INVESTMENT SELECTION CRITERIA:
An examination of the theory of the internal rate of return and of the investment discount rate under conditions of uncertainty.

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by

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CONTENTS

SUMMARY

The thesis is concerned with the problems which confront the practitioner when seeking to translate capital budgeting theory into operational terms. It argues that the lack of widespread acceptance in industry of the discounting model can be attributed, in part at least, to the complexity and subjectivity of conventional recommended procedures. In particular, the practitioner is presented with two central issues, the uncertainty about whether to use the Internal Rate of Return or the Net Present Value method, and the apparently almost insurmountable problem of selecting the correct discount rate. The thesis seeks to establish:

(1) that the internal rate of return is incorrect in principle for choosing between alternative projects, and should be discarded as a primary criterion,

(2) that the current yield on Government bonds is a satisfactory discount rate to use for evaluating any project, irrespective of

(a) the project's riskiness,
(b) the method of financing,
(c) the expected rate of inflation,
(d) the tax deductibility of interest payments,
(e) the firm's dividend policy,
(f) the flotation costs incurred in raising the finance.

Contribution of the thesis

Internal rate of return:

1. Demonstrating that the reinvestment opportunities for intermediate cash flows are irrelevant when assessing projects under nonrationing conditions.

2. Indicating the theoretical irrelevance of a rate of return internal to a project for assessing its desirability.

3. Proposing an alternative ratio to supplement the NPV which is consistent with the latter.

The required rate of return:

1. Demonstrating that the operation of the +ve NPV rule

   (a) invalidates all empirical studies designed to ascertain ex ante required returns from analyses of ex post returns;

   (b) invalidates the assumption that maximisation of the share price is equivalent to maximisation of the value of the firm.

2. Demonstrating

   (a) that rejection of the assumption of a risk-premium for equity investors is theoretically and empirically defensible;

   (b) that the cost of a corporate bond is independent of the bond's coupon rate;

   (c) that the practice of reducing the cost of debt by the rate of corporation tax to reflect the tax deductibility of interest payments can lead to incorrect solutions, and

   (d) that the practice of adjusting the cost of capital to allow for flotation costs leads to incorrect solutions.

INTRODUCTION

"While theorists recommend the IRR (or NPV) criterion of investment appraisal, this study confirms the prevalence of the payback period and the accounting profit criteria in practice. The theorists must identify the reason why financial executives prefer these alternative criteria and modify the IRR (or NPV) method to make it more generally applicable."

James C. T. Mao Journal of Finance, May 1970, p.359

The selection of capital projects is among the most critical decisions that businessmen are required to make. The pace and direction of the firm's growth and, in turn, the economic welfare of the nation, are dependent, in part, on the quality of the investment appraisal procedures employed. Yet it appears that a high proportion of firms in the U.K. employ theoretically inadequate methods of assessment. This thesis is concerned with the problem of making the correct appraisal techniques acceptable and comprehensible to the significant number of firms which have not yet adopted them, without a material sacrifice of theoretical rigour.

The management of capital expenditures comprises a number of steps which were best summarised by Joel Dean, as follows: (1)

1. A creative search for investment opportunities
2. Long-range plans and projections for the company's future development
3. A short-range capital budget
4. A correct yardstick of economic worth
5. Estimation of the economic worth of individual projects

(1) Dean, 1954

6. Screening and selection of proposals

7. The control of authorised outlays

8. Post-completion audit

9. Disposal of the project

10. Forms and procedure

Capital budgeting theory is primarily concerned with step 4, the search for a correct measure of economic worth, and it is clear that if unsatisfactory yardsticks are used to evaluate potential investments there may be little incentive to carry out the other steps with any degree of thoroughness. The effort needed to search for new investment opportunities, for example, or to improve the quality and flow of information needed to evaluate them, may appear futile if their effectiveness is destroyed by inadequate assessment. But, at the same time, it needs to be emphasised that whatever yardstick is used, it should be perceived as no more than a tool whose function is to give the decision-maker an insight into an investment's desirability. It seems at times that the emphasis which academic writers have given to this stage of the investment decision process is disproportionately large. To be effective, the selection criterion must not only be theoretically acceptable, it must also be practicable, and be capable of being understood, both by those who use it, and, if possible, by those whose contribution to the investment process is affected by it. These goals of validity and simplicity may at times conflict with one another, particularly, when the desire by theoreticians to construct a completely satisfactory theoretical framework leads to the development of solutions which are inaccessible to the majority of practitioners. It must be apparent to readers of journals such as the Journal of Finance, the Journal of Financial and Quantitative Analysis, and the Journal of Business Finance and Accounting etc., that the conflict between the two goals may have already come to a head. The investment

appraisal literature demands from executives the capacity to integrate a variety of management science techniques, including utility analysis, mathematical programming, probability and statistical methods, whilst at the same time, it exposes and leaves unresolved a series of conceptual and operational difficulties which combine to give the recommended procedures a high susceptibility to wide margins of error. There is an increasing danger that decision-makers may pay no more than a lip service to the newer techniques, whilst seeking refuge in the traditional more familiar rules of thumb for their primary guidance. There is some evidence(2) that even among those firms which have adopted the discounted cash flow approach, there are many which still employ payback and the accounting rate of return as supportive if not primary criteria of selection. A large number of firms do not even pay lip service to the discounting methods,(3) presumably because they are ignorant of them, or because they believe that the derivation of data to be analysed involves such a range of possible errors that only a rudimentary economic analysis is justified.

If the economic principles which underly the quantitative techniques employed are incapable of being interpreted by the nonspecialist, the problem of balancing the qualitative and quantitative aspects of the decision will be magnified to a degree that may allow the quantitative measures to dictate the decision rather than to inform the decision-maker. It becomes increasingly important to guard against the danger that as the elegance and sophistication of the techniques increase, the investment decision will be assumed to be capable of resolution entirely on a quantitative basis, with the result that effective authority for making investment decisions could be transferred from

(2) e.g. see Klammer
(3) see Cooper 1975 page 198

the management team to the specific personnel responsible for manipulating the data. If we accept the premise that the decision to undertake other than routine investments is too important a matter and has too many nonfinancial implications to be encompassed in a mathematical formula, however elegant, it becomes a matter of importance that this trend be resisted. To some extent, at least, the tools recommended by scholars must be tempered to match management's capacity to use them.

The purpose of this thesis, therefore, is to search for an approach to the analysis of capital expenditure, which is fundamentally consistent with current theory and, at the same time, sufficiently straightforward to give the discounting technique a wider appeal, and a genuine significance for the non finance specialist. The search derives its inspiration from the conviction that the recent emphasis by theoreticians on the need to evaluate projects not as isolated financial events but as constituents of a portfolio of risky assets of unspecified range, is as potentially capable of providing a framework in which the selection criterion is reducible to a meaningful expression within the real understanding of the nonspecialist, as it is capable, if not harnessed, of alienating him entirely.

One of the dangers of striving for a simplified framework to facilitate the task of practitioners in real world decisions is that of appearing to wish to make light of the contribution which theory has to make. In no sense is that the intention of the present study. Indeed, it is precisely as a result of the very significant theoretical strides that have been made in recent years that the proposed framework finds its basis. Part of the task, of course, is to question a number of technicalities that have emerged in the literature and which appear not to justify their usefulness. And, by definition, the search for

simplification involves the rejection of certain refinements as being inappropriate to a basic model which is designed to be acceptable to a broad spectrum of corporate users. But the writer is aware that in a field which is expanding as rapidly as corporate finance, further significant developments can be expected in the future, and, therefore, no solution can be presented which is not amenable to improvement. It is important, therefore, that the framework presented retains significant flexibility to allow refinements to be incorporated where appropriate, without, at the same time destroying the practicability of the basic model.

Although the subject of the thesis concerns the practicability of recommended investment techniques, the research procedure is essentially theoretical in character, apart from some empirical testing of investors' attitudes to risk, the findings of which are reported in Chapter 5. A considerable proportion of the study consists of seeking to identify errors in accepted theory which have created unnecessary complications for the student and practitioner of capital budgeting, and the remainder consists of an attempt to develop a selection criterion which meets the test of practicability with the minimum sacrifice of theoretical rigour and integrity. In particular, the thesis is concerned with the nature and validity of the internal rate of return criterion as an apparent alternative to the net present value approach to the evaluation of investment proposals, and with exploring the possibility of justifying the use of a readily observable market-determined rate of interest as the appropriate discount or cut-off rate.

The first chapter consists of an exposition of the fundamental tools recommended by academics to practitioners for the analysis and evaluation of investment proposals. Commencing with a brief account

of the historical development of accepted decision procedures, the chapter proceeds to identify the two principal theoretical issues with which practitioners are confronted, the choice between using the IRR and NPV approaches, and the derivation of the relevant cost of capital for use as the discount rate. The chapter identifies and discusses at length the unresolved issues associated with these controversial topics.

The second chapter focuses on the IRR, and seeks to establish that current theory has offered incorrect reasons for its failure to produce consistently correct signals, and concludes that the IRR is incorrect in principle, and has no effective relevance as a criterion for optimal investment decisions. An alternative profitability ratio is offered as a supplement to the NPV measure, which is consistent with the economic rationale of the latter.

The third and subsequent chapters are concerned with the theoretical and practical issues involved in the derivation of the firm's cost of capital. The cost of equity and the cost of debt are considered in turn, together with the impact of the firm's capital mix and dividend policy. Clearly, these issues present enormous conceptual problems which may never be capable of being fully resolved, and it is far beyond the scope of this thesis to seek to do so. However, they are issues which create significant obstacles for the practitioner in his efforts to find operational rules for applying the correct decision procedures, and the respective chapters address themselves principally to the task of finding compromise solutions acceptable to practitioners and theoreticians alike. The penultimate chapter considers the accounting implications of the previous chapters and seeks to demonstrate the dependency of the proposed solution on adequate publication of

relevant data. The final chapter draws together the findings of the previous sections, and after analysing their significance, presents the recommended decision framework.

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David J. Cooper — "Rationality and Investment Appraisal", Accounting and Business Research, Summer, 1975

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Chapter One

THE ISSUES

The objective of the Investment Decision

The theory of business finance in general, and of capital budgeting in particular, is concerned with the impact of the firm's financial decisions on the welfare of one group of participants, the shareholders or residual owners of the firm. This is not to say that finance writers fail to recognise the existence of other goals or imply that firms do not have responsibilities other than to shareholders. But in a free enterprise society, it is desirable at times to abstract from the wider responsibilities of the firm in order to focus on the implications of the firm's decisions as they affect those who supply the risk capital, and with whom traditionally rests the right to initiate or discontinue business operations. Whilst it may be possible in the future to construct a theory which encompasses the firm's responsibilities to all interested groups including society at large, theoreticians have not yet succeeded in achieving an agreed operational framework for the shareholder group. It is unlikely that we will be able to develop the proper perspective needed to undertake the wider synthesis until we have successfully formulated the claims of the individual participating groups. Therefore, even in a mixed economy, the apparently narrow focus of capital budgeting theory can be validated.

Shareholders' welfare is generally defined in terms of wealth maximisation or maximisation of the net worth of the firm.(1) Wealth

(1) e.g. See Van Horne, page 6 and Weston and Brigham page 11

rather than profits is the relevant goal because the former reflects both the risk dimension of the firm's financial decisions and the time value of money. It is therefore in the context of this goal that the subsequent chapters are developed.

In order to achieve a valid measure of wealth, two ingredients are necessary, an appropriate income stream and a discounting procedure to convert the income stream into present value terms. The relevant income stream is now almost universally acknowledged by theoriticians to be cash, because it is cash rather than profit which is needed to service capital. The main controversies are concerned with identifying a discounting procedure which conceptually and operationally is capable of achieving the agreed objective.

Although compound interest as a mechanism for reflecting the time value of money dates from the Old Babylonian Period in Mesopotamia 1600 B.C.,(2) prior to the 19th century the application of discounting was substantially restricted to the evaluation of loans and life insurance. It was not until there was a significant increase in the magnitude of investment outlays, in particular with the coming and development of the railways, that the importance of incorporating some allowance for time in the evaluation of nonfinancial investments became recognised. Since the 1950s especially the sophistication of capital budgeting techniques has increased enormously. Yet certain fundamental issues remain unresolved, not least the fact that there are two distinct methods of implementing the discounting principle,

(2) For a brief history of the discounting approach cf. Parker 1969 Chapter 3.

the internal rate of return (IRR) and the net present value approach (NPV). The IRR is the rate of interest which equates the present value of future cash receipts to the initial capital cost of the project, namely the solution r to the equation

$$\sum_{t=1}^{N} \frac{A_t}{(1+r)^t} - C = 0 \qquad (1)$$

where A_t is the net cash flow at the end of year t; C is the capital outlay; and N is the expected life of the project. The IRR criterion states that a project should be accepted if its internal rate of return is above k the cost of capital to the firm.

The equation for the net present value formula is

$$\sum_{t=1}^{N} \frac{A_t}{(1+k)^t} - C \qquad (2)$$

The NPV criterion, therefore, states that a project should be accepted if the value of its net present value is greater than zero, where the excess represents the contribution to the firm's present value.

The IRR is variously known as the Time Adjusted Rate of Return; the Investor's Method; the Yield: and (misleadingly) the Discounted Cash Flow Method. It is identical to Keynes' marginal efficiency of capital[3] but not identical, as Keynes erroneously assumed,[4] to Irving Fisher's marginal rate of return over cost.[5]

(3) Keynes, page 140
(4) For a discussion see Alchian 1955
(5) Fisher, page 155

The latter ratio is essentially related to the comparison of alternative streams, whilst the former is the rate which equates the present worth of an income stream with its expense stream. The ranking of projects according to their NPVs depends in part on the rate of interest used to discount the cash flows. The Fisher rate is the rate which brings the NPVs of two investments into equality. It is the turning-point interest rate, and for rates above the Fisher rate, the ranking of the two projects is different from that given by rates below it. The Fisher rate is mentioned here because, although it has not been widely used or recommended for use in the text-books, it has recently been cited in the journal literature(6) and by at least one text-book(7) to provide support for the IRR in the IRR v NPV controversy. The significance of the Fisher rate will be discussed in a later section of this chapter and again in Chapter 2.

IRR v NPV

Under conditions of certainty the NPV measure is obtained simply by discounting the net cash flows at the pure rate of interest, and the IRR is compared with the same rate. For simple accept or reject decisions it is a matter of indifference which of the two methods is used because they both provide consistently correct signals. In reality, of course, conditions are not so simple and significant problems arise fundamentally for two reasons (a) the investment decision is frequently not a simple one of 'accept or reject', but involves a choice between

(6) Carlton Dudley, pp 909 - 913
(7) See Mao, Quantitative Analysis of Financial Decisions, page 234

projects, and (b) the future is not known with certainty, and some allowance must be made for the degree of risk which projects are perceived to have.

Before outlining the conceptual problems which the decision-maker must face in practice when choosing between the two methods, it should be stated that it cannot be argued that having two methods rather than one is in itself a source of strength. Even if the two approaches consistently produced correct solutions, there appears little merit in presenting practitioners with both, if by agreeing upon one of them, the decision process could be simplified. If in fact one method consistently produces correct solutions, and the other not, then clearly the former would appear to be the appropriate choice. This conclusion would seem even more forceful if the second (inconsistent) method gave rise to a number of theoretically complex issues which caused the decision-process to appear considerably more complicated than if the first method alone were used. In practice the two methods do produce conflicting solutions and because there is strong support for each it remains to be established whether both have a valid role to play.

Although much of the subsequent discussion will be concerned with the theoretical validity of each of the two techniques, it should be noted that the arguments proposed in support of the respective approaches are frequently practical in character. Thus the IRR being a ratio of profitability is claimed to be understood by businessmen accustomed to thinking in terms of percentages.(8)

(8) Merrett and Sykes, The Finance and Analysis of Capital Projects, Page 123

Again, since by definition the IRR is the rate internal to the project, the ranking it gives is independent of the cost of capital and it is claimed this has the advantage that the investment analysis can be delegated to lower management without the necessity of the latter being involved in the derivation of the firm's cost of capital. Moreover the IRR is alleged to avoid the need to be specific about the cost of capital[9] the derivation of which is a difficult process. Those who prefer the NPV method argue that the method measures the monetary contribution which a project makes to the value of the firm, and is therefore more meaningful than a ratio of profitability. They argue also that there is no real advantage in ranking projects independently of the cost of capital since the relative desirability of a project frequently depends on the cost of capital. The NPV has also fewer problems of implementation than the IRR. On the other hand it is an important feature of the controversy that however strongly a particular writer might favour one of the two approaches, he invariably acknowledges a valid role for the alternative method under certain conditions.

The reinvestment assumption

The most common explanation for the failure of the IRR to rank projects consistently in accordance with the NPV method is that the IRR assumes that the intermediate cash flows can be reinvested at the internal rate.[10] This is in contrast to the assumption inherent in the NPV method that the cash flows are reinvested at

(9) Ibid, page 124
(10) Weston and Brigham, page 292

the cost of capital. When the decision-maker elects to use one method rather than the other, it should be on the basis that the reinvestment assumption of the chosen method is more appropriate in the circumstances than the other.[11] The decision is further complicated by the fact that not all writers agree about the validity of this reinvestment hypothesis. Merrett and Sykes,[12] for example, hold that the IRR carries no assumption about reinvestment any more than the rate of overdraft interest carries an assumption about a bank's reinvestment opportunities. More recently, Dudley[13] and Mao[14] have argued that the relevant reinvestment rate is not the IRR but the Fisher rate, and that one project's superiority over another depends on whether the intermediate cash flows are reinvested at a rate above or below the Fisher intersection. In chapter 2 it will be argued that this reinvestment controversy is in fact quite misplaced because for the normal selection process the attractiveness of a project is independent of the reinvestment opportunities for its intermediate cash flows.

Multiple yields etc.

Not every investment has a unique internal rate of return, and this fact is used frequently by those seeking to discredit the IRR. Teichroew, Robichek and Montalbano[15] have distinguished between simple investments whose net cash outlays are restricted to the initial period, and nonsimple investments which involve

(11) Van Horne, page 81
(12) Merrett and Sykes Capital Budgeting and Company Finance
(13) Dudley, op.cit., page 913
(14) Mao, op.cit., page 234
(15) Teichroew, Robichek and Montalbano, page 395 - 403

cash outlays in years subsequent to years of net cash inflows. Simple investments always have a unique internal rate of return, but nonsimple investments may not have a rate of return internal to the project, that is a rate which is independent of the cost of capital. For this purpose a further classification of nonsimple investments can be made[16] into pure and mixed investments, a classification which Mao holds is "critical for understanding the meaning of the IRR." A pure investment is one in which the outstanding capital invested at any point of time computed at the project's IRR is either zero or negative. A mixed investment is one which at a certain stage overdraws on its return and is a 'liability' to the project.

Only in the case of a pure investment is the IRR independent of the cost of capital to the firm. A mixed investment is partly an investment and partly an overdraft or source of finance, and the return therefore varies with the cost of capital.

An example of a mixed investment which technically has a unique IRR, but which requires a different interpretation from the conventional sense of the term is cited by Mao,[17] as follows

Year	0	1	2
Cash flow	-£10	+£40	-£40

The solution to the equation

$$-£10 + \frac{£40}{(1 + r)} - \frac{£40}{(1 + r)^2} = 0$$

is r = 100%. It is clear, however, that if the cost of capital is 0, the project has a negative net present value of -£10, and likewise if the cost of capital is ∞. The project, in fact,

(16) Mao, op.cit., page 199
(17) Mao, ibid, page 201

will not have a positive value for any value of k and the IRR is the maximum value of the NPV function as depicted in figure 1.

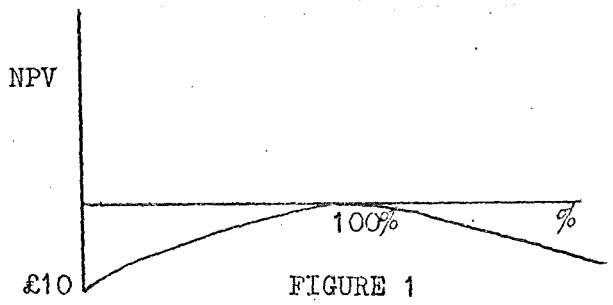


FIGURE 1

The best known characteristic of mixed investments, however, is they are capable of producing multiple yields. The rate of return r, it will be recalled is a solution to the equation

$$\frac{a_o}{(1+r)^o} + \frac{a_1}{(1+r)^1} + \frac{a_2}{(1+r)^2} + \ldots\ldots \frac{a_n}{(1+r)^n} = 0 \qquad (3)$$

where a_n is the cash flow in year n. If we multiply through (3) by q^n, where $q = (1 + r)$, we obtain

$$a_o q^n + a_1 q^{(n-1)} + a_2 q^{(n-2)} + \ldots\ldots a_n = 0 \qquad (4)$$

Equation (4) is a polynomial equation of degree n and the IRR is found from the roots of this polynomial when the NPV = 0. According to Descartes' Theorem[18] there are as many positive real roots as there are changes in sign of the coefficients, or less by an even number. If there are two changes of sign there may be, though not necessarily, multiple roots.

The possibility of obtaining more than one rate of return has been the source of much confusion in the capital budgeting literature, it led Weston and Brigham[19] to conclude that in

(18) e.g. see Haley and Schall, page 69
(19) Weston and Brigham, 3rd edition page 206

order to interpret the two rates of return, "the rates are simply examined, one is judged to be 'unreasonable' and the other is selected as being the appropriate one. 'Unreasonable' means that the cash flows from the project cannot be reinvested at so high a rate." Van Horne on the other hand concludes[20] that "neither rate is correct, because neither is a measure of investment worth." Others[21] have even suggested that the IRR method is invalidated by the possibility of multiple rates, since clearly no project can have two or more rates of return.

Finally, it is possible that a perfectly respectable and clearly desirable cash flow pattern has no internal rate of return such as

Year	0	1	2
Cash flows	+£1,000	-£3,000	+£2,500

The graph of the NPV function has the shape depicted in figure 2.

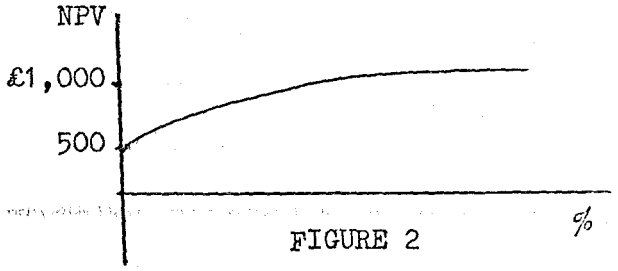


FIGURE 2

Attempts to vindicate the IRR

Those writers who take the view that the advantages of using a profitability ratio rather than an absolute measure of investment worth are such as to make the IRR fundamentally superior to the NPV method or at least a desirable supplement to it, have frequently

(20) Van Horne, page 95
(21) e.g. see Halford, page 269 - 270

gone to great lengths to deal with the problems which it presents. These salvage attempts have taken the form of attempting to identify the circumstances where the IRR is either inappropriate or should be used with some degree of caution, and of proposing some modifications for those conditions where it fails to give a correct or unambiguous solution. Thus the IRR is alleged to be capable of being used to compare two or more projects with a reasonable degree of confidence, provided (a) the cash flow patterns of the individual projects are of the pure investment type (b) the projects have equal outlays[22] (c) the projects have equal lives[23] and (d) the projects have not dissimilar cash flow patterns.[24] When any one of these conditions is absent, it is necessary either to abandon the IRR or to modify it. Some of the modifications or techniques which have been proposed include the Extended Yield, the Incremental Yield, and Fisher's Intersection. These will now be examined briefly.

The Extended Yield

(22) Bierman and Smidt, page 42
(23) Weston and Brigham 5th edition, page 272
(24) Ibid, page 272

The Extended Yield

The extended yield has been proposed by Merrett and Sykes[25] and others to provide a solution for projects whose cash flow pattern corresponds to the 'mixed investment' category. The method consists of finding

> "The point from which the future cash flows (discounted at the yield rate) are negative. These cash flows are then all discounted at the normal cost of capital to bring them back in time to the point at which they are largely absorbed by the preceding positive cash flows. A revised yield is then performed on the cash flows modified in this way."[26]

The technique is demonstrated in an example:

Years	0	1 - 7	8	9	10	Yield
Cash flows	-£4,277	£1,000	£1,000	£1,000	-£2,000	15%
Adjusted cash flows	-£4,277	£1,000	£188	0	0	14.6%

For the adjusted cash flows in line 2 above, a cost of capital of 7% is assumed at which rate the 'liability' of £2,000 is discounted to year 9, giving again a negative value of -£869, and this sum in turn is further discounted to year 8, providing a positive cash flow for that year of £188.

Merrett and Sykes define the extended yield as the yield of a project "based on the period over which the project is an asset and after making provision for meeting the future liabilities associated with the project."

The rationale of the method is that in order to be able to meet the outflow of £2,000 in year 10, a 'sinking fund' is established in

(25) The Finance and Analysis of Capital Projects, pages 135 - 139
(26) Ibid, page 135

year 8 of £812 and in year 9 of £1,000, which will amount to £2,000 in year 10 if invested at the cost of capital 7%. Merrett and Sykes reject the apparent IRR of 15%, because it is based on the assumption that the sinking fund earned 15% which is unrealistically high in relation to the cost of capital. They conclude that it is invalid to compare the 15% to the cost of capital 7% in order to gauge the degree of safety which the project has in relation to variations in the cost of capital. The conventional yield must be revised in accordance with the above procedure to provide a more realistic comparison.

The extended yield corresponds to the RIC (return on invested capital) described by Mao[27] and the PIR (project investment return) of Teichroew, Robichek and Montalbano.[28] It is, of course, not a rate of return 'internal' to the project, because its magnitude depends on the external cost of capital.

The Incremental Yield

One suggested solution to the situation where the IRR fails to give a correct ranking of competing projects is the incremental yield approach.[29] All projects are ranked according to the size of their initial investment starting with the smallest. The first project is chosen as the defender and the next alternative becomes the challenger. The cash flows of the second are deducted from those of the first, and the rate of return on the difference in the cash flow and outlay streams is computed. If the resulting incremental yield exceeds the required rate of return, then the

(27) op.cit. pages 201 - 211
(28) Teichroew, Robichek and Montalbano
(29) e.g. Bierman and Smidt, page 43

challenger is preferred and itself becomes the defender to a new challenger, the third alternative. This procedure is continued down the list of potential projects until the victor is found.

Fisher's rate

The incremental yield is of course equal to the rate which has already been identified as Fisher's intersection. It is the crossover point at which one project's NPV goes from a position of being greater than to being smaller than that of another. Mao[30] and Dudley[31] argue that Fisher's intersection indicates the minimum reinvestment rate which must be assumed if project A's NPV is to exceed that of project B in figure 3.

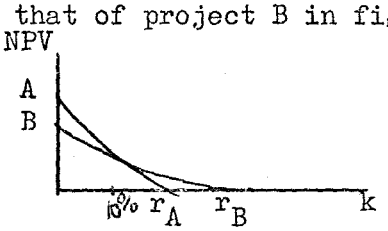


FIGURE 3

If project A's intermediate cash flows can be reinvested at a rate higher than 10%, then project A will be superior to B, and the ranking given by the IRR will be correct and accord with that given by their NPVs. Mao concludes[32] that this finding conflicts with the conventional claim that the IRR assumes that the reinvestment rate is equal to the IRR. The relevant assumption is that the reinvestment funds can be reinvested at a return higher than Fisher's return over cost.

However this approach has its limitations. When two projects are so compared, they do not necessarily have any intersection, whilst others may have multiple intersections.

(30) Mao, op.cit., page 234
(31) Dudlas, op.cit., page 913
(32) Mao, op.cit., page 234

Mao delineates the conditions for the existence and uniqueness of the Fisher rate for pairs of investments:

There will be no Fisher intersection in the interval $(0, r_m)$ where r_m = the smaller of the two rates of return, if

(1) a) A's NPV $>$ B's NPV at zero discount rate, and

b) A's NPV decreases at a greater rate than B's, in response to a given increase in k

c) A's IRR $>$ B's IRR.

(2) a) A's NPV $>$ B's NPV at zero discount rate, and

b) A's NPV decreases at a lesser rate than B's in response to a given increase in k.

There will be a unique intersection between the two NPV functions where:

a) A's NPV $>$ B's NPV at zero discount rate, and

b) A's IRR $<$ B's IRR

c) A's NPV decreases at a greater rate than B's in response to a given increase in k.

There may be multiple intersections if the NPV functions of the two projects do not satisfy the above conditions, in which case the difficulties of making generalisations about the Fisher's intersection are such as to cause Mao to recommend for ranking purposes "the use of the NPV criterion or the method of mathematical programming."(33)

Thus the IRR presents a number of problems which make it far from being a simple decision tool, and bearing in mind that one of

(33) Ibid, page 240

its alleged advantages is its understandibility to businessmen, the desirability of making the above refinements must give rise to some doubts by even the most ardent disciple of the yield approach. Moreover, the fact that the incremental yield gives rise to cash flow patterns that frequently produce the multiple yield problem which has so often been used to undermine the reputation of the discounting technique must also give cause for concern. The question has to be examined, therefore, not simply whether the claim that the IRR is readily understood by businessmen is illusory, but whether, in fact the method can make any valid contribution to the investment selection process which cannot be more simply and more effectively achieved by the NPV method alone.

Certainly one of the greatest obstacles to the acceptance of the NPV approach is the argument that it requires a precise derivation of the cost of capital. This, of course is a spurious argument, because it misleadingly implies that the IRR can validly rank without reference to the cost of capital, which in the next chapter will be shown to be false. On the other hand, the problem of computing the cost of capital is a real one, and whilst the literature has made significant advances in recent years in identifying the issues involved, little progress has been made in developing an operational measure of the cost of capital. Irwin Friend of the University of Pennsylvania, in his Presidential Address to the American Finance Association, 1972, remarked on the 'deplorable state of the arts in corporation finance' and added(34)

"The measurement of even the average cost of capital to say nothing of the marginal cost of capital has not advanced greatly in recent years. One of the most substantial difficulties here is the absence of a satisfactory measure of the required rate of return on the market portfolio, or equivalently a measure of the risk differential between the risk-free rate and the average required return on common stocks."

(34) Irwin Friend, page 270

THE DISCOUNT RATE

The hurdle rate which is most widely advocated for measuring investment profitability is the firm's cost of capital, or equivalently the return required by its suppliers of capital. Under conditions of certainty and with perfect capital markets, differences between firm's methods of raising finance would have no significance with respect to cost, and all firms and all projects would have the same hurdle rate, namely the prevailing rate of interest.

In the absence of certainty, the expected cash flows from both a company and its projects are not known with complete assurance, and must therefore be derived by computing the mean of the probability distribution of cash flows. Depending on the degree of variability about the mean, that is to say, the variability which investors are unable to diversify away in their personal portfolios, investors are assumed[35] to require some compensation. The greater the variability, the greater the compensation.

This compensation for risk can be incorporated into the analysis by adopting one of two approaches. The first, the certainty-equivalent method consists of adjusting the numerator of the present value equation by a factor which reduces the cash flows to their certainty equivalent values.

(35) e.g. See Sharpe, page 425

The equation for the net present value formula becomes, then

$$NPV = \sum_{t=1}^{N} \frac{\alpha \, At}{(1+i)^t} - C$$

where i is the riskless rate of interest.

The alternative method, known as the risk-adjusted discount rate approach, consists of adjusting the denominator of the present value equation to incorporate a discount rate which includes a premium representing investors' compensation for risk. This latter approach is the most widely used model in theory and in practice[36] although it has been shown[37] to suffer from the serious deficiency of assuming that risk is a function of time by treating the futurity of the cash flows and their variability in the one process.

Given that the purpose of this section is to examine the problems which practitioners face when trying to measure the compensation for risk necessary to satisfy the suppliers of capital, this deficiency of RADR must be borne in mind. On the other hand, most of the difficulties which do exist apply whichever method is adopted, and the subsequent analysis will proceed by focusing on the specific problems of RADR in view of the fact that it appears to have a greater intuitive appeal than the other, and is the one which most text-books favour.

The most frequently recommended technique for estimating the appropriate discount rate is by measuring the firm's weighted average cost of capital (WACC). Most writers[38] take the view

(36) See Klammer, page 391
(37) Robichek and Myers, pp 79 - 93
(38) e.g. Weston and Brigham, page 595

that it is fallacious to argue that the appropriate cost of capital is the explicit cost of the specific funds used to finance the project, since this would imply that a project financed by 'cheap' debt required a lower acceptance criterion than an identical project financed by equity. Whatever particular financial instruments are issued to finance an investment, it is the weighted average or composite cost which is the relevant discount rate.

There are three steps required to compute this composite rate:

(a) Identification of the components of the capital structure

(b) Measurement of the costs of the individual components

(c) Combination of the costs to produce the WACC.

Identifying the components

This might appear to be a straightforward process, but in fact there are a number of unresolved issues. Only three categories of funds are usually identified, Debt, Equity, and Preference Shares, but there is lack of agreement as to what is included in debt. Merrett and Sykes,[39] for example, include short-term sources of capital. Mao[40] recommends that only interest-bearing debt should be included, and that noninterest-bearing debt should be deducted from the gross value of the investment. Others recommend that only long-term sources should be considered.

Not every source of capital is explicitly represented in the capital structure of the firm's balance sheet. Leasing, for example, can be a significant source of funds and should be logically included as a component of the WACC, yet it does not normally appear as a source of capital in the firm's balance sheet.

(39) The Finance and Analysis of Capital Projects, page 95
(40) op.cit., page 378

Likewise depreciation-generated funds may provide a material proportion of the firm's annual budget, but it remains a matter of controversy whether such funds should be perceived as an ingredient of the capital structure. Merrett and Sykes[41] argue that they should be included whilst others[42] take the view that since the alternative to reinvestment is to redistribute the depreciation-generated funds to shareholders and debtholders in the proportions in which they have financed the firm's assets, the effective cost of the funds is substantially equal to the average cost of capital, and therefore their inclusion in the cost of capital calculation is superfluous. This latter argument, however, presupposes that the cost of capital is unaffected by the size of the capital budget.

Measuring the individual costs

Having identified the relevant components of the capital structure, the next operation is to assign costs to the individual items. The cost of equity is undoubtedly the most difficult of these to measure.

The cost of equity

The conventional assumption is that all investors are averse to risk and seek to be rewarded in the form of higher ex ante returns. There are broadly two approaches to estimating the return required by investors for a specific company's shares. The first is to try to solve the equation for the well known dividend valuation model

$$k_e = \frac{D}{P} + g$$

where g is the expected rate of growth in earnings, and dividends, D the current dividend, and P the market price of the shares.

(41) The Finance and Analysis of Capital Projects, page 114
(42) e.g. Van Horne, page 117

The price of the firm's shares is observable in the market, as is the current dividend, and the solution therefore is a matter of estimating g. This approach is frequently recommended in the investment manuals[43] but it suffers from certain defects:

(1) It is oversimplistic in that it assumes that the company will grow at a fixed rate forever. Certainly for unseasoned companies, it may be unreasonable to assume a constant growth rate, and the above equation would need to be modified to accommodate a varied growth pattern. A complete and highly complex model has been proposed by Mao.[44]

(2) In order to estimate g for the 'normal' company in 'normal' times, a number of texts recommend simply extrapolating past growth rates into the future. However, apart from the obvious difficulty of defining and identifying normal times, it will be demonstrated in chapter 3 that even for the normal company, ex post returns cannot be used as a basis for estimating ex ante required returns. For 'abnormal' times or for 'abnormal' companies, the past is clearly a defective source, and the decision-maker is advised to draw upon the security analyst to obtain estimates of investors' growth expectations.[45]

The second approach to estimating the cost of equity is to proceed in the framework of the capital asset pricing model of Sharpe[46] and Lintner.[47] According to this approach, the fact

(43) e.g. Abdelsammad, page 110
(44) Mao, op.cit., page 402
(45) Weston and Brigham, page 605
(46) see Sharpe 1964
(47) see Laitner 1965

that the cost of equity comprises two elements, the risk-free rate and a premium for risk, is specifically recognised. The risk premium is determined by reference to the firm's correlation with the market's returns. All other risk can be eliminated by diversification and is therefore irrelevant. The expected return for a share j is derived from the following equation(48)

$$\bar{k}_j = i + \frac{(\bar{k}_m - i)}{\sigma^2 m} (r_{jm} \sigma_j \sigma_m)$$

where i = the riskless rate of interest

$\bar{k}_m$ = return on the market portfolio

σ_j = standard deviation of the probability distribution of possible returns for security j

σ_m = standard deviation of the probability distribution of possible returns for the market portfolio

r_{jm} = correlation of rates of return for j and the market

$$\text{If } \beta_j = \frac{r_{jm} \sigma_j \sigma_m}{\sigma^2_m}$$

then

$$\bar{k}_j = i + \beta_j(\bar{k}_m - i)$$

The beta coefficient is a measure of the share's systematic or nondiversifiable risk, and since $(\bar{k}_m - i)$ = the risk premium for the market as a whole, then the term $\beta_j(\bar{k}_m - i)$ represents the risk premium appropriate to the share. Therefore, in order to measure the risk premium for a particular company's shares it is necessary to be able to

(1) identify the 'market portfolio'. Most empirical studies identify the market as the New York stock exchange or the U.K. market, but as will be argued in chapter 4, the relevant

(48) For development of this equation see Francis and Archer, chapter 5.

market must be perceived in much wider terms to include the world market of risky assets.

(2) measure the market return. Apart from the obvious difficulties of measuring the return for a market which cannot be clearly defined, it will be argued that just as the realised returns of an individual security cannot be used to discover the ex ante required return, nor can the realised return of the market portfolio be used to discover the market required return.

(3) measure the covariance of the individual security's returns with those of the market.

Clearly then the reputation which the cost of equity has as being an exceptionally difficult task is well deserved. However, contrary to common belief, other securities present theoretical problems which are scarcely less complicated.

