V A R I O U S   P A P E R S.

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B Y

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B O O K I.

E X P L A N A T O R Y.

This collection of papers is submitted as fairly representative of the Author's work during the period 1920-1922; and a few remarks in explanation of the various items may be given.

The greater part of the time has been spent on the general subject of nozzle flow; and, within the period referred to, the Author has had the honour of collaborating with Professor A. L. Mellanby, D.Sc., in the experimental and analytical work, and in the composition, of the following published papers on this subject:-

"Steam Action in Simple Nozzle Forms"
British Association, Section G, Aug. 1920.
"Engineering", Sept. 3. 1920.

"Pressure-Flow Experiments on Steam Nozzles".
Proc.Inst.Engrs.& Shipdrs. in Scot., Nov.1920.
"Engineering", March 4, 1921.

"On the Losses in Convergent Nozzles".
Proc. N-E. Coast Inst.Engrs.& Shipdrs., Feby.1921.

"The Supersaturated Condition as shown by Nozzle Flow".
Proc.Inst.Mech.Engrs., Paris Meeting, June 1922.
"Engineering", June 30, 1922.

The first paper of the present series also deals with this subject and constitutes a study of the various loss effects in nozzles. It is thought that the treatment given carries this particular matter definitely beyond the point which it had previously reached. The first two sections of the paper utilise experimental data contained in the publications mentioned; and the third section is primarily concerned with an analysis of some results provided by Dr Morley's reaction experiments on air nozzles (Proc. Inst. Mech. Engrs., Jany. 1916). The Author can, therefore, only admit a partial responsibility for the experimental figures used in this particular paper, but must accept the full responsibility for the manner and results of the various discussions.

The different sections into which this "Study of Nozzle

Explanatory. (contd.)

Losses" is divided are characteristic of the nature and difficulties of the subject. Section A, on "The Frictional Effect", deals with a loss type that is readily appreciated. The treatment given is fairly complete and leads, finally, to the establishment of coefficients, and the formulation of methods of calculation, that should be of service. Section B, on "The Entrance Loss", examines a rather mysterious effect, and is mainly speculative as to causes. It introduces new aspects of the matter in the various points of view discussed, and in the relation of certain sets of data. The losses in jet compression receive attention in Section C; but this is merely an attempt to obtain some idea of the general magnitude of a loss effect not hitherto isolated; and the data are rather inadequate. The mode of treatment and certain general findings may be of interest. Section D comprises remarks on various kinds of loss, and provides an opportunity for summary, illustration and suggestion.

Some experimental work has also been carried out on an impulse turbine, and the next two papers deal with certain aspects of this subject. The first of these, on "Jet Action in Blading", deals mainly with certain peculiar variations observed in the angle of outflow from blades. So far as the matter is dealt with here the data have been obtained with the turbine wheel locked; but the effects observed seemed to require some explanation before proceeding with the examination of the moving wheel. The paper represents an endeavour to provide this explanation, but also contains some tentative considerations of the problem of the moving blade. The full experimental investigation promises to be very prolonged, and the discussion given may be held as representing only the first phase.

The other paper originating in the turbine experiments is on "Turbine Wheel Friction". This subject was previously treated by the Author in a paper in 1912, entitled:-

"The Steam Friction of Turbine Wheels"
Proc. R.T.C. Scientific Soc., Dec., 1912.
"Engineering", Aug. 22, 1913.

and the present attack was prompted by the apparent inconsistency between the results contained in that paper and those of the recent tests. It was found necessary to re-examine the whole subject, and the treatment now given is of fair generality. The outcome of the investigation represents probably the most complete result yet attained for full scale bladed wheels: and some conclusions arrived at regarding the mode of treatment of these involved resistance effects may deserve the emphasis accorded them.

The three papers just described are concerned with matters that have been experimentally examined, and are recorded as "Papers on Experimental Subjects". At various intervals, however, time has been occupied with short investigations of a more limited aim, and a selection of these is included, listed under the heading "Miscellan-eous Investigations". The main papers show the treatment of general problems arising out of research: the others illustrate methods of dealing with specific problems arising out of design, and the list is typical rather than exhaustive. The treatment in each case of the latter is brief, but sufficient to explain the motive and show the result.

The first on "High Blade Stresses at Low Powers" is an endeavour to present the underlying cause of a somewhat unusual type of blade failure. The conditions are rather unexpected; but a very definite case of the kind actually occurred in a naval turbine set, and it is thought that this short discussion of the problem may be of some interest.

The second article of this group, entitled "Stresses in a Special Form of Turbine Rotor", undertakes the application of disc stress theory to a particular form of rotor that has come into use in the course of developments in high speed marine turbines. As this type is now frequently employed the reduction to a direct and easy calculation is probably useful.

The remaining two articles are illustrations of processes developed for the purpose of evading the more complicated, but more correct, standard methods. The one dealing with "Reheat Factors"

Explanatory. (contd.)

simply presents a method for calculation of the essentials of a cumulative heat curve. The method does not require any high degree of accuracy in use; nor is it complicated by change of field; and these are advantages that will appeal to those who have to make such calculations. The final article on "The Provisional Determination of Critical Speed" endeavours to replace - for certain purposes - the usual lengthy and tedious graphical computation of a critical speed, by a straightforward calculation. The result obtained is shown to be in good agreement with actual cases covering a very wide range of size, which fact demonstrates its usefulness.

The Author's chief activities, apart from his association with Professor Mellanby on nozzle work, must be taken as represented by the three papers of the first group. The minor investigations comprising the second set are not advanced as part of any scheme of research, but as accidental subjects leading, it is hoped, to useful results, but included in this compilation only to show a particular aspect of the work which has been carried through.

The Author desires here to record his great indebtedness to Professor Mellanby, under whose general direction the various researches have been conducted, and who has, throughout, been generous in granting all the necessary facilities, and ready with helpful suggestion. His thanks are also gratefully extended to Mr D. S. Anderson, B.Sc., A.R.T.C., who willingly supplied those figures from his own experimental data to which reference is duly made in the text; and to Mr T. W. F. Brown, B.Sc., A.R.T.C., and Mr J. Cameron, B.Sc., for their capable assistance with some of the turbine tests.

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December, 1922.

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PAPERS ON EXPERIMENTAL SUBJECTS:—

MISCELLANEOUS INVESTIGATIONS:—

PAPERS ON EXPERIMENTAL SUBJECTS.

NO. 1.

A STUDY OF NOZZLE LOSSES

A STUDY OF NOZZLE LOSSES

A : — THE FRICTIONAL EFFECT.

Introductory: It has always been recognised that a fluid expanding in a nozzle is subject to frictional loss: but, in spite of the very considerable amount of experimental work that has been carried out on nozzles, there is a remarkable dearth of quantitative infor::mation on this essential point. It is certain that the more usual experimental methods, involving flow, impact or reaction observations cannot permit of accurate deductions regarding the losses, since they only provide evidence concerning the aggregate effect of any detrimental processes at work within the expansion length. Consequently, a very large proportion of the experimental data on this subject is useless as direct information on friction loss; the more so that the overall facts contained therein do not help, in any way, to resolve the doubt as to whether the total loss arises from friction alone or is the summation of several effects.

In an expanding fluid the factors on which any kind of loss will depend are subject to rapid change; and, in order that satisfactory conclusions may be drawn from any actual observations, it is essential that examination be made at different stages in the jet development. This detail information is entirely lacking in the usual data, and the weakness resulting therefrom is well shown by the general habit of applying total efficiencies with practically no consideration of the modifying influences present in any specific case.

The frictional resistances created by high speed fluid flow within confining surfaces ~~has~~ been extensively investigated; but while the speed range hitherto explored well covers the usual limits of pipe flow it falls far short of the speeds common in nozzle expansion. There is, therefore, no absolute certainty that full agreement will exist when the step up to nozzle speeds is taken.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

The principle of dynamical similarity, on which the general law of frictional resistance depends, is of such comprehensiveness that we should naturally expect very close agreement; but this can hardly be said to be apparent if the total losses, as disclosed by most experimental results, are supposed chargeable to friction only.

It would seem desirable, then, that the nature and extent of the agreement with theory in this matter should be demonstrated by means of actual friction results for nozzles. This demands, as an essential, the isolation ofthe frictional loss incurred in the expansion; which, in turn, entails observation of the whole expan: :sion range, since only thereby can we hope to exclude effects not primarily due to friction.

This necessary detail study has been the main function of the recent researches of Mellanby and Kerr on elementary nozzles; and later work, along the same lines, carried out on some special forms by Anderson has added to the essential data. So far these various investigations have been content with a demonstration of the values of friction in terms of a special loss factor, without undertaking the reduction to more definite forms or the detail comparison with full theory; but the very important matter of the isolation of the friction totals has been achieved.

The main duty of the present article is to conduct a special review of the friction values disclosed by this recent work with intent to place the actual facts on a clear and rational basis, and to establish the requisite constants in a simple and suitable method of calculation. Such an exposition is certainly required, as the faulty process of working with a velocity coefficient - which does not seem to be affected by nozzle size or condition - is almost universally employed.

While the essential purpose is as described the general consideration to be given to the frictional effect provides a convenient opportunity for the treatment of a special point of some importance. This relates to the controversial question as to the true form of the efficiency curve, and a few explanatory remarks on this matter may be made here in justification of the attention it receives later.

In many reputable nozzle experiments efficiency curves have been obtained that show a falling tendency with limitation of the expansion range, thus apparently indicating that the best steam speed of operation is high. In fact, it is a common deduction from tests that the best pressure range for a nozzle is the maximum range possible to its particular form. This would seem to show the superiority of high speeds in turbines but, in practice, it is apparently found that turbines with low steam speeds are in every way as efficient; and, indeed, it is customary to add a few stages to a standard design to meet any demand for improved consumption. It is, therefore, inferred in many cases that the falling characteristic of the efficiency curve is wrong; and a somewhat ridiculous situation arises, in which experimental workers on nozzles present coefficient curves of one form, whilst designers prefer and employ curves of a different type.

In the Author's opinion the argument seemingly derived from turbine practice is invalid. Nozzle action is certainly an important item in the operation of a turbine, but it is by no means the whole matter; there are entrance, gap, blading and other effects, all or any of which may obscure the issue. It is impossible to get true and indubitable evidence regarding pure nozzle action from complete turbine tests; and no test engineer could possibly claim that his results are of such fineness as to determine a point only clearly shown in very careful stationary nozzle experiments. Reference to other papers in this collection will show that there are several occurrences in turbines not by any means certain in their influence, and likely to preclude the possibility of sound detail deductions being made from their total effects.

In the light of the discussion in Section E of this paper there would seem to be little difficulty in achieving a correct view of this matter. It has been explained there that the ordinary frictional loss is separable from an additional loss apparently inherent in the expansion effect. This latter is of such an order as necessitates a falling coefficient. Although this fact offers a satisfactory explanation, the mystery in which the mechanism of the loss is enshrouded and the tendency to ignore evidence pointing in this particular direction combine to prevent a free acceptance of the result.

Now, if this second type of loss were considered to be effaced from the list the friction effect alone would - admittedly - govern the nature of the efficiency curve; and there is no directly apparent reason to suppose that it should then have a definite falling tendency. It is, however, proposed in a later sub-section of this article to demonstrate that this characteristic still persists even when the chief cause of the fall in the curve is, for the sake of argument, discarded. The treatment given shows that, under frictional conditions alone, there is no possible reason to expect other than a depression of the efficiency value with restriction of the expansion range. The tendency is quite clear and such a result should, at least, serve to indicate that the usual exper: :imental curves for nozzles operating on fair ranges are rational in form. It may also be considered a demonstration that the differ: :ent kind of curve is fallacious; and, when account is taken of the fact that the other indicated sources of loss are entirely neglected, the conclusion as to the general trend of the efficiency curve must be recognised as definite.

The Law of Friction Loss: In dealing with the resistance of fluids at speeds commensurate with that of sound the complete expression for frictional force may be written:-

$$S = \rho \cdot \frac{u^2}{2g} \cdot d^2 \cdot \phi\left\{\frac{\mu}{\rho \cdot u \cdot d}, \frac{u}{u_s},\right\}.$$

In this g is inserted to give the usual units; ρ is the density; u is the speed; d is a characteristic dimension; μ is the coefficient of viscosity; u_s is the acoustic velocity; and ϕ denotes an unknown function of the given ratios, of which the first is the well known Reynold's number, and the other simply expresses the ratio of the actual and the acoustic velocities. This latter is easily introduced into the general equation by assuming that the elasticity of the fluid is of importance in the resistance effects.

Changing the expression to read energy loss per unit volume per unit time, instead of resistance, there is obtained:-

$$E = \frac{\rho \cdot u^3}{2g \cdot d} \cdot \phi\left\{\frac{\mu}{\rho \cdot u \cdot d}, \frac{u}{u_s},\right\} \text{------} A(1).$$

If we employ specific volume in place of density, and choose for d the hydraulic mean depth A/p, where A is the area open to the flowing fluid, and p the boundary perimeter at A, then:-

$$E = \frac{1}{V} \cdot \frac{u^3}{2g} \cdot \frac{p}{A} \cdot \phi\left\{\frac{\mu}{\rho \cdot u \cdot d}, \frac{u}{u_s},\right\} \text{----} A(2)$$

Again, if there is a mass flow of M lb. per sec. then:-

$$M = A \cdot u / V.$$

and the energy loss per unit of mass in an element of length dx becomes:-

$$de = \frac{E}{M} A \cdot dx = \frac{u^2}{2g} \cdot \frac{p}{A} \cdot \phi\left\{\left(\frac{\mu V}{u} \cdot \frac{p}{A}\right), \left(\frac{u}{u_s}\right)\right\} dx. \text{ -} A(3).$$

For the unknown function we may write, as the fullest expression that can probably be examined by experimental means:-

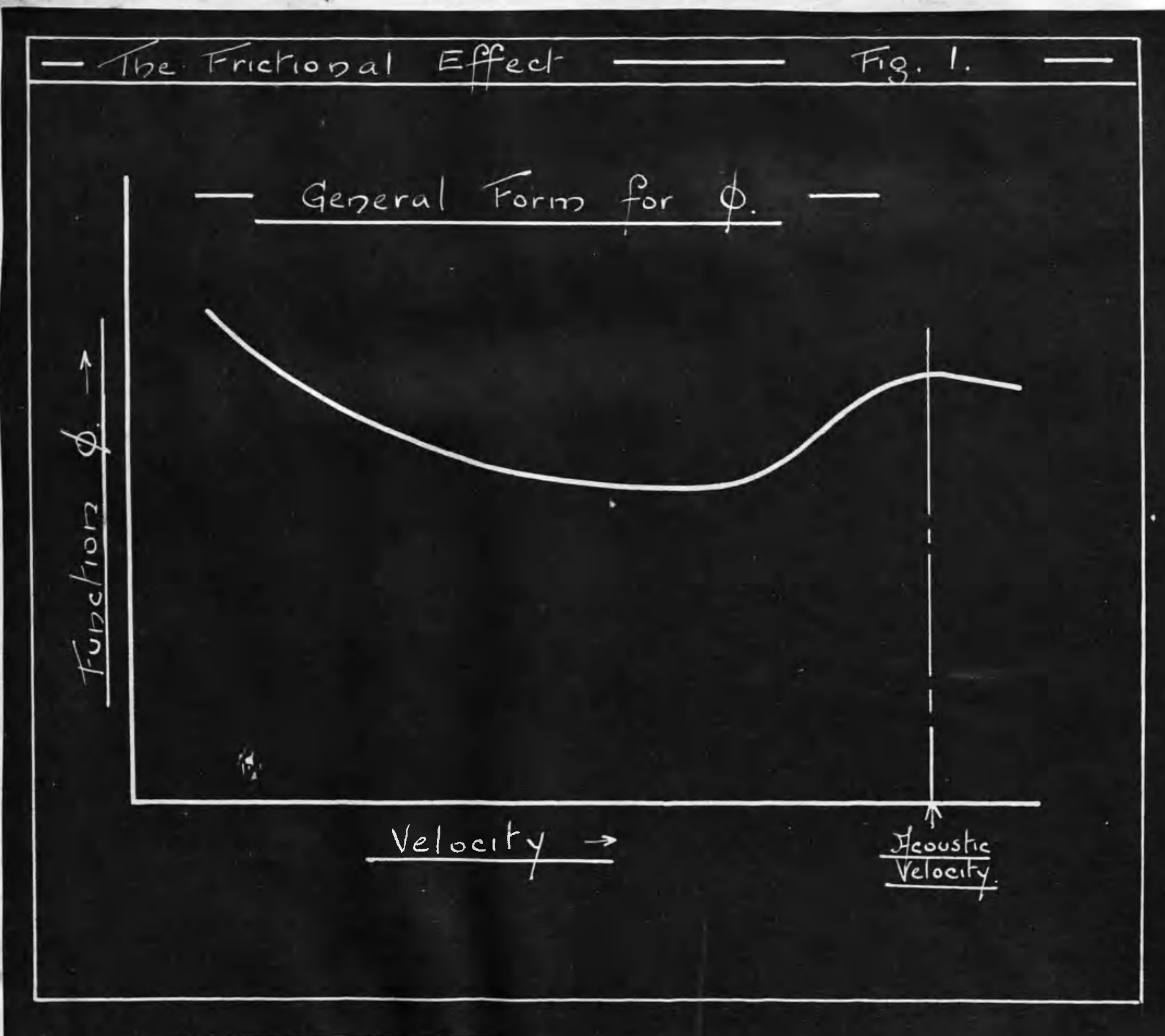
$$\phi\left\{\left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A}\right), \left(\frac{u}{u_s}\right)\right\} = a_1\left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A}\right)^\beta + b_1\left(\frac{u}{u_s}\right)^\delta$$

and, hence, the general expression for the space rate of energy loss per unit mass of flowing fluid becomes:-

$$\frac{de}{dx} = \frac{u^2}{2g} \cdot \frac{p}{A} \left\{a_1\left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A}\right)^\beta + b_1\left(\frac{u}{u_s}\right)^\delta\right\} \text{-} A(4).$$

If the part within the bracket in (4) is assumed constant the usual simple (velocity)^2 law appears.

The two separate terms may conveniently be referred to as the "viscosity" and "elasticity" terms. The former marks an energy loss in maintaining the régime established by the viscous forces



The Frictional Effect — Fig. 1.

while the latter arises from wave production; each has, in general, its own range of special influence.

Consider the case of a solid moving through a fluid of fixed conditions and relatively great extent. As the speed increases the viscosity factor diminishes; but uniform increments of speed will mean diminishing decrements in the value of the factor which will, therefore, tend towards a limit. This will be appreciated from the low value that β usually has - about 0.2 - but it also follows naturally when we realise that this factor measures the scale effect of the forces of viscosity. We must expect, therefore, that at the higher speeds the viscosity factor will cease to influence the resistance. At these higher speeds, however, the elasticity term enters into account, since it is only when the ratio u/u_s approaches unity that extensive wave production need be expected. Hence, with the entrance of this effect the resistance coefficient will again rise, and tend towards a maximum at the velocity of sound. This would give a curve for the general variation of the function ϕ somewhat as shown by fig. 1, when plotted on a base of speed and, of course, on the assumption that μ, V, d and u_s are constant.

The point to be specially emphasised for our present pur: :poses is that, if the elasticity term is of importance, it should give the characteristic rise in the curve of the function ϕ near the acoustic velocity.

It may be said at once that there is no evidence of this particular effect in nozzle flow. For the purpose of indicating this we may employ the friction constants given by Mellanby and Kerr* for the case of a convergent-parallel nozzle. These investigators express the friction loss in terms of a special factor, thus:-

$$dk = c\left(1 - k - r^{\frac{n-1}{n}}\right)\frac{p}{A}\,dx \text{ ------ } B(5).$$

where k is the measure of loss, and r is the ratio of expansion.

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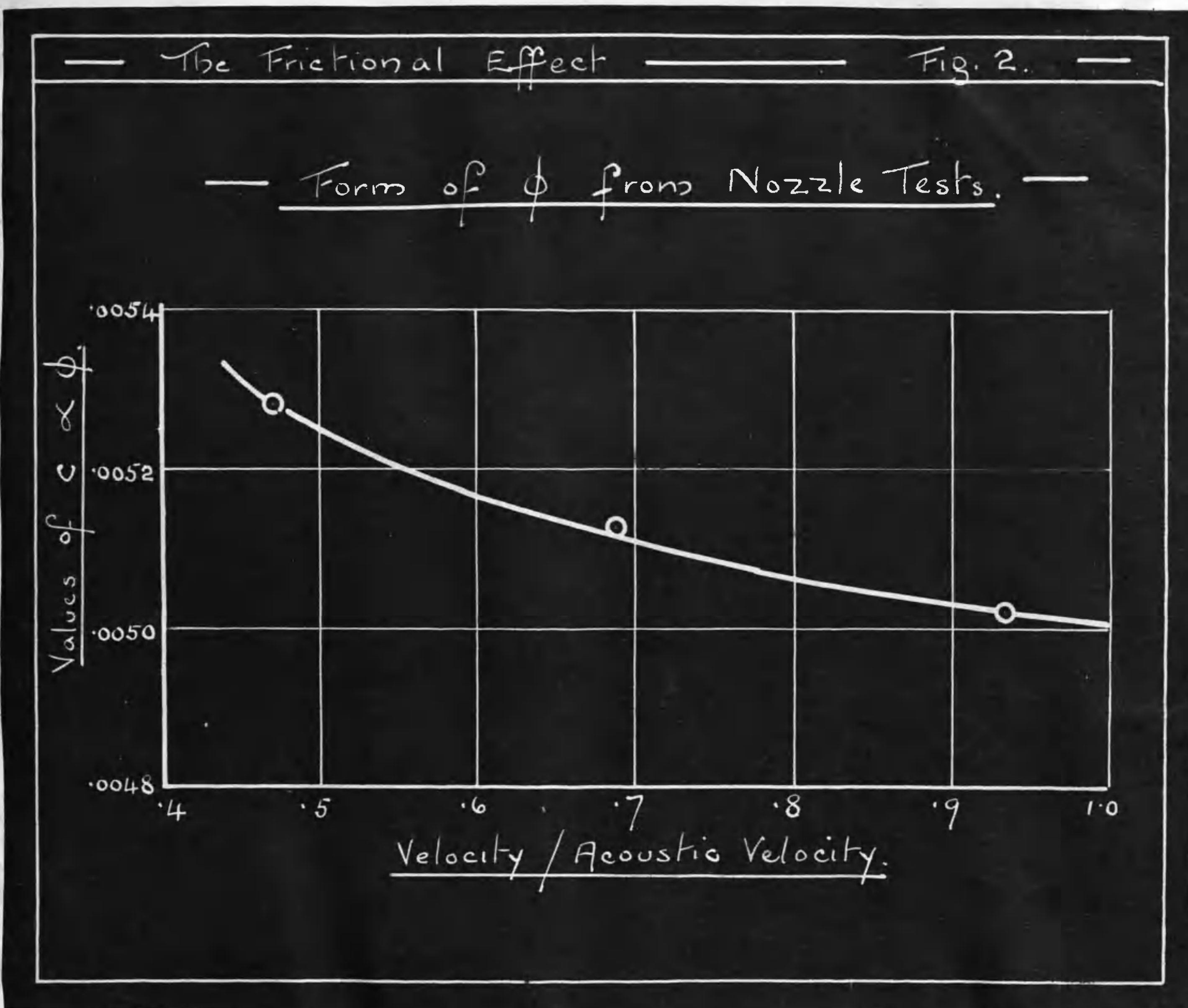
* "On the Losses in Convergent Nozzles" - Proc. N-E Coast Inst. Engrs. & Shipdrs., Feby. 1921.



The Frictional Effect
Fig. 2.
Form of φ from Nozzle Tests.
Values of C ∝ φ
·0054
·0052
·0050
·0048
·4
·5
·6
·7
·8
·9
1·0
Velocity / Acoustic Velocity.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

But the quantities are such that :-

$$\frac{de}{u^2} = \frac{dk}{2g\left(1 - k - r^{\frac{n-1}{n}}\right)} \text{ - - - - - - - - A⑥.}$$

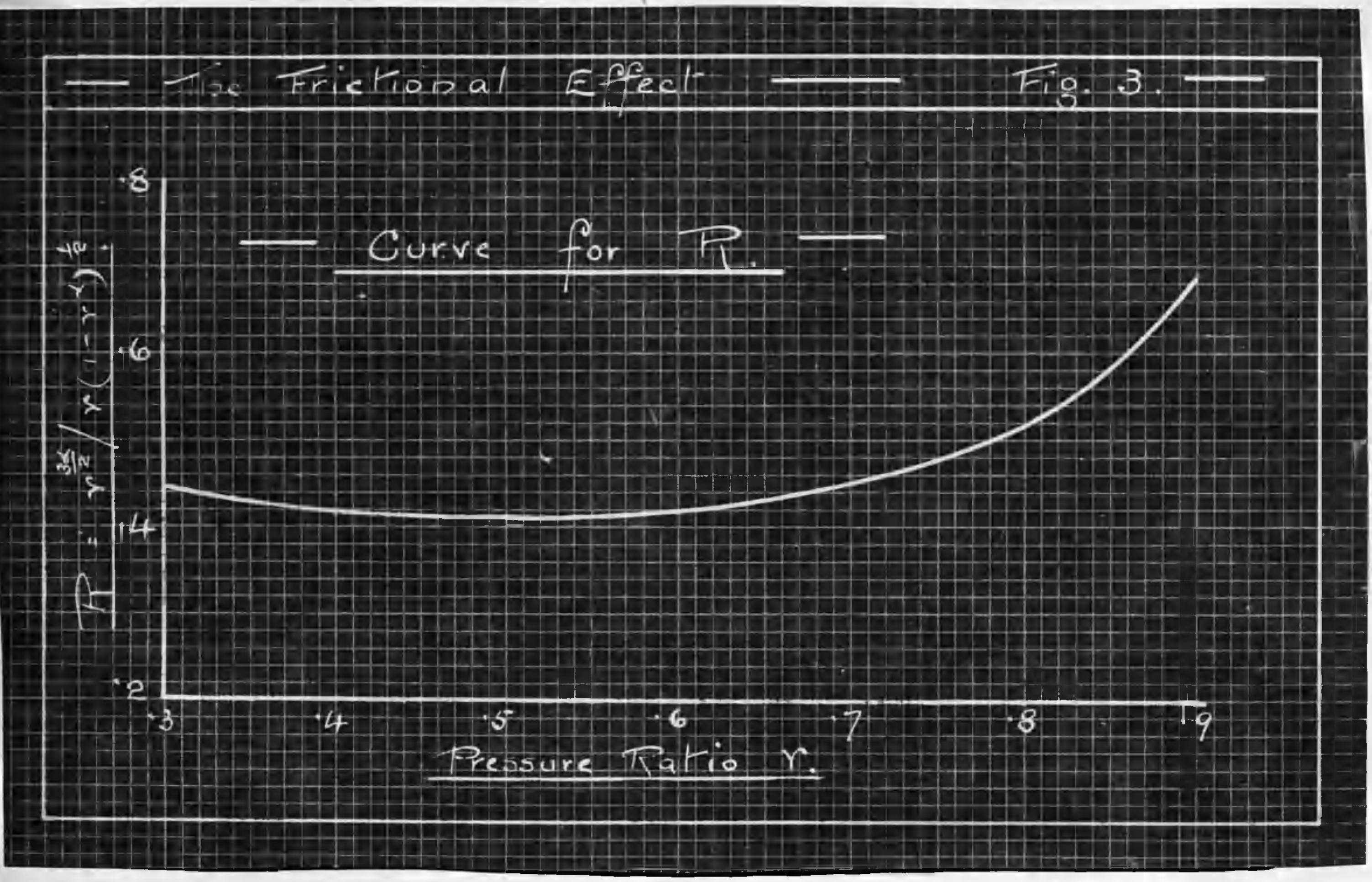
and, hence, the c in ⑤ represents the function ϕ. While c was actually used as a constant in the work referred to, its observed variation for three distinctly different expansion ranges was noted. When these given values are plotted against the average speeds in the respective expansions the form in fig. 2 is obtained; and this shows a falling value of ϕ practically up to the acoustic velocity. The data are admittedly meagre, but clear so far as they go; and there would, therefore, seem to be little necessity for considering the presence of the elasticity term in the general law of loss.

The negligibility of this term might, however, have been decided upon quite apart from the consideration of any actual results. Obviously of fair importance where wave effects may be freely pro:duced, as in the high speed motion of solids in a fluid atmosphere, it can hardly have an outstanding value where fluids flow in very narrow channels. The constant b_1 in the elasticity term will - as between different systems - be largely dependent on space ratios which must be exceedingly small in nozzles, when compared with the corresponding descriptive figures for, say, the motion of projectiles. (It is in ballistics that the influence of the elasticity term is mainly evident.)

From all points of view, then, there is little to be gained by including the second term of ④ in the law of loss for nozzle flow. Of course, this only applies to that part of the jet which is controlled by the nozzle form: it is quite possible that the term enters into the consideration of free jet effects, but that we are not here concerned with.

It follows from this that a fair representation of the rate of frictional loss is given by :-

$$\frac{de}{dx} = \frac{u^2}{2g} \cdot \frac{p}{A} \cdot a_1 \left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A}\right)^{\beta} \text{ - - - - - - A⑦.}$$


The Frictional Effect
Fig. 3.
Curve for R.
·8
·6
·4
·2
·3
·4
·5
·6
·7
·8
·9
Pressure Ratio r.

and from any available values of :-

$$(de/d_f)\,/\,(u^2 \cdot p/A).$$

the probable magnitudes of a_1 and β can be indicated. For this it is necessary to deal with a single nozzle, since small variations of surface condition as between different nozzles would prevent mixed data from being really useful.

It should be noticed that, whereas in pipe flow the vis::cosity factor is dependent simply on u, nozzle flow in even a parallel channel involves changes in each item of $\mu V/u$. As the numerator of this ratio is subject to increase while the denominator rises in value, the rate of change of the ratio is probably less pronounced in nozzle action than in the simpler case.

For the determination of the likely values of a_1 and β only the three previously instanced figures for c in equation ⑤ are really available. From the statements in ⑤, ⑥ and ⑦ it is clear that :-

$$c = a_1 \left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A} \right)^{\beta} \text{ --------- } A⑧.$$

but in nozzle calculation it is desirable to change the variables in the $\mu V/u$ ratio, and to make this primarily dependent on the import::ant pressure ratio r.

Since the variation of μ along a jet is comparatively small and, in any case, not very well known for the temperatures that obtain in such work, we may put as a first approximation :-

$$\mu \propto T^{\frac{1}{2}} \propto (PV)^{\frac{1}{2}}.$$

where P is pressure and V is specific volume. If, further, we neglect the small effect on the viscosity factor due to the losses, there follows, as a reasonable approximation :-

$$\frac{\mu \cdot V}{u} \propto \frac{(PV)^{\frac{1}{2}} \cdot V_1 \cdot r^{\alpha}}{(P_1 V_1)^{\frac{1}{2}} \cdot r \cdot (1-r^{\alpha})^{\frac{1}{2}}} \propto V_1 \cdot R \text{ say.}$$

where :- V_1 = initial specific volume.
r = jet pressure/initial pressure = P/P_1.
α = $(n-1)/n$; n = adiabatic index.

The values of:-

$$R = r^{\frac{3\alpha}{2}} / r(1-r^{\alpha})^{\frac{1}{2}}.$$

for different pressure ratios are as plotted in fig. 3.

— The Frictional Effect — Table I. —

— Values of β and a. —

Outlet Pressure Ratio	Average $V_1 \cdot R \cdot P/A$.	c.	β.	a.
·498	1130	·00502	·15	·00175
·717	1270	·00513	·15	·00175
·852	1660	·00528	·15	·00174

The equation given above as (7) now changes to :-

$$\frac{de}{dx} = a \cdot \frac{u^2}{2g} \cdot \frac{p}{A} \left(V_1 \cdot R \cdot \frac{p}{A} \right)^{\beta} \text{ - - - - - - - - - - - - A(9).}$$

and the viscosity factor can be readily determined for any specified case. There is a certain sacrifice of purity in making this trans::formation as the bracketed factors no longer constitute a dimen::sionless number, but this is no disadvantage in the calculation of effects if the constant a is made to suit the usual units for V, p and A.

From the investigation of the convergent-parallel nozzle carried out by Mellanby and Kerr we have initial conditions, pressure ratio curves, nozzle dimensions and the values of c in (8). From these facts it is simple to determine the changes of $\left(V_1 \cdot R \cdot \frac{p}{A}\right)$ along the tail length and, hence, the average of this quantity for each case in which c is known. The value of β which will give a steady figure for the constant a can then be found. The results of the calculation are given in table I.

The somewhat limited experimental facts are, therefore, well met by $\beta = 0.15$, and $a = 0.00175$; and it will be noticed that the β value is in fairly close agreement with corresponding figures derived from slow speed flow.

It would seem, then, that we may take the following as a general expression for the rate of energy loss at any point in a nozzle where the pressure ratio is r and the speed u :-

$$\frac{de}{dx} = a \cdot \frac{u^2}{2g} \cdot \frac{p}{A} \left(V_1 \cdot R \cdot \frac{p}{A} \right)^{\cdot 15} \text{ - - - - - - - - - - - A(10).}$$

in which:-
- e = energy loss - ft. lb. per lb. fluid.
- x = length - ins.
- u = velocity - ft. per sec.
- p = perimeter - ins.
- A = area - sq. ins.
- V_1 = initial volume - cub. ft. per lb.
- R = $r^{3/2} / \{ r(1-r^{\lambda})^{\frac{1}{2}} \}$ - see fig. 3.
- a = a constant depending principally on nozzle surface = $\cdot 0.00175$ for a machined brass surface.

It will readily be conceded that (10) is a rational general expression for the frictional loss rate; and it is, therefore, of some interest and value to discuss the efficiency curve form that results from the proper application of (9) to the important case of the convergent-parallel nozzle.

The Trend of the Efficiency Curve: It is customary to demonstrate the supposed constancy of the efficiency curve for a convergent nozzle somewhat after the following fashion. For the resistance wehave:-

$$S = \frac{K}{2g} \cdot d^2 \cdot \frac{u^2}{V} \left(\frac{\mu \cdot V}{u \cdot d} \right)^{\beta} \qquad K \text{ a constant.}$$

which, for energy loss per unit mass in a length l , becomes :-

$$e = \frac{K}{2g} \cdot \frac{p}{A} \cdot l \cdot u^2 \left(\frac{\mu \cdot V}{u \cdot d} \right)^{\beta}$$

and, hence, the efficiency, considering the losses to be small, is nearly :-

$$\eta = 1 - \frac{e}{u^2/2g} = 1 - K \cdot \frac{p}{A} \cdot l \left(\frac{\mu \cdot V}{u \cdot d} \right)^{\beta}.$$

If the index β has an appreciable positive value the effect of the viscosity factor is to diminish the efficiency with decreasing speed; if it is negligible the efficiency would appear to be constant; as it is known to be very small it would seem that the efficiency must be practically constant over the range in which the stated friction law applies.

The effect of the probable value of β is, however, of no moment; but the above line of examination is distinctly faulty as the tacit assumption is made that the velocity changes within the nozzle do not affect the result. This is obviously incorrect; and it may now be shown that the mere existence of a frictional loss entails a definite fall of the efficiency with speed of efflux; but the proper occurrences within the nozzle must be kept in view throughout the process. The general form of the result arrived at below would appear whether the complete or simplified form of the law of loss were used, but the general equation (10) will be used here for the sake of completeness.

Consider a convergent-parallel nozzle and allow that the

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

losses to the end of the convergence are negligible. This is equiv:
:alent to the assumption that any frictional loss to the so-called throat is not sufficient to affect the constancy of the efficiency, and confines the treatment of loss, for simplicity, to the straight tail length in which the friction loss is certain.

From equation (10) it is seen that, if the total length is l, the total loss is given by :-

$$e = \frac{a}{2g} \cdot \frac{p}{A} \int_0^l u^2 \left(V_1 \cdot R \cdot \frac{p}{A}\right)^{\beta} dx$$

Let the theoretical velocity of efflux for the condition of working be u_0'; then the fractional loss per lb. of fluid is :-

$$\varepsilon = \frac{e}{u_0'^2/2g} = a \cdot \frac{p}{A} \int_0^l \left(\frac{u}{u_0'}\right)^2 \left(V_1 \cdot R \cdot \frac{p}{A}\right)^{\beta} dx \text{ ---- A (11).}$$

With any specified and fixed initial conditions, and an outlet pressure ratio r_0, the kinetic energy of outflow will be proportional to :-

$$(1 - \varepsilon)(1 - r_0^{\alpha}).$$

If the pressure ratio at the throat is r_1, the corresponding ex:
:pression for the energy there is :-

$$(1 - r_1^{\alpha}).$$

The specific volume at the throat will be proportional to:

$$r_1^{\alpha - 1}$$

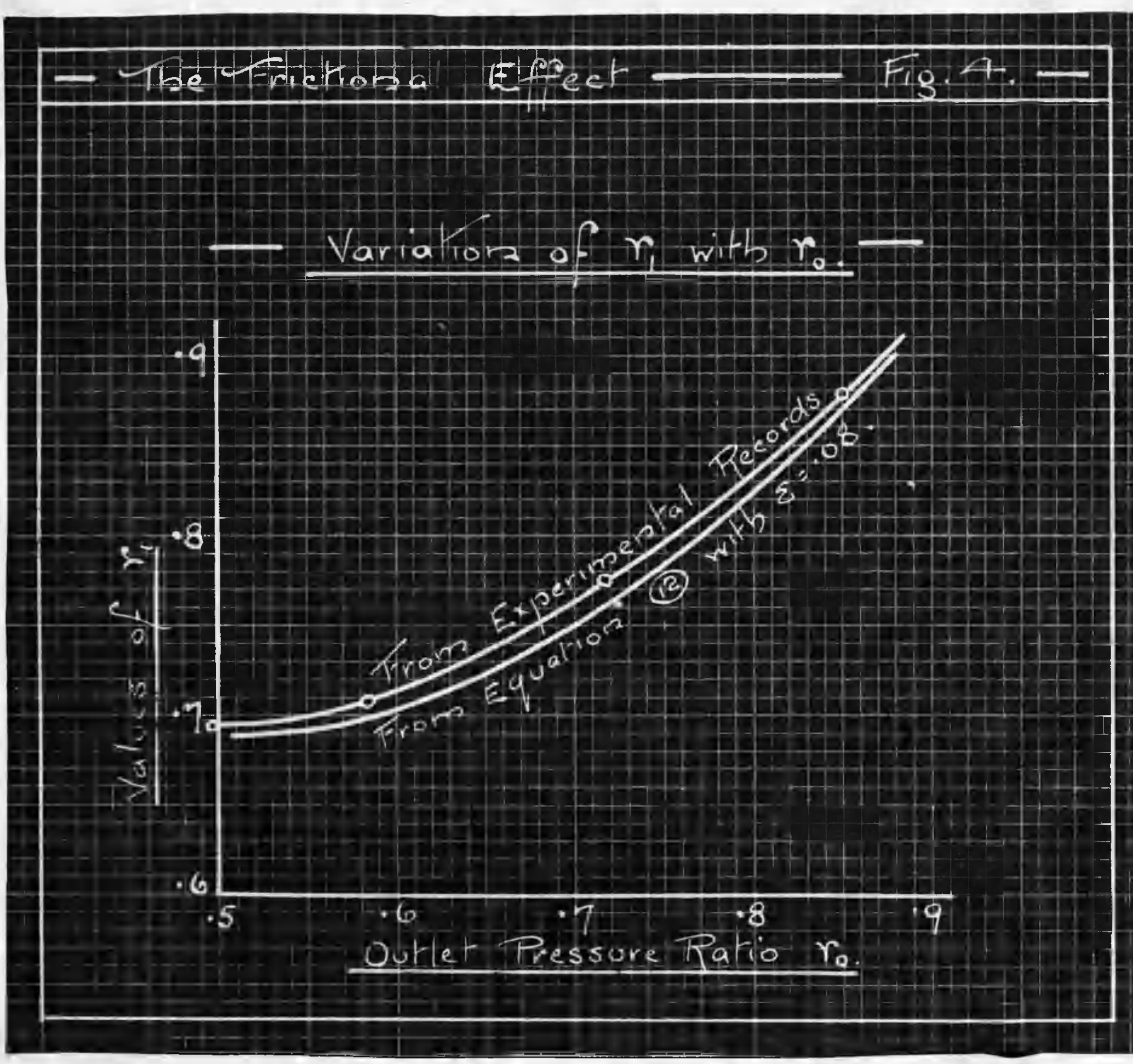
and that at the outlet to :-

$$\frac{\varepsilon(1 - r_0^{\alpha}) + r_0^{\alpha}}{r}$$

and, hence, the flow quantity is proportional to :-

$$\frac{r_0 (1 - \varepsilon)^{\frac{1}{2}} (1 - r_0^{\alpha})^{\frac{1}{2}}}{\varepsilon(1 - r_0^{\alpha}) + r_0^{\alpha}} = \frac{(1 - r_1^{\alpha})^{\frac{1}{2}}}{r_1^{\alpha - 1}} \text{ ----------- A (12).}$$

From this it is clear that the pressure at the throat is dependent, in a somewhat involved fashion, on the loss and the out:
:let ratio. Since the velocity gradient along the tail depends on the change of pressure ratio, the percentage range of increase of velocity will be subject to alteration with r_0. We may assume ε to be constant, as any change in this value which may ultimately be


— The Frictional Effect — Fig. 4. —
— Variation of r_1 with r_0. —
Values of r_1
·9
·8
·7
·6
·5
·6
·7
·8
·9
Outlet Pressure Ratio r_0.
From Experimental Records
From Equation (12) with ε = ·08.

shown will hardly affect the relative pressure ratios. From (12) the ratio of velocities at throat and outlet is :-

$$\left(\frac{1}{1-\varepsilon}\right)^{\frac{1}{2}}\left(\frac{1-r_1^{\alpha}}{1-r_0^{\alpha}}\right)^{\frac{1}{2}} = \frac{r_0^{\alpha}\cdot r_1^{\alpha-1}}{\varepsilon\left(1-r_0^{\alpha}\right)+r_0^{\alpha}}$$

and the throat velocity is obviously not a constant fraction of that at the outlet, even with ε constant.

It is obvious from (11) that, under the conditions of operat::ion, the variation of ε is :-

$$\varepsilon \quad \propto \quad \int_0^{l}\left(\frac{u}{u_0}\right)^{2}\cdot R^{\beta}\cdot dx \quad \text{--------} \quad (13).$$

Since the integrand over the length l is entirely dependent on the change of r, whereas, so far, we have only established the throat and outlet ratios, it is necessary to achieve a definition of the pressure ratio curve or, otherwise, replace the integral by the direct product of l and the arithmetic mean of the integrand. The latter process shows the effect clearly enough and would serve; but the other method is the more correct and will be better for the present purpose.

Mellanby and Kerr have shown that the assumption of a con::stant rate of loss along a parallel tail gives an essentially correct pressure curve, and by a process which these Authors have given it is possible to determine the value of r at any point of the length for known loss and pressure limits. This is exactly the problem in the present instance, and it is therefore possible to determine, graph::ically, the value of the essential integral to a very close approx::imation, after equation (12) has disclosed the necessary ratios..

Taking, for purposes of illustration, ε = .08, and using equation (12) with various assumed values of r_0, the relationship bet::ween r_0 and r_1 is as shown in fig. 4. In addition, fig. 4 gives a corresponding curve derived from experimental records on a simple convergent-parallel nozzle, $\frac{1}{4}$" dia. and 1" long; from which it should be appreciated that the effect under review is not merely a matter of theoretical reasoning.

The subsequent stages of the calculation will be fairly obvious and the details may be omitted. The variation of ε is determined by means of (13) and, taking .08 as the figure for the full range, the variation of :-

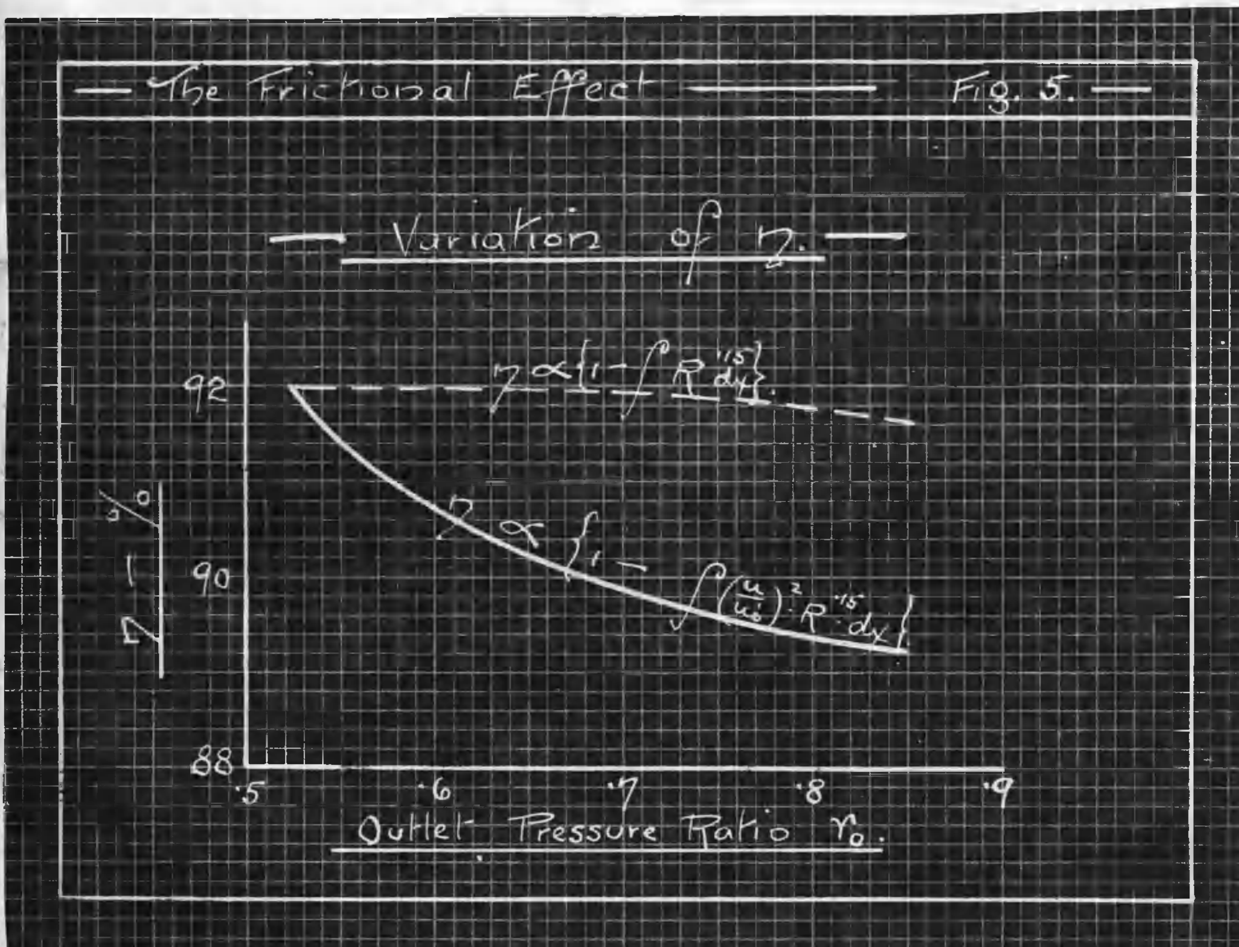
$$\eta = (1 - \varepsilon)$$

is as shown by the curve in fig. 5. To the fall of efficiency thus defined both the $(u/u_0')^2$ and the R^β factors contribute - according to (13). ; but the latter is really of little account. Thus the dotted curve in fig. 5 indicates the influence of R^β if acting alone, and it will be seen to be entirely negligible in comparison with the total effect.

The magnitude of the efficiency drop demonstrated in fig.5 is rather surprising as a result of the seemingly fine point inves::tigated. Its actual value is, however, dependent to some extent on the assumptions made in the calculation; as shown, it is the max::imum possible for ε = .08 and no loss in the entrance portion. It will vary slightly with ε and with any allowance for friction loss in the inlet curvature; but for the convergent-parallel type of nozzle the efficiency clearly falls with falling speed.

We see, then, that the proper application of the friction loss law demonstrates a definite downward slope of the efficiency curve. It is clear from this that any diagram of the kind which does not show this falling characteristic must be faulty, or must receive a different explanation to that usually advanced.

It must be clearly understood that the considerations here given to this matter are for the purpose of demonstrating that,under no conceivable frictional conditions can the coefficient for a con::vergent nozzle be expected to show a rising, or even a steady, form with increase of pressure ratio - at least within the range in which the resistance depends on the square of the speed. In order to make the demonstration the more convincing the loss effect discussed in Section B of this paper has been entirely neglected; but it still remains the chief reason why the coefficient does fall to the extent indicated by many experiments.


The Frictional Effect
Fig. 5.
Variation of η.
92
90
88
η %
$\eta \propto \{1 - \int R^{\cdot 15} dx\}$
$\eta \propto \{1 - \int (\frac{u}{u_0})^2 R^{\cdot 15} dx\}$
·5
·6
·7
·8
·9
Outlet Pressure Ratio r_0.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

Comparison with Pipe Flow : In the discussion of the friction loss which has resulted in equation (10) the examination was based entirely on the fundamental theory, and utilised figures from nozzle tests only. It is now desirable that a comparison with the well established facts of ordinary pipe flow should be carried out since, if good general agreement can be shown to exist, the figures available from this latter source become of service.

The fullest investigation of the frictional effects at the lower speeds is probably that carried out by Stanton and Pannell ⊕ who employed smooth brass pipes varying from .3 cm. to 12 cm., and speeds up to about 5000 cm. per sec. using both air and water. This work has been carefully analysed by Lees * who, using other data in conjunction, advances the following very accurate general expression for the resistance per unit area :-

$$F = \rho \cdot v^2 \left\{ \cdot 0009 + \cdot 0765 \left(\frac{\mu}{\rho \cdot d \cdot v} \right)^{\cdot 35} \right\}.$$

The symbols are as in the original, and the stated form obviously refers to full bore flow in circular pipes. The resistance as given is in absolute units - on any system - but, for the purpose of com: :parison, it is advisable to transform this equation to read in line with the previously derived expression. Changing the symbols where necessary, this gives as a comparable form to (10) :-

$$\frac{de}{dx} = \frac{u^2}{2g} \cdot \frac{p}{A} \left\{ \cdot 0018 + \cdot 0945 \left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A} \right)^{\cdot 35} \right\} \quad \text{-- A (14).}$$

Hence, if the general law for slow speeds applies to the high speeds of nozzle expansion we ought to have :-

$$\cdot 00175 \left(V_1 \cdot R \cdot \frac{p}{A} \right)^{\cdot 15} = \cdot 0018 + \cdot 0945 \left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A} \right)^{\cdot 35}.$$

when the several factors are chosen to represent the fluid conditions in the nozzle. This equality assumes the pipe and nozzle surfaces to be identical in condition, which is hardly possible; but both are equally well described by the term "smooth brass pipe" so that

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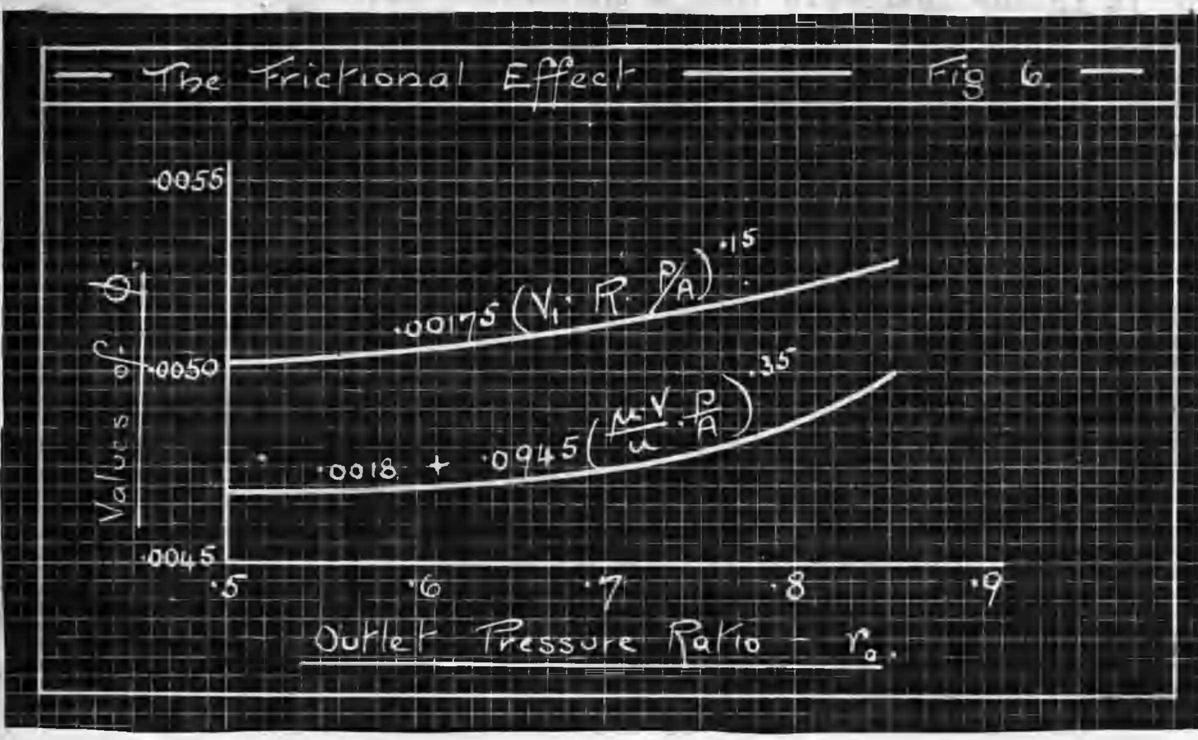
⊕ Phil. Trans. Roy. Soc., A, Vol. CXIV., 1914.

* Roy. Soc. Proc., A, Vol. XCI., 1914.



The Frictional Effect — Fig 6.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

the discrepancy should be slight. The difference between the two sides of the last expression as regards the index of the viscosity factor is more apparent than real, since the presence of the constant term in Lees' form necessitates a fall of the "equivalent" index as the value of $\mu V/\mu$ decreases towards the high speeds.

It seems rather difficult to get full information regarding the variation of μ for steam, but taking that, in C.G.S. units :-

$$\mu = \cdot 00013(1 + \cdot 00366t) \qquad t = {}^\circ C.$$

we have an equation which suits a usual figure for steam near 0° C., and embodies the rate of variation for air as given by Clerk-Maxwell. Errors in this are not of very great importance owing to the small powers involved.

Taking the cases specified in table I for which the volume, speed and temperature conditions can be obtained from the experimental data, and evaluating Lees' function, using the specified rule for μ, we get the relationship expressed by fig. 6.

Considering the extreme difference in flow conditions, and the probability of a difference in surface effect, it will be admitted that the agreement is quite good; the general formulae for the slow speeds of pipe flow would appear to be directly applicable to the case of expansion in nozzles. This certainly holds for speeds up to the acoustic velocity, since that is the limit for convergent type nozzles.

For the still higher speeds attained in divergent nozzles there should be little reluctance in using the established results, since the generality of the whole process is strongly confirmed by the outcome of the above comparison. The correctness of the method in such cases is shown, with fair clearness, by the application of the constants defined by convergent nozzle operation to the observed results with divergent forms. Thus Anderson,⊕ using a very long

⊕ "Investigation of the Losses in a De Laval Turbine". "Greenock Research Scholarship" Report. R.T.C. June 1922 divergent nozzle of the usual finish, and in which expansion proceed: :ed to a fairly low figure, measured the flow and the outlet ratio; and, with the Mellanby and Kerr formula :-

$$dk = c(1 - k - r^2) \cdot \frac{P}{A} \cdot dx$$

he has shown that $c = .0050$ gives practically correct results. This general agreement is found in all cases, so far, in which divergent types have been examined for friction. It must be said, however, that the exact and direct investigation of the low range nozzle, along the lines that have been successfully employed for the parallel type, is not easily carried out. The pressures in the tail length are much less stable and the readings much less reliable - except towards the outlet. It is probably this difficulty that underlies the lack of exact frictional data for nozzles, as the divergent nozzle has hitherto been quite undeservedly popular with investigators.

Effect of Surface : It is now desirable to obtain some guidance as to the effect of surface condition. So far only a limited examin: :ation of this kind has been carried out on actual nozzles, but the results are important and, by adding a few figures from other sources, it is possible to provide a fair guidance.

Obviously, in studying the relative effects of different surfaces in the approximate fashion at present possible, it will be quite sufficient to employ the average constants given by the simple law of loss - such as the c value already frequently referred to.

From the figures previously presented it is clear that for an ordinary turned brass nozzle - say a "smooth finish" - we have $c = .0050$ about. This constant has also been investigated by Anderson⊕ who used nozzles of rectangular section with well polished surfaces; and a fair value for c in such circumstances would appear to be about .0035. Anderson repeated his tests after having

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⊕ "Losses in Nozzles of Rectangular Section" - Greenock Engineering Society. March 1922.

spoiled the surface finish of the nozzles by roughening with a file. The expected effect on the pressure curves was very clearly shown, and the value of c rose to about .0080. No direct nozzle test res: :ults are available for more extreme cases of roughness, or for the various degrees of finish met with in practice; but it is desirable to note the kind of figures that obtain for the usual surfaces in hydraulics.

The simple hydraulic formula for head lost in pipe friction may be written :-

$$h = \frac{f}{2g} \cdot \frac{p}{A} \cdot l \cdot u^2.$$

where f varies somewhat with speed,size and surface. Now this is exactly the same as the simple form of our main expression for, changing the above equation to read energy loss per unit surface per unit time, we get :-

$$\frac{f}{2g} \cdot \delta \cdot u^3.$$

where δ is density of water. In the same terms the general equation gives :-

$$\frac{c}{2g} \cdot \frac{1}{V} \cdot u^3.$$

Hence the c for the nozzle corresponds to the f for water resis: :tance. Both of these really stand for the function ϕ but compar: :ison of averages is justifiable if roughly the same value of the variable in ϕ is taken.

Establishing a value of $\mu V/ud$ from the conditions in a nozzle which we know have given c = .0050 for a smooth finish, and deducing therefrom the corresponding pipe conditions in water flow, a value of f of about .0080 seems to be representative for a "new cast iron pipe".* For an old incrusted iron pipe this would appar: :ently rise to about .0160; and for a cleaned pipe about .0090.

These figures seem fairly consistent with those resulting from nozzle tests. In view of the fact that nozzle constants seem somewhat higher than pipe constants for corresponding conditions, it

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* e. g. see Gibson's "Hydraulics and its Applications". p. 202.

The Frictional Effect — Table II.

Values of Surface Constants.

Kind of Surface	Value of c.	Constant in:- $a(V_1 \cdot R \cdot P/A)^{\cdot 15}$	Constants in:- $a' + b'\left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A}\right)^{\cdot 35}$	
		a.	a'.	b'.
Well Polished.	·0035	·00123.	·00135.	·071.
Machined.	·0050	·00175.	·00193.	·102.
Plate.	say ·0070.	·00245.	·00270.	·142.
Roughened.	·0080	·00280.	·00310.	·163.
Cast.	·0090 upwards.	·00315 +	·00348+	·183.+

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

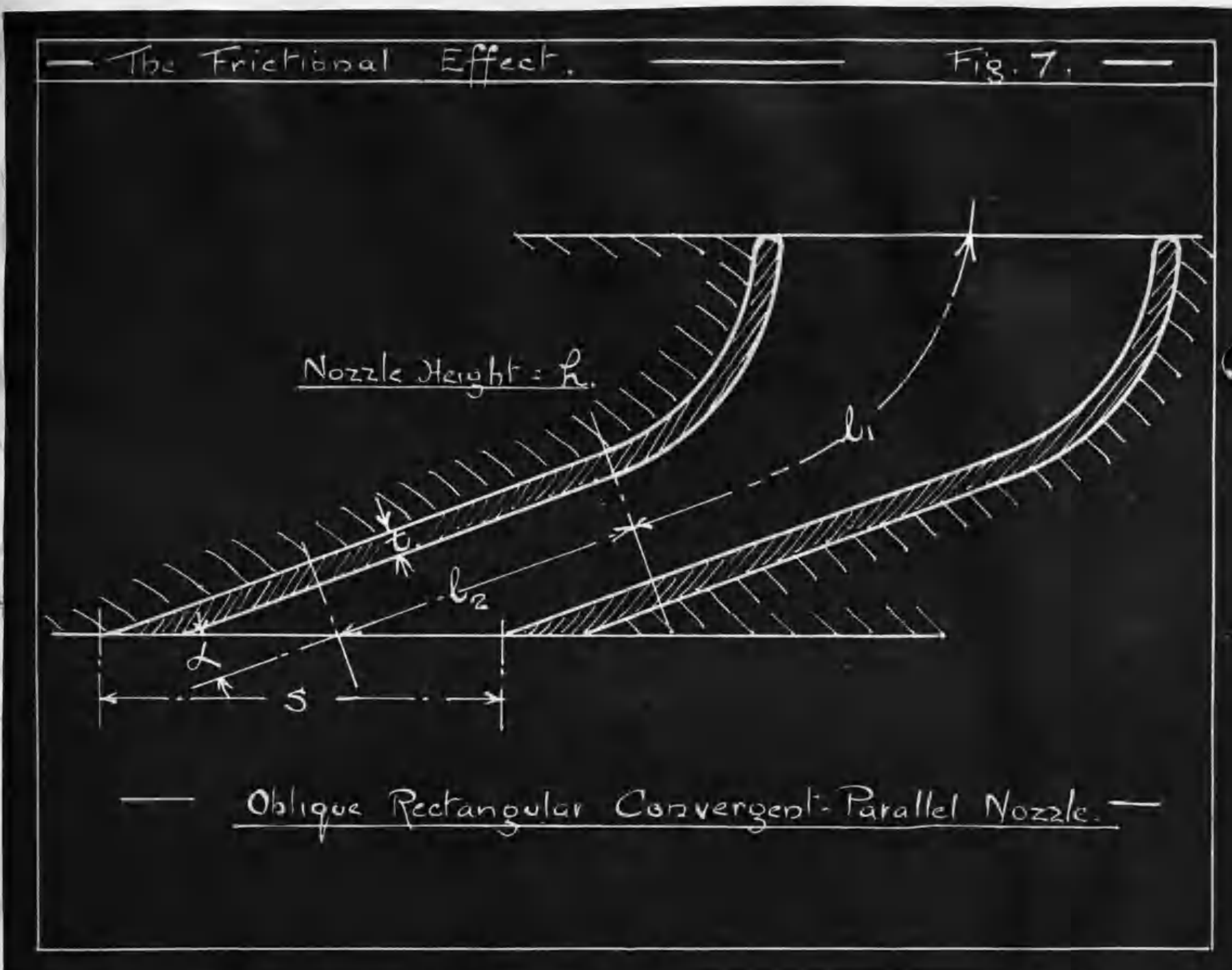
might be best to take a figure of from .0090 upwards for nozzle cast surfaces. What would be called a clean cast surface in fair sized pipes is rather difficult to get in small nozzle castings; and although an attempt is usually made to smooth up the surface it is difficult to get well back into the nozzle, and the total result is not, generally, too good. Higher figures than .0090 would appear to be quite possible in cast nozzle work.

In practice the commonest type of turbine nozzle has usually a cast surface on two sides and a plate surface on the other two. The plates may be moderately smooth, although the "casting-in" process tends to spoil the surface. An endeavour is also made to improve the cast faces but not always with complete success. It is questionable whether the whole arrangement is any better than a good cast surface when we have, say, equal perimeters of the two kinds; but as the relationship between the surfaces is altered, different values will be required, and it will be found necessary, in calculation, to deal with each part separately - as explained later.

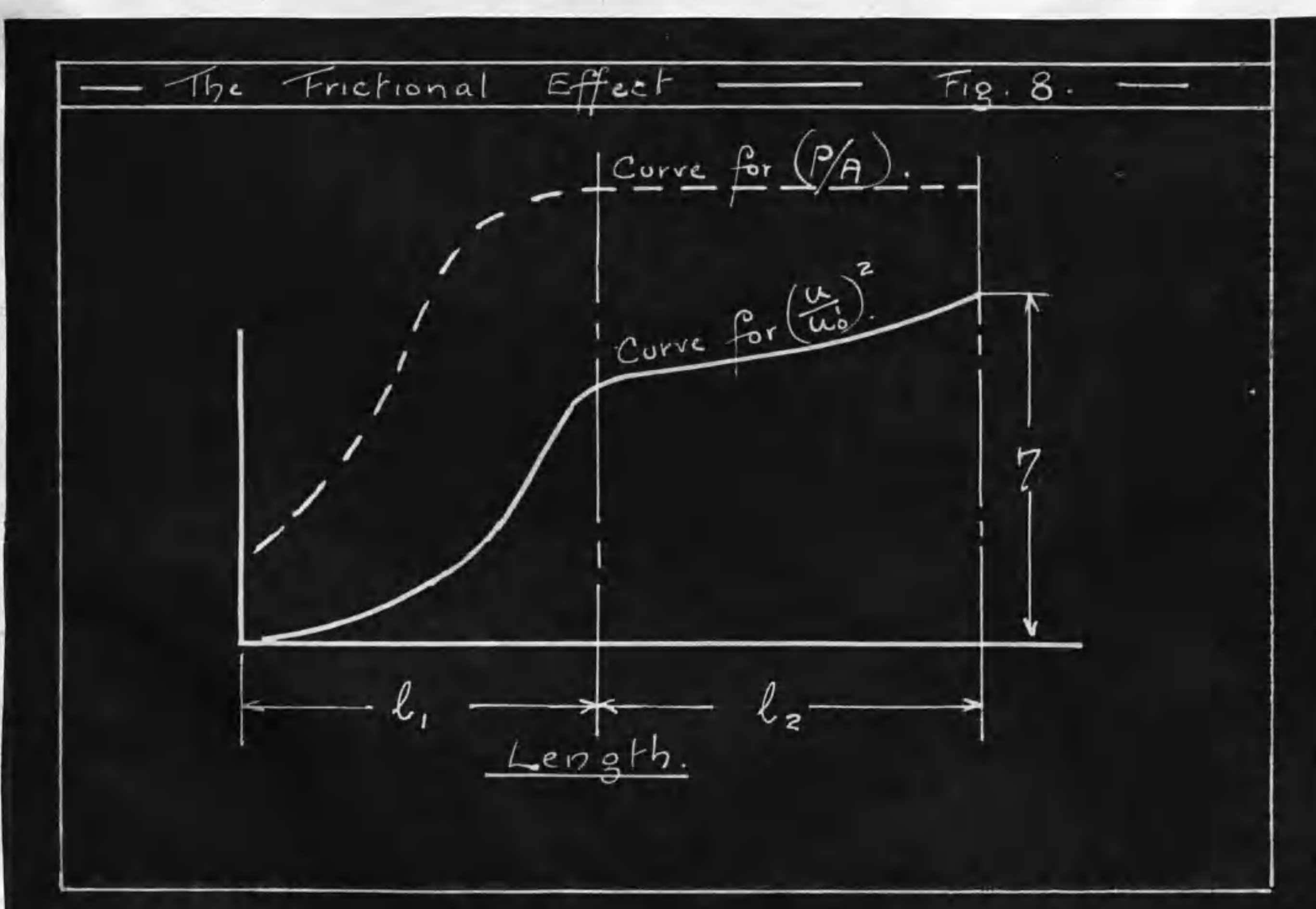
Naturally, of course, when special pains are taken to finish the nozzle surfaces suitable lower coefficients can be used.

