

STRUCTURAL JOINT CHARACTERISTICS.

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AN EXPERIMENTAL INVESTIGATION
OF THE PROPERTIES OF MULTI-ROW JOINTS
UNDER STATIC LOADING
AND IN VIBRATION.

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by

H. L. McBROOM, B.Sc., A.R.T.C.

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PART I.

GENERAL REVIEW.

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REVIEW OF EXPERIMENTAL WORK.

The process of riveting has been employed since the earliest days of engineering, and it is to be expected that the bibliography of the subject should be an extensive one. Tests on riveted joints have been numerous, but despite this, it would appear that certain aspects of their behaviour under load still remain obscure. With regard, for example, to the friction grip inherent in such a method of construction and the manifestation of "slip" which must naturally be associated with friction, the experimental evidence and the opinions advanced by certain investigators are in definite conflict.

The uncertainty existing in the present knowledge of the properties of riveted joints is reflected in the nature and scope of the extensive researches at present being conducted by the "Steel Structures Research Committee". These include the experimental study of the extent of the rigidity of joints, the distribution of stresses among a series of rivets, the comparative behaviour of rivets, fitted bolts and black bolts, the effect of vibration on black bolted connections, and generally the exact part played by friction and shear.

RIVET TENSION.

Before summarising the diverse opinions expressed by various experimenters, it may be advisable to consider the/

the evidence provided by recent developments in connection with the application of tensile loads to rivets. Until a few years ago, a clause commonly occurring in specifications stated that rivets should not be used in positions where they were subjected to tension. In many countries, modification of this clause has resulted from tests carried out in Germany, Canada and America, demonstrating conclusively that the strength of a rivet in tension is equal to the normal ultimate strength of the material of the rivet. These tests have been supplemented by extensometer measurements showing that, as the applied rivet tension is gradually increased, no extension of the rivet occurs till a load value is reached comparable with that given by German investigators as the estimated tension in a rivet due to cooling. After this load is reached, extension is normal.

These tests provide definite evidence of a cooling tension in the rivets, necessarily accompanied by an equal pressure between the plates or elements of the joint. The application of an external tensile load to the rivet merely relieves the pressure between the plates without affecting rivet tension, so long as the applied load does not exceed the cooling tension in the rivet. In association with this evidence of pressure between the plates, the simple laws of friction insist on the existence in any joint of a definite slip resistance which may, or may not, be exceeded/

exceeded by the working load for the joint.

One of the earliest expressions of opinion on the subject of friction in riveted joints was made by Considère, and subsequently endorsed by Sir Benjamin Baker, as follows, "That in all constructions in which riveted portions have "not already commenced sliding, the rivets hold solely by "longitudinal tension; and the adherence to which this "tension gives rise between the plates constitutes the "sole resistance". This view is, of course, in harmony with the simple friction considerations already outlined, and experimental evidence in support of it is not lacking.

MONTGOMERIE'S EXPERIMENTS.

Any attempt to analyse the mass of published experimental data leads inevitably to the rejection of a large amount of it on account of lack of refinement in strain measurement; but the results of a series of tests carried through in 1919 by Dr. James Montgomerie¹ on behalf of the Committee of Lloyd's Register of Shipping are worthy of note. The test specimens were full scale treble riveted and quadruple riveted lap joints using 0.44" and 0.58" plates with $\frac{3}{4}$ " and $\frac{7}{8}$ " diameter rivets respectively. They were extended in a testing machine, and an accurate form of strainmeter was used.

Generally, the results indicated that up to working loads no slip occurred between the plates. The joint/

joint behaved as an elastic solid, the load was transmitted entirely by the friction grip arising from the tension in the rivets, and the rivets were not subject to shear.

Slip observations on similar joints with different numbers of rivets showed that the slip load was proportional to the number of rivets and gave in all cases a nominal shear stress on the rivets of $7\frac{1}{2}$ - 8 tons/in². As a result of the distortion of the rivet holes, the further statement was made that the outer rows of rivets carried a much higher proportion of the load than the inner rows.

This investigation was subsequently extended by the same author², using $\frac{3}{4}$ " and 1" plates with 1" and $1\frac{1}{8}$ " rivets respectively. With each increase in plate thickness slip occurred at a lower rivet stress value, but the experimental evidence gave definite indication that this was due to the tremendous difficulty of securing uniform contact between the plates. Special precautions in making one joint in 1" plates were successful in increasing the rivet nominal shear stress at slip from 5 to 7 tons/in², the latter figure being comparable with the value obtained from thin plates.

REED'S EXPERIMENTS.

It is interesting to note that exactly similar results were obtained from experiments carried out at Pembroke Dockyard³ in 1869. In comparison with the experiments already described, the method of slip detection/

detection was decidedly lacking in sensitivity, visual indication being obtained by an ordinary magnifying glass. This probably accounts for the slightly higher values quoted in all cases for the rivet shear stress at slip. Slip load was shown to be proportional to the number of rivets, and the rivet shear stress at slip loads decreased as the plate thickness increased.

BATHO'S EXPERIMENTS.

The foregoing evidence suggests that riveted joint behaviour may be explained by the existing simple laws of friction, yet mathematical theories developed in connection with riveted joints neglect friction entirely and assume rivet deflection proportional to rivet load. Such theories are given by Batho⁴ and by Hovgaard⁵, and the experimental work carried out by the former in testing the accuracy of his analysis provides conclusions in marked contrast to those adduced above. The test specimens were multi-row double butt strap joints using only a single line of rivets. Strain measurements were made on the outside of the cover plates by an extensometer measuring accurately to 0.00001".

The results show that extensometer readings taken on the outside of the cover plate give accurate indications confirming the calculated load partition among the various rivets, and from this the statement is made, "If there is any frictional hold between the plates, it acts only over those/

"those portions in the immediate neighbourhood of the rivets. "All the experiments tend to show that friction does not "play an important part". An additional feature of the strain measurements which would appear to support this view arises from the strain being minimum in the line of the rivet centres, indicating that the deformation of the holes is not prevented by the frictional grip of the rivet heads. Still further experimental support is claimed at a later date by Batho from investigations carried out in Germany.

GAYHART'S EXPERIMENTS.

Another research programme commanding attention has been carried out in America by E. L. Gayhart⁶ for the U.S. Bureau of Construction and Repair of the Navy Department. Accurate strain measurements were made to determine load partition, and the results may be summarised as follows. With medium steel, slip load is proportional to the number of rivets. Inferior slip resistance is shown by joints with four or more rows of rivets, by unsymmetrical joints, and by high tensile or special steel plates. For low plate stresses, load partition depends on frictional resistance; but the frictional grip fails at such low stresses that for practical purposes load partition depends on the bearing and shear resistance of the rivets.

CRITICAL SUMMARY.

The experimental results described above present contrasts/

contrasts that are almost self-evident and require little emphasis. One range of experiments indicates that no slip whatever occurs in riveted joints at working loads; also the outer rows carry the greater proportion of the load. If the first statement be accepted, then the second overlooks the obvious fact that with a rigid friction grip the outer rows would carry the whole load.

One other point is disregarded. Since slip load is stated to be proportional to the total rivet tension, it would appear that simple friction considerations apply. If so, since the outer rows carry the greater proportion of the load, slip must occur at the outer rows as a preliminary to slip of the joint as a whole. No accurate indication seems to have ever been obtained of this "partial" slip; but its existence is suggested by a statement of Professor Haigh's that, "Riveted joints are liable to show "permanent set under very moderate loads".

The other range of experiments provides similar conclusions regarding load partition, but suggests that friction plays no important part in the transmission of the load. In addition to being in direct opposition to other results from similar experiments, the latter statement is at variance with the existing evidence of a definite cooling tension in rivets.

In riveted joint literature, numerous other test results are available. Those quoted have been selected as typical/

typical and representative. They show a curious diversity of opinion associated with a form of construction that can only be regarded as essentially simple and elementary.

The persistence in design calculations of methods which are generally admitted to be erroneous can be ascribed largely to the uncertainty arising from these results, the contradictory features of which have effectively prevented the adoption of a more rational procedure.

It would appear that the simplicity of construction offers a marked contrast to the difficulties of experimental investigation. The same suggestion is contained in the one statement wherein all research workers in this subject attain unanimity, to the effect that "further experimental work is required". Despite these difficulties, and making due allowance for the fact that present methods of design have led to no serious troubles, the existence of such confusion is regrettable and can only be regarded as a serious reflection on the standard of research in certain branches of engineering.

REVIEW OF THEORY.

The existing mathematical theories developed by Batho and Hovgaard in connection with riveted joints are already noted. In Batho's theory, the presence of friction in the joint is ignored and the strain energy in a rivet is/

is assumed proportional to the square of the load on the rivet. Consideration will show that this is equivalent to the assumption that the relative deflection of the plates under a rivet is proportional to the load on the rivet.

An expression for the total strain energy in the joint under these conditions is obtained, and the principle of minimum energy is applied. Differentiating the total energy with respect to each unknown rivet load gives a number of simultaneous equations corresponding to the number of rivet rows in the joint. The solution of these is left to be completed in each particular case.

Hovgaard's solution is more suitable for application to welded joints or to riveted joints where the number of rivet rows is very large. At any point in the length of the joint the relative displacement of the plates is assumed to be proportional to the shear stress in the rivets or weld, and friction is neglected. The subsequent analysis is much more complicated than Batho's. An expression for the total strain energy in the joint is obtained, and is differentiated to find the condition for minimum energy. The solution of the resulting equation requires recourse to the Calculus of Variations, and finally an equation is developed giving the shear stress at any point in the joint.

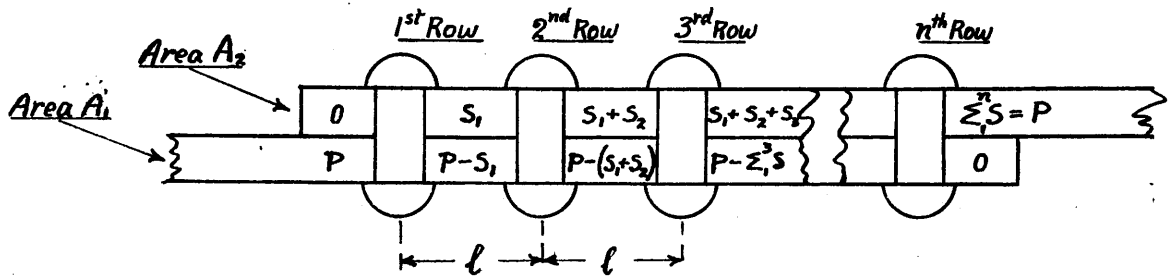
Both theories are needlessly involved. The employment of advanced mathematical methods in a straightforward investigation may be justifiable when definite/

definite advantages accrue in shorter developments and more convenient solutions; but otherwise their use only obscures the fundamental nature and simplicity of the problem, and is to be deplored. It is shown below that, starting with the same assumptions, Batho's results can be written down by using nothing more involved than the simple stress-strain relationship. Further, Hovgaard's "new theory" can be developed in exactly the same way, and differs from Batho's only in the presentation of the results as a continuous function instead of a number of simultaneous equations.

A better understanding of riveted joint action based on the subsequent experimental work would render the theory obsolete. It is presented here simply to demonstrate the similarity of the two methods, and because in the form of a continuous function the solution may still be applicable to a number of practical cases of welding construction.

SHORT RIVETED JOINTS.

The sketch shows the loads in the various sections of a multi-row riveted joint with "n" rows of rivets. Friction between the plates is neglected, and the load transmitted by any rivet is assumed proportional to the relative displacement of the plates at that point.



i.e. if rivet load = S ,

then rivet deflection = relative movement of plates
= kS

where "k" is a numerical constant.

Between the first and second rows of rivets,

$$\text{Extension of plate 1} = \frac{(P - S_1) l}{A_1 E}$$

$$\text{Extension of plate 2} = \frac{S_1 l}{A_2 E}$$

$$\text{Deflection of rivet 1} = k S_1$$

$$\text{Deflection of rivet 2} = k S_2$$

and the principle of consistent deflections gives the equation

$$\frac{(P - S_1) l}{A_1 E} - k S_1 + k S_2 = \frac{S_1 l}{A_2 E}$$

Similarly, between the second and third rows,

$$\frac{\{P - (S_1 + S_2)\} l}{A_1 E} - k S_2 + k S_3 = \frac{(S_2 + S_3) l}{A_2 E}$$

or/

or, generally,

$$\frac{(P - \sum_1^p S) l}{A_1 E} - k S_p + k S_{p+1} = \frac{(\sum_1^p S) l}{A_2 E}$$

where "p" is any integer between 1 and (n - 1).

This gives (n - 1) equations for the solution of the unknowns

S_1, S_2, \dots, S_n .

The nth equation is obviously

$$\sum_1^n S = P$$

The general equation may be written, if preferred, as

$$P - \sum_1^p S - K(S_p - S_{p+1}) = C \sum_1^p S$$

$$\text{where } C = \frac{A_1}{A_2} \text{ and } K = \frac{k A_1 E}{l}$$

or finally,

$$P - (1 + C) \sum_1^p S - K(S_p - S_{p+1}) = 0$$

and $P = \sum_1^n S$

If $A_1 = A_2$, and the joint is symmetrical in every other respect, only half of it need be considered.

$$\text{Then } P - 2 \sum_1^{n/2} S - K(S_p - S_{p+1}) = 0$$

$$\text{where } K = \frac{A_1 E}{l}$$

and "p" is an integer varying from 1 to ($\frac{n}{2} - 1$).

The additional equation to make a total of $\frac{n}{2}$ is obviously

$$\sum_1^{n/2} S = \frac{P}{2}$$

If/

If "n" is an odd number, then as above,

$$P - 2 \sum_1^n S - K(S_n - S_{n+1}) = 0$$

where "p" has now any value from 1 to $(\frac{n-1}{2})$

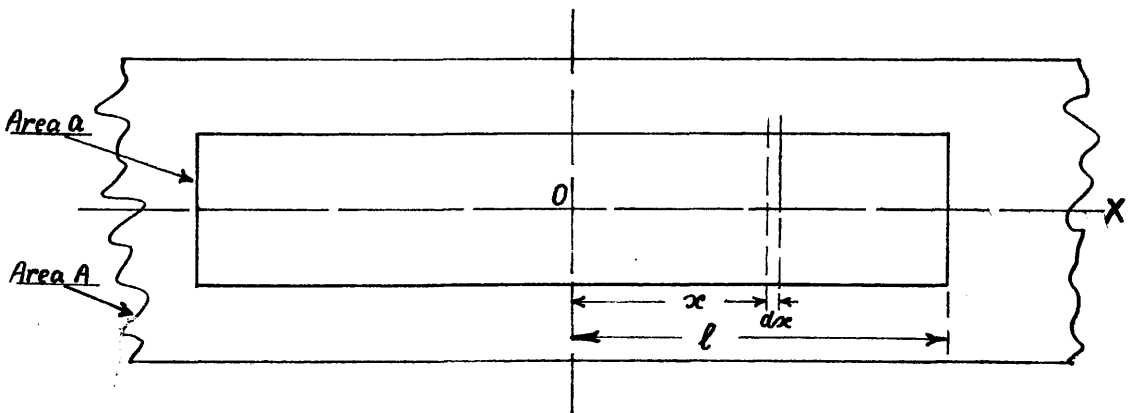
and the additional equation is

$$P - \sum_1^{\frac{n-1}{2}} S = S_{\frac{n+1}{2}}$$

= load on central row.

WELDED OR LONG RIVETED JOINTS.

A plate of area A , under a stress f , has a bar of area a attached to it by welding or riveting. Friction is neglected, and it is assumed that the relative



displacement of bar and plate at any section is proportional to the shear stress in the weld or rivets at that section.

Instead of considering the deformations occurring in a rivet pitch length as in the previous case, attention is confined to an infinitely short length dx distant x from/

from an origin, 0 at the centre of the joint.

Let α = sectional area of weld per unit length,

q = shear stress in weld at section x ,

μq = displacement of bar relative to plate at x ,

μ being a "displacement coefficient".

$$\text{Extension of length } dx \text{ of bar} = \frac{dx}{a E} \left(\int_x^l q \cdot \alpha \cdot dx \right)$$

$$\text{Extension of length } dx \text{ of plate} = \frac{dx}{A E} \left(p A - \int_x^l q \cdot \alpha \cdot dx \right)$$

$$\text{Extension of length } dx \text{ of weld} = \mu \cdot dq.$$

The principle of consistent deformations gives the equation

$$\mu \frac{dq}{dx} = \frac{1}{A E} \left(p A - \int_x^l q \cdot \alpha \cdot dx \right) - \frac{1}{a E} \left(\int_x^l q \cdot \alpha \cdot dx \right) \dots (1)$$

which on differentiation, since $\frac{d}{dx} \left(\int_x^l q \cdot dx \right) = -q$, gives

$$\begin{aligned} \frac{d^2 q}{dx^2} &= \frac{1}{\mu E} \left(\frac{\alpha}{A} q + \frac{\alpha}{a} q \right) \\ &= \frac{\alpha (A + a)}{\mu E A a} q \end{aligned}$$

$$\text{or} \quad \frac{d^2 q}{dx^2} - n^2 q = 0$$

$$\text{where } n^2 = \frac{\alpha (A + a)}{\mu E A a}$$

This is a standard type of differential equation, for which the well known and easily verified solution is

$$q = A \cosh nx + B \sinh nx$$

The known condition that $q = 0$ when $x = 0$ gives $A = 0$

The further condition that $q = Q$ when $x = l$ gives $Q = B \sinh nl$

$$\text{or } B = \frac{Q}{\sinh nl}$$

$$\text{Hence } q = Q \frac{\sinh nx}{\sinh nl}$$

To evaluate Q , use the particular form assumed by equation (1) when $x = l$, i.e.

$$\mu \frac{dq}{dx} = \frac{p}{E}$$

Substituting for q , or rather for $\frac{dq}{dx}$, gives

$$\frac{\mu n Q \cosh nl}{\sinh nl} = \frac{p}{E}$$

$$\text{or } Q = \frac{p}{\mu n E} \tanh nl$$

$$\text{and therefore } q = \frac{p}{\mu n E} \frac{\sinh nx}{\cosh nl}$$

If desired, the stress in the bar may now be written

$$\begin{aligned} p_x &= \frac{1}{a} \int_x^l q \cdot \alpha \cdot dx \\ &= \frac{\alpha}{a} \cdot \frac{p}{\mu n^2 E} \left(1 - \frac{\cosh nx}{\cosh nl} \right) \end{aligned}$$

and the stress in the plate becomes

$$p'_x = p - \frac{a}{A} p_x$$

The/

The analysis may be extended to cases where the stress f varies along the length of the plate. If f can be expressed as a function of x , the solution of the resulting differential equation may be completed without difficulty by the inclusion of a particular integral.

The method used in the above analysis will be found to be much more elementary, much shorter and simpler in every respect than Professor Hovgaard's, the final equations being exactly the same. Provided the elastic properties of welds conform to the assumption made regarding the displacement (the experimental supporting evidence is rather scanty), the equations may be applied to numerous cases of attachment of brackets and similar fittings by welding.

PURPOSE OF PRESENT INVESTIGATION.

The preceding review indicates that further research work must aim at the elimination of the controversial aspects of the subject and the establishment of sound methods of design. It becomes necessary to establish more definitely the exact effect of friction in multi-row joints, to investigate the possibility of slip at the various rows, and to define the resulting load distribution.

Brief consideration of the volume and standard
of/

of the work already executed makes it clear that, for effective extension, the development of new methods may be regarded as a necessity. The static tests described in Part II constitute an attempt, by utilising a new method, to throw further light on the obscure features of joint behaviour, and to find a direction in which further experimental work may be profitably prosecuted.

The universal use of riveted and bolted joints renders it impossible to avoid situations where such connections are subject to load fluctuations which may be either unidirectional or reversible. In certain cases, a knowledge of the frictional damping influences exerted by such joints during vibration would undoubtedly be advantageous; but in the investigation of vibration phenomena generally, experimental and analytical difficulties abound, and it is not altogether surprising to find that no data whatever are available with regard to frictional damping influences in individual joints.

The vibration experiments of Part III were initiated primarily with the intention of throwing further light on the aspects of joint friction demonstrated by the static tests, but also with the hope of providing definite information regarding the nature and magnitude of the damping forces arising from the presence of such joints in a vibrating structure.

PART II.

JOINT INVESTIGATIONS BY STATIC LOADING.

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EXPERIMENTAL METHODS.

CONTROLLING FACTORS.

In almost any contemplated investigation, general considerations indicate the desirability of conducting preliminary experiments on a small scale if possible. These preliminary attempts will in many cases elucidate the points requiring attention, and the mode of action and type of results to be expected. The most direct and economical full scale work may then follow.

In designing the apparatus described below, regard had to be paid to the fact that it was intended to supplement a series of static measurements by subsequent vibration tests. The problems and difficulties associated with the latter are detailed elsewhere; but generally it may be assumed that very small vibrations lead to difficulty in measurement, while the production of vibrations involving full scale riveted joints would require heavy and expensive plant.

A certain amount of guidance may be derived also from the experiences of previous experimenters. Practically all previous efforts to demonstrate riveted joint properties have been made on what may be termed full scale joints, and the experimental procedure has emphasised certain difficulties which might possibly be wholly or partly eliminated by adopting alternative constructional arrangements.

One of the most prominent relates to the use of simple joints, in which the bending effects due to eccentric loading are so large that little importance can be attached to many of the other aspects of the results. The assumption of uniform stress distribution across the width of a joint may be badly upset by the uncertain effects of distortion and slip in the grips of the testing machine. In order to eliminate this latter effect as far as possible, it was found necessary in Batho's experiments to remove and reapply the load several times before taking strain readings, a condition that can only be characterised as most undesirable. Also, in a heavy testing machine with a beam weighing several tons, where slip observations are concerned the inertia of the beam may be sufficient to produce irregularities in the readings large enough to obscure important features of the results.

SUBSTITUTION OF BOLTS FOR RIVETS.

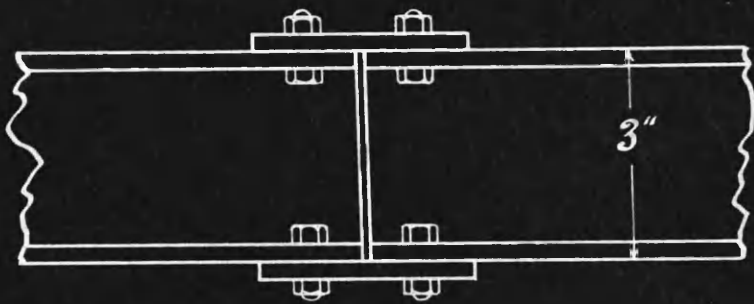
In considering the possibility of using small scale joints, it has to be admitted that with very small rivets the excessive rate of cooling, and the liability to bend or buckle, render it doubtful whether such rivets will exhibit the same characteristics as the larger sizes. In such circumstances the use of bolts in place of rivets provides an attractive alternative. Whereas the use of rivets means that for the investigation of any one feature of joint action a large number of similar specimens is necessary/

necessary, the use of bolts allows a single specimen to be tested, dismantled and reassembled for subsequent tests, so long as overstraining is avoided.

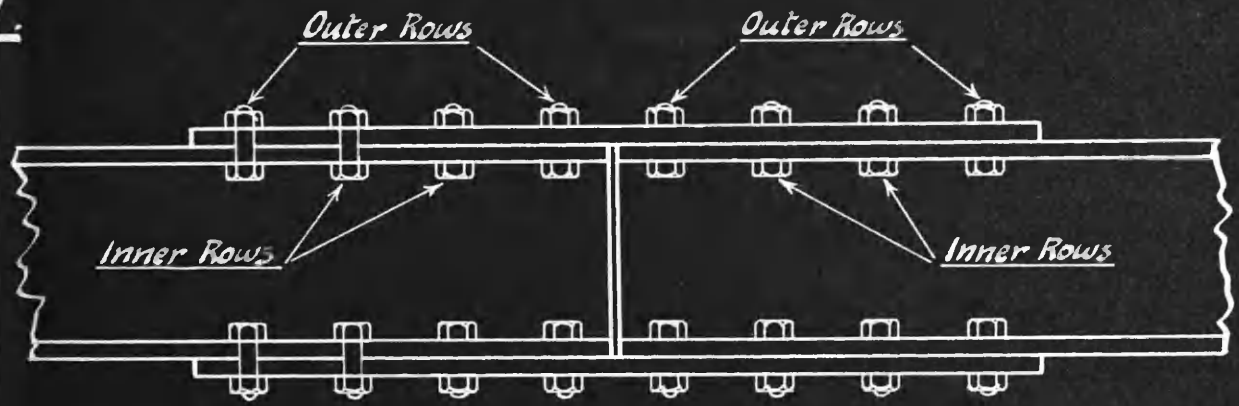
The necessary justification for the use of bolts in place of rivets has been provided by Professor Batho who, from a comprehensive series of tests on joints with rivets, fitted bolts and black bolts, makes the following statement⁷, "Turned and fitted bolts behave very much like rivets, except for differences arising from the smaller initial tension; with black bolts, slip naturally becomes of importance".

The principal source of doubt regarding the use of bolts in an investigation intended to refer to riveted joints is in the common belief that a rivet fills the rivet hole perfectly. In applying this idea to deflection calculations, the effect of the rivet holes on the moment of inertia is estimated for the tension flange only; the compression flange is assumed to be solid metal. Such doubts are effectively dispelled by an investigation made in the works of Sir William Arrol & Co. Specimens were riveted up and subsequently dissected by a fine milling cutter. The cutting of sections at right angles to the longitudinal centre line of the rivet revealed that in all cases the part of the rivet between the sections conformed to the usual idea of a push fit, i.e. easily moved axially but with no excess clearance, the actual diametral clearance/

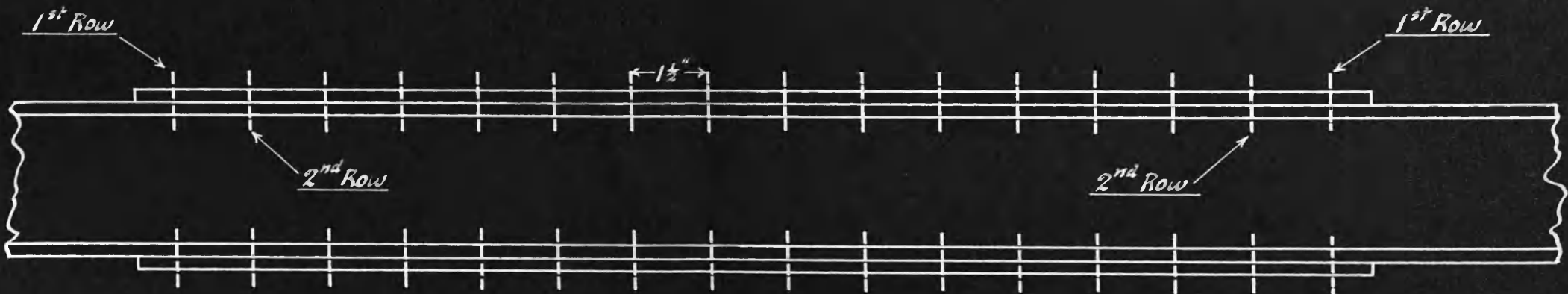
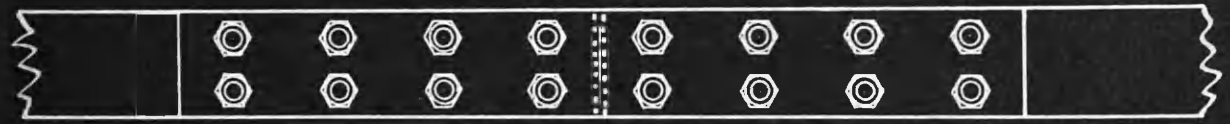
FIG. 1.



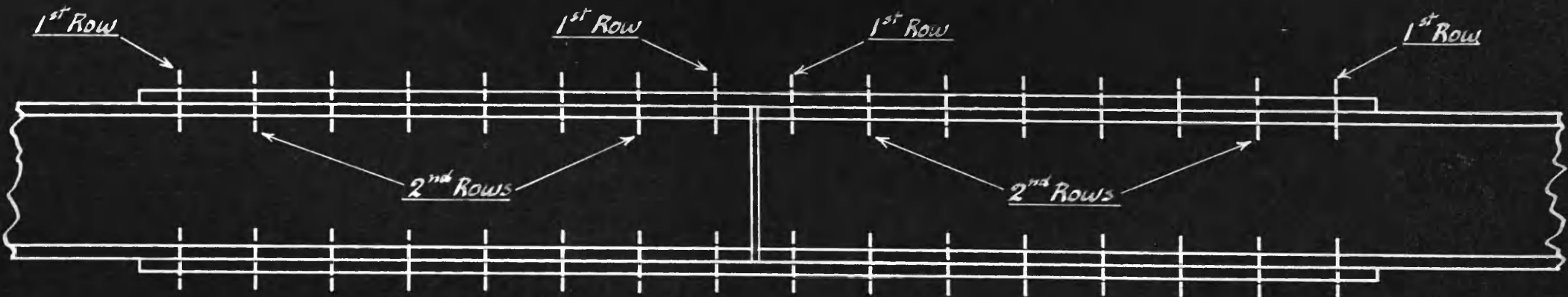
Single Row Joint



Four Row Joint



Added Flange Plates



Eight Row Joint

clearance being approximately .001".

CONSTRUCTIONAL DETAILS.

The above considerations have resulted in the unusual features of the following joint tests. The joints are made in the flanges of a standard rolled beam section by means of cover plates, and are loaded by bending the beam. The distortion which usually occurs in a simple lap joint under tension is thereby avoided. The construction of the test joints is shown in Fig. 1. It will be observed that the "single row" joint is really an arrangement of four independent and exactly similar single row lap joints between flange and cover plate; and slip in any one of these produces a beam deflection which is superimposed on the ordinary deflection due to bending.

Obviously the actual joint slip is greatly magnified, and the span of the beam is chosen so that the central deflection due to joint slip is large enough for easy and accurate measurement. The accuracy is in no way affected by the fact that this deflection is associated with a much larger deflection due to bending, since in the analysis of the results the two deflections are not separated. An additional feature of the construction lies in the application of tensile and compressive loads to the joints. These may be analysed separately, if desired, by cutting only one flange (and the web) of a beam and jointing the other.

The four row and eight row joints are similarly arranged and require no special description. The remaining specimen consists of a complete beam with added flange plates. Actually the last specimen was tested first, and by cutting the beam through the centre the eight row joint was formed.

In regarding each individual joint in any one beam as a simple lap joint between two plates, the beam is visualised as an arrangement of two flange plates under simple tension and compression respectively. The effect of the web is small, and may be accounted by a small allowance to be included in the "equivalent area" of each flange. In the multi-row arrangements, complete symmetry is attained as nearly as possible by machining the inside of each flange (at the joint only) to such dimensions that the equivalent area of each flange is the same as the area of the cover plate. This machining, incidentally, provides also a flat seating for the bolt heads. To ensure that each joint carries the total flange load, the ends of the beam are not butted.

BOLT TENSIONING.

In all cases, the tensioning of the bolts is performed by a specially constructed spring-loaded "spanner" which applies a definite torque to the nut. Its operation will be understood by referring to Fig. 3. The spring is set/

FIG. 2
Static Loading Arrangement

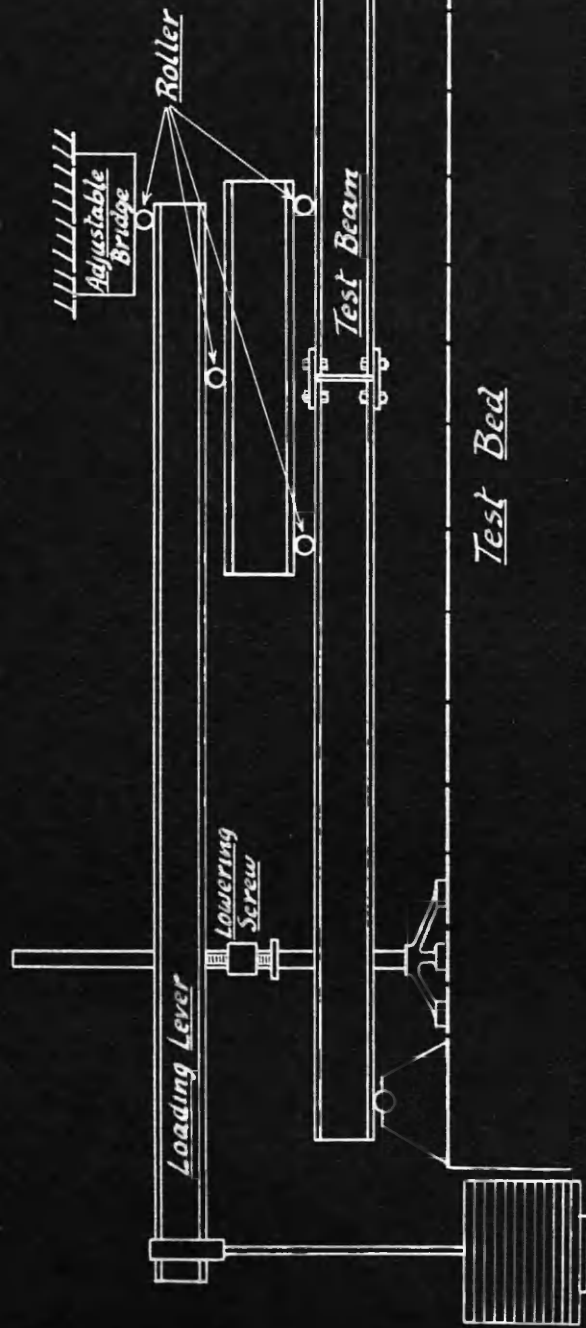
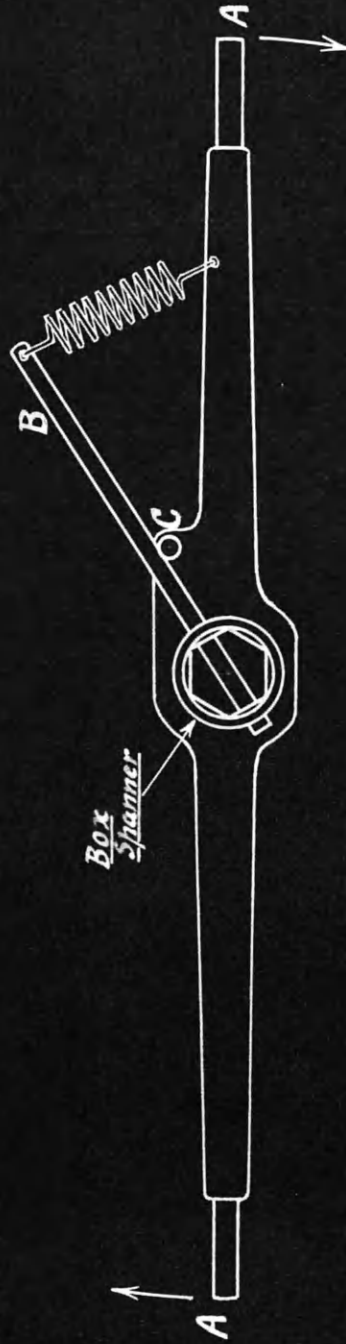


FIG. 3

Bolt Tensioning Device



set to a predetermined tension, which merely pulls the bar B against the stop C. When the box spanner is applied to a nut, the effort at A is gradually increased till the bar just leaves the stop and the full torque produced by the spring is applied to the nut. The use of finished bolts, nuts and washers, ensures that variations in the resulting tension shall be small, and the effect of such variation is further reduced by the use of two bolts per row.

