Four essays in international macroeconomics

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Dedicated to my wife, Bingqing.

Declaration

I declare that, except where explicit reference is made to the contribution of others, this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signature:

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Abstract

Chapter 1: We propose an integral correction mechanism to model real exchange rate dynamics. In estimation, we also allow a Harrod-Balassa-Samuelson effect on real exchange rate long-run equilibrium. Using data from 19 OECD countries, we find the integral correction mechanism fitting in-sample data significantly better than the popular smooth transition autoregression model. The special dynamics of the integral correction mechanism help explain the PPP puzzle by distinguishing mean-reversion speeds in the long- and short- run. However, the integral correction mechanism shows a significant out-of-sample forecast gain over the random walk in only few cases. Though the gain is robust across forecast horizons and quite large at long horizons.

Chapter 2: This chapter evaluates the ability of a standard IRBC model augmented with an input adjustment cost of imported goods to explain different aspects of the real exchange rate like the standard deviation, the autocorrelation function, the spectrum and the integral correction mechanism. I find that the simple IRBC model with an appropriate calibration can well capture all features of the real exchange rate. The input adjustment cost plays the key role. As compared to the standard model, it implies a reversed impulse response of the real exchange rate with a fast speed going back to steady state and introduces a long-run cyclical movement in most macroeconomic variables. I find that this particular impulse response helps explain the PPP puzzle.

Chapter 3: I study optimal unconventional monetary policy under commitment in a two-country model where domestic policy entails larger spillovers to foreign countries. Equity injections into financial intermediaries turn out to be more efficient than discount window lending and the large-scale asset purchases that have been employed in many countries. Due to precautionary effects of future crises, a central bank should exit from its policy more slowly than the speed of deleveraging in the financial sector. The optimal policy can be changed considerably if cross-country policy cooperation is not imposed. In this case, interventions tend to be too strong in one country but too weak in the other. Gains from cooperation become positive if using unconventional monetary policy is costly enough, then correlates positively with the cost.

Chapter 4: I consider implementation of optimal unconventional monetary policy outlined in chapter 3. I find the Ramsey policy characterised by a simple rules responding to gaps in asset prices. However, it requires knowledge of asset prices that would be realized in a world free of financial friction so cannot be used to guide unconventional monetary policy in practice. The best practical simple rule responds to credit spread with inertia.

Contents

Foreword

The purchasing power parity (PPP) puzzle is one of the six major puzzles in the field of international macroeconomics identified by Obstfeld and Rogoff (2000), and is still puzzling to economists today. The literature approaches this puzzle from two angles. Empirical researchers take a stance that real factors, such as preference and productivity, cannot explain the high volatility of real exchange rates. They aim to show that the degree of persistence in the real exchange rate is consistent with the time frame in which nominal shocks can have substantial real effects. A lower degree of persistence has been found using, for example, nonlinear models in which real exchange rates revert to their long-run equilibrium faster in the presence of larger deviation. Though it is not clear if the degree of persistence is low enough to be explained by nominal shocks. On the other hand, theorists add channels for the propagation of either real or nominal shocks such that a theoretical model can generate volatility and persistence close to data. However, they mainly focus on the business cycle frequencies.

The success of the literature is imperfect, and the two strands of literature are to a large extent isolated. In chapter one and two, I contribute to the theory and the empirics of exchange rate determination separately. I propose an empirical model called integral correction mechanism (ICM) to capture real exchange rate dynamics. It implies that the real exchange rate responds to an unidentified shock non-monotonically. The real exchange rate reverts to its long-run equilibrium quickly in the short-run but persistently move around the equilibrium. Hence the PPP puzzle can be understood by distinguishing between degrees of persistence in the short-run and the long-run. I formally evaluate both the in-sample and out-of-sample performance of ICM taking the popular smooth transition autoregression (STAR) model as a benchmark. For the in-sample fitness, the ICM does significantly better than the STAR for most countries considered. The ICM exhibits an out-of-sample forecast gain for only a few countries. However, when it does, the gain is substantial. It is therefore worth considering the ICM forecast as a candidate at least.

In the second chapter, I aim to obtain a deeper understanding the ICM. I show that a standard two-country dynamic stochastic general equilibrium (DSGE) model augmented with an input adjustment cost can generate real exchange rate dynamics similar to the ICM. The micro-foundation underlying this type of dynamics is a time-varying price elasticity of tradable goods such that international trade is less responsive to terms of trade in the short-run. Calibration exercises show that the DSGE model can capture all the time series properties of real exchange rates at all frequencies. However, the calibrated parameters implies a short-run trade elasticity smaller than that suggested by micro-evidence. The second chapter also makes a contribution by bridging the empirical and theoretical literature.

The third and fourth chapters are separated from the previous ones and focus on optimal cooperative and non-cooperative unconventional monetary policy (UMP). Since the 2007-2008 financial crisis, the so-called unconventional monetary policy has been employed by many major central banks to provide liquidity and affect credit conditions at a large scale. For example, the Federal Reserve mainly used an expanded discount window in the early stage of the 2007-2008 crisis. After the Lehmann failure, the Fed started its asset purchase programs (quantitative easing, or QE) and injected equity into the financial system. The Fed started to taper the QE at the end of 2013, and ceased it in October 2014, after which the Federal Reserve has kept the size of its balance sheet constant by buying just enough to replace maturing securities. The European Central Bank (ECB) employed two series of Targeted longer-term refinancing operations in June 2014 and March 2016 respectively. The ECB initialized its asset purchase programs at a relatively small scale, slightly later than the Federal Reserve. The ECB formally introduced QE in 2015 and further increased the amount purchased in late 2016. The ECB also used equity injections during this period.

UMP could entail large cross-country spillovers with internationally integrated financial markets. If quantitative easing focuses on an asset that is either itself traded internationally or closely substitutive to internationally traded assets, foreign credit conditions are also affected. The spillovers make cross-country policy cooperation potentially welfare improving. There has been much work devoted to evaluating the effectiveness of UMP. However, the nominative perspective of UMP theory is still blank. In the third chapter, I study Ramsey UMP and draw two conclusions. First, effective UMP should be designed to relaxes financial constraints that banks face. Otherwise, the UMP largely crowds out private funds and will be painfully long-lasting. However, regardless of the policy design, a central bank should exit slowly due to a precautionary effect. Second, welfare gain of policy cooperation is a weakly increasing function of the intervention cost. There is no gain if the intervention cost is small. Increasing the intervention cost to a certain point generates positive gains. In the noncooperative equilibrium, the interventions are too strong in one country but too weak in the other.

Ramsey policy is silent regarding implementation. In the fourth chapter, I consider policy implementations via simple rules. The main contribution is to design the best practical policy guided by the optimal results. In the literature, a naive policy focusing on credit spreads is employed. I find that the optimal policy is characterised by a rule responding to asset price gaps. However, it requires knowledge of asset prices that would be realized in a world free of financial friction so cannot be used to guide unconventional monetary policy in practice. The best practical simple rule responds to credit spread with inertia. I discuss the intuition of this rule, which is different than that argued in the literature.

Chapter 1

Understanding the PPP puzzle: dynamics of real exchange rates towards their time-varying equilibrium

This chapter is co-authored with my supervisor Prof Gabriel Talmain

1.1 Introduction

The first prediction of real exchange rate movements is given by the well-known purchasing power parity (PPP) hypothesis, justified by frictionless international goods arbitrage. The PPP hypothesis suggests that real exchange rates should be stationary time series with a constant unconditional mean. Although much evidence supporting stationary real exchange rates has been found (Taylor, 2009), economists are surprised by their extremely slow mean-reversion, at least when the reversion speeds are measured by standard methods which assume constant real exchange rate equilibrium and a (proportionally) constant reversion speed. Rogoff (1996) asks how to reconcile the slow mean-reversion with a high real exchange rate volatility, which is known as the PPP puzzle.

In the literature, there are at least two reasons why true mean-reversion speeds can be faster than those measured by standard methods. First, low-frequency movements of real exchange rates can be explained by time-varying equilibrium, which is driven by real shocks, such as productivity shocks, according to the Harrod-Balassa-Samuelson effect (Balassa, 1964; Harrod, 1933; Samuelson, 1964, HBS). The key to the HBS effect is a sector of nontradable goods subject to no international arbitrage. Second, dynamics of real exchange rates towards their equilibrium can be nonlinear such that mean-reversion is faster in the presence of larger deviations, like the smooth transition autoregression (STAR) model of (Taylor et al., 2001). Therefore, estimates of mean reversion speeds are biased under the assumption of constant equilibrium and a constant reversion speed.

In this chapter, we revisit the PPP puzzle by proposing a new type of dynamics referred to as integral correction mechanism (ICM). The ICM features persistent oscillations around the equilibrium with quick mean-reversion in the short-run. It suggests that a shock (possibly a real one) can generate enormous short-term volatility and damp out at extremely slow rates, so the PPP puzzle is less puzzling. Furthermore, the ICM suggests paying attention to the entire impulse response function or the entire autocorrelation function. By contrast, the common practice in the literature is to summarize real exchange rate dynamics by half-lives of shocks and the first few orders of autocorrelation. As shown by Jiang (2017), the ICM can be understood by time-varying trade elasticities such that import and export goods are less responsive to their prices in the short-run than in the long-run. In estimating the ICM, we also allow the HBS effect, i.e., time-varying equilibrium. We employ the version of Bordo et al. (2017) in which the HBS effect works through a direct channel and a terms of trade channel.

Using data from the US and 18 OECD countries during the post-Bretton-Woods period, we estimate the ICM model and a benchmark STAR model that is popular in the literature. Our findings are threefold. First, we establish that the ICM model fits in-sample data statistically better than the STAR model. The origin of this superiority is that the ICM can capture autocorrelation functions (ACF) of real exchange rates very well. Second, regarding the HBS effect, we find the terms of trade channel statistically significant in more countries than the direct channel. This is expected because of our relatively short span of data. At last, we evaluate the out-of-sample performance of both models. The ICM beats random walk forecasts for only 4 out of 18 real exchange rates. However, when it does, the ICM can predict real exchange rates at both short and long horizons. The STAR, on the other hand, can beat random walk forecasts for 17 real exchange rates, though not always significantly. The two models seem complementary to each other in the sense that the ICM often shows significant forecast gain when the STAR does not, and vice versa. Incorporating time-varying equilibrium often helps forecast using both models.

Our main contribution to the literature is proposing the ICM model as a statistically and economically better representation of real exchange rates than the STAR model. The STAR model, its variants, and other similar regime-switching models have been prevailing in the literature in the last two decades.[1] Roughly speaking, early work of Obstfeld and Taylor (1997), Engel and Kim (1999) and Taylor et al. (2001) popularise threshold, Markov, and smooth transition regime-switching models, respectively.[2] In STAR, mean reversion speeds are faster with relatively rare, larger shocks. Norman (2010) concludes that the reversion speeds faster than the consensus are observed frequently enough to support STAR as a solution to the PPP puzzle. However, Yoon (2010) applies a nonparametric measure of serial dependence to real exchange rates, which threshold and smooth transition models struggle to replicate. This is where this chapter comes in.

Our modelling strategy generally follows the recent literature of modelling nonlinear mean reversion speeds and time-varying equilibrium jointly.[3] While smooth transition are routinely employed to capture the mean reversion process, real exchange rate equilibrium is modelled in various ways. For example, Lothian and Taylor (2008) and Peltonen et al. (2011) focuses on the HBS effect. Paya and Peel (2006) and Boero et al. (2015) further allow a capital account effect that currency tends to appreciate in real terms

[1]The common justifications for these types of models include: 1) the presence of the transaction cost causing a greater goods arbitrage when the real exchange rate misalignment grows (Dumas, 1992; Obstfeld and Taylor, 1997; Parsley and Wei, 1996; Sarno et al., 2004; Taylor and Kim, 2009); 2) A growing degree of consensus concerning the appropriate or likely direction of a nominal exchange rate movement among traders (Kilian and Taylor, 2003); 3) A greater likelihood of the occurrence and success of the intervention by authorities to correct a strongly misaligned exchange rate (Reitz and Taylor, 2008; Sarno et al., 2004; Taylor, 2004). Ahmad et al. (2013) examine if a real exchange rate governed by STAR can be generated by a medium-scale two-country DSGE model. Their model features incomplete international financial markets, local currency pricing, home bias and nontradable goods.

[2]They are, of course, not the first to apply these models on exchange rates. Engel and Hamilton (1990) and Engel (1994) apply Markov switching models on nominal exchange rates. Michael et al. (1997) use a exponential smooth transition model to test unit roots. Followers of these models are plenty. Bergman and Hansson (2005) focuses on the forecast performance of Market switching models. Kruse et al. (2012) run a competition between Markov switching and smooth transition. It's often believed that threshold models are more appropriate than smooth transition models for sectoral data. Juvenal and Taylor (2008) use a threshold model to test the Law Of One Price (LOOP). However, there are also examples of mixed match of models and data, see Nakagawa (2010) and Kim and Moh (2011).

[3]There is a stand-alone literature focusing on real exchange rate equilibrium only. A incomplete list: Chinn (2000), Lane and Milesi-Ferretti (2002), Chen and Rogoff (2003), Lee and Tang (2003), Lane and Milesi-Ferretti (2004), MacDonald and Ricci (2005), Choudhri and Khan (2005), MacDonald and Ricci (2007), Thomas and King (2008), Chong et al. (2012), Ricci et al. (2013). Overall, evidence supporting the HBS effect is mixed (Taylor and Taylor, 2004), but more evidence is found using long-span data over centuries.

in the presence of accumulation of net foreign assets. Specifically, Boero et al. (2015) find a long-run relationship between real exchange rates, productivity differentials, real interest rate differentials and the capital account. Since real exchange rate equilibrium can be affected by a wide range of variables, Taylor and Kim (2009) and Kim (2012) summarise information in those variables by a few unobservable factors. Béreau et al. (2010) consider an alternative so-called Behavioral Equilibrium of exchange rates.

This chapter also contributes to the literature of real exchange rate predictability, particularly at short horizons. Results in the literature are mixed. Indeed, as pointed out by Rogoff and Stavrakeva (2008) and Rossi (2013), this is partially due to different data, models, evaluation methods employed in different studies. Early work of Kilian and Taylor (2003) and Rapach and Wohar (2006) find that, for post-Bretton Woods period data, STAR model can outperform simple linear autoregressive models and a random walk in terms of out-of-sample forecasting at short but not at long horizons (2 to 3 years). Buncic (2012) finds no forecast gains of STAR models at any horizon. The author demonstrates that the nonlinearity in the conditional means of STAR models decreases as the forecast horizon increases. López-Suárez and Rodríguez-López (2011) find results in favour of STAR even at short horizons using a panel smooth transition error-correction model. Pavlidis et al. (2012) also find STAR favourable in terms of forecasting for long-span dollar-sterling real exchange rates. Kim (2012) argue that the poor out-of-sample forecasting performance of STAR model in the literature may be caused by the lack of properly modelled real exchange rate equilibrium. We confirm this argument in this chapter.

The rest of the chapter is organised as follows. The next section presents a modern version of the HBS effect. Section 1.3 discusses real exchange rate equilibrium, the ICM and STAR model, separately. Data is described in section 1.4. Then, we report in-sample results in section 1.5 and discuss economic significance of the ICM in section 1.6. Section 1.7 evaluates out-of-sample performance. The last section concludes.

1.2 The Harrod-Balassa-Samuelson effect

The intuition of the standard HBS hypothesis is as follows. Consider a two-sector economy. There is a tradable sector where the the Law Of One Price (LOOP) holds and a nontradable sector subject to no international arbitrage. The standard HBS hypothesis assumes that productivity in the tradable sector grows faster than in the nontradable sector. Since capital flows across countries but labour can only transfer between domestic sectors, growth of the tradable sector productivity pushes up the wages in both sectors. While prices of tradable goods do not necessarily rise, nontradable goods become more expensive as compared to tradable goods. Consequently, the overall price index rises and appreciates the real exchange rate.

In this chapter, we employ a variety of the standard HBS effect proposed by Bordo et al. (2017). This variety features production specialisation of each country and monopolistic competition, which are fairly standard in the literature of two-country dynamic stochastic general equilibrium (DSGE) models. Consider a world economy that consists of two countries, Home and Foreign. Each country contains two sectors: the nontradable sector, the country-specialized tradable sector. There are n_N, n_H (n_F) firms producing differentiable goods in each sector. LOOP holds for tradable sectors. We focus on Home equations in the following and denote Foreign variables with a superscript "*". Goods are aggregated as follows:

$$C = \frac{C_N^{\gamma} C_T^{(1-\gamma)}}{\gamma^{\gamma} (1-\gamma)^{(1-\gamma)}},$$

$$C_T = \left[(\chi_H C_H)^{\frac{(\eta-1)}{\eta}} + (\chi_F C_F)^{\frac{(\eta-1)}{\eta}} \right]^{\frac{\eta}{(\eta-1)}},$$

$$C_N = \left[\int_{i=1}^{n_N} C_N(i)^{\frac{(\sigma-1)}{\sigma}} di\right]^{\frac{\sigma}{(\sigma-1)}},$$

$$C_H = \left[\int_{i=1}^{n_H} C_H(i)^{\frac{(\sigma-1)}{\sigma}} di\right]^{\frac{\sigma}{(\sigma-1)}},$$

$$C_F = \left[\int_{i=1}^{n_F} C_F(i)^{\frac{(\sigma-1)}{\sigma}} di\right]^{\frac{\sigma}{(\sigma-1)}},$$

where $\gamma \in (0,1)$ is the share of nontraded goods in total expenditure, $\chi_H \in (0,1)$ and $\chi_F \in (0,1)$ are preference parameters for Home and Foreign goods, η and σ are the positive elasticity of substitution, C, C_T, C_H, and C_F are appropriate aggregate goods, and the index i indicates varieties in each sector. Minimizing the cost of consuming one unit of aggregate goods results in the corresponding price indexes:

$$P = P_N^{\gamma} P_T^{(1-\gamma)},$$

$$P_T = \left[\left(\frac{P_H}{\chi_H}\right)^{1-\eta} + \left(\frac{\tau P_F}{\chi_F}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

$$P_N = \left[\int_{i=1}^{n_N} P_N(i)^{1-\sigma} di\right]^{\frac{1}{(1-\sigma)}} = n_N^{\frac{1}{(1-\sigma)}} P_N(i),$$

$$P_H = \left[\int_{i=1}^{n_H} P_H(i)^{1-\sigma} di\right]^{\frac{1}{(1-\sigma)}} = n_H^{\frac{1}{(1-\sigma)}} P_H(i),$$

$$\tau P_F = \left[\int_{i=1}^{n_F} (\tau P_F(i))^{1-\sigma} di\right]^{\frac{1}{(1-\sigma)}} = n_F^{\frac{1}{(1-\sigma)}} \tau P_F(i),$$

where P, P_T, P_N, P_H, and τP_F are appropriate real price indexes, the second equation of P_N, P_H, and τP_F uses symmetry across firms, and$\tau > 1$ is an iceberg cost of international trade. Real exchange rate q is the relative price of C in terms of $C*$, $q \equiv \frac{P}{P^*}$. Terms of trade is the relative price of export in terms of import $tot \equiv \frac{P_H}{\tau P_F}$.

The production function for each goods variety is $Y_j(i) = A_j L_j(i), j = N, H$ where A_j is sector-wide productivity, and $L_j(i)$ is labour input. Each firm maximises its profit and sets the price with a markup over the marginal cost, $P_j(j) = \frac{\sigma}{\sigma-1}\frac{w}{Aj}$, where w is the real domestic wage rate.

Substituting the definitions of price indexes and the monopolistic pricing conditions into the definition of the real exchange rate gives

$$q = \frac{\left[n_N^{\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w}{A_N}\right]^{\gamma} \left[\left(\frac{n_H^{\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w}{A_H}}{\chi_H}\right)^{1-\eta} + \left(\frac{\tau n_F^{\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w^*}{A_F^*}}{\chi_F}\right)^{1-\eta}\right]^{\frac{1-\gamma}{1-\eta}}}{\left[n_N^{*\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w^*}{A_N^*}\right]^{\gamma} \left[\left(\frac{\tau n_H^{\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w}{A_H}}{\chi *_H}\right)^{1-\eta} + \left(\frac{n_F^{\frac{1}{(1-\sigma)}} \frac{\sigma}{\sigma-1}\frac{w^*}{A_F^*}}{\chi_F^*}\right)^{1-\eta}\right]^{\frac{1-\gamma}{1-\eta}}}.$$

Then applying log linearisation, we obtain [4]

$$\begin{aligned} \hat{q} &= \left[\gamma\left(\hat{A_H} - \hat{A_N}\right) - \gamma^*\left(\hat{A_F^*} - \hat{A_N^*}\right)\right] \\ &+ \left[\gamma\left(\hat{n_H} - \hat{n_N}\right) - \gamma^*\left(\hat{n_F^*} - \hat{n_N^*}\right)\right] / (\sigma - 1) \\ &+ \left[\gamma\left(1 - \chi_H^{\eta-1}\right) + \gamma^*\left(\frac{\chi_H^*}{\tau}\right)^{\eta-1} + \left(1 - \chi_H^{\eta-1}\right) - \left(\frac{\chi_H^*}{\tau}\right)^{\eta-1}\right] \hat{tot} \end{aligned}, \tag{1.1}$$

[4]The full model can be found in the NBER working paper version of Bordo et al. (2017).

where variables with a hat are in log deviations from their steady states. The first term in equation 1.1 captures the standard HBS effect. The second term shows due to monopolistic competition. The third term captures a terms of trade channel. While the LOOP always holds for each variety of tradable goods, PPP does not hold for price indexes in the short-run due to the imperfect substitution between Home and Foreign tradable goods, and the preference toward domestic specialised goods represented by χ_H. Bordo et al. (2017) show that the terms of trade channel can diminish or even reverse the standard HBS effect, depending on the relative degree of preference toward domestic specialised goods $\chi_H^{\eta-1} - \left(\frac{\chi_H^*}{\tau}\right)^{\eta-1}$. Furthermore, the endogenous entry & exit (movement in the number of firms) of firms due to monopolistic competition magnifies the standard HBS effect.

1.3 The econometric models

1.3.1 Real exchange rate equilibrium

We model real exchange rate equilibrium as a log linear function of cross-country differentials of real GDP per capital, $rgdp_t$, terms of trade, tot_t, and a trade balance to GDP ratio tb_t:

$$\ln q_t^{EQ} = a_0 + a_1 \ln rgdp_t + a_2 \ln tot_t + a_3 tb_t. \tag{1.2}$$

The first two explanatory variables captures the HBS effect, as motivated in the last section. As standard in the literature, we use real GDP per capita as a proxy for productivity mainly because productivity data is not available for all countries in our sample. [5] The fluctuates in terms of trade reflect not only supply side shocks, as in the HBS hypothesis, but possibly also demand side factors such as income-elasticity of nontradable consumption. Incorporating terms of trade could be particularly important for countries relying on exporting primary commodity (Chen and Rogoff, 2003). While equation 1.1 is derived from a static model, dynamic general equilibrium models such as those developed in Benigno and Thoenissen (2003) and Lane and Milesi-Ferretti 2002 suggest including the trade balance in the equilibrium relationship. Intuitively, a country in debt needs a trade surplus to service its external liabilities. In the long-run equilibrium, the real exchange rate must depreciate to improve the competitiveness of this country. There are possibly other variables determining real exchange rate in the long-run. But these three variables are the most commonly employed ones in the literature, see the literature review in the introduction.

Denoting real exchange rate misalignment as $err_t = \ln q_t - \ln q_t^{EQ}$, we proceed to describe dynamics that govern movements of the misalignment. The complete models are described by equation 1.2 and the equations that specify the dynamics.

1.3.2 The integral correction mechanism

The ICM [6] is given by

$$err_t = \sum_{j=1}^{p} \beta_j^{ecm} err_{t-j} + \beta^{icm} ie_{t-1} + \varepsilon_{t,icm}, \tag{1.3}$$

[5] We take the ratio between Home's and Foreign's real GDP per capita and let it enter the equation as a single explanatory variable. As an alternative, Home's and Foreign's real GDP per capita may enter the equation separately. If doing so, we cannot reject the null hypothesis that the coefficients on each of the two variables have the same value with opposite signs; hence, we keep the specification in equation 1.2.

[6] This mechanism is first proposed by Hendry and von Ungern-Sternberg (1981) and employed by Abadir and Talmain (2012) in a different context.

where $ie_t = \sum_{i=0}^{t} err_i$ denotes integral error, $\beta^{icm} \leq 0$ determines the correction of integral error, β_j^{ecm} satisfies the stationary condition for an AR(p) process, and $\varepsilon_{t,icm} \sim N\left(0, \sigma_{icm}^2\right)$ is the residual. The $\beta_j^{ecm} err_{t-j}$ terms introduce long memory in a parsimonious way. The integral error implies that the force driving the real exchange rate back to its equilibrium is stronger when the real exchange rate is away from its equilibrium for a longer period of time. The history dependent real exchange rate could be a result of incomplete international financial market. To understand the ICM dynamics, consider a positive shock. If β_j^{ecm} implies a stationary AR(P) process, the misalignment will be corrected gradually and meanwhile, the integral error is built up at a decreasing rate. As $\beta^{icm} < 0$, the real exchange rate misalignment reverts to zero more quickly than a pure AR process. The integral error reaches it maximum when the real exchange rate returns to the equilibrium. However, the integral error keeps driving the real exchange rate to the other side of its equilibrium, so that the real exchange rate is undervalued. The dynamics stop when both ie_t and err_t are zero. The ICM is similar to a physical pendulum. Let the real exchange rate be a massive bob hanged by a rod, the bottom position being the equilibrium. Given a shock, the real exchange rate moves back and forth around its equilibrium periodically because ie_t stores the "momentum" of movement until the momentum is drained by air friction. β_j^{ecm} captures the role of air friction.

