Essays on estimating and calibrating the effects of macroeconomic policy over the business cycle

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Abstract

This thesis consists of three chapters on post-reform Chinese business cycles and alternative methods for solving non-linear rational expectations models.

Using quarterly data for the period 1980-2009, Chapter 1 examines the effects of aggregate demand and supply shocks on aggregate fluctuations in China. It further decomposes demand shocks into money supply, money demand and fiscal shocks as in the IS-LM-PC model by applying both long- and short-run restrictions in the context of the structural VAR proposed by Galí (1992). The results show that the estimated impulse responses, in terms of the supply and the three demand shocks, match well with the predictions of the theory. However, as the forecast error variance decompositions show, supply shocks are the main source of fluctuations, accounting for about 89% of output variations in the short-run. Given the nature of this transition economy, this may indicate that there are still institutional obstacles due to incomplete economic reform which prevents the market mechanism from working fully. Despite the overall dominance of supply shocks, the historical decomposition of the five cycles in output between 1983 and 2009 detects important roles played by various demand shocks in some sub-periods. The above results are robust to alternative choices of data for money and interest rate.

In Chapter 2, an RBC model with utility generating government consumption and productive public capital is calibrated to annual Chinese data for the post-reform period 1978-2006. The main findings are: (i) the model generates a reasonable overall account of the business cycles in the Chinese economy; (ii) TFP shocks mainly contribute to the good fit of the model, whilst the two fiscal policy shocks help to further improve

ticular, the generated price dispersions are significantly different across solution methods. The accuracy evaluations in terms of Judd's criteria and Marcet's statistical test show that the PEA performs better than the other two methods, particularly when solving the price-adjustment equation. This result is robust to a number of alternative calibrations.

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Declaration

The material contained in this thesis has not been previously submitted for a degree in this or any other university.

Introduction

This thesis consists of three related chapters comprised of both empirical and theoretical analyses which contribute to the study of business cycles in the post-reform Chinese economy and to solving non-linear rational expectations models more generally. This introduction briefly describes the research background including discussion of the relevant literature and the research objectives of the thesis.

Research background and objectives

This thesis aims to address the following questions:

1. What are the sources of business cycle fluctuations in post-reform China and are all cycles all alike?

2. Can the transmission mechanisms of economic disturbances, particularly monetary and fiscal policy shocks in China, be explained by theories developed for industrialized countries?

3. Can the real business cycle model (RBC) augmented to capture fiscal policy be usefully used to model the Chinese economy?

4. How important is the choice of solution method when solving a non-linear rational expectation model?

Answering the first two questions, in Chapter 1, requires a formal empirical study of the sources of business cycles and transmission mechanisms of various economic disturbances. In empirical studies of both developed and developing countries, the most widely used framework is the structural vector autoregressive (SVAR) model. There have been several studies (Zhang and Wan (2004), Wang (2004) and Siklos and Zhang (2007), etc.) which use the SVAR technique for investigating the sources of business cycles in China. However, these studies have several shortcomings. In particular, they have mainly focused on using long-run restrictions such as those proposed by Blanchard and Quah (1989) and only identified sources of economic shocks into aggregate supply (AS) and aggregate demand (AD) shocks. None of these studies has further investigated the components of AS and AD shocks. This has prevented them from identifying the effects of the sources of AS and AD shocks and thus made analysis of the effects of monetary and fiscal policy impossible. Moreover, their results are relatively mixed, partly due to different data frequency and the sample period employed. There is, therefore, a substantial gap in the existing literature regarding the source of economic shocks in China. This forms the first objective of this thesis - to construct and estimate a SVAR model such that the residuals can be identified as the components of AS and AD shocks which can be interpreted as the structural disturbances in a coherent framework.

Addressing the third question, in Chapter 2, requires that the applicability of the RBC model for the Chinese economy be assessed. There have been numerous models for explaining economic fluctuations. Among these models, the real business cycle (RBC) model has been one of the most influential theoretical frameworks. The aim of the study is to assess the suitability of the RBC model for Chinese data. This is motivated by several considerations. First, the RBC model emphasises the importance of supply shocks, which seems to be supported by existing empirical research on China (Zhang and Wan (2004) and Xu (2007)). Second, the usefulness of the RBC has been further illustrated by its applications to several developing countries (see e.g., Kydland and Zarazaga (2002), Kehoe (2003), Bergoeing *et al.* (2002a, 2002b), Aguiar and Gopinath (2007) and Angelopoulos *et al.* (2010)). In the case of China, application of the RBC model has been rare. The only exception is Hsu and Zhao (2009). However, this study did not provide a full set of assessments of the RBC model, focusing instead only on the volatility moments of output. Other important business cycle features such as persistence and cross-correlations among variables have been ignored. Moreover, the assumed structure of the public sector is quite simple and unable to match the important role played by the Chinese government in the actual economy. Based on the above discussion, conducting a comprehensive set of assessment of an appropriate RBC model constitutes the second objective of this thesis.

The final question concerning the broad yet critical issue of the accuracy of the solutions obtained from rational expectations models is addressed in Chapter 3. The literature has developed various approximation methods for solving non-linear rational expectations models. The accuracy of a solution matters since different solutions imply different optimal decision rules for the economic agents populating the model. These can in turn generate not only different model dynamics but can also affect welfare comparisons for policy analysis. While accuracy of model solution has been extensively examined in the existing literature (Taylor and Uhlig (1990), Christiano (1990) and recently Fernandez-Villaverde and Rubio-Ramirez (2003), Heer and Maussner (2004) and Pichler (2005)), some important omissions remain.

The first such omission is that nearly all of these studies were conducted using the stochastic growth model. They all neglected to examine the accuracy of solutions for more complicated models with market frictions which deviate from the classical assumptions. Surprisingly, despite their popularity, New Keynesian (NK) models have not been previously examined. The second omission is that the majority of the literature, especially the early work, does not provide formal checks for accuracy. Most of it focuses only on simple comparisons of model generated data. These two gaps in the literature motivate the third objective of this thesis - to solve the benchmark NK model using linear and non-linear solution methods and formally evaluate the accuracy of solution methods.

Chapter 1: Sources of business cycle fluctuations in China

1.1 Introduction

Since 1978, economic reform in China has brought about massive changes in both it's economic structure and in the way it conducts economic policy. During this period, the Chinese economy has not only experienced rapid economic growth but also significant aggregate fluctuations. In an attempt to understand the latter, following Galí (1992), this chapter estimates and identifies a structural vector autoregressive (SVAR) model and examines whether the predictions of the SVAR cohere with a simple theoretical framework broadly based on the IS-LM-PC framework which underpins modern New Keynesian models.

Developing theories for research on business cycle fluctuations has been of crucial importance and highly debated in macroeconomics. Notably, regardless of the differences across theories, there have been two broad consensuses reached by researchers. The IS-LM-PC model popularized in the 1970s as the compromise between Classical economics, Monetary economics and the Keynesian economics and more recently the New-Neoclassical-Synthesis model as a hybrid of the New Keynesian and New Neoclassical economics. Both consensuses share the same spirit that they represent synthesis type of models which are particularly useful for addressing aggregate economic issues. In this sense, the NNCS has been considered by many researchers (see e.g., Galí (2000)) as a modern counterpart of the IS-LM-PC model.

Synthesis type models have important potential use for understanding and modeling economic fluctuations in transition economies. Economic fluctuations in transition economies are considered to be related with both economic reforms and economic policies[1]. On the one hand, economic reforms induce changes in factors that effect productivity and efficiency through changing institutions and economic structure. This results in a macroeconomy that relies more on market mechanisms. Thus, the factors

[1] See for example, Fischer and Sahay (2000) and Dibooglu and Kutan (2001) for their explanations that both economic reforms and stabilization policies are responsible for fluctuations in transition economies. In the case of China, researchers such as Naughton (1995) and Imai (1996) attribute fluctuations during the early stage of reform to some old traditions from the central-planned era such as the government's commitment to high state investment. Some other authors, see e.g., Yu (1997) and Fung *et al.* (2000) emphasize the role of credit channel and monetary policy on macroeconomic instability. The line of research such as Yusuf (1994), Oppers (1997), Qian and Roland (1998), Brandt and Zhu (2000) and Feltensteina and Iwata (2005) provide explanations of cycles taking various institutional features of reform, macro control and fiscal and monetary policy into account.

related to reforms are important sources of fluctuations which can broadly be considered as supply side shocks to the economy. On the other hand, transition economies also incur a strong desire for economic policies due to the imperfect working of the market mechanism. These macroeconomic policies represent demand shocks to the economy which also have effects on economic fluctuations[2]. These considerations suggest that a synthesis type model like the IS-LM-PC model is a suitable candidate for studying economic fluctuations in a transition economy.

The empirical framework of this study is based on the SVAR model which has been widely used in both developed countries[3] and developing countries[4]. Similar formal empirical studies on the Chinese economy are very few. Three exceptions are Zhang and Wan (2004), Wang (2004) and Siklos and Zhang (2007). Zhang and Wan (2004) estimated an output-price VAR using quarterly data 1985Q2-2000Q4 in the traditional AS-AD model by assuming that only AS shocks have long-run effect on output (see, Blanchard and Quah (1989)). Their variance decompositions show that AS shocks are only slightly more important than AD shocks for accounting for short-run output fluctuations, whilst AD shocks account for almost all of the fluctuations in inflation. Wang (2004) constructed a three variable VAR model consisting of relative output, real effective exchange rate and relative prices to examine the sources of fluctuations in real exchange rate. By applying the restriction that nominal shocks do not have long-run effect on real exchange rate (see Clarida and Galí (1994)), they decomposed the

[2]There have been positive findings on the effectiveness of fiscal and monetary policies and the existence of money demand relationship in post-reform China. For example, the VAR studies such as in Xie (2004), Qin *et al.* (2005), Geiger (2006) and Dickinson and Liu (2007) have reported positive findings on the effectiveness of monetary policy. Few studies, e.g., Ducanes *et al.* (2006) and Jha *et al.* (2010) also find that there is effective fiscal policy in the Chinese economy. A strand of literature, e.g., Xu (1998), Huang (1994), Gerlach and Kong (2005) and Mehrotra (2006) have found empirical evidence that there exists a stable money demand function in the post-reform economy.

[3]Excellent examples can be found in Blanchard and Watson (1986), Blanchard and Quah (1989), Shapiro and Watson (1988), Galí (1992) and Clarida and Galí (1994) for studying the sources of business cycles in the post-war US economy. It has also been intensively used for studying the Great Depressions in the US economy (see e.g., Cecchetti and Karras (1994)) and in other developed countries (see e.g., Karras (1994) for Germany, France and UK). Recently, the SVAR model has also been used for generating stylized facts about the effects of monetary policy shocks (see e.g., Christiano, Eichenbaum and Evans (2005)), fiscal policy shocks (see e.g., Mountford and Uhlig (2009)) and technology shocks (see e.g., Galí (1999) and recently Galí and Rabanal (2004), Uhlig (2004) and Francis and Ramey (2005)) for contrasting theoretical models.

[4]A number of studies, e.g., Bayoumi and Eichengreen (1994), Morling (2002) and Omar H M N (2009) have employed SVAR models for investigating the responses of key macroeconomic variables such as output and prices to supply and demand shocks in developing countries. Many researchers have also used open economy SVAR models to analyze the impacts of external factors domestic fluctuations in developing countries. See e.g., Ying and Kim (2001) for detecting the sources of capital flows in Korea and Mexico; Dibooglu and Kutan (2001) for examining sources of real exchange rate fluctuations in Poland and Hungary; and Canova (2005), Mackowiak (2007) and Sato, Zhang and McAleer (2009) for studying the impacts of various US shocks on emerging economies.

changes in the three variables into components attributable to AS, real and nominal AD shocks. The results show a dominant role for AS shocks in causing output and inflation fluctuations, while changes in real effective exchange rate are mostly related to real AD shocks.

Siklos and Zhang (2007) conducted a similar analysis as in Zhang and Wan (2004) but also considered two extensions of the benchmark Blanchard and Quah identification: (i) Blanchard and Quah identification with correlated AS-AD shocks as in Cover, Enders and Hueng (2006); and (ii) Blanchard and Quah identification in a trivariate (output-price-money) VAR model as in Bordo and Redish (2003). They examined the sources of inflation and output fluctuations using quarterly data from 1990Q1 to 2004Q3. Their results show substantial differences depending on which identification scheme is used. There are several shortcomings of their strategies. The first point is that they did not justify the use of each strategy, with no empirical evidence to support correlated AS-AD shocks. Second, some long-run restrictions in their third strategy are unusual. For example, their third identification relies on the assumption that there is no long-run impact of demand shocks on prices which is only valid for small open economies as considered in Bordo and Redish (2003) but not for China[5]. Third, their empirical framework is still unable to provide a suitable basis for testing economic theory and analyzing the components of demand shocks such as fiscal and monetary policies.

Compared with the literature, this study offers three main innovations for analyzing fluctuations in the Chinese economy. Firstly, this work represents the first attempt to further decompose AD shocks into money supply, money demand and IS shocks[6]. Long-run restrictions of Blanchard and Quah (1989) and short-run restrictions of Galí (1992) are applied to identify the economic disturbances as the four structural shocks found in the IS-LM-PC model. This allows an evaluation of the consistency of fiscal and monetary effects with those predicted in market economies as a gauge of the progress of economic reform. Secondly, a relatively long sample size of high frequency data is used (quarterly data from 1980Q1 to 2009Q3). The advantage to the research is that the relatively long sample size better justifies the long-run restriction while the quarterly frequency justifies the use of short-run restrictions. Thirdly, there is a formal

[5]For example, this assumption is obviously at odds with the empirical studies mentioned earlier which support the existence of long-run money demand.

[6]Due to the lack of data on producing input of output such as employment, it is not possible to conduct further decomposition of supply shocks such as the one proposed in Shapiro and Watson (1988).

and complete analysis of the sources of economic fluctuations over the sample period. The overall importance of each shock is inferred from the variance decompositions and the contribution of each shock in different periods is learned from the historical decompositions. Due to the restriction of small sample size, the previous study, i.e., Zhang and Wan (2004) was only able to characterize part of the cycles recognized in the Chinese economy. Taking the advantage of the long sample size, we are able to provide the first complete characterization of all the five cycles from 1983 to 2009.

The remainder of this chapter is as follows: Section 1.2 gives a brief description of the sources and construction of the data used in this study. Section 1.3 provides a short description of the institutional background of the Chinese economy. Section 1.4 describes the SVAR model and illustrates the identification schemes in terms of long- and short-run restrictions. Section 1.5 presents the empirical results of the estimated impulse responses to examine if the IS-LM-PC fits the post-reform Chinese data. The importance of each economic shock is investigated in Section 1.6 by computing the variance decompositions and the historical decompositions for the period 1983-2009. Section 1.7 considers alternative measures of variables to check the robustness of results and an alternative specification of the VAR to examine the sources of the unit root in nominal variables. Section 1.8 presents the conclusions.

1.2 Data

1.2.1 Description of data

The data used for estimation includes[7]: quarterly real GDP, the consumer price index (CPI), the one-year bank lending rate and the nominal money ('money plus quasi-money') from 1980Q1 to 2009Q3[8].

Real GDP was chosen as a measure of the output since it is a more comprehensive index for measuring output than other related series such as industrial production or consumption. For example, these related series can not capture the changes in services which have gained substantial increases proportional to GDP in recent years. Moreover, data such as industrial production has missing values even in the 1990s.

The CPI is chosen as a index for the price level due to the fact that the data for GDP deflator has never been published in China. It is impossible either, to infer the implicit GDP deflator in our full data sample since the quarterly nominal GDP data

[7]Although high frequency data is preferred, it is not possible to use monthly data since it is of poor quality and suffers from missing values even for recent years.

[8]The sources and construction of data for each variable are provided in Appendix (1.9.1).

is only available since 1992. Moreover, other price indices such as retail price index (RPI) and producer price index (PPI) also have much shorter data lengths.

The measure of nominal interest rate is the bank lending rate. The bank deposit rate will also be used in estimation but only as a robustness check.

The data of 'money plus quasi-money' is used as the measure of the nominal money holdings. The term 'money plus quasi-money' refers to a measure of money that is wider than M1 (currency plus checkable deposits) but narrower than M2 (currency plus overall deposits). It has been used by the People's Bank of China (the central bank of China, PBC, hereafter) since 1979. The possible alternative, the narrow money M1 alone is also considered at the end of this study as a robustness check.

One inevitable issue in the literature concerning the empirical research for the Chinese economy is the credibility of data. While doubts and criticisms on the official Chinese statistics can be found for example, in Rawski (2001), Young (2003) and Holz (2004), positive views have also been found in Chow (1985), Klein and Ozmucur (2002), Chow and Shen (2004), Holz (2005) and Chow (2006). To ensure the accuracy and consistency of the data, this study makes use of the most recent data from the database of the National Bureau of Statistics of China (NBSC, hereafter)[9]. The use of Chinese statistics are supported by the view in Chow (2006) that the Chinese statistics are 'by and large reliable'.

1.2.2 Preliminary tests

Some preliminary tests related to the long-run properties of data are required prior to the specification and estimation of the VAR model below. The data series to be tested are : the log of real GDP, y, the one-year lending rate, i, the log of the CPI in levels, p, the log of money plus quasi-money, m, the real interest rate, $ri = i - \triangle p$, and the log of real money, $rm = m - p$.

Following Engle and Granger (1987), the degree of integration of each data series and the existence of cointegration relationships must be examined so as to determine the order of differencing of data. Additionally, since the Chinese data at hand are not seasonally adjusted, we also need to seasonally adjust the data before conducting standard unit root tests. That is, unit root tests at seasonal frequencies are also required. In what follows, the preliminary tests are conducted in two steps.

[9] Given the intensive debates on the overestimation of real GDP growth rate during 1997-2002 and the underestimation of it from 2003 upwards, the NBSC has been revising its GDP statistics since 2004.

Firstly, there is the test for the existence of unit roots at seasonal frequencies for quarterly data using the procedure proposed in Hylleberg, Engle, Granger and Yoo (1990) (HEGE, hereafter). There are four seasonal unit roots considered here for quarterly data $\{1, -1, \pm i\}$, at frequencies $\left\{0, \pi, \frac{\pi}{2}\right\}$. The ordinary least squares (OLS) regression which the tests are based on, takes the following form:

$$\varphi^{*}(L)\, s_{4t} = \pi_1 s_{1t-1} + \pi_2 s_{2t-1} + \pi_3 s_{3t-2} + \pi_4 s_{3t-1} + d_t + \beta t + \varepsilon_t \tag{1}$$

where $\varphi^{*}(L)$ is the lag polynomial defined in HEGE equation (3.2), d_t and t are the seasonal dummies and the time trend respectively, and s_{1t}, s_{2t}, s_{3t} and s_{4t} are the four different seasonal differences of the original data x_t, i.e.:

$$\begin{aligned} s_{1t} &= \left(1 + L + L^2 + L^3\right) x_t \\ s_{2t} &= -\left(1 - L + L^2 - L^3\right) x_t \\ s_{3t} &= -\left(1 - L^2\right) x_t \\ s_{4t} &= \left(1 - L^4\right) x_t. \end{aligned}$$

The null hypothesis of the seasonality test is that there are unit roots at all frequencies. Therefore, the absence of unit root at any seasonal frequency requires that π_1, π_2, π_3 and π_4 are all different from zero. Moreover, since the third and fourth unit roots are complex numbers with the same root, a jointly test for π_3 and π_4 can be used. Therefore, the seasonal unit root tests involve two t tests with respect to the null hypothesis of $\pi_1 = 0$ and $\pi_2 = 0$ and a F test with the null hypothesis that both π_3 and π_4 are zero.

The results of the seasonal unit root tests are reported in Table 1.1A below. A seasonal dummy is always included in the OLS regression. Both cases with and without a time trend in the regressions are considered. The test statistics are then compared with the critical values taken from HEGE Tables 1a and 1b. It is shown that none of the data series can be considered as stationary since not all the seasonal unit root tests reject the null hypothesis of unit roots. If the focus is on seasonal frequencies π and $\frac{\pi}{2}$, only the nominal interest rate does not have any seasonal pattern. These results imply that all the data need to be seasonally adjusted except nominal interest rate. The program TRAMO-SEATS is used to seasonally adjust all the raw data except the nominal interest rate[10]. In the process of seasonal adjustment, a pre-test

[10] The TRAMO-SEATS package is developed by Goez and Maravall (1996) for the Central Bank of Spain and has been widely used by the EU countries. Another popular program in seasonal adjustment

for the log/level specification is always used and the adjustment automatically detects for outliers and accounts for trading day and Leap year effects.

Table 1.1A: Seasonal Unit Root Tests

Sample Period: 1980Q1 to 2009Q3

Variable	Trend	π_1	π_2	F_{π_3,π_4}
Real GDP (y)	No	−0.52	−1.54	4.37
	Yes	−3.38	−1.60	4.10
Prices (p)	No	−2.06	-3.06^{*}	5.93
	Yes	−0.87	-3.05^{*}	5.86
Nominal Interest Rate (i)	No	−1.28	-3.74^{*}	17.11^{**}
	Yes	−2.15	-3.71^{*}	16.92^{**}
Real Interest Rate	No	−1.46	−2.53	6.58
($i - \triangle p$)	Yes	−2.08	−2.51	6.65
Nominal Money (m)	No	−1.22	−2.27	4.48
	Yes	−1.38	−2.28	5.15
Real Money	No	0.35	−2.62	4.48
($rm = m - p$)	Yes	-3.61^{*}	−2.78	5.15
Frequency of unit root		0	π	$\frac{\pi}{2}$

$*$ →Significant at the 5% level $**$ →Significant at the 1% level

After the seasonal adjustment, we check the adjusted data series again and test for the seasonality to make sure that all the seasonal patterns in data have been removed.

is the X-12 which is used by the United States Bureau of Labor Statistics. It uses the X-12-ARIMA method to seasonally adjust data series. This study has applied both TRAMO-SEATS and X-12 to do the seasonal adjustment. The resulting series turn out to be very similar. Moreover, the results of the structural VAR analyses using the two different seasonally adjusted series are also very similar.

These are reported in Table 1.1B:

Table 1.1B: Seasonal Unit Root Tests for Seasonally Adjusted data

Sample Period: 1980Q1 to 2009Q3

Variable	Trend	π_1	π_2	F_{π_3,π_4}
Real GDP (y)	No	-0.44	-4.36^{**}	21.61^{**}
	Yes	-3.52^{*}	-4.61^{**}	24.31^{**}
Prices (p)	No	-2.15	-3.77^{**}	26.34^{**}
	Yes	-0.86	-3.75^{*}	26.04^{**}
Nominal Interest Rate (i)	No	-1.28	-3.74^{*}	17.11^{**}
	Yes	-2.15	-3.71^{*}	16.92^{**}
Real Interest Rate	No	-1.88	-4.22^{**}	20.72^{**}
$(i-\triangle p)$	Yes	-2.36	-4.19^{**}	20.39^{**}
Nominal Money (m)	No	-1.21	-3.94^{**}	14.06^{**}
	Yes	-1.68	-3.98^{*}	15.41^{**}
Real Money	No	0.28	-4.30^{**}	14.06^{**}
$(drm = \triangle m - \triangle p)$	Yes	-3.86^{*}	-4.57^{**}	15.41^{**}
Frequency of unit root		0	π	$\frac{\pi}{2}$

$*$ →Significant at the 5% level $**$ →Significant at the 1% level

Table 1.1B shows that the seasonal components in the raw data have been successfully removed. All the statistical tests reject the null of seasonal unit roots at frequencies π and $\frac{\pi}{2}$ and almost all of them are significant at the 1% level. Moreover, since the seasonal adjustment only removes the unit root at seasonal frequency, most of the t tests for π_1 cannot reject the null hypothesis of unit root at zero frequency.

The second step is to test the unit roots at zero frequency for the seasonally adjusted data. Here we consider two types of tests, i.e., the augmented Dickey Fuller (ADF) test and the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) test[11]. The former is based on a OLS regression of the form, $\triangle x_t = \alpha + \beta t + \phi x_{t-1} + \delta(1-L)x_{t-1} + \varepsilon_t$ and tests the null hypothesis of unit root, $H_0 : \phi = 0$. The KPSS test on the other hand, tests the null hypothesis that a time series x_t is stationary. It starts with the regression: $x_t = \alpha + \beta t + \mu_t + \varepsilon_t$, with $\varepsilon_t \rightarrow I(0)$ and $\mu_t = \mu_{t-1} + \epsilon_t$, $\epsilon_t \backsim WN(0, \sigma_\epsilon^2)$. The construction of the KPSS statistic is then the Lagrange multiplier (LM) statistic for testing the variance of the random walk process being zero: $H_0 : \sigma_\epsilon^2 = 0$. The

[11]Another widely used unit test, i.e., the Phillips-Perron test is not conducted in this study. Some researchers, e.g., Schwert (1989) point out that Phillips-Perron test is likely to be more biased than the ADF test when the data process takes an ARIMA representation.

results of the ADF test and the KPSS test for the seasonally adjusted data series are shown in Table 1.2A (the last two columns). The results of the same tests for the first-differenced data series are shown in Table 1.2B (the last two columns). The length of the time lag we choose is 4 in both the ADF and the KPSS tests.

Table 1.2A: Unit Root Tests For Seasonally Adjusted Variables[12]

Sample Period: 1980Q1 to 2009Q3

Variable	Trend	ADF test	KPSS test
Log Real GDP (y)	No	-0.65	2.47^{**}
	Yes	-4.48^{**}	0.11
Log Prices (p)	No	-2.02	2.32^{**}
	Yes	-0.94	0.56^{**}
Nominal Interest Rate (i)	No	-1.31	1.13
	Yes	-2.08	0.39
Real Interest Rate	No	-2.75	0.72^{*}
($ri = i - \triangle p$)	Yes	-3.21	0.35^{**}
Log Nominal Money (m)	No	-1.26	2.47^{**}
	Yes	-1.00	0.54^{**}
Log Real Money	No	0.35	2.48^{**}
($rm = m - p$)	Yes	-4.47^{**}	0.10

$*$ →Significant at the 5% level $**$ →Significant at the 1% level

[12] The critical values of the ADF test at the 1% and the 5% significance level are -3.49 and -2.89 respectively when only constant is included in regressions. The same change to -4.04 and -3.45 respectively when both a constant and a time trend is included in regressions. On the other hand, the critical values of the KPSS test at the 1% and the 5% level are 0.74 and 0.46 respectively when only a constant is included in regressions. The same changes to 0.22 and 0.15 when both a constant and a time trend is included in regressions.

Table 1.2B: Unit Root Tests For First-differenced Variables

Sample Period: 1980Q1 to 2009Q3

Variable	Trend	ADF test	KPSS test
Differenced Real GDP ($\triangle y$)	No	-3.62^{**}	0.099
	Yes	-3.55^{*}	0.073
Differenced Prices ($\triangle p$)	No	-3.28^{*}	0.70^{*}
	Yes	-3.87^{*}	0.11
Differenced Nominal Rate ($\triangle i$)	No	-4.09^{**}	0.15
	Yes	-4.09^{**}	0.11
Differenced Real Rate ($\triangle ri$)	No	-6.15^{**}	0.06
	Yes	-6.12^{**}	0.04
Differenced Nominal Money ($\triangle m$)	No	-2.91^{*}	0.68^{*}
	Yes	-3.32	0.18^{*}
Differenced Real Money	No	-4.12^{**}	0.05
($\triangle rm = \triangle m - \triangle p$)	Yes	-4.10^{**}	0.05

$*$ →Significant at the 5% level $**$ →Significant at the 1% level

The unit root tests for the seasonally-adjusted data in Table 1.2A suggest that real GDP is either difference-stationary or trend stationary. For example, the ADF test without trend in regression cannot reject the null of unit root and the KPSS test rejects the null of stationarity at 1% level. On the other hand, the ADF test with trend in regression rejects the null of unit root at the 1% level and the KPSS test cannot reject the null of stationarity. However, the trend-stationary results might be the Type I error. It can be seen from Table 1.2B that the unit root tests for the first-differenced real GDP reject the unit root at the 1% level for both with trend and without trend cases. If the true data process of real GDP was trend stationary, a first-differencing would introduce a unit root in it and thus should result in non-rejection of the ADF test and a rejection of stationarity in the KPSS test. However, this is obviously not the case. Therefore the conclusion is that the log of real GDP data, y, is $I(1)$. This is also consistent with the findings in literature (see for example, Zhang and Wan (2004) and Wang (2004)).

The results also suggest that the seasonally adjusted prices and nominal money are also $I(1)$ processes. The ADF tests for the two variables in level cannot reject the unit root while the ADF tests for the two variables in first-differences reject the null of unit root at the 5% level. The KPSS tests also come to the same conclusion. Given the stationarity in differenced nominal money and prices, the differenced real money should be also stationary, which is confirmed by the unit root tests for differenced real money in Table 2B.

Finally, the specification of the covariance stationary vector process then depends on the long-run property of the nominal and real interest rates. Apparently, the ADF and KPSS tests in Table 1.2A suggest that both nominal and real interest rates are not stationary. The same tests for the first-differenced data imply that the nominal and real interest rates are all $I(1)$ processes. However, these results should be rejected based on the following considerations. First, the nominal interest rate has been administratively fixed in the short-run. It is then adjusted with a shift to a new level according to the climate of economy. Although it still shows variations in the long-run, the short-term fixed feature amounts to introduce many structural breaks in both nominal and real interest rates. By consequence, the unit root tests might be biased in favor of a unit root process (Type II error). Second, the existence of a unit root in the real interest rate is hardly reconciled with economic theory which makes our analysis difficult to interpret. Given the identified $I(1)$ process for prices, the stationarity of real interest rate requires that the nominal interest rate is also stationary. Therefore, based on these two considerations both the nominal and real interest rates are treated as stationary processes.

Based on the discussions above, the plausible long-run properties of the data are given by: $y \to I(1)$, $m \to I(1)$, $p \to I(1)$ and $i \to I(0)$. Accordingly, the covariance stationary process would be $[\triangle y, i, \triangle p, \triangle m]$. Note that this specification has important implications for the long-run property of nominal variables. For example, the absence of unit root in the nominal interest rate means that there is no money-real interest rate relationship (thus an LM equation) in the long-run. Moreover, the stationary money growth implies that any monetary intervention is absent in the long-run.

1.3 Some Institutional Background

Prior to illustrating the structural model used in this study, this section provides an overview of the institutional background of the Chinese economy. In particular, since much of the work is on decomposing the demand shocks in the IS-LM-PC framework, the focus will shift to the institutional changes in the financial and fiscal reforms and the conduct of monetary and fiscal policy.[13]. This section will serve two purposes.

[13]It is argued that other aspects of the economic reform such as enterprise reform, labour reform and reform in trade and openness mainly have supply side effects on the economy through changing

First, it helps to justify the methodology of this study, especially the identification strategy used for the SVAR model. Second, it provides a context for understanding and interpreting the empirical results.

1.3.1 Financial reform and monetary policy

Since 1979, the financial market has been built and reformed to fulfil its function for allocating financial resources and for providing the environment for the conduct of monetary policy. Studies of the financial reform can be seen among others, in De Wulf and Goldsbrough (1986), Brandt and Zhu (1995) and Lardy (1998). Here we focus on three aspects of the financial reform which are closely related with our purpose: the emergence of the new banking system, the financial decentralization and the conduct of monetary policy.

A summary of the institutions of the financial market and the macro management in the pre-reform era is as follows. Before 1979, the allocation of financial resources was totally controlled by the central government. Accordingly, the conduct of monetary policy was in the form of credit plan mostly controlled by the central government. The credit was extended to enterprises in the form of budgetary grants. The decisions of amount, distribution and purpose of these funds, were made by the Ministry of Finance, a department of the State Council. On the other hand, the role of the PBC, currently the central bank of China, was only accommodating the central plan and issuing bank credit to enterprises for very specific production plans. Therefore, there was no 'market' or formal monetary policy replying on market in the pre-reform era.

After the economic reform in 1979, the government started to build a financial market where financial resources can be allocated by the market mechanism. The highly centralized financial system was reformed from 1979 to 1984 when a set of specialized commercial banks[14] were founded and the State Council assigned the PBC the function of a modern central bank. This resulted in a new banking system which has gained increasing importance for allocating financial resources. For example, the PBC issued bank credits to commercial banks which were then extended to enterprises. Although other financial markets were also established in the 1980s such as the bond market and the stock market, most of the allocation of financial resources had still been through

technological progress and efficiency. Thus, we do not provide a detailed review on these market reforms since we are not focusing on decomposing the supply shocks.

[14] There were four specialized commercial banks established during 1979-1984: the Industrial and Commercial Bank, the Agricultural Bank of China, the People's Construction Bank of China and the Bank of Communications.

the banking system. Today, the financial reforms are still ongoing. The incomplete reform makes the financial market different from those in industrialized countries. For example, the bank interest rates (deposit rate and lending rate) were fixed in the short-run. In fact, the interest rates were used mainly for attracting households' savings by the commercial banks which were then used for financing investment. Many observers (see for example, Naughton (1987)) have seen rapid increases in bank deposits since the 1980s.

A key feature of the financial reform was the decentralization of decision-making. Due to the setup of branches of the PBC at the provincial level, the branches were granted the autonomy to decide how much credit to be extended to the state-owned commercial banks. The local branches of the state-owned commercial banks then in turn obtained the discretion in issuing loans to enterprises. For example, the commercial banks were allowed to retain some of their profits gained from lending money to enterprises. Also, they were also allowed to decide the distribution of loans between state and non-state sectors.

At the start of the 1990s, more decentralization occurred. The inter-bank market was established where a more flexible nominal interest rate was used to facilitate bank businesses between the commercial banks. The decentralization in financial system has been found to be crucial for the reallocation of bank credits across different sectors of the economy. In particular, many researchers (see for example, Chai (1997) and Brandt and Zhu (2000)) have observed that this financial decentralization has directed more funds from less productive state owned enterprises (SOEs) to more productive non-state owned enterprises (non-SOEs) and thus contributed to economic growth.

The newly-built banking system has also changed the way in which the monetary policy was conducted[15]. Although the target of the PBC has long been the stability of price level, the way it achieves this target has changed after the reform of the financial system. Compared with the monetary policy prior to 1979 when the allocation of financial resources were controlled by the state council through credit plan, now the PBC act as the central bank and sets monetary targets for the economy. This sort of bank credit is usually referred as the indicative plan since it relies mostly on the banking system itself rather than the administrative controls. Two characteristics are noteworthy. First, the monetary policy was not fully independent of the central

[15] Studies on the implementation of monetary policy during the post-reform period can be seen for example, in Chow (1987), Naughton (1987), Chen (1989), Yusuf (1994), McKinnon (1994) and Xie (2004).

government especially during the early reform period 1979 - 1998. Instead, the monetary policy had been characterized by both indicative credit plans and administrative controls[16]. This makes the monetary policy difficult to implement independently and effectively. In 1995, 'central bank law' was issued which stating the PBC as the independent institution for conducting monetary policy. As a result, the administrative credit plan has rarely been used since 1996. Instead, the indicative credit plan has played a more important role in the conduct of monetary policy. Second, unlike the monetary policy practice in market economies where central banks set interest rates, a fundamental feature of the conduct of monetary policy in China is that the PBC has targeted the money aggregate. For example, the PBC has announced clear monetary targets for money supply during the period 1998-2002 (see, Xie (2004)). The use of money aggregate rather than interest rate for monetary target is mainly due to the incomplete reform in financial market.

1.3.2 Fiscal reform

Another important factor in the changes in the public sector is fiscal reform. Compared with the vast studies on the financial reform and the monetary policy, there has been far less on fiscal reform in China.

Two features have been identified as worthy of interest. The first feature of fiscal reform is the fiscal decentralization recognized, among others, by Bell *et al.* (1993), Tseng *et al.* (1994), Hofman (1993), Lardy (1998) and Ma (1997). The central feature of the fiscal reform is the decentralization of decision-making between the central government and the local governments. For example, during the 1980s, the local governments were allowed to retain most of their revenues and make their own decisions on spending. The new tax system was adopted in 1994 to categorize the local tax revenues into central government tax revenues, local government tax revenues and revenues shared by both. The budget plan was also made according to different administrative levels. The intention of the fiscal decentralization was to enforce the control of the central government on tax and stimulating investment of local governments. However, it resulted in enormous decreases in the budget revenue of the central government. The fall in budget revenues has continued until 2000 when the government enforced fiscal centralization through the tax system.

[16] Typical examples of the government intervention were observed during economic overheating (such as in 1981, 1989-90 and 1993-94) where the government forced the PBC to apply tight monetary policy to restrict the amount and distribution of funds to the commercial banks. Declines in output and inflation in the following periods were observed.

The second feature of fiscal reform is that the central government has discriminated its policies between SOEs and non-SOEs. In particular, it is shown that the central government still has a commitment to the state sector due to the old tradition that it takes the majority of it's support from the workers in the SOEs. Thus maintaining employment and wage level in the SOEs is important for political considerations. Some researchers (see e.g., Brandt and Zhu (2000)) show that the central government has made transfers to the state sector during output growth. Moreover, when the tax revenue is not enough to finance the transfers, the central government has to collaborate with the PBC and finance the transfers through money creation. This in turn, affects inflation. Thus, the fiscal reform also has important influence on the macro economy.

1.4 The Structural VAR Model

The general approach of the SVAR model is to identify the economic disturbances from the residuals in an estimated VAR model by applying a set of economic restrictions. Specifically, this study applies the methodology used in Galí (1992) and adopts both long- and short-run restrictions in the IS-LM-PC framework[17]. By applying Galí's approach, the economic disturbances in the VAR can be identified as the four structural disturbances as in the IS-LM-PC model.

The long-run restriction that aggregate demand shocks do not have permanent effect on output is used to isolate the supply shocks from demand shocks. This long-run restriction was introduced by Blanchard and Quah (1989) and has been widely used in empirical studies such as Shapiro and Watson (1988), Bayoumi and Eichengreen (1994), Clarida and Galí (1994), Galí (1999), Christiano, Eichenbaum and Vigfusson (2003), Francis and Ramey (2004) and Galí and Rabanal (2004).

The three types of demand shocks, i.e., money supply shocks, money demand shocks and IS shocks as in the IS-LM-PC model are identified by applying three short-run restrictions on the contemporaneous impact of the money shocks on output and the contemporaneous reaction of output and prices to money shocks. These short-run restrictions are built on the work by Blanchard and Watson (1986), Bernanke (1986) and Sims (1986) and more applications in Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (2005)[18].

[17] This strategy of using both long-run and short-run restrictions has also been used in empirical studies such as Cechetti and Karras (1994) and Karras (1994) for their study of causes of US economic fluctuations during the Great Depression.

[18] Other identification schemes in literature include Kim and Roubini (2000) and Cushman and Zha (1997) for open economy SVAR and Uhlig (1999), Faust (1998), Canova and de Nicolo (2002) and

It is noteworthy that Galí's approach provides an advantage that the long- and short-run restrictions are imposed in a sequential order and thus are independent of each other. That is, even if one or two restrictions fail to work, other restrictions are still valid in identifying other structural shocks. This also enables a comparison of the results in each step of the identification to the result found in the literature. The following will illustrate the main predictions of the IS-LM-PC model, the specification of the SVAR and the implementation of the identifying restrictions.

1.4.1 The IS-LM-PC model

Before estimating the SVAR, the main predictions of the Phillips Curve augmented IS-LM-PC model are highlighted so that they can be compared with the estimated dynamics of the SVAR[19]. The structure of the basic IS-LM-PC model can be summarized as:

$$\begin{aligned}
y_t &= \alpha + \mu_{s,t} - \sigma\left(i_t - E_t \Delta p_{t+1}\right) + \mu_{is,t} && \text{(IS equation)} \\
m_t - p_t &= \phi y_t - \lambda i_t + \mu_{md,t} && \text{(LM equation)} \\
\Delta m_t &= \mu_{ms,t} && \text{(Money supply)} \\
\Delta p_t &= \Delta p_{t-1} + \beta\left(y_t - \mu_{s,t}\right) && \text{(Phillips Curve)}
\end{aligned}$$

where y, i, m and p are the endogenous variables denoting the log of output, the nominal interest rate, the log of the money supply and the log of the price level respectively. The subscripted variables μ_s, μ_{ms}, μ_{md} and μ_{is} represent the four structural shocks in the economy, i.e., aggregate supply, money supply, money demand and government spending which are assumed to follow stochastic processes. The first difference operator Δ is used to calculate money growth Δm and inflation Δp. The real interest rate is given by $(i - E\Delta p_{t+1})$ where E is the expectations operator.

Although the IS-LM-PC model has been criticized for missing micro-foundations (see e.g., Lucas (1972)) especially on the supply side, it is still useful in providing stylized predictions on the dynamics of the economy[20]. For example, the IS equation

Mountford and Uhlig (2005) for sign restrictions on the impulse response functions.

[19] The IS-LM model is proposed by Hicks and its Phillips Curve augmented extensions are made by Dornbusch (1990), Romer (2000) and Taylor and Moosa (2002).

[20] Recent developments in macroeconomics (either in terms of the Real Business Cycle (RBC) model or the New Keynesian model) have built a new consensus which is broadly related to this framework. For example: (i) the evolution of output gap in the New Neo-Classical Synthesis (NNCS) maps with the IS curve; (ii) the demand for real balances maps with the LM curve; and (iii) the traditional Phillips Curve evolves to the New Keynesian Phillips Curve (NKPC). Some authors (see for example, Carlin and Soskice (2005) and Benigno (2009)) also show that the NNCS has a graphical exposition as in the traditional IS-LM-PC model. Finally, Galí (2000) suggests that recent development of NNCS as a return of the IS-LM-PC framework.

captures the negative relationship between the real interest rate and output, the latter is also positively affected by favourable supply side shocks and public spending. The LM equation defines how the money demand for real balances is satisfied given output and nominal interest rate. The money supply equation assumes that monetary authority conducts monetary policy by controlling the money aggregate. While this is unlikely the case in most of the developed countries, it is a good description of how monetary policy is conducted in China. Finally, the IS-LM-PC model is complemented with a Phillips Curve to reveal how prices in the short-run evolve along with the output gap, i.e., the difference between output and its potential value, μ_s.

With this structure in place, the following predictions relating to the money-interest-output transmission mechanism are of particular interest:

- Aggregate demand shocks can have short-run effects on real variables due to nominal rigidities such as slow adjustments of prices.

- The money-real interest rate - output channel - Money shocks change real interest rate given price rigidity, which in turn affect output.

Examining the empirical validity of the above predictions is crucial for understanding roles of demand and supply shocks and how they generate aggregate fluctuations.

1.4.2 Specification of the SVAR

The SVAR can be represented in its moving average form as:

$$\mathbf{x} = \mathbf{C}\left(\mathbf{L}\right)\boldsymbol{\varepsilon} \tag{2}$$

where $\mathbf{x}$ is a $n \times 1$ covariance stationary vector which contains the dependant variables of interest, $\boldsymbol{\varepsilon}$ consists of a $n \times 1$ vector of structural disturbances which are serially uncorrected and $\mathbf{C}\left(L\right) = C_0 + C_1 L + C_2 L^2 + ...$denotes the $n \times n$ matrix of the current and lagged effects of structural disturbances on dependant variables with L as the normal lag operator. For simplicity, the time script has been suppressed.

To make the SVAR comparable with the IS-LM-PC model discussed above, we consider four structural shocks in $\boldsymbol{\varepsilon}$: aggregate supply, money supply, money demand and IS. The matrix $\mathbf{x}$ contains the transformed variables of output, real or nominal interest rate, prices and money with the type of transformation depending on the data property. It is noteworthy that the inclusion of the real or nominal interest rate in the VAR might cause problems due to the fact that the nominal interest rate in China has been administratively fixed in the short-run. Nonetheless, we still stick with the four-variable VAR specification due to two considerations. First, although the nominal interest rate is fixed in the short-run, it is flexible in the medium- and long-term. In fact, it has been adjusted by the PBC based on different conditions of the economy. For example, during economic overheating, the PBC usually tightens the money supply to raise interest rates, providing that higher nominal interest rate increases the borrowing cost of bank loans. Also, some recent studies (see for example, Koivu (2009)) find that the Chinese macroeconomic variables have become increasingly responsive to changes in the real interest rates. These suggest that the changes in nominal interest rate might correctly reflect economic conditions, which can be examined in our SVAR framework. Second, the inclusion of the real interest rate provides a four-variable VAR that is suitable for distinguishing money supply from money demand shocks using short-run restrictions as in Galí (1992). It is impossible, on the other hand, to identify the two money shocks in the IS-LM-PC model using alternative VAR models with less than four variables.

Based on the data analysis in section 1.2, we adopt the following specification as the benchmark case[21]:

$$\begin{pmatrix} \Delta y \\ i \\ \Delta p \\ \Delta m \end{pmatrix} = C\left(L\right) \begin{pmatrix} \varepsilon_{as} \\ \varepsilon_{ms} \\ \varepsilon_{md} \\ \varepsilon_{is} \end{pmatrix} \tag{3}$$

where Δy denotes the growth rate (log-difference) of output, i is the nominal interest rate, $i - \Delta p$ represents the real interest rate and $\Delta m - \Delta p$ is the growth rate of real balances of money holdings. The disturbances ε_{as}, ε_{ms}, ε_{md} and ε_{is} are the aggregate supply, money supply, money demand and IS shocks respectively. According to the IS-LM-PC model, the two money shocks and the IS shock are together categorized as aggregate demand shocks.

1.4.3 Blanchard-Quah and Galí identification scheme

Since the structural disturbances are not observable, the structural model in (3) and the polynomial matrix $\mathbf{C}\left(\mathbf{L}\right)$ can not be directly estimated from data. One can then estimate the reduced form VAR model which, in moving average form, is given by:

$$\mathbf{x} = \mathbf{B}\left(\mathbf{L}\right)\mathbf{v} \tag{4}$$

[21] The analysis based on the alternative specification allowing a unit root in nominal variables will be conducted below using the same identification strategy.

where $\mathbf{B}(\mathbf{L}) = B_0 + B_1 L + B_2 L^2 + ...$ is the $n \times n$ matrix of polynomial lag of the estimated coefficients and $\mathbf{v}$ is the $n \times 1$ residual matrix. Once $\mathbf{B}(\mathbf{L})$ is known, one needs to identify $\mathbf{C}(\mathbf{L})$ so that the structural impacts of shocks in $\boldsymbol{\varepsilon}$ on $\mathbf{x}$ can be also found. To do this, substitute the reduced form moving average process (4) back into the structural moving average process (2) to obtain,

$$\mathbf{B}(\mathbf{L})\mathbf{v} = \mathbf{C}(\mathbf{L})\boldsymbol{\varepsilon}. \tag{5}$$

To proceed, note that the unknown structural disturbances in $\boldsymbol{\varepsilon}$ are assumed to be linear combinations of the reduced VAR residuals in $\mathbf{v}$:

$$\mathbf{v} = \mathbf{S}\boldsymbol{\varepsilon} \tag{6}$$

where $\mathbf{S}$ is a $n \times n$ full rank matrix. Substituting above result back into (5) yields,

$$\begin{aligned} \mathbf{B}(\mathbf{L})\mathbf{S}\boldsymbol{\varepsilon} &= \mathbf{C}(\mathbf{L})\boldsymbol{\varepsilon} \\ &\Rightarrow \mathbf{C}(\mathbf{L}) = \mathbf{B}(\mathbf{L})\mathbf{S}. \end{aligned} \tag{7}$$

Now the identification of the structural coefficients matrix $\mathbf{C}(\mathbf{L})$ is transformed to the identification of the matrix $\mathbf{S}$.

To find the $n \times n$ matrix $\mathbf{S}$, one needs to find $n^2 = 16$ equations to exactly identify its 16 elements. A first set of equations can be naturally obtained from the assumption that the four structural disturbances are not correlated[22] and therefore their covariance matrix is an identity matrix, $\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}' = I$. This can be achieved by taking the variance of both side of equation (6):

$$\mathbf{v}\mathbf{v}' = \mathbf{S}\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'\mathbf{S}'$$

where

$$\widehat{\boldsymbol{\Sigma}} = \mathbf{S}\mathbf{S}'$$

is the covariance matrix of the VAR residuals which is also known after estimation. Since this covariance matrix $\widehat{\boldsymbol{\Sigma}}$ is symmetric, it provides $n(n+1)/2 = 10$ equations for identifying $\mathbf{S}$. Therefore an additional number of $n(n-1)/2 = 6$ restrictions are needed to just identify $\mathbf{S}$.

This study applies the identification scheme in Blanchard and Quah (1989) and Galí (1992) to provide six additional restrictions concerning both the long-run and short-run behavior of the structural shocks. The design and implementation of the

[22] See for example, Cover, Enders and Hueng (2006) for a discussion of the restrictions allowing correlated aggregate demand and aggregate supply shocks.

IS-LM-PC identification is carried out as follows. First, following Blanchard and Quah (1989), we use a long-run restriction that demand shocks do not affect long-run output to identify aggregate supply shocks from aggregate demand shocks, i.e.:

- **Restrictions 1-3:** None of the three demand shocks has long-run effect on the level of output. As a result, three long-run restrictions are put on the matrix $\mathbf{C}(1)$ which governs the long-run dynamics of the model: $\mathbf{C}_{12}(1) = \mathbf{C}_{13}(1) = \mathbf{C}_{14}(1) = 0$. Given $\mathbf{B}(\mathbf{1})\mathbf{S} = \mathbf{C}(\mathbf{1})$, these restrictions imply that:

$$\mathbf{B}_{11}(\mathbf{1}) S_{12} + \mathbf{B}_{12}(\mathbf{1}) S_{22} + \mathbf{B}_{13}(\mathbf{1}) S_{32} + \mathbf{B}_{14}(\mathbf{1}) S_{42} = 0 \quad (8)$$

$$\mathbf{B}_{11}(\mathbf{1}) S_{13} + \mathbf{B}_{12}(\mathbf{1}) S_{23} + \mathbf{B}_{13}(\mathbf{1}) S_{33} + \mathbf{B}_{14}(\mathbf{1}) S_{43} = 0 \quad (9)$$

$$\mathbf{B}_{11}(\mathbf{1}) S_{14} + \mathbf{B}_{12}(\mathbf{1}) S_{24} + \mathbf{B}_{13}(\mathbf{1}) S_{34} + \mathbf{B}_{14}(\mathbf{1}) S_{44} = 0 \quad (10)$$

  which amounts to restricting a set of sums of structural coefficients. This long-run restriction is controversial. For example, since the coefficient matrix $\mathbf{B}(\mathbf{1})$ is estimated with error, deriving the elements in $\mathbf{C}(\mathbf{1})$ using above equations might not be accurate (see for example, Hansen and Sargent (1991), Faust and Leeper (1997) and Cooley and Dwyer (1998)). Moreover, many demand disturbances do have long-run impacts on output. Examples include the changes in the social saving rate (stemming from changes in the discount rate of household or the investment rate of the government) which affect the long-run level of capital stock and thus of output. This is the case for many overlapping generations models. In the case of China, a consideration that deserves attention is that a large proportion of GDP is fixed investment. The changes in fixed investment may also have long-lasting effects on output through the accumulation of capital. However, as Blanchard and Quah (1993) argue, even if these permanent demand shocks exist, their effects are expected to be small. Also, we use near 30 years quarterly data so that most of the long-lasting impacts of demand disturbances die out sooner or later. Therefore, we consider the long-run restrictions as reasonable.

Second, Galí (1992) further puts three short-run restrictions to identify the three components of the demand shocks. To separate IS shocks from the two money shocks, two assumptions on the contemporaneous effects of money shocks on output are imposed, i.e.:

- **Restriction 4:** No contemporaneous effect of money supply shocks on output;

- **Restriction 5:** No contemporaneous effect of money demand shocks on output.

Since the short-run contemporaneous relationship between the structural shocks and the variables of interest is given by

$$\begin{pmatrix} \Delta y_t \\ i_t \\ \Delta p_t \\ \Delta m_t \end{pmatrix} = C(0) \begin{pmatrix} \varepsilon_{as,t} \\ \varepsilon_{ms,t} \\ \varepsilon_{md,t} \\ \varepsilon_{is,t} \end{pmatrix}, \tag{11}$$

this set of short-run restrictions actually concerns $\mathbf{C}(0)$ which is the matrix of the contemporaneous coefficients of structural shocks on the dependant variables. Particularly in our case, $\mathbf{C}(0)$ is equal to $\mathbf{S}$ and these two short-run restrictions imply that,

$$S_{12} = 0 \tag{12}$$

$$S_{13} = 0. \tag{13}$$

These two restrictions are a result of the 'outside lags' assumption that aggregate demand is not affected in the short-run if the money shocks do not change the financial conditions such as real interest rate and real exchange rate. Supportive evidence of this short-run restriction on money shocks has been found in developed countries[23]. Although there is no such evidence for China, it is argued that this restriction is also reasonable. This is based on two considerations. First, it is argued that for a financial market in reform, money shocks take even more time to affect output due to the inefficiency of the financial transmission channels. Second, it is true that, during the early stage of reform, the PBC has occasionally employed direct administrative controls on the supply and demand of money. However, it is still very likely that even these direct controls also need time to take effect on output. Also, the use of these direct controls have greatly decreased over the data sample. These discussions imply that the short-run impact of money shocks is expected to be very small. Thus, these two short-run restrictions are justified.

There is one last restriction left to separate the two money shocks. Following Galí (1992), we consider three possible short-run restrictions that concern the contemporaneous impacts of dependant variables on structural shocks, i.e.:

[23] Supportive evidence in the developed countries can be found from the effect of money shocks on investment as in Shapiro (1986) and also on the effect of exchange rate on trade flows as in Wilson and Takacs (1979).

- **Restriction 6:** Contemporaneous prices do not enter the money supply rule (thus restricts the monetary authority not to respond to short-run prices changes);
- **Restriction 7:** Contemporaneous GDP does not enter the money supply rule (thus restricts the monetary authority not to respond to short-run GDP changes);
- **Restriction 8:** Contemporaneous prices do not enter the money demand rule (thus imposes homogeneity in the money demand function).

Since these restrictions deal with the contemporaneous impacts of dependent variables on structural shocks, we rewrite the system in (11) to obtain,

$$\mathbf{C}(0)^{-1}\begin{pmatrix}\Delta y_t\\ \Delta i_t\\ \Delta p_t\\ \Delta m_t\end{pmatrix}=\begin{pmatrix}\varepsilon_{as,t}\\ \varepsilon_{ms,t}\\ \varepsilon_{md,t}\\ \varepsilon_{is,t}\end{pmatrix}. \tag{14}$$

These three restrictions then concern the inverse of the matrix $\mathbf{C}(0)$,

$$\mathbf{C}(0)^{-1}=\mathbf{S}^{-1}=\overline{\mathbf{S}}$$

where $\overline{\mathbf{S}}$ denotes the inverse of the matrix $\mathbf{S}$. For restriction 6 where the relationship between Δp_t and $\varepsilon_{ms,t}$ is considered, it implies that,

$$\overline{S}_{23}=0. \tag{15}$$

For restriction 7 where the relationship between Δy_t and $\varepsilon_{ms,t}$ is considered, it implies that,

$$\overline{S}_{21}=0. \tag{16}$$

Finally for restriction 8 where the relationship between Δm_t, Δp_t and $\varepsilon_{md,t}$ is considered, it implies that,

$$\overline{S}_{33}-\overline{S}_{34}=0. \tag{17}$$

Employing any restriction from 6-8 together with the restrictions 1-5 discussed above will provide 16 equations to just-identify S.

The choice of restrictions 6-8 deserves a further discussion. The first two restrictions, i.e., R6 and R7 are associated with the 'inside lags' of the PBC in response to changes in the economy. Thus, the choice of restrictions R6-R7 depends on which one is more consistent with the monetary policy practice in China. In fact, R6 and R7 concern how much weight the government has put on output gap and inflation in the short-run. Unfortunately, it is difficult to choose between R6 and R7 given the monetary practice in China. On one hand, the government has used both indirect economic and direct administrative tools to control the bank loans and money supply during an inflation hike. This indicates that using R6 to identify the money shocks might be risky[24]. On the other hand, China as a developing country has also put much emphasis on GDP growth rate every year. Thus it is also risky to use R7 to identify the two money shocks. In an attempt to further understand the implications of these restrictions we conducted an empirical analysis using both R6 and R7. The results show that both R6 and R7 failed to distinguish money supply shocks from money demand shocks (see Appendix (1.9.2) for details). Therefore, we discard R6-7 and turn to R8.

Now we turn to R8 which amounts to assuming that changes in demand for nominal money move one-for-one with changes in prices. This assumption is usually to imply that there is no cost of adjusting nominal money holdings. This assumption of homogeneity in money demand seems to be supported by a number of empirical studies on money demand in China (see e.g., Hafer and Kutan (1994)). In fact, recent studies on money demand in China (see e.g., Xu (1998) and Mehrotra (2006)) often impose the homogeneity assumption directly. We then use R8 to conduct the analysis. The results turn to be more plausible with theory than the results obtained using R6 and R7. Based on the above discussion, the following will proceed with R8 to identify the two money shocks in our SVAR.

1.4.4 General issues with specification and identification of SVAR

Although the restrictions discussed above enable us to mathematically solve for the unknown structural coefficients, the accuracy of this methodology in correctly estimating the effects of economic shocks has been questioned in literature. While early criticism on the identification technique can be found on the misuse of long-run restrictions discussed above, some more general issues have been acknowledged in the literature. Recent work by Chari, Kehoe and McGrattan (2005) claims that the structural VAR models are likely to be misspecified. Their estimated impulse response functions of hours to technology shocks using structural VAR based on the simulated data from the Real Business Cycle model are at odds with the theoretical ones. However, Christiano, Eichenbaum, and Vigfusson (2006) also makes use of the RBC model as data

[24]Besides, the awareness of the danger of price volatility (such as the Tiananmen Square Incident) leads the government to issue the Central Bank Law in 1995 which states the price stability as the primary objective of the PBC.

generating process but arrive in the opposite conclusion that although the long-run restriction generate wider confidence intervals than short-run restriction due to bigger sampling uncertainty, it is still accurate as long as the technology shock accounts for at least one percent variance in hours worked. Particularly, Gali and Rabanal (2004) re-examined the critics in Chari, Kehoe and McGrattan (2005) and pointed out that their striking results are due to misspecification and misidentification in their use of the structural VAR, not to the flaws of the technique itself. The structural VAR is still extremely useful in identifying responses of economic shocks and discriminating theoretical models.

Some researchers have investigated the performance of structural VARs in a more general way. For example, Ravenna (2007) evaluated the performance of the finite order VAR approximations to the exact infinite order VAR representation of a theoretical model. It is found that even if there is no identification bias, the truncated VAR can still deliver largely inconsistent estimates of the structural coefficients. The problem lies in the fact that finite order VAR is truncated VAR approximation that might not be the appropriate approximation of the infinite order VAR representation of the theoretical model. To overcome this problem, one way is to include more variables in the finite truncated VAR (see, e.g., Erceg, Guerrieri and Gust (2005)) and test the robustness of the results. To do such a robustness check, more data is required (which might be a difficulty in the case of China) and the identification scheme illustrated above needs to be revised accordingly. While this has not been undertaken in the current thesis, this deserves a further examination in future work.

1.5 How well does the IS-LM-PC model fit post-reform Chinese data?

The next two sections report the empirical results of the SVAR model specified in Section 1.4. Firstly there are the estimated impulse response functions which generate model dynamics that can be compared with the predictions in the IS-LM-PC model. This is used as a way to examine the fit of the theoretical model to the Chinese data. Following, the standard forecast error variance decomposition was calculated in Section 1.6 to examine the contribution of each economic shock to output fluctuations over the data sample. Additionally the historical forecast error decomposition is presented together with the recognized economic events so as to answer the question that how the role of each economic shock changes over time.

Firstly the question is how successful the IS-LM-PC model is in explaining the Chinese data. This is done by comparing the estimated effects of economic shocks on the economy with those predicted by the IS-LM-PC model summarized in Section 1.4.1. The estimated impulse response functions are presented which describe how the variables of interest react to a one period change in the structural shocks. For example, from the moving average form of the structural model (3), the impact of a particular structural shock on a particular dependant variable is given by:

$$\mathbf{x}_{i,t} = \mathbf{C}_{i,j}\left(\mathbf{L}\right)\boldsymbol{\varepsilon}_{j,t} \tag{18}$$

where $i, j \in 1, 2, 3, 4$ correspond to the four dependant variables in $[\triangle y, \triangle i, ri, \triangle rm]$ and the four structural shocks in $[as, ms, md, is]$ respectively, and they select the (i, j) entry of the $\mathbf{C}\left(\mathbf{L}\right)$ matrix. Expressing above result explicitly and leading it h period ahead gives:

$$\begin{aligned}
\mathbf{x}_{i,t} &= \mathbf{C}_{i,j}(0)\boldsymbol{\varepsilon}_{j,t} + \mathbf{C}_{i,j}(1)\boldsymbol{\varepsilon}_{j,t-1} + \mathbf{C}_{i,j}(2)\boldsymbol{\varepsilon}_{j,t-2} + ... \\
\mathbf{x}_{i,t+1} &= \mathbf{C}_{i,j}(0)\boldsymbol{\varepsilon}_{j,t+1} + \mathbf{C}_{i,j}(1)\boldsymbol{\varepsilon}_{j,t} + \mathbf{C}_{i,j}(2)\boldsymbol{\varepsilon}_{j,t-1} + ... \\
\cdots &= \cdots \\
\mathbf{x}_{i,t+h} &= \mathbf{C}_{i,j}(0)\boldsymbol{\varepsilon}_{j,t} + \mathbf{C}_{i,j}(1)\boldsymbol{\varepsilon}_{j,t+h-1} + ...\mathbf{C}_{i,j}(h)\boldsymbol{\varepsilon}_{j,t} + ...
\end{aligned}$$

which implies that the impulse responses of $\mathbf{x}_{i,t+h}$ to the structural shock $\varepsilon_{j,t}$ is given by:

$$\mathbf{x}_{i,t+h} = \mathbf{C}_{i,j}(h)\boldsymbol{\varepsilon}_{j,t} \tag{19}$$

which is the general result for calculating the impulse response functions for variables in levels such as the real interest rate. For variables in first difference, such as the real GDP, it is easily to show that the impulse response function is just the cumulative sum of the (weighted) structural shock which is given by:

$$\mathbf{x}_{i,t+h} = \sum_{l=1}^{h} \mathbf{C}_{i,j}(l)\boldsymbol{\varepsilon}_{j,t}. \tag{20}$$

The impulse responses of different variables of interest to the four structural shocks considered in the IS-LM-PC framework are reported below.

1.5.1 Impulse responses - Supply shock

The impulse responses of variables to a favourable one-standard deviation aggregate supply shock are shown in Figure 1.1. The initial impact of aggregate supply shock on GDP is about 0.62 percent which roughly matches the one estimated in Galí (1992) for US data. The increase in output growth grows larger in the following 10 quarters and stabilizing around 2 percent, double of the same for US data. As the economic theory predicts a favourable supply shock dampens prices. An initial 0.15 percent fall in prices is observed. This decrease in prices is small and short-lived - vanishing after three quarters. From the fourth quarter, the impact of the initial supply shock becomes inflationary. Prices continue to climb up and reach a peak 8 quarters after the shock. After that, the inflation takes more than four years to disappear.

The marked surges of money supply observed are largely responsible for the mid- to long-run inflationary impacts of the supply shock. It is shown that, the nominal money responds by an over 0.5 percent increase in its growth rate, which is large. This explains the small decrease in prices in the first three quarters. As found in many developed countries, this can be seen as a monetary accommodation by the central bank as a way to offset the falling prices. However, as shown in Figure 1.1, the demand for money seems to be soon 'over' accommodated - the increase in money growth remains large and persistent (although shows some irregular variations) in the following two to four years. This soon gives rise to inflationary pressures and induces a long-lived inflation.

The responses of the nominal interest rate are considerable with substantial persistence. Since the adjustments in prices are relatively small, the responses of the real rate display similar pattern with the nominal interest rate. However, the dynamics of the nominal interest rate and the real balances are largely consistent with the LM equation: their adjustments within four quarters show a different shape with output since the nominal rate decreased considerably. After six quarters when the output gradually stabilizes, the adjustments of real balances and nominal rate show adverse directions. The embedded estimate of the income elasticity of real balances, φ, varies over time. Its short-run value is about 1.1, bigger than the same estimate for the US, 0.3. Its the long-run value is about 0.9, smaller than the same estimate for US, 1.5.

Compared with similar studies of responses of variables to supply shocks, two features in our results are noteworthy. First, as discussed above, the reaction of output after a supply shock in the medium-run is about twice as big as the one estimated in Galí (1992) for US. Although this is not surprising given China as a developing country, it does highlight the importance of economic reforms in China. It implies that the efficiency improvement through economic reform is crucial for the development of the economy. This point has been commonly acknowledged in the developing economies literature and will be further discussed in the next section by calculating the importance of supply shocks.

The second important feature in the Chinese economy regarding the results in Figure 1.1 is the exceptional large monetary ‘over’ accommodation. The money supply goes up together with output growth after a supply shock. The reason for this behavior deserves a further discussion. That is, what has induced the large and persistent monetary responses? As mentioned above, the discussion of institutional background in Section 1.3 might shed light on this issue. It suggests that institutional factors related with the economic reform and the conduct of monetary policy must account for the odd monetary responses. In fact, there have been attempts of researchers which explain macroeconomic performance with these institutional factors[25]. Among these attempts, the explanation of Brandt and Zhu (2000) seems to be most suitable for understanding our results. Brandt and Zhu (2000) argued that the money creation and inflation can be a natural result of two institutions in China: the decentralization and the commitment of the government to the state sector. Economic decentralization allows more productive non-SOEs access more resources and thus contribute to economic growth. However, as the gap between the SOEs and the non-SOEs becomes large, the government has to make fiscal transfers to the SOEs due to its commitment to the latter and its inability of redirecting bank credit to the state sector under the decentralized banking system. Since its fiscal revenue is declining due to the fiscal decentralization, the fiscal transfers have to rely on money creation which causes inflation. This explanation fits the result well. The observed significant and persistent increases in money growth just reflect the dilemma of the government - the faster the output grows, the more gap of productivity and wage between SOEs and non-SOEs and thus more need to compensate the less productive state sector and to increase money supply. This situation continues, until the economy is overheating with rising inflation. The government has to adopt strict administrative controls on bank credit and cuts money supply. This cools both the output growth and inflation down.

[25]Examples of these attempts include Yusuf (1994), Naughton (1995), Imai (1996), Yu (1997), Oppers (1997), Fung *et al.* (2000), Qian and Roland (1998) and Brandt and Zhu (2000).

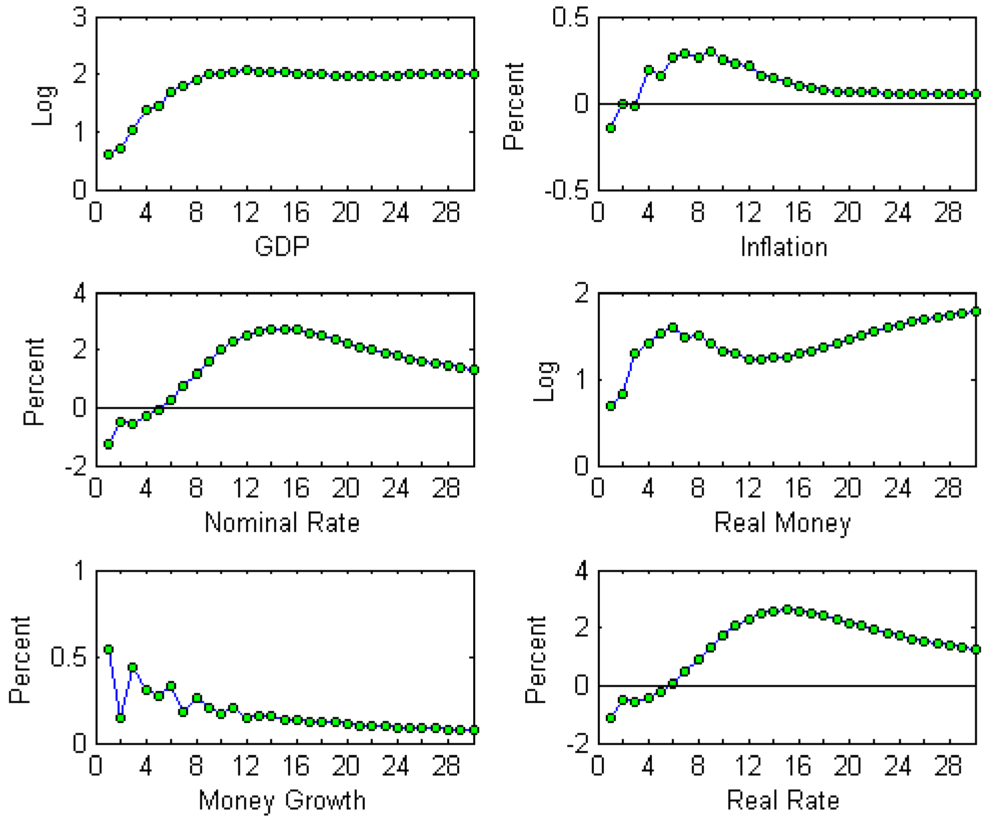

Figure 1.1: Impulse Responses - Supply Shock

1.5.2 Impulse responses - Money supply shock

Figure 1.2 shows the responses of different variables to a one-standard deviation shock in money supply. We first observe an initial jump of 0.75 percent in nominal money, followed by some irregular behavior in the same variable which eventually returns to its initial value. The money-real interest rate-output transmission mechanism in the IS-LM-PC model works well: the increase in money growth induces an immediate increase in real balances since the increase in inflation is less. Both nominal and real interest rates fall considerably, indicating that the liquidity effect[26] far outweighs the Fisher effect due to smaller adjustment of prices. Output, although assumed not to respond to demand shocks in the first quarter, gradually goes up due to lower real interest rate in a way consistent with the IS equation. However, compared with the substantial drop in real interest rate, the effect on output is very small, indicating that the real interest rate effect is weak in China. As output growth increases, inflation and nominal interest rate also go up in a way consistent with the Phillips Curve and the

[26] Considering the fact that the nominal interest rate is administratively fixed in the short-run, this liquidity effect might reflect the fact that the PBC cuts interest rate to reduce the borrowing cost of the commercial banks and enterprises when an expansionary policy is implemented.

LM equation. It is noteworthy that the adjustment of nominal interest rate is quite fast. It becomes positive only after 4 quarters, and so does the real interest rate since the increase in prices remain small. In consequence, output growth slows down and turns slightly negative after 12 quarters, leading inflation to gradually return to zero.

Since there is no unit root in nominal variables under this specification of the model, inflation, nominal rate and money growth eventually return to their initial values in the long-run. Accordingly, the LM equation which measures the relationship between the real balances and the real interest rate finally disappears after infinite horizons. However, since the adjustments of the two variables are slow, it is still possible to measure the short- to mid-term LM equation: the point estimate of the interest-semi-elasticity, λ, is about 0.5 in 12 quarters and is close to 0.9 after 29 quarters. In the long-run, only the real balances are permanently affected (since the level of money is lower than the price level in the long-run). The working of the IS-LM-PC transmission mechanism of money supply shock is one of the most striking results of this study. This indicates that as economic reform goes deep, market mechanisms gain more importance. This gives the monetary authority more room to conduct its policy and reply more on the working of the market mechanism. This finding is consistent with the observation that the PBC has adopted more and more indirect instruments than direct controls in the post-reform period.

The responses of output and inflation to the money supply shock in Figure 1.2 can be compared with the those to the supply shock in Figure 1.1. It is shown that the impact of the money supply shock on output in Figure 1.2 is much smaller than the increase in output growth observed in Figure 1.1: The maximum of response of output to money supply shock is 0.22 and it soon vanishes and even turns negative after 12 quarters. On the other hand, the impact of the money supply shock on inflation in Figure 1.2 is much bigger than the jump in inflation in Figure 1.1: The maximum response of output to money supply shock is almost double of the same in supply shock case and it persists for 12 quarters. Taking the results together, it is concluded that the responses of output in Figure 1.1 is mostly caused by supply shocks while the induced money expansion is mainly responsible for the increases in inflation.

The estimated impulse responses of variables after a money supply shock show various differences with the estimates for the US data. The biggest difference lies in that the effect of the money supply shock on output is much smaller in the case of China while the effect on prices is much bigger. In other words, a positive money supply shock leads to smaller increases in GDP and is more likely to cause inflation. This has been reflected in the estimates of the medium to long-run interest-semi-elasticity. For instance, the estimate for λ after 29 quarters for US data is close to 2, much bigger than the estimate as in Chinese data. This difference in the impact of monetary expansion has important implication. It indicates that there are still obstacles in the Chinese economy which prevents the monetary transmission mechanism from working fully. This is not difficult to understand given the incomplete reform in financial markets and the conduct of monetary policy discussed in the institutional background section 1.3.1.

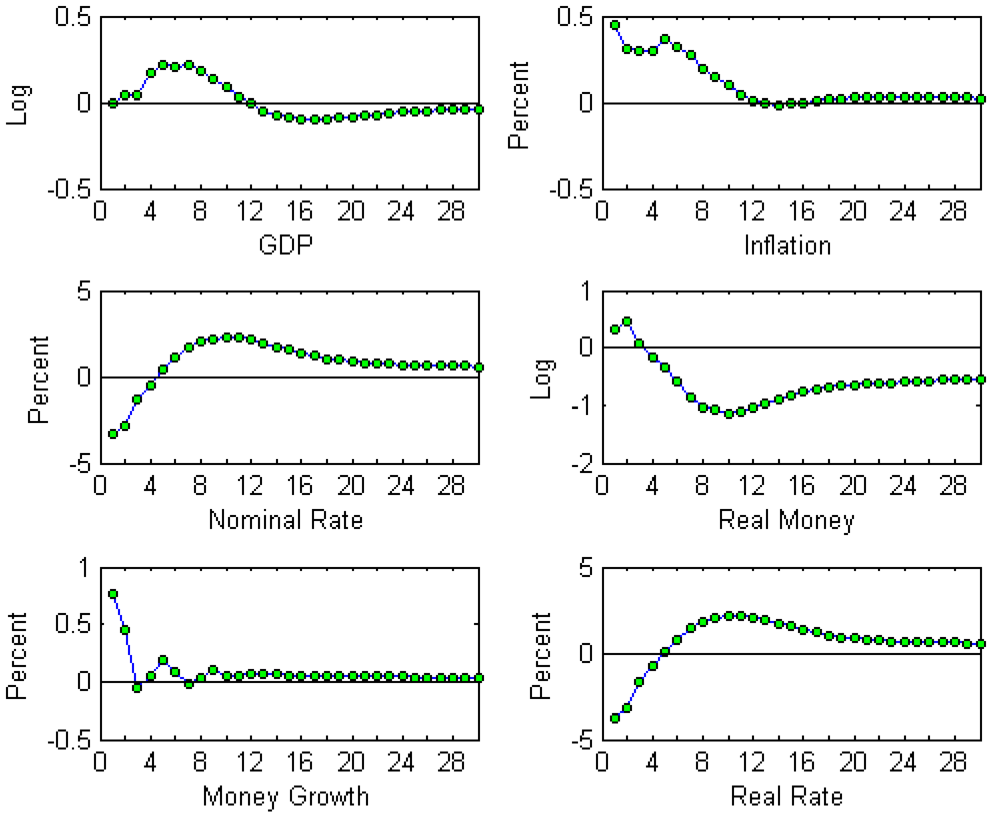

Figure 1.2: Impulse Responses to Money Supply Shock

1.5.3 Impulse responses - Money demand shock

The responses of variables to an one-standard deviation increase in real balances demanded are shown in Figure 1.3. At a first glance, they appear qualitatively as a mirror image of the impulse responses to the money supply shock. There are two noteworthy exceptions. The first exception is that the responses of output within 8 quarters take an erratic pattern. There appears a short term positive correlation between output and real interest rate, which is thus inconsistent with the IS equation. It disappears after

8 quarters and is replaced by a negative relationship between output and real interest rate consistent with the IS equation. The second exception concerns the observed inflation instead of deflation as would be predicted by general equilibrium models. This is due to the strong monetary accommodation to the increase in money demand: Money growth increases 2 percent which is even more than the increase of the same variable to a money supply shock. The vast liquidity seems to induce excess demand over supply, entailing a jump in prices. Since the jump in money supply is one-off, the jump in inflation is also short-lived. It is important to note that, the positive correlation between output growth and inflation suggests that the Phillips Curve is still valid. Overall, the responses of variables to money demand shock still favors the reasonableness of the IS-LM-PC model.

In the long-run, only the real balances are permanently affected. Although prices adjust quickly after the jump, the slow adjustment of the nominal interest rate (again, due to its short-run 'fixed' feature) leads to large persistence in real interest rate and thus a long-lived effect on output.

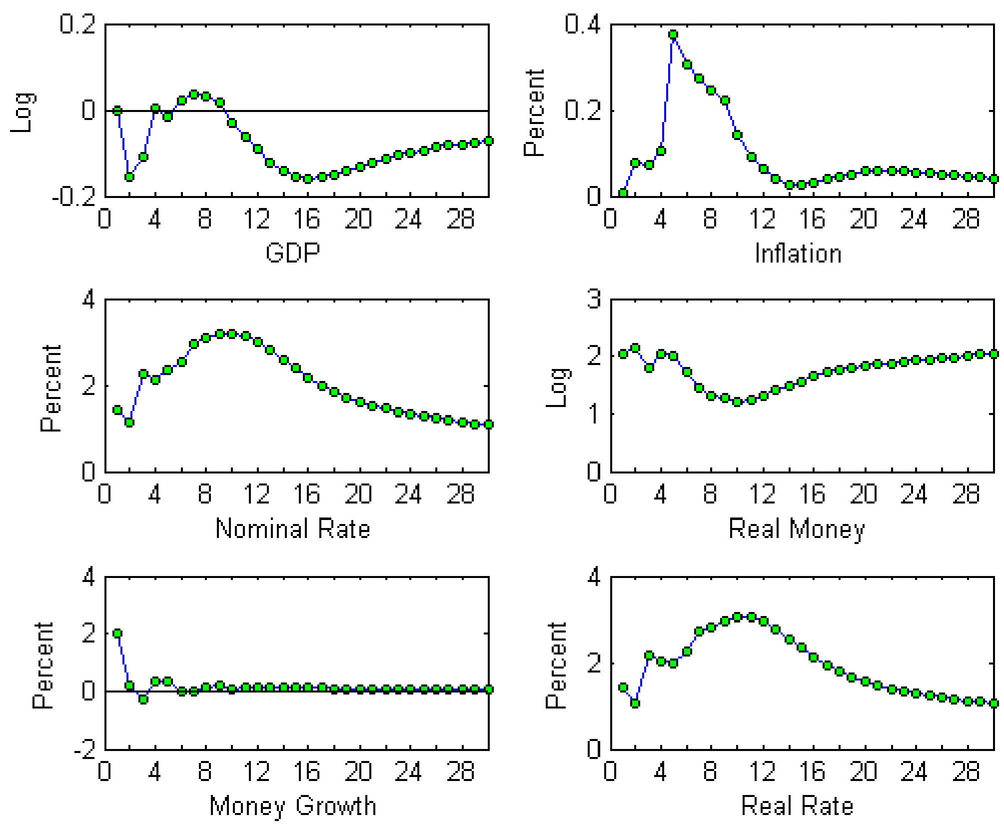

Figure 1.3: Impulse Responses to Money Demand Shock

1.5.4 Impulse responses -IS shock

The effect of a positive one-standard deviation shock in public spending is shown in Figure 1.4. The results vividly display the dynamics of the nominal and real variables and the interaction between the fiscal and monetary policies. Firstly, the fiscal expansion leads to an immediate increase of 0.3 percent in real GDP. The nominal interest rate goes up considerably in a way consistent with the LM equation. Prices also respond fast but moderately. As a result, the real interest rate jumps up in a shape similar with the nominal interest rate. The quick response of the real interest rate dampens the increase in output in a way consistent with the IS equation. The downward pressure of output and inflation however, is not only from the crowding-out effect - the monetary authority cuts money supply in response to the increase in fiscal spending. Real balances immediately go negative after the monetary contraction, generating more upward pressure on real interest rates. After the real interest rate reaches a peak in 7 quarters, output falls and the positive impact of the initial fiscal spending dies out. As a result, the positive impacts of fiscal expansion on output disappear after 8 quarters.

In the long-run, money growth by assumption returns to zero. All the variables except for the real balance variable return to their initial values. The estimated semi-elasticity of real interest rate in the LM equation, λ, is 1.5 in 12 quarters and is about 3.3 in 20 quarters. These estimates are higher than in the case of money supply shock.

The result of the estimated responses of variables to an IS shock highlight two interesting findings. First, it turns out that the monetary policy is complementary with fiscal policy. This is clearly an important difference compared with the result for the US where money supply increases together after an IS shock. This might have reflected the fiscal policy design of the Chinese government probably in fear of overheating problem in the Chinese economy. The latter again can be due to institutional reasons. Second, since nominal money is assumed to be I(1), there is no monetary intervention in the long-run. In many developed countries, the intention of raising money supply in the medium and long-run of the monetary authority usually lead to inflation in the long-run. In the case of China however, there is no such attempt and therefore no inflation in the long-run.

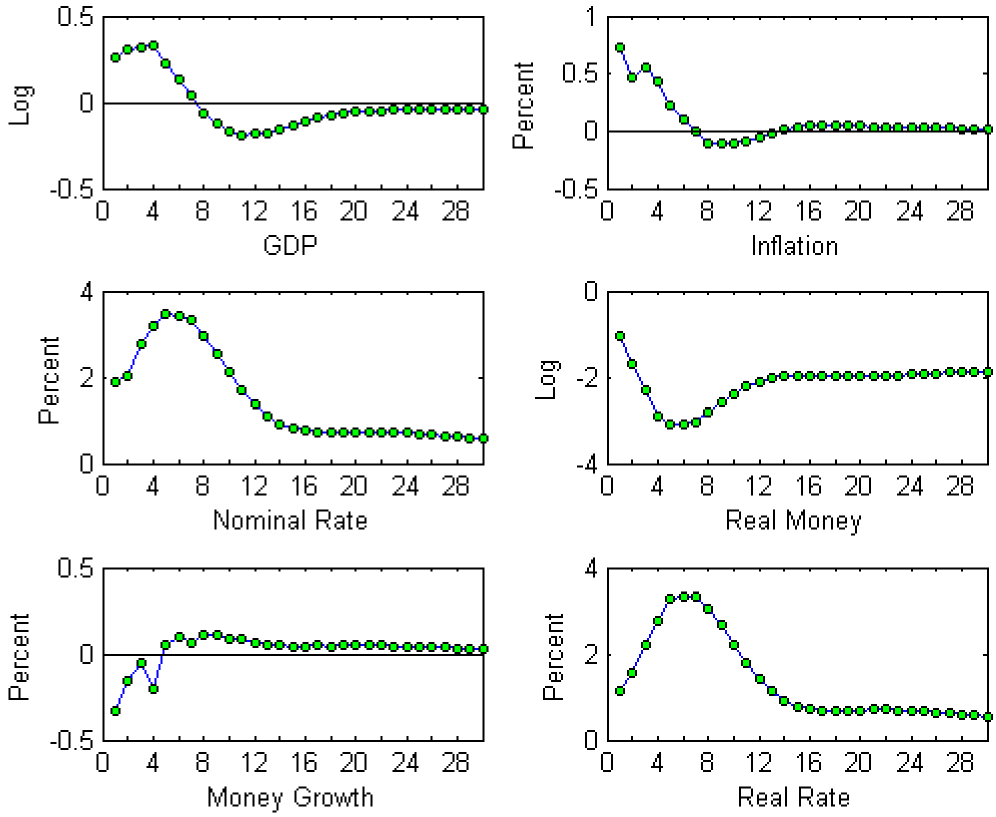

Figure 1.4: Impulse Responses - IS Shock

In summary, the estimated responses of variables to the four structural shocks are all consistent with the dynamics predicted by the IS-LM-PC model. This implies that the market mechanisms during economic reform are gaining increasing importance. Not only are supply shocks such as changes in technology and efficiency work in consistent with the theory, but also that the fiscal and monetary policies work more through market channels. The results confirm the positive changes in the economic structure in the post-reform Chinese economy. This is one of the most important findings in this study.

1.6 Sources of China's post-reform business cycles

1.6.1 Variance decompositions

Another important question is that of the sources of output fluctuations. One might have inferred the importance of each structural shock from the magnitudes of the responses of variables to different shocks discussed above. To bolster the inferences two more standard examinations were conducted, i.e., the forecast error variance decomposition (FEVD) and the historical forecast error decomposition (HFED) to precisely gauge the contribution of each structural shock to output fluctuations. The FEVD assigns the output variance over the data sample to proportions accounted by the four structural shocks and thus can be considered as reflecting the overall importance of each shock in accounting for output variations. The HFED decomposes the time series of the variations in output into structural shocks components providing a historical view on the importance of each shock.

To see the derivation of these two decompositions, note that the forecast error of variables in $\mathbf{x}$ based on the estimated SVAR is given by:

$$\begin{aligned} & \mathbf{x}_{t+h} - \widehat{\mathbf{x}}_{t+h} \\ = & \ \mathbf{C}(L)\boldsymbol{\varepsilon}_{t+h} \\ = & \sum_{l=0}^{\infty} \mathbf{C}(l)\boldsymbol{\varepsilon}_{t+h-l} \end{aligned} \tag{21}$$

where $\mathbf{x}_{t+h}$ is the matrix consists of the realizations of the variables of interest, $\widehat{\mathbf{x}}_{t+h}$ is the fitted value based on the estimated structural coefficient matrix $\mathbf{C}(L)$ and h is the forecast horizon. The historical decomposition of this forecast error with respect to each structural shock is then given by:

$$\mathbf{x}_{i,t+h} - \widehat{\mathbf{x}}_{i,t+h} = \sum_{l=0}^{h-1} \mathbf{C}_{i,j}(l)\boldsymbol{\varepsilon}_{j,t+h-l} \tag{22}$$

where as defined before the subscripts i and j correspond to the ith variables in $\mathbf{x}$ and jth shock in $\boldsymbol{\varepsilon}$ and they select the (i,j) entry of the structural matrix $\mathbf{C}(L)$. Again it is noteworthy that the above calculation is for variables in first difference such as the real interest rate. The calculation for variables in levels such as the output is slightly different and is given by:

$$\mathbf{x}_{i,t+h} - \widehat{\mathbf{x}}_{i,t+h} = \sum_{l=0}^{h-1} \mathbf{D}_{i,j}(l)\boldsymbol{\varepsilon}_{j,t+h-l} \tag{23}$$

where $\mathbf{D}_{i,j}(l)$ is now a cumulative sum, $\mathbf{D}_{i,j}(l) = \sum_{s=0}^{l} \mathbf{C}_{i,j}(s)$.

The results of the FEVDs for the four dependant variables are reported in Table 1.3 and plotted in Figure 1.5. These results confirm the findings in the previous impulse responses analysis. Aggregate supply shocks account for an average of 89% of the short-run (two years) real GDP fluctuations[27]. After twenty quarters, almost all of the

[27] Note that the monetary expansion along with the supply shock also plays an important role in driving the dynamics of the economy. However, as discussed in Section (1.5.2), its impact is mainly on inflation while its impact on output is very small.

GDP fluctuations are due to supply shocks. The demand shocks on the other hand, only account for a small part of GDP fluctuations. Specifically, the fiscal spending shock accounts for about 17% of GDP fluctuations in the first quarter and 10% in the first year. Its impact vanishes quickly in two years. Also, although the impulse responses of output under all shocks are affected by the responses in money growth, the contributions of the two money shocks on GDP fluctuations are almost negligible. The result of the dominant contribution of supply shocks to output fluctuations is much larger than those found by Zhang and Wan (2004) and Silkos and Zhang (2008)[28]. This result is also consistent with the findings for developed countries (for example, Blanchard and Quah (1989) and Galí (1992) for US data).

It is very useful to compare the results of the FEVDs with those estimated for other developed and developing countries. For example, early studies using similar VAR specification and identification strategy (e.g., Shapiro and Watson (1988) and Galí (1992)) found smaller fraction (around 70%) of output contributable to supply shocks in the short-run for the US economy. However, recent studies using only long-run restrictions (e.g., Galí (1999), Galí and Rabanal (2004), Francis and Ramey (2004), Chari, Kehoe and McGrattan (2005) and Christiano, Eichenbaum and Vigfusson (2006)) have found this fraction to be substantially smaller (from 7% to 37%, overall not over 40% in all VAR specifications). Also, similar structural VAR studies for the european countries (e.g., Karras (1994)) also found much smaller (not over 45%) importance of supply shocks. On the other hand, the dominant role of supply shocks are consistent with the findings in most developing countries. For example, the study of Morling (2005) using Blanchard and Quah identification reported a average fraction of 87% of supply components for developing countries in Asia, Africa, Middle East and Western Hemisphere. Some other studies using alternative identification schemes (e.g., Plessis *et al.* (2008) and Santo, Zhang and McAleer (2009)) also found quite high percentage of output fluctuations due to supply shocks.