TEST PROCEDURE.

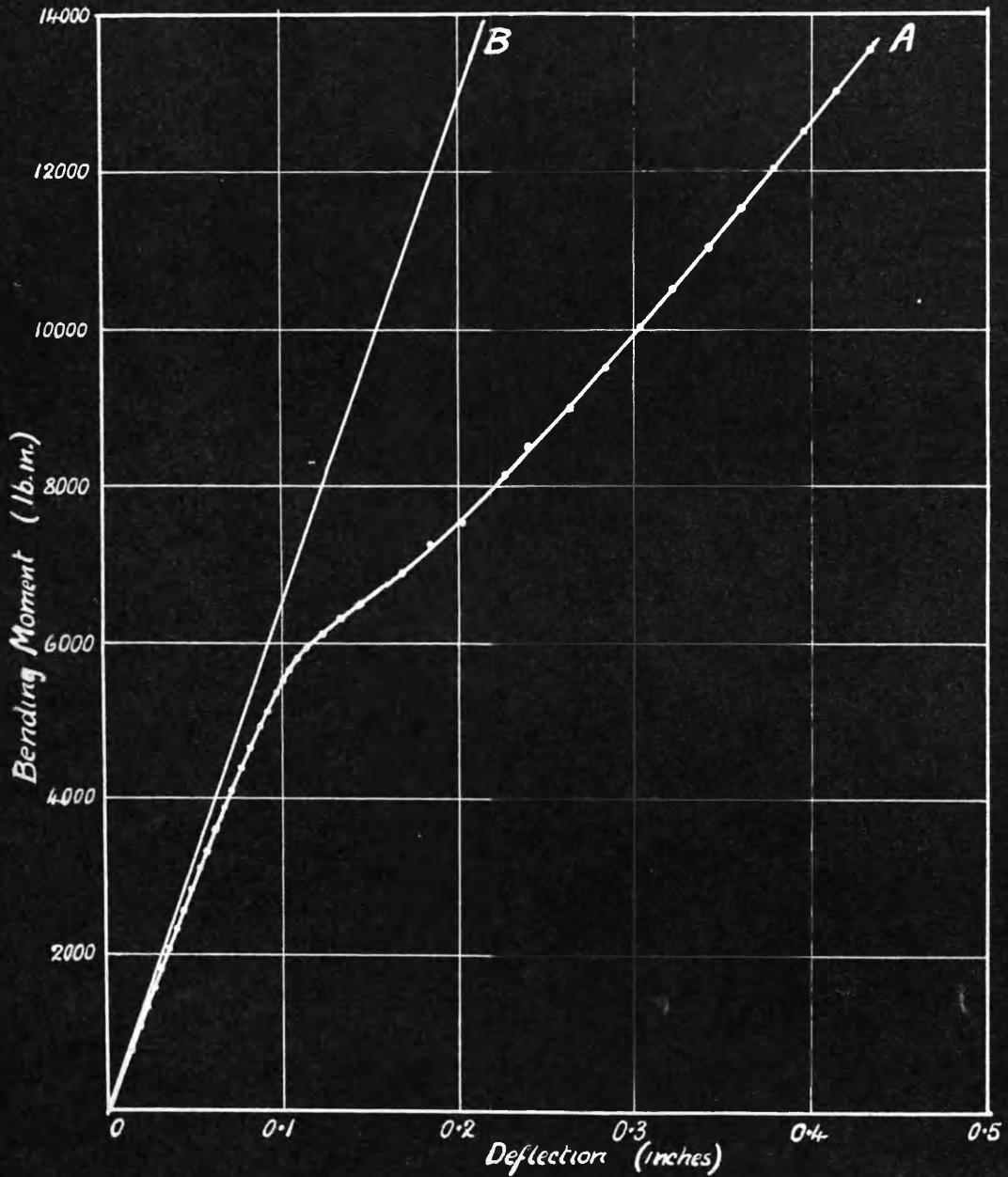
In carrying out the experiments, the joints were arranged at the centre of a simply supported span of 8 feet, and the loading was applied by deadweights on a simple lever as indicated in Fig. 2. It will be observed that over the length of the joint the bending moment is constant and the shearing force zero. Dynamic effects were avoided by using a fine pitch screw to lower the lever to its equilibrium position, and after each deflection reading the lever was returned to the horizontal by a screw adjustment of the height of the bridge. The beam deflections were measured at the centre of the span by an ordinary internal hand micrometer, estimating to the nearest tenth of a division, i.e. to .0001".

RESULTS/

FIG. 4.

A - Single Row Joint.

B - Jointless Beam.



RESULTS.

SINGLE ROW JOINT.

The study of the behaviour of a single row joint is an obvious preliminary to an investigation of the properties of more complicated constructions. The characteristics of a single row joint, in which the bolts had a diametral clearance in the holes of .002", are shown in Fig. 4 by plotting joint bending moment on a base of central beam deflection.

During the experiment it was noted that beyond a bending moment value of 5500 lb.in. the occurrence of bodily slip in the joints was marked by a jerky deflection of the loading lever after each increase in load, and an additional indication was provided by the appearance of a creep effect with time. The slip is clearly shown in the graph by the sudden increase in the rate of deflection; it continued till bolt clearance was taken up, and thereafter the new slope of the graph gives evidence of bolt deflection due to shear. It is interesting to note that the applied bending moment was more than sufficient to shear all the bolts, yet due to friction in the joints the bolts were quite unmarked.

For comparison, the graph obtained by similarly loading an unjointed beam is given on the same diagram; it is, of course, a straight line. It will immediately be/

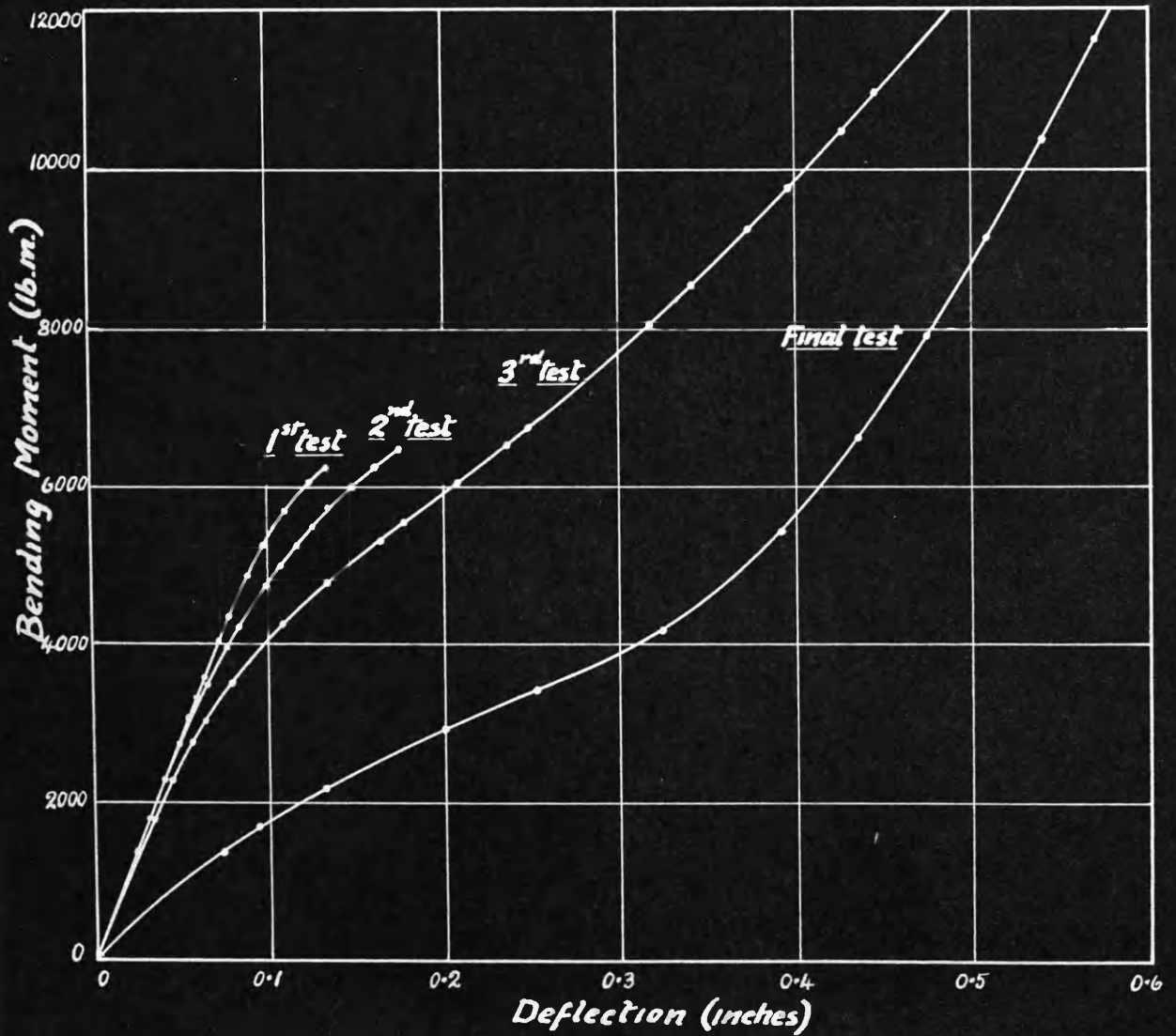
be observed that, even below slip load, the joint characteristic exhibits a slight but definite curvature. The fact that the pressure exerted by the bolts is not uniformly distributed and therefore gives rise to stress concentration seems hardly a sufficient explanation of this feature. In addition, it would seem to indicate the gradual breakdown of the friction grip, with the consequent development of the effect of the bolt holes on the deflection, before actual bodily slip takes place. The stress (or strain) in the flanges gradually "eats in" under the bolts as the load is increased, till it finally spreads right through at slip.

This effect was clearly demonstrated by subsequent tests which showed the slight curvature below slip to be due to friction. The application and removal of any load less than the slip load actually produced a small but easily measurable permanent set in the beam, though no overstraining of any kind was present. Reapplication of the load produced the same total or absolute deflection as was obtained by the first application.

EFFECT OF SLIP ON BOLT TENSION.

One other aspect of this test is deserving of notice, being a point which may possibly have considerable practical significance, though the effect has been eliminated from the remaining tests of this series. At the conclusion of the test, the bolts were found to have lost a considerable proportion/

Fig. 5.
Effect of Surface Oxide



proportion of their initial tension. Retensioning them required a rotation of the nut amounting to almost 10° . Opening the joints disclosed the presence on the surface of the beam of a quantity of black powder due to the mechanical disintegration of the film of oxide with which all rolled surfaces are covered. Removing the powder and reassembling the joint led to exactly the same test result, and once again the bolts were found to have lost tension though not to the same extent.

The effect was further investigated by preparing a fresh joint in a similar beam and subjecting it to test. The load was increased till slip occurred, then removed. The beam was inverted and reloaded till slip in the reverse direction occurred. A third slip was produced by loading once more in the original direction. Thereafter the beam was subjected to eight more unrecorded slips, i.e. four in each direction, and a further final test was noted.

The results are shown in Fig. 5, and the progressive disappearance of the friction grip is obvious. The final test shows the friction to be negligible, slip starting immediately the load is applied. The tension remaining in the bolts was found to be very small indeed, and a nut rotation of approximately 30° was necessary to restore it. Opening the joints displayed a quantity of loose black powder on the beam surfaces.

It/

It was found that retightening the bolts without dismantling the joints restored the friction grip evidenced by the first test. Consequently, since the effect of the presence of the powder appears to be negligible, the curves may also be taken to represent the effect of progressive reductions in bolt tension. Reduced tension causes a much more gradual development of slip, and the use of a smaller tension would probably have entirely obscured the important aspects of joint behaviour which are shown to perfection by the subsequent tests.

It might be imagined that in actual riveted joint construction the pressure applied during the process of closing a rivet would be sufficient to cause disintegration of the oxide film, but it must be remembered that the phenomenon of "pulling loose" is a recognised one in riveted joint practice. In these experiments the bolts were stressed to full working load. In fact, the process of tensioning them was frequently accompanied by fracture, especially if there was the slightest irregularity in the seating resulting in eccentric loading. It is therefore exceedingly doubtful if the pressure intensities existing in riveted joints are really any greater than in the joints under consideration.

Montgomerie's experiments, already discussed, showed that in riveted joints with plate thicknesses up to/
to/

to .6" slip occurred when the nominal shear stress on the rivets was 8 tons/in². The first tests on the present joints showed a joint bending moment at slip of 6000 lb.in., giving a load of 2000 lb. on each pair of bolts, or a nominal shear stress of nearly 9 tons/in². It must be concluded therefore that the occurrence of actual bodily slip in a joint may be a serious matter with thin plates, resulting as it does in the disappearance of the friction grip. The effect will become less marked with increasing plate thicknesses, and may even become negligible.

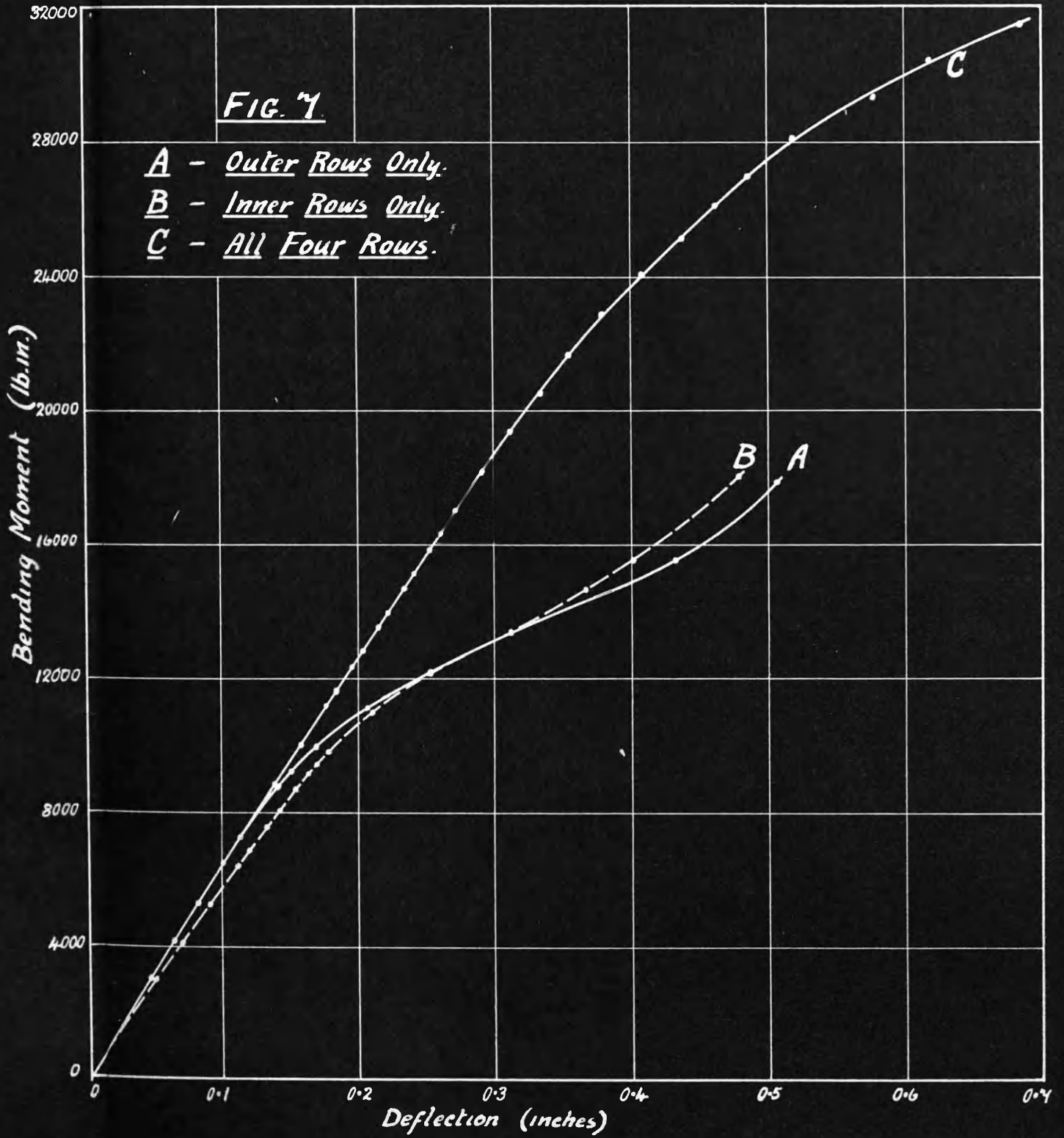
While the loss of friction grip may be an important practical aspect of the subject, it was not considered a desirable element in the present series of tests; and in all further tests it has been eliminated either by subjecting the joints to a sufficient number of slips or by removal of the oxide from the surfaces by scraping. This process naturally resulted in a somewhat lower friction coefficient, but in the effective demonstration of friction effects in joints that has proved no serious disadvantage.

EFFECT OF BOLT CLEARANCE.

It has been explained that, in using bolts in place of rivets, the only feature wherein the joints differ from the more practical riveted type is the magnitude of the clearance in the holes. In order that the characteristics displayed in the remaining tests may be related to riveted joints/

FIG. 7.

- A - Outer Rows Only.
- B - Inner Rows Only.
- C - All Four Rows.



joints, it will be desirable to show the effect of reducing the clearance. Fig. 6 shows the results obtained from a single row joint with the bolt clearance reduced as nearly as practicable to zero. The jerky deflection of the loading lever which occurred at slip in the first test was absent here, and the graph merely changed its slope when the friction grip broke down.

FOUR ROW JOINT.

Due to the symmetry produced by the equal (effective) areas of flange and cover plate, it will be understood that a two row joint should display the same features exactly as the single row joint, but the load values will all be doubled. This is confirmed in the next step of the development, the study of the four-row joint arrangement, the results from which are presented in Fig. 7.

Curve A is obtained with only the outer rows of bolts in position, and curve B with only the inner rows. Both curves therefore are for two-row joints, and in comparison with the single row joint results they show the expected similarity. The cover plates increase the moment of inertia of the section over the length of the beam between the bolts. This length is different in the two cases, consequently below slip loads the curves exhibit a difference in slope. The occurrence of slip in each case was marked during the experiment by jerky deflections of the loading lever, accompanied by sharp cracking noises.

Curve/

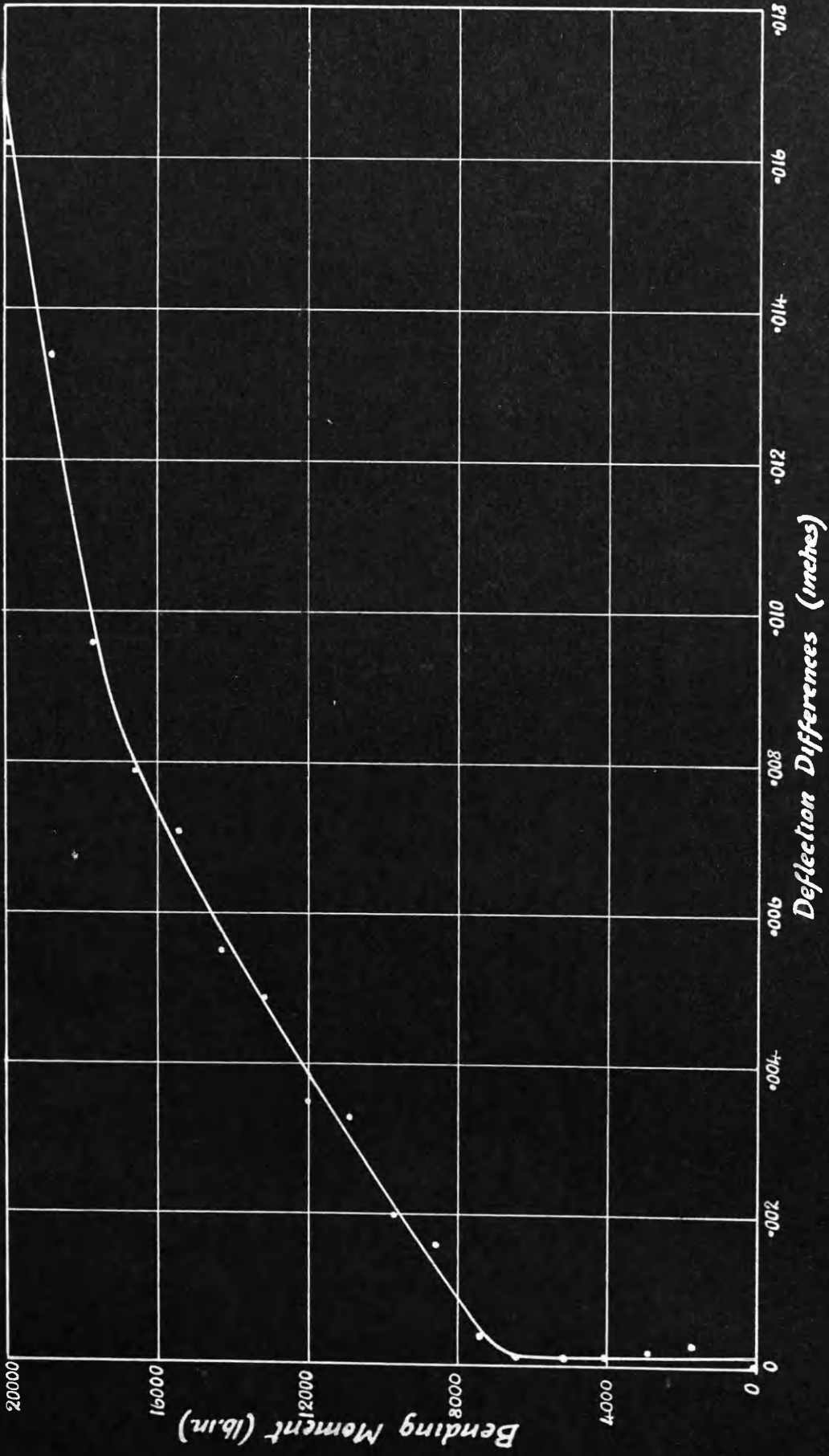
Curve C was obtained with all the bolt rows in position, and the first remarkable feature of the results is the exact coincidence of this curve with curve A for end rows only, this coincidence extending up to the point where slip is indicated by curve A. This in itself is a conclusive indication that, up to a load value corresponding to slip in a two-row joint, the load is transmitted entirely by the friction grip at the outer rows. The inner rows are inactive, and none of the bolts in the joint is under shear.

Above this critical load, since there is clearance in the outer bolt holes, the excess load will be transmitted almost wholly to the inner rows. Further increase in the load causes the friction grip at the inner rows in turn to break down, and slip of the joint as a whole is produced, this occurring in a jerky fashion and accompanied by loud reports.

DETECTION OF PARTIAL SLIP.

Such an account of the action of a four-row joint suggests a possible method of obtaining further supporting evidence. Up to a bending moment of approximately 8000 lb.in., the friction grip at the outer rows carries the whole flange load. Beyond this load, slip will be in progress at the outer rows, and slip at the outer rows must necessarily be accompanied by corresponding beam deflections. There should therefore be some change, however small, in the/

FIG. 8.



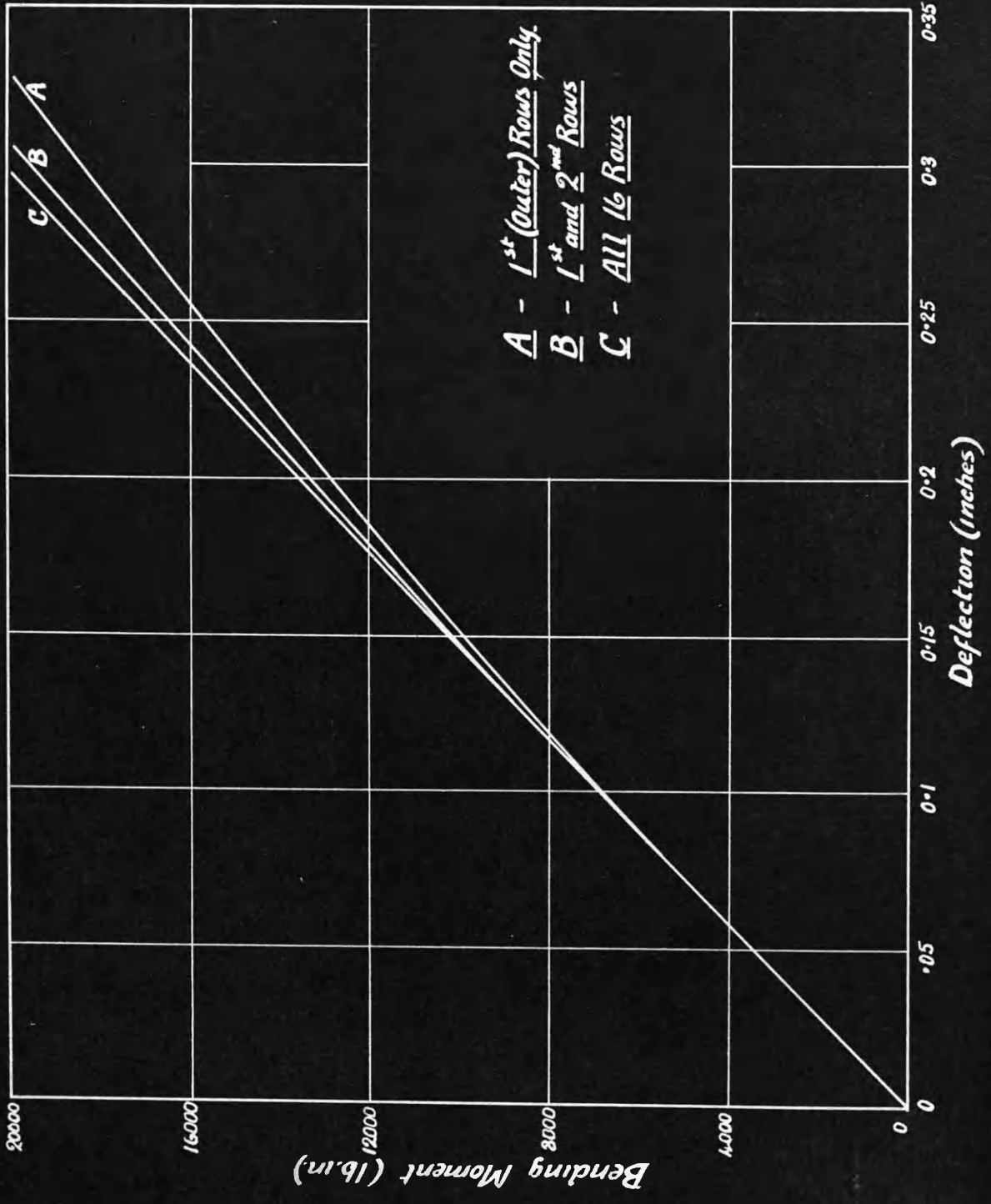
the slope of the deflection curve for the beam at the point where slip begins at the outer rows.

Casual inspection of the numerical results conveyed the impression that this might be so, but afforded no very definite confirmation, appearing rather to indicate a gradually increasing curvature in the graph. To test the matter further, it was decided to investigate the differences in abscissae between the curve and a straight line with a slope almost the same as that of the lower part of the curve.

Fortunately the load readings included a series of loads with regular increments, consequently it was only necessary to perform the easy and accurate arithmetical process of subtracting from the existing deflection readings a series of regularly increasing deflections. The remaining small differences were plotted against the load values, giving Fig. 8., and the graph brings out in a most unexpectedly striking fashion the changes in slope which exist in curve C (Fig. 7).

Naturally the deflection readings from which curve C was drawn contain unavoidable experimental errors, but actually the accuracy of deflection measurement was much greater than strictly necessary for the construction of the curve, and errors were not noticeable. In plotting the differences, however, the huge reduction in the values of the abscissae causes a corresponding increase in the relative/

FIG. 9.



relative magnitude of the experimental errors involved in the readings, and the points no longer fall exactly on a smooth curve.

Despite this factor, the changes in slope at loads corresponding to slip in the outer rows and slip in the inner rows are quite unmistakable, and provide a graphic representation of a feature of multi-row joint action which has not hitherto been effectively demonstrated.

ADDED FLANGE PLATES.

The addition of flange plates to a complete beam does not form an arrangement of four similar lap joints. In the other test specimens, cutting the beam flange ensures that the cover plates transmit to each joint the full flange load. With added flange plates and clearance in the bolt holes, the loads in the cover plates are entirely dependent on the friction grip due to bolt pressure.

The various load-deflection curves are given in Fig. 9. It has already been mentioned that the deflection measurements were effected with greater accuracy than was necessary for drawing these graphs. In this case the numerical results indicated a gradual divergence of the deflection curves while the differences were still too small to be appreciable on the graphs. Consideration will show that such an effect might be expected, since the friction grip of a joint exhibits a slight departure from perfect rigidity as already evidenced in Fig. 4. The lack of/

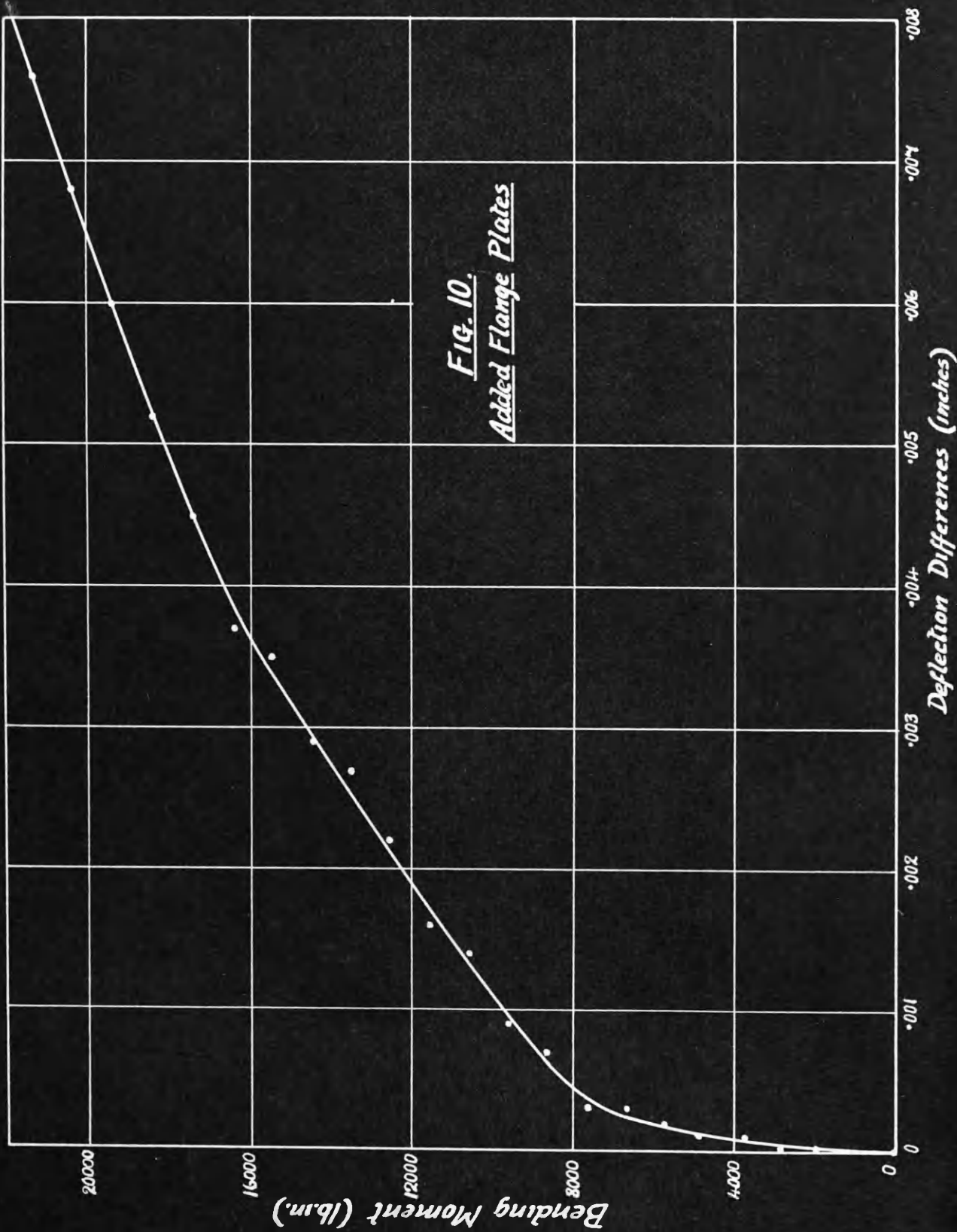
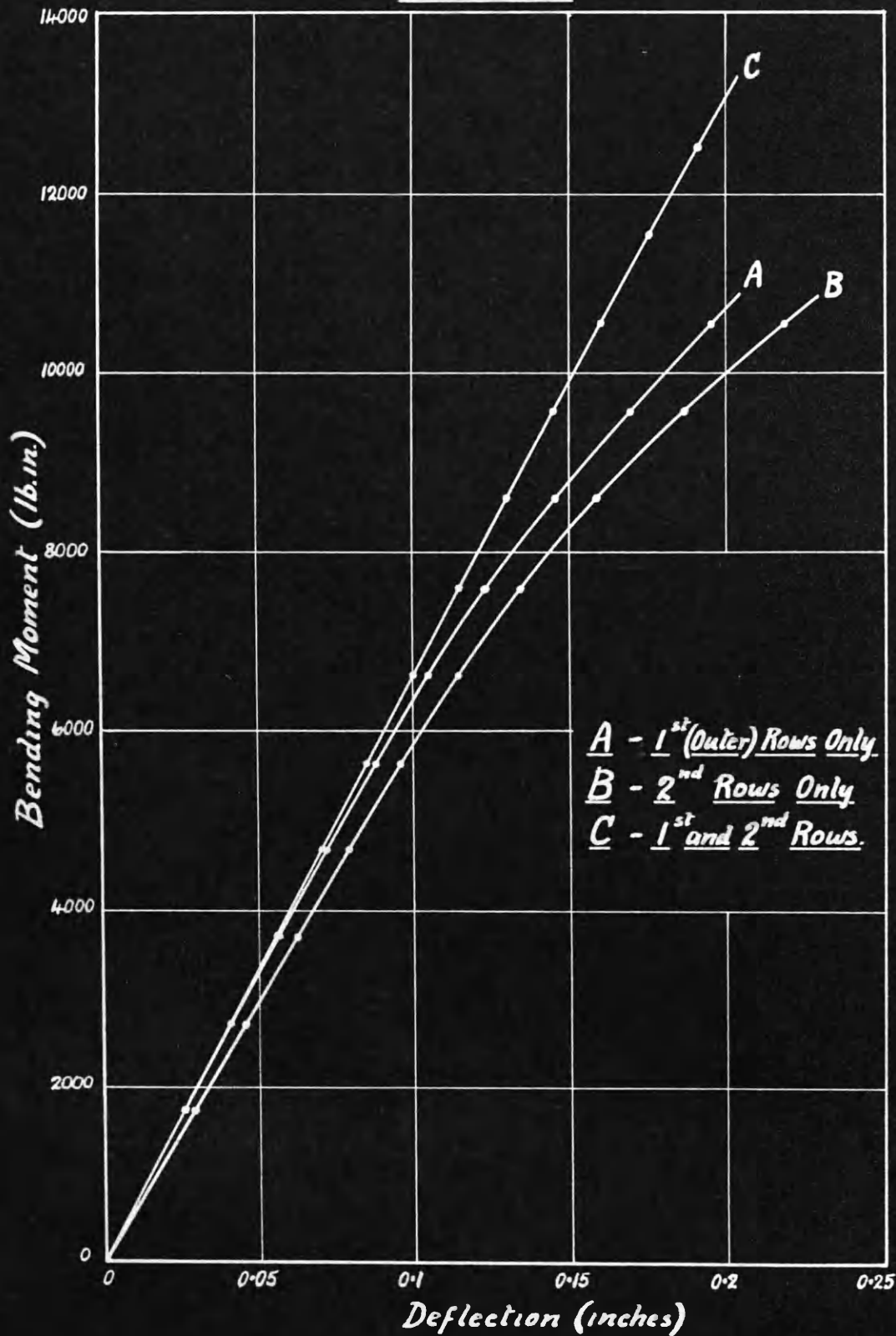


FIG. 11
8-Row Joint



of rigidity will produce a small difference between the flange and cover plate stresses, decreasing the former and increasing the latter, an effect which will be modified by the addition of further bolt rows.

