Essays on Optimal Monetary Policy under Rule-of-thumb Behaviour by Price Setters

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Submitted in fulfilment of the requirements for the

Degree of Doctor of Philosophy

Department of Economics

Faculty of Law, Business and Social Sciences

University of Glasgow

December 2009

Abstract

The aim of this thesis is to study the effects of inflation persistence due to rule-of-thumb behaviour by price setters on optimal monetary policy. We start with a canonical log-linearised New Keynesian model, which we extend by allowing a fraction of price setters to follow a rule-of-thumb when setting a new price. We consider different specifications for the rule-of-thumb. In all models, steady-state distortions are assumed to be small so to guarantee the feasibility of optimal monetary policy analysis within a linear-quadratic framework. We derive utility-based objective functions for the monetary authority and analyse a range of optimal commitment policies. We perform welfare analysis in order to rank the range of optimal commitment policies. We analytically derive the optimal steady-state inflation rates associated with each commitment policy. We show that rule-of-thumb behaviour by price setters generates an incentive for positive steady-state inflation. A type of timeless perspective commitment policy is also capable of delivering positive steady-state inflation, even in the absence of rule-of-thumb behaviour by price setters. The optimal steady-state inflation rates are directly proportional to the gap measuring the steady-state distortions and turn out to be small in magnitude.

We depart from the assumption of small steady-state distortions and consider the case of a largely distorted steady state within a nonlinear medium-scale model, which adds both nominal rigidities and real rigidities to the basic New Keynesian model. We extend the model by allowing a fraction of price setters to follow a rule-of-thumb when posting a new price. We numerically characterise the optimal rate of inflation in the Ramsey steady state. We find that rule-of-thumb behaviour implies optimal positive inflation only in the absence of transactional frictions. We find that the gap reflecting steady-state distortions is only slightly larger than in the case of small steady-state distortions. Finally, we study Ramsey dynamics and the implementation of optimal monetary policy via simple interest-rate rules, which we expand to explore the importance of welfare-relevant output gaps.

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Acknowledgements

I would like to express my gratitude to my supervisors, Professor Campbell Leith and Doctor Ioana Moldovan, for their guidance and advice.

No word can express my gratitude to Eleni and my family.

Declaration

The material contained in this thesis has not been previously submitted for a degree in this or any other university.

CHAPTER 1

Introduction

The purpose of this thesis is twofold. First, it aims to study how rule-of-thumb behaviour by price setters affects optimal monetary policy in an otherwise basic small-scale log-linearised New Keynesian model. In doing so, we follow Woodford (2003) as we derive a utility-based policy objective function and subsequently analyse the policy problem in a linear-quadratic framework. Second, maintaining the presence of rule-of-thumb price setters, it aims to the extend the analysis of optimal monetary policy to the medium-scale economy developed in Altig et al. (2005). We characterise Ramsey-optimal monetary policy using the methodology and algorithms developed in Schmitt-Grohé and Uribe (2004b, 2007).

New Keynesian economics embeds nominal rigidities and imperfect competition into the dynamic general equilibrium framework of the Real Business Cycle paradigm. In its basic formulation, a New Keynesian model features one nominal rigidity modelled in terms of a constraint on the frequency of price setting. Three main models of nominal price rigidity are used in the literature: the quadratic price adjustment cost model of Rotemberg (1982), the random price adjustment signal model of Calvo (1983), and the model of staggered contracts of Taylor (1980). The Calvo model is the most widely used in the literature for reasons of tractability and we maintain the assumption of sticky prices à la Calvo (1983) throughout this thesis. However, the analysis carried out in this thesis applies equally well to the other models of nominal price rigidity since, as shown in Rotemberg (1987) and Roberts (1995), they all lead, up to a log-linear approximation, to the same form of aggregate-supply relation.

In its basic formulation, a New Keynesian model derived from a discrete-time version of the Calvo price setting model is purely forward-looking[1]. Inflation dynamics are described by what Roberts (1995) labels the New Keynesian Phillips curve (NKPC henceforth)[2]. The NKPC relates inflation today to a measure of excess demand and expected future inflation. On the one hand, the appealing features of the NKPC are well known. First, it is microfounded in the idea that monetary non neutrality is due to nominal price rigidities. Second, it recognises the importance of inflationary expectations in the determination of inflation today as firstly stressed by Friedman (1968) and Phelps (1968). Third, it is simple enough to be useful for theoretical monetary policy analysis. As a result, the New Keynesian model derived from a discrete-time version of the Calvo price setting model has become the workhorse for much research on monetary policy and it has been described by McCallum (1997) as "*the closest thing there is to a standard specification*"[3].

On the other hand, the failures of the NKPC are equally well-known as discussed in Mankiw (2001). First, as initially pointed out by Ball (1994), it implies costless disinflation, namely no short-run trade-off between output and inflation. Second, as pointed out by Fuhrer and Moore (1995) it fails to capture the empirical fact that inflation is highly persistent. These two problems imply that a disinflation of any size could be achieved costlessly and immediately by a central bank that could commit to set the path of future output gaps equal to zero. Third, evidence from VAR studies also show that the response of inflation to shocks is “hump-shaped” rather than front loaded as prescribed by the NKPC. Studies which seek to estimate the NKPC find that it fits the data poorly (e.g. Fuhrer and Moore (1995), Fuhrer (1997), Nelson (1998), Galì and Gertler (1999), Roberts (2005), and Sbordone (2005)). These studies frequently reject the NKPC in favour of a

[1]The first use of a discrete-time version of Calvo’s model of price setting, in the context of dynamic general equilibrium model, is in the work of Yun (1996). Other early applications of the same device inlcude King and Watson (1996), King and Wolman (1996), and Goodfriend and King (1997).

[2]Aggregate-supply relation and Phillips curve are used interchangeably in this thesis.

[3]The survey article by Clarida et al. (1999) and the landmark work by Woodford (2003) are only two examples among many others.

hybrid Phillips curve, which entails that current inflation depends on both inflationary expectations and past inflation, although the estimated relative values of the forward-looking component and the backward-looking component vary greatly between studies.

To accommodate the persistence in inflation data, two main variants of the basic New Keynesian model have been put forward in the literature. Both variants generate the dependence of current inflation not only on expected future inflation but also on lagged inflation by making an additional assumption about the price setting mechanism. The first variant is due to Galì and Gertler (1999). They assume that a proportion of the firms randomly assigned to reoptimise their prices in the Calvo model do not behave rationally but follow a rule-of-thumb[4]. Specifically, the rule-of-thumb prices are a weighted average of the optimal forward-looking prices set in the previous period plus an adjustment based on lagged inflation. The second variant is due to Christiano et al. (2005). They assume that the firms not assigned to reoptimise their prices will instead index their prices to lagged inflation.

In this thesis we consider backward-looking rule-of-thumb behaviour by price setters.

Rule-of-thumb behaviour by price setters is appealing for at least five reasons. First, it involves virtually no computational burden: all that is needed is for rule-of-thumb price setters to observe last period's output and/or price setting decisions. Second, it involves passive learning of the behaviour of forward-looking optimising price setters. Third, it implies convergence among individual choices once the effects of all shocks are eliminated from the economy. Fourth, Galì and Gertler (1999) provide empirical evidence of the presence of rule-of-thumb behaviour in price setting[5]. Fifth, using backward-looking price indexation as a modeling strategy is less appealing as it implies that all prices are revised at every point in time, which not only contradicts empirical evidence that some

[4]An earlier example of the utilization of this type of assumption in order to better explain the deviations of actual behavior from the predictions of models which assume fully rational agents is Campbell and Mankiw (1989). They use this type of assumption to explain the relation between consumption and income.

[5]Once we take the theoretical economies to the data, we carefully review the empirical evidence provided by Galì and Gertler (1999).

prices are fixed for a certain amount of time in nominal terms (e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)) but also clashes with the rationale as to why models with nominal price rigidity were developed. Moreover, while the implications of backward-looking indexation are thoroughly analysed in Woodford (2003), the consequences of rule-of-thumb behaviour on the optimal long-run inflation rate have, to the best of our knowledge, not been analysed previously in the literature[6].

In characterising optimal monetary policy under rule-of-thumb behaviour by price setters, we depart from the widespread practice in New Keynesian economics of restricting the attention to models in which the deterministic steady state is efficient. The Pareto efficiency of the deterministic steady state is achieved by assuming the existence of subsidies which eliminate the steady-state distortions originating from monopolistic competition[7]. This widespread practice has two potential shortcomings. First, the instrument necessary to eliminate steady-state distortions (i.e. subsidies financed by lump-sum taxation) is empirically uncompelling. Second, it is ex ante not clear whether a policy that is optimal for an economy with an efficient steady state remains optimal for an economy where the steady state is distorted.

For these reasons, we do not make the efficient-steady-state assumption but instead work with models whose steady state is distorted. This implies that three equilibrium levels of output coexist in the model: 1) the actual level output, which obtains in the presence of both nominal rigidities and monopolistic competition; 2) the natural level of output, which obtains in the presence of monopolistic competition and in the absence of nominal rigidities; and 3) the efficient level of output, which obtains in the absence of both nominal rigidities and monopolistic competition.

[6]Rule-of-thumb behaviour has been extensively used to investigate various issues: responses to supply shocks (Steinsson (2003)), monetary policy rules (Amato and Laubach (2003)), uncertainty (Kimura and Kurozumi (2007)), and open economy (Kirsanova et al. (2007)).
[7]Steady-state and long-run are used interchangeably in this thesis.

For the purpose of the first aim of this thesis, we consider the case of small steady-state distortions as discussed in Woodford (2003). The degree of inefficiency of the deterministic steady state is assumed to be minimal so that it can be treated as an expansion parameter. This in turn guarantees that it suffices to approximate the equilibrium of the model to first order to obtain a second-order accurate measure of welfare. Steady-state distortions introduce a gap between the the natural level of output and the efficient level of output. This wedge is constant and invariant to shocks so that the "*divine coincidence*" in Blanchard and Galì (2007) holds[8]. However, allowing for steady-state distortions matters for the optimal average levels of inflation and output, namely for the deterministic description of optimal monetary policy. This is because, from a welfare point of view, the constant-over-time gap between the natural level of output and the efficient level of output appears in the central bank's utility-based loss function. In other words, the combination of general equilibrium foundations and steady-state distortions provides a microfoundation for targeting a level of output above the natural level of output.

Following the theoretical literature on optimal monetary policy, we assume that the central bank's policy instrument is the short-term nominal interest rate. The combination of a cashless economy and the central bank's control of the nominal interest rate implies that the economy is fully described by the aggregate-supply relation and the central bank's objective function. This model of central bank behaviour then allows determining analytically the long-run inflation rate associated with a given policy. In particular, as stressed by Woodford (2008) "*The fact that the equations are log-linearized does not mean that one simply assumes an average inflation rate; the equations allow one to derive the average inflation rate corresponding to a given policy*". We consider three theoretical economies: 1) the purely forward-looking New Keynesian model; 2) the model with rule-of-thumb behaviour à la Galì and Gertler (1999); and 3) the model with rule-of-thumb behaviour à la Steinsson (2003). All models are small-scale New Keynesian models as they feature

[8]That is, percentage changes in the natural level of output correspond in the log-linear approximation to percentage changes in the efficient level of output.

only one nominal rigidity (i.e. price stickiness) and one real rigidity (i.e. monopolistic competition in product markets). We consider different types of optimal commitment policy that have been proposed in the literature: the zero-optimal policy, the timeless perspective commitment policy in Woodford (1999), and the alternative timeless perspective commitment policy put forward by Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008). Our preference for commitment is based on our focus on analysing the optimal long-run inflation rate when the deterministic steady state is distorted[9].

For the purpose of the first aim of this thesis, three results stand out from the literature concerning optimal monetary policy in log-linearised New Keynesian models. First, rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), breaks the optimality of zero long-run inflation found in New Keynesian models. Indeed, within New Keynesian economics, the optimality of a monetary policy that aims at zero inflation is surprisingly robust. Full price stability is optimal despite the inefficiency of the nonstochastic steady state and the existence of a positively sloped long-run Phillips-curve trade-off. Moreover, as shown in Woodford (2003), zero long-run inflation is also robust to the presence backward-looking price indexation. Rule-of-thumb behaviour, regardless of its specification, implies that the stimulative effect of higher inflation is greater than the output cost of higher inflation thus generating a long-run incentive for positive inflation under an optimal commitment. Second, a type of timeless perspective commitment policy is also capable of delivering positive steady-state inflation, even in the purely forward-looking New Keynesian model. Third, all the optimal long-run inflation rates are directly proportional to the gap between the natural level of output and the efficient level of output. Hence, what we show here is that the widespread assumption of an efficient steady state is not innocuous: a policy that is optimal for an economy with an efficient steady state does not remain optimal in an economy where the

[9] Of course, discretionary conduct of monetary policy would result in the well-known inflation bias stressed by Kydland and Prescott (1977) and Barro and Gordon (1983).

steady state is distorted. Moreover, the positive long-run inflation rates turn out to be small in magnitude for empirically realistic values of the models' structural parameters.

For the purpose of the second aim of this thesis, we consider the medium-scale model developed in Altig et al. (2005). This model emphasises the importance of combining nominal as well as real rigidities in explaining business-cycle fluctuations. Specifically, the model features four nominal rigidities, sticky prices, sticky wages, a transactional demand for money by households, and a cash-in-advance constraint on the wage bill of firms, and four real rigidities, investment adjustment costs, variable capacity utilisation, habit formation, and imperfect competition in product and labour markets. We extend the model by allowing a fraction of price setters to behave in a rule-of-thumb manner à la Galì and Gertler (1999). We depart from the assumption of small steady-state distortions and consider the case of a largely distorted steady state. We characterise both the Ramsey steady state and Ramsey dynamics and address the question of implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules. In doing so, we use the algorithms developed in Schmitt-Grohé and Uribe (2004b, 2007). Specifically, large steady-state distortions imply that to obtain a second-order accurate measure of welfare it does not suffice to approximate the model's equilibrium conditions up to first order. In characterising interest-rate rules, we use the methodology and the algorithm developed in Schmitt-Grohé and Uribe (2004b) for second-order approximations to policy functions of dynamic and stochastic models.

As for the Ramsey steady state, the key policy problem faced by the central bank is the trade-off between the stabilisation of the degree of price dispersion and the stabilisation of transactional frictions, which calls for the Friedman rule, namely a deflation which is consistent with a zero nominal interest rate. We find that the results in Schmitt-Grohé and Uribe (2007), who consider the possibility of backward-looking price indexation, generally hold. Rule-of-thumb behaviour by price setters does not alter the high sensitivity of the long-run inflation rate with respect to the degree of price stickiness: the optimal long-run inflation is always negative and it varies between the level implied by the Friedman rule and a level close to price stability. We depart from the analysis in Schmitt-Grohé and Uribe (2007) and consider the case of a cashless medium-scale macroeconomic model[10]. Indeed, we seek to establish a link between the analysis of optimal monetary policy in a basic log-linearised New Keynesian model and its counterpart in a much richer nonlinear theoretical economy. We find that, in the absence of transactional frictions, rule-of-thumb behaviour by price setters entails that the optimal long-run inflation in the steady state of the Ramsey equilibrium is positive. We thus obtain the same result that we analytically derive in the linear-quadratic framework. Indeed, the optimal long-run inflation rate is not only positive but also small in magnitude. In the linear quadratic framework the inflation rate is directly proportional to the gap between the natural level of output and the efficient level of output. Hence, we solve the social planner problem in the medium-scale economy so to compare the steady-state gap between the social planner level of output and the Ramsey level of output with the steady-state efficiency gap in the log-linearised small-scale economies. We find that the difference between the two is in fact rather small.

We study Ramsey dynamics. In doing so, we are interested in addressing two issues. First, we want to assess whether the zero lower bound on the nominal interest rate constitutes an impediment to optimal monetary policy. Indeed, one argument against setting a negative inflation rate, as recommended by the model in the presence of money demand by households and firms, or a near-zero inflation rate, as recommended by the cashless model, is that at negative or near-zero rates of inflation the risk of incurring in the zero lower bound on nominal interest rate would restrict the central bank's ability to stabilise the economy. We find that this argument is of no relevance in the context of both the model with money and its cashless counterpart. The reason for this is that under the Ramsey-optimal policy, the zero lower bound poses an impediment to monetary policy only in the case of an adverse shock that forces the interest rate to be roughly 8 standard

[10]Schmitt-Grohé and Uribe (2007) go on to analyse the optimal long-run rate of inflation by taking into account the fiscal side of the optimal policy problem. They do so by replacing the assumption of lump-sum taxes with the assumption of distortionary income taxes. The optimal long-run inflation, although remaining always negative, is then found to be much closer to price stability.

deviations below its mean. The probability of this happening is so small that the zero lower bound on the nominal interest rate does not impose an economically important constraint on the conduct of optimal monetary policy. Second, we characterise the Ramsey-optimal impulse responses to the three shocks that drive aggregate fluctuations. Specifically, we present the responses of key macroeconomic variables and we focus on how the Ramsey planner uses monetary policy to respond to each of the three shocks. We show how the Ramsey-optimal stabilisation policy is robust to the presence/absence of money in the model.

Finally, we consider the implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules, using the methodology and the algorithm developed in Schmitt-Grohé and Uribe (2004b) for second-order approximations to policy functions of dynamic and stochastic models. Initially, we show how the implementation of optimal monetary policy is virtually unaffected by the presence-absence of money. We characterise the operational interest-rate rule, which is defined exactly as in Schmitt-Grohé and Uribe (2007), in both the medium-scale model with money and its cashless counterpart. In both cases, the optimal operational interest-rate rule is confirmed to be active in price and wage inflation, mute in output growth and moderately inertial. We also consider a modification of the operational interest-rate rule, which prescribes a concern not for output growth per se but for stabilisation of output around a welfare-relevant measure of output, namely the gap between the Ramsey level of output and the efficient level of output. We find that the optimal operational interest-rate rule remains active in price and wage inflation and moderately inertial, but also implies a positive coefficient on output stabilisation. Regardless of the presence/absence of money in the model, it is optimal for a central bank to stabilise output gap, namely the log-difference between the level of output and the efficient level of output.

The remainder of the thesis is organised as follows.

Chapter 2 lays out a basic New Keynesian model that we extend by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner, specified either à la Galì and Gertler (1999) or à la Steinsson (2003).

Chapter 3 presents utility-based objective functions for the central bank. We extend the analysis in Woodford (2003) by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner. The backward-looking behaviour is specified in two ways: in the manner of Galì and Gertler (1999) and in the manner of Steinsson (2003).

Chapter 4 studies what constitutes optimal monetary policy in the three theoretical economies. We consider different types of optimal commitment policy that have been proposed in the literature: the zero-optimal policy and two types of timeless-perspective policy. Our preference for commitment is based on our focus on analysing the optimal long-run inflation rate when the steady-state is distorted.

Chapter 5 discusses the calibration of the models' structural parameters, evaluates the optimal long-run inflation rates, and studies welfare under the alternative commitment policies.

Chapter 6 characterises the optimal steady-state inflation rate of the Ramsey planner in the medium-scale macroeconomic model developed in Altig et al. (2005), which we extend by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner à la Galì and Gertler (1999).

Chapter 7 studies Ramsey dynamics and address the question of implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules.

Chapter 8 provides concluding remarks.

CHAPTER 2

Basic New Keynesian Model

In this chapter, we lay out a basic New Keynesian model that we extend by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner. This results in a Phillips curve where current inflation depends on both expected future inflation and on lagged inflation, namely a hybrid Phillips curve. Backward-looking rule-of-thumb behaviour is specified in two ways. First, following Galì and Gertler (1999) we allow the rule-of-thumb price setters to index their prices to lagged inflation. Second, following Steinsson (2003) we allow the rule-of-thumb price setters to index their prices to both lagged output gap and lagged inflation.

The case of rule-of-thumb behaviour à la Steinsson (2003) contains an original contribution to the literature[1]. Specifically, we correct the hybrid Phillips curve reported in the original paper. The mistake in Steinsson (2003) relates to the coefficient on the term in current output gap in the hybrid Phillips curve. It has been acknowledged by the Steinsson in the *Erratum to Optimal Monetary Policy in an Economy with Inflation Persistence*, available on the author's webpage.

The theoretical economy is assumed closed and there is no capital accumulation. The model consists of households that supply labour and purchase goods for consumption and firms that hire labour and produce and sell differentiated goods in monopolistically competitive markets. Households and firms behave optimally: households maximise utility and firms maximise profits.

The model is basic in that it features only one real rigidity and one nominal rigidity. The real rigidity stems from monopolistic competition in the goods' markets, which is

[1] The hybrid Phillips curve in the case of backward-looking rule-of-thumb behaviour à la Galì and Gertler (1999) coincides with the one reported in Amato and Laubach (2003).

modelled as in Dixit and Stiglitz (1977). The nominal rigidity is given by staggered price adjustment as in Calvo (1983).

Specifically, the New Keynesian model laid out here is the basic neo-Wicksellian model in Woodford (2003). Woodford (2003) calls models of this kind neo-Wicksellian in order to stress the importance of a monetary policy transmission mechanism in which interest rates affect intertemporal spending decisions. Yet, following Clarida et al. (1999) among others, the terminology "New Keynesian" has become common place. We share the basic neo-Wicksellian model's assumptions and general formalism. Appendix A reports a detailed derivation of the hybrid Phillips curve that obtains in the presence of rule-of-thumb à la Steinsson (2003).

2.1. Households and Market Structure

There is a continuum of households of size one. The representative household seeks to maximise a discounted sum of utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t;\xi_t\right) - \int_0^1 v(h_t(i);\xi_t)di \right] \tag{2.1}$$

where $0 < \beta \leq 1$ is the discount factor, C_t is an aggregate of the household's consumption of a continuum of individual goods which are indexed by i over the unit interval, ξ_t is a vector of exogenous real shocks, namely exogenous shocks to household's impatience to consume and to the household's willingness to supply labour, and $h_t(i)$ is the supply of type i labour.

Following Dixit and Stiglitz (1977), the consumption aggregate is defined as

$$C_t = \left[\int_0^1 c_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{2.2}$$

where $c_t(i)$ is the consumption of good i and $\theta > 1$ is the constant elasticity of substitution between goods. For any given realisation of ξ_t, the period utility function, $u\left(C_t;\xi_t\right)$, is assumed to be concave and strictly increasing in C_t whereas the period disutility of supplying labour of type i, $v(h_t(i);\xi_t)$, is assumed to be convex and increasing in $h_t(i)$. We assume specific labour markets in the sense that type i labour is only used in the production of good i. Moreover, the representative household is assumed to simultaneously supply all types of labour. Considering differentiated labour inputs, as we shall see below, has the advantage of delivering a model with labour markets that is equivalent to the frequently used yeoman farmers model, in which households are assumed to supply goods directly. Moreover, if one were to replace specific labour markets with a single homogenous labour market, our results would not change qualitatively but only quantitatively. On the one hand, the assumption on the structure of the labour market affects the way in which the output gap enters the Phillips curve. On the other hand, as we shall see below, what matters for our results is that the aggregate-supply relation implies the existence of a positively sloped long-run trade-off between inflation and the output gap. As shown in Woodford (2003, Ch. 3), a positively sloped Phillips-curve long-run trade-off obtains under both assumptions about the structure of the labour market.

Financial markets are assumed to be complete, such that, through risk sharing, households face the same budget constraint, which is given by

$$\int_0^1 p_t(i)c_t(i)di + E_t\left[Q_{t,t+1}B_{t+1}\right] \leq B_t + \int_0^1 W_t(i)h_t(i)di + \int_0^1 \Pi_t(i)di - T_t \tag{2.3}$$

where $p_t(i)$ is the price of good i, B_t is the nominal value of financial wealth brought into the period, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs, T_t is net nominal tax collection by the government, $W_t(i)$ is the nominal wage for labour of type i, and $\Pi_t(i)$ is the nominal profits from sales of good i. The government is not modelled endogenously and it is assumed to run a balanced budget at all times. G_t is the exogenous process that describes government purchases of the aggregate good. Specifically, denoting with $g_t(i)$ the government's consumption of good i, G_t takes the Dixit and Stiglitz (1977) aggregate form, namely

$$(2.4) \qquad G_t = \left[\int_0^1 g_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)}$$

The budget constraint states that, in any period, financial wealth carried into the subsequent period plus consumption cannot be worth more than the value of financial wealth brought into the period plus after-tax nonfinancial income earned during the period. Note that we assume that every household owns an equal share of all the firms operating in the economy. The assumption of complete financial markets implies that the assumed firms' ownership and the assumption that the representative household supplies all types of labour directly are innocuous; dropping these assumptions would not change the conditions that determine equilibrium prices and quantities.

The household's optimal behaviour is described by three sets of conditions.

First, households face a decision in each period about how much to consume of each individual good. They adjust the share of a particular good in their consumption bundle so to exploit any differences in relative prices. Minimising the level of total expenditure, given the consumption aggregate in (2.2), yields the demand for each individual good

$$(2.5) \qquad c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} C_t$$

where the aggregate price level, P_t, is given by

$$(2.6) \qquad P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{1/1-\theta}$$

This specification of the price index has by construction the property that $P_t C_t$ gives the minimum price for which an amount C_t of the aggregate consumption can be purchased.

Market clearing implies that the total nonfinancial income, that is the economy-wide sales revenues, can be written as $P_t Y_t$. Here, Y_t is the economy's total output, which is the Dixit and Stiglitz (1977) aggregate of the quantities supplied of the various differentiated goods, denoted with $y_t(i)$, namely

$$Y_t = \left[\int_0^1 y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{2.7}$$

The household's budget constraint can thus be rewritten as

$$P_t C_t + E_t \left[Q_{t,t+1} B_{t+1} \right] \leq B_t + P_t Y_t - T_t \tag{2.8}$$

The absence of arbitrage opportunities implies that there exists a unique stochastic discount factor, $Q_{t,t+1}$. The riskless short-term nominal interest rate, i_t, has a simple representation in terms of the stochastic discount factor, namely

$$\frac{1}{1+i_t} = E_t \left[Q_{t,t+1} \right] \tag{2.9}$$

A complete description of the household's budget constraint requires ruling out Ponzi schemes. The implied constraint for financial wealth carried into the subsequent period, B_{t+1}, is given by

$$B_{t+1} \geq -\sum_{T=t+1}^{\infty} E_{t+1} \left[Q_{t+1,T} \left(P_t Y_t - T_t \right) \right] < \infty \tag{2.10}$$

with certainty, that is, in each state of the world that may be reached in the subsequent period. Here $Q_{t,T}$ discounts nominal income received in period T back to period t, $Q_{t,T} = \prod_{s=t+1}^{T} Q_{s-1,s}$. Equation (2.10) implies that a household's debt in any state of the world is bounded by the present value of future after-tax nonfinancial income, which is assumed to be finite. Furthermore, preventing unlimited consumption also requires that the nominal interest rate satisfies the zero lower bound, $i_t \geq 0$, at all times: a negative nominal interest rate would in fact allow households to finance unbounded consumption by selling enough bonds. The entire infinite series of flow budget constraints and borrowing constraints in turn defines the lifetime budget constraint for the household

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[P_t C_t\right] \leq B_0 + E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[(P_t Y_t - T_t)\right] \tag{2.11}$$

We can now complete the description of optimal household behaviour. Maximising utility (2.1) subject to the intertemporal budget constraint (2.11) yields the optimal allocation of consumption across time

$$\frac{u_c\left(C_t;\xi_t\right)}{u_c\left(C_{t+1};\xi_{t+1}\right)} = \frac{\beta}{Q_{t,t+1}} \frac{P_t}{P_{t+1}} \tag{2.12}$$

and the optimal supply of labour of type i

$$\frac{v_h(h_t(i);\xi_t)}{u_c\left(C_t;\xi_t\right)} = \frac{W_t(i)}{P_t} \tag{2.13}$$

where u_c and v_h denote respectively the partial derivative of u with respect to the level of consumption and the partial derivative of v with respect to the supply of labour. Substituting for the riskless short-term nominal interest rate, as given by (2.9), in the optimal intertemporal allocation of consumption delivers the familiar Euler equation for consumption

$$\beta E_t \left[\frac{u_c\left(C_{t+1};\xi_{t+1}\right)}{u_c\left(C_t;\xi_t\right)} \frac{P_t}{P_{t+1}}\right] = \frac{1}{1+i_t} \tag{2.14}$$

Allowing for tilting due to interest rates differing from the household's discount factor, rational consumers are thus attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods.

2.2. Firms

We assume that each good i has the linearly homogeneous production function

$$y_t(i) = A_t h_t(i) \tag{2.15}$$

where A_t is a time-varying exogenous technology factor. It follows that the nominal marginal cost of supplying a quantity $y_t(i)$ of good i is given by

$$MC_t(i) = W_t(i)A_t^{-1} \tag{2.16}$$

Note that the assumption of specific labour markets does not imply that each price setter is a monopsonist in their labour market. The possibility of firms having any market power in their labour market is ruled out by assuming that price setters that change their prices at the same time also hire labour from the same market[2]. Specifically, this is achieved by assuming a double continuum of differentiated goods, indexed by $(I\ ,\ j)$ with an elasticity of substitution of θ between any two goods. Goods belonging to the same industry I are then assumed to change their prices at the same time and to be produced using the same type of labour, namely type I labour. The fact that now a continuum of price setters demand type I labour eliminates the possibility of market power in their labour market without any change for the degree of market power of each price setter in their product market.

Combining the optimal supply of labour of type i (2.13) and the nominal marginal cost specification (2.16), the real marginal cost is given by

$$mc(y_t(i); C_t; \widetilde{\xi}_t) = \frac{v_h(y_t(i)/A_t; \xi_t)}{u_c\left(C_t; \xi_t\right) A_t} \tag{2.17}$$

where labour is expressed in terms of output and $\widetilde{\xi}_t$ denotes the vector of exogenous disturbances, which includes exogenous real shocks to technology, to household's impatience to consume, and to the household's willingness to supply labour.

2.3. Market Clearing

Market clearing requires, for each good i and at all times

$$y_t(i) = c_t(i) + g_t(i) \tag{2.18}$$

[2]The Calvo lottery is over industries' prices rather than goods' prices.

equivalently, in aggregate terms

$$Y_t = C_t + G_t \tag{2.19}$$

Substituting the market clearing condition into the Euler equation for consumption (2.14) yields

$$\beta E_t \left[\frac{\widetilde{u}_c(Y_{t+1}; \widetilde{\xi}_{t+1})}{\widetilde{u}_c(Y_t; \widetilde{\xi}_t)} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1+i_t} \tag{2.20}$$

with

$$\widetilde{u}(Y_t; \widetilde{\xi}_t) = u(Y_t - G_t; \xi_t) \tag{2.21}$$

Equivalently, substituting the market clearing condition into the real marginal cost specification (2.17) yields

$$mc(y_t(i); Y_t; \widetilde{\xi}_t) = \frac{\widetilde{v}_y(y_t(i); \widetilde{\xi}_t)}{\widetilde{u}_c(Y_t; \widetilde{\xi}_t)} \tag{2.22}$$

with

$$\widetilde{v}(y_t(i); \widetilde{\xi}_t) = v(y_t(i)/A_t; \xi_t) \tag{2.23}$$

Equations (2.21) and (2.23) are the indirect utility functions[3]. The former, which is increasing and concave in Y_t for each possible realisation of vector $\widetilde{\xi}_t$, indicates the utility flow to the representative household as a function of its aggregate demand for resources, where aggregate demand adds the household's share of government purchases to the household's private consumption. Under the assumption of G_t being exogenously determined, variations in the level of government expenditure are simply another source of exogenous

[3]Note that we use the same notation as in Woodford (2003). Subscript c denotes partial derivatives of the indirect utility function $\widetilde{u}$ with respect to the level of production, as these derivatives are identical to the partial derivatives of the direct utility function with respect to the level of aggregate consumption.

variation in the Euler equation for consumption[4]. The latter, which is increasing and convex in $y_t(i)$ for each possible realisation of vector $\widetilde{\xi}_t$, converts the household's disutility of supplying labour used for the production of good i into the household's disutility of directly supplying good i. Accordingly, the model laid out here is identical to the one that obtains under the assumption of a single yeoman farmer (i.e. continuum of yeoman farmers), which is used in both Steinsson (2003) and Amato and Laubach (2003). The representative household's discounted sum of utility can in fact be rewritten solely as a function of all $y_t(i)$

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\widetilde{u}(Y_t; \widetilde{\xi}_t) - \int_0^1 \widetilde{v}(y_t(i); \widetilde{\xi}_t) di \right] \tag{2.24}$$

with the period utility U_t being concave in the level of production of each of the differentiated goods.

2.4. Price Setting Behaviour

We now turn to the description of price setting behaviour. Following Calvo (1983), we assume that only a fraction $1 - \alpha$ of industries' prices are reset in each period. The probability of not resetting the price in each period, $0 < \alpha < 1$, is independent of both the time that has gone by since the last price revision and the misalignment between the actual price and the price that would be optimal to charge, namely pricing decisions in any period are independent of past pricing decisions. Furthermore, we assume that profits are discounted using a stochastic discount factor that equals on average β.

We now depart from full rationality by introducing backward-looking rule-of-thumb behaviour by price setters. Following Galì and Gertler (1999), we assume that only a fraction $1 - \omega$ of industries behave optimally (i.e. in a forward-looking manner) when setting the price, the remaining fraction of industries use the same backward-looking rule-of-thumb when revising their prices. The degree of rule-of-thumb behaviour, $0 \leq \omega < 1$,

[4]Henceforth, the vector $\widetilde{\xi}_t$ includes exogenous real shocks to technology, to Government purchases, to household's impatience to consume, and to the household's willingness to supply labour.

is thus constant over time and price setters cannot switch between backward-looking and forward-looking behaviour.

If follows that in each period all forward-looking price setters will set the same price, which we denote with p_t^f, and all backward-looking price setters will as well charge a common price, which we denote with p_t^b. The aggregate price level in (2.6) hence evolves according to

$$P_t = \left\{(1-\alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right\}^{\frac{1}{1-\theta}} \tag{2.25}$$

where

$$p_t^* = (1-\omega)p_t^f + \omega p_t^b \tag{2.26}$$

denotes the overall reset price.

The firms allowed to change their price at time t choose p_t^f so to maximise expected future profits subject to the demand they face. The price setter's objective is given by

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Pi(p_t(i), p_{t+s}^I, P_{t+s}, Y_{t+s,} \widetilde{\xi}_{t+s}) \tag{2.27}$$

The price setter's nominal profit function, Π, is linearly homogeneous in its first three arguments (i.e. good's price, industry's price, p_t^I, and aggregate price level) and, for any value of the industry price and the aggregate price level, single-peaked for some positive value of the good's price[5]. The common forward-looking reset price, p_t^f, is implicitly defined by the relation

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s,} \widetilde{\xi}_{t+s}) = 0 \tag{2.28}$$

[5]Under $y_t(i) = A_t h_t(i)$, the nominal profit function is given by

$$\Pi(p_t(i), p_t^I, P_t, Y_t, \widetilde{\xi}_t) = p_t(i)\left(\frac{p_t(i)}{P_t}\right)^{-\theta} Y_t - \frac{v_h((p_t^I/P_t)^{-\theta} Y_t/A_t); \xi_t)}{u_c(C_t; \xi_t)} P_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta} \frac{Y_t}{A_t}$$

Here $\Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s}, \widetilde{\xi}_{t+s}) = 0$ implicitly defines what Woodford (2003, Ch. 3) labels the notional short-run aggregate supply curve, which indicates how relative prices would be set if there was no constraint on the frequency of price setting.

The common rule-of-thumb backward-looking reset price, p_t^b, is specified as in Steinsson (2003)

$$p_t^b = p_{t-1}^* \frac{P_{t-1}}{P_{t-2}} \left(\frac{Y_{t-1}}{Y_{t-1}^n} \right)^{\delta} \tag{2.29}$$

where Y_{t-1}^n denotes the natural level of output, which we precisely define below, and $0 \leq \delta \leq 1$ is the degree of indexation to past demand conditions. Rule-of-thumb price setters thus index the previous period overall reset price to past inflation, fully, and past demand conditions, according to δ. In the case of zero indexation to past demand conditions, the rule-of-thumb collapses to the specification in Galì and Gertler (1999), $p_t^b = p_{t-1}^* \frac{P_{t-1}}{P_{t-2}}$.

2.5. Flexible Price Equilibrium and Efficient Equilibrium

We now consider an environment in which all firms can adjust prices optimally each period, taking the path of aggregate variables as given. Profit-maximising behaviour under perfectly flexible prices then implies that firms will operate at the point at which the relative price is a mark-up over the real marginal cost

$$\frac{p_t(i)}{P_t} = mc(y_t(i); Y_t; \widetilde{\xi}_t)\mu \tag{2.30}$$

where $\mu = \theta/(\theta - 1) > 1$ is the desired constant mark-up, which is common to all firms. The relative supply of good i thus satisfies

$$\left(\frac{y_t(i)}{Y_t} \right)^{-1/\theta} = mc(y_t(i); Y_t; \widetilde{\xi}_t)\mu \tag{2.31}$$

Given identical prices and demand conditions the equilibrium under perfectly flexible prices is symmetric: each good is produced and consumed in the same quantity. Within New Keynesian economics, the natural level of output in Friedman (1968), which we denote with $Y_t^n(\widetilde{\xi}_t)$, has a precise meaning: it is simply the equilibrium level of output under perfectly flexible prices. It follows from (2.31) that the natural level of output is implicitly defined by

$$mc(Y_t^n; Y_t^n; \widetilde{\xi}_t) = \mu^{-1} \tag{2.32}$$

In the case of fully flexible prices, equilibrium output thus equals the natural level of output at all times, $Y_t = Y_t^n(\widetilde{\xi}_t)$. The natural level of output in turn depends only on the exogenous real shocks, which entails that the equilibrium output under perfectly flexible prices is completely independent of monetary policy.

In the case of sticky prices, equilibrium output can instead differ from the natural level of output. The concept of output gap in fact plays a major role in New Keynesian economics, both as a force bringing about fluctuations in inflation as well as a target for monetary policy. Specifically, the output gap, which we denote with x_t, has a precise meaning: it is the deviation of actual output from the natural level of output, namely $x_t = \widehat{Y}_t - \widehat{Y}_t^n = \log(Y_t/Y_t^n)$.

The natural steady-state level of output is the equilibrium level of output that obtains in the presence of fully flexible prices and in the absence of exogenous real shocks, that is $\widetilde{\xi}_t = 0$ at all times. We denote a variable's value at steady state with a bar. The natural steady-state level of output, $\overline{Y}$, is implicitly defined by

$$mc(\overline{Y}; \overline{Y}; 0) = \mu^{-1} \tag{2.33}$$

Note that we do not use the superscript n in denoting the natural steady-state level of output. This is because, if $\widetilde{\xi}_t = 0$ and $Y_t = \overline{Y}$ at all times, (2.25) has a solution with zero inflation at all times. In other words, in the presence of zero steady-state inflation, the steady state of the economy with sticky prices coincides with the natural steady-state level of output. Indeed, we later log-linearise the model around a steady-state with zero inflation and the natural steady-state level of output, $\overline{Y}$.

We must stress that the natural level of output is not Pareto efficient. The efficient level of output is in fact the equilibrium level of output under both perfectly flexible prices and perfect competition. The efficient level of output, $Y_t^*(\widetilde{\xi}_t)$, is thus implicitly defined by

$$mc(Y_t^*; Y_t^*; \widetilde{\xi}_t) = 1 \tag{2.34}$$

Accordingly, the efficient steady-state level of output, $\overline{Y}^*$, is implicitly defined by

$$mc(\overline{Y}^*; \overline{Y}^*; 0) = 1 \tag{2.35}$$

2.6. Log-linearised Model

Log-linearising requires choosing the steady state around which the log-linear approximation is taken. We log-linearise the model around a steady-state with zero inflation and the natural steady-state level of output.

We denote a variable's log-deviation from its steady-state value, which is denoted with a bar, with a hat. Taking a first-order Taylor series expansion to $\widetilde{u}_c\left(Y_t; \xi_t\right)$ yields

$$\widehat{\widetilde{u}}_c\left(Y_t; \widetilde{\xi}_t\right) = \frac{\widetilde{u}_{cc}\overline{Y}}{\widetilde{u}_c}\left(\widehat{Y}_t + \frac{\widetilde{u}_{c\xi}}{\widetilde{u}_{cc}\overline{Y}}\widetilde{\xi}_t\right) \tag{2.36}$$

where $\widehat{Y}_t = \log(Y_t/\overline{Y})$ and all partial derivatives are evaluated at steady state[6]. Similarly, taking a first-order Taylor series expansion to $\widetilde{v}_y(y_t(i); \widetilde{\xi}_t)$ gives

$$\widehat{\widetilde{v}}_y\left(y_t(i); \widetilde{\xi}_t\right) = \frac{\widetilde{v}_{yy}\overline{Y}}{\widetilde{v}_y}\left(\widehat{y}_t(i) + \frac{\widetilde{v}_{y\xi}}{\widetilde{v}_{yy}\overline{Y}}\widetilde{\xi}_t\right) \tag{2.37}$$

where $\widehat{y}_t(i) = \log(y_t(i)/\overline{Y})$.

Log-linearising the Euler equation for consumption (2.20) yields

$$\widehat{i}_t = \widehat{r}_t + E_t\pi_{t+1} \tag{2.38}$$

[6]In what follows we use the notation in Woodford (2003). subscript ξ denotes partial derivatives of the indirect utility functions with respect to all exogenous real disturbances in vector $\widetilde{\xi}_t$.

where $\widehat{i}_t = \log[(1+i_t)/(1+\bar{i})]$ and $\pi_t = \widehat{P}_t - \widehat{P}_{t-1} = \log(P_t/P_{t-1})$. Here the log-deviation in the ex ante short-term real interest rate, $\widehat{r}_t$, is a process given by

$$\widehat{r}_t = \sigma^{-1}\left[E_t(\widehat{Y}_{t+1} - g_{t+1}) - (\widehat{Y}_t - g_t)\right] \tag{2.39}$$

where the constant coefficient

$$\sigma = -\frac{\widetilde{u}_c}{\widetilde{u}_{cc}\overline{Y}} > 0 \tag{2.40}$$

measures the intertemporal elasticity of substitution of aggregate expenditure and the disturbance term

$$g_t = -\frac{\widetilde{u}_{c\xi}}{\widetilde{u}_{cc}\overline{Y}}\widetilde{\xi}_t \tag{2.41}$$

indicates the percentage variation in output required to keep the marginal utility of expenditure at its natural steady-state level, given shocks to government purchases and to household's impatience to consume. Equation (2.38) can be reformulated as an intertemporal expectational IS relation of the form

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \sigma\left[\widehat{i}_t - E_t\pi_{t+1} - \sigma^{-1}(g_t - E_t g_{t+1})\right] \tag{2.42}$$

The expectational IS relation, (2.42), can be expressed in terms of output gap as

$$x_t = E_t x_{t+1} - \sigma\left(\widehat{i}_t - E_t\pi_{t+1} - \widehat{r}_t^n\right) \tag{2.43}$$

where

$$\widehat{r}_t^n = \sigma^{-1}\left[(g_t - \widehat{Y}_t^n) - E_t(g_{t+1} - \widehat{Y}_{t+1}^n)\right] \tag{2.44}$$

is the natural real rate of interest, namely the real interest rate that would obtain if all prices were perfectly flexible.

Log-linearising the real marginal cost specification (2.22) yields

$$\widehat{mc}_t(i) = \varpi\,(\hat{y}_t(i) - q_t) + \sigma^{-1}(\widehat{Y}_t - g_t) \tag{2.45}$$

where $\widehat{mc}_t(i) = \log(mc_t(i)/\mu)$. Here the constant coefficient

$$\varpi = \frac{\widetilde{v}_{yy}\overline{Y}}{\widetilde{v}_y} > 0 \tag{2.46}$$

measures the elasticity of real marginal cost with respect to own output and the disturbance term

$$q_t = -\frac{\widetilde{v}_{y\xi}}{\widetilde{v}_{yy}\overline{Y}}\widetilde{\xi}_t \tag{2.47}$$

indicates the percentage variation in output required to keep the marginal disutility of labour supply at its natural steady-state level, given shocks to technology and to the household's willingness to supply labour.

Under perfectly flexible prices, (2.45) reduces to

$$\log\left(\frac{\mu^{-1}}{\mu^{-1}}\right) = \varpi\left(\widehat{Y}_t^n - q_t\right) + \sigma^{-1}(\widehat{Y}_t^n - g_t) \tag{2.48}$$

Solving for $\widehat{Y}_t^n = \log(Y_t^n/\overline{Y})$ yields

$$\widehat{Y}_t^n = \frac{\varpi q_t + \sigma^{-1}g_t}{\varpi + \sigma^{-1}} \tag{2.49}$$

Using (2.45), percentage fluctuations in the efficient level of output are given by

$$\widehat{Y}_t^* = \frac{\varpi q_t + \sigma^{-1}g_t}{\varpi + \sigma^{-1}} \tag{2.50}$$

which is the same as (2.49). On the one hand, percentage fluctuations in the efficient level of output are equal, to second order, to percentage fluctuations in the natural level of output. On the other hand, monopolistic competition brings about a constant wedge between the natural steady-state level of output and the efficient steady-state level of output. The natural steady-state level of output, $\overline{Y}$, can in fact be rewritten as

$$mc(\overline{Y};\overline{Y};0) = \mu^{-1} = 1 - \Phi_y \tag{2.51}$$

where the parameter Φ_y summarises the distortions in the natural steady-state level of output due to monopolistic competition. When Φ_y is small enough, the steady-state efficiency gap, $x^* = \log(\overline{Y}^*/\overline{Y})$, can be log-linearised as

$$\log(\overline{Y}^*/\overline{Y}) = \frac{\Phi_y}{\varpi + \sigma^{-1}} + O\left(\|\Phi_y\|^2\right) \tag{2.52}$$

2.7. Hybrid Phillips Curves

We can now turn to the aggregate supply function. Under rule of thumb behaviour à la Steinsson (2003), the inflation rate and the output gap in any period satisfy an aggregate supply relation of the form

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_2 x_t + \kappa_3 x_{t-1} \tag{2.53}$$

Appendix A reports a detailed derivation of the hybrid Phillips curve. The coefficients on the terms in inflation are given by

$$\chi_f = \frac{\alpha}{\phi};\ \chi_b = \frac{\omega}{\phi};\ \phi = \alpha + \omega - (1-\beta)\omega\alpha \tag{2.54}$$

The coefficients on the terms in output gap are given by

$$\kappa_2 = \frac{(1-\omega)\alpha\kappa - (1-\alpha)\alpha\beta\omega\delta}{\phi};\ \kappa_3 = \frac{(1-\alpha)\omega\delta}{\phi} \tag{2.55}$$

with

$$\kappa = \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\varpi)}{(1+\varpi\theta)\,\alpha} \tag{2.56}$$

The parameters reported in Steinsson (2003) (p. 1451-1452) are the same as above apart from κ_2. As acknowledged by the author, the mistake in the coefficient on the term in current output gap is due to an incorrect specification for the elasticity of the notional short-run aggregate supply curve. This mistake implies that, once the possibility of rule-of-thumb behaviour by price setters is ruled out (i.e. $\omega = 0$), the hybrid Phillips curve in Steinsson (2003) does not collapse to the NKPC in Woodford (2003, 2.12 and 2.13, p. 187), namely

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{2.57}$$

where κ is given as in (2.56). On the contrary, absent rule-of-thumb behaviour by price setters, our specification for the hybrid Phillips curve, namely (2.53), is easily seen to collapse to the NKPC in Woodford

If the fraction ω is reset according to backward-looking rule-of-thumb behaviour à la Galì-Gertler (1999) (i.e. $\delta = 0$ in (2.53)), the aggregate supply relation is of the form

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t \tag{2.58}$$

The parameters on the terms in inflation are defined as in (2.54). The parameter on the term in output gap is given by

$$\kappa_1 = \frac{(1-\omega)\alpha\kappa}{\phi} \tag{2.59}$$

where κ is defined as in (2.56). Of course, absent rule-of-thumb behaviour by price setters, (2.58) collapses to the NKPC in Woodford (2003), namely (2.57).

Rule-of-thumb behaviour is indeed capable of providing a rationale for the dependence of current values of inflation on past as well as expected inflation conditions. Under rule-of-thumb behaviour, current inflation depends on a convex combination of expected future inflation and lagged inflation. Moreover, the weights on the expected future inflation and lagged inflation are functions of the model's structural parameters.