The above discussion seems to suggest that supply shocks play dominant role in developing countries. This finding has important implications. First, the dominant role of supply shocks highlights the importance of economic reforms undergone developing countries. For developing countries, the efficiency improvement is another source of

[28] Possible reasons why these researchers obtain smaller role of supply shocks might be that Zhang and Wan (2004) use a two-variable VAR model, which might ignore the supply-induced money effect on output. Moreover, their data length is also smaller. Silkos and Zhang (2008) apply a different identification strategy where supply and demand shocks are correlated, which might assign a large portion of supply side effect to demand side.

economic growth besides technolgical progress. Second, given this dominant role of supply shocks, diagnosing the sources of supply shocks is critical for understanding economic fluctuations. A further decomposition is very meaningful. Applying another identification scheme (see example, Shapiro and Watson (1988)) to further investigate the components of supply shocks requires data for the producing inputs of output such as employment. However, this is not possible for China since the supply side data are not available, especially at quarterly frequency. This leaves areas for future research once more data becomes available. Third, the dominance of supply shocks for causing economic fluctuations also has important implication for macroeconomic modeling. It indicates that theoretical models for explaining the working of the Chinese economy should mainly build on real side of the economy. In this sense, modeling paradigm starting from a real business cycle (RBC) model would be preferred. Fourth, in the case of China, despite the working of the fiscal and monetary policies shocks analyzed in the impulse responses, the overall contributions of them to economic fluctuations are small. This indicates that the incomplete reform and related institutional structures discussed before might have built obstacles for monetary shocks to affect the economy. In other words, the working of the fiscal and monetary policies might have still partially replied on non-market channels such as credit controls. Thus, further reforms are needed to allow the financial market to work fully.

Finally, although the emphasis of this study is on output fluctuations, the FEVDs for other variables are also summarized as follows. The fluctuations in nominal interest rate are mainly accounted by IS shocks and money supply shocks in the short-run. For, the long-run, fluctuations are caused by all of the four shocks with more important responsibility given by money supply shocks and IS shocks. This again confirms that, all of the shocks are sources of unit root in nominal variables. Nonetheless, the FEVDs further reveal that money supply shocks and IS shocks are most responsible. The short-run fluctuations in inflation are most accounted by the money supply and the IS shocks. However, money demand shocks and supply shocks gain increasingly importance over time. The former becomes the main source of inflation fluctuations in the long-run. The variations in money growth are mainly determined by money supply shocks. In the long-run, all the shocks are responsible for inflation fluctuations with money demand shocks playing the most important role.

Table 1.3: Decomposition of Forecast Error Variance

Component:	Supply	Money Supply	Money Demand	IS
GDP				
1 quarter	0.83	0.00	0.00	0.17
4 quarters	0.88	0.01	0.01	0.10
8 quarters	0.96	0.01	0.00	0.03
12 quarters	0.97	0.01	0.00	0.02
20 quarters	0.98	0.00	0.01	0.01
Nominal Rate				
1 quarter	0.03	0.26	0.00	0.71
4 quarters	0.00	0.27	0.01	0.72
8 quarters	0.02	0.36	0.08	0.54
12 quarters	0.09	0.39	0.13	0.39
20 quarters	0.19	0.36	0.17	0.28
Inflation				
1 quarter	0.09	0.61	0.11	0.19
4 quarters	0.04	0.35	0.21	0.40
8 quarters	0.3	0.20	0.30	0.47
12 quarters	0.10	0.2	0.35	0.35
20 quarters	0.21	0.19	0.34	0.26
Money Growth				
1 quarter	0.09	0.61	0.11	0.19
4 quarters	0.04	0.35	0.20	0.41
8 quarters	0.03	0.20	0.30	0.47
12 quarters	0.09	0.21	0.35	0.35
20 quarters	0.22	0.18	0.34	0.26

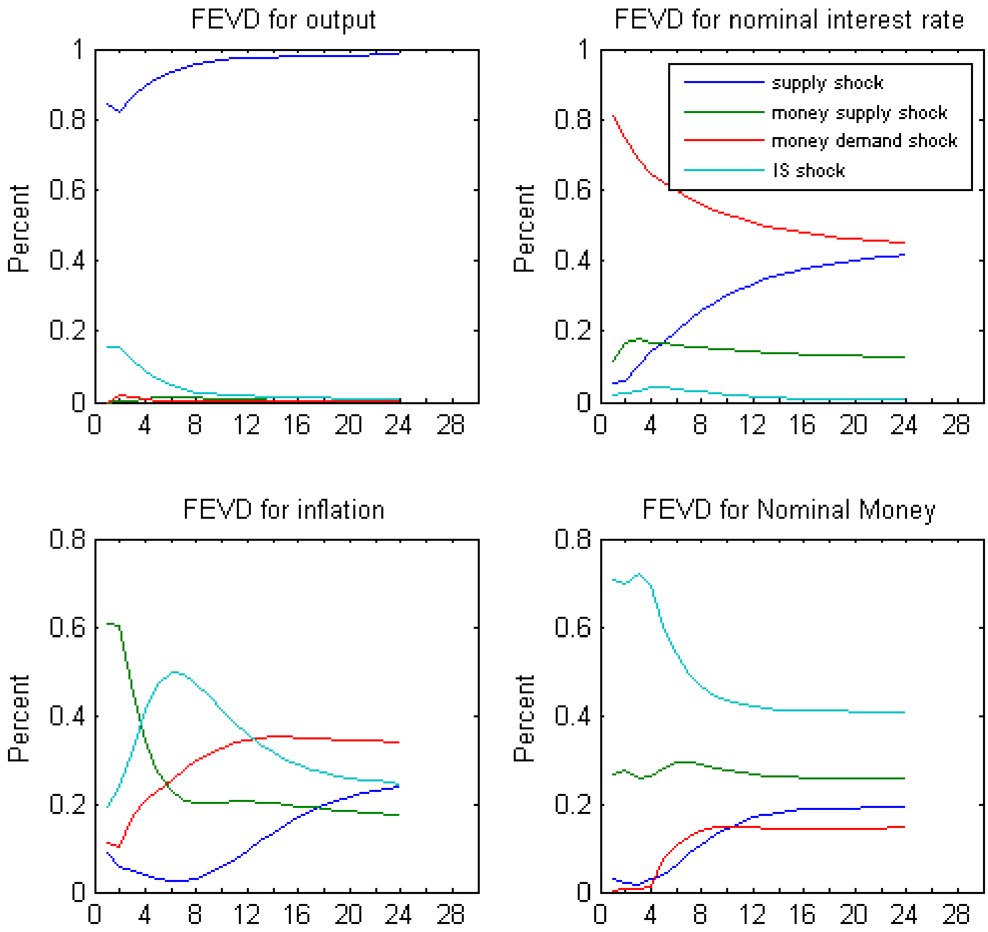

Figure 1.5: Decomposition of Forecast Error Variance

1.6.2 Are business cycles all alike?

The above discussion of the FEVDs can be considered as an exposition of the *overall* contribution of each structural shock in accounting for fluctuations. It does not tell us however, the importance of each shock at a specific time. Although the variance decompositions implies that the historical contributions of the three demand shocks on average are also small, a historical examination might tell different stories. Therefore, this section presents the HFED of the real GDP series and examines the sources of the business cycles in the sample period 1983Q1-2009Q3[29].

The calculation of the FEVD is very similar. The FEVD is measured as the ratio of the contribution of a particular structural shock to the variance of h-step forecast error of a given variable over the variance of the h-step forecast error of this variable. Since the variance-covariance matrix of the structural shocks is just identity matrix,

[29] The first three years, 1980-1982, are lost due to differencing variables (one year lost for first-differencing and two years lost due to the two-year forecast horizon).

the variance of the forecast error is just given by:

$$\sum_{l=0}^{h-1} \mathbf{C}(l)\mathbf{C}(l)'$$

and the decomposition of this variance according to each structural shock is given by:

$$FEVD_{i,h} = \frac{\sum_{l=0}^{h-1} \mathbf{C}_{i,j}(l)^2}{\sum_{l=0}^{h-1} \mathbf{C}(l)\mathbf{C}(l)'} \tag{24}$$

where the subscripts i, j as defined before represent the contribution of the jth shock on the variance of the forecast error of the ith variable. They select the (i, j) entry of the structural matrix $\mathbf{C}(L)$. This representation is for variables in level such as the nominal rate. For variables in first difference such as output, the FEVD is given by:

$$FEVD_{i,h} = \frac{\sum_{l=0}^{h-1} \mathbf{D}_{i,j}(l)^2}{\sum_{l=0}^{h-1} \mathbf{D}(l)\mathbf{D}(l)'} \tag{25}$$

where $\mathbf{D}_{i,j}(l)$ is now a cumulative sum, $\mathbf{D}_{i,j}(l) = \sum_{s=0}^{l} \mathbf{C}_{i,j}(s)$.

Before starting the analysis, we characterize the chronology of China's business cycles from 1983Q1 to 2009Q3 by examining the de-trended series derived by the HP filter (Hodrick and Prescott (1997)) plotted in Figure 1.6. The Figure demonstrates broad co-movement between detrended GDP and inflation (i.e., the peaks and the troughs) roughly map each other, implying that the occurrences of economic overheating over the sample period. Another significant feature is that both the amplitudes of the fluctuations in GDP and inflation decrease after 1996. This feature is usually referred as the 'soft landing' given that prices are largely reduced at a small cost of output reduction. The swings of GDP and inflation emerge again from 2004 but the amplitude of inflation fluctuation is much smaller than previous periods. After an inspection of the output fluctuations, we classify five business cycles for Chinese economy[30]: 1983Q1—1985Q4, 1986Q1—1989Q4, 1990Q1—1997Q4, 1998Q1—2004Q3 and 2004Q4—2009Q3.

For illustration purpose, a green (solid) line is used to represent a peak in real GDP and a red (dashed) line to indicate a trough. The period starting from a red line and ending with a green line represents an expansion. The period starting from a green

[30] We depict a cycle that begins with increasing output and ends with decreasing output after a peak.

line and ending with a red line refers to a recession or downturn[31]. The time periods where two red lines appear in the downturns (such as 1988Q4-1989Q4, 1997-1998 and 2008Q3-2009Q3) indicate severe recessions.

The first four cycles are roughly consistent with the ones identified in literature (for example see, Khor (1992), Yu(1997), Oppers (1997), Zhang and Wan (2004) and Laurenceson and Dobson (2008)). The first three cycles and the last cycle are quite complete, in the sense that they start from a trough, then reach the peak and then end with another trough. The fourth cycle on the other hand, only displays small fluctuations around the trend, representing a great stability of the economy.

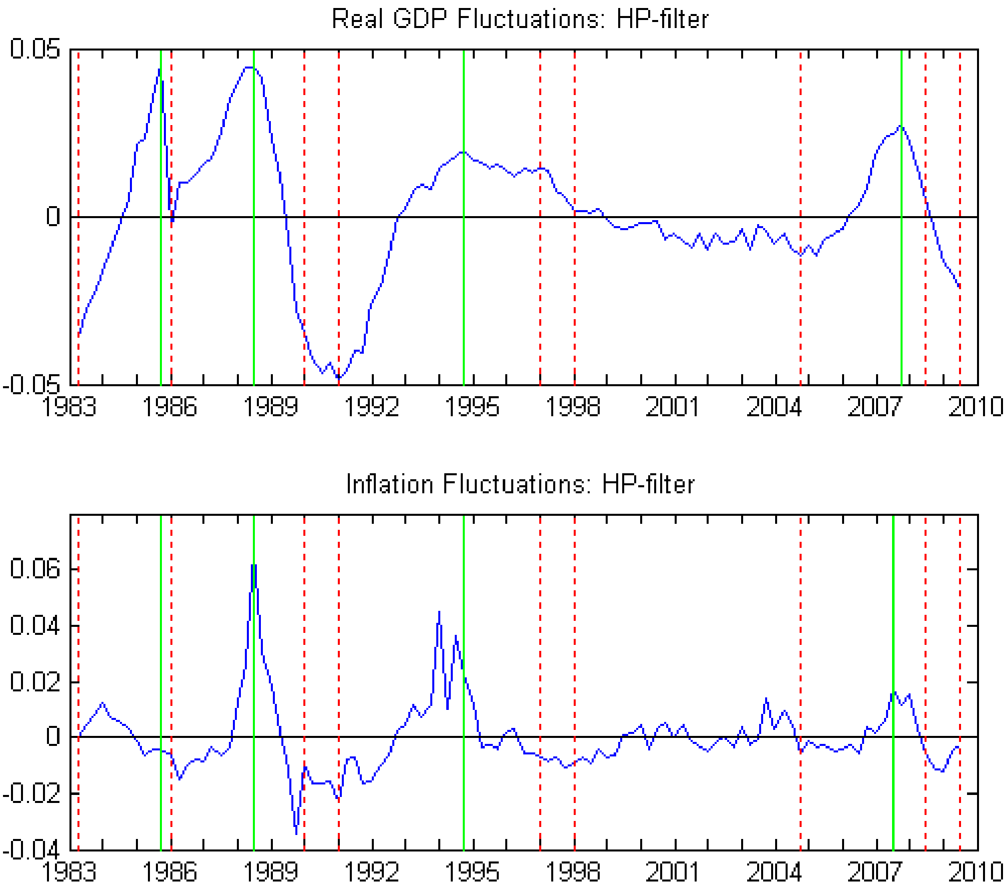

Figure 1.6: China's Business Cycles: 1983Q2 - 2009Q3

By using a forecasting horizon of 8 quarters, the following will discuss the sources of business cycles in the five sub-periods by using the HFED of real GDP. These results of the HFED are shown in Figure 1.7A and 1.7B. For the analysis of each sub-period, a description of the contribution of each shock based on our decompositions. The results

[31] Obviously, our terminology 'recession' refers to deviations below the HP-trend rather than negative output growth.

are associated with the historical events such as the economic reforms there to better understand the sources of these disturbances.

a. 1983Q1—1985Q4.

AS-AD decomposition: The first cycle starts with significant surges in aggregate demand components. Then the supply components also go up with a magnitude that gradually becomes much bigger than the demand components and thus mainly account for the boom of the economy in 1984 and 1985. The demand components on the other hand, show a considerable drop during 1985Q4-1986Q1, bearing the responsibility for the recession. It is also noteworthy that, the demand components move ahead with a direction that the supply components follow.

IS-LM-PC decomposition: The movements of the IS components dominate other demand components. In particular, the main source of the surge in demand components which is crucial for the recovery of the economy is related with the IS shocks. The IS components also account for the sharp decline of demand in 1986. The money supply moves inversely with the IS components. The role of money demand is very limited.

Events: The main event in the 1983Q1-1984 period is the ongoing economic reform. Key elements of the reform include: The 12th Plenum of the Communist Central Committee in 1984 announced the plan to push the reform to a new phase; successful reforms in both rural (setup of rural exchange market) and urban areas (enterprises reform giving more autonomy and incentives to managers); development of non-SOEs which created more job opportunities and attracted more labour forces flowed from rural area to urban area; prices liberalizations; increased imports (73 percent in 1984) resulting in a significant trade deficit. The economic reform has both demand and supply side effects. On the one hand, it improves the efficiency in production and gives positive supply shocks. Our results show that these supply shocks due to reform have fast impact on the economy and contribute the most to the boom. On the other hand, the economic reform also releases the demand of people suppressed during the central planning era. The initial swing in aggregate demand confirms this. This demand is accommodated only gradually in the following years.

The key events in the remaining time of this subperiod were the economic overheating showed up in 1985 and the tightened economic policies in late 1985. These events have negative demand effects which are confirmed by the observed immediate substantial drop of demand especially the IS component.

b. 1986Q1—1990Q4.

AS-AD decomposition: Aggregate demand bears the full responsibility of the recovery of the economy from in 1986. Both supply and demand components are responsible for the boom in 1987 and the recession during 1989-1990, with the contribution of the former five times larger than the latter.

IS-LM-PC decomposition: The recovery and the peak of demand components again emphasize the role of the IS components. The large decline in demand components in 1989 are due to a sharp decrease in IS components while the money supply components are increasing. Both the roles of money supply and money demand are very limited except that the further drop of demand components in 1990 is largely due to the large decline in money supply.

Events: The central government decided to launch further reform following the bad economic situation in 1985-86. More reforms were conducted in SOEs, banking system and trade system. The price liberalization was implemented in April 1988, resulting in climbing prices of consumption goods. As a result, an increase in aggregate supply components was also observed during this period. At the same time, the bank credit and monetary policy were eased again, resulting in rapid growth of broad money (30 percent in the first three quarters of 1987). Consequently, a surge in aggregate demand was observed around 1988 due to this proactive monetary policy. In June 1989, the economic heating plus the corruption problem of the government induced the 'Tiananmen Square Incident'. It resulted in a drop in hours and production. Meantime, the government tightened the monetary policy in 1989 and postponed the economic reform. As a result, both aggregate supply and demand went downhill, bearing the responsibility of the recession in 1989.

c. 1991Q1—1997Q4.

AS-AD decomposition: The supply components continued to gain significant developments from 1991 to 1995, bearing the responsibility for the recovery of the economy. In contrast the demand components only fluctuate around the trend. The period from 1993Q2 to 1995Q1 represents an exception where the demand components contribute to almost all of the increases in real GDP while the supply components experience a decline. However, the demand components soon jump down in 1994Q4, bearing the responsibility for starting the downturn. The supply component jump again in 1995Q4 but soon start the decline from 1996. Both supply and demand components are responsible for the recession during 1996-1997. However, the decline in real GDP is quite moderate compared with the downturns of the previous two cycles. Since the prices are largely reduced at a small cost of output reduction, this downturn in 1996-1997 has been referred as the 'soft landing' from previous overheating years.

IS-LM-PC decomposition: The money supply components account for most of the increases in demand components at the beginning of this cycle. The short-lived decline in demand in 1993 is caused by the money demand. There would have been a further decline of demand in 1995 following by the jump in IS components, if there were no increase in money supply. The money supply and IS components are responsible for the large decline of demand components in 1996 and in 1997 respectively. It is noteworthy that the overall volatility of the IS components has greatly reduced in this cycle.

Events: The economic downturn and the political pressure did not lead the government to turn back to central planning. Rather, several further reforms were undertaken to cure the recession. The key aspects include further price liberalization, deeper reform in SOEs, establishment of more commercial banks and the reform in trade sector to promote export (trade balance turned to surplus in 1990 after a depreciation of the currency in the same year). A marked historical event was the Deng Xiaoping's Southern Tour and the 14th National Congress of the Communist Party held in 1992 which set the backbone for deeper reforms and the aim to build a socialist market economy. These events sent significant signals of deeper reform to people. As can be seen in the Figures, they generated large increases in aggregate supply components responsible for the recovery of the economy.

The fiscal and monetary policies during 1992-1993 were very proactive, leading to an investment boom. That corresponds to the surges of supply and IS shocks during 1992-1993. There was then a short period in 1993 where the authorities slowed down the reform and tightened the monetary policy, however, both fiscal and money policy became loose again in early 1994. The Chinese currency was greatly depreciated in late 1994, leading to significant increases of trade surplus and marketable inflows of foreign direct investment (FDI). As a result, although the money demand dropped in 1993 probably due to high prices, the increases in IS and supply components maintain output at a high level. Thus, the soft landing of the economy might be due to the moderate decline in supply and IS components. Possible reasons for this moderate declines might be the continued effect of the reform, the moderate changes in monetary and fiscal policy and the increasing trades surplus and the inflow of FDI.

From late 1994, the PBC had adopted tight monetary policy in fear of economic overheating. The Asian Currency Crisis occurred in 1997 and added more downward pressure on the economy. These result in the quick and large decline of aggregate demand (i.e., money supply and IS components) during 1996 - 1997.

d. 1998Q1—2004Q3.

AS-AD decomposition: Both supply and demand components bottom out in 1998Q3. However, supply components take the following six years to return to the trend, whilst demand components return to their trend faster. The overall volatility of both supply and demand components is much smaller than for the previous cycles. Especially, the movements of demand components in the following years in 2001-2005 are extremely smooth. In fact, this sub-period has not shown a standard business cycle but rather macroeconomic stability. It is noteworthy that, the movements in supply components and demand components seem to be on the opposite during this period. Thus they equally contribute to the fluctuations in real GDP.

IS-LM-PC decomposition: The money demand and IS components are very stable during this cycle. Only the money supply components show some small variations. It is noteworthy that at the end of this sub-period until mid 2004, the jumps in the two money components are mainly responsible for the increase in GDP growth rather than the supply and IS components.

Events: The most important feature of events during 1998-2002 is the moderation of both economic reform and policy. For example, as a policy reaction to the economic downturn in 1997 and to overcome the negative impact of the Asian Currency Crisis, the tight macroeconomic management now is replaced by an moderately expansionary monetary and fiscal policies. Meantime, the overall speed of reform has been slowed down compared with the last decade. The only significant reform is the SOEs structure reform proposed by Zhu Rongji, the new premier of the State Council. The moderation in reform and policy seems to affect the economy in the same way. The overall volatility of both supply and the three demand components is much smaller than for the previous cycles as summarized above. Given the slow development in the supply side[32], the moderate monetary and fiscal policies explain the macroeconomic stability during this period.

[32] Retrieving to the institutional discussion in Section (1.2), the restructure of the state-owned enterprises reduces the burden of the central government. Less fiscal transfer and money expansion are needed to subsidize their deficits, which also helps macroeconomic stability.

There are two other important events occurred during this subpeirod: China became a member of the World Trade Organization (WTO) in November 1999 and the Severe Acute Respiratory Syndromes (SARS) occurred in 2003. However,

Since the recovery of the economy has been slow and is still below the trend, the expansionary policy continued until early 2004. However, as the results show, their effects on the economy have been very limited and short-lived.

e. 2004Q4—2009Q3.

AS-AD decomposition: From 2004Q4, output entered a new boom with the main contribution coming from the supply components. Demand components respond negatively in the next two years and then increase in 2006Q3 following the same pattern of the supply components. The peak in 2007Q3 was not long-lived with the recession starting from 2007Q3 to 2009Q3. The demand components are as important as the supply components in driving the economy to the recession. Overall, the supply and demand components move in the same direction.

IS-LM-PC decomposition: The increases in demand components in the beginning of this cycle are due to the two money components. However, the boom of the demand components in 2007 is due to the IS shocks since the money supply components on the other has been falling from 2005 to 2008Q3. The IS components then fall considerably during 2008Q3 - 2009Q3, bearing the responsibility for the recession together with the supply components. Over this period, the money demand components have continued its smoothness since last cycle and have hardly moved.

Events: The monetary policy became loose and loose from 2004 to early 2005, maybe due to the relative stability of the economy during the last subperiod. The government has committed to a neutral fiscal policy before 2007. The Olympic Games were successfully held in August 2008. The loose economic policies and the pre-Olympic investment tide explain the boom of the economy in 2007.

The economic reform on the other hand, entered a phase with no significant changes in economic structure or institutions. Therefore, the surge in supply components might not related with efficiency improvement in reform but related with other factors. These factors might include technological progress gained during the reform and openness and also the delayed supply effects of joining the WTO.

In 2007, monetary policy was tightened. The outbreak of the International Financial Crisis occurred in 2008Q3. The government launched a set of fiscal and monetary measures from December 2008 to combat the recession[33]. By consequence, the International Financial Crisis has greatly affected output, indicating that the Chinese economy has been more integrated with the world economy. The stimulus package of the government helps the recovery of the economy. Our results show a turning point of the demand components in 2008Q4 and a turning point of the supply components in 2009Q2. A further detection shows that the expansionary policies lifted money components first, then came the IS components and finally reached the supply components. This reflects that the proactive fiscal policy has been effective not only in expanding aggregate demand but also supply. This may be due to the fact that most of these stimulus packages have been spent on basic industries such as infrastructure and health care, which benefit production. It is also noteworthy that the money supply components has not increased much compared with the IS components during the recovery from 2009Q1. This might indicate that the monetary policy has been quite moderate.

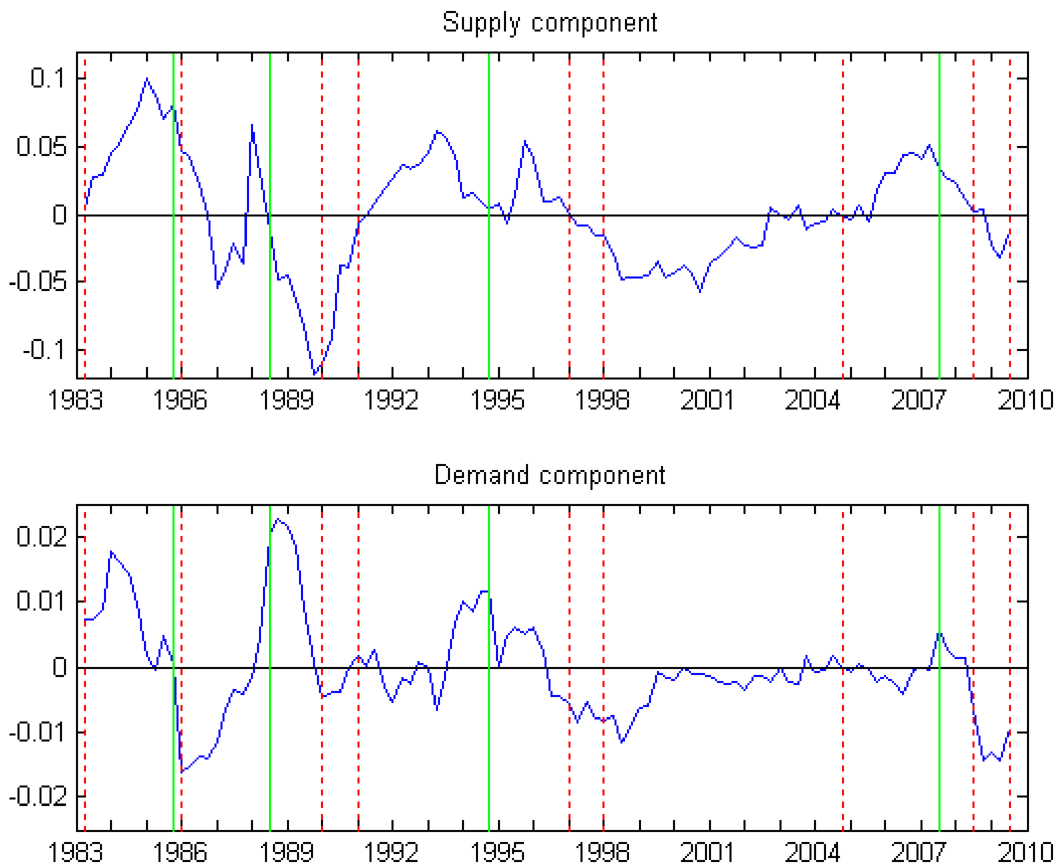

Figure 1.7A: Historical Decomposition of GDP Series

[33]The key elements of these measures include a 4000 billion RMB stimulus package, a 500 billion RMB tax-cut package and an 850 billion RMB package on health care. The stimulus package has been financed by the fiscal revenue of the central government, by selling bonds and by utilizing the revenues from the local governments.

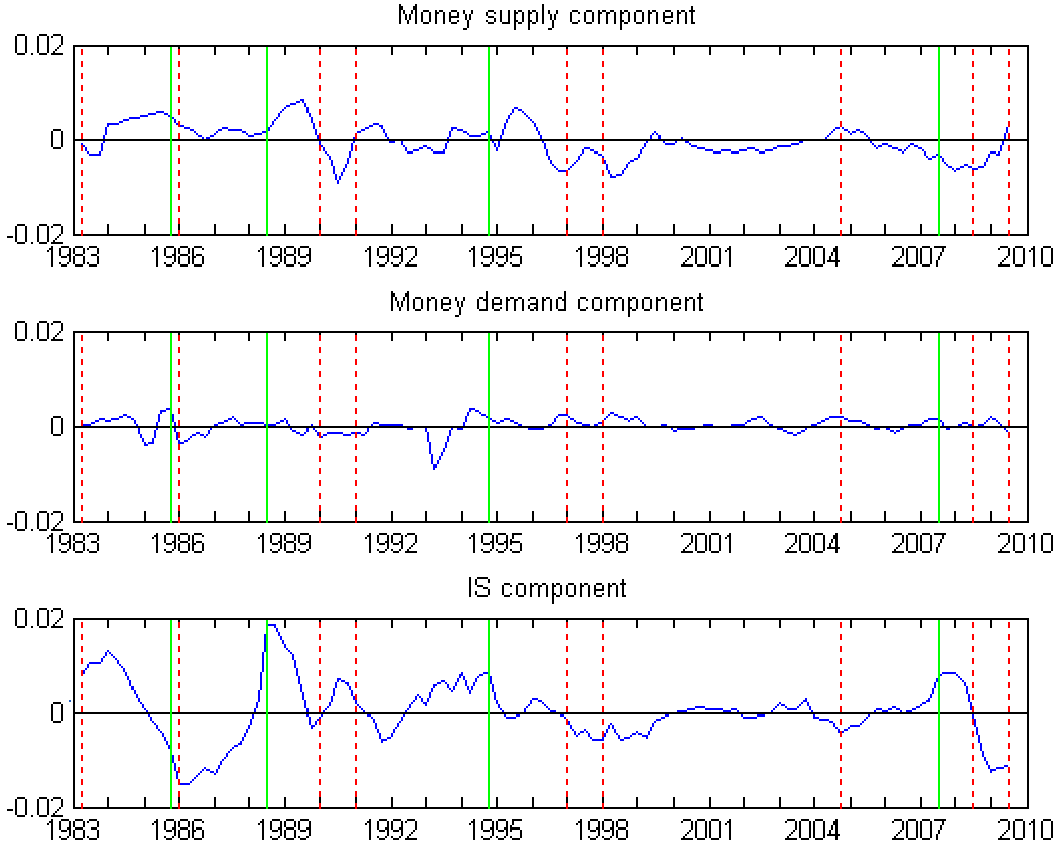

Figure 1.7B: Historical Decompositions of GDP series

1.7 Robustness of results

To check the robustness of the evidence above, the same empirical analysis was made using other choices of the dependant variables in the VAR model. In particular, the alternatives included using 'money' rather than 'money & quasi-money' and using deposit rate rather than lending rate. The results are presented in Appendix (1.9.3).

1.7.1 Deposit rate

The main finding is that when lending rate is replaced with deposit rate, all the results above (i.e., the impulse responses, the variance decomposition and the historical decomposition) remain similar. The only slight differences are quantitative, not qualitative. For example, the responses of real interest rates to supply and IS shocks are higher than in the benchmark model. The responses of money growth and inflation to IS shocks are also higher. For the variance decomposition, a lower proportion of GDP fluctuations are accounted for by supply shocks whilst a greater proportion is accounted by the IS shocks. The historical decomposition changes very little. The conclusion, therefore, is that the results discussed in previous sections are robust to alternative measure of interest rate.

1.7.2 Narrow money

When the series of 'money' is used in estimation, the impulse responses to supply remain qualitatively identical. However, the responses of variables to money shocks are sensitive to the choice of money measures. In particular, the response of money growth to a money supply shock is negative in the first quarter. Furthermore, even in the short-run, the LM equation is violated in the money supply shock case. This might imply that the money shocks are incorrectly identified. The reason for this might be that the narrow money is not adequate for identifying the effect of monetary policy.

In recent years the PBC has announced targets for broad money growth rather than narrow money growth. Therefore, the narrow money might not be a suitable choice of dependant variable in our VAR analysis. Interestingly, the results of the variance decomposition and the historical decomposition do not show significant differences with the benchmark case. Therefore the results of the estimated decompositions, discussed in the main context can be considered as particularly robust.

1.7.3 Alternative specification and the sources of the unit root in nominal variables

The specification of the VAR relies on the unit root tests where all the nominal variables, i.e., nominal interest rate, money growth and inflation, are stationary. This implies that impacts of the structural shocks on nominal variables only exist in the short-run. However, it is well-known that, in general, unit root tests have low power. For example, Schwert (1987) argued that the ADF test is biased if the data series takes an ARIMA representation with large (negative) MA coefficient. Also, the above unit root tests perform poorly when the true data process is high persistent and close to being $I(1)$. In fact, the unit root tests do not show a 1% rejection of unit root in the first-differenced inflation and money growth series. Given the small sample size available, the stationarity of inflation and money growth might not be guaranteed. Based on these discussions, it is possible to consider the alternative specification that a unit root is present in nominal variables. This allows an examination in the adjustments of nominal variables in longer horizons. Since the results found that the responses of the real variables are very similar (and qualitatively identical) under both specifications, the following will focus on the responses of nominal variables.

Supply shock The impulse responses of variables to a favourable aggregate supply shock under the alternative specification are shown in Figure 1.8. The responses of GDP and real balances show little difference with the benchmark specification. However, the release of the stationarity of nominal variables results in substantial and sustainable increases in money growth and inflation. Therefore, the supply shock is responsible for the unit root in nominal variables. Since the nominal interest rate still adjusts slowly in the short-run, the surge in prices results in a small decrease in interest rate. As a result, the long-run level of output is a little higher than the same under the second specification.

Two important features remain unchanged. The first feature is that the dynamics of the nominal rate and real balances are still consistent with the LM equation, not only in the short-run, but also in the long-run. The estimated short- and long-run income elasticity, ϕ, is 1.4 and 0.5 respectively. Also, as output increases, it creates upward pressure on inflation in a manner consistent with the Phillips Curve. Secondly, the substantial changes of output and inflation are mixed effects of the supply shock and the monetary expansion. Again, they can be explained by the hypothesis of institutional accounting of fluctuations[34].

[34] Note that, for example, according to Brandt and Zhu (2000), a positive supply shock usually occurs when the government adopts a loose indicative credit plan with less resort to money creation. Thus the negative correlation between aggregate supply and money growth is possible when output growth is low.

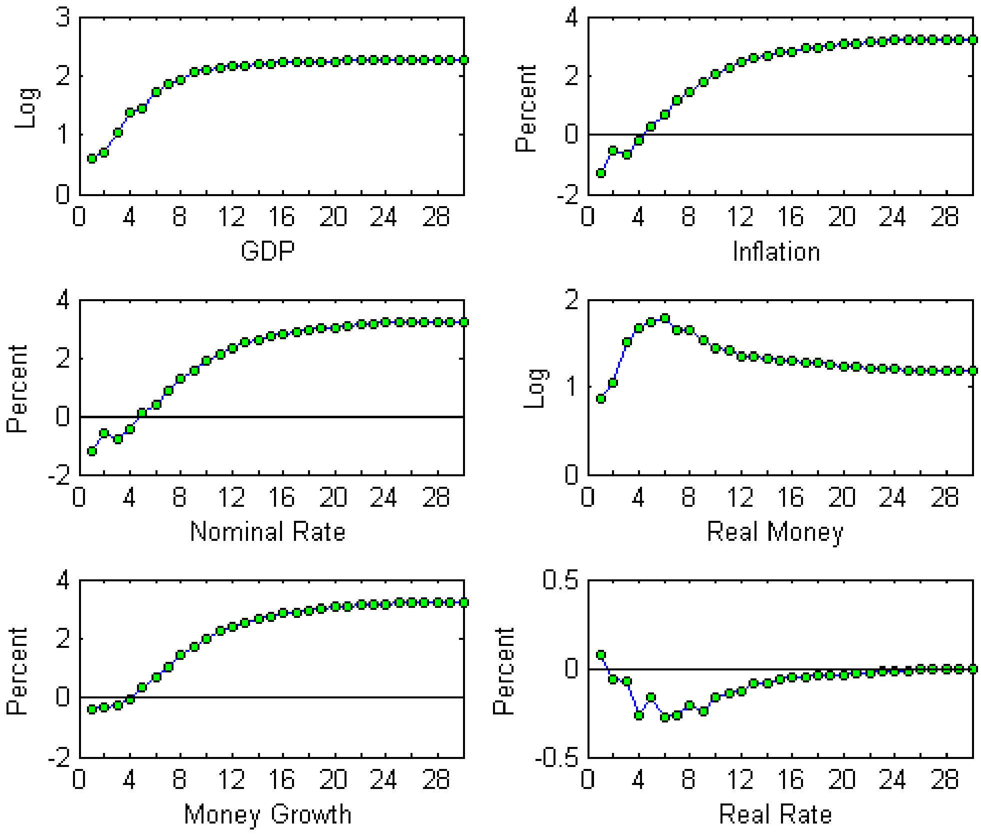

Figure 1.8: Impulse Responses - Supply Shock

Money supply shock Figure 1.9 shows the responses of different variables to a money supply shock. The dynamics of real variables are again similar, while the responses of nominal variables make the differences. Money supply only increases slightly and then becomes negative for two quarters. The same variable then gradually increases and reaches a much higher level after 12 quarters. The institutional background provides an insight into the small and even negative response of money growth in the initial periods: The money supply is raised normally when output growth is high and prices are temporarily low. In fact, the negative response of money growth is indeed related to the drop of prices within the first 3 quarters, as shown in figure 6. As money growth increases and output growth rises, prices go up. Therefore, it seems that the money supply shock is also one source of unit root in nominal variables.

The money-interest rate-output transmission mechanism in the IS-LM-PC model now works both in the short-run and in the long-run. The estimated semi-elasticity of nominal interest rate, λ, is 0.7 and 0.2 respectively. Again, the effect of the money supply shock on output is still very small compared with the supply shock. The decrease in interest rate is much smaller than the same in the second specification. This indicates that the money expansion itself is not effective in raising output due to small changes in interest rate. On the other hand, the money supply shock has substantial impact on prices. The magnitudes of changes in money growth and inflation in figure 1.9 also match strikingly well with those in the case of the supply shock in Figure 1.8. Therefore, it could be argued that the output increases in figure 1.8 are mainly due to the supply shock, while the increases in inflation are mostly accounted for by the induced money expansion.

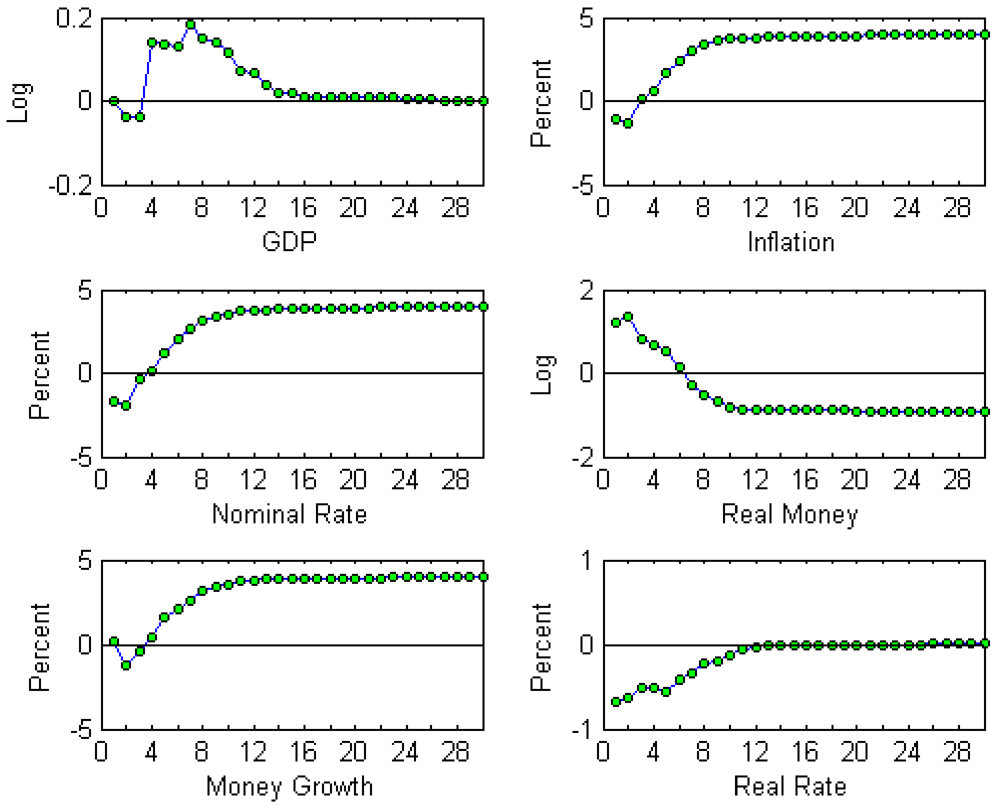

Figure 1.9: Impulse Responses to Money Supply Shock

Money demand shock

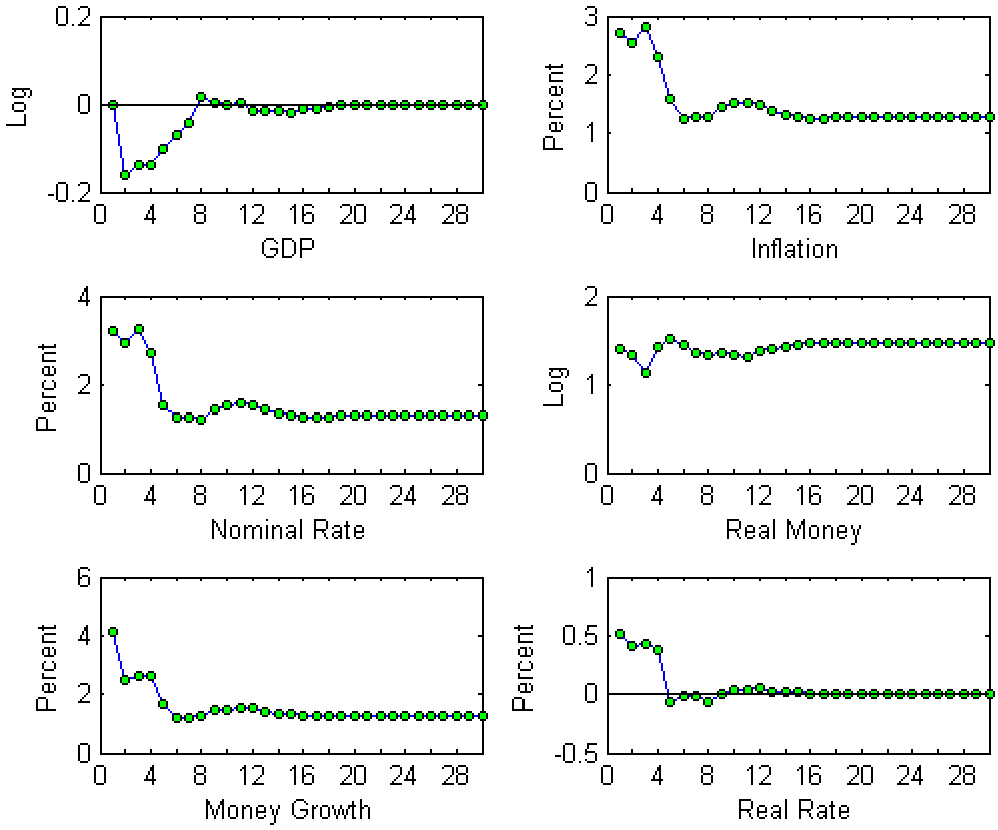

Figure 1.10: Impulse Responses to Money Demand Shock

The responses of variables to a money demand shock shown in Figure 1.10 appear as the qualitative mirror image of the results in Figure 1.9 in the case of a money supply shock. The differences come from the rapid adjustment of prices, interest rates and output. Moreover, the results here are almost qualitatively identical with the same under the second specification in Figure 1.3. The difference is again in the speed of adjustments of nominal variables and, additionally, in their long-run values. Here all nominal variables are permanently raised up, suggesting that the money demand shock is another source of the unit root in nominal variables.

IS shock The responses of variables to a fiscal spending shock are given in Figure 1.11. At first glance, all the variables show identical responses except the money growth and the initial responses of the interest rate. In fact, the monetary policy is no longer complementary to fiscal policy. Since the nominal rate is raised up through the LM equation, the PBC seems to raise the money supply to dampen the upward pressure on the nominal interest rate. However, since the inflation jumps higher than the nominal interest rate, the interest rate jumps down about 0.5 percent. As a result, the initial increase in output is higher than the same in the benchmark specification. In mid-term, from 5 to 12 quarters, the interest rate goes up and the crowding-out of the fiscal expansion drives output down in a way consistent with the IS curve. Nominal rate and inflation also decrease in a manner consistent with the LM equation and the Phillips Curve. The estimated short-run and long-run interest elasticity, λ, is 0.7 and 3.1 respectively. The short-run estimate is nearly the same as the one obtained in the money supply shock case, while the long-run estimate differs significantly. The non-stationarity of money growth seems to indicate that the PBC adopts the monetary accommodation again after 13 quarters to avoid recession. As a result, money growth remains positive in the long-run with permanent increases in nominal rate and inflation. Only output and interest rate therefore return to zero in the long-run. The IS shock appears as another source of the unit root in nominal variables.

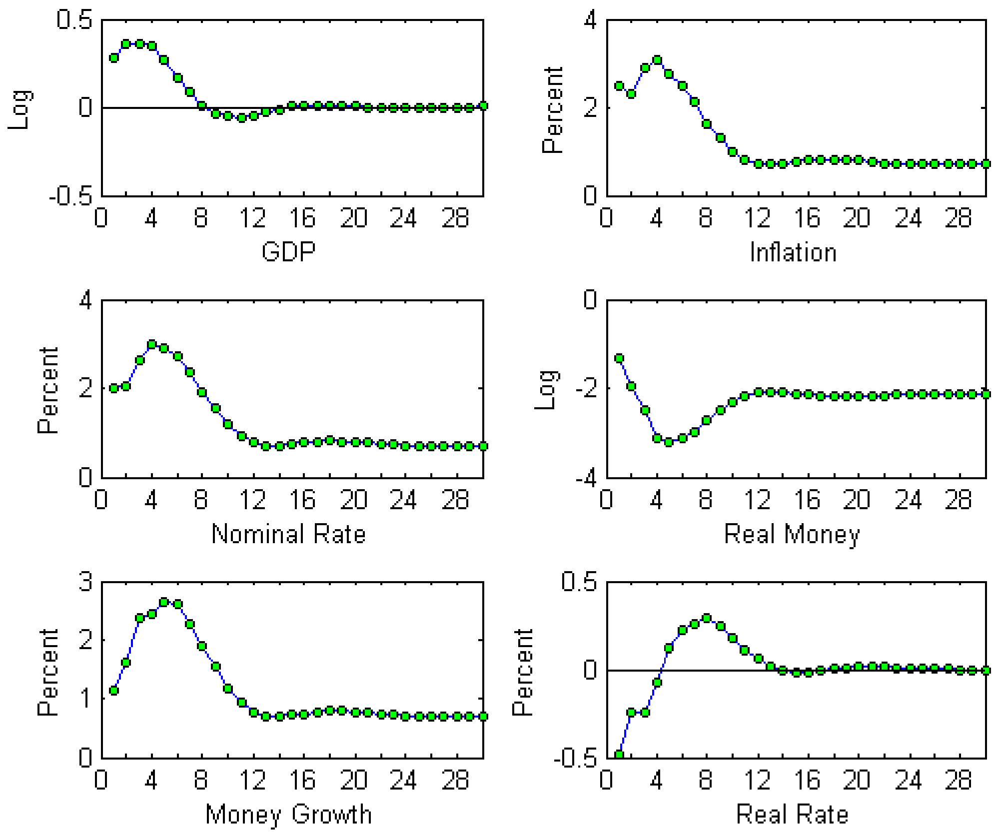

Figure 1.11: Impulse Responses - IS Shock

The main finding of the above examination is that all four structural shocks are responsible for the unit root in nominal variables. This is confirmed in the plot of the joint responses of nominal variables, i.e., inflation, money growth and nominal, in Figure 1.12. In the long-run, the impact of money supply shock is the biggest while the same of IS shocks is the smallest. However, as discussed above, since the permanent changes in nominal variables under supply and money demand shocks coincide with the induced money growth, it can be concluded that the contribution of the unit root in nominal variables is mainly due to the fiscal spending and the active monetary policy.

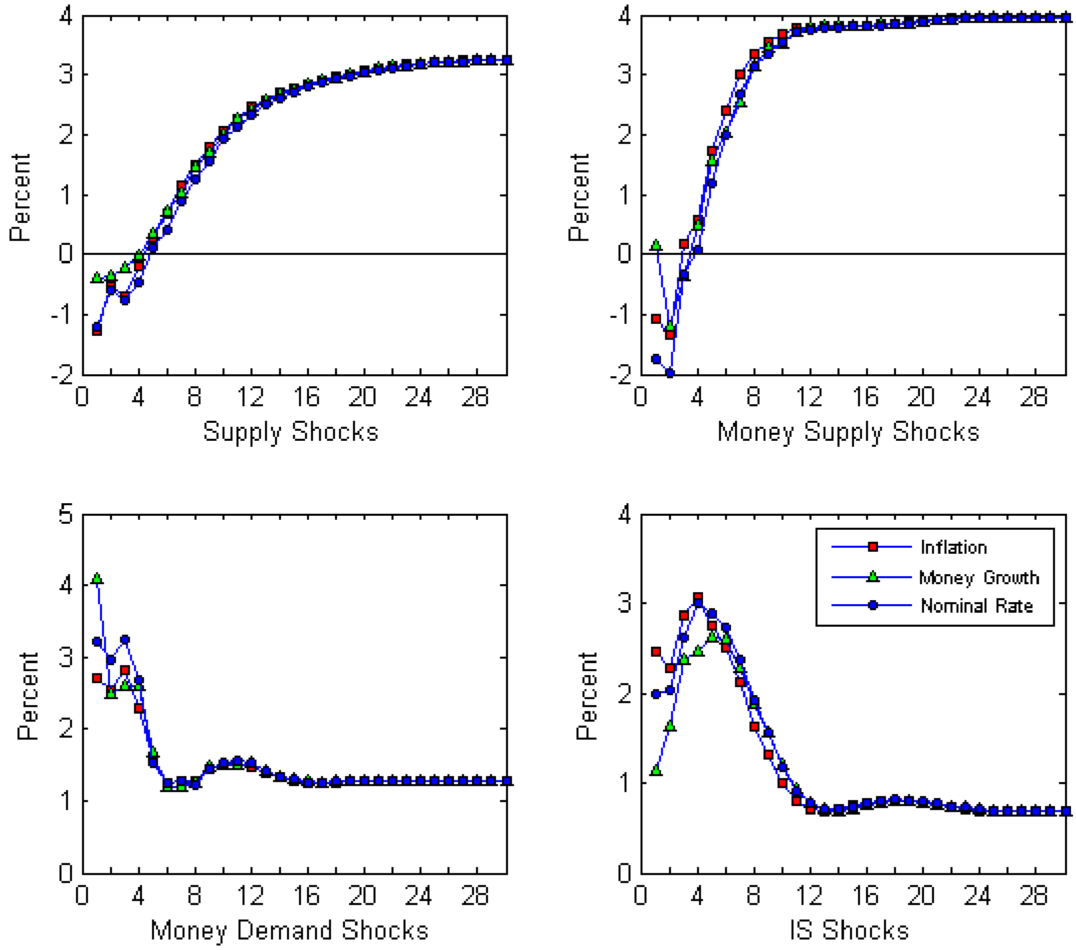

Figure 1.12: The Unit Root in Nominal Variables

1.8 Concluding remarks

The present study examines the sources of China's economic fluctuations in the post-reform period 1980-2009. Based on the popular IS-LM-PC model, the sources of fluctuations are accounted for by the four driving forces, i.e., aggregate supply shocks, money supply shocks, money demand shocks and IS shocks. The joint behaviors of GDP, prices, money and interest rate are then estimated in a four-variable VAR model using quarterly data. By applying the identification strategy proposed in Galí (1992), four structural shocks are identified so that they can be interpreted as the four driving forces in the IS-LM-PC model. This is achieved by adopting economic restrictions relating to different long-run and short-run dynamics of the economy. After the identification, the estimated effects of the structural shocks are compared with those predicted in the IS-LM-PC model.

The results show that the estimated responses of variables to all the four structural shocks in the SVAR model match strikingly well with those predicted by the IS-LM-PC model. In particular, the working of the three types of demand shocks are evidence that as reforms goes deeper and deeper, the market mechanism has gained growing importance, allowing the market channels to work in terms of fiscal and monetary policies. The fit of the IS-LM-PC model for explaining the Chinese economy and the consistent responses of the economy to the three demand shocks are new findings. Second, there is strong evidence from the variance decomposition that supply shocks associated with technology progress, efficiency and institutional changes in reform account for almost all fluctuations in output. This suggests that the role of fiscal and monetary policy shocks are minor. Whist this is not a surprising result for China as a transition economy, it might also suggest that the working of fiscal and monetary policy might be through non-market mechanisms such as direct management and other administrative controls found in literature. This implies that further reform in economic structures and institutions, such as financial liberalization, are needed to remove the obstacles for a workable economic policy. Based on the current progress of reform, it is suggested that theoretical models should first be built on the real side of the economy. Second, this study provides a historical decomposition of the forecast error of the SVAR model to examine the sources of GDP fluctuations in the five business cycles over the post-reform period, 1983-2009. This is performed not only from a traditional AS-AD perspective but also a decomposition of the forecast error related to the three demand shocks. Our findings show that supply shocks are the main sources of output fluctuations while demand shocks, especially the fiscal forces, can play important roles in different sub-periods. Third, it is shown that the above results are robust to alternative specification of the VAR with alternative measures of interest rate and money. Finally, by allowing integrated processing in nominal variables, it is found that all the four structural shocks account for the unit root in nominal variables.

While the methodology used in this study has been widely used by researchers for developed countries, it has seldom been applied for developing countries. For a transitional and developing country like China, the working of the IS-LM-PC model is less well familiar. Therefore, the main contribution of this study is that it provides a first attempt to examine the fit of this influential theoretical model to the Chinese economy and the sources of demand-side contributions to fluctuations. In particular, the working of the IS-LM-PC model, especially in the case of fiscal and monetary shocks, are new findings. Moreover, since there has been no such empirical research that covers time-series data as extensive as ours, our study is also a first attempt to empirically examine the underlying sources of fluctuations for the whole post-reform period. This is crucial for understanding the performance of the Chinese economy, having taken reforms and openness into account. The data sample has also allowed us to shed light into issues relating to the possible explanations for the macro-stability of the Chinese economy during 1998-2003 and the influences of the external shocks to the Chinese economy such as the Asian Currency Crisis in 1997-1998 and the Financial Crisis from 2008 to date.

Given the dominant role of estimated supply shocks in generating output fluctuations, a further decomposition of the components of the supply shocks is particular interesting. There are several possible components in supply shocks recognized by researchers: technology shock, capital utilization shock, labour input shock and reforms and institutional shocks. There have been studies using long-run restrictions to identify non-reform related shocks. For example, Shapiro and Watson (1988) specified a output-hours-price-interest rate VAR model and applied the restriction of exogeneity of long-run labour input to isolate technology shocks from labour input shocks. However, empirical framework designed to identify reform related supply shocks have not been developed well in the literature. Given that reforms play crucial role in developing countries like China, it is particularly interesting to devise a econometric model and a identification scheme to disentangle different supply shocks. This might be done by introducing more variables charactering the effect of reforms. To develop an appropriate identification scheme, one can make use of long-run restrictions as in Shapiro and Watson (1988) but can also restrict short-run behaviour of particular variables of interest. Of course, short-run restrictions reply on using high frequency data which might not be available in the case of China. In all, a further decomposition of supply shocks is a promising direction for future research.

Some limitations also apply to this study. For example, since the Chinese economy has been in transition during the sample period, the economic structure and the conduct of economic policies might have also changed. It is thus constructive to estimate a time-varying VAR analysis. Second, bearing in mind the Lucas's Critique (Lucas (1972)), it would be useful to construct a theoretical model with micro-foundations for China. Given the dominant role of supply side disturbances, this study suggests that theoretical models explaining Chinese economic fluctuations should be built on the real side of the economy. Therefore, the line of real business cycle (RBC) models could be used to explain Chinese economic fluctuations. The theoretical model could be calibrated to Chinese data and its performance examined by conducting a set of assessments developed in the literature.

1.9 Appendices

1.9.1 Sources and construction of data

The main sources of the quarterly data are the databases of the National Bureau of Statistics (NBSC) and the International Financial Statistics (IFS). The former is the official agency directly under the State Council for statistics and economic accounting in China. The latter is a database founded by the International Money Fund (IMF). The sample period of the quarterly data is from 1980 Quarter 1 to 2008 Quarter 3. The sources and the construction of data for each of the four variables used in this study are described below.

Real GDP There are two difficulties in constructing Chinese quarterly real GDP data. The first difficulty is that neither nominal nor real GDP data is available until 1992. One needs to estimate the quarterly GDP data using annual GDP data for the period 1980-1991. Secondly, there is no data for GDP deflator in China ever since. One might overcome this problem by instead using another price index to deflate the nominal GDP. However, as found by other researchers, deflating the nominal GDP data by other price indices such as the Consumer Price Index (CPI) or the Producer Price Index (PPI) does not give the correct real GDP data that match the official figures. Based on this situation, we construct our real GDP data by two sequences: The first sequence of real GDP data from 1980Q1 to 1991Q4 is taken from the estimated real GDP data reported in Abeysinghe and Rajaguru (2004). In their estimation, the real GDP growth rate was interpolated using the Chow and Lin method based on annual real GDP growth rate data and taking money stock M1 and trade as related series. The real GDP data in levels were then recovered by taking 1997 as the base year (since they found the quarterly real GDP growth rate and nominal growth rate are the same in 1997). The second sequence of real GDP data from 1992Q1 to 2008Q3 is calculated using the cumulative year-on-year real GDP growth rate data which is available from the NBSC. We believe the real GDP growth rate data from NBSC is the most reliable data, since the NBSC has adjusted the real GDP growth rate since 2004 for consistency of data. For example, there have been debates on the overestimation of real GDP growth rate during 1997-2002 and underestimation of it from 2003 upwards. The NBSC has adjusted these possible biases based on economic surveys, such as the National Economic Census in 2004.

Prices The quarterly Consumer Price Index data from 1987Q1 to 2008Q3 is taken from the IFS database. This data has the best possible length compared with other price indicators and we have crossed checked its accuracy by comparing the recent CPI data published in other sources including the NBSC. One particular issue is that the quarterly CPI data from IFS (or other sources that were checked) is year-on-year rate of change in CPI. Therefore, to recover the prices in level, we need to find a base year where the quarter-to-quarter CPI data can be compared. Fortunately, the month-to-month CPI data is available from 2001 with the price level in 2000 as the base year. Therefore, we first calculated the month-to-month prices in level in such a way that the resulting price level in 2000 is equal to 100. Following this, we derived the quarter-to-quarter prices in level for 2001 by taking a seasonal average from the calculated month-to-month prices. After this, the other price levels from 1987Q1 to 2000Q4 and from 2002Q1 to 2008Q3 can be obtained using the quarter-to-quarter CPI data taken from the IFS database.

The quarterly CPI data from 1980Q1 to 1986Q4 is unfortunately unavailable. Therefore, we interpolated these missing values by following the Chow and Lin method using annual CPI data, taking the money stock M1 as related series. The sample size of the interpolation regression is from 1980Q1 to 2008Q3 and we have checked the quality of the interpolated data by comparing the overlapping period between the interpolated values and the true observations. Finally, with the estimated price levels from 1980Q1 to 1986Q4 and the prices calculated above using IFS data, the whole series of prices from 1980Q1 to 2008Q4 can be constructed.

Nominal interest rate The interest rate data are the bank lending rates taken from the IFS. The same data taken from the NBSC was found to be the same. Better interest rate data such as the inter-bank rate or the bank reservation rate are only available from the late 1990s.

Money stock The data for the common statistics of money such as M0, M1 and M2 is/was not available since the PBC has used different measures of money until 2006. Therefore the data for money used in our study is the 'money plus quasi-money' data taken from the IFS database. It is a measure of money that is wider than M1 but narrower than M2. We believe this is a good measure of money that provides information that is no worse than M1 and M2. Moreover, at the conclusion of our study, we also considered the data of 'money' (seasonally adjusted) from the IFS database, which is close to M1. The results show that the two money measures make little change in the SVAR estimation.

1.9.2 Results under restriction 6 and 7

Impulse responses The results of impulse responses to supply and IS shocks are the same across restrictions, so only the results in the case of the two money shocks are presented.

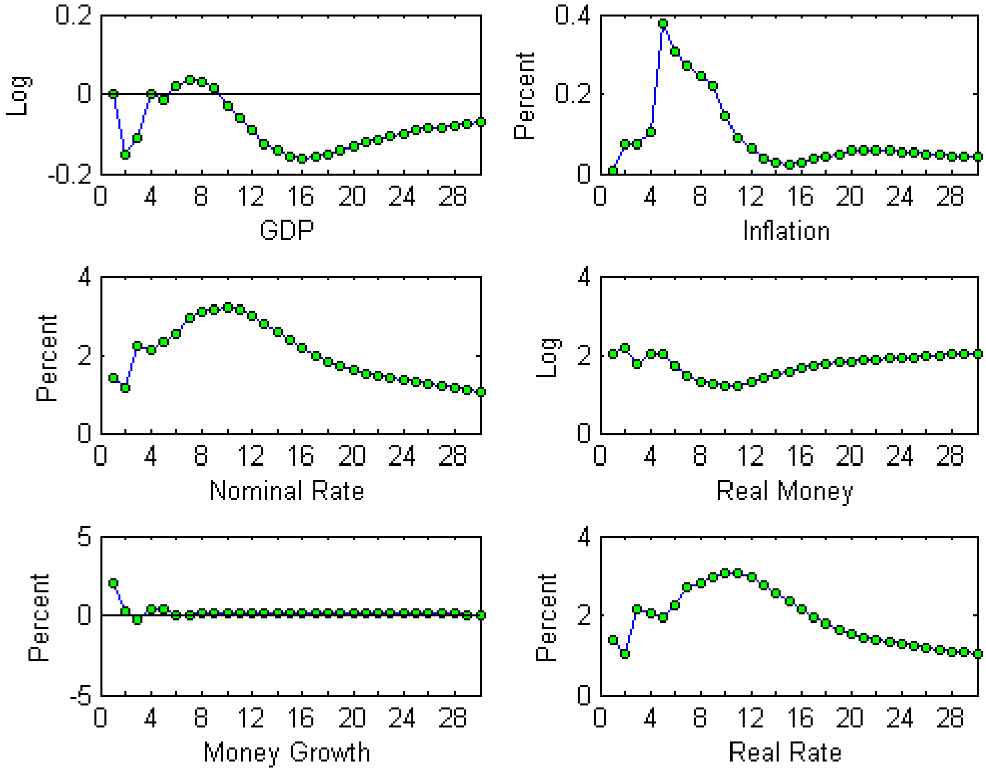

Figure 1.13: Impulse Responses to Money Supply Shock - R6

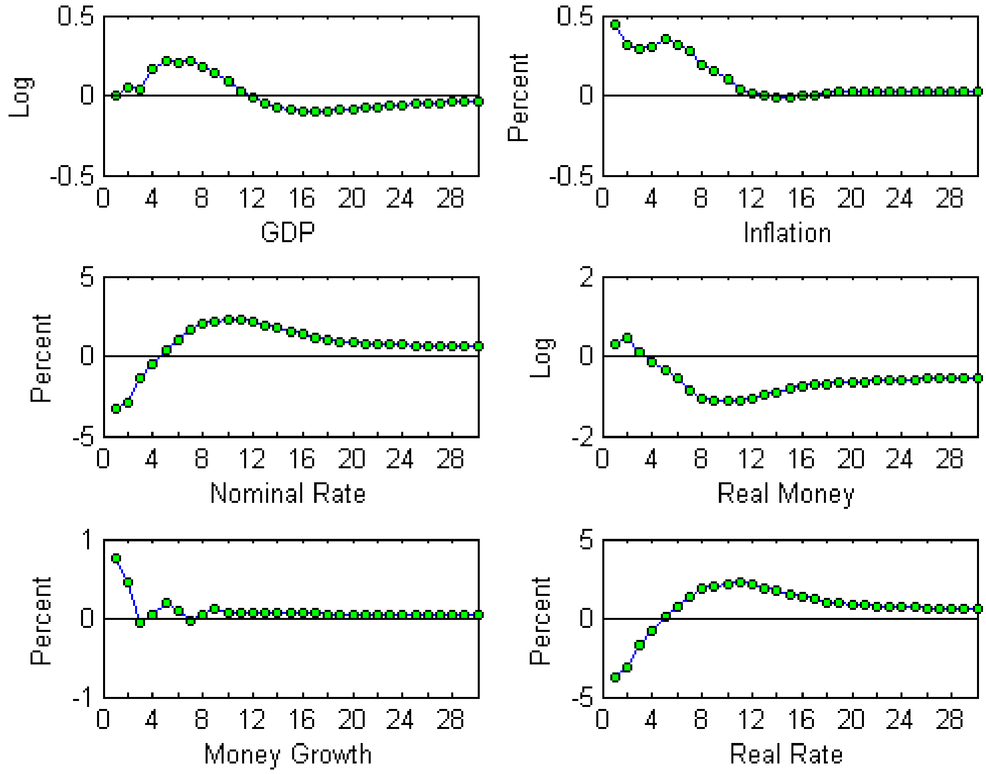

Figure 1.14: Impulse Responses to Money Demand Shock - R6

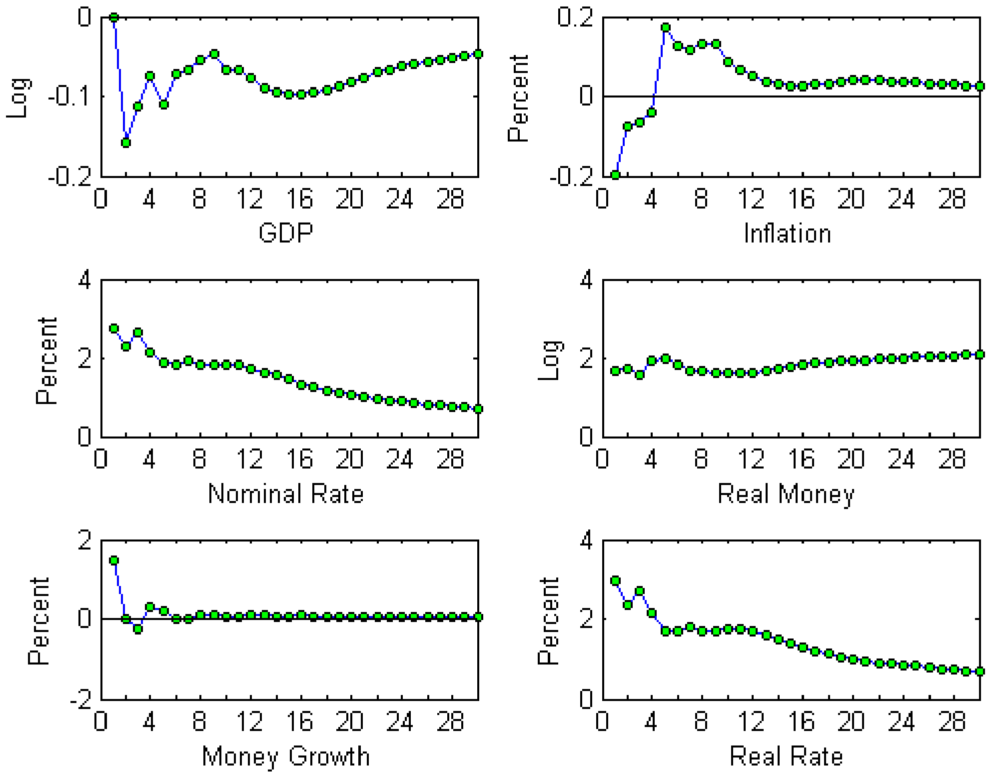

Figure 1.15: Impulse Responses to Money Supply Shock - R7

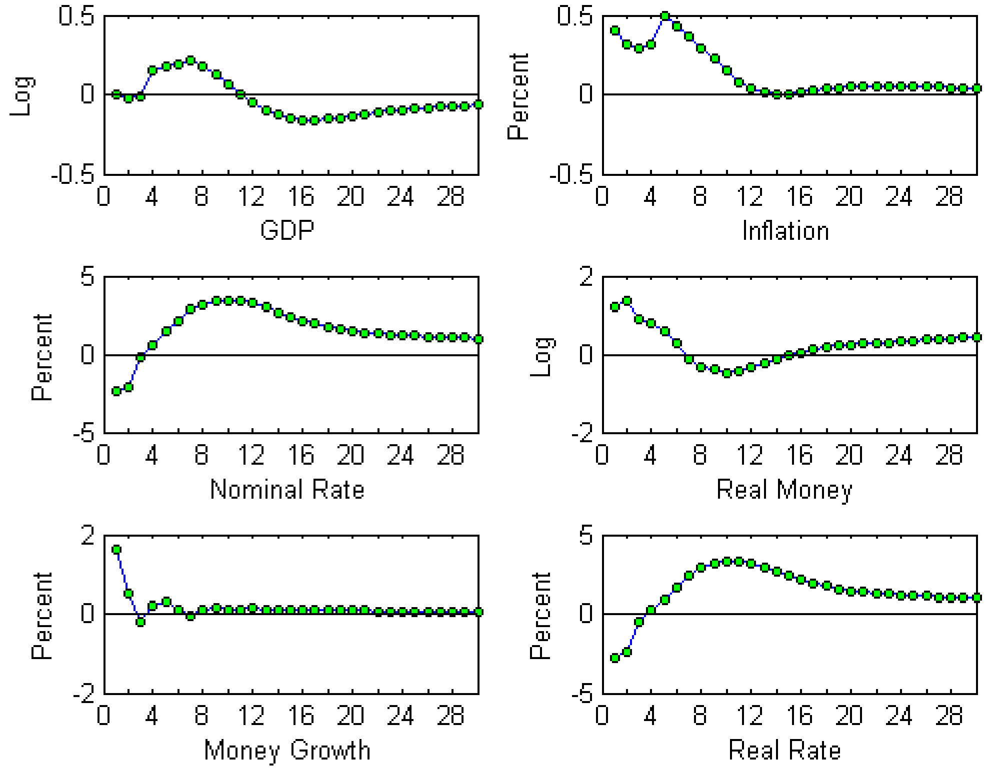

Figure 1.16: Impulse Responses to Money Demand Shock - R7

Variance decompositions The results of decompositions of forecast error to supply and demand shocks are the same across restrictions, so here only present the decompositions with respect to the three types of demand shocks.