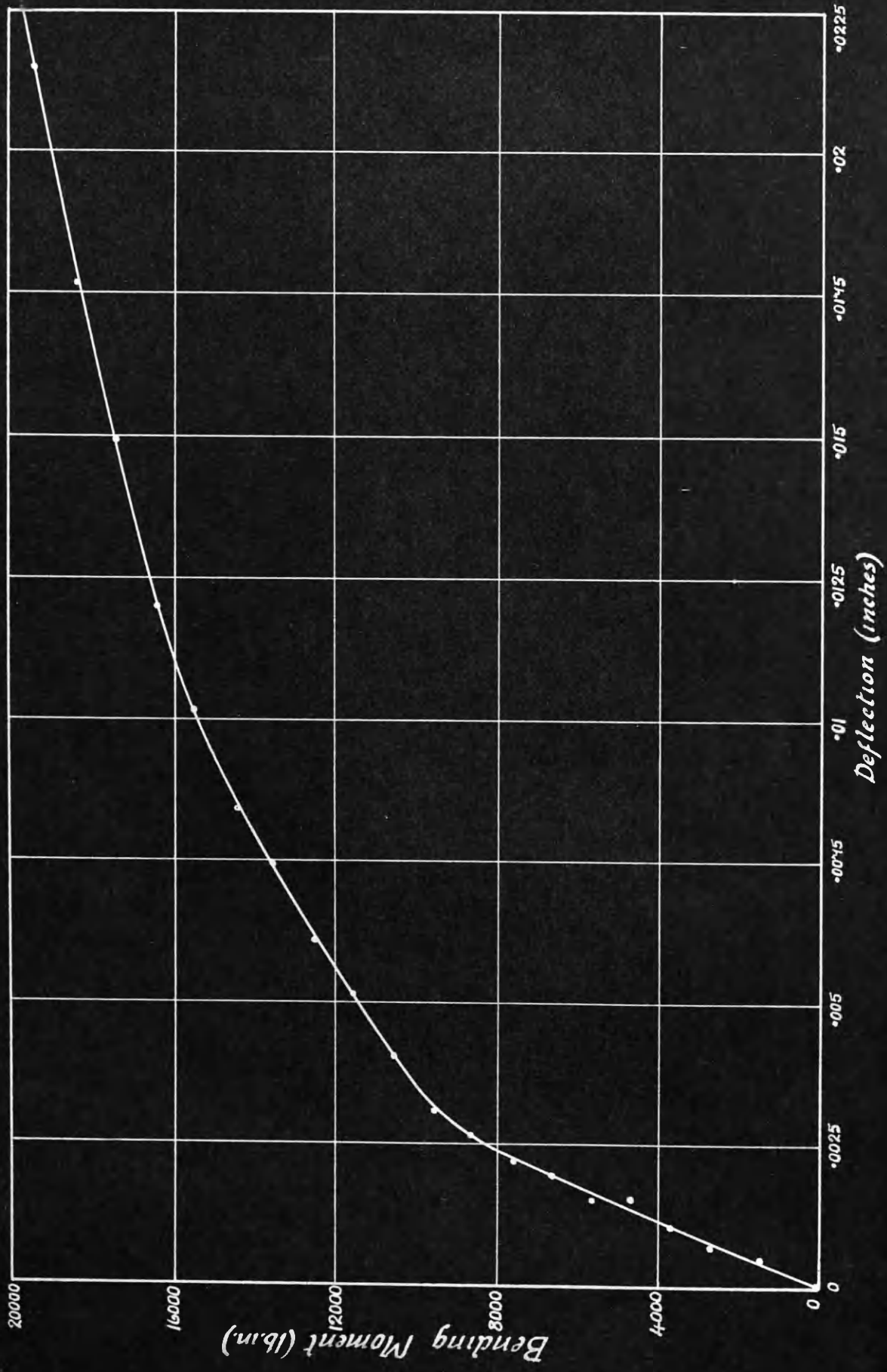
Although the differences between the curves are small, they obscure the points at which the divergence might be taken to indicate slip; but an application of the same process as in the previous case - the "method of differences" - is once more successful in showing the occurrence of slip in the individual rows at definite moment values. The resulting graph of deflection differences is given in Fig.10; the slope changes indicating the beginning of slip in the first rows and in the second rows occur at bending moment values which are practically the same as in the case of the four-row joint.

EIGHT ROW JOINT.

The first series of tests applied to the eight-row joint was similar to that already described for the four-row. Tests were carried out with only the first (outer) bolt rows, then with only the second bolt rows, and finally with first and second rows together. The results appear in Fig. 11. For each of the two-row joint arrangements, the change in curvature of the graph shows the breakdown of the friction grip at the usual load value.

The larger scale used here shows that there is in this/

FIG. 12.
8-Row Joint - 1st and 2nd Rows Only.



this case also a gradual divergence of the curves obtained with outer rows only and with the two outer rows together; hence in order to demonstrate the occurrence of slip from the latter graph, it becomes necessary to employ again the "method of differences". The resulting deflection differences reproduce once more the features already shown in the other cases. Slip in the individual rows is indicated by sudden changes of slope in the "differences" curve (Fig. 12) at bending moment values which are practically the same as previously.

The consistency with which this demonstration has been obtained with the various specimens is probably sufficiently convincing in establishing the operation of the simple laws of friction in multi-row joints. The satisfactory completion of the evidence requires only one final confirmation, the nature of which may be ascertained by analysing on a basis of the above evidence the behaviour of the eight-row joint during loading and unloading.

REVERSAL OF SLIP DURING UNLOADING.

Fig. 13 shows half of an eight-row joint, of which there are four in the beam; XX is a section through the centre of the joint. The joint is presumed to be loaded by a tensile pull applied to the flange at one end and to the cover plate at the other. Bending effects due to eccentricity of loading may be ignored since they are not permitted to develop in the actual arrangement. The load necessary/

FIG. 13

Stresses in 8-row Joint - Loading and Unloading.

		P	P	P	P	
$\sum 2P$		P	P	P	P	A
		P	2P	2P	2P	
$\sum 4P$		3P	2P	2P	2P	B
		P	2P	3P	3P	
$\sum 6P$		5P	4P	3P	3P	C
		P	2P	3P	4P	
$\sum 8P$		7P	6P	5P	4P	D
		0	P	2P	3P	
$\sum 6P$		6P	5P	4P	3P	E
		-P	0	P	2P	
$\sum 4P$		5P	4P	3P	2P	F
		-P	-P	0	P	
$\sum 2P$		3P	3P	2P	P	G
		-P	-2P	-P	0	
$\sum 0$		P	2P	P	0	H

necessary to produce slip in a single row of bolts is taken as P .

The load on the joint may be imagined to increase gradually, till at the value of $2P$ the loads in the various sections of the joint will be as marked on diagram A. The first row is now on the point of slipping, since the friction grip there cannot communicate to the cover plate a pull greater than P .

As the load is further increased, continuous slip proceeds at the first row; and the conditions corresponding to a total load of $4P$ are shown in diagram B. Since the second bolt row is now contributing its full quota P to the cover plate, it is on the point of slipping. Further increase of load to a total of $6P$ therefore produces continuous slip at the first and second rows, and the loads in the various sections become as given in diagram C.

Increasing the load to $8P$ gives the conditions indicated in diagram D, where the first three rows have slipped and the fourth row is on the point of slipping. It will be appreciated, of course, that since the joint is symmetrical an exactly similar process has taken place at the other end, and consequently the joint as a whole is now on the point of slipping. Bodily slip will begin if the load is further increased.

From this point, however, it may be imagined that the load is gradually reduced to zero. Successive stages in/
in/

in the reduction are shown in diagrams E, F, G, H. During the removal of load, no slip will occur at any row unless that row becomes subject to a load P . This does not occur till the total load is reduced to the value $4P$ (diagram G). It will be observed that between the first and second rows the cover plate is now under tension of magnitude P , and further reduction of load will result in a corresponding extension of the joint with the development of slip in the reverse direction at the end rows.

The subsequent stages may be followed out step by step and it will be found that, while slip continues at the end row till the load is entirely removed, no slip will occur at any other row. At zero load the condition of the joint is such that the application of a load in the reverse direction, i.e. a compressive load, however small, will produce slip at the second row; and this slip will continue, along with slip at the first row, as the compressive load is increased.

The important point arising from the analysis is that in an eight-row joint, provided the loading is carried up to or beyond the point where slip begins at the third row, the development of slip at the end row while unloading should be indicated by a definite change in slope of the deflection curve obtained during the process.

Before subjecting the eight-row joint to this procedure, a preliminary test on a complete beam similarly drilled/

FIG. 14.

8-Row Joint - All Rows
(Loading and Unloading)

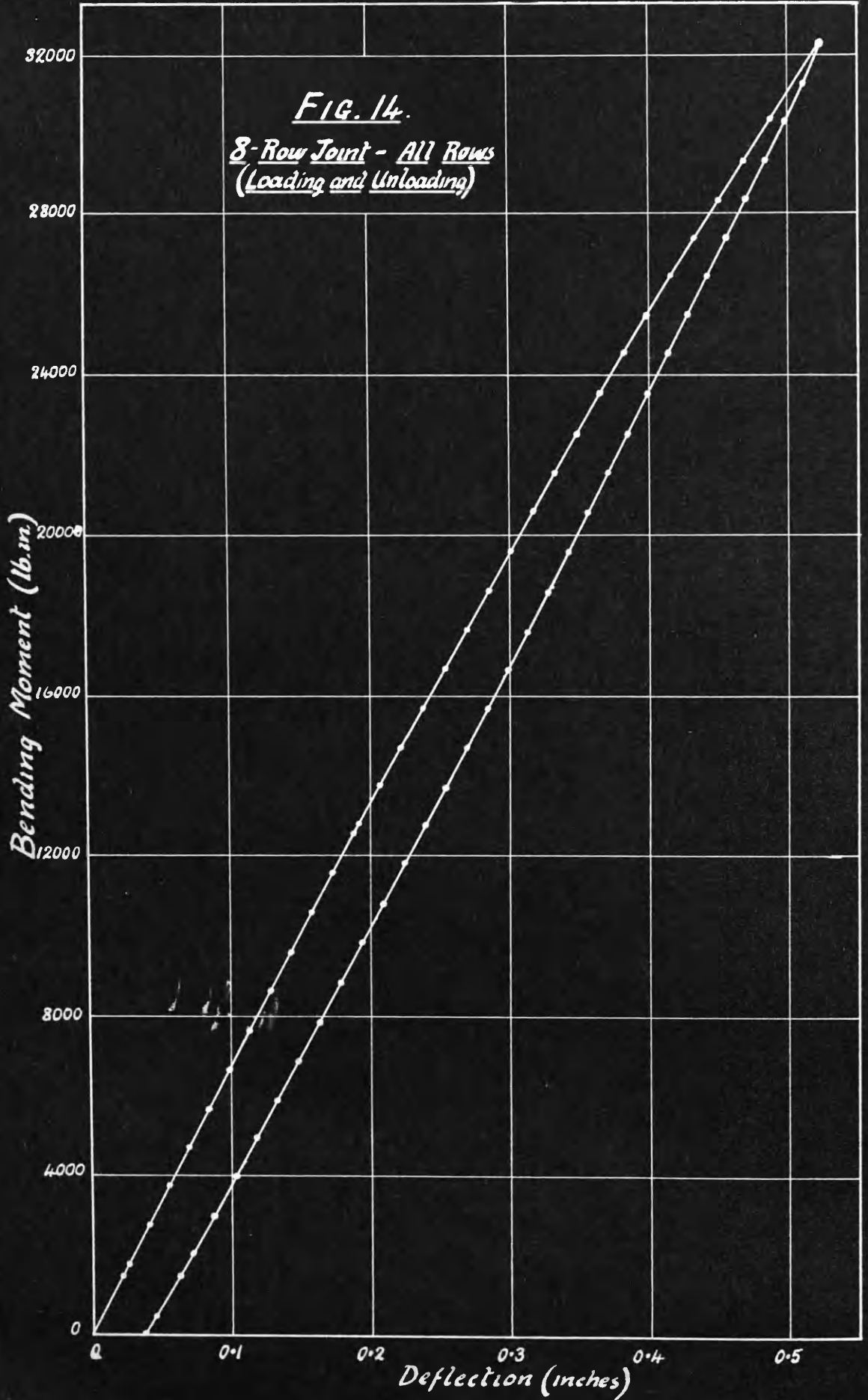
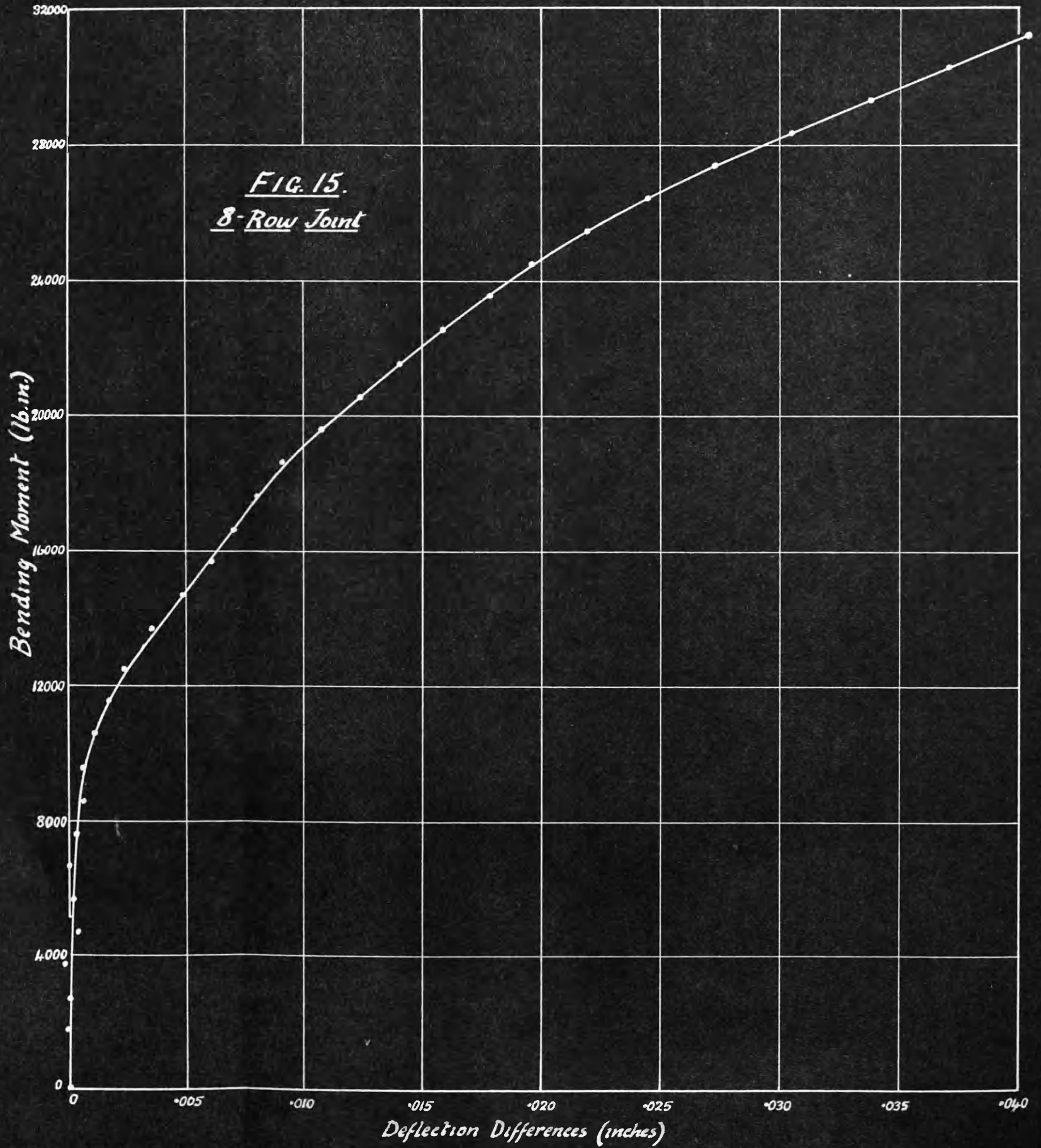
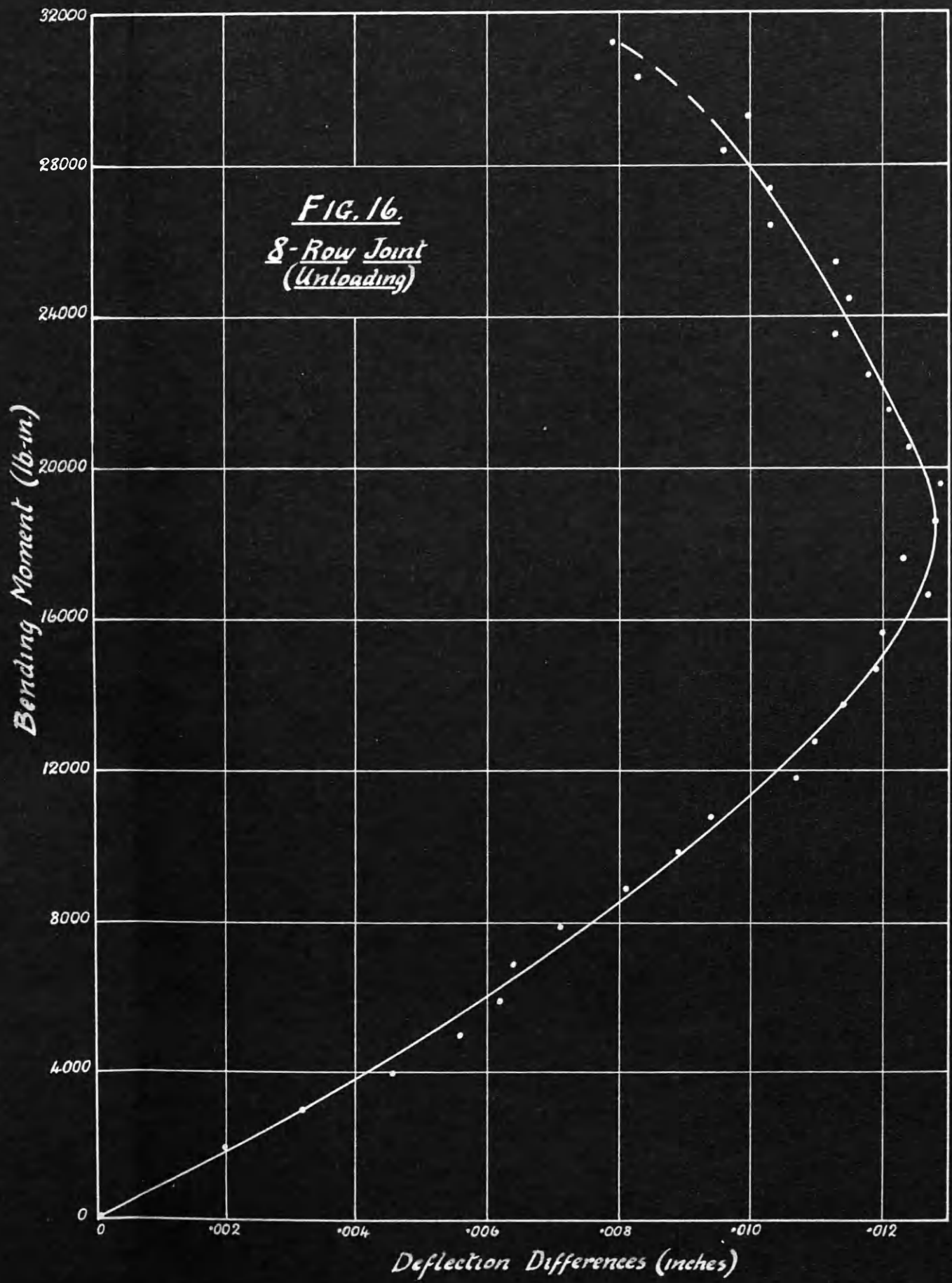


FIG. 15.
8-Row Joint





drilled and machined was carried to the point where local yield was indicated by the departure of the deflection line from the straight. A knowledge of this point enabled the actual joint to be subjected in the usual way to the maximum possible load value while avoiding permanent set in the material of the beam, deflection readings being taken for small regular load increments during the whole process of loading and unloading. The curve obtained is shown in full in Fig. 14.

Testing the loading part for sudden changes in slope by the usual method gave Fig. 15, and the unloading part gave Fig. 16. Fig. 15 shows very clearly the changes in slope indicating the beginning of slip at the first, second and third rows respectively. It is evident that loading has been carried practically up to the point where slip of the fourth row and therefore bodily slip of the whole joint is about to develop. Fig. 16 completes the investigation by indicating very definitely the development of slip in the reverse direction at the first row during unloading, this occurring very nearly at the load value suggested by the analysis.

GENERAL DISCUSSION OF BOLTED JOINT TESTS.

As a whole, the foregoing results constitute an almost perfect demonstration of the operation of the simple laws of friction in multi-row joints. The proximity of the various rows to each other makes it seem little short of amazing/

amazing to find the progress of slip from row to row marked in such a definite fashion.

Actually the method owes its success to each and all of the following precautions - the gradual application of each load increment by means of a fine screw and the consequent avoidance of any inaccuracy due to "overshooting" - the measurement of the beam deflections with a degree of accuracy much in excess of what was strictly necessary for drawing the usual load-deflection curves - the use of regular load increments. The last factor is by no means negligible, as with irregular load increments the use of a slide rule to give proportionate deflections would have obscured the results entirely.

The changes in slope on which the method depends appear to become progressively less definite after the first, but at the same time the experimental errors become less noticeable. Both effects are relative, and are due to the increasing magnitudes of the differences. Any one of the changes in slope can be shown quite as definitely as the first by selecting a suitable "zero line" from which to measure the differences. For a given deflection curve, the form of the "differences" graph will naturally vary with every chosen straight line. It may therefore be noted that no significance attaches to the actual slope of the differences graph; only the location of the points where the slope/

slope suddenly changes is important.

The first single row joint tests show the highest friction coefficient. The bending moment on the beam at slip is 5500 lb.in., giving a load of 1830 lb. on each joint, or a nominal shear stress on the bolts of 8.35 tons/in², a value at least as great as has been obtained with riveted joints. The lower coefficient exhibited by the subsequent tests is due to the treatment accorded to the joint surfaces with the object of eliminating the effects of oxide.

The four-row joint was subjected to a number of bodily slips in each direction with the bolts fully tensioned and maintained at full tension. The eight-row joint surfaces were tapped with a light hammer and finally scraped to remove the oxide before assembly. This treatment, and possible unequal distribution of the total bolt pressure, probably account for such variations as are shown in the numerical results.

SLIP LOAD VALUES.

Slip in the outer rows of the multi-row joints occurs at bending moment values varying from 7000 to 9000 lb.in.; for the second rows the values range from 15500 to 18500, and for the third rows 25000 lb.in. (one test only). The average value of the bending moment necessary to produce slip in a two-row joint therefore emerges as 8000 lb.in., giving a nominal bolt shear stress of 6.1 tons/in². The assumption that clearance in the bolt/

bolt holes would prevent any appreciable increase in the load carried by the first row after slip had occurred there is borne out by the fact that the quoted bending moment values are practically even multiples of 8000. Any increase in the load carried by the outer rows after slip would have meant a corresponding increase in the total load necessary to cause slip in the remaining rows.

The sharpness of the slope changes in the graphs of deflection differences provides evidence that the friction grip, and therefore also the bolt pressure, are not distributed over the joint but are localised in the immediate vicinity of the bolts. One further suggestion arises from the same feature. The definite location of slip shows that all four joints slip together. It would appear that joints under compression exhibit exactly the same features as joints under tension, and therefore lateral expansion and contraction in the material produce no appreciable alteration in bolt tension.

JOINT RIGIDITY.

It has already been pointed out that the single row joint, despite the definite absence of frictional slip, is not an absolutely rigid construction when compared with the unjointed beam. The same lack of rigidity is still evident in the other joints. The following deflection curve slopes, expressed in thousandths of an inch per 100 lb. load, apply to the four-row joint but may be accepted as typical of the properties/

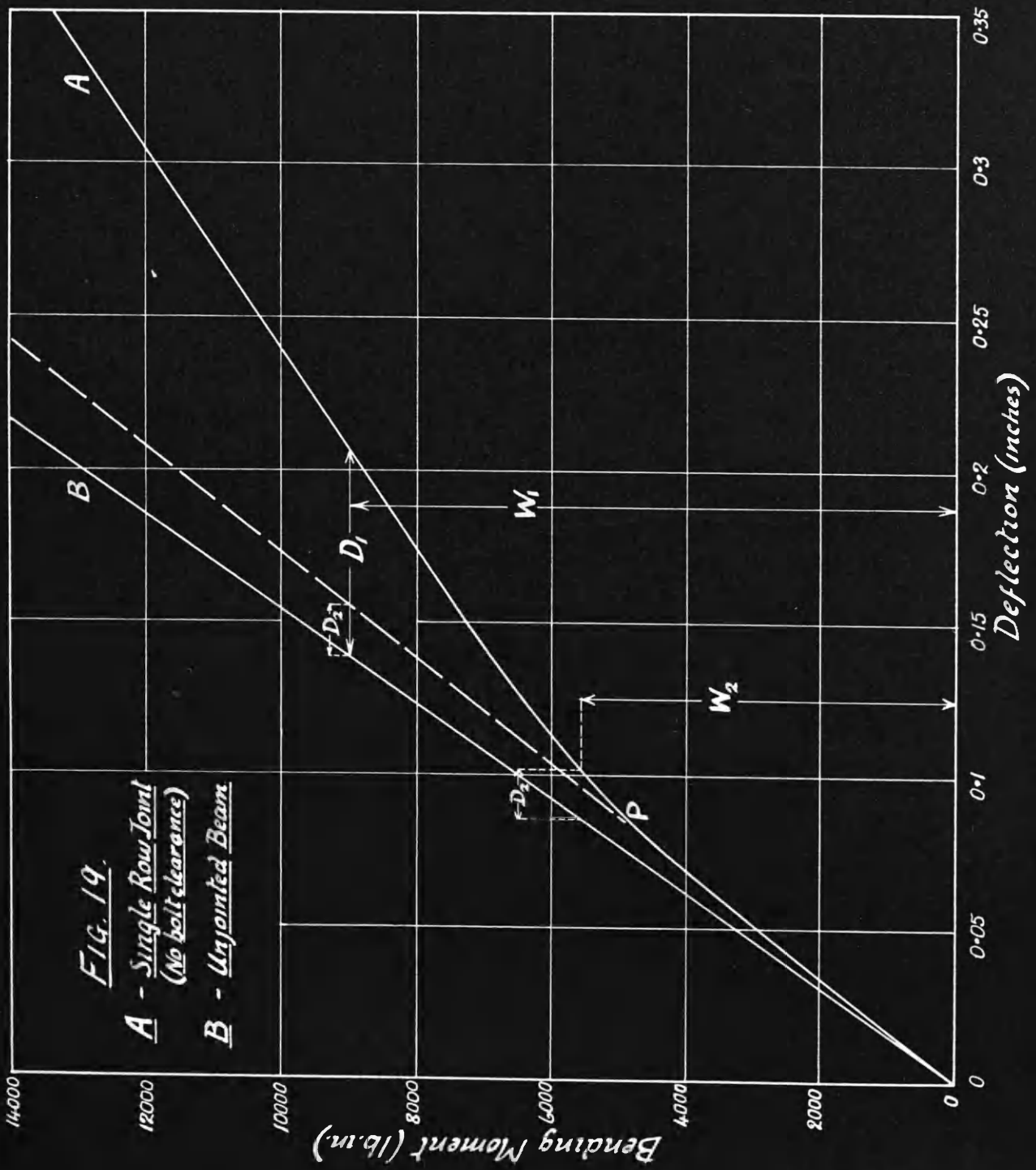


FIG. 19.

A - Single Row Joint
(No bolt clearance)

B - Unjointed Beam

Bending Moment (lb.in.)

Deflection (inches)

properties of the others:-

Unjointed beam.	- - - 34.9.
Calculated value for rigid four-row joint, the parts of the cover plates between the bolt rows being considered as integral with the beam.	- - - 30.2.
Actual four-row joint.	- - - 32.1.
Four-row joint after slip begins in outer rows.	- - - 33.8.

A further indication of this departure from perfect rigidity was obtained while testing the multi-row joints, first with end rows only and subsequently with second rows added. It was observed that generally the addition of the second rows had a slight stiffening effect on the beam, even where no end row slip had occurred. The difference produced in the slope of the deflection curve was small but measurable. Unfortunately, repetitions of any one test showed that slope differences of comparable magnitude were produced by support effects arising from the twisted and warped condition of standard rolled beams, and the joint effect could not be accurately calculated. An estimate was obtained from the following reasoning.

Fig. 19 gives the deflection curve for the single row joint contrasted with the deflection of the unjointed beam. At any load below slip load, the difference in abscissae between the two graphs represents a joint distortion which is related to the joint load. With the load scale doubled/

doubled, presumably the same joint distortion values will apply to a two-row joint, since each carries half the total load; they will therefore be applicable to a multi-row joint with only end rows present.

Any effect which modifies this distortion must modify the load on the end rows. Hence, from the change produced in the slope of the deflection curve by the addition of the second rows, the reduction in end row load may be estimated. From this reasoning emerges the suggestion that, before end row slip occurs in a multi-row joint, stress concentrations at the end rows allow the "leakage" to the second rows of a very small load whose magnitude appears to be not greater than 3 per cent. of the total.

Since Fig. 19 was obtained from a joint without appreciable bolt clearance, the same reasoning may be extended to an estimate of the characteristics of riveted joints. If the load scale be doubled, then the joint distortion in a two-row joint is represented by the differences in abscissae as already indicated. These distortions apply to any load above or below slip, and to multi-row joints with only end rows in position. In the latter case, a known joint distortion corresponds to a definite end row load; the distortion is D_1 for a load W_1 .

With the second rows inserted, it is known that the change in slope at end row slip is very small; consequently the/

the joint distortion will be approximately represented by the difference in abscissae between the straight line and the dotted line, i.e. for the same load W_1 when the second rows are also in position the joint distortion is now D_2 . Associating D_2 with the original curve shows that the end row load is W_2 , only slightly greater than its slip value.

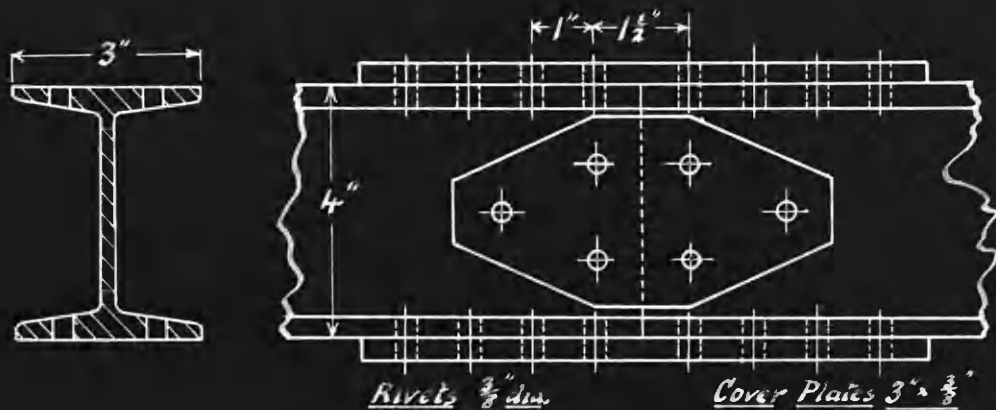
Without a detailed numerical calculation, the magnitude of the change in slope of the graph at point P indicates that in a multi-row riveted joint the end row load will not increase to any great extent beyond its slip value. For practical calculations, it is probable that no serious error will be introduced by assuming similar characteristics for bolted and riveted joints.

RIVETED JOINTS.

CONDITIONS OF TEST.

As a whole, the preceding tests provide a singularly convincing and complete exposition of bolted joint action, and all that is necessary to establish its practical importance and applicability is definite evidence of the same action in riveted joints. In all the records of experimental work carried out on riveted joints, only one published test of similar type to the foregoing has been found. It is a test of a riveted joint in a straight beam⁸, carried out by Professor Haigh at the Royal Naval College/

FIG. 14 - Riveted Joint Test



Load (lb.)	Actual Deflection (1000")	Deflection Proportional To load	Deflection Difference	Load (lb.)	Actual Deflection (1000")	Deflection Proportional To load	Deflection Difference
0	0	- 4.8	4.8	3800	48.2	48.4	- 0.2
200	2.2	- 2.0	4.2	4000	51.1	51.2	- 0.1
400	4.4	+ 0.8	3.9	4200	53.9	54.0	- 0.1
600	6.4	3.6	3.3	4400	56.4	56.8	- 0.4
800	9.1	6.4	2.4	4600	59.5	59.6	- 0.1
1000	11.9	9.2	2.4	4800	62.3	62.4	- 0.1
1200	14.1	12.0	2.1	5000	65.8	65.2	+ 0.6
1400	16.4	14.8	1.6	5200	68.5	68.0	0.5
1600	19.0	17.6	1.4	5400	71.2	70.8	0.4
1800	21.0	20.4	0.6	5600	74.2	73.6	0.6
2000	23.1	23.2	- 0.1	5800	77.6	76.4	1.2
2200	26.1	26.0	+ 0.1	6000	80.8	79.2	1.6
2400	28.8	28.8	0.0	6200	83.1	82.0	1.1
2600	31.9	31.6	+ 0.3	6400	86.7	84.8	1.9
2800	34.3	34.4	- 0.1	6600	89.0	87.6	1.4
3000	37.0	37.2	- 0.2	6800	92.5	90.4	2.1
3200	39.7	40.0	- 0.3	7000	95.9	93.2	2.7
3400	42.2	42.8	- 0.6	7200	98.2	96.0	2.2
3600	45.8	45.6	+ 0.2	7400	101.6	98.8	2.8

College, Greenwich. The writer is very much indebted to Professor Haigh for kindly supplying on request full details of the tests and the actual numerical results obtained.

