A STUDY OF LACTATION AS AFFECTED BY HEREDITY AND ENVIRONMENT.

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by

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CONTENTS.

PART I. THE MEASUREMENT OF THE SHAPE OF THE LACTATION CURVE.

PART II. THE LACTATION CURVE AS AFFECTED BY ENVIRONMENT.

PART III. THE LACTATION CURVE AS AFFECTED BY HEREDITY.

2. Stability of the Lactation Curve.

3. Methods of Measuring the Heritability of Individual Differences.

4. Correlation between the Production of Dam and Daughter.

INTRODUCTION.

It is well known that the total milk yield of a cow is affected both by her heredity and her environment. Distinct genetic differences in milk production have, for instance, been found between different breeds of cows and between individual cows within the same breed, while the influence of various environmental factors, such as feeding, management, age and season of calving has also been demonstrated. The general subject of the mode of inheritance of milk yield has been ably reviewed by Smith and Robison (1933).

Speculations have been made as to the number of factors (genes) involved. The earlier workers (Wilson, 1911, 1925, Hansen, 1917a, b, and V. Patow, 1926, 1930) have assumed that the number of genes is small, and that the inheritance of milk yield is little different from that of any other simple Mendelian character, e.g. colour. Modern workers (Cole, 1925 , Gowen, 1924) agree, however, that the inheritance of milk yield is far more simple and that, as with quantitative characters in plants, the number of genes involved is large.

The actual nature of genes and their mode of action is not yet clearly understood. Their existence is recognised only through their effects, and these must clearly be achieved through some modification of the physiological processes involved.

Rather than to attempt to determine directly the exact number and nature of the genes responsible for milk production, it would seem more profitable to differentiate and study the various components of the lactation yield.

This yield is not a single entity. In reality it represents the area of the lactation curve, which consists of two segments, (i) a rising segment, and (ii) a declining segment. For some time after calving a cow's milk yield increases. Animals differ in the period during which this rise lasts, and in the rate at which it occurs. Some reach their maximum production within a few days of calving, others do not do so until much later in the lactation. Lactations in which the yield has continued to increase slightly for 10-12 months after calving have in fact been reported (Gaines 1926a), though with most animals the maximum production is reached within three to eight weeks. Following this period of rising milk production the declining phase sets in. Differences are, however, again found in the rate at which the decline occurs. The less "persistent" animals decline in production very suddenly after the period of peak production, and rapidly 'dry off', whereas the more "persistent" animals show little or no decline over a prolonged period. These points are illustrated in Figure 1, where the lactation curves of certain selected cows have been graphed.

The total milk yield over a complete lactation thus depends on (a) the area of the rising segment of the lactation curve, which in turn depends on the duration of the rising period and the maximum yield attained, and (b) the area of the declining segment which depends on the subsequent rate of decline. Two cows may give identical total yields in a lactation, yet the shapes of their lactation curves may be markedly different. One may have a curve with a high maximum and a steep decline, while the other may have a lower maximum but only a slight decline. The latter animal will thus compensate for her lower maximum by a longer and more persistent lactation.

If high maximum yield and high persistency are distinct hereditary characters and are not physiologically incompatible, it should be possible to combine these in the same individual and thus obtain even higher yields. The validity of this contention can, however, only be determined by a study of the interrelationships of the different components of the lactation curve. This in turn must entail the differentiation of the effects of heredity from those of environment. Information regarding the latter point is supremely important, since only the genetic part of the variance of a character (Fisher, 1930) can be transmitted from the parent to offspring, and therefore be stabilised by breeding. The present thesis is devoted to a general study of these interrelationships.

It should not be concluded that maximum yield and persistency are the only variable components of the lactation curve. Either or both of them may be dependent on more than one physiological function. Existing knowledge of the physiology of lactation is not however yet sufficiently complete to justify any final resolution of the lactation curve into all its possible components. Until such information becomes available, the study of maximum yield and persistency may clearly be undertaken as a first step.

The need for studies of this nature has been repeatedly emphasised by various writers (Turner, 1926b; Asdell, 1935; Smith, 1935), for our present knowledge of the heritability of milk production (and particularly of persistency), as well as of the interrelationships of heredity and environment, is extremely meagre.

Note on the Statistical Method used.

In obtaining the results detailed in the following pages common statistical methods as described by Yule and Kendall (1937), Fisher (1938), and Snedecor (1938) have been employed. Sheppard's correction for grouping has been used throughout. To judge the significance of an observed result standard error rather than probable error has been used. Results significant at 5% probability level of significance have been interpreted as significant.

All the computations were made with the help of a calculating machine. Barlow's tables were freely used for finding the square roots, cubes and higher powers, reciprocals, etc. Castles' five-figure logarithm tables and the statistical tables by Fisher and Yates (1938) were used in the process of curve-fitting.

Throughout the tables and occasionally in the text, calculated figures have been given to a larger number of decimal places than would be included in a published paper. This was done to ensure accuracy in computation.

PART I. THE MEASUREMENT OF THE SHAPE OF THE LACTATION CURVE.

1. Review of Previous Methods.

Before the shape of the lactation curve can be studied, it is necessary to give a quantitative expression to this shape. By the ordinary "visual measurement" method (Bonnier, 1935) it is no doubt possible to compare the curves of a few cows, but for statistical studies a quantitative measure is essential.

Of the two components of the lactation curve, i.e. maximum and persistency, the former is relatively easy to determine. Its value is given by the maximum ordinate of the curve. Because the day-to-day variability of the milk yield of a cow is considerable even when all conditions of management are kept as uniform as possible (Bartlett, 1929), this value may be slightly too high if based on the yield of a single day. Where information regarding day-to-day milk yields is available, this defect may, however, be overcome by using instead Gavin's revised maximum, i.e. the maximum daily yield reached or exceeded three times in a week (1912).

Persistency is more difficult to determine. Persistency defines the slope of the curve and measures the relative* rate at which the maximum yield, when

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*For difference from "absolute" rate see Fisher (1939 p.27-30) and Brody (1927a) once reached, declines. In actual practice, this relative rate may vary from period to period or may be roughly constant over the whole course of the lactation. In the latter case the average slope for the various periods of the lactation will measure the persistency for the whole curve. To find the slope is not a difficult matter mathematically. It is readily obtained by substracting the natural logarithm of the production of each period from that of the preceding period. The average of these values for the successive periods of the lactation will give the average value for the whole lactation. If the persistency is determined from daily or even from weekly yields, the high random day-to-day variability of the yield affects this value much more than the maximum. This drawback may, however, be overcome by averaging the yields over reasonably long periods, say a month. Again, the variation of the slope from period to period may, for various reasons, be too high and systematic rather than random. In such cases the average value of the slope for the whole curve may not give a true value of the rate of decline. It was the realisation of this fact that led Bonnier (1935) to propose his "visual measurement" method.

As early as 1886 Sturtevant pointed out that each month's production is roughly a constant percentage of the production of the preceding month. In mathematical terms this means that the course of the lactation curve is exponential. This latter fact was independently pointed out by Brody, Ragsdale and Turner (1922) in their study of the shape of the lactation curve of cows of various breeds. In order to describe the average lactation curve of a large number of cows of different breeds, they employed the equation

$$M_t = M_o e^{-kt}$$

where M_t = milk production during any month t, M_o is the value of the initial theoretical rate of milk flow at the time of parturition (not attained in actual practice), k is the constant measuring the relative rate of decline, and e the base of natural logarithms. They found that the equation fitted the data remarkably well. On the basis of this work they formulated what they termed the law relating milk secretion with the advance of period of lactation, i.e. "that each month's production after the second month is a constant percentage of the preceding month's production". (1923). Turner (1927a) subsequently qualified the conditions governing this law by stating that "when all other conditions are uniform, the monthly milk or fat percentage during the lactation period, after the maximum is passed, is a constant percentage of the preceding month's production". Under average conditions of herd management, the first qualification can hardly be realised in actual practice. Arguing that, since the course of a lactation curve and that of a monomolecular chemical reaction are both defined by the same type of exponential curve, Brody, Ragsdale and Turner (1923) suggested that the phenomenon of milk production is governed by a monomolecular reaction. Gaines (1926a), though strongly disagreeing with such a chemical interpretation, used the exponential curve in his studies of persistency (1927a, 1931). He worked out graphic (1927a) and algebraic (1927b) methods for determining the constants of the equation, and conjointly with Palfrey (1932) evolved an ingenious least square curve fitting machine capable of determining the constants mechanically and so obviating the mathematical labour of curve fitting. In their studies Davydov (1933a,b) Kartha (1934a,b) and Gooch (1935) have also employed the exponential equation to define the lactation curve.

Numerous other measures have, however, been employed by various workers to evaluate persistency.

McCandlish _et al_.(1919) expressed each month's yield as a percentage of the first month's production and compared graphically the slopes of the different curves thus obtained. This method suffers from the same defect as Bonnier's "visual measurement" method (1935).

Sanders (1923, 1930) suggested a ratio method to express the shape of the lactation curve according to the relationship

Total lactation yield = maximum yield x persistency

or persistency = total lactation yield/maximum yield.

He gave the name "shape figure" to his measure of persistency. The value of the "shape figure" is however, largely affected by the calving interval (Ostergaard, 1931). Sanders tried to allow for this by correcting the "shape figure" for the "service period", which in turn affects the length of lactation. However, as in actual practice the length of the calving interval is not altogether determined by the service period (for instance, in some of the writer's data, the cessation of milk recording for a lactation period was observed to synchronise with the close of the calendar year rather than the actual drying off of the cow), the corrected value of the "shape figure" is still affected by the calving interval - at least in some cases. Further, this measure fails to distinguish a rising from a declining curve and, as Sanders himself says, "gives no detail as to the shape of the lactation curve".

Turner (1926a) suggested a ratio method similar to that of Sanders, in which persistency was determined from the ratio of the maximum to the total yield. Turner, however, used the milk yield for a definite period of time (12 months) instead of the total lactation yield. This obviated the necessity of correcting for service period and is thus an improvement on Sander's method. Turner also presented a graph for the expression of this ratio in terms of persistency percentage. A similar graph, but slightly more accurate from theoretical considerations, was given by Brody (1927b).

The ratio method, as devised by Sanders and Turner, has the advantage of speed and simplicity. It is therefore specially suitable for statistical studies of persistency where a large volume of data have to be handled. Gaines (1927b) has shown, however, that there is some question as to the trustworthiness of this method of measuring persistency. The values obtained by this method are slightly different from those determined algebraically, i.e. by fitting the curve to the observed yields by the method of least squares. Although Turner used this method in some of his earlier studies (1927, 1934) he now considers it unsuitable for students of persistency. For a scientific study of the shape of the lactation curve, he considers the exponential curve method to be most appropriate (1938).

Becker and Mcgilliard (1928) employed the method of "visual measurement" of differences in the shape of lactation curves of different animals by representing each curve graphically.

Ostergaard (1931) measured persistency by the percentage which the average daily milk yield for the whole lactation constituted of the highest daily milk yield. As the average daily yield depends upon the length of the lactation, the value of persistency obtained by this method is not comparable for cows with lactations of varying lengths.

Johannsen (1939), has also used a ratio method, his measure of persistency being the ratio of the milk yield of the second 100 days of the lactation to that of the first 100 days. The drawback of this method is that the first 100-day period, which denotes the maximum physiological capacity of the cow for milk production, is too long to reveal differences between cows differing in their capacities to maintain yield during this crucial period. Two cows may give the same yield in the first 100 days of their lactation, yet they may have vastly different initial maximum yields, and therefore persistency values, for this period.

Fohrman and Graves (1939) expressed the amount of milk produced in each 30-day period as a percentage of the total yield for 360 days. As a basis for comparison the lactation curve was assumed to be a straight line on the supposition that during each 30-day period 8.33% of the total yield would be produced. The deviations in percentage from the straight line for the different periods of each curve were summated and the total was used to express persistency.

Csukas (1939) defined the shape by the "degree of declination" (?) in milk production during 10 weeks after calving of those cows conceiving in that time. This is obviously too short a period to give any idea of the real differences as regards persistency between individual cows.

In a quite recent study Pontecorvo (1940) has used the exponential equation to determine the rate of decline. Instead of using the whole lactation curve, however, he has used what he terms "the middle part of the lactation curve" to determine the persistency. He regards this portion as a distinct physiological component of the lactation curve, and holds that its use in place of the whole lactation curve constitutes a definite improvement in method. There would certainly be some justification for this choice if our knowledge of the physiology of lactation were so definite as to guide us in the division of the curve into distinct portions. Such knowledge is however lacking. We do not, for instance, know why the milk yield rises for some time after calving. Pontecorvo considers this upward trend to be the combined result of the development of the glandular system *before* parturition (under the direct influence of oestrogen and other associated pregnancy hormones) and the intensity of the stimulating factors *after* parturition. However, the general view is that the proliferation of the mammary gland is complete *at parturition*. Moreover Reece et al. (1939) have shown that the secretion of lactogen (the pituitary hormone which stimulates milk secretion) is at a maximum then, so that these cannot be the causes of the upward trend *subsequent to parturition*. Moreover the rate and duration of this rise is not similar in all animals. Neither is it exactly alike for the same cow in different lactations. Even the point of termination of this period is not sharply marked in most lactations. The point of commencement of Pontecorvo's "middle period" would, therefore vary more or less arbitrarily from cow to cow and even in the same cow from lactation to lactation. His procedure cannot therefore be expected to give comparable results for different animals.

As a result of the above review of literature it was decided that the exponential equation probably provided the best measure of a lactation curve. In the preliminary studies detailed below this method was therefore adopted.

2. Collection and Tabulation of Data.

The present study is based on the records of milk production of Ayrshire cows in six tuberculin tested pedigree herds, all of which are situated in Ayrshire. The animals of five of these herds were of the "milk" type and those of the sixth were of the "vessel-bred" type*. The milk records used were those recorded officially by the Scottish Milk Recording Association the yields being determined by the Association's own officials. The Association publishes the records of production of selected cows annually. In the past most of the work done on the inheritance of milk yield, both in this country and abroad, is based only

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*The Ayrshire breed consists of two distinct types: (i) "vessel" type, the cows of which have tight, flat vessels and small corky perpendicularly-hung teats, Greater emphasis is laid on the conformation of the "vessel" - udder - rather than the milk yield in this type. (ii) "milk" type in which not so much attention is paid to the actual shape of the udder as to the milking propensities of the animal.

on such <u>published</u> records, and therefore on selected data. The results of such work are consequently of limited application. It was decided to avoid this criticism in the present study. Original milk record books were therefore obtained from the herd owners and <u>all</u> normal records of milk production of the cows were abstracted. Preliminary examination of the records showed that, owing to the changes in policy in herd management, significant changes had taken place in the milk yields of cows during the post-war period as compared with earlier years. Although milk recording had been continuously carried on since 1910 in five out of the six herds (records therefore being actually available for a period of about 30 years), it was decided to restrict the study to the post-war years only. This selection was considered necessary in order to secure homogeneous data which would be as free as possible from the disturbing influence of the changes in herd management. This obviated the necessity of attempting to correct the milk records for the year-to-year variations. Lush (1936) and Lortscher (1937) have pointed out that such year-to-year variability of milk yield in a herd is not entirely due to differences in herd management, as was supposed by V. Patow (1930), but is also the result of the differential genetic constitution of the herd. As it is difficult to separate the genetic from the purely environmental variability, any attempt to introduce correction factors to allow for year-to-year variations in management would clearly be most undesirable.

Data regarding the number of records provided by each herd and the years to which these records belong are given in Table I.

The milk records of the Scottish Milk Recording Association are based on measurements of milk yield which are carried out personally by the official Recorders, who visit each farm for the purpose once in every 14-28 days. The milk yield is determined by actual weighing over a period of 24 hours, and the results are entered in the milk records book. The day of visit is regarded as the middle day of the period covered by the test, and the total yield for the period is estimated by multiplying the observed yield by the number of days covered by the test. As the duration of this period may vary from visit to visit, the milk yield entries in the milk records book may represent varying periods. In the present study it was essential that the yields should be for periods of constant duration. After copying out the milk records, the yield for each successive 30-day period of every record was computed. It was observed that in many cases the first test was not made until about 30-40 days after calving. To overcome this difficulty the first 15 days of every record were omitted when fixing the limits of the different periods. The computed yield for the first 30-day period thus covered from the fifteenth to the forty-fifth day after calving.

It will be understood that the copying of records from the records books and the computation of the 30-day yields, involved a mass of routine work. Much of this was carried out by clerical assistants who worked under the writer's supervision. To ensure accuracy, all calculations were arranged so that they were self-checking. Records of individual lactations were transferred to printed forms, and special ready reckoner tables were prepared for the assistants' use. Subsequent computations were usually made with the use of a calculating machine.

The details of the information collected for each lactation and the system of computing the yields for the 30-day periods are shown in Table 2.

3. Fit of the Exponential Curve.

(a) Method of Curve Fitting.

In the herds under study the average calving interval was found to be twelve months. Since the average gestation period of cows is 280 days, this indicates that the cows were served approximately three months after calving. The work of Gavin (1913a)

Sanders (1923 and 1927c), Gaines (1926b) and Ostergaard (1931) has shown that gestation appreciably affects the milk production some five months after conception. To obviate the necessity of correcting for the influence of varying service periods it was decided to use only that part of the lactation curve which was free from the influence of pregnancy. For most records, the milk yield for the first eight 30-day periods was used, though for records in which the cow conceived earlier than three months after calving, a shorter part of the lactation curve was used.

The process of curve fitting was straight forward. As already pointed out, Gaines has described a graphic (1927a) and an algebraic method (1927b) for doing this. The former method, although quicker, naturally does not give as accurate results as the latter. It was at first hoped that it would be possible to shorten the work by using the least-square curve fitting machine described by Gaines and Palfrey (1932) and Professor Gaines kindly loaned the machine for this purpose. From the point of view of the present investigation the machine has, however, two disadvantages. It is designed to determine the constants only when the curve has to be fitted to an odd number of observed values (9 or 11), whereas the number of observed values in this study was more frequently even (8) than odd. It is also specially adapted to give the constants for "fat-corrected" (Gaines, 1923) instead of "raw" milk yields. Smith (1933, 1939) has questioned the advisability of fat-correcting the observed milk yields in studies of the inheritance of milk production.

For these reasons the use of the curve fitting machine was abandoned, and the curves were fitted algebraically. The method described by Kartha (1934a) was adopted. The final value of persistency was expressed on percentage basis, i.e. $100 \times e^{-k}$, for each curve.

(b) Discussion of Results.

As a result of this preliminary work of curve fitting it was soon found that, whereas the exponential equation gave a reasonably good fit with some lactation curves, the fit was poor in others. Table 3 and Figure 2 show the fits for some selected curves, which are typical of the general results obtained.

As will be observed by a comparison of the observed with the estimated yields given in this Table, the fit is fairly close with curve one, though it is poor with the remaining three curves. This fact is confirmed by reference to the Σd^2 values given for each curve. In particular the estimated yields of curve two diverge widely from the observed yields throughout the whole course of the lactation. Whereas the actual maximum daily yield of this cow did not exceed 52.4 lbs she had, according to the fitted exponential curve, the capacity to yield 71.8 lbs one month after calving and 92.15 lbs immediately after parturition. Undoubtedly the theoretical curve gives a very exaggerated estimate of her capacity for maximum milk production. The curve of cow four, on the other hand, is of the reverse type. Her yield according to the theoretical curve is 33.98 lbs immediately after calving and 31.6 lbs a month later, though she actually produced 37.5 lbs during the latter period. Her capacity for maximum milk production is thus definitely understated by the exponential curve. Again with curve three the actual maximum production was obtained in the second period, and not, as is indicated by the exponential curve, in the first.

Let us examine the causes of this poor fit. Within the range of t taken, the exponential curve $y_t = M_0 e^{kt}$ has its maximum (or minimum) ordinate at zero. The values fall off (or increase) exponentially thereafter, i.e. the rate of relative decline (or rise) is constant. For such a curve to give a good fit, these conditions must be satisfied by the data. As has already been pointed out, curve three has its maximum ordinate at the second period, so that the requisite condition that the maximum ordinate should be attained at the start is not satisfied.

Neither is the relative rate of decline from month to month constant with curves,two, three and four. With curve two, for instance, the yield was nearly constant during the first three periods. There was indeed a slight increase. However, this was followed by a sudden decline, the relative rate of which continued to accelerate from period to period. With curve four, on the other hand, the fall in yield was too abrupt at the commencement of the lactation. The curve flattened out four months later, and there was little subsequent decline, It is significant to point out that curves of type two were found to be rather characteristic of cows calving in late spring, and those of type four typical of heifers calving in autumn.

It will be clear from these examples that individual lactation curves may not conform to the exponential type even approximately and that, if the latter type of theoretical curve is used to represent such data, one may fail to distinguish real differences between the shapes of the curves, - even when the curves are as dissimilar as the "observed" and "estimated" shown as type two in Figure 2. No doubt, the type two curve exhibits a rather extreme divergence, In general, the differences between "observed" and "estimated" values though well-marked, are not so great.

The Missouri workers, Brody et al. (loc.cit.), have considered these variations from the "type" to be entirely environmental and by representing the data with an exponential equation, have ignored them. In the absence of definite supporting evidence, their view is hardly justified. It is quite conceivable that the responses of individual animals to the various environmental influences may themselves vary according to their inherent constitution, so that a part of the divergence from "type" may in reality be genetic. In any event it is clearly preferable to measure such variations, since these can then be correlated with the possible causative agencies to determine how far they are environmental and how far they are hereditary.

It has been suggested that a better fit can be obtained with the exponential curve if the declining segment only is used in fitting (Turner, 1939). This procedure does not appear justifiable, however, since it not only fails to distinguish between curves with rising segments of varying duration, but also involves the comparison of these curves for varying periods. Moreover, the fit will still be poor with curves of types two and four.

It may be noted that the exponential curve does give a good fit for the <u>average</u> lactation curve. This is clear from curve five in Table 3, which was obtained by averaging 100 lactation curves of individual cows.

The reason for this better fit is, of course, that the differences of curves two and four from the exponential are compensating in nature. The fact that the exponential curve gives a good fit with averages of lactation curves is no proof that the rate of decline of milk yield with the advance of lactation is normally exponential.

As far as can be ascertained no results have been reported in the literature which show how far the exponential curve fits the lactation curves of individual cows. Gaines (1927b) has compared the accuracy of his graphic and of Turner's ratio methods with the least-square method for fitting the exponential curve, but he has not shown how well the latter fits the curves of individual cows. In one of his papers (1931) he has instanced two lactation curves; these conform to types two and four of Table 3. Gooch (1935) determined the root-mean-square error* for each curve in order to measure how far the fitted curve diverged from the actual data. Her work only shows, however, the extent of the errors involved in estimating yields from the fitted curve. It too does not show if the exponential is really the best fitting curve.

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* i.e. square root of sum of squares of the difference between logarithms of observed and calculated y's divided by the degrees of freedom.

4. Fit of the Parabolic Exponential Curve.

(a) Method of Curve Fitting.

In view of the considerable discrepancies which were found to exist between the actual lactation curves and the fitted exponential values, an attempt was made to see whether some other type of mathematical curve would represent the lactation curve better. As the course of the latter is parabolic, a trial of the parabolic exponential curve, $y = Ae^{b't+c't^2}$, was suggested by Dr R.A. Robb. This was studied by means of data secured from the Institute's own herd. 100 lactation records were taken, and the milk yield of each worked out for successive 28-day periods. Unlike the previous data these records were based on the day-to-day recording of the milk yields of each cow throughout her lactation.

The parabolic exponential equation $y = Ae^{b't+c't^2}$ may in ordinary logarithms be written as

$$\log_{10}y = \log_{10}A + bt + ct^2$$

$$\text{where} \quad b = b'\log_{10}e$$

$$c = c'\log_{10}e$$

The fitting of such a regression line may be conveniently carried out by the method of Orthogonal polynomials described by Fisher and Yates (1938). The linear constant b' in the above equation then corresponds to constant k of the exponential equation $y = Ae^{kt}$.

As the fitting is carried out in two stages it is possible to examine separately the fit of the linear and the parabolic equations to the data, and to judge whether the contribution of the parabolic constant is significant in the reduction of variation. Where the parabolic term is significant it shows that the linear constant alone does not account for all the systematic variations of the data, i.e. that the exponential curve does not give the best possible fit.

(b) Discussion of Results.

This work confirmed the previous finding that the goodness of fit of the linear term (i.e. its contribution in the reduction of sum of squares) varied considerably in the individual records. The sum of squares which the linear term accounted for varied from 33.92 to 94.48% of the total in individual curves, the mean being 85.06±1.43% and the standard deviation 14.28±1.01%. This contribution was significant at 1% level in 91 curves, not significant at 1% but significant at 5% level in another four curves, and not significant even at 5% level in the remaining five curves. The milk yield data of the latter five curves are graphed in figure 3. It will be observed that the milk yields of successive periods in these five lactations are too erratic to be satisfactorily graduated by any simple mathematical curve. The equation gives, of course, a value of b (which is a measure of the average slope of the curve) for each of these lactation curves, but owing to the highly erratic slope at different points of these curves, this average value has little meaning. In the remaining records the linear term is undoubtedly the preponderating term and represents the greater part of the systematic variation of rate of milk production with advance in lactation.

The above facts do not indicate , however, that in these records the exponential curve gives as good a fit as it is possible to obtain, for, if this were so, the contribution of the additional constants to the reduction of variation would be insignificant. It was in fact observed that in 48 of the 100 lactations the parabolic term brought about a significant reduction in the sum of squares, - in 23 cases significant at 1% level and in the remaining 25 at 5% level. In the remaining curves either the linear term accounted for ~~the~~ most of the systematic variation and the value of the parabolic term was therefore too low to bring about any further marked reduction of variation, or the residual variation was too high, owing to irregular yields, to allow the contribution of the parabolic term to be statistically significant. The fit of the parabolic equation was with almost all lactations better than that of the exponential, although the extent of this improvement was in some instances not high enough to justify the sacrifice of a degree of freedom by the addition of another constant. Table 4 shows the relative fit obtained with the two types of theoretical curves. It will be seen that the parabolic term is quite important in curves one, two and three; in curve four, however, its contribution is not significant. The observed values of curve five are so variable that neither the linear nor the parabolic term are significant.

It has already been pointed out that the shape of individual curves is too variable to be satisfactorily represented by a rigid curve of the exponential type. Table 4 shows that the parabolic exponential curve is in many cases elastic enough to take care of this variability. It is sufficiently adaptable to take into account satisfactorily both the varying position of the maximum ordinate and the varying rate of change in milk production in successive periods, provided that the data are reasonably regular. The latter condition is important, as is clear from curve five in Table 4.

In the parabolic logarithmic equation

$$\log y = \log a + bt + ct^2$$

b = Uniform rate at which log of milk yield is changing every month, i.e. the average relative slope of the curve.

c = 1/2 log of the rate of change of decline rate (b) per month per month. If positive it means that the rate of decline is retarding, and if negative that it is accelerating with the advance of lactation.

This equation thus not only gives an average measure of the rate of decline (persistency) for the whole curve, but also measures how, on an average, this decline varies from period to period. In this respect it has a distinct advantage over the exponential equation.

It is possible to estimate the value of the theoretical maximum by the differentiation of this equation. This estimated value may be realised at any stage during the lactation or, according to the equation, may be supposed to occur even before the start of the lactation (-t). In the latter case the value, as with the exponential equation, will be hypothetical and extrapolated. This extrapolation is not safe (Snedecor, 1938 p.316). It has already been shown how very different the theoretical maximum (extrapolated from the exponential curve) may be from the actual maximum. The latter will, in the writer's opinion, provide the safer and truer measure of the cow's physiological capacity for maximum milk production, in spite of Gaines (1926a) brilliant inhibition hypothesis (by which he seeks to explain why A (the theoretical maximum) is not realised in practice), and Davydov's (1933a) conclusion that A and the actual maximum are highly correlated. It may be noted that Gaines' hypothesis is in any event in the nature of a surmise and must always remain so, since no experiment can be set up to prove or disprove it.

Gaines (1926a, 1927a) observed that 5% of the curves in his data differed from the <u>exponential</u> curve in as much as they were of the increasing rather than the usual decreasing type, - at least for the period for which the milk yields were examined (10-12 months). As Gaines himself pointed out, it "would be absurd to suppose that the lactation curve could continue to ascend for more than a limited time" (1926a). The rather abnormal rise is bound to result in a decline sooner or later. The exponential curve cannot take account of this fact though the parabolic exponential does so.

From the foregoing it will be clear that the shape of the lactation curve of individual cows conforms more closely to the parabolic exponential than to the exponential type. The use of the former is therefore to be preferred in graduating data for any genetical study of the shape of lactation curve. Apart from the advantage of the better fit, the parabolic exponential curve provides two separate constants, - the linear, which measures the average slope of the curve, and the parabolic, which describes how on an average the rate of this slope varies from period to period. In any study of persistency it is clearly of advantage to be able to separate these two constants, so that they can be correlated independently with any relevant environmental or hereditary factors.

5. The Frequency Distribution and Constants of the Various Characteristics of the Lactation Curve.

In view of the results reported in the previous section it was decided to use the parabolic exponential curve to graduate the data of this investigation. However, by this time, the exponential curve had been fitted to nearly the whole of the data of one herd. The parabolic exponential curve was therefore fitted to the data of the remaining herds only, comprising 1900 records.

After the work of curve-fitting was completed, the results were transferred to special cards. One card was used for each cow's data, and bore on it the following information:-

1. Name and Herd Book No. of Cow.
2. Her Pedigree.
3. For each of her lactations
   - (a) Date of calving
   - (b) Total milk yield for 8 months.
   - (c) b (linear) Constant
   - (d) c (parabolic) Constant
   - (e) b Constant expressed as percentage persistency
   - (f) Maximum Yield

A total of 2,828 milk records derived from 863 different cows were thus tabulated. Of these 863 cows, 215 had only one record each. For the remaining 648 animals two or more records were known. It was noticed that there was a far smaller variability in the persistency of different cows on the basis of their heifer than their later records. It was therefore decided to exclude all cows which had only one record. This restriction was considered necessary in order to secure more dependable results and it was felt that it did not cause any appreciable or undesirable selection of the data. This procedure left 2,613 records, 2392 of which came from herds A to E ("milk" type), and the remaining 221 from herd F ("vessel" type). It was considered advisable to keep the "milk" herds data separate from the "vessel" herd data, as there are considerable differences in the systems of management of these two types of herds. The study of the environmental factors was made from the "milk" herds only. The frequency diagrams illustrating the distribution of persistency parabolic constant, maximum yield and total yield of the 2,392 "milk" type records are given in Figure 4. The main statistical constants of these data are given in Table 5.

It will be observed that the distribution of all the four constants is slightly asymmetrical, The mode is higher than the mean in the case of persistency and parabolic constant, and lower than the mean in the case of maximum yield and total yield. The skewness is negative in the case of persistency and parabolic constant and positive in the case of maximum yield and total yield. All the four curves are leptokurtic, the curve for parabolic constant showing this peaked effect to the greatest degree. The values of k indicate that the curves for persistency and maximum yield conform to type one, and the curves for total yield and parabolic constant to type four of Pearsonian curves (Elderston, 1906, p.50). There exist very great differences between the variability of parabolic constant and persistency as compared with total yield and maximum yield. The value of coefficient of variation is the lowest in the case of persistency and the highest in the case of parabolic constant. In fact, the value of standard deviation is actually higher than the mean in the latter, which gives it a coefficient of variation which is more than 100. This shows that the parabolic constant is by far the most variable constant of the lactation curve.

PART II. THE LACTATION CURVE AS AFFECTED BY ENVIRONMENT.

1. Introduction.

The milk production of a cow is an expression of her heredity as modified by environment. The heredity of an individual is fixed at birth and remains constant throughout its life. The environment may, and in actual practice does, change from period to period. That is why the milk production of a cow is apt to vary from time to time.

The observed differences between the production of individual cows may be partly hereditary and partly environmental. For the correct estimation of the former, it is important to allow for the effects of the latter. To eliminate the effect of environmental variations altogether (e.g. for genetical researches) Smith (1935) has emphasised the great value of the method of collecting data for milk production from animals kept under conditions of uniform environment and has pointed out that he is attempting to follow this procedure at the experimental farms of the Institute of Animal Genetics, Edinburgh. The number of environmental factors affecting milk production is, however, so large, that the complete experimental control of environment appears impracticable. The nature of food supply is bound to differ from year to year, and still more in different seasons of the same year. All cows do not calve at the same time of the year, neither do they all calve in the same state of health. Changes may take place in the byre staff, with consequent changes in the methods of handling stock. Under such conditions the records for different periods are seldom directly comparable. As a result it becomes necessary to exercise a statistical control and allow for these differences of environment. Even this, however, is only possible where the nature of the environmental differences are known, and where their effects can be estimated. When the influence of an environmental factor is known, it is possible to "correct" or "standardise" the record to what it would have been under the conditions of the standard environment. The determination of such "correction" factors is a pre-requisite to all genetical researches.

It is perhaps desirable at this stage to point out that correction factors measure only the <u>average</u> influence of a particular factor in the whole population. As the responses of different individuals to an environmental stimulus are not necessarily identical, the records of individual animals even after "standardisation" may differ from what they would have attained under the standard environment. For this reason each standardised record is still liable to error. However, if the correction factors are valid for the population as a whole, the errors in individual records after "standardisation" are as likely to be positive as negative. When several "standardised" records of a cow are available, such errors are likely to counterbalance one another.

Of the various environmental factors, feeding and management are perhaps the most potent in determining differences in the production of individual animals. It has not been possible, however, to examine the influence of these factors statistically in the present investigation, since the differences in this respect between individuals and even between herds are not known.

Hammond and Sanders (1923) and Sanders (1927a) have shown that the season of the year in which a cow calves, her age at the time of calving, the length of the service period (i.e. the interval between parturition and the following gestation), and the length of the dry period (i.e. the period for which she is kept dry before her next lactation starts), are four important factors that affect milk production. Turner (1927c), Sikka (1931), Ostergaard (1931) have shown the importance of the length of the lactation, and Copeland (1935) the number of times a day the cow is milked. Matson (1929) has studied the influence of the length of one lactation on the milk production of the succeeding lactation.

In the present investigation all the cows were milked twice daily, so that there were no differences in this respect between the individual records. The segment of the lactation curve used was restricted to the first eight and a half months, and the average service period was about three months. In view of the researches of Gavin (1913a) Hammond and Sanders (1923), Sanders (1927b) and Ostergaard (1931), no correction has therefore been considered necessary for the influence of pregnancy or for the length of lactation. Moreover, since a part of the observed variation in the length of lactation of individual animals may be due to genetic differences between the animals, Smith & Robison (1933) have in any event questioned the advisability of correcting the records of production for the length of lactation in genetical studies.

It was not possible to study the influence of dry period since, owing to the prevailing practice of stopping the recording of milk of certain animals with the close of the calendar year, the length of this period could not be correctly ascertained in all records. Only two important factors remained for investigation, i.e. month of calving and age. The influence of these factors was therefore studied, using the total data of the four "milk" herds (2,392 records) and is detailed in the following sections. It is possible that the influence of these factors may vary from herd to herd. Such intra-herd variation could not be studied.

One further point should be mentioned. Table 6 shows the number and ages of cows calving in each month of the year. It will be observed that the proportion of animals calving in different months varies markedly. In general many more cows calve during the spring and autumn than during the summer. The incidence of calving is lowest for July. It increases until October, but falls during November and December. There is a sudden rise in January, and the higher rate is more than maintained during the following month. There is again a very steep rise in March when the incidence of calving is highest. This is followed by a steep fall, which continues until July. This seasonal variation in the incidence of calving is shown graphically in Figure 5.

The important point is, however, that a distinct correlation exists between the age of the cow and the month in which she calves. This is clear from the last column of Table 6, which gives the mean age of the cows calving in each month. It will be observed that the means are lowest in the autumn months and are highest in mid-summer, the seasonal trends being very regular and consistent. A very much higher proportion of younger cows calve, therefore, in the autumn and winter than in the spring or summer.

2. General Method of Study.

In studying the influence of the month of calving and of age on the different constants of the lactation curve it is necessary to allow for the correlation reported in the preceding paragraph. The method of multiple regression would be ideal for the purpose, if the regressions were linear. However, as will be shown later, the regressions are distinctly non-linear. This method is therefore not practicable.

It was found that in the data available for the present study there were a number of animals which calved in the same month during two or more successive lactations. The variation of the different constants of the lactation curve with age was determined from the data furnished by such animals by the "paired-lactation" method, the details of which will be explained later. Such constants will be free from the influence of month of calving. The correction factors thus obtained were used to correct all records for the influence of age. The influence of month of calving was then studied from the corrected records and correction factors for month of calving worked out. These were in turn applied to the original data and from the thus corrected data the effect of age was studied. The weakness of this method is that the preliminary age correction factors were obtained from rather limited data. The correction factors for month of calving may therefore be subject to slight errors, the magnitude of these errors being dependent on the extent of the differences between the preliminary and the final age correction factors. It will however be shown later that the influence of the month of calving was very small in comparison with that of age, especially in the case of maximum and total yield. The main results therefore cannot have been seriously affected.

The variation of the parabolic constant was studied from the raw data, i.e. without allowing for the correlation observed between month of calving and age.

Throughout the present thesis the constants of the lactation curve are (unless otherwise stated) defined as follows:- (i) Persistency is the linear constant of the exponential curve expressed on a percentage basis, and represents the average relative slope of the lactation curve; (ii) the parabolic constant is the third term of the parabolic exponential equation, and represents the rate of change of decline rate; (iii) the maximum yield is the highest 30-day yield calculated from the official milk records, and (iv) the total yield is the yield of milk for the portion of the lactation curve studied (i.e. from 15 days to 8½ months after calving) calculated as shown in Table 2.

3. Influence of Month of Calving.

(a) Variation of the Constants of the Lactation Curve with the Month of Calving.

The means of the four constants of the lactation curves of cows calving in different months (after correction for age) are given in Table 7. Figure 6 shows these results graphically.

(i) Persistency.

The analysis of variance of persistency with the month of calving is given below:-

Source of Variation	d/f	Sum of Squares	Mean Square
Between means of calving months	11	4857.8231	441.6203
Within calving months	2380	43805.9945	18.4059
Total	2391	48663.8176	20.3529

$$F = 441.6203/18.4059 = 23.9934.$$

The observed value of F (the variance ratio) is highly significant, i.e. well beyond the 1% level. This shows that there are real differences between the persistency of cows calving in the different months. These differences account for

$$\frac{20.3529 - 18.4059}{20.3529} \times 100 = 9.566\%$$

of the total variance of persistency.

An examination of the mean persistency values given in Table 7 shows that these have a very consistent seasonal trend. The cows calving in May have the lowest mean, and are therefore the least persistent. The value increases regularly during the succeeding months until August, remains more or less constant until November, and then diminishes. As compared with the mean of all months, the means for the period July to January are higher, and those for the remaining period of the year lower.

(ii) Parabolic constant.

The analysis of variance of the parabolic constant with the month of calving is given below:-

Source of Variation	d/f	Sum of Squares	Mean Square
Between means of calving months	11	24572.4125*	2233.8556*
Within calving months	1667	204643.1415	122.7613
Total	1678	229215.5540	136.6004

* in units of class interval.

$$F = 2233.8556/122.7613 = 18.1967$$

The observed value of F is again highly significant, showing that the means of the parabolic constant for different months vary significantly among themselves. Their variation accounts for

$$\frac{136.6004 - 122.7613}{136.6004} \times 100 = 10.1310\%$$

of the total variance of the parabolic constant.