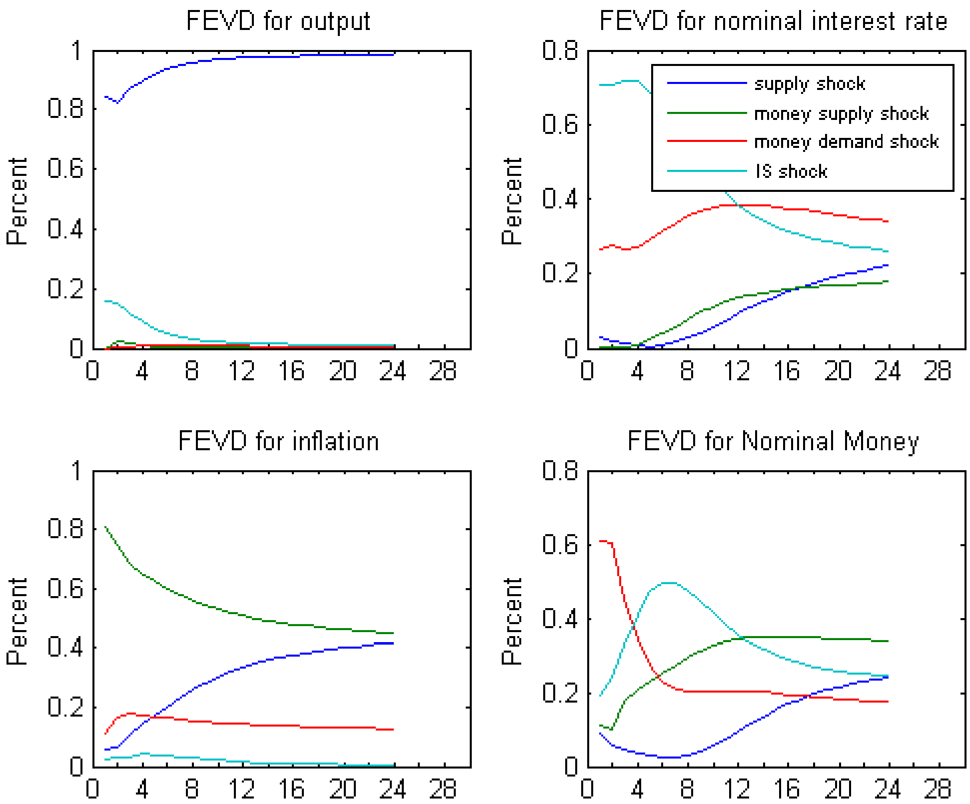

Figure 1.17: Decomposition of Forecast Error Variance - R6

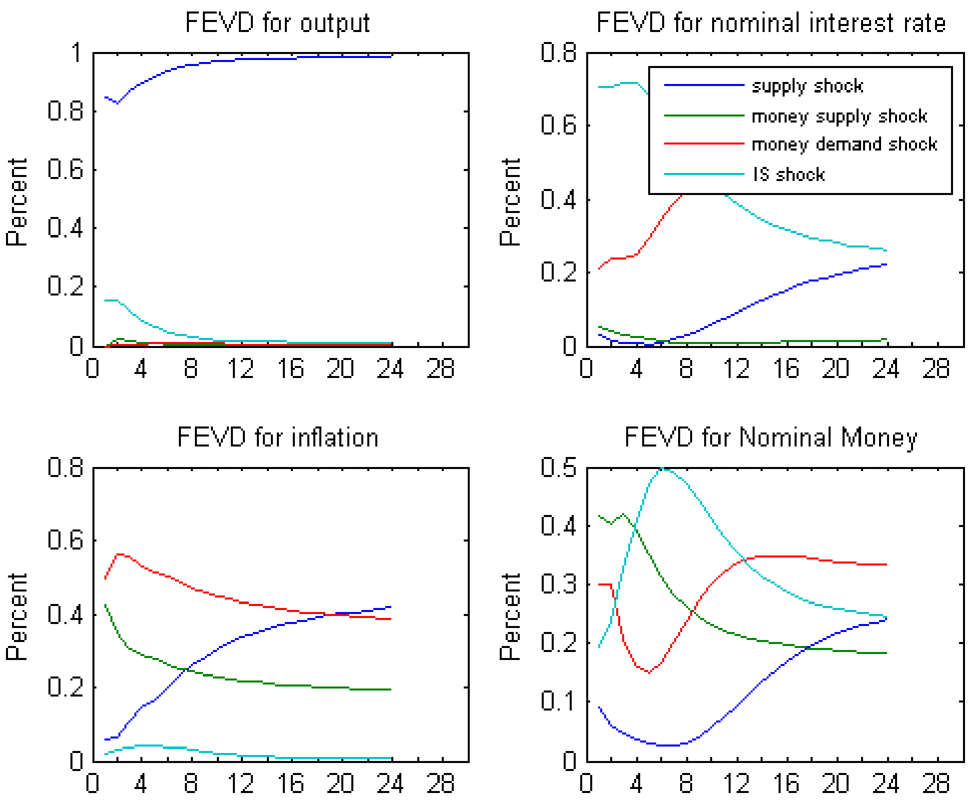

Figure 1.18: Decomposition of Forecast Error Variance - R7

Historical decompositions

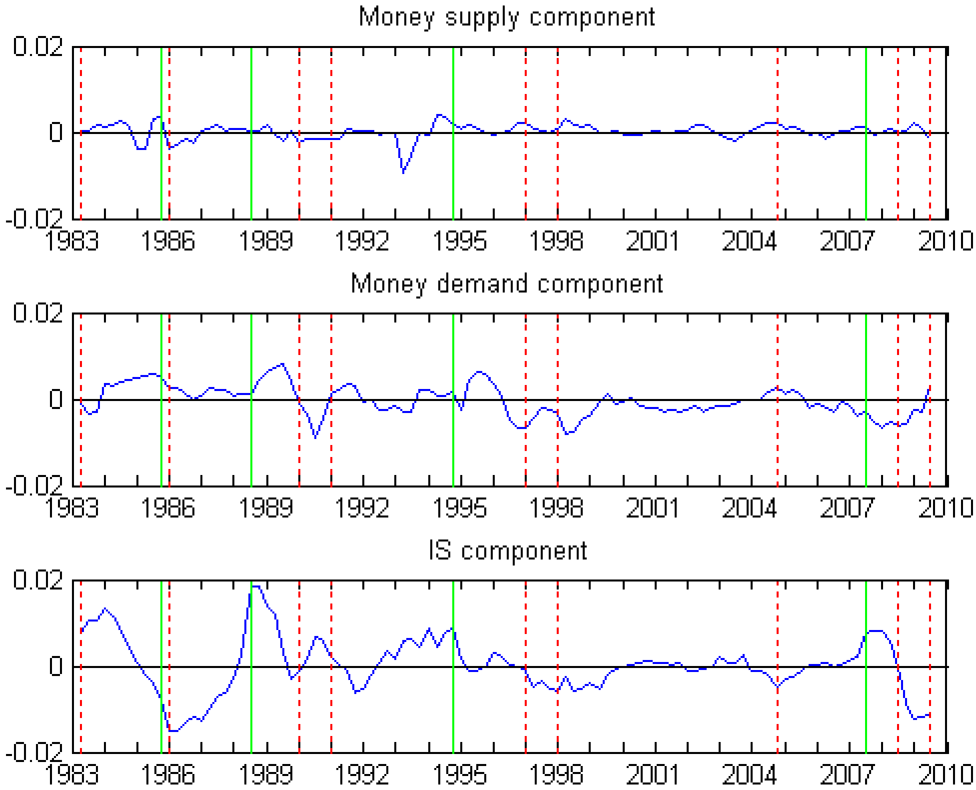

Figure 1.19: Historical Decompositions - R6

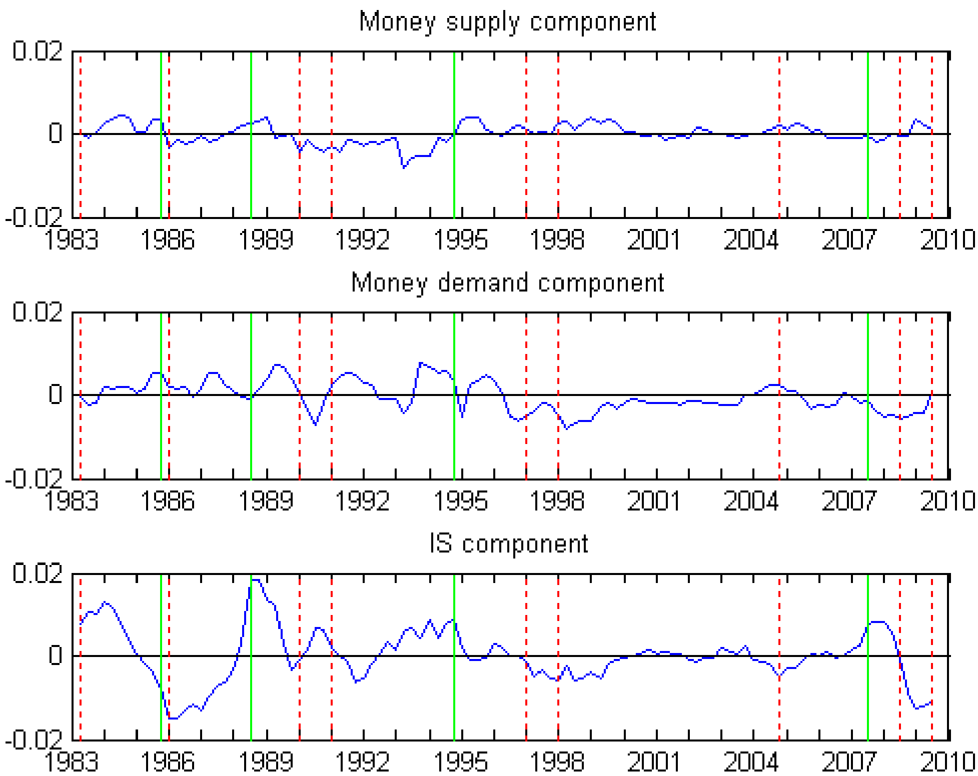

Figure 1.20: Historical Decompositions - R7

1.9.3 Results using alternative choice of dependent variables

Deposit rate

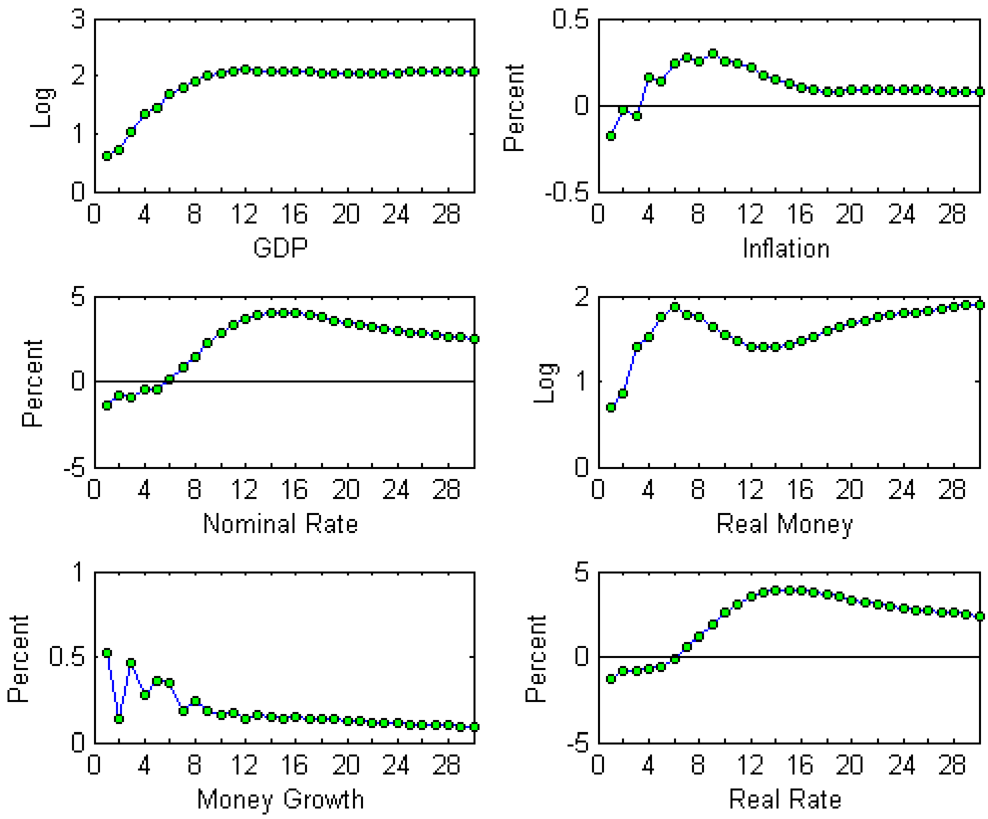

Figure 1.21: Impulse Responses to A Supply Shock

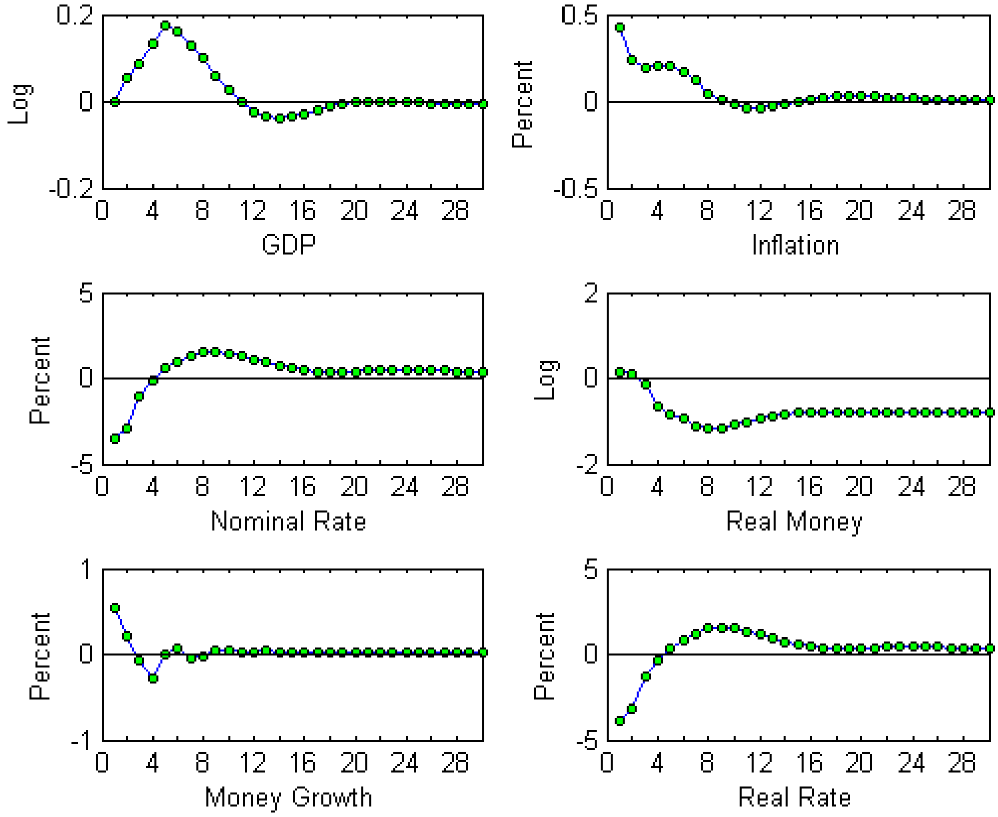

Figure 1.22: Impulse Responses to A Money Supply Shock

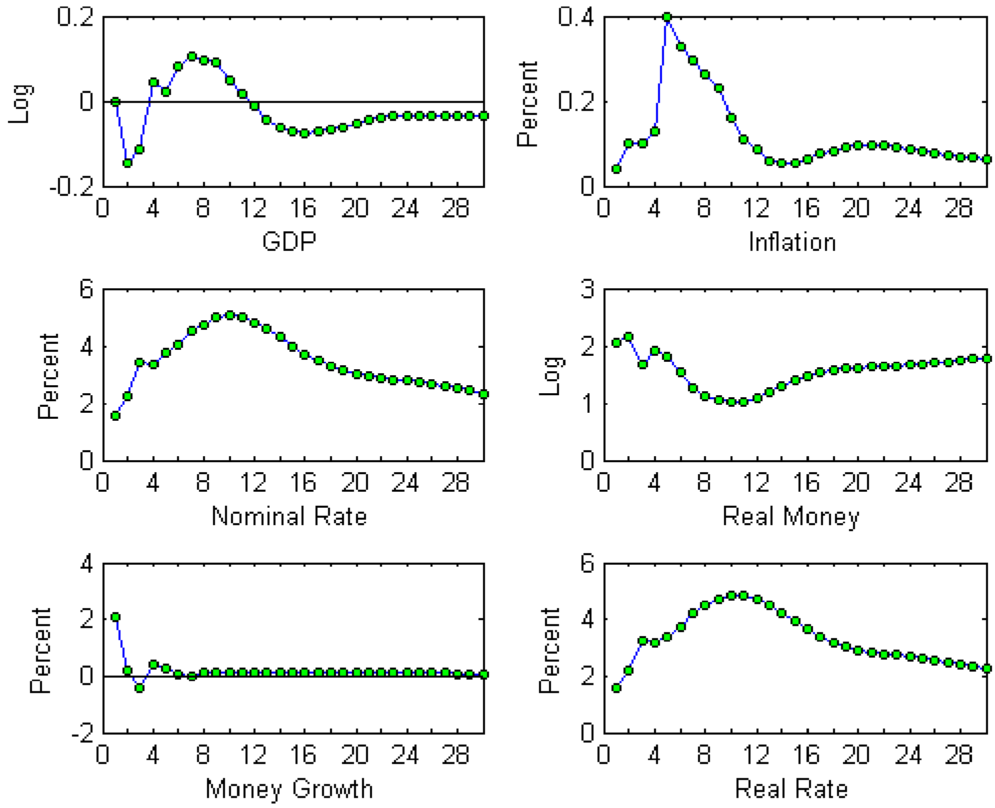

Figure 1.23: Impulse Responses to A Money Demand Shock

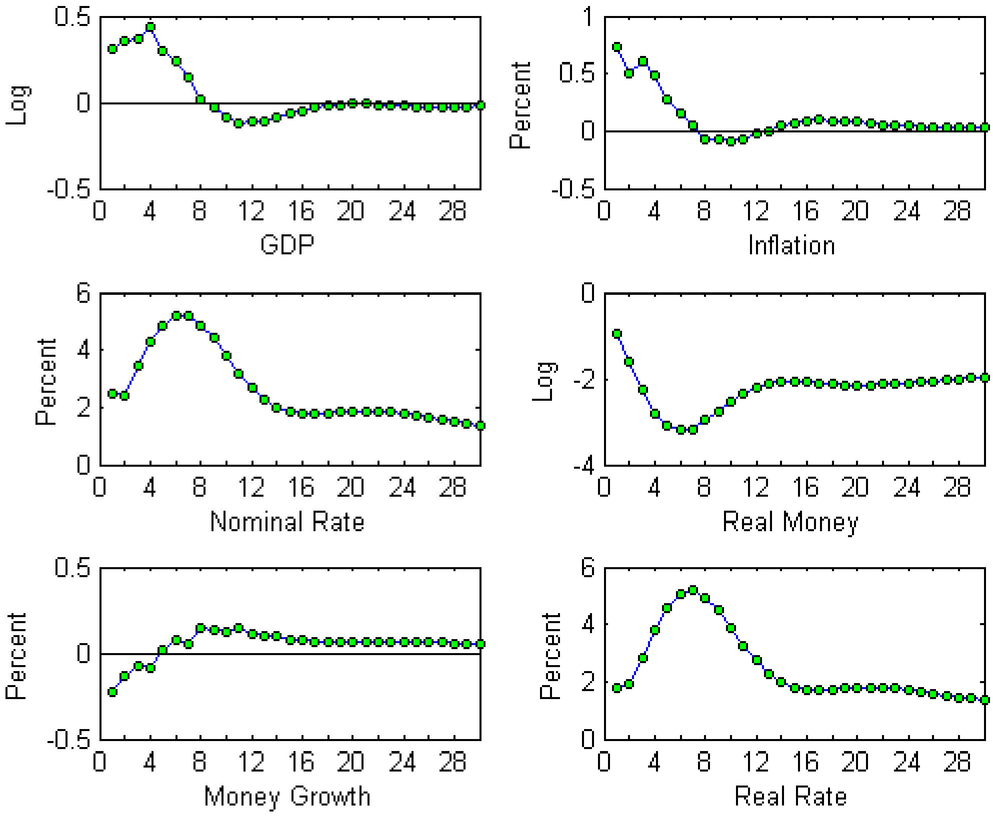

Figure 1.24: Impulse Responses to An IS Shock

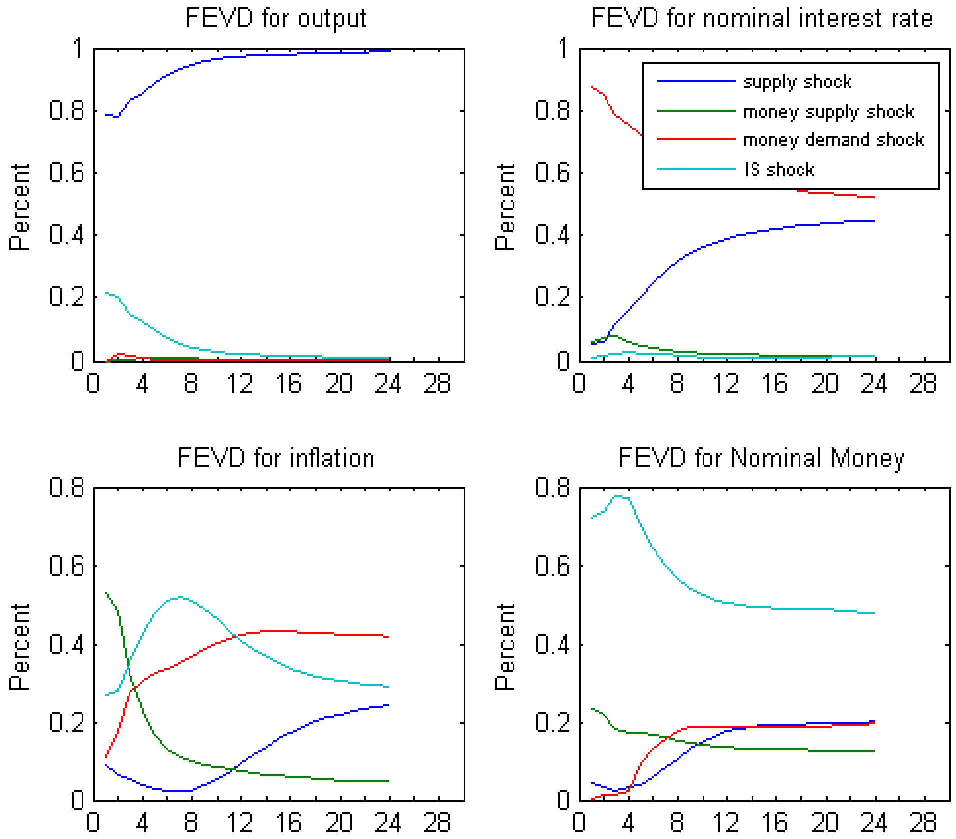

Figure 1.25: Decomposition of Forecast Error Variance

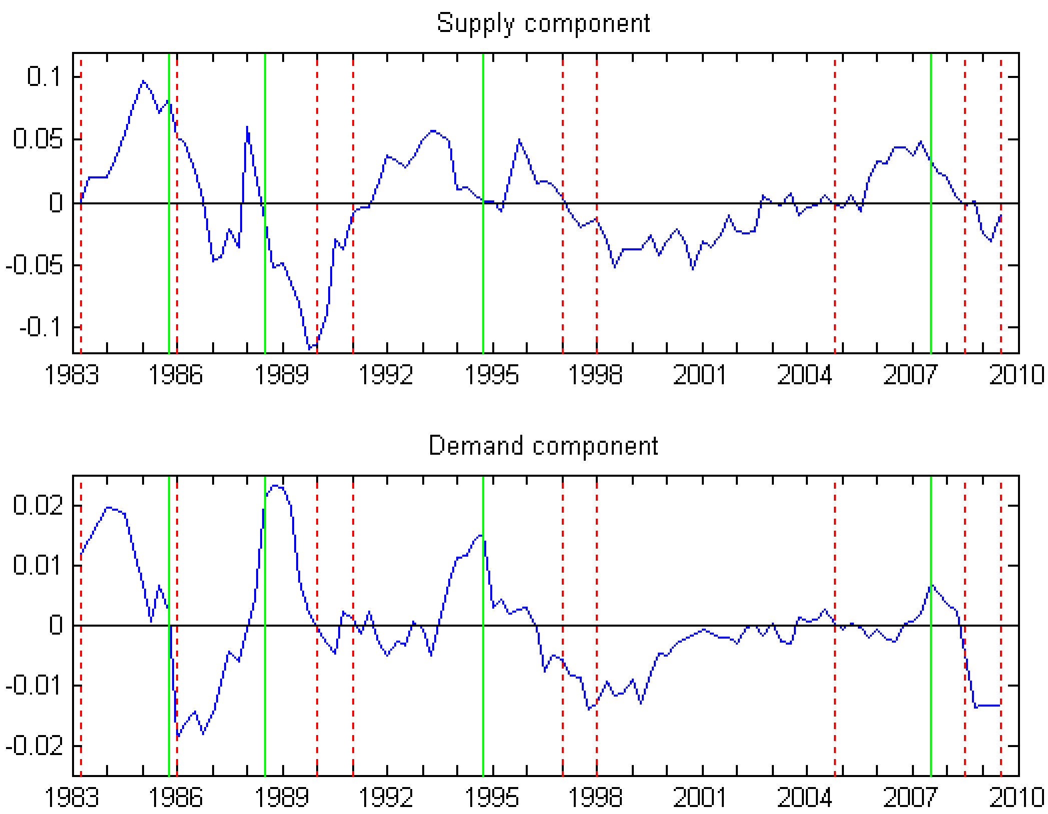

Figure 1.26: Historical Decomposition of Forecast Error

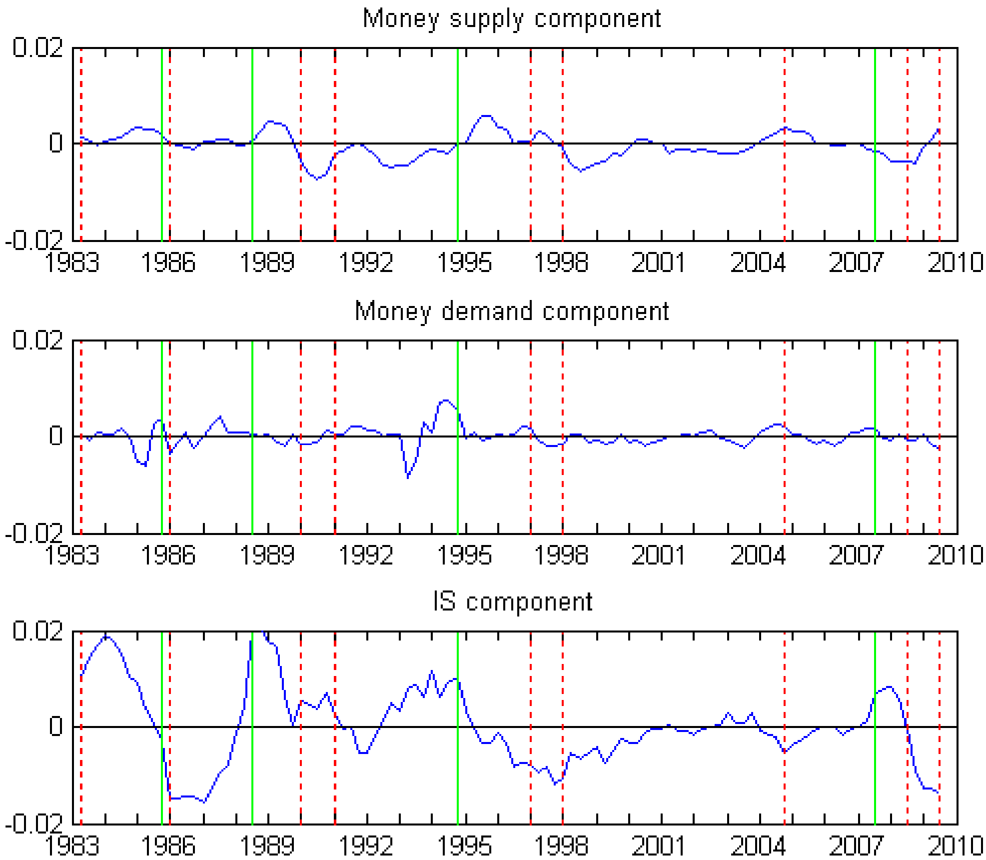

Figure 1.27: Historical Decomposition of Forecast Error

Narrow money

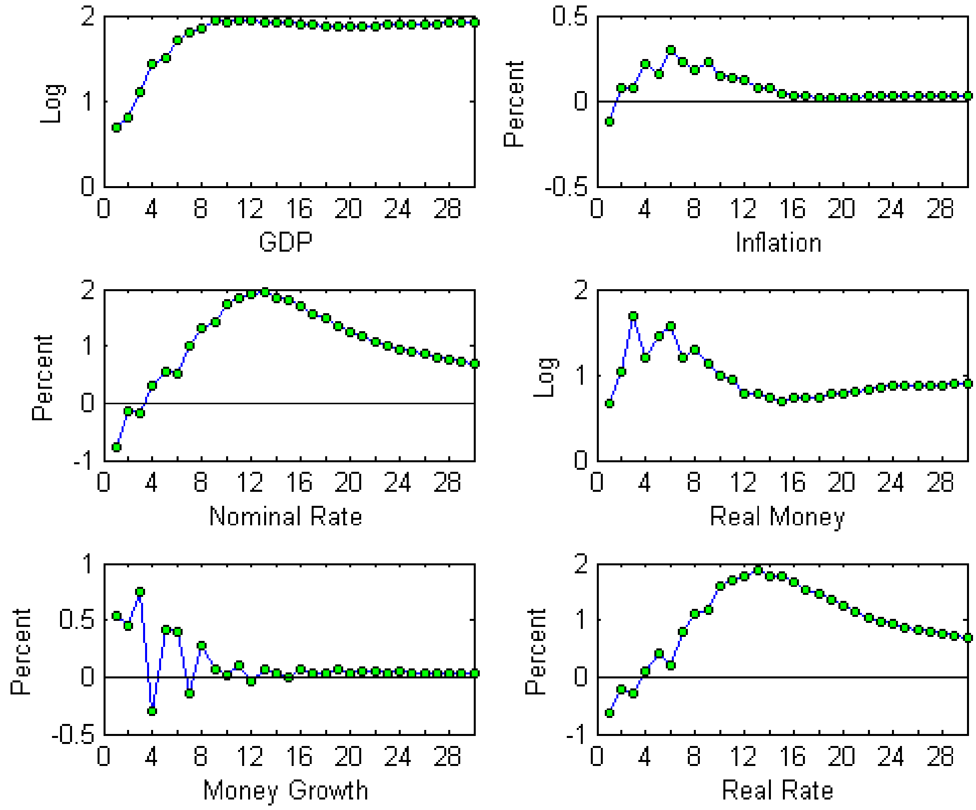

Figure 1.28: Impulse Responses to A Supply Shock

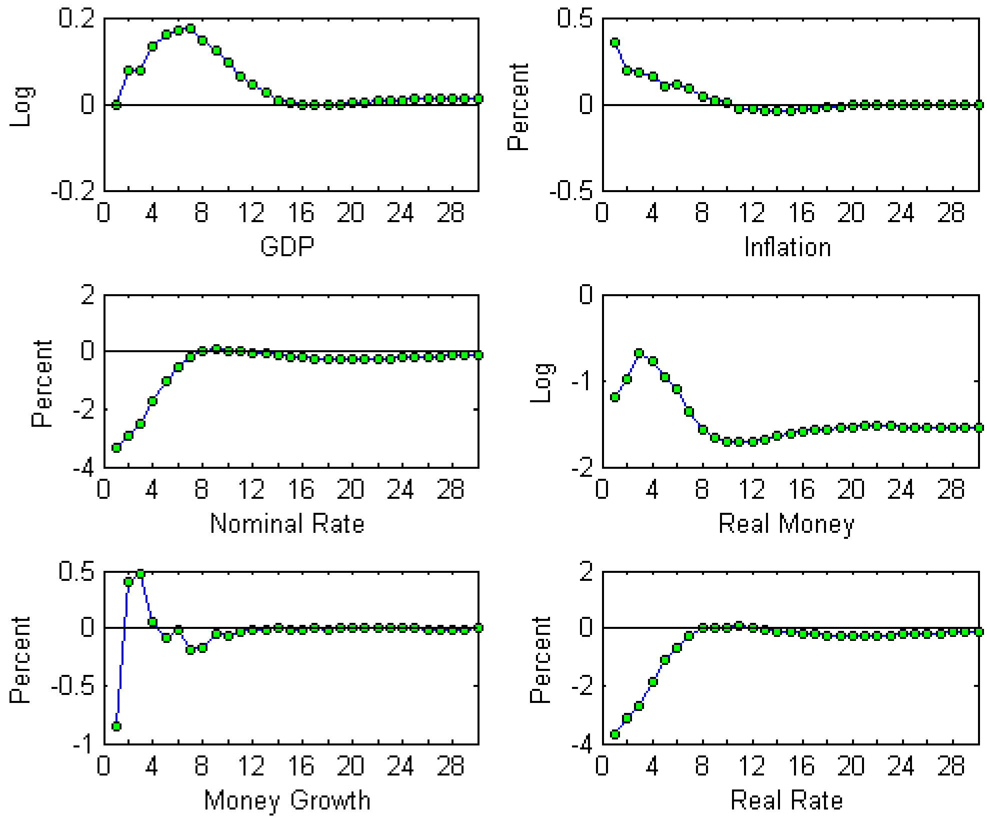

Figure 1.29: Impulse Responses to A Money Supply Shock

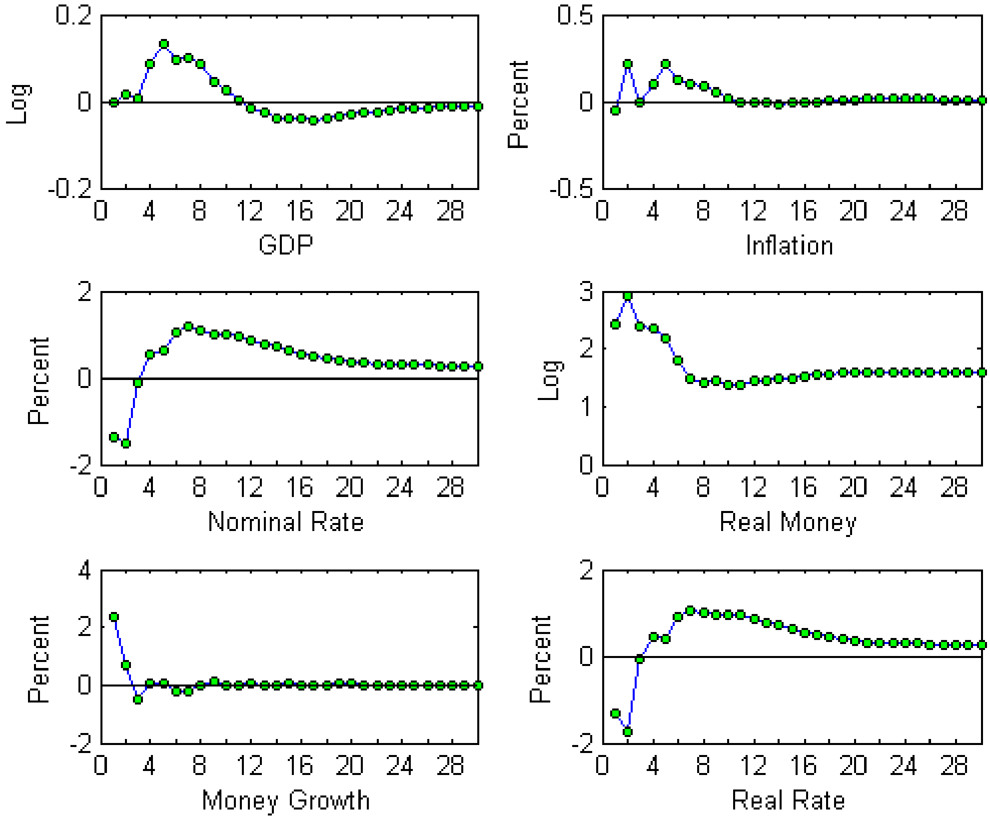

Figure 1.30: Impulse Responses to A Money Demand Shock

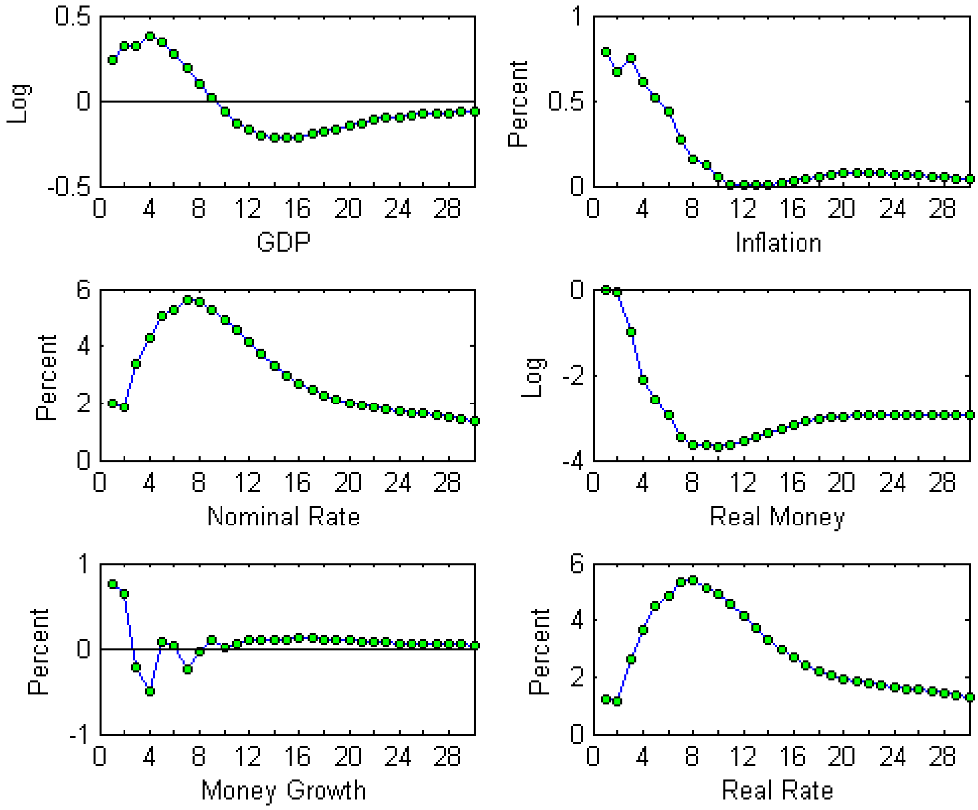

Figure 1.31: Impulse Responses to An IS Shock

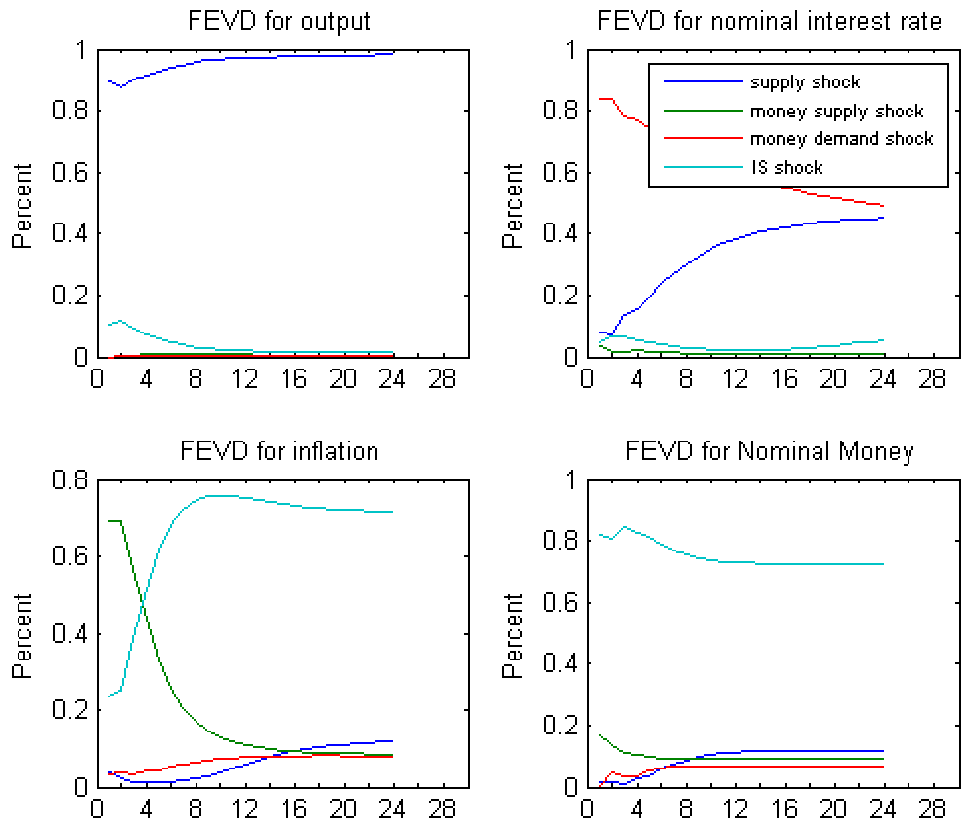

Figure 1.32: Decomposition of Forecast Error Variance

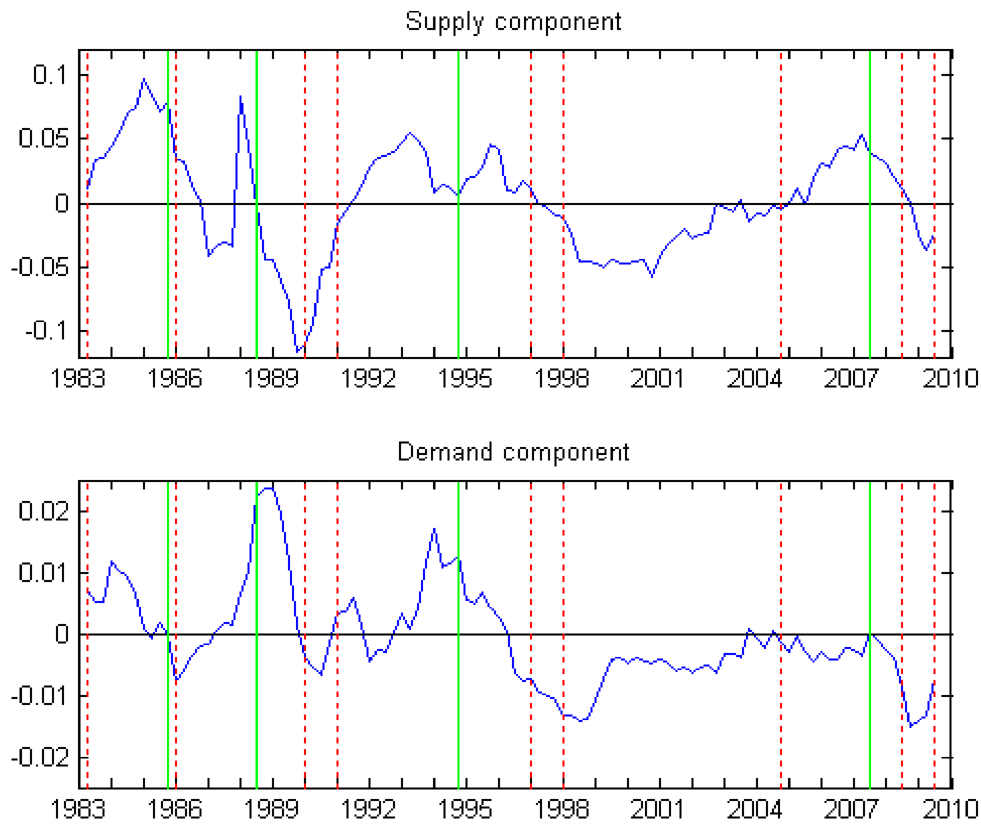

Figure 1.33: Historical Decomposition of Forecast Error

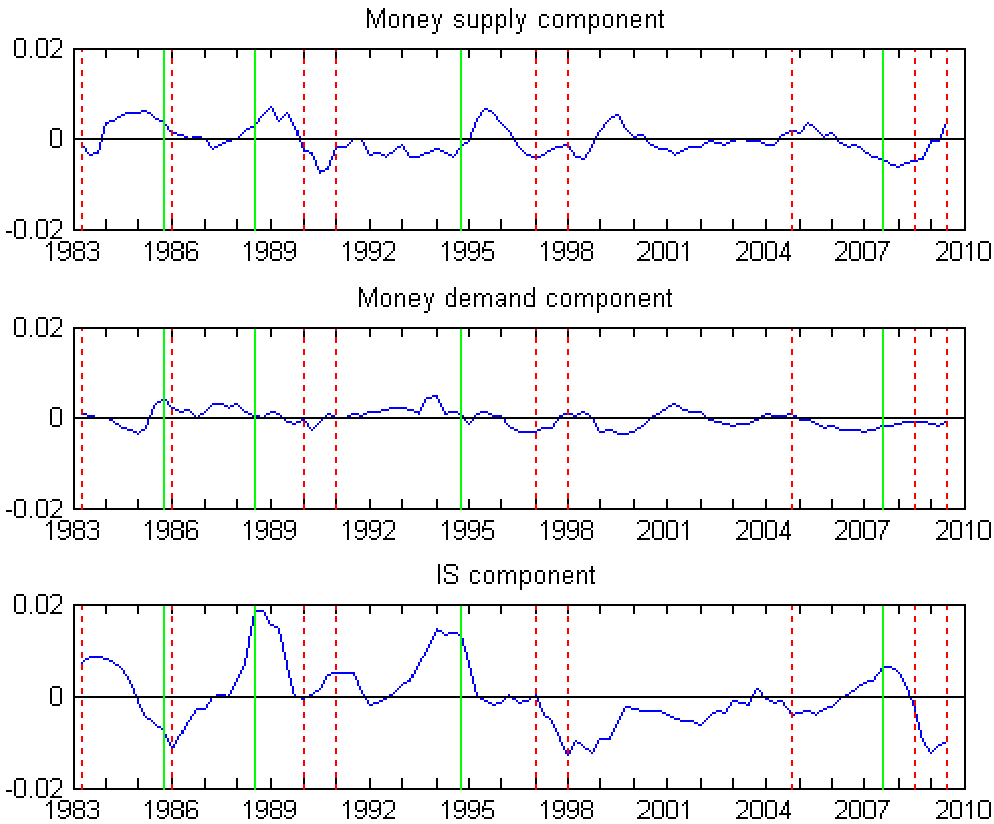

Figure 1.34: Historical Decomposition of Forecast Error

Chapter 2: Productivity, fiscal policy and aggregate fluctuations: An RBC model for China

2.1 Introduction

The real business cycle (RBC) model has long served as the baseline for research on economic fluctuations in developed countries. Recently, its popularity has been further illustrated by several studies for developing countries. Despite differences in institutions and policy transmission mechanisms, these studies found the basic model quite useful. For example, the baseline model has been successfully used for explaining the depressions of developing countries (see, examining Argentina's depressions in the 1980s and during the period 1998-2002 (Kydland and Zarazaga (2002) and Kehoe (2003)); explaining the economic decline of Mexico and Chile in the 1980s and 1990s (Bergoeing *et al.* (2002a, 2002b)); and understanding the Brazilian Depression in the 1980s and 1990s (Bugarin *et al.* (2002))). Moreover, several key studies have illustrated that the standard RBC model can perform better if it is extended to reflect specific features of the economy of interest. For example, Aguiar and Gopinath (2007) show that the standard RBC model can match some special features in the Mexican economy such as high countercyclical current accounts, consumption volatility and 'sudden stops' in capital inflow when the shocks to the trend growth rate is included. Angelopoulos *et al.* (2010) find that by accounting for a particular institutional factor, i.e. weak property rights, the basic RBC model can better match the Mexican data. These findings are encouraging and beg the question on whether the RBC model may be useful for explaining economic fluctuations in larger developing countries such as China? This chapter examines this question by assessing the performance of a calibrated three-sector RBC-DSGE model for matching the Chinese business cycle fluctuations during the reform period 1978-2006.

Several empirical studies on the Chinese economy have also informed us, i.e., there seems to be a overwhelming support for the importance of supply side factors in driving Chinese economic fluctuations. Econometric studies using the structural VAR models mostly come to the conclusion that, supply side shocks are the main sources of output fluctuations[35]. In particular, our empirical study in Chapter 1 using the structural VAR model shows that supply side shocks account for about 89% of output fluctuations, the rest of which is mostly related to fiscal shocks. Moreover, although using a different

[35] See Chapter 1 for a brief review about the structural VAR studies on the sources of Chinese business cycle fluctuations.

strategy, empirical research using the business cycle accounting (BCA) approach (see Chari, Kehoe, and McGrattan (2007)) also points out the importance of real shocks in the economy[36]. A common finding in the BCA literature is that efficiency changes (or what they term, efficiency wedges) related with technology shocks and institutional changes account for most of the variations in output[37]. This evidence points to the importance of exploiting the transmission mechanisms of the real side of the economy and the effects of real shocks. Our study thus responds to these empirical findings and employs an RBC model for China with an extension of the government sector.

Previous research on developing and assessing DSGE models for explaining business cycles in the Chinese economy has been limited except for some recent studies. Notably, Hsu and Zhao (2009) examine the factors that account for volatility changes of fluctuations using a RBC model with exogenous government spending. They calibrate the model to the Chinese data for the period 1954-2006 and the two sub-periods 1954-1977 and 1978-2006. Their results show that total factor productivity (TFP) shocks explain most of the economic fluctuations over the whole sample. The lower standard deviations of TFP can explain the great moderation of economic fluctuations in the post-reform period. On the other hand, the estimated government consumption and government investment shocks can supplement the model for explaining relative volatility changes in the post-reform period.

Relative to the above research, our study offers a wider scope and provides the first complete evaluation of an RBC model for the post-reform Chinese economy. In particular our intended value-added for China is as follows. First, we aim to provide a comprehensive set of assessments of the theoretical model rather than focusing on only one aspect of business cycle comparison. For example, not only do we examine the volatility of variables, but also compare other business cycle features such as persistence and cross-correlations among variables. These features are important for understanding the business cycle moments of the data and have been ignored in the above literature.

Second, we calibrate the theoretical model to annual Chinese data from 1978 to 2006

[36] For example, Xu (2007) evaluate the sources of business cycle fluctuations in a calibrated neoclassical growth models with government using BCA procedure. Their results show that the estimated efficiency wedges are responsible for most of output fluctuations. The falls in efficiency wedges explain the moderation of output fluctuations since 1992. The same BCA exercise is applied in a small open economy neoclassical DSGE model by He *et al.* (2009) which confirm the importance of efficiency wedges in driving output fluctuations for the post-reform Chinese economy. It is also shown that labour frictions such as those arising from wage rigidities explain the movement of labour force.

[37] See Christiano and Davis (2006) for a discussion of some pitfalls of the BCA procedure. For example, since the wedges in the model are bundles of fundamental economic shocks which are not identified, the BCA is unable to account for spillover effects across wedges.

only. We do not apply the model to the pre-reform period when the Chinese economy was simply central planned. This avoids the potential risk that not only the policy regime changes from pre- to post-reform periods, but also the structural parameters characterizing people's preferences, expectations and consumption patterns might also differ.

Third, we specify a richer structure of the public sector to match the important role played by the Chinese government in the actual economy. This is done by allowing for utility-generating government consumption and productive public capital in the aggregate production function. The former is designed to capture the possible co-movement between public consumption, private consumption and output. The latter allows government investment to contribute to the accumulation of productive public capital which helps catch possible supply side effect of the government spending.

Fourth, we offer careful calibration of the model to data. Our calibration satisfies that all the steady-state ratios of the components of output are consistent with their long-run averages in the data. Also, instead of simply setting a high persistence of the TFP shocks as in Hsu and Zhao (2009), we estimate both the persistence and the standard deviation of TFP using the detrended Solow residual. For the fiscal instruments, we assume the shares of government consumption and investment to output to follow AR(1) processes rather than specifying stationary processes for the variables themselves. This stems from the observation in the actual data that not only the government spending items themselves but also their shares vary over time. Thus this specification can better capture the variations in fiscal policy.

Finally, besides the standard assessments of the fit of the model to data, we also conduct a sensitivity analysis to see if our results are robust to alternative calibrations of the model. We also show the how the calibrated DSGE model can be useful for understanding the effects of fiscal policy on the Chinese economy.

The rest of this chapter is organized as follows: Section 2.2 provides some stylized facts about the post-reform Chinese economy in terms of economic fluctuations and economic growth. The theoretical model is presented in Section 2.3. Model calibration is carried out in Section 2.4. Section 2.5 presents the quantitative results including the assessments of the fit of the model to data, counterfactual experiments and a sensitivity analysis. Section 2.6 examines the impacts of economic shocks especially the fiscal shocks on the dynamics of the model. Finally, Section 2.7 concludes.

2.2 Stylized facts about China's aggregate activity

In this section we start by discussing the data employed and then describe the main empirical features of the Chinese economy relating to volatility, persistence, co-movement between key aggregates and long-run ratios. We conclude by discussing the modelling implications of these empirical regularities.

2.2.1 Data

We require real per capita data on output and its spending components (such as consumption and investment) and production inputs (such as labour and capital). For output and its spending components, the data of annual nominal and real GDP, nominal and real (growth rate of) private consumption and nominal government consumption are available from annual issues of the China Statistical Yearbook (CSY) published by the National Bureau of Statistics of China (NBSC). The nominal government investment[38] and transfers data are available from the annual issues of the Finance Yearbook of China (FYC) published by Ministry of Finance of the P.R.C. The implicit consumer price index is then used to deflate the nominal government consumption data. The investment price index is inferred by using the nominal gross capital formation data and the growth rate of the real gross capital formation data in the national accounts from CSY. It is then used to deflate the nominal government investment data. Since there is no explicit and consistent data for private investment[39], we derive it indirectly from the following national accounting identity in real terms:

$$\text{real total investment} = \text{real}\, GDP - \text{real total consumption} - \text{real net exports}$$

where the real net exports data is obtained by deflating the nominal net export data from the CSY by the implicit GDP deflator. Real private investment is the difference between the real total investment and the real government investment. It turns out that the resulting real gross private investment data is smaller than the official fixed gross capital formation data and is bigger than the fixed assets data which only covers the net increases in fixed assets.

For the producing inputs of output, the data on working hours is not fully available in our sample period. Thus data of employment is used for a measuring labour

[38] The government investment data is collected from the government spending budget under the construction account.

[39] For example, the nominal gross capital formation data covers a range of capital series (fixed assets and inventories) which is wider than the gross investment economists normally use. Another statistic 'fixed assets' is available from CSY since 1981. But its coverage is too narrow and does not account for financial assets, land and inventories.

input which is taken from annual issues of China labour Statistics Yearbook (CLSY) published by NBSC. It is notable that there was a jump in the employment in 1990 due to the change of data collection method from firm survey to household survey in that year. Thus we adjust the pre-1993 employment data[40]. The data for private and public capital are not available. The estimation of capital stock in China has been controversial. In fact, different researchers have employed different methods to construct capital stock series using different investment data[41]. Given this situation, we use the time-to-build evolution rule to construct series for both private and public capital:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{26}$$

$$k_{t+1}^g = (1 - \delta^g) k_t^g + g_t^i \tag{27}$$

where the derivation of data for the real gross private investment i_t and the real gross government investment g_t^i have been explained above. To initiate the calculation, we set the capital stock in 1978 to 14111.993 (100 million Yuan) which is taken from Chow (1993). We then assign a proportion of this value to be private capital with the proportion determined by the average ratio of private investment to total private investment from 1952 to 1977. The depreciation rate used in deriving these two capital series is 0.1. The data for the market prices, i.e., real wage and the interest rate are taken from the CLSY and the IMF's International Financial Statistics (IFS) database respectively.

All the above real data are derived at 1978 prices. Finally, the data of population is available from CSY and is used to transform the data (except the wage and interest rates) to per capita terms.

2.2.2 Aggregate fluctuations

We start our study by identifying the stylized facts about the Chinese economy in terms of the economic fluctuations at business cycle frequency and some great ratios characterizing the long-run. Following the approach widely used in the studies of business cycles for developed countries (see for example, Kydland and Prescott (1990)

[40] The method used adjust the pre-1994 employment before is as follows. We first assume that the growth rate of employment from 1988 to 1989 is the same as the growth rate from 1989 to 1990 so as to calculate the employment in 1990 as if the data collection method was not changed. Then the gap between this estimated employment and the actual figure in 1990 are calculated as a proportion of the employment in 1989. Finally, we assume that this proportion as a degree of underestimation is the same from 1978 to 1989 and adjust the data accordingly. See Appendix (2.8.5) Figure 2.13 for the plots of the original and adjusted employment data.

[41] See for example, Chow (1993), Chow and Li (2002), Bai *et al.* (2006) and Holz (2006) for their estimations of capital stock in China.

and King and Rebelo (1999) for the US economy), we employ the HP filter[42] (see for example, Hodrick and Prescott (1980)) to decompose the Chinese aggregate data into a trend component and a cyclical component.

Since we will be studying the working of the RBC model, we only present the cyclical movements of variables in real terms. Nominal variables are ignored. All variables (except employment and the real wage and interest rates) are per capita terms and have been detrended with the HP filter after taking natural logarithms (with the exception of the real wage and interest rates). Additionally, we also display the cyclical movements of the three underlying driving forces of the economy, i.e. TFP and the shares of government consumption and investment in output, hypothesized by the RBC model below.

The cyclical components for Chinese major aggregate variables for the period 1978-2006 are plotted in Figure 1-3 below. In each graph, the cyclical component of output (blue solid line) is always plotted to contrast the cyclical components of other variables (magenta dashed line) in terms of their co-movements and relative volatilities. Some selected statistics summarizing the business cycles moments are provided in Table 2.1. The following will first generalize the stylized facts of the business cycles in the Chinese economy by focusing on three types of statistics: the volatility and persistence of detrended variables and the co-movements between macroeconomic aggregates.

Volatility By examining the magnitudes of fluctuations in Figure 2.1-2.3 and the summary statistics of volatility and relative volatility in Table 2.1, we characterize the volatility of these business cycles of key macroeconomic aggregates as follows:

- Typical business cycle fluctuations observed in most developed countries also appear in the Chinese economy. The span of output fluctuations is approximately five to six years[43].

[42] The HP filter is a non-linear smoothing procedure which identifies the cyclical component of a time series as the difference between its current values x_t and a trend component $\widetilde{x}_t$. The trend component is derived by minimizing:

$$\underset{\widetilde{x}_t}{Min} \sum_{t=0}^{T} (x_t - \widetilde{x}_t)^2 + \lambda \sum_{t=0}^{T} \left[(\widetilde{x}_{t+1} - \widetilde{x}_t) - (\widetilde{x}_t - \widetilde{x}_{t-1}) \right]^2$$

where T is the length of the data and λ is a smoothing parameter. We use a standard value 100 for λ. There are other detrending tools such as the band-pass (BP) filter developed by Baxter and King (1995) that also have some attractive features. However, we only present results derived by the HP filter since both filters deliver very similar results.

[43] Please also see Chapter 1 for a more traditional classification of these cycles based on quarterly data.

- Private consumption is slightly less volatile than output (as shown in Figure 2.1, first panel)

- Private investment is almost three times more volatile than output, a result in common with developed countries (as shown in Figure 2.1, second panel and Table 2.1, third column).

- All the three types of government expenditures are more volatile than output. However, government consumption is only slightly more volatile while the volatilities of government investment and transfers are about four and then times bigger than output respectively (as shown in Figure 2.1, panel 3-5).

- Both private and public capital are as volatile as output[44] (as shown in Figure 2.2, panel 1-2). This result is obviously different from most developed countries where capital stock is much less volatile than output. One possible explanation is that the initial capital stock is relatively low in China. Also, the high capital volatility might be a result of institutional changes during the sample period[45].

- Employment is only about one fifth as volatile as output (as shown in Figure 2.2, third panel and Table 2.1, third column). This is also in contrast with most developed countries[46].

- The interest rate is much less volatile than output while real wage rate is more volatile than output (as shown in Figure 2.2, panel 4-5).

- TFP[47] is slightly less volatile as output (as shown in Figure 2.3, first panel); the government consumption and consumption shares are much more volatile than output. Their volatilities match those of government consumption and investment quite well (as shown in Figure 2.3, panel 2-3 and Table 2.1, third column).

Among the volatility stylized facts of the Chinese economy, the close to unity relative volatility of consumption is the most special feature compared with both developed

[44] The relative volatility of capital stock is consistent with the one derived using the capital data constructed in Chow and Li (2002) for the period 1978-1998. It is even higher, 1.5, if derived using the capital data in Bai *et al.* (2006).

[45] For example, possible evidence is that the volatility of private capital is about 0.6 before the Asian Financial Crisis in 1998. It increases to about 1.4 from 1999 to 2006 when the crisis ended and the economic reform entered a new phase.

[46] Another important measure of labour supply, the working hours data is only available for limited years in China. Hsu and Zhao (2009) provide a tentative estimate of working hours per person and find much more volatility.

[47] The calculation of the TFP series using aggregate data is shown below in section 2.4.5.

and developing countries. It is in stark contrast to most developed countries where consumption is much less volatile than output and is also different from developing countries where consumption is much more volatile. This feature has also been acknowledged by other researchers in the literature (e.g., Xu (2008)). There are several explanations of this phenomenon which deserve a further discussion. First, Chadwick and Nelson (2010) claimed that the low volatility (compared with other developing countries) of consumption in China is a result of precautionary saving behaviour of households due to weak intra-national consumption risk sharing. Empirical evidence (e.g., Xu (2008)) suggests that the provincial consumption risk sharing is very low in China. Thus, more precautionary saving is generated leading less volatility of consumption. Second, the low volatility of consumption can be a result of income disparity due to economic reforms. In particular, the SOEs reform leads to wage differences in urban area. Moreover, the income gap between rural and urban areas has always existed during the reform period. The inequality of income also dampens consumption volatility. Third, precautionary saving can be a natural result under the under-developed social security system in China. The welfare and insurance systems have only been setup recently. During most of the sample period, workers have no cover of illness and other risks during working. The health system is also lagged behind the economic development. Many households need to accumulate their income against risk or save income for their children. This also creates precautionary savings which makes consumption not as volatile as other developing countries.

Persistence As the fourth column of Table 1 shows, all the detrended variables show some degree of first-order sequential persistence. For comparison, public capital and employment are the most persistent variables. Government transfers, the share of government consumption and government consumption have the lowest persistence. The persistence of output, government investment, private capital and TFP are close and in the middle. Compared with the persistence statistics of the US economy (see e.g., King and Rebelo (1999), Table 1), the persistence of macroeconomic aggregates of China are overall smaller.

Co-movement The co-movements of output with other variables can be seen from the graphs in Figure 2.1-2.3 by gauging the mapping of cycles of these variables (such as the timings of peaks and troughs) with output. As an important measure of such co-movement, the contemporaneous correlations of variables with output are calculated and reported in the last column of Table 2.1. It is seen that the spending components of output, i.e., (private and public) consumption and investment are all procyclical, although the correlations of private consumption and investment with output are higher than their public counterparts. The correlation of TFP with output is the highest, 0.86, even bigger than the correlations of the components of output. Private capital, real wage and the interest rate are also procyclical. There are several variables, i.e., government transfers, the share of government consumption, public capital, the share of government investment and employment, are negatively correlated with output. In particular, the strong negative correlations between government transfers and the share of government consumption and output indicate a countercyclical fiscal policy in the post-reform period.

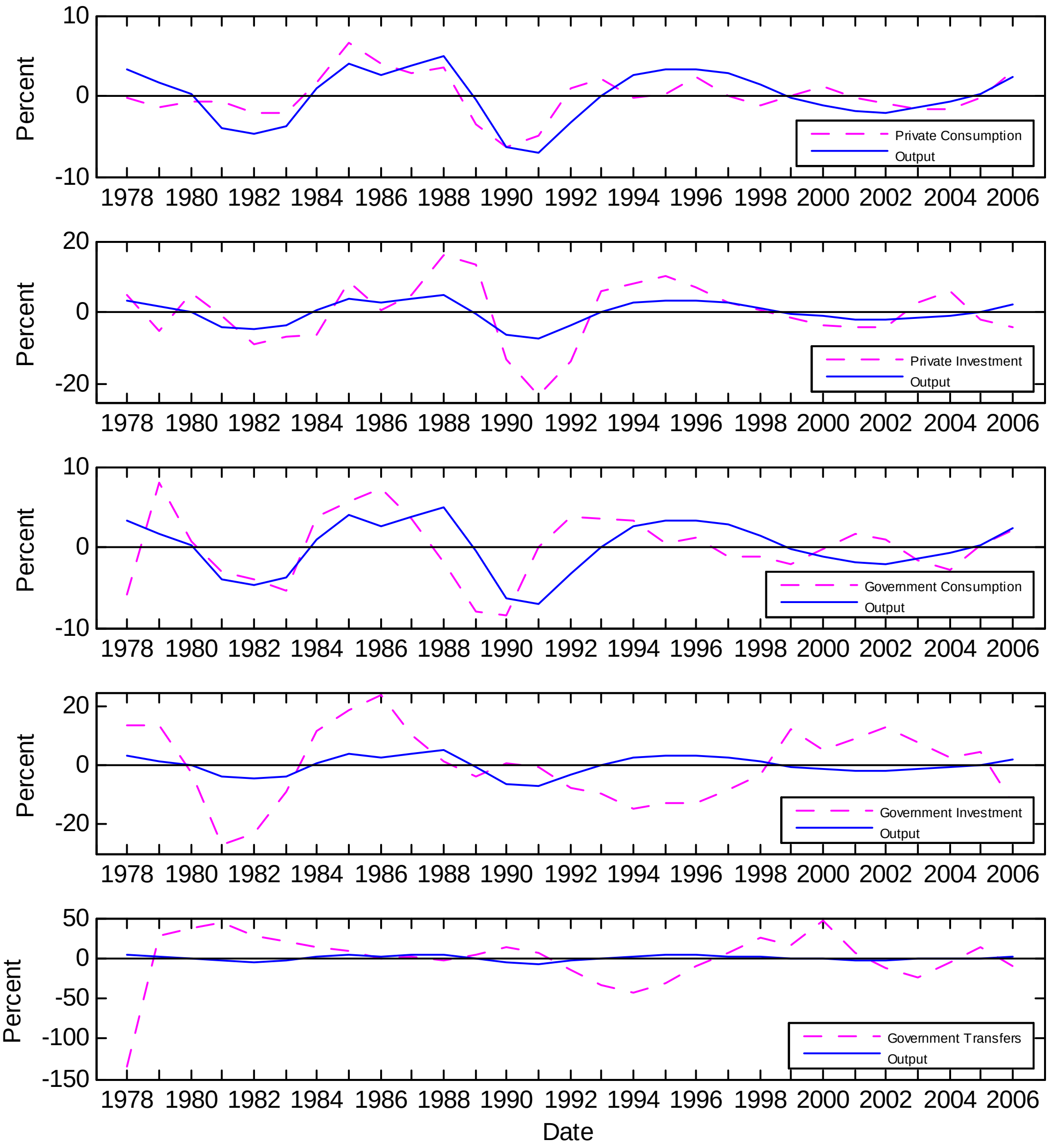

Figure 2.1: China's business cycles: Output and its components

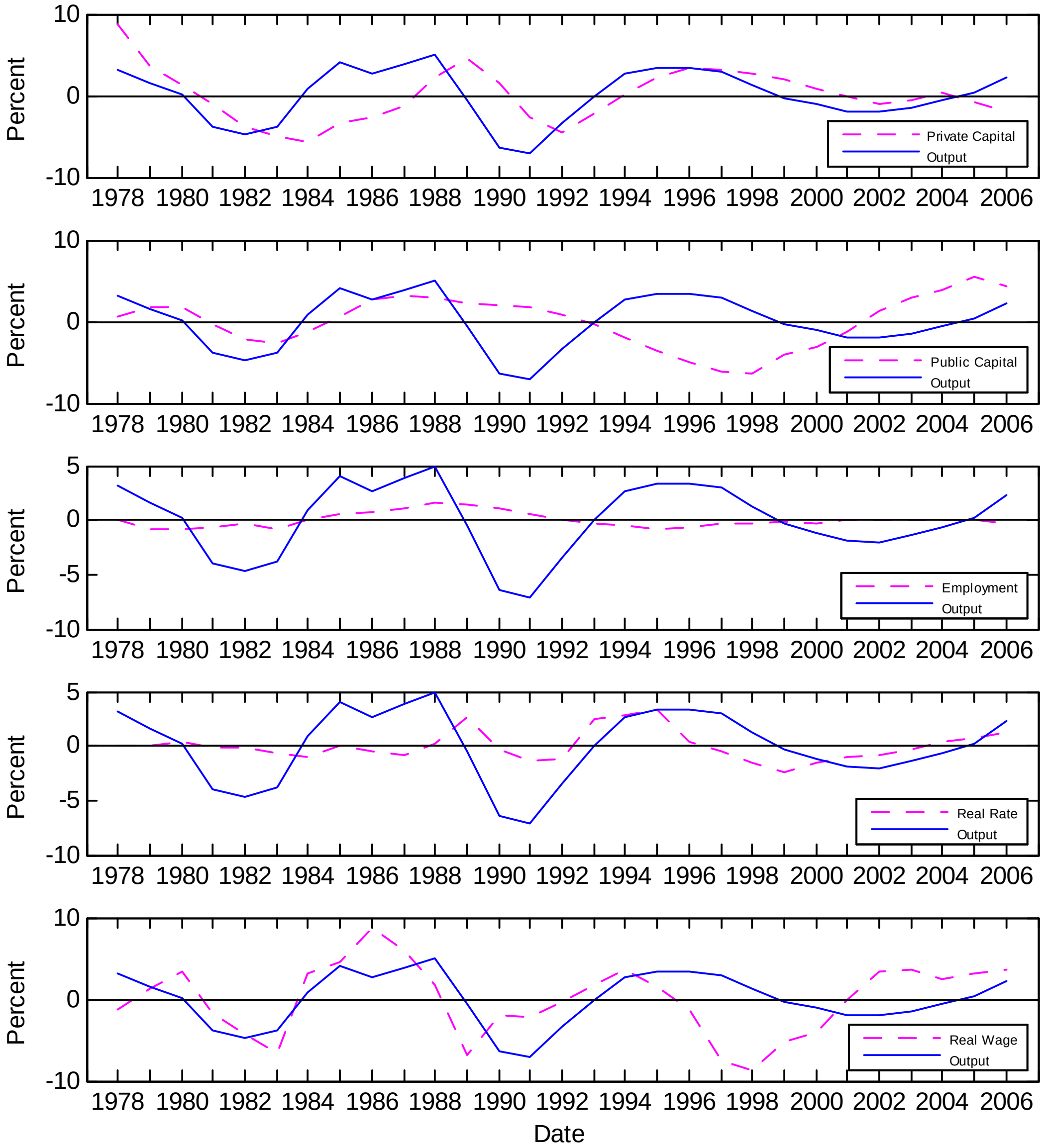

Figure 2.2: China's business cycles: Output and its producing inputs

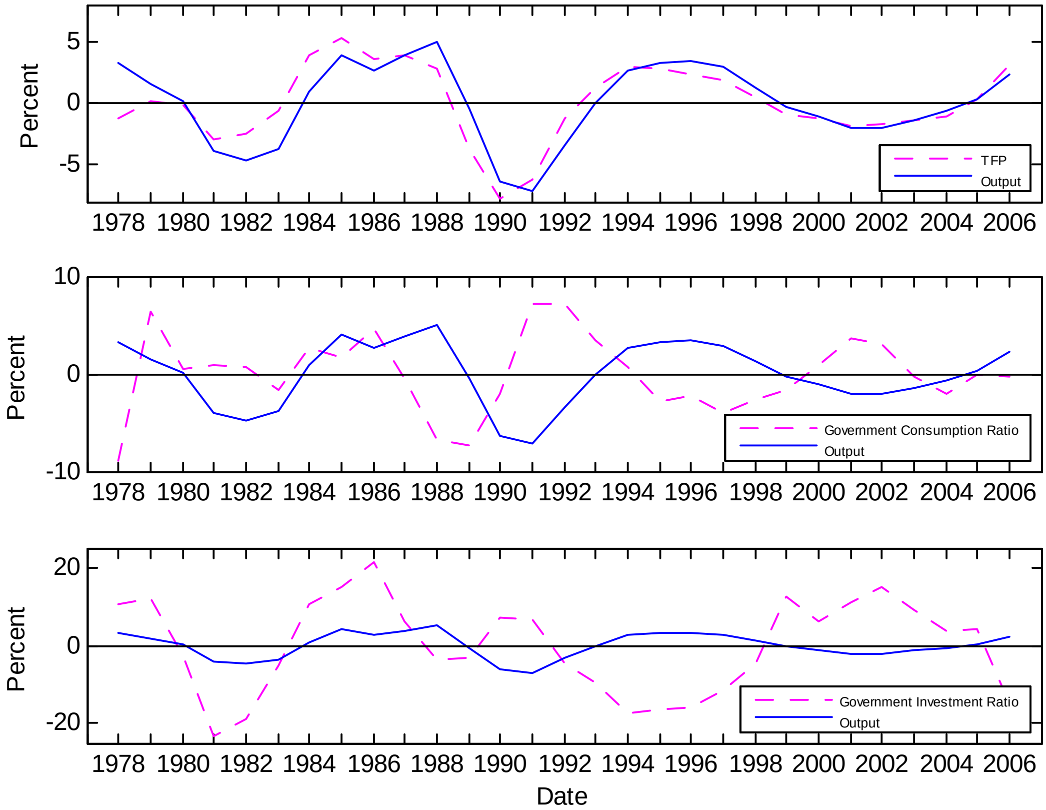

Figure 2.3: China's business cycles: Output and driving processes

Table 2.1: Business cycle statistics for the Chinese economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with Y
Y	3.20	1.00	0.68	1.00
C	2.68	0.84	0.53	0.74
I	8.55	2.67	0.45	0.77
G^c	4.07	1.27	0.36	0.42
G^i	12.54	3.91	0.65	0.26
G^t	33.83	10.56	0.20	−0.34
K	3.22	1.01	0.64	0.37
K^G	3.17	0.99	0.87	−0.07
L	0.66	0.21	0.83	0.03
w	4.34	1.36	0.61	0.35
r	1.42	0.44	0.55	0.31
A	3.04	0.95	0.69	0.86
G^c/Y	3.98	1.24	0.28	−0.37
G^i/Y	12.1	3.78	0.63	0.01

To clarify our notation, Y, C and I represent output, private consumption and investment respectively; G^c, G^i and G^t are the government consumption, investment and transfers; K and K^G are private and public capital stock; L is employment; w and r are the real wage rate and real bank deposit rate; A is total factor productivity; G^c/Y and G^i/Y are shares of government consumption and investment to output.

2.2.3 The great ratios

Studies on the growth facts of many countries usually suggest the existence of stable ratios of some macroeconomics variables to output (see for example, Kuznets (1973) and King, Plosser, Stock and Watson (1991)). This finding has been an important reason for many RBC models which specify a common growth trend for all variables. For the same reason, we also examine some ratios of the Chinese economy. Figure 2.4 plots the ratios of the four spending components of output and also the ratio of government transfers. It is shown that, the ratios of private consumption and government transfers to output have been quite stable - they roughly fluctuate around some constant means. The ratios of private investment, government consumption and private investment to output show more variations (from 10 to 20 percent) in our data sample. Although these variations are not big, they do indicate that not only the levels of these variables change over time but also do their shares.

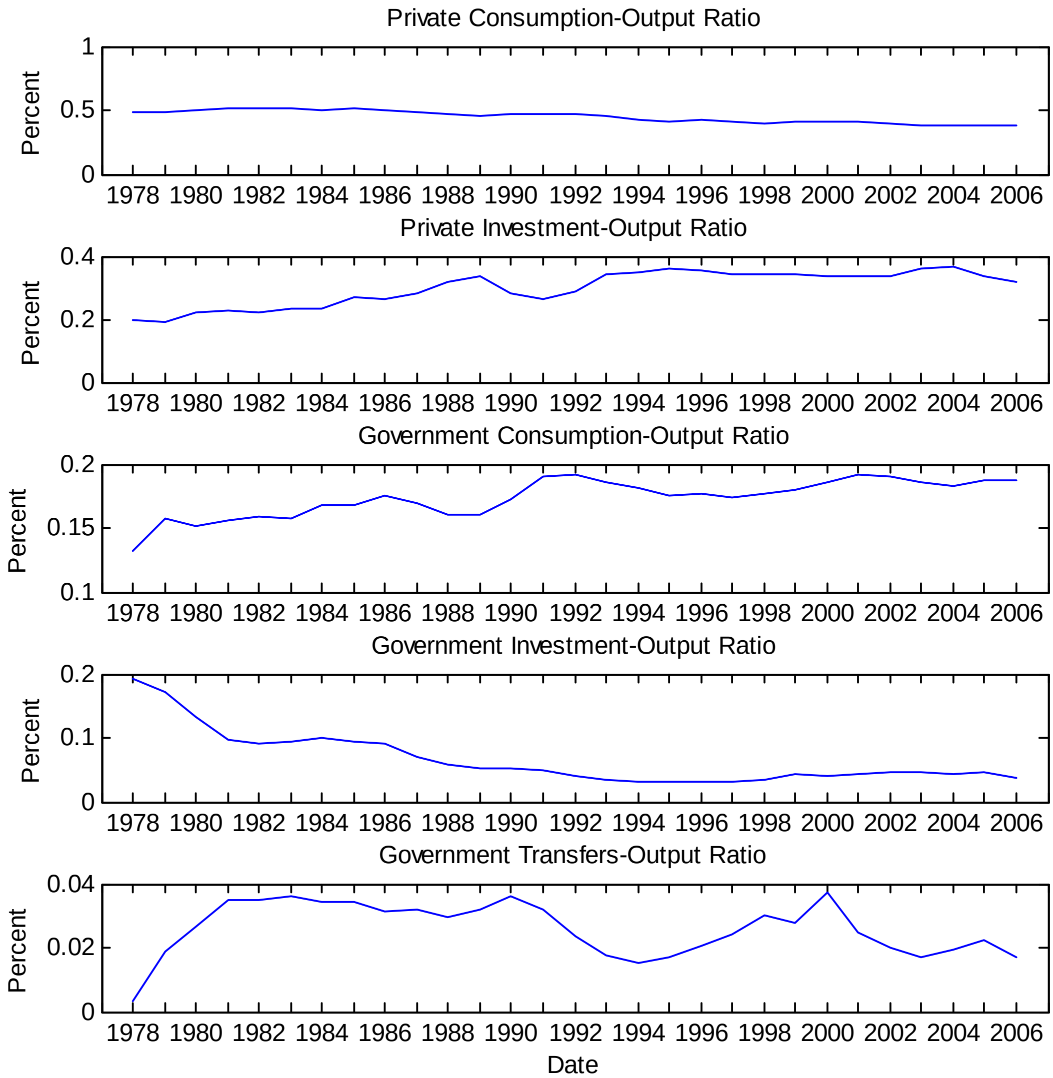

Figure 2.4: Ratios of output components

2.2.4 Implications for macroeconomic modeling

The above stylized facts about the cyclical components and the long-run ratios of China's major macroeconomic aggregates have important implications for macroeconomic modeling. For example, the much lower volatility of private consumption relative to output and investment is consistent with the typical consumption smoothing phenomenon as depicted in the permanent income hypothesis for many developed countries. It implies that Chinese households have similar consumption smoothing behavior which needs to be considered in our model. Moreover, the high (positive or negative) correlations between government spending components (and their shares) and output suggest a close relationship between fiscal policy and output movement. Fiscal innovations might be another source of aggregate fluctuations besides TFP shocks. Also, considering the big share of government expenditure in output (almost one fourth according to data average), suggests the inclusion of government sector in our model. Furthermore, the ratios of government spending items to output suggest that not only the government spending themselves but their shares in output also change over time. Thus, a proper specification of fiscal instruments should involve assuming a process for the shares of government spending rather than the variables themselves.

2.3 The theoretical model

The theoretical model used in the study is a RBC model with government. The model economy is populated by a large number of households who maximize their life-time utilities, a large number of firms who are profit maximizers and a central government who finances its purchase by lump-sum taxes. Given the important role of the Chinese government in the economy, we incorporate utility-yielding government consumption and productive public capital in the model. Both private capital and public capital are augmented in a standard one year time-to-build fashion. The production of firms is affected by technological progress which follows a first-order autoregressive process. The government conducts its fiscal policy by varying the shares of its consumption and investment to output which also take the form of first-order autoregressive processes. For convenience, perfect competitions are assumed in goods, capital and labour markets. There is no trend growth in this model. All variables will be represented in a decentralized competitive equilibrium (DCE) such that economic fluctuations are interpreted as the deviations from this DCE due to exogenous shocks. The specification and derivation of the model are illustrated below.

2.3.1 Households

There is a large number of identical infinitely-lived households indexed by the superscript h, where $h = 1, 2, ...N^h$. The population $N^h > 1$, is assumed to be constant and exogenous. Each representative household maximizes a time-separable utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^h, h_t^h, \overline{G}_t^c\right) \tag{28}$$

where E_0 is the expectations operator; β represents the subjective rate of time preference, C_t^h is the private consumption of household; $0 < h_t^h < 1$ is the household's labour input or working hours when the total time endowment is normalized to 1 and $(1-h)$ measures leisure time; and $\overline{G}_t^c = G_t^c/N^h$ is the average (per household) government services or the total amount of public goods *per capita*. The idea of incorporating government consumption in the households utility function follows Christiano and Eichenbaum (1992), Baxter and King (1993) and Bouakez and Rebei (2007).

The instantaneous utility function is assumed to be CRRA (constant rate of risk aversion) and is separable in consumption, labour and government consumption:

$$u_t = \frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{1-\sigma} \tag{29}$$

where $\mu_1(1-\sigma)-1$ is the constant rate of risk aversion in consumption[48]; and $\mu_1, \mu_2, 1-\mu_1-\mu_2$ control the weights of the three components respectively. Note that, when the value of the government consumption weight $(1-\mu_1-\mu_2)$ is smaller than the value of $\frac{1}{\sigma}$, government and private consumption are complements, generating a co-movement of the two. On the opposite, when $(1-\mu_1-\mu_2) > \frac{1}{\sigma}$, government and private consumption are substitutes, resulting in an additional channel amplifying the crowding-out effect of public spending[49].

The household also has accesses to capital and labour markets. It receives wage income, $w_t h_t^h$, from supplying labour and gains capital income, $r_t K_t^h$, from holding capital. Since both capital and labour markets are perfectly competitive, w_t and r_t are market real wage rate and market gross (before tax and excluding depreciation) real rate of return to capital respectively. Each household is assumed to run a firm and to receive profit, Π_t^h. Moreover, each household receives average transfers, $\overline{G}_t^t = G_t^t/N^f$, from the government. The household spends part of the above income on consumption and saves the remainder for investment. The intertemporal budget constraint of the

[48] The risk aversion in consumption is defined as $-c\frac{u_{cc}}{u_c}$, where u_c and u_{cc} are the first and second derivatives of the utility function with respect to consumption respectively. Also, it can be shown that $\frac{1}{\mu_1(1-\sigma)-1}$, $\frac{1}{\mu_2(1-\sigma)-1}$ and $\frac{1}{(1-\mu_1-\mu_2)(1-\sigma)-1}$ are the elasticity of intertemporal substitution for consumption, leisure and average government consumption respectively.

[49] To see this, we derive the marginal utility of private consumption as

$$\frac{\partial u_t}{\partial C_t^h} = \mu_1\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\right]^{-\sigma}\left(1-h_t^h\right)^{\mu_2}\left(C_t^h\right)^{\mu_1-1}\left(\overline{G}_t^c\right)^{1-(1-\mu_1-\mu_2)\sigma}.$$

It implies that when $1-(1-\mu_1-\mu_2)\sigma > 0$, an increase in government consumption raises the marginal utility of private consumption, generating a co-movement of the two. In this sense, they are complements. On the opposite, when $1-(1-\mu_1-\mu_2)\sigma < 0$, government and private consumption are substitutes since an increase in government reduces the marginal utility of private consumption and makes the latter decrease.

household is given by:

$$\left(1+\tau^{c}\right) C_{t}^{h}+I_{t}^{h}=\left(1-\tau^{k}\right) r_{t} K_{t}^{h}+\left(1-\tau^{h}\right) w_{t} h_{t}^{h}+\Pi_{t}^{h}+\overline{G}_{t}^{t} \tag{30}$$

where τ^c, τ^k and τ^h are the tax rates on private consumption, capital income and labour income respectively. The evolution of private capital stock is given by:

$$I_{t}^{h}=K_{t+1}^{h}-(1-\delta) K_{t}^{h} \tag{31}$$

where $0<\delta<1$ is a constant depreciation rate.

Since the labour and capital markets are competitive, each representative household takes market prices (r_t, w_t) and policy instruments $(\tau^c, \tau^k, \tau^h, \overline{G}_t^c, \overline{G}^t)$ as given. The household's problem is then to maximize the life-time utility (28) by choosing $\left\{C_t^h, h_t^h, K_{t+1}^h\right\}_{t=0}^{\infty}$ subject to its budget constraints (30) and (31). By substituting the household investment in (31) into (30), this optimization problem is given by:

$$\begin{gathered}
\underset{C_t^h, h_t^h, K_{t+1}^h}{\operatorname{Max}} \; E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{1-\sigma} \\
\text{s. t. } \left(1-\tau^k\right) r_t K_t^h+\left(1-\tau^h\right) w_t h_t^h+\Pi_t^h+\overline{G}_t^t \\
-\left(1+\tau^c\right) C_t^h-K_{t+1}^h+(1-\delta) K_t^h=0 .
\end{gathered}$$

By combining the three first-order conditions (FOCs) with respect to C_t^h, h_t^h and K_{t+1}^h, we obtain

$$w_t=\frac{\mu_2\left(1+\tau^c\right) C_t^h}{\mu_1\left(1-\tau^h\right)\left(1-h_t^h\right)} \tag{32}$$

which is the labour supply function of the household and

$$\begin{gathered}
\frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{C_t^h}= \\
\beta E_0\left\{\frac{\left[\left(C_{t+1}^h\right)^{\mu_1}\left(1-h_{t+1}^h\right)^{\mu_2}\left(\overline{G}_{t+1}^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}\left(1-\delta+\left(1-\tau^k\right) r_{t+1}\right)}{C_{t+1}^h}\right\}
\end{gathered} \tag{33}$$

which is the Euler equation for household consumption.

2.3.2 Firms

There is large number of identical firms indexed by the superscript f, where $f = 1, 2, ...N^f$ and $N^f > 1$. Each representative firm hires labour, h_t^f, supplied by households and rent both its own capital, K_t^f, and public capital, $\overline{K}_t^g$, to produce goods,

Y_t^f. The term $\overline{K}_t^g = K_t^g/N$ is average (per household) government capital or the total amount of government capital *per capita*. We assume that the firm's production function is CRS (constant returns to scale) in private capital and labour and IRS (increasing return to scale) in total inputs[50]. The production function takes a CES (constant elasticity of substitution) form which is given by:

$$Y_t^f = A_t \left(K_t^f\right)^{\alpha_1} \left(h_t^f\right)^{1-\alpha_1} \left(\overline{K}_t^g\right)^{\alpha_2} \tag{34}$$

where $\alpha_1 < 1$, $\alpha_2 < 1$ are the productivity of private and public capital respectively; and $(1 - \alpha_1)$ is the productivity of labour. When $\alpha_2 = 0$, the production function collapses to the standard case where the public capital is unproductive. When $0 < \alpha_2 < 1$, the public capital becomes productive. The technology progress A_t is assumed to follow a AR(1) process:

$$\log A_{t+1} = (1 - \rho_a) \log A_0 + \rho_a \log A_t + \varepsilon_{a,t+1} \tag{35}$$

where A_0 is a constant, ρ_a represents the persistence and $\varepsilon_{a,t+1} \sim N\left(0, \sigma_a^2\right)$ is the normally distributed disturbances.

It is assumed that firms act competitively in the goods market. Each representative firm earns gross income, Y_t^f, and pays interest, r_t, and wages, w_t, to capital and labour. Therefore, the profit of the representative firm is given by[51]:

$$\Pi_t^f \equiv Y_t^f - r_t K_t^f - w_t h_t^f. \tag{36}$$

The static optimization problem for each representative firm is then that it chooses $\left\{K_t^f, h_t^f\right\}_{t=0}^{\infty}$ to maximize profits (36) subject to the technology constraint (34):

$$\underset{h_t^f, K_t^f}{\text{Max}} \left\{Y_t^f - r_t K_t^f - w_t h_t^f\right\}$$
$$s.t.\ Y_t^f - A_t \left(K_t^f\right)^{\alpha_1} \left(h_t^f\right)^{1-\alpha_1} \left(\overline{K}_t^g\right)^{\alpha_2} = 0.$$

The FOC with respect to K_t^f is:

$$r_t = \frac{\alpha_1 Y_t^f}{K_t^f} \tag{37}$$

[50]Some researcher see e.g., Aschauer (1989), Munnell (1990), Ai and Cassou (1995) and Lansing (1998) suggest CRS production function in all inputs for developed countries. However, here we allow for IRS production function for China as a developing country.

[51]We assume that any tax or subsidies on Y_t^f is absent.

which is the return to capital stock. Similarly, the FOC with respect to h_t^f is:

$$w_t = \frac{(1-\alpha_1) Y_t^f}{h_t^f} \tag{38}$$

which is the return to labour supply.

The profit of each firm is zero given the perfect competition in goods market and the specification of the production function. This can be seen by substituting the returns to private capital and labour (37) and (38) into the profit function (36):

$$\Pi_t^f \equiv Y_t^f - \frac{\alpha_1 Y_t^f}{K_t^f} K_t^f - \frac{(1-\alpha_1) Y_t^f}{h_t^f} h_t^f = 0. \tag{39}$$

2.3.3 Government

There is a central government which spends its income on public consumption, investment and transfers. Public consumption delivers positive utility to households. Public investment is used in augmenting public capital stock to serve for production. The residuals of the government's expenditure are the transfers given to households. It is assumed that the government expenditure is financed by means of taxes on private consumption, capital and labour income. The government's budget constraint is then given by:

$$G_t^c + G_t^i + G_t^t = N^h \left[\tau^c C_t^h + \tau^k r_t K_t^h + \tau^h w_t h_t^h\right]. \tag{40}$$

The evolution of public capital stock takes the same law of motion as the private capital:

$$G_t^i = K_{t+1}^g - (1-\delta^g) K_t^g \tag{41}$$

Government consumption and investment are assumed to respond to both exogenous factors and endogenous output. This is achieved by assuming that the government consumption share, $\frac{\overline{G}_t^c}{Y_t}$, and investment share, $\frac{\overline{G}_t^i}{Y_t}$, follow AR(1) processes:

$$\log\left(\frac{\overline{G}_{t+1}^c}{Y_{t+1}}\right) = (1-\rho_{g^c}) \log\left(\frac{\overline{G}_0^c}{Y_0}\right) + \rho_{g^c} \log\left(\frac{\overline{G}_t^c}{Y_t}\right) + \varepsilon_{g^c,t+1} \tag{42}$$

$$\log\left(\frac{\overline{G}_{t+1}^i}{Y_{t+1}}\right) = (1-\rho_{g^i}) \log\left(\frac{\overline{G}_0^i}{Y_0}\right) + \rho_{g^i} \log\left(\frac{\overline{G}_t^i}{Y_t}\right) + \varepsilon_{g^i,t+1} \tag{43}$$

or

$$\log\left(\overline{G}_{t+1}^{cy}\right) = (1-\rho_{g^c}) \log\left(\overline{G}_0^{cy}\right) + \rho_{g^c} \log\left(\overline{G}_t^{cy}\right) + \varepsilon_{g^c,t+1} \tag{44}$$

$$\log\left(\overline{G}_{t+1}^{iy}\right) = (1-\rho_{g^i}) \log\left(\overline{G}_0^{iy}\right) + \rho_{g^i} \log\left(\overline{G}_t^{iy}\right) + \varepsilon_{g^i,t+1} \tag{45}$$

where

$$\overline{G}_t^{cy} = \frac{\overline{G}_t^c}{Y_t} \tag{46}$$

$$\overline{G}_t^{iy} = \frac{\overline{G}_t^i}{Y_t}. \tag{47}$$

2.3.4 Aggregate consistency condition

The aggregate consistency condition of the economy is that the quantity of commodities desired by the households and the government is equal to the quantity of commodities supplied by the firms. In our model setup, this implies that the aggregate consumption and investment of households and the government are equal to the aggregate production of firms:

$$N^h C_t^h + N^h I_t^h + G_t^c + G_t^i = N^f Y_t^f. \tag{48}$$

2.3.5 Decentralised Competitive Equilibrium (DCE)

We now describe the DCE which will serve our analysis on economic fluctuations. Given the realizations of the exogenous technological progress and policy instruments $\left\{A_t,\, G_t^{cy},\, G_t^{iy},\, \tau^c,\, \tau^k,\, \tau^h\right\}_{t=0}^{\infty}$ and a set of initial values of the state variables $\left[A_0,\ G_0^{cy},\ G_0^{iy},\ \overline{K}_0^g,\ I\right.$ the DCE is defined to be a sequence of market prices $\{w_t,\, r_t\}_{t=0}^{\infty}$ and economic allocations $\{Y_t,\ C_t,\ I_t,\ h_t,\ K_t,\ K_t^g\}_{t=0}^{\infty}$ where: i) all markets, i.e., the goods, capital and labour markets, clear; ii) households maximize their lifetime utilities, which gives two optimality conditions; iii) firms maximize their expected profits, which gives the production function and another two optimality conditions; iv) the government balances its budget, which solves $\{g_t^t\}_{t=0}^{\infty}$ residually; and v) the evolutions of private and public capital and the resource constraint are satisfied in each time period. As a result, we will have nine equations with nine unknowns and three equations with three exogenous variables in the DCE[52]

To summarize the DCE, we first note that the market-clearing condition in the capital market is that the demand of capital of firms is equal to the supply of capital by households:

$$N^f K_t^f = N^h K_t^h. \tag{49}$$

Similarly, the demand of hiring labour of firms is satisfied by the labour supply of households, clearing the labour market:

$$N^f h_t^f = N^h h_t^h. \tag{50}$$

[52] Since the profits of firms are zero, we ignore Π_t^h and Π_t^f in the DCE.

Note that in our model setup, each household runs a firm, i.e., the number of households is equal to the number of firms:

$$N^h = N^f = N, \tag{51}$$

the above market-clearing conditions (49) and (49) imply that $K_t^f = K_t^h$ and $h_t^f = h_t^h$. Thus the superscripts f and h for population N, private capital stock K and labour h can be removed. For simplicity, the superscripts f and h can also be removed for C_t^h, Y_t^f and Y_t^f.

Moreover, since $N^h = N^f = N$, the aggregate consistency condition (48) can be written in per capita form as:

$$C_t^h + I_t^h + \overline{G}_t^c + \overline{G}_t^i = Y_t^f \tag{52}$$

where $\overline{G}_t^i = \frac{G_t^i}{N}$. Similarly, the budget constraint (40) and the evolution of public capital (41) can be written in per capita form as:

$$\overline{G}_t^t = \tau^c C_t + \tau^k r_t K_t + \tau^h w_t h_t - \overline{G}_t^c - \overline{G}_t^i \tag{53}$$

$$\overline{G}_t^i = \overline{K}_{t+1}^g - (1 - \delta^g)\, \overline{K}_t^g. \tag{54}$$

Finally, note that since there is no growth in this system and all variables are stationary we will use lower case variables. Thus, the non-linear DCE is given by:

$$w_t = \frac{\mu_2 (1 + \tau^c)\, c_t}{\mu_1 (1 - \tau^h)(1 - h_t)} \tag{55}$$

$$\frac{\left[(c_t)^{\mu_1} (1 - h_t)^{\mu_2} (g_t^c)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{c_t} =$$
$$\beta E_0 \left\{ \frac{\left[(c_{t+1})^{\mu_1} (1 - h_{t+1})^{\mu_2} \left(g_{t+1}^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma} \left(1 - \delta + \left(1 - \tau^k\right) r_{t+1}\right)}{c_{t+1}} \right\} \tag{56}$$

$$i_t = k_{t+1} - (1 - \delta)\, k_t \tag{57}$$

$$r_t = \frac{\alpha_1 y_t}{k_t} \tag{58}$$

$$w_t = \frac{(1 - \alpha_1)\, y_t}{h_t} \tag{59}$$

$$y_t = a_t (k_t)^{\alpha_1} (h_t)^{1-\alpha_1} (k_t^g)^{\alpha_2} \tag{60}$$

$$g_t^i = k_{t+1}^g - (1 - \delta^g)\, k_t^g \tag{61}$$

$$c_t + i_t + g_t^c + g_t^i = y_t \tag{62}$$

$$g_t^t = \tau^c c_t + \tau^k r_t k_t + \tau^h w_t h_t - g_t^c - g_t^i. \tag{63}$$

2.3.6 Processes for technology and the fiscal instruments

The processes of the exogenous variables are specified as follows. We assume that the log of technology is a first-order autoregressive process:

$$\log(a_{t+1}) = (1-\rho_a)\log(a_0) + \rho_a \log(a_t) + \varepsilon_{a,t+1} \tag{64}$$

where $a_0 > 0$ is a constant, $0 < \rho_a < 1$ governs the persistence and $\varepsilon_{a,t+1} \sim iid\,(0, \sigma_a^2)$ is the random shocks to technology.

We next specify the processes of the fiscal instruments. The data suggests that the government spending items are hardly seen as stationary processes. They are usually correlated with output over time. Therefore, we follow the literature and assume that the shares of government spending items relative to output are stationary and follow first-order autoregressive processes:

$$\log\left(g_{t+1}^{cy}\right) = \left(1-\rho_{g^c}\right)\log\left(g_0^{cy}\right) + \rho_{g^c}\log\left(g_t^{cy}\right) + \varepsilon_{g^c,t+1} \tag{65}$$

$$\log\left(g_{t+1}^{iy}\right) = \left(1-\rho_{g^i}\right)\log\left(g_0^{iy}\right) + \rho_{g^i}\log\left(g_t^{iy}\right) + \varepsilon_{g^i,t+1} \tag{66}$$

where $g_t^c = g_t^{cy} y_t$ and $g_t^i = g_t^{iy} y_t$ are the government consumption and investment as a share of output respectively; ρ_{g^c} and ρ_{g^i} are the autoregressive parameters; $\varepsilon_{g^c,t+1} \sim iid\left(0, \sigma_{g^c}^2\right)$ and $\varepsilon_{g^c,t+1} \sim iid\left(0, \sigma_{g^{iy}}^2\right)$ represent the random shocks to government consumption and investment as a share of output. Note that the above specification of the fiscal policy instruments implies that both the government spending and its composition change over time.

2.4 Calibration

Our analysis will be based on calibration of the model to annual Chinese data. This methodology follows Kydland and Prescott (1982).

We calibrate our model to the data described above at annual frequency. Compared with the calibration for industrialized economies which has been quite standard for most parameter values, the calibration for the Chinese economy hardly reaches any consensus in the literature. This is due to the problem of limited quality data and the lack of relevant empirical estimates of macroeconomic parameters. Here we will make use of both our data set and the steady-state[53] relationships of the model to set parameter values.

[53]See Appendix (2.8.1) for the derivation of the steady-state of the model.

2.4.1 Private and public capital

We calibrate the ratio of private capital to output in the steady-state at $k/y = 1.675$, which is the average ratio of the private capital to GDP in the data from 1978 to 2006. Similarly, the steady-state public capital to GDP ratio is set at $k^g/y = 0.706$ which is also the historical average in the data. Moreover, the ratios of private and public investment to output in the steady-state are set at $i/y = 0.298$ and $g^i/y = 0.066$ respectively. Given these ratios and the steady-state relationships of investment and capital, the implied depreciation rates of private and public capital are $\delta = \frac{i/y}{k/y} = 0.178$ and $\delta^g = \frac{g^i/y}{k^g/y} = 0.094$ respectively. Note that these depreciation rates are slightly bigger than the ones used in Chow and Li (2002), Hsu and Zhao (2009) and Bai *et al.* (2006).

2.4.2 Production function

There have been debates on the shares of capital and labour in the production function. For example, the capital shares used in Xu (2007), He, *et al.* (2009) and Hsu and Zhao (2009) are 0.65, 0.5 and 0.456. Bai *et al.* (2006) estimate the capital share by using provincial data of labour income share. Their average capital share for the period 1978-2006 is 0.48. Given this uncertainty, we will treat α_1 as a free parameter and test several values for it. An easy way to derive a benchmark case is to make use of the estimate of the rate of return to capital in Bai *et al.* (2006). That is, we set the capital share according to the marginal product of capital in the steady-state, $\alpha_1 = \frac{rk}{y} = 0.503$ where $r = 0.3$ is the return to capital estimated by Bai *et al.* (2006)[54]. This benchmark marginal product of capital is high compared with developed countries. However, it is reasonable since China is still a growing economy. Given the constant return to scale in capital and labour, the share of labour is simply $1-\alpha_1$. After setting the benchmark value for capital share, we also consider $\alpha_1 = 0.45$ and $\alpha_1 = 0.55$ as alternative values.

There is no empirical study on the share of public capital for the Chinese economy. Also, the estimates on the share of public capital for industrialized countries have been reached no consensus as well. For example, Aschauer (1989) reports 0.39 in his estimation of log-linear production function, while other studies, e.g., Holtz-Eakin (1994) and Kamps (2004) find no significant effect of public capital on production. Given the diverse views on the value of the public capital share, we follow Baxter

[54] Note that, in our model $r = \frac{\alpha_1}{k/y}$ is the *gross* rate of return to capital, while Bai *et al.* (2006) estimate a rate of return excluding depreciation, $\widetilde{r} = \frac{\alpha_1}{k/y} - \delta = 0.2$. Since the depreciation rate they use is $\delta = 0.1$, we should set $r = 0.3$.

and King (1993) and set the public capital share to the average ratio of government investment to GDP in the data, i.e. $\alpha_2 = 0.0662$.

2.4.3 Tax rates

There is no explicit tax rate on capital income. We set τ^k to 30% which is consistent with a number of studies on the Chinese capital market[55]. In practice, the wage income tax in China varies from 5% to 45% depending on different wage levels. For example, the 10% tax rate applies to monthly wage income over 1600 Yuan. The data on wage income shows us that it is only recently that the average wage income exceeds 1600 Yuan, implying a value for the average tax rate between 5% and 10%. We then set the wage income tax rate, τ^h, to 7.5%. Finally, the tax rate on consumption also varies, from 5% to 50%. It is set using the government budget constraint in the steady state, i.e.,

$$g^t = \tau^c c + \tau^k r k + \tau^h w h - g^c - g^i.$$

Dividing the above expression by steady-state output gives,

$$\frac{g^t}{y} = \tau^c \frac{c}{y} + \tau^k r \frac{k}{y} + \tau^h (1 - \alpha_1) - g^{cy} - g^{iy}$$

or

$$\tau^c = \frac{\frac{g^t}{y} - \tau^k r \frac{k}{y} - \tau^h (1 - \alpha_1) + g^{cy} + g^{iy}}{\frac{c}{y}} \tag{67}$$

where $\frac{c}{y}$, g^{cy}, g^{iy} and $\frac{g^t}{y}$ are the average ratios of private consumption to GDP, government consumption to GDP, government investment to GDP and government transfers to GDP in the data respectively. Note that since g^{cy} and g^{iy} will be set to data later, the calibration of τ^c amounts to ensuring that $\frac{c}{y}$ and $\frac{g^t}{y}$ are consistent with data[56].

2.4.4 Preferences and utility

The parameter σ which affects the elasticity of the intertemporal substitution of consumption and leisure is set to 2. This is a standard choice in the literature. The time preference parameter of households, β is set to 0.969 which is calculated using the consumption Euler equation in the steady-state:

$$r = \frac{\frac{1}{\beta} - 1 + \delta}{(1 - \tau^k)}$$

[55] For example, Bai *et al.* (2006) report that the before tax rate of return to capital, $\frac{\alpha_1}{k/y} - \delta$ is 0.2 to 0.25 and the after tax rate of return to capital $\frac{\alpha_1}{k/y}\left(1 - \tau^k\right) - \delta$, is 0.10 to 0.15. Given the (average) capital share $\alpha_1 = 0.503$, capital ratio $k/y = 1.675$ and $\delta = 0.1$ in their study, it is inferred that the value of capital income tax falls in the region [0.28, 0.33].

[56] Note that $\frac{c}{y} = 0.462$ in our calibration is slightly higher than in the data which is 0.4486. This is because in our model, $\frac{c}{y} = 1 - \frac{i}{y} - g^{cy} - g^{iy}$, which does not consider net exports.

or

$$\beta = \frac{1}{r\left(1-\tau^k\right)+1-\delta}. \tag{68}$$

Note that this value is also not far away from 0.9602, a value calculated using $\beta = \frac{1}{1+0.414}$ with 0.414 being the annual *ex post* real interest rate.

We next need to calibrate the weights of consumption and leisure in the household's utility function, μ_1 and μ_2. Here we make use of the labour supply function in the steady-state to derive their values:

$$h = \frac{\mu_1\left(1-\tau^h\right)(1-\alpha_1)}{\mu_2\left(1+\tau^c\right)\left(1-g^{cy}-g^{iy}-\frac{\delta\alpha_1}{r}\right)+\mu_1\left(1-\tau^h\right)(1-\alpha_1)}. \tag{69}$$

We make two additional assumptions to solve μ_1 and μ_2. The first assumption is that we set the steady-state value of labour as

$$h = 1/3 \tag{70}$$

which indicates that households work 8 hours per day. This is a common choice in the literature. Second, we impose another assumption that the weight of households consumption in the utility function is four times bigger than the weight of the government consumption they receive on average[57]:

$$\frac{\mu_1}{1-\mu_1-\mu_2} = 4. \tag{71}$$

Given (70) and (69), the labour supply function (71) can be rearranged to solve μ_1 and μ_2 out as:

$$\mu_1 = \frac{\frac{\mu_1}{1-\mu_1-\mu_2}\left(1+\tau^c\right)\frac{c}{y}}{\left(\frac{1}{h}-1\right)\frac{\mu_1}{1-\mu_1-\mu_2}\left(1-\tau^h\right)(1-\alpha_1)+\left(1+\frac{\mu_1}{1-\mu_1-\mu_2}\right)\left(1+\tau^c\right)\frac{c}{y}} \tag{72}$$

$$\mu_2 = 1-\mu_1-\frac{\mu_1}{\frac{\mu_1}{1-\mu_1-\mu_2}}. \tag{73}$$

This yields $\mu_1 = 0.34$ and $\mu_2 = 0.58$. Note that these two numbers imply a ratio $\frac{\mu_2}{\mu_1} \equiv 1.71$ which is consistent with the ones used in a number of studies on the Chinese economy (see for example, Xu (2007) and Hsu and Zhao (2009)). Also note that under the above calibrated utility function, we have $(1-\mu_1-\mu_2)-\frac{1}{\sigma} = -0.42$ which indicates a medium degree of complementarity between government consumption and private consumption.

[57] This ratio of the weight of private consumption to the weight of government consumption in the utility function is consistent with the ones used in a number of studies (see e.g., Bouakez and Rebei (2007) and Leeper *et al.* (2009)) for industrialized countries.