The dimensions of the beam and joint are given in Fig. 17. As in the bolted joint tests, the flange equivalent area is the same as the area of the cover plate, and the arrangement therefore gives an assembly of four symmetrical four-row joints. It is not quite so fortunate that a web splice using three rivets has been included; but as this is so weak and flexible in comparison with the rigid flange joint, it is probable that no noticeable effect will be produced.

In assembling the joint, the ends of the beam were faced and tightly butted together by taper pins driven through flanges and cover plates. Several holes were reamed and the rivets closed. Removal of the taper pins then permitted the remaining holes to be reamed and the riveting completed. The test was carried out by mounting the beam in a testing machine on a 4 ft. span with the joint at the centre, applying central loads with equal 200 lb. increments and measuring the central deflection by a dial gauge reading to .001" and estimating to the nearest .0001".

There are certain marked differences between the conditions of this test and those of the preceding investigation, all of which are unfavourable to the successful detection of slip at the various rows. The rivets/

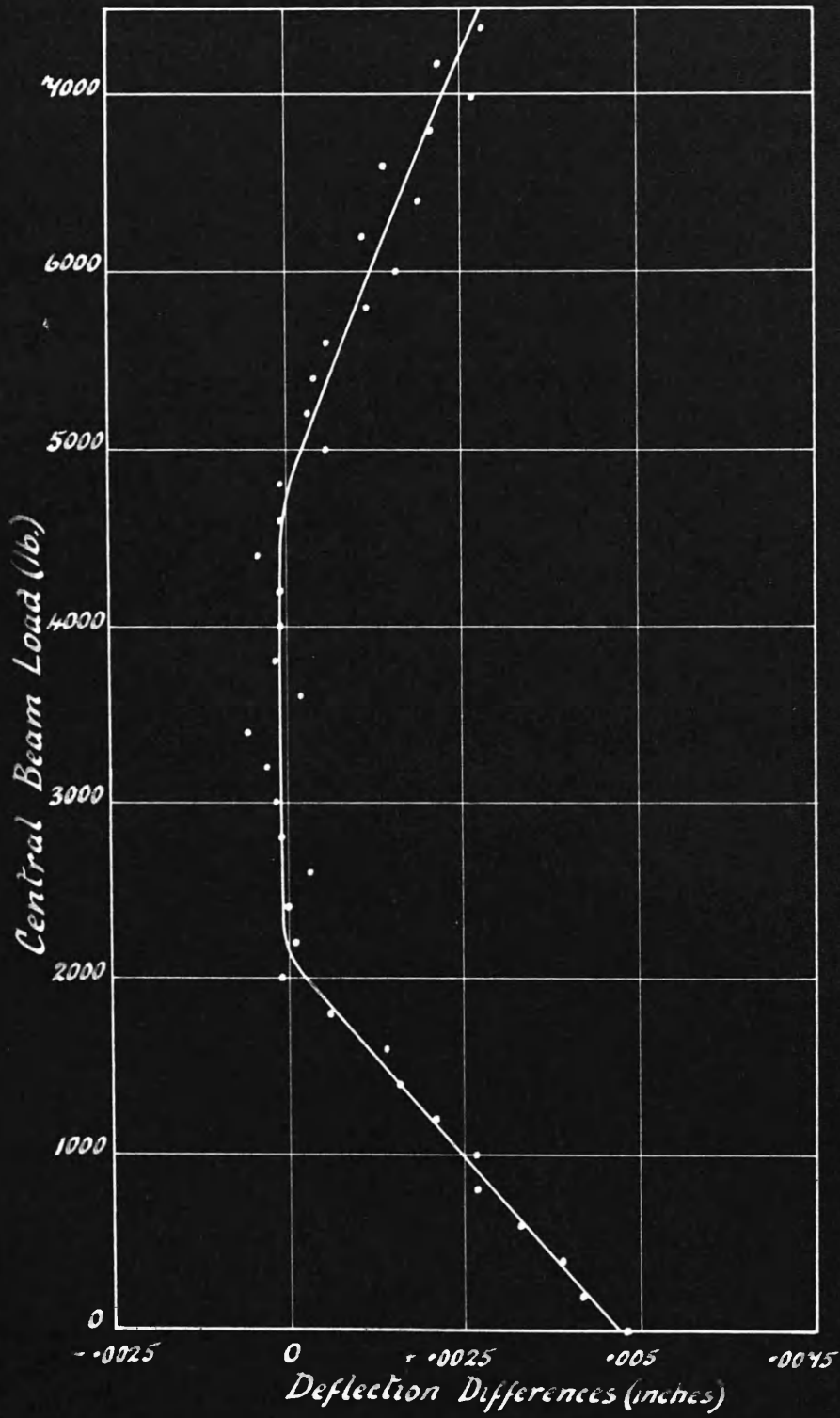
rivets are of $\frac{3}{8}$ " diameter with a pitch of only 1" between the rows; the comparatively short pitch will reduce the magnitude of the slope changes in the deflection curve. A similar effect will be produced by the butting of the faced ends of the beam; as the action of the joints in the compression flange is thereby entirely prevented, only the joints in the tension flange are subject to load and the magnitude of the slip deflection is halved.

In addition, the overall length of the joint is approximately 9", and with a central load there will be some variation of bending moment along the length of the joint, a condition which tends to upset the symmetry of the individual joints by unbalancing the loads on the end rows. Finally, during the progress of the test the load was periodically reduced to 400 lb. to ascertain the magnitude of the permanent set; this operation tends to introduce variations which are rather upsetting in using a method of analysis depending on small differences.

ANALYSIS OF RESULTS.

The examination of the deflection curve has been made by employing the method of differences in the usual manner, and the numerical values for this particular test are detailed in tabular form in Fig. 17. The analysis has not been carried beyond a load value of 7400 lb., because at this load the nominal stress in the undrilled flanges/

FIG. 18
Riveted Joint Test.



flanges is 10.7 tons/in^2 and further investigation is liable to be obscured by the occurrence of local overstressing.

Despite the unfavourable conditions noted above, the graph of differences (Fig. 18) shows the usual familiar features. First row slip is indicated at a load of 2000 lb., giving a bending moment of 24000 lb. in. and a total load in each flange of 6000 lb. Considering only the end row rivet area since the end rows carry all the load, the nominal shear stress on the rivets at slip is 6.1 tons/in^2 . The second change of slope occurs at a load of 4600 lb. and is quite definite although not so well marked as the first. Over the whole joint, this load gives a nominal rivet shear stress at slip of 7 tons/in^2 , which agrees very well with Montgomerie's figures and the results of the bolted joint tests.

It may be noted that second row slip occurs at a load which is slightly more than twice the value producing end row slip; but one isolated experimental value can hardly be considered a reliable indication of a shearing load on the first row rivets when slip is developing at the second rows. It does indicate, however, that the proportion (if any) of the excess load retained as a shearing force on the first row rivets is comparatively small, and thereby confirms the inference already drawn from the results of the bolted joint tests.

The recorded figures show that permanent set was negligible/

negligible up to a load of 2000 lb. and thereafter it increased slowly but definitely. It will be observed that the permanent set is due to slip in the outer rows of rivets in the individual joints.

An indication of the value of the actual coefficient of friction is provided by the results of tests (unpublished) carried out by Professor Haigh to determine rivet tensions. The load on a $\frac{1}{2}$ " diameter rivet was found to be in the region of 3000 lb., the corresponding stress being 6.8 tons/in². If the same stress value be assumed applicable to the $\frac{3}{8}$ " diameter rivet the coefficient of friction is approximately 0.9, a value which is not unreasonable for surfaces with a commercial rolled finish.

With regard to rigidity, the riveted joint shows the same features as the bolted joint. The deflections of an unjointed length of the same rolled section were measured, and the comparison of deflection curve slopes, expressed in thousandths of an inch deflection per 100 lb. of load, is as follows:-

Unjointed beam	- - -	1.23.
Jointed beam with parts of the cover plates between the rows calculated as integral with the beam.	- - -	1.00.
Riveted joint	- - -	1.17.
Riveted joint after slip at outer rows	- -	1.39.

CONCLUSIONS/

CONCLUSIONS.

In a multi-row joint, the simple frictional relationship which has previously been shown to exist between total rivet pressure and slip load is also applicable to slip at the individual rows.

The friction grip at the outer rows of the joint transmits the whole load up to the point where limiting friction is reached and slip develops. This occurs when the nominal rivet shear stress (assuming the whole load to be carried by the outer rows only) reaches a value between 6 and 7 tons/in².

Thereafter, in a bolted joint with clearance in the bolt holes, the additional load passes to the second rows. Should the total load exceed the limiting friction for the first two rows, the excess will pass to the third rows.

In a riveted joint, when the applied load exceeds the friction grip at the end rows, part of the excess load may possibly be retained as an actual shearing load on the end row rivets while the remainder passes to the second rows. The results indicate that the actual shearing load on the end row rivets is comparatively small even if the rivet clearance is assumed negligible. Since the rivet clearance is definitely measurable, no very serious error will result from the neglect of the shearing load in practical calculations.

The/

The tests described here show slip occurring at well defined load values. The pressures exerted by the individual bolts or rivets are therefore concentrated in the immediate vicinity of the bolts or rivets. The records of previous investigators show that with plate thicknesses in excess of $\frac{5}{4}$ " , uniform contact between the plates can only be secured with difficulty; in such cases slip manifestations may not be quite so definite and regular.

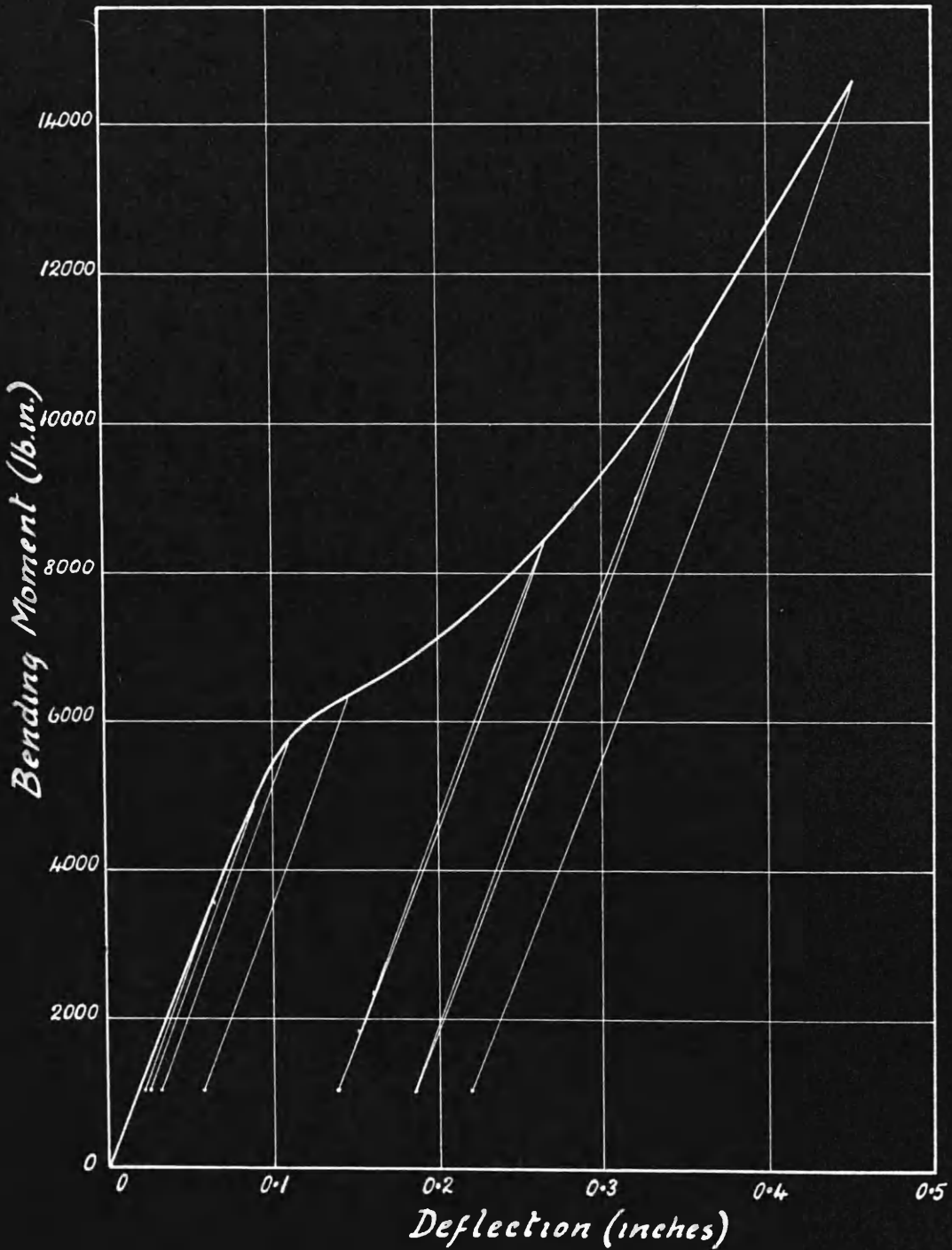
The stress values suggest that generally in riveted joints slip will not take place, but in joints with three or more rows it may develop in the outer rows. Where it does occur, it will probably be accompanied by a decrease in rivet tension due to disintegration of surface oxide on the plates. The effect may be negligible where slip in one direction is concerned, but it would appear that reversal of slip should be avoided at all costs.

Where plates of different thicknesses are united, the usual conceptions of load partition in riveted joints suggest dissymmetry in the stress distribution and in the loads on the various rows. It is worthy of note that on a basis of the explanation presented above, such dissymmetry cannot exist.

The assumption of elastic deformation of the rivets hitherto made in theoretical investigations should be modified to take account of the friction grip of the individual rows. But/

FIG. 20.

Hysteresis Loops - Single Row Joint



But in view of the restricted number of rows in which slip can occur at working stresses, and since the actual shearing load on the rivets appears to be negligible, a mathematical theory may be considered superfluous.

Finally, it may be pointed out that the method used in this paper for the investigation of the characteristics of riveted joints could easily be extended to include the elastic properties of fillet welds. Using welds instead of rivets, deflection curves for short (unit) lengths of weld could be compared with similar curves for various lengths of weld. The accurate determination of the deflection curve slopes would enable the elastic properties of welds to be completely explored.

HYSTERESIS LOOP TESTS.

As a preliminary step towards the investigation of the characteristics of multi-row joints under vibration, a number of static measurements of friction "hysteresis" was attempted. A few representative results are given, principally because they throw further light on the behaviour of joints under repeated loading.

UNIDIRECTIONAL LOADING.

The deflection curve for a single row joint with clearance in the holes is now a familiar one. The same curve is shown in Fig. 20 but at a number of points during the/

FIG. 21.

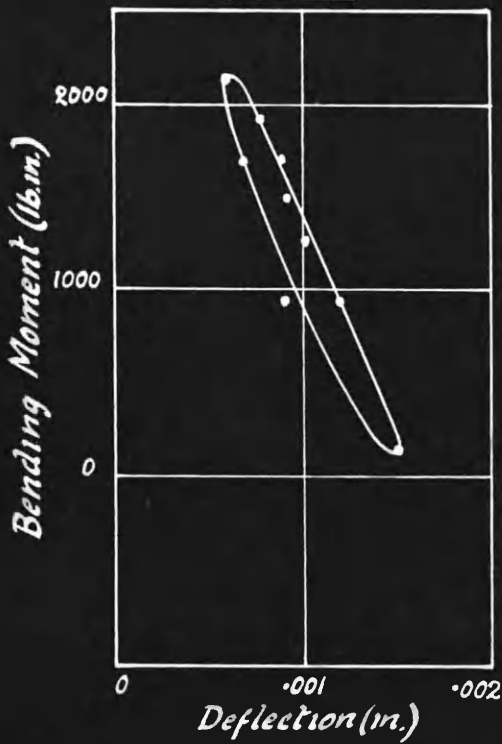
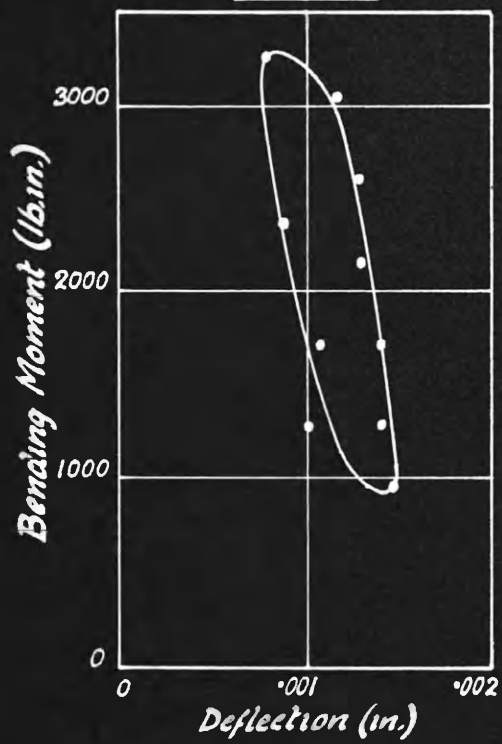


FIG. 22.



Hysteresis Loops

FIG. 23.

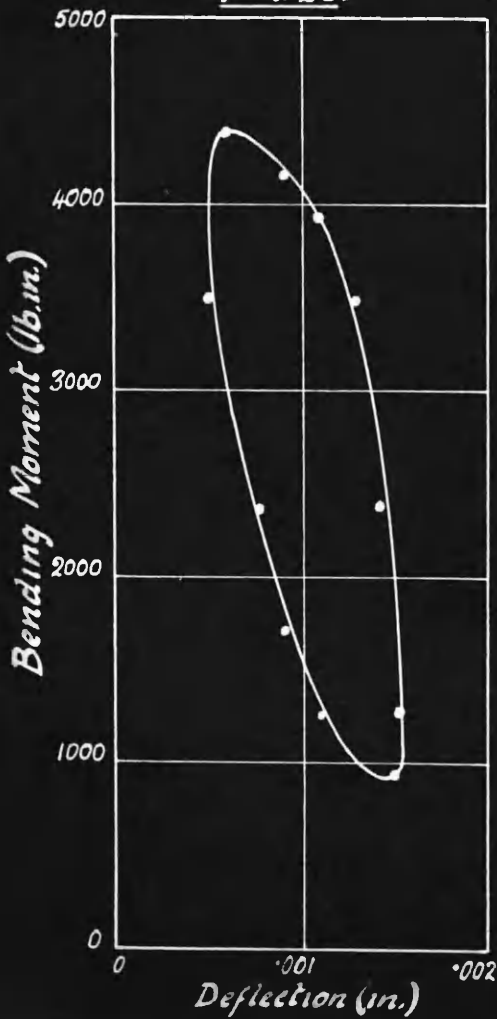
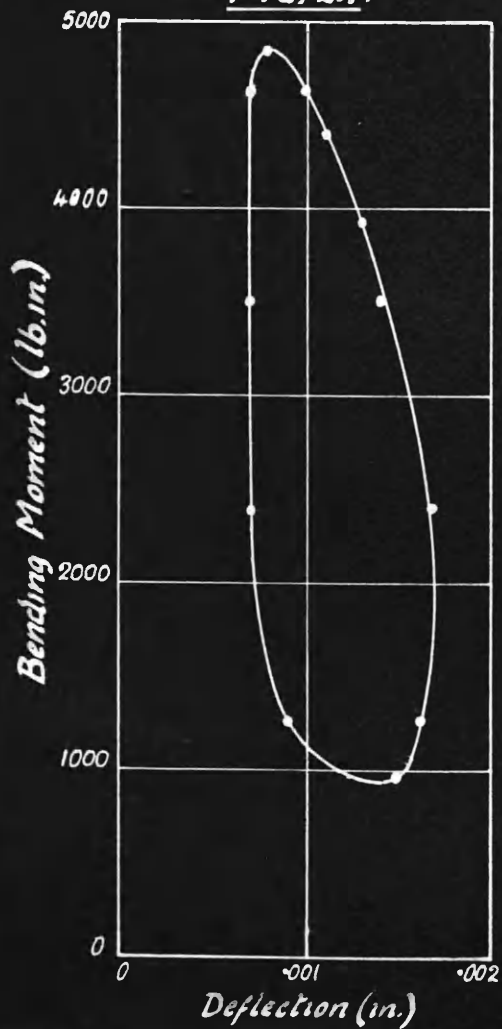


FIG. 24.



Hysteresis Loops

FIG. 25.

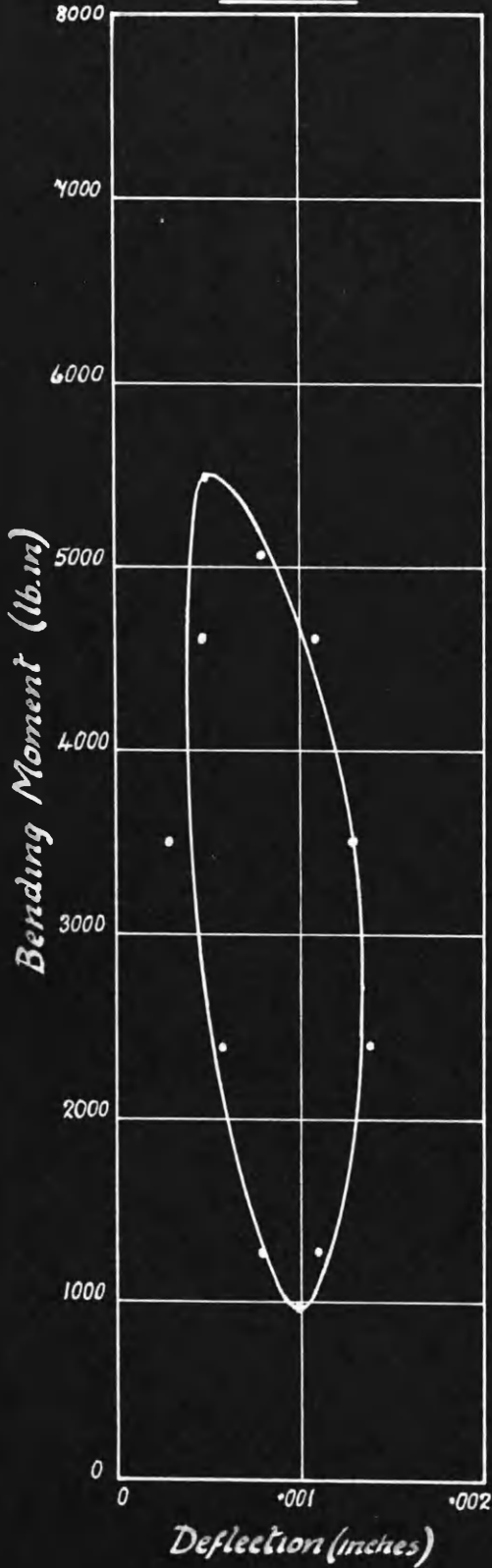


FIG. 26.

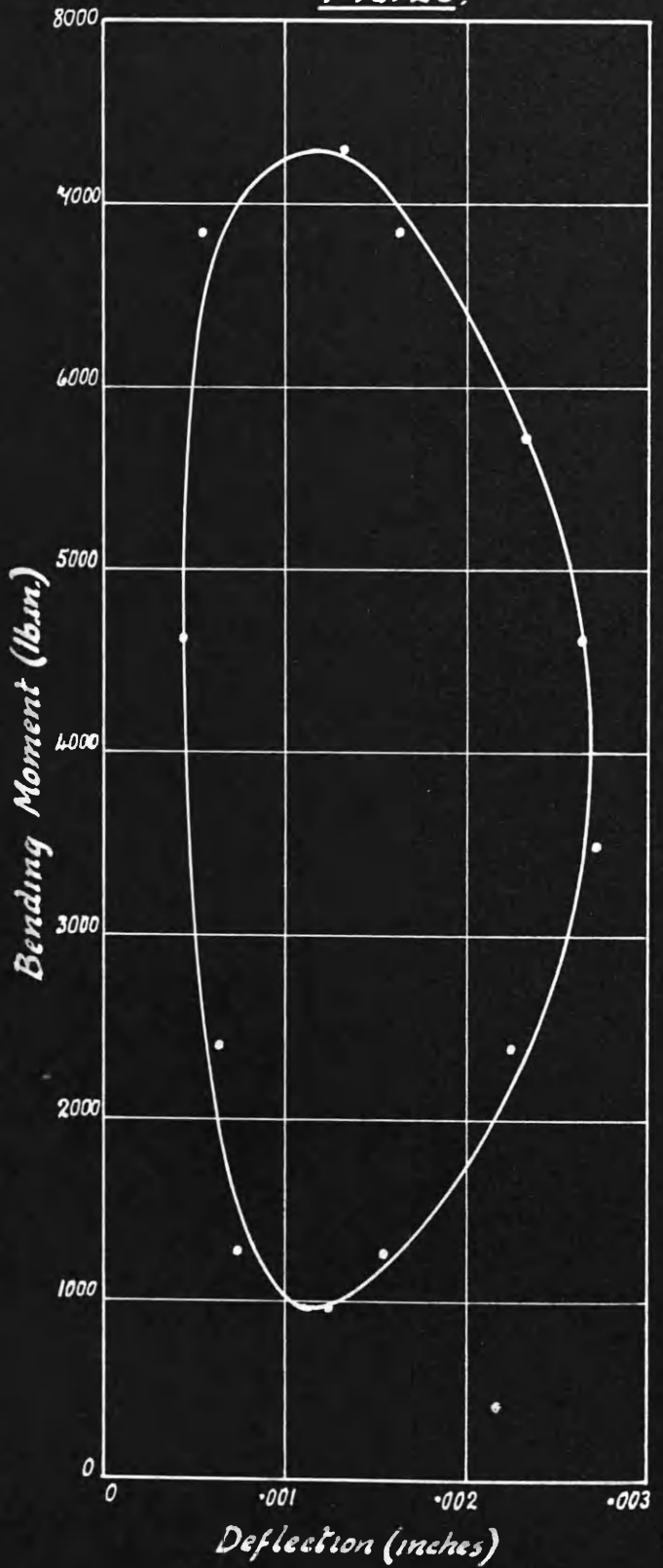


FIG. 27.

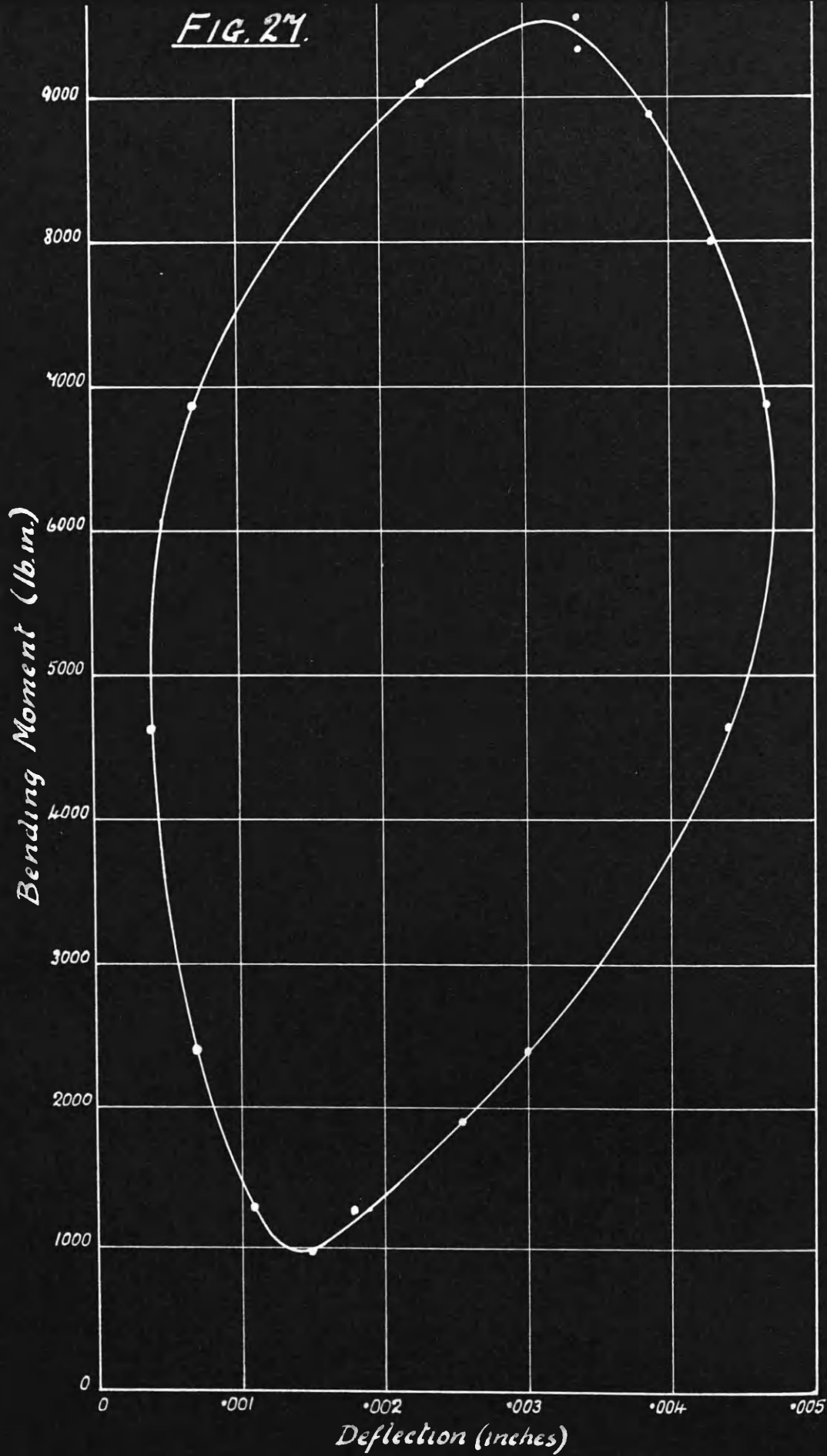
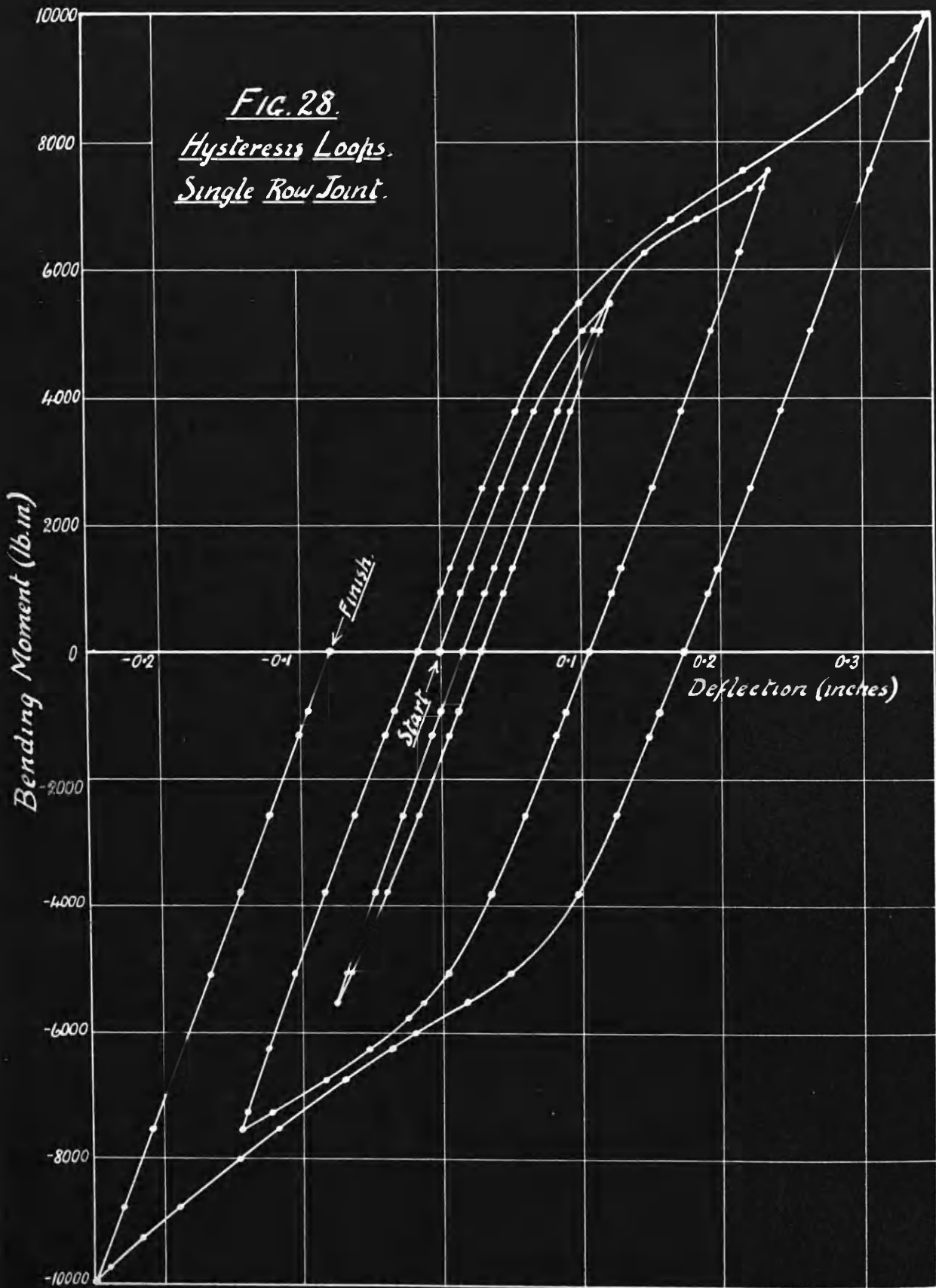


FIG. 28.
Hysteresis Loops.
Single Row Joint.



the test the load was reduced almost to zero and reapplied. In this way a number of unidirectional or 'half' loops was obtained. The exact form and magnitude of the loops were investigated by using the method of differences already explained. The small differences in abscissae between the loops and a chosen straight line were obtained and plotted to a much larger scale. The loops are shown in Fig.21 - 27.

Subsequent tests showed that these loops did not assume a regular form even after several cycles had been performed. The smaller loops suffered from insufficient accuracy in deflection measurement, while the larger loops were affected by the existence of a small creep effect with time.