The cost of debt

Assigning a cost to the company's bonds is normally assumed to be a straightforward undertaking. For a bond issued and maturing at par, the interest rate is taken to be the cost. For a bond issued or selling above or below par, the effective yield is the cost as measured approximately from the formula[49]

$$k_B = \frac{I + D/n}{(P + MV)/2}$$

where k_B = yield to maturity

I = interest income

D = the amount of discount

n = years to maturity

P = current market price of the bonds

MV = the maturity value of the bond

(49) See Archer and D'Ambrosio, page 182

Although most textbooks do not consider the cost of debt to be anything more complicated than that implied by the above formula, some controversy has developed about whether I and MV are the relevant ingredients to be included. A distinction is sometimes made[50] between the promised cost of debt (the solution to k_B in the above equation) and the expected cost which, for a risky bond, must be less than k_B. Haley and Schall[51] for example recommend the use of the expected cost, whilst Brennan[52] argues that it is the promised cost of debt which is relevant. This controversy will be taken up again in chapter 6 and will be shown to have a fairly significant bearing upon the measurement of the cost of capital.

One further issue which appears to be ignored in the literature is whether the cost of debt can properly be perceived as dependent on the maturity of the bond. If short term rates happen to be low and long term rates high, is it valid to consider a company which has financed its assets by short term bonds to have a lower cost of capital than one which has used long term bonds?

Other components

Apart from differences in taxation effects, preference shares are not dissimilar to bonds and the issues which apply to the one apply usually to the other, except that preference shares are normally issued in perpetuity. Hybrid varieties of preference

(50) See Boness, pp 99 - 106
(51) Haley and Schall, page 295
(52) Brannan, page 27

shares and bonds (that is, convertible securities and securities issued with warrants attached.) need to be considered. Convertibles, for example, possess characteristics of both equity and debt, and assume the theoretical problems associated with each. In addition, the cost, if relevant, of depreciation-generated funds, of short term sources without explicit costs such as trade credit, and non balance sheet items such as leasing present individual problems.

Combining the individual costs

Once the individual components have been identified and their explicit costs determined, the next stage of the process is to introduce some mechanism for combining the individual costs into a composite rate. The usual recommendation(53) is to weight the individual costs according to the capital market values of the component securities. Market values are preferred because the individual costs are computed by reference to the capital market returns expected by investors. However Lewellen(54) and many of the management investment manuals(55) suggest that book values should be used. Some argue that book values for debt and preferred stock should be used, with market values for equity.(56) The arguments favouring book values tend to be influenced by practical considerations, and the market value approach appears to be theoretically more soundly based. With both approaches there is lack of agreement whether the weights should be determined by reference to the capital structure which the firm perceives to be optimal and aspires to achieve in the

(53) See Archer and D'Ambrosio, page 197
(54) Lewellen, page 87
(55) e.g. Wright, page 156
(56) See Mao, page 197

future. Finally, the literature offers little guidance to practitioners in relation to the choice of market values to be used in the weighting process, whether, that is, in a volatile market the market values should be computed by reference to an average of the preceding six months or year, or whether they should be the most recent values available at the time of the decision.

The validity of WACC

The three steps outlined above of identifying the correct components, measuring their costs and combining them into a composite rate are essential to the process of determining the firm's WACC, and it would not be surprising if practitioners are sceptical about the reliability of the discount rate which emerges from such an undertaking. This uncertainty is further accentuated by the fact that although the WACC approach is advocated in most textbooks, the usefulness of the technique has been seriously challenged on the grounds that it depends for its validity on a number of restrictive assumptions. These are

(1) that the project under review does not change the firm's risk characteristics(57)

(2) that the project consists of a constant perpetual stream of cash flows(58)

(3) that the adoption of the project will not change the debt-equity ratio(59)

(4) that the firm's existing assets are expected to generate a constant perpetual stream of cash(60)

(57) Haley and Schall, page 320
(58) Arditti, page 1004
(59) Myers, 1974, page 12
(60) Myers, ibid, page 12

The first of these assumptions is in itself one of the most serious limitations of the WACC concept. The underlying premise of the risk-premium hypothesis is that each security has a risk premium appropriate to its systematic risk. But the systematic risk of the firm is a function of the average risk of the assets owned by the firm. For the WACC to be a valid discount rate for a specific project, it would be necessary to assume that the project has the same risk characteristics as the firm's average existing assets. But this is exceedingly unlikely for most modern companies which tend to be engaged in more than one product or industry. A new project is more likely to have its own risk characteristics or to have the risk characteristics of one of the firm's existing projects rather than that of the average. Hence an adjustment to the WACC may frequently be required to satisfy the conditions of the risk-premium framework. This leads to the fundamental question whether it is necessary to compute the WACC in the first place, because if an adjustment needs to be made and the mechanism is available for determining the appropriate adjustment, it should be possible to derive the final discount rate without undertaking the intermediate step.

Lindsay and Sametz[61] have advocated as an alternative to WACC a marginal-sequential costing-of-funds approach, in which the marginal cost curve is constructed by reference to individual sources of finance, beginning with the cheapest first (debt) followed by retained earnings and then new equity. The marginal cost curve is then smoothed and plotted against the marginal

(61) Lindsay and Sametz 1967, page 327

efficiency-of-investment curve on the same graph, as in figure 4

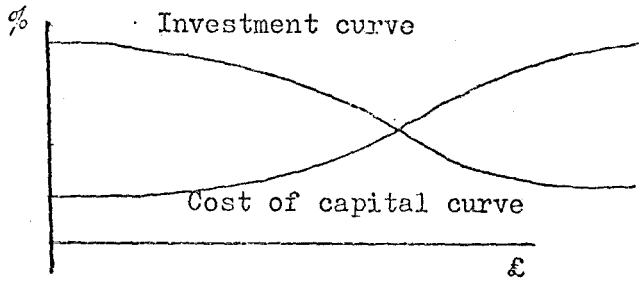


FIGURE 4

The intersection of the two curves determines the cut-off point for investments. The objection to this approach is that it does not lend itself to present value analysis, and the measure of profitability is confined to the internal rate of return.(62)

More recently Stewart Myers(63) has proposed an alternative procedure called the Adjusted Present Value approach which purports to avoid some of the problems raised by the traditional weighted average cost approach. It consists of calculating the project's basic contribution, by assuming that the firm is all-equity financed, with the result that the after-tax cash flows are discounted at the rate appropriate to a pure equity company of the risk class. To this base value is added the present value of the contribution which the project makes to the firm's debt capacity. Finally, from the sum of these two values is deducted the present value of transaction costs of planned equity issues and, if relevant, other penalty costs such as bankruptcy costs or tax penalties associated with dividend payments. In effect, the objective of the Myers approach is to separate the various aspects

(62) See Van Horne, 1st edition, page 134
(63) op.cit., pp 1 - 25

of the investment decision into different segments, and to ascertain in present value terms the significance of each segment rather than to seek to capture the interactions of the financing and investment decisions in a single step by attempting to construct an all-inclusive discount rate. The Myers' approach has clearly interesting possibilities. For example, in a subsequent paper[64] Myers and Pogue developed a linear programming model based on the APV framework. However, Myers admits[65] that the extra complications of the APV rule do not make it suitable for decision-makers concerned with run-of-the-mill projects. He also concedes that the technique contains certain inherent deficiencies which must be balanced against the disadvantages of using the traditional capital budgeting rules.

Inflation

A further complication has been highlighted in recent years namely the impact of inflation on the investment decision process. Market rates of interest are assumed to contain an element which is designed to compensate investors for expected changes in the value of money. If the view is taken that the cash flows of a project should be expressed in a stabilised monetary unit[66] then the discount rate should likewise be expressed without an inflationary element. Thus whatever method is used to derive the firm's cost of capital, a further adjustment would be necessary to 'deflate' the rate to arrive at the net-of-inflation discount rate. Most writers,[67] however, recommend using both nominal cash

(64) May 1974
(65) Myers, March 1974, page 22
(66) e.g. see Bromwich, page 39
(67) e.g. see Wilkes, pp 46 - 53

flows and nominal discount rates, and do not advocate an adjustment of the nature described.

Definition of risk and risk-free

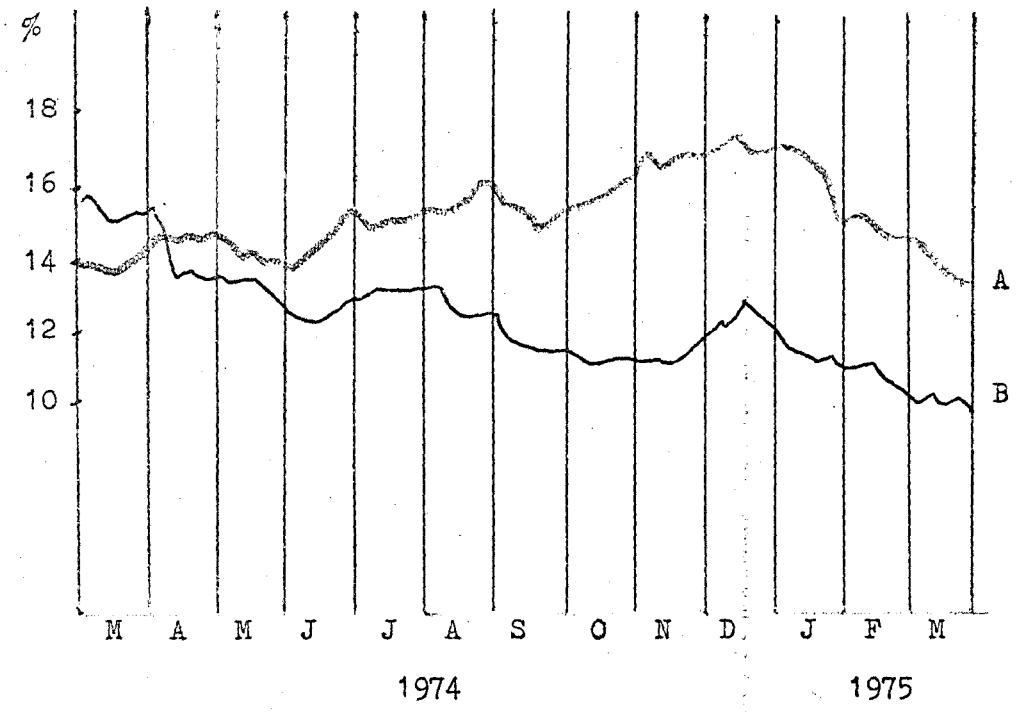
We have so far considered some of the difficulties which confront management in deciding what discount rate to use to evaluate prospective investments. A number of factors have been identified which appear to make the 'going rate of interest' criterion of the perfect market model unacceptable. Of these factors risk undoubtedly presents the greatest challenge. Whichever approach is used to derive the appropriate discount rate, the underlying problem springs from the assumption that the return required from a financial security depends on the variability of the returns from the assets which it is used to finance, and possibly on the relationship which the financial security has to other securities in the capital structure. The problems of measuring the premium for risk are so fundamental that there is not even universal agreement about the definition of risk. Although the standard deviation of return is the most frequently advocated measure, particularly because of its use in portfolio analysis and because of its susceptibility to mathematical manipulation, other definitions have been proposed, for example semi-variance as suggested by Markowitz(68) on the grounds that only below average returns are risky; semi-interquartile deviation, because it is not affected by extreme values; Baumol's lower confidence limit;(69) Sharpe's beta coefficient as an index of systematic or nondiversifiable risk.

(68) see Markowitz, chapter 9
(69) Baumol, 1963

Indeed there is not even agreement as to what is the appropriate risk-free rate of interest to which the risk premium is added. In capital asset pricing theory, the risk-free rate is commonly held to be the short term government bond rate[70] but it is far from clear that the required equity return should be perceived as a short term rate plus, if relevant, a premium for risk.

If management does decide to undertake the recommended procedures for computing the risk-adjusted discount rate outlined in this chapter, it has to be borne in mind that because the risk-free rate is constantly changing, so therefore presumably is the risk-adjusted rate. Indeed the relationship of the risk-premium with the risk-free rate may reasonably be assumed to vary with the level of the latter, and with changes in investors' psychology. It is not unlikely that when interest rates are high the risk-premium has a relatively different magnitude than when interest rates are low. It follows that the heavy dependence of the recommended procedures on analysis of ex post returns is unlikely to yield a rate which is very relevant to current needs. But even if one can avoid the dependency on the past, the fact that the risk-free rate is observed to change frequently necessitates that the risk-adjusted rate itself be calculated frequently, however laborious the process, if the discount rate is to maintain its relevance. To illustrate this point, figure 5 depicts the movement in interest rates during the twelve months period to March 1975. A represents the rate of interest on three-month

(70) e.g. See Weston and Brigham, page 669



Interest rates 1974-75

Figure 5

money market deposits over the period and B the yield on undated Government stocks. Whichever rate is considered to be the appropriate base upon which to append a risk-premium it is clear that a significant variation in the resulting discount rate must be assumed to take place within the twelve month period. It follows that the procedures which are advocated in the text books for deriving the investment discount rate need to be undertaken regularly for an authentic rate reflecting underlying interest rate movements to be achieved.

Capital budgeting techniques in practice

The following conclusions appear to emerge from analysis of available studies relating to the practice of capital budgeting techniques

(1) A large number of firms do not use discounting techniques in any form or use them only as secondary standards of selection. Klammer,(71) for example, found in 1970 that of a sample of 369 fairly large firms with sizable and continuing capital expenditure programmes and which "would be expected to make relatively heavy use of the more sophisticated capital budgeting techniques," only 57% used discounting as a primary standard of evaluation, and more than half of these used in addition other theoretically less satisfactory criteria such as payback.

(2) Firms which do use discounting techniques tend to use the IRR in preference to the NPV approach(72)

(71) Klammer, page 393
(72) e.g. See Bower and Lessard, page 323

(3) Few firms make use of a probabilistic framework for investment analysis[73]

(4) Only a small minority of firms treat risk formally in the evaluation process[74] and for the few which employ a risk-adjusted discount rate approach, the discount rate is formulated without reference to investors' ability to diversify through their own portfolios.[75] Klammer found that only 3% of firms interviewed took account of the covariance of projects.[76]

(5) Few of the firms which do compute a cost of capital update their calculations more frequently than annually[77]

(6) The range of overall after-tax costs of capital employed by firms in selected industries appears to vary significantly. For example, in the U.S. textile industry, the after-tax cost of capital computed by firms appeared to range between 2% and 18% during 1970.[78]

It would seem therefore that considerable progress has still to be made in translating the 'correct' decision procedures into terms which businessmen will accept. Whilst the writer is of the opinion that the practical and theoretical problems (multiple yields, reinvestment assumption etc.) primarily associated with

(73) Mao "Survey", page 356
(74) David Cooper, page 198 and Bower and Lessard
(75) Mao, "Survey", page 352
(76) Ibid, page 391
(77) e.g. See Abdelsammad, page 106
(78) Ibid, page 115

the IRR must operate as a significant deterrent to businessmen from wholeheartedly accepting the discounting approach, it is ironic that the IRR is the method preferred by those who use discounting. But this is partly due to the fact that investment manuals frequently present the IRR as the primary method[79] possibly because the authors believe that businessmen will be attracted by a ratio of profitability, and because they believe the IRR relieves the decision maker of the need to make a precise estimation of the cost of capital. Certainly the evidence suggests that efforts to derive the appropriate cut-off rate appear to be arbitrary and crude, and to have little relationship with the theoreticians 'pure rate of interest plus premium for systematic risk.' Weston and Brigham observe[80] that the capital asset pricing model as a framework for measuring the cost of capital is today where the discounted cash flow budgeting techniques were about twenty years ago. Hopefully, however, the practice of perceiving investors' required returns in a portfolio context will more quickly gain widespread acceptance in industry. As noted in the introduction, this broader perception of the cost of capital has the potentiality of making the discount model appear even more remote to the majority of firms, but equally it has the potentiality of providing the basis for a more simplified approach.

(79) e.g. See Wright, Chapter 2
(80) Weston and Brigham, page 676

Conclusion

In the first section of this chapter some of the issues arising from having two distinct methods of incorporating the time value of money into the investment decision process were examined. In the immediately following chapter it will be argued that the IRR is not a valid alternative to the NPV, that it is incorrect in principle for choosing between investments, and should be relegated to a minor role.

In the second section, the principal factors which have been variously held to necessitate modification of the market rate of interest were identified. In the succeeding chapters these factors will be separately analysed. It will be shown that some of the factors such as dividend policy can in fact be validly assumed to have no affect. Others, such as the tax deductibility of interest payments will be shown to produce incorrect solutions if reflected in the discount rate, as is the conventional practice, rather than reflected directly in the cash flows. Others, yet again, such as inflation will be shown to be more conveniently assumed to be already reflected in the market rate of interest and to require no further adjustment. But the one factor more than any which makes some adjustment to the market rate of interest appear to be necessary is risk, and the impact of risk on the investment discount rate will therefore occupy a major portion of the subsequent analysis.

It has become axiomatic in finance that projects have different risk characteristics and that this fact has a direct bearing on the returns required by investors. This belief rests on one of two assumptions, that the amount of a project's risk which investors

cannot diversify away in their personal portfolios is significant or that investors care sufficiently about differential risks to the extent that they would refuse to provide capital to finance an investment which had only a moderately higher degree of relevant risk than alternative investments with comparable expected returns.

These assumptions will be closely examined in the succeeding chapters. It will not be argued that all risk can in fact be diversified away, or that investors are totally risk-indifferent, but rather that with improved market facilities for diversification there is reason to doubt that the amount of relevant risk which cannot be diversified away is significant and that investors' aversion to risk is sufficiently pronounced to necessitate the kind of refinements to the market rate of interest contemplated by theoriticians. Modification of the pure rate of interest is not a step which can be taken 'moderately' or 'partially'. It is one which involves forfeiting absolutely the objectivity of an observable market rate of interest, and can only be desirable if it can be clearly demonstrated that the quality of the resulting investment decision is likely to be higher than could be achieved if the rate were left in its pure form. Before advocating departure from the pure market rate, therefore, it is essential that the evidence for the existence of a market premium for risk is persuasive, that the size of the risk premium is material to the efficiency of the selection procedure and that the operational disadvantages of having to undertake the exceptionally difficult task of measuring the premium appropriate to a specific project are outweighed by the benefits.

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Chapter Two

THE INTERNAL RATE OF RETURN

Much of the criticism that has been made against the Internal Rate of Return has taken the form of identifying the situations in which that ratio fails to give a correct solution to the optimal selection of investment proposals, or fails altogether to give a single unambiguous solution. Having identified such situations, most text-books nonetheless concede that, subject to these 'exceptional' cases, the IRR is a useful measure of investment worth, and although inferior to the NPV method, has the merit of being more easily understood by businessmen, who, it is alleged, think in terms of 'profitability' rather than of 'contributions to wealth'. The IRR is represented by its adherents as a valid measure of project worth, to be treated with caution in well-defined circumstances, and by its critics as a defective measure of project worth, but nonetheless a useful guideline in well-defined circumstances.

This chapter seeks to demonstrate that the arguments which are commonly advanced to explain the inadequacy of the IRR are partly misconceived and, to the extent that they fail to attack the fundamental concept of an internal rate of return, they are misdirected, and likely to harden the resistance of the business community to the discounted cash flow approach. That resistance will not successfully be broken down as long as theorists propose, under the present value umbrella, two conflicting methods of investment appraisal, one (NPV) because it is theoretically in accordance with the assumed objective of wealth maximisation and the other (IRR) because it is in accordance with the way in which businessmen are alleged to think. It is paradoxical that the IRR should be offered as the more appealing of the two approaches,

when in fact, most of the controversial conceptual issues concern it rather than the NPV method.

The circumstances in which the two techniques are likely to produce conflict are generally identified as when

(a) the cash flow of one project increases over time and that of the other decreases

(b) the projects have different lives

(c) the projects have different outlays.

The common explanation for the differences in ranking is summarised by Weston and Brigham,(1) one of the most widely used text books in finance:

"The net present value method discounts all proceeds at the firm's cost of capital, thus implicitly assuming that these proceeds can be reinvested at this rate. The internal rate of return method on the other hand, implicitly assumes that cash flows can be reinvested at the IRR. The correct choice of methods for the firm to use thus depends upon which reinvestment rate is closest to the rate that the firm will be able to earn on the cash flows generated by its projects".

It will be argued, in this chapter that

(1) the reinvestment opportunities for intermediate and terminal cash flows are irrelevant under conditions of nonrationing, and

(2) the internal-rate-of-return is invalid not because of any implicit reinvestment assumption but because it is irrelevant to the objective of maximising the firm's wealth.

Investment decisions are taken in any of three conditions relating to the supply of capital.

(a) Nonrationing and nonsurplus of capital, where the firm's potential supply of capital exceeds the amount needed to finance investment opportunities.

(b) Rationing of capital, where the investment opportunities available exceed the supply of capital needed to finance them.

(1) Weston and Brigham, 4th edition, page 151.

(c) Capital surplus, where the firm has more cash than is needed to finance investment opportunities and is precluded from paying back the capital at will.

The significance of the reinvestment opportunities associated with a project's intermediate and terminal cash flows, and the relevance of the internal-rate-of-return under each of the above conditions will be considered in turn.

NONRATIONING

Under nonrationing conditions, firms are assumed to undertake all projects whose prospective returns exceed the cost of raising funds, all projects, that is, which are not mutually exclusive with alternative opportunities. As a signal of acceptability, the IRR always accords with the NPV method, although it will be argued subsequently that as a measure of desirability in the accept-reject decision, it is misleading. For the time being, we are concerned with its function when choosing among conflicting proposals.

It has been noted that, for projects with broadly similar cash flow patterns, economic lives and initial outlays, no conflict between the two methods is likely to present itself. The fact that conflict does arise, however, requires explanation, because on the explanation depends one's view whether the two methods are truly alternative approaches, or whether one of the methods, perhaps, is fundamentally at odds with the assumed objectives.

Differing cash flow patterns

In Table I two conflicting projects, A and B are ranked differently by the IRR and the NPV methods. The conventional solution is to make a judgement about which, if any, of the two reinvestment assumptions is most realistic and to choose accordingly.

Table I

Period	Project A	Project B	Project C
0	-£100	-£100	£-
1	-	+100	-100
2	+144	+ 30	+124
IRR	20%	24%	24%
NPV at 10%	19	16	

The implication is that if it could be known with certainty that the £100 arising in year 1 could be reinvested in a third, independent, project, C, available at the end of year 1, with a return equal to B's internal-rate-of-return (24%), B would have a terminal value of £154 and therefore, be judged superior to project A whose terminal value is £144.

Before examining this argument it is essential to bear in mind a distinction between

(a) Reinvestment opportunities which are physically, logically or sequentially dependent upon a project, whatever the source of capital used to finance these opportunities.

(b) Reinvestment opportunities financed by the cash flows arising from a project.

Every investment appraisal must take account of the first category of investments, namely those which are contingent upon or excluded by it. To undertake an investment involves committing the firm to a certain course of action and the consequence of pursuing that course must be evaluated when assessing the project. Now, since, by definition, C does not fall into that category of dependent investments the fact that it happens to be financed by the £100 arising from project B does not represent grounds for associating it exclusively with project B. Project C can also be undertaken in conjunction with project A and financed by new capital at the cost of 10%.

The terminal value would then be £158:

Terminal value of A	£144
Terminal value of C	124
	£268
Less Loan and interest on C	110
	£158

Other things being equal, then, project A is always superior to project B, no matter how profitably B's cash flow may be reinvested, provided the reinvestment projects are independent. The destiny of the cash flows as such is never relevant. If there are dependent projects, they are relevant because they are dependent and not because they happen to be financed by particular cash flows. If, instead, project C had been dependent on project A, the assessment of A would have to include an evaluation of project C, notwithstanding the absence of cash flows in period 1.

Unequal lives

The comparison of projects with unequal lives is frequently specified as a special case requiring particular assumptions about the profitability of projects available to succeed the shorter of the projects. In fact the presence of unequal lives does not create a circumstance which calls for treatment different from comparison of projects with equal lives. The reinvestment opportunities for the terminal cash flow of the shorter project are no more relevant to the selection than are the opportunities for the intervening cash flows of either. The only significance of one life being shorter than the other is that the termination of the shorter project might make it possible to undertake a third project that had been excluded by it. But as previously noted, the assessment of any project must take account of all other projects contingent

upon or excluded by it.

Table 2

Time	Project D	Project E	Project F
0	-£100	-£100	-£100
1	121	-	-
2	-	144	-
3	-	-	169
NPV at 10%	10	19	27
IRR	21%	20%	19%

Thus, the problem of ranking Projects D, E, and F in Table 2, is fundamentally no different from that of ranking Projects A and B in Table 1. Ignoring the question of risk, Project F is preferred because it contributes most to the firm's value. There is no need to consider the destiny of the £121 in year 1 of Project D, or the £144 in year 2 of Project E, because, even if these sums could be used to finance additional projects rather than be applied to the repayment of the 'original loan', such additional projects might equally be financed by new capital raised for that purpose, if Project F were undertaken.

The cessation of Project D might make it possible to undertake at the end of year 1, a further Project G, which was precluded by the adoption of D, E or F, and, therefore, the benefit of being able to undertake Project G a year or two earlier than otherwise possible would have to be evaluated in assessing Project D. But it is not the receipt of the terminal cash flow that makes Project G possible or relevant, but the fact of Project D ceasing to be an obstacle. To emphasise the point even further, if Project E terminated (physically) in period 1, but the terminal cash flow was not receivable until period 2, Project G could be undertaken, and this would negative the relative advantage of Project D in relation

to E, in which case the terminal cash flows would be seen to be irrelevant.

Therefore, in the absence of mutually exclusive or dependent projects, the order of preference is FED irrespective of how profitably the terminal flows of D and E can be reinvested.

If the reinvestment opportunities for intermediate and terminal cash flows are irrelevant to the choice between two or more competing projects the question remains why it is that a project can have a 'higher return' than another but have a lower NPV.

The explanation is that one project may have a higher rate of profit per unit of capital invested than another but, if it has fewer units of capital invested in it, it may make a smaller contribution to the wealth of the firm. The objective is to maximise the firm's wealth rather than the rate of profitability per unit of capital invested since the latter goal could be achieved by rejecting all but the most highly profitable projects. If two projects, then, have different amounts of capital invested they cannot validly be differentiated by their respective rates of return per unit of capital any more than a choice between two products can validly be made on the basis of the rate of profit per unit sold without reference to the number of units involved.

Unequal Outlays

This fact is universally recognised when projects with different initial outlays are compared. Even its most ardent supporters admit that the IRR is invalid for the purposes of discriminating between projects whose outlays differ, because different outlays imply projects of different sizes and the contribution a project makes is a function of both rate of return and size. It is self-evident that a project costing £1 with an IRR of 100% is not thereby more desirable

than a project costing £100 with an IRR of 20% if the cost of capital is 10%. But that is because the differences in scale of the two projects are immediately apparent by virtue of the different outlays.

However, the size of a project is not determined by its initial outlay alone. For projects with different outlays the difference in size is established at the outset and increased or decreased by the subsequent pattern of cash flows. For projects with identical outlays, the difference in size is determined by the pattern of subsequent cash flows only but is no less significant for that fact.

Project A and B in Table I appear to be projects of equal length because their terminal flow happen to coincide. But Project A's outlay has 'earned' its 20% over two years, and project B, for the most part, its return of 24% only over one year. Likewise, the projects appear to have the same capital outlays, but the emphasis on the initial year is misleading since the significant fact is that B has a smaller average capital invested over the two years.

The practice of categorising projects into those of unequal lives or initial outlays is invalid because it is based on the misconception that it is otherwise possible to identify projects as having the same size. In fact no two projects have the same size unless they have identical cash flows, and it follows that a rate of return per unit of capital invested can never in principle be used as a criterion for choosing between projects.

To deal with this problem of different initial outlays Merrett and Sykes (2) recommended the incremental yield approach to be used, which "simply consists of subtracting the net cash flows of the

(2) Merrett and Sykes, 1973, page 127.

cheaper alternative from those of the dearer, to establish the incremental cash flows which result from accepting the alternative involving the higher initial capital outlays." In effect, this exercise seeks to convert a ranking problem into an accept-or-reject decision. Merrett limits this technique to comparison of projects with different initial outlays but Bierman and Smidt suggest the technique could be applied to comparison of any two projects whether of equal outlays or unequal lives. For example, by subtracting the flows of Project B in Table I from those of Project A an 'incremental yield' can be calculated, namely 14% on the additional capital invested over period 2 of Project A. However, they do not make clear how the decision-maker should assess the significance at the present time of an 'investment with an IRR of 14%' which does not commence until a future date.

Moreover, only trial and error can determine which project should be deducted from the other, and when there are a number of projects to choose from, the procedure could be unnecessarily complicated. In addition, the net flows would frequently be of a pattern that would give multiple yields. Since the NPV method provides an immediate solution, this complicated refinement is unnecessary. Finally, the technique assumes that it is desirable to use the IRR for an accept-or-reject decision, and this assumption will be questioned in a later section.

The conclusion that the NPV is the only correct criterion under nonrationing conditions is, of course consistent with the findings of a number of writers. But the reasons which have been proposed are different from those usually given, being totally independent of the reinvestment rate assumed to be appropriate for the intermediate cash flows. As a consequence, a source of

confusion associated with the conventional approach is eliminated. For example, Dudley, (3) who argues that neither the IRR nor the NPV techniques make any implicit assumption about the reinvestment opportunities for intermediate cash flows but that the assumption is implicit in the decision to use one or other of the two techniques and not to make any estimate of the possible return on reinvestment of intermediate cash flows, concludes "if financial capital is freely available at any point in time, the reinvestment rate should be the marginal cost of these alternative funds, i.e. the firm's cost of capital. In such a case, the present value criterion is to be preferred." Now, if, as has been already demonstrated, the reinvestment opportunities for the cash flows are irrelevant, it is misleading to base the support for the NPV criterion on the assumption that the cash flows are reinvested at the marginal cost of capital. Thus, if a practitioner believed that a particular intermediate cash flow from a project would in fact be reinvested at 20%, even although the cost of capital is 10%, he could be excused for being confused about whether the 10% or the 20% was the relevant rate. On the other hand, if he perceived the essential defectiveness of the IRR technique as having nothing to do with the reinvestment of the intermediate cash flows, then no such confusion need arise.

CAPITAL RATIONING

It is, more than anything, the need to provide a guide for firms operating under capital rationing conditions which has been used to lend credence to the rate of return criterion. When cash is scarce, the rate of return per unit of capital invested might appear to be a significant factor in the selection procedure. In addition, the opportunities for reinvestment of a project's intermediate

(3) Dudley, page 914.

cash flows become relevant when other sources of capital are restricted. Nonetheless it will be argued that the IRR remains fundamentally incorrect in principle, and only the NPV approach is valid.

It is not a matter of importance to the discussion whether capital rationing is a common condition experienced by companies. As individual savers, we are all subject to capital rationing, and take it for granted, but it is far from clear that a similar condition is experienced by firms in their transformation of savings into productive resources. Apart from the apparent irrationality of failing to raise capital, however costly, when the rate of return from its use is expected to be greater than the cost, it is doubtful, from everyday observations, whether the average company has a superfluity of profitable projects which outweigh its capacity to finance them. However, for the purposes of discussion, the fact that such a condition can exist will be assumed, because all that concerns us is whether the IRR can make a valid contribution to the selection of projects when firms are so constrained.

When operating under financial constraints, the basic objective is no different from that which is appropriate for nonrationing conditions, namely to select the combination of projects which, in aggregate, effect the greatest contribution to the wealth of the firm. The selection, however, is more complex with rationing because, in addition to having to take account of physical interdependencies amongst projects, a problem which exists whether rationing is present or not, cash interdependency is introduced. Projects, otherwise independent, become interrelated when the feasibility of the one depends on the cash flows generated by the other, and the process of choice can become exceedingly complicated. (4)

(4) For development of this subject, see Weingartner.

However, the fundamental objection to a rate of return measure to discriminate between projects of different sizes (different, that is, even with identical initial outlays) remains valid. The need to make some allowance for the reinvestment opportunities available for intermediate cash flows calls for a modification of the straightforward NPV approach, but in no sense does it validate the IRR. The modification to NPV can be in two forms, either by changing the discount rate from the cost of capital to the opportunity reinvestment rate, or by calculating the terminal value of each alternative programme of investment after taking into account the cash flows generated by projects financed by each inflow of cash, and then, if desired, discounting the terminal value at the cost of capital.

The first approach has serious limitations:

(a) it is only valid if the relevant rate is constant throughout the life of the project. The reason is that the method operates by penalising cash flows for the lost opportunity of not being generated in an earlier period. If the opportunity rate changed from year to year the effect would be to penalise most severely those cash flows which would create the greater opportunities for the firm.

(b) it presupposes that none of the cash flows is used to pay interest or dividends on the capital raised to finance the project until the project is terminated.

Whichever method is used, the conclusion is that, under rationing, the simple NPV approach is invalid, i.e. when the project's own cash flows are discounted at the cost of capital. Let this simple NPV be denoted by NPV (k_o). $NPV(k_o)$, even under rationing, indicates the contribution a project makes to the wealth of the firm in respect of its <u>own</u> cash flows. But the significance

of capital rationing is that the contribution which a project makes in isolation from the projects financed by its cash flows ceases to be the relevant criterion of choice. If it were valid to assume a constant reinvestment rate then the NPV derived by discounting the cash flows at the reinvestment rate = $NPV(k_r)$ would indicate the relative (not the absolute) desirability of alternative investment programmes.

Dudley [5] and others have observed that if a firm is presented under rationing with two or more projects having identical outlays, and if the ranking given by the simple $NPV(k_o)$ conflicts with the ranking given by the IRR, then, provided the reinvestment rate is less than the Fisher rate [6] (defined as the rate which brings the present value of the two income streams into equality), the $NPV(k_o)$ will rank correctly, and if the reinvestment rate is greater than the Fisher rate, the IRR will rank correctly. This has, surprisingly, been presented as evidence of the indeterminacy of the relative superiority of the NPV and the IRR approaches. It has, however, been shown above that, under rationing, the single $NPV(k_o)$ is itself invalid in principle, and therefore the so-called Fisher rate is no more than an arithmetic device to indicate the point at which the invalidity of $NPV(k_o)$ actually manifests itself in terms of the ranking of the projects. If the rate of reinvestments is less than the Fisher rate, the ranking given by $NPV(k_o)$ is correct in fact though incorrect in principle. If the rate is greater than the Fisher rate, the ranking is incorrect in principle and incorrect in fact. But above the Fisher rate, the IRR, which remains incorrect

(5) Dudley, page 914
(6) See Alchian, 1955.

incorrect in principle throughout, becomes correct in fact because above that rate the consequences of the invalidity are insufficient to cause the ranking to differ from the correct order. Even when they do achieve correct rankings, the IRR and NPV(k_o) are misleading, because, being calculated without reference to the relevant reinvestment rate, they imply that the relative desirability of alternative projects is independent of that rate.

The fact is that, when rationing exists, the relevant discount rate is the reinvestment rate and NPV(k_r) produces the correct ranking (given the right conditions) whether the reinvestment rate is above or below the Fisher rate. If the conditions which allow the use of NPV(k_r) are absent, then NPV(k_o) and IRR will not even rank correctly using the Fisher rate device, and their claim for consideration could not even be presented. Only the second of the NPV modifications will then yield correct rankings. It is not a valid criticism of the NPV approach that the ranking given by NPV(k_o) is not universally correct in circumstances where the relevant NPV criterion is not NPV(k_o) but NPV(k_r) or the terminal value approach. Still less is it a validation of the IRR technique if, in very limited circumstances, it is capable of indicating the correct order of preference, that is (1) when there are no other sources of funds (2) when the projects' initial outlays are equal (3) the servicing of capital is passive (4) the reinvestment rate is constant throughout the lives of the projects, and (5) the Fisher rate is known and understood. Even in these restricted circumstances, it has been shown that the IRR remains invalid in principle, and gives the correct ranking merely because its degree of irrelevance is insufficient to affect the ranking. Payback can effect correct rankings for projects having a protracted series of equal cash flows but remains nonetheless incorrect in principle.

CAPITAL SURPLUS

There is one possible set of circumstances in which the reinvestment

opportunities could have some bearing on the validity of the net present value and internal rate of return, not only in ranking projects, but even in the accept-or-reject decision. There is an implicit assumption in both methods that the capital raised can be repaid as the cash flows are received, failing which, that the cash flows can be reinvested to earn a return at least equal to the cost of capital. (Repayment would include direct repayment, purchasing the company's loan stock on the market, paying out increased dividends, and if applicable, purchasing the company's own shares. Reinvestment opportunities would include the purchase of shares or loan stock in other companies in the same risk class.) This assumption does not imply the relevance of reinvestment opportunities in a general sense, but merely requires the recognition of costs associated with holding surplus cash.

Although it is reasonable to suppose that the circumstances outlined are rarely if ever met, if they do exist the application of either the IRR or the NPV method in its pure form could lead to suboptimal decisions and even the acceptance of unprofitable investments.

The correct solution is to advance all cash flows receivable in a period of capital surplus at a compound rate of interest equal to the maximum available rate of interest on investments (Government stock, etc.) to the first period in which capital surplus conditions do not apply (which may be before or even after the project has ceased) and to discount the resulting cash flows at the cost of capital to their present value. The effect is thus to incorporate the cost of carrying cash directly in the NPV computation, with the result that the basic validity of the NPV approach remains intact.

RISK

Apart from when capital is rationed, the other consideration

which gives a ratio of profitability a more intuitive appeal than any measure of the absolute contribution which the project makes, is in evaluating the project's riskiness. The impact of risk on the evaluation process will be taken up in the succeeding chapters, but, for the present, it is sufficient to say that if the IRR is inappropriate for discriminating between projects under conditions of certainty it cannot be validated by the introduction of uncertainty. A higher rate than normal may be required to compensate for a given level of risk, but the choice between projects can still not be made on the basis of their respective rates of return. This must again be made on the basis of the net present values after adjusting for risk through the discount rate or after applying certainty-equivalent coefficients to the prospective cash flows.