2.4.5 Exogenous processes

We follow the standard approach in the RBC literature and estimate the technology process using the Solow residual calculated from data. According to our model, the formula is:

$$\log(SR_t) = \log(y_t) - \alpha_1 \log(k_t) - \alpha_2 \log(h_t) - (1-\alpha_1-\alpha_2)\log(k_t^g). \quad (74)$$

To extract the technology process, we first fit a linear trend to $\log(SR_t)$,

$$\log(SR_t) = \zeta t + \varepsilon_{sr,t} \quad (75)$$

and take the residuals $\varepsilon_{sr,t}$, i.e., the detrended Solow residual, as an estimate of the stationary technological progress[58]:

$$\log(\widehat{a}_t) \equiv \varepsilon_{sr,t}. \quad (76)$$

Since $\log(a_t)$ is assumed to follow AR(1) processes,

$$\log(a_t) = (1-\rho_a)\log a_0 + \rho_a \log(a_{t-1}) + \sigma_a \varepsilon_{a,t}, \quad (77)$$

the constant term, a_0, the persistence, ρ_a and the standard deviation, σ_a, are then estimated by running an OLS regression of $\log(\widehat{a}_t)$ on its own past values $\log(\widehat{a}_{t-1})$ with a constant,

$$\log(\widehat{a}_t) = \widehat{\beta}_0 + \widehat{\beta}_1 \log(\widehat{a}_{t-1}) + \widehat{\varepsilon}_{a,t}. \quad (78)$$

and then using the following estimates[59]:

$$\begin{aligned} \widehat{a}_0 &= \exp\left(\frac{\widehat{\beta}_0}{1-\widehat{\rho}_a}\right) \\ \widehat{\rho}_a &= \widehat{\beta}_1 \\ \widehat{\sigma}_a &= std(\widehat{\varepsilon}_{a,t}). \end{aligned} \quad (79)$$

The constant terms of the AR(1) processes for the ratios of government consumption and investment to output, $\frac{g_0^c}{y_0}$ and $\frac{g_0^i}{y_0}$, are set to 0.173 and 0.066 respectively, which are the shares of government consumption and investment in GDP in the data. The persistence and standard deviations of these two AR(1) process, ρ_{g^c}, ρ_{g^i}, σ_{g^c}, and σ_{g^i}, are calibrated similarly with the technology shocks. That is, we first de-mean g^{cy} and

[58] The ADF test (although not shown here) assures us that the detrended Solow residual is stationary with one time lag. Thus a AR(1) process can be fitted to the data process.

[59] Note that since the value of a_0 only affects the scale of the technology shock and the economy, it makes little difference if we simply normalize it to 1.

g^{iy}, and then use the residuals to estimate ρ_{g^c}, ρ_{g^i}, σ_{g^c}, and σ_{g^i}. This is done by running the following OLS regressions of the de-meaned g^{cy} and g^{iy}:

$$\log\left(g_t^{cy}\right) = \widehat{\gamma}_1 \log\left(g_{t-1}^{cy}\right) + \widehat{\varepsilon}_{g^{cy},t} \tag{80}$$

$$\log\left(g_t^{iy}\right) = \widehat{\gamma}_2 \log\left(g_{t-1}^{iy}\right) + \widehat{\varepsilon}_{g^{iy},t} \tag{81}$$

and then using the following estimates[60]:

$$\begin{aligned}
\widehat{\rho}_{g^c} &= \widehat{\gamma}_1 \\
\widehat{\sigma}_{g^{cy}} &= std\left(\widehat{\varepsilon}_{g^{cy},t}\right) \\
\widehat{\rho}_{g^i} &= \widehat{\gamma}_2 \\
\widehat{\sigma}_{g^{iy}} &= std\left(\widehat{\varepsilon}_{g^{iy},t}\right).
\end{aligned} \tag{82}$$

[60] The ADF tests suggest that the de-meaned government consumption and investment shares are stationary only at 15% level with one time lag. However, we still fit a AR(1) process to them since the power of the ADF test might be small due to the small sample size of data.

The calibrated parameter values are summarized in Table 2.2 below.

Table 2.2: Calibration of the model

Parameter	Value	Definition
α_1	0.503	Productivity of private capital
α_2	0.066	Productivity of public capital
β	0.969	Rate of time preference of household
σ	2.000	Elasticity of intertemporal substitution
μ_1	0.338	Consumption weight in household's utility
μ_2	0.577	Leisure weight in household's utility
δ	0.178	Depreciation rate on private capital
δ^g	0.094	Depreciation rate on public capital
τ^c	0.170	Indirect tax rate on household's consumption
τ^k	0.300	Tax rate on household's capital income
τ^h	0.075	Tax rate on household's labour income
a_0	1.012	Steady-state technology progress
g_0^{cy}	0.173	Steady-state government consumption - GDP ratio
g_0^{iy}	0.066	Steady-state government investment - GDP ratio
ρ_a	0.784	Persistence of technology shock
ρ_{g^c}	0.689	Persistence of government consumption shock
ρ_{g^i}	0.885	Persistence of government consumption shock
σ_a	0.024	Standard deviation of technology shock
σ_{g^c}	0.039	Standard deviation of government consumption shock
σ_{g^i}	0.103	Standard deviation of government consumption shock

2.5 Quantitative results

The above model will be solved numerically to conduct various model evaluations and policy analysis. Following the RBC literature, we use the log-linearization method to obtain a numerical solution at first-order accuracy. This method takes two steps. The first step is to approximate the non-linear system of the model by log-linearizing it around the steady-state. Second, the resulting linear rational expectational model is solved using the method proposed by Klein (2000) (see Appendix (2.8.3) for details).

Before we analyze the model dynamics and policy implications, we first examine the fit of the model to the Chinese data. That is, to check if this calibrated DSGE model can predict the main features of the business cycles experienced by the economy summarized in Section 2.2. Here we employ two standard assessments that have been used by the traditional RBC literature. First, we simulate the model and compare the population moments of the artificial data, i.e., standard deviation, relative deviation to output, first-order autocorrelation and contemporaneous correlation with output, with those of the actual data. To calculate the population moments, multiple simulations are conducted with random numbers employed as the innovations to the three exogenous shocks in each simulation. In all cases, we let private and public capital start from the steady-state. Second, we depict the historical business cycles of the Chinese economy together with the simulated cyclical movements using the innovations extracted from the actual Solow residual and government spending data.

2.5.1 Moment matching

Table 2.3 reports the moment comparisons of the actual data with the artificial data generated by the model. Both the model generated data and the actual data are in natural logarithms (with the exception of real wage and interest rate) and have been detrended using the HP filter. To reduce sampling uncertainty, we follow Prescott (1986) and King and Rebelo (1999) and conduct multiple simulations of the model and compute the means of the moments across simulations.

By comparing the moments of the model and those of the data, we have the following findings. First, the model generates output series that is as volatile as in actual Chinese economy. The Kydland-Prescott variance ratio of output is 0.8 which implies that, in their interpretation, the model explains 80% of variations in output. This result is similar with studies using the same methodology for the US economy. Second, the standard deviations of the spending components of output in the model are also consistent with those in the data. The only exception is that the private consumption and investment are only more than half as volatile than in the data. Third, the volatilities of the supply side components of output are less consistent with those of data. The variation in the simulated capital series is smaller than the data, partly due to the low capital-output ratio and the low volatility of investment. The cyclical movement of employment, on the other hand, is more volatile in the model than in the data. Fourth, the volatilities of the market prices, i.e. real wage and the interest rate, also display discrepancies with data. One explanation for the discrepancy in the interest rate is that the nominal interest rate is fixed in short term in China. The difference in real wage volatility is due to its extraordinary volatility in the data. Obviously, exploration of this phenomenon deserves further research in the labour market of China.

The relative volatility moments also match the data quite well. For example, the model correctly depicts that private consumption is much less volatile than output while private investment is much more volatile than output. The statistics of the model and data are not exactly the same but not far away. Bigger differences emerge in the relative volatilities of private capital and employment. For example, the model predicts that private capital is only half volatile as output (as in most cases of developed countries) while the data shows almost the same volatility for the two variables. The market prices again show differences with the data. The real wage are much less volatile than in the data and the interest rate is too volatile in the model.

Not surprisingly, the persistence of the model generated data and the actual data do not match well. For example, the first-order autocorrelations of output, government consumption and employment show big differences with the data. On the other hand, private consumption and investment, government transfers, private and public capital match data fairly well. Given the absence of adjustment costs and market imperfections, an obvious explanation for the lack of persistence for some series such as output is that, the three exogenous processes of the model economy also show very low persistence. As pointed out by Cogley and Nason (1995) and King and Rebelo (1999), the persistence of the endogenous variables rely on the persistence of the shocks and the persistence of capital accumulations, governed by the depreciation rates used.

The co-movement statistics are presented in Table 2.3 under the contemporaneous correlation with output column. It is shown that the co-movements of most variables with output are well captured by the model. For example, the contemporaneous correlations of the components and the TFP with output are consistent with those of the data. In particular, the model mimics the co-movements of private consumption and public capital strikingly well. On the other hand, the model mimics the co-movements between output and private capital, market prices and the two fiscal shocks less precisely.

Table 2.3: Moments of the model and data

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	2.56	3.20	1.00	1.00	0.35	0.68	1.00	1.00
C	1.47	2.68	0.58	0.84	0.46	0.53	0.75	0.74
I	4.77	8.55	1.87	2.67	0.30	0.45	0.92	0.77
G^c	4.25	4.07	1.70	1.27	0.19	0.36	0.65	0.42
G^i	10.72	12.54	4.33	3.91	0.35	0.65	0.33	0.26
G^t	36.42	33.83	14.74	10.56	0.24	0.20	−0.16	−0.34
K	1.32	3.22	0.52	1.01	0.68	0.64	0.07	0.37
K^G	1.74	3.17	0.70	0.99	0.72	0.87	−0.05	−0.07
L	1.15	0.66	0.46	0.21	0.29	0.83	0.83	0.03
w	1.72	4.34	0.67	1.36	0.42	0.61	0.93	0.35
r	2.76	1.42	1.08	0.44	0.27	0.55	0.88	0.31
A	2.10	3.04	0.83	0.95	0.29	0.69	0.96	0.86

The number of simulation used for generating these population moments is 1000.
The time length of each simulation is 29 which is the sample size of the data.
The smoothing parameter used in HP filtering both data is 100.

2.5.2 Simulation of business cycles

Some of discrepancies between model generated and actual data may reflect, instead of model misspecification, the fact that artificial shocks were used to generate the model data. To reduce this potential source of bias, this section provides another useful set of comparisons where the model is simulated with the actual innovations to the Solow residual and government spending processes in data rather than using random numbers. This simulation strategy follows King and Rebelo (1999) which allows us to focus on examining the transmission mechanism of the shocks in the model. The cyclical movements of endogenous variables of the model are plotted against the actual cyclical movements extracted by the HP filter in Figure 2.5A and 2.5B. A summary of key statistics are reported in Table 2.4.

A clear finding in the simulated results in Figure 2.5A is that the model generates a very good account of the variations in output and its spending components. The simulated cyclical movements in output almost overlap with the cyclical movements in the data, with only a few small discrepancies in some sub-periods. The recessions in the data for the periods 1980-1981, 1989-1990 and the booms for the periods 1984-1988, 1993-1997, are all correctly replicated by the model. The relative smoothness of output fluctuations after 1992 is also well captured by the model. The overall contemporaneous correlation of simulated data with actual data is 0.96. These results are also confirmed by the statistics presented in Table 2.4 where all the moments of the variables in the model are close to those in the actual data. It is worth noting that now we do not have any persistence problem as in the population moments comparisons in Table 2.3. This again confirms that the internal persistence of the model is weak. The performance of the model relies on large and persistent shocks as illustrated before.

For the individual spending components of output, the two government spending shocks are modeled near perfectly given the goodness of fit of output fluctuations (the contemporaneous correlations are 0.99 and 0.998 respectively). The fits of simulated private consumption and investment are overall good (as also reflected in Table 2.4), with slightly lower volatility in the model. However, the co-movements of these two variables with output differ slightly from data. There are also sizable differences between the persistence of simulated private consumption and investment and the persistence in the data. All these explain why the simulated series of the two variables look more smoothed and the simulated period-to-period variations do not correspond to actual data fully (as a result, the contemporaneous correlations are 0.43 and 0.70 respectively). The simulated government transfers show similar volatilities with data but differ significantly in the co-movements with output (as shown in Figure 2.5A last panel and Table 2.4). There is little correspondence in the period-to-period movements between simulated government transfers and the actual data (as a result, the contemporaneous correlation is -0.18).

Turning to the supply side components of output, we find that the simulated variations in private capital are very close to those observed in the data in all aspects (the contemporaneous correlation is 0.90). The simulated public capital variations on the other hand, show more inconsistencies with data both in volatility and co-movements with output (as shown in Figure 2.5B second panel and Table 2.4; also, the contemporaneous correlation is 0.39). The discrepancy between simulated labour input and actual employment data is large - the simulated series is much more volatile and procyclical than the data is (the contemporaneous correlation is 0.01, indicating a strong failure of the model). Again, this might be the problem of using employment as a measure of total hours worked.

Finally, the simulation results of the two market prices are shown in the last two panels of Figure 2.5B. The simulated real wage series seems to provide a good account of the volatility and persistence in the data. However, it cannot generate correct co-movements with output (as shown in Table 2.4; also, the contemporaneous correlation is 0.21). The interest rate does not cohere well in terms of volatility and co-movements with output (the contemporaneous correlation is 0.16).

Overall, when simulating the model with real shocks extracted from the actual economy, the model mimics the cyclical movements in most aggregate variable even better. The propagation mechanism of the model correctly captures the effects of the shocks on the main endogenous variables, making a reasonable account of their cyclical variations.

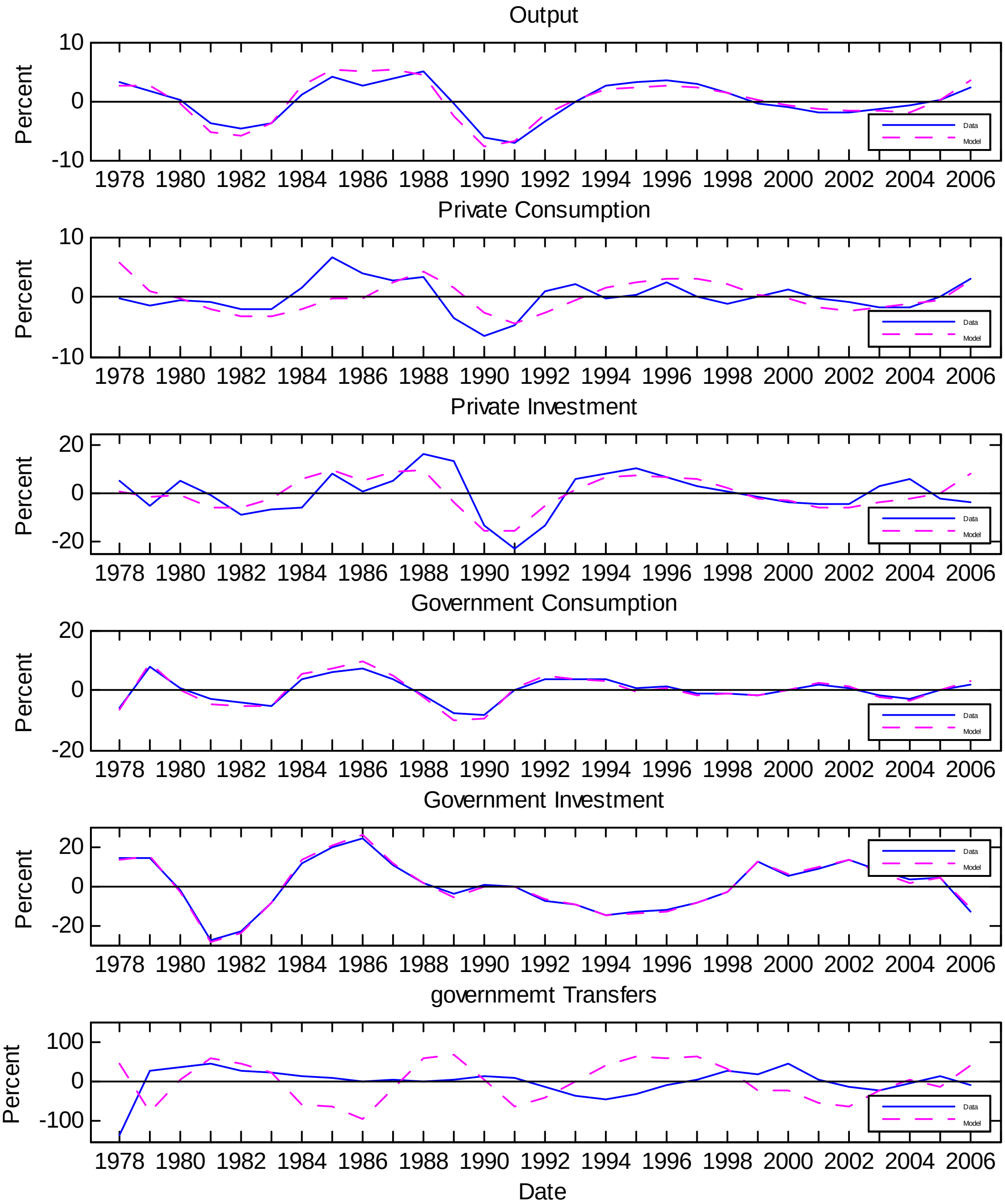

Figure 2.5A: Simulated business cycles: Output and its components

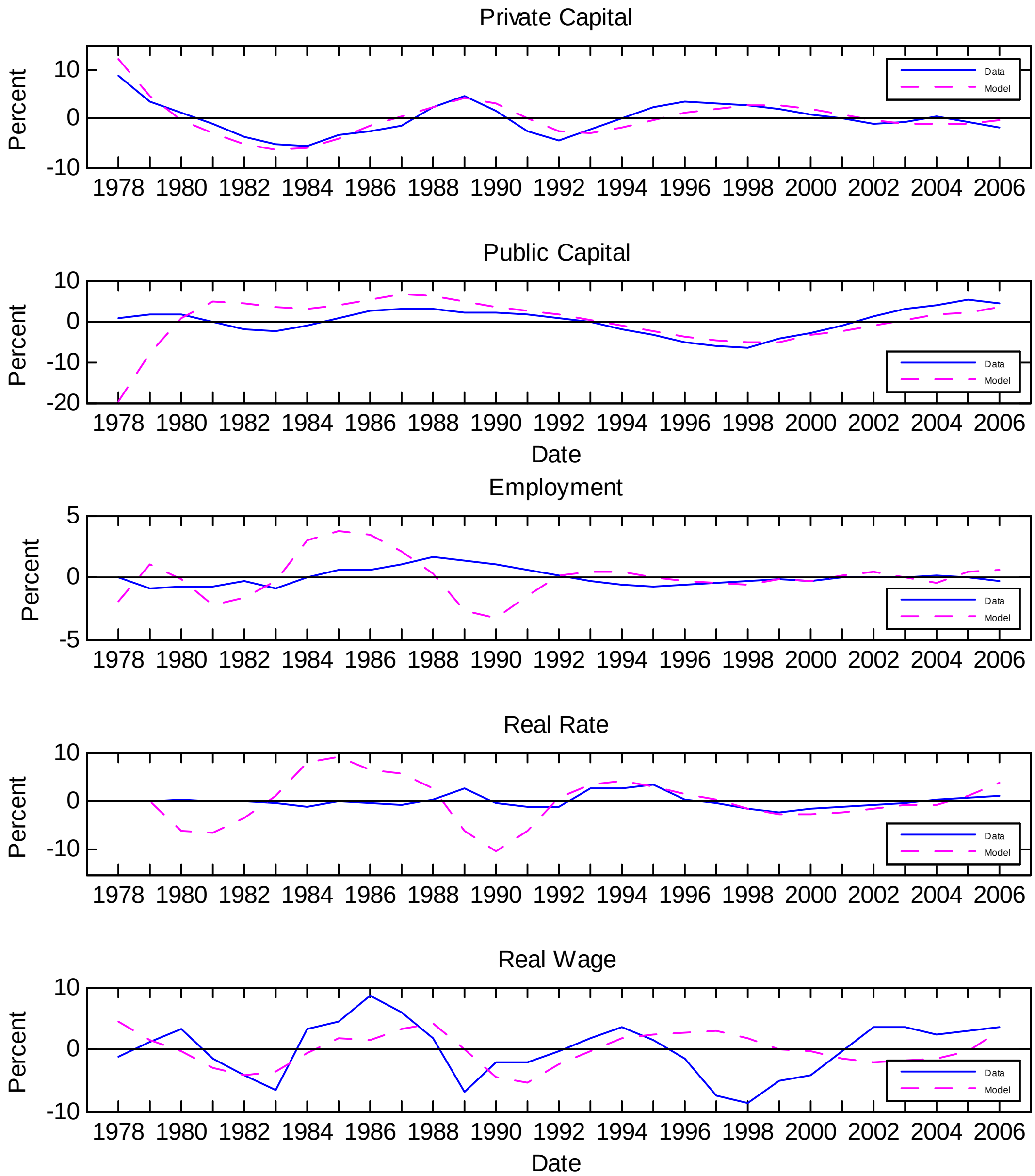

Figure 2.5B: Simulated business cycles: Output and its producing inputs

Table 2.4: Moments of the model and data (1 simulation)

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	3.61	3.20	1.00	1.00	0.68	0.68	1.00	1.00
C	2.47	2.68	0.68	0.84	0.59	0.53	0.73	0.74
I	6.88	8.55	1.90	2.67	0.67	0.45	0.92	0.77
G^c	4.93	4.07	1.37	1.27	0.39	0.36	0.60	0.42
G^i	13.17	12.54	3.64	3.91	0.66	0.65	0.42	0.26
G^t	49.15	33.83	13.61	10.56	0.48	0.20	−0.05	−0.34
K	3.70	3.22	1.02	1.01	0.62	0.64	0.16	0.37
K^G	5.42	3.17	1.50	0.99	0.60	0.87	−0.18	−0.07
L	1.64	0.66	0.45	0.21	0.63	0.83	0.73	0.03
w	2.66	4.34	0.74	1.36	0.64	0.61	0.91	0.35
r	4.72	1.42	1.31	0.44	0.71	0.55	0.83	0.31

Note that the moments of the three driving processes are not presented since they are the same in the model and in the data.

Further discussion on Chinese labour market A short summary of the above findings in Section 2.5.1 and 2.5.2 seems to be that the theoretical model performs overall well in matching output and both its spending components and capital items. However, the model falls extremely short of fitting the labour market data. For example, the model produces almost twice the volatility of employment than the data and much higher correlation between employment and output than in the data. Moreover, it also fails dramatically in matching the low correlation of real wage with output in the data. Since these failures are substantial, it is necessary to explore the underlining factors that drive this result.

About the mis-match in employment data, it can be argued that this is primarily due to the imperfect mapping between the labour input concept in the model and in the data. That is, the labour input means hours worked per worker while the data we use as a measurement is only employment. Thus it is not surprising that employment is much less volatile. In fact, as discussed earlier in Section 2.2 footnote, some researchers (e.g., Hsu and Zhao (2009)) have found that the estimated (although very imprecise) data of hours per worker is much more volatile than employment (although the hours data is not fully available). Thus, the actual total hours worked should be much more volatile and more comparable with our simulated series.

For the failure of the model in matching real wage series, it might be due to the fact that the labour market is not fully market oriented. The wages have not been always determined by market forces. Instead, the labour market has been under reform for two decades. The wage was simply fixed in the beginning of our data sample. It was then relaxed only gradually during reform. For example, in 1980s the fixed wage system was replaced with floating wage system taking the popular form of wage-plus-bonus. Some authors (e.g., Meng and Kidd (1997)) argued that this wage system was still far away from the real flexible wage determination in market economies. In 1990s, more flexible wage systems were implemented in the urban area and in non-SOEs. However, there are still less flexible wage settings in SOEs. Given the imperfect market imperfect wage determination and the perfect market assumption in the model, it is not surprising to see the failure of the model in match real wage series.

To overcome the failure of the model in fitting labour market data, one needs to further explore the labour market reform in China. In fact, it can be very likely that even if the hours data is available, it could still be hardly approximated by the model due to unaccounted institutions during the labour market reform in China. There are many reforms happened from late 1970s to 1990s ((Meng (2000), Brooks and Ran (2003)) such as the wage reform mentioned above and also the flexibility in labour allocation in 1980s and the laid-off of workers in SOEs in the 1990s. Thus, future extensions of the model should focus on the effects of these reforms and try to find a proper way to account for these factors. For example, a exogenous variable can be introduced into the model to capture these institutional effects. Since the reforms have been implemented in a piecemeal pattern, this exogenous variable can contain a stochastic trend. For another, the model structure can be extended by discriminating the SOEs sector and the non-SOEs sector at least for the labour market given their differences in the Chinese economy.

2.5.3 Counterfactual experiment

The above simulation results suggest that, economic fluctuations in the Chinese economy are reasonably well explained by our model. However, it is not clear which shock or shocks account for most of the fluctuations. To distinguish the impacts of the three economics shocks on the performance of the model, a counterfactual analysis is conducted. That is, we undertake the same exercise in Section 2.5.1 and 2.5.2 but turn off one or both of the government spending shocks. The first experiment assumes that TFP shocks are the only shocks in the economy holding government spending shocks constant at their steady-state values. The second experiment shocks both TFP and the government consumption ratio while holding the government investment ratio shocks constant at its steady-state value.

The population moments and the simulated business cycle series under the first experiment are presented in Table 2.5 and Figure 2.6A-2.6B[61] below[62]. When the two government spending shocks are absent, we obtain a slightly worse result compared with the full shocks case in Table 2.3 and Figure 2.5A-2.5B. First, for the population moments, almost all variables are now less volatile, persistent and show less consistent contemporaneous correlation with output with data. For example, the volatility of output now falls to 2.53. The same holds for private consumption and private investment shows even bigger falls to 1.29 and 4.64 respectively. Private consumption now also shows much stronger procyclical pattern. There is one improvement in the volatility of labour supply which now falls from 1.15 as in Table 5 to 0.93, more consistent with data. Second, turning to the simulated series as shown in Figure 2.6A-2.6B, the simulated output variations using only real TFP shocks now show some discrepancies with data, especially during the recession period 1980-1983 and the boom period 1986-1988. Private consumption shows a weaker fit to data after 2000 and is less volatile over the whole sample. Private investment shows large differences with data from 1978 to 1992. There are several improvements of model fit for several variables such as private consumption, employment and the two market prices. For example, the contemporaneous correlations of private consumption and employment with output rise from 0.43 to 0.7 and from 0.01 to 0.07 respectively. This implies that the inclusion of government spending shocks dampens the model's ability in matching these variables with data. In sum, despite several small differences explained above, the model with only TFP shocks still explain most of business cycle fluctuations as observed in the actual data. Therefore, our results are largely consistent with the empirical research which suggests technology shocks as the main sources of economic fluctuations of the Chinese economy.

[61] See Appendix (2.8.6) for a summary of moments comparisons for counterfactual simulated business cycle series using actual shocks.

[62] Since only TFP shocks are active, all government spending ratios to output are constant (at steady-state level) and government consumption and government investment simply follow the dynamics of output. Therefore, there is no need to report the results for government spending or their ratios.

Table 2.5: Moments of the model and data: TFP shocks

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	2.53	3.20	1.00	1.00	0.35	0.68	1.00	1.00
C	1.29	2.68	0.51	0.84	0.49	0.53	0.94	0.74
I	4.64	8.55	1.83	2.67	0.31	0.45	0.99	0.77
K	1.29	3.22	0.51	1.01	0.69	0.64	0.07	0.37
L	0.93	0.66	0.37	0.21	0.28	0.83	0.95	0.03
w	1.68	4.34	0.66	1.36	0.42	0.61	0.98	0.35
r	2.73	1.42	1.08	0.44	0.27	0.55	0.88	0.31

The number of simulation used for generating these population moments is 1000. The time length of each simulation is 29 which is the sample size of the data. The smoothing parameter used in HP filtering both data is 100.

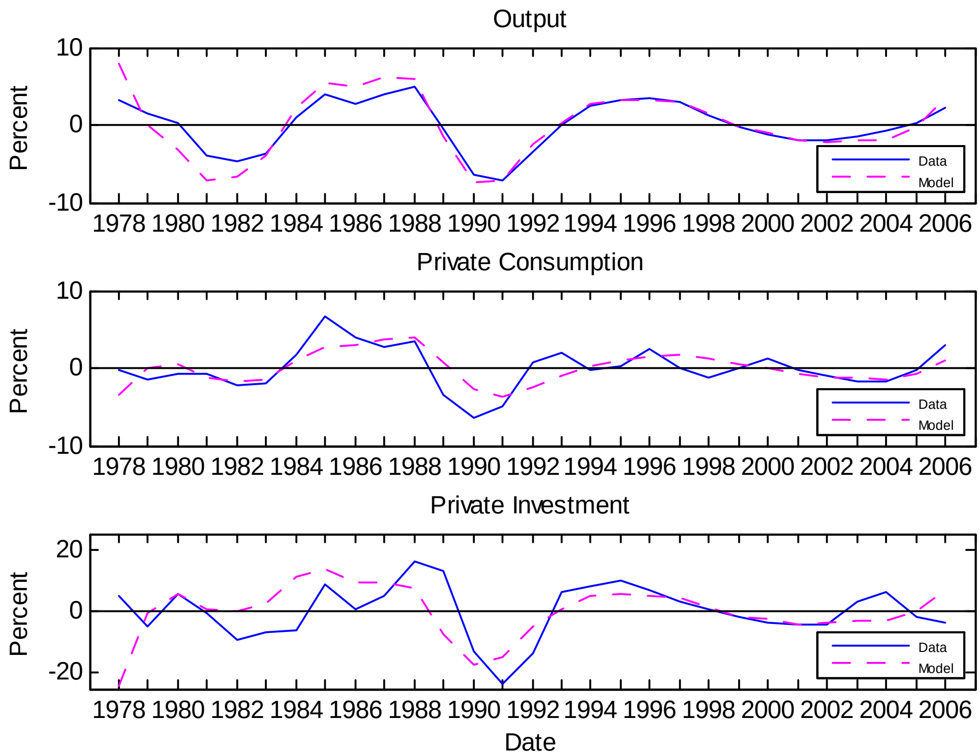

Figure 2.6A: Simulated business cycles with only TFP shocks: Output and its components

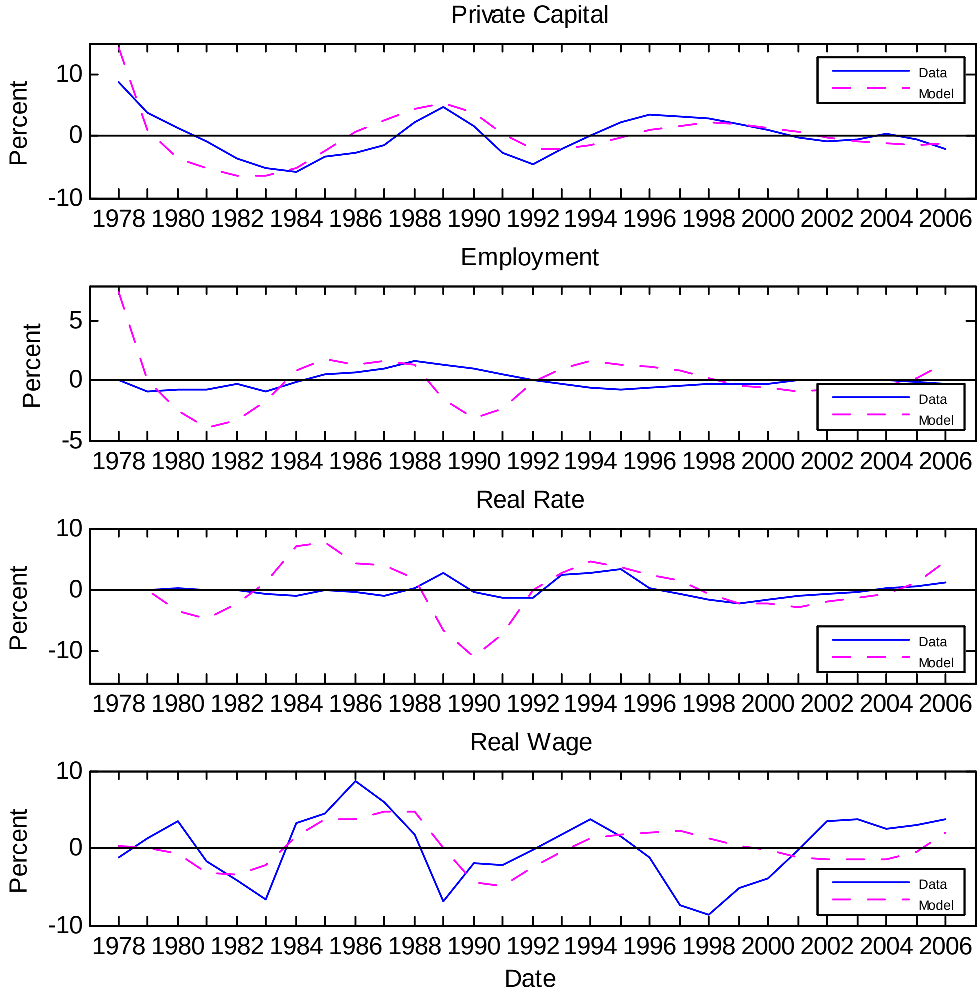

Figure 2.6B: Simulated business cycles with only TFP shocks: Output and its producing inputs

The population moments of the second experiment with both TFP shocks and government consumption ratio shocks are summarized in Table 2.6. The simulated series of the model are plotted in Figure 2.7A-2.7B. These results are both comparable with the sole TFP shocks case in Table 2.5 and Figure 2.6A-2.6B, and the benchmark case in Table 2.3 and Figure 2.5A-2.5B. Useful implications can be drawn regarding the two government spending shocks. For example, if the fit of model with both TFP and government consumption ratio shocks (as shown in Table 2.6 and Figure 2.7A-2.7B) is better than the sole TFP shocks case (as shown in Table 2.5 and Figure 2.6A-2.6B) but worse than the full shocks case (as shown in Table 2.3 and Figure 2.5A-2.5B), we can conclude that the inclusion of both government consumption and investment ratios shocks raises the explanatory power of the model. For another, if the result in Table 2.6 is better than the result in Table 2.5 and also better than the result in Table 2.3, we can deduce that only government consumption ratio shocks are helpful. On the other hand, if the fit of two shocks case is worse than the sole TFP shocks case and the full shocks case, we can say that the government consumption ratio shocks are not helpful while the government investment ratio shocks are.

The findings for the second counterfactual experiment can be summarized as follows. First, for the population moments, almost all of the business cycle moments (as shown in Table 2.6) are improved compared with the sole TFP shocks case (as shown in Table 2.5). For example, output and its spending components are now more volatile and more consistent with data. Private capital variations are also slightly improved. The two market prices also show more consistent with data in terms of all the four types of moments. The only exception is the moments of labour supply which now displays too much volatility and is too persistent and procyclical with output (again subject to data problem). However, these improved results under two shocks case in Table 2.6 are still worse than the full shocks case in Table 2.4. This indicates that both government consumption ratio and government investment ratio shocks are helpful for explaining business cycles. Second, for the simulated series using real shocks, the results are plotted in Figure 2.7A-2.7B. Also, the summarized contemporaneous correlation between simulated series (under all cases) and the data are reported in Table 2.7. Comparing the simulated series in Figure 2.7A-2.7B with those plotted in Figure 2.6A-2.6B and 2.5A-2.5B, we can see that the fit of model to data is better than the sole TFP shocks case but still slightly worse than the full shocks case. For example, the larger discrepancies of variations in output, private consumption and investment are now much smaller as shown in Figure 2.7A. The same is also true for the producing inputs of output, especially for private capital and real wage as shown in Figure 7B. These simulated series using two shocks are however, still inferior to the full shocks case. Thus, one can infer that both government consumption ratio and government investment ratio shocks help capture the business cycles in the Chinese economy. Again, the fit of several variables such private consumption, labour supply and the two market prices are reduced compared with the single TFP shocks case. In particular, the contemporaneous correlation of model generated labour supply with data turns to a negative number -0.25 which indicates a significant discrepancy. This indicates that the inclusion of the two government spending shocks worsens the model's ability to replicate the variations of these variables in data.

Based on the above findings, two important conclusions can be drawn. First, the simplified model with only TFP shocks as the driving forces explains the overall fit of the model to output. Second, the inclusion of both government consumption and investment ratio shocks improve the fit of the model to most variables such as output and its components. At the same time, the government spending shocks do not help explain the variations in some variables such as labour supply and the two market prices.

Table 2.6: Moments of the model and data: TFP + g_t^{cy} shocks

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	2.54	3.20	1.00	1.00	0.35	0.68	1.00	1.00
C	1.34	2.68	0.52	0.84	0.47	0.53	0.89	0.74
I	4.73	8.55	1.87	2.67	0.30	0.45	0.95	0.77
K	1.31	3.22	0.51	1.01	0.68	0.64	0.07	0.37
L	0.99	0.66	0.39	0.21	0.27	0.83	0.91	0.03
w	1.69	4.34	0.66	1.36	0.42	0.61	0.97	0.35
r	2.74	1.42	1.08	0.44	0.27	0.55	0.88	0.31

The number of simulation used for generating these population moments is 1000.
The time length of each simulation is 29 which is the sample size of the data.
The smoothing parameter used in HP filtering both data is 100.

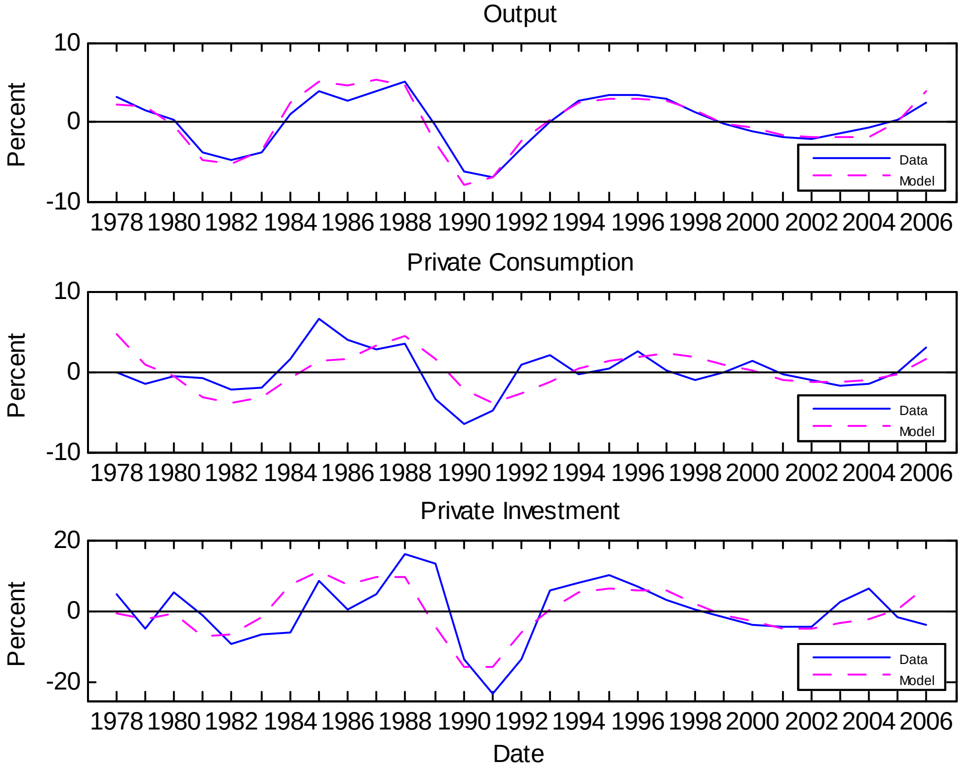

Figure 2.7A: Simulated business cycles with only TFP + g_t^{cy} shocks: Output and its components

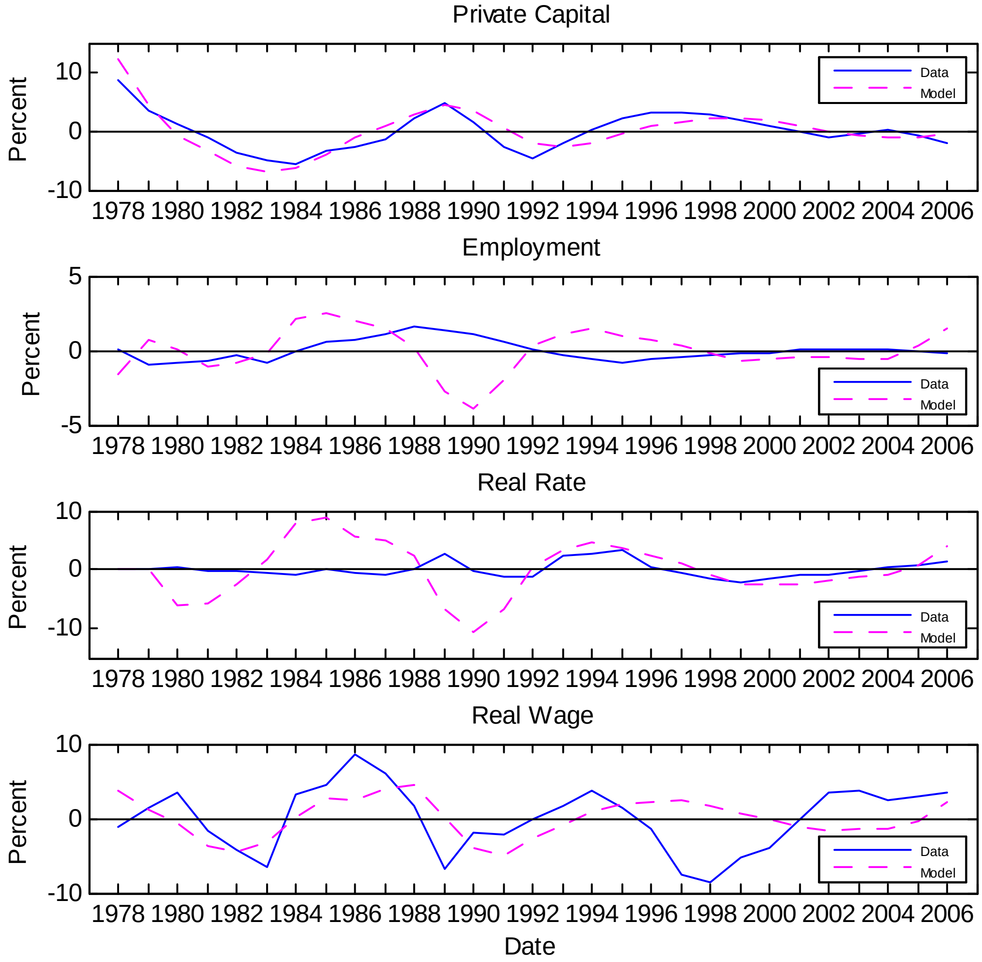

Figure 2.7B: Simulated business cycles with only TFP + g_t^{cy} shocks: Output and its producing inputs

Table 2.7: Contemporaneous correlation of model with data: Simulated series

Variables	TFP shocks	TFP + g_t^{cy} shocks	All shocks
Y	0.94	0.97	0.99
C	0.70	0.52	0.43
I	0.40	0.67	0.70
K	0.80	0.88	0.90
L	0.07	-0.25	0.01
w	0.35	0.27	0.21
r	0.20	0.18	0.16

This table summarizes the correlations of the simulated data with actual data using different set of shocks to gauge the fit of the model. All data are logged and have been detrended using HP filter.

2.5.4 Sensitivity analysis

Our analysis so far has hinged on the use of calibration of the model to data. Although we have justified the parameter values used in the benchmark analysis according to the data at hand and the relevant literature, there are still uncertainties on the calibration of several parameters. For example, the capital share in the aggregate production function varies from one study to another, within the range [0.45, 0.6]. For another, there is limited microeconomic study on the leisure weight of utility. We have circumvented this problem by assuming that the weight of private consumption is four times bigger than the weight of government consumption. Moreover, the aggregate production function is chosen such that public capital is assumed to be increasing return to scale with other inputs. There is however, also limited empirical evidence to justify this assumption.

In light of the above, we need to check the robustness of our results by varying the calibration of these uncertain parameters. In the following, we will conduct a sensitivity analysis which considers alternative values for capital share and leisure weight in utility and constant return to scale public capital in the production function. For simplicity, we only present the population moments comparisons for each case.

The population moments generated by lower ($\alpha_1 = 0.45$) and higher ($\alpha_1 = 0.55$) capital share are reported in Table 2.8 below[63]. These results are comparable to the benchmark case with $\alpha_1 = 0.50$ in Table 2.3. We summarize the findings with respect to the volatility, persistence and correlation (with output) of the variables. First, varying capital share produces very interesting results in the volatilities of variables. Output and its components become less volatile when capital share shrinks and more volatile when capital share increases. The volatility of consumption is an exception where the smaller the capital share, the smoother the consumption variations. Note that the volatilities of the three inputs to production are increasing in the capital share while the volatility TFP is decreasing.[64]. This implies that the increase (decrease) in output volatilities are mainly associated with the increase (decrease) in capital and labour volatilities, not with TFP. Moreover, the relative volatility changes are mixed. Finally, it is easy to gauge that the differences of volatilities under different capital

[63]Note that, to ensure the great ratios of the model, some parameters need to be adjusted. For example, the capital return rate is 0.27 (when $\alpha_1 = 0.45$) and 0.33 (when $\alpha_1 = 0.55$) and the capital tax rate is 0.22 (when $\alpha_1 = 0.45$) and 0.36 (when $\alpha_1 = 0.55$).

[64]The same random numbers are used as the innovations of the three shocks. However, since capital share changes, the calculation of the Solow residual also changes. Thus, the persistence and standard deviations of TFP might also change.

shares are small (not over 5%). Thus, changing the capital share does not the change the explanatory power of the model in matching the volatility of data. Second, the persistence of variables under two extreme capital share cases are very close to each other and to the benchmark case in Table 2.3. Given the persistence of the three exogenous variables are almost the same, this is not surprising. Thus, the variation of capital share does not help to address the persistence problem we have in the benchmark case. Third, the components of output show little differences in the contemporaneous correlations with output. They differ more for private consumption and investment, government investment and transfers and labour supply. The difference in the cyclical behavior of government transfers is the biggest, -0.24 compared with -0.09, indicating a much more countercyclical fiscal spending in the smaller capital share case. Again, these findings do not overturn the conclusion in the benchmark case that the model correctly predicts the cyclical movements of variables with output.

Table 2.8: Moments of the model and data (alternative capital share)

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
α_1	0.45	0.55	0.45	0.55	0.45	0.55	0.45	0.55
Y	2.63	2.51	1.00	1.00	0.34	0.36	1.00	1.00
C	1.44	1.50	0.55	0.60	0.46	0.46	0.74	0.76
I	4.97	4.61	1.90	1.85	0.29	0.30	0.93	0.92
G^c	4.30	4.21	1.67	1.71	0.19	0.19	0.66	0.64
G^i	10.77	10.69	4.23	4.40	0.35	0.35	0.34	0.32
G^t	38.55	34.72	15.17	14.34	0.25	0.24	−0.24	−0.09
K	1.36	1.29	0.52	0.51	0.68	0.69	0.06	0.08
K^G	1.74	1.73	0.69	0.71	0.72	0.72	−0.04	−0.05
L	1.21	1.10	0.46	0.45	0.29	0.29	0.85	0.81
w	1.72	1.73	0.65	0.69	0.41	0.42	0.93	0.93
r	2.85	2.69	1.09	1.08	0.27	0.28	0.88	0.88
A	2.08	2.13	0.80	0.85	0.29	0.29	0.96	0.95
G^c/Y	3.22	3.22	1.26	1.33	0.10	0.10	0.07	0.07
G^i/Y	10.16	10.16	4.00	4.19	0.35	0.35	0.11	0.09

The number of simulation used for generating these population moments is 1000. The time length of each simulation is 29 which is the sample size of the data. The smoothing parameter used in HP filtering both data is 100.

The population moments generated by much lower ($\frac{\mu_1}{1-\mu_1-\mu_2} = 2$) and higher ($\frac{\mu_1}{1-\mu_1-\mu_2} = 8$) ratios of private consumption weight to government consumption weight in the utility function are reported in Table 2.9[65]. Again, we compare the results with the benchmark case in Table 2.3 with $\alpha_1 = 0.50$. First, with smaller private consumption weight in utility, all variables (except the real wage) are more volatile. This might be due to the fact that private consumption is much smoother than government consumption and leisure. The relative volatilities of variables are mixed and can be been in Table 2.9. However, compared with the benchmark case in Table 3, the differences in volatilities are quite small (also smaller than the varying capital share case as in

[65] Note that, according to our calibration scheme, $\frac{\mu_1}{1-\mu_1-\mu_2} = 2$ implies that $\mu_1 = 0.31$, $\mu_2 = 0.53$ and $\frac{\mu_1}{1-\mu_1-\mu_2} = 8$ implies that $\mu_1 = 0.35$, $\mu_2 = 0.60$. Again, to ensure the great ratios of the model, some parameters such as the capital return rate (r) and tax rate (τ^k) are also adjusted.

Table 2.8). Second, the persistence of the model again shows little changes. This is because the persistence of the three driving forces is the same as before. Third, the contemporaneous correlations of variables with output are also very close. Even the government spending items also do not differ much. All these findings suggest that varying the weights of the components of utility does not help improve the model's ability to generate closer business cycles with the actual data.

Table 2.9: Moments of the model and data (alternative leisure weight)

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
$\frac{\mu_1}{1-\mu_1-\mu_2}$	2	8	2	8	2	8	2	8
Y	2.58	2.54	1.00	1.00	0.35	0.35	1.00	1.00
C	1.48	1.47	0.58	0.58	0.45	0.46	0.70	0.77
I	4.83	4.74	1.88	1.87	0.31	0.30	0.94	0.91
G^c	4.30	4.22	1.70	1.69	0.19	0.19	0.66	0.64
G^i	10.73	10.72	4.30	4.35	0.35	0.35	0.33	0.33
G^t	36.75	36.24	14.73	14.74	0.24	0.25	−0.18	−0.15
K	1.35	1.31	0.52	0.51	0.68	0.68	0.07	0.07
K^G	1.74	1.74	0.70	0.70	0.72	0.72	−0.05	−0.05
L	1.22	1.11	0.48	0.44	0.28	0.30	0.82	0.83
w	1.71	1.73	0.66	0.68	0.42	0.41	0.91	0.93
r	2.79	2.74	1.08	1.08	0.27	0.27	0.88	0.88
A	2.10	2.10	0.82	0.83	0.29	0.29	0.96	0.96

The number of simulation used for generating these population moments is 1000.
The time length of each simulation is 29 which is the sample size of the data.
The smoothing parameter used in HP filtering both data is 100.

The population moments generated by the model with constant return to scale (CRS) in all producing inputs ($\alpha_1 + \alpha_1 + \alpha_1 = 1$) are reported in Table 2.10 below[66]. To distinguish these results with the benchmark case with increasing return to scale (IRS) in public capital ($\alpha_1 + \alpha_1 + \alpha_1 > 1$), we also include the results in Table 2.3 into Table 2.10. The results are even more interesting than the above two exercises. First, all variables except TFP are now less volatile compared with the IRS case[67]. This

[66] Accordingly, CRS in all inputs implies that the labor share reduces to 0.43. Also, to ensure the great ratios of the model, some parameters such as the capital return rate (r) and tax rate (τ^k) are also adjusted.

[67] The persistence and standard deviation might change due to the change of production function and the way to calculate the Solow residual.

indicates a worsening of model performance with respect to matching the volatility of actual data. However, as the fourth and the fifth column of Table 2.10 results show, the relative volatilities of all variables are now higher than the benchmark case, indicating an improvement of the model performance in matching data. Again, all these differences are not big and do not change the performance of the model dramatically. Second, the persistence of the model economy does not change at all. This is again not surprising given that the persistence of the driving forces is still the same as before. Neither the change of production function form improves the persistence of capital accumulation process. This proves that the CRS technology does not improve the persistence problem of the model. Third, the CRS model economy gives very similar predictions about the co-movements between output and its components. In other words, there is no gain from moving from IRS to CRS production technology.

In a short summary, the above experiments have considered some variations of the model, i.e., varying the capital share, changing the weights of the components of households' utility and employing CRS production function form. These variations of the model do not change the performance of the benchmark model much. Instead, the benchmark model provides a medium case which these two experiments are close to. Therefore, our results of the assessments of the model's fit to data are robust to these variations.

Table 2.10: Moments of the model and data (constant return to k^g)

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
$y\,(k^g)$	CRS	IRS	CRS	IRS	CRS	IRS	CRS	IRS
Y	2.48	2.56	1.00	1.00	0.35	0.35	1.00	1.00
C	1.46	1.47	0.59	0.58	0.46	0.46	0.74	0.75
I	4.66	4.77	1.88	1.87	0.30	0.30	0.92	0.92
G^c	4.19	4.25	1.72	1.70	0.19	0.19	0.63	0.65
G^i	10.67	10.72	4.44	4.33	0.35	0.35	0.31	0.33
G^t	36.16	36.42	15.08	14.74	0.24	0.24	−0.14	−0.16
K	1.30	1.32	0.52	0.52	0.68	0.68	0.08	0.07
K^G	1.73	1.74	0.72	0.70	0.72	0.72	−0.05	−0.05
L	1.12	1.15	0.46	0.46	0.29	0.29	0.82	0.83
w	1.69	1.72	0.68	0.67	0.42	0.42	0.92	0.93
r	2.68	2.76	1.08	1.08	0.28	0.27	0.88	0.88
A	2.11	2.10	0.85	0.83	0.29	0.29	0.96	0.96

The number of simulation used for generating these population moments is 1000.
The time length of each simulation is 29 which is the sample size of the data.
The smoothing parameter used in HP filtering both data is 100.

2.6 Impacts of TFP and government spending

After assessing the fit of the model to data, we now illustrate the usefulness of the model for policy analysis by studying the dynamics of the model in response to the exogenous shocks. That is, how the three exogenous shocks are amplified to generate aggregate fluctuations through the propagation mechanism of the model. The differences in the impacts of the TFP shock and the two fiscal shocks will be summarized and analyzed.

2.6.1 TFP shock

The responses of endogenous variables to a one standard deviation TFP shock are reported in Figure 2.8. The one period increase in productivity affects the dynamics of the model through a standard wealth effect, accompanied by substitution effects induced by the changes in the two market prices. To see this, we first note that the initial increase in TFP implies that more wealth is now generated (through the production function) and households have to choose between consumption and investment. Since the marginal utility of consumption is decreasing, households only consume a small fraction of this increased output and save most of it for investment (as shown in Figure 8 second row panel). This is a clear result of consumption smoothing which naturally presents in this model. Since there is no such decision problem for the government, the public consumption and investment just increase proportionally with output[68]. At the same time, the initial increase in TFP raises the marginal product of labour and capital. As a result, both real wage and the interest rate jump up initially. The increases in real wage substantially increases the opportunity cost of taking leisure, inducing households to work more. The wealth effect of the increase in real wage is thus relatively small given the fact that households understand that the current wage is abnormally high and future wages will return to its steady-state level. As the capital stock is built up quickly, the interest rate falls below its steady-state level. This low interest rate together with the persistent productivity explains the persistence in consumption and investment in next 20-40 years. In the long run, both the interest rate and real wage gradually return to their steady-state values. It is noted that all variables show persistence in dynamics except the labour supply. In fact, the positive substitution effect of high real wage on labour supply dies out quickly, indicating that it is soon offset by the wealth effect caused by the same variable. A summary of the business cycle features in this impulse response analysis is that, the TFP is transmitted with similar volatility to most variables and is amplified to investment and output. Furthermore, all variables (except labour supply) show more persistent dynamics than the original TFP shock, indicating some degree of inherent persistence propagation mechanism of the model.

[68]Note that government transfers are not a component of output. It is simply the residual of government tax income minus government spending. Thus, the following will not analyze the effects of shocks on government transfers.

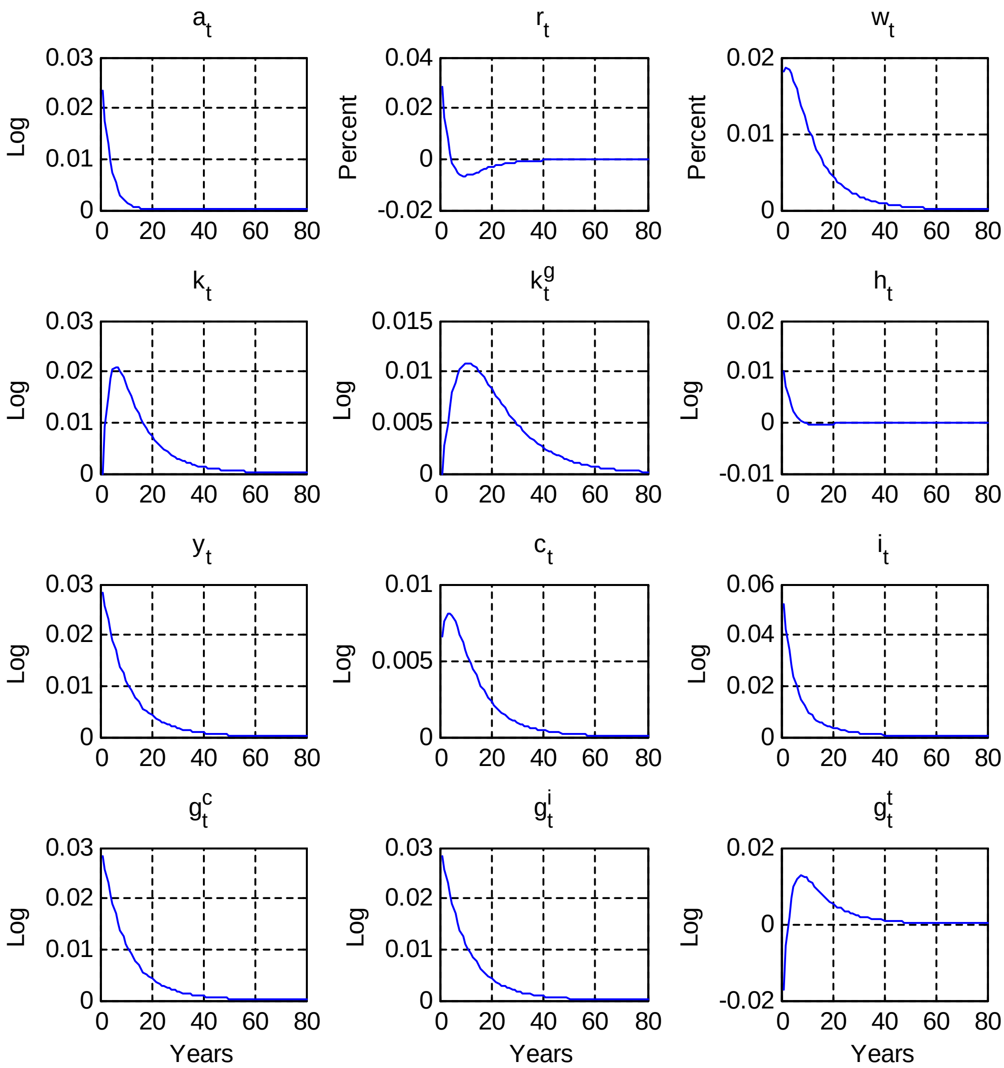

Figure 2.8: Impulse responses to TFP shock

2.6.2 Government consumption shock

The RBC model also enables us to examine the dynamic responses of variables to government spending shocks. Studies using the standard RBC model to analyze the impacts of government spending shocks on industrialized economies have been rich. Early research on the effects of the government spending shocks, e.g., Baxter and King (1993) and Edelberg, Eichenbaum and Fisher (1999)[69], usually found significant 'crowding-out' effect and negative wealth effect on the US economy. That is, *ceteris paribus*, the increase in government consumption implies fewer resources that are available in the economy for private agents. It thus crowds out both consumption and

[69] See also, e.g., Ramey and Shapiro (1998), Edelberg, Eichenbaum and Fisher (1999) for a literature review on other early studies on government spending.

investment of households. Meanwhile, if the government spending is financed by lump-sum taxes, as in our case, there will be also a negative wealth effect due to the decrease in the disposal income of households. This negative wealth effect suppresses private consumption but stimulates private investment. In both cases of 'crowding-out' effect and negative wealth effect, private consumption is reduced and labour supply is raised. Also, the interest rate goes up and most models observe a decrease in real wage. Recently, some studies, e.g., Leeper *et al.* (2009), show that models with government consumption in the utility function of households also create an effect characterizing the movements between private consumption and government consumption. For example, as illustrated in the model setup in Section 2.3, when $(1 - \mu_1 - \mu_2) < \frac{1}{\sigma}$ which is the case under our calibration, the government consumption and private consumption co-move with each other.

Research on the fiscal policy using DSGE models for the Chinese economy have been absent in the literature. We thus provide the first examination of the effects of government spending shocks using our calibrated model above. Figure 2.9 shows the model dynamics to a standard deviation shock in the ratio of government consumption to output. It is seen that the dynamics of the model to a government consumption shock are roughly comparable with those from the standard RBC models calibrated for the US economy. The one period increase in government consumption ratio drags down private consumption and raises labour supply. This indicates that the negative wealth effect dominates. The increases in labour supply drives real wage down. The negative wealth effect also raises the interest rate. A short period jump in government investment is observed but this is just because it moves proportionally with output given that its ratio to output is constant. Given the short period jump in labour supply and public capital, output only increases in the first several years. It quickly goes down and becomes negative when both private and public capital fall due to the reductions in private and public investment.

In the long-run, only the responses of the two capital series and market prices display some degree of persistence. The responses of other variables on the other hand, are very short-lived. This result is due to the weak persistence of the government consumption ratio shock, but is also due to the model's weak internal persistence discussed above.

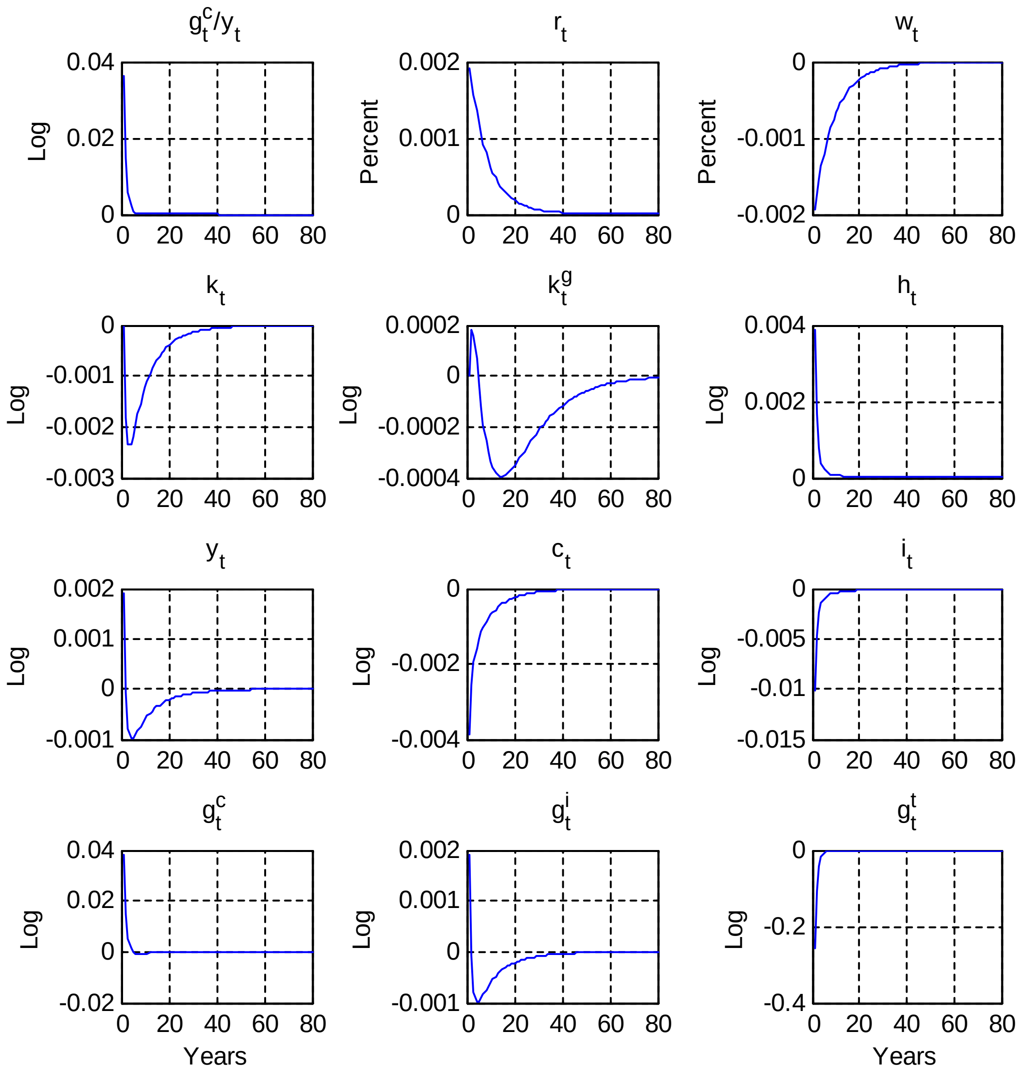

Figure 2.9: Impulse responses to government consumption - output ratio shock

We have not analyzed how the co-movement between government consumption and private consumption affect the dynamics of variables above. To see this, we also provide a counterfactual analysis of the responses of variables when public consumption and private consumption are less complementary. In particular, we consider the extreme case that, $(1 - \mu_1 - \mu_2) < \frac{1}{\sigma}$, i.e., the government consumption and private consumption are substitutes rather than complements. The results are plotted in Figure 2.10. It is shown that, in this extreme case, the dynamics of the model are mainly driven by the negative wealth effect. Since government consumption and private consumption are now substitutes, private consumption decreases more than in the benchmark case in Figure 2.9. Since the resources are the same, more reductions in private consumption enable households to increase their investment as a way to earn income. As a result, we observe decreases in private consumption and real wage but increases in labour supply, private investment and the interest rate, a result consistent with dominant negative wealth effect. Given these findings, output responds positively rather than negatively as in the benchmark case in Figure 2.9.

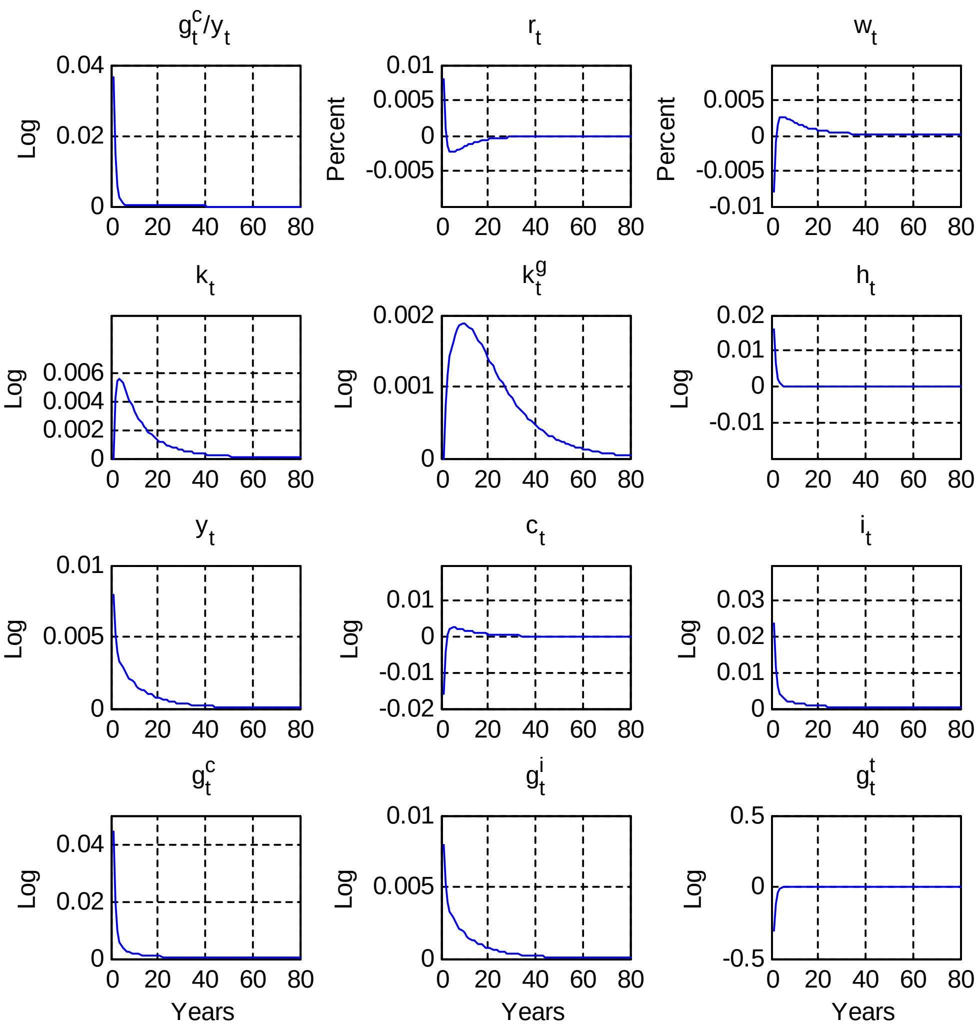

Figure 2.10: Impulse responses when $\frac{\mu_1}{(1-\mu_1-\mu_2)} = 0.5$ such that $(1-\mu_1-\mu_2) > \frac{1}{\sigma}$.

2.6.3 Government investment shock

Before we illustrate the effect of the shock to the government investment ratio to output, we briefly discuss two additional effects of the changes in government investment on the economy. These effects have been noted early in Baxter and King (1993) and have been well known in recent studies on public investment such as Leeper *et al.* (2009). The first effect is a direct productivity effect or positive wealth effect due to the increase in productive public capital in the production function. Given the accumulation rule of public capital, this effect is subject to a one year lag. The second effect is associated with the improvement in the marginal product of private capital and labour supply. This corresponds to the result that the interest rate and real wage increase and households gradually consume more, work more and invest less. Moreover, these effects can take impact even in the very short-run since agents have expectations today about the increase in future social wealth tomorrow.

The impulse responses to a one standard deviation shock to the ratio of government investment to output are reported in Figure 2.11 below. It is clearly seen that, compared with the government consumption ratio shock case, the government investment ratio shock induces a quite different model dynamics due to the two positive effects discussed above. For example, in the first year after the shock, the negative wealth effect of public spending still dominates, leading to reductions in private consumption and investment and an increase in labour supply (resulting in a small increase in output). However, as the public investment soon accumulates productive public capital from the second year, direct positive wealth effects are generated. People observe more wealth available today and have expectations for more wealth in near future and thus increase their consumption and investment (as shown in the same figure). Furthermore, the increase in public capital gradually raises the marginal product of private capital and labour, which leads to sustained increases in the quantity of private investment and labour supply. As a result, we observe a long-lived positive effect on private investment, labour supply and output growth. This is clearly in contrast with the model dynamics in the case of government consumption ratio shock in Figure 2.9. In the long-run, output gradually returns to its original level as do its components. Overall, the responses of variables to the government investment ratio shock are similar with the standard results found in developed countries (see e.g., Boxer and King (1993) and Leeper *et al.* (2009) for the U.S.). These positive impacts of the government investment ratio shock on the economy possibly explain the effectiveness of the stimulating fiscal policy by increasing government investment advocated by the Chinese government especially for the period 1996-2006.

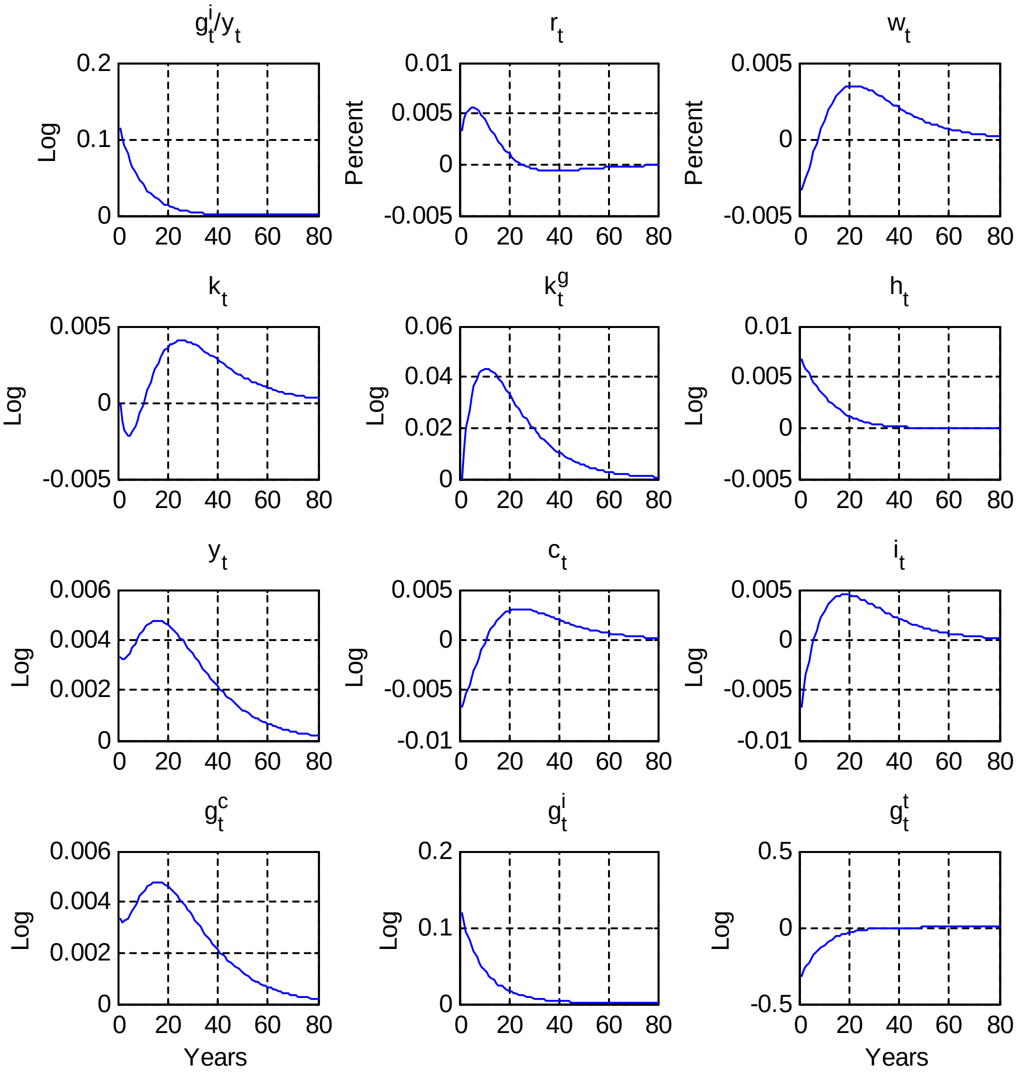

Figure 2.11: Impulse responses to government investment - output ratio shock

It is also helpful to see how the results will change when the public capital is unproductive. Figure 2.12 provides the impulse responses of variables under a counterfactual case that $\alpha_3 = 0$. It is shown that the positive wealth effect disappears. Instead, the negative wealth effect make the dynamics of the model similar with the government consumption ratio shock case in Figure 2.9.

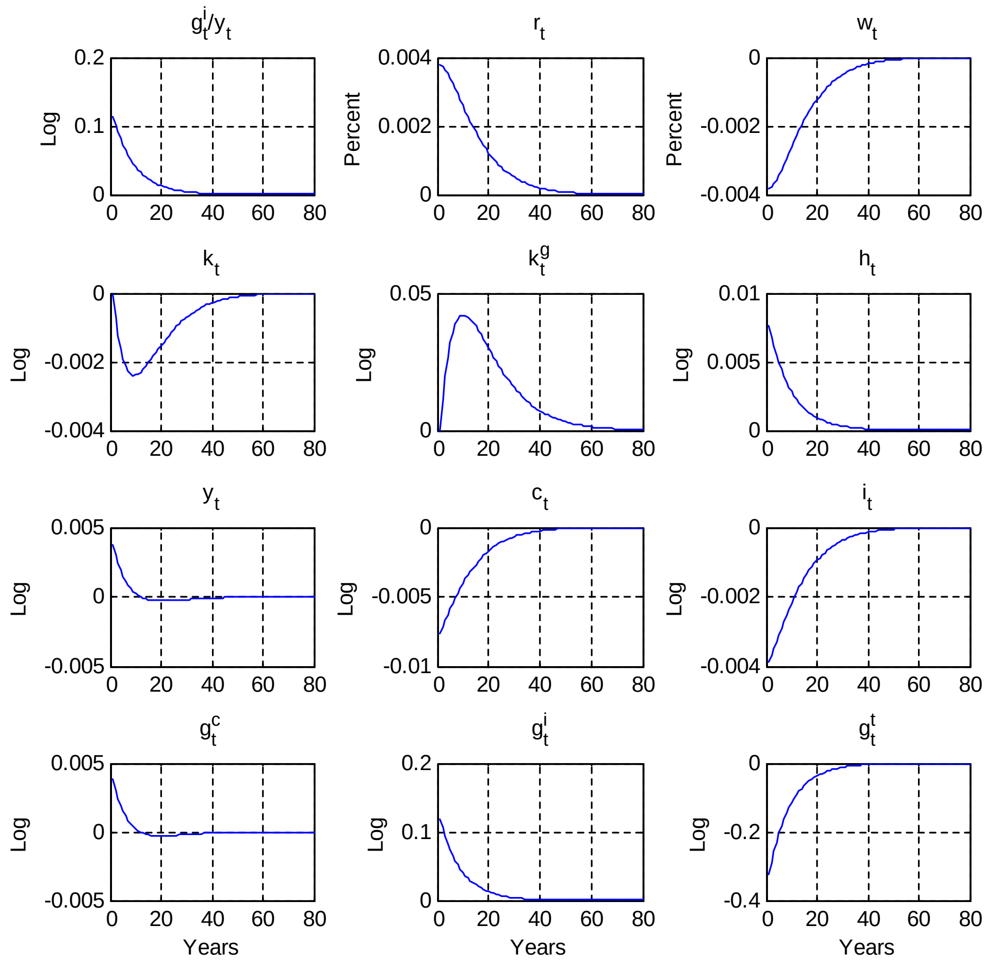

Figure 2.12: Impulse responses when public capital is un-producitive ($\alpha_3 = 0$).

2.6.4 Some caveats

The above fiscal policy analysis is based on the assumption that the government budget constraint is always balanced by government lump-sum residual policy instrument, i.e., government transfers. This assumption however, is only realistic for the pre-reform period and the early reform period in the 1980s. Before the fiscal reform in 1994, the spending of the government was strictly under central plan. In other words, the tax revenue is equal to the government spending, government consumption and government transfers. After the fiscal reform in 1994, difficulties emerged in balancing the budget of the government as a result of decentralization of decision making between local and central governments. The central planning was cancelled and more autonomy was given to local governments on spending. The bonds market was set up to enable the government to have debt to the economy. Reforms in tax system also introduced various types of taxes. Therefore, our assumption of lump-sum residual policy instrument is not realistic after the fiscal reform. To relax this assumption, different budget constraint and government behaviour need to be specified. For example, more distortionary taxes can be introduced into the model. It is also possible to derive optimal endogenous tax rates as well but this might be realistic since the fiscal policy in China is not fully based on consumption utility maximization. Adding financial variables such as government bonds in the government budget constraint is also necessary to better capture the government's attempt to reduce its deficit over time. These are possible extensions of the public sector in future work.

2.7 Concluding remarks

In light of the recent successful applications of the RBC model for developing countries and the empirical structural VAR study in Chapter 1, this chapter examined China's post-reform economic fluctuations using an RBC model. This theoretically coherent model enabled us to understand business cycles in a dynamic stochastic general equilibrium framework. The model incorporating exogenous fiscal policy was calibrated to annual Chinese data for the post-reform period 1978-2006. Its performance was evaluated by conducting a set of assessments including moment matching, simulation of business cycles, counterfactual experiment and a sensitivity check. The effects of economic shocks on the dynamics of the model were illustrated by analyzing the impulse response functions.

Our main findings are: (i) the model generates a reasonable overall account of the business cycles in the Chinese economy; (ii) TFP shocks mainly contribute to the good fit of the model, whilst the two fiscal policy shocks help to further improve the model's performance; (iii) our results are robust to alternative calibrations such as high and low capital shares, weights of components in utility and constant return to scale aggregate production function in public capital; (iv) shocks to the ratio of government consumption to output delivers a dominant negative wealth effect, whilst the shock to the ratio of government investment to output can generate significant positive wealth effects in both the short- and long-run.

Given the overall reasonable performance of the model and the dominant role of TFP, it is suggested that further investigation in productivity in China deserves future research. When the RBC model was advocated and accepted as the baseline for business cycle research, Prescott (1998) claimed that a theory of TFP is needed for developed countries. The same also applies to a developing country such as China.

The difference might be that, while the TFP shocks are mainly associated with technological progress in developed countries such as the U.S., the same for China might be more related with other factors such as institutional changes. For example, the actual labour market has been in reform through out our data sample. In fact, there are still barriers that prevent the free flow of labour forces from rural to urban area. For another, the nominal interest rate has been administratively controlled for a long time which prevents it from clearing the capital market. There are other related reforms of institutions as well such as the financial liberalization and stock market reform. Indeed some research has suggested that the poor quality of institutions has been one of the most important factors responsible for economic fluctuations, see, e.g. Acemoglu *et al.* (2003), Bergoeing *et al.* (2002a, 2002b) and Angelopoulos *et al.* (2010). Thus it appears that the analysis of issues related to the effects of evolving institutions on aggregate fluctuations in China would be a very worthwhile extension to the current research.

2.8 Appendices

2.8.1 Solving optimization problems

The Lagrangian for the household's utility maximization problem is:

$$L_t^h = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[\left(C_t^h\right)^{\mu_1} \left(1-h_t^h\right)^{\mu_2} \left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2} \right]^{1-\sigma}}{1-\sigma} \right. \\ \left. +\lambda_t \left[\begin{array}{c} \left(1-\tau^k\right) r_t K_t^h + \left(1-\tau^h\right) w_t h_t^h + \Pi_t^h + \overline{G}_t^t \\ -\left(1+\tau^c\right) C_t^h - K_{t+1}^h + (1-\delta) K_t^h \end{array} \right] \right\}. \tag{83}$$

The first-order condition (FOC) with respect to C_t^h is:

$$\frac{(1-\sigma)\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}\mu_1}{(1-\sigma)\,C_t^h} - \left(1+\tau^c\right)\lambda_t = 0$$

or

$$\lambda_t = \frac{\mu_1}{\left(1+\tau^c\right)} \frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{C_t^h}. \tag{84}$$

The FOC with respect to h_t^h is:

$$\frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}(1-\sigma)(-\mu_2)}{(1-\sigma)\left(1-h_t^h\right)} + \left(1-\tau^h\right) w_t \lambda_t = 0$$

or

$$w_t \lambda_t = \frac{\mu_2\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{\left(1-\tau^h\right)\left(1-h_t^h\right)}. \tag{85}$$

Since K_{t+1} appears in both time t and $t+1$, the FOC with respect to K_{t+1} is :

$$-\beta^t \lambda_t + E_0 \beta^{t+1} \lambda_{t+1} \left((1-\delta) + \left(1-\tau^k\right) r_{t+1}\right) = 0$$

or

$$\lambda_t = \beta E_0 \lambda_{t+1} \left(1 - \delta + \left(1-\tau^k\right) r_{t+1}\right). \tag{86}$$

To simplify the three first-order conditions, we lead (84) one period ahead and take expectations to obtain:

$$E_0 \lambda_{t+1} = \frac{\mu_1}{\left(1+\tau^c\right)} E_0 \frac{\left[\left(C_{t+1}^h\right)^{\mu_1}\left(1-h_{t+1}^h\right)^{\mu_2}\left(\overline{G}_{t+1}^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{C_{t+1}^h}. \tag{87}$$

Substituting λ_t in (84) into (85) gives,

$$w_t \frac{\mu_1}{\left(1+\tau^c\right)} \frac{\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{C_t^h} \\ = \frac{\mu_2\left[\left(C_t^h\right)^{\mu_1}\left(1-h_t^h\right)^{\mu_2}\left(\overline{G}_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{\left(1-\tau^h\right)\left(1-h_t^h\right)}$$

or

$$w_t = \frac{\mu_2 (1 + \tau^c) C_t^h}{\mu_1 (1 - \tau^h) (1 - h_t^h)} \tag{88}$$

which is the labour supply function of the household in the main text.

Similarly, substituting λ_t and λ_{t+1} in (84) and (87) into (86) gives,

$$\frac{\left[(C_t^h)^{\mu_1} (1 - h_t^h)^{\mu_2} (\overline{G}_t^c)^{1-\mu_1-\mu_2} \right]^{1-\sigma}}{C_t^h} =$$

$$\beta E_0 \left\{ \frac{\left[(C_{t+1}^h)^{\mu_1} (1 - h_{t+1}^h)^{\mu_2} (\overline{G}_{t+1}^c)^{1-\mu_1-\mu_2} \right]^{1-\sigma} \left(1 - \delta + (1 - \tau^k) r_{t+1}\right)}{C_{t+1}^h} \right\} \tag{89}$$

which is the Euler equation for household consumption in the main text.

For the firm's profit maximization problem, we first substitute the output, Y_t^f, in the production function (34) into the firm's profits (36) in the main text to obtain:

$$\Pi_t^f = A_t \left(K_t^f\right)^{\alpha_1} \left(h_t^f\right)^{1-\alpha_1} (\overline{K}_t^g)^{\alpha_2} - r_t K_t^f - w_t h_t^f. \tag{90}$$

The FOC with respect to K_t^f is:

$$\alpha_1 \left(K_t^f\right)^{\alpha_1 - 1} A_t \left(h_t^f\right)^{1-\alpha_1} (\overline{K}_t^g)^{\alpha_2} - r_t = 0$$

or

$$\begin{aligned} r_t &= \alpha_1 \left(K_t^f\right)^{\alpha_1 - 1} A_t \left(h_t^f\right)^{1-\alpha_1} (\overline{K}_t^g)^{\alpha_2} \\ &= \frac{\alpha_1 Y_t^f}{K_t^f} \end{aligned} \tag{91}$$

which is the return to capital stock in the main text.

The FOC with respect to h_t^f is:

$$(1 - \alpha_1) \left(h_t^f\right)^{-\alpha_1} A_t \left(K_t^f\right)^{\alpha_1} (\overline{K}_t^g)^{\alpha_2} - w_t = 0$$

or

$$\begin{aligned} w_t &= (1 - \alpha_1) A_t \left(K_t^f\right)^{\alpha_1} \left(h_t^f\right)^{-\alpha_1} (\overline{K}_t^g)^{\alpha_2} \\ &= \frac{(1 - \alpha_1) Y_t^f}{h_t^f} \end{aligned} \tag{92}$$

which is the return to labour supply in the main text.