1.3.3 The smooth transition autoregression

We use the popular STAR model (Granger and Teräsvirta, 1993; Teräsvirta, 1994) as a benchmark. Real exchange rate dynamics governed by the $STAR\,(p,d)$ can be written as

$$err_t = \sum_{j=1}^{p} \beta_j^{ecm} err_{t-j} + \left[\sum_{j=1}^{p} \beta_j^{ecm*} err_{t-j} \right] \times G\,(\theta, d) + \varepsilon_{t,star}, \tag{1.4}$$

where $\varepsilon_{t,star} \sim N\left(0, \sigma_{star}^2\right)$, and $G\,(\theta, d)$ controls a smooth transition between two regimes represented by β_j^{ecm} and β_j^{ecm*}. The transition may take one of two forms. The model exhibits exponential STAR (ESTAR) dynamics if

$$G\,(\theta, d) = \left[1 - \exp\left[\theta\,(err_{t-d})^2\right]\right], \tag{1.5}$$

and exhibits logistic STAR (LSTAR) dynamics if

$$G\,(\theta, d) = \left[1 + \exp\left[\theta err_{t-d}\right]\right]^{-1}, \tag{1.6}$$

where θ determines the speed of the transition. The STAR model can be considered as an AR model for real exchange rate misalignment with time-varying coefficients governed by the function $G\,(\theta, d)$:

$$err_t = \sum_{j=1}^{p} \beta_{t,j} err_{t-j}. \tag{1.7}$$

If the model follows the ESTAR, the time-varying AR coefficients $\beta_{t,j}$ vary between β_j when $err_{t-d} = 0$ and $\beta_j + \beta_j^*$ when $err_{t-d} = \pm\infty$. In contrast, if the model follows the LSTAR, $\beta_{t,j}$ varies between β_j when $err_{t-d} = +\infty$ and $\beta_j + \beta_j^*$ when $err_{t-d} = -\infty$ and equals $\beta_j + \frac{\beta_j^*}{2}$ when $err_{t-d} = 0$. Therefore, the ESTAR implies symmetric dynamics while the LSTAR implies asymmetric dynamics.

1.4 Data

We estimate both the ICM model and the STAR model using quarterly data over the period 1973Q2-2013Q4[7] for 18 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland and the United Kingdom. Ideally, the data should be collected for each of the 18 OECD countries and their trading partners. Then, the data from the trading partners is weighted averaged by the trade share to represent the Foreign country. Since data on bilateral trade shares is not always available, we choose the United States as the reference country.

Data is collected from IMF's International Financial Statistics (IFS) database and OECD's Outlook database. The real exchange rate, q, is defined as units of the US goods per unit of home country goods and calculated using CPI. The terms of trade, tot, is calculated as the ratio between export and import deflators for goods and services. The trade balance to GDP ratio, tb, is defined as the net export of goods and services as a proportion of GDP. Throughout this chapter, all variables are expressed in natural logarithm except tb. Detailed data sources can be found in appendix A.3.

1.5 Empirical analysis

1.5.1 Stationarity tests

Before taking data to the econometric models, we test the stationarity of the real exchange rate or its misalignment, err_t. The results are reported in table 1.1. We first apply traditional tests. The Augmented Dicky-Fuller test (ADF) test only rejects nonstationary for the UK-US real exchange rate. For all other countries, we test cointegration between real exchange rates with the three variables[8] that explain the real exchange rate equilibrium, using the Engle-Granger cointegration test (EG-tau and EG-z). We only find weak evidence of cointegration for Canada and New Zealand. However, due to the well-known low power of these tests, not rejecting the nonstationarity should be interpreted as these series at least being very persistent. Next, we test global stationarity in the STAR model against a random walk. We use the t(ESTAR) test of Kılıç (2011), which is claimed to have a stronger power as compared to alternatives such as that of Kapetanios et al. (2003). Overall, the t(ESTAR) test finds strong evidence in favour of the stationary STAR model. The only exception is Portugal, for which we test cointegration in the STAR framework by Kapetanios et al. (2006, KSS)'s test[9] and the null of no cointegration can be rejected.

Ideally, we would also like to test stationarity in the ICM model against a random walk. As no such test is available, we obtained some sense of stationarity in the ICM model by using the method proposed in Abadir and Talmain (2012). To be specific, the dynamics represented by the ICM can also be represented by a highly flexible parametric autocorrelation function (ACF) given as

$$\rho_\tau = \frac{1 - a\left[1 - \cos\left(\omega\tau\right)\right]}{1 + b\tau^c}, \tag{1.8}$$

where ρ_τ is the autocorrelation of order τ, and a, b, c, and ω are parameters. Abadir et al. (2013) find that this ACF is flexible enough to capture the dynamics of a wide range of macroeconomic variables

[7]Using long-span data, Lothian and Taylor (2008) and Bergin et al. (2006) argue that real exchange rate volatility and the HBS effect are time-varying. By focusing on the post-Bretton-Woods period, we are satisfied to assume that constant volatility and the HBS effect.

[8]Only those variables for which the ADF test cannot reject unit-roots are included in the cointegration test.

[9]The tests of Kılıç (2011) and Kapetanios et al. (2006, KSS) are designed for ESTAR in particular. The KSS test can only be used to test stationarity in the LSTAR in certain contexts, as suggested by the authors. According to our knowledge, no unit-root test is available for LSTAR.

Table 1.1: Unit root tests for real exchange rates

Country	ADF	EG-tau	EG-z	t(ESTAR)	KSS
Australia	-1.87	-2.96	-16.76	-2.20*	-
Austria	-2.63*	-2.76	-15.40	-2.82**	-
Belgium	-2.04	-3.59	-23.00	-2.35*	-
Canada	-1.86	-3.87*	-23.80	-4.01***	-
Denmark	-2.10	-2.86	-15.88	-3.63***	-
Finland	-2.26	-2.85	-14.69	-2.48**	-
France	-2.38	-2.45	-11.41	-2.74**	-
Germany	-2.44	-2.05	-8.792	-2.91**	-
Italy	-2.03	-3.01	-16.97	-2.76**	-
Japan	-2.18	-1.92	-7.443	-3.10***	-
Netherlands	-2.32	-2.27	-10.50	-3.13***	-
New Zealand	-2.04	-3.72*	-22.44*	-3.64***	-
Norway	-2.84*	-2.86	-15.65	-2.98***	-
Portugal	-0.95	-2.01	-11.14	-1.24	-4.80***
Spain	-2.24	-3.62	-18.56	-2.62**	-
Sweden	-2.41	-2.69	-14.18	-3.00***	-
Switzerland	-2.49	-2.84	-15.97	-2.90**	-
United Kingdom	-2.94**	-	-	-3.74***	-

Numbers are statistics of each test.
*, **, *** denote statistically significant at 10%, 5% and 1%, respectively.

and aggregate financial variables. The stationary conditions[10] are $b > 0$ and $c > 0$. We use maximum likelihood to jointly estimate the parameters in equation 1.8 and in the following equation[11]

$$q_t = a_0 + a_1 \ln rgdp_t + a_2 \ln tot_t + a_3 tb_t + err_t, \tag{1.9}$$

where the ACF of err_t follows equation 1.8. Figure 1.1 reports estimates of ρ_τ, and the empirical ACF of err_t. Equation 1.8 seems to capture the dynamics of the real exchange rate very well, and we find the stationarity conditions to be satisfied for all real exchange rates. The ACFs slump to a negative level and move cyclically around zero with a decaying magnitude. This particular shape of the ACF suggests that the real exchange rate is transitory in the short run but persistent in the long run, which could be the reason why standard tests struggle to reject a unit root. As we will show shortly, our ICM model can capture the ACF very well while other models cannot.

1.5.2 Model specifications

Before estimating models, we select the order of autoregression p, the lag of smooth transition d, and choose between LSTAR and ESTAR.

Following Lothian and Taylor (2008), we pick a p based on partial autocorrelation functions (PACF) of real exchange rates and the real exchange rate misalignment. Most real exchange rates have no significant PACF beyond the first order. The exceptions are Canada, Portugal, and the UK. However, the real exchange rate misalignment, err_t, of these three countries has no significant PACF beyond the first order.[12] Therefore, we take $p = 1$ for all countries and omit the index j.

The conventional economic intuition suggests a smaller value of d and a symmetric ESTAR. It is hard to imagine that real exchange rates move asymmetrically around the equilibrium and wait a long time before switching regimes. Recently, Ahmad et al. (2013) examine the STAR model in an open economy

[10] To be precise, these conditions are for a concept that is slightly more general than covariance stationary as this concept allows a time-varying unconditional mean

[11] See appendix A.4 for technical details. We also apply this method with constant q_t^{EQ} and find similar results.

[12] We obtain err_t by estimating equations 1.8 and 1.9.

Figure 1.1: Estimated and empirical ACF of err_t

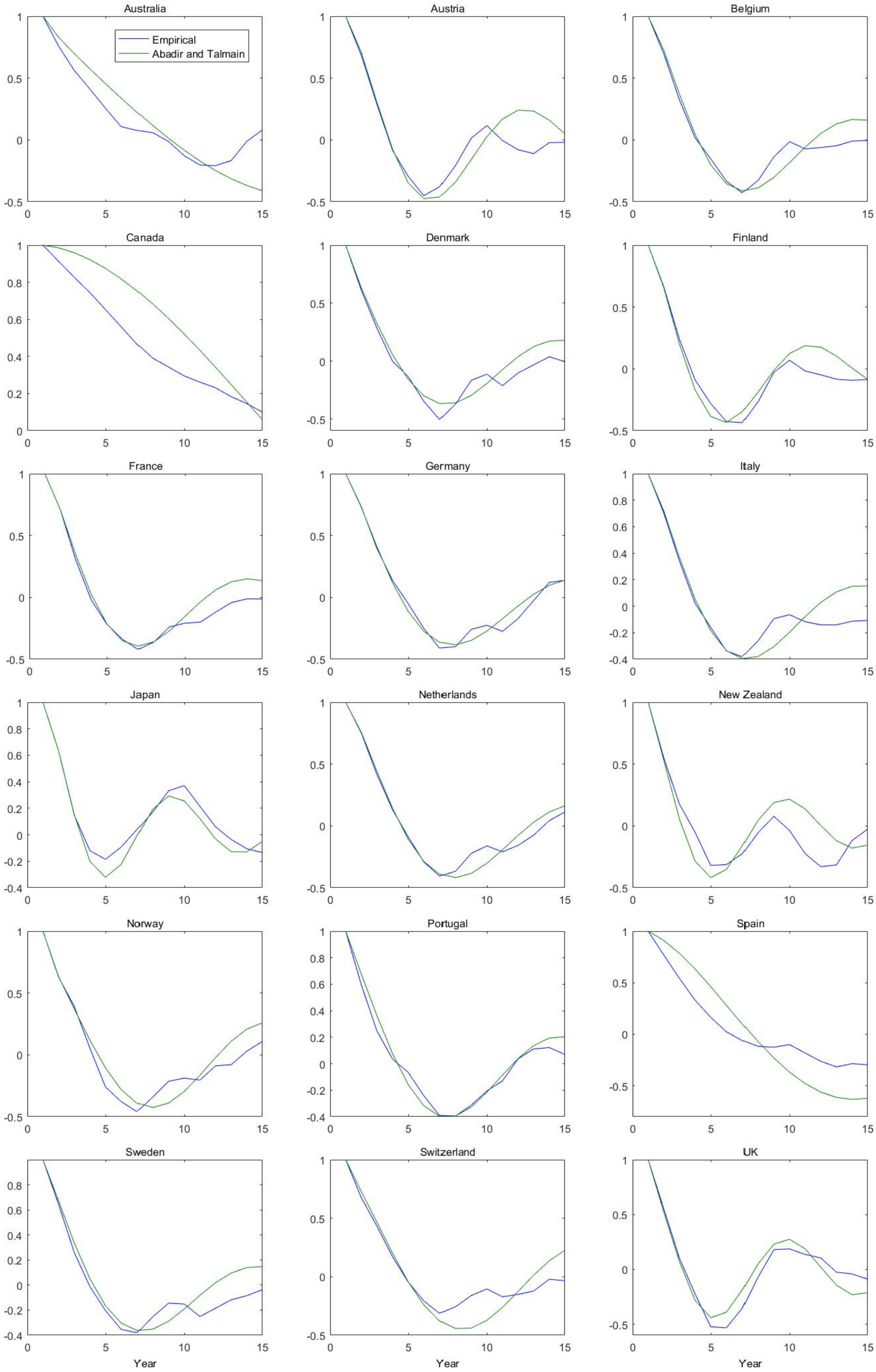


Table 1.2: Model estimation for the UK

LSTAR (1,8)		ICM	
a_0	0.53 {0.05}	a_0	0.48 {0.13}
a_1	-0.01 {0.98}	a_1	0.01 {0.97}
a_2	0.08 {0.75}	a_2	0.03 {0.89}
a_3	0.80 {0.14}	a_3	0.69 {0.15}
β^{ecm}	1 (fixed)	β^{ecm}	0.92 (29.28)
θ	-8.46 (-0.92) {0.14}	-	
β^{ecm*}	-0.16 (-2.36)	β^{icm}	-0.03 (-5.00) {0.00}
σ_{star}	0.05	σ_{icm}	0.05
R^2	0.83	R^2	0.85
Diagnostic: AR(1)	{0.01}	Diagnostic: AR(1)	{0.31}
Diagnostic: ARCH(1)	{0.97}	Diagnostic: ARCH(1)	{0.42}
Diagnostic: nonlinearity	{0.40}	Diagnostic: nonlinearity	{0.56}

Note: number in () and {} are t-statstics and P-value, respectively.
Diagnostic: AR(1), ARCH(1), nonlinearity are Lagrange Multiplier tests for neglected autocorrelation, heteroscedasticity and nonlinearity, respectively.

DSGE model. They find that the DSGE model with home bias and a nontradable sector yields real exchange rate consistent with ESTAR. Introducing local currency pricing may generate a real exchange rate that is described by the LSTAR. A large delay parameter d is found in the DSGE model with both local currency pricing and an incomplete financial market. Formally, we go through the specification produces described in Teräsvirta (1994) and find that the LSTAR describes most real exchange rates and d ranges from 1 to 8.

Finally, the literature (Lothian and Taylor, 2008; Taylor et al., 2001) usually imposes constraints such as, $\beta^{ecm} = 1$, and $\beta^{ecm*} = -1$, and find no evidence to reject them. These constraints make the ESTAR model easy to interpret because the real exchange rate misalignment switches between a random walk and a white noise. Less restrictive constraints are sometime imposed, such as $\beta^{ecm} = -\beta^{ecm*}$ (Paya and Peel, 2006), or $\beta^{ecm} = 1$ (Kılıç, 2011). In this chapter, we impose $\beta^{ecm} = -\beta^{ecm*}$ on the ESTAR model and $|\beta^{ecm}| \leq 1$ and $|\beta^{ecm} + \beta^{ecm*}| \leq 1$ on the LSTAR model.

1.5.3 Estimation results

We estimate the ICM model (equations 1.2 and 1.3) and the STAR model (equations 1.2 and 1.4) for each country separately by maximum likelihood. The results for the UK are reported in table 1.2. To save space, other results are reported in table A.1 and table A.2 in the appendix.

The key parameters that govern the dynamics of the STAR model and the ICM model are θ and β^{icm}, respectively. If they equal zero, the model reduces to a standard AR. To test their statistic significant, we construct the distribution of their t-statistics by Monte Carlo simulations as in Lothian and Taylor (2008). Under the null hypothesis of $\theta = 0$, and the assumption of $\beta^{ecm} = 1$, the model is nonstationary so the standard distribution does not apply.. Under the null of $\beta^{icm} \geq 0$, the model is either a standard AR or explosive so we expect the distribution of β^{icm} to be right skewed. To be specific, we simulate the estimated model under the null by 5000 times. Each time we generate a real exchange rate, initilised at 0, of length 100+ the number of observations in data. The first 100 points are then discarded. We estimate the STAR or the ICM for each simulation and form a empirical distribution for the t-statistics considered. We cannot reject $\theta = 0$ for most countries[13] with the exceptions of Denmark, Italy, Sweden

[13] Teräsvirta (1994) devises another test for the same purpose, by which we can reject $\theta = 0$ for more countries. However, Kilic (2004) and Sandberg 2008 suggest that the size of Teräsvirta (1994)'s test can be distorted. When the test is applied to a linear but nonstationary or a highly persistent data generating process, the null hypothesis of linearity is rejected too often. Moreover, Ahmad et al. (2013) find that Teräsvirta (1994)'s test suffers from an omitted exogenous variables problem, which may mislead the results as we included an exogenous time-varying equilibrium of the real exchange rate.

and Switzerland. Nevertheless, this result is sensitive to the delay parameter d. We can reject $\beta^{icm} \geq 0$ for most countries, namely Belgium, Finland, France, Italy, Portugal, Sweden and the UK at a 5% level of significance, and additionally Australia, Denmark, Japan, Netherlands, New Zealand, Norway, and Switzerland at a 10% level of significance.

The estimated coefficients in equation 1.2 are similar in the ICM model and the STAR model. Consistent with the literature (Taylor and Taylor, 2004), the real GDP per capita differential has a small coefficient in $[-0.4, +0.4]$. As discussed in section 1.2, the overall HBS effect can be reduced or even reversed by the terms of trade channel. Nevertheless, given that we have controlled the terms of trade, the coefficient a_1 should mainly capture the direct HBS effect. In addition, the standard t-test cannot reject the null hypothesis of no direct HBS effect for all countries. This could be due to the fact that our data span is not long enough to capture the long-run HBS effect. Furthermore, Paya and Peel (2006) suggest that the standard t-test gives spurious results if the explanatory variables are very persistent. This is indeed our case. Following Paya and Peel (2006), we construct small sample distributions by bootstrap for a_1, a_2, and a_3. Some evidence for the direct HBS effect can be found since a_1 is significant in 9 out of 18 countries. The terms of trade channel of the HBS effect, i.e., the coefficient a_2, is significant in 14 out of 18 countries. The coefficient of the trade balance is usually negative and significant in 17 out of 18 countries, which suggests that the real exchange rate also tends to move in such a direction that the value of Home's external liabilities decreases in terms of domestic goods.

Finally, we notice that, with a few exceptions, there is no further autocorrelation, heteroscedasticity, and nonlinearity in the residual of the STAR model. However, in the ICM model, we detect a neglected nonlinearity for Denmark, Italy and Norway, which suggests a missing smooth transition mechanism. We investigate this possibility in the next subsection.

1.5.4 In-sample fitness

We compare in-sample fitness of ICM and STAR by likelihood ratio tests. To begin, we construct a model nesting both, referred to as ST-ICM:

$$err_t = \sum_{j=1}^{p} \beta_j^{ecm} err_{t-j} + \left[\sum_{j=1}^{p} \beta_j^{ecm*} err_{t-j} \right] \times G(\theta, d) + \beta^{icm} ie_{t-1} + \varepsilon_{t,sticm}.$$

Then, we test if this model is superior to the ICM and the STAR in terms of log likelihood. If the ICM is the true data generating process, we expect to find the ST-ICM not significantly superior to the ICM but significantly superior to the STAR. As reported in table 1.3,[14] we find the results consistent with our expectation for 12 countries. For the remaining countries, Denmark, Italy, Norway, and Sweden show that the ST-ICM is significantly better than both the STAR and the ICM. This result suggests some nonlinearity similar to STAR in their true data generating processes. In fact, we detect neglected nonlinearity in ICM residuals using data from these countries. For Austria and Canada, the ST-ICM is not significantly better than either the STAR or the ICM. This is not surprising as their real exchange rate dynamics look quite different than others, indicated by figure 1.1.

To investigate the possible origin of superiority of the ICM, we report ACFs calculated from the data and models for the UK in figure 1.2. The ICM's ACF fits the data remarkably well, in particular the cyclical pattern. On the other hand, the STAR's ACF decreases monotonically and is roughly in line with long-run "trend" of the data's ACF. By design, STAR only allows nonlinear changes in mean-reversion speeds but not the mean-reversion directions.

[14] The results in this table are robust to specifications of the STAR model, such as ESTAR versus LSTAR and different values of d, and using information criteria.

Table 1.3: P-value of likelihood ratio tests

Country	Australia	Austria	Belgium	Canada	Denmark	Finland
ST-ICM v.s. ICM	{0.72}	{0.82}	{1.00}	{0.68}	{0.01}	{0.29}
ST-ICM v.s. STAR	{0.01}	{0.19}	{0.00}	{0.12}	{0.04}	{0.00}
Country	France	Germany	Italy	Japan	Netherlands	New Zealand
ST-ICM v.s. ICM	{0.25}	{0.72}	{0.00}	{0.07}	{0.13}	{0.40}
ST-ICM v.s. STAR	{0.00}	{0.01}	{0.00}	{0.00}	{0.01}	{0.01}
Country	Norway	Portugal	Spain	Sweden	Switzerland	the UK
ST-ICM v.s. ICM	{0.01}	{0.50}	{1.00}	{0.01}	{0.25}	{1.00}
ST-ICM v.s. STAR	{0.02}	{0.00}	{0.05}	{0.00}	{0.01}	{0.00}

Figure 1.2: Autocorrelation function

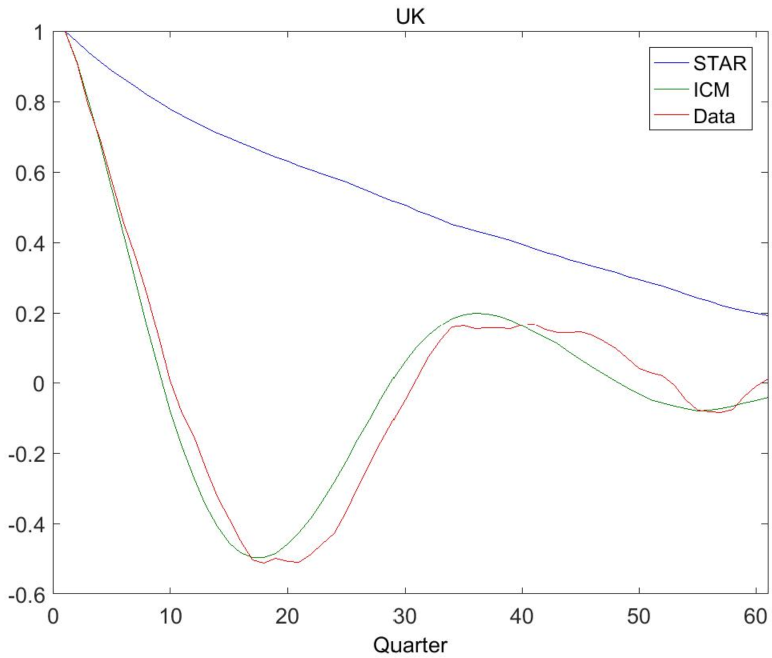
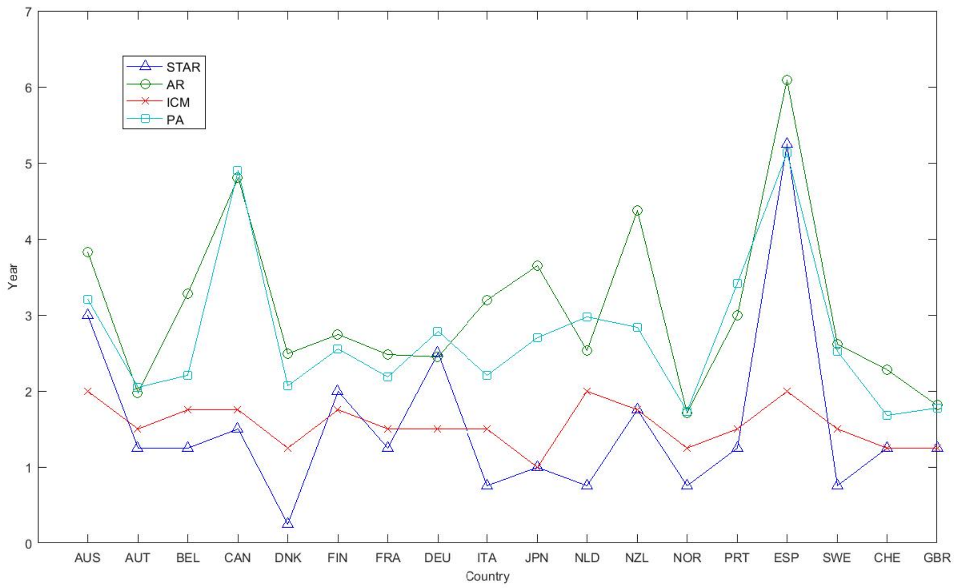


Figure 1.3: Half-lives



1.6 Revisiting the PPP puzzle

In this section, we study the PPP puzzle by investigating mean-reverting properties of the estimated models. For this purpose, it is useful to consider two extra models: an AR model and a partial adjustment model (PA). The AR model contains both constant real exchange rate equilibrium and a constant mean-reversion speed. It is commonly used in the DSGE literature to measure persistence of real exchange rates. The PA model allows time-varying equilibrium but keeps the mean-reversion speed constant. Put in another way, the PA model is the ICM model with $\beta^{icm} = 0$ or the STAR model with $\theta = 0$.

To maintain comparability with the literature, we calculate half-lives implied in each model. Figure 1.3 reports half-lives for all 18 countries. A half-live is defined as the number of periods for half of a shock to die out; hence, it can be used to measure the mean-reversion speed.[15] The AR model implies half-lives ranging from 2-6 years, which is consistent with the literature (Taylor and Taylor, 2004). By allowing time-varying equilibrium, the PA model reduces the half-lives in most countries. However, the half-lives are still too big to resolve the puzzle. The ICM model implies much shorter half-lives with an average equal to 1.5 years. Notably, the half-live of Spain-US real exchange rate in the ICM model is much smaller than in all other models. At last, in the STAR model, half-lives depends on shock sizes. In figure 1.3, we consider a very large shock that drives the real exchange rate 40% away from its equilibrium so that the average half-live equals 1.5 years, same as in the ICM model. However, as such big shocks would be very rare, the estimated STAR model still indicate very persistent real exchange rates.