Accept or Reject Decisions

For accept-or-reject decisions, the IRR and the NPV always give the same solution, but since the IRR is invalid for all project comparisons, to justify its retention for the nonranking problem it must be shown to possess some advantage over the NPV. Bierman and Smidt who prefer the present value method as being "simpler, safer, easier and more direct" (7) concede, however, that the yield method may be useful "to dramatise the relative desirability of an investment" and that the "relative desirability of the investment may be better judged if we know that the yield is 60% or if we know the yield is 10%." (8)

For an investment project to be desirable, it must make a worthwhile contribution to the value of the firm. The contribution a project makes is a function of the average amount of capital

(7) Bierman and Smidt, 4th edition, page 57.
(8) Ibid., 2nd edition, page 49. In the 4th edition they do not even concede this much.

invested and the lifespan of the project. A project involving an outlay of £10 in period 0 and an inflow of £16 in period 1 has an IRR of 60%, but is clearly undesirable in the sense of making any significant contribution to the firm. Therefore the IRR does not indicate the desirability of a project in an absolute sense. Presumably what is meant by being a measure of desirability in a relative sense, is that any project is more desirable for having a higher rate of return than a lower one whatever its size. It has been shown that size should not be determined by the initial outlay but by the average capital invested, taking account of the amount and pattern of cash flows. Since the IRR is determined by the pattern of cash flows, a project cannot have a different IRR unless it has a different size, and it follows that the above statement is equivalent to saying 'a project is more desirable for having a high rate of return with one size than it is with a low rate of return with a different size.' But this latter statement is true only if 'different' means smaller. If 'different' means some flows are smaller and others greater it may or may not be true. In other words, as has already been demonstrated in the ranking problem, the IRR is incapable of discriminating between two patterns of cash flows.

It is, therefore, as incorrect to say that a project is more desirable for having an IRR of 10% rather than 9% as it is to say that it is more desirable than another project with 9%. Assume, for example, Project B in Table I were presented to a businessman, not in competition with A, but as an independent project requiring an accept-or-reject decision only. And assuming that it were possible for the cash flows to be regulated without increasing the risk, in such a way that the cash flows in year 1 could be delayed until year 2 and that the effect would be to produce £144 in that year

(as in Project A), then the businessman might be misled into taking the wrong course of action by assuming that the project was more desirable for having an IRR of 24% rather than 20%. Another way of expressing the same idea is that if A and B were independent projects, with identical risks, requiring accept-or-reject decisions and the businessman imposed a cut-off of 21% for projects of that class, to represent his minimum level of desirability, he would accept B and reject A despite the fact that A is superior.

The only circumstances in which a high rate of return can unequivocally be said to be preferable to a lower rate of return is when the cost of capital falls between the two, which simply means that a project which is acceptable is categorically preferable to a project which is unacceptable. Thus if a project has an IRR of 24% it can validly be stated that such an IRR is superior to 20% provided the cost of capital falls between 20% and 24%. If, in fact, the cost of capital is less than 20%, however, this observation ceases to be relevant, because both IRRs are acceptable and it is quite invalid to conclude that a relationship which exists under certain hypothetical conditions continues to exist in the absence of those conditions.

It has previously been observed that under perfect conditions no project would have an IRR greater than the cost of capital, that is, greater than the return required by the suppliers of capital, because in perfect equilibrium the suppliers of capital would 'require' the return that was obtainable. The existence of a project with an IRR which is greater than the cost of capital therefore indicates a state of disequilibrium (imperfect competition, or artificially low interest rates, as discussed more fully in the next chapter) and the rate of return can be perceived as being a measure of the degree

of disequilibrium. A high IRR unequivocally indicates a higher state of disequilibrium than does a lower IRR, but this is far from implying that a higher state of disequilibrium is necessarily more desirable than a lower state. All that we can say is that whilst a state of disequilibrium must exist to produce a benefit to the investor (a +ve NPV), the extent of the benefit is not directly a function of the degree of disequilibrium.

It follows then that, although the IRR gives a correct signal that a project can be undertaken without loss, it is not a number in a scale against which the businessman can represent the level of his 'satisfaction'. It does not indicate how desirable a project is, far less whether that project is preferable to any other. Therefore, if its continued use is to be advocated in the accept-or-reject situation, it can only be on the basis that businessmen constantly bear in mind that even with the same risk a project is not necessarily better for having a higher rather than a lower rate-of-return. But in practice this can only be misleading in contrast to the NPV method, where the businessman knows that for the same risk, £10,000 is unambiguously more desirable than £5,000.

The maximum acceptable cost of capital

In the introductory chapter some very cumbersome efforts to make the IRR a meaningful statistic when non-simple investments are under review were examined. These efforts result from perceiving the IRR as a profitability ratio and the need to resolve the confusion which arises when some investments which are clearly desirable have no rate of return, when others have two or more rates of return, and others require the external cost of capital to be incorporated before the internal rate can be computed. It has been shown that even when these salvage efforts are carried out, the end result is a rate of return per unit of capital invested, a measure which remains basically

irrelevant for selecting investments which have varying and unspecified amounts of capital invested in them. On the other hand, if the IRR is perceived not as a profitability statistic but as a cost of capital statistic, two advantages emerge, an immediate description of the restricted function which the ratio has, and a better insight into the significance of the ratio for non-simple investments. It is suggested therefore that if the ratio were interpreted as the breakeven cost of capital, a considerable amount of unnecessary conceptualising could be spared. For the simple investment, the ratio, therefore, represents the maximum cost of capital which could be paid without incurring a loss. For a quasi-borrowing situation, on the other hand, such as:

Time	0	1
Flow	+ £100	-£110

the ratio represents the minimum cost of capital which makes that particular source of finance preferable to alternative sources.

This interpretation is also more consistent with the logic of discounting. The effect of discounting is to impose a penalty on cash flows occurring in the future, and since the extent of the penalty is a function of the size of the discount rate, then the IRR method, which by definition discounts future flows at the IRR, would imply, if perceived as an index of profitability, that the more valuable a series of cash flows, the greater would be the penalty attached to these flows. That is, a cash flow of £1,000 generated in period t with an IRR of 25% would be deemed to have a lower present value than a £1,000 generated in period t of a project with an IRR of 15%. The discount rate applied to future flows should not penalise them because they are profitable, but because the capital used to generate them has a cost, whether an actual cost, an opportunity cost or a notional cost. In any given

period the actual cost or opportunity cost cannot be both 15% and 25%. Therefore the IRR is a notional cost, namely the maximum cost a project is capable of supporting.

Multiple roots

It is now possible to approach the problems of multiple roots and imaginary roots with some degree of understanding.

The most celebrated example of the problem is the Lorie and Savage pump project (9) which involves the outlay of £1,600 in Year 0 to extract the oil more quickly and advance the receipt of £10,000 from Year 2 to Year 1. The problem is represented in Table 3, where the application of the IRR method yields two answers, 25% and 400%.

Solomon's assessment of the problem is as follows:

"Neither of these rates is a measure of investment worth, neither has relevance to the profitability of the project under consideration, and neither, therefore, is correct. The fault lies in the incorrect application of the "usual prescription" for finding the rate of return.

The correct solution for the investment worth of the project is simple. But it requires an explicit answer to a relevant question: "What is it worth to the investor to receive £10,000 one year earlier than he would have otherwise received it?" This is actually all that the installation of the larger pump achieves. If the investor expects to be able to put the £10,000 to work at a yield of x per cent per annum, then getting the money a year earlier is worth £100x. If x is 23 per cent, for example, getting £10,000 a year earlier is worth £2,300. In other words, if he spent £1,600 on the larger pump now (at that t), he would end up at time t having £2,300 more than he otherwise would have had. This can be stated as an equivalent "rate of return," which in this case would be about 20 per cent (£1,600 at 20 per cent per annum would amount to £2,304 at the end of two years). Using this approach, a unique and meaningful rate of return can always be found for any set of cash inflows and outflows. (10)

The implication of Solomon's solution is that the pump project should be accepted if the cost of capital is less than his "unique

(9) Lorie and Savage, pp. 235-38.
(10) Solomon, page 79 of Management of Corporate Capital, Macmillan, 1964.

and meaningful rate of return". In fact the correct solution is that the project should be rejected. Solomon's line of argument implies that even in non-rationing conditions a source of finance is acceptable simply because the funds can be invested at a rate higher than the cost of capital. But the presence of a profitable investment opportunity is a signal only that finance should be raised and is not a justification for any particular source of funds. That justification comes from comparison with alternative sources of finance.

Solomon's unique return of 20% is presumably compared with the firm's cost of capital, say 10%, and judged acceptable. But if the investor has an opportunity to invest £10,000 in year 1 at 23% he should borrow the £10,000 at 10% and thereby finance the project from normal sources, to produce a terminal value in Year 2 of:

Terminal value	£12,300
Less borrowed funds and interest	11,000
	£ 1,300

This compares with Solomon's Solution

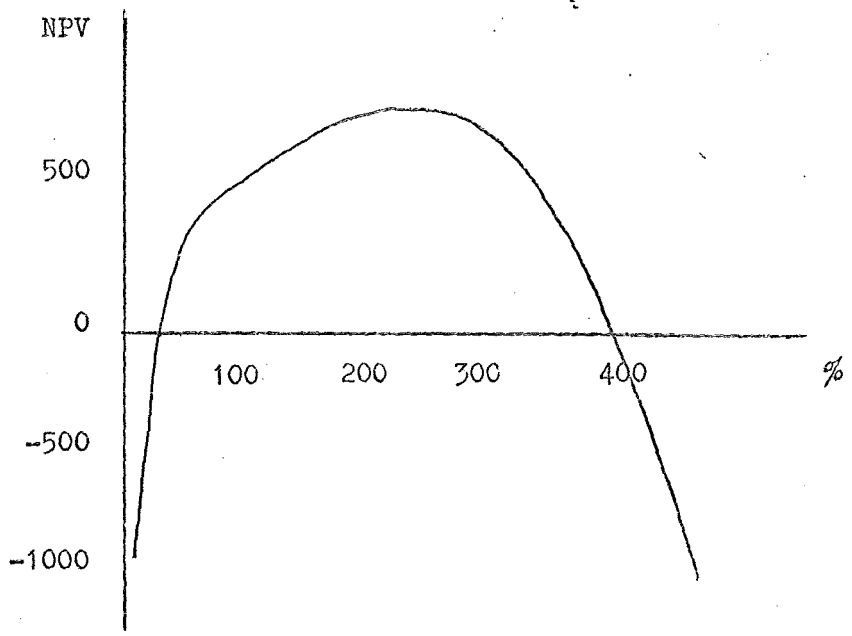
Terminal value	£ 2,300
Less borrowed funds and interest for two years	1,936
	£ 364

The problem, then, consists of determining whether it is worthwhile undertaking a particular project to provide a source of funds. The evaluation should be on the basis of a comparison with other sources of funds and not on the profitability of investment opportunities, although the existence of such opportunities must be ascertained before the decision to raise funds can be made.

The two rates of return represent, then, a range of interest

Table 3

$$NPV = -£1,600 + \frac{£10,000}{(1 + r)} - \frac{£10,000}{(1 + r)^2}$$



rates within which it would be to the advantage of the company to raise the £10,000 by means of buying a pump rather than by calling on its normal sources of finance.

Why, it may be asked, is it cheaper to buy the pump when alternative sources are above 25% and more expensive when they exceed 400%? The answer is that the use of the £10,000 cash in year 1 has two costs:

1. A fixed charge of £1,600 in year 0
2. The interest charge of borrowing the £1,600.

For interest rates under 25% it is better for the company to raise the £10,000 through the normal channels and pay the year's interest on that sum rather than pay the fixed charge of £1,600 and the interest on the £1,600.

For interest rates between 25% and 400%, it is worthwhile paying the £1,600 and the related costs because the interest on the larger sum of £10,000 would be an even greater burden.

For rates greater than 400% the interest burden on the £1,600 becomes so great that it will, for all higher rates, be less expensive to raise the £10,000 directly rather than undertake the outlay of £1,600 and related interest costs in the previous period.

The solution to the multiple-root situation in conditions of non-rationing is to discount the flows at the cost of capital; the resulting net present value will represent the contribution to the firm's value of selecting an unconventional method of raising finance.

Curiously, therefore, far from being meaningless, multiple roots have more relevance to the investor in the context in which they arise, than has a unique rate of return in the normal investment appraisal. When confronted with a financing opportunity (which is essentially what the cash flow pattern represents in the multiple root situation)

the investor is concerned with a comparison of costs with alternative financing sources.

The multiple roots indicate clearly to him the range of costs within which the best alternative source must fall to make this project preferable. He can, furthermore, by studying the curve which lies above the horizontal axis, observe that if the best alternative was at 70%, 80% or 350%, the pump would be worth purchasing, but that although the pump would be more acceptable if alternative costs were 80% rather than 70% it would be less attractive if the alternative rose to 350%. In other words, the diagram represents a sensitivity analysis, which is perhaps a useful supplement to the single NPV figure proposed as the solution. If discounted at 100% the NPV indicates the contribution the project makes but it does not indicate whether the contribution would be more or less if the cost of capital were 101%.

Imaginary roots

Certain cash flow patterns consistent with 'respectable investment options' (11) have been identified as having no real internal rate (i.e. where the present value equation has only imaginary roots). As with multiple roots, this phenomenon has been used to support the view that the IRR is ambiguous. Examples of investment pattern having no real rates of return are contained in Table 4.

(11) Hirshleifer, page 468 of Book of Reading, Archer and D'Ambrosio, Macmillan, 1967.

Table 4

Period	H	J
0	-1	+1
1	+3	-3
2	$-2\frac{1}{2}$	$+2\frac{1}{2}$
IRR	None	None
NPV	Always - ve	Always + ve

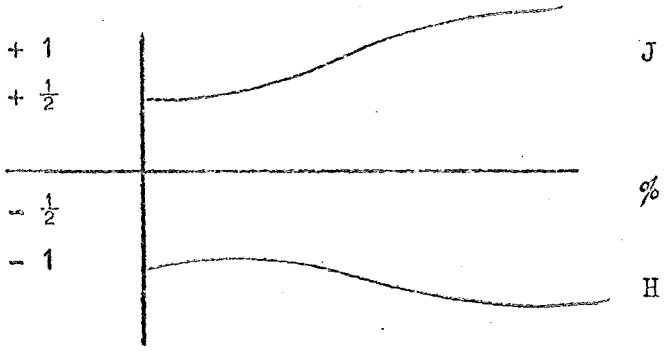
Those who perceive the Internal Rate of Return as an index of profitability understandably take the view that if no internal rate can be found an investment such as J, which is clearly capable of being profitable, such an index cannot be relied on with any confidence. But, once again, it is in these circumstances in which the Internal Rate of Return has been dismissed as ambiguous or meaningless that the benefit of perceiving it as a 'maximum cost of capital' is most apparent. The absence of a discount rate which equates the present value of the inflows of a project with the present value of the outflows signifies that there is no constant cost of capital which would make that project acceptable, or, depending on the pattern of cash flows, unacceptable. Thus, Table 5 illustrates that there is no cost of capital that would make the 'financing-project' H an acceptable source of capital, or investment J an unacceptable investment.

It would always be more advantageous to borrow £3 in period 1 at any cost of capital than to undertake project H and borrow £1 in period 0 at the same cost of capital, and incur an outlay of £$2\frac{1}{2}$ in period 2. Conversely, undertaking project J is always a cheaper method of generating £1 in period 0 and £$2\frac{1}{2}$ in period 2, than borrowing these amounts directly at whatever cost.

Therefore, the fact of having no internal rate of return is not

in itself meaningless or ambiguous. It indicates for example, that project J is desirable at any cost of capital. But that is an 'accept-or-reject' decision. The defect of the measure is, once again, that it fails to discriminate between project J and any other mutually exclusive project. To make that selection it is necessary to ascertain which project makes the greater contribution to the firm. This can be achieved only by the Net Present Value method.

Table 5



Measuring the 'Size' of a Project

In Table 6, a series of projects are presented which differ from one another in duration, in cash flow pattern, and in the amount of the initial outlay. The order of preference is given by their respective net present values rather than by their respective rates of return. We have established that the explanation is simply that each project has a different scale from the others and however profitable a project may be in relation to its size, it may not make as significant a contribution to the value of the firm as an alternative project of a greater size, even although the latter is less profitable. It is the monetary contribution which a project makes which is the primary consideration in the ranking of alternatives, and a valid

Table 6

Net Cash Flows

Year	M	N	P	Q	R
0	£2,000	£2,000	£2,000	£2,000	£1,000
1	+ 10	+2,100	+2,500	+2,200	-
2	-	-	-	-	-
3	+3,109	+ 329	-	-	+1,728
4	-	-	-	+ 483	-
NPV	£345	£156	£272	£330	£297
IRR	16%	17%	25%	23%	20%

selection can never be achieved, other than by chance, on the basis of a ratio of profitability. Table 7 demonstrates how the scale of a project might be measured. The resulting figure of £5,981 represents the equivalent amount of capital employed for one year in project M, taking into account the effect of the cash flows on the capital outstanding in each year. Table 8 contains the 'scale' of number of units of capital employed as calculated by the same method for each of the five projects. Project R cannot have a higher NPV than project N and a lower NPV than project M ~~because~~ of its lower initial outlay. It is more 'profitable' than both M and N but M has more units of capital profitably employed and succeeds in producing a greater increase in the firm's wealth. N has more capital initially invested in it but has a smaller total investment of capital when seen in perspective over the three years. To specify its cash flow pattern as the explanation for its superiority to N, and its longer life for its greater acceptability than P, and its lower initial outlay for its inferiority to M is simply to recognise the symptoms and not the cause.

Ascertaining the scale of a project does not in itself have any practical value. The merit of the NPV method is that it measures the contributions which a project makes without the necessity of computing the number of capital units involved. But if we are to accept the observation that practitioners are reluctant to accept the NPV without some method of relating it to the size of the project, then it may be useful to be able to provide some index of relative profitability to supplement the absolute monetary measure, even although it has to be borne in mind that the supplementary ratio has no authority beyond that provided by the primary measure. But such an index should satisfy certain criteria before its use as a supplementary ratio is warranted; it should be simple to understand; it should be consistent with the assumptions underlying the NPV method;

Table 7

Calculation of number of units of capital employed

Year		Project M			Project N		
			PV Factor			PV Factor	
1	Invested Capital	£2,000	1.00	£2,000	£2,000	1.00	£2,000
2	Interest at 10%	200			200		
		2,200			2,200		
	Cash flow	10			2,100		
	Invested Capital	2,190	.909	1,991	100	.909	91
3	Interest at 10%	219			10		
		2,409	.826	1,990	110	.826	91
	Units of Capital Employed			£5,981			£2,182

Table 8

	M	N	P	Q	R
Units of Capital Employed	5981	2182	2000	2000	3000
NPV	345	156	272	330	297
IRR	16%	17%	25%	23%	20%
PI	1.172	1.078	1.136	1.165	1.297
ERI	5.8%	7.2%	13.6%	16.5%	9.9%

Ranking:

NPV	1	5	4	2	3
IRR	5	4	1	2	3
PI	2	5	4	3	1
ERI	5	4	2	1	3

and since the object is to supplement the NPV and not to provide an alternative, the index should relate the NPV directly to some measure of the size or scale of the project.

The two existing profitability ratios are, of course, the IRR and the PI (the present value of the future returns divided by the investment outlay). The IRR, however, fails to meet any of the above criteria. It has the unwarranted reputation of being readily comprehensible to businessmen, but it is doubtful if the significance of a rate of return internal to a project is fully understood. It may be interpreted as the rate of growth of each unit of capital initially invested but without any apparent means of specifying the number of units invested for a single year only, or two years etc. Alternatively, it may be perceived as the annual rate of return for each unit of capital invested in the total life of the project, again without any obvious means of quantifying the number of units of capital employed. In order to measure the number of units employed according to the method described in Table 7, the rate used to discount the annual outstanding capital could no longer be equal to the cost of capital since that would imply that the resulting number of units of capital had succeeded in achieving the internal rate. But to discount at the IRR itself would imply that the amount of capital invested in any particular year was partly a function of the cash flows received after that year since these in part determine the IRR, and this would be clearly confusing, if not absurd. In addition, how does one conceptualise an IRR of 17½% against a cut-off rate of 15% and evaluate the project's desirability in relation to an alternative project with an IRR of 19% and a cut-off rate of 16?

In other words, the IRR suffers from a fundamental inconsistency with the NPV method namely that it takes no account of the cost of capital. What is alleged to be a source of strength, the fact that

the IRR is independent of the cost of capital, is in fact a basic weakness, since both the NPV and the size of the project are a function of the cost of capital. Finally, the possibility of multiple returns etc. provides an unnecessarily burdensome framework for both specialist and non-specialist seeking to acquire an insight into the economic rationale of the investment process.

The Profitability Index (or Benefit-Cost Ratio) does have the merit of avoiding many of the technical problems associated with the IRR and of seeking to place the NPV in a meaningful context. But it suffers from the serious defect of equating the size of a project with its initial outlay, ignoring the time scale or the amount of capital outstanding throughout the life of the project. It is apparent from table 8 for example that the PI misleading suggests that project R is substantially more 'profitable' than any of the other projects, as a result of the disproportionate emphasis on the initial outlay.

A new ratio is therefore proposed which for want of a better term is called the Excess Return Index. It is illustrated in Table 8. The denominator is equal to the total capital employed in the project as calculated by the method described in Table 7, and the numerator is the net present value. The ratio is consistent with the underlying logic of the primary selection procedure and incorporates the NPV directly. One cannot arrive at or comprehend the ratio without first being made aware of the absolute contribution which the project makes to the wealth of the firm. In addition, it directs the attention of the user to the size of the project, thus highlighting the fundamental reason why profitability ratios cannot be used in isolation to discriminate between alternative proposals. It is relatively easy to conceptualise, the capital employed being the equivalent amount of

capital invested for one year at the required return, and the NPV being the immediate increase in the firm's value as a result of employing that capital. In Table 8 the ranking produced by the ERI is compared to that of the IRR and the PI. Not one of the methods ranks consistently in agreement with the NPV method, but, as previously observed, that is an inevitable concomitant of the scalar differences between the projects. The NPV remains the only valid criterion. But of the three indices of relative performance, the ERI alone puts the NPV in correct perspective and provides the user with a valid measure of the project's effectiveness in relation to its size.

CONCLUSION

The alleged preference for a ratio rather than an absolute monetary measure to describe an investment may be an understandable desire to extend the familiar technique employed in the securities market. The IRR is the accepted yardstick for measuring the relative merits of government or corporate bonds, and its application to productive investments is clearly an inviting possibility. But what distinguishes financial securities from productive investments is that there are effectively no constraints on the amount of capital which can be invested in the former. The choice is not normally between investing £10,000 in I.C.I. or £12,000 in General Motors stock but between the investment of an unspecified number of units in either, with the result that the rate of return per unit of capital is a legitimate criterion for making the choice. The approach, however, cannot be extended to the selection of real investments, which require a specific amount of capital for a predetermined period of time. Productive projects come in 'parcels' and the selection of one parcel from another of a different size can only validly be effected by a comparison of the amounts by which each project increases the value

of the firm, and not the percentage return which each parcel yields per unit of capital invested. When applied to real investments the IRR is capable of giving an apparently correct solution. It is also capable of giving incorrect solutions. What the text-books fail to point out is that there is no method of knowing when it will give a correct solution other than by determining the NPV. Without the NPV the only situation in which two investments can confidently be said to be ranked correctly by the IRR is when they are identical in every respect! As for an accept-or-reject decision there is no occasion when a high IRR can in itself be said to be better than a lower one, because that would be to say that a given set of cash flows in better for being as it is, rather than different, and the IRR can only be confidently relied upon to discriminate between two patterns of cash flows when the patterns are not different.

It has been proposed that, if it is perceived as the highest cost of capital that could be paid for the funds used to finance a project, the limitations of the IRR might be more readily recognised, and the controversies about its significance when there is no unique solution to the present value equation finally set to rest. In addition, the business community might be better disposed to accepting the validity of the present value concept without fear of being mystified by the confusion that has surrounded it.

SUMMARY

The essential problem in capital budgeting is to select the combination of projects which exploits to the maximum the imperfections of the market. If competition amongst firms were perfect and the securities market totally efficient, companies would not be able to achieve returns in excess of that required by the suppliers of capital. Prospective projects would have zero NPVs, or, equivalently, IRRs

equal to the required return, and the selection of projects would be relatively straightforward. It is the existence of imperfect markets which allows projects to be undertaken with expected returns in excess of the required return, which, in turn, gives rise to the difficulties in the selection procedure. The source of the problem in choosing real investments is their indivisibility, which precludes the selection procedures appropriate to the securities market and which necessitates that the expected excess returns be expressed in absolute monetary terms in order to make the optimum selection possible. The IRR was found to be invalid in principle even under the limited circumstances in which it happens to produce correct rankings. If users find it desirable to relate the project's NPV to the scale of the project, this might be achieved by using, as a supplementary measure, the Excess Return Index described in this chapter which, unlike the IRR and PI ratios, reflects the essential difference between choosing a financial security and a productive investment.

NOTE. In discussion, it is sometimes convenient to speak of a project having a high or low rate of return. This is not a contradiction of the principle that the yield approach is invalid for choosing investments. To say that a project has a high yield is a convenient way of emphasising that there are excess returns available to be exploited. It remains true, nonetheless, that to measure the significance of the excess returns only the NPV approach is valid. Therefore, if in the subsequent chapters a project is described as having a high rate of return, this is simply to avoid the more cumbersome but more technically correct expression 'a high positive net present value.'

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Chapter Three

THE NET PRESENT VALUE RULE

The traditional assumption that rational investors are averse to risk has led to the widely accepted proposition that the minimum return acceptable to investors in risky assets must be greater than the default-free rate. This in turn has led to the decision rule that firms should:

(a) discount the expected values of the annual cash flows at a risk-adjusted rate, or

(b) discount the certainty-equivalent cash flows at the risk-free rate, and should accept only projects which have a positive net present value. Of course, precisely because projects are risky, then accepting those which are expected to yield more than the required return is no guarantee that such returns will be achieved. Some will earn considerably less than the minimum acceptable return and others will earn more than their expected return (their IRR), but by the law of large numbers, if there is no systematic bias on the part of corporate decision-makers' estimates, the average achieved returns from the aggregate of projects undertaken in the market as a whole should exceed the average minimum acceptable return of investors financing those projects.

Equity investors, it seems therefore, have two incentives to entice them into a portfolio of equities in preference to a default-free security: A) the fact that management seeks a return on their capital which purports to compensate them for their aversion to risk, and B) the fact that the company will not undertake subsequent expansion unless the returns from the new projects are expected to

exceed the return required by the new suppliers of capital, with the result that the existing shareholders should benefit by a corresponding increase in their wealth.

Since the incentive offered by A alone is, by definition, sufficient to attract investors, the additional returns arising from the subsequent wealth-increments produced by B must be a supplementary bonus. That is, B cannot be explained away as the reward for risk if projects are discounted at the rate which is the reward for risk.

In present value terms this excess return for a particular project is represented by its net present value, and when the firm undertakes a new project, the value of the firm should arise, ceteris paribus, by an amount equal to the net present value. This accretion in value accrues entirely to the firm's shareholders and its existence demands that there be a disparity between the return required by the shareholders and the prospective return from the project. A positive net present value signifies one of three events:

(1) A transfer of wealth to one company from others as a result of the former securing some competitive advantage

(2) A creation of new wealth as a result of some technological advance, increased productive efficiency, or the discovery of new resources.

(3) Disequilibrium between the return required by the suppliers of capital and the opportunity return from productive investments.

If all +ve NPVs were attributable to the first of these events, that is if one company's +ve NPV implied by necessity a proportional reduction in the value of other competing firms, this would suggest

that the pursuit by individual companies of the classical wealth-maximising goal could not secure any benefit to the holder of the market portfolio, since this would not be increased in aggregate but merely changed with respect to its distribution between the components of the portfolio. In a portfolio context then a positive net present value would be of no significance to shareholders, and it would be a matter of indifference that a firm selected a project with a low NPV in preference to one with a high NPV.

Indeed, if there were perfect competition between firms, no such NPVs would arise because the competitive advantage of the one company would be instantly eliminated by the competitive process. In these circumstances all projects would have zero NPVs and it would not be possible to add to the wealth of shareholders either at the individual company level or at the aggregate portfolio level.

To have any real economic significance to the portfolio holder the +ve NPV rule and wealth-maximising goal must imply the possibility of adding real wealth to the aggregate portfolio as well as to the individual company, whether by the discovery of additional resources or the development of some more effective production technique. Since the present study is operating within the wealth-maximising framework of conventional theory, the possibility that new wealth can be created for shareholders will be assumed hereafter.

The third source of positive net present value calls for special explanation. The wealth arising is not new wealth or a transfer of wealth from one company to another but a differential

wealth caused by the failure of the market rate of interest to reach a level which reflects the opportunities available from real productive investments. The cost of equity equals the market rate of interest plus, if relevant, a premium for risk.(1) If the market rate of interest is below its equilibrium level, then the cost of equity will likewise be below its equilibrium level. It follows that a project yielding a return equal to the equilibrium rate of interest will produce a +ve NPV if its returns are discounted at a rate below the equilibrium rate.

It is necessary to consider how it is possible for the rate of interest to be below its equilibrium level. Before doing so, however, it should be noted that the possibility of achieving profits in excess of the requirements of the suppliers of risk capital can be only partially attributed to inventiveness, technological advances, managerial flair, etc. If the market's competitive forces operate relatively efficiently, one would expect the opportunities to achieve surplus returns of this nature to be restricted in number, at least to the extent of making it impracticable for the prospect of achieving surplus returns to be the primary criterion for acceptance of investment projects. Yet the positive net present value rule (where the NPV is computed by discounting the expected cash flows at the 'risky' rate) states precisely that, namely that the achievement of returns surplus to the required 'risky' rate should be a prerequisite for new investments. Finance text-books tend to state the rule as accept projects if

$$\sum_{t=0}^{n} \frac{A_t}{(1+k)^t} \geqslant 0$$

(1) Van Horne, page 110.

but in practice, imply [2] that there is no incentive to accept unless the NPV >0, since no contribution is otherwise made to the wealth of the existing shareholders, which is the assumed objective.

The fact that the rule is advocated even for routine investments implies a factor other than abnormal profits. It implies the existence of a force which causes the equilibrium rate of interest (and, by derivation, the firm's cost of capital) to be less than the marginal productivity of capital, a force, in effect, which produces a divergence between the 'money rate' prevailing in the loan market and the 'natural rate' [3] attainable from marginal productive investments. According to the classical theory of interest, the rate of interest is dependent on the supply of savings and the demand for loanable funds, and the long-run equilibrium rate of interest in the loan market is one in which the money rate coincides with the natural rate or, equivalently, with the marginal productivity of capital. But this equilibrium state is essentially a long-run concept and does not preclude short-run discrepancies between the money and natural rates. Indeed, this possibility is recognised by the classical school.[4] For example, if for any reason the quantity of money is increased, causing the bond demand curve to be shifted to the right and the supply curve to the left, there will be a decline in the money rate of interest which will produce a discrepancy between the market and natural rates of interest. This decline, it is argued, will be a transitory one, because the discrepancy "automatically generates equilibrating forces" [5] which bring the money and

(2) Van Horne, page 74, and Weston and Brigham, page 267.
(3) These terms were originated by Wicksell.
(4) Patinkin, page 369.
(5) Patinkin, page 368.

natural rates to equality. The inflationary effects of an increase in the money supply eventually bring about a reversal in the downward movement of the interest rate and a return to the equilibrium level. But because the equilibrating force is not an instantaneous process, then during the period of adjustment, borrowers will be able to raise finance at less than the equilibrium price. Irving Fisher, whose theory of interest supported the notion of an interest rate which adjusts completely to anticipated rates of inflation, conceded in the face of evidence to the contrary, that 'when prices are rising, the rate of interest tends to be high but not so high as it should be to compensate for the rise,' (6) and, again, that 'men are unable or unwilling to adjust at all accurately and promptly the money interest rates to changed price levels. The erratic behaviour of real interest is evidently a trick played on the money market by the 'money illusion' when contracts are made in unstable money.' (7) If the money rate exceeds the natural rate in periods of deflation, there is no corresponding adverse effect on productive borrowers (borrowers for consumption purposes will, of course, suffer) other than the temporary disappearance of opportunities to exploit the imperfections of the bond market. The net present value rule should signal to corporate borrowers a rejection of all marginal projects (which will yield a negative net present value at the money rate of interest) until the interest rate falls below its natural level.

There are, doubtless, other forces in addition to increases in the quantity of money which operate to produce a divergence between the market and the equilibrium rate of interest, and in

(6) Fisher, page 368.
(7) Ibid., page 415.

particular which act to push the rate below its natural level. Governments and central banks are under social pressures to intervene [8] to dampen down the money rate of interest when the rate is high in anticipation of price-level changes, either by issuing additional currency to meet the needs of borrowers at a reasonable rate, or by fiscal policy. [9] Also, management appear to some extent to be conditioned to exert influence in the market to produce a discrepancy in the two rates and to create a climate of opinion in which low interest rates are thought to be intrinsically desirable. In a study by Merrett and Sykes [10] in 1966, a comparison was made between the ex post results of undated Government Stock ($2\frac{1}{2}$ Consols) and those of a widely based portfolio of U.K. securities. For the period 1949-1966 the return on the Consols on a year to year basis averaged -4.6% in real terms compared to +7.4% for equities. This would suggest a persistent failure on the part of fixed-interest investors to predict the trend of inflation, or a persistent inability to raise interest rates to a level which compensated them for inflation.

Whatever the cause might be, if the interest rate is below its 'natural' level, then, assuming the interest rate directly governs the cost of equity capital (whether or not there is a risk-premium added), the latter will also be proportionately less than its 'natural' level. As a consequence, a firm's weighted-average cost of capital will be less than the return on its marginal projects, and the marginal projects will, when discounted at the cost of capital, have a net present value which is positive notwithstanding

(8) For example, in the U.K., the period 1932-1951 was marked by a 'cheap money' policy, inaugurated by the conversion of War Loan from a 5% to $3\frac{1}{2}$% basis in 1932, and later intensified by Dalton's ultra-cheap money drive in 1946. Throughout the period the Bank Rate was unchanged at 2%.

(9) Hirshleifer, page 137.

(10) See Solomons for example, pages 61-62.

their marginality.

There are, therefore, a combination of forces which operate to establish positive net present values or surplus returns for the equity investor. When he purchases a portfolio of equities he can expect to achieve, on average, a return equal to his required minimum plus the right to participate in the future series of wealth increments arising from projects undertaken and financed by capital raised after his acquisition of the portfolio. Hence the rationale of the classical shareholder-wealth-maximising objective.

Four ex ante returns can be identified for a company within a given risk class, assuming for the present that all companies within the class are financed by equity only and there are no taxes:

k_e = the minimum return acceptable to investors in the firm's equity capital

k_r = the average return expected to be earned by investors from holding the firm's equity capital

k_a = the average return expected by investors from the firm's projects

k_i = the default-free rate

In perfect market conditions $k_e = k_r = k_a$, and under conditions of certainty, or risk-indifference, $= k_i$. But, if projects are to be capable of yielding +ve NPVs, it is necessary that k_a be greater than k_e and k_i. And since over the life of the firm k_r should on average equal k_a, then k_r should also be greater than k_e and k_i. The implications of the generally accepted assumption that the expected return and the minimum required return on equity are substantially synonomous are presumably that investors

(a) require the return which they expect to obtain, or

(b) bid up the price of securities expected to yield more than is required until their value is such as to yield only the return which investors require.

It has been shown, however, that if a number of acceptable projects of similar risks have IRRs of varying magnitudes, the required return, k_e, must be less than the expected average return, k_a, because k_e is a marginal rate whilst k_a is the average of expected rates above the margin. Therefore, investors cannot eliminate the disparity between k_e and k_r simply by raising k_e to the level of k_r, because, since k_e is the investment cut-off rate used by management, all projects with IRRs less than k_a ($= k_r$) would be rejected, resulting in a new k_a which was again greater than the new k_e, a process which would continue until k_e = the IRR of the single most profitable project in the class, and a major recession takes place.

The relationship between the ex-ante and the ex-post returns on equity securities is illustrated in Figure 1. P_jM_j = the total set of projects in risk class j available in the market. For simplicity, to begin with, the problem of leverage is ignored, and each firm engaged in the production of projects in risk class j is assumed to be all-equity financed such that the required minimum return, and the long-run ex-post return on the firm's equities equal respectively the required minimum return, the expected actual return and the long-run ex-post return on the firm's productive investments. If $p_jk_e^j$ = the set of acceptable projects in risk class j, then k_e^j = the minimum acceptable rate for projects of the class. If some projects in class j have IRRs k_e^j as is implied by the shape of the curve P_jM_j, then the average ex-ante IRR of

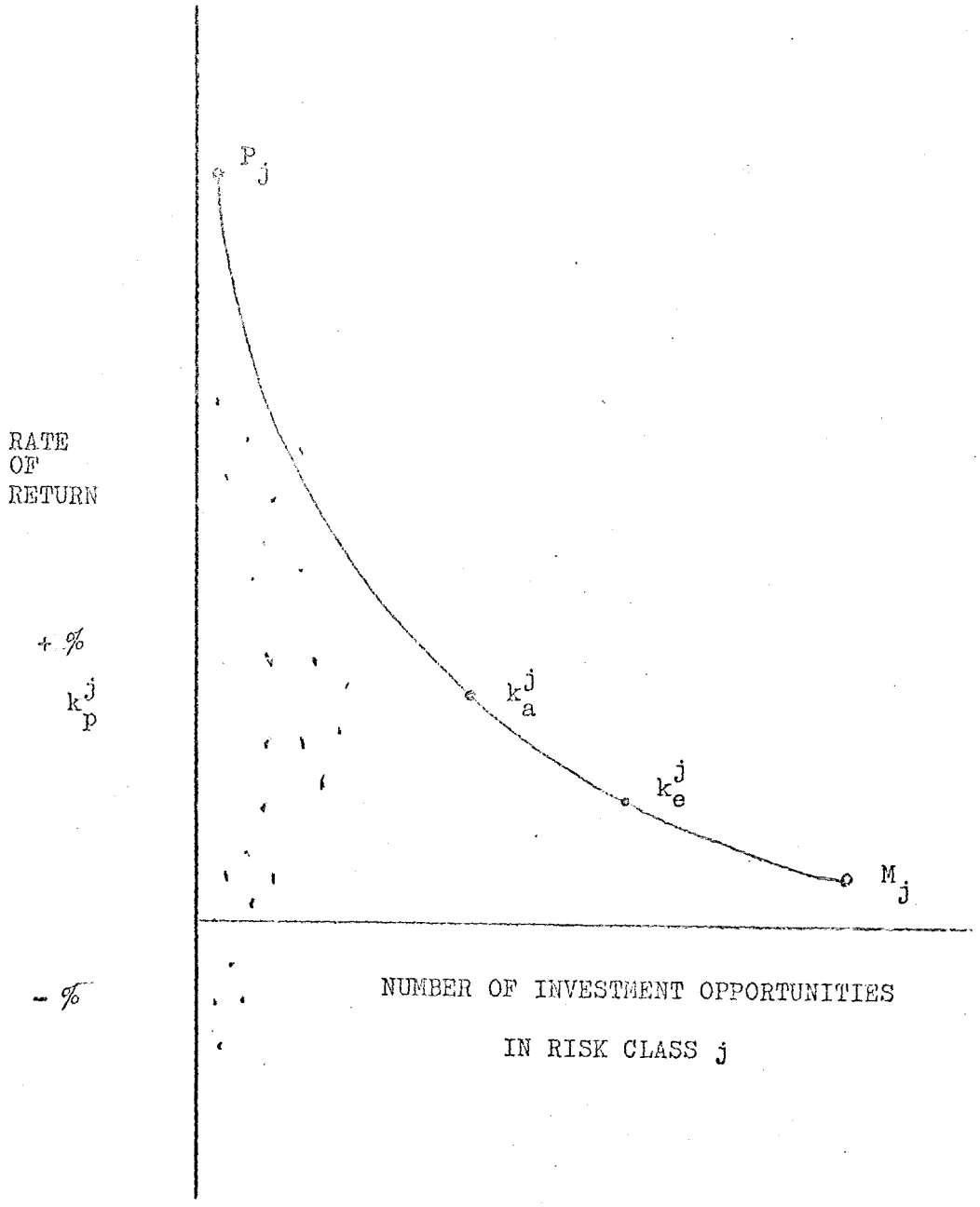
RATE
OF
RETURN
+ %
k_p^j
- %
P_j
k_a^j
k_e^j
M_j
NUMBER OF INVESTMENT OPPORTUNITIES
IN RISK CLASS j

FIGURE 1

the total set of acceptable productive investments, k_a^j, will be greater than k_e^j, and, if as is likely, assuming no systematic bias in the cash flow estimates at the corporate level, $k_a^j = k_p^j$, where k_p^j = the average ex-post rate of return on productive investments of class j, then k_p^j will also be greater than k_e^j. The ex-post rates of return (of which k_p^j is the average) are denoted by the dots. The expected cash flows from the shares of all firms in the class will be discounted in the market at k_e^j.