Steady-state of the DCE We can suppress time-subscripts to derive the non-linear steady-state system:

$$w = \frac{\mu_2 (1+\tau^c) c}{\mu_1 (1-\tau^h)(1-h)} \tag{93}$$

$$1 = \beta \left(1 - \delta + \left(1-\tau^k\right) r\right) \tag{94}$$

$$i = \delta k \tag{95}$$

$$r = \frac{\alpha_1 y}{k} \tag{96}$$

$$w = \frac{(1-\alpha_1) y}{h} \tag{97}$$

$$y = a (k)^{\alpha_1} (h)^{1-\alpha_1} (k^g)^{\alpha_2} \tag{98}$$

$$g^i = \delta^g k^g \tag{99}$$

$$c + i + g^c + g^i = y \tag{100}$$

$$g^t = \tau^c c + \tau^k r k + \tau^h w h - g^c - g^i. \tag{101}$$

Exogenous processes:

$$a = a_0 \tag{102}$$

$$g^{cy} = g_0^{cy} \tag{103}$$

$$g^{iy} = g_0^{iy} \tag{104}$$

where

$$g^c = g^{cy} y \tag{105}$$

$$g^i = g^{iy} y. \tag{106}$$

Analytical solution:

$$a = a_0 \tag{107}$$

$$g^{cy} = g_0^{cy} \tag{108}$$

$$g^{iy} = g_0^{iy} \tag{109}$$

$$r = \frac{\frac{1}{\beta} - 1 + \delta}{(1-\tau^k)} \tag{110}$$

$$h = \frac{\mu_1 \left(1-\tau^h\right)(1-\alpha_1)}{\mu_2 (1+\tau^c)\left(1 - g^{cy} - g^{iy} - \frac{\delta \alpha_1}{r}\right) + \mu_1 (1-\tau^h)(1-\alpha_1)} \tag{111}$$

$$\begin{aligned} r &= \frac{\alpha_1 y}{k} \rightarrow k = \frac{\alpha_1 y}{r} \\ g^{iy} y &= \delta^g k^g \rightarrow k^g = \frac{g^{iy} y}{\delta^g}, \end{aligned}$$

substituting k and k^g above into the production function gives,

$$\begin{aligned}
y &= a\left(\frac{\alpha_1 y}{r}\right)^{\alpha_1}(h)^{1-\alpha_1}\left(\frac{g^{iy}y}{\delta^g}\right)^{\alpha_2} \\
y &= a\left(\frac{\alpha_1}{r}\right)^{\alpha_1}(y)^{\alpha_1}(h)^{1-\alpha_1}\left(\frac{g^{iy}}{\delta^g}\right)^{\alpha_2}(y)^{\alpha_2} \\
y^{1-\alpha_1-\alpha_2} &= a\left(\frac{\alpha_1}{r}\right)^{\alpha_1}(h)^{1-\alpha_1}\left(\frac{g^{iy}}{\delta^g}\right)^{\alpha_2} \\
y &= \left[a\left(\frac{\alpha_1}{r}\right)^{\alpha_1}(h)^{1-\alpha_1}\left(\frac{g^{iy}}{\delta^g}\right)^{\alpha_2}\right]^{\frac{1}{1-\alpha_1-\alpha_2}}. \qquad (112)
\end{aligned}$$

Other variables can be derived as:

$$k = \frac{\alpha_1 y}{r} \qquad (113)$$

$$i = \delta k \qquad (114)$$

$$w = \frac{(1-\alpha_1)\,y}{h} \qquad (115)$$

$$g^c = g^{cy}y \qquad (116)$$

$$g^i = g^{iy}y \qquad (117)$$

$$k^g = \frac{g^i}{\delta^g} \qquad (118)$$

$$c = y - i - g^c - g^i \qquad (119)$$

$$g^t = \tau^c c + \tau^k rk + \tau^h wh - g^c - g^i. \qquad (120)$$

2.8.3 Model solution

Log-linearization

$$\begin{aligned}
w_t &= \frac{\mu_2(1+\tau^c)\,c_t}{\mu_1(1-\tau^h)(1-h_t)} \\
\ln(w_t) &= \ln\mu_2 + \ln(1+\tau^c) + \ln(c_t) \\
&\quad - \ln\mu_1 - \ln\left(1-\tau^h\right) - \ln(1-h_t) \\
\frac{d\ln(w_t)}{dt} &= \frac{d\left[\ln(c_t) - \ln(1-h_t)\right]}{dt} \\
-\frac{1}{w}\frac{dw_t}{dt} &= \frac{1}{c}\frac{dc_t}{dt} + \frac{1}{1-h}\frac{dh_t}{dt} \\
-\widehat{w}_t &= \widehat{c}_t + \frac{h}{1-h}\widehat{h}_t \qquad (121)
\end{aligned}$$

$$\frac{\left[(c_t)^{\mu_1}(1-h_t)^{\mu_2}\left(g_t^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}}{c_t} =$$

$$\beta E_0\left\{\frac{\left[(c_{t+1})^{\mu_1}(1-h_{t+1})^{\mu_2}\left(g_{t+1}^c\right)^{1-\mu_1-\mu_2}\right]^{1-\sigma}\left(1-\delta+\left(1-\tau^k\right)r_{t+1}\right)}{c_{t+1}}\right\}$$

$$\mu_{1(1-\sigma)-1}\ln(c_t)+(1-\sigma)\mu_2\ln(1-h_t)+(1-\sigma)(1-\mu_1-\mu_2)\ln\left(g_t^c\right)$$

$$=\ln\beta+E_0\left\{\begin{array}{c}\mu_1((1-\sigma)-1)\ln(c_{t+1})+(1-\sigma)\mu_2\ln(1-h_{t+1})+\\(1-\sigma)(1-\mu_1-\mu_2)\ln\left(g_{t+1}^c\right)+\ln\left(1-\delta+\left(1-\tau^k\right)r_{t+1}\right)\end{array}\right\}$$

$$\mu_{1(1-\sigma)-1}\frac{d\ln(c_t)}{dt}+(1-\sigma)\mu_2\frac{d\ln(1-h_t)}{dt}+(1-\sigma)(1-\mu_1-\mu_2)\frac{d\ln\left(g_t^c\right)}{dt}$$

$$=E_0\left\{\begin{array}{c}\mu_{1(1-\sigma)-1}\frac{d\ln(c_{t+1})}{dt}+(1-\sigma)\mu_2\frac{d\ln(1-h_{t+1})}{dt}+\\(1-\sigma)(1-\mu_1-\mu_2)\frac{d\ln\left(g_{t+1}^c\right)}{dt}+\frac{d\ln\left(1-\delta+\left(1-\tau^k\right)r_{t+1}\right)}{dt}\end{array}\right\}$$

$$\mu_{1(1-\sigma)-1}\widehat{c}_t+(1-\sigma)\mu_2\frac{h}{1-h}\widehat{h}_t+(1-\sigma)(1-\mu_1-\mu_2)\widehat{g}_t^c$$

$$=E_0\left\{\begin{array}{c}\mu_{1(1-\sigma)-1}\widehat{c}_{t+1}+(1-\sigma)\mu_2\frac{h}{1-h}\widehat{h}_{t+1}+(1-\sigma)(1-\mu_1-\mu_2)\widehat{g}_{t+1}^c\\+\left(\frac{1-\tau^k}{1-\delta+\left(1-\tau^k\right)r_{t+1}}\right)\widehat{r}_{t+1}\end{array}\right\} \tag{122}$$

$$\begin{aligned}
i_t &= k_{t+1}-(1-\delta)k_t\\
\ln(i_t) &= \ln(k_{t+1}-(1-\delta)k_t)\\
\frac{d\ln(i_t)}{dt} &= \frac{d\ln(k_{t+1}-(1-\delta)k_t)}{dt}\\
\frac{1}{i}\frac{di_t}{dt} &= \frac{1}{i}\frac{d(k_{t+1}-(1-\delta)k_t)}{dt}\\
\widehat{i}_t &= \frac{k}{i}\widehat{k}_{t+1}-\frac{k}{i}(1-\delta)\widehat{k}_t
\end{aligned} \tag{123}$$

$$\begin{aligned}
g_t^i &= k_{t+1}^g-(1-\delta^g)k_t^g\\
\widehat{g}_t^i &= \frac{k}{g^i}\widehat{k}_{t+1}^g-\frac{k}{g^i}(1-\delta^g)\widehat{k}_t^g
\end{aligned} \tag{124}$$

$$\begin{aligned}
r_t &= \frac{\alpha_1 y_t}{k_t}\\
\widehat{r}_t &= \widehat{y}_t-\widehat{k}_t
\end{aligned} \tag{125}$$

$$\begin{aligned}
w_t &= \frac{(1-\alpha_1)y_t}{h_t}\\
\widehat{w}_t &= \widehat{y}_t-\widehat{h}_t
\end{aligned} \tag{126}$$

$$\begin{aligned}
y_t &= a_t(k_t)^{\alpha_1}(h_t)^{1-\alpha_1}\left(k_t^g\right)^{\alpha_2}\\
\widehat{y}_t &= \widehat{a}_t+\alpha_1\widehat{k}_t+(1-\alpha_1)\widehat{h}_t+\alpha_2\widehat{k}_t^g
\end{aligned} \tag{127}$$

$$\begin{aligned}
c_t + i_t + g_t^c + g_t^i &= y_t \\
\ln\left(c_t + i_t + g_t^c + g_t^i\right) &= \ln\left(y_t\right) \\
\frac{1}{y}\frac{d\left(c_t + i_t + g_t^c + g_t^i\right)}{dt} &= \frac{1}{y}\frac{dy_t}{dt} \\
\frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{g^c}{y}\widehat{g}_t^c + \frac{g^i}{y}\widehat{g}_t^i &= \widehat{y}_t \qquad (128)
\end{aligned}$$

$$\begin{aligned}
g_t^t &= \tau^c c_t + \tau^k r_t k_t + \tau^h w_t h_t - g_t^c - g_t^i \\
\ln\left(g_t^t\right) &= \ln\left(\tau^c c_t + \tau^k r_t k_t + \tau^h w_t h_t - g_t^c - g_t^i\right) \\
\widehat{g}_t^t &= \frac{\tau^c c}{g^t} c_t + \frac{\tau^k r k}{g^t}\left(\widehat{r}_t + \widehat{k}_t\right) + \frac{\tau^h w h}{g^t}\left(\widehat{w}_t + \widehat{h}_t\right) \\
&\quad - \frac{g^c}{g^t}\widehat{g}_t^c - \frac{g^i}{g^t}\widehat{g}_t^i. \qquad (129)
\end{aligned}$$

Exogenous processes:

$$E_0 \widehat{a}_{t+1} = \rho_a \widehat{a}_t \qquad (130)$$

$$E_0 \widehat{g}_{t+1}^{cy} = \rho_{g^c} \widehat{g}_t^{cy} \qquad (131)$$

$$E_0 \widehat{g}_{t+1}^{iy} = \rho_{g^c} \widehat{g}_t^{iy} \qquad (132)$$

where

$$\widehat{g}_t^c = \widehat{g}_t^{cy} + \widehat{y}_t \qquad (133)$$

$$\widehat{g}_t^i = \widehat{g}_t^{iy} + \widehat{y}_t. \qquad (134)$$

Solve the linearized system Rewrite the linearized system in first-order form:

$$-\widehat{w}_t = \widehat{c}_t + \frac{h}{1-h}\widehat{h}_t \qquad (135)$$

$$\begin{aligned}
& E_0 \left\{ \begin{array}{c} \mu_{1(1-\sigma)-1}\widehat{c}_{t+1} + (1-\sigma)\,\mu_2 \frac{h}{1-h}\widehat{h}_{t+1} + \\ (1-\sigma)\left(1-\mu_1-\mu_2\right)\widehat{g}_{t+1}^c + \left(\frac{1-\tau^k}{1-\delta+\left(1-\tau^k\right) r_{t+1}}\right)\widehat{r}_{t+1} \end{array} \right\} \\
= \; & \mu_{1(1-\sigma)-1}\widehat{c}_t + (1-\sigma)\,\mu_2 \frac{h}{1-h}\widehat{h}_t + (1-\sigma)\left(1-\mu_1-\mu_2\right)\widehat{g}_t^c \qquad (136)
\end{aligned}$$

$$\widehat{i}_t = \frac{k}{i}\widehat{k}_{t+1} - \frac{k}{i}\left(1-\delta\right)\widehat{k}_t \qquad (137)$$

$$\widehat{g}_t^i = \frac{k}{g^i}\widehat{k}_{t+1}^g - \frac{k}{g^i}\left(1-\delta^g\right)\widehat{k}_t^g \qquad (138)$$

$$\widehat{r}_t = \widehat{y}_t - \widehat{k}_t \qquad (139)$$

$$\widehat{w}_t = \widehat{y}_t - \widehat{h}_t \qquad (140)$$

$$\widehat{y}_t = \widehat{a}_t + \alpha_1 \widehat{k}_t + \left(1-\alpha_1\right)\widehat{h}_t + \alpha_2 \widehat{k}_t^g \qquad (141)$$

$$\frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{g^c}{y}\widehat{g}^c_t + \frac{g^i}{y}\widehat{g}^i_t = \widehat{y}_t \tag{142}$$

$$\widehat{\Pi}_t = \widehat{y}_t \tag{143}$$

$$\widehat{g}^t_t = \frac{\tau^c c}{g^t}c_t + \frac{\tau^k rk}{g^t}\left(\widehat{r}_t + \widehat{k}_t\right) + \frac{\tau^h wh}{g^t}\left(\widehat{w}_t + \widehat{h}_t\right) - \frac{g^c}{g^t}\widehat{g}^c_t - \frac{g^i}{g^t}\widehat{g}^i_t \tag{144}$$

$$\widehat{g}^c_t = \widehat{g}^{cy}_t + \widehat{y}_t \tag{145}$$

$$\widehat{g}^i_t = \widehat{g}^{iy}_t + \widehat{y}_t. \tag{146}$$

$$E_0\widehat{a}_{t+1} = \rho_a\widehat{a}_t \tag{147}$$

$$E_0\widehat{g}^{cy}_{t+1} = \rho_{g^c}\widehat{g}^{cy}_t \tag{148}$$

$$E_0\widehat{g}^{iy}_{t+1} = \rho_{g^c}\widehat{g}^{iy}_t \tag{149}$$

Matrix form:

$$\mathbf{A}\mathbf{X}_{t+1} = \mathbf{B}\mathbf{X}_t \tag{150}$$

where $X = \left[\widehat{k}_t,\ \widehat{k}^g_t,\ \widehat{a}_t,\ \widehat{g}^{cy}_t,\ \widehat{g}^{iy}_t,\ \widehat{c}_t,\ \widehat{h}_t,\ \widehat{w}_t,\ \widehat{r}_t,\ \widehat{i}_t,\ \widehat{y}_t,\ \widehat{g}^c_t,\ \widehat{g}^i_t,\ \widehat{g}^t_t\right]'$ is the 14×1 matrix which contains the variables of the model. Klein (2000) shows that the above system can be solved using the *generalized* Schur or 'QZ' decomposition on the coefficient matrix **A** and **B**. The Matlab function to solve the above linear system is *solab.m*[70]. The format of this function is: $[\mathbf{F}, \mathbf{P}] = solab\,(\mathbf{A}, \mathbf{B}, nk)$. The inputs are matrices **A**, **B** defined above and $nk = 5$ which is the number of state variables. The outputs are the coefficient matrices **P** and **F** which solve the linearized system so that the predetermined and non-predetermined variables can be represented in the following form:

$$\widehat{\mathbf{X}}^p_t = \mathbf{P}\widehat{\mathbf{X}}^p_t \tag{151}$$

$$\widehat{\mathbf{X}}^f_t = \mathbf{F}\widehat{\mathbf{X}}^f_t \tag{152}$$

where $\widehat{\mathbf{X}}^p_t$ includes the predetermined variables $\widehat{k}_t$, $\widehat{k}^g_t$, $\widehat{a}_t$, $\widehat{g}^{cy}_t$ and $\widehat{g}^{iy}_t$, and $\widehat{\mathbf{X}}^f_t$ consists of the non-predetermined endogenous variables $\widehat{c}_t$, $\widehat{h}_t$, $\widehat{w}_t$, $\widehat{r}_t$, $\widehat{i}_t$, $\widehat{y}_t$, $\widehat{g}^c_t$ and $\widehat{g}^i_t$.

Simulate the model To simulate the model, one requires a sequence of normally distributed disturbances, $\{\boldsymbol{\varepsilon}_t\}_{t=0}^T$ for the three exogenous shocks with sample size T, the initial values of the endogenous predetermined variables, k_0, k^g_0, and the evolution

[70] Note that, to use *solab.m* properly, one needs to write the variables in **X** in an order in which the predetermined variables $\left[\widehat{k}_t,\ \widehat{k}^g_t,\ \widehat{a}_t,\ \widehat{g}^{cy}_t,\ \widehat{g}^{iy}_t\right]$ come first.

of the endogenous non-predetermined variables in the form of the model solution given in (151) and (152). The simulation is conducted using the following form:

$$\widehat{\mathbf{X}}^p_{t+1} = \mathbf{P}\widehat{\mathbf{X}}^p_t + \mathbf{S}\boldsymbol{\varepsilon}_{t+1} \tag{153}$$

$$\widehat{\mathbf{X}}^f_t = \mathbf{F}\widehat{\mathbf{X}}^p_t + \mathbf{D}\boldsymbol{\varepsilon}_t \tag{154}$$

where $\widehat{\mathbf{X}}^f$, $\widehat{\mathbf{X}}^p$, $\mathbf{F}$, $\mathbf{P}$ have been defined before. In our case, the matrix $\mathbf{S} =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $\mathbf{D}$ is just a 15×5 zero matrix. Based on the above system, the function *dlsim.m* can be used in simulating the model in Matlab. It takes the form: $[\widehat{\mathbf{X}}^f, \widehat{\mathbf{X}}^p]$=**dlsim**($\mathbf{P}$, $\mathbf{S}$, $\mathbf{F}$, $\mathbf{D}$, **ee**), where the inputs and outputs have been defined above.

2.8.4 Impulse response function

The calculation of impulse responses (IRs) using the linear solution is straight forward. It can be derived using the above first-order form in (153) and (154) as well. The function *dimpulse.m* written by Little (1985) can be used for generating IRs in Matlab. The format of *dimpulse.m* is [**en**, **enp**] = *dimpulse*($\mathbf{P}$, $\mathbf{S}$, $\mathbf{F}$, $\mathbf{D}$, *shock*, *hor*) where the inputs $\mathbf{P}, \mathbf{S}, \mathbf{F}, \mathbf{D}$ have been defined above. The input *shock* selects the kind of shock which hits the economy and the input *hor* chooses the time horizon. The output matrix **en** contains the IRs of the jump variables defined in $\widehat{\mathbf{X}}^f$ and the output matrix **enp** contains the IRs of the predetermined variables defined in $\widehat{\mathbf{X}}^p$.

2.8.5 Adjustment of the employment data

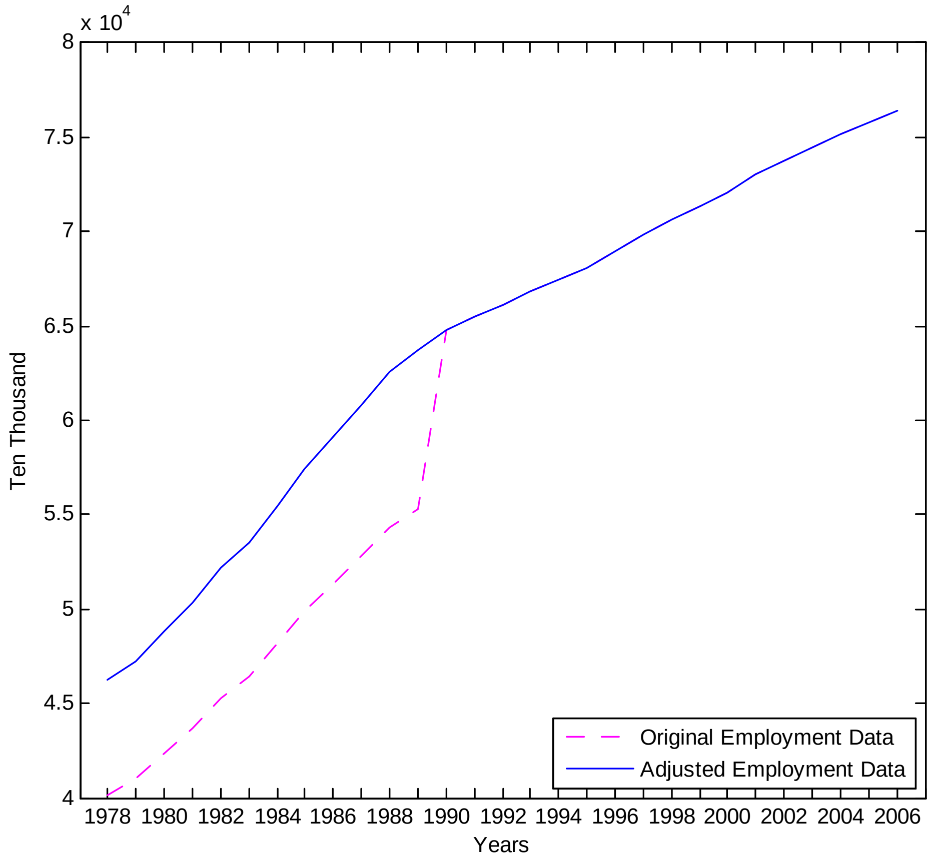

Figure 2.13: Adjustment of the employment data

2.8.6 Moments comparison from the counterfactual experiment using actual shocks

Table 2.11: Moments of the model and data: TFP shocks

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	4.25	3.20	1.00	1.00	0.63	0.68	1.00	1.00
C	1.98	2.68	0.47	0.84	0.67	0.53	0.68	0.74
I	8.32	8.55	1.96	2.67	0.51	0.45	0.41	0.77
K	4.04	3.22	0.95	1.01	0.51	0.64	0.46	0.37
L	2.17	0.66	0.51	0.21	0.42	0.83	0.89	0.03
w	2.50	4.34	0.59	1.36	0.73	0.61	0.92	0.35
r	4.35	1.42	1.02	0.44	0.71	0.55	0.82	0.31

Table 2.12: Moments of the model and data: TFP + g_t^{cy} shocks

	Standard deviation		Relative standard deviation		First-order autocorrelation		Contemporaneous correlation with Y	
	Model	Data	Model	Data	Model	Data	Model	Data
Y	3.59	3.20	1.00	1.00	0.68	0.68	1.00	1.00
C	2.29	2.68	0.64	0.84	0.65	0.53	0.82	0.74
I	6.89	8.55	1.92	2.67	0.70	0.45	0.95	0.77
K	3.78	3.22	1.05	1.01	0.62	0.64	0.14	0.37
L	1.43	0.66	0.40	0.21	0.62	0.83	0.79	0.03
w	2.60	4.34	0.72	1.36	0.67	0.61	0.94	0.35
r	4.73	1.42	1.32	0.44	0.71	0.55	0.83	0.31

Chapter 3: Comparing alternative approximation methods for solving the benchmark New Keynesian model

3.1 Introduction

The New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model[71] (see for example, Goodfriend and King (1997), Woodford (2003) and Galí (2008)) which features imperfect competition and nominal rigidities has been widely used in policy analysis in recent years. One important issue related with such analysis is to solve the policy function which governs the optimal behavior of agents in terms of their decision rules. Since this policy function is nonlinear, a popular approach in the literature is to obtain solutions via local approximation methods[72], i.e., the linear method and the second order approximation method. A question with this approach is then whether these approximation methods provide an accurate solution of the model. It can be argued that, both linear and second-order approximation methods are based on certain assumptions of the model, e.g., the certainty equivalence imposed by linear solutions and that, the model does not deviate much too often from its stationary steady-state. The approximation error might become non-negligible when these assumptions do not hold. Since the accuracy of the solution to the model is crucial for understanding the dynamics of the model, it should be examined before using the model. This chapter examines the accuracy of the approximation methods used in solving the NK model by comparing them with a non-linear method, i.e., the parameterized expectations algorithm (PEA).

Comparisons between different linear and non-linear solution methods have been rich. The first attempt was made by Taylor and Uhlig (1990) who compared ten solution methods (e.g., linear-quadratic method of Kydland and Prescott (1982), backward solving method of Sims (1984, 1989), Euler equation grid of Coleman (1990)) in solving the stochastic optimal growth model as in Brock and Mirman (1972). Their results show substantial differences across solution methods but no recommendation was made on how to choose among methods. Christiano (1990) assessed the performances of the

[71] This model is also referred as 'New Neo-Classical Synthesis' (NNCS) or 'New New-Keynesian Synthesis' (NNKS).

[72] For the first order approximation of the policy function in the NK model, see for example, Clarida, Gali and Gertler (1999) and Woodford (2003). Recently, some other researchers, see for example, Judd and Guu (1997), Sims (2000, 2002), Collard and Juillard (2001) and Schmitt-Grohe and Uribe (2004), have also applied a second-order approximation to the policy function to better capture the decision rules of agents.

basic log-linear method and the value function iteration and found that the two methods provided similar answers in terms of the growth model. The log-linear method was contrasted again with PEA in solving the growth model with human capital by Baranano, Iza and Vazquez (2002). They concluded that PEA captures more cross correlations of variables when the technology shocks are strong. Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2003) conducted a numerical experiment where the stochastic growth model is solved using linear and log-linear methods, finite elements method, second and fifth order perturbation methods and projection methods. Their results show that higher order perturbation methods, finite elements method and projection method provide superior solutions than linear and log-linear methods. Similar result was also found by Pichler (2005) who showed significant improvement of accuracy by moving from linear to non-linear methods, i.e., the perturbation method, PEA and the projection method. However, the work done by Novales and Perez (2004) and Heer and Maussner (2004) which solved the standard business cycles model using log-linear, extended path, value function iteration, PEA and projection methods, argued that log-linear method gives solutions as good as PEA and Projection method.

Despite the above comprehensive research on solution comparisons, in all cases they have only been conducted using the simple optimal growth model. None of them has examined the accuracy of solutions in the context of the NK-DSGE model. Moreover, most of them, especially the early work, do not provide a strict accuracy check. To fill the gap and overcome the limitations in the literature, this study will proceed as follows. First, this study starts with specifying the basic NK model in Walsh (2003), i.e., the simple model without capital evolution, money demand and fiscal expenditure. Sticking with the basic model helps deliver a benchmark case in which the performance of solution methods can be compared. The only deviation from the basic model is that we add a cost-push shock in the price-adjustment to allow a trade-off between the stabilization of the output gap and inflation. Second, the NK model is solved by applying the linear approximation, the second order approximation and the PEA. Third, we conduct a set of comparisons of the dynamics of the model generated using the three different solution methods. This includes the plots of the derived decision rules, moments and distributions and the impulse response functions. To provide an evaluation of the solution accuracy, we compare the Euler equation error generated by different methods using the technique of Judd (1992) and a statistical test proposed by Den Haan and Marcet (1994). Fourth, we do the same accuracy evaluations using alternative values of the structural parameters. For example, we also consider the cases of non-zero steady-state inflation, high risk aversion of households, a large degree of firm market power, high nominal stickiness in price setting and large uncertainty of shocks. These exercises can be seen as a robustness examination of the results obtained in the benchmark case.

The results are highlighted as follows. First, the three solution methods demonstrate quantitatively small differences in simulated data for all variables except for the price dispersion. For example, the calculated population moments and sample distributions show significant differences in the simulated price dispersion but not in other variables in the model. Moreover, the solution generated model dynamics display more differences for all variables. For example, the plotted policy functions generated by the three solutions display various differences. Also, the impulse response functions of output gap, nominal interest rate and price dispersion differ considerably. In particular, the impulse responses of price dispersion from PEA solution are shown to be significantly different from linear and second-order approximations for all three exogenous shocks. Second, the Judd measure and Marcet's J-test show similar results that PEA is more accurate than the linear and second-order solution methods. The biggest difference of accuracy emerges in solving the price-adjustment equation. The Judd measures between linear and second-order approximations are close. Third, the differences across solution methods become larger when alternative calibrations are considered. For example, non-zero steady-state inflation, higher degree of nominal rigidity, risk aversion of households, market power of firms and volatility of technology enlarge differences in summary statistics and accuracy evaluations. However, the PEA continues to perform quite well in all alternative calibrations. Therefore, the main result from benchmark model that PEA is more accurate is robust.

The main contribution of this research is that, it shows, for the first time, how linear and non-linear methods are compared in terms of solving the NK model. According to our results, the PEA algorithm delivers more reliable solutions compared with the linear and second order approximation methods. This result is robust to a number of alternative parameterization of the NK model. Therefore, it is necessary to examine the accuracy of solving the NK model and choosing between alternative solution methods before using the model. Also, more comparisons in terms of welfare can also be conducted in future research.

The remainder of the chapter is organized as follows: Section 3.2 describes the benchmark NK model with imperfect competition, nominal rigidity and a monetary policy rule. The three solution methods are discussed in Section 3.2, and then the benchmark NK model is solved using a linear solution method, second-order approximation and the PEA algorithm in section 3.3. Section 3.4 compares the simulated data from three solution methods in terms of the model generated moments and impulse response functions. Section 3.5 illustrates how the performances of the three solution methods can be compared in terms of accuracy evaluations using the Judd's criteria and the Marcet's J test. then Section 3.6 concludes.

3.2 The benchmark NK model

This section sets out the structure of the benchmark NK model in Woodford (2003) and Walsh (2003)[73]. The model features a decentralized economy with a large population of representative households, imperfectly competitive firms and a central bank. Microfoundations have been well developed both for demand and supply sides according to households' and firms' optimal behavior. Particularly, the elasticity of substitution between goods leads to market power of firms who set their prices in the manner described in Calvo (1983). This then gives rise to nominal rigidity in the price level. The implementation of monetary policy is not by means of controlling aggregate money but by setting the nominal interest rate. A simple Taylor rule is introduced to close the model. This benchmark model is designed for short-run analysis, thus there is no capital evolution or investment in the model. Additionally, this model assumes complete financial market and flexible wages.

The only deviation of this study from the benchmark model is the inclusion of a cost-push shock. This remedies the difficulty that, there is no trade-off between stabilizing output and stabilizing inflation (see for example, Clarida, Galí and Gertler (1999) and Walsh (2003)) when supply side shock is absent. The model structure and agents' optimization problems are illustrated below.

3.2.1 Households

Cost minimization The NK economy is populated by a large number of identical households indexed by j. Each of the households consumes a basket of goods[74],

[73] For earlier work, see Yun (1996), Goodfriend and King (1997) and Rotemberg and Woodford (1997), which combine nominal rigidity in a dynamic stochastic general equilibrium framework with microfoundations.

[74] Note that many authors assume there is a final good producer who combines the intermediate good c_{jt} to make the final good c_t. Therefore, this CES basket is also the demand function of the final good producer for an individual good j.

$$c_t = \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \tag{155}$$

where c_t is the consumption of household at time t and c_{jt} represents an individual good produced by firm j, and $\int$ is the integral operator over all goods in the household's basket in the continuum $j \in [0,1]$. The above basket is in the so-called CES (constant elasticity of substitution or Dixit-Stiglitz CES) form, and $\theta > 1$ is the constant price elasticity of individual goods.

Prior to utility maximization, the household considers a cost minimization problem. That is, the household has to minimize the cost of purchasing the consumption basket by choosing the combination of individual goods.

The cost minimization problem of the household is then given by

$$\begin{aligned} &\underset{c_{jt}}{\text{Min}} \int_0^1 p_{jt} c_{jt} dj \\ \text{subject to } c_t \; &= \; \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \end{aligned}$$

where p_{jt} is the price of an individual good produced by firm j, and $\int_0^1 p_{jt} c_{jt} dj$ represents the cost of buying all individual goods in the consumption basket (155).

The first-order condition (FOC) with respect to c_{jt} gives the demand function of household for good j:

$$c_{jt} = \left[\frac{p_{jt}}{P_t} \right]^{-\theta} c_t \tag{156}$$

where P_t is the aggregate price index[75],

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{157}$$

Equation (156) shows that the household chooses the level of consumption of an individual good j based on the price relative to all other goods given the demand elasticity θ[76]. This implies the demand function for good j.

Utility maximization The household derives utility from consumption of the basket of goods and suffers dis-utility from supplying labour n_t. If the utility of the household is time-separable, the objective function can be written in discrete time as:

[75] Equation (157) is obtained by setting final good producer's proft to zero, as a result of perfect competition in final good market.

[76] As $\theta \to \infty$, we move towards perfect competition.

$$E_t \sum_{i=0}^{\infty} \beta^i U\left(c_{t+i}, n_{t+i}\right)$$

where E_t is the conditional expectations operator, $0 < \beta < 1$ is the subjective rate of time preference, and $i \in [0, \infty]$ is the time horizon. If we further assume the utility function is CRRA (constant rate of risk aversion) both in consumption and labor supply, then the objective function of the household becomes,

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{c_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{e_t n_{t+i}^{1+\eta}}{1+\eta} \right\} \tag{158}$$

where $\sigma^{-1}, \eta^{-1} > 0$ are the the intertemporal rates of substitution of consumption and labour supply respectively, σ also represents the degree of risk aversion. As defined above, c_{t+i} is a basket of goods the household chooses at time $t+i$ and n_{t+i} is the labor supply of the household at time $t+i$. The cost-push shock, e_t, affects the marginal rate of substitution between consumption and leisure and the real wage. Therefore, e_t alters the wage markup of firms (see for example, Clarida, Galí and Gertler (1999 and 2001))[77].

At each period, the household consumes a basket of goods and buys bond. The income of the household in each period comes from the labour income (wages), the income from the financial asset B (growing at a rate of nominal interest rate from last period) and a profit it received from firms. Therefore, the budget constraint of the household can be written in real terms as:

$$c_t + \frac{B_t}{P_t} = \frac{W_t n_t}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + \Pi_t \tag{159}$$

where P_t is the price level at time t, $\frac{B_t}{P_t}$ represents the real bonds holding of the household at time t, R_{t-1} is the gross nominal rate of return of bonds last period, W_t is nominal wage at time t, and Π_t is the real profits from the imperfectly competitive firms redistributed to the household. The left side of the budget constraint is the total spending at time t, while the right side is the net income flow at time t. Note that this simple budget constraint takes the assumption that the financial market is complete, the household can borrow whatever it needs from the financial market to finance its consumption in finite horizon.

[77] For other ways of introducing cost-push shocks, see for example, Steinsson (2003) which makes the price elasticity θ stochastic so that it affects the firm's mark up. However, it can be shown that this does not make much difference in causing policy trade-offs between output stabilization and inflation stabilization.

Combining the FOC with respect to c_t, n_t and B_t yields the intertemporal consumption of household,

$$c_t^{-\sigma} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} \right) c_{t+1}^{-\sigma} \tag{160}$$

and the intertemporal labor supply of household,

$$\frac{\chi e_t n_t^{\eta}}{c_t^{-\sigma}} = \frac{W_t}{P_t}. \tag{161}$$

The Euler Equation for consumption (160) shows that consumption increases when expected real interest rate is bigger than the time preference, *vice versa*. This can bee seen by noting that $\beta = \frac{1}{1+\rho}$ with ρ as the discount rate and inflation $\frac{P_t}{P_{t+1}} = \frac{1}{\pi_{t+1}}$. Then the Euler equation can be re-written as $c_t^{-\sigma} = E_t \frac{1}{1+\rho} \left[\frac{R_t}{\pi_{t+1}} \right] c_{t+1}^{-\sigma} = E_t \left(\frac{1+r_t}{1+\rho} \right) c_{t+1}^{-\sigma}$, where $r_t = \frac{R_t}{\pi_{t+1}} - 1$ is the real interest rate.

The labour supply function of household (161) shows that the labour supply is increasing in the real wage ($\frac{W_t}{P_t}$) because the household becomes more willing to work when facing higher real wages. But labour supply is falling in consumption, as the level of consumption increases, the household becomes more willing to substitute consumption with more leisure/less working hours.

3.2.2 Firms

There are a large number of imperfectly competitive intermediate goods producing firms who hire labour (supplied by households) to produce an individual good y_{jt} using the following technology:

$$y_{jt} = z_t n_{jt} \tag{162}$$

where n_{jt} is labour used by the firm j at time t and z_t is the technology which is the same for all firms. Additionally, we assume a first-order auto-regressive process for the technology as

$$z_{t+1} = z_t^{\rho_z} e^{\varepsilon_{t+1}}, \ 0 < \rho_z < 1, \ \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$$

which in natural log form is,

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{t+1}. \tag{163}$$

The objective of firm is to minimize the cost of production and maximize the expected profits over time, which will be illustrated below.

Cost minimization Each intermediate firm faces a cost minimization problem of the following form:

$$Min\left(\frac{W_t}{P_t}\right)n_{jt}+\varphi_t\left(y_{jt}-z_tn_{jt}\right)$$

where φ_t represents the firm's real marginal cost. The FOC with respect to n_{jt} gives,

$$\frac{W_t}{P_t}-\varphi_t z_t=0$$

$$\Rightarrow\varphi_t=\frac{W_t/P_t}{z_t} \tag{164}$$

which implies the marginal cost of each firm is equal to the real wage over technology. Note this marginal cost function is common for all firms.

Profit maximization The intermediate goods market is assumed to be imperfectly competitive and firms have market power to set their prices. However, a key feature in the NK model is that they are not allowed to change prices in each time period. More specifically, the type of 'Calvo contract' used assumes that in any period, only a constant proportion of $(1-\omega)$ of firms are able to change their prices at random time intervals, while the remaining proportion of ω firms keep their prices unchanged. Therefore, ω captures the degree of price rigidity.

Two aspects of "Calvo Contract" are noteworthy. First, those firms who are able to change prices set the same price. This is because they have the same constraints (i.e., same demand curve of household and technology). Second, although the newly set price at each time period is the same, the proportion $(1-\omega)$ of firms is randomly chosen. This implies varieties of existing prices at each time period.

In a NK setup with 'Calvo Contract', firm chooses an optimal price, p^*_{jt}, for the product it produces at time t to maximize discounted future profits. Since only $(1-\omega)$ of firms are able to reset their prices at each period, when they reset prices at time t, they must accept the fact that the price they set today might remain the same in subsequent periods at probability ω^i. Therefore, the firm's objective function in discrete representation is:

$$E_t\sum_{i=0}^{\infty}\omega^i\Delta_{i,t+i}\left[\left(\frac{p^*_{jt}}{P_{t+i}}\right)y_{j,t+i}-\varphi_{t+i}y_{j,t+i}\right] \tag{165}$$

where p_{jt} is the optimal price chosen by the firm at time t, and $\Delta_{i,t+i}$ is the stochastic discount factor.

The constraints for the profit maximization problem are those given by technology described in (162) and the demand curve in (156). Substituting these into (165) gives the price-setting firm's objective function in terms of the aggregate price level P_t, aggregate consumption y_{t+i} and its own optimal price p_{jt}:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_t^*}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\theta} \right] y_{t+i}.$$

The reason for removing the subscript j for p_{jt}^* and using P_t^* is that those firms who are able to change prices set the same price, implying $p_{jt}^* = P_t^*$.

The FOC with respect to P_t^*, after some substitution (see Appendix (3.7.1) for details), is:

$$\frac{P_t^*}{P_t} = \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}} \tag{166}$$

which reveals the price setting behavior of firms with price rigidity. The firm has to take into account the expected future marginal cost and the price level to choose an optimal price today to maximize discounted future profits. If we allow all firms to change their prices every period, i.e., $\omega = 0$, price rigidity is eliminated, equation (166) simplifies to:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \varphi_t \tag{167}$$

which is the same result as in standard imperfectly competitive models, where $\left(\frac{\theta}{\theta-1}\right)$ is the firm's mark-up over marginal cost.

The price-setting equation (166) contains infinite sums and is not in aggregate terms. To facilitate the PEA solution later, it is convenient to split it into two parts to obtain a recursive form as follows (see appendix (3.7.1) for details):

$$\frac{P_t^*}{P_t} = \frac{p_t^N}{p_t^D} \tag{168}$$

$$p_t^N = \left(\frac{\theta}{\theta - 1}\right) (c_t)^{-\sigma} y_t \varphi_t + \omega \beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N \tag{169}$$

$$p_t^D = (c_t)^{-\sigma} y_t + \omega \beta E_t (\pi_{t+1})^{\theta-1} p_{t+1}^D. \tag{170}$$

3.2.3 Evolution of price level

Recall that the price level is given by,

$$P_t = \left[\int_0^1 P_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

and is composed of both surviving contracts and new prices. Suppose the surviving contract at time $t-i$ is p_{t-i}, then the probability that p_{t-i} survives at time t is ω^i. Therefore the probability that p_{t-i} survives from time $t-i$ but ends at the end of period t is $(1-\omega)\omega^i$. Further note that, the proportion of firms resetting their prices is always randomly chosen among all firms, which means that the aggregate price level is just the average of all surviving contracts occurring at probability $(1-\omega)\omega^i$, i.e.,

$$P_t = \left[\sum_{i=0}^{\infty}(1-\omega)\omega^i(P_{t-i}^*)^{1-\theta}di\right]^{\frac{1}{1-\theta}}.$$

In recursive form, the previous expression can be written as (see Appendix (3.7.1) for details),

$$P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}, \qquad 0<\omega<1$$

which shows that the price level, P_t, is equal to the weighted average of past price level P_{t-1} with probability ω (those who do not change their prices) and newly set price p_t^* with probability $(1-\omega)$ (those who optimally choose new prices). Deflating above result gives,

$$1 = (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1} \qquad 0<\omega<1. \tag{171}$$

3.2.4 Monetary policy rule

The interest rate rule used in this research is a Taylor rule with nominal interest rate responds to changes in current inflation and output gap:

$$\widehat{R}_t = \delta_\pi\widehat{\pi}_t + \delta_x\widehat{x}_t + \widehat{v}_t \tag{172}$$

where the monetary policy shock follows an AR(1) process,

$$\ln v_{t+1} = \rho_v \ln v_t + \varepsilon_{v,t+1},\ \varepsilon_{v,t+1} \sim N(0,\sigma_{\varepsilon_v}^2). \tag{173}$$

To see the implication of this Taylor rule, note that output gap in our model is defined as (See Appendix (3.7.2)),

$$\widehat{x}_t = \widehat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\widehat{z}_t + \left(\frac{1}{\sigma+\eta}\right)\widehat{e}_t \tag{174}$$

substituting this result into the Taylor rule (172) gives

$$\widehat{R}_t = \delta_\pi\widehat{\pi}_t + \delta_x\left(\widehat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\widehat{z}_t + \left(\frac{1}{\sigma+\eta}\right)\widehat{e}_t\right) + \widehat{v}_t \tag{175}$$

which indicates that the central bank needs to know not only information relating to changes in output, inflation and past interest rate, but also realizations (and therefore distributions) of the driving forces of the economy over time.

Also, the interest rate rule (175) in exponential form can be written as (see Appendix (3.7.1) for details),

$$R_t = R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}\left\{\left(\frac{y_t}{y}\right)\left(\frac{z_t}{z}\right)^{-m_z}\left(\frac{e_t}{e}\right)^{m_e}\right\}^{\delta_x}\left(\frac{v_t}{v}\right) \tag{176}$$

where

$$\begin{aligned} m_z &= \left(\frac{1+\eta}{\sigma+\eta}\right) \\ m_e &= \left(\frac{1}{\sigma+\eta}\right) \end{aligned}$$

and R and π are the steady-state of nominal interest rate and inflation respectively.

3.2.5 Price dispersion

So far, the production function, $y_{jt} = z_t n_{jt}$, is for an individual firm. We next aggregate it in terms of all goods $j \in [0, 1]$,

$$\int_0^1 y_{jt} dj = \int_0^1 z_t n_{jt} dj.$$

The right hand side of the above equation is just $z_t n_t$ since there is no imperfection in the labor market. However, we have the problem that $\int_0^1 y_{jt} dj \neq Y_t$. We then insert the demand function $y_{jt} = \left[\frac{p_{jt}}{P_t}\right]^{-\theta} y_t$ into the integral to obtain

$$\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} y_t dj = \int_0^1 z_t n_{jt} dj.$$

Rearranging the above expression gives,

$$\left(\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} dj\right) y_t = z_t n_t \tag{177}$$

where the part $\left(\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} dj\right)$ is shown to be the price dispersion in this NK model due to relative prices caused by the price stickiness. As shown in literature (see for example, Woodford (2003), Ch 6), price dispersion can be represented as a first-order autoregressive process[78],

$$pd_t = (1-\omega)\left(p_t^*\right)^{-\theta} + \omega\pi^\theta pd_{t-1}. \tag{178}$$

To transform this price dispersion in first-order form, we next add a link variable

$$zp_{t+1} = pd_t \tag{179}$$

[78] See the Appendix (3.7.1) for derivation of price dispersion process.

so that the price dispersion can be rewritten as

$$pd_t = (1-\omega)(p_t^*)^{-\theta} + \omega\pi^{\theta} zp_t. \tag{180}$$

Also, the aggregate production function (177) now becomes

$$pd_t y_t = z_t n_t. \tag{181}$$

3.2.6 System of non-linear equilibrium conditions

Since there is no capital evolution (and thus investment) in the model, aggregate demand is just $y_t = c_t$[79]. Also, we deflate the economy by the price level P_t, for example, $w_t = \frac{W_t}{P_t}$ and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$. Therefore, the non-linear system of the benchmark NK model can be written as:

$$\frac{\chi e_t n_t^{\eta}}{c_t^{-\sigma}} = w_t \tag{182}$$

$$c_t^{-\sigma} = \beta R_t E_t(\frac{1}{\pi_{t+1}})c_{t+1}^{-\sigma} \tag{183}$$

$$\varphi_t = \frac{w_t}{z_t} \tag{184}$$

$$p_t^* = \frac{p_t^N}{p_t^D} \tag{185}$$

$$p_t^N = \left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t \varphi_t + \omega\beta E_t(\pi_{t+1})^{\theta} p_{t+1}^N \tag{186}$$

$$p_t^D = (c_t)^{-\sigma} y_t + \omega\beta E_t(\pi_{t+1})^{\theta-1} p_{t+1}^D \tag{187}$$

$$1 = (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1} \tag{188}$$

$$pd_t = (1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^{\theta} zp_t \tag{189}$$

$$zp_{t+1} = pd_t \tag{190}$$

$$pd_t y_t = z_t n_t \tag{191}$$

$$y_t = c_t \tag{192}$$

$$R_t = R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}(x)^{\delta_x}\left(\frac{v_t}{v}\right) \tag{193}$$

$$x_t = \left(\frac{y_t}{y}\right)\left(\frac{z_t}{z}\right)^{-m_z}\left(\frac{e_t}{e}\right)^{m_e} \tag{194}$$

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{z,t+1},\ \varepsilon_{z,t+1} \sim N(0,\sigma_{\varepsilon_z}^2) \tag{195}$$

$$\ln(v_{t+1}) = \rho_v \ln(v_t) + \varepsilon_{v,t+1},\ \varepsilon_{v,t+1} \sim N(0,\sigma_{\varepsilon_v}^2) \tag{196}$$

$$\ln(e_{t+1}) = \rho_e \ln(e_t) + \varepsilon_{e,t+1},\ \varepsilon_{e,t+1} \sim N(0,\sigma_{\varepsilon_e}^2). \tag{197}$$

[79]It can also be verified that by substituting the firm's profit function (165) into the household's budget constraint (159), the aggregate demand function $y_t = c_t$ holds.

3.3 Solution methods

The non-linear system in section 3.2.6 can be expressed more succinctly as:

$$F\left\{E_t\left[H\left(Y_{t+1}, X_{t+1}, Y_t, X_t\right)\right], Y_t, X_t, Z_t\right\} = 0 \tag{198}$$

where E_t is the conditional expectations operator, Y, X and Z are the matrices that consist of the non-predetermined (endogenous) variables $\{x$, n, c, w, R, π, φ, p^*, p^N, p^D, $pd\}$, the predetermined variable $\{zp\}$[80] and the exogenous variables $\{z, v, e\}$ respectively. The function H describes the model and depends on the variables of which we take expectations (i.e. those transformed by the function H) and those not involving expectations.

As will be shown below, various linear and non-linear approximating methods have been developed for solving the model, with the former approximating the policy function around a particular equilibrium and the latter finding a fixed point in a contraction algorithm. This section shows how to solve the above standard NK model using three popular methods: the log-linear method, the second-order approximation method and the PEA algorithm. The rationale of each method is briefly explained and the application of each method to solve the NK model is illustrated.

3.3.1 Linear approximation method

The linear approximation method assumes that the model economy does not deviate far away very often from a particular equilibrium. The model can then be represented around this equilibrium up to first-order accuracy. The first sort of linear methods is due to the work of Kydland and Prescott (1982) which approximates the policy function by a linear quadratic function. Since linearizing a DSGE model is always convenient and costless, methods of solving linearized expectational model are then developed by Blanchard and Kahn (1980), Uhlig (1999), Klein (2000) and Sims (2002). Uhlig's method is a general log-linearization method that solves the linear policy function by undetermined coefficients. Blanchard and Kahn's method, Klein's method and Sims' method find the solution by decomposing the coefficient matrix of the linear first-order autoregressive expectational system. This section chooses Klein's method to solve the linearized model since it is more general[81] than other linear methods.

[80] Since the initial value of pd is given, we treat it as a predetermined variable in the NK model.

[81] For example, Klein's method works fine with complex eigenvalues of the coefficient matrix while the other two methods do not.

First, the non-linear model in (198) is log-linearized around the steady-state equilibrium[82]. Defining a 'hat' above a variable as the percentage deviation from its steady-state value, leaving the technical details in Appendix (3.7.2), we derive the log-linearized model as follows:

$$\eta\widehat{n}_t + \widehat{e}_t = -\sigma\widehat{c}_t + \widehat{w}_t \tag{199}$$

$$-\sigma\widehat{c}_t = \widehat{R}_t - \sigma E_t\widehat{c}_{t+1} - E_t\widehat{\pi}_{t+1} \tag{200}$$

$$\widehat{\varphi}_t = \widehat{w}_t - \widehat{z}_t \tag{201}$$

$$\widehat{p}_t^* = \widehat{p}_t^N - \widehat{p}_t^D \tag{202}$$

$$\widehat{p}_t^N = \left(1 - \omega\beta\pi^\theta\right)\left(-\sigma\widehat{c}_t + \widehat{y}_t + \widehat{\varphi}_t\right) + \omega\beta E_t\left(\pi\right)^\theta\left(\theta\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^N\right) \tag{203}$$

$$\widehat{p}_t^D = \left(1 - \omega\beta\pi^{\theta-1}\right)\left(-\sigma\widehat{c}_t + \widehat{y}_t\right) + \omega\beta E_t\left(\pi\right)^{\theta-1}\left(\left(\theta - 1\right)\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^D\right) \tag{204}$$

$$(1 - \omega\pi^{\theta-1})\widehat{p}_t^* = \omega\pi^{\theta-1}\widehat{\pi}_t \tag{205}$$

$$pd\widehat{pd}_t = -\theta(1-\omega)\left(p^*\right)^{-\theta}\widehat{p}_t^* + \omega\pi^\theta zp\left(\theta\widehat{\pi}_t + \widehat{zp}_t\right) \tag{206}$$

$$\widehat{zp}_{t+1} = \widehat{pd}_t \tag{207}$$

$$\widehat{pd}_t + \widehat{y}_t = \widehat{z}_t + \widehat{n}_t \tag{208}$$

$$\widehat{y}_t = \widehat{c}_t \tag{209}$$

$$\widehat{R}_t = \delta_\pi\widehat{\pi}_t + \delta_x\widehat{x}_t + \widehat{v}_t \tag{210}$$

$$\widehat{x}_t = \widehat{y}_t - m_z\widehat{z}_t + m_e\widehat{e}_t \tag{211}$$

$$E_t\widehat{v}_{t+1} = \rho_v\widehat{v}_t \tag{212}$$

$$E_t\left(\widehat{z}_{t+1}\right) = \rho_z\left(\widehat{z}_t\right) \tag{213}$$

$$E_t\left(\widehat{e}_{t+1}\right) = \rho_e\left(\widehat{e}_t\right) \tag{214}$$

The second step is to apply the Klein's method to solve this linear system. Klein (2000) shows that above linearized system can be transformed to a first-order autoregressive system of the form[83]

$$\mathbf{A}\mathbf{E}_t\widehat{\mathbf{X}}_{t+1} = \mathbf{B}\widehat{\mathbf{X}}_t + \mathbf{C}\widehat{\mathbf{Z}}_t. \tag{215}$$

[82] Additionally, we impose no restriction on stead-state value inflation so that it can be $\pi = 1$ or $\pi > 1$.

[83] See Appendix (3.7.2) for how the New Keynesian model in matrix form can be transformed to this first-order system.

The variables in $\widehat{\mathbf{X}}_{t+1}$ can then be solved by applying a generalized Schur decomposition to the coefficients matrix A, B and C[84]. Finally the solution of the whole system takes the following state space form:

$$\begin{aligned}
\widehat{X}_{t+1}^{b} &= P\widehat{X}_{t}^{b} + Q\widehat{Z}_{t} \\
\widehat{Z}_{t+1} &= \rho\widehat{Z}_{t} + \varepsilon_{t+1} \\
\widehat{X}_{t}^{f} &= M\widehat{X}_{t}^{b} + N\widehat{Z}_{t} \\
\widehat{Y}_{t} &= S\widehat{X}_{t}^{b} + T\widehat{Z}_{t}
\end{aligned}$$

where the matrices P, Q, M, N represents the approximate policy function which describes the decision rule of agents. This solution is recursive in that the non-predetermined variables $\widehat{X}_{t}^{f}$ (e.g., consumption) and the one period ahead predetermined variables X_{t+1}^{b} (e.g., price dispersion at time $t+1$) are determined by the predetermined variables X_{t}^{b} (e.g., price dispersion at time t) and the exogenous variables $\widehat{Z}_{t}$ (e.g., technology, monetary policy and cost-push shocks). Finally the static variables of interest[85] Y_t are represented by predetermined variables X_{t}^{b} and the exogenous variables $\widehat{Z}_{t}$.

3.3.2 Second-order approximation method

The second-order approximation is also referred as the second-order perturbation method which was introduced in economic applications by Judd and Guu (1997). It is based on a second-order Taylor series expansion of the policy function around a particular equilibrium of the model (usually the steady-state, the same as in the log-linear method). Applications of the second-order approximation to DSGE models can be found in Sims (2000), Collard and Juillard (2001) and Schmitt-Grohe and Uribe (2004). Here we use the method of Schmitt-Grohe and Uribe (2004).

The second-order approximation assumes that the solution of the system in (198) is in the following form[86]:

$$\begin{aligned}
Y_t &= g\left(X_t, \sigma\right) \\
X_{t+1} &= h\left(X_t, \sigma\right) + \eta\sigma Z_{t+1}
\end{aligned} \tag{216}$$

where $g\left(X_t, \sigma\right)$ and $h\left(X_t, \sigma\right)$ are the policy functions of the endogenous non-predetermined and predetermined variables respectively. Substituting policy functions in (216) into

[84] See Klein (2000) for details of this decomposition and how to solve jump and predetermined variables seperately.

[85] These static variables are often the variables of interest in measurement equations. In our setup of the linear solution, we consider output, optimal price and interest rate as the static variables.

[86] The notation is taken from Schmitt-Grohe and Uribe (2004).

the original non-linear system (198) gives

$$F\left\{E_t\left[g\left(h\left(X_t,\sigma\right)+\eta\sigma Z_{t+1},\sigma\right),g\left(X_t,\sigma\right)\right],h\left(X_t,\sigma\right)+\eta\sigma Z_{t+1},X_t,Z_t\right\}=0 \qquad (217)$$

and then the task is to approximate these two policy functions $g\left(X_t,\sigma\right)$ and $h\left(X_t,\sigma\right)$ up to second-order around steady-state value of X and $\sigma=0$.

For a simple example in the NK model, the second-order approximation works, for example, for policy functions of consumption and price dispersion as follows:

$$\begin{aligned}
c_t &= g\left(x_t,\sigma\right)\simeq g\left(\overline{x},0\right)+\left[g_x\left(\overline{x},0\right)\left(x-\overline{x}\right)\right]_{zp,z,v,e}+\left[g_\sigma\left(\overline{x},0\right)\sigma\right]_{zp,z,v,e} \\
&+\frac{1}{2}\left[g_{xx}\left(\overline{x},0\right)\left(x-\overline{x}\right)\left(x-\overline{x}\right)\right]_{zp,z,v,e}+\frac{1}{2}\left[g_{x\sigma}\left(\overline{x},0\right)\left(x-\overline{x}\right)\sigma\right]_{zp,z,v,e} \\
&+\frac{1}{2}\left[g_{\sigma x}\left(\overline{x},0\right)\left(x-\overline{x}\right)\sigma\right]_{zp,z,v,e}+\frac{1}{2}\left[g_{\sigma\sigma}\left(\overline{x},0\right)\sigma^2\right] \qquad (218)
\end{aligned}$$

$$\begin{aligned}
zp_t &= h\left(x_t,\sigma\right)+\eta\sigma Z_{t+1}\simeq h\left(\overline{x},0\right)+\left[h_x\left(\overline{x},0\right)\left(x-\overline{x}\right)\right]_{zp,z,v,e}+\left[h_\sigma\left(\overline{x},0\right)\sigma\right]_{zp,z,v,e} \\
&+\frac{1}{2}\left[h_{xx}\left(\overline{x},0\right)\left(x-\overline{x}\right)\left(x-\overline{x}\right)\right]_{zp,z,v,e}+\frac{1}{2}\left[h_{x\sigma}\left(\overline{x},0\right)\left(x-\overline{x}\right)\sigma\right]_{zp,z,v,e} \\
&+\frac{1}{2}\left[h_{\sigma x}\left(\overline{x},0\right)\left(x-\overline{x}\right)\sigma\right]_{zp,z,v,e}+\frac{1}{2}\left[h_{\sigma\sigma}\left(\overline{x},0\right)\sigma^2\right]+\eta\sigma Z_{t+1} \qquad (219)
\end{aligned}$$

where the sub-scripts zp,z,v,e stand for (first or second) derivatives with respect to four predetermined variables zp,z,v,e respectively. It is easy to see that $g\left(\overline{x},0\right)=\overline{c}$ and $h\left(\overline{x},0\right)=\overline{zp}$. The first-order derivatives such as $g_x\left(\overline{x},0\right)$ and $h_x\left(\overline{x},0\right)$ are solved using linear algorithms such as Klein's method. For the second-order terms, Schmitt-Grohe and Uribe (2004) prove that the cross derivatives $g_{x\sigma}\left(\overline{x},0\right),g_{\sigma x}\left(\overline{x},0\right),h_{x\sigma}\left(\overline{x},0\right)$ and $h_{\sigma x}\left(\overline{x},0\right)$ are just zero. Moreover, to identify unique solutions for $g_{xx}\left(\overline{x},0\right),g_{\sigma\sigma}\left(\overline{x},0\right),h_{xx}\left(\overline{x},0\right)$ and $h_{\sigma\sigma}\left(\overline{x},0\right)$, it must be the case that $g_\sigma\left(\overline{x},0\right)=0$ and $h_\sigma\left(\overline{x},0\right)=0$. This greatly simplifies the calculation of the above problem.

3.3.3 Parameterized Expectations Algorithm

While there are at least more than ten non-linear methods developed in literature, three of them (besides the perturbation method) have gained the most popularity: The dynamic programming and value function iteration method (see e.g., Bellman (1957), Lucas, Stokey and Prescott (1989) and Judd and Solnick (1994)) finds the value function that satisfies the Bellman equation. The PEA algorithm (see e.g., Marcet (1988), Den Han and Marcet (1990), Marcet and Sargent (1992) and Marcet and Marshall (1994), Marcet and Lorenzoni (1998)) solves the expectation function in the Euler equation by a prior given polynomial function, then estimates the undetermined parameters in the polynomial by non-linear least squares regressions using simulated data. The projections/minimum weighted residual method (see e.g., Judd (1992) and McGrattan (1999)) is similar to PEA, but estimates the undetermined parameters using some orthogonality conditions rather than simulated data.

The study chooses PEA as our non-linear method to solve the NK model for its several advantages. First, the PEA algorithm is attractive for its ease of use for solving DSGE models and can be a good starting point for using non-linear solution methods. Second, the performance of PEA is also good. Marcet and Marshall (1994) argue that the PEA algorithm is accurate. Also, Marcet and Marshall (1992) suggest that the PEA algorithm is expected to converge for most DSGE models as long as the model to be solved does not have multiple equilibria and the initial condition is properly set. Third, as the following illustrates, the PEA algorithm can be interpreted as a learning algorithm which reveals the agents' decision rule by a rule-of-thumb. Fourth, the PEA algorithm has been extended in recent years to increase its practical applicability and speed. For example, Christiano and Fisher (2000) illustrate how to combine projection method with PEA to solve a economy with binding constraints. Their work also introduces many methods to speed up the PEA algorithm such as using a grid of state variable vector other than simulating times series data. To improve the convergence of PEA, Perez (2004) proposes a log-linear homotopy approach to provide better initial conditions. Also, Maliar and Maliar (2003) put moving bounds on certain endogenous variables. To reduce the computation time, Maliar and Maliar (2005) propose parameterization on labor function instead of consumption Euler equation. Creel (2008) shows that PEA can be parallelized to run on multiple CPUs to speed up computations. For other extensions, Maliar and Maliar (2005) incorporate the value function iteration method in the PEA framework which offers a more efficient alternative to the standard PEA algorithm.

Some drawbacks of the PEA and all other non-linear solutions still apply. For example, the PEA might have difficulty in converging to the solution if the initial guess is not good. Therefore, researchers usually use the guess from the linear solution as the initial condition of the PEA. Moreover, retaining full non-linearity of the model implies the computation time may be very slow for simulating models that do not have a closed form solution. This is actually the case in our NK model where all the endogenous variables can only be solved simultaneously. Finally, multicollinearity may also reduce the accuracy of PEA solution if higher order terms are used in approximating the polynomial function. Therefore, in this research, we only use first-order polynomials.

The idea of PEA is to replace the expectation function in the policy function of a dynamic model by a given polynomial function with undetermined coefficients. The unknown coefficients are then estimated by a econometric approach (i.e., a non-linear least squares regression) using simulated data. When an estimate is obtained, new data is simulated to yield another estimate of the coefficients. This algorithm ends when the old estimate and new estimate of the coefficients converge to a fixed point.

A general case can be illustrated using our non-linear system in (198):

$$F\left\{E_t\left[H\left(y_{t+1}, x_{t+1}, y_t, x_t\right)\right], y_t, x_t, \varepsilon_t\right\} = 0.$$

The basic idea of the PEA is to replace the unknown expectations function $E_t H\left(.\right)$ with a polynomial function $\Psi\left(x_t, b\right)$ such that the model can be approximated by

$$F\left\{\Psi\left(x_t, b\right), y_t, x_t, \varepsilon_t\right\} = 0. \tag{220}$$

With an initial guess of the polynomial function Ψ, the above expression can be used to simulate the model. Then using simulated data, the unknown parameters in the polynomial are estimated by minimizing the expectational difference:

$$\widehat{b} \in \min \frac{1}{T} \sum_{t=0}^{T} \left[E_t H\left(y_{t+1}, x_{t+1}, y_t, x_t\right) - \Psi\left(x_t, b\right)\right]^2 \tag{221}$$

which amounts to a non-linear least squares regression. Note that, in the NK model we have three polynomials to approximate the three expectation functions. So we will have three non-linear regressions and get three estimates namely $\widehat{b_1}$, $\widehat{b_2}$ and $\widehat{b_3}$.

A convergence scheme is needed to find the solution. For example, in each iteration the new estimates are updated using:

$$\chi^{i+1} = \psi\hat{\chi}^i + \left(1 - \psi\right)\chi^i \tag{222}$$

where $\hat{\chi}^i$ contains the new estimates $\widehat{b_1}$, $\widehat{b_2}$ and $\widehat{b_3}$ at iteration i and ψ is a smoothing parameter. For a less non-linear model with good initial condition, a higher value of ψ (such as values close to 1) saves computational time in reaching the convergence. On the other hand, if the model is highly non-linear or the initial condition is not good, a lower value of ψ helps convergence at a cost of more computational time. In practice, researchers might test several values for ψ in terms of both ensuring convergence and saving computational time.

The estimates at iteration $i+1$ is updated as a weighted sum of the old estimate χ^i and the new estimate $\hat{\chi}^i$. The algorithm stops when:

$$\left|\chi^{i+1} - \chi^i\right| < \eta \tag{223}$$

where $\eta = 1e - 6$ is a sufficiently small number chosen as a criterion of convergence. That is, we compare the new estimates with the old ones using a rule of thumb. If $|\chi^{i+1} - \chi^i| < \eta$ then stop and take $\hat{\chi}^i$ as the PEA solution. Otherwise, go back to step 2 and simulate data again.

While the implementation details of PEA to the NK model are provided in Appendix (3.7.4), several points are explained here. First, the initial condition used by PEA for the NK model is a second-order approximation. Linear solution also works as initial condition for benchmark calibration but it takes much more time for PEA to converge. Secondly, PEA uses a 20,500 sample size to simulate the model, with an initial draw of 500 series. This sample size is widely used in literature (see for example, Collard (2006)) to insure the quality of the final solution. Finally, the smoothing parameter used in the convergence scheme is 0.75 and the convergence criterion used is 1-6e.

3.4 Comparing characteristics of solutions

This section compares both quantitative and qualitative features of the NK model in terms of three solution methods. Specially, we will examine the differences in model generated moments and distributions as many authors would do in literature. Next, we focus on the policy function which is the core of solutions and compare decision rules of agents. And finally we will compare the impulse response functions which have both qualitative and quantitative implications about how the economy reacts according to different types of shocks.

3.4.1 Calibration

Given the fact that most properties of the model, e.g., the degree of non-linearity, the distance of model economy from its steady-state, are governed by the structural parameters, this study will consider various calibrations of the NK model to make the accuracy comparisons more useful. A benchmark model with calibrations widely used in literature and consistent with data is evaluated to provide a baseline. Then alternative calibrations especially related with more frictions, high non-linearity and uncertainties will also be considered as a robustness check of our numerical findings. The calibrations are illustrated below:

Model 1 (Benchmark model):

Most parameter values, for example, $\beta = 0.99$, $\sigma = 1$, $\eta = 1$, $\rho_v = 0.5$, $\sigma_z = \sigma_v = 0.01$, are taken from Walsh (2003) and Galí (2008). The price elasticity in the

CES production function (or the market power of firms) θ varies according to different studies[87], and here we take $\theta = 6$ as the benchmark case. The degree of price rigidity ω is set to be $2/3$ which is consistent with estimates of Blinder (1994), Galí and Gertler (2001), Smets and Wouters (2003) and Leith and Malley (2005) for US data. The elasticity of labor η^{-1} is set to be 1 which is taken from the RBC literature (see for example, Prescott (1986)). The persistence of technology ρ_z is set to be 0.95 which is consistent with empirical studies (see for example, Lansing (1998)). Since has been uncertainty of the process of the cost-push shock, its persistence and volatility are set the same as the policy shock. The parameter that determines the response of the central bank to inflation, δ_π, is set to be 3 to ensure the determinacy of the model. Finally, we set steady-state value of inflation to be $\overline{\pi} = 1$ and $\overline{\pi} = 1.03^{\frac{1}{4}}$. The latter is taken from the US quarterly inflation data.

Table 3.1: Calibration: Model 1 (Benchmark)

β	σ	θ	ω	η	δ_π	δ_x	ρ_z	ρ_v	ρ_e	σ_z	σ_v	σ_e	$\overline{\pi}$
0.99	1	6	2/3	1	3	0.5	0.95	0.5	0.5	0.01	0.01	0.01	1

Alternative calibrations:

This study will consider alternative values for the steady-state value of inflation, $\overline{\pi}$, the degree of price rigidity, ω, the risk aversion of households, σ, the market power of firms, θ and the size of technology shock[88], σ_z. These alternative calibrations of the NK model are summarized below in Table 2. The reason for considering positive steady-state inflation lies in the feature of the NK model that, zero inflation eliminates the cost-push shock in the log-linearized Phillips Curve and the price dispersion[89]. Our PEA exercises also show the increased difficulty in finding the equilibrium solution as in positive inflation case. The reason for varying risk aversion and volatility of technology shock is then that the curvature and uncertainty are expected to affect the performance of solution methods. These variations have been common considerations in literature

[87] See for example, Erceg *et al.* (2000) who set $\theta = 4$ and also check the robustness by setting $\theta = 21$. Gali *et al.* (2001) estimated a hybrid NK model conditional on $\theta = 11$.

[88] High volatility of policy shock and cost-push can also be considered. However, it can be argued that they enter the polynomial in the PEA in the same way as the technology shock, and as such, their effects on the performance of the PEA should be similar.

[89] See for example, discussions in Walsh (2003) that the cost-push shock does not appear in the right side of the log-linearized Phillips Curve if steady state inflation is zero. Therefore no real trade-offs between output gap and inflation. Also, zero inflation makes the log-linearized price dispersion zero. In other words, positive inflation gives an extra distortion to the model besides imperfect competition and price rigidity.

when comparing solution methods (see for example, Aruoba *et al.* (2006)).

Table 3.2: Alternative calibrations:

	ω	θ	σ	σ_z	$\overline{\pi}$
Model 2	2/3	6	1	0.01	$1.03^{\frac{1}{4}}$
Model 3	0.8	6	1	0.01	1
Model 4	0.8	6	1	0.01	$1.03^{\frac{1}{4}}$
Model 5	2/3	6	2	0.01	1
Model 6	2/3	6	2	0.01	$1.03^{\frac{1}{4}}$
Model 7	2/3	4	1	0.01	1
Model 8	2/3	4	1	0.01	$1.03^{\frac{1}{4}}$
Model 9	2/3	6	1	0.03	1
Model 10	2/3	6	1	0.03	$1.03^{\frac{1}{4}}$

3.4.2 Distributions and moments

One way to check the differences of solution methods is to compare the model generated data which conveys rich information about the dynamics of the endogenous variables. The characteristics of the data are summarized by their moments and distributions. The first four moments, i.e., sample mean, variance, skewness and kurtosis as well as median and mode are included. Skewness measures the degree of asymmetry in the shape of a distribution and kurtosis measures the shape of the 'peak' and 'tails'. For example, the negative Skewness means left-skewed and positive Skewness means right-skewed and a bigger value of kurtosis means a sharper peak and fatter tails. They are calculated by first simulating the model using Monte-Carlo and summarizing the vector of moments. Then the moments are evaluated by taking the average of the moment vector across simulations. The number of simulations and sample size for each simulation should be large enough, whereas in this study we use 1000 simulations with a sample size of 500 for each simulation. The calculation of the empirical distributions of variables is also based on simulations. Then the density functions are calculated using the normal kernel function method in Matlab where the densities are evaluated at 100 equally spaced points that cover the range of the simulated data. Finally, to make these comparisons meaningful, all the simulations use the same exogenous shocks.

The distributions of consumption, inflation, the nominal interest rate and price dispersion for the benchmark calibration are plotted in Figure 3.1. A simple comparison reveals that all three solutions generate distributions that follow log-normal distributions. This is not surprising since the logs of the exogenous variables are simulated using normally distributed innovations. Moreover, the shapes of distributions of the three approximation methods are very close for consumption, inflation and nominal interest rate. They are almost on top of each other. Nonetheless, differences are found in the distributions of the price dispersion. Under Model 1, the linear method only generates price dispersion values all equal to one while the second-order and the PEA generate values equal to and above one. We should not be surprised at the constant price dispersions generated using linear solution, since by log-linearization the hatted values of price dispersion are all zero. However, it is noteworthy that these qualitatively and quantitatively significant differences in simulated price dispersions only induce tiny differences in other variables. This may suggest that the role of price dispersion is quantitatively small in the dynamics of the model. In other words, although linear method makes bigger approximation error in solving for price dispersion, the error in solving for other variables overall might not be big.

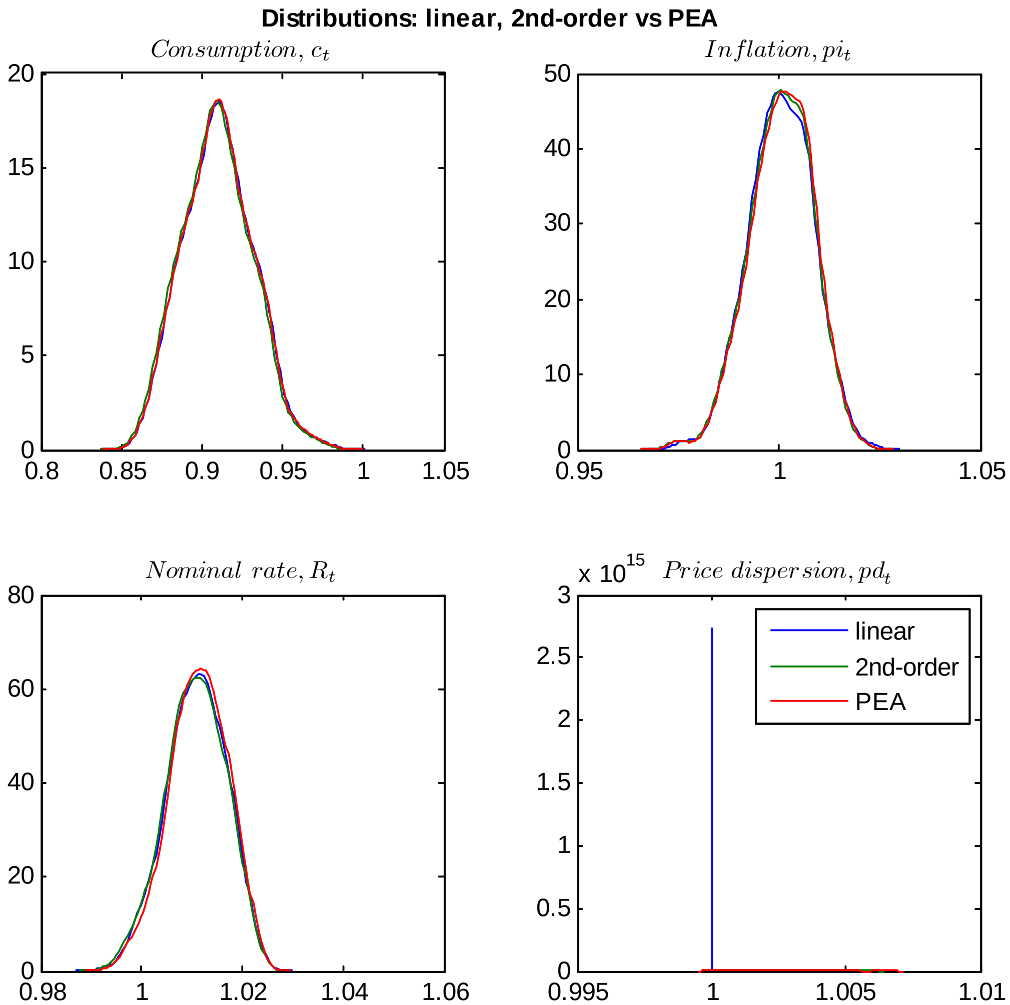

Figure 3.1: Distributions of key variables: Model 1 (benchmark model)

The same distributions for model 2 with non-zero steady-state inflation are plotted in Figure 3.2. Compared with model 1, the results are similar. All model generated distributions are very close except the price dispersion. The linear solution can also generate price dispersion around one. However, as acknowledged in literature (see for example, Damjanovic and Nolan (2006)), the price dispersion should be always equal to or greater than one. Therefore, linear solution makes large error in solving for price dispersion. However, again we may consider the differences in price dispersion as relatively trivial in affecting the model dynamics given that the distributions of other variables are very close. Another difference in distributions can be found that, linear solution generates less sharp distributions than the PEA and the second-order method.

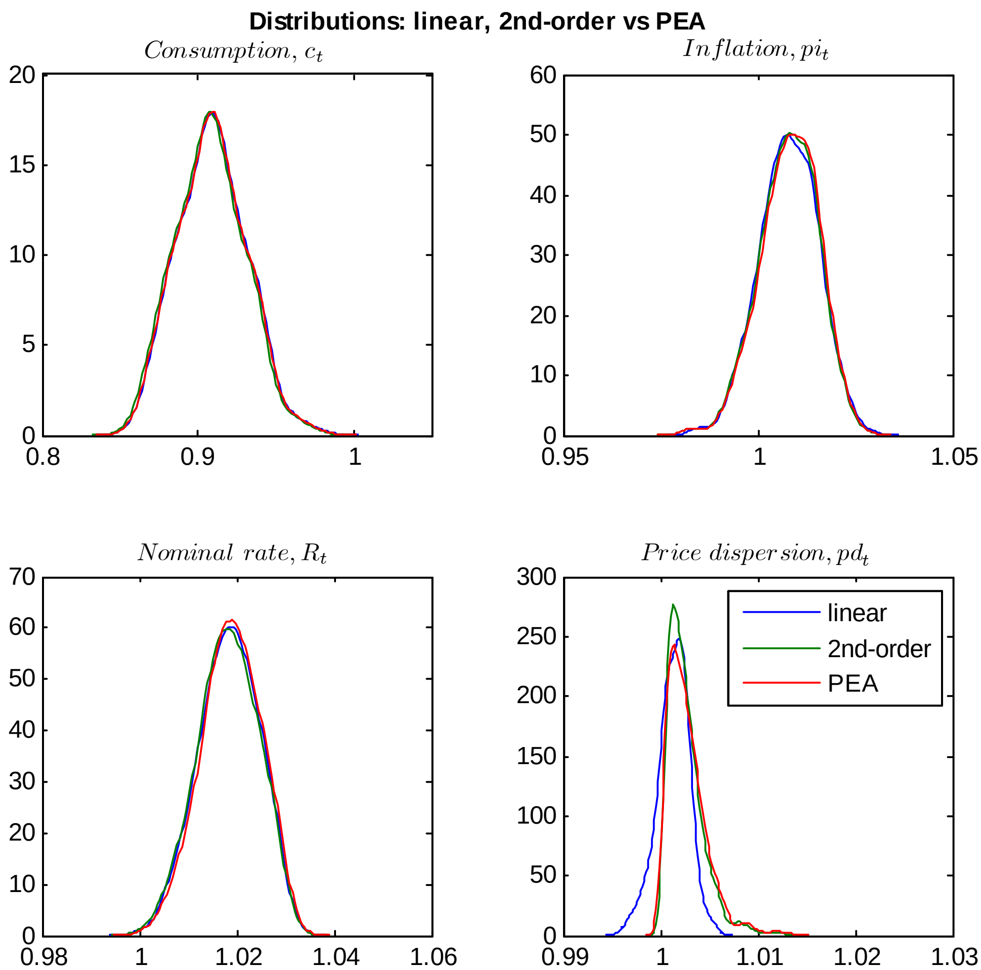

Figure 3.2: Distributions of key variables: model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

The moments generated from the three solutions under model 1 and 2 are reported in Table 3.3 and Table 3.4. Several points are noteworthy. First, under both model

1 and model 2, the three solution methods generate very close levels (as reflected by mean, median and mode) of consumption, inflation, nominal interest rate and the price dispersion. This again may be due to the fact that the limited importance of the price dispersion under the model setup leads to small quantitative differences in simulations of other variables of interest. Second, the volatility of the simulated data is very similar across the three solution methods. The only exception is that PEA and the second-order approximation generate more volatile price dispersions than the linear approximation. In fact, the linear solution fails to capture the variations in price dispersion especially under zero steady-state inflation. Third, the skewness and kurtosis of the three solutions are also different. However, the skewness and kurtosis from PEA and the second-order approximation are very close, whilst they differ significantly from the linear solution. This explains the similarity between the shape of distribution from PEA solution and the shape of distribution from the second-order solution. Particularly, there is no skewness and kurtosis for price dispersion from the linear solution under zero steady-state inflation due to the fact that the price dispersion is a constant. Fourth, simple t-tests (although not shown in this table) show that, under model 1, only the differences in Skewness and Kurtosis moments from second-order approximation and PEA are significant. On the other hand, more moments such as mode generated from the second-order approximation and PEA are significantly different under model 2.

Table 3.3: Moments Comparison: Model 1 (Benchmark Model)

Method	Consumption			Inflation			Nominal Rate			Price Dispersion		
	Lin	2nd	PEA	Lin	2nd	PEA	Lin	2nd	PEA	Lin	2nd	PEA
Mean	0.913	0.912	0.913	1.000	1.000	1.000	1.010	1.010	1.011	1.000	1.001	1.001
Median	0.913	0.911	0.913	1.000	1.000	1.001	1.010	1.010	1.011	1.000	1.001	1.001
Mode	0.853	0.851	0.852	0.977	0.975	0.975	0.993	0.992	0.993	1.000	1.000	1.000
Std	0.022	0.022	0.022	0.008	0.008	0.008	0.006	0.006	0.006	0.000	0.001	0.001
Max	0.977	0.975	0.977	1.023	1.021	1.022	1.028	1.028	1.028	1.000	1.006	1.006
Min	0.853	0.851	0.852	0.977	0.975	0.975	0.993	0.992	0.993	1.000	1.000	1.000
Skewness	0.064	0.039	0.053	0.025	-0.14	-0.16	0.017	0.050	0.034	*nan*	2.059	2.092
Kurtosis	2.961	2.955	2.958	2.988	3.009	3.008	2.953	2.965	2.963	*nan*	9.226	9.399

Table 3.4: Moments Comparison: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

Method	Consumption			Inflation			Nominal Rate			Price Dispersion		
	Lin	2nd	PEA	Lin	2nd	PEA	Lin	2nd	PEA	Lin	2nd	PEA
Mean	0.912	0.911	0.912	1.007	1.007	1.008	1.018	1.017	1.018	1.001	1.002	1.003
Median	0.912	0.911	0.912	1.007	1.008	1.008	1.018	1.017	1.018	1.001	1.002	1.002
Mode	0.850	0.847	0.849	0.985	0.983	0.983	0.999	0.999	1.000	0.997	1.000	1.000
Std	0.022	0.022	0.022	0.007	0.007	0.007	0.006	0.006	0.006	0.002	0.002	0.002
Max	0.979	0.976	0.978	1.030	1.028	1.028	1.036	1.037	1.037	1.006	1.012	1.013
Min	0.850	0.847	0.849	0.985	0.983	0.983	0.999	0.999	1.000	0.997	1.000	1.000
Skewness	0.066	0.031	0.054	0.023	−0.17	−0.20	0.019	0.073	0.046	0.002	1.682	1.612
Kurtosis	2.962	2.954	2.959	2.985	3.017	3.027	2.956	2.968	2.963	2.930	6.560	6.255

3.4.3 Policy functions

The policy function delivered by a solution is a function that reveals agents optimal behavior in terms of their decision rule. More explicitly, the policy function or the decision rule describes how agents forecast the endogenous variables as a function of the predetermined and exogenous variables. Therefore comparison of the decision rules provides a natural check on how three solutions imply different policy functions in the model and different dynamic behavior of agents. In our case, for example, the decision rules for consumption and price dispersion, relate c_t/zp_{t+1} to the state variables zp_t, z_t, v_t and e_t. In our analysis, we consider two kinds of decision rules. One is the stochastic decision rule which is obtained by simulating the model using one particular solution where zp starts from the steady-state and evolves along with the exogenous shocks z_t, v_t and e_t given random innovations $\varepsilon_{z,t+1}$, $\varepsilon_{v,t+1}$ and $\varepsilon_{e,t+1}$. The other one is the deterministic decision rule calculated by holding the exogenous shocks constant (e.g., steady-state) and choosing only a set of values zp around its steady-state. Therefore the deterministic decision rule captures the dynamics of the model when the uncertainty is absent.

The plots of deterministic decision rules for next period price dispersion, labor supply, consumption and inflation are shown in Figure 3.3 and 3.4. We summarize two main findings. First, in all cases, the deterministic decision rules for all the three solutions have the same slope. For example, the next period price dispersion, current labor supply and inflation are increasing in current price dispersion while the *vice versa* for consumption. The signs of these slopes confirm some findings in literature (see for example, Damjanovic and Nolan (2006)). Second, the deterministic decision rules vary mainly in magnitudes across three solutions. For example, given the same level of price dispersion, the PEA solution generates higher levels of labor and inflation and medium level of consumption under Model 1. Under Model 2, the PEA solution generates higher levels of labor and consumption, lower level of inflation and medium level of next period price dispersion. These differences are consistent with the findings in moment comparisons.

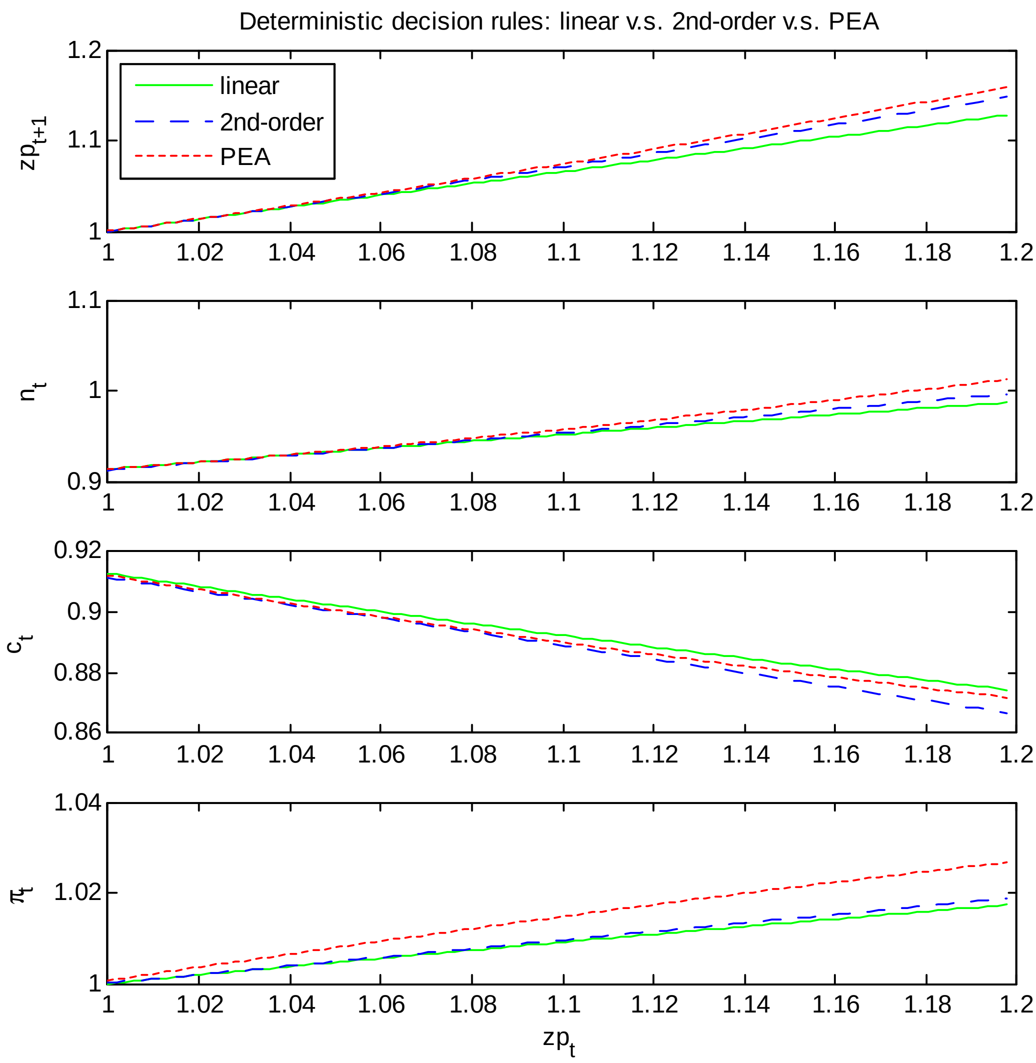

Figure 3.3: Deterministic decision rules (Model 1)

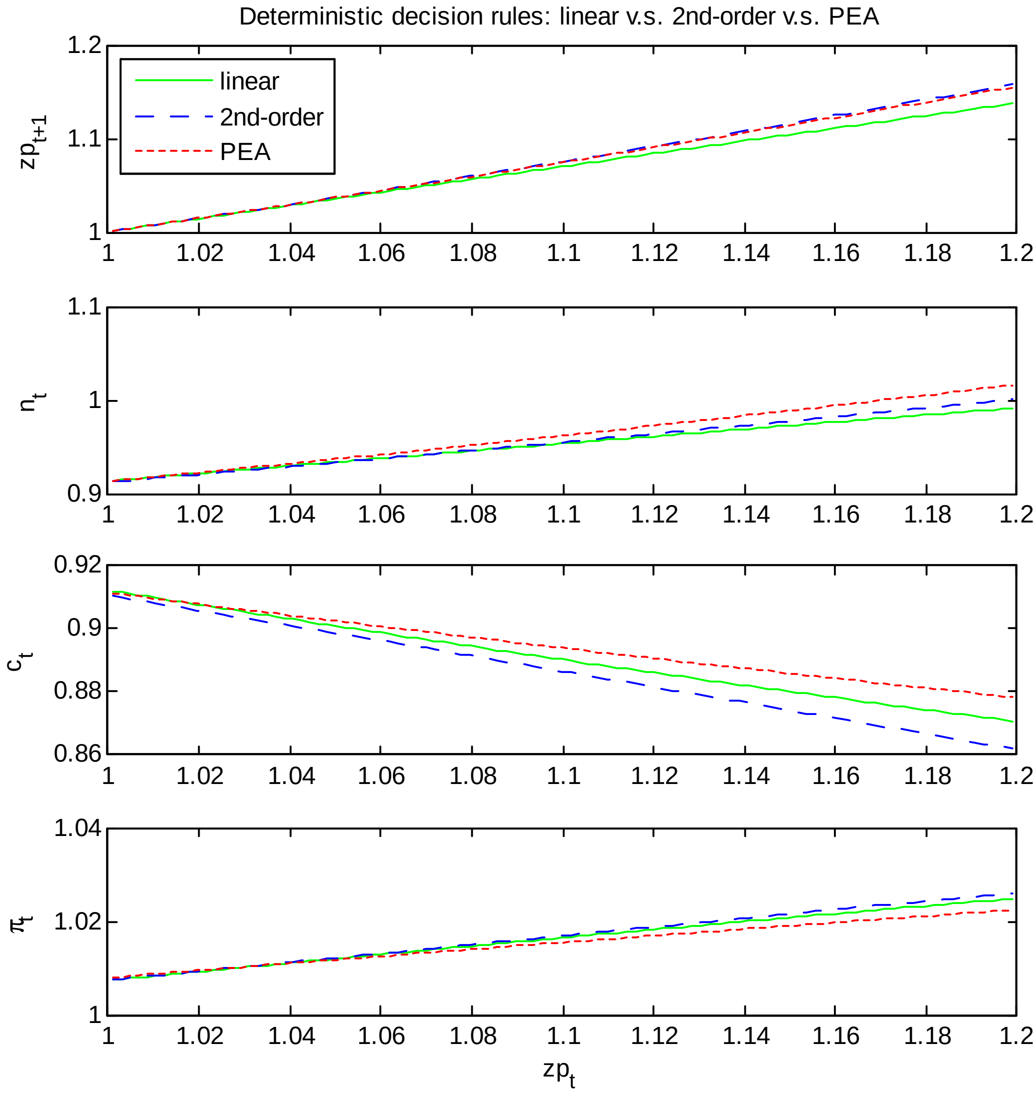

Figure 3.4: Deterministic decision rules (Model 2)

The plots of stochastic decision rules for next period price dispersion, labor supply, consumption and inflation are shown in Figure 3.5 and 3.6 for Model 1 and Model 2. The sample size used in simulation is 500. It can be seen from Figure 3.5 that, the linear approximation method shows dramatic differences for stochastic decision rules of all endogenous variables compared with the second-order and the PEA solution methods. The linear approximation method forces the price dispersion to be 1 in every time period. This is because the hatted values of price dispersion during log-linearization are all zero. When we go to model 2 in Figure 3.6 under positive inflation, the linear approximation method generates price dispersion values that fall very often below one where those generated with the second-order approximation and the PEA method are mostly greater than one. This again indicates the imprecision of the linear solution since from the non-linear model. Therefore the linear approximation makes huge mistake in simulating the price dispersion dynamics. On the other hand, the stochastic decision rules of the second-order approximation and the PEA are very close.