Apart from the hysteresis loops, the experiment shows clearly that repeated loading beyond the slip load does not produce repeated slip. The slip remains at the value corresponding to the maximum load applied.

ALTERNATING LOADING.

A series of complete loops for the same joint is shown in Fig. 28, the loads being carried beyond the slip value. It will be observed that slip in one direction remains on removal of load; further, on reversal of load the slip is not affected till the reversed load attains its slip value. This applies, of course, only to the single row joint.

A series of full loops on the four-row joint is shown/

FIG. 29.
Hysteresis Loops
Four Row Joint.

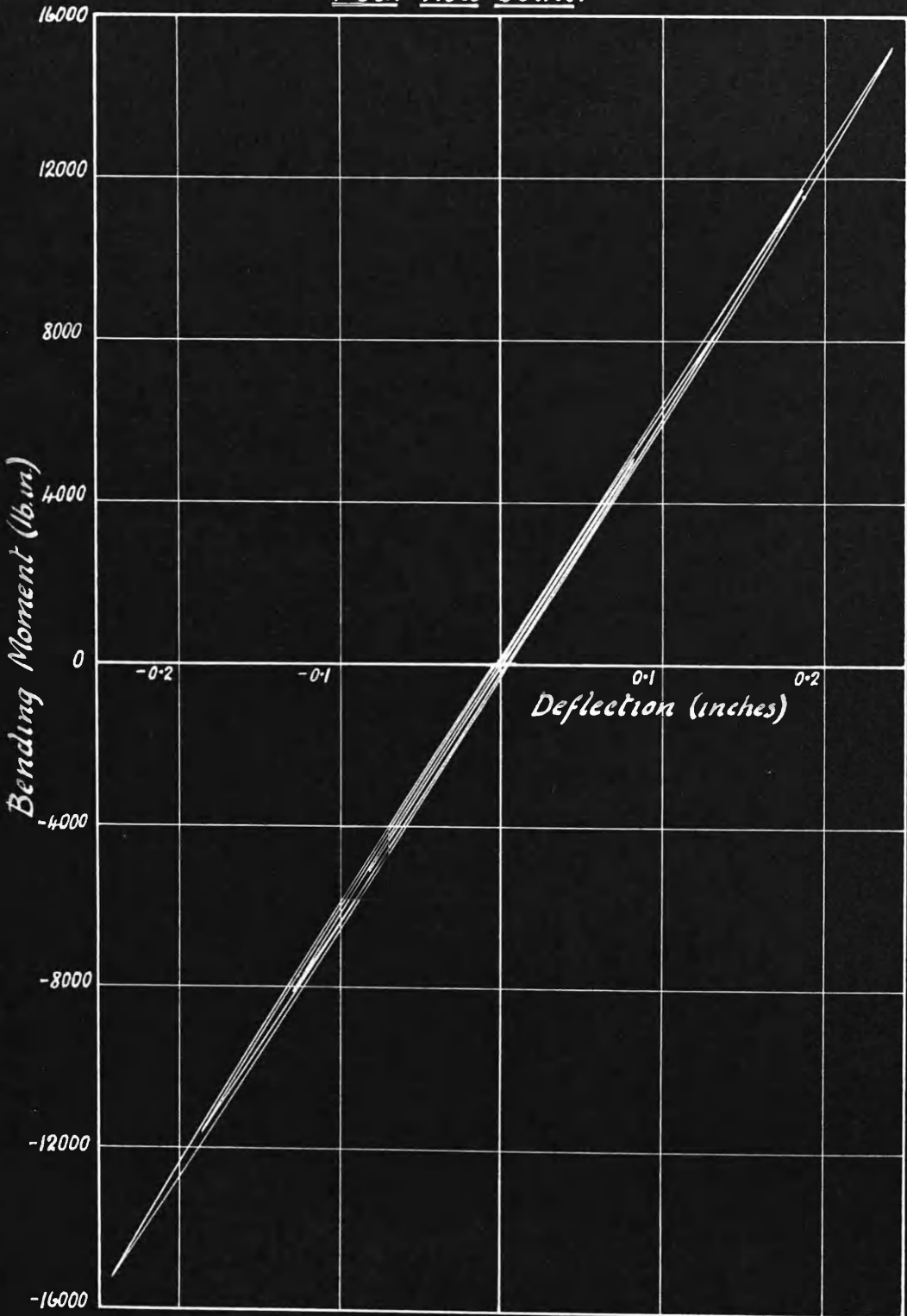
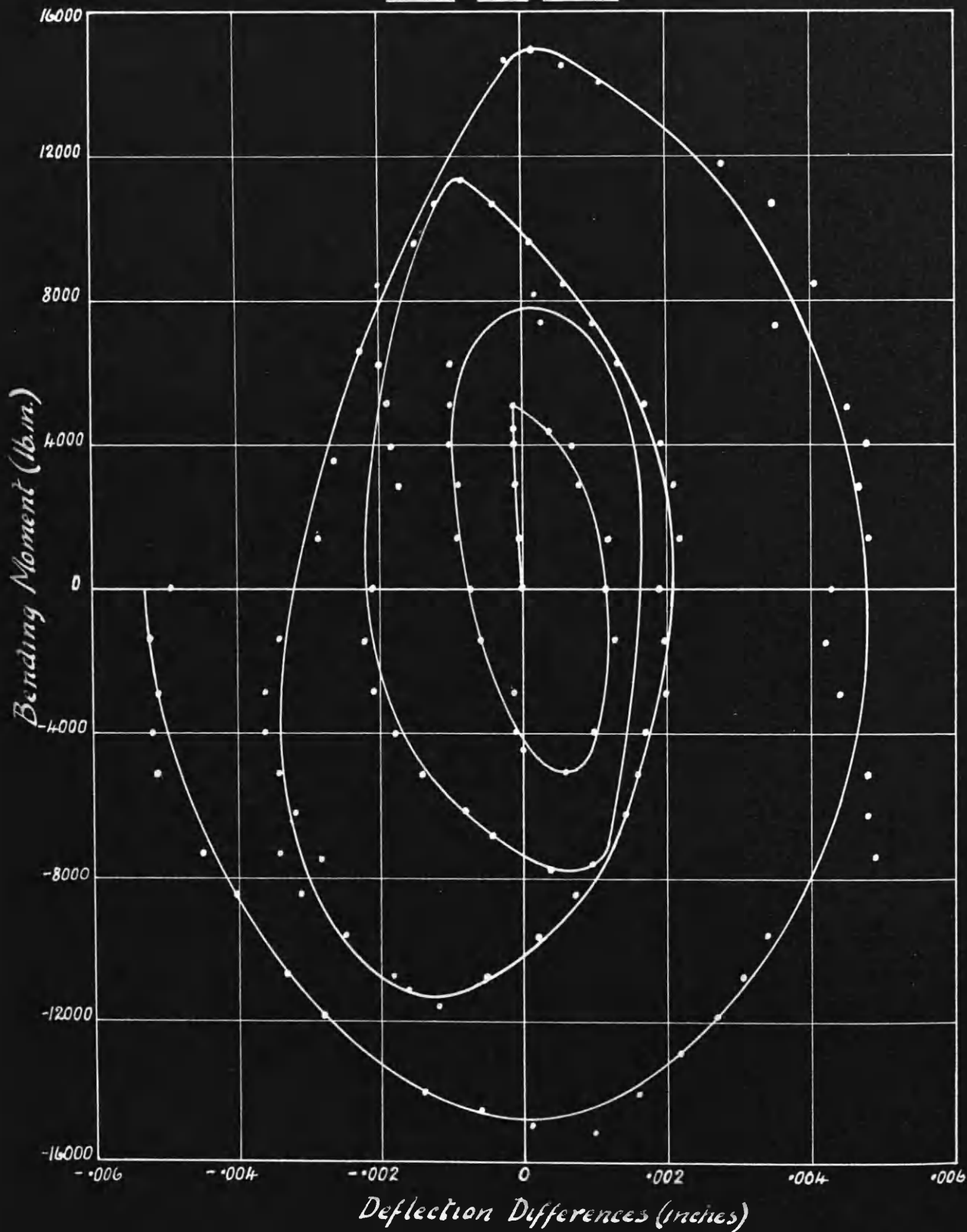


FIG. 30.
Hysteresis Loops
Four Row Joint.



shown using the normal scale in Fig. 29. Their development by the method of differences, instead of leading to a number of symmetrical loops, gives the erratic continuous curve of Fig. 30.

Actually it was not found possible under any circumstances to determine a characteristic form for the hysteresis loops. In addition, comparison of the energy represented by the loop area in the case of the half loops showed that this energy was from 100 to 1000 times greater than the work lost per cycle in the subsequent vibration experiments. For the full loops the ratio was much greater. The further description of static loops is therefore omitted.

PART III.

JOINT INVESTIGATIONS BY VIBRATION TESTS.

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RELEVANT VIBRATION THEORY.

It has been considered desirable to include here for convenient reference certain extracts from standard vibration theory. No extensions are made, but the extracts have been adapted to give a concise presentation of the theory used in the subsequent calculations. The application of the theory is shown to be justified by the results of other theoretical and practical investigations.

For a mass W lb. with one degree of freedom, vibrating under the action of an elastic restoring force together with a resisting force proportional to the velocity, the equation of motion may be written:-

$$\frac{W}{g} \ddot{x} = - R\dot{x} - Fx$$

where F = restoring force per unit displacement,
and R = resisting force per unit velocity.

$$\text{or } M\ddot{x} + R\dot{x} + Fx = 0$$

$$\text{where } M = \frac{W}{g}$$

If, in addition, a force with a maximum value P and a simple harmonic variation be applied, the equation becomes

$$M\ddot{x} + R\dot{x} + Fx = P \cos pt$$

the well known solution of which may be written in the form

$$x = \frac{P}{h \sqrt{R^2 + \left(Mh - \frac{F}{h}\right)^2}} \sin(pt - \alpha)$$

$$\text{where } \tan \alpha = \frac{Mh - \frac{F}{h}}{R}$$

When $f = n$ (resonance), $\alpha = 0$, $x = \frac{P}{fR} \sin ft = \frac{P}{fR} \cos\left(ft - \frac{\pi}{2}\right)$

The displacement lags 90° behind the impressed force at resonance.

$$\text{Maximum displacement} = x_m = \frac{P}{fR}$$

WORK DONE PER CYCLE.

$$\text{Force} = P \cos ft$$

$$\text{Velocity} = \frac{P}{R} \cos ft$$

(Force and velocity are in phase at resonance)

$$\begin{aligned} \text{Work done per cycle} &= 2 \int_0^{\frac{\pi}{f}} P \cos ft \cdot \frac{P}{R} \cos ft \cdot dt. \\ &= \frac{2P^2}{2R} \int_0^{\frac{\pi}{f}} (1 + \cos 2ft) dt. \\ &= \frac{\pi P^2}{Rf} = \pi P x_m \end{aligned}$$

FORM OF HYSTERESIS LOOP.

Alternatively, with the force as $F = P \cos \theta$

the displacement may be written as $x = A \sin \theta + B \cos \theta$

Eliminating θ from these equations gives

$$\left\{ x - B \left(\frac{F}{P} \right) \right\}^2 = A^2 \left(1 - \frac{F^2}{P^2} \right)$$

$$\text{or} \quad x^2 - \frac{2B}{P} Fx + \frac{A^2 + B^2}{P^2} F^2 - A^2 = 0$$

This/

This is the equation of an ellipse, the presence of the Fx term indicating that the axes of the ellipse are inclined to the vertical and horizontal axes of F and x .

At resonance, $(f = n)$, $B = 0$

and the equation becomes

$$\frac{x^2}{A^2} + \frac{F^2}{P^2} = 1$$

the equation of an ellipse with semi-axes A and P vertical and horizontal.

Area of ellipse = πPA

where A = maximum amplitude value = $\frac{P}{hR} = x_m$

APPLICATION TO PRACTICAL CASES.

In practical cases the damping often differs widely from the simple type assumed above; but because of the mathematical complication resulting from other types of damping, an approximate solution is usually obtained on a basis of suitable simplifying assumptions.

For any type of damping, analysis of the damping forces is conveniently carried out from the logarithmic decrement shown by the curve of free damped vibration. Records taken on riveted steel structures generally show mixed damping, with solid damping and viscous damping as the principal constituents. An exact analytical solution for this particular case of mixed damping has recently been propounded/

propounded and checked experimentally by J. P. Den Hartog¹⁰.

The analysis shows that, where the damping forces are not heavy enough to distort the vibration form to any great extent, the phase angle between impressed force and displacement at resonance is so near to 90° that the cosine may be taken as unity without appreciable error. The small departure from 90° is due to the presence of solid friction. The analysis by L. S. Jacobsen¹¹ of systems damped by forces proportional to the n^{th} power of the velocity shows the phase angle to be always 90° at resonance.

Consequently in all cases where the vibration form is approximately a sine wave, it would appear that the assumption of a 90° phase angle between impressed force and displacement at resonance may be made with safety. The work per cycle is then πPA as in the simple theory, and the use of forced vibration methods gives a convenient and accurate estimate of the energy loss per cycle.

APPARATUS AND EXPERIMENTAL METHODS.

ARRANGEMENT OF VIBRATING SYSTEM.

Broadly speaking, the vibration experiments were intended to utilise the existing apparatus as far as possible, with a method of approach somewhat similar to that already developed in the static tests. For any one joint/

FIG. 31

Vibration Test Arrangement

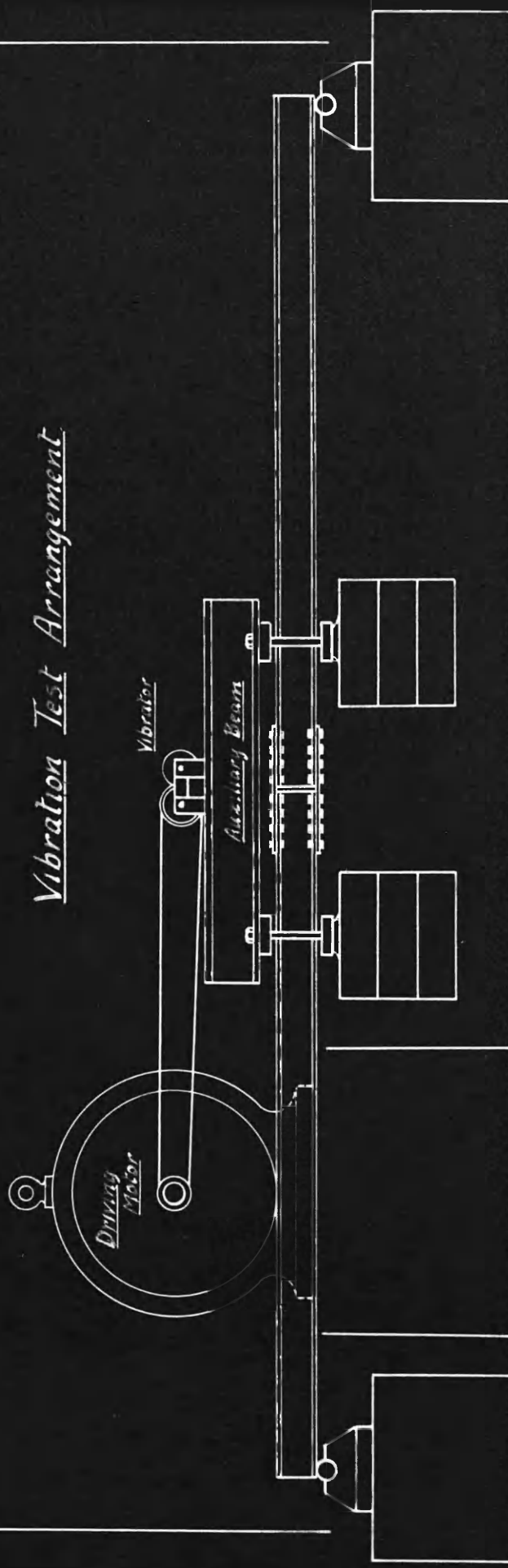
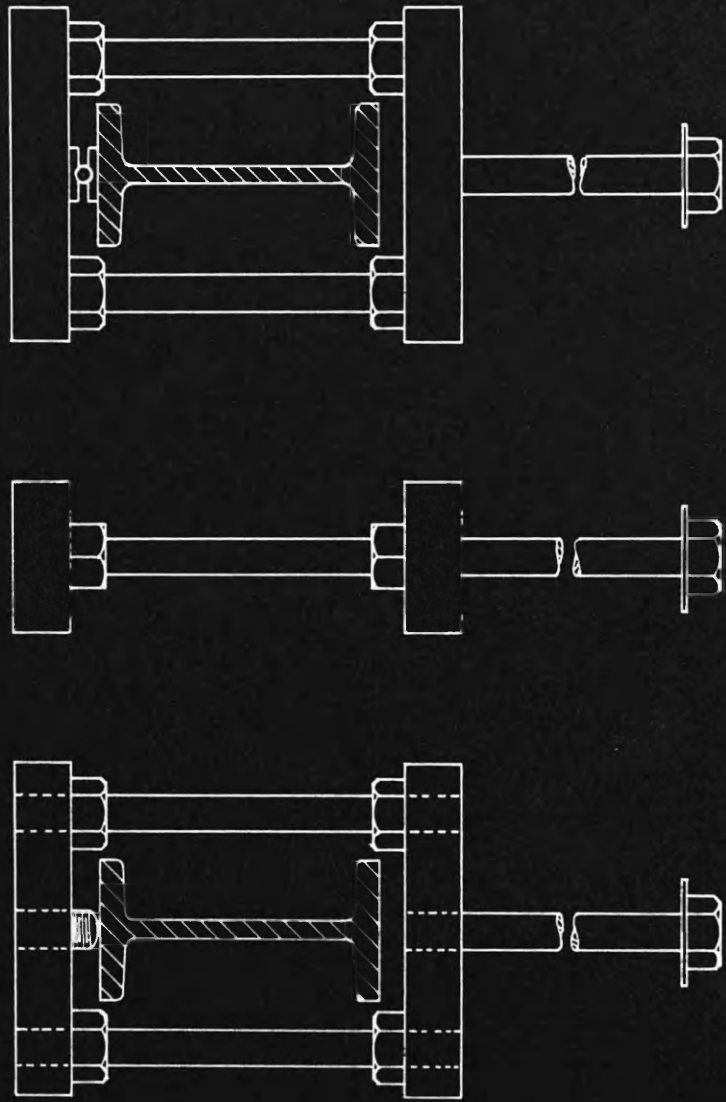


FIG. 32
Load Carriers



joint arrangement, vibration tests with end rows only and also with additional rows would show the effect of the additional rows. Tests on jointed and unjointed beams, provided a suitable basis of comparison could be found, would give the absolute value of the damping forces in the joint.

Apart from the fact that the introduction of vibrations required certain modifications in the apparatus, the process would appear to be easy and straightforward. In reality, the difficulties experienced in attempting to secure consistency in the results proved so great as to appear for a time almost insuperable. The final details of the experimental arrangements, shown generally in Fig. 31, were evolved as a result of a long series of preliminary tests, and exhibit certain features which may appear peculiar at first sight. The reasons for these are enumerated in the subsequent chapters.

The beams were supported at each end of the span on a simple roller, and a relatively massive and rigid foundation was secured by utilising the table of a planing machine as a convenient test bed. The machine in turn was carried on a heavy concrete foundation. The loading was accomplished by deadweights, tightly clamped together in specially constructed carriers (Fig. 32) and freely suspended from the beam in such a way that, for each load, contact with the/

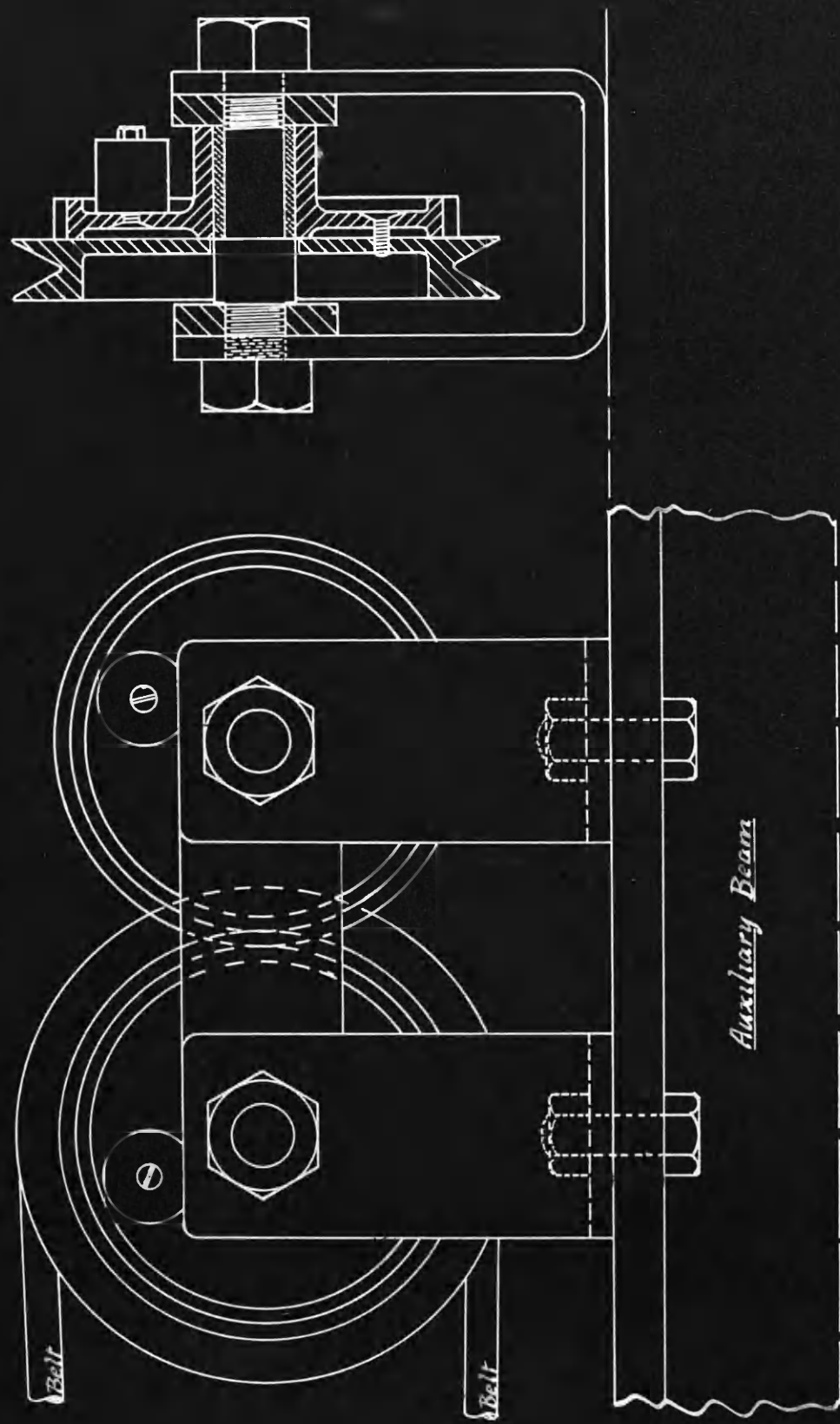
the beam occurred at one point only on the top surface.

A short length of a heavier section beam was rigidly attached to both carriers, this "auxiliary beam" with the two load carriers forming a rigid unit. The rigidity thus introduced necessitated some modification of the support conditions in order to avoid the application of a constraint to the beam flange. The necessary flexibility was introduced by arranging a ball and suitable hard steel grooved races between one of the carriers and the beam flange as shown in Fig. 32. The auxiliary beam formed a suitable and convenient mounting for the devices used in producing and measuring the vibrations.

VIBRATION PRODUCTION.

The preliminary experiments included the recording of a number of vibration decay curves for the loaded beams, using a suitable optical magnification system and a light-tight box fitted with a shutter and containing a slowly revolving drum carrying a photographic plate. While records were successfully obtained by this means, the results presented difficulties in interpretation. It was considered also that the possible existence of transient conditions might be one of the contributory factors in producing the inexplicable inconsistencies displayed. In addition, the time required for the development and analysis of each plate constituted a serious drawback. Preference was therefore given to the method whereby forced vibrations were produced by/

FIG. 33
Vibrator - Full Size



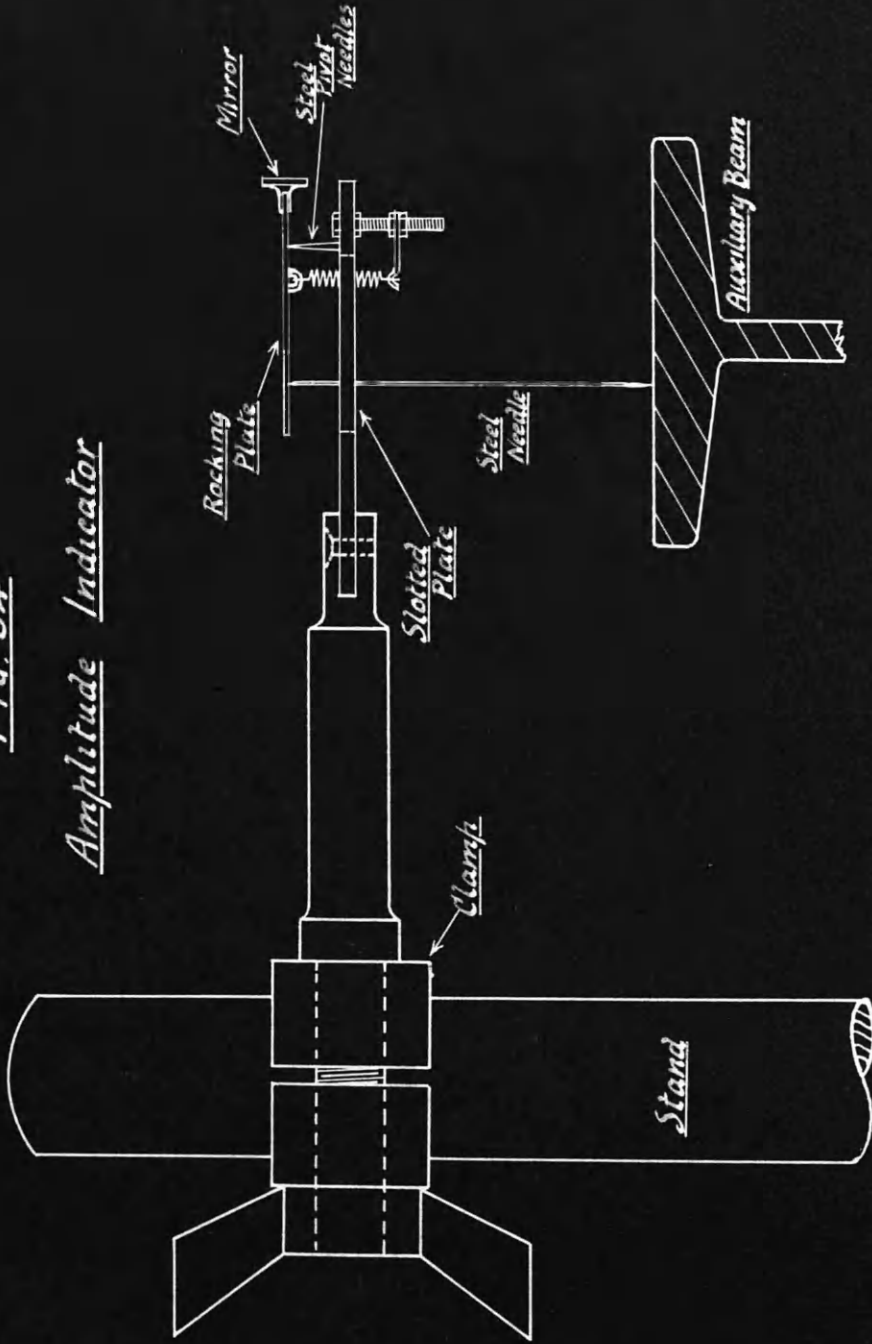
by an impulsive force whose frequency was adjusted to give the condition of resonance, and the resulting amplitudes were measured.

VIBRATOR.

The design of a suitable instrument for vibration production required some preliminary investigation and estimate of the magnitude of the impulsive forces required. The "vibrator" finally took the form of the Lanchester balancing device, and consists of two wheels geared together and running in opposite directions. Each wheel carries a small weight attached in such a position that both weights pass simultaneously through the vertical centre lines of the wheels. Combining the inertia forces of the two weights for any position of the wheels shows that the resultant inertia force has no horizontal component but is a purely vertical force with a simple harmonic variation. The size of the vibrator was determined by the weights which it had to accommodate. These in turn were dependent on the range of frequencies and the energy lost per cycle in the various beams. The dimensions and general arrangement are shown in Fig. 33.

The wheels are driven from an electric motor by a horizontal belt operating in a V-grooved pulley attached to one of the wheels. The actual weights used being small, in order to eliminate error it was necessary to form an estimate/

FIG. 34
Amplitude Indicator



estimate of the unbalanced effect of the instrument. For this purpose, it was mounted on the free end of a light rod arranged as a cantilever, and a small "exploring" mass was attached to each wheel. The masses were arranged at a number of angular positions on the wheels, but always in such a way that only a simple harmonic vertical force was produced. The corresponding beam amplitude readings disclosed the existence of a minimum amplitude, and thereby located the unbalance in the wheels. With the masses arranged to give this minimum amplitude, it was a simple matter to alter their values to make the amplitude zero, and the wheels were accurately balanced.

VIBRATION MEASUREMENT.

The ease with which joint distortion could be measured as a result of the method of construction has already been explained. Similar advantages are obtained in the measurement of vibration amplitudes; but since the important aspects of the results were concerned with the differences between jointed and jointless beams, it was still necessary to effect such measurement with the greatest possible accuracy.

In the arrangement finally adopted (Fig. 34), the vertical motion of the beam was communicated to a light plate, producing a rocking motion of the plate about a horizontal axis. A ray from a concentrated source of light was/

was focussed on a small mirror mounted on the rocking plate, and the reflected spot of light was intercepted on a vertical scale approximately 6 feet distant. A vibration of the beam caused a corresponding vibratory motion of the spot of light, producing virtually a vertical line of light whose length was a measure of the beam amplitude. The usual magnification was in the region of 300.

The introduction of frictional damping influences in the measuring device itself was avoided by supporting the rocking plate on the points of chrome steel needles. Two needles at one end were fixed, and formed the pivot on which the plate rocked. The single needle at the other end was held in position between the rocking plate and the vibrating beam by the tension of a light steel wire helical hair spring.

To enable the instrument to include all the necessary amplitude measurements in its range while keeping the angular displacement of the rocking plate within reasonable limits, alternative positions for the vibrating needle were provided at varying distances from the support needles. The whole instrument was mounted on a stand in such a way that its vertical position could be quickly and conveniently adjusted to suit any beam. Suitable precautions had to be taken during the experiment to prevent the development of resonant vibrations in the stand itself.

TEST/

FIG. 35.

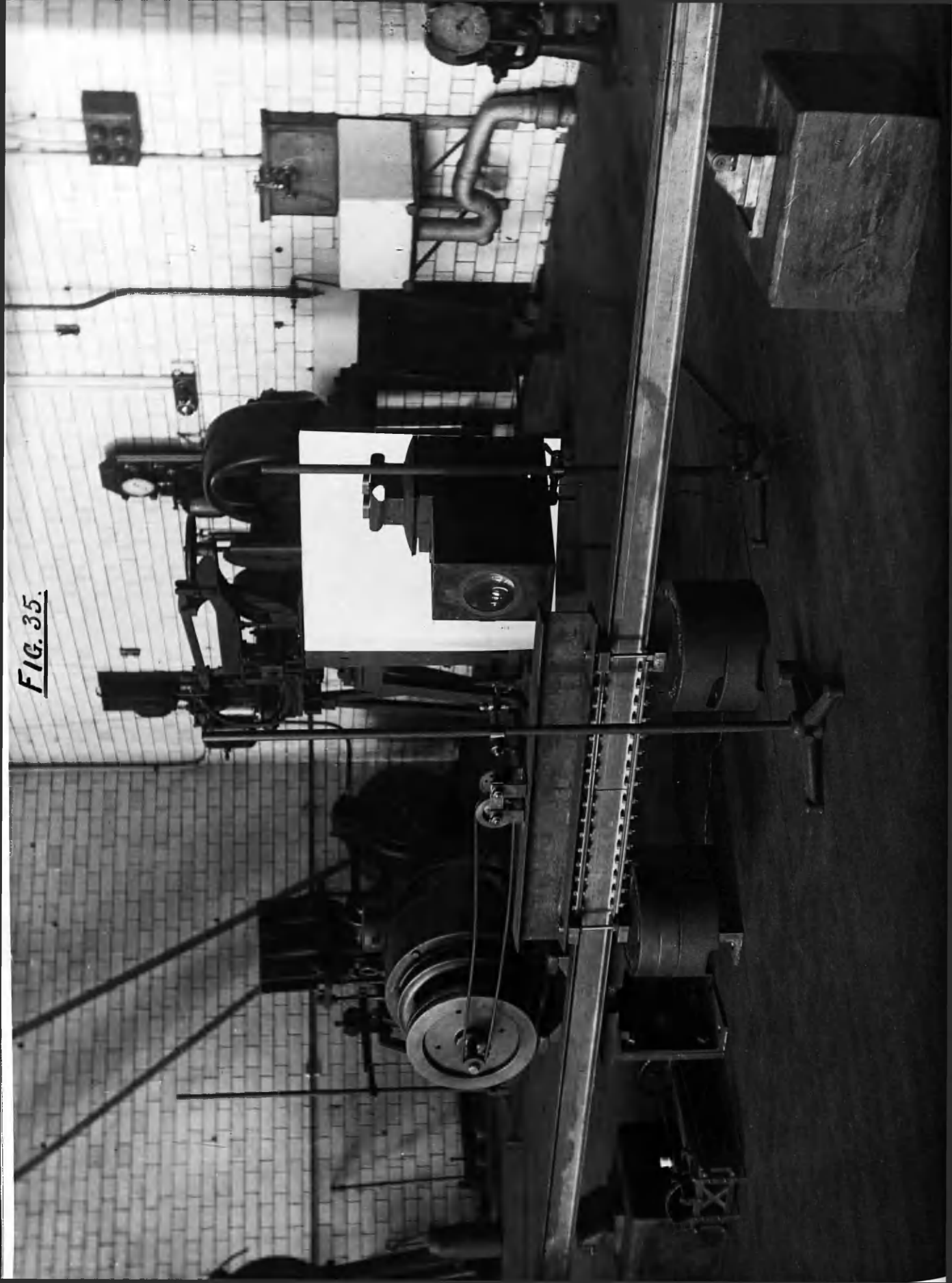
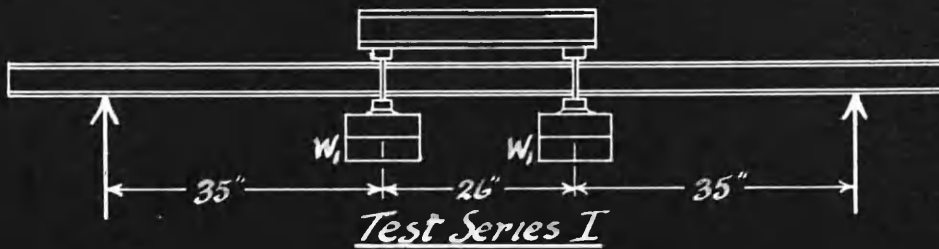


FIG. 36 - Vibration Test Arrangements

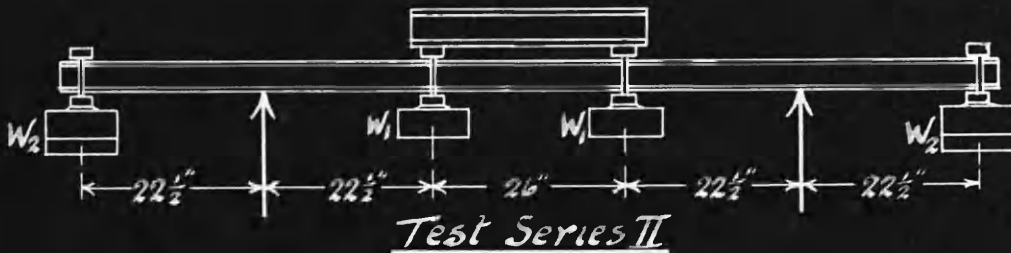


Equivalent Loads (including beam inertia allowance)

Loading	A	$W_1 = 184 \text{ lb.}$
"	B	$W_1 = 132 \text{ lb.}$
"	C	$W_1 = 81 \text{ lb.}$

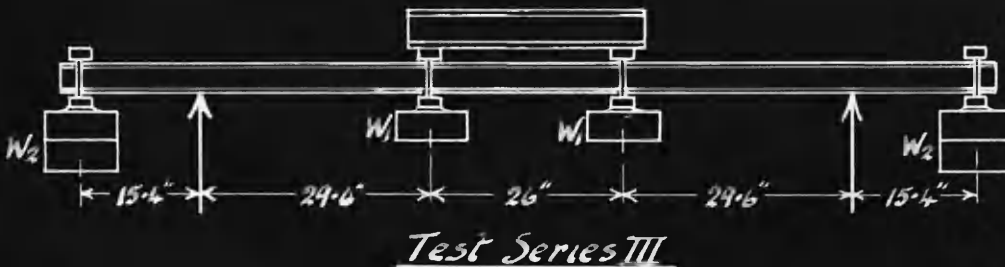
Resonant Frequencies per minute

Loading	A	B	C
1-row Joint	574	682	840
4-row Joint	604	714	911
8-row Joint	640	755	965
Flange Plates	652	770	-



Equivalent Loads (including beam inertia allowance) - $W_1 = W_2 = 82 \text{ lb.}$

Test Specimen	Plain	1-row	4-row	8-row
Frequency per minute	848	840	905	944



Equivalent Loads (including beam inertia allowance) - $W_1 = 81 \text{ lb.}, W_2 = 158 \text{ lb.}$

Test Specimen	Plain	1-row	4-row	8-row
Frequency per minute	744	753	793	840

TEST PROCEDURE.