The foregoing consideration of the effect of surface leads, then, to figures of the kind shown in table II. Of the specified constants those for polished, machined and roughened surfaces may be considered fairly definite; that given as a lower limit for cast surfaces is consistent therewith, and supported by data from hydraulics; while the interpolated figure for plate surfaces may be held a reasonable assumption. The table gives values of a, for use in equation (10), which correspond to those chosen for c; and also includes the probable constants for use in Lees' form of the function ϕ.

It is of course, appreciated that the index .15 in equation (10) may itself be subject to variation with surface, but the point is not open to discussion at present; and the tentative and approximate nature of the proposed constants make any possible variation in this of little relative importance.



The Frictional Effect.
Fig. 7.
Nozzle Height = h.
l_1
t.
l_2
α
s
Oblique Rectangular Convergent-Parallel Nozzle.



The Frictional Effect
Fig. 8.
Curve for (P/A).
Curve for $\left(\frac{u}{u_0'}\right)^2$.
7
l_1
l_2
Length.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

Methods of Calculation : For the determination of the rate of loss per lb. of fluid we may use, with fair accuracy, the equation :-

$$\frac{de}{dx} = a \cdot \frac{u^2}{2g} \cdot \frac{p}{A} \left(V_1 \cdot R \cdot \frac{p}{A} \right)^{\cdot 15} \text{ - - - - - - - - - A (10).}$$

The quantities are measured as previously explained, and the values of the surface factor a are as given in table II. Also, as already shown, this becomes for the fractional loss per lb. of fluid :-

$$\mathcal{E} = a \int_0^l \left(\frac{u}{u_0'}\right)^2 \left(\frac{p}{A}\right) \left(V_1 \cdot R \cdot \frac{p}{A} \right)^{\cdot 15} dx \text{ - - - - - - - A (15).}$$

This quantity $\mathcal{E}$ is always the required figure as, when known, it may be used on the total available energy - in any units - to give the energy lost in friction throughout the expansion within the nozzle. In the actual use of (15) it would be necessary to determine the probable pressure ratio curve for the nozzle; deduce curves of the various factors involved; and integrate the product as indic::ated.

If it were preferred to deal with the general type of equation that is also suitable for pipe flow then Lees' expression could be used in the form :-

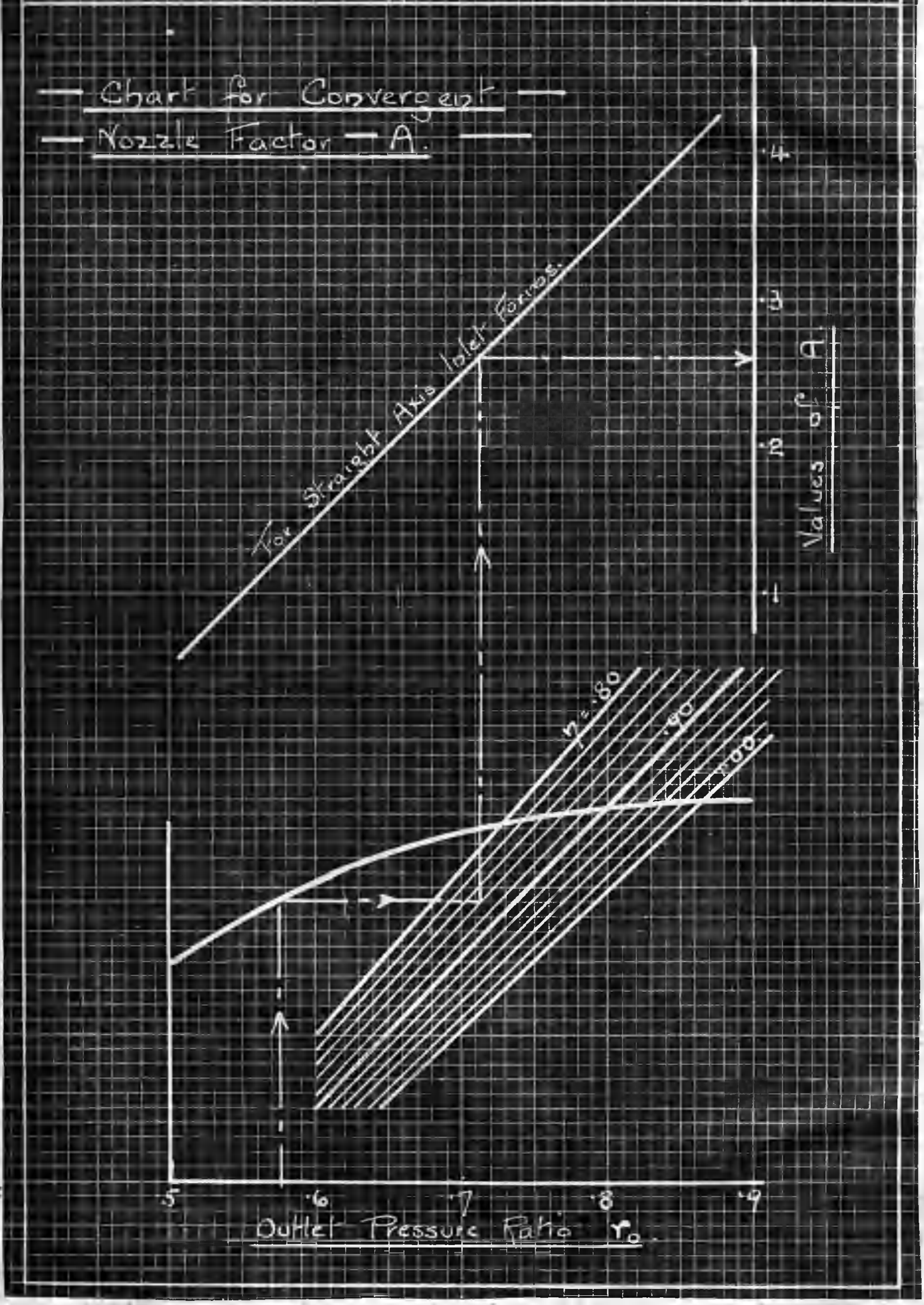
$$\mathcal{E} = \int_0^l \left(\frac{u}{u_0'}\right)^2 \left(\frac{p}{A}\right) \left\{ a' + b' \left(\frac{\mu \cdot V}{u} \cdot \frac{p}{A} \right)^{\cdot 35} \right\} dx \text{ - A (16).}$$

There is an uncertainty here as to whether the change of surface would affect a' and b' in the same ratio, or would only influence a'. Assuming, however, that both factors reflect surface variation in equal ratio, suitable a' and b' values would be as given in table II. The process of calculation would be the same as before and, in general, (15) and (16) would give very similar results. These should be fairly accurate, but the methods involved are quite unsuit::able for practical calculation and simplification is demanded.

One obvious simplification is to neglect the variation caused by the viscosity factor and write (15) as :-

$$\mathcal{E} = c \int_0^l \left(\frac{u}{u_0'}\right)^2 \left(\frac{p}{A}\right) dx \text{ - - - - - - - - - A (17).}$$

The Frictional Effect — Fig. 9.



where c is the average constant, already discussed, and to be taken as stated in table II. The necessary avoidance of integration req::uires the reasonable determination of the average $(u/u_0')^2(p/A)$ for any given example.

There are two general types, (1) the convergent-parallel nozzle and (2) the convergent-divergent nozzle; and it is preferable to deal with these separately.

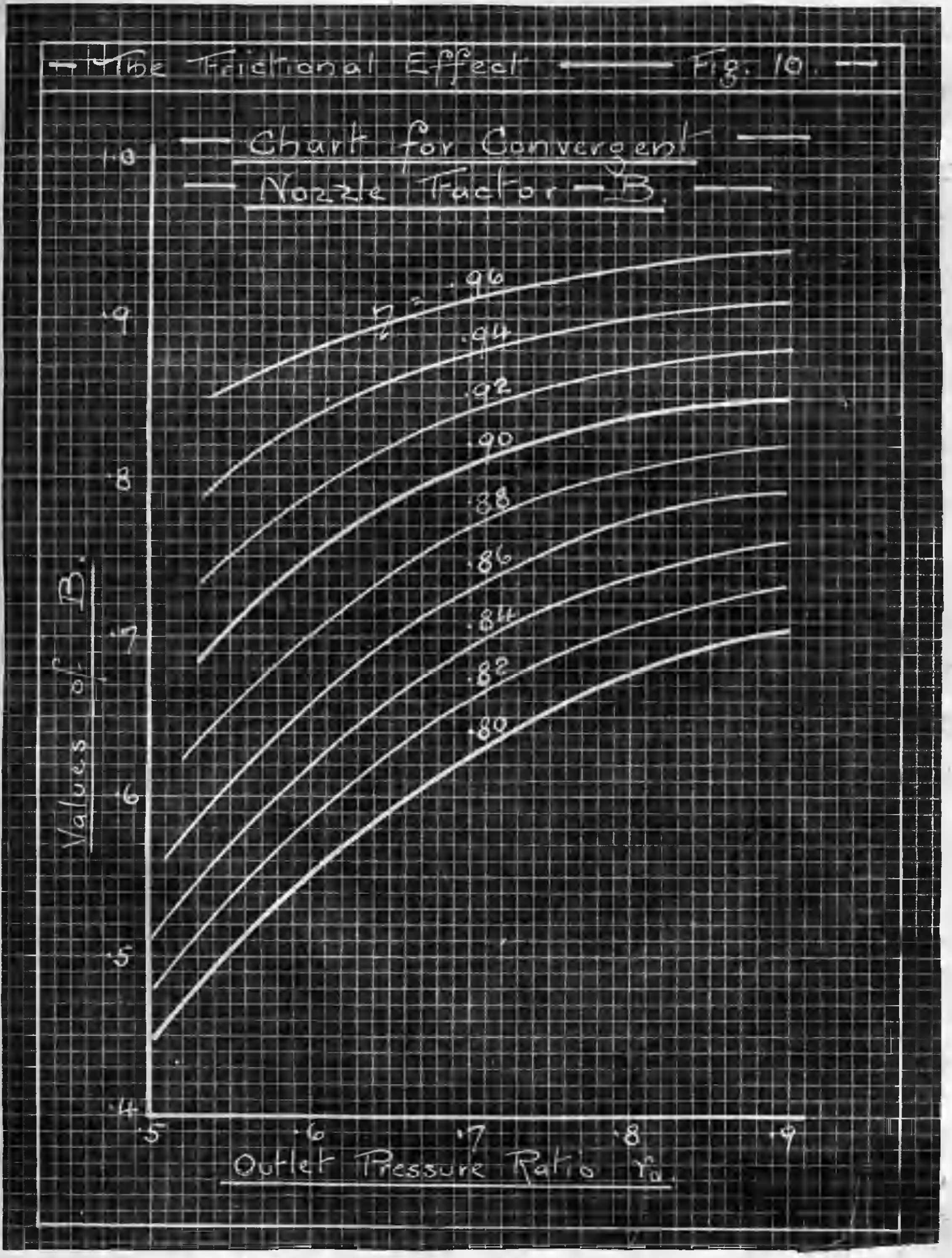
The Convergent-Parallel Type : We may suppose the nozzle length divided into two parts, viz., the inlet length l_1 and the tail length l_2. Fig. 7 shows this division for the case of the rectangular convergent-parallel nozzle with oblique axis and curved entrance; other forms are generally simpler and the application will be clear without further diagrams.

The fractional loss $\mathcal{E}$ multiplies the total available energy to give the friction in the full expansion. Now $(u/u_0')^2$ is simply the ratio of the transformed and total available energies. The curves of this ratio and of (p/A) along the kind of nozzle that is being considered are somewhat as indicated in fig. 8, from which it is clear that we may write as a fair approximation :-

$$\mathcal{E} = c\left(\frac{p}{A}\right)_0 \left\{ A'\left(\frac{u_0}{u_0'}\right)^2 l_1 + B'\left(\frac{u_0}{u_0'}\right)^2 l_2 \right\}$$

$$= c\left(\frac{p}{A}\right)_0 \left\{ A l_1 + B l_2 \right\} \text{ -------------- } A (8).$$

in which A and B are special factors to cover mainly the several variations indicated by fig. 8. It is clear that both are affected in some degree by efficiency and by pressure range; and that, in general, A will be less than $\frac{1}{2}$ and B more than $\frac{1}{2}$. The value of $(p/A)_0$ is that at the outlet, the figure for A being supposed to cover the fact that the actual value varies in the convergence. It may therefore appear that A has to deal with rather many variations, but the total frictional effect in the entrance is usually relatively small, so that the stated approximation may be allowed. In any case when the form of the nozzle is established A can, by means of a


The Frictional Effect — Fig. 10.
Chart for Convergent
Nozzle Factor — B.
Values of B.
1.0
.9
.8
.7
.6
.5
.4
η = .96
.94
.92
.90
.88
.86
.84
.82
.80
.5
.6
.7
.8
.9
Outlet Pressure Ratio r_a

fuller preliminary investigation, be fixed very closely.

If the nozzle has two different kinds of surface - say of perimeters ρ' and ρ'', and surface constants c' and c'' - then it should be more accurate to take :-

$$\mathcal{E} = \left\{ c'\left(\frac{\rho'}{A}\right)_0 + c''\left(\frac{\rho''}{A}\right)_0 \right\} \left\{ A\ell_1 + B\ell_2 \right\} \text{------} A(19).$$

since it is practically impossible to choose a single value of c that will represent the varied relationships of the different surfaces, caused by changes in nozzle height.

The value of (ρ/A) will vary with the form of nozzle but can readily be calculated from the particulars, thus :-

$(\rho/A)_0$ ... For circular nozzle.....diameter d.... $= 4/d$.
" ... " square " ....side s.... $= 4/s$.
" ... " rectangular " ....width w, height h $= 2(w+h)/wh$.
" ... " the general form of fig. 7 $= 2(h + s\cdot\sin\alpha - t)/h(s\cdot\sin\alpha - t)$.

For the factors A and B the charts in figs. 9 and 10 may be employed. Fig. 9 for A has been mainly determined from the few experimental or calculated values available for the friction in ent: :rance forms. It can hardly be considered definitive as the data are too scanty, but the agreement with straight axis inlets is quite good and the variation of A does not conflict with theoretical require: :ments. It is possible that modification of the coefficient will be necessary for curved inlets where the curved length is taken; to allow for this the chart has been so arranged that additional lines embodying such modifications can easily be introduced when sufficient data exists to define them. It is unlikely that the divergence of any new lines from that given for the simple case will be great, and the diagram may be used as given for most examples with fair confidence.

Fig. 10 for B is much more definite than the correspond: :ing chart for A, since its dependence on pressure range and efficiency, and the fact that (ρ/A) in the tail is constant, make it more amenable to theoretical determination. Actually, in fixing this chart the curves were first decided upon by calculation from

The Frictional Effect — Table III.

Comparison of Observed and Calculated ε Values for Convergent Type Nozzles.

Nozzle Description.			Observed Results				Calculated
Type	Sizes	Surface	Authority	r	c	ε	ε
Circular Convergent with Search Tube	¼" dia × ¼" long	Turned.	Mellanby & Kerr	·575	·0051	·0132	·0131
				·710	"	·0150	·0148
				·843	"	·0154	·0151
Circular Convergent-Parallel with Search Tube (Convergence only).	¼" dia × 1" long	Turned.	"	·695	·0051	·0130	·0131
				·780	"	·0140	·0135
				·895	"	·0141	·0135
Circular Convergent-Parallel with Search Tube (Whole Length).	¼" dia × 1" long	Turned	"	·498	·00502	·0890	·0860
				·717	·00513	·1090	·1020
				·852	·00528	·1100	·1080
Square Convergent-Parallel (Convergence on 4 Sides) with Search Tube	·28" sq. × 1¼" long	Polished.	Anderson	·510	·0035	·0550	·0570
				·690	"	·0660	·0660
				·840	"	·0650	·0650
— D° —	"	Rough.	"	·490	·0082	·1090	·1100
				·680	"	·1320	·1320
				·830	"	·1370	·1380
Square Convergent-Parallel (Convergence on 2 Sides) with Search Tube	"	Polished.	"	·530	·0040	·0616	·0615
				·680	"	·0707	·0670
				·830	"	·0690	·0690
— D° —	"	Rough	"	·510	·0080	·1070	·1050
				·680	"	·1310	·1250
				·830	"	·1330	·1310
Rectangular Convergent-Parallel (Convergence on 2 Sides) Straight Axis at Inlet.	3/8" × 1/4" × 1 3/8" long	Smooth but Rusted	"	·550	·0050	·0395	·0430
				·650	"	·0456	·0460
				·850	"	·0460	·0480
Rectangular Convergent-Parallel (Convergence on 1 Side only).	"	Polished	"	·550	·0035	·0318	·0310
				·650	"	·0350	·0330
				·850	"	·0380	·0350
Rectangular Convergent-Parallel Curved Axis at Inlet.	3/8" × 1/4" × about 1¾" long	Polished	"	·550	·0035	·0357	·0340
				·650	"	·0402	·0370
				·850	"	·0460	·0380

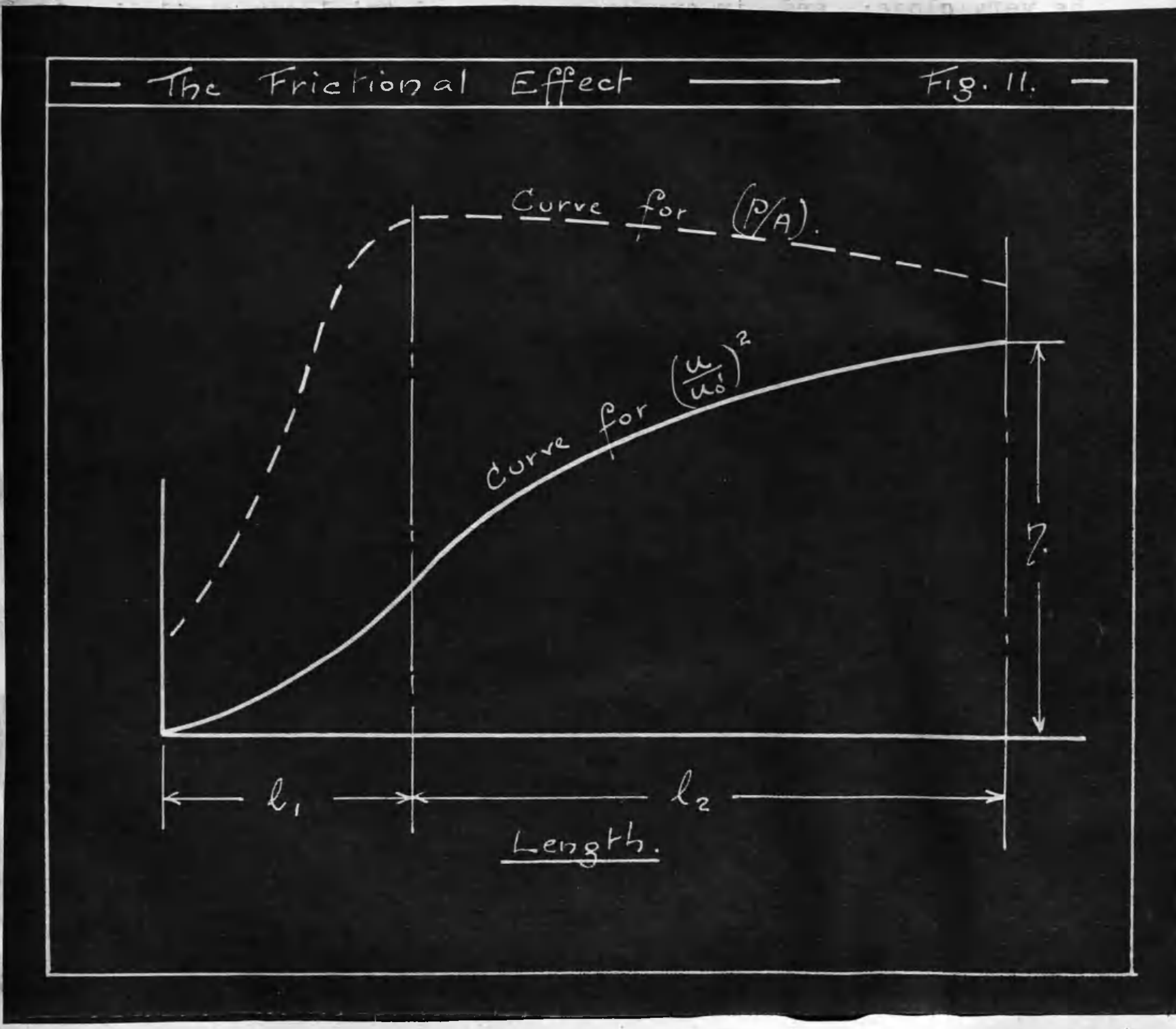
the mean condition obtaining in the parallel length under any speci: :fied Υ and η values, and then adjusted the necessary amount to give the best agreement with the available experimental figures. The reading for B, therefore, may be considered fairly certain.

It will be observed that, in finding either A or B, an estimate of efficiency has to be made, but this will not require to be very close; and, in any case, a second trial can easily be made if the assumed efficiency disagrees greatly with the ultimately cal: :culated ε. It should be remembered, however, that owing to the existence of other loss effects η will, in general, be somewhat less than $(1-\varepsilon)$ as determined from the friction chart.

To show the agreement between the results of the chart system of calculation and the experimental facts for convergent nozzles, table III has been prepared. In this the observed values of c and ε are tabulated and the calculated ε's stated alongside for comparison. It may be remarked that a few of the experimental results show slight inconsistences, and the full amount of the dis: :crepancy existing in places is not, perhaps, chargeable to the cal: :culation. On the whole, however, the agreement seems quite good in view of the diversity of form and surface, and the outlined process of calculation would appear to be quite serviceable.

<u>The Convergent-Divergent Type :</u> The curve of transformed energy in this case is somewhat as indicated in fig. 11, and (p/A) now varies along the tail length, being principally dependent on the pressure ratio. The conditions at the throat are, however, fairly definite in this form as they are fixed by the critical drop and are practic: :ally unaffected by the total nozzle losses. This is of considerable assistance in simplifying the treatment.

We may again suppose the complete length divided into the two parts, l_1 to the throat and l_2 from throat to outlet, as shown by fig. 7 for the simpler type. In dealing with the effect in the ent: :rance we can make use of the critical condition corrected by a fac: :tor for which guidance may be obtained from the previous values for



The Frictional Effect — Fig. 11.

Study of Nozzle Losses:-
The Frictional Effect.(contd.).

the convergent type. For the tail length the throat and outlet con::ditions must both be taken into consideration if a workable average tail effect is to be derived.

Let the suffix 1 refer to throat and the suffix 0 to outlet conditions, then consideration of fig. 11 will show that we may write as an approximation :-

$$\begin{aligned} \mathcal{E} &= c\left(\tfrac{p}{A}\right)_1\left(\tfrac{u_1}{u_0'}\right)^2 D' l_1 + m\cdot c\left\{\left(\tfrac{p}{A}\right)_1\left(\tfrac{u_1}{u_0'}\right)^2 + \left(\tfrac{p}{A}\right)_0\left(\tfrac{u_0}{u_0'}\right)^2\right\} l_2 \\ &= c\left[D\left(\tfrac{p}{A}\right)_1 l_1 + m\left\{E\left(\tfrac{p}{A}\right)_1 + \eta\left(\tfrac{p}{A}\right)_0\right\} l_2\right] \text{------} A\ (20). \end{aligned}$$

in which factors D and E are mainly dependent on pressure ratio, and m represents an averaging multiplier. Thus if the curve of $(u/u_0')^2(p/A)$ were straight m would be $\frac{1}{2}$, but since at the full exp::ansion ranges this curve usually becomes convex upwards, m increases above this figure at the lower ratios. This is shown in fig. 12. Reasonable values for D can be determined by theoretical calculat::ion, if the limiting figure at the critical ratio is taken as given in the chart for the convergent nozzle. This results in the curve marked D in fig. 12. The curve for E in fig. 12 is very simply determined, being practically unaffected by efficiency.

Equation (20) with the curves of fig. 12 may be used for any divergent nozzle in which the surfaces on all sides are in like con::dition. If, however, we are to deal with a convergent-divergent form with different surfaces on the opposite sides - not a very usual construction in this type - it would be necessary to make a modific::ation similar to that made in (18), viz., $c(p/A)$ must be replaced by:-

$$c'\left(\frac{p'}{A}\right) + c''\left(\frac{p''}{A}\right).$$

and this changes (20) to :-

$$\mathcal{E} = D\left\{c'\left(\tfrac{p'}{A}\right)_1 + c''\left(\tfrac{p''}{A}\right)_1\right\} l_1 + m\left[E\left\{c'\left(\tfrac{p'}{A}\right)_1 + c''\left(\tfrac{p''}{A}\right)_1\right\} + \eta\left\{c'\left(\tfrac{p'}{A}\right)_0 + c''\left(\tfrac{p''}{A}\right)_0\right\}\right] l_2 \text{--} A\ (21).$$

the curves of fig. 12 being used as before.

If the nozzle form is such that all cross sections are sim::ilar then a simplification of (20) is made possible. This occurs in examples where the same taper is employed on all sides, and gives a

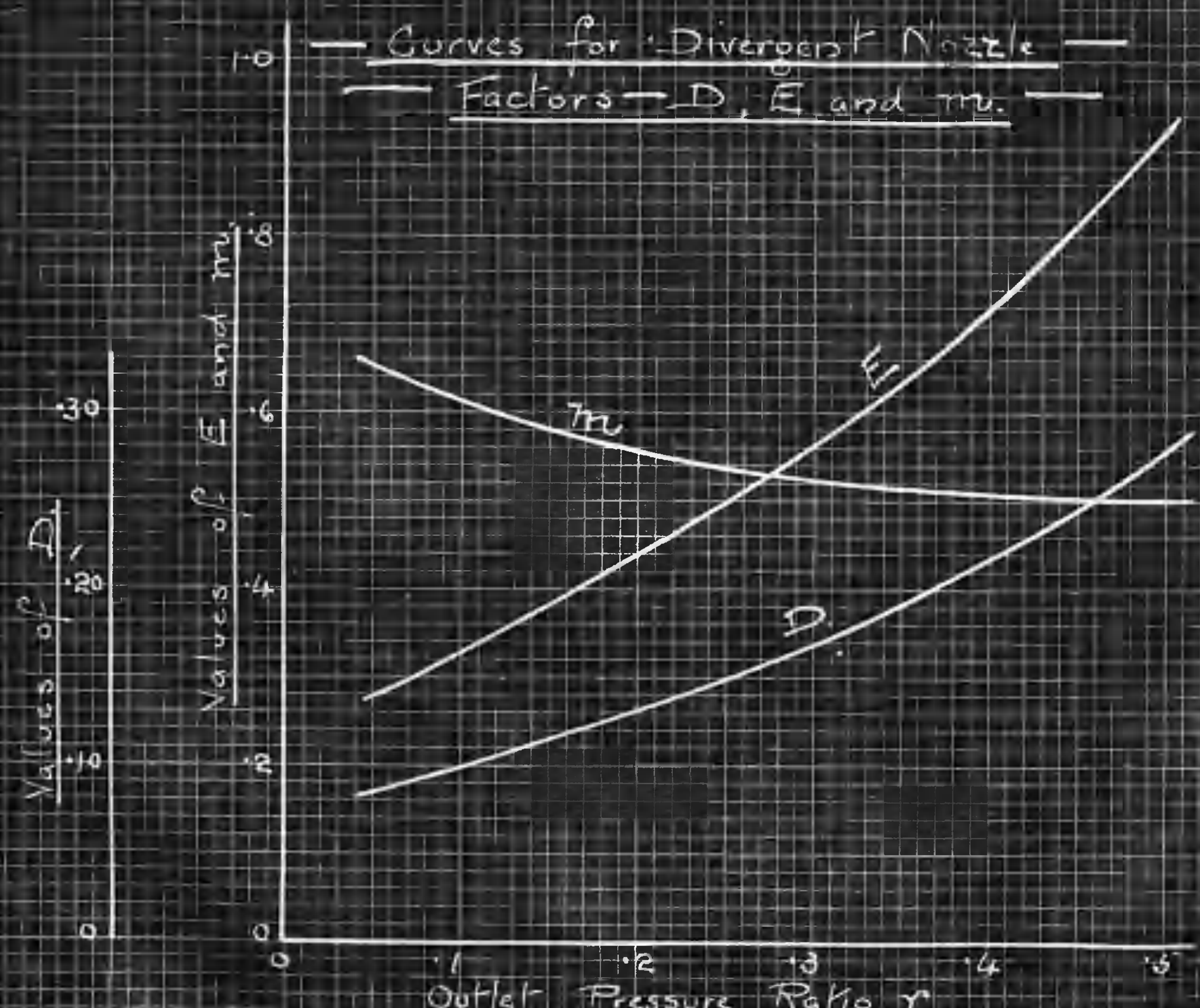
The Frictional Effect — Table IV.

Comparison of Observed and Calculated E Values for Divergent Type Nozzles.

Nozzle Description		Observed Results				Calculated E.
Type	Approx. Sizes	Conditions	r.	c.	E.	
Circular Convergent-Divergent with Search Tube	$\frac{1}{4}"$-$\frac{5}{16}"$ dia × 1" long.	Turned Surface Full Pressures and Flow Measurements.	·200	·0051	·083	·079
Circular Convergent Divergent.	$\frac{1}{4}"$-$\frac{3}{8}"$ dia × $3\frac{3}{4}"$ long	Turned Surface End Pressure and Flow Measurements.	·139	·0050	·166	·164
— D° —	$\frac{3}{16}"$-$\frac{7}{16}"$ dia × $2\frac{3}{4}"$ long	Calculation from Assumed Press. Ratio Curve	·040	·0050	·109	·111

The Frictional Effect — Fig. 12.

Curves for Divergent Nozzle Factors — D, E and m.



relation between $(p/A)_1$ and $(p/A)_0$, viz.:-

$$\frac{(p/A)_1}{(p/A)_0} = \left(\frac{A_0}{A_1}\right)^{\frac{1}{2}}$$

and, hence, (20) becomes :-

$$\begin{aligned} \mathcal{E} &= c\left(\frac{p}{A}\right)_1\left[D\ell_1 + m\left\{E + \eta\left(\frac{A_1}{A_0}\right)^{\frac{1}{2}}\right\}\ell_2\right] \\ &= c\left(\frac{p}{A}\right)_1\left\{D\ell_1 + N\ell_2\right\} \text{------------ } A(22) \end{aligned}$$

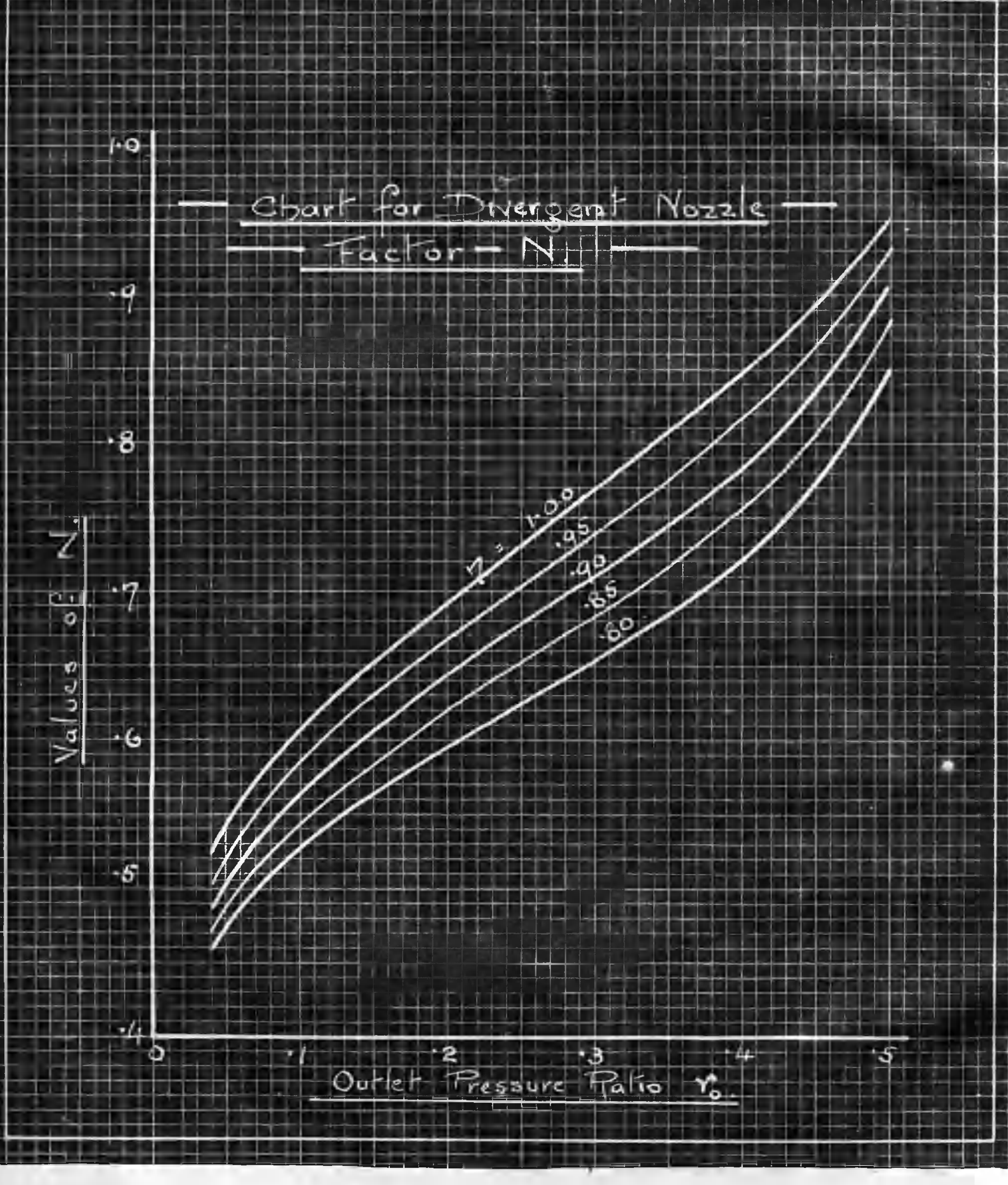
which gives a form of expression as simple as that for the convergent type. The factor N is affected by several quantities and requires a curve series for its definition. This series can be built up from the m, E, η values with inclusion of the effect of $(A_1/A_0)^{\frac{1}{2}}$ which depends both on r and η. The chart for N, as so determined, is given in fig. 13.

The curves for the various factors required to deal with the divergent type are hardly established with the certainty of the corresponding coefficients for the convergent forms. This is mainly due to the scarcity of data against which to carry out a full check, but it should be noticed that the process of fixing curve forms is more definitely controlled by theoretical methods, owing to the invariable nature of the throat conditions. These forms are, therefore, likely to be quite sound.

In table IV a comparison is made between the values calculated as described above and those derived from a full examination of a few divergent nozzles. The true values have been determined by complete detail calculation of the nozzles and are in line with the observed performances. The agreement in these few cases, in conjunction with the rational nature of the factors employed in the calculation system, should give assurance of fair accuracy in the general use of the method.

<u>Influence of Steam Condition :</u> It will have been observed from the form of the proposed expressions for the direct calculation of $\mathcal{E}$ that, apparently, this value is independent of steam quality and density. Since the main friction constants have been shown to be applicable

The Frictional Effect — Fig. 13.



with practically equal accuracy to water or dry steam, the independence with steam quality would appear to be genuine, i. e., the calculated ε for a given nozzle should apply for frictional effect whether the steam is superheated or wet.

This does not, however, cover the influence of the viscosity factor neglected in the simplified theory, and which may have an appreciable effect for wide changes in steam condition. This quantity has the form :-

$$\left(V_1 \cdot R \cdot \frac{p}{A}\right)^{\cdot 15} \propto \left(\frac{\mu V}{u} \cdot \frac{p}{A}\right)^{\cdot 15}$$

so that altered conditions on any particular nozzle will alter $\mu V/u$; but assuming the same pressure ratio under the new conditions we get, for any specified position on the nozzle length :-

$$\frac{\mu \cdot V}{u} \propto \frac{\mu \cdot A}{M} \propto \mu\left(\frac{V_1}{P_1}\right)^{\frac{1}{2}}$$

since, even for extreme changes of initial state on any one nozzle, a close approximation is always :-

$$\frac{M}{A} \propto \left(\frac{P_1}{V_1}\right)^{\frac{1}{2}}$$

Allowing now that for these wide changes under constant pressure ratio expansion :-

$$\mu \propto (P_1 V_1)^{\frac{1}{2}}$$

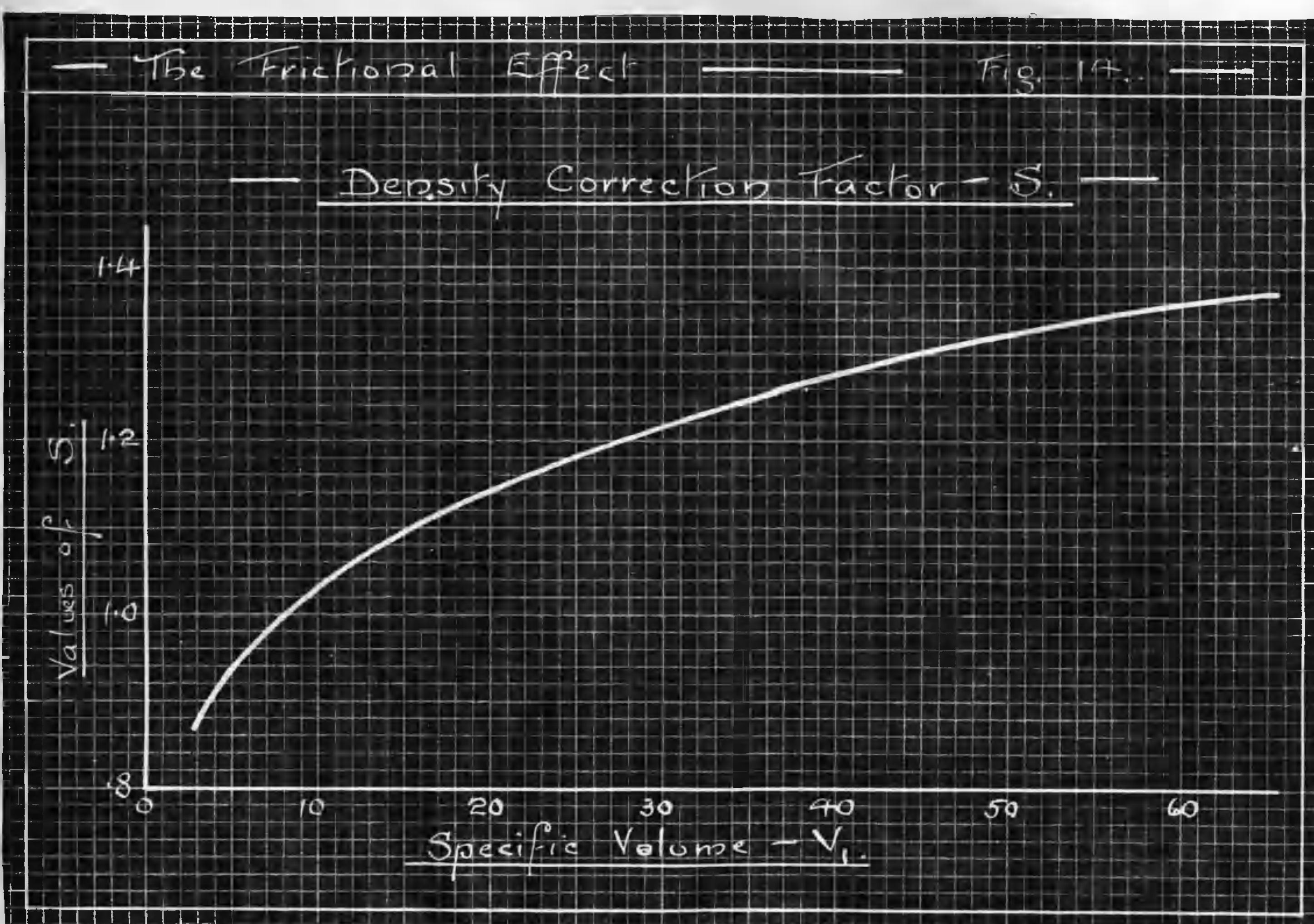
we obtain finally :-

$$\frac{\mu \cdot V}{u} \propto V_1 \qquad \text{--------------- A(23).}$$

as an approximation quite sufficiently good considering the small powers of this number involved.

The apparent elimination of the influence of R here must not be misunderstood. This R is essential in the general form to cover the different ratio values; but since in changing steam conditions we may alter the field in which expansion takes place, it would seem necessary to embody the effect of R in any correction. This has, however, been done indirectly by introducing :-

$$\frac{M}{A} \propto \left(\frac{P_1}{V_1}\right)^{\frac{1}{2}}.$$



The Frictional Effect
Fig. 14.
Density Correction Factor - S.
Values of S.
1·4
1·2
1·0
·8
0
10
20
30
40
50
60
Specific Volume - V1.

which is a relationship of great service, and satisfactorily covers the change of field. Hence the given method of correction is legit:imate so long as the figure to be modified, for change of initial conditions, is primarily determined for the correct pressure ratio by means of the given rules.

The coefficients put forward in the various methods of cal:culation have been mainly determined by examination of data involving values of V_1 of the order of 8 cub. ft. per lb.; hence from the above it would appear necessary to correct any approximate figure for $\mathcal{E}$ by means of the multiplier :-

$$S = (V_1/8)^{\cdot 15} \qquad (24)$$

and a curve of this correcting factor is given in fig. 14.

It should be clearly recognised that the correction just discussed is not for the quality of the steam but for the density. If the simple law making the loss dependent only on the square of the speed were exact, there would be no correction of this kind. Nearly all curves of velocity coefficients embody variations for wetness and superheat, but it is obvious that, so far as frictional effect is concerned, the correction is necessitated by the influence of the Reynold's number in the accurate expression, and is not affected by change of quality except when such is accompanied by change of density. The demonstration given earlier that the ultimate friction constants are practically steady whether we deal with the liquid or the gas should make it clear that they remain steady for any mixture of liquid and gas in the accurate treatment. The values of fig. 14 are, therefore, only to be used in conjunction with the simplified rules of procedure, and are not required in connection with the full expressions shown by equations (15) or (16).

The methods that have been given serve to determine $\mathcal{E}$ for any case. Usually nozzle calculations are based on the total heat available in the expansion - say DH_ϕ - and the reheating effect due to the friction in the nozzle is then $\mathcal{E} \cdot DH_\phi$ heat units.

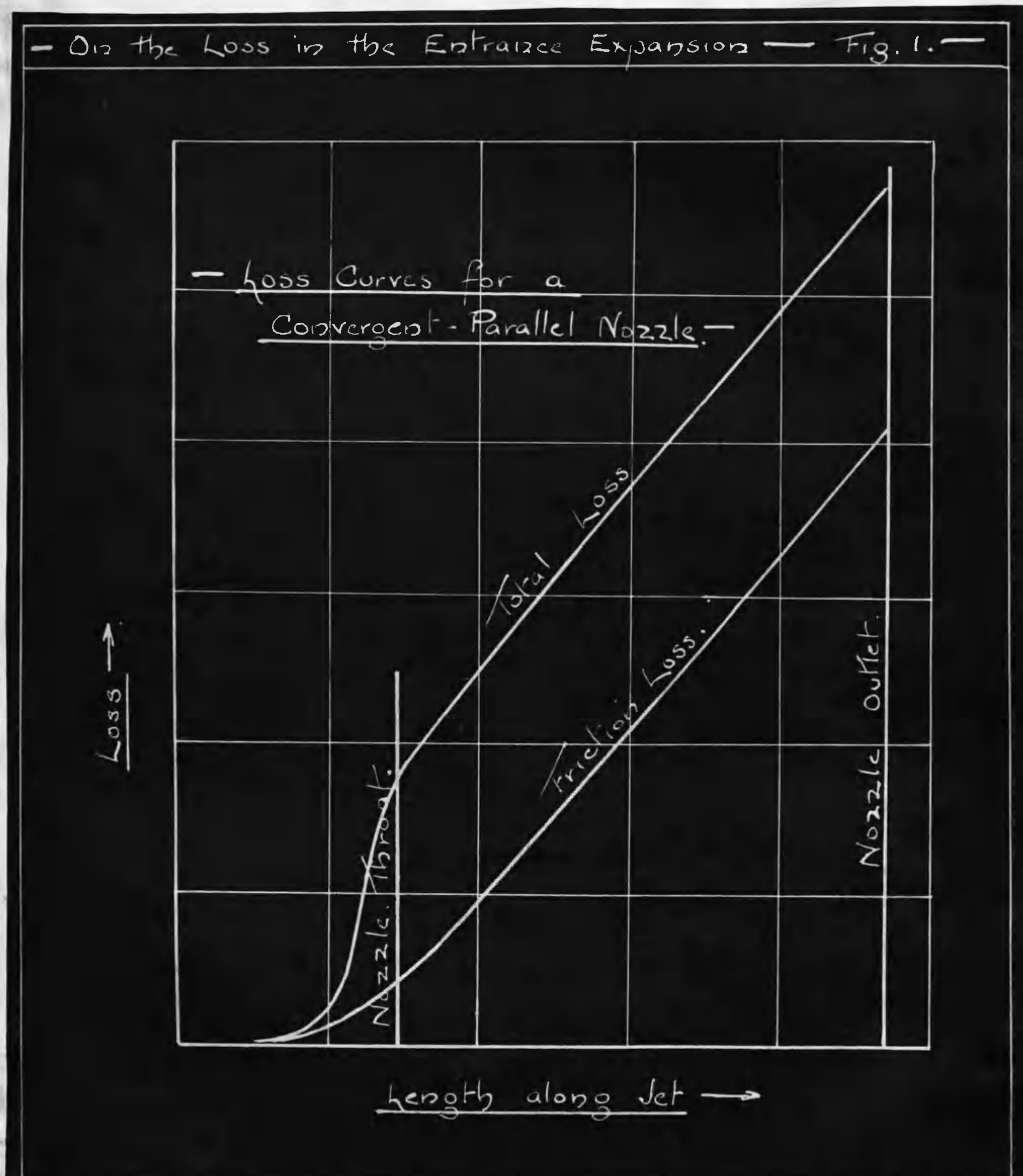
A STUDY OF NOZZLE LOSSES

B:— ON THE LOSS IN THE ENTRANCE EXPANSION

Introductory: In the search tube method of examining nozzle phenomena it is found that the most accurate and certain results are obtained with the convergent type of nozzle. Of this class the form provided with a fair length of parallel tail, following on an ample inlet curvature, is the most illuminating; and the main value of Mellanby and Kerr's recent experimental work on nozzles lies in the detail points which they have been able to deduce from the examination of expansion in such types. The analysis to which they have subjected their search tube and flow records has resulted in the appearance of two kinds of loss; one of the frictional type and the other an accompaniment, or a result, of the expansion in the convergent entry portion. The former is a rational and expected effect and has been studied in fair detail in Section A of this paper. The latter is less easily envisaged - and somewhat indefinite - but it is proposed to devote the present article to its discussion.

The point hitherto reached in this matter of the entrance expansion loss may be briefly explained. It would appear definite that a loss effect exists in a convergent nozzle which is not chargeable to the frictional action, and it seems fairly clear that the source of this energy loss is the expansion in the convergent part. For this reason Mellanby and Kerr have designated it the "convergence loss". Its appearance is well shown by the curves in fig. 1,* which represent the growth of energy loss along a convergent parallel nozzle; the upper curve gives the total loss as deduced from the experimental data, while the lower represents the

* "On the Losses in Convergent Nozzles" -- Proc. N.E. Coast Inst. Engrs. & Shipdrs., February 1921.



On the Loss in the Entrance Expansion — Fig. 1.

integration of the frictional loss along the full length. It is to be noticed that the difference between these is established by the time the end of the convergence is reached, and is maintained at that constant value along the tail, i.e., the loss, as disclosed by the search tube measurements, is a feature of the rapid expansion and is not shown in the slow expansion region.

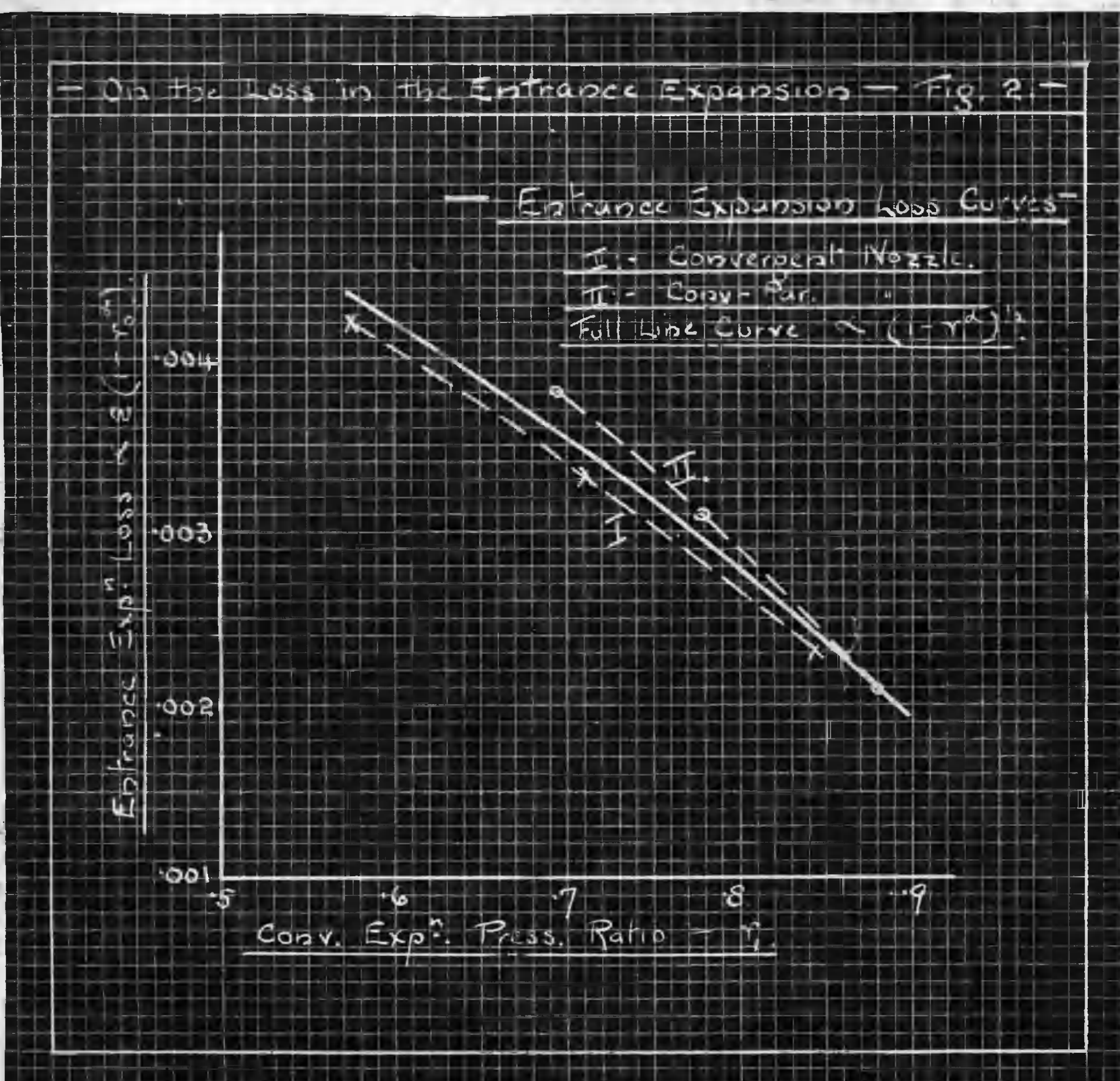
From the examination of several differences of the kind shown by fig. 1, it has been demonstrated that the loss per lb. of fluid is approximately as the fluid speed established in the entrance length. Failing a more rational explanation it was advanced that the effect might arise from an excessive agitation in the convergence proportional to density and to the rate of energy transformation. This suffices to cover the observed relationship, but is rather artificial and hardly convincing, since there is no obvious reason why agitation in the entrance should give a loss effect exceeding the usual result of turbulent flow. Besides which, it is well known that turbulence - as commonly understood - is much more difficult to establish in convergent than in parallel flow. It remains, however, that the values, as exhibited by the narrow range of available data, are roughly of the order indicated; and are sufficiently above the frictional losses to show up clearly in the examination of totals.

The data on which this matter has hitherto rested are entirely of the kind referred to above but, lately, as a minor feature of some experiments, undertaken for a different purpose, a further manifestation of this effect was noticed. This occurred in the determination of flow curves for convergent and convergent-parallel nozzles with increasing superheat at constant pressure. It was found that the flow for the simpler form fell away quite considerably from the condition of constant coefficient of discharge while, with the tail length added, the difference was much less noticeable. If a friction loss alone were active neither curve should diverge to any appreciable degree from that demanded by a constant fractional loss, and the difference actually observed is, therefore, significant of some other effect. Since, again, the discrepancy diminishes with reduced entrance expansion range it would seem necessary to search this part for an explanation. The way in which these flow curves confirm the previously expressed belief in an entrance loss is altogether unexpected, but the fact is of some value since the observation is of a direct kind and not, as in the previous case, dependent on a detail jet examination which offers a small residue as the main evidence. Since, however, this original evidence is supported by the new flow curve indications its value is clearly increased.

It is not possible to undertake a full quantitative discussion of this subject at present as the only data available are for one particular form of entry; but it would seem of interest to consider shortly some possible lines of explanation of the effect in the light of the double set of facts provided by the different results mentioned. The actual information obtained from the experimental work may first be clearly presented.

The Experimental Facts: We have seen in fig. 1 how the difference between the total loss and the friction loss is demonstrated, and by the same means its value may be approximately determined. Hence, by carrying out such investigations for different ranges the variation of the loss with expansion ratio may be derived. This variation has been given by Mellanby and Kerr* for a short convergent nozzle, 1/4" diar. by 1/4" long with 7/32" radius of inlet, and is given in fig. 2 by the curve marked I, plotted on outlet ratio, i.e., practically the ratio at the end of the curvature. From the same source we can obtain figures for the case of a 1/4" diar. by 1" long nozzle with a similar inlet form, and when the

* "On the Losses in Convergent Nozzles". - Proc. N-E. Coast Inst. Engrs. & Shipdrs., February 1921.


— On the Loss in the Entrance Expansion — Fig. 2 —
— Entrance Expansion Loss Curves —
I :- Convergent Nozzle.
II :- Conv-Par. "
Full Line Curve
Entrance Exp^n. Loss
·004
·003
·002
·001
·5
·6
·7
·8
·9
Conv. Exp^n. Press. Ratio — r.
I
II

losses for this case are plotted, on the base of pressure ratio at the commencement of the parallel length, the curve marked II in fig. 2 is obtained. Considering that the nozzles are different, and that the results presented are the outcome of a considerable amount of reduction of essential data, the agreement may be held as reasonable.

The approximate relation that seems to hold between the loss and the velocity established in the entrance is shown by means of the full line curve in fig. 2, which represents a fair average, and has been plotted on the assumption that the loss varies as:-

$$(1 - r_1^{\alpha})^{1/2}.$$

where r_1 is the pressure ratio at the "throat", and α has the usual meaning. Hence it would appear that this entrance loss was directly as the velocity at the "throat".

This represents the result achieved by Mellanby and Kerr, who further assumed that the effect was probably dependent on the absolute velocity and, therefore, contained the factor $(P_1 V_1)^{1/2}$ as a necessary correction.

It is not, of course, in any way certain that the initial conditions only enter into the matter in this way since this loss, being only apparently dependent on velocity, does not have the invariability with initial conditions which is essential in the case of friction. We only know that, within the narrow range examined by the foregoing experiments, it appears that loss varies directly as velocity. It is, however, extremely probable that the initial conditions, or the fluid density, enter in such a way as to make the $P \cdot V$, or the V, factor much more active in its influence than is expressed by $(P \cdot V)^{1/2}$

Considering this we might write the total energy loss in such nozzles in the form:-

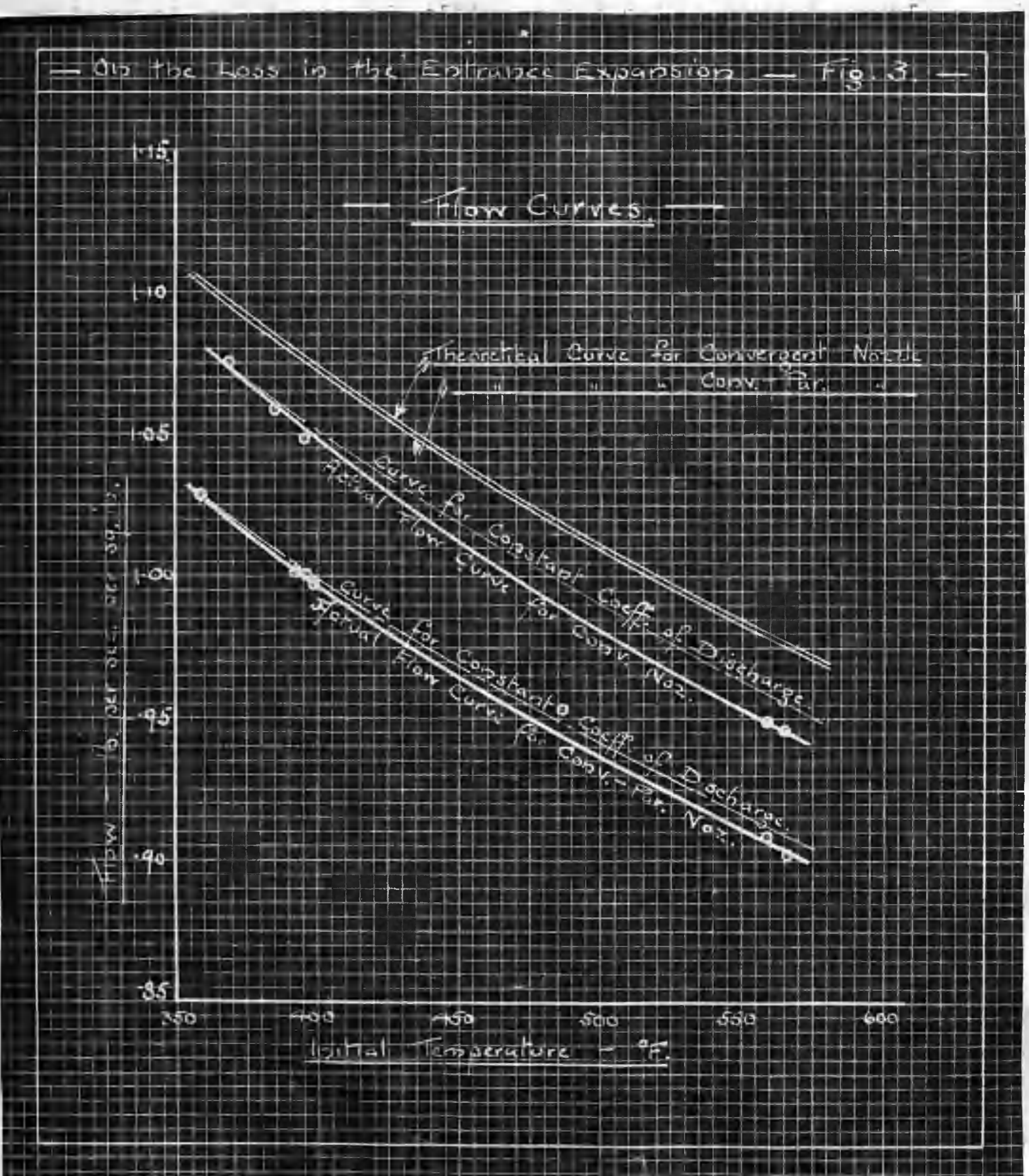
$$e = c\int_0^{l} u^2 \cdot \frac{p}{A} \cdot dy + b \cdot (P_1 V_1)^{x} (1 - r_1^{\alpha})^{1/2}.$$

or, expressed as a fractional loss:-

$$\mathcal{E} = c\int_0^l \left(\frac{u}{u_0}\right)^2 \left(\frac{\rho}{A}\right) dx + b(P_1 \cdot V_1)^x \frac{(1-r_1^\alpha)^{1/2}}{(1-r_0^\alpha)} \quad \text{-------} B\,(1).$$

in which the first term represents the friction loss in the form already treated in Section A; while the second term is meant to cover the entrance loss, with r_1 and r_0 as "throat" and outlet ratio respectively, and $(P_1 \cdot V_1)^x$ introduced for the unknown influence of the fluid conditions.

By means of (1) we get more definite ideas. Consider it from the point of view of loss variation in the two cases which have already provided evidence on the entrance effect, viz., (i), a simple convergent nozzle, and (ii), a relatively long convergent-parallel nozzle. In (i) we have a very small friction loss and a comparatively large inlet loss; in (ii) the frictional loss is very much greater and the other is definitely reduced on account of the fact that r_1 becomes greater than r_0. If the second term of equation (1) is really dependent on initial conditions, or on density, to any appreciable degree, the fact that it diminishes with increase of tail length should mean that the total loss tends to become more constant with the presence of a parallel outlet. The first term is practically constant for moderate changes in initial conditions; so that if both nozzles were operated with the same, or nearly the same, r_0 value the only matter that compels a divergence from the condition of constant efficiency with, say, increase of V_1 is the influence of V_1 itself on the entrance loss. Now, any such influence must be in the same proportion in all cases, so that with a falling loss we have a nearer approach to constant efficiency. Hence if both nozzles were run at same r_0 and same P_1, but with V_1 increasing with the superheat used, the flow curves should fall away from the condition of constant discharge coefficient (or constant efficiency); and the greater departure should be shown by the shorter nozzle.


— On the Loss in the Entrance Expansion — Fig. 3. —
— Flow Curves. —
Theoretical Curve for Convergent Nozzle
" " " Conv.-Par. "
Curve for Constant Coeff. of Discharge.
Actual Flow Curve for Conv. Noz.
Curve for Constant Coeff. of Discharge.
Actual Flow Curve for Conv.-Par. Noz.
Flow — lb. per sec. per sq. in.
1.15
1.10
1.05
1.00
.95
.90
.85
350
400
450
500
550
600
Initial Temperature — °F.

Study of Nozzle Losses:-
The Loss in the Entrance Expansion (contd.)

The actual flow for two such nozzles as determined by a few careful experiments are shown in fig. 3. These were obtained by flow measurements under changing superheat at an inlet pressure of about 76 lb. per sq. in. (absolute), the nozzles in both cases being run at their maximum ranges of expansion, which are known from search tube readings taken during the tests. The experiments were not made from any belief that they could throw light on the entrance loss effect; they are merely test results obtained for a different purpose*, together with the few results already considered in Mellanby and Kerr's treatment — the nozzles being the same two as they employed. It was the peculiar difference in the "lie" of the curves that led to their consideration from the present point of view. It will be observed that only a range well above the saturation point is included, this being necessitated by the awkward complications introduced into the flow curve by supersaturated effects at low initial superheats.

The curves in fig. 3 obviously imply the influence already explained. It is clear that the efficiency of the long parallel nozzle is less disturbed by change of temperature (or volume) than is the case with the short curved form, and would thus apparently indicate a very certain effect of fluid condition on the loss in the entrance expansion.

The actual forms of fig. 3 do not permit of any exact search. It would require a great many careful tests between the limits employed to define these curves with the necessary correctness for full analysis; but, meantime, the general features of the difference in the two cases may be held as demonstrated.

It seems on the whole, therefore, that the experimental facts of this matter are, (a), a rough indication that under constant supply conditions the loss is dependent on the range of

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* "The Supersaturated Condition as shown by Nozzle Flow" - Mellanby and Kerr. Proc. Inst. Mech. Engrs. Paris Meeting, June 1922.

expansion in the inlet and (b), a general indication that, with constant expansion range, the loss is appreciably influenced by initial temperature or volume, at least. The former appears to demand a loss effect per lb. fluid varying with the velocity attained in the entrance curvature; while the latter, so far as it may be possible to deduce a quantitative value, hints at a dependence on a power of V_1 of the order of the square.

Neither relationship can be considered very definite; the experimental data are as yet too meagre; but it must be emphasised that the two general facts taken together make out a very much stronger case for the existence of an entrance loss of important amount than either of them considered alone. Again, the evidence is from two distinctly different sources the one result arising from the detail examination by search tube, and the other from direct flow measurement in no sense dependent on the search tube readings or method. The kind of observational errors that might negative the first have no influence on the second, and vice versa. Consequently, we may feel assured that we are dealing with a real effect and some consideration of the matter along general lines, at least, seems called for.

On Stodola's "Turbulence" Factor: In a paper in 1919 * Professor Stodola carried out a study of pressure and velocity conditions in different kinds of nozzles, in the course of which he gives an interesting discussion of what he calls "turbulence work". His idea in this particular investigation was to undertake a fairly general examination of nozzle action on a poly-dimensional basis, apparently on the belief that we have now learned all that it is possible to deduce from simpler principles.

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* "Strömung in Düsen und Strahlvorrichtungen, mehrdimensional betrachtet". - Zeit. d. Ver. Deut. Ing. Jany., Feby., 1919.

Study of Nozzle Losses:-
The Loss in the Entrance Expansion (contd.)

The essential results of this examination of "turbulence" may be stated in terms of the summary as given by Stodola.

"The velocity of a steam jet issuing from a nozzle or orifice is uniform over the greater area of its cross section and only drops to zero close to the outer rim."

"The equalisation of the axial kinetic energy is the result of turbulence, the intensity of which is approximately proportional to the distance from the axis."

"The turbulence in jets is deduced from the observations of Diebold and Trüpel. This leads to the conclusion that the turbulence can be replaced by a form of friction of the outer layer which depends on the radial velocity in the same manner as the friction of viscosity, but is enormously larger."

In the investigation of turbulence Stodola considers the motion of an initially cylindrical element undergoing change of shape of an erratic order; and turbulence is defined as the quantity of momentum carried through each unit of surface of the cylinder. It is clear from this definition that the effect can be expressed - as is done - by:-

$$R = \gamma \cdot df \cdot \frac{dw}{dr}$$

where df is an element of surface; w is axial velocity and r is radius. Obviously the coefficient γ is analagous to the ordinary coefficient of viscosity, but it has a very much higher value than this well established figure, - as it must have to cover any noticeable effect in nozzle flow.

The point that we must notice here is, however, that the effect is based on a conception that seems to be simply a large scale viscosity, and is obviously by its form applicable to all parts of a jet. In fact, Stodola is primarily concerned with its application to the straight parts.

Study of Nozzle Losses:-
The Loss in the Entrance Expansion (contd.)

Under these circumstances the losses in parallel - or nearly parallel flow should reflect the influence of this, but there seems no reason why this complication should be introduced into a case where the pressure effects appear to be adequately explained by the ordinary frictional conception.

Since we have really a rational explanation of the general effects observed in a tail length we may ignore the application in such a case; but, on the other hand, such distortion rates as dw/dr must have real values at all parts of a convergent expansion and the idea could be held applicable in this part. We would then simply be assuming that the "turbulence work" is measurable in a convergence and negligible in the straight. In this way the "convergence loss" might be met by the idea of "turbulence work" which is, after all, not very different in physical basis from the "excessive agitation" used in explanation by Mellanby and Kerr. It must be pointed out, however, that when we pass to the entrance form dw/dr becomes only one of several distortion rates that must enter into the question.