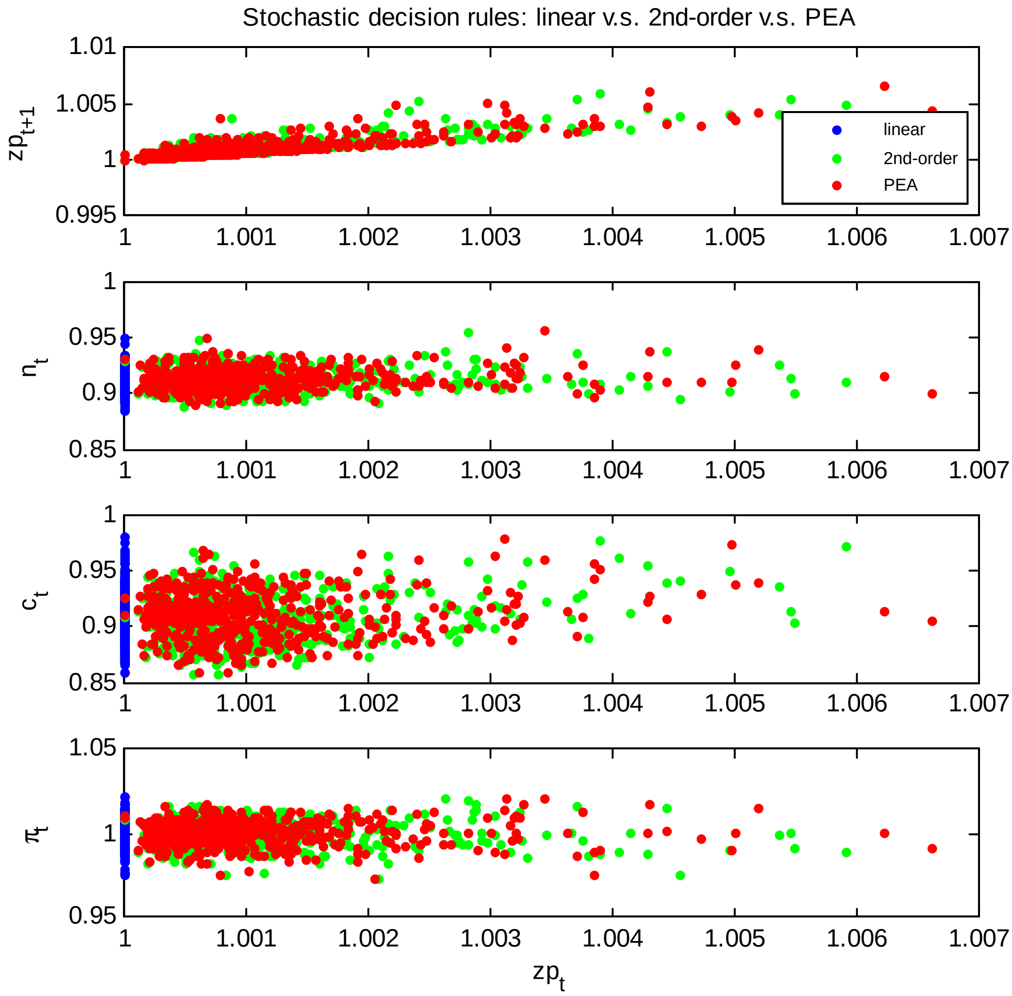

Figure 3.5: Stochastic decision rules (Model 1)

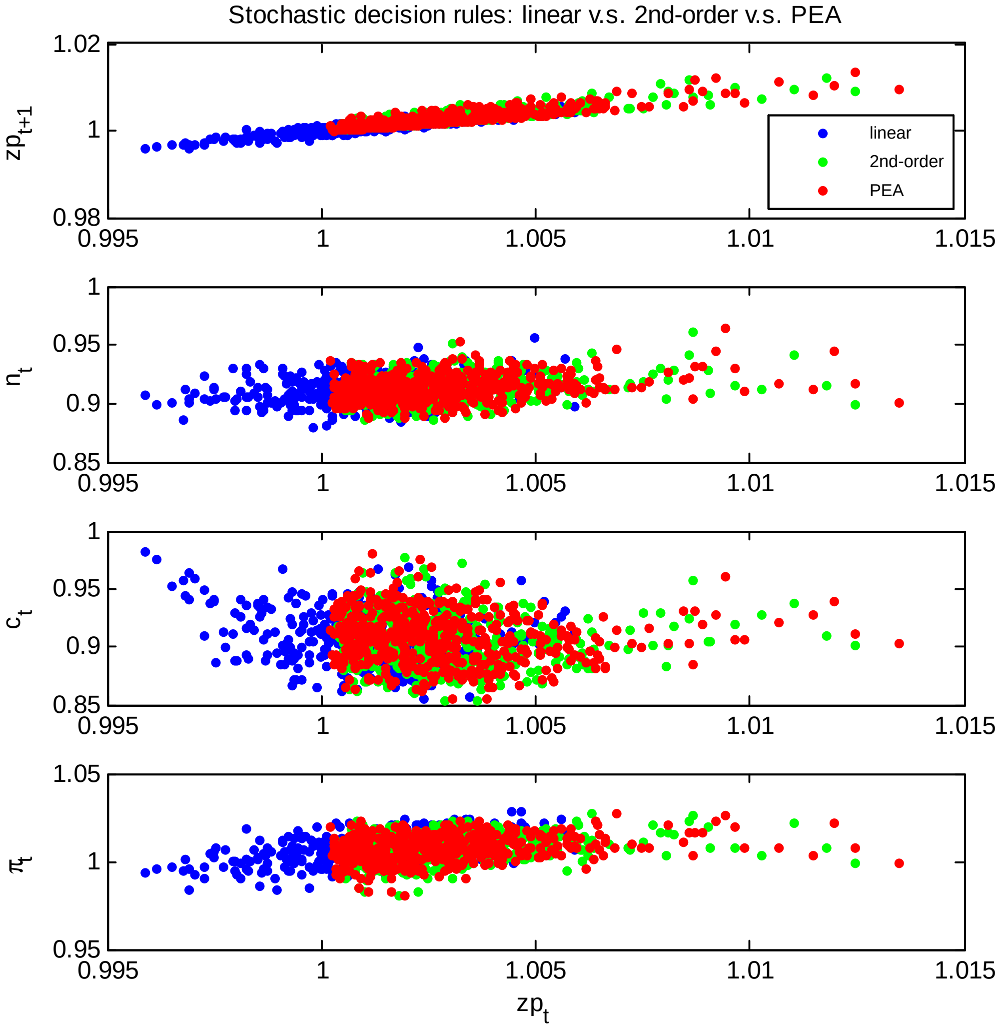

Figure 3.6: Stochastic decision rules (Model 2)

3.4.4 Impulse response functions

Impulse response functions are a useful tool to help understand the dynamics of the model. Comparing the responses of endogenous variables to exogenous shocks reveal different dynamics of the model generated by different solution methods. In particular, examining the responses of variables to the policy shocks is important for understanding the transmission mechanisms of monetary policy in the NK model.

The impulse response function of linear solution is straightforward. The model is represented in a state space form and the impulse response function is analytical and embedded in the coefficient matrix of the moving average representation. The impulse response functions of the second-order approximation method and the PEA algorithm on the other hand, do not have a closed form. They are therefore again calculated using Monte-Carlo simulations. This research makes use of the method proposed by

Koop, Pesaran and Potter (1996). For example, define the IRF as

$$IRF(h, shock) = \frac{E[y_{t+h} \,|shock, \Omega_0|] - E[y_{t+h} \,|\Omega_0|]}{E[y_{t+h} \,|\Omega_0|]} \tag{224}$$

where y is the endogenous variable, $shock$ is the magnitude of shock, t is the time horizon and Ω_0 is the information set at time 0. Intuitively, $E[y_{t+h} \,|shock, \Omega_0|]$ represents the shocked data and $E[y_{t+h} \,|\Omega_0|]$ represents the base data. The details of simulations and computations of the second-order approximation and PEA solution are provided in Appendix (3.7.3 and 3.7.4).

The impulse response functions for the three solution methods are reported in Figure 3.7-3.12. The results can be summarized as follows. First, all the three solutions correctly produce reasonable impulse response functions according to the transmission mechanism of the NK model. For example, a positive monetary policy shock raises nominal and real interest rate through the interest rule, which then induces a reduction in consumption and output gap through the Euler equation of households. The marginal cost of firms goes down due to a lower wage resulting from households supplying more labor. Thus inflation falls via the price-adjustment equation. The effects of the technology and cost-push shocks are also reasonable. Second, responses of most of the key macroeconomic variables such as output (under sticky prices) and inflation are very similar both under the benchmark model and model 2 with non-zero steady-state inflation. This is not a surprising result and is echoed by other studies (see for example, Collard (2001)). This is also due to the fact that the impulse response functions do not display much difference in higher order terms. Third, some variables, such as output gap (to flexible price equilibrium), nominal interest rate and price dispersion do react differently according to different solutions. For example, the nominal interest rate reacts more to a monetary policy shock under linear and second-order approximations than under PEA solution. Also, the output gap reacts more to a technology shock under linear and second-order approximations. Particularly, price dispersion always reacts differently for three solution methods under three exogenous shocks. Furthermore, the difference in price dispersion involve not only quantitative but also qualitative measure. For example, under benchmark model calibration, the price dispersion decreases to a cost-push shock under PEA solution while increases under linear and second-order approximations. The difference in price dispersion dynamics is thus one of the main findings of this chapter.

When we turn to alternative calibrations, the differences in impulse response function change (In some cases, the signs of responses even change). A summary is that, the differences between three solutions become larger under higher risk aversion and higher volatility of technology shock. This result confirms the recent findings in literature. On the other hand, these differences become smaller under non-zero steady-state inflation, lower market power and higher nominal rigidity. This is new to literature and can be explained according to the model structure. For example, the price dispersion term disappears under log-linearization under zero inflation, so linear solutions should make more error than non-linear methods. Lower market power and higher nominal rigidity both make the model less non-linear, so approximation error of linear and second-order also decreases. Finally, in all cases impulse responses are closer between linear and second-order solutions than PEA solution.

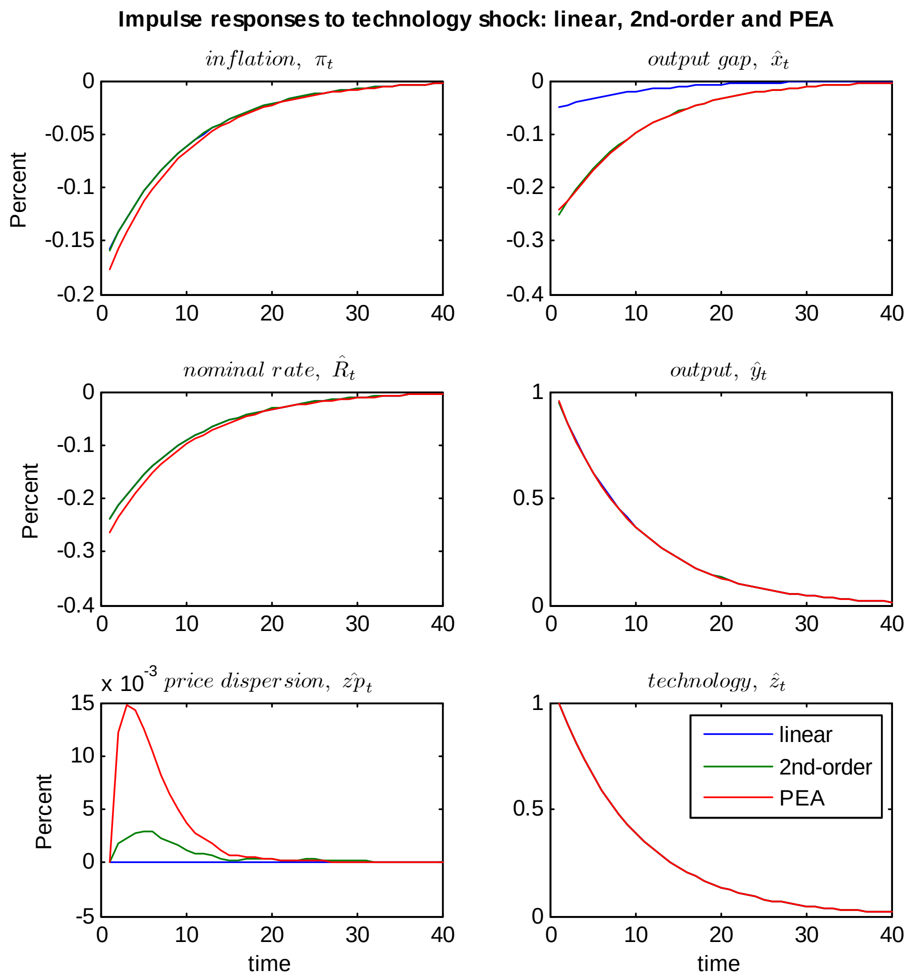

Figure 3.7: Impulse response functions: Model 1 (benchmark model)

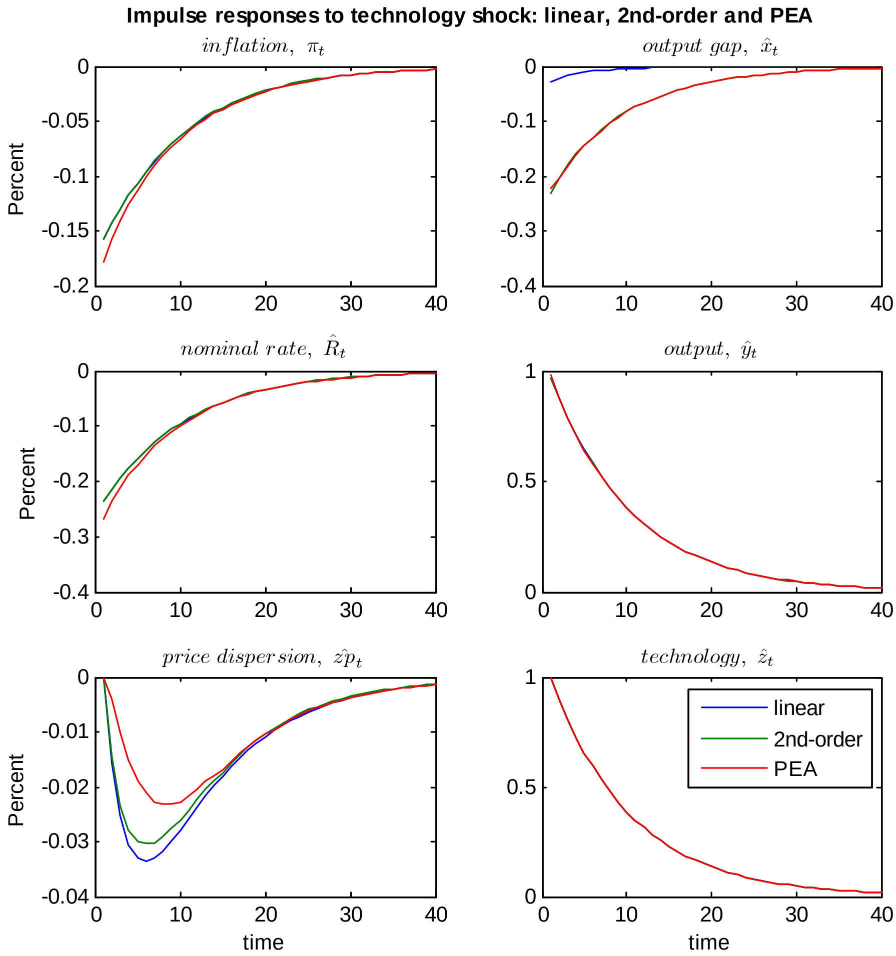

Figure 3.8: Impulse response function: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

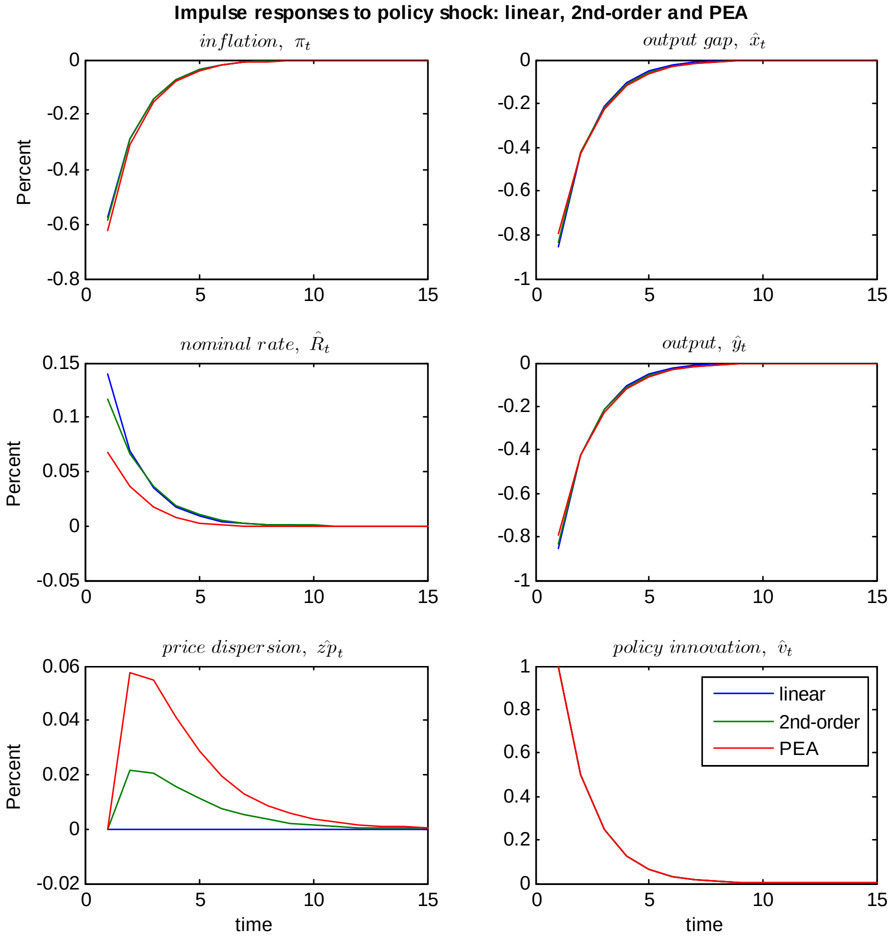

Figure 3.9: Impulse response functions: Model 1 (benchmark model)

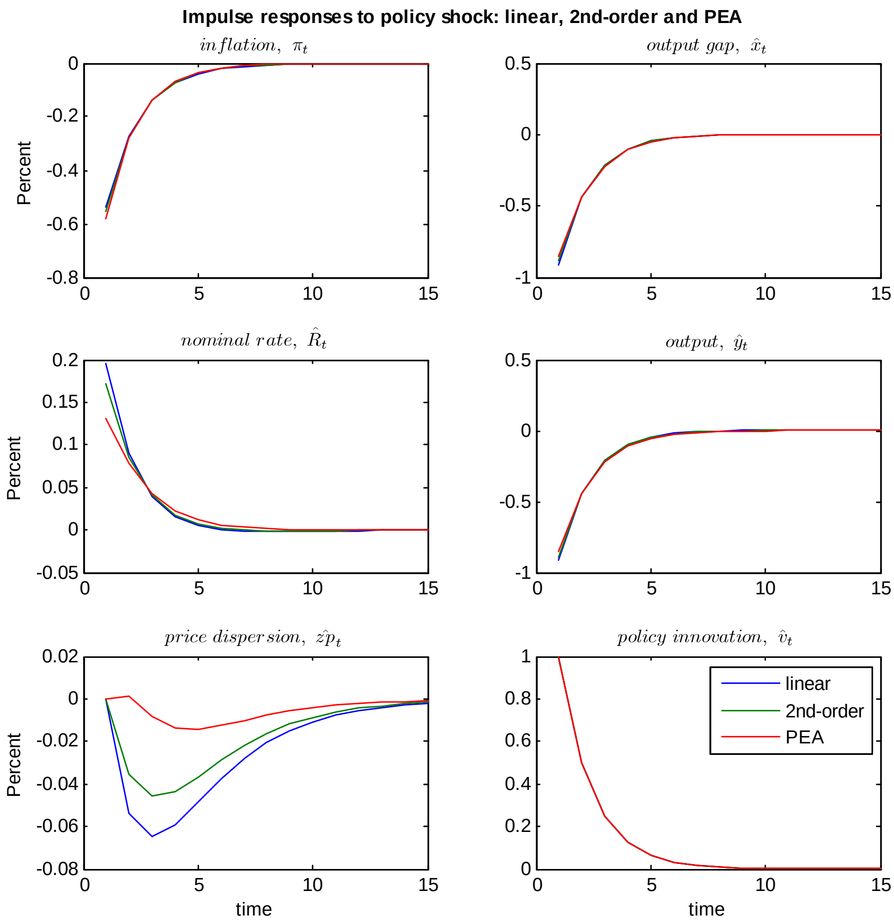

Figure 3.10: Impulse response functions: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

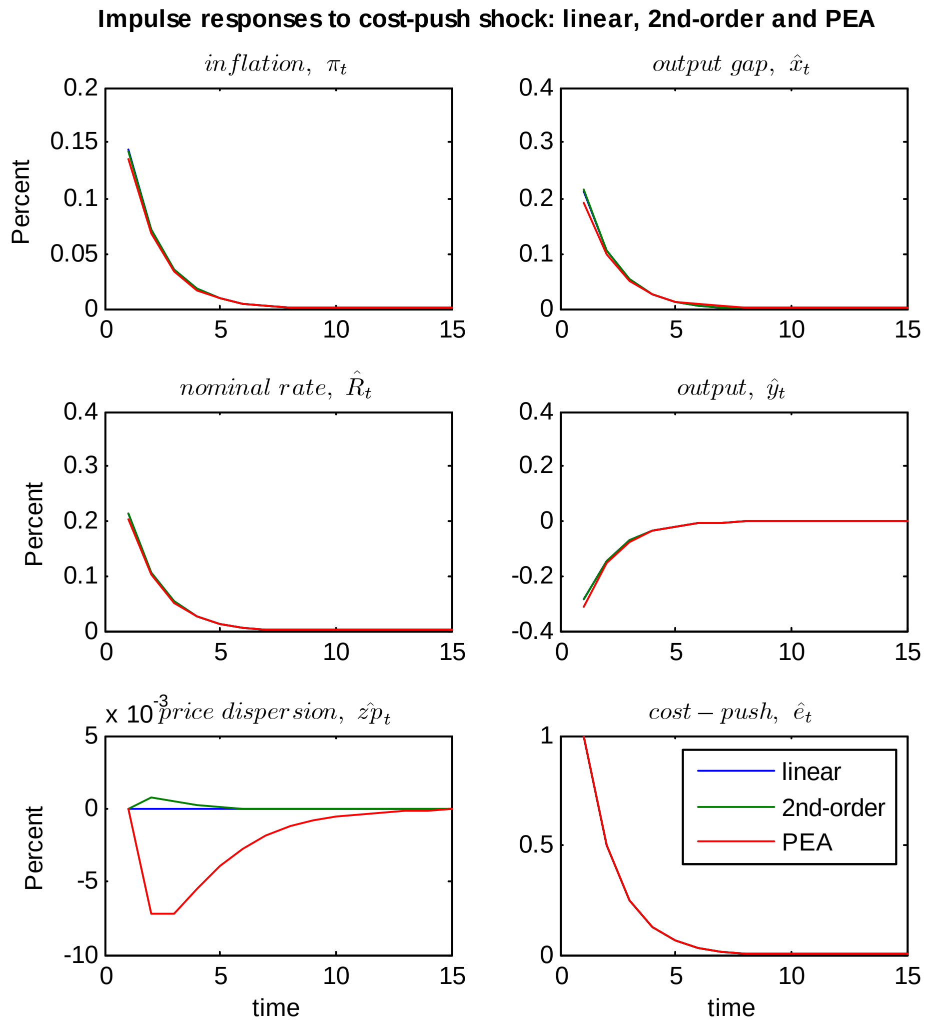

Figure 3.11: Impulse response functions: Model 1 (benchmark model)

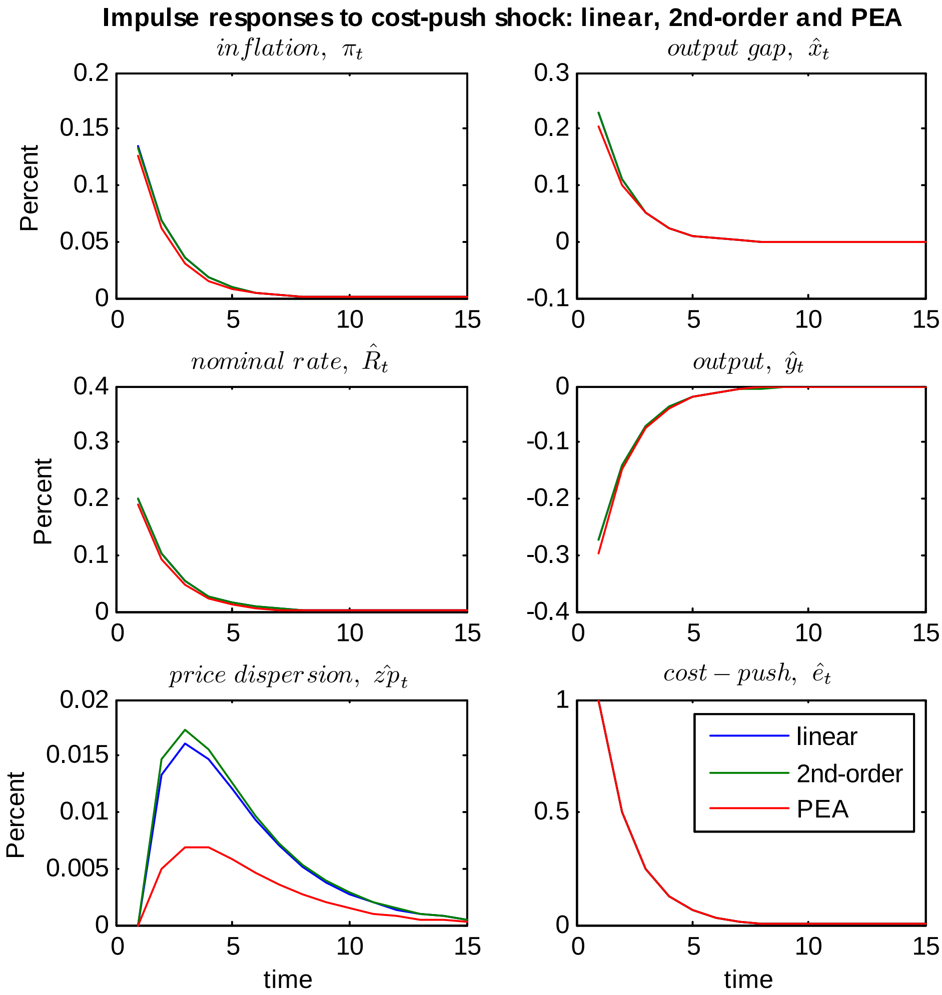

Figure 3.12: Impulse response functions: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

3.5 Assessing the accuracy of solutions

Although the last section shows the difference between the simulations based on three solutions, it does not address the issue of accuracy. This section makes use of two commonly used techniques to assess the accuracy of three solutions. These accuracy checks are based on measuring the Euler equation residuals as a way to evaluate the approximation error. The first method concerns a mathematical measurement of the Euler equation error and the second method conducts a statistical accuracy test of the rationality condition of agents. The standard and alternative calibrations, the implementation and results of the evaluations are illustrated below.

3.5.1 Measuring the Euler equation error

The first approach to measure the magnitude of approximation error is proposed by Judd (1992). It starts from the idea that an accurate solution should make the Euler equation equal. Accordingly, any approximation error can be inferred from the Euler equation residuals. The smaller the Euler residual, the more accurate a solution. Judd (1992) suggests that one can calculate the Euler residual and represent it as a fraction of the level of a particular variable of interest so that it can be normalized as an unit-free measurement of the approximation error.

In terms of the consumption Euler equation in the NK model, the expectation error in the consumption Euler equation is represented by

$$u_{c,t} = c_t - \left\{ \beta R_t \left(\frac{1}{\pi_{t+1}} \right) c_{t+1}^{-\sigma} \right\}^{-\frac{1}{\sigma}}$$

and in the price-adjustment equation is represented by

$$u_{p,t} = p_t^* - \frac{\left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t \varphi_t + \omega\beta \left(\pi_{t+1}\right)^{\theta} p_{t+1}^N}{(c_t)^{-\sigma} y_t + \omega\beta \left(\pi_{t+1}\right)^{\theta-1} p_{t+1}^D}.$$

Judd (1992) proposed to use the error-consumption ratio u_t/c_t as a measure of the expectation error and compute the following criteria

$$\varepsilon_{c,1} = \log_{10} \left(E_t \left| \frac{u_{c,t}}{c_t} \right| \right) \tag{225}$$

$$\varepsilon_{c,2} = \log_{10} \left(E_t \left\| \frac{u_{c,t}}{c_t} \right\|^2 \right) \tag{226}$$

$$\varepsilon_{c,3} = \log_{10} \left(\max \left| \frac{u_{c,t}}{c_t} \right| \right) \tag{227}$$

where $\varepsilon_{c,1}$ can be seen as the average expectation error, $\varepsilon_{c,2}$ is also a measure of expectation error but containing the information of volatility, $\varepsilon_{c,3}$ is the maximum of the expectation error. Take $\varepsilon_{c,1}$ as an example, if $\varepsilon_{c,1}$ turns out to be -6, this tells us that the agent would make £1 mistake in a consumption of £1,000,000. This seems acceptable to most researchers –therefore the solution can be seen as accurate.

For the expectation error in the price adjustment equation, we propose to calculate the criteria as

$$\varepsilon_{p,1} = \log_{10} \left(E_t \left| \frac{u_{p,t}}{p_t^*} \right| \right) \tag{228}$$

$$\varepsilon_{p,2} = \log_{10} \left(E_t \left\| \frac{u_{p,t}}{p_t^*} \right\|^2 \right) \tag{229}$$

$$\varepsilon_{p,3} = \log_{10} \left(\max \left| \frac{u_{p,t}}{p_t^*} \right| \right) \tag{230}$$

where $\varepsilon_{p,1}, \varepsilon_{p,2}$ and $\varepsilon_{p,3}$ can be seen as the average, volatility and maximum error respectively. The intuition of these criteria, for example $\varepsilon_{p,1}$, is that if $\varepsilon_{p,1}$ turns out to be -6, this tells us that the firm would make £1 mistake in when setting a price of

£1,000,000. Similarly as the consumption criteria, this also can be seen as acceptable to most researchers.

The calculations of above criteria involve approximation of conditional expectations which can be achieved either by numerical (Gaussian Hermite) integration or Monte-Carlo integration. Since there are three exogenous shocks with three independent distributions, the Monte-Carlo integration would be preferred in terms of simplicity of use. The implementation of Judd's criteria is provided in Appendix (3.7.5).

The results are summarized in Table 3.5 and 3.6. It can be seen that, under both model 1 and 2, the PEA algorithm outperforms linear and second-order methods. That is, the bigger the absolute value of the criteria is, the less approximation error it makes. It provides solutions that generate less consumption Euler equation error (for all criteria). For the price-adjustment equation error, although linear and second-order solutions also generate similar error with the PEA solution, the maximum error they generated is much bigger. The PEA algorithm on the other hand, still generates similar error as for the consumption error. It has been argued by many researchers (see for example, Judd and Guu (1997) and Aruoba *et al.* (2006)) that, the maximum error is an import indicator of poor solutions. Therefore, PEA is more accurate than the alternatives. The only striking result is that, sometimes, second-order approximation method performs even worse than linear method. For example, the maximum error the second-order solution makes is huge for the price-adjustment equation error (1.1568 and 1.3009 means that the forecasted price is more than ten times higher than the true value). This may be due to the fact that the benchmark NK model is set up in a log linear form.

When we turn to alternative calibrations (see Appendix (3.7.8)), the main results in the benchmark model remain valid. The main findings are as follows. First, overall PEA still performs better than the alternatives under higher nominal rigidity, market power, risk aversion and volatility of shock. There are some cases, where the average error in price-adjustment equation PEA makes is slightly bigger than linear and second-order solutions. However, the same time, linear and second-order solutions make much bigger maximum error. Second, no conclusion can be drawn regarding second-order solution is better than linear solution or not. As in the benchmark case, the second-order approximation is even worse than linear solutions. The overall performances of them are fairly close. Third, the accuracy of all the three solutions decreases when the nominal rigidity, market power, degree of risk aversion and volatility of shock are higher.

Table 3.5: Judd's criteria: Model 1 (Benchmark model)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-2.039	-2.039	-2.077	-2.080	-2.089	-2.088
ε_2	-4.079	-4.079	-4.154	-4.160	-4.178	-4.177
ε_3	-1.268	-1.265	-1.288	-0.818	-1.157	-1.198

Table 3.6: Judd's criteria: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-2.030	-2.032	-2.065	-2.054	-2.056	-2.060
ε_2	-4.060	-4.063	-4.129	-4.108	-4.112	-4.120
ε_3	-1.258	-1.255	-1.266	-0.729	-0.989	-1.181

3.5.2 Accuracy test on the rationality

A solution can be considered as accurate if it yields a decision rule that satisfies the rational expectation hypothesis. In other words, the Euler residual must follow a martingale difference. Since the 'true' expectations are unknown, all solution methods must have expectational error. For example, in the Euler equation for consumption,

$$u_{c,t+1} = \left(R_t \beta E_t (\frac{1}{\pi_{t+1}}) c_{t+1}^{-\sigma} \right)^{-\frac{1}{\sigma}} - c_t \tag{231}$$

and in the price-setting equations,

$$u_{p,t+1} = E_t \frac{\left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t \varphi_t + \omega\beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N}{(c_t)^{-\sigma} y_t + \omega\beta E_t (\pi_{t+1})^{\theta-1} p_{t+1}^D} - p_t^*. \tag{232}$$

Den Haan and Marcet (1994) proposed a statistical accuracy test which states that, if the rational expectations hypothesis is satisfied, the residual at time $t+1$ in an Euler equation should be orthogonal to the state variables at time t,

$$E\left[u_{t+1} \otimes h(x_t)\right] = 0$$

where u_{t+1} is the Euler residual at time $t+1$, $h(x_t)$ is function of x_t which contains the model's q state variables at time t.

In simulations, its counterpart is,

$$g = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_{t+1} \otimes h(\hat{x}_t)$$

where T is the sample size, a 'hat' denotes simulated data of u_{t+1} and x_t.

A GMM type J-statistic is built to test whether g is significant from zero: Under null hypothesis,

$$J_T = Tg'\Omega_T^{-1}g \sim \chi_q^2 \tag{233}$$

where $\Omega_T = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_{t+1}^2 h(\hat{x}_t)h(\hat{x}_t)'$, is a consistent estimate of the variance-covariance matrix of $\hat{u}_{t+1}$ and $\hat{x}_t$.

Repeat the test many times using Monte-Carlo, and report the percentage that J_T falls in the rejection area, i.e. higher than 95% and lower than 5%. If the expectational error is indeed orthogonal to state variables, we should expect only 5% of J-tests fall into rejection area too. Otherwise too many outcomes fall in rejection area indicates a rejection of the null hypothesis of orthogonality. The detailed steps of calculating the Marcet's J-test are provided in Appendix (3.7.6).

The J-test results for the benchmark model and model 2 with non-zero steady-state inflation are reported in Table 3.7 and 3.8. A number of 1000 simulations are used in performing Monte-Carlo simulation with 1000 sample size for each simulation. It can be seen clearly that PEA solution performs better than linear and second-order approximations. The p-values of the J-test of PEA are always close to 5% level for the consumption Euler equation, with a marginal rejection (8.5%) for the price-adjustment equation. On the contrary, the p-values of linear and second-order approximations are more below 5% level for the consumption Euler equation. Particularly, linear and second-order approximations produce densities that deviate dramatically from the theoretical ones for the price-adjustment equation. This indicates the difficulty of linear and second-order approximations in giving accurate solutions for the price-adjustment equation. This is also consistent with the findings in the Judd measure which shows huge maximum approximation error for linear and second-order approximations. Given the result that the linear and second-order approximation are again close to each other in J-tests with the latter only slightly better, a conclusion is drawn that, a second-order approximation may not be enough to solve the price-adjustment equation in the NK model. Finally, since PEA also performs worse for the price-adjustment equation than the Euler equation, it is suggested that, the PEA still needs to be refined to improve the accuracy[90].

Table 3.7: Marcet's J-test: (Benchmark model)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.031	0.029	0.043	0	0	0.022
higher 5%	0.031	0.032	0.055	0.404	0.197	0.085

Table 3.8: Marcet's J-test: Model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.031	0.027	0.038	0	0	0.019
higher 5%	0.033	0.031	0.052	0.422	0.263	0.085

The findings in Table 3.7 and Table 3.8 are confirmed in the plots of the empirical cumulative density functions (CDFs) of three solutions together with the theoretical χ^2_q CDF in Figure 13-16[91]. It is obvious that PEA continues to deliver accurate solutions both for solving the consumption Euler equation with a slightly inaccurate solution for solving the price-adjustment equation. The linear and second-order approximations on the other hand, deliver empirical CDFs close to the theoretical CDFs for the consumption Euler equation (even look better than PEA solution) but far away for the price-adjustment equation. Again the linear and second-order approximations produce similar results with the former the worst and the latter only slightly better.

[90] As mentioned in the introduction, one advantage of nonlinear method such as PEA is that, in case the solution is not accurate, the algorithm can be refined (for example, changing the choice of state variables or function forms of the polynomial function), while the linear and second order solution methods can not.

[91] The empirical CDFs are calculated using Matlab function cdfplot.m, and the degree of freedom for the theoretical CDF is 9.

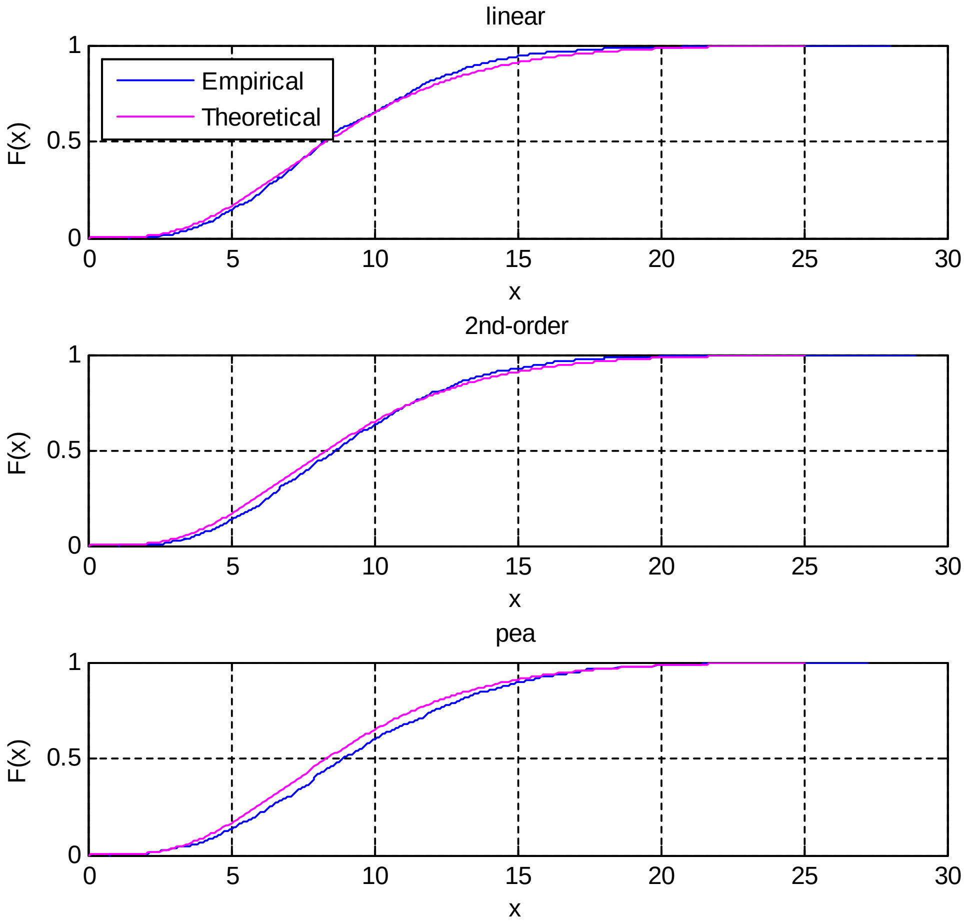

Figure 3.13: χ^2 specification test: consumption equation, model 1 (benchmark model)

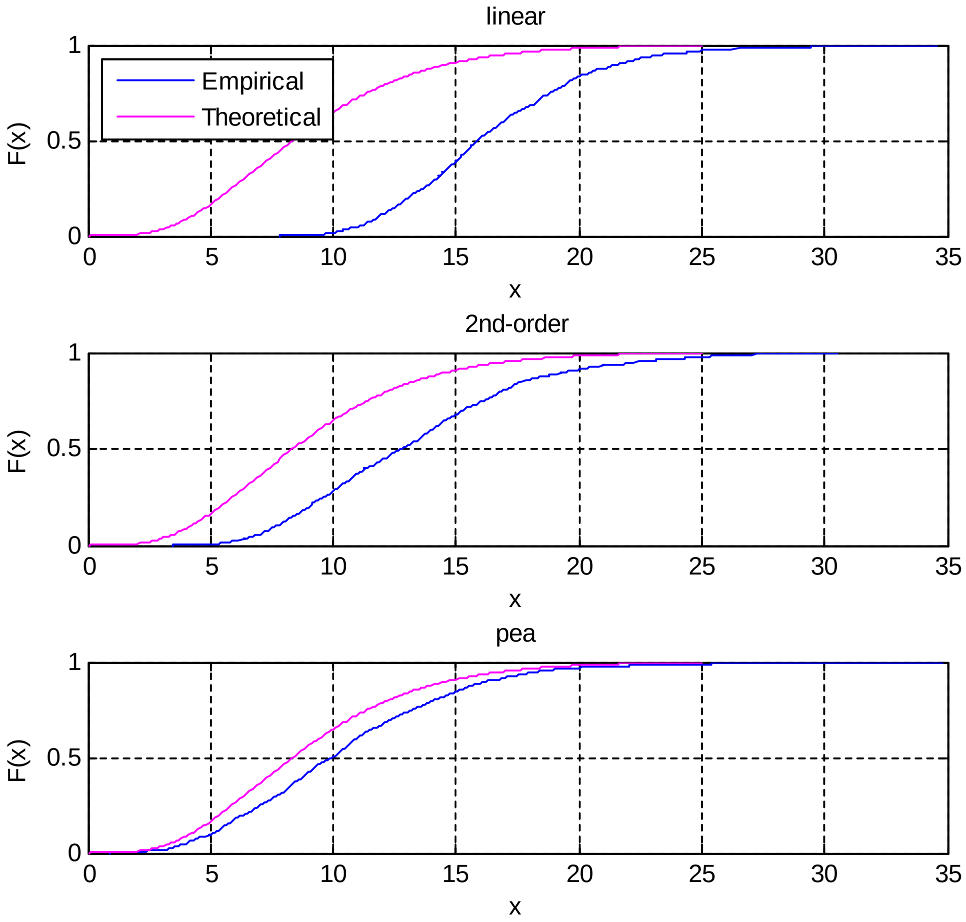

Figure 3.14: χ^2 specification test: price-adjustment equation, model 1 (benchmark model)

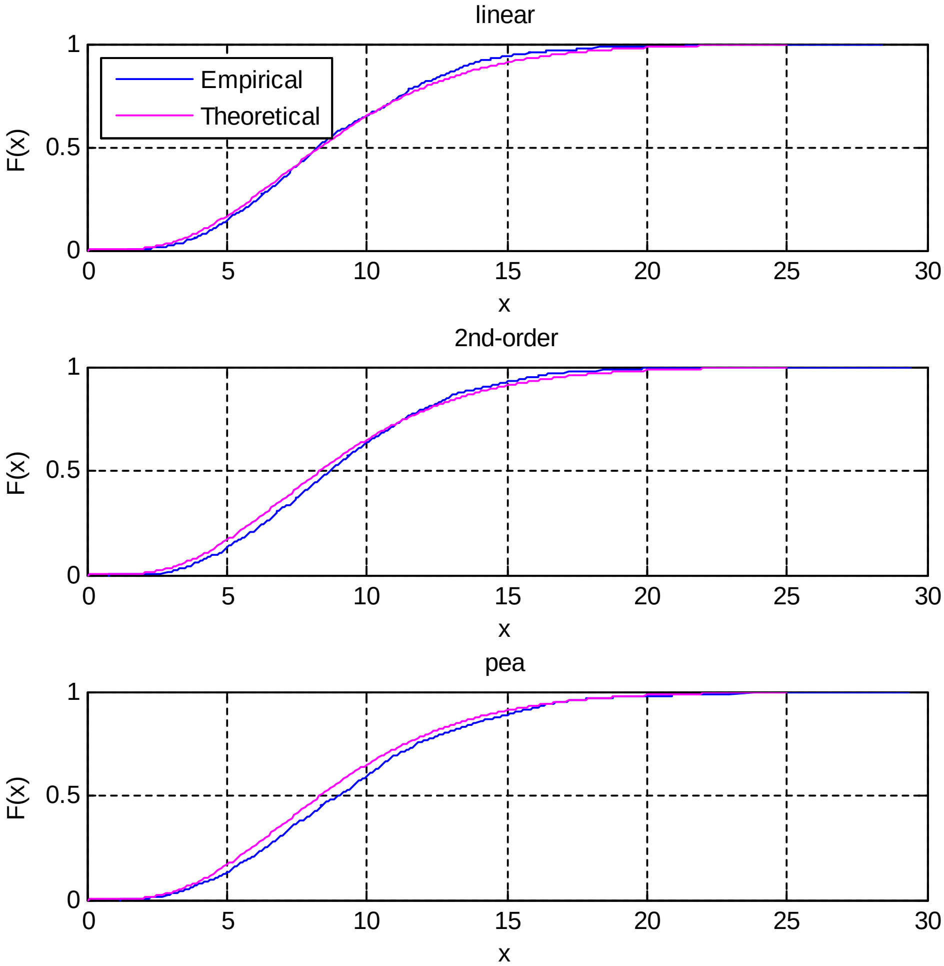

Figure 3.15: χ^2 specification test: consumption equation, model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

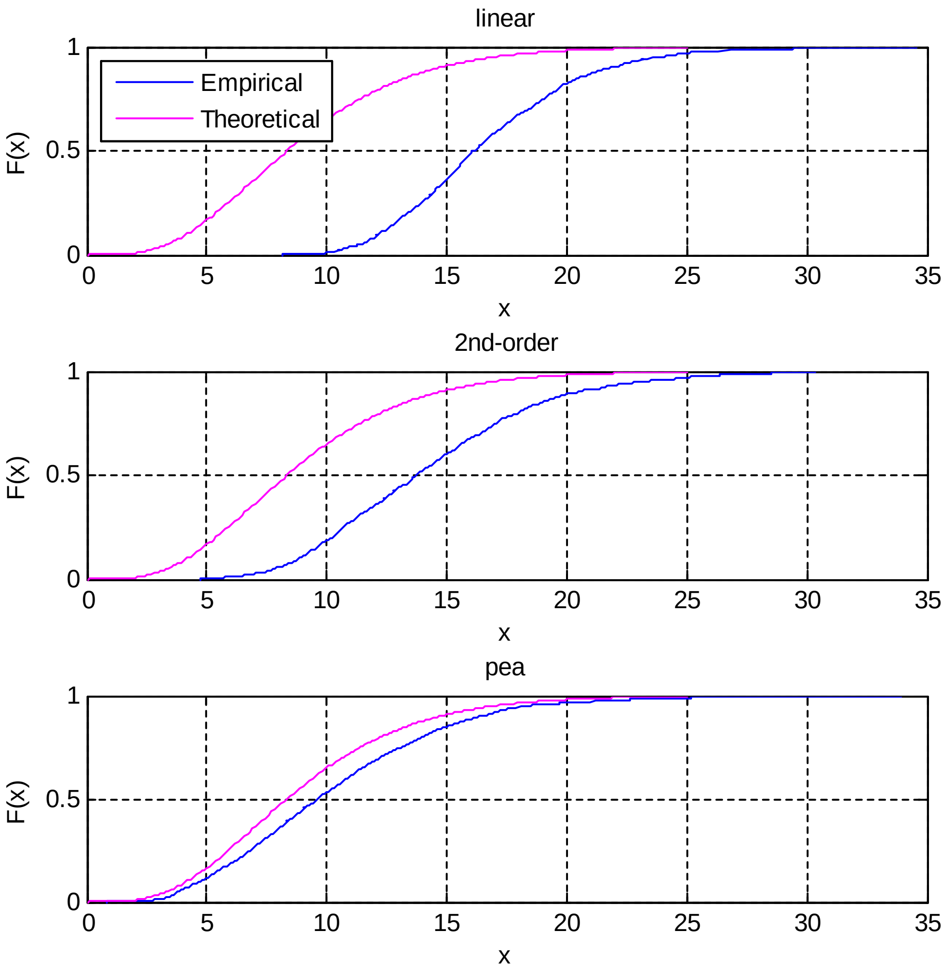

Figure 3.16: χ^2 specification test: price-adjustment, model 2 ($\overline{\pi} = 1.03^{\frac{1}{4}}$)

The results of the J-test and plots of CDFs for alternative calibrations are provided in Appendix (3.7.8) given the space constraint. It is again found that, the solution for the price-adjustment equation is much less accurate than the solution for the consumption Euler equation. The linear and second-order approximations make much larger error than PEA and the empirical CDFs depart dramatically from the theoretical CDFs for solving the price-adjustment equation. Furthermore, the results show that the higher degree of nominal rigidity, risk aversion, market power and volatility of shock significantly reduce the accuracy of linear and second-order approximations, especially for the price-adjustment equations. On the other hand, changing parameter values has only small effects on PEA accuracy. And particularly for the price-adjustment equation, PEA even gives more accurate solution under higher non-linearity and uncertainty. This suggests that as a non-linear solution method, PEA captures more of the non-linearity and uncertainty than linear and second-order approximations. Therefore, alternative calibrations do not violate the main conclusion drawn from the benchmark model that PEA provides a better solution.

3.6 Concluding remarks

Despite the popularity of the use of linear and second-order approximation methods for solving the NK model and evaluating policy analysis, the accuracy of these methods has not been well researched. This study solves the benchmark NK model by applying the linear approximation, second-order approximations and the PEA algorithm as an alternative. After that, the simulated model economies using the three solution methods are compared. Moreover, the accuracy of the three solutions is evaluated by comparing the Euler equation error generated by each solution methods using the Judd's measure and the Marcet's statistical test. Various alternative calibrations of the structural parameters are considered to check the robustness of the results.

The main results are highlighted as follows. First, the three solution methods display small differences in simulated data in terms of sample distributions, population moments, policy functions and impulse response functions of most variables. However, the results for the price dispersion and some other variables display significant differences. For example, the distributions and population moments of the price dispersion from the three solution methods are qualitatively different. This also leads to large differences in plotted policy functions of variables. Also, the impulse responses of output gap, nominal interest rate and the price dispersion differ dramatically across solution methods. The differences in the responses of price dispersion are shown to be robust across all exogenous shocks. Second, the Judd measure and the Marcet's J-test demonstrate the same results that PEA performs better than the linear and second-order solution methods as far as accuracy is concerned. The biggest difference of accuracy emerges in solving the price-adjustment equation where all solution methods become less accurate with PEA still performing the best. Third, our alternative calibrations with non-linear steady-state inflation, higher degree of nominal rigidity, higher risk aversion of households, larger degree of firm market power and bigger volatility of technology shock make all the solutions more different in summary statistics and accuracy evaluations. However, the main result from benchmark model that PEA performs better than the other solution methods remains robust across all alternative calibrations. Therefore, attentions need to be paid to checking the accuracy of solving the NK model and choosing between alternative solution methods before using the model.

Several directions for future work are also proposed. First, since the approximation error is still big in solving the price-adjustment equations for all solution methods, it is suggested that PEA needs to be refined to obtain a better solution of the NK model. The linear solution and the second-order approximation on the other hand cannot be refined. The PEA algorithm can be re-implemented by using higher order polynomial functions and by using different combinations of state variables. Second, more non-linear solution methods can be applied to the NK model for comparison purposes. Third, since the three solution methods generate significantly different Euler equation error, it is meaningful to examine how these differences affect the welfare of the economy. For example, Santos (2000) argues that the change in welfare is in square order of the Euler equation error. It is then suggested that, the line of welfare analysis (see for example, Woodford (2003)) can be conducted in the NK model as to how the welfare can change with respect to different model dynamics implied by different solution methods. Moreover, these welfare comparisons can be conducted by using an *ad hoc* policy via a simple Taylor rule but also the optimal commitment policy. Finally, so far the simulated data using the three solution methods have not been compared with actual data. It is then suggested to compare the solution methods in terms of fitting the model to data. This will include both moment matching, e.g., comparing moments of different solutions with ones in the data and also the autocorrelation functions comparisons. These also leave for future work.

3.7 Appendices

3.7.1 The benchmark NK model

Household's problem

Cost minimization:

$$\underset{c_{jt}}{\text{Min}} \int_0^1 p_{jt} c_{jt} dj$$
$$\text{s.t. } c_t = \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}}.$$

Form the Lagrangian:

$$L_t = \int_0^1 p_{jt} c_{jt} dj + \psi_t \left[c_t - \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} \right].$$

The first-order condition (FOC) respect to good j is:

$$p_{jt} - \psi_t \left[\left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} \right] = 0. \tag{234}$$

Combining equation (234) with the CES form, $c_t = \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}}$, gives,

$$p_{jt} - \psi_t c_t^{\frac{1}{\theta}} c_{jt}^{-\frac{1}{\theta}} = 0,$$

or

$$c_{jt} = \left[\frac{p_{jt}}{\psi_t} \right]^{-\theta} c_t. \tag{235}$$

Next we need to solve for the multiplier ψ_t. To see this, substitute (235) into the CES form, $c_t = \left\{ \int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}}$ to obtain,

$$c_t = \left\{ \int_0^1 \left[\left[\frac{p_{jt}}{\psi_t} \right]^{-\theta} c_t \right]^{\frac{\theta-1}{\theta}} dj \right\}^{\frac{\theta}{\theta-1}} = \left[\frac{1}{\psi_t} \right]^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{\theta}{\theta-1}} c_t$$

or rearranging for ψ_t yields,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \tag{236}$$

Note that the right side of the above result is equal to the aggregate price, P_t[92],

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{237}$$

[92] Note that equation (237) is obtained by setting final good producer's proft to zero, implying perfect competition in final good market.

which implies that

$$\psi_t = P_t. \tag{238}$$

Substituting (238) back into (235) yields the demand function given by (156) in the main text,

$$c_{jt} = \left[\frac{p_{jt}}{P_t}\right]^{-\theta} c_t. \tag{239}$$

Utility Maximization

$$\underset{c_{t+i}, n_{t+i}, B_{t+i}}{\text{Max}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{e_t n_{t+i}^{1+\eta}}{1+\eta}\right]$$
$$\text{s.t.} \quad \left(c_{t+i} + \frac{B_{t+i}}{P_{t+i}} - R_{t+i-1}\frac{B_{t+i-1}}{P_{t+i}}\right) - \frac{W_{t+i}}{P_{t+i}} n_{t+i} - \Pi_{t+i}.$$

The Lagrangian is:

$$\begin{aligned} L_{t+i} = E_t \sum_{i=0}^{\infty} \Bigg\{ \beta^i \left[\tfrac{c_{t+i}^{1-\sigma}}{1-\sigma} - \chi \tfrac{e_t n_{t+i}^{1+\eta}}{1+\eta}\right] \\ + \beta^i \lambda_{t+i} \left[\left(c_{t+i} + \tfrac{B_{t+i}}{P_{t+i}} - R_{t+i-1}\tfrac{B_{t+i-1}}{P_{t+i}}\right) - \tfrac{W_{t+i}}{P_{t+i}} n_{t+i} - \Pi_{t+i}\right] \Bigg\}. \end{aligned} \tag{240}$$

The FOC with respect to c_{t+i} is:

$$E_t \beta^i c_{t+i}^{-\sigma} + \beta^i E_t \lambda_{t+i} = 0.$$

Or, for $i = 0$,

$$c_t^{-\sigma} + \lambda_t = 0. \tag{241}$$

Similarly, the FOC with respect to n_{t+i} is:

$$-\beta^i E_t \chi e_t n_{t+i}^{\eta} - E_t \lambda_{t+i} \frac{W_{t+i}}{P_{t+i}} = 0$$

or

$$-E_t \chi e_t n_t^{\eta} - E_t \lambda_t \frac{W_t}{P_t} = 0 \ for \ i = 0. \tag{242}$$

Since B_{t+i} appears in both time $t+i$ and $t+i+1$, the FOC with respect to B_{t+i} is :

$$\lambda_{t+i} - \beta E_t \lambda_{t+i+1} R_{t+i} \frac{P_{t+i}}{P_{t+i+1}} = 0$$

or

$$\lambda_t - \beta r_t E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \ for \ i = 0. \tag{243}$$

Having obtained the three FOCs, we can now combine them to get the underlying optimal behavior of labour supply and consumption. Firstly from (241) we get

$$\lambda_t = -c_t^{-\sigma} \tag{244}$$

then substituting this into (242) to eliminate λ_t gives,

$$-\chi e_t n_t^\eta + c_t^{-\sigma} \frac{W_t}{P_t} = 0$$

or

$$\frac{\chi e_t n_t^\eta}{c_t^{-\sigma}} = \frac{W_t}{P_t} \tag{245}$$

which is the labour supply function of the household.

Next, substituting (244) into equation (243) gives,

$$-c_t^{-\sigma} + \beta R_t E_t c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} = 0$$

or

$$c_t^{-\sigma} = \beta R_t E_t (\frac{P_t}{P_{t+1}}) c_{t+1}^{-\sigma} \tag{246}$$

which is the Euler Equation for consumption.

Firm's price-setting problem The firm's real profit function is:

$$\Pi_{jt} = \left(\frac{p_{jt}}{P_t} \right) y_{jt} - \frac{W_t n_{jt}}{P_t}. \tag{247}$$

Substituting the marginal cost, $\varphi_t = \frac{W_t/P_t}{z_t}$, and the production function, $y_{jt} = z_t N_{jt}$, into the previous expression gives,

$$\Pi_{jt} = \left(\frac{p_{jt}}{P_t} \right) y_{jt} - \varphi_t y_{jt}$$

where y_{jt} is the good produced by the firm j and φ_t is the marginal cost common to all firms at time t.

In a NK setup with "Calvo contracts", firms choose an optimal price P_t^* at time t to maximize discounted future profits. Since only $(1-\omega)$ of firms are able to reset their prices in each period, when they reset prices at time t, they must accept the fact that the price they set today may be remain the same in subsequent periods at probability ω^i. Therefore, the firm's objective is to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right) y_{j,t+i} - \varphi_{t+i} y_{j,t+i} \right] \tag{248}$$

by choosing an optimal price p_{jt} at time t. The stochastic discount factor $\Delta_{i,t+i}$ takes the following form,

$$\Delta_{i,t+i} = \beta^i \left(\frac{y_{t+i}}{y_t} \right)^{-\sigma} \tag{249}$$

where $y_{t+i}^{-\sigma} = \lambda_{t+i}$ is the marginal utility of households with respect to the real profit, Π_{t+i}. It is derived by taking first derivative of the Lagrangian of households' utility maximization problem in (240) with respect to Π_{t+i}. Therefore, $\left(\frac{y_{t+i}}{y_t}\right)^{-\sigma}$ measures the utility of future consumption compared with current consumption of household.

The constraint for the profit maximization problem is the household's demand curve (239):

$$y_{j,t+i} = \left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta} y_{t+i}. \tag{250}$$

Substituting (250) into the objective function (248) yields,

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right)\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta} y_{t+i} - \varphi_{t+i}\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta} y_{t+i}\right].$$

Rearranging gives the price-setting firm's objective function in terms of the price level, P_t, aggregate consumption y_{t+i} and its own optimal price p_{jt}:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\theta} - \varphi_{t+i}\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\theta}\right] y_{t+i}.$$

This optimization problem here is to find the optimal price firms choose to set. Importantly, because firms all have the same production technology and face the same demand curve, they are assumed to choose the same price when they are able to change their prices. Then we can simply define the same optimal price as P_t^*.

The FOC with respect to P_t^* is:

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta)\left(\frac{P_t^*}{P_{t+i}}\right)^{-\theta}\frac{1}{P_{t+i}} + \theta\varphi_{t+i}\left(\frac{P_t^*}{P_{t+i}}\right)^{-\theta-1}\frac{1}{P_{t+i}}\right] y_{t+i} = 0$$

or

$$E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[(1-\theta)\frac{P_t^*}{P_{t+i}} + \theta\varphi_{t+i}\right]\left(\frac{P_t^*}{P_{t+i}}\right)^{-\theta}\frac{y_{t+i}}{P_t^*} = 0. \tag{251}$$

Then substituting $\Delta_{i,t+i} = \beta^i \frac{\lambda_{t+i}}{\lambda_t} = \beta^i \left(\frac{c_{t+i}}{c_t}\right)^{-\sigma}$ into above equation and solving for P_t^* yields,

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{c_{t+i}}{c_t}\right)^{-\sigma} y_{t+i}\left[(1-\theta)\frac{P_t^*}{P_{t+i}} + \theta\varphi_{t+i}\right]\left(\frac{P_t^*}{P_{t+i}}\right)^{-\theta}\frac{1}{P_t^*} = 0$$

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{c_{t+i}}{c_t}\right)^{-\sigma} y_{t+i}\left[(1-\theta)\frac{P_t^*}{P_{t+i}} + \theta\varphi_{t+i}\right]\left(\frac{1}{P_{t+i}}\right)^{-\theta}(P_t^*)^{-\theta-1} = 0.$$

Note that since c_t and P_t^* are independent of i, the above result can be simplified to

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i}\left[(1-\theta)\frac{P_t^*}{P_{t+i}} + \theta\varphi_{t+i}\right]\left(\frac{1}{P_{t+i}}\right)^{-\theta} = 0.$$

This can be decomposed as,

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i}(\theta-1)\frac{P_t^*}{P_{t+i}}\left(\frac{1}{P_{t+i}}\right)^{-\theta} = E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i}\theta\varphi_{t+i}\left(\frac{1}{P_{t+i}}\right)^{-\theta}$$

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i}(\theta-1)P_t^*(P_{t+i})^{\theta-1} = E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i}\theta\varphi_{t+i}(P_{t+i})^{\theta}.$$

Solving for P_t^* gives,

$$P_t^* = \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} (P_{t+i})^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i} (P_{t+i})^{\theta-1}}. \tag{252}$$

Dividing the above result by P_t to both sides yields,

$$\frac{P_t^*}{P_t} = \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}}. \tag{253}$$

To facilitate the application of the PEA, we let

$$p_t^N = E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} \tag{254}$$

$$p_t^D = E_t \sum_{i=0}^{\infty} \omega^i \beta^i (c_{t+i})^{-\sigma} y_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta-1} \tag{255}$$

and obtain

$$\frac{P_t^*}{P_t} = \frac{p_t^N}{p_t^D}. \tag{256}$$

Recursive Form It is helpful to note that P_t^N and P_t^D themselves have recursive forms which link P_t^N, P_t^D to next period's variables P_{t+1}^N and P_{t+1}^D. To see this, we first note that the full representation of p_t^N is

$$p_t^N = E_t \sum_{i=0}^{\infty} \omega^{t+i-t} \beta^{t+i-t} \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} \tag{257}$$

i.e., β is the discount rate at time $t+i$. Now we lead t in (257) one period to define P_{t+1}^N

$$\begin{aligned} p_{t+1}^N &= E_{t+1} \sum_{i=0}^{\infty} \omega^{(t+1)+i-(t+1)} \beta^{(t+1)+i-(t+1)} \left(\tfrac{\theta}{\theta-1}\right) \left(c_{(t+1)+i}\right)^{-\sigma} y_{(t+1)+i} \varphi_{(t+1)+i} \left(\tfrac{P_{(t+1)+i}}{P_{t+1}}\right)^{\theta} \\ p_{t+1}^N &= E_{t+1} \sum_{i=0}^{\infty} \omega^i \beta^i \left(\tfrac{\theta}{\theta-1}\right) \left(c_{(t+1)+i}\right)^{-\sigma} y_{(t+1)+i} \varphi_{(t+1)+i} \left(\tfrac{P_{(t+1)+i}}{P_{t+1}}\right)^{\theta} \end{aligned} \tag{258}$$

this can be equivalently represented by

$$p_{t+1}^N = E_{t+1} \sum_{i=1}^{\infty} \omega^{i-1} \beta^{i-1} \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i} \varphi_{t+i} \left(\frac{P_{t+i}}{P_{t+1}}\right)^{\theta}. \tag{259}$$

Note that P_t^N (257) can be rewritten equivalently as

$$\begin{aligned}
p_t^N &= E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} \\
&= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t + E_t \sum_{i=1}^{\infty} \omega^i \beta^i \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\theta}
\end{aligned}$$

or

$$\begin{aligned}
p_t^N &= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t \\
&\quad + E_t \sum_{i=1}^{\infty} (\omega\beta)\, \omega^{i-1} \beta^{i-1} \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+1}}{P_t}\right)^{\theta} \left(\frac{P_{t+i}}{P_{t+1}}\right)^{\theta} \\
p_t^N &= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t \\
&\quad + E_t (\omega\beta) \left(\frac{P_{t+1}}{P_t}\right)^{\theta} \sum_{i=1}^{\infty} \omega^{i-1} \beta^{i-1} \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+i}}{P_{t+1}}\right)^{\theta}.
\end{aligned}$$

From the property of expectation operator we know that

$$E_t E_{t+1} \equiv E_t.$$

Using this property and inserting E_{t+1} into above result gives

$$\begin{aligned}
p_t^N &= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t \\
&\quad + E_t (\omega\beta) \left(\frac{P_{t+1}}{P_t}\right)^{\theta} E_{t+1} \sum_{i=1}^{\infty} \omega^{i-1} \beta^{i-1} \left(\frac{\theta}{\theta-1}\right) (c_{t+i})^{-\sigma} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+i}}{P_{t+1}}\right)^{\theta}.
\end{aligned}$$

Now we know that the part $E_{t+1} \sum_{i=1}^{\infty} \omega^{i-1}\beta^{i-1} \left(\frac{\theta}{\theta-1}\right) \frac{\lambda_{t+i}}{\lambda_{t+1}} y_{t+i}\varphi_{t+i} \left(\frac{P_{t+i}}{P_{t+1}}\right)^{\theta}$ is equal to p_{t+1}^N as seen in (259). Thus we obtain

$$\begin{aligned}
p_t^N &= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t + E_t (\omega\beta) \left(\frac{P_{t+1}}{P_t}\right)^{\theta} p_{t+1}^N \\
&= \left(\frac{\theta}{\theta-1}\right) (c_t)^{-\sigma} y_t\varphi_t + \omega\beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N. \qquad (260)
\end{aligned}$$

Similarly we undertake the same steps to get a recursive form for p_t^D as

$$p_t^D = (c_t)^{-\sigma} y_t + \omega\beta E_t (\pi_{t+1})^{\theta-1} p_{t+1}^D. \qquad (261)$$

Recursive form of the price level Recall that the price level is,

$$P_t = \left[\sum_{i=0}^{\infty} (1-\omega)\omega^i (P_{t-i}^*)^{1-\theta} di\right]^{\frac{1}{1-\theta}}.$$

In recursive form, it is,

$$P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}\ , \qquad 0 < \omega < 1 \tag{262}$$

since

$$\begin{aligned}\sum_{i=1}^{\infty}(1-\omega)\omega^i(P_{t-i}^*)^{1-\theta}di &= \omega\sum_{i=0}^{\infty}(1-\omega)\omega^i(P_{t-i-1}^*)^{1-\theta}di \\ &= \omega P_{t-1}^{1-\theta}.\end{aligned}$$

Deflating (262) by P_t gives,

$$1 = (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1}\ , \qquad 0 < \omega < 1. \tag{263}$$

Monetary policy rule in exponential form

$$\widehat{R}_t = \delta_\pi\widehat{\pi}_t + \delta_x\widehat{x}_t + \widehat{v}_t. \tag{264}$$

Returning to its log-difference form gives,

$$(\ln R_t - \ln R) = \delta_\pi\left(\ln \pi_t - \ln \pi\right) + \delta_x\left(\ln x_t - \ln x\right) + \left(\ln v_t - \ln v\right)$$

or

$$\ln\left(\frac{R_t}{R}\right) = \delta_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \delta_x \ln\left(\frac{x_t}{x}\right) + \ln\left(\frac{v_t}{v}\right).$$

Finally taking the exponential of both sides of above expression yields,

$$R_t = R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}\left(\frac{x_t}{x}\right)^{\delta_x}\left(\frac{v_t}{v}\right). \tag{265}$$

Price dispersion and its first-order autoregressive form All equations in our model are now in aggregate form except that the production function, $y_{jt} = Z_t N_{jt}$. Integrating both sides of the production function gives,

$$\int_0^1 y_{jt}dj = \int_0^1 z_t n_{jt}dj.$$

The right hand side of above equation is just $z_t n_t$ since there's no imperfections in labor market. However, we have the problem that $\int_0^1 y_{jt}dj \neq Y_t$. We then make use of the demand function, $y_{jt} = \left[\frac{p_{jt}}{P_t}\right]^{-\theta} y_t$, and insert it into the integral to obtain

$$\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} y_t dj = \int_0^1 z_t n_{jt} dj$$

or

$$\left(\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} dj\right) y_t = z_t n_t. \tag{266}$$

The above expression (266) still has an individual term p_{jt}. One way to solve this problem is to define another price level[93],

$$\left(\int_0^1 p_{jt}^{-\theta} dj\right)^{-\frac{1}{\theta}} = \bar{P}_t \tag{267}$$

and represent it in aggregate term as

$$\bar{P}_t^{-\theta} P_t^{\theta} y_t = z_t n_t$$

or

$$\left(\frac{\bar{P}_t}{P_t}\right)^{-\theta} y_t = z_t n_t. \tag{268}$$

The above new price level (267) can be written in recursive form as,

$$\bar{P}_t^{-\theta} = (1-\omega)(P_t^*)^{-\theta} + \omega \bar{P}_{t-1}^{-\theta}\ , \qquad 0 < \omega < 1. \tag{269}$$

Deflating gives

$$\overline{p}_t^{-\theta} = (1-\omega)(p_t^*)^{-\theta} + \omega \pi^{\theta} \overline{p}_{t-1}^{-\theta}\ , \qquad 0 < \omega < 1 \tag{270}$$

since we define $\overline{p}_t = \frac{\overline{P}_t}{P_t}$.

Another way to aggregate (266) is to focus on the integral $\int_0^1 \left[\frac{p_{jt}}{P_t}\right]^{-\theta} dj$, and define the price dispersion as

$$pd_t = \int_0^1 \left[\frac{P_{jt}}{P_t}\right]^{-\theta} dj. \tag{271}$$

This price dispersion is caused by the relative prices present in the NK model as a result of imperfect competition. Next we represent the price dispersion in the same way for the price level as

$$pd_t = \sum_{i=0}^{\infty} (1-\omega)\omega^i \left(\frac{P_{t-i}^*}{P_t}\right)^{-\theta}$$

which implies that it is composed by all surviving contracts. Finally, writing above result in recursive form yields,

$$pd_t = (1-\omega)\left(p_t^*\right)^{-\theta} + \omega \pi^{\theta} pd_{t-1} \tag{272}$$

which is the price dispersion we use in this chapter.

[93] Note that this price level is defined similar to the previous one: $P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$. However, introducing this price level is only to overcome the aggregation problem.

Non-linear equilibrium conditions Since there is no capital in the model, aggregate demand is simply $y_t = c_t$[94]. Also, we deflate the economy by the price level P_t, for example, $w_t = \frac{W_t}{P_t}$ and additionally $\pi_{t+1} = \frac{P_{t+1}}{P_t}$. Therefore, the non-linear system of the benchmark NK model is:

$$\frac{\chi e_t n_t^\eta}{c_t^{-\sigma}} = w_t \tag{273}$$

$$c_t^{-\sigma} = \beta R_t E_t(\frac{1}{\pi_{t+1}})c_{t+1}^{-\sigma} \tag{274}$$

$$\varphi_t = \frac{w_t}{z_t} \tag{275}$$

$$p_t^* = \frac{p_t^N}{p_t^D} \tag{276}$$

$$p_t^N = \left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t \varphi_t + \omega\beta E_t (\pi_{t+1})^\theta p_{t+1}^N \tag{277}$$

$$p_t^D = (c_t)^{-\sigma} y_t + \omega\beta E_t (\pi_{t+1})^{\theta-1} p_{t+1}^D \tag{278}$$

$$1 = (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1} \tag{279}$$

$$pd_t = (1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^\theta zp_t \tag{280}$$

$$zp_{t+1} = pd_t \tag{281}$$

$$pd_t y_t = z_t n_t \tag{282}$$

$$y_t = c_t \tag{283}$$

$$R_t = R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}\left(\frac{x_t}{x}\right)^{\delta_x}\left(\frac{v_t}{v}\right) \tag{284}$$

$$\frac{x_t}{x} = \left(\frac{y_t}{y}\right)\left(\frac{z_t}{z}\right)^{-m_z}\left(\frac{e_t}{e}\right)^{m_e} \tag{285}$$

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{z,t+1},\ \varepsilon_{zt+1} \sim N(0,\sigma_{\varepsilon_z}^2) \tag{286}$$

$$\ln v_{t+1} = \rho_v \ln v_t + \varepsilon_{v,t+1},\ \varepsilon_{v,t+1} \sim N(0,\sigma_{\varepsilon_v}^2). \tag{287}$$

$$\ln(e_{t+1}) = \rho_e \ln(e_t) + \varepsilon_{e,t+1},\ \varepsilon_{e,t+1} \sim N(0,\sigma_{\varepsilon_e}^2) \tag{288}$$

Note that the expression for output gap x_t will be defined below in 8.3.3.

[94] It can also be verified that by substituting the firm's profit function (247) into the household's budget constraint (240), the aggregate demand function $y_t = c_t$ holds.

The deterministic Steady-state Since inflation can be present in the steady-state of the model, we assign two values for π:

$$\pi = 1 \tag{289}$$

and

$$\pi = 1.03^{1/4} \tag{290}$$

where $1.03^{1/4}$ is the inflation rate adjusted for quarterly US data.

The rest of the steady-state variables can be solved analytically as follows:

$$p^* = \left(\frac{1-\omega\pi^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} \tag{291}$$

$$pd = \left(\frac{1-\omega}{1-\omega\pi^{\theta}}\right)(p^*)^{-\theta} \tag{292}$$

$$\varphi = p^* \left(\frac{\theta}{\theta-1}\right)^{-1} \left(\frac{1-\omega\beta\pi^{\theta}}{1-\omega\beta\pi^{\theta-1}}\right) \tag{293}$$

$$w = \varphi \tag{294}$$

$$c = \left(\frac{w}{\chi pd^{\eta}}\right)^{\frac{1}{\sigma+\eta}} \tag{295}$$

$$y = c \tag{296}$$

$$n = pdy \tag{297}$$

$$R = \frac{1}{\beta}\pi \tag{298}$$

$$p^D = \frac{(c)^{-\sigma} y}{1-\omega\beta(\pi)^{\theta-1}} \tag{299}$$

$$p^N = p^* p^D \tag{300}$$

$$zp \equiv pd \tag{301}$$

$$x = 1 \tag{302}$$

$$z = 1 \tag{303}$$

$$v = 1 \tag{304}$$

$$e = 1 \tag{305}$$

3.7.2 Linear approximation

Log-linearizing the non-linear system Here we log-linearize the non-linear system by representing the variables as percentage deviations from their constant steady-state values. The strategy used here is that we first take natural logs of all the equations in the non-linear system and then differentiate the resulting logged equations at the steady-state with respect to time. The derivations of the log-linearized equations are shown below:

$$
\begin{aligned}
\frac{\chi e_t n_t^{\eta}}{c_t^{-\sigma}} &= w_t \\
\ln \chi + \ln(e_t) + \eta \ln(n_t) + \sigma \ln(c_t) &= \ln(w_t) \\
\frac{d\left[\ln \chi + \ln(e_t) + \eta \ln(n_t) + \sigma \ln(c_t)\right]}{dt} &= \frac{d \ln(w_t)}{dt} \\
0 + \frac{1}{e}\frac{de_t}{dt} + \frac{\eta}{n}\frac{dn_t}{dt} + \frac{\sigma}{c}\frac{dc_t}{dt} &= \frac{1}{w}\frac{dw_t}{dt} \\
\eta \widehat{n}_t + \widehat{e}_t &= -\sigma \widehat{c}_t + \widehat{w}_t \qquad (306)
\end{aligned}
$$

$$
\begin{aligned}
c_t^{-\sigma} &= \beta R_t E_t\left(\frac{1}{\pi_{t+1}}\right) c_{t+1}^{-\sigma} \\
-\sigma \ln(c_t) &= \ln \beta + \ln(R_t) - E_t \ln(\pi_{t+1}) - \sigma \ln(c_{t+1}) \\
-\sigma \frac{d \ln(c_t)}{dt} &= \frac{d\left[\ln \beta + \ln(R_t) - E_t \ln(\pi_{t+1}) - \sigma \ln(c_{t+1})\right]}{dt} \\
-\frac{\sigma}{c}\frac{dc_t}{dt} &= 0 + \frac{1}{R}\frac{dR_t}{dt} - E_t \frac{1}{\pi}\frac{d\pi_{t+1}}{dt} - \frac{\sigma}{c}\frac{dc_{t+1}}{dt} \\
-\sigma \widehat{c}_t &= \widehat{R}_t - \sigma E_t \widehat{c}_{t+1} - E_t \widehat{\pi}_{t+1} \qquad (307)
\end{aligned}
$$

$$
\begin{aligned}
\varphi_t &= \frac{w_t}{z_t} \\
\ln(\varphi_t) &= \ln(w_t) - \ln(z_t) \\
\frac{d \ln(\varphi_t)}{dt} &= \frac{d \ln(w_t)}{dt} - \frac{d \ln(z_t)}{dt} \\
\frac{1}{\varphi}\frac{d\varphi_t}{dt} &= \frac{1}{w}\frac{dw_t}{dt} - \frac{1}{z}\frac{dz_t}{dt} \\
\widehat{\varphi}_t &= \widehat{w}_t - \widehat{z}_t \qquad (308)
\end{aligned}
$$

$$
\begin{aligned}
p_t^* &= \frac{p_t^N}{p_t^D} \\
\ln(p_t^*) &= \ln\left(p_t^N\right) - \ln\left(p_t^D\right) \\
\frac{d \ln(p_t^*)}{dt} &= \frac{d\left[\ln\left(p_t^N\right) - \ln\left(p_t^D\right)\right]}{dt} \\
\frac{1}{p^*}\frac{dp_t^*}{dt} &= \frac{1}{p^N}\frac{dp_t^N}{dt} - \frac{1}{p^D}\frac{dp_t^D}{dt} \\
\widehat{p_t^*} &= \widehat{p_t^N} - \widehat{p_t^D} \qquad (309)
\end{aligned}
$$

$$p_t^N = \left(\tfrac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t\varphi_t + \omega\beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N$$
$$\ln\left(p_t^N\right) = \ln\left[\left(\tfrac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t\varphi_t + \omega\beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N\right]$$
$$\tfrac{d\ln\left(p_t^N\right)}{dt} = \tfrac{d\ln\left[\left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t\varphi_t + \omega\beta E_t(\pi_{t+1})^{\theta} p_{t+1}^N\right]}{dt}$$
$$\tfrac{1}{p^N}\tfrac{dp_t^N}{dt} = \tfrac{1}{p^N dt}\left[\begin{array}{c}\left(\frac{\theta}{\theta-1}\right)(-\sigma)(c)^{-\sigma-1} y\varphi dc_t + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma}\varphi dy_t + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y d\varphi_t \\ +\omega\beta\theta(\pi)^{\theta-1} E_t p^N d\pi_{t+1} + \omega\beta E_t(\pi)^{\theta} dp_{t+1}^N\end{array}\right]$$
$$\tfrac{1}{p^N}\tfrac{dp_t^N}{dt} = \tfrac{1}{p^N dt}\left[\begin{array}{c}\left(\frac{\theta}{\theta-1}\right)(-\sigma)(c)^{-\sigma} y\varphi \frac{dc_t}{c} + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi\frac{dy_t}{y} + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi\frac{d\varphi_t}{\varphi} \\ +\omega\beta\theta(\pi)^{\theta} p^N E_t\frac{d\pi_{t+1}}{\pi} + \omega\beta(\pi)^{\theta} p^N E_t\frac{dp_{t+1}^N}{p^N}\end{array}\right]$$
$$\tfrac{1}{p^N}\tfrac{dp_t^N}{dt} = \tfrac{1}{p^N dt}\left[\begin{array}{c}\left(\frac{\theta}{\theta-1}\right)(-\sigma)(c)^{-\sigma} y\varphi\widehat{c}_t + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi\widehat{y}_t + \left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi\widehat{\varphi}_t \\ +\omega\beta\theta(\pi)^{\theta} p^N E_t\widehat{\pi}_{t+1} + \omega\beta(\pi)^{\theta} p^N E_t\widehat{p}_{t+1}^N\end{array}\right]$$
$$\widehat{p}_t^N = \tfrac{1}{p^N}\left(\tfrac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi\left(-\sigma\widehat{c}_t + \widehat{y}_t + \widehat{\varphi}_t\right) + \omega\beta(\pi)^{\theta} E_t\left(\theta\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^N\right)$$

Since in the steady-state we have $\left(1-\omega\beta\pi^{\theta}\right) = \frac{1}{p^N}\left(\frac{\theta}{\theta-1}\right)(c)^{-\sigma} y\varphi$, we can then simplify the above expression as

$$\widehat{p}_t^N = \left(1-\omega\beta\pi^{\theta}\right)\left(-\sigma\widehat{c}_t + \widehat{y}_t + \widehat{\varphi}_t\right) + \omega\beta E_t(\pi)^{\theta}\left(\theta\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^N\right). \tag{310}$$

$$\begin{aligned}
p_t^D &= (c_t)^{-\sigma} y_t + \omega\beta E_t(\pi_{t+1})^{\theta-1} p_{t+1}^D \\
\ln\left(p_t^D\right) &= \ln\left[(c_t)^{-\sigma} y_t + \omega\beta E_t(\pi_{t+1})^{\theta-1} p_{t+1}^D\right] \\
\frac{d\ln\left(p_t^D\right)}{dt} &= \frac{d\ln\left[(c_t)^{-\sigma} y_t + \omega\beta E_t(\pi_{t+1})^{\theta-1} p_{t+1}^D\right]}{dt} \\
\frac{1}{p^D}\frac{dp_t^D}{dt} &= \frac{1}{p^D dt}\left[\begin{array}{c}(-\sigma)(c)^{-\sigma-1} y dc_t + (c)^{-\sigma} dy_t \\ +\omega\beta(\theta-1)E_t(\pi)^{\theta-2} p^D d\pi_{t+1} + \omega\beta E_t(\pi)^{\theta-1} dp_{t+1}^D\end{array}\right] \\
\frac{1}{p^D}\frac{dp_t^D}{dt} &= \frac{1}{p^D}\left[\begin{array}{c}(-\sigma)(c)^{-\sigma} y\frac{dc_t}{c} + (c)^{-\sigma} y\frac{dy_t}{y} \\ +\omega\beta(\theta-1)E_t(\pi)^{\theta-1} p^D\frac{d\pi_{t+1}}{\pi} + \omega\beta E_t(\pi)^{\theta-1} p^D\frac{dp_{t+1}^D}{p^D}\end{array}\right] \\
\widehat{p}_t^D &= \frac{1}{p^D}(c)^{-\sigma} y\left(-\sigma\widehat{c}_t + \widehat{y}_t\right) + \omega\beta E_t(\pi)^{\theta-1}\left((\theta-1)\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^D\right)
\end{aligned}$$

Since in the steady-state we have, $\left(1-\omega\beta\pi^{\theta-1}\right) = \frac{1}{p^D}(c)^{-\sigma} y$, we can then simplify the above expression as

$$\widehat{p}_t^D = \left(1-\omega\beta\pi^{\theta-1}\right)\left(-\sigma\widehat{c}_t + \widehat{y}_t\right) + \omega\beta E_t(\pi)^{\theta-1}\left((\theta-1)\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^D\right). \tag{311}$$

$$
\begin{aligned}
1 &= (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1} \\
\ln(1) &= \ln\left[(1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1}\right] \\
0 &= \frac{d\ln\left[(1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1}\right]}{dt} \\
0 &= \frac{1}{dt}\left[(1-\omega)(1-\theta)\,dp_t^*(p^*)^{-\theta} + \omega(\theta-1)\,d\pi_t\pi^{\theta-2}\right] \\
0 &= \frac{1}{dt}\left[(1-\omega)(1-\theta)\frac{dp_t^*}{p^*}(p^*)^{1-\theta} + \omega(\theta-1)\frac{d\pi_t}{\pi}\pi^{\theta-1}\right] \\
0 &= \frac{1}{dt}\left[(1-\omega)\frac{dp_t^*}{p^*}(p^*)^{1-\theta} - \omega\frac{d\pi_t}{\pi}\pi^{\theta-1}\right] \\
0 &= (1-\omega)(p^*)^{1-\theta}\widehat{p}_t^* - \omega\pi^{\theta-1}\widehat{\pi}_t
\end{aligned}
$$

Since in steady-state we have $(1-\omega)(p^*)^{1-\theta} = (1-\omega\pi^{\theta-1})$, we can then simplify the above expression as

$$0 = (1-\omega\pi^{\theta-1})\widehat{p}_t^* - \omega\pi^{\theta-1}\widehat{\pi}_t$$

or

$$(1-\omega\pi^{\theta-1})\widehat{p}_t^* = \omega\pi^{\theta-1}\widehat{\pi}_t. \tag{312}$$

$$
\begin{aligned}
pd_t &= (1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^{\theta}zp_t \\
\ln(pd_t) &= \ln\left[(1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^{\theta}zp_t\right] \\
\frac{d\ln(pd_t)}{dt} &= \frac{d\ln\left[(1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^{\theta}zp_t\right]}{dt} \\
\frac{1}{pd}\frac{dpd_t}{dt} &= \frac{1}{pd}\left[\frac{d(1-\omega)(p_t^*)^{-\theta}}{dt} + \frac{d\omega\pi_t^{\theta}zp_t}{dt}\right] \\
\widehat{pd}_t &= \frac{1}{pd}\left[\frac{(1-\omega)(-\theta)(p^*)^{-\theta-1}dp_t^*}{dt} + \frac{\omega\theta\pi^{\theta-1}zpd\pi_t}{dt} + \frac{\omega\pi^{\theta}\pi dzp_t}{dt}\right] \\
pd\widehat{pd}_t &= -\theta(1-\omega)(p^*)^{-\theta}\,\widehat{p}_t^* + \omega\pi^{\theta}zp\left(\theta\widehat{\pi}_t + \widehat{zp}_t\right) \qquad (313)
\end{aligned}
$$

$$
\begin{aligned}
zp_{t+1} &= pd_t \\
\ln(zp_{t+1}) &= \ln(pd_t) \\
\frac{d\ln(zp_{t+1})}{dt} &= \frac{d\ln(pd_t)}{dt} \\
\frac{1}{zp}\frac{dzp_{t+1}}{dt} &= \frac{1}{pd}\frac{dpd_t}{dt} \\
\widehat{zp}_{t+1} &= \widehat{pd}_t \qquad (314)
\end{aligned}
$$

$$
\begin{aligned}
pd_t y_t &= z_t n_t \\
\ln(pd_t) + \ln(y_t) &= \ln(z_t) + \ln(n_t) \\
\frac{d\ln(pd_t)}{dt} + \frac{d\ln(y_t)}{dt} &= \frac{d\ln(z_t)}{dt} + \frac{d\ln(n_t)}{dt} \\
\frac{1}{pd}\frac{dpd_t}{dt} + \frac{1}{y}\frac{dy_t}{dt} &= \frac{1}{z}\frac{dz_t}{dt} + \frac{1}{n}\frac{dn_t}{dt}
\end{aligned}
$$

$$\widehat{pd}_t + \widehat{y}_t = \widehat{z}_t + \widehat{n}_t \tag{315}$$

$$\begin{aligned}
y_t &= c_t \\
\ln(y_t) &= \ln(c_t) \\
\frac{d\ln(y_t)}{dt} &= \frac{d\ln(c_t)}{dt} \\
\frac{1}{y}\frac{dy_t}{dt} &= \frac{1}{c}\frac{dc_t}{dt} \\
\widehat{y}_t &= \widehat{c}_t
\end{aligned} \tag{316}$$

$$\begin{aligned}
R_t &= R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi} v_t \\
\ln(R_t) &= \ln R + \delta_\pi \ln(\pi_t) - \delta_\pi \ln(\pi) + \ln(v_t) \\
\frac{d\ln(R_t)}{dt} &= \frac{d\ln R}{dt} + \frac{\delta_\pi d\ln(\pi_t)}{dt} - \delta_\pi \frac{d\ln(\pi)}{dt} + \frac{d\ln(v_t)}{dt} \\
\frac{1}{R}\frac{dR_t}{dt} &= 0 + \delta_\pi \frac{1}{\pi}\frac{d\pi_t}{dt} - 0 + \frac{1}{v}\frac{dv_t}{dt} \\
\widehat{R}_t &= \delta_\pi \widehat{\pi}_t + \widehat{v}_t
\end{aligned} \tag{317}$$

$$\begin{aligned}
\ln(z_{t+1}) &= \rho_z \ln(z_t) + \varepsilon_{z,t+1},\ \varepsilon_{zt+1} \sim N(0, \sigma^2_{\varepsilon_z}) \\
E_t \ln(z_{t+1}) &= \rho_z \ln(z_t) \\
E_t \frac{d\ln(z_{t+1})}{dt} &= \rho_z \frac{d\ln(z_t)}{dt} \\
E_t \frac{1}{z}\frac{dz_{t+1}}{dt} &= \frac{\rho_z}{z}\frac{dz_t}{dt} \\
E_t(\widehat{z}_{t+1}) &= \rho_z(\widehat{z}_t)
\end{aligned} \tag{318}$$

$$\begin{aligned}
\ln(v_{t+1}) &= \rho_v \ln(v_t) + \varepsilon_{v,t+1},\ \varepsilon_{vt+1} \sim N(0, \sigma^2_{\varepsilon_v}) \\
E_t \ln(v_{t+1}) &= \rho_v \ln(v_t) \\
E_t \frac{d\ln(v_{t+1})}{dt} &= \rho_v \frac{d\ln(v_t)}{dt} \\
E_t \frac{1}{v}\frac{dv_{t+1}}{dt} &= \frac{\rho_v}{v}\frac{dv_t}{dt} \\
E_t(\widehat{v}_{t+1}) &= \rho_v(\widehat{v}_t)
\end{aligned} \tag{319}$$

$$\begin{aligned}
\ln(e_{t+1}) &= \rho_e \ln(e_t) + \varepsilon_{e,t+1},\ \varepsilon_{et+1} \sim N(0, \sigma^2_{\varepsilon_e}) \\
E_t \ln(e_{t+1}) &= \rho_e \ln(e_t) \\
E_t \frac{d\ln(e_{t+1})}{dt} &= \rho_e \frac{d\ln(e_t)}{dt} \\
E_t \frac{1}{e}\frac{de_{t+1}}{dt} &= \frac{\rho_e}{e}\frac{de_t}{dt} \\
E_t(\widehat{e}_{t+1}) &= \rho_e(\widehat{e}_t)
\end{aligned} \tag{320}$$