A general view of the apparatus assembled ready for test is given in the photograph, Fig. 35. With definite weights attached to the vibrator, the motor was started and adjusted nearly to the resonant condition. The speed of the motor was varied several times up and down through the critical value as slowly as possible, till it lingered on the critical speed long enough to allow the maximum amplitude to develop. Measurement of this critical speed enabled the actual value of the impulsive force to be calculated and associated with the corresponding amplitude reading in the construction of force-amplitude curves.

The process was repeated with different weights on the vibrator, each weight giving a point on the curve. A series of such readings was obtained for the unjointed beam under different load arrangements, and repeated for each jointed beam using end rows only and all rows together.

TEST SEQUENCE.

Three series of tests were carried out, with the load systems shown in Fig. 36. In Series I, the amplitude of the vibrations was of necessity always less than the static deflection, consequently no reversal of stress in the joint could occur. In Series II, the static loading was altered so that the bending moment on the joint was zero. In vibration, therefore, the joints were subjected to an alternating load with a mean value zero. Series III was

a repetition of Series 11, but an attempt was made to obtain larger amplitudes by using a different arrangement of the load system.

EXPERIMENTAL DIFFICULTIES.

A description of the experimental procedure is apt to convey a misleading impression, and for the guidance of anyone who may consider applying similar methods to full scale joints, it has been considered advisable to include a description of the unexpected difficulties that had to be overcome before the following results could be obtained.

The preliminary attempts in vibration measurement were marked by an entire lack of consistency. The individual readings appeared to be quite haphazard and could not be related to each other in any way. The factors contributing to this failure were numerous enough to prevent any one of them being definitely identified, and several months intensive effort was devoted to discovering an arrangement which would give dependable results. The test of the success of any method of mounting and loading the beam was taken to be its ability to produce the same series of readings on two separate occasions, between which the apparatus (except the joints) was dismantled and reassembled.

The first peculiar source of inaccuracy was found in the supports. The supporting rollers rested in ordinary/

ordinary type journal bearings carried in rigid metal mountings which were elevated to the required height above the test bed by placing them on solid blocks of metal. All the surfaces in contact were machined to a smooth finish, and were as truly plane as ordinary machining processes could make them. Nevertheless, it was discovered that, although no relative movement of any kind could be detected at the supports, the application of any additional pressure to the roller mountings altered the indicated amplitude of vibration.

Investigation showed that the amplitude was unaccountably sensitive to support conditions, but the effect could be practically eliminated by applying a heavy pressure to the roller mountings; these were therefore glanded down. Curiously enough, the additional pressure caused a decrease in the recorded amplitude, the only obvious explanation being that a slight departure of the machined surfaces from the truly plane condition permitted a rocking movement too small to be detected but with an appreciable influence on the roller damping effect. Similar but smaller effects were discovered at other points - the deadweights used in loading, for example, instead of resting on top of each other in a rigid carrier, had to be tightly clamped together.

Still greater difficulties arose in driving the vibrator. The motor first employed was of $\frac{1}{4}$ H.P. rating, and/

and although the power input to the beam at resonance was negligible it was sufficient to affect the motor speed appreciably. In approaching the resonant condition by altering the motor speed as slowly as possible, it was found that when resonance was approximately attained the motor speed suddenly slipped through the critical value and increased beyond it. The converse process of speed reduction showed an exactly similar effect.

To eliminate this regulation difficulty a much more heavily built motor rated at 1 H.P. was utilised. This appeared moderately successful in obtaining consistent results with light beam loads and high frequencies, the motor then running at full speed of 2500 r.p.m. With heavier beam loads the motor speed had to be reduced by resistances connected in the armature circuit, and the inferior regulation characteristics of such an arrangement allowed the previous trouble to reappear.

In a final attempt to eliminate driving troubles, a heavy 4 H.P. motor was mounted, and to ensure slow speed changes a heavy flywheel was fitted to the shaft. Specially made pulleys enabled the motor to be run at the highest possible speed in order to obtain the maximum benefit from the inertia of the system. The elimination of regulation and inertia effects, however, discovered a source of trouble previously obscured, viz., fluctuations in the voltage of the public supply mains.

This/

This last feature constituted the most serious difficulty of all. The resulting speed fluctuations were small and would pass unnoticed in ordinary running; but they were effective in preventing attainment of the very low rate of speed change on which the successful measurement of a resonant amplitude depended. At odd intervals the effects were small enough to permit of readings being taken, but generally this was impossible. One attempt to secure a reading, represented by a single point in a curve, lasted for one and a half hours and had finally to be abandoned. The results were secured ultimately by confining the experimental readings to periods of industrial inactivity in the late evenings and at week-ends.

The experience gained in the preliminary experiments indicated that the complete elimination of damping would be undesirable. For a vibrating system with small damping forces, the curve of amplitude on a base of frequency assumes such a sharply pointed form that it is a matter of great difficulty to secure exact adjustment of the impressed frequency to the resonant value. The requisite damping exists in the load and beam suspensions to make the amplitude determination comparatively easy while still permitting a sufficiently accurate estimate of the difference in the damping effects exhibited by the jointed and unjointed beams.

RESULTS/

RESULTS.DAMPING FORCES IN UNJOINTED BEAM.

In mounting the loads and other attachments on the beam, care was taken to avoid the heavy and uncertain friction effects consequent on the clamping of an unstressed surface to another undergoing stress fluctuations. Apart from the damping influences in the material of the beam and in the joints, the only sources of friction occur in rolling contacts at the supports and at the points of suspension of the loads. It must be understood, of course, that at the support points the roller motion is so small that no bearing friction develops; the usual bearing clearance ensures that the rollers merely oscillate about a point at the bottom of the bearing.

For very small rolling motions of this type, simple pendulum vibration experiments show that the energy dissipation is proportional to the square of the amplitude. The hysteresis loss in the material of the beam has been shown by Kimball and Lovell⁹ to be proportional to the square of the stress range, and therefore conforms to the same law. It follows that for the unjointed beam the total friction losses are proportional to the square of the vibration amplitude.

The same relationship applies to the energy loss in the case of viscous damping, but the shape of the hysteresis loop is different. With the small amount of damping present in/

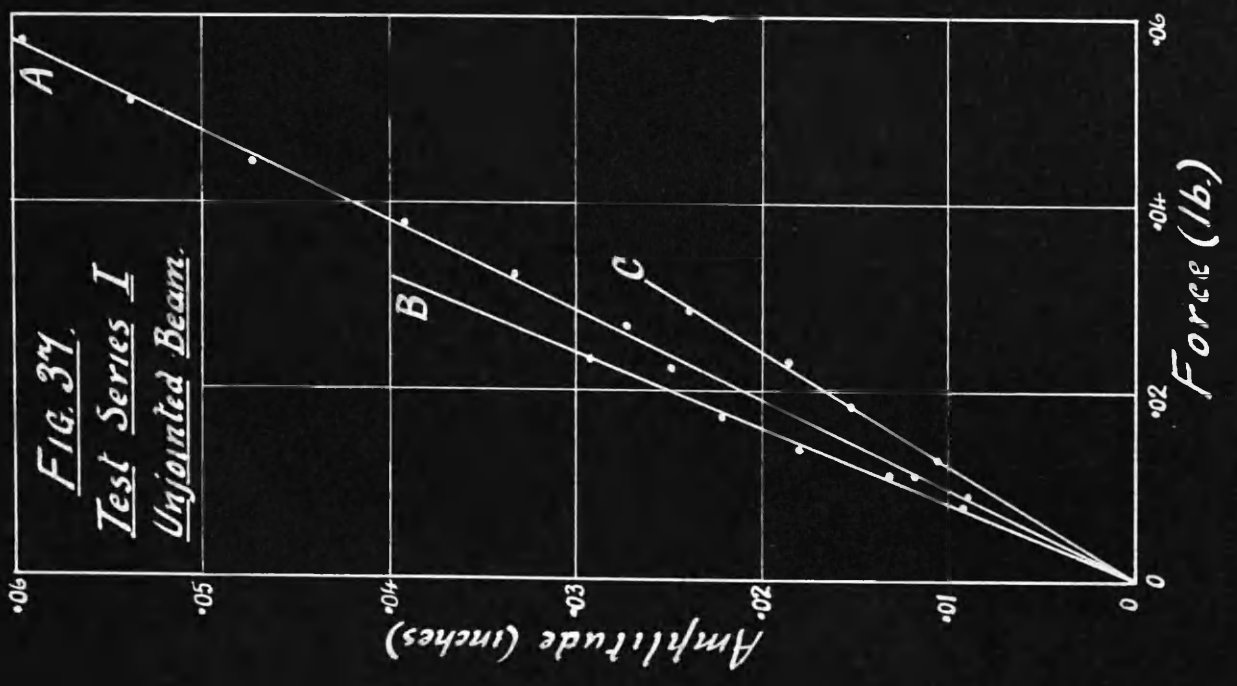
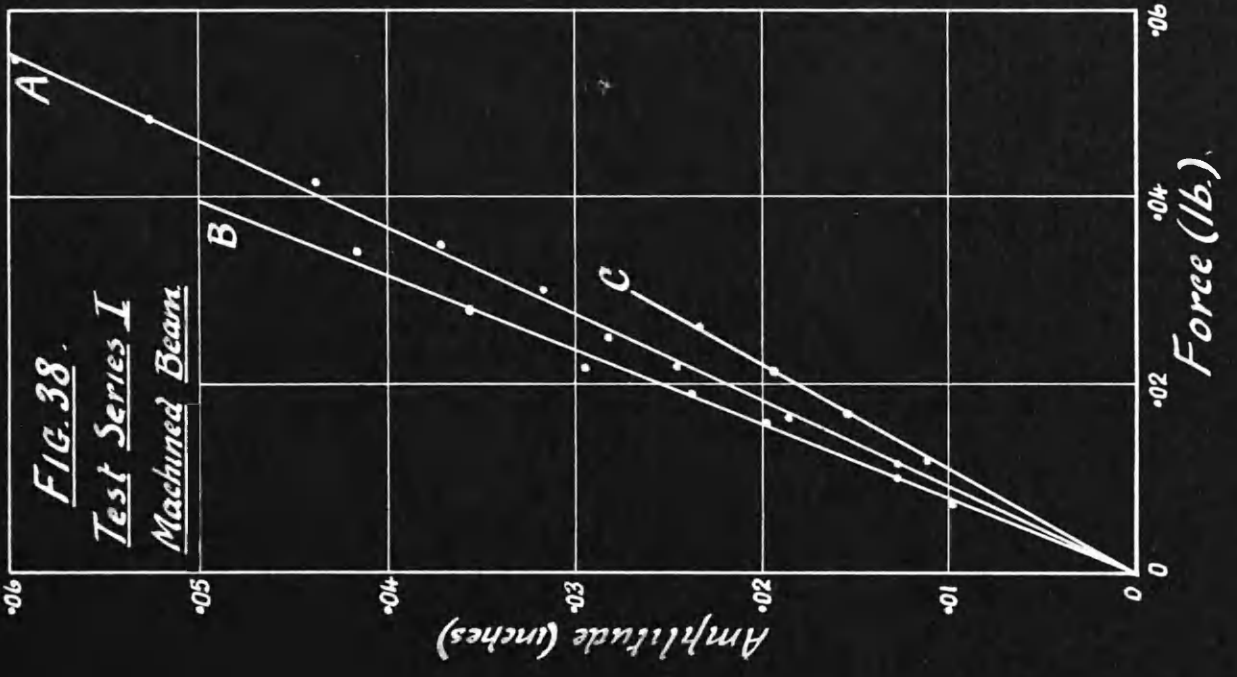


FIG. 39.
Single Row Joint.

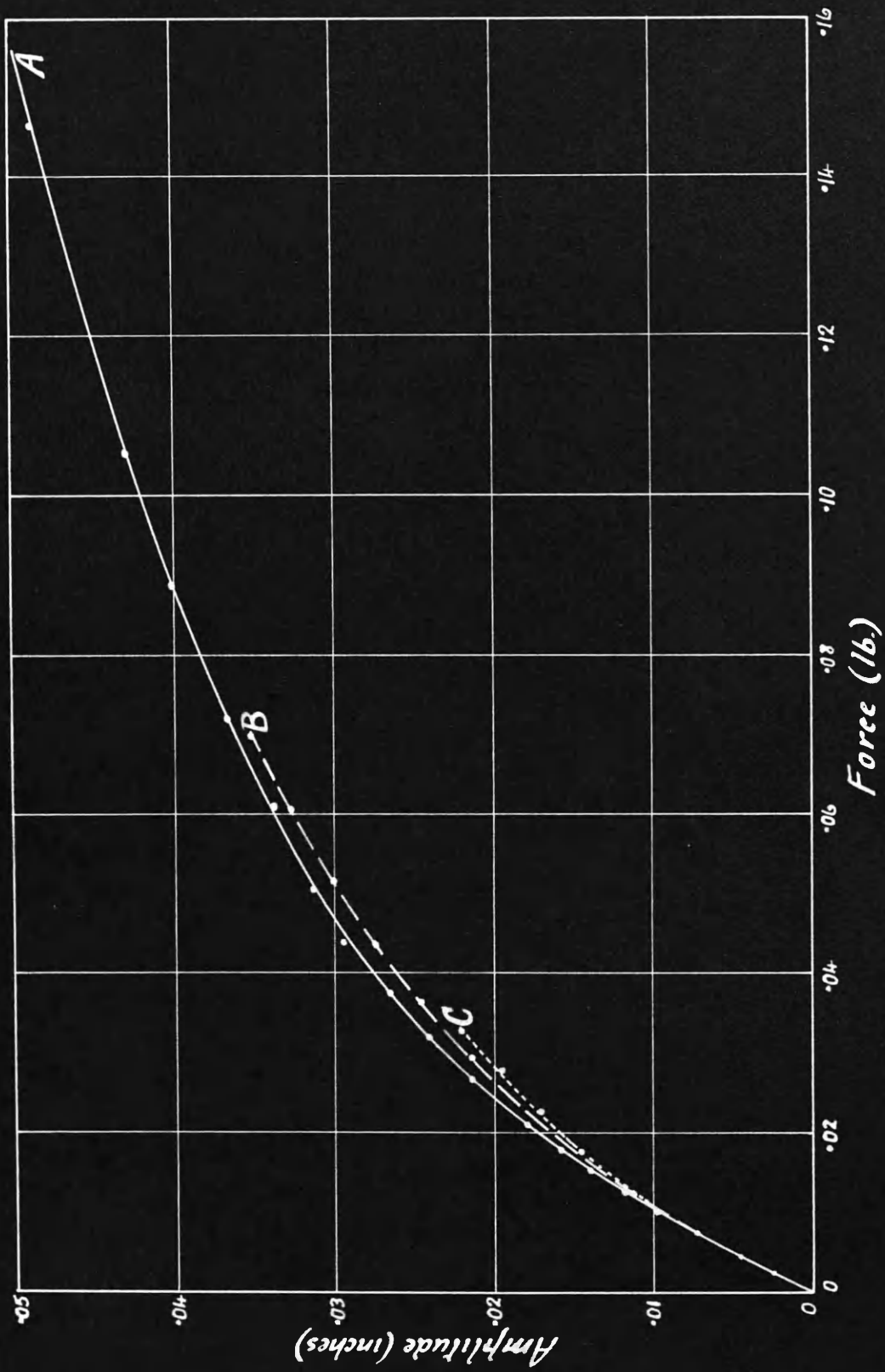


FIG. 40

A-Row Joint - End Rows

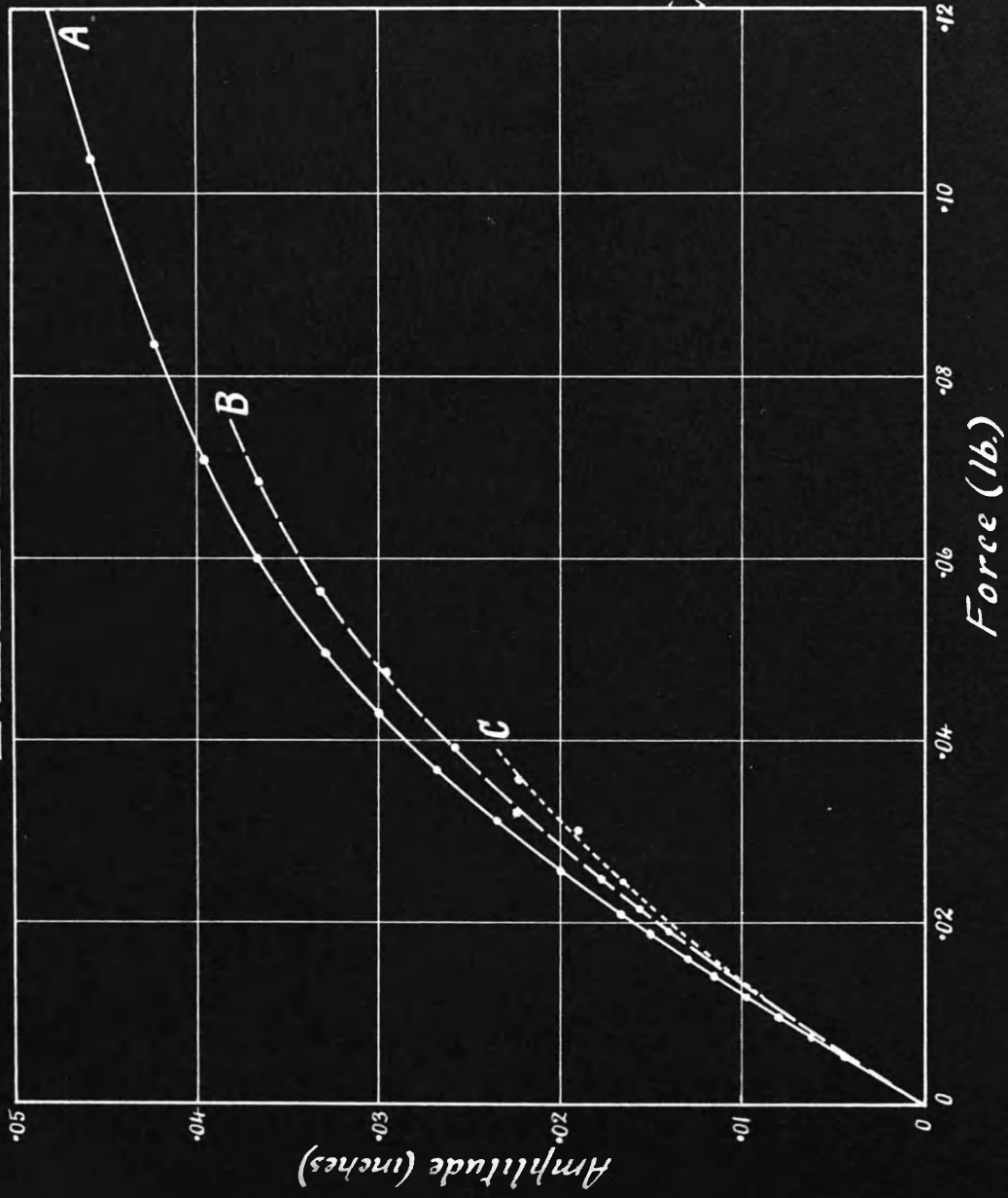


FIG. 41
4-Row Joint - All Rows

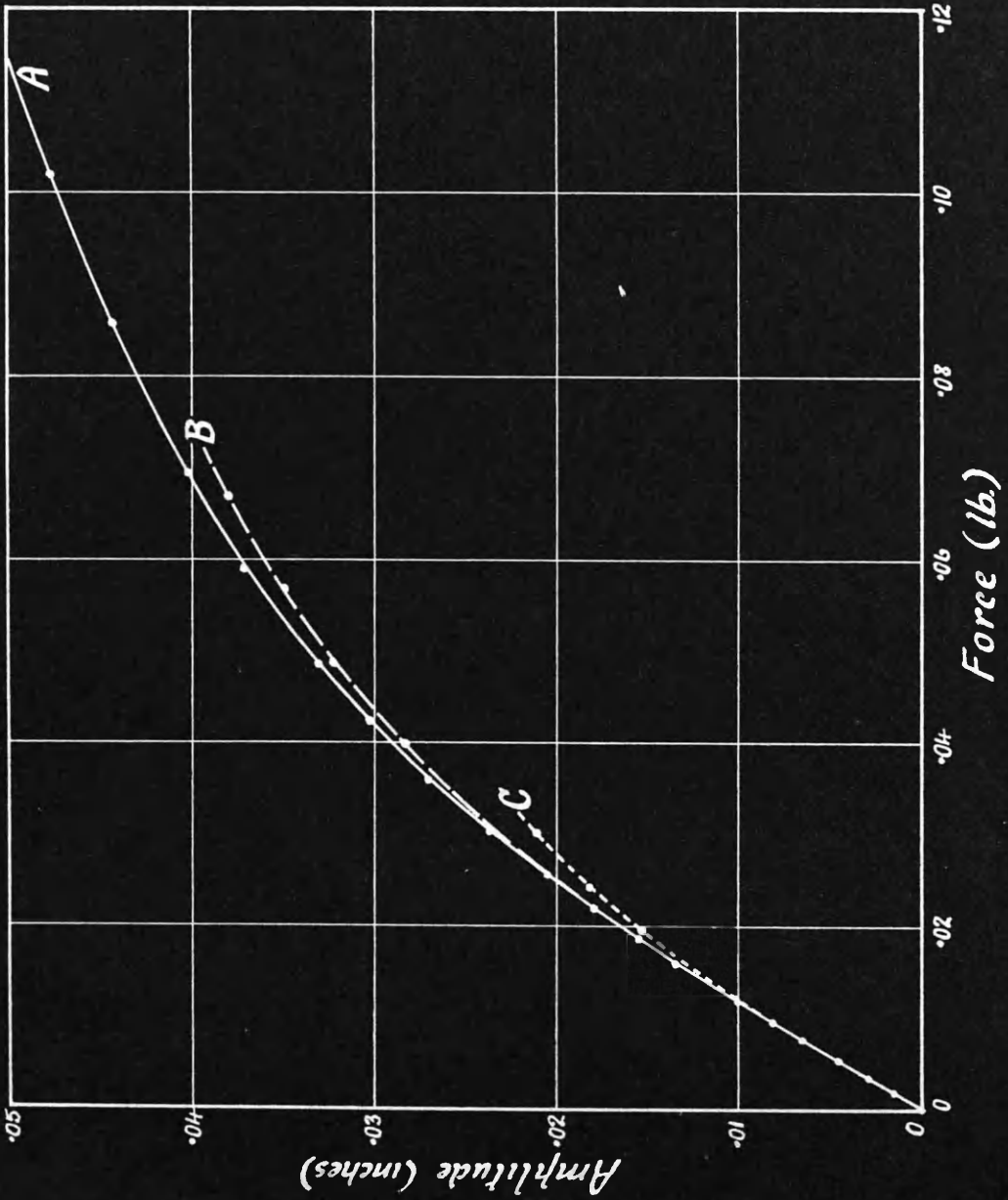


FIG. 42.
8-Row Joint - End Rows

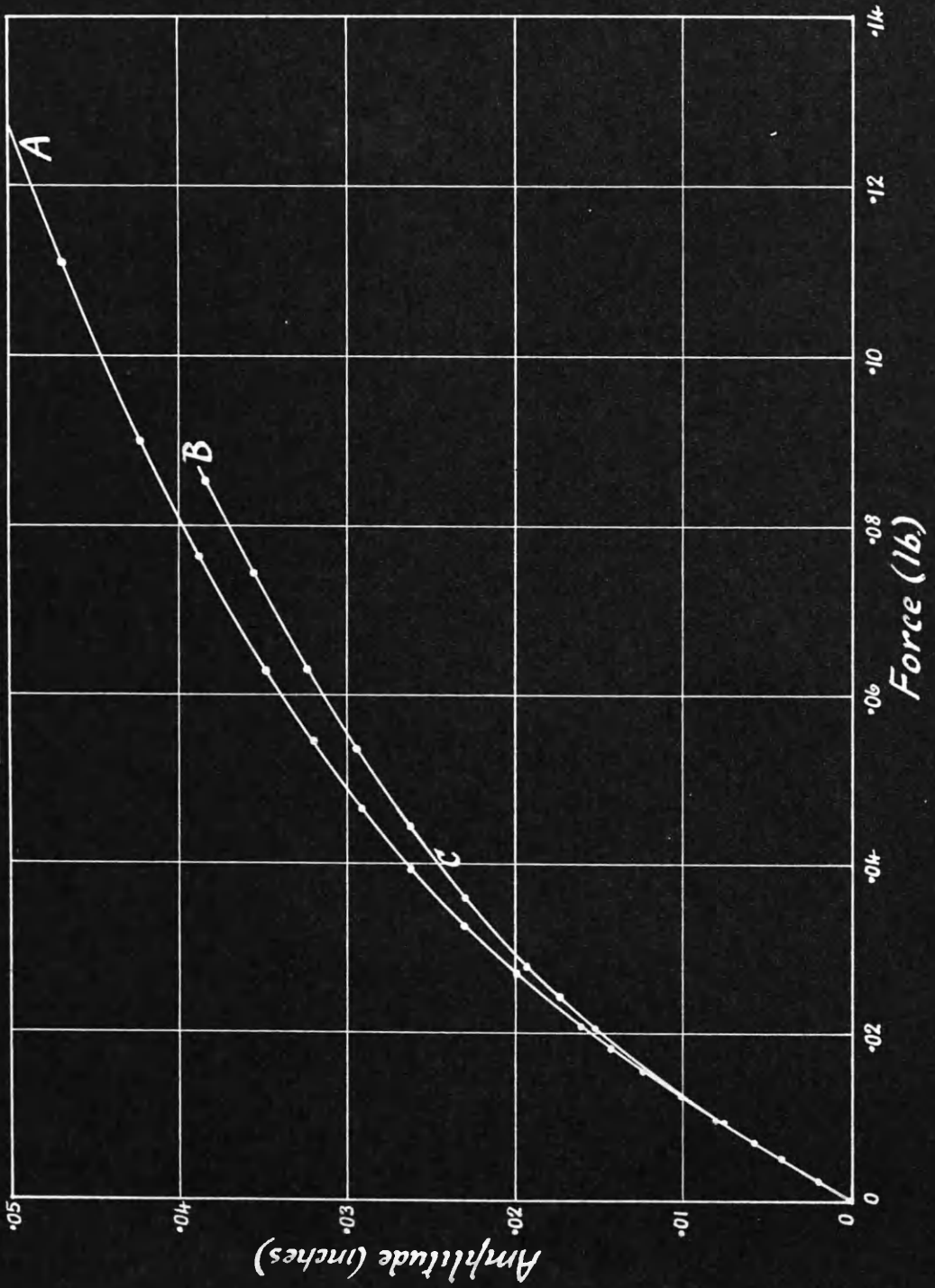


FIG. 43.

8-Row Joint - All Rows

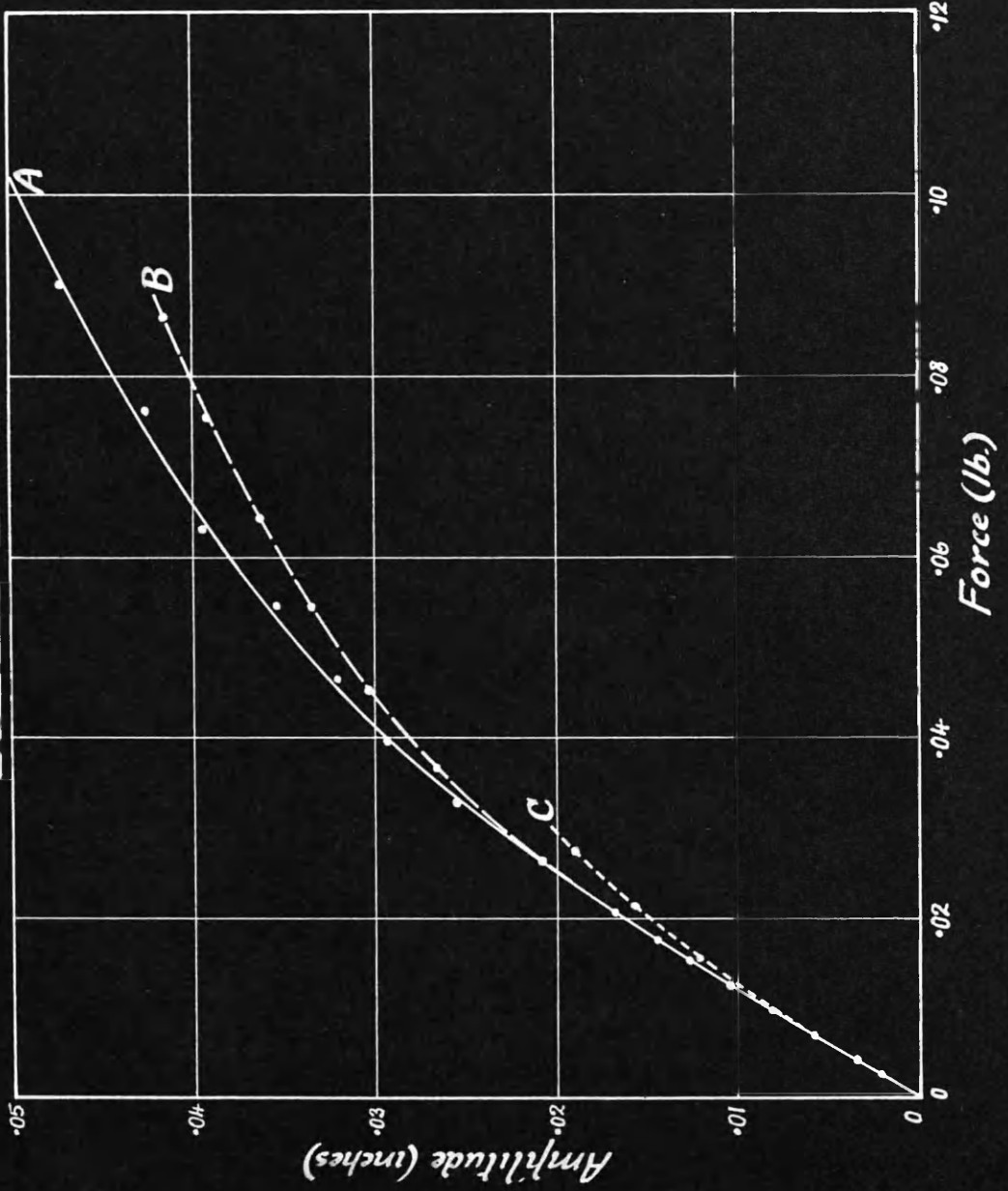


FIG. 44
Flange Plates - End Rows

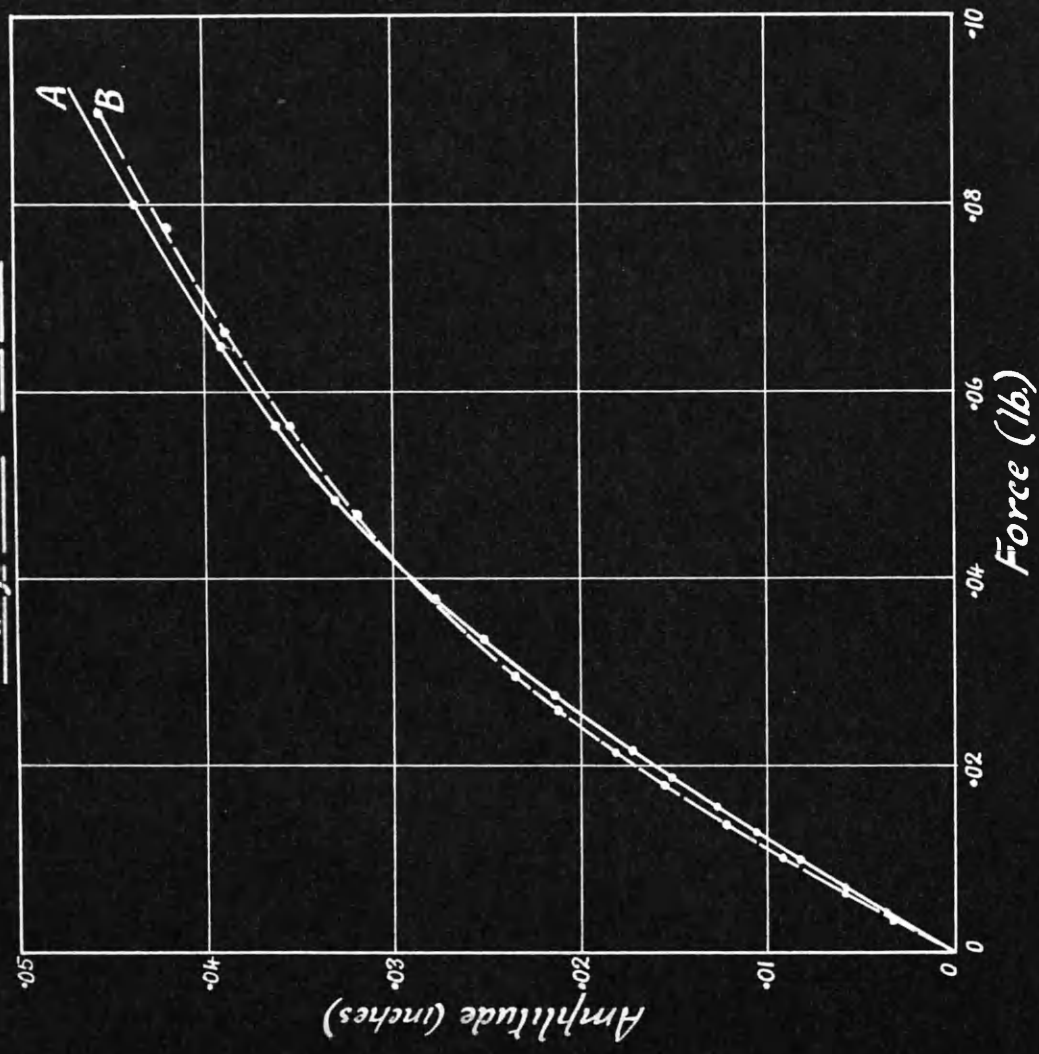
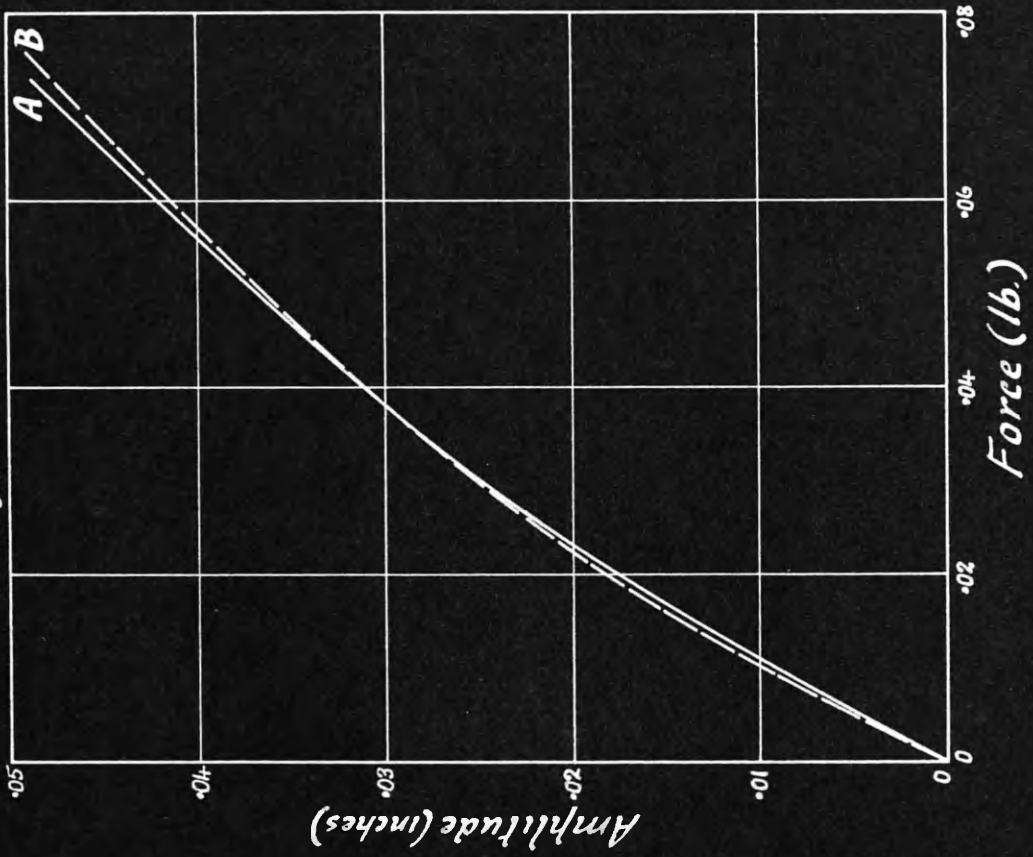


FIG. 45

Flange Plates - All Rows



in this instance, the displacement of the beam is not altered from the sine wave form, and on the assumption that the phase angle between force and displacement is 90° at resonance the work done by an impressed force with a maximum value P and a simple harmonic variation is given by:-

$$\text{Work per cycle} = \pi P x$$

where x is the amplitude of motion.

$$\text{But energy loss} = \text{const.} \times x^2$$

Equating these gives $x = \text{const.} \times P$

i.e. for any one system of loads the amplitude is proportional to the impressed force.