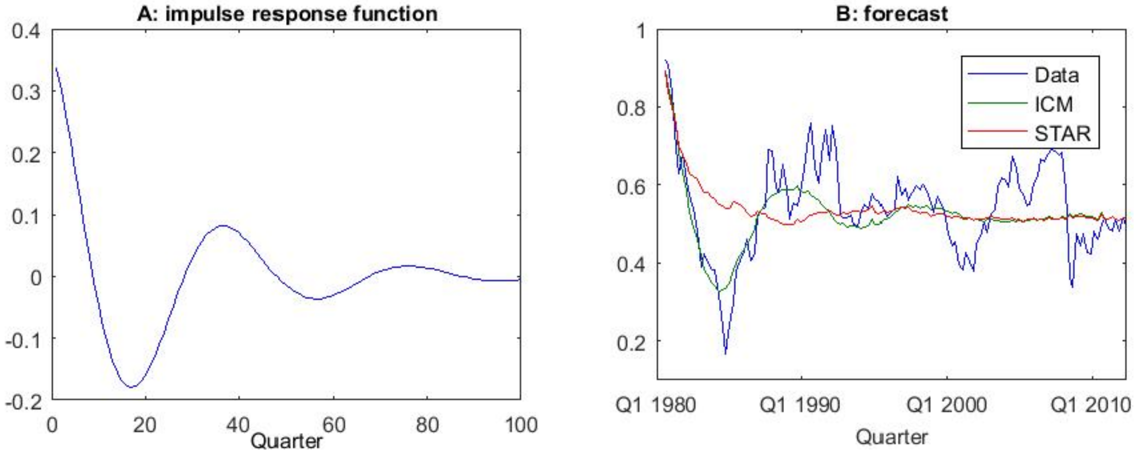
Half-lives provide very limited information. To obtain full information of mean-reversion processes, we look at the entire impulse response function. In panel A of figure 1.4, we plot a impulse response

[15]We calculate half-lives by impulse responses. For a nonlinear model like the STAR model, we need to use the Generalized Impulse Response Function (GIRF) according to Koop et al. (1996):

$$GIRF_h(h, \phi, \omega_{t-1}) = E(y_{t+h} \mid u_t = \phi, \omega_{t-1}) - E(y_{t+h} \mid u_t = 0, \omega_{t-1})$$

where $h = 1, 2, \ldots$, denotes horizon, u_t is a shock that occurs at time t, and ω_{t-1} is the history of a time series. $GIRF_h$ is a random variable and its analytic expression is generally not available for $h > 1$. To calculate the GIRF numerically, we conduct two Monte Carlo forecasts for 200 periods ahead using the estimated STAR model. The first has a shock occurring in the first period and the second does not. Both forecasts condition on $q_0 = q_0^{EQ}$. The difference between the two forecasts gives the $GIRF$.

Figure 1.4: ICM dynamics of the UK-US real exchange rate



function of the ICM model estimated using the UK data, which looks similar to the ACF reported in figure 1.2. This non-monotonic impulse response function states that a half-life only measures the short-term persistence. Likewise, measuring persistence by the first few orders of autocorrelation, as do many papers in the literature, gives misleading results. Clearly, the impulse response function suggests that real exchange rates are persistent only in the long run due to oscillation around the equilibrium. This oscillation explains a large proportion of the real exchange rate volatility at low frequencies. Therefore, our ICM model answers Rogoff's question by distinguishing between different degrees of persistence in the short-run and the long-run. In addition, the ICM model features a mean-reversion speed reaching its maximum when the real exchange rate passes through the equilibrium. By contrast, the STAR model features a slow mean-reversion speed in the neighbourhood of the equilibrium. To see how this difference matters, we compare in-sample forecasts of the ICM and the STAR. Forecasts of the UK-US real exchange rate is plotted in panel B of figure 1.4. The forecasts are made conditional on 1980Q3. The special ICM dynamics match the large depreciation data in the mid 1980s.

1.7 Forecasting performance

In-sample fitness does not necessarily correlate with out-of-sample predictability. We now formally test predictability of real exchange rate based on the ICM and the STAR model separately. There are large variances in the literature regarding forecast evaluation methods, on which the conclusion depends a lot (Rossi, 2013). Here I choose a random walk without a drift as our benchmark model because beating the random walk forecasts can be interpreted as exchange rate predictability, and adding a drift worsen the results (Engel et al., 2007). We consider forecast horizons from 1 to 12 quarters, covering those typically considered in the literature. We estimate the models and calculate forecast error using rolling windows. The first window runs from 1973Q2 to 2003Q1-h where h denotes the forecast horizon, then we make h-step forecasts for 2003Q1. We treat out-of-sample real exchange rate equilibrium equal to the last in-sample observation. For the nonlinear STAR model, forecasts are based on 1000 Monte Carlo simulation. The second window runs from 1973Q3 to 2003Q2-h, then the third window etc. The number of forecasts $f = 44$ at each horizon is constant. Given our 163 observations, the out-of-sample portion accounts roughly one third of the full sample. We find our results generally robust to other partitions.

After rolling window forecasting, the mean squared forecast error (MSFE) of a given horizon is calculated as $MSFE_m = \frac{1}{f}\sum_{t=2003Q1-h}^{2013Q4-h}\left(\epsilon_{t+h|t}\right)^2$ where m denotes a model, i.e., ICM, STAR, or random walk. There are many ways to compare forecasting performance. Following the suggestion of Rogoff and

Stavrakeva (2008), we employ Theil's U-statistic, $U_{ICM} = \frac{MSFE_{ICM}}{MSFE_{RW}}$ and $U_{STAR} = \frac{MSFE_{STAR}}{MSFE_{RW}}$. If the U statistics is smaller than 1, it indicates real exchange rate predictability based on the ICM or the STAR model. Significance of the predictability can be tested against the null hypothesis of $U_m \geq 1$ by bootstrap. According to Rogoff and Stavrakeva (2008), this test is more powerful and better sized than popular alternatives. The U-statistics and their P-values are reported in table 1.4. We use "+" to indicate a U-statistic smaller than 1, in which occasions P-values are reported in braces. We use "*" to indicate statistical significance at 10%.

The ICM beats random walk forecasts in only 4 countries, namely, Canada, Denmark, Italy, and Norway. However, in most of these cases, the predictability is significant. The ICM tend to provide better forecasts at longer horizons, except Canada showing the opposite. In fact, the forecast gain of the ICM at long horizons is quite strong with lowest U-statistic being 0.29 (Italy, 12 quarters ahead). The STAR beats random walk forecasts in 17 countries, and significantly in 10 countries.The forecast gain varies across horizons and countries. The forecast accuracy is better at shorter horizons in France and Netherlands, while the opposite holds true in Germany, Japan, Spain, Sweden, and the UK. In Denmark, Norway, and Switzerland, the forecast gain is larger at medium horizons. We also find that time-varying real exchange rate equilibrium helps forecast. If assuming constant equilibrium, the number of cases in which the ICM or the STAR beats random walk forecasts drops by about a quarter.

At last, we note that the two models seem complementary to each other. When the ICM shows significant forecast gain, the STAR does not, and vice versa. However, there are cases in which neither models perform well, e.g., Belgium.

1.8 Conclusion

In this chapter, we model real exchange rates between the US and 18 OECD countries by an innovative dynamic process called ICM and time-varying equilibrium. We use the popular STAR model as a benchmark. After estimating the models, we find evidence of integral correction mechanism and smooth transition nonlinearity in data. We further establish that the ICM fits in-sample data both statistically and economically better than the STAR model. Superiority of the ICM originates from its ability to capture ACFs of real exchange rates. The ICM model implies that real exchange rates revert to their equilibrium very quickly in the short-run, but moves back and forth around the equilibrium in the long-run. By distinguishing the degree of persistence at different horizons, the ICM model helps us understand the PPP puzzle. At last, we evaluate out-of-sample performance of both models. The ICM beats random walk forecasts for only 4 out of 18 real exchange rates. However, when it does, the ICM can predict real exchange rate well at both short and long horizons. Real exchange rate predictability, though not always significant, is found for 17 real exchange rates using the STAR model.

Table 1.4: Out-of-sample performance

Horizon	U_{ICM}	U_{STAR}	U_{ICM}	U_{STAR}	U_{ICM}	U_{STAR}
	Austrialia		Austria		Belgium	
1	1.847	0.994{0.220}+	1.111	0.988{0.129}+	2.362	1.053
2	2.401	0.976{0.234}+	1.195	1.023	3.399	1.045
3	2.993	0.956{0.203}+	1.297	0.989{0.177}+	4.51	1.05
4	3.799	0.943{0.204}+	1.339	0.984{0.146}+	5.254	1.096
5	4.641	1.039	1.431	0.984{0.160}+	6.208	1.11
6	5.199	1.038	1.398	0.991{0.232}+	7.016	1.128
7	5.441	1.058	1.254	0.992{0.214}+	6.61	1.581
8	1.396	Inf	1.145	1.006	6.326	1.495
9	5.623	1.06	1.126	1.025	6.336	1.437
10	5.76	1.043	1.215	0.983{0.183}+	6.321	1.399
11	5.911	1.037	1.128	1.001	6.423	1.507
12	5.837	1.008	1.082	1.015	6.475	1.531
	Canada		Denmark		Finland	
1	0.954{0.036}+*	1.029	0.958{0.037}+*	0.983{0.116}+	1.083	0.955{0.042}+*
2	0.937{0.069}+*	1.012	0.897{0.036}+*	1.009	1.13	0.957{0.101}+
3	0.912{0.049}+*	0.981{0.196}+	0.837{0.021}+*	0.979{0.138}+	1.219	0.926{0.096}+*
4	0.937{0.090}+*	0.979{0.239}+	0.770{0.016}+*	0.755{0.012}+*	1.24	0.950{0.173}+
5	0.993{0.135}+	0.981{0.223}+	0.704{0.016}+*	0.794{0.032}+*	1.298	0.942{0.135}+
6	1.06	0.974{0.227}+	0.643{0.025}+*	0.878{0.089}+*	1.363	0.972{0.252}+
7	1.125	0.996{0.334}+	0.619{0.017}+*	0.877{0.099}+*	1.331	0.939{0.184}+
8	1.182	1.008	0.584{0.016}+*	0.853{0.101}+	1.298	0.953{0.221}+
9	1.204	0.994{0.332}+	0.542{0.012}+*	0.860{0.109}+	1.308	0.941{0.214}+
10	1.193	0.991{0.315}+	0.490{0.015}+*	0.883{0.127}+	1.296	0.900{0.196}+
11	1.138	1	0.457{0.015}+*	0.895{0.154}+	1.324	0.924{0.229}+
12	1.131	1.002	0.418{0.010}+*	0.918{0.172}+	1.368	0.905{0.203}+
	France		Germany		Italy	
1	1.344	0.915{0.010}+*	2.225	0.864{0.000}+*	0.953{0.027}+*	1.004
2	1.566	0.923{0.060}+*	3.003	0.816{0.006}+*	0.868{0.014}+*	0.999{0.293}+
3	1.798	0.821{0.017}+*	3.685	0.779{0.008}+*	0.801{0.018}+*	0.988{0.227}+
4	1.857	0.824{0.038}+*	4.031	0.827{0.044}+*	0.691{0.008}+*	0.998{0.283}+
5	1.96	0.872{0.093}+*	4.492	0.858{0.061}+*	0.572{0.005}+*	1.003
6	2.024	0.924{0.144}+	4.788	0.778{0.044}+*	0.482{0.002}+*	1.009
7	1.86	0.914{0.155}+	4.458	0.749{0.043}+*	0.434{0.004}+*	1.004
8	1.773	0.898{0.162}+	4.289	0.713{0.029}+*	0.406{0.002}+*	1.006
9	1.744	0.911{0.181}+	4.257	0.708{0.037}+*	0.385{0.004}+*	0.954{0.229}+
10	1.699	0.899{0.208}+	4.261	0.732{0.078}+*	0.358{0.002}+*	0.939{0.224}+
11	1.683	0.939{0.230}+	4.38	0.729{0.074}+*	0.329{0.005}+*	1.054
12	1.672	0.989{0.327}+	4.485	0.732{0.088}+*	0.290{0.003}+*	1.001

Note: "+" indicates a U-statistic smaller than 1, "*" indicates statistical significance at 10%, P-values are reported in braces.

Table 4: Out-of-sample performance (continue)

Horizon	U_{ICM}	U_{STAR}	U_{ICM}	U_{STAR}	U_{ICM}	U_{STAR}
	Japan		Netherlands		New Zealand	
1	3.315	0.989{0.171}+	1.272	0.998{0.276}+	1.221	1.014
2	5.847	0.981{0.157}+	1.448	0.989{0.219}+	1.287	0.974{0.168}+
3	7.338	0.940{0.101}+	1.591	0.979{0.180}+	1.359	0.965{0.124}+
4	8.615	0.858{0.057}+*	1.619	1.028	1.412	0.946{0.137}+
5	9.572	0.792{0.045}+*	1.706	1.098	1.439	0.922{0.120}+
6	9.586	0.776{0.050}+*	1.797	1.12	1.46	1.169
7	9.809	0.745{0.044}+*	1.756	1.118	1.463	1.09
8	10.383	0.716{0.046}+*	1.754	1.09	1.459	1.194
9	11.072	0.670{0.042}+*	1.785	1.065	1.468	1.198
10	11.64	0.678{0.046}+*	1.79	1.068	1.463	1.273
11	11.882	0.651{0.033}+*	1.846	1.156	1.448	1.301
12	12.186	0.638{0.052}+*	1.931	1.172	1.388	1.305
	Norway		Portugal		Spain	
1	0.994{0.113}+	1.128	1.16	1.01	2.972	0.937{0.027}+*
2	0.985{0.118}+	1.045	1.223	0.992{0.158}+	4.621	0.802{0.005}+*
3	0.987{0.117}+	0.766{0.005}+*	1.323	1.012	6.402	0.754{0.005}+*
4	0.960{0.095}+*	0.709{0.003}+*	5.11	0.993{0.221}+	7.689	0.783{0.023}+*
5	0.937{0.093}+*	0.736{0.010}+*	1.526	0.993{0.209}+	8.95	0.717{0.008}+*
6	0.915{0.097}+*	0.922{0.153}+	6.594	0.999{0.393}+	9.953	0.709{0.020}+*
7	0.882{0.086}+*	0.965{0.215}+	1.53	1.015	9.533	0.677{0.017}+*
8	0.860{0.079}+*	1.043	1.527	0.997{0.317}+	9.209	0.711{0.039}+*
9	0.839{0.096}+*	1.034	1.92	1	9.273	0.627{0.018}+*
10	0.791{0.086}+*	0.982{0.272}+	1.886	1	9.181	0.595{0.018}+*
11	0.760{0.062}+*	1.122	7.2	0.986{0.130}+	9.117	0.595{0.021}+*
12	0.720{0.067}+*	1.239	7.528	0.995{0.290}+	9.057	0.565{0.014}+*
	Sweden		Switzerland		UK	
1	1.779	0.955{0.057}+*	2.244	1.075	1.239	0.947{0.024}+*
2	2.235	0.852{0.011}+*	3.479	0.964{0.153}+	1.329	0.872{0.018}+*
3	3.145	0.838{0.033}+*	4.929	0.969{0.212}+	1.44	0.842{0.026}+*
4	4.125	0.802{0.040}+*	5.241	0.867{0.065}+*	1.435	0.819{0.038}+*
5	5.266	0.786{0.048}+*	6.184	0.903{0.168}+	1.415	0.813{0.059}+*
6	6.044	0.803{0.080}+*	6.854	0.902{0.178}+	1.352	0.797{0.074}+*
7	6.167	0.798{0.086}+*	6.437	0.885{0.191}+	1.263	0.780{0.056}+*
8	6.193	0.761{0.069}+*	6.357	0.907{0.234}+	1.224	0.760{0.060}+*
9	6.256	0.746{0.077}+*	6.318	0.844{0.176}+	1.192	0.746{0.060}+*
10	7.166	0.742{0.090}+*	6.475	1.038	1.159	0.745{0.079}+*
11	7.971	0.798{0.145}+	6.564	1.107	1.132	0.758{0.070}+*
12	8.892	0.824{0.203}+	6.534	1.109	1.073	0.767{0.115}+

Note: "+" indicates a U-statistic smaller than 1, "*" indicates statistical significance at 10%, P-values are reported in braces.

Appendix A

A.1 Estimation results of STAR

Table A: Estimation Results of STAR

Country	Australia	Austria	Belgium
Specification	ESTAR (1,1)	LSTAR (1,3)	LSTAR (1,7)
a_0	-0.21 {0.16}	-2.59 {0.00}	-3.45 {0.00}
a_1	-0.21 {0.63}	-0.40 {0.51}	0.11 {0.83}
a_2	0.21 {0.17}	1.20 {0.01}	3.11 {0.001}
a_3	-2.00 {0.002}	-1.26 {0.11}	-2.31 {0.04}
β^{ecm}	$= -\beta^{ecm*}$	1 (fixed)	1 (fixed)
θ	-0.40 (-0.42) {0.33}	-1.18 (-0.17) {0.19}	0.48 (0.09) {0.40}
β^{ecm*}	-0.98 (-14.06)	-0.18 (-1.31)	-0.15 (-2.11)
σ_{star}	0.05	0.07	0.06
R^2	0.92	0.85	0.91
Diagnostic: AR(1)	{0.37}	{0.87}	{0.58}
Diagnostic: ARCH(1)	{0.75}	{0.94}	{0.44}
Diagnostic: nonlinearity	{0.78}	{0.95}	{0.70}

Throughout Section A.1 and Section A.2:

Numbers in () and {} are t-statistics and P-value, respectively.

Diagnostic: AR(1), ARCH(1), nonlinearities are Lagrange Multiplier tests to neglect autocorrelation, heteroscedasticity and nonlinearity, respectively.

Table A continued: Estimation Results of STAR

Country	Canada	Denmark	Finland
Specification	LSTAR (1,5)	LSTAR (1,1)	LSTAR (1,8)
a_0	-0.09 {0.05}	-2.36 {0.001}	-1.43 {0.00}
a_1	-0.34 {0.13}	0.38 {0.35}	0.10 {0.75}
a_2	0.98 {0.00}	0.72 {0.05}	0.12 {0.54}
a_3	-0.90 {0.00}	-1.24 {0.02}	-0.34 {0.17}
β^{ecm}	1 (fixed)	0.97 (31.98)	0.95 (3.06)
θ	4.55 (1.55) {0.12}	-12.70 (-2.06) {0.02}	-31.16 (-0.43) {0.33}
β^{ecm*}	-0.13 (-1.92)	-0.51 (-2.05)	-0.03 (-0.49)
σ_{star}	0.03	0.06	0.05
R^2	0.95	0.878	0.88
Diagnostic: AR(1)	{1.00}	{0.44}	{0.76}
Diagnostic: ARCH(1)	{1.00}	{0.16}	{0.98}
Diagnostic: nonlinearity	{1.00}	{0.95}	{0.99}

Table A continued: Estimation Results of STAR

Country	France	Germany	Italy
Specification	LSTAR (1,4)	LSTAR (1,4)	LSTAR (1,2)
a_0	-1.75 {0.00}	-0.58 {0.002}	-7.51 {0.00}
a_1	-0.28 {0.47}	-0.34 {0.29}	-0.36 {0.09}
a_2	0.45 {0.17}	0.59 {0.03}	0.78 {0.00}
a_3	-0.34 {0.73}	1.28 {0.06}	-0.05 {0.93}
β^{ecm}	0.96 (28.11)	0.95 (15.58)	0.97 (2.29)
θ	-45.71 (-0.72) {0.23}	6.20 (0.25) {0.40}	-19.58 (-1.67) {0.05}
β^{ecm*}	-0.16 (-1.72)	-0.02 (-0.16)	-0.37 (-1.90)
σ_{star}	0.06	0.06	0.05
R^2	0.88	0.88	0.90
Diagnostic: AR(1)	{0.37}	{0.91}	{0.94}
Diagnostic: ARCH(1)	{0.68}	{0.43}	{0.92}
Diagnostic: nonlinearity	{0.62}	{0.39}	{1.00}

Table A continued: Estimation Results of STAR

Country	Japan	Netherlands	New Zealand
Specification	LSTAR (1,5)	LSTAR (1,4)	LSTAR (1,7)
a_0	-5.14 {0.001}	-0.32 {0.04}	-0.57 {0.00}
a_1	0.13 {0.69}	0.13 {0.73}	-0.17 {0.42}
a_2	0.58 {0.00}	1.38 {0.001}	0.39 {0.04}
a_3	-1.68 {0.09}	-1.84 {0.00}	-0.71 {0.01}
β^{ecm}	1 (fixed)	1 (fixed)	1 (fixed)
θ	-1.65 (-1.31) {0.15}	-18.38 (-1.08) {0.16}	3.80 (0.87) {0.12}
β^{ecm*}	-0.19 (-2.51)	-0.28 (-1.92)	-0.11 (-1.75)
σ_{star}	0.05	0.06	0.06
R^2	0.93	0.89	0.91
Diagnostic: AR(1)	{0.83}	{1.00}	{1.00}
Diagnostic: ARCH(1)	{0.75}	{1.00}	{1.00}
Diagnostic: nonlinearity	{0.98}	{1.00}	{1.00}

Table A continued: Estimation Results of STAR

Country	Norway	Portugal	Spain
Specification	LSTAR (1,2)	LSTAR (1,2)	ESTAR (1,1)
a_0	-0.67 {0.39}	-3.45 {0.00}	-5.99 {0.00}
a_1	-0.43 {0.17}	1.17 {0.01}	0.02 {0.96}
a_2	0.15 {0.01}	0.06 {0.88}	0.22 {0.14}
a_3	-0.70 {0.002}	4.73 {0.00}	-0.44 {0.35}
β^{ecm}	0.96 (3.44)	1 (fixed)	$= -\beta^{ecm*}$
θ	-26.42 (-1.02) {0.15}	-2.60 (-1.14) {0.19}	-0.03 (-0.91) {0.18}
β^{ecm*}	-0.41 (-1.96)	-0.15 (-1.92)	-1.04 (-59.21)
σ_{star}	0.06	0.07	0.05
R^2	0.84	0.95	0.97
Diagnostic: AR(1)	{1.00}	{0.76}	{0.84}
Diagnostic: ARCH(1)	{0.72}	{0.61}	{0.80}
Diagnostic: nonlinearity	{1.00}	{0.18}	{0.81}

Table A continued: Estimation Results of STAR

Country	Sweden	Switzerland	the UK
Specification	ESTAR (1,1)	ESTAR (1,1)	LSTAR (1,8)
a_0	-1.24 {0.25}	-0.03 {0.93}	0.53 {0.05}
a_1	-0.36 {0.50}	-0.25 {0.70}	-0.01 {0.98}
a_2	0.87 {0.01}	1.01 {0.08}	0.08 {0.75}
a_3	-2.72 {0.00}	-0.63 {0.33}	0.80 {0.14}
β^{ecm}	$= -\beta^{ecm*}$	$= -\beta^{ecm*}$	1 (fixed)
θ	-2.95 (-2.81) {0.003}	-1.66 (-2.08) {0.02}	-8.46 (-0.92) {0.14}
β^{ecm*}	-1.08 (-20.74)	-1.03 (-10.38)	-0.16 (-2.36)
σ_{star}	0.06	0.06	0.05
R^2	0.91	0.87	0.83
Diagnostic: AR(1)	{0.08}	{0.98}	{0.01}
Diagnostic: ARCH(1)	{0.38}	{0.51}	{0.97}
Diagnostic: nonlinearity	{0.78}	{0.81}	{0.40}

A.2 Estimation results of ICM

Table B: Estimation Results of ICM

Country	Australia	Austria	Belgium
a_0	0.34 {0.36}	-2.84 {0.00}	-3.06 {0.00}
a_1	-0.21 {0.64}	-0.41 {0.48}	-0.15 {0.76}
a_2	0.20 {0.19}	1.16 {0.01}	1.50 {0.11}
a_3	-1.93 {0.002}	-1.41 {0.06}	-0.85 {0.39}
β^{ecm}	0.94 (28.82)	0.89 (22.72)	0.94 (33.62)
β^{icm}	-0.006 (-2.75) {0.06}	-0.002 (-0.55) {0.72}	-0.011 (-3.54) {0.006}
R^2	0.92	0.85	0.91
Diagnostic: AR(1)	{0.72}	{0.57}	{0.95}
Diagnostic: ARCH(1)	{0.66}	{0.71}	{0.32}
Diagnostic: nonlinearity	{0.36}	{0.12}	{0.90}

Table B continued: Estimation Results of ICM

Country	Canada	Denmark	Finland
a_0	0.28 {0.009}	-1.84 {0.02}	-1.44 {0.00}
a_1	-0.41 {0.19}	-0.03 {0.95}	0.06 {0.83}
a_2	0.96 {0.00}	1.22 {0.002}	0.15 {0.41}
a_3	-0.88 {0.009}	-1.32 {0.07}	-0.15 {0.09}
β^{ecm}	0.92 (30.19)	0.91 (25.85)	0.94 (33.69)
β^{icm}	-0.002 (-1.97) {0.27}	-0.012 (-2.79) {0.06}	-0.010 (-3.40) {0.008}
R^2	0.95	0.88	0.89
Diagnostic: AR(1)	{0.61}	{0.08}	{0.49}
Diagnostic: ARCH(1)	{0.96}	{0.52}	{0.74}
Diagnostic: nonlinearity	{0.18}	{0.01}	{0.51}

Table B continued: Estimation Results of ICM

Country	France	Germany	Italy
a_0	-1.52 {0.00}	-0.05 {0.86}	-7.93 {0.00}
a_1	-0.41 {0.12}	-0.42 {0.20}	-0.62 {0.009}
a_2	0.40 {0.25}	0.38 {0.27}	0.66 {0.002}
a_3	-0.56 {0.59}	0.82 {0.19}	0.09 {0.88}
β^{ecm}	0.93 (31.23)	0.92 (28.73)	0.93 (30.23)
β^{icm}	-0.013 (-3.86) {0.003}	-0.008 (-1.937) {0.28}	-0.011 (-3.33) {0.01}
R^2	0.88	0.89	0.91
Diagnostic: AR(1)	{0.59}	{0.72}	{0.54}
Diagnostic: ARCH(1)	{0.58}	{0.78}	{0.86}
Diagnostic: nonlinearity	{0.22}	{0.31}	{0.01}

Table B continued: Estimation Results of ICM

Country	Japan	Netherlands	New Zealand
a_0	-5.81 {0.00}	-0.68 {0.01}	-0.71 {0.06}
a_1	0.13 {0.64}	-0.07 {0.86}	-0.31 {0.13}
a_2	0.62 {0.00}	1.22 {0.002}	0.54 {0.004}
a_3	-1.85 {0.06}	-1.40 {0.003}	-0.67 {0.02}
β^{ecm}	0.86 (21.05)	0.95 (33.71)	0.95 (27.51)
β^{icm}	-0.006 (-2.77) {0.06}	-0.009 (-2.76) {0.06}	-0.012 (-2.80) {0.06}
R^2	0.94	0.89	0.92
Diagnostic: AR(1)	{0.15}	{0.50}	{0.54}
Diagnostic: ARCH(1)	{0.61}	{0.97}	{0.38}
Diagnostic: nonlinearity	{0.77}	{0.26}	{0.87}

Table B continued: Estimation Results of ICM

Country	Norway	Portugal	Spain
a_0	-1.93 {0.02}	-2.41 {0.002}	-5.72 {0.00}
a_1	-0.02 {0.95}	1.12 {0.02}	0.02 {0.96}
a_2	0.23 {0.007}	1.04 {0.00}	0.29 {0.06}
a_3	-0.62 {0.007}	4.98 {0.00}	-0.67 {0.20}
β^{ecm}	0.91 (25.63)	0.94 (35.25)	0.93 (37.71)
β^{icm}	-0.011 (-2.76) {0.06}	-0.012 (-4.49) {0.00}	-0.003 (-1.91) {0.29}
R^2	0.84	0.95	0.97
Diagnostic: AR(1)	{0.31}	{0.38}	{0.23}
Diagnostic: ARCH(1)	{0.60}	{0.48}	{0.69}
Diagnostic: nonlinearity	{0.03}	{0.69}	{0.86}

Table B continued: Estimation Results of ICM

Country	Sweden	Switzerland	the UK
a_0	-1.62 {0.11}	0.14 {0.68}	0.48 {0.13}
a_1	-0.07 {0.86}	0.002 {0.99}	0.01 {0.97}
a_2	0.63 {0.007}	1.07 {0.01}	0.03 {0.89}
a_3	-2.33 {0.00}	-0.21 {0.72}	0.69 {0.15}
β^{ecm}	0.92 (28.90)	0.91 (25.41)	0.92 (29.28)
β^{icm}	-0.013 (-3.28) {0.01}	-0.013 (-2.70) {0.07}	-0.03 (-5.00) {0.00}
R^2	0.91	0.88	0.85
Diagnostic: AR(1)	{0.63}	{0.24}	{0.31}
Diagnostic: ARCH(1)	{0.63}	{0.90}	{0.42}
Diagnostic: nonlinearity	{0.26}	{0.44}	{0.56}

A.3 Data source

We mainly collect CPI and nominal exchange rate data from IMF's International Financial Statistics (IFS) database, except Germany's CPI available at the OECD's Main Economic Indicators database. The bilateral nominal exchange rates between Euro area countries and the US are converted from the Euro/USD exchange rate by the official Euro conversion rates.

