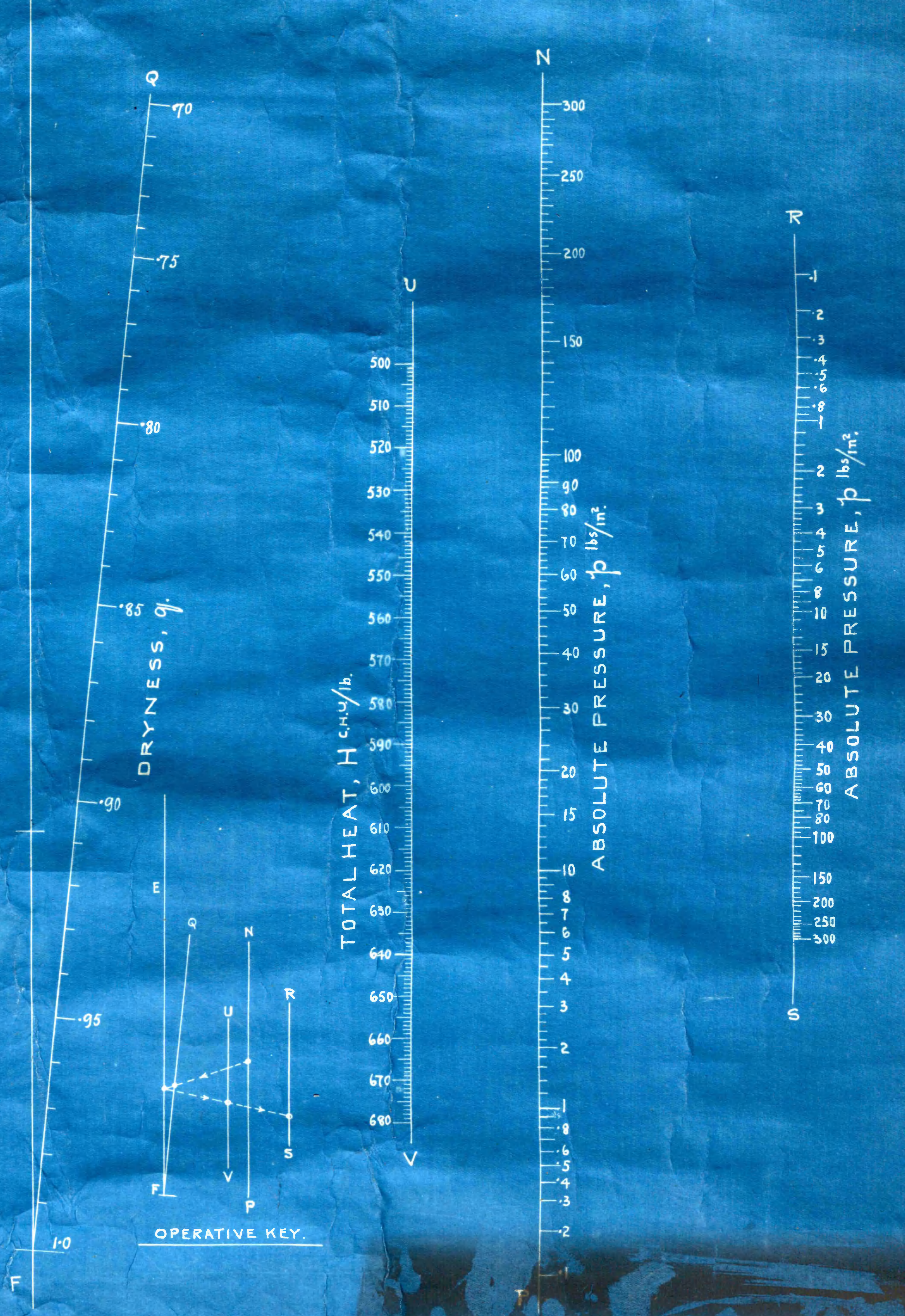
OPERATIVE RULE

DRY STEAM

LAY A STRAIGHT EDGE ACROSS UNIT DRYNESS ON AXIS FQ, AND THE GIVEN PRESSURE ON AXIS RS, AND READ THE TOTAL HEAT WHERE IT CUTS THE AXIS U.V.

WET STEAM

LAY A STRAIGHT EDGE ACROSS THE GIVEN PRESSURE ON AXIS NP, AND DRYNESS ON AXIS FQ, AND MARK THE POINT OF INTERSECTION ON E.F. LAY THE EDGE ACROSS THIS POINT AND THE GIVEN PRESSURE ON AXIS RS, AND READ THE TOTAL HEAT WHERE IT CUTS THE AXIS U.V.



Matter for D Sc Thesis

These two sheets of Illustrations returned on 2nd May 1919, are re-submitted, in accordance with the request contained in the Clerk of Senate's letter of 15th May 1919.

W. J. Goudie

19th May 1919

The Illustrations refer to the Supplementary note to Chapt V of the Steam Turbine Book.