TEST SERIES I.

The results obtained from the unjointed beam are given in Fig. 37. The small amount of damping gives a relatively sharp resonance peak, but although the readings are slightly erratic the graph is recognisable as a straight line. For checking purposes the results of the same tests applied to a similar beam machined and drilled ready for the addition of flange plates are given in Fig. 38, and are substantially the same.

The inclusion in the beam of the various joint arrangements gives the results shown in Fig. 39 - 45. As might be expected, the joints cause a departure from the linear relationship between P and x . The increased damping leads to greater accuracy, and the points lie on smooth curves/

curves.

ESTIMATE OF ENERGY LOSS IN JOINTS.

It now becomes necessary to estimate the energy loss per cycle for the jointed beams. The nature of the damping effects is not known, but an indication of their magnitude is afforded by the fact that the form of the vibration appeared in all cases indistinguishable from a sine wave. It has been pointed out (page 56) that when this condition applies, the phase angle between the applied force and the displacement at resonance may be taken as 90° without appreciable error. Consequently the energy input to the system per cycle is $\pi P x$ as given by the simple vibration theory.

In estimating the difference in the energy losses in the plain and jointed beams, it has to be recognised that the frequencies differ, and the effect of frequency on the plain beam losses requires consideration. The experiments of Kimball and Lovell show that the elastic hysteresis loss in the material is independent of the frequency over a wide range. The same is approximately true of the friction loss due to very small rolling motions. Hence it may be assumed that for a given loading the dissipation of energy in the beam and supports is unaltered at the frequency of the jointed beams, and for any chosen amplitude the energy loss in a joint is given by

$$\text{Loss per cycle} = \pi x (P_1 - P_0)$$

where/

FIG. 46.
Energy Loss per Cycle
Test Series I - Loading A

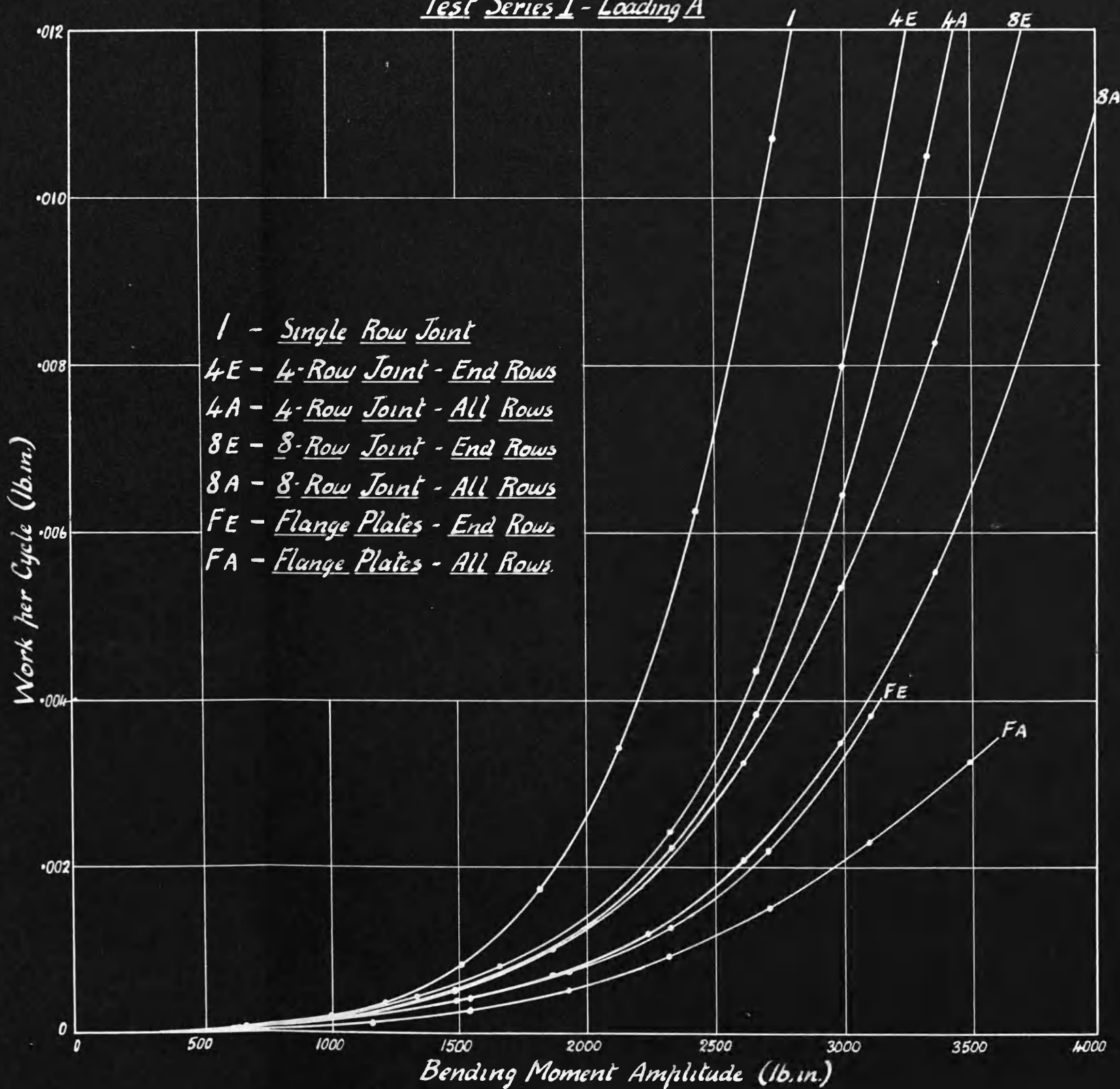


FIG. 47.

Energy Loss per Cycle
Test Series I-Loading B.

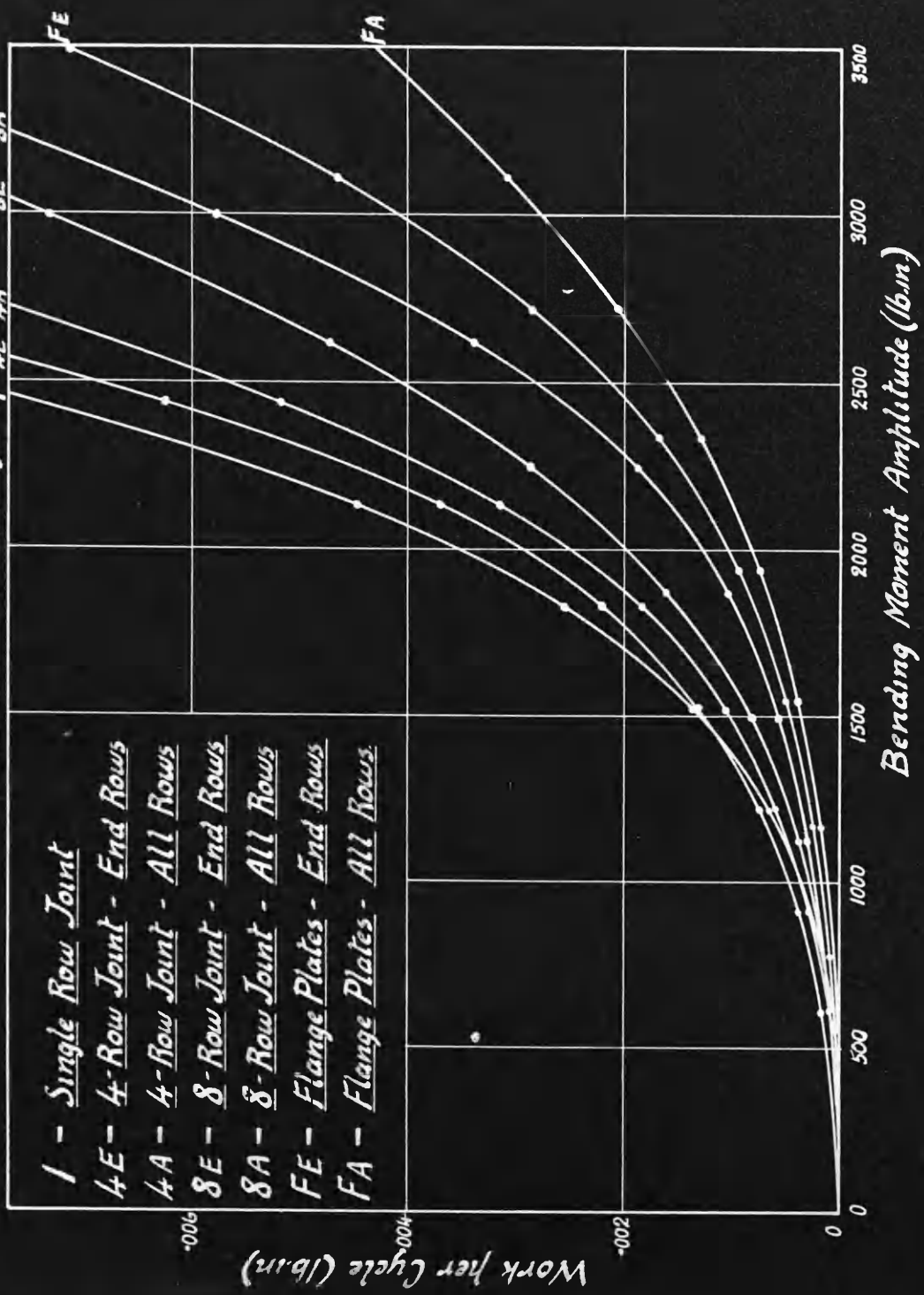


FIG. 4-8.

Series II - Unjointed Beam

1 - Outer Load Amplitude

2 - Inner Load Amplitude

3 - Joint Amplitude

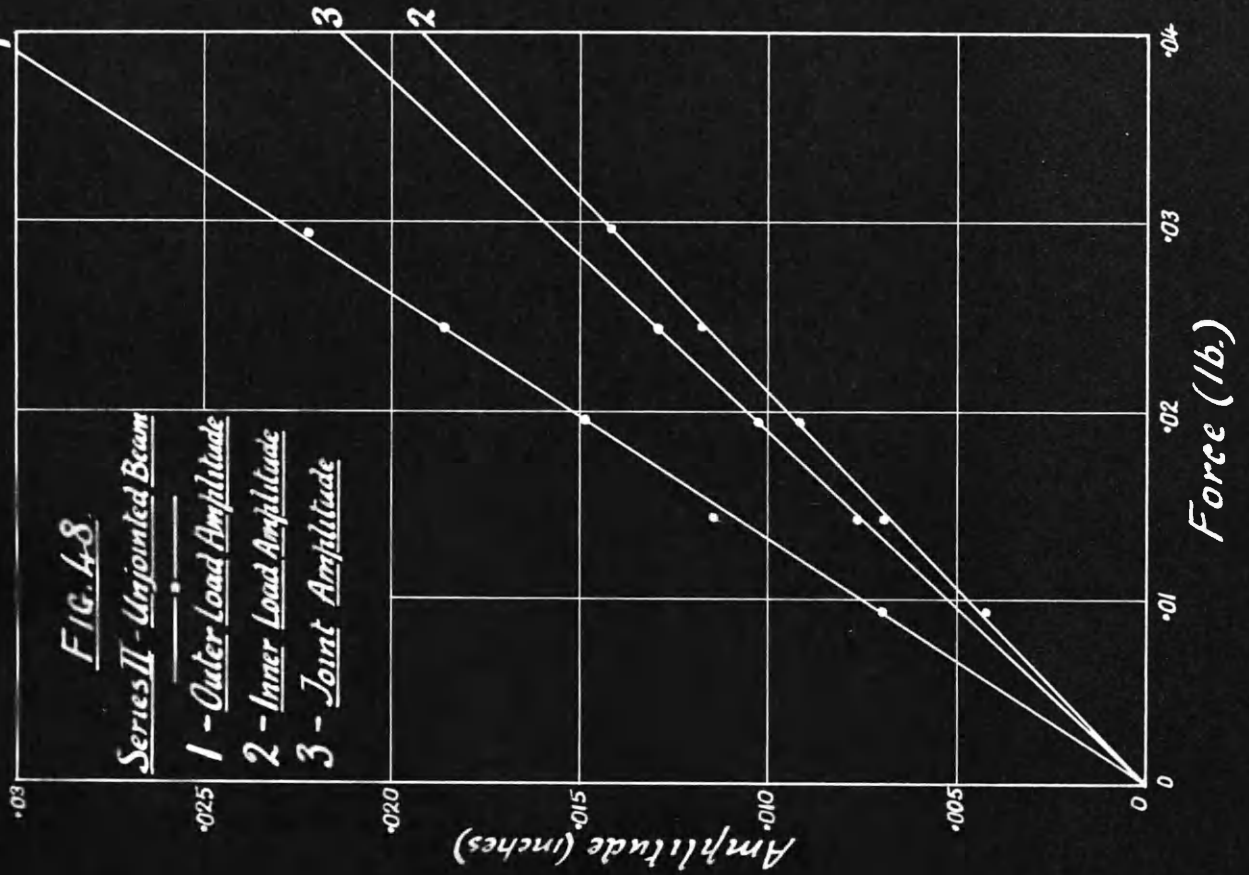


FIG. 49.

Series II - Single Row Joint

1 - Outer Load Amplitude

2 - Inner Load Amplitude

3 - Joint Amplitude

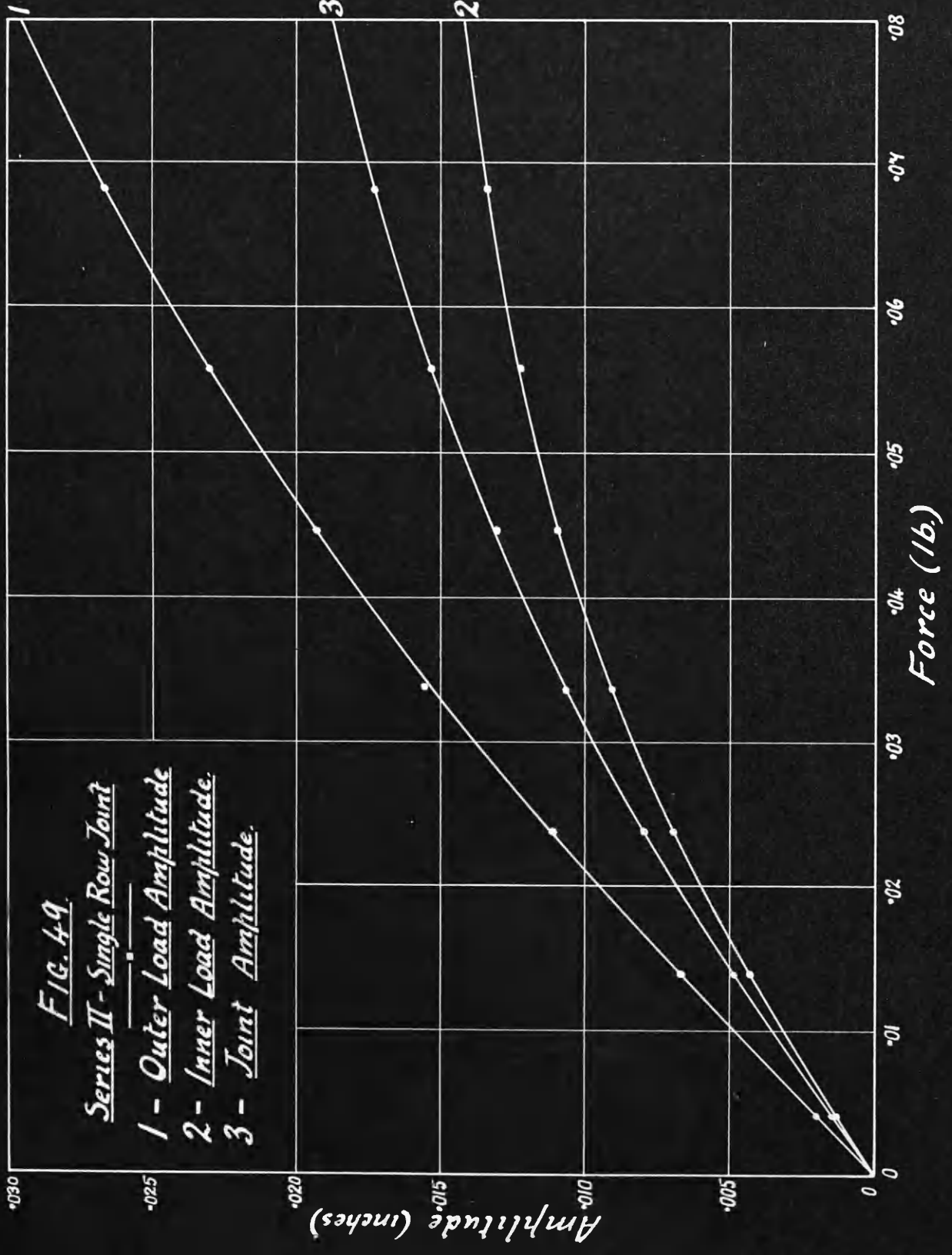


FIG. 50

Series II - 4-Row Joint - End Rows

1 - Outer Load Amplitude

2 - Inner Load Amplitude

3 - Joint Amplitude

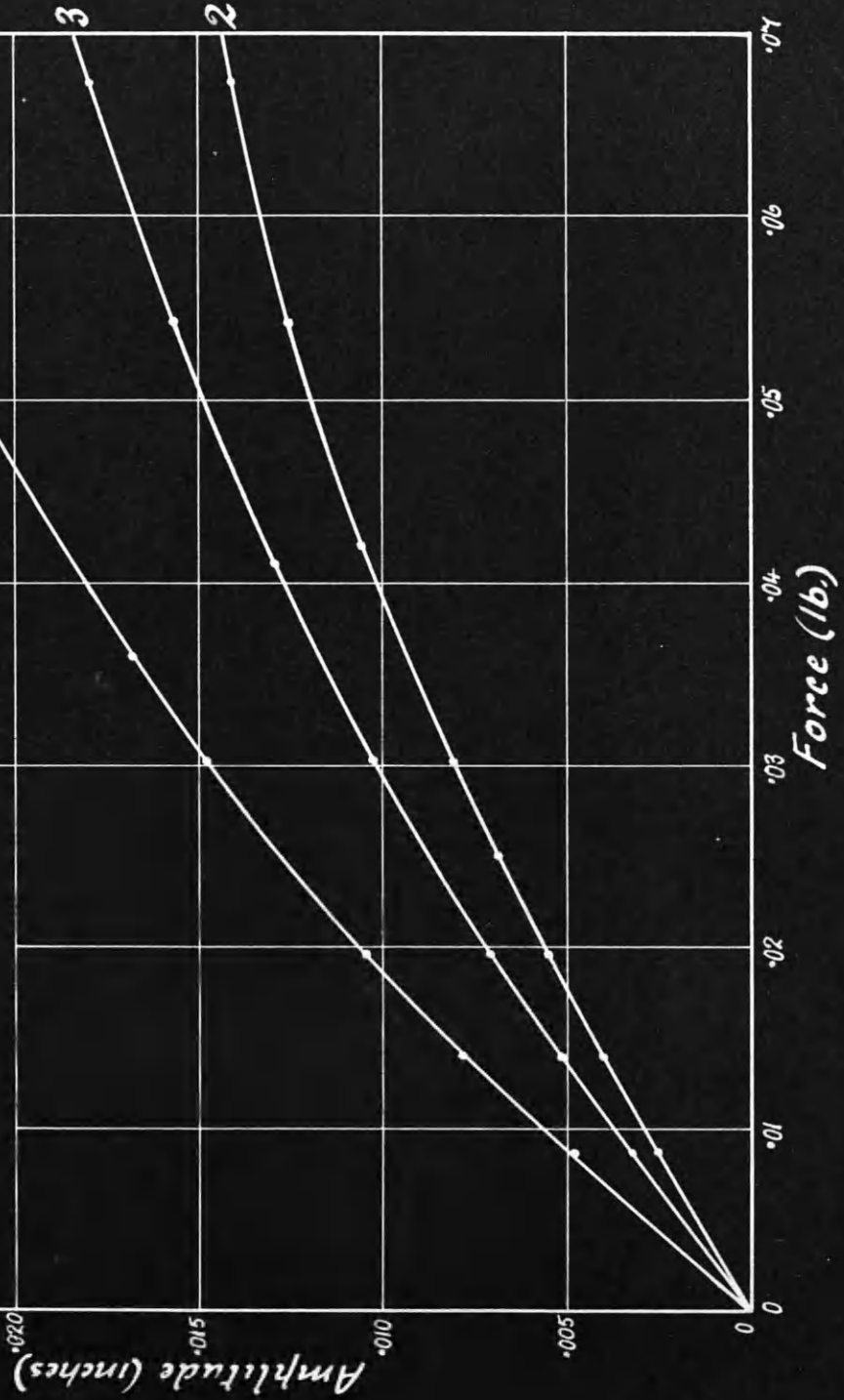


Fig. 51

Series II - 4-Row Joint - All Rows

1 - Outer Load Amplitude

2 - Inner Load Amplitude

3 - Joint Amplitude

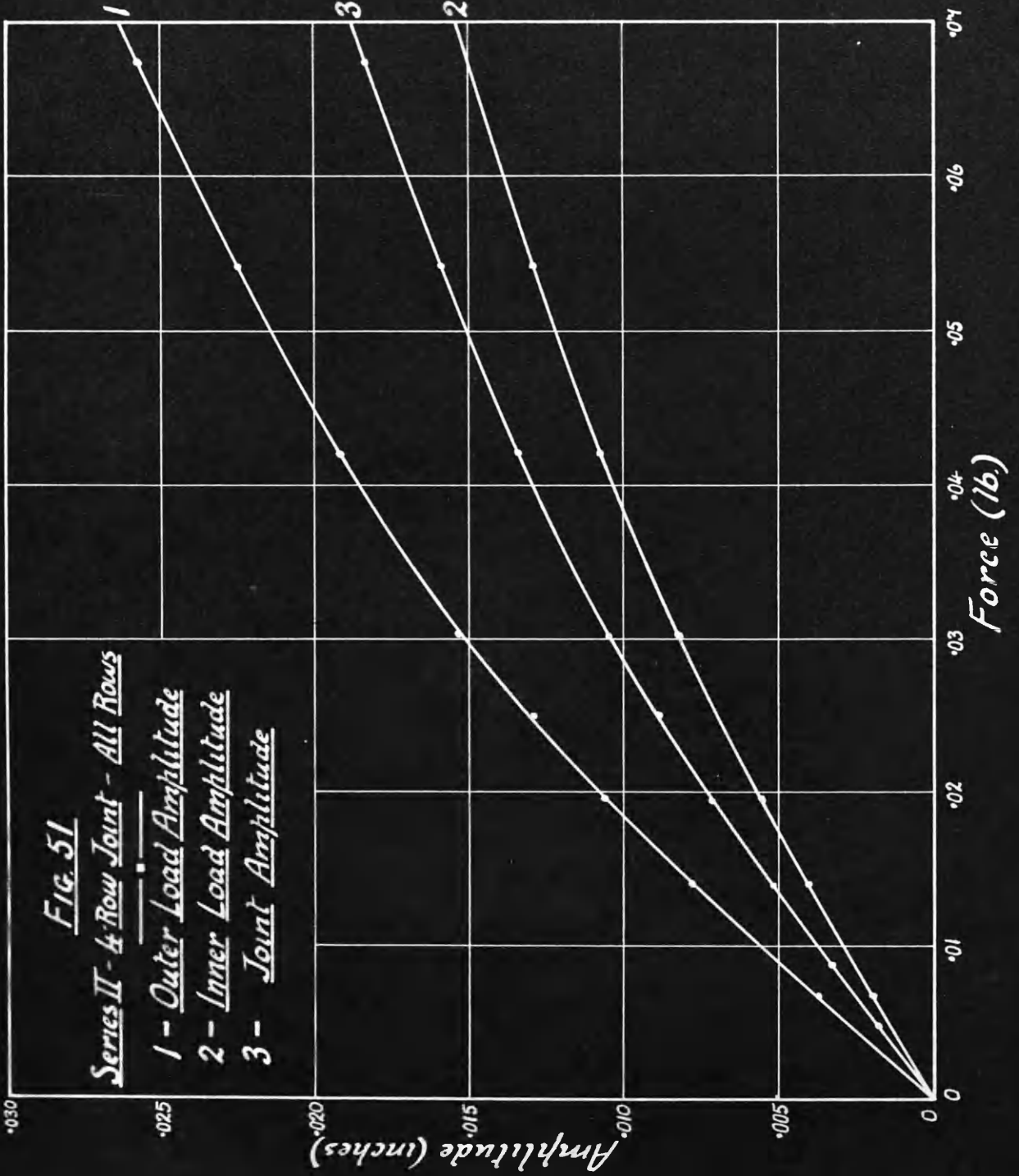


FIG. 52

Series II - 8-Row Joint - End Rows

1 - Outer Load Amplitude

2 - Inner Load Amplitude

3 - Joint Amplitude

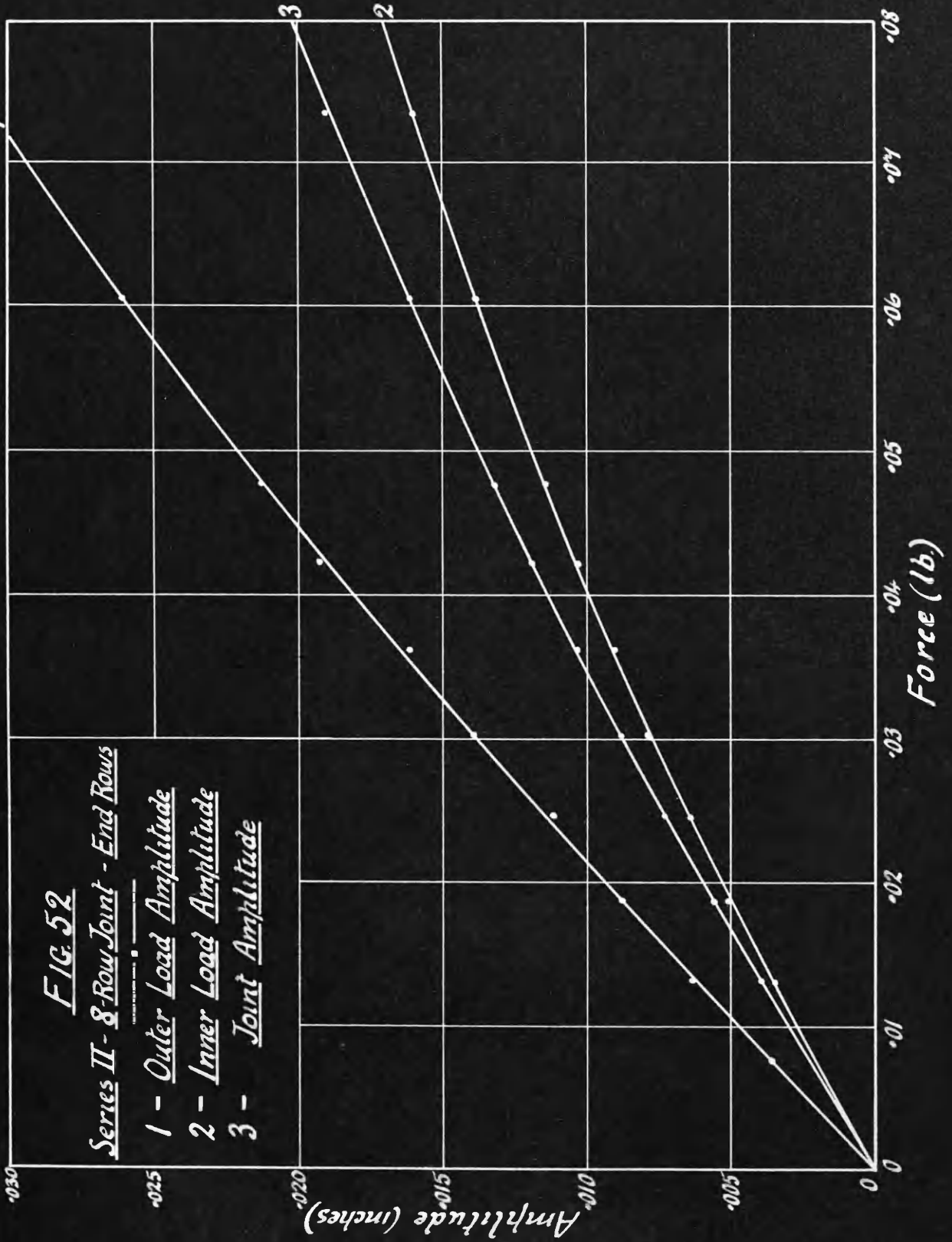
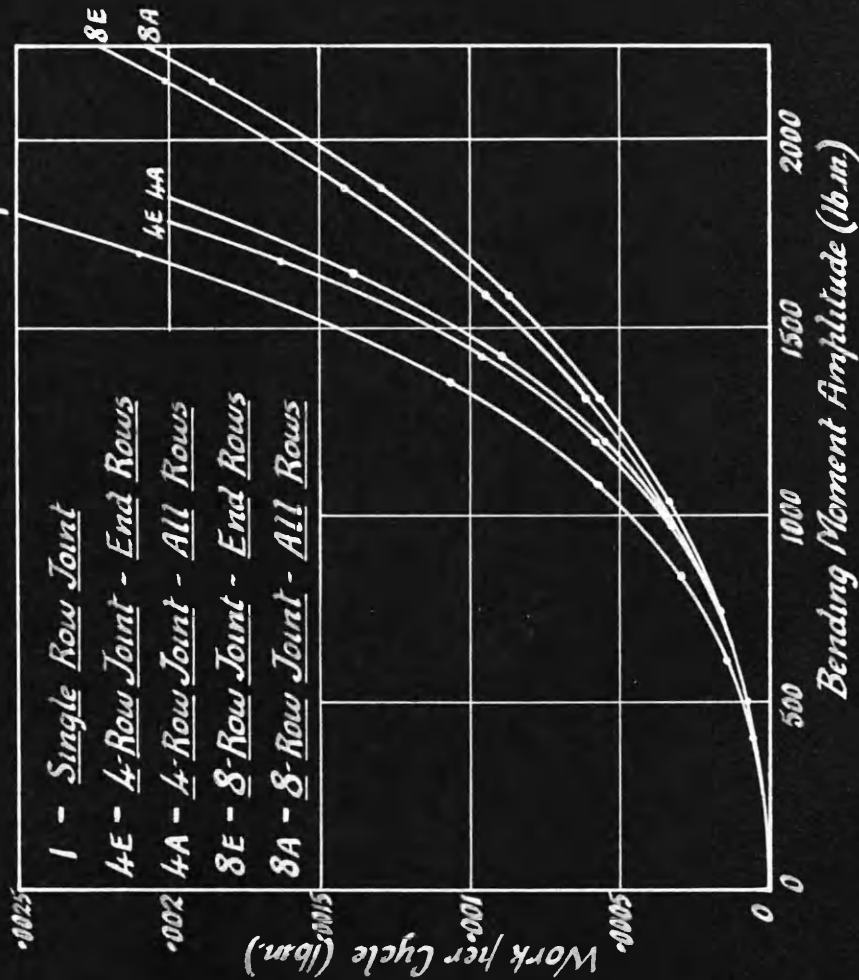


FIG. 54

Test Series II

Energy Loss per Cycle.



where P_j is the impressed force producing amplitude α in the jointed beam, and P_o is the force producing amplitude α in the plain beam.