If $k_p^j \neq k_a^j$, an analysis of ex-post returns will not indicate anything except to give an incorrect estimate of k_a^j. If k_p^j is assumed to be equal to k_a^j, then an analysis of k_p^j will yield a value for k_a^j but indicate nothing of k_e^j unless the relationship between k_e^j and k_a^j is known. If it were true that investors related their required returns directly to the performance of shares in the past, then k_e^j would be raised to the level of k_p^j with the result that k_a^j would be located at a higher point on the curve P_jM_j a process which would continue until the values of k_e^j, k_a^j and k_j^p coincided at the highest point on the curve, implying (1) a continually declining aggregate supply of investible funds as the prospective rate of return increased or (2) the disappearance of the imperfect competition which made it possible for the curve P_jM_j to be other than horizontal.

It follows, then, that the minimum acceptable rate for a given risk class will, on average, be less than return expected to be achieved. If a similar analysis is carried out for all risk classes in the market, then it could be shown that the weighted average ex-post return of all classes would be less than the weighted average ex-ante required return for all classes.

Let us assume for the moment that the cost of capital is constant, that the market is not volatile, and that all earnings are paid out in dividends. Apart from increases in the amount of capital invested from time to time arising from new savings, the value of the market portfolio will rise periodically as a result of the increases in wealth represented by the NPVs associated with new technology etc. If we look back to try to find out the return required by investors, this could be achieved by relating the average earnings to the average market value of the portfolio. For this purpose the increase in market value of the portfolio resulting from the generation of new wealth could be treated as an investment of new capital rather than as a windfall return to the preceding portfolioholders. In effect, an investor's required return would never be earned in the form of a capital gain but only in dividends. All capital gains would be surplus returns.

In practice, however, the cost of capital is not constant, companies do not pay out all earnings in dividends and share values are in fact volatile. Therefore the distinction between the returns which are required by investors and the returns which are surplus to their requirements is blurred and cannot be discerned from a retrospective analysis. An investor's actual returns are in the form of dividends and capital gains, and there is no way of being able to identify the capital gains brought about by the reinvestment of earnings and the capital gains created by surplus wealth (NPVs).

It follows from this that the traditional practice of measuring past equity returns to learn what equity holders required as their minimum acceptable return will lead to biased estimates because it ignores the fact that in a dynamic economy the realised returns from equities will include periodic wealth increments.

If the capitalist system had evolved differently it might have

been that when any wealth increment arose it would have been divided proportionately between equity debt and labour, with the result that debtholders would obtain periodic windfall increases in their wealth surplus to their required return. In practice, however, the shareholder wealth-maximising rule implies that the excess goes entirely to the residual owners, the shareholders. Economists have for long regarded this entitlement to all excess returns as the reward for risk-taking, but it is clear that this cannot be correct if the NPV by definition is the accretion to wealth after discounting at a rate which includes the risk-adjusted return required by shareholders. Therefore it follows that if the required return for equity is the same as that for debt, the average realised returns for equities are likely to be greater than the required and realised returns from debt.

Share prices and +ve NPVs

It has been suggested by a colleague that the disparity between the ex post yield from equities and the ex ante minimum required return would be eliminated in the market if the price of securities were bid up to take account of all future surpluses. In this section, however, it will be argued that

(1) even if this bidding-up process took place it would not alter the fact that over the life of the company the suppliers of the equity capital would on average earn more than their required minimum,

(2) if the bidding-up process took place as a once-and-for-all financial event the effect would be to make the classical wealth-maximising goal an illusory objective, since all future wealth would be impounded in the current price of shares.

A distinction has to be made between specific surplus returns which are foreseeable consequences of a firm's current and planned

operations and the nonspecific surplus returns which, in an expanding economy, are expected to arise in the future, but which cannot yet be associated with particular firms. Investors will bid up the shares to discount specific surpluses, but the extent to which future non-specific surpluses are included in the price of a security is a matter to be resolved between one generation of equity investors and the next. If the price of the market portfolio of equities were bid up so that all future nonspecific surpluses were discounted in the price, then it is true that, for subsequent purchasers, k_e would be equal to k_r. But there would have to be a point of time in which this anticipation of future surplus returns and the consequent increase in share prices became effective. If the period during which $k_r > k_e$ is to be minimised then the bidding-up process would take place at the inception of the company so that the equity capital subscribed to the new company immediately acquired an enhanced market value, reflecting the share of surplus returns which the company <u>might</u> earn in the future. For subsequent purchasers of the shares k_e would equal k_r, but for the original subscribers, k_r would exceed k_e by a significant and immediately realisable amount. Thus if, at the time of the issue of the shares of the average firm in class j all future expansion programmes by the firm are foreseen by the market, together with all associated future NPVs, and if the expected rate of return from the firm's is k_a^j, then the market value of the shares will be bid up by the factor $\frac{k_a^j}{k_e^j}$ such that k_e^j will equal the ex-ante and the ex-post return for subsequent purchasers of the shares. But the return on the capital invested in the firm by the original subscribers to the equity, I, will be, on average,

$$\frac{k_a^j}{k_e^j} \times k_e^j = k_a^j, \text{ plus the immediate windfall of}$$

$$I \times \frac{k_a^j - k_e^j}{k_e^j}$$

Therefore the proposition remains fundamentally true that over the life of the average company $k_r > k_e$. What is less obvious is whether that inequality persists throughout the life of the company, or whether it exists for one generation of investors only. If all future NPVs were impounded in existing share prices this presumably would mean that every company in the market was credited with its potential share of future growth given that the specific companies which in time will generate the NPVs cannot be identified. The objective of wealth-maximisation in the portfolio context would cease to have its accepted meaning. The object of business enterprise would be simply to fulfil the expectations of the first generation of equity investors. As each new project became specific, the rise in value of the relevant company would be matched by a corresponding decrease in the value of all other companies so that the total value of the portfolio remained unchanged. In addition, with each new company that is formed there would, it seems, be an immediate accretion in value representing its potential share of future growth matched again by a proportional decrease in the value of existing companies. It would follow that because all the benefits from the operation of the NPV rule accrued to the first generation of equity investors, there would be a definite bias in favour of forming new companies to finance expansion in order to avail of the immediate enhancement in value. However, if part of the attraction of investing in a portfolio of equities is to participate in the future growth of the economy, it is unlikely that the second and subsequent generations of investors would be prepared to pay a price which

discounted all future growth. As we will see from the empirical evidence in the next chapter, there is good reason to believe that the participation in growth is an ongoing process rather than a once-and-for-all phenomenon.

The minimum acceptable rate of return

Let us assume that the default-free rate is 10%, and that equity investors are prepared to accept it as their minimum return. It has been shown that if all firms seek to achieve 10%, or more, from their assets, the probability is that, on average, they will earn more than 10%. If experience showed that the average return from all assets is, say, 12%, then it has been argued that, although each firm with projects yielding more than 10% will be valued so that the yield on the firm's securities equals 10%, the portfolio of market securities will not be valued to discount all future excess returns which might be earned because this would lead to the creation by arbitrageurs of new companies to undertake each subsequent expansion. But if the expected average return from the market portfolio of securities is 12%, should not a firm use this rate as the minimum acceptable return from its new investments, since that rate of return could be achieved by investing in the market portfolio? Why should the firm accept a project yielding 11% on the grounds that its yield exceeds 10%, the cost of capital, if 12% can be attained from the market? And if it should not accept the project, does not the market return effectively become the relevant criterion of acceptance? When presented with the apparently simple choice between 11% and 12%, then, of course it is difficult to be persuaded that anything lower than the 12% could be acceptable. But, in fact, the choice is not a straightforward one of 11% or 12%, because the two rates do not mean precisely the same thing. By accepting the project offering 11%, the value of

the firm should rise by the amount of the +ve NPV which the project generates, where the cash flows are discounted at 10%. This rise, if not immediate, should occur once the economic significance of the project is communicated to the market. Thus, whilst the 11% is the average return over the life of the asset, the element of the return which is excess to the required minimum is translated into an immediate increase in wealth for the existing shareholders. The 12% from the market portfolio, however, is an average expected return, but no more than that. It comprises the 10% required minimum return which is expected to be attained from existing and predictable projects, plus the 2% excess return which is the average, expressed in percentage terms, of the benefits produced by the series of NPVs expected to arise in the future, when new projects are undertaken and become identified with specific companies. But there is no immediate increment in wealth created by an investment in the market portfolio. The 12% arises because various companies, as yet unidentified by the market, are expected, in the future, to undertake projects yielding 10%, 11% 12%, 13% etc. and which have an average yield of 12%. Projects offering 11% are as much an integral part of the expected average of 12% as projects offering 13% or more. By rejecting the project yielding 11%, the firm is rejecting the opportunity to increase the firm's wealth in the immediate future. Thus, if a project costing £1,000 were expected to produce a perpetuity of £110, the present value of the project would be £1,100, increasing the firm's worth by £100. The shareholders could, if they wished, sell their shares and invest in the market portfolio to achieve an expected return of 12%, that is, a perpetuity of £132, compared to the £120 which they would have obtained had the original capital of £1,000 been invested directly in the market portfolio.

A company which has hitherto succeeded in achieving the average

market return of 12% may well seek to continue to achieve or improve upon that rate, but nonetheless would be misguided in making that target the minimum standard of acceptance, and so rejecting projects which happen not to yield 12% or more. The use of return-on-capital criteria as a guide to investment decisions is known to favour the rejection of marginally profitable investments because of their adverse effect on the average rate of return. (11) Indeed, to use the percentage return of previous projects as the minimum acceptable return for future projects could be disastrous for a company which was fortunate enough to find an initial project which happened to yield 25%. The minimum standard, therefore, is the cost of capital irrespective of how successful the company has been in the past or expects to be in the future in earning more than the minimum. It may be counterintuitive to accept that shareholders may expect 12% when they require 10%, or that firms can have a minimum cut-off rate for new projects of 10%, but have an expected average return of 12%. But when one bears in mind the import of the NPV rule, and the fact that it amounts, in effect, to a mandate issued by the suppliers of capital to the firm's management, advising them that 'our minimum required return is 10%, therefore, do not accept any projects unless you expect to earn more than that rate,' it becomes less difficult to accept that a disparity between the required minimum return and the expected average return for both real investments and equity securities is other than a logical and virtually inevitable consequence.

(11) See Solomons, for example, pp. 61-62.

SUMMARY

The significance of the positive net present value concept has been examined and it was contended that if companies pursue the NPV rule there must inevitably be a disparity between the minimum acceptable return (the cost of capital) for new investments and the average realised return from equities. This leads to the important conclusion that the conventional practice of measuring historical returns from equities both at the individual and the market level to estimate the minimum return required by equity investors is ill-founded. It cannot therefore be assumed because equities have earned on average a higher return than that earned on bonds that equity investors had a minimum acceptance return higher than that of bonds.

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Chapter Four

THE COST OF EQUITY CAPITAL 1

In the last chapter we came to the important conclusion that the cost of equity cannot be discerned from past equity returns, not simply because the past is an unsatisfactory guide to the present but because past realised returns are average returns and the cost of equity is a minimum return. If we cannot use the past as a guide it becomes a very difficult, if not impossible task to decide what the cost of equity actually is. We know that, in general terms, it is held to be equal to the rate of interest on debt plus or minus a premium or discount for risk. It is only in recent years that the possibility of a risky project meriting a discount rate less than the risk-free rate has been recognised, by virtue of the fact that in the context of the capital asset pricing model, if a project's returns, however risky in themselves, are negatively correlated with the returns of the market portfolio, the project has the effect of reducing the riskiness of the portfolio, a feature which should, it is argued, be reflected in the discount rate. As a result, the cost of equity is generally defined as the market rate of interest plus or minus some adjustment for risk, although the magnitude and direction of the adjustment is a matter of difficult judgment, and in practice, it appears, a matter of arbitrary rule or guesswork.

In this chapter it is contended that rather than accept as axiomatic the need for corporate managers to undertake the task of estimating the cost of equity, it must first be demonstrated that there are material benefits to be derived frommaking any adjustment at all to the market rate of interest. It has already been pointed

out that since the market rate of interest itself changes weekly if not daily, then the theoretically correct cost of equity can be assumed to change at least as frequently given that the magnitude of the adjustment to the rate of interest is itself susceptible to changes in investors' psychology and perception of risk. Hence, however difficult a task it might be to compute the cost of equity, the inherent logic of the concept would require that the computation be carried out frequently if the discount rate derived is to be of relevance to current economic conditions.

There are three reasons for proposing that practitioners should disregard the risk premium concept when selecting investment projects, and need make no adjustment to the objectively determined and readily observable market rate of interest:

(1) Equity securities are not materially risky in a portfolio context

(2) Government bonds are not in fact free of risk but are subject to money and purchasing power risk

(3) Equity investors appear not to be as averse to volatility as theoreticians assume, and may have a 'risk-tolerance' level within which a measure of volatility is acceptable without a higher expected return.

The Market Portfolio

If we compare a highly volatile project with a government bond, few would gainsay that the former is significantly riskier than the latter and that some measurable difference in expected return would be required by most investors. But, although the risk premium hypothesis was first advanced at a time when this kind of perspective

was considered to be the appropriate one, in modern capital budgeting theory the individual risk of the project is no longer regarded as relevant. For the difference in required returns to be material it must be shown that the volatility of an efficiently constructed portfolio of risky assets, including the project under review is, material. But what is that portfolio? Capital market theorists refer frequently to the 'market portfolio' and for the purpose of their statistical studies this usually means the N.Y.S.E. or the U.K. market. But it is now recognised that shareholders' capacity to diversify, if necessary through the medium of investment trusts etc., extends beyond a single nation's securities market, and indeed beyond financial securities into other media.(1) In practice there is little difficulty in divising and operating a portfolio which transcends international boundaries and includes several investment media as proposed by Cohn and Pringle(2) and others. If a portfolio of risky assets is capable of including U.S., U.K., Canadian, Australian, German and Japanese stocks, together with investments in land, even the investor's private home, it is the volatility of such a portfolio which is relevant. The problems of specifying the components of an international, multi-media portfolio and then of measuring its riskiness are, of course, substantial. For example, it will be argued in chapter 7 that the relevant 'market' portfolio for measuring undiversifiable risk should include corporate bonds in the proportion in which they are issued by the respective companies. But, in order to justify the proposition that the minimum return required from this portfolio should be clearly distinguished from the default-free interest rate, it would need to be a reasonable assumption that

(1) C. Robichek, Cohn and Pringle, 1972
(2) Cohn and Pringle, 1973

the nondiversifiable risk of the portfolio was significantly greater than the purchasing power and money risks of the default-free security.

Inflation and the relative riskiness of bonds versus equities

A Government bond cannot be classified as risk-free if inflation is taken into account. Conventional measures of variability, however, are computed in relation to cash flows which are unadjusted for changes in the purchasing power of money and, therefore, ignore a significant source of risk. If it can be assumed that a portfolio of 'risky' assets is to some extent protected from the consequences of inflation and that the real rate of return on fixed-interest securities is variable with the rate of inflation, then, unless the expected cash flows of each are expressed in a common monetary unit, the stability of 'risk-free' securities and the variability of 'risky' assets will both be overstated.

Assume, for simplicity, that all variability risk can be diversified away, and that only inflation risk remains. Assume also, for the purposes of illustration, that the market portfolio of equity securities is perfectly correlated with the price index. Given a net-of-inflation required rate of return of 4%, a nominal 'risk-free' rate of 10%, and that the expected change in money values is 0.5 probability of 4% and 0.5 probability of 8%, then the expected monetary returns from the equity portfolio would be a 0.5 probability of 8% and a 0.5 probability of 12%(3) and from the 'risk-free' asset a 1.0 probability of 10%. But the expected real net-of-inflation returns for equities would be 1.0 probability of 4% and for the

(3) More precisely these rates should be (4% + 4% x 1.04) = 8.16%, and (8% + 4% x 1.08) = 12.32%.

risk-free security a 0.5 probability of 2% and a 0.5 probability of 6%. Conventional measures of volatility would classify the equity portfolio as risky and the government bond as riskless. When measured in a stabilised unit of purchasing power it is the equity portfolio which is riskless and the government bond which is risky. The implication therefore, is not only that the framework in which the risk premium hypothesis has developed, that is the standard Capital Asset Pricing Model, is in need of respecification[4] to take account of purchasing power risk, but that by ignoring inflation the market price of risk has been overstated in the traditional model. The subject of inflation will be taken up again in chapter 10.

Investors' attitude to risk

We have argued that unless it can be shown that the variability risk of an international portfolio is materially greater than the purchasing power risk of a Government bond, the required return from the market portfolio can be reasonably assumed not to be distinguishable from the yield of the latter. This would imply that the average risky project with a variability similar to that of the market should be discounted at a rate equal to the Government bond yield. But it would be arguable within the framework of the Capital Asset Pricing Model that nonaverage projects with variability characteristics significantly different from those of the market should be discounted at a higher or lower rate. This however presupposes that investors' aversion to risk extends indiscriminately to all levels of volatility and that investors are not prepared to accept any degree of volatility without a higher expected return. We will now examine the conventional risk-aversion hypothesis which it is contended is based on a fairly

(4) see J. Lintner 1975, page 278

simplistic view about investors' attitudes to variability. An investor knows his portfolio will vary in value over its lifetime, as will the value of the default-free stock. It may, in fact, never rise above the price he paid for it, but unless his time-horizon is very short or unless he has no control over the date of disposal, he will not regard every possible depression with proportionately equal distaste. His attitude to a projected series of volatile movements in market values is likely to be related to his time-horizon and investment latitude, varying from volatility-aversion when subject to short time-horizons and inflexible disinvestment opportunities, to volatility-partiality when operating with longer time-horizons and flexible disinvestment opportunities. Investors who have no tactical latitude are unlikely to be attracted to the equity market even with a risk premium and will presumably seek fixed-interest securities which match their maturity preferences. Those who are attracted to the equity market presumably expect to be able to exercise some control over the timing of their exit from the market. But the conventional approach to measuring the riskiness of a portfolio of equities is to give every market movement equal weighting, implying that the probability of the portfolio falling in value to a particular level at a particular time is equal to the probability of the investor having to sell his portfolio at that level and time, and does not allow for the latitude which most investors undoubtedly expect to have in being able to ride many of the bad patches and of being able, to some extent, to select the point of his departure from the market. Indeed, if one portfolio is predictably more

volatile than another, it may be for that reason as attractive to the aggressive investor who is free to choose when to sell, as it would be unattractive to the investor who lacks that freedom.

In addition, the widely held assumption that investors require a higher expected return for a higher level of risk may be reasonable for significant levels of risk, but unrealistic in relation to the volatility of a well-diversified portfolio of assets. It is sometimes implied that because investors insure their car at a costly premium, despite the fact that the car may have a lesser value and less risk than their security portfolios, is evidence of an aversion to risk. But this presupposes that investors do not distinguish between the kind of situation which typifies an insurable event, that is, an event which is capable of involving a once-and-for-all, substantial and irrecoverable loss, and the variability-over-time pattern which characterises the cyclical movement of the world's securities markets. Hirshleifer notes(5) that the existence of gambling at unfavourable odds requires an explanation in the face of the risk-aversion assumption underlying the theory of choice amongst risky assets. He observes that gambling on a scale likely to impoverish is rarely encountered in the middle or upper classes, and that, except for the poor, it is more "like a consumption good than an investment good - an activity engaged in for pleasure rather than with the intention of changing one's wealth status."(6) Pleasure-oriented gambling exists all along the income scale and is a

(5) J. Hirshleifer page 230
(6) Ibid, page 230

potential source of enjoyment which will be normally "characterised by repetitive miniscule bets practically guaranteed not to drastically change wealth status." He concludes that because such an activity can be distinguished from an investment or wealth-oriented decision, the latter can be represented by a concave utility function, implying an aversion to all levels of risk.(7) If, however, Hirshleifer's observation that gambling is more like a consumption good than an investment good is conceded, it does not follow that the two goods cannot coexist in the one activity. The wealth-oriented impulse may cause an investor to construct his equity portfolio so as to minimise the possibility of a drastic reduction in his wealth, whilst the pleasure-oriented impulse may cause him to be tolerant of, if not positively sympathetic to, a measure of volatility over time. The individual investor may not compartmentalise his pleasure and wealth-oriented activities as implied by Hirshleifer in the sense of being willing to assume some risk at unfavourable odds in one context and of being unwilling to accept any risk without favourable odds in an other. Indeed, for many, the securities market may be the most satisfactory if not the only outlet for the pleasure-oriented impulse, by providing a superficially more rational and socially acceptable context than the tables of Las Vegas. The success of government lotteries testifies to the willingness of investors to accept an investment with a very high probability of a negative net present value for the sake of obtaining a very low probability of a high net present value, with an expected average return, if anything, marginally less than the market rate of interest.

(7) Ibid. page 231

Paradoxically, therefore, the complexity of the decision-making process of choosing between risky investments may have been magnified by an oversimplified specification of investors' preferences and behaviour patterns. A greater recognition of the complexity of investors' motivations and preferences might, it is argued, justify a more neutral status for the variability of returns and lead to a simpler decision rule, one in which the expected return alone could be perceived as the primary criterion.

It is certainly not a firm foundation for the conventional risk-premium hypothesis to assume, in the face of a widespread acceptance of risk in one context for an actuarial return which is not only less than the risk-free rate but which is actually negative, that investors in securities are unwilling to tolerate a moderate degree of volatility from their portfolios for a return which on average is not expected to be less than the default-free rate.

The intuitive basis for the risk premium concept

There is undoubtedly a strong intuitive appeal for the risk-premium concept and the desirability of modifying the required return to suit the risk characteristics of the individual project. There is also presumably a widespread belief that however difficult it might be to put the theory into practice, management's judgment and experience can be depended upon to make a reasonably satisfactory attempt. Indeed, at no time since the risk-premium approach was first conceived could it be said that its implementation was not

heavily dependent upon managerial judgment. However the scenario in which that judgment was traditionally expected to operate has been significantly changed by the developments which we have observed to have taken place in capital budgeting theory. When one proceeded from the assumptions first that there existed a truly risk-free rate which could be observed in the market and secondly that it was the total risk of a project as it appeared to the manager in isolation from projects outside his company which was the relevant risk, it might be a reasonable expectation that management could operate the risk-premium approach relatively effectively. The manager could be confident, for example, that if the project was materially riskier than the risk-free security he should make an upward adjustment to the risk-free rate. In addition, if one project was clearly riskier than another, the upward adjustment for the former should be greater than that of the latter. His main concern would be to determine the magnitude of the adjustment. The validity of this relatively straightforward intuitive framework has however been significantly undermined by the developments discussed in this chapter, namely in portfolio and capital market theory and the recognition that government bonds are subject to purchasing power risk. To begin with, a much more penetrating perspective is needed to estimate the size of the adjustment to the market rate in a portfolio context. But, more important, the correct direction of the adjustment, upwards or downwards, is now uncertain. Rather than use as the base rate some notional risk-free rate which cannot be observed in the market, it is clearly preferable to proceed from the government bond rate which is objectively determined and readily observable.

We have argued that because of inflation and because of shareholders' capacity for diversification, the required return from the market portfolio must be closer to the government bond yield than has been traditionally assumed, and that if they are broadly the same, it would imply that average run-of-the-mill projects should be discounted at the government bond yield, leaving only non-run-of-the-mill projects to be discounted at an adjusted rate to satisfy the risk premium framework. But such adjustment is as likely to be downwards from the bond yield as it is to be upwards. It follows that the intuitive basis for measuring the price of risk has effectively been destroyed, and that whilst the absolute significance of project risk has materially declined, the scope for error in operating the risk premium refinement has increased significantly.

In the next chapter we will consider the evidence to support the claim that the derivation and application of differential discount rates by managers are not an essential prerequisite of effective present value analysis and that the benefits of simplicity are too great to be traded for a refinement of doubtful practical utility.

SUMMARY

In this chapter we questioned the validity of the risk premium hypothesis in a portfolio context with specific reference to the cost of equity. It was argued that since only the proportion of a project's risk which cannot be diversified away is relevant, it is far from apparent that the relevant risk of an internationally diversified portfolio comprising corporate bonds and equities and other risky assets is significantly greater, and is perceived by investors to be significantly greater in real terms than the money and purchasing power risk of a default-free security.

In addition, it was contended that the conventional risk-aversion hypothesis is postulated on a naive and simplistic description of investors' attitude to volatility, which states that investors are unwilling to accept any level of volatility without a measurably higher expected return. The concept of a risk-tolerance level was introduced which is compatible with investors' observed insurance and speculation behavioural patterns, and which proposes that within certain limits of volatility investors may not make fine distinctions between different risk-return combinations.

It was also noted in the previous chapter that investors have an incentive to select the market portfolio in preference to a default-free bond without any risk premium, given the nature of the +ve Net Present Value rule which directs management to select only projects which are expected to yield more than the minimum needed to satisfy equity investors.

Finally, it was noted that the intuitive basis for the traditional risk-premium approach in which it could be assumed that all risky projects should have a minimum return higher than the yield on bonds and that more risky projects should have a higher minimum return than less risky projects has been undermined by the implications of modern capital market theory and the recognition of purchasing power risk. It follows that implementation of the approach places considerably more demands on managers and has a much higher susceptibility to error than is generally assumed.

In the following chapter we will examine the empirical evidence relating to the risk premium concept and present data which raises sufficient doubt about the materiality and even the existence of a market premium for risk to justify the expediency that management should use only the pure time value of money to evaluate risky projects.

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CHAPTER FIVE

THE COST OF EQUITY CAPITAL II

The empirical evidence

The required returns on equity securities cannot be directly observed in the market. One approach has been to study the ex post returns from shares to find out what previous required returns have been, on the assumption that realised returns are good estimates of ex ante expectations, but this approach has been observed to suffer from the fundamental defect that it presumes the kind of market conditions in which the +ve NPV rule would have no practical validity. The realised returns from a share include dividends and capital gains, and since there is no available method of distinguishing between the proportion of the returns which are needed to meet shareholders' minimum requirements and the excess returns which arise at each phase of expansion, a retrospective analysis will at best indicate investors' expected average returns and not their required minimum return. A number of statistical studies, in particular those of Ellis(1) Blume and Friend(2) and Norgaard(3) have revealed that the difference between the realised returns on equities in the New York Stock Exchange and the yield on corporate bonds was much greater than could be accounted for by the risk differential. This difference between the certainty-equivalent bond yield and the certainty-equivalent equity yield mirrors the disparity between

(1) C. Ellis, chapter 3
(2) Blume and Friend, 1973
(3) Norgaard, 1974

the minimum acceptable yield for equities and the realised yield. Moreover, according to Norgaard's study, the unwarranted difference between bond and equity yields (an average of 5.5% compound yield) remained relatively stable throughout the period 1926-1969, suggesting that the higher returns accruing to equities by virtue of the NPV rule are not entirely discounted in the price at the company's formation but are spread over the company's life as the events which give rise to the +ve NPVs occur and become known to the market. In each of the above studies, the authors concluded that there was a positive advantage for investors to choose a portfolio of equities rather than a portfolio of bonds. One explanation suggested was "segmentation of markets between stocks and bonds"(4) but this would imply that the equity required return was independent of the bond yield, a conclusion which would undermine the foundation of the 'risk-free rate plus risk premium' hypothesis. The present study finds the +ve NPV effect a more defensible explanation, implying as it does that the minimum equity return is directly related to the bond yield. The significant implication of both explanations, however, is that the risk premium, if any, appropriate to equities cannot be defined as the difference between the risk-free rate and the average return earned by equities.

Before examining the available evidence relating to risk premiums, it is necessary first to develop further the concept of a risk-tolerance level introduced in the previous chapter in order to understand how the nonexistence of a risk premium is quite compatible with the existence of risk.

(4) Blume and Friend, 1973, page 31

The context in which risk premiums are determined according to the conventional capital asset pricing model is depicted graphically in Figure 1. The shaded area represents the opportunity set containing all possible combinations of risky assets. Only portfolios lying on the curve PMQ are efficient in the sense of dominating all other portfolios in the opportunity set. However, if investors can lend or borrow at i, then everyone will hold the same portfolio of risky assets, M, the point of tangency with the straight line drawn from i, and will supplement their holding in portfolio M, by borrowing or lending at i to achieve a combination which lies somewhere along the capital market line. Therefore, the investor whose indifference curves lie to the left of M will hold both the risk-free security and portfolio M, while the more aggressive investor whose indifference curves lie to the right of M will place all his capital in portfolio M, having borrowed at i to augment his holding. The significant fact is that both investors will hold the same portfolio of risky assets, i.e. the market portfolio containing all risky assets in exactly the proportion they are supplied in the market in equilibrium.

The shape of the curve PMQ results from the assumption that the market is composed of risk averse investors who positively trade off risk for return, and the slope of the CML is a function of the market's price for risk. In perfect market conditions, where returns in excess of the required minimum would not be available, and where a zero rather than a +ve NPV rule would operate, the angle of the CML slope would depend solely on the distance of M from i, and investors'

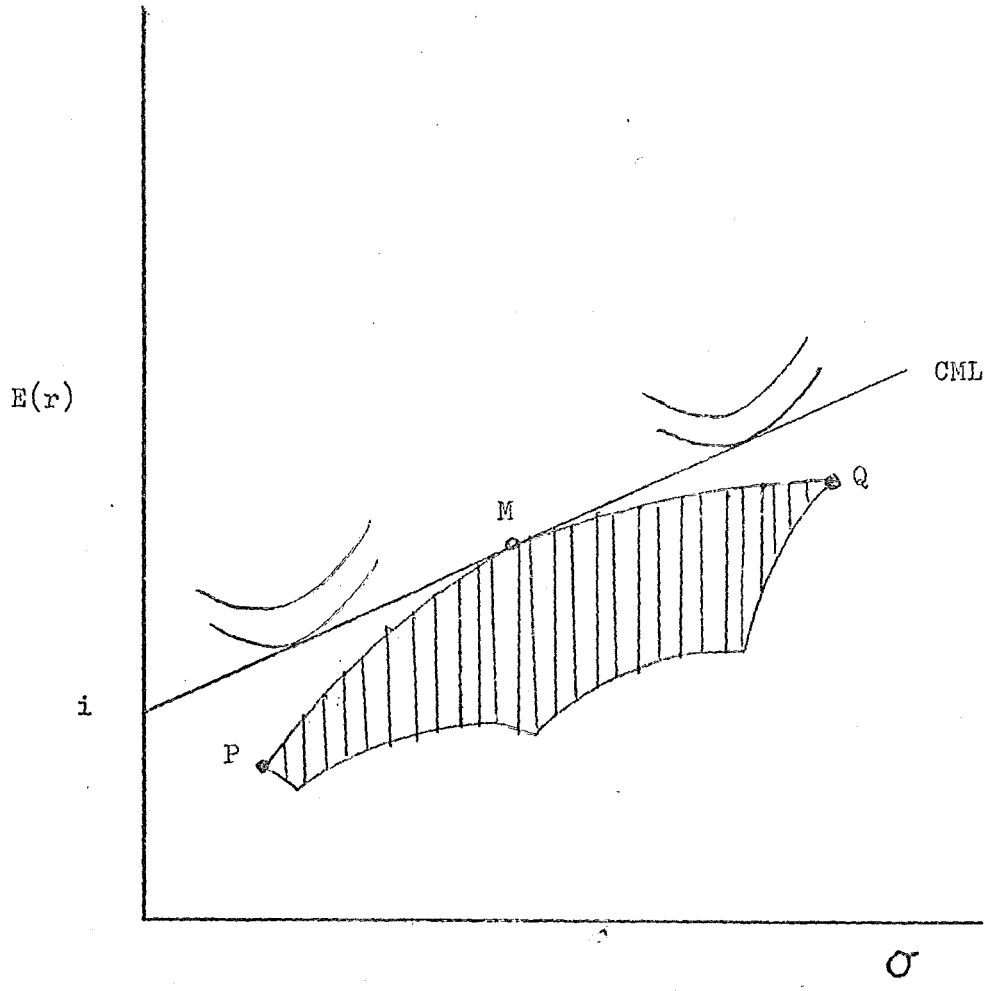

FIGURE 1

The opportunity set in risk-return space

attitude to risk. If the distance between M and i is insignificant, that is if most risk can be diversified away, then whatever the attitude of investors to risk, the CML would be approximately horizontal. If the distance between i and M is not insignificant, the slope of the line will depend on investors' attitude to volatility.

In the previous chapter, it was argued that because fixed-interest securities are especially vulnerable to purchasing power risk, and because of the enhanced opportunities to diversify on a world-wide basis, investors are unlikely to perceive the systematic risk of the market portfolio as being significantly greater than that of a government bond. It was also argued that even if investors were able to distinguish between the risks of the portfolio and the bond, the incremental risk is not necessarily at a level to which they are averse. In conventional utility analysis, investors are generally assumed to fall into one of three main categories with respect to risk. Thus Figure 2 represents the utility functions for a risk-averse, risk-neutral, and risk-loving investor respectively. For the least degree of variance, the risk averse investors is assumed to demand a higher return to compensate him. The risk-lover, on the other hand, is prepared to suffer a fall in his expected return in order to have the opportunity to be exposed to variance. The risk-neutral investor is concerned with expected return only, and is indifferent to the variability of the returns.

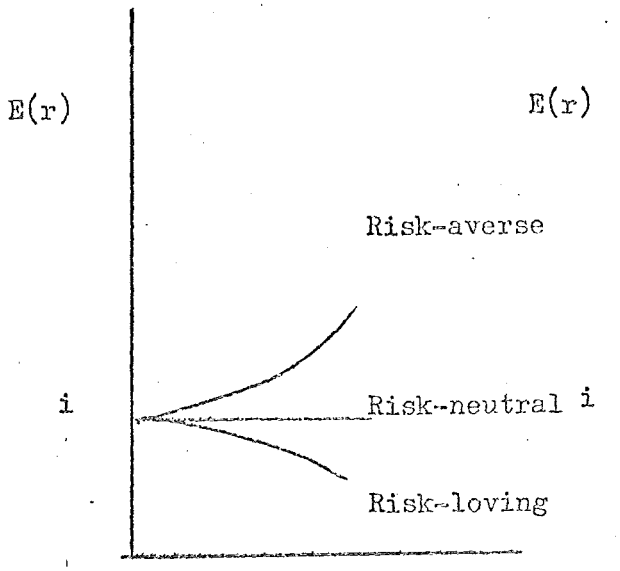

FIGURE 2

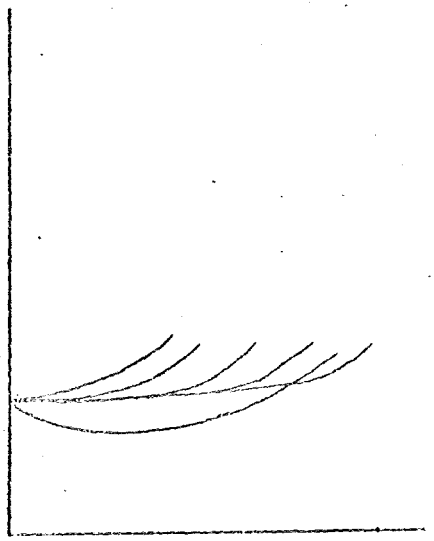

FIGURE 3

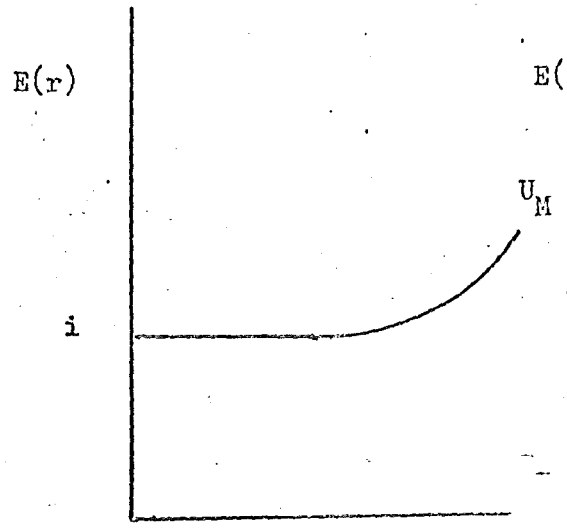

FIGURE 4

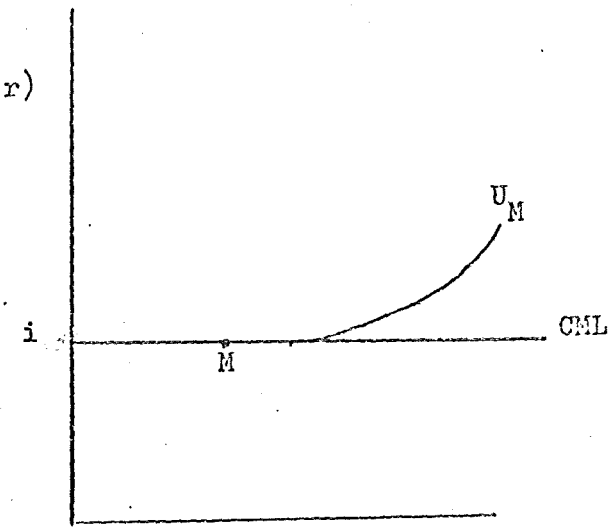

FIGURE 5

It has already been argued that because it is an observable fact that many investors insure against risk on the one hand, and take part in lotteries on the other, the categorisation of investors as either exclusively risk-averse or risk-seeking is unrealistic and that most investors display a tolerance if not a possible liking for risk up to a certain level, and thereafter become increasingly averse to it. The level of risk tolerance will vary with each individual according to his initial wealth and temperament, ranging from nil tolerance for the very timid to a high level of tolerance for speculators. The majority could be expected to fall somewhere in between. Thus Figure 3 depicts the range of indifference curves which investors with ambivalent attitudes to volatility might have, with Figure 4 representing the average market curve. The flat-bottomed shape of the market utility curve does not negate the general substance of the conventional risk hypothesis. It simply denotes that whilst people generally insure against the possibility of major losses, they are prepared to undertake and sometimes even seek some variance without any specific compensation other than the challenge and satisfaction of participating in the dynamic of the market.