It will be observed from Table 7 that the trend of means shows a remarkable regularity from month to month. This indicates that the underlying causal influence for the observed variation is regular in its action.

The means are positive in sign for the period August to November and negative for the remaining months of the year. However, of the positive means only that for October is more than twice its standard error, and may therefore be considered to be significant. The remaining positive means are not significantly different from zero. The values of all the negative means are, however, more than twice their standard errors, and are therefore significant.

The low positive value of the mean for October calvers shows that their rate of decline of milk yield retards with advancing lactation, i.e. they maintain their milk yield better in the second half of their lactation than in the first half.

The negative value for the months December to July indicates that the rate of decline of the curves of cows calving in this period accelerates with the advance in lactation, i.e. their production declines more rapidly in the latter half than in the first half of the lactation. This phenomenon of high persistency in the first half but tendency to quick drying off in the latter half of the lactation was exhibited to a varying extent by the cows calving in different months. The spring calvers, i.e. cows calving between the period February to April, showed this to the greatest extent. Animals calving in months immediately preceding or following this period showed it to a lesser degree.

Figure 7 illustrates very clearly the differences in the shape of the lactation curve between autumn and spring calvers.

(iii) Maximum Yield.

The analysis of variance of the maximum yield with the month of calving is given below:-

Source of variation	d/f	Sum of Squares	Mean Square
Between means of calving months	11	2319.5530	210.8685
Within calving months	2380	152035.1256	63.8803
Total	2391	154354.6786	64.5565

$$F = 210.8685/63.8803 = 3.3010$$

The observed value of F is again significant, showing that the means of different calving months differ significantly between themselves. These differences in the month of calving account for

$$\frac{64.5565 - 63.8803}{64.5565} \times 100 = 1.047\%$$

of the total variance of the maximum yield. This is very much lower than for the persistency or the parabolic constant, showing that the maximum yield of the cows in these data is less influenced by the effect of seasonal variations than the linear or parabolic terms.

A study of the means given in Table 7 shows that these are below the average for the months July to September, and above the average for the three months immediately preceding this period. May and June seem to provide the conditions most conducive to the production of the highest daily yields, whereas August appears to be the month when conditions are least favourable in this respect.

(iv) Total Yield.

The analysis of variance of the total yield with the month of calving is given below:-

Source of Variation	d/f	Sum of Squares	Mean Square
Between means of calving months	11	1134.6337*	103.1485*
Within calving months	2380	94180.4323	39.5716
Total	2391	95315.0660	39.8641

*in units of class interval.

$$F = 103.1485/39.5716 = 2.6066$$

The observed value of F is significant beyond 1% level, showing that the means of different calving months differ significantly among themselves. However, these differences between means account for only

$$\frac{39.8641 - 39.5716}{39.8641} \times 100 = 0.734\%$$

of the total variance of milk yield. From this fact it appears that the month of calving is only a minor cause of the variation of the total milk yield. This is largely due to two causes: (i) The variation of persistency and maximum, the two determinants of total milk yield, in some months is inverse, i.e. for months which have a relatively higher persistency the maximum is comparatively low and vice versa; and (ii) the number of calvings is the lowest when the yield is also the lowest. In spite of this the means are above the average for the period October to January and below the average for the months April to September. The mean for January is the highest and that for July the lowest.

(b) Alternative Method of Study and Calculation of Correction Factors.

The validity of the foregoing results relating the variation of the constants of the lactation curve with the month of calving might be questioned on two grounds: (i) that the observed differences may be partly due to the fact that the variation of other environmental factors which affect the shape of the lactation curve is not random from month to month; and (ii) that the observed differences may have arisen as a result of the tendency for the best and poorest yielding cows to calve in different periods of the year.

It was not found possible to test the first objection, and it was assumed that none of the other environmental factors (e.g. service period or dry period) except age was correlated with the month of calving. To make sure that the observed differences were not attributable to the second factor, the variations of the different constants of the lactation curve with the month of calving were studied afresh by comparing the age-corrected records of the same cow falling in different calving months. It is evident that such intra-cow comparisons would preclude the possibility of any of the observed differences arising from cause (ii) above.

(i) Persistency.

The mean persistencies for the different calving months obtained from this intra-cow comparison are given in Table 8. The first two columns compare the mean persistency of cows when they calved in January with their persistency when they calved in the other months of the year. The next two columns give the February comparisons, and similar comparisons are given for subsequent months in the remaining columns.

It will be observed that the means of the cows calving in the late summer and autumn months are higher than those of cows calving in the spring or early summer months. The significance of these differences will become clearer, however, if the means of different months are compared after standardisation to a common basis. It will then be easier to determine whether the results given in the different columns are consistent and mutually confirmatory. This has been done in Table 9 by taking the means for January as the standard and giving them the arbitrary value of 100.

The first column of Table 9 gives the relative values of the persistency of cows calving in January in one lactation and any other month in another lactation. It will be observed that the values for February, September and November are little different from 100, whereas those for August and October are higher and for the remaining months of the year lower than 100.

Let us now examine how far these results are supported by those of other columns of Table 8. This can be done by taking the relative value for each month (as given in the first column of Table 9) as a standard, and calculating the comparative persistency of the different months from the results given in the remaining columns of Table 8. The values thus obtained can then be compared among themselves. Such values are given in the remaining columns of Table 9.

It will be observed that, although there are a few anamolies, the values for the various months show reasonable consistency. The values have, therefore, been averaged to determine the mean variation of persistency for each month. The weighted means thus obtained are given in Table 9 and are shown graphically in Figure 8. For comparison the relative values calculated from the first method (~~see Table 7~~) are also given.

It will be seen that in general the values obtained by the two methods agree closely. Only those for February, July and August calvers differ by more than 1%.

According to the results obtained by the second method, there appears to be little difference in the persistency of cows calving from December to February. The March calvers do not however maintain their yield so well. This decrease in persistency continues during the following two months. The persistency of cows calving in May is, in fact, the lowest for the year. Some improvement takes place with the June and July calvers and there is a steep rise in August, when the persistency is the highest for the year. The value declines again in September, though it still remains slightly above the January level. It rises during the next two months.

These results are in accord with the researches of other workers (Sanders, 1927a); Gaines, 1927a; Gooch, 1935; Pontecorvo 1940).

It is important to note that the foregoing results only give the variation of <u>the average value of persistency</u>. This average value does not show how the <u>rate of decline</u> of the same curve varies from month to month. The parabolic constant measures the rate of decline, and it has already been shown that there are significant differences in the value of this constant in different months, autumn calvers having a low positive value, and spring calvers a high negative value.

Considering the variation of both the constants together, it appears that cows calving in the spring months have more or less ideal conditions for the maintenance of their milk yield in the first half of lactation, but that these conditions become increasingly unfavourable in the second half. The reverse appears to be true for autumn calvers, though the difference in the rate of decline in the two halves of lactation is less marked. The spring calvers are undoubtedly able to maintain their yields better in the first half of lactation than the autumn calvers, but the rate of decline in the latter half is so much more rapid that over the whole lactation their persistency is considerably lower.

Note on the Relation of Persistency to Feeding and Management.

The feeding and management of cows differ considerably according to the season of the year. During spring and summer the animals are kept in the open and live largely on pasture. On the other hand during the late autumn and winter period they are more or less confined to the byre and are entirely stall-fed. Can the observed differences in persistency be correlated with these different conditions of feeding and management?

The rate of growth of herbage in the pastures is highly seasonal. Watson (1939) gives a curve showing this variation for the different months in England. According to this curve, maximum growth takes place in May. The rate of growth diminishes rapidly until July, when it is only about 25% of the peak rate. This low rate is maintained till the middle of August, after which the second flush period occurs. After the middle of September the rate falls off steeply, and there is little growth in November and the remaining winter months. The shape of the growth curve of pasture grass will probably vary somewhat from locality to locality, and from year to year. However, Watson's results seem to give a good picture of the seasonal variation observed in most parts of Great Britain in normal years. Data from the Hannah Institute Farm (unpublished 1937) supports this view.

Woodman et al. (1926) have shown that the dry matter of grazed pasture herbage in early summer has the character of a protein concentrate. It has a high protein content and a high digestibility. However, this high nutritive value diminishes considerably with the onset of drought conditions during the mid-summer period when, owing to the lignification of tissues, the herbage becomes coarser, has a lower protein content and a lower digestibility. Similar changes occur if the grass is not grazed but is left on to mature.

Crampton and Forshaw (1940) have shown that the feeding value of the herbage of any single species changes during the growing season. Herbage grown during spring and fall when plant growing conditions are favourable is held to be of excellent feeding value. On the other hand, herbage grown during mid-summer is stated to be of lower nutritive value, due apparently to a reduction in the availability to the animal of the carbohydrate fractions. It may further be noted that Morris et al.(1937) observed that whereas the proteins of the spring grass have a high biological value, those of the autumn grass are distinctly inferior in this respect.

In the light of these results an explanation may be offered for the observed seasonal variation in persistency. The initial flat curve of the spring calvers may be attributed to the copious supply of the highly nutritious spring grass. The young, succulent herbage available at this time stimulates milk production and maintains it at a high pitch. This supply is, however, too abundant to be used up before it gets woody. This maturing, combined with the mid-summer drought, causes a definite deterioration in the quality of herbage available for grazing after June, so that milk yield cannot be maintained at the high spring level. The diminishing biological value of the grass proteins as Autumn approaches probably accelerates this rate of decline. The higher rate of decline in the earlier part of the lactation of October calvers is also attributable to this effect, and the retardation of their rate of decline later on is presumably due to their more adequate stall feeding as soon as they are permanently housed for the winter, and to the gradually improving pasture conditions with the approach of spring.

It is evident that from July onwards pasture alone is not sufficient to keep up a high rate of milk flow. Grazing must be supplemented by stall-feeding at this time in order to achieve the best results. The exact time when the need for stall-feeding will arise will, of course, vary from year to year and from place to place. Any observed acceleration in the rate of decline of yield should indicate to the herd owner the need for immediate supplementary feeding.

Correction Factors for Persistency. From the weighted means given in Table 9 it is possible to derive the correction factors required to standardise the records fob the effect of the month of calving*. Taking the persistency of January calvers as the standard, such factors are given below:-

Month of Calving	Correction Factor (persistency for January 1.000)
February	1.004
March	1.016
April	1.031
May	1.040
June	1.037
July	1.024
August	0.979
September	0.995
October	0.990
November	0.985
December	1.002

It will be observed that the factors are higher than unity for the period February to July, when the

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* The correction factor for any month is given by the ratio the value of the standard month is to the value of that particular month.

persistency is lower than that of January, and lower than unity for the remaining months when it is higher than that of January. The observed value of persistency for any month must be multiplied by the appropriate factor to standardise it to the January basis. Ward and Campbell (1939) have questioned the advisability of employing a ratio method of correction. A full discussion of their views will be deferred until the influence of age is considered. The ratio method is, it is true, defective, inasmuch as it assumes, without proof, that the effect of month of calving is in proportion to the observed value of persistency, although it is quite conceivable that less persistent animals may differ from the more persistent animals in this respect. However, the effect of month of calving is so small that the standardised values obtained by the ratio method would not be materially in error if the correction factors, instead of being proportional, differed somewhat with varying levels of persistency.

(ii) Maximum Yield.

The mean maximum yields for the different calving months obtained by the second method are given in Table 10, and Table 11 shows these results when the yield of January calvers is taken as the standard and rated at 100. As with persistency, the results obtained by the first method are also shown in the latter table for comparison.

It will be observed that the difference between the two sets of results is more marked than with persistency. The first method seems to have exaggerated the actual difference between the maximum of late summer calvers and of other cows. This difference is reduced by the second method.

According to the second method, the February calvers appear to give a slightly lower maximum than the January calvers. The difference may not, however, be significant. Conditions seem to be more favourable for higher production with March calvers, and improvement continues until June, when conditions seem to be more or less ideal. The maximal production of June calvers is followed by a steep decline in July calvers,when the yield falls to the January level. This decline reaches its lowest level with August calvers. The yield recovers during the next two months, when it is more or less on a level with that of the January calvers. There is slight fall in November followed by recovery in December, but it is doubtful if these latter changes are significant. These variations are shown graphically in Figure 8.

The above results confirm those of earlier workers (Sanders, 1927a; Gaines, 1927a; Gooch, 1935) in showing that the maximum daily yield of a cow may be influenced to some extent by the time of the year when she calves.

It may be noted that in general the trend of the curve for maximum yield appears to be opposite to that for persistency. During the months when the maximum yield is high that of the persistency is low, and vice versa. This phenomenon will be referred to again when considering the variation in total yield.

The high maximum yield of the spring calvers is undoubtedly due to the copious supply of highly nutritious young spring grass. The poorer production of the autumn calvers, on the other hand, is probably the result of the inferior quality of pasture then available.

Correction Factors for Maximum Yield. The following table gives the correction factors necessary to standardise the maximum yields for the effect of month of calving. As with persistency, the production for the month of January has been taken as the standard.

Month of Calving	Correction Factor (January, 1.000)
February	1.008
March	0.989
April	0.967
May	0.947
June	0.936
July	1.000
August	1.049
September	1.000
October	1.000
November	1.012
December	1.007

(iii) Total Yield.

Table 12 gives the mean total yields for the different calving months as determined by the second method, and Table 13 shows these results when the yield for January calvers is taken as the standard and rated at 100. To facilitate comparison the relative yields for the different months as obtained by the first method are also included in this Table.

The results obtained by the second method differ somewhat from those given by the first method. In general, less variability is noticeable in the relative values of different months according to the former than the latter method.

The results given by the second method show that the cows calving in July give the lowest milk yield, about 10% lower than those calving in January. Conditions seem to be more favourable for production for cows calving during the following month, when the value increases by about 5%. This higher rate is maintained in September calvers. The October calvers show a further increase, their milk yields being very similar to those of January calvers. The November calvers give the highest yield for the year, i.e. 100.6% of the standard, though there is a distinct decrease with December calvers. The total yields of cows calving between January and July decrease progressively.

These results are presented graphically in Figure 8, which also gives the curves for the seasonal variation of persistency and maximum. A study of these curves shows that for cows calving during the spring and early summer months the total [illegible] milk yield is lower, in spite of the higher daily maximum, than for cows calving in the autumn and winter months, when the daily maximum is lower. The explanation undoubtedly lies in the differences in persistency between the two periods. This re-emphasises the importance of persistency in the determination of total yield.

It will also be clear that the variations of maximum yield with month of calving are in the opposite direction to those of persistency, at any rate during some months. This has the effect of reducing the seasonal variations in total yield. Such variations would have been far greater if the periods of high maximum yield had synchronised with those of high persistency. In this connexion it is important to point out that the foregoing variations have been observed with milk yields covering only an eight month period. If, instead, the full lactation yield had been used, the superiority of the autumn calvers would have become still clearer, because of their higher persistency and therefore longer lactations as compared with the spring calvers.

The variation of the total milk yield with the month of calving has been extensively investigated. McDowell (1922), Hammond and Sanders (1923), Sanders (1927a), Sikka (1931), Ostergaard (1931), Cannon (1933) and Gooch (1935) have all studied the effect of this factor. As the influence of month of calving inevitably varies according to the conditions of feeding and management which are liable to vary from one locality to another, no attempt will be made to review the results of these researches in detail. It will be sufficient to state that in general they bear out the main conclusions of this study.

One further point may be mentioned. The curve showing the percentage of cows calving in the different months is given in Figure 8. It will be observed that during the period when the total yields are lowest the number of cows calving is also the lowest. Does this prove that the herd owners recognize that this is the worst calving period for milk production? There can be little doubt that the best chance of securing maximum production lies in ensuring a high maximum with a high persistency. Nature has made the attainment of this possible by providing highly nutritious grass in spring. Only if the farmer can duplicate these conditions in the other seasons of the year by more adequately supplementing grazing with stall-feeding, can the total yields of his cows be maintained at maximum level.

Correction Factors for Total Yield. The following Table gives the correction factors necessary to standardise the total milk yields for the effect of month of calving. The production for the month of January has again been taken as the standard.

Month of Calving	Correction Factor (January, 1.000)
February	1.021
March	1.026
April	1.039
May	1.050
June	1.057
July	1.098
August	1.044
September	1.050
October	1.006
November	0.994
December	1.020

4. Influence of Age.

(a) Variation of Constants with Age.

The influence of the month of calving on the four constants of the lactation curve was discussed in the last section, and correction factors were given which enabled these to be standardised for differences in the month of calving. The influence of age was next studied from the data thus standardised. The present section reports the results of this study.

It will be convenient to deal with each constant under three heads: determinations of variation by correlation and by the analysis of variance and covariance, the influence of selection, and determinations of variation by the "paired-lactation" method. It will be noted that no study has been included of the variation of the parabolic constant with age. It has already been shown that the linear term has the preponderating influence in determining persistency, and it was therefore decided to concentrate meantime on this aspect of persistency. It has already been shown in Table 5, moreover, that the parabolic term shows great variability, and that its use in genetic investigations would therefore probably involve special difficulties. The time available to the writer did not permit him to undertake a full study of the significance of the parabolic constant in relation to the inheritance of persistency.

(i) Persistency.

Correlation and Analysis of Variance and Covariance.

A correlation table was drawn up to determine the variation of persistency with age. Table 14 gives the more important statistical constants as derived from this Table:

Table 14 showing correlation of Persistency with Age.

	Age	Persistency
Mean	3.39±0.046	90.79±0.093
Standard deviation	2.26±0.033	4.54±0.067
Coefficient of variation	66.78±1.642	5.00±0.0725
Correlation coefficient	-0.15997±0.01993	

The mean age is 3.39 lactations. Taking the age at first calving as 2.5 years, the average age of cows therefore works out to 5.89 years. Actually this is rather higher than the true value, since only those cows which had been kept for at least two lactations were included in the study. If allowance is made for this the mean age will actually be lower than 5.89 years. This is in close agreement with the results of

Smith and Robison (1931a).

A comparison of the above persistency constants with those of the "raw" data (as given in Table 5) shows that as a result of standardisation the value of the mean has been increased and that of the standard deviation and coefficient of variation slightly though significantly diminished. The value of the correlation coefficient, though less, is still highly significant. Its negative sign indicates that the variation of persistency is inverse to that of age.

Table 15 shows the analysis of variance and covariance of persistency with age.

Table 15 showing the analysis of variance and covariance of persistency with age.*

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between means of age classes	12	8963.3	746.9416	44.0798
Within age classes	2378	40295.9	16.9452	
Total	2390	49259.2	20.6105	
Linear regression	1	1260.6	1260.6	74.3927
Deviations from linear regression	11	7702.7	700.2455	41.3241
Total	12	8963.3		

The observed value of F for differences between means is highly significant showing that the means of different age classes differ significantly among themselves. Their differences account for

$$\frac{20.6105 - 16.9452}{20.6105} \times 100 = 17.7836\%$$

of the total variance of persistency.

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*Based on 2,391 records.

The analysis of the covariance shows that, though the linear regression accounts for a significant portion of the observed variation between the means of different age classes, ($F = 74.3927$), the course of the curve showing the variation of persistency with age is distinctly non-linear (F for deviations from linear regression is 41.3241, which is highly significant). This non-linearity of the regression is the cause of the low value of the correlation coefficient already reported.

The mean, standard deviation and coefficient of variation of each age class are tabulated in Table 16.

It will be observed that the mean is the highest for the first lactation; it diminishes in value considerably at the second calving, but there is little difference between the means of second and third lactation. After this the value increases slowly but consistently with further advance in age. The polynomial regression line given by the equation

$$\log y = 1.9567504+0.00272286x-0.00047279x^2-0.00012192x^3 +0.00002667x^4$$

where y = Percentage perisstency for any age (in lactations) X

and $x = X-\bar{x} = X-6.5$

describes this variation for the first twelve age classes satisfactorily. The fit of this theoretical curve to the observed means is shown in Figure 9, and the means estimated from it are given in Table 20.

According to this equation the persistency reaches its minimum value at the age of 3.65 lactations.

The value of the standard deviation also varies with age, showing that persistency with age is heteroscedastic. The value of standard deviation is lowest with first calvers, increases up to the third lactation, and then diminishes till the eighth lactation. The subsequent trend is irregular and uncertain owing to the paucity of data. The fourth degree polynomial regression line

$$y = 3.818002-0.223298x+0.081387x^2+0.009922x^3-0.012032x^4$$

where y = Standard deviation of persistency for any age (in lactations) X

and $x = X-\bar{X} = X-6.5$

describes satisfactorily this variation of the standard deviation with age. This is clear from Figure 10, where the fit of the theoretical curve to the observed values is shown. The differentiation of this equation gives 3.85 lactation as the age when the value of the standard deviation is maximal.

The curve showing the change of coefficient of variation with age follows the same course as the standard deviation curve. The values increase up to the third lactation, after which they decrease till the eighth lactation. The subsequent means are, of course, again based on too few observations, and their trend is erratic.

The foregoing results agree with those of Gaines (1927a) in that they show that persistency is highest with first calvers, diminishes for some time thereafter and then increases with advancing age. However, like Gaines' results, they are based on the "lumped-lactation" method, and are fallacious in as much as the limited population included in the older age class is definitely selected. With lumped data no allowance can be made for such selection. It is therefore clearly desirable to determine the extent of selection in the present data.

Extent of Selection: The influence of selection may be examined in two ways: (i) By comparing the mean persistency for the same age class of cows kept for different number of lactations. If there has been no selection, there will be no difference between the means of cows kept for a shorter or longer time in the herd and vice versa. (ii) By comparing for a particular age the standard deviation of persistency of cows kept for different numbersof lactations. The effect of the selection will be to reduce variability, so that the value of standard deviation for any age class will be lower for animals kept for a longer period than for animals kept for a shorter period. Both these methods were explored, with the following results:-

The variations of means for each age class in relation to the number of lactations for which the cows were retained in the herd are shown in Table 17. Each column in this Table shows the persistency for a particular age for cows kept for varying numbersof lactations, the latter being shown on the left. The first column, for instance, compares the mean persistency for the first lactation for cows kept for two, three or more lactations, similar comparisons for the second and subsequent lactations are given in the remaining columns of the table. If there has been no selection, the intra-age-class means will not show any significant change with the increase in age. There will be an increase if the selection is positive and a decrease if it is negative. The means for each age class may either be examined from Table 17, or alternatively this comparison may be made from Figure 11, in which the means for each age class are shown graphically. In the latter case if the means for the same age class show no differences between the older and the younger cows, the graph for each age will run parallel to the base line.

A study of Figure 11 shows that the trend of the first five curves is definitely upward up to the eighth lactation and is somewhat irregular thereafter. All the remaining curves except the tenth run more or less parallel to the base. The trend of the tenth curve is slightly downward. These results show that positive selection has taken place up to the fifth lactation. The same point is equally clear from Table 17. It will be seen that up to the sixth lactation the means are in every case higher for the animals kept for an extra lactation than for those of the original animals. Thus in the second column the mean for animals kept for two lactations only is 89.43 whereas that for animals kept for three lactations is 89.94. The difference, 0.51±0.295, is 1.73 times its standard error and, though not by itself statistically significant, is unlikely to be a chance result in view of the similar differences observed in the other age classes.

The variations of the standard deviations for each age class with respect to the number of lactations for which the cows were retained in the herd are shown in Table 18. These results are represented graphically in Figure 12. It will be observed that in general the course of these curves is not as consistent as those showing the mean persistencies. However, indications of the diminution of value with the older cows (up to sixth lactation) are quite distinct. This supports the above conclusion that the population included in the older age classes is definitely selected.

Parallel results for the coefficients of variation are shown in Table 19 and Figure 13. The trend of these curves is very similar to those of the standard deviations.

From the foregoing it is clear that the animals included in the older age classes are not as representative of the population as those included in the younger age classes. Selection has been proceeding in every lactation up to the fifth, only the better animals being retained. It is thus unjustifiable to compare the animals of the earlier age classes with those of the later without allowing for such selection. It is significant to note that most workers in the past have ignored this point. Their results do not therefore give a true picture of this variation.

Unfortunately, there is no ideal method of allowing for such selection. Fohrman's method (1926b) of only using the original entry records and of discarding all retest data does not take into account the fact that the population in the later age classes is definitely more selected than in the earlier age classes. The method used by Kay and McCandlish (1929) of restricting their study to data obtained from animals kept for at least five lactations is unsuitable for two additional reasons (i) since the selection is taking place at all ages the animals which escaped "weeding" for five lactations would be definitely selected; and (ii) such a restriction greatly limits the data that can be used for such studies. Tocher (1928), in his study of the milk yield of Ayrshire cows, employed Pearson's method (1914) of determining the whole of a frequency curve when a part is known in order to ascertain how far selection influenced the correlation observed by him between age and milk yield. This method assumes, without proof, that the frequency curve of the milk yield of a population conforms to a particular type.

Roberts (1928), Sanders (1928), and Sikka (1931) have used the "paired-lactation" method of building up a composite curve to study the variation of milk yield with age. In this method the records which the same cow made at different ages are directly compared. However, since the population in the older age classes is limited and highly selected even this method fails to eliminate completely the influence of selection. The results obtained are strictly applicable to such selected individuals only.

To obtain results entirely free from the influence of selection we need data of a population from which there has been no culling. It is evident that such a condition is almost impossible in commercial herds, which have from time to time to weed out all unprofitable animals. Nevertheless there is no doubt that the "paired-lactation" method does remove the influence of selection in so far as this is practicable. Moreover, it also makes full use of the available data. With data from commercial herds it is therefore the most useful method for determining the true variation of milk yield with age.

"Paired-lactation" Method. In the present data there were 503 cows whose persistency for the first and second lactations was known. The mean persistency for the first lactation of these cows was 94.23±0.143% and for the second lactation 89.43±0.199%. Similarly there were 363 cows whose persistency for the second and third lactation was known. The mean for the second lactation was 89.94±0.218% and for the third 89.32±0.244%. Knowing the relation between the means of first and second lactations, it was possible to determine from the observed means for second and third lactations the true values of the mean for the third lactation, as

$$\frac{89.32 \times 89.43}{89.94} = 88.81,$$

thus giving the persistency values 94.23, 89.43 and 88.81 for first, second and third lactations respectively. This process was repeated for all other ages. The mean persistency values thus obtained are shown in the second column of Table 20, and represented graphically in Figure 14.

It will be observed that the persistency diminishes in value at an ever diminishing rate till the sixth lactation, after which it is more or less constant. Compare these results with those obtained by the "lumped-lactation" method, which are shown in the third column. No increase is noticeable with the "paired-lactation" method after the third lactation, proving that this increase according to the "lumped-lactation" method was fallacious and was the result of selection. The comparison is also illustrated diagramatically in Figure 15 which provides a very interesting picture of this selection. It shows how by selection the herd owner has been able to adjust his policy in such a way as to prevent the natural decline of persistency with age from occurring. Whether such selection is voluntary or involuntary it is not possible to say. Differences of persistency between individual cows are not easily recognised, so that the observed positive selection for persistency may in fact be mainly an indirect result of selection for total yield.

Having determined the true relation of persistency with age, which is given by the mean value for each age in the second column of Table 20 , it is only necessary to smooth these values. The logarithmic curve given by the polynomial

$$\log y = 1.9435168 - 0.0017648x - 0.0000627x^2 + 0.0002219x^3 + 0.0000245x^4 + 0.0000113x^5.$$

where y = Persistency for any age (in lactations) X.

and $x = X - \bar{x} = X - 6$

fits the observed means satisfactorily, as shown in Figure 14. The estimated means from this theoretical curve are given in the third column of Table 20. If the mean of the fourth lactation is taken as the standard and rated at 100, the relative persistency of the other lactations is shown in the fifth column. It will be observed that the value of the first lactation is 106.6% of the standard. There is a decline of about 5% at the second lactation and another 1.5% at the third lactation. Thereafter the value is practically constant.

The foregoing results agree with those of other workers. Turner (1927a) found that the persistency was highest in the first lactation. After this the value decreased rapidly at first but the decline slowed up as maturity was reached. Sanders (1930) observed that the value of the "shape figure" was highest for the first lactation. There was a steep decline in value at the second lactation, after which there was a slow decrease with advance of age. Ostergaard (1931) reported that the value diminished from the first to the third lactation and was approximately constant thereafter. Gooch (1935) found that the younger cows were more persistent than the older cows. A distinct decrease in the value of persistency was observed up to the age of 3½ years. The changes after this were rather erratic, probably due to paucity of data.

Pontecorvo (1940) found that the persistency of first calvers was higher and had a greater variability than that of subsequent lactations, though from the second lactation onwards no very definite trend was noted. His results are in line with the present work in that they show that the first calvers are the most persistent . However the results reported in the present thesis prove conclusively that the variability (as measured by the value of both standard deviation and coefficient of variation) is actually lowest for the first calvers and not, as Pontecorvo reports, highest. It may be noted that Pontecorvo did not present any results in support of his claim.

(ii) Maximum Yield.

Correlation and Analysis of Variance and Covariance. From a correlation table relating age and maximum yield the statistical constants given in Table 21 were obtained. A comparison of these constants with those given in Table 5 shows that the process of standardisation for the month of calving has not materially altered their values. The mean, standard deviation and coefficient of variation have diminished in value but slightly.

Table 21 showing correlation of Maximum Yield with Age.

	Age	Maximum Yield
Mean	3.39±0.046	45.82±0.197
Standard deviation	2.26±0.033	9.66±0.142
Coefficient of variation	66.78±1.642	21.08±0.318
Correlation coefficient	+0.42363±0.01678	

The value of the correlation coefficient is moderate but highly significant, showing that the maximum yield of a cow is to a considerable extent determined by her age.

The analysis of variance and covariance of maximum yield with age is shown in Table 22.

Table 22 showing the analysis of variance and covariance of maximum yield with age.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between means of age classes	12	70234.9	5852.9083	91.1537
Within age classes	2378	152689.5	64.2092	
Total	2390	222924.4	93.2738	
Linear regression	1	40007.3	40007.3	623.0773
Deviations from linear regression	11	30227.6	274.7964	4.4173
Total	12	70234.9		

The result of the analysis of variance shows that the means of different age classes differ significantly among themselves, the value of F being 91.1537. The differences between means account for

$$\frac{93.2738 - 64.2092}{93.2738} \times 100 = 31.1605\%$$

of the total variance of maximum yield.

The results of the analysis of covariance prove that, although the linear regression line takes account of a major part of the covariance of maximum yield and age (F = 623.0773) the deviations from the linear regression line are quite significant (F = 4.4173), so that the relationship between age and maximum yield is far from linear. Figure 16, which shows this relationship, confirms this result. But for this non-linearity of the regression line, the value of the correlation coefficient between the two variables would have been higher than that observed, i.e. +0.42363±0.01678.

The mean, standard deviation and coefficient of variation of the maximum yield at each age are given in Table 23.

It will be noted that the value of the mean increases up to the seventh lactation, after which it diminishes somewhat. The rate of increase is highest between the first and second lactation and diminishes with advance of age until the maximum is reached. Thus, whereas an increase of about 9 lbs occurred between the first and second lactations, the total increase during the next five lactations was only about 6½ lbs. The regression line given by the polynomial equation

$$y = 51.08581+0.31911x-0.10446x^2-0.04775x^3-0.00556x^4 +0.00250x^5.$$

where y = Maximum yield for any age (in lactations) X

and $x = X-\bar{x} = X-6.5$

fits the data of observed means satisfactorily. The fit of this theoretical curve is shown in Figure 16 and the means estimated from it are given in Table 27. According to this equation the highest value of the maximum is reached at the age of 7.42 lactations.

Taking the age at first calving to be 2.5 years and the average length of lactation one year, this corresponds to an actual age of 9.92 years.

The value of the standard deviation is seen to change significantly with age, showing that the maximum yield and age are heteroscedastic. The value increases up to the fifth lactation. The general trend thereafter is definitely diminishing, although individual values are somewhat erratic. The logarithmic curve given by the polynomial equation

$$\log y = 0.94336-0.019712x-0.0036435x^2+0.0008323x^3$$

where y = standard deviation of the maximum yield of any age (in lactations) X

$$x = X-\bar{X} = X-6.5$$

fits the observed variation of the standard deviation as satisfactorily as may be expected in view of the irregular values of the later ages. The fit of this theoretical curve is shown in Figure 17. According to this curve the standard deviation is the highest at the age of 4.79 lactations. The values of the coefficient of variation do not seem to show any significant trend with age.

The foregoing results were obtained by the "lumped-lactation" method. Such results, it will be recalled, are liable to be complicated by the influence of selection. This point was therefore next examined.

Influence of Selection. The variations in the mean maximum yields for each age class with respect to the number of lactations for which the cows are retained in the herd are given in Table 24. Figure 18 illustrates this information graphically. It will be seen that the means given in the first two columns of the table show a significant decrease in value in passing from the younger to the older ages. This definite negative selection can hardly be voluntary. Does it, then, indicate that only the relatively poorer animals could survive to an old age? Or is the larger proportion of poorer animals in the older ages due to some other adventitious factor? If the results are in fact due to the first cause, they support the views of Loeb and Lewis (1902), Pearl (1922) and Brody (1939) who claim that longevity is determined by the rate of living. The present data are, however, too meagre to ensure that no adventitious factor is operating. It would not be safe, therefore, to stress the rôle of the first cause on these results alone.

No significant change is visible in the means given in the third column, though some positive selection is noticeable in the following four columns. On the whole, the selection seems to be less stringent in the case of maximum than in the case of persistency.

Similar information with regard to the variations of standard deviation is shown in Table 25, and Table

26 gives the results for the coefficient of variation. Figures 19 and 20 respectively illustrate these results graphically. No significant changes are noticeable in the values of either of these constants in the same age class as between younger and older cows.

"<u>Paired-Lactation" Method.</u> The technique of the "paired-lactation" method has already been described. The variations of maximum yield with age according to this method are shown in the second column of Table 27. The curve given by the logarithmic polynomial

$$\log y = 1.6904227-0.0039082x+0.0001660x^2-0.0000801x^3 -0.0001360x^4+0.0000251x^5$$

where y = Maximum yield for any age (in lactations) X

and $x = X-\bar{x} = X-6$

describes this variation remarkably well. This is clear from Figure 21 as well as from Table 27, where the values estimated from the theoretical curve are compared with the observed means. The differentiation of this equation gives 3.91 lactations as the age of highest maximum yield. It will be recalled that according to the "lumped-lactation" method the maximum yield was not found to be reached until the seventh lactation. The present results show that this latter result must have been largely due to the influence of selection. It is clear that animals actually attain their mature production very much earlier.

If the mean of the fourth lactation is taken as a standard and rated at 100, the relative valuesfor the other lactations, as found by the two methods, are compared in the last two columns of Table 27. It will be observed that the "paired-lactation" method not only shows an earlier age of maturity, but also a greater subsequent decline in production with the advance of age. A similar comparison is made graphically in Figure 15, which shows incidentally that the herd owner, by making use of selection, has been able to keep the maximum yield of his animals considerably above the normal value.

These results, which indicate a positive selection for maximum yield throughout the whole course of the curve, are opposed to the results of Sanders (1930), who observed a negative selection after the second lactation. Sanders considered this negative selection to be caused by an assumed higher incidence of disease in the better milkers which, he considered, would more than nullify the effect of any voluntary selection. Ward (1939) has, however, recently shown that there are no indications of a higher incidence of disease in the better cows, while the results of the present study demonstrate that such differences, even if they did exist, would not have been sufficient to nullify completely the influence of conscious selection on the part of the herd owner.

The variations of maximum yield with age have been studied by several workers in the past. Gavin's (1913b) work is probably the earliest in this field. He found that cows of the Shorthorn and Friesian breeds attained their maximum production in the sixth lactation. More recently Turner (1926b) has employed correction factors for total yield in order to standardise the maximum yield for age. He evidently considered that the error involved in this procedure would be slight. However, as will be shown later, this view is erroneous, there being considerable differences between the variations of total yield and of maximum yield with age. Gaines (1927a) found that the level of maximum yield varied with age according to the logarithmic equation $y = a+bx+cx^2+d\log x$, and that the highest yield was given at the age of 8.89 years by Guernsey cows and 8.53 years by Friesians. Sanders (1930) found that the maximum increased up to the age of 6.58 lactations (by the "lumped-lactation" method) or 6.88 lactations (by the "paired-lactation" method), and then diminished. The mature yield was 68% higher than the yield for the first lactation. Ostergaard (1931) observed that the maximum yield increased steadily until the sixth lactation, when the value was about 50% higher than the first lactation. A subsequent decrease took place with the onset of old age. Gooch (1935) found that in Jersey cows the highest yield was reached at about 6½ years. The means of the various age classes were subsequently somewhat irregular, though their trend was decreasing.

(iii) Total Yield.

Correlation and Analysis of Variance and Covariance. The constants given in Table 28 were derived from a correlation table relating age with total yield. It will be seen that the process of standardisation for month of calving has raised the mean by about 2% (compare Table 5). This is because the mean of the standard month is higher than the mean of all months. The values of the other constants have not altered significantly. The value of the correlation coefficient is moderate but highly significant. It would have been still higher if, as will be shown later, the regression were not curvi-linear. This shows that the total milk yield of a cow is influenced to a considerable extent by her age.

Table 28 showing the correlation of Total Yield with Age.

	Age	Total Yield.
Mean	3.39±0.046	8316.9 ±36.0
Standard deviation	2.26±0.033	1760.1 ±25.8
Coefficient of variation	66.78±1.642	21.16± 0.319
Correlation coefficient	+0.35335±0.01790	

The analysis of variance and covariance of total yield with age is shown in Table 29.

Table 29 showing the analysis of Variance and Covariance of Total Yield with Age.

Source of Variation	d/f	Sum of Squares	Mean Square	F.
Between means of age class	12	17890.5*	1490.8750*	35.2519
Within age classes	2378	100570.4	42.2920	
Total	2390	118460.9	49.5652	
Linear regression	1	14790.8	14790.8	349.7304
Deviations from linear regression	11	3099.7	281.7909	6.6630
Total	12	17890.5		

*in units of class interval used.

The result of the analysis of variance shows that the means of different age class differ very significantly among themselves, the observed value of variance ratio being 35.2579. The differences between the means of different age classes account for

$$\frac{49.5652 - 42.2920}{49.5652} \times 100 = 14.6740\%$$

of the total variance of milk yield.

The results of the analysis of covariance show that the linear regression line fitted to the mean yields of different age classes takes account of the greater part of the covariance of maximum yield and age (F = 349.7304). That this is not the best fitting line, however, is clear from the significant value of variance ratio (6.6630) for the deviations from linear regression. This is further clear from

Figure 22, which represents graphically the change of the total milk yield with age. It will be observed that the regression line describing this variation is distinctly curvilinear. The means increase in value at first with the advance of age and then diminish.

Table 30 gives the main statistical constants of total milk yield of each age class.

It will be observed that the means increase in value, though at a diminishing rate, until the sixth lactation. There is then a sudden jump, and the yield is the highest for the seventh lactation. After this, as a result of paucity of data and (as will be shown later) the influence of selection, the variation is somewhat erratic though the general tendency of the means is to decrease. The standard errors of the means of these older age classes are very high, so that their differences from the maximal yield are not significant.