If Stodola's conception is taken to mean that there is, in jets, a molar interflow analagous to the molecular communications that create viscosity the present Author is inclined to concur, since the introduction of a loss coefficient varying somewhat like viscosity, but much larger, is exceedingly helpful in meeting the facts. The idea should, however, be confined to the convergence, where there is some reason to expect actions calling for an effect of the kind.

It may be remarked that Mellanby and Kerr, using the dissipation function of hydrodynamic theory, have shown that the convergence effects as derived by them are roughly covered in magnitude and general variation if μ is given a value several hundred times greater than the usual viscosity coefficient. In fact the value of μ, as so obtained, is in fair agreement with the corresponding coefficient in Stodola's work, but the two figures are derived by different methods and apply to different actions. The Authors referred to were sceptical of the genuineness of such a coefficient derived in such a way; but Stodola apparently considers his treatment and coefficient to be rational.

We need not carry the consideration of this particular matter further, but we leave it with the general impression that a "large scale" viscosity coefficient may be a reasonable conception in nozzle action; with the proviso, however, that it is only of moment in rapid expansions where alone the factors - that render important any effect of viscosity, or any effect analagous to viscosity - are sufficiently large.

Influence of the Rate of Expansion: In a fluid the pressure in one direction at a point differs from the mean pressure at that point by an amount dependent on the distortion rates. It is usual to assume that the differences are linear functions of these rates and, hence, the expressions for the "principal stresses" are obtained as:-

$$\left.\begin{aligned} p_1 &= -p' + \lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + 2\mu\cdot\frac{\partial u}{\partial x}. \\ p_2 &= -p' + \lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + 2\mu\cdot\frac{\partial v}{\partial y}. \\ p_3 &= -p' + \lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + 2\mu\cdot\frac{\partial w}{\partial z}. \end{aligned}\right\} \text{---- B(2).}$$

where p' is the mean pressure, λ and μ are physical constants; and u, v and w are the velocities in the x, y, z directions respectively.

Allowing the linearity, it is obvious that (2) is complete or not, depending on whether we suppose that p' is or is not the same as the static pressure defined by the characteristic equation. If we maintain the generality and consistency of the argument it seems necessary to write:-

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.).

$$p' = p - \gamma\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \text{------} B.③.$$

and, hence, the mean pressure established in flow conditions may differ from that defined by the equation of state by an amount depending on a new physical constant and the rate of volume distortion.

It is clear that if this coefficient γ is considered non-existent then we have simply:-

$$p_1 + p_2 + p_3 = -3p' = -3p.$$

and:-

$$3\lambda + 2\mu = 0.$$

But, if γ be considered to have a real value, there seems no necessity to differentiate between γ and λ since there is no loss of generality by writing:-

$$\left.\begin{array}{l} p_1 = -p + \gamma\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + 2\mu\cdot\frac{\partial u}{\partial x} \\ \text{etc.} \end{array}\right\} \text{---} B④.$$

where γ and μ are both considered to be real values denoting certain physical properties, but independent of each other.

It is customary in mathematical physics to neglect γ and, writing:- $\lambda = -2\mu/3$, to state the equations correspondingly, but the procedure seems to arise from a desire to simplify rather than from any real evidence as to γ. Kinetic theory appears, however, to point to the fact that:-

$$3p' = 3p$$

and, hence, the usual process is considered reasonable.

It must, however, be said that in no case has the neglect of γ been justified by experiment, since it is only in such expansions as hold in nozzle work that we reach the high rates of volume distortion at which any γ value would make itself obvious. In all experimental determinations of μ, for instance, we establish rates of distortion of the kind du/dy, and eliminate the volume rates; thus concentrating on one of the fluid constants, and neither proving nor disproving the existence of the other.

It may, then, be taken that (4) represents the more general form of the stress equations when no governing conditions as to the mean pressures are introduced, hence maintaining the independence of y and μ. In this matter the Author does not mean to imply that hydrodynamic theory is in error but, having reached the conclusion that there are coefficients at work of the type of viscosity, and recognising also that the effects disclosed by experiment are inherent in rapid expansion, it seems desirable to explore any formal line of thought that appears to connect these two quantities naturally. If the investigation leads to inadequate conclusions we shall, at least, have demonstrated the important fact that the high volume rates are not mainly responsible.

The rate at which the stresses are doing work on unit volume is:-

$$p_1 \cdot \frac{\partial u}{\partial x} + p_2 \cdot \frac{\partial v}{\partial y} + p_3 \cdot \frac{\partial w}{\partial z}.$$

which gives:-

$$-p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + y\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2 + 2\mu\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}.$$

of which the first term represents the intrinsic energy increase and the other two denote energy losses. As here we are primarily concerned with the volume rates, we may neglect the term containing the ordinary coefficient of viscosity, and, hence, as a rough measure of the variation of energy loss due to volume distortion we may write:-

$$y\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2.$$

Now:-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = \frac{1}{V}\cdot\frac{dV}{dt}.$$

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

but, since :- $dx/dt = u$, we get:-

$$y\left(\frac{u}{V}\cdot\frac{dV}{dx}\right)^2\cdot A\cdot dx$$

for the energy loss per sec. in a volume element $A\cdot dx$. Introducing the rate of mass flow:-

$$M = A\cdot u/V.$$

there follows for the space rate of increase of energy loss per lb. fluid:-

$$\frac{de}{dx} = V\cdot u\cdot y\left(\frac{1}{V}\cdot\frac{dV}{dx}\right)^2 \text{ ---------- } B(5).$$

Now let us suppose that y is dependent on temperature in much the same way as an ordinary viscosity coefficient, viz.,:-

$$y \propto T^{1/2} \propto (PV)^{1/2}$$

and that the theoretical outlet speed is u_0', corresponding to expansion to a ratio r_0. Then, using the theoretical relationships between the various quantities and the pressure ratio r, substituting in (5) and simplifying out, there finally appears for the fractional loss of energy:-

$$\mathcal{E} = \frac{b\cdot V_1}{1-r_0^{\alpha}}\int_0^{l} r\left\{\frac{1}{r^{\alpha}}-1\right\}^{\frac{1}{2}}\left\{\frac{d}{dx}(r^{\alpha-1})\right\}^2 dx \text{ ----- } B(6).$$

where b is a constant.

This equation gives a loss which, for constant initial conditions, is dependent on the expansion range; and for constant overall ratio of expansion (which for a given nozzle involves a definite ratio curve and, therefore, a constant value of the integral) varies with initial volume.

Equation (6) then, approaches the order of change of the effects actually observed; but when the attempt is made to compare it with these effects a very definite disagreement is discovered. The rate of change of $\mathcal{E}$ with r at constant V_1 is much too rapid to meet, even approximately, the observed variations as recorded in fig. 2; while the dependence on V_1, at constant r_0, seems somewhat too slow to give reasonable agreement with the special features of fig. 3.

This result, then, does not provide a suitable explanation. It has been included in the discussion to show one of the possible lines of attack; one, indeed, that is very reasonable in conception in view of the apparent limitation of the loss to rapid expansion zones. While it is not satisfactory, it demonstrates two points of some value, viz., that the loss seems to be dependent on some slower effect than the total volume distortion rate, and that the essential coefficient would appear to have a more active dependence on temperature than the usual viscosity coefficient.

Influence of Spin: It is well known that in expanding through a curved entry a spin effect is set up in the fluid. This is particularly noticeable with water nozzles; but it can also be observed with purely convergent steam nozzles, more especially when the nozzle section is not symmetrical about the central axis. Anderson has made a note somewhere which indicates that when water is passed through a simple convergent nozzle the rotation of the jet is conspicuous, but if the nozzle is of the convergent parallel type no spin can be detected at the outlet. This, however, does not preclude the existence of a spin during the initial expansion, since the progress along the confining parallel channel will naturally damp out the rotation.

That a rotation is almost inevitable, at least in the first stages of development, can easily be recognised when the essential form of the jet is considered. On the outer boundary there is a rapid curvature which reduces to zero at the centre of the jet, and again at the "throat". Any fluid element in an outer layer is not merely acquiring forward speed in expanding but must be subject to lateral acceleration, which will depend on the curvature. Since there is no reason to believe that the lateral speeds will simply be radial we must presume that, in general, there are components in both directions at right angles to the forward velocity, and these together will give the element a circumferential velocity. Since the curvature is greater at the greater radii the lateral speeds will increase in some way with the distance from the axis. This is all that is necessary to create what would appear as a mass spin. Obviously, there must be only a small tendency in this respect towards the completion of the entrance expansion and, hence, any spin present at the throat would be rapidly damped out in the subsequent parallel flow.

It would seem, then, that spin of the fluid is a natural accompaniment of convergent expansion. Search tube records only provide a guide to forward velocities. Any spin is, therefore, an energy form included in the loss effects at the end of the convergence and, as such, deserves a little attention in the present discussion of these.

We may consider the matter by means of dimensional theory. Naturally it would be imagined that energy loss due to the present cause would involve the density and, hence, we would choose as the main variables density, velocity and spin. Writing:-

$$E = f(\rho, l, u, \theta)$$

where E is energy loss per unit volume per unit time; ρ is density; l is a linear dimension; u is the forward velocity; and θ is the rotational velocity. Equating dimensions we have:-

$$M \cdot L^{-1} \cdot T^{-3} = M^{a} \cdot L^{-3a} \cdot L^{c} \cdot L^{d} \cdot T^{-d} \cdot T^{-e}$$

and, therefore:-

$$a = 1, \quad c = e - 1, \quad d = 3 - e.$$

This gives:-

$$E = \frac{\rho . u^3}{l} \cdot f\left(\frac{l\theta}{u}\right)$$

and reducing to rate of energy loss per lb. fluid - according to the process already frequently illustrated - we get:-

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

$$\frac{de}{dx} = \frac{u^2}{l} f\left(\frac{l\theta}{u}\right) \quad \text{B (7).}$$

If this is to meet the condition of loss varying with velocity, it would be necessary to write it with the first power of the dimensionless number as the unknown function. The spin would then require to be a constant for the nozzle; size would have no effect; and the fluid conditions would hardly influence the loss. This particular process of examination would appear, then, to be rather fruitless as the results are not merely unsatisfactory, they are almost irrational.

But consider the corresponding line of argument with density omitted and a coefficient like viscosity in its place. Whereas in the last case we assumed the spin loss to depend on the fluid density, this suggested mode of attack would mean that the spin itself was to some extent governed by a physical coefficient, just as the form of turbulence may be controlled by the fluid viscosity.

This would give:-

$$E = f(y, l, u, \theta).$$

which, on equating dimensions, becomes:-

$$E = y \cdot \theta^2 \cdot f\left(\frac{l\theta}{u}\right).$$

or:-

$$\frac{de}{dx} = y \cdot \theta^2 \cdot \frac{V}{u} \cdot f\left(\frac{l\theta}{u}\right) \quad \text{B. (8).}$$

and it is only necessary to allow that θ varies directly as u, and that y involves the fluid conditions, to obtain a very satisfactory result. Thus with:-

$$y \propto (P.V), \quad \text{and} \quad \theta \propto u.$$

we would obtain a representation of loss showing about the required variation with r for constant supply conditions, and a dependence on $V_1^{3/2}$ for constant expansion range and initial pressure. Clearly, also, if the unknown function has a real significance,

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

there is an effect of size.

This result represents the first fair agreement with fact that we have so far reached. It cannot, however, be considered altogether acceptable. It is hardly credible that a real loss due to spin can be dissociated from density and grouped with a viscosity influence in preference. Although, therefore, the apparently necessary deduction that spin varies with velocity is fairly rational, the main basis of the whole result would appear to be im-perfect. The fluid may expand in such a way as to create a spin in the inlet form, but it does not seem possible that the loss in the convergence can be imagined as the rotational energy of spin.

We cannot, therefore, be content with (8) ; but the fact that it does give agreement leads naturally to a closer scrutiny of the quantities contained therein; for, since the structure results from equating dimensions, any quantities of like units will give a corresponding form and, perhaps, an equally satisfactory agreement. The fact is then recognised that θ has exactly the same dimension as a rate of deformation, viz., T^{-1}, and hence there is, in all probability, some such rate that, in association with γ and u, will approximately explain the results. The point of view of a previous section proved conclusively enough that the rate of volume distortion is too rapid; so that the guidance afforded by both demonstrations leads naturally to the next mode of attack.

Influence of the Rate of Convergence: It has already been explain-ed that in the convergent expansion the fluid must, in general, possess velocities in the directions perpendicular to that of the main axial flow. Now these velocities are generated of necessity in the outer layers, because of the curvature of the flow lines there; and they are on the whole directed towards regions of lesser curvature. They must, therefore, be damped out, or transformed to axial velocity, since they cannot be appreciable either at the centre of the jet or at the end of the convergence. In the earlier stages of the expansion, and in the outer portions of the jet, these lateral velocities must be comparable with the main flow velocity and, so, should exceed the ordinary "velocity of turbulence".

The reduction or transformation of the lateral velocities will result in energy losses showing by part change, at least, from useless flow motion to heat motion. There will be no corresponding loss of the kind in connection with the main flow, since the same conditions do not exist therein. In effect, the argument denies "symmetry of the fluid loss actions" if such an expression may be permitted; and, in this respect, is in keeping with the usual experimental methods which measure one velocity only as useful. We must admit that if a fluid element has three mutually perpendicular velocities, and only that in one of these directions can ultimately exist, the other two must give rise to excessive losses in comparison; since neither elimination nor transformation can be perfectly carried out.

We may, then, examine the matter from the point of view that only the lateral deformation rates are important. Although borrowing in a crude way from theoretical hydrodynamics, it must be recognised that we are dealing with effects on a much larger scale. If, then, v and w are the velocities in the y and z directions, perpendicular to the main flow velocity, u, in the x direction, we might put:-

$$E = \gamma\left\{\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\}.$$

for the energy loss rate per unit volume.

Now, from the equation of continuity:-

$$M = A \cdot u / V.$$

we obtain:-

$$\frac{dA}{dx} = \frac{A}{V}\cdot\frac{dV}{dx} - \frac{A}{u}\cdot\frac{du}{dx}.$$

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

or:-
$$\frac{u}{A}\cdot\frac{dA}{dx} = \frac{u}{V}\frac{dV}{dx} - \frac{du}{dx}$$

Again, since:-

$$\frac{u}{V}\cdot\frac{dV}{dx} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.$$

it follows that we may say:-

$$\frac{u}{A}\cdot\frac{dA}{dx} = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}.$$

and, allowing symmetry in the two lateral directions, we have:-

$$\left\{\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\} \propto \left(\frac{u}{A}\cdot\frac{dA}{dx}\right)^2$$

Hence we may write:-

$$E = y\left(\frac{u}{A}\cdot\frac{dA}{dx}\right)^2$$

or, by reduction in the usual way:-

$$\frac{de}{dx} = y\cdot V\cdot u\left(\frac{1}{A}\cdot\frac{dA}{dx}\right)^2 \text{---------} B.(9).$$

which might have been deduced directly from (5), ante, if we had merely assumed that, since the volume rate was excessively high, the area rate might be more suitable.

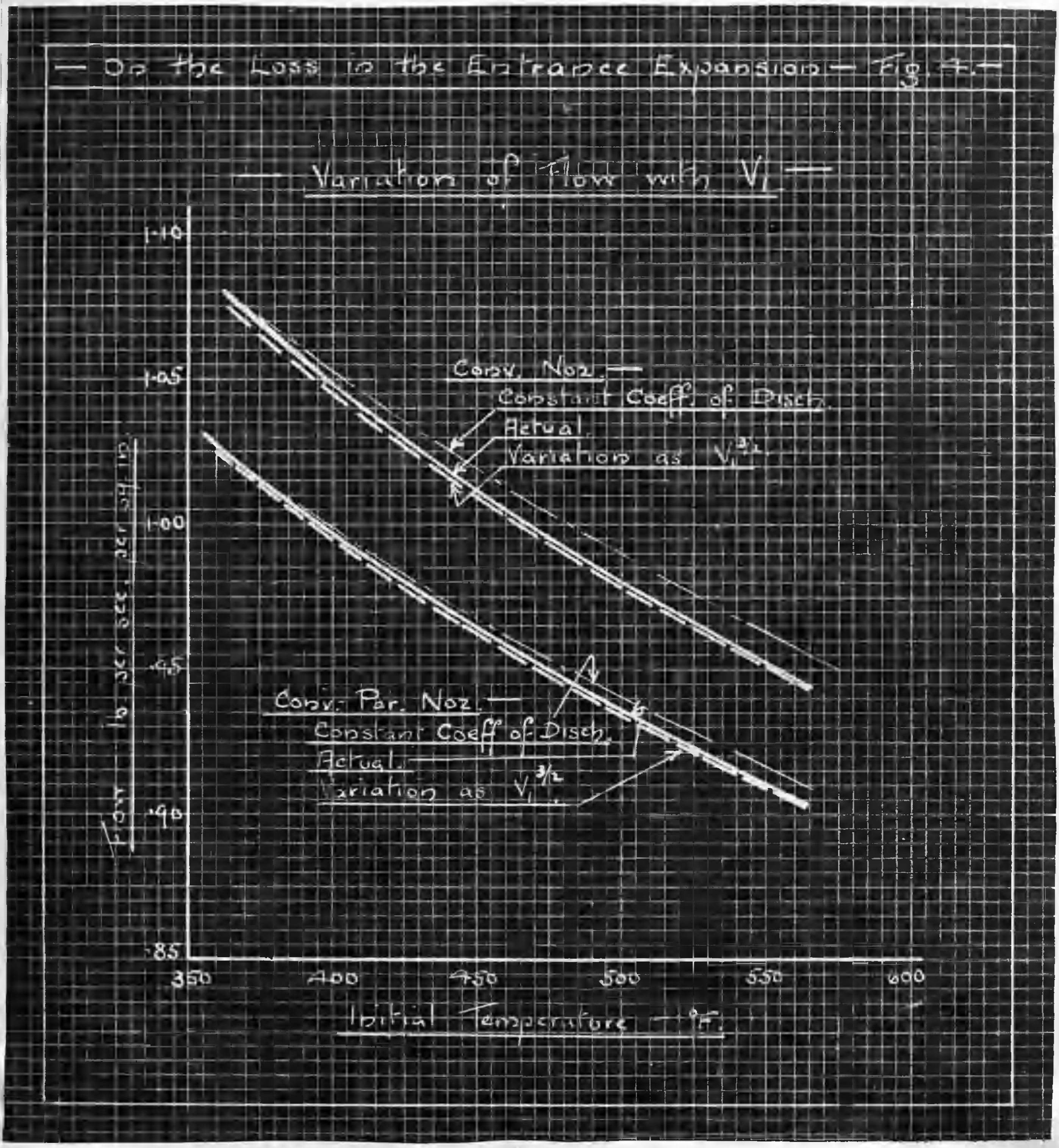
Let us suppose that:-

$$y \propto T \propto (P.V.)$$

since it is clear from the attempted explanation that the molecular energy must enter very definitely into the matter. Then, developing (9) by means of the relations between the various quantities, the ratio of expansion, and the initial conditions, we obtain for the fractional loss per lb. fluid expanding to an outlet ratio r_o :-

$$\mathcal{E} = b\cdot P_1^{\frac{1}{2}}\cdot V_1^{\frac{3}{2}}\int_0^l r^{2\alpha-1}\frac{(1-r^\alpha)^{\frac{1}{2}}}{(1-r_o^\alpha)}\left(\frac{1}{A}\cdot\frac{dA}{dx}\right)^2 dx \text{-----} B.(10).$$

This equation gives the loss as varying with ratio of expansion, as dependent on the inlet form, and as definitely influenced by the initial conditions of the fluid. The expression $(1/A)(dA/dx)$ is worthy of remark. It represents a very suitable


— On the Loss in the Entrance Expansion — Fig 7. —
— Variation of Flow with V_1 —
Conv. Noz. —
Constant Coeff. of Disch.
Actual.
Variation as $V_1^{3/2}$
Conv. Par. Noz. —
Constant Coeff of Disch.
Actual.
Variation as $V_1^{3/2}$
Flow — lb per sec per sq in
1·10
1·05
1·00
·95
·90
·85
350
400
450
500
550
600
Initial Temperature — °F.

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

method of measuring the inlet form, since the area reduction rate per unit of area is clearly a reasonable figure wherewith to express the general acuteness of the entrance curvature. The data in hand do not permit of any real check of the validity of this factor, since both nozzles for which we have information have almost exactly the same entrance form. It is clear, however, that (10) does not involve any absolute influence of size, as it is possible to have the same value of $(1/A)(dA/dx)$ for different values of A, although this constancy will be the exception rather than the rule.

Since, for the actual cases, $(1/A)(dA/dx)$ is the same for the cases considered its influence even in the integral will not be appreciable, so that under constant initial conditions we have approximately:-

$$\varepsilon(1-r_0^{\alpha}) \propto \int r^{2\alpha-1}(1-r^{\alpha})^{\frac{1}{2}}\, dx$$

Now $r^{2\alpha-1}$ is very much less rapid in its variation than $(1-r^{\alpha})^{\frac{1}{2}}$ and, owing to the form of the r curves in general, it is even less effective when involved in the integral. Consequently the loss $\varepsilon(1-r_0^{\alpha})$ will depend very largely on $(1-r^{\alpha})^{1/2}$, which is almost exactly the result derived from the actual figures - see fig. 2. Actually (10) gives a slightly greater rate than that due to velocity, but the difference is inconsiderable when compared with the normal uncertainty of the effect.

With the expansion ratio fixed the pressure curves for the inlet are not noticeably affected by the steam conditions. The integral is therefore practically a constant in such circumstances, so that with constant r_0 and P_1 the equation indicates:-

$$\varepsilon \propto V_1^{3/2}.$$

This may be compared with the flow curves of fig. 3 for, knowing both entrance and friction losses for these nozzles we can determine the total losses and, therefore, how the coefficient of discharge should vary. The comparison is shown by the heavy dotted line in fig. 4, in which the flow curves of fig. 3 are reproduced. The effects observed are of course small but the curves indicate at least that such a variation as is required by (10) is not in conflict with the particular facts established by the flow experiments.

For general changes in fluid conditions at constant pressure ratio (10) gives:-

$$\mathcal{E} \propto (P_1 \cdot V_1)\left(\frac{V_1}{P_1}\right)^{\frac{1}{2}}.$$

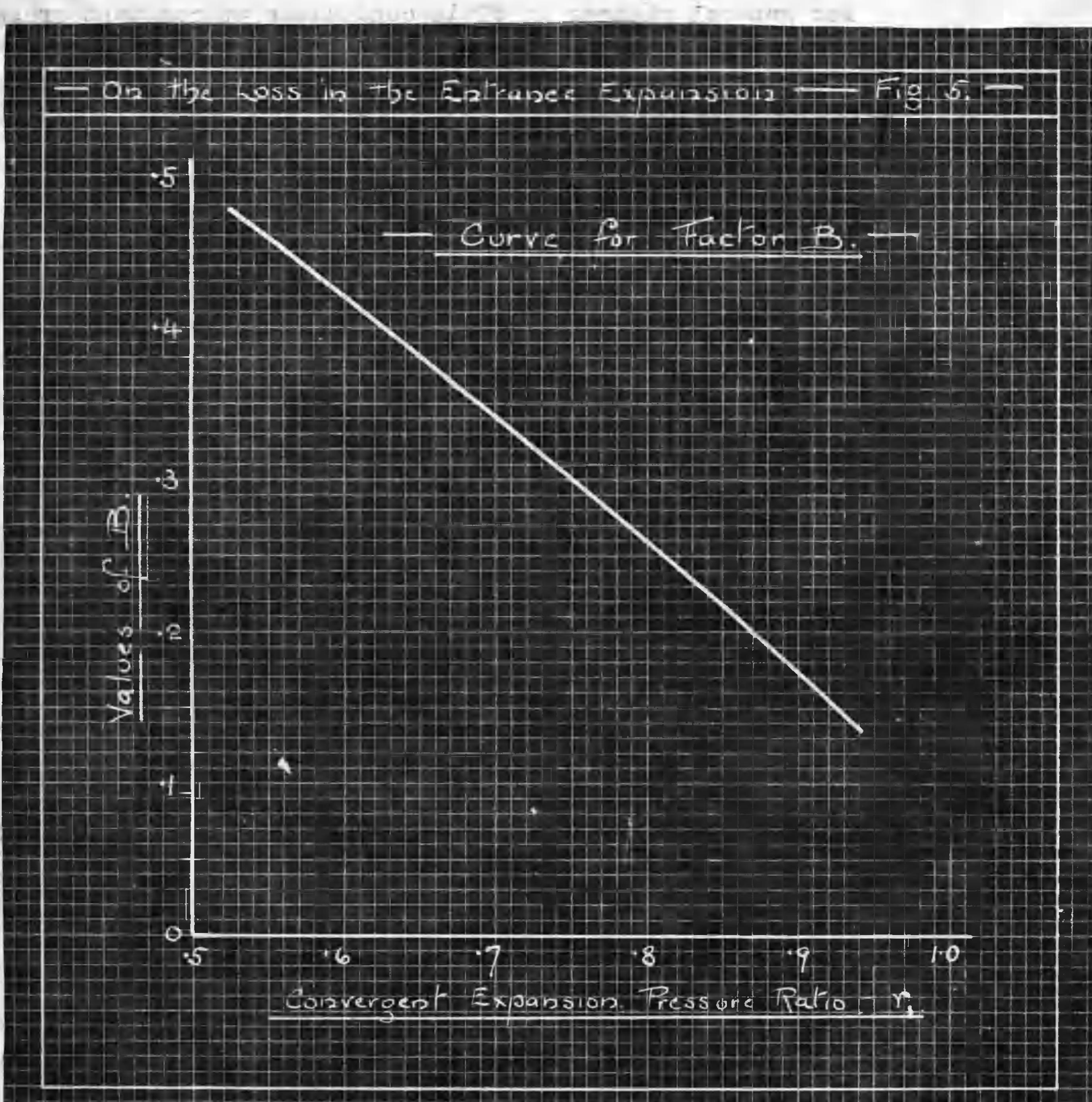
which for any one nozzle means, practically, a variation directly as absolute temperature and ~~xxxxxxxx~~ inversely as the mass flow per sec.

<u>Conclusion</u>: Considering the imperfections of the data, on which the evidence of this entrance loss rests, the general agreement achieved by equation (10) may be held as fair. In view of this, and the comparatively rational basis of the explanation which has resulted in this equation, we may accept the idea put forward in the last discussion as a useful conception of the loss action in the convergence. The points of view developed in the previous sections have been introduced to show the different lines of thought which have been followed out in the attempt to achieve a rational outlook. Both rapidity of expansion and spin are characteristic of convergent expansions and, therefore, worthy of notice; but, in reality, they have been of value mainly on account of the guidance they afford for the establishment of a fuller explanation.

It is not a very satisfactory procedure to reduce (10) to a rough form for the purpose of making estimates of this effect but, in view of the unsuitable form of the main equation for such calculations, the following may be tentatively advanced:-

$$\mathcal{E} = \frac{3 \cdot 5}{10^6} \cdot \frac{B}{1 - r_0^{\alpha}} \cdot \ell_1 \cdot P_1^{1/2} \cdot V_1^{3/2} \left(\frac{1}{A} \cdot \frac{dA}{dx}\right)^2 \text{ – – – – – – B.(11).}$$

in which:-



On the Loss in the Entrance Expansion
Fig. 5.
Curve for Factor B.
Values of B.
·5
·4
·3
·2
·1
0
·5
·6
·7
·8
·9
1·0
Convergent Expansion Pressure Ratio r_1

Study of Nozzle Losses:-
The Loss in the Entrance Expansion. (contd.)

ϵ. = fractional loss of energy in the full expansion due to entrance effect.
P_1. = initial pressure - lb. per sq. in. (abs.).
V_1. = " volume - cub. ft. per lb.
r_0. = pressure ratio at outlet.
α. = $(n-1)/n$, with n = 1.3.
l_1. = length of convergent expansion - inches.
B. = factor covering the influence of the convergent expansion range, a curve for which is given in fig.5.
$\left(\frac{1}{A}\cdot\frac{dA}{dx}\right)$ = convergence factor, which should be estimated from that part of the inlet form at which the pressure curve has its maximum gradient or, otherwise, at about the mid length; A and x in inch units.

The most that can be said for this expression is that it is in rough agreement with the variation demanded by (10), and with the observed figures for the investigated cases.

It will be difficult, however, to estimate the convergence factor properly and this may upset the result. If the form of the pressure curve is known, the value of $(1/A)(dA/dx)$ may be most accurately determined by using the "jet function" of Mellanby and Kerr's system of nozzle analysis. It is easy to show that:-

$$\frac{1}{A}\cdot\frac{dA}{dx} = -\frac{1}{F}\cdot\frac{dF}{dx}.$$

where F is, approximately:-

$$\frac{r\left(1-r^{\alpha}\right)^{1/2}}{r^{\alpha}}$$

and the value obtained for the factor in this way, by using the main pressure range in the inlet, should be a fair measure of the quantity.

The curve for B in fig. 5 gives practically the theoretical variation within the range usually possible in convergent forms.

A STUDY OF NOZZLE LOSSES.

C :- COMPRESSION LOSSES IN DIVERGENT JETS.

Introductory: In convergent type nozzles the expansion is continuous throughout the length and, according to Mellanby and Kerr,* it is possible, by the analysis of data from search tube experiments, to indicate the nature and value of the loss effects accompanying this expansion. When the attention is directed, however, to the action in any divergent jet form, it is at once found that the condition of a continuous expansion along the full length no longer generally applies. In order to meet a back pressure in excess of the normal the jet compresses itself with great facility; and the many disturb:ing effects produced by this occurrence make the main features of the flow somewhat difficult to analyse.

In the deduction of nozzle efficiencies from search tube and flow observations, Mellanby and Kerr⊕ have explicitly assumed that the outflow area of any nozzle is always completely filled. This assumption is certainly justified in all cases of continuous expan:sion, no matter what the type of nozzle may be; and although experiment actually hints at a contraction of area in a recompression effect, it could hardly be supposed that this would be sufficient to affect the results appreciably. Detail consideration of the case of the divergent nozzle shows, however, that the assumption is inaccur:ate where a recompression of the fluid takes place within the tail length, and this condition requires further attention.

Now, it is an essential matter in the search tube method of experiment - and, consequently, in any analytical system resting on this - that the area filled by the flowing fluid should be clearly defined. In continuous expansion in any normal nozzle this is the

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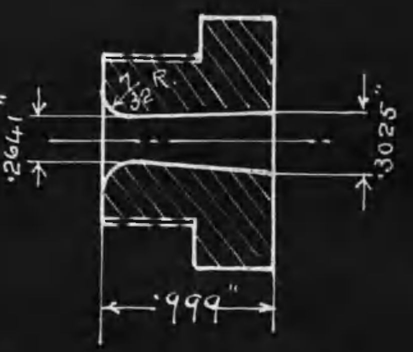
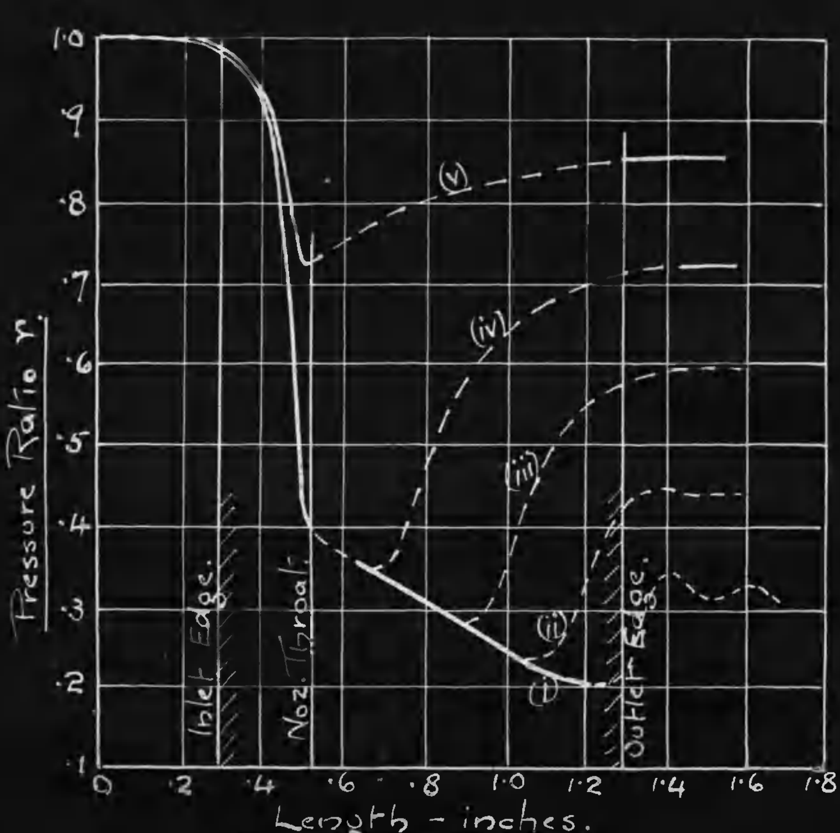
* "On the Losses in Convergent Nozzles" - Proc. N-E Coast Inst. Engrs. & Shiprs., Feby. 1921.

⊕ "Pressure Flow Experiments on Steam Nozzles" - Proc. Inst. Engrs. & Shiprs. in Scotland., Nov. 1920.

— Compression Losses in Divergent Jets. —



— Fig. 1 :- Convergent-Divergent Nozzle. —



— Fig. 2 :- Pressure Ratio Curves. —

area marked out by the nozzle boundary; but if, in a recompressing fluid, the boundary ceases to control, the true area is not directly measurable. It then follows that the search tube method is, by itself an insufficient guide to the jet action during compression and, consequently, in the study of such effects additional data must be sought elsewhere.

It is proposed here to investigate the somewhat meagre data available on this subject, with a view to the establishment of the few results that can be deduced regarding compression effects in divergent nozzles. For the sake of clearness and completeness it would seem best to present first, and briefly, the result of a treatment of divergent nozzle data on the basis of filled areas throughout. This serves to show the main error in the assumption, and makes the subsequent developments clearer; and, on this score, the following section, which recapitulates to some extent, may be excused.

Results of the Direct Treatment: In the study of nozzle losses the following expression is of great service:- *

$$F = c\left(\frac{V_1}{P_1}\right)^{\frac{1}{2}}\frac{G}{A} = \frac{r\left(1-k-r^{\alpha}\right)^{\frac{1}{2}}}{k+r^{\alpha}} \quad --- \quad C\,(1).$$

in which:-

- P_1 & V_1 are initial press.(lb./sq.in.) and volume(ft^3/lb.).
- G = massflow - lb./sec.
- A = flow area - sq.in.
- r = pressure ratio = P/P_1
- k = loss factor.
- α = $(n-1)/n$, where n is the adiabatic index.
- n = 1.3 for superheated steam, and 1.4 for air.
- c = 0.718 " " " " , " 0.800 " " .

For present purposes the chief value of expression (1) lies in the ease with which a graphical representation of a nozzle performance may be obtained by means of a chart, covering the possible variations of the function F on a base of r ; and including a

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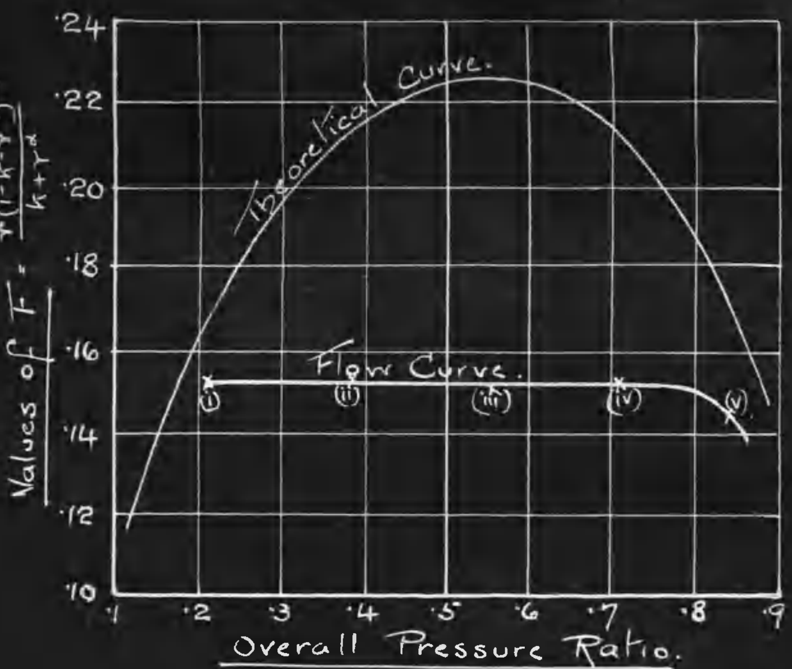
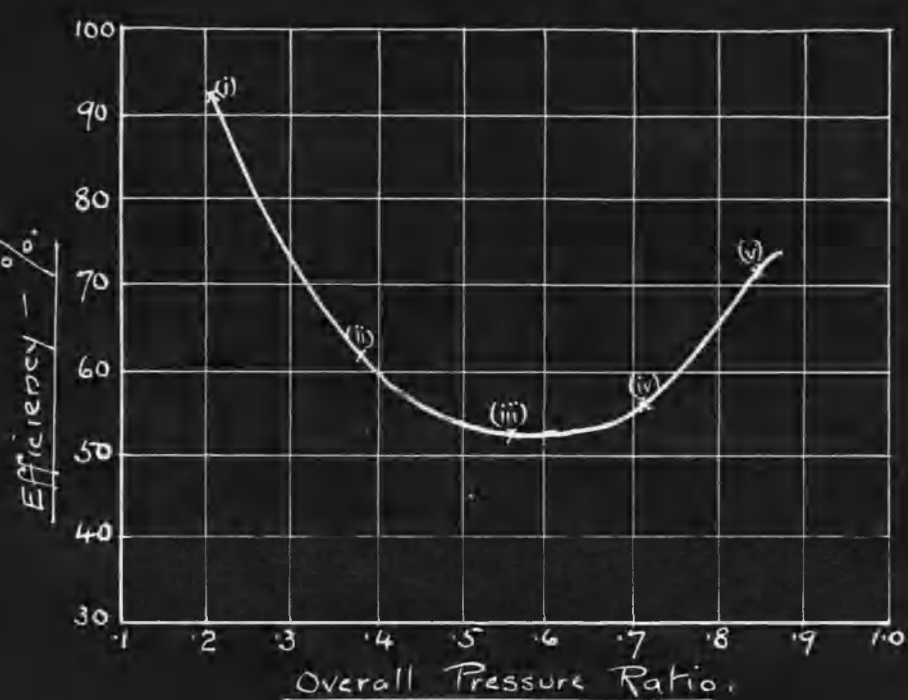
* "Steam Action in Simple Nozzle Forms" - Mellanby and Kerr. British Association, Section G., August 26, 1920. or "Engineering", Sept. 3, 1920.

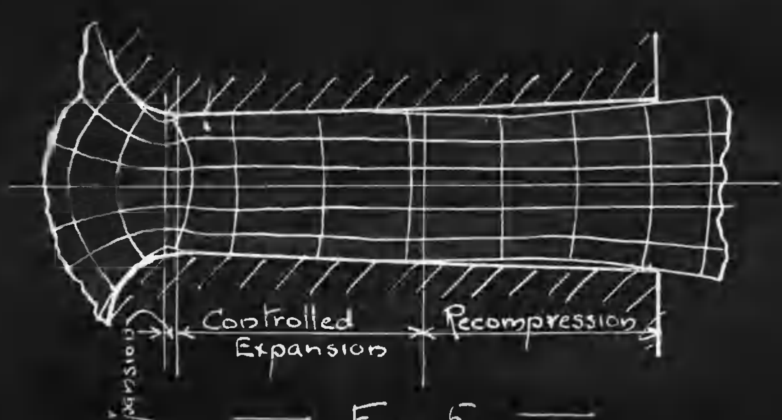
— Compression Losses in Divergent Jets. —



— Fig. 3 :- Flow Curve. —



— Fig. 4 :- Efficiency Curve. —



— Fig. 5. —

series of curves of constant loss. From experimental results -with A known - F is readily calculated and, hence, k is directly deter:mined; and, if A and r are known for all parts of the expansion the curve of loss can be derived.

If attention is concentrated on the outflow section the variation of F on the standard chart shows the variation of flow with different outlet conditions. Such a curve is known as a "flow curve". with the k value at the end section then known the effic:iency of the nozzle is easily obtained from:-

$$\eta = 1 - \frac{k}{(1 - r^{\alpha})} \qquad (2).$$

Again, if in dealing with the experimental data use is made of the change of pressure ratio and section along the nozzle, the resulting curve of the function F represents the "nozzle char:acteristic" for the given range of operation. The flow curve is simply the locus of the terminal points of the full series of char:acteristics.

Figs. 2,3 and 4 are reproductions of diagrams* for the case of the convergent-divergent nozzle shown in fig. 1. Fig. 2 shows the pressure ratio curves for the different ranges and, in this, the recompression effects are clearly brought out. On the assumption of a filled outflow area fig. 3 is readily obtained from (1).; while calculation, by means of equation (2), gives the corres:ponding efficiency curve (fig. 4) on the same assumption.

So far there is no very obvious fault in the manner of investigation for, while the efficiency falls considerably, the minimum values are not excessively low, and the actual variation is somewhat as would be expected.

In fig. 2 parts of the pressure lines are shown dotted. This procedure is meant to indicate the ranges in which the pressure

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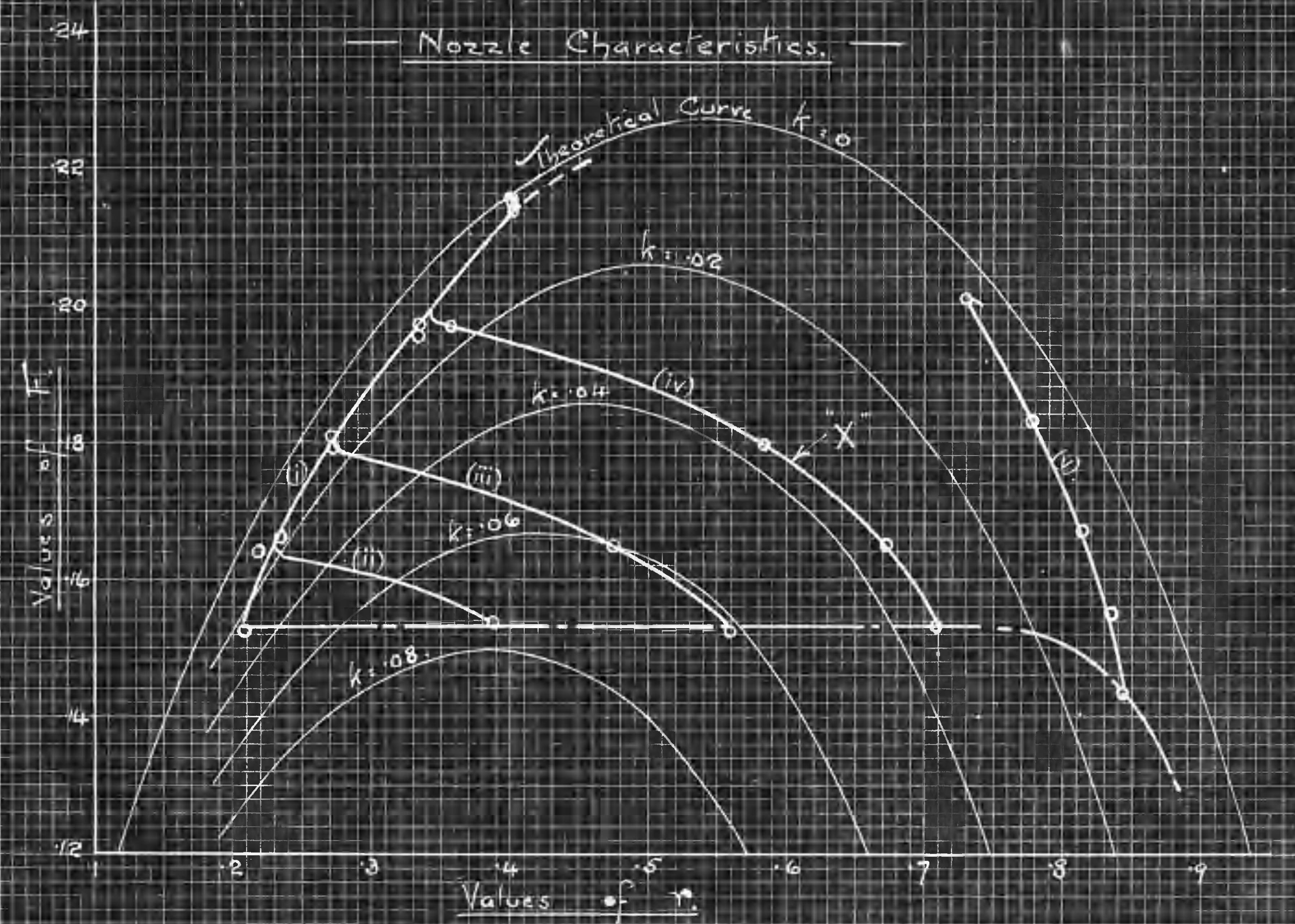
* "Pressure Flow Experiments on Steam Nozzles" - Proc. Inst. Eng[rs]. & Ship[rs]. in Scotland, Nov. 1920.

Compression Losses in Divergent Jets — Fig. 6.

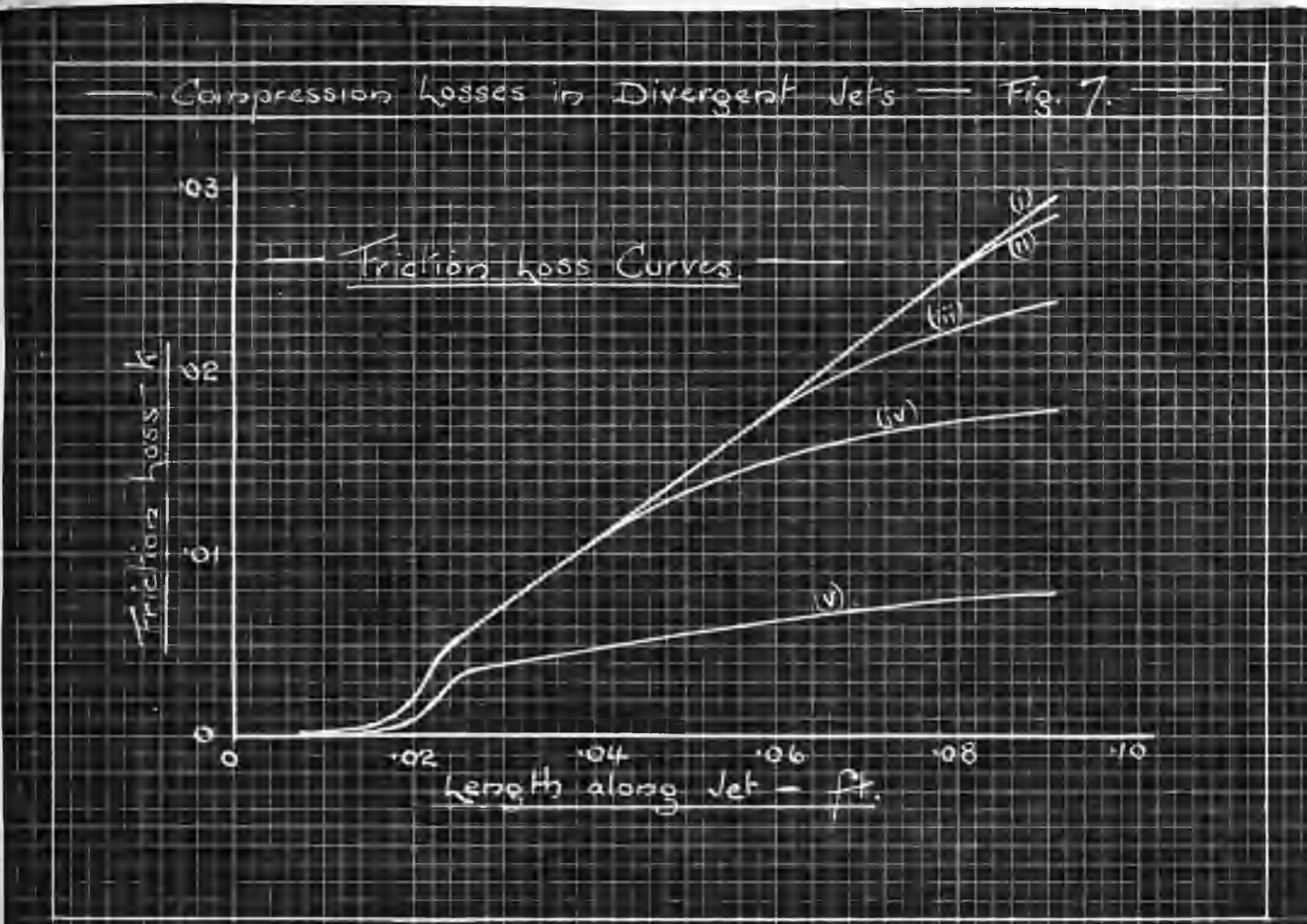


readings were unsteady; and, from other considerations, it was deduced that this resulted from a breakaway of the jet from the nozzle walls. In fact, fig. 5 was advanced as a representation of the actual occurrence. It will be seen that, while jet contraction was thus envisaged in the initial stages of compression, it was not thought to be of such an extent as to prohibit the use of the full nozzle sections near the end of the compression.

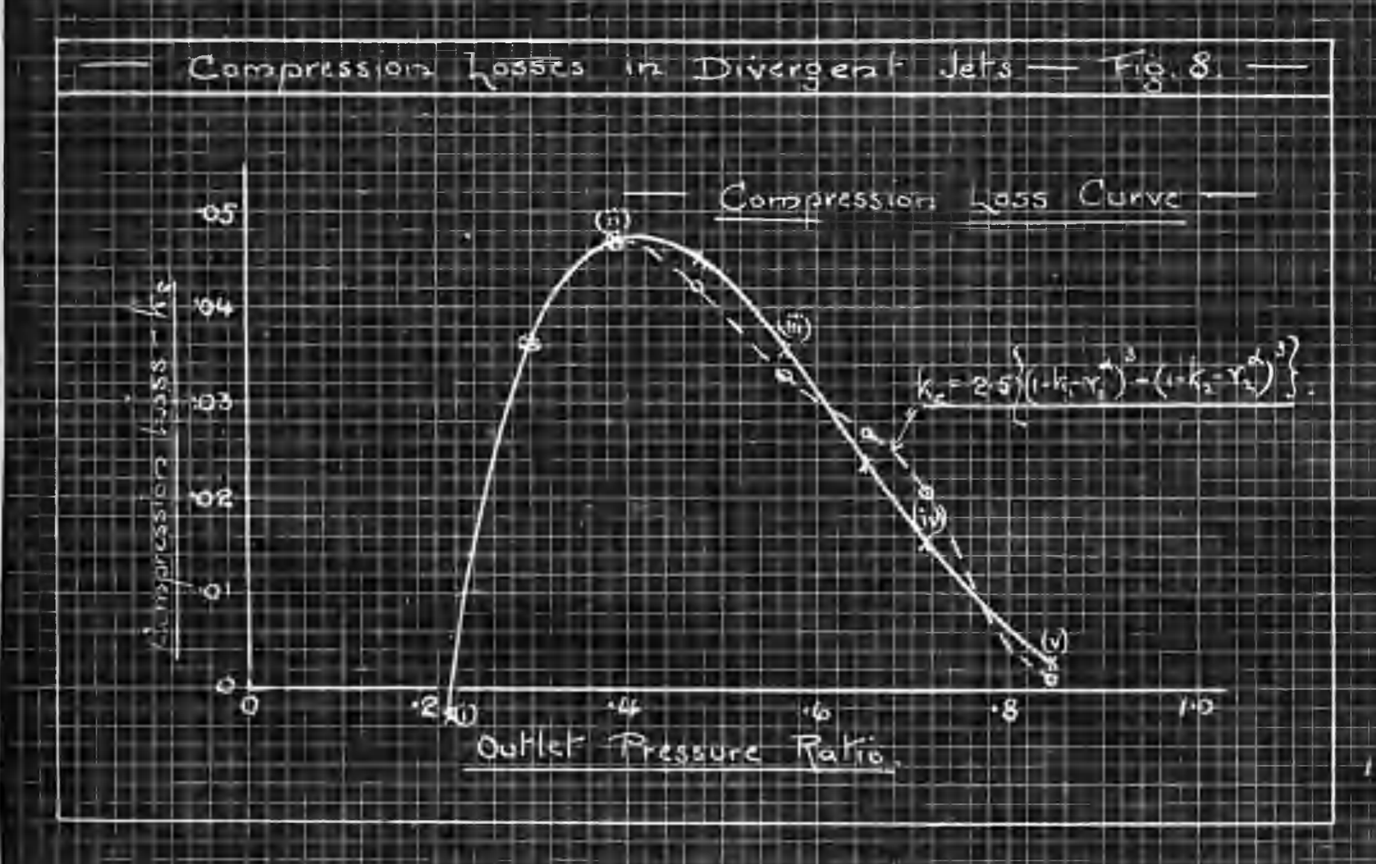
On a continuation of the study, beyond the stage of publish::ed records, by detailed determination of the nozzle character::istics, the diagram in fig. 6 is evolved. This assumes filled areas throughout and, while it thus traverses the above finding for the conditions at the start of the compression, it was supposed that the examination of the <u>complete</u> losses during compression would still be fairly reasonable.

A definite discrepancy appears, however, in these nozzle characteristics. It is an obvious condition that, since the expan::sion and compression are suffering continuous loss, the k value must progressively increase; or, stated explicitly, the nozzle characteristic can only cross any single k line once. This condit::ion is seen to be violated by such a curve as "X" (fig. 6) for instance, where the k value at a position well before the end of the expansion is slightly higher than that given by the terminal point. This is the first really certain indication of an error in the main line of reasoning.

With less confidence the examination may be carried through to a finish. The losses due to friction and to the ent::rance expansion may be determined by methods that have already been treated. This results in fig. 7. Again, from the end points of the characteristic curves the apparent total losses are obtain::able. The difference between these and the total amounts in fig.7 represent the losses that must be charged to the compressions. When these are plotted on a base of overall pressure ratio the full line curve in fig. 8 is established.



Compression Losses in Divergent Jets — Fig. 7.



Compression Losses in Divergent Jets — Fig. 8.

Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

These losses are seen to show a high value for a large compression range occurring at high speeds of flow; and fall away rapidly, either for small ranges at high speeds, or for large ranges at slow speeds. It seems obvious that the speed is the main influ:ence; and, assuming that they depend on some power of the speed, the losses should be given by:-

$$k_c \propto \left\{ (1 - k_1 - r_1^{\alpha})^{x/2} - (1 - k_2 - r_2^{\alpha})^{x/2} \right\} \quad \text{---} (3).$$

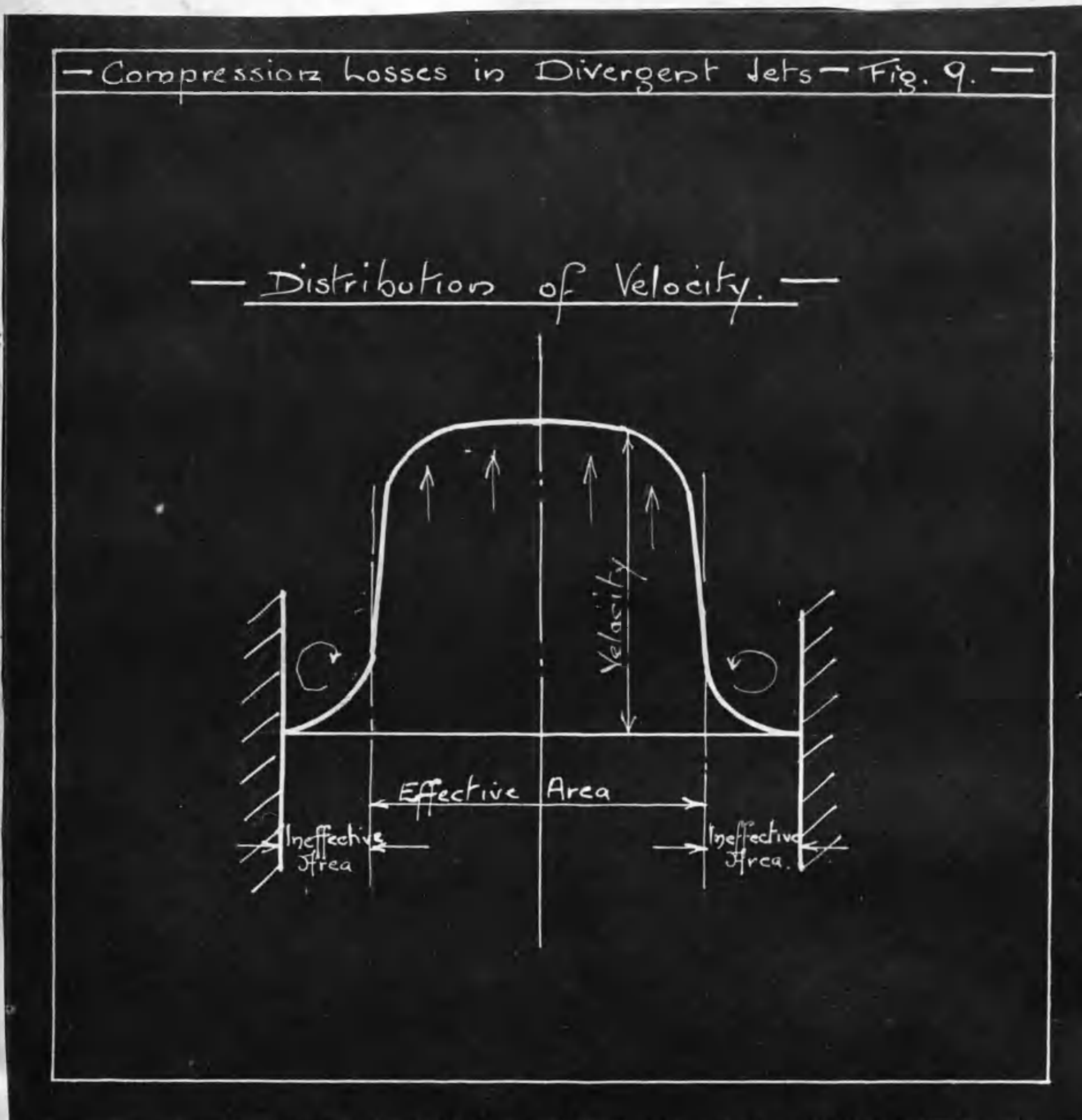
where r_1 and r_2 are the ratios, and k_1 and k_2 the total losses at start and finish of compression respectively. The power of speed involved is then given by x .

The best agreement on this basis is shown by fig. 8, the dotted curve representing the equation (3) with $x = 6$; thus appar:ently indicating that the loss in compression varies with the sixth power of the speed.

In conjunction with previous uncertainties this result gives definite pause, as it represents an extraordinary order of loss. The maximum range of expansion of the particular nozzle under review is not sufficiently high to demonstrate this with the greatest clearness; but it will readily be disclosed on consider:ation of the results that would follow from a similar attack on nozzles with large area ratios, say, four or five to one.

It will be seen, then, that there is a definite fault in the assumption of filled areas in compression ranges. The assump:tion is correct for cases of continuous expansion; it is also true for compressions commencing at the nozzle throat, as will be seen later; but where the compression starts at an intermediate point in the tail length it forms its own throat independently of the nozzle boundary. It follows that for all compressions commen:cing beyond the throat of a divergent nozzle the flow areas are uncertain.

The direct method of attack for compression losses, as outlined above, therefore, fails. It has been pointed out, however, that the other losses are rationally covered by the usual methods


— Compression Losses in Divergent Jets — Fig. 9. —
— Distribution of Velocity. —
Velocity
Effective Area
Ineffective Area
Ineffective Area.

Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

If, then, we suppose these known to the start of the compression, it only requires a knowledge of the total overall k value to be able to determine the total compression loss. The determination of this k is not possible to the search tube method when the areas are unknown; but it can be made by means of reaction results. Hence for the study of compression losses, data from reaction exper:iments on divergent nozzles seem essential; and, for full detail research on this subject, a combination of the search tube and reaction methods is indicated. For the purposes of the present treatment a special analysis is made, in a later section, of the results of Morley's* reaction experiments with air nozzles.

The condition of affairs in a jet compressing within a divergent nozzle may be visualised somewhat as in fig. 9. In this we imagine a central core of fluid moving as if confined by a solid boundary set to its own natural form. The annular space surrounding it may be supposed filled with eddies that add practically nothing to the flow, but detract somewhat from the full jet energy. From one point of view this amounts simply to an extreme case of irreg:ular velocity distribution across the nozzle section; but, in effect, we have a fairly normal jet that occupies only a part of the area provided.

Consideration of the Flow Curve: In the previous treatment of the flow curve for the convergent-divergent nozzle, the form of the curve results in a simple way from the assumption of filled areas. This has been shown to be wrong, but the curve form is still useful as the expression of a limiting condition. It is of value to study the opposite limit so that, by means of an intermediate type, the probable true form may be indicated.

Consider the section of an "F" chart shown in fig. 10, on

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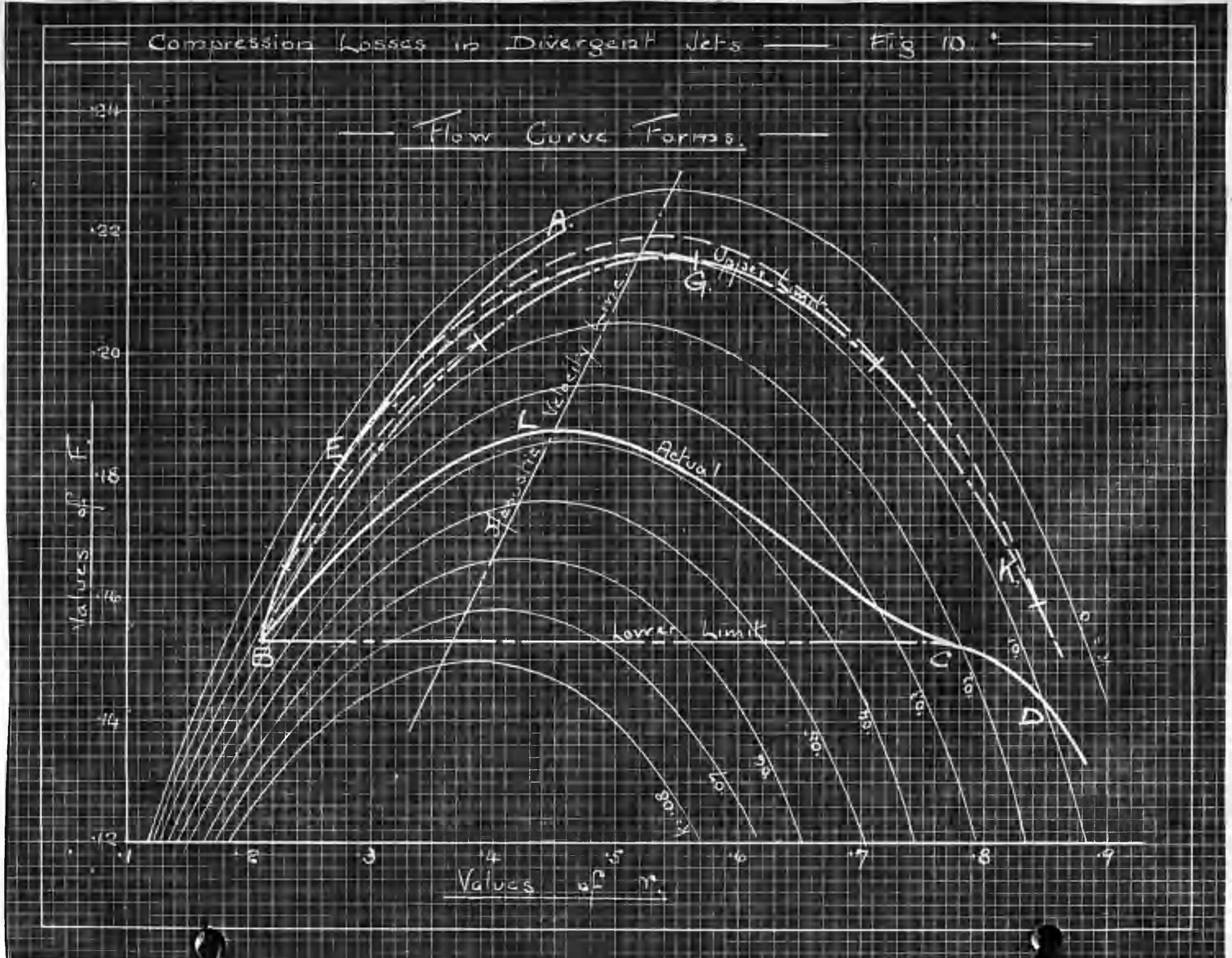
* "The Flow of Air through Nozzles" - Proc. Inst. Mech. Engrs. January 21, 1916.



Compression Losses in Divergent Jets
Fig 10.
Flow Curve Forms.
A.
E
G.
Upper Limit
L
Actual
Lower Limit
B
C
D
K.
Velocity Curve
Values of F.
Values of r.
·12
·14
·16
·18
·20
·22
·24
·1
·2
·3
·4
·5
·6
·7
·8
·9
k=·08

which the curve AB represents the case of continuous expansion for the nozzle previously considered. Then the curve BCD represents, as before, the locus of the terminals of all the nozzle characteristics on the assumption of filled areas. It is seen that this curve begins to fall in value at C, where the outlet ratio is .77.

Now, on reference to fig. 2, it will be noticed that, as the back pressure increases, the point at which compression starts moves back along the nozzle; so that there is some particular outlet pressure at which the nozzle throat pressure is affected. In this case the value is .77 and, at this condition, the compressing jet is filling the nozzle at all points; since the required minimum section is at the actual minimum section where the critical pressure exists. This is indicated, geometrically, by fig. 10; for, since the start of compression is at the highest possible point, any compression effect must entail larger areas. If the jet were able to compress freely it would compress very quickly, but the presence of the tail prevents this, and the compression follows the nozzle outline. The point C must, therefore, be a point on the true flow curve.

Also the point D must lie on the actual curve, since the areas must be filled for all back pressures higher than that at which the throat condition is first affected.

The same method of argument shows that continuous expansion must result in full areas, since the presence of the nozzle wall means that the naturally rapid expansion of a free jet is slowed down to suit the restriction of the boundary. It therefore follows that B is on the real flow curve.