Both the hybrid Phillips curves (2.53) and (2.58) have the property that they nest the NKPC. Additionally, Steinsson (2003) shows how (2.53) has the property that it also nests a purely backward-looking Phillips curve (i.e. $\pi_t = \pi_{t-1} + (1-\alpha)\delta x_{t-1}$) in the limit when $\omega \to 1$.

On the one hand, the property displayed by (2.53) of collapsing to the purely backward-looking Phillips curve is certainly appealing. Nonetheless, empirical studies (e.g. Fuhrer and Moore (1995), Fuhrer (1997), Galì and Gertler (1999), and Roberts (2005), and Sbordone (2005)) are not only able to reject the purely forward-looking Phillips curve but also the purely backward-looking Phillips curve. On the other hand, an implication, not previously noted in the literature, of allowing rule-of-thumb price setters to index their prices to lagged output gap is that the long-run Phillips-curve trade-off is affected.

Indeed, all Phillips curves imply a positively sloped long-run Phillips-curve trade-off. Denoting with $\overline{\pi}$ and $\overline{x}$ the steady-state values of inflation and output gap, the NKPC (2.57) implies an upward sloping relation of the form

$$\overline{x} = \frac{(1-\beta)}{\kappa}\overline{\pi} \tag{2.60}$$

The long-run relation between inflation and output gap is in fact due to the smaller coefficient on the expected-inflation (i.e. β) term relative to that on current inflation (i.e. 1).

On the one hand, rule-of-thumb behaviour à la Galì and Gertler (1999) desirably implies that all price setters behave identically once shocks are eliminated from the economy. In other words, Galì-Gertler's backward-looking rule-of-thumb behaviour does not alter the steady state that would obtain under forward-looking behaviour by all price setters. The long-run Phillips-curve trade-off is in fact not affected by the presence of rule-of-thumb price setters: (2.58) evaluated at steady state results in (2.60). On the other hand, the same is not true under rule-of-thumb behaviour à la Steinsson (2003). The fact that rule-of-thumb price setters index their prices to the lagged output gap alters the long-run Phillips-curve trade-off that obtains in the purely forward-looking model. Indeed, the hybrid Phillips curve (2.53) evaluated at steady state yields

$$\overline{x} = \frac{(1-\beta)(1-\omega)\alpha}{(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta}\overline{\pi} = \Gamma\overline{\pi} \tag{2.61}$$

which collapses to (2.60) under $\delta = 0$. As we shall see below, this will have important consequences once we consider what constitutes the optimal inflation rate under a particular type of commitment policy.

CHAPTER 3

Policy Objective Functions

In this chapter, we specify objective functions for the central bank. The general equilibrium foundations of New Keynesian models allow deriving the objective a central bank should pursue starting from the utility of the representative household. Woodford (2003, Ch. 6) provides a detailed analysis of the utility-based framework for the evaluation of monetary policy. Accordingly, the central bank's objective is the discounted sum of utility of the representative household, which is approximated to second order by the discounted sum of central bank's single-period loss function. The central bank's single-period loss function in turn depends on the details of the price setting.

We extend the analysis in Woodford (2003) by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner. The backward-looking behaviour is specified in two ways. First, following Galì and Gertler (1999) we allow the rule-of-thumb price setters to index their prices to lagged inflation. Second, following Steinsson (2003) we allow the rule-of-thumb price setters to index their prices to both lagged output gap and lagged inflation.

The case of rule-of-thumb behaviour à la Steinsson (2003) contains an original contribution to the literature[1]. Specifically, we correct the utility-based objective function of the central bank reported in the original paper. The mistake in Steinsson (2003) relates to his reported λ_4 coefficient in the single-period central bank's loss function. It has been acknowledged by the Steinsson in the *Erratum to Optimal Monetary Policy in an Economy with Inflation Persistence*, available on the author's webpage.

[1]The hybrid Phillips curve in the case of backward-looking rule-of-thumb behaviour à la Galì and Gertler (1999) coincides with the one reported in Amato and Laubach (2003).

Correcting the mistake allows us to show how the quadratic terms that stem from the presence of rule-of-thumb behaviour by price setters can be combined in a single quadratic term. As a result, rule-of-thumb behaviour, regardless of its specification, can now be seen as introducing a single extra term in the central bank's single-period loss function relative to the case of purely forward-looking price setting. Interestingly, this single extra term has a precise economic interpretation: it penalises variations in the difference between general inflation and rule-of-thumb price increases.

We first review the conditions that guarantee the validity of the utility-based approach to monetary policy analysis in the presence of a linear approximation to the model's structural relations. We then proceed to compute a second-order approximation to the period utility of the representative household. Finally, we specify the objective functions for the central bank. Appendix B reports a detailed derivation of the objective function that obtains in the presence of rule-of-thumb à la Steinsson (2003).

3.1. Theoretical Background

The problem at hand is how to evaluate expected utility, $E[U(x,\xi)]$, under alternative policies on the basis of a log-linear approximation to fluctuations in the endogenous variables. Here, x denotes a vector of endogenous variables, ξ is a vector of stochastic exogenous shocks, and the utility function $U(x,\xi)$ is concave for each realisation of ξ and at least twice differentiable.

The model's structural equations are log-linearised around the deterministic steady state, $\overline{x}$, which is the vector of steady-state values of the endogenous variables in the absence of shocks, $\xi = 0$. The model is then closed by an approximate policy rule, which can either be assumed or, as we will do in the next chapter, optimally derived by specifying a loss function for the central bank.

Supposing that the linear approximate model is determinate[2], a first-order approximation to the equilibrium fluctuations of the endogenous variables for any given policy rule is given by

$$x = x^0 + A\xi + O\left(\|\xi, \varrho\|^2\right) \tag{3.1}$$

where x^0 is the policy-driven deterministic steady state, A is a matrix of coefficients, and the second-order residual is a function of the bound on the size of the exogenous shocks, $\|\xi\|$, and of the bound on the policy-driven deterministic steady state, $\|\varrho\|$.

The deterministic steady state $\overline{x}$ does not need to correspond to x^0. The parameter vector ϱ in fact indexes aspects of the policy rule that affect the steady state. A single linear approximate general equilibrium model can thus be used for the evaluation of any policy provided that the policy-driven deterministic steady state is close enough to the steady state around which the model's structural equations are log-linearised. Precisely, the residual in (3.1) is of order $O\left(\|\xi, \varrho\|^2\right)$ independently of the policy rule only if $\widetilde{x}^0 = x^0 - \overline{x} = O\left(\|\varrho\|\right)$.

Taking a second-order approximation to the utility function delivers

$$U(x, \xi) = \overline{U} + U_x \widetilde{x} + U_\xi \xi + \frac{1}{2}\widetilde{x}' U_{xx} \widetilde{x} + \widetilde{x}' U_{x\xi} \xi + \frac{1}{2}\xi' U_{\xi\xi} \xi + O\left(\|\xi, \varrho\|^3\right) \tag{3.2}$$

where $\overline{U} = U(\overline{x}, 0)$, $\widetilde{x} = x - \overline{x}$, and all partial derivatives of U are evaluated at steady state. The third-order residual stems from (3.1), which implies that $x - \overline{x} = x^0 - \overline{x} + A\xi = O\left(\|\xi, \varrho\|\right)$. Given that the shocks are normalised to zero, namely $E\left[\xi\right] = 0$, taking the expected value of the second-order approximation to the utility function (3.2) yields the

[2]The issue of whether a linear approximate general equilibrium model is determinate, namely the system has a unique bounded solution in the case of bounded disturbances, is discussed in Blanchard and Kahn (1980), Klein (2000), and Soderlind (1999).

approximate welfare criterion

$$E[U(x,\xi)] = \overline{U} + U_x E\left[\widetilde{x}\right] + \frac{1}{2} tr\left\{U_{xx} var\left[\widetilde{x}\right]\right\} + tr\left\{U_{x\xi} cov(\xi, x)\right\} + \frac{1}{2} tr\left\{U_{\xi\xi} var\left[\xi\right]\right\} + O\left(\|\xi, \varrho\|^3\right) \tag{3.3}$$

where $E\left[\widetilde{x}\right]$ is the expectation of the random vector $\widetilde{x}$, $var\left[\widetilde{x}\right]$ and $var\left[\xi\right]$ are respectively the variance-covariance matrices of the random vectors $\widetilde{x}$ and ξ, and $cov(\xi, x)$ is the matrix of covariances between the random vectors $\widetilde{x}$ and ξ.

We can now consider the validity of the welfare criterion for a given policy rule satisfying (3.1). Substituting the first-order approximation to equilibrium fluctuations (3.1) in $\widetilde{x}$ delivers $\widetilde{x} = \widetilde{x}^0 + A\xi$, which implies that $E\left[\widetilde{x}\right] = \widetilde{x}^0$. Accordingly, the approximate welfare criterion (3.3) is now given by

$$U^0 = \overline{U} + U_x \widetilde{x}^0 + \frac{1}{2} tr\left\{U_{xx} var\left[\widetilde{x}\right]\right\} + tr\left\{U_{x\xi} cov(\xi, x)\right\} + \frac{1}{2} tr\left\{U_{\xi\xi} var\left[\xi\right]\right\} + O\left(\|\xi, \varrho\|^3\right) \tag{3.4}$$

Comparing the approximate welfare criteria, (3.4) and (3.3), we note that

$$E[U(x,\xi)] - U^0 = U_x E\left[\widetilde{x} - \widetilde{x}^0\right] + O\left(\|\xi, \varrho\|^3\right) \tag{3.5}$$

Equation (3.5) implies that U^0 cannot correctly rank alternative policies, even in the case of a small bound on the amplitude of the disturbances, unless

$$U_x E\left[\widetilde{x} - \widetilde{x}^0\right] = O\left(\|\xi, \varrho\|^3\right) \tag{3.6}$$

Condition (3.6) shows that the problem at hand here is one of a general nature. That is the inaccuracy of evaluating welfare in terms of a second-order approximation to utility on the basis of a linear approximation to the model structural equations given that the residual in a linear approximation is only of second order and not of third order. Indeed, the problem arises from the presence of the linear term in the second-order approximation to the utility function: there may be second-order terms that are left as part of the second order-residual in a linear approximation but that may make a nonzero second order contribution to the left-hand side of condition (3.6) thus spoiling the validity of the approximate welfare criterion (3.3). Moreover, the issue whether such terms are nonzero or not cannot generally be addressed unless considering a second-order approximation for the model's structural equations[3].

Nonetheless, the inaccuracy of the linear approximation for the purpose of policy evaluation can also be overcome without having to discard the linear approximation to the model's structural relations. The validity of welfare ranking on the basis of log-linear approximation to the model structural equations can be gained by restricting U_x so to make condition (3.6) hold.

The easiest restriction on U_x one can think of is the zero restriction

$$U_x(\overline{x}, 0) = 0 \tag{3.7}$$

so that condition (3.6) necessarily holds and the approximate welfare criterion (3.3) does not contain linear terms. Accordingly, comparisons of the value of U^0 provide a correct welfare ranking of alternative policies under the conditions that assure the determinacy of rational expectation equilibrium. Equation (3.7) guarantees that in order to avoid the inaccuracy of the log-linearised equations one should linearise the model's structural equations around the Pareto efficient steady state. Indeed, this is the widespread practice in the New Keynesian literature: the steady-state distortions due to monopolistic competition in the goods' markets are eliminated by assuming the existence of a subsidy to production. This assumption is uncompelling for two main reasons. First, there is no evidence of the existence of the kind of subsidies needed to assume away steady-state distortions. Second, it is not clear whether a policy that is optimal for an economy with an efficient steady

[3]Second-order solution methods are discussed in Benigno and Woodford (2005), Collard and Juillard (2001), Jin and Judd (2002), and Schmitt-Grohè and Uribe (2004b).

state remains optimal in an economy where the steady state is distorted. With this latter respect, we will indeed show how maintaining steady-state distortions has important consequences on the characterisation of the optimal inflation rate for monetary policy.

The accuracy of first-order approximations to correctly evaluate welfare up to second order does not necessarily require for the steady state to be efficient. Condition (3.7) can in fact be weakened by imposing only that

$$U_x(\overline{x},0) = O\left(\|\Phi_y\|\right) \tag{3.8}$$

where the parameter Φ_y, introduced in Chapter 2, summarises the distortions in the natural steady-state level of output due to monopolistic competition. If condition (3.7) provides a benchmark for the steady state in order to gain the accuracy of the linear approximation for the purpose of welfare evaluation, condition (3.8) simply requires the steady state be near such a benchmark, namely not to be too inefficient. Imposing condition (3.8) thus amounts to binding the equilibrium inefficiency. The additional bound introduced in turns guarantees that

$$U_x E\left[\widetilde{x} - \widetilde{x}^0\right] = O\left(\|\Phi_y, \xi, \varrho\|^3\right) \tag{3.9}$$

which entails that the approximate welfare criterion (3.3) does not contain linear terms, namely

$$E[U(x,\xi)] = U^0 + O\left(\|\Phi_y, \xi, \varrho\|^3\right) \tag{3.10}$$

To summarise, it is possible to use first-order approximations to evaluate welfare accurately up to second order as long as: I) the exogenous disturbances are small enough, II) the policy-driven steady state is close enough to the steady-state around which the log-linearisations are taken, and III) the steady-state distortions are small enough.

3.2. Utility-based Welfare Criterion

We consider the scenario of small steady-state distortions discussed in the previous section. Substituting the indirect utility functions, the period utility of the representative household in (2.1) can be rewritten as a function solely of all $y_t(i)$ as given by (2.24). For convenience, we report here the period utility

$$U_t = \widetilde{u}(Y_t; \widetilde{\xi}_t) - \int_0^1 \widetilde{v}(y_t(i); \widetilde{\xi}_t) di \tag{3.11}$$

This change of variables matters because the small inefficiency of the steady state is never in terms of the endogenous variables in the utility function per se. In other words, neither the marginal disutility of labour nor the marginal utility of consumption is small enough at steady state. The small inefficiency of the steady state is rather in terms of the structural relationship relating labour to consumption. This structural relationship characterises feasible consumption-work outcomes as a result of the production function. Accordingly, we need to express utility as a function of the level of production, an endogenous variable for which the marginal utility is close enough to zero.

A second-order Taylor approximation of the first term in the period utility (3.11) is given by

$$\widetilde{u}(Y_t; \widetilde{\xi}_t) = \overline{u} + \widetilde{u}_c \widetilde{Y}_t + \widetilde{u}_\xi \widetilde{\xi}_t + \frac{1}{2}\widetilde{u}_{cc}\widetilde{Y}_t^2 + \widetilde{u}_{c\xi}\widetilde{Y}_t\widetilde{\xi}_t + \frac{1}{2}\breve{\xi}_t \widetilde{u}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\widetilde{\xi}, \varrho\right\|^3\right) \tag{3.12}$$

where $\overline{u} = \widetilde{u}(\overline{Y}; 0)$ and $\widetilde{Y}_t = Y_t - \overline{Y}$. $\widetilde{Y}_t$ is related to $\widehat{Y}_t$ through the second-order Taylor approximation, $Y_t = \overline{Y}(1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2) + O(\left\|\widetilde{\xi}, \varrho\right\|^3)$. Substituting accordingly for $\widetilde{Y}_t$ yields

$$\begin{aligned} \widetilde{u}(Y_t; \widetilde{\xi}_t) \quad = \quad & \overline{u} + \widetilde{u}_c\overline{Y}(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2) + \widetilde{u}_\xi\widetilde{\xi}_t + \frac{1}{2}\widetilde{u}_{cc}\overline{Y}^2\left(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2\right)^2 \\ & + \widetilde{u}_{c\xi}\overline{Y}(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2)\widetilde{\xi}_t + \frac{1}{2}\breve{\xi}_t\widetilde{u}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\widetilde{\xi}, \varrho\right\|^3\right) \end{aligned} \tag{3.13}$$

Dropping the terms that are higher than second order and collecting terms that are independent of policy in the term $t.i.p$, we obtain

$$\widetilde{u}(Y_t;\widetilde{\xi}_t) = \overline{Y}\widetilde{u}_c\left\{\widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1}g_t\widehat{Y}_t\right\} + t.i.p + O\left(\left\|\widetilde{\xi},\varrho\right\|^3\right) \tag{3.14}$$

where σ and g_t are respectively given by (2.40) and (2.41).

The second term in the period utility (3.11) can be approximated to second order by

$$\widetilde{v}(y_t(i);\widetilde{\xi}_t) = \overline{v} + \widetilde{v}_y\widetilde{y}_t(i) + \widetilde{v}_\xi\widetilde{\xi}_t + \frac{1}{2}\widetilde{v}_{yy}\widetilde{y}_t^2(i) + \widetilde{v}_{y\xi}\widetilde{y}_t(i)\widetilde{\xi}_t + \frac{1}{2}\widetilde{\xi}_t^{\smile}\widetilde{v}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\widetilde{\xi},\varrho\right\|^3\right) \tag{3.15}$$

where $\overline{v} = \widetilde{v}(\overline{Y};0)$ and $\widetilde{y}_t(i) = y_t(i) - \overline{Y}$. $\widetilde{y}_t(i)$ is related to $\widehat{y}_t(i)$ through the second-order Taylor approximation, $y_t(i) = \overline{Y}(1+\widehat{y}_t(i)+\frac{1}{2}\widehat{y}_t^2(i)) + O(\left\|\widetilde{\xi},\varrho\right\|^3)$. Substituting accordingly for $\widetilde{y}_t(i)$ yields

$$\begin{aligned}\widetilde{v}(y_t(i);\widetilde{\xi}_t) \quad = \quad & \overline{v} + \widetilde{v}_y\overline{Y}(\widehat{y}_t(i) + \frac{1}{2}\widehat{y}_t^2(i)) + \widetilde{v}_\xi\widetilde{\xi}_t + \frac{1}{2}\widetilde{v}_{yy}\overline{Y}^2(\widehat{y}_t(i) + \frac{1}{2}\widehat{y}_t^2(i))^2 \\ & + \widetilde{v}_{y\xi}\overline{Y}(\widehat{y}_t(i) + \frac{1}{2}\widehat{y}_t^2(i))\widetilde{\xi}_t + \frac{1}{2}\widetilde{\xi}_t^{\smile}\widetilde{v}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\widetilde{\xi},\varrho\right\|^3\right)\end{aligned} \tag{3.16}$$

Dropping the terms that are higher than second order and collecting terms that are independent of policy in the term $t.i.p$, we obtain

$$\widetilde{u}(Y_t;\widetilde{\xi}_t) = \overline{Y}\widetilde{v}_y\left\{\widehat{y}_t(i) + \frac{1}{2}(1+\varpi)\widehat{y}_t(i)^2 - \varpi q_t\widehat{y}_t(i)\right\} + t.i.p + O\left(\left\|\widetilde{\xi},\varrho\right\|^3\right) \tag{3.17}$$

where ϖ and q_t are respectively given by (2.46) and (2.47). Using the real marginal cost specification (2.22) evaluated at steady state and the definition of the natural steady-state level of output as given by (2.51), it is possible to substitute $\widetilde{u}_c(1-\Phi_y)$ for $\widetilde{v}_y$, with Φ_y being an expansion parameter. Accordingly, equation (3.17) can be rewritten as

$$\widetilde{v}(y_t(i);\widetilde{\xi}_t) = \overline{Y}\widetilde{u}_c\left\{\begin{array}{c}(1-\Phi_y)\widehat{y}_t(i) \\ +\frac{1}{2}(1+\varpi)\widehat{y}_t(i)^2 - \varpi q_t\widehat{y}_t(i)\end{array}\right\} + t.i.p + O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{3.18}$$

Integrating this over the differentiated goods i gives

$$\int_0^1 \widetilde{v}(y_t(i);\widetilde{\xi}_t)di = \overline{Y}\widetilde{u}_c \left\{ \begin{array}{c} (1-\Phi_y)E_i\widehat{y}_t(i) - \varpi q_t E_i\widehat{y}_t(i) \\ +\frac{1}{2}(1+\varpi)\left[(E_i\widehat{y}_t(i))^2 + var_i\widehat{y}_t(i)\right] \end{array} \right\} + t.i.p + O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{3.19}$$

where $E_i\widehat{y}_t(i)$ and $var_i\widehat{y}_t(i)$ denote respectively the mean value and the variance of $\widehat{y}_t(i)$ across all differentiated goods i at date t. Using a second-order approximation to the Dixit-Stiglitz output index, $\widehat{Y}_t = E_i\widehat{y}_t(i) + \frac{1}{2}(1-\theta^{-1})var_i\widehat{y}_t(i) + O(\left\|\widetilde{\xi},\varrho\right\|^3)$, to substitute for $E_i\widehat{y}_t(i)$ yields

$$\int_0^1 \widetilde{v}(y_t(i);\widetilde{\xi}_t)di = \overline{Y}\widetilde{u}_c \left\{ \begin{array}{c} (1-\Phi_y)\widehat{Y}_t + \frac{1}{2}(1+\varpi)\widehat{Y}_t^2 - \varpi q_t\widehat{Y}_t \\ +\frac{1}{2}(\theta^{-1}+\varpi)var_i\widehat{y}_t(i) \end{array} \right\} + t.i.p + O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{3.20}$$

Putting back together the second-order approximations to the two terms entering the period utility function, (3.14) and (3.20), a second-order approximation to the period utility function is given by

$$U_t = -\frac{\overline{Y}\widetilde{u}_c}{2} \left\{ \begin{array}{c} (\sigma^{-1}+\varpi)\left[-2x^*\widehat{Y}_t + \widehat{Y}_t^2 - 2\widehat{Y}_t^n\widehat{Y}_t\right] \\ +(\theta^{-1}+\varpi)var_i\widehat{y}_t(i) \end{array} \right\} + t.i.p + O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{3.21}$$

where we use equations (2.49) and (2.52) to express the period utility function in terms of the percentage fluctuations in the natural level of output, $\widehat{Y}_t^n$, and the steady-state efficiency gap, x^*.

Recalling that each supplier faces a constant elasticity demand as given by (2.5) , it follows that the variance of $y_t(i)$ across all differentiated goods i at date t is related to the variance of prices across all differentiated goods i at date t according to $var_i \log y_t(i) = \theta^2 var_i \log p_t(i)$. Using this and noting that $(x_t - x^*)^2 = \widehat{Y}_t^2 - 2x^*\widehat{Y}_t + -2\widehat{Y}_t^n\widehat{Y}_t + t.i.p$, we obtain

$$U_t = -\frac{\overline{Y}\widetilde{u}_c}{2}\left[\begin{array}{c}(\sigma^{-1}+\varpi)(x_t - x^*)^2 \\ +(1+\varpi\theta)\theta var_i \log p_t(i)\end{array}\right] + t.i.p + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{3.22}$$

which coincides with Woodford (2003, 2.13, p. 396).

There are two main points to take from (3.22). First, absent the steady-state distortions due to monopolistic competition (i.e. $x^* = 0$), the welfare-relevant measure of output gap is precisely the same that enters the aggregate supply curve. The monetary authority should stabilise the output gap, x_t, that is it should stabilise the actual level of output around the natural level of output, which would be efficient. In the presence of steady-state distortions, the monetary authority should instead try to stabilise the output gap around the steady-state efficiency gap, x^*. In other words, the combination of general equilibrium foundations and steady-state distortions provides a microfoundation for targeting a level of output above the inefficient natural level of output.

Second, it is also appropriate for monetary policy to aim to curb price dispersion. In the presence of sticky prices, price dispersion is costly because it reduces the utility of the representative household. This happens for two reasons. First, the representative household's utility depends on the consumption of an aggregate good. Faced with dispersion of prices for the individual goods produced in the economy, the household consumes more of the relatively cheaper goods and less of the relatively more expensive goods. Given diminishing marginal utility, the loss in utility due to consuming less of the relatively more expensive goods is greater than the increase in utility due to consuming more of the relatively cheaper goods. Second, the representative household's utility depends on the supply of each individual labour type. Given increasing marginal disutility of supplying labour, the loss in utility due to producing more of the relatively more expensive goods is greater than the increase in utility due to producing less of the relatively cheaper goods. For these reasons, price dispersion reduces utility. When prices are sticky, price dispersion is caused by inflation. However, how fluctuations in the general price level affect price dispersion, hence the central bank's objective function, depend upon the details of price setting.

3.3. Utility-based Objective Functions

We can now specify the utility-based objective functions for monetary policy. The central bank's objective is the discounted sum of utility of the representative household, which is approximated to second-order by the discounted sum of central bank's single-period loss function. The central bank's single-period loss function, denoted with L_t, depends on the details of price setting. Denoting with W the central bank's objective function, we have

$$W = \sum_{t=0}^{\infty} \beta^t U_t = -\Omega \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + O\left(\left\| \Phi_y, \widetilde{\xi}, \varrho, \Delta_{-1}^{1/2} \right\|^3 \right) \tag{3.23}$$

where the constant Ω is given by $\Omega = \overline{Y}\widetilde{u}_c(\sigma^{-1} + \varpi)\theta/2\kappa$. Appendix B reports a detailed derivation of the central bank's objective function in the presence of rule-of-thumb behaviour à la Steinsson (2003).

Under rule-of-thumb behaviour à la Steinsson (2003), the single-period central bank's loss function takes the form

$$L_t = \pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1}) \right]^2 \tag{3.24}$$

The coefficients λ_1 and λ_2 are given by

$$\lambda_1 = \frac{\kappa}{\theta} \text{ and } \lambda_2 = \frac{\omega}{(1-\omega)\alpha} \tag{3.25}$$

where κ is defined as in (2.56).

The single-period central bank's loss function (3.24) constitutes an original contribution to the literature[4]. As acknowledged by Steinsson, the mistake in his derivation relates to

[4]The hybrid Phillips curve in the case of backward-looking rule-of-thumb behaviour à la Galì and Gertler (1999) coincides with the one reported in Amato and Laubach (2003).

the coefficient he denotes with λ_4. Correcting the mistake allows us to show, as we detail in Appendix B, how the quadratic terms that stem from the presence of rule-of-thumb behaviour by price setters can be combined in a single quadratic term. As a result, rule-of-thumb behaviour by price setters à la Steinsson (2003), can now be seen as introducing a single extra term in the central bank's single-period loss function relative to the case of purely forward-looking price setting. Interestingly, as we discuss below, we can give a clear and intuitive interpretation to this single extra term.

In the presence of backward-looking rule-of-thumb behaviour à la Galì-Gertler (1999) (i.e. $\delta = 0$), the single-period central bank's loss function is given by

$$L_t = \pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 \left(\pi_t - \pi_{t-1}\right)^2 \tag{3.26}$$

with λ_1 and λ_2 defined as in (3.25).

Absent rule-of-thumb behaviour, $\omega = 0$, (3.24) and (3.26) collapse to the loss function in Woodford (2003, 2.22, p. 400), namely

$$L_t = \pi_t^2 + \lambda_1 (x_t - x^*)^2 \tag{3.27}$$

with λ_1 defined as in (3.25).

A loss function similar to (3.27) has indeed been widely assumed in the literature on optimal monetary policy evaluation. Walsh (2003, Ch. 8) provides a survey of earlier works on optimal monetary policy that assume a quadratic loss function closely related to (3.27). As discussed in Woodford (2003, Ch. 6) the main advantage of the utility-based approach to monetary policy analysis lies in providing a theoretical justification, namely a microfoundation, for such widely used loss function. There are two critical differences between the utility-based loss function (3.27) and the nonmicrofounded loss functions assumed by the earlier literature on optimal monetary policy. First, the output gap is measured relative to the natural level of output, namely the equilibrium level of output under perfectly flexible prices. In the earlier literature, the output variable was instead interpreted as being output relative to trend or output relative to an unspecified natural level of output. Second, as described above, inflation enters the loss function because of sticky prices: price dispersion reduces the utility of the representative household and, when firms cannot adjust their prices every period, price dispersion is brought about by inflation. Moreover, the relative weights on inflation and output gap stabilisation are again determined by the model's structural parameters. Specifically, the relative weight on output variations depends linearly on the slope of the purely forward-looking Phillips curve.

An evident implication of allowing for rule-of-thumb behaviour is that it affects the objective that monetary policy seeks to pursue. Rule-of-thumb behaviour implies that the single-period central bank's loss function includes an extra term with respect to the single-period loss function in the purely forward-looking model.

Indeed, the extra terms in the single-period loss function due to the presence of backward-looking rule-of-thumb à la Steinsson (2003) can now be combined in a single quadratic term. Interestingly, this extra term can now be seen as penalising variations in the difference between general inflation and rule-of-thumb price increases. In the presence of Galì-Gertler's rule-of-thumb behaviour, rule-of-thumb price setters index their prices to lagged inflation, which is reflected in the term in inflation acceleration, $\pi_t - \pi_{t-1}$. In the presence of Steinsson's rule-of-thumb behaviour, rule-of-thumb price setters index their prices to both lagged inflation and lagged output gap, which is reflected in the term $\pi_t - (\pi_{t-1} + (1 - \alpha)\delta x_{t-1})$.

Moreover, an implication of allowing rule-of-thumb price setters to index their prices to lagged output gap is that the steady-state loss function is affected. Galì and Gertler (1999) rule-of-thumb behaviour does not alter the steady-state loss function that would obtain under forward-looking behaviour by all price setters as the term in inflation acceleration in (3.26) does not matter at steady state. The same is not true under rule-of-thumb behaviour à la Steinsson (2003): the fact that rule-of-thumb price setters index their prices to the lagged output gap entails that the additional, with respect to (3.27), term in (3.24) matters at steady state. As we shall see below, this will have important consequences once we consider what constitutes the optimal inflation rate under a particular type of commitment policy.

CHAPTER 4

Optimal Monetary Policy: Optimal Long-run Inflation Rates

In this chapter, we study what constitutes optimal monetary policy in our theoretical economies, which are all characterised by the existence of a long-run Phillips-curve trade-off and by an inefficient natural level of output. Specifically, we concentrate on what constitutes the optimal long-run inflation rate. Throughout the chapter we assume that the central bank is able to act under commitment. We consider different types of optimal commitment policy that have been proposed in the literature: the zero-optimal policy, the timeless perspective commitment policy in Woodford (1999), and the alternative timeless perspective commitment policy put forward by Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008). Our preference for commitment is based on our focus on analysing the optimal long-run inflation rate when the steady-state is distorted[1].

The analysis we carry out in this chapter is entirely about steady-state outcomes. Certainty equivalence guarantees that the results we obtain in the purely deterministic setting hold in the presence of random disturbances. While we initially characterise the steady state of our theoretical economies, the dynamic nature of the three models derived in the previous chapters is later employed for the analysis of stochastic outcomes, such as the dynamic analysis of responses to shocks and the welfare costs due to the stabilisation of shocks.

Within New Keynesian literature, Woodford (2003) studies the optimal long-run rate of inflation in the basic NK model and its extension where inflation inertia is due to backward-looking price indexation by price setters, which is considered by, among others, Christiano et al. (2005) and Smets and Wouters (2003). This chapter provides the first derivation of

[1]Of course, discretionary conduct of monetary policy would result in the well-known inflation bias stressed by Kydland and Prescott (1977) and Barro and Gordon (1983).

the optimal long-run inflation rate in a small New Keynesian model where inflation inertia is due to backward-looking rule-of-thumb behaviour by price setters specified either à la Galì and Gertler (1999) or à la Steinsson (2003).

In the standard New Keynesian framework, the optimality of a monetary policy that pursues complete price stability is well-known. Price stability is complete as it characterises both the deterministic and the stochastic component of optimal monetary policy: zero inflation is optimal in the steady state and inflation should not vary in response to the shocks buffeting the economy. The intuition for zero OLIR, as firstly stressed by Goodfriend and King (1997) is neat. The welfare-theoretical loss function reflects the two distortions in a basic NK model: distortions due to monopolistic competition in products markets and distortions due to relative-price distortions. Under the widespread assumption of a subsidy to production aimed at eliminating the long-run distortions originating from monopolistic competition, zero steady-state inflation is optimal as it allows to fully stabilise the distortions due to relative-price distortions. Zero long-run inflation remains optimal even in the presence of the steady-state distortions due to monopolistic competition: the central bank finds optimal to fully stabilise the distortions due to relative-price distortions whereas it does not intervene on the distortions due to monopolistic competition. Woodford (2003) thus concludes "*It is sometimes supposed that the existence of a long-run Phillips-curve trade-off, together with an inefficient natural rate, should imply that the Phillips curve should be exploited to some extent, resulting in positive inflation forever, even under commitment. But here that is not true because the smaller coefficient on the expected inflation-term relative to that on current inflation-which results in the long-run trade-off- is exactly the size of the shift term in the aggregate supply that is needed to precisely eliminate any long-run incentive for nonzero inflation under an optimal commitment.*" Woodford (2003, p. 415). Moreover, as shown in Woodford (2003), zero OLIR is robust to inflation inertia due to backward-looking price indexation by price setters, regardless of the assumption about the distortions originating from monopolistic competition in products market[2].

This is the first study of the optimal long-run inflation rate in a small NK model where inflation inertia is due to backward-looking rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003).

We show how backward-looking rule-of-thumb behaviour, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), breaks the optimality of zero long-run inflation. In other words, backward-looking rule-of-thumb behaviour brings about a long-run incentive for nonzero inflation. This is because the stimulative effect of higher inflation is generally greater than the output cost of higher inflation. Optimal steady-state inflation collapses to zero in the absence of backward-looking rule-of-thumb behaviour, in the absence of a long-run Phillips-curve trade-off, and in the absence of steady-state distortions.

Positive optimal long-run inflation also obtains in the purely forward-looking New Keynesian model under a type of timeless perspective commitment policy that has recently been proposed in the literature. Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008) propose considering a timeless perspective policy which is based on the optimisation of the unconditional value of the central bank's objective function. Moreover, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour, when this is characterised as in Galì and Gertler (1999), with the optimal long-run inflation rate being invariant to the degree of rule-of-thumb behaviour.

Overall, of the six optimal long-run inflation rates we derive, five are positive. In all the theoretical cases considered, optimal steady-state inflation collapses to zero in the absence of a long-run Phillips-curve trade-off and in the absence of steady-state distortions. Taking together the basic message of our results is that the widespread practice in the New Keynesian literature of restricting the attention to the case of an efficient natural

[2]The only consequence of introducing inflation inertia due to backward-looking price indexation is that the existence of a positive rate of inflation before the adoption of the optimal commitment policy affects the rate at which the central bank brings inflation back to the zero long-run optimal target.

level of output is not innocuous. A policy that is optimal for an economy with an efficient steady state differs from what is optimal in an economy where the subsidies that achieve Pareto efficiency are unavailable.

We first review different types of optimal commitment policy that have been proposed in the literature. We then proceed to set up the policy problem faced by the central bank. Finally, we evaluate what constitutes optimal long-run inflation in our theoretical economies. Table 1 summarises the six optimal long-run inflation rates we derive.

4.1. Different Perspectives on Optimal Monetary Policy

The past few years have been characterised by a large body of literature on the topic of optimal monetary policy. However, there is disagreement as to which is the appropriate perspective for monetary policy optimality. Following the work of Kydland and Prescott (1977), the literature has focused on two main approaches to monetary policy analysis: commitment and discretion. These two approaches correspond to two different assumptions about central bank behaviour. The difference of the two approaches in fact lies in the central bank's ability to precommit about its future actions. As Kydland and Prescott (1977) firstly pointed out, commitment is not time consistent. That is, the behaviour which the central bank would like to commit itself to carrying out at a future date does not generally remain optimal for the bank when that future date actually arrives. Conversely, discretion is time consistent as the central bank is free to choose at any date the best policy given the conditions existing in the economy.

On the one hand, the New Keynesian literature has emphasised that discretionary conduct of policy leads, in addition to the well-known inflation bias stressed by Kydland and Prescott (1977) and Barro and Gordon (1983), to the so-called stabilisation bias[3]. Woodford (1999, 2003, Chapter 7) and Clarida et al. (1999) discuss how a central bank that is able to credibly commit can influence private sector expectations in a way that leads to more favorable responses to shocks. In particular, Woodford (1999, 2003, Chapter 7)

[3]Earlier papers which discuss this effect include Jonsson (1997) and Svensson (1997).

shows that optimal policy under commitment is history dependent whereas discretionary policy is purely forward-looking. The logic behind the optimality of history dependence is quite intuitive. In an economy where private sector expectations are formed rationally, commitment by the central bank can influence these expectations only if the central bank's earlier commitments are sustained in later periods. Hence, successful steering of private sector expectations requires that the central bank's conduct in later periods depends not only on the current state of the economy but also on the state of the economy in earlier periods.

On the other hand, within the commitment class of policy, the literature has proposed three types of policy strategy, which represent different perspectives on the concept of optimal monetary policy.

First, there is full commitment on the basis of the initial conditions at an arbitrary date zero, when the policy is implemented. Following the terminology in Woodford (2003), we refer to this type of strategy as being the zero-optimal commitment. This strategy entails that existing expectations need not to be fulfilled as they are taken as given at date zero. The exploitation of existing expectations implies that zero-optimal policy is not time invariant. The central bank's behaviour is in fact not described by a time invariant rule but rather by a set of rules: one rule for date zero and one rule for all subsequent dates. In other words, the policy chosen at later dates is not a continuation of the policy selected at date zero. With this respect, we must stress that by rule we mean an optimal targeting rule, namely a target criterion for inflation that is derived by combining the optimality conditions with respect to inflation and output gap.

Second, there is timeless perspective commitment, which, originally introduced by Woodford (1999), has subsequently received a great deal of attention. This strategy seeks to overcome the lack of continuation that characterises the zero-optimal policy. Indeed, the timeless perspective reflects a type of commitment that, unlike the zero-optimal commitment, constraints the central bank's rule to be time invariant. It does so by relying upon optimality conditions that would have been chosen under a commitment regime if this had been adopted in the distant past. In other words, by ignoring the temptation to exploit the expectations existing in the economy, the initial conditions prevailing at date zero are not the ones utilised, but the economy's initial evolution is constrained to be the the one associated with the policy. The zero-optimal policy and the timeless perspective policy in fact differ only with respect to the central bank's posited behaviour in the initial period. Given the intertemporal nature of the aggregate-supply relation, it follows that the two policies imply different transition paths for inflation and output gap to the same optimal long-run values.

Third, Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008) argue that timeless perspective commitment, as usually described, is not thoroughly timeless. The timeless perspective developed by Woodford is in fact based on optimality conditions obtained from a conditional optimality calculation. The authors thus propose considering an alternative timeless perspective policy which is based on optimisation of the unconditional value of the central bank's objective function. In this sense, the rule they obtain is globally optimal in the sense of Taylor (1979). Taylor (1979) proposes adopting a monetary policy that, given complete knowledge, in terms of both structural equations and exogenous shock processes, of the structure of an economy characterised by rational expectations, is optimal on average. Monetary policy is optimal on average if it yields the smallest unconditional expectation of the central bank's objective function. The rule implied by this third perspective is both timelessly optimal and globally optimal with respect to the unconditional variance. In what follows, we refer to this third approach as implying an alternative timeless perspective policy to the standard timeless perspective policy in Woodford (1999).

Here we concentrate on the optimal long-run inflation rates entailed by these alternative commitment policies. We consider the forward-looking canonical New Keynesian economy and its alterations due to backward-looking rule-of-thumb behaviour by a fraction of price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003). In so doing, we consider the empirically realistic case of an economy where the deterministic steady state is inefficient as the distortions due to monopolistic competition are not offset by subsidies to production.

4.1.1. The Policy Problem

Before analysing the optimal long-run inflation rates under the different commitment policies, we present the problem faced by the central bank. Following the theoretical literature on optimal monetary policy, we assume that the central bank's policy instrument is the short-term nominal interest rate. The combination of cashless economy, which entails that there are no costs associated with varying the nominal interest rate, and central's bank control of the nominal interest rate implies that the intertemporal expectational IS relation imposes no real constraint on the central bank. Given the central bank's optimal choices for inflation and output gap, the expectational IS equation simply determines the path of nominal interest rate necessary to achieve the optimal path for the output gap. As a consequence, it is more convenient to treat output gap as if it were the central bank's policy instrument.

The economy is thus fully described by the aggregate-supply relation and the central bank's objective function. This model of central bank behaviour allows determining the long-run inflation rate corresponding to a given policy. In particular, as stressed by Woodford (2008) "*The fact that the equations are log-linearized does not mean that one simply assumes an average inflation rate; the equations allow one to derive the average inflation rate corresponding to a given policy*".

As described in the previous chapters, both the central bank's single-period loss function, L_t, and the aggregate-supply relation depend upon the details of price setting. However, regardless of the details of price setting, a central bank able to precommit faces a constrained minimisation problem. That is, the central bank chooses a path for current and future inflation and a path for current and future output gap to minimise its objective function subject to the aggregate-supply relation. Specifically, the utility-based central bank's objective function at an arbitrary time $t = 0$, is here taken to be the expected discounted sum of central bank's loss function. In other words, we drop the constant $-\Omega$, which multiplies the discounted sum of central bank's loss function in (3.23), as it does not matter for the constrained minimisation problem.

The commitment policies then differ as for expected value of the central bank's objective function being unconditional or conditional on information available at date 0.

A central bank acting under zero-optimal or timeless perspective commitment faces the problem of minimising the expected value of its objective function conditional on information available at date 0, namely

$$E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{4.1}$$

where E_0 denotes the expectation operator conditional on information available at date 0. The two commitment policies then differ because the timeless perspective policy ignores the conditions actually prevailing in the economy at the policy's implementation date.

Conversely, a central bank acting under the alternative timeless perspective commitment faces the problem of minimising the unconditional expected value of its objective function. Denoting with E the unconditional expectation operator, the law of iterated expectations implies that

$$E\left(E_0 \sum_{t=0}^{\infty} \beta^t L_t\right) = (1-\beta)^{-1} E(L_t) \tag{4.2}$$

Except for discounting, the unconditional expectation of the expected value of the central bank's objective function conditional on information available at date zero corresponds to the unconditional expectation of the single-period central bank's loss function, $E(L_t)$. The optimality conditions under the alternative timeless perspective policy are obtained as done in Blake (2001). Blake (2001) describes the rule under the alternative timeless perspective policy as being implied by the expected undiscounted minimisation problem conditional on information available at date zero, under which the central bank minimises $E_0 \sum_{t=0}^{\infty} L_t$.

In what follows, superscripts FL, GG, and S denote respectively the purely forward-looking New Keynesian model, the model with rule-of-thumb behaviour à la Galì and Gertler (1999), and the model with rule-of-thumb behaviour à la Steinsson (2003). Superscript $Z0$ denotes the zero-optimal commitment policy, superscript TP designates the standard timeless perspective policy, and superscript ATP indicates the alternative timeless perspective policy. We can now proceed to characterise each commitment policy in terms of target criterion and optimal long-run inflation rate.

4.2. Basic New Keynesian Model

A central bank acting under commitment faces the problem of choosing paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the expected discounted sum of central bank's loss function, with the single period-loss given by (3.27), conditional on information available at date zero subject to the constraint that the sequences must satisfy (2.57) each period. The Lagrangian associated with this problem is of the form

$$\mathcal{L}_0^{FL} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \varphi_t \left[\pi_t - \beta\pi_{t+1} - \kappa x_t\right] \right\} \tag{4.3}$$

where φ_t is the Lagrangian multiplier associated with period t aggregate-supply relation. Differentiating with respect to π_t and x_t, we get the optimality conditions

$$\frac{\partial \mathcal{L}_0^{FL}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \varphi_t - \varphi_{t-1} = 0 \tag{4.4}$$

$$\frac{\partial \mathcal{L}_0^{FL}}{\partial x_t} = 0 \Rightarrow \lambda_1(x_t - x^*) - \kappa\varphi_t = 0 \tag{4.5}$$

Under zero-optimal commitment policy, there is no fulfillment of the expectations existing at the time of the policy implementation, that is (4.4) in period 0 holds with $\varphi_{-1} = 0$. Hence, zero-optimal commitment policy is characterised by the output gap optimality condition (4.5) for all $t \geq 0$ and two inflation optimality conditions: one for period zero and one for all subsequent periods

$$\pi_t + \varphi_t = 0 \qquad t = 0 \tag{4.6}$$

$$\pi_t + \varphi_t - \varphi_{t-1} = 0 \qquad t \geq 1 \tag{4.7}$$

Combining the optimality conditions (4.5) and (4.6) delivers the central bank's target criterion in period 0

$$\pi_0 = -\frac{\lambda_1}{\kappa}(x_0 - x^*) \tag{4.8}$$

whereas combining the optimality conditions (4.5) and (4.7), the central bank in any period $t \geq 1$ behaves according to the rule

$$\pi_t = -\frac{\lambda_1}{\kappa}(x_t - x_{t-1}) \tag{4.9}$$

Woodford (2003) hence concludes "*Thus it is optimal (from the point of minimizing discounted losses from date zero onward) to arrange an initial inflation, given that the decision to do so can have no effect upon expectations prior to date zero (if one is not bothered by the non-time-consistency of such a principle of action). The optimal policy involves positive inflation in subsequent periods as well, but there should be a commitment to reduce inflation to its optimal long-run value of zero asymptotically*" Woodford (2003, p. 414−5).

Despite the inefficiency of the nonstochastic steady state, namely $x^* > 0$, and the existence of a positively sloped long-run Phillips-curve trade-off, as implied by (2.60) evaluated at steady state, there is an advantage for having positive inflation only in period 0, whereas there is no long-run incentive for positive inflation. This is because the increase in output in any period caused by higher inflation in the same period, φ_t, is exactly offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. The steady-state efficiency gap thus enters (4.8), but it does not appear in (4.9). Hence, the optimal long-run inflation rate is zero.

Alternatively, the same result can be illustrated without having to rely on the optimality conditions of the minimisation problem. Integrating forward the NKPC (2.57) entails that, regardless of policy, the expected discounted sum of future output gaps conditional on information available at date 0 can be rewritten as a function solely inflation at time 0, $\pi_0 = \kappa E_0 \sum_{t=0}^{\infty} \beta^t x_t$. Accordingly, the central bank's objective function in (4.1) can be rewritten as

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t \right\} = \lambda_1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t x^{*2} \right\} - \frac{2x^*}{\theta} \pi_0 + E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_1 x_t^2 \right] \right\} \tag{4.10}$$

The first term is purely a function of steady-state efficiency gap, x^*. The other terms are minimised by choosing $\pi_t = 0$ each period, which, given the NKPC, implies $x_t = 0$ each period, except the one that is function of the initial rate of inflation π_0. The presence of this term implies a welfare gain from an initial positive rate of inflation, but because it only applies to inflation in the initial period, it is optimal to commit to zero long-run inflation. Moreover, the linear term in π_0 affects the zero-optimal commitment policy for periods later than 0 as the NKPC implies an intertemporal linkage between current inflation and future inflation. The welfare gain resulting from positive π_0 can be obtained with less increase in period 0 output gap, x_0, thus resulting in less increase in $\lambda_1 x_0^2$, if it is associated with an increase in expected inflation at date one, $E_0\pi_1$. Given that the loss associated with $E_0\pi_1$ occurs later in time, and is thus weighted less strongly, the transition to zero-optimal inflation lasts for more than one period.

Woodford (1999) argues that zero-optimal commitment policy is not attractive as it is not time invariant. As an alternative to zero-optimal policy, Woodford (1999) puts forward another commitment policy, which he labels timeless perspective. The policy proposal is simple to outline. What makes the zero-optimal commitment policy not time invariant is the separate treatment of initial period and all other periods. At time 0, the central bank sets inflation according to the rule (4.8) and promises to follow the rule (4.9) at any later date. Yet, if a central bank reoptimised in any later period, it would find optimal to set inflation according to (4.8), updated to that period. By ignoring the conditions existing in the economy at the policy's implementation, commitment policy is in fact timeless as it can be thought of as a policy rule that was chosen in the distant past. The current values of inflation and output gap are the values chosen form that earlier perspective to satisfy the two optimality conditions (4.4) and (4.5). Woodford's timeless perspective commitment policy thus ignores the start-up condition (4.6) and the central bank's rule in all periods $t \geq 0$ is given by (4.9). Hence, despite the steady-state distortions and the existence of long-run Phillips-curve trade-off, there is never advantage from having positive inflation.