The regression line given by the polynomial equation

$$y = 9252.41 \pm 5.3123x - 37.8552x^2 + 3.3067x^3$$

where y = Total yields for any age (in lactations) X

and $x = X - \bar{X} = X - 6.5$

fits the observed means satisfactorily. This is clear from Figure 22. According to this theoretical equation the age of maximum production is 7.24

lactations. The heteroscedasticity already pointed out in the case of age and persistency and age and maximum yield is also noticeable in the case of total yield. It will be seen from Table 30 that the values of the standard deviation increase up to the fifth lactation and decrease thereafter. The only exceptions to this are the values for the tenth and twelfth lactations, which show an increase over the value of the preceding age class. Neither of these increases is, however, statistically significant. The logarithmic curve given by the polynomial equation

$$\log y = 3.2321235-0.0104546x-0.0022140x^2+0.0003857x^3$$

where y = standard deviation of total yield for any age (in lactations) X

and $x = X - \bar{x} = X - 6.5$

describes satisfactorily the observed variation of standard deviation of milk yield with age. The fit of this curve is shown in Figure 23. According to this theoretical curve the highest value occurs at the age of 4.85 lactations.

The variations of the coefficient of variation with age are irregular. The differences observed between the values of successive lactations are erratic and none of them are statistically significant, though there is a tendency for a slight decrease in value after the maximal value has been reached.

Influence of Selection. The above results are based on the "lumped-lactation" method and are thus influenced by the effect of selection. The effect of this selection may be seen from Table 31, which gives the mean total yield for each age class of animals retained in the herds for varying numbers of lactations. Figure 24 illustrates these results graphically. It will be noted that, with the exception of eighth column, the mean is always higher for the animals retained for another lactation than for those constituting the original population. For instance, in the second column it will be observed that the mean total yield of cows kept for two lactations was 7947.5 lbs for the second lactation, though the mean of animals that were kept for another lactation was 8075 lbs. The difference between the two means when compared with its standard error is not actually significant, but since such differences occur consistently throughout the table one can place greater confidence in their values than is warranted on the basis of their standard errors. Moreover the differences of some of the means in columns four to six are in fact more than twice their standard errors, and are therefore statistically significant. It appears that the stringency of the selection has increased with age up to the sixth lactation, but there are no indications of any selection after the seventh lactation. The yield of cows kept for nine or ten lactations is, in general, lower than the average, though the reason is not apparent.

Lush (1939b) has reported a similar selection for the butter-fat yield in the Iowa and Kansas Cow-Testing Association's herds. In the words of Lush "Many cows die or are sold for reasons not under the owner's control, yet in every year studied, the cows which left the herd during the following year averaged lower in production than those which were kept for at least another year. The net effect of this culling on the whole dairy population would be to raise the average level of production somewhere between one half pound and a pound and a half of fat per year, if management were unchanged and if the bulls were out of average cows. While this rate of improving the dairy population by cow culling may appear low, it can produce considerable improvement if projected 10, 20 or more years".

Table 32 compares for the different ages the standard deviations of animals retained in the herd for a varying number of lactations. Figure 25 illustrates these results graphically. Like the means in Table 31, the values of standard deviations are lower for the animals kept a lactation longer than of animals which constituted the original population.

The only exception to this is the second column. This again clearly indicates selection, even though the observed differences are not statistically significant.

Similar results for the coefficient of variation are given in Table 33 and Figure 26. These results differ little from those of standard deviation.

"Paired-Lactation" Method. The "paired-lactation" method was again adopted as a check on the above results. The variations of the total milk yield with age according to this method are shown in Table 34.

The observed means are given in column two, and the smoothened values according to the best-fitting theoretical curve in column three, The equation of this theoretical curve is

$$y = 8485.52 - 146.6356x - 40.3246x^2 + 7.4574x^3$$

where y = Milk yield for any age (in lactations) X

and $x = X - \bar{x} = X - 6$

The fit of the curve is shown in Figure 27. It will be observed that the agreement between the observed and estimated values is very close indeed. According to this theoretical curve the age of maximum total yield is 4.67 lactations. This is considerably lower than that given by the "lumped-lactation" curve, i.e. 7.24 lactations. The means for the different ages as estimated from this latter curve are given in the fourth column of Table 34. It will be observed that after the second lactation they are definitely higher than those given by the "paired-lactation" curve, and that the differences continue to increase with advance of age.

If the mean for the fourth lactation is taken as a standard and rated at 100, the relative values of the mean yields for the different ages by the two methods will be as shown in the last two columns of Table 34. Figure 15 compares the actual means graphically. It again shows that by disposing of the relatively poorer producing cows the herd owner is able to maintain the milk yield of his animals at a much higher level than would be possible without such selection.

The influence of age on milk yield has been studied by many workers both in this country and abroad. [Spier (1909), Tocher (1919, 25, 28), Gowen (1920), Hammond and Sanders (1923), Ragsdale et al. (1924), Fohrman (1926), Sanders (1928), Davidson (1928), Roberts (1928), Kay and McCandlish (1929), Glen and McCandlish (1930), Sikka (1931), Tuff (1931), Ostergaard (1931), Ward and Campbell (1939), Lortscher (1937)]. Most workers have employed the "lumped-lactation" method. A complete review of the results of all the above investigations would be outside the scope of the present study. However, Tocher (1928), Kay and McCandlish (1929), and Glen and McCandlish (1930) used the Scottish Milk Recording Association's data for their investigations, and it will not be without interest to compare their results with those reported here.

Tocher (1928), working with the published records of the Association for the years 1911 and 1912, found that the variation of the total yield with age was described by a parabolic regression line. The maximum yield occurred at the age 13½ and 12½ years in the 1911 and 1912 data respectively. It will be noted that these results are entirely different from those reported above. The reason for this lies in the defect of Tocher's method of study, i.e. his failure to allow for the highly selected nature of the population of the older age classes.

It has already been stated that Kay and McCandlish (1929) and Glen and McCandlish (1930) studied the influence of age on total milk yield by using data from Ayrshire cows which had completed at least five lactations. They reported their results in two different ways; (i) by measuring age in years, and (ii) by measuring it in lactations. Their results by the second method are thus directly comparable to those of the present study, and are shown in the following table:-

Age in Lactations	Relative milk production of different ages (5th lactation =1.00)	
	Glen and McCandlish	Present data
1	1.16	1.179
2	1.13	1.079
3	1.06	1.027
4	1.02	1.003
5	1.00	1.000

It will be seen that the agreement between the two sets of results is reasonably close, though the writer's figures indicate a quicker rate of attainment of maturity. The difference is probably due to the different methods of study and, as already pointed out, to the selected nature of Glen and McCandlish's data.

(b) Standardisation of Milk Records for Age.

Having determined the variation of the different constants of the lactation curve with age, it should be possible to standardise the records of cows made at different ages to a standard age. To do this, nearly all workers in the past have used what Ward and Campbell (1938) call "percentage" correction factors. According to this method the correction factor for a given age is simply the ratio (on a percentage basis) of the value of standard age to that of the given age. For instance, if

$\bar{y}_s$ = mean milk yield of standard age s

$\bar{y}_x$ = mean milk yield of any other age x

b_x = correction factor for age x

then $b_x = \frac{\bar{y}_s}{\bar{y}_x} \times 100$ ....(1)

the values of $\bar{y}_s$ and $\bar{y}_x$ being given by the curve showing the variation of milk yield with age.

The standardised value (Y_s) corresponding to any observed value (y_x) is then given by the equation

$$Y_s = b_x . y_x \qquad \text{..........(2)}$$

It will be realised that this method postulates that the change of the yield with age is strictly in proportion to its value; in other words, that the milk yield of a relatively higher yielder shows a greater absolute change than that of a comparatively poorer yielder.

Tuff (1931), however, concluded that "the increase in milk yield from young to full-grown age of an individual cow can neither be summarised by a constant addition nor by a percentage addition alone" and suggested instead that the real relation was of the type

$$Y_s = a+b_x' y_x \qquad \ldots\ldots\ldots\ldots(3)$$

Ward and Campbell (1938) have recently reiterated this point and have emphasised the fact that insufficient attention has hitherto been directed to the operation, under normal herd conditions, of age correction factors determined according to equation (1) above. They argue that this system assumes (a) that the change of milk yield is strictly in proportion to its value, and (b) that the correlation between the production of different ages is perfect. In the light of their results, which show a distinct regression, they consider the latter assumption to be unwarranted. They therefore recommend that the relationship as given by equation (3) above should be used to standardise milk yields for age. As will be shown later, however, these authors have really misunderstood the importance of the major cause of the observed regression, i.e. the influence of variable environment at different ages. It will therefore be useful to examine the implications of their method. For this purpose it will first be necessary to determine the correlation between the milk production at different ages. Incidentally such a study will furnish results regarding the comparative reliability, as measures of her heredity, of the milk yield of an animal at different ages. Such results will be of great practical utility, since they will show which of all the different records of a cow has the highest repeatability and is therefore the best measure of her phenotypic constitution, - so far as this latter is measured by her life-time milk production. As far as can be ascertained no information regarding the repeatability value of persistency at different ages has hitherto been available, though Gowen (1920, 1923) Fohrman (1926a) Sikka (1933) Copeland (1938) and Berry and Lush (1939) have all published results relating to total yield and Gavin (1913b) relating to maximum yield.

The present data are particularly suitable for such a study as they consist of life-time milk records of cows kept in the same herds.

(i) Relative Repeatability of the Lactation Curve at Different Ages.

(a) Persistency.

Correlation Coefficients. Figures showing the correlation between the persistency of one lactation and that of another for the first ten lactations are given in the bottom left half of Table 35*.

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*As these values would repeat themselves in the right half of the table they have been omitted there.

To judge the significance of an observed value of a correlation coefficient the common method has been to compare the observed value with its estimated standard error. As Fisher (1938 p.198) has pointed out, however, such a comparison is valid only with large samples and moderate or small correlations, and is unjustified with small samples. To test the significance of an observed correlation determined from a small sample the t test should be applied.

$$t = \frac{r}{\sqrt{1 - r^2}} \times \sqrt{n' - 2}$$

$$n = n' - 2$$

Alternatively, the significance may be judged by transforming the value of r into z, and by comparing the latter with its standard error. The transformation may be made by the formula

$$z = \tfrac{1}{2}[\log_e(1+r) - \log_e(1-r)]$$

or with the help of the special table given by Fisher. The standard error of z is given by the formula

$$S.E._z = \frac{1}{\sqrt{n' - 3}}$$

where n' = number of pairs of observations upon which the value of r is based.

The advantage of the transformation of r into z lies in the fact that the random sampling distribution of z is more nearly normal than that of r, while its standard error can be readily determined since, unlike r, it does not involve any unknown parameters of the population of which the data is supposed to be a sample.

The values of r in table 35 are consequently given without their standard errors. In the top right half of the table the values of z corresponding to those of r are given for the different ages. The transformations of r into z were actually made with the help of Fisher's table*.

The values of z which are less than twice their standard errors (and are therefore not significant) and the corresponding values of r are underlined.

It will be observed that out of the 45 values, 23 are not significant. Of the correlations between the first and other lactations, only those with the second, third and fourth lactations are significant. Of those between the second and other lactations only the first four are significant. In general, the correlation between the persistency of ages that do not differ by more than three years are significant, whereas the values for ages which differ by more than this period are uncorrelated. The values of r are highest for successive lactations and diminish as the lapse of time between the records correlated increases:

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* As pointed out by Fisher (1938, p.211) in calculating the value of z, the value of r found without using Shappard's adjustment should be used. In this study, however, where the correlation was determined by arranging the data into a correlation table, Shappard's correction had been used, and the value of z was determined from such values of r. This procedure is not strictly correct, but the error introduced in the value of z is only in the third or fourth decimal place.

such diminutions in value are in most cases significant. For instance, the value of r for the first and second lactations is significantly different from that of the first and third or first and fourth lactations, e.g. the difference between the values of first and second and first and third lactations is 0.164±0.06912, and that between the first and second and first and fourth is 0.2569±0.07715, both of which are more than twice their standard errors. The correlations between <u>successive</u> lactations, on the other hand, does not show any definite change with age up to six lactations. A slight increase in value is noticeable from the first to the second lactation, and again from the fourth to the fifth lactation. However, neither of these increases is significant. This shows that the repeatability value of the observed persistency for the first six lactations is very nearly the same.

The results given in the preceding paragraph are highly interesting in that they show how greatly the persistency is influenced by environment, and how large have been the changes in environment in these herds. If persistency were determined completely by heredity (or environment which had remained constant during the whole life of the animal) the correlation between the values of different lactations of the same animal should have been perfect. The fact that the observed values are considerably lower than unity shows that they are greatly influenced by environment.

Again, if the environment of the animal had remained unaltered during its lifetime, the correlations between the persistency of different ages should have been the same. That they are not so proves that the environment has altered considerably during the lifetime of each animal. It is not proposed at this stage to discuss the relative influence of heredity and environment on persistency. This point will be discussed in detail in a later section. It is clear, however, that because of the diminishing value of the correlation with increase in the lapse of time between two lactations, the value of an earlier lactation in predicting the persistency of a future lactation also diminishes. The best measure of a cow's expected performance in any future lactation is, therefore, her performance in the lactation immediately preceding it.

Regression Equations. The common method used in the past to test whether the regression line is or is not linear has been to determine the correlation ratio (η) and compare the value η^2-r^2 with its standard error. Fisher (1938) has strongly criticised this procedure. He recommends the use of the method of analysis of variance for this purpose.

The results of the test of linearity of regression lines of ages with at least 100 pairs of observations, i.e. up to sixth lactation, by this better method are shown in Table 36. It will be noted that of the 16 regression lines, only two, i.e. those for the third and fourth lactations on the second lactation, have a significant value of variance ratio for deviations from linear regression. For the remaining 14 regression lines the values of F are not significant, showing that these are linear.

Assuming, on the basis of these results, that the relation between the persistency at the different ages is linear, the regression equations (which are necessary for estimating the expected persistency of a cow for a future age, - up to 10 lactations, - from the observed value of an earlier age) are given in Table 37.

The use of these equations may be illustrated by the following example. The values of persistency of two cows are observed to be 90% and 85% for the first lactation. What may be expected to be their probable persistency in the second and fourth lactations?

The regression equation describing the relationship between the persistency of first and second lactation, as given in Table 37, is

$$Y_2 = 28.06+0.65126y_1$$

Now y_1 for the first cow is 90% and for the second cow 85%. Therefore their expected values for the second lactation will be

$$\text{1st cow} = 28.06+0.65126 \text{x} 90 = 86.67\%$$

$$\text{2nd cow} = 28.06+0.65126 \text{x} 85 = 83.42\%$$

Similarly the regression equation giving the relationship between the first and fourth lactation is

$$Y_4 = 52.72+0.39317y_1$$

Therefore the expected values for the fourth lactation of these cows will be

$$\text{1st cow} = 52.72+0.39317\text{x}90 = 88.11\%$$

$$\text{2nd cow} = 52.72+0.39317\text{x}85 = 86.14\%$$

These estimates of the expected value have a standard error which is given by the formula

$$\text{S.E. of estimate} = \frac{\sqrt{S(y-Y)^2}}{\sqrt{n'-2}}$$

According to this formula the standard error of the estimate for the second lactation is ±2.95 and for the fourth lactation ± 4.29.

The value of the regression coefficient in each of these equations gives a measure of the reliance that may be placed in the observed persistency (Yp) of any age in predicting the value of a future lactation. The higher this value is, the greater the reliance, and vice versa. Where the value is low the expected value in a future lactation will regress to the mean of the population, whatever the observed persistency.

It will be observed that the regression coefficients are very much higher for successive lactations than for non-successive lactations. The values for ages differing by more than three lactations, are, in fact, in nearly all cases too low to be statistically significant.

For instance, the regression coefficients of the first five equations of the 1st lactation have the following standard errors:

Regression Equation	Regression coefficient	Standard error	t
1st	0.65126	0.05493	11.86
2nd	0.54324	0.08311	6.54
3rd	0.39317	0.09713	4.05
4th	0.13031	0.14185	-
5th	0.12392	0.14185	-

The t values show that the first three regression coefficients are significant. However, the observed values of fourth and fifth coefficient are even lower than their standard errors so that they are definitely not significant.

The above discussion shows that (at any rate with the data used in this study) the persistency at ages not differing by more than three lactations can be predicted with some accuracy from the observed value of an earlier lactation. When the gap between lactations is greater than this, the expected value roughly equals the mean of the population.

The regression coefficients of successive lactations are all about equal in value. The decrease from the first to the second lactation (0.09266±0.07514) is not significant. This supports the previous conclusion that the observed persistency for all ages up to six lactations is equally accurate in predicting the probable performance during the immediately succeeding lactation.

Finally, it is significant to point out that the observed values of regression coefficients are too low to make possible a very accurate prediction of the future performance of a cow for any age. This is clear from the following results, which compare the observed standard deviation of the age the persistency of which is predicted, with the standard error of the estimated value.

	Standard deviation of y_p	Standard error of estimate.	$\frac{\text{S.E.x100}}{Y_p}$
Y_2 from y_1	4.46	3.95	88.56
Y_3 from y_2	4.66	4.05	86.91
Y_4 from y_3	4.51	3.91	86.70
Y_5 from y_4	4.48	3.92	87.50
Y_6 from y_5	4.08	3.40	83.33
Y_3 from y_1	4.68	4.42	94.44
Y_4 from y_1	4.42	4.29	97.06
Y_5 from y_1	4.42	4.42	100.00
Y_6 from y_1	4.18	4.18	100.00

It will be noted that, when the persistency of a future lactation is predicted from the observed value of the lactation immediately preceding it is the standard error of the estimate is roughly 87% of the observed standard deviation, the gain in accuracy in prediction being therefore approximately 13%. It will be agreed that this is a poor result. However, as will be clear from the results of the non-successive lactations given above, there is no gain whatever in accuracy when the ages of the two lactations differ by more than three years.

(b) Maximum Yield.

Correlation Coefficients. Figures showing the correlation between the maximum yield of one lactation and that of another for the first ten lactations are given in the bottom left half of Table 38. As with persistency, the values of the correlation coefficients are given without their standard errors. Their significance may be judged, however, from the transformed values of z which are given in the top right half of the table. All those values of z which are not significant (i.e. are less than twice their standard error) and the corresponding values of r have,been underlined.

It will be noted that of the 45 values 16 are not significant. All the correlations between the first and seventh lactations are significant. The correlations of first, second, third and fourth lactations with eighth, ninth and tenth lactations are not significant, as also are those of fifth,and sixth lactations with the tenth/ and the seventh and eighth lactations with the ninth lactation.

The values of r for maximum yield are invariably higher than the corresponding vales for persistency. Further, most of these differences are significant. This shows that in predicting the expected performance of one lactation from the observed performance of another, the prediction is more accurate with maximum yield than with persistency. In other words, maximum yield is much less influenced by environment than persistency.

As observed with persistency, the correlations for maximum yield are higher between consecutive than non-consecutive lactations, though the differences in this respect are much less marked than those reported for persistency, - at any rate over the first seven lactations. The values of r for consecutive lactations are approximately the same up to the fifth lactation, but they then diminish. For instance, the difference between the values of z for the fifth and sixth, and sixth and seventh lactations is 0.2306±0.1363 which, though not statistically significant, appears definite. This shows that up to the fifth lactation the maximum yield for each age is of equal value in predicting the probable maximum of the succeeding lactation. After this there is a marked decline in accuracy of prediction.

Gavin (1913b) has reported the following results regarding the repeatability of his "revised maximum":-

Lactation	r with highest "revised maximum".
1st	+0.394±0.031
2nd	+0.452±0.030
3rd	+0.506±0.028
4th	+0.605±0.024
5th	+0.762±0.016

These results show a distinct increase in the value of r with the increase in age. As these correlations are between the "revised maximum" of each lactation and the highest "revised maximum", they are not directly comparable with the present results. However, a part of this increase in the value of r with age is probably due to the fact that there is smaller difference between the older ages than between the younger ages and the age of highest "revised maximum".

Regression Equations. Table 39 shows the nature of the regression lines for maximum yield for the first six lactations. It will be observed that of these 16 regression lines 12 are linear and only four are significantly different from linear, the probability significance level for three of the latter being 5% and for only one 1%. This shows that no great error would be involved if the linear regression line were to be used to describe the variation of maximum yield of one age with that of another age.

The equations necessary to estimate the future performance of a cow for any age, up to ten lactations, from the observed yield of an earlier lactation are given in Table 40. It will be noted that the values of the regression coefficient in these equations are considerably higher than those reported for persistency, a fact which again shows that the maximum yield of one lactation is a more accurate measure of the performance of another lactation than persistency. However, some of these values (those for the lactations for which r is not significant) are not statistically significant, e.g. those relating the eighth, ninth and tenth lactations to the first lactation. This is clear from the following figures:

	Regression Coefficient	Standard error	t
8th on 1st	+0.52815	±0.26813	1.97
9th on 1st	+0.00625	±0.25077	..
10th on 1st	+0.37905	±0.23880	1.58

The value of the regression coefficients is the highest for consecutive lactations and some decrease is noticeable with the increase in the lapse of time. The probable maximum yield of a cow in any lactation can therefore be more accurately predicted from the observed performance of the preceding lactation than from a lactation further removed. This is further clear from the results given in Table 41.

It will be seen that for successive lactations the standard error of the estimate is roughly 75% of the observed standard deviation. This represents a gain in accuracy of about 25%, which is double that observed for persistency. However, the gain is definitely of a still lower order for non-successive lactations, the greater the difference between the ages the higher being the relative standard error of estimate (i.e. expressed as percentage of the standard deviation).

(c) Total Yield.

Correlation of Coefficients. Figures showing the correlation between the total yield of one lactation and that of another for the first ten lactations are given in the bottom left half of Table 42. The corresponding values of z are given in the top right half of the same table. As before, those values of z and r which are not significant are underlined.

The very striking similarity of these results to these reported for maximum yield will be noted. Out of the 45 values of r, 17 are not significant. All the correlations between the first and seventh lactations are significant, although the correlations between these and the ninth and tenth lactations are not significant. The correlations between the first, third, and sixth lactations and the eighth lactation are similarly not significant, while those of second, fourth and seventh lactations are significant. The correlations between eighth, ninth and tenth lactations are significant.

As with maximum yield, the value of r for total yield is invariably higher than the corresponding value observed for persistency. The differences between values for total yield and for maximum are irregular. The former are usually higher, but in a few instances they are lower. However, the differences are in no case significant.

The correlations are higher for consecutive than non-consecutive lactations. The values of r are practically the same for the first five consecutive lactations, after which they decrease.

These results are in agreement with those of previous workers. Fohrman (1926a), for instance, has reported "a slight tendency for the correlations to decline as the lapse of time between initial and re-test records increases". More recently Berry and Lush (1939) have presented indirect evidence indicating that the correlations between non-consecutive records average somewhat lower than those between consecutive records. Gowan (1920) found that the values of r were in general higher for the younger than for the older ages. Copeland's results (1938) point to a similar decrease in the value of correlation with age.

Regression Equations. Table 43 shows the nature of the regression lines for the total milk yield for the first six lactations. It will be observed that of the 16 regression lines only five are non-linear, the probability significance level of three of these being 5% and of the two others 1%. In the remaining eleven the linear regression line gives a good fit.

Assuming that in every case the regression lines are in fact linear, the equations needed for estimating the expected performance of a cow in a future lactation (up to the age of ten lactations) from the observed milk yield of an earlier lactation are given in Table 44.

It will be observed that the values of the regression coefficients for total yield are very much higher than those reported for persistency, though about equal to those for maximum yield. The general trend of these values for total yield actually differs but little from that of maximum yield, so that in order to avoid repetition the subject will not be discussed in detail. Results similar to those given in Table 41 for maximum yield are given in Table 45afor total yield.

These results confirm the previous conclusion that, in predicting the performance of the following lactation, the value of the first four lactations is about the same, though with the fifth and subsequent lactations there is a distinct diminution in value. The data tend to show, moreover, that, for the first four lactations, a slightly more accurate estimate of the total yield of a succeeding lactation is possible from the observed yield of the preceding lactation than is true for maximum yield.

(ii) General Discussion.

From the above results the following conclusions may be drawn:- (i) The correlation between the production (whether measured in terms of total or maximum yield or persistency) of one lactation with that of another is far from perfect. (ii) The observed correlation is lowest for persistency and about equal for maximum and total yield. (iii) The correlation between non-consecutive lactations is definitely poorer than between consecutive lactations. In general, the diminution in the value of r is in proportion to the lapse of time between lactations. This phenomenon is most marked in the case of persistency. (iv) Because of the imperfect correlation observed, the prediction of a future lactation when estimated from the observed performance of an earlier lactation is subject to considerable error. This error is the least in the case of maximum and total yield and greatest in the case of persistency. Again it is minimum with consecutive and maximum with non-consecutive lactations. (v) The first four or five lactations are of about equal value in predicting the future performance of a cow.

How greatly the record of production of a cow may be affected by the various environmental influences is too well known to need any emphasis. The state of health of the animal, the nature of the food supply and the conditions of management all affect her performance. The observed differences in the milk production of the same cow at different ages are not only the result of the physiological changes in her body due to advancing age (i.e. growth and senesence), but are also caused to a marked extent by environmental factors which vary from year to year. The conditions of environment are as likely to be favourable as unfavourable in any given year. When conditions are favourable, production will be normal or above normal.

On the other hand, comparatively unfavourable conditions in a following year will depress production. Such alternating conditions of environment will naturally affect to some extent the regression of the milk yield of one lactation on that of another. Yet if the influence of age on milk yield is studied by the correlation method (as Ward and Campbell (1938) have recommended), the whole of the observed regression will be ascribed to the influence of age.

On the other hand certain environmental changes, instead of being of the fluctuating type, may be permanent in nature. For instance, the management policy of the herd may have to be changed completely for economic reasons, or disease may permanently impair the functional ability of a cow or a group of cows. The effect of such changes will be to bring about a permanent and more pronounced change in milk production. Environmental changes of this type are likely to influence milk yields over a relatively long period of time, so that the consecutive records of a cow will be less influenced than non-consecutive records. This is why the correlation between consecutive records is highest and why there is a greater regression with non-consecutive than consecutive records. This latter point is particularly important, as it indicates one great weakness of Ward and Campbell's method of standardisation for age, - a weakness which they seem to have overlooked.

It has already been pointed out that when the correlation between two lactations is low, i.e. the regression is high, the predicted milk yield approximates more nearly to the mean of the population than to the observed yield. This implies that, whether a cow has given a low or a high yield, her standardised milk record approximates to the mean of the whole population. Where the observed low yield is due to some accidental factor, such as unfavourable environment, the standardisation may raise the predicted yield to a more correct level. On the other hand, where the poor production is due to the genetic make up of the animal, the result is highly misleading. The method is thus apt to confuse real differences between individuals and is therefore unsuited for genetical investigations. Ward and Campbell have attributed the observed regression to supposed differences between high and low yielders, and have argued that high yielders may not show as much increase in yield as they mature as low yielders. As they themselves have pointed out, such differences, if they exist, have not been studied. Sanders (1928), however, has shown the difficulties and fallacies involved in the evaluation of such differences. In the writer's view, any differences which do exist in this respect are likely to be but slight, and will in no case account for the whole or even for a major part of the regression reported. Further, it will be shown later how some allowance may be made for this supposed regression.

Obviously Ward and Campbell have confused the standardisation of records for the influence of age with the prediction of the probable performance of a cow. Gowen (1924) has clearly pointed out the difference between the two procedures. Where the probable performance of a future lactation is to be estimated the regression method is more appropriately used, as this corrects not only for the differences in age but also for the influence of varying environments. On the other hand, where the object is simply to standardise for age, this should be done with the help of the curve showing the variation of milk yield with age. In this connexion it is important to re-emphasise the fact that the correction factors are based on average results, and that great accuracy is therefore not possible in the standardisation of individual records. Neither is such accuracy attainable by the use of regression equations, since these also are based on average results. Moreover, since the environmental differences may vary from herd to herd, the regression equations of one set of data may not be applicable to another. This definitely limits the utility of the method for use by practical breeders. There is no evidence to show, on the other hand, that the results from the mean curve are not equally applicable to all the different individuals within a breed.

(iii) Correlations with Combined Records.

Correlations, Simple and Multiple. We have so far discussed the correlation of one lactation record with another. It has been observed that the highest correlation is obtained between the production of consecutive lactations. Even then, however, the value of r does not exceed +0.74 for total yield, +0.69 for maximum yield and +0.54 for persistency. The values for non-consecutive lactations are considerably lower. The use of these correlations in predicting the probable performance of a future lactation has been indicated. It has been shown that the predicted value has a high standard error. This error is highest with persistency and very much lower with both maximum or total yield.

Each milk record of a cow may be looked upon as a measure of her phenotypic constitution as modified by environment. It is from such modified phenotypic values that we have to estimate the genotypic differences between cows. It is unfortunate that the repeatability of these phenotypic values is far from perfect. This means that, because of the confusing effects of environment, we cannot make an exact estimate even of the phenotype, let alone the genotype.

It was thought that it might be useful to determine whether the repeatability value of different records could be enhanced by a combination of two or more records. Obviously such an increase in correlation, if attainable in practice, would make possible a more accurate prediction of the future performance of the cow.

It is scarcely necessary to point out that such an increase in the value of the correlation is possible on theoretical grounds. It is easy to show, for instance, that if r is the value of the correlation coefficient between the milk yields of any two of the different lactations, 1, 2, 3.... n, and σ the value of standard deviation of each lactation then the value of the multiple correlation coefficient R between the yield y_1 and y_2, y_3, y_4.....y_n is given by the formula

$$R = r\sqrt{\frac{n-1}{1+r(n-2)}}$$

Thus if

$$r_{y12} = r_{y13} = r_{y14} = r_{23} = r_{24} = r_{34} = +0.5$$

and $\sigma y_1 = \sigma y_2 = \sigma y_3 = \sigma y_4$

Then $R_{1(23)}$ will be +0.577

and $R_{1(234)}$ will be +0.612

It will be instructive to see empirically how far this theoretical increase in the value of the correlation is actually realised. It is possible to do this by the method of multiple regression from the results already reported in the last section. However, to obviate the change of the results being affected by the weeding out of cows at different ages, it was considered more appropriate to study the whole subject afresh from results which were free from this defect.

It was found that there were 221 cows whose records for the first four lactations were available. The data furnished by these cows was therefore used. The study was confined to total yield and persistency only. It will be obvious that the results for total yield will apply equally to maximum yield, since the correlations are very similar for both.

The important statistical constants of these data are given in Table 45.

A comparison of these results with those already reported for the whole data shows that the total yield of the fourth lactation is the same in both. The yield of the first lactation is slightly lower and that of the second and third lactations slightly higher with the selected data. Similarly the persistency is higher for the first three lactations and practically the same for the fourth lactation. The values of the standard deviation and coefficient of variation do not differ materially. However, based as the present results are on comparatively limited data, too much reliance cannot be placed on these small observed differences.

The values for the correlations between the production of different lactations is shown in Table 46.

It will be observed that the correlation is higher for successive than non-successive lactations both in the case of persistency and total yield. The differences between the three values of successive lactations are not significant, but those between successive and non-successive lactations are very definite and generally significant. These results thus support the conclusions of the previous section.

Now let us compare the multiple correlation coefficients with the ordinary correlation coefficients. It will be observed that the value of R is invariably higher than the corresponding values of r, e.g. $r_{13} = +0.76556$ though $R_{1(23)} = +0.79218$. This would be anticipated. However, it is important to note that this increase is in every case very slight as compared with the value of r between the successive lactations. For instance, take the case of $R_{1(234)}$, the value of which for total yield is +0.74296. The value of r_{34} is +0.72267, so that the increase in correlation is hardly appreciable. This means that the yield of the fourth lactation can be predicted nearly as accurately from the third lactation alone as from all the first three lactations together, or, in other words, where the yield of the preceding lactation is known, there is little gain in considering the production of any of the other past lactations to estimate the probable yield in a future lactation. This conclusion is further confirmed by the values of regression coefficients given in the next paragraph.

Regression Equations. Table 47 shows the nature of regression line for each of the six comparisons. It will be observed that the only regression lines which are not linear are those for the third lactation on the second in the case of total yield and for the fourth on the second lactation in the case of persistency. The probability significance level with both is 5%. All the remaining regression lines are linear.

The equations of the various regression lines and the standard errors of estimate for each are given in Table 48. These results are completely in accord with those reported in the last section, and call for no remark. They are given here chiefly for later comparison with the results of partial regression equations.

Partial Regression Coefficients. The values of the various betas (partial regression coefficients) necessary to determine the probable performance of a cow from the earlier observed yields are given in Table 49.

It will be observed that in every case the value of beta is highest for the lactations nearest to the one whose yield is to be estimated, and vice versa. In fact all the betas for the first lactation in the case of persistency and all except one in the case of of total yield are not significant. The one that is significant in the case of total yield is $\beta_{Y3y1} \cdot y2$ i.e. where the yield of the third lactation is to be determined from the observed yield of the first and second lactations.

The regression equations needed to determine the probable yields of subsequent lactations from the observed yields of earlier lactations, and the standard errors for each, are given in Table 50. A comparison of the standard errors of estimate for the different regression lines given in this table with those given in Table 48 shows that the gain in accuracy in predicting the probable performance of a future lactation from the observed value of <u>all</u> the previous lactations is but slight as compared with prediction from the immediately preceding lactation. For instance, the standard error of the estimated yield of the third lactation is 1070.8 lbs when estimated from the second lactation alone and 1018.1 lbs when this is done from both first and second lactations. Similarly the standard error of the fourth lactation yield is 1258.8 lbs when estimated from the third lactation as against 1228.5 and 1208.7 lbs when estimated from the second and third, and the first, second and third lactations respectively. The results for persistency are very similar. If anything, the advantage is even less marked.

Discussion. These results are disappointing in that they show that in practice the correlation between the performance of any one lactation and the average performance of a number of other lactations is little higher than between two successive lactations. With the data employed the value of this correlation was found to be approximately +0.7 for total and maximum yield and +0.5 for persistency. This means that no one lactation record is a perfect measure of the phenotype of the cow.

Copeland (1938) obtained a correlation of +0.92± 0.008 between the highest corrected milk yield of any lactation and the average corrected production of five records. He has consequently emphasised the great value of the highest yield as the measure of a cow's real producing ability. As Berry and Lush (1939) have shown, however, this high correlation is spurious, having arisen as a result of ex post facto selection, so that the highest yield in any lactation is a no more accurate measure of a cow's phenotype than any other single unselected yield.

If, however, the average lifetime production of a cow is taken to represent her true phenotype, it is clear that the record of production of each lactation will help in making a truer estimate of this value. The observed correlation between two variates is, after all, due to the elements common in them being simply the ratio of the common to non-common elements*.

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*For a full discussion see Snedecor (1938) pages 128-131.

The larger the number of lactations on which the average is based, the greater will be the correlation of this average with the average lifetime production of the cow.

(iv) Correction Factors for Age.

The smoothed values of the means of each age for the persistency, maximum and total yield have already been given in Tables 20, 27 and 34 respectively. It is possible to correct the production of any age x to standard age S from these values by using the formula

$$Y_S = \bar{y}_S + (y_x - \bar{y}_x) \quad \ldots\ldots(4)$$

where $\bar{y}_s$ = mean of the standard age s

$\bar{y}_x$ = mean of the observed age x

and y_x = observed yield at age x

However, this does not take into account the heteroscedasticity already reported (Holzinger, 1924, Arthur, 1924). It is important to allow for this. This can be done by the formula

$$Y_s = \bar{y}_s + (y_x - \bar{y}_x)\,\sigma s/\sigma x \quad \ldots.(5)$$

where σ_s = mean standard deviation of the standard age s

and σ_x = mean standard deviation of the age x.

The regression curves showing the variation of the standard deviation of each constant with age have already been described and smoothed. These latter curves give the following values for the different ages, which may therefore be used for standardising for heteroscedasticity:-

Table 51 showing the variation of standard deviation with age.

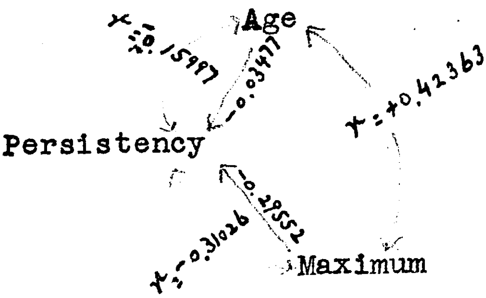
Age (in lactations)	Persistency %	Maximum lbs	Total Yield lbs
1	3.105	6.354	1440.3
2	4.333	7.628	1581.8
3	4.720	8.551	1679.0
4	4.612	9.054	1731.4
5	4.287	9.160	1742.9
6	3.949	8.958	1724.9
7	3.728	8.565	1684.2
8	3.684	8.098	1632.1
9	3.806	7.661	1578.2
10	4.007	7.335	1530.9
11	4.132	7.190	1497.2
12	3.950	7.298	1485.2

It will be observed that this method of correction postulates a constant increase or decrease in value for any age with cows of different grades of production. It is consequently different from the "ratio" method which was described earlier and was criticised by Ward and Campbell. Although no experimental evidence has been advanced to show that the use of this method is more warranted than the ratio method, it has been used because the writer considers it to be fairer to animals of different grades of production. Since, according to this method, all animals are assumed to show the same absolute increase or decrease of production with change of age, the relative rate of change is very much higher with low than with high yielders.

Gaines (1927a) standardised persistency in relation to maximum yield rather than to age. He adopted this procedure as he found that, although persistency and age and persistency and maximum yield were both negatively correlated, the correlation between age and persistency vanished when maximum yield was held constant. He thus considered that the variation of persistency with age was due entirely to the variation of maximum yield with age. There are, however, two objections to Gaines' procedure. In the first place his regressions are distinctly non-linear, and the partial correlation method is therefore not applicable. The present data confirmed his result <u>provided that this non-linearity was ignored.</u> For example, if persistency = p, maximum = m, and age = a, then

$$r_{ap} = -0.15997 \pm 0.01993$$
$$r_{am} = +0.42363 \pm 0.01678$$
$$r_{pm} = -0.31026 \pm 0.01849$$
$$r_{ap.m} = -0.03313 \pm 0.02044$$
$$r_{mp.a} = -0.27119 \pm 0.01895$$

It is clear that $r_{ap.m}$ (the correlation between age and persistency with maximum yield constant) is not significant. Using Wright's method of path coefficients (1921a) to measure the comparative influence of the different causal factors, the causation of the variation of persistency by age and maximum is represented diagramatically as follows:-



In this figure the values of the observed correlations are represented along the curved lines. The straight lines indicate the *direct paths* to persistency from age and maximum yield, the values of these paths being given by the standard regression coefficients (betas). There is, however, another *indirect path* from age to persistency via maximum yield, the value of which is given by $r_{am}\beta_{pm.a}$ and which equals 0.12520. It will be noted that the value of this indirect path is very much higher than that of the direct path showing that the variation of maximum yield as a result of the change of age is *the* important cause of change of persistency with age.

However, as has already been pointed out, the regression lines of both maximum yield and persistency on age are definitely curvi-linear, so that neither the partial correlation method nor the method of path coefficients is tenable.

In the second place, as will be shown in the next section, although there exists a negative correlation between persistency and maximum yield within age-corrected records of the same cow, *no such correlation exists between different cows*. There is, therefore, no valid reason for correcting the persistency of different cows for the maximum yield. Moreover both persistency and maximum are separate physiological entities and should therefore be studied separately. It is only by such means that we can ascertain how far it is possible by breeding to combine in the same cow high maximum and high persistency.