Consider now the compression shown by (iii) in fig. 2, and suppose that this takes place entirely without loss between the recorded pressure limits. This means that the corresponding characteristic would follow a constant loss line, and take the form shown by EG. Under such imaginary circumstances the changes in the F value shown by this curve indicate that the jet would first

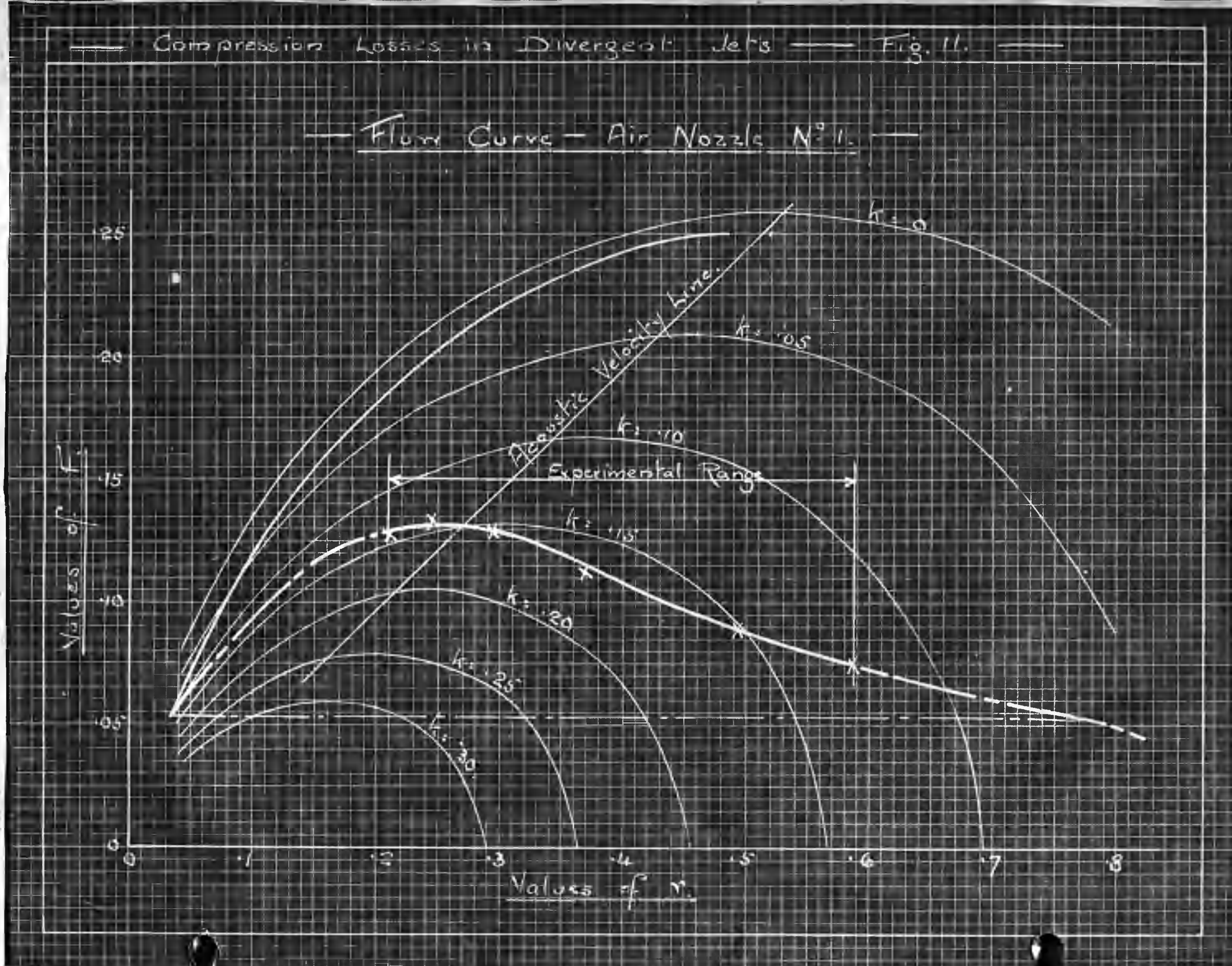
contract by about 16% and then expand very slightly as the compress:
:ion finishes until, at the outlet, its area would be about 70% of the nozzle section.

It should be particularly noticed that the form evolved for the representation of any one compression in this way shows a maximum at the acoustic velocity condition. This is , of course, a general result, as under any conditions of flow the point at which the acoustic velocity is reached entails a minimum area.

The main value of this lies in the opportunity it affords of deducing an important feature of the flow curve. Since there is a minimum area at the acoustic velocity on any jet, it is reasonable to suppose that we have the minimum outlet jet area when the outlet velocity is equal to the velocity of sound for the outlet conditions. This is not - as might at first sight be supposed - an entirely rigorous deduction, as it is possible to conceive such a variation of loss with compression range as would prevent it from being stric:
:tly correct; but it is not possible that, under even the most extreme conditions, it can be far from the truth. Hence, in general, the actual flow curve should show a maximum at, or very near to, the acoustic velocity line. A line of this kind on the chart is, therefore very useful in facilitating the study of the action.

If the other compressions shown in fig. 2 are also treated on the basis of perfect compression, the series of dotted character:
:istics shown in fig. 10 is obtained. The curve BGK, joining up their terminal points, then represents the flow curve on the assumption of no compression losses; and, of course, it fails to meet the points C and D that must lie on the real curve.

It is now a simple matter to realise the form that the actual flow curve should take. It must be of the type indicated by BLCD, which agrees with the terminal point B for the continuous expansion with filled areas; meets the point C at which the flow first begins to fall away on the same conditions; and has a maximum value on or near the acoustic velocity line.



Compression Losses in Divergent Jets — Fig. 11.
Flow Curve — Air Nozzle No. 1.
Acoustic Velocity Line.
Experimental Range
k = 0
k = .05
k = .10
k = .15
k = .20
k = .25
k = .30
Values of r.
Values of
0
.05
.10
.15
.20
.25
.1
.2
.3
.4
.5
.6
.7
.8

Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

The difference between the upper limiting curve and the actual is the result of the losses caused by the compression. These losses require some examination, as really nothing whatever is known regarding them. As has been pointed out, reaction experimental data are necessary for the study of their overall values, and we now proceed to the discussion of certain data of the kind.

Analysis of Reaction Data: In order to examine compression effects fully, combined search tube and reaction observations are necessary. At the present stage this combination is lacking, and the best that can be done is to examine reaction data from one source, and compare the deductions with search tube results from another. For the latter the steam flow experiments on the convergent-divergent nozzle already considered will serve, while the reaction figures may be taken from Morley's tests on air nozzles.

Morley tested a fair range of nozzles but, unfortunately, only a few of these are suitable. The compression losses in practically all of them might have been studied if the internal frictional effects could be accurately covered; but this is impossible without pressure readings, and without a knowledge of the surface conditions. Under these circumstances we are restricted to cases in which the compression losses are exceptionally severe, since the possible errors in the assumed frictional values are then proportionately small. Besides the uncertainty regarding the frictional effects, the more highly efficient nozzles would be definitely affected by a systematic experimental error that seems to have been present, as efficiencies greater than 100% are recorded in some cases of high pressure ranges.

The nozzles of lowest efficiencies are Nos. 1, 3 and 4, but these are so alike in their sizes and performances that, for the purposes of the present investigation, only one need be considered. That chosen for full treatment is No. 1. Nozzles marked 4A and 5 are also used for a particular purpose later, but complete

— Compression Losses in Divergent Jets — Table I. —

— Total Losses — Air Nozzle N° 1. —

Supply Press P_1	$r_2 = \frac{14.7}{P_1}$	$C_v = \frac{R_a}{C_d \cdot R_t}$	$C_v^2 = \eta$	$1 - r_2^{\alpha}$	$1 - k_2 - r_2^{\alpha}$	Total Loss k_2	F.
70.	·210	·785	·617	·360	·2220	·1380	·1270
60	·245	·747	·558	·332	·1852	·1468	·1290
50	·294	·675	·455	·296	·1348	·1612	·1250
40	·368	·555	·308	·250	·0770	·1730	·1110
30	·491	·405	·164	·182	·0298	·1522	·0878
25	·588	·328	·108	·140	·0151	·1249	·0734

analyses in these cases would be open to the objections already advanced.

Fig. 11 shows an "F" chart for air on which essential pro::cedure can readily be followed.

Nozzle No. 1 is .1935" diameter at throat, .42" diameter at outlet, and 2.7" long. Tests were made with supply pressures varying between 70 and 25 lb. per sq. in.(absolute), the air passing out to atmosphere. In what follows the atmospheric pressure is taken at 14.7 lb.per sq. in.

The stated coefficient of discharge for this nozzle is .96, which makes the F value at the throat approximately .2500. From this and the dimensions, the F value at any section is easily derived from:-

$$F \propto 1/A.$$

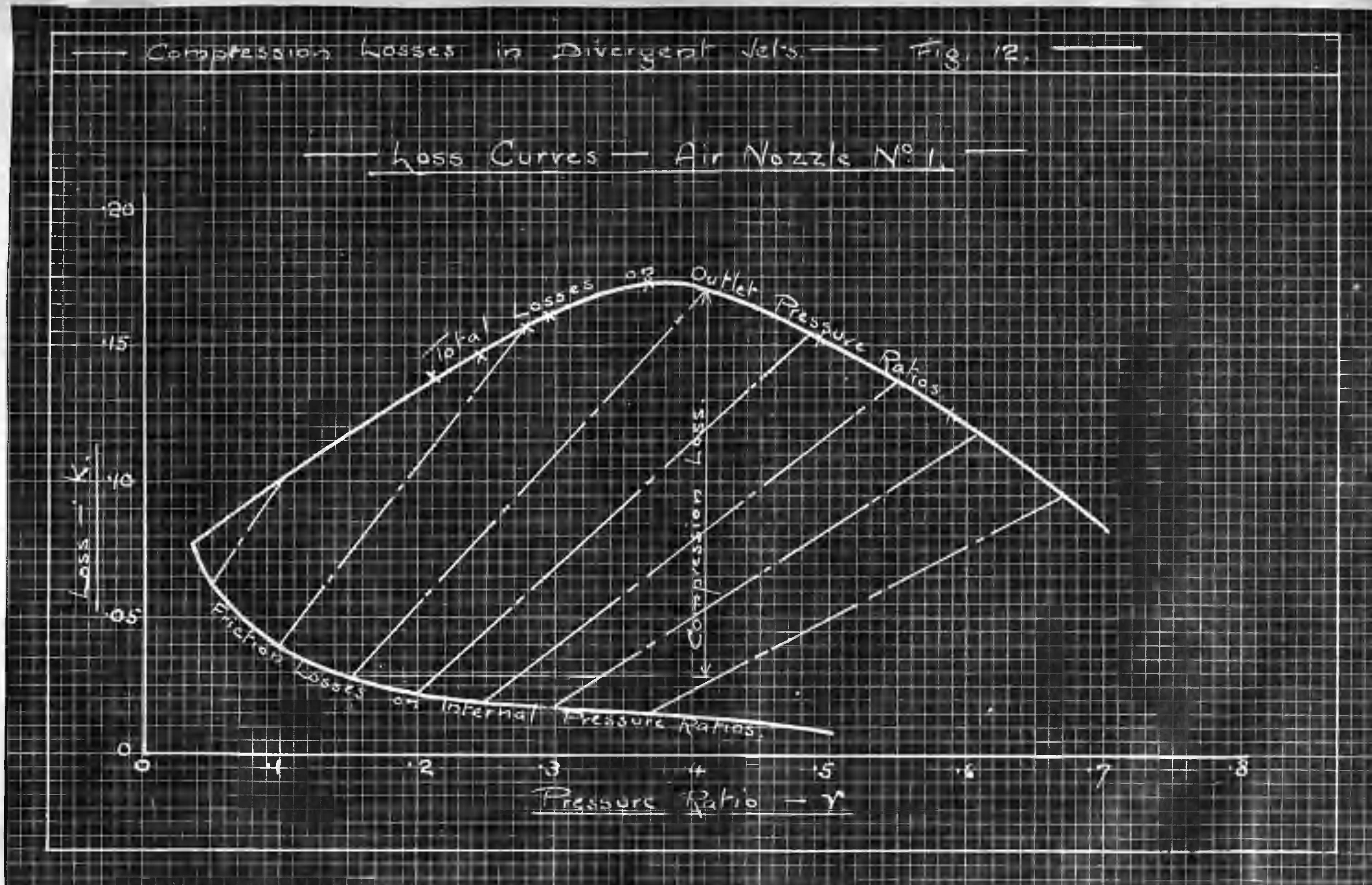
By means of these F values it is possible to decide provisional r and k values for any point on the nozzle length in the case of continuous expansion; and then a close estimate of the frictional loss is obtained from:-

$$k_b = \cdot 005 \int_0^l \frac{p}{A}\cdot(1 - k - r^{\alpha})\cdot dx. \quad \text{------} (4).$$

where p/A represents the hydraulic mean depth, and dx is an elem::ent of length. The friction constant has been assumed at .005 as being fairly probable; the surface condition of the nozzle is not specified.

The integration of (4) allows of the determination of the frictional effect to any pressure ratio occurring in the normal expansion. By the addition of a small amount for entrance effects, the data necessary to plot the probable characteristic for this case is obtained. This curve is shown in fig. 11; and the actual losses, on a base of pressure ratio, are given by the lower curve in fig. 12.

We now use the reaction results to determine the total


Compression Losses in Divergent Jets — Fig. 12.
Loss Curves — Air Nozzle No. 1.
Total Losses of Outlet Pressure Ratios
Friction Losses of Internal Pressure Ratios.
Compression Loss.
Loss — K
·20
·15
·10
·05
0
·1
·2
·3
·4
·5
·6
·7
·8
Pressure Ratio — r

losses to the outlet. If R_a is the actual reaction, R_t the theo::retical reaction, and c_d the discharge coefficient, then the vel::ocity coefficient is:-

$$c_v = \frac{R_a}{c_d \cdot R_t} \qquad (5)$$

Since the necessary figures can be obtained, on a base of supply pressure, from Morley's curves, it is a simple matter to determine the values of c_v corresponding to different outlet pressure ratios. With r known, the kinetic energy at outlet is proportional to:-

$$c_v^2 (1 - r_2^{\alpha}) = (1 - k_t - r_2^{\alpha}) \qquad (6)$$

and, hence, k_t - the total loss to the outlet - is determined. The steps in the calculation are easily followed from table I.

With k_t and r_2 known the corresponding values of F, for the experimental cases, are found from:-

$$F = \frac{r_2 (1 - k_t - r_2^{\alpha})^{\frac{1}{2}}}{(k_t + r_2^{\alpha})} \qquad (7)$$

and these have been plotted in fig. 11.

The range covered by the experiments does not serve to define the full length of the flow curve; but the important feature of a maximum value at the acoustic line emerges very clearly. The tendency of the curve is to complete a form such as is indicated by the dotted continuations and, obviously, it satisfies the require::ments of a true flow curve in all particulars.

From the values of k_t determined by (6) the upper curve in fig. 12 is obtained. It must be noted that this total loss curve is plotted on outlet pressure ratio, whereas the lower friction curve is based on internal pressure ratio.

If now, for any compression, we have the ratios at the start and at the outlet, it is possible to determine from fig. 12 the total loss occurring therein. At this point the lack of search tube data becomes noticeable, as pressure readings would settle this matter at once and with accuracy.

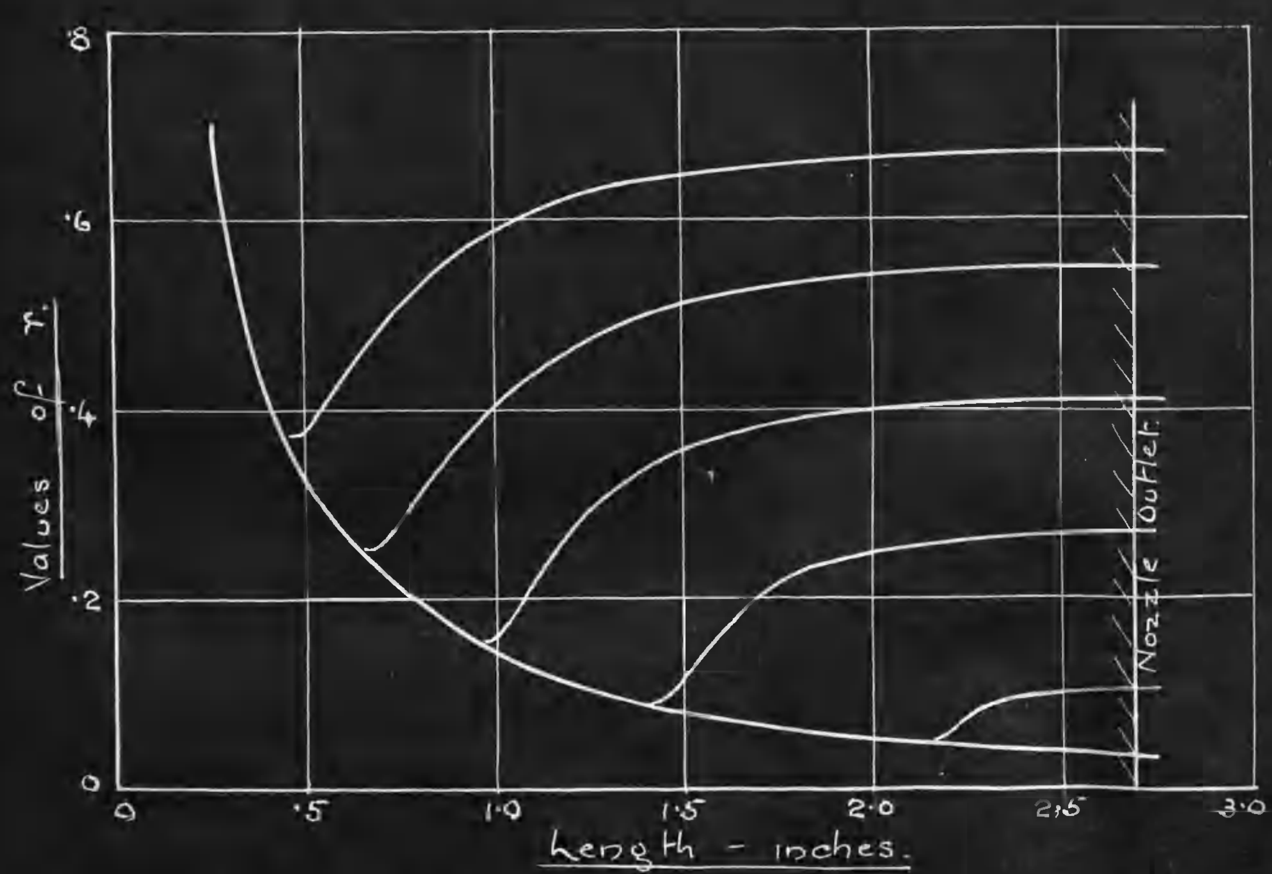
— Compression Losses in Divergent Jets — Table II. —

— Compression Losses - Air Nozzles. —

Nozzle No.	Compression Ratios At Start r_1	Compression Ratios At Outlet r_2	Compression Loss k_c	$E = r_2^{\alpha} - r_1^{\alpha}$
1	·05	·10	·038	·091
"	·10	·27	·118	·172
"	·15	·41	·143	·196
"	·20	·485	·133	·184
"	·25	·55	·117	·172
"	·30	·61	·101	·161
"	·37	·67	·080	·138
4A	·21	·50	·063	·182
5	·31	·60	·024	·149

— Compression Losses in Divergent Jets — Fig. 13. —

— Pressure Ratio Curves — Air Nozzle No 1. —



Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

From the friction investigation already made a pressure ratio curve for the normal expansion can be drawn. When a series of probable compression curves is added we get the full set shown in fig. 13. Such a method of fixing compression is hardly convincing but it would seem the only course open in the circumstances and, so far as the losses are concerned, the results should not be much out.

With definite ranges of compression decided in this way, the various "contour" lines shown in fig. 12 may be drawn; and the difference of the two end readings of any one such line gives the loss and the corresponding pressure range. The actual values are given in table II.

The next step is to determine the probable relation between the compression and the loss. With the data in hand it is only possible to examine a variation with fluid speed, or with the energy of conversion, thus:-

$$k_c \propto \left\{(1 - k_1 - r_1^{\alpha})^{x/2} - (1 - k_2 - r_2^{\alpha})^{x/2}\right\} \text{ --- } C(8).$$

or:-

$$k_c \propto \left\{(1 - k_1 - r_1^{\alpha}) - (1 - k_2 - r_2^{\alpha})\right\}^{y} \text{ --- } C(9).$$

Without entering into the details of a variety of attempts it may be said that the second basis provides the best agreement. The speed does not appear to be quite suitable as the independent variable, and the closest results are found by using the adiabatic energy in place of the actual as expressed in (9). The adiabatic energy of conversion is proportional to:-

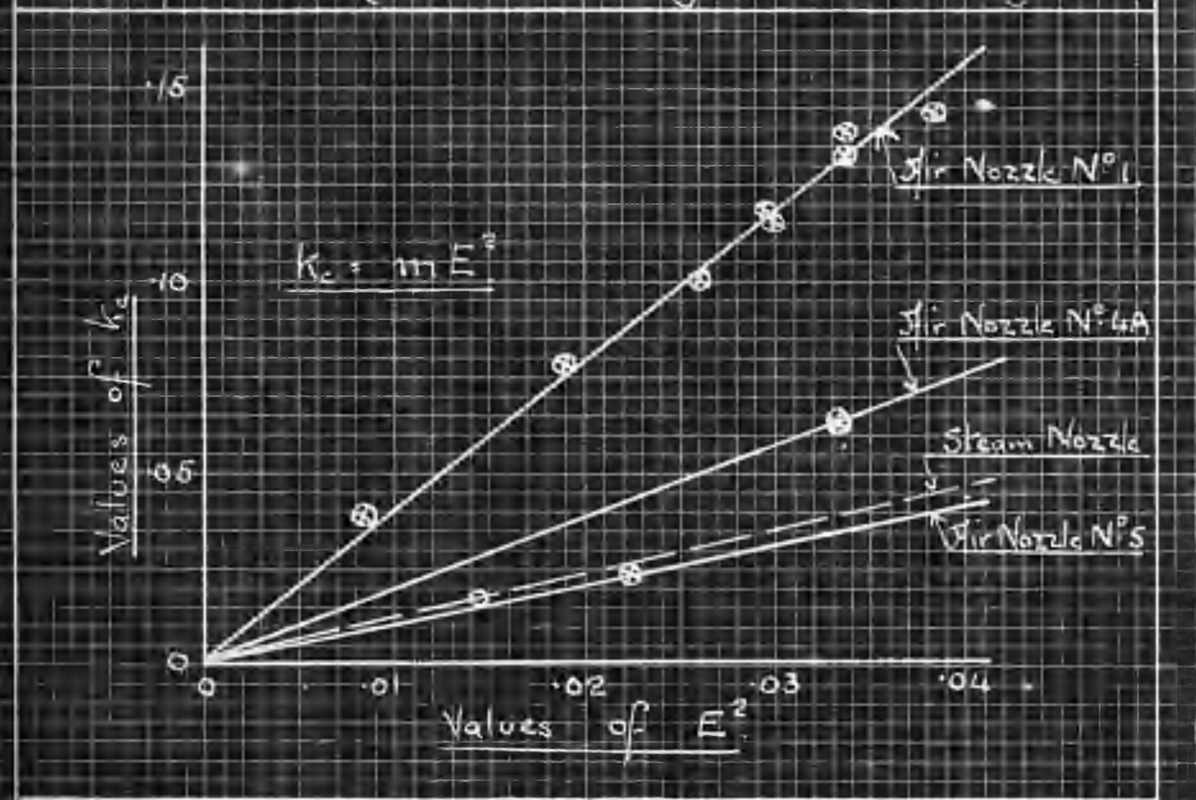
$$E = (1 - r_1^{\alpha}) - (1 - r_2^{\alpha})$$
$$= (r_2^{\alpha} - r_1^{\alpha})$$

and the equation of loss, derived from the data of Nozzle 1, is:-

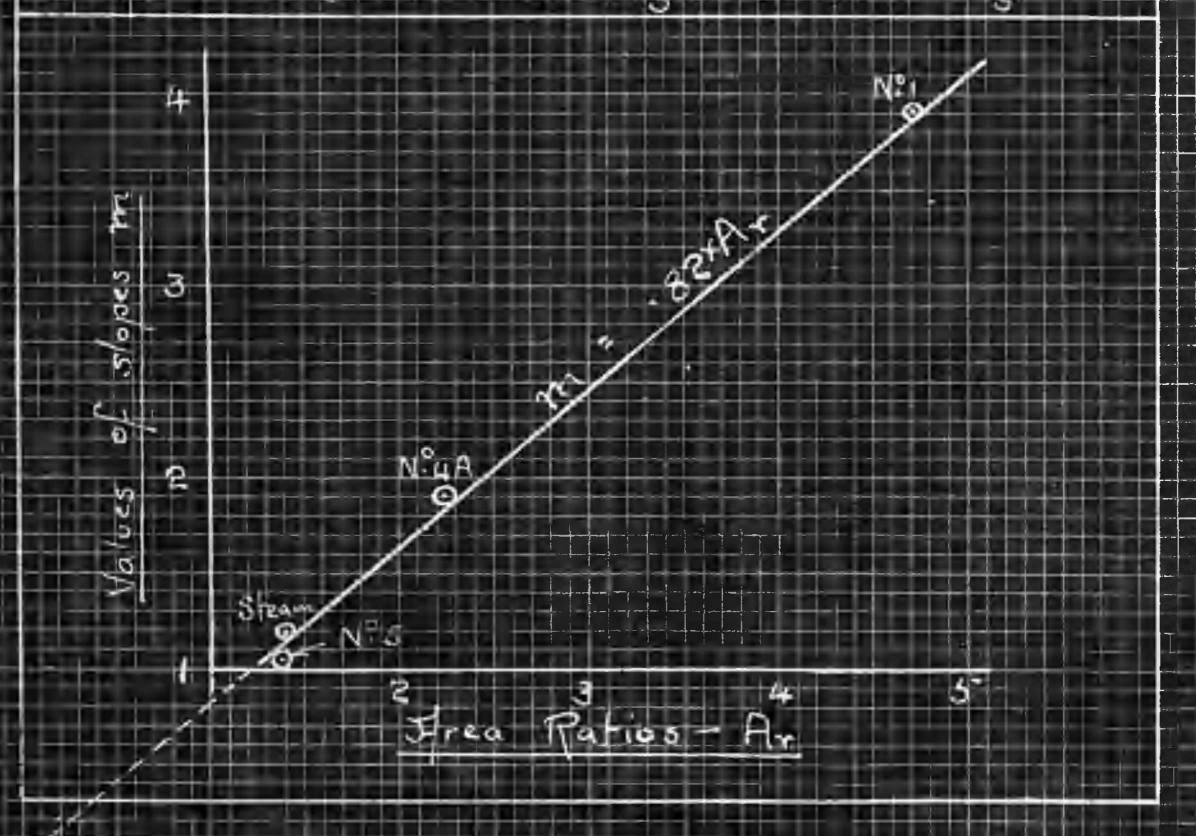
$$k_c / E = 3{\cdot}9\,E \text{ ---------- } C(10).$$

The nature of the agreement is shown by the top line in fig. 14; and it may be remarked that the severe compression losses in this nozzle should prevent the unavoidable inaccuracies, due to

— Compression Losses in Divergent Jets — Fig 14 —



— Compression Losses in Divergent Jets — Fig. 15 —



the reduction operations, from having a very serious effect on the general result.

On further consideration it will be clear that the loss expression should contain some factor representing the nozzle itself. It would seem certain that the length or area ratio should enter into account. For the purpose of examining this point lower area ratio nozzles used by Morley have been considered, but the uncer: :tainty of the friction effects, and of the efficiencies at the lower pressure ratios, make the results less satisfactory than in the case of Nozzle 1.

Nozzles marked 4A and 5 have been chosen. The former is .193" diameter at throat, .290" diameter at outlet and 1.12" long; while the taper of the divergence (1 in 8) is the same as Nozzle 1. No. 5 is .1955" diameter at throat, .230" diameter at outlet and 2.3" long, with the very small divergence of 1 in 64.

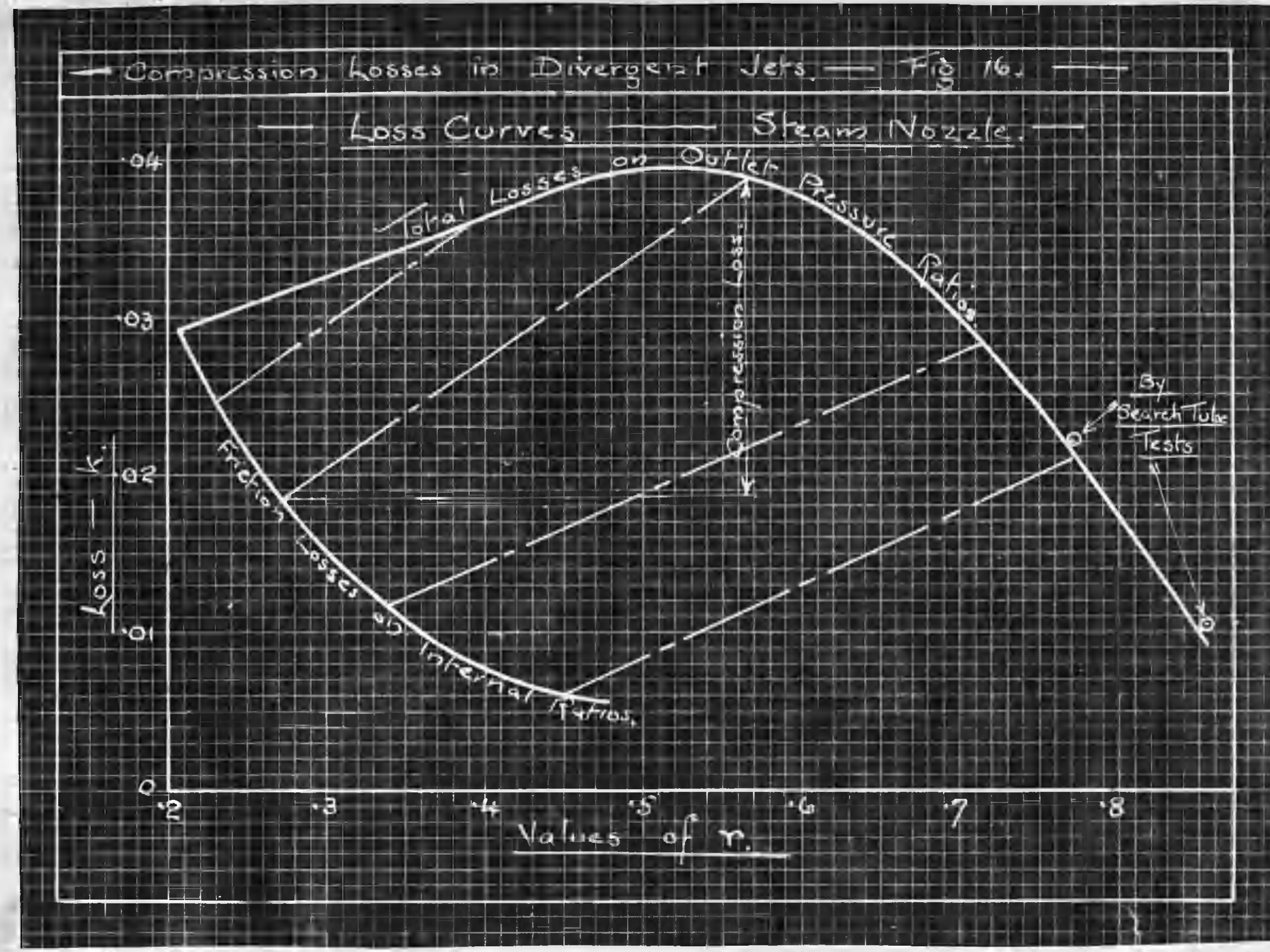
In both cases reduction of the friction constant and of the efficiency values at the low ratios would seem called for, in order to get rational curve series. However, the assumption may be made that a relation as in (10) applies, and only the maximum comp: :ression losses need then be considered. The figures for these are given in table II, and the corresponding lines are included in fig.14.

We have thus three relations of the type of (10), which represent three nozzles differing in length, taper and area ratio. Of these the results for the fully investigated case must be consid: :ered fairly good, while the other two may be looked on as probable. In fig. 15 the slopes of the lines in fig. 14 are plotted against the corresponding area ratios, and are found to fall approximately on a straight line through the origin; so that the general express: :ion for the loss would appear to be of the type:-

$$k_c / E = a \cdot A_r \cdot E \quad \text{---------} \quad (11).$$

in which:-

$$A_r = \frac{\text{Nozzle Area at Outlet}}{\text{Nozzle Area at Throat}}$$


Compression Losses in Divergent Jets. — Fig 16.
Loss Curves — Steam Nozzle.
Total Losses on Outlet Pressure Ratios.
Friction Losses on Internal Ratios.
Compression Loss.
By Search Tube Tests
Loss — λ.
·04
·03
·02
·01
0
·2
·3
·4
·5
·6
·7
·8
Values of r.

Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

Considering the nature of the data any attempts at further refinement would be somewhat ridiculous; and it may, therefore, be taken that the results of Morley's series of reaction experiments point, on the whole, to a rule of the following type for compression losses:-

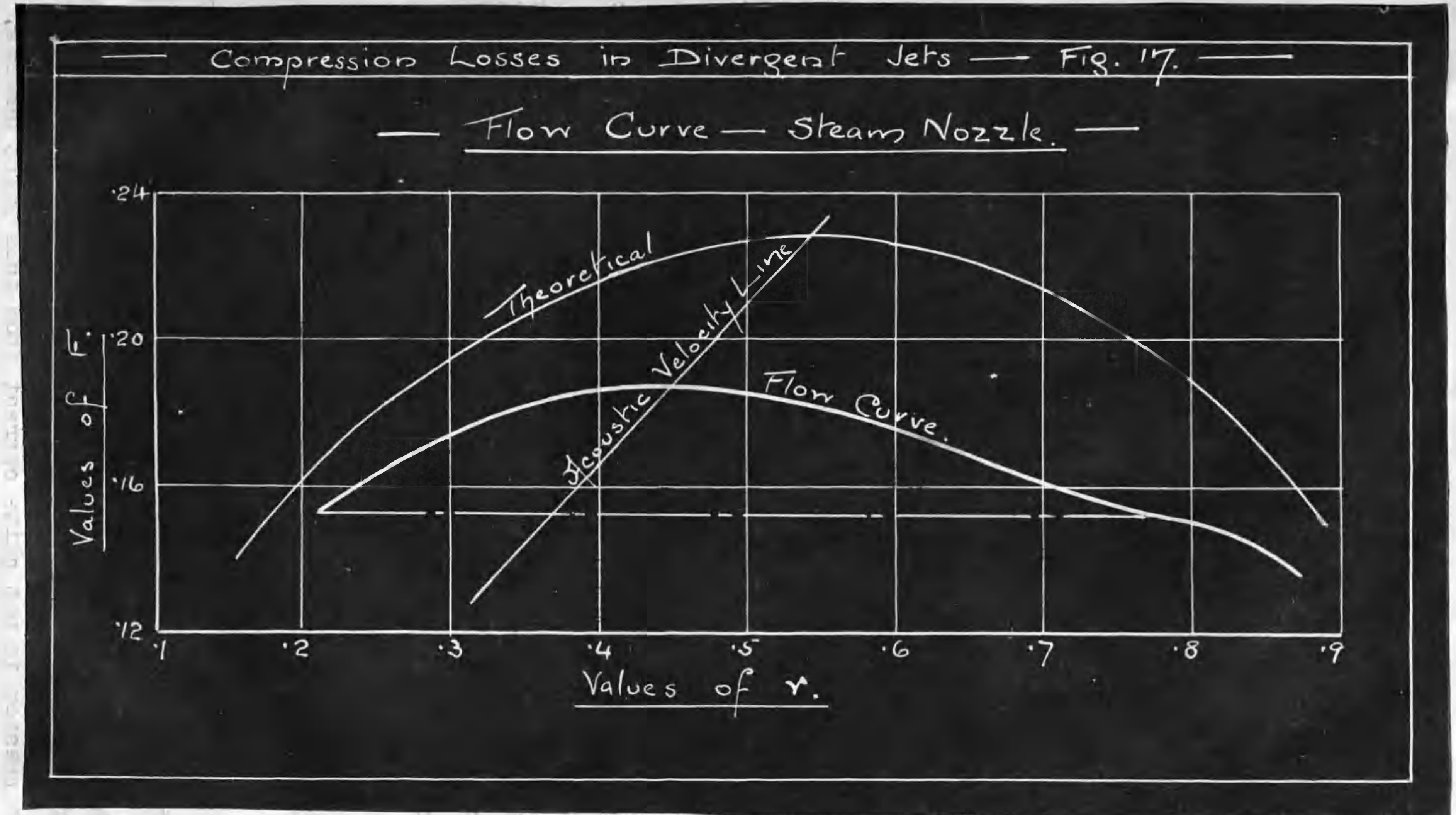
$$k_c / E = \cdot 82 \times A_r \times E \qquad (12)$$

in which k_c is a loss factor measuring the effect. But it should be noticed that k_c/E is the fractional loss, and can be used with any system of energy units. After it has been calculated from (12), it may be directly applied, for instance, to the adiabatic energy expressed in heat units.

Application to the Steam Nozzle Tests: There is, of course, no reason to expect any very serious change in the law of loss due to change of fluid, so that the expression derived from the preceding analysis should give rational results for the steam nozzle tests.

Since for this nozzle we have the compression ranges from the pressure records it is a relatively simple matter to establish the loss diagram corresponding to fig. 12. This is given by fig. 16; the friction curve being obtained from the calculation previously made, and the compression losses determined from equation (12) and the known pressure ranges.

Search tube and flow results can, however, provide one check on the applicability of (12), if observation is made of the conditions at which the flow per sq. in. of nozzle area first begins to fall off. The check possible with such a single result is not very exact as the compression loss is not high, and the method followed of charging the total loss in the compression length against the compression is not quite correct from this point onwards. But the calculation may be made in order to show that there is no serious discrepancy.


Compression Losses in Divergent Jets — Fig. 17.
Flow Curve — Steam Nozzle.
Theoretical
Acoustic Velocity Line
Flow Curve.
Values of F.
·24
·20
·16
·12
·1
·2
·3
·4
·5
·6
·7
·8
·9
Values of r.

Study of Nozzle Losses:-
Compression Losses in Divergent Jets.(contd.).

It has been previously shown that the condition referred to is established for this nozzle at an outlet pressure ratio of about .77, and a special test carried out with this back pressure value gave the ratio at the start of compression as .45. In this case, of course, compression commences at the nozzle throat so that the effect occupies the full tail length and the nozzle areas are filled throughout. (This value of .45 is considerably below the critical ratio but the throat readings for this particular nozzle have always been low owing to overexpansion in the convergence). With the known pressures and filled areas the total loss between the stated limits is readily determined as:-

$$K_c = \cdot 016.$$

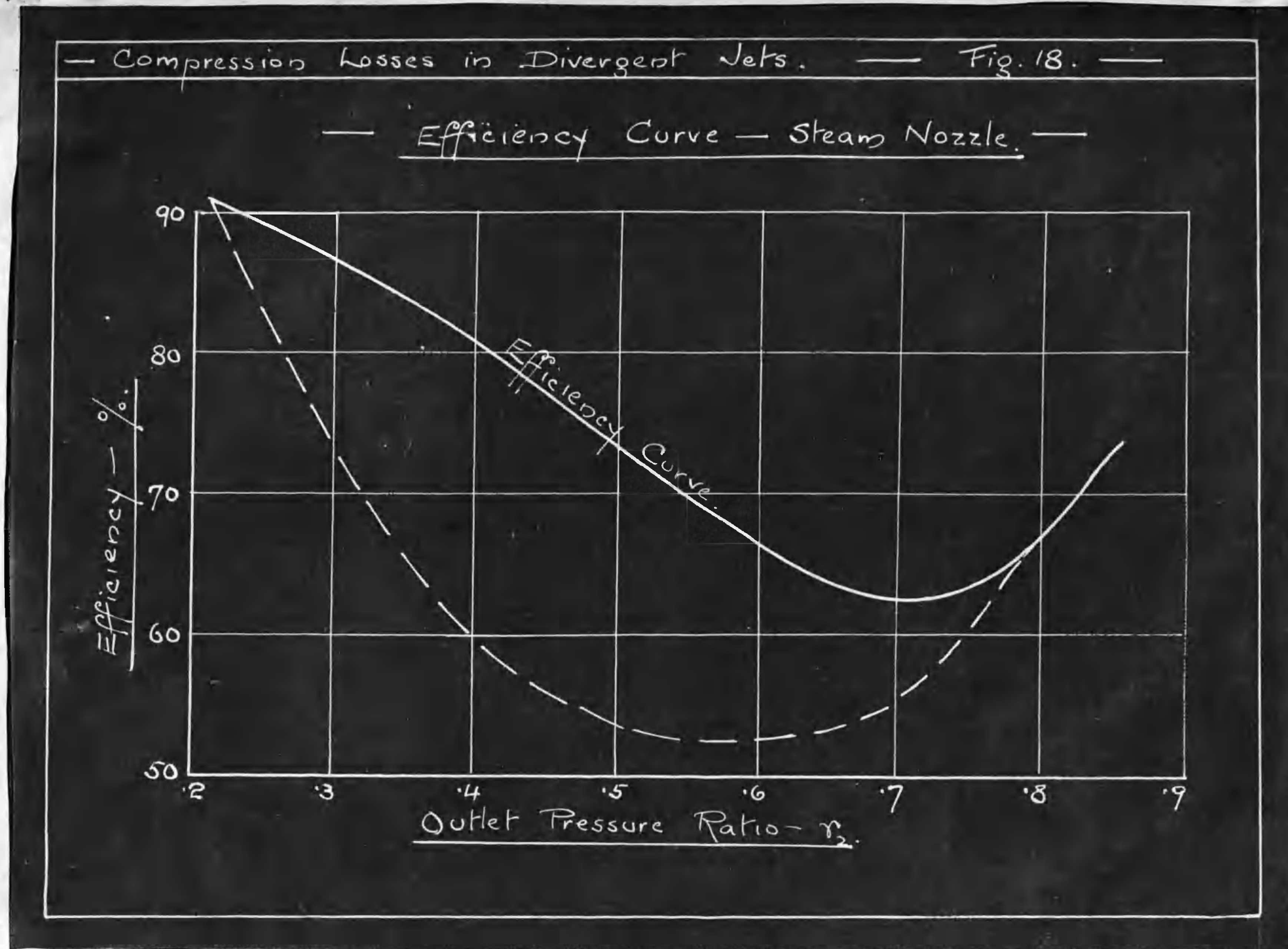
Using this single value we may, as before, draw a radial line in fig. 14 on the assumption that:-

$$K_c / E \propto E.$$

This gives the dotted line shown in fig. 14; and when its slope is plotted against the area ratio in fig. 15 the point obtained is found to be fairly close to the line representing the air nozzle results. So far as it goes, then, this special result of the search tube tests is not in any serious conflict with the general findings of the previous analysis; although, of course, acting alone as a check it makes a rather insignificant item.

From the loss and ratio values in fig. 16 the actual flow curve for the nozzle is readily derived. This is shown in fig. 17 and it is seen to possess all the typical features that have already been emphasised. It indicates that the minimum outlet jet section is about 80% of the outlet nozzle section.

The efficiency curve resulting from this new determin:ation of the losses is given in fig. 18 on which the original curve of fig. 4 is also shown for comparison. It will be seen that the general form of the curve is not greatly altered, but the new curve shows definitely higher efficiencies than are obtained on the assumption of filled areas; while the minimum efficiency occurs at


— Compression Losses in Divergent Jets. — Fig. 18. —
— Efficiency Curve — Steam Nozzle. —
Efficiency Curve.
Efficiency — %.
90
80
70
60
50
·2
·3
·4
·5
·6
·7
·8
·9
Outlet Pressure Ratio – r₂.

a much higher outlet ratio than before. It is quite clear, however, that a divergent nozzle efficiency falls away very rapidly as the back pressure rises above that particular lower limit required to maintain continuous expansion.

Clearly the steam nozzle results could not, alone, be held as establishing any certainty in regard to the losses; but, so far as they go, they provide confirmatory evidence of the general deductions that have been made. Perhaps the simple way in which the area ratio appears in equation (12) is hardly convincing, but the final form so given represents the best that can be done with the available data. In view of the fact that no definite quantit::ative law of compression loss has, hitherto, been advanced, equation (12) may be considered serviceable for approximate estimates, until more complete experimental information permits of a fuller examin::ation and a more rigorous discussion.

A STUDY OF NOZZLE LOSSES.

D: GENERAL REMARKS.

The Friction Loss: This effect has been fully dealt with in Section A. The experimental data which have been utilised in the treatment are almost entirely due to Mellanby and Kerr and to Anderson; the former being responsible for those finer variations of loss that have determined the form in which the friction law has been given; while the latter provides the information for fixing several of the surface coefficients in that law. In general, the agreement which has been demonstrated between nozzle values and those for slow flow fluid is significant of the correctness of the whole matter; and it is believed that the main equation is a close representation of the actual effect. No doubt the various numerical coefficients can only be taken, at present, as approximately correct; final and definite values really await the collection of a considerable amount of experimental data both on the effect of surface and on the influence of the Reynold's number.

The reduction of the general equation to give suitable methods for direct calculation introduces the possibility of error; but this need not be considered serious, and is more than compensated by the handiness of the forms evolved. The various curve series in aid of these calculations may be held as fairly well verified for straight axis convergent nozzles, but require further checking for oblique axis and for divergent types. Generally, however, they are so connected with theory that the forms should be rational, although the actual values may suffer change as the data whereon they rest are extended.

The various results may be briefly summarised. For exact calculation for any nozzle form we must apply:-

$$\varepsilon = a\int_0^l \left(\frac{u}{u_0'}\right)^2 \left(\frac{p}{A}\right)\left(V_1 \cdot R \cdot \frac{p}{A}\right)^{\cdot 15} dy \quad \text{---------- A.(15)}$$

using the constants in table II, page 24. To use this requires a knowledge of the pressure ratio curve and involves integration. If we are to avoid these complications the calculation for convergent types is by:-

$$\mathcal{E} = c\left(\frac{p}{A}\right)_0 \left\{ A\cdot l_1 + B\cdot l_2 \right\} \text{ ---------- } A.(18)$$

using the charts in figs. 9 and 10. For the divergent forms we have:-

$$\mathcal{E} = c\left\{ D\cdot\left(\frac{p}{A}\right)_1 \cdot l_1 + m\left[E\left(\frac{p}{A}\right)_1 + 7\left(\frac{p}{A}\right)_0\right] l_2 \right\} \text{ ------- } A.(20)$$

for cases where the taper is not the same on all sides; and:-

$$\mathcal{E} = c\left(\frac{p}{A}\right)_1 \left\{ D\cdot l_1 + N\cdot l_2 \right\} \text{ -------- } A.(22)$$

where the taper is constant. Figs. 12 and 13 are used in conjunction with equations (20) and (22). In all cases the presence of two different kinds of surface in the one nozzle involves the substitution:-

$$\left\{ c'\left(\frac{p'}{A}\right) + c''\left(\frac{p''}{A}\right) \right\} \text{ for } c\left(\frac{p}{A}\right).$$

These practical methods have been established by neglecting the influence of the Reynold's number. This will not be great for moderate changes of state but, for wide changes, a correction may be made by means of the graph in fig. 14(A). In all probability the index of the Reynold's number is subject to some alteration with change of surface. If it is desired to take this into account, then all that can be done at present is to note that, for the rougher surfaces, the correction will tend towards unity.

It is to be recognised that the general equation form, and the constants as deduced from the experimental facts, appear to be adequate to cover all the effects observed in the tail length by search tube methods; and this applies to continuous expansion in any type whether parallel or with a normal taper. Therefore the application to the action in the straight part of a convergent-parallel nozzle to obtain an indication of how the efficiency should vary with pressure ratio is legitimate; and it has been shown in the special subsection entitled "The Trend of the Efficiency Curve" that friction alone entails a certain fall of efficiency with fall of outlet speed. This result is of importance, as any other curve form must involve loss effects - either inside or outside the nozzle - which are of an order distinctly different from friction.

In conclusion, the friction loss may be illustrated by an example or two. As representative of a high pressure nozzle take the following:-

Nozzle angle 14°; pitch of partition plates $2\frac{3}{8}''$; thickness of plates 14 L.S.G; nozzle height $\frac{5}{8}''$; mean curved length to end of convergence say $1''$; length of straight part say $1\frac{1}{2}''$; cast and plate surfaces.

Allowing c = .0090; r at outlet = .65; and initial volume $V_1 = 4.0$; we obtain:-

Friction Loss = ε = 9.5%.

Again, take the following particulars for a low pressure nozzle:-

Nozzle Angle 20°; pitch of plates $2\frac{1}{8}''$; thickness of plates 9 L.S.G; nozzle height 7" ; mean curved length to end of convergence say $1\frac{1}{2}''$; length of straight part say $2''$; cast and plate surfaces.

With c = .0080; r at outlet = .60; and initial volume V_1 = 40.0; we find:-

Friction Loss = ε = 7.9%.

These two examples may be taken as roughly indicating the kind of figures likely with practical forms of the convergent type.

Consider now the following divergent nozzle:-

Size at throat $\frac{1}{2}'' \times \frac{3}{4}''$; at outlet $\frac{5}{8}'' \times \frac{3}{4}''$; length to throat $\frac{1}{2}''$; length throat to outlet $3.0''$; cast surfaces.

Assuming c = .0090; r at outlet = .25; initial volume V_1 = 4.0; we find:-

Friction Loss = ε = 11.5%.

Again, taking the particulars:-

Study of Nozzle Losses:-
General Remarks. (contd.)

Size at throat $\frac{3}{16}'' \times \frac{1}{2}''$; at outlet $\frac{9}{16}'' \times \frac{11}{16}''$;
length to throat $\frac{1}{2}''$; length throat to outlet $3{\cdot}0''$;
cast surfaces:

and allowing c and V_1 as before, but with $r_0 = .04$; there follows:-

Friction Loss $= \varepsilon = 17{\cdot}8\%$.

The last two examples compare a divergent nozzle for large flow and small expansion with one for small flow and large expansion Since the ε values operate on distinctly different heat drops, it is clear that the larger range nozzle is responsible for a very large reheating effect compared with the other. If we look on these two cases as representing the "main" and "cruising" nozzles of a naval turbine type at one time common, we must realise the probability of the main nozzles at cruising power being as efficient as the special cruising nozzles themselves. It is simply a question as to whether the free expansion with the former would cost more than the difference in friction losses. The case as given helps to show the value of sound comparative estimates of frictional effects, although the specific problem may be somewhat out-of-date.

The Entrance Loss: Section B has been devoted to a somewhat speculative discussion of this matter, but it contains practically all the real data bearing clearly on the subject.

A special loss in the entrance is indicated by search tube examination of convergent nozzles, and this has received further confirmation from the flow curves obtained with increasing superheat The fact that these two entirely different results support each other in the general assumption that the loss measured is due to the entrance expansion, is the main feature of the evidence that has been submitted.

Since the effects in the tail are adequately covered by a simple frictional conception, it would appear that the entrance expansion possesses characteristics of a special order. Now, the convergent expansion is distinguished from the tail action by its very rapid volume expansion, by the creation of fluid spin, and by the convergence of the fluid streams. Only the last gives results in fair agreement with fact and rational in conception and, hence, the loss is conceived as mainly due to this factor.

The convergent form is imagined to set up lateral velocities which are rapidly "damped" out in the inward and forward directions. To a certain extent these "useless flow velocities" are of somewhat the same order as the main axial speed and will, therefore, give rise to a loss effect having a distinctly higher value than that arising from normal turbulence. This is a noticeable feature of the recorded quantities.

The coefficient involved in the effect is of the type of a viscosity, but of a much higher order than that representing the usual physical quality, since the actual occurrence is due to molar rather than molecular, motions. In the idea of a coefficient of the kind there is a resemblance to Stodola's "turbulence factor" - as the present Author understands it. The application is, however, quite different. The coefficient considered is dependent only on the existence of a convergence, and may actually show its influence where turbulence is not present. It is, in fact, well known that turbulence is difficult to establish in a convergent form.

Since the loss is due to the convergence it is largely determined by shape and the suitable measure of this shape discloses itself as $(1/A)(dA/dx)$. If this is correct it seems probable that in long well curved inlet forms, the loss will be inappreciable. On the other hand, the maximum values should be encountered in sharp edged inlet types where expansion takes place at the maximum free rate.

The equation derived as giving a rough measure of the effect is:-

$$\varepsilon = \frac{3\cdot 5}{10^6}\cdot\frac{B}{1-r_0^2}\cdot \ell_1\cdot P_1^{1/2}\cdot V_1^{3/2}\left(\frac{1}{A}\cdot\frac{dA}{dx}\right)^2 \quad\text{------ B.(11)}$$

but there is great difficulty in deciding on the value of a truly representative $(1/A)(dA/dx)$ unless the actual pressure curve for the inlet is known. This equation meets the only available data fairly well and embodies the convergence factor in an apparently reasonable way.

In the discussion in Section B we have only been concerned to demonstrate all the facts and to establish the fairest point of view. It must be admitted that there are almost insurmountable difficulties in isolating this peculiar effect experimentally with any degree of exactitude. Since, then, the actual figures are likely to be uncertain a rational conception of the matter becomes a necessity.

The Compression Loss: It has been shown in Section C that the search tube method fails to deal with compressions in a divergent form, because the jet breaks away from the nozzle wall at the point of compression. Hence, if we desire to obtain some general idea of the losses incurred, it is necessary to know the loss to the commencement of compression, and the total loss to the outlet, separately. This obviously demands the kind of data obtained from combined search tube and reaction experiments.

In the treatment given of this matter the reaction tests made by Morley on air nozzles have been analysed, with a view to showing the order of the effect; and it has been found necessary to deal with those nozzles which have excessive losses. Because of the magnitude of the losses on which the examination is based it is thought that the main result should be fairly serviceable for large and high speed compression ranges.

The equation derived is:-

$$\frac{k}{E} = \cdot 82 \, A_r \left(r_2^{\alpha} - r_1^{\alpha} \right) \text{\_\_\_\_\_\_\_\_} C.(12).$$

where (k/E) is the fractional loss on the adiabatic energy transformation in the compression range r_1 to r_2 ; and A_r is the

Study of Nozzle Losses:-
General Remarks. (contd.)

nozzle area ratio. To express this in our usual form of a fractional loss in the net expansion we have:-

$$\varepsilon = \frac{.82\, A_r \left(r_2^{\alpha} - r_1^{\alpha}\right)^2}{1 - r_2^{\alpha}} \qquad \text{------------ D.①.}$$

The main difficulty lies in the influence of the nozzle size or ratio; there is insufficient evidence to demonstrate this with clearness; but owing to the entire absence of any guidance on this matter the above equation may be of some service.

While the outcome of the investigation is frankly empirical certain aspects of it are worthy of remark. It may be held as conclusively proved that a compressing jet need not, in any way, be confined by the nozzle boundary. It can establish and maintain a definite form of its own. Again, by fairly general considerations the form of the flow curve for a divergent nozzle - as expressed on the Mellanby and Kerr "F" chart - is clearly defined. This form is adequately supported by the evidence obtained from Morley's results, and may be taken as general.

An interesting deduction is also possible. The outlet pressure ratio at which a divergent nozzle flow first begins to fall off is very high; but it must be largely governed by the loss in compression. If so, a simple method of studying the loss incurred in slow confined compression, at the limiting condition, is offered by the employment of nozzles of constant divergence and varying area ratio, and by the examination of the change of the limiting pressure therein. If, for instance, the law of loss is as has been given there should be a characteristic variation of this pressure as the area ratio increases. This can be readily deduced from the rule and the observed change - which could be studied by simple flow experiments - would be a sensitive check on the rule and a probable guide to a more correct representation.

The form of the study in Section C is probably of more interest and value than the result of it; and it might, perhaps, be remarked that the process of investigation affords an excellent example of those methods of examination and analysis which have been developed by Mellanby and Kerr in their nozzle researches.

The Loss by Overexpansion: In a divergent nozzle there is one particular low value of the outlet pressure at which continuous expansion along the full length is possible. Any lowering of the back pressure below this figure merely introduces free expansion beyond the nozzle; but any attempt to operate at a higher back pressure causes a recompression in the jet towards the outlet. This effect is the result of "overexpansion" since, if the desire is to meet the actual exhaust condition used, an unnecessarily low pressure has been reached in the process of expanding. Obviously the overexpansion, with its resulting jet compression, is the outcome of employing too limited an expansion range for the nozzle area ratio; or, conversely, it arises from the fitting of too large a ratio for the intended pressure values.

It is clear that the loss incurred by overexpansion is largely dependent on the cost of the recompression, and its determination becomes a fairly direct matter when the general magnitude of compression losses is understood. Consequently, the result of the investigation of compression effects in Section C is of value in this respect.

Consider a divergent nozzle that, under continuous expansion conditions, has an outlet ratio r_o . Suppose the loss and pressure ratio curves are known for this condition - since they can be determined very closely by methods already given. If this nozzle is operated with a back pressure ratio r_2, which is higher than r_1, then all losses, incurred beyond the point at which r_1 is first reached in normal expansion, are the result of "overexpansion" and are chargeable to this effect. These losses will arise from the overexpansion and the subsequent recompression.

In the usual cases of this occurrence the compression ranges are not great and, hence, owing to the difficulty of deciding the lowest pressure reached within the nozzle during overexpansion we may assume that the compression range is from r_0 to r_2. This will overestimate the compression loss slightly, but in such circumstances this is hardly a fault.

Allow that the distance from the first r_2 value to the outlet is l ; that the outlet perimeter is p ; that the outlet area is A ; and that the area ratio is A_r ; then, as an approximation to the total loss due to the faulty operation, we have:

$$\mathcal{E} = c\cdot\left(\frac{p}{A}\right)\cdot l + \cdot 82\, A_r\left\{\frac{r_2^{\alpha} - r_0^{\alpha}}{(1 - r_2^{\alpha})^{\frac{1}{2}}}\right\}^2 \quad \text{---- D.(2).}$$

where $\mathcal{E}$ here stands for the fractional loss in the net expansion caused by continuing the action too far; and where c is the usual surface constant.

This equation is simply based on the assumption that to meet the r_2 value correctly, the nozzle tail length would require to be reduced by an amount l. Since this is not done the jet actions, consequent on the presence of this length, are accompanied by losses roughly as stated; and the fractional loss so calculated is, strictly speaking, an unnecessary and avoidable addition to that naturally incurred by continuous expansion to r_2 in a nozzle of the same divergence.

The only data hitherto available on this matter are apparently due to Steinmetz; and Professor Goudie has presented these in a convenient form in his treatise on "Steam Turbines".* The loss as there given would appear to be mainly dependent on the extent to which the actual area of the nozzle departs from that required by the real outlet pressure. The variable is expressed by Goudie as A_{o1}/A_o, where "A_{o1} is the actual outlet area and A_o the correct area for complete expansion; with $A_{o1} > A_o$ there is overexpansion". This would make the loss quite in-

* "Steam Turbines" 2nd edn. p. 255.

independent of the normal range of the nozzle, which can hardly be correct. The loss given by the above equation would seem to be more rational in its variation; but the values as obtained by the two different methods are not in agreement.

In this question of overexpansion we have simply to recognise the existence of a recompression in order to appreciate the loss effect that must ensue. Obviously, the true law of compression loss must be applicable, but it is possible that the empirical rule deduced in Section C is hardly suitable for the relatively small ranges that are usually involved in overexpansion occurrences. For the most accurate method of calculation, or for large compression ranges, it will be necessary to employ the processes of examination used and described in Section C.

The Loss by Supersaturation: On this we have very little direction except that afforded by theory. Mellanby and Kerr have shown,* by an examination of flow curve variations, that, although the steam is certainly in a supersaturated condition within the nozzle, there is evidence of a 12 - 20% reversion during the rapid expansion in the inlet. Since, then, a fractional reversion appears possible under such circumstances, it would seem reasonable to conclude that the full reversion occurs before the blading action is completed. That is, the steam will have reached a condition of stability before proceeding to any further expansion and, consequently, the loss due to supersaturation may be included in the nozzle effects.

If a nozzle expands to such a condition that the end state point falls between the dry line and the "Wilson" line on the heat chart, we are to assume that this is a supersaturated condition and that there is a loss of available energy due to ultimate reversion to the stable state. This is not a loss of the ordinary type. It

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* "The Supersaturated Condition as shown by Nozzle Flow" Proc. I.M.E., Paris Meeting, June 1922.

Study of Nozzle Losses:-
General Remarks. (contd.)

does not mean a destruction of transformed energy, but simply represents an inability to utilise a certain small fraction of the heat that would be made available if the expansion took place under conditions of thermal equilibrium. If it were certain that the steam could remain fully supersaturated throughout any range of expansion there would be no necessity to deal with this loss effect, as the dry steam heat drop could then be taken as the standard. In the actual circumstances it is necessary to take the adiabatic heat drop corresponding to the natural and stable fluid conditions as the ideal, and any part of this not utilised must be recorded as a loss.

If, then, we imagine that, for any given expansion within the range of influence, there is ultimately full reversion to the wet condition, we may proceed as follows to obtain a measure of the resulting loss.

Let H_1 be the initial heat, and H_2' the heat at the end of the adiabatic on the supersaturated expansion line. Then the usual losses - determined by methods already described - are written:-

$$\varepsilon'(H_1 - H_2') = \varepsilon' \cdot DH_\phi'.$$

and, hence, the actual end heat value is:-

$$H_2 = H_2' + \varepsilon' \cdot DH_\phi'.$$

Now the loss of available heat, due to reversion at constant total heat, is that representing the difference between the adiabatic end points at the supersaturated and equilibrium conditions respectively. Hence if H_2'' represents the latter, and ϕ_1 is the entropy value of the adiabatic, then:-

$$H_2'' = T_2 \cdot \phi_1 - G_2.$$

where T_2 is the saturation temperature at the end pressure P_2, and G_2 is Gibb's function at the same temperature. It follows directly that the fractional loss of total available energy is:-

$$\varepsilon'' = \frac{H_2' - (T_2\phi_1 - G_2)}{H_1 - (T_2\phi_1 - G_2)} = 1 - \frac{DH_\phi'}{H_1 - (T_2\phi_1 - G_2)} \quad \text{------} \quad D.(3).$$

Study of Nozzle Losses:-
General Remarks. (contd.)

The quantities involved in this equation can be determined from Callendar's tables so that the calculation is direct.

If the initial condition of the steam should be only q_1 dry, then we suppose the supersaturated condition to apply to this fraction, which would give approximately:-

$$\mathcal{E}'' = q_1 - \frac{q_1 \cdot DH_\phi'}{H_1 - (T_2\phi_1 - G_2)} \quad \text{------- D. (4).}$$

If $\mathcal{E}'$ covers all the other known losses in the usual way then the total fractional loss, on the full equilibrium adiabatic, would be:-

$$\mathcal{E} = q_1 - \frac{DH_\phi'(q_1 - \mathcal{E}')}{H_1 - (T_2\phi_1 - G_2)} \quad \text{------ D. (5).}$$

Again, if the steam condition should actually cross the "Wilson" line in the process of expansion we may suppose, for simplicity, that the reversion is sudden and complete at the pressure corresponding to the point of intersection. Representing the conditions at this pressure by means of symbols without suffix, we have the loss given by:-

$$H' - (T\phi_1 - G)$$

and, hence, the fractional loss would be, for initial conditions H_1 and q_1 :-

$$\mathcal{E}'' = \frac{q_1 \cdot H' - q_1(T\phi_1 - G)}{H_1 - (T_2\phi_1 - G_2)} \quad \text{------- D. (6).}$$

and this would be added to the $\mathcal{E}'$ value, as ordinarily determined, to give the total fractional loss in the full range.

The treatment given is for the simple purpose of providing a guide to the calculation of this particular effect. The methods advanced do not pretend to be exact but they are probably in that respect well ahead of the general ideas on this subject, which are distinctly vague as to the mode, extent and point of reversion. If we take, as the governing principle, that full reversion is possible after expansion, then the equilibrium adiabatic is the natural standard and the described methods are reasonable. It should be noticed

Study of Nozzle Losses:-
General Remarks. (contd.)

that the calculations are based on Callendar's system and values: and, since they involve rather small differences, they require to be carried out with some care.

On The Free Space Effects: The various losses occurring **within** the nozzle have been discussed as well as the information in hand allows. That information has, on the whole, been derived from search tube and flow data. In the case of compression losses reaction data have been employed; while the only guidance in connection with supersaturation is that given by theory. When attention is directed, however, to what is going on in the free space beyond the nozzle outlet it must be confessed that there is no guidance whatsoever. The search tube method ceases to be adequate in this region for, while it gives an indication of pressure effects, these are unaccompanied by other essentials, and the losses are hidden. The impact method would appear to be the most suitable for an examination of the kind but, as it is absolutely necessary to eliminate the nozzle occurrences, it would seem that the search tube would have to be used in conjunction. No completed experimental work provides data of this kind, and so the free space effects cannot be demonstrated apart. We may, however, remark on the probable occurrences, if only to show the nature of the complications.

If the fluid is fully expanded to the back pressure at the nozzle outlet then the unconfined flow to the objective should, presumably, be fairly free from loss. There are, however, certain actions at work which may have a widely varying influence in different cases.

If the free path is fairly long dispersion of the jet may have become active by the time it is completed. The "velocity of dispersion", when once commenced, grows rapidly, and the action on the jet objective will, no doubt, depend to some extent on the opportunity offered for this growth. If, however, the jet is fully expanded on delivery from the nozzle and the free path is short this effect should not be very appreciable.

All jets leaving a nozzle show a greater or less depression of pressure over a certain length. This is a definite manifestation of the "ejector action" that is so general a feature of fluid jets, and demonstrates with clearness that this well known effect is not merely a frictional entrainment of the surrounding atmosphere but a bodily absorption of the extraneous fluid. Now this must entail energy loss since the entering fluid must be accelerated and the necessary momentum can only be imparted at a cost of some dissipation of energy. This loss is probably small in good jet forms with moderate pressure ranges; but the effect can hardly be negligible in cases of high speed flow and unsymmetrical forms. For instance, with any particular range, rectangular jet sections show the "ejector pressures" much more clearly than circular forms.

Again, if a nozzle underexpands there is a drop to the fixed back pressure in the free space. If the speeds are moderate and the extra expansion small this takes place smoothly but at high speeds the free expansion establishes pressure waves that persist for a fair length. This can hardly occur without loss and all pressure curves of the kind show a damping effect that may represent a measure of the loss.

These few considerations are probably sufficient to show that the occurrences between the nozzle outlet and the jet target are not necessarily unimportant in their incidence on total effects. They are certainly obscure; but it is quite conceivable that they may tend to upset general conclusions drawn from results that include them without separate definition.

The difficulties of experimental examination are obvious. The impact plate might be helpful in some cases but only if a definitive method of covering the nozzle effects were available. Generally, however, the difference between jet and nozzle totals will not be amenable to such a procedure.

Study of Nozzle Losses:-
General Remarks (contd.)

For the purpose of arriving at a fair point of view in this matter we might consider that the gap effects, in combination with the nozzle outflow conditions, produce a jet that, at its working point, is not merely possessed of a certain energy but is also characterised by a certain quality. In effect, the energy function is qualitative as well as quantitative, This would appear a useful conception and may be allowed a short special notice.

On Jet Quality: The ideal jet leaving a nozzle would be parallel, in form, symmetrical in section, clearly defined, and solidly directed without tendency to dispersion. In many cases of calculation we are almost guilty of assuming these qualities; the usual "velocity coefficient" method of working being rather undiscriminating.

Actually, however, the jet at outflow may show features that very certainly distinguish it from the ideal. It may, for instance, possess spin; may be undergoing a compression action; or freely expanding; it may establish an appreciable ejector effect; it may have been developed at an excessive rate of divergence; or in a curved path with the probability of lateral velocities. In its passage through the free space or "gap" it naturally attains a "velocity of dispersion" and this, when once established, has a tendency to rapid growth. It is quite clear that this tendency is not helped by any of the conditions enumerated.

The importance of the question becomes more obvious when we realise that the actual working operation of a jet is not truly represented by the action on an impact plate . The jet is to work throughout the length of a blade passage; and, hence, there is a very full opportunity for the development of any flaw in "quality" which may be inherent in the jet as supplied. Clearly questions of form are also involved; whether the jet as delivered suits the passage as arranged; and the inadvisability of using a circular jet in conjunction with a rectangular passage is simply an outstanding but particular, instance of the general matter considered.