More recently, it has been recognised that the use of (4.9) in all periods $t \geq 0$ is not optimal within the class of time-invariant policy rules. Specifically, Blake (2001), Jensen (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008) show that there is a slightly different policy rule that is not only timeless but also globally optimal with respect to the unconditional expectation of the central bank's objective function. As discussed above, the optimality conditions under this alternative timeless perspective policy can in fact be found by considering the expected undiscounted minimisation problem conditional on information available at date zero. That is, the Lagrangian (4.3) becomes

$$\mathcal{L}^{FL}_{0,Undis} = E_0 \sum_{t=0}^{\infty} \left\{ \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \varphi_t \left[\pi_t - \beta\pi_{t+1} - \kappa x_t\right] \right\} \tag{4.11}$$

where the subscript $Undis$ indicates that the undiscounted minimisation problem is considered. Differentiating with respect to π_t and x_t, the output gap optimality condition (4.5) is unaffected but instead of the inflation optimality condition (4.4) we obtain

$$\frac{\partial \mathcal{L}^{FL}_{0,Undis}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \varphi_t - \beta\varphi_{t-1} = 0 \tag{4.12}$$

From a timeless perspective, the central bank sets policy according to optimality conditions (4.5) and (4.12) in all periods $t \geq 0$. Combining these optimality conditions, the central bank's rule is given by

$$\pi_t = -\frac{\lambda_1}{\kappa}(x_t - \beta x_{t-1}) + \frac{(1-\beta)\lambda_1}{\kappa}x^* \tag{4.13}$$

Comparing the policy rule under the alternative timeless perspective policy (4.13) with the policy rule under the standard timeless perspective policy (4.9), we note that the alternative timeless perspective brings about an incentive for committing to positive inflation. Specifically, evaluating (4.13) at steady state delivers

$$\overline{\pi} + \frac{(1-\beta)\lambda_1}{\kappa}\overline{x} = \frac{(1-\beta)\lambda_1}{\kappa}x^* \tag{4.14}$$

Taking into account the positively sloped relationship between steady-state output gap, $\overline{x}$, and steady-state inflation, $\overline{\pi}$, implied by the NKPC (2.60), the alternative timeless perspective policy entails positive steady-state inflation of the form

$$\overline{\pi}^{FLATP} = \frac{(1-\beta)\kappa}{\theta\kappa + (1-\beta)^2}x^* \tag{4.15}$$

Given $k > 0$, $\overline{\pi}^{FLATP}$ is positive and collapses to zero in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$) or in the absence of steady-state distortions (i.e. $x^* = 0$).

The logic behind this result is quite intuitive. If the central bank shares the discount factor of the private sector, the cost resulting from the anticipation of higher inflation occurs earlier in time and it is thus weighted more strongly (by a factor $1/\beta > 1$) than the benefit stemming from higher inflation (weighted by a factor 1). However, expected future inflation enters the NKPC with a coefficient β that is smaller than the unitary coefficient on actual inflation. Hence, as in (4.9), the increase in output in any period caused by higher inflation in the same period, φ_t, is offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. Accordingly, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero.

Under the alternative timeless perspective policy, the private sector's discount factor appears in the model's structural equations, thus resulting in the long-run Phillips curve trade-off. On the other hand, the central bank now equally weighs the increase in output in any period caused by higher inflation in the same period and the cost of the reduction in output in the previous period as a result of expected higher inflation. Hence, the stimulative effect of higher inflation on output is greater than the output cost of higher inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

The result can be interpreted as highlighting the effects of discounting on monetary policy choices. With this respect, Henry et al. (2006) show how the result derived by Bean (1998) depends on the central bank not discounting the future. Bean (1998) shows that the outcomes of monetary policy, in terms of the variances of inflation and output, are very similar for a wide range of central bank's preferences with respect to inflation and output stability[4].Conversely, Henry et al. (2006) show that when the monetary authority discounts the future, the outcomes of monetary policy become more sensitive to the central bank's preferences.

What we show here is that the effects of discounting on the optimal target for monetary policy are remarkable. If the central bank shares the same discount factor of the private sector, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero. Conversely, if the central bank does not discount the future, positive steady-state inflation emerges under commitment even in the purely forward-looking model.

4.3. Rule-of-thumb Behaviour

We now proceed to compare the three alternative commitment policies when the Phillips curve becomes hybrid due to the presence of backward-looking rule-of-thumb price setters. We consider first the rule-of-thumb behaviour à la Galì and Gertler (1999) and subsequently turn our attention to Steinsson's (2003) rule-of-thumb behaviour. In both cases, we formalise the policy problem and we characterise the three commitment policies in terms of target criterion and optimal long-run inflation rate.

[4]The model considered by Bean (1998) is a closed-economy monetary policy model that prescribes that monetary policy acts with a lag. The same model is also considered in Ball (1999) and Svensson (1997).

4.3.1. Rule-of-thumb Behaviour à la Galì and Gertler

A central bank acting under commitment faces the problem of choosing paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the expected discounted sum of central bank's loss function, with the single-period loss given by (3.26), conditional on information available at date zero subject to the constraint that the sequences must satisfy (2.58) each period. The Lagrangian associated with this problem is of the form

(4.16)
$$\mathcal{L}_0^{GG} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 (\pi_t - \pi_{t-1})^2 \right] + \varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_1 x_t \right] \right\}$$

Differentiating with respect to π_t and x_t, we get the optimality conditions

$$\frac{\partial \mathcal{L}_0^{GG}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \lambda_2 (\pi_t - \pi_{t-1}) - \beta \lambda_2 (\pi_{t+1} - \pi_t) + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} = 0 \tag{4.17}$$

$$\frac{\partial \mathcal{L}_0^{GG}}{\partial x_t} = 0 \Rightarrow \lambda_1 (x_t - x^*) - \kappa_1 \varphi_t = 0 \tag{4.18}$$

Under zero-optimal commitment policy, there is no fulfillment of the expectations existing at the time of the policy implementation, that is (4.17) in period 0 holds with $\varphi_{-1} = 0$. Hence, zero-optimal commitment policy is characterised by the output gap optimality condition (4.18) for all $t \geq 0$ and two inflation optimality conditions: one for period zero and one for all subsequent periods

$$\pi_t + \lambda_2 (\pi_t - \pi_{t-1}) - \beta \lambda_2 (\pi_{t+1} - \pi_t) + \varphi_t - \beta \chi_b \varphi_{t+1} = 0 \qquad t = 0 \tag{4.19}$$

$$\pi_t + \lambda_2 (\pi_t - \pi_{t-1}) - \beta \lambda_2 (\pi_{t+1} - \pi_t) + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} = 0 \qquad t \geq 1 \tag{4.20}$$

Combining the optimality conditions (4.18) and (4.19) delivers the central bank's target criterion in period 0

$$\pi_0 = \frac{1}{1+\lambda_2(1+\beta)}\left\{\lambda_2\pi_{-1} + \beta\lambda_2\pi_1 + \frac{\lambda_1}{\kappa_1}\left[\beta\chi_b x_1 - x_0 + (1-\beta\chi_b)x^*\right]\right\} \tag{4.21}$$

whereas combining the optimality conditions (4.18) and (4.20), the central bank in any period $t \geq 1$ behaves according to a rule of the form

$$\pi_t = \frac{1}{1+\lambda_2(1+\beta)}\left\{\begin{array}{c} \lambda_2\pi_{t-1} + \beta\lambda_2\pi_{t+1} \\ +\frac{\lambda_1}{\kappa_1}\left[\beta\chi_b x_{t+1} + \chi_f x_{t-1} - x_t + (1-\beta\chi_b-\chi_f)x^*\right] \end{array}\right\} \tag{4.22}$$

Under the standard timeless perspective commitment policy, the start-up condition (4.19) is ignored and the central bank's rule in all periods $t \geq 0$ is given by (4.22). Given $x^* > 0$, there is an advantage for having positive long-run inflation. Indeed, evaluating (4.22) at steady state delivers

$$\overline{\pi} = -\frac{(1-\alpha)(1-\beta)\omega}{(1-\omega)\alpha\theta}(\overline{x} - x^*) \tag{4.23}$$

Note that here, and in what follows, all parameters in the hybrid Phillips curve are rewritten in terms of structural parameters (keeping κ implicit).

Rule-of-thumb behaviour à la Galì and Gertler (1999) desirably implies that all price setters behave identically once shocks are eliminated from the economy. As we have seen in the previous chapters, Galì-Gertler's backward-looking rule-of-thumb behaviour does not alter the steady state that would obtain under forward-looking behaviour by all price setters. Specifically, the long-run Phillips-curve trade-off is not affected by the presence of rule-of-thumb price setters: (2.58) evaluated at steady state results in (2.60). Using this to eliminate $\overline{x}$ from (4.23), the optimal long-run inflation rate, which equally obtains under zero-optimal and the standard timeless perspective policy, is given by

$$\overline{\pi}^{GGZOTP} = \frac{(1-\alpha)(1-\beta)\omega\kappa}{(1-\omega)\alpha\theta\kappa + (1-\alpha)(1-\beta)^2\omega}x^* \tag{4.24}$$

Given $k > 0$ and $0 < \alpha < 1$, $\overline{\pi}^{GGZOTP}$ is positive and collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips-curve trade-off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$).

The reason behind the optimality of positive steady-state inflation is intuitive. The only difference implied by Galì-Gertler's rule-of-thumb behaviour, with respect to the purely forward-looking model, is in terms of the first-order condition with respect to inflation, (4.17). Substituting for χ_f and χ_b in terms of structural parameters yields

$$\pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta\lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \frac{\alpha}{\phi}\varphi_{t-1} - \frac{\beta\omega}{\phi}\varphi_{t+1} = 0 \tag{4.25}$$

Higher inflation in any period results in output increase in the same period, φ_t, and reduction in output in both the previous period as a result of expected higher inflation, $(\alpha/\phi)\varphi_{t-1}$, and the subsequent period, $(\beta\omega/\phi)\varphi_{t+1}$. Recalling that $\phi = \alpha + \omega\left[1 - \alpha(1-\beta)\right]$, (4.25) evaluated at steady state delivers

$$\overline{\pi} + \left[1 - \frac{\alpha + \beta\omega}{\alpha + \omega\left[1 - \alpha(1-\beta)\right]}\right]\overline{\varphi} = 0 \tag{4.26}$$

Checking the relationship between the stimulative effect of higher inflation on output and the output cost of higher inflation amounts to solve the inequality

$$1 \geq \frac{\alpha + \beta\omega}{\alpha + \omega\left[1 - \alpha(1-\beta)\right]} \tag{4.27}$$

The solution is given by

$$\omega(1-\beta)(1-\alpha) \geq 0 \tag{4.28}$$

Backward-looking rule-of-thumb behaviour results in the stimulative effect of higher inflation on output being generally greater than the output cost of higher inflation. The stimulative effect of higher inflation equals the output cost of higher inflation in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$) or in the absence of long-run

Phillips curve trade off (i.e. $\beta = 1$). Otherwise, there exists a long-run incentive for positive inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

The optimality conditions under the alternative timeless perspective policy can be found by considering the expected undiscounted minimisation problem conditional on information available at date zero. That is, the Lagrangian (4.3) becomes

(4.29)
$$\mathcal{L}^{GG}_{0,Undis} = E_0 \sum_{t=0}^{\infty} \left\{ \frac{1}{2} \left[\pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 (\pi_t - \pi_{t-1})^2 \right] + \varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_1 x_t \right] \right\}$$

Differentiating with respect to π_t and x_t, the output gap optimality condition (4.18) is unaffected but instead of the inflation optimality condition (4.17) we obtain

$$\text{(4.30)} \quad \frac{\partial \mathcal{L}^{GG}_{0,Undis}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \chi_f \beta \varphi_{t-1} - \chi_b \varphi_{t+1} = 0$$

From a timeless perspective, the central bank sets policy according to optimality conditions (4.18) and (4.30) in all periods $t \geq 0$. Combining these optimality conditions, the central bank's target criterion is given by

(4.31)
$$\pi_t = \frac{1}{1 + 2\lambda_2} \left\{ \lambda_2 \pi_{t-1} + \lambda_2 \pi_{t+1} + \frac{\lambda_1}{\kappa_1} \left[\chi_b x_{t+1} + \chi_f \beta x_{t-1} - x_t + (1 - \chi_b - \chi_f \beta) x^* \right] \right\}$$

Evaluating the central bank's rule (4.31) at steady state, we obtain the same steady-state target criterion that is implied by the alternative timeless perspective in the purely forward-looking model, namely (4.14). This is because the terms in inflation acceleration in the inflation optimality condition (4.30) do not matter at steady state. Additionally, the different coefficient on the steady-state Lagrange multiplier, $\overline{\varphi}$, simplifies once the output gap optimality condition, (4.18), is taken into account[5].

[5] The optimality condition for inflation (4.30) implies that at steady state $\overline{\pi} + [(1-\beta)(1-\omega)\alpha/\phi]\overline{\varphi} = 0$. The optimality condition for output gap (4.30) implies that at steady state $\lambda_1(\overline{x} - x^*) - \kappa_1 \overline{\varphi} = 0$. Given the definition of $\kappa_1 = (1-\omega)\alpha\kappa/\phi$, it follows that combining (4.30) and (4.30) at steady state yields (4.14).

Moreover, as discussed in Chapter 2, the long-run Phillips-curve trade-off is not affected by the introduction of rule-of-thumb behaviour à la Galì and Gertler (1999). Therefore, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour, when this is characterised as in Galì and Gertler (1999), with the optimal long-run inflation rate being invariant to the degree of rule-of-thumb behaviour, namely

$$\overline{\pi}^{FLATP} = \overline{\pi}^{GGATP} \tag{4.32}$$

4.3.2. Rule-of-thumb Behaviour à la Steinsson

A central bank acting under commitment faces the problem of choosing paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise the expected discounted sum of central bank's loss function, with the single-period loss given by (3.24), conditional on information available at date zero subject to the constraint that the sequences must satisfy (2.53) each period. The Lagrangian associated with this problem is of the form

$$\mathcal{L}_0^S = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \frac{\lambda_2}{2}\left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2 \\ +\varphi_t\left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_2 x_t - \kappa_3 x_{t-1}\right] \end{array} \right\} \tag{4.33}$$

Differentiating with respect to π_t and x_t, we get the optimality conditions

$$\frac{\partial \mathcal{L}_0^S}{\partial \pi_t} = 0 \Rightarrow \left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} \\ +\lambda_2\left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] \\ -\beta\lambda_2\left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \tag{4.34}$$

$$\frac{\partial \mathcal{L}_0^S}{\partial x_t} = 0 \Rightarrow \left\{ \begin{array}{c} \lambda_1(x_t - x^*) - \kappa_2\varphi_t - \beta\kappa_3\varphi_{t+1} \\ -\beta\lambda_2(1-\alpha)\delta\left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \tag{4.35}$$

Under zero-optimal commitment policy, there is no fulfillment of the expectations existing at the time of the policy implementation, that is (4.34) in period 0 holds with $\varphi_{-1} = 0$.

Hence, zero-optimal commitment policy is characterised by the output gap optimality condition (4.35) for all $t \geq 0$ and two inflation optimality conditions: one for period zero and one for all subsequent periods

$$\left\{ \begin{array}{c} \pi_t + \varphi_t - \beta\chi_b\varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] \\ -\beta\lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \qquad t = 0 \tag{4.36}$$

$$\left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f\varphi_{t-1} - \beta\chi_b\varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] \\ -\beta\lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0 \qquad t \geq 1 \tag{4.37}$$

Combining the optimality conditions (4.35) and (4.36) delivers the central bank's target criterion in period 0

$$\pi_0(1+\lambda_2) = \lambda_2\left(\pi_{-1} + (1-\alpha)\delta x_{-1}\right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_0 - x^*) \tag{4.38}$$

$$+\underbrace{(\chi_b - \frac{\kappa_3}{(1-\alpha)\delta}}_{0})\beta\varphi_1 - (\frac{\kappa_2}{(1-\alpha)\delta} + 1)\varphi_0 \tag{4.39}$$

whereas combining the optimality conditions (4.35) and (4.37), the central bank in any period $t \geq 1$ behaves according to a rule of the form

$$\pi_t = \frac{1}{1+\lambda_2}\left\{ \begin{array}{c} \lambda_2\left(\pi_{t-1} + (1-\alpha)\delta x_{t-1}\right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_t - x^*) \\ -\left(\frac{\kappa_2}{(1-\alpha)\delta} + 1\right)\varphi_t + \chi_f\varphi_{t-1} \end{array} \right\} \tag{4.40}$$

It is interesting to note that, in any period, the Lagrangian multiplier associated with the subsequent period aggregate-supply relation does not enter the target criterion. This is because, as shown in (4.38), the coefficient on the Lagrangian multiplier associated with the subsequent period hybrid Phillips curve is constantly equal to zero.

However, given the intertemporal nature of the output gap optimality condition (4.35), it is cumbersome to express the central bank's rule as a function of inflation and output gap only. Hence, when we analyse monetary policy in the presence of rule-of-thumb behaviour à la Steinsson (2003) we consider the two optimality conditions separately. In other words, the Lagrangian multiplier becomes an additional endogenous variable and we have three equations for the determination of the three endogenous variables.

Under the standard timeless perspective commitment policy, the start-up condition (4.36) is ignored and the central bank's rule in all periods $t \geq 0$ is given by (4.40). Given $x^* > 0$, there is an advantage for having positive long-run inflation. The two optimality conditions, (4.34) and (4.35), can be simultaneously satisfied only if

$$(4.41) \qquad \overline{\pi} = \frac{(1-\alpha)(1-\beta)(1-\omega)(\delta\theta-1)\alpha\omega\kappa}{(1-\omega)\alpha\theta\left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]}\overline{x} + \frac{(1-\alpha)(1-\beta)\omega\kappa}{\theta\left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]}x^*$$

As discussed in Chapter 2, rule-of-thumb behaviour à la Steinsson (2003) does not imply that all price setters behave identically once shocks are eliminated from the economy. Specifically, the fact that rule-of-thumb price setters index their prices to lagged output gap alters the long-run Phillips-curve trade-off that obtains in the purely forward-looking model. Using the long-run Phillips-curve trade-off in (2.61) so to eliminate $\overline{x}$ from (4.41), the optimal long-run inflation rate, which equally obtains under zero-optimal and the standard timeless perspective policy, is given by

(4.42)

$$\overline{\pi}^{SZOTP} = \frac{(1-\alpha)(1-\beta)\kappa\theta^{-1}\omega\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]}{\left\{\begin{array}{c}(1-\omega)(1-\alpha)(\theta^{-1}-\delta)(1-\beta)^2\alpha\omega\kappa+\\ \left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]\end{array}\right\}}x^* = \Psi x^*$$

Given $k > 0$ and $0 < \alpha < 1$, $\overline{\pi}^{SZOTP}$ is positive and collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips-curve trade-off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$).

The reason behind the optimality of positive inflation is the same we have described above in the case of rule-of-thumb behaviour à la Galì and Gertler (1999). The reason for this is quite intuitive. On the one hand, the fact that rule-of-thumb price setters index their prices to lagged output gap under Steinsson's rule-of-thumb behaviour alters the relationship between inflation and output gap implied by the optimality conditions. In particular, the output gap enters the inflation optimality condition and inflation enters the output gap optimality condition. On the other hand, the indexation to past output gap does not affect the way in which current inflation is related to the Lagrange multipliers through the inflation optimality condition. In other words, the Lagrange multipliers enter the inflation optimality condition in the same way as under rule-of-thumb behaviour à la Galì and Gertler (1999). This can be clearly seen by comparing the inflation optimality conditions (4.17) and (4.34).

It follows that backward-looking rule-of-thumb behaviour, regardless of its specification, results in the stimulative effect of higher inflation on output being generally greater than the output cost of higher inflation. The stimulative effect of higher inflation equals the output cost of higher inflation in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$) or in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$). Otherwise, there exists a long-run incentive for positive inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

The optimality conditions under the alternative timeless perspective policy can be found by considering the expected undiscounted minimisation problem conditional on information available at date zero. That is, the Lagrangian (4.33) becomes

$$(4.43) \qquad \mathcal{L}^{S}_{0,Undis} = E_0 \sum_{t=0}^{\infty} \left\{ \begin{array}{c} \frac{1}{2}\pi_t^2 + \frac{\lambda_1}{2}(x_t - x^*)^2 + \frac{\lambda_2}{2}\left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right]^2 \\ +\varphi_t\left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_2 x_t - \kappa_3 x_{t-1}\right] \end{array} \right\}$$

Differentiating with respect to π_t and x_t, both optimality conditions are affected. The inflation optimality condition is now given by

$$(4.44) \qquad \frac{\partial \mathcal{L}^S_{0,Undis}}{\partial \pi_t} = 0 \Rightarrow \left\{ \begin{array}{c} \pi_t + \varphi_t - \chi_f \beta \varphi_{t-1} - \chi_b \varphi_{t+1} \\ +\lambda_2 \left[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})\right] \\ -\lambda_2 \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0$$

whereas the output gap optimality condition takes the form

$$(4.45) \qquad \frac{\partial \mathcal{L}^S_{0,Undis}}{\partial x_t} = 0 \Rightarrow \left\{ \begin{array}{c} \lambda_1 (x_t - x^*) - \kappa_2 \varphi_t - \kappa_3 \varphi_{t+1} \\ -\lambda_2 (1-\alpha)\delta \left[\pi_{t+1} - (\pi_t + (1-\alpha)\delta x_t)\right] \end{array} \right\} = 0$$

From a timeless perspective, the central bank sets policy according to optimality conditions (4.44) and (4.45) in all periods $t \geq 0$. Combining the first order conditions, the central bank's target criterion is given by

$$(4.46) \qquad \pi_t = \frac{1}{1+\lambda_2} \left\{ \begin{array}{c} \lambda_2 \left(\pi_{t-1} + (1-\alpha)\delta x_{t-1}\right) + \frac{\lambda_1}{(1-\alpha)\delta}(x_t - x^*) \\ -\left(\frac{\kappa_2}{(1-\alpha)\delta} + 1\right) \varphi_t + \chi_f \beta \varphi_{t-1} \end{array} \right\}$$

Given $x^* > 0$, there is an advantage for having positive long-run inflation. The two optimality conditions, (4.44) and (4.45), can be simultaneously satisfied only if

(4.47)
$$\overline{\pi} = -\frac{\left[(1-\omega)\alpha\kappa + (1-\alpha)^2\delta^2\theta\omega\right](1-\beta)}{\theta\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right]}\overline{x} + \frac{(1-\beta)(1-\omega)\alpha\kappa}{\theta\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right]}x^*$$

Using the long-run Phillips-curve trade-off (2.61) to eliminate $\overline{x}$ from (4.47), the alternative timeless perspective policy implies positive steady-state inflation of the form[6]

$$(4.48) \qquad \overline{\pi}^{SATP} = \frac{\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right](1-\beta)(1-\omega)\alpha\kappa}{\left\{ \begin{array}{c} \theta\left[(1-\omega)\alpha\kappa + (1-\alpha)(1-\alpha\beta)\omega\delta\right]^2 \\ +\left[(1-\omega)\alpha\kappa + (1-\alpha)^2\delta^2\theta\omega\right](1-\beta)^2(1-\omega)\alpha \end{array} \right\}} x^* = \Theta x^*$$

[6]Under $\delta = 0$, $\overline{\pi}^{SBJM}$ collapses to $\overline{\pi}^{FLBJM}$.

Given $k > 0$ and $0 < \alpha < 1$, $\overline{\pi}^{SATP}$ is positive and collapses to zero in the absence of long-run Phillips-curve trade-off (i.e. $\beta = 1$) and in the absence of steady-state distortions (i.e. $x^* = 0$). If the rule-of thumb is characterised as in Steinsson (2003), the optimal long-run inflation rate under the alternative timeless perspective commitment policy ceases to be the same as in the purely forward-looking New Keynesian model.

4.4. Discussion

Table 1 summarises the results obtained for the optimal long-run inflation rates.

Model/Policy	Timeless Perspective	Alternative Timeless Perspective
Purely Forward-Looking	$\overline{\pi}^{FLZOTP} = 0$	$\overline{\pi}^{FLATP} > 0$
Galì-Gertler rule-of-thumb	$\overline{\pi}^{GGZOTP} > 0$	$\overline{\pi}^{GGATP} = \overline{\pi}^{FLATP}$
Steinsson rule-of thumb	$\overline{\pi}^{SZOTP} > 0$	$\overline{\pi}^{SATP} > 0$

Table 4.1. Optimal Long-run Inflation Rates

Zero long-run inflation is the optimal target for monetary policy only in the purely forward-looking model under the standard timeless perspective policy (or the zero-optimal policy).

In all the other cases, it is optimal for the central bank to target a positive inflation rate. Two different reasons emerge as to why the combination of a long-run Phillips curve trade-off and steady-state distortions results in positive inflation forever, even under commitment.

First, backward-looking rule-of-thumb behaviour, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), entails that the stimulative effect of higher inflation on output is generally greater than the output cost of higher inflation. The stimulative effect of higher inflation equals the output cost of higher inflation in the absence of backward-looking rule-of-thumb behaviour or in the absence of a long-run Phillips-curve trade-off. Otherwise, there exists a long-run incentive for positive inflation. The long-run Phillips curve trade-off is then exploited and it is optimal for the central bank to commit to positive steady-state inflation.

Second, the alternative timeless perspective policy put forward by Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008) is also capable of delivering optimal positive steady-state inflation. The result can be interpreted as highlighting the effects of discounting on monetary policy choices. If the central bank shares the same discount factor of the private sector, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero (i.e. $\overline{\pi}^{FLZOTP}$). Conversely, if the central bank does not discount the future, positive steady-state inflation emerges under commitment even in the purely forward-looking model (i.e. $\overline{\pi}^{FLATP}$). In particular, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour, when this is characterised as in Galì and Gertler (1999), with the optimal long-run inflation rate being invariant to the degree of rule-of-thumb behaviour.

In all theoretical economies, optimal steady-state inflation collapses to zero in the absence of a long-run Phillips-curve trade-off and in the absence of steady-state distortions. Taking together the basic message of our results is that the widespread practice in the New Keynesian literature of restricting the attention to the case of an efficient natural level of output is not innocuous. A policy that is optimal for an economy with an efficient steady state differs from what is optimal in an economy where the subsidies that achieve Pareto efficiency are unavailable.

CHAPTER 5

Quantitative Analysis and Welfare Analysis

In this chapter we begin by discussing the calibration of the models' structural parameters. The models' primitives are the average duration that an individual price is fixed, namely the degree of price stickiness, α, and the fraction of firms that reset prices in a backward-looking manner, that is the degree of rule-of-thumb behaviour, ω. Available empirical estimates of the degree of price stickiness vary greatly according to whether they are based on macro or micro data. Similarly, available empirical estimates of the degree of rule-of-thumb behaviour span over a large range. Hence, we consider ample ranges for both the average duration that an individual price is fixed and the fraction of firms that reset prices in a backward-looking manner.

We proceed by evaluating the optimal long-run inflation rates derived in Chapter 4. All optimal long-run inflation rates turn out to be small in magnitude. On the one hand, the optimal long-run inflation rates we derive are not capable of explaining the observed inflation rates. On the other hand, the policy-driven steady state is very close to the steady state around which the models are log-linearised, which is characterised by zero inflation. It follows that, as discussed in Chapter 3, we can use first-order approximations to evaluate welfare accurately up to second order.

We conclude by evaluating welfare under the alternative commitment policies. We initially characterise welfare on the basis of the deterministic equilibrium to establish whether steady-state inflation is welfare enhancing with respect to a policy of zero steady-state inflation. However, welfare analysis is typically conducted in a stochastic environment so to quantify the welfare costs due to the stabilisation of shocks. We thus employ the dynamic nature of the three models derived in the previous chapters. As discussed in Chapter 2, the monetary authority, in our models, should not respond to movements in output which are caused by preference shocks or shocks to productive capabilities. This is because the movements in output brought about by those shocks are efficient, namely they represent variations in the efficient level of output. Following Clarida et al. (1999), we hence augment the aggregate-supply relations with an inefficient shock (i.e. a cost-push shock) and analyse the welfare costs due to the stabilisation of the cost-push shock. Similarly, if we were to drop the assumed complete and efficient financial markets we would then be able to consider the stabilisation, and the welfare costs associated with it, of financial shocks.

In performing welfare analysis, our main objective is, as in Jensen and McCallum (2002), to simply rank the alternative commitment policies. We present robustness analysis for ample ranges of two structural parameters rather than, as in Blake (2001) and Jensen and McCallum (2002), coefficients that are functions of structural parameters..

On the basis of the deterministic equilibrium, the zero-optimal commitment policy ranks first followed by the alternative timeless perspective policy and the standard timeless perspective policy. Moreover, steady-state inflation is found to be welfare enhancing with respect to a policy of zero steady-state inflation. The reason for this is that positive steady-state inflation, by bringing about positive output gap, allows eliminating some of the steady-state loss due to monopolistic competition. This conclusion is only slightly affected in the presence of rule-of-thumb behaviour. Precisely, we find that the alternative timeless perspective policy is always superior to a policy of zero inflation at all times whereas the same it is not always true under the standard timeless perspective policy: unrealistically high levels of the degree of rule-of-thumb behaviour would imply that having positive steady-state inflation, hence a positive degree of price dispersion, would only add to the steady-state loss due to monopolistic competition.

On the basis of the stochastic equilibrium, we consider both an unconditional welfare measure and a measure of welfare conditional on initial conditions. When considering unconditional welfare, the alternative timeless perspective policy ranks first followed by the standard timeless perspective policy and the zero-optimal policy. When considering welfare conditional on initial conditions, the zero-optimal policy ranks first followed by the standard timeless perspective policy and the alternative timeless perspective policy.

5.1. Calibration

Before proceeding to evaluate the optimal long-run inflation rates and welfare under the alternative commitment policies, we discuss how we calibrate the model. The time unit is a quarter. Table (5.1) presents our benchmark calibration together with a description of the structural parameters.

The purely forward-looking model contains five structural parameters (α, β, θ, ϖ, and σ^{-1}), for which values must be specified. Allowing for rule-of-thumb behaviour, introduces the additional parameter ω. Finally, under rule-of-thumb behaviour à la Steinsson (2003), there is the further need to calibrate the parameter δ.

Given that the purely forward-looking model considered here is exactly the basic neo-Wicksellian model in Woodford (2003), it follows that it is natural to consider the benchmark calibration in Woodford (2003, p. 431). The calibration stems from the estimation results in Rotemberg and Woodford (1997), which presents estimates based on quarterly data for a purely forward-looking New Keynesian model using a moment-matching approach. Consequently, (β, θ, ϖ, and σ^{-1}) are given by: $\beta = 0.99$, $\theta = 7.88$, $\varpi = 0.47$, and $\sigma^{-1} = 0.16$. The parameter that summarises steady-state distortions, Φ_y, is implied by the definition of the natural steady-state level of output, as in equation (2.51): given $\theta = 7.88$, it follows that $\Phi_y = 0.127$. Accordingly, the definition of the the steady-state efficiency gap, as given by equation (2.52), entails that x^* is equal to 0.2, which, of course, is the same value used by Woodford (2003).

In the absence of an empirical estimate for the degree of indexation to lagged output gap, δ, by rule-of-thumb price setters, we follow Steinsson (2003) and set it to 0.052. Steinsson (2003) obtains this value by imposing that the coefficient on x_{t-1} in the purely backward-looking Phillips curve, which as discussed above is implied in the limit when $\omega \rightarrow 1$, is equal to the coefficient on x_t in the hybrid Phillips curve (2.53) when $\omega = 0$.

The remaining two structural parameters are in fact the key model's primitives: the average duration that an individual price is fixed, namely the degree of price stickiness, α, and the fraction of firms that reset prices in a backward-looking manner, that is the degree of rule-of-thumb behaviour, ω. Galì and Gertler (1999) report estimates of ω between 0.077 and 0.552, with 3 of their 6 estimates between 0.2 and 0.3. As for the degree of price stickiness, empirically realistic values of the average price duration based on macroeconomic data vary between 2 and 5 quarters, namely $0.5 \leq \alpha \leq 0.8$. Evidence on price stickiness based on microeconomic data suggest a much higher frequency of price changes than the evidence based on macro data. Available empirical estimates using microeconomic data, as in Bils and Klenow (2004) and Golosov and Lucas (2007), suggest in fact a lower average price duration of around 1.5 quarters, that is a value of α of about 0.33.

In what follows we want to assess the robustness of our results with respect to alternative values for these two parameters. We thus consider $0.33 \leq \alpha \leq 0.8$ and extend the range for the degree of rule-of-thumb behaviour up to 0.7, namely $0.01 \leq \omega \leq 0.7$. This is because $\omega = 0.7$ implies that the hybrid Phillips curve under rule-of-thumb behaviour, regardless of its specification, puts equal weight on future expected inflation and lagged inflation.

We must stress that, once we assess the robustness of the welfare ranking in the presence of rule-of-thumb behaviour by price setters, we pick $\alpha = 0.66$, namely an average price duration of 3 quarters, and $\omega = 0.3$ as our benchmark values This is because they both sit in the middle of the respective range of available empirical estimates. The results presented below are unaffected if one were to consider different benchmark estimates for both α and ω. Specifically, we have considered limiting values for ω (i.e. $\omega = 0.01$ and $\omega = 0.7$) when analysing robustness with respect to α and limiting values for α (i.e. $\alpha = 0.33$ and $\alpha = 0.8$) when studying robustness with respect to ω.

Parameter	Value	Description
β	0.99	Subjective discount factor
θ	7.88	Elasticity of substitution between varieties of goods
ϖ	0.47	Elasticity of real marginal cost with respect to own output
σ^{-1}	0.16	Intertemporal elasticity of substitution of aggregate expenditure
x^*	0.2	Steady-state efficiency gap
δ	0.052	Degree of indexation to past output gap
α	$0.33 - 0.8$	Degree of price stickiness
ω	$0 - 0.7$	Degree of rule-of-thumb behaviour

Table 5.1. Benchmark Calibration

5.2. Optimal Long-run inflation rates

We now proceed to evaluate the positive optimal long-run inflation rates. We throughout present the annualised percentage optimal long-run inflation rates.

Figure (5.1) reports the annualised percentage optimal steady-state inflation which is implied by the alternative timeless perspective policy in both the purely forward-looking model and in the model with rule-of-thumb behaviour à la Galì and Gertler (1999). Figure (5.2) presents the annualised percentage optimal steady-state inflation which is implied by the standard timeless perspective policy in the model with rule-of-thumb behaviour à la Galì and Gertler (1999). Figure (5.3) shows the annualised percentage optimal steady-state inflation which is implied by the standard timeless perspective policy in the model with rule-of-thumb behaviour à la Steinsson (2003). Finally, Figure (5.4) displays the annualised percentage optimal steady-state inflation which is implied by the alternative timeless perspective policy in the model with rule-of-thumb behaviour à la Steinsson (2003).

There are two main observations to take from Figures (5.1)-(5.4).

First, the deviation from zero inflation is observed to be minimal. On the one hand, the optimal long-run inflation rates we derive are thus not capable of explaining the observed

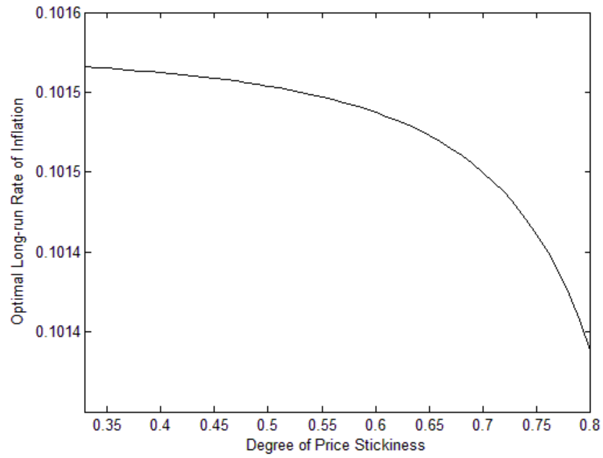

Figure 5.1. Purely forward-looking model and model with rule-of-thumb behaviour à la Galì and Gertler (1999). Annualised percentage optimal steady-state inflation implied by the alternative timeless perspective policy.

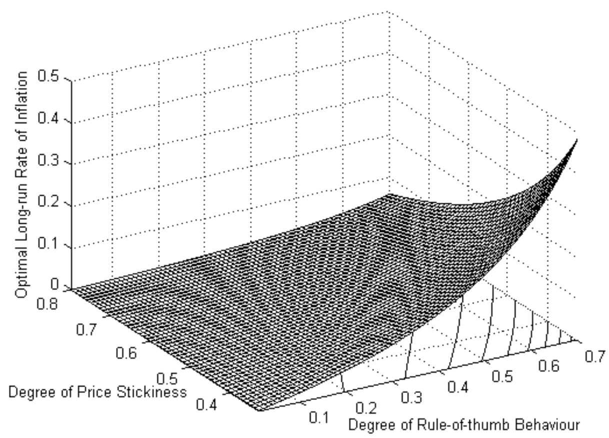

Figure 5.2. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Annualised percentage optimal steady-state inflation implied by the standard timeless perspective policy.

inflation rates. In effect, in developed countries inflation rates vary between 2% and 4% per year whereas slightly higher targets are observed in developing countries. On the other hand, the policy-driven steady state is, both in terms of inflation and output gap, very close to the steady state around which the models are log-linearised, which is characterised by zero inflation and zero output gap. It follows that we can rest assured that it is possible to use first-order approximations to evaluate welfare accurately up to second order. Indeed,

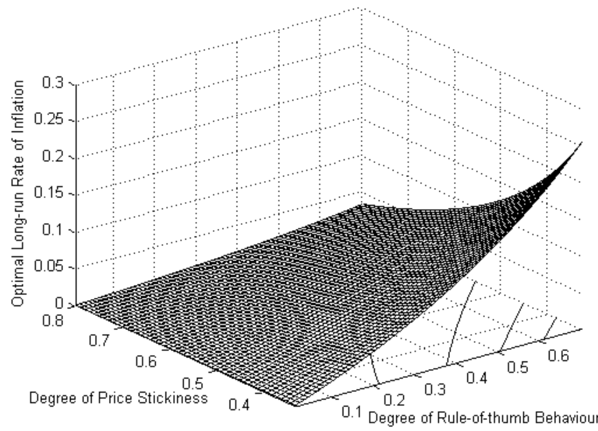

Figure 5.3. Model with rule-of-thumb behaviour à la Steinsson (2003). Annualised percentage optimal steady-state inflation implied by the standard timeless perspective policy.

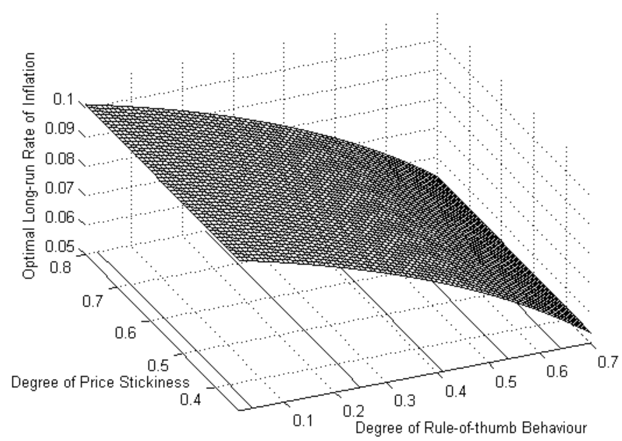

Figure 5.4. Model with rule-of-thumb behaviour à la Steinsson (2003). Annualised percentage optimal steady-state inflation implied by the alternative timeless perspective policy.

as we shall see below, the different commitment policies rank in terms of welfare in line with the intuition.

Second, the behaviour with respect to the structural parameters is quite robust across the different optimal long-run inflation rates. Table (5.2) presents comparative statics. The optimal long-run inflation rates are observed to be monotonically decreasing in the structural parameters. It is only under the standard timeless perspective policy that an increasing degree of rule-of-thumb behaviour, regardless of its specification, is associated with an increasing inflation rate.

inflation rate	Monotonically Decreasing	Monotonically Increasing
$\overline{\pi}^{FLATP} = \overline{\pi}^{GGATP}$	α, β, θ, ϖ, and σ^{-1}	
$\overline{\pi}^{GGZOTP}$	α, β, θ, ϖ, and σ^{-1}	ω
$\overline{\pi}^{SZOTP}$	α, β, θ, ϖ, σ^{-1}, and δ	ω
$\overline{\pi}^{SATP}$	α, β, θ, ϖ, σ^{-1}, δ, and ω	

Table 5.2. Comparative Statics

5.3. Welfare Analysis

We evaluate the alternative commitment policies both on the basis of the deterministic equilibrium and on the basis of the stochastic equilibrium, which stems from augmenting the aggregate-supply with a cost push shock. In so doing, our main objective is, as in Jensen and McCallum (2002), to simply rank the alternative commitment policies. We present robustness analysis for ample ranges of two structural parameters rather than, as in Blake (2001) and Jensen and McCallum (2002), coefficients that are functions of structural parameters.

The natural welfare criterion is the discounted sum of utility of the representative household, which, as shown in Chapter 3, is approximated to second-order by the discounted sum of central bank's single-period loss function. In other words, the welfare criterion is given by the central bank's objective function (3.23), which we report here for convenience

$$W = -\Omega \sum_{t=0}^{\infty} \beta^t L_t \tag{5.1}$$

The constant Ω is given by $\Omega = \overline{Y}\widetilde{u}_c(\sigma^{-1} + \varpi)\theta/2\kappa$. Following Erceg et al. (2000), we express welfare as a proportion of steady-state level of output. Moreover, we present welfare in percent terms. Expressing welfare as a proportion of steady-state level of output implies manipulating Ω. Dividing Ω by $\overline{Y}\widetilde{u}_c$, welfare would be expressed as a proportion of one period's steady-state level of output. Applying the perpetuity formula, welfare as a proportion of steady-state level of output implies that the constant Ω is given by

$$\Omega = \frac{(1-\beta)(\sigma^{-1}+\varpi)\theta}{2\kappa} \tag{5.2}$$

The level of welfare that obtains in the deterministic equilibrium, denoted with $\overline{W}$, takes the form

$$\overline{W} = -\frac{\Omega\overline{L}}{(1-\beta)} = -\frac{(\sigma^{-1}+\varpi)\theta}{2\kappa}\overline{L} \tag{5.3}$$

where $\overline{L}$ is the steady-state loss function.

On the basis of the stochastic equilibrium, we report two measures of welfare. First, we consider welfare conditional on information available at date zero, which is given by

$$W = -\Omega E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

with Ω given as in (5.2). In evaluating welfare conditional on information available at date zero, we assume that the economy's initial condition is the steady state implied by the policy under consideration.

Second, we report average values of the central bank's objective function so to abstract from initial conditions. That is, we evaluate alternative policies by taking the unconditional expectation of the welfare criterion in (5.1). The unconditional welfare measure is the most commonly employed in the literature. Opposite to the conditional welfare measures, that is conditional on the initial point, the unconditional one by design 'integrates away' the role of the initial state. Denoting with E the unconditional expectation operator, the law of iterated expectations then implies that welfare is proportional to the unconditional expectation of the single-period loss function.

$$E(W) = E\left(-\Omega\sum_{t=0}^{\infty}\beta^t L_t\right) = -\Omega(1-\beta)^{-1}E(L_t) = -\frac{(\sigma^{-1}+\varpi)\theta}{2\kappa}E(L_t) \tag{5.4}$$

Here, as in Woodford (2003, Ch. 7), the measure of variability of any random variable z entering the single-period loss function is given by $V\left[z\right] = (1-\beta)\sum_{t=0}^{\infty}\beta^t var(z_t)$.

5.3.1. Welfare on the basis of the Deterministic Equilibrium

We first consider welfare on the basis of the deterministic equilibrium. The purposes of this exercise are twofold.

First, we want to establish the ranking of the alternative commitment policies. Intuitively, the zero-commitment policy should rank first followed by the two timeless perspective policies. The intuition follows from two observations. First, it is only under the zero-optimal policy that we observe a transition toward the steady state whereas under the timeless perspective policies the economy is already at steady state and remains there forever. Second, the transition toward the steady state is always welfare-enhancing. The presence of the linear term in the steady-state efficiency gap, x^*, in the single-period central bank's loss function implies a welfare gain from rate of inflation that, at least initially, differs from the steady-state rate of inflation. This welfare gain can only be achieved if there is transition toward the steady-state rate of inflation, which only happens under zero-optimal policy.

Second, we can compare whether a positive long-run inflation improves welfare relative to a policy of zero inflation at all times. In other words, we want to assess whether steady-state inflation is welfare reducing due to its effects on relative price dispersion. In the purely forward-looking model the comparison is straightforward as the standard timeless perspective policy implies zero steady-state inflation whereas the alternative timeless perspective policy implies positive steady-state inflation. In the models with rule-of-thumb behaviour, the steady-state loss function can instead be seen as the sum of the loss that obtains in the presence of zero inflation at all times and the loss attributable to positive optimal long-run inflation. Hence, in all our theoretical models, we are able to address whether positive long-run inflation improves welfare relative to a policy of zero inflation at all times.

The results we obtain are in line with the a priori beliefs. First, in all our theoretical economies, zero-optimal commitment policy delivers the highest level of welfare on the basis of the deterministic equilibrium followed by the alternative timeless perspective policy and the standard timeless perspective commitment.

Second, steady-state inflation is welfare enhancing with respect to a policy of zero steady-state inflation. A positive but small level of inflation, which generates positive steady-state price dispersion, is thus preferable to a policy of zero inflation at all times, which implies zero steady-state price dispersion. The reason for this is that positive steady-state inflation, by bringing about positive output gap, allows eliminating some of the steady-state loss due to monopolistic competition. This conclusion is only slightly affected in the presence of rule-of-thumb behaviour. Precisely, we find that the alternative timeless perspective policy is always superior to a policy of zero inflation at all times whereas the same is not always true under the standard timeless perspective policy. In other words, under the alternative timeless perspective policy a small but positive level of inflation, which brings about a positive level of price dispersion, invariably allows eliminating some of the steady-state loss due to monopolistic competition. Conversely, under the standard timeless perspective policy, unrealistically high levels of the degree of rule-of-thumb behaviour would imply that having positive steady-state inflation, hence a positive degree of price dispersion, would only add to the steady-state loss due to monopolistic competition.

5.3.1.1. Basic New Keynesian Model. Woodford (2003, Ch. 6) shows that under the zero-optimal commitment policy there exists a unique nonexplosive solution for the Lagrange multiplier associated with the NKPC. This solution, which is consistent with the zero-optimal commitment policy not fulfilling the period-minus-one NKPC (i.e. $\varphi_{-1} = 0$

in (4.3)), is of the form

$$\varphi_t = -(1 - u_1^{t+1})\frac{\lambda}{\kappa}x^* \tag{5.5}$$

where $u_1 < 1$ depends on the model's structural parameters, namely $u_1 = \left[\gamma - (\gamma^2 - 4\beta)^{0.5}\right]/2\beta$ with $\gamma = 1 + \beta + (\kappa^2/\lambda)$. Given the optimal path for inflation (i.e. (4.4)), this solution for the multiplier implies that inflation under the zero-optimal commitment policy evolves according to

$$\pi_t = (1 - u_1)\frac{\lambda}{\kappa}u_1^t x^* \tag{5.6}$$

Similarly, given the optimal path for output gap (i.e. (4.5), output gap under the zero-optimal commitment policy evolves according to

$$x_t = u_1^{t+1} x^* \tag{5.7}$$

The single-period loss function, (3.27), can thus be rewritten solely as a function of the model's structural parameters

$$L_t^{FLZO} = \left[1 + (1 - u_1)^2 \frac{\lambda}{\kappa^2} u_1^{2t} + u_1^{2(t+1)} - 2u_1^{t+1}\right]\lambda x^{*2} \tag{5.8}$$

which implies that welfare on the basis of the deterministic equilibrium under the zero-optimal commitment policy is of the form

$$\overline{W}^{FLZO} = -\Omega\lambda x^{*2} E\sum_{t=0}^{\infty}\beta^t\left[1 + (1 - u_1)^2\frac{\lambda}{\kappa^2}u_1^{2t} + u_1^{2(t+1)} - 2u_1^{t+1}\right] \tag{5.9}$$

Given $\beta < 1$ and $u_1 < 1$, all the terms entering the sums to infinity are converging geometric series. Hence, it is possible to eliminate the infinite sums, so that (5.9) becomes

(5.10)

$$\overline{W}^{FLZO} = -\frac{(\sigma^{-1} + \varpi)}{2}x^{*2} - \frac{(1 - \beta)(\sigma^{-1} + \varpi)}{2}x^{*2}\left\{\frac{1}{1 - \beta u_1^2}\left[u_1^2 + \frac{(1 - u_1)^2\lambda}{\kappa^2}\right] - \frac{2u_1}{1 - \beta u_1}\right\}$$

Under the alternative timeless perspective policy, it is optimal to have positive steady-state inflation. Given the long-run trade-off implied by the NKPC (i.e. (2.60)), combining the steady-state loss function, $\overline{L}^{FL} = \overline{\pi}^2 + \lambda(\overline{x} - x^*)^2$, with the optimal positive long-run inflation rate (i.e. (4.15)), yields

$$\overline{L}^{FLATP} = \frac{1}{[1 + (1-\beta)^2(\theta\kappa)^{-1}]}\lambda x^{*2} \tag{5.11}$$

which implies that welfare on the basis of the deterministic equilibrium under the alternative timeless perspective policy is of the form

$$\overline{W}^{FLATP} = -\frac{(\sigma^{-1} + \varpi)}{2\,[1 + (1-\beta)^2(\theta\kappa)^{-1}]}x^{*2} \tag{5.12}$$

Under the standard timeless perspective commitment policy, it is optimal to have zero inflation. The steady-state loss function takes the form

$$\overline{L}^{FLTP} = \lambda x^{*2} \tag{5.13}$$

which implies that welfare on the basis of the deterministic equilibrium under the standard timeless perspective is invariant to the degree of price stickiness, that is

$$\overline{W}^{FLTP} = -\frac{(\sigma^{-1} + \varpi)}{2}x^{*2} \tag{5.14}$$

Welfare on the basis of the deterministic equilibrium under the alternative timeless perspective policy is thus seen to be always better than the one under the standard timeless perspective policy, namely

$$\frac{\overline{W}^{FLATP}}{\overline{W}^{FLTP}} = \frac{1}{[1 + (1-\beta)^2(\theta\kappa)^{-1}]} < 1 \tag{5.15}$$

In other words, steady-state inflation is welfare enhancing with respect to a policy of zero steady-state inflation. A positive but small level of inflation, which generates positive steady-state price dispersion, is thus preferable to a policy of zero inflation at all times, which implies zero steady-state price dispersion. The reason for this is that positive steady-state inflation, by bringing about positive output gap, allows eliminating some of the steady-state loss due to monopolistic competition.