Solving the linearized system using Klein's (2000) method

Matrix form of the linearized model

Static measurement variables:

$$\left\{\begin{array}{c}\widehat{y}_t=\widehat{c}_t\\ \widehat{p}_t^{*}=\widehat{p}_t^{N}-\widehat{p}_t^{D}\\ \widehat{R}_t=\delta_\pi\widehat{\pi}_t+\delta_x\widehat{x}_t+\widehat{v}_t\\ \widehat{x}_t=\widehat{y}_t-m_z\widehat{z}_t+m_e\widehat{e}_t\end{array}\right\} \tag{321}$$

or

$$\mathbf{N}_{mm}\mathbf{Y}_t=\mathbf{N}_{mn}\mathbf{X}_t+\mathbf{N}_{mx}\mathbf{Z}_t \tag{322}$$

$$\overbrace{\begin{bmatrix}1&0&0&0\\0&1&0&0\\-\delta_x&0&1&0\\-1&0&0&1\end{bmatrix}}^{\mathbf{N}_{mm}}\overbrace{\begin{bmatrix}\widehat{y}_t\\\widehat{p}_t^{*}\\\widehat{R}_t\\\widehat{x}_t\end{bmatrix}}^{\mathbf{Y}_t}=\overbrace{\begin{bmatrix}0&1&0&0&0&0&0&0&0\\0&0&0&0&0&1&-1&0&0\\0&0&0&\delta_\pi&0&0&0&0&0\\0&0&0&0&0&0&0&0&0\end{bmatrix}}^{\mathbf{N}_{mn}}\overbrace{\begin{bmatrix}\widehat{n}_t\\\widehat{c}_t\\\widehat{w}_t\\\widehat{\pi}_t\\\widehat{\varphi}_t\\\widehat{p}_t^{N}\\\widehat{p}_t^{D}\\\widehat{pd}_t\\\widehat{zp}_t\end{bmatrix}}^{\mathbf{X}_t}$$

$$+\overbrace{\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\\-\delta_x m_z&0&\delta_x m_e\end{bmatrix}}^{\mathbf{N}_{mx}}\overbrace{\begin{bmatrix}\widehat{z}_t\\\widehat{v}_t\\\widehat{e}_t\end{bmatrix}}^{\mathbf{Z}_t}$$

Dynamic endogenous variables:

$$\left\{\begin{array}{c}
\eta\widehat{n}_t+\widehat{e}_t=-\sigma\widehat{c}_t+\widehat{w}_t \\
-\sigma\widehat{c}_t=\widehat{R}_t-\sigma E_t\widehat{c}_{t+1}-E_t\widehat{\pi}_{t+1} \\
\widehat{\varphi}_t=\widehat{w}_t-\widehat{z}_t \\
\widehat{p}_t^N=\left(1-\omega\beta\pi^{\theta}\right)\left(-\sigma\widehat{c}_t+\widehat{y}_t+\widehat{\varphi}_t\right)+E_t\omega\beta\left(\pi\right)^{\theta}\left(\theta\widehat{\pi}_{t+1}+\widehat{p}_{t+1}^N\right) \\
\widehat{p}_t^D=\left(1-\omega\beta\pi^{\theta-1}\right)\left(-\sigma\widehat{c}_t+\widehat{y}_t\right)+\omega\beta E_t\left(\pi\right)^{\theta-1}\left(\left(\theta-1\right)\widehat{\pi}_{t+1}+\widehat{p}_{t+1}^D\right) \\
(1-\omega\pi^{\theta-1})\widehat{p}_t^*=\omega\pi^{\theta-1}\widehat{\pi}_t \\
pd\widehat{pd}_t=-\theta(1-\omega)\left(p^*\right)^{-\theta}\widehat{p}_t^*+\omega\pi^{\theta}zp\left(\theta\widehat{\pi}_t+\widehat{zp}_t\right) \\
\widehat{zp}_{t+1}=\widehat{pd}_t \\
\widehat{pd}_t+\widehat{y}_t=\widehat{z}_t+\widehat{n}_t
\end{array}\right\} \tag{323}$$

or

$$\mathbf{M}_{nn}^1\mathbf{E}_t\mathbf{X}_{t+1}+\mathbf{M}_{nm}^1\mathbf{E}_t\mathbf{Y}_{t+1}+\mathbf{M}_{nx}^1\mathbf{E}_t\mathbf{Z}_{t+1}=\mathbf{M}_{nn}^0\mathbf{X}_t+\mathbf{M}_{nm}^0\mathbf{Y}_t+\mathbf{M}_{nx}^0\mathbf{Z}_t, \tag{324}$$

$$\overbrace{\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\omega\beta\left(\pi\right)^{\theta}\theta & 0 & -\omega\beta\left(\pi\right)^{\theta} & 0 & 0 & 0 \\
0 & 0 & 0 & -\omega\beta\left(\pi\right)^{\theta-1}\left(\theta-1\right) & 0 & 0 & -\omega\beta\left(\pi\right)^{\theta-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}}^{\mathbf{M}_{nn}^1} \overbrace{E_t\begin{bmatrix}
\widehat{n}_{t+1} \\ \widehat{c}_{t+1} \\ \widehat{w}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{\varphi}_{t+1} \\ \widehat{p}_{t+1}^N \\ \widehat{p}_{t+1}^D \\ \widehat{pd}_{t+1} \\ \widehat{zp}_{t+1}
\end{bmatrix}}^{\mathbf{X}_{t+1}}$$

$$+\overbrace{\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}}^{\mathbf{M}_{nm}^1} E_t\overbrace{\begin{bmatrix}
\widehat{y}_{t+1} \\ \widehat{p}_{t+1}^* \\ \widehat{R}_{t+1} \\ \widehat{x}_{t+1}
\end{bmatrix}}^{\mathbf{Y}_{t+1}}+\overbrace{\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}}^{\mathbf{M}_{nx}^1} E_t\overbrace{\begin{bmatrix}
\widehat{z}_{t+1} \\ \widehat{v}_{t+1} \\ \widehat{e}_{t+1}
\end{bmatrix}}^{\mathbf{Z}_{t+1}}=$$

$$
\underbrace{\begin{bmatrix}
-\eta & -\sigma & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -\left(1-\omega\beta\pi^{\theta}\right)\sigma & 0 & 0 & \left(1-\omega\beta\pi^{\theta}\right) & -1 & 0 & 0 & 0 \\
0 & -\left(1-\omega\beta\pi^{\theta-1}\right)\sigma & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & \omega\pi^{\theta-1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \omega\pi^{\theta} zp\theta & 0 & 0 & 0 & -pd & \omega\pi^{\theta} zp \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}}_{\mathbf{M}^0_{nn}}
\underbrace{\begin{bmatrix}
\widehat{n}_t \\ \widehat{c}_t \\ \widehat{w}_t \\ \widehat{\pi}_t \\ \widehat{\varphi}_t \\ \widehat{p}^N_t \\ \widehat{p}^D_t \\ \widehat{pd}_t \\ \widehat{zp}_t
\end{bmatrix}}_{\mathbf{X}_t}
$$

$$
+\underbrace{\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\left(1-\omega\beta\pi^{\theta}\right) & 0 & 0 & 0 \\
\left(1-\omega\beta\pi^{\theta-1}\right) & 0 & 0 & 0 \\
0 & -(1-\omega\pi^{\theta-1}) & 0 & 0 \\
0 & -\theta(1-\omega)\left(p^{*}\right)^{-\theta} & 0 & 0 \\
0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{bmatrix}}_{\mathbf{M}^0_{nm}}
\underbrace{\begin{bmatrix}
\widehat{y}_t \\ \widehat{p}^{*}_t \\ \widehat{R}_t \\ \widehat{x}_t
\end{bmatrix}}_{\mathbf{Y}_t}
+\underbrace{\begin{bmatrix}
0 & 0 & -1 \\
0 & 0 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}}_{\mathbf{M}^0_{nx}}
\underbrace{\begin{bmatrix}
\widehat{z}_t \\ \widehat{v}_t \\ \widehat{e}_t
\end{bmatrix}}_{\mathbf{Z}_t}
$$

Dynamic exogenous variables:

$$
\left\{\begin{array}{l}
\widehat{z}_{t+1}=\rho_z\widehat{z}_t+\varepsilon_{z,t+1} \\
\widehat{v}_{t+1}=\rho_v\widehat{v}_t+\varepsilon_{v,t+1} \\
\widehat{e}_{t+1}=\rho_e\widehat{e}_t+\varepsilon_{e,t+1}
\end{array}\right\} \tag{325}
$$

or

$$
\mathbf{E}_t\mathbf{Z}_{t+1}=\boldsymbol{\rho}\mathbf{Z}_t \tag{326}
$$

$$
E_t\overbrace{\begin{bmatrix}
\widehat{z}_{t+1} \\ \widehat{v}_{t+1} \\ \widehat{e}_{t+1}
\end{bmatrix}}^{\mathbf{Z}_{t+1}}
=\overbrace{\begin{bmatrix}
\rho_z & 0 & 0 \\
0 & \rho_v & 0 \\
0 & 0 & \rho_e
\end{bmatrix}}^{\boldsymbol{\rho}}
\overbrace{\begin{bmatrix}
\widehat{z}_t \\ \widehat{v}_t \\ \widehat{e}_t
\end{bmatrix}}^{\mathbf{Z}_t}
$$

Transformation To use Klein's method, the above three equations need to be combined together and transformed as,

$$\mathbf{A}\mathbf{E}_t\mathbf{X}_{t+1} = \mathbf{B}\mathbf{X}_t + \mathbf{C}\mathbf{Z}_t \tag{327}$$

where

$$\mathbf{A} = \left(\mathbf{M}^1_{nn} + \mathbf{M}^1_{nm}\mathbf{N}^{-1}_{mm}\mathbf{N}_{mn}\right)$$

$$\mathbf{B} = \left(\mathbf{M}^0_{nn} + \mathbf{M}^0_{nm}\mathbf{N}^{-1}_{mm}\mathbf{N}_{mn}\right)$$

and

$$\mathbf{C} = \left[\mathbf{M}^0_{nm}\mathbf{N}^{-1}_{mm}\mathbf{N}_{mx} + \mathbf{M}^0_{nx}\right] - \left[\mathbf{M}^1_{nm}\mathbf{N}^{-1}_{mm}\mathbf{N}_{mx} + \mathbf{M}^1_{nx}\right]\boldsymbol{\rho}.$$

Applying Klein's code The Matlab script to solve the above linear system (327) is *solvek.m*. The format of Klein's function is [**M**, **N**, **P**, **Q**]=solve **k**(**A**, **B**, **C**, **RHO**, **ns**). The inputs are matrices **A**, **B**, **C** and $\boldsymbol{\rho}$ discussed above. The outputs are the coefficient matrices that solve the linearized system in the following form:

$$\widehat{\mathbf{X}}^b_{t+1} = \mathbf{P}\widehat{\mathbf{X}}^b_t + \mathbf{Q}\widehat{\mathbf{Z}}_t \tag{328}$$

$$\widehat{\mathbf{Z}}_{t+1} = \boldsymbol{\rho}\widehat{\mathbf{Z}}_t + \boldsymbol{\varepsilon}_{t+1} \tag{329}$$

$$\widehat{\mathbf{X}}^f_t = \mathbf{M}\widehat{\mathbf{X}}^b_t + \mathbf{N}\widehat{\mathbf{Z}}_t \tag{330}$$

$$\widehat{\mathbf{Y}}_t = \mathbf{V}\widehat{\mathbf{X}}^b_t + \mathbf{W}\widehat{\mathbf{Z}}_t. \tag{331}$$

where $\widehat{\mathbf{X}}^b_t$ includes the predetermined endogenous variable $\widehat{zp}_t$, $\widehat{\mathbf{Z}}_t$ includes the predetermined exogenous variables $\widehat{z}_t$, $\widehat{v}_t$ and $\widehat{e}_t$, $\widehat{\mathbf{X}}^f_t$ consists of the non-predetermined endogenous variables $\widehat{n}_t$, $\widehat{c}_t$, $\widehat{w}$, $\widehat{\pi}_t$, $\widehat{\varphi}_t$, $\widehat{p}^N_t$, $\widehat{p}^D_t$, $\widehat{pd}_t$ and $\widehat{\mathbf{Y}}_t$ contains the variables of interest which are $\widehat{y}_t$, $\widehat{p}^*_t$ and $\widehat{R}_t$.

Simulating the model using linear solution The above solution can be used to simulate the model in a recursive way. For example, given a draw of innovations of exogenous shocks ε_{t+1} and the starting value (we use the steady-state values) of the predetermined variables ($\widehat{\mathbf{X}}^b_t$ and $\widehat{\mathbf{Z}}_t$), the next period predetermined variables can be simulated using (328) and (329),

$$\widehat{\mathbf{X}}^b_{t+1} = \mathbf{P}\widehat{\mathbf{X}}^b_t + \mathbf{Q}\widehat{\mathbf{Z}}_t$$

$$\widehat{\mathbf{Z}}_{t+1} = \boldsymbol{\rho}\widehat{\mathbf{Z}}_t + \boldsymbol{\varepsilon}_{t+1}$$

or

$$\begin{bmatrix} \widehat{\mathbf{X}}^b_{t+1} \\ \widehat{\mathbf{Z}}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{Q} \\ \mathbf{0}_{3\times 1} & \boldsymbol{\rho} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{X}}^b_t \\ \widehat{\mathbf{Z}}_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I}_{3\times 3} \end{bmatrix} \begin{bmatrix} 0 \\ \boldsymbol{\varepsilon}_{t+1} \end{bmatrix}$$

or

$$\mathbf{S}_{t+1} = \mathbf{MSSS}_t + \mathbf{MSE}\boldsymbol{\varepsilon}_{t+1} \tag{332}$$

with $\mathbf{MSS} = \begin{bmatrix} \mathbf{P} & Q \\ \mathbf{0}_{3\times1} & \boldsymbol{\rho} \end{bmatrix}$ and $\mathbf{MSE} = \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}$.

The non-predetermined endogenous variables are then simulated using (330),

$$\widehat{\mathbf{X}}_t^f = \mathbf{M}\widehat{\mathbf{X}}_t^b + \mathbf{N}\widehat{\mathbf{Z}}_t$$

which in matrix form is

$$\widehat{\mathbf{X}}_t^f = \begin{bmatrix} \mathbf{M} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{X}}_t^b \\ \widehat{\mathbf{Z}}_t \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{0}_{3\times3} \end{bmatrix} \begin{bmatrix} 0 \\ \boldsymbol{\varepsilon}_{t+1} \end{bmatrix}$$

or more succinctly,

$$\widehat{\mathbf{X}}_t^f = \boldsymbol{\Gamma}\mathbf{S}_t + \mathbf{D}\boldsymbol{\varepsilon}_{t+1} \tag{333}$$

with $\Gamma = \begin{bmatrix} \mathbf{M} & \mathbf{N} \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 0 & \mathbf{0}_{3\times3} \end{bmatrix}$.

The variables of interest in the measurement equation are next generated using (331),

$$\widehat{\mathbf{Y}}_t = \mathbf{V}\widehat{\mathbf{X}}_t^b + \mathbf{W}\widehat{\mathbf{Z}}_t$$

which in matrix form is

$$\widehat{\mathbf{Y}}_t = \begin{bmatrix} \mathbf{S} & \mathbf{T} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{X}}_t^b \\ \widehat{\mathbf{Z}}_t \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{0}_{3\times3} \end{bmatrix} \boldsymbol{\varepsilon}_{t+1}$$

or more succinctly,

$$\widehat{\mathbf{Y}}_t = \boldsymbol{\Phi}\boldsymbol{\Gamma}_t + \mathbf{G}\boldsymbol{\varepsilon}_{t+1} \tag{334}$$

with $\boldsymbol{\Phi} = \begin{bmatrix} \mathbf{S} & \mathbf{T} \end{bmatrix}$ and $\mathbf{G} = \begin{bmatrix} 0 & \mathbf{0}_{3\times3} \end{bmatrix}$.

The above simulation of the linearized model can also be done using the matlab function *dlsim.m* which takes the form: $[\widehat{\mathbf{X}}^{\mathbf{f}}, \boldsymbol{\Gamma}]$=**dlsim**(**MSS**, **MSE**, Γ, **D**, **ee**) where the inputs have been defined above. The output $\widehat{\mathbf{X}}^f$ contains simulated series of the non-predetermined endogenous variables (in our case, $\widehat{n}_t$, $\widehat{c}_t$, $\widehat{w}$, $\widehat{\pi}_t$, $\widehat{\varphi}_t$, $\widehat{p}_t^N$, $\widehat{p}_t^D$ and $\widehat{pd_t}$) and Γ contains simulated series of the predetermined variables (in our case, $\widehat{zp}_t$ $\widehat{z}_t$, $\widehat{v}_t$ and $\widehat{e}_t$). Note that, the solution of $\boldsymbol{\Phi}$ (i.e., $\mathbf{S}$ and $\mathbf{T}$) for the variables of interest are not directly given in this function. They can be obtained from the measurement equation (322) as $\mathbf{V} = \mathbf{N}_{mm}^{-1}\mathbf{N}_{mn}$ and $\mathbf{W} = \mathbf{N}_{mm}^{-1}\mathbf{N}_{mx}$.

Flexible price equilibrium of log-linearized system and the output gap In the NK economy, policy makers are not only concerned with the inefficiency caused by the imperfect competition of firms but also the slow adjustment of nominal prices. Particularly, given the assumption many authors usually put that the inefficiency of the imperfect competition can be removed by applying an offsetting fiscal policy, the focus of policy analysis in the NK model falls in designing an optimal monetary policy to minimize the gap between output under flexible prices and sticky prices. This concept of the output gap is illustrated and mathematically derived as follow.

Firstly we start with the equilibrium of the model when all prices are flexible. Note that flexible prices imply that all firms are able to change their prices each time period, so we have

$$\omega = 0.$$

Therefore equation (312) simplifies to

$$\widehat{p}_t^{*f} = 0. \tag{335}$$

Substituting this result into (206) and (208) gives the price dispersion and output under flexible prices,

$$\widehat{pd}_t^f = 0 \tag{336}$$

and

$$\widehat{y}_t^f = \widehat{z}_t + \widehat{n}_t^f. \tag{337}$$

Also since $\widehat{p}_t^{*f} = 0$, the log-linearized price-setting equations (202), (203) and (204) now become

$$\begin{aligned} 0 &= \widehat{p}_t^N - \widehat{p}_t^D \\ \widehat{p}_t^N &= (1-0)\left(-\sigma\widehat{c}_t + \widehat{y}_t + \widehat{\varphi}_t\right) \\ \widehat{p}_t^D &= (1-0)\left(-\sigma\widehat{c}_t + \widehat{y}_t\right) + 0. \end{aligned}$$

We then combine them to get

$$\widehat{\varphi}_t^f = 0. \tag{338}$$

Substituting this result into the marginal cost equation (201) gives

$$\widehat{w}_t^f - \widehat{z}_t = 0. \tag{339}$$

We can now summarize the following flexible price equilibrium conditions including the production function, the marginal cost, the consumption and labor decision and the aggregate demand as:

$$\begin{aligned}
\widehat{y}_t^f &= \widehat{z}_t + \widehat{n}_t^f \\
\widehat{w}_t^f - \widehat{z}_t &= 0 \\
\eta \widehat{n}_t^f + \widehat{e}_t &= -\sigma \widehat{c}_t^f + \widehat{w}_t^f \\
\widehat{y}_t^f &= \widehat{c}_t^f.
\end{aligned}$$

Combining the above conditions to eliminate $\widehat{\varphi}_t^f, \widehat{n}_t^f, \widehat{c}_t^f, \widehat{w}_t^f$ gives the output level under flexible prices,

$$\begin{aligned}
\widehat{y}_t^f &= \widehat{z}_t + \frac{-\sigma \widehat{c}_t^f + \widehat{w}_t^f - \widehat{e}_t}{\eta} \\
\widehat{y}_t^f &= \widehat{z}_t + \frac{-\sigma \widehat{y}_t^f + \widehat{z}_t - \widehat{e}_t}{\eta} \\
\eta \widehat{y}_t^f &= \eta \widehat{z}_t - \sigma \widehat{y}_t^f + \widehat{z}_t - \widehat{e}_t \\
(\eta + \sigma) \widehat{y}_t^f &= (1 + \eta) \widehat{z}_t - \widehat{e}_t \\
\widehat{y}_t^f &= \left(\frac{1+\eta}{\sigma+\eta} \right) \widehat{z}_t - \left(\frac{1}{\sigma+\eta} \right) \widehat{e}_t \qquad (340)
\end{aligned}$$

which is a result slightly different from the benchmark model in that both technology and cost-push shocks affect flexible price equilibrium output.

Finally the output gap is then defined as the difference between output under sticky prices and output under flexible prices as:

$$\begin{aligned}
\widehat{x}_t &= \widehat{y} - \widehat{y}_t^f \\
&= \widehat{y} - m_z \widehat{z}_t - m_e \widehat{e}_t \qquad (341)
\end{aligned}$$

where $m_z = \left(\frac{1+\eta}{\sigma+\eta} \right)$ and $m_e = \left(\frac{1}{\sigma+\eta} \right)$.

3.7.3 Second-order approximation

The second-order approximation method used in this study is taken from Schmitt-Grohe and Uribe (2004). The basic idea of this method has been discussed in the main text. The aim of this section is to explain the application of their Matlab code to the NK model. There are two groups of scripts in Schmitt-Grohe and Uribe's package: One group contains functions that describe the model economy, i.e., *NK_par.m* and *NK_ss.m* define the parameter values and steady-state in the model and *NK_mod_f.m* sets out the model equilibrium condition equations. The script *NK_mod_f.m* makes use of the symbolic math toolbox embedded in Matlab and needs to be evaluated given parameter values of the model. Another group of scripts include *gx_hx.m* and *gxx_hxx.m* which solve for the first and second derivatives of the policy function *g(.)* and *h(.)* The calculation of **gx** and **hx** actually utilizes Klein's code since they are linear solutions to the model. The functions *gxx.m* and *hxx.m* are calculated according to the algorithm explained earlier in the main text.

3.7.4 PEA solution

Expectations functions to approximate According to the non-linear system, we have three expectation functions to approximate using PEA embedded in equation (274), (277) and (278). Since we have four state variables z_t v_t, e_t and zp_t, we approximate the three expectation functions as following:

First of all, we let

$$E_t\left(\frac{1}{\pi_{t+1}}\right)c_{t+1}^{-\sigma}\equiv\phi_1\left(z_t,v_t,e_t,zp_t;b_1\right)$$

and rewrite equation (274) as,

$$c_t^{-\sigma}=\beta r_t\phi_1\left(z_t,v_t,e_t,zp_t;b_1\right) \tag{342}$$

where

$$\begin{aligned}&\phi_1\left(z_t,v_t,e_t,zp_t;b_1\right)\\ =\;&\exp\left(b_{11}+b_{12}\log\left(z_t\right)+b_{13}\log\left(v_t\right)+b_{14}\log\left(e_t\right)+b_{15}\log\left(zp_t\right)\right)\end{aligned}$$

is the 1st order exponential polynomial function to approximate the expectation function[95].

Next we let

$$E_t\left(\pi_{t+1}\right)^{\theta}p_{t+1}^{N}\equiv\phi_2\left(z_t,v_t,e_t,zp_t;b_2\right)$$

and rewrite equation (277) as,

$$p_t^N=\left(\frac{\theta}{\theta-1}\right)\left(c_t\right)^{-\sigma}y_t\varphi_t+\omega\beta\phi_2\left(z_t,v_t,e_t,zp_t;b_2\right) \tag{343}$$

where

$$\begin{aligned}&\phi_2\left(z_t,v_t,e_t,zp_t;b_2\right)\\ =\;&\exp\left(b_{21}+b_{22}\log\left(z_t\right)+b_{23}\log\left(v_t\right)+b_{24}\log\left(e_t\right)+b_{25}\log\left(zp_t\right)\right).\end{aligned}$$

[95] Increasing the order of the polynomial potentially helps improve the accuracy PEA solution. However, convergence problem and multicollinearity problem might also emerge. Therefore we start with a first-order function form of the polynomial and a simple combination of the state variables.

Next similarly we let

$$E_t\left(\pi_{t+1}\right)^{\theta-1}p_{t+1}^D\equiv\phi_3\left(z_t,v_t,e_t,zp_t;b_3\right)$$

and rewrite equation (278) as,

$$p_t^D=\left(c_t\right)^{-\sigma}y_t+\omega\beta\phi_3\left(z_t,v_t,e_t,zp_t;b_3\right) \tag{344}$$

where

$$\begin{aligned}&\phi_3\left(z_t,v_t,e_t,zp_t;b_3\right)\\=\;&\exp\left(b_{31}+b_{32}\log\left(z_t\right)+b_{33}\log\left(v_t\right)+b_{34}\log\left(e_t\right)+b_{35}\log\left(zp_t\right)\right).\end{aligned}$$

Now the non-linear system becomes:

$$\frac{\chi e_t n_t^\eta}{c_t^{-\sigma}}=w_t \tag{345}$$

$$c_t^{-\sigma}=\beta R_t\phi_1\left(z_t,v_t,e_t,zp_t;b_1\right) \tag{346}$$

$$\varphi_t=\frac{w_t}{z_t} \tag{347}$$

$$p_t^*=\frac{p_t^N}{p_t^D} \tag{348}$$

$$p_t^N=\left(\frac{\theta}{\theta-1}\right)\left(c_t\right)^{-\sigma}y_t\varphi_t+\omega\beta\phi_2\left(z_t,v_t,e_t,zp_t;b_2\right) \tag{349}$$

$$p_t^D=\left(c_t\right)^{-\sigma}y_t+\omega\beta\phi_3\left(z_t,v_t,e_t,zp_t;b_3\right) \tag{350}$$

$$1=(1-\omega)(p_t^*)^{1-\theta}+\omega\pi_t^{\theta-1} \tag{351}$$

$$pd_t=(1-\omega)(p_t^*)^{-\theta}+\omega\pi_t^\theta zp_t \tag{352}$$

$$zp_{t+1}=pd_t \tag{353}$$

$$pd_t y_t=z_t n_t \tag{354}$$

$$y_t=c_t \tag{355}$$

$$R_t=R\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}\left(x_t\right)^{\delta_x}\left(\frac{v_t}{v}\right) \tag{356}$$

$$x_t=\left(\frac{y_t}{y}\right)\left(\frac{z_t}{z}\right)^{-m}\left(\frac{e_t}{e}\right)^n \tag{357}$$

$$\ln\left(z_{t+1}\right)=\rho_z\ln\left(z_t\right)+\varepsilon_{z,t+1},\;\;\varepsilon_{z,t+1}\sim N(0,\sigma_{\varepsilon_z}^2) \tag{358}$$

$$\ln\left(v_{t+1}\right)=\rho_v\ln\left(v_t\right)+\varepsilon_{v,t+1},\;\;\varepsilon_{v,t+1}\sim N(0,\sigma_{\varepsilon_v}^2) \tag{359}$$

$$\ln\left(e_{t+1}\right)=\rho_e\ln\left(e_t\right)+\varepsilon_{e,t+1},\;\;\varepsilon_{e,t+1}\sim N(0,\sigma_{\varepsilon_e}^2) \tag{360}$$

Steps of simulations and iterations

1. Defining T as the sample size of the simulation, we generate draws from an *i.i.d.* distribution for the technology shock $\{\varepsilon_{z,t}\}_{t=1}^{T}$, the monetary policy shock $\{\varepsilon_{v,t}\}_{t=1}^{T}$ and the cost-push shock $\{\varepsilon_{e,t}\}_{t=1}^{T}$. Then we generate series for $\{z_t\}_{t=1}^{T}$, $\{v_t\}_{t=1}^{T}$ and $\{e_t\}_{t=1}^{T}$ using:

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{z,t+1},\ \ \varepsilon_{z,t+1} \sim N(0, \sigma_{\varepsilon_z}^2)$$

$$\ln(v_{t+1}) = \rho_v \ln(v_t) + \varepsilon_{v,t+1},\ \ \varepsilon_{v,t+1} \sim N(0, \sigma_{\varepsilon_v}^2)$$

$$\ln(e_{t+1}) = \rho_e \ln(e_t) + \varepsilon_{e,t+1},\ \ \varepsilon_{e,t+1} \sim N(0, \sigma_{\varepsilon_e}^2).$$

2. Let the predetermined variable zp_t start from the steady-state,

$$zp_1 = zp. \tag{361}$$

By using the first polynomial, the Euler equation now becomes

$$c_t^{-\sigma} = \beta R_t \phi_1$$

also

$$y_t = c_t$$

so we have

$$y_t^{-\sigma} = \beta R_t \phi_1.$$

Substituting the interest rate rule (356) and the output gap (357) into the above output equation gives,

$$y_t^{-\sigma} = \beta R \left(\frac{\pi_t}{\pi}\right)^{\delta_\pi} \left\{ \left(\frac{y_t}{y}\right) \left(\frac{z_t}{z}\right)^{-m_z} \left(\frac{e_t}{e}\right)^{m_e} \right\}^{\delta_x} \left(\frac{v_t}{v}\right) \phi_1. \tag{362}$$

Next, dividing (349) by (350) gives,

$$\begin{aligned} \frac{p_t^N}{p_t^D} &= \frac{\left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t \varphi_t + \omega\beta\phi_2}{(c_t)^{-\sigma} y_t + \omega\beta\phi_3} \\ &= \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma} \varphi_t + \omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3}. \end{aligned}$$

From (348) we also have

$$p_t^* = \frac{p_t^N}{p_t^D}$$

so we get

$$p_t^* = \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma} \varphi_t + \omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3}. \tag{363}$$

Next, from the FOCs of labor and marginal cost,

$$\frac{\chi e_t n_t^\eta}{c_t^{-\sigma}} = w_t$$

$$\Rightarrow \frac{\chi e_t n_t^\eta}{y_t^{-\sigma}} = w_t$$

$$\varphi_t = \frac{w_t}{z_t}.$$

Eliminating w_t gives

$$\varphi_t = \frac{\chi e_t n_t^\eta}{y_t^{-\sigma} z_t}. \tag{364}$$

Then (363) becomes

$$p_t^* = \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma}\left(\frac{\chi e_t n_t^\eta}{y_t^{-\sigma} z_t}\right) + \omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3}. \tag{365}$$

Next, we find n_t from the production function as

$$n_t = \frac{pd_t y_t}{z_t}$$

also find pd_t from (352) as

$$pd_t = (1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^\theta z p_t. \tag{366}$$

Therefore n_t is represented as

$$n_t = \left((1-\omega)(p_t^*)^{-\theta} + \omega\pi_t^\theta z p_t\right)\frac{y_t}{z_t}. \tag{367}$$

Substituting n_t into equation (365) gives,

$$p_t^* = \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma}\left[\left((1-\omega)(p_t^*)^{-\theta} + \omega\pi^\theta z p_t\right)\frac{y_t}{z_t}\right]^\eta \frac{\chi e_t}{y_t^{-\sigma} z_t} + \omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3}.$$

Finally from the price evolution,

$$1 = (1-\omega)(p_t^*)^{1-\theta} + \omega\pi_t^{\theta-1}$$

we have

$$p_t^* = \left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}}. \tag{368}$$

Substituting into above result yields

$$\begin{aligned}\left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} &= \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma}\left[\left((1-\omega)\left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{-\theta}{1-\theta}} + \omega\pi_t^\theta z p_t\right)\frac{y_t}{z_t}\right]^\eta \frac{\chi e_t}{y_t^{-\sigma} z_t}}{(y_t)^{1-\sigma} + \omega\beta\phi_3} \\ &+ \frac{\omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3}\end{aligned} \tag{369}$$

3. For each period, we use $\{z_t, v_t, e_t, zp_t, b_1, b_2, b_3\}$ as inputs, and solve y_t and π_t from the two equations below:

$$y_t^{-\sigma} = \beta R \left(\frac{\pi_t}{\pi}\right)^{\delta_\pi} \left\{ \left(\frac{y_t}{y}\right) \left(\frac{z_t}{z}\right)^{-m_z} \left(\frac{e_t}{e}\right)^{m_e} \right\}^{\delta_x} \left(\frac{v_t}{v}\right) \phi_1 \tag{370}$$

$$\left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} = \frac{\left(\frac{\theta}{\theta-1}\right)(y_t)^{1-\sigma}\left[\left((1-\omega)\left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{-\theta}{1-\theta}} + \omega\pi_t^{\theta} zp_t\right)\frac{y_t}{z_t}\right]^{\eta} \frac{\chi e_t}{y_t^{-\sigma} z_t}}{(y_t)^{1-\sigma} + \omega\beta\phi_3} + \frac{\omega\beta\phi_2}{(y_t)^{1-\sigma} + \omega\beta\phi_3} \tag{371}$$

We can solve y_t and π_t numerically from above equation using *fsolve.m* in Matlab.

4. Conditional on y_t and π_t, we can solve other variables accordingly. Particularly, we need to generate zp_{t+1} as an input for *fsolve.m* to simulate the model in next period. This can be done by using equation (368), (366) and (353):

$$p_t^* = \left(\frac{1-\omega\pi_t^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} \tag{372}$$

$$pd_t = (1-\omega)(p_t^*)^{-\theta} + \omega\left(\pi_t\right)^{\theta} zp_t \tag{373}$$

$$zp_{t+1} = pd_t. \tag{374}$$

5. As we repeat step 2-4 for each period, we can obtain a vector of all variables of length T. We can then run three non-linear least square regressions to update parameters in the three polynomial functions.

Regression (1):

$$\begin{aligned} &\log\left(\left(\frac{1}{\pi_{t+1}}\right)c_{t+1}^{-\sigma}\right) \\ = \;& b_{11} + b_{12}\log(z_t) + b_{13}\log(v_t) + b_{14}\log(e_t) + b_{15}\log(zp_t) \end{aligned}$$

which generates new estimate

$$\widehat{b_1} = \left[\begin{array}{ccccc} \widehat{b}_{11} & \widehat{b}_{12} & \widehat{b}_{13} & \widehat{b}_{14} & \widehat{b}_{15} \end{array}\right].$$

Regression (2):

$$\begin{aligned} &\log\left(\left(\pi_{t+1}\right)^{\theta} p_{t+1}^{N}\right) \\ = \;& b_{21} + b_{22}\log(z_t) + b_{23}\log(v_t) + b_{24}\log(e_t) + b_{25}\log(zp_t) \end{aligned}$$

which generates new estimate

$$\widehat{b_2} = \left[\begin{array}{ccccc} \widehat{b}_{21} & \widehat{b}_{22} & \widehat{b}_{23} & \widehat{b}_{24} & \widehat{b}_{25} \end{array}\right].$$

Regression (3):

$$\begin{aligned}
& \log\left(\left(\pi_{t+1}\right)^{\theta-1} p_{t+1}^{D}\right) \\
= \; & b_{31} + b_{32}\log\left(z_t\right) + b_{33}\log\left(v_t\right) + b_{34}\log\left(e_t\right) + b_{35}\log\left(zp_t\right)
\end{aligned}$$

which generates new estimate

$$\widehat{b_3} = \left[\begin{array}{ccccc} \widehat{b_{31}} & \widehat{b_{32}} & \widehat{b_{33}} & \widehat{b_{34}} & \widehat{b_{35}} \end{array}\right].$$

6. Since steps 2-5 are iterative, i.e., if we substitute the new estimates $\widehat{b_1}$, $\widehat{b_2}$ and $\widehat{b_3}$ into step 2, we can simulate the model again and obtain another group of new estimates. Therefore we need a convergence scheme to stop the algorithm. We employ the following updating scheme,

$$\chi^{i+1} = \varphi\hat{\chi}^{i} + (1-\varphi)\,\chi^{i} \tag{375}$$

where χ^{i+1} is the updated as an weighted sum of the estimate $\hat{\chi}$ and the old estimate χ. In each iteration, we compare the updated estimate χ^{i+1} with the new estimates $\hat{\chi}$. The algorithm stops when

$$\left|\chi^{i+1} - \hat{\chi}^{i}\right| < \eta \tag{376}$$

where $\eta = 1e-6$ is a small number chosen as a criterion of convergence. If $|\chi^{i+1} - \chi^{i}| < \eta$ then stop, otherwise go back to step 2 and perform the algorithm again.

Note that since we have three groups of estimates $\widehat{b_1}$, $\widehat{b_2}$ and $\widehat{b_3}$, we need to employ the same updating scheme to each group of estimates and only stop when all the three groups of estimates converge.

3.7.5 Implementation of Judd Criteria

Judd (1992) proposes to calculate the rational expectational error embedded in the Euler equation as a way to judge the magnitude of solution. Recall that the expectational error is represented by the consumption Euler error

$$u_{c,t} = c_t - \left\{\beta R_t\left(\frac{1}{\pi_{t+1}}\right) c_{t+1}^{-\sigma}\right\}^{-\frac{1}{\sigma}} \tag{377}$$

and price-adjustment equation error

$$u_{p,t} = p_t^{*} - \frac{\left(\frac{\theta}{\theta-1}\right)\left(c_t\right)^{-\sigma} y_t\varphi_t + \omega\beta\left(\pi_{t+1}\right)^{\theta} p_{t+1}^{N}}{\left(c_t\right)^{-\sigma} y_t + \omega\beta\left(\pi_{t+1}\right)^{\theta-1} p_{t+1}^{D}}. \tag{378}$$

Since the magnitude of the expectational error not only depends on the solution one uses but also the choice of state variables, it should be evaluated with a large set of chosen state variables. In the following, we will calculate the expectational error subject to a range of the predetermined variable and a distribution of the exogenous shocks. The detailed steps of the implementation of Judd criteria are illustrated below:

1. First of all, specify the range of state variables over which we are evaluating the expectational error. For example, given a sample size T, we select a range of price dispersion values $\{zp_t\}_{t=1}^{T}$: $zp_t \in [zp\min,\ zp\max]$, with $zp\min=1$ (since zp_t should not be smaller than its steady-state value 1) and $zp\max = 1.2 * zp$. The lower and upper bounds of exogenous shocks is set as follows. We firstly generate random numbers that cover 99.9999% of the theoretical distribution of the innovations of the exogenous shocks[96]. For example, for the technology shock:

$$\log z_{t+1} = \rho \log z_t + \phi_{t+1}, \quad \phi_{t+1} \backsim N\left(0, \sigma_\phi^2\right) \tag{379}$$

we define the upper bound and lower bound of ϕ_{t+1} as the upper and lower critical values of a normal distribution at confidence level 99.9999%. We can derive these bounds in Matlab as

$$[\phi^U, \phi^L] = \text{norminv}([0.00005, 0.99995], 0, \sigma_\phi).$$

Then we restrict the series of ϕ_{t+1} within the above bounds. This amounts to re-scaling the series. We use the following transformation equation:

$$\phi_{t+1} = \frac{\phi^U - \phi^L}{\max\phi - \min\phi}\phi_{t+1} + e_z^U - \frac{\phi^U - \phi^L}{\max\phi - \min\phi}\max\phi.$$

thus we can use series of $\left\{\phi_{t+1}\right\}_{t=0}^{T-1}$ to generate a range of technology process using (379). We then get the minimum and maximum values of the simulated series of technology shocks: $[z\min, z\max]$. Using a similar strategy, we can also obtain the lower and upper bounds of v and z as $[v\min, v\max]$ and $[e\min, e\max]$.

2. Using the above domains, generate a range of state variables: $zp_t \in [zp\min,\ zp\max]$, $z_t \in [z\min,\ z\max]$, $v_t \in [v\min,\ v\max]$ and $e_t \in [e\min,\ e\max]$. Note that the calculation of the expectation error is subject to a number of T^2 combinations

[96] Other choices of the state variables are also possible, for example, choosing values of zp from the empirical distribution that generated by a particular solution. See for example, Aruoba *et al.* (2006) which uses the distribution of state variables generated from value function iteration method as an approximate of the true distribution in order to calculate an average expectational error.

of state variables $[zp, z, v, e]$ if we do not distinguish the combination of exogenous shocks.

3. Before starting to calculate the expectational error in (377) and (378), we need to calculate the next period variables. However, since calculating c_{t+1} for example, basically requires values of $\varepsilon_{z,t+1}, \varepsilon_{v,t+1}$ and $\varepsilon_{e,t+1}$ which are unknown, we need to approximate this conditional expectation Ec_{t+1} by making use of the distribution of the innovations. Popular methods to evaluate the uncertainty include Monte-Carlo integration and numerical integration. Here we use the former due to the fact that we have three exogenous variables and the application of numerical integration can become cubersome. We continue as follows:

4. Given a combination of $[zp_t, z_t, v_t, e_t]$, $t \in T^2$, calculate the current non-predetermined variables, e.g., consumption c_t and inflation π_t and one period ahead predetermined variable zp_{t2} based on the current price dispersion zp_t and the three exogenous shocks v_t, z_t and e_t. For example, the current consumption can be generated using
$$c_t = f\left(zp_t, v_t, z_t, e_t; b\right) \tag{380}$$
where f represents a solution method. Therefore we obtain matrices of data for all current endogenous variables.

5. Draw three vectors of length S for the innovations of the three exogenous shocks $\left\{\varepsilon^i_{z,t2}\right\}_{i=1}^S, \left\{\varepsilon^i_{v,t2}\right\}_{i=1}^S$ and $\left\{\varepsilon^i_{e,t2}\right\}_{i=1}^S$ from normal distribution. These innovations are then used to calculate three vectors of the exogenous shocks $\{z^i_{t2}\}_{i=1}^S, \{v^i_{t2}\}_{i=1}^S$ and $\{e^i_{t2}\}_{i=1}^S$. Then, the matrices of data for one period ahead endogenous variables, e.g., $\{c^i_{t2}\}_{i=1}^S, \{\pi^i_{t2}\}_{i=1}^S$ and $\{e^i_{t2}\}_{i=1}^S$, can be obtained.

6. Calculate the expectational error for each draw i, for example, the consumption Euler equation as,
$$u^i_t = c_t - \left\{\beta R_t \left(\frac{1}{\pi^i_{t2}}\right)\left(c^i_{t2}\right)^{-\sigma}\right\}^{-\frac{1}{\sigma}}. \tag{381}$$
Also calculate the ratio
$$\varepsilon^i_t = \log_{10}\left(\left|\frac{u^i_t}{c}\right|\right).$$

7. The expectation error is then evaluated by taking the mean,
$$u_t = mean\left(u^i_t\right),\ i = 1, 2, ..., S. \tag{382}$$

And the Judd measure is then obtained by taking the mean,

$$\epsilon_i = mean\left(\varepsilon_t^i\right),\ i = 1, 2, ..., S. \tag{383}$$

8. Do step $4-7$ for all possible combinations of $[zp_t, z_t, v_t, e_t]$, $t \in T^2$ and obtain a number of T^2 expectational error $\{\epsilon_t\}_{t=1}^{T}$. We can then report the average, square and maximum of the expectational error which are $\epsilon_1, \epsilon_2, \epsilon_3$ respectively.

3.7.6 Implementation of Marcet's test

To implement the J test proposed by Marcet, Monte-Carlo simulations are used to calculate a set of J test statistics. The detailed steps are as follows:

1. Draw nS vectors of i.i.d. sequences for the three innovations $\{\varepsilon_{z,t+1}\}_{t=1}^{nT}$, $\{\varepsilon_{v,t+1}\}_{t=1}^{nT}$ and $\{\varepsilon_{e,t+1}\}_{t=1}^{nT}$, where nS is the number of simulations and nT is the length of each simulation. Then a $nSxnT$ matrix of the technology shock, the monetary policy shock and the cost-push shock can be generated using the following law of motion:

$$\ln(z_{t+1}) = \rho_z \ln(z_t) + \varepsilon_{z,t+1},\ \varepsilon_{z,t+1} \sim N(0, \sigma_{\varepsilon_z}^2) \tag{384}$$

$$\ln(v_{t+1}) = \rho_v \ln(v_t) + \varepsilon_{v,t+1},\ \varepsilon_{v,t+1} \sim N(0, \sigma_{\varepsilon_v}^2) \tag{385}$$

$$\ln(e_{t+1}) = \rho_e \ln(e_t) + \varepsilon_{e,t+1},\ \varepsilon_{e,t+1} \sim N(0, \sigma_{\varepsilon_e}^2). \tag{386}$$

2. Use the above $nSxnT$ matrix of shocks, the predetermined variable zp_t which starts from steady-state and one particular approximation solution to generate nS x nT matrices of data for endogenous variables. Also generate the Euler residuals $\{\hat{u}_{t+1}\}_{t=1}^{nT}$ as,

$$u_{c,t+1} = \left(R_t \beta E_t(\frac{1}{\pi_{t+1}}) c_{t+1}^{-\sigma}\right)^{-\frac{1}{\sigma}} - c_t \tag{387}$$

and

$$u_{p,t+1} = E_t \frac{\left(\frac{\theta}{\theta-1}\right)(c_t)^{-\sigma} y_t \varphi_t + \omega\beta E_t (\pi_{t+1})^{\theta} p_{t+1}^N}{(c_t)^{-\sigma} y_t + \omega\beta E_t (\pi_{t+1})^{\theta-1} p_{t+1}^D} - p_t^*. \tag{388}$$

where $u_{c,t+1}$ and $u_{p,t+1}$ are the Euler residuals in the consumption Euler equation and price adjustment equation respectively. Note that, to make comparisons useful, the same vectors of shocks will be used to generate endogenous variables for all solutions.

3. Calculate the J_T test statistic nS times using

$$J_T = Tg'\Omega_T^{-1}g \sim \chi_q^2$$

where

$$g = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_{t+1} \otimes h(\hat{x}_t)$$

and $\Omega_T = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_{t+1}^2 h(\hat{x}_t)h(\hat{x}_t)'$, $\hat{u}_{t+1}$ is the Euler residual at time $t+1$ and $h(x_t)$ is function containing the number of q state variables at time t and before.

4. Finally, compute the percentage that J_T falls in the rejection area, i.e., lower than the 5% confidence interval and higher than the 95% confidence interval. The Matlab function *chi2inv.m* is used to give the percentile of the χ_q^2 distribution. The plots of the cumulative density functions are calculated using Matlab function *cdfplot.m*. They are compared with the theoretical cumulative density function which is output of the Matlab function *chi2cdf.m*.

3.7.7 Calculation of IRs of non-linear approximations

The impulse response functions of second-order approximation and PEA solution are calculated using Monte-Carlo integration as follows:

1. First of all, suppose the information set Ω is composed by the state variables x and innovation vector ε. We draw S simulations with sample size h, an innovation vector $\{\varepsilon_h\}^s$, $s = 1, ..., S$.

2. For given initial value of x_0 and the T realizations for innovations $\{\varepsilon_h\}^s$, we can recover S simulations of endogenous variables $\{y_h(x_0, \varepsilon_h)\}^s$ for $s = 1, ..., S$. We consider this as the base data.

3. We then compute T realizations of $\{y_h(x_0, \varepsilon_h, shock)\}^s$ using the same procedure as in step 2 but with one additional shock of size σ. This produces the shocked data.

4. If the data simulated is in levels (e.g., the PEA case), then compute the IRF as

$$IRF^s(h,\sigma) = \frac{\{y_h(x_0, \varepsilon_h, \sigma)\}^s - \{y_h(x_0, \varepsilon_h)\}^s}{\{y_h(x_0, \varepsilon_h)\}^s} \tag{389}$$

for S simulations. If the data simulated is in logs (i.e., the second-order approximation case), then compute the log-difference as

$$IRF^s(h,\sigma) = \log\{y_h(\log(x_0), \varepsilon_h, \sigma)\}^s - \log\{y_h(\log(x_0), \varepsilon_h)\}^s. \tag{390}$$

5. Finally average the IRs across S simulations:

$$IRF(h,\sigma) = \frac{1}{S}\sum_{s=1}^{S} IRF^{s}(h,\sigma). \tag{391}$$

3.7.8 Results under alternative Calibrations

Judd's criteria

Table 3.9: Judd's criteria: Model 3 (high price rigidity)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	−2.048	−2.047	−2.092	−2.008	−2.008	−1.984
ε_2	−4.096	−4.094	−4.184	−4.015	−4.016	−3.968
ε_3	−1.265	−1.268	−1.297	−0.424	−1.152	−1.170

Table 3.10: Judd's criteria: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	−2.019	−2.022	−2.063	−1.975	−1.967	−1.948
ε_2	−4.038	−4.044	−4.126	−3.949	−3.934	−3.897
ε_3	−1.244	−1.247	−1.272	−0.417	−0.962	−1.152

Table 3.11: Judd's criteria: Model 5 (high risk-aversion of households)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	−2.243	−2.245	−2.287	−2.088	−2.097	−2.080
ε_2	−4.487	−4.490	−4.573	−4.176	−4.193	−4.160
ε_3	−1.478	−1.478	−1.475	−1.060	−1.164	−1.225

Table 3.12: Judd's criteria: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-2.231	-2.234	-2.257	-2.052	-2.056	-2.060
ε_2	-4.461	-4.468	-4.513	-4.104	-4.111	-4.119
ε_3	-1.463	-1.463	-1.449	-0.942	-1.173	-1.205

Table 3.13: Judd's criteria: Model 7 (high market power of firms)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-2.039	-2.0390	-2.078	-2.086	-2.089	-2.089
ε_2	-4.078	-4.078	-4.156	-4.171	-4.179	-4.179
ε_3	-1.267	-1.266	-1.289	-0.978	-1.053	-1.208

Table 3.14: Judd's criteria: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-2.034	-2.032	-2.071	-2.063	-2.073	-2.072
ε_2	-4.069	-4.064	-4.142	-4.127	-4.146	-4.144
ε_3	-1.262	-1.260	-1.276	-0.926	-1.050	-1.200

Table 3.15: Judd's criteria: Model 9 (high volatility of technology shock)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-1.623	-1.623	-1.612	-1.891	-1.890	-1.859
ε_2	-3.247	-3.246	-3.225	-3.782	-3.781	-3.719
ε_3	-0.869	-0.861	-0.791	-0.818	-0.957	-1.041

Table 3.16: Judd's criteria: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution methods	Linear	2nd-order	PEA	Linear	2nd-order	PEA
ε_1	-1.607	-1.604	-1.588	-1.844	-1.844	-1.816
ε_2	-3.214	-3.208	-3.176	-3.688	-3.689	-3.633
ε_3	-0.856	-0.844	-0.764	-0.730	-0.873	-0.990

Marcet's J-test under alternative calibrations

Table 3.17: Model 3 (high price rigidity): Marcet's J-test

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.033	0.042	0.041	0	0	0.035
higher 5%	0.032	0.038	0.066	0.366	0.140	0.073

Table 3.18: Marcet's J_test: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.035	0.035	0.038	0	0	0.037
higher 5%	0.038	0.034	0.061	0.472	0.201	0.073

Table 3.19: Marcet's J_test: Model 5 (high risk-aversion of households)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.033	0.033	0.041	0	0	0.027
higher 5%	0.028	0.027	0.055	0.609	0.403	0.081

Table 3.20: Marcet's J_test: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.036	0.031	0.035	0	0	0.024
higher 5%	0.029	0.027	0.053	0.659	0.485	0.086

Table 3.21: Marcet's J_test: Model 7 (high market power of firms)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.031	0.031	0.041	0	0.008	0.028
higher 5%	0.031	0.032	0.055	0.315	0.051	0.068

Table 3.22: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$): Marcet's J-test

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.031	0.028	0.037	0	0.013	0.027
higher 5%	0.032	0.030	0.052	0.326	0.058	0.072

Table 3.23: Marcet's J_test: Model 9 (high volatility of technology shock)

Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.035	0.016	0.048	0	0.001	0.026
higher 5%	0.032	0.055	0.070	0.783	0.352	0.130

Table 3.24: Marcet's J_test: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)						
Type of error	Consumption Euler			Phillips Curve		
Solution method	Linear	2nd-order	PEA	Linear	2nd-order	PEA
lower 5%	0.036	0.018	0.041	0	0.001	0.019
higher 5%	0.032	0.057	0.069	0.841	0.453	0.150

Plots of CDFs of J_statistics - Consumption Euler equation

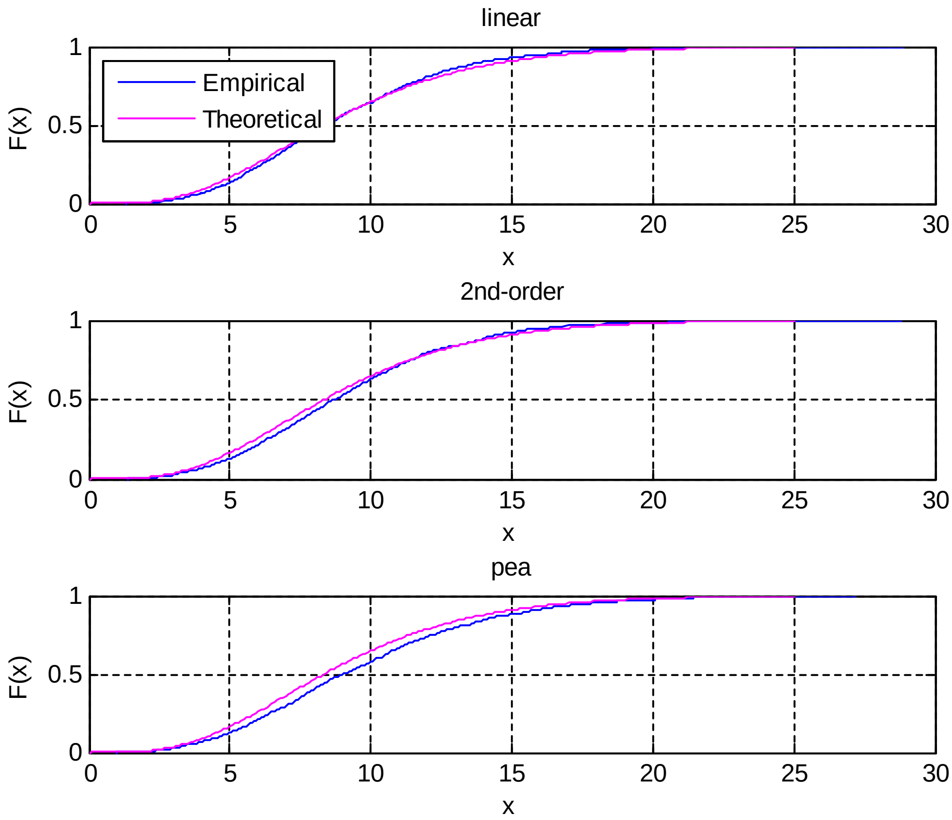

Figure 3.17: χ^2 specification test: Model 3 (high price rigidity)

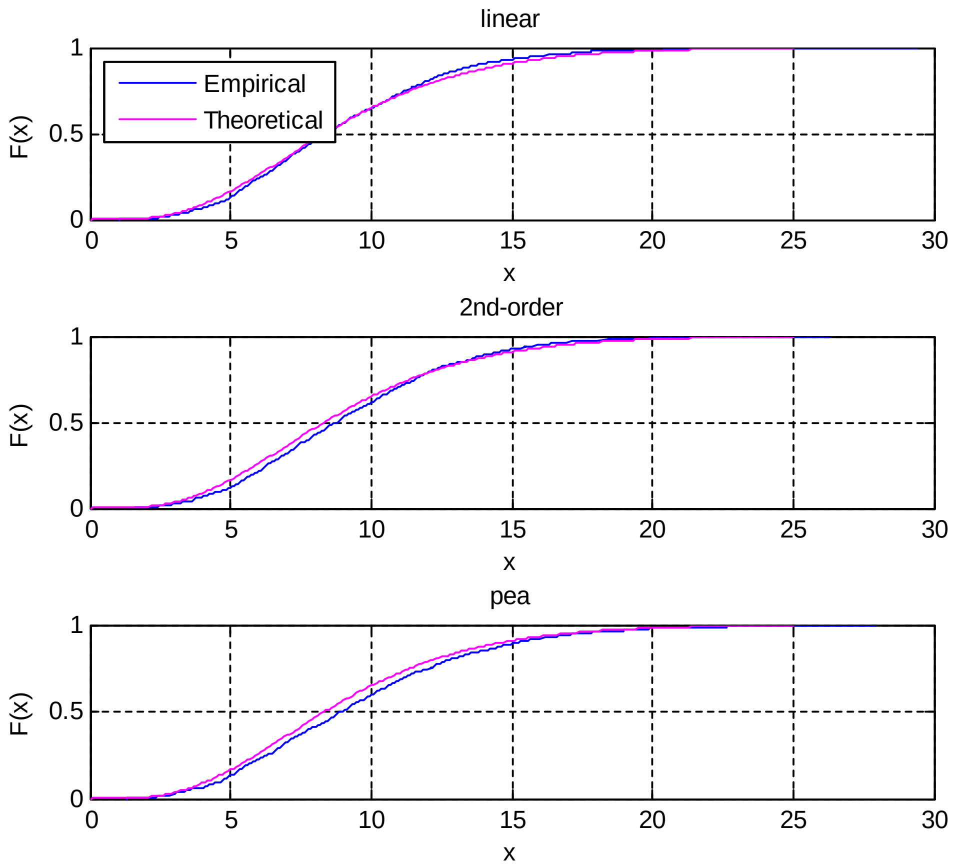

Figure 3.18: χ^2 specification test: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

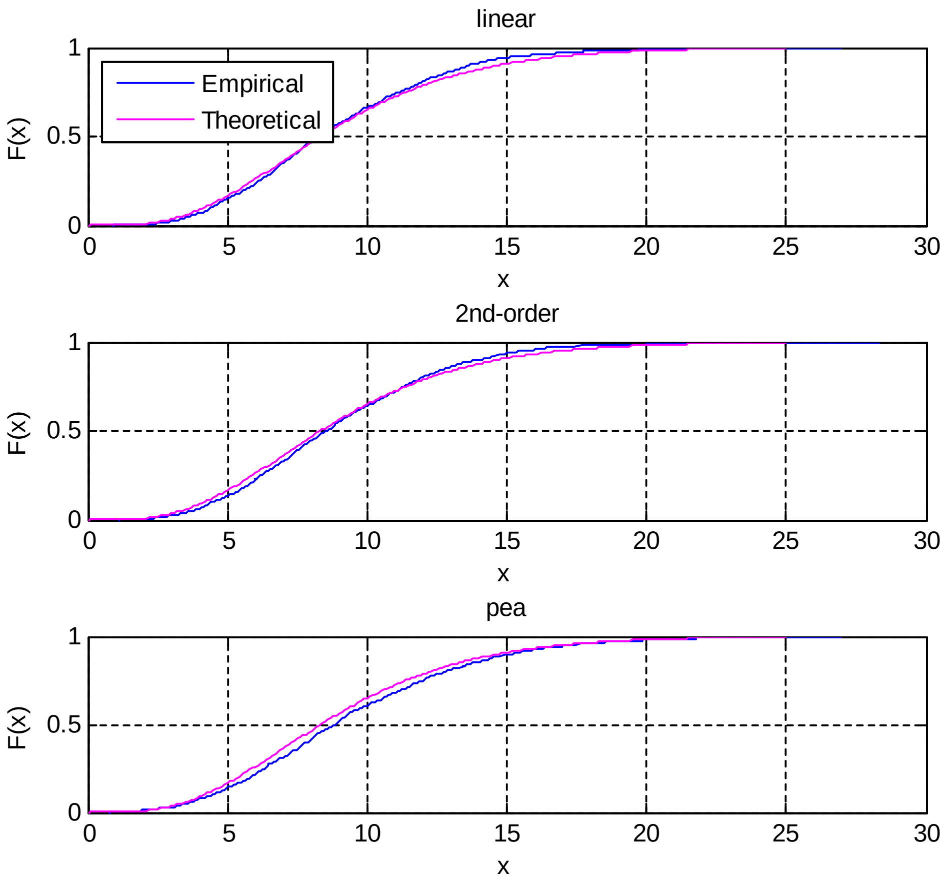

Figure 3.19: χ^2 specification test: Model 5 (high risk-aversion of households)

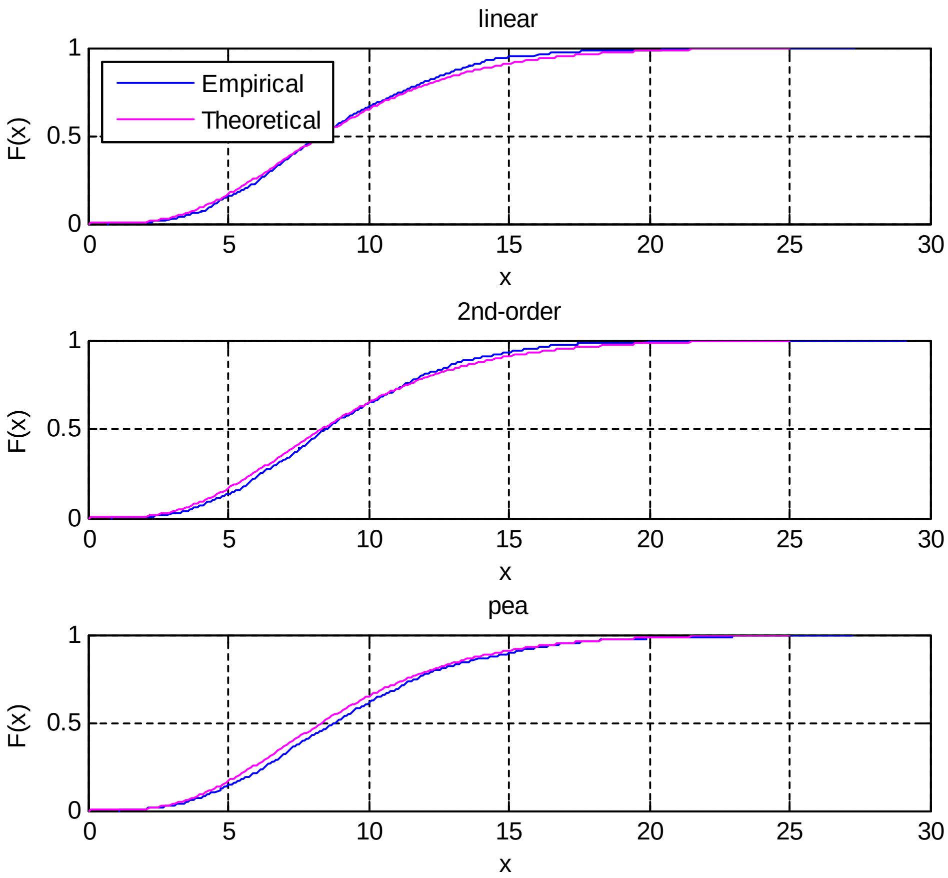

Figure 3.20: χ^2 specification test: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

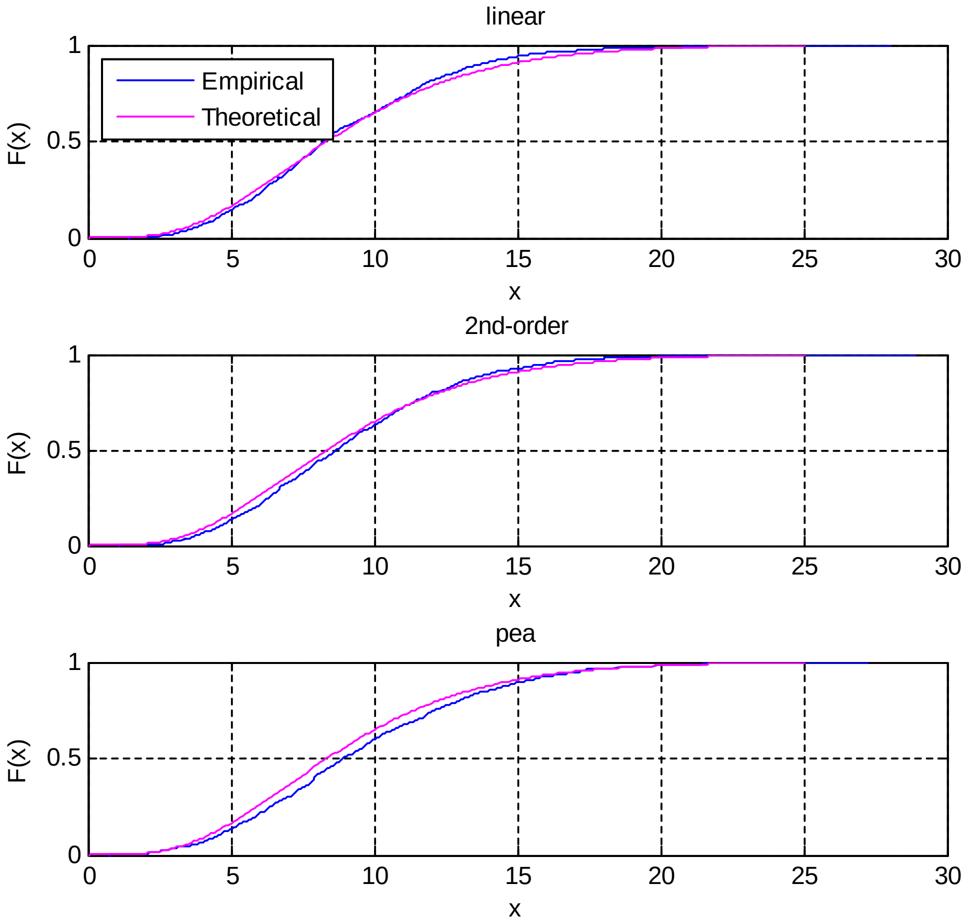

Figure 3.21: χ^2 specification test: Model 7 (high market power of firms)

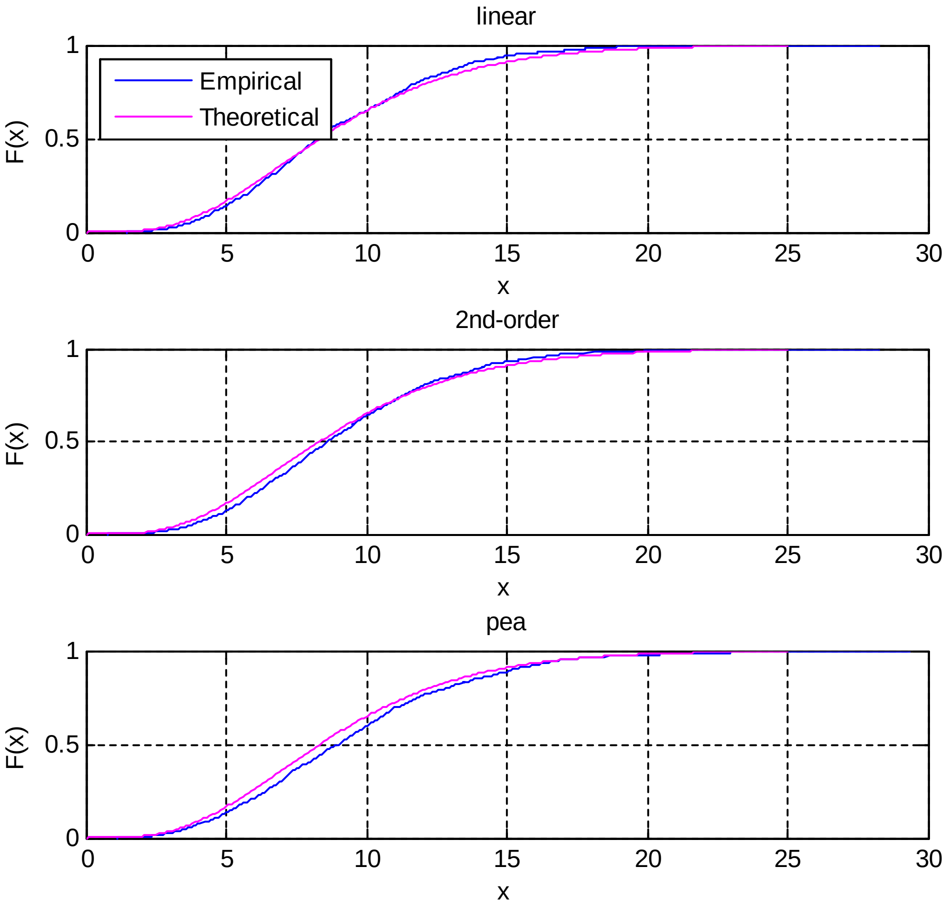

Figure 3.22: χ^2 specification test: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

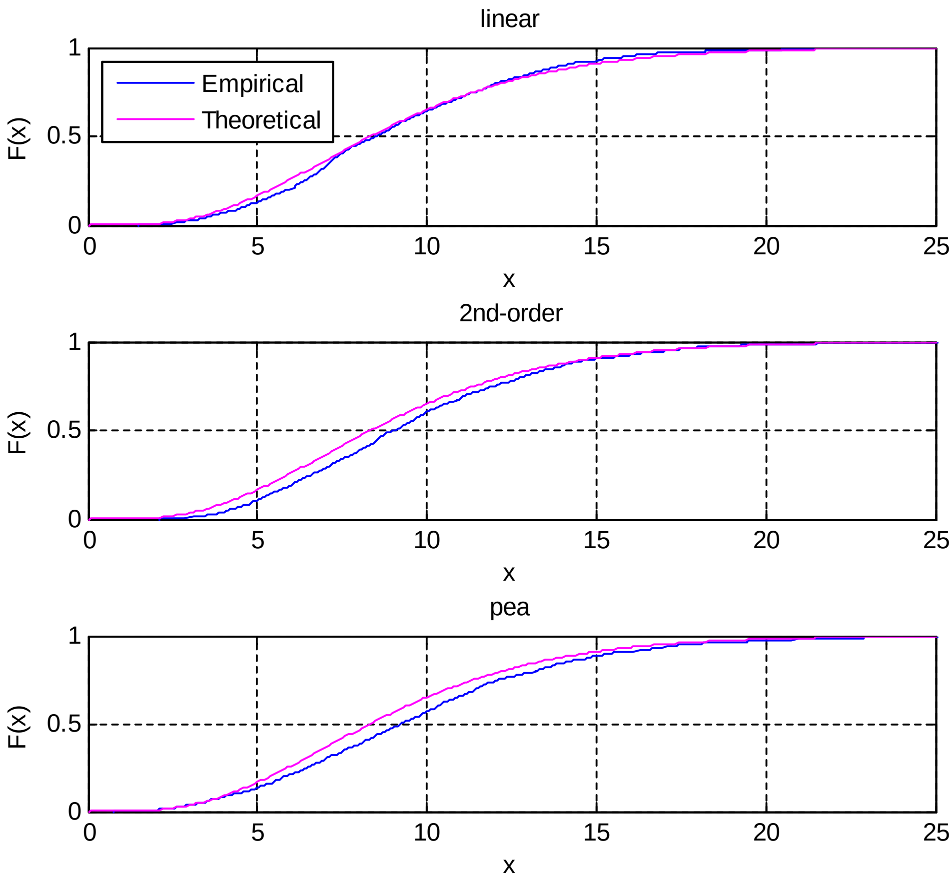

Figure 3.23: χ^2 specification test: Model 9 (high volatility of technology shock)

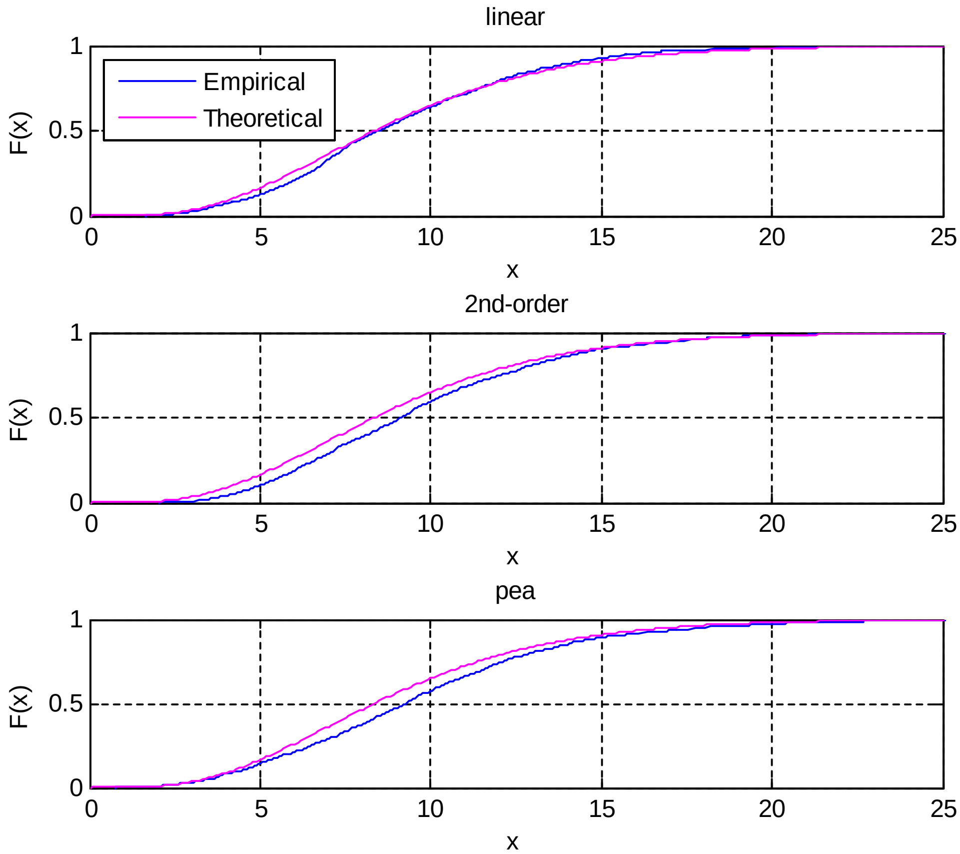

Figure 3.24: χ^2 specification test: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Plots of CDFs of J_statistics - Price-adjustment equation

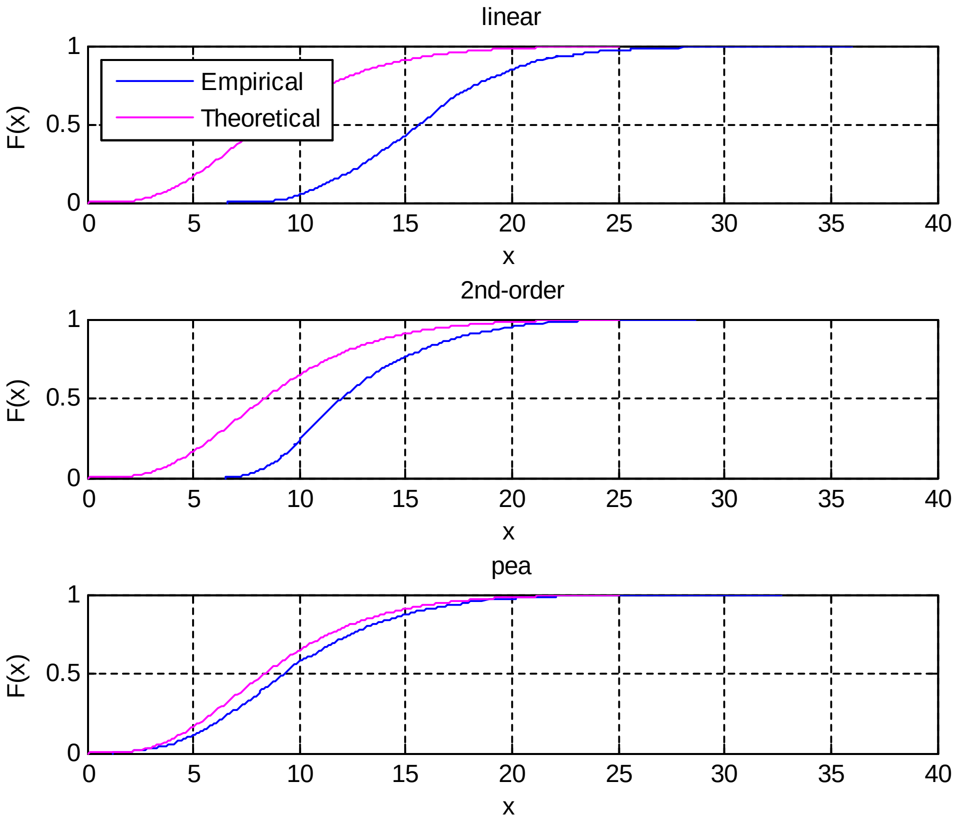

Figure 3.25: χ^2 specification test: Model 3 (high price rigidity)

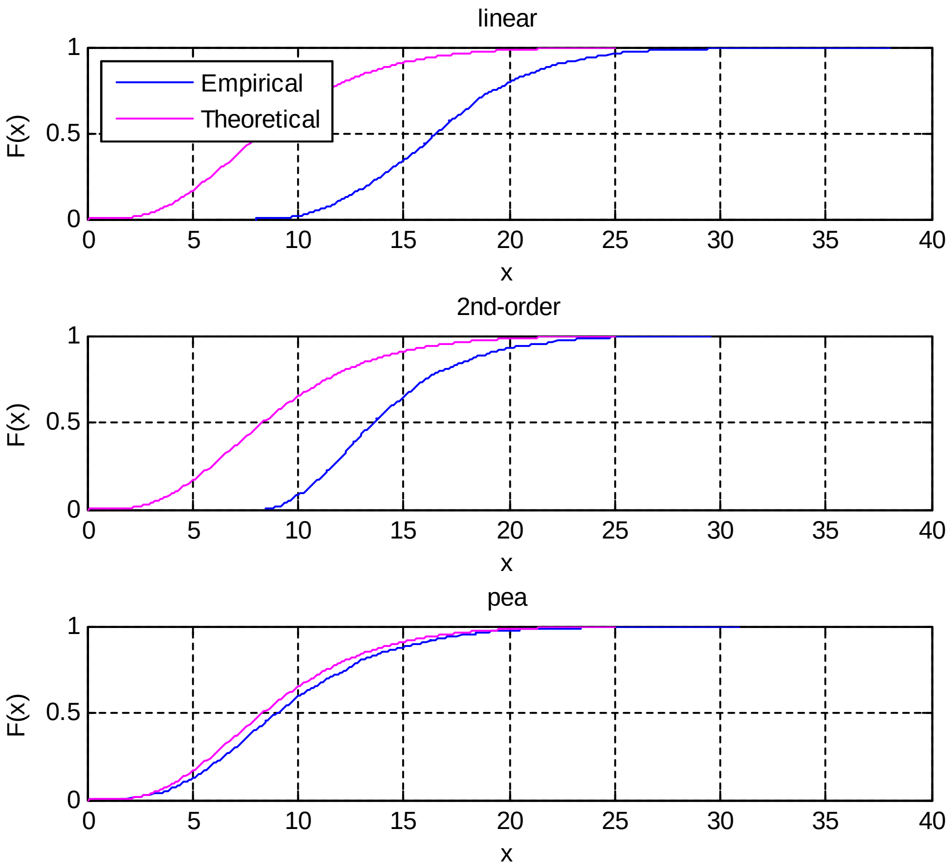

Figure 3.26: χ^2 specification test: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

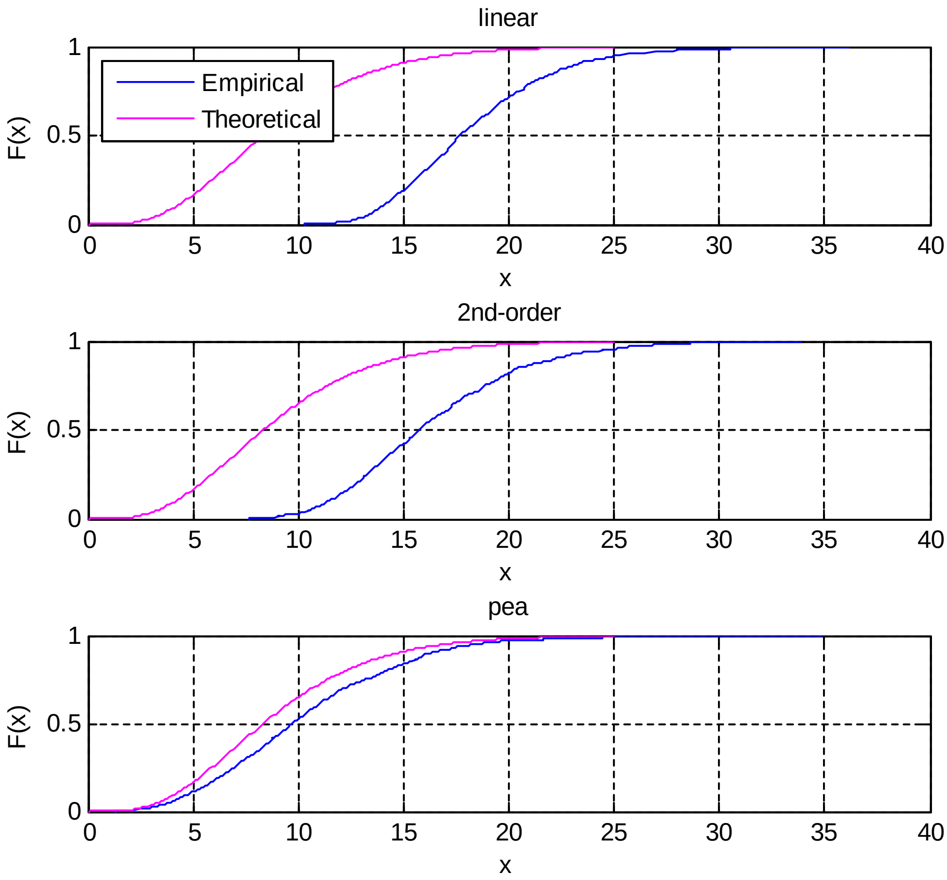

Figure 3.27: χ^2 specification test: Model 5 (high risk-aversion of households)

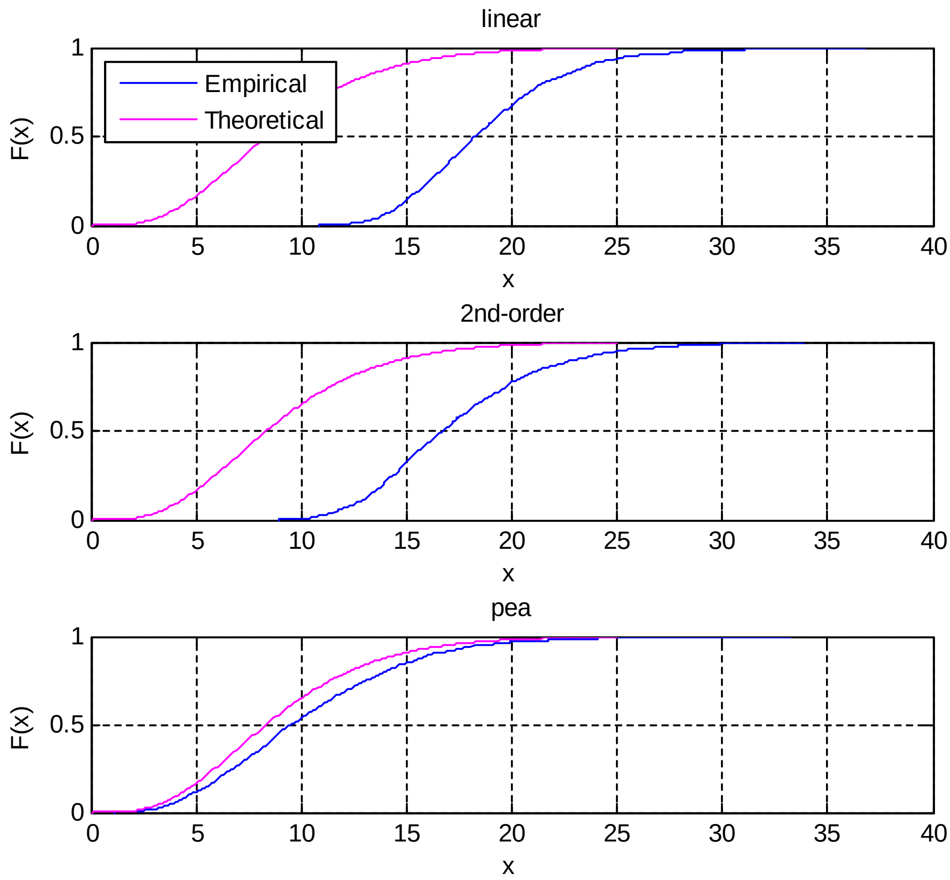

Figure 3.28: χ^2 specification test: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

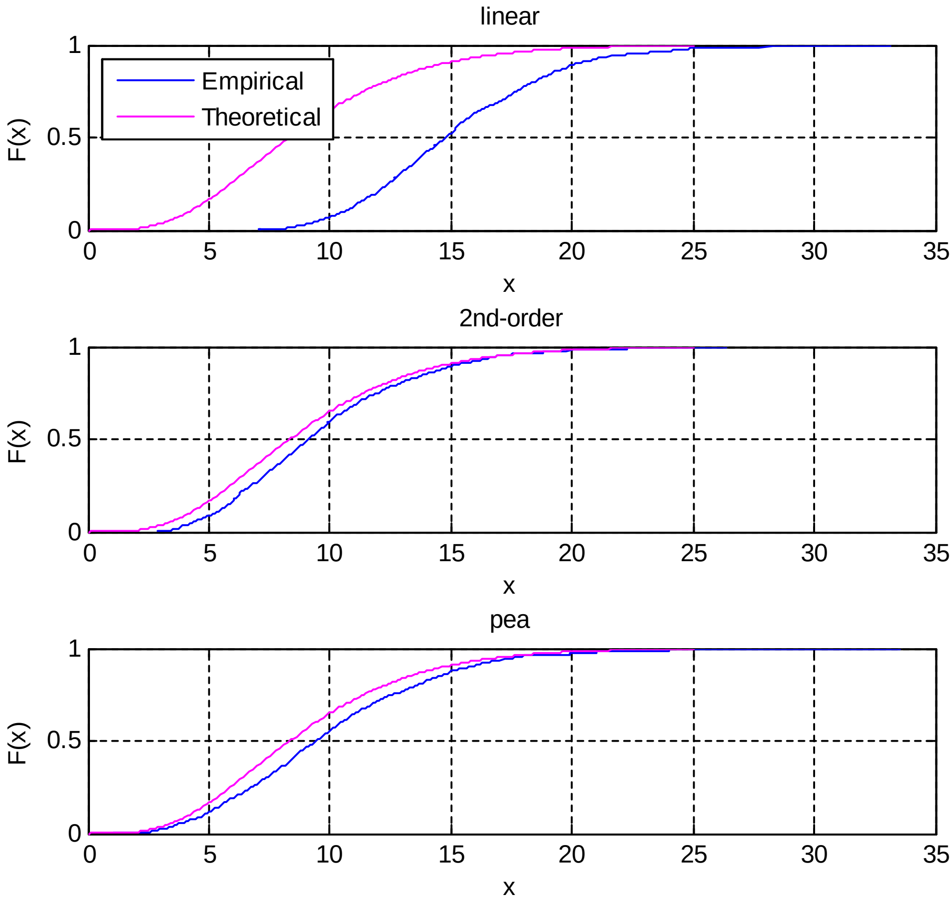

Figure 3.29: χ^2 specification test: Model 7 (high market power of firms)