The necessity for having a risk premium for even the riskiest of assets depends on whether the level of variance at which the market as a whole ceases to be neutral is greater than the variance of the portfolio of risky assets. Graphically, it depends on whether the point where the market's utility curve begins to rise in Figure 4 is the left or right of M. Therefore, two conditions

must be met to justify a risk premium:

(a) the distance between M and i must be significant, and

(b) the horizontal section of the market utility curve must not extend beyond the point M.

If neither of these conditions is met, the CML will be horizontal as in Figure 5.

It is not necessary that each individual investor has the same degree of risk tolerance. Those who have a low tolerance level but nonetheless wish to purchase a portfolio of risky assets may choose a portfolio composed of low-risk, defensive securities whose combined variance is less than M's, and at a level which is tolerable to them. More aggressive investors will be satisfied with a portfolio composed of more volatile securities, even if their portfolio is not expected to earn on average more than the defensive one. This, of course, contradicts the separation theorem of the conventional capital asset model which states that all investors in risky assets will hold the same portfolio. If there are different levels of risk tolerance amongst investors, then one group may perceive one portfolio as inefficient which another group finds efficient, in the sense that the latter group may derive a positive satisfaction from holding a volatile portfolio even although its volatility could be further reduced without a corresponding reduction in the average expected return. Indeed research has shown that most households(5) do not hold well diversified portfolios, implying that investors have heterogeneous expectations or have a risk-tolerance level which the traditional framework fails to make allowance for.

(5) See Blume and Friend, 1975

To summarise, therefore, it has been argued that in order to challenge the utility of incorporating risk premiums into the capital budgeting framework, it is not necessary to show that nondiversifiable risk does not exist, nor that investors are indifferent to risk. All that is required is to throw doubt on the assumption that the level of nondiversifiable risk which does exist is regarded by investors as being of a significant level.

The market evidence

Although it has been shown that the superior performance of equities is not acceptable evidence that the minimum return required by equity investors exceeds the bond rate, it remains necessary to consider the evidence indicating a risk-return pattern as between one equity security and another.

A number of empirical studies have indicated a relationship between the level of a security's systematic risk, or Beta factor, and its realised excess return. This has been assumed to imply that investors are averse to volatility and that they seek a higher return for those securities which contribute more to the portfolio's volatility.

Two points, however should be noted,

(1) It has been shown that the realised returns on equity securities reflect at best the expected returns from the firms' productive assets, and not the minimum return required on the firms' securities. It may be that the risk-return relationship of firms' productive assets has conformed partially to the traditional capital asset pricing model, causing, in turn, the pattern of the ex post risk-return relationship of the firm's securities to

assume a similar pattern, and so giving a false impression of the ex ante required relationship. Thus, the risk-return tradeoff for one industry as against another will be influenced, in part, by management's perception of risk, and if management generally are more averse to risk than investors are, or if they ascribe a higher degree of aversion to investors than is warranted, then the level of competition in more risky industries will tend to be less severe than in less risky ones. To the extent that some diversification is characteristic of most leading companies in the market, the competition might vary inversely with the degree of systematic risk. The returns from productive assets could then be expected to conform, partially at least, to the traditional asset pricing model, and this could explain the observed pattern of returns on financial assets. If shareholders' required return were less than the cut-off rate used by corporate managers, then the shares of those 'risky' firms would be bid-up as the super-normal profits were communicated to the market. But the overall performance of the shares would, in a retrospective analysis, be superior to that of shares in less risky industries, and this might be taken to imply a higher required return by investors. Yet, without any method of analysing the results so as to identify the proportion of the returns which were excess to shareholders' requirements, it would not be possible to assume the existence of a risk premium let alone measure it.

(2) Whether or not investors have the same level of risk-aversion as managers, one would expect the degree of aversion of both communities to decline as the capacity for both to diversify internationally has increased over the years. Hence, any risk-return pattern derived from observations over long periods will tend to lack relevance for current conditions. The evidence of recent periods will be more relevant than long-term averages, subject to the reservation that during short periods less reliability can be placed upon the evidence of realised returns to estimate ex ante expectations.

The findings of a recent study conducted by Black Jensen and Scholes[6] are of particular importance in this context insofar as they provide a very significant challenge to the relevance of the risk-premium refinement under modern market conditions. The results of the study are summarised in tables 1 and 2.

(6) Black, Jensen and Scholes, 1972

Summary of Statistics for Time Series Tests, Entire Period (January, 1931 - December, 1965)
(Sample Size for Each Regression = 420)

Item*	Portfolio Number										M_M
	1	2	3	4	5	6	7	8	9	10	
$\hat{\beta}$	1.5614	1.3838	1.2483	1.1625	1.0572	0.9229	0.8531	0.7534	0.6291	0.4992	1.0000
$\bar{R}$	0.0213	0.0177	0.0171	0.0163	0.0145	0.0137	0.0126	0.0115	0.0109	0.0091	0.0142

$\hat{\beta}$ = Estimates systematic risk of the portfolio

* $\bar{R}$ = Average monthly excess returns

TABLE 1

SUMMARY OF COEFFICIENTS FOR THE SUBPERIODS

PORTFOLIO NUMBER

Item	Sub-Period ≠	1	2	3	4	5	6	7	8	9	10	M_M
$\hat{B}$	1	1.5416	1.3993	1.2620	1.1813	1.0750	0.9197	0.8569	0.7510	0.6222	0.4843	1.0000
	2	1.7157	1.3196	1.1938	1.0861	0.9697	0.9254	0.8114	0.7675	0.6647	0.5626	1.0000
	3	1.5427	1.3598	1.1822	1.1216	1.0474	0.9851	0.9180	0.7714	0.6547	0.4868	1.0000
	4	1.4423	1.2764	1.1818	1.0655	0.9957	0.9248	0.8601	0.7800	0.6614	0.6226	1.0000
$\bar{R}$	1	0.0412	0.0326	0.0317	0.0272	0.0230	0.0197	0.0166	0.0127	0.0115	0.0099	0.0220
	2	0.0233	0.0183	0.0165	0.0168	0.0136	0.0147	0.0134	0.0122	0.0126	0.0098	0.0149
	3	0.0126	0.0112	0.0120	0.0126	0.0117	0.0109	0.0115	0.0110	0.0103	0.0075	0.0112
	4	0.0082	0.0082	0.0081	0.0087	0.0096	0.0095	0.0088	0.0101	0.0092	0.0092	0.0088

≠ Subperiod 1 = January 1931 - September 1939; 2 = October 1939 - June 1948; 3 = July 1948 - March 1957;
4 = April 1957 - December 1965.

TABLE 2

The statistics cover the 35 years from 1931 to 1965, and consist of monthly returns for each of ten portfolios composed of securities grouped according to their estimated risk category ($\hat{\beta}$), where portfolio 1 contains the highest-risk securities and portfolio 10 the lowest-risk securities. The overall results summarised appear to accord with the conventional theory that the excess returns should increase with the portfolio's beta factor. However, further analysis by BJS revealed that high-risk securities almost consistently earned less and low-risk securities consistently more than predicted by the traditional form of the asset pricing model.

But of even greater relevance to the present issue are the results summarised in Table 2, which were prepared by BJS to test the stationarity of the empirical relations. The 35 year interval was divided into four equal subperiods of 105 months each, and the results indicated that, although the risk coefficients were fairly stationary, the pattern of low earnings R for high-beta assets and high earnings for low-beta assets became progressively more marked in recent years, until in the latest of the four subperiods (1957-65), the risk or beta factor had no apparent influence whatsoever. Thus note how in Table 2 the very significant conventional risk-return relationship which existed in the 1930s (line 1 of band $\bar{R}$) contrasts with the equally significant stability of returns irrespective of risk in the 1960s (line 4 of band $\bar{R}$) The fact that the average risk-return relationship throughout the whole period 1931-1965, as in Table 1, conformed approximately to the conventional risk-premium theory is irrelevant, because the conditions of the market and the perceptions of investors have necessarily changed since the thirties and the recent period is the most relevant one to current conditions. The fact that the relationship existing

in the last decade is in line with the trend of the whole period also implies that the former is not unrepresentative or anomalous.

It might be asked if this data has been available since 1972 why their significance was not earlier recognised. It must however be remembered that the returns of the equities, albeit indistinguishable amongst themselves in respect of risk, are on average significantly greater than the returns obtained from government bonds, and to any interpreter who has not considered the significance of the +ve NPV effect outlined in Chapter 3 and the necessary disparity between equity realised and ex ante returns, this would appear to corroborate the basic risk premium theory. He would simply search for some other explanation to account for the findings of Table 2.

Thus in their own analysis BJS did not comment on the significance of the reported trends from the aspect of investors' attitudes to volatility. But in private correspondence with Professor Michael Jensen, the writer asked whether he would agree that the results of his study were consistent with the hypothesis that investors' aversion to relevant risk had, over the period, shown a steady decline and that latterly investors appeared to be indifferent to the existing level of undiversified risk and to accept the expected return as the sole criterion of value. He replied that the results were in fact consistent with this hypothesis, although he preferred to seek some other explanation, in view, for example, of the continuing 'demand for insurance'.

It should however be emphasised again that the observation that shareholders appear to be willing to accept the existing level of nondiversifiable risk does not carry with it an assumption, as Professor Jensen seems to imply, that shareholders have become indifferent to risk. It is quite consistent with the proposition that investors are averse to significant risk and are observed to insure themselves against significant risk, to assume that once the significant risks are insured against (in the context of investments by the process of diversification) they are willing to accept the residue of risk.

Therefore, while it is not clear to what extent the results relating to earlier periods reflect only investors' risk preferences or are influenced, in addition, by management's risk aversion, the overall results of the BJS study, nonetheless, strongly suggest that investors have not, in fact, displayed the degree of intolerance to variance predicted by a capital asset pricing model based on conventional theories of risk aversion, and more important, that the trend in recent decades has suggested that nondiversifiable variance is within the risk-tolerance level of investors, and has ceased to play any significant role in the valuation process.

The market evidence to support the risk premium theory in capital budgeting is therefore very weak indeed, and it is unrealistic to expect even financially sophisticated managers to implement it effectively if its very validity is so clearly open to doubt.

In order to gain additional insights into investors' attitudes to risk and to test whether they appear to be consistent with the above interpretation of the BJS findings, it was decided to undertake the more direct and apparently untried method of questioning a sample of investors and inviting them to describe their attitudes.

The Survey

The aim of the survey was to give investors the opportunity to express their attitudes to three aspects of risk:

(1) Whether, taking into account the effects of inflation, investors perceive the variance of the market portfolio as being significantly in excess of the variance of a default-free bond; in graphical terms, whether they perceive the distance between M and i to be significant.

(2) Whether investors display any tolerance of risk, and if so, whether the level of variance which investors associate with the market portfolio is within their tolerance level; graphically, whether the extreme point of the horizontal section of their indifference curve is to the right or left of M.

If markets are perfect, it is necessary that both these conditions be satisfied, namely, that the risk which cannot be diversified away is both significant and beyond the market's tolerance level, to necessitate a risk premium in the ex ante required return. But it has been argued, in the previous chapter, that under imperfect markets, in which there are surplus returns available to equity investors, a third condition is necessary, that is, that the prospect of participating in those surplus returns is inadequate compensation for investors to be attracted to the equity market. Therefore, the third aspect with

which the survey is concerned is,

(3) Whether, given equity holders' residual right to profits in excess of the minimum acceptable return, investors would be attracted to a well-diversified portfolio of risky assets where each component of the portfolio is selected on the basis of expected return only, without regard to variance; in effect, whether they would accept the default-free rate as the minimum cut-off rate for corporate capital budgeting decisions.

Procedure

Clearly, a number of problems presented themselves, not least the problem of defining risk. It was not possible to explain to interviewees the implication of the capital asset pricing model, or to distinguish explicitly between systematic and non-systematic risk. Therefore, the questions were framed in such a way that the investors were compelled to formulate their replies from the standpoint of a holder of a very well-diversified portfolio, even more diversified than most might ever consider desirable. A few were, in fact, puzzled by the practicability of investing so widely with a relatively small amount of funds, but accepted that it could be effected through the agency of an investment trust. Although the international portfolio specified in the questionnaire was intended to suggest maximum diversification, the U.K. portfolio was included as an alternative to give an option for those who might conceivably take the view that the inclusion of foreign securities would add to rather than diminish the total risk of the portfolio. This fear was borne out by the replies of a few interviewees, but for the main

part most respondents commented on how well diversified the international portfolio was.

Because of the unavoidable complexity of the questions, and therefore the probability of a low and unreliable response to a postal questionnaire, it was decided to conduct personal interviews in order to be able to deal with any difficulties raised by the respondents, with the result that the size of the sample was limited to 50, and the method of selection largely a matter of convenience, namely, colleagues and acquaintances of colleagues. The sample consisted of ten representatives of each of five occupational groups living in Glasgow, accountants, businessmen and industrialists, academics (business studies), investment analysts or advisers, and other professionals. Each interviewee was asked as a screening device, if he had ever invested or taken an interest in the stock market, and whether his technical knowledge was sufficient to understand terms such as "the yield on undated loan stock" etc., unless, of course, from his occupation such expertise could be assumed. The default-free security chosen for the purpose was the familiar $3\frac{1}{2}\%$ War Loan, that is, an undated security, on the grounds that whatever time horizon equity investors in undated loan stock might have, neither investment medium is dominated by the prospects of an early maturity. An investor may not perceive his period of investment extending beyond a 5 or 10 year horizon, but since the terminal value of his portfolio will be affected by the discount rates appropriate to the cash flows of succeeding periods, he must take account of the long-term average of future short-term rates of return, and the best estimate of the

average of future short-term rates of interest is provided by the yield on undated stock.

Each subject was asked during the interview to complete the questionnaire in the writer's presence. A copy of the questionnaire is included in the appendix. The respondents were invited to raise any difficulties they might have in understanding the questions. It was emphasised to each when he approached question 4 that he was being asked to rank the portfolios in order of their riskiness and not their desirability. His attention was drawn to the fact that No. 1 signified the most risky etc. In question 8 it was made clear that a project's expected yield was no more than the mean of a range of possible yields. In question 9, the purpose of stating portfolio A so laboriously rather than in the form of a simple bank deposit was to minimise the effect of any possible prejudice against fixed-interest securities. It was also explained to each interviewee that to have a 2, 5, or 10 year time horizon was in no way a guarantee that there might not arise circumstances which would require a forced sale. It was merely an expectation that the funds would not be required for other purposes during the period.

Results

It was clear from the interview that the respondents found it difficult to think of the risk of an investment in isolation from its desirability. It was also obvious that the tentative definition of risk given in the introduction to the questionnaire conflicted with many of the respondents' concept of risk. One respondent,

for example, remarked that "in your sense of risk, undated stock is not particularly risky because the investor is virtually assured of losing money." Furthermore, at the time of the interview (May 1974), the economic situation could not be described as normal. The rate of inflation was expected to exceed 15% in 1974, War Loan was yielding nearly 14½%, and the F.T. Ordinary Share Index was at its lowest point for seven years. There was general disillusionment expressed about equities for their failure in recent years to keep pace with inflation, but it is probably fair to say that the depressed conditions produced an even greater disillusionment with and prejudice against fixed interest stock. Nonetheless the results of the study, as indicated in Table 1 appear to corroborate the BJS study and hypotheses presented in the last chapter, namely,

(1) That undated default-free loan stock is perceived by investors as having a substantially equivalent level of riskiness as an efficiently diversified portfolio of risky assets.

The general dissatisfaction with the definition of risk has already been noted, and it was clear that most respondents were replying to the questions according to their own perception of risk. However the fact that as many as 46 out of 50 took the view that the loan stock was actually riskier (in whatever sense) in the medium or long term must cast doubt about any generalised assumption that a default-free security is less risky. Inflation, of course, was frequently cited as the main consideration inarriving at their decision.

(2) That the risk-aversion hypothesis is not a valid assumption for all levels of risk

As many as 48 out of the 50 investors questioned were willing to invest at least some of their surplus funds in a volatile portfolio without

any expectation of a higher average return than a non-volatile security. To what extent this readiness to accept some volatility without compensation implied risk-tolerance on the part of the investors, or a belief in their ability to time their exit from the market under favourable conditions was not clear. Neither reason, of course, may conform to the theoretical concept of rational investor behaviour. But it is how investors actually behave which matters and not how they ought to behave, and it would seem that few investors would welcome the disappearance of all variability.

(3) That the yield on default-free stock is a reasonable proxy for the minimum acceptable return on the market portfolio of risky assets.

All members of the sample indicated that they would be prepared, for the medium and long term, to invest at least some of their surplus funds, and 48 out of the 50 said all of their surplus funds, in a portfolio of equities, if management adopted the yield on undated stock as the minimum acceptance criterion.(7)

(7) The responses used in table (1) to test these three hypotheses were (1) 1cii (92%) (2) 3b (96%) and (3) 2ii (100%). In each case a test of the null hypothesis that the probability of response in the population is below 50% is rejected at the 5% level, using the Normal approximation to the Binomial distribution. For cases (1) and (2), approximate confidence intervals for the probabilities of response can be calculated using the Poisson approximation to the Binomial distribution. The intervals are:

(1) (85%, 97%)
(2) (91%, 99%)

	Number (Total 50)	Percentage of response
(1) Irredeemable default-free loan stock is:		
a) moderately or significantly safer than a well-diversified portfolio of risky assets		
i) in the short term	7	14%
ii) in the medium term	1	2%
iii) in the long term	1	2%
b) only marginally safer than a well-diversified portfolio of risky assets		
i) in the short term	4	8%
ii) in the medium term	3	6%
iii) in the long term	2	4%
c) riskier than a well diversified portfolio of risky assets		
i) in the short term	39	78%
ii) in the medium term	46	92%
iii) in the long term	47	94%

	All surplus funds	At least some surplus funds	All surplus funds	At least some surplus funds
(2) Investors preferring to invest some or all surplus funds in a portfolio of equities of companies which use the default-free yield as the minimum cut-off point for selecting risky projects rather than to invest in the default-free stock itself				
i) for the short term	42	47	84%	94%
ii) for the medium term	48	50	96%	100%
iii) for the long term	49	50	98%	100%
(3) Investors preferring to invest some or all surplus funds in a volatile portfolio of risky assets rather than in a non-volatile security which yields 10% with certainty				
a) even if the long-term expected yield on the volatile portfolio was only 9%	33	47	66%	94%
b) if the long-term expected yield on the volatile portfolio equalled 10%	37	48	74%	96%
c) if the long-term expected yield on the volatile portfolio was 11%	37	49	74%	98%

TABLE 3

It is a defect of the survey that the groups interviewed were arbitrarily defined and therefore do not conform to any recognisable representative sample of the investing population. On the other hand, it is not clear that those who determine risk premiums can be assumed to be representative of the general body of investors. The original intention had been to ascertain whether any clear differences of attitude could be identified between the different occupational groups, but because of the unexpectedly high degree of consensus among the respondents, it was clearly not fruitful to analyse the results into sub-groups. Nonetheless, it is significant that of those who could be described as professionals in the finance field, the accountants and brokers and analysts, all twenty were not only tolerant of some risk, but were actually prepared to pay a premium to assume a degree of risk, as indicated by their willingness to invest in a volatile portfolio whose expected average return was 9% in preference to obtaining a certain 10%. Furthermore, all twenty stated that the default-free yield was an acceptable yardstick for companies seeking a minimum cut-off rate.

It was of interest to find quite a few of the respondents lighting upon the concept of nondiversifiable risk. For example, when they replied, as all of them in fact did, that they would be willing to accept the default-free rate as the minimum cut-off rate for new projects undertaken by the constituent companies of the portfolio, it was emphasised to them that the rate would be used even for projects which <u>in themselves</u> were very risky. The

typical reply was that it would hardly matter how risky a particular project was if the portfolio was as widely diversified as the one envisaged.

There also appeared to be an implicit recognition of the effect of the +ve NPV rule. When asked why they would choose to invest in the portfolio rather than the loan stock, if the yield of the latter was the yardstick for the former, they invariably replied that they would be attracted by the prospect of earning a higher return on average than the default-free rate if the latter were used as the minimum criterion.

In conclusion, then, the survey revealed that investors' attitudes appear to conform to the hypothesis presented in chapter 4, and to the interpretation offered of the BJS findings, namely that under modern capital market conditions, there is insufficient evidence to establish the necessity or even the desirability for managers seeking to apply the present value technique to depart from the market rate of interest.

SUMMARY

The evidence examined in this chapter and the preceding one indicated that:

(1) Equities have consistently earned more than bonds by an amount greater than can be accounted for by the risk differential.(8) This supports the contention of chapter 4 that as a result of the operation of the +ve NPV rule, ex post equity yields cannot be used to estimate ex ante required minimum rates of return.

(2) The internationalisation of equity portfolios reduces relevant risk significantly below that associated with domestic markets.(9)

(3) With the increased efficiency and internationalisation of capital markets investors appear to have become progressively less concerned about the differences in variability of securities and in recent decades they have appeared to base their selection on expected return only.(10)

The survey carried out to test investors' attitudes to risk revealed that

(a) When inflation is taken into account, investors do not perceive an efficiently diversified portfolio of risky assets to be any riskier than a fixed-interest default-free security.

(b) Investors have a greater tolerance of volatility once the major risks have been diversified away than the conventional theory of risk aversion implies.

(c) Investors would find it acceptable if companies used the market-determined Government bond yield as the cut-off rate for new investments whatever the variability characteristics of the individual project.

(8) Horgaard
(9) Cohn and Pringle
(10) Black, Jensen and Scholes, 1972

Thus the fact that equities have earned more than debt securities does not necessarily imply the existence of a premium for risk and can be explained by the effect of the +ve NPV rule as discussed in chapter 3. To support the risk-premium theory it would need to be demonstrated that the required returns amongst risky assets vary according to their relevant risk. Again because of the +ve NPV rule effect we cannot observe the minimum return required on equities but only the average achieved returns. If achieved returns have varied according to the riskiness of the industries involved, this may have been due as much to management's aversion to risk as to investors'. But the fact that in recent decades achieved returns have been observed not to vary at all with risk provides very compelling evidence that variance has ceased to play any significant role in the valuation process.

It is not suggested, of course, that the evidence demonstrates conclusively that the differential risk of the world market portfolio is not a material consideration to investors, but in view of the very considerable operational difficulties of implementing the risk-premium hypothesis, it is argued that it is merely sufficient to raise serious doubts about the materiality of nondiversifiable risk to be justified in questioning the utility of introducing the risk premium refinement into the capital budgeting framework. It would appear that a high proportion of the relatively small number of companies who even attempt to adjust the discount rate for risk do no more than apply an arbitrary set of rules which produce rates that are not even responsive to changes in the market rate of interest.(11) It is concluded therefore that because of the doubts raised by the evidence relating to risk premiums, and because of both the significant problems of implementation and the possible consequences

(11) See, for example, Abdelsammad, pp 105 - 106 and Klammer, page 391

of defective or arbitrary solutions it is reasonable to assume that the quality of investment decisions in the market as a whole would not be adversely affected and might well be significantly improved if the pure market rate of interest were adopted as the relevant discount rate. There are, of course other considerations which have been alleged to necessitate departure from the pure market rate of interest, and these will be examined in the succeeding chapters.

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Chapter Six

THE COST OF DEBT

In order to justify the proposition that the relevant discount rate for capital projects can be taken for practical purposes to be that given by the government bond yield, it is necessary to explain the fact that corporate bond yields not only vary according to their respective maturities but apparently according to their degree of riskiness.

A significant proportion of the literature in recent years has been devoted to the problem of measuring the cost of equity capital in a capital-asset pricing model context, and this has earned the reputation of being the principal stumbling block in the derivation of an operational measure of the weighted average cost of capital. On the other hand, the typically brief text-book treatment of the procedure for computing the cost of debt gives the impression of the absence of any significant controversial issues. Thus, the cost of debt is usually defined as the discount rate which equates the present value of the interest plus principal payments with the net proceeds of the debt issue, that is, the solution for k in the following equation:

$$D = \frac{I}{(1+k)^1} + \frac{I}{(1+k)^2} + \cdots \frac{I}{(1+k)^n} + \frac{B}{(1+k)^n} \cdots \quad (1)$$

where D = net proceeds of bond issue, or market price of bond

I = annual interest payments

B = maturity value of the bond

n = number of years to maturity.

For the purposes of project analysis, where the cash flows are

expressed on an after-tax basis, the relevant cost of debt is usually defined as $k_B(1 - t)$. Therefore, in the simplest case, where the bond is issued at par and matures at par, the after-tax cost of debt is equal to the coupon rate less the rate of corporation tax.

In this chapter it will be demonstrated that this widely accepted definition is an oversimplification, and that although the scope for error in computing the cost of debt is restricted, the problems of measuring it are potentially as complex as those associated with calculating the cost of equity. It will be argued that, in order to derive the overall cost of capital, the relevant cost of a particular debt security is not the promised yield to maturity, but the expected time-adjusted average cost of the series of successive securities of which the particular debt security is one component. In effect, the cost of debt is shown to be independent of the coupon rate of interest and, on the assumption that lenders are capable of diversifying efficiently, it is argued that the cost of a bond of any maturity should approximate the yield of a default-free security. Thus one of the most convincing arguments for the validity of the risk premium concept, albeit in the nonequity market, namely the hierarchy of interest rates for bonds which vary only in their degree of risk, is in fact, illusory.

THE COST OF DEBT AND THE LENGTH OF TIME TO MATURITY

It is an observable fact that rates of interest differ for equivalent-grade securities according to differences in the length of time to maturity. The relationship between yield and maturity will be influenced by a number of factors, but, in particular, by the market's expectations of the future course of interest rates.

A two year bond may have a yield of 5% at the same time that an

equivalent-grade, seven-year bond may yield 10%. The two year bond cannot, however, be said to be cheaper per se than the seven-year one. Presumably the market's expectations are that after two years, interest rates will rise so that, on average, over the seven year period, a purchaser of a series of short term bonds will earn approximately 10%. There may, of course, be other factors which operate to put the average of the short term rates above or below the 10%, for instance, investors' liquidity preferences or segmentation between the short and medium markets. But, nonetheless, it remains true that the relationship of the two yields cannot be assumed, prima facie, to reflect fundamental differences in cost, otherwise, a simple decision rule of issuing only lower yield securities, irrespective of maturity, would commend itself to firms.

For convenience, the subsequent analysis proceeds within the framework of the traditional Modigliani and Miller assumptions of efficient markets and no taxes etc. where the overall cost of capital can be shown to be unaffected by the degree of leverage. However, this framework is not essential to the argument, and the principles involved are equally valid within a more conventional framework.

Assume there are two firms, M and N, of an identical risk class, which are expected to earn a perpetuity of cash earnings, $\bar{X}$, such that each firm has a market value, V. In addition, both firms have identical capital structures except that the maturity of M's debt is short-term and that of N, long-term, and the interest rate, r_n on N's debt exceeds that on M's debt, r_m. Therefore

$$V = S_m + D_m = S_n + D_n$$

where S = the market value of the equity, and D = the market value of the debt, and $S_m = S_n$, and $D_m = D_n$.

In the absence of corporate taxes, the weighted average cost of capital for each firm is

$$w = \frac{\bar{X}}{S_u}$$

where S_u = the market value of the equity of an unlevered firm of the same class. Therefore

$$w_m = k_s^m \left(\frac{S_m}{S_m+D_m}\right) + k_B^m \left(\frac{B_m}{S_m+D_m}\right) = \frac{\bar{X}}{S_u}$$

and $$w_n = k_s^n \left(\frac{S_n}{S_n+D_n}\right) + k_B^n \left(\frac{B_n}{S_n+D_n}\right) = \frac{\bar{X}}{S_u}$$

Where k_s and k_B are the relevant costs of the respective equity and debt capitals.

Now, if $k_s^m = k_s^n$

then, $$k_s^m \left(\frac{S_m}{S_m+D_m}\right) = k_s^n \left(\frac{S_n}{S_n+D_n}\right)$$

and therefore

$$k_B^m \left(\frac{D_m}{S_m+D_m}\right) = k_B^n \left(\frac{D_n}{S_n+D_n}\right)$$

and $$k_B^m = k_B^n$$

but since $r_m < r_n$, then it follows that r cannot, per se, be the relevant cost of debt in determining the firm's cost of capital.

If $k_B^m \neq k_B^n$, then either

(a) $w_m < w_n$, or

(b) $k_s^m > k_s^n$

The first of these alternatives is rejected because it conflicts with the Modigliani and Miller theorem that the cost of capital is independent of the form of financing and because the resulting disparity in firm values would be eliminated by their now celebrated arbitrage process. The second alternative implies that the cost of equity articulates perfectly with the cost of debt, or equivalently that the implicit cost of debt (including explicit and implicit costs) is independent of the debt's yield to maturity. If therefore, in practice, the cost of equity is computed on the basis of the firm's debt-equity market value ratio, without reference to the maturity structure of the firm's debt capital, then, to compensate, the cost of debt should be computed independently of its length of time to maturity. That is, k_B does not equal r but, rather, equals the weighted-average of the expected return from the bond and the expected return from the security or securities which subsequently replace it. Therefore

(1) If the firm intends to replace its dated debt security by another debt security or by a series of debt securities, so as to maintain its existing capital structure, then the relevant cost of the dated security is not its own cost but the expected average cost of the entire series of debt instruments. Unless the management considers itself competent to predict the future course of interest rates, then the best estimate of that expected average is given by the yield of equivalent-grade long-term bonds.

(2) If the firm expects to replace its existing dated debt by equity capital, then in the context of the conventional assumption that the cost of equity is distinguishable from the cost of debt, the manager is faced with the difficult task of computing the average cost of the debt-equity series. It has to be borne in mind that if the current market yield-maturity relationship for debt

securities indicates an expectation of a future increase or decrease in interest rates, then it is reasonable to assume a corresponding increase or decrease in the cost of equity capital, if the latter is perceived as being equal to the rate of interest plus a premium for risk.

(3) If the firm intends not to replace the dated debt with any security, then the relevant cost will be the average of the cost of the existing debt and of the weighted average cost of capital expected to prevail after the debt has been repaid.

This conclusion is based on the observation that if the conventional method of calculating a firm's cost of capital produces solutions which vary according to the maturity structure of the component securities, then the cost of capital as a decision tool must be redefined, so that it can be expressed in maturity terms which are consistent with the needs of the decision. Either cash flows should be discounted at the rate specifically appropriate to the year in which they are receivable, in which case the cost of capital should be perceived as a short-term rate to be computed for each year of the period under review, or cash flows should be discounted at a single time-adjusted average rate, in which case the cost of capital should be expressed as a long-term average rate. Since the cost of equity is generally defined in terms of an indefinite stream of income, and since most theoreticians favour the convenience of using a single rate for discounting prospective cash flows, then the latter approach is preferred. But whether one or the other is adopted, the yield to maturity is clearly not the relevant cost for inclusion in the weighted average calculation.

THE PROMISED VERSUS THE EXPECTED RATE OF INTEREST

The second aspect of the cost of debt capital to be examined

concerns the distinction sometimes made in the finance literature between the promised rate of return and the expected rate of return.[1] The promised rate is the rate which solves equation (1), whilst the expected rate is the rate which debt-holders actually expect to receive. If the debt is riskless, then these two rates are synonomous. If the debt is risky, then the promised rate is the maximum rate within the range of possible returns receivable by the bondholder, and, therefore, must be higher than the expected rate since the latter allows for the possibility of default. Some controversy exists about which of the two rates is the relevant one for inclusion in the firm's cost of capital computation. Haley and Schall,[2] for example, argue that it is the expected return which is the relevant one, whilst, in a recent paper,[3] Brennan argued that, with bankruptcy costs and corporate taxes, it is the promised not the expected cost which is relevant, and, in fact, most textbooks consider only the promised rate. The issue is whether investment projects should be accepted if their expected rate of return exceeds the rate needed to provide the suppliers of capital with the rate which they expect on average to earn, or whether it should at least equal the rate which the firm commits itself to pay to the suppliers of capital.

The problem is more easily put in perspective if we initially make the convenient, albeit unlikely, assumption that a company could be financed entirely by debt, again in the context of the Modigliani and Miller assumption that the capital structure is irrelevant to the firm's overall cost of capital. The issues raised, however, once again, do not depend upon acceptance of the MM framework, but are nonetheless more readily placed in perspective within that context.

(1) For example, see Boness.
(2) See Haley and Schall, page 160.
(3) Brennan, page 27.

The distinguishing features of debt capital as distinct from equity are that,

(a) the returns are more predictable, and

(b) there is a higher degree of stability in the pattern of returns.

The predictability is achieved by establishing a contractual rate of interest per annum, so that the debtholder can ascertain the most likely pattern of cash flows. The variability of that pattern is minimised by attaching to debt a prior claim upon the firm's income, plus a right to look to the firm's assets should the firm fail to meet the contractual rate of interest. The higher the proportion of debt in a firm's capital structure, the more does a debt security lose its predictability and protection from risk, and the more does it assume the characteristics of equity. For a pure debt company, the returns are neither more predictable nor more secure than those of a pure equity company, and, therefore, the concept of a pure debt company is highly unrealistic. On the other hand, it is a useful artefact for the purposes of focusing upon the similarities and differences between the cost of debt and the cost of equity.

Perfect Markets

By perfect markets, it is assumed that there are no bankruptcy costs, and that competition is so effective, both amongst firms for capital projects, and between the debt and equity sections of the securities market, that the firm's overall cost of capital is substantially equal to the expected rate of return from acceptable projects. That is, no firm can find projects whose IRR $>$ k, or equivalently, which have a positive NPV when the cash flows are discounted at k, where k is the cost of capital appropriate to the

risk class. All acceptable projects have NPVs = 0. The cost of capital for a pure debt firm should equal the cost of capital of a pure equity firm of the same class, but unless the firm's asset returns are free of risk, the promised rate for the debt company would be higher than the return expected by the bondholders, and, therefore, by the same token, higher than the return expected by investors in the pure equity company. The minimum equilibriun rate for the promised rate of interest is the highest return included in the market's probability distribution of possible returns from the firm's projects. If it were any less than the highest possible return from the projects then the expected return for the debt would be less than the mean expected return from the projects. Thus, in order to have an expectation of achieving a required return of, say, 15%, the promised rate for the pure debt company might have to be 50%, or even 500%, if there were the remotest possibility of the projects' actual returns deviating to that extent from the expected return of 15%. Indeed, to be assured that every possible return were included, the nominal rate for a pure debt company might be set as high as ∞%, when of course the debt is indistinguishable from equity. But whatever the appropriate promised rate might be, the relevant rate to guide investment decisions would be the expected rate. In perfect markets, lenders would not exercise their right to declare a firm bankrupt for failure to meet the nominal rate of interest as long as there existed the expectation of achieving, on average, the required return. If they sued for failure to earn the nominal rate, no firm would be able to survive for any length of time, because, by definition, it would not be possible to find any projects which offered an ex ante average return higher than the return required by the suppliers of capital. The promised rate, therefore, is clearly no more than a

mechanism to give bondholders the opportunity of achieving on average the return required by them. It is a device for capturing possible deviations from the mean return, and cannot, therefore, be used as a screening criterion for selecting projects. The effective decision criterion is the expected cost of debt, and the acceptability of projects depends on this rate only. It cannot be observed in the market, and must be assumed to be, like equity, equal to the default-free rate plus, if necessary, a premium for risk. The size of the premium, if any, depends on the degree of the firm's nondiversifiable risk, whilst the differential between the default-free rate and the promised rate depends on the total variability of the company. Van Horne (4) defines the risk premium on corporate bonds as "the differential between the promised yield and the actual yield on a comparable risk-free security." It is clear, however, that this is not the case, and that the risk premium is the differential between the expected rate on the risky bond and the risk-free yield. If it were possible for bondholders to diversify all risk away, then bonds should have no risk premium, and the expected rate would equal the default-free rate. But the promised rate would nonetheless exceed the expected rate, because the promised rate is a function of the total variability of the company's returns, whilst the expected rate is a function of nondiversifiable risk only.