Figure (5.5) plots $\overline{W}^{FLZO}$, $\overline{W}^{FLTP}$, and $\overline{W}^{FLATP}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$, with $\overline{W}^{FLTP}$ not depending on α. Zero-optimal commitment policy delivers the highest level of welfare given that, as discussed above, the transition to the steady state is always welfare-enhancing. Welfare on the basis of the deterministic equilibrium under both zero-optimal commitment policy and the alternative timeless perspective commitment policy is monotically increasing in the degree of price stickiness. This is consistent with the results presented in Blake (2001) and Jensen and McCallum (2002), which show that a greater weight on output fluctuations is welfare-worsening. In the present framework, the output gap coefficient, κ, hence the coefficient on output fluctuation λ, is monotically decreasing in the degree of price stickiness. Therefore, a higher average price duration results in better welfare on the basis of the deterministic equilibrium.

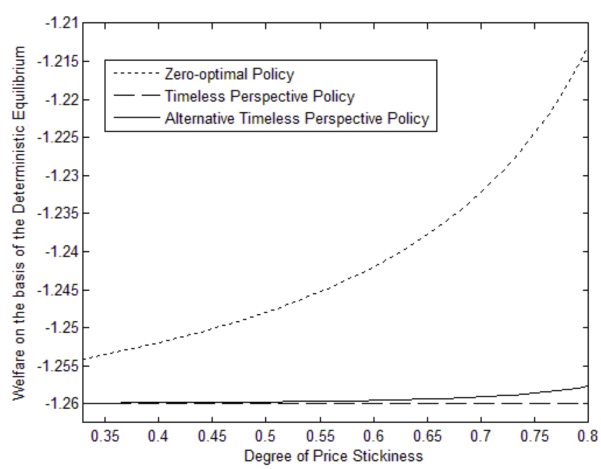

Figure 5.5. Purely forward-looking model. Welfare on the basis of the deterministic equilibrium.

It is relevant to note that using Soderlind's (1999) method, with the time horizon set to 1000 periods[1], to derive welfare under the zero-optimal policy delivers the same level of welfare that obtains under the analytical solution, as given by (5.10), up to the seventh decimal figure, namely up to the fifth decimal figure when, as done here, welfare levels are expressed in percent terms.

5.3.1.2. Rule-of-thumb Behaviour à la Galì and Gertler. We do not attempt to analytically derive the evolution of inflation and output gap under the zero-optimal commitment policy, hence welfare on the basis of the deterministic equilibrium, $\overline{W}^{GGZO}$, but we resort to Soderlind's (1999) method. In doing so, we assume that economy is at the steady state.

Under both timeless perspective policies, the steady-state loss function, namely $\overline{L}^{FL} = \overline{L}^{GG}$, can be seen as the sum of the loss that obtains in the presence of zero inflation at all times, $\overline{L}^{FLTP}$, and the loss attributable to positive optimal long-run inflation , $\overline{\pi}^2 + \lambda_1 \overline{x}^2 - 2\lambda_1 x^* \overline{x}$. Hence, we are able to address whether positive long-run inflation improves welfare relative to a policy of zero inflation at all times, which implies $\overline{W}^{FLTP}$.

As described above, the alternative timeless perspective policy is robust to the introduction of backward-looking rule-of thumb behaviour à la Galì and Gertler (1999). It follows that $\overline{W}^{GGATP} = \overline{W}^{FLATP}$, which in always better than $\overline{W}^{FLTP}$.

Under the standard timeless perspective policy, it is also optimal to have positive steady-state inflation. Given the long-run trade-off implied by the hybrid Phillips curve (i.e. (2.60)), combining the steady-state loss function, $\overline{L}^{GG}$, with the optimal positive long-run inflation rate (i.e. (4.24)), yields

$$\overline{L}^{GGTP} = \lambda_1 x^{*2} + \lambda_1 \frac{(1-\alpha)(1-\beta)^2\omega \begin{Bmatrix} \theta\kappa\left[\omega + \alpha(\omega-2)\right] \\ -(1-\alpha)(1-\beta)^2\omega \end{Bmatrix}}{\left[(1-\omega)\alpha\theta\kappa + (1-\alpha)(1-\beta)^2\omega\right]^2} x^{*2} = \lambda_1 x^{*2}(1+\Delta) \tag{5.16}$$

[1]Whenever we employ Soderlind's method, the time horizon is set to 1000 periods.

which implies that welfare on the basis of the deterministic equilibrium is of the form

$$\overline{W}^{GGTP} = -\frac{(\sigma^{-1} + \varpi)}{2}(1+\Delta)x^{*2} \tag{5.17}$$

The standard timeless perspective policy is hence not always superior to a policy of a policy of zero steady-state inflation. The condition that guarantees $\overline{W}^{GGW} > \overline{W}^{FLW}$ is easily seen to be $\Delta < 0$, where the sign of Δ is determined by the term in curly brackets in (5.16). Solving in terms of ω yields

$$\omega\left[(1+\alpha)\theta\kappa - (1-\alpha)(1-\beta)^2\right] \leq 2\alpha\theta\kappa \Leftrightarrow \omega \leq \frac{2\alpha\theta\kappa}{[(1+\alpha)\theta\kappa - (1-\alpha)(1-\beta)^2]} \tag{5.18}$$

If one plotted condition (5.18), holding with equality, for the full range of the degree of price stickiness, $0.01 \leq \alpha \leq 0.99$, the values of ω that imply $\overline{W}^{GGTP} < \overline{W}^{FLTP}$ would be observed to be well outside the estimates of the degree of rule-of-thumb behaviour reported in Galì and Gertler (1999), especially when limiting the range of the degree of price stickiness to $0.33 \leq \alpha \leq 0.8$.

The alternative timeless perspective policy is thus always superior, in terms of welfare on the basis of the deterministic equilibrium, to a policy of zero inflation at all times whereas the same it is not always true under the standard timeless perspective policy. In other words, under the alternative timeless perspective policy a small but positive level of inflation, which brings about a positive level of price dispersion, invariably allows eliminating some of the steady-state loss due to monopolistic competition. Conversely, under the standard timeless perspective policy, unrealistically high levels of the degree of rule-of-thumb behaviour would imply that having positive steady-state inflation, hence a positive degree of price dispersion, would only add to the steady-state loss due to monopolistic competition.

Figure (5.6) plots $\overline{W}^{GGZO}$, $\overline{W}^{GGTP}$, and $\overline{W}^{GGATP}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Given the relationship between α and κ, welfare on the basis of the deterministic equilibrium under all commitment policies is increasing in the degree of price stickiness. Zero-optimal commitment policy ranks first. The alternative timeless perspective commitment policy entails welfare on the basis of the deterministic equilibrium that is always better than the one implied by the standard timeless perspective policy.

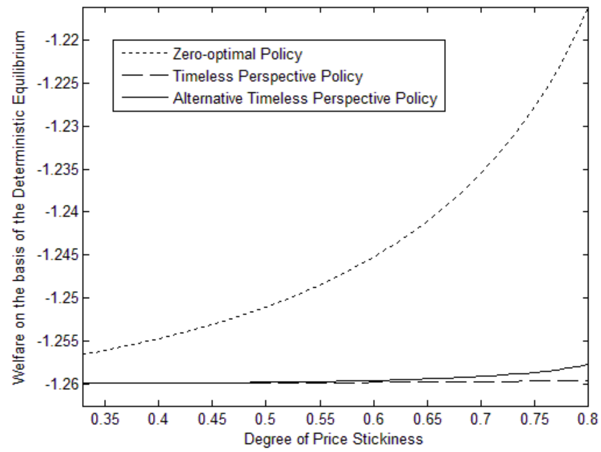

Figure 5.6. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Welfare on the basis of the deterministic equilibrium for alternative values of the degree of price stickiness.

As for the relation between welfare on the basis of the deterministic equilibrium and the degree of backward-looking rule-of-thumb behaviour, Figure (5.7) plots $\overline{W}^{GGZO}$, $\overline{W}^{GGTP}$, and $\overline{W}^{GGATP}$ for $0.01 \leq \omega \leq 0.7$. The alternative timeless perspective policy implies the same level of welfare that obtains in the purely forward-looking model, which is invariant to the degree of rule-of-thumb behaviour. Welfare on the basis of the deterministic equilibrium under zero-optimal policy is observed to be monotonically decreasing in the degree of rule-of-thumb behaviour. The standard timeless perspective policy instead implies that a larger fraction of firms resetting prices in a backward-looking rule-of-thumb manner is initially welfare-enhancing, although never delivering better welfare levels than the alternative timeless perspective commitment policy, and subsequently becomes welfare-worsening. Once again, zero-optimal policy is better than the two timeless perspective policies, with the alternative timeless perspective policy being always superior to the standard timeless perspective policy.

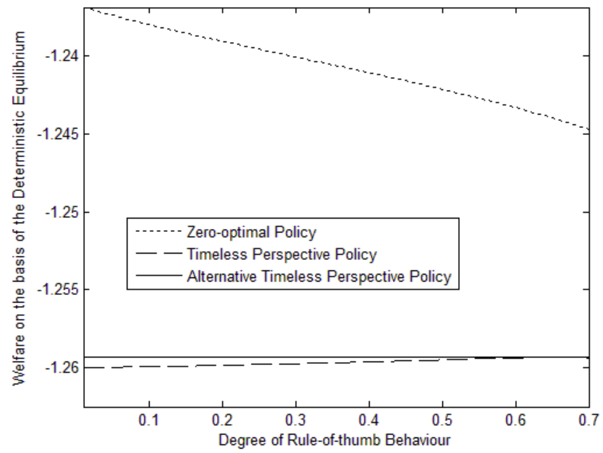

Figure 5.7. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Welfare on the basis of the deterministic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

5.3.1.3. Rule-of-thumb Behaviour à la Steinsson. We do not attempt to analytically derive the evolution of inflation and output gap under the zero-optimal commitment policy, hence welfare on the basis of the deterministic equilibrium, $\overline{W}^{SZO}$, but we resort to Soderlind's (1999) method. In doing so, we assume that economy is at the steady state.

Steinsson's (2003) rule-of-thumb behaviour alters both the long-run trade-off between output gap and inflation and the steady-state loss function (i.e. $\overline{L}^{S} = \overline{L}^{FL} + \lambda_2(1-\alpha)^2\delta^2)\overline{x}^2)$ that obtain in the purely forward-looking New Keynesian model. Under both timeless perspective policies, the steady-state loss function can again be seen as the sum of the loss that obtains under zero inflation at all times, $\overline{L}^{FLTP}$, and the loss due to positive long-run inflation, $\overline{\pi}^2 + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\overline{x}^2 - 2\lambda_1 x^* \overline{x}$.

Given the long-run trade-off implied by the hybrid Phillips curve (i.e. (2.61)) and the optimal positive long-run inflation rate (i.e. (4.42)), the standard timeless perspective policy involves a steady-state loss function of the form

$$\overline{L}^{STP} = \lambda_1 x^{*2} + \Psi x^{*2}\left[\Psi - 2\lambda_1\Gamma + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\Gamma^2\Psi\right] \tag{5.19}$$

which implies that welfare on the basis of the deterministic equilibrium is given by

$$\overline{W}^{STP} = -\frac{(\sigma^{-1}+\varpi)\theta}{2\kappa}\overline{L}^{STP} \tag{5.20}$$

Similarly, the alternative timeless perspective policy, under which the optimal positive long-run inflation rate is given by (i.e. (4.48), entails a steady-state loss function of the form

$$\overline{L}^{SATP} = \lambda_1 x^{*2} + \Theta x^{*2}\left[\Theta - 2\lambda_1\Gamma + (\lambda_1 + \lambda_2(1-\alpha)^2\delta^2)\Gamma^2\Theta\right] \tag{5.21}$$

which implies that welfare on the basis of the deterministic equilibrium is given by

$$\overline{W}^{SATP} = -\frac{(\sigma^{-1}+\varpi)\theta}{2\kappa}\overline{L}^{SATP} \tag{5.22}$$

As under Galì-Gertler's rule-of-thumb behaviour, we can compare welfare under positive steady-state inflation at all times vis-a-vis welfare under a policy of zero steady-state inflation at all times. The derivation of the condition that guarantees an increase in welfare on the basis of the deterministic equilibrium is cumbersome. However, quantitative analysis confirms that the alternative timeless perspective is always superior to a policy of zero inflation at all times, whereas the same it is not always true under the standard timeless perspective policy. Under the alternative timeless perspective policy a small but positive level of inflation, which brings about a positive level of price dispersion, invariably allows eliminating some of the steady-state loss due to monopolistic competition. Conversely, under the standard timeless perspective policy, unrealistically high levels of the degree of rule-of-thumb behaviour would imply that having positive steady-state inflation, hence a positive degree of price dispersion, would only add to the steady-state loss due to monopolistic competition.

Figure (5.8) plot respectively $\overline{W}^{SZO}$, $\overline{W}^{STP}$, and $\overline{W}^{SATP}$ for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Given that the coefficient on output fluctuation is monotically decreasing in the degree of price stickiness, welfare on the basis of the deterministic equilibrium under all commitment policies is increasing in the degree of price stickiness. Zero-optimal commitment policy ranks first. The alternative timeless perspective commitment policy is confirmed to entail welfare on the basis of the deterministic equilibrium that is always better than the one implied by the standard timeless perspective policy.

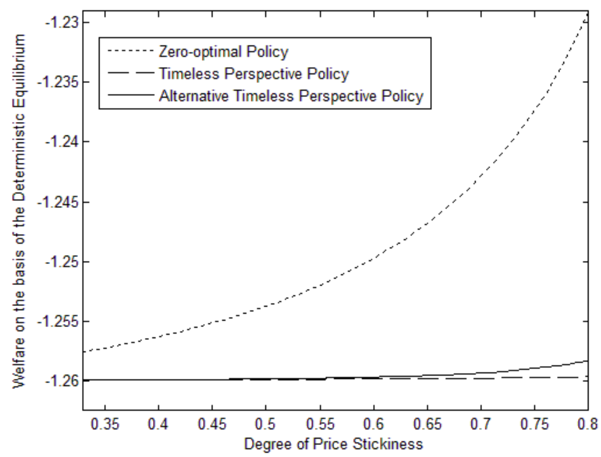

Figure 5.8. Model with rule-of-thumb behaviour à la Steinsson (2003). Welfare on the basis of the deterministic equilibrium for alternative values of the degree of price stickiness.

As for the relation between welfare on the basis of the deterministic equilibrium and the degree of backward-looking rule-of-thumb behaviour, Figure (5.9) plot respectively $\overline{W}^{SZO}$, $\overline{W}^{STP}$, and $\overline{W}^{SATP}$ for $0.01 \leq \omega \leq 0.7$. Welfare on the basis of the deterministic equilibrium under both zero-optimal policy and the alternative timeless perspective policy is observed to be monotonically decreasing in the degree of rule-of-thumb behaviour. The standard timeless perspective policy implies that a larger fraction of firms resetting prices in a backward-looking rule-of-thumb manner is associated initially with improvements in welfare and subsequently becomes welfare-worsening. Once again, zero-optimal policy is better than the the two timeless perspective policies, with the alternative timeless perspective policy being always superior to the standard timeless perspective policy.

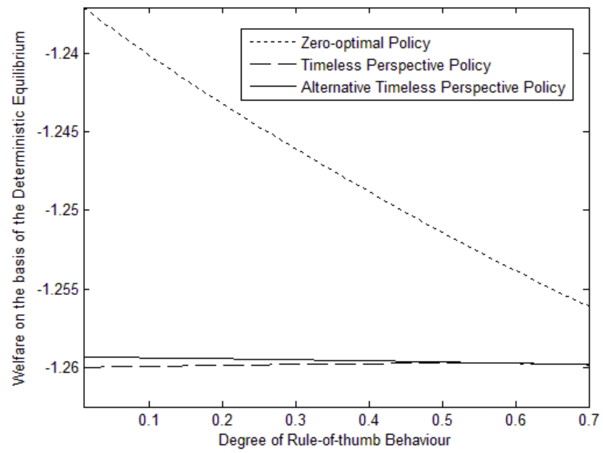

Figure 5.9. Model with rule-of-thumb behaviour à la Steinsson (2003). Welfare on the basis of the deterministic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

5.3.2. Welfare on the basis of the Stochastic Equilibrium

Having shown how the alternative commitment policies unequivocally rank on the basis of the deterministic equilibrium, we proceed to describe the performance of these policies in the face of shocks. Indeed, we now evaluate policies on the basis of the stochastic equilibrium. As discussed in Chapter 2, the monetary authority, in our models, should not respond to movements in output which are caused by preference shocks or shocks to productive capabilities. This is because the movements in output brought about by those shocks are efficient, namely they represent variations in the efficient level of output. Following Clarida et al. (1999), we hence augment the aggregate-supply relations with an inefficient shock (i.e. a cost-push shock) and analyse the welfare costs due to the stabilisation of the cost-push shock. which stems from augmenting the aggregate-supply relation with a cost-push shock. For instance, the NKPC is now given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t \tag{5.23}$$

where, using the terminology in Clarida et al. (1999), μ_t represents a cost-push shock, which is assumed to be autoregressive of order one with AR parameter ρ and innovation shock ϵ_t being i.i.d, namely $\mu_t = \rho\mu_{t-1} + \epsilon_t$[2]. We calibrate the standard deviation of the cost-push shock innovation to 0.016, which is the value estimated in Smets and Wouters (2003) and we set the AR parameter, ρ, to 0. The same remark in Jensen and McCallum (2002) applies here, changing the standard deviation of the mark-up shock innovation would only scale welfare values up or down proportionately[3].

Augmenting the aggregate-supply relation with a cost-push shock does not alter the central bank's target criterion, but it implies that the monetary authority should react to movements in output which are caused by a cost-push shock while it should not react to movements in output which are caused by any other shocks.

The experiment we undertake in order to rank the alternative commitment policies on the basis of the stochastic equilibrium is simple to illustrate. We consider a draw of 100 cost-push shocks, which we maintain across all the theoretical cases studied[4]. For each (α, ω) pair we calculate welfare as the mean value that obtains across the 100 shocks. From this initial level of welfare we then subtract the corresponding welfare on the basis of the deterministic equilibrium. The levels of welfare on the basis of the stochastic equilibrium we report thus abstract from consideration of steady-state outcomes as they do not take into account steady-state welfare. We do so as we want to analyse welfare that is purely due to the stabilisation of the cost-push shock.

It is important to note that Soderlind's (1999) method solves for the evolution of endogenous variables under zero-optimal commitment policy. However, it can be used for the evolution of the endogenous variables under timeless perspective policy on the provision that a dummy control variable is introduced into the system. That is, while under zero-optimal policy the output gap is the control variable and the central bank's only constraint is the aggregate-supply relation, under timeless perspective policy the control

[2]Of course, (2.58) and (2.53) are also augmented with μ_t.

[3]The results we present are robust to the possibility of a positive AR parameter. Specifically, we have considered $\rho = 0.5$ and $\rho = 0.8$.

[4]The results we report are not altered when considering a larger number of shocks. Specifically, we have considered a draw of 1000 positive cost-push shocks.

variable equals zero at all times and the central bank is constrained by the aggregate-supply relation and the target criterion[5].

We report two measures of welfare on the basis of the stochastic equilibrium. We consider both unconditional welfare and welfare conditional on information available at date zero. When the criterion is unconditional welfare, the a priori beliefs suggest that the alternative timeless perspective policy should rank first followed by the standard timeless perspective policy and the zero-optimal policy. This is because, by construction, the alternative timeless-perspective commitment policy optimises the unconditional expectation of the central bank's objective function. Similarly, the standard timeless-perspective commitment policy is expected to perform better than the zero-optimal policy as the latter responds best to one particular shock. In evaluating welfare conditional on information available at date zero, we assume that the economy's initial condition is the steady state implied by the policy under consideration. Second, we report average values of the central bank's objective function so to abstract from initial conditions. When considering welfare conditional on information available at date zero, the a priori beliefs suggest that the zero-commitment policy should rank first followed by the two timeless perspective policies. This is because, as discussed in the previous chapter, the zero-optimal policy is truly the optimal policy from a conditional perspective. Indeed, timelessness imposes an extra condition on the optimal evolution of inflation and output gap so as to obtain continuation of policy.

The results we obtain are in line with the a priori beliefs. First, in all our theoretical economies, zero-optimal commitment policy delivers the highest level of welfare conditional on information available at date zero followed by the alternative timeless perspective policy and the standard timeless perspective commitment. Second, in all our theoretical

[5] As described above, under rule-of-thumb behaviour à la Steinsson (2003), the target criterion is replaced by the first-order conditions with respect to inflation and output gap

economies, the alternative timeless perspective policy delivers the highest level of unconditional welfare followed by the standard timeless perspective policy and the zero-optimal policy.

5.3.2.1. Basic New Keynesian Model. Figure (5.10) plots unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. This is contrary to the relation between welfare on the basis of the deterministic equilibrium and degree of price stickiness, but the logic for this result is quite intuitive. In an economy where price setting is staggered, a higher degree of price stickiness implies a higher degree of price dispersion, which is in fact costly as it brings about dispersion of output levels across goods[6]. Hence, a higher average price duration results in larger losses associated with the stabilisation of the cost-push shock. The policies rank according to the a priori beliefs. Zero-optimal policy ranks last. The two timeless perspective policies imply nearly the same welfare levels, but if one plotted the difference in welfare levels between the two policies, the alternative timeless perspective policy would invariably deliver better welfare than the standard timeless perspective policy.

Figure (5.11) plots conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. The three policies imply nearly the same welfare levels. Yet, if one plotted the difference in welfare levels between any pair of policies, the zero-optimal policy would invariably rank first, followed by the standard and the alternative timeless perspective policies.

[6]The degree of price dispersion at time t in the purely forward-looking model is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$, which is easily seen to be increasing in α.

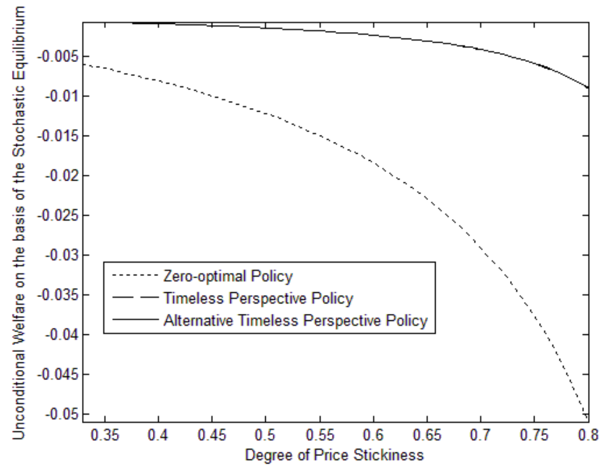

Figure 5.10. Purely forward-looking model. Unconditional welfare on the basis of the stochastic equilibrium.

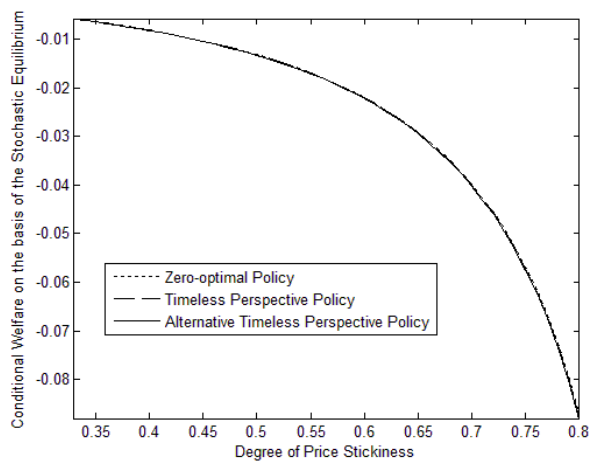

Figure 5.11. Purely forward-looking model. Conditional welfare on the basis of the stochastic equilibrium.

5.3.2.2. Rule-of-thumb Behaviour à la Galì and Gertler. Figure (5.12) plots unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. The ranking across the alternative commitment policies is univocal. The alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and the zero-optimal policy.

Figure (5.13) plots conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. The three policies imply nearly the same welfare levels. Yet, if one plotted the difference in welfare levels between any pair of policies, the zero-optimal policy would invariably rank first, followed by the standard and the alternative timeless perspective policies.

Figure (5.14) plots unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. A larger fraction of firms resetting prices in a backward-looking is indeed associated with higher degree of price dispersion, hence larger losses associated with the stabilisation of the cost-push shock[7]. The ranking across the alternative commitment policies is univocal. The alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and the zero-optimal policy.

Figure (5.15) plots conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. The three policies imply nearly the same welfare levels. Yet, if one plotted the difference in welfare levels between any pair of policies, the zero-optimal policy would invariably rank first, followed by the standard and the alternative timeless perspective policies.

Note that backward-looking rule-of-thumb behaviour implies inferior welfare on the basis of the stochastic equilibrium than in the purely forward-looking model. Intuitively, backward-looking rule-of-thumb behaviour invariably increases the degree of price dispersion, which results in additional welfare losses associated with the stabilisation of the cost-push shock.

5.3.2.3. Rule-of-thumb Behaviour à la Steinsson. Figure (5.16) plots unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in

[7]The degree of price dispersion at time t in the model with rule-of-thumb à la Galì and Gertler (1999) is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}(\pi_t - \pi_{t-1})^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$, which is easily seen to be increasing in both α and ω.

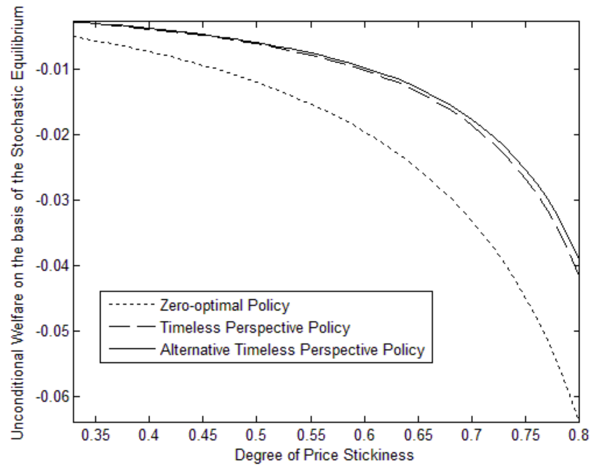

Figure 5.12. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness.

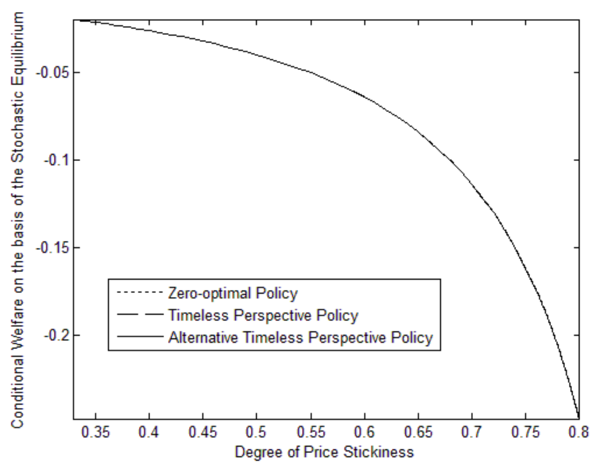

Figure 5.13. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness.

the degree of price stickiness. The ranking across the alternative commitment policies is univocal. The alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and the zero-optimal policy.

Figure (5.17) plots conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness, $0.33 \leq \alpha \leq 0.8$. Under all policies, welfare is monotonically decreasing in the degree of price stickiness. The three policies imply nearly the same welfare levels. Yet, if one plotted the difference in welfare levels

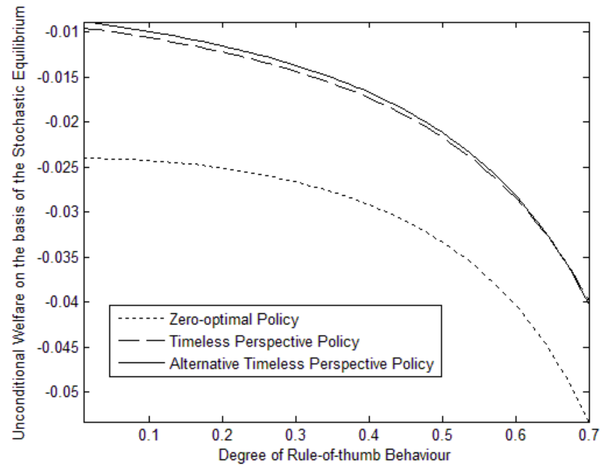

Figure 5.14. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

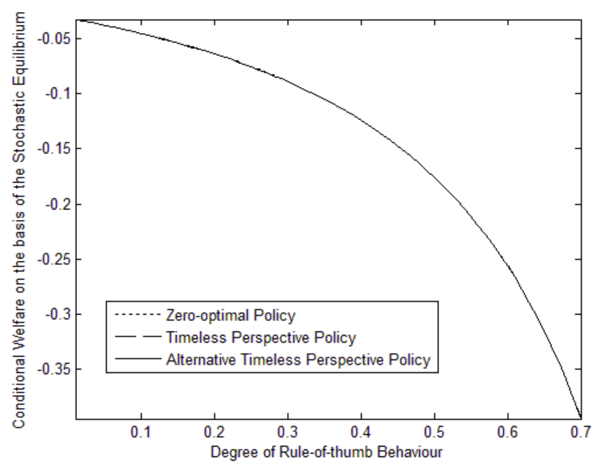

Figure 5.15. Model with rule-of-thumb behaviour à la Galì and Gertler (1999). Conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

between any pair of policies, the zero-optimal policy would invariably rank first, followed by the standard and the alternative timeless perspective policies.

Figure (5.18) plots unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. A larger fraction of firms resetting prices in a backward-looking is indeed associated with higher degree of price dispersion, hence larger losses associated with the stabilisation of the cost-push shock[8]. The ranking between the alternative commitment policies is univocal. The ranking across the alternative commitment policies is univocal. The alternative timeless perspective policy ranks first, followed by the standard timeless perspective policy and the zero-optimal policy.

Figure (5.19) plots conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour, $0.01 \leq \omega \leq 0.7$. Under all policies, welfare is monotonically decreasing in the degree of rule-of-thumb behaviour. The three policies imply nearly the same welfare levels. Yet, if one plotted the difference in welfare levels between any pair of policies, the zero-optimal policy would invariably rank first, followed by the standard and the alternative timeless perspective policies.

Note that backward-looking rule-of-thumb behaviour à la Steinsson (2003) implies superior welfare on the basis of the stochastic equilibrium than under rule-of-thumb behaviour à la Galì and Gertler (1999). Intuitively, indexation to lagged output gap curbs the degree of price dispersion, which results in smaller welfare losses associated with the stabilisation of the cost-push shock.

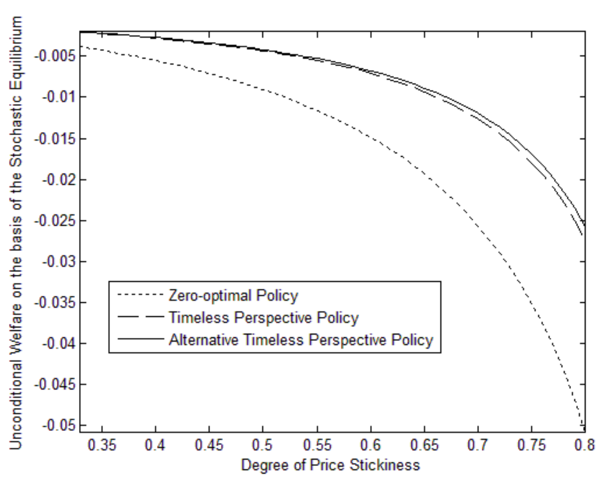

Figure 5.16. Model with rule-of-thumb behaviour à la Steinsson (1999). Unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness.

[8]The degree of price dispersion at time t in the model with rule-of-thumb à la Galì and Gertler (1999) is given by $\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}(\pi_t - \pi_{t-1})^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$, which is easily seen to be increasing in both α and ω.

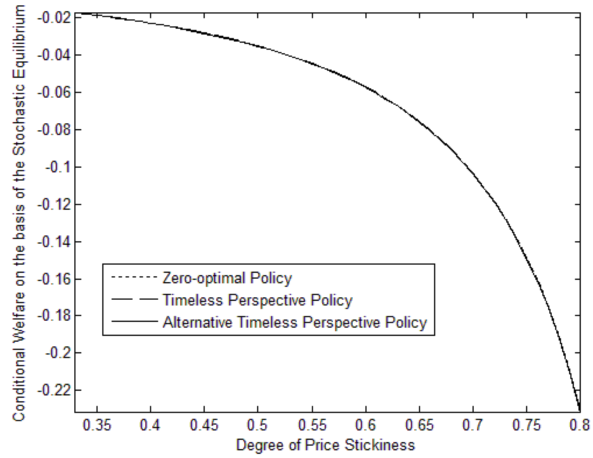

Figure 5.17. Model with rule-of-thumb behaviour à la Steinsson (1999). Conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of price stickiness.

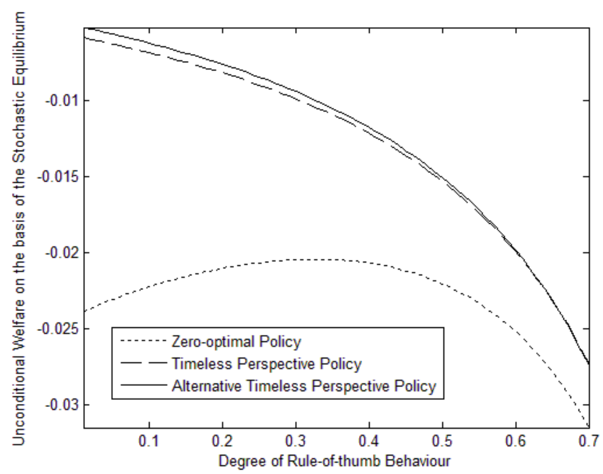

Figure 5.18. Model with rule-of-thumb behaviour à la Steinsson (2003). Unconditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

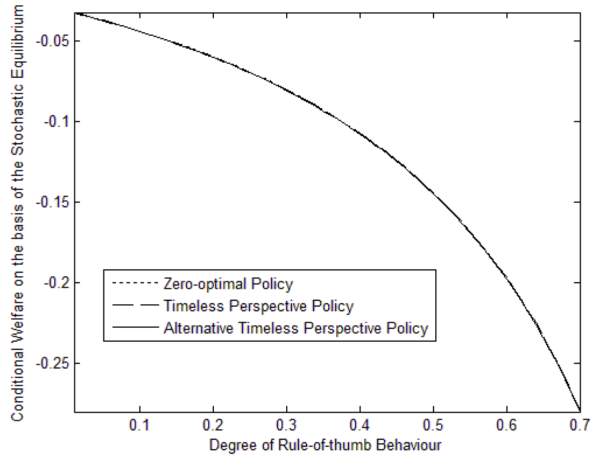

Figure 5.19. Model with rule-of-thumb behaviour à la Steinsson (2003). Conditional welfare on the basis of the stochastic equilibrium for alternative values of the degree of rule-of-thumb behaviour.

CHAPTER 6

Optimal inflation rate in a Medium-Scale Macroeconomic Model with Rule-of-thumb Price Setters

In this chapter we characterise the optimal steady-state inflation rate of the Ramsey planner in the medium-scale macroeconomic model developed in Altig et al. (2005). The model emphasises the importance of combining nominal as well as real rigidities in explaining business-cycle fluctuations. Specifically, the model features four nominal rigidities, sticky prices, sticky wages, a transactional demand for money by households, and a cash-in-advance constraint on the wage bill of firms, and four real rigidities, investment adjustment costs, variable capacity utilisation, habit formation, and imperfect competition in product and labour markets. We extend the model by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner. In other words, we extend the analysis in Schmitt-Grohé and Uribe (2007) to an economy where inflation persistence is due to rule-of-thumb behaviour by price setters à la Galì and Gertler (1999), rather than backward-looking price indexation.

The qualification Ramsey is worthy to note. The origin of the qualification traces back to the seminal work by Ramsey (1927). The policy problem in Ramsey's study takes the form of an allocation problem, in which the policymaker can be thought of choosing directly a feasible allocation subject to those constraints that summarise the evolution of the economy. Studies of optimal policy in dynamic economies (e.g. Atkinson and Stiglitz (1976), Lucas and Stokey (1983), Chari, Christiano and Kehoe (1992)) have employed the same approach, labelling it Ramsey-type approach. Specifically, the Ramsey planner, maximises the household's utility subject to a resource constraint, to the constraints describing the equilibrium in the private sector economy, and via an explicit consideration of all the distortions that characterise the economy. In what follows, we conform to this practice in the literature. We employ a Ramsey-type approach to characterise optimal monetary policy and we use the qualification Ramsey for both steady-state outcomes (i.e. Ramsey steady state) and dynamic outcomes (i.e. Ramsey impulse response functions). We must stress that the linear-quadratic approach, which we have employed in the first part of this thesis, and the Ramsey-type approach rely on the same kind of intertemporal optimisation by the policymaker. However, while in the linear-quadratic approximation the objective of the policymaker is a quadratic approximation to the utility of the representative household, the Ramsey planner is concerned with the maximisation of the utility of the representative household per se.

Recently, there has been a resurgence of interest for a Ramsey-type approach in dynamic general equilibrium models with nominal rigidities. Khan at al. (2003) analyse optimal monetary policy in a closed economy where the relevant distortions are imperfect competition, staggered price setting and monetary transaction frictions. Schmitt-Grohé and Uribe (2004a, 2005), and Siu (2004) focus on the joint optimal determination of monetary and fiscal policy in an economy with sticky prices, imperfect competition,and money demand. The key policy problem faced by the central bank in setting the optimal rate of inflation is the trade-off between the stabilisation of the degree of price dispersion, which calls for zero inflation, and the stabilisation of transactional frictions, which calls for the Friedman rule, namely a deflation which is consistent with a zero nominal interest rate. These studies find that the Friedman prescription for deflation governs the optimal steady-state inflation: the average level of the nominal interest rate should be sufficiently low so that there should be deflation on average. Specifically, the level of optimal inflation varies with the degree of price stickiness: the Ramsey-optimal steady-state inflation can range from close to the Friedman rule to close, but below, price stability.

Our main goal in characterising the optimal rate of inflation in the Ramsey steady state is the investigation of whether the features that deliver optimal positive long-run inflation in the linear-quadratic framework are capable to overturn the Friedman rule. In characterising the optimal inflation rate, we in fact consider again the case of an inefficient deterministic steady state. Nonetheless, the steady-state distortions are assumed not to be small as in the linear-quadratic framework. Considering a model with large steady-state distortions implies that, as we discussed in Chapter 3, to obtain a second-order accurate measure of welfare it does not suffice any longer to approximate the model's equilibrium conditions up to first order[1]. In this chapter, we characterise the long-run state of the Ramsey equilibrium in an economy without uncertainty, namely the Ramsey steady state. Given the complexity of the model, the Ramsey steady state cannot be characterised analytically. We thus employ the algorithm in Schmitt-Grohé and Uribe (2007). The algorithm characterises and numerically solves the Ramsey steady-state in medium-scale macroeconomic models. Specifically, it yields an exact numerical solution for the Ramsey steady-state

We find that the results in Schmitt-Grohé and Uribe (2007) generally hold. Rule-of-thumb behaviour by price setters does not alter the sensitivity of the long-run inflation rate with respect to the degree of price stickiness. Indeed, the optimal long-run inflation is always negative and it varies between the level implied by the Friedman rule and a level close to price stability.

We depart from the analysis in Schmitt-Grohé and Uribe (2007) and consider the case of a cashless medium-scale macroeconomic model. The motivation for this is twofold. First, maintaining the cashless qualification of the economy, we seek to establish a link between the analysis of optimal steady-state inflation carried out in the previous chapters within a basic New Keynesian model and its counterpart in a much richer theoretical nonlinear economy. Second, we want to study the case of large steady-state distortions in order to assess whether dropping the assumptions of small steady-state distortions is capable of delivering larger positive inflation rates.

[1]Indeed, when we consider the implementation of optimal monetary policy in the next chapter we are forced to solve the model up to second order. We use the methodology and the algorithm developed in Schmitt-Grohé and Uribe (2004b) for second-order accuarate approximations to policy functions of dynamic and stochastic models.

We find that the Ramsey-optimal steady-state inflation in the cashless model with rule-of-thumb behaviour by price setters is positive. However, as found in the linear-quadratic framework, the inflation rate is still observed to be small. Moreover, the inflation rate is again observed to be monotonically decreasing in the degree of price stickiness and monotonically increasing in the degree of rule-of-thumb behaviour.

We proceed to analyse the social planner allocation. The social planner decides how to allocate the consumption and the production of goods within the economy regardless of the details of the price and wage mechanisms and the nature of the factors' markets and goods' markets. Indeed, in chapter 4 we have shown that in the linear-quadratic framework the steady-state inflation rate is directly proportional to the steady-state efficiency gap, which is the constant gap between the steady-state level of output and the efficient steady-state level of output. Solving the social planner's problem allows us to derive the efficient steady-state level of output. We subsequently compute its log-difference with the Ramsey steady-state level of output so to obtain a measure of the gap between the two steady-state levels of output. We find that this steady-state gap is only slightly larger than in the case of small steady-state distortions assumed in the linear quadratic framework. Specifically, while the steady-state efficiency gap in the linear-quadratic framework is equal to 0.2 under benchmark calibration, the steady-state efficiency gap in the medium-scale model is found to be in the region of 0.26 both in the model with money and in its cashless counterpart.

We first present the theoretical model. We then describe how to solve for the Ramsey steady state and present the calibration of the model. We characterise the Ramsey steady state in both the model with money and its cashless counterpart. Finally, we characterise the social planner allocation.

6.1. The model

The theoretical economy is the neoclassical growth model augmented with a number of real and nominal frictions developed in Altig et al. (2005), which is taken in its setup from Schmitt-Grohé and Uribe (2007). This model has been estimated econometrically and shown to account fairly well for business-cycle fluctuations in the postwar United States. We extend the model by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner, specified as in Galì and Gertler (1999). The equilibrium conditions of the model are presented in their nonlinear recursive form.

The theoretical framework emphasizes the importance of combining nominal as well as real rigidities in explaining the propagation of macroeconomic shocks. The nominal rigidities include price and wage stickiness à la Calvo (1983) and money demands by both households and firms. On the one hand, differently from Schmitt-Grohé and Uribe (2007), prices are not indexed to past inflation, but two types of price setters are assumed to exist. Given Calvo-type constraints on price setting, one type of firms acts rationally, the other type sets prices according to a backward-looking rule-of-thumb. On the other hand, wages are still assumed to be indexed to past inflation. The real frictions stem from internal habit formation in consumption, monopolistic competition in both factors' and goods' markets, investment adjustment costs, and variable costs of adjusting capacity utilisation.

Aggregate fluctuations are driven by three shocks: a permanent neutral technology shock, a permanent investment-specific technology shock, and temporary variations in government spending. Altig et al. (2005) and Christiano et al. (2005) argue that the theoretical economy for which we study optimal monetary policy is in fact capable of explaining the observed responses of inflation, real wages, nominal interest rates, money growth, output, investment, consumption, labor productivity, and real profits to neutral and investment-specific productivity shocks and monetary shocks in the postwar United States. The model we derive in this Chapter thus differs, relative to the small-scale New Keynesian models employed in the first part of this thesis, for one important respect: the ability to replicate business cycle fluctuations. The medium-scale model retains all the features of the basic New Keynesian framework, but builds on it so as to improve its empirical fit. While the basic New Keynesian model has become the workhorse for the analysis of optimal policy and welfare, the models of last generation improve on the basic framework in the direction of better empirical realism and are thus more suitable for an explicit consideration of the business cycle fluctuations. Indeed, in the next Chapter, we employ the medium-scale model to study business cycle dynamics and we study the impulse response functions to the three macroeconomic shocks driving aggregate fluctuations.

6.1.1. Households and Market Structure

The economy is assumed to be populated by a large representative household with a continuum of members that are identical as for consumption and hours worked. The representative household seeks to maximise a discounted sum of utility with the period utility depending on per capita consumption, c_t, and per capita labour effort, h_t, namely

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left(c_t - bc_{t-1}; h_t \right) \tag{6.1}$$

where E_0 is the mathematical operator that denotes expectation conditional on information available at time 0, $0 < \beta < 1$ measures the subjective discount factor, and $0 \leq b < 1$ denotes the degree of internal habit formation in consumption. The period utility function, U, is assumed to be strictly increasing in c_t, strictly decreasing in h_t, and strictly concave. Following Dixit and Stiglitz (1977), per capita consumption is defined in terms of a composite good made of a continuum of differentiated goods indexed by i over the unit interval

$$c_t = \left[\int_0^1 c_{it}^{1-1/\eta} di \right]^{1/(1-1/\eta)} \tag{6.2}$$

where c_{it} is the consumption of good i and $\eta > 1$ measures the constant elasticity of substitution between different varieties of consumption goods.

The household faces a decision in each period about how much to consume of each variety of consumption goods. Denoting the nominal price of good i with P_{it}, the household adjusts the share of each differentiated good in the consumption bundle so to exploit any relative price differences. Minimising the level of total expenditure, $\int_0^1 P_{it} c_{it} di$, given the consumption aggregate in (6.2), yields the demand for each differentiated good

$$c_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} c_t \tag{6.3}$$

where the aggregate price level, P_t, is given by

$$P_t = \left[\int_0^1 P_{it}^{1-\eta} di\right]^{1/(1-\eta)} \tag{6.4}$$

This specification of the price index has by construction the property that $P_t c_t$ gives the minimum expenditure for which an amount c_t of the composite consumption good can be purchased.

The labour input used in the production of good i, h_{it}, is correspondingly assumed to be a composite quantity made of a continuum of differentiated labour inputs indexed by j over the unit interval

$$h_{it} = \left[\int_0^1 h_{it}^{j\ 1-1/\widetilde{\eta}} dj\right]^{1/(1-1/\widetilde{\eta})} \tag{6.5}$$

where $\widetilde{\eta} > 1$ measures the constant elasticity of substitution between different varieties of labour inputs and the aggregate labour demand, h_t^d, satisfies $h_t^d = \int_0^1 h_{it} di$. Denoting the nominal wage paid to labour of variety j with W_t^j, the firm adjusts the share of each differentiated labour service in the composite labour input so to exploit any relative wage differences. Minimising the level of total labour cost, $\int_0^1 W_t^j h_{it}^j dj$, given the labour aggregate in (6.5), yields the demand for each differentiated labour service

$$h_{it}^j = \left(\frac{W_t^j}{W_t}\right)^{-\widetilde{\eta}} h_{it} \tag{6.6}$$

where the aggregate nominal wage level, W_t, is given by

$$W_t = \left[\int_0^1 W_t^{j\ 1-\widetilde{\eta}} dj\right]^{1/(1-\widetilde{\eta})} \tag{6.7}$$

This specification of the wage index has by construction the property that $W_t h_{it}$ gives the minimum cost for which an amount h_{it} of the composite labour input can be hired.