From the known inertia loading on each beam, the amplitudes have been converted to corresponding bending moment variations at the joint, and the energy losses plotted on this bending moment as base are shown graphically in Fig. 46 - 47.

Comparison of the force-amplitude curves for the plain and jointed beams reveals a slight peculiarity. In the plain beam, by changing from loading A to loading B with a corresponding increase in frequency, a higher amplitude is obtained for the same impressed force. In the jointed beams the reverse occurs. As a result of this, Fig. 46 and 47 show that a reduction from loading A to loading B increases the joint loss. On further reduction to loading C, however, the losses revert to the values for loading A. Since the range of values available for loading C is small, these losses have not been shown separately.

TEST SERIES II.

Only one load system was used in these tests. The force-amplitude curves are given in Fig. 48 - 53, which show the amplitudes of all the masses and of the centre points of the beams. The vibration frequencies and the amplitudes of the loads enabled the inertia loading on the beam and hence the joint bending moment variation to be determined. The method/

FIG. 55

Series III - Unjoined Beam

1 - Outer Load Amplitude

2 - Inner Load Amplitude

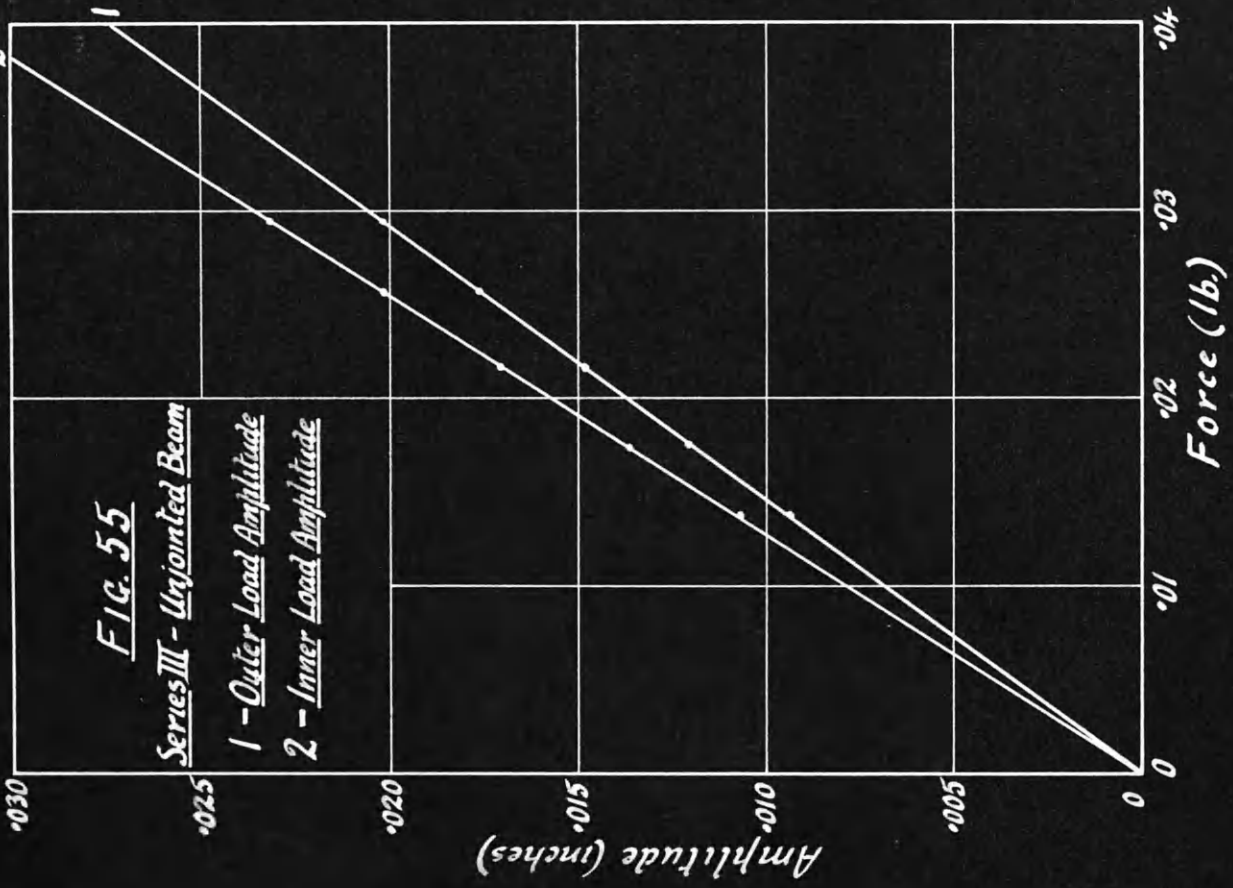


FIG. 56

Series III - Single Row Joint

1 - Outer Load Amplitude

2 - Inner Load Amplitude

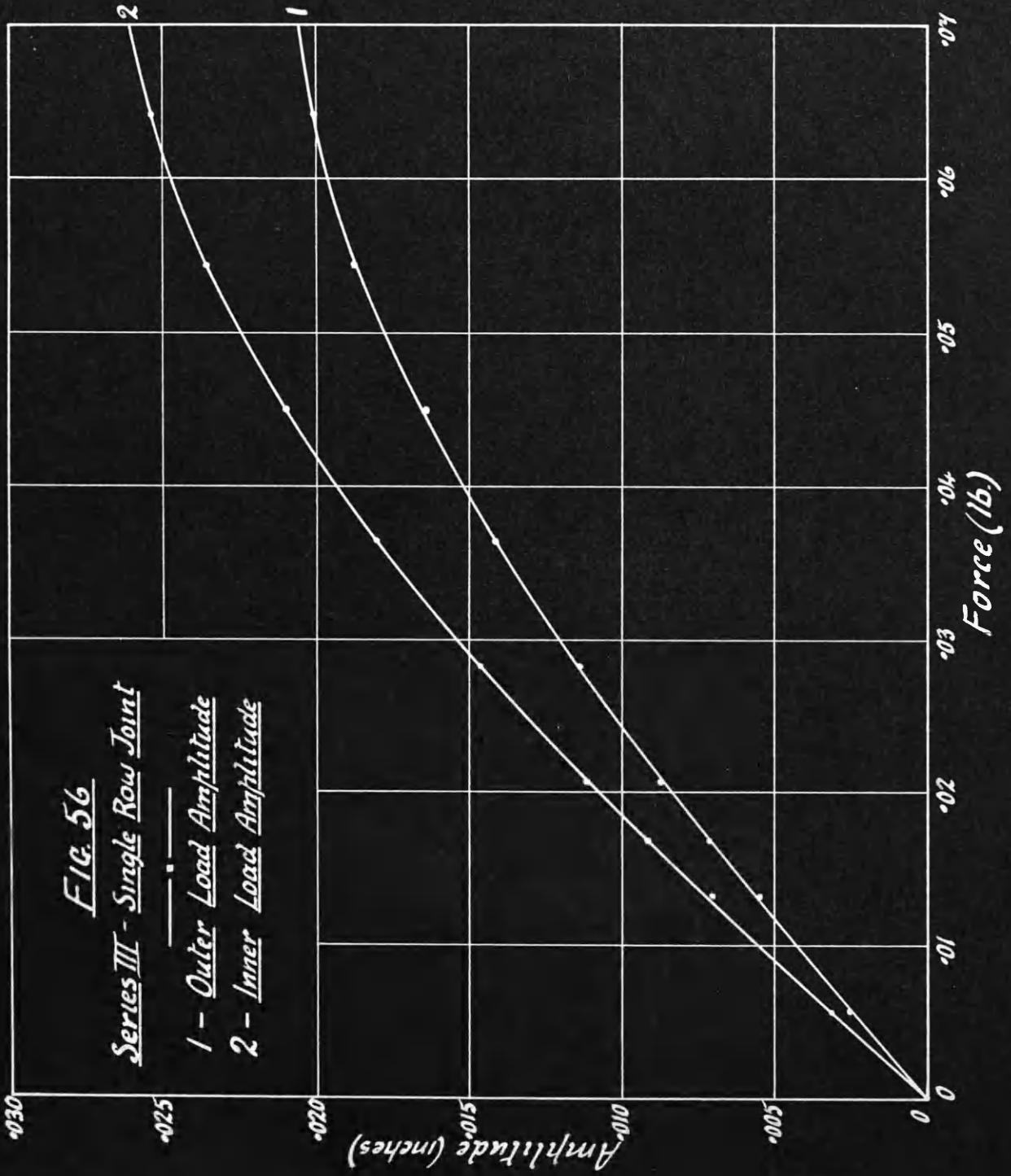


FIG. 57

Series III - 4-Row Joint - End Rows

1 - Outer Load Amplitude

2 - Inner Load Amplitude

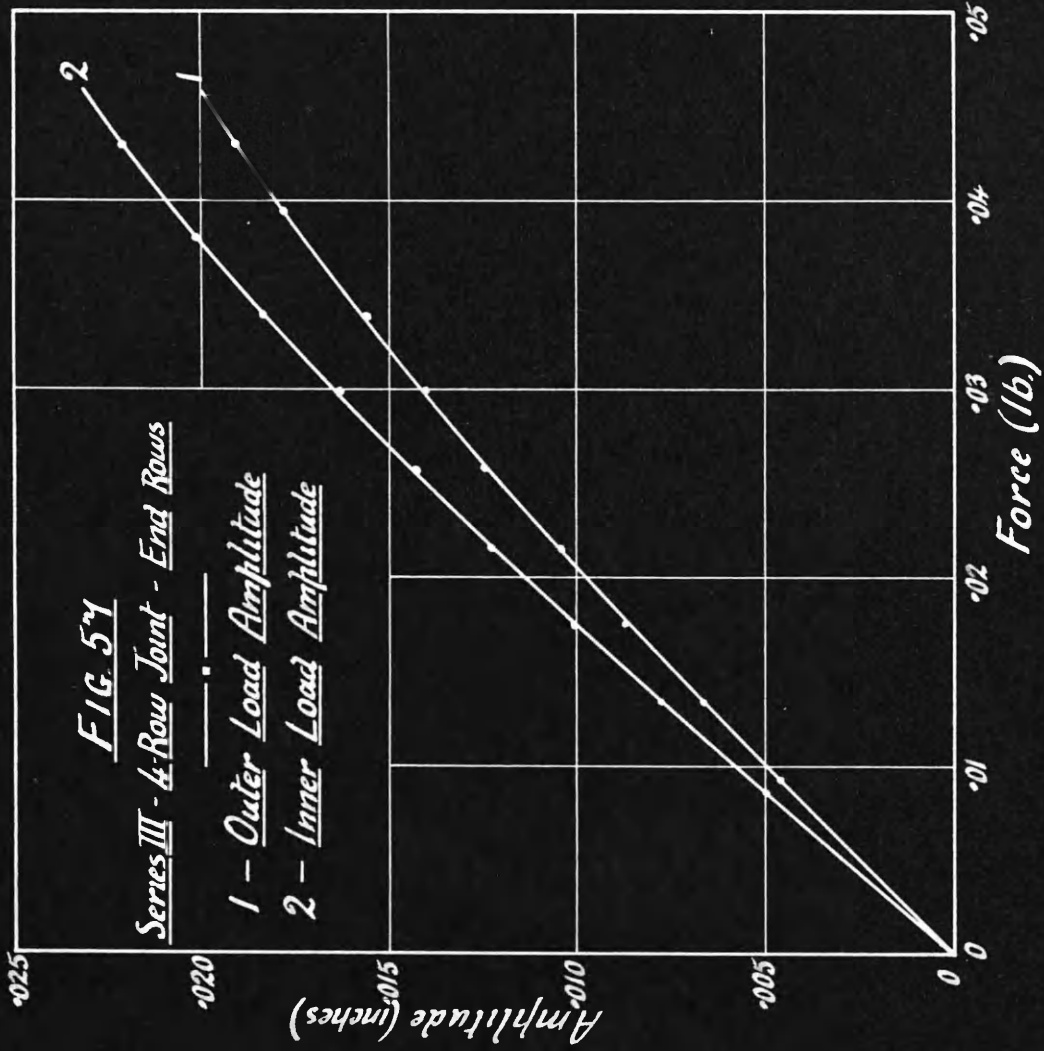
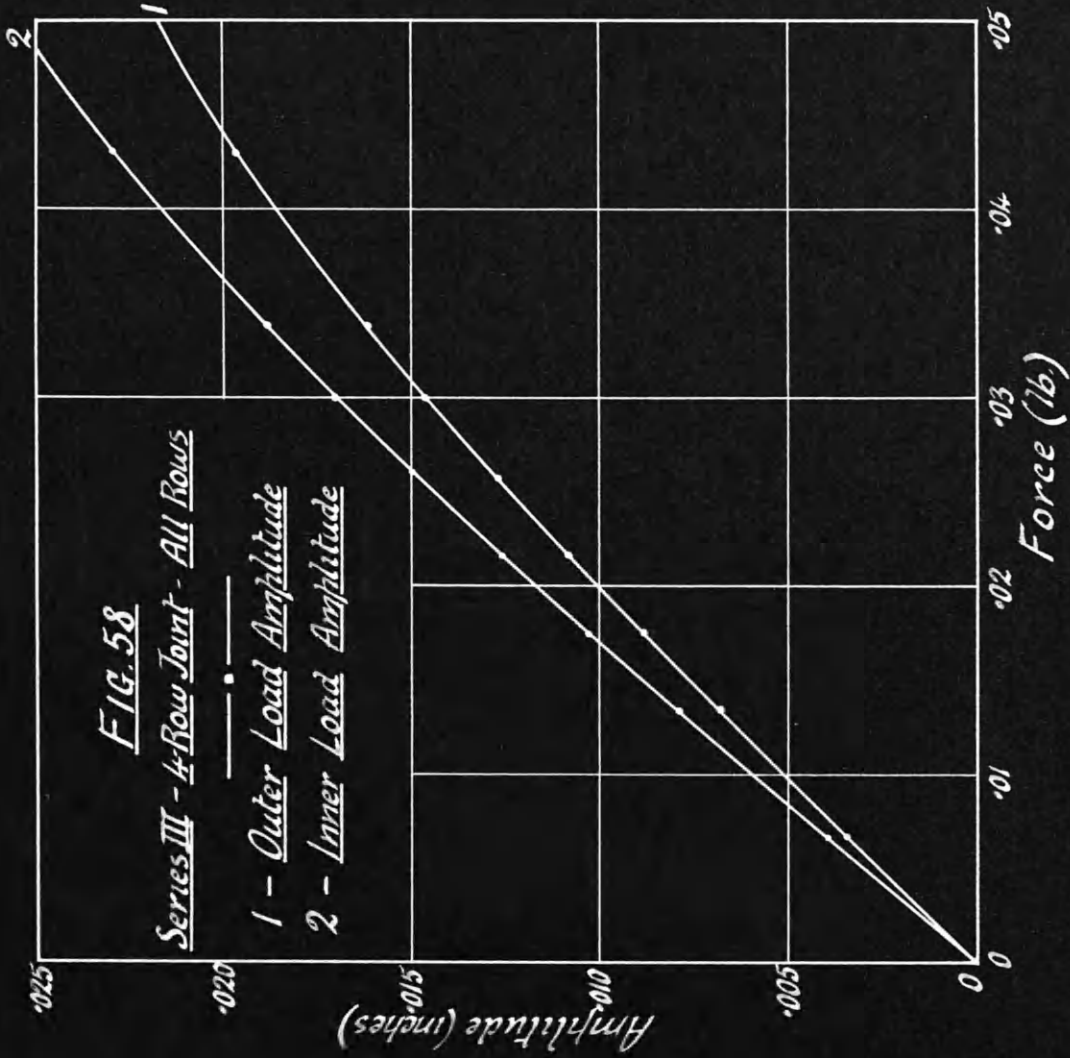


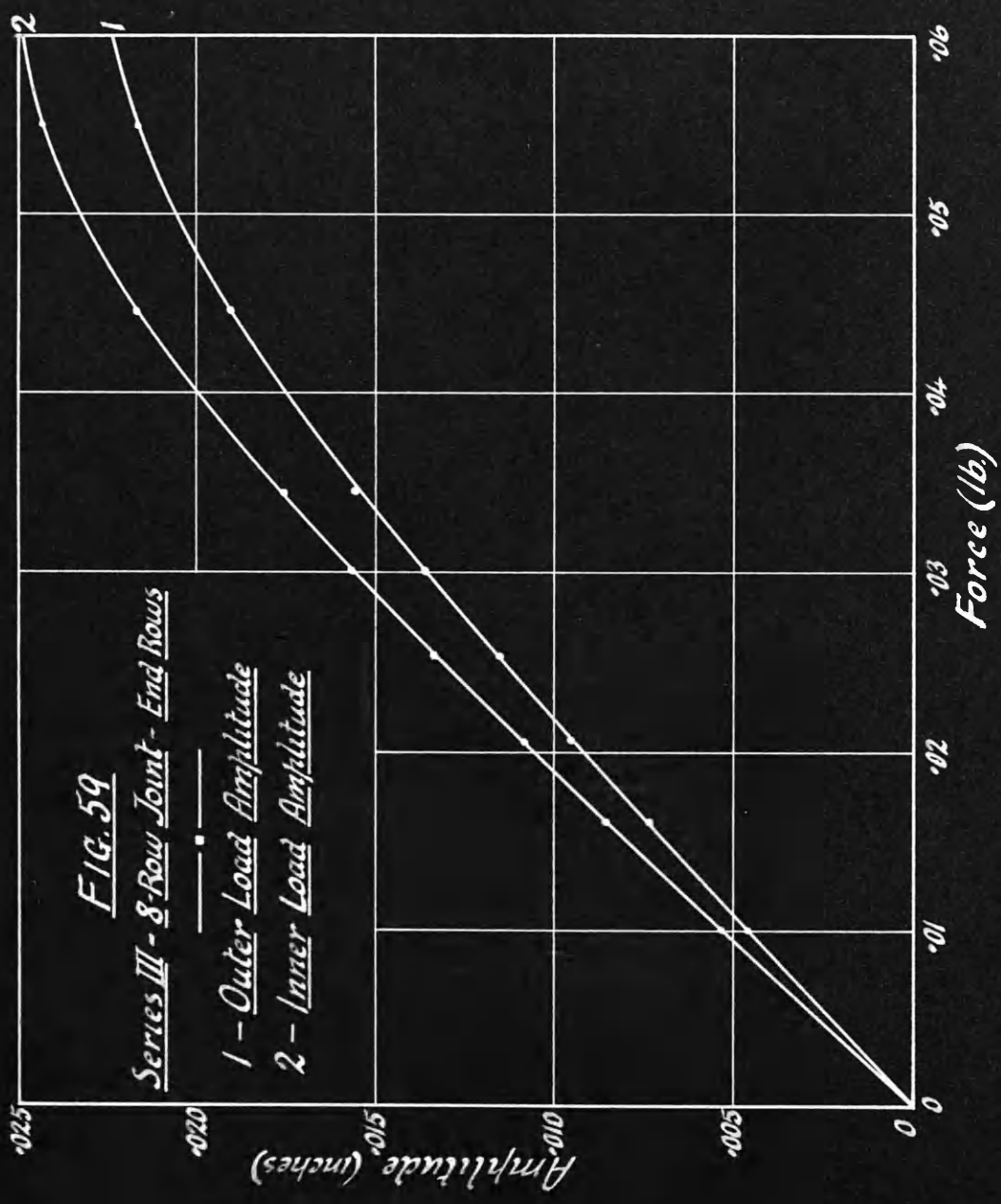
FIG. 58

Series III - 4-Row Joint - All Rows

1 - Outer Load Amplitude

2 - Inner Load Amplitude





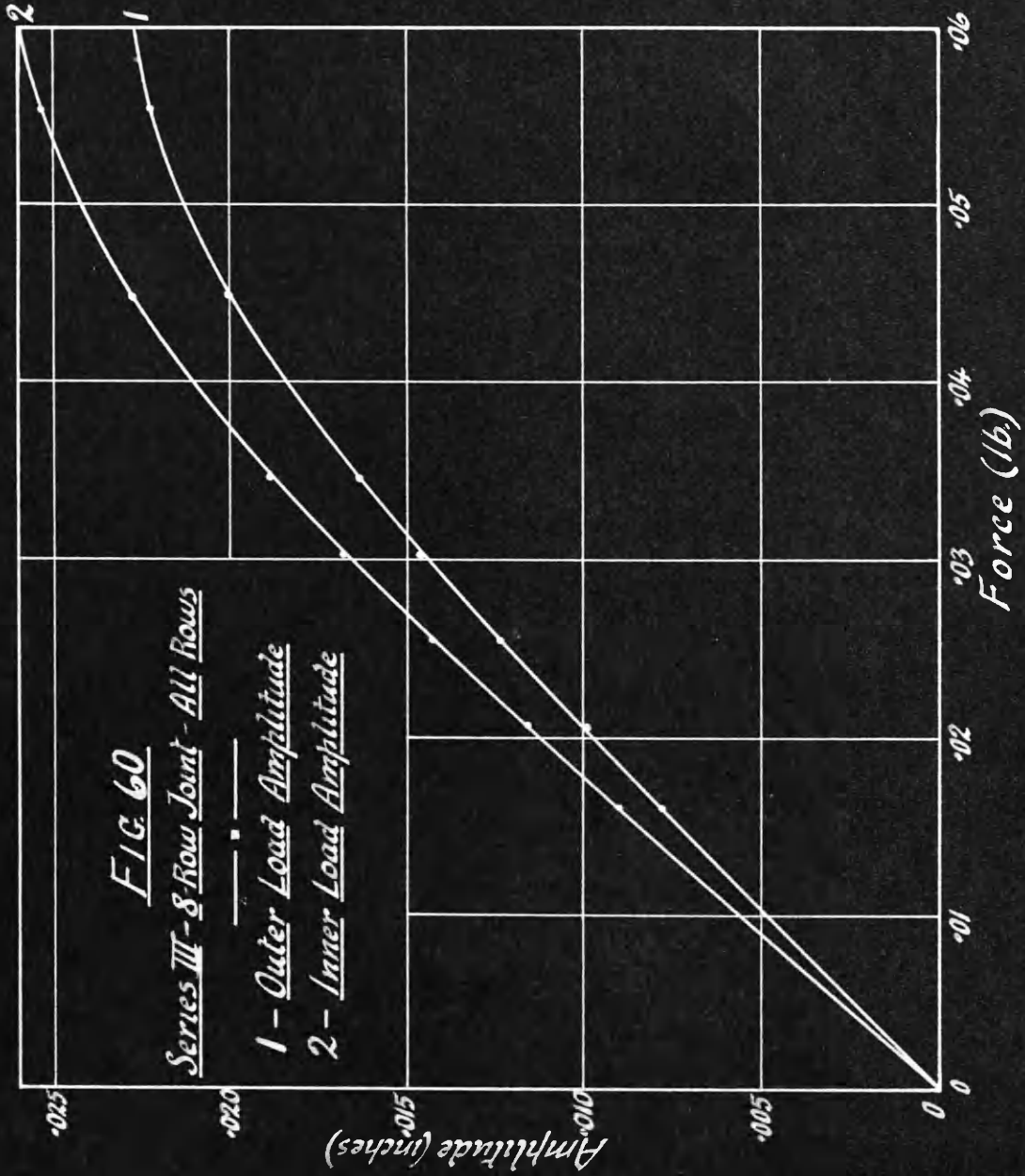
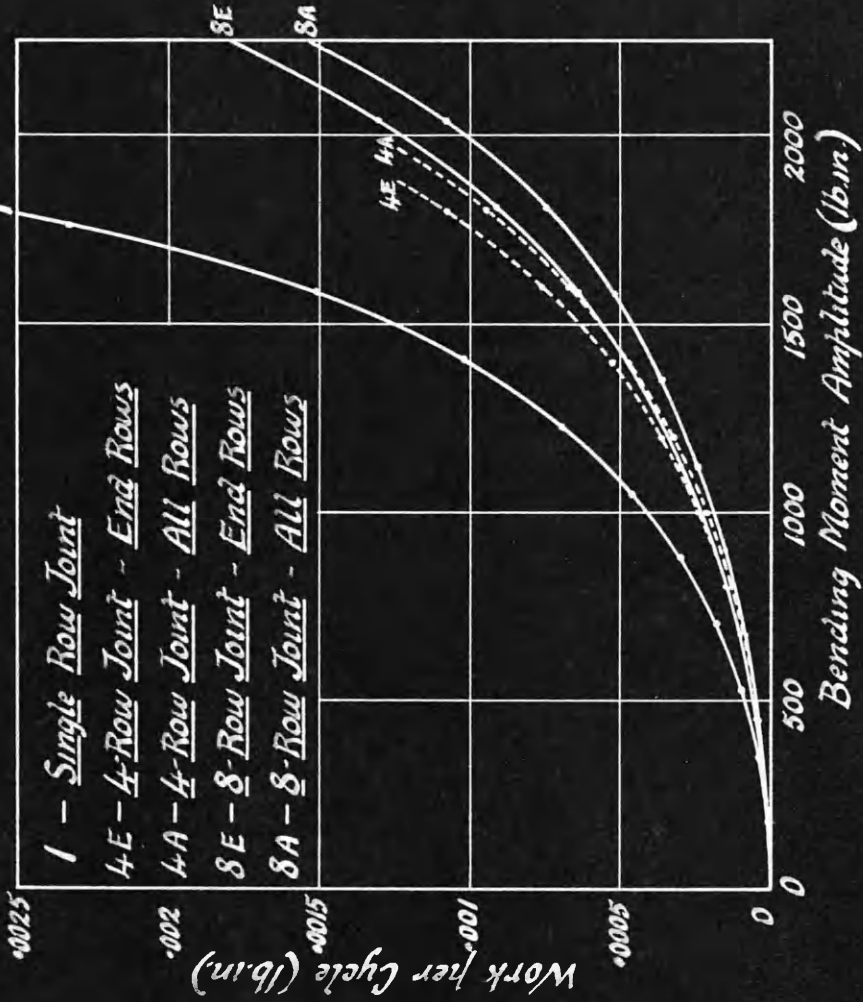


FIG. 61

Test Series III

Energy Loss per Cycle



method of estimating the energy losses in the joints was similar to that used for the results of Series I. The losses are given in Fig. 54.

TEST SERIES III.

This was a repetition of Series II with a different arrangement of the loading. The force-amplitude curves are shown in Fig. 55 - 60, and the joint losses in Fig. 61.

GENERAL DISCUSSION.

INFLUENCE OF NUMBER OF ROWS ON DAMPING.

In relation to the joint characteristics already established by the static tests, the outstanding feature of the vibration results is that in every joint, without exception, the energy loss with only end bolt rows present is reduced by the insertion of additional rows. In order to establish this point beyond doubt, later tests were carried out in the first instance with all bolt rows in position. The bolts with the exception of the end rows were then removed without disturbing the loading and support arrangements, and the experiments were repeated. The results remained quite definitely as stated.

The trend of the subsequent discussion makes it desirable to emphasise at this point that the removal of the inner rows produced no measurable effect on the natural frequency of the beam and therefore on the inertia loading. The test conditions with end rows only are exactly the same as/

as with additional rows. Consideration will show that if any slip movement, partial or complete, occurred at the second or subsequent rows when the bolts were inserted there, additional friction losses would result. There is therefore no appreciable load fluctuation on the second or subsequent bolt rows.

It has already been pointed out that a bolted or riveted joint is not a definitely rigid construction; and the static test results indicate the "leakage", due to stress concentrations at the first bolt rows, of a negligibly small load to the second rows. In vibration, the strains or distortions associated with such leakage give rise to friction losses which, though small, are by no means negligible in comparison with the losses from other sources. The insertion of the second bolt rows imposes a check on the development of these minute distortions, resulting in a reduction of the friction loss and the imposition of a very small load on the second rows.

INFLUENCE OF SLIP ON DAMPING.

In the unidirectional loading tests, the vibration characteristics were not affected in any way by the occurrence of slip. Slip only altered the static deflection form or mean vibration form of the beam. With the single row joint, the production of an amplitude which took the beam beyond its slip deflection was accompanied by a static deflection, during the occurrence of which the beam vibration continued quite/

quite unaffected.

With less than six rows per joint, periodic slip under one or more bolt rows at vibration frequency cannot be produced by any condition of unidirectional loading. Consequently, although during the vibration tests static slip developed in the single and four row joints and in certain rows of the other specimens, no evidence of this is afforded by the energy loss curves.

INFLUENCE OF JOINT LENGTH ON DAMPING.

A striking feature of the curves of Fig. 46 and 47 is the relatively large decrease in energy loss per cycle as the stiffness of the beam construction with regard to bending is increased by the greater length of joint. If attention be confined to those cases with only end rows present, consideration of the origin of the friction losses suggests that the energy loss should be independent of the distance between the rows, i.e. should be independent of the total length of the joint. The only other quantity which differs for the various joints is the natural frequency of beam vibration.

Unfortunately the exact effect of frequency on the energy loss in the joints cannot be determined from the results presented here. Comparison, for example, of the curves in Fig. 46 and 47 applicable to the same joint cannot give the frequency effect since the mean joint load is also different in each case. Such indications as can be obtained suggest/

suggest that the large differences in the energy losses for the various joints cannot possibly be ascribed to the difference in frequency. It would appear that they are due to an indirect effect of frequency, and the following explanation is suggested.

Fig. 46 and 47 are plotted on a base of joint bending moment amplitude. For a given load system, a more rigid joint construction is associated with a higher natural frequency; consequently for a given amplitude the inertia forces are higher. Conversely, the attainment of a specified joint bending moment variation requires a smaller beam amplitude with a long joint than with a short one. The smaller beam amplitudes entail smaller losses because of the effect of curvature in the joints.

The moment necessary to bend the individual cover plates to the same curvature as the beam is a negligible proportion of the total bending moment on the beam, and in the static tests it does not obscure in any way the effect of the frictional attachment of the cover plate to the beam flange. But this moment requires the application to the cover plate of forces which may be distributed along the cover plate in an irregular fashion but which usually entail force concentrations at the ends of the plate or at the centre.

These forces are superimposed on the ordinary bolt pressure distribution, and their situation is favourable to/

to the development of friction damping when the beam vibrates. Such relative motion as does take place between the elements of a joint occurs outside the end rows, i.e. at the points where the bending forces on the cover plates are concentrated; hence increased curvature will involve increased friction losses. For a specified bending moment, curvature is least for the longest joints, and the energy loss is smaller in such joints.

NATURE OF DAMPING FORCES.

With regard to the type of damping present, it is possible to state quite definitely that, although the losses originate in solid friction, the latter is not manifested in the usual way. With solid friction, joint movement would not occur till the joint load was sufficient to overcome the friction; consequently with small beam amplitudes there would be no energy loss in the joints since no motion would occur in them. The energy loss at any amplitude depends on the $(P_1 - P_0)$ difference shown by the force amplitude curves. Superimposing the plain beam results on the jointed beam curves makes it evident that these differences persist right down to zero amplitude.

For any joint, a simple type of analysis enables the energy loss to be expressed in terms of powers of the amplitude, the lowest power being the second; but since it would appear that the losses are due more to bending effects in/
in/

in the joints than to direct tension and compression effects, the equations are of no direct interest in the present investigation. The results may be regarded as a particular illustration of a general tendency - the more rigid the joint, the smaller the energy dissipation in vibration. A statically stronger structure may be subject to much larger vibration stresses.

CYCLIC SLIP WITH ALTERNATING LOADS.

The intention in Series II was to arrange masses with phase angles differing by 180° , so that the inertia forces would balance to some extent and prevent the beam supporting forces being reduced to zero at any point in the cycle. By this means it was hoped to obtain vibration amplitudes large enough to cause cyclic slip in the single row joint and possibly also in the end rows of the multi-row specimens. This condition, however, could not be attained. Beyond a definite amplitude, an increase in the impressed force caused the beam to travel about bodily on its supports just as if the supporting forces were periodically zero.

A rearrangement of the loads to secure better balance of the inertia forces led to Series III, which was no more successful in its object. The amplitude refused to increase beyond the value which would produce cyclic slip. Possibly no other result could be expected, since the energy input necessary to maintain such slip would be relatively enormous.

In Series II and Series III, the curves obtained from the/

the four-row joint are rather widely different, due probably to the badly twisted and distorted form of that particular beam; but generally the curves confirm the decrease in damping with increased rigidity, already indicated by Series I. They also show that the friction loss is greater with a zero mean bending moment than with unidirectional loading. The change from half loops (unidirectional loading) to complete loops would naturally be accompanied by a large increase in energy loss, but this increase is partly obscured by the effects of curvature in the joints.

CONCLUSIONS.

In the static tests, the slight bending of the joint due to beam deflection proved to be no disadvantage. In the vibration tests, this bending resulted in the production of dissipative forces of much greater magnitude than those due to simple tension and compression in the joints; the latter effects are thereby largely obscured.

Despite the bending effect, it has been definitely shown that in any multi-row joint with only end rows present, the insertion of additional rows produces a small decrease in damping. This confirms the information provided by the static tests with regard to stress concentration and consequent leakage of a very small load to the second rows. Practically, however, it may be assumed that in all cases, even/

even where slip has occurred, the fluctuation of load is carried entirely by the end rows.

With unidirectional loading, the occurrence of slip is a static effect and has no influence on vibration damping. In actual riveted joints the occurrence of cyclic slip is a practical impossibility, since the stresses produced would destroy the joint.

With alternating loading, as might be expected, the normal energy loss is somewhat higher than with unidirectional loading. The amplitude may be carried up to the point where cyclic slip is about to develop, but does not increase beyond it even if the impulsive force is increased.

Although the losses originate in solid friction, none of the usual effects of solid friction is manifested in any way. The energy loss for any one joint shows no component proportional to the amplitude.

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