5. The Frequency Distribution and Constants of the Various Characteristics of the Lactation Curve after Standardisation.

The influence of age having been studied, all the records were standardised for age with the correction factors previously reported. The fourth lactation was chosen as the standard age for this purpose. The standardised constants for each cow were then entered in her card. Figure 28 illustrates the frequency distribution of the 2,392 records for persistency, maximum yield and total yield, and Table 52 gives their important statistical constants.

It will be observed that all three curves are slightly asymmetrical. The mode is lower than the mean in the case of total yield and maximum yield, and higher than the mean in the case of persistency. All three curves are leptokurtic, the degree of kurtosis being highest with persistency. The K values indicate that the curve for maximum yield conforms to type one and the curves for total yield and persistency to type four of Pearsonian curves.

The process of standardisation has significantly increased the means for total yield and maximum yield and significantly diminished the mean for persistency (See Table 5). The value of standard deviation of of maximum yield and of coefficient of variation of total yield and maximum yield have significantly diminished. The diminution has, however, been higher in the case of maximum yield than total yield, so that the value of the coefficient of variation of the former is significantly lower than that of the latter.

6. Correlation between Persistency, Maximum and Total Yield.

Having corrected the data for the two environmental factors, month of calving and age, let us study the relationship between persistency, maximum and total yield. The correlation between the first two will show how far they are compatible, and the correlation between each of them and the total yield, their relative importance in determining the latter.

In examining these correlations, it is essential to realise that the available data consist of records of cows belonging to different herds, and that the correlation determined from the total data is a compound value dependent on (a) correlation "between herds" and (b) correlation "within herds". The latter is again a compound value being dependent on (a) correlation "between cows" and (b) correlation "within cows". Since it is possible that the values of these component correlations may differ between themselves, it is important that the method of study should be such as to permit their separate determination, in addition to their total. Fortunately such a method is available in the analysis of variance and covariance.

It should be noted that past studies have not taken this point into account. For instance, Gaines (1927a) and Gooch (1935) have both reported a negative correlation between maximum and persistency. Neither of these authors has shown, however, whether this negative correlation in the total data actually exists between the means of records of different cows, as distinct from a negative correlation between the means of various records of the same cow. It is clearly important to secure such information for, if the means of persistency and maximum of different cows are found to be truly negatively correlated, it will show that it would be difficult to combine a high maximum yield with high persistency in the same individual.

(a) Relation between Persistency and Maximum Yield.

The detailed results of the analysis of variance and covariance for persistency and maximum yield (using the data from the "milk" herds, i.e. 2,392 records) are given in Table 53. For the present purpose the chief interest lies in the covariance of these variates. The discussion of the results of the analysis of variance will be deferred to the next section.

The value of r for the total data is -0.08045. The corresponding value of z is -0.08650±0.02046 which, though very low, is significant. This indicates that there is a slight negative correlation between these two variates when the data are taken as a whole.

The "between herd" value of r is -0.63468, with a corresponding value for z of -0.74930±0.70711. This correlation is not significant, which means that between different herds the persistency is independent of the maximum yield.

Let us next examine the intra-herd correlations. From the intra-herd values of r given in Table 53 the values of z given in Table 54 were obtained.

Table 54 showing values of z for maximum and persistency (intra-herd).

Herd	"Within Herd"	"Between Cows"	"Within Cows".
A	-0.14122 ±0.04740	*-0.09091 ±0.09853	-0.23146 ±0.05431
B	-0.09573 ±0.04725	*-0.05991 ±0.09449	-0.15028 ±0.05480
C	*-0.04896 ±0.03753	*+0.08249 ±0.07372	-0.26124 ±0.04373
D	*-0.06593 ±0.06415	*-0.03198 ±0.12403	*-0.13156 ±0.07559
E	*+0.00456 ±0.04340	*+0.06370 ±0.09167	*+0.05604 ±0.04945
Total	-0.06596 ±0.02046	*+0.00093 ±0.04099	-0.16666 ±0.02420

*not significant

The "intra-herd" value of z for the total data is -0.06596±0.02046 which, though very low, is still significant. However, only two of the five individual herds have such a significant negative value. In the remaining three the observed values are not significant. In no herd are the "between cows" correlations significant, though the "within cows" correlations are significant in three of the five herds. Similarly the "within cows" correlation for all herds taken together is highly significant.

The results thus show that the negative correlation between maximum yield and persistency which is observed in the total data is really due to the "within cows" effect. A cow can have a high maximum yield and a high or low persistency. However, her capacity in these respects seems to be fixed for life, so that when the maximum is higher in any lactation then what may be called her <u>optimum capacity</u>, the persistency is diminished, and vice versa. There are of course distinct differences in this respect between the different herds, some showing a more pronounced effect than others.

Sanders (1930), using an altogether different method, concluded that the variations of the standardised milk yield of the same cow from one lactation to another were negatively correlated with changes in standardised persistency. He has not, however, presented any results to show whether a similar relation existed between different cows.

Kartha (1934), by standardising persistency for the initial rate of yield instead of for age, found that the rate of decline of milk production increased with increase of maximum yield. This result is not unexpected since the persistency is highest in the first lactation, when the maximum yield is lowest, and diminishes thereafter with increasing maximum yield.

Comparing pedigree Sahiwal cows with Indian Crossbreds, Kartha found that, whereas the former had a lower maximum than the latter, they had a slightly higher persistency. In view of the positive correlation observed between maximum yield and rate of decline, he forecast that "the pedigree Sahiwal would lose her reputation for higher persistency when her level of production equals that of Crossbreds". Since the present study shows that between cows persistency is independent of maximum yield (if allowance has been made for the variations of both with age), there seems to be no cause for this pessimism.

(b) Relation between Persistency and Total Yield.

The detailed results for the analysis of variance and covariance of persistency and total yield are given in Table 55. The value of r for the total data is +0.40021 and that of z +0.42390±0.02046. This moderate but highly significant positive correlation shows that the total yield of a cow is partly determined by her persistency, i.e. that the relatively persistent cows give higher yields than those which are less persistent, and vice versa. These results are contrary to those reported by Gooch (1935), who found no evidence of any correlation between persistency and total yield ($r = -0.050 \pm 0.026$) though they agree with those of Gaines (1927a) and of Sanders (1930), both of whom observed a similar positive correlation.

The value of r for "between herds", i.e. -0.48702, is not significant, the value of z being -0.53215±0.70711.

The transformed values of the various intra-herd correlations, are given in Table 56.

Table 56 showing values of z for the Total Yield and Persistency (intra-herd)

Herd	"Within herd"	"Between cows"	"Within cows"
A	+0.25022 ±0.04740	+0.26869 ±0.09853	+0.23629 ±0.05431
B	+0.53234 ±0.04725	+0.54980 ±0.09449	+0.51100 ±0.05480
C	+0.54875 ±0.03753	+0.62067 ±0.07372	+0.43334 ±0.04373
D	+0.50858 ±0.06415	0.58464 ±0.12403	+0.36775 ±0.07559
E	+0.44555 ±0.04340	+0.49967 ±0.09167	+0.39118 ±0.04945
Total	+0.45359 ±0.02046	+0.49703 ±0.04099	+0.39273 ±0.02420

It will be observed that all the values are positive and significant. The "between cows" values are in every case the highest and the "within cows" values the lowest, though the only difference between these two sets of values which is actually significant is that between the values of herd C, i.e. 0.18733 ±0.08571. The negative "within cows" correlation between maximum yield and persistency, which has already been reported, is the cause of this lower "within cows" correlation between total yield and persistency. There are significant differences between the correlations observed in the different herds, showing that the role of persistency in determining the total yield varies from herd to herd, e.g., persistency seems to be more important in herd C than herd A.

(c) Relation between Maximum and Total Yield.

The detailed results of the analysis of variance and covariance of maximum and total yield are given in Table 57.

The value of r for the whole data is +0.81101, and the corresponding value of z is +1.12985±0.02046. This extremely high positive correlation shows the important role of maximum yield in determining the total yield. Gaines (1927a), Sanders (1930), and Gooch (1935) have all reported a similar high correlation between these two variates.

It will also be noted that this correlation is far higher than that reported for persistency and total yield. The difference between the two values of z is 0.70595±0.02893, which is highly significant. Maximum yield thus appears to be a much more important factor in determining the variation of total yield than persistency. The relative importance of these two determinants of total yield will, however, be discussed again.

The value of r for "between herds" is +0.95726 which is significant, z being +1.91206±0.70711. This shows that the herds with the higher maximum yields also had the higher total yields, and vice versa,

The values of z for the various intra-herd correlations are given in Table 58. It will be seen

Table 58 showing the values of z for Maximum and Total Yield (intra-herd).

Herd	"Within herd"	"Between cows"	"Within cows"
A	+1.39931 ±0.04740	+1.61299 ±0.09853	+1.06459 ±0.05431
B	+0.95670 ±0.04725	+1.14050 ±0.09449	+0.71966 ±0.05480
C	+0.99874 ±0.03753	+1.27106 ±0.07372	+0.68145 ±0.04373
D	+0.99898 ±0.06415	+1.07963 ±0.12403	+0.89145 ±0.07559
E	+1.17952 ±0.04340	+1.36099 ±0.09167	+1.01876 ±0.04945
Total	+1.10682 ±0.04099	+1.30407 ±0.04099	+0.85946 ±0.02420

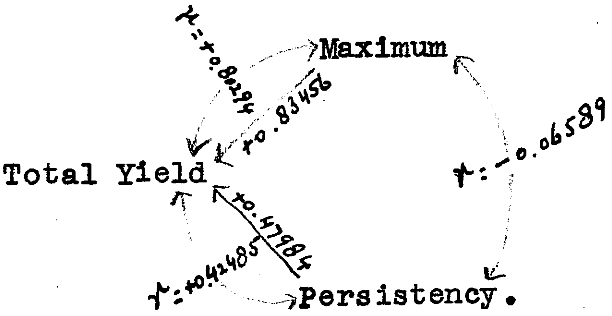
that all the values are positive and highly significant. As with persistency, the "between cows" values are the highest and the "within cows" values the lowest. Further many of the differences between these two sets of values are significant. The values also differ between different herds. Herd A, which showed the lowest correlation between persistency and total yield, showed the highest correlation between maximum and total yield.

A comparison of these results with those given for persistency shows that the correlation of maximum with total yield is in every case significantly higher than that of persistency.

(d) Relative Importance of Persistency and Maximum Yield in determining Total Yield.

The relative importance of persistency and maximum yield in determining total yield may now be examined in detail. There is no doubt that the relative influence of these two factors varies from herd to herd. However, such individual herd differences have not been explored and only the average results from the combined data will be discussed. The relative influence has been measured by Wright's method of path coefficients (1921a).

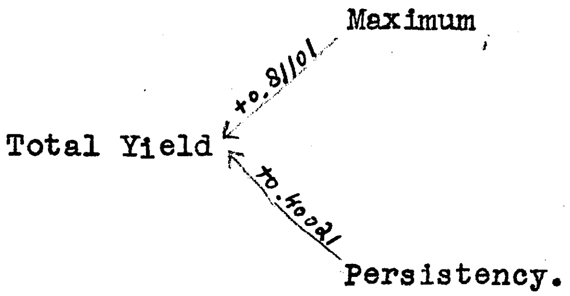
Results "within herd". The relative magnitude of the "within herds" differences in total yield due to persistency and to maximum yield respectively is shown diagramatically in the following figure. The straight lines in this diagram indicate the direct paths, the direction of the line indicating the direction of the paths, while the curved lines indicate the correlations between the different variables. The indirect paths are not shown.



The values of the direct paths (i.e. the standard regression coefficients) from maximum and persistency to total yield are +0.83456 and +0.47984 respectively. The value of the indirect path from maximum to total yield via persistency is -0.05499 and that of indirect path from persistency through maximum is -0.03162. These results show that maximum yield is just three times as important as persistency in determining the "within herds" variance of milk yield.

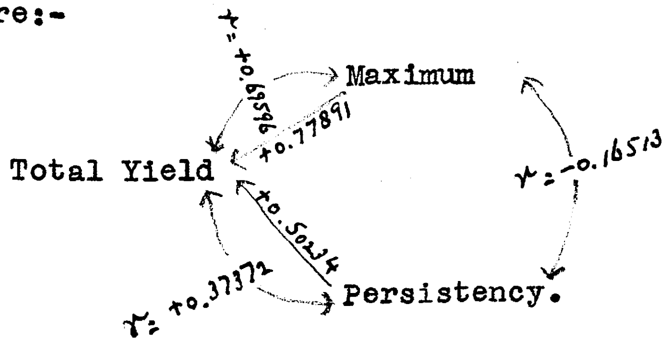
Sanders (1930) has also reported certain findings on this point, although his method of study is not free from criticism. He used the square of the observed value of the correlation to measure the relative influence of each factor, although, as Wright (1921a) has shown, in the case of correlated characters, the real contribution of a particular factor is represented by the value of the path coefficient and not the correlation coefficient. Since the value of the latter is the sum of the values of all path coefficients, it may be deceptively large or small depending on the values of these paths (Snedecor, 1938, p.281-282). In spite of this technical error Sanders' results agree with those now recorded in showing that maximum yield is far more potent than persistency in determining lactation yield.

Results "between cows". The following figure gives a diagramatic representation of the relative influence of maximum yield and persistency on the "between cows" differences in total yield.



It will be recalled that no correlation exists between maximum yield and persistency when compared on a "between cows" basis. The values of the direct paths are consequently given merely by the correlation coefficients, there being no indirect paths. The square of the path coefficient from maximum to total yield is 4.11 times the square of the path coefficient from persistency to total yield, showing that maximum yield is over four times as important as persistency in determining the "between cows" differences.

Results "within cows". The "within cows" results are represented diagramatically in the following figure:-



It will be observed that the value of the direct path to the total yield is +0.77891 from maximum yield and +0.50234 from persistency. The values of the indirect paths are -0.12862 and -0.08295 respectively. These results indicate that maximum yield is 2.4 times as potent as persistency in determining the "within cows" variation of milk yield.

Finally, it seems desirable to emphasise once again that the foregoing results were calculated from the "lumped" data of all the herds, and that there are considerable differences between individual herds in this respect.

PART III. THE LACTATION CURVE AS AFFECTED BY HEREDITY.

1. Introduction.

It is well recognised that heredity influences milk production. The practice of selective breeding which is adopted by every progressive milk-stock breeder rests on the belief that there exist hereditary differences between the milking capacities of individual animals which will be transmitted from parent to offspring according to the fundamental law of heredity that "like begets like".

Numerous researches have been directed in the past to the study of the mode of inheritance of milk yield. Smith and Robison (1933) have already summarised the results of the various workers, and it would therefore serve no useful purpose to review the whole subject afresh. The general view is that the mode of inheritance is mendelian and that it is, like any other quantitative character, of the multiple-factor type. Further, it is claimed that both the parents are equally important in transmitting their qualities to the offspring (Gowen 1924, 1934), though the Edinburgh school is inclined to believe that some of the genes involved are sex-linked (Smith _et al_., 1930; Smith and Robison, 1931; Smith, 1937). Marchlewski (1939) has also presented some evidence supporting this latter view.

If there is sex-linkage in the inheritance of milk yield, it would mean that a male offspring would get all his sex-linked inheritance from his dam and none from his sire, while the female would inherit from both sire and dam. However, the differences due to this phenomenon are in any event unlikely to be very important. Of the thirty pairs of chromosomes (Krallinger, 1931) which cattle possess, twenty-nine pairs are autosomes and only one pair heterosomes. If, therefore, all the various pairs of chromosomes are assumed to carry the same proportion of genes, the ratio of heterosomal to autosomal genes will be 1:29. Unless the genes borne on the sex chromosomes are very much more important (which is, of course, possible), or unless they constitute a very much higher proportion of the total number of chromosomes, the effect of sex-linkage will inevitably be slight. The fact that the practical breeder does not attach any greater importance to one side of the pedigree than to the other lends further support to this view. Nevertheless, it must be confessed that the point cannot be settled definitely by such arguments or the statistical method of approach so far resorted to. As Smith (1937) himself has pointed out, the latter is useful for reconnaissance only. Final and crucial evidence must be derived from planned experimentation involving reciprocal crosses. Such work is already in progress at Edinburgh, so that in due course, we may hope to get definite experimental evidence for or against sex-linkage.

Another important aspect of the inheritance of milk yield is the degree to which observed differences in individual animals are hereditary as distinct from environmental. If, for instance, the total variance of milk yield observed in any population is represented by σ^{2*} and the parts of this variance due to hereditary and environmental causes by σ_H^2 and σ_E^2 respectively, we do not know yet what is the value of $\frac{\sigma_H^2}{\sigma^2}$. There can be no doubt that such knowledge would prove most valuable to the herd owner in that it would enable him to estimate what improvement might be expected as a result of the action of a certain selection differential. In the absence of such information he is inclined to believe that <u>all</u> the observed differences between his animals are genetic. Accordingly he mates the best animal to the best, only to find that the progeny may show distinct regression. The cause of such regression lies largely in the fact that, in reality, only a part of the observed superiority of the parents is due to genetic causes, i.e. the progeny only inherit that part (σg^2) of the selection differential which is genetic.

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* The observed variance of (σ^2) for any character may be really composed of three fractions: (i) due to hereditary differences σ_H^2; (ii) due to environmental differences, σ_E^2, and (iii) due to the combined influence of heredity and environment, σ_{HE}^2.

The fraction σ_H^2 may again consist of: (i) the additive genetic variance which Fisher (1930) calls genetic variance and Wright represents by σg^2; (ii) variance due to dominant deviations from the additive scheme, σd^2, and (iii) variance due to epistatic deviations from the additive scheme σI^2.

So far few attempts have been made to investigate this point. Gowen's work (1934) shows that in cattle included in the American Jersey Register of Merit 50-70% of the variation of milk yield and 75-85% of fat percentage is due to genetic causes. Wright (1931a) has estimated from Gowen's data of American Guernsey and Holstein-Friesian cattle that heredity accounts for 75% of the observed differences. Plum (1935), however, reports that in cows of the Iowa Cow Testing Association not more than 40% of the observed differences are hereditary (in fact he states that the value may be as low as 20%), and emphasises the great desirability of further work to determine the true value.

It is clear that the value of σ_H^2 will vary from population to population, and that in any given population it can be changed by either breeding or by altering the environment. For instance, continuous inbreeding will, within a few generations, reduce the genetic variability considerably, thus increasing the proportion of σ_E^2. This point is well demonstrated by Wright's work with guinea pigs (1920). The results from a particular set of data are thus rigidly applicable only to that data alone and it is therefore not surprising that Plum's results were so different from Gowen's. Gowen's data were much more selected than Plum's, and referred to animals which were kept under markedly different conditions of management.

No work on the relative rôles of heredity and environment in the variation of milk yield of Ayrshire cows has been so far reported in this country. It will be interesting therefore to study these data from this standpoint.

Existing knowledge regarding the heritability of persistency and maximum yield is extremely meagre. This is particularly true of persistency. Although it is the general experience of the breeders that individual cows differ in their ability to maintain milk yield, and that animals of the beef type have a very much shorter lactation than those of the dairy type, doubts have been expressed by some workers as to whether persistency is an inherited character. Other workers hold the opposite view. It will therefore be useful to give a brief summary of past work.

McCandlish _et al_.(1919) compared the persistency of scrub, grade and purebred dairy cows, and found the latter type to be the most persistent, while grades were intermediate and scrub cows the least persistent.

Sanders (1923) reported that the variability of the "shape figure" of different lactations of the same cow was less than that of all cows considered together, and interpreted this as evidence of the fact that persistency was dependent on the individuality of the cow. He subsequently confirmed these findings from more extensive data (1930). Sanders' interpretation is open to the objection that the resemblance between the different lactations of the same cow may have been due to the effect of the common environment. He did not produce any supporting evidence, such as correlations between the persistencies of animals related by a common line of descent, to prove that this was not the case.

Cole (1925) reported that the milk production of Jersey and Angus crosses was intermediate between that of the two original breeds, with high production tending to be dominant. The higher production of the Jersey was found to be the result of a higher maximum yield coupled with a higher persistency. The cross-breds resembled the Jersey rather than the Angus parent in regard to both these characteristics.

Turner (1926a) has expressed the view that each cow inherits a definite maximum persistency which, though it cannot be increased by overfeeding, is liable to be lowered by unsatisfactory feeding and management. In a later paper Turner (1927b) reported certain comparative values of the persistency of fat secretion of the daughters of Guernsey sires, from which he concluded that high maximum yield of fat is not necessarily associated with persistency, and that within groups of sires having similar daughter averages, an increase in a dam's persistency is accompanied by little increase in her daughters' persistency.

Gaines (1927a) from a study of records of Guernsey and Holstein cows, concluded that "persistency of lactation is nearly as definite an individual character as is initial rate of yield, judged by the fact that with the herd constant the correlation between two lactations records of the same cow is r =0.370 for persistency and r =0.400 for initial rate". However, this view was not substantiated by the results of correlations between animals related by a common line of descent. For instance, whereas he observed a significant correlation between the persistency of dam and daughter and of maternal half sisters, no correlation was found between full sisters or paternal half sisters. These anamolous results led Gaines to doubt if "there is really any genetic difference between individuals within either of the two breeds with respect to persistency of production", and he went on to say that "as compared with initial rate of yield, or rate of yield at later stages of lactation, persistency is a much less important factor in determining the ordinary lactation yield". Finally he concluded by stating that "it is a question for serious consideration whether we may not progress as well or better in breeding and selecting high-yielding cows on the basis of a short-time yield soon after calving, as we may on the basis of a short-time test later in the lactation or on the basis of the lactation yield itself. Given high initial rate of yield, and regular frequent reproduction, <u>persistency of lactation seems to be of minor</u>

importance in the problem of breeding and selecting efficient cows".

Again in another paper, Gaines (1931) reported "the stability or permanency of the persistency character is very low, the correlation between first and second records in different herds being 0.208±0.031. *Persistency as such, seems therefore to be a questionable character*. Primary rate is evidently much more definite, the corresponding correlation being 0.420±0.027. The 10-month's yield, from these two, primary rate and persistency, shows still more stability, r =0.565±0.022. *From these observations the conclusion might be drawn that the lactation yield is a better measure of milking capacity than either of the lactation curve constants. Before such a conclusion is drawn, however, it is well that the inheritance of these characters be investigated*".

Becker and McGilliard (1928) have concluded from rather limited data that both maximum yield and persistency are inherited characters. The time of attaining the maximum was found to be inherited, the scrub cows attaining it earlier in the lactation period than the pure-bred Holsteins and Jerseys. The *absolute* rate of decline was more rapid in the higher producing cows after the maximum had been attained; the milk flow of scrub cows declined less rapidly (lbs per day) but they had shorter lactation periods.

Finally, both sire and dam contributedto the inheritance of persistency of lactation of their progeny.

Bonnier (1935) in a paper entitled "Is the shape of the lactation curve genetically determined?", reported the inability of over-feeding to alter the shape of the lactation curve, and quoted this in support of the view that the shape is constitutionally determined.

Gooch (1935), from a study of Jersey cows, found a greater variation between the different lactations of the same cow than between the lactations of different cows. She interpreted this as being due to the fact that the particular group of cows studied was probably genetically homogeneous.

Copeland (1937) reported that persistency, though largely influenced by environmental factors, seemed to be an inherited character.

Fohrman and Graves (1939) compared the persistencies of the two-year old milk records of the daughters of a bull with the persistencies of their dams. Only in seven of the 18 daughter-dam pairs were the daughters more persistent. This was taken to indicate that this particular sire tended to decrease persistency. Mention was also made of evidence (not detailed) which they showed that persistency is also transmitted from dam to daughter.

Pontecorvo (1940), from a study of persistency in a herd of Ayrshire cows, concluded "that persistency is to a great extent an expression of an animal's individuality and that the physiological causes determining the repeatability of persistency in the different lactations of the same cow are approximately of the same importance as those which tend to diversify it from one lactation to another". In the limited data examined, no correlation was found between the persistency of dam and daughter (r = 0.166±0.17).

It is clear from the foregoing summary of previous work that the views regarding the heritability of persistency are conflicting. In reality, however, the controversy is an idle one, for strictly speaking every character is hereditary in the sense that, however suitable the environment, it cannot develop without the presence of the necessary genes. On the other hand the genes cannot show their proper effect in the absence of a suitable environment. The point at issue really is, therefore, not whether persistency is a hereditary character, but (as with milk yield) *how far the observed differences in a particular population are hereditary and how far they are environmental.* Thus, even if it were shown that in a particular population the observed variance was entirely environmental, this latter result might still be either the outcome of the breeding system practised or the effect of particular environment.

Finally it may be stated that existing knowledge.

regarding the heritability of maximum yield is itself very meagre, and there is again no information available as to the relative influence of heredity and environment in causing observed differences between individuals.

It appears, therefore, that two major points require to be investigated: (i) the relative stability of the different constants of the lactation curve, and (ii) the portion of the total variance of each constant which is due to hereditary causes. These points are dealt with in the succeeding sections.

2. Stability of the Lactation Curve.

The first point worth examining is the variation of the different constants of the lactation curve in the different years and herds. A study may then be made of the intra-herd differences to determine how far they are due to differences "between cows" as distinct from differences "within cows". These latter results will give the requisite information regarding the stability of the different constants in these data, as these will show to what extent a cow's performance is more apt to repeat itself in different lactations than it is likely to resemble that of its other herd-mates chosen at random.

It will be recalled that the data for the present study was derived from six herds, five being of the "milk" type and one of "vessel" type. Since the conditions of management vary between the herds of these two types, the results from the "vessel" herd will be presented separately.

(a) Variance due to differences between years.

(i) Persistency.

The annual variations in persistency of the five "milk" herds are shown graphically in Figure 29. It will be observed that no significant trend is noticeable in herd C, the variations being of the fluctuating type. On the other hand, in the remaining four herds a distinct rise occurred about the period 1932-33, which was later followed by a definite fall. The period of rise synchronises with the inauguration of the Milk Marketing Board, so that the cause of this rise is probably environmental. The herd owners, being assured of a better market for their milk, will presumably have improved their method of feeding and management. On the other hand, if this does represent the actual cause of the increase in persistency, it is not clear why the value fell at a later date. It may be noted that these variations are of a temporary nature, and that no permanent change in the value of persistency was noticeable in any of the herds during the complete period of 20 years covered by this study.

The analysis of variance of persistency due to the year in which a record was made is shown in Table 59. The mean squares for both "between years" and

Table 59 showing analysis of variance of persistency due to the year of record (Total data "milk" herds).

Source of Variation	d/f	Sum of Squares	Mean Square	F
Within years	2322	48527.7296	22.6184	
Between herds	4	644.1334	161.0333	*7.119
Between years	14	2622.3333	187.3095	*8.281
Herd year interaction	51	2285.7171	44.8180	*1.982
Total	2391	54080.5372		

*significant

"herd year interaction" are highly significant. This shows that the means of the different years differed significantly among themselves, and also that the variations of persistency in the various herds in any particular year were different.

The analysis of variance of individual herds is given in Table 60. It will be noted that herds D and E show the greatest "between year" effect, about 11% of the total variance of persistency in these herds being due to this cause. This corresponds to a correlation of 0.11 between the persistency of two records chosen at random from the same year and herd. On the other hand, herds A and C did not show the "between year" effect to any marked extent, only 3% of their total variance being due to this cause. Herd B was intermediate, the differences "between years" explaining about 8% of its variance.

(ii) Maximum Yield.

The annual variation of the maximum yield of each of the five herds is shown graphically in Figure 30. It will be seen that, unlike those for persistency, these graphs exhibit a distinctly regular trend, the year-to-year fluctuations being practically absent. In herd C a slow but steady increase is noticeable throughout the whole period studied. In Herds A, B and E a definite increase took place between 1924-27. This was followed by a period of more or less constant production up to 1931, after which a steep rise occurred up to 1934. The yield diminished slightly during the following period. Herd D also experienced a similar increase up to 1934 and a slight decrease thereafter. The greatest increase occurred in herd A, the order of the remaining herds in this respect being B, E, C and D.

The analysis of variance of maximum yield due to the year in which a record was made is shown in Table 61. As with persistency, the mean squares for both

Table 61 showing analysis of variance of persistency due to the year of record. (total data "milk" herds).

Source of Variation	d/f	Sum of Squares	Mean Square	F
Within years	2322	135497.30	58.3537	
Between herds	4	11735.78	2933 .9450	*50.279
Between years	14	1732.62	123. 7585	* 2.121
Herd year interaction	51	47265 .84	926.7812	*15.882
Total	2391	196231.54		

*significant

"between years" and "herd year interaction" are significant, showing that the means of the different years differed significantly among themselves, and that the variations of the yield of the different herds in a particular year are not similar.

Table 62 presents the results of analysis of variance of individual herds. It will be observed that there were very great differences between the different herds with regard to the "between year" effect. In herd A, which shows a maximal effect, 56.5% of the variance was due to this cause. This corresponds to a correlation of 0.56 between two records selected at random from the same year in this herd. In the remaining herds the effect was of a lower order, viz. in herd B 22.7% of the total variance was accounted for by this cause, in herd E 16.7% and in herds C and D 6.4% and 4.6% respectively.

(iii) Total Yield.

Figure 31 shows graphically the annual variation of total milk yield of each of the five herds. It will be seen that the variations in each herd in this respect are very similar to those already reported for maximum yield. In herds A, B, D and E the highest production was reached in the year 1934, and the yield then diminished to a varying extent. In herd C, however, the highest yield occurred in 1931, diminished slightly the following year, and then remained constant; the increase in 1938 was probably not significant owing to the small number of animals included in that year.

The analysis of variance of total yield due to the year in which a record was made is shown in Table 63. The mean squares for both "between herds" and "herd year interaction" are again highly significant, showing that the means of the different years differed significantly among themselves and that the variations of the different herds in any year are not similar.

Table 63 showing analysis of variance of total yield due to the year of record. (Total data "milk" herds).

Source of Variation	d/f	Sum of Squares	Mean Square	F
Within years	2322	5444711676	2344837	[illegible]
Between herds	4	362207160	90551790	*38.617
Between years	14	917951165	65567940	*27.963
Herd year interaction	51	727822137	14278865	* 6.089
Total	2391	7452692138		

*significant.

The results of analysis of variance of individual herds are presented in Table 64. Herd A again shows the most pronounced "between year" effect, the differences between yearly means accounting for 57% of the total variance in this herd. In the remaining herds the effect was again of a lower order, viz. in herd E 18%, in herd B 10.8%, in herd D 10.0% and in herd C 2.7%.

(iv) Discussion.

The "between year" effect may be the result of genetic and /or environmental causes. The genotypes of the cows constituting the populations in the different years will change as a result of selection (if operative), or through the periodic change of bulls. Similarly the conditions of feeding usually vary somewhat in different years. The management policy may also be altered for economic reasons, thus increasing any year-to-year variations.

It would clearly be desirable to separate variations due to environmental changes from those due to genetic causes, since it would then be possible to determine to what extent the production of a particular herd had been permanently altered during the period studied.

The German school, led by Von Patow (1930), have proposed the "byre average" (stalldurchschnitt) method to allow for the influence of year-to-year variations in management. As Lush (1936) has already pointed out, however, this method regards the annual changes of production as being entirely due to environmental influences and thus ignores the genetic causes of the change. Lortscher's researches (1937) have substantiated Lush's view.

There can be little doubt that a fairly large part of the improvements in maximum and total yield which are recorded above are due to environmental causes. Conditions of feeding and management have definitely improved within the past 20 years as a result of a more efficient advisory service, of improved prices for milk, and of better marketing facilities. But to assume that the whole of this improvement has been due to such factors and that selection and breeding have played no part is just as unjustifiable as to credit the improvement solely to genetic causes.

Judged by the average value for the different years no permanent improvement of persistency is noticeable in any of the herds. If anything, the average for the last four years of this period is lower than the value in the first four-year period.

(b) Variance due to differences between herds.

(i) Persistency.

The mean and standard deviation of the persistency in each herd are given in Table 64a.

Table 64a showing the variation of persistency of different herds.

Herd	Mean	Standard Deviation
A	89.80±0.207	4.37±0.146
B	88.45±0.229	4.87±0.162
C	89.32±0.185	4.94±0.131
D	88.96±0.315	4.94±0.223
E	88.88±0.195	4.51±0.138
F	87.66±0.361	5.36±0.255
Average (weighted)* A to E	89.33±0.097	*4.76±0.070

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*The slight difference from values given in Table 52 is due to the differences in the method of calculation. These values given now were obtained without grouping the data.

The mean of herd F, which is of the "vessel" type, is the lowest and its standard deviation is the highest, the differences of these values from the weighted average values of the remaining five herds being sifnificant. The differences between the various "milk" herds, although much less marked, are nevertheless significant. This is clear from the value of the mean square for differences "between herds" which has already been given in Table 59.

As will be noted from Table 67, which gives the analysis of variance of persistency due to herd and cow, the "herd" effect accounts for only 1.02% of the total variance of the "milk" herds.

(ii) Maximum Yield.

Table 65 gives the mean and standard deviation of the maximum yield of each herd.

Table 65 showing the variation of maximum of different herds.

Herd	Mean	Standard deivations
A	51.73±0.490	10.37±0.347
B	54.65±0.396	8.42±0.280
C	48.76±0.290	7.76±0.205
D	53.66±0.561	8.80±0.397
E	50.21±0.387	8.95±0.274
F	39.47±0.448	6.65±0.317
Average (weighted) A to E	*51.26±0.185	9.06±0.131

*See footnote under Table 64a.

Both the mean and the standard deviation of the "vessel" herd F are very significantly lower than either the "weighted average" values of the "milk" herds or the values of any individual "milk" herd. The means and standard deviations of the "milk" herds also differ significantly among themselves, though these differences are not so marked as with herd F. The value of the mean square for differences "between herds", which is given in Table 61, is significant. The very much higher standard deviation of herd A is largely due to the large "year" effedt which has already been reported.

It will be observed from Table 69, which gives the analysis of variance of maximum yield due to herd and cow, that 5.82% of the total variance of "milk" herds is due to herd differences.

(iii) Total Yield.

The mean and standard deviation of the total yield of each herd are given in Table 66.

Table 66 showing the variation of total yield of different herds.

Herd	Mean	Standard deviation
A	9121.7±90.7	1919.7±64.1
B	9292.4±79.1	1679.6±55.9
C	8394.4±60.2	1608.4±42.6
D	9320.5±114.8	1801.4±81.2
E	8580.4±73.5	1697.9±52.0
F	6542.5±88.5	1315.4±62.5
Average (weighted) A to E	*8836.7±36.1	*1765.5±25.5

*see footnote under Table 64a

As already reported for persistency and maximum yield the mean and standard deviation of the "vessel" herd are both considerably lower than either the weighted average values of the "milk" herds or the individual values of any of these herds. Similarly the values of the various "milk" herds differ significantly among themselves, the value of the mean squares for "between herds" as given in Table 63, being significant. The differences between the means of herds A, B and D are not significant, though the difference of each of them from the mean of C or E is significant. The standard deviation of herd A is significantly higher than that of herds B, C and E, though its difference from the value of herd D is not significant. Similarly the differences between the standard deviations of herds B, C, D and E are not significant.

Table 71, which gives the analysis of variance of total yield due to herd and cows, shows that the herd differences account for 4.70% of the total variance of "milk" herds.

(iv) Discussion.

It will be seen from the foregoing results that, though there are significant differences among the means of different herds, the "herd" effect is not an important cause of the variance of either persistency, maximum or total yield in the "milk" herds. This means that these five herds constitute a nearly homogeneous population.

Plum (1935), working on the fat yields of cows belonging to Iowa Cow Testing Associations, found that the influence of this "herd" effect was so strong that it accounted for 33% of the total variance. Plum's data were, however, derived from 95 herds containing animals of different breeds. The strong "herd" effect noted by him is therefore not surprising. The animals in the present study all belonged to one breed and one locality. Their owners work in relatively close cooperation and to some extend depend on each other for their bulls. Their systems of feeding and management are also similar. Under such conditions a pronounced "herd" effect cannot be expected.

(c) Variance due to Differences between and within cows.

(i) Persistency.

Table 67 shows, for the "milk" herd data, the analysis of variance due to herd and cow.

Table 67 showing analysis of variance of persistency due to herd and cow ("milk" herds data)

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	644.1334	161.0333	*12.550
Within herds	2387	53436.4038	22.3864	
Total	2391	54080.5372	22.6184	
Between cows within herd	593	30417.9184	51.2950	* 3.998
Between records of the same cow	1794	23018.4854	12.8308	

*significant.

Portion of total variance due to differences between:-

$$\text{Herds}\quad \frac{22.6184 - 22.3864}{22.6184} \times 100 = 1.025\%$$

$$\text{Cows}\quad \frac{22.6184 - 12.8308}{22.6184} \times 100 = 43.273\%$$

Portion of intra-herd variance due to differences between cows:

$$\frac{22.3864 - 12.8308}{22.3864} \times 100 = 42.685\%$$

The part of the variance which is due to differences between cows may be expressed in terms of the total or of intra-herd variance. It will be observed that 42.7% of the intra-herd variance is due to differences between cows. This means that there is a squared multiple correlation of 0.427 between the persistency of a cow as the dependent variable and all those causes which affect her persistency in different lactations alike, but are different for the other cows in the same herd. These causes which make two records of a cow in the same herd resemble each other may be both hereditary and environmental. If they are entirely hereditary then the value of σ_H^2 in these data is 42% in the case of persistency. If on the other hand, some of this resemblance is the result of the common environment in different years, then the value of σ_H^2 will be still lower. This point will be examined again in the next section.

If the influence of the individual herds is ignored and the data of all herds are lumped together, there is a correlation of 0.433 between the records of the same cow. This is slightly higher than the intra-herd value reported in the last paragraph (0.427) because of the small herd effect which is included.

Since the herd and cow differences together account for 43.3% of the total variance, the balance (56,7%) is due to differences between the records of the same cow. There can be many causes for this intra-cow variation, though these are all environmental, the term environment being used in a broad sense to include variations due to the errors in estimating persistency besides those caused by definite environmental factors, such as conditions of feeding and management, state of health of animal, etc.

The above results pertain to the combined data from all the "milk" herds. The value of the intra-herd correlation for different records of the same cow varies, however, from herd to herd. The analysis of variance of individual herds has already been given in Table 53. The results are summarised in Table 68 in such a way as to show this variation.

The results are self explanatory and need little comment. The value of the correlation between the different records of the same cow varies from 0.336 in herd E to 0.538 in herd D. The value in the "vessel" herd F nearly equals the average value of the "milk" herds, i.e. 0.427.

(ii) Maximum Yield.

Table 69 shows the analysis of variance of maximum yield due to herd and cow.

Table 69 showing the analysis of variance of maximum due to herd and cow.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	11735.78	2933.945	*75.858
Within herds	2387	184495.76	77.2919	
Total	2391	196231.54	82.0709	
Between cows within herd	593	115109.99	194.1147	* 5.019
Between records of the same cow	1794	69385.77	38.6766	

*significant

Portion of total variance due to differences between:-

$$\text{Herds} \quad \frac{82.0709 - 77.2919}{82.0709} = 5.823\%$$

$$\text{Cows} \quad \frac{82.0709 - 38.6766}{82.0709} = 52.874\%$$

Portion of intra-herd variance due to differences between cows:-

$$\frac{77.2919 - 38.6766}{77.2919} = 49.960\%$$

It will be observed that the differences between cows account for about 50% of the intra-herd and 53% of the total variance. The former result corresponds to a correlation of 0.50 between records of the same cow in a population of cows all belonging to the same herd, and the latter to a correlation of 0.53 between records of the same cow in the total population of cows kept in these five herds. Since the herd and cow differences together account for nearly 53% of the total variance, the balance of 47% is due to differences within the records of the same cow.