The labour decisions are assumed to be made by a union within the household, which monopolistically supplies labour to the continuum of labour markets. Note that we assume that the household supplies all types of labour. In each labour market, the union is here regarded not to be powerful enough as to influence aggregate labour variables. Taking W_t and h_t^d as given, the union is assumed to supply enough labour in each market j, h_t^j, to satisfy demand, namely

$$h_t^j = \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\tilde{\eta}} h_{it} di = \left(\frac{w_t^j}{w_t} \right)^{-\tilde{\eta}} h_t^d \tag{6.8}$$

where $w_t^j = W_t^j / P_t$ and $w_t = W_t / P_t$ denote respectively the real wage paid to labour of variety j and the aggregate real wage level. Moreover, the aggregate labour supply, h_t, satisfies $h_t = \int_0^1 h_t^j dj$, which, combined with the supply of labour of type j in (6.8), implies that aggregate labour supply and aggregate labour demand are related through

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\tilde{\eta}} dj \tag{6.9}$$

The household is assumed to own physical capital, k_t, which accumulates according to

$$k_{t+1} = (1 - \delta) k_t + i_t \left[1 - S \left(\frac{i_t}{i_{t-1}} \right) \right] \tag{6.10}$$

where i_t denotes gross investment and δ measures the rate at which physical capital depreciates. The function S introduces investment adjustment costs, which are assumed to be zero up to first order in the neighbourhood of the deterministic steady state, namely $S = S' = 0$ and $S'' > 0$.

As in Fisher (2006) and Altig et al. (2005), investment is subject to exogenous permanent investment-specific shocks, which are denoted with Υ_t. Permanent investment-specific shocks are shown by Fisher (2006) to account for a large share of business cycle fluctuations in the United States of America in the period after World War II. Goods can either be consumed or transformed in investment goods. Specifically, Υ_t^{-1} units of the composite consumption good yield one unit of investment good, which is accordingly assumed to be a composite good made of a continuum of differentiated goods as in (6.2). It follows that the demand for each differentiated good for investment is given by $i_{it} = (P_{it}/P_t)^{-\eta} \Upsilon_t^{-1} i_t$. Denoting with μ_Υ the growth rate of Υ_t in the deterministic steady state, the percentage deviation of the gross growth rate of Υ_t, $\mu_{\Upsilon,t} = \Upsilon_t/\Upsilon_{t-1}$, from its steady-state value, $\widehat{\mu}_{\Upsilon,t} = \ln\left(\mu_{\Upsilon,t}/\mu_\Upsilon\right)$, is assumed to be autoregressive of order one with AR parameter ρ_{μ_Υ} and innovation shock $\varepsilon_{\mu_\Upsilon,t}$, namely $\widehat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon}\widehat{\mu}_{\Upsilon,t-1} + \varepsilon_{\mu_\Upsilon,t}$.

The household can control the intensity at which physical capital is utilised with μ_t measuring the capacity utilisation. Using the stock of capital with intensity μ_t is assumed to imply a cost of $\Upsilon_t^{-1} a\left(\mu_t\right) k_t$ units of the composite consumption good. The function a is assumed to satisfy $a(1) = 0$, $a'(1) > 0$, and $a''(1) > 0$, meaning that overutilising physical capital entails a strictly convex cost in terms of composite consumption good. The household rents physical capital to firms at the real rental rate r_t^k per unit of capital, which implies that the total real revenue that accrues to the household from the rental of capital is $r_t^k \mu_t k_t$.

Demand for money by the household is rationalised by assuming that purchasing consumption goods entails a proportional transaction cost that is increasing in the velocity of money, v_t, which is of the form

$$v_t = \frac{c_t}{m_t^h} \tag{6.11}$$

Money velocity is thus based on consumption, being the ratio of consumption to real money balances in the hands of the household, denoted by m_t^h. Specifically, purchasing a unit of composite consumption good implies a cost given by $\ell\left(v_t\right)$, where the transaction cost function satisfies four assumptions. First, $\ell\left(v\right)$ is non negative and twice continuously differentiable, which means that money changing hands does not generate resources. Second, there exists a positive finite value of money velocity, $0 < \underline{v} < \infty$, such that

$\ell(\underline{v}) = \ell'(\underline{v}) = 0$. This ensures that the level of real money balances associated with $\underline{v}$ satiates the household's demand for money so that the Friedman rule, namely a zero nominal interest rate, is not connected with an infinite demand for money. Third, $(v - \underline{v})\,\ell'(v) > 0$ for $v \neq \underline{v}$, which guarantees that in equilibrium money velocity is never smaller than the satiation level $\underline{v}$. Fourth, $2\ell'(v) + v\ell''(v) > 0$ for all $v \geq \underline{v}$, which implies that demand for money is decreasing in the nominal interest rate.

The household can access a complete set of nominal state-contingent assets. Formally, consumers in any period $t \geq 0$ can purchase any nominal state-contingent payment in the subsequent period, denoted by X_{t+1}^h. Denoting with $r_{t,t+1}$ the stochastic nominal discount factor for one period ahead payoffs, the cost of purchasing X_{t+1}^h is thus given by $E_t r_{t,t+1} X_{t+1}^h$. The absence of arbitrage opportunities in financial markets implies that there exists a unique stochastic nominal discount factor and the gross riskless short-term nominal interest rate, R_t, has a simple representation in terms of $r_{t,t+1}$, namely $E_t\left[r_{t,t+1}\right] = 1/R_t$. The household's budget constraint expressed in real terms is then of the form

$$\begin{array}{ccc} E_t r_{t,t+1} x_{t+1}^h + m_t^h + c_t[1 + \ell(v_t)] & & \phi_t + \frac{x_t^h + m_{t-1}^h}{\pi_t} + r_t^k u_t k_t \\ & = & \\ +\Upsilon_t^{-1}\left(i_t + a(u_t)\,k_t\right) & & +\int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\widetilde{\eta}} h_t^d dj - \tau_t \end{array} \tag{6.12}$$

where $x_t^h = X_t^h / P_t$ denotes the real payoff of a state-contingent payment bought in the previous period, ϕ_t measures the real dividends from the ownership of firms, $\pi_t = P_t / P_{t-1}$ is the gross inflation rate, and τ_t indicates real lump-sum taxes. The budget constraint states that, in any period, financial wealth carried into the subsequent period plus consumption, either genuine or for investment purposes, cannot be worth more than the value of financial wealth brought into the period plus after-tax nonfinancial income earned during the period.

Following Calvo (1983), in each period, the union within the household can optimally set the nominal wage in a fraction $1 - \widetilde{\alpha}$ of randomly chosen labour markets. The probability of not resetting the nominal wage in each period, $0 < \widetilde{\alpha} < 1$, is independent of both the time that has gone by since the last nominal wage revision and the misalignment between the actual wage and the wage that would be optimal to charge, namely wage decisions in any period are independent of past wage decisions. In those $\widetilde{\alpha}$ markets, the nominal wage is postulated to be indexed to both average real wage growth, μ_{z^*}, and to the previous period's inflation rate, according to $W_t^j = W_{t-1}^j \left(\mu_{z^*}\pi_{t-1}\right)^{\widetilde{\chi}}$, where $0 \leq \widetilde{\chi} \leq 1$ measures the degree of wage indexation.

The household chooses processes for c_t, x_{t+1}^h, h_t, k_{t+1}, i_t, m_t^h, u_t, and w_t^j so to maximise the discounted sum of utility (6.1) subject to (a) (6.9)-(6.12); (b) the wage stickiness; and (c) a no-Ponzi-scheme condition, taking as given (a) the processes w_t, r_t^k, h_t^d, $r_{t,t+1}$, π_t, ϕ_t, and τ_t; and (b) the initial conditions x_0^h, k_0, and m_{-1}^h. Combining (6.11) with (6.12) and denoting with $\lambda_t w_t/\widetilde{\mu}_t$, $\lambda_t q_t$, and λ_t the Lagrangian multipliers associated with constraints (6.9), (6.10), and (6.12), we form the following Lagrangian

$$\mathcal{L}^h = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} U\left(c_t - bc_{t-1}, h_t\right) + \lambda_t w_t/\widetilde{\mu}_t \left[h_t - h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\widetilde{\eta}} dj \right] \\ +\lambda_t q_t \left[(1-\delta) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] - k_{t+1} \right] \\ +\lambda_t \left[\begin{array}{c} \frac{x_t^h + m_{t-1}^h}{\pi_t} + \phi_t + + r_t^k u_t k_t + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\widetilde{\eta}} dj - \tau_t \\ -r_{t,t+1} x_{t+1}^h - m_t^h - c_t \left[1 + \ell\left(\frac{c_t}{m_t^h}\right)\right] - \Upsilon^{-1}\left(i_t + a\left(u_t\right) k_t\right) \end{array} \right] \end{array} \right\}$$

The first-order conditions are given by

$$\text{(6.13)} \quad \frac{\partial \mathcal{L}^h}{\partial c_t} = 0 \Rightarrow U_c\left(c_t - bc_{t-1}, h_t\right) - b\beta E_t U_c\left(c_{t+1} - bc_t, h_{t+1}\right) = \lambda_t \left[1 + \ell\left(v_t\right) + v_t \ell'\left(v_t\right)\right]$$

$$\text{(6.14)} \quad \frac{\partial \mathcal{L}^h}{\partial x_{t+1}^h} = 0 \Rightarrow \lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}}$$

$$\text{(6.15)} \quad \frac{\partial \mathcal{L}^h}{\partial h_t} = 0 \Rightarrow -U_h\left(c_t - bc_{t-1}, h_t\right) = \frac{\lambda_t w_t}{\widetilde{\mu}_t}$$

$$\text{(6.16)} \quad \frac{\partial \mathcal{L}^h}{\partial k_{t+1}} = 0 \Rightarrow \lambda_t q_t = \beta E_t \lambda_{t+1} [r_{t+1}^k u_{t+1} - \Upsilon_{t+1}^{-1} a(u_{t+1}) + q_{t+1}(1-\delta)]$$

$$\frac{\partial \mathcal{L}^h}{\partial i_t} = 0 \Rightarrow \frac{\lambda_t}{\Upsilon_t} = \left\{ \begin{array}{c} \lambda_t q_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right) S'\left(\frac{i_t}{i_{t-1}}\right)\right] \\ +\beta E_t \lambda_{t+1} q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right) \end{array} \right\} \tag{6.17}$$

$$\frac{\partial \mathcal{L}^h}{\partial m_t^h} = 0 \Rightarrow v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \tag{6.18}$$

$$\frac{\partial \mathcal{L}^h}{\partial u_t} = 0 \Rightarrow r_t^k = \Upsilon_t^{-1} a'(u_t) \tag{6.19}$$

$$\frac{\partial \mathcal{L}^h}{\partial w_t^j} = 0 \Rightarrow w_t^j = \begin{cases} \widetilde{w}_t \text{ if } w_t^j \text{ is set optimally in } t \\ w_{t-1}^j \left(\mu_{z^*} \pi_{t-1}\right)^{\widetilde{\chi}} / \pi_t \text{ if } w_t^j \text{ cannot be reset in } t \end{cases}$$

Writing $E_t[r_{t,t+1}] = 1/R_t$, the optimality condition for real state-contingent payments (6.14) can be rewritten as a standard Euler equation for pricing nominal riskless assets

$$\lambda_t = \beta R_t E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{6.20}$$

Combining this with the optimality condition for money holdings (6.18), yields

$$v_t^2 \ell'(v_t) = 1 - R_t^{-1} \tag{6.21}$$

On the one hand, $1 - R_t^{-1}$ measures the opportunity cost of holding money, which is clearly increasing in the nominal interest rate. On the other hand, given the assumptions on the transaction cost function ℓ, $v_t^2 \ell'(v_t)$ is increasing in the consumption-based velocity of money. It follows that the optimality condition for money holdings, (6.18), rewritten in terms of R_t defines a liquidity preference function, which is decreasing in the nominal interest rate and unit elastic in consumption.

The variable $\widetilde{w}_t$ is the real wage in the $1 - \widetilde{\alpha}$ labour markets in which the union can optimally set at time t the nominal wage, accordingly denoted with $\widetilde{W}_t$. Indeed, we assume that $\widetilde{W}_t$ is identical across all the $1 - \widetilde{\alpha}$ labour markets where the union is allowed to optimally reset nominal wage. Given that the demand for labour faced by the union is identical across all labour markets, as given by (6.5), and because the wage paid is postulated to be the same in all the $1-\widetilde{\alpha}$ markets, it follows that the supply of labour, denoted with $\widetilde{h}_t = (\widetilde{w}_t/w_t)^{-\widetilde{\eta}} h_t^d$, is also identical across all labour markets in which nominal wage is reset optimally in period t. Hence, the only distinction that matters is the one between the $1-\widetilde{\alpha}$ labour markets and the remaining $\widetilde{\alpha}$ markets. In any labour market where the nominal wage is set optimally in period t, the real wage is $\widetilde{w}_t$. If in the subsequent period the union cannot reoptimise the nominal wage in that labour market, the new real wage, denoted with $\widetilde{w}_{t,1}$[2], is given by $\widetilde{w}_{t,1} = \widetilde{W}_t \left(\mu_{z^*}\pi_t\right)^{\widetilde{\chi}} / P_{t+1} = \widetilde{w}_t \left(\mu_{z^*}\pi_t\right)^{\widetilde{\chi}} / \pi_{t+1}$. This is because we have postulated that in the $\widetilde{\alpha}$ markets the nominal wage is indexed, according to $\widetilde{\chi}$, to both average real wage growth, μ_{z^*}, and to the previous period's inflation rate. Generally, if s periods go by without the union being allowed to reoptimise the nominal wage in a given labour market, the real wage in that labour market is $\widetilde{w}_{t,s} = \widetilde{W}_t \prod_{k=1}^{s} \left(\mu_{z^*}\pi_{t+k-1}\right)^{\widetilde{\chi}} / P_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \left[\frac{\left(\mu_{z^*}\pi_{t+k-1}\right)^{\widetilde{\chi}}}{\pi_{t+k}}\right]$. It follows from the specification of the aggregate nominal wage level in (6.7) that the aggregate real wage level evolves according to

$$w_t^{1-\widetilde{\eta}} = (1-\widetilde{\alpha})\left(\widetilde{w}_t\right)^{1-\widetilde{\eta}} + \widetilde{\alpha} w_{t-1}^{1-\widetilde{\eta}} \left(\frac{\left(\mu_{z^*}\pi_{t-1}\right)^{\widetilde{\chi}}}{\pi_t}\right)^{1-\widetilde{\eta}} \tag{6.22}$$

The union is here assumed to set $\widetilde{w}_t$, taking W_t, h_t^d, and π_t as given. To this end, it is convenient to report the parts of the household's problem that are relevant for the wage

[2]The first subscript refers to the period in which the union is allowed to optimally reset the nominal wage. The second subscript indicates the number of periods that elapsed since the last optimal updating of the nominal wage.

setting problem, namely

$$\begin{aligned}
\mathcal{L}^h_{\widetilde{w}_t} &= E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t h^d_t \left\{ \begin{array}{c} \int_0^1 w^j_t \left(\frac{w^j_t}{w_t}\right)^{-\widetilde{\eta}} dj \\ -\frac{w_t}{\widetilde{\mu}_t} \int_0^1 \left(\frac{w^j_t}{w_t}\right)^{-\widetilde{\eta}} dj \end{array} \right\} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t h^d_t w^{\widetilde{\eta}}_t \left\{ \begin{array}{c} \widetilde{w}^{1-\widetilde{\eta}}_t \\ -\frac{w_t}{\widetilde{\mu}_t} \widetilde{w}^{-\widetilde{\eta}}_t \end{array} \right\} \\
&= E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s \lambda_{t+s} h^d_{t+s} w^{\widetilde{\eta}}_{t+s} \left\{ \begin{array}{c} \widetilde{w}^{1-\widetilde{\eta}}_t \prod_{k=1}^{s} \left[\frac{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}{\pi_{t+k}}\right]^{1-\widetilde{\eta}} \\ -\frac{w_{t+s}}{\widetilde{\mu}_{t+s}} \widetilde{w}^{-\widetilde{\eta}}_t \prod_{k=1}^{s} \left[\frac{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}{\pi_{t+k}}\right]^{-\widetilde{\eta}} \end{array} \right\} \\
&= E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s \lambda_{t+s} h^d_{t+s} w^{\widetilde{\eta}}_{t+s} \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}\right]^{\widetilde{\eta}} \left\{ \begin{array}{c} \widetilde{w}^{1-\widetilde{\eta}}_t \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}\right]^{-1} \\ -\frac{w_{t+s}}{\widetilde{\mu}_{t+s}} \widetilde{w}^{-\widetilde{\eta}}_t \end{array} \right\}
\end{aligned}$$

where the discount factor accounts for the probability of not being able to optimally reset the nominal wage, $\widetilde{\alpha}$. The first-order condition with respect to $\widetilde{w}_t$ is given by

$$0 = E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s \lambda_{t+s} h^d_{t+s} w^{\widetilde{\eta}}_{t+s} \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}\right]^{\widetilde{\eta}} \left\{ \frac{\widetilde{\eta}-1}{\widetilde{\eta}} \underbrace{\widetilde{w}_t \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}\right]^{-1}}_{\widetilde{w}_{t,s}} - \frac{w_{t+s}}{\widetilde{\mu}_{t+s}} \right\}$$

where $\widetilde{\eta}/(\widetilde{\eta}-1)$ is the markup of wages over marginal cost of supplying labour that would prevail in a world of perfectly flexible wages. Using the optimal supply of labour in (6.15), the first-order condition can be rewritten as

$$0 = E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s \lambda_{t+s} h^d_{t+s} w^{\widetilde{\eta}}_{t+s} \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\widetilde{\chi}}}\right]^{\widetilde{\eta}} \left\{ \frac{\widetilde{\eta}-1}{\widetilde{\eta}} \widetilde{w}_{t,s} - \frac{-U_h\left(c_{t+s} - bc_{t+s-1}, h_{t+s}\right)}{\lambda_{t+s}} \right\}$$

The first-order condition states that, in setting $\widetilde{w}_t$, the union tries to equate future expected average marginal revenue to future expected average marginal cost of supplying labour. On the one hand, the marginal revenue s periods after the last nominal wage reoptimisation is simply the marked up real wage s periods after the last nominal wage reoptimisation. On the other hand, the marginal cost of supplying labour is the marginal rate of substitution between consumption and leisure, $-U_h\left(c_{t+s} - bc_{t+s-1}, h_{t+s}\right)/\lambda_{t+s} = w_{t+s}/\widetilde{\mu}_{t+s}$. $\widetilde{\mu}_t$ thus represents a wedge between the disutility of supplying labour and the aggregate real wage. Hence, $\widetilde{\mu}_t$ can be regarded as the average markup that the union imposes in the labour markets. If relaxing the period-by-period budget constraint by one unit entails a λ_t increase in utility, supplying an extra unit of labour then achieves a $\lambda_t w_t / \widetilde{\mu}_t$ increase in utility. The wage-setting equation can be rewritten in recursive form, namely

$$f_t^1 = f_t^2 \tag{6.23}$$

with

$$\begin{aligned} f_t^1 &= \left(\frac{\widetilde{\eta}-1}{\widetilde{\eta}}\right)\widetilde{w}_t E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s \lambda_{t+s} h_{t+s}^d \left(\frac{w_{t+s}}{\widetilde{w}_t}\right)^{\widetilde{\eta}} \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{\left(\mu_{z^*}\pi_{t+k-1}\right)^{\widetilde{\chi}}}\right]^{\widetilde{\eta}-1} \\ &= \left(\frac{\widetilde{\eta}-1}{\widetilde{\eta}}\right)\widetilde{w}_t \lambda_t h_t^d \left(\frac{w_t}{\widetilde{w}_t}\right)^{\widetilde{\eta}} + \widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{\left(\mu_{z^*}\pi_t\right)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}-1} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t}\right)^{\widetilde{\eta}-1} f_{t+1}^1 \end{aligned} \tag{6.24}$$

and

$$\begin{aligned} f_t^2 &= -\widetilde{w}_t^{-\widetilde{\eta}} E_t \sum_{s=0}^{\infty} (\widetilde{\alpha}\beta)^s h_{t+s}^d w_{t+s}^{\widetilde{\eta}} U_h\left(c_{t+s} - b c_{t+s-1}, h_{t+s}\right) \prod_{k=1}^{s} \left[\frac{\pi_{t+k}}{\left(\mu_{z^*}\pi_{t+k-1}\right)^{\widetilde{\chi}}}\right]^{\widetilde{\eta}} \\ &= -\left(\frac{w_t}{\widetilde{w}_t}\right)^{\widetilde{\eta}} h_t^d U_h\left(c_t - b c_{t-1}, h_t\right) + \widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{\left(\mu_{z^*}\pi_t\right)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}} \left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t}\right)^{\widetilde{\eta}} f_{t+1}^2 \end{aligned} \tag{6.25}$$

6.1.2. The Government

We denote government's consumption of the composite good with $g_t = \left[\int_0^1 g_{it}^{1-1/\eta} di\right]^{1/(1-1/\eta)}$. The government faces a decision in each period about how much to consume of each variety of consumption goods. The government, like the household, adjusts the share of each differentiated good in g_t so to exploit any relative price differences. Minimising the level of total public expenditure, $\int_0^1 P_{it} g_{it} di$, given the consumption aggregate g_t, yields the government's demand for each differentiated good, namely $g_{it} = \left(P_{it}/P_t\right)^{-\eta} g_t$.

Recollecting that demand for good i for consumption purposes is given by (6.3), it follows that aggregate demand for good i, y_{it}, is of the form $y_{it} = \left(P_{it}/P_t\right)^{-\eta} y_t$, where aggregate demand, $y_t = \left[\int_0^1 y_{it}^{1-1/\eta} di\right]^{1/(1-1/\eta)}$, adds government's consumption of the composite good to the household's consumption, either genuine or for investment purposes, namely

$$y_t = c_t \left[1 + \ell\left(v_t\right)\right] + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right) k_t\right] + g_t \tag{6.26}$$

We postulate that along the balanced-growth path the share of government expenditure in total output, g_t/y_t, is constantly equal to s_g, namely $\lim_{j\to\infty} E_t g_{t+j}/y_{t+j} = s_g$. Formally, we impose that $g_t = z_t^* \overline{g}_t$ where z_t^* denotes a permanent shock, which we precisely define below, and $\overline{g}_t$ represents temporary exogenous variations in government expenditure. Denoting with $\overline{g}$ the level of government expenditure in the deterministic steady state, the percentage deviation of $\overline{g}_t$ from its steady-state value, $\widehat{\overline{g}}_t = \ln\left(\overline{g}_t/\overline{g}\right)$, is assumed to be autoregressive of order one with AR parameter $\rho_{\overline{g}}$ and innovation shock $\varepsilon_{g,t}$, namely $\widehat{\overline{g}}_t = \rho_{\overline{g}} \widehat{\overline{g}}_t + \varepsilon_{\overline{g},t}$.

The variable m_t^f denotes demand for real money balances by firms, which we rationalise below. The monetary authority is assumed to issue enough money, with m_t measuring real money balances supply, so to satisfy demand, that is

$$m_t = m_t^f + m_t^h \tag{6.27}$$

It follows that seigniorage in real terms is of the form $m_t - m_{t-1}/\pi_t$. For simplicity, we postulate that our theoretical economy starts with a zero level of government debt and we further assume that government never contracts debt. This latter assumption implies that the fiscal authority levies real lump-sum taxes, τ_t, so to achieve a balanced government budget in any period, namely

$$g_t = \tau_t + m_t - m_{t-1}/\pi_t \tag{6.28}$$

6.1.3. Firms

Each differentiated good i is produced by a single firm in a monopolistically competitive environment by means of labour and physical capital. Firms are assumed to rent capital and hire labour from centralised markets. We assume that each variety of goods has the linearly homogeneous production function $F(k_{it}, z_t h_{it}) - \psi z_t^*$, where k_{it} denotes physical capital services, h_{it} denotes labour services, and z_t is a permanent neutral technological shock. For any given realisation of z_t, the production function F is assumed to be concave and strictly increasing in both capital services and labour services. The parameter $\psi > 0$ measures fixed costs of operating a firm in each period, which entails that the production function displays increasing returns to scale. Following Altig et al. (2005), we postulate that fixed costs are subject to permanent shocks, z_t^*, and that fixed costs do not disappear along the balanced-growth path, namely $z_t^*/z_t = \Upsilon_t^{\frac{\theta}{1-\theta}}$. Denoting with μ_z the growth rate of z_t in the deterministic steady state, the percentage deviation of the gross growth rate of z_t, $\mu_{z,t} = z_t/z_{t-1}$, from its steady-state value, $\widehat{\mu}_{z,t} = \ln(\mu_{z,t}/\mu_z)$, is assumed to be autoregressive of order one with AR parameter ρ_{μ_z} and innovation shock $\varepsilon_{\mu_z,t}$, namely $\widehat{\mu}_{z,t} = \rho_{\mu_z}\widehat{\mu}_{z,t-1} + \varepsilon_{\mu_z,t}$.

On the one hand, each firm expects to sell a quantity of their good in any period given by y_{it}. On the other hand, each firm produces their own good according to the same linearly homogeneous production function. The linear homogeneity assumption implies that aggregate output, y_t, can be expressed in terms of aggregate labour, h_t, and the stock of capital, $k_t = \int_0^1 k_{it} di$, as $y_t = F(k_t, z_t h_t) - \psi z_t^*$. Given that z_t^* is postulated to enter the specification of g_t, it follows that government expenditure and aggregate output are cointegrated. Moreover, both factor inputs are understood to be homogeneous across all firms. Every firm can thus produce an additional unit of output according to the same production technology by hiring labour at the real wage w_t per unit of labour and renting capital at the real rate r_t^k per unit of capital. It follows that in any period the cost of production of an extra unit of output is identical across all firms in the economy. We denote the common marginal cost with mc_{it}[3].

Demand for money by firms is rationalised by assuming that wages are paid before the firm cashes in on the sale of goods. Specifically, we postulate a cash-in-advance constraint

[3]Indeed, given the postulated common production function, the subscript i could be dropped. We maintain it here and drop it once we consider market clearing.

on wage payments of the form

$$m_{it}^{f} = vw_t h_{it} \tag{6.29}$$

where m_{it}^{f} measures the demand for real money balances by firm producing good i and $v \geq 0$ denotes the fraction of wage payments that must be sustained by monetary assets. Of course, holding money entails forgoing the riskless nominal interest rate. Recollecting that the opportunity cost of holding money, $1 - R_t^{-1}$, is increasing in the nominal interest rate, R_t, the financial cost incurred by firms is thus given by $\left(1 - R_t^{-1}\right) m_{it}^{f}$, namely $\left(1 - R_t^{-1}\right) vw_t h_{it}$ when m_{it}^{f} is substituted for according to (6.29) . Denoting with ϕ_{it} the real profits that firm i distributes to the shareholders, the budget constraint of firm i expressed in real terms is of the form

$$\begin{aligned} E_t r_{t,t+1} x_{it+1}^{f} + m_{it}^{f} &= \frac{x_{it}^{f} + m_{it-1}^{f}}{\pi_t} + \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{it} \\ &\quad - w_t h_{it}\left[1 + \left(1 - R_{t+s}^{-1}\right) v\right] - \phi_{it} \end{aligned} \tag{6.30}$$

where $E_t r_{t,t+1} x_{it+1}^{f}$ denotes the real cost of uniperiod state-contingent that the firm carries into the subsequent period. The budget constraint states that, in any period, firm producing good i can carry into the subsequent period financial wealth that is worth as much as the financial wealth brought into the period plus after-dividend nonfinancial income earned during the period. The firm is understood to be committed to supply whatever quantity buyers may wish to purchase at the posted price, namely supply of good i at least equals aggregate demand for good i

$$F\left(k_{it}, z_t h_{it}\right) - \psi z_t^{*} \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t \tag{6.31}$$

The firm chooses processes for P_{it}, h_{it}, k_{it}, x_{it}^{f}, and m_{it}^{f} so to maximise the expected present discounted value of profits, namely $E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \phi_{it+s}$, subject to (6.30) and (6.31), and a no-Ponzi-scheme condition. Here, $r_{t,t+s} = \prod_{k=1}^{s} r_{t+k-1,t+k}$ for $s \geq 1$ denotes the stochastic nominal discount factor for s periods ahead payoff, and $r_{t,t} = 1$. Indeed, given that $r_{t,t+s}$ represents both the firm's stochastic discount factor and the market's pricing kernel for financial assets and because the firm's budget constraint is linear in assets holdings, any assets accumulation plan that satisfies the no-Ponzi game condition must be optimal. Postulating, without loss of generality, that the firm manages their assets so to have a nil financial position at the beginning of each period, that is $x^f_{it+1} + m^f_{it} = 0$ at all dates $t \geq -1$ and in all states, implies that the firm's budget constraint can be rewritten as

$$\phi_{it} = \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r^k_t k_{it} - w_t h_{it}\left[1 + \left(1 - R^{-1}_{t+s}\right)v\right] \tag{6.32}$$

Denoting the Lagrangian multiplier associated with constraint (6.31), holding with equality, with $r_{t,t+s}P_{t+s}mc_{it+s}$, we form the following Lagrangian

$$\mathcal{L}^f = E_t \sum_{s=0}^{\infty} r_{t,t+s}P_{t+s}\left\{\begin{array}{c} \left(P_{it+s}/P_{t+s}\right)^{1-\eta} y_{t+s} - r^k_{t+s}k_{it+s} - w_{t+s}h_{it+s}\left[1 + \left(1 - R^{-1}_{t+s}\right)v\right] \\ +mc_{it+s}\left[F\left(k_{it+s}, z_{t+s}h_{it+s}\right) - \psi z^*_{t+s} - \left(P_{it+s}/P_{t+s}\right)^{-\eta} y_{t+s}\right] \end{array}\right\}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}^f}{\partial h_{it}} = 0 \Rightarrow mc_{it}z_tF_2\left(k_{it}, z_th_{it}\right) = w_t\left[1 + v\frac{R_t - 1}{R_t}\right] \tag{6.33}$$

$$\frac{\partial \mathcal{L}^f}{\partial k_{it}} = 0 \Rightarrow mc_{it}F_1\left(k_{it}, z_th_{it}\right) = r^k_t \tag{6.34}$$

$$\frac{\partial \mathcal{L}^f}{\partial P_{it}} = 0 \Rightarrow P_{it} = \left\{\begin{array}{c} P^f_t \text{ if } P_{it} \text{ is set optimally in } t \\ P^b_t \text{ if } P_{it} \text{ is set according to rule-of-thumb behaviour in } t \\ P_{it-1} \text{ if } P_{it} \text{ cannot be reset in } t \end{array}\right.$$

The first order-conditions (6.33) and (6.34) are the standard cost minimisation conditions, which state that the ratio of factors' prices is equal to the marginal rate of technical substitution between the two factors. The postulated cash-in-advance constraint on wage payments implies that hiring labour also entails a financial cost, which is increasing in the nominal interest rate.

Following Calvo (1983), we assume that only a randomly chosen fraction $1-\alpha$ of goods' nominal prices are reset in each period. The probability of not resetting the price in each period, $0 \leq \alpha < 1$, is independent of both the time that has gone by since the last price revision and the misalignment between the actual price and the price that would be optimal to charge, namely pricing decisions in any period are independent of past pricing decisions.

We depart from full rationality by introducing backward-looking rule-of-thumb behaviour by price setters. Following Galì and Gertler (1999), we assume that only a fraction $1-\omega$ of the $1-\alpha$ firms behave optimally (i.e. in a forward-looking manner) when setting the price, the remaining ω fraction of firms use the same backward-looking rule-of-thumb when revising their prices. The degree of backward-looking rule-of-thumb behaviour, $0 \leq \omega < 1$, is thus constant over time and price setters cannot switch between backward-looking and forward-looking behaviour.

We thus need to distinguish both between the $1-\alpha$ goods' markets and the remaining α markets and, within the $1-\alpha$ goods' markets, between the $1-\omega$ goods' markets and the remaining ω markets. In any good's markets where the nominal price cannot be reoptimised in period t, firms keep on charging the price posted in period $t-1$, P_{it-1}. In any good's market where the nominal price is set optimally in period t, the nominal forward-looking reset price is denoted with P_t^f, with the real price being given by p_t^f. In any good's market where the nominal price is set according to rule-of-thumb behaviour in period t, the nominal backward-looking price is denoted with P_t^b, with the real reset price being price is p_t^b. Hence, the overall reset real price at time t, denoted with $\widetilde{p}_t$, is given by

$$\widetilde{p}_t = (1-\omega)p_t^f + \omega p_t^b \tag{6.35}$$

It follows that the aggregate price level in (6.4) evolves according to

$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\,\widetilde{p}_t^{1-\eta} \tag{6.36}$$

Following Galì and Gertler (1999), rule-of-thumb price setters are postulated to set P_t^b equal to the previous period overall nominal reset price, $\widetilde{P}_{t-1}$, fully indexed to past inflation, namely $P_t^b = \widetilde{P}_{t-1}\pi_{t-1}$. It follows that p_t^b is of the form

$$p_t^b = \widetilde{p}_{t-1}\frac{\pi_{t-1}}{\pi_t} \tag{6.37}$$

The price P_t^f is set so to maximise the expected present discounted value of profits, namely

$$\mathcal{L}_{P_t^f}^f = E_t \sum_{s=0}^{\infty} \alpha^s r_{t,t+s} P_{t+s} \left\{ \begin{array}{c} \left(P_t^f / P_{t+s}\right)^{1-\eta} y_{t+s} - r_{t+s}^k k_{it+s} - w_{t+s} h_{it+s}\left[1 + \left(1 - R_{t+s}^{-1}\right) v\right] \\ + mc_{it+s}\left[F\left(k_{it+s}, z_{t+s} h_{it+s}\right) - \psi z_{t+s}^* - \left(P_t^f / P_{t+s}\right)^{-\eta} y_{t+s}\right] \end{array} \right\}$$

where the discount factor accounts for the probability of not being able to optimally reset the nominal price, α. The first-order condition with respect to P_t^f is given by

$$0 = E_t \sum_{s=0}^{\infty} \alpha^s r_{t,t+s} \left(\frac{P_t^f}{P_{t+s}}\right)^{-\eta-1} y_{t+s} \left[\frac{\eta-1}{\eta}\left(\frac{P_t^f}{P_{t+s}}\right) - mc_{it+s}\right]$$

where $\eta/(\eta-1)$ is the markup of prices over marginal cost of supplying goods that would prevail in a world of perfectly flexible prices. The first order-condition states that, in setting P_t^f, the firm tries to equate future expected average marginal revenue to future expected average marginal cost. The price-setting equation can be rewritten in recursive form, namely

$$x_t^1 = \frac{\eta-1}{\eta} x_t^2 \tag{6.38}$$

with[4]

$$x_t^1 = E_t \sum_{s=0}^{\infty} \alpha^s r_{t,t+s} \left(\frac{P_t^f}{P_{t+s}}\right)^{-\eta-1} y_{t+s} mc_{t+s} = \left\{ \begin{array}{c} \left(\frac{P_t^f}{P_t}\right)^{-\eta-1} y_t mc_t \\ +\alpha r_{t,t+1} \left(\frac{P_t^f}{P_{t+1}^f}\right)^{-\eta-1} E_t x_{t+1}^1 \end{array} \right\}$$

$$(6.39) \quad = \left(p_t^f\right)^{-\eta-1} y_t mc_t + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta-1} \pi_{t+1}^{\eta} x_{t+1}^1$$

and

$$x_t^2 = E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s \left(\frac{P_t^f}{P_{t+s}}\right)^{-\eta-1} y_{t+s} \left(\frac{P_t^f}{P_{t+s}}\right) = \left\{ \begin{array}{c} \left(\frac{P_t^f}{P_t}\right)^{-\eta} y_t \\ +\alpha r_{t,t+1} \left(\frac{P_t^f}{P_{t+1}^f}\right)^{-\eta} E_t x_{t+1}^2 \end{array} \right\}$$

$$(6.40) \quad = \left(p_t^f\right)^{-\eta} y_t + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta} \pi_{t+1}^{\eta-1} x_{t+1}^2$$

where we use (6.14) to substitute $\beta\lambda_{t+1}P_t/\lambda_t P_{t+1}$ for $r_{t,t+1}$.

6.1.4. Market Clearing

Market clearing in the goods' markets requires, for each good i and at all times that supply equals demand, namely

$$(6.41) \qquad F\left(k_{it}, z_t h_{it}^d\right) - \psi z_t^* = \left\{c_t \left[1 + \ell\left(v_t\right)\right] + g_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right) k_t\right]\right\} \left(\frac{P_{it}}{P_t}\right)^{-\eta}$$

Equivalently, in aggregate terms, we obtain

$$(6.42) \qquad F\left(u_t k_t, z_t h_t^d\right) - \psi z_t^* = \left\{c_t \left[1 + \ell\left(v_t\right)\right] + g_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right) k_t\right]\right\} s_t$$

with

$$(6.43) \qquad mc_t z_t F_2\left(u_t k_t, z_t h_t\right) = w_t \left[1 + v \frac{R_t - 1}{R_t}\right]$$

[4]Note that we drop the subscript i.

and

$$mc_t F_1 (k_t, z_t h_t) = r_t^k \tag{6.44}$$

where the production function is expressed in terms of the aggregate effective level of capital, $u_t k_t$, and (6.43) and (6.44) are the cost minimisation conditions (6.33) and (6.34) at the aggregate level. The state variable s_t measures the degree of price dispersion in the economy brought about by stickiness in the adjustment on goods' nominal prices, namely

$$\begin{aligned} s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di = (1-\alpha)\left(\frac{\widetilde{P}_t}{P_t}\right)^{-\eta} + (1-\alpha)\,\alpha\left(\frac{\widetilde{P}_{t-1}}{P_t}\right)^{-\eta} + ... \\ &= (1-\alpha)\sum_{s=0}^{\infty}\alpha^s\left(\frac{\widetilde{P}_{t-s}}{P_t}\right)^{-\eta} = (1-\alpha)\left(\widetilde{p}_t\right)^{-\eta} + \alpha\pi_t^{\eta} s_{t-1} \end{aligned} \tag{6.45}$$

The state variable s_t thus summarizes the resource costs stemming from inefficient price dispersion, with s_{-1} given. Indeed, as shown in Schmitt-Grohé and Uribe (2007), price dispersion is always a costly distortion in this model, namely s_t is bounded below by 1. Denoting with $\overline{s}$ the level of price dispersion in the deterministic steady state, we accordingly denote with $\widehat{s}_t$ the percentage deviation of s_t from its steady-state value, that is $\widehat{s}_t = \ln(s_t/\overline{s})$. In an economy with zero inflation in the nonstochastic steady state, that is a unitary level of gross inflation, $\widehat{s}_t$ follows, up to first order, the univariate autoregressive process $\widehat{s}_t = \alpha\widehat{s}_{t-1}$. Hence, restricting the analysis to linear approximations to the equilibrium conditions justifies neglecting the price dispersion only if the model features no price dispersion in the deterministic steady state. Indeed, that is always the case in a model log-linearised around the perfectly flexible prices equilibrium, which, trivially, implies absence of price dispersion across goods. Yet, s_t matters up to first order when the deterministic steady state features movements in relative prices across goods varieties. More importantly, even if relative prices are stable in the deterministic steady state, price dispersion must be taken into account if one is interested in approximations to the equilibrium conditions that are of order higher than linear.

Market clearing in the labour markets requires, for each type of labour j and at all times that supply equals demand, as implied by (6.8). Equivalently, in aggregate terms, we have equation (6.9). Moreover, as discussed above, the supply of labour is identical across all labour markets in which nominal wage is reset optimally in period t, namely $\widetilde{w}_t^{\widetilde{\eta}}\widetilde{h}_t = w_t^{\widetilde{\eta}} h_t^d$. Combining this with (6.8) and (6.9), yields

$$h_t = (1-\widetilde{\alpha})\, h_t^d \sum_{s=0}^{\infty} \widetilde{\alpha}^s \left(\frac{\widetilde{W}_{t-s} \prod_{k=1}^{s} \left(\mu_{z^*}\pi_{t+k-s-1}\right)^{\widetilde{\chi}}}{W_t} \right)^{-\widetilde{\eta}} \tag{6.46}$$

Letting $\widetilde{s}_t = (1-\widetilde{\alpha}) \sum_{s=0}^{+\infty} \widetilde{\alpha}^s \left(\widetilde{W}_{t-s} \prod_{k=1}^{s} \left(\mu_{z^*}\pi_{t+k-s-1}\right)^{\widetilde{\chi}} / W_t \right)^{-\widetilde{\eta}}$ denote the degree of wage dispersion across different types of labour, we obtain

$$h_t = \widetilde{s}_t h_t^d \tag{6.47}$$

with

$$\begin{aligned} \widetilde{s}_t &= (1-\widetilde{\alpha}) \sum_{s=0}^{\infty} \widetilde{\alpha}^s \left(\frac{\widetilde{w}_{t-s}}{w_t} \right)^{-\eta} \left(\frac{\pi_{t+s}}{\left(\mu_{z^*}\pi_{t+s-1}\right)^{\widetilde{\chi}}} \right)^{\widetilde{\eta}} \\ &= (1-\widetilde{\alpha}) \left(\frac{\widetilde{w}_t}{w_t} \right)^{-\widetilde{\eta}} + \widetilde{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{-\widetilde{\eta}} \left(\frac{\pi_t}{\left(\mu_{z^*}\pi_{t-1}\right)^{\widetilde{\chi}}} \right)^{\widetilde{\eta}} \widetilde{s}_{t-1} \end{aligned} \tag{6.48}$$

The state variable $\widetilde{s}_t$ thus summarizes the resource costs stemming from inefficient wage dispersion, with $\widetilde{s}_{-1}$ given. Indeed, wage dispersion is always a costly distortion in this model, namely $\widetilde{s}_t$ is bounded below by 1. Denoting with $\overline{\widetilde{s}}$ the level of wage dispersion in the deterministic steady state, we accordingly denote with $\widehat{\widetilde{s}}_t$ the percentage deviation of $\widetilde{s}$ from its steady-state value, namely $\widehat{\widetilde{s}}_t = \ln\left(\widetilde{s}/\overline{\widetilde{s}}\right)$. In an economy with zero wage dispersion in the nonstochastic steady state, $\widehat{\widetilde{s}}_t$ follows, up to first order, the univariate autoregressive process $\widehat{\widetilde{s}}_t = \widetilde{\alpha}\widehat{\widetilde{s}}_{t-1}$. Hence, restricting the analysis to linear approximations to the equilibrium conditions justifies neglecting the wage dispersion only if the model features no wage dispersion in the deterministic steady state. Indeed, that is always the case in a model log-linearised around the perfectly flexible wages equilibrium, which, trivially, implies absence of wage dispersion across types of labour. Yet, $\widetilde{s}_t$ matters up to first order when the deterministic steady state features movements in relative wages across labour varieties. More importantly, even if relative wages are stable in the deterministic steady state, wage dispersion must be taken into account if one is interested in approximations to the equilibrium conditions that are of order higher than linear.

Finally, it follows from profits at the firm's level, (6.32), that aggregate profits are given by

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d \left[1 + \left(1 - R_t^{-1}\right) v\right] \tag{6.49}$$

and, equations (6.29) and (6.27), imply that real money balances in equilibrium are of the form

$$m_t = v w_t h_t^d + m_t^h \tag{6.50}$$

6.2. Solving the Model

Given the complexity of the theoretical economy, the long-run state of the Ramsey equilibrium in an economy without uncertainty, that is the Ramsey steady state, cannot be characterised analytically, but we use the algorithm used in Schmitt-Grohé and Uribe (2007). The algorithm numerically solves the Ramsey steady state in medium-scale macroeconomic models. Specifically, it yields an exact numerical solution for the Ramsey steady state. The only inputs that need to be provided are the set of equilibrium conditions and the steady-state level of the model's variables and parameters in the competitive equilibrium.

The solution for the Ramsey steady state is obtained in five steps. First, we need to specify functional forms for utility, technology, the investment adjustment cost function, the transaction cost function, and the cost of higher capacity utilisation. We use the same functional forms in Schmitt-Grohé and Uribe (2007). Second, some variables are not stationary along the balanced-growth path as the theoretical economy displays two types of permanent shocks. This requires rescaling those variables so that the model's equilibrium conditions are functions of stationary variables only, which we denote with the corresponding capital letters. The nonstationary variables and the respective scaling factors are the same as in Schmitt-Grohé and Uribe (2007). Appendix C presents the functional forms and reports the complete set of equilibrium conditions in terms of stationary variables.

Third, we need to define the competitive equilibrium. A stationary competitive equilibrium is formally defined as being a set of stationary processes C_t, M_t^h, M_t, W_t, $\widetilde{W}_t$, Y_t, G_t, Φ_t, X_t^1, X_t^2, T_t, K_{t+1}, I_t, F_t^1, F_t^2, Q_t, R_t^k, Λ_t, p_t^f, p_t^b, $\widetilde{p}_t$, u_t, v_t, mc_t, h_t, h_t^d, s_t, $\widetilde{s}_t$, and π_t satisfying (6.10), (6.11), (6.13), (6.15)-(6.26), (6.28), (6.35)-(6.45), and (6.47)-(6.50) written in terms of stationary variables, given I) the exogenous stochastic processes for $\mu_{\Upsilon,t}$, $\mu_{z,t}$, and $\overline{g}_t$, II) the policy process R_t, and III) initial conditions c_{-1}, w_{-1}, s_{-1}, $\widetilde{s}_{-1}$, π_{-1}, i_{-1}, and κ_0. Fourth, we need to derive the steady state of the competitive equilibrium. In our economy, the equilibrium conditions in terms of stationary variables contain 29 equations and 29 variables, which are listed above. In addition the equilibrium conditions feature 29 parameters: ϕ_1, ϕ_2, ϕ_3, ϕ_4, γ_1, γ_2, θ, κ, b, β, δ, η, $\tilde{\eta}$, α, $\tilde{\alpha}$, ω, $\tilde{\chi}$, ν, ψ, μ_I, μ_z, μ_Υ,σ_{μ_z}, σ_{μ_Υ}, ρ_{μ_z}, ρ_{μ_Υ}, $\rho_{\overline{g}}$, $\sigma_{\varepsilon_{\overline{g}}}$, $\overline{g}$. In order to obtain values for the steady-state levels of all variables and for the structural parameters, we thus need to impose 29 restrictions. These 29 restrictions come from using the same calibration in Schmitt-Grohé and Uribe (2007), which draws on the estimation results in Altig at al. (2005). In other words, we fix all structural parameters and we then find the steady-state levels of all variables as a function of the structural parameters. In particular, this entails that, apart from the equations describing the price setting, the specification of the steady state of the competitive equilibrium is the same as the one presented in the technical appendix to Schmitt-Grohé and Uribe (2007) (i.e. the one that obtains when all structural parameters are known).

Fifth, we can define the Ramsey equilibrium**.** Specifically, we assume that a $t = 0$ the Ramsey planner has been operating for an infinite number of periods. In choosing the optimal policy, namely $\{R_t\}_{t=0}^{\infty}$, the Ramsey planner is assumed to sustain commitments made in the past. In other words, we study commitment from a timeless perspective. The Ramsey equilibrium is formally defined as being a set of stationary processes C_t, M_t^h, M_t,

W_t, $\widetilde{W}_t$, Y_t, G_t, Φ_t, X_t^1, X_t^2, T_t, K_{t+1}, I_t, F_t^1, F_t^2, Q_t, R_t^k, Λ_t, p_t^f, p_t^b, $\widetilde{p}_t$, u_t, v_t, mc_t, h_t, h_t^d, s_t, $\widetilde{s}_t$, π_t, and R_t for $t \geq 0$ that maximises

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(z_0^* \prod_{k=1}^{t} \mu_{z^*,s}\right)^{(1-\phi_4)(1-\phi_3)} \left[\left(C_t - b\frac{C_{t-1}}{\mu_{z^*,t}}\right)^{1-\phi_4} (1-h)^{\phi_4}\right]^{1-\phi_3} - 1}{1-\phi_3}$$

subject to I) the competitive equilibrium conditions (6.10), (6.11), (6.13), (6.15)-(6.26), (6.28), (6.35)-(6.45), and (6.47)-(6.50) written in terms of stationary variables, and II) $R_t \geq 1$ for $t > -\infty$ and given: I) the exogenous stochastic processes for $\mu_{\Upsilon,t}$, $\mu_{z,t}$, and $\overline{g}_t$, II) values of the variables listed above dated $t < 0$ and III) values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$. The fact that we consider commitment from a timeless perspective implies that the optimality conditions associated with the Ramsey equilibrium are time-invariant. Conversely, under the standard Ramsey equilibrium definition, the equilibrium conditions in the initial periods are different from the ones applying in later periods. Before characterising the Ramsey steady state, we describe the calibration of the model.