Data on real GDP does primarily come from the OECD's Outlook database (OECD code: GDPV) except for Germany, data for which comes from the IFS database (calculated by GDP and GDP deflator, IFS code: 13499B.CZF, 13499BIRZF). The IFS data is in billion Deutsche mark at the quarterly level and was converted to million Euro at the annual level, in line with OECD data. Annual data on population is from IFS (IFS code: 99Z—ZF).

Data on the terms of trade is from OECD's Outlook database (OECD code: TTRADE) and is calculated by the ratio between the export deflator and the import deflator of goods and services (OECD code: PXGS, PMGS). The exceptions are Austria and Germany, for which the data is collected from IFS's import and export price index (IFS code: 76-ZF, 76-X-ZF).

Data on the trade balance for all countries is also from OECD's Outlook database (OECD code: FBGSD), defined as net exports of goods and services.

In the OECD databases, data for the Euro area countries is now expressed in Euro, so pre-1999 data was converted from the national currency using official euro conversion rates. Thus, "national currency" always refers to the Euro for the Euro area countries. IFS data on the terms of trade and the CPI is not seasonally adjusted at the source and was therefore harmonized with the X11 procedure, which is the standard method of seasonal adjustment for many statistical agencies.

A.4 Regression with parameterized ACF

This appendix presents the quasi maximum likelihood estimation procedure in the estimation of equations 1.8 and 1.9, which are repeated here:

$$\rho_\tau = \frac{1 - a\left[1 - \cos\left(\omega\tau\right)\right]}{1 + b\tau^c},$$

$$q_t = a_0 + a_1 \ln rgdp_t + a_2 \ln tot_t + a_3 tb_t + err_t.$$

We rewrite the second equation as

$$q = q^{EQ} + err,$$

where $q \equiv (q_1, \ldots, q_T)$, $q^{EQ} \equiv (a_0 + a_1 \ln rgdp_1 + a_2 \ln tot_1 + a_3 tb_1, \ldots, a_0 + a_1 \ln rgdp_T + a_2 \ln tot_T + a_3 tb_T) = \beta X$, and $err \sim (0, \Sigma)$ are $(1 \times T)$ vectors. Assuming err_t mean-reversion, the autocorrelation matrix of err has a symmetric Toeplitz structure

$$R \equiv \begin{matrix} \rho_0 & \cdots & \rho_{T-1} \\ \vdots & \ddots & \vdots \\ \rho_{T-1} & \cdots & \rho_0 \end{matrix}$$

where $\rho_\tau \equiv cov(err_t, err_{t-\tau})/\sqrt{(var(err_t)var(err_{t-\tau})}$. If err_t follows an AR(1) process, we have $\rho_\tau = \rho^\tau$ and one parameter, ρ, to estimate. Given equation 1.8, there are four parameters in the ACF to estimate. To estimate parameters in q^{EQ} and Σ, we note that Σ is proportional to R and applying Cholesky decomposition gives $\Sigma = \sigma^2 LL'$. L^{-1} is a lower triangular matrix that removes autocorrelation from u and takes the form $L^{-1} \equiv \begin{matrix} A & 0 \\ -\alpha' & 1 \end{matrix}$ where $\alpha' \equiv (\alpha_{T-1}, \ldots, \alpha_2, \alpha_1)$ and A is a $T-1$ dimension lower-triangular block matrix. Pre-multiplying equation 1.9 by L^{-1} gives $L^{-1}q = L^{-1}q^{EQ} + \varepsilon$, $\varepsilon \sim D(0, \sigma^2 I_T)$. The transformed residual ε is free of autocorrelation. This transformed model has several implications:

1. The transformed residual, ε, is effectively a linear projection of the original residual. Hence, the last row of L^{-1} has an interpretation as coefficients in an AR(T-1) representation of ε_T by which we can calculate the impulse response function. This interpretation is also justified by Cramer's Decomposition. It states that any non-explosive process, whether nonlinear and/or nonstationary, can be represented by an invertible MA with time-varying coefficients, which explains the time-varying AR representations implied by the last rows of L^{-1}. The ESTAR model also allows a time-varying AR coefficient but the order of AR is very limited. Hence, its power to capture the dynamic is limited.

2. Since q^{EQ} is exogenous and L^{-1} is a lower triangular matrix, $L^{-1}q^{EQ}$ is a linear combination of only current and past values. Hence, it is also exogenous.

3. The constant in q^{EQ}, after transformation by L^{-1}, is no longer a constant vector. We assumed that the data has been de-meaned before being transformed. Having a nonzero sample mean in the data would have introduced a common factor of a transformed constant vector in all these transformed variables, which may dominate these series and produce some seemingly common factor that causes

multicollinearity and other unnecessary numerical instabilities. If a constant is required in the regression, it should be transformed separately and then added to the transformed regression. The theorem of Frisch and Waugh (1933) proves that the resulting point estimates would be identical with or without removing the mean.

The log maximum likelihood function of the model is

$$-\frac{T}{2}log\left(2\pi\right)-\frac{T}{2}log\left|\Sigma\right|-\frac{T}{2}\left(q-q^{EQ}\right)'\Sigma^{-1}\left(q-q^{EQ}\right).$$

The maximum likelihood estimators are

$$\hat{\beta}=(X'A'AX)^{-1}X'A'Aq,$$

$$\hat{\sigma}^2=\frac{1}{T}\left(q-X\hat{\beta}\right)'(LL')^{-1}\left(q-X\hat{\beta}\right).$$

And the estimators of the ACF's parameters are obtained by numerically maximizing

$$-log\left|\left(q-X\hat{\beta}\right)'R^{-1}\left(q-X\hat{\beta}\right)R\right|+Tlog\left(T\right)-T.$$

Chapter 2

Can DSGE Models Explain Real Exchange Rate Facts at All Frequencies?

2.1 Introduction

Movement of real exchange rates has been an important topic in international economics. One of the reasons is that, for example, monetary and exchange rate policy should correct the misalignment in international relative prices, e.g., real exchange rates. Real exchange rates are expected to be stationary time series according to the Purchasing Power Parity (PPP) hypothesis, which states that, once converted into a common currency, two countries should have the same price level in the long run. In the short run, real exchange rates are generally not constant. In particular, real exchange rates have been found to exhibit long memory as evidenced by either half-lives (about 2-5 years) or first-order autocorrelation[1] (roughly 0.91-0.97 for quarterly data), and high volatility (three times more volatile than GDP).

A question is asked by Rogoff (1996): If we take that liquidity shocks drive the high volatility of real exchange rates, then the real exchange rates would be expected not to be very persistent. This problem is known as the PPP puzzle. There are two strands of literature addressing the puzzle from different angles. First, theoretical work employs dynamic stochastic general equilibrium (DSGE) models to generate highly volatile and/or persistent real exchange rates (Chari, Kehoe, and McGrattan, 2002; Corsetti, Dedola, and Leduc, 2008; Rabanal, Rubio-Ramirez, and Tuesta, 2011; Rabanal and Tuesta, 2013; Steinsson, 2008, among others). These models typically focus on business cycle (BC) frequencies, targeting statistics after applying a H-P filter or other detrending methods on data. Another strand of literature employs reduced-form empirical models in which real exchange rates are governed by nonlinear dynamics. This literature argues that the standard measures of persistence, e.g. the first-order autocorrelation, implicitly assume linearity and are subject to bias. One of the most popular nonlinear dynamics called smooth transition (ST) autoregression features slower mean reversion when real exchange rates are closer to their long-term equilibrium level. Taylor, Peel, and Sarno (2001), Paya and Peel (2006) and Lothian and Taylor (2008) among many others compare ST autoregression models with the standard linear autoregression models and find evidence in favour of the former. They also find shorter half-lives implied by the ST autoregression.

[1] The partial autocorrelation typically suggests that the real exchange rate is a first-order process so only the first-order autocorrelation is typically employed.

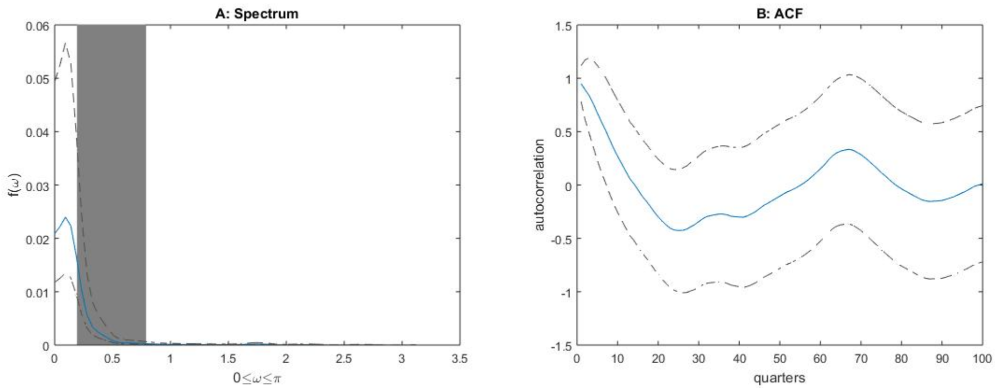
So far, these two strands of literature are to a large extent isolated. DSGE models are routinely calibrated to match standard statistics and ignore the shorter half-lives and nonlinear dynamics discovered in the recent empirical literature. Additionally, DSGE models are typically designed to capture real exchange rate at business cycle frequencies while empirical models use un-filtered data, i.e., capturing real exchange rate all all frequencies. This chapter makes two contributions to the literature. First, I bridge these two distinct strands of literature above by studying time-series properties of real exchange rates generated by DSGE models. In doing so, I employ the DSGE model studied by Rabanal and Rubio-Ramirez (2015) which can capture real exchange rates at all frequencies by fitting their spectrum. This is important because a spectral analysis indicates that business cycle frequencies only account for about 25% of real exchange rates variance, and about 70% of the variance are assigned to frequencies lower than business cycle frequencies. Therefore, the literature of business cycle models misses a large part of the story. Furthermore Rabanal and Rubio-Ramirez (2015) argue that conventional DSGE models, by focusing on business cycle frequencies, assign too little real exchange rate variance to frequencies lower than the business cycle frequencies when the model is calibrated to un-filtered data. This is referred to as "excess persistence of the RER" puzzle. My choice of Rabanal and Rubio-Ramirez (2015)'s international real business cycle (IRBC) model is also justified by Gehrke and Yao (2017). They use a structural vector autoregression model and decompose real exchange rate variance across both shocks and frequencies. They find supply shocks to be the main driver of real exchange rates at low frequencies. Given the DSGE model, I investigate if it can generate the integral correction mechanism (ICM) studied in chapter 1. Chapter 1 suggests that the ICM is a better representation of real exchange rates than the popular ST autoregression model because the ST autoregression model also exhibits the "excess persistence" problem.[2] The second contribution of this chapter is to give structural interpretations to the ICM and the PPP puzzle, using the DSGE model. In doing so, I also make comments on the micro-foundation of the key feature, i.e. a input adjustment cost, of Rabanal and Rubio-Ramirez (2015)'s model. The input adjustment cost depends on changes in the ratio between domestic and imported goods. It makes imported good less responsive to it price and hence makes the trade elasticity time-varying.

Specifically, I examine if the DSGE model can generate several features of real exchange rates at all frequencies, which are robust across pairs of countries. These features include the standard deviation, the autocorrelation function (ACF), the spectrum and the ICM. The findings are threefold. First, I find that the model employed by Rabanal and Rubio-Ramirez (2015) overfits the spectrum in the sense that it misses other features of the real exchange rate. A minor modification fixes this problem so that the model captures all features of the real exchange rate fairly well. However, the final calibration requires a input adjustment cost that is larger than that suggested by micro evidence. Second, I study impulse response functions in the DSGE model by varying the size of the input adjustment cost. The size of the input adjustment cost makes nonmonotone impacts on model dynamics. The minor modification mentioned above turns out to make a significant difference on the impulse responses. I interpret this difference by highlighting the inter-temporal distortion of the input adjustment cost. Third, the DSGE model provides a structural interpretation of the ICM representation which helps us better understand the PPP puzzle. With the final calibration, the input adjustment cost introduces a long lasting cyclical impulse response in most of the macroeconomic variables including the real exchange rate, given a persistent positive shock to home-country productivity. Surprisingly, I find that the initial response of the real exchange rate is negative and reverting to its steady state quickly.

The rest of this chapter is organized as follows. Section 2.2 discusses the empirical features of the real exchange rate that a successful DSGE model should be able to explain. The next section introduces

[2] I do not study the smooth transition dynamics because Ahmad et al. (2013) have shown that this type of nonlinearity can be detected in a rich, yet standard, IRBC model with nontradable goods and sticky price. And as argued by Chapter 1, the smooth transition is unlikely to be the key feature of the real exchange rate.

Figure 2.1: Spectrum and ACF of U.S. real exchange rate



Notes:
1. The gray area in panel A represents BC frequencies, defined from 8 to 32 quarters. Frequencies lower than BC will be referred to as low frequencies.
2. The dotted lines represent 90% confidence intervals.

the IRBC model. In Section 2.4, I investigate whether the IRBC model is able to generate the empirical features of the real exchange rate. Section 2.5 gives an intuition about how the new mechanism in the IRBC model brings it closer to data. Section 2.6 discusses the quantitative performance of this model on other dimensions. The last section concludes the chapter.

2.2 Data features

This section describes the source of the data and reports features of the real exchange rate from different aspects. I study the quarterly U.S. effective data. The real exchange rate is collected from the federal reserve board's major index. This index is a weighted average of the foreign exchange values of the U.S. dollar against the currencies circulated widely outside the issuing country, including the Euro, the Canada dollar, the Japanese yen, the British pound, the Swiss franc, the Australian dollar and the Swedish krona. These seven countries are referred to as the rest of world (R.W.). The U.S. is treated as the home country and the R.W. is treated as the foreign country. The rest of the data is collected from the U.S. Bureau of Economic Analysis, except the Solow Residual taken from Rabanal et al. (2011). The data spans from 1973q2 to 2006q4 during which period the estimate of the Solow Residual is available.

The real exchange rate is known to be highly volatile and persistent. The standard deviation of the sample is about 0.1095. As shown in the left-hand panel of Figure 2.1 on page 29, about 75% of the variance are at a low frequency. The standard deviation at BC frequencies[3] is 0.051, or 3.3 times that of GDP. The degree of persistence is usually summarized by the first-order autocorrelation, which is about 0.95. Standard tests for the unit root such as Augmented Dickey-Fuller cannot reject the unit root. Chapter 1 suggest that not only the first-order autocorrelation but the autocorrelation function (ACF) are investigated, which is natural because the autocorrelation function corresponds to the spectrum one by one. As shown in the right-hand panel of Figure 2.1 on page 29, the ACF goes down very fast[4] to a negative level and cyclically moves around zero with a decaying magnitude. The ACF suggests that the real exchange rate is persistent but not in a monotone way. The autocorrelations higher than order 7 are insignificant according to the 90% confidence interval. However, it may not be a good idea to take higher order autocorrelation simply as zero because, in this case, the fast decay of ACF suggests that

[3]The business cycle is obtained by applying an H-P filter throughout this chapter.

[4]I construct two AR(1) processes for the real exchange rate by estimation, and by assigning the sample first-order autocorrelation as the AR coefficient. Either of them implies an ACF that goes down slower than the one shown in panel B during the first 20 periods.

the real exchange rate is transitory. Actually, the higher-order autocorrelation drives the estimates of a traditional reduced form model, such as an AR model or a smooth transition AR model, such that the model implies a persistent process. The higher order autocorrelation is also important for understanding the shape of the spectrum. The low frequencies can account for a significant proportion of the volatility only if the real exchange rate is autocorrelated in the long run.

To study the hidden features of the real exchange rate, I estimate a time series model with an integral correction mechanism (ICM). The general form of such a model is

$$err_t = \sum_{j=1}^{p} \beta_j^{ecm} err_{t-j} + \beta^{icm} ie_{t-1} + \varepsilon_t, \tag{2.1}$$

where the integral error (ie) is defined by

$$ie_{t-1} = \sum_{i=0}^{t-1} err_i, \tag{2.2}$$

and err_t is defined by

$$err_{t-1} = \ln q_{t-1} - \ln q_{t-1}^{EQ}, \tag{2.3}$$

with q^{EQ} representing the "equilibrium" value of the real exchange rate (will be defined shortly), $\beta^{icm} \leq 0$, and β_j^{ecm} implies a stationary AR(p) process. The key parameter of this model is β^{icm}, which determines the emergence of the integral correction mechanism. When $\beta^{icm} < 0$, the model features a physical pendulum-like behaviour. Let err_t be a massive bob hung by a rod, the bottom position is the equilibrium in this model where $err_t = 0$. Given a shock, err_t will periodically move around this equilibrium position because the term IE_t can store the "momentum" until all momentum is drained by air friction. β_j^{ecm} plays the role of air friction. The trajectory of the massive bob is the impulse response of err_t. To better understand the dynamics of ICM, see 2.2 for the impulse response of the real exchange rate when the model is estimated using U.S. data. In this model, β_j^{ecm} controls the "conventional persistence", e.g. how fast does the real exchange rate revert to the equilibrium position. It also determines the period of the cyclical part of the movement. β^{icm} is analogous to the length of rope and gravity. It controls the magnitude of the cyclical part of the movement, which is the "cyclical persistence". I note that this impulse response looks like that of the AR model with some negative coefficients. However, no such estimates have been found in the literature even when the number of lags is allowed to be relatively high; see for example Steinsson (2008).

When estimating the model, I must define the "equilibrium" value of the real exchange rate q^{EQ} first. Following Chapter 1 and the references therein, I define $\ln q_t^{EQ} = a_0 + a_1 \ln rSR_t + a_2 \ln tot_t + a_3 tb_t$. rSR is the Solow residual ratio between the U.S. and the R.W., capturing the Harrod-Balassa-Samuelson (HBS) effect. tb is the trade balance to output ratio and tot is the terms of trade. Finally, I select $p = 1$ based on first-order partial autocorrelation. All parameters in equations 2.1 and 2.3 are estimated simultaneously by maximum likelihood. I construct bootstrap distributions of each parameter using a sample size of 5000. This will be used later to facilitate the comparison between ICM estimated from data and ICM generated from the IRBC model. The estimates of parameters in equation 2.1 are reported in Table 2.1 on page 31.[5] A significant β^{icm} is found for the U.S. real exchange rate, indicating the

[5]The model does not pass the diagnostic test of function form. This problem may be caused by the missing nonlinearity in the form of, for example, smooth transition. Adding a smooth transition mechanism allows the model to pass the diagnostic test but does not change the estimate of other parameters.

Figure 2.2: IRF of U.S. real exchange rate under ICM

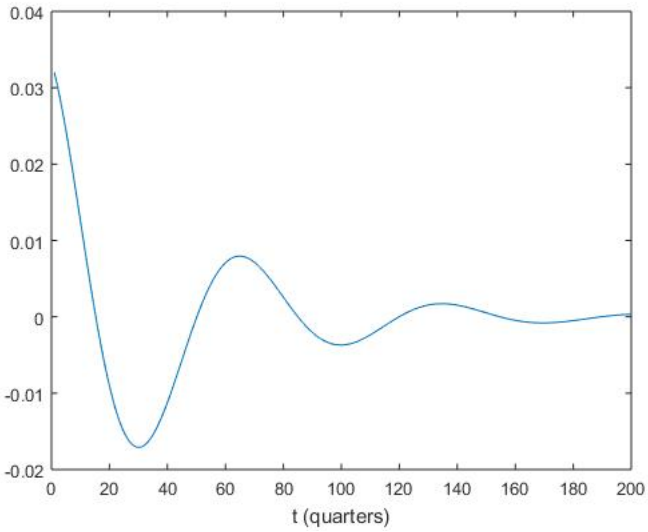


Table 2.1: Time series model estimation

Estimated ICM

a_0	β^{ecm}	β^{icm}	σ
4.55	0.96	-0.008	0.03
(1161.05)	(31.20)	(-3.25)**	
[4.54, 4.56]	[0.88, 0.98]	[-0.017, -0.006]	
R^2=0.91	LL= 270.60	ICM={46.44%}	
AR(1)={0.60}	ARCH(1)={0.90}	Function form={0.01}	

Notes:

1. Numbers in parentheses are t-ratio, numbers in brackets are the bootstrap 90% interval, numbers in braces are P-values.
2. Stars next to the parentheses under β^{icm} represent significantly different from zero at the 10%, 5% and 1% level. The critical value is obtained by Monte Carlo experiments because $\beta^{icm} > 0$ makes the model explosive and the distribution of t-statistics under the null is non-standard.
3. AR(1), ARCH(1) and function form represent the diagnostic score tests of neglected first-order serial correlation, first-order autoregressive heteroskedasticity and general nonlinearity, respectively.
4. ICM denotes the frequency of a significant β^{icm} in the bootstrap sample.

appearance of ICM. The significance of β^{icm} is tested in each bootstrap sample and the frequency of significant β^{icm} is about 46%.

2.3 The DSGE framework

The DSGE framework I will use is that developed by Rabanal and Rubio-Ramirez (2015). The world consists of two countries with one referred to as Home and the other as Foreign. The two countries are symmetric unless otherwise indicated. Each country produces a tradable intermediate good using local labour and capital. Home and Foreign intermediate goods are used to produce nontradable final goods. The international financial market is incomplete with only one asset: a real riskless bond. To save space, this section only presents the problems of Home country agents. The problems for the Foreign country can be constructed in a similar way. Variables in the Foreign country are denoted with superscript "*". I collect equilibrium conditions of the model in appendix B.

2.3.1 Households

There is a unit mass of infinitely-lived households in each country. The representative household maximizes ots life-time discounted utility

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} \frac{\left[C_j^{\tau} \left(1-L_j\right)^{1-\tau}\right]^{1-\sigma}}{1-\sigma},$$

subject to the following budget constraint

$$P_t \left(C_t + X_t\right) + P_{H,t} Q_t D_t \leq P_t \left(W_t L_t + R_t K_{t-1}\right) + P_{H,t} \left[D_{t-1} - \Phi\left(D_t\right)\right],$$

and the law of motion for capital

$$K_t = (1-\delta) K_{t-1} + X_t - \frac{\psi}{2} K_{t-1} \left(\frac{X_t}{K_{t-1}} - \delta\right)^2.$$

The notation follows Rabanal and Rubio-Ramirez (2015): $\beta \in (0,1)$ is the subjective discount factor, $\tau \in (0,1)$ is the weight on consumption in the utility function, $\sigma > 0$ is the coefficient of relative risk aversion, $\delta \in (0,1)$ is the depreciation rate of capital and $\psi \geq 0$ controls the size of the capital adjustment cost.

The representative household provide L_t units of labour at real wage W_t. At the beginning of each period, the household receive capital income $R_t K_{t-1}$ with real rental rate R_t and interest by owning a net stock of international assets $P_{H,t} D_{t-1}$. One unit of international asset purchased at price $P_{H,t} Q_t$ in period t pays one unit of Home intermediate goods next period, the price of which is $P_{H,t+1}$. On the expenditure side, the household purchases final goods at price P_t for consumption (C_t) and investment (X_t). The capital accumulation exhibits a adjustment costs. As in Schmitt-Grohé and Uribe (2003), to induce stationarity of the model, the Home country is subjected to a portfolio adjustment cost that depends on its net foreign asset. The adjustment cost is measured by $\Phi\left(D_t\right) = \frac{\phi}{2} A_t \left(\frac{D_t - \bar{D}}{A_t}\right)^2$ in units of Home intermediate goods where A_t is Home country productivity.

Solving the maximization problem gives the following first-order condition with respect to C_t, L_t, X_t, K_t and D_t:

$$U_{C_t} = \lambda_t,$$

$$-\frac{U_{L_t}}{U_{C_t}} = W_t,$$

$$\lambda_t = \mu_t \left[1 - \psi\left(\frac{X_t}{K_{t-1}} - \delta\right)\right],$$

$$\mu_t = \beta E_t \left[\lambda_{t+1} R_{t+1} + \mu_{t+1}\left(1 - \delta + \frac{\psi}{2}\left(\frac{X_t^2}{K_{t-1}^2} - \delta^2\right)\right)\right],$$

$$Q_t = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t}\frac{P_{H,t+1}/P_{t+1}}{P_{H,t}/P_t}\right) - \frac{\partial \Phi}{\partial D_t},$$

where U_L and U_C are derivatives of single period utility with respect to labour and consumption, respectively, λ_t is the Lagrange multiplier of the budget constraint and μ_t is the Lagrange multiplier of the capital law of motion. Due to the investment adjustment cost, $\mu_t/\lambda_t = \left[1 - \psi\left(\frac{X_t}{K_{t-1}} - \delta\right)\right]$ forms the shadow price (Tobin's Q) of capital, i.e., the price of installed capital relative to a real good. The second equation states the intratemporal trad-offs between labour disutility and labour income, and determines labour supply. The last two equation states the intertemporal trad-offs between consumption today and consumption next period financed by capital and international asset returns, and determines the interest rate. The last Euler equation and its foreign counterpart define international risk-sharing.