The results for individual herds, as summarised from the detailed results given in Table 53, are given in Table 70.

It will be observed that the "between cows" differences account for 61% of the total variance of herd A, about 50% in the case of herds B, C, and D, and only 40% in the case of herds E and F. Herd E it will be recalled had the lowest value for persistency as well.

(iii) Total Yield.

Table 71 gives the analysis of variance of total yield due to herd and cow.

Table 71 showing the analysis of variance of total yield due to herd and cow. ("milk" herds data).

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	362207160	90551790.0	*69.137
Within herds	2387	7090484978	2970458.7	
Total	2391	7452692138	3116977.0	
Between cows within herd	593	4740794182	7994593.9	* 6.104
Between records of the same cow	1794	2349690796	1309749.6	

*significant.

Portion of total variance due to differences between:

Herds $\dfrac{3116977.0 - 2970458.7}{3116977.0} = 4.701\%$

Cows $\dfrac{3116977.0 - 1309749.6}{3116977.0} = 57.980\%$

Portion of intra-herd variance due to differences between cows:

$$\frac{2970458.7 - 1309749.6}{2970458.7} = 55.907\%$$

The differences between cows are responsible for nearly 56% of the intra-herd and 58.0% of the total variance of milk yield. This means that there is a correlation of 0.56 between the milk yield in different lactations of the same cow in a population of cows all coming from the same herd and of 0.58 if the total population of the five herds is lumped together.

Since the herd and cow differences together are responsible for 58% of the total variance, the balance of 42% is the result of the variations between the different records of the same cow due to environmental influences.

The results for individual herds in this respect have been summarised from the detailed data given in Table 55 and are shown in Table 72. These results are very similar to those already reported for maximum yield, the only difference being that the portion of total variance due to differences between cows is higher than with maximum yield. This increase is particular marked in herd C.

(d) Relative Stability of the Different Constants of the Lactation.Curve.

The foregoing results show that the correlation between the performance of the same cow during different lactations varies from herd to herd with all three characters, i.e. persistency, maximum and total yield. The results also show that no one herd has the highest values for all three characters, although herd E invariably has the lowest. In every individual herd, except D, the correlation for persistency is the lowest, that for maximum yield is intermediate, and that for total yield is the highest. In herd D the value for persistency is higher than that for maximum yield, though the difference between the two is very slight. Considering the average value for the data from the "milk" herds, the intra-herd correlations are 0.42, 0.49 and 0.56 for persistency, maximum and total yield respectively. This shows that total yield is more stable throughout the life time of the cow than either persistency or maximum. Gaines (1931) has reported similar results.

The greater consistency in maximum yield as compared with persistency is easily explained. The measurement of persistency extends over the complete lactation, while the maximum yield is realised in a comparatively short period. It is obvious that, in comparing different lactations of the same cow, there will be a greater probability of chance variations over the complete lactation than over part of it.

The above results definitely disprove Gaines' view that persistency is a questionable character. The repeatability value of 0.42 for the persistency of the different lactations of the same cow is as high, for instance, as that reported by Plum (1935) for the fat yield of cows in Iowa Cow Testing Associations, so that in the present data persistency seems to be as definite a character as the fat yield in Plum's data.

It has already been pointed out that the correlation between the performance of different lactations of the same cow may be associated with both hereditary and environmental factors. The above results show that the value of σ_H^2 for persistency, maximum and total yield is no higher than 0.42, 0.49 and 0.56 respectively. It may actually be less if this resemblance between the different records is, to any extent, due to the intra-herd correlations between the environment to which the cow has been subjected in the different years and between the cows' environment and her heredity. This question of the relative influence of heredity and environment will be examined in the next section.

3. Methods of Measuring the Heritability of Individual Differences.

(a) General Discussion.

The relative importance of heredity and environment in determining the observed differences between the different individuals of a population may be best studied experimentally. It is well known that no genetic differences exist among the individuals constituting a pure line, all variation in them being of the random fluctuating type and thus environmental. This was first shown experimentally by Johannsen in his studies of the Princess bean (Babcock and Clausen, 1927, p.207-211). The observed variance in a mixed population, on the other hand, may be due to both environmental and genetic differences. If, therefore, the variance observed in a mixed population is compared with that present within pure lines, we can obtain an estimate of the part of the total variance which is due to genetic differences. Pure lines exist normally in autogamous species of plants. Even in allogamous species, which are generally cross-pollinated, it is pure lines which can be quickly established, provided that it is possible to subject them to self-pollination. No pure lines exist normally among the higher animals, which too are allogamous, although it is possible to establish strains showing a high degree of homozygosity by practising continuous inbreeding for a number of generations. Ten generations of brother-sister mating,will, for instance, reduce the heterozygosity to 5.7% (Pearl, 1914, Jenning, 1914, Wright, 1921). However, this procedure is impracticable with such slow breeding animals as cattle.

Isogenic lines are encountered in animals in the form of monozygotic twins, and it should be possible to study the relative influence of heredity and environment by comparing the variance in the total population with that observed within such lines. However, the number of twin births in dairy cattle is only one in 50-60, and, as,estimated by Johannson, only 6.0±1.2% of twin births are of the monozygotic type (Lush, 1938, p.325) so that, on an average, one identical twin is born in every 1000 births. This rate of birth would clearly provide insufficient animals for reliable results.

Another experimental method of approach is to practise selection,both in the plus and minus direction for a number of generations. The difference produced in this way between the means of the population divided by the extent of selection practised gives a a measure of the true genetic variance present in that population. This result is obvious since, as already pointed out, the resultant shift in the value of the mean due to selection is σ_g^2 (H^2 in the narrow sense) of the selection differential. If, therefore, the value of σ_E^2 is zero, the difference produced as a result of certain degree of selection will nearly equal the selection differential. On the other hand, if σ_g^2 is zero then σ_E^2 will approximate in value to σ^2 and no permanent change will be possible through selection.

The foregoing method of selection has been used with both plants and animals to study the extent to which selection is effective in producing changes in a population. The more important examples are the work with maize carried out by the Illinois School and Castle's experiments regarding the hooded pattern in rats (Babcock and Clausen, 1927, p.221-228).

Both the above experimental methods are of limited application with such slow-breeding animals as dairy cattle. The alternative statistical method of determining the heritability from the observed value of correlation between ~~the~~ individuals related due to a common line of descent, though not as accurate, is the only practicable one. This method which was first used by Galton and his associates to measure the "strength of heredity" between different relationships, postulates that the observed correlation between related individuals with regard to a particular trait is due to the influence of common causes. The identical genes possessed by related animals is an important, though not the only, cause of this correlation. For instance, a parent gives a sample half of its germ-plasm to each of its progeny who must, as a result, show a certain resemblance in their traits. For the same reason there should be a correlation between the parent and offspring. The more identical are the genes possessed by two such individuals the higher is the correlation and vice versa. The values of the genotypic correlations between individuals showing different degrees of relationship, both direct and collateral, under a system of random mating are given in Table 73.

Table 73 showing value of correlation between related individuals.

Parents and Offspring	0.50
Grandparents and Offspring	0.25
Full sibs	0.50
Half sibs	0.25

The values given in Table 73 are for genotypic correlations under conditions of random mating. If the variation of a trait is not determined completely by hereditary causes the value of the correlation will be lower. Fisher (1930) has suggested the method of "dividing the square of parental correlation by grandparent correlation to get a good estimate of the fraction of the total observable variance of the measurement which may be regarded as genetic variance".

Wright (1921) has expressed the correlation between parent and offspring by the equation:

$$r_{po} = \frac{1}{2}h^2$$

under conditions of random mating, no dominance, no correlation between heredity and environment, and complete determination by heredity. It is, therefore, possible to determine the part of variance that is due to hereditary causes by the use of this formula on the observed values of the parent-offspring correlations, if the necessary conditions for the application of the formula are satisfied.

However, the influence of environment may not be the only factor responsible for the diminution of the observed value of correlation, for both dominance and epistasis have the same effect. The parent-offspring correlation measures the truly additive part of the genetic variance, - what Fisher (1930) calls the "genetic variance" and Wright (1930) denotes by the fraction g^2 of the total hereditary variance ($h2$), (the latter including, in addition to g^2, the fraction (d^2) which is due to dominance deviations from the additive scheme, and (I^2) due to epistatic deviations). The influence of dominance has been studied by Fisher (1922), who reports that under conditions of random mating, and with complete dominance and no epistasis, the value of g^2 for the effectsof each gene pair is

$$\frac{2\ (1-q),}{2 - q}$$

where q represents the gene frequency of the dominant gene. This gives the value $\frac{q}{2-q}$ for d^2, i.e. the portion of h^2 which is due to dominance and which Fisher calls "dominance ratio". He regards $\frac{1}{3}$ as the most typical value for d^2 for most characteristics. Knowing this relationship, it is possible to determine h^2 from the observed value of g^2. Wright (1931b), however, considers that the "dominance ratio" is more near $\frac{1}{5}$ than $\frac{1}{3}$, and that the dominance is in general not so complete as Fisher holds. If Wright's view is correct, then h^2 is $\frac{5}{4}g^2$. Nothing so definite is known regarding the influence of epistasis in lowering the value of observed correlation.

Again, the observed value of correlation may be too high for one or both of two reasons: (i) the breeding, instead of being random, may be assortive, and (ii) there may exist a correlation between the related individuals due to a common environment.

As regards the first point, Wright (1921b) has shown that, if there is assortive mating, whether on genetic resemblance such as inbreeding, or on somatic resemblance, the value of the genotypic correlations is higher than that given in Table 73. For instance, as a result of single generation of brother-sister mating the value of the correlation between parent and offspring is 0.671, instead of the value of 0.5 as for random mating. Similarly under conditions of perfect assortive mating, based on somatic resemblance (m = correlation between mates

= 1.0), the value is 0.816.

As regards the second point, an increase in value results if there exists a correlation between the environments of the related individuals. Such a correlation is normally present in most data, and its value varies with the relationship of individuals. Generally, the greater the lapse of time between the periods to which the records of the related individuals belong, the lower will be this correlation. Accordingly, there is a lower correlation due to this cause between the production of parent and offspring than between the sibs*. Of the latter, paternal half-sisters, who are generally contemporaries, have a greater resemblance due to environment than maternal half-sisters or full sisters. Again animals kept in the same herd resemble one another more than those kept in different herds.

Fisher (1939, p.193-197), has stressed the importance of taking into the account the probable effect of such environmental correlations when estimating the "strength of heredity" from the observed correlation between related animals.

As Lush (1939a) has pointed out, when heritability is being estimated from the resemblance between relatives, the most dependable estimates are based upon the closest relationships, because the

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* i.e. collateral relation.

sampling errors are thereby kept relatively small. For this reason relatives more remote than half-sibs are rarely of use for estimating heritability. Further, (in Lush's own words) "the correlation between parent and offspring is generally the most useful approach, if environmental correlations can be discounted adequately. The correlation between dam and offspring, within groups of offspring all by the same sire, is a very useful way of automatically discounting most environmental correlations and deviations from random mating".

Finally it must be mentioned that since the character of milk secretion is manifested by only the female sex, direct correlations are possible only between females. The correlation between sire and daughter can only be measured indirectly from the observed resemblance between paternal half sisters. Gowen (1924) and Smith et al. (1930) have employed this latter method.

(b) Measure of Inherent Milking Capacity Employed.

Many different measures have been used to denote the innate milking capacity of a cow. Gowen (1920), Turner (1927c), Copeland (1938), and Fohrman and Graves (1939), for instance, have argued that the highest age-corrected milk record of a cow is the best measure of her physiological capacity for milk production, and have therefore recommended its use for genetical studies. Copeland (loc.cit.) has also claimed that the highest milk record of a cow is very closely correlated with her average milk production for a number of lactations, although Berry and Lush (1939) have shown that this high correlation is spurious and is the result of ex post facto selection. In his later investigations Gowen (1924) used the milk record nearest to the age of maximum productivity, i.e. eight years, after correcting it for age. Smith et al. (1930) and Smith and Robison (1931), on the other hand, employed the average of all corrected records of an animal to measure her milk yielding capacity. Plum (1935) used the first available record of each cow. Lush and Shultz (1938) and Seath and Lush (1940) used the highest corrected record for some animals and the average of a number of corrected records for others.

As has been pointed out already, each unselected record of a cow is a measure of her phenotype as modified by environment. No one record is perfectly correlated with another record or with the average of a number of other records, so that no one lactation is a perfect measure of a cow's phenotype. In general, the environmental conditions are as likely to be favourable as unfavourable in any lactation. If the highest corrected record is chosen to represent a cow's milking capacity one is selecting a record made under the most favourable conditions of environment. When cows, all of which have not completed the same number of lactations, are compared on the basis of their highest corrected record, the animals with a larger number of known lactations obtains an advantage over those with fewer lactations, in as much as they have had greater chance to make a high record.

In the writer's opinion the two alternative procedures which are above criticism are either (i) to use the record for a stipulated age for all animals regardless of the magnitude of that record, if otherwise normal; or (ii) to use the average of all available, normal age-corrected records. Since one record is more liable to be influenced by the vagaries of environment, it will be preferable to use the average of all available normal age-corrected records. Accordingly, this latter procedure was adopted in the present study, and the values noted in the following sections for persistency, maximum and total yield therefore represent the mean values for all recorded lactations.

4. Correlation between the Production of Dam and Daughter.

The correlation between the production of dam and daughter was obtained by two methods: (i) 338 dam-daughter pairs were included in the "milk" herds data, and the correlation was obtained from these total data; (ii) the correlation between dam and daughter within groups of daughters all by the same bull was studied for those bulls which had at least five recorded daughters. This latter method furnished

238 daughter-dam pairs, the daughters being the progency of 25 different bulls. Except for a few daughter-dam pairs, the records of both dam and daughter were made in the same herd. Similarly nearly all the different daughters of a bull came from the same herd, so that it was possible to determine the correlation for each herd separately.

The results obtained by the two methods will be presented separately.

(a) Dam-Daughter Correlations, Total data.

(i) Persistency.

From a correlation table drawn from the values of persistency of dams and daughters the statistical constants shown in Table 74 were obtained.

Table 74 showing the correlation between the persistency of dam and daughter.

	Dam	Daughter.
Mean	89.79±0.209	88.21±0.234
Standard deviation	3.46±0.148	3.80±0.166
Coefficient of variation	3.85±0.148	4.31±0.166

r +0.20162
z +0.20440±0.05464

It will be observed that the mean of dams is significantly higher than that of daughters. The standard deviation of the daughters, on the other hand, is slightly, though not significantly, higher. The value of z is significant, showing that the observed positive correlation is significant. This means that the dams with persistency higher than the average had daughters whose persistency was also higher than the average, and vice versa. The observed value of r is, however, very much lower than the genotypic value of 0.5 which would be expected if the parent-offspring correlation were solely due to heredity. This shows that a major part of these differences is due to environmental causes.

Table 75 gives the results regarding the nature of regression line of daughters' persistency on dams' persistency.

Table 75 showing results about test of regression line of daughters' persistency on dams' persistency.

Source of Variation	d/f	Sum of Sqaires	Mean Square
Between arrays	18	544.2	
Within arrays	319	4325.2	13.5586
	337	4869.4	
Linear regression	1	198.0	
Deviations from linear regression	17	346.2	20.3647
Total	18	544.2	

$$F = 1.502$$

The observed value of F for the mean square of deviations from the linear regression is not significant at the 5% level, showing that the regression line describing the relation of daughters' persistency to dams' persistency is linear. The equation describing this regression line is

$$P_o = 68.30+0.22175p_d$$

where P_o = expected persistency of daughter.

and p_d = observed persistency of dam.

The standard error of the estimate of daughters' persistency according to the foregoing equation is ±3.72%, which is little lower than the observed standard deviation of the daughters, i.e. 3.80. This shows the very strong regression of the persistency of individual daughters towards the mean of the total population of daughters.

(ii) Maximum Yield.

From the correlation table drawn for the maximum yields of dams and daughters, the statistical constants given in Table 76 were obtained.

Table 76 showing the correlation between the maximum yield of dams and daughters.

	Dams	Daughters
Mean	50.56±0.378	52.21±0.409
Standard deviation	6.96±0.267	7.52±0.289
Coefficient of variation	13.76±0.539	14.41±0.565
r	+0.36097	
z	+0.37801±0.05464	

Both the mean and the standard deviation of the daughters are significantly higher than those of the dams, although the difference between the value of the coefficient of variation is not significant. The value of z is highly significant, showing that the positive value of r is not a chance result. The value of r is nearly twice that reported for persistency. However, as will be shown later this difference is largely due to the effect of assortive mating.

The regression line describing the relation of daughters' to dams' maximum yield is linear. This is clear from the results given in Table 77, where the value of F for the mean square of deviations from the linear regression line is not significant at the 5% level.

Table 77 showing the results about the test of linearity of regression line of daughters' maximum on dams' maximum.

Source of Variation	d/f	Sum of Squares	Mean Square
Between arrays	34	4323.59	48.6845
Within arrays	303	14751.40	
Total	337	19074.99	
Linear regression	1	2485.43	
Deviations from linear regression	33	1838.16	55.7018
Total	34	4323.59	

$$F = 1.1441$$

The equation of this regression line is

$$M_o = 32.47+0.39042m_d$$

where M_o = expected maximum of daughter

and m_d = observed maximum of dam

The standard error of the estimate of daughters' maximum yield according to this equation is ±7.03 lbs as against the observed standard deviation of 7.52 lbs.

(iii) Total Yield.

From a correlation table drawn for the total yields of dams and daughters, the statistical constants given in Table 78 were obtained.

Table 78 showing the correlation between dams' total yield and daughters' total yield.

	Dam	Daughter
Mean	8886.1±75.0	8835.0±79.8
Standard deviation	1378.1±53.0	1467.6±56.4
Coefficient of variation	15.51± 0.611	16.61± 0.656
r	+0.31521	
z	+0.32634±0.05404	

There are no significant differences between the values of mean, standard deviation or coefficient of variation of dams and daughters. The value of r, however, is highly significant. The correlation of total yield is slightly lower than that reported for maximum yield, but higher than that for persistency, - although its difference from these two correlations is not significant.

The regression line describing the relation between the total yield of daughters and dams is linear. This is clear from the results given in Table 79, where it will be seen that the mean square of the deviations from the linear regression line is actually lower than that within arrays.

Table 79 showing results of the test of linearity of regression line of daughters' total yield on dams' total yield.

Source of Variation	d/f	Sum of Squares*	Mean Square*
Between arrays	28	1930.92	
Within arrays	309	9682.28	31.3342
Total	337	11613.20	
Linear regression	1	1153.86	
Deviations from linear regression	27	777.06	28.7800
Total	28	1930.92	

* In units of class interval used.

The equation of this regression line is

$$Y_o = 5852.2+0.33568y_d$$

where Y_o = expected total yield of daughter

and Y_d = observed total yield of dam.

The standard error of estimate of the daughters' total yield according to this equation is ±1394.8 lbs, as compared with the observed standard deviation of 1467.6 lbs.

(b) Intra-Sire-Dam-Daughter Correlations.

We have seen that the daughter dam correlations for persistency, maximum and total yield are 0.20162, 0.36097 and 0.31521 respectively. If these results are free from the influence of assortive mating and common environment, then according to Wright's formula

$$r_{po} = \frac{1}{2}h^2$$

the value of h^2g^2 for persistency, maximum and total yield is 40.324, 72.194 and 63.042% respectively. However, be fore any reliance can be placed on these values, it must be established that the mating was really random and that the observed correlation is not due to any extent to the influence of common environment.

The breeding practised may not be random as a result of either inbreeding or of assortive mating based on individual performance or the performance of near relatives. The extent of inbreeding practised in the Ayrshire breed is slight. Fowler (1932) estimated it to be 4.6% for the whole breed. It is unlikely that the value differed markedly from this in any of the herds under study. The observed value of parent-offspring correlation cannot, therefore, be affected greatly by this cause. The same does not, however, hold for assortive mating. In fact various workers in the past have shown that the system of breeding practised within a breed is definitely assortive in this respect. From the data of Guernsey sires Goodale (1926) has, for instance, reported that "those sires whose daughters average the largest yield of butter fat were mated to cows of superior average butter-fat production, while sires whose daughters averaged the least, were mated to cows whose average was comparatively low". Gowen (1934) has shown that the system of assortive mating practised in Jersey cattle gives a correlation of +0.20±0.02 between the milk yields of a sire's mate and his dam, and of +0.12±0.03 between a sire's dam and his mate's dam. Lush (1935), discussing the correlation arising as a result of assortive mating, has stated that "it is not unreasonable either from the genetic principles involved or from the published studies of dairy data to suppose that this correlation within dairy breeds might well be as large as +0.15 or 0.20".

To remove the influence of this assortive mating from dam-daughter correlations Turner (1927c) adopted the procedure of correlating daughter-dam production within groups of sires having the same average progeny performance. This method postulates, however, that sires having the same average progeny performance have similar transmitting ability, and ignores the influence of the dam in determining the progeny performance of the sires. Turner himself has admitted this, as is clear from the following statement "It is quite obvious that the fault with any classification of sires using the progency performance as a measure of their transmitting ability is the difficulty of equitably evaluating the dam's contribution. On the other hand, to determine the relation between dams and daughters some measure of the sire is necessary. It is a vicious circle for which as yet no entirely satisfactory solution has been found".

The method proposed by Lush (1939a) of determining the daughter-dam correlation within groups of daughters, all by the same sire, eliminates the influence of assortive mating, if present. Since the sire is constant for the various daughters of a bull this intra-sire correlation measures the average contribution of the dam to the daughter for any character.

The results obtained by this method are detailed below.

(i) Persistency.

The correlations between the persistency of the mates and daughters of each of the 25 bulls are given in Table 80. It will be observed that the value of r for individual sires varies considerably, the lowest positive value being 0.04889 and the highest 0.92465. Four out of the 25 correlations are negative, showing a strong regression of daughter's persistency towards the mean. The average intra-sire correlation and the value of h^2g^2 for the different herds is given in Table 81.

Table 81, showing the intra-sire daughter-dam correlation for persistency in the case of individual herds.

Herd	r	z	h^2g^2 %
A	+0.26897	+0.27577±0.12403	53.794
B	+0.27487	+0.28214±0.12217	54.974
C	+0.18610	+0.18833±0.16440	37.220
D	+0.35525	+0.37144±0.21822	71.050
E	+0.07122	+0.07133±0.17408	14.244
All	+0.23764	+0.24228±0.06523	47.528

Because of the small number of observations upon which each correlation is based, the standard errors of z are very high, so that the differences between the individual values of z are not significant. The average intra-herd value of r must therefore be taken as a measure of parent-offspring correlation. It is significant to note, however, that, except for herd C, the order of values of r of individual herds is the same as that already reported in Table 68 for the repeatability of persistency of the same cow in different lactations. Herd D, which has the highest repeatability value, has the highest value of r, and herd E is the lowest in both respects.

From the average value of r, the value of h^2g^2 works out at 47.5%. Increasing the latter value by $\frac{1}{3}$ and $\frac{1}{5}$, (according to Fisher's and Wright's estimates respectively for value of d^2), gives the values of

71.3 and 59.4% for h^2. These results are based upon the assumption that the observed dam-daughter resemblance is entirely due to genetic causes, and that there exists no correlation between the environment of dam and daughter. This assumption is, however, erroneous, since it has already been shown in Table 67 that 42.7% of the total variance is due to differences between cows. The value of h^2 cannot therefore be higher than this. The inevitable conclusion is that much of the observed resemblance between dam and daughter is due to the influence of common environment, for which, unfortunately, no allowance can be made. Even if it were assumed that there is no dominance, i.e. that d^2 is zero, the estimated value of h^2g^2 is slightly higher than the maximum possible value of h^2. There exists, however, definite experimental evidence, in the Jersey x Angus crossbreeding work reported by Cole (1925), that high persistency is partially dominant to low persistency, so that absence of dominance cannot be the cause of the high value of h^2g^2.

(ii) Maximum Yield.

Table 82 gives the correlations between maximum yield of mates and daughters of the individual bulls. It will be observed that the value of r varies with individual sires, the lowest positive value being 0.04343 and the highest 0.92698, and is negative for six bulls. The average intra-sire correlation and the estimated value of h^2g^2 for each herd are given in Table 83.

Table 83 showing intra-sire daughter-dam correlation for maximum yield in the case of individual herds.

Herd	r	z	h^2g^2 %
A	+0.25706	+0.26297±0.12403	51.412
B	+0.24657	+0.25178±0.12217	49.314
C	+0.31335	+0.32428±0.16440	62.670
D	+0.35830	+0.37494±0.21822	71.660
E	-0.07988	-0.08080±0.17408	15.976
All	+0.22262	+0.22644±0.06523	44.524

The negative value of r of herd E is not significant, neither are the differences between the values of r of individual herds. The intra-herd value calculated for the total data must therefore be taken as an estimate of the correlation.

From this average correlation the value of h^2g^2 works out at 44.5%, and the values of h^2 at 66.8 and 55.7% according to Fisher's and Wright's estimate of d^2 respectively. Both these latter values are higher than 50%, the portion of total intra-herd variance of maximum yield due to differences between cows which was reported in Table 69. This shows that either the observed value of r_{po} is higher than the real value, due to the influence of common environment, or that the value of d^2 is zero, i.e. the influence of dominance is lacking.

(iii) Total Yield.

The correlations between the total yield of mates and daughters of individual bulls are given in Table 84. It will be observed that, as with persistency and maximum yield, the value of r for individual sires varies considerably and is negative in three cases. The average intra-sire correlation, together with the estimated value of h^2g^2 for each herd are given in Table 85.

Table 85 showing the intra-sire dam-daughter correlation for total yield in the case of individual herds.

Herd	r	z	h^2g^2 %
A	+0.23967	+0.24444±0.12403	47.934
B	+0.24484	+0.24994±0.12217	48.968
C	+0.56112	+0.63452±0.16440	112.224
D	+0.22376	+0.22764±0.21822	44.752
E	+0.07665	+0.07682±0.17408	15.330
All	+0.26314	+0.26951±0.06523	52.628

It will be observed that, except for the difference between the values of z in herds C and E, none of the differences between the different herds is significant. No reason can be given for the abnormally high value of herd C except that it may be the result of environmental influences. The average intra-sire correlation for the total data gives the value 52.6% for h^2g^2, and 78.9% and 65.8% for h^2 according to Fisher's and Wright's estimate of d^2 respectively. Since the maximum possible value of h^2 as already reported in Table 71 (on the basis of that portion of intraherd variance which is due to differences between cows) is 55.9%, both these estimates of h^2 are too high. This shows either that the value of h^2g^2 is too high on account of the contribution of the common environment to the dam-daughter correlation (for which no allowance is made) or that there is no dominance. Gowen's experiments with dairy and beef crosses (1920b) and extensive crossbreeding work carried out in India have shown definitely that high yield is partially dominant to low yield, so that lack of dominance cannot be the cause of the high value of h^2g^2. This must therefore presumably be due to the influence of environment.

(c) Discussion.

The intra-sire values of dam-daughter correlation for persistency, maximum and total yield are 0.23764, 0.22262 and 0.26314 respectively, as compared with 0.20162, 0.36097 and 0.31521 respectively obtained from the total data. The difference between the two sets of values is due to the influence of assortive mating. The intra-sire values which are free from the effect of assortive mating are all about equal, their differences from one another being not significant.

All the intra-sire correlations are too high, owing to the correlation between the environment of dam and daughter for which no allowance can be made. As a result it is not possible to determine the exact value of h^2 for these data. All that can be done is to indicate the maximum values that are possible. If the influence of common environment is equal in all three correlations, these maximum values are approximately 43,50 and 56% for persistency, maximum yield and total yield respectively. It is possible, however, that the influence of environment may not be identical, in which case these values would require modification.

From the foregoing results, it will probably not be incorrect to state that, of the total variance observed in these data for each of the three characters, only 40-45% is due to genetic causes (h^2), There are indications that this value is very much lower for herd E, where probably not more than 15-20% of the total variance is hereditary. The value of h^2g^2 for the total data is approximately 30%, if there is complete dominance. Since it is the additive genetic part (h^2g^2) of variance that is gained by selection, the results show that, corresponding to a selection differential of 100 lbs in any generation the real advance will be 30 lbs. The average interval between generations in the case of dairy cattle is four to four and a half years (Lush, 1938). This means that the annual rate of advance will be approximately 7% of the selection differential. This can hardly be termed an encouraging rate of progress.

5. Influence of Sire and Correlation between Paternal Half-Sisters.

In the last section the correlation between dam and daughter with regard to the different constants of the lactation curve was discussed. Let us next examine whether any genetic differences exist between the different sires used in the herds under study.

Obviously the method employed in the last section, i.e. a direct correlation between the performance of parent and offspring, is not applicable. Reliance must therefore be placed on the indirect method of study i.e. the relation between paternal half-sisters.

(a) Persistency.

(i) Influence of Sire. The mean persistency of the daughters and mates of individual bulls (those with at least five daughters) used in these herds have been given in Table 80. It is quite clear from these results that there exist distinct differences between the mean persistency of the daughters of different bulls, and that these differences are not entirely due to the differences between the persistency of the mates of different bulls. Bulls bred to mates having a persistency above the average begot daughters below the average, and vice versa. The influence of the sire is thus substantiated, as also are the differences between the transmitting abilities of the different bulls. This is equally well shown in Table 86, which gives the analysis of variance of the persistency of the daughters of different bulls.

Table 86 showing the analysis of variance of persistency of daughters due to bulls.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	194.0505	48.5126	*4.259
Within herds	233	2957.9876	12.6952	
Total	237	3152.0381	13.2997	
Between means of daughters of different sires	20	531.6685	26.5834	*2.334
Within daughters of different sires	213	2426.3191	11.3912	
Total	233	2957.9876		

*significant.

Portion of total variance of daughters due to differences between

Herds $\frac{13.2997 - 12.6952}{13.2997} = 4.545\%$

Bulls $\frac{13.2997 - 11.3912}{13.2997} = 14.350\%$

Portion of intra-herd variance of daughters due to differences between

Bulls $\frac{12.6952 - 11.3912}{12.6952} = 10.27\%$

It will be observed that the mean squares for both "between herds" and "between sires" are significant. This shows that there are significant differences between the means of daughters belonging to different herds, and between the means of daughters of different bulls belonging to the same herd. The "within daughters" mean square is 10.3% lower than that "within herds". This corresponds to a correlation of 0.103 between the persistency of paternal half-sisters in a population of paternal half-sisters belonging to the same herd. This point will be discussed again, however, in the light of results obtained from more extensive data.

That the differences reported in the preceding paragraph between the daughters of different bulls are not due to differences in the mates of different bulls is clear from the results given in Table 87, which shows the analysisnof variance of persistency of mates of different bulls.

Table 87 showing analysis of variance of persistency of mates due to bulls.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	74.1589	18.5397	1.501
Within herds	233	2876.7314	12.3465	
Total	237	2950.8903	12.4510	
Between means of mates of different sires	20	246.0295	12.3015	
Within mates of different sires	213	2630.7019	12.3507	
Total	233	2876.7314		

Neither the "between herds" nor the intra-herd "between sires" mean squares are significant. The latter, in fact, is less than the "intra-herd within mates" mean square.

The last column of Table 80 shows the persistency "index" of each sire calculated according to formula

Sire index = 2 x daughter's performance - dam's performance

With minor modifications this formula has been recommended by Hansson(1913), Schmidt (1919), Yapp (1924), Wright (1931a), Rice (1933), and Lush (1933). The index for each bull measures his transmitting ability. It will be observed that there are very great differences between the different bulls in this respect.

(ii)Correlation between paternal half-sisters. We have discussed so far the results from sires with at least five daughters. There were only 25 sires which satisfied this condition, although there were 88 sires who had two or more daughters each. The study of the correlation between paternal half-sisters was made from this latter material.

It must be emphasised at the outset that the correlation between sibs is of the intra-class type, as distinct from the inter-class type that exists between parent and offspring. The general method so far employed to determine the intra-class correlation has been first to construct a symmetrical correlation table (Gowen, 1924, 1933, 1934), and then to determine the correlation from this table according to the well-known formulae. In this study, however, we used Fisher's method of analysis of variance to accomplish the same.

It has already been mentioned that there were 88 bulls with two or more daughters. The number of families of half-sisters was therefore, 88. These 88 families came from five different herds, and the total number of paternal half-sisters included in these families was 479. Table 88 shows the results of the analysis of variance of persistency of these paternal half-sisters.

Table 88 showing the analysis of variance of persistency of paternal half-sisters.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	235.6429	58.9107	*4.877
Within herds	474	6752.2472	14.2452	
Total	478	6987.8901	14.6190	
Between families	83	2029.3223	24.4497	*2.024
Within families	391	4722.9249	12.0791	
Total	474	6987.8901		

*significant.

Portion of total variance of persistency of paternal half-sisters due to differences between

$$\text{Herds} \quad \frac{14.6190 - 14.2452}{14.6190} = 2.557\%$$

$$\text{Families} \quad \frac{14.6190 - 12.0791}{14.6190} = 17.374\%$$

Portion of intra-herd variance of paternal half-sisters due to differences between families

$$\frac{14.2452 - 12.0791}{14.2452} = 15.206\%$$

It will be observed that the mean square "between herds" is significant showing that the means for persistency of half-sisters coming from different herds differ significantly among themselves. The differences in this respect account for 2.6% of the total variance. Similarly the "intra-herd"between families" mean square is highly significant. The "intra-herd within families"mean square is lower by 17.4% and 15.2% than the "total" and "intra-herd" mean squares respectively. The former results corresponds to a correlation of 0.174 between paternal half-sisters in the lumped data for all five herds, and the latter result to a correlation of 0.152 in a population of half-sisters belonging to the same herd. The latter result also represents a correlation of 0.39 between the "sire" and daughters on an intra-herd basis, if the term "sire" is used to denote not only the sire himself, but also all other factors which are common to paternal half-sisters belonging to the same herd. It will be observed that this value is considerably higher than the dam-daughter correlation already reported. This is mainly because the correlation due to the common environment is higher between paternal half-sisters, who are generally contemporaries, than between dams and daughters, whose records for the same age are separated by a period of at least three years.

So far the combined results from all the five herds have been considered. A study of the results of individual herds, however, revealed distinct differences between the values of half-sister correlations. This is clear from Table 89, which summarises the results for individual herds. It will be observed that the mean square "between families" is significant only in herds B and C. In the other three herds it is not significant even at 5% level. In herds B and E only 6% of the total variance is due to differences between the means of families, and in herd A this value is only 10%. This indicates that the transmitting abilities of the sires of most of the daughters in these three herds, particularly in herds D and E, were almost identical so far as persistency is concerned.

(b) Maximum Yield.

(i) Influence of Sire. Table 82 gave the mean maximum yield of the mates and daughters of each of the 25 bulls who have at least five daughters. It will be observed that the means of the daughters of different bulls differ considerably, and that these differences are not altogether due to the differences between the means of mates of different bulls. Bulls bred to mates having a maximum yield below the average begot daughters whose maximum was above the average, and vice versa. This proves conclusively the important and independent rôle of the sire in determining the maximum yield of his daughters, as also the fact that there are distinct genetic differences between the different bulls in their transmitting ability with regard to maximum yield. These conclusions are further supported by the results given in Tables 90, 91, and 92.

Table 90 showing the analysis of variance of maximum of daughters of different bulls.

Source of Variation	d/f+	Sum of Squares	Mean Square	F
Between herds	4	1612.74	403.1850	*10.476
Within herds	233	11820.08	50.7299	
Total	237	13432.82	56.6786	
Between means of daughters of different bulls	20	3622.65	181.1325	* 4.707
Within means of daughters of different bulls	213	8197.43	38.4856	
Total	233	11820.08		

*significant.

Portion of total variance of daughters due to differences between

$$\text{Herds} \quad \frac{56.6786 - 50.7299}{56.6786} = 10.495\%$$

$$\text{Bulls} \quad \frac{56.6786 - 38.4856}{56.6786} = 32.098\%$$

Portion of intra-herd variance of daughters due to differences between bulls

$$\frac{50.7299 - 38.4856}{50.7299} = 24.136\%$$

It will be observed that the mean squares for "between herds" and "between sires" are both significant. However, as is clear from Table 91, there are similar significant differences between the mates of different bulls. There is a correlation of 0.168 between the different mates of a bull in a population of mates all belonging to the same herd.

Table 91 showing the analysis of variance of mates of different bulls.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	878.53	219.6325	*5.724
Within herds	233	10751.58	46.1441	
Total	237	11630.11	49.0722	
Between mates of different bulls	20	2578.35	128.9175	*3.360
Within mates of different bulls	213	8173.23	38.3720	
Total	233	10751.58		

*significant

Portion of total variance of mates due to differences between

$$\text{Herds} \quad \frac{49.0722 - 46.1441}{49.0722} = 5.967\%$$

$$\text{Bulls} \quad \frac{49.0722 - 38.3720}{49.0722} = 21.805\%$$

Portion of intra-herd variance of mates due to differences between bulls

$$\frac{46.1441 - 38.3720}{46.1441} = 16.843\%$$

We must, therefore, determine whether the observed differences between the mean performance of daughters of different bulls are explained by the differences between the mates of the different bulls, or whether they are due to the independent influence of each bull. This examination can be made by the method of analysis of covariance after adjusting the means of daughters of different bulls for differences with respect to the mates. Table 92 gives the results of such an examinatiom.

The highly significant value of F leads to the conclusion that the differences between the mates do not explain the differences between the bulls; after these differences have been adjusted to a common basis so far as the yield of mates is concerned, they still differ significantly. It is therefore clear that the differences between the daughters are due to the sires.

The "index" of maximum yield for each sire is given in the last column of Table 82. A comparison of these values with those already reported for persistency shows that the two characters are quite independent.

(ii) Correlation between paternal half-sisters. As with persistency, this was studied from the total data by the method of analysis of variance. The results are given in Table 93.

Table 93 showing the analysis of variance of maximum of paternal half-sisters.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	2620.60	655.1500	*18.748
Within herds	474	22529.48	47.5305	
Total	478	25150.08	52.6152	
Between families	83	8865.70	106.8157	* 3.057
Within families	391	13663.78	34.9457	
Total	474	22529.48		

*significant.

Portion of total variance of half-sisters due to differences between

$$\text{Herds} \quad \frac{52.6152 - 47.5305}{52.6152} = 9.664\%$$

$$\text{Families} \quad \frac{52.6152 - 34.9457}{52.6152} = 33.582\%$$

Portion of intra-herd variance of half-sisters due to differences between

$$\text{Families} \quad \frac{47.5305 .- 34.9457}{47.5305} = 26.477\% .$$

It will be observed that the mean squares both "between herds" and "between families" are highly significant. The herd differences account for 9.7% of the total variance of half-sisters. The mean square "between families" is 33.6% lower than the gross mean square and 26.5% lower than the intra-herd mean square. This indicates that there is a correlation of 0,336 between paternal half-sisters in the population of half-sisters in the lumped data for all five herds, and a correlation of 0.265 between half-sisters in the population of half-sisters all belonging to the same herd. The latter correlation corresponds to the sire-daughter correlation of 0.51, which is considerably higher than the value reported for dam and daughter in the last section. This higher value is due to two causes: (i) the influence of assortive mating, as indicated by the correlation of 0.1684 between the mates of the same bulls, and (ii) the fact that the correlation due to the common environment is higher between paternal half-sisters than between dam and daughter.