Indeed, for the pure debt company, the promised rate has to do with the upper tail of the probability distribution of the firm's project returns rather than the dispersion about the mean. Thus the promised rate for a pure debt company, A, may be higher than that of another pure debt company, B, even although A has a lower total risk than B. Table 1, for example, gives the distribution

(4) Van Horne, page 104.

of possible internal rates of return for the productive assets held by companies A and B. B is the riskier of the two, but A's assets have, nonetheless a small possibility of achieving 19% as against B's maximum of 18%. The expected rate for each is assumed to be 10%. If no risk premium is required because of the opportunities for diversification, then the 10% will also equal the risk-free rate. But the promised rate for A's debt is 19% and for B's only 18%. At each of these levels the respective bondholders can expect to receive an average of 10%, as required. The promised rate has, therefore, no economic significance other than to provide an umbrella under which the return expected by bondholders can be realised, given the particular configuration of the probability distribution of returns from the firm's projects.

TABLE 1

A		B	
IRR	Probability	IRR	Probability
1%	.01	2%	.20
10%	.98	10%	.60
19%	.01	18%	.20

Perfect Markets with partial leverage

When firms operate under partial leverage in perfect markets, the promised rate of debt will vary proportionately with the degree of leverage and with the variability of the firm's assets. The promised rate will continue to be higher than the expected rate in order to give bondholders the right to receive deviations in returns above the expected rate up to the promised rate, to counterbalance possible deviations below the expected return. Paradoxically, if the realised returns from the firm's assets exactly equal their

expected returns, the realised return on the firm's debt will exceed the bondholders' expected rate, and the realised return on the firm's equity will be less than the return expected by the shareholders. Therefore, in order that a portfolio of levered equities should achieve the ex-ante expected return, it is necessary that the spread of the realised returns be substantially as broad as the spread of the ex-ante probability distribution of returns from the firm's assets. Just as the necessity for a differential between the promised rate of debt and the expected rate is caused by the variability of the prospective returns from the firm's productive assets, so the possibility of achieving the returns expected from a portfolio of levered shares depends on the actual occurrence of those deviations. Fixed income securities actually benefit from and equity securities suffer from a higher degree of stability ex post than was expected ex ante. Thus, an equity investor who holds shares in two levered companies, each with a significant degree of spread in its ex-ante probability distribution of returns, will, in fact, fail to achieve his required return if each company earns its expected return, even if the returns of the companies were perfectly negatively correlated. He will fare better if one firm does bery badly and the other very well, provided, of course, the average return is no less than the expected return.

To illustrate this counter-intuitive effect of achieving the ex-ante expected return from the firm's assets, assume that P and Q are two companies whose returns are perfectly negatively correlated, and have probability distributions as follows, for one year hence:

	P		Q	
Event	Probability	Cash Flow	Probability	Cash Flow
w_1	.2	£530	.2	£1,670
w_2	.6	1,100	.6	1,100
w_3	.2	1,670	.2	530

Each company has equity of £500 and debt of £500, the latter having a promised rate of 11%. The required return for both the equity and debt are assumed to be 10%. The expected returns for a portfolio of debt and for a portfolio of equity are, therefore, as presented in Table 2.

TABLE 2

Event		P	Q	Portfolio PQ	Probability	Equity	Debt
w_1	Equity	0	1,115	1,115	.2	223	
	Debt	530	555	1.085	.2		217
w_2	Equity	545	545	1,090	.6	654	
	Debt	555	555	1,110	.6		666
w_3	Equity	1,115	0	1,115	.2	223	
	Debt	555	530	1,085	.2		217
						1,100	1,100

It is clear that if each firm earns the expected return of £1,100 (event w_2) the equity portfolio will earn less than the required return, and the debt portfolio more, namely a return equal to the promised rate. Conversely, if P or Q does badly with the other doing well, the return on the equity portfolio will exceed both the return from the debt portfolio and the minimum return required by the equityholders.

Imperfect Markets

In imperfect markets, the firm's cost of capital and the expected return from the firm's projects are not identical, and indeed it is a condition of acceptance, under the positive NPV rule, that the latter exceeds the former. The promised rate of interest is, once again, the rate which the market estimates to be necessary to give bondholders an expectation of obtaining their required return, but unlike the perfect market situation, the promised rate for a pure debt company would be less than the maximum possible return included in the probability distribution of returns for the firm's projects, otherwise the debtholders could expect to achieve a return equal to that expected from the firm's projects, which, by definition, under the operation of the NPV rule, would be greater than the return required by them.

If a pure debt company accepts a project which is expected to earn more than the 'cost of capital', i.e. more than the debtholders' required return, then the question may be asked, to whom does the resulting NPV or excess return belong? If the project's IRR is greater than k_B but less than r, then because debtholders are entitled to receive any returns up to the level of the nominal rate, r, it might seem that the excess returns would accrue to the debtholders. Indeed, it has already been noted that if every firm achieved the return expected from its productive assets, the debtholders would earn a return equal to r, and would, therefore, secure to themselves the full benefit of the excess returns. However, looking at the whole market portfolio of levered firms, it is extremely improbable that each component firm will earn precisely the return expected from its productive assets, but rather that the pattern of realised returns will conform somewhat to the ex-ante spread of returns which was contemplated when the market level for r was determined. Each pure

debt company has the possibility of earning a return higher than r, and the excess return, if it does materialise, must accrue to the residual owners, who, in the circumstances are the 'noncontributory' equity owners. In other words, where disequilibrium returns are possible, a company which starts off as a pure debt company, in the sense that all the funds needed initially to purchase its plant and machinery are supplied by bondholders, ceases to be a pure debt company as soon as it accepts a project whose expected IRR $> k_B$. This paradox is a direct result of the residual nature of equity's claim in market conditions which create a divergence between the expected IRRs of acceptable projects and the required rates of return for the capital instruments used to finance those projects.

The significant point for present purposes, however, is that even under imperfect conditions, the relevant cost of capital for determining the contribution which a project makes to the value of the firm remains the expected rather than the promised rate. On the other hand, it is a reasonable assumption that the nominal rate is used by lenders in their evaluation of a firm's capacity to pay the expected rate, although we do not know precisely what criteria are used by them in deciding whether bankruptcy proceedings are justified. It has been argued that under perfect conditions the probability of bankruptcy would be no greater for a pure debt company than for a pure equity company of the same class, because investors in both sections of the security market would be guided only by the prospects of achieving the expected return, which per unit of risk, would be the same for each type of security. If under imperfect conditions, the probability of bankruptcy appears to be increased by the degree of leverage, the implication is that leverage leads to premature bankruptcy, because lenders use failure to achieve the promised rate as the signal for action, or alternatively that

equityholders are, in fact, dilatory in their response to the appropriate signals and permit their firm to continue operating beyond the point where it is justified on economic grounds. Whichever explanation is correct, the promised rate may acquire a more significant role than has hitherto been identified, and a risk-averse management, interested in survival, may use the nominal rate as the relevant hurdle for new investments to minimise the possibility of bankruptcy. But if management wishes to shield the firm against the consequences of bankruptcy, the use of the promised rate as the minimum acceptance rate does not thereby make it the correct rate for measuring the project's contribution to the value of the firm. It is no more than one of a series of measures which they might adopt, such as employing a low level of leverage, or engaging in firm diversification, and which may lead to a suboptimal position in relation to shareholders' welfare. For example, firm diversification, in itself, produces no benefit for shareholders which they could not have achieved through their own portfolios, and may lead to the rejection of otherwise profitable projects which might upset the balance of the portfolio. A low debt-equity ratio might reduce the value of the firm as a result of the loss of tax benefits. Lastly, the use of the promised rate as the acceptance criterion might lead to the rejection of projects capable of contributing to the value of the shareholders' portfolios., Nonetheless, it cannot be said that anyone of the above strategies necessarily affects the return required by bondholders. Whatever screening device is employed by management to determine the acceptability of projects, the magnitude of the contribution which a project makes to the value of the firm can only be measured by discounting the project's cash flow at the return required by the suppliers of capital.

Bankruptcy costs

Bankruptcy costs

So far, it has been assumed that no costs are incurred by the firm at the point of bankruptcy. Brennan argues (5) that such costs, together with the tax deductibility of interest payments, make the promised rather than the expected rate the relevant one. This can only be correct, however, if the promised rate is equivalent to the expected rate plus a premium sufficient to cover the expected costs of bankruptcy, and it is argued that this could be so only if the bonds are riskless, that is, that bankruptcy can be assured of being declared before the value of the firm, after payment of bankruptcy costs, falls below a level sufficient to give bondholders their required return. But a risky bond is one which is capable of yielding, after all costs have been paid, a return which is less than the minimum required by the bondholders. Therefore, to compensate, the lender must be capable of earning a rate of return greater than the expected rate plus any bankruptcy premium, otherwise he cannot hope, on average, to achieve the expected rate. It follows that the promised rate must exceed the relevant rate for capital budgeting purposes, and that its function is fundamentally no different than when bankruptcy costs are assumed not to exist, namely to provide an umbrella to capture deviations from the mean. By stating that a rate higher than the bondholders' expected rate is the appropriate one for management to discount the firm's cash flows in order to cover expected bankruptcy costs, Brennan is merely suggesting that the best way to allow for bankruptcy costs is not to include them in the probability distribution of cash flows, but to adjust the expected rate upwards so that sufficient returns will be earned to meet the additional costs. Now, that may be a useful practical device, but it is no more theoretically valid than omitting the project's installation expenses and then applying a higher discount

(5) Brennan, page 27

rate to the subsequent inflows to compensate for the omission. Bankruptcy costs, like the installation costs of a project, or the chairman's salary, should be included in the probability distribution of the firm's cash flows, and, therefore, affect the expected value of the firm, but, per se, they do not change bondholders' required return, which is the rate applicable to the residue of cash flows available to the bondholders from the firm's projects, after deducting all prior expenses. The costs affect the value of the terminal cash flow which the bondholders are entitled to receive, but they do not affect the discount rate which the bondholders' apply to that cash flow. Therefore, even with bankruptcy costs, the relevant decision criterion for measuring a project's worth is the expected or required rate.

Taxes

Brennan also argues that the promised cost is the more relevant cost with corporate taxes, "since the tax savings from debt issuance depend upon the promised return and not the expected return to bondholders." But since the promised return is the maximum which the bondholder can hope to receive, then the tax savings associated with that return are the maximum which the firm can hope to achieve. The present value of the tax savings from a bond issued at par is either the value of the promised cash benefits discounted at the promised rate, $\frac{trD}{r} = tD$, or, equivalently, the value of the expected cash benefits discounted at the expected rate, $\frac{tk_BD}{k_B} = tD$.

If we perceive the stream of cash benefits from a project as being composed of two elements, namely

(1) the after-tax flows expected to arise from the project computed on the basis that the project is purely equity financed, and

(2) the tax savings expected to arise from the use of debt capital to finance the project

it is clear that the first stream is unaffected by the tax deductibility of interest payments, and should therefore be discounted at the expected rate, whilst the present value of the second stream equals tB, which has been shown to equal the value of the expected tax benefits discounted at the expected rate of interest. It follows that the relevant rate, even with corporate taxes, is the expected rate and not the promised rate.

SUMMARY

The conventional definition of the cost of debt as the yield to maturity was examined and found deficient for two reasons. Firstly, the yield to maturity is an expression of the promised rate of interest which is no more than an umbrella rate to ensure that bondholders have the expectation of earning the return required by them. The promised rate is the maximum rate which can be earned by bondholders, whilst the relevant criterion for measuring the value of new projects and for discriminating between them is the minimum rate acceptable to bondholders, namely the default-free rate, plus, if appropriate, a premium to compensate for nondiversifiable risk. The cost of debt therefore, like the cost of equity, cannot be directly observed in the market and depends for its precise derivation on the decision-maker's ability to measure the market price of risk.

Secondly, the weighted average cost of capital as a decision tool in investment appraisal, is either one of a series of short-term rates calculated for each year under review or a single long-term average of the future short-term rates expected to prevail during the whole period under review. It cannot be composed of a mixture of redemption yields of securities of different maturities. If the single long-term average approach is adopted, then the relevant cost of a debt security of any maturity is the time-weighted average of its own cost to maturity and of the cost of the securities which are expected to succeed it. If the firm intends to maintain its existing debt equity ratio, then the effective cost of debt is the expected return from a long-term security of the same class.

It was argued that the existence of bankruptcy costs and tax

benefits for debt do not directly affect the return required by bondholders. Therefore, given the opportunities to construct a well diversified portfolio of bonds, the degree of nondiversifiable risk for any particular bond should be insignificant, and for practical purposes the return expected may be assumed to be equal to the default-free rate. In that event, the effective cost of debt may be taken to be equal to the yield of a long-term Government bond, whatever the coupon rate of the individual debt security or its length of time to maturity.

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Chapter Seven

THE DEBT-EQUITY MIX AND THE COST OF CAPITAL

In chapter 6, we examined the cost of debt and how it responded to varying degrees of leverage. It was argued that although the rate of interest could be expected to increase as the proportion of debt in the capital structure increased, it did not follow that the cost of debt proper, that is, the return expected to be earned by lenders, increased given the opportunities to diversify the risk away.

In chapters 4 and 5, we examined the cost of equity and concluded that there was no firm evidence that in current markets a risk-premium needs to be added to the pure rate of interest to attract equity investors into a well diversified portfolio of risky assets.

This chapter is concerned with the effect which the debt-equity mix in the firm's capital structure has on the cost of equity. If a pure equity stream of a given class is subject to risk, the effect of introducing leverage into the capital structure is to magnify the riskiness of the levered equity stream. If the market places a price on risk, then, as leverage increases, the cost of equity rises correspondingly. The traditional view[1] implies that the rise in the cost of equity initially for moderate amounts of debt is not such as to cancel out all benefits of debt, and that leverage can therefore produce a reduction in the overall cost of capital. Modigliani and Miller[2] argue that since equity investors can engage in homemade leverage, the rise in the cost of equity is directly proportional to the increase in leverage and therefore the overall cost of capital will be unaffected. Therefore, whilst

(1) e.g. See Durand, page 108 of The Theory of Business Finance. A Book of Readings, by Archer and D'Ambrosio

(2) Modigliani and Miller, 1958

the two views diverge about the precise effect of leverage on the cost of equity, both views imply that the cost of equity rises with leverage. In this chapter, whilst the above principle is accepted that if a pure equity stream has a capitalisation rate which includes a risk premium, the introduction of leverage will magnify the risks and therefore magnify the premium, it will be argued that if the pure equity stream has a degree of risk which is insignificant and does not necessitate a risk premium, the introduction of leverage will not necessitate a risk premium for the equity even if the risk of the residual equity stream is magnified to a degree which causes it to become significant. To establish this proposition, it will be argued that

(a) because investors can, in their personal portfolios, 'undo' corporate leverage by lending to the company in the proportion in which the company issues debt, if the level of risk for an unlevered stream is insignificant, then the combined debt-equity investment in the levered company will also be insignificant.

(b) the benefit to shareholders resulting from being entitled to receive a project's excess returns (+ve NPVs) increases proportionally with the level employed.

Undoing leverage

The risk class of a company is determined by its business risk, that is, the character of its investments. Leverage affects the risk components of the capital structure but does not affect the total business risk. Shareholders can, if they wish, avoid the effect on the components of the capital structure by investing

in those components in the proportions in which they are issued. Assume that there are two companies of the same risk class, with identical gross earnings X, except that company A is purely equity-financed and company B has a mix of debt D_B and equity S_B. If company 'A''s shares are not considered to be sufficiently risky when included in a well diversified portfolio to justify a risk premium, that is $k_e = k_i$, the cost of debt, then it can be shown that for company B, $k_e = k_i = k_o$, and that, ignoring bankruptcy costs, the market values of the two companies, V_A and V_B must be the same. Thus, if, as a result of leverage, B's equity capitalisation rate k_e^B is higher than k_e^A then $V_B < V_A$. A shareholder in company A could sell the proportion α of company A's shares S which he owns and with the proceeds he could undo B's leverage by purchasing the shares and bonds of B as follows: shares αS_B; bonds $\alpha(S_A - S_B)$. His new return will be

$$\begin{aligned} &\alpha\left[X - k_i D_B) + k_i(S_A - S_B)\right] \\ &= \alpha X + \alpha k_i(S_A - (S_B + D_B) \\ &= \alpha X + \alpha k_i(V_A - V_B) \end{aligned}$$

which, if $V_A > V_B$, is clearly greater than αX, the investor's previous income from holding A's shares.

Since the risk of the two portfolios is the same and investors gain income from switching from company A's shares into the mixture of company B's shares and bonds, then, in equilibrium, V_A must equal V_B. It follows that, in order to justify the risk-premium

hypothesis, it is not sufficient to demonstrate that an internationally diversified portfolio of marketable equity securities is significantly riskier than a default-free bond. It is necessary to establish that such a portfolio would be significantly riskier even if each component company had no debt in its capital structure. Therefore, even if every company adopted the Modigliani and Miller recommendation of employing extremely high levels of debt to exploit the associated tax advantages, the fact that the resulting portfolio of levered shares might be highly volatile would not be sufficient to induce the portfolio-holder to seek a risk premium, since by adjusting his portfolio to include loans to the constituent companies he could construct a portfolio which would be identical in risk-return terms to one composed of unlevered shares. Hence, if the volatility of the purely unlevered portfolio were insignificant or within the tolerance level of investors, the volatility of a portfolio composed of levered shares and loans to the levered companies would be equally insignificant in total, even although the equity segment of the latter portfolio might in itself be significantly volatile.

Thus the yield of a levered share according to Modigliani and Miller's Proposition II[3] is as follows:

$$k_e = k_e^* + (k_e^* - k_i) \frac{D}{S}$$

where k_e^* is the required return from the share in the absence of leverage. If the required return from a pure equity stream k_e^* is insignificantly distinguishable from k_i, then, however large $\frac{D}{S}$ is, k_e remains unaffected.

The benefits of leverage

(3) Ibid, page 134 of the The Theory of Business Finance. A Book of Readings, by Archer and D'Ambrosio.

The benefits of leverage

This section seeks to demonstrate that the operation of the +ve NPV rule should cause leverage to be positively beneficial to shareholders, without in any way destroying the Modigliani and Miller proposition that corporate leverage is irrelevant to the value of the firm.

Assume, once again, that there are two companies in the same risk class, but A has only equity, S_A, in its capital structure, and B has a capital structure $S_B + D_B$, where S and D equal the respective market values of the equity and debt. If both companies are presented with identical projects, each of which has a +ve NPV or wealth increment Z, then the market values of the two firms will rise to $S_B + D_B + Z$ and $S_A + Z$ respectively, assuming that the financial data concerning the new projects is fully communicated to the market. The wealth increments are identical for the two firms because the cost of capital is assumed to be independent of the differences in the capital structure, as implied by the Modigliani and Miller framework. Since the benefit of Z is entirely attributed to the equity holders, the shares of the levered company will rise by the ratio $\frac{S_B + Z}{S_B}$ ~~and those of the unlevered company by~~ $\frac{S_A + Z}{S_A}$ and those of the unlevered company by $\frac{S_A + Z}{S_A}$. Given that $S_B < S_A$, then the former ratio is greater than the latter. It is true that if Z turns out to be negative, the adverse effect on S_B will be greater than on S_A, but the fact that the balance of probabilities is that Z will on average be positive if firms as a rule accept only projects which are expected

to yield a return in excess of the cut-off rate. Thus, if the price of the shares in the two firms prior to accepting the projects was $\frac{S_B}{n}$ and $\frac{S_A}{2n}$ where n and 2n are the respective number of shares issued in the companies, and where $\frac{S_B}{n} = \frac{S_A}{2n}$, then after undertaking the projects with wealth increments of Z, the price of the respective shares will rise to $\frac{S_B + Z}{n}$ and $\frac{S_A + Z}{2n}$ so that $\frac{S_B + Z}{n} > \frac{S_A + Z}{2n}$.

The price of the levered share has increased by a greater amount than that of the unlevered share, although the increase in the total values of the two firms, Z, is the same. It is important to note that the superior performance of the levered shares is <u>not</u> the reward for the additional risk of leverage, since Z is the wealth increment after discounting at the weighted average of B's equity and debt. Thus, even if one postulates a risk premium for both companies' overall cost of capital, the relevant wealth increment, Z, is determined after discounting at the risk-adjusted rate, and therefore Z would be surplus to the rewards needed to satisfy investors for accepting risk. The superior performance of the levered shares, in fact, highlights the disequilibrium nature of the +ve NPV rule, and emphasises the advantage to investors to make use of borrowed funds to finance projects which are expected to yield returns higher than can be accounted for by their riskiness. The constant equality of the values of the two firms, on the other hand, confirms the Modigliani and Miller theorem that the advantage of leverage can be achieved by personal borrowing as effectively as it can by corporate borrowing.

Table 1 contains a numerical example of this effect on the share price of adopting a project with a positive net present value. Assume that an unlevered company A and a levered company B have been formed, each with a total capital of £100, to search for property development sites in the city of London, a business activity for which the market's minimum acceptable rate of return is taken to be $12\frac{1}{2}\%$.(4)

(4) For the moment, the existence of a risk-premium over the riskless rate of (say) 10% is assumed for the purpose of illustrating that the effect of the NPV rule on the price of levered shares applies even given the conventional risk premium hypothesis.

TABLE 1

SHARE PRICE BEHAVIOUR UNDER THE NET PRESENT VALUE RULE

	Company A	Company B
Capital structure at formation	£	£
Ordinary shares of £1	100.00	50.00
Debt capital at 10%		50.00
Total market value (pre-project)	£100.00	£100.00
NPV of project costing £100	20.00	20.00
Total market value (post-project)	£120.00	£120.00
Market value of equity (post-project)	120.00	70.00
Market value of debt (post-project)		50.00
	£120.00	£120.00
Share price (pre-project)	£ 1.00	£ 1.00
Share price (post-project)	£ 1.20	£ 1.40
Gain	20%	40%
Personal leverage		
Market value (post-project) of two shares	2.40	
Less loan(5) at 10% to finance additional share	1.00	
	£1.40	
Gain	40	

(5) Note that the prospective income streams of the two portfolios are identical

		Company A	Company B
Earnings per share -	original share	12½p	15p
	additional share	12½p	
		15p	25p
Less interest on personal loan at 10%			10p
		15p	15p

Given the usual assumptions about the absence of corporate taxes, that the business risk remains constant, and that all earnings are distributed in full, then, in accordance with the Modigliani and Miller proposition that the overall cost of capital is unaffected by the capital structure, the capitalisation rate for the shares of company A is 12½%, and for company B, 15%, such that the overall cost of capital remains at 12½%.

If both companies happen to be presented after formation with projects which are identical in respect of risk and earnings, and which promise a perpetuity of £15 per annum for an outlay of £100, then the present value of each of the two projects, when discounted at 12½% is £120, and, therefore, the market value of each firm will, ceteris paribus, rise by £20. The increase will accrue to existing shareholders of each company but the effect will be more favourable on the levered shares which will rise to £1.40 compared to a rise of £1.20 for the unlevered shares. At these prices, the prospective yields will be 14.3% for company B's shares (reduced from 15% in response to the new debt-to-equity ratio) and 12½% for company A's shares, both of which rates represent the market's required return from the shares in equilibrium. Although, the performance of B's shares is superior to that of A's, the same effect could have been achieved by A's shareholders if they had engaged in personal leverage as illustrated in the table. The conclusion, therefore, is not that corporate leverage has an advantage over homemade leverage, but that when 'excess' profits are available, it is beneficial to engage in any kind of leverage, whether homemade or corporate. If, in the future, the companies further expand their activities, then, however the expansion is financed, any associated increase in wealth will again cause the respective share prices to be bid up to a higher level, with the levered shares rising more steeply than the unlevered shares. It follows

that the aggregate of levered shares in the market can be expected, on average, to outperform corresponding unlevered shares by an amount which exceeds that needed to compensate for risk.

Conclusion

Two important conclusions can be drawn from the preceding analysis. The first has no direct bearing on the main issue, that is, the derivation of the cost of capital, but is nonetheless worthy of note. It concerns the commonly held assumption that the Modigliani and Miller theorem which states that the value of the firm is independent of the firm's capital structure implies equally that the value of the firm's shares is independent of the capital structure. Thus, Van Horne states that "the financing decision does not matter from the standpoint of our objective of maximising market price per share."(6) It has been demonstrated that this assumption is unwarranted, and that even within the MM framework, the price of a firm's share can be shown to vary directly with the degree of leverage employed by the firm. It follows that the text-books are incorrect in equating share-price maximisation with maximisation of the value of the firm. The

(6) Van Horne, page 236

former objective would imply an advantage to corporate over personal leverage and must, therefore, be treated with reserve as an index of managerial effectiveness. However, the behaviour of the price of the levered shares does bear witness to the benefit which equity-holders receive in having the exclusive right to all returns which are surplus to the required rate.

The second conclusion has important implications for the empirical findings of chapter 5. In the survey carried out, investors were invited to express their attitude to the volatility of the market portfolio as they knew it to be. But the market's volatility is presumably, if anything, greater than it would be if all firms were purely equity-financed. Since all that is required to validate the proposition that a risk premium is unnecessary is to show that investors do not perceive a pure equity portfolio as being significantly more risky than debt, then the findings of the survey were all the more persuasive, given that the portfolio presented in the questionnaire consisted of levered shares. It follows that those who find it difficult to accept that a well diversified portfolio of shares is not significantly more risky in real terms than a government bond, must bear in mind that the relevant volatility to be considered is not that which confronts them in the market but the volatility which would exist if all companies in the portfolio were without debt in their capital structure, or equivalently bear in mind that the relevant portfolio is one which contains every security in the market, including all corporate bonds in the proportions in which they are issued by the respective companies.

SUMMARY

The chapter considered the effect of introducing debt into the capital structure. It was argued that for the purposes of determining whether the nondiversifiable risk of the market portfolio is or is not significant, the portfolio should be envisaged as containing a proportion of all marketable equity capital in the world assuming that none of the companies employed any leverage, or equivalently as one containing every marketable security in the world including all corporate bonds in the proportion in which they are issued by the constituent companies. If the relevant risk of such a portfolio is not considered to be sufficiently material to warrant forfeiting the objectivity of the market rate of interest when operating the present value criterion, then whatever the degree of leverage employed by a company, the cost of equity and the cost of debt can each be assumed to be unaffected, since the investor is able to 'undo' such leverage as has been created.

It was also observed that whilst the market value of the company is independent of the capital structure the market value of the company's shares is not, and it was concluded, therefore, that share price maximisation is not equivalent to maximisation of the value of the firm.

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Chapter Eight

DIVIDEND POLICY AND THE COST OF CAPITAL

This chapter is concerned with the impact which the firm's dividend policy has on the cost of capital. (1) Dividends clearly affect shareholders' wealth, but the extent to which dividend policy does is not clear. Moreover, even if dividend policy can be shown to affect shareholders' wealth, it does not follow that it necessarily affects the cost of capital. Thus, if dividend policy is capable of affecting the value of the firm it must be because one policy rather than another causes investors

(1) to raise (or lower) the rate of return which they require from the firm's investments, and or

(2) to lower (or raise) their estimate of the rate of return which is expected to be earned from the firm's investments.

Modigliani and Miller (2) have demonstrated, under conditions of certainty, that the firm's dividend decision is not relevant because it cannot affect the return required by investors or the rate of return earned on the firm's assets. In the absence of certainty, however, Gordon (3) and others have argued that the dividend decision is relevant on the grounds that investors value a pound of dividends more highly than a pound of retained earnings.

The purpose of this chapter is to challenge the proposition that investors, when confronted with uncertainty, adjust their required return according to the firm's dividend policy. It will be argued that, although the dividend decision may be an important

(1) Taxation considerations are deferred until the next chapter.
(2) Miller and Modigliani, 1961.
(3) Gordon, 1963.

consideration for shareholders' welfare, the financial manager need not concern himself with dividend policy, as such, when faced with the problem of establishing the cut-off rate for new projects.

Gordon's argument rests on the proposition that investors are not indifferent between cash payments and increases in the market value of their shares, and that, with uncertainty, future dividends are discounted at a rate which increases with the distance in the future.

Gordon represents (4) the value of a share as

$$P_o = \frac{Y_o}{(I + k_1)} + \frac{Y_o}{(I + k_2)^2} + \frac{Y_o}{(I + k_3)^3} \cdots \frac{Y_o}{(I + k_t)^t} + \cdots \quad \cdots (I)$$

where

P_o = Share price

Y_o = Annual earnings and payout

$k_t > k_t - 1$

The return required by shareholders = k, which is 'an average of the k_t with Y_o, the weight assigned to each item'. If the company retains $Y_1 = Y_o$ and invests it to earn kY_o per period in perpetuity, equation (I) becomes

$$P_o' = \frac{0}{(I + k_1)^1} + \frac{Y_o + kY_o}{(I + k_2)^2} + \frac{Y_o + kY_o}{(I + k_3)^3} + \cdots \frac{Y_o + kY_o}{(I + k_t)^t} + \cdots \quad \cdots (2)$$

He concludes that 'the shareholder gives up Y_o and gets kY_o in perpetuity, but the latter is now discounted at the rates k_t, $t = 2 \rightarrow \infty$ and it can be shown that kY_o, so discounted is less than Y_o. Hence $P_o' < P_o$ and dividend policy influences share price.'

(4) Gordon, page 368.

Gordon's assertion that the discount rate used by investors with an aversion to risk will rise with the distance in the future can be interpreted in any one of three ways:

1. Investors discount at time t = o the dividends expected in t = I, t = 2, t = 3, ... at the rates k_1, k_2, k_3..., and at t = I the dividends expected in t = 2, t = 3 ... at the rates k_2, k_3 ... and at t = 2 ... the dividends expected in t = 3 ... at the rate k_3 ... This would imply more than simply that investors perceive risk as an increasing function of time, but that risk in fact does increase over time. It would follow that whatever the dividend policy of the company might be, the company would cease to exist in time when k_t reached higher levels than reinvestment opportunities would justify.

2. Investors discount at t = o the dividends expected in t = I, t = 2 and t = 3, at the rates k_1, k_2 and k_3 but at t = I the discount rate for t = 2 and t = 3 will drop to k_1 and k_2, and at t = 2 the discount rate for t = 3 will drop to k_1. In other words, if Gordon's assumption is correct that the discount rate is initially perceived as an increasing function of time, it follows that over the life of the company the rate applied to any specific expected dividend decreases with the passage of time. As a result, although kY_o in perpetuity is worth less than Y_o at time o by reason of being discounted at the rates k_t, $t = 2 \rightarrow \infty$, its value will increase to Y_o in time I, because at time t = I the perpetuity of kY_o is discounted at the rates k_t, $t = I \rightarrow \infty$. Therefore the announcement of the dividend reduction will cause only a temporary dislocation of the share price which will 'right' itself after a year. But this temporary dislocation arises not from the fact that dividends matter but from Gordon's assumption of an increasing

discount rate. The dislocation would equally result from the announcement at t = c that a rights issue of an amount equal to Y_o was going to be made at t = I to finance the proposed investment. If a rights issue is made at t = I to raise a sum of money Y_o, the market value of the associated share issued at t = I will be equal to Y_o, being the perpetuity of kY_o discounted at k_t, t = I $\rightarrow \infty$. But if the rights issue (to be made at t = I) is announced at t = o, then according to Gordon's assumption of increasing discount rates, the present value at t = o, of the future perpetuity of kY_o, and therefore of the future new share, will be less than $Y_o/(I + k_1)$. But the present value at t = o of the subscription price will equal $Y_o/(I + k_1)$ and therefore the existing share price will fall to P_o at time t = o to reflect the difference between the present value of the prospective subscription price and the prospective new share. However, as with the dividend-reduction proposal, the share price P_o at t = o will rise to P_o at t = I when the 'value' of the perpetuity is restored to Y_o.

If maximization of the present value of the shareholders' wealth is assumed to be the objective, then it is apparent from Table I that the optimal policy would be for the company to embark on single period investments with a return $\geqslant$ k and to repay the entire capital at the end of the period (and presumably start again). Companies A, B and C are identical in every respect except that their respective share capitals are invested in projects lasting 1, 2 and 3 years respectively. If the capital is repaid at the end of each project's life, then the present value of the shares will be inversely related to the life of the project.

Table 1

Table 1

Time	Discount rate (%)	A	B	C
o		PV = £100	PV = £98.3	PV = £95.4
		Flows	Flows	Flows
1	10	+ 110	+ 10	+ 10
2	11		+ 110	+ 10
3	12			+ 110

3. In practice, shareholders apply a flat rate k to discount all dividends emanating from an investment financed by retentions, in contrast to k the rate applicable to dividends from investments financed by rights issues or new issues. MM would argue that this would be irrational because the present value of the investment is independent not only of the type of security used to finance it, but presumably also of the method of raising that security (rights, retentions, or new issues). It is true that the riskiness of an investment remains unaffected by the method of finance, but it is not necessarily true that the market's perception of the riskiness of the investment should be so unaffected. MM's assumption of a perfect capital market implies that when a new project is being undertaken, there is no distinction made by management or investors about the quality of information required under different methods of raising the associated finance. In that respect, the market is distinctly imperfect. Stock Exchange regulations impose severe minimum disclosure requirements for share issues to assist prospective investors to assess the future. For investments financed by retentions, no such information is required and company annual reports typically provide scant information about the intended uses of retained funds. It follows, therefore, that although the method of raising the finance cannot affect the attractiveness of

an investment proposal, the extent of investors' knowledge may vary with the method used. Thus if a positive dividend policy has the effect of causing management to retain funds which cannot be productively employed, or which tend not to be subjected to the same critical investment appraisal analysis as funds from more formal sources, the result may be to cause low payout firms to perform unfavourably by comparison with high payout firms. Or, even if management does apply the normal rigorous criteria for retained funds, if it is the practice not to communicate to the market the same quality of information concerning the intended uses of the retained funds as is required for external issues of capital, it is possible that investors tend to assume a lower reinvestment rate than is actually the case, and to cause the low payout firm, temporarily, at least, to have a lower market value than equivalent high payout firms.

These considerations alone are sufficient to make the dividend decision an important one for management. But they do not, in themselves, affect the investment criterion which the firm should use for assessing new projects. Rather, they indicate the importance of applying the existing criteria more effectively to projects financed by retained earnings, and to the need by firms to support their decision to retain funds with adequate relevant information about the intended uses of the funds. But in order to sustain the argument that dividend policy can effect the cut-off rate itself, it would need to be shown that rational investors seek a higher rate of return from projects financed by retained earnings than from projects financed by rights issues.

This is not to say that shareholders are indifferent to the firms' payout policy. But it is a reasonable assumption that

SUMMARY

The empirical tests which have been conducted to support a particular theory of dividends have been observed [6] to contain a number of statistical biases, and such evidence as there is to indicate a relationship between the firm's dividend policy and the market value of its shares is insubstantial. It has been argued in this chapter that if there appears to be a preference for a high payout policy, this preference can be attributed to a reaction against the informational deficiencies associated with internally financed projects, and to the tendency by some firms to regard retained funds as merely the residue after payout, and, as such, not requiring to be subjected to the same rigorous tests as funds from other sources. The danger of pursuing a policy of maintaining a regular, slowly growing, dividend payout policy is that earnings may be retained within the firm without any economic justification other than that they are the residue after the dividend is paid. If there are firms which are prepared to retain what is 'left over' after the desired trend of dividend payout has been achieved, whether or not there are profitable opportunities for reinvestment, it would not be surprising if such firms' performance over time is inferior to that of others which retain no earnings unless they have a profitable outlet. It is not dividend policy as such which is the cause of the inferior performance, but the decision to apply capital (whether obtained internally or externally) to nonprofitable projects or other uses which have inadequate returns.

It is concluded, therefore, that however important the dividend decision might be indirectly to shareholders' wealth, the issues

(6) Friend and Puckett, 1964.

involved are not ones which need concern management when determining the appropriate criterion for investment selection. When abstracted from taxation considerations, the choice of investment cut-off rate can be assumed to be independent of the firm's dividend policy.

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Chapter Nine

TAXATION AND THE COST OF CAPITAL

This chapter is concerned with the problems of combining the costs of debt and equity, given the tax advantages of the former, to establish what has come to be known as the weighted average cost of capital (WACC). The WACC has been traditionally used for two purposes first as a tool to choose the firm's optimal capital structure, and secondly to act as a cut-off rate in planning the firm's investments. The discussion will focus primarily on WACC as the investment cut-off rate.

Writers have failed to agree upon the best definition of WACC, and there appear to be three possible candidates:

(1) The weighted average of the cost of debt and of the before-tax cost of equity

$$w = K_s(1 - \theta) + K_i\theta$$

where K_s = shareholders' required return before corporate taxes

K_i = cost of debt

$$\theta = \frac{D}{S + D}$$

S = market value of the equity

D = market value of the debt

(2) The weighted average of the cost of debt and of the after-tax cost of equity

$$w_{vt} = K_e(1 - \theta) + K_i\theta$$

where $K_e = K_s(1 - t)$

(3) The weighted average of the after-tax cost of equity and of the cost of debt as adjusted by the corporate tax rate

$$w_{dt} = K_e (1 - \vartheta) + K_i (1 - t)\theta$$

Few writers support the use of w, although Arditti[1] recommends its use in choosing the optimal capital structure. w_{dt} is the generally accepted definition, and is almost universally advocated in finance textbooks. The purpose of this chapter is to propose w_{vt} as being the more readily comprehensible and theoretically valid definition.

In order to demonstrate the behaviour of each of the three WACCs, a numerical example is developed[2] in tables 1 and 2, based initially on the conventional assumption that the cost of equity is distinguishable from the cost of debt. We will then examine the significance of the findings in the context of the hypothesis that for practical purposes there is only one cost of capital. A project is expected to generate an annual cash flow of $\bar{X}$ = £200. The cost of a pure equity stream of the class, K_u, = 10%, the riskless rate, i = 6%, and t = 50%.

(1) See Arditti
(2) Note the example is developed within the Modigliani and Miller framework that taxes apart, capital structure is irrelevant. This is for convenience and the argument in no way depends on this assumption.