Study of Nozzle Losses:-
General Remarks (contd.)

The argument has also another aspect. If a curved blade passage be used as a nozzle and the energy of the outflowing jet measured in one of the usual ways, is it to be expected that the variations characteristic of the more definite straight forms will be shown ? If, as seems probable, the curved passage be held the more liable to produce disturbing velocities, then the jet as delivered will be of a poor quality, and will display this quality in its ultimate action.

Altogether, it should be clear that besides the question of the necessary energy to do work there is that of the ability to surrender this energy effectively. This will depend on such a variety of factors that separation or discrimination will be impossible; and we may employ the term "jet quality" as a general mode of reference. Hence, when we deal with a jet on the point of starting its working operation we should know more about it than is represented by the term velocity; and a blading coefficient should reflect the "quality" of the working jet as well as the speed at which it operates.

VARIOUS PAPERS.

(1920 - 1922)

BY

WILLIAM KERR, A.R.T.C.

BOOK II.

PAPERS ON EXPERIMENTAL SUBJECTS.

NO. 2.

ON JET ACTION IN BLADING

WITH

SHORT ADMISSION ARCS.

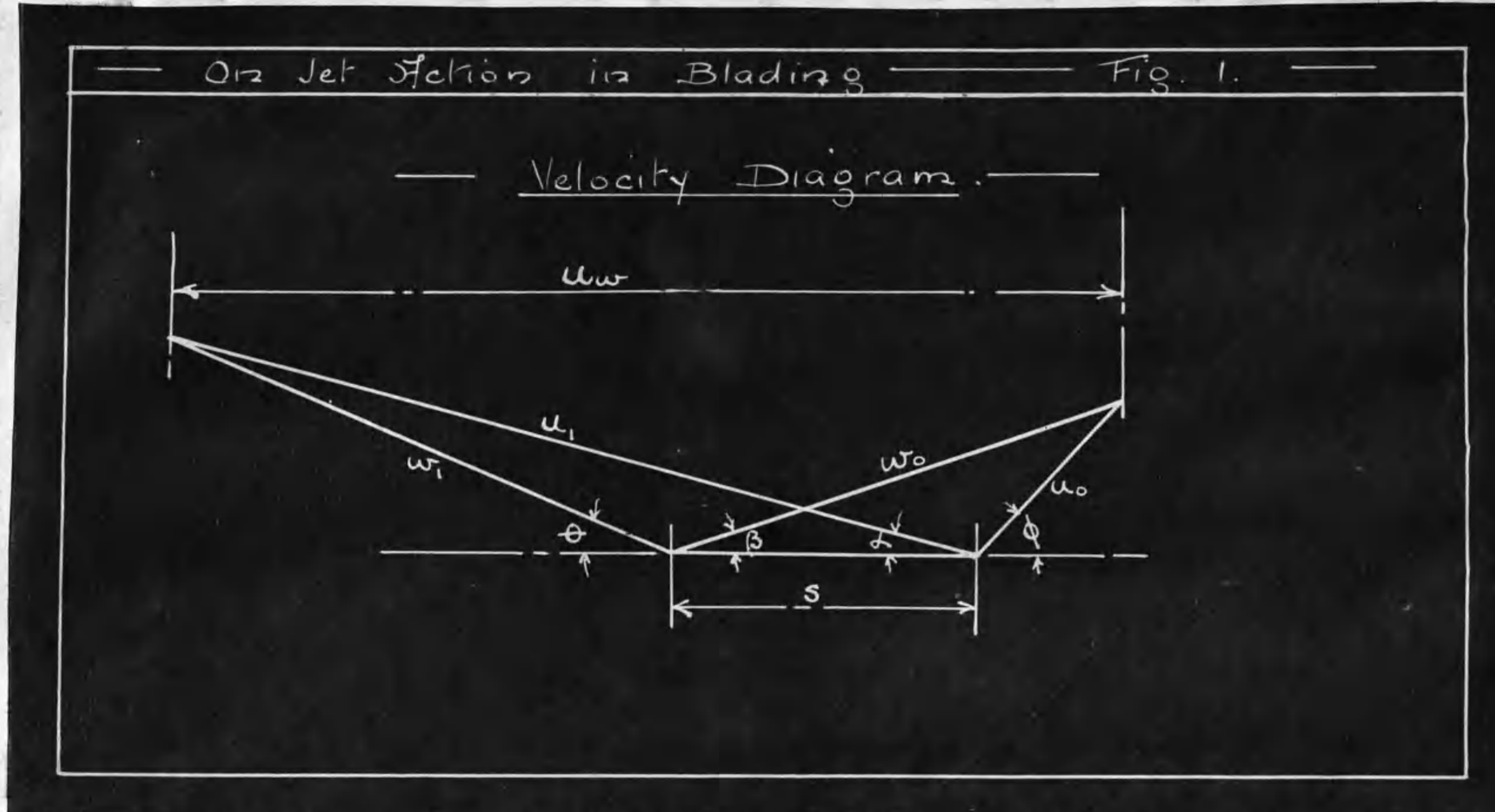
ON JET ACTION IN BLADING WITH SHORT ADMISSION ARCS.

Introductory: It will be clear that one main function of a detail study of nozzle losses is to permit the exclusion of this particular factor in the general consideration of the complex nozzle-gap-blading arrangement which makes up a turbine element. If a reasonable estimate of the nozzle effect can be made, the separate examination of the blade action is brought more definitely within the bounds of possibility.

At the best, however, such an investigation cannot be so readily carried out as that on a stationary nozzle, since there is practically no simple way of reproducing true blade effects. Even if the desire were only to examine the occurrences with the blade at rest, and under the normal action of the jet, it would be more or less essential to use an actual turbine; and, when the blade shape and the casing conditions are taken into consideration, it will be readily seen that any of the usual methods of examining fluid flow must be difficult of application, and rather unfruitful in results. When it is further recognised that the general examination must involve a rotating blade row the difficulties almost become imposs--ibilities.

Investigations have, at various times, been carried out on the pressure variations in stationary blade passages. In this respect we have a method more or less analagous to the search tube process in nozzle examination; but there is nothing like the same degree of exactitude, since, with the awkward forms, there is no certainty regarding flow areas. Furthermore, such results do not lead to much useful knowledge regarding the working operation: they show, as it were, only secondary features of the main action, and are not, in any sense, closely related to the fundamental principle.

The method of observation which has provided the results

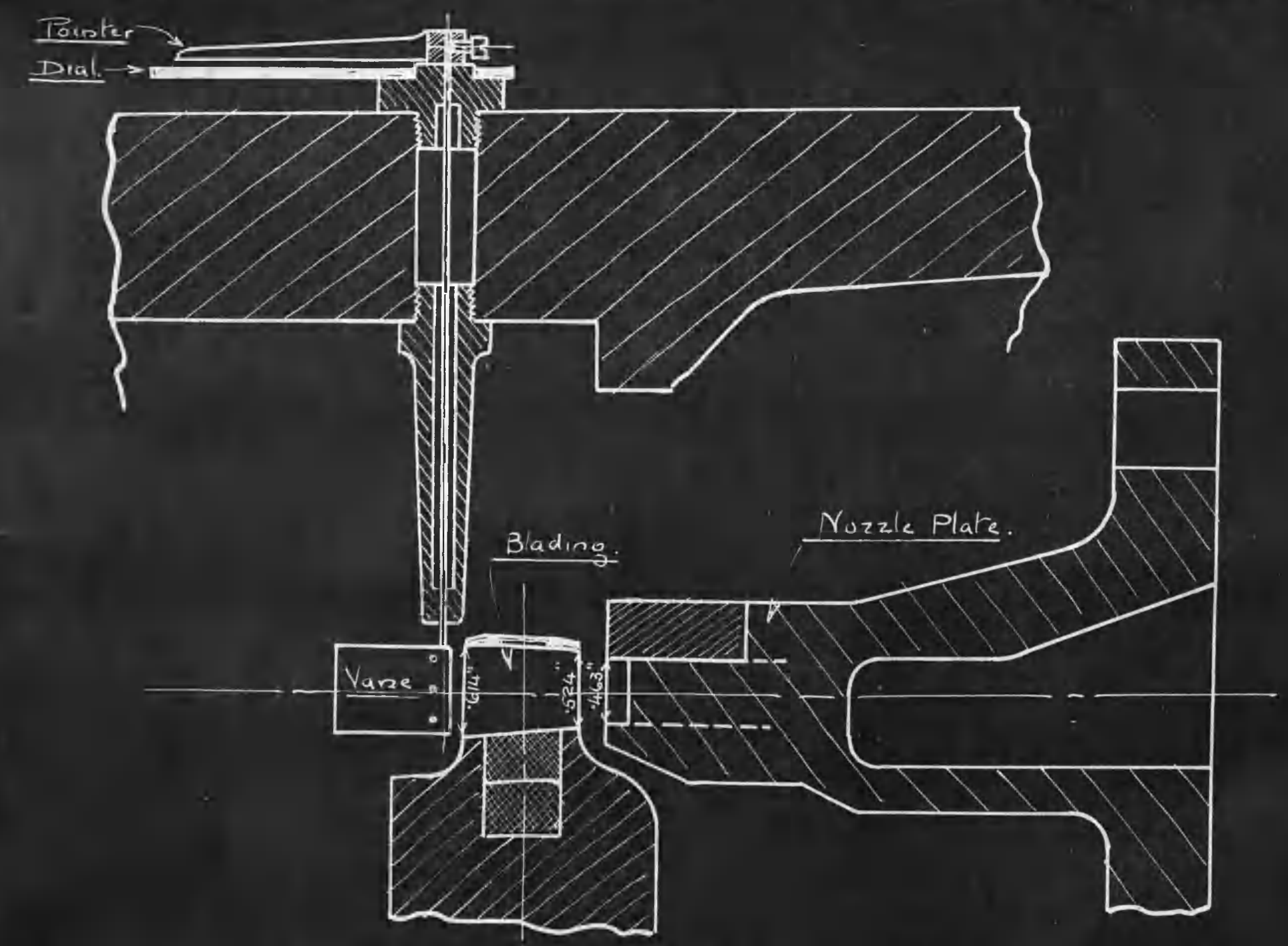


On Jet Action in Blading — Fig. 1.

to be considered in this paper is, perhaps, the only really simple procedure open to the investigator. Its derivation is easily seen on consideration of the principle on which turbine blading is supposed to operate. Thus, taking fig. 1 as ordinary velocity triangles for a blade row, it is clear that power test results of the usual kind will define the important tangential component of the change of velocity shown by u_w. On the supposition that the nozzle conditions are known, the value of u_1 is fixed and, if the blade angle β is definite, the apex of the "outlet triangle" is positively determined. The absolute outlet jet angle ϕ then follows. It is clear from all this that the directions of flow are, on the whole, almost as important as the fluid speeds, in so far as the jet action on the blade is concerned: and it would appear at least necessary to make certain that β was a definite angle and that the actual ϕ agreed with its derived value. If the velocity triangles as drawn represent completely the action on the blades, the value of ϕ becomes a very adequate check on the whole process: if the method is not entirely representative, then its shortcomings will, of necessity, be indicated by unexpected variations of ϕ. Since, then, the actual measurement of this particular angle provides a check of correctness, or indicates a fault, its importance is manifest.

Now the direction ϕ is obviously a quantity that may be measured in the actual turbine. It only requires a vane set in the fluid stream, just beyond the outlet edge of the blade row, and carried on a fine spindle passing through the casing, to register direction on a dial plate outside. It can be used with the wheel moving or at rest and, hence, serves as a method of search for variation in either the angle of efflux from a stationary blade, or in the absolute jet angle from a moving blade; and, therefore, both β and ϕ are open to observation. It should be remarked, however, that vanes cannot be used on the separate blade rows of a velocity compounded wheel, as the spaces are quite inadequate; but this should be unnecessary as its use with the single row will lead, one way or the other, to general conclusions.

On Jet Action in Blading. — Fig. 2.



Arrangement of Vane Fitting

On Jet Action in Blading. (contd.)

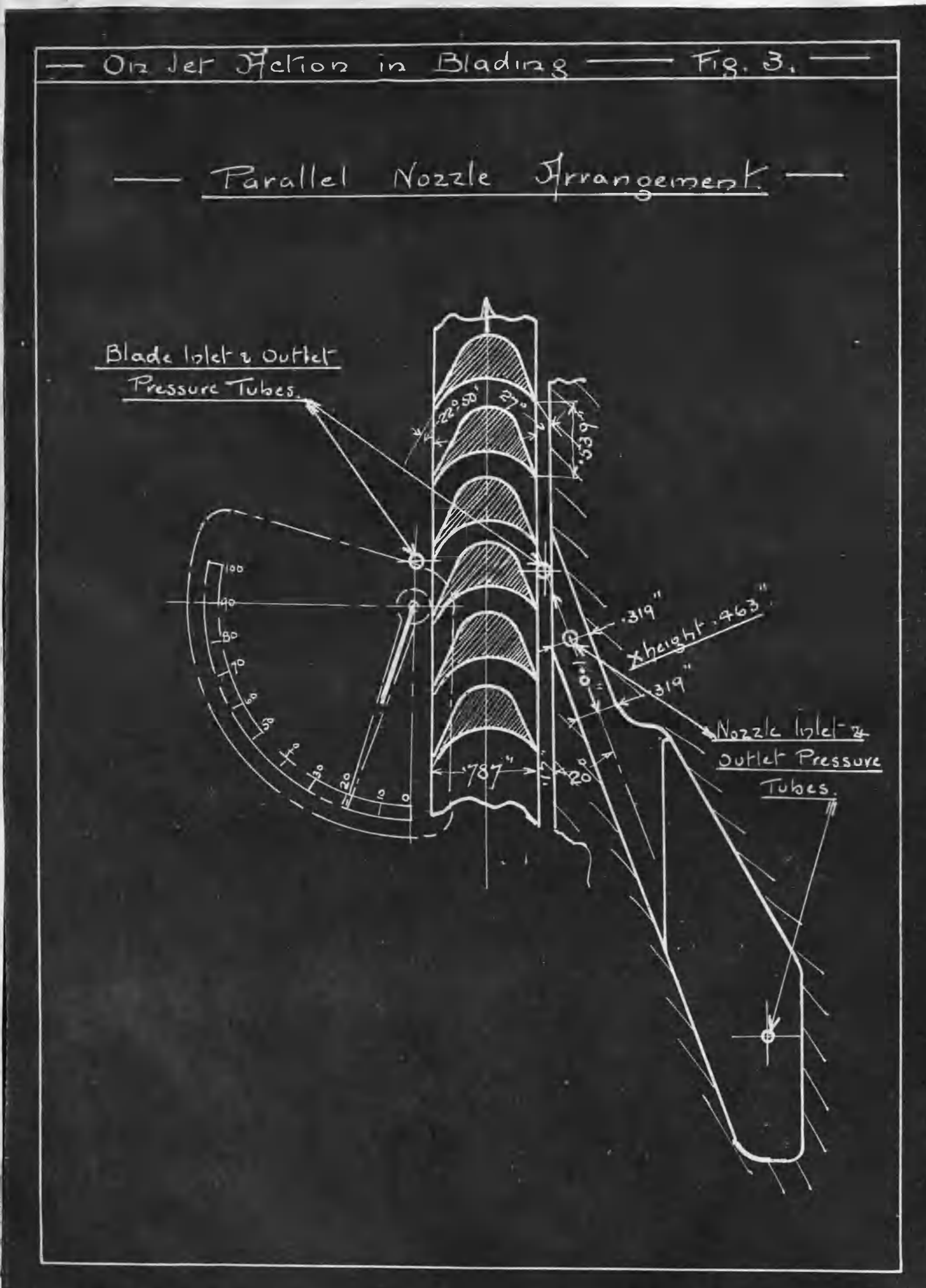
From considerations such as those outlined it appeared that the "vane method" offered the simplest and most direct method of making observations on the total blade effects. When first used the tests were carried out on the moving wheel: but it was soon apparent that little could be done until more fundamental facts were established, and the "stationary wheel" tests, with which this paper is mainly concerned, were then undertaken.

Vane Experiments with Stationary Wheel: The turbine on which these tests were carried out is that illustrated by fig. 3 of the paper on "Turbine Wheel Friction" included in this collection. It is a one row wheel, but really represents the first row of the 250 K.W. Impulse Turbine in the Mechanical Engineering Laboratory of the Royal Technical College. The turbine has, however, shed most of those internal arrangements which fix the "250 K.W", as not only have the guide blades and the second and third blade rows been removed, but the original nozzles have been replaced by a special nozzle plate, carrying accurately finished examples of a parallel and a divergent nozzle.

The cross section of fig. 2 shows clearly enough the arrangement of nozzle plate blade row and vane; while the plan views of figs. 3 and 4 illustrate the different nozzle forms and relative vane positions. Both nozzles have the same radial height and throat sizes, and the divergent nozzle has taper on the width only. The essential particulars of the nozzles and blading are as follows:-

Nozzles:-	Parallel	Divergent
Width at Throat	.319"	.319"
Width at Outlet	.319"	.890"
Height	.463"	.463"
Length from Throat to Outlet	.920"	3.340"
Nozzle Angle	20°	20°

Blading:-	
Height at Inlet	.524"
Angle of Inlet	27°-0'
Height at Outlet	.614"
Angle of Outlet	22°-50'
Mean Pitch	.539"
Thickness at Outlet	.015"


On Jet Action in Blading
Fig. 3.
Parallel Nozzle Arrangement
Blade Inlet & Outlet Pressure Tubes.
Nozzle Inlet & Outlet Pressure Tubes.
22°30'
27°
.539"
.319"
x height .463"
.319"
.787"
20°
100
90
80
70
60
50
40
30
20
10
0

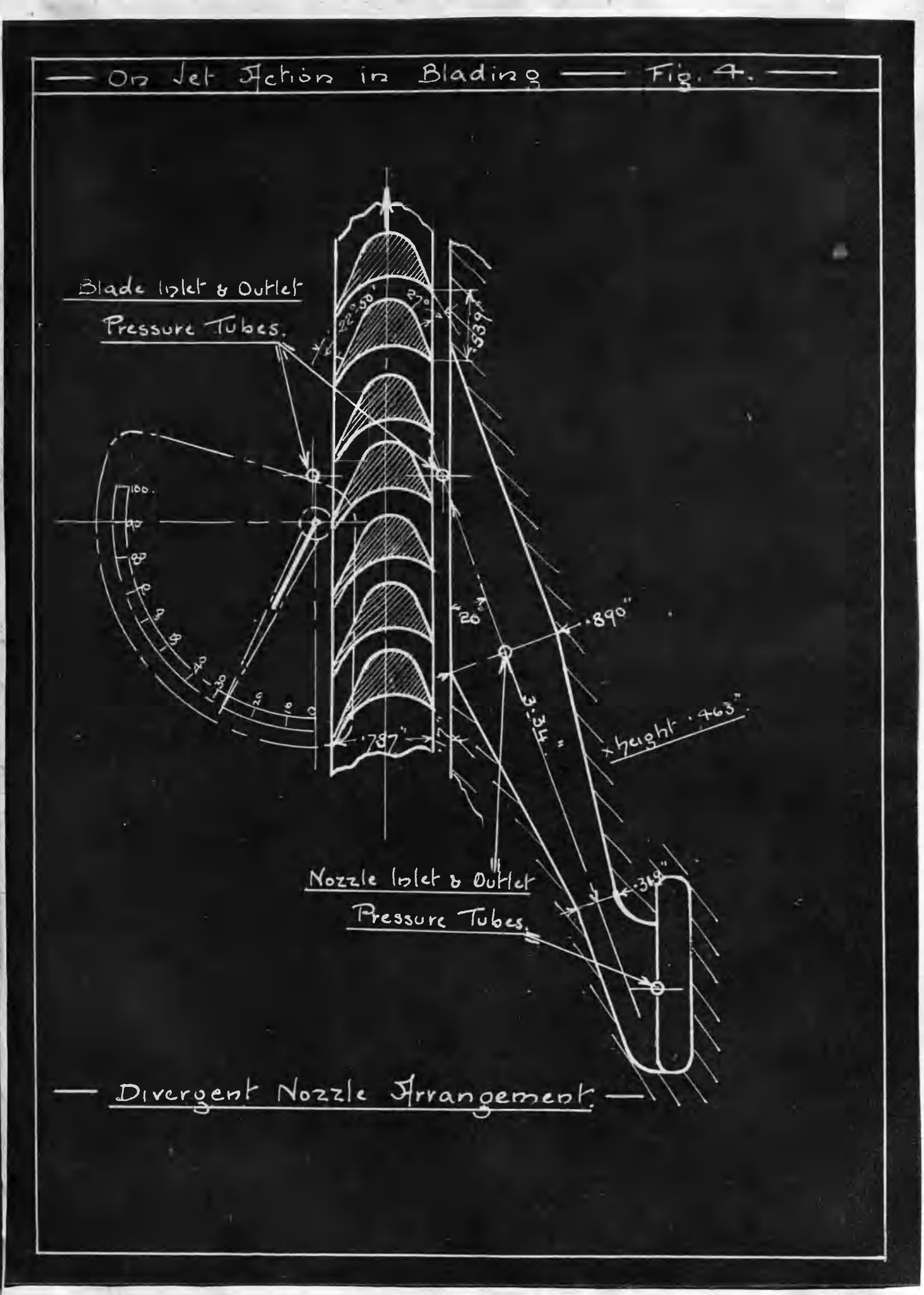
Pressure tubes are led from the four positions indicated in figs.3 and 4 through the casing walls to gauges on the instrument board. It is thus possible to read, for both cases, the nozzle inlet and outlet pressures, and those on either side of the blade row.

The procedure in these tests was simply to lock the wheel in some position by means of a key on the coupling bolts, and then to note the vane angles registered under different steam conditions. Either the chamber pressure was fixed and the supply pressure alter-ed or vice versa; thus making every test an examination of the variation of outflow angle under changing pressure ratio of expan-sion.

In actual use it is found that, except with the very lowest and the very highest expansion ranges, the vanes record the angle with definiteness. There are occasional cases in which the vane seems to be somewhat "looser" than in others; but, generally, within the main ranges, the observations are certain, and the "kick" of the vane when removed from its true position very marked. Even with the wheel running there is no difficulty in determining the true angle as, although vibration of the pointer may exist, it is not excessive and the equilibrium value is readily fixed. Indeed, in actual runn-ing of the turbine the vane shows itself so sensitive that it is possible, in many cases, to detect changes of speed before they can be observed on the revolution counter.

Table I is presented as a typical set of test readings. A full tabular statement of all the various experimental records is quite unnecessary, as the plotted data given below shows all that is required.

The tests divide into two different series; one in which the setting of the rotor was arbitrarily changed from test to test with more or less convenient pressure ranges in each. This series gives a general view of the bounds within which the outflow angle may vary. The second series includes several tests at one definite



gives a general view of the blade within which the outflow angle may vary. The second series included the tests at one definite position on the rotor in which the widest possible variations of pressure were used, thus giving a clear definition of the influence of the pressure ratio.

In practically all cases the difference of pressure on the two sides of the blade row is negligible, and the pressure ratio used in plotting is:-

$$r = \frac{\text{Pressure on Outlet Side of blade.}}{\text{Pressure at Inlet to Nozzle.}}$$

The first test series with varying positions is represented by the confused sets of curves in figs. 5 and 6; the former giving the vane readings for the parallel nozzle jet and the latter for the divergent nozzle. It will be seen from fig. 5 that, for any fixed position, the vane reading is nearly constant down to a value of r of about .3 to .4 and then rises rapidly; becoming, in fact, very high (and somewhat indefinite) at very low r values. Fig. 6, on the other hand, discloses a general curve form falling to a minimum at ratios of about .10 to .15 and then rising rapidly as in the prev-ious case. Further, it is readily seen from these diagrams that, while the curve forms have fairly definite features, the actual values are obviously - within certain limits - dependent on rotor position; and, since the positions have been erratically chosen, the variations due to this cause appear very irregular.

We now show by fig. 7 the typical and clear results for one fully examined position. This fig. records both vane results. The previously noticed influence of the pressure ratio is now definitely established. The parallel jet vane gives readings rising rapidly for values of r below about .4; the divergent jet vane readings fall more and more rapidly to a sharp minimum at an r value of .14, and then rise rapidly with further fall of the ratio. It is also clearly brought out, by a general comparison of all the results, that the outlet angle is distinctly different for the two jets at the same pressure ratio, and that neither of them is really very close to the geometrical angle of the blade. Within the steady ranges the diverg-ent jet angle is above the geometrical, while the parallel jet angle

— On Jet Action in Blading —— Table I. —

— Experimental Results for Test Nº 9. —

Instrument		Reading									
Nº of Test.		9.									
Rotor Position		Position 2.									
Date		4. V. 22.									
Steam Conditions.		Varying Press: Constant Vac.									
Barometer		29·43" = 14·4 lb/in².									
Parallel Nozzle:-											
Noz. Inlet Press.	lb. abs.	54·0	63·9	74·1	83·3	94·1	103·7	113·4	122·6	133·4	144·1
" Outlet "	"	27·4	33·7	38·2	42·0	47·3	50·5	54·1	56·3	57·7	60·9
Blade Inlet "	"	13·0	13·0	13·0	13·0	12·8	12·1	11·9	12·2	12·2	11·1
" Outlet "	"	13·2	13·2	13·3	13·4	13·5	13·1	13·2	13·1	13·0	13·1
Press. Ratio		·245	·207	·179	·161	·144	·126	·116	·107	·097	·091
Vane Angle.	degrees	20·0	22·7	25·2	27·2	29·5	32·3	27·0	34·5	39·0	34·5
Divergent Nozzle:-											
Noz. Inlet Press	lb. abs.	54·0	63·9	74·1	83·3	93·7	102·7	112·2	121·4	132·5	143·1
" Outlet "		13·7	13·7	13·7	13·7	13·7	13·7	13·7	13·7	13·7	13·7
Blade Inlet "		13·2	13·2	13·2	13·2	13·2	13·0	13·1	13·1	13·2	13·2
" Outlet "		13·2	13·2	13·2	13·2	13·2	13·0	13·1	13·6	13·2	13·8
Press. Ratio		·245	·207	·178	·159	·141	·127	·117	·112	·099	·096
Vane Angle	degrees.	25·5	24·0	23·0	21·4	21·0	22·8	22·8	25·8	28·8	31·0
Remarks.		Both Vanes Steady	"	"	"	"	Mean Value P.N.V. defin. D.N.V. Steady	P.N.V. "loose" D.N.V. Steady	Mean Value P.N.V. definit. D.N.V. Steady	"	P.N.V. "loose" D.N.V. Steady

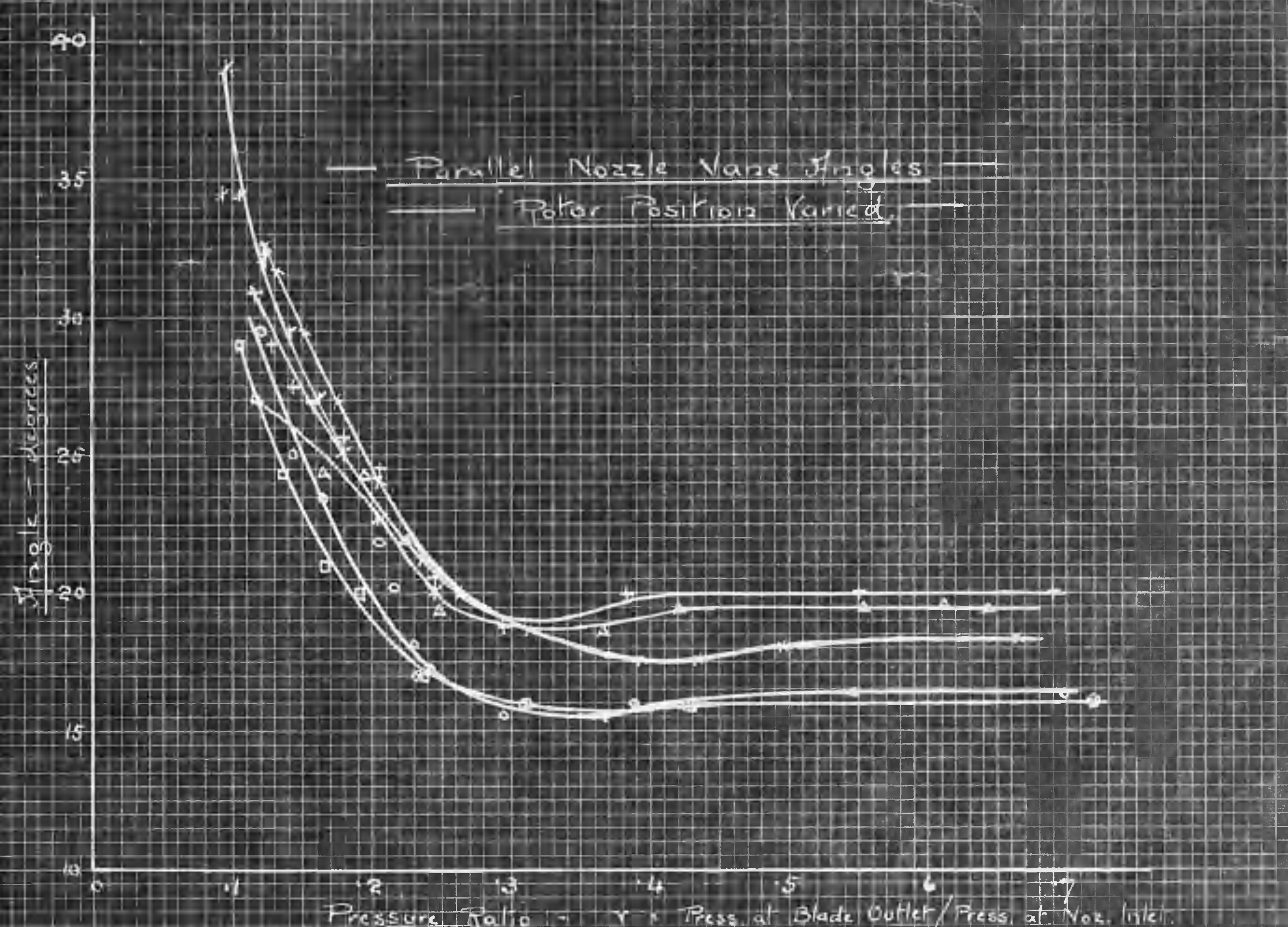
is below; the minimum values exhibited by fig. 7 are, however, not greatly dissimilar.

The variations shown by these different results are rather unexpected but they are apparently characteristic and, therefore, of some importance. Obviously any method of examination of blade action which ignores the consistent variation with pressure ratio must be distinctly faulty. It should be understood that the same pressure ratio may represent widely different absolute pressure values. Fig. 7, for instance, includes results with supply pressures varying between 25 and 85 lb. per sq. in. absolute and chamber pressures ranging from 2.5 to 35 lb. per sq. in. absolute; from which it is clear that the reading is dependent on ratio of expansion, and not on absolute pressure or mass flow.

The Variation of Outlet Direction: Stated concisely, the experimental results obtained with the wheel stationary show, (i), that for any given pressure ratio the outflow angle may vary within 4° or 5° limits with the position of the rotor, and (ii), that for a fixed rotor position the variation of the angle with pressure ratio gives a characteristic curve form which does not appear to be much restricted by the geometrical blade angle.

The first point is at present much less important than the second, and may be quickly dealt with. It will be readily recognised that with a changing rotor position the configuration of jet, blade section and vane will alter. Now as the jet is supplied by a single nozzle it is of no great width and, hence, it is easily possible to have the blade passage in such a relative position to the jet that there is only partial filling on the inlet area. Clearly, the flow through the passage, and the outlet direction, will be influenced to some extent by the degree of filling, and whether the jet spills over the face or the back of a blade. In fact, the variation of angle by position is the result of the very short jet arc used. For instance, the divergent jet extends over a somewhat

On Jet Action in Blading — Fig. 5.



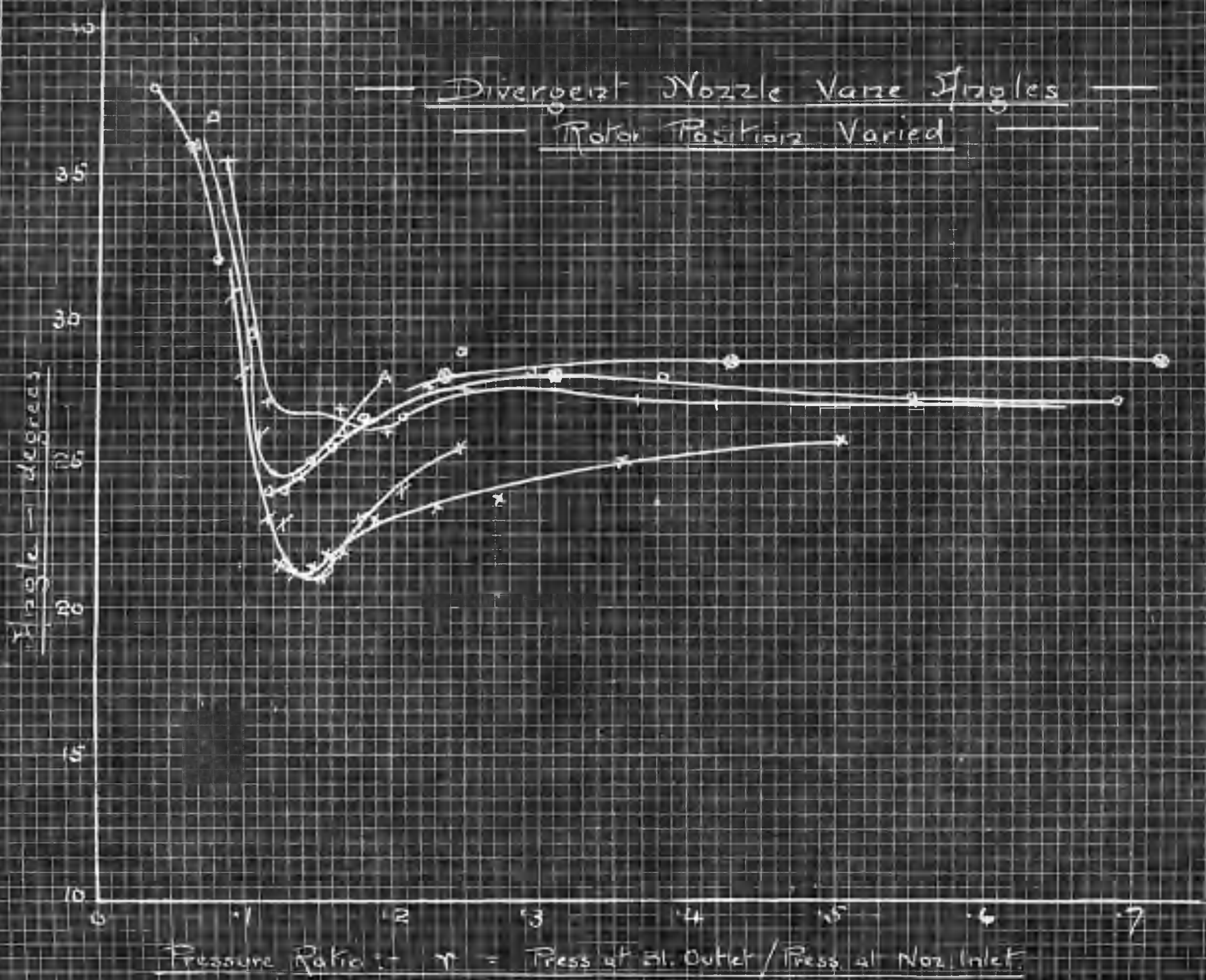
longer arc than that from the parallel nozzle; and this explanation would require a reduced variation of angle at any one position for the longer arc. On comparison of figs. 5 and 6 this general tendency will be recognised, as the various curves are clustered together more closely in the latter diagram than in the former. It may, then be taken that, if the admission arc were long enough to ensure that all the blade passages in the vicinity of the vane were quite clear of the jet fringes, the variation of angle with rotor position would not be shown.

Since, however, the change of outlet direction with pressure ratio is shown for all rotor positions, we must believe that this is a genuine manifestation of the jet action, and it must receive a more careful consideration. It is not, of course, known which particular curve will represent the case of the correctly situated blade passage; but, since we are mainly concerned with the nature of the changes, fig. 7 may be held as representing the true variation of angle after elimination of the accidental effect of position.

Consider, then, the case of a blade passage fully covered by the operating jet. The steam that enters must pass through and out. The mean onward flow velocity naturally suffers diminution and the outlet flow area must pass the entered steam at the exit velocity. Notice that the essential conditions involve a difference of inlet and outlet speeds, and a sufficient outlet area.

Suppose the steam enters with a speed w_1; and suffers a drop to $x \cdot w_1$; then the relationship of areas must at the least be as $1 : 1/x$. Now, if the jet has always the same outlet angle, it follows that the available areas must maintain a constant ratio and, therefore, x must be invariable for the blade, i.e., the usual idea of a definite outlet direction implicitly involves the conception of a nearly constant velocity coefficient. The further assumption of the geometrical angle is equivalent to taking a value for this.

On Jet Action in Blading — Fig. 6



On Jet Action in Blading. (contd.)

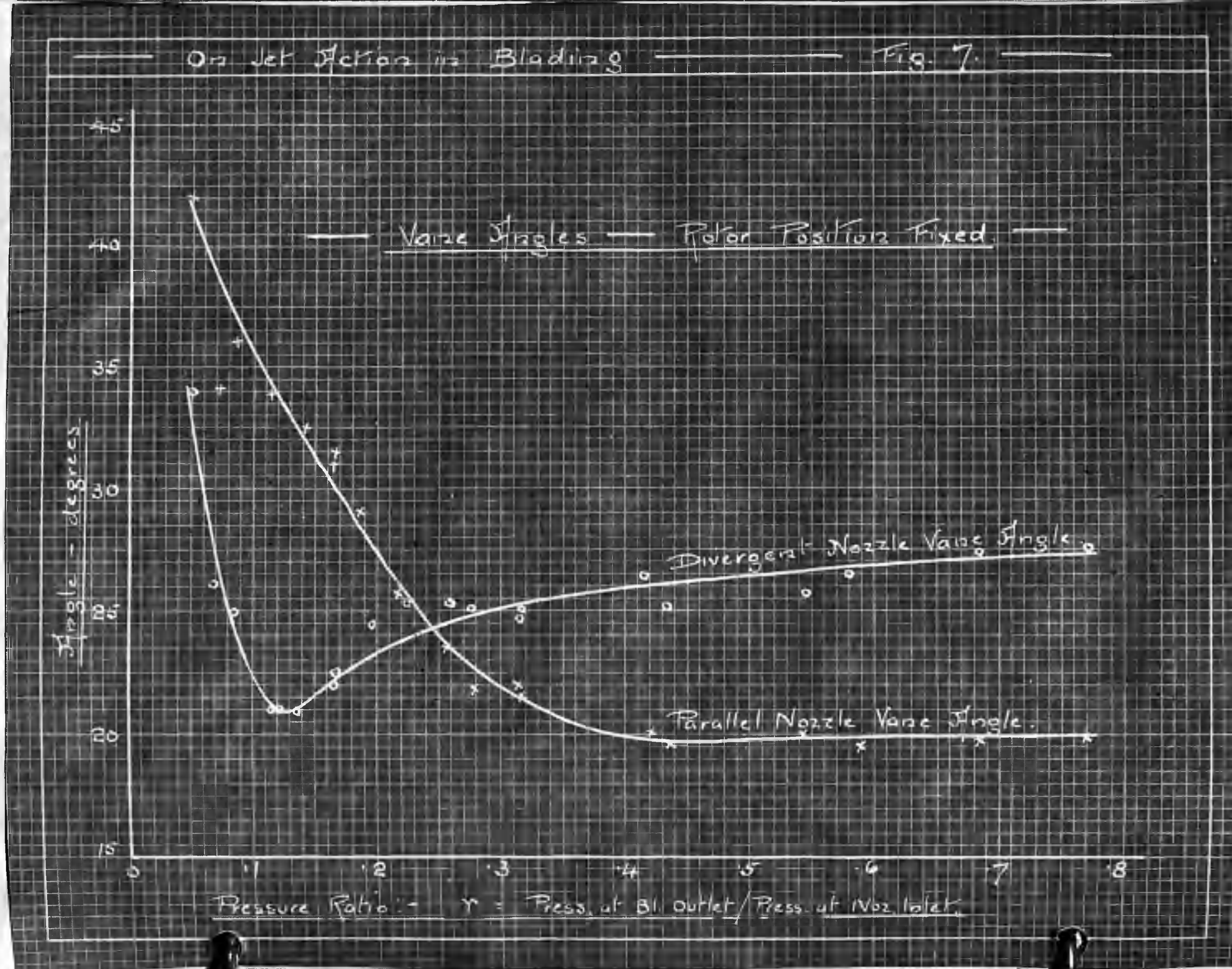
If, then, we suppose that the usual blade form effects a complete control of jet direction, it will be necessary to accept the above finding in its general form; and, therefore, to believe that the blade shape influences the flow entirely by reacting on the inlet conditions to the passage!

Carried thus to its logical conclusion the argument for an invariable angle of outlet flow hardly seems reasonable. It would appear necessary to envisage a varying α value and, hence, the possibility of a changing area ratio for the blade. Now the inlet area to a blade passage is really dependent on the nozzle jet height and angle and is, therefore, independent of the blade shape itself; so that any alteration in α, with a given jet supply, can only be met by changing outlet conditions. In other words if α is a variable the outlet direction is not fixed.

That it is not definitely fixed by blade form will be admitted on inspection of any standard turbine blade; the jet is obviously not compelled to follow the geometrical angle. It is well-known that a small variation is possible in the jet line from a straight axis nozzle with oblique outlet face, and it might natur-ally be expected that in a blade this variation could be much wider.

The point brought forward, then, is that a jet working in a blade passage can adapt its own outlet direction, within fairly wide limits, to suit the conditions in the passage. The geometrical angle is only roughly a guide to a jet angle; it is more probably a measure of the latitude of possible variation than a control of the actual angle itself.

We may obtain guidance relationships between flow area and direction by reference to the sketches in fig. 8. The simple ass-umption is made that, for angles different from the geometrical, the jet "hangs" on to one blade edge or the other, depending on whether the effective jet direction is more or less acute than the tangent to the blade curve. Thus for angles β', exceeding the form angle β_o, we have, for flow area:-


On Jet Action in Blading
Fig. 7.
Vane Angles — Rotor Position Fixed
Divergent Nozzle Vane Angle.
Parallel Nozzle Vane Angle.
Angle - degrees
45
40
35
30
25
20
15
0
·1
·2
·3
·4
·5
·6
·7
·8
Pressure Ratio - r = Press. at Bl. outlet / Press. at Noz. Inlet.

$$A_o' = l_o \times \overline{OB} = l_o \cdot p_o \cdot \sin\beta_o / \cos(\beta' - \beta_o) \text{ ------ } (1).$$

and for angles β'' less than β_o :-

$$A_o'' = l_o \times \overline{QC} = l_o \cdot p_o \cdot \sin\beta_o / (\sin\beta_o / \sin\beta'') \text{ ----- } (2).$$

where:-

A_o', A_o'' = outlet areas, - sq. ins.
l_o = blade height - ins.
p_o = mean distance between consecutive blade edges - ins.
= $\bar{p} - t/\sin\beta_o$, where $\bar{p}$ is mean blade pitch and t is blade thickness.

Hence the flow area, as defined by shape, may be increased or dimin--ished in the respective ratios:-

$$\frac{1}{\cos(\beta' - \beta_o)}, \quad \frac{\sin\beta''}{\sin\beta_o}.$$

The effective area at inlet to blade is really decided by the jet angle. Suppose this to be α and that the jet only occupies fraction a of the inlet blade height; then, on the supposition that the jet covers the full width of passage, we have:-

$$A_1 = a \cdot l_1 \cdot p_1 \cdot \sin\alpha. \text{ ----------- } (3).$$

where the symbols have similar meanings to those used for outlet conditions.

The ratio of inlet and outlet area is, therefore, roughly of the order:-

$$\left.\begin{aligned} A_r &= a\left(\frac{l_1}{l_o}\right)\left(\frac{p_1}{p_o}\right)\left(\frac{\sin\alpha}{\sin\beta_o}\right)\cos(\beta' - \beta_o) \\ \text{or:-} \quad A_r' &= a\left(\frac{l_1}{l_o}\right)\left(\frac{p_1}{p_o}\right)\left(\frac{\sin\alpha}{\sin\beta_o}\right)\left(\frac{\sin\beta_o}{\sin\beta''}\right) \end{aligned}\right\} \text{ ----- } (4).$$

as suits the particular case

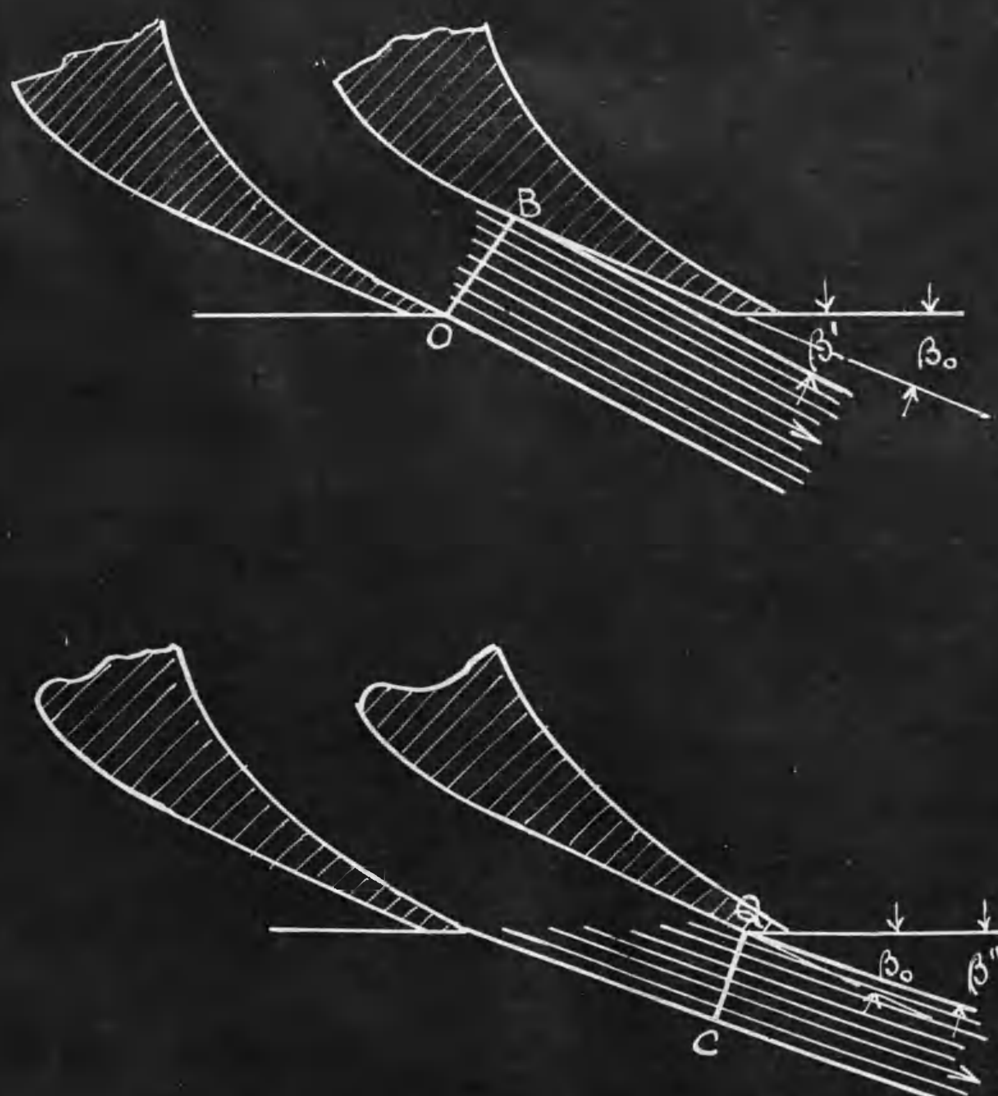
If we assume that, with variation of pressure ratio, the jet angle α is practically unaltered, then the area ratio is propor--tional to:-

$$A_r \propto a \cdot \cos(\beta' - \beta_o), \text{ or } \propto a \cdot \sin\beta_o / \sin\beta'' \text{ --- } (5).$$

The trigonometric ratios contained in this are not sub--mitted as exact expressions of the variations due to angle, but as

On Jet Action in Blading — Fig. 8.

Flow Area and Direction.



rough indications of the probable order. They serve to bring out the fact that increase beyond the geometrical is comparatively small in its effect, whereas a fall below the standard is more significant than it appears. These conclusions are probably fairly sound, but the discontinuity indicated by the change of trigonometric form can hardly be accurate. Having established some little guidance in the matter, however, we may proceed to discuss the probable cause and effect somewhat more fully.

The Variation of the Coefficient: Both inlet and outlet effective areas must satisfy the equation of continuity, hence:-

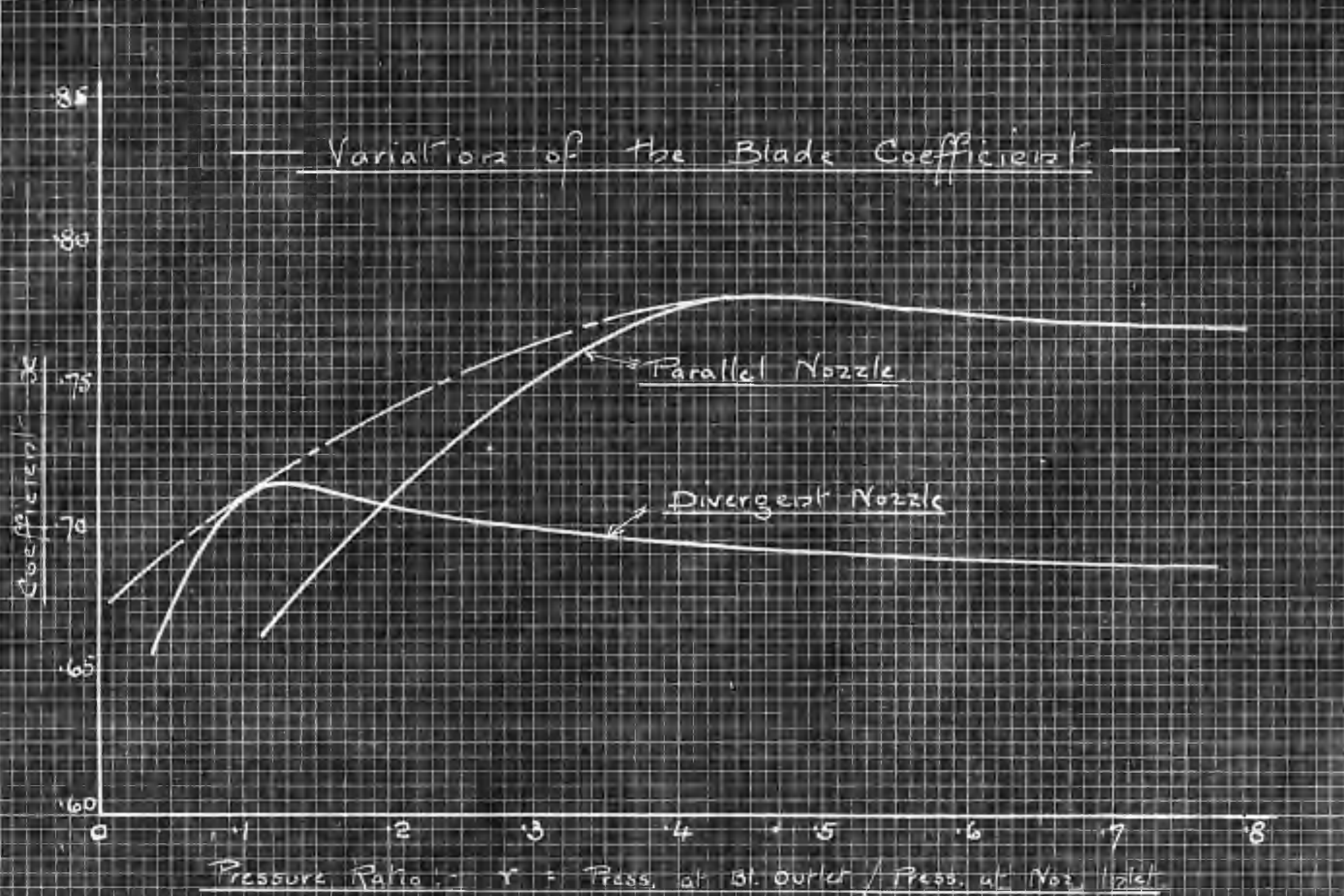
$$\frac{w_1 \cdot A_1}{V_1} = \frac{w_0 \cdot A_0}{V_0}$$

where w represents velocity and V specific volume. If $w_0 = x \cdot w_1$, then:-

$$x = \frac{A_1 \, V_0}{A_0 \, V_1} = A_r \frac{V_0}{V_1} \qquad (6)$$

Generally V_0/V_1 will simply depend on the reheating effect with--in the blades, which will make V_0 slightly greater than V_1; but this influence on the ratio is of a secondary order only. The pressure values may also alter, however, and V_0/V_1 would then reflect pressure changes as well as reheat effects. While it is quite possible that the pressures are not uniform, it is certain that the differences are of small account; there is no evidence, for instance, of any real pressure changes in the very close agree--ment shown between inlet and outlet pressures in these tests. Unless the changes were large they could not greatly influence the ratio x; and, although we may admit that there are secondary effects of reheat and pressure, it is quite reasonable to assume that, in general, with an impulse blade, the value of x is given to a first approximation by the ratio of areas. The obvious diffi--culty is, of course, that we can never really be sure of the effective areas; although a step has been taken towards this necess--ary determination by the present examination of outflow angles.

On Jet Action in Blading — Fig 9.



If, then, we may assume that x is approximately fixed by the area ratio and, further, that equations (4) give an idea of this, it follows that a rough estimate of the value and variation of x is possible from the curves in fig. 7. Allowing that, since the nozzles have no radial divergence and the spread in this direction is not likely to be great, the value of a may be taken as constant at the ratio of nozzle and blade inlet heights; and, again, that it is permissible to smooth out the curve form expressed by relations (5) to avoid the sharp change at the geometrical angle; then, applying the known dimensions, and equations (5), to the main values of fig. 7, we finally obtain the curves shown in fig. 9.

While admitting that several approximations of a rough order have been used in the various steps of reduction it must be emphasised that the general order of effects has, at all times, been retained. Hence fig. 9 must be considered a fair expression of the kind of variation that x must undergo if the observed data on angle changes are to be met.

Appreciation of the special features of these curves and their meaning will be assisted by recognition of the following facts.

(i). The parallel nozzle can expand completely to a ratio of about $r = .5$. The expansion is also complete within this nozzle for all ratios above .5; but for ratios below this a free expansion occurs beyond the outlet.

(ii). The divergent nozzle expands completely and continuously for an outlet ratio slightly below $r = .1$. For all ratios above this there is recompression within the nozzle, and the closer the ratio is to .1 the nearer the rapid part of the recompression is to the nozzle outlet. For ratios below .1 there is free expansion beyond the outlet. It should also be realised that the fully expanded jet issuing from this nozzle possesses definite divergence of profile.

(iii). No matter what rotor position is taken the minimum outflow angle for the divergent nozzle jet is always greater than the corresponding minimum angle for the parallel jet - figs. 5 and 6.

(iv). It has been explained in another paper of this collection* that the "quality" of a jet delivered from a nozzle must be affected by such occurrences as compression, divergence, free expansion etc.; and that by "quality" is meant the general tendency to dispersion, caused by the state of the jet action; the greater this tendency the poorer the "quality".

It should be understood that we do not require to consider nozzle efficiencies in any way; the effect portrayed in fig. 9 is purely a blade effect and would be practically uninfluenced by nozzle efficiency, even if one nozzle attained 95% and the other only 75%. The effect of efficiency on speed is secondary to that of expansion ratio; and the general influence of speed - if any - can readily be deduced from the latter. Efficiency might, to some extent, reflect "quality" and so have a slight significance, but there is, of course no general relation between the two; the latter must mainly depend on the type of nozzle and the ratio of expansion. It is clear, then, that r is the essential variable, and the curves of fig. 9 are correctly based.

It is to be noticed that both curves give a maximum reasonably near the respective best ratios for the nozzles. Further, since in these particular cases the assumption made as to the value of a is quite sound, it follows that these maxima are the most definite results on the curves. At the high ratios the parallel nozzle expansion is always complete and the curve shows only a very slight drop below the maximum. The "envelope" curve shown in fig.9 might, therefore, be taken as representing, generally, the effect of speed when the jet concerned is fully expanded in the nozzle. This would mean that, for such "proper" jets, the blading coefficient

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* "A Study of Nozzle Losses" - Section D.

would appear to be a maximum at or near the acoustic velocity; fall-ing very slightly for lower speeds and more rapidly for higher speeds. The drop in the latter case is certain - as it follows from the general result stated in (iii) above - but whether it is a true effect of speed, or a result of the divergence entailed in the generation of that speed it is impossible to say exactly.

Consider the form of the divergent nozzle curve for ratios above the best. It is seen to show a definite fall, and we know that, in these ratios, the jet suffers compression towards the nozzle outlet. It is, therefore, supplied to the blading while diverging, and just after compression. It seems reasonable, therefore, to believe that the low values recorded in this range are due to the poorness of the jet itself. Notice that, if the variation of α were entirely dependent on jet speed, this part of the curve considered should follow out the line of the "envelope".

Again, consider those parts of both curves for pressure ratios below the best in each. The known conditions here are that the jets suffer from overexpansion, and the results deduced indicate a lowering of the coefficient; and, again, we reach the conclusion that the jet condition is the main factor at work is determining the variation of α .

In these various cases of lowering of the coefficient it should be observed that the reductions are clearly demanded by the actual changes of outflow angle as experimentally determined, although the fall in values shown in fig. 9 may not be accurate in view of the vagueness of several factors required in calculation. The sharp minimum and the curve form for the divergent nozzle, in fig. 7, should, in particular, be noticed as definitely substantiating the emphasised effects.

In view of all the facts it must be admitted that the hypothesis regarding the influence of the "quality of the jet" is well maintained, and it appears highly probable that the coefficient for any given blading varies mainly on account of this. It should be understood that the blade coefficient, as herein examined, is a figure only applicable to the actions occurring strictly within the passage; gap and edge effects are entirely outwith its scope, and have to be otherwise covered.

While the explanation advanced may serve for the variations of the coefficient below the maximum value there is, of course, always the question as to the cause of the main fall below unity. This investigation is not really concerned with this particular point as it rests entirely on the evidence of change of value; but one aspect might be remarked upon. It would naturally be supposed that friction was the main cause, the more so that the parallel nozzle curve form to the acoustic velocity condition is well in keeping with the requirements of friction, while the fall at the higher speeds - which is not in harmony with a frictional effect - may, as has been noted, be due to divergence. It must be confessed however, that if the whole of the range between the maximum, as recorded in fig. 9, and unity is due to friction, the coefficient involved is exceptionally high. Thus we could write:-

$$\frac{w_1^2}{2g}(1-b^2) = c \cdot \frac{p}{A} \cdot \frac{w_1^2}{2g} \cdot L \qquad (7)$$

as a fair approximation for the frictional loss of energy per lb. fluid in the blade passage, of length L, and for which A/p is a representative hydraulic mean depth; c is, of course, a friction coefficient of the usual type. Clearly:-

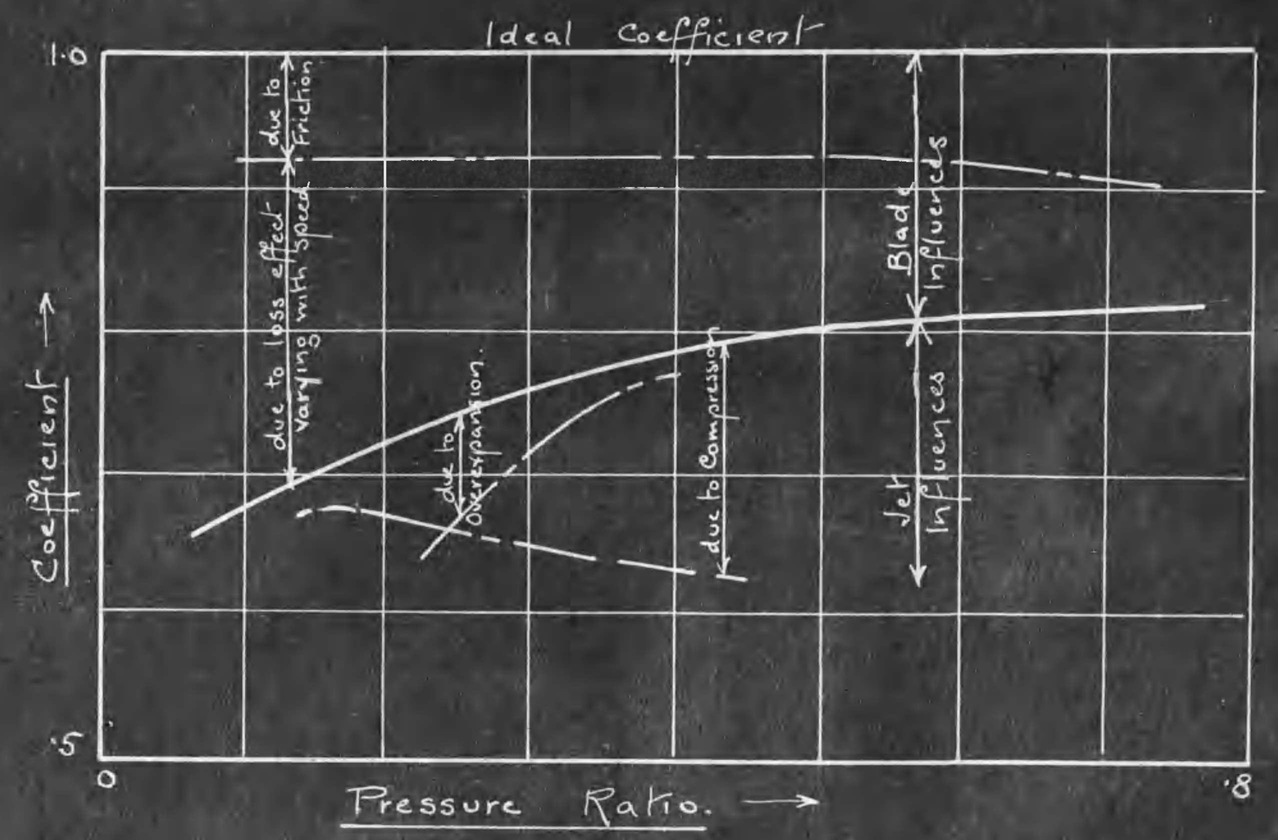
$$c = \frac{(1-b^2)}{L \cdot p/A}$$

and, allowing reasonable values as defined by the blading sizes and results, there follows:-

$$c = .035.$$

Now this coefficient would have been expected to appear as something of the order of .005, so that the apparent loss effect in the curved blade passage - and resulting from the fluid flow in that passage - is several times greater than would be demanded by a frictional effect of the usual type. It is obviously improbable

On Jet Action in Blading — Fig. 10.



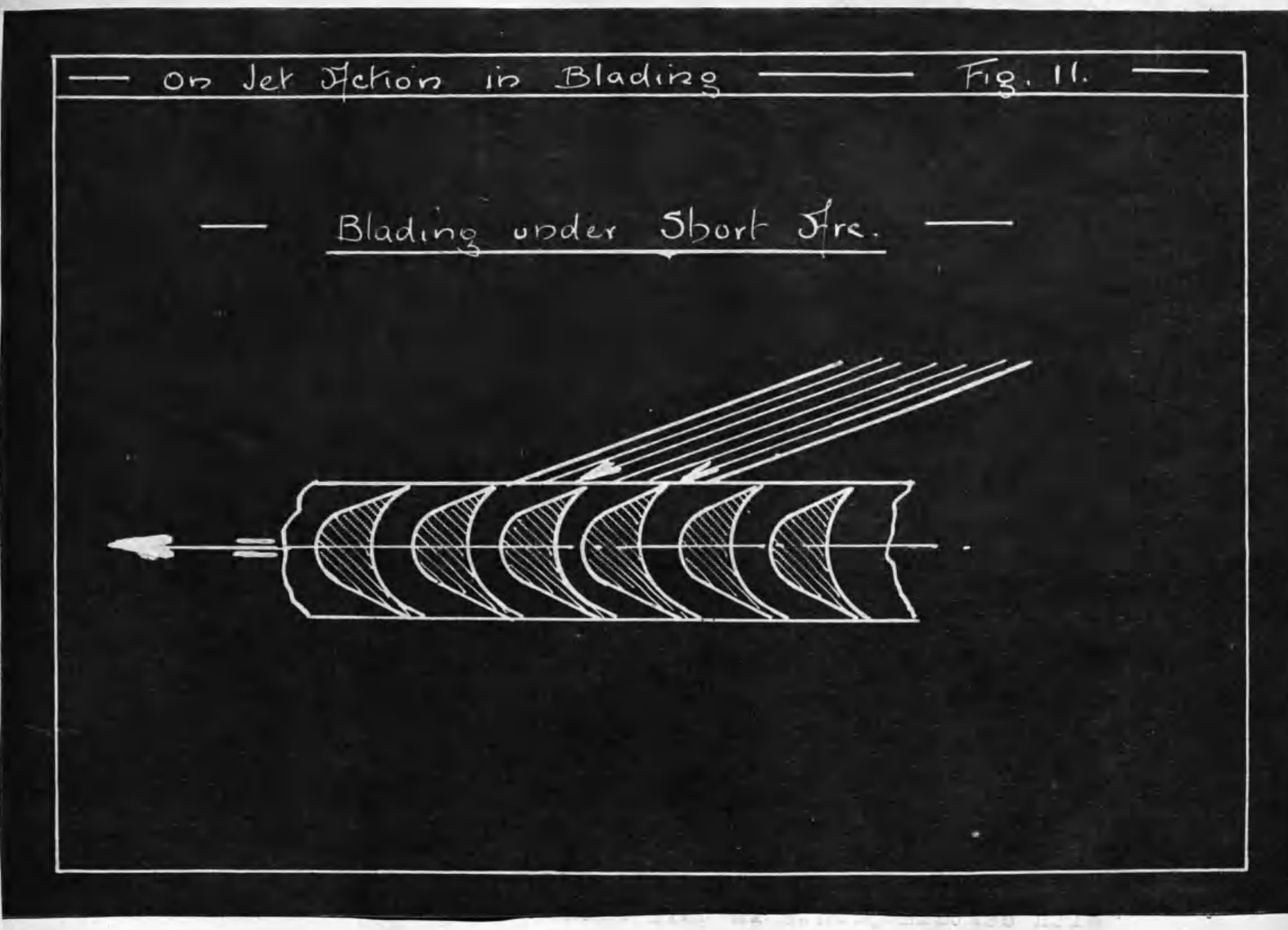
Factors Influencing the Blade Coefficient.

that friction is a true explanation; and, although the absolute value of x which has been employed in this demonstration cannot be taken as exact, the value that would be necessary to make friction an adequate explanation is quite impossible. We reach the conclusion therefore, that besides friction - which must exist - there is another loss which is characteristic of blade passages. This loss might, for instance, be dependent on speed in such a way as to make the fall of the coefficient at the high speeds a rational effect of the fluid flow, rather than of the jet condition of divergence, as has been suggested.