2.3.2 Intermediate goods firms

The market of domestic intermediate goods firms is perfectly competitive. The representative intermediate goods producer uses domestic labour and domestic capital in order to produce intermediate goods sold to both the Home and Foreign producers of final goods. The firms maximize period-by-period profits by solving

$$\max_{L_t, K_{t-1}} P_{H,t}\left(Y_{H,t} + Y_{H,t}^*\right) - P_t\left(W_t L_t + R_t K_{t-1}\right),$$

subject to the production function

$$Y_{H,t} + Y_{H,t}^* = A_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{1-\alpha},$$

where $Y_{H,t}$ and $Y_{H,t}^*$ denote the Home intermediate goods demanded by Home and Foreign markets, respectively, $\alpha \in (0,1)$ is the capital share of output and A_t is Home country productivity cointegrated with Foreign country productivity A_t^*

$$\begin{bmatrix} \Delta \ln A_t \\ \Delta \ln A_t^* \end{bmatrix} = \begin{bmatrix} A \\ A^* \end{bmatrix} + \begin{bmatrix} \kappa \\ -\kappa \end{bmatrix}\left(\ln A_{t-1} - \ln A_{t-1}^*\right) + \begin{bmatrix} \epsilon_{A,t} \\ \epsilon_{A^*,t} \end{bmatrix}, \tag{2.4}$$

$$\begin{bmatrix} \epsilon_{A,t} \\ \epsilon_{A^*t} \end{bmatrix} \sim N\left(0, \Sigma\right).$$

The first-order conditions of this problem are

$$W_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right) A_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{-\alpha},$$

$$R_t = \alpha\left(\frac{P_{H,t}}{P_t}\right) A_t^{1-\alpha} K_{t-1}^{\alpha-1} L_t^{1-\alpha}. \tag{2.5}$$

where production factors are paid for their marginal product.

2.3.3 Final goods firms

The domestic and imported intermediate goods ($Y_{H,t}$ and $Y_{F,t}$) are combined by perfectly competitive final goods firms. The key element in this model is the input adjustment cost in the production function introduced by Erceg et al. (2005) and later used by Rabanal and Rubio-Ramirez (2015), which takes the form

$$Y_t = \left[\omega^{\frac{1}{\sigma_{HF}}} Y_{H,t}^{\frac{\sigma_{HF}-1}{\sigma_{HF}}} + (1-\omega)^{\frac{1}{\sigma_{HF}}} \left(\varphi_t Y_{F,t}\right)^{\frac{\sigma_{HF}-1}{\sigma_{HF}}}\right]^{\frac{\sigma_{HF}}{\sigma_{HF}-1}},$$

where $\omega \in (0,1)$ denotes the degree of home bias or the degree of openness, $\sigma_{HF} > 0$ is a parameter which determines the long-run elasticity of substitution between domestic and imported intermediate goods, $\varphi_t = \left[1 - \frac{\iota}{2}\left(\frac{\frac{Y_{F,t}}{Y_{H,t}}}{\frac{Y_{F,t-1}}{Y_{H,t-1}}} - 1\right)^2\right]$ is the input adjustment cost with ι being the size of the cost. This specification has two implications. Given the time varying φ_t, the real cost of producing one unit of final goods will generally be time varying. Therefore, it distorts the inter-temporal decisions made by households. Whenever the final goods firms, or equivalently the households, find it optimal to change the ratio between domestic and imported goods, they must take the expected input adjustment cost in the next period into account. On the other hand, the input adjustment cost implies that the imported goods share in consumption is relatively unresponsive to the changes in its price in the short run. Therefore, the short-run price elasticity of imported goods will be lower than σ_{HF} and intra-temporal decisions are distorted. This is supported by the recent literature that estimates the trade elasticity; see, for example, **?**. In addition, note that the cost is due to the change in the ratio between domestic and imported goods, which suggests that the level of imported goods is allowed to jump costlessly in response to changes in the overall consumption demand. To give a quantitative example, when $\iota = 900$, a change of 1% in the ratio will result in $\varphi_t = 0.955$ so that Foreign intermediate goods are 4.5% less efficient in production.

Denote $\Omega_{t,j} = \beta^{t-j}\frac{\frac{\lambda_j}{P_j}}{\frac{\lambda_t}{P_t}}$ as the stochastic discount factor and the final goods firm maximizes the expected discount profits given by

$$E_t \sum_{j=t}^{\infty} \Omega_{t,j} \left(P_j Y_j - P_{H,j} Y_{H,j} - P_{F,j} Y_{F,j}\right),$$

subject to the production function. The first-order conditions with respect to $Y_{H,t}$ and $Y_{F,t}$ are

$$P_t \frac{\partial Y_t}{\partial Y_{H,t}} + E_t\left(\Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{H,t}}\right) = P_{H,t},$$

$$P_t \frac{\partial Y_t}{\partial Y_{F,t}} + E_t\left(\Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{F,t}}\right) = P_{F,t},$$

where the term $E_t\left(\Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{H,t}}\right)$ and $E_t\left(\Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{F,t}}\right)$ appear due to the input adjustment cost. These two terms distort the standard demand equations for $Y_{H,t}$ and $Y_{F,t}$ up to the point that $Y_{H,t}$ and $Y_{F,t}$ also affect output next period via φ_t. In a standard model ($\iota = 0$), substituting the two first-order conditions into the production function yields the price index of final goods

$$P_t = \left[\omega P_{H,t}^{1-\sigma_{HF}} + (1-\omega) P_{F,t}^{1-\sigma_{HF}}\right]^{\frac{1}{1-\sigma_{HF}}},$$

and , given the Law of One Price, the real exchange rate

$$RER_t = \frac{\left[\omega P_{F,t}^{1-\sigma_{HF}} + (1-\omega)\, P_{H,t}^{1-\sigma_{HF}}\right]^{\frac{1}{1-\sigma_{HF}}}}{\left[\omega P_{H,t}^{1-\sigma_{HF}} + (1-\omega)\, P_{F,t}^{1-\sigma_{HF}}\right]^{\frac{1}{1-\sigma_{HF}}}} = \frac{\left[\omega + (1-\omega)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\sigma_{HF}}\right]^{\frac{1}{1-\sigma_{HF}}}}{\left[\omega\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\sigma_{HF}} + (1-\omega)\right]^{\frac{1}{1-\sigma_{HF}}}}. \tag{2.6}$$

In this case, the movement of the real exchange rate is due to home bias $\omega > 0.5$ and the volatility is partially governed by non-unit elasticity of substitution σ_{HF}. The input adjustment cost allows time-varying elasticity of substitution and hence introduces new dynamics into the real exchange rate. In particular, it reallocates real exchange rate volatility across frequencies.

2.3.4 Market clearing conditions

Finally the model is closed by the final goods market clearing condition

$C_t + X_t = Y_t,$

and the international bond market clearing condition

$D_t + D_t^* = 0.$

2.3.5 Benchmark calibration

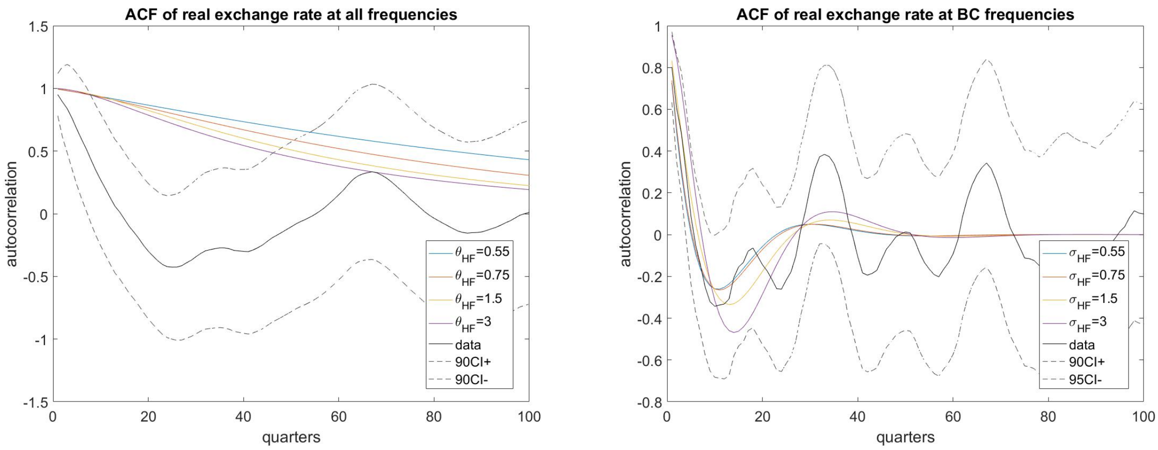
The parameters of the cointegration process are taken from the estimates by Rabanal et al. (2011) in which $A = 0.001$, $A^* = 0.006$, $\kappa = -0.007$, $\sigma_A = 0.0108$ and $\sigma_{A^*} = 0.088$. The subjective discount factor β, the consumption share in the utility function τ, the risk aversion σ, the depreciation rate δ and the capital share in production α are set to the standard value in the literature and they are equal to 0.99, 0.34, 2, 0.025 and 0.36, respectively. There is no standard value for the cost of holding international asset ϕ as the literature uses different function forms of the cost. I set this to 0.03 so that the model gets the standard deviation of the trade balance approximately right. The rest of the parameters are calibrated to fit the spectrum of the real exchange rate through the strategy of Rabanal and Rubio-Ramirez (2015). I set the low home bias $\omega = 0.8$ and a high long-run elasticity of substitution $\sigma_{HF} = 3$, the same as Rabanal and Rubio-Ramirez (2015). The parameter of the input adjustment cost ι is set to 140.

2.4 Main Experiments

The main experiments conducted in this chapter are to investigate whether the features of real exchange rate data reported in Section 2.2 can be generated in the model described in Section 2.3. The standard deviation, the ACF, and the spectrum can be calculated analytically from the IRBC model and they are expected to be reasonably close to their counterparts from the data. The integral correction mechanism is uncovered from the IRBC model in the following way. The IRBC model is solved up to the first-order approximation.[6] The artificial real exchange rate and other variables are simulated for the same length as the data. The parameters in equations 2.1 and 2.3 are estimated by the artificial variables. The procedure is repeated 5000 times, which results in a Monte Carlo sample of each parameter in equations 2.1 and 2.3. When comparing the ICM generated by the IRBC model to the ICM estimated in the data, I compare the Monte Carlo sample to the Bootstrap sample described in Section 2.2.

[6] However, all the results are robust to a higher order approximation.

Figure 2.3: Fit of ACF at different frequencies by baseline model



Note:
1. The 90CI denotes the upper and lower 90% confidence interval of the sample ACF. It is calculated by the asymptotic normal distribution and standard deviation $SE(\rho_h) = \sqrt{\frac{1+2\sum_{i=1}^{h-1}\rho_i^2}{N}}$ where ρ_h denotes h order autocorrelation and N is the sample size.

2.4.1 The baseline results

To facilitate the comparison, I use a baseline model in which the input adjustment cost is shut down by setting $\iota = 0$. The home bias is reset to 0.9 to match the import to output ratio in the steady state. It is well known that the substitution between domestic and imported goods is critical for the real exchange rate. To see this point, consider the case that the substitution between the home and foreign good is perfect so $\sigma_{HF} = \infty$. In this case, the real exchange rate will be a constant. Two values for the elasticity of substitutionσ_{HF} are considered. When $\sigma_{HF} = 0.55$, the model does a good job at the business cycle frequencies. It closely fits the standard deviation of the H-P filtered real exchange rate relative to that of GDP[7] (3.26 in the model and 3.3 in the data) although the standard deviation at all frequencies is 0.22, twice the size of 0.1095 in the data. In terms of the persistence at BC frequencies, the right-hand panel of Figure 2.3 on page 36 plots the ACF of the real exchange rate at BC frequencies for the data against the model. The data shows a fast initial decay to a negative value. The higher order of ACF is very noisy and insignificant but the ACF of order 10 has a significant negative value. It is surprising to find that those unusual features of ACF at BC frequencies can be generated in the baseline model. However, the estimate of ICM on the model side only detects significant β^{icm} in 8% of the Monte Carlo sample. This probability is only slightly larger than the size of the test, suggesting a no or very small integral correction mechanism in the baseline model.

The other relevant value of σ_{HF} is 0.75, which allows the model to fit the standard deviation of the real exchange rate at all frequencies (0.1073 in the model and. 0.1095 in the data). It also increases the frequency of the ICM appearance to 16%. However, it only explains 30% of the standard deviation at BC frequencies (0.015 in the model and 0.051 in the data) and assigns most of the standard deviation to frequencies lower than BC. This problem is referred to as the excess persistence of the RER in Rabanal and Rubio-Ramirez (2015). The left-hand panel of Figure 2.3 on page 36 shows that too much persistence is generated by the model at all frequencies, although the excess persistence problem is slightly eased as compared to $\sigma_{HF} = 0.55$ by noting that the ACF decreases faster when σ_{HF} increases. This experiment gives a general lesson that a model may completely miss the evolution of persistence, even if it gets the first-order autocorrelation about right.

[7]The statistics generated from the IRBC model at BC frequencies is obtained by applying an H-P filter to the model.

Table 2.2: ICM in Baseline model

	β^{ecm} 90% int.	β^{icm} 90% int.	$p\,(icm)$
data	[0.8795, 0.9854]	[-0.0172, -0.0056]	46%
$\sigma_{HF}=0.55$	[0.9083 1.0000]	[-0.0067 -0.0003]	8%
$\sigma_{HF}=0.75$	[0.9239 1.0052]	[-0.0060 -0.0003]	16%
$\sigma_{HF}=1.5$	[0.9660 1.0208]	[-0.0056 -0.0000]	63%
$\sigma_{HF}=3.0$	[0.9793 1.0312]	[-0.0060 -0.0000]	90%

Notes: $p\,(icm)$ is the frequency of significant ICM in the Monte Carlo or Bootstrap sample.

Using ACF instead of the spectrum, I replicate the results discussed in Rabanal and Rubio-Ramirez (2015) which suggest that the performance of the baseline model faces a trade off between low frequencies and BC frequencies. In order to fit the real exchange rate features at BC frequencies, the model generates extra persistence and is volatile at low frequencies. On the other hand, if the model fits the features at all frequencies, the model underestimates the persistence and the volatility at BC frequencies. Therefore, the model is unable to fit the shape of the spectrum. The new finding here is the relationship between the substitution σ_{HF} and the frequency of the ICM appearance. To investigate this relationship further, I run another two experiments using σ_{HF} equal to 1.5 and 3, respectively. Table 2.2 on page 37 reports the estimates of ICM under different values of σ_{HF}. First, note that the frequency of ICM occurrence ($p\,(icm)$) is increasing along the σ_{HF}. However, given that the estimate of β^{icm} remains at a small magnitude, the increase of $p\,(icm)$ is only caused by the smaller standard deviation in the residual. On the other hand, more volatility of the real exchange rate is explained by a higher β^{ecm}. Using the interpretation of β^{ecm} and β^{icm} discussed in Section 2.2, a higher substitution between domestic and imported intermediate goods increases the volatility of the real exchange rate by increasing its conventional persistence while the cyclical persistence remains at a small magnitude.

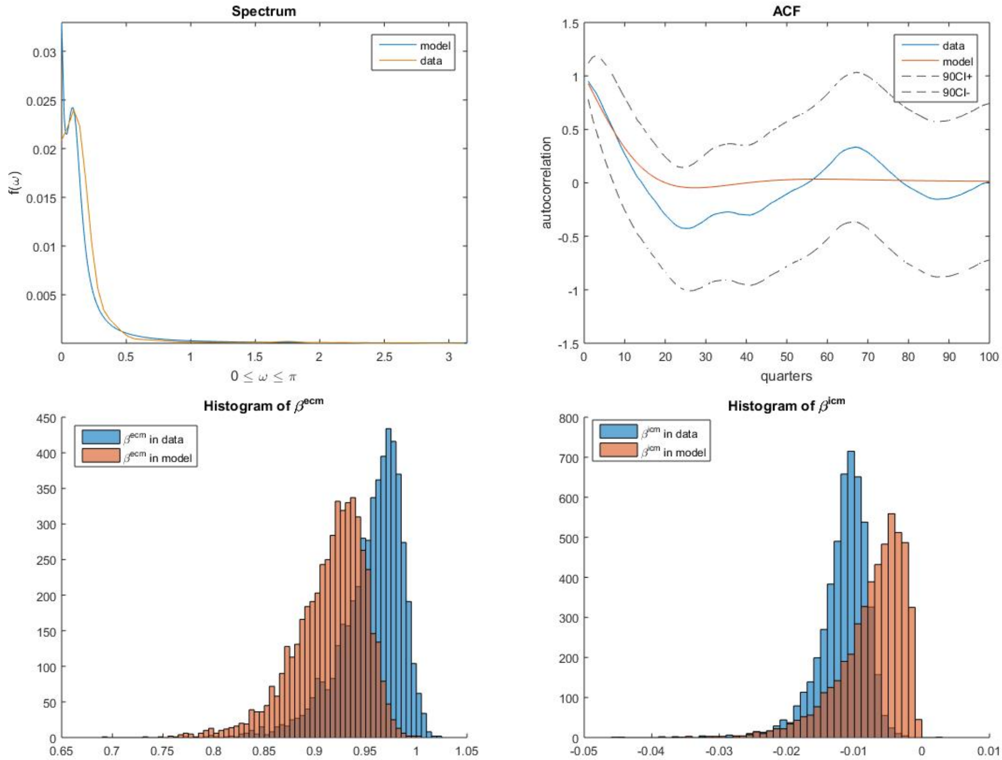
2.4.2 The workhorse model

Adding the input adjustment cost helps the model reallocate the variance of the real exchange rate at different frequencies. The upper half of Figure 2.4 on page 38 shows the fit of the spectrum and ACF. Using the benchmark calibration, this model can fit the spectrum very well. The ACF of the model at all frequencies now lies well within the confidence interval of the data, fixing the excess persistence problem in the baseline model. It also has the cyclical movement. However, the magnitude of the cyclical movement is negligible as compared to the data. Together with the initial fast decay, the ACF suggests the real exchange rate to be a very smooth process.

I now turn to the estimates of ICM using simulations of the workhorse model. The 90% confidence interval of β^{icm} is [-0.0170, -0.0017] in the Monte Carlo sample, covering the point estimate in the data -0.0084. The 90% confidence interval of β^{ecm} [0.8493, 0.9621] covers the point estimate in data 0.96 at the margin. The lower panel of Figure 2.4 on page 38 compares the distribution of β^{icm} and β^{ecm} in the IRBC Monte Carlo sample and the data Bootstrap sample. It shows that the model explains the volatility of the real exchange rate by underestimating the magnitude of cyclical persistence β^{icm} and overestimating the magnitude of conventional persistence β^{ecm}. The frequency at which ICM is found statistically significant is only 18% in the model while this prequency is 46% in the data. Nevertheless, the workhorse model is an improvement over the baseline model.

Can we push the workhorse model further? To answer this question, I first argue that the perfect fit of the spectrum like Rabanal and Rubio-Ramirez (2015)'s strategy is likely to suffer from "over-fitting". Unlike the population ACF and spectrum, the sample ACF does not correspond exactly to the sample spectrum. Hence, fitting exactly one of them may miss the information that is contained in the other. I propose a calibration strategy that fits both the spectrum and ACF reasonably well in the sense that

Figure 2.4: Compare real exchange rate features in the data to the workhorse model with benchmark calibration



the IRBC model captures the basic shape of the spectrum and ACF and lie in the 90% interval. Given that the sample ACF and the spectrum contain similar but slightly different information, this strategy should ease the over-fitting problem. In my final calibration, I substantially increase the ι to 900. The home bias ω and the portfolio adjustment cost ϕ are slightly adjusted to 0.82 and 0.05, respectively. This calibration generates excess volatility for a wide range of variables. Therefore, I replace the cointegration productivity process by a stationary VAR(1) process,

$$\begin{bmatrix} logA_t \\ logA_t^* \end{bmatrix} = \begin{bmatrix} 0.97 & 0.025 \\ 0.025 & 0.97 \end{bmatrix} \begin{bmatrix} logA_{t-1} \\ logA_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix},$$

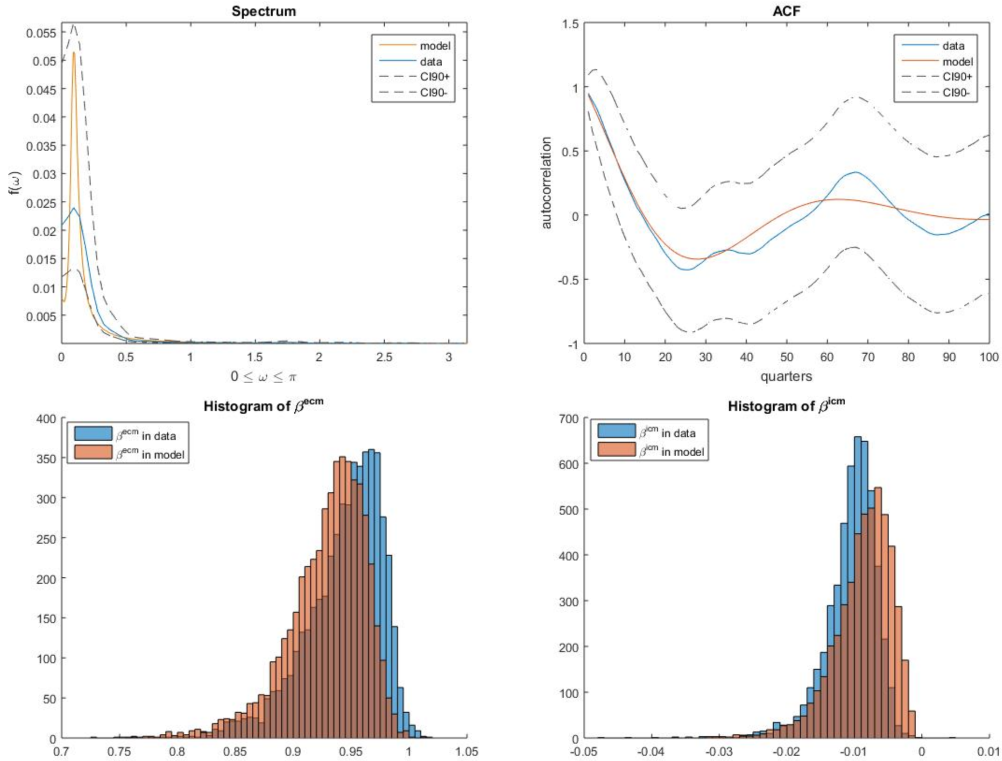
$$\begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix} \sim N\left(\begin{array}{c} 0 \\ 0 \end{array}, \Sigma \right),$$

$$StD\left(\epsilon_t\right) = 0.0073, StD\left(\epsilon_t^*\right) = 0.0044, Corr\left(\epsilon_t, \epsilon_t^*\right) = 0.29,$$

estimated by Heathcote and Perri (2002).[8] The standard deviation of the real exchange rate is now 0.1075, the same as the benchmark calibration and data. Figure 2.5 on page 39 reports the good fits of ACF and the spectrum. The frequency of the ICM occurrence is 41.68%. The magnitude of β^{ecm} and β^{icm} is much closer to the data. As compared to the benchmark calibration, the 90% interval of β^{icm}

[8]The difference between the stationary VAR(1) technology and the cointegration technology is discussed by Rabanal et al. (2011). They find that the cointegration technology is preferred; however, their analysis focuses on business cycle frequencies only. There is no evidence supporting cointegration technology if the aim is to explain the real exchange rate at all frequencies. However, one may wonder why the real exchange rate at all frequencies should be explained by a model that either does not incorporate trends or does not seriously capture the trends (in the case of productivity containing unit roots, for example). In the real word, the real exchange rate of a pair of countries is usually stationary serious even if the growth rates of these two economies are persistently different. Therefore, I assume that the properties of the real exchange rate at all frequencies have nothing to do with the trend.

Figure 2.5: Compare real exchange rate features in the data to the workhorse model with a modified calibration

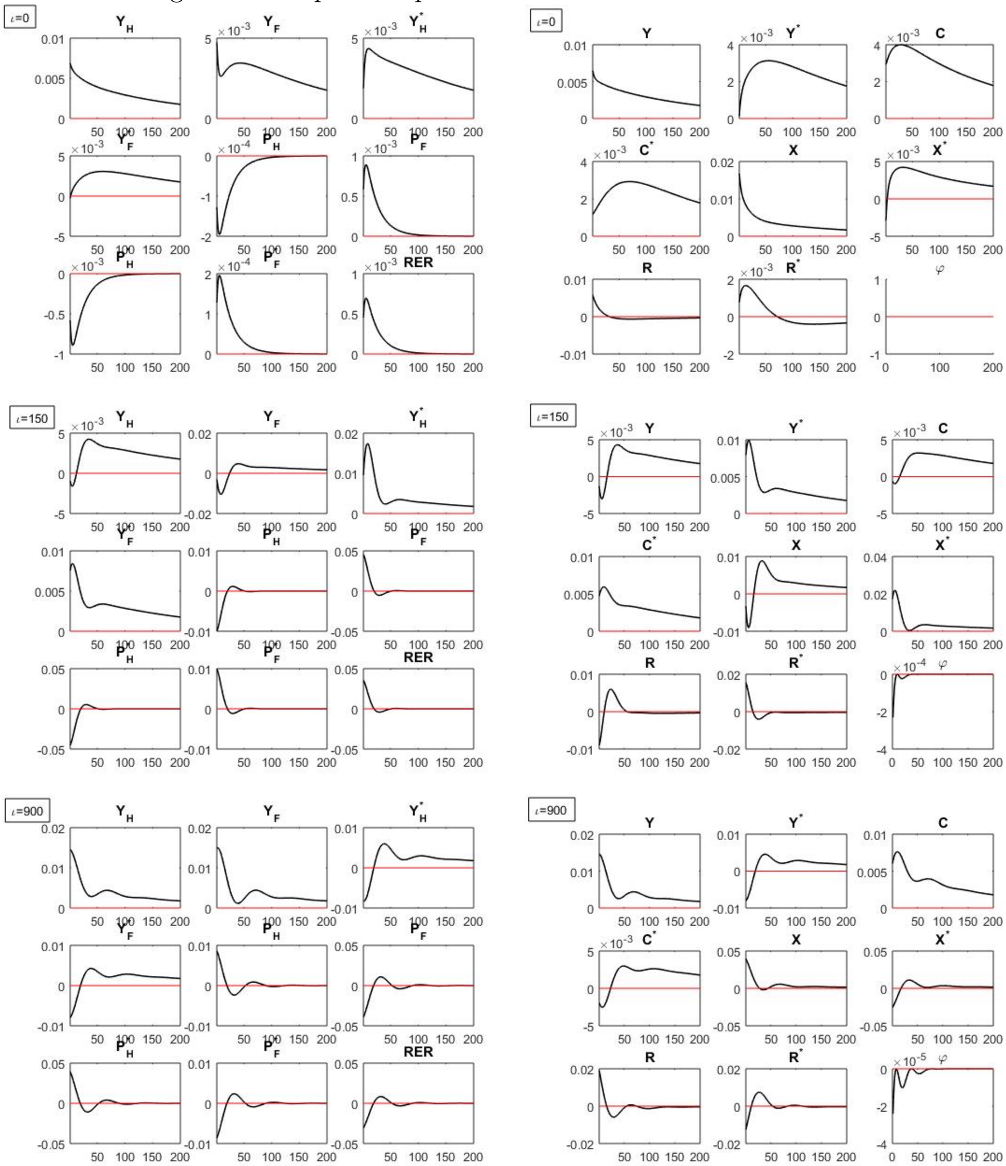


increases (in absolute) to [-0.0166, -0.0031] and the 90% interval of β^{ecm} increases to [0.8637, 0.9728]. Hence, the new calibration explains the volatility real exchange rate using a larger conventional and cyclical persistence and a smaller standard deviation in the residual. To summarize, the model developed by Rabanal and Rubio-Ramirez (2015) is able to capture a wide range of real exchange rate features with appropriate calibration and stationary productivity.