The respective values of K_e and K_i for varying levels of θ are given in table 1,(3) together with the relevant values of w_{vt}, w and w_{dt}. In table 2, the present value of the perpetuity of £200 is given for each of the three definitions of WACC for different values of θ. Because each definition produces the same present value, it appears to be a matter of indifference which of them is used. However, it will be argued in a later section that both w and w_{dt} are valid only on the assumption that the firm's tax obligation is paid concurrently with the relevant income. In this section the primary concern is which of the three methods is most consistent with the economic objective and is most likely to have intuitive appeal to the non-specialist decision-maker.

With w, the project's cash flows are overstated by being computed as if there were no corporate taxes and are then discounted at a discount rate which is proportionally overstated to compensate for this assumption. With w_{dt}, the traditional method, the cash flows are understated by not reflecting the tax advantages of debt financing and are then discounted at a rate of return which, by way of compensation, understates the market required rate of return. With w_{vt} on the other hand, the relevant cash flows

(3) How K_i varies with θ is of course unknown, and is assumed here to vary according to the equation proposed by Haley and Schall page 304

$$K_i = i + (K_u - i)\theta^2, \text{ and}$$

$$K_e = \frac{(1 - t\theta)K_u - (1 - t)\theta K_i}{1 - \theta}$$

θ	K_i%	K_c%	w_{vt}%	w%	w_{dt}%
0.0	(6.0)	10	10	20	10
0.1	6.04	10.2	9.8	18.96	9.5
0.5	7.0	11.5	9.25	15	7.5
0.7	8.0	12.3	9.29	13	6.5
0.9	9.2	13.6	9.64	11	5.5
1.0	10.0	(14.0)	10	10	5.0

Table 1 Values of w_{vt}, w, and w_{dt} for varying levels of leverage θ

θ	$\frac{\bar{X}(1-t)+t\bar{R}}{W_{vt}}$			$\frac{\bar{X}}{W}$			$\frac{\bar{X}(1-t)}{W_{dt}}$		
0	$\frac{100}{.10}$	=	£1,000	$\frac{200}{.20}$	=	£1,000	$\frac{100}{.10}$	=	£1,000
.1	$\frac{103.17}{.098}$	=	£1,053	$\frac{200}{.189}$	=	£1,053	$\frac{100}{.095}$	=	£1,053
.5	$\frac{123.29}{.0925}$	=	£1,333	$\frac{200}{.15}$	=	£1,333	$\frac{100}{.075}$	=	£1,333
.7	$\frac{143}{.0929}$	=	£1,540	$\frac{200}{.13}$	=	£1,540	$\frac{100}{.065}$	=	£1,540
.9	$\frac{175.25}{.0964}$	=	£1,818	$\frac{200}{.11}$	=	£1,818	$\frac{100}{.055}$	=	£1,818
1.0	$\frac{200}{.10}$	=	£2,000	$\frac{200}{.10}$	=	£2,000	$\frac{100}{.05}$	=	£2,000

$\bar{X}$ = £200, t = .50, K_u = .10

Table 2 Present value of perpetuity of £200 for varying levels of leverage and for each of three definitions of WACC

are the actual net of tax cash flows expected to be received, taking into account the tax-deductibility of the incremental interest payments, and the relevant discount rate is the rate of return required by the suppliers of capital. When one considers the hesitancy with which DCF criteria have been adopted in industry and the desirability, once adopted, of being able to present the results in a form which is meaningful to a broad spectrum of decision-makers, it is difficult to understand why w_{dt} has become the generally accepted method. The validity of the present value approach rests on the simple principle that the cash flows from the firm's investments should, after deducting all costs including taxes, be sufficient to meet the returns required by those who supply the capital. It is hardly conducive to a better understanding of the principle if the data is presented in the form of a series of cash flows which exclude the tax benefits of debt and which are evaluated by reference to a cut-off rate which is less than the market required return.

Indeed this method of presentation has misleadingly given rise to the notion that the cost of debt is reduced by the tax deductibility of the interest payments. In table 2, for example, the cost of capital for $\theta = 1$ is presented as 5%, when the reality is that lenders continue to require 10%. Even although debt carries with it tax advantages, the cost of debt in the pure sense of the rate of return required by lenders is independent of the firm's tax obligation. But because it is conventional to

express the after-tax flows of a project as if the project were financed by equity only, it has become necessary to capture the tax advantages of the relevant debt inputs by modifying the lenders' required return by the factor $(1 - t)$. This, in turn, has led to the practice of describing $K_i(1 - t)$ as the after tax cost of debt, when, in fact, it is no more than an arithmetic device to correct the bias in the cash flows. $K_i(1 - t)$ is the equivalent cost which debt would need to have if interest payments were not deductible for tax, provided, once again, the payment of tax is simultaneous with the receipt of the income. We are, however, unlikely to persuade unsophisticated users of the value of the DCF approach if it is necessary to accompany the conventional procedure with an explanation why it is useful to discount £100 per annum when, in fact, we expect to receive £200 at a discount rate of 5% when the market required return is 10%.

It follows then that w_{vt} is the more logically consistent of the three approaches and reflects more faithfully the underlying economic process. Its single disadvantage is that it requires an estimate of the actual tax benefits which the financing inputs of the project are expected to generate, and, counterintuitively, these cannot be determined until the project's contribution has been assessed, and which itself, of course, depends upon the magnitude of the tax benefits. To illustrate this point, assume that the project evaluated in table 2 had an initial outlay of £1,000. If the firm has a debt-equity ratio of .5 and wishes to maintain that ratio, it must finance the project outlay of £1,000 by raising, not £500, but £667 debt.

The essence of the +ve NPV rule is that a project is unacceptable unless it increases the market value of the firm's equity by more than the value of the equity employed to finance the project. Therefore, the ratio of the debt-equity inputs employed in financing the project's initial outlay must inevitably differ from the expected debt-equity ratio resulting from the acceptance of the project. It is not the incremental capital mix(4) which is the appropriate one to base the discount rate calculation upon but rather the target ratio which the incremental capital is intended to bring about.

This conclusion is not simply a repetition of the usual warning against the famous "Liquigas" fallacy of assuming that if a firm raises equity one year to finance a project, it is the cost of equity which is the appropriate discount rate, and if it raises debt the succeeding year, the discount rate is the cost of debt. The Liquigas fallacy ignores the implication of raising capital in proportions which clearly disturb the desired ratio. The point that is being made above, however, is that when finance is raised in proportions which are designed to achieve the desired ratio, the incremental proportions can seldom, if ever, be the same as those of the desired overall ratio. The +ve NPV criterion in essence implies that a project is undesirable unless it would change the capital structure which preceded it, if financed in the same proportions.

(4) Another way of expressing the same argument consistently with the intuitively appealing assumption that it is the incremental capital which is the relevant base for weighting, is to perceive the firm as raising £667 debt, explicitly to finance the project, and £666 equity, of which £333 is explicit and £333 is consequential on acceptance of the project in the form of an expected increase in the market value of the equity.

Therefore, if a firm has a certain debt-equity ratio which it means to maintain, and raises capital in the same proportions to finance the outlay of the project, its debt-equity ratio on average will be different from the desired ratio, and the WACC will be incorrect in relation to its new capital structure. Likewise, if the firm raises its marginal capital in the proportions necessary to maintain its existing debt-equity ratio, and computes its WACC in relation to the marginal mix, the WACC will again be incorrect. The correct solution is to compute the WACC in relation to the target ratio and to raise the capital in the proportions needed to secure the target ratio, which will invariably be in different proportions. The firm's financial strategy (Θ) precedes the investment decision, but the financing inputs for a particular project depend on the expected outcome of the project.

This, then, is the problem of applying w_{vt}, namely that the cash flows for which it is appropriate must include the tax benefits of the debt inputs. To overcome the problem, it is proposed that once the firm has established its optimum capital structure, it should calculate the 'effective' rate of tax on operating income and use that rate to compute the after tax cash flows.

The effective rate of tax

Although the rate of tax, t, is established for taxable income after interest payments have been deducted, the relevant income flow for most purposes in finance theory is the operating

income which, whether before or after tax, is measured before deducting interest. It has never been the practice in capital budgeting to deduct the interest payments from the annual cash flows of a project in order to assess the present value of the flow. When the government decides that the rate of tax for the taxable income of all companies is t, this is a convenient method of stating that the effective rate of tax, t_m, on operating income is different for each company according to its debt level. The process of requiring each firm to determine the proportion of its operating income which is taxable, and then applying to that proportion the universal tax rate t is clearly an essential exercise for the purpose of measuring the firm's tax obligation, but it is not necessarily the most relevant approach for the individual company seeking to evaluate its operating income flows. When $\theta = 1$ for example, the effective rate of tax is nil, and from the firm's point of view this is the relevant rate for decision-making purposes. It does not make sense, if $\theta = 1$, to state the cash flows from a project as being $\bar{X}(1 - t)$, as is the conventional practice, and to discount them then at $k_i(1 - t)$ instead of stating them to be what they are, $\bar{X}$, and discounting them at k_i. If the effective rate of tax were known before undertaking the assessment, then the after tax flows could be readily calculated and expressed in a form which reflects the realities of the situation. Thus

if t_m = the effective rate of tax on operating income

$$t_m = \frac{t(\bar{X} - y)}{\bar{X}}$$

Where y = the annual interest payments ($K_i D$) allowed for tax.

If V = the present value of the project

$$y = \theta V K_i$$

and $$(1 - \theta) V = (1 - t) \frac{(\bar{X} - y)}{K_e}$$

$$\frac{\theta K_i}{(1 - \theta)} = \frac{y K_e}{(1 - t)(\bar{X} - y)}$$

$$\theta K_i (1 - t) \bar{X} - y \left[\theta K_i (1 - t)\right] = y(K_e(1 - \theta))$$

$$y = \frac{\theta K_i (1 - t) \bar{X}}{K_e(1 - \theta) + \theta K_i (1 - t)} \qquad (1)$$

But $$t_m = \frac{t(\bar{X} - y)}{\bar{X}}$$

$$= t\left[1 - \frac{\theta K_i (1 - t)}{K_e(1 - \theta) + K_i(1 - t)}\right] \qquad (2)$$

$$= t\left[1 - \frac{K_i (1 - t)}{K_e(1/\theta - 1) + K_1(1 - t)}\right]$$

			Rate of company taxation			
θ	K_d%	K_e%	30%	40%	50%	60%
.1	6.04	10.2	28.5	38.5	48.5	58.5
.5	7.0	11.5	21.0	29.0	38.5	48.0
.7	8.0	12.3	14.7	20.8	28.5	38.0
.9	9.2	13.6	5.5	8.5	12.5	17.5

Table 3 Effective rates of tax on operating income for K_u = 10%

Using Eq (2), a table can be constructed such as table 3 to provide approximate effective rates of tax for varying values of θ, t and k_u. Now if the firm is presented with the project in table 2 and has a target $\theta = .5$, the present value can be calculated as follows:

Cash flows in perpetuity	£200
Effective rate of tax, as per table, 38.5%	77
	123
Cost of capital, per Table I	.925
Present value	£1,333

This compares with the conventional method of discounting the incorrect after tax cash flows (£100) at an artificially reduced required rate of return (7.5%). Having computed the present value, the relevant proportion of debt to be raised can readily be calculated, i.e. θ (£1,333) = £667.

Timing of tax payments

The superiority of w_{vt} over alternative approaches is more than just a matter of logical consistency. Where there is a time lag between the receipt of the relevant income flows and the payment of the associated tax w and w_{dt} cease to provide correct solutions. In the United States, prior to 1952, corporations paid their tax liabilities in instalments in the year following the year in which the income was earned. Since that date the tax is paid in instalments during the relevant year on the basis of the estimated liability. Therefore, the tax payments effectively coincide with the receipt of the income. In the U.K., however, there is a very definite time lag.

If the relevant income is assumed to be earned on average half-way during the firm's accounting year, payment of tax is deferred[5] at least one and possibly two years, with significant consequences in present value terms. Assume, for example, that the payment of tax is delayed one year. If $\theta = .5$ and $t = .5$, the after tax cash flows for the project will be £200, 123, 123, ... etc., with the result that the present value of the project is increased by £77 $(1.0925)^{-1}$ = £70 (the present value of the corresponding terminal outflow of £77 is assumed to be insignificant). According to the w method, the present value of the stream remains unchanged at $\frac{£200}{.15}$ = £1,333, because the method takes no cognizance of the delay in payment and therefore fails to capture the benefit.

In applying the traditional approach, w_{dt}, the user is recommended to express the cash flows in terms which ignore the tax deductibility of the interest payments on the grounds that adjusting the cost of debt by the factor $(1 - t)$ is intended to take care of this benefit. If the tax payments are delayed a year, the stream of after tax cash flows will be perceived by the decision-maker as £200, £100, £100, ... etc. and when discounted at 7.5% will produce a present value of $\frac{£100}{.75}$ + £100 $(1.075)^{-1}$ = £1,333 + 93. Therefore, because the conventional method is based on an overstatement of the tax obligation, the effect is to overstate the advantage of deferring the payment of the obligation, the overstatement being magnified by the use of a discount rate which is itself understated. The correct solution, on the other hand, is reached by the use of the 'effective' rate of tax to establish the actual expected cash flows and then by discounting the flows at the

(5) Apart from Advance Corporation Tax on dividends paid.

market rate of return w_{vt}.

Personal taxes

So far, nothing has been said about the implications of shareholders' personal tax position when seeking to determine the required rate of return. Under the classical system of corporate taxation, two schools of thought have developed. The one[6] contends that the cost of retained earnings should reflect the difference between shareholders' marginal rate of income tax and the capital gains tax rate. The other school[7] emphasises the practical problems of determining shareholders' marginal rates of tax and the difficulty of resolving differences in shareholders' tax rates, and advocates an 'external yield' criterion. That is, the opportunity cost for retained funds is held to be provided by the rate of return which could be achieved by investing in another company with a similar level of risk. Thus, even if there is a tax advantage for some shareholders when funds are retained in the business to finance new projects, the minimum acceptable rate of return for retained earnings should not be perceived as being less than k_e, the cost of subscribed equity, because this return can be expected to be achieved externally in another enterprise. In effect, the problem of tax differentials for investors is not relevant. This second approach is the one which has been implicitly adopted in this study.

However, a further complication is created in the U.K. by the fact that the classical system of tax which operates in the United States, and which operated in the U.K. between 1965 and 1973, has

(6) Merrett and Sykes, page 64
(7) Van Horne, page 112

since been replaced by the imputation system, which, unfortunately, does not fit comfortably into the capital budgeting framework. Under the imputation system, if the earnings after corporate taxes are distributed to the shareholders, they are deemed to be net of the basic rate of income tax in the hands of the recipients. That is to say, the effective income before tax to the shareholder is the amount which he receives from the company multiplied by the factor $\frac{100}{100 - t_d}$ where t_d = the basic rate of income tax for shareholders. This has sometimes been interpreted to mean that shareholders who are liable only to the basic tax rate, t_d, are indifferent to the distribution-retention decision, since for them no additional tax liability arises on distribution. This, of course, ignores the fact that a potential tax liability arises for them on retention. If earnings are retained and (presumably) become reflected in the price of the share, they become a chargeable gain for capital gains tax on any subsequent disposal of the shares. If, on the other hand, they are distributed as a dividend and immediately reinvested in a rights issue of the company, they become a new investment, that is a chargeable asset and, therefore, an allowable expense on any subsequent disposal of the shares.

The reality is, of course, that shareholders' personal tax positions range from tax-exemption through to very high marginal tax rates, and that the effect of the imputation system on shareholders cannot be generalised in any way. If it is assumed that all earnings are paid out to shareholders then the relevant

return for all investors can be represented as the grossed up value of the distributed earnings, whether the shareholders are liable to additional tax thereafter or to a tax rebate. Under these circumstances, the cost of equity is the required rate of return, k_e, reduced by the factor $\frac{100 - t_d}{100}$. On the other hand, if all profits are retained by the company, the return to shareholders (now exclusively in the form of capital gains) is subject to capital gains tax. As a result, the effective cost of equity will begin to approximate k_e, depending on the differences between the basic rate of tax and the capital gains rate, and on shareholders' marginal rate of tax. For companies whose payout ratio falls between 0 and 1, the effective cost of equity depends on the payout ratio, the basic rate of tax, the capital gains rate of tax, and shareholders' assumed marginal rate of tax.

Clearly, the difficulties of estimating the appropriate factor to apply to k_e are enormous, whether the risk-premium hypothesis is accepted or rejected. All that can be said is that in most cases the minimum acceptable certainty-equivalent rate of return from projects after corporate taxes will be lower than the rate of return from the default-free perpetuity. If, therefore, the decision-maker chooses to disregard the personal taxation issue and uses the default-free rate without adjustment as the relevant discount rate, he is erring on the 'safe' side.

Conclusion

The analysis in this chapter has focused on the conventional framework in which, taxes apart, the WACC needs to be calculated to take account of the individual costs of the different segments of the capital structure. However, in the context of the proposed framework in which it is argued that corporate-decision makers are justified in recognising only one cost of capital, the need to compute a WACC is eliminated. The significance of this chapter's findings are that the presence of corporate taxes and the existence of differential tax treatments for varying classes of securities does not affect the cost of capital. A methodology has been provided in which a project's after tax cash flow can be readily computed and where the market-determined discount rate remains undisturbed. Hence if $K_e = K_i$

eq. (2) reduces to

$$t_m = t\left[1 - \frac{K(1-t)}{K\left(\frac{1}{\theta} - 1\right) + K(1-t)}\right]$$

$$= t\left[1 - \frac{1-t}{\frac{1}{\theta} - t}\right] \qquad (3)$$

t_m is, therefore, independent of the cost of capital, and a relatively simple reference table, as in table 4, can be prepared to show the effective rates of tax for varying degrees of leverage.

RATE OF COMPANY TAXATION

θ	20%	30%	40%	50%	60%
.1	18.5	28.0	37.5	47.5	57.5
.2	17.0	25.5	35.0	44.5	54.5
.3	15.0	23.0	32.0	41.0	51.0
.4	13.0	20.5	28.5	37.5	47.5
.5	11.0	17.5	25.0	33.0	43.0
.6	9.0	14.5	21.5	28.5	37.5
.7	7.0	11.5	16.5	23.0	31.0
.8	5.0	8.0	12.0	16.5	23.0
.9	2.5	4.0	6.0	9.0	13.0

Table 4 Effective rates of tax on operating income for varying degrees of leverage (θ)

SUMMARY

The conventional practive of reducing the cost of debt by the factor (1 - t) for discounting purposes is based on the premise that a project's after tax cash flows should be measured as if they derive no benefit from the tax-deductibility of interest payments, and on the fiction, by way of compensation, that the return required by lenders is lower than it actually is by an amount equal to the corporate rate of tax. Apart from the difficulty of communicating the rationale of this device to non-specialist users of DCF measurements, the method fails to produce correct solutions if, as in the U.K., there is a time lag between the payment of the firm's tax obligation and the receipt of the relevant income flows. The concept of an 'effective rate of tax' for operating income was presented in order to make it possible to use the intuitively more appealing and theoretically superior approach of discounting the actual cash flows expected to be received at the return actually required by the suppliers of capital. In effect, the cost of capital and therefore the investment discount rate are unaffected by the tax benefits of debt financing.

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Chapter Ten

OTHER ASPECTS OF THE COST OF CAPITAL

We have argued that the effective cost of both equity and debt for the purposes of project selection can be taken by firms to be a rate equal to the government bond yield, and that there is for practical purposes a single cost of capital for all companies which is either one of a series of short-term rates expected to operate during the decision-maker's time horizon or, more simply, a single rate comprising the average of short-term rates expected to operate in the future. Even differences in corporate tax privileges between interest payments and dividends should not be used to differentiate between the costs of debt and equity. The tax benefits associated with the former should be reflected in the stream of cash flows expected from the project rather than in an artificial modification of the cost of debt.

There remain, however, two final aspects of the problem which need to be considered before the market cost of capital can be said to provide the decision-maker with an immediately relevant discount rate.

1. The yield on a government bond is a nominal or money rate not a real inflation-free rate of interest, and it has to be resolved whether the appropriate discount rate for selecting projects should be denominated in real or nominal terms.
2. The observed yield on government bonds does not take into account the transaction costs incurred by the investor in buying his securities or the flotation costs incurred by the company in raising new capital. The conventional practice is to adjust the basic cost of capital upwards to allow for these costs.(1)

(1) See, for example, Merrett and Sykes, page 66.

I. Inflation

In the last decade few matters can have caused such concern to the financial and business community as inflation. Surprisingly, however, significantly more research efforts have been directed by accountants to the problem of reporting financial events under inflation than have been undertaken by finance theorists to the problem of assessing the effect of inflation on the firm's project selection criterion. For example the 1975 edition of Weston and Brigham (915 pages), one of the most widely used text books in the United States, makes no mention of inflation.

It was not until 1974 that any significant attempt was made to integrate uncertainty regarding inflation into the Sharpe-Lintner-Mossin capital asset pricing model.(2) Inevitably, the effect of this modification to the traditional model is to increase the complexity of the project selection procedure under the risk premium hypothesis and to make implementation of the traditional framework more difficult. But it also has the effect of increasing the doubts expressed in this study about the necessity of postulating a risk premium over the market rate of interest.

In their paper Chen and Boness develop a project selection criterion for a value-maximising firm under uncertain inflation as follows: (3)

$$E(\tilde{R}_z) > k^*_z = R_f + \lambda^*\left[\text{Cov}\ (\tilde{R}_z\tilde{R}_m) - g\ \text{Cov}\ (\tilde{R}_z\tilde{R}_a)\right]$$

(2) See Chen and Boness, pp. 469-483.
(3) Ibid., pp. 474-475.

where $\tilde{R}_z$ = the random internal rate of return of project z

k^*_z = the cost of capital appropriate to project z under uncertain inflation

R_f = the nominal risk-free rate of return on bonds

λ^* = the price of risk in the market under uncertain inflation

$\tilde{R}_m$ = the random return on the market

$\tilde{R}_a$ = the random rate of inflation

g = W/S

W = the aggregate investable wealth

S = the aggregate market value of all stocks.

It is clear that the cost of capital under the traditional capital asset pricing model, namely

$$R_f + \lambda \text{Cov}\,(\tilde{R}_z \tilde{R}_m)$$

is incomplete, and, of some significance for our purposes, overstates the market premium for risk if uncertain inflation is expected for two reasons:

(a) because $\left[\text{Cov}\,(\tilde{R}_z \hat{R}_m) - g\text{Cov}\,(\hat{R}_z \tilde{R}_m)\right] < \text{Cov}\,(\hat{R}_z \hat{R}_m)$ implying that under uncertain inflation relevant risk is overstated, and

(b) because $\lambda^* < \lambda$ as demonstrated by Chen and Boness,[4] implying that the market price of risk is overstated.

Under conditions of inflation k the return required from any financial security is conventionally assumed to be composed of three elements, e the pure time rate of money assuming prices are stable, α the rate needed to compensate for future changes in the price level and ϕ the premium needed to compensate the holder for nondiversifiable risk.

(4) Ibid., page 476.

$e + \alpha$ of course = i the nominal market rate of interest. (5) In earlier chapters we argued that because ϕ is composed of two elements, variability risk and purchasing power risk, and since it cannot be demonstrated that the purchasing power risk of a 'risk-free' bond is materially distinguishable from the undiversifiable risk of an equity security, we cannot expect corporate decision makers to attempt to distinguish between the cost of equity and the cost of debt. In this chapter we will therefore focus on the two elements e and α and proceed on the assumption that

$$k = e + \alpha$$

On the other hand, most of what is to be said is equally relevant even in the context of the traditional risk-premium model. The question we are concerned with is whether, for the purposes of measuring a project's contribution to the value of the firm, the nominal or real return is the appropriate discount rate.

Before the present value formula can be correctly specified the assumptions about inflation must be clearly stated, that is it must be made clear whether forecasts of future sales and costs allow for price level changes. There are three possible approaches:

1. To estimate future cash flows in current monetary units on the assumption that future costs and revenues move in step with the general change in the price level, and then to discount them at the pure time rate e, excluding the inflation element .

2. To estimate future cash flows in the monetary unit of the year in which the flows are received, and then to stabilise the figures by deflating at an inflation discount rate. The resulting flows are then discounted at the pure time rate e.

(5) Strictly speaking $e + \alpha + e\alpha$, but for convenience $e\alpha$ will be disregarded.

3. To estimate future cash flows in nominal terms, that is in the monetary unit of the year of receipt, but without making any adjustment to the flows to convert them to a common monetary unit. The cash flows are then discounted at the composite nominal rate k, which includes both e and α.

In the unlikely event that the project's cash flows move precisely in step with the general rate of inflation, method 1 appears to have the virtue of simplicity. But, in fact, it involves expressing the projected cash flows in a form which does not permit them to be compared readily with the actual cash flows ultimately received. More important, however, it requires a departure from the market rate of interest. If, on the other hand, the third method is adopted, the cash flows are expressed in inflated terms and can readily be compared with subsequent flows received, and no adjustment needs to be made to the market discount rate.

If the project's cash flows are not expected to keep in step with the general level of inflation, the choice is between methods 2 and 3. There are however certain positive disadvantages to method 2, without any significant compensating advantages:

(a) It is quite possible when the inflation element α is removed from k that i acquires a negative value. We have noted in chapter 3 that there is strong evidence that during periods of inflation the market rate of interest is not permitted to rise to a level which fully reflects the expected rate of inflation. It follows under these circumstances that the expected 'real' cost of capital may be negative, and that if the cash flows are deflated at the expected rate of inflation α, the deflated cash flows would need to be discounted at a negative rate. This would clearly be quite misleading to all but the most sophisticated decision-makers.

(b) Only the effects of inflation as they affect the cash flows of the project need be calculated if method 3 is adopted. If method 2 is used, it is necessary to analyse the market rate of interest into its two segments e and α, which of course are not observable. This added complexity can be avoided if the market rate is left undisturbed and used to discount the 'inflated' cash flows.

Underlying the double discounting approach there appears to be an implied assumption that a project should have a positive real rate of return. But no such requirement is in fact necessary to make a project worthwhile. It is sufficient that the project has a higher expected return that alternative investments, and of course the opportunity return from alternative non-equity investments is captured in the market discount rate. Therefore, if the market rate of interest is 'low' in relation to expected rates of inflation, for whatever reason, even to the extent that the real return is negative, it remains worthwhile to undertake projects which have a greater nominal return than the rate of interest notwithstanding that the project fails to protect the investor fully from the effects of inflation. There is no absolute minimum acceptable real return which must be earned to make a project desirable. The only absolute minimum requirement is that a project should have a positive nominal rate of return.

Few finance text-books actually deal explicitly with the effects of inflation on the investment discount rate, but by implication they can be assumed to favour the third approach by failing to recommend making any adjustment to the discount rate. Merrett and Sykes [6] appear to be sympathetic with the

(6) Op. cit., pp. 166-167.

double discounting method, as does Bromwich. (7) Wilkes, (8) on the other hand, firmly rejects it in favour of the conventional approach on the grounds of the "computational efficiency, clarity and minimal likelihood of misinterpretation" of the latter.

Scholefield, McBain and Bagwell (9) also favour the use of a nominal cost of capital rather than double discounting, but recommend that companies should adjust the market rate of interest upwards or downwards if they feel that the market is being too optimistic or pessimistic about future inflation. But, as already suggested, if interest rates are low in relation to the company's expectation of future inflation, it is a matter of indifference whether the reason is that the market is over-optimistic or that Government and social pressures have prevented the rate of interest from rising to their full level. If management has its own expectations about future inflation these expectations should be reflected in their estimates of the project's cash flows. But the discount rate is a market-determined standard and it is irrelevant that management is of the opinion that the market standard is too high or too low. If interest rates are too high it is in the interest of investors that projects yielding less than the rate of interest should be rejected since investors could earn the rate of interest by investing in debt securities. If interest rates are too low, it is better that management should accept projects which have positive NPVs when discounted at the market rate of interest, even although, according to management's estimates, the

(7) Bromwich. 1969
(8) Wilkes, page 46.
(9) Scholefield McBain and Bagwell, page 45.

projects will fail to protect the invested capital against inflation, because the inflation protection, such as it is, is at least greater than that provided by the return on fixed-interest securities.

A numerical example will illustrate this argument. Assume a market money rate of interest of 10% during a period in which the general price level is expected by management to change by 15%, and that a company is presented with an investment costing £100 at time 0 which is expected to pay £110 in the following year, time 1, then, in real terms, the £110 has a value in time 1 of $\frac{110}{1.15}$ = £95.6. The present value, however, at time 0 is nonetheless £100. In effect, the time value of real money is approximately -4.5%. It is much more convenient to derive that present value by using the composite market rate of 10% than to attempt to break the rate down into its component parts.

A parallel can be drawn from the traditional risk premium framework. The conventional risk adjusted discount approach advocates discounting prospective cash flows at $(i + \phi)$. Abstracting from the risk adjusted discount rate versus certainty-equivalent approach as to whether it is valid to use a discount rate to allow for risk unless risk is perceived to be a function of time, nowhere is it suggested that it is desirable first to reduce the cash flows by the discount rate ϕ to their certainty equivalents and then to reduce the certainty equivalents to their present value by discounting at i. The intermediary step is unnecessary because expressing the components of the cash flow series in CE terms serves no purpose other than to provide a base for conversion into present value, a conversion which can be immediately achieved by discounting

the expected cash flows at the composite rate $(i + \phi)$. Likewise, reducing the money flows to their real value before discounting them at the pure time rate only imposes an unnecessary burden on the decision-maker of analysing the market rate into its constituent parts e and α.

The question may be asked why it is thought by some to be appropriate to adjust historic accounting reports to a single monetary unit as a means of reporting the consequences of the firm's investments decisions if it is desirable to base the decisions on a stream of cash flows denominated in a variety of monetary units. The difference of course is that in the one case the relevant income flow is profit and in the other case it is cash. The purpose of measuring annual profit is, among other things, to represent the effectiveness of the firm's use of its resources during a particular period. To appreciate the significance of the measure it is useful to be able to compare it with performances in other periods and therefore conversion into a common monetary unit may facilitate comparison. But at the original decision-making stage, establishing the series of cash flows which a project is expected to generate is no more than a step in the evaluation process. The cash flows computed for a particular period are not measures of the project's performance during that period, and there is no purpose in seeking to compare one year's flows with another's. The only economic significance of the series for decision-making purposes is its present value, which is effectively and conveniently arrived at by discounting the actual cash expected to be received at the market money rate.

In conclusion, then, inflation is a consideration which cannot be ignored in the investment selection process. It affects both

the discount rate and the projected cash flows. But its effect on the former is determined in the market place and it is unnecessary to depart from that rate. The relevant decision variable for management is the significance of inflation as it affects the particular cash flows of the project, and although this is clearly a difficult decision, it is not one which involves modification of the basic selection procedure.

II. Flotation costs

In this section we consider the impact of investors' transaction costs and company flotation costs on the cost of capital. It will be argued that the conventional practice of revising the basic cost of capital upwards to take account of transaction and flotation costs is invalid and leads to incorrect solutions.

Assume that a company raises £1,000 to finance a project costing £1,000 and that the company incurs £50 flotation costs. Assume also that the basic cost of capital is 10%. Textbooks (10) recommend that the basic cost of capital k should be increased to reflect the percentage which the flotation costs bear to the capital sum raised. If c = the percentage cost of flotation expenses, then the adjusted cost of capital k* is such that

$$k^* = \frac{k}{1-c} = \frac{.10}{1-.05} = 10.5\%$$

(10) Merrett and Sykes, page 66; and Weston and Brigham, page 603. Note: Weston and Brigham mistakenly state that $k^* = \frac{D}{P_o(1-c)} + g$, implying that the lower the current dividend is, the lower is the cost of equity. But if P_n = the net price received by the firm, then $P_n = P_o(1-c)$. Weston and Brigham fail to observe that if P_o is expected to grow by g then P_n must be expected to grow by a greater rate, viz. $\frac{g}{1-c}$ therefore,

$$k^* = \frac{1}{(1-c)}\left(\frac{D}{P_o} + g\right) = \frac{k}{1-c}$$

Thus if the project were expected to generate £105 in perpetuity the NPV would be $\frac{£105}{.105}$ - £1,000 = 0, implying that sufficient flows have been generated to meet the annual cost of servicing the capital raised plus the amount needed to recover the flotation costs of £50.

An alternative procedure, which will be shown subsequently to be the only valid one is to treat the flotation costs as additional outlay and to leave the cost of capital undisturbed. Thus:

Initial Outlay (£1,000 + £50)		£1,050
Annual cash flows	£105	
Discount rate	.10	
Present value		£1,050
	NPV	0

It would appear to be a matter of arithmetic indifference which of the two approaches is adopted. However the former traditional approach is valid only under the very restrictive assumption that the project has a NPV = 0. This can be illustrated in the above example if we assume that the project generates a perpetuity of £157.5. The NPV according to the traditional approach is

$$\frac{£157.5}{.105} - £1,000 = £500$$

According to the alternative method proposed the NPV is

$$\frac{£157.5}{.10} - £1,050 = £525$$

The NPV under the second method is the correct solution. The reason is that adjustment of the discount rate is merely an arithmetic device which ceases to be valid if the present value of the cash flows differs from the initial outlay, that is if the NPV is positive or negative. Raising the discount rate has the effect of penalising the cash flows. In the example the rationale of the device is to

penalise the flows to the extent of £50 in present value terms, namely 5% of £1,000. But because the cash flow stream is such as to have a present value greater than £1,000 then the upwards adjustment of 0.5% on the basic rate will exact a penalty greater than £50. In the example the excess penalty is £25 which is 5% of the NPV £500.

To use the traditional device correctly it would be necessary to compute c by relating the flotation costs of £50 to the present value of the project, £1,575, that is 3.2% approximately. Using this rate to discount the cash flows the present value = $\frac{£157.5}{.1032}$ = £1,525. Since however, the adjusted discount rate can only be calculated by reference to the present value calculated on the basis of the unadjusted market rate, it is clearly impractical to use the adjusted discount rate approach. It follows that flotation costs (11) should be dealt with as additions to the capital outlay and do not necessitate departure from the observed market rate of interest. This is consistent with the practice recommended in earlier chapters of treating bankruptcy costs and the tax benefits of debt financing within the cash flow stream rather than by adjusting the discount rate.

(11) For this purpose, brokerage fees etc. incurred by the investor can be dealt with in the same way as flotation costs incurred by the company, that is, as an addition to the capital outlay.

SUMMARY

Inflation adds significantly to the problems of forecasting future cash flows, although it is not necessarily a greater source of difficulty than forecasting other uncertain events. In recent years it has in fact dominated financial affairs, and it is not possible to examine its full implications within a single chapter. But it has not been the purpose of the chapter to consider the causes of inflation or its cures, or to examine the problems of reporting financial events which have been affected by it. The purpose has simply been to consider whether the existence of inflation calls for a refinement of the market rate of interest in establishing a yardstick for measuring investment worth.

It was concluded that no such adjustment is required and that the conventional practice of expressing a project's cash flows in the monetary units of the year in which they are received, and then discounting them at the market rate, including the inflation element, is well-founded. The alternative procedure of expressing future cash flows in a common monetary unit and of discounting these stabilised flows at what is perceived to be the 'real' rate of interest was rejected. Furthermore, the rate determined by the market mechanism was found to be appropriate even if the company's decision-makers take the view that such a rate indicates an under- or over-estimation by the market of the future rate of inflation. The NPV criterion is concerned with the contribution to value in relation to the actual return by the suppliers of capital and not with the return which might have been required by them had circumstances been different.

The chapter also considered the validity of the accepted practice of adjusting the market rate of interest to take account

of flotation and transaction costs incurred in raising capital. It was concluded that this leads to incorrect solutions and that if instead such costs are treated as an integral part of the capital outlay the correct NPV can be found by discounting the cash flows at the market rate without making any adjustment.

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Chapter Eleven

CORPORATE DISCLOSURE POLICY AND THE COST OF CAPITAL

Corporate managers are intermediaries between the holders of the company's financial securities and the investment of capital in real productive assets. As far as possible, it would seem desirable that the knowledge and future expectations of the one group should be communicated on an ongoing basis to the other. That is to say the flow of financial data from the management group reporting past and future estimated returns from the firm's projects should be reciprocated by a communication from the shareholder group of their required cut-off rate for marginal projects. In reality, of course, a constantly changing heterogeneous body of shareholders is unable to articulate its required returns in precise terms and to communicate them to management, and therefore the information flow tends for the main part to be one way.

The preceding chapters have been largely concerned with finding a solution to management's problems of discovering investors' required return when no formal communication system exists, and by necessity this has involved searching for a method of interpreting observable market signals which does not place unreasonable demands on management's analytical ability. The hypothesis was presented that the interest rate on Government bonds has a valid claim to be the most effective yardstick for this purpose. This chapter is concerned with the contribution which management can make to the validity of this claim in particular and to the investment process in general by providing investors with the data relevant to their needs.

It is recognised that the holders of the firm's securities are only one of a number of user groups, and the observations which are made are not intended to be general but rather to focus on the effect that the supply of information can have on investors' requirements since it is their needs which are relevant to the determination of the discount rate. The term investors in this context includes nonequity investors, because the provision of adequate information to them is no less important in influencing their required return than it is for equity investors.

A number of factors operate to inhibit management from implementing fully its information-provision role, but two factors are especially relevant. Managers are exhorted to pursue a policy of maximising the value of the company's shares, and it is understandable that at times they should hesitate to disclose data, however relevant, which might have the effect of adversely affecting the price of the shares. The hesitancy, for example, of the great majority of companies to present statements indicating the effect of changes in the purchasing power of money during periods in which such changes have been substantial must in part be due to the fear of a possible depressing effect on the share price of reporting lower earnings. The second factor is that because information is addressed to a heterogeneous group of shareholders, some assumption about the level of sophistication in financial matters needs to be made. Traditionally, the weight of opinion has appeared to favour restricting the quantity and quality of information to the level of interpretative skill of the 'reasonably informed investor' (1) rather than the expert. (2)

(1) See, for example, Eldon S. Hendriksen, page 561.
(2) However, see the Report of the Committee on Research Methodology in Accounting, pp. 426-427.