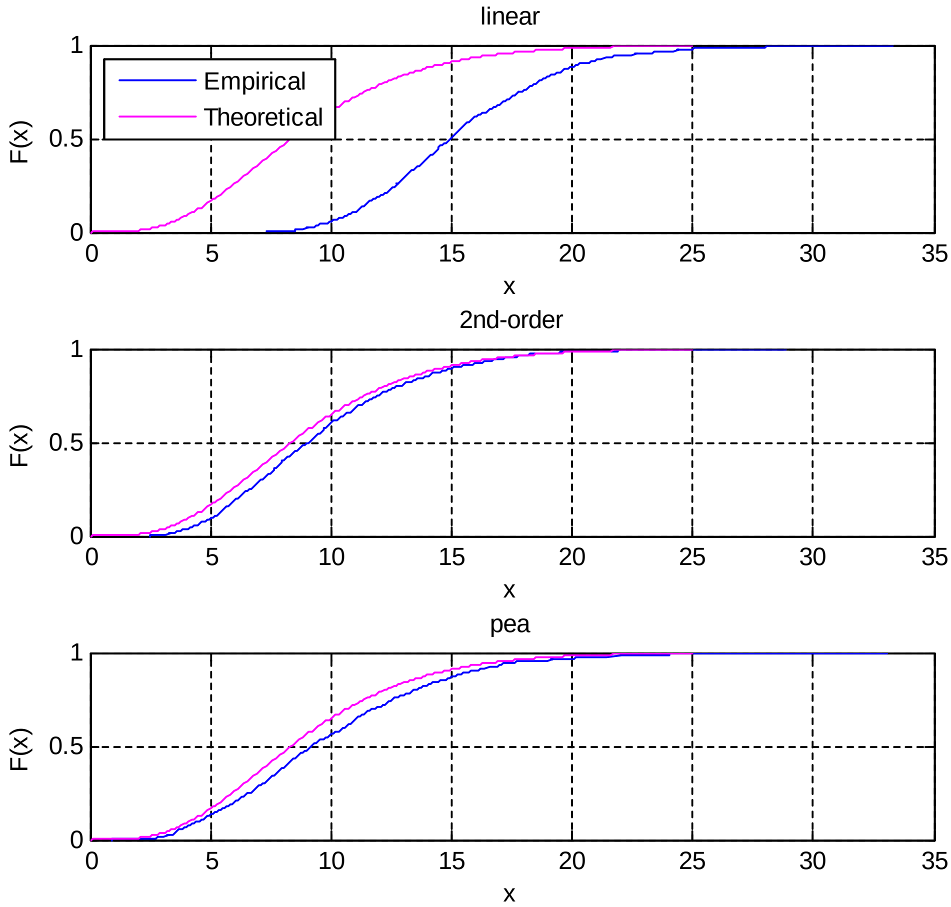

Figure 3.30: χ^2 specification test: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

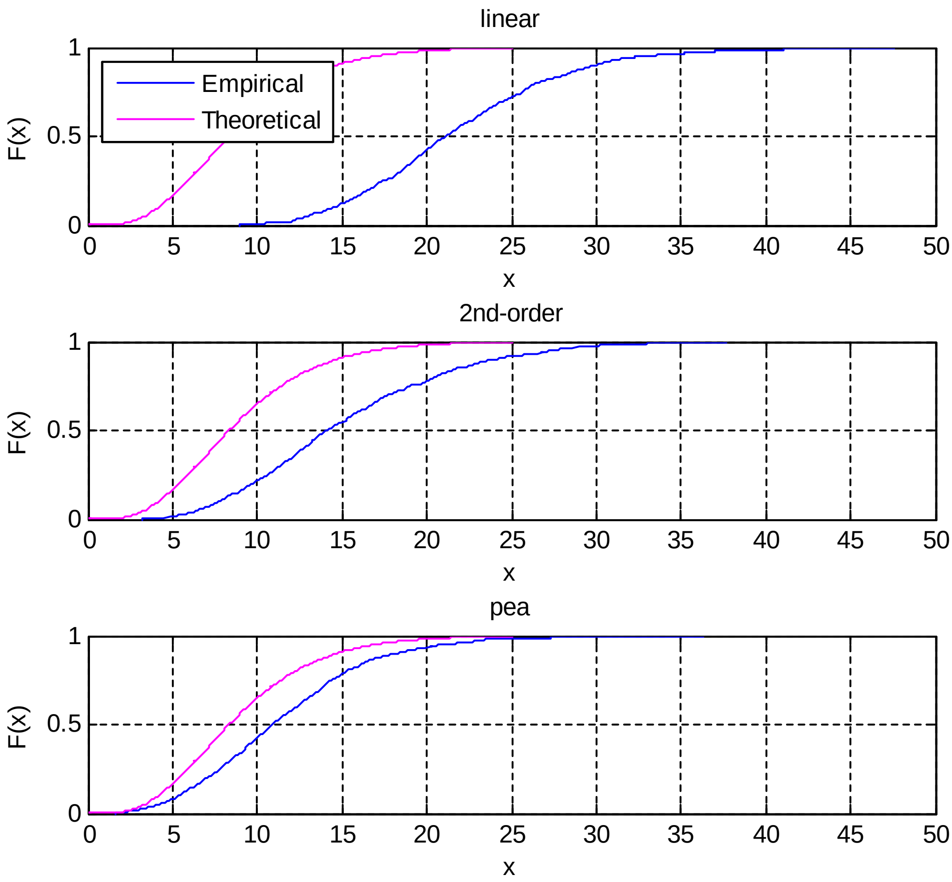

Figure 3.31: χ^2 specification test: Model 9 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

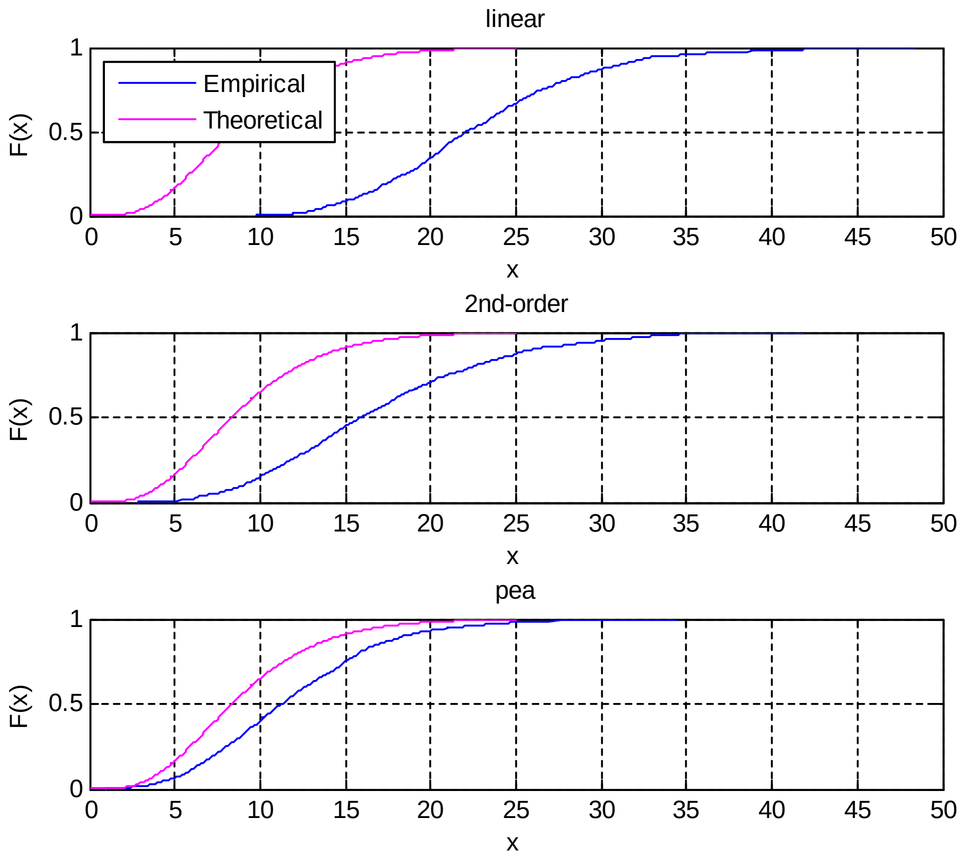

Figure 3.32: χ^2 specification test: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Impulse response functions

Technology shock

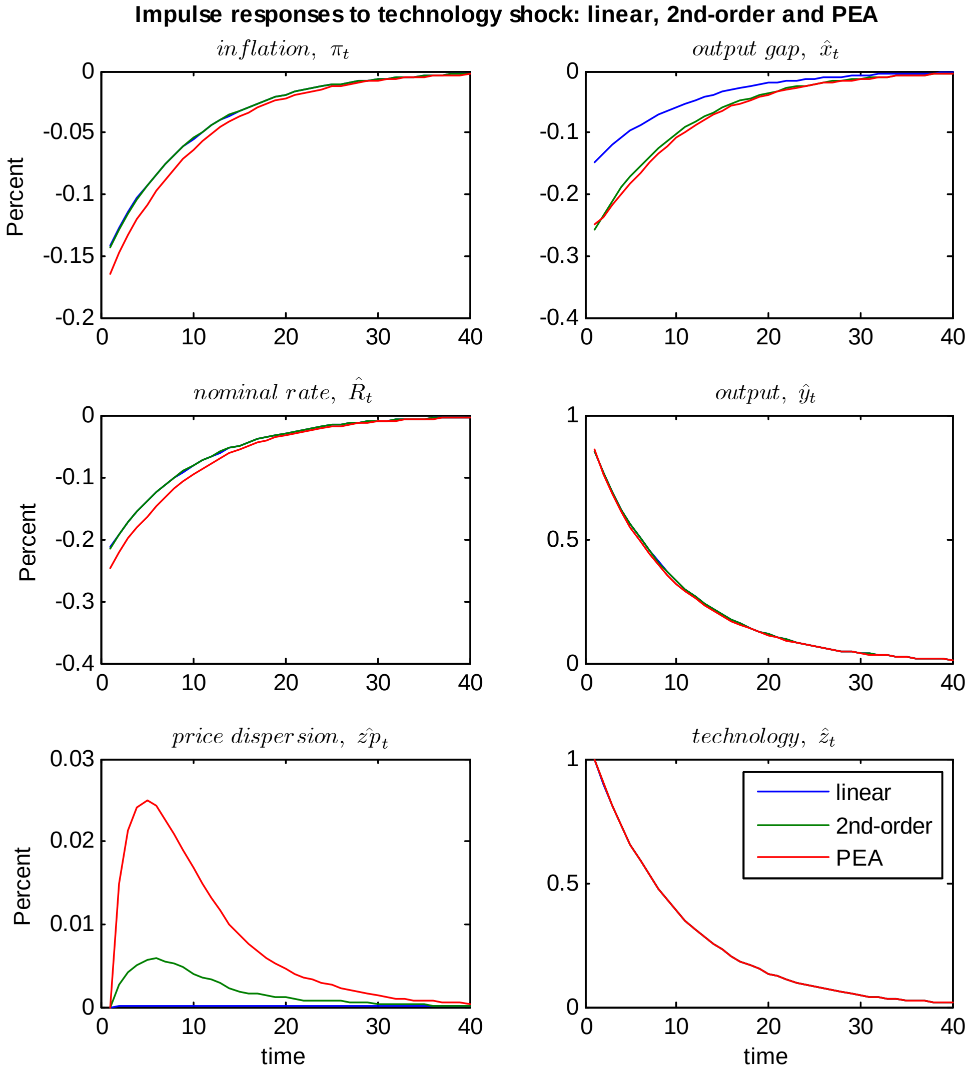

Figure 3.33: Model 3 (high price rigidity)

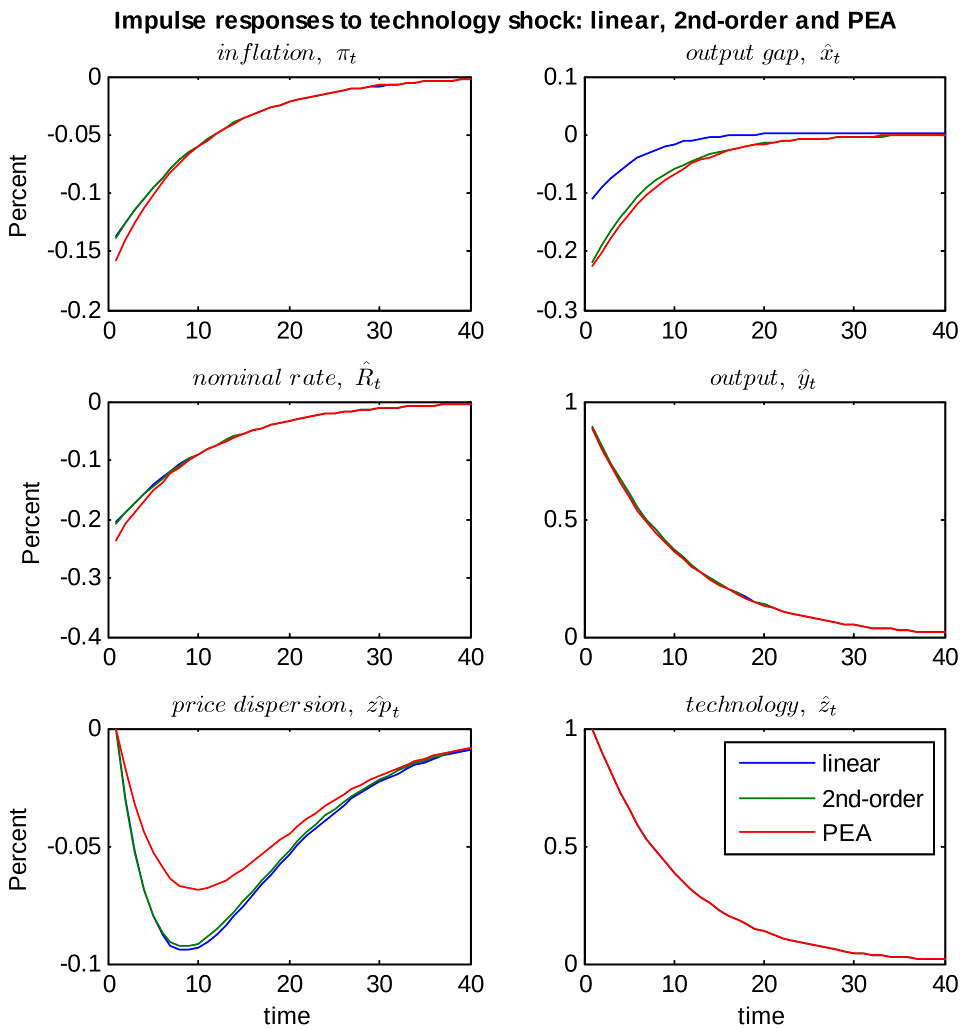

Figure 3.34: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

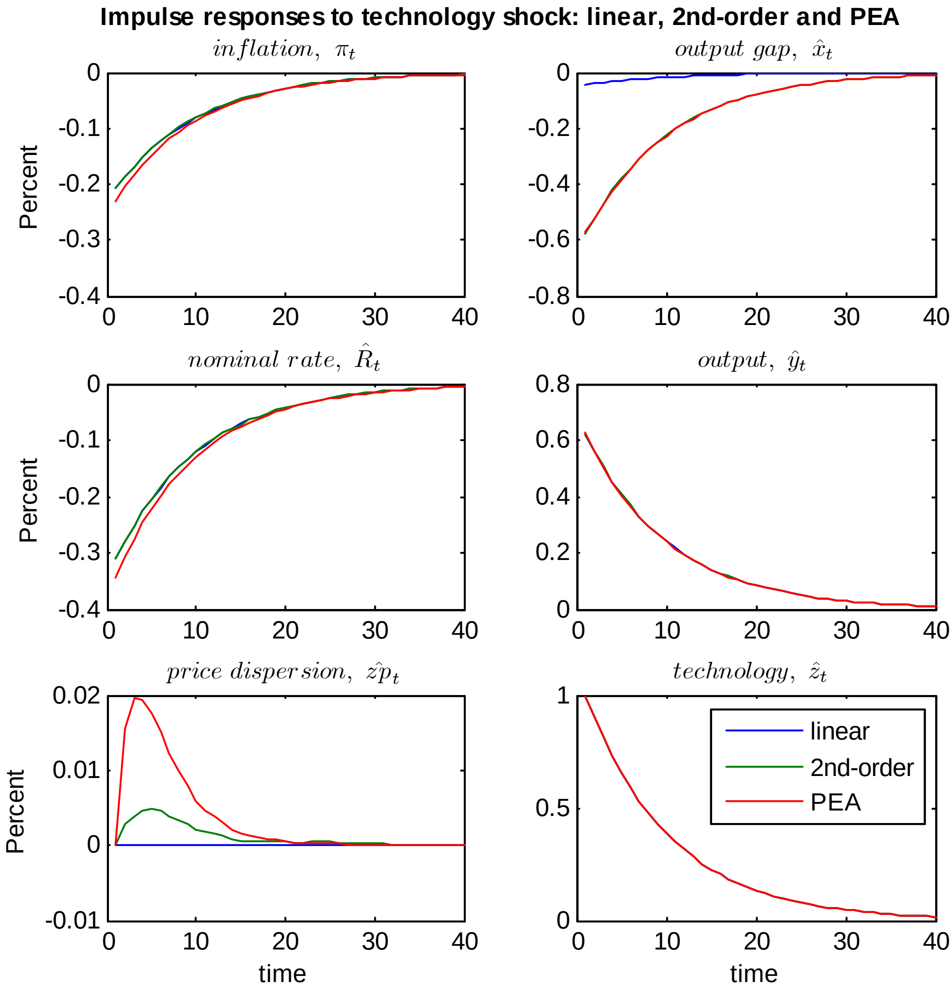

Figure 3.35: Model 5 (high risk-aversion of households)

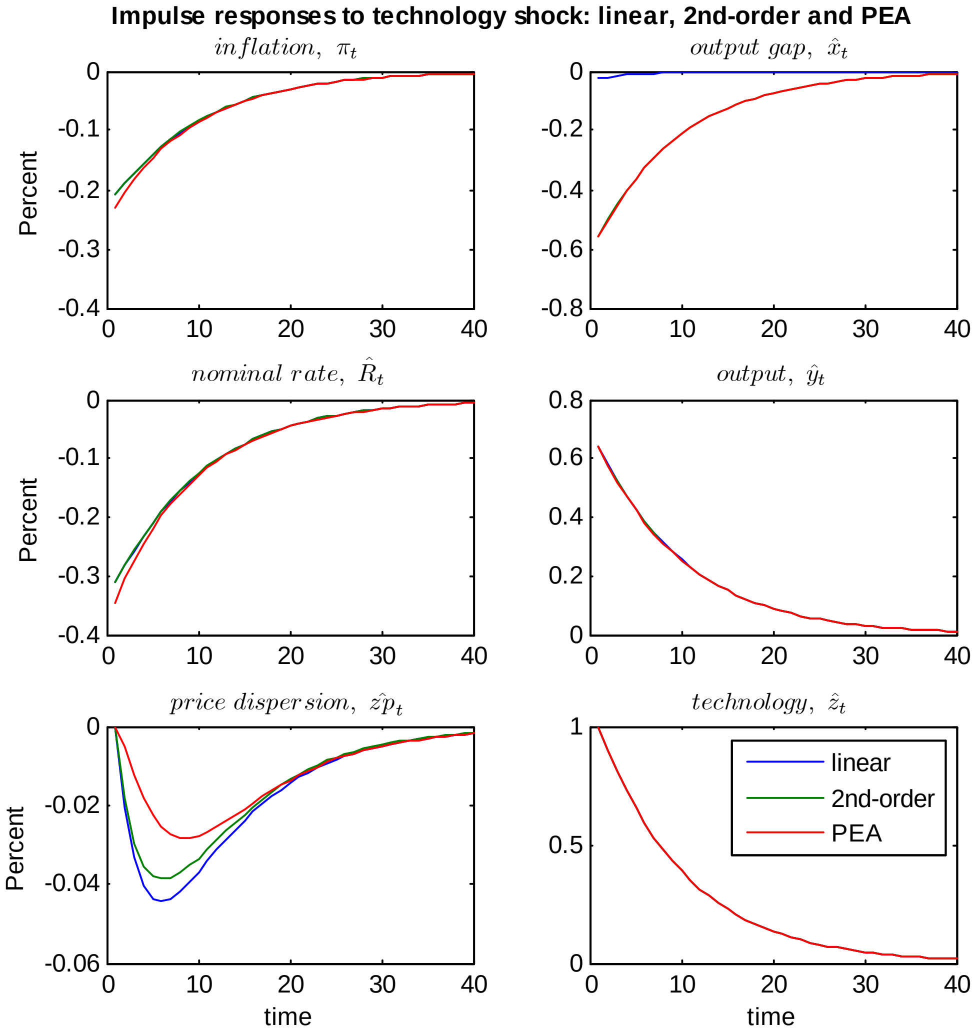

Figure 3.36: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

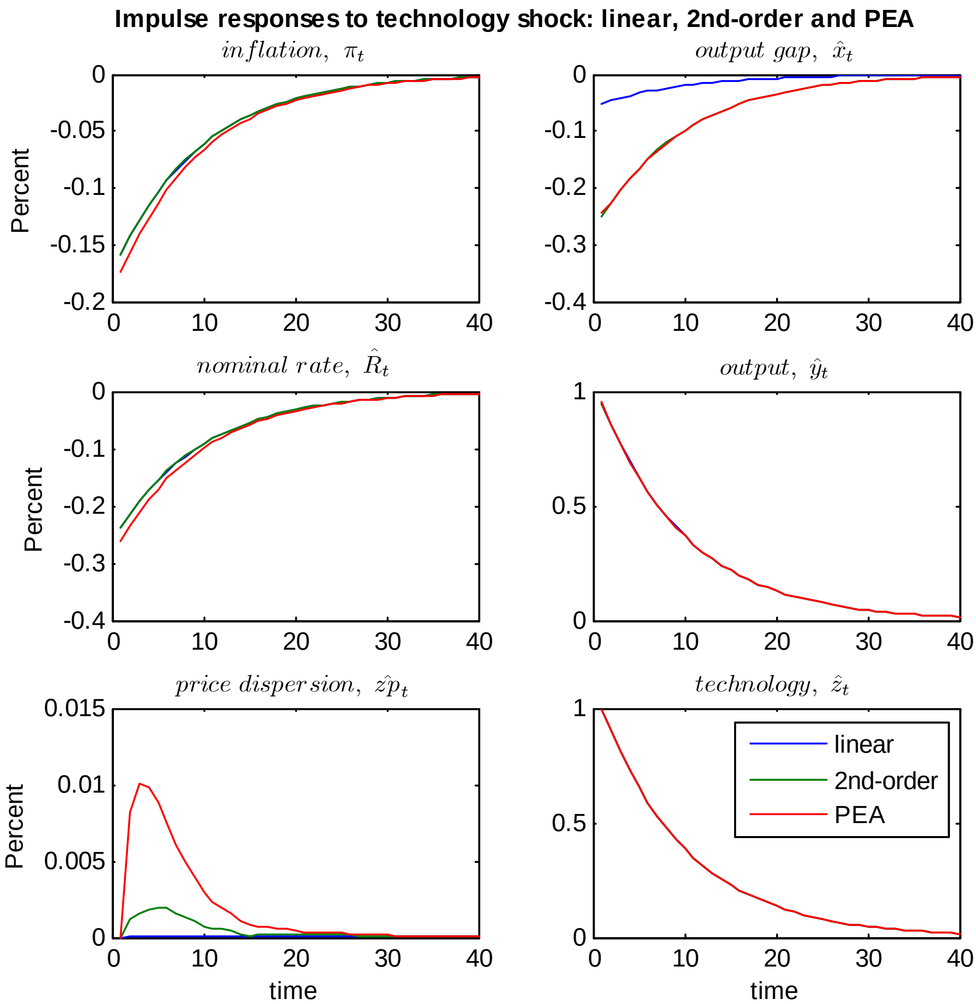

Figure 3.37: Model 7 (high market power of firms)

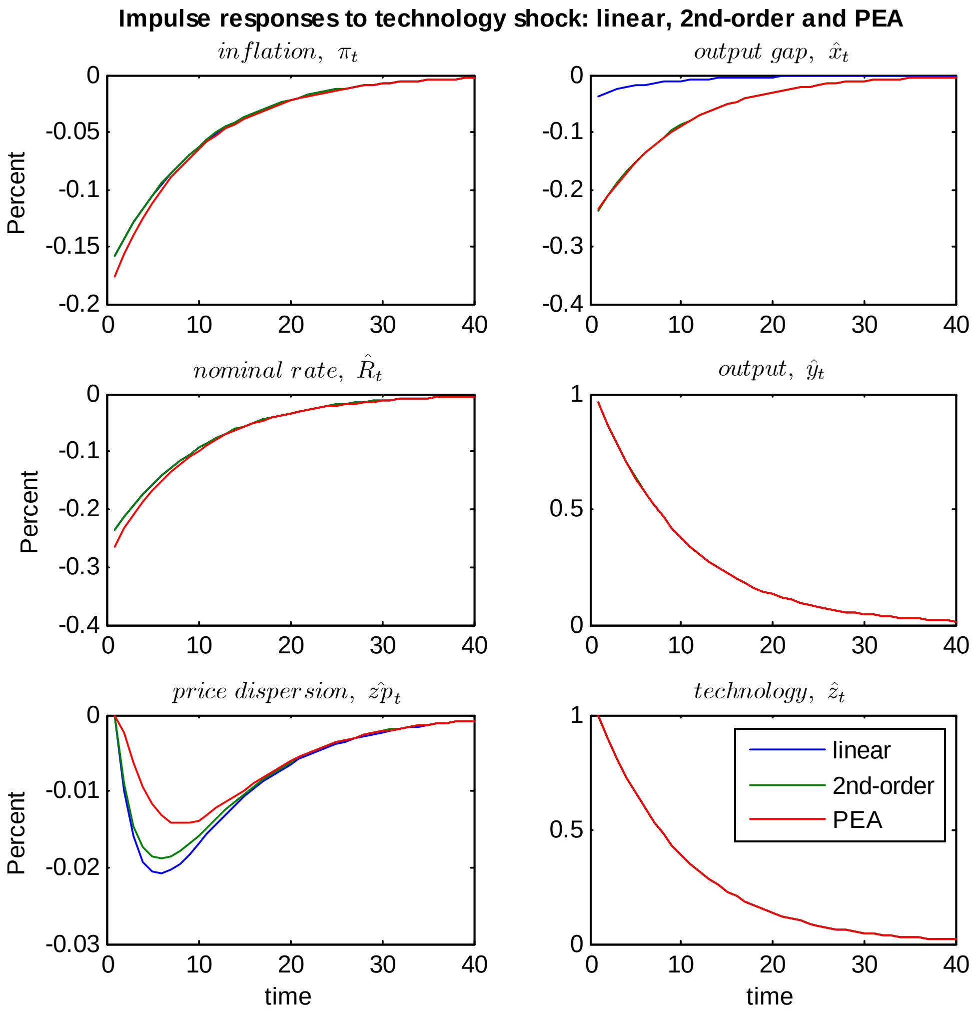

Figure 3.38: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

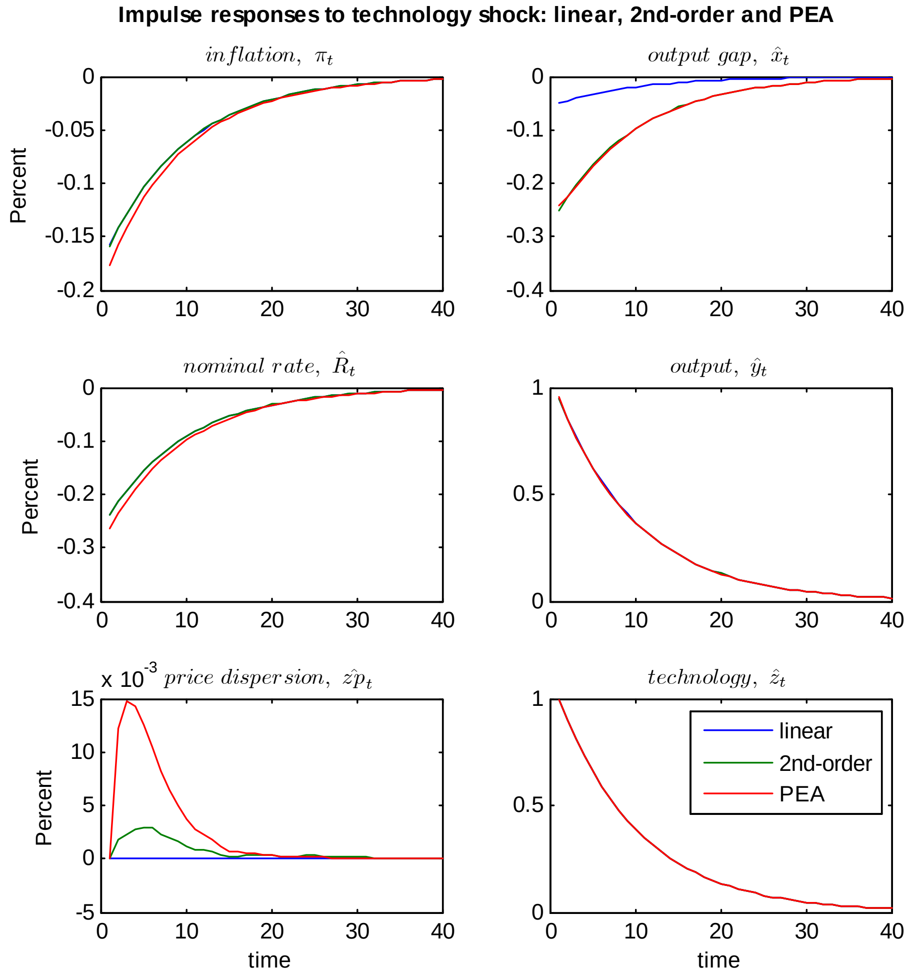

Figure 3.39: Model 9 (high volatility of technology shock)

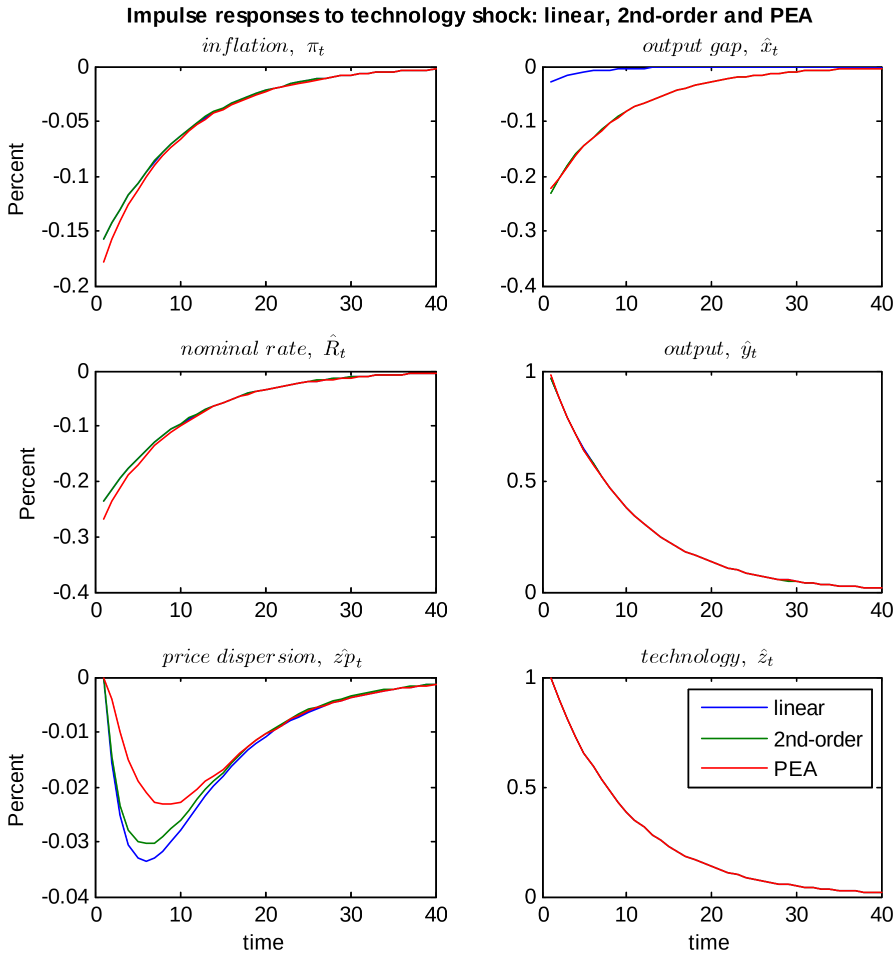

Figure 3.40: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Monetary policy shock

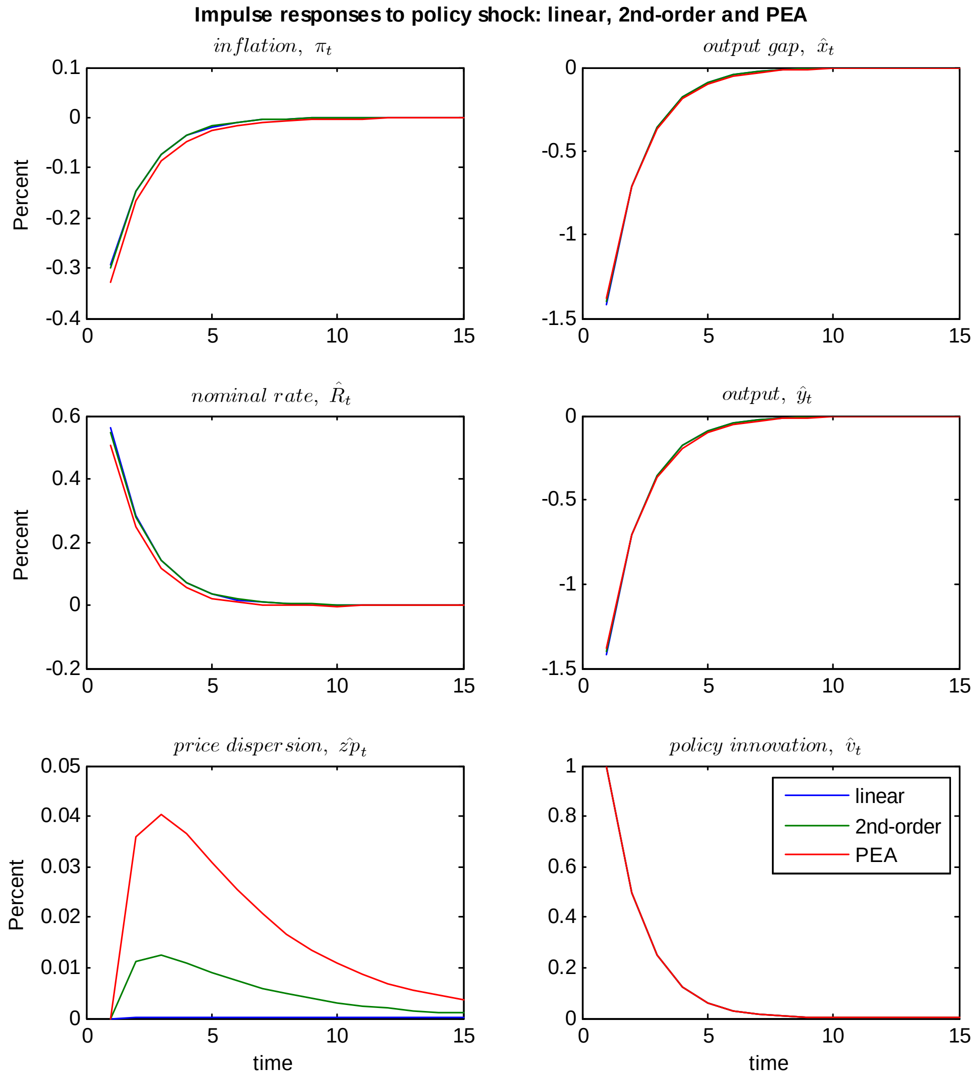

Figure 3.41: Model 3 (high price rigidity)

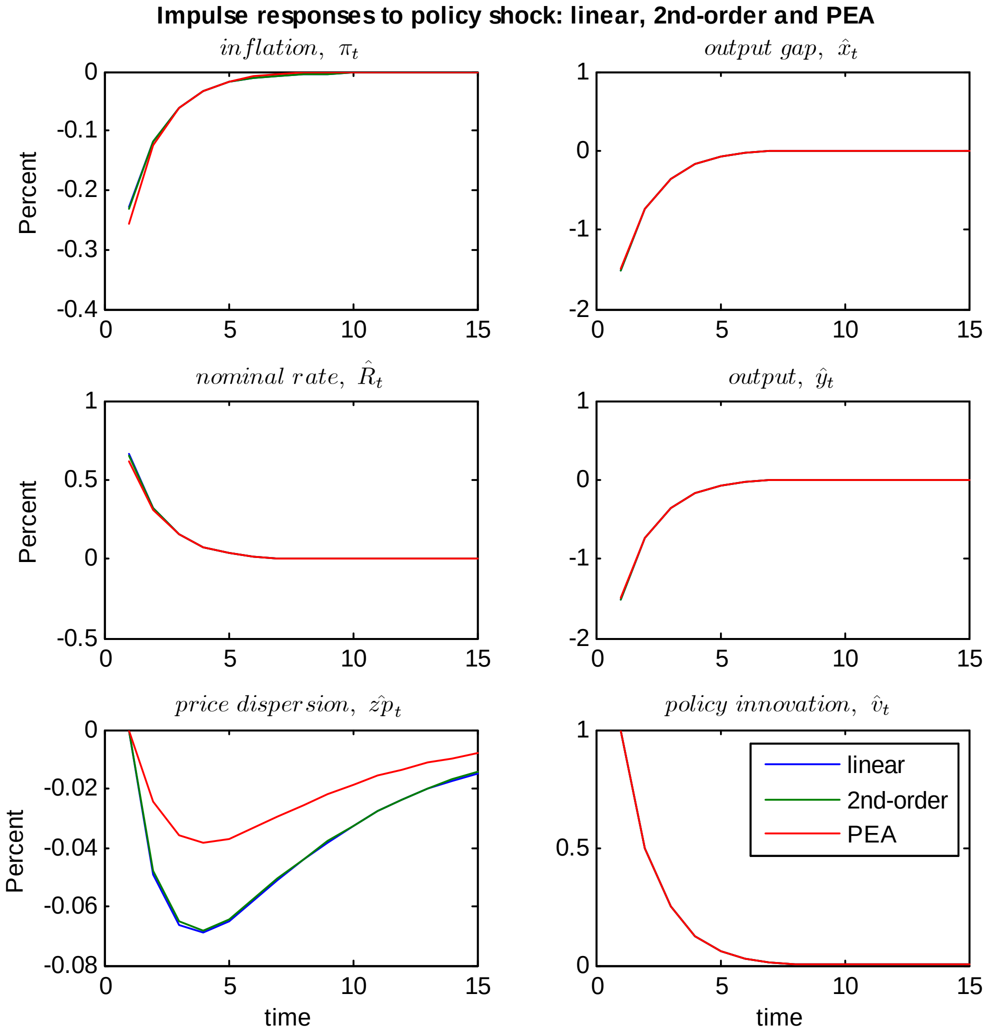

Figure 3.42: Model 4 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

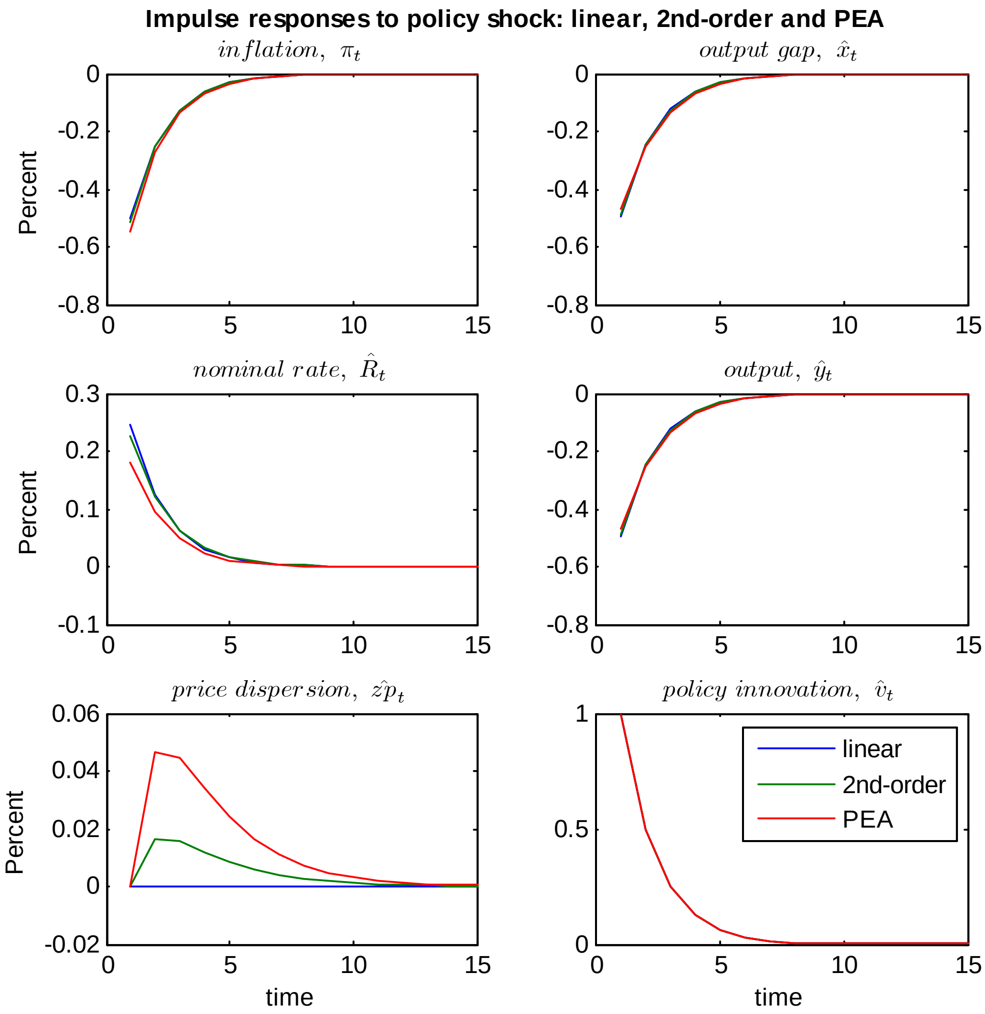

Figure 3.43: Model 5 (high risk-aversion of households)

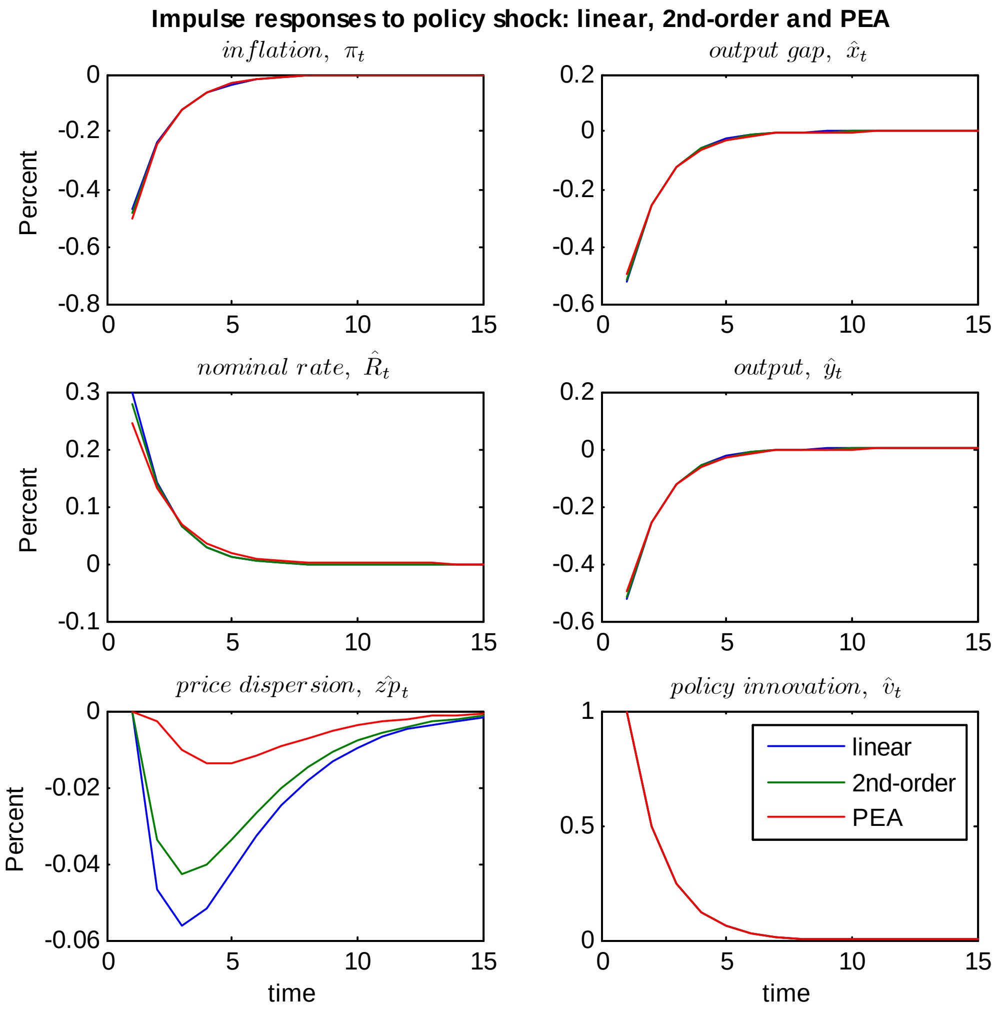

Figure 3.44: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

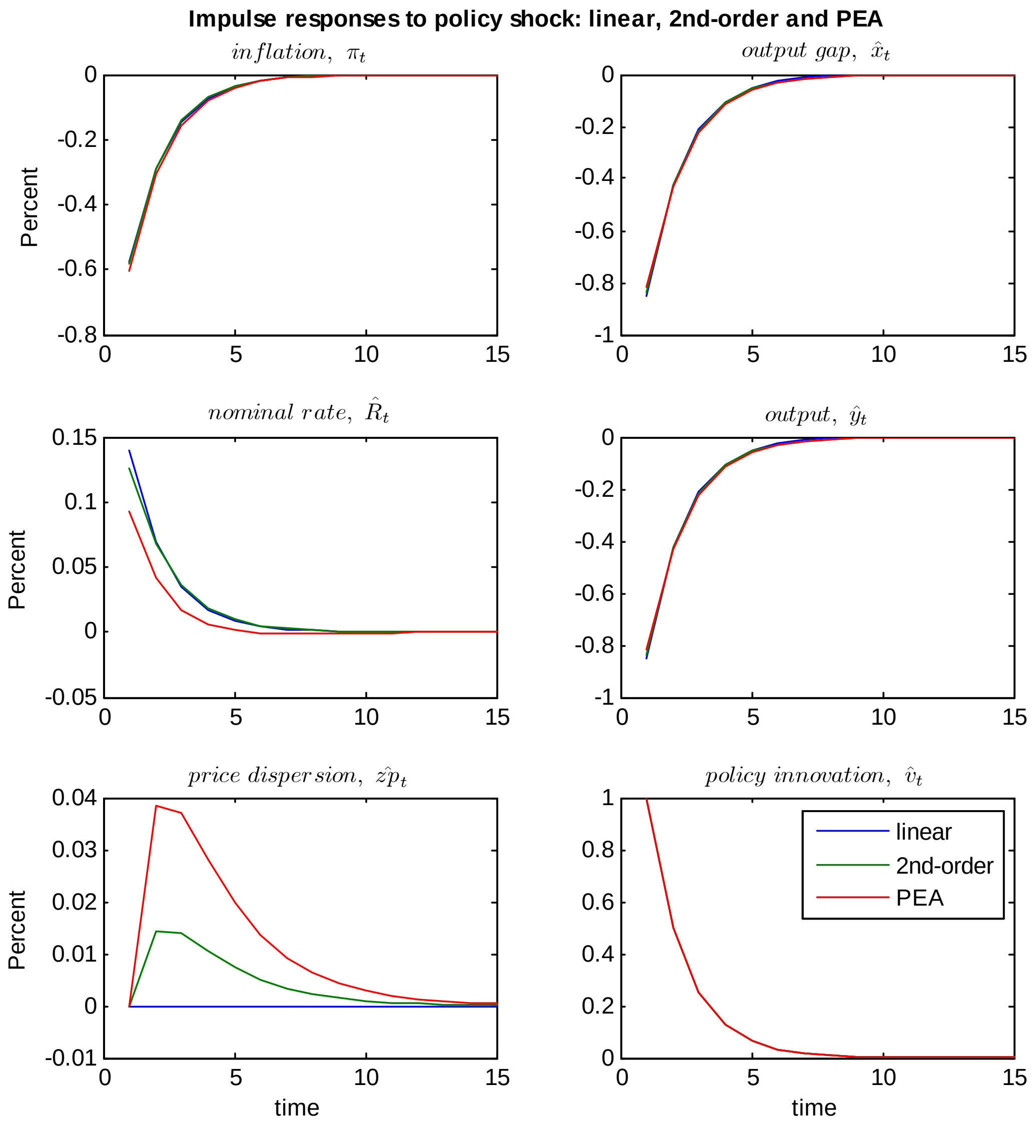

Figure 3.45: Model 7 (high market power of firms)

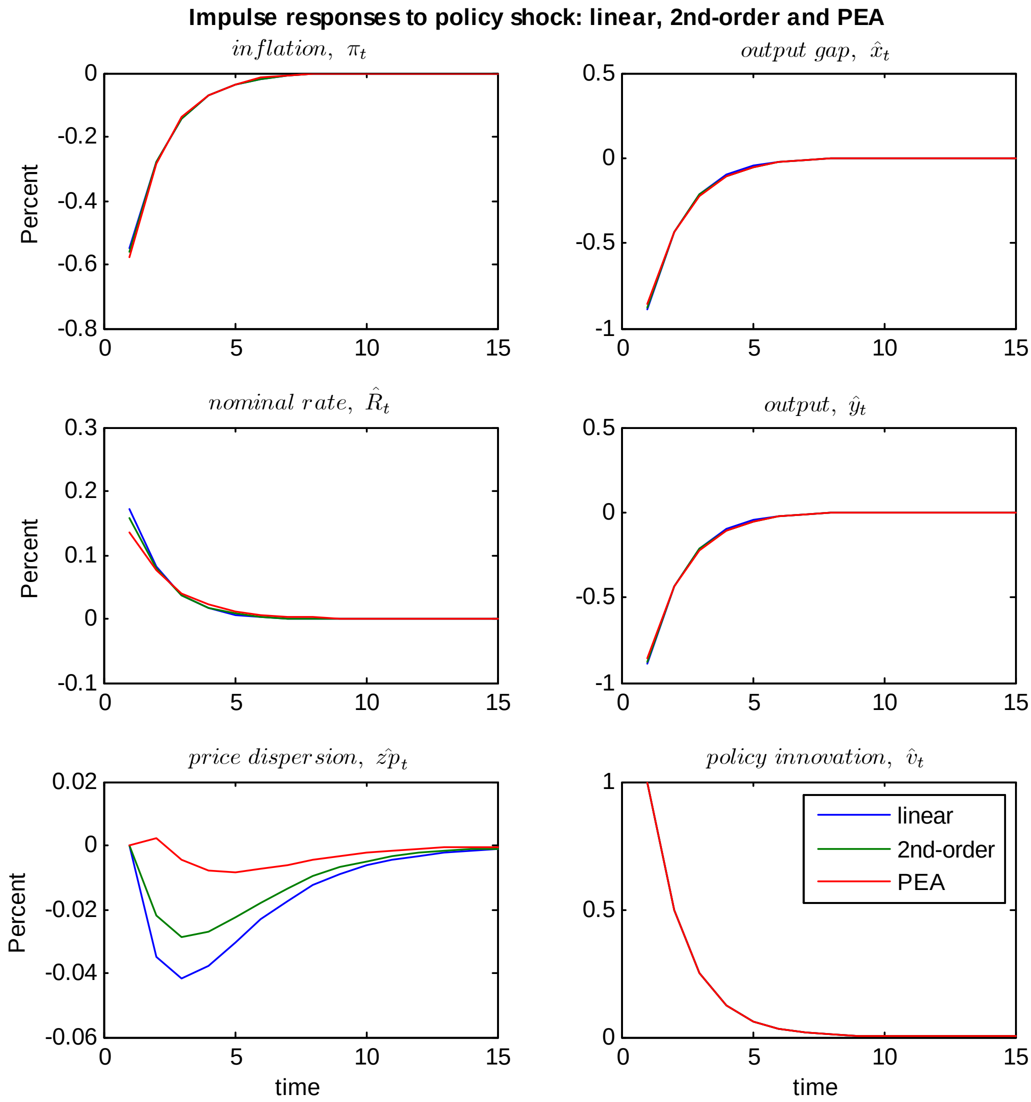

Figure 3.46: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

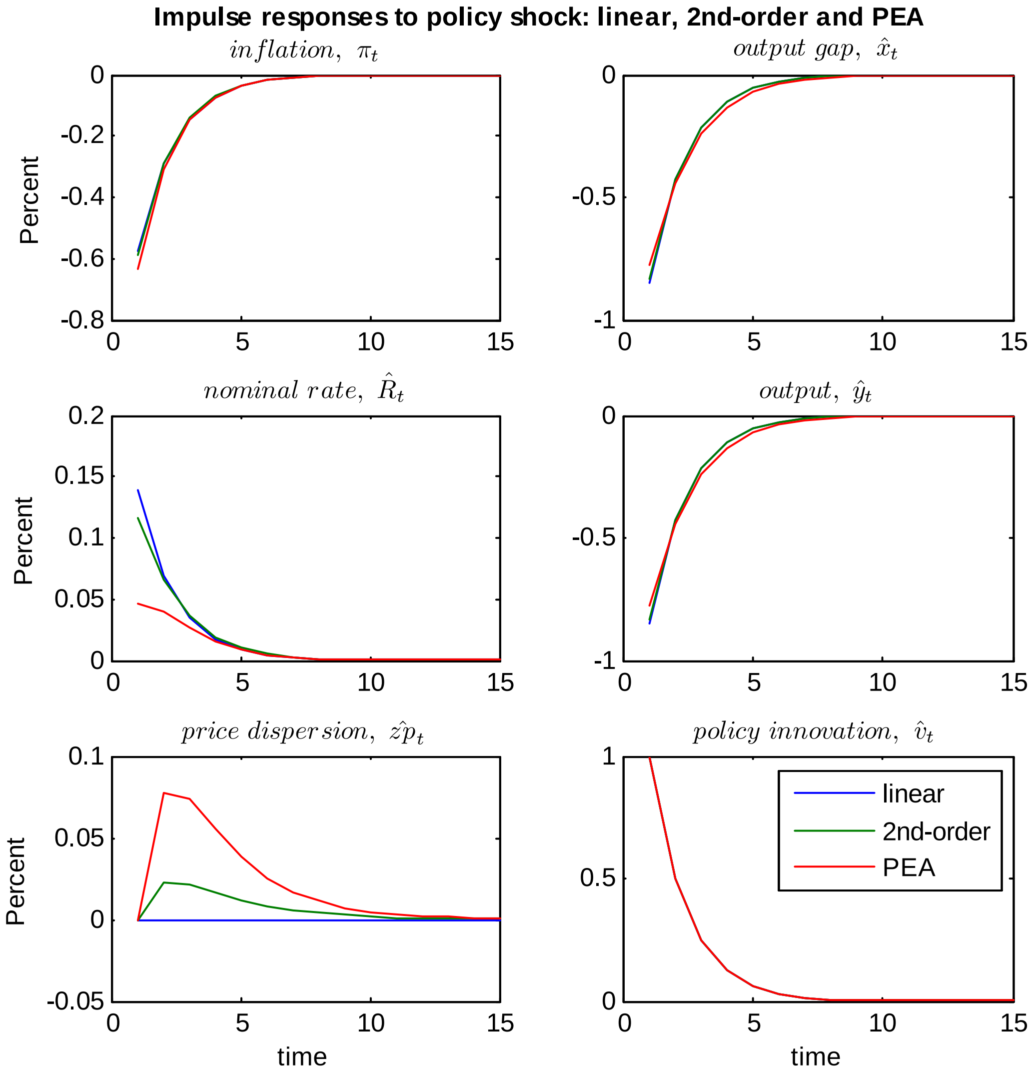

Figure 3.47: Model 9 (high volatility of technology shock)

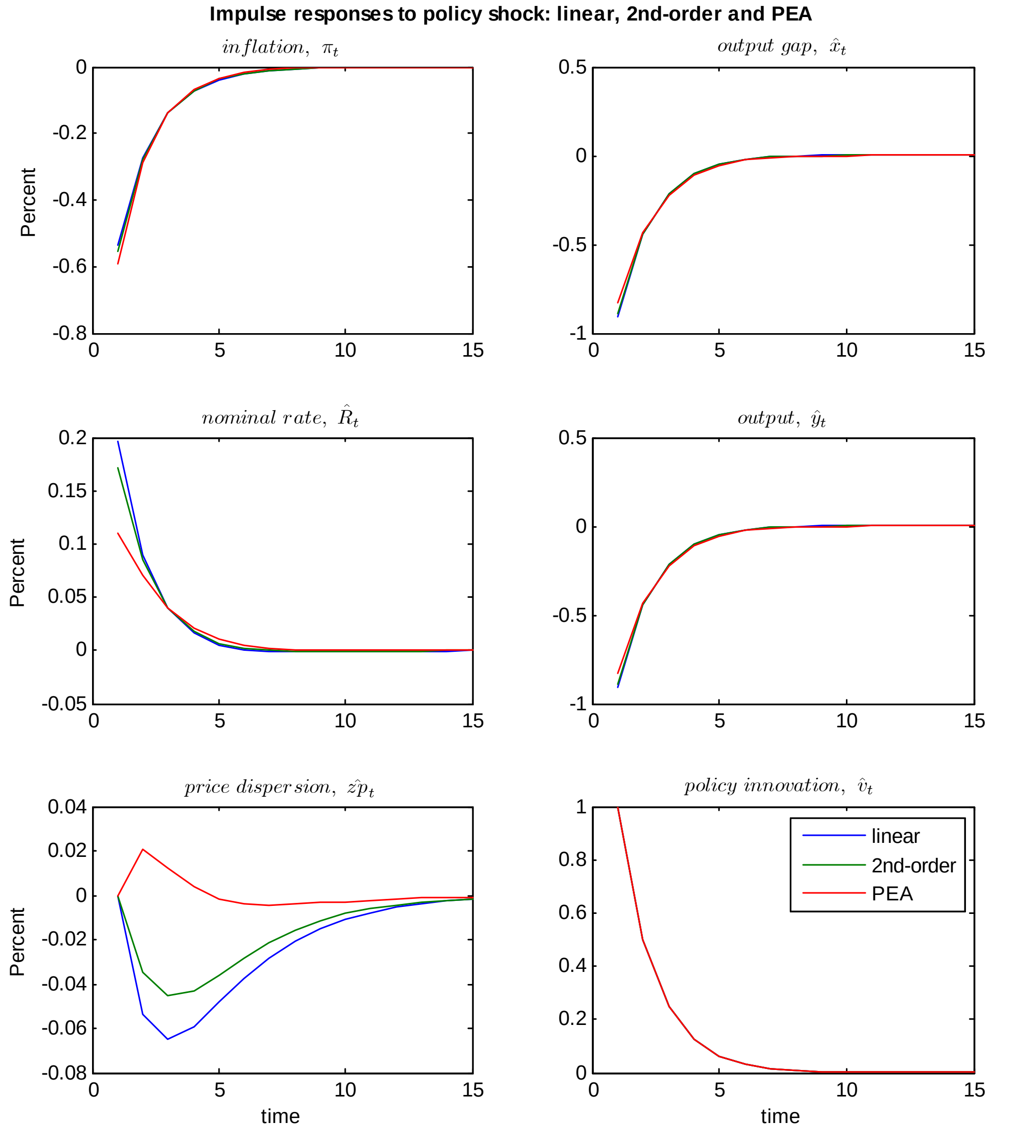

Figure 3.48: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Cost-push shock

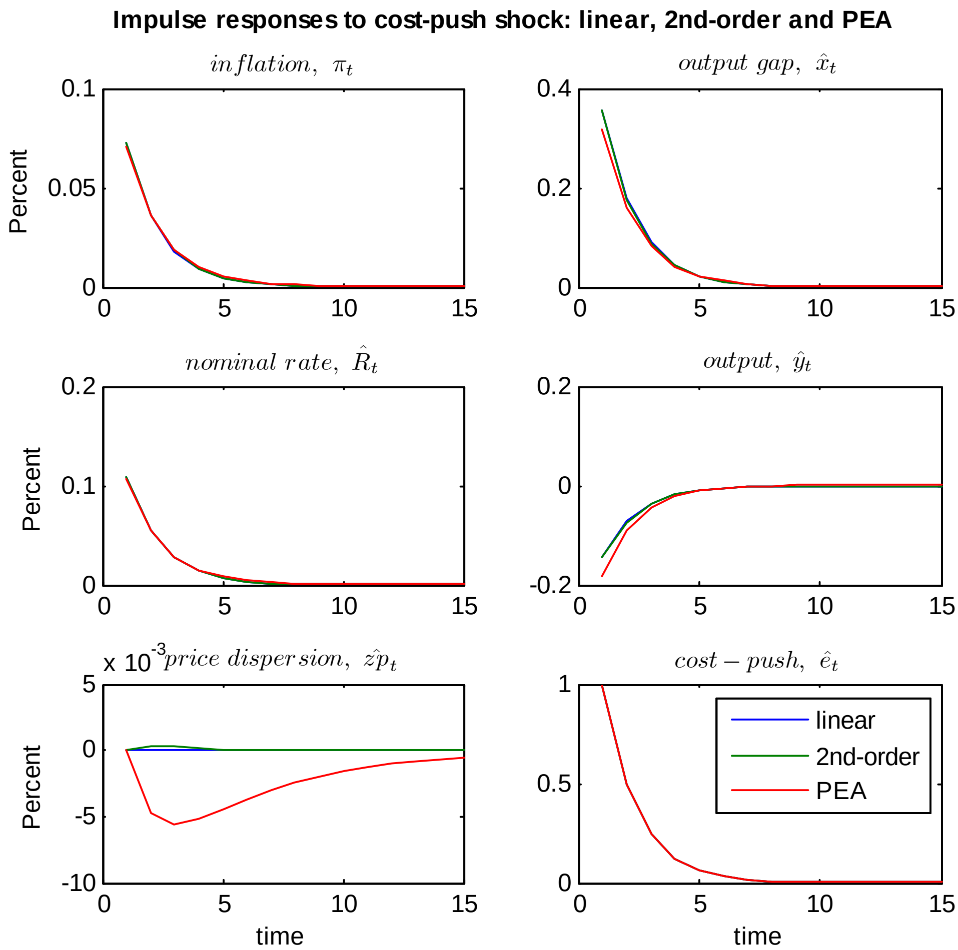

Figure 3.49: Model 3 (high price rigidity)

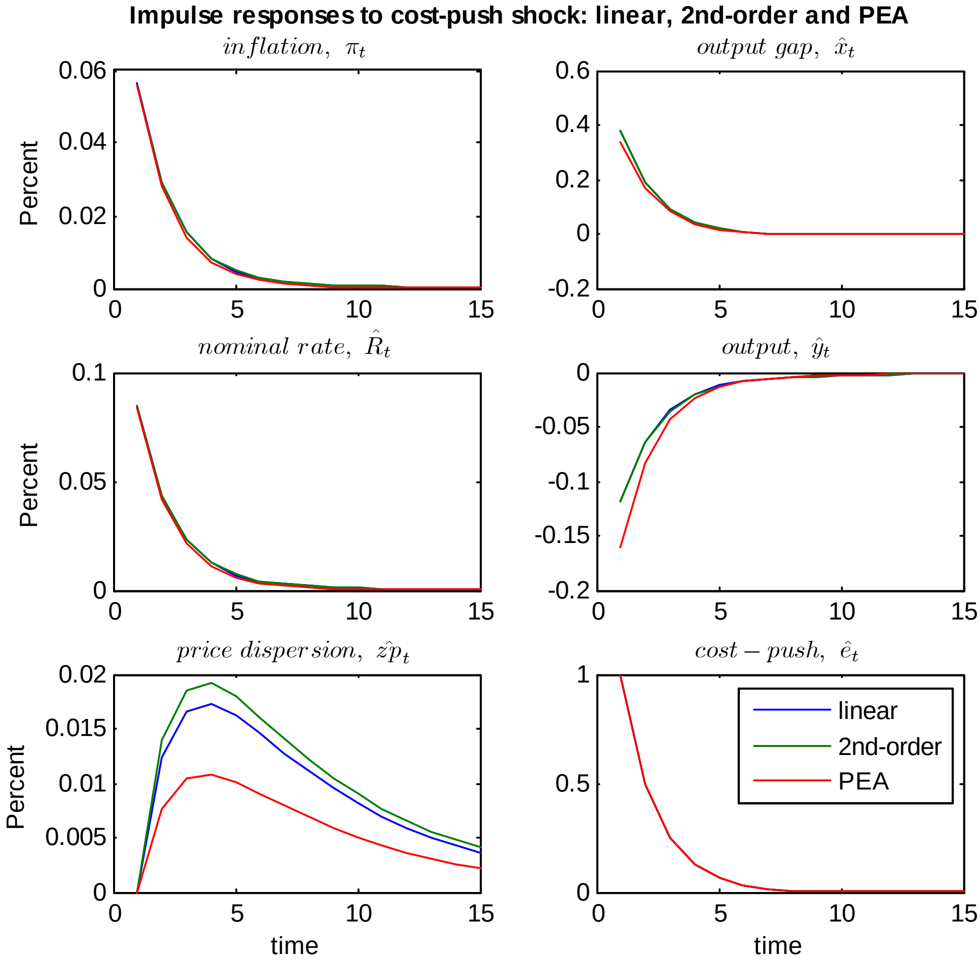

Figure 3.50: Model 3 (high price rigidity + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

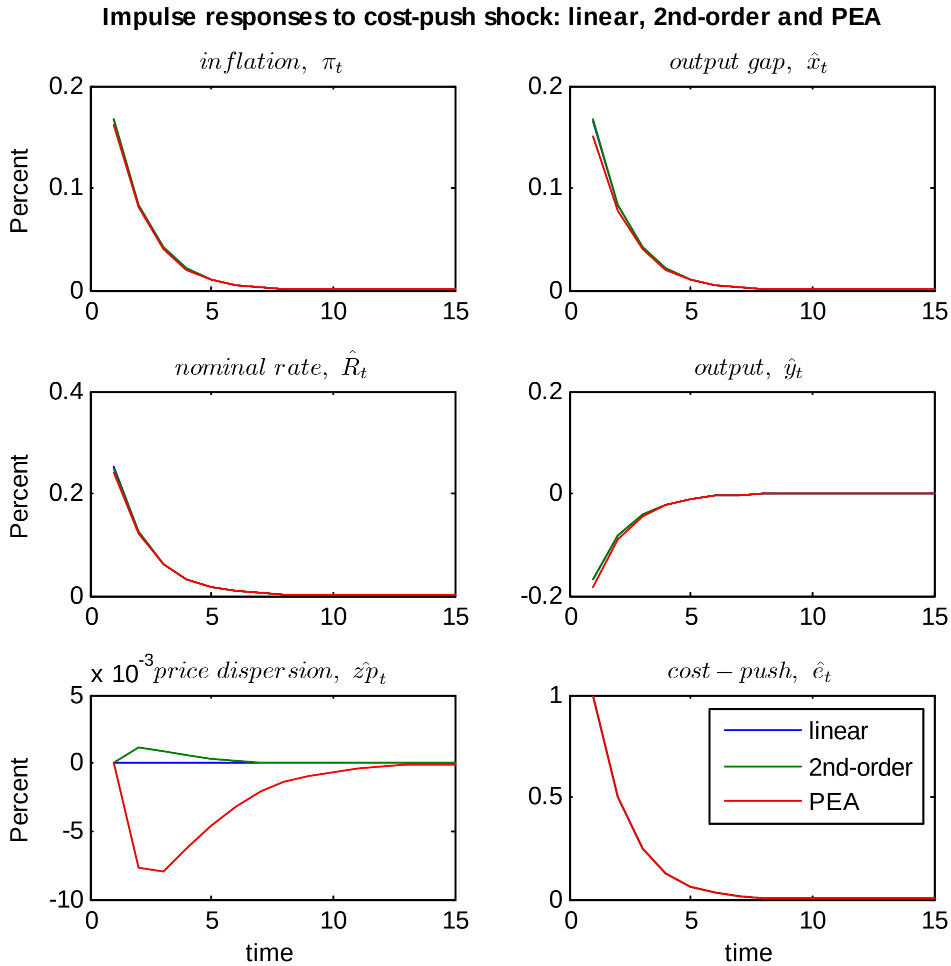

Figure 3.51: Model 5 (high risk-aversion of households)

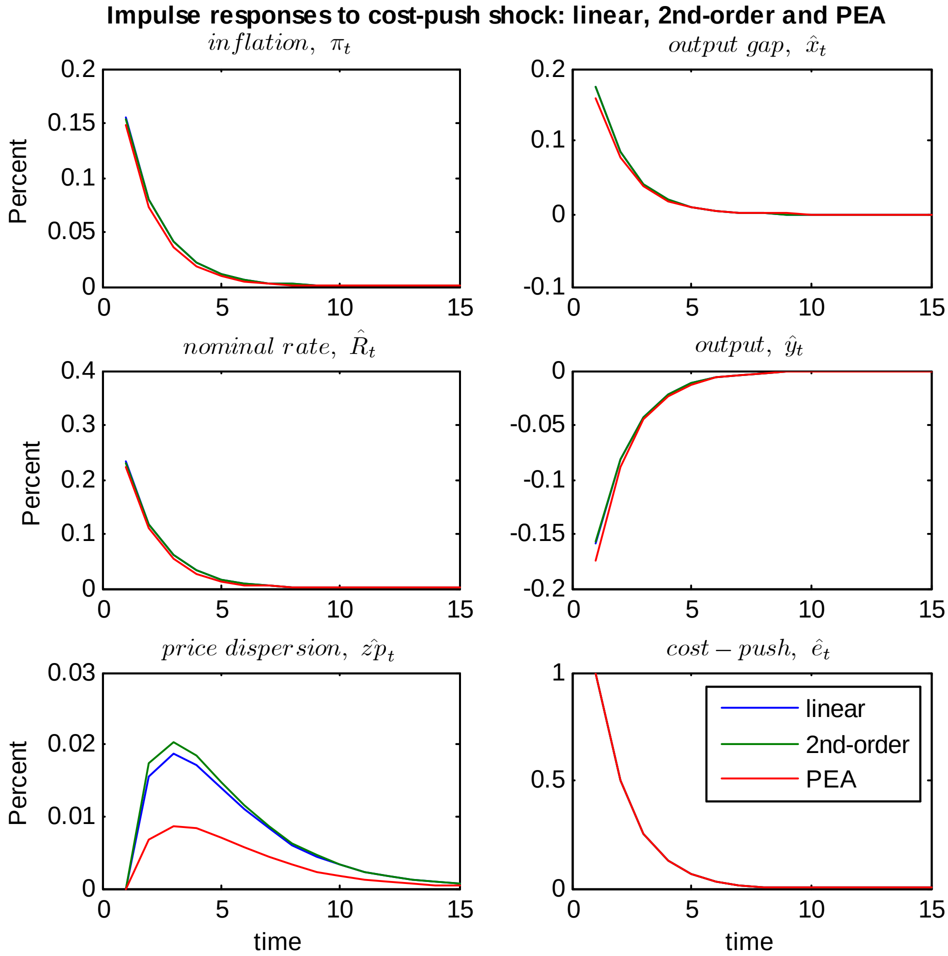

Figure 3.52: Model 6 (high risk-aversion of households + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

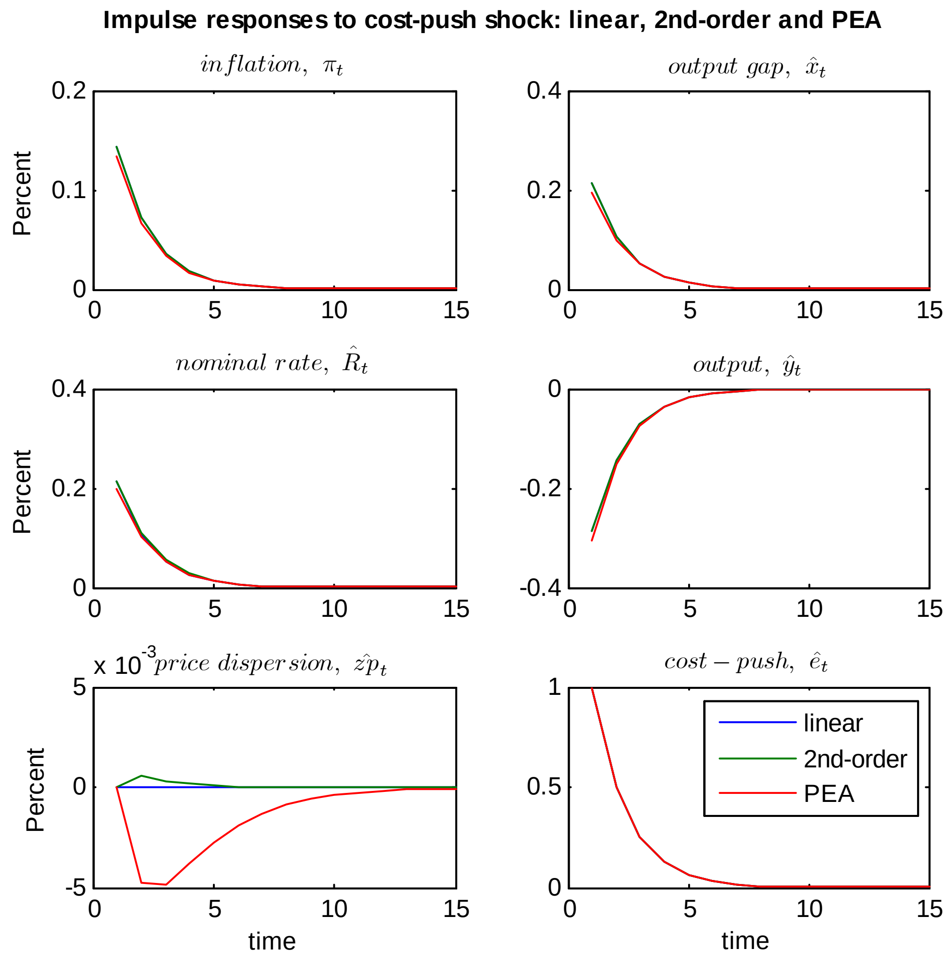

Figure 3.53: Model 7 (high market power of firms)

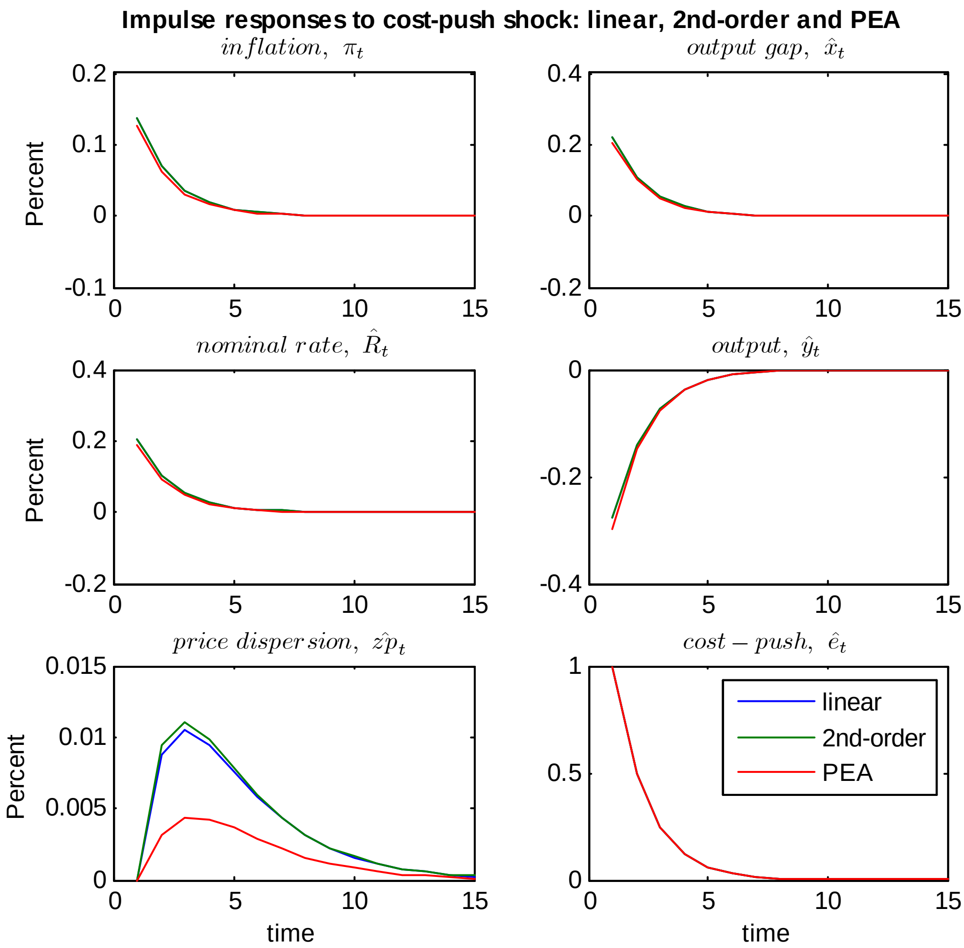

Figure 3.54: Model 8 (high market power of firms + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

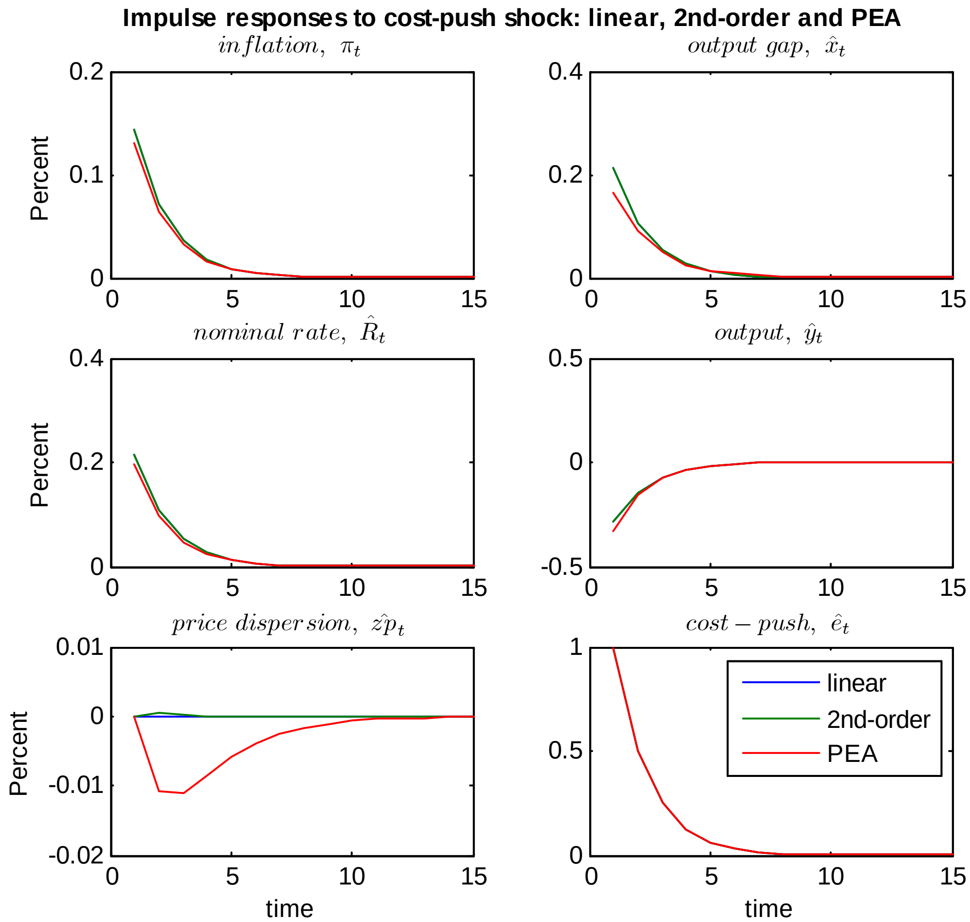

Figure 3.55: Model 9 (high volatility of technology shock)

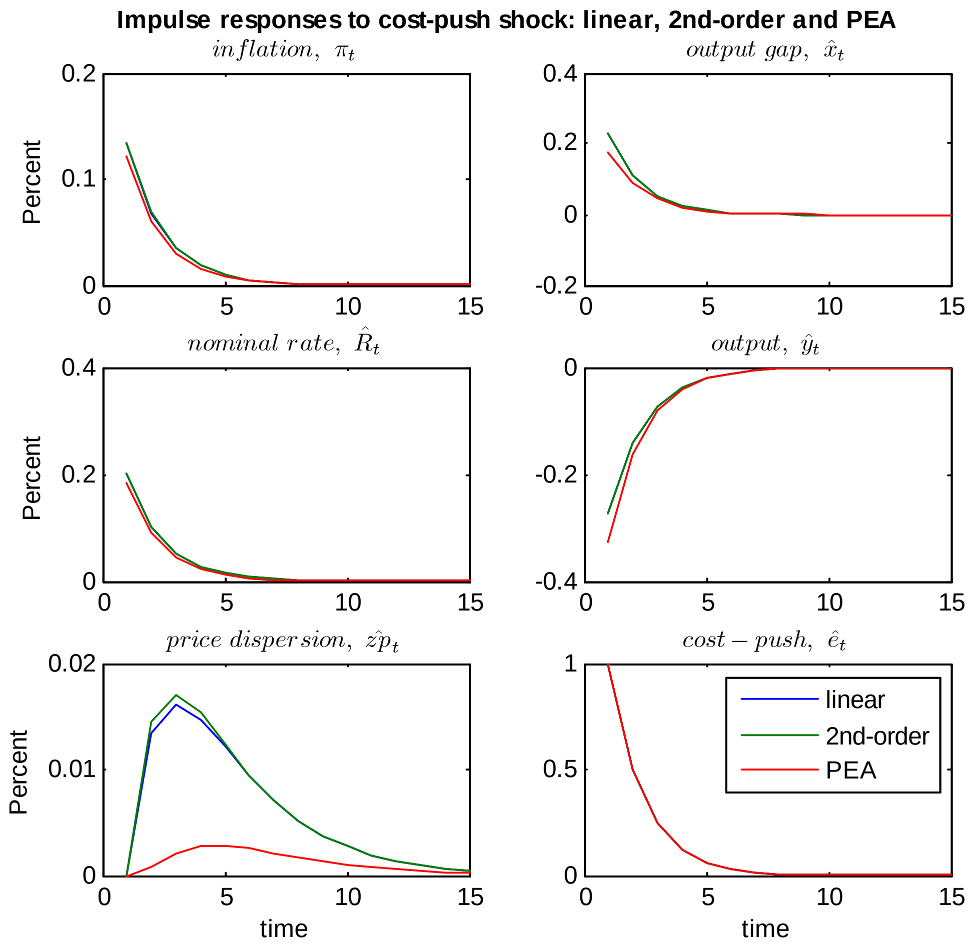

Figure 3.56: Model 10 (high volatility of technology shock + $\overline{\pi} = 1.03^{\frac{1}{4}}$)

Summary and conclusions

This thesis has undertaken empirical and theoretical analyses of business cycles in post-reform China and has examined the accuracy of alternative methods for solving non-linear rational expectations models. Chapter 1 estimated a VAR model using quarterly data and identified economic disturbances by applying both long- and short-run restrictions proposed by Blanchard and Quah (1989) and Galí (1992) so that they could be interpreted as the four structural shocks found in the IS-LM-PC model. Chapter 2 then constructed an RBC model augmented with fiscal policy for the Chinese economy and conducted a full set of assessments of the performance of the model. Chapter 3 examined a broad yet critical issue: the accuracy of solving non-linear rational expectations models. This was done by comparing the linear approximation method, the second order approximation method and the PEA algorithm for solving the benchmark NK model.

Main findings (Chapter 1)

1. The estimated responses of variables to all the four structural shocks in the SVAR model match strikingly well with those predicted by the IS-LM-PC model. In particular, the working of the three types of demand shocks are evidence that as reform goes deeper and deeper, the market mechanism has gained growing importance, allowing the market channels to function in terms of fiscal and monetary policies.

2. There is strong evidence from the variance decomposition that supply shocks associated with technology progress, efficiency and institutional changes in reform account for almost all fluctuations in output. This suggests that the roles of fiscal and monetary policy shocks are minor. Whist this is not a surprising result for transition economy such as China, it might also suggest that the working of fiscal and monetary policy might be through non-market mechanisms such as direct management and other administrative controls found in literature. This implies that further reform in economic structures and institutions such as financial liberalization are needed to remove the obstacles preventing economic policies from functioning fully. Moreover, based on the current progress of reform, it is suggested that theoretical models should first be built on the real side of the economy.

3. Over the post-reform period, 1983-2009, supply shocks are the main sources of output fluctuations while demand shocks, especially the fiscal forces, can play important roles in different sub-periods.

4. The above results are robust to alternative specification of the VAR with alternative measures of interest rate and money. Moreover, by allowing integrated processes in nominal variables, it is found that all the four structural shocks account for the unit root in nominal variables.

Main findings (Chapter 2)

1. The three-sector RBC DSGE model delivers an overall good account of the Chinese economy. The population moments of the cyclical movements of output and its components in the calibrated model roughly match the sample moments of the data well. The simulated cyclical components of most variables, using real shocks in the data, mimic the real cyclical movements experienced by the actual economy reasonably closely.

2. Several aspects where the model does not perform successfully were also found. For example, the volatility of the model cannot match all the variables in the data. Furthermore, the model generates far lower persistence for some simulated variables such as output. The model also fails to generate comparable cyclical movements of labour supply and real interest rate. This implies that the simple assumptions of market mechanisms in both labour and capital markets might be problematic in the case of China.

3. The counterfactual experiments allowing only one or two shocks to work in the model suggest that TFP shocks contribute the most to the fit of the model to data. The two fiscal shocks, on the other hand, improve the performance of the model to varying degrees

4. The sensitivity analysis shows that the main results in the benchmark case are quite robust to alternative values of capital share, weights of utility components and constant return to scale production function in public capital. This indicates that the model can usefully serve as a benchmark model for studying the Chinese economy.

5. The calibrated DSGE model is useful for studying the effects of fiscal policy in

China. The impulse responses of variables show that the shock in the ratio of government consumption to output has a strong negative wealth effect. Conversely, the shock to the ratio of government investment to output can generate mid- to long-run positive wealth effects, leading to positive and persistent impacts on output.

Main findings (Chapter 3)

1. The linear, second order and non-linear solution methods demonstrate significant differences in simulated data reflected by the policy functions, moments and distributions. The impulse response functions of most endogenous variables are generally similar for all solutions. However, those of the output gap, nominal interest rate and price dispersion differ considerably. In particular, the impulse responses of price dispersion from the PEA solution are shown to be significantly different from linear and second-order approximations for all three exogenous shocks.

2. The Judd measure and Marcet's J-test show similar results that the PEA is more accurate than the linear and second-order solution methods. The biggest difference of accuracy emerges in solving the price-adjustment equation. The Judd measures of linear and second-order approximations are close.

3. The differences across solution methods become larger when alternative calibrations are considered. For example, non-linear steady-state inflation, higher degree of nominal rigidity, risk aversion of households, market power of firms and volatility of technology enlarge differences in summary statistics and accuracy evaluations. However, the PEA continues to perform reasonably well in all alternative calibrations. Therefore, the main result from the benchmark model - that the PEA is more accurate - is robust.

Contributions

Compared to the existing empirical studies on the sources of business cycle fluctuations in China, Chapter 1 represents the first attempt to further decompose the components of AD shocks. In particular, the identification of monetary and fiscal shocks has important implications. It enables the analysis on the effects of monetary and fiscal policy on the economy, which is in turn taken as a gauge of the progress of the reform and the growing of market mechanisms. Moreover, since the study has used high frequency data with long sample size, both long- and short-run restriction assumptions have been justified. Furthermore, the study offers a complete analysis of the sources of economic fluctuations for the period 1998-2009.

Chapter 2 provides the first complete evaluation of the RBC model for the post-reform Chinese data. The existing literature only looked at the volatility changes before and after the reform, whilst this study also examined other important features of the business cycles such as persistence of variables and cross-correlations between variables. The RBC model used in the study was also specified with richer structure in the government sector. Particularly, the utility-yielding government consumption and productive public capital are incorporated in the model to better reflect the important role played by the government in the Chinese economy. Additionally, this study offered a superior benchmark calibration of the model to data. Moreover, a sensitivity analysis allowing alternative calibrations has also been conducted as a means of checking the robustness of the main results.

Chapter 3 contributes to the body of literature focusing on solving non-linear rational expectations by formally examining the accuracy of the NK model solution. This was performed by specifying a benchmark NK model with cost-push shocks and solving it using the linear approximation method, the second order approximation method and the PEA algorithm. The comparisons of solutions obtained from the three different methods have been comprehensive. Not only did they make use of the comparisons of model generated data, they also used formal economic and statistical evaluation techniques on accuracy. The robustness of the results was also examined by conducting the same evaluations under alternative parameter values of the model.

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