2.5 Microeconomic foundation of the integral correction mechanism

So far, it is clear that the baseline model can only generate a highly volatile real exchange rate by increasing its conventional persistence so it is unable to capture the cyclical persistence. The workhorse model fixes this problem by adding an input adjustment cost, which makes the trade elasticity time-varying. To investigate the micro-foundation of the integral correction mechanism and the DSGE model, it is useful to use micro evidence of trade elasticities to regulate the key parameter ι and σ_{HF}. As Imbs and Mejean (2015) suggest, one-sector macroeconomic model should be calibrated using weighted average of micro elasticities (sector level elasticities) instead of macro elasticities (elasticities estimated from aggregate data) to avoid odd predictions. Imbs and Mejean (2016) further estimate macro elasticities that are equivalent to the micro ones in terms of welfare implications. The estimates range from 1.5 to 5, which justifies $\sigma_{HF} = 3$. Other papers using more standard econometric methods (e.g. Bahmani-Oskooee and Kara, 2005, and working papers of the Global Trade Analysis Project) typically report smaller macro elasticities due to heterogeneity bias, argued by Imbs and Mejean (2015). However, There are only few papers that distinguish between short- and long- term elasticities. For example, Gallaway et al. (2003)

Figure 2.6: Impulse response function under a different value of ι



Note: The notations follow Section 2.3 except that prices in this graph are defined relative to the final good price.

estimate micro elasticities, and Hooper et al. (2000) estimate macro elasticities. Both papers conclude that the short-term elasticities are roughly half or below half of the long-term ones.

In the DSGE model, it is difficult to form an analytical expression of short-run elasticity. Instead, I define a "pseudo" elasticity as $\sigma_{HF,k}^{pseudo} = -\frac{\hat{y}_{H,t+k}-\hat{y}_{F,t+k}}{\hat{p}_{H,t+k}-\hat{p}_{F,t+k}}$, where a hat denotes the log deviation from the steady state following a shock at t. The short-term elasticity can be treated as $\sigma_{HF,1}^{pseudo}$ or a average over the first few periods, depending on how the short-term elasticity is measured in empirical research. Given this definition, the short-term elasticity is half of the long-term elasticity when ι is around 10, or comparable to the micro evidence in level when ι is around 125. For the final calibration $\iota = 900$, the short-term elasticity is about 0.05. Overall, the input adjustment cost that is required to match features of the real exchange rate is relative stronger than that suggested by micro evidence.

In the following, I study how the integral correction mechanism is generated from the DSGE model. I calculate impulse responses to a positive Home productivity shock in the workhorse model with final calibration and analyse the sensitivity by varying ι. Three domains of ι are relevant. The first is of course 0, by which the cost is shut down. The second domain is (0, 245) during which the sensitivity analysis shows that a larger size of the cost increases the volatility of the real exchange rate. However, the volatility of the real exchange rate starts to decrease when ι>245. In addition, the basic shape of impulse responses remains within each domain but varies across domains. The benchmark calibration falls into the second domain and the final calibration falls into the third. Figure 2.6 on page 40 reports the impulse responses of workhorse models by 3*2 blocks. Each row of the blocks is reported for ι equal to 0 (case 1), 150 (case 2) and 900 (case 3), respectively. The left-hand column of blocks contains the intra-temporal variables and the right-hand column contains inter-temporal variables. All prices are defined relative to the local final goods price.

Let us start with a baseline model in which $\iota = 0$ shuts down the cost and the standard results of the IRBC model apply. On the impact of the shock, the right-hand panel shows that both the capital return and the interest rate increase in the Home country. Households in the Home country smooth their consumption by increasing today's consumption. The higher return in the Home country induces more investment which, together with increasing consumption, push up aggregate demand y. Since the financial markets are integrated, households in the Foreign country find it beneficial to shift resources to the more productive location, the Home country. They do so by lending and exporting to the Home country. Consequently, investment in the Foreign country decreases but consumption increases because the Foreign country enjoys a wealth effect due to lending and a terms of trade effect as follows.

The terms of trade effect can be seen from the left-hand panel, which plots the components of aggregate demand. Higher capital return (together with the associated higher wage) and higher productivity in the Home country have opposite effects on the marginal cost of producing Home intermediate goods. Since the perfect competitive intermediate goods firms set price P_H equal to the marginal cost, the decrease in P_H indicates that the effect of a higher capital return is smaller than the effect of higher productivity. P_F increases because of the higher foreign capital rent and a zero or mild increase in productivity. It turns out that the Home country terms of trade $\frac{P_F}{P_H}$ and, according to equation 2.6, RER depreciate (increase). The real exchange rate also features a hump shape because the spillover of Home country productivity to the Foreign country has a lag, which causes asynchronous movements between the capital return in the Home and Foreign country, see the discussion of Steinsson (2008).

When the input adjustment cost is introduced, it has two effects on the model. First, it makes the final goods more expensive when the households want to change the ratio between Home and Foreign goods. Therefore, the inter-temporal substitution of final goods is distorted. Second, the cost changes the intra-temporal substitution between Home and Foreign goods because domestic goods are not subject to this cost.

When ι is in the second domain, the world households still find it optimal to shift more resources to the Home country. The Foreign country lends and exports to the Home country as in case 1. This behavior causes a surge in the input adjustment cost (in the absolute, as below) so that economic activities in the Home country, including consumption and investment, actually go down. However, households expect that the input adjustment cost will be diminishing fast as they need not change the domestic to imported goods ratio to any considerable extent in the future. Several periods later, households in the Home country can start enjoying their higher productivity and raise consumption and investment. This expectation explains why world households would shift resources to the home country in the first place. Although Home country households suffer from a decrease in consumption in the first few periods, the net benefit during their life time is still positive. On the other hand, given this expectation, the interest rate is low on the impact of the shock, which helps us reduce the marginal cost and the price of the Home good. It is clear from the figure that the initial response of P_H is much larger than in case 1. As a consequence, the response of terms of trade and the real exchange rate is much larger. The standard deviation of the real exchange rate increases in case 2. Another feature of the real exchange rate in this case is that it reverts more quickly to steady state. Since the interest rate difference between two countries is larger, the real exchange rate has to move fast in order to conform to the uncovered interest rate parity. Moreover, note that the hump shape response of the real exchange rate disappears because of the more synchronized (although in a different direction) movement of the interest rate in each country.

When the input adjustment cost is rapidly disappearing as expected, the difference in productivity in the two countries still exists. The new round of resource shifting once more increases the input adjustment cost although the magnitude is much smaller this time. The cyclical movement of the input adjustment cost is transmitted to all macroeconomic variables, including the real exchange rate as captured by ICM.

If ι is in the third domain, the cost is so large that it is no longer optimal to shift resources from the Foreign country to the Home country. Instead, Home country households lend and export to the Foreign country. Consequently, the direction of the real exchange rate response is reversed. The Home economy in case 3 behaves just like the Foreign country in case 2. The difference is that the exporter in case 3 has a larger response on impact than the exporter in case 2 because the former also enjoys a positive productivity shock. Also note that the initial response of φ_t is smaller than in case 2, which suggests that households have to reduce the input adjustment cost by keeping the changes in $\frac{Y_{F,t}}{Y_{H,t}}$ smaller. This is why increasing ι in the third domain makes the volatility of the real exchange rate smaller. On the other hand, the cyclical movement of φ_t is more persistent, which is transmitted to all other macroeconomic variables. The input adjustment cost is not the only mechanism implying a persistent oscillations of all variables. Researchers highlighting self-fulfilling expectation shocks, also referred to as "sunspots" shocks, also find this type of behaviour. In particular, see a two-country model studied by Xiao (2004).

To summarize this, the impulse responses of the real exchange rate are very close to those implied by ICM and resemble the shape of ACF. This helps to explain the PPP puzzle further because, in the presence of the input adjustment cost, the impulse response of the real exchange rate to a persistent AR(1) productivity process features a fast reversion to steady state in the short run and a cyclical movement in the long run. This particular dynamics reduces the variance in the short run (higher frequencies) and increases the variance in the long run (low frequencies). In contrast, the conventional belief is that high volatility is mainly driven by nominal shocks and the persistence is caused by persistent real shocks. Another finding in this section is that the size of the cost can alter the direction of the initial response of the real exchange rate.

2.6 Other dimensions of the model

Before conclude, I comment on the performance of the model in other dimensions, including how the model can capture the domestic and international business cycle, by comparing the second moments of H-P filtered key macroeconomic variables.

The results are reported in Table 2.3 on page 44 where I report the (relative) standard deviation of GDP (Y), the trade balance to GDP ratio (TB/Y), consumption (C), investment (X) and employment (L) as well as the relevant domestic and international correlations. The upper block of the table suggests that introducing an input adjustment cost does not sacrifice the ability to capture domestic business cycle features. As compared to the standard model, it actually makes some minor improvement. An important implication of the input adjustment cost, regardless of the size of the cost, is that GDP and the real exchange rate respond in opposite directions to a productivity shock. This is reflected in the negative correlation between the real exchange rate and GDP, which is consistent with the data. However, the correlation in the model is too high in absolute value. On the other hand, the trade balance becomes pro-cyclical and again independent of the size of the input adjustment cost.[9] Turning to the bottom block of the table, it is well known that international quantitative properties of standard international business cycle models are at odds with the data. A model with homogeneous goods across countries (substitution σ_{HF} goes to infinity) typically generates a low or negative cross-correlation of output, a higher consumption cross-correlation than that of output, and a negative cross-correlation of investment and employment (Backus et al., 1992; Baxter, 1995, among others). Allowing heterogeneous but substitutable goods gives quantitative results that are closer to the data but do not fix the "anomalies" named by Backus et al. (1993); see the row labelled $\iota = 0$. The main drawback of my model with an input adjustment cost is the strong negative cross-country correlation among all main variables, which makes the problem even worse. The negative cross-correlation of output is closer to the homogeneous goods model than to the heterogeneous goods model. Given the time varying short-run substitution between domestic and foreign goods in my model, this feature seems odd. Fortunately, a vast literature has tried to fix the international anomalies by introducing new features into the models. In particular, financial integration via a nontrivial banking sector is promising in increasing the cross-country comovement. Adding those new features may help the model here but it is beyond the scope of this chapter.

2.7 Summary

This chapter aims at examining the ability of IRBC models to explain several features of the real exchange rate, in particular the integral correction mechanism found by chapter 1. I document that the US effective real exchange rate features high volatility, a cyclical movement of ACF, a positively skewed spectrum, and a significant integral correction mechanism representation. I find that the standard IRBC model is able to fit one or two features but faces a trade off to fit all of them. A workhorse model can closely match all features seen in the data by introducing an input adjustment cost of imported goods in the final goods production. However, the required input adjustment cost is relative stronger than that suggested by micro evidence. I argue that the input adjustment cost helps resolve the PPP puzzle with three new features in the impulse response of the real exchange rate to a persistent real shock: 1) The initial response to a positive shock is negative, 2) The deviation from steady state decays very fast in the short run, and 3) There are persistent cyclical movements in the long run.

Further work on real exchange rate modelling at low frequencies should be carried out in at least two ways. First, this chapter only suggests the input adjustment cost as a candidate mechanism to

[9] The model in this chapter does not incorporate a capital adjustment cost, which should fix this problem as suggested by **?**.

Table 2.3: Moment Matching
it is easy to show that

	StD(Y)	StD(TB/Y)	StD(C)/StD(Y)	StD(X)/StD(Y)	StD(L)/StD(Y)
data	1.58	0.45	0.76	4.55	0.75
$\iota = 0$	1.12	0.19	0.54	2.51	0.31
$\iota = 900$	1.50	0.80	0.60	4.10	1.00
	Correlation				
	Corr(Y,C)	Corr(Y,X)	Corr(Y,L)	Corr(Y,TB)	Corr(Y,RER)
data	0.84	0.91	0.87	-0.36	-0.14
$\iota = 0$	0.91	0.97	0.97	-0.65	0.82
$\iota = 900$	0.87	0.99	0.96	0.92	-0.95
	Corr(Y,Y*)	Corr(C,C*)	Corr(X,X*)	Corr(L,L*)	
data	0.44	0.36	0.28	0.40	
$\iota = 0$	0.33	0.81	-0.05	-0.05	
$\iota = 900$	-0.77	-0.75	-0.95	-0.98	

Note: The statistics of the data is taken from Rabanal et al. (2011). The data ranges from 1973:1 to 2006:4, which is an extension of the standard statistics used in the literature, see Backus et al. (1994) and Heathcote and Perri (2002) among others. The row labelled$\iota = 0$ is a standard two-country two-goods model used in 2.4.1, which is close to the one studied by Heathcote and Perri (2002). The row labelled $\iota = 900$ is the model with an input adjustment cost and final calibration. All statistics (including both data and model) are based on logged (except for net exports) and Hodrick-Prescott-filtered data with a smoothing parameter of 1,600.

explain real exchange rates at all frequencies, in particularly at frequencies lower than business cycle. I make no attempt to estimate the key parameter ι against other possible mechanisms. Since the input adjustment cost alone falls short in explaining data, it is helpful to examine the input adjustment cost in a richer model. For example, Drozd et al. (2017) propose another way to disconnect the short- and the long-run trade elasticity by introducing searching friction. They show that this feature helps to resolve the trade-comovement puzzle, partially through a risk-sharing channel. Second, the model studied in this chapter either has no trend or has a exogenous trend. This kind of models has a chance to shed lights on real exchange rates at low frequencies only if we are willing to assume that economic growth is to a large extent independent to real exchange rates, which is unlikely to be true according to, for instance, Harrod-Balassa-Samuelson effect. Comin and Gertler (2006) point out a medium-frequency cycle that is typically filtered out together with the trend. In their endogenous growth model, medium-term cycle is driven by standard high-frequency shocks that propagates through the economy via technological change and R&D. Therefore, a more appropriate way to study real exchange rate at all frequencies could be to allow endogenous productivity in the spirit Comin and Gertler (2006), technology diffusion and catch-up in a multi-country model. The low frequency real exchange rate can be explained by at least two channel, i.e., a endogenous Harrod-Balassa-Samuelson effect and the medium-frequency cycle.

Appendix B

This appendix collects the equilibrium conditions of the workhorse model. Since P_t and P_t^* are not determined without specifying monetary policy, I define $\tilde{P}_{H,t} = \frac{P_{H,t}}{P_t}$,$\tilde{P}_{F,t} = \frac{P_{F,t}}{P_t}$, $\tilde{P}^*_{F,t} = \frac{P^*_{F,t}}{P^*_t}$, $\tilde{P}^*_{H,t} = \frac{P^*_{H,t}}{P^*_t}$. Assuming Law of One Price gives $\tilde{P}_{H,t} = \tilde{P}^*_{H,t} RER_t$ and $\tilde{P}_{F,t} = \tilde{P}^*_{F,t} RER_t$ where RER_t is the real exchange rate.

Marginal utility of consumption:

$$\frac{\tau}{C_t}\left[C_t^{\tau}\left(1-L_t\right)^{1-\tau}\right]^{1-\sigma} = \lambda_t,$$

$$\frac{\tau}{C_t^*}\left[C_t^{*\tau}\left(1-L_t^*\right)^{1-\tau}\right]^{1-\sigma} = \lambda_t^*$$

Labour supply:

$$\frac{1-\tau}{\tau}\frac{C_t}{1-L_t} = W_t,$$

$$\frac{1-\tau}{\tau}\frac{C_t^*}{1-L_t^*} = W_t^*,$$

Capital accumulation:

$$K_t = (1-\delta)K_{t-1} + X_t - \frac{\psi}{2}K_{t-1}\left(\frac{X_t}{K_{t-1}} - \delta\right)^2,$$

$$K_t^* = (1-\delta)K_{t-1}^* + X_t^* - \frac{\psi}{2}K_{t-1}^*\left(\frac{X_t^*}{K_{t-1}^*} - \delta\right)^2.$$

Capital investment decisions:

$$\frac{\lambda_t}{\left[1-\psi\left(\frac{X_t}{K_{t-1}}-\delta\right)\right]} = \beta E_t\left[\lambda_{t+1}R_{t+1} + \frac{\lambda_{t+1}}{\left[1-\psi\left(\frac{X_{t+1}}{K_t}-\delta\right)\right]}\left(1-\delta+\frac{\psi}{2}\left(\frac{X_t^2}{K_{t-1}^2}-\delta^2\right)\right)\right],$$

$$\frac{\lambda_t^*}{\left[1-\psi\left(\frac{X_t^*}{K_{t-1}^*}-\delta\right)\right]} = \beta E_t\left[\lambda_{t+1}^*R_{t+1}^* + \frac{\lambda_{t+1}^*}{\left[1-\psi\left(\frac{X_{t+1}^*}{K_t^*}-\delta\right)\right]}\left(1-\delta+\frac{\psi}{2}\left(\frac{X_t^{*2}}{K_{t-1}^{*2}}-\delta^2\right)\right)\right].$$

Euler equations:

$$Q_t = \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\tilde{P}_{H,t+1}/P_{t+1}}{\tilde{P}_{H,t}/P_t}\right) - \phi D_t,$$

$$Q_t = \beta E_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{\tilde{P}_{H,t+1}/RER_{t+1}}{\tilde{P}_{H,t}/RER_t}\right).$$

Intermediate goods production:

$$Y_{H,t}+Y^{*}_{H,t}=A^{1-\alpha}_{t}K^{\alpha}_{t-1}L^{1-\alpha}_{t},$$

$$Y_{F,t}+Y^{*}_{F,t}=A^{*1-\alpha}_{t}K^{*\alpha}_{t-1}L^{*1-\alpha}_{t}.$$

Labour demand:

$$W_{t}=(1-\alpha)\,\tilde{P}_{H,t}A^{1-\alpha}_{t}K^{\alpha}_{t-1}L^{-\alpha}_{t},$$

$$W^{*}_{t}=(1-\alpha)\,\tilde{P}^{*}_{F,t}A^{*1-\alpha}_{t}K^{*\alpha}_{t-1}L^{*-\alpha}_{t}.$$

Capital demand:

$$R_{t}=\alpha\tilde{P}_{H,t}A^{1-\alpha}_{t}K^{\alpha-1}_{t-1}L^{1-\alpha}_{t},$$

$$R^{*}_{t}=\alpha\tilde{P}^{*}_{F,t}A^{*1-\alpha}_{t}K^{*\alpha-1}_{t-1}L^{*1-\alpha}_{t}.$$

Final goods production:

$$Y_{t}=\left[\omega^{\frac{1}{\sigma_{HF}}}Y_{H,t}^{\frac{\sigma_{HF}-1}{\sigma_{HF}}}+(1-\omega)^{\frac{1}{\sigma_{HF}}}\left(\varphi_{t}Y_{F,t}\right)^{\frac{\sigma_{HF}-1}{\sigma_{HF}}}\right]^{\frac{\sigma_{HF}}{\sigma_{HF}-1}},$$

$$Y^{*}_{t}=\left[\omega^{\frac{1}{\sigma_{HF}}}Y_{F,t}^{*\frac{\sigma_{HF}-1}{\sigma_{HF}}}+(1-\omega)^{\frac{1}{\sigma_{HF}}}\left(\varphi^{*}_{t}Y^{*}_{H,t}\right)^{\frac{\sigma_{HF}-1}{\sigma_{HF}}}\right]^{\frac{\sigma_{HF}}{\sigma_{HF}-1}}.$$

Impute adjustment costs:

$$\varphi_{t}=\left[1-\frac{\iota}{2}\left(\frac{\frac{Y_{F,t}}{Y_{H,t}}}{\frac{Y_{F,t-1}}{Y_{H,t-1}}}-1\right)^{2}\right],$$

$$\varphi^{*}_{t}=\left[1-\frac{\iota}{2}\left(\frac{\frac{Y^{*}_{H,t}}{Y^{*}_{F,t}}}{\frac{Y^{*}_{H,t-1}}{Y^{*}_{F,t-1}}}-1\right)^{2}\right].$$

Demand for Home intermediate goods:

$$P_{t}\frac{\partial Y_{t}}{\partial Y_{H,t}}+E_{t}\left(\Omega_{t,t+1}P_{t+1}\frac{\partial Y_{t+1}}{\partial Y_{H,t}}\right)=P_{H,t},$$

$$P^{*}_{t}\frac{\partial Y^{*}_{t}}{\partial Y^{*}_{H,t}}+E_{t}\left(\Omega^{*}_{t,t+1}P^{*}_{t+1}\frac{\partial Y^{*}_{t+1}}{\partial Y^{*}_{H,t}}\right)=P^{*}_{H,t}.$$

Demand for Foreign intermediate goods:

$$P_{t}\frac{\partial Y_{t}}{\partial Y_{F,t}}+E_{t}\left(\Omega_{t,t+1}P_{t+1}\frac{\partial Y_{t+1}}{\partial Y_{F,t}}\right)=P_{F,t},$$

$$P^{*}_{t}\frac{\partial Y^{*}_{t}}{\partial Y^{*}_{F,t}}+E_{t}\left(\Omega^{*}_{t,t+1}P^{*}_{t+1}\frac{\partial Y^{*}_{t+1}}{\partial Y^{*}_{F,t}}\right)=P^{*}_{F,t}.$$

Net foreign assets:

$$Q_{t}D_{t}=Y^{*}_{H,t}-\tilde{P}_{F,t}Y_{F,t}+D_{t-1}-\frac{\phi}{2}D^{2}_{t}.$$

Market clearing conditions:

$$C_t + X_t = Y_t,$$

$$C_t^* + X_t^* = Y_t^*.$$

Steady-state variables are solved by a numerical solver. However, since the input adjustment cost equals to 1 in steady state, the steady-state real exchange rate is given by equation 1.2, which equals to one if assuming zero net foreign asset in the first period.

Chapter 3

Optimal Cooperative and Non-Cooperative Unconventional Monetary Policy Under Commitment

3.1 Introduction

The recent financial crisis involved a significant disruption to financial intermediation, as evidenced by limited access to credit (e.g. Ivashina and Scharfstein, 2010) and high credit spreads. Such disruption can propagate internationally via integrated financial markets. To stabilize the financial system, fiscal and monetary authorities in major economies acted jointly and introduced new policy tools. These new tools included the provision of large-scale liquidity, and resulted in balance sheets of some central banks expanding 20 percent to 30 percent of GDP. The new tools break away from conventional monetary policy, which may have reached its effective lower bound, and are commonly known as unconventional monetary policies (UMPs).[1] In the last ten years, different sets of UMPs have been employed at different stages. For example, the Federal Reserve mainly used an expanded discount window in the early stage of the 2007-2008 crisis. After the Lehmann failure, the Fed started its asset purchase programs (quantitative easing, or QE) and injected equity into the financial system. The Fed started to taper the QE at the end of 2013, and ceased it in October 2014, after which the Federal Reserve has kept the size of its balance sheet constant by buying just enough to replace maturing securities. The European Central Bank (ECB) employed two series of Targeted longer-term refinancing operations in June 2014 and March 2016 respectively. The ECB initialized its asset purchase programs at a relatively small scale, slightly later than the Federal Reserve. The ECB formally introduced QE in 2015 and further increased the amount purchased in late 2016. The ECB also used equity injections during this period.

By employing UMP, policymakers hope to reduce long-term interest rates, boost lending, and stimulate real activity. In the meantime, conventional monetary policy can play an active role if its transmission channel via credit and financial markets is restored (Altavilla et al., 2016). As a probably unintended consequence, domestic interventions into markets of intentionally traded assets also affect financial conditions in foreign countries. Although much work has been devoted to evaluating the effectiveness of

[1] These policies may not be strictly monetary policy. For example, Kollmann et al. (2013) consider government support for banks as fiscal policy. I use the terms unconventional policy and unconventional monetary policy interchangeably.

UMP,[2] a normative analysis is still missing, namely, what is the optimal way of conducting unconventional policy, especially when foreign countries may take a free-ride on domestic policy. A normative analysis does not only provide a natural benchmark against which we can evaluate certain policy implementations but also sheds some light on critical policy decisions.

In this chapter, I study optimal unconventional policy under commitment (Ramsey policy) and aim to address three questions. First, which of the three unconventional policies, namely public asset purchases, discount window lending, and equity injections into banks, is more effective. More generally, what are the factors that affect policy effectiveness. Second, how should the optimal policy respond to a foreign or domestic shock that may trigger a global financial crisis. Then, how should a central bank exit from its policy. Since UMP may not be permanent, the exit from policy is particularly interesting after a decade of the crisis because central banks start to discuss shrinking their large balance sheet. For instance, in a blog article, Bernanke (2017) argues from a policy communication perspective that the shrinkage should be done in a passive and predictable way. Third, what is the difference between cooperative policy coordinated across countries and non-cooperative policy conducted strategically by independent central banks.

The literature has identified several channels through which UMP works. One is a signalling channel (Bauer and Rudebusch, 2014; Christensen and Rudebusch, 2012), which means an announcement of interventions lowers market expectations about future short-term rates, and therefore current long-term rates. Christensen and Gillan (2017) argue that introducing the central bank as a large committed buyer to financial markets lowers liquidity premiums of targeted assets. However, Kandrac (2014) finds evidence that this effect can be negative when trading among private participants decreases too much. With segmentation of the market for reserves, Christensen and Krogstrup (2016) demonstrate that reserve expansions associated with UMP can affect long-term rates through a portfolio channel even in the absence of interventions in the long-term asset market. (Unconventional) monetary policy may affect banks willingness take on risk exposures, and hence affect financial conditions via the risk-taking channel (Angeloni et al., 2015; Borio and Zhu, 2012; Bruno and Shin, 2015; Coimbra and Rey, 2017). Chakraborty et al. (2017) discuss an origination channel that is specific to the Fed's purchases of mortgage-backed securities.