6.2.1. Calibration

The time unit is a quarter. The calibration in Schmitt-Grohé and Uribe (2007) draws most parameters from Altig et al. (2005). To simplify the presentation of the calibration we partition the parameters' set into three groups: preferences, technology and shocks and the market structure. A complete presentation of the calibration is shown in Table (6.1).

6.2.1.1. Preferences. The discount factor β is set equal to $1.03^{-1/4}$ so to match an annualised real interest rate of 3 percent. Preferences are assumed to be separable in consumption and labour and logarithmic in habit-adjusted consumption, which implies that $\phi_3 = 1$. ϕ_4 is calibrated to match a unit elasticity of labour supply in the competitive steady state, chosen in this way $\phi_4 = 0.53$. The parameters governing the demand for money (ϕ_1 and ϕ_2) are calibrated assuming that the steady-state share of money held by households is 0.44 percent and that the annualized interest rate semi-elasticity of money demand is -0.81, as estimated by Altig et al. (2005). This implies that $\phi_1 = 0.0458$ and $\phi_2 = 0.1257$. Additionally, it also follows that the share of the wage bill subject to a cash-in-advance constraint is 60%, that is $\nu = 0.6011$. As for the degree of habit formation, measured by the parameter b, Altig et al. (2005) estimate it to be 0.69.

6.2.1.2. Technology and Shocks. Altig et al. (2005) assume a steady-state share of capital income equal to 36 percent (i.e. $\theta = 0.36$) and 10 percent rate of depreciation of capital per year (i.e. $\delta = 0.025$ per quarter). The fixed cost parameter in the aggregate production function, ψ, is set so that in the competitive steady state there are zero profits (i.e. $\psi = 0.2503$). The parameter in the investment adjustment cost function, κ, has been estimated by Altig et al. (2005) to be equal to 2.79. The parameters in the capital utilisation cost function are calibrated assuming that in competitive steady state the capital utilisation is full, namely $u = 1$. This results in γ_1 and γ_2 being respectively given by 0.0412 and 0.0601[5]. As for the steady-state growth rate of investment, μ_I, we follow Schmitt-Grohé and Uribe (2007) in setting it so that in steady state adjustment costs are nil, chosen in this way $\mu_I = 1.028$.

The exogenous stochastic processes for the investment specific and the neutral technology shocks are calibrated using results in Altig et al. (2005). That is, the parameters of the exogenous stochastic process for the investment specific shock, $\mu_{\Upsilon,t}$, and the neutral technology shock, $\mu_{z,t}$, are respectively $(\mu_\Upsilon, \sigma_{\mu_\Upsilon}, \rho_{\mu_\Upsilon}) = (1.0042, 0.0031, 0.20)$ and $(\mu_z, \sigma_{\mu_z}, \rho_{\mu_z}) = (1.00213, 0.0007, 0.89)$. The exogenous stochastic process for the government spending shock is calibrated as in Ravn (2005), namely $(G, \sigma_{\varepsilon_{\overline{g}}}, \rho_{\overline{g}}) = (0.2141, 0.008, 0.9)$.

6.2.1.3. Market Structure. Following Altig at al. (2005), we set the steady-state mark-up of wages over the marginal rate of substitution between leisure and consumption to 5 percent, which implies a value for the elasticity of substitution between labour services, $\tilde{\eta}$, equal to 21. The steady state mark-up in product markets is assumed to be 20 percent, which implies a value for the elasticity of substitution between goods, η, equal to 6.

[5]Schmitt-Grohé and Uribe (2005) experiment other functional forms for the capacity utilisation cost function and various calibrations in a model similar to the one in this paper. They find that the results about optimal monetary policy are not affected by these parameters.

The average duration of wage contracts, about three quarters, has been estimated by Altig et al. (2005) and implies a value for $\tilde{\alpha}$ of 0.69. In those labour markets in which households can not set an optimal wage, the old wage is fully indexed to past inflation ($\tilde{\chi} = 1$).

As in the linear-quadratic framework, we consider ample ranges for the average duration that an individual price is fixed, namely the degree of price stickiness, α, and the fraction of firms that reset prices in a backward-looking manner, that is the degree of rule-of-thumb behaviour, ω. Galì and Gertler (1999) report estimates of ω between 0.077 and 0.552, with 3 of their 6 estimates between 0.2 and 0.3. As for the degree of price stickiness, empirically realistic values of the average price duration based on macroeconomic data vary between 2 and 5 quarters, namely $0.5 \leq \alpha \leq 0.8$. Evidence on price stickiness based on microeconomic data suggest a much higher frequency of price changes than the evidence based on macro data. Available empirical estimates using microeconomic data, as in Bils and Klenow (2004) and Golosov and Lucas (2007), suggest in fact a lower average price duration of around 1.5 quarters, that is a value of α of about 0.33. In what follows we want to assess the robustness of our results with respect to alternative values for these two parameters. We thus consider $0.33 \leq \alpha \leq 0.8$ and extend the range for the degree of rule-of-thumb behaviour up to 0.7, namely $0.01 \leq \omega \leq 0.7$. This is because, as discussed in Chapter 5, $\omega = 0.7$ implies that the hybrid Phillips curve under rule-of-thumb behaviour puts equal weight on future expected inflation and lagged inflation.

Parameter	Value	Description
β	$1.03^{-1/4}$	Subjective discount factor
θ	0.36	Share of capital in value added
ψ	0.2503	Fixed cost parameter
δ	0.025	Depreciation rate
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	$0.33 - 0.8$	Probability of not setting a new price each period
$\tilde{\alpha}$	0.69	Probability of not setting a new wage each period
ω	$0 - 0.7$	Share of rule-of-thumb price setters
b	0.69	Degree of habit persistence
ϕ_1	0.0458	Transaction cost parameter
ϕ_2	0.1257	Transaction cost parameter
ϕ_3	1	Preference parameter
ϕ_4	0.5300	Preference parameter
ν	0.6011	Share of wage bill subject to CIA constraint
κ	2.48	Investment adjustment cost parameter
$\tilde{\chi}$	1	Degree of wage indexation
γ_1	0.0412	Capital utilisation cost function parameter
γ_2	0.0601	Capital utilisation cost function parameter
μ_{Υ}	1.0042	Growth rate of investment-specific technology shock
$\sigma_{\mu_{\Upsilon}}$	0.0031	Stad. Dev. of the innovation to the investment-specific tech. shock
$\rho_{\mu_{\Upsilon}}$	0.20	Autoregressive parameter in the investment specific tech. shock

μ_z	1.0213	Growth rate of neutral technology shock
σ_{μ_z}	0.0007	Stad. Dev. of the innovation to the neutral tech. shock
ρ_{μ_z}	0.89	Autoregressive parameter in the neutral tech. shock
μ_I	1.028	Steady-state growth rate of investment
$\overline{g}$	0.2141	Steady-state value of government consumption
$\sigma_{\varepsilon_{\overline{g}}}$	0.008	Stad. Dev. of the innovation to the government spending process
$\rho_{\overline{g}}$	0.9	Autoregressive parameter in the government spending process

Table 6.1. Benchmark Calibration

6.3. The Ramsey Steady State

In this section, we characterise the long-run state of the Ramsey equilibrium in an economy without uncertainty, namely the Ramsey steady state. We first characterise the Ramsey steady state in the model with money and subsequently proceed to analyse how the Ramsey steady state is affected when demand for money by both households and firms is assumed away.

In the presence of money, we find that the results in Schmitt-Grohé and Uribe (2007) generally hold. Figure (6.1) shows the relationship between the optimal long-run rate of inflation, which is throughout expressed in annualised percentange points, and the degree of price stickiness. Rule-of-thumb behaviour by price setters does not to alter the high sensitivity of the long-run inflation rate with respect to the degree of price stickiness. In the absence of rule-of-thumb behaviour, the optimal rate of inflation coincides with the one presented in Schmitt-Grohé and Uribe (2007). Introducing rule-of-thumb behaviour does not affect the shape of the relationship between the long-run rate of inflation and the degree of price stickiness. Specifically, relatively low values for the degree of rule-of-thumb behaviour imply nearly the same levels for the optimal rate of inflation. If, as we do in Figure (6.1), a relatively high value for the degree of rule-of-thumb behaviour is considered, the optimal rate of inflation would be only slightly affected for values of the degree of price stickiness that are in line with macroeconomic evidence. However, the optimal long-run inflation is always negative and it is found to vary between the level implied by the Friedman rule and a level close to price stability. Moreover, the relationship between the optimal long-run rate of inflation and the degree of price stickiness is still observed to be steep for the range of α values that are in line with macroeconomic evidence.

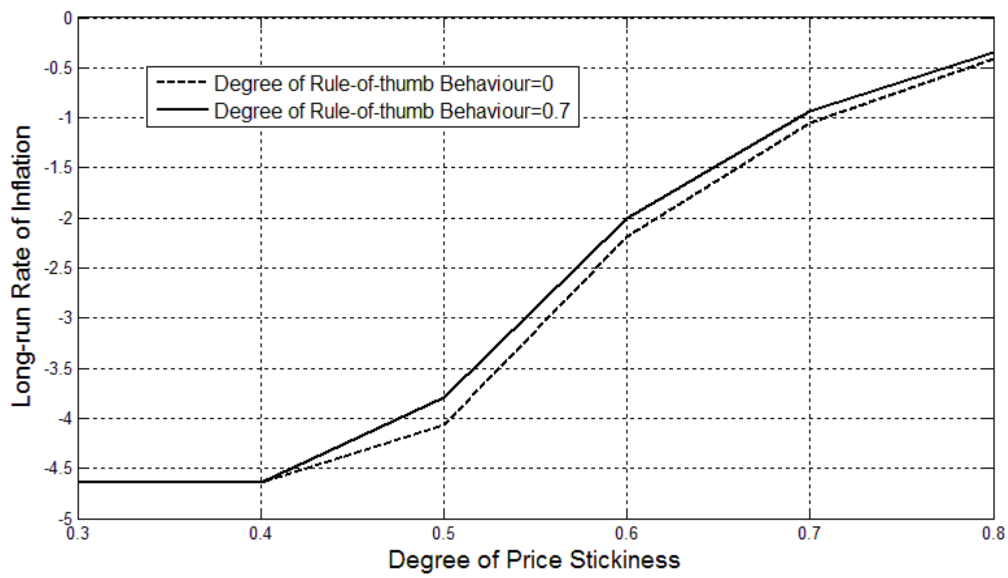

Figure 6.1. Model with money: degree of price stickiness and the Ramsey steady-state rate of inflation.

As for the relationship between the optimal long-run rate of inflation and the degree of rule-of-thumb behaviour, Figure (6.2) shows how a larger fraction of firms resetting their prices in a backward-looking manner is associated with a less negative optimal inflation rate. In particular, the relationship is observed to be nearly flat for relatively low values of the degree of rule-of-thumb behaviour.

Finally, to complete the analysis of the Ramsey steady state in the presence of money, we study the relationship between the long-run rate of infation and the parameters determining the demand of money. Figure (6.3) displays the optimal rate of inflation as a function of the two structural parameters underpinning the demand of money by households, ϕ_1 and ϕ_2 for two alternative values of the degree of price stickiness (i.e. $\alpha = 0.66$

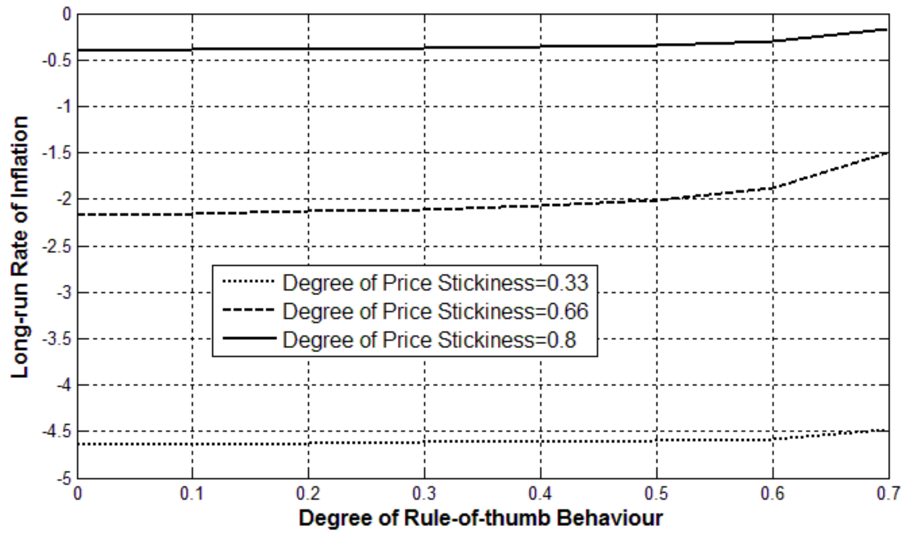

Figure 6.2. Model with money: degree of rule-of-thumb behaviour and the Ramsey steady-state rate of inflation.

and $\alpha = 0.8$)[6]. The results we obtain are again very similar to the ones in Schmitt-Grohé and Uribe (2007). Considering an average duration of prices of 5 quarters (i.e. $\alpha = 0.8$), the benchmark value of 0.05 for the parameter ϕ_1 implies that the optimal rate of inflation is -0.38 percent per year and money demand is 18 percent of annual output. If ϕ_1 increases by a factor of 10 to 0.5, the optimal rate of inflation is -1 percent per year, but the demand for money increases to 37 percent of annual output. Finally, if ϕ_1 increases by a factor of more than 150 to around 8, the optimal rate of inflation is close to the level implied by the the Friedman rule. However, at this value of ϕ_1, the demand for money is larger than one entire annual output. As stressed by Schmitt-Grohé and Uribe (2007), the sensitivity of the optimal long-run rate of inflation relative to the parameter ϕ_1 hinges on the importance of price stability. This is clearly shown in the first panel of Figure (6.3). If we consider a lower degree of price stickiness (i.e. $\alpha = 0.66$) the optimal rate of inflation converges faster to the value called for by the Friedman rule. A similar message emerges as one varies the other transaction cost parameter, ϕ_2. Considering an average duration of prices of 5 quarters (i.e. $\alpha = 0.8$), the benchmark value of 0.126 for the parameter ϕ_2 implies that the optimal rate of inflation is -0.4 percent per year and money demand is

[6] In producing Figure 3, we set the degree of rule-of-thumb behaviour equal to 0.5. Considering alternative values for the degree of rule-of-thumb behaviour does not affect the results presented.

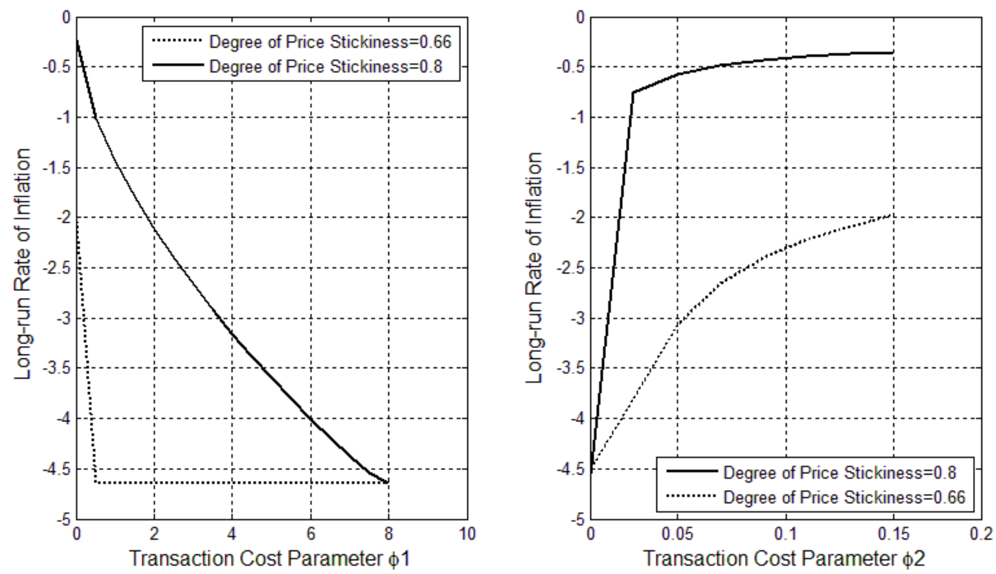

Figure 6.3. Model with money: transaction cost parameters and the Ramsey steady-state rate of inflation.

16 percent of annual output. If ϕ_2 decreases by a factor of 5 to 0.025, the optimal rate of inflation is -0.7 percent per year, but the demand for money increases to 39 percent of annual output. Finally, if ϕ_2 decreases by a factor of 10 to 0.0, the optimal rate of inflation is close to the level implied by the the Friedman rule. However, at this value of ϕ_2, the demand for money is larger than one entire annual output. Moreover, the sensitivity of the optimal long-run rate of inflation relative to the parameter ϕ_2 again hinges on the importance of price stability. This is clearly shown in the second panel of Figure (6.3). If we consider a lower degree of price stickiness (i.e. $\alpha = 0.66$) the optimal rate of inflation converges faster to the value called for by the Friedman rule.

6.3.1. The Cashless Model

Schmitt-Grohé and Uribe (2007) go on to analyse the optimal long-run rate of inflation by taking into account the public finance aspect of the optimal policy problem. They do so by replacing the assumption of lump-sum taxes with the assumption of distortionary income taxes. Specifically, they consider the theoretical economy in Schmitt-Grohé and Uribe (2005) and analyse the possibility of a social planner setting jointly monetary policy and fiscal policy in a Ramsey-optimal fashion. The optimal long-run inflation, although remaining always negative, is then found to be much closer to price stability. Moreover, the high sensitivity of the long-run inflation rate with respect to the degree of price stickiness disappears thus making mild deflation robust to the uncertainty about the exact degree of price stickiness. The intutition for this resul is quite neat. The fiscal authority can finance government's consumption through seignorage or distortionary taxation. A higher rate of inflation then allows the social planner to trade revenue due to distortionary taxation for seignorage revenue. It follows that the trade-off between price stability and the Friedman rule is resolved in favour of price stability.

We depart from the analysis in Schmitt-Grohé and Uribe (2007) and consider the case of a cashless medium-scale macroeconomic model. The reasons for this are primarily two. First, maintaining the cashless qualification of the economy, we seek to establish a link between the analysis of optimal monetary policy carried out in the previous chapters within a canonical log-linearised New Keynesian model and its counterpart in a much richer theoretical nonlinear economy. Second, we want to study the case of large steady-state distortions in order to assess whether dropping the assumptions of small steady-state distortions is capable of delivering larger positive inflation rates.

In the model at hand money is demanded both by households for transactional reasons and by firms given the assumed cash-in-advance constraint on wage payments. Considering the possibility of a cashless medium-scale macroeconomic model thus amounts to drop both assumptions. The consequences on the equilibrium conditions of the model are as follow.

First, (6.11) ceases to apply. The household's budget constraint expressed in real terms is then of the form

$$E_t r_{t,t+1} x_{t+1}^h + c_t + \Upsilon_t^{-1}\left(i_t + a\left(u_t\right)k_t\right) = \frac{x_t^h}{\pi_t} + \phi_t + r_t^k u_t k_t + \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\widetilde{\eta}} h_t^d dj - \tau_t \tag{6.51}$$

which implies that I) (6.18) ceases to apply; II) (6.14)-(6.17) are unaffected; III) (6.16) becomes

$$\frac{\partial \mathcal{L}^h}{\partial c_t} = 0 \Rightarrow U_c\left(c_t - bc_{t-1}, h_t\right) - b\beta E_t U_c\left(c_{t+1} - bc_t, h_{t+1}\right) = \lambda_t\left[1 + \ell\left(v_t\right) + v_t \ell'\left(v_t\right)\right] \tag{6.52}$$

Second, the aggregate resource constraint is now given by

$$y_t = c_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right)k_t\right] + g_t \tag{6.53}$$

which implies that (6.42) becomes

$$F\left(u_t k_t, z_t h_t^d\right) - \psi z_t^* = \left\{c_t + g_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right)k_t\right]\right\} s_t \tag{6.54}$$

Third, the government budget constraint is now given by

$$g_t = \tau_t \tag{6.55}$$

Fourth, the absence of a cash-in-advance constraint on the wage payments implies that the cost minimisation condition with respect to labour becomes

$$mc_t z_t F_2\left(u_t k_t, z_t h_t\right) = w_t \tag{6.56}$$

which implies that aggregate profits are now given by

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d \tag{6.57}$$

The complete set of equilibrium conditions in the cashless model is then given by 26 equations, namely (6.10), (6.15)-(6.17), (6.19)-(6.25), (6.35)-(6.40), (6.44), (6.45), (6.47), (6.48), (6.52), (6.53), (6.54), (6.55), (6.56), and (6.57). We use the same functional forms as in the model with money. Moreover, the nonstationary variables and the respective scaling factors are the same as in the model with money. The complete set of equilibrium conditions in terms of stationary variables is reported in Appendix C. On the one hand, a stationary competitive equilibrium is a set of stationary processes C_t, W_t, $\widetilde{W}_t$, Y_t, G_t, Φ_t, X_t^1, X_t^2, T_t, K_{t+1}, I_t, F_t^1, F_t^2, Q_t, R_t^k, Λ_t, p_t^f, p_t^b, $\widetilde{p}_t$, u_t, mc_t, h_t, h_t^d, s_t, $\widetilde{s}_t$, and π_t satisfying (6.10), (6.15)-(6.17), (6.19)-(6.25), (6.35)-(6.40), (6.44), (6.45), (6.47), (6.48), (6.52), (6.53), (6.54), (6.55), (6.56), and (6.57) written in terms of stationary variables, given I) exogenous stochastic processes for $\mu_{\Upsilon,t}$, $\mu_{z,t}$, and $\overline{g}_t$, II) the policy process R_t, and III) initial conditions c_{-1}, w_{-1}, s_{-1}, $\widetilde{s}_{-1}$, π_{-1}, i_{-1}, and κ_0. The equilibrium conditions in terms of stationary variables contain 26 equations and 26 variables, which are listed above. In addition the equilibrium conditions feature 26 parameters: ϕ_1, ϕ_2, ϕ_3, ϕ_4, θ, κ, b, β, δ, η, $\tilde{\eta}$, α, $\tilde{\alpha}$, ω, $\tilde{\chi}$, ψ, μ_I, μ_z, μ_Υ,σ_{μ_z}, σ_{μ_Υ}, ρ_{μ_z}, ρ_{μ_Υ}, ρ_G, σ_g, G. In order to obtain values for the steady-state levels of all variables and for the structural parameters, we thus need to impose 26 restrictions. These 26 restrictions come from using the same calibration in Schmitt-Grohé and Uribe (2007), which is reported in Table (6.1). In other words, as we do above for the model with money, we fix all structural parameters and we then find the steady-state levels of all variables as a function of the structural parameters. In particular, this entails that we only need to modify the specification of the steady state of the competitive equilibrium presented in the technical appendix to Schmitt-Grohé and Uribe (2007) (i.e. the one that obtains when all structural parameters are known).

On the other hand, the Ramsey equilibrium is formally defined as being a set of stationary processes C_t, W_t, $\widetilde{W}_t$, Y_t, G_t, Φ_t, X_t^1, X_t^2, T_t, K_{t+1}, I_t, F_t^1, F_t^2, Q_t, R_t^k, Λ_t, p_t^f, p_t^b, $\widetilde{p}_t$, u_t, mc_t, h_t, h_t^d, s_t, $\widetilde{s}_t$, π_t, and R_t for $t \geq 0$ that maximises

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(z_0^* \prod_{k=1}^{t} \mu_{z^*,s})^{(1-\phi_4)(1-\phi_3)} \left[\left(C_t - b\frac{C_{t-1}}{\mu_{z^*,t}} \right)^{1-\phi_4} (1-h)^{\phi_4} \right]^{1-\phi_3} - 1}{1-\phi_3}$$

subject to I) (6.10), (6.15)-(6.17), (6.19)-(6.25), (6.35)-(6.40), (6.44), (6.45), (6.47), (6.48), (6.52), (6.53), (6.54), (6.55), (6.56), and (6.57) written in terms of stationary variables, and II) $R_t \geq 1$ for $t > -\infty$ and given: I) the exogenous stochastic processes for $\mu_{\Upsilon,t}$,

$\mu_{z,t}$, and $\overline{g}_t$, II) values of the variables listed above dated $t < 0$ and III) values of the Lagrange multipliers associated with the constraints listed above dated $t < 0$.

Figure (6.4) shows the relationship between the optimal long-run rate of inflation and the degree of price stickiness. The optimal long-run inflation is observed to be positive. Specifically, the optimal long-run inflation is observed to spike for extremely low values of α, which are empirically unrealistic. Within the range of empirically realistic values of the degree of price stickiness, the optimal long-run inflation is instead small.

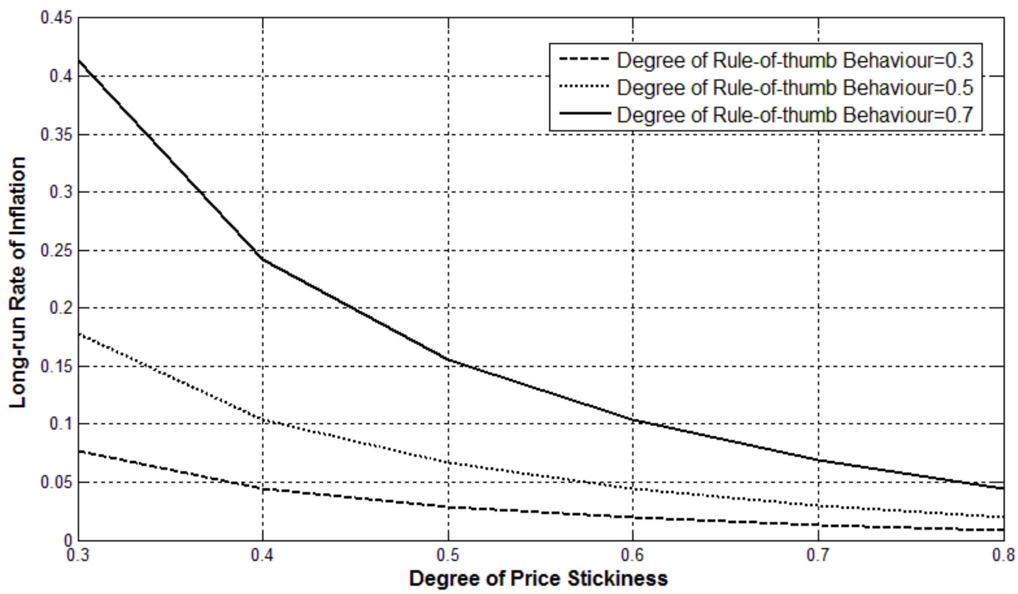

Figure 6.4. Cashless model: degree of price stickiness and the Ramsey steady-state rate of inflation.

As for the relationship between the optimal long-run rate of inflation and the degree of rule-of-thumb behaviour, Figure (6.5) shows how a larger fraction of firms resetting their prices in a backward-looking manner is associated with an increasingly positive optimal inflation rate.

We thus find that the Ramsey-optimal steady-state inflation in the cashless model with rule-of-thumb behaviour by price setters is positive. However, as found in the linear-quadratic framework, the inflation rate is still observed to be small. Moreover, the inflation

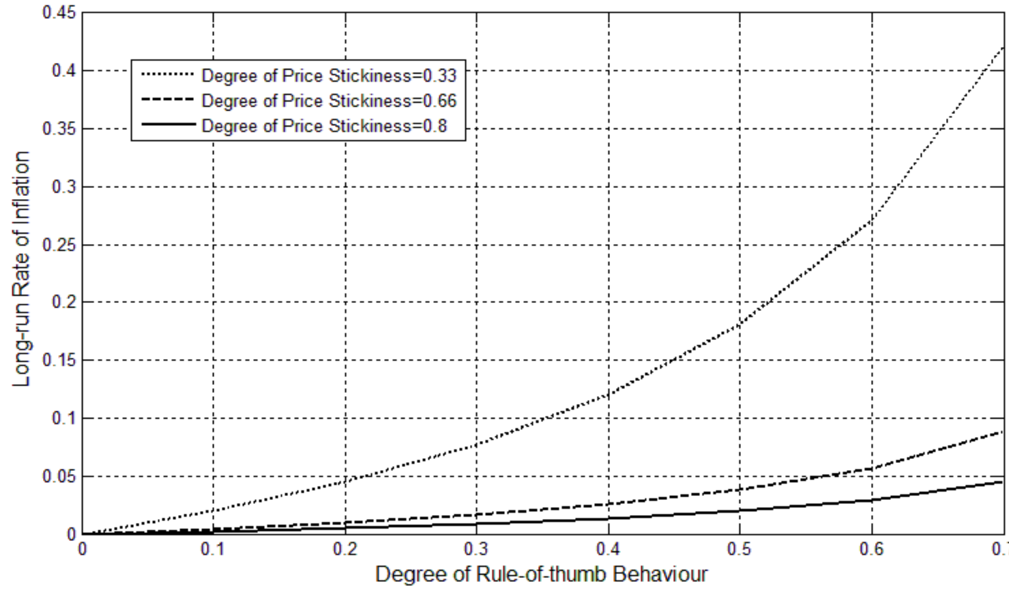

Figure 6.5. Cashless model: degree of rule-of-thumb behaviour and the Ramsey steady-state rate of inflation.

rate is again observed to be monotonically decreasing in the degree of price stickiness and monotonically increasing in the degree of rule-of-thumb behaviour.

6.4. The Social Planner Allocation

The social planner decides how to allocate the consumption and the production of goods within the economy regardless of the details of the price and wage mechanisms and the nature of the factors' markets and goods' markets.

The purpose of this analysis is twofold. First, in chapter 4 we have shown that in the linear-quadratic framework the steady-state inflation rate is directly proportional to the steady-state efficiency gap, which is the constant gap between the steady-state level of output and the efficient steady-state level of output. Solving the social planner's problem allows us to derive the efficient steady-state level of output. We subsequently compute its log-difference with the Ramsey steady-state level of output so to obtain a measure of the gap between the two steady-state levels of output. We find that this steady-state gap is only slightly larger than in the case of small steady-state distortions assumed in the linear quadratic framework. Specifically, while the steady-state efficiency gap in the linear-quadratic framework is equal to 0.2 under benchmark calibration, the steady-state efficiency gap in this model is found to be in the region of 0.26 both in the model with money and in its cashless counterpart. Moreover, we perform sensitivity analysis in terms of the degree of price stickiness and the degree of rule-of-thumb behaviour. The steady-state efficiency gap is observed to be highly stable with respect to the degree of rule-of-thumb behaviour and the degree of price stickiness. Indeed, the differences in the steady-state efficiency gap for varying degrees of rule-of-thumb behaviour and price stickiness are so small that the scale of the differences may be simply due to numerical approximation noises related to the numerical solution of the Ramsey steady-state level of output.

Within the economy developed in Altig et al. (2005) and presented above, the social planner decides how to allocate the consumption and the production of goods within the economy regardless of the details of the price and wage mechanisms and the nature of the factors' and goods' markets. Here, we report the equations that are relevant for the social planner problem. The social planner is constrained by the production technology

$$y_t = F\left(u_t k_t, z_t h_t\right) - \psi z_t^* \tag{6.58}$$

the aggregate resource constraint

$$y_t = c_t + g_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right) k_t\right] \tag{6.59}$$

and the law of motion of physical capital

$$k_{t+1} = \left(1-\delta\right) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{6.60}$$

Subject to these constraints, the social planner chooses processes for c_t, h_t, k_{t+1}, i_t, and u_t so to maximise the discounted sum of utility. The Lagrangian is then given by

$$\mathcal{L}^{SP} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} U\left(c_t - bc_{t-1}, h_t\right) + \lambda_t q_t \left[\left(1-\delta\right) k_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] - k_{t+1}\right] \\ +\lambda_t \left[F\left(u_t k_t, z_t h_t\right) - \psi z_t^* - c_t - g_t - \Upsilon_t^{-1}\left[i_t + a\left(u_t\right) k_t\right]\right] \end{array} \right\}$$

The first-order conditions are given by

$$\frac{\partial \mathcal{L}^h}{\partial c_t} = 0 \Rightarrow U_c\left(c_t - bc_{t-1}, h_t\right) - b\beta E_t U_c\left(c_{t+1} - bc_t, h_{t+1}\right) = \lambda_t \tag{6.61}$$

$$\frac{\partial \mathcal{L}^h}{\partial h_t} = 0 \Rightarrow -U_h\left(c_t - bc_{t-1}, h_t\right) = \lambda_t z_t F_2\left(u_t k_t, z_t h_t\right) \tag{6.62}$$

$$\frac{\partial \mathcal{L}^h}{\partial k_{t+1}} = 0 \Rightarrow \lambda_t q_t = \beta E_t \lambda_{t+1}[u_{t+1} F_1\left(u_{t+1} k_{t+1}, z_{t+1} h_{t+1}\right) - \Upsilon_{t+1}^{-1} a(u_{t+1}) + q_{t+1}\left(1-\delta\right)] \tag{6.63}$$

$$\frac{\partial \mathcal{L}^h}{\partial i_t} = 0 \Rightarrow \frac{\lambda_t}{\Upsilon_t} = \lambda_t q_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right) S'\left(\frac{i_t}{i_{t-1}}\right)\right] + \beta E_t \lambda_{t+1} q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right) \tag{6.64}$$

$$\frac{\partial \mathcal{L}^h}{\partial u_t} = 0 \Rightarrow k_t F_1\left(u_t k_t, z_t h_t\right) = \Upsilon_t^{-1} a'\left(u_t\right) k_t \tag{6.65}$$

The complete set of equilibrium conditions is given by 8 equations, namely (6.58)-(6.65). The solution to the social planner's problem is obtained in five steps. First, we need to specify functional forms for utility, technology, the investment adjustment cost function, and the cost of higher capacity utilisation. We use the same functional forms considered for the Ramsey steady state, which come from Schmitt-Grohé and Uribe (2007). Second, some variables are not stationary along the balanced-growth path as the economy displays two types of permanent shocks. This requires rescaling those variables so that the model's equilibrium conditions are functions of stationary variables only, which we denote with the corresponding capital letters. The nonstationary variables and the respective scaling factors are the same as in the Ramsey economy. Appendix D presents the functional forms and reports the complete set of equilibrium conditions in terms of stationary variables. The equilibrium conditions in terms of stationary variables contain 8 equations (not counting the law of motion of the exogenous variables) and 8 variables (not counting the exogenous variables): C_t, Y_t, K_{t+1}, I_t, Q_t, Λ_t, u_t, and h_t. The equilibrium conditions feature 11 parameters (ϕ_3, ϕ_4, γ_1, γ_2, θ, κ, b, β, δ, ψ, μ_I). The three exogenous processes feature 9 parameters (μ_z, μ_Υ,σ_{μ_z}, σ_{μ_Υ}, ρ_{μ_z}, ρ_{μ_Υ}, ρ_G, σ_g, G). This means that in order to obtain values for the efficient steady-state levels of all variables and deep structural parameters we need to impose 20 calibration restrictions, namely we have 8 equations and 28 between variables and parameters. The 20 restrictions come from imposing that all the deep structural parameters are identical to those that obtain in the competitive steady state of the theoretical economy, which are given Table (6.1). Given the 20 restrictions, we can proceed to analytically derive the steady state of the social planner equilibrium.

Under the restriction $\phi_3 = 1$, the nonstochastic steady state of the efficient equilibrium is described by

$$Y = \left(uK\mu_I^{-1}\right)^{\theta}(h)^{1-\theta} - \psi \tag{6.66}$$

$$Y = C + \left[I + a(u)\,K_t\mu_I^{-1}\right] + G \tag{6.67}$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right) \tag{6.68}$$

$$\phi_4 (1-h)^{-1} = \Lambda(1-\theta)(uK\mu_I^{-1})^{\theta}(h)^{-\theta} \tag{6.69}$$

$$(1-\phi_4)\left[C(1-b/\mu_{z^*})\right]^{-1}(1-b\beta\mu_\Lambda) = \Lambda \tag{6.70}$$

$$Q = \beta \frac{\mu_\Lambda}{\mu_\Upsilon} \left[\begin{array}{c} \theta \mu^\theta (K \mu_I^{-1})^{\theta-1} (h)^{1-\theta} \\ -a(u) + Q(1-\delta) \end{array} \right] \tag{6.71}$$

$$Q = 1 \tag{6.72}$$

$$\theta (u K \mu_I^{-1})^{\theta-1} (h)^{1-\theta} = \gamma_1 \tag{6.73}$$

Equation (6.72) determines Q. Substituting (6.72) and (6.73) in (6.71), delivers

$$1 = \beta \frac{\mu_\Lambda}{\mu_\Upsilon} \left[\mu \gamma_1 - a(u) + (1-\delta) \right] \tag{6.74}$$

In the competitive steady state, the numerical value for the parameter γ_1 was chosen such that $1 = \beta \frac{\mu_\Lambda}{\mu_\Upsilon} \left[\gamma_1 + (1-\delta) \right]$. Hence, equation (6.74) entails that $u = 1$: in both the steady state of the efficient equilibrium and the competitive steady state capacity utilisation is full. On the one hand, the steady-state values for Q and u in the steady state of the efficient equilibrium are the same that obtain in the competitive steady state. On the other hand, the steady-state values for C, Y, K, I, Λ, and h differ as the social planner is not constrained by the details of the price and wage mechanisms and the nature of the factors' and goods' markets. To determine the steady-state values for C, Y, K, I, Λ, and h we have 6 equations, namely (6.66)-(6.70) and (6.73), which under $u = 1$ take the form

$$Y = \left(K \mu_I^{-1} \right)^\theta (h)^{1-\theta} - \psi \tag{6.75}$$

$$Y = C + I + G \tag{6.76}$$

$$I = K\left(1 - \frac{(1-\delta)}{\mu_I}\right) \tag{6.77}$$

$$\phi_4 (1-h)^{-1} = \Lambda(1-\theta)(K\mu_I^{-1})^{\theta}(h)^{-\theta} \tag{6.78}$$

$$(1-\phi_4)\left[C(1-b/\mu_{z^*})\right]^{-1}(1-b\beta\mu_\Lambda) = \Lambda \tag{6.79}$$

$$\theta(K\mu_I^{-1})^{\theta-1}(h)^{1-\theta} = \gamma_1 \tag{6.80}$$

Rewriting equation (6.80), the capital to labour ratio is given by

$$\frac{K}{h} = \left(\frac{\gamma_1}{\theta}\right)^{\frac{1}{\theta-1}} \mu_I \tag{6.81}$$

Accordingly, equations (6.75) and (6.78) can be rewritten as

$$Y = \left(\frac{\gamma_1}{\theta\mu_I}\right) K - \psi \tag{6.82}$$

$$\phi_4 (1-h)^{-1} = \Lambda(1-\theta)\left(\frac{\gamma_1}{\theta}\right)^{\frac{\theta}{\theta-1}} \tag{6.83}$$

Substituting equation (6.79) in equation (6.83) and solving for C yields

$$C = \frac{(1-\phi_4)(1-b\beta\mu_\Lambda)(1-\theta)\left(\frac{\gamma_1}{\theta}\right)^{\frac{\theta}{\theta-1}}}{\phi_4(1-b/\mu_{z^*})}(1-h) \tag{6.84}$$

Using (6.81), the steady-state consumption can be rewritten as

$$C = \frac{(1-\phi_4)(1-b\beta\mu_\Lambda)(1-\theta)\left(\frac{\gamma_1}{\theta}\right)^{\frac{\theta}{\theta-1}}}{\phi_4(1-b/\mu_{z^*})}\left(1 - K\left(\frac{\gamma_1}{\theta}\right)^{-\frac{1}{\theta-1}} \mu_I^{-1}\right) \tag{6.85}$$

Substituting equations (6.77), (6.82), and (6.85) into equation (6.76), we note that we can now find K as a function solely of the deep structural parameters

$$
\begin{aligned}
(6.86)\ \left(\frac{\gamma_1}{\theta\mu_I}\right)K-\psi \;\; &= \;\; \frac{(1-\phi_4)(1-b\beta\mu_\Lambda)(1-\theta)\left(\frac{\gamma_1}{\theta}\right)^{\frac{\theta}{\theta-1}}}{\phi_4(1-b/\mu_{z^*})}(1-K\left(\frac{\gamma_1}{\theta}\right)^{-\frac{1}{\theta-1}}\mu_I^{-1}) \\
&\quad +K\left(1-\frac{(1-\delta)}{\mu_I}\right)+G
\end{aligned}
$$

Solving for K delivers

$$
(6.87)\quad K=\frac{(1-\phi_4)(1-b\beta\mu_\Lambda)(1-\theta)\left(\frac{\gamma_1}{\theta}\right)^{\frac{\theta}{\theta-1}}+(G+\psi)\phi_4(1-b/\mu_{z^*})}{(1-\phi_4)(1-b\beta\mu_\Lambda)(1-\theta)\mu_I^{-1}+\phi_4(1-b/\mu_{z^*})(\gamma_1\theta^{-1}\mu_I^{-1}-1+(1-\delta)\mu_I^{-1})}
$$

Equation (6.87) gives the steady-state value of the level of capital in the social planner equilibrium as a function solely of the deep structural parameters. Given K, the steady-state values of C, Y, I, and h are respectively given by equations (6.85), (6.82), (6.77), and (6.81). Given C, the steady-state value of Λ is given by equation (6.79).

We proceed to compare the log-difference between the level of output in the social planner steady state and the level of output in the Ramsey steady state. We consider both the cashless economy and the economy with money. Figure (6.6) and Figure (6.7) plot the steady-state output gap in the model with money for alternative values of the degree of price stickiness and the degree of rule-of-thumb behaviour. Figure (6.8) and Figure (6.9) plot the steady-state output gap in the cashless model for alternative values of the degree of price stickiness and the degree of rule-of-thumb behaviour.

There are two main points to take from these figures. First, the steady-state output gap is larger than its equivalent in the linear-quadratic framework, which is obtained under the assumption of small steady-state distortions, although the difference between the two is observed to be small. Specifically, the steady-state output gap is observed to be in the region of 0.26 whereas the steady-state output gap in the linear-quadratic framework, as given in equations (2.51) and (2.52), is found to be equal to 0.2. Second, the steady-state efficiency gap is observed to be highly stable with respect to both the degree of

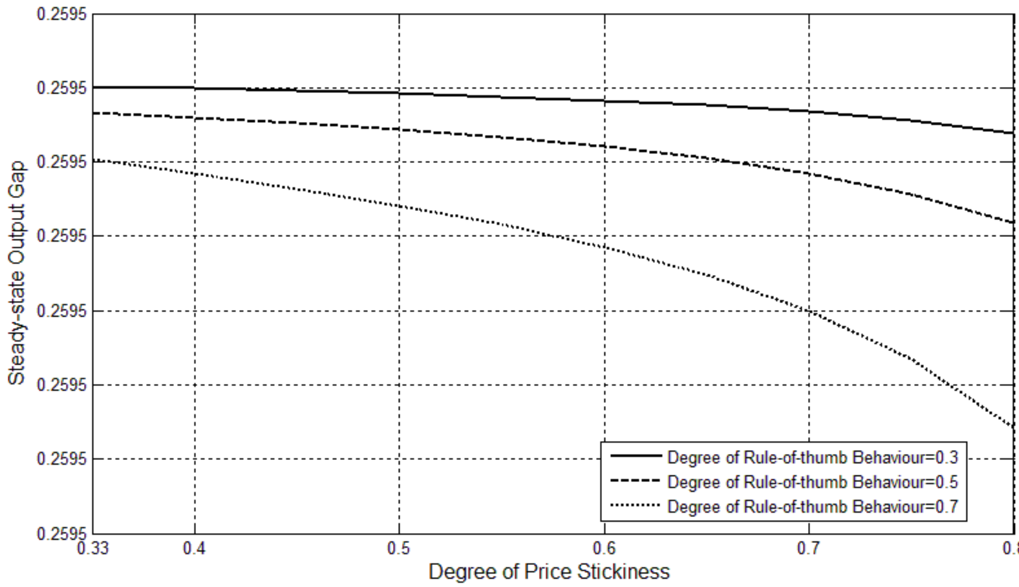

Figure 6.6. Model with money: the steady-state output gap for alternative values of the degree of price stickiness.

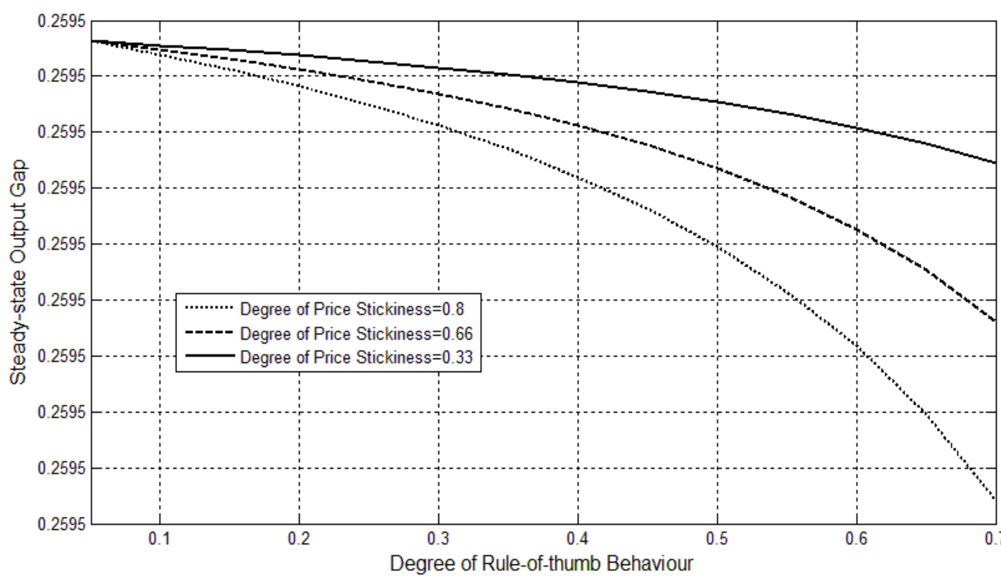

Figure 6.7. Model with money: the steady-state output gap for alternative values of the degree of rule-of-thumb behaviour.

rule-of-thumb behaviour and the degree of price stickiness. Indeed, the differences in the steady-state efficiency gap for varying degrees of rule-of-thumb behaviour and price stickiness are so small that the scale of the differences may be simply due to numerical

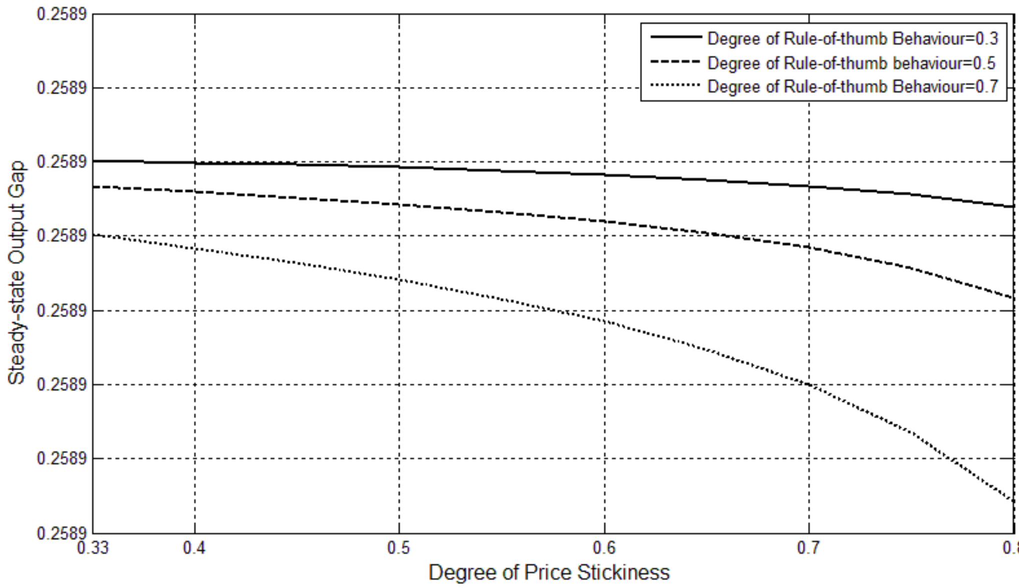

Figure 6.8. Cashless model: the steady-state output gap for alternative values of the degree of price stickiness.