There is therefore a potential source of conflict between the fundamental need to provide investors with the information they need to select their portfolios, and the desire by management to protect investors from falls in the market value of their shares or of being disadvantaged in relation to more sophisticated users. The significance of this potential conflict has, however, considerably declined in the light of modern capital market theory, and it will now be contended that the desire to protect investors from the effects of unfavourable or excessive disclosure can no longer validly be used as a reason for withholding relevant data. As a precondition, however, it is essential to recognise that it is no longer possible to pursue shareholders' welfare effectively on the assumption that their interests are exclusively tied up with the destiny of one company's shares. It is, of course, not practicable and would probably not be useful for management to ascertain precisely how the portfolios of their shareholders are constituted. But if we assume that investors have some degree of aversion to risk, it is sensible to operate on the basis that the shareholders hold diversified portfolios composed potentially of every security in the market. It follows that management must recognise that shareholders' welfare is dependent upon the efficiency of the market. In reality, of course, investors tend not to have portfolios so widely based, partly because they may not be as averse to volatility as the conventional framework assumes and partly because it is possible to achieve a high proportion of the diversification effect with a relatively small number of securities.(3) Therefore to construct a surrogate market portfolio it is important for the investor to be able to make a judgement about the desirability of potential candidates, in particular to satisfy himself that

(3) See Francis and Archer, page 155.

(1) the price of the shares is fair and does not overstate their value in relation to expected returns,

(2) the company's dividend policy meets his particular income preferences

(3) the covariance of the share's returns with other potential components is such as to achieve his desired goal

(4) the social and environmental policies of the company are acceptable to him to the extent that he would wish to be associated with the company.

It is naive to assume that once an investor has formed his portfolio his primary interest is to see the value of his shares raised even beyond their 'intrinsic' value. In order to maintain the balance of his portfolio, the investor needs periodically to revise his holdings and to sell securities which have disproportionately increased in size and correspondingly to augment his holding in other securities to restore the balance. This periodic adjustment process makes it important to him that the exchange price is fair. He is both a buyer and a seller, and because the share price is the entry value for the purchaser in addition to being the exit value for the seller it is in the investor's best interests that the capital market is adequately served with the information needed to establish a fair value, and not with information designed to create as high an entry price as possible to the newcomer. The only investors who can reasonably object to that criterion are those who propose to disinvest and to cease to be participants in the capital market.

The desire to be fair to all investors has undoubtedly influenced the development of accounting disclosure practices and deterred managers at times from disclosing data which might yield an advantage to the skilled interpreter. In order to demonstrate that it serves

shareholders' interests better that companies publish all information which contributes to the efficiency of the market rather than to withhold data which might be beyond the understanding even of the reasonably informed investor, it is necessary to take account of the evidence in support of what has come to be known as the efficient market hypothesis.

The efficient-market hypothesis

There is a strong body of empirical evidence to support the hypothesis that the market for securities is efficient in the 'semi-strong' form [4] that is, that all publicly available information is instantaneously impounded in the share price, and that the analysis of published information cannot permit an investor to achieve returns in excess of a naive buy-and-hold strategy, using a dart for selection. There are no superior trading rules and therefore the search for undervalued or overvalued securities will not yield a higher than average return, except by short runs of good luck. Only if investors are privy to inside information can abnormal returns be achieved over the long term.

There has been and undoubtedly will continue to be a marked resistance, outside the academic world, to accepting the evidence of the efficient-market hypothesis. Indeed the securities market's infrastructure has been erected on the basic assumption of the market's inefficiency in the semi-strong form, insofar as the vast number of fundamental analysts, chartists, stock-market periodicals etc. all bear witness to the entrenched belief that the collection and skilful appraisal of published data can assist investors to discover and profit from transacting in under and overvalued stocks.

(4) For example, see Fama, 1970.

Despite several studies published over a period of years which indicate that investment institutions are unable to achieve superior returns over the long term, the fact that over short periods of time some institutions (and individuals) must inevitably perform better than others makes the acceptance of the long term statistical evidence difficult for market participants. It requires a heroic effort on their part to approach with the necessary perspective and independence of mind a body of evidence which supports the view that their judgement can do no better than chance.

For the purposes of this chapter, the findings of the efficient-market surveys are accepted as being substantially validated. That is not to say that it is essential that one should believe that no individual exists or might exist who can on average outguess the market in identifying undervalued or overvalued securities without the benefit of insider information, but simply that the evidence points firmly to the rarity and exceptional nature of such individuals, if they do exist.

Thus although corporate financial reports are conventionally addressed to the company's shareholders, their primary function is to provide the market with data needed to establish the price of the shares. The conventional notion of providing information for investors to evaluate the shares and to compare their valuation with the market price to produce a basis for their buy-sell hold decisions has ceased to have any real significance in the face of the efficient market evidence. By the 'market' in this context is meant the stock exchange jobbers who adjust the share price in response to new information, and the sophisticated investors who act instantaneously on publication of the data. Since their

analysis and response effectively adjusts the price to its 'intrinsic' value, it follows that they are the primary users of accounting data. It might be asked what advantage is there for investors to press for more disclosure of information if it will be instantaneously reflected in the share price. Clearly to the investment analyst seeking to outguess the market there is no advantage, and to the privileged insider there is a positive disadvantage. By providing more complete information more quickly the market price of the shares at any time will represent not only the 'intrinsic' value in relation to all published information, but more closely to the value given by all potential information. As the published data becomes more technical and complex, the gulf which separates the naive outsider from the informed outsider is widened. But, equally, as it becomes more complete, the gulf which separates the privileged insider from the informed outsider is narrowed. There is no advantage for the naive outsider to demand that published information should be tailored to suit his limited understanding. The more efficiently the price-setting mechanism operates, the greater assurance has he that the market price is fair, and the less need for him to consider whether the share is over or undervalued in relation to other securities.

If it could be demonstrated that the market was not so efficient as to cause prices to adjust instantaneously to new information, then the publication of complex technical data could be viewed as conceding an advantage to the specialist investor. But such an advantage would be preferable from the point of view of the ordinary investor than the alternative of maintaining an advantage for the unscrupulous insider. Withholding relevant data which because of its technical nature might have given privileged and profitable insights into the intrinsic value of the share to a few specialist investors cannot

actually benefit less skilled investors or enable them to make more useful evaluations of share prices. Their welfare would still depend on the allocational efficiency of the capital market. The granting of a reward to investors with specialist knowledge legitimately acquired is a small price to promote that efficiency. The long term interests of the ordinary investor are best served by an improvement in the price-setting mechanism, combined with a clear recognition of his own analytical limitations and that if he wishes to engage in serious investment rather than speculative ventures he should do so under the guidance of an informed adviser.

Since it is the sophisticated interpreter who determines market share prices it follows that it is his level of comprehension which is relevant. Any effort directed towards making accounting statements understandable to the average or even to the well-informed investor (for example, by providing simplistic data such as earnings per share) may do a disservice to him by implying that there is a shortcut method of carrying out the highly complex process of valuation more efficiently than the market.

There is evidence to suggest that information processors' judgement is diminished beyond a certain level of complexity. H. Miller, (5) for example, has expressed concern that if tests carried out by behavioural scientists can be shown to be applicable to the financial reporting environment, there may be an optimal loading of financial data beyond which there is tendency for the user to resort to fixed decision rules such as price-earnings ratios with a consequent deterioration in decision-making. If it can be assumed that the optimal level of complexity varies with the decision-maker's capabilities, there are significant implications for disclosure policies. Miller observes, (6)

(5) H. Miller, 1972
(6) Miller, page 35

"If the theorist is to design an optimal environment for financial reports and optimal levels vary for different processors, which optimal level is he to choose as his goal? The choices range from the most concrete to the most abstract processor with perhaps some average as an inviting possibility."

Miller's choice of processor fortunately accords with the implications of the efficient market hypothesis, and the most skilled investor is selected. "It seems intuitively wise to select the most abstract investor's decision model for financial reports. The selection of a more concrete processor .... would deprive the sophisticated investor of information which might be crucial to his decision" and because of the widespread use of analysts' advice "it appears likely that the creation of an information environment that is optimal for the analyst will serve other investors as well."(7)

Miller concludes that because more sophisticated users of financial statements have made repeated requests for additional information we have yet to exceed the analyst's optimal conceptual level and can therefore probably follow a data expansion policy now. However, he warns, "we will eventually be limited and continual haphazard expansion will inevitably cause superoptimality and a reduction in the usefulness of financial reports for decision-making."(8)

Whilst endorsing Miller's concern about the dangers of haphazard expansion, doubt must be expressed about his anxiety concerning the provision of excessive relevant data. The only effective evidence that relevant data is excessive is to show that market efficiency has diminished as a result of the increased information load, and that market share prices have ceased to approximate the intrinsic values as determined by published data. But that would presuppose the existence of a sufficient number of sophisticated analysts who could process the increased informational load to determine the

(7) Miller, page 36.
(8) Miller, page 37.

'correct' intrinsic value and so secure an insight from which they could profit and, in so doing, eliminate the disparity between the market price and the intrinsic value. It is true that as the informational load is increased some processors of a lower conceptual level than others would cease to rank among the body of 'efficient' conceptual processors, but provided the price of shares continues to be determined by the most efficient information processors, the loss in status by the former processors would affect only their responsibility to recognise their limitations. If the increased load diminished market efficiency in the sense that there developed a time-lag in what had previously been an instantaneous adjustment to new information, then, as stated earlier, such a concession to skilled analysts would be preferable to the alternative of maintaining a privilege for informed insiders and of reducing the efficiency of resource allocation.

Conclusion

In the preceding chapters it was contended that because investors are to a large extent capable of exercising control over their risk-exposure by constructing and monitoring their portfolio so as to reflect the degree of volatility which they desire, it follows that their role in the company's investment decision process is more active than has been traditionally assumed. It is only a matter of a few years since text-books (9) taught that one of the goals of financial management should be to combine its investments in such a way as to minimise the overall return required by its shareholders. This exercise

(9) For example, see J.C. Van Horne, 1st edition 1968, page 483.

is no longer seen to produce any benefit for shareholders who are now assumed to be able to carry out the diversification more efficiently. (10) There may of course be other good reasons why a measure of firm diversification is desirable, but the fact remains that the impact of diversification on the investment discount rate is determined by what shareholders do, or more precisely can do in their portfolios rather than what management do.

It follows that management has a responsibility to disclose any information (other than commercial secrets) which is material and relevant to shareholders' portfolio needs, that is to provide the market with all data, however technical or apparently incomprehensible to the 'ordinary investor', (11) which affect the market value of the shares and the allocation of capital to the company, and to provide the individual investor or his adviser with

(a) a statement of the company's future dividend policy,

(b) information which will enable the investor to place the shares in a 'covariance' category. This might include a breakdown of sales into product lines, an analysis of the firms activities into geographical or economic regions, and possibly a probabilistic analysis of future returns,

(c) a profile of the firm's social and environmental policies.

There are, of course, practical difficulties in implementing these recommendations, but it is on the basis of this kind of

(10) See Haley and Schall, page 195.

(11) This conflicts with the Accounting Standards Steering Committee's publication 'The corporate report' 1975, which indicates that the information should be directed at the 'reasonably instructed reader' (page 29), although elsewhere (page 22) it is recognised that the existence of an analyst-adviser group fulfilling an interpretative function will 'lead to a demand for more elaborate information than otherwise would be the case'.

information flow that the investor is best equipped to select a portfolio which he finds acceptable in relation to his time-horizon and risk-tolerance level.

SUMMARY

The fundamental lesson of portfolio theory is that rational investors without inside information or special insights will hold well-diversified portfolios. As a result, corporate shareholder objectives need to be framed in terms which take account of the fact that the company's shares are a constituent element of a portfolio. The implication of the previous chapters is that shareholders have a more positive role in the investment process than has hitherto been attributed to them insofar as the risk dimension of investment decisions can be left to them to regulate within their own portfolios, thus enabling management to select the firm's investments on the basis of expected return only. But in order to implement this role effectively investors have a corresponding right to receive from management the information needed by them or their advisers to evaluate existing and potential candidates for their portfolios, in particular to be able to form a judgment about the covariance characteristics of the company's expected returns. If necessary, this may require the publication of probabilistic data and details of the firm's various activities. The disclosure of information should not be inhibited in any way by the fear of placing less sophisticated investors at a disadvantage, or of adversely affecting the market value of the shares, because in the framework of an efficient market where information is observed to be impounded instantaneously in the share price, portfolio-holders' welfare is better served by having an adequate flow of relevant information from companies to the market to enable it to perform its dual function of establishing a fair exchange price for the shares, and of providing a vehicle for allocating capital to the

most profitable users. The more efficient corporate disclosure practices are the more effectively can investors set about constructing portfolios which accord with their individual income and volatility preferences, and in so doing carry out their function of simplifying the investment selection process for management.

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Chapter Twelve

CONCLUSION

The original purpose of this thesis was stated to be concerned with the communication of the economic principles of investment appraisal to those who have the responsibility of translating them into operational terms. It has been argued that, although theorists have developed an elegant framework for making investment decisions which seeks to be exhaustive, significant aspects of the procedures recommended can only be applied on the basis of guesswork or arbitrary rules of thumb, with the result that the economic principles are vulnerable to outright rejection by those who take the view that the proposed procedures are in fact inoperable. It has been the specific objective, therefore, to try to develop guidelines which can be substantially reconciled with theoretical and empirical data, and which at the same time are readily understood and capable of implementation without undue difficulty. This final chapter, therefore, seeks to integrate and summarise the findings of the previous chapters without, as far as possible, resorting to financial jargon or mathematical notations, in an attempt to convey the basic principles of the argument in a form which is at once acceptable to the specialist and, at the same time, intuitively accessible to the nonspecialist, and which will enable the latter to participate meaningfully in the decision-making process.

SUMMARY

Under conditions of certainty, the return required by those who supply the capital to finance industry and commerce would be a single rate of interest, observable in the market, and common to every security whatever characteristics the security might have. Differences in security types would be essentially related to differences in the manner in which the return is achieved. Management, therefore, would use this unique rate of interest to evaluate prospective investments, and projects would be accepted if the yield which they offered equalled the interest rate, or, equivalently, if the contribution (net present value) which the project made to the firm was non-negative. One of the consequences of the assumption of perfect certainty is that if some innovation is undertaken which appears to give the innovator the opportunity to achieve excess returns, that is, to obtain a +ve NPV, competition from other sources, the unions or competing firms, would eliminate the excess returns instantaneously. The opportunity, therefore, to make excess returns would not present itself, and the basic criterion of selection would be that the internal rate of return equals the required rate of interest, k, or the project's net present value equals 0. It would be a matter of indifference which of the two techniques, IRR or NPV, were used to make the selection. Moreover, the goal of business enterprise would be simply to achieve the required return for the suppliers of capital, and not, as is generally assumed, the maximisation of wealth (in the sense of maximising projects' +ve NPVs)

Under conditions of uncertainty, perfect competition does not in fact exist, and the possibility of discrepancies between the returns required by lenders versus equity-holders, the suppliers of capital versus labour, and one firm versus another, becomes a reality. Since virtually no business can operate without capital, the western capitalist system has developed on the basis that, although there are a number of groups which are interested in the outcome of business enterprise the primary goal of business is concerned with making profit. It might be that society is in the process of change to the extent that the profit goal will cease to be a significant consideration, in which case a different framework from that presented by finance theorists may need to be developed. But on the assumption that the decision to bring a business into existence, or to terminate its life, rests with the suppliers of capital, investment decision theory has developed on the basis that the primary goal should be directed towards furthering the interests of those who supply the residual capital. One of the consequences of imperfect competition is that the division of operating 'profits' between labour, creditors and equity cannot be achieved with perfect harmony. The equity-holder has been perceived to have a unique relationship to the other factors of production in that he can propose not to advance his capital to any business unless the return expected is at least equal to a certain minimum after satisfying the claims of the other claimants. Hence the rule of acceptance has evolved which states that those who are entrusted with equity capital, namely

management, should not accept projects unless their return is expected to exceed the amount necessary to satisfy all factors of production, that is, unless the IRR $>$ K or NPV $>$ 0. In effect, the decision rule states that projects are unacceptable unless they are expected to yield more than is necessary to satisfy the suppliers of equity capital. Writers have presented this rule as appropriate even under conditions of certainty, but it is clear that the rule depends on uncertainty to have any significance. The rule has also been elevated by theorists into the assumed objective of business enterprise, namely the maximisation of shareholders's wealth. But this too clearly depends on the existence of uncertainty and imperfect markets, because the opportunity to earn excess returns and therefore to add wealth (+ve NPVs) to the value of the firm would not exist with certainty.

Given uncertainty and imperfect markets, therefore, the wealth-maximisation objective and the excess-return decision rule have become accepted as axiomatic in finance theory. However, even if the overall required return were known, it ceases to be a matter of indifference whether the IRR or NPV approach is used to select projects. Either method can be used to signal that a project is acceptable, but if ever a choice has to be made between projects, the NPV method alone is valid. If one perceives the IRR as the highest cost of capital which a project could sustain without making a loss, it begins to become apparent that a project with a higher IRR than another is not necessarily preferable.

It may be an interesting statistic to know that if the rate of interest had been much higher, the project would still have been acceptable, but the relevant question is, given that K_o has the value it has, what contribution does the project actually make to the value of the firm. It is true that the IRR technique is used in the stock exchange to differentiate between securities, but there is an essential difference. It is possible to invest virtually any desired amount of capital in any security for any length of time, and therefore the rate of return per unit of capital invested initially can be used as a guide to the relative desirability of the alternative investments. But with real investments in machinery, buildings, etc the amount of capital is not a matter of choice, nor is the lifespan or pattern of the cash flows. Alternative real investments nearly always require different inputs of capital, and £(p) invested more 'profitably' than £(p+q) is not necessarily more desirable. What is more, the amount of capital invested in a project cannot be determined by reference to the initial outlay alone, but depends on the pattern of the cash flows and the length of time involved. If a project differs from another in relation to any one of these aspects, the projects can be assumed to have different amounts of capital invested in them, and the IRR ceases to be valid. It follows that if the goal is to contribute wealth to the shareholder, the selection of projects should be made on the basis of the project or combination of projects which make the greatest contribution (+ve NPV).

Application of the NPV technique, however, is not the straightforward process implied under conditions of certainty. It is first necessary to apply a probabilistic approach to determine the expected stream of cash flows. Secondly, and a much more problematic issue, the pure rate of interest may not be the appropriate discount rate for an uncertain stream of cash flows, if investors require some form of compensation to induce them to accept the risk of variation from the expected return. On the other hand, it must be borne in mind that the implication of the +ve NPV rule is that acceptable projects are expected to earn more than the discount rate and it may be that the simple prospect of achieving more than the pure rate of interest is sufficient to induce investors to purchase equity securities.

If there is reason to believe that the prospect of participating in the excess returns is not sufficient inducement, it may be necessary to adjust the pure rate of interest for each project under review, according to the project's risk characteristics. In practice, however, it is a very difficult task to decide precisely what adjustment should be made, because the price of risk cannot be observed in the market. One recommended solution is to examine the returns achieved by companies engaged in a similar operation, but this presents a number of problems. Firstly, it is unlikely that there exists a company whose share returns are determined by a single project. Secondly, if management accepts only projects which are expected to earn more than the minimum required return, it would seem that the only assumption that could reasonably be made about the returns actually achieved in the past is that they exceed the required returns.

Thirdly, it may not be that what investors wanted, on average, in the past is relevant to what they want today. For example, in view of the significant increases in the default-free rate of interest in recent years, it is doubtful whether analysis of the returns of any securities in previous years is relevant to today's conditions. In addition, the capital markets have undergone significant changes in recent years, which may have affected investors' attitude to risk. Not only is the portfolio effect more widely recognised as a risk-reducing operation, but the opportunities to diversify have been enormously enhanced by the development of intermediary institutions such as unit trusts, pension funds etc. Investors may not always choose to avail of this facility to diversify so widely, but if they do not, it is reasonable to assume that it is because they do not fully appreciate the portfolio effect, or because they prefer not to diversify all risk away. If we accept the proposition that the only risk which matters to an investor is the risk which he cannot diversify away as distinct from the risk which he chooses not to, then it is probable that investors' perception of risk is undergoing constant change.

Clearly, then, the derivation of the correct discount rate is an exceedingly complicated issue, so much so that it has become the central issue of finance theory. Because the solution is so inaccessible, the fundamental question has to be raised whether it is necessary or even responsible for theorists to expect practitioners to undertake the search, when so little practical guidance is available to them. If theorists cannot agree on the size of

risk premium for the market portfolio, may it not be that its size, if it exists at all, is insufficiently significant to merit imposing on practitioners the burden of searching for it? For the risk premium to be significant, it must be shown that a world-wide portfolio of risky assets, including shares, bonds, land, property, etc., is significantly riskier than a government bond, bearing in mind that risk must be perceived in terms which take account of inflation. Moreover, when making this judgment, it is necessary to be certain that investors are intolerant of any degree of risk without compensation.

The evidence for the existence of risk premiums is, in fact, far from conclusive. In the bond market it is generally assumed that because interest rates appear to vary according to the risk status of the borrower, this bears witness to the existence of a premium for risk. This, however, is an unwarranted conclusion. The promised rate of interest does vary with risk, and must do so, even if it can be shown that lenders are indifferent to risk. It can, however, be demonstrated that the expected rate is independent of the nominal interest rate charged. Given that lenders have substantial opportunity to diversify risk away and that they receive sufficient data to determine the appropriate promised rate of interest, there is no reason to believe that the return required by lenders significantly exceeds the default-free rate.

The evidence in the equity market is no less inconclusive. Theorists have based their belief in the existence of risk premiums on the observation that shares have on average earned more than the default-free return. But it has already been noted that this superior performance of shares is virtually an inevitable effect of the NPV rule, where management accepts only projects which are expected to exceed the default-free rate. A number of empirical studies have indicated that higher returns can be expected from shares with a high beta or risk factor, that is, a high correlation with the market portfolio of shares. This has been interpreted to confirm the risk premium hypothesis. Apart from the fundamental deficiencies of past data observations as outlined above, two additional points need to be made. One recent major empirical study(1) analysed portfolio performances into subperiods and the results indicated a steady decline in the significance of the beta factor. Indeed, beta appeared to have no influence at all in recent decades, implying that differences in volatility do not enter into investors' calculations. Secondly, the fact that achieved returns appear to have been consistent with the beta factor does not establish that this relationship exists for ex ante required returns. It is probable that management has a higher degree of risk aversion than investors have, because of the former's inferior opportunities for diversification. If this is the case, the degree of competition amongst firms in industries which have a high degree of risk may be less severe

(1) F. Black, M. Jensen and M. Scholes, see chapter 5

than in less volatile industries. Hence the returns obtained from companies in the risky group might be expected to be greater than in the safer group. If shareholders' required returns were in fact unaffected by differences in risk characteristics, the typical project of the risky firm would have a higher +ve NPV than the projects of the safer firms. Ex post analysis, however, reveals only the total earnings including the basic required element and the excess element. Since there is no recognised method of distinguishing between the two elements, the fact that the total returns of the risky firms are observed to be higher is insufficient evidence that the basic required element itself is higher. To the extent that companies have become larger and more internationally diversified, the level of undiversified risk is likely to have declined for management as well as shareholders. Hence the apparent significance of beta and its subsequent decline can equally be attributed to management's attitudes to risk as they can to shareholders attitudes.

Finally, for the purposes of the present study, interviews were conducted with a group of investors to find out whether their perception of risk conformed to the conventionally assumed pattern. The results strongly supported the view that investors do not perceive a well diversified portfolio of equity securities as being any riskier in real terms than a default-free bond. The investors, moreover, expressed a willingness to absorb a certain amount of volatility without any risk premium, and indicated that they would accept the default-free rate as the minimum cut-off rate for risky projects, especially if they held a well diversified portfolio.

The relevant default-free rate

There is, unfortunately, no single default-free yield which provides a totally satisfactory rate for discounting capital projects. A wide range of default-free rates can be observed to exist, varying from short-term to perpetuity rates, and therefore there is a wide range of potential discount rates. Likewise, although it is the practice to speak of the return on the market portfolio, there is in fact no single market return. The return required by investors in the year ahead is unlikely to be the same as the return required for the two year period ahead, and still less likely to be the same as the ten year period. If any single bond rate can be said to correspond to 'the market portfolio' rate it is the consol or perpetuity rate because that alone reflects the average of all future short, medium and long term rates expected to operate during the presumed perpetual life of the market portfolio.

On the other hand, when translating the market required returns into discount terms for evaluating particular projects, account must be taken of the fact that most projects occupy only a finite period in the market's perpetual flow. The only theoretically correct approach is to discount each cash flow of the series at the one year bond rate expected to operate during that year. This approach has, of course, never been strongly advocated because of the very onerous task of predicting future short-term rates. Hence, the use of a single average rate is almost universally recommended. In chapter 6, however, we rejected the conventional practice of

computing such an average on the basis of the maturity structure of the financial instruments which the company happens to have issued because such an approach implies that a project's desirability depends on the individual company's financial strategy rather than on an extraneous, objective yardstick. The choice must therefore be between using the yield of a government bond which matches the maturity of the project, or choosing a single bond rate which represents the expected average maturity of all projects, presumably the long-term bond yield if we assume that the average capital investment is long-term.

Both these approaches of course suffer from the disadvantages associated with any averaging process and have the effect of penalising early cash flows and of favouring later flows during periods of expected interest rate rises and vice-versa during periods of expected interest rate falls. The use of a matching bond yield has an intuitive appeal, but it involves selecting a different bond yield for every project with the result that the cash flows in year 1, for example, of two competing projects will be discounted at separate rates according to the length of time over which the subsequent cash flows happen to be extended. In addition, the selection of the appropriate bond must be made with care to avoid a yield which is distorted by capital gains tax considerations. The choice of a single long-term bond yield for all projects is crude but has the virtue of simplicity and of being relatively freer from capital gains tax distortions.

Whichever approach is adopted, it must be borne in mind that the problems present themselves equally within the risk-premium framework, although they are obscured by its overall complexity.

Indeed the problems are magnified if an appropriate risk-premium has to be calculated for each maturity class. The issues therefore do not affect the fundamental thesis that the default-free rate is the appropriate one for evaluating risky projects.

RECOMMENDED PROCEDURE

The following are the basic steps necessary to evaluate a capital investment proposal:

(1) The initial cost outlay of the project should be measured including the flotation costs incurred in raising the funds needed to finance the project.

(2) The expected net cash flows resulting from the project and from other projects uniquely associated with it should be computed by

   (a) estimating the after-tax returns, taking into account the level of leverage employed, under different states of the economy (including the state of bankruptcy with its associated costs) where the returns are denominated in the monetary unit of the year in which the cash returns are expected to be received

   (b) assigning a probability to each state's occurrence, and

   (c) obtaining the weighted average of the possible states to arrive at the expected value of the cash flows.

(3) The expected net cash flows should then be discounted at a rate equal to the relevant government bond yield. The market rate should be used in its pure state without modification to take account of the project's risk characteristics or the decision-makers' estimate of the future general rate of inflation

or the firm's dividend policy, or any of the other items underlined above, each of which has variously been held in the capital budgeting literature to necessitate adjustment of the market rate, but which for reasons advanced in the text are best dealt with in the cash flow stream or (on the grounds of irrelevance or immateriality) disregarded entirely.

The net present value so calculated represents the increase in value to the firm which the project should produce if the market had access to the same information and shared the same expectations as the firm's decision-makers. A project is acceptable if the net present value is positive. Where a conflict arises, the project or combination of projects which produces the highest net present value should be chosen.

CONCLUSION

The conventional capital budgeting framework presents practitioners with two distinct approaches which are capable of producing conflicting solutions, namely the relative profitability approach as characterised by the IRR and the absolute contribution approach as represented by the NPV method. Writers, conscious of the limitations of the former, have, in fact, tended to give incorrect reasons to explain them, and have sought to devise a complex set of rules to assist in identifying the conditions in which the IRR can be assumed to give the same solutions as the NPV method. They have failed to observe, however, that even when

the IRR does provide a correct solution, it is never correct in principle. Furthermore, efforts to validate the IRR have been largely counterproductive by making the selection procedure appear considerably more complex than it need be. It is concluded that the only relevant criterion for measuring a project's contribution is provided by the NPV approach, and that the IRR is no more than a ratio indicating the project's sensitivity to changes in the required rate of return, and should be accorded no higher a status than that.

In addition, the search for the correct discount rate has not progressed much in practical terms since the discounting approach was first applied to business decisions. Because there is insufficient evidence that a market risk premium exists, or if it exists that it is significant, it is argued that there is no virtue in exhorting practitioners to use a discount which cannot be precisely defined and which cannot be observed in the market whilst there remains serious doubt whether departing from the market rate of interest is in fact necessary. It is proposed, therefore, that until the academic literature succeeds in firmly justifying and establishing an operational solution to the risk-premium hypothesis, the contribution which a project makes to shareholders' wealth can most effectively be measured by using the yield on Government bonds as the relevant discount rate. The flotation costs, taxation and bankruptcy implications of the financing strategy employed should be incorporated into the project's cash flows, with the result that the discount rate should require no further adjustment.

The contribution which this thesis is intended to make to the decision-making process terminates at this point. However, it is not argued that, in practice, the decision to undertake one investment rather than another should be made wholly on the basis of the criterion which has been proposed. The theory of capital budgeting has traditionally focused almost exclusively on the significance of financial decisions from the point of view of those who supply the equity capital funds, and the procedures outlined above have sought to remain within that framework. But this narrow focus fits uncomfortably into the modern conception of corporate responsibility which reflects a shift away from the shareholder-centred emphasis to the wider task of balancing the interests of all groups which participate in or are affected by the business enterprise. The mandate issued to management is today perceived as being far more complex than the simple goal of maximising the wealth of one single group. Nonetheless, it is in the interests of all participants that the scarce resource of capital entrusted to management should be used effectively, and to do this, management must have a yardstick by which to judge that effectiveness. Free enterprise cannot be expected to survive if the suppliers of capital are unable to expect to achieve at least the return which the state is prepared to pay for borrowed funds, and the criterion which has been proposed seeks to measure the contribution which a project makes above that minimum standard of effectiveness. But there are other considerations which will influence management's selection of investments and which may conflict with, and in practice override, the preferences indicated by the basic economic model. The decision to undertake a particular investment may have social costs which should properly be excluded from the cash flows

when evaluating the project's potential contribution to shareholders' wealth, but which should be taken into account when management exercises its judgment about the overall desirability of the project. Of even more immediate significance to management, indeed, is that the choice of investment may have far-reaching consequences for the economic survival of the company and for the security of those who are employed by it. When the principles of investment decision theory were first conceived, the risk of the business unit and that of the owners were perceived to be broadly equivalent. But the development of portfolio theory and capital asset theory has destroyed that equivalence, and whereas the opportunities for diversification are extensive for the suppliers of capital, the opportunities open to management are far more restricted. Theorists, indeed, argue that firm diversification provides no benefit to shareholders and that the diversification process is best left to the latter on the grounds that they can regulate it more effectively. But this recommendation ignores the fact that the relevant risk to managers and their fellow employees is not the risk which shareholders are unable to diversify away, but the risk which the corporate entity has not diversified away. It follows that, although the basic model implies that differences in project variability should be ignored, that is not to say that risk has ceased to be a relevant dimension in the decision process. But the risk which management must evaluate is the risk to which the company is exposed and those whose livelihood, etc., is dependent upon it. The incidence of risk has shifted substantially away from the portfolio-holder

to the employees of the corporate units which comprise the portfolio, and in the process the character of relevant risk has altered. The measures which are appropriate for the one are not necessarily so for the other. Therefore, when the shareholders' contribution has been assessed, it remains for management to measure the social costs and benefits of the decision and to balance the claims of the various participating groups. Precisely how the effect of these benefits and costs should be incorporated into the decision process is not clear. Whether they should be expressed as a value to be added or deducted from the basic economic net present value, and whether the resulting measurement should be used as the basis of selection is outside the scope of this study. Much research requires to be done before it is possible to quantify the social costs and benefits. In the meantime, management must exercise its judgment about the desirability of adhering to or departing from the indications given by the basic model.

The usefulness of the recommended measure, therefore, is not that it provides a compelling basis of selection, but that it indicates to management the basis of selection which best promotes the interests of the shareholder group. For the run-of-the-mill project this basis may be sufficient. If other factors are brought to bear which lead management to select otherwise, they have, at least, a measure of the cost to shareholders of choosing the alternative path.

In conclusion, this thesis has argued that, on balance, the benefits of a wider acceptance of the discounting model which could

be expected to result from adopting the simplified approach, and the advantages for effective allocation of capital in having an objective, observable yardstick rather than what at best must be a crude attempt to implement the risk-premium hypothesis, should prevail over the doctrine that practitioners must make some adjustment to the market rate of interest even if they have no realistic guidance as to how much the adjustment should be.

What is certainly clear is that if future efforts are directed towards improving the quality of forecasting techniques within companies, and towards improving the quality of the informational flow from companies to the world's capital markets, and if progress in facilitating the flow of capital between nations continues to be made, the hypothesis that project variability has ceased to be a significant factor in evaluating the impact on the suppliers of capital of the firm's investment decisions will become progressively a more appealing and defensible proposition.

APPENDIX

Questionnaire (Chapter 5)

Note: The term 'risky' will be used in the following questionnaire. When an investment is described as 'risky' it means that there is more than one possible return that can be achieved from the investment. If we define the 'expected return' as the average (or mean) of the range of possible returns, (whether from dividends, interest received, or capital gains) then the riskiness of the investment is related to the dispersion of the possible returns about this average (or mean). The total riskiness of a portfolio of investments is less than the aggregate risk of each individual investment, provided the returns from each investment do not depend entirely on the same set of events. Therefore the chances of doing exceptionally badly are diminished as the number of securities in the portfolio is increased, and of course the chances of doing exceptionally well are also diminished.

In the course of the questionnaire you will be asked to rank or to state a preference for one of a number of investment alternatives. It may be that you happen to believe that at the present time one section of the stock market is undervalued in relation to another, and your decision would therefore be influenced by the desire to exploit such an imperfection. Since, however, the object is not to examine your views about a particular market situation but to learn something about your general attitude to different media, please try to give your answers on the basis that you have no reason to suppose that any section of the market is temporarily under or overvalued in relation to other sections.

Ignore transaction costs and taxation aspects

Please tick opposite appropriate answer

1. Have you ever
   (a) purchased equity securities for yourself? yes / no
   (b) recommended the purchase of equity securities for someone else or on behalf of some fund? yes / no

2. Have you ever
   (a) purchased fixed interest securities for yourself? yes / no
   (b) recommended the purchase of fixed interest securities for someone else or on behalf of some fund yes / no

3. If your answer to 1(a) was yes, what approximately was the fewest number of companies in which you had an investment at any point of time.

   Approximately for how long a period did your portfolio consist of that number?

4. Assume that you had say £5,000 which you did not immediately need for any special purpose, and your choice of investment was limited to one of the following portfolios

   A = 3½% War Loan

   B = A portfolio composed of shares in every company quoted in the U.K. Stock Exchange

   C = A portfolio composed of a professionally selected representative cross section of companies quoted in the U.K., U.S. and Australian Stock Exchanges together with some investments expertly selected in land, property and some works of art.

   Taking into account all uncertainties including future inflationary trends, state the order of riskiness (1, 2 or 3)* which you consider the portfolios have in relation to their expected real net-of-inflation returns:

   (a) assuming that you could invest for no more than two years but could sell the portfolio at any time, if you believed it were to your advantage A / B / C

   (b) assuming that you could invest for up to five years but could sell the portfolio at any time if you believed it to your advantage A / B / C

* 1 = most risky, 2 = second most risky, 3 = least risky

(c) assuming that you could invest for ten years or more, but could sell the portfolio at any time, if you believed it were to your advantage.

A
B
C

Do you believe that your choice of the second most risky (No. 2) in each of the above circumstances is significantly or just marginally more risky than the least risky (No.3)

(a)

Marginally
Moderately
Significantly

(b)

Marginally
Moderately
Significantly

(c)

Marginally
Moderately
Significantly

5. Which portfolio would you select using the notional £5,000 that you had and did not require immediately?

(a) assuming that you could invest for no more than two years

A
B
C
Combination of A & B
Combibation of A & C

(b) assuming that you could invest for up to five years

A
B
C
Combination of A & B
Combination of A & C

(c) assuming that you could invest for ten or more years.

A
B
C
Combination of A & B
Combination of A & C

6. Indicate your reasons for selecting the above portfolios

7. Are there any circumstances in which your selection would be different? If so, indicate briefly.

8. Again assume that you have £5,000 which you do not need in the near future, and you have a choice of the following portfolios

A = 3½% War Loan

B = a portfolio composed of shares in every company quoted in the U.K. Stock Exchange, assuming that the management of each company accepted projects only which were expected to yield a return equal to the yield on 3½% War Loan or more.

C = a portfolio composed of a representative cross section of companies quoted in the U.K., U.S. and Australian Stock Exchanges, together with investments professionally selected in land, property, and works of art, assuming that the management of each company accepted only projects which were expected to yield a return equal to the return on 3½% War Loan or more. The land etc. was also expected to yield at least the yield on 3½% War Loan.

Which portfolio would you select

a) assuming that you could invest for no more than two years

A
B
C
Combination of A & B
Combination of A & C

b) assuming that you could invest for up to five years

A
B
C
Combination of A & B
Combination of A & C

c) assuming that you could invest for ten years or more?

A
B
C
Combination of A & B
Combination of A & C

9. Again, assuming that you had £5,000 that you did not expect to have to call on for any purpose for several years and your choice was as follows:

A = a portfolio of shares so constructed that all ups and downs were expected to balance each other out. The return to be obtained could be expected to be equal to 10% per annum with virtual certainty, that is, whatever booms might occur in the market or whatever depressions, this portfolio would not be affected and could be depended upon to yield 10%, whether from dividends or capital gains, or both.

B = a portfolio of shares which would broadly move with the Financial Times Share Index, that is, which would rise with economic booms and go down during depressions, but <u>on average</u> over the long term might be expected to yield 10% whether from dividends, or capital gains or both.

Which portfolio would you choose, bearing in mind that you would expect to be free to sell the portfolio whenever you think it to your advantage.

A

B

Combination of A and B

10. How would your choice in question 9 be affected if the return on portfolio B was expected to be

(a) 9%

(b) 11%

11. Would you describe yourself as

(a) High-risk taker

(b) Moderate-risk taker

(c) Low-risk taker

(d) Risk-avoider

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