As a first step toward a comprehensive analysis of optimal unconventional policy, however, this chapter focuses on a capital gain channel, which has received much attention and been well tested in data yet not fully understood in terms of optimality.[3] This channel is particular relevant in a multi-country context because, with financial market integration, asset returns are synchronised internationally. To this aim, I consider a simple two-country model where each country has a stylized multinational banking

[2] It is relatively well established that UMP reduces long-term interest rates. See, among many others, Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011) for the Federal Reserve's QE, and Joyce et al. (2011) and Christensen and Rudebusch (2012) for the Bank of England's QE. However, UMP can have insignificant or unintended real effects through the bank lending channel, as shown by Chakraborty et al. (2017) and Acharya et al. (2017). Negro et al. (2017) evaluate the Fed's QE using a DSGE model with a borrowing constraint and a resaleability constraint. Mouabbi and Sahuc (2016) estimate a DSGE model in which the macroeconomic effects of the ECB's UMP are equivalent to that of a negative shadow short-term rate. Borio and Zabai (2016) provide a review of a border range of issues related to UMP, such as a taxonomy of unconventional measures, diminishing returns and long-term effectiveness of UMP.

[3] It could complicate the model significantly to consider other channels. For example, the signalling channel and policies such as forward guidance requires information frictions. Solving Ramsey optimal policy in such as complicated model is very difficult. Using a framework similar to that of Bernanke et al. (1999), it is possible to incorporate the risk-taking channel into a simple model. I choose not to do so because it is not clear whether taking more risks is harmful or beneficial to the economy. Encouraging risk-taking is modeled as costs of policy in some papers (Coimbra and Rey, 2017) but as benefits of policy in others (Brunnermeier and Koby, 2017; Pancost and Robatto, 2017). The answer to this question demands some empirical evidence and could be country specific.

sector similar to that in Gertler and Kiyotaki (2010).[45] Banks face a balance sheet constraint (financial constraint) derived from an agency problem between the banks and their depositors. The constraint is slack in normal times but binds endogenously in a financial crisis, which constitutes the systemic risk in this model.[6] Given their high leverage, these banks are vulnerable to shocks having negative impact on the value of their assets and to financial shocks that tighten their balance sheet. When the balance sheet constraint binds, banks have difficulties of rolling over their short-term debts, which leads to a collapse in asset prices and investment. With multinational banks, the deteriorated balance sheet condition has a global impact, which also means that there will be large spillovers of unconventional policy from one country to another. The basic mechanism of the capital gain channel is as follows. Unconventional policy provides liquidity to support asset prices. Banks holding these assets have an improved balance sheet condition. Consequently, the policy leads to more lending to the non-financial sector.[7]

My main findings are as follows. First, unconventional policy effectively crowds out deposits received by banks. The crowding-out effect is mitigated if the policy relaxes the financial constraint that banks face. I find public asset purchases, while constitute the main policy that has been employed in many countries, have relatively larger crowding-out effect. The most efficient policy, i.e. equity injections, has the smallest crowding-out effect. Second, domestic and foreign policy respond asymmetrically to shocks. The degree of asymmetry depends on the nature of the shock, the cost of interventions, and the bank's portfolio. After a relatively strong response in same period when a shock hits (i.e., in a non-prudential manner), the central bank exits from its interventions in accordance with the deleveraging of the banks, the speed of which depends on the crowding-out effect. Overall, due to the precautionary effects of the occasionally binding constraints, the exit is slow and lasts even after the economy has escaped from the financial constraints. Third, the difference between cooperative and noncooperative policy is a weakly increasing function of the intervention cost. There is no cooperation gain if the intervention cost is small. Increasing the intervention cost to a certain point generates positive gains to cooperation. In the noncooperative equilibrium, the interventions are too strong in one country, but too weak in the other.

I contribute to the literature that examines the capital gain channel of unconventional policy. The literature has focused on simple rules or a particular policy scheme. The work of Dedola et al. (2013), which is most closely related to this chapter, studies the international dimension of public asset purchases in an economy where the financial constraint in each country is always binding. In their consideration of a credit spread rule, the lack of policy cooperation reduces the policy responses in both countries, which is in sharp contrast to my result. My discussion about the exit problem is linked to Foerster (2015) who also suggests slowly unwinding the central bank's balance sheet. I find that, due to a precautionary effect, the exit speed is even slower than what Foerster (2015) suggests. He and Krishnamurthy (2013) compare multiple policies: borrowing subsidies, equity injections, and public asset purchases. They also find that equity injections lead to the fastest recovery. Their comparison is based on a particular policy scheme that corresponds to the actual policy employed during the recent crisis. Ellison and Tischbirek (2014) and Quint and Rabanal (2017) investigate if asset purchase programs could be valuable in normal times.

[4] Earlier, i.e. before the recent crisis, models with financial frictions do not seem to have very big effects. For example, Linde et al. (2016) shows that the financial accelerator of Bernanke et al. (1999) has only a modest quantitative effect on the impulse response functions. Kocherlakota (2000) provide similar findings for the Kiyotaki and Moore (1997) type credit constraints. Brzoza-Brzezina and Kolasa (2013) compare the empirical performance of the standard New Keynesian DSGE model with variants that incorporate financial frictions proposed pre-crisis. They find no significant improvement on the performance of the benchmark model, either in terms of marginal likelihoods or impulse response functions. So, the earlier frameworks are unlikely to be suitable for the purpose of this chapter.

[5] Evidence for increasingly global banking and integrated financial markets can be found in Devereux and Yetman (2010), Perri and Quadrini (2011), Fillat et al. (2015), and Bank for International Settlements' international banking statistics.

[6] While the banks in this model will never actually default, Coimbra and Rey (2017) precisely define systemic risk as a state in which generalized solvency issues take place.

[7] In models with imperfectly substitute assets, especially assets with different maturities, this channel is also referred to as the portfolio balance channel. And the effect of interventions on asset prices is reflected by the yield curve. See, for example, Negro et al. (2017) and Quint and Rabanal (2017).

The former paper jointly optimises the parameters in the interest rate and the asset purchase rules while the latter optimises the asset purchase rules conditional on a estimated Taylor rule.

Implications made in this chapter could be relevant to recent work on other channels of UMP, and the effective lower bound and negative interest rates of monetary policy. For example, Brunnermeier and Koby (2017) build a model where a effective lower bound of the conventional monetary policy is determined partially by financial constraints that banks face. They suggest that, due to the crowding-out effect, QE should only be employed when conventional monetary policy hits its effective lower bound. My result implies that equity injection policy could be used more freely thanks to its very small crowding-out effect. Moreover, banks' risk-taking behaviours are constrained by their net worth, which is improved by UMP via the capital gain channel. So this chapter is related to the risk-taking literature.

This chapter also relates to the literature that works on occasionally binding constraints (OBCs) as a source of nonlinearity. The benefits of this setting is to introduce asymmetry such that we can capture the sudden and discrete nature of a financial crisis, and eliminate the financial accelerator mechanism during normal times (Del Negro et al., 2016; Swarbrick et al., 2017). In this chapter, the OBC setting also means that unconventional policy may be conducted only when the constraint is binding. More importantly, the risk of the constraint being binding in the future has strong precautionary effects on the Ramsey policy.

The rest of the chapter is organized as follows. The next section presents a two-country model with occasionally binding financial constraints. After describing my numerical method in section 3.3, sections 3.4 and 3.5 report the main results for cooperative and noncooperative policy, respectively. The last section concludes.

3.2 The Model

The model mostly follows Dedola et al. (2013), which extends the two-country real business cycle model (Backus et al., 1992) by including a Gertler and Karadi (2011) style financial friction. The world economy consists of two countries, Home and Foreign, that are symmetric before being hit by a shock. In each country, domestic labour and capital are used to produce homogeneous goods, which can be used for consumption and capital production. To finance their capital, goods producers borrow from banks. Banks receive deposits from households in both countries and lend to goods producers in both countries. [8]I use the term "non-financial sector" to refer to households and producers of goods and capital, and the term "financial sector" to refer to banks. The problem facing each agent in the Home economy is described in this section. Foreign variables are denoted by "*". Lower case letters denote individual variables and real prices while upper case letters denote aggregate variables and nominal prices.

3.2.1 Households

There is a unit-continuum of infinitely lived households. Households consume homogeneous goods, supply labour, and save. The menu of assets available to households includes a deposit in domestic banks, $d_{h,t}$, a deposit in foreign banks, $d_{f,t}$, and a domestic government bond, b_t. Without loss of generality, only domestic citizens can hold their own government's bonds. All these assets are risk-free one-period bonds denominated in terms of the issuing country's goods and paying gross real return, r_t or, r_t^*. Households also hold shares in non-financial firms.

Each household consists of workers and bankers who pool consumption risk perfectly. Workers provide labour to goods producers and bring wages to the household. Bankers manage a bank and transfer the

[8]Deposit market segmentation would change the results mildly up to a misallocation of household savings. On the other hand, the security market integration is a key channel for shock propagation. In a slightly different context, Brunnermeier and Sannikov (2015) consider the case of an integrated deposit market but an separated security market.

profits to the household when they exit from the business. It is convenient to assume that households do not save in their own banks. Complete consumption insurance allows me to express the problem facing the consolidated representative household. The representative household chooses consumption, c_t, labour supply, l_t, and end-of-period wealth, consisting of domestic bonds, $d_{h,t} + b_t$ and foreign bonds, $d_{f,t}$ to maximize their expected discounted life-time utility, taking the wage rate and the interest rate as given:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[\frac{\left(c_{t+j} - hc_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} \right],$$

where $h \in [0,1)$ is the habit parameter, $\sigma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the inverse of the (Frisch) elasticity of labour supply, $\chi \geq 0$ is the relative utility weight on labour, and $\beta_{t,t+j}$ is the subjective discount factor from period t to $t+j$. To induce stationarity with incomplete financial markets, the discount factor is assumed to depend on aggregate consumption relative to aggregate income, $\beta_{t,t+1} = \bar{\beta} + \psi_\beta \log\left(\frac{C_t}{Y_t}\right)$, following Kollmann (2016). Denoting Π_t as the net profit distribution that the household earns from its ownership of banks and non-financial firms, w_t as the real wage rate, $\tau_{w,t}$ as a tax rate on wages, T_t as a lump-sum tax, the household faces the budget constraint

$$c_t + d_{h,t} + b_t + d_{f,t} = \left(1 - \tau_{w,t}\right) w_t l_t + \Pi_t + \left(d_{h,t-1} + b_{t-1}\right) r_{t-1} + d_{f,t-1} r_{t-1}^* - T_t.$$

Let $\Xi_{t,t+1} \equiv \beta_{t,t+1} \frac{(c_{t+1}-hc_t)^{-\sigma} - \beta_{t+1,t+2} h (c_{t+2}-hc_{t+1})^{-\sigma}}{(c_t-hc_{t-1})^{-\sigma} - \beta_{t,t+1} h (c_{t+1}-hc_t)^{-\sigma}}$ be the stochastic discount factor. The first-order conditions for the household problem are fairly standard:

$$w_t \left(1 - \tau_{w,t}\right) = \frac{\chi l_t^{\varphi}}{\left(c_t - hc_{t-1}\right)^{-\sigma} - \beta_{t,t+1} h \left(c_{t+1} - hc_t\right)^{-\sigma}},$$

$$\mathbb{E}_t \left[\Xi_{t,t+1} r_t\right] = 1, \tag{3.1}$$

$$\mathbb{E}_t \left[\Xi_{t,t+1} r_t^*\right] = 1. \tag{3.2}$$

Equations 3.1 and 3.2 imply that the risk free rate is equalized across countries thanks to deposit market integration.

3.2.2 Non-financial firms

There are two types of non-financial firms: capital producers and goods producers.

3.2.2.1 Goods producers

Goods producers hire workers and purchase capital from capital producers to produce final goods that are homogeneous across countries. They operate in markets for goods, capital, and labour that are perfectly competitive. The production technology is a standard Cobb-Douglas function $y_t = A_t \left(\xi_t k_{t-1}\right)^{\alpha} l_t^{1-\alpha}$ where α is the capital share, A_t is total factor productivity, and k_t is the capital stock at the end of period t. Using δ to denote the depreciation rate, and ξ_t to govern the quality of capital, a goods producer acquires additional capital $i_t = k_t - (1-\delta) \xi_t k_{t-1}$ at a given price q_t. To finance its physical investment, the goods producer borrows from banks by issuing securities

$$q_{t,t}^s \left(s_t - s_{t-1}\right) = i_t q_t, \tag{3.3}$$

where s_t denotes the number of securities issued at the end of period t and $q^s_{T,t}$ is the period T price of security issued at period t. Each security is a state-contingent claim to the future return from one unit of investment: z_{t+1}, $(1-\delta)\,\xi_{t+1}z_{t+2}$, $(1-\delta)^2\,\xi_{t+1}\xi_{t+2}z_{t+3}$, ... with z_t denoting the gross profit per unit of capital.

The problem faced by the representative goods producer is

$$\max_{\{l_{t+j},k_{t+j},s_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \Xi_{t,t+j} \\ \times \left[(1-\tau_{y,t})\,y_{t+j} - w_{t+j}l_{t+j} - i_{t+j}q_{t+j} + q^s_{t+j,t+j}\,(s_{t+j} - s_{t+j-1}) - z_{t+j}s_{t+j-1}\right],$$

subject to equation 3.3, the production function, and the capital accumulation equation. $\tau_{y,t}$ is a sales tax. Let the multiplier associated with equation 3.3 be λ^{nf}_t, the first-order conditions for the goods producer's problem are then

$$w_t = (1-\alpha)\,\frac{y_t\,(1-\tau_{y,t})}{l_t},$$

$$q_t\left(1+\lambda^{nf}_t\right) = \mathbb{E}_t\Xi_{t,t+1}\left[\frac{\partial y_{t+1}}{\partial k_t} + (1-\delta)\,\xi_{t+1}q_{t+1}\left(1+\lambda^{nf}_{t+1}\right)\right], \tag{3.4}$$

$$q^s_t\left(1+\lambda^{nf}_t\right) = \mathbb{E}_t\Xi_{t,t+1}\left[z_{t+1} + q^s_{t+1}\left(1+\lambda^{nf}_{t+1}\right)\right]. \tag{3.5}$$

It is important to assume that investment is fully financed by securities, i.e., $\lambda^{nf}_t \neq 0$. Otherwise, firms can effectively borrow directly from households by paying a negative dividend, which makes the banking sector trivial. Using equations 3.4 and 3.5, it is easy to show that the time T price of security issued at time t is $q^s_{T,t} = q_T\,(1-\delta)^T \prod_{j=1}^{T} \xi_{t+j}$, and $s_t = k_t$, given $s_0 = k_0$. I can define the return of holding security issued in period t from t to $t+1$ as

$$r_{k,t+1} = \frac{z_{t+1} + (1-\delta)\,\xi_{t+1}q_{t+1}}{q_t},$$

where the gross profit per unit of capital is obtained using the zero profit condition

$$z_t = \frac{y_t - w_tl_t}{k_{t-1}} = \alpha\frac{y_t\,(1-\tau_{y,t})}{k_{t-1}}.$$

3.2.2.2 Capital producers

Given the demand for new capital i_t and the market price q_t, capital producers maximize their expected profit discounted by the household's stochastic discount factor

$$\max_{\{i_{t+j}\}_{j=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \Xi_{t,t+j}\left[q_{t+j}i_{t+j} - f\left(k_{t+j-1}, i_{t+j}\right)\right],$$

subject to the cost function

$$f\left(\cdot\right) = i_t + \frac{\eta}{2}\left(\frac{i_t}{\delta k_{t-1}} - 1\right)^2 \delta k_{t-1},$$

where $\eta \geq 0$. The first-order condition for the production decision pins down the market price of new capital

$$q_t = 1 + \eta \left(\frac{i_t}{\delta k_{t-1}} - 1 \right).$$

3.2.3 Banks

Banks receive deposits amounting to $d_{h,t}$ and $d^*_{h,t}$ from domestic and foreign depositors, respectively, and purchase $s_{h,t}$ and $s_{f,t}$ units of securities from domestic and foreign goods producers. The lending channel from banks to goods producers is frictionless. If banks have difficulty raising deposits, they have the option of borrowing $d_{g,t}$ from the central bank's discount window at the interest rate $r_{g,t}$, or they may receive equity injections from their government. By holding long-term risky securities funded by short-term risk free deposits, banks in this model act as investment banks as well as commercial banks, which is a stylized fact of the recent financial crisis. For the same reason, the literature often refers to such banks as financial intermediaries.

The balance sheet of a representative bank is given by

$$\omega_t \equiv q_t s_{h,t} + q^*_t s_{f,t} = d_{h,t} + d^*_{h,t} + d_{g,t} + e_{h,t} + e_{g,t}, \tag{3.6}$$

where ω_t denotes the total assets of the bank, and $e_{h,t}$ and $e_{g,t}$ are equity held by households and the government, respectively. Total bank profits, referred to as net worth n_t, is given by

$$e_{h,t} + e_{g,t} = n_t \equiv q_{t-1} s_{h,t-1} r_{k,t} + q^*_{t-1} s_{f,t-1} r^*_{k,t} - d_{h,t-1} r_{t-1} - d^*_{h,t-1} r^*_{t-1} - d_{g,t-1} r_{g,t-1}. \tag{3.7}$$

I refer to deposits and household equity as private funds and refer to discount window lending and government equity as public funds. Bank leverage may be defined as assets financed by deposits and private equity divided by private equity

$$\phi_t = \frac{\omega_t - d_{g,t} - e_{g,t}}{e_{h,t}}.$$

Following Gertler and Karadi (2011), I assume that the bank shuts down with probability $r_{n,t}$ at the end of each period and will distribute its net worth evenly to all equity. The probability of shutting down can be exogenous stochastic. Then, the banker becomes a worker. In the meantime, a similar number of workers from the same household randomly become new bankers. New bankers receive "start-up" funds from their household at a proportion ϖ of the total assets owned by a representative incumbent plus the value of the central bank's asset purchase program.[9] The probability of a shutdown has two roles. First, an infinitely lived bank will sooner or later accumulate enough net worth to finance its investment without borrowing from households. In this case, the financial constraint that I will detail shortly plays no role. Second, the probability enters the bank's stochastic discount factor, which ensures that the bank is always "less patient" than households so that funds always flow from households to banks. The

[9] As I will show in impulse responses, a central bank's asset purchase crowds out banks. I assume the value of the government's asset purchase program enters here so that these purchases do not lead to fewer start-up funds received by new banks. If the start-up funds can be interpreted as new equity, data shows counter-cyclical and positively skewed equity issuance. However, there is no evidence suggesting that equity issuance should be affected by, and only by, asset purchase policy. Therefore this assumption ensures that public asset purchases do not suffer from artificial disadvantage comparing to other policies.

notation of $r_{n,t}$ follows the suggestion of Swarbrick et al. (2017) that the probability of shutting down can be interpreted as an exogenous dividend rate.

The bank chooses $s_{h,t}$, $s_{f,t}$, $d_{h,t}$, $d^*_{h,t}$, and $d_{g,t}$, given prices and rates of returns, to maximize the expected present value of net worth paid upon closure

$$\begin{aligned} V_t(n_t) &= \max \mathbb{E}_t \sum_{j=0}^{\infty} r_{n,t+j,t+j}(1 - r_{n,t,t+j-1}) \Xi_{t,t+j+1}(n_{t+1+j}) \\ &= \max \mathbb{E}_t \Xi_{t,t+1}[r_{n,t,t} n_{t+1} + (1 - r_{n,t,t}) V_{t+1}(n_{t+1})] \\ &= \nu_{n,t} n_t, \end{aligned} \tag{3.8}$$

where the third equality follows a conjecture that the value function is linear in net worth and $(1 - r_{n,i,j})$ is the probability that the bank operates until the end of period j conditional on the bank operating at the beginning of period i. The bank's ability to raise deposits is restricted up to an incentive constraint (or financial constraint)

$$OBC_t \equiv \nu_{n,t}(n_t - e_{g,t}) - [\theta_t(\omega_t - \theta_g d_{g,t}) - \nu_{n,t} e_{g,t}] \geq 0 \tag{3.9}$$

where OBC_t stands for an occasionally binding constraint and measures the distance of this constraint from binding, $\theta_t \in [0,1]$ is exogenous stochastic, and $\theta_g \in [0,1]$ is a parameter. The intuition behind this constraint is the following. Banks are able to declare bankruptcy and exit. In this case, the banker diverts to his or her family a proportion θ_t of the divertable assets, $\omega_t - \theta_g d_{g,t}$, minus government equity. The creditors can reclaim only the un-diverted funds. Therefore, creditors are willing to lend to a bank only if the bank has no incentive to default, i.e., the value of the private equity is larger than the value of default. The fact that $\theta_g d_{g,t}$ is un-divertable and government equity are fully secured indicates that the central bank has superior power to enforce repayment.

For convenience, the decision on $s_{h,t}$ and $s_{f,t}$ can be written in terms of the total assets ω_t and the portfolio $\alpha_{p,t} = \frac{q_t^* s_{f,t}}{\omega_t}$. Denoting the multiplier associated with inequality 3.9 by $\lambda_t \geq 0$, the necessary conditions of the maximization include the slackness condition of inequality 3.9, and the first-order conditions with respect to the total assets (ω_t), the portfolio $(\alpha_{p,t})$, and the borrowing from the government $(d_{g,t})$

$$\begin{aligned} \mathbb{E}_t \Xi_{t,t+1}(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1})(r_{k,t+1} - r_t) &\equiv \nu_{\omega,t} \\ &= \frac{\lambda_t}{1+\lambda_t} \theta_t \end{aligned}, \tag{3.10}$$

$$\begin{aligned} \mathbb{E}_t \Xi_{t,t+1}(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1}) \left(r_{k,t+1} - r^*_{k,t+1} \right) &\equiv \nu_{\alpha_p,t} \\ &= 0 \end{aligned}, \tag{3.11}$$

$$\begin{aligned} \mathbb{E}_t \Xi_{t,t+1}(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1})(r_{g,t} - r_t) &\equiv \nu_{d_g,t} \\ &= \frac{\lambda_t}{1+\lambda_t} \theta_t \theta_g \end{aligned}, \tag{3.12}$$

where in equation 3.10 I use the fact that $\nu_{\alpha_p,t} = 0$ for all t thanks to market integration. Given $\nu_{n,t+1} \geq 1$, the extra term $(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1})$ multiplying the stochastic discount factor suggests that banks are generally less patient than households. The unknown time-varying coefficient in the value function can be solved using the first-order conditions and the financial constraint:

$$\nu_{n,t} = \nu_t \left(\frac{\nu_{\omega,t}}{\theta_t - \nu_{\omega,t}} + 1 \right), \tag{3.13}$$

where $\nu_t \equiv \mathbb{E}_t \Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \, \nu_{n,t+1} \right) r_t$ is defined similarly to $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$, and $\nu_{d_g,t}$. Because $\nu_{n,t}$ is independent of the bank's decision variables, equation 3.13 verifies the earlier conjecture that the value function is linear.

The properties of the bank's problem have been well discussed in Gertler and Kiyotaki (2010) . Here I only underline some key results related to the occasionally binding constraint. To begin, use equations 3.6, 3.7, and 3.8 to write the value function as

$$\nu_{n,t} n_t = \nu_{\omega,t} \omega_t - \nu_{\alpha_p,t} \omega_t \alpha_{p,t} - \nu_{d_g,t} d_{g,t} + \nu_t n_t.$$

Then, $\nu_{n,t}$, $\nu_{\omega,t}$, $\nu_{\alpha_p,t}$, $\nu_{d_g,t}$, ν_t can be conveniently interpreted as the expected marginal value of net worth, total assets, portfolio, borrowing from the government, and deposits, respectively. If the financial constraint is not binding, then $\lambda_t = 0$ and the first-order conditions imply that having one extra unit of ω_t by borrowing from households or the government does not raise the bank's value. In addition, equation 3.13 becomes $\nu_{n,t} = \nu_t \approx 1$, meaning that net worth and deposits are equally valued at the margin. If the financial constraint is binding, $\lambda_t > 0$ implies $\nu_{\omega,t} > 0$, $\nu_{d_g,t} > 0$, and $\nu_{n,t} > \nu_t$. Securities, borrowing from the government, and equity are more valuable than deposits because they also help relax the financial constraint. In addition, $\nu_{\omega,t} > 0$ indicates a credit spread between returns on securities and returns on deposits. The spread, $spread_t \equiv \mathbb{E}_t \left(r_{k,t+1} - r_t \right)$, is a convenient measure of financial friction. Note that, unless the model is solved with certainty equivalence, the spread is positive even when the financial constraint is not binding. Positive spread suggests insufficient investment due to imperfect financial intermediation. So the economy subject to the financial friction is too small relative to the one subject to no such friction, i.e., the social optimum.

3.2.4 Government and unconventional policies

Following the standard approach in the public finance literature, the specific agency that implements the unconventional policies is abstracted in the model. The consolidated government budget is given by

$$G_t + \Gamma_{i,t} + r_t B_{t-1} + AP_{i,t} = T_t + \tau_{w,t} w_t L_t + \tau_{y,t} Y_t + B_t + \mathcal{L}_{i,t},$$

where G_t and T_t are lump-sum government spending and tax, respectively, B_t are government bonds, $\Gamma_{i,t}$, $AP_{i,t}$, and $\mathcal{L}_{i,t}$ are resource costs, aggregate spending, and the gross profit of unconventional policy. The government can provide liquidity to the economy in three ways, namely, public asset purchases, lending to banks, and equity injections, indexed by $i = 1, 2, 3$. The aggregate spending is therefore the total asset purchased, the total discount window lending, and the value of government equity, respectively. All policies can be financed by government bonds (or reserves, the liabilities of the central bank), a lump-sum tax, a labour income tax, or a sales tax. As argued in Del Negro and Sims (2015), in order to avoid central bank insolvency, it would be appropriate for a central bank conducting unconventional policies to receive fiscal backing from the fiscal authority. I now proceed to describe each policy.

Using public asset purchases, the government lends directly to domestic goods producers.[10] I assume that the government does not purchase foreign securities for political reasons or due to a very high cost of evaluation and monitoring.[11] The government acts like a financial intermediary but faces no constraint in addition to its budget. By reducing the amount of securities available on the market, the policy pushes up asset prices and gives banks a capital gain. The policy also relaxes the constraint 3.9. As alternative policies, the government can provide liquidities to banks. Replacing one unit of deposits by

[10] Quint and Rabanal (2017) show how the model can be modified slightly such that the asset purchases are applied to long-term government bonds. These modifications should not change the main implications of this chapter.