Taking all the points that have been discussed together, we might conceive the general factors determining a true blade coefficient as represented in the scheme shown by fig. 10. This diagram is self explanatory and embodies the main deductions. We appear to be guided to a discrimination between "blade passage effects" and "jet quality effects"; the former being natural to the blading, and the latter showing the influence of the method of jet development.

The matter that has been dealt with refers only to the stationary blade but the results are rather more general than could have been expected, and appear to show that such experiments are of considerable value. Although it is not intended to include herein any tests on the moving blade, it may be permitted to deal briefly with certain points in this rather difficult problem, that come naturally within our present point of view.

The Problem of the Moving Blade: It is well known that small turbines are much less efficient than large ones; and the remark is frequently made that results from the former are in no sense applicable to the latter. This is true so long as by results we mean such overall figures as efficiency, but it certainly cannot be correct if the results are of a fundamental type. Clearly detail experimental work carried out - as usually it must be - on small


On Jet Action in Blading
Fig. 11.
Blading under Short Arc.

plants can only be of use if the differences between the conditions in the two cases are fully appreciated.

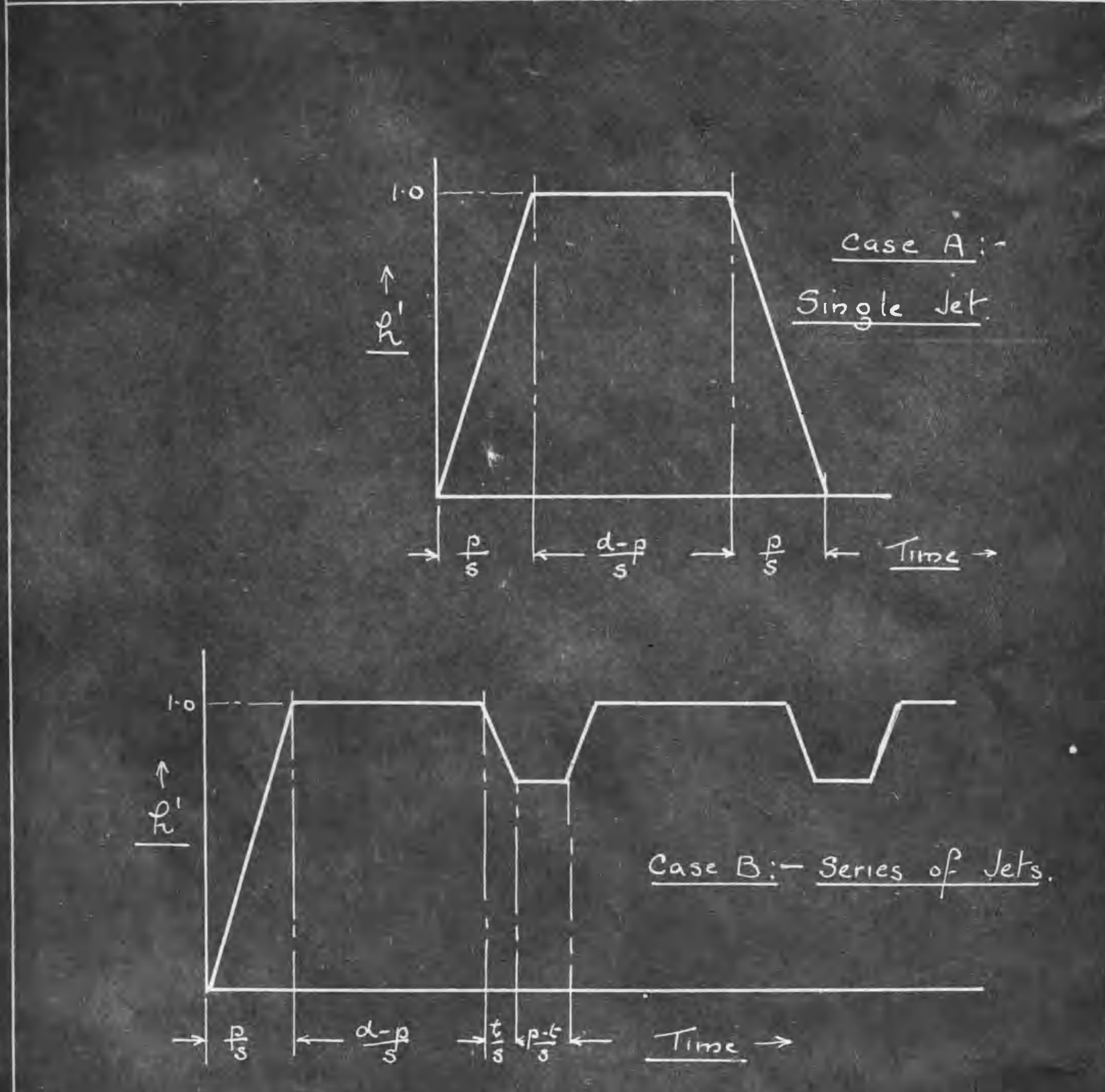
Neglecting those differences that can readily be covered, such as bearing friction, windage, etc., we have as the main features distinguishing the small plant from the large, (a), finer dimen--sions of nozzles and blades, (b), nozzle forms that are generally not so correct for their duty, and (c), short admission arcs. The difference in blade speed/steam speed ratio is not fundamental, and is a point that should be adequately covered by any theory.

The dimensional differences cease to be difficulties when the loss effects are fully known; and the smaller size of turbine should really facilitate their determination. That the nozzles are usually of divergent, or underexpanding, forms obviously involves imperfect actions in the blading, of the kind already discussed. With short arcs, however, we reach a point not generally taken into account, but one that is clearly of importance in any experimental turbine which is to be made the medium of fundamental investigation; and particularly so when the admission arc contains one nozzle only. We may, therefore, discuss the general features of the action in blading under very short arcs in order, mainly, to establish a basis method equally applicable to all cases.

With the blades in motion over a short arc it is obvious that there is a variable action in the passage from the instant that steam first spills over the blade edge, through the period of full flow in the passage, to the final instant at which the flow ceases. The time of action will depend on the blade speed, pitch and jet width. The greater speeds will mainly affect the time of action, without direct influence on the mode.

Thus fig. 11 shows a short jet operating on three blade passages; one filling, one full, and one emptying. Now suppose this jet is from a single nozzle passing M lb. steam per sec.; that the height of the jet is l, and its width parallel to the line of blading is d. Then the full blade passage is receiving:-

On Jet Action in Blading — Fig. 12.



Variation of Flow into a Blade Passage.

On Jet Action in Blading. (contd.)

$$\frac{p}{d} \cdot M \ldots \text{lb. per sec.}$$

if l is less than l_1, the blade inlet height. If l is greater than l_1, it would be necessary to write:-

$$\frac{p}{d} \cdot \frac{l_1}{l} \cdot M \ldots \text{lb. per sec.}$$

We may, however, assume that the first form is sufficiently general. At any instant of time, however, within the period in which the blade is under the jet action, the quantity entering will be at the rate:-

$$k' \cdot \frac{p}{d} \cdot M \ldots \text{lb. per sec.}$$

with k' rapidly increasing to unity at first and falling rapidly from unity at the end; but remaining at its maximum so long as the full passage is within the jet limits. If s is the blade speed then:-

time of filling	=	p/s sec.
" filled	=	$(d-p)/s$ "
" emptying	=	p/s "

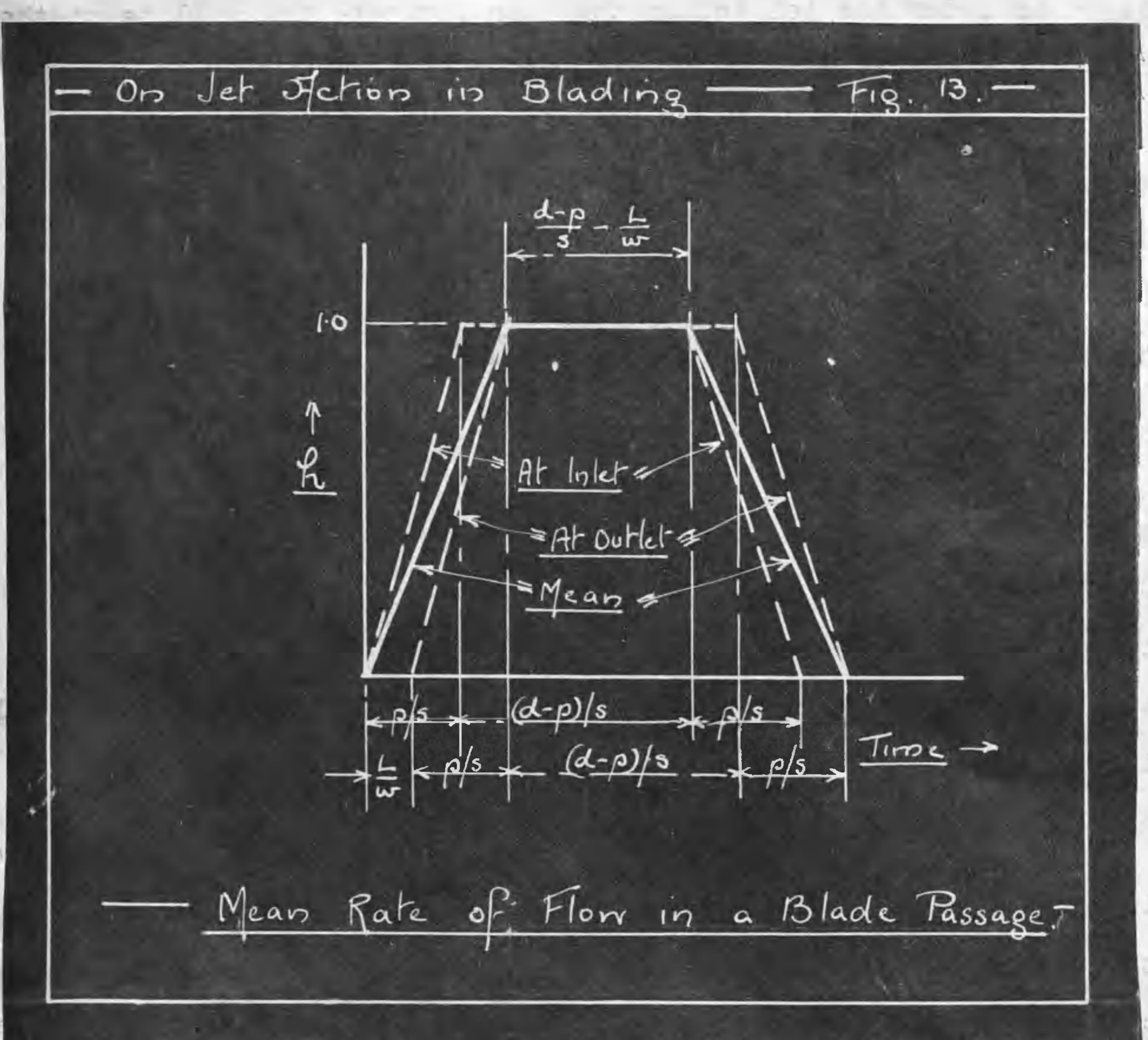
so that the curve for k' is somewhat as indicated in fig. 12A. Obviously for longer total arcs, divided into separate sections by the nozzle partition plates or bars, the curve will show irregularities corresponding thereto, and as marked in fig. 12B. These should in most cases, be small in relation to the end effects.

Suppose that the length of flow through the blade passage is L, and that relative speed of steam in blade is w, then the time of passage through the blade is L/w secs.; and it is clear that the flow conditions at the outlet blade area are similar to those at the inlet, but lag behind by a time period L/w secs. If we include this fact in the general representation, fig. 12 changes to fig. 13; and we may take it that the mean rate of flow in the blade at any instant is given by:-

$$k \cdot \frac{p}{d} \cdot M \ldots \text{lb. per sec.}$$

where k is shown by the mean curve marked. It will be noticed also that the period of time during which a steady flow exists in the blade is reduced to:-

$$\frac{d-p}{s} - \frac{L}{w} = \frac{1}{s}\left\{d - p - \frac{Lb}{b_1}\right\}.$$


— On Jet Action in Blading —— Fig. 13. —
$\frac{d-p}{s} - \frac{L}{w}$
1·0
h
At Inlet
At Outlet
Mean
p/s
(d-p)/s
p/s
L/w
p/s
(d-p)/s
p/s
Time →
Mean Rate of Flow in a Blade Passage.

On Jet Action in Blading. (contd.)

where:- $b = s/u_1$
$b_1 = w/u_1$
u_1 = jet speed.

In contradistinction to this, we would have the steady rate over the full length for long jet arcs having a constant delivery rate per unit length.

It will be clear that, except during the period expressed above, the blade passage is not dealing with a full jet and the action is liable to serious faults since, even allowing the jet to be perfect as supplied, it is really impossible to believe that a partly filled inlet and a varying flow rate can give the best efficiency. It will be noticed that we are emphasising here a jet condition, created within the blade passage, which is akin to that already shown to have important influences when present in the delivered jet, viz., freedom of dispersion.

On reference to the usual kind of velocity triangles - for instance, see fig. 1 - it is clear that, for the force acting on the blade at any instant we have:-

$$h \cdot \frac{p}{d} \cdot \frac{M}{g} \cdot w_1' \left(\cos\theta' + x' \cdot \cos\beta' \right) \quad . . . \text{lb.}$$

where the accented symbols denote instantaneous values. It is easily seen that the total distance travelled while steam is actually passing is:-

$$d + p + Lb/b_1$$

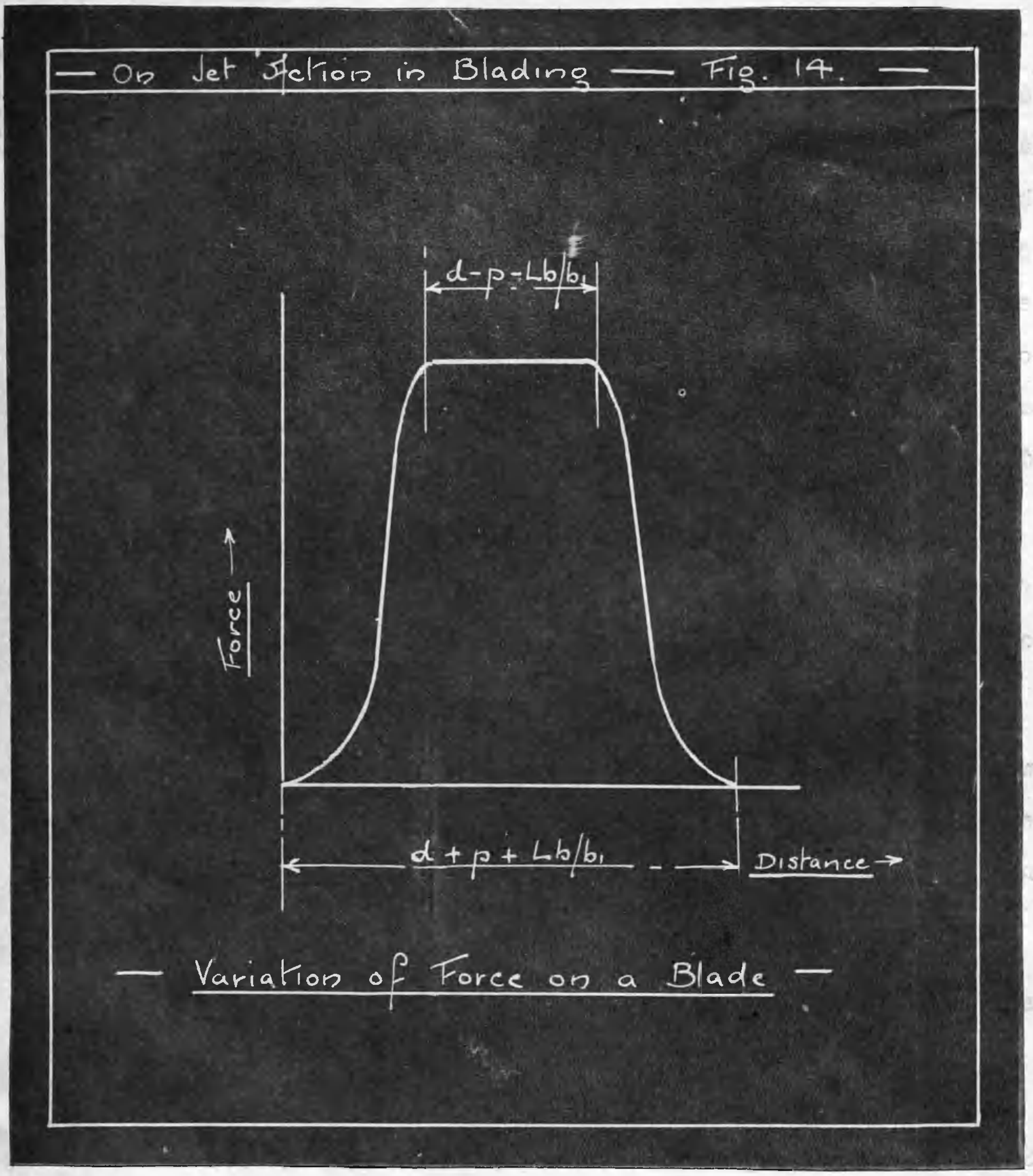
so that the work done on one blade is:-

$$\frac{p}{d} \cdot \frac{M}{g} \int_0^{d+p+Lb/b_1} h \cdot w_1' \left(\cos\theta' + x' \cdot \cos\beta' \right) dc \quad . . . \text{ft.-lb.}$$

Since the total number of blades operated on per sec. is s/p, it follows that the work done per sec. is:-

$$W = \frac{M s}{g} \cdot \frac{1}{d} \int_0^{d+p+Lb/b_1} h \, w_1' \left(\cos\theta' + x' \cdot \cos\beta' \right) dc \quad \text{————} \quad (8).$$

Obviously the condition in which there is a steady rate of flow must be made the standard, as it is only then that the probable x and β values can be determined with any certainty or,



— On Jet Action in Blading — Fig. 14. —

— Variation of Force on a Blade —

in general, are of any value. Also, it is only in this condition that we have similarity for all lengths of arc. Hence we may write

$$\int_0^{d+p+Lb/b_1} h \cdot w_1'\left(\cos\theta' + x'\cdot\cos\beta'\right) dx \;=\; w_1 \cdot f\left(\cos\theta + x\cdot\cos\beta\right)\left(d+p+Lb/b_1\right) \quad \text{(9)}$$

where θ, x and β are for the filled passage. The coefficient f may then be described as the "form factor" of the force curve. It is naturally low for relatively sharp types of curves, such as are necessarily given by low values of the ratio:-

$$\left(d - p - Lb/b_1\right) \Big/ \left(d + p + Lb/b_1\right).$$

and it becomes unity for continuous admission when the curve form is rectangular. This "form factor" therefore relates the short and long arcs and, consequently, is essential in the general problem. It has to cover several variations since it is certain that, with h changing, x, β, w and θ will all alter, to a greater or less extent; in fact, while fig. 13 may fairly represent the variation of h, the probable changes in x alone would result in a force curve of the form shown by fig. 14. The point to be clearly recognised is that, with the varying flow conditions in the blade passage, there is an undesirable fall away from the ideal conditions and this must seriously reflect on the very short arc arrangement. The disadvantage may, however, quickly disappear as the arc length is increased, but when tests are carried out with the shortest single nozzle arcs it would seem quite probable that changes of out-let angle under running conditions may be influenced by the unavoid-able disturbances of flow. It has previously been explained that, under stationary conditions, the outlet vane can record real var-iations of flow direction corresponding to different jet-blade-vane configurations, but it does not possess quite the same facility with the wheel running; as obviously it must be within a certain definite length and this length narrows somewhat with blade speed.

Suppose that the usual theory were applied to tests giving the power; then for the velocity of whirl we have:-

$$u_w = w_1\left(\cos\theta + x\cdot\cos\beta\right) = W \Big/ \frac{M\cdot s}{g} \quad \text{(10)}$$

but from (9) we see that the fuller treatment gives:-

$$u_w = w_1(\cos\theta + x\cdot\cos\beta) = W/\left\{\frac{M\cdot s}{g}\cdot\frac{f}{d}(d+p+Lb/b_1)\right\} \quad \text{---} (11)$$

The two methods will not, in general, lead to the same values for this essential quantity. The first case, expressed by (10), gives an average whirl velocity on the assumption of steady flow at all times; the second case, according to (11), gives the actual during the real period of steady flow. Clearly for the longer arcs the value of:-

$$\frac{f}{d}(d+p+Lb/b_1).$$

approaches unity, and there is no difference between the two cases; but for single nozzles the difference must be appreciable.

Again, x and β are mutually dependent and any occurr--ence which tends to reduce x will increase β. Actually, under running conditions, it is the absolute outflow angle ϕ which is read, but this should follow from x and β directly. Since higher blade speeds mean more rapidly fluctuating conditions in the blade passage, it does not seem improbable that x will fall some--what as the speed is increased, and as, beyond a certain point, the outflow angles are very sensitive to change of x, it would follow that β may attain quite high values at the full speeds.

It should be clear, then, that if x and β may, in general, be considered determined by stationary tests - with perhaps slight modification to meet the measured ϕ value in actual running - it follows that the factor f is reasonably decided by power tests of the usual type. This factor permits the elimination of the detrimental feature of the small plant tests and so rationalises blade coefficients; since, without it, the small plant would show exceedingly low values, that would have no obvious relation to those applicable in the more usual cases.

Conclusion: The stationary blading tests have shown quite clearly that the outflow angle from a given blade is not really fixed within narrow limits by the geometrical form, as is commonly assumed. Consideration of the actual data demonstrates that, under constant jet conditions, there are changes of angle that really reflect the variable action in the blading, due to the employment of very short admission arcs: and such changes are, therefore, the result of the testing arrangement and, as it were, adventitious. The variations with different jet conditions are, however, quite fundamental, and wide limits of angle appear possible. It has been advanced that these changes are due to the quality of jet which modifies the blade coefficient, with consequent influence on the outlet angle. It appears, both from the tests and from geometrical considerations, that small angle variations below the standard show fair improvements in the coefficient but that above the standard the angle is very sensitive to change of the coefficient. We are left with a general impression that the blade coefficient is influenced both by the passage and by the nature of the jet - as, for instance, shown by the sketch in fig. 10 - but the general point should not be missed that this important coefficient is easily affected by any lack of steadiness in the action, with a corresponding influence on the angle of outflow.

The angle variations, primarily due to the short arc of admission, suggest that the running action in such cases is not quite so simple as the ordinary theory supposes. The treatment given of this matter is mainly for the purpose of permitting the deduction of general results from small experimental plants using short arcs. It is clear that unless this is done there can be no real harmony in blade coefficients deduced from power tests. It is suggested that short arc effects should be covered by means of a factor which allows for the influence of the variable force curve on a blade, and automatically becomes unity with large arcs of admission. Strictly speaking it can never really be unity as long arcs are really composed of a close series of single nozzles, and the divis--ions must be responsible for a certain irregularity.

It would appear that accurate experimental work on stationary blading, with measurement of the outflow angle as a main method of search, will throw considerable light on the value and variation of the true blade coefficient; and it is only this co--efficient which has general value. It is extremely probable that these variations of outlet angle from the blade at rest will serve to explain the inconsistences in the absolute outflow direction from the moving blade, which generally appear if a constant blade angle is used for all conditions. But the method of checking recorded outflow angles against the theory is faulty because, with short arcs, the usual theory is not really complete, and it would appear necessary for general purposes of examination to envisage a power expression of the following type:-

$$W = \frac{M}{g} \cdot b \cdot b_1 \cdot u_1^2 \cdot \frac{f}{d}\left(d + p + Lb/b_1\right)\left(\cos\theta + x\cos\beta\right) \qquad (12).$$

in which:-

- W = ft. lb./sec.
- M = flow rate lb./sec.
- b = blade speed/jet speed = s/u_1
- b_1 = inlet speed to blade/jet speed = w_1/u_1
- u_1 = jet speed - ft. per sec.
- d = circumferential length of jet - ins.
- p = mean width of blade passage - ins.
- L = mean length of flow in blade passage - ins
- θ = angle of inflow of jet
- β = angle of outflow of jet
- x = blade velocity coefficient
- f = form factor of the curve of force.

The value of this lies in the fact that it relates the work done to the actions during the period of steady flow in the blade passage. In actual application the data from stationary and running tests would be required; and while the procedure is by no means simple and straightforward, it must be conceded that the use of the vane method of observation has led to useful ideas and a slight simplification of a rather difficult subject.

PAPERS ON EXPERIMENTAL SUBJECTS.

NO. 3.

ON

TURBINE WHEEL

FRICTION

ON TURBINE WHEEL FRICTION

INTRODUCTORY:

In 1912 the Author conducted a series of tests on the steam frictional losses of a three row velocity compounded impulse wheel. In a paper ⊕ dealing with these tests the results were analysed; and an attempt was made to establish an expression generally applicable to turbine wheels. To effect this latter, use was also made of formulae given by Stodola and by Lasche. The final equation form was held to represent, as far as was possible at that time, the best of the available data regarding the friction of full scale bladed wheels; and it has been very widely used since.

A short time after the publication of the paper referred to an important theoretical discussion of the subject of wheel friction appeared. This treatment was given by Buckingham* who applied the dimensional theory of fluid resistance to all the available experimental data in an attempt to establish some degree of order therein. The general incompleteness of the experiments and the many incomprehensible variations in the different results told rather heavily against the effort; and, while Buckingham succeeded in smoothing out several apparent discrepancies, his main achievement lies in the establishment of a rational outlook on the subject. As a result of his study he gives - tentatively - an expression for single row wheels which could, perhaps, be accepted as rational in its main form but which is hardly quantitatively correct. The fundamental theory as treated by Buckingham and the elements of the equation finally developed by him are, however, sufficient to show that the Author's original equation is slightly irrational in form; and correction in this respect would seem desirable if generality is aimed at.

While the necessity to revise the original equation, in order to bring it more into line with a sound theoretical outlook,

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⊕ "The Steam Friction of Turbine Wheels" - Proc. R.T.C. Scientific Soc., Dec. 7, 1912. or "Engineering". Aug. 22, 1913.

* " The Windage Resistance of Steam Turbine Wheels" - Bulletin of the Bureau of Standards. July 25, 1913 or "Engineering" Mar 13, 1914.

On Turbine Wheel Friction: (Continued)

has been clearly recognised the data have been too limited to make a re-examination of the matter of much advantage hitherto, as there are differences of detail as well as of form. The latter is admitted and could be rectified in a fairly simple fashion; but the former raises questions of more importance, in a practical sense, and necessitates discussion.

Recently some tests have been carried out on a single row wheel under conditions similar to those holding for the three row wheel of the first tests; and these additional data with the facts and peculiarities embodied therein seem to indicate that a new attack on the evidence might be profitably undertaken, with a view to carrying this confused subject a stage further towards clearness.

These new tests have been carried out on the same turbine as before, viz,, an impulse turbine in the mechanical engineering laboratory of the Royal Technical College. The change from a three row to a one row wheel is rendered possible by the special construction of the turbine rotor in which separate discs carry the different blade rows - as shown in fig. 1. The experiments were an essential preliminary of a different kind of investigation but their bearing on the three row wheel friction results and on theory give them a value apart from this.

The differences between the two sets of results were, on first sight, very disconcerting; but it was thought that both series were fairly sound and, therefore, that the differences were genuine. That being so an attempt to correlate the results seemed called for; and it mst be admitted that the elimination of the discrepancies will be a potent argument on behalf of any method of examination that effects it.

The purpose of the present paper is, then, to show how the two different sets of results may be brought into line with each other; and this without traversing the essential theory of the subject. It is further desired to discuss the details of the general equation in the light of what is established by this analysis in the hope of developing useful, rational, and satisfactory rules for the calculation of steam frictional losses.

Before discussing the later figures and analyses it might be best, for the sake of clearness and completeness, to recapitulate, the essential points of Buckingham's work and the main features of the Author's earlier paper on this subject.

OUTLINE OF BUCKINGHAM'S TREATMENT:

By ordinary dimensional methods of examination it is easily shown that the power dissipated by the friction of a rotating wheel may be written, generally, as:-

$$N = n^3 \cdot d^5 \cdot \rho \cdot f\left(\frac{\mu}{\rho \cdot n \cdot d^2}\right)$$

in which f represents an unknown function of the given argument, N is horsepower; n is revs. per min.; d is diameter; ρ is the density of the fluid; and μ the coefficient of viscosity.

Obviously this may be written:-

$$N = n^3 \cdot d^5 \cdot \rho \cdot \sum A \left(\frac{\mu}{\rho \cdot n \cdot d^2}\right)^{\delta} \quad \text{------- (1).}$$

where δ is an unknown exponent and A is a constant.

If rotation is at such a low speed that pure stream line motion exists then μ must appear as a first power, i.e., $\delta = 1$, and:-

$$N = A \cdot n^2 \cdot d^3 \cdot \mu \quad \text{------------ (2).}$$

If, however, in the opposite extreme turbulence is established of such a nature that the viscosity of the fluid is altogether negligible in its effect, then $\delta = 0$, and:-

$$N = A \cdot n^3 \cdot d^5 \cdot \rho \quad \text{------------ (3).}$$

Most actual cases will lie nearer (3) than (2) in those regions, at least, where frictional losses are of any practical

On Turbine Wheel Friction: (Continued)

importance; and hence the general equation will be as given by (1) with δ a comparatively small number.

If in the various terms under the summation sign in (1) the index δ has the same value, then:-

$$N = A \cdot n^{3-\delta} \cdot d^{5-2\delta} \cdot \rho^{1-\delta} \cdot \mu^{\delta} \text{ ------------ } (4).$$

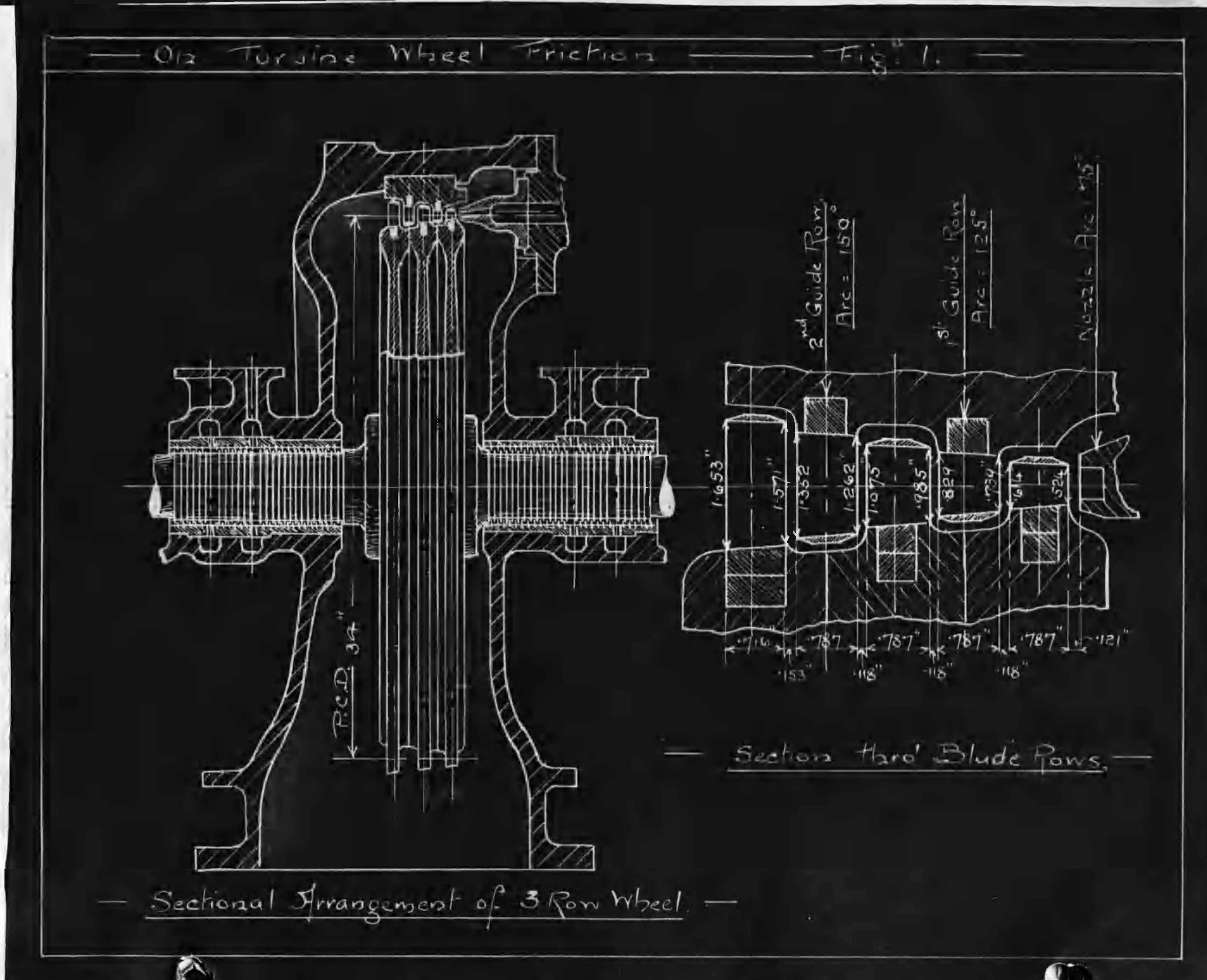
and a study of one variable with the others constant should serve for the determination of δ .

If on the other hand δ is small enough to be neglected then (3) applies and the purpose of experimental data is to establish the form of A .

Now A is a mere number but it will depend on a series of length ratios for any particular case; it will, however, only be possible to take cognisance of the most important of these. Obviously in the specific instance of a bladed wheel the outstanding ratio is l/d, where l is the blade height and d the wheel diameter. The clearance ratio should enter into account as next in importance but the general indefiniteness of the figure in experimental records makes it necessary to ignore this influence. The general effects of clearance are understood from Stodola's experiments but their definition is not sufficiently acute to allow of inclusion of the ratio in expressions for power.

It folows, therefore, that the blade height-diameter ratio is practically the only definite value that can enter into the form of A . There are, of course. other ratios of shape besides those mentioned but they are less important than the clearance ratio and still less definite in their influence. If, then, only the l/d value is used to express A the equation for power can only be theoretically of the correct form for wheels running free or with ample clearances.

Buckingham's study of the exponents of (4) and of the reasonable representation of A is very interesting; and exhaustive so far as the data at his command allowed. It is unnecessary


Oil Turbine Wheel Friction
Fig. 1.
P.C.D. 34"
Sectional Arrangement of 3 Row Wheel.
2nd Guide Row, Arc = 150°
1st Guide Row Arc = 125°
Nozzle Arc = 75°
1·653"
1·571"
1·352
1·262
1·075
·985
·829
·734
·614
·524
·116
·787"
·153
·118"
·121"
Section thro' Blade Rows.

here to consider the details of that study but a remark on the conclusions reached is desirable.

It is found that (4) is competent to express all the reliable data on the subject and that with $\delta = 0{\cdot}1$ probably the best approximation consistent with that data is obtained. The quantity A - called the "shape coefficient" - may be written:-

$$A = a + b\left(\frac{l}{d}\right)^2$$

for wheels run in the open or with fairly large clearances.

This gives a presumably general form:-

$$N = n^{3-\delta} \cdot d^{5-2\delta} \cdot \rho^{1-\delta} \cdot \mu^{\delta} \left\{ a + b\left(\frac{l}{d}\right)^2 \right\} \quad \text{------} \quad (5).$$

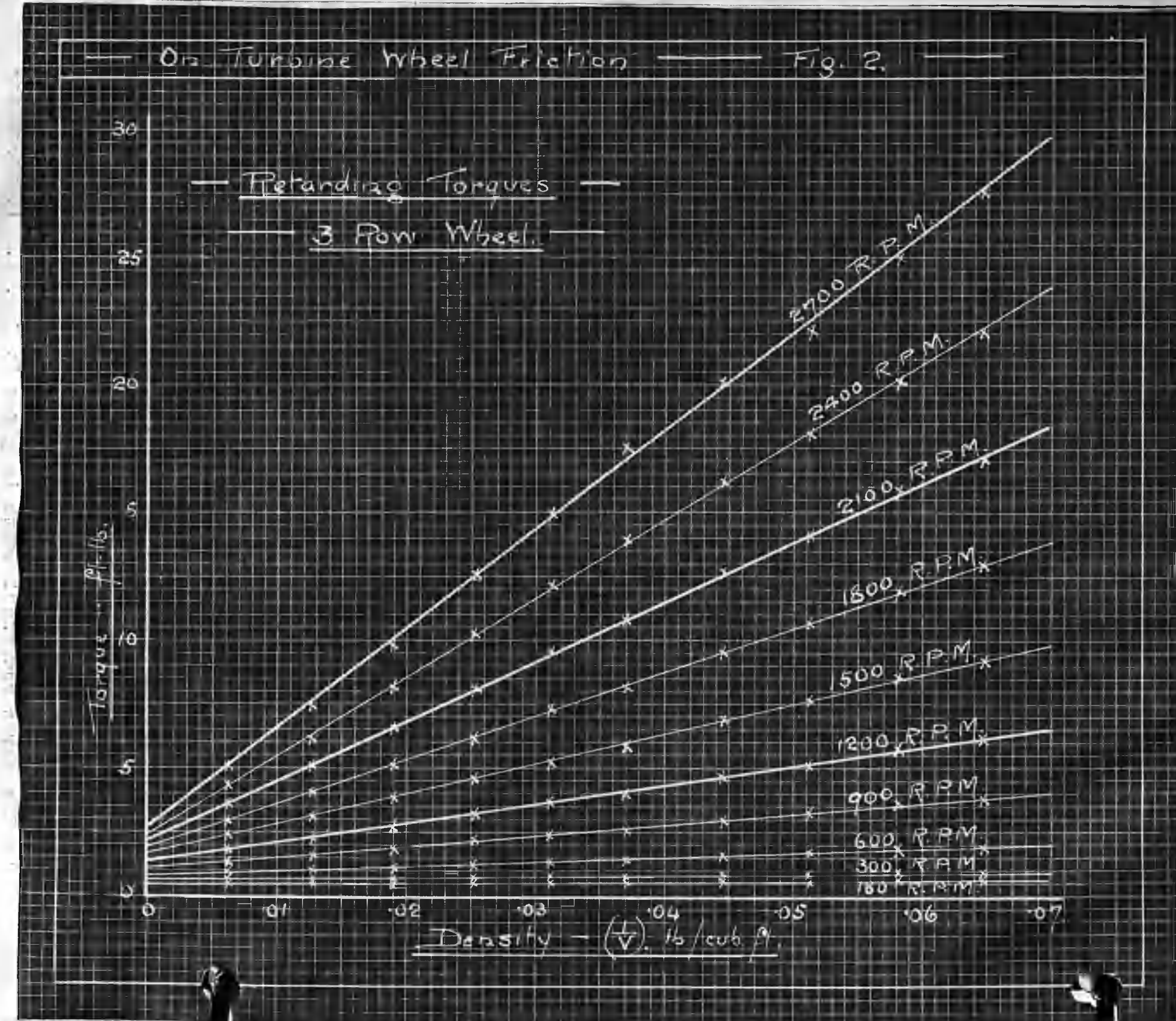
but, neglecting the small value of δ and working principally on Stodola's figures, the following simple equation with numerical coefficients is obtained as suitable for calculation of a probable maximum:-

$$N = 10^{-16} \cdot n^3 \cdot d^5 \cdot \rho \left\{ 1 + 590\left(\frac{l}{d}\right)^2 \right\} \quad \text{------} \quad (6).$$

Equations (4) and (6) may be taken respectively as representing, according to Buckingham's treatment, the indication of theory and the rough average of experiment. If the form given for the shape coefficient A is reasonably correct then (5) may be considered a fairly definite theoretical equation; and all experimental results should be reducible thereto. The proposed equation (6) is, however, open to criticism as the balance of terms is incorrect; and this invalidates the exponent of the blade height-diameter ratio. This matter will be discussed later.

EPITOME OF THE 1912 PAPER:

A sectional sketch of the rotor and casing as arranged for the three row wheel tests is given in fig. 1. It shows the usual blade grouping for partial admission. The axial blade clearances between moving and fixed rows are normal but the casing-wheel spaces are large as compared with corresponding forms in


On Turbine Wheel Friction
Fig. 2.
Retarding Torques
3 Row Wheel.
Torque - ft-lbs.
30
25
20
15
10
5
0
Density - (1/V). lbs/cub. ft.
0
.01
.02
.03
.04
.05
.06
.07
2700 R.P.M.
2400 R.P.M.
2100 R.P.M.
1800 R.P.M.
1500 R.P.M.
1200 R.P.M.
900 R.P.M.
600 R.P.M.
300 R.P.M.
180 R.P.M.

multistage turbines. Considering the moderate arcs occupied by the fixed blades, the large exhaust space on the bottom and the full clearance around the wheel it could be taken that the case is one of "ample" clearances. (In this connection it should also be understood that to get a noticeable effect from fine clearances it is necessary to shroud both sides of a blade row closely; in practice, generally, the nozzle ring may act somewhat as a cover on one side but the other side is fairly free).

The test procedure was very simple. The turbine was run up to a high speed; steam was then shut off quickly and records of speed and casing conditions were taken at short intervals while the rotor slowed down to rest under the combined action of steam and bearing friction. By varying the casing conditions from high vacua to pressures above atmosphere speed-time curves for a fair range of steam densities were obtained. Differentiation of these curves and the introduction of the moment of inertia of the rotor gave the torque-speed-density series reproduced in fig. 2.

The experimental points marked in fig. 2 make it clear that for any given speed the resisting torque is nearly a linear function of the density. Examination of the lines drawn for the higher speed values will show that a very slight curvature might be allowed, but so long as this series is alone available the straight lines may be considered a sufficient approximation.

The torque values at zero density must represent bearing friction and these values permit the necessary elimination of this effect. The corresponding bearing friction coefficients are shown in fig. 5, and the agreement with Stribeck's coefficient curves for similar bearings show that this process is quite reasonable.

The steam friction torques are then represented by a radiating series of straight lines; and logarithmic plotting of torque against speed defines the important speed index.

On Turbine Wheel Friction: (Continued)

The plot gave:-

$$N \propto n^{3.64} \cdot \rho .$$

but by reducing the index to 3.0 for simplicity, taking an average constant, and introducing blade speed in place of revs. per min. the following equation was obtained:-

$$N = 3.016 \left(\frac{S}{100}\right)^3 \left(\frac{1}{V}\right) \text{ - - - - - - - - - - - } (7).$$

in which:-

N = horse power lost in steam friction.
S = mean blade speed - ft. per sec.
V = specific volume of steam - cub. ft. per lb.

This equation (7) gives the essential result of the tests. It should be noted that the experimental records are regular and consistent throughout and, excepting the error deliberately introduced by the reduction of the speed index, there is little doubt that the given expression covers the facts with reasonable accuracy.

By consideration of Stodola's and Lasche's equations and values the general equation - here somewhat altered in form - was established:-

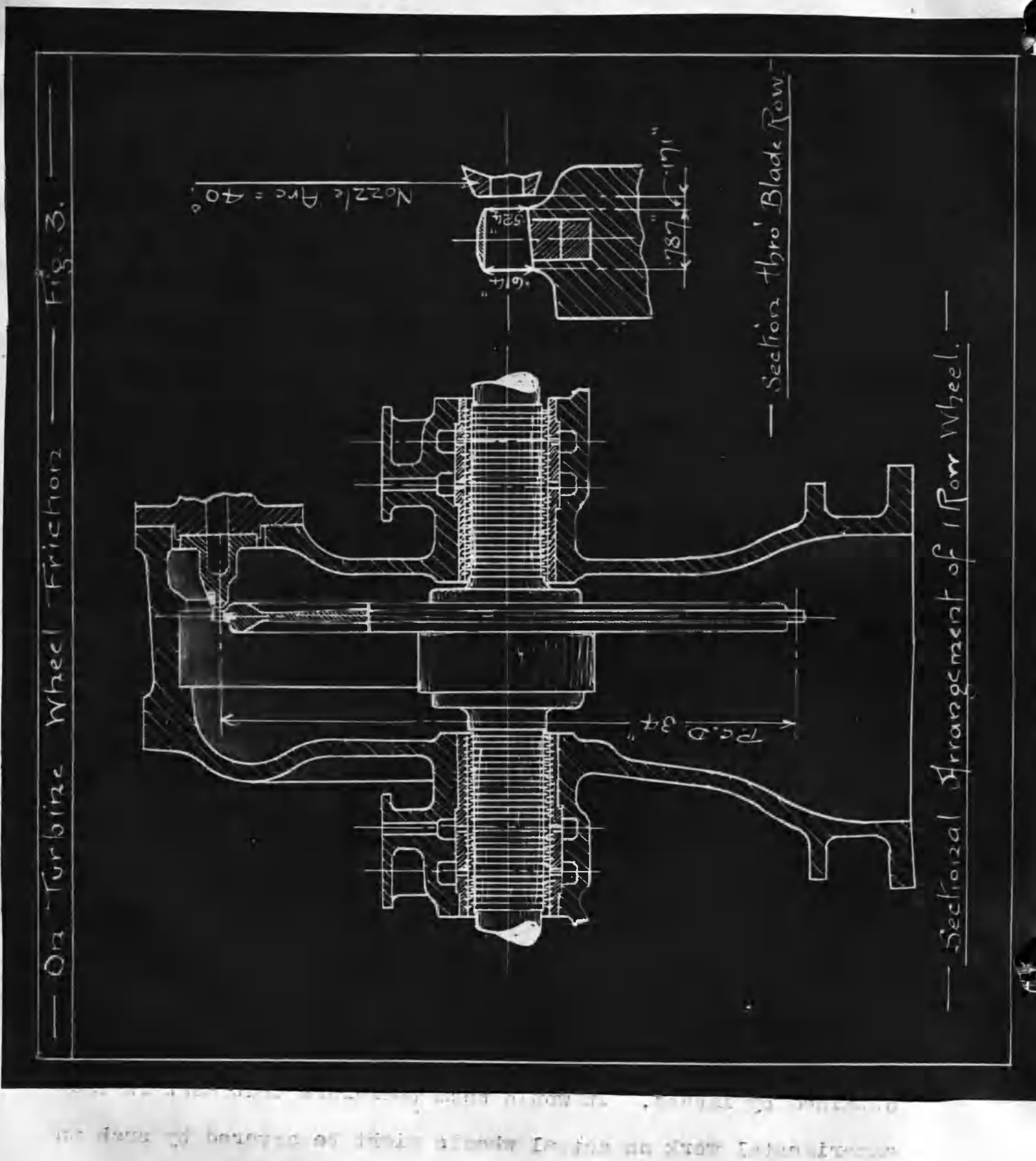
$$N = \left(\frac{S}{100}\right)^3 \left(\frac{d}{10}\right)^2 \left(\frac{1}{V}\right) \left\{ \cdot 042 + 3.82\, m \left(\frac{l^{1.5}}{d}\right) \right\} \text{ - - - - - - - } (8)$$

in which:-

l = mean blade height - inches.
d = mean blade diameter - inches.
m = 1.0 for single row wheel.
= 1.8 for three " "

This equation reduces nearly to (7) when proper values of l, m and d are inserted. It follows the form given by Stodola, and the m values meet, as nearly as possible, the results obtained by Lasche. It would seem, therefore, that most of the experimental work on actual wheels might be covered by such an expression.

If however comparison is made with the simplest form of


On Turbine Wheel Friction — Fig. 3.
Nozzle Arc = 40°
.524"
.614"
.787"
.171"
Section thro' Blade Row.
P.C.D. 34"
Sectional Arrangement of 1 Row Wheel.

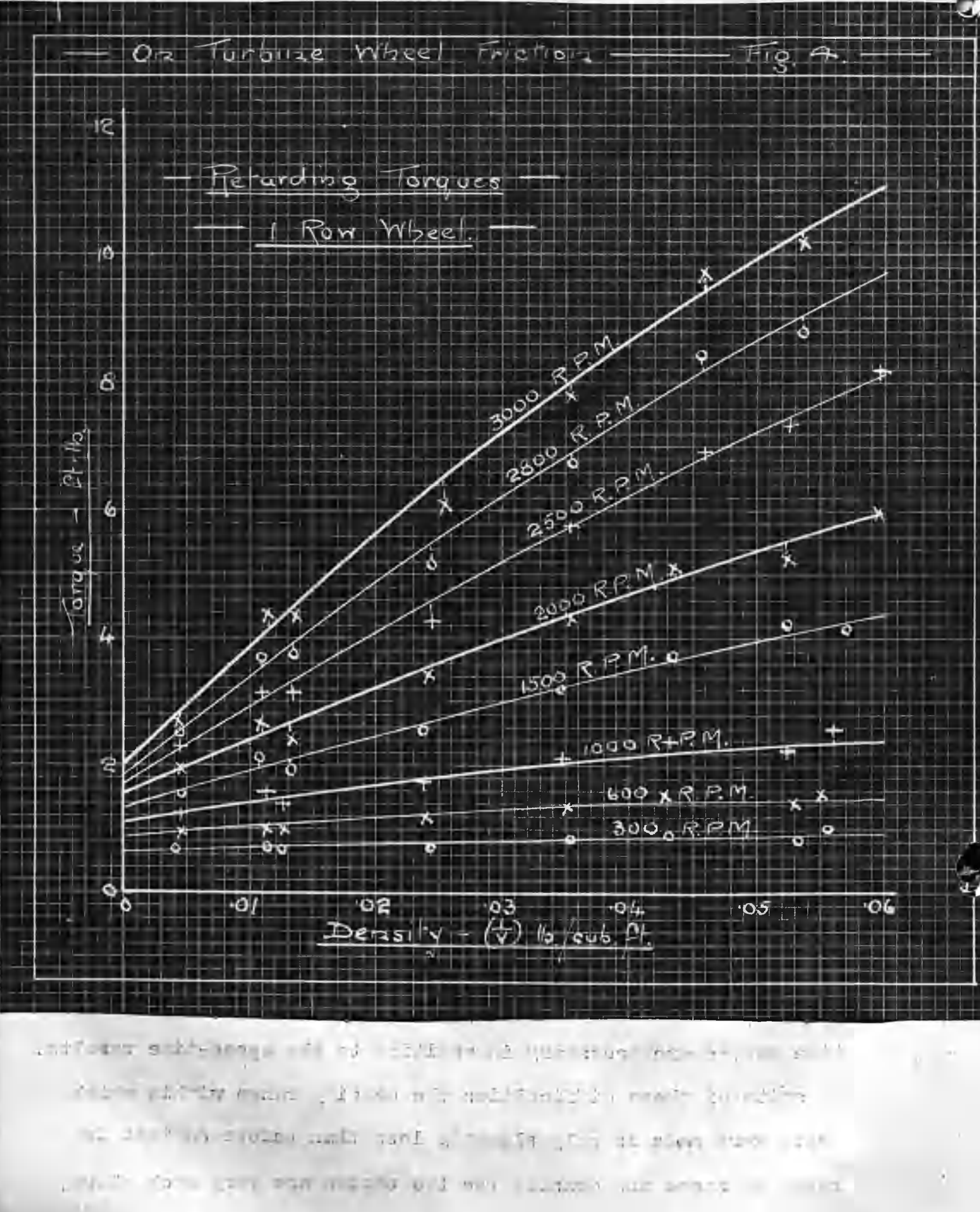
rational expression for bladed wheels - equation (6) - it will be seen that the manner of introduction of the blade height in (8) is faulty, since the terms within the brackets ought, strictly, to be dimensionless.

THE ONE ROW WHEEL TESTS:-

In dealing with these new experiments descriptive details are unnecessary as the method of operation and the first steps in the development follow the same lines as in the earlier tests. A full explanatory treatment is given in the original paper.

The rotor arrangement is as shown in fig. 3. The two partial rows of fixed blades and the ring which carries these have been removed so that, except over the very short length in which the nozzle plate covers the inlet side of the blade row, the wheel is definitely free and unshrouded. The one row tests are, therefore, under nearly "open" conditions and in this respect should be fairly comparable with the three row wheel tests.

In carrying out the tests the same procedure as before was followed. The conditions are however not quite so favourable. The moment of inertia of the rotor is reduced from 24.17 (ft. lb. sec. units) in the three row case to 8.27 in the present series. This reduction means that the retardation effects are more pronounced, which makes the speed-time curves much steeper and less easily dealt with in differentiation. There is also some difficulty in maintaining steady conditions in the casing; the larger spaces apparently have a considerable influence on this - particularly with pressures above atmosphere. This difficulty prevents the averaging of the density values throughout a test, and density-time curves are necessary in addition to the speed-time results. In spite of these difficulties the density range within which tests were made is only slightly less than before so that, in range of speed and density, the two series are very much alike.

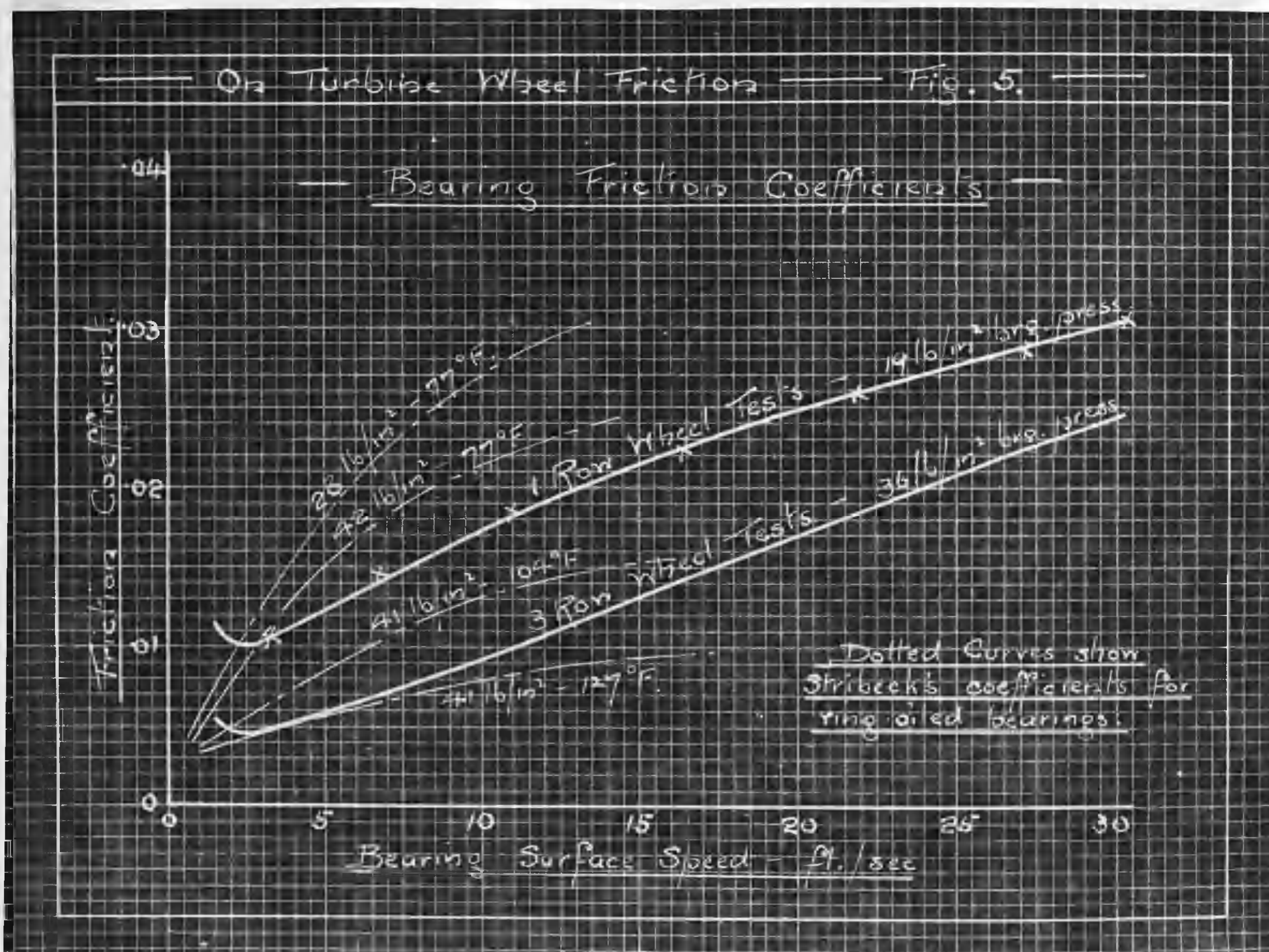

On Turbine Wheel Friction
Fig. 4.
Retarding Torques
1 Row Wheel.
3000 R.P.M.
2800 R.P.M.
2500 R.P.M.
2000 R.P.M.
1500 R.P.M.
1000 R.P.M.
600 R.P.M.
300 R.P.M.
Torque – Ft. lbs.
Density – $\left(\frac{1}{V}\right)$ lb./cub. ft.
0
2
4
6
8
10
12
·01
·02
·03
·04
·05
·06

On Turbine Wheel Friction: (Continued)

The general similarity of method and conditions does not, unfortunately, give results of equal smoothness and consistency. Considerable difficulty was encountered in getting repeat tests of later date to agree with the main series, and the discrepancies are greater than can be covered by the difficulties of experimenting remarked upon above. The main cause of these inconsistencies is thought to be the variability of bearing friction effects. The journals are not in the best condition, whereas in the earlier tests they were in excellent order; in addition, the bearing temperatures and oil conditions are liable to fair change as between different tests; and, in view of the very light bearing load these several causes may be effective in producing friction variations. Actually the differences in derived torques show themselves by the formation of a band of torque density curves at any particular speed in place of a well defined mean curve; and this would be, naturally, a result of change in the constant term, i.e. the bearing friction.

If this reading of the observed effects is correct then the irregularities do not have a correspondingly objectionable influence on the steam friction values since the bearing friction is eliminated for purposes of analyses.

In fig. 4 the torque-speed-density series for the one row wheel tests is shown. This is derived in the same way as before and the examination has been made upon the first full set of systematic experiments. Later tests - as has been pointed out - do not give a curve series coincident with fig. 4, but a similar and roughly parallel set of curves displaced upwards by a small amount; from which it would seem clear that only the intercept values have changed. Obviously the points - even in this main series - are not so smoothly in line as the corresponding values in fig. 2. Several of the marked results in fig. 4 represent the means of a number of tests at the same conditions and are plotted as means in order to get the best curve definition possible.


On Turbine Wheel Friction — Fig. 5.
Bearing Friction Coefficients
Friction Coefficient, f.
·04
·03
·02
·01
0
0
5
10
15
20
25
30
Bearing Surface Speed – ft./sec
28 lb/in² – 77°F.
42 lb/in² – 77°F.
1 Row Wheel Tests – 19 lb/in² brg. press.
41 lb/in² – 104°F
3 Row Wheel Tests – 36 lb/in² brg. press.
41 lb/in² – 127°F.
Dotted Curves show
Stribeck's coefficients for
ring oiled bearings.

On Turbine Wheel Friction: (Continued)

On the whole the lines drawn in fig. 4 may be accepted as a fair approximation to the steam frictional effects of the one row wheel, the above remarks merely indicating that the experimental accuracy is not quite so high as in the original three row tests.

Examination of fig. 4 at once shows that the linear relation between retarding torque and density is no longer a close approximation, the lines being certainly curved. It is this peculiarity that makes the main point of difference between the two sets of results and, obviously, marks an objection against the use of the general equation (8) originally established; while just as obviously it seems to call for an equation form such as (4) developed from theory.

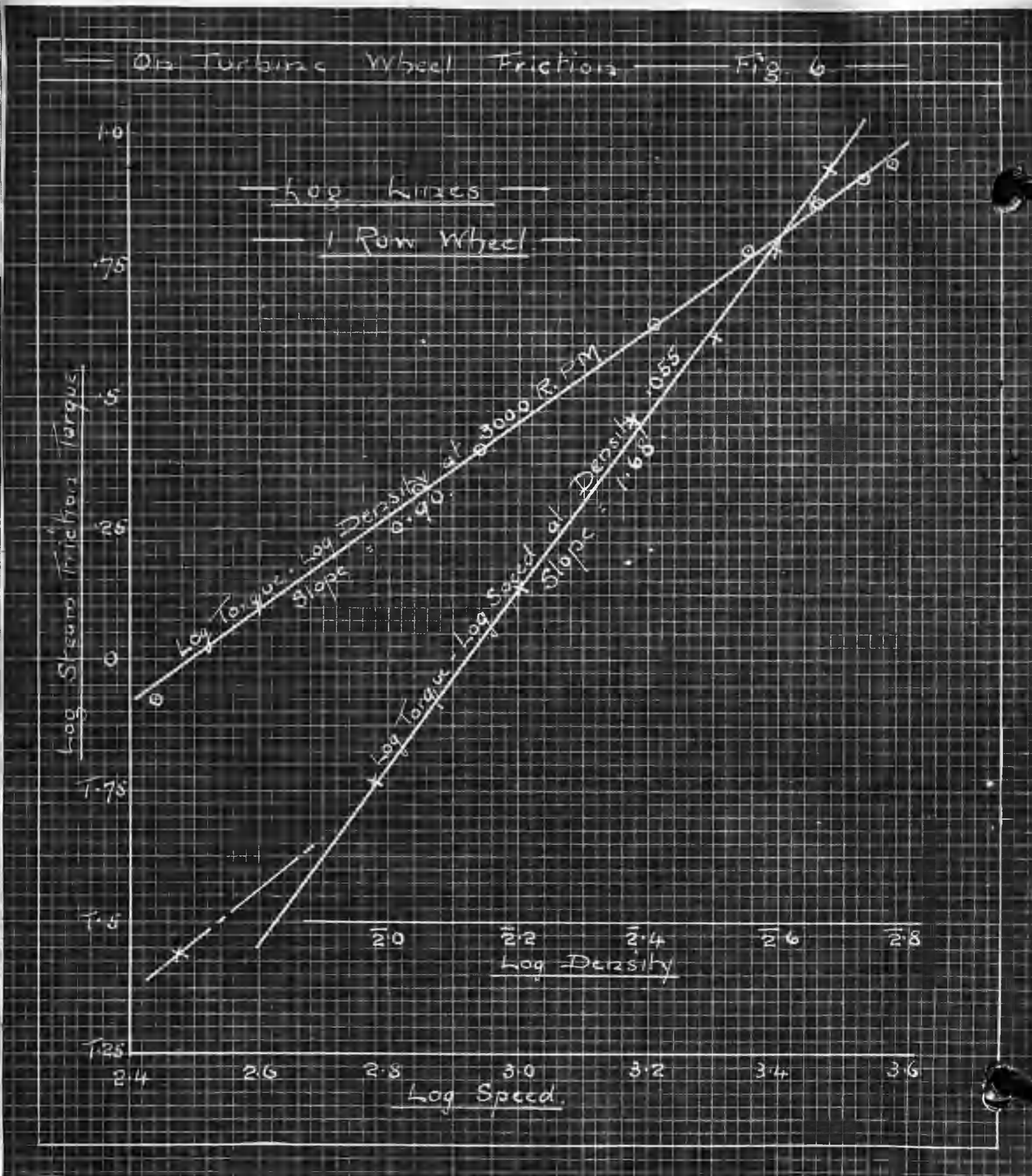
The intercepts of the torque curves on the axis of zero density are taken to represent the bearing friction. Reducing these to coefficients of friction and plotting on a base of surface velocity the line shown on fig. 5 is obtained. Otherwise this figure is much the same as in the previous paper. The difference indicated in bearing friction by the lines for the two cases is hardly more than can be accounted for by the reduction of load intensity; and deterioration of the bearing surfaces may easily cause the variations and further increases seemingly indicated by different test results.

By the elimination of the intercept values the curves in fig. 4 offer the opportunity of studying the effects of speed and density separately; the steam temperatures do not differ greatly so that viscosity is practically a constant. Equation (4), viz.:-

$$N = A \cdot n^{3-\delta} \cdot d^{5-2\delta} \cdot \rho^{1-\delta} \cdot \mu^{\delta}$$

shows that the separate logarithmic plotting of torque against speed and density should give indices that are related to each other as ($2-\delta$) and ($1-\delta$).

In order to obtain the clearest results values giving


On Turbine Wheel Friction
Fig. 6
Log Lines
1 Row Wheel
Log Steam Friction Torque
1·0
·75
·5
·25
0
1̄·75
1̄·5
1̄·25
Log Torque : Log Density at 3000 R.P.M.
Slope : 0.90
Log Torque : Log Speed at Density ·055
Slope : 1·68
2̄·0
2̄·2
2̄·4
2̄·6
2̄·8
Log Density
2·4
2·6
2·8
3·0
3·2
3·4
3·6
Log Speed

the largest torques are chosen. Thus fig. 6 shows the log. torque-log. speed line at a constant density of .055 and the log. torque-log density line at a constant speed of 3000 revs. per min. The lines are fairly well defined and result in:-

$$N \propto n^{2.68} . \rho^{.9} \qquad (9)$$

Since these exponents are not nearly in the ratio $(2-\delta)/(1-\delta)$ there is definite disagreement in the essential magnitudes; the tendency is in the correct direction to meet the theoretical form but the difference is too great to be satisfactory. Even if it is argued that the shape of the curves in fig. 4 is not definitely fixed by the somewhat irregular points, it must be noted that any allowable alteration of this shape would increase the index of density instead of reducing it as is required by the necessary relation between the indices. It is also to be observed that the speed index is well below 3, so that in all the important points there is a clear departure from the previous result as expressed by .

Since the two sets of experiments have been carried out on the same turbine, and under similar conditions, it seems necessary to achieve some closer agreement between them. The facts of a conspicuous change in the speed index and a glaring discrepancy between the speed and density exponents are not to be explained by a casual reference to experimental inaccuracies. The problem of reconciling the results requires a little further examination of the theoretical matter, and some consideration of the legitimacy of logarithmic plotting in such cases as these.

FURTHER THEORETICAL CONSIDERATIONS:

Referring to equation (4) :-

$$N = A \cdot n^{3-\delta} \cdot d^{5-2\delta} \cdot \rho^{1-\delta} \cdot \mu^{\delta} \qquad (4)$$

it will be readily appreciated that when only one variable is

On Turbine Wheel Friction: (Continued)

allowed to alter direct logarithmic plotting of the results is an exact method of examination. This, however, would assume that (4) was a complete representation of the theoretical facts; but it is only a simplified form of the general equation:-

$$N = n^3 \cdot d^5 \cdot \rho \sum A \left(\frac{\mu}{\rho \cdot n \cdot d^2}\right)^{\delta} \text{ - - - - - } (1).$$

being obtained on the assumption that δ is invariable in all the terms involving the Reynold's number:-

$$\left(\mu / \rho \cdot n \cdot d^2\right).$$

There is however no a priori reason why δ should not vary from term to term and between relatively wide limits. Thus the general equation may be written:-

$$N = n^3 \cdot d^5 \cdot \rho \left[A_1\left(\frac{\mu}{\rho \cdot n \cdot d^2}\right)^{\delta_1} + A_2\left(\frac{\mu}{\rho \cdot n \cdot d^2}\right)^{\delta_2} + \text{- - -}\right] \text{ - - - - - } (1)'$$

and there are no restrictions on the δ values; δ_1 might, for instance be zero and $\delta_2 = .5$. Under such circumstances logarithmic plotting would cease to be valid, since N would be expressible as a somewhat confused power series of the variable concerned.

This point would seem to be of importance and deserving of special attention in all cases in which dimensional theory is applied to the resistance of bodies of awkward shape. Such forms can only be defined by a number of length ratios involving a fair number of terms in the general expression. Each term will have its own special power of the Reynold's number as a factor. All the δ's might conceivably be alike, in which case logarithmic plotting would be a true guide; they might be unlike but with one particular term predominating in numerical value when logarithmic plotting would probably give a sufficient indication; but if they are unlike and two or more terms have nearly equal influences then the usual direct use of a logarithmic graph is misleading.