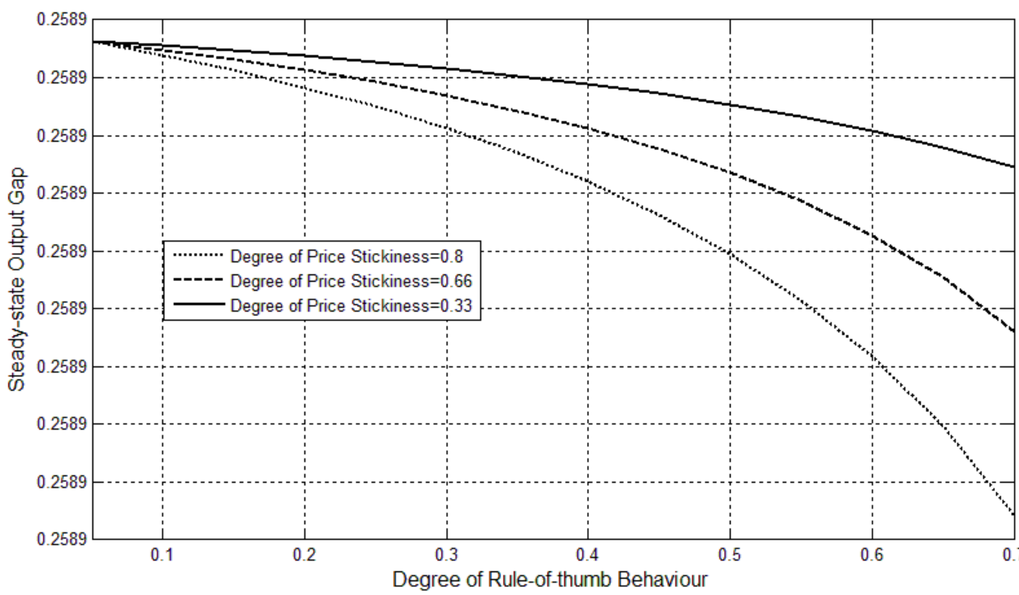

Figure 6.9. Cashless model: the steady state output gap for alternative values of the degree of rule-of-thumb behaviour.

approximation noises related to the numerical solution of the Ramsey steady-state level of output.

On the one hand, as shown in Chapter 4, the inflation rates in the linear-quadratic framework are directly proportional to the steady-state output gap. On the other hand, the steady-state output gap in the medium-scale economy is found to be only slightly larger that its counterpart in the linear-quadratic framework. This could explain the reason as to why the inflation rate in the cashless medium-scale model is again positive but only slightly differs from the one that obtains in the linear-quadratic framework. Indeed, the steady-state level of inflation in the Ramsey equilibrium is greater than the the optimal steady state-state inflation that obtains in the linear-quadratic framework, as given by (4.24), albeit the difference between the two inflation rates is small.

CHAPTER 7

Ramsey Dynamics and Optimal Operational Interest-rate Rules

In this chapter, we study Ramsey dynamics and address the question of implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules.

First, we study the business cycle dynamics that arise in the stochastic steady state of the Ramsey equilibrium. In doing so, we are interested in addressing two issues. First, we want to assess whether the zero lower bound on the nominal interest rate constitutes an impediment to optimal monetary policy. Indeed, one argument against setting a negative inflation rate, as recommended by the model in the presence of money demand by households and firms, or a near-zero inflation rate, as recommended by the cashless model, is that at negative or near-zero rates of inflation the risk of incurring in the zero lower bound on nominal interest rate would restrict the central bank's ability to stabilise the economy. We find that this argument is of no relevance in the context of both the model with money and its cashless counterpart. The reason for this is that under the Ramsey-optimal policy, the zero lower bound poses an impediment to monetary policy only in the case of an adverse shock that forces the interest rate to be more that 8 standard deviations below its mean. The probability of this happening is so small that, as in Schmitt-Grohé and Uribe (2007), the zero lower bound on the nominal interest rate does not impose an economically important constraint on the conduct of optimal monetary policy. We proceed to characterise the Ramsey-optimal responses to the three shocks that drive aggregate fluctuations: the permanent neutral technological shock, the permanent investment-specific technological shock, and temporary variations in government expenditure. Specifically, we present the responses of key macroeconomic variables and we focus on how the Ramsey planner uses monetary policy to respond to each of the three shocks. We show how the

Ramsey-optimal stabilisation policy is robust to the presence/absence of money in the model.

Second, we study the implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules. Insofar as equilibrium distortions are neither nil nor small, it no longer suffices to approximate the equilibrium of the model to first order to obtain a second-order accurate measure of welfare. We use the methodology and the algorithm developed in Schmitt-Grohé and Uribe (2004b) for second-order approximations to policy functions of dynamic and stochastic models. Initially, we show how the implementation of optimal monetary policy is virtually unaffected by the presence-absence of money. We characterise the operational interest-rate rule, which is defined exactly as in Schmitt-Grohé and Uribe (2007), in both the medium-scale model with money and its cashless counterpart. In both cases, the optimal operational interest-rate rule is confirmed to be active in price and wage inflation, mute in output growth and moderately inertial. However, in the cashless economy the coefficients on price and wage inflation are greater than in the model that features transactional frictions.

Finally, we also consider a modification of the operational interest-rate rule, which prescribes a concern not for output growth per se but for stabilisation of output around a welfare-relevant measure of output, namely the gap between the Ramsey level of output and the efficient level of output. We find that the optimal operational interest-rate rule remains active in price and wage inflation and moderately inertial, but also implies a positive coefficient on the output gap. This is true in both the model with money and its cashless counterpart.

We first characterise Ramsey dynamics in both the model with money and its cashless counterpart. We then proceed to study optimal operational interest-rate rules.

7.1. Ramsey Dynamics

In this section, we describe the business cycle dynamics that arise in the stochastic steady state of the Ramsey equilibrium, namely the Ramsey dynamics. The Ramsey dynamics are approximated by solving a first-order approximation to the Ramsey equilibrium conditions. The literature provides evidence that first-order approximations to the Ramsey equilibrium conditions deliver dynamics that are close to those implied by the exact solution. For instance, Schmitt-Grohé and Uribe (2004c) compute the exact solution to the Ramsey dynamics in a model characterised by flexible prices and monopolistic competition. Schmitt-Grohé and Uribe (2004b) instead compute Ramsey dynamics in the same model using a first-order approximation to the Ramsey equilibrium conditions. They conclude that the dynamics implied by the exact solution and the ones obtained under the first-order approximation are not significantly different. Moreover, Benigno and Woodford (2006) show that, within the context of optimal taxation in the standard Real Business Cycle model, the first-order approximation to the Ramsey equilibrium conditions implies second moments that are very similar to the second moments computed from the exact solution.

In addressing the characterisation of Ramsey dynamics we are concerned with two issues. First, we want to study how the Ramsey planner resolves the stabilisation of volatility in the endogenous variables. In doing so, we want to analyse whether the zero lower bound on nominal interest rate imposes an economically important constraint on the conduct of monetary policy. With this respect, an argument against setting a negative inflation rate, as recommended by the model in the presence of money demand by households and firms, or a near-zero inflation rate, as recommended by the cashless model, is that at negative or near-zero rates of inflation the risk of incurring in the zero lower bound on nominal interest rate would restrict the central bank's ability to stabilise the economy. This argument is in fact advocated by Summers (1991), among others, as the main reason for setting a positive inflation rate. Second, we want to characterise the Ramsey-optimal responses to the three shocks that drive aggregate fluctuations: the permanent neutral technological shock, the permanent investment-specific technological shock, and temporary variations in government expenditure. Specifically, we focus on how the Ramsey planner uses monetary policy to respond to each of the three shocks. In addressing these questions we consider both the model with money and its cashless counterpart[1].

In the model with money, the Ramsey planner faces a three-way trade-off in determining the optimal degree of volatility in the endogenous variables. First, the distortion due to sticky prices calls for minimising inflation volatility. Second, the distortion due to transactional frictions calls for minimising volatility in the nominal interest rate. Third, the distortion due to sticky wages implies that minimisation of wage inflation net of lagged price inflation, given full indexation to past price inflation, is also Ramsey optimal. Table (7.1) reports the standard deviations, measured in percentage point per year, of some macroeconomic variables under the Ramsey-optimal policy in the model with money. Table (7.1) shows that the three-way trade-off is resolved in favour of price stability. Indeed, inflation volatility is observed to be much smoother than any other endogenous variable's volatility over the business cycle. Moreover, in order to assess how sensitive inflation stability is with respect to the size of the sticky wage distortions, we also consider the case of a higher degree of wage stickiness (i.e. $\widetilde{\alpha} = 0.9$), which implies that wages are reoptimised only every ten quarters. In this case, as shown in column 2 of Table (7.1), the optimal volatility of price inflation increases and the optimal volatility of wage inflation decreases. However, price inflation remains much smoother over the business cycle than wage inflation. In this sense, the Ramsey pursues a policy of inflation targeting. As for the importance of the zero lower bound on nominal interest rate, Table (7.1) shows that the standard deviation of the nominal interest rate is 0.4 percentage points at an annual rate. On the other hand, the mean of the nominal interest rate in the Ramsey stochastic steady state is 3.4 percent. This implies that for the nominal interest rate to reach the zero lower bound it must fall by more than 8 standard deviations below its target level. The likelihood of this taking place is so small that within the theoretical economy, the

[1]All structural parameters of the model take the values shown in Table (6.1) and we consider a degree of price stickiness of 0.66 and a degree of rule-of-thumb behaviour of 0.3.

zero lower bound on the nominal interest rate does not impose an economically strong constraint on the conduct of optimal monetary policy.

Variable	$\alpha = 0.66 \;\; \widetilde{\alpha} = 0.69$	$\alpha = 0.66 \;\; \widetilde{\alpha} = 0.9$
Nominal interest rate	0.4	0.4
Price Inflation	0.2	0.4
Wage Inflation	1.1	0.9
Output Growth	0.7	0.7
Consumption Growth	0.4	0.4
Investment Growth	1.2	1.4

Table 7.1. Model with Money: Standard Deviations under the Ramsey Optimal Stabilisation Policy

The same message carries on to the case of a cashless economy. The Ramsey planner now faces a two-way trade-off as the absence of the distortion due to transactional frictions implies that minimising volatility of the nominal interest rate is not Ramsey optimal. Table (7.2) reports the standard deviations, measured in percentage point per year, of inflation and other macroeconomic variables under the Ramsey-optimal stabilisation policy. On the one hand, the table shows that the standard deviation of the nominal interest rate is 0.3 percentage points at an annual rate. On the other hand, the mean of the nominal interest rate in the Ramsey stochastic steady state is 2.8 percent. This implies that for the nominal interest rate to reach the zero lower bound it must fall by more than 8 standard deviations below its target level. Hence, the zero lower bound on the nominal interest rate does not impose a strong constraint in the conduct of monetary policy. Moreover, the policy trade-off faced by the Ramsey planner is again resolved in favour of price stability. In the case of a higher degree of wage stickiness, as shown in column 2 of Table (7.2), the optimal volatility of price inflation increases and the optimal volatility of wage inflation decreases. However, price inflation remains much smoother over the business cycle than wage inflation.

Variable	$\alpha = 0.66 \quad \widetilde{\alpha} = 0.69$	$\alpha = 0.66 \quad \widetilde{\alpha} = 0.9$
Nominal interest rate	0.35	0.35
Price Inflation	0.1	0.3
Wage Inflation	1.2	1.0
Output Growth	0.6	0.6
Consumption Growth	0.4	0.4
Investment Growth	1.1	1.3

Table 7.2. Cashless Model: Standard Deviations under the Ramsey Optimal Stabilisation Policy

We now proceed to characterise the Ramsey-optimal impulse responses to the three shocks that drive aggregate fluctuations: the permanent neutral technological shock, the permanent investment-specific technological shock, and temporary variations in government expenditure. Specifically, we present the responses of key macroeconomic variables and we focus on how the Ramsey planner uses monetary policy to respond to each of the three shocks. We consider both the model with money and its cashless counterpart. In characterising impulse response functions, we follow Schmitt-Grohè and Uribe (2007). The nominal interest rate and the inflation rate are expressed in levels in percent per year. Output, wages, investment, and consumption are expressed in cumulative growth rates in percent. Hours and capacity utilisation are expressed in percentage deviations from their steady-state values. The two models indeed imply very similar stabilisation policies. Moreover, the dynamic responses in both economies closely resemble the ones presented in Schmitt-Grohé and Uribe (2007), who consider backward-looking price indexation.

Figure (7.1) and Figure (7.2) show the response to a one percentage increase in the growth rate of the neutral technology shock in the model with money and its cashless counterpart respectively. The Ramsey planner raises nominal interest rate and allows inflation to fall. By doing so, the Ramsey planner is trying to replicate the allocation that would prevail in an economy characterised by flexible prices and flexible wages. In a flexible-price and flexible-wage economy, the real interest rate should rise in response to a positive shock to the growth rate of technology. When prices are sticky, the Ramsey planner finds optimal to induce a rise in the real interest rate without having to rely on costly movements in the rate of inflation. Since the real interest rate is simply the difference between the nominal interest rate and the inflation rate, the Ramsey-optimal policy is to raise nominal interest rates by more or less the same amount that real interest rates would rise in the flexible-price and flexible-wage economy. This is true in both the model with money and its cashless counterpart, although the nominal interest rate is tightened to avoid deflation in the model with money whereas is tightened to avoid inflation in the cashless model. A permanent increase in technology raises the demand for capital and labour. Specifically, hours decline on impact in response to a permanent increase in neutral technology. This is line with the finding in Galì (1999) whereas it runs contrary to what found in Altig et al. (2005). The intuition for the initial decline in hours is linked to the Ramsey planner's intent of replicating the allocation that would prevail in the flexible-price and flexible-wage economy. Given that the Ramsey planner finds optimal to induce a strong increase in the real interest rate on impact, the initial increase in consumption is small. It follows that, at least initially, the positive wealth effect brought about by the increase in technology results in an increase in consumption of leisure. After the initial period, higher real wages make households substitute current work for future leisure. Capital is used more intensively which generates additional changes in hours given that the two factors are complementary in the production of goods. Overall, both models are capable of explaining a strong rise in equilibrium hours, output, consumption, investment and real wage.

Figure (7.3) and Figure (7.4) show the response to a one percent innovation in government expenditure in the model with money and its cashless counterpart respectively. Ramsey-optimal policy calls for a contraction in monetary policy, which is in line with conventional wisdom, and implies a higher nominal interest rate that reverts the initial increase in the rate of inflation. An exogenous and temporary increase in government

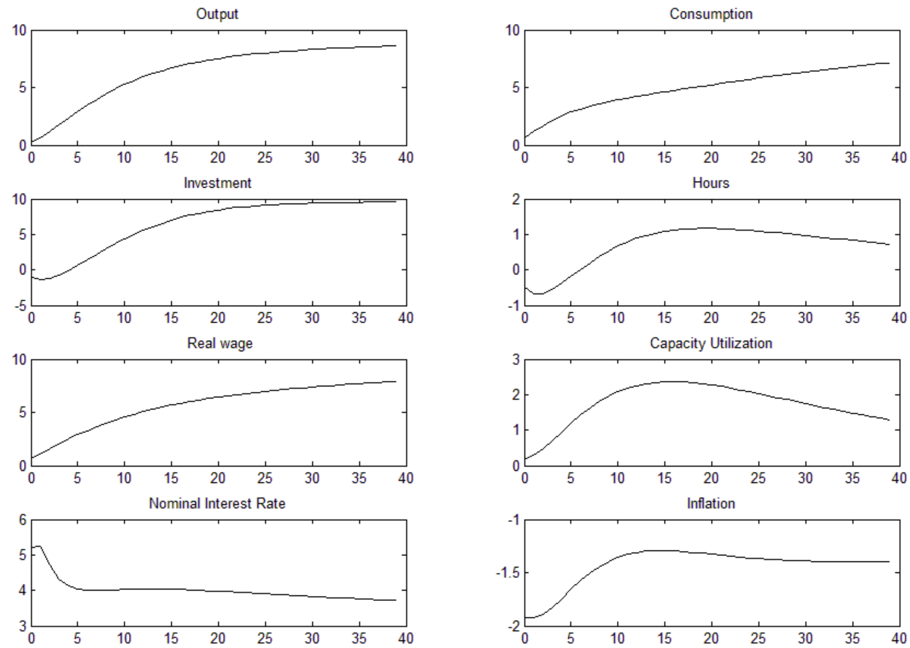

Figure 7.1. Model with money: Ramsey response to a neutral productivity shock.

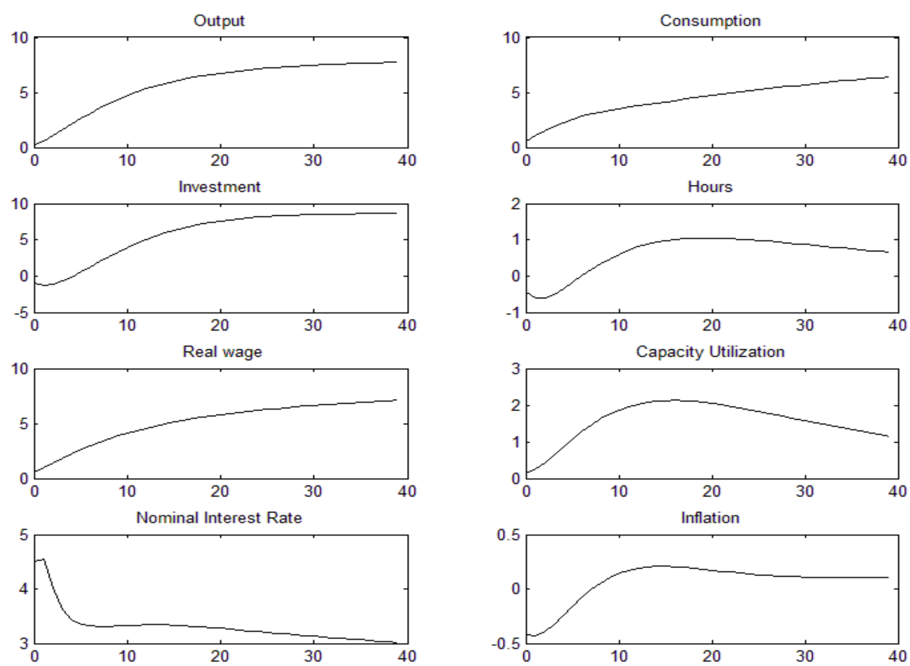

Figure 7.2. Cashless model: Ramsey response to a neutral productivity shock.

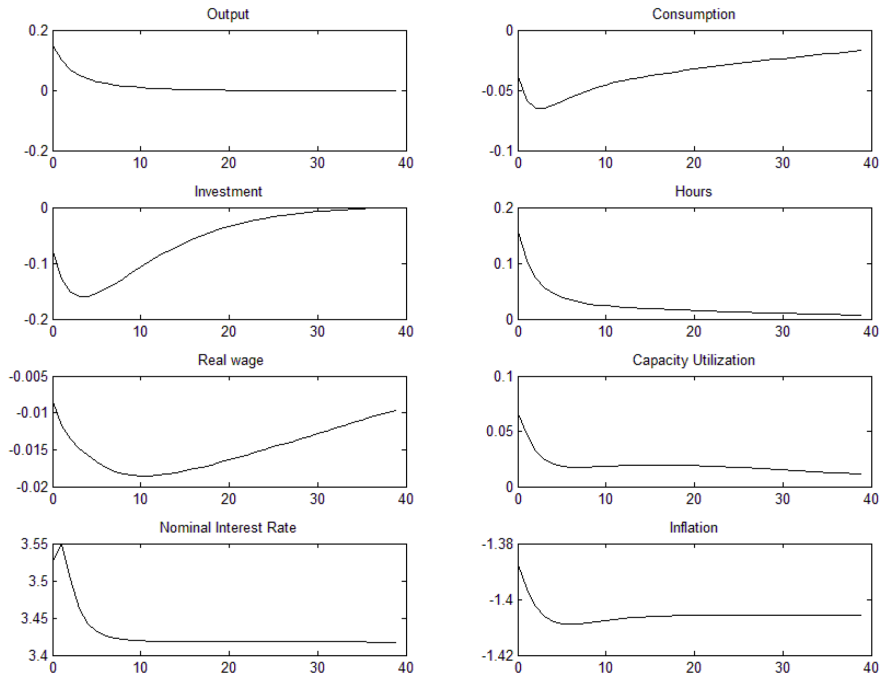

Figure 7.3. Model with money: Ramsey response to a government expenditure shock.

expenditure reduces the present value of households' after-tax income which results in an increase in the labor supply which leads to temporarily higher equilibrium employment and output and a decrease in real wage. However, there is still a crowding out effect of private consumption and investment. Given the complementarity between capital and labour in production, the increase in hours determines an increase in the degree of capital utilisation.

Figure (7.5) and Figure (7.6) show the response to a one percentage point increase in the growth rate of the investment-specific technological shock in the model with money and its cashless counterpart respectively. The Ramsey planner finds optimal to ease monetary policy. The intuition is that the Ramsey planner replicates what would prevail in the flexible-price and flexible-wage economy. In the absence of stickiness in prices and wages, the real interest rate would fall. The Ramsey planner thus lowers nominal rates so to achieve a fall in real rates without putting pressure on the rate of inflation. This is true in both the model with money and its cashless counterpart. A permanent increase in

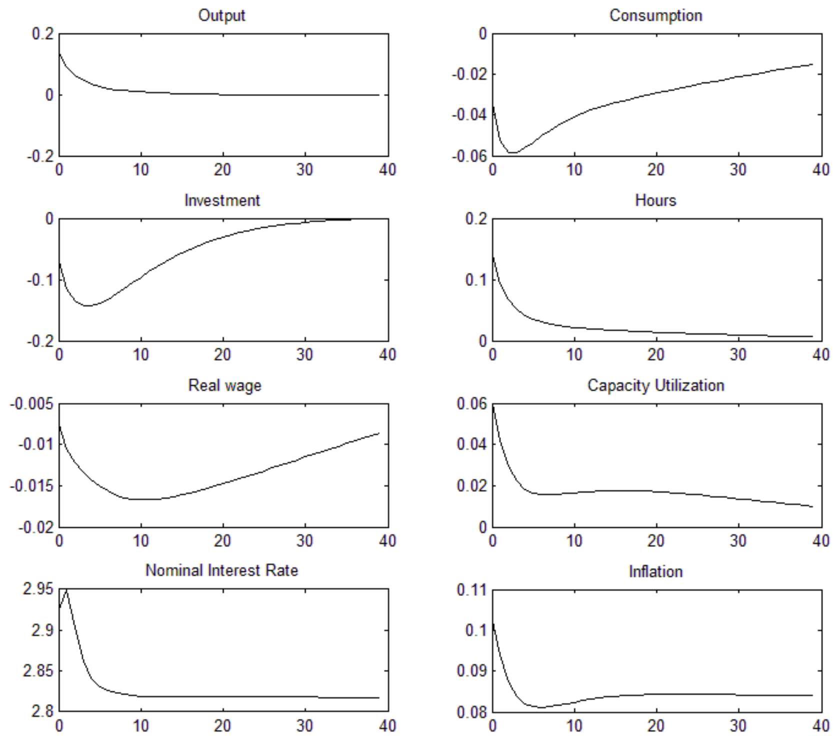

Figure 7.4. Cashless model: Ramsey response to a government expenditure shock.

investment-specific technology raises the demand for capital and labour. Hours rise on impact in response to a permanent increase in investment-specific technology. On impact, the increase in investment-specific technology results in a decrease in consumption and investment. Capital is used more intensively which generates additional changes in hours given that the two factors are complementary in the production of goods. Overall, both models are capable of explaining a strong rise in equilibrium hours, output, consumption, investment and real wage.

7.2. Optimal Operational Interest-rate Rules

In this section, we analyse the issue of the implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules. On the one hand, the solution to the Ramsey problem defines the equilibrium behaviour of the policy variable, namely the nominal interest rate. On the other hand, the equilibrium process of the

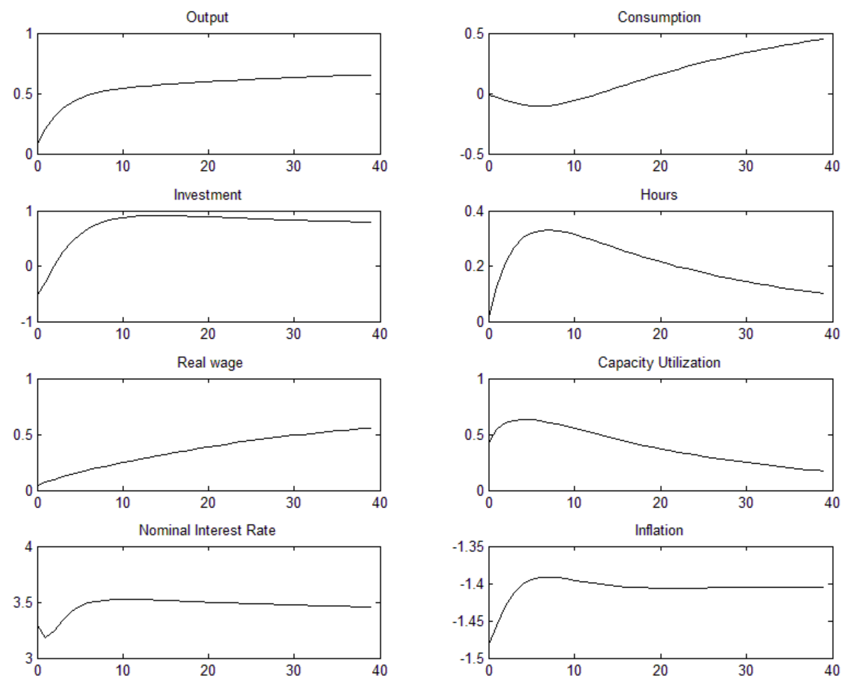

Figure 7.5. Model with money: Ramsey response to an investment-specific productivity shock.

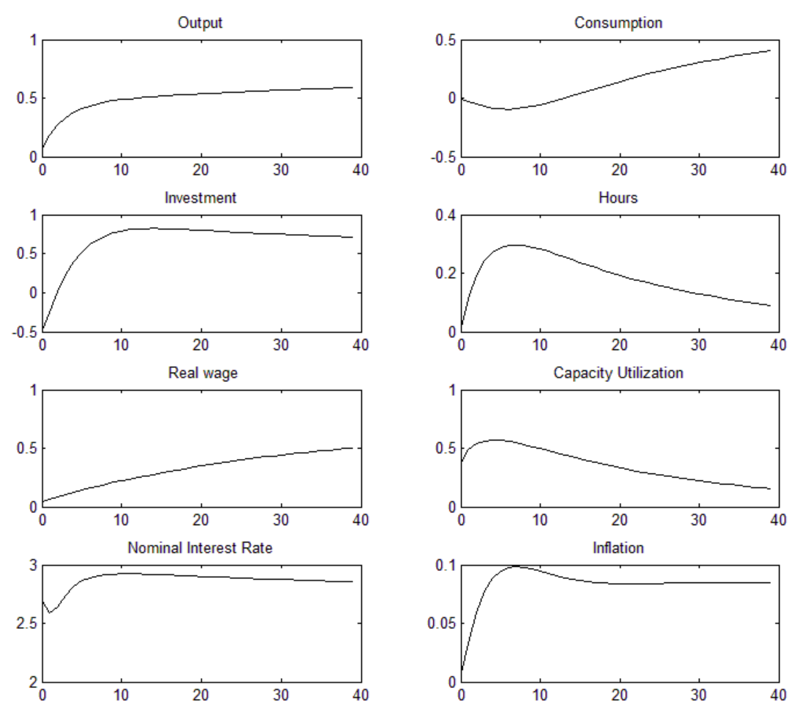

Figure 7.6. Cashless model: Ramsey response to an investment-specific productivity shock.

nominal interest rate depends on all the state variables of the Ramsey equilibrium, which include all the exogenous driving forces and all the endogenous predetermined variables. Most of these variables, such as the past values of the Lagrange multipliers associated with the constraints of the Ramsey problem, are not readily available nor easily controllable in reality. Moreover, even if the policy maker could observe all these variables, using the equilibrium process of the nominal interest rate as the policy regime could bring about multiplicity of equilibria.

The first example of operative rules for the nominal interest rate is the so-called Taylor Rule, introduced in the seminal paper of Taylor (1993). Much of the recent literature has concentrated on simple instrument rules that can be considered extensions of the original specification by Taylor[2]. In our analysis we first follow Schmitt-Grohé and Uribe (2007) as we characterise simple interest-rate feedback rules of the form

$$\ln\left(\frac{R_t}{R^*}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi^*}\right) + \alpha_W \ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) + \alpha_y \ln\left(\frac{y_t}{y_{t-1}\mu_y^*}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R^*}\right) \tag{7.1}$$

The nominal interest rate thus depends linearly on its own lag, price inflation, wage inflation, and the growth rate of output. The target values R^*, π^*, π^{W*} and μ_y^* are postulated to be the Ramsey steady-state values of the respective endogenous variables. Beyond simplicity, as in Schmitt-Grohé and Uribe (2007), we also require for the the rule to be operational. That is, the rule must induce a unique rational expectation equilibrium and the associate path for the nominal interest rate rule must not violate the zero bound[3]. Moreover, we study optimal operational interest-rate rule as we look for rules that maximise welfare. In other words, the four policy parameters (α_π, α_W, α_y, α_R) are chosen so as to maximise welfare. Welfare is given by the conditional expected value of the representative household lifetime utility. Specifically, welfare is conditional on the initial state

[2]For instance, among others, McCallum and Nelson (1999) and Taylor (1999). Clarida et al. (1998) show how Taylor rules provide a good approximation for the actual conduct of monetary policy over the past forty years.

[3]Specifically we follow Schmitt-Grohé and Uribe (2005) requiring the target value for the nominal interest rate be greater than two times the standard deviation of the nominal interest rate (formally $ln(R^*) \geq 2\sigma_{\hat{R}_t}$).

of the economy being the Ramsey steady state. We solve the model with the perturbation method, and the algorithm that implements it, developed in Schmitt-Grohé and Uribe (2004b). The algorithm approximates the conditional welfare measure to second-order accuracy by using a second-order approximation to the policy function.

We consider both the model with money and the cashless model. With this respect, it is important to stress that the target values in the two rules are different, as discussed in the previous sections[4]. The optimal operational interest-rate rule in the model with money is of the form

$$(7.2) \quad \ln\left(\frac{R_t}{R^*}\right) = 4.98\ln\left(\frac{\pi_t}{\pi^*}\right) + 1.63\ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) - 0.13\ln\left(\frac{y_t}{y_{t-1}\mu_y^*}\right) + 0.43\ln\left(\frac{R_{t-1}}{R^*}\right)$$

The policy parameters are observed to be nearly identical to the ones reported in Schmitt-Grohé and Uribe (2007), namely ($\alpha_\pi = 5$, $\alpha_W = 1.6$, $\alpha_y = -0.1$, $\alpha_R = 0.4$). The rule displays a greater-than-unity response to both price and wage inflation (i.e. it is an active rule) and basically no response to the growth rate of output. Furthermore, it prescribes that the optimal policy is inertial, since the autoregressive parameter on the lagged value of the interest rate is significantly greater than zero but lower then one. As stressed in Schmitt-Grohé and Uribe (2007) "*the optimal interest-rate rule can indeed be interpreted as a pure inflation targeting rule*". The same result obtains in the cashless economy. In this case the optimal operational interest-rate rule is given by

$$(7.3) \quad \ln\left(\frac{R_t}{R^*}\right) = 11.35\ln\left(\frac{\pi_t}{\pi^*}\right) + 4.00\ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) - 0.10\ln\left(\frac{y_t}{y_{t-1}\mu_y^*}\right) + 0.48\ln\left(\frac{R_{t-1}}{R^*}\right)$$

There are virtually no differences between the coefficients on the output gap and on lagged interest rate of this rule and the one in the model with money. However, the coefficients on price and wage inflation are greater than their counterparts shown in (7.2). In other

[4]In characterising the optimal operational interest-rate rules we set the degree of price stickiness, α, to 0.66 and the degree of rule-of-thumb behaviour, ω, to 0.3. The interest-rate rules are robus to alternative specifaction for α and ω. Specifically, we have considered combining limiting values for ω (i.e. $\omega = 0.01$ and $\omega = 0.7$) with limiting values for α (i.e. $\alpha = 0.3$ and $\alpha = 0.8$). The coefficients in the rule change slightly, but the rule is always found to be active in price and wage inflation, mute in output growth and moderately inertial.

words, the message given by these rules is the same (i.e. a pure inflation targeting rule), with a stronger evidence in the case of the cashless economy[5].

Finally, we consider optimal operational interest-rate rules that replace output growth with a measure of output stabilisation. Specifically, we consider the possibility of an interest-rate rule that replaces the growth rate of output with a measure of output gap, namely the log-difference between the level of output in the Ramsey economy and the social planner level of output. In other words, the interest-rate rule prescribes that the nominal interest rate depends linearly on the log-difference between the Ramsey level of output and the social planner level of output, denoted with y_t^{SP}.

$$\ln\left(\frac{R_t}{R^*}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi^*}\right) + \alpha_W \ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) + \alpha_y \ln\left(\frac{y_t}{y_t^{SP}}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R^*}\right) \tag{7.4}$$

The optimal operational interest-rate rule in the model with money is of the form

$$\ln\left(\frac{R_t}{R^*}\right) = 4.65\ln\left(\frac{\pi_t}{\pi^*}\right) + 1.55\ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) + 0.17\ln\left(\frac{y_t}{y_t^{SP}}\right) + 0.42\ln\left(\frac{R_{t-1}}{R^*}\right) \tag{7.5}$$

whereas the optimal operational interest-rate rule in the cashless model is given by

$$\ln\left(\frac{R_t}{R^*}\right) = 4.87\ln\left(\frac{\pi_t}{\pi^*}\right) + 1.61\ln\left(\frac{\pi_t^W}{\pi^{W*}}\right) + 0.19\ln\left(\frac{y_t}{y_t^{SP}}\right) + 0.44\ln\left(\frac{R_{t-1}}{R^*}\right) \tag{7.6}$$

There are two main observations to take from the interest-rate rules (7.5) and (7.6). First, the optimal policy reacts positively to the gap between the level of output in the Ramsey economy and the social planner level of output. Regardless of the presence/absence of money in the model, the optimal interest-rate rule remains active in both price and wage inflation and inertial, but also prescribes a concern for stabilising the level of output around the efficient level of output. Second, the rules are virtually identical across the models. In other words, if demand of money is seen to matter when the interest-rate rule implies a concern for the rate of growth of output, the same is not true once the measure of output stabilisation that enters the operational interest-rate rule is

[5]Note however that the relative importance of price and wage inflation in the two rules, i.e. the ratio α_π/α_w, is almost the same.

the gap between the level of output in the Ramsey economy and the social planner level of output.

CHAPTER 8

Conclusions

The main contribution of this thesis is the investigation of the effect of inflation persistence due to rule-of-thumb behaviour by price setters on optimal monetary policy. The analysis takes place in New Keynesian models where the supply-side of the economy is characterised by monopolistically competitive firms that face rigidity in the setting of prices. A fraction of price setters does not behave rationally when setting a new price, but follows a rule-of-thumb. This introduces persistence in the evolution of inflation. The effects of inflation persistence due to rule-of-thumb behaviour by price setters are studied when the monetary authority acts under commitment. The thesis addresses what constitutes optimal monetary policy in:

1) The basic purely forward-looking New Keynesian model which we extend by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner. This results in a Phillips curve with both a forward-looking term and a backward-looking term. Backward-looking rule-of-thumb behaviour is specified in two ways. First, following Galì and Gertler (1999) we allow the rule-of-thumb price setters to index their prices to lagged inflation. Second, following Steinsson (2003) we allow the rule-of-thumb price setters to index their prices to both lagged output gap and lagged inflation. In all models, steady-state distortions are assumed to be small so that it suffices to approximate the equilibrium of the model to first order to obtain a second-order accurate measure of welfare. We derive utility-based objective functions for the monetary authority and analyse a range of optimal commitment policies that have been proposed in the literature: the zero-optimal policy and two types of timeless perspective commitment policy

2) The medium-scale New Keynesian model developed by Altig et al. (2005) which we extend by allowing a fraction of price setters to behave in a backward-looking rule-of-thumb manner à la Galì and Gertler (1999). We depart from the assumption of small steady-state distortions and consider the case of a largely distorted steady state. We characterise optimal monetary policy with a Ramsey-type approach. We describe the Ramsey steady state and the Ramsey dynamics using the algorithms developed in Schmitt-Grohé and Uribe (2007). We study the implementation of optimal monetary policy by characterising optimal, simple, and implementable interest-rate rules. Large steady-state distortions imply that to obtain a second-order accurate measure of welfare it does not suffice to approximate the model's equilibrium conditions up to first order. In characterising interest-rate rules, we use the methodology and the algorithm developed in Schmitt-Grohé and Uribe (2004b) for second-order approximations to policy functions of dynamic and stochastic models.

Regarding (1):

a) Rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), breaks the optimality of zero long-run inflation found in New Keynesian models. A key implication of the purely forward-looking New Keynesian model is that zero long-run inflation is the optimal target of monetary policy. Woodford (2003) writes "*It is sometimes supposed that the existence of a long-run Phillips-curve trade-off, together with an inefficient natural rate, should imply that the Phillips curve should be exploited to some extent, resulting in positive inflation forever, even under commitment. But here that is not true because the smaller coefficient on the expected inflation-term relative to that on current inflation-which results in the long-run trade-off- is exactly the size of the shift term in the aggregate supply that is needed to precisely eliminate any long-run incentive for nonzero inflation under an optimal commitment.*" Woodford

(2003, p. 415). Moreover, as shown in Woodford (2003), zero long-run inflation is also robust to the presence backward-looking price indexation. Rule-of-thumb behaviour, regardless of its specification, implies that the stimulative effect of higher inflation is greater than the output cost of higher inflation thus generating a long-run incentive for positive inflation under an optimal commitment. Optimal steady-state inflation collapses to zero in the absence of backward-looking rule-of-thumb behaviour, in the absence of a long-run Phillips-curve trade-off, and in the absence of steady-state distortions.

b) Positive optimal long-run inflation also obtains in the purely forward-looking New Keynesian model under a type of timeless perspective commitment policy that has recently been proposed in the literature (i.e. Blake (2001), Jensen and McCallum (2002), and Damjanovic et al. (2008)) and is based on the optimisation of the unconditional value of the central bank's objective function. The intuition for this result hinges on the discount factor. If the central bank shares the same discount factor of the private sector, there is no long-run incentive for positive inflation and optimal steady-state inflation is zero. Conversely, if the central bank does not discount the future, positive steady-state inflation emerges under commitment even in the purely forward-looking model. Moreover, the alternative timeless perspective commitment policy is robust to the introduction of backward-looking rule-of thumb behaviour, when this is characterised as in Galì and Gertler (1999), with the optimal long-run inflation rate being invariant to the degree of rule-of-thumb behaviour.

c) All optimal positive long-run inflation rates turn out to be small in magnitude. We perform sensitivity analysis for ample ranges of two key structural parameters, namely the degree of price stickiness and the degree of rule-of-thumb behaviour. On the one hand, the optimal long-run inflation rates we derive are not capable of explaining the observed inflation rates. On the other hand, the policy-driven steady state is very close to the steady state around which the models are log-linearised, which is characterised by zero inflation.

d) The different commitment policies rank in terms of welfare in line with the intuition. We consider both welfare the on the basis of the deterministic equilibrium and on the basis of the stochastic equilibrium, which stems from augmenting the aggregate-supply with a cost-push shock. Our main objective is, as in Jensen and McCallum (2002), to rank the alternative commitment policies. On the basis of the deterministic equilibrium, the zero-optimal commitment policy ranks first followed by the alternative timeless perspective policy and the standard timeless perspective policy. Moreover, steady-state inflation is found to be welfare enhancing with respect to a policy of zero steady-state inflation. A positive but small level of inflation, which generates positive steady-state price dispersion, is thus preferable to a policy of zero inflation at all times, which implies zero steady-state price dispersion. The reason for this is that positive steady-state inflation, by bringing about positive output gap, allows eliminating some of the steady-state loss due to monopolistic competition. On the basis of the stochastic equilibrium, we consider both an unconditional welfare measure and a measure of welfare conditional on initial conditions. In both cases the different commitment policies rank according to the a priori belief. When considering unconditional welfare, the alternative timeless perspective policy ranks first followed by the standard timeless perspective policy and the zero-optimal policy. When considering welfare conditional on initial conditions, the zero-optimal policy ranks first followed by the standard timeless perspective policy and the alternative timeless perspective policy.

Regarding (2):

a) Rule-of-thumb behaviour by price setters, specified à la Galì and Gertler (1999) implies optimal positive inflation in the Ramsey steady state only in the absence of transactional frictions. Otherwise, the Friedman prescription for deflation governs the optimal steady-state inflation: the average level of the nominal interest rate should be sufficiently low so that there should negative inflation on average. As in the linear-quadratic framework, the positive inflation rate in the Ramsey steady state target is found to be small for ample ranges of the degree of price stickiness and the degree of rule-of-thumb behaviour. Moreover, the inflation rate is again observed to be monotonically decreasing in the degree of price stickiness and monotonically increasing in the degree of rule-of-thumb behaviour. In the linear-quadratic framework the steady-state inflation rate is directly proportional to the steady-state efficiency gap, which is the gap between the steady-state level of output and the efficient steady-state level of output. We proceed to analyse the social planner allocation so to compute the social planner steady state. We find that the steady-state efficiency gap in the medium-scale economy is only slightly larger than in the case of small steady-state distortions assumed in the linear quadratic framework. This may explain the reason as to why the inflation rate in the cashless medium-scale model is again positive but it is only slightly larger than the one that obtains in the linear-quadratic framework.

b) Ramsey dynamics are not affected by the presence/absence of money in the theoretical economy. The zero lower bound on the nominal interest rate does not constitute an impediment to optimal monetary policy. The reason for this is that under the Ramsey-optimal policy, the zero lower bound poses an impediment to monetary policy only in the case of an adverse shock that forces the interest rate to be more that 8 standard deviations below its mean. The probability of this happening is so small that, as found by Schmitt-Grohé and Uribe (2007) in the case of backward-looking price indexation, the zero lower bound on the nominal interest rate does not impose an economically important constraint on the conduct of optimal monetary policy. We study Ramsey-optimal responses to the three shocks that drive aggregate fluctuations: the permanent neutral technological shock, the permanent investment-specific technological shock, and temporary variations in government expenditure. The Ramsey-optimal stabilisation policy to all three shocks is robust to the presence/absence of money in the model.

c) Optimal operational interest-rate rules are not affected by the presence/absence of money in the theoretical economy. We characterise the operational interest-rate rule, which is defined exactly as in Schmitt-Grohé and Uribe (2007), in both the medium-scale model with money and its cashless counterpart. In both cases, the optimal operational interest-rate rule is confirmed to be active in price and wage inflation, mute in output growth and moderately inertial. However, in the cashless economy the coefficients on price and wage inflation are greater than in the model that features transactional frictions. We consider a modification of the operational interest-rate rule, which prescribes a concern not for output growth per se but for stabilisation of output around a welfare-relevant measure of output, namely the gap between the Ramsey level of output and the efficient level of output. We find that the optimal operational interest-rate rule remains active in price and wage inflation and moderately inertial, but also implies a positive coefficient on the output gap. This is true in both the model with money and its cashless counterpart.

Taking together the basic message of our results is that

- The widespread practice in the New Keynesian literature of restricting the attention to the case of an efficient natural level of output is not innocuous. A policy that is optimal for an economy with an efficient steady state differs from what is optimal in an economy where the subsidies that achieve Pareto efficiency are unavailable.
- The two most popular sources of inflation persistence, namely rule-of-thumb behaviour and price indexation, have different consequences as for the optimal

steady-state inflation rate. In this sense, the results we obtain in the linear-quadratic framework conflict with Woodford (2003).

- The results in the linear-quadratic framework carry on to the case of a medium-scale model only when the theoretical model does not feature transactional frictions. Interestingly, while the presence of money matters for the Ramsey steady state, it does not seem to affect the Ramsey-optimal stabilisation policy and the implementation of optimal monetary policy via simple interest-rate rules.

Our results are sensitive to both the assumption of inflation persistence being caused by rule-of-thumb behaviour by price setters and the Calvo's (1983) assumption of a constant probability of price adjustment, irrespective of the duration of prices. Future research based on this thesis may address the consequences of relaxing these two assumptions. Different sources of inflation persistence have in fact been put forward in the literature. The importance of these ideas is discussed further in Woodford (2007). For instance, Milani (2005, 2007) shows how a process of adaptive learning by agents explains some persistence. In a similar vein, Paloviita (2006) and Roberts (1997) account for inflation persistence by arguing that it results from inflation expectations not being formed rationally. Moving away from Calvo's (1983) assumption of a constant probability of price adjustment can also be achieved in different ways. For instance, Goodfriend and King (1997) suggest that an upward-sloping probability of changing price would be a more appropriate assumption than the constant probability of the Calvo model. Sheedy (2007a) builds on this suggestion and derives a Phillips curve that exhibits intrinsic inflation persistence. Inflation persistence is intrinsic in the sense that inflation determination is partially backward-looking even though all agents remain forward-looking[1]. Mankiw and Reis (2002) replace the assumption of sticky prices with that of sticky information and show how inflation persistence is also implied by agents' limited ability to update or absorb information. Perhaps, the most natural extension of this thesis would be an investigation of the effects of

[1]Sheedy (2007b) goes on to analyse optimal monetary policy in response to shocks, but the steady state he considers is Pareto efficient.

these different reasons for inflation persistence on the optimal target for monetary policy in economies where the steady state is not assumed to be Pareto efficient.

CHAPTER 9

Appendices

9.1. Appendix A. Detailed Derivation of the Hybrid Phillips Curve under Rule-of-thumb Behaviour à la Steinsson (2003)

We begin by log-linearising the aggregate price level (2.25)

$$\widehat{P}_t = (1-\alpha)\widehat{p}_t^* + \alpha \widehat{P}_{t-1} \tag{9.1}$$

A log-linearisation to the overall reset price (2.26) is given by

$$\widehat{p}_t^* = (1-\omega)\widehat{p}_t^f + \omega \widehat{p}_t^b \tag{9.2}$$

As shown in Woodford (2003, Chapter 3), a log-linearisation of the notional short-run aggregate supply curve, which is implicitly defined by $\Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s}, \widetilde{\xi}_{t+s}) = 0$, takes the form

$$\log(p_t^f / P_t) = \zeta x_t \tag{9.3}$$

where ζ is the elasticity of the notional short-run aggregate supply curve curve. Under the assumption of specific labour markets, ζ is given by

$$\zeta = \frac{(\sigma^{-1} + \varpi)}{(1 + \varpi\theta)} > 0 \tag{9.4}$$

Combining the definition of the forward-looking reset price (2.28) with the log-linearisation of the notional short-run aggregate supply curve (9.3) yields

$$\widehat{p}_t^f = (1-\alpha\beta)E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \left[\widehat{P}_{t+s} + \zeta x_{t+s}\right] \tag{9.5}$$

Quasi-differencing (9.5), we obtain the log-linearised forward-looking reset price

$$\widehat{p}_t^f = (1-\alpha\beta)\zeta x_t + (1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t \widehat{p}_{t+1}^f \tag{9.6}$$

Log-linearising the backward-looking reset price under rule-of-thumb behaviour à la Steinsson, (2.29), delivers

$$\widehat{p}_t^b = \widehat{p}_{t-1}^* + \pi_{t-1} + \delta x_{t-1} \tag{9.7}$$

Combining the log-linearised aggregate price level (9.1) with the log-linearised overall reset price (9.2) we obtain the evolution of the aggregate inflation rate

$$\pi_t = \frac{1-\alpha}{\alpha}\left[(1-\omega)(\widehat{p}_t^f - \widehat{P}_t) + \omega(\widehat{p}_t^b - \widehat{P}_t)\right] \tag{9.8}$$

In order to derive the hybrid Phillips curve we need to find: I) the difference between the log-linearised forward-looking reset price and the log-linearised aggregate price level, namely $\widehat{p}_t^f - \widehat{P}_t$ and II) the difference between the log-linearised backward-looking reset price and the log-linearised aggregate price level, that is $\widehat{p}_t^f - \widehat{P}_t$.