Table 94 gives the results for individual herds. It will be observed that the mean square for herds A and B are significant, whereas those for herds C, D and E are not significant, even at 5% level. The

The portions of total variance of paternal half-sisters which are due to differences between families are about 8, 5 and 6% respectively in the latter herds. This shows that, so far as maximum yield is concerned, there is little genetic variability between the various bulls used in these herds. On the other hand, the correlation between half-sisters is too high in herds A and B, where $r = 0.5446$ and 0.3984 respectively, instead of 0.25 which is the value of the genotypic correlation between half-sibs. This points to the large influence of common environment and assortive mating in determining these correlations.

(c) Total Yield.

(i) Influence of Sire. The mean total yield of the mates and daughters of each of the 25 bulls having five or more daughters has been given in Table 84. It will be observed that the means of both daughters and mates of different bulls differ considerably. However, the daughter differences are not entirely due to the differences between the mates. Bulls bred to mates having total yields above the average begot daughters whose total yield was below the average, and vice versa. This again proves the important and independent rôle of the sire in determining the total yield of his daughters. It also shows that there exist definite hereditary differences between the various bulls and their transmitting abilities with regard to total yield. These conclusions are further

supported by the results given in Tables 95, 96, and 97.

Table 95 showing the analysis of variance of total yield of daughters of different bulls.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	55162614	13790653	*9.214
Within herds	233	456554922	1959463	
Total	237	511717536	2159146	
Between means of daughters of different bulls	20	137772151	6888625	*3.516
Within means of daughters of different bulls	213	318782771	1496633	
Total	233	456554922	1959463	

*significant

Portion of total variance of daughters due to differences between

$$\text{Herds} \quad \frac{2159146 - 1959463}{2159146} = 9.248\%$$

$$\text{Bulls} \quad \frac{2159146 - 1496633}{2159146} = 30.684\%$$

Portion of intra-herd variance of daughters due to differences between bulls

$$\frac{1959463 - 1496633}{1959463} = 23.620\%$$

It will be observed that the means squares "between herds" and "between sires" are both significant. The differences between herds account for about 9% of the total variance of the daughters, whereas the differences between bulls account for about 24%. As will be clear from Table 96, however, there are also significant differences between the production of the mates of different bulls. This assortive mating is sufficient to cause a correlation of 0.154 between the different mates of a bull on an intra-herd basis.

Table 96 showing the analysis of variance of total yield of mates of different bulls.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	32516107	8129027	*5.167
Within herds	233	433049549	1858582	
Total	237	465565656	1964412	
Between means of mates of different bulls	20	97979862	4898993	*3.114
Within means of mates of different bulls	213	335069687	1573097	
Total	233	433049549		

*significant

Portion of total variance of total yield of half-sisters due to differences between

$$\text{Herds}\quad \frac{1964412 - 1858582}{1964412} = 5.387\%$$

$$\text{Bulls}\quad \frac{1964412 - 1573097}{1964412} = 19.921\%$$

Portion of intra-herd variance of total yield of half-sisters due to differences between bulls

$$\frac{1858582 - 1572097}{1858582} = 15.36\%$$

That the observed differences between the daughters of different bulls are not entirely due to the differences of mates of different bulls is shown in Table 97, which gives the results of the analysis of covariance after adjusting the means of daughters of different bulls for the differences with respect to the mates. It will be noted that the value of F is highly significant. This proves the independent influence of the sire in determining the production of his daughters.

The last column of Table 84 gives the "index" of each sire for total yield. It will be seen that these indexes have a large range, the lowest value being 5858 lbs and the highest 12,170 lbs. This demonstrates the great differences between the transmitting abilities of the various bulls used in the herds under study.

(ii) Correlation between paternal half-sisters. The results of the analysis of variance of total yield of paternal half-sisters derived from the total data are given in Table 98.

Table 98 showing the analysis of variance of total yield of paternal half-sisters.

Source of Variation	d/f	Sum of Squares	Mean Square	F
Between herds	4	89531539	22382885	*13.772
Within herds	474	951814288	2008047	
Total	478	1041345827	2178548	
Between families	83	316350886	3811456	* 2.345
Within families	391	635463402	1625226	
Total	474	951814288		

*significant.

Portion of total variance of paternal half-sisters due to differences between

$$\text{Herds} \quad \frac{2178548 - 2008047}{2178548} = 7.826$$

$$\text{Families} \quad \frac{2178548 - 1625226}{2178548} = 25.399\%$$

Portion of intra-herd variance of paternal half-sisters due to differences between families

$$\frac{2008047 - 1625226}{2008047} = 19.064\%$$

It will be noted that both the mean squares "between herds" and "between families" are highly significant. The herd differences account for 7.8% of the total variance, and the family differences for roughly 25.4% of the total variance and 19.1% of the intra-herd variance. These latter results correspond to a correlation between paternal half-sisters of 0.254 in the lumped data for all five herds, and of 0.191 on intra-herd basis. The intra-herd correlation gives the value 0.43 for sire-daughter correlation which, for the reasons already explained in the case of persistency and maximum, is again considerably higher than the daughter-dam correlation. Plum (1933) studying the data of milk yield of a large Jersey herd observed a correlation of 0.48 between daughters and sire.

The results for individual herds are given in Table 99. It will be observed that the only significant mean squares are for herds A and B. The means square for herd E is not significant at 5% level, whereas for herds C and D the mean square "within families" is higher than that "between families". This indicates that in these two herds paternal half-sisters varied slightly more than unrelated cows, i.e. that the individual sires did not cause any differentiation of their progeny into distinct families each with a different level of production. The "sire" effect, is however, very strong in herds A and B, the correlation between the paternal half-sisters being 0.526 and 0.300 respectively in these herds. This, as already pointed out in the case of maximum yield, is due to assortive mating and the influence of environment.

(d) Discussion.

Under conditions of random mating with no dominance and complete determination by heredity, the correlation between half-sisters (r_{oo}) may be represented as

$$r_{oo} = \tfrac{1}{4}h^2$$

If, however, a part of the resemblance between half-sisters is due to the influence of common environment, then

$$r_{oo} = \tfrac{1}{4}h^2 + e^2$$

Wright (1921b) has evolved alternative formulae to allow for the influence of dominance and assortive mating, if present. Gowen (1933, 1934) has quoted and employed these formulae in his studies. For this purpose Gowen (1934) assumed (i) that the observed value of the dam-daughter correlation was entirely the result of genetic resemblance, and (ii) that the effect of the common environment was the same with full sisters as with half-sisters. From these assumptions and from the observed values of correlation between sire and dam, dam and daughter, full-sisters and half-sisters, Gowen was able to separate approximately the part of variance which was due to hereditary causes from that which was due to environmental causes common to both the full and half-sisters.

The results reported in the last section definitely prove that the first assumption does not apply to these data. Similarly there seems little justification for the second assumption. Paternal half-sisters are generally contemporaries, and therefore the correlation between them is influenced to a greater degree by environment than the correlation between maternal half-sisters or full sisters.

Since we do not know the exact value of h^2 as determined from parent-offspring correlation, it is not possible to separate from the observed correlation between half-sisters, that part of their resemblance which is due to the effect of common environment. However, from the results now reported it is quite clear that definite genetic variability does exist between the bulls used in the different herds, and, when each herd is considered individually, between the bulls used in herds A and B in regard to maximum and total yield and in herds B and C in regard to persistency.

GENERAL SUMMARY.

1. A review of the existing knowledge regarding the inheritance of milk yield indicated the need for a more extensive investigation of the subject, particularly with reference to (i) the inheritance of the various components of the lactation yield, and (ii) the differentiation of hereditary from environmental variations.

2. In the present thesis an account is given of such an investigation, the requisite data being obtained from six prominent herds of tuberculin tested pedigree Ayrshire cattle. The Official milk records of all the recorded cows in these herds were used as a basis of study, (i.e. the investigation was not limited to published records and therefore to selected data). The available data comprised a total of over 2,800 lactation period milk records. The portion of the lactation curve studied was between the period 15 days to eight and a half months after calving and the milk production of each record in successive 30-day intervals over this period was computed.

3. The first step was to find a satisfactory mathematical expression to describe the shape of the lactation curve. After reviewing the previous work, the use of the exponential curve was decided upon. As a result of preliminary work, however, it was discovered that, although this curve gave a good fit

with some lactations, the fit was extremely poor for others. The parabolic exponential curve was next tried and it was found that with most cows the addition of the extra constant considerably improved the fit of the exponential curve. As a result of this work it became clear that, to describe the shape of individual curves satisfactorily, two constants were required: (i) the linear (exponential), which measures the average slope of the curve, and (ii) the parabolic, which describes how on an average the rate of this slope varies from period to period. With most lactations the former term was, however, found to have a preponderating influence in graduating the time-change of milk yield, and in view of this fact and of the very high variability exhibited by the parabolic term, the latter was not included in the subsequent work (except in the study of the effect of time of calving).

4. Because of the poor fit of the exponential curve in some lactations, the maximum yield as given by the fitted theoretical curve was very different from the observed maximum. It was, therefore, decided to use the latter instead of the theoretical value to measure the maximum yield of any lactation.

5. A special study was made of two environmental factors which are known to influence milk production and on which quantitative data were available, namely month of calving and age.

6. Two methods were adopted in studying the

influence of month of calving, one of which was based on the lumped data for all lactations and the other confined to intra-cow comparisons. The results obtained by the two methods are discussed in the text and an explanation offered of the observed variations. Correction factors were worked out for persistency, maximum yield and total yield.

7. The influence of age was next investigated from data corrected for the effect of month of calving, two alternative methods again being employed. The influence of selection was studied and allowed for in working out the correction factors for persistency, maximum yield, and total yield. A study was also made of the correlation between the production of a cow at different ages. The bearing of these latter results on the question of predictability of the future performance of a cow, the value of each production record as a measure of a cow's genetic constitution, and the method of standardisation of records for the influence of age was indicated.

8. The predictability of the performance of a cow in a future lactation on the basis of her production in all the earlier lactations was studied by the method of multiple regression. It was found that the performance during the lactation immediately preceding that to be predicted gives nearly as good an estimate of the probable production as the yield

for all the previous lactations.

9. The correlation between persistency, maximum yield and total yield was determined from data standardised for the influence of age. It was found that, though there existed a slight negative correlation between persistency and maximum yield on an intra-cow basis, the persistency was independent of the maximum yield when the means of different cows were correlated. The value of this result in practical breeding was indicated.

The relative rôle of maximum yield and persistency in determining the variations of total yield both "between cows" and "within cows" was investigated.

10. In studying the observed variations of the standardised data, special attention was devoted to a determination of the relative parts played by heredity and environment. The portions of total variance due to differences between years, herds and cows were determined and the relative stability of the different constants of the lactation curve investigated. Different methods of measuring the heritability of a particular character were reviewed, and it was shown that in the case of farm animals the only practical course was the use of the method of correlation between animals related by a common line of descent. The major pitfalls in the interpretation of results obtained by this method were indicated. The parent-offspring correlations and the correlation between

paternal half-sisters were reported, and it was shown that the genetic differences (in the broad sense) between the animals studied could not account for more than 40-45% of the observed variance of persistency, maximum yield or total yield. in the "lumped" data.

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Bound Volume of large Tables and
Diagrams to accompany Thesis

submitted by

LALL CHAND SIKKA, B.Sc., (Agric.).

entitled

"A STUDY OF LACTATION AS AFFECTED BY HEREDITY AND ENVIRONMENT".

April, 1940.

[redacted]

Table I. showing No. of Records for each herd and the Year to which these Records belonged.

Herd	Before 1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	Total
A	45	13	18	22	24	26	25	29	29	27	24	27	23	26	23	24	26	26	28		485
B				1	17	25	32	32	28	35	30	33	31	44	31	36	37	31	34	1	478
C	3	1	2	2	3	42	61	55	45	47	65	64	56	62	62	67	71	61	12		781
D										4	18	25	30	31	32	38	36	35	25		274
E					3	11	26	25	43	39	30	51	45	46	54	49	62	50	33		567
F	12	8	10	12	19	16	20	17	20	20	15	18	20	16	8	5	4	3			243
Total	60	22	30	37	66	120	164	158	165	172	182	218	205	225	210	219	236	206	132	1	2828

[redacted]

Table 2.

Name	Tattoo Markings	Herd Book No.	Born March 1919
Sire	Herd Book No.	Dam	Herd Book No.
Date of Calving 8/3/23.	No. of Lactation 2nd	Date of Last Service 4/6/23	Service Period 88 days
Date of Last Calving 3/10/21.	Date of Drying Last Lactation 20/12/22	Dry Period Before Current Lactation 78 days	Sex of Calf Heifer.
No. of Times Milked. Twice			

Year 1923 Dates	Milk in lbs Daily	Gallons of Milk for days	Per cent of Fat	Lbs. of Butter Fat for ....days	No. of Days	MILK RECORD BY PERIODS.								
						No. of Period	No. of Days	Daily Milk Yield lbs.	Total Milk Yield lbs.	Per cent of Fat	Fat Yield lbs.	Total Milk Yield lbs.	Total Fat Yield lbs.	Average Daily Milk Yield for each Period.
18/3	47.5	95	3.6	342	20		15	47.5	712.5	3.6	25.6	712.5	25.6	
5/4	40.5	73	4.1	299	18									
21/4	41.5	66	3.6	238	16	1	5	47.5	237.5	3.6	8.6			
/5	39.5	67	4.0	268	17		18	40.5	729.0	4.1	29.9			
28/5	46.0	92	3.6	331	20		7	41.5	290.5	3.6	10.4	1257.0	48.9	41.9
16/6	42.5	81	3.8	308	19	2	9	41.5	373.5	3.6	13.3			
8/7	37.0	81	4.1	332	22		17	39.5	671.5	4.0	26.8			
27/7	32.0	61	4.1	250	19		4	46.0	184.0	3.6	6.5	1229.0	46.6	41.0
17/8	31.0	65	3.9	254	21	3	16	46.0	736.0	3.6	26.6			
4/9	24.5	44	4.3	189	18		14	42.5	595.0	3.8	22.8	1331.0	49.4	44.4
27/9	20.5	47	5.0	235	23	4	5	42.5	212.5	3.8	8.0			
15/10	16.5	30	4.8	144	18		22	37.0	814.0	4.1	33.2			
3/11	14.0	27	5.0	135	19		3	32.0	96.0	4.1	4.1	1122.5	45.3	37.4
22/11	18.5	35	4.6	161	19	5	16	32.0	512.0	4.1	20.9			
8/12	15.5	25	4.4	110	16		14	31.0	434.0	3.9	16.8	946.0	37.7	31.5
22/12	14.0	20	4.8	96	14	6	7	31.0	217.0	3.9	8.6			
							18	24.5	441.0	4.3	18.9			
							5	20.5	102.5	5.0	5.0	760.5	32.5	25.3
						7	18	20.5	369.0	5.0	18.5			
							12	16.5	198.0	4.8	9.6	567.0	28.1	18.9
						8	6	16.5	99.0	4.8	4.8			
							19	14.0	266.0	5.0	13.5			
							5	18.5	92.5	4.6	4.1	457.5	22.4	15.2
						9	14	18.5	259.0	4.6	12.0	(*7670.5)		
							16	15.5	248.0	4.4	11.0	507.0	23.0	
						10	14	14.0	196.0	4.8	9.6	196.0	9.6	
Total		909	4.07	3692	299		299		9086.0		369 .1	9086.0	369. 1	

*Total for eight periods.

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Table 3 showing fit of exponential curve.

	1.		2.		3.		4.		5.	
Month of Lactation	Observed Yield	Estimated Yield	Observed Yield	Estimated Yield	Observed Yield	Estimated Yield	Observed Yield	Estimated Yield	Observed Yield	Estimated Yield
1st	38.8	39.9	51.2	71.8	44.4	54.2	37.5	31.6	41.98	45.55
2nd	32.5	34.6	51.9	56.0	50.3	47.5	32.2	29.4	41.14	40.49
3rd	32.8	30.0	52.4	43.6	47.7	41.7	25.1	27.3	37.57	35.95
4th	28.3	26.0	42.3	34.0	40.1	36.6	23.2	25,4	33.35	32.00
5th	21.5	22.5	34.7	26.5	30.3	32.1	20.6	23.6	29.33	28.45
6th	19.4	19.5	22.4	20.6	31.3	28.2	18.8	22.0	25.77	25.29
7th	16.1	16.9	16.8	16.1	24.5	24.7	20.1	20.5	22.39	22.48
8th	14.1	14.6	9.5	12.5	19.2	21.7	18.5	19.0	18.85	20.03
9th	13.3	12.7					21.7	17.7		
Σd^2	21.01		661.27		171.27		92.10		19.42	
Equation of fitted curve	$46.01e^{-.14314t}$		$92.15e^{-.24949t}$		$61.82e^{-.13106t}$		$33.98e^{-.07245t}$		$51.24e^{-.11771t}$	

Table 4 comparing the fit of the Exponential and Parabolic Exponential Curves.

	1.			2.			3.			4.			5.		
Month	Observed Yield	Estimated Yield		Observed Yield	Estimated Yield		Observed Yield	Estimated Yield		Observed Yield	Estimated Yield		Observed Yield	Estimated Yield	
		Exponential	parabolic		Exponential	parabolic		Exponential	parabolic		Exponential	parabolic		Exponential	parabolic
1	51.2	71.8	50.6	55.9	75.9	56.5	37.5	31.6	38.3	32.0	27.8	29.9	33.4	38.3	36.2
2	51.9	56.0	53.2	65.4	62.1	59.5	32.2	29.4	30.8	24.9	25.4	25.6	40.9	37.1	36.8
3	52.4	43.6	50.6	49.6	50.7	57.6	25.1	27.3	25.9	20.9	23.2	22.5	39.2	35.9	36.8
4	42.3	34.0	43.6	53.0	41.5	51.2	23.2	25.4	22.6	18.8	21.1	20.1	34.4	34.8	36.2
5	34.7	26.5	33.9	43.0	33.9	41.9	20.6	23.6	20.6	18.4	19.3	18.3	34.0	33.7	35.1
6	22.4	20.5	23.9	31.9	27.7	31.4	19.8	22.0	19.6	19.5	17.6	17.1	30.4	32.7	33.5
7	16.8	16.1	15.3	21.9	22.6	21.7	20.1	20.5	19.4	17.2	16.1	16.2	35.2	31.7	31.4
8	8.5	12.5	8.3	13.5	18.5	13.8	19.5	19.0	20.0	14.4	14.7	15.8	28.4	30.7	29.0
9	-	-	-	-	-	-	21.7	17.7	21.7	-	-	-	-	-	-
Σd^2		661.3	12.2		670.3	104.02		92.19	5.0		34.2	20.2		72.4	51.4

	1.	2.	3.	4.	5.
Total sum of squares	.5749685	.3856327	.09121	.0788819	.0189474
Sum of squares accounted for by linear term	.4930738	.3224912	.0594110	.0660121	.0079366
" " "parabolic "	.0784648	.0566964	.0248721	.0033488	.0019354
" " "Residual "	.0034299	.0064451	.0069306	.0095210	.0090754
Mean Square "	.000686	.001289	.001155	.001904	.001815
Variance Ratio - Linear term..	*718.8	*250.19	*51.7	*34.670	4.37
Variance Ratio parabolic term.	*114.4	* 43.984	*21.5	1.76	1.07

*Significant.

Table 5 showing the statistical constants of the "Raw" data.

	Total milk yield	Maximum Yield	Persistency	Parabolic Constant
Mean	8133.6±35.23	46.28±0.201	90.08±0.101	-0.00370±0.00014
Standard Deivation	1723.0±24.91	9.84±0.142	4.95±0.072	0.00584±0.00010
Coefficient of variation	21.18±0.325	21.27±0.319	5.45±0.08	157.84 ±6.66
μ_2	47.4999392	96.9130118	24.5427581	136.4357587
μ_3	+135.8747580	+333.4199274	-50.2975554	-730.3805169
μ_4	+7708.2970062	+29328.5318858	+1908.4340773	69316.3835376
$\beta 1$	0.1722655	0.1221341	0.1711290	0.2100457
$\beta 2$	3.4164285	3.1226704	3.1683301	3.7237411
$\gamma_1 = \sqrt{\beta_1}$	+0.4150488	+0.3494767	-0.4136774	-0.4583074
$\gamma_2 = (\beta_2 - 3)$	+0.4164285	+0.1226704	+0.1683301	+0.7237411
Skewness	0.1889134	0.1819329	0.2194112	0.184337
Mode	7808.1	44.993	91.170	0.00478
k	+0.42664	-0.77982	-0.75747	+0.20361
Type of curve	4	1	1	4

[redacted]

Table 6 showing the Variation of Month of Calving with Age.

Month of Calving	Age in Lactations													Total No. of cows	Percentage of Total number	Mean age in Lactations
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th			
January	67	64	52	47	19	13	7	2	2	2	1	1	-	277	11.58	3.03
February	50	74	55	43	30	17	9	11	3	4	1	-	-	297	12.42	3.42
March	39	99	99	70	70	34	23	14	6	-	2	1	-	437	18.27	3.84
April	24	76	68	56	32	30	19	16	7	7	3	3	-	341	14.25	4.13
May	16	23	25	15	22	12	9	5	7	4	3	-	-	140	5.85	4.36
June	11	9	4	3	5	8	5	4	1	2	1	-	1	53	2.22	4.60
July	3	5	7	4	1	2	2	2	3	-	-	1	-	30	1.25	4.53
August	7	10	8	7	6	6	5	1	1	1	1	-	-	53	2.22	4.11
September	72	22	17	10	11	3	4	1	1	1	1	-	-	143	5.98	2.42
October	111	49	25	10	3	6	4	4	5	3	1	-	-	221	9.24	2.36
November	90	53	21	10	7	12	6	3	4	2	1	1	-	209	8.74	2.76
December	54	60	32	12	15	8	6	3	-	1	-	-	-	191	7.98	2.71
Total	534	533	403	290	221	151	98	66	40	27	15	7	1	2392	100.000	3.39

Table 7 showing the variation of the different constants of the lactation curve with month of calving.

Month of Calving	Persistency %	Parabolic Constant	Maximum Yield lbs	Total Yield lbs
January	90.85 ±0.265	-0.00338 ±0.00030	45.76 ±0.485	8323.5 ± 90.5
February	89.43 ±0.284	-0.00607 ±0.00036	45.03 ±0.443	7843.9 ± 90.9
March	88.88 ±0.216	-0.00757 ±0.00032	45.94 ±0.401	7940.2 ± 78.8
April	88.16 ±0.239	-0.00616 ±0.00036	46.80 ±0.400	7800.9 ± 79.4
May	87.78 ±0.372	-0.00301 ±0.00048	48.34 ±0.753	7837.5 ±138.1
June	88.20 ±0.570	-0.00161 ±0.00077	48.63 ±1.233	7775.9 ±216.5
July	90.23 ±0.918	-0.00163 ±0.00080	44.23 ±0.986	7525.0 ±214.9
August	91.58 ±0.638	+0.00036 ±0.00096	43.12 ±0.882	7634.4 ±194.3
September	91.30 ±0.323	+0.00069 ±0.00043	45.13 ±0.646	7836.5 ±127.5
October	91.64 ±0.269	+0.00102 ±0.00037	45.79 ±0.532	8119.4 ±108.3
November	91.97 ±0.267	+0.00033 ±0.00039	45.76 ±0.557	8224.3 ±108.7
December	91.11 ±0.260	-0.00124 ±0.00039	46.02 ±0.611	8176.1 ±122.4
Mean of all months	89.92 ±0.092	-0.00370 ±0.00014	45.98 ±0.164	7964.3 ± 32.2

Table 8 comparing the persistency (corrected for age) of the same cow when calving in different months.

	Comparison with January		Comparison with February		Comparison with March		Comparison with April		Comparison with May		Comparison with June		Comparison with July		Comparison with August		Comparison with September		Comparison with October		Comparison with November	
	Mean when calving month is January	Mean when calving month is as on the left	Mean when calving month is February	Mean when calving month is as on the left	Mean when calving month is March	Mean when calving month is as on the left	Mean when calving month is April	Mean when calving month is as on the left	Mean when calving month is May	Mean when calving month is as on the left	Mean when calving month is June	Mean when calving month is as on the left	Mean when calving month is July	Mean when calving month is as on the left	Mean when calving month is August	Mean when calving month is+ as on the left	Mean when calving month is+ September	Mean when calving month is as on the left	Mean when calving month is October	Mean when calving month is as on the left	Mean when calving month is November	Mean when calving month is as on the left
February	90.999	90.656	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
March	91.435	89.318	89.517	88.927	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
April	91.650	88.180	89.984	88.658	89.630	88.789	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
May	92.375	88.771	90.414	87.586	90.192	88.933	88.433	88.088	..	..	..	..	..	..	..	..	..	..	..	..	..	..
June	91.273	88.664	89.212	88.181	89.940	87.320	88.826	88.672	87.552	87.651	..	..	..	..	..	..	..	..	..	..	..	..
July	92.173	89.447	91.090	88.440	90.223	91.457	87.986	90.510	90.259	89.885	90.675	93.228	..	..	..	..	..	..	..	..	..	..
August	92.240	93.432	90.718	94.930	89.787	94.801	86.841	94.290	89.180	94.557	91.072	90.932	90.084	90.296	..	..	..	..	..	..	..	..
September	91.319	90.980	90.124	91.221	88.629	91.016	88.017	92.813	88.316	92.737	91.690	91.370	90.416	91.381	92.063	92.387	..	..	..	..	..	..
October	91.492	92.270	89.912	92.192	89.039	92.553	86.688	92.373	89.008	91.554	89.711	93.541	94.236	93.316	91.065	92.450	91.820	91.858	..	..	..	..
November	91.265	91.506	89.350	92.135	89.354	92.461	87.770	93.581	89.899	93.631	90.053	91.725	90.632	92.684	91.736	95.650	91.067	92.714	91.987	92.567	..	..
December	92.152	91.310	88.864	90.802	89.164	90.906	88.976	91.368	89.397	91.599	89.471	90.439	87.970	91.220	90.404	90.584	91.885	91.973	92.557	91.832	92.374	92.288

Table 9 comparing the Persistency for different months of Calving.

(Value of January calvers = 100.00)

Month of Calving	January	February	March	April	May	June	July	August	September	October	November	December	Weighted Mean	Comparative value by 1st Method
January	100.000	..	..	..	..	..	..	..	..	..	..	..	100.000	100.000
February	99.623	99.623	..	..	..	..	..	..	..	..	..	..	99.623	98.437
March	97.684	98.965	97.684	..	..	..	..	..	..	..	..	..	98.443	97.832
April	96.213	98.155	96.767	96.213	..	..	..	..	..	..	..	..	97.006	97.039
May	96.098	96.507	96.320	95.837	96.098	..	..	..	..	..	..	..	96.112	96.621
June	97.141	98.471	94.837	96.046	96.207	97.141	..	..	..	..	..	..	96.457	97.083
July	97.042	96.724	99.019	98.972	95.699	99.876	97.042	..	..	..	..	..	97.628	99.318
August	101.292	104.247	103.139	104.465	101.892	96.991	97.270	101.292	..	..	..	..	102.182	100.804
September	99.628	100.835	100.315	101.455	100.908	96.801	98.077	101.648	99.628	..	..	..	100.490	100.495
October	100.850	102.148	101.539	102.523	98.846	101.288	96.094	102.832	99.669	100.850	..	..	100.963	100.870
November	100.264	102.727	101.080	102.582	100.087	98.944	99.239	105.613	101.429	101.485	100.264	..	101.497	101.233
December	99.086	101.795	99.592	98.799	98.465	98.191	100.627	101.494	99.723	100.059	100.170	99.086	99.832	100.286

Table *10* comparing the Maximum Yield (corrected for age) of the same cow when calving in different months.

	Comparison with January		Comparison with February		Comparison with March		Comparison with April		Comparison with May		Comparison with June		Comparison with July		Comparison with August		Comparison with September		Comparison with October		Comparison with November 6	
	Mean when calving month is January	Mean when calving month is as on the left	Mean when calving month is February	Mean when calving month is as on the left	Mean when calving month is March	Mean when calving month is as on the left	Mean when calving month is April	Mean when calving month is as on the left	Mean when calving month is May	Mean when calving month is as on the left	Mean when calving month is June	Mean when calving month is as on the left	Mean when calving month is July	Mean when calving month is as on the left	Mean when calving month is August	Mean when calving month is as on the left	Mean when calving month is September	Mean when calving month is as on the left	Mean when calving month is October	Mean when calving month is as of the left	Mean when calving month is November	Mean when calving month is as on the left
February	45.12	44.74	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
March	44.58	46.04	45.28	45.52	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
April	44.02	46.57	44.81	44.79	45.65	46.26	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
May	45.38	47.33	46.43	46.65	46.82	47.47	46.47	48.01	..	..	..	..	..	..	..	..	..	..	..	..	..	..
June	45.72	50.71	48.23	47.93	46.16	47.63	44.98	47.47	46.54	47.29	..	..	..	..	..	..	..	..	..	..	..+	..
July	47.93	45.86	47.64	44.73	50.35	41.40	48.40	47.12	44.98	45.17	47.48	45.18	..	..	..	..	..	..	..	..	..	..
August	44.95	42.46	43.95	43.38	46.28	43.10	47.49	43.61	52.53	43.08	57.20	46.00	51.78	51.76	..	..	..	..	..	..	..	..
September	48.68	45.19	43.19	47.32	47.80	45.92	50.26	46.75	45.69	42.69	~~54.20~~	48.62	42.38	46.80	42.52	44.77	..	..	..	..	..	..
October	47.44	46.37	47.57	47.87	47.19	47.21	49.61	46.62	50.26	47.44	47.77	47.16	38.76	43.05	41.40	43.14	43.22	45.14	..	..	..	..
November	46.69	46.02	46.72	46.82	48.60	47.98	49.71	46.83	47.64	44.58	48.15	48.04	48.16	40.36	40.87	41.44	43.92	45.36	48.61	46.28	..	..
December	46.89	46.41	46.38	46.09	46.22	45.89	47.89	43.94	47.12	46.46	49.11	47.80	48.98	50.81	43.21	45.13	45.77	50.53	48.59	48.24	47.02	45.93

Table 11 comparing Maximum Yields for different Months of Calving.

(Value of January Calvers = 100.00)

Month of calving	January	February	March	April	May	June	July	August	September	October	November	December	Weighted Mean	Comparative value by 1st Method
January	100.000	..	..	..	..	..	..	..	..	..	..	..	100.000	100.000
February	99.157	99.157	..	..	..	..	..	..	..	..	..	..	99.157	98.405
March	103.275	99.683	103.275	,.	..	..	..	..	..	..	..	..	101.104	100.393
April	105.792	99.112	104.655	105.792	..	..	..	..	..	..	..	..	103.414	102.273
May	104.297	99.626	104.708	109.297	104.297	..	..	..	..	..	..	..	105.627	105.638
June	110.914	98.539	106.563	111.648	105.977	110.914	..	..	..	..	..	..	106.787	106.272
July	95.681	93.099	84.917	102.994	104.737	105.540	95.681	..	..	..	..	..	100.104	96.656
August	94.660	97.871	96.178	97.148	85.534	89.196	95.644	94.660	..	..	..	..	95.354	94.231
September	92.830	108.638	99.212	98.403	97.449	99.494	105.660	99.668	92.830	..	..	..	99.799	98.623
October	97.744	99.782	103.318	99.415	98.445	109.498	106.271	98.638	96.954	97.744	..	..	99.958	100.066
November	98.565	99.369	101.957	99.662	97.597	110.660	80.183	95.980	95.873	99.179	98.565	..	98.778	100.000
December	98.976	98.536	102.538	97.065	102.836	107.955	99.256	98.866	102.483	97.039	96.279	98.976	99.284	100.568

Table 12 comparing the Total Yield (corrected for age) of the same cow when calving in different months.

	Comparison with January		Comparison with February		Comparison with March		Comparison with April		Comparison with May		Comparison with June		Comparison with July		Comparison with August		Comparison with September		Comparison with October		Comparison with November	
	Mean when calving month is January	Mean when calving month is as on the left	Mean when calving month is February	Mean when calving month is as on the left	Mean when calving month is March	Mean when calving month is as on the left	Mean when calving month is April	Mean when calving month is as on the+ left	Mean when calving month is May	Mean when calving month is as on the left	Mean when calving month is June	Mean when calving month is as on the left	Mean when calving month is July	Mean when calving month is as on the left	Mean when calving month is August	Mean when calving month is as on the left	Mean when calving month is September	Mean when calving month is as on the left	Mean when calving month is October	Mean when calving month is+ as on the left+	Mean when calving month is November	Mean when calving month is as on the left
February	8117	7951	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••
March	8080	8024	7941	7814	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••	••
April	7975	7813	7919	7527	7929	7763	••	••	••	••	••	••	••••	••	••	••	••	••	••	••	••	••
May	8288	7853	8264	7657	8303	8010	7766	7718	••	••	••	••	••	••	••	••	••	••	••	••	••	••
June	8351	8204	8267	7731	8310	7555	7744	7693	7424	7513	••	••	••	••	••	••	••	••	••	••	••	••
July	8902	7829	8507	7377	9237	7581	8176	8084	7681	7589	8321	8192	••	••	••	••	••	••	••	••	••	••
August	8484	7696	7980	8253	8202	8158	7892	8207	8473	8421	9666	8031	7848	7679	••	••	••	••	••	••	••	••
September	8817	7711	7784	7934	8183	7777	8237	8326	7188	7742	9198	8245	7228	8033	7471	7738	••	••	••	••	••	••
October	8525	8470	8504	8629	8233	8626	8085	8539	8507	8317	7868	8584	7250	8297	7173	7862	7476	8005	••	••	••	••
November	8373	8204	8257	8605	8428	8801	8113	8823	8272	8339	8153	8315	7202	7469	7219	8045	7736	8437	8101	8330	••	••
December	8485	8312	8135	8299	8066	8211	8123	7998	8338	8680	8271	7907	8016	8724	7445	8073	8055	9095	8841	8619	8541	8337

Table 13 comparing Total Yield for different Months of Calving.

(Value of January calvers = 100.00)

Month of Calving	January	February	March	April	May	June	July	August	September	October	November	December	Weighted Mean	Comparative value by 1st Method
January	100.000	..	..	..	..	..	..	..	..	..	..	..	100.000	100.000
February	97.954	97.954	..	..	..	..	..	..	..	..	..	..	97.954	94.237
March	99.306	96.387	99.306	..	..	..	..	..	..	,,	..	..	97.435	95.395
April	97.968	93.104	97.227	97.968	..	..	..	..	..	..	..	..	96.280	93.721
May	94.751	90.758	95.801	97.362	94.451	..	..	..	..	..	..	..	95.264	94.161
June	98.239	91.603	90.283	97.322	95.886	98.239	..	..	..	..	..	..	94.597	93.421
July	87.946	84.942	81.502	96.865	93.616	96.715	87.946	..	..	..	..	..	91.040	90.407
August	90.711	101.305	98.773	101.878	94.169	81.622	86.052	90.711	..	..	..	..	95.763	91.721
September	87.456	99.842	94.378	99.026	102.053	88.060	97.741	93.952	87.456	..	..	..	95.265	94.149
October	99.354	99.393	104.046	103.469	92.634	107.179	100.646	99.424	93.644	99.354	..	..	99.436	97.548
November	97.981	102.082	103.700	106.541	95.518	100.190	91.206	101.090	95.380	102.162	97.981	..	100.611	98.808
December	97.961	99.928	101.091	96.460	98.637	93.915	95.713	98.362	98.747	96.868	95.640	97.961	98.083	98.229

Table 16 showing the variation of Persistency with Age.

Age in Lactations	No. of Animals	Mean	Standard Deviation	Coefficient of Variation
1	534	94.31±0.134	3.09±0.095	3.28±0.100
2	533	89.48±0.191	4.41±0.135	4.93±0.151
3	403	89.35±0.231	4.64±0.163	5.19±0.183
4	296	89.78±0.261	4.50±0.185	5.01±0.206
5	220	89.81±0.301	4.47±0.213	4.98±0.238
6	151	90.07±0.324	3.98±0.229	4.41±0.254
7	98	90.66±0.379	3.75±0.268	4.14±0.296
8	66	90.73±0.434	3.52±0.307	3.88±0.339
9	40	91.07±0.568	3.59±0.402	3.95±0.442
10	27	91.06±0.879	4.57±0.622	5.02±0.684
11	15	91.63±0.970	3.76±0,686	4.10±0.750
12	7	91.21±1.523	4.03±1.077	4.42±1.183
13	1	91.50	-	-

Table 17 showing variation of Mean Persistency (Percentage) with age.

Age in Lactations.

Age in Lactations.		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
	1st	..	89.43 ±0.199	89.26 ±0.246	89.66 ±0.275	89.66 ±0.329	90.13 ±0.389	91.14 ±0.442	91.15 ±0.488	91.68 ±0.928	92.53 ±0.927
	2nd	94.23 ±0.143	..	89.32 ±0.244	89.71 ±0.274	89.84 ±0.330	89.97 ±0.373	90.82 ±0.467	90.68 ±0.555	91.83 ±0.624	92.29 ±1.108
	3rd	94.71 ±0.149	89.94 ±0.218	..	89.62 ±0.274	89.59 ±0.330	90.00 ±0.380	90.95 ±0.464	91.12 ±0.501	91.35 ±0.845	92.52 ±1.189
	4th	93.94 ±0.172	90.33 ±0.242	90.01 ±0.253	..	89.66 ±0.314	90.02 ±0.354	90.86 ±0.415	90.79 ±0.460	91.39 ±0.677	91.90 ±1.062
	5th	95.33 ±0.190	90.64 ±0.274	90.04 ±0.296	90.10 ±0.292	..	90.04 ±0.346	90.92 ±0.368	90.84 ±0.450	91.55 ±0.674	92.08 ±0.894
	6th	95.13 +0.257	90.77 +0.304	90.22 +0.364	90.24 +0.359	90.51 +0.316	..	90.75 +0.385	90.70 +0.435	91.20 +0.656	90.81 +1.011
	7th	94.88 ±0.374	90.76 ±0.424	90.31 ±0.484	90.46 ±0.442	91.01 ±0.394	90.70 ±0.387	..	90.75 ±0.446	91.18 ±0.623	91.28 ±0.993
	8th	94.85 ±0.526	90.80 ±0.569	90.59 ±0.567	90.92 ±0.494	91.27 ±0.431	90.34 ±0.460	90.73 ±0.429	..	91.07 ±0.591	91.16 ±1.032
	9th	94.51 ±0.879	90.42 ±0.759	91.06 ±0.781	90.57 ±0.668	91.38 ±0.590	90.91 ±0.627	91.32 ±0.547	90.96 ±0.610	..	91.26 ±0.930
	10th	94.46 ±1.083	90.01 ±1.006	90.99 ±0.834	89.80 ±0.873	91.05 ±0.681	90.07 ±0.824	90.95 ±0.649	90.68 ±0.774	91.14 ±0.656	

Table 16 showing Variation of Standard Deviation of Persistency with Age.

Age in Lactations.

Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	4.46 ±0.141	4.68 ±0.174	4.42 ±0.195	4.42 ±0.233	4.18 ±0.275	3.45 ±0.312	3.24 ±0.393	4.04 ±0.656	3.59 ±0.655
2nd	3.21 ±0.095	..	4.66 ±0.173	4.43 ±0.194	4.42 ±0.233	4.05 ±0.264	3.85 ±0.330	3.20 ±0.345	3.06 ±0.442	4.57 ±0.783
3rd	2.85 ±0.106	4.15 ±0.154	..	4.51 ±0.194	4.55 ±0.233	4.65 ±0.304	3.85 ±0.328	3.36 ±0.354	4.05 ±0.598	4.61 ±0.841
4th	2.76 ±0.122	3.92 ±0.171	4.16 ±0.179	..	4.48 ±0.222	4.05 ±0.250	3.67 ±0.294	3.31 ±0.325	3.58 ±0.479	4.51 ±0.751
5th	2.54 ±0.134	3.68 ±0.194	4.08 ±0.209	4.17 ±0.206	..	4.08 ±0.245	3.40 ±0.184	3.37 ±0.318	3.75 ±0.477	3.90 ±0.632
6th	2.76 ±0.182	3.30 ±0.215	4.00 ±0.257	4.11 ±0.254	3.72 ±0.223	..	3.69 ±0.272	3.26 ±0.308	3.82 ±0.464	4.63 ±0.715
7th	2.92 ±0.264	3.50 ±0.300	4.02 ±0.342	3.90 ±0.312	3.63 ±0.279	3.72 ±0.274	..	3.56 ±0.315	3.74 ±0.441	4.76 ±0.702
8th	3.07 ±0.372	3.73 ±0.403	3.80 ±0.401	3.56 ±0.349	3.22 ±0.304	3.44 ±0.326	3.43 ±0.303	..	3.60 ±0.418	4.84 ±0.730
9th	3.83 ±0.621	3.72 ±0.537	3.75 ±0.553	3.54 ±0.472	3.28 ±0.417	3.66 ±0.444	3.28 ±0.387	3.71 ±0.431	..	4.65 ±0.658
10th	4.20 ±0.766	4.15 ±0.712	3.23 ±0.590	3.71 ±0.617	2.97 ±0.482	3.78 ±0.583	3.11 ±0.459	3.63 ±0.547	3.28 ±0.464	..

Table 19 showing Variation of Coefficient of Variation of Persistency with Age.

	Age in Lactations.									
Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	4.99 ±0.158	5.24 ±0.195	4.93 ±0.218	4.93 ±0.260	4.63 ±0.306	3.79 ±0.343	3.57 ±0.434	4.41 ±0.717	3.88 ±0.710
2nd	3.40 ±0.139	..	5.22 ±0.194	4.95 ±0.217	4.92 ±0.260	4.50 ±0.294	4.24 ±0.364	3.51 ±0.379	3.33 ±0.481	4.95 ±0.851
3rd	3.01 ±0.112	4.61 ±0.171	..	5.03 ±0.217	5.08 ±0.265	4.65 ±0.304	4.24 ±0.361	3.69 ±0.389	4.44 ±0.656	4.98 ±0.911
4th	2.94 ±0.130	4.34 ±0.190	4.63 ±0.200	..	5.00 ±0.248	4.51 ±0.279	4.04 ±0.324	3.65 ±0.358	3.92 ±0.524	4.90 ±0.827
5th	2.67 ±0.141	4.06 ±0.214	4.53 ±0.236	4.63 ±0.230	..	4.54 ±0.273	3.74 ±0.287	3.71 ±0.351	4.10 ±0.522	4.23 ±0.688
6th	2.90 ±0.191	3.64 ±0.237	4.43 ±0.290	4.56 ±0.282	4.11 ±0.247	..	4.07 ±0.300	3.59 ±0.340	4.19 ±0.509	5.10 ±0.789
7th	3.08 ±0.279	3.86 ±0.331	4.45 ±0.380	4.31 ±0.346	3.99 ±0.307	4.10 ±0.302	..	3.93 ±0.348	4.10 ±0.484	5.22 ±0.771
8th	3.24 ±0.393	4.11 ±0.444	4.20 ±0.443	3.92 ±0.384	3.53 ±0.334	3.81 ±0.361	3.78 ±0.335	..	3.95 ±0.460	5.31 ±0.803
9th	4.05 ±0.659	4.11 ±0.594	4.16 ±0.401	3.90 ±0.522	3.59 ±0.457	4.02 ±0.489	3.59 ±0.424	4.08 ±0.475	..	5.10 ±0.722
10th	4.44 ±0.812	4.61 ±0.792	3.55 ±0.649	4.13 ±0.689	3.26 ±0.530	4.19 ±0.648	3.42 ±0.505	4.00 ±0.605	3.60 ±0.510	..

Table 20 showing the variation of Persistency with Age.

Age in Lactations	"Paired-Lactation" Method		"Lumped-Lactation" method	Comparative Values (Persistency of 4th lactation = 10	
	Observed	Estimated from theoretical curve	Estimated from theoretical curve	"Paired-Lactation" method	"Lumped-Lactati method
1	94.23	94.12	93.79	106.64	105.25
2	89.43	89.80	90.57	101.74	101.64
3	88.81	88.51	89.25	100.28	100.16
4	88.42	88.26	89.11	100.00	100.00
5	87.99	88.11	89.57	99.83	100.52
6	87.52	87.81	90.22	99.49	101.25
7	87.58	87.48	90.78	99.12	101.87
8	87.59	87.41	91.09	99.04	102.22
9	87.70	87.68	91.14	99.34	102.28
10	87.81	87.97	91.04	99.67	102.17
11	87.33	87.27	91.04	98.88	102.17

Table 23 showing the variation of Maximum Yield with Age.

Age in Lactations	No. of Animals	Mean	Standard deviations	Coefficient of variation
1	534	36.40±0.274	6.34±0.194	17.41±0.549
2	533	45.29±0.337	7.78±0.238	17.18±0.541
3	403	48.57±0.424	8.51±0.300	17.53±0.636
4	296	49.77±0.500	8.59±0.353	17.27±0.730
5	220	50.41±0.646	9.58±0.457	19.00±0.938
6	151	50.77±0.719	8.83±0.508	17.39±1.030
7	98	51.90±0.948	8.43±0.670	16.25±1.191
8	66	51.08±1.086	8.82±0.768	17.27±1.547
9	40	49.90±1.073	6.79±0.759	13.60±1.548
10	27	49.43±1.646	8.55±1.164	17.30±2.560
11	15	48.63±1.603	6.21±1.134	12.77±2.368
12	7	49.07±2.910	7.70±2.058	15.69±4.295
13	1	49.500	-	-

Table 24 showing variation of Mean Maximum Yield (lbs) with Age.

Age in Lactations.

Age in Lactations.	1st	2nd	3th	4th	5th	6th	7th	8th	9th	10th
1st	..	45.41 ±0.345	48.81 ±0.446	49.87 ±0.548	50.52 ±0.735	50.31 ±0.871	51.40 ±1.143	52.74 ±1.640	47.89 ±1.409	48.27 ±1.478
2nd	36.29 ±0.284	..	48.68 ±0.454	49.74 ±0.539	50.51 ±0.744	50.80 ±0.809	51.65 ±1.011	51.76 ±1.394	48.46 ±1.143	41.80 ±1.670
3rd	36.40 ±0.337	45.52 ±0.415	..	49.79 ±0.536	51.03 ±0.710	50.62 ±0.842	51.37 ±1.059	51.34 ±1.298	48.90 ±1.152	48.13 ±1.567
4th	36.10 ±0.397	45.29 ±0.486	48.83 ±0.524	..	50.43 ±0.688	50.77 ±0.805	51.85 ±0.952	51.38 ±1.275	48.11 ±1.103	47.07 ±1.338
5th	35.12 ±0.460	44.83 ±0.564	48.72 ±0.640	50.54 ±0.615	..	50.79 ±0.762	52.15 ±0.926	51.803 ±1.191	48.48 ±1.058	47.91 ±1.446
6th	34.42 ±0.603	43.98 ±0.682	47.96 ±0.838	51.11 ±0.777	50.93 ±0.792	..	52.00 ±0.872	52.282 ±1.174	48.57 ±1.024	48.16 ±1.484
7th	34.19 ±0.861	43.71 ±0.887	47.51 ±1.158	51.83 ±1.019	51.28 ±1.051	52.29 ±0.912	..	51.453 ±1.076	48.89 ±0.997	48.50 ±1.451
8th	34.35 ±1.021	44.06 ±1.042	48.79 ±1.462	52.46 ±1.315	52.23 ±1.327	53.54 ±1.114	52.98 ±0.996	..	49.50 ±1.133	48.24 ±1.492
9th	32.65 ±1.344	43.12 ±1.384	47.52 ±1.970	50.71 ±1.327	50.09 ±1.383	50.66 ±0.931	51.91 ±1.245	50.473 ±1.230	..	50.46 ±1.600
10th	31.73 ±1.576	41.80 ±1.670	46.25 ±2.746	50.29 ±1.983	48.37 ±2.142	48.700 ±1.082	49.64 ±1.304	49.927 ±1.551	50.50 ±1.344	..

Table 25 showing variation of Standard Deviation of Maximum Yield with Age.

	Age in Lactations.									
Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	7.74 ±0.243	8.49 ±0.315	8.78 ±0.388	9.87 ±0.520	9.35 ±0.616	8.93 ±0.808	9.56 ±1.160	6.14 ±0.996	5.73 ±1.045
2nd	6.38 ±0.200	..	8.66 ±0.321	8.73 ±0.381	9.99 ±0.526	8.78 ±0.572	8.34 ±0.715	9.14 ±0.986	5.60 ±0.808	6.11 ±1.047
3rd	6.42 ±0.238	7.90 ±0.293	..	8.81 ±0.379	9.78 ±0.502	9.26 ±0.595	8.80 ±0.749	8.71 ±0.918	5.52 ±0.814	6.07 ±1.108
4th	6.35 ±0.281	7.86 ±0.344	8.61 ±0.370	..	9.83 ±0.487	9.22 ±0.569	8.41 ±0.673	9.19 ±0.902	5.84 ±0.780	5.68 ±0.946
5th	6.17 ±0.325	7,56 ±0.398	8.83 ±0.453	8.78 ±0.435	..	8.99 ±0.539	8.54 ±0.655	8.91 ±0.842	5.89 ±0.748	6.30 ±1.022
6th	6.47 ±0.426	7.41 ±0.482	9.22 ±0.592	8.90 ±0.550	9.33 ±0.560	..	8.37 ±0.617	8.79 ±0.830	5.97 ±0.724	6.80 ±1.049
7th	6.73 ±0.609	7.31 ±0.627	9.62 ±0.819	9.00 ±0.721	9.69 ±0.743	8.75 ±0.645	..	8.61 ±0.761	5.98 ±0.705	6.96 ±1.026
8th	5.60 ±0.722	6.84 ±0.737	9.81 ±1.034	9.48 ±0.930	9.93 ±0.938	8.34 ±0.788	7.97 ±0.704	..	6.89 ±0.801	7.00 ±1.055
9th	5.86 ±0.950	6.78 ±0.979	9.45 ±1.393	7.02 ±0.938	7.70 ±0.978	5.43 ±0.658	6.75 ±0.795	7.48 ±0.870	..	8.00 ±1.132
10th	6.10 ±1.114	6.88 ±1.181	10.64 ±1.942	8.41 ±1.40	9.34 ±1.515	4.96 ±0.765	6.25 ±0.922	7.28 ±1.097	6.72 ±0.950	..

Table 26 showing variation of Coefficient of Variation of Maximum Yield with age.

Age in Lactations.

Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	17.04 ±0.553	21.88 ±0.850	17.60 ±0.801	19.53 ±1.068	18.58 ±1.266	17.37 ±1.620	18.13 ±2.270	12.82 ±2.114	11.86 ±2.195
2nd	17.57 ±0.571	..	17.79 ±0.681	17.55 ±0.790	19.77 ±1.082	17.29 ±1.159	16.15 ±1.420	17.66 ±1.963	11.55 ±1.690	12.89 ±2.247
3rd	17.65 ±0.675	17.36 ±0.663	..	17.70 ±0.785	19.18 ±1.034	18.29 ±1.215	17.13 ±1.500	16.96 ±1.839	11.30 ±1.687	12.61 ±2.338
4th	17.59 ±0.801	17.37 ±0.781	17.63 ±0.782	..	19.50 ±1.001	18.15 ±1.158	16.22 ±1.333	17.89 ±1.810	12.14 ±1.645	12.06 ±2.039
5th	17.58 ±0.955	16.86 ±0.914	18.12 ±0.974	17.37 ±0.885	..	17.69 ±1.094	16.37 ±1.289	17.21 ±1.673	12.15 ±1.565	13.16 ±2.171
6th	18.79 ±1.282	16.84 ±1.127	19.22 ±1.280	17.41 ±1.108	18.33 ±1.135	..	16.09 ±1.216	16.81 ±1.633	12.30 ±1.513	14.12 ±2.222
7th	19.68 ±1.849	16.73 ±1.474	20.25 ±1.793	17.37 ±1.432	18.91 ±1.500	16.73 ±1.267	..	16.73 ±1.519	12.23 ±1.487	14.34 ±2.158
8th	17.34 ±2.164	15.52 ±1.713	20.11 ±2.203	18.08 ±1.830	19.01 ±1.860	15.57 ±1.506	15.04 ±1.359	..	13.92 ±1.649	14.51 ±2.233
9th	17.94 ±3.002	15.72 ±2.325	19.88 ±3.045	13.84 ±1.885	15.38 ±1.999	10.71 ±1.314	13.00 ±1.557	14.83 ±1.761	..	15.86 ±2.298
10th	19.23 ±2.156	16.47 ±2.900	23.00 ±4.415	16.73 ±2.865	19.31 ±3.247	10.18 ±1.587	12.60 ±1.887	14.58 ±2.243	13.31 ±1.915	..

Table 27 showing variation of maximum yield with age.

Age in Lactations	**"Paired-Lactation" Method** Observed lbs	"Paired-Lactation" Method Estimated from theoretical curve lbs	"Lumped-Lactation" Method Estimated from theoretical curve lbs	Comparative Yield (4th lactation = 100) "Paired-Lactation" method	Comparative Yield (4th lactation = 100) "Lumped-Lactation" method
1	36.28	36.37	36.49	71.81	73.07
2	45.51	45.01	45.04	88.86	90.19
3	48.56	48.83	48.62	96.41	97.36
4	49.51	50.65	49.94	100.00	100.00
5	49.41	49.48	50.50	97.69	101.12
6	49.27	49.03	50.91	96.80	101.94
7	49.00	48.58	51.21	95.91	102.54
8	47.58	48.00	51.14	94.77	102.40
9	46.67	47.11	50.49	93.01	101.10
10	46.63	46.04	49.32	90.90	98.76
11	45.39	45.54	48.35	89.91	96.82

Table 30 showing the variation of Total Yield with Age.

Age in Lactations	No. of Animals	Mean	Standard Deviation	Coefficient of variation
1	534	7290.7± 62.7	1448.5± 44.3	19.87±0.635
2	533	7923.3± 68.9	1590.6± 48.7	20.08±0.639
3	403	8520.1± 80.0	1607.0± 56.6	18.86±0.688
4	296	8861.5±102.1	1757.2± 72.2	19.83±0.846
5	220	9006.8±124.4	1844.8± 87.9	20.48±1.017
6	151	9098.5±137.0	1683.8± 96.9	18.51±1.101
7	98	9433.6±167.4	1657.1±118.4	17.57±1.293
8	66	9359.8±199.0	1616.5±140.7	17.27±1.547
9	40	9131.2±245.6	1553.0±173.6	17.01±1.955
10	27	8921.3±312.4	1623.2±220.9	18.19±2.556
11	15	9158.3±372.3	1441.9±263.3	15.74±2.945
12	7	8910.7±565.4	1496.0±399.8	16.79±4.612
13	1	9375.0	-	-

Table 31 showing variation of Mean Total Yield (lbs) with Age.

Age in Lactations.

Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	7947.5 ± 70.38	8551.3 ± 84.68	8863.3 ±108.66	8983.3 ±136.67	9007.6 ±160.17	9329.9 ±218.89	9625.0 ±299.58	8870.4 ±400.49	9028.5 ±366.90
2nd	7268.1 ± 65.01	..	8536.1 ± 87.29	8844.4 ±110.91	9015.3 ±141.81	9065.7 ±154.45	9367.6 ±208.61	9537.8 ±247.36	9020.8 ±279.46	8870.4 ±395.06
3rd	7363.3 ± 75.45	8075.4 ± 83.66	..	8850.0 ±109.28	9076.3 ±138.24	9054.7 ±154.51	9375.0 ±203.67	9475.0 ±229.39	8892.5 ±319.71	9114.3 ±403.82
4th	7350.6 ± 89.95	8111.6 ± 98.21	8652.8 ± 98.96	..	8997.5 ±132.50	9079.2 ±151.34	9375.0 ±188.41	9409.6 ±229.13	8892.8 ±312.01	8700.5 ±361.53
5th	7212.5 ±106.38	8130.5 ±118.08	8676.3 ±120.86	9035.5 ±120.22	..	9107.0 ±143.54	9519.5 ±188.09	9495.5 ±219.45	9019.0 ±277.12	8845.8 ±395.65
6th	7053.2 ±135.02	7968.2 ±143.83	8649.8 ±158.48	9146.0 ±155.20	9254.5 ±154.36	..	9437.5 ±175.38	9526.8 ±216.53	8922.6 ±266.86	8702.4 ±347.04
7th	6899.6 ±191.89	7812.5 ±181.71	8548.9 ±214.41	9317.3 ±210.73	9389.7 ±200.96	9529.9 ±167.83	..	9425.8 ±198.94	9041.2 ±257.53	8882.8 ±334.02
8th	7029.4 ±231.87	7904.0 ±212.98	8736.1 ±242.39	9586.5 ±275.06	9589.3 ±241.83	9767.8 ±211.43	9691.4 ±181.67	..	9019.2 ±259.51	8782.0 ±341.75
9th	6600.0 ±265.05	7677.1 ±259.32	8470.7 ±344.69	9080.3 ±291.21	9177.6 ±225.99	9280.9 ±218.13	9728.5 ±240.77	9280.4 ±249.21	..	9105.0 ±303.76
10th	6330.2 ±282.75	7496.3 ±311.47	8262.3 ±379.02	8866.0 ±392.50	8805.5 ±289.54	8770.0 ±218.55	9202.0 ±304.33	9146.7 ±296.05	9245.0 ±283.96	..

Table 32 showing variation of Standard Deviation of Total Yield with Age.

Age in Lactations.

Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	1578.5 ± 49.77	1613.4 ± 59.88	1738.5 ± 76.83	1833.6 ± 96.64	1717.6 ±113.26	1709.6 ±154.78	1746.8 ±211.83	1745.7 ±283.19	1421.0 ±259.43
2nd	1458.0 ± 45.97	..	1663.0 ± 61.72	1795.2 ± 78.42	1902.6 ±100.27	1677.8 ±109.21	1720.2 ±147.51	1622.1 ±174.91	1369.1 ±197.61	1628.9 ±279.35
3rd	1437.6 ± 53.35	1594.0 ± 59.16	..	1795.7 ± 77.27	1905.5 ± 97.75	1699.6 ±109.26	1691.9 ±144.02	1538.8 ±162.20	1533.3 ±226.07	1564.0 ±285.84
4th	1439.2 ± 63.60	1589.7 ± 69.45	1626.1 ± 69.97	..	1892.4 ± 93.69	1732.2 ±107.01	1664.0 ±133.23	1652.3 ±162.02	1651.0 ±220.62	1533.9 ±255.64
5th	1427.2 ± 75.22	1584.3 ± 83.50	1666.0 ± 85.46	1717.0 ± 85.01	..	1692.3 ±101.50	1734.1 ±133.00	1642.2 ±155.17	1543.0 ±195.99	1724.6 ±279.77
6th	1447.9 ± 95.47	1562.4 ±101.70	1743.2 ±112.06	1776.3 ±109.74	1819.9 ±109.15	..	1673.1 ±123.34	1620.4 ±153.11	1556.1 ±188.69	1590.4 ±245.39
7th	1498.7 ±135.69	1498.5 ±128.49	1781.9 ±151.61	1861.1 ±149.01	1852.8 ±142.10	1601.0 ±118.03	..	1591.5 ±140.67	1545.2 ±182.10	1601.9 ±236.19
8th	1352.0 ±163.96	1396.6 ±150.60	1626.0 ±171.40	1983.5 ±194.50	1809.7 ±171.00	1582.2 ±149.50	1453.4 ±128.46	..	1578.6 ±183.50	1603.0 ±241.65
9th	1155.4 ±187.42	1270.4 ±183.36	1653.1 ±243.73	1540.9 ±205.91	1258.3 ±159.79	1271.9 ±154.24	1444.6 ±170.25	1515.9 ±176.21	..	1518.8 ±214.79
10th	1095.1 ±199.93	1284.2 ±220.24	1468.0 ±268.01	1665.2 ±277.54	1262.1 ±204.73	1001.5 ±154.54	1459.5 ±215.19	1388.6 ±209.34	1419.8 ±200.79	..

Table 33 showing variation of Coefficient of Variation of Total Yield with Age.

Age in Lactations.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	19.86 ±0.651	18.87 ±0.725	19.62 ±0.900	20.41 ±1.120	19.07 ±1.302	18.32 ±1.714	18.15 ±2.272	19.68 ±3.314	15.74 ±2.944
2nd	20.06 ±0.657	..	19.48 ±0.750	20.30 ±0.923	21.10 ±1.161	18.51 ±1.245	18.36 ±1.626	17.01 ±1.886	15.18 ±2.240	18.36 ±3.253
3rd	19.52 ±0.752	19.74 ±0.761	..	20.29 ±0.908	20.99 ±1.123	18.77 ±1.248	18.05 ±1.586	16.24 ±1.757	17.24 ±2.617	17.16 ±3.224
4th	19.58 ±0.898	19.60 ±0.889	18.79 ±0.837	..	21.03 ±1.086	19.08 ±1.221	17.75 ±1.465	17.56 ±1.774	18.57 ±2.565	17.63 ±3.028
5th	19.79 ±1.083	19.49 ±1.065	19.20 ±1.020	19.00 ±0.974	..	18.58 ±1.152	18.22 ±1.443	17.30 ±1.682	17.11 ±2.235	19.50 ±3.281
6th	20.53 ±1.409	19.61 ±1.325	20.15 ±1.347	19.42 ±1.244	19.67 ±1.224	..	17.73 ±1.347	17.01 ±1.653	17.44 ±2.178	18.28 ±2.913
7th	21.72 ±2.057	19.18 ±1.704	20.83 ±1.849	19.97 ±1.662	19.73 ±1.571	16.80 ±1.273	..	16.88 ±1.534	17.09 ±2.072	18.03 ±2.744
8th	19.23 ±2.417	17.67 ±1.964	18.61 ±2.029	20.69 ±2.114	18.87 ±1.846	16.20 ±1.759	15.00 ±1.355	..	17.36 ±2.078	18.25 ±2.842
9th	17.51 ±2.925	16.55 ±2.453	19.52 ±2.985	16.97 ±2.332	13.71 ±1.774	13.70 ±1.693	14.85 ±1.788	16.33 ±1.949	..	16.68 ±2.424
10th	17.30 ±3.251	17.13 ±3.023	17.77 ±3.344	18.78 ±3.239	14.33 ±2.372	11.42 ±1.785	15.86 ±2.400	15.18 ±2.341	15.36 ±2.223	..

Table 34 showing the variation of total yield with age.

Age in Lactations	"Paired-Lactation" Method		"Lumped-Lactation" Method	Comparative Yield (4th lactation = 100)	
	Observed	Estimated from theoretical curve	Estimated from theoretical curve	"Paired-Lactation" method	"Lumped-Lactation" method
	lbs	lbs	lbs		
1	7268.1	7281.3	7280.4	85.05	82.37
2	7945.5	7952.5	7958.1	92.89	90.04
3	8401.0	8364.1	8470.8	97.70	95.84
4	8592.5	8560.8	8838.4	100.00	100.00
5	8556.5	8587.3	9080.6	100.31	102.74
6	8420.2	8488.4	9217.4	99.15	104.29
7	8338.7	8308.9	9268.5	97.06	104.87
8	8110.2	8093.5	9253.9	94.54	104.70
9	7881.8	7887.0	9193.3	92.13	104.02
10	7762.3	7734.0	9106.5	90.34	103.03
11	7658.5	7679.3	9013.6	89.70	101.98

Table 35 showing the Correlation between Persistency of Different Ages.

Age in Lactations.

Age in Lactations.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	+0.5077 ±0.04472	+0.3437 ±0.05270	+0,2508 ±0.06287	+0.0751 ±0.07516	+0.0815 ±0.09449	+0.1810 ±0.1313	+0.1439 ±0.1796	+0.2129 ±0.2500	-0.2058 ±0.2887
2nd	+0.4681	..	+0.5459 ±0.05270	+0.4663 ±0.06214	+0.4250 ±0.07516	+0.1199 ±0.09325	+0.0296 ±0.1240	+0.0031 ±0.1581	-0.3995 ±0.2182	-0.1357 ±0.2673
3rd	+0.3308	+0.4974	..	+0.5488 ±0.06120	+0.4632 ±0.07313	+0.2775 ±0.09206	+0.3168 ±0.1231	+0.1582 ±0.1543	-0.0354 ±0.2236	+0.1499 ±0.2887
4th	+0.2457	+0.4352	+0.4991	..	+0.5336 ±0.07053	+0.4105 ±0.08839	+0.3042 ±0.1155	+0.1671 ±0.1429	+0.1389 ±0.2000	+0.2699 ±0.2582
5th	+0.0750	+0.3140	+0.4327	+0.4884	..	+0.6007 ±0.08575	+0.3595 ±0.1104	+0.4512 ±0.1374	+0.1635 ±0.1890	-0.1577 ±0.2500
6th	+0.0813	+0.1193	+0.2643	+0.3889	+0.5375	..	+0.5449 ±0.1060	+0.3557 ±0.1374	+0,1296 ±0.1796	+0.2511 ±0.2357
7th	+0.1791	+0.0296	+0.3066	+0.2951	+0.3448	+0.4967	..	+0.2002 ±0.1280	+0.2209 ±0.1741	+0.7377 ±0.2236
8th	+0.1429	+0.0031	+0.1569	+0.1656	+0.4229	+0.3414	+0.1976	..	+0.4388 ±0.1715	+0.4702 ±0.2294
9th	-0.2098	-0.3796	-0.0354	+0.1380	+0.1620	+0.1289	+0.2174	+0.4126	..	+0.7889 ±0.2085
10th	-0.2034	-0.1349	+0.1488	+0.2635	-0.1564	+0.2459	+0.6277	+0.4384	+0.6578	..

Table 36 showing the Results of the Test of Linearity of Regression Lines of Different Ages for Persistency.

Regression line for	Degrees of Freedom		Mean Square*		Variance Ratio for Deviations from Linear regression	Remarks.
	Within arrays	Deviations from Linear regression	Within arrays	Deviations from Linear regression		
y_2^* on y_1	483	18	15.4896	17.6333	1.1384	
y_3 on y_1	345	16	19.3936	22.9687	1.1843	
y_4 on y_1	240	15	18.7979	11.9666	..	
y_5 on y_1	165	13	18.7085	29.9077	1.5986	
y_6 on y_1	100	13	17.1350	21.2000	1.2372	
y_3 on y_2	338	23	15.1715	34.2086	2.1768	F significant at 1% level.
y_4 on y_2	239	21	15.1983	25.8761	1.7026	F significant at 5% level.
y_5 on y_2	160	18	17.3681	21.0611	1.2126	
y_6 on y_2	100	16	15.0520	24.3125	1.6152	
y_4 on y_3	249	19	15.1787	16.9368	1.1158	
y_5 on y_3	170	18	16.2488	22.9944	1.4151	
y_6 on y_3	101	18	17.2623	19.8166	1.1480	
y_5 on y_4	182	20	14.9703	19.2650	1.2869	
y_6 on y_4	111	18	13.0594	20.0888	1.5383	
y_6 on y_5	121	16	11.5727	14.8312	1.2816	

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* The ~~suffix~~ subscript indicates the age in lactations.

Table 37 showing the regression equations to estimate the expected Persistency (Yp) for any Age from that of observed Persistency (yp) for Another Age.

Age in Lactations for which the Expected Persistency (Yp) is required.	Age in Lactations for which the Persistency (yp) is observed. 1	2	3	4	5	6	7	3	9	10
1	..									
2	$Y_2 = 28.06+0.65126y_1$	..								
3	$Y_3 = 37.81+0.54324y_1$	$Y_3 = 39.08+0.5586y_2$	..							
4	$Y_4 = 52.72+0.39317y_1$	$Y_4 = 45.25+0.49224y_2$	$Y_4 = 41.02+0.53991y_3$	..						
5	$Y_5 = 77.24+0.13031y_1$	$Y_5 = 55.63+0.37735y_2$	$Y_5 = 46.17+0.48224y_3$	$Y_5 = 42.31+0.52547y_4$	..					
6	$Y_6 = 78.35+0.12392y_1$	$Y_6 = 76.68+0.14645y_2$	$Y_6 = 65.05+0.27645y_3$	$Y_6 = 55.46+0.38299y_4$	$Y_6 = 35.75+0.58981y_5$	..				
7	$Y_7 = 71.04+0.21185y_1$	$Y_7 = 93.78-0.03262y_2$	$Y_7 = 64.41+0.29384y_3$	$Y_7 = 65.77+0.27729y_4$	$Y_7 = 61.59+0.32228y_5$	$Y_7 = 44.53+0.50963y_6$	..			
8	$Y_8 = 76.37+0.1508y_1$	$Y_8 = 90.90+0.00273y_2$	$Y_8 = 78.52+0.13915y_3$	$Y_8 = 76.78+0.15424y_4$	$Y_8 = 50.50+0.44197y_5$	$Y_8 = 61.53+0.32289y_6$	$Y_8 = 72.13+0.2052y_7$	..		
9	$Y_9 = 112.61-0.22150y_1$	$Y_9 = 119.99-0.31142y_2$	$Y_9 = 94.84-0.03834y_3$	$Y_9 = 78.73+0.13981y_4$	$Y_9 = 77.09+0.18528y_5$	$Y_9 = 78.96+0.13468y_6$	$Y_9 = 68.56+0.24769y_7$	$Y_9 = 49.88+0.45278y_8$	..	
10	$Y_{10} = 108.89-0.17324y_1$	$Y_{10} = 105.65-0.14848y_2$	$Y_{10} = 73.22+0.21218y_3$	$Y_{10} = 63.12+0.32048y_4$	$Y_{10} = 110.79-0.20542y_5$	$Y_{10} = 63.64+0.30158y_6$	$Y_{10} = 3.96+0.96013y_7$	$Y_{10} = 38.18+0.584337y_8$	$Y_{10} = 6.26+0.93261y_9$	..

Table *38* showing the Correlation between Maximum of Different Ages.

Age in Lactations.

Age in Lactations.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	+0.8490 ±0.4472	+0.7131 ±0.05270	+0.5226 ±0.06287	+0.5345 ±0.07516	+0.4341 ±0.09449	+0.8239 ±0.1313	+0.3417 ±0.1796	+0.0062 ±0.2500	+0.4285 ±0.2887
2nd	+0.6906	..	+0.8270 ±0.05270	+0.6453 ±0.06214	+0.5415 ±0.07516	+0.3041 ±0.09325	+0.6757 ±0.1240	+0.3102 ±0.1581	-0.0131 ±0.2182	+0.2346 ±0.2673
3rd	+0.6126	+0.6789	..	+0.7655 ±0.06120	+0.6184 ±0.7313	+0.4794 ±0.09206	+0.7032 ±0.1231	+0.1796 ±0.1543	+0.3525 ±0.2236	+0.2659 ±0.2887
4th	+0.4797	+0.5685	+0.6443	..	+0.7765 ±0.07053	+0.5704 ±0.08839	+0.5288 ±0.1155	+0.1442 ±0.1429	+0.2785 ±0.2000	+0.2750 ±0.2582
5th	+0.4888	+0.4941	+0.5500	+0.6507	..	+0.8113 ±0.08575	+0.6914 ±0.1104	+0.3526 ±0.1374	+0.4006 ±0.1890	+0.4271 ±0.2500
6th	+0.4087	+0.2950	+0.4457	+0.5157	+0.6703	..	+0.5707 ±0.1060	+0.3315 ±0.1374	+0.3842 ±0.1796	+0.4583 ±0.2357
7th	+0.6772	+0.5887	+0.6064	+0.4845	+0.5989	+0.5159	..	+0.6037 ±0.1280	+0.2688 ±0.1741	+0.6003 ±0.2236
8th	+0.3290	+0.3006	+0.1779	+0.1432	+0.3441	+0.3199	+0.5396	..	+0.2303 ±0.1715	+0.7358 ±0.2294
9th	+0.0062	-0.0131	+0.3386	+0.2715	+0.3805	+0.3664	+0.2625	+0.2263	..	+1.1029 ±0.2085
10th	+0.4040	+0.2304	+0.2598	+0.2683	+0.4029	+0.4287	+0.5372	+0.6266	+0.7670	..

Table 39 showing the Results of the Test of Linearity of Regression Lines of Different Ages for Maximum Yield.

Regression line for	Degrees of Freedom		Mean Square*		Variance Ratio from Deviations from Linear regression	Remarks.
	Within arrays	Deviations from Linear regression	Within arrays	Deviations from Linear regression		
y_2^* on y_1	467	34	31.4931	30.1235	..	
y_3 on y_1	328	33	43.0899	65.8818	1.5289	F significant at 5% level.
y_4 on y_1	225	29	59.2182	64.0034	1.0808	
y_5 on y_1	151	27	74.5781	74.2296	..	
y_6 on y_1	89	24	64.0325	108.087	1.6880	F significant at 5% level.
y_3 on y_2	323	38	40.1207	44.0236	1.0973	
y_4 on y_2	222	38	49.4495	65.0868	1.3162	
y_5 on y_2	146	32	77.6356	67.4875	..	
y_6 on y_2	87	29	74.1723	61.7344	..	
y_4 on y_3	230	38	44.5421	52.0526	1.1686	
y_5 on y_3	150	38	63.0220	83.3815	1.3230	
y_6 on y_3	87	32	56.4459	104.2625	1.8471	F significant at 5% level.
y_5 on y_4	162	40	35.9302	137.3975	3.8240	F significant at 1% level.
y_6 on y_4	92	37	60.7260	68.0793	1.1211	
y_6 on y_5	99	38	40.3464	56.3289	1.3961	

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* The Subscript ~~suffix~~ indicates the age in lactations.

Table 40 showing the regression equations to estimate the expected Maximum Yield (Yp) for any Age from that of Observed Maximum Yield (yp) for another Age.

Age in Lactations for the Expected Maximum Yield (Yp) is required	Age in Lactations for which the Maximum Yield (yp) was observed. 1	2	3	4	5	6	7	8	9	10
1	••									
2	$Y_2 = 14.81+0.84336y_1$	••								
3	$Y_3 = 19.34+0.80979y_1$	$Y_3 = 14.81+0.74389y_2$	••							
4	$Y_4 = 25.94+0.66298y_1$	$Y_4 -= 21.17+0.63083y_2$	$Y_4 = 17.58+0.65962y_3$	••						
5	$Y_5 = 23.07+0.78145y_1$	$Y_5 = 21.25+0.65256y_2$	$Y_5 = 21.33+0.60959y_3$	$Y_5 = 13.59+0.72897y_4$	••					
6	$Y_6 = 29.98+0.59058y_1$	$Y_6 = 35.42+0.34986y_2$	$Y_6 = 29.15+0.44773y_3$	$Y_6 = 23.47+0.53415y_4$	$Y_6 = 17.93+0.64532y_5$	••				
7	$Y_7 = 21.47+0.89882y_1$	$Y_7 = 22.30+0.67141y_2$	$Y_7 = 25.0275+0.5544y_3$	$Y_7 = 28.38+0.45266y_4$	$Y_7 = 25.10+0.52742y_5$	$Y_7 = 26.19+0.4935y_6$	••			
8	$Y_8 = 34.59+0.52815y_1$	$Y_8 = 34.05+0.40188y_2$	$Y_8 = 44.7505+0.15779y_3$	$Y_8 = 44.87+0.14771y_4$	$Y_8 = 35.67+0.30895y_5$	$Y_8 = 34.22+0.33737y_6$	$Y_8 = 20.57+0.58277y_7$	••		
9	$Y_9 = 47.69+0.00625y_1$	$Y_9 = 48.92-0.01078y_2$	$Y_9 = 34.49+0.19796y_3$	$Y_9 = 36.66+0.22580y_4$	$Y_9 = 33.91+0.29093y_5$	$Y_9 = 28.14+0.40327y_6$	$Y_9 = 37.73+0.21501y_7$	$Y_9 = 38.98+0.20834y_8$	••	
10	$Y_{10} = 36.24+0.37905y_1$	$Y_{10} = 38.83+0.20430y_2$	$Y_{10} = 41.27+0.14822y_3$	$Y_{10} = 37.97+0.18107y_4$	$Y_{10} = 34.75+0.27191y_5$	$Y_{10} = 19.52+0.58811y_6$	$Y_{10} = 18.84+0.59754y_7$	$Y_{10} = 18.15+0.60264y_8$	$Y_{10} = 4.33+0.91337y_9$	••

Table 41 comparing the regression coefficients of different ages.

	Regression Coefficient	Standard Deviation	Standard error of estimate	$\frac{\text{S.E.} \times 100}{Y_p}$
Y_2 from y_1	+0.84336±0.03924	7.74	5.60	72.39
Y_3 from y_2	+0.74389±0.04357	8.66	6.37	73.52
Y_4 from y_3	+0.65962±0.04783	8.81	6.75	76.62
Y_5 from y_4	+0.72897±0.05983	9.83	7.48	76.12
Y_6 from y_5	+0.64532±0.06106	8.99	6.69	74.47
Y_3 from y_1	+0.80979±0.05999	8.49	6.72	79.15
Y_4 from y_1	+0.66298±0.07087	8.78	7.73	88.08
Y_5 from y_1	+0.78145±0.10451	9.87	8.63	87.48
Y_6 from y_1	+0.59085±0.12398	9.34	8.57	91.67

Table 42 showing the Correlation between Total Yield of Different Ages.

Age in Lactations.

Age in Lactations.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1st	..	+0.9558 ±0.04472	+0.8239 ±0.05270	+0.5744 ±0.06287	+0.5221 ±0.07516	+0.4941 ±0.09449	+0.7317 ±0.1313	+0.3397 ±0.1796	-0.0491 ±0.2500	-0.0305 ±0.2887
2nd	+0.7423	..	+0.9278 ±0.05270	+0.7634 ±0.06214	+0.6283 ±0.07516	+0.4052 ±0.09325	+0.5169 ±0.1240	+0.5864 ±0.1581	+0.0142 ±0.2182	-0.0921 ±0.2673
3rd	+0.6772	+0.7296	..	+0.9117 ±0.06120	+0.6906 ±0.07313	+0.5781 ±0.09206	+0.6554 ±0.1231	+0.3061 ±0.1543	+0.1171 ±0.2236	+0.0621 ±0.2887
4th	+0.5186	+0.6431	+0.7219	..	+0.9380 ±0.07053	+0.6306 ±0.08839	+0.5568 ±0.1155	+0.3115 ±0.1429	+0.0258 ±0.2000	+0.0172 ±0.2582
5th	+0.4828	+0.5499	+0.5984	+0.7343	..	+0.7545 ±0.08575	+0.7181 ±0.1104	+0.4958 ±0.1374	+0.2316 ±0.1890	+0.2676 ±0.2500
6th	+0.3749	+0.3844	+0.5213	+0.5584	+0.6378	..	+0.6168 ±0.1060	+0.2627 ±0.1374	+0.2558 ±0.1796	+0.2988 ±0.2357
7th	+0.6241	+0.4753	+0.5753	+0.5056	+0.5498	+0.5489	..	+0.5860 ±0.1280	+0.0774 ±0.1741	+0.3579 ±0.2236
8th	+0.3272	+0.5273	+0.2969	+0.3018	+0.4588	+0.2568	+0.5270	..	+0.4218 ±0.1715	+0.4590 ±0.2294
9th	-0.0491	+0.0142	+0.1166	+0.0258	+0.2275	+0.2501	+0.0772	+0.3984	..	+0.9895 ±0.2085
10th	-0.0305	-0.0919	+0.0620	+0.0172	+0.2614	+0.2902	+0.3434	+0.4293	+0.7572	..