Suppose for example that the index of speed varies somewhat as between a smooth surface and a rough surface and consider

On Turbine Wheel Friction: (Continued)

a body of simple shape but partly rough and partly smooth in finish. Each part might be defined by a simple ratio, but the general expression would require at least two terms to represent the effects, thus:-

$$N = \left\{A_1 \cdot n^{3-\delta_1} \cdot d^{5-2\delta_1} \cdot \rho^{1-\delta_1} \cdot \mu^{\delta_1}\right\} + \left\{A_2 \cdot n^{3-\delta_2} \cdot d^{5-2\delta_2} \cdot \rho^{1-\delta_2} \cdot \mu^{\delta_2}\right\} \text{-----} (10).$$

and clearly the value deduced by the simple logarithmic determination of say $(3-\delta)$ would depend upon the relative magnitudes of A_1 and A_2; and would vary between $(3-\delta_1)$ and $(3-\delta_2)$ according as one or the other became of small account. Naturally, if both terms were of importance the log line obtained by plotting as if δ were constant could not be quite straight; but, generally, the curvature would be very slight and would in all likelihood be dismissed as due to experimental inaccuracy.

Some such line of argument would seem necessary for the case of a bladed turbine wheel. The disc is usually smooth in finish and with a surface continuous in the direction of motion. In contradistinction to this the blade row is composed of many separate items of awkward form that produce a surface aspect of extreme irregularity. It is natural to suppose that, at least, two terms are necessary to express the loss in such a case; and it is not difficult to believe further that the index values must be different in these two terms. Greater refinement than is allowed by the double term would seem impossible at present on account of ignorance as to the effects and importance of the secondary aspect ratios.

Accepting the above, introducing the useful blade speed in place of speed of rotation and the reciprocal of the specific volume for density, the general equation for a bladed wheel could be written:-

$$N = \left(\frac{s}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)\left[A_1\left\{\frac{\mu}{\left(\frac{s}{100}\right)\left(\frac{d}{10}\right)\left(\frac{1}{V}\right)}\right\}^{\delta_1} + A_2\left\{\frac{\mu}{\left(\frac{s}{100}\right)\left(\frac{d}{10}\right)\left(\frac{1}{V}\right)}\right\}^{\delta_2}\right] \text{---} (11).$$

On Turbine Wheel Friction: (Continued)

The steam friction formulae hitherto used are represented by the limiting case of this when $\delta_1 = \delta_2 = 0$.

Now it is natural to believe that the "broken" surface of a blade row will involve the highest speed index, i.e. the lowest value of δ ; and if in any instance δ has a value of importance it might be expected in the case of a smooth disc. Hence, where the frictional loss is almost entirely due to blading the speed index on a direct logarithmic plot should be practically 3 ; but where the disc loss is not relatively unimportant a lower value could be anticipated.

This is what is actually shown by relations (7) and (9) for the three row and one row wheels respectively; and an examination of figs. 1 and 3 will indicate that the relative values of the two separate parts of the loss must be as above outlined.

It may therefore be taken that in the blade term the exponent of the Reynold's number is zero, while in the disc term it is a small but important fraction.

In calculations of the type under consideration - which at the best can only be roughly approximate - the necessity to deal with the absolute value of the viscosity coefficient would be a trouble giving no special gain in accuracy. It may therefore be replaced by the square root of the absolute temperature ($\sqrt{T}$) as a sufficiently good approximation over the range of steam temperatures that occurs in problems of the kind. This point is of no moment in the particular series of results that are to be subject to this analysis, but the proposed modification may be permitted for the sake of applicability to widely different temperature values. It also aids the arithmetic.

The smoothness and continuity of the disc is supposed covered by the value of δ_1 ; nothing further regarding the disc is specified except the diameter. Hence A_1 in the equation considered should be a steady constant applicable generally to wheels of this order. In strictness, of course, this constant is influenced by dimensional ratios that are neglected herein.

In dealing with the blading only the blade height-diameter ratio has been allowed to enter the discussion. Hence A_2 should depend on some function of this ratio, i.e.:-

$$A_2 = b \cdot f\left(\frac{l}{d}\right).$$

Generally the data are of such a kind that it is only possible to put:-

$$f\left(\frac{l}{d}\right) = \left(\frac{l}{d}\right)^x.$$

and the search is for a suitable value of x. Buckingham makes $x = 2$, but no empirical relation representing any one set of experiments introduces such a high power of the blade height.

These various considerations lead to the following:-

$$N = \left(\frac{S}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)\left[a\left\{\frac{\sqrt{T}}{\frac{S}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^\delta + b\cdot f\left(\frac{l}{d}\right)\right] \quad \text{-----} \quad (12).$$

as a probably suitable form for bladed wheels. The blade term has so far been left with an indefinite function, but this indefiniteness does not forbid the trial of (12) on the actual test data, since in each set of tests the blading is fixed.

The essential point of the above discussion is that, for such bodies as bladed turbine wheels, the usual procedure of logarithmic plotting of torque against speed and against density is not valid; although in special cases it may give nearly correct results. Even in simpler forms, where several aspect ratios of nearly equal importance are involved, the method is misleading, and only in those cases in which one measure of shape is dominant does it result in sound values. Such considerations may serve to explain the extreme variation of speed index found by the usual logarithmic analysis of experimental work on disc friction - below the figure of 3.0. They do not, however, apply to higher values than this as there is no reason to believe that δ can be negative; although there is nothing in the strict dimensional theory that prohibits this. In the present connection it is interesting to note that Buckingham explains Odell's high values for paper discs as due to "fluttering" at high speed.

It is now necessary to examine the results already given in the light of equation (12).

RECONSIDERATION OF TEST RESULTS:

Since the actual test results have been presented in the form of torque-speed-density curves for both cases, equation (12) may be changed to read torque; and this gives for separate application to each set:-

$$Q = 2{\cdot}29\left(\frac{S}{100}\right)^2\left(\frac{d}{10}\right)^3\left(\frac{1}{V}\right)\left\{a\left[\frac{\sqrt{T}}{\frac{S}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right]^{\delta} + B\right\} \quad \text{------} \quad (13).$$

where Q represents torque in ft. lb; B must, of course, have a constant value for each blading arrangement.

The problem, then, is to determine the values of a, B and δ that best suit the actual curves, and this is largely a matter of trial and error. The best results obtained from the study of the figures are shown in figs. 7 and 8.

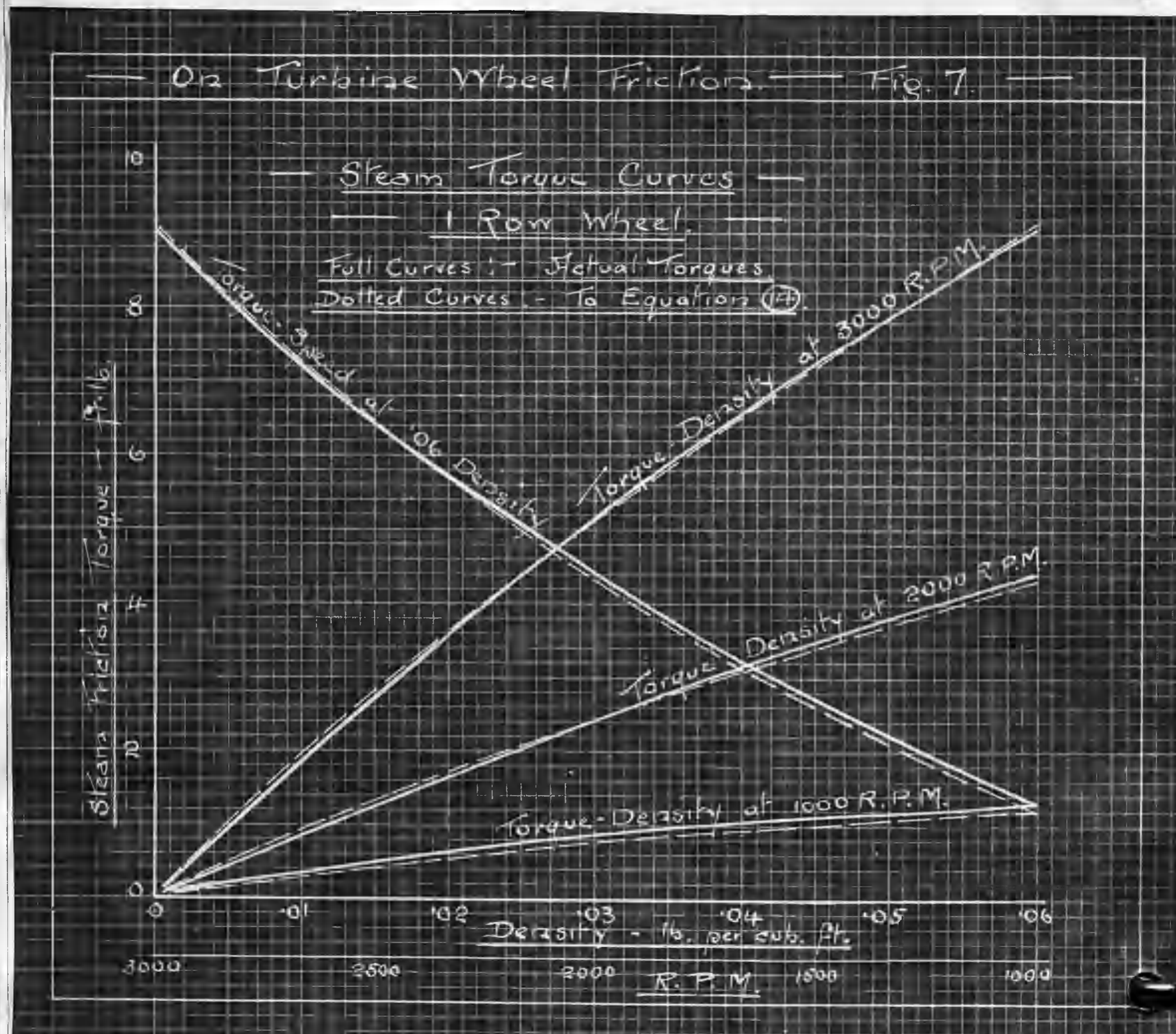
In both of these diagrams the full lines reproduce experimental results already given in figs. 2 and 4, while the dotted curves represent the equations of form (13) that most nearly agree therewith.

Fig. 7 gives the curves for the one row wheel, and the equation there used is:-

$$Q = 2{\cdot}29\left(\frac{S}{100}\right)^2\left(\frac{d}{10}\right)^3\left(\frac{1}{V}\right)\left[{\cdot}020\left\{\frac{\sqrt{T}}{\frac{S}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{{\cdot}25} + {\cdot}040\right] \quad \text{---} \quad (14).$$

The three row wheel curves are shown in fig. 8, and the corresponding equation is:-

$$Q = 2{\cdot}29\left(\frac{S}{100}\right)^2\left(\frac{d}{10}\right)^3\left(\frac{1}{V}\right)\left[{\cdot}0175\left\{\frac{\sqrt{T}}{\frac{S}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{{\cdot}25} + {\cdot}227\right] \quad \text{---} \quad (15).$$



— On Turbine Wheel Friction — Fig. 7 —
— Steam Torque Curves —
— 1 Row Wheel —
Full Curves :- Actual Torques
Dotted Curves - To Equation (14)
Torque-Speed at ·06 Density
Torque-Density at 3000 R.P.M.
Torque-Density at 2000 R.P.M.
Torque-Density at 1000 R.P.M.
Steam Friction Torque - ft.lb.
0
2
4
6
8
10
0
·01
·02
·03
·04
·05
·06
Density - lb. per cub. ft.
3000
2500
2000
1500
1000
R.P.M.

that δ can be negative; although there is nothing in the strict dimensional theory that prohibits this. In the present connection it is interesting to note that Buckingham explains Odell's high values for paper discs as due to "fluttering" at high speed.

It is now necessary to examine the results already given in the light of equation (12).

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Since the actual test results have been presented in the form of torque-speed-density curves for both cases, equation (12) may be changed to read torque; and this gives for separate application to each set:-

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where Q represents torque in ft. lb; B must, of course, have a constant value for each blading arrangement.

The problem, then, is to determine the values of a, B and δ that best suit the actual curves, and this is largely a matter of trial and error. The best results obtained from the study of the figures are shown in figs. 7 and 8.

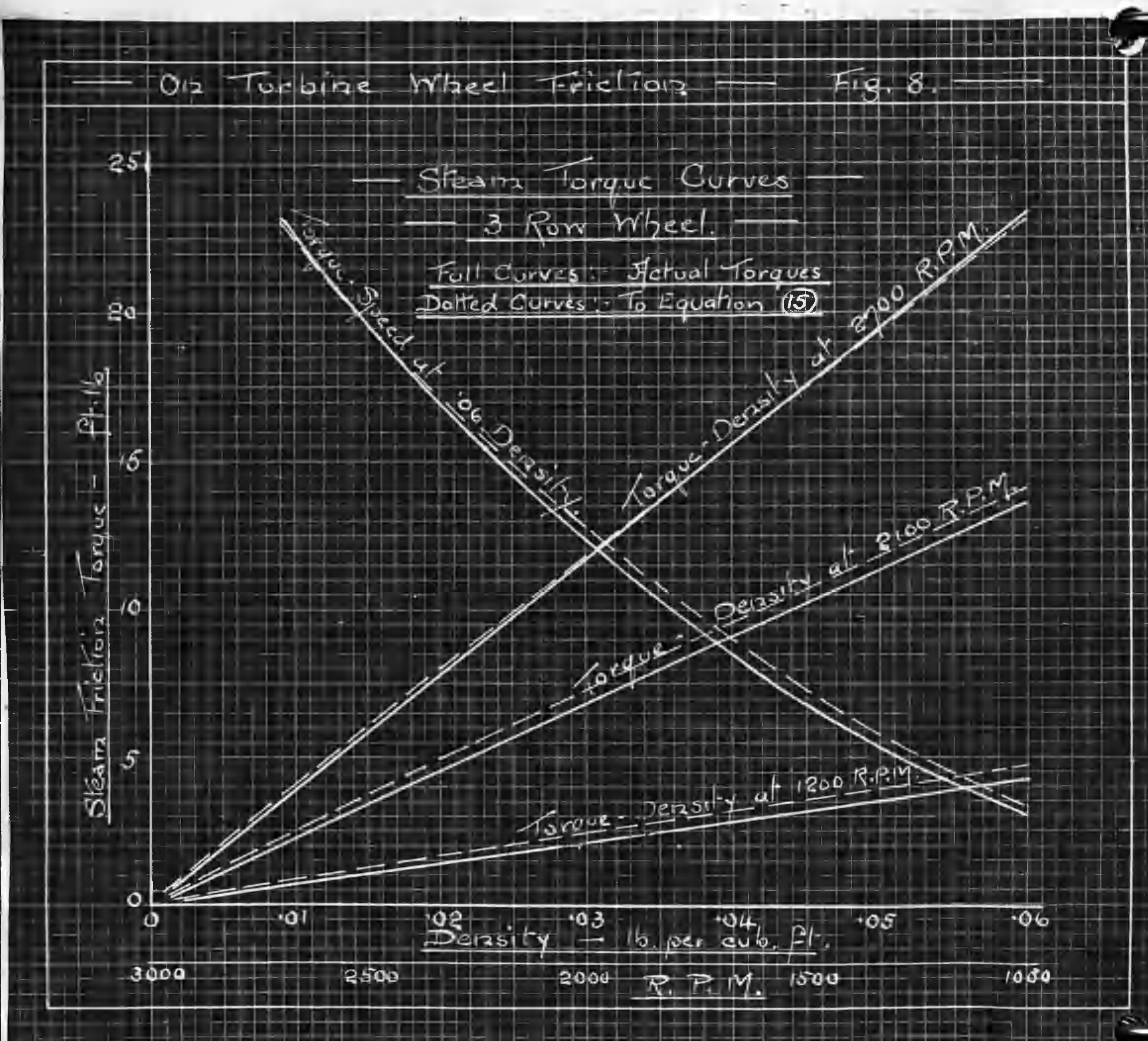
In both of these diagrams the full lines reproduce experimental results already given in figs. 2 and 4, while the dotted curves represent the equations of form (13) that most nearly agree therewith.

Fig. 7 gives the curves for the one row wheel, and the equation there used is:-

$$Q = 2.29\left(\frac{s}{100}\right)^2\left(\frac{d}{10}\right)^3\left(\frac{1}{V}\right)\left[.020\left\{\frac{\sqrt{T}}{\frac{s}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{.25} + .040\right] \quad \text{---} \quad (14).$$

The three row wheel curves are shown in fig. 8, and the corresponding equation is:-

$$Q = 2.29\left(\frac{s}{100}\right)^2\left(\frac{d}{10}\right)^3\left(\frac{1}{V}\right)\left[.0175\left\{\frac{\sqrt{T}}{\frac{s}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{.25} + .227\right] \quad \text{---} \quad (15).$$


On Turbine Wheel Friction — Fig. 8.
Steam Torque Curves
3 Row Wheel.
Full Curves: Actual Torques
Dotted Curves: To Equation (15)
Torque - Speed at .06 Density.
Torque - Density at 2700 R.P.M.
Torque - Density at 2100 R.P.M.
Torque - Density at 1200 R.P.M.
Steam Friction Torque - ft. lb.
25
20
15
10
5
0
0
.01
.02
.03
.04
.05
.06
Density - lb. per cub. ft.
3000
2500
2000
1500
1000
R. P. M.

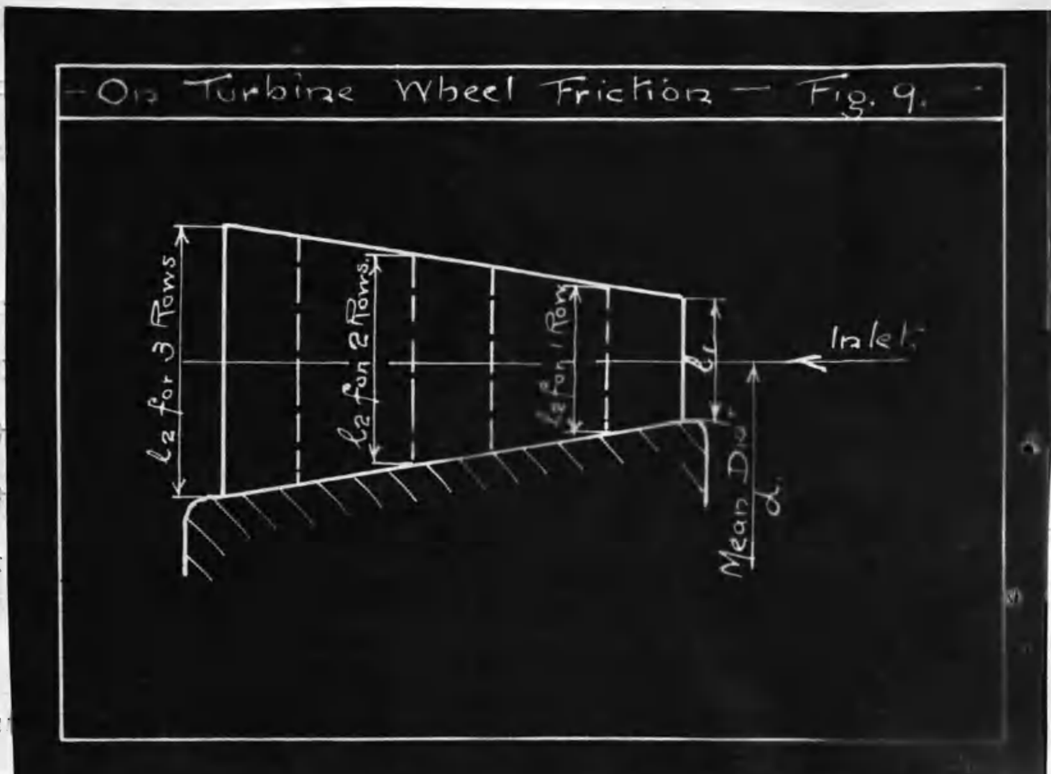
On Turbine Wheel Friction: (Continued)

In view of the nature of the problem and the data, the agreement portrayed by figs. 7 and 8 must be considered very fair. The differences at the lower speeds in the case of the three row wheel (fig.8) could not be eliminated by any further adjustment of an equation of the above form unless negative δ values were envisaged. The agreement is almost exactly the same as in the case of the simple expression (7) already stated as the direct result of the three row wheel tests. In fact, the small discrepancy shown by fig. 8 is equivalent to the error introduced in the previous paper by simplifying the index 3.04 to 3.0. There is, therefore, no sacrifice of accuracy in applying the new form of equation to the old tests, while fig. 7 shows that the new tests are adequately covered by it.

In both cases, then, the experimental results may be taken as well suited by the expressions given; so that, although at first sight they appeared widely different in their nature, they have now been shown to be in satisfactory, and almost equally close, agreement with an equation form that strictly accords with theory.

It follows, therefore, that Buckingham's conclusion to the effect that all the data on this subject is competently expressed by the theoretical equation given by (4) is not justified in the present instance; since the step required to relate two complementary sets of experiments has been found in developing the theory beyond the stage at which it is expressible by (4).

The equations (14) and (15) represent the essential facts of the particular data in hand and one main duty of the present paper may be considered fulfilled by their establishment. It remains to discuss the details of the disc and blade terms and to extend the applicability of the equation by generalising the constants. For this purpose the disc loss and blade loss may be considered separately.



On Turbine Wheel Friction — Fig. 9.

On Turbine Wheel Friction: (Continued)

THE DISC TERM:

Reference to equations (14) and (15) will show that there is a noticeable difference in the disc constants as there given. A distinction of this kind seems uncalled for; but the task of meeting the actual curves has shown that it is real; and that it is not possible to get an equally good agreement with the same constant in both equations. It seems necessary, therefore, to consider what differences in the discs may affect this point.

It will be seen that the constant for the one row disc is nearly 15% higher than the other. In fig. 3 it is shown that a distance piece of fair diameter takes the place of the discs that have been removed. This might possibly entail a small extra effect tending to make the loss in this case high; but it could not possibly account for the difference actually found; a very small percentage at the most should cover it.

The diameter d appearing in the equations so far is the mean blade ring diameter. The real disc diameter is however $(d - \ell)$ and the disc loss should, strictly, be written in terms of the diameter and of the rim speed corresponding thereto. This is of no moment in the study of the effects of a particular wheel and blade, since the correction is constant; but it must be taken into account for different wheels, and is of very considerable importance in the case of varying blade heights on the same mean diameter.

In both of the test series under review the blade heights are small; but they are different, and the influence of these different heights on the effective disc faces should be considered.

The sketch in fig. 9 illustrates the form common to both cases - as all the blades are tapered axially. To deal

On Turbine Wheel Friction: (Continued)

with all cases in the same way it may be taken that the diameters of the effective disc faces are determined by the heights of the inlet edge of the first row and the outlet edge of the last row respectively. If, then, the effect of the different amounts of exposed rim breadth is supposed included in the blade term - as would seem natural - the difference in the disc constants in (14) and (15) should show a distinct tendency to vanish when $(d - \ell_1)$ and $(d - \ell_2)$ are used in place of d. It should be noted in this connection that the blade speed involves the diameter; and the apparently small correction on diameter is necessary because the loss is thus dependent on the fifth power.

Making the necessary correction in such a way as to retain the blade speed and mean diameter in the expression for loss, the true constant should be given by:-

$$a\left\{\left(1-\frac{\ell_1}{d}\right)^{4.5} + \left(1-\frac{\ell_2}{d}\right)^{4.5}\right\} = \cdot 020 \text{ for the one row wheel.}$$

$$\text{"} \qquad = \cdot 0175 \text{ " " three " "}$$

When the proper ratios are inserted there results:-

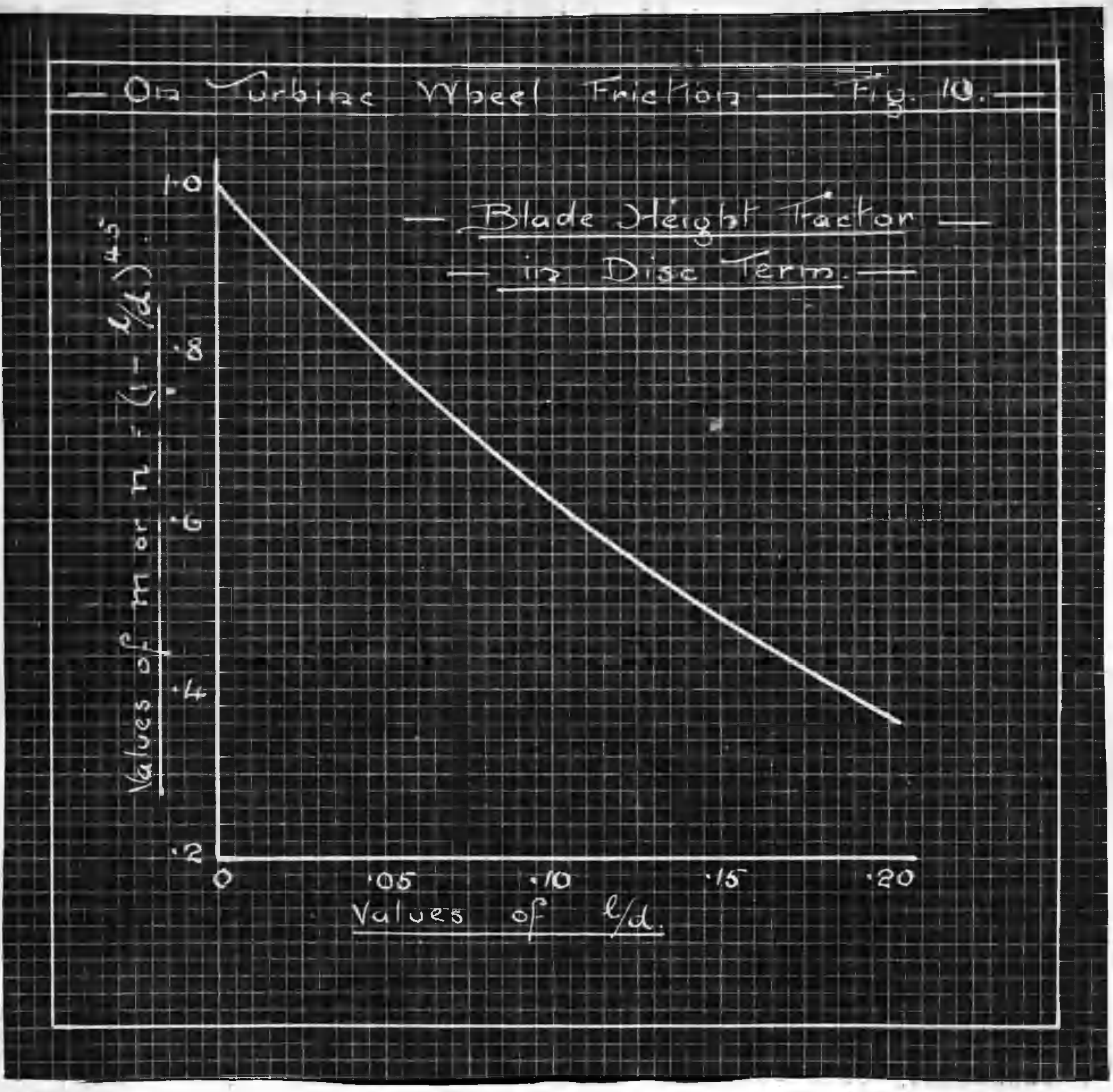
$$a = \cdot 0107 \text{ for the one row wheel.}$$

$$= \cdot 0102 \text{ " " three " "}$$

so that by this process of examination a discrepancy of 15% is reduced to one of 5%; and this last is of the order of uncertainty of the main figures.

It could be taken, then, that the average of .0105 is a satisfactory disc constant for practical wheels running enclosed in steam, slightly superheated or dry; and when the correct disc diameter is used.

It is of interest to compare this figure with that deduced by similar calculation from a test of Stodola's on a bladeless disc run in the open air. The disc was of unmachined boiler plate 21.14" diameter and when run at 2000 revs. per min. absorbed .147 H.P. Assuming air at about 70° F. the constant is obtained as:-


On Turbine Wheel Friction — Fig. 10.
Blade Height Factor
in Disc Term.
Values of m or n = (1 - l/d)^4.5
1·0
·8
·6
·4
·2
0
·05
·10
·15
·20
Values of l/d.

On Turbine Wheel Friction: (Continued)

$$a = \frac{\left(\frac{\cdot 147 \times 13\cdot 3}{1\cdot 84^3 \times 2\cdot 114^2}\right)}{\cdot 298 \times 2\left(\frac{\sqrt{530} \times 13\cdot 3}{1\cdot 84 \times 2\cdot 114}\right)^{\cdot 25}} = \cdot 0117.$$

The increase above the previous average value of .0105 is not greater than can readily be charged to the different surface and medium, and the more open conditions.

It seems reasonable, then, to advance the following as a suitable expression for the disc loss of a turbine wheel:-

$$N_d = \left(\frac{s}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)\left[\cdot 0105\left\{\frac{\sqrt{T}}{\frac{s}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{\cdot 25}(m + n)\right] \text{------} (16).$$

in which:-

N_d = disc loss due to steam friction - horse power.
s = mean blade speed - ft. per sec.
d = mean blade ring diameter - inches.
V = specific volume of steam - cub. ft. per lb.
T = steam temperature - °F absolute.
$m = (1 - l_1/d)^{4\cdot 5}$ where l_1 is inlet edge blade height inches
$n = (1 - l_2/d)^{4\cdot 5}$ " l_2 " outlet " " " "

The values of m and n based on l/d can be readily taken from fig. 10.

THE BLADE TERM:

The blading figures in (14) and (15) are probably fairly sound for the particular cases investigated, but they cannot serve to show how the height-diameter ratio should enter into the blade term for general use. They would have been rather more serviceable in this respect if the blade heights had been more extreme. If, however, it is possible from other considerations to establish the influence of the important ratio then the figures that have been obtained from the test analysis should fix, satisfactorily, the values of the constants to be used.

It was the Author's primary intention to use the square of the height-diameter ratio (as already employed by Buckingham) - see equation (6) - and simply determine the constants for one and three row wheels from his own figures. But on examination this process has been discarded as the exponent used by Buckingham hardly seems justified, even by the main facts on which it was based, viz., Stodola's results.

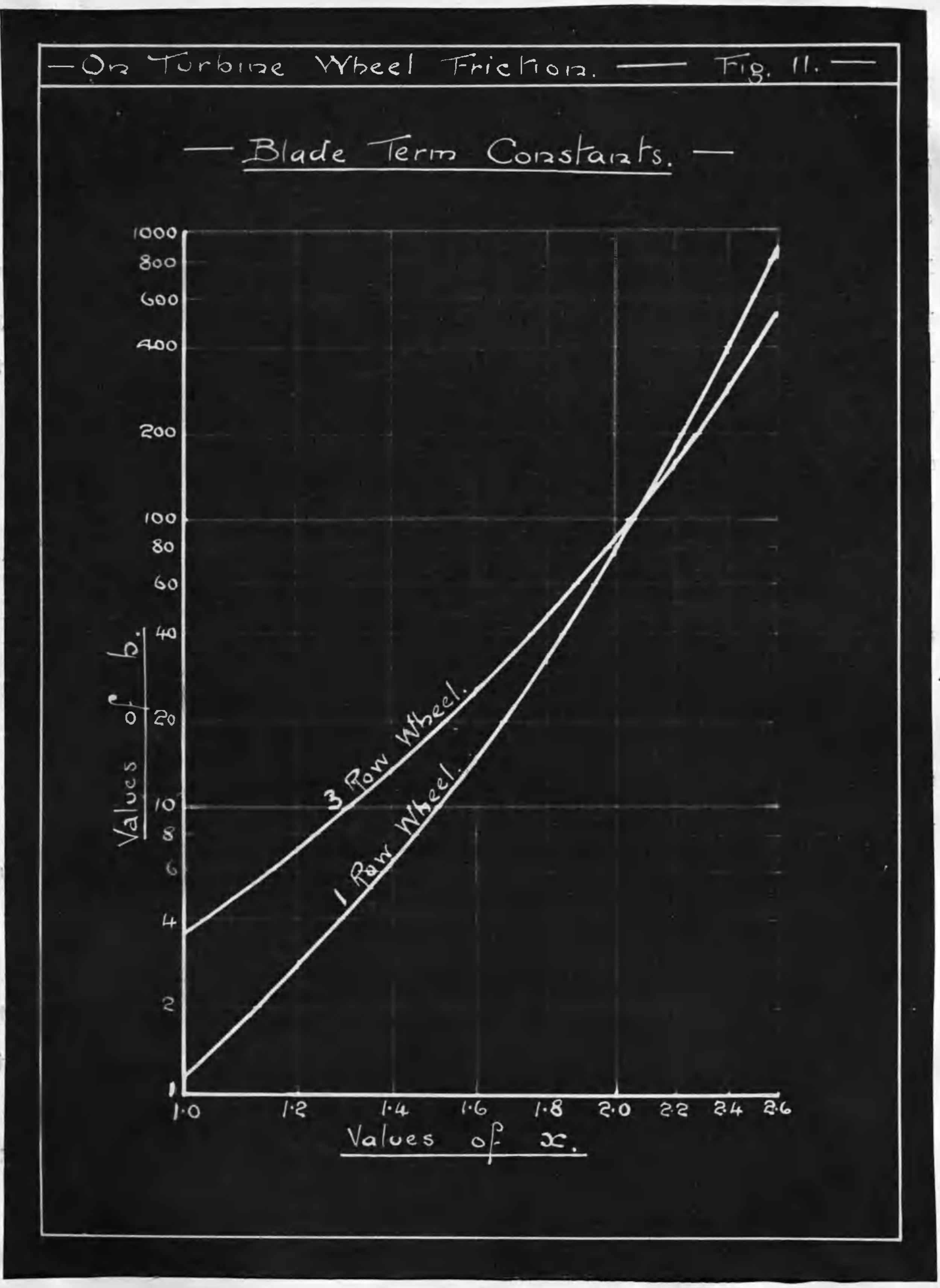
The line of argument to be followed may be briefly stated. It is first shown from the present test analyses that the second power of the ratio is very unlikely; and on the high side. Stodola's results are then employed to show that Buckingham's value of 2.0 is much too high. A probable value is then taken, more or less in agreement with accepted figures, and a new test which gives a valuable result for an extreme case is used in support. Finally, the equation so established is compared with the only other extant tests on multi-row wheels, viz., Lasche's experiments, and good agreement is found. In this way it is thought that the general balance of the evidence is in favour of the proposed form.

It has already been indicated that the blading figure should be in the form of:-

$$b \cdot f\left(\frac{l}{d}\right).$$

and that the best that can be done considering the facts, is to use a simple power of the ratio for the function. Again, since two blade height ratios have been used to determine the disc sizes the same procedure should be employed, in the general case, for the blading; since with multi-row wheels the first and last blade heights are radically different. Reference to fig. 9 will show that by using the first row inlet height and the last row outlet height, the trapezium form by which the blading is enveloped is fairly defined. This should be superior to the use of a mean blade height since the shape of the blading arrangement is more clearly specified; and it is almost certain that the exponent of the ratio is higher than unity.

This point of view would give:-



— On Turbine Wheel Friction. — Fig. 11. —

On Turbine Wheel Friction: (Continued)

$$b\left[\left(\frac{l_1}{d}\right)^x + \left(\frac{l_2}{d}\right)^x\right] = \cdot 040 \quad \text{for the one row wheel.}$$
$$\text{"} \qquad = \cdot 227 \quad \text{" " three " " .}$$

Fig. 11 shows how the value of b alters if different assumptions are made for x. It is readily seen that the b values are impossible for $x > 2.0$, since the constant for the three row wheel would then be less than for the one row wheel. The constants are nearly alike at $x = 2.0$, and this would be a significant result if the blading function were known to be complete and correct. This can hardly be so since it does not even contain a reference to number of rows; so that 2.0 is too high to be acceptable unless well supported otherwise.

Again at $x = 1.0$ the ratio of constants is practically 3 : 1 ; but it is not possible that the constant will vary so rapidly as the number of rows since the intermediate blading is relatively unimportant. Consequently $x = 1.0$ is certainly too low.

The most that this particular data can give is, then, that the index is not so high as 2.0 and not so low as 1.0; and the nature of the figures is such as to indicate that it is not really close to either of these extremes.

Turning now to the Stodola results on which Buckingham mainly based his finding that $x = 2.0$, it is found that the disc term in Buckingham's equation is much too severe. The equation is:-

$$N = 10^{-16} \cdot n^3 \cdot d^5 \cdot \rho \left\{1 + 590 \cdot \frac{l}{d}\right\} \text{ --------- } (6).$$

where d = disc diameter - inches.

and, in establishing it, no direct use has been made of the only bladeless disc test of the series. Thus in Table 1 the first 8 columns and the first 5 lines reproduce the essentials of Buckingham's comparison between equation (6) and Stodola's facts. This comparison shows that in only one case is there a radical

On Turbine Wheel Friction. —— Table 1. ——

Case	Wheel Type.	Mean Dia.r d.	Height Dia.r Ratio ℓ/d.	Revs. per Min.	Powers. Actual.	By Eq.n (6).	Diff.ce %	Power. Actual.	By Eq.n (17)	Diff.ce %
1	Bladed Wheel.	35·935	·030	1600	2·306	2·315	−0·4	2·306	2·120	+8·1
2	" "	27·460	·035	2200	1·778	1·736	+2·4	1·778	1·860	−4·6
3	" "	20·680	·038	2200	0·536	0·446	+18·0	0·536	0·510	+4·8
4	" "	47·638	·0455	980	2·895	3·000	−3·5	2·895	3·230	−11·5
5	" "	22·205	·106	2100	1·850	1·833	+0·9	1·850	1·820	+1·6
6	Bladeless Disc.	21·142	0	2000	0·147	0·234	−59·2	0·147	0·149	−1·3

On Turbine Wheel Friction: (Continued)

disagreement, viz. 18%. But when the 6th line is added for the plain disc it is seen at once that there is a serious discrepancy; and it must be contended that all six results should be reasonably within any equation form made to represent the series. With the disc term overcharged the index of the height-diameter ratio is misleading.

To show that a better agreement can be obtained with quite a different index the last columns in Table 1 have been calculated by means of an equation of the type:-

$$N = \left(\frac{S}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)\left[a\left\{\frac{\sqrt{T}}{\frac{S}{100}\cdot\frac{d}{10}\cdot\frac{1}{V}}\right\}^{\cdot 25}(m+n) + b\left\{\left(\frac{\ell_1}{d}\right)^x + \left(\frac{\ell_2}{d}\right)^x\right\}\right] \quad \text{-----} (7).$$

using the following values:-

a = .0117, b = 5.0, x = 1.3, V = 13.3.

and it will be seen that there is no really serious discrepancy in any of the six values. If a better balance can be obtained in this way with an index of 1.3 there is certainly little justification for using 2.0!

On the whole it does not seem possible to do better than take x = 1.5 as a generally applicable figure. This value has been much used as an exponent of blade height in the irrational forms of equations hitherto employed. Some experiments might certainly indicate a slightly higher value; but it has been shown that 2.0 is impossibly high, while it is now seen that 1.3 fits one important set of results.

Accepting x = 1.5 it follows that, for the Author's test results:-

$$b\left\{\left(\frac{\ell_1}{d}\right)^{1\cdot 5} + \left(\frac{\ell_2}{d}\right)^{1\cdot 5}\right\} = \cdot 040 \text{ for the one row wheel.}$$

" = ·227 " " three " "

from which:-

b = 9.3 - for the one row wheel.

= 17.9 - for the three row wheel.

The best test of the general suitability of the index chosen would be for an extreme case of a bladed wheel, such as a

On Turbine Wheel Friction: (Continued)

very small diameter with large height-diameter ratio. The Author is fortunate in having seen the result of a recent determination of the loss in a small De Laval wheel which satisfies these conditions. This test was made by Mr D. S. Anderson* on a wheel 3.97" mean diameter with a blade height ratio of .149. The wheel was run in "open air" conditions and at 27000 revs. per min. the horsepower absorbed is about 1.6 (± 6%).

In making a calculation of the loss, equation (17) was used with the following constants:-

$$a = .0117. \quad b = 10.3, \quad x = 1.5, \quad V = 13.3.$$

These constants allow a 12% increase for "open air" conditions over "enclosed steam" conditions, and the result is a calculated value of 1.5 horsepower. This shows an excellent agreement with the actual 1.6 considering the special nature of the case.

The equation covering Lasche's experiments on multi-row wheels is ⊕ :-

$$N = 10^{-10} \cdot \alpha \cdot \left(\frac{d}{10}\right)^4 \cdot l \cdot n^3 \cdot \frac{1}{V} \quad \text{------------} \quad (18)$$

in which:-

α = .4 for one row wheel.

= .64 for three row wheels.

This equation is claimed to hold between the limits 35" to 47" diameter, and from ¾" to 2" blade height. It is obviously wrong in form and can only have a narrow range of accuracy. Assuming a 40" diameter with 1½" blade height as a mean condition, for which the equation is sound, comparison may be made with (17). Using

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* "Investigation of the Losses in a De Laval Turbine" - D. S. Anderson, B.Sc.
"Greenock Research Scholarship" Report. R.T.C. June 1922.

⊕ This equation differs from that ascribed to Lasche in the Author's 1912 Paper. The latter was taken from Stodola "Die Dampfturbinen" (4th edn.) p. 129 but a correspondent (K. Shogenji, Japan) has pointed out ("Engineering" June 26, 1914) that there is a misprint involved in the German original and gives the correct form - on which equation (18) has been based.

On Turbine Wheel Friction: (Continued)

the steam constants:-

a = .0105, b = 9.3 for one row wheel

x = 1.5 , b = 17.9 for three row wheel.

and, taking S = 500 and $1/V$ = .06 for the purposes of the special factor in the disc term, the following results are obtained:-

Values of $N/\left(\frac{S}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)$.		
	By Lasche's Equation	By Equation (17).
One Row Wheel	·180	·173
Three Row Wheel.	·288	·298.

The agreement is good enough to show that (17) with the constants that have been deduced does not controvert the findings of this purely empirical relation.

On the whole the various values are well supported and the following expression may be accepted as a reasonable approximation to the blading loss in practical wheels running enclosed in steam and with "full" clearances:-

$$N_b = \left(\frac{S}{100}\right)^3\left(\frac{d}{10}\right)^2\left(\frac{1}{V}\right)\left\{ b\left[\left(\frac{l_1}{d}\right)^{1.5} + \left(\frac{l_2}{d}\right)^{1.5}\right]\right\} \text{-------- (19).}$$

in which:-

N_b = blade loss due to steam friction - horsepower.
S = mean blade speed - ft. per sec.
d = mean blade ring diameter - inches.
V = specific volume of steam - cub. ft. per lb.
l_1 = blade height of first row inlet side - inches.
l_2 = blade height of last row outlet side - inches.
b = 9.3 for one row wheels.
= 12.5 for two row wheels.
= 17.9 for three row wheels.

The value of the constant for the two row wheel is simply a reasonable estimate suitably intermediate between the other known constants, and guided to some extent by the corresponding figures due to Lasche.

The blade loss equation may be approximately modified by special factors to suit the conditions of partial admission or reverse running. Thus for partial admission the loss as given by (19) should be multiplied by:-

$$(1 - r)$$

where γ = admission arc/ full circumference.

The effect of reverse running is difficult to assess. With blade rows well shrouded there is practically no difference between forward and backward losses; but, generally, the clearances will be such as to make the increase on reversal quite appreciable. The loss calculated from (19) could be multiplied by a figure which will probably lie between 3 and 5; and the value to be chosen will depend on the nature of the clearances. Judging from astern sets in marine turbines the figure is fairly high, as it is difficult otherwise to account for the excessive superheat set up in these casings when the astern turbines are rotated in the ahead direction.

SUMMARY:

By the nature of the subject the discussion of details and evidence is rather laborious, and the following summary of the essential points may be of value.

A theoretical treatment of the subject of wheel friction due to Buckingham (1913) shows that the equation for frictional loss proposed by the Author in 1912 is somewhat irrational in the blade term. Direct correction of this point was easily possible, but certain other differences suggested that a fuller re-examination of the matter was necessary.

These differences centre in two main conclusions reached by Buckingham. The first is that all wheel friction results can be expressed by an equation of the form:-

$$N = A \cdot n^{3-\delta} \cdot d^{5-2\delta} \cdot \rho^{1-\delta} \cdot \mu^{\delta} \text{ --------- } (4).$$

and the second is that the "shape coefficient" A is of the type:-

$$A = a + b\left(\frac{l}{d}\right)^2.$$

This latter result seems to be mainly established by means of Stodola's tests. It is an important point, as it makes the loss increase much more rapidly with blade height than has been hitherto supposed possible.

Equation (4) is of a form permitting direct logarithmic plotting of results - which is the usual method of examination of fluid resistance effects. But the test analyses show that for the complex form of a bladed wheel (4) is insufficiently complete, and logarithmic plotting is really faulty. The double test series are only adequately met by a sum of two such terms as in (4), with different δ values in each term.

By the evolution of the separate terms the balance of the disc and blade losses is indicated. The disc effect seems very closely covered by the expression obtained for it; and the separate consideration given to the blade term shows that the second power of the (ℓ/d) ratio is very improbable, and a value of 1.5 is taken as being more completely justified.

The full consideration of the subject leads to the general equation:-

$$N = \left(\frac{S}{100}\right)^3 \left(\frac{d}{10}\right)^2 \left(\frac{1}{V}\right) \left[a \left(\frac{\sqrt{T}}{\frac{S}{100} \cdot \frac{d}{10} \cdot \frac{1}{V}} \right)^{\delta} \left\{ \left(1 - \frac{\ell_1}{d}\right)^{5-2\delta} + \left(1 - \frac{\ell_2}{d}\right)^{5-2\delta} \right\} + b \left\{ \left(\frac{\ell_1}{d}\right)^{x} + \left(\frac{\ell_2}{d}\right)^{x} \right\} \right] \quad \text{--- (20)}$$

and to the conclusion that, with:-

a = .0105, δ = .25, x = 1.5

and b = 9.3, 12.5, 17.9 for 1, 2, and 3 row wheels respectively the case of enclosed wheels, running with full clearances in dry or slightly superheated steam, is well fitted; while an increase of 12% in the constants a and b would approximately meet the conditions for wheels rotating in the open air.

There are several points of difference between (20) and the Author's original equation due mainly to the various refinements made possible by the present discussion; but, apart from the fault due to the use of blade height instead of blade height-diameter ratio, the earlier form seems to have been as close a representation of the available data as was possible at the time. Even yet its handier form may make it more acceptable for the practical calculation of normal cases.

MISCELLANEOUS INVESTIGATIONS.

NO. 1.

HIGH BLADE STRESSES AT LOW POWERS IN NOZZLE - CONTROLLED IMPULSE TURBINES.

- HIGH BLADE STRESSES AT LOW POWERS -

IN

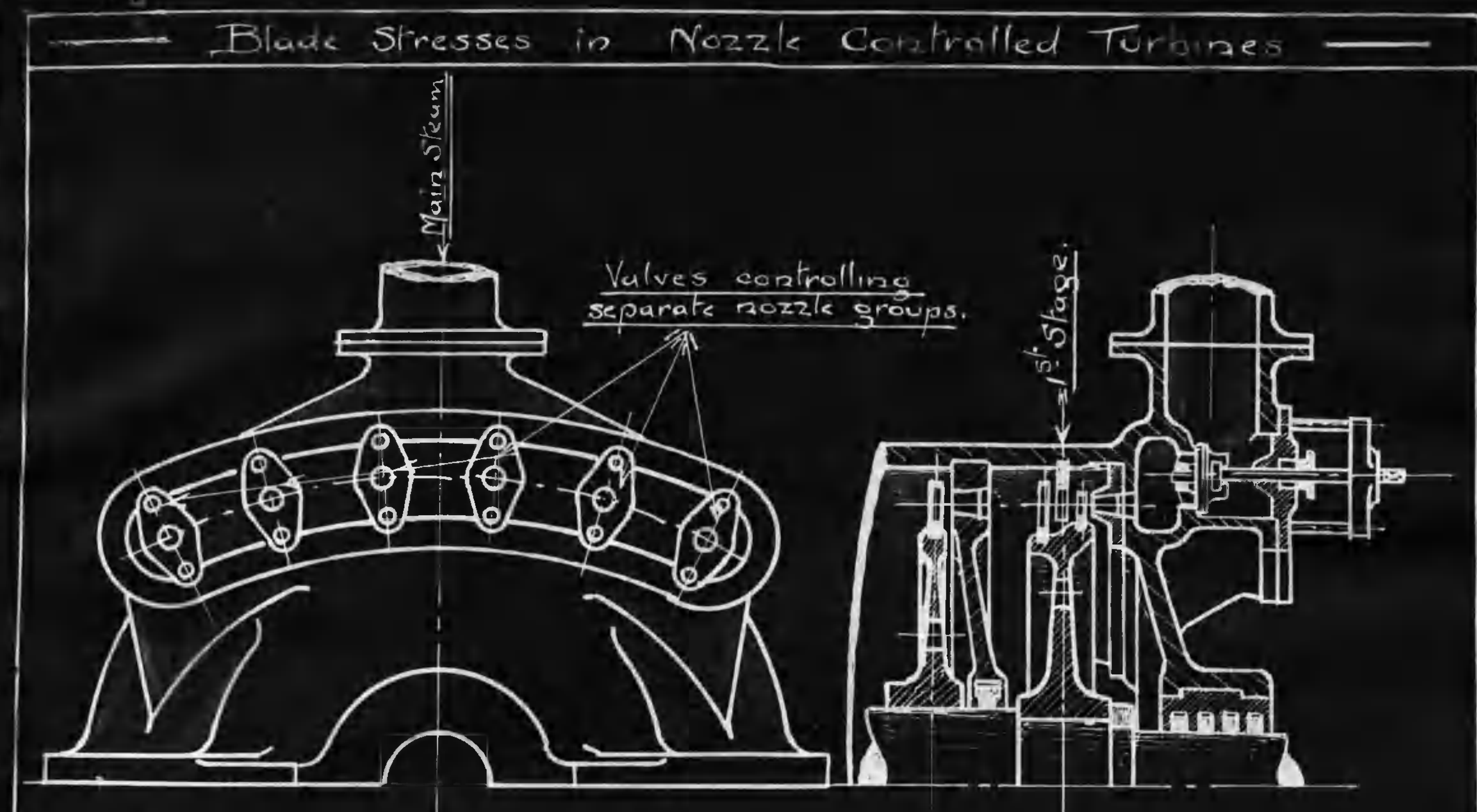
- NOZZLE CONTROLLED IMPULSE TURBINES -

In many large impulse turbines power regulation is attained by controlling the number of nozzles supplying the first stage, and the succeeding stage areas remain unaltered. This means that regulation of power results in corresponding changes in the first stage chamber pressure; whereas the control of the first stage nozzles maintains the full supply pressure. Hence the first stage heat drop is subject to wide variations, which might have important influences on the blading stresses.

The point is one that applies particularly to large power naval sets in which "cruising" conditions are to be met by nozzle control of the main turbines. Here an extreme variation in the first stage conditions of working takes place, and it is really necessary to consider this change in the blading design.

With reduction of power under such circumstances there is reduction in steam flow, with practically proportionate decrease in the arc of admission and, therefore, of the number of blades taking the driving load. But this reduction is accompanied by increased heat drop and, therefore, by greatly increased steam jet speeds. The inference is obvious that the blading loads and, consequently, the bending stresses will be increased. The point is, however, not usually appreciated.

If, in any particular case, power regulation by such means is made at constant speed, the centrifugal stresses are constant throughout the range; and any augmentation of bending stress is an addition to the total stress value; but if, as in marine cases, power and speed changes both occur the centrifugal stresses are reduced with the power and this may provide some margin for increase of bending stress. In all those cases, however, where small high speed H.P. turbines are designed for large powers the blading dimensions,


Blade Stresses in Nozzle Controlled Turbines
Main Steam
Valves controlling separate nozzle groups.
1st Stage.

— Fig. 1. —

even in the first stage, are not inconsiderable and, therefore, at full power the bending stresses may not be entirely negligible; and, if it can be shown that low power working by nozzle control gives large ratio increase in bending stress, it is conceivable that the stress values at low powers may become excessive.

The result pointed out appears almost paradoxical at first sight, but the matter is worthy of definite discussion. An indication of the nature of the effects may be given in a general fashion, as follows.

Fig. 1 shows the kind of arrangement that creates the conditions under review. The nozzles are arranged in groups of varying numbers, so that the possible permutations will allow of the development of almost any power with the full supply pressure. Clearly, the small steam flow through a small group, when operating alone, will mean a low chamber pressure and, therefore, the expansion in the nozzle and gap will be over a wide pressure range.

If M lb. per sec. represents the steam flow under any conditions; P the supply pressure-supposed constant at all powers; and P_2 the chamber pressure; then to a very close approximation:-

$$M = a \cdot P_2. \qquad (a = \text{constant}).$$

and the pressure ratio for the first stage is:-

$$r = P_2 / P.$$

Suppose that DH_ϕ represents the adiabatic heat drop for the ratio r, and f is the stage efficiency, then:-

$$\text{Work per sec.} = M \cdot f \cdot DH_\phi \text{ --- heat units.}$$

If F_t is the total driving force on the blades in lbs, and S the blade speed in feet per sec, then also:-

$$\text{Work per sec.} = F_t \cdot S / 778 \text{ --- heat units.}$$

Now, on the supposition that we are dealing with a one row wheel with n blades in action on the admission arc, it follows that:-

$$n = c \cdot M. \qquad (c = \text{constant})$$

Blade Stresses in Nozzle Controlled Turbines: (Continued)

since the pressure drop will in general be about, or greater than, the critical.

Hence:-

$$\text{Force per blade} = F = \frac{F_t}{n}.$$

$$= \frac{778 \cdot M \cdot f \cdot DH_\phi}{c \cdot M \cdot s}$$

i.e.:-

$$F \propto \frac{f \cdot DH_\phi}{s}.$$

Since the blading dimensions are fixed the bending stresses are as the blade forces; and, hence, denoting full power conditions by accented symbols, and lower power conditions by unaccented symbols, we have:-

$$\frac{F}{F'} = \frac{f}{f'} \cdot \frac{DH_\phi}{DH_\phi'} \cdot \frac{s'}{s}.$$

or, if the speed ratio, blade speed/jet speed = b, then:-

$$\frac{F}{F'} = \frac{f}{f'} \cdot \left(\frac{DH_\phi}{DH_\phi'}\right)^{\frac{1}{2}} \cdot \frac{b'}{b} \quad \text{--------} \quad (1)$$

Equation (1) is directly applicable to a one row wheel, or to the 1st moving row of a two row wheel, if the assumption is made that the one row wheel efficiency applies to the 1st row, - which is reasonable. To apply the result to the 2nd row of a two row wheel necessitates the use of $(e - f)$ in place of f, where e is the efficiency of the two row wheel. Thus:-

$$\frac{F}{F'} = \left(\frac{e - f}{e' - f'}\right) \cdot \left(\frac{DH_\phi}{DH_\phi'}\right)^{\frac{1}{2}} \cdot \frac{b'}{b}. \quad \text{--------} \quad (2)$$

for the blading of a second row. We need not carry the matter further than this since the two row wheel is the most usual first stage type.

The force ratio expressed by (1) or (2) is calculable when the component ratios are known. In any actual case the accented values will be known; hence a knowledge of the variations of the essential quantities with power is required.

For all practical purposes the flow M can be related to the power N by a linear law -- this being a very close approximation in all cases. Since M is directly as P_2, it follows that r is a linear function of N readily determinate in any particular case, i.e.:-

$$r = \frac{P_2}{P} = \ell \cdot N + k. \qquad (\ell \; \& \; k \text{ constants}).$$

Now, by Callendar's equations it may be shown that the adiabatic heat drop - for either saturated or superheated steam - can be expressed as:-

$$DH_\phi = (H_1 - B)(1 - r^m).$$

where H_1 & B are dependent on supply conditions.

Hence:-

$$\frac{DH_\phi}{DH'_\phi} = \frac{1 - r^m}{1 - r'^m} \quad \text{-------------------} \quad (3).$$

Also, since:-

$$DH_\phi = \left(\frac{1 - r^m}{1 - r'^m}\right) DH'_\phi$$

and DH'_ϕ is known, the value of the jet speed u is readily calculated for any value of r.

The blade speed may be either a constant or a variable. If the latter it must be expressible as a function of the power. In the marine case, for instance, it is given very closely by:-

$$s \propto N^{1/3}.$$

Since s and u are both calculable the value of

$$b = \frac{s}{u}. \quad \text{-------------------} \quad (4).$$

is readily obtained for any particular power.

For variation of efficiency we may use the well known parabolic law; and for the 1st row, if we take the maximum possible efficiency as f_1 at ratio b_1, then:-

$$f = f_1 \cdot \frac{b}{b_1}\left(2 - \frac{b}{b_1}\right).$$

and, consequently:-

$$\frac{f}{f'} = \frac{b}{b'}\left\{\frac{2 - b/b_1}{2 - b'/b_1}\right\} \quad \text{-------------} \quad (5).$$

For the two rows together we have, in a similar way:-

$$e = e_2 \cdot \frac{b}{b_2}\left(2 - \frac{b}{b_2}\right).$$

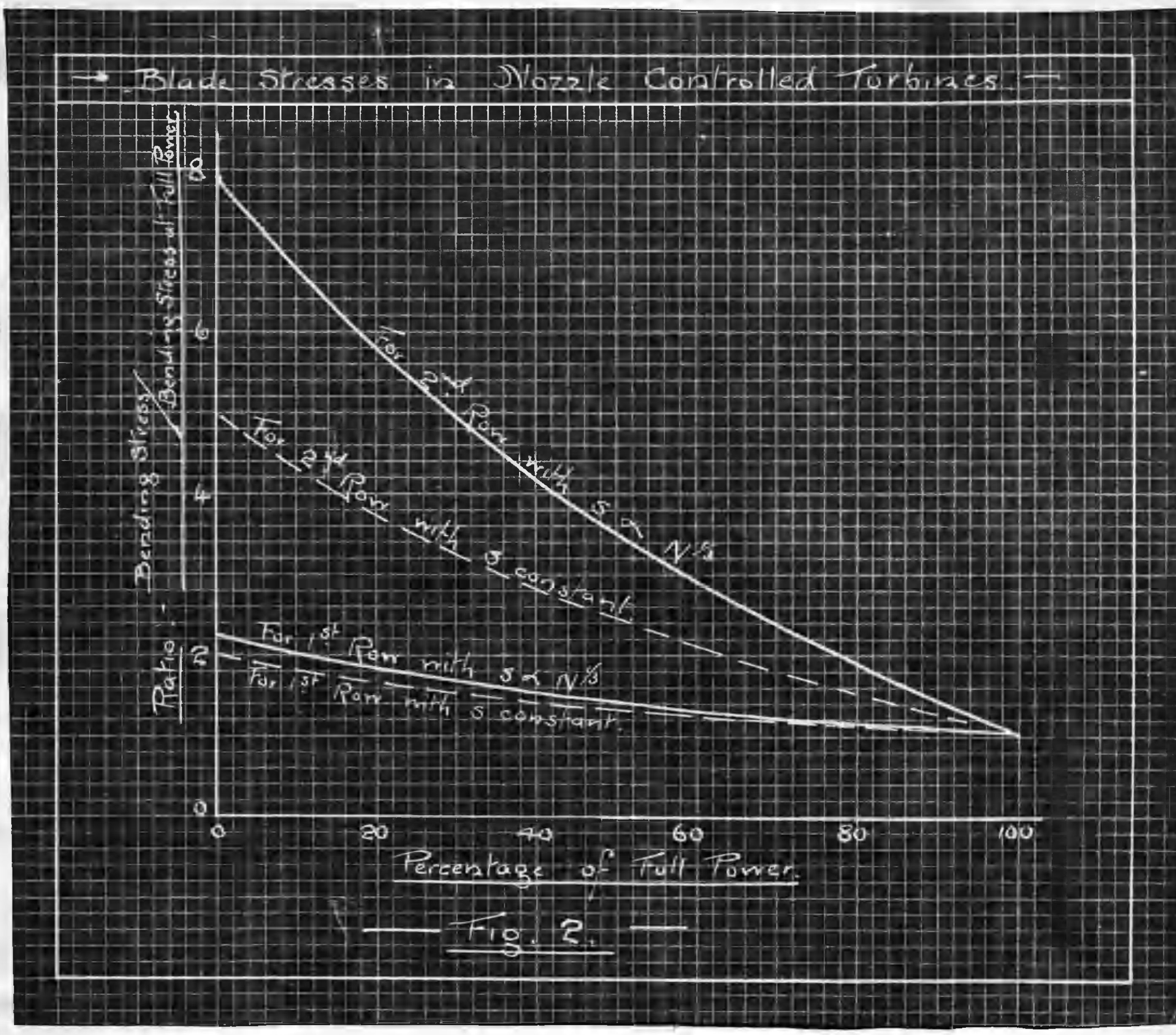

Blade Stresses in Nozzle Controlled Turbines
Ratio - Bending Stress / Bending Stress at Full Power
0
2
4
6
8
0
20
40
60
80
100
Percentage of Full Power
For 2nd Row with s ∝ N⅓
For 2nd Row with s constant
For 1st Row with s ∝ N⅓
For 1st Row with s constant

Fig. 2.

and hence:-

$$\frac{(e-f)}{(e'-f')} = \frac{e_2 \cdot \frac{b}{b_2}\left(2-\frac{b}{b_2}\right) - f_1 \cdot \frac{b}{b_1}\left(2-\frac{b}{b_1}\right)}{e_2 \cdot \frac{b'}{b_2}\left(2-\frac{b'}{b_2}\right) - f_1 \cdot \frac{b'}{b_1}\left(2-\frac{b'}{b_1}\right)} \quad \text{------} \quad (6)$$

The equations (1) and (2) may, therefore, be solved by means of the individual results obtained through equations (3) to (6).

Fig. 2 shows curves for a case so calculated. In this the following values have been taken:-

P = 200 lb per sq. in. abs., Steam initially dry
P_2' = 90 " " " " ", M' = 30 lb/sec.
f_1 = ·82, b_1 = ·45,
e_2 = ·72, b_2 = ·22, $b' = b_2$.

While definite values are necessary to effect calculation it must be noticed that, since only ratios are involved, the results cannot be greatly affected by any reasonable change of conditions. The curves in fig. 2 must, therefore, be applicable with fair generality.

It will be seen from the curves that, while the bending stresses in the first row may increase to twice the full power value, those in the second row may increase to about eight times this figure. Naturally if the full power stress is only a few hundred lb. per sq. inch the variation is of no moment; but if it should be as much as a few thousand lb. per sq. inch the variation is critical.

This possible increase of stress must, therefore, be taken into account in any case in which bending stresses are not entirely negligible at full power; and other objectionable features arise if this increase is great. Thus, let us take the following case in illustration:—

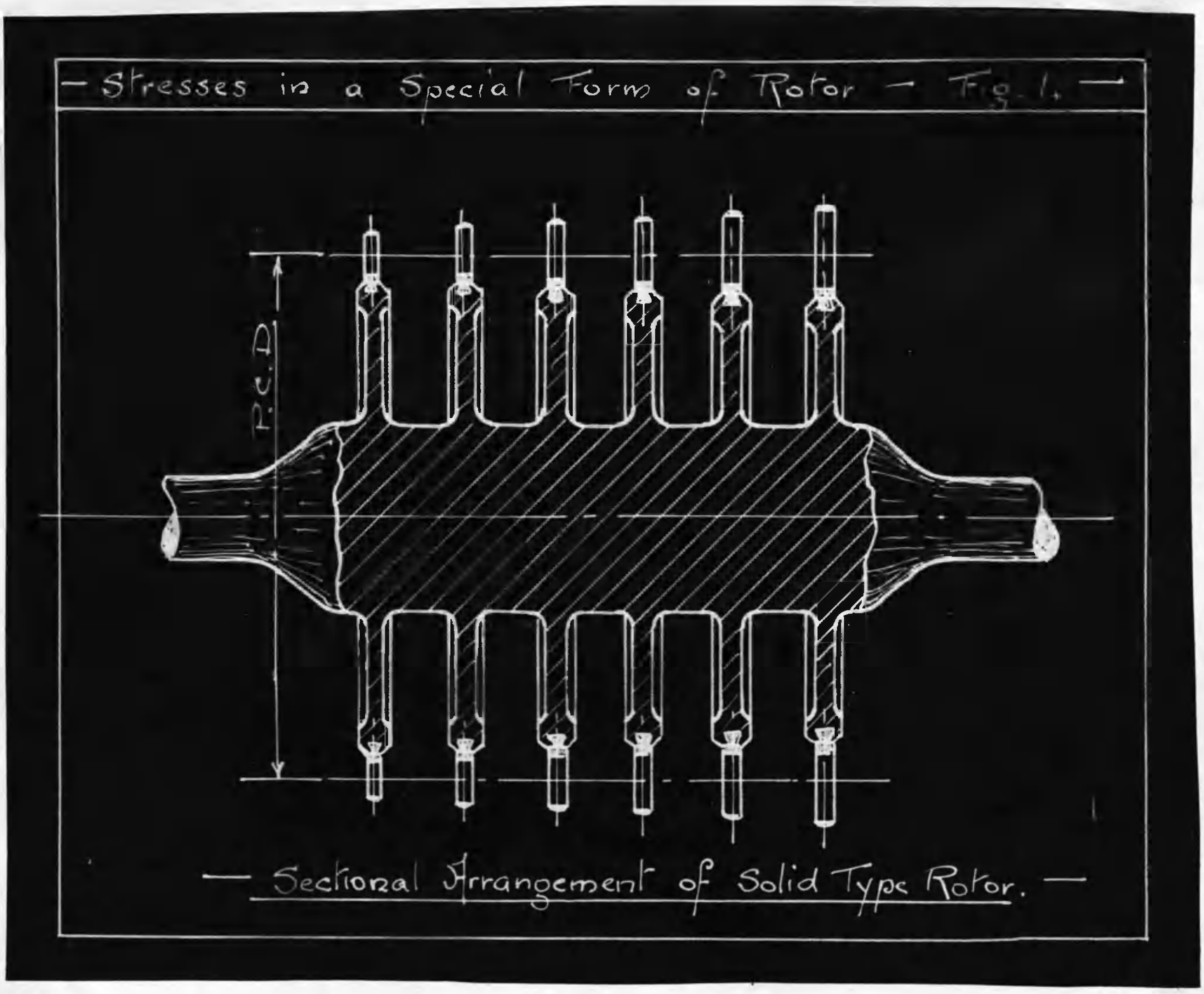
A turbine set developes 10% of full power with 25% of full power flow at 47% of full speed (which is 3000 r.p.m.). To meet this requires the opening of, say, two small groups which are not consecutively arranged. If the second row blades of the first stage had a bending stress of - say - 2500 lb. per sq. in. at full power, then the low power condition would mean a bending stress of about 17500 lb. per sq. in. Now this stress is not merely large; it is fluctuating between zero and the full value some 50 times per sec; and, moreover, this severe action may take place at temperatures at which the blading suffers from reduction of strength.

It is obvious, therefore, that where such circumstances arise severe fatigue action may be created and failure may occur. It is admitted that there is little likelihood of the effect in low power sets where the first stage blade heights are small, and where nozzle grouping is not too fine; but those acquainted with naval designs of high speed H.P. impulse turbines will appreciate the point; and the Author is aware of at least one bad failure that was certainly due to this cause.