Using the log-linearised aggregate price level (9.1) at time $t-1$ to substitute for $\widehat{p}_{t-1}^*$ in the log-linearised backward-looking reset price (9.7), $\widehat{p}_t^b - \widehat{P}_t$ takes the form

$$\widehat{p}_t^b - \widehat{P}_t = \frac{1}{1-\alpha}\pi_{t-1} - \pi_t + \delta x_{t-1} \tag{9.9}$$

Rewriting the log-linearised forward-looking reset price (9.6) in terms of $\widehat{p}_t^f - \widehat{P}_t$ yields

$$\widehat{p}_t^f - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \alpha\beta E_t(\widehat{p}_{t+1}^f - \widehat{P}_t) \tag{9.10}$$

We thus need to find the expected value of the difference between the log-linearised forward-looking reset price at time $t+1$ and the log-linearised aggregate price level at time t, namely $\widehat{p}_{t+1}^f - \widehat{P}_t$. Combining the log-linearised overall reset price, as given by (9.2) at $t+1$, with the log-linearised backward-looking reset price, as implied by (9.7) at $t+1$, gives

$$\widehat{p}^*_{t+1} - \widehat{P}_t = (1-\omega)(\widehat{p}^f_{t+1} - \widehat{P}_t) + \omega(\widehat{p}^*_t - \widehat{P}_{t-1} + \delta x_t) \tag{9.11}$$

where we subtract $\widehat{P}_t$ from both sides. Rewriting the log-linearised price level (9.1) allows finding an expression for the difference between the log-linearised overall reset price and the log-linearised aggregate price level in the previous period, that is

$$\widehat{p}^*_t - \widehat{P}_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} \tag{9.12}$$

Using this to substitute for both $\widehat{p}^*_t - \widehat{P}_{t-1}$ and $\widehat{p}^*_{t+1} - \widehat{P}_t$ in (9.11) and taking the expected value at t, $E_t(\widehat{p}^f_{t+1} - \widehat{P}_t)$ is given by

$$E_t(\widehat{p}^f_{t+1} - \widehat{P}_t) = \frac{1}{(1-\alpha)(1-\omega)}E_t(\pi_{t+1} - \omega\pi_t) - \frac{\omega\delta}{(1-\omega)}x_t \tag{9.13}$$

We can now go back to equation (9.10) for $\widehat{p}^f_t - \widehat{P}_t$. Substituting $E_t(\widehat{p}^f_{t+1} - \widehat{P}_t)$, as given by (9.13), in (9.10), $\widehat{p}^f_t - \widehat{P}_t$ takes the form

$$\widehat{p}^f_t - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \frac{\alpha\beta}{(1-\alpha)(1-\omega)}E_t(\pi_{t+1} - \omega\pi_t) - \frac{\alpha\beta\omega\delta}{(1-\omega)}x_t \tag{9.14}$$

Having found both $\widehat{p}^f_t - \widehat{P}_t$ and $\widehat{p}^f_t - \widehat{P}_t$ as functions of inflation and output gap only, the hybrid Phillips curve is the solution to the system of equations (9.8), (9.9), and (9.14). Combining them and solving for inflation delivers

$$\frac{\alpha + \alpha\beta\omega + (1-\alpha)\omega}{\alpha}\pi_t = \left\{ \begin{array}{l} \frac{\alpha\beta}{\alpha}E_t\pi_{t+1} + \frac{\omega}{\alpha}\pi_{t-1} + \frac{(1-\alpha)\omega\delta}{\alpha}x_{t-1} \\ + \left[\frac{(1-\alpha)(1-\omega)(1-\alpha\beta)\zeta - (1-\alpha)\alpha\beta\omega\delta}{\alpha}\right]x_t \end{array} \right\} \tag{9.15}$$

Normalising on π_t and taking into account the specification of ζ in (9.4), we obtain the hybrid Phillips curve under rule-of-thumb behaviour à la Steinsson in the main text (i.e. (2.53) with the coefficients defined as in (2.54) and (2.55)).

9.2. Appendix B. Detailed Derivation of the Utility-based Objective Function under Rule-of-thumb Behaviour à la Steinsson (2003)

Under Calvo (1983) staggered price setting and backward-looking rule-of-thumb behaviour by price setters, the distribution of prices in any period, $\{p_t(i)\}$, consists of α times the distribution of prices in the previous period, $\{p_{t-1}(i)\}$, an atom of size $(1-\alpha)(1-\omega)$ at the forward-looking reset price, p_t^f, and an atom of size $(1-\alpha)\omega$ at the rule-of-thumb backward-looking reset price, p_t^b

$$\{p_t(i)\} = \alpha\{p_{t-1}(i)\} + (1-\alpha)(1-\omega)p_t^f + (1-\alpha)\omega p_t^b \tag{9.16}$$

Let $\Delta_t = var_i \log p_t(i)$ denote the degree of price dispersion and $\overline{P}_t = E_i\{\log p_t(i)\}$ denote the average price, hence $\overline{P}_t - \overline{P}_{t-1} = E_i\left[\log\{p_t(i)\} - \overline{P}_{t-1}\right]$. Recalling the overall reset price (9.2), which implies that $\log p_t^* = (1-\omega)\log p_t^f + \omega \log p_t^b$, and using the distribution of prices in (9.16), $\overline{P}_t - \overline{P}_{t-1}$ can be rewritten as

$$\overline{P}_t - \overline{P}_{t-1} = \overbrace{\alpha E_i\left[\{\log p_{t-1}(i)\} - \overline{P}_{t-1}\right]}^{0} + (1-\alpha)(\log p_t^* - \overline{P}_{t-1}) \tag{9.17}$$

Similarly, Δ_t can be rewritten as

$$\begin{aligned} \Delta_t &= E_i\left\{\left[\log\{p_t(i)\} - \overline{P}_{t-1}\right]^2\right\} - \left[E_i \log\{p_t(i)\} - \overline{P}_{t-1}\right]^2 \\ &= \alpha E_i\left\{\left[\log\{p_{t-1}(i)\} - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)(1-\omega)(\log p_t^f - \overline{P}_{t-1})^2 \\ &\quad +(1-\alpha)\omega(\log p_t^b - \overline{P}_{t-1})^2 - (\overline{P}_t - \overline{P}_{t-1})^2 \end{aligned} \tag{9.18}$$

$\overline{P}_t$ is related to the Dixit-Stiglitz price index through the log-linear approximation

$$\overline{P}_t = \log P_t + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{9.19}$$

the second-order residual follows from the fact that the equilibrium inflation process (as the equilibrium output process) satisfies a bound of second order $O(\left\|\widetilde{\xi}, \varrho\right\|^2)$ together with a second-order bound on the initial degree of price dispersion, Δ_{-1}. Note that, as in

Woodford (2003), Δ_{-1} is assumed to be of second order (that is why it enters the second-order residual in (9.19) to the power of 1/2). It then follows that this measure of price dispersion continues to be only of second order in the case of first-order deviations of inflation from zero.

In order to derive the utility-based objective function we need to find: I) the log-difference between the forward-looking reset price and the lagged aggregate price level, namely $\log p_t^f - \overline{P}_{t-1}$ and II) the log-difference between the backward-looking reset price and the lagged aggregate price level, that is $\log p_t^b - \overline{P}_{t-1}$.

Recalling the backward-looking reset price under rule-of-thumb behaviour à la Steinsson (9.7), which implies that $\log p_t^b = \log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}$, and using the log-linear approximation (9.19), $\log p_t^b - \overline{P}_{t-1}$ is given by

$$\log p_t^b - \overline{P}_{t-1} = \log p_{t-1}^* - \overline{P}_{t-2} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{9.20}$$

Similarly, using I) the overall reset price in (9.2), II) the backward-looking reset price under rule-of-thumb behaviour à la Steinsson (9.7), and III) the log-linear approximation (9.19), we can rewrite $\log p_t^f - \overline{P}_{t-1}$ as

$$\begin{aligned} \log p_t^f - \overline{P}_{t-1} \quad &= \quad \frac{1}{1-\omega}(\log p_t^* - \overline{P}_{t-1}) - \frac{\omega}{1-\omega}(\log p_{t-1}^* - \overline{P}_{t-2}) \\ &\quad - \frac{\omega\delta}{1-\omega}x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \end{aligned} \tag{9.21}$$

Using the log-linear approximation (9.19), the distribution of prices in (9.17) can be rewritten as

$$\pi_t = (1-\alpha)(\log p_t^* - \overline{P}_{t-1}) + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{9.22}$$

Accordingly substituting in the expressions for $\log p_t^b - \overline{P}_{t-1}$, (9.20), and for $\log p_t^f - \overline{P}_{t-1}$, (9.21), we obtain

$$\log p_t^b - \overline{P}_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{9.23}$$

$$(9.24)\quad \log p_t^f - \overline{P}_{t-1} = \frac{1}{(1-\omega)(1-\alpha)}(\pi_t - \omega\pi_{t-1}) - \frac{\omega\delta}{(1-\omega)}x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)$$

Having found both $\log p_t^b - \overline{P}_{t-1}$ and $\log p_t^f - \overline{P}_{t-1}$ as functions of inflation and output gap only, the degree of price dispersion (9.18), by taking into account the log-linear approximation (9.19), can be rewritten as

$$(9.25)\quad \begin{aligned}\Delta_t &= \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2 \\ &\quad + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)\end{aligned}$$

Integrating forward, starting from any small initial degree of price dispersion, Δ_{-1}, the degree of price dispersion in any period $t \geq 0$ is given by

$$(9.26)\quad \begin{aligned}\Delta_t &= \sum_{s=0}^{\infty}\alpha^{t-s}\left[\begin{array}{c}\frac{\alpha}{(1-\alpha)}\pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2\end{array}\right] \\ &\quad + \alpha^{t-1}\Delta_{-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^3\right)\end{aligned}$$

The term $\alpha^{t-1}\Delta_{-1}$ is independent of monetary policy. Taking the discounted value of the degree of price dispersion (9.26) over all periods $t \geq 0$ gives

$$(9.27)\quad \begin{aligned}\sum_{t=0}^{\infty}\beta^t\Delta_t &= \frac{1}{1-\alpha\beta}\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}\frac{\alpha}{(1-\alpha)}\pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)}\left[\begin{array}{c}\pi_t - \pi_{t-1} \\ -(1-\alpha)\delta x_{t-1}\end{array}\right]^2\end{array}\right] \\ &\quad + t.i.p + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^3\right)\end{aligned}$$

Taking the discounted value of the second-order approximation to the period utility function (3.22) over all periods $t \geq 0$ delivers

$$(9.28)\quad \sum_{t=0}^{\infty}\beta^t U_t = -\frac{\overline{Y}\widetilde{u}_c}{2}\left[\begin{array}{c}(\sigma^{-1} + \varpi)\sum_{t=0}^{\infty}\beta^t(x_t - x^*)^2 \\ +(1+\varpi\theta)\theta\sum_{t=0}^{\infty}\beta^t\Delta_t\end{array}\right] + t.i.p + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right)$$

Combining (9.27) with (9.28) and normalizing on inflation, we obtain the utility-based objective function under rule-of-thumb behaviour à la Steinsson in the main text (i.e. (3.23) with the single-period loss function as in (3.24) and the coefficients defined as in (3.25)).

9.3. Appendix C. Functional Forms and Equilibrium Conditions in Stationary Variables

9.3.1. Functional Forms

We use the same functional forms in Schmitt-Grohé and Uribe (2007). The period utility function is given by

$$U=\frac{\left[\left(c_t-bc_{t-1}\right)^{1-\phi_4}\left(1-h_t\right)^{\phi_4}\right]^{1-\phi_3}-1}{1-\phi_3} \tag{9.29}$$

This entails that equilibrium conditions (6.13), (6.15), and (6.25) become respectively

(9.30)
$$\left\{\begin{array}{c}\left(1-\phi_4\right)\left(c_t-bc_{t-1}\right)^{(1-\phi_3)(1-\phi_4)-1}\left(1-h_t\right)^{\phi_4(1-\phi_3)}\\ -b\beta E_t(1-\phi_4)\left(c_{t+1}-bc_t\right)^{(1-\phi_3)(1-\phi_4)-1}\left(1-h_{t+1}\right)^{\phi_4(1-\phi_3)}\end{array}\right\}=\lambda_t\left[1+\ell\left(v_t\right)+v_t\ell'\left(v_t\right)\right]$$

$$\phi_4\left(c_t-bc_{t-1}\right)^{(1-\phi_3)(1-\phi_4)}\left(1-h_t\right)^{\phi_4(1-\phi_3)-1}=\frac{\lambda_t w_t}{\widetilde{\mu}_t} \tag{9.31}$$

(9.32)
$$f_t^2=\left[\phi_4\left(c_t-bc_{t-1}\right)^{(1-\phi_3)(1-\phi_4)}\left(1-h_t\right)^{\phi_4(1-\phi_3)-1}\right]\left(\frac{w_t}{\widetilde{w}_t}\right)^{\widetilde{\eta}}h_t^d+\widetilde{\alpha}\beta E_t\left(\frac{\pi_{t+1}}{\left(\mu_z*\pi_t\right)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}}\left(\frac{\widetilde{w}_{t+1}}{\widetilde{w}_t}\right)^{\widetilde{\eta}}f_{t+1}^2$$

The production function takes the Cobb-Douglas the form

$$F\left(k,h\right)=k^{\theta}h^{1-\theta} \tag{9.33}$$

This implies that equilibrium conditions (6.42), (6.43), and (6.44) become respectively

$$(9.34)\qquad \left(u_t k_t\right)^{\theta}\left(z_t h_t^d\right)^{1-\theta} - \psi z_t^* = \left\{c_t\left[1+\ell\left(v_t\right)\right] + g_t + \Upsilon_t^{-1}\left[i_t + a\left(u_t\right)k_t\right]\right\} s_t$$

$$(9.35)\qquad mc_t z_t(1-\theta)(u_t k_t)^{\theta}(z_t h_t^d)^{-\theta} = w_t\left[1 + v\frac{R_t - 1}{R_t}\right]$$

$$(9.36)\qquad mc_t\theta(u_t k_t)^{\theta-1}(z_t h_t^d)^{1-\theta} = r_t^k$$

The investment adjustment cost function is assumed to be the same as in Christiano et al. (2005), namely

$$(9.37)\qquad S\left(\frac{i_t}{i_{t-1}}\right) = \frac{k}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2$$

with μ_I being the steady-state growth rate of investment. This implies that equilibrium conditions (6.10) and (6.17) become respectively

$$(9.38)\qquad k_{t+1} = (1-\delta)\,k_t + i_t\left[1 - \frac{k}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2\right]$$

(9.39)

$$\frac{\lambda_t}{\Upsilon_t} = \lambda_t q_t\left[1 - \frac{k}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2 - \left(\frac{i_t}{i_{t-1}}\right)k\left(\frac{i_t}{i_{t-1}} - \mu_I\right)\right] + \beta E_t\lambda_{t+1}q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 k\left(\frac{i_{t+1}}{i_t} - \mu_I\right)$$

The transaction cost function takes the functional form used in Schmitt-Grohé and Uribe (2004a), that is

$$(9.40)\qquad l\left(v\right) = \phi_1 + \phi_2/v - 2\sqrt{\phi_1\phi_2}$$

which implies that demand for money is given by

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1} \frac{R_t - 1}{R_t} \tag{9.41}$$

Finally, the costs of higher capacity utilisation take the form

$$a(u) = \gamma_1 (u - 1) + \frac{\gamma_2}{2} (u - 1)^2 \tag{9.42}$$

9.3.2. Stationary Variables

The theoretical economy displays two permanent shocks. Hence, some variables are not stationary along the balanced-growth path. The nonstationary variables and the respective rescaling factors are the same as in Schmitt-Grohé and Uribe (2007). Specifically, I) variables c_t, m_t^h, m_t, w_t, $\widetilde{w}_t$, y_t, g_t, ϕ_t, x_t^1, x_t^2, and τ_t are rescaled by z_t^*; II) variables k_{t+1} and i_t are rescaled by $\Upsilon_t z_t^*$; III) variables f_t^1 and f_t^2 are rescaled by $z_t^{*(1-\phi_3)(1-\phi_4)}$; IV) variables q_t and r_t^k are rescaled by $1/\Upsilon_t$; and V) variable λ_t is rescaled by $z_t^{*(1-\phi_3)(1-\phi_4)-1}$. The remaining variables, namely p_t^f, p_t^b, $\widetilde{p}_t$, v_t, u_t, mc_t, h_t, h_t^d, s_t, $\widetilde{s}_t$, and π_t, are instead stationary.

Dividing the nonstationary variables by the associated scaling factor, we obtain the stationary variables, which we denote with the corresponding capital letters. That is:

$$C_t = c_t / z_t^* \tag{9.43}$$

$$M_t^h = m_t^h / z_t^* \tag{9.44}$$

$$M_t = m_t / z_t^* \tag{9.45}$$

$$W_t = w_t / z_t^* \tag{9.46}$$

$$\widetilde{W}_t = \widetilde{w}_t / z_t^* \tag{9.47}$$

$$Y_t = y_t / z_t^* \tag{9.48}$$

$$G_t = g_t / z_t^* \tag{9.49}$$

$$\Phi_t = \phi_t / z_t^* \tag{9.50}$$

$$X_t^1 = x_t^1 / z_t^* \tag{9.51}$$

$$X_t^2 = x_t^2 / z_t^* \tag{9.52}$$

$$T_t = \tau_t / z_t^* \tag{9.53}$$

$$K_{t+1} = k_{t+1} / \Upsilon_t^{\frac{1}{1-\theta}} z_t \text{ given } z_t^* / z_t = \Upsilon_t^{\frac{\theta}{1-\theta}} \tag{9.54}$$

$$I_t = i_t / \Upsilon_t^{\frac{1}{1-\theta}} z_t \tag{9.55}$$

$$F_t^1 = f_t^1 / z_t^{*(1-\phi_3)(1-\phi_4)} \tag{9.56}$$

$$F_t^2 = f_t^2 / z_t^{*(1-\phi_3)(1-\phi_4)} \tag{9.57}$$

$$Q_t = q_t \Upsilon_t \tag{9.58}$$

(9.59)
$$R_t^k = r_t^k \Upsilon_t$$

(9.60)
$$\Lambda_t = \lambda_t / z_t^{*(1-\phi_3)(1-\phi_4)-1}$$

9.3.3. Equilibrium Conditions in terms of Stationary Variables

We report the complete set of equilibrium conditions in the model with money, namely (6.10), (6.11), (6.13), (6.15)-(6.26), (6.28), (6.35)-(6.45), and (6.47)-(6.50), written in stationary form. To this end, we define $\mu_{z^*,t} = z_t^*/z_{t-1}^*$. Recalling that $I)$ $z_t^*/z_t = \Upsilon_t^{\frac{\theta}{1-\theta}}$, $II)$ $\mu_{z,t} = z_t/z_{t-1}$, and $III)$ $\mu_{\Upsilon,t} = \Upsilon_t/\Upsilon_{t-1}$, it follows that $\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{\theta}{1-\theta}} \mu_{z,t}$.

(9.61)
$$K_{t+1} = (1-\delta)\frac{K_t}{\mu_{I,t}} + I_t\left[1 - \frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2\right] \text{ with } \mu_{I,t} = \mu_{\Upsilon,t}\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{1}{1-\theta}}\mu_{z,t}$$

(9.62)
$$v_t = \frac{C_t}{M_t^h}$$

(9.63)
$$\left\{\begin{array}{c} (1-\phi_4)\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)-1}(1-h_t)^{\phi_4(1-\phi_3)} \\ -b\beta E_t(1-\phi_4)\left(C_{t+1}\mu_{z^*,t+1} - bC_t\right)^{(1-\phi_3)(1-\phi_4)-1}(1-h_{t+1})^{\phi_4(1-\phi_3)} \end{array}\right\} = \Lambda_t\left[1 + \ell(v_t) + v_t\ell'(v_t)\right]$$

(9.64)
$$\phi_4\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1} = \frac{\Lambda_t W_t}{\widetilde{\mu}_t}$$

(9.65)
$$\Lambda_t Q_t = \beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}[R_{t+1}^k u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)] \text{ with } \mu_{\Lambda,t} = \mu_{z^*,t}^{(1-\phi_3)(1-\phi_4)-1}$$

$$\Lambda_t = \left\{ \begin{array}{c} \Lambda_t Q_t \left[1 - \frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2 - \left(\frac{I_t}{I_{t-1}}\mu_{I,t}\right) k \left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right) \right] \\ +\beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}} \Lambda_{t+1} Q_{t+1} \left(\frac{I_{t+1}}{I_t}\mu_{I,t+1}\right)^2 k \left(\frac{I_{t+1}}{I_t}\mu_{I,t+1} - \mu_I\right) \end{array} \right\} \tag{9.66}$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{\Lambda,t+1} \frac{1}{\pi_{t+1}} \tag{9.67}$$

$$R_t^k = a'(u_t) \tag{9.68}$$

$$\Lambda_t = \beta R_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}} \mu_{\Lambda,t+1} \tag{9.69}$$

$$W_t^{1-\widetilde{\eta}} = (1-\widetilde{\alpha}) \widetilde{W}_t^{1-\widetilde{\eta}} + \widetilde{\alpha} \left(W_{t-1}/\mu_{z^*,t}\right)^{1-\widetilde{\eta}} \left(\frac{\left(\mu_{z^*}\pi_{t-1}\right)^{\widetilde{\chi}}}{\pi_t} \right)^{1-\widetilde{\eta}} \tag{9.70}$$

$$F_t^1 = F_t^2 \tag{9.71}$$

$$F_t^1 = \left\{ \begin{array}{c} \left(\frac{\widetilde{\eta}-1}{\widetilde{\eta}}\right) \widetilde{W}_t \Lambda_t h_t^d \left(\frac{W_t}{\widetilde{W}_t}\right)^{\widetilde{\eta}} \\ +\widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}-1} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\mu_{z^*,t+1}\right)^{\widetilde{\eta}-1} F_{t+1}^1 \mu_{\Lambda,t+1} \mu_{z^*,t+1} \end{array} \right\} \tag{9.72}$$

$$F_t^2 = \left\{ \begin{array}{c} \left[\phi_4 \left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)} (1-h_t)^{\phi_4(1-\phi_3)-1} \right] \left(\frac{W_t}{\widetilde{W}_t}\right)^{\widetilde{\eta}} h_t^d \\ +\widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\mu_{z^*,t+1}\right)^{\widetilde{\eta}} F_{t+1}^2 \mu_{\Lambda,t+1} \mu_{z^*,t+1} \end{array} \right\} \tag{9.73}$$

$$Y_t = C_t\left[1 + \ell\left(v_t\right)\right] + \left[I_t + a\left(u_t\right)K_t\mu_{I,t}^{-1}\right] + G_t \tag{9.74}$$

$$G_t = T_t + M_t(1 - R_t^{-1}) \tag{9.75}$$

$$\widetilde{p}_t = (1-\omega)p_t^f + \omega p_t^b \tag{9.76}$$

$$1 = \alpha\pi_t^{\eta-1} + (1-\alpha)\,\widetilde{p}_t^{1-\eta} \tag{9.77}$$

$$p_t^b = \widetilde{p}_{t-1}\frac{\pi_{t-1}}{\pi_t} \tag{9.78}$$

$$X_t^1 = \frac{\eta-1}{\eta}X_t^2 \tag{9.79}$$

$$X_t^1 = \left(p_t^f\right)^{-\eta-1} Y_t mc_t + \alpha\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\mu_{\Lambda,t+1}\left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta-1}\pi_{t+1}^{\eta}X_{t+1}^1\mu_{z^*,t+1} \tag{9.80}$$

$$X_t^2 = \left(p_t^f\right)^{-\eta} Y_t + \alpha\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\mu_{\Lambda,t+1}\left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta}\pi_{t+1}^{\eta-1}X_{t+1}^2\mu_{z^*,t+1} \tag{9.81}$$

$$\left(u_tK_t\mu_{I,t}^{-1}\right)^{\theta}\left(h_t^d\right)^{1-\theta} - \psi = \left\{C_t\left[1+\ell\left(v_t\right)\right] + G_t + \left[I_t + a\left(u_t\right)K_t\mu_{I,t}^{-1}\right]\right\}s_t \tag{9.82}$$

(9.83)
$$mc_t(1-\theta)(u_tK_t\mu_{I,t}^{-1})^{\theta}(h_t^d)^{-\theta} = W_t\left[1+v\frac{R_t-1}{R_t}\right]$$

(9.84)
$$mc_t\theta(u_tK_t\mu_{I,t}^{-1})^{\theta-1}(h_t^d)^{1-\theta} = R_t^k$$

(9.85)
$$s_t = (1-\alpha)\left(\widetilde{p}_t\right)^{-\eta} + \alpha\pi_t^{\eta}s_{t-1}$$

(9.86)
$$h_t = \widetilde{s}_t h_t^d$$

(9.87)
$$\widetilde{s}_t = (1-\widetilde{\alpha})\left(\frac{\widetilde{W_t}}{W_t}\right)^{-\widetilde{\eta}} + \widetilde{\alpha}\left(\frac{W_{t-1}}{W_t\mu_{z^*,t}}\right)^{-\widetilde{\eta}}\left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\widetilde{\chi}}}\right)^{\widetilde{\eta}}\widetilde{s}_{t-1}$$

(9.88)
$$\Phi_t = Y_t - R_t^k u_t K_t \mu_{I,t}^{-1} - W_t h_t^d\left[1+\left(1-R_t^{-1}\right)v\right]$$

(9.89)
$$M_t = vW_th_t^d + M_t^h$$

9.3.3.1. Cashless model. We report the complete set of equilibrium conditions in the cashless model, namely (6.10), (6.15)-(6.17), (6.19)-(6.25), (6.35)-(6.40), (6.44), (6.45), (6.47), (6.48), (6.52), (6.53), (6.54), (6.55), (6.56), and (6.57), written in terms of stationary variables.

(9.90)
$$K_{t+1} = (1-\delta)\frac{K_t}{\mu_{I,t}} + I_t\left[1-\frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t}-\mu_I\right)^2\right] \text{ with } \mu_{I,t} = \mu_{\Upsilon,t}\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{1}{1-\theta}}\mu_{z,t}$$

$$(9.91)\qquad \left\{ \begin{array}{c} (1-\phi_4)\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)-1}(1-h_t)^{\phi_4(1-\phi_3)} \\ -b\beta E_t(1-\phi_4)\left(C_{t+1}\mu_{z^*,t+1} - bC_t\right)^{(1-\phi_3)(1-\phi_4)-1}(1-h_{t+1})^{\phi_4(1-\phi_3)} \end{array} \right\} = \Lambda_t$$

$$(9.92)\qquad \phi_4\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)}(1-h_t)^{\phi_4(1-\phi_3)-1} = \frac{\Lambda_t W_t}{\widetilde{\mu}_t}$$

(9.93)

$$\Lambda_t Q_t = \beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}} \Lambda_{t+1}[R^k_{t+1}u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)] \text{ with } \mu_{\Lambda,t} = \mu_{z^*,t}^{(1-\phi_3)(1-\phi_4)-1}$$

$$(9.94)\qquad \Lambda_t = \left\{ \begin{array}{c} \Lambda_t Q_t\left[1 - \frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2 - \left(\frac{I_t}{I_{t-1}}\mu_{I,t}\right)k\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)\right] \\ +\beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}Q_{t+1}\left(\frac{I_{t+1}}{I_t}\mu_{I,t+1}\right)^2 k\left(\frac{I_{t+1}}{I_t}\mu_{I,t+1} - \mu_I\right) \end{array} \right\}$$

$$(9.95)\qquad R^k_t = a'(u_t)$$

$$(9.96)\qquad \Lambda_t = \beta R_t E_t \frac{\Lambda_{t+1}}{\pi_{t+1}}\mu_{\Lambda,t+1}$$

$$(9.97)\qquad W_t^{1-\widetilde{\eta}} = (1-\widetilde{\alpha})\widetilde{W}_t^{1-\widetilde{\eta}} + \widetilde{\alpha}\left(W_{t-1}/\mu_{z^*,t}\right)^{1-\widetilde{\eta}}\left(\frac{(\mu_{z^*}\pi_{t-1})^{\widetilde{\chi}}}{\pi_t}\right)^{1-\widetilde{\eta}}$$

$$(9.98)\qquad F^1_t = F^2_t$$

$$F_t^1 = \left\{ \begin{array}{c} \left(\frac{\widetilde{\eta}-1}{\widetilde{\eta}}\right) \widetilde{W}_t \Lambda_t h_t^d \left(\frac{W_t}{\widetilde{W}_t}\right)^{\widetilde{\eta}} \\ +\widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}-1} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\mu_{z^*,t+1}\right)^{\widetilde{\eta}-1} F_{t+1}^1 \mu_{\Lambda,t+1}\mu_{z^*,t+1} \end{array} \right\} \tag{9.99}$$

$$F_t^2 = \left\{ \begin{array}{c} \left[\phi_4 \left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)} \left(1-h_t\right)^{\phi_4(1-\phi_3)-1}\right] \left(\frac{W_t}{\widetilde{W}_t}\right)^{\widetilde{\eta}} h_t^d \\ +\widetilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t}\mu_{z^*,t+1}\right)^{\widetilde{\eta}} F_{t+1}^2 \mu_{\Lambda,t+1}\mu_{z^*,t+1} \end{array} \right\} \tag{9.100}$$

$$Y_t = C_t + \left[I_t + a\left(u_t\right) K_t \mu_{I,t}^{-1}\right] + G_t \tag{9.101}$$

$$G_t = T_t \tag{9.102}$$

$$\widetilde{p}_t = (1-\omega)p_t^f + \omega p_t^b \tag{9.103}$$

$$1 = \alpha \pi_t^{\eta-1} + (1-\alpha)\,\widetilde{p}_t^{1-\eta} \tag{9.104}$$

$$p_t^b = \widetilde{p}_{t-1}\frac{\pi_{t-1}}{\pi_t} \tag{9.105}$$

$$X_t^1 = \frac{\eta-1}{\eta} X_t^2 \tag{9.106}$$

$$X_t^1 = \left(p_t^f\right)^{-\eta-1} Y_t mc_t + \alpha\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\mu_{\Lambda,t+1}\left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta-1} \pi_{t+1}^{\eta} X_{t+1}^1 \mu_{z^*,t+1} \tag{9.107}$$

$$X_t^2 = \left(p_t^f\right)^{-\eta} Y_t + \alpha\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \mu_{\Lambda,t+1} \left(\frac{p_t^f}{p_{t+1}^f}\right)^{-\eta} \pi_{t+1}^{\eta-1} X_{t+1}^2 \mu_{z^*,t+1} \tag{9.108}$$

$$\left(u_t K_t \mu_{I,t}^{-1}\right)^{\theta} \left(h_t^d\right)^{1-\theta} - \psi = \left\{C_t + G_t + \left[I_t + a\left(u_t\right) K_t \mu_{I,t}^{-1}\right]\right\} s_t \tag{9.109}$$

$$mc_t(1-\theta)(u_t K_t \mu_{I,t}^{-1})^{\theta}(h_t^d)^{-\theta} = W_t \tag{9.110}$$

$$mc_t \theta (u_t K_t \mu_{I,t}^{-1})^{\theta-1}(h_t^d)^{1-\theta} = R_t^k \tag{9.111}$$

$$s_t = (1-\alpha)\left(\widetilde{p}_t\right)^{-\eta} + \alpha \pi_t^{\eta} s_{t-1} \tag{9.112}$$

$$h_t = \widetilde{s}_t h_t^d \tag{9.113}$$

$$\widetilde{s}_t = (1-\widetilde{\alpha})\left(\frac{\widetilde{W}_t}{W_t}\right)^{-\widetilde{\eta}} + \widetilde{\alpha}\left(\frac{W_{t-1}}{W_t \mu_{z^*,t}}\right)^{-\widetilde{\eta}} \left(\frac{\pi_t}{\left(\mu_{z^*}\pi_{t-1}\right)^{\widetilde{\chi}}}\right)^{\widetilde{\eta}} \widetilde{s}_{t-1} \tag{9.114}$$

$$\Phi_t = Y_t - R_t^k u_t K_t \mu_{I,t}^{-1} - W_t h_t^d \tag{9.115}$$

9.4. Appendix D. The Social Planner Allocation: Equilibrium Conditions in Stationary Variables

We maintain the same functional forms for utility, technology, investment adjustment cost, and costs of higher capacity utilisation described in Appendix C. The economy displays two permanent shocks. Hence, some variables are not stationary along the balanced-growth path. The nonstationary variables and the respective rescaling factors are the same as in Schmitt-Grohé and Uribe (2007). Specifically, I) variables c_t, y_t, and g_t are rescaled by z_t^*; II) variables k_{t+1} and i_t are rescaled by $\Upsilon_t z_t^*$; III) variable q_t is rescaled by $1/\Upsilon_t$; and IV) variable λ_t is rescaled by $z_t^{*(1-\phi_3)(1-\phi_4)-1}$. The remaining variables, namely u_t and h_t, are instead stationary.

Dividing the nonstationary variables by the associated cointegrating factor, we obtain the stationary variables, which we denote with the corresponding capital letters. That is:

$$C_t = c_t/z_t^* \tag{9.116}$$

$$Y_t = y_t/z_t^* \tag{9.117}$$

$$G_t = g_t/z_t^* \tag{9.118}$$

$$K_{t+1} = k_{t+1}/\Upsilon_t^{\frac{1}{1-\theta}} z_t \text{ given } z_t^*/z_t = \Upsilon_t^{\frac{\theta}{1-\theta}} \tag{9.119}$$

$$I_t = i_t/\Upsilon_t^{\frac{1}{1-\theta}} z_t \tag{9.120}$$

$$Q_t = q_t \Upsilon_t \tag{9.121}$$

$$\Lambda_t = \lambda_t/z_t^{*(1-\phi_3)(1-\phi_4)-1} \tag{9.122}$$

9.4.1. Equilibrium Conditions in terms of Stationary Variables

We report the complete set of equilibrium conditions, namely (6.58)-(6.65), written in terms of stationary variables. To this end, we define $\mu_{z^*,t} = z_t^*/z_{t-1}^*$. Recalling that I) $z_t^*/z_t = \Upsilon_t^{\frac{\theta}{1-\theta}}$, II) $\mu_{z,t} = z_t/z_{t-1}$, and III) $\mu_{\Upsilon,t} = \Upsilon_t/\Upsilon_{t-1}$, it follows that $\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{\theta}{1-\theta}} \mu_{z,t}$.

$$Y_t = \left(u_t K_t \mu_{I,t}^{-1}\right)^{\theta} \left(h_t^d\right)^{1-\theta} - \psi \quad \text{with } \mu_{I,t} = \mu_{\Upsilon,t}\mu_{z^*,t} = \mu_{\Upsilon,t}^{\frac{1}{1-\theta}} \mu_{z,t} \tag{9.123}$$

$$Y_t = C_t + \left[I_t + a\left(u_t\right) K_t \mu_{I,t}^{-1}\right] + G_t \tag{9.124}$$

$$K_{t+1} = (1-\delta)\frac{K_t}{\mu_{I,t}} + I_t\left[1 - \frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2\right] \tag{9.125}$$

$$\left\{\begin{array}{c} (1-\phi_4)\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)-1}\left(1-h_t\right)^{\phi_4(1-\phi_3)} \\ -b\beta E_t(1-\phi_4)\left(C_{t+1}\mu_{z^*,t+1} - bC_t\right)^{(1-\phi_3)(1-\phi_4)-1}\left(1-h_{t+1}\right)^{\phi_4(1-\phi_3)} \end{array}\right\} = \Lambda_t \tag{9.126}$$

$$\phi_4\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{(1-\phi_3)(1-\phi_4)}\left(1-h_t\right)^{\phi_4(1-\phi_3)-1} = \Lambda_t(1-\theta)(u_t K_t \mu_{I,t}^{-1})^{\theta}(h_t)^{-\theta} \tag{9.127}$$

$$\Lambda_t Q_t = \beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}\left[\begin{array}{c} \theta u_{t+1}^{\theta}(K_{t+1}\mu_{I,t+1}^{-1})^{\theta-1}(h_{t+1})^{1-\theta} \\ -a(u_{t+1}) + Q_{t+1}\left(1-\delta\right) \end{array}\right]\mathrm{w}\mu_{\Lambda,t} = \mu_{z^*,t}^{(1-\phi_3)(1-\phi_4)-1} \tag{9.128}$$

$$\Lambda_t = \left\{\begin{array}{c} \Lambda_t Q_t\left[1 - \frac{k}{2}\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)^2 - \left(\frac{I_t}{I_{t-1}}\mu_{I,t}\right)k\left(\frac{I_t}{I_{t-1}}\mu_{I,t} - \mu_I\right)\right] \\ +\beta E_t \frac{\mu_{\Lambda,t+1}}{\mu_{\Upsilon,t+1}}\Lambda_{t+1}Q_{t+1}\left(\frac{I_{t+1}}{I_t}\mu_{I,t+1}\right)^2 k\left(\frac{I_{t+1}}{I_t}\mu_{I,t+1} - \mu_I\right) \end{array}\right\} \tag{9.129}$$

$$\theta(u_t K_t \mu_{I,t}^{-1})^{\theta-1}(h_t)^{1-\theta} = \gamma_1 \tag{9.130}$$

References

[1] Altig, D., L. Christiano, M. Eichenbaum and J. Lindé (2005). Firm-specific Capital, Nominal Rigidities, and the Business Cycle. *NBER Working Paper*, No. 11034.

[2] Amato, J. and T. Laubach (2003). Rule-of-thumb Behaviour and Monetary Policy. *European Economic Review* 47, 791 − 831.

[3] Atkinson, A. and J. Stiglitz (1976). The Design of Tax Structure: Direct Versus Indirect Taxation. *Journal of Public Economics*, 6, 55 − 75.

[4] Ball, L. (1994). Credible Disinflation with Staggered Price Setting. *American Economic Review*, 84, 282 − 289.

[5] Ball, L. (1999). Efficient Rules for Monetary Policy. *International Finance*, 2, 63 − 83.

[6] Barro, R. and D. Gordon (1983). A Positive Theory of Monetary Policy in a Natural-rate Model. *Journal of Political Economy* 91, 589 − 610.

[7] Bean C. (1998). The New UK Monetary Arrangements: A View from the Literature. *Economic Journal*, 108, 1795 − 1809.

[8] Benigno, P. and M. Woodford (2005). Inflation Stabilization and Welfare: the Case of a Distorted Steady State. *Journal of the European Economic Association*, 3, 1185 − 1236.

[9] Benigno, P. and M. Woodford (2006). Optimal Taxation in an RBC Model: A Linear Quadratic Approach. *Journal of Economic Dynamics and Control* , 30, 1445 − 1489.

[10] Bils, M. and P. Klenow (2004). Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy*, 112, 947 − 985.

[11] Blake, A. (2001). A 'Timeless Perspective' on Optimality in Forward-Looking Rational Expectations Models. *NIESR Discussion Papers,* No. 188.

[12] Blanchard, O. and C. Kahn (1980). The Solution of Linear Difference Models Under Rational Expectations. *Econometrica*, 48, 1305 − 11.

[13] Blanchard, O. and J. Galí (2007). Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, 39, 35 − 65.

[14] Calvo, G. (1983). Staggered Prices in a Utility-maximising Framework. *Journal of Monetary Economics* 12, 383 − 398.

[15] Campbell, J and G. Mankiw (1989). Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence. *NBER Macroeconomic Annual,* 185 − 216.

[16] Chari, V., L. Christiano, and P. Kehoe (1991). Optimal Fiscal and Monetary Policy: Some Recent Results. *Journal of Money, Credit and Banking*, 23, 519 − 539.

[17] Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113, 1 − 45.

[18] Clarida, R., J. Galì and M. Gertler (1998). Monetary Policy Rules in Practice: Some International Evidence. *European Economic Review*, 42, 1033 − 1067.

[19] Clarida, R., J. Galì and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 37, 1661 − 1707.

[20] Collard, F. and M. Juillard (2001). Perturbation Methods for Rational Expectations Models. Unpublished manuscript, CEPREMAP.

[21] Damjanovic, T., V. Damjanovic, and C. Nolan (2008). Unconditionally Optimal Monetary Policy. *Journal of Monetary Economics*, 55, 491 − 500.

[22] Dixit, A. and J. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67, 297 − 308.

[23] Fisher, J. (2006). The Dynamic Effects of Neutral and Investment-specific Technology Shocks. *Journal of Political Economy* 114, 413 − 451.

[24] Friedman , M. (1968). The Role of Monetary Policy. *American Economic Review* 58, 1 − 17.

[25] Fuhrer, J. (1997). The (Un)Importance of Forward-Looking Behavior in Price Specifications. *Journal of Money, Credit and Banking*, 29, 338 − 50.

[26] Fuhrer, J. and G. Moore (1995). Inflation Persistence. *Quarterly Journal of Economics*, 110, 127 − 160.

[27] Galì, J. and M. Gertler (1999). Inflation Dynamics: a Structural Econometric Analysis. *Journal of Monetary Economics* 44, 195 − 222.

[28] Galì, J. (1999). Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *American Economic Review* 89, 249 − 271.

[29] Golosov, J. and R. Lucas (2007). Menu Costs and Phillips Curves. *Journal of Political Economy*, 115, 171 − 199.

[30] Goodfriend, M. and R. King (1997). The New Neoclassical Synthesis and the Role of Monetary Policy. *NBER Macroeconomics Annual*, 231 − 283.

[31] Henry, S.G.B., M. Satchi and D. Vines (2006). The Effect of Discounting on Policy Choices in Inflation Targeting Regimes. *Economic Journal*, 116, 266 − 282.

[32] Jensen, C. and B. McCallum (2002). The Non-Optimality of Proposed Monetary Policy Rules under Timeless-Perspective Commitment. *Economics Letters*, 77, 163 – 168.

[33] Jin, H. and K. Judd (2002). Perturbation Methods for General Dynamic Stochastic Models. Unpublished manuscript, Stanford University.

[34] Jonsson, G. (1997). Monetary Politics and Unemployment Persistence. *Journal of Monetary Economics* 39, 303 – 325.

[35] Khan, A., R. King and A. Wolman (2003). Optimal Monetary Policy. *Review of Economic Studies*, 70, 825 – 860.

[36] Kimura, T. and T. Kurozumi (2007). Optimal Monetary Policy in a Micro-founded Model with Parameter Uncertainty. *Journal of Economic Dynamics and Control* 31, 399 – 431.

[37] King, R. and M. Watson (1996). Money, Prices, Interest Rates and the Business Cycle. *Review of Economics and Statistics*, 78, 35 – 53.

[38] King, R. and A. Wolman (1996). Inflation Targeting in a St. Louis Model of the 21st Century. Federal Reserve Bank of St. Louis Review, 78, 83 – 107.

[39] Kirsanova, T., M. Satchi, D. Vines, and S. Wren-Lewis (2007). Optimal Fiscal Policy Rules in a Monetary Union. *Journal of Money, Credit and Banking*, 39, 1759 – 1784.

[40] Klein, P. (2000). Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model. *Journal of Economic Dynamics and Control*, 24, 1405–1423.

[41] Kydland, F. and E. Prescott (1977). Rules rather than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85, 473 – 491.

[42] Lucas, R. and N. Stokey (1983). Optimal Fiscal and Monetary Policy in an Economy Without Capital. *Journal of Monetary Economics*, 12, 55 – 93.

[43] Mankiw, G. (2001). The Inexorable and Mysterious Tradeoff between Inflation and Unemployment. *Economic Journal*, 111, 45 – 61.

[44] Mankiw, G. and R. Reis (2002). Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve. *Quarterly Journal of Economics*, 117, 1295 – 1328.

[45] Milani, F. (2005). Adaptive Learning and Inflation Persistence. *Working paper*, Princeton University.

[46] Milani, F. (2007). Expectations, Learning and Macroeconomic Persistence. *Journal of Monetary Economics*, 54, 2065 – 2082.

[47] McCallum, B. (1997). Comment. *NBER Macroeconomics Annual*, 355 – 359.

[48] Nakamura, E. and J. Steinsson (2008). Five Facts about Prices: A Reevaluation of Menu Cost Models. *Quarterly Journal of Economics*, 123, 1415 – 1464.

[49] Nelson, E. (1998). Sluggish Inflation and Optimizing Models of the Business Cycle. *Journal of Monetary Economics*, 42, 303 – 322.

[50] Ramsey, F. (1927). A contribution to the Theory of Taxation. *Economic Journal*, 37, 47 – 61.

[51] Roberts, J. (1995). New Keynesian Economics and the Phillips Curve. *Journal of Money, Credit and Banking,* 27, 975 – 984.

[52] Roberts, J. (1997). Is inflation sticky? *Journal of Monetary Economics*, 39, 173 – 196.

[53] Roberts, J. (2005). How Well Does the New Keynesian Sticky-price Model Fit the Data? *Contributions to Macroeconomics*, 5, Iss. 1, Art. 6.

[54] Rotemberg, J. (1982). Monopolistic Price Adjustment and Aggregate Output. *Review of Economic Studies*, 44, 517 – 531.

[55] Rotemberg, J. (1987). The New Keynesian Microfoundations. *NBER Macroeconomics Annual*, 69 – 104.

[56] Rotemberg, J. and M. Woodford (1997). An optimizing-based econometric framework for the evaluation of monetary policy. *NBER Macroeconomics Annual*, 297 – 346.

[57] Paloviita, M. (2006). Inflation Dynamics in the Euro Area and the Role of Expectations. *Empirical Economics* 31, 847 – 860.

[58] Phelps E. (1968). Money-Wage Dynamics and Labor Market Equilibrium. *Journal of Political Economy*, 76, 678 – 711.

[59] Sbordone, A. (2005). Do Expected Future Marginal Costs Drive Inflation Dynamics? *Journal of Monetary Economics*, 52, 1183 – 1197.

[60] Schmitt-Grohé, S. and M. Uribe (2004a). Optimal Fiscal and Monetary Policy under Sticky Prices. *Journal of Economic Theory* 114, 198 – 230.

[61] Schmitt-Grohé, S. and M. Uribe (2004b). Solving Dynamic General Equilibrium Models Using a Second-order Approximation to the Policy Function. *Journal of Economic Dynamics and Control* 28, 755 – 775.

[62] Schmitt-Grohé, S. and M. Uribe (2004c). Optimal Fiscal and Monetary Policy under Imperfect Competition. *Journal of Macroeconomics* 26, 183 – 209.

[63] Schmitt-Grohé, S. and M. Uribe (2005). Optimal Fiscal and Monetary Policy in a Medium-scale Macroeconomic Model. *NBER Macroeconomics Annual*, 175 – 206.

[64] Schmitt-Grohé, S. and M. Uribe (2007). Optimal Inflation Stabilization in a Medium-scale Macroeconomic Model. *Monetary Policy Under Inflation Targeting*, Central Bank of Chile, 125 – 186.

[65] Sheedy, K. (2007a). Intrinsic Inflation Persistence. *Working paper*, London School of Economics.

[66] Sheedy, K. (2007b). Robustly Optimal Monetary Policy. *Working paper*, London School of Economics.

[67] Siu, H. (2004). Optimal Fiscal and Monetary Policy with Sticky Prices. *Journal of Monetary Economics*, 51, 575 – 607.

[68] Smets F. and R. Wouters (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* 1, 1123 – 1175.

[69] Soderlind, P. (1999). Solution and Estimation of RE Macromodels with Optimal Policy. *European Economic Review* 43, 813 – 823.

[70] Steinsson, J. (2003). Optimal Monetary Policy in an Economy with Inflation Persistence. *Journal of Monetary Economics* 50, 1425 – 1456.

[71] Svensson, L (1997). Inflation Targeting: Implementing and Monitoring inflation rates. *European Economic Review*, 41, 1111 – 1146.

[72] Svensson, L. (1997). Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts. *American Economic Review*, 87, 98 – 114.

[73] Summers, L. (1991). How Should Long-term Monetary Policy Be Determined? *Journal of Money, Credit and Banking*, 23, 625 – 631.

[74] Taylor, J. (1979). Estimation and Control of a Macroeconomic Model with Rational Expectations. *Econometrica*, 47, 1267 – 86.

[75] Taylor, J. (1980). Aggregate Dynamics and Staggered Contracts. *Journal of Political Economy*, 88, 1 – 22.

[76] Taylor, J. (1993). Discretion Versus Policy Rules in Practice. *Carnegie-Rochester Series on Public Policy*,North-Holland, 39, 195 – 214.

[77] Taylor, J. (ed.) (1999). *Monetary Policy Rules*. University of Chicago Press for NBER.

[78] Walsh, C. (2003). *Monetary Theory and Policy*. MIT Press.

[79] Woodford, M. (1999). Commentary: how should monetary policy be conducted in an era of price stability? *New Challenges for Monetary Policy*. Federal Reserve Bank of Kansas City.

[80] Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

[81] Woodford, M. (2007). Interpreting inflation persistence: comments on the conference on 'Quantitative Evidence on Price Determination'. *Journal of Money, Credit and Banking* 39, 203 – 210.

[82] Woodford, M. (2008). How Important Is Money in the Conduct of Monetary Policy? *Journal of Money, Credit and Banking.* Forthcoming

[83] Yun, T. (1996). Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles. *Journal of Monetary Economics* 37, $345 - 370$.