Table 43 showing the Results of the Test of Linearity of Regression Lines of Different Ages for Total Yield.

Regression line for	Degrees of Freedom		Mean Square*		Variance Ratio for Deviations from Linear regression	Remarks.
	Within arrays	Deviations from Linear regression	Within arrays	Deviations from Linear regression		
y_2^{+} on y_1	467	34	17.7989	19.8235	1.1137	
y_3 on y_1	330	31	22.1721	27.3193	1.2321	
y_4 on y_1	225	29	37.0564	23.3517	..	
y_5 on y_1	150	28	40.5100	46.7178	1.1532	
y_6 on y_1	89	24	41.6831	38.1250	..	
y_3 on y_2	330	31	19.9481	29.3290	1.4703	
y_4 on y_2	230	30	30.3117	30.6833	1.0122	
y_5 on y_2	150	28	37.3193	58.3535	1.5636	F Significant at 5% level.
y_6 on y_2	92	24	32.1804	63.7583	1.9813	F Significant at 1% level approximately
y_4 on y_3	235	33	23.7068	32.5666	1.3737	
y_5 on y_3	158	30	35.3360	48.8370	1.3821	
y_6 on y_3	91	28	29.0098	49.9785	1.7228	F Significant at 5% level
y_5 on y_4	172	30	25.1209	34.6434	1.3791	
y_6 on y_4	102	27	29.0450	49.3222	1.6981	F Significant at 5% level.
y_6 on y_5	108	29	22.3555	46.1068	2.0624	F Significant at 1% level.

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* In units of class interval.

\+ The ~~suffix~~ Subscript indicates the age in lactations.

Table 44 showing the Regression equations to estimate the expected milk yield (Yp) for any age from that of Observed yield (yp) for another age.

Age in Lactations foe which the expected Total Yield (YP) is required.	Age in Lactations for which the Total Yield (yp) was observed. 1	2	3	4	5	6	7	8	9	10
1	..									
2	$Y_2 = 2106.3+0.80368y_1$	..								
3	$Y_3 = 2955.3+0.75998y_1$	$Y_3 = 2389.5+0.76116y_2$	..							
4	$Y_4 = 4258.0+0.62652y_1$	$Y_4 = 2953.5+0.72623y_2$	$Y_4 = 1952.3+0.79717y_3$	..						
5	$Y_5 = 4509.4+0.62028y_1$	$Y_5 = 3645.7+0.66042y_2$	$Y_5 = 3138.0+0.68443y_3$	$Y_5 = 1685.1+0.8093y_4$	..					
6	$Y_6 = 5870.9+0.44472y_1$	$Y_6 = 5775.7+0.41289y_2$	$Y_6 = 4658.7+0.50823y_3$	$Y_6 = 5946.3+0.5727y_4$	$Y_6 = 3618.5+0.59306y_5$	..				
7	$Y_7 = 4418.0+0.71191y_1$	$Y_7 = 5104.5+0.54658y_2$	$Y_7 = 4703.4+0.54646y_3$	$Y_7 = 5163.3+0.45203y_4$	$Y_7 = 4687.6+0.51460y_5$	$Y_7 = 3970.9+0.57363y_6$	..			
8	$Y_8 = 6653.6+0.42271y_1$	$Y_8 = 4696.9+0.61245y_2$	$Y_8 = 7020.4+0.28097y_3$	$Y_8 = 6999.7+0.25139y_4$	$Y_8 = 5503.4+0.41631y_5$	$Y_8 = 6965.7+0.26219y_6$	$Y_8 = 3833.6+0.57702y_7$	..		
9	$Y_9 = 9360.2-0.07421y_1$	$Y_9 = 8903.2+0.01532y_2$	$Y_9 = 7976.5+0.10813y_3$	$Y_9 = 8641.4+0.02769y_4$	$Y_9 = 6458.6+0.27898y_5$	$Y_9 = 6083.1+0.30595y_6$	$Y_9 = 3237.7+0.08259y_7$	$Y_9 = 5241.3+0.41485y_8$	..	
10	$Y_{10} = 9278.9-0.03955y_1$	$Y_{10} = 9743.9-0.11652y_2$	$Y_{10} = 8584.6+0.06411y_3$	$Y_{10} = 8560.5+0.01581y_4$	$Y_{10} = 5700.8+0.55718y_5$	$Y_{10} = 4657.9+0.46080y_6$	$Y_{10} = 5556.7+0.36146y_7$	$Y_{10} = 4249.1+0.49558y_8$	$Y_{10} = 1616.8+80997y_9$	..

Table 45a comparing the regression coefficient of different ages.

	Regression Coefficient	Standard Deviation	Standard error of estimate	$\frac{\text{S.E. x 100}}{Y_p}$
Y_2 from y_1	+0.80368±0.03098	1578.5	1012.1	64.12
Y_3 from y_2	+0.76116±0.03755	1663.0	1138.8	68.48
Y_4 from y_3	+0.79717±0.04672	1795.7	1245.0	69.33
Y_5 from y_4	+0.80930±0.06081	1892.4	1287.7	68.04
Y_6 from y_5	+0.59306±0.06003	1692.3	1283.1	75.82
Y_3 from y_1	+0.75998±0.04346	1613.4	1188.7	73.68
Y_4 from y_1	+0.62652±0.06482	1738.5	1489.3	85.66
Y_5 from y_1	+0.62028±0.08431	1833.6	1610.3	87.82
Y_6 from y_1	+0.44472±0.10351	1717.6	1599.2	93.11

Table 45 showing statistical constants of milk yield and persistency.

		Age in Lactations.			
		1.	2.	3.	4.
Mean	Total Yield lbs	7099.0 ±98.0	8180.4 ±105.6	8697.4 ±111.7	8861.4 ±120.6
	Persistency %	94.93± 0.193	90.26± 0.272	90.02± 0.280	89.55± 0.304
Standard Deviation	Total Yield lbs	1457.2 ±69.3	1569.6 ± 74.7	1660.6 ± 79.0	1793.5 ± 85.3
	Persistency %	2.87± 0.136	4.04± 0.192	4.17± 0.198	4.52± 0.215
Coefficient of Variation	Total Yield lbs	20.53± 1.017	19.19± 0.946	19.09± 0.955	20.24± 1.001
	Persistency %	3.02± 0.144	4.48± 0.214	4.63± 0.221	5.05± 0.241

Table 46 showing the correlation between the production of different lactations.

Lactations correlated	Total Yield		Persistency	
	r	z	r	z
1st and 2nd	+0.76230	+1.00165±0.06773	+0.45289	+0.48836±0.06773
1st and 3rd	+0.71541	+0.89818±0.06773	+0.24930	+0.25468±0.06773
1st and 4th	+0.52653	+0.58532±0.06773	+0.23416	+0.23859±0.06773
2nd and 3rd	+0.76556	+1.00943±0.06773	+0.52187	+0.58760±0.06773
2nd and 4th	+0.65462	+0.78337±0.06773	+0.45206	+0.48732±0.6773
3rd and 4th	+0.72267	+0.91327±0.06773	+0.51159	+0.56485±0.06773
$R_1(23)$	+0.79218		+0.52207	
$R_1(24)$	+0.65600		+0.45326	
$R_1(34)$	+0.72279		+0.52330	
$R_2(34)$	+0.73286		+0.55569	
$R_1(234)$	+0.74296		+0.55638	

Table 47 showing the results of the tests of Linearity of regression line of Different Ages for Total Yield and Persistency.

Regression line	Degrees of Freedom				Mean Square				Variance Ratio	
	Total Yield		Persistency		Total Yield+		Persistency			
	Within arrays	Deviations from linear regression	Within arrays	Deviations from linear regression	Within arrays	Deviations from linear regression	Within arrays	Deviations from linear regression	Total Yield	Persistency
Y_2 on y_1	190	29	204	15	16.3405	18.2068	13.0318	13.3200	1.1142	1.0221
Y_3 on y_1	190	29	204	15	22.1094	18.5586	16.7495	10.9733	..	..
Y_4 on y_1	190	29	204	15	39.6131	22.6689	19.8250	15.0000	..	..
Y_3 on y_2	190	29	198	21	16.8500	28.1551	12.5737	13.7666	*1.6709	1.0949
Y_4 on y_2	190	29	198	21	30.2121	25.1896	15.2465	27.3857	..	*1.7962
Y_4 on y_3	187	32	200	19	23.9486	29.0968	14.9505	18.1421	1.2150	1.2135

+ In units of class interval used.

* Significant at 5% level.

Table 48 giving the regression equations to determine probable yield (Y_p) from the Observed Yield (y_p

Regression Line	Total Yield	Persistency	Standard Error of Estimate	
			Total Yield	Persistency.
Y_2 from y_1	$Y_2=2351.5+0.82109y_1$	$Y_2=29.50+0.63907y_1$	1018.2	3.61
Y_3 from y_1	$Y_3=2909.6+0.81529y_1$	$Y_3=55.61+0.36253y_1$	1162.9	4.04
Y_4 from y_1	$Y_4=4260.9+0.64806y_1$	$Y_4=54.40+0.37034y_1$	1548.4	4.42
Y_3 from y_2	$Y_3=2071.5+0.80997y_2$	$Y_3=41.48+0.53781y_2$	1070.8	3.56
Y_4 from y_2	$Y_4=2742.3+0.74802y_2$	$Y_4=43.82+0.50666y_2$	1376.8	4.05
Y_4 from y_3	$Y_4=2073.4+0.78047y_3$	$Y_4=39.46+0.55640y_3$	1258.8	3.90

Table 49 showing the values of betas (partial regression coefficients).

	Total Yield			Persistency		
	$^{+B}Y_{3}y_{1}\cdot y_{2}$		$^{B}Y_{3}y_{2}\cdot y_{1}$	$^{B}Y_{3}y_{1}\cdot y_{2}$		$^{B}Y_{3}y_{2}\cdot y_{1}$
Y_3 on y_1 and y_2	+0.31468±0.06386		+0.52566±0.06386	*+0.01629±0.06479		+0.51449±0.06479
	$^{B}Y_{4}y_{1}\cdot y_{2}$		$^{B}Y_{4}y_{2}\cdot y_{1}$	$^{B}Y_{4}y_{1}\cdot y_{2}$		$^{B}Y_{4}y_{2}\cdot y_{1}$
Y_4 on y_1 and y_2	*+0.06567±0.07898		+0.60456±0.07898	*+0.03702±0.06771		+0.43529±0.06771
	$^{B}Y_{4}y_{1}\cdot y_{3}$		$^{B}Y_{4}y_{3}\cdot y_{1}$	$^{B}Y_{4}y_{1}\cdot y_{3}$		$^{B}Y_{4}y_{3}\cdot y_{1}$
Y_4 on y_1 and y_3	*+0.01950±0.06699		+0.70872±0.06699	*+0.11368±0.05959		+0.48324±0.5959
	$^{B}Y_{4}y_{2}\cdot y_{3}$		$^{B}Y_{4}y_{3}\cdot y_{2}$	$^{B}Y_{4}y_{2}\cdot y_{3}$		$^{B}Y_{4}y_{3}\cdot y_{2}$
Y_4 on y_2 and y_3	+0.24490±0.07085		+0.53518±0.07085	+0.25435±0.06601		+0.37885±0.06601
	$^{B}Y_{4}y_{1}\cdot y_{2}y_{3}$	$^{B}Y_{4}y_{2}\cdot y_{1}y_{3}$	$^{B}Y_{4}y_{3}\cdot y_{1}y_{2}$	$^{+B}Y_{4}y_{1}\cdot y_{2}y_{3}$	$^{B}Y_{4}y_{2}\cdot y_{1}y_{3}$	$^{B}Y_{4}y_{3}\cdot y_{1}y_{2}$
Y_4 on y_1y_2 and y_3	*-0.11419±0.07401	+0.30413±0.08038	+0.57153±0,07445	*+0.03086±0.06328	+0.24066±0.07184	+0.37830±0.06613

+ B stands for beta (β)

*not significant.

Table 50 giving the regression equations.

	Total Yield		Persistency	
	Regression equation	Standard error of estimate	Regression equation	Standard error of estimate
Y_3 from y_1 and y_2	$1602.3+0.35861y_1+0.55612y_2$	1018.1	$39.92+0.02369y_1+0.53019y_2$	3.57
Y_4 from y_1 and y_2	$2635.4+0.08083y_1+0.69082y_2$	1377.8	$40.07+0.05841y_1+0.48675y_2$	4.06
Y_4 from y_1 and y_3	$2033.7+0.02400y_1+0.76544y_3$	1245.1	$25.32+0.17938y_1+0.52437y_3$	3.88
Y_4 from y_2 and y_3	$1545.0+0.27984y_2+0.57801y_3$	1228.5	$26.87+0.28442y_2+0.41109y_3$	3.78
Y_4 from y_1 y_2 and y_3	$1647.7-0.14055y_1+0.34752y_2+0.61727y_3$	1208.7	$23.69+0.04869y_1+0.26910y_2+0.41051y_3$	3.79

Table 52 showing the statistical constants of the "standardised" data.

	Total Milk Yield	Maximum Yield	Persistency
Mean	8832.6±36.02	51.22±0.185	89.33±0.098
Standard deviation	1761.8±25.47	9.06±0.131	4.79±0.069
Coefficient of variation	19.95± 0.30	17.68±0.263	5.36±0.078
μ_2	49.6612691	82.0004682	22.9234828
μ_3	+125.2410568	+315.8901155	-47.2867415
μ_4	8268.4593952	21778.8727557	1855.0033541
β_1	0.1280678	0.1809769	0.1856252
β_2	3.3526558	3.2389387	3.5300715
γ_1	+0.3578656	+0.4254138	-0.4308420
γ_2	+0.3526558	+0.2389387	+0.5300715
Skewness	0.18250	0.21724	0.18665
Mode	8546.3	49.75	90.22
k	+0.3089	-2.1809	+0.1866
Type of Curve	4	1	4

Table 53 showing analysis of Variance and Covariance of Maximum (M) and Persistency (P) corrected for month of calving and age.

Herd	Total Records	Total			Between herds			Within Herds												Correlation Coefficient		
								Total				Between cows				Within cows						
		SM²	SP²	SMP	(SM)²	(SP)²	(SM)(SP)	d/f	sm²	sp²	smp	d/f	sm²	sp²	smp	d/f	sm²	sp²	smp	Within herd	Between cows	Within cows
A	448	1246959.31	3621389.3114	2078319.261	1198851.03	3612833.6001	2081165.364	447	48108.28	8555.7113	-2846.103	105	338131.76	4543.0226	-1124.821	342	14276.52	4012.6887	-1721.282	-0.14029	-0.09073	-0.22742
B	451	1378960.92	3539264.9019	2178427.413	134705965	3528588.0394	2180187.740	450	31901.27	10676.8625	-1760.327	114	20144.49	6049.5262	- 660.754	336	11756.78	4627.3363	-1099.573	-0.09539	-0.05986	-0.14908
C	713	1738227.74	5705114.8845	3103985.594	1695400.81	5687742.1999	3105318.459	712	42826.93	17372.6846	-1332.865	186	26346.77	10565.6831	1373.075	526	16480.16	6807.0015	-2705.940	-0.04886	+0.08230	-0.25548
D	246	727242.13	1952740.1046	1173543.092	708281.95	1946754.3813	1174244.860	245	18960.18	5985.7233	- 701.768	67	12313.32	3977.0408	- 223.740	178	6646.86	2008.6825	- 478.028	-0.06587	-0.03197	-0.13082
E	534	1388962.76	4325027.9044	2410083.654	1346263.66	4314182.4823	2409984.874	533	42699.10	10845.4221	98.780	121	22473.65	5282.6457	693.298	412	20225.45	5562.7764	- 594.518	+0.00459	+0.06363	-0.05605
Total	2392	6480352.86	19143537.1068	10944359.014	6295857.10	19090100.7030	10950901.297	2387	184495.76	53436.4038	-6542.283	593	115109.99	30417.9184	+ 57.058	1794	69385.77	23018.4854	-6599.341	-0.06589	+0.00096	-0.16513
orrection		6284121.32	19089456.5696	10952646.307	6284121.32	19089456.5696	10952646.307															
		196231.54	54080.5372	-8287 .293	11735.78	644.1334	-1745.010															

	r	z
Total data	-0.08045	-0.08650±0.02046
Between herds	-0.63468	-0.74930±0.70711

Table 55 showing analysis of variance and covariance of Persistency (P) and Total Yield (Y) corrected for Month of Calving and Age.

Herd	Total Records	Total			Between herds							Within			
								Total				Between cows			
		SP^2	SY^2	SPY	$(SP)^2$	$(SY)^2$	(SP)(SY)	d/f	sp^2	sy^2	spy	d/f	sp^2	sy^2	spy
A	448	3621389.3114	38923572365	367897967.95	3612833.6001	37276195567	366977781.17	447	8555.7113	1647376798	920186.78	105	4543.0226	1233453408	621200.36
B	451	3539264.9019	40212474015	372486920.79	3528588.0394	38942939315	370698390.67	450	10676.8625	1269534700	1793530.12	114	6049.5262	818264640	1113297.79
C	713	5705114.8845	52083662283	537393199.81	5687742.1999	50241749877	534567227.74	712	17372.6846	1841912406	2825972.07	186	10565.6831	1270581290	2021082.85
D	246	1952740.1046	22165520691	204991118.82	1946754.3813	21370517746	203968500.14	245	5985.7233	795002945	1022618.68	67	3977.0408	539068201	770206 .36
E	534	4325027.9044	40851385090	413545915.92	4314182.4823	39314726961	411838446.91	533	10845.4221	1536658129	1707469.01	121	5282.6457	879426643	995492.76
Total	2392	19143537.1068	194236614444	1896315123.29	19090100.7030	187146129466	1888045346.63	2387	53436.4038	7090484978	8269776.66	593	30417.9184	4740794182	5521280.12
Correction		19089456.5696	186783922306	1888280586.60	19089456.5696	186783922306	1888280586.60								
		54080.5372	7452692138	8034536.69	644.1334	362207160	-235239.97								

	r	z
Total data	+0.40021	+0.42390±0.02046
Between herds	-0.48702	-0.53215±0.70711

nce of Persistency (P) and Total Yield
and Age.

Within Herd									
Between cows			Within Cows				Correlation Coefficient		
sp^2	sy^2	spy	d/f	sp^2	sy^2	spy	Within Herd	Between cows	Within cows
4543.0226	1233453408	621200.36	342	4012.6887	413923390	298986.42	0.24510	0.26242	0.23199
6049.5262	818264640	1113297.79	336	4627.3363	451270060	680232.33	0.48715	0.50038	0.47073
10565.6831	1270581290	2021082.85	526	6807.0015	571331116	804889.22	0.49957	0.55161	0.40814
3977.0408	539068201	770206 .36	178	2008.6825	255934744	252412.32	0.46878	0.52603	0.35204
5282.6457	879426643	995492.76	412	5562.7764	657231486	711976.25	0.41825	0.46186	0.37236
0417.9184	4740794182	5521280.12	1794	23018.4854	2349690796	2748496.54	0.42485	0.45978	0.37372

Table 57 showing analysis of variance and covariance of Maximum (M) and Total Yield (Y) corrected for Month of Calving and Age.

Herd	Total Records	Total			Between herds			Within												Correlation Coefficient		
								Total				Herd										
												Between cows				Within cows						
		SM²	SY²	SMY	(SM)²	(SY)²	(SM)(SY)	d/f	sm²	sy²	smy	d/f	sm²	sy²	smy	d/f	sm²	sy²	smy	Within herd	Between cows	Within cows
A	448	1246959.31	38923572365	219277249.8	1198851.03	37276195567	211396795.9	447	48108.28	1647376798	7880453.9	105	33831.76	1233453408	5966341.9	342	14276.52	413923390	1914112.0	.88521	.92360	.78740
B	451	1378960.92	40212474015	233765800.0	1347059.65	38942939315	229038124.3	450	31901.27	1269534700	4727675.7	114	20144.49	818264640	3307179.0	336	11756.78	451270060	1420496.7	.74288	.81458	.61671
C	713	1738227.74	52083662283	298615508.7	1695400.91	50241749877	291855963.5	712	42826.93	1841912406	6759545.2	186	26346.77	1270581290	4941580.4	526	16480.16	571331116	1817964.8	.76107	.85408	.59246
D	246	727242.12	22165520691	126001904.6	708281.95	21370517746	123029832.5	245	18960.18	795002945	2972022.1	67	12313.32	539068201	2043230.2	178	6646.86	255934744	928791.9	.76550	.79306	.71211
E	534	1388962.76	40951385090	236762138.2	1346263.66	39314726961	230060885.8	533	42699.10	1536658129	6701302.4	121	22473.65	879426643	3896278.7	412	20225.45	657231486	2805023.7	.82730	.87642	.76936
Total	2392	6480352.86	194236614444	1114422601.3	6295857.10	187146129466	1085381602.0	2387	184495.76	7090484978	29040999.3	593	115109.99	4740794182	20154610.2	1794	69385.77	2349690796	8886389.1	.80294	.86276	.69596
Correction		6284121.32	186783922306	1083407970.2	6284121.32	186783922306	1083407970.2															
		196231.54	7452692138	31014631.1	11735.78	362207160	1973631.8															

	r	Z
Total data	+0.81101	+1.12985±0.02046
Between herds	+0.95726	+1.91206±0.70711

Table 60 showing analysis of variance of persistency of individual herds due to the year of record.

Herd	Degrees of Freedom			Mean Square			Portion of total variance due to differences between years.
	Total	Between years	Within years	Total	Between years	Within years	%
A	447	14	433	19.1403	*38.2750	18.5216	3.232
B	450	14	436	23.7263	*82.0448	23.5922	7.892
C	712	14	698	24.3998	*64.6686	21.7310	3.310
D	245	9	236	24.4315	*95.2449	21.8538	11.053
E	533	14	519	20.3479	*104.3577	18.0817	11.137

*significant.

Table 62 showing analysis of variance of maximum of individual herds due to the year of record.

	Degrees of Freedom			Mean Square			
Herd	Total	Between years	Within Years	Total	Between years	Within years	Portion of total variance du to differences between years %
A	447	14	433	107.6248	*1988.41	46.8140	56.502
B	450	14	436	70.8917	* 572.57	54.7827	22.723
C	712	14	698	60.1501	* 252.19	56.2983	6.404
D	245	9	236	77.3884	* 171.21	73.8104	4.623
E	533	14	519	80.1109	* 576.64	66.7169	16.719

*significant

Table 64 showing analysis of variance of total yield of individual herds due to the year of the record.

Herd	Degrees of Freedom			Mean Square			Portion of total variance of total yield due to differences between years.
	Total	Between years	Within years	Total	Between years	Within years	%
A	447	14	433	3685406.7	*68679553.1	1583979.3	57.020
B	450	14	436	2821188.2	*12291184.1	2517105.7	10.779
C	712	14	698	2586955.6	*6031466 .1	2517868.0	2.671
D	245	9	236	3244910.0	*11726790.2	2921448.4	9.968
E	533	14	519	2883035.9	*23014381.7	2339993.8	18.836

*significant

Table 68 summarising results of analysis of variance of individual herds.

Herd	Degrees of Freedom		Mean Square			Portion of total persistency due to differences	
	Between cows	Within cows	Total	Between cows	Within cows	Between cows %	Within cows %
A	105	342	19.1403	*43.2669	11.7330	38.700	61.300
B	114	336	23.7264	*53.0668	13.7718	41.955	58.045
C	186	526	24.3998	*56.8047	12.9411	46.962	53.038
D	67	178	24.4315	*59.3588	11.2847	53.811	46.189
E	121	412	20.3479	*43.6582	13.5019	33.645	66.355
F	53	167	28.7486	*66.3324	16.8208	41.490	59.510

*Significant

Table 70 summarising results of analysis of variance of maximum of individual herds.

Herd	Degrees of Freedom		Total	Mean Square		Portion of total variance of maximum due to differences	
	Between cows	Within cows		Between cows	Within cows	Between cows %	Within cows %
A	105	342	107.6248	*322.2072	41.7442	61.2132	38.7868
B	114	336	70.8917	*176.7061	34.9904	50.6424	49.3576
C	186	526	60.1502	*142.6493	31.3311	47.9118	52.0882
D	67	178	77.3885	*183.7809	37.3419	51.7474	48.2526
E	121	412	80.1109	*185.7326	49.0909	38.7213	61.2787
F	53	167	44.2875	*101.9808	25.9777	41.3432	58.6568

*significant.

Table 72 summarising the results of analysis of variance of individual herds.

Herd	Degrees of Freedom		Total	Mean Square		Portion of total variance due to differences	
	Between cows	Within cows		Between cows	Within cows	Between cows %	Within cows %
A	105	342	3685406.7	*11747175.3	1210302.3	67.159	32.841
B	114	336	2821188.2	*7177760.0	1343065.6	52.394	47.606
C	186	526	2586955.6	* 6831082.2	1086180.8	58.013	41.987
D	67	178	3244910.0	* 8045794.0	1437835.6	55.689	44.311
E	121	412	2883035.9	* 7267988.8	1595222.0	44.669	55.331
F	53	167	1730327.4	* 4329543.8	905426.4	47.673	52.327

*Significant.

Table 80 showing the intra-sire correlation between the persistency of dam and daughter and persistency "index" of each bull.

Herd	Bull	Dam and Daughter pairs	Mean persistency		Mean Square		r	Regression equation daughters persistency from dams persistency	Bulls' index
			Dam %	Daughter %	Dam	Daughter			
A	1	18	89.17	90.18	12.44	11.58	+0.19839	73.11+0.19144x	91.19
	2	13	89.04	90.81	8.42	1.36	-0.45767	107.19-0.18393x	92.58
	3	7	91.74	85.81	6.27	21.56	+0.18923	53.63+0.35085x	79.88
	4	7	89.67	87.52	6.45	18.61	+0.62625	-7.90+1.06416x	85.37
	5	7	89.33	88.76	13.77	7.76	+0.16627	77.61+0.12482x	88.91
	6	6	89.91	87.85	15.11	16.58	+0.41150	48.09+0.43110x	85.79
	7	5	89.25	86.59	22.71	8.59	+0.92465	42.31+0.49613x	83.93
	8	5	92.64	90.97	1.82	4.57	+0.60880	1.50+0.96581x	89.30
	All	68	89.80	89.02	10.89	12.53	+0.26897	65.19+0.26542x	
B	1	9	88.40	87.84	19.44	13.44	+0.38417	59.36+0.32189x	87.19
	2	8	87.79	88.83	6.51	8.99	+0.19650	68.55+0.23095x	89.87
	3	10	88.94	84.62	4.66	14.23	-0.26925	126.46-0.47040x	80.30
	4	8	89.82	84.80	11.70	9.06	+0.44451	49.67+0.39113x	79.78
	5	6	90.03	91.21	3.12	19.60	+0.12528	62.96+0.31377x	92.39
	6	29	89.06	87.37	20.16	15.41	+0.34358	60.61+0.30045x	85.68
	All	70	88.99	87.24	13.46	16.33	+0.27487	62.93+0.27319x	
C	1	8	90.08	88.54	10.99	7.59	+0.37055	80.80+.30796x	87.00
	2	8	89.99	88.21	10.02	10.16	+0.78345	17.22+0.78892x	86.43
	3	7	89.11	89.71	26.72	7.94	-0.22212	100.50-0.12108x	90.31
	4	10	91.51	86.62	3.32	25.03	+0.07276	68.34+0.19978x	81.73
	5	7	92.47	90.42	7.99	4.14	+0.27331	72.24+0.19660x	88.38
	All	40	90.67	88.53	11.24	12.67	+0.18610	70.87+0.19477x	
D	1	5	92.05	87,84	24,82	4.43	+0.90395	52.68+0.38196x	83.63
	2	19	89.20	86.44	9.47	10.10	+0.25351	62.98+0.26303x	83.66
	All	24	89.79	86.73	13.12	9.08	+0.35525	59.18+0.30681x	
E	1	17	89.33	88,85	9.75	8.38	+0.02628	86.67+0,02438x	88.37
	2	7	92.22	89.39	2.61	12.19	-0.49755	188.58-1.07551x	86.56
	3	5	88.96	88.62	22.31	6.46	+0.69439	55.37+0.37371x	88.28
	4	7	89.30	89.67	27.89	8.39	+0.04889	87.27+0.02682x	90.04
	All	36	89.83	89.08	13.66	8.24	+0.07122	83,87+0.05795x	
	Total	238	89.71	88.19	12.45	13.30	+0.23764	66.87+0.22822x	

Table 82 showing the intra-sire correlation between the maximum yield of dam and daughter and maximum yield "index" of each bull.

Herd	Bull	Dam and Daughter pairs	Mean Maximum Dam	Mean Maximum Daughter	Mean Square Dam	Mean Square Daughter	r	Regression equation daughters maximum yield for dams maximum yield	Bulls' Index.
A	1	18	51.84	58.12	43.24	55.20	+0.39863	34.77+0.45042x	64.40
	2	13	50.12	54.81	25.22	85.04	+0.29565	27.60+0.54293x	59.50
	3	7	59.39	64.70	51.76	18.26	+0.48959	47.43+0.29077x	70.01
	4	7	42.43	45.49	19.10	13.28	+0.04343	45.49+0.03621x	48.55
	5	7	44.69	44.34	18.30	16,75	+0.34092	30.27+0.32611x	44.99
	6	6	59.43	57.17	23.23	43.98	-0.49496	97.64-0.68105x	54.91
	7	5	42.84	56.08	3.54	19.01	+0.21187	35.04+0.49118x	69.32
	8	5	45.12	53.48	5.43	4.37	-0.67454	80.79-0.60525x	
	All	68	50.10	54.92	58.36	71.60	+0.25706	39.22+0.31332x	
B	1	9	52.39	53,58	42.41	41.41	+0.41419	32.14+0.40924x	54.77
	2	8	54.92	63.32	22.02	17.66	+0.07209	59.77+0.06456x	71.72
	3	10	55.81	55.34	30.80	50.79	+0.22967	38.90+0.29492x	54.87
	4	8	52.16	47.56	44.99	26.25	+0.32666	34.54+0.24957x	42.96
	5	6	55.18	52.32	67.01	35.62	+0.72909	21.48+0.55896x	49.46
	6	29	51.12	57.72	57.39	35.58	+0.11854	52.95+0.09333x	64.32
	All	70	52.86	55.87	47.45	50.61	+0.24657	44.58+0.21351x	
C	1	8	50.99	48.02	44.75	19.39	+0.45477	32.76+0.29935x	45.05
	2	8	47.03	48.81	18.70	31.36	+0.66332	8.41+0.85898x	50.59
	3	7	49.53	51.40	37.75	34.99	-0.10234	56.28-0.09853x	53.27
	4	10	48.09	47.02	61.07	11.60	+0.03795	46.22+0.01654x	45.95
	5	7	49.56	51.53	22.28	29.43	+0.92698	-1.27+1.06537x	53.50
	All	40	48.96	49.13	36.65	25.07	+0.31335	37.00+0.24772x	
D	1	5	51.90	52.26	6.36	11.07	+0.76653	-0.41+1.01100x	52.62
	2	19	55.20	53.92	57.18	61.39	+0.34575	34.16+0.35801x	52.64
	All	24	54.52	53.57	45.85	49.90	+0.35830	33.19+0.37377x	
E	1	17	48.42	51.08	31.79	47.69	+0.11309	45.15+0.12248x	53.74
	2	7	50.63	49.43	22.38	23.30	-0.10875	56.70-0.11095x	48.23
	3	5	49.92	51.84	15.82	55.93	-0.26175	76.41-0.49218x	53.76
	4	7	49.04	50.04	51.03	40.79	-0.43304	69.03-0.38717x	51.04
	All	36	49.18	50.66	29.71	39.84	-0.07988	55.23-0.09297x ?	
	Total	238	51.03	53.44	49.07	56.68	+0.22262	42.06+0.22298x	

Table 84 showing the intra-sire correlation between the total yield of dam and daughter and total yield "index" of each bull.

Herd	Bull	Dam and daughter pairs	Mean Yield		Mean Square		r	Regression equation Daughter total Yield for dams total yield	Bulls' index (20-D)
			Dam	Daughter	Dam	Daughter			
A	1	18	9124.3	10456.7	1545520	1179708	+0.18379	8949.0+0.16524x	11789.1
	2	13	8625.5	9815.1	881832	3305105	-0.09834	11457.3-0.19039x	11004.7
	3	7	10902.6	10611.3	1590364	2434058	+0.84310	-760.4+1.04303 x	10320.0
	4	7	7467.3	7519.7	817573	1150055	+0.67670	2247.6+0.7062x	7572.1
	5	7	7848.6	7675.1	1051009	948521	+0.02004	7525.7+0.01904x	7501.6
	6	6	10634.8	9797.0	599773	1521231	+0.15728	7133.1+0.25049x	8959.2
	7	5	7471.6	9336.0	909701	1162350	+0.87522	1944.2+0.98932x	11200.4
	8	5	8423.2	9545.8	245807	758854	-0.30794	14103.3-0.54107x	10668.4
	All	68	8870.3	9553.7	2188473	2701963	+0.23967	6896.2+0.30174x	
B	1	9	8944.4	9171.4	1731598	1440469	+0.49034	5171.2+0.44723x	9398.4
	2	8	9467.3	10818.7	1081063	2236182	+0.49594	4066.3+0.71327x	12170.1
	3	10	9717.5	8587.2	1514587	1755689	+0.38665	4541.9+0.41629x	7456.9
	4	8	9301.1	7579.6	1141404	486528	+0.19039	6423.5+0.12430x	5858.1
	5	6	9781.2	9383.2	549598	1995675	+0.54541	-782.4+1.03930x	8985.2
	6	29	8823.0	9624.6	3832071	3842455	+0.09245	8880.2+0.08437x	10426.2
	All	70	9176.8	9300.2	1700400	2290514	+0.24484	7050.3 +0.24517x	
C	1	8	8961.2	8010.7	2179701	1598291	+0.72657	2435.3+0.62217x	8010.7
	2	8	8448.1	8337.0	1642441	1000016	+0.79263	3111.7+0.61852x	8225.9
	3	7	8515.1	8732.4	2743072	856475	+0.44074	6635.4+0.24627x	8949.7
	4	10	8431.4	7808.5	1411369	952031	+0.04732	7480.9+0.03886x	7135.6
	5	7	9135.0	9089.4	1284010	993705	+0.94237	1516.3+0.82902x	9043.8
	All	40	8678.5	8340.5	1631277	970705	+0.56112	4584.0+0.43285x	
D	1	5	9815.0	8660.6	414408	79[illegible]4	+0.78448	-1987.7+1.08490x	7506.2
	2	19	9717.5	8921.0	3096657	152[illegible]75	+0.19383	7600.1+0.13593x	8124.5
	All	24	9737.8	8866.7	2495541	132[illegible]34	+0.22376	7276.1+0.16334x	
E	1	17	8160.1	8499.6	1308750	147[illegible]	+0.06994	7868.8+0.07421x	8839.1
	2	7	8952.7	8220.7	598952	36[illegible]	+0.50304	4707.6+0.39241x	7488.7
	3	5	8352,4	8689.2	803040	153[illegible]	-0.15762	10508.9-0.21786x	9026.0
	4	7	8496.0	8429.7	22969[illegible]	12[illegible]	0.08229	7914.8+0.06060x	8429.7
	All	36	8406.2	8458.1	1277688	114[illegible]	0.07665	7830.7+0.07464x	
	Total	238	8945.5	9040.2	1964412	215[illegible]	0.26314	6744.2+0.25666x	

Table 89 showing the analysis of variance of persistency of paternal half-sisters. (Individual herds).

Herd	Degrees of Freedom		Mean Square			F	Portion of total variance due to differences
	Between families	Within families	Total	Between families	Within families		Between families
A	14	72	11.6383	17.8585	10.4288	1.712	10.392
B	13	81	15.2265	36.9512	11.7399	*3.147	22.898
C	34	117	17.8795	28.1014	14.9091	*1.885	16.613
D	10	36	12.5079	15.5079	11.9005	1.303	6.182
E	12	85	10.6882	15.7009	9.9805	1.573	6.621
All	83	391	14.2452	24.4497	12.0791	*2.024	15.206

*significant

Table 92 showing the analysis of covariance and test of significance of adjusted means of different bulls. (Maximum Yield).

					Errors of Estimate			
Source of Variation	d/f	$Sx^{2\phi}$	Sy^{2+}	Sxy	Sum of Squares	d/f	Mean Square	F
Total	237	11630.11	13432.82	4403.87				
Herds	4	878.53	1612.74	783.62				
Bulls	20	2578.35	3622.65	1798.00				
Within bulls	213	8173.23	8197 .47	1822.25	7791.15	212	36.7507	
Bulls + Within Bulls	233	10751.58	11820.08	3620.25	10601.08	232		
Difference for testing adjusted bull means					2809.93	20	140.4965	*3.823

ϕ mates + daughters

* significant

Table 94 showing analysis of variance of maximum of paternal half-sisters (individual herds).

	Degrees of Freedom		Mean Square				
Herd	Between families	Within families	Total	Between families	Within families	F	Portion of total variance due to differences between families %
A	14	72	72.8129	276.7557	33.1574	*8.3467	54.462
B	13	31	46.2715	161.1215	27.8388	*5.7877	39.836
C	34	117	36.0603	46.0900	33.1457	1.3905	8.083
D	10	36	44.8715	53.6100	42.4442	1.2631	5.409
E	12	85	45.4521	66.1150	42.5349	1.5544	6.418
All	83	391	47.5305	106. 8157	34.9457	*3.0566	26.477

*significant

Table 97 showing the analysis of covariance and test of significance of adjusted means of different bulls. (Total Yield).

Source of Variation	d/f	Sx2ϕ	Sy2+	Sxy	Sum of Squares	d/f	Mean Square	F
Total	237	465565656	511717536	155388725				
Herds	4	32516107	55162614	17058519				
Bulls	20	97979862	137772151	52330285				
Within bulls	213	335069687	318782771	85999921	296709791	212	1399574	
Bulls + within bulls	233	433049549	456554922	138330206	412367725	232		
Difference for testing adjusted bull means					115657934	20	5782896	*4.132

ϕ mates　　*significant.　　+ daughters.

Table 99 showing the analysis of variance of total yield of **paternal** half-sisters.

Herd	Degrees of Freedom		Mean Square			F	Portion of total variance due to differences between families
	Between families	Within families	Total	Between families	Within families		%
A	14	72	2690979	9964029	1276775	*7.804	52.553
B	13	81	1935512	5558988	1353967	*4.106	30.046
C	34	117	1847709	1719971	1884830	..	..
D	10	36	1915709	1621304	1997488	..	..
E	12	85	1766240	2491298	1663878	1.497	5.795
All	83	391	2008047	3811456	1625226	*2.345	19.064

*significant,