MISCELLANEOUS INVESTIGATIONS.

NO. 2.

STRESSES IN A SPECIAL FORM OF TURBINE ROTOR.


- Stresses in a Special Form of Rotor - Fig. 1. -
P.C.D.
- Sectional Arrangement of Solid Type Rotor. -

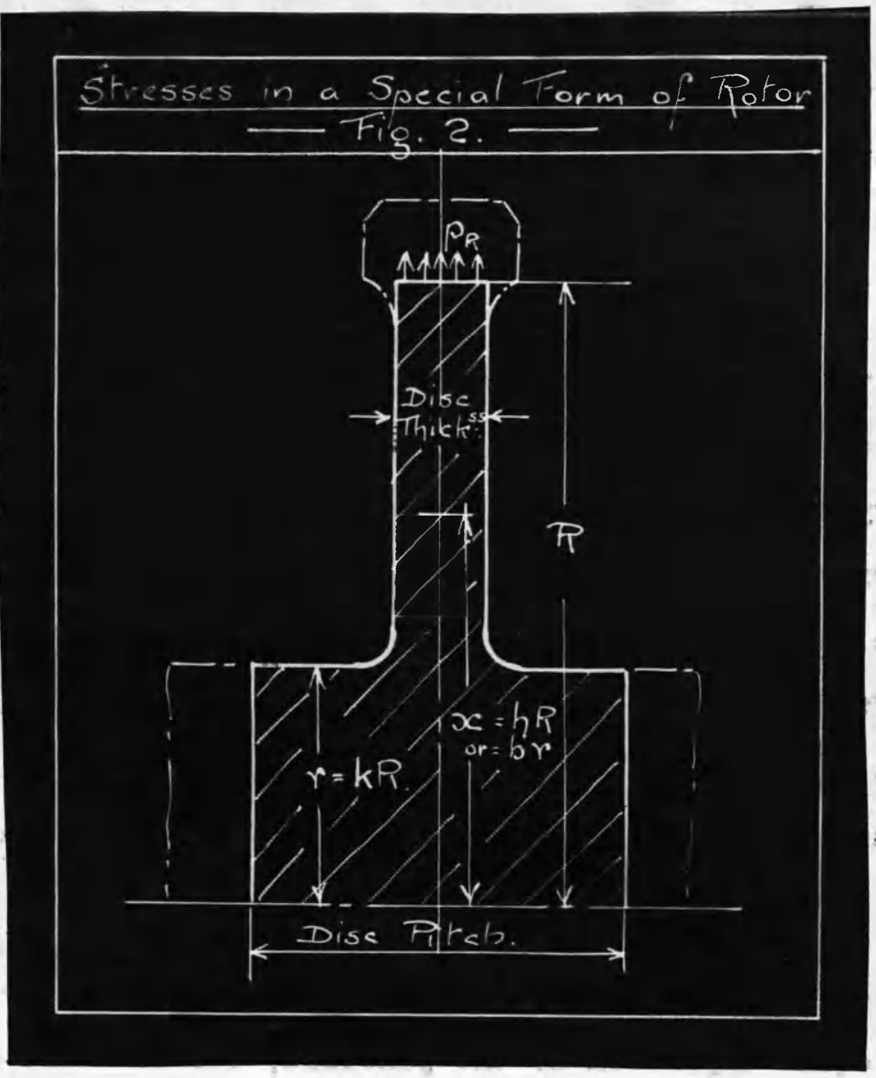
STRESSES IN A SPECIAL FORM OF TURBINE ROTOR

In the development of high power turbine plants for superheated steam the tendency is - particularly in marine work - to employ several units in series. With large reduction gearing the turbine speeds become fairly high and the dimensions are correspondingly reduced. The consequence is that the high pressure and intermediate pressure rotors are quite small, and the ordinary construction of separate wheels forced on to a shaft may be discarded in favour of a single forging, on which the discs are formed by machining. Such a rotor is illustrated by fig. 1.

In the design of single wheels the stress determination in the material of the wheels considers a radial stress on the outer disc periphery due to blade loading, but no radial stress on the inner hub periphery. With the type here considered, however, both kinds must be taken into account, since the disc is in continuity with the shaft. Moreover, the disc and shaft at their point of junction must undergo the same expansion by stress and, therefore, the conditions in the shaft itself have a very important influence.

In the consideration of this case the problem is complicated by the existence of the two different stresses, and by the condition of equal expansion at the junction; but somewhat simplified by the fact that the disc is usually cut of uniform thickness. The investigation is carried out by the usual methods for stresses in rotating wheels and shafts, but the great desirability of basing the design on a single - and maximum - stress value leads to the development of a final form giving this in the shape of an equation for the radial stress at the junction of disc and shaft.

The disc and shaft must first be considered separately, and the results combined by means of the junction condition.



Stresses in a Special Form of Rotor
Fig. 2.

Stresses in a Special Form of Rotor: (Continued)

A uniform disc having central hub and carrying radial stresses on the internal and external peripheries represents the condition for each collar on the rotor; while a shaft length carrying external radial stress represents the corresponding portion of the rotor body.

The problem is thus reduced to the elucidation of the stress in such a form as is given in fig. 2.

Considering the disc portion there is obtained, in the usual way,* by integration of the fundamental differential equations:-

$$\left.\begin{aligned} u &= Ax + \frac{B}{x} - \frac{w}{g}\cdot\omega^2\cdot\frac{m^2-1}{8Em^2}\cdot x^3 \\ \frac{u}{x} &= A + \frac{B}{x^2} - \frac{w}{g}\cdot\omega^2\cdot\frac{m^2-1}{8Em^2}\cdot x^2 \\ \frac{du}{dx} &= A - \frac{B}{x^2} - \frac{3w}{g}\cdot\omega^2\cdot\frac{m^2-1}{8Em^2}\cdot x^2 \end{aligned}\right\} \text{-------} (1).$$

in which:-

u. = expansion at any radius.
A & B. = integration constants.
w. = specific weight of material.
ω. = angular velocity of rotation.
E. = Young's modulus.
m. = 1/ Poisson's ratio.

If we choose the following values for the constants (in order to reduce the repetition of symbols as much as possible):-

w. = $\cdot 283$ — lb per cub. inch.
g. = $32\cdot 2 \times 12$ — ins. per sec. per sec.
E. = 30×10^6 — lb per sq. inch.
m. = $3\cdot 33$.
ω. = $2\pi N/60$ — where N = revs per min.

then:-

$$\left.\begin{aligned} u &= Ax + \frac{B}{x} - \frac{\cdot 304}{10^{13}}\cdot N^2\cdot x^3 \\ \frac{u}{x} &= A + \frac{B}{x^2} - \frac{\cdot 304}{10^{13}}\cdot N^2\cdot x^2 \\ \frac{du}{dx} &= A - \frac{B}{x^2} - \frac{\cdot 912}{10^{13}}\cdot N^2\cdot x^2 \end{aligned}\right\} \text{--------} (1)'.$$

The radial stress may be determined from (1)' in the usual way, and we obtain:-

Radial Stress = $f_r = aA - bB/x^2 - cx^2$ ---- (2).

where:-

$a. = 9\cdot 9 \times 10^6 \times 4\cdot 33.$
$b. = 9\cdot 9 \times 10^6 \times 2\cdot 33.$
$c. = 9\cdot 9 \times 10^6 \times 3\cdot 344\, N^2/10^{13}.$

---

* e.g., see Morley's "Strength of Materials".

Stresses in a Special Form of Rotor: (Continued).

Now let the radial stress at the outer periphery be p_R, and at the inner periphery p_r, then:-

$$\left.\begin{aligned} p_R &= aA - b\frac{B}{R^2} - cR^2 \\ p_r &= aA - b\frac{B}{r^2} - cr^2 \end{aligned}\right\} \quad \text{-------------- (3).}$$

Allowing:-

$$r/R = k \;, \quad \text{and} \quad x/R = h.$$

and solving equations (3), we get:-

$$\left.\begin{aligned} aA &= p_R + ck^2R^2 - \frac{k^2}{k^2-1}(p_R - p_r) + cR^2. \\ bB &= ck^2R^4 - \frac{k^2R^2}{k^2-1}(p_R - p_r). \end{aligned}\right\} \text{------ (4).}$$

and hence from (2):-

$$f_r = p_R\left\{1 - \frac{k^2(h^2-1)}{h^2(k^2-1)}\right\} + p_r \cdot \frac{k^2(h^2-1)}{h^2(k^2-1)} + cR^2\left\{k^2 - h^2 - \frac{k^2}{h^2} + 1\right\} \text{----- (5).}$$

which expresses the value of the radial stress in the disc at any radius x given by $h = x/R$.

Also:-

$$u. = Ax + \frac{B}{x} - lx^3.$$

where:- $\quad l = \cdot 304\, N^2/10^{13}.$

and, by means of (4), this reduces to:-

$$u. = p_R R\left\{\frac{h}{a} - \frac{k^2}{k^2-1}\left(\frac{h}{a} + \frac{1}{bh}\right)\right\} + p_r \cdot R\left\{\frac{h}{a} + \frac{1}{bh}\right\}\frac{k^2}{k^2-1} + cR^3\left\{\frac{h}{a}k^2 + \frac{h}{a} + \frac{1}{hb}k^2 - \frac{l}{c}h^3\right\} \text{--- (6).}$$

The equation for tangential stress in the disc is determined from (1)' and (4), as:-

$$f_t = a \cdot A + b \cdot \frac{B}{x^2} - dx^2.$$

where:- $\quad d = 9.9 \times 10^6 \times 1.922\, N^2/10^{13}.$

hence:-

$$f_t = p_R\left\{1 - \frac{k^2(h^2+1)}{h^2(k^2-1)}\right\} + p_r \cdot \frac{k^2(h^2+1)}{h^2(k^2-1)} + cR^2\left\{k^2 + \frac{k^2}{h^2} + 1 - \frac{d}{c} \cdot h^2\right\} \text{---- (7).}$$

Examination of equations (5) and (7) will show that both kinds of stresses are affected by the imposed peripheral radial stresses and, also, by the speed - which is embodied in the factor c. We know, from ordinary disc equations, that the tangential stress at the inner radius is the higher, so that we require to notice the influence of the inner radial stress in this particular case.

Stresses in a Special Form of Rotor: (Continued)

Now the middle term in f_r is positive, but the corresponding term in f_t is negative, by reason of the negative factor in the denominator; hence the influence of ρ_r is opposite in the two cases. This indicates that the tangential stress tends to lose its importance as the major stress, if ρ_r is really large. The physical conditions are alone sufficient to make clear that ρ_r must be a large figure in any particular example, since both ρ_R and the speed will contribute to it; and, obviously, the binding effect of the shaft must tend to eliminate the tangential stresses near the root. Hence it may be taken that the tangential stresses are in no case so great as the radial stresses. This is, of course, as regards maximum values; it is possible that near the outer periphery equality is approached.

The fact that the tangential stress is unimportant in relation to the radial stress can readily be seen by calculation of any specific case.

From (5) it will be seen that - neglecting the 3rd term - f_r will increase continuously as h diminishes, so long as $\rho_r > \rho_R$, for:-

$$y = \frac{k^2(h^2-1)}{h^2(k^2-1)}.$$

has no turning value, and the terms containing ρ_R and ρ_r give:-

$$\rho_R\cdot(1-y) + \rho_r\cdot y$$

and, if $\rho_r = m\cdot\rho_R$, this becomes:-

$$\rho_R\{1 + y(m-1)\}.$$

Again, the 3rd term in (5) has a maximum for:-

$$h = \sqrt{k}.$$

and is zero both for $h = k$ and $h = 1$. Its total range of value is however not great unless N is excessively high, and consequently the influence of this maximum is not sufficient to change the position of the highest value, due to the first two terms, away from the internal radius r.

It follows, therefore, in a fairly general way that, so long as we are dealing with this class of disc rotor within the limits of size and speed that usually obtain, the maximum stress may be taken as the value of the radial stress, p_r, at the junction of the disc and the shaft.

With the constants evaluated, the equations for the disc are:-

$$\left.\begin{aligned} u &= \frac{p_R \cdot R}{43\times10^6}\left\{\ell - \frac{k^2(\ell^2+1.86)}{\ell(k^2-1)}\right\} + \frac{p_r \cdot R}{43\times10^6}\cdot\frac{k^2(\ell^2+1.86)}{\ell(k^2-1)} + \frac{7.72 \cdot N^2 \cdot R^3}{10^6\times10^6}\left\{\frac{k^2(\ell^2+1.86)}{\ell} + \ell(1-.395\ell^2)\right\} \\ f_r &= p_R\left\{1 - \frac{k^2(1-\ell^2)}{\ell^2(1-k^2)}\right\} + p_r\frac{k^2(1-\ell^2)}{\ell^2(1-k^2)} + \frac{3.3\,N^2 \cdot R^2}{10^6}\left\{k^2 - \frac{k^2}{\ell^2} + 1 - \ell^2\right\} \\ f_t &= p_R\left\{1 + \frac{k^2(1+\ell^2)}{\ell^2(1-k^2)}\right\} - p_r\frac{k^2(1+\ell^2)}{\ell^2(1-k^2)} + \frac{3.3\,N^2 \cdot R^2}{10^6}\left\{k^2 + \frac{k^2}{\ell^2} + 1 - .576\ell^2\right\} \end{aligned}\right\} \text{---} (8)$$

and these give the full expressions for expansion and stresses at any radius in a uniform disc with central hole and applied radial stresses at inner and outer radii.

The case of a solid shaft rotating at N revs per min. and carrying externally applied stress gives the equations* :-

$$\left.\begin{aligned} u' &= \frac{.072\,N^2 \cdot r^3}{10^6\times10^6}\left\{b - .3465\,b^3\right\} + \frac{.0234}{10^6}\cdot p_r' \cdot b \cdot r \\ f_r' &= \frac{3.46\,N^2 r^2}{10^6}(1-b^2) + p_r' \end{aligned}\right\} \text{------} (9)$$

where $b = x/r$, and the accented symbols mark those of similar meaning to the corresponding disc quantities.

It is now necessary to relate p_r and p_r', the radial stresses in disc and shaft respectively at r.

The rotor element in fig. 2. indicates that the radial stress on the shaft at or near r will be reduced well below that active on the inner disc section. Suppose these stresses have the same relation as the disc pitch and the disc thickness - which must be nearly correct at a short distance below the surface - then:-

$$s = \frac{\text{disc thickness}}{\text{disc pitch}}$$

and:-

$$p_r' = s \cdot p_r \text{ ------------------ } (10)$$

* "Centrif. Stresses in Rotors" - Kerr - Proc. R.T.C. Scientific Soc. 1914.

For the necessary junction condition of equal expansions we have:-

$$u' = u, \quad \text{at} \quad x = r.$$

Noting that at this radius:-

$$h = k.$$
$$b = 1.$$
$$r = k \cdot R.$$

substituting in the first of the equations in (8) and (9), equating these, reducing and solving for p_r, we get:-

$$p_r = \frac{N^2 R^2}{10^6} \cdot \left\{ \frac{\cdot 183 (k^2 + 4\cdot 7)(1-k^2)}{5(1-k^2) + (k^2 + 1\cdot 86)} \right\} + p_R \cdot \left\{ \frac{2\cdot 86}{5(1-k^2) + (k^2 + 1\cdot 86)} \right\} \text{-----} (11).$$

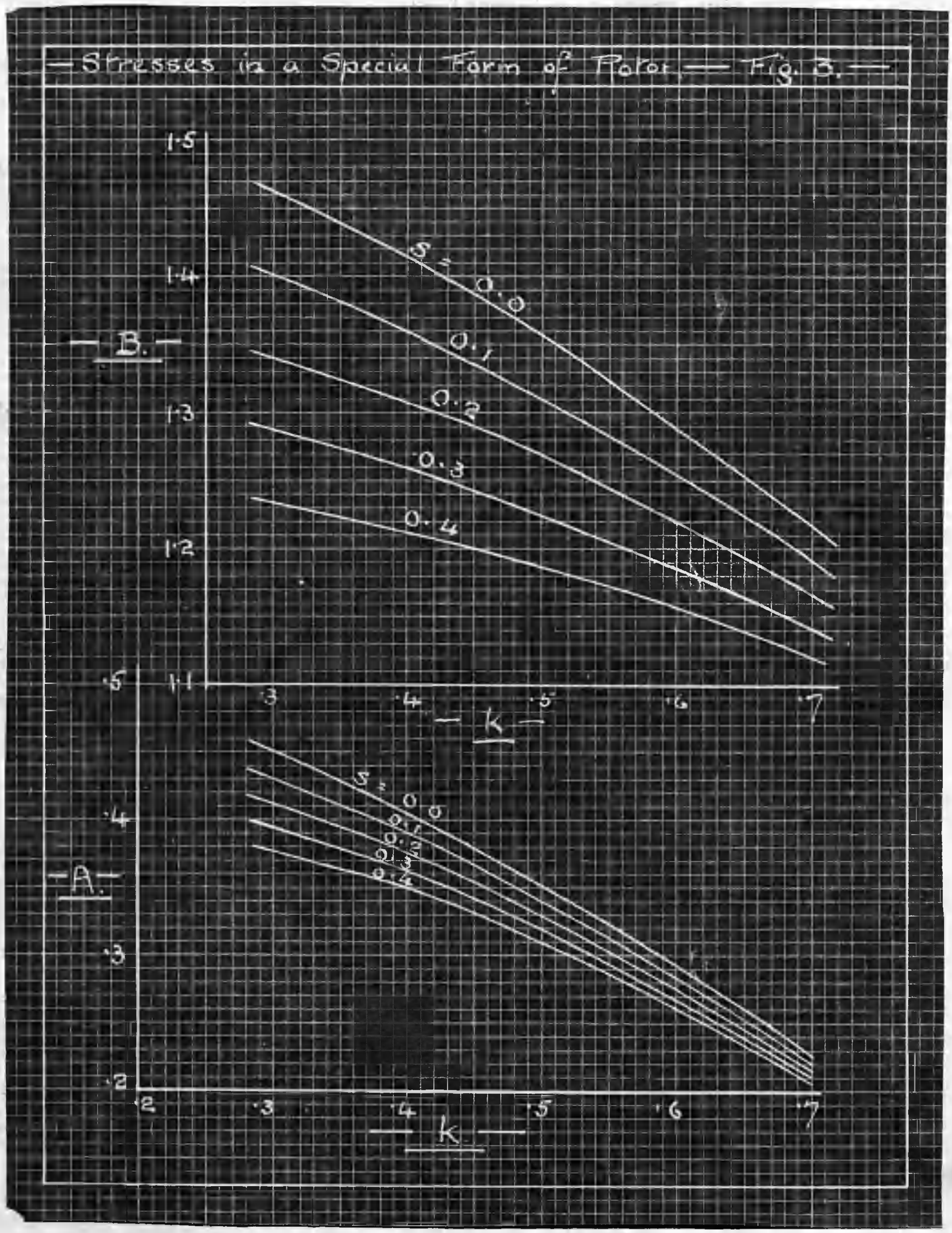
This equation expresses the value of the stress acting between the disc and the shaft and shows it to depend on speed, outer peripheral stress, and relation between shaft and disc radii. All these are known in any given case, since the calculation of p_R due to blade and rim loads is the first step in the determination of a disc stress.

The previous argument has shown that this stress is, in all probability, the maximum for the arrangement and, hence, (11) represents the essential result of the investigation. If it were desired in any case to study the variation of stress (11) would be used first, and the value of p_r substituted in (8) and (9) and these solved for different values of h and b.

In fixing stresses for this type of rotor it is advisable to realise that at the position of this maximum stress value we are well into the body of the forging, and if it were originally of any great size the quality of the material may not be very high. It is, therefore, desirable to keep the stress well below what is allowed for the usual disc wheels - say 50% to 60% of that figure.

Equation (11) may be arranged for rapid calculation by writing it:-

$$p_r = A \cdot \frac{N^2 R^2}{10^6} + B \cdot p_R \text{-----------------} (12).$$


— Stresses in a Special Form of Rotor. — Fig. 3. —
1·5
1·4
1·3
1·2
1·1
— B. —
S = 0.0
0.1
0.2
0.3
0.4
·3
·4
·5
·6
·7
— k —
·5
·4
·3
·2
— A. —
S = 0.0
0.1
0.2
0.3
0.4
·2
·3
·4
·5
·6
·7
— k —

and using graph systems based on k and s, which give A and B directly.

Such a set of curves is given in fig. 3; and by means of this graph the maximum stress in this type of rotor is very readily determined.

MISCELLANEOUS INVESTIGATIONS.

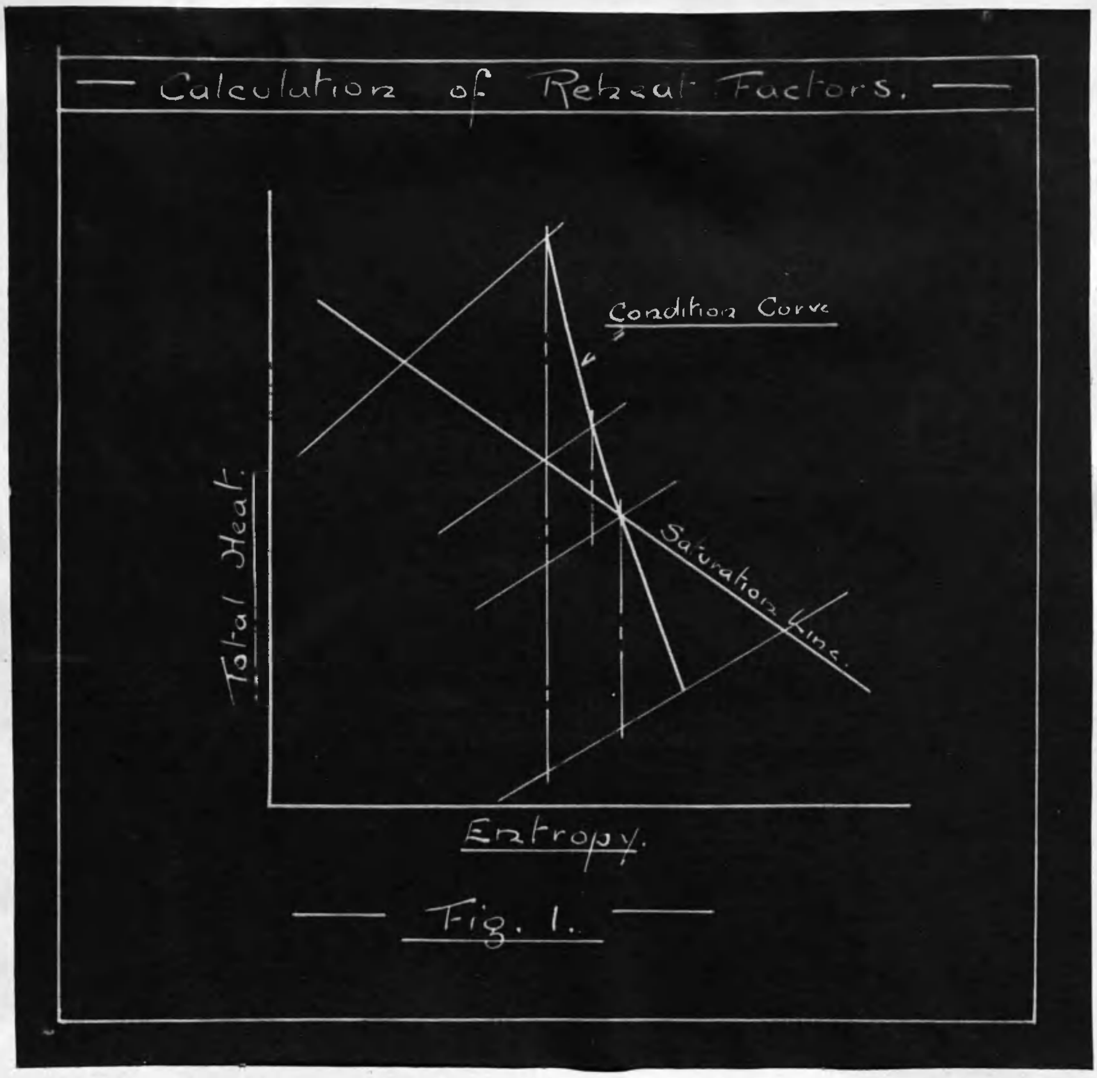
NO. 3.

A

METHOD FOR CALCULATION

OF

REHEAT FACTORS.



— Calculation of Reheat Factors. —

— Fig. 1. —

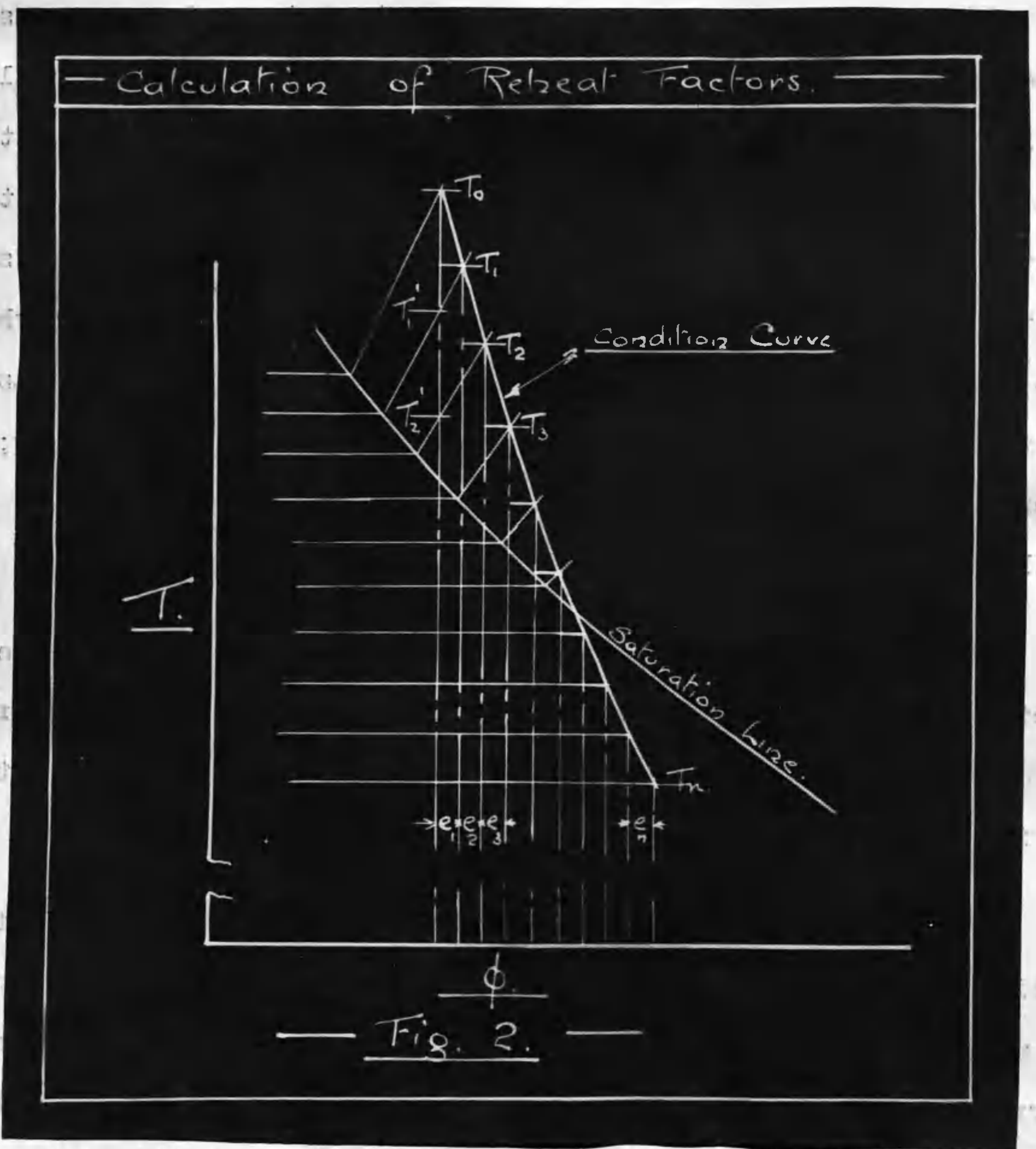
- A METHOD FOR CALCULATION -
OF
---- REHEAT FACTORS ----

In the general and most usual turbine conditions the adiabatic between the initial state point of the steam and the final pressure lies partly in the superheat field and partly in the wet field. The additions on the available heat due to reheating effects are not very readily determined in such a case, as the reheat factors - usually based on pressure ratios - are widely different in the two fields. Moreover, the attempt to isolate the two parts results in an intermediate section for which reheat factors are in no sense definite. The three section subdivision of the range is shown in fig. 1. and it will be seen that, while the reheat factors may be quite certain for the first and last sections, they must be applied to different basis adiabatics; and, even with this elaboration, there remains a central part that is rather awkward to deal with.

Besides this, the use of the ordinary tabulated reheat factors tacitly assumes that a sufficient number of stages are used to make the difference between the actual reheat factor and that for an infinite number of stages negligible.

Both points remarked upon represent weaknesses in the ordinary employment of reheat factors; the first means a complication of work hardly justified by the percentage effect of the reheat corrections; the second is in several instances not applicable. Full range high pressure turbines are now being built with quite a small number of stages; while astern marine turbines never have more than three.

It would seem possible, therefore, that there is a use for an approximate method alike applicable to small and large numbers of stages, and that is practically unaffected by the dual field. One such method - quite accurate enough for all practical design purposes - is outlined here.


Calculation of Reheat Factors.
T_0
T_1
T_1'
T_2
T_2'
T_3
Condition Curve
Saturation Line.
T_n
e_1 e_2 e_3 e_n
T.
φ.
Fig. 2.

Calculation of Reheat Factors: (Continued).

Consider the $T-\phi$ diagram in fig. 2. The condition curve for a stage efficiency f might be as shown; this f value may, as usual, be supposed applicable to every stage, with all stages assumed to have the same available heat.

The stage temperatures are then $T_1, T_2, T_3, ------ T_n$, in degrees absolute, and the reheats added in the different stages may be supposed represented by the rectangular strips shown. These separate additions cause entropy steps $e_1, e_2, e_3 ----- e_n$, and the summation of the strip areas above the lowest temperature line gives the heat made available in the full expansion in excess of that for the perfect Rankine cycle.

If DH_ϕ is the total Rankine heat drop for the full range, and $\mathcal{E}$ the efficiency ratio, then:-

$$(1 - \mathcal{E})\cdot DH_\phi$$

is the extra heat rejected due to the inefficiency.

But since:-

$$f(DH_\phi + h) = \mathcal{E}\cdot DH_\phi$$

where h is the additional available heat, it follows that:-

$$(1-f)(DH_\phi + h) = h + (1-\mathcal{E})DH_\phi$$

i.e.:-

$$h = (1-f)(DH_\phi + h) - (1-\mathcal{E})DH_\phi \quad ------- (1).$$

Now the last term in (1) is - from fig. 2 - simply:-

$$(e_1 + e_2 + e_3 + ---- e_n)T_n.$$

and:-

$$e_r = (1-f)\frac{DH_\phi + h}{n\cdot T_r}$$

which represents the addition of entropy at any stage r, and for the condition of equal drops per stage. (It is obvious that entropy effects will not be greatly influenced by unequal drops per stage since the larger drop will mean practically a correspondingly larger entropy).

Hence:-

$$h = (1-f)(DH_\phi + h) - (1-f)\cdot\frac{DH_\phi + h}{n}\left\{\frac{1}{T_1} + \frac{1}{T_2} + --- \frac{1}{T_n}\right\}T_n.$$

$$\therefore\quad h = (1-f)(DH_\phi + h)\left\{1 - \frac{T_n}{n}\cdot\sum\frac{1}{T}\right\}.$$

Calculation of Reheat Factors: (Continued).

$$\therefore \frac{h}{DH_\phi + h} = (1-f)\left\{1 - \frac{T_n}{n}\Sigma\frac{1}{T}\right\}$$

or
$$\frac{h}{DH_\phi} = \frac{(1-f)\left\{1 - \frac{T_n}{n}\Sigma\frac{1}{T}\right\}}{1 - (1-f)\left\{1 - \frac{T_n}{n}\Sigma\frac{1}{T}\right\}} \quad \text{------ (2)}$$

The reheat factor R is, therefore,:-

$$R = 1 + \frac{h}{DH_\phi} = \frac{1}{1 - (1-f)\left\{1 - \frac{T_n}{n}\Sigma\frac{1}{T}\right\}} \quad \text{---- (3)}$$

although it is better to use at all times the fractional increment h/DH_ϕ expressed in (2).

These results are obviously of a very simple nature and, although approximate, the extent of the approximations is apparent; and not too severe for practical purposes. If T is correctly determined the method as given is exact for the wet field; in the superheat field it is the more correct the larger the number of stages and, therefore, in this respect it has the same feature as the established processes.

Obviously, accuracy depends on the summation of the temperature reciprocals, and since we are dealing with absolute temperatures and with a series of these between definite limits even quite considerable errors in the values of temperature chosen will have quite an inappreciable effect on the final result. Hence we have the important simplification - from the point of view of actual computation - that values read off from the heat-entropy or temperature-entropy chart may be quite rough. This is the direct antithesis of any method based on heat drops, since in most cases it is impossible to deal with these to a sufficient degree of accuracy by means of the chart.

The method is, then, extremely serviceable and quite reliable for the determination of the total reheat factor for expansion from any condition <u>to a final condition in the wet field</u>, and only requires a rough estimate of temperature values. It is not accurate however for expansion in the superheat field alone, unless a certain modification is introduced.

Calculation of Reheat Factors: (Continued).

Let us imagine the problem as it most frequently appears, viz., to determine the reheat factor for any portion of a given expansion range. In the present method it would seem, from the mathematical forms, that it is only necessary to give n the successive values 1, 2, 3, etc. in order to determine the fractional increment or reheat factor for the corresponding range and, therefore, to fix the extra available heat at any specified stage. This is perfectly true so long as that stage has a condition in the wet field, no matter what may have been the starting condition of the expansion; but it is not true as it stands for terminal superheat conditions. This will be seen when it is recognised that the constant pressure lines in the superheat field are not horizontal on the temperature entropy diagram. But for nearly all cases they may be considered straight lines and hence, instead of T_n in the formula:-

$$(1-f)\left\{1-\frac{T_n}{n}\Sigma\frac{1}{T}\right\}.$$

we must use:-

$$(T_n + T_n')/2.$$

where T_n' is the temperature, at the same pressure, on the adiabatic. This is quite a simple matter if temperatures are fixed from inspection of a chart. It must be understood clearly, however, that this mean value is not included in the summation series since that involves stage temperatures only.

Clearly, since T_n and T_n' are the same in the wet field the process can be made perfectly general by writing:-

$$\frac{h}{DH_\phi} = \frac{(1-f)\left\{1-\frac{T_n+T_n'}{2n}\Sigma\frac{1}{T}\right\}}{1-(1-f)\left\{1-\frac{T_n+T_n'}{2n}\Sigma\frac{1}{T}\right\}} \quad ------- (2)'$$

or

$$R = \frac{1}{1-(1-f)\left\{1-\frac{T_n+T_n'}{2n}\Sigma\frac{1}{T}\right\}} \quad ------ (3)'$$

so that by using (2)' and (3)' we have equations that will give fractional increment or reheat factor for any part of a complete expansion range, whether expansion is superheated, supersaturated or wet, or any combination of these; and the method has the special

Calculation of Reheat Factors. —— Table 1.

Case No.	Conditions.	No. of Stages.	Press. abs.		Stage Efficiency	Reheat Factors.			Remarks.
			Initial	Final		By Eqn (3)	Martin Inf. St.	By Calcn	
1.	In Wet Field only. Steam initially dry	23.	200	1	75%	1.047	1.046	-	Stage divisions quite unequal.
2.	In Sup. Field only. Steam initially 300°F sup.	10	100	10	60%	1.096	1.103	-	Temperatures determined from a str. line condition curve drawn for an efficiency ratio of 63%
3.	From Sup. Field to Wet Field. Steam initially 50°F sup.	3.	160.	1.5	70%	1.033	-	1.035	Division into three stages performed very roughly.

merit of being but little affected by approximate determination of the temperature values. Of course, it will be appreciated that there is a liability of a somewhat greater error in superheat field than in wet field calculations, but in no case need the differences cause doubt of the accuracy of the method in practical calculations.

Table I gives a few results showing the agreement achieved with roughly approximate assumptions as to the division into stages.

MISCELLANEOUS INVESTIGATIONS.

NO. 4.

THE PROVISIONAL DETERMINATION OF CRITICAL SPEED.

-- THE PROVISIONAL DETERMINATION --
OF
------ CRITICAL SPEED ------

One of the most important items in turbine design is the determination of the critical speed of the rotor. This is, however, rather a laborious process and demands, for accuracy, a full detail design of the rotor. Now, at the stage at which some knowledge of safety in this respect is required, the design is far from being complete in detail; and a method which can give a provisional figure for the critical speed is highly acceptable. Moreover, in most cases of preliminary designs for estimating purposes the time available entirely forbids any accurate calculation of the standard type, and the suitability of a barely outlined design is frequently judged by simple comparison with previously worked out cases. This is hardly a satisfactory process; and the desirability of having approximate methods allowing of rapid calculation will be obvious.

In preliminary design work the quantities usually approximated to are the speed, stages, spaces and weights. The fundamental questions of blading and nozzle dimensions are usually roughly dealt with. The problem at this point is really to obtain assurance that the speed and weight are permissible for safe running, and give a suitable starting point for the detail calculations.

The treatment here given only envisages the case of the two bearing rotor. This is however, by far the most important example. The development of approximate methods must be accompanied by data which substantiates them, and since the Author is only able to produce data for the two bearing design he considers it necessary to limit the treatment correspondingly.

The critical speed of a two bearing rotor loaded in any fashion is taken in practice as given by:-

$$N_c = \frac{\text{Constant}}{\sqrt{y_m}} \quad \text{---------- (1).}$$

where:-

y_m = maximum static deflection
N_c = critical speed r.p.m.

Provisional Determination of Critical Speed: (Continued)

The critical speed is closely akin to the frequency of free vibrations; in fact, neglecting the rotatory inertia of the wheel masses the two are alike. Since, therefore, it is permissible to use the elastic curve of static loading in the determination of the natural period it should be allowable to employ the same characteristic for critical speed. The use of the static deflection is, therefore quite sound although originally introduced into this calculation in quite an empirical way; earlier methods used the loading and curves due to centrifugal action, which is strictly the correct method. The justification for the use of the static process rests, however, on theoretical grounds, as it comes from a general principle in the theory of vibrations which states that the period is stationary for small changes in the mode of vibration.

The use of equation (1) is, then, a great simplification, and means that a critical speed calculation is reduced to a deflection determination. Let us examine here the values that the constant in (1). would have for the simplest cases.

Consider a uniform shaft loaded centrally with a single mass W. If the deflection under the load is y and the force of restitution is $F\cdot y$ then, for rotation at ω radiams per sec., it is simple to show that y becomes infinite for:-

$$\omega = \sqrt{\frac{F\cdot g}{W}}.$$

Hence:-

$$N_c = \frac{30}{\pi}\sqrt{\frac{F\cdot g}{W}}.$$

Now for this case the static deflection under the load, i.e., the maximum static deflection is:-

$$y_m = \frac{W}{F}.$$

$$\therefore\ N_c = \frac{30}{\pi}\sqrt{\frac{g}{y}} = \frac{187}{\sqrt{y_m}} \text{ - - - - - - - - - - } (2).$$

So that the constant for the single centrally placed load is 187.

Consider now a uniformly loaded shaft and apply the condition that the critical speed is the same as the frequency of free transverse vibrations. Then, equating kinetic energy in passing through the unstrained state to the strain energy at the full amplitude, we get:-

$$N_c = 60 \times 1 \cdot 57 \sqrt{\frac{g \cdot E \cdot I}{w \cdot l^4}}$$

where the symbols have the usual meanings.

But the maximum static deflection is:-

$$y_m = \frac{5}{384} \cdot \frac{w \cdot l^4}{E \cdot I}.$$

Hence:-

$$N_c = 60 \times 1 \cdot 57 \sqrt{\frac{5}{384} \cdot \frac{g}{y_m}}$$

$$= \frac{210}{\sqrt{y_m}} \quad \text{-----------------} \quad (3).$$

Although these two cases are really very simple, all practical cases lie between the limits so established. Hence we should expect that the constant applicable generally to practical constructions lies between 187 and 210. This is shown by the following values which represent the constants used by different builders.

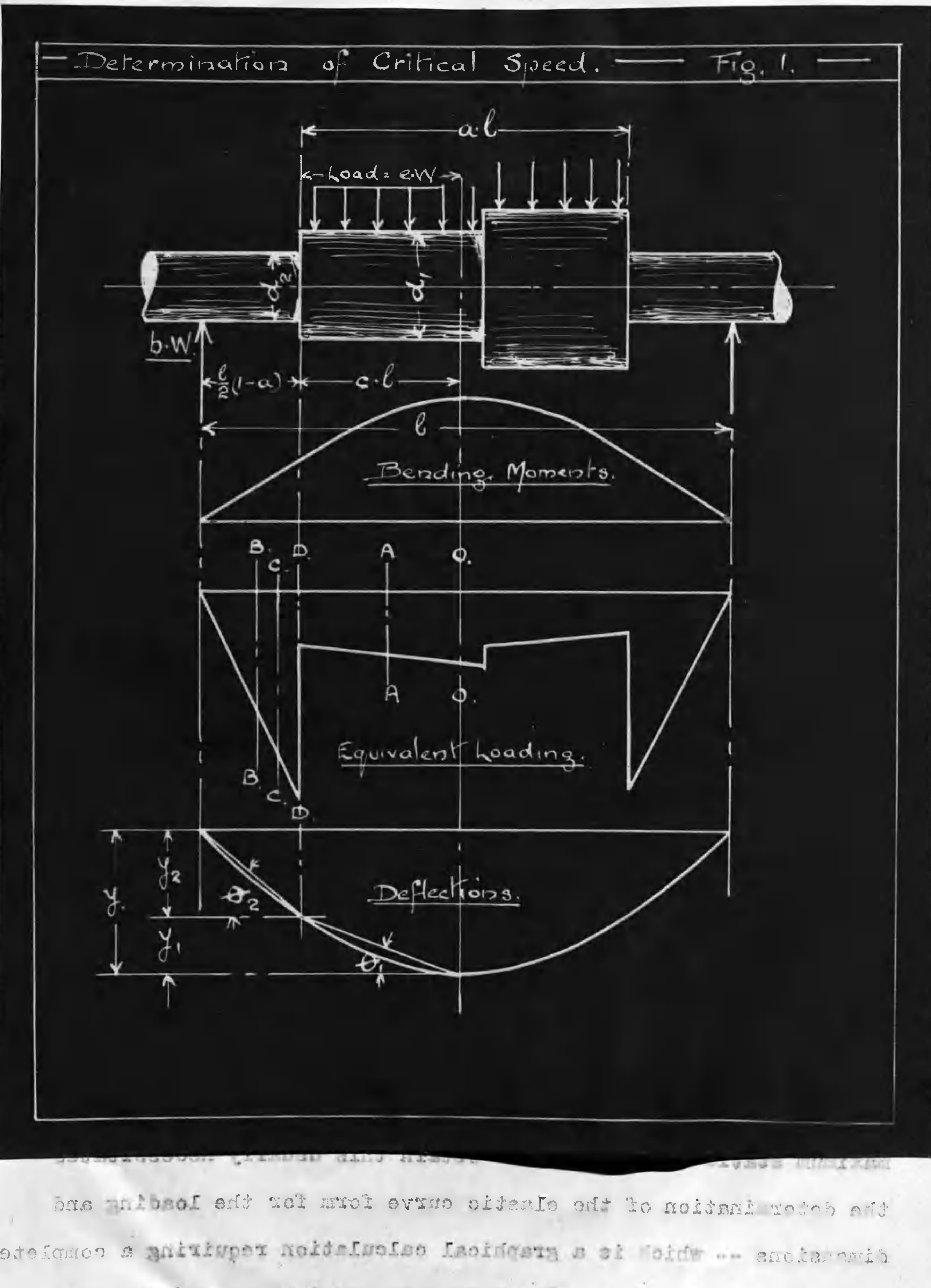
Mr K. Baumann (Metropolitan-Vickers):	Constant	=	195.
Parson & Co.	"	=	200.
Brown-Curtis	"	=	187.

The first of these should be the most suitable, as Mr Baumann established it by direct comparison with correctly investigated cases. The others have probably been influenced by the results of such simple calculations as are outlined above.

It is therefore permissible to calculate the critical speed from the formula:-

$$N_c = \frac{195}{\sqrt{y_m}} \quad \text{---------------} \quad (4).$$

in all cases and, consequently, the only figure required is the maximum static deflection. To obtain this usually necessitates the determination of the elastic curve form for the loading and dimensions -- which is a graphical calculation requiring a complete knowledge of all sizes. The method to be given here assumes a

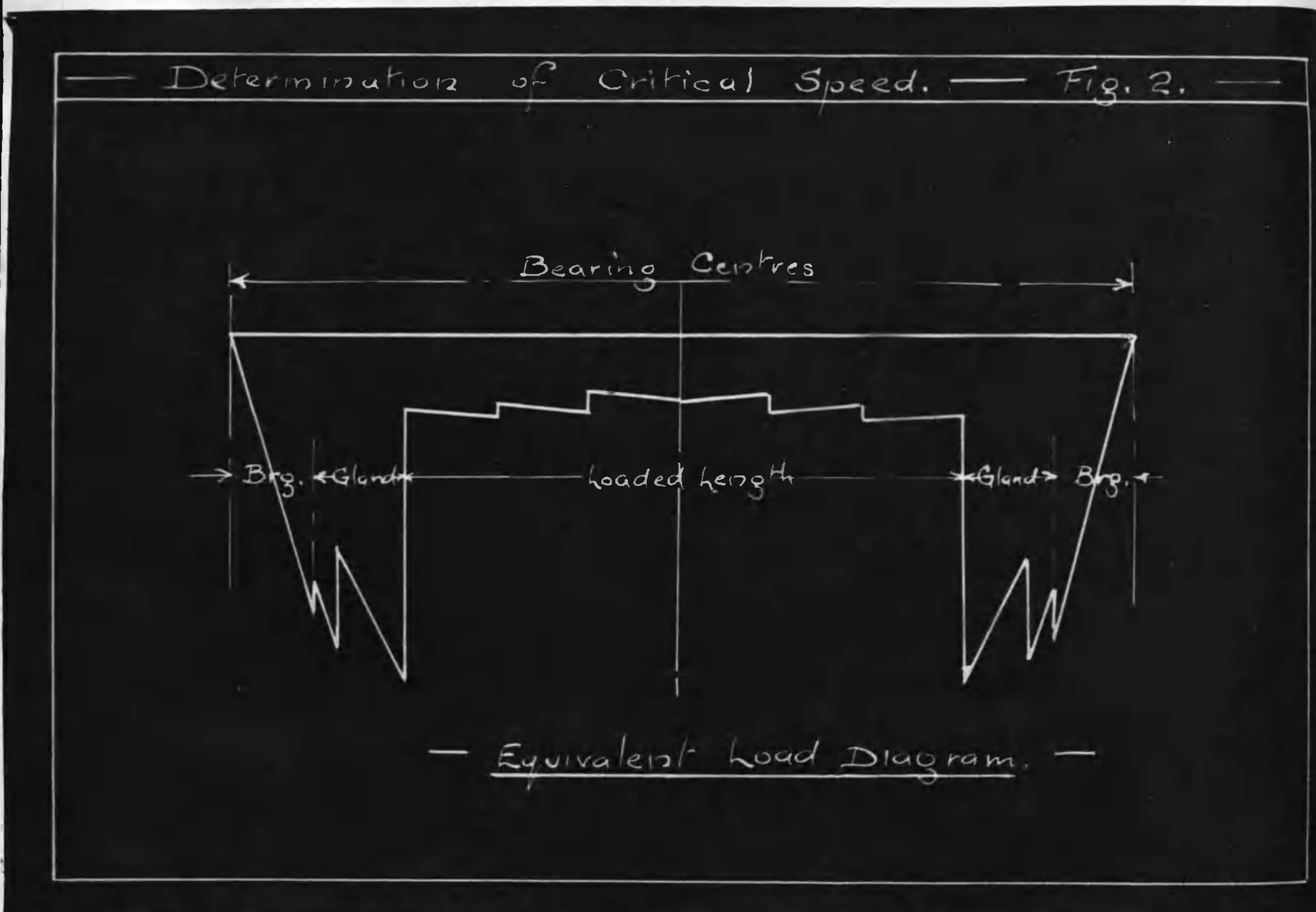

Determination of Critical Speed. — Fig. 1.
a·l
Load = e·W
d2
d1
b·W
l/2(1-a)
c·l
l
Bending Moments.
B. C. D. A O.
A O.
B. C. D.
Equivalent Loading.
Deflections.
y.
y2
y1
θ2
θ1

general knowledge of overall weights and lengths, and certain type factors which will depend on the constructional form. From these an approximation to y_m is deduced, and it should be noticed that even a 10% error in y_m only gives a 5% error in N_c. Since the running speed must give a margin of 25-30% on the critical speed, errors of this magnitude are permissible in a provisional calculation. The main desire is to ensure that the general features of the design are suitable before proceeding to details; and that any discrepancy in the final and accurately determined figure can be met by moderate adjustments.

Different turbine types vary somewhat in the forms of the rotor bodies but, generally, the loading of the shaft is partial only, being set on a long central length of relatively large size. Outside this central section there are unloaded, and nearly equal, lengths of small diameter extending through the gland and bearing spaces.

Variations in type are shown mainly by the distribution of the loading and diameters in the central part. The impulse turbine can in most cases be readily reduced to a single central diameter loaded very uniformly; the reaction turbine may show two or more entirely different diameters and load intensities.

Suppose, then, that we consider the rotor reduced to the form shown by fig. 1, in which a loaded part of $a \cdot l$ in length carries the total load W on two diameters. Generally, the position of maximum deflection will not depart much from the central point of the span and may be taken as represented in fig. 1. We are to assume that for any given type this position has been roughly established by the factor c, which gives the distance from the end of the loaded section as $c l$. The weight over the same distance is $e \cdot W$ and the reaction from the bearing on the same side is $b \cdot W$. The factors b, c and e are, therefore, the type factors, and it would seem necessary to know these for any given type of body. Actual use of the method to be developed will show, however, that



Determination of Critical Speed. — Fig. 2.

<u>Provisional Determination of Critical Speed</u>: (<u>Continued</u>).

these factors need not be studied with any great accuracy; the assumption of symmetry in the rotor about the central plane gives results that are surprisingly close, even in rotors that depart considerably from such a condition.

The factors referred to are related to the position of maximum deflection for the following reason.

By ordinary bending theory:-

$$\frac{dy}{dx} \propto \int \frac{M}{I} \cdot dx$$

or, the slope of the elastic curve at any point is proportional to the area of the "equivalent loading" diagram between that point and the position of zero slope. Obviously, therefore, the natural origin to work from is that at which the slope is zero, i.e., that at the maximum deflection.

Referring now to fig. 1 it will be seen that the total deflection may be considered as made up of the two parts, y_1, and y_2; and that the slope must undergo a quick change in passing from the heavy to the light section. These partial deflections may be supposed known, with sufficient accuracy, when the average slopes θ_1 and θ_2 in the corresponding lengths are obtained, since :-

$$y_1 = \theta_1 \times c \cdot l.$$
$$y_2 = \theta_2 \times \frac{l}{2}(1 - a).$$

The slopes may be determined by proportionality with the equivalent loading areas embraced between axes OO and AA, and between axes OO and BB; the axes AA and BB being taken at the middle of the respective lengths. Judged only from fig. 1. it might seem more accurate to take these axes at positions which would bisect the <u>areas</u> in order to get closer to the mean slopes; but reference to fig. 2 which shows an equivalent loading diagram of the more usual form will demonstrate that the procedure chosen is the better in the circumstances. The matter is studied from fig. 1 for simplicity, but fig. 2 has to be kept in mind as the reality.

Provisional Determination of Critical Speed: (Continued).

The necessary development may now be made as follows. The bending moments at AA and at CC are:-

$$M_A = b\cdot W\left\{\frac{l}{2}(1-a) + \frac{c\cdot l}{2}\right\} - \left(\frac{e\cdot W}{2}\times\frac{c\cdot l}{4}\right).$$

$$= \frac{W\cdot l}{2}\left\{b(1-a) + c\left(b-\frac{e}{4}\right)\right\}.$$

$$M_C = \frac{W\cdot l}{2}\left\{\frac{3}{4}\cdot b(1-a)\right\}.$$

in which W is the total load on the span length l. The value of the "equivalent" loads at the same positions are, therefore:-

$$\left(\frac{M}{I}\right)_A = \frac{W\cdot l}{2\cdot I_1}\left\{b(1-a) + c\left(b-\frac{e}{4}\right)\right\}.$$

$$\left(\frac{M}{I}\right)_C = \frac{W\cdot l}{2\cdot I_2}\left\{\frac{3}{4}\cdot b(1-a)\right\}.$$

in which I_1 and I_2 are themoments of inertia corresponding to the diameters d_1 and d_2 in fig. 1. The various areas required are, then, approximately:-

$$\overline{OOAA} = \left(\frac{M}{I}\right)_A\times\frac{c\cdot l}{2} = \frac{W\cdot l^2}{4\cdot I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\}.$$

$$\overline{OODD} = \left(\frac{M}{I}\right)_A\times c\cdot l = \frac{W\cdot l^2}{2\cdot I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\}.$$

$$\overline{DDBB} = \left(\frac{M}{I}\right)_C\times\frac{l}{4}(1-a) = \frac{W\cdot l^2}{8\cdot I_2}\left\{\frac{3}{4}\cdot b(1-a)^2\right\}.$$

$$\overline{OOBB} = \overline{OODD} + \overline{DDBB}.$$

$$= \frac{W\cdot l^2}{2\cdot I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\} + \frac{W\cdot l^2}{8 I_2}\left\{\frac{3}{4}b(1-a)^2\right\}.$$

Hence:-

$$\theta_1 \propto \frac{W\cdot l^2}{4 I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\}.$$

$$\theta_2 \propto \frac{W\cdot l^2}{2\cdot I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\} + \frac{W\cdot l^2}{8 I_2}\left\{\frac{3}{4}\cdot b(1-a)^2\right\}.$$

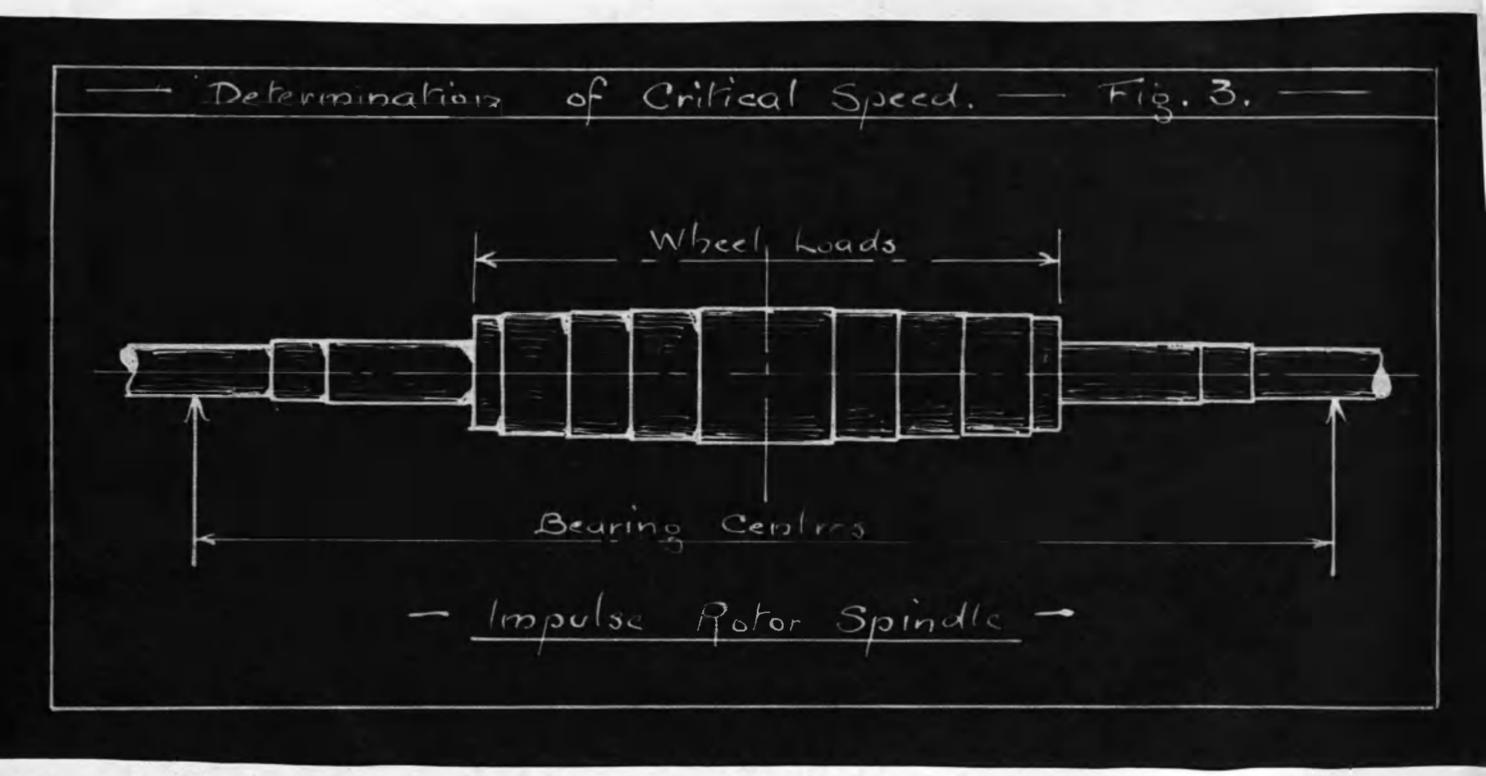
and, therefore, for the partial deflections:-

$$y_1 \propto \frac{W\cdot l^3}{4\cdot I_1}\left\{c^2\cdot b(1-a) + c^3\left(b-\frac{e}{4}\right)\right\}.$$

$$y_2 \propto \frac{W\cdot l^3}{4\cdot I_1}\left\{c\cdot b(1-a) + c^2\left(b-\frac{e}{4}\right)\right\}(1-a) + \frac{W\cdot l^3}{16\cdot I_2}\left\{\frac{3}{4}\cdot b(1-a)^3\right\}.$$

If $I_1 = n\cdot I_2$, then - on simplification - we may write for the maximum deflection:-

$$y_m \propto \frac{W\cdot l^3}{32\cdot I_2}\left[\frac{8\cdot c}{n}(c-a+1)\left\{b(1-a) + c\left(b-\frac{e}{4}\right)\right\} + \frac{3}{2}\cdot b(1-a)^3\right] \quad \text{---} (4)$$



— Determination of Critical Speed. — Fig. 3. —

— Impulse Rotor Spindle —

Provisional Determination of Critical Speed: (Continued).

In general, therefore, we may write:-

$$y_m = \frac{k}{I_2} \cdot W \cdot l^3 \left(\frac{A}{n} + B \right). \quad ⑤.$$

where:-

k = constant independent of type of central section.
$A = 8c(c-a+1)\{b(1-a) + c(b - e/4)\}$.
$B = \frac{3}{2} b (1-a)^3$.

The value of a will be naturally decided in the preliminary determination of lengths. The values of b, c and e will depend on the nature of the central section, and are supposed fixed for any particular type; as a matter of fact, they must always be near the values:- $b = \frac{1}{2}$, $c = \frac{a}{2}$, $e = \frac{1}{2}$

and, if definite values are unknown for any particular construction complying with the general form considered, these approximations may be employed with confidence.

There is, however, one particular type of rotor for which the above values of b, c and e are correct, viz., the impulse rotor loaded throughout the central section at a nearly uniform rate per unit length; and having diameters throughout this length that do not differ widely. In that case - which is shown by fig. 3 - the substitution of the above values gives the following:-

$A = a(2-a)(1 - 5a/8)$.
$B = \frac{3}{4}(1-a)^3$.

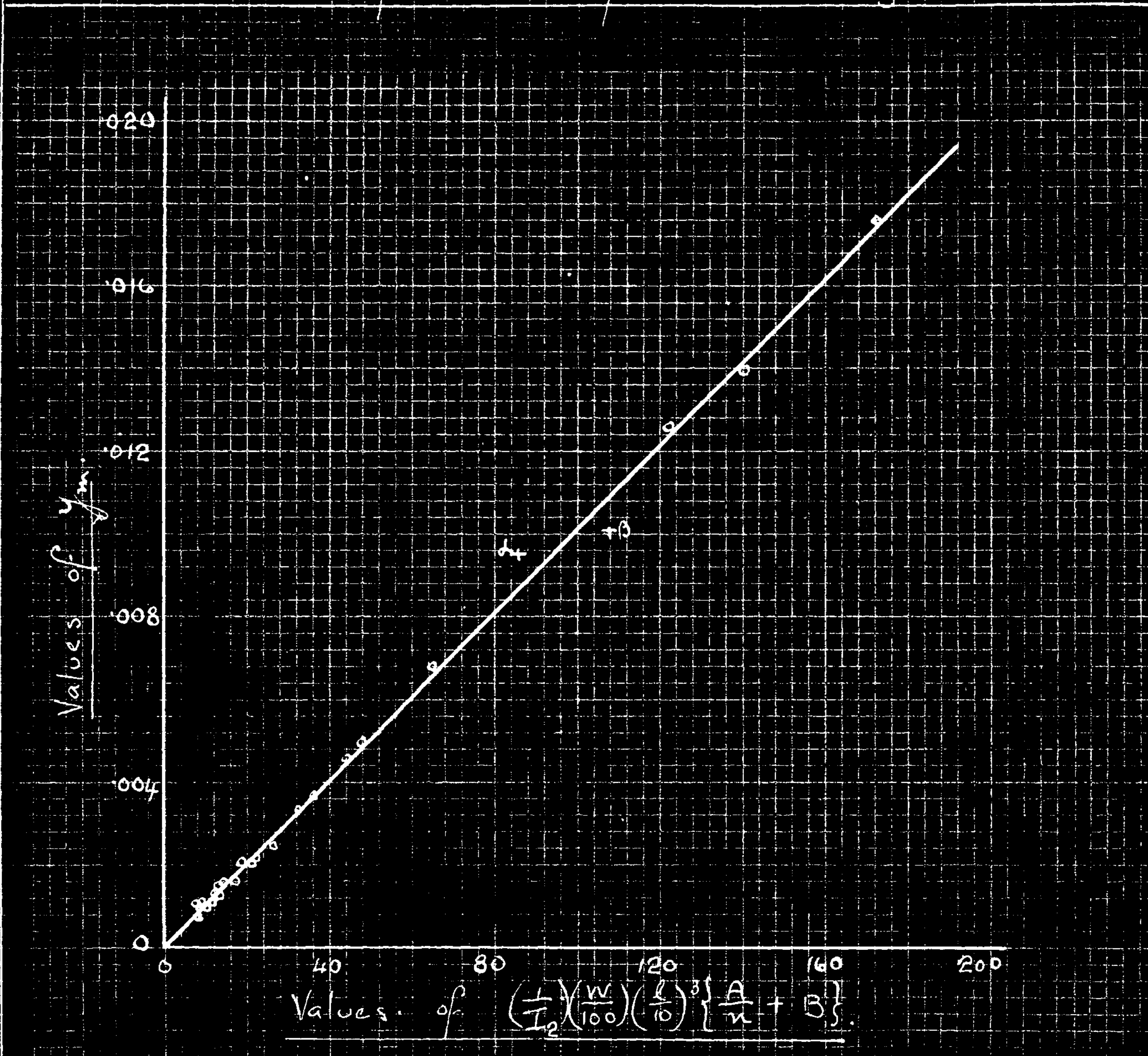
and this is an important case for which verification of the formula is forthcoming.

For convenience in calculation the equation may be written:

$$y_m = \frac{k}{I_2} \cdot \frac{W}{100} \cdot \left(\frac{l}{10} \right)^3 \left(\frac{A}{n} + B \right) \quad ⑥.$$

in which:- y_m = maximum static deflection in inches.
W = total weight of rotor between bearing centres-lb.
l = bearing centres--ins.
n = I_1/I_2.
$A = 8c(c-a+1)\{b(1-a) + c(b - e/4)\}$ - for unsymmetrical central parts.
$B = \frac{3}{2} \cdot b(1-a)^3$ " " " "
$A = a(2-a)(1 - 5a/8)$. for symmetrical " "
$B = \frac{3}{4}(1-a)^3$ " " " "
k = constant applicable to either type.

— Determination of Critical Speed — Fig. 4. —



Provisional Determination of Critical Speed: (Continued).

The value of k may readily be determined by plotting a few results derived from accurate graphical calculation of examples of one type. Fig. 4 shows quite a large number of values so determined for fairly symmetrical impulse rotors; the deflections obtained from the graphical processes being plotted against the expression in equation (6) with the simpler A and B values.

The agreement, it will be admitted, is exceedingly close, and holds over a very wide range as the plotted points in fig. 4 are for rotors varying in weight between about 400 lbs and 16 tons. This graph gives:-

$$k = 1/10^4.$$

and this may be taken as generally applicable.

There are two points in fig. 4 - marked α and β - which are definitely off the line. These points represent the results of applying the simpler form of (6) to reaction turbine rotors of unsymmetrical type. Both cases had loaded lengths comprised of two entirely different diameters and carrying loads of different intensities. The value of an equivalent uniform I_1 for the central section was roughly fixed by means of:-

$$I_1 = 2 \cdot I_1' \cdot I_1'' / (I_1' + I_1'').$$

where I_1' and I_1'' are the moments of inertia of the two different portions. It is clear that this only amounts to a roughly approximate use of an approximate equation for deflection, and yet the error in the critical speed derived from the deflection so calculated would not be more than 4 - 5%. Obviously, therefore, the method which has been developed is quite useful, and must be fairly close when the type factors are known with some certainty. Naturally, the determination of these factors for any particular form must be left to those who are dealing with that form, but the use of (6) for rotors that are arranged in fair symmetry about the central plane requires no special knowledge.

The applicability of the method is further illustrated by the agreement with the actual deflections plotted in fig. 4,

Provisional Determination of Critical Speed: (Continued).

as the points represent cases of high pressure turbines loaded regularly along the central section; intermediate pressure turbines which carry a heavy astern wheel near one end with a short unloaded gap between the ahead and astern sections; and low pressure turbines with a long unloaded gap between the ahead and astern wheels. From this it follows that the maximum deflection is not greatly affected by considerable departure from the condition of uniform loading rate on the central length. This fact is also shown by the rough agreement reached in the case of the reaction rotors since at no stage of the approximations involved has any attention been given to the different rates of loading.

In the application of (6) it is only necessary to realise that we must reduce to single diameters in the loaded and unloaded sections respectively. Generally, there are certain changes in size in each part - such as the step up from a bearing diameter to a gland diameter; or the continuous increase in the body diameter to facilitate wheel fitting. The reduction may usually be made by inspection although in the unloaded section the nature of the investigation will show - if considered - that it is advisable to give rather more weight to the gland diameter than to the journal diameter in fixing the mean value. A few applications to any one type will soon give facility in the necessary reduction.

This investigation provides a good example of how a very serviceable rule results from an approximate method of attack, provided the method rests on a sound process and does not ignore essentials other than those that can be covered by suitable factors, or are reasonably ineffective in their variations.