[11] There is a natural upper bound that the government cannot buy more assets than those available on the market. We are unlikely to hit this bound with reasonable calibration.

one unit of government lending[12] relaxes the financial constraint, thanks to the government's superior power of enforcement. Consequently, the banks can expand their investment to the extent allowed by the relaxed financial constraint. Government equity stabilises the financial sector in a similar way. The constraint-relaxing effect of one unit of equity, held either by households or the government, is multiplied by the marginal value of net worth $\nu_{n,t}$, which is high in a financial crisis. While private equity can only be accumulated slowly, the stock of government equity is freely adjustable.

As the government collects tax from households and provides liquidity to banks or goods producers, unconventional policy serves as a bypass for the financial friction between households and banks. Then the optimal policy problem can be interpreted as the optimal size of the government's balance sheet relative to that of banks given some policy cost and the fact that the economy exits from unconventional policy in the long-run. Clearly, these three policies may work against each other. For example, banks that receive equity injections would wish to expand their asset holding. However, they may not able to do so if the government also conducts a large scale asset purchase program. In this chapter, I consider one policy at a time.

Before proceeding, I briefly compare the policies discussed above to those in the literature. I model public asset purchases and lending to banks following Gertler and Karadi (2011), but I model equity injections differently. Gertler and Karadi (2011) assume that a unit of government equity has the same payout stream as a unit of security. The government is willing to pay a higher price than the prevailing market price of securities. They also assumed that government equity is non-divertable. Due to these assumptions, equity injections are effectively public asset purchases with a lump-sum transfer to banks, and hence, they have very similar effects on the economy. By contrast, I assume that government equity are identical to private equity in nature, which makes this policy similar to its counterpart in He and Krishnamurthy (2013). Negro et al. (2017) consider two types of assets. The private assets are illiquid and can be sold up to a certain fraction of holding in each period. Government bonds and money, on the other hand, are liquid and not subject to this constraint. Therefore, the unconventional policy in their paper is to sell liquid assets and buy illiquid assets, roughly in line with the evolution of the asset side of the Federal Reserve balance sheet during the crisis. Illiquid assets are similar to securities in this chapter. Thus, the asset swap policy in Negro et al. (2017) can be seen as a mix of discount window lending and public asset purchases in this chapter.

So far, using unconventional policy is costless to the economy. Any level of interventions between just offsetting the financial friction and fully crowding out private funds is equally optimal. In the literature, policy costs are either abstracted in the analysis (Negro et al., 2017; Quint and Rabanal, 2017) or modeled in a reduced form (Dedola et al., 2013; Foerster, 2015; Gertler and Karadi, 2011). To the best of my knowledge, there is only one paper (Kandrac, 2014) that attempts to evaluate the potential costs of the Fed's QE in the sense that the Federal Reserve, as a dominant buyer, may deteriorate the financial market functioning. To form policy trade-offs, I follow the literature in the main text and assume that the government must pay a reduced form resource cost on its holding of securities, equity, and its lending to banks:

$$\Gamma_{i,t} = \tau AP_{i,t}^2. \tag{3.14}$$

This cost represents inefficient public activism in private financial markets or the cost of strengthened financial surveillance.[13] Then the policy trade-offs are between efficient financial intermediation and

[12] The policy I refer to as discount window lending can also be interpreted as the ECB's longer-term refinancing operations.

[13] Dedola et al. (2013) also add a linear term to the cost but they only find the coefficient on the quadratic term playing an important role. In my context, the linear term implies a positive marginal cost regardless of the level of intervention so the Ramsey policy is nonzero even when the financial constraints are not binding. So I choose to make the cost pure quadratic.

GDP losses. To facilitate a comparison across policies, I assume the same τ for each policy. However, we should keep in mind that the intervention costs are arguably smaller for high-grade instruments like commercial papers, agency debt and mortgage backed securities (Gertler and Kiyotaki, 2010). Sensitivity of the optimal policy to this cost will be discussed in section 3.5.

In appendix C.3, I consider the policy cost as a distorting effect of the tax that is necessary to finance at least a proportion of the policy spending. The trade-off is between efficient financial intermediation and inefficient labour market (if the tax is a labour income tax). I conclude that the distortionary tax is too expensive to finance the unconventional policy. A comprehensive investigation of how unconventional policy is financed and the associated costs is left for future work.

3.2.5 Aggregation and the market clearing conditions

The law of motion for the aggregate equity held by households is given by

$$\begin{aligned} E_{h,t} &= \frac{E_{h,t-1}}{E_{h,t-1}+E_{g,t-1}}\left(1-r_{n,t,t}\right)\left(q_{t-1}S_{h,t-1}r_{k,t}+q^*_{t-1}S_{f,t-1}r^*_{k,t}-D_{h,t-1}r_{t-1}-D^*_{h,t-1}r^*_{t-1}-D_{g,t-1}r_{g,t-1}\right) \\ &+ \varpi\left(\omega_{t-1}+AP_{1,t-1}\right), \end{aligned}$$

where the last term is the start-up funds received by new banks. Finally, the model is closed by market clearing conditions on the goods and security markets

$$Y_t+Y^*_t=C_t+C^*_t+G_t+G^*_t+\Gamma_t+\Gamma^*_t+f\left(K_{t-1},I_t\right)+f\left(K^*_{t-1},I^*_t\right),$$

$$q_tS_t=q_t\left(S_{h,t}+S^*_{h,t}\right)+AP_{1,t},$$

$$q^*_tS^*_t=q^*_t\left(S_{f,t}+S^*_{f,t}\right)+AP^*_{1,t}.$$

3.3 The numerical method

3.3.1 Simulation method

3.3.1.1 Dealing with the occasionally binding constraints (OBCs)

Stochastic models with OBCs are typically simulated using global methods. However, the model I describe above and the corresponding model to solve the Ramsey policy contain too many state variables to be solved even by methods that are explicitly designed to deal with large state spaces, such as that of Maliar and Maliar (2015). Guerrieri and Iacoviello (2015) provide a fast algorithm based on piecewise linearization which, however, gives certainty equivalent results. I employ the approach proposed by Holden (2016a,b). This approach supports second-order approximation to evaluate welfare and captures the risk of the constraint binding in the future. DynareOBC[14] created by the same author is a toolkit to implement this approach, which roughly consists of the following steps. First, the model is Taylor approximated up to a chosen order around the deterministic steady state. All inequalities are ignored during the approximation, but enter the approximated model. Then, the approximated model with OBCs can be solved under perfect foresight using Holden (2016b)'s algorithm. We can simulate a stochastic version of the model using the idea of the extended path (EP) algorithm of Fair and Taylor (1983). For a model that is linear apart from the OBCs (due to first-order approximation), the simulation is certainty equivalent. For a model that is non-linear apart from the OBCs (due to higher-order approximation), the simulation captures the risk stemming from non-OBC nonlinearity so that the slopes of variables' responses change at the bound. To further capture the risk of hitting the bound in the future, Holden

[14]DOI: http://dx.doi.org/10.5281/zenodo.50132.

Table 3.1: Parameterization

Description	Parameter	Value
Steady-state discount factor	$\bar{\beta}$	0.99
Elasticity of discount factor	ψ_β	-0.001
Habit	h	0.815
Risk aversion	σ	1.5
Weight on disutility of labour	χ	3.4
Inverse elasticity of labour supply	φ	0.276
Capital share	α	0.33
Inverse elasticity of investment to the capital price	η	1.728
Depreciation rate	δ	0.025
Steady-state survival probability of banks	$1 - \bar{r}_n$	0.972
Transfer rate from households to new banks	ϖ	0.0045
Steady-state fraction of divertable assets	$\bar{\theta}$	0.2457
Fraction of un-divertable discount window borrowing	θ_g	1
Reduced form policy costs	τ	0.0001
Persistence of financial shock	ρ_θ	0.8
Standard deviation of financial shock	σ_θ	0.1
Persistence of capital quality shock	ρ_ξ	0.66
Standard deviation of capital quality shock	σ_ξ	0.05
Persistence of productivity shock	ρ_A	0.95
Standard deviation of productivity shock	σ_A	0.0044

(2016a) applies a modified version of the stochastic extended path (SEP) algorithm of Adjemian and Juillard (2013). To form expectations, the SEP algorithm involves integrating the model over a certain number of periods of future uncertainty. I integrate over 50 periods and find no considerable change from integrating over longer periods. I refer to the solutions based on the EP and the SEP algorithm as EP alike and SEP alike solutions, respectively. I will compare these two solutions to show the precautionary motives to avoid the bound.

3.3.1.2 Dealing with the indeterminate portfolio

An issue related to the perturbation based method is indeterminacy of the equilibrium portfolio $\alpha_{p,t}$. According to Devereux and Sutherland (2011), a second (third, fourth, ...) order approximation of the model is generally enough to pin down up to zero (first, second, ...) order term(s) of the portfolio, while up to the first (second, third, ...) order terms of the portfolio are relevant for the second (third, fourth, ...) order approximated model. The zero-order term is the deterministic steady state. Devereux and Sutherland (2011) propose a general solution as follows. Conjecturing $\alpha_{p,t}$ as a (N-1)th order polynomial of the model's state variables, we can use this conjecture to replace equation 3.11. Then, we can simulate the Nth order approximated model and search for parameters in the conjecture such that the (N+1)th order approximation of equation 3.11 is satisfied.

As in Dedola et al. (2013), I only solve the zero-order portfolio. This is sufficient when I focus on dynamics and calculate a first-order approximation to the model. In the evaluation of welfare, the first-order terms of the portfolio are neglected as it is very demanding to compute portfolio dynamics in a model with OBCs.

3.3.2 Parameterization

Table 3.1 shows the parameterization of the economy based on the second-order approximated model where no policy is employed. The behaviour of such a economy is reported in appendix C.2. The quantitative results are similar to that of Dedola et al. (2013). However, I show in the next section how the OBC setting matters for the optimal policy.

Parameters concerning the non-financial sector are standard in the literature and are borrowed from Dedola et al. (2013). I depart from the literature by choosing a steady state in which the financial constraints are slack.[15] The constraints may bind endogenously due to adverse shocks. I define a financial crisis as the occasion when the financial constraints are tight such that the credit spread is two standard deviations above its mean. Since 1983 in the U.S., this definition corresponds to the early 21st century recession and the 2007 - 2008 financial crisis. The unconditional probability of a financial crisis is 5.28% under my calibration.

There are three parameters in the financial sector. Following Gertler and Kiyotaki (2010), I choose a survival rate implying that, on average, bankers survive for around 8 years. Next, I set the steady-state leverage ratio to 4. Given that the financial constraint is slack in the steady state, this pins down $\varpi = \left(1 - \frac{1-\bar{r}_n}{\beta}\right)/\bar{\phi}$. The steady-state proportion of divertable assets $\bar{\theta}$ is chosen such that the financial constraint is close to being binding in the steady state.

There are three exogenous variables in each country, namely productivity, A_t, capital quality, ξ_t, and the fraction of divertable assets, θ_t. Each of them follows an uncorrelated AR(1) process. Parameters for the productivity are taken from the estimate of Heathcote and Perri (2002). Parameters for the capital quality follow Gertler et al. (2012), the working paper version of which provides the microfoundations in its appendix. I choose a standard deviation of θ_t to make the mean of the annualized spread about 2.35%.[16] However, without features such as liquidity premia and true default risk,[17] I inevitably overestimate the standard deviation of the spread. Or I would underestimate the spread mean if I calibrated the model to match the standard deviation.

As a robustness check, I also consider other relevant calibrations. For example, in an alternative parameterization, $\bar{r}_n$ is set to match a dividend rate of 5.15% made by the largest 20 U.S. banks during 1965–2013.[18] Quint and Rabanal (2017) use GMM to estimate a similar model with nominal frictions, a Taylor rule, and an always binding financial constraint. They find a much larger steady-state leverage of 16. This is probably not very surprising as Gertler and Kiyotaki (2010) consider the leverage of 4 as an average across sectors with vastly different financial structures. All alternative calibrations change my results quantitatively but do not change the main conclusions.

There are two policy-related parameters. For the proportion of un-divertable discount window lending, I consider two values. When $\theta_g = 1$, discount window lending cannot be diverted. This choice makes the three policies more comparable because the funds of the two other policies are effectively also non-divertable. However, as noted by Gertler and Kiyotaki (2010), there is likely to be a capacity constraint on the central bank's ability to retrieve funds. Interest rate data of the ECB's longer-term refinancing operations is higher than the rate banks can borrow, which also suggests $\theta_g < 1$. Thus, I assume $\theta_g = 0.8$. The second policy parameter is the intervention cost, τ. Since it is difficult to measure the inefficiency of

[15] With integrated security markets and a steady state in which the financial constraints are not binding, there is indeterminacy between the bank's total assets, ω_t and ω_t^*. To pin down these variables, it is sufficient to introduce an asset adjustment cost to the bank's balance sheet:

$$\frac{\psi_\omega}{2}\left(\omega_t + AP_{1,t} - SteadyState\right)^2,$$

where $\psi_\omega = 10^{-5}$ in practice.

[16] This figure is calculated from quarterly data of Moody's seasoned Bbb corporate bond yield relative to the yield on 10-year treasury constant maturity, 1983q1-2017q1, retrieved from FRED, Federal Reserve Bank of St. Louis. By contrast, Gertler and Kiyotaki (2010) and Dedola et al. (2013) use 1%, which is roughly the mean of Moody's seasoned Aaa corporate bond yield relative to the yield on 10-year treasury constant maturity.

[17] Recently Gertler et al. (2017) are working on a similar model where banks can default on their debts possibly due to a bank run.

[18] This number is calculated by Swarbrick et al. (2017) using the Baron (2017) dataset.

public activism in private financial markets, I set it to a number small enough to allow me to focus on the benefits of interventions. I consider larger intervention costs in section 3.5.

3.4 The Ramsey cooperative policy

In this section, I focus on a benchmark case in which the Home and Foreign governments conduct cooperative policy, and the policy is financed by government bonds (or equivalently a lump-sum tax) and only entails small reduced form cost.

The government in each country jointly maximizes a single objective function - the life-time utilities averaged across countries - by committing to a state-contingent plan of one of the three policies discussed previously. Policy makers solve the following problem:

$$\min WEL_g \equiv$$
$$-\mathbb{E}_t \sum_{j=t}^{\infty} \beta^g_{t,t+j} \frac{1}{2} \left[\frac{(c_{t+j} - hc_{t+j-1})^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} + \frac{\left(c^*_{t+j} - hc^*_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{*1+\varphi}}{1+\varphi} \right]$$

subject to all the equilibrium conditions of private agents,[19] where $\beta^g_{t,t+1}$ is identical to the households' discount factor. In solving for the optimal policy, I follow the “timeless” perspective advocated by Woodford (2003). However, the resulting system is very difficult to simulate when I employ the SEP alike solution. Thanks to the small policy cost, I can approximate the true Ramsey problem arbitrarily well by a slightly simplified problem. Specifically, I make the following assumption

ASSUMPTION 1: The governments conduct policy subject to $\lambda_t = \lambda_t^* = 0$, instead of $\lambda_t \geq 0$ and $\lambda_t^* \geq 0$.

According to equation 3.10, the positive multiplier implies a positive spread. In the true Ramsey problem, the governments can tolerate positive spreads (roughly at 10^{-6} given my main calibration) to the extent that the marginal benefits of reducing the spreads equal the marginal costs of interventions. Hence, we have $\lim_{\tau \to 0^+} \lambda_t = \lim_{\tau \to 0^+} \lambda_t^* = 0$ in the Ramsey equilibrium, which justifies assumption 1 when τ is small. The qualitative results in this section remain unchanged if I do not make assumption 1.

3.4.1 Impulse response analysis

Following the literature, I consider policy responses to two shocks. First, I consider a negative capital quality shock ξ_t in Home. The impacts of this shock can be decomposed into two stages. In the first stage, this shock has real impacts similar to those of a productivity shock. Specifically, the return on securities is low, which reduces the banks' net worth by a multiplier of their leverage. Consequently, the financial constraints may be binding and the second stage “financial accelerator” effects take place. In this case, banks must firesell their assets. Since banks take asset prices as given, it is their externality that the firesale depresses asset prices and further impairs their net worth. As a result, the financial constraint binds even tighter. In the second stage, banks are inefficient financial intermediaries. The second shock θ_t , referred to as a financial shock, tightens the financial constraints directly (Dedola and Lombardo, 2012; Negro et al., 2017; Perri and Quadrini, 2011), and hence, only has the second-stage effects. How the economy responds to these shocks under no policy intervention can be found in appendix C.2.

Figures 3.1 and 3.2 plot the one standard deviation impulse responses for three variables, namely banks' assets financed by private funds, private equity, and policy spending as a fraction of domestic asset value. Other variables are not shown because they behave as if there were no financial constraint,

[19] These conditions include two inequalities, $\lambda_t \geq 0$, $OBC_t \geq 0$, a slackness condition $\lambda_t OBC_t = 0$, and their Foreign counterparts. It can be verified that $\lambda_t \geq 0$ is a redundant constraint. Intuitively, $\lambda_t \geq 0$ roughly implies $r_{k,t+1} - r_t \geq 0$ according to equation 3.10, which a benevolent policy maker would never violate.

Figure 3.1: Cooperative policies under Home capital quality shock

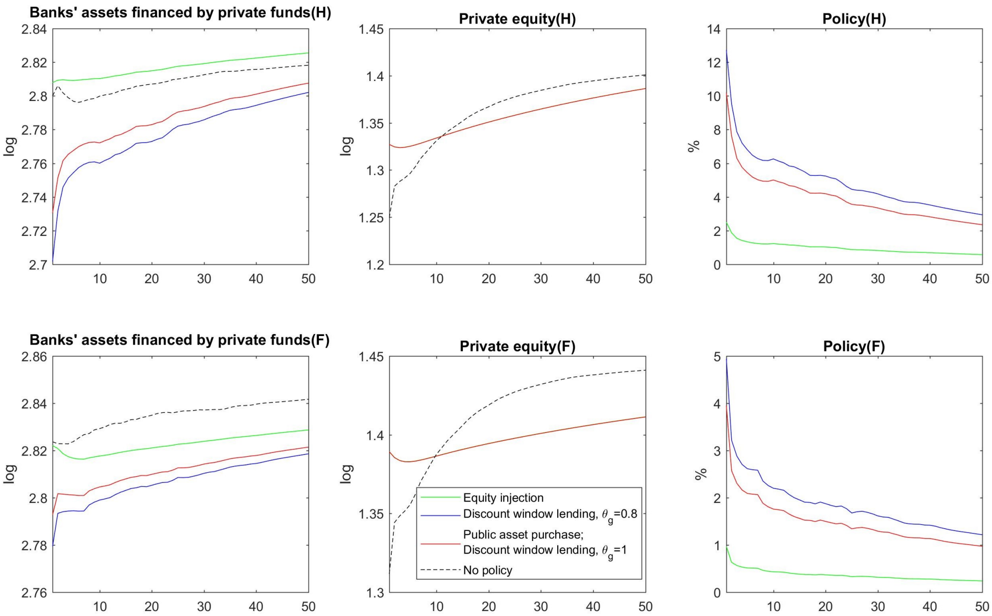
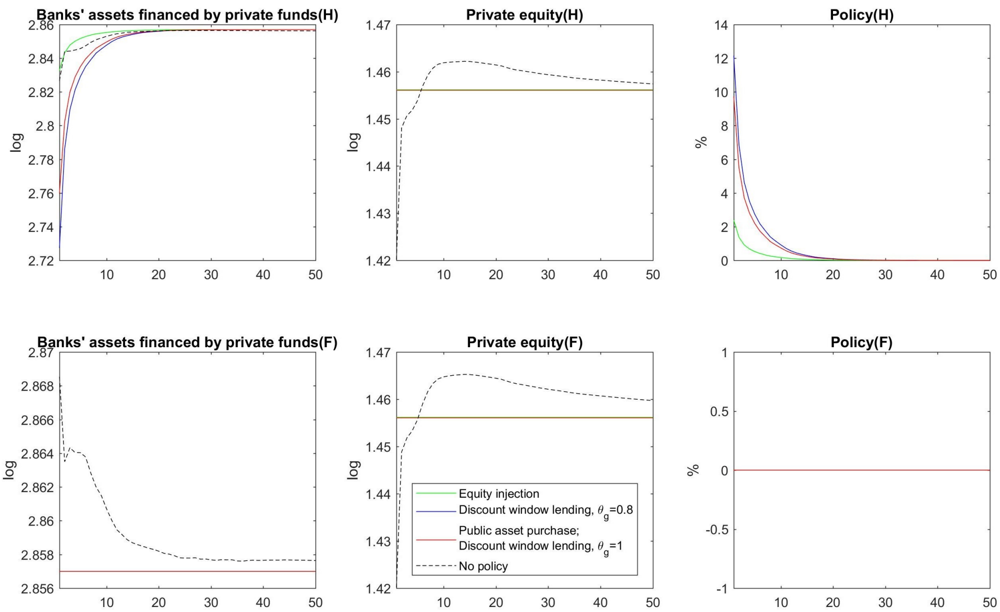


Figure 3.2: Cooperative policies under Home financial shock



i.e., like the black broken lines in Figures C.1 and C.2 in the appendix. The results obtained with public asset purchases, discount window lending, equity injections, and no policy are shown in red, blue, green, and black, respectively.

By design, all unconventional polices address only the second stage effect of a shock. They share two common roles. First, policy provides public funds to the economy when banks are constrained to borrow from households and hence are constrained to lend. The public funds push up asset prices, give banks a capital gain and improve their balance sheet. This effect undo the bank's externality. Therefore, upon the impact of a shock, the losses on net worth is smaller under policy interventions than the case in which there is no intervention, as shown by the middle column of the figures. All policies imply the same path of private equity because public equity, discount window lending, and securities (purchased by the government as an opportunity cost for banks) are equally valued when the financial constraints do not or just bind.

On the other hand, however, the left column suggests that public funds crowd out private funds. This crowding-out effect results in slow growth of private equity. To stabilize the financial sector, the government must also exit slowly from the policy until banks accumulate enough private equity. The exit path needs to be consistent with the path of deleveraging. Evidently, the crowding-out effect is much smaller under equity injections than under the other two policies. This difference must be explained by the fact that, given equity injections, banks can raise more deposits thanks to more relaxed financial constraints. The second role of policy is to relax the financial constraint 3.9, which reduces the crowding-out effect. The smaller crowding-out effect allows banks to depend less on policy so less interventions are required.

To summarise, Equity injections clearly constitute the most efficient policy. Discount window lending is at most as efficient as public asset purchases in the extreme case of $\theta_g = 1$. The fact that it is hard to tell in prior which policy has smaller crowding-out effect makes the analysis in this section nontrivial even under assumption 3.4.

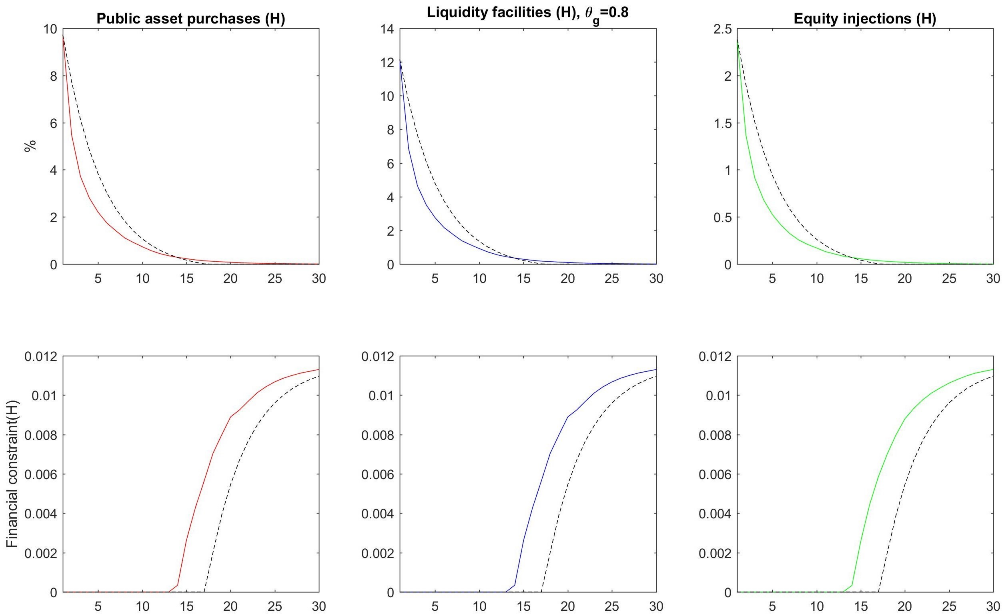
On the international dimension, the policy responses are asymmetric. Following a Home capital quality shock, Foreign interventions are roughly half as strong as Home interventions. This is because banks hold a portfolio that consists of more domestic assets ($\alpha^p = 0.4$). Since Home banks are more affected by the shock, they benefit more from the purchases of Home assets. However, discount window lending and equity injections are not immune to this portfolio effect and these two policies affect the whole portfolio by design. Following a Home financial shock, however, Foreign need not intervene at all.[20] A financial shock does not have the first-stage real impacts. Foreign banks would only be affected by depressed asset prices worldwide if there were no policy response. Home interventions fully stabilize asset prices in both countries so the Foreign country can enjoy a free ride.

3.4.2 Precautionary effects and the exit from policy

I have argued that the exit from policy must in line with the speed of deleveraging. To be more specific, the exit must be slower due to a precautionary effect arising from the future risk of binding financial constraints. In figure 3.3, I compare the EP alike solution (black broken lines) and the SEP alike solution (coloured solid lines) of the impulse responses to a Home financial shock. The SEP alike solution is also employed in the previous subsection. If no risk of future bonding constraints is taken into account, unconventional policy ends in the same period when the economy escapes from the financial constraint. By contrast, if this risk is predicted by both private agents and the government, the policy is precautionary in the sense that it is relatively stable and persistent even when the economy has escaped from the constraint sooner than in the former case. Intuitively, the policy should give some precautionary protection to the

[20] Without assumption 1, foreign interventions approach zero as the policy cost approaches zero

Figure 3.3: Cooperative policies with and without precautionary effects



Note: Coloured solid lines are SEP alike results and black broken lines are EP alike results.

economy for a few periods after a crisis, during which the banks, though having enough net worth to escape from the financial constraints, are vulnerable to another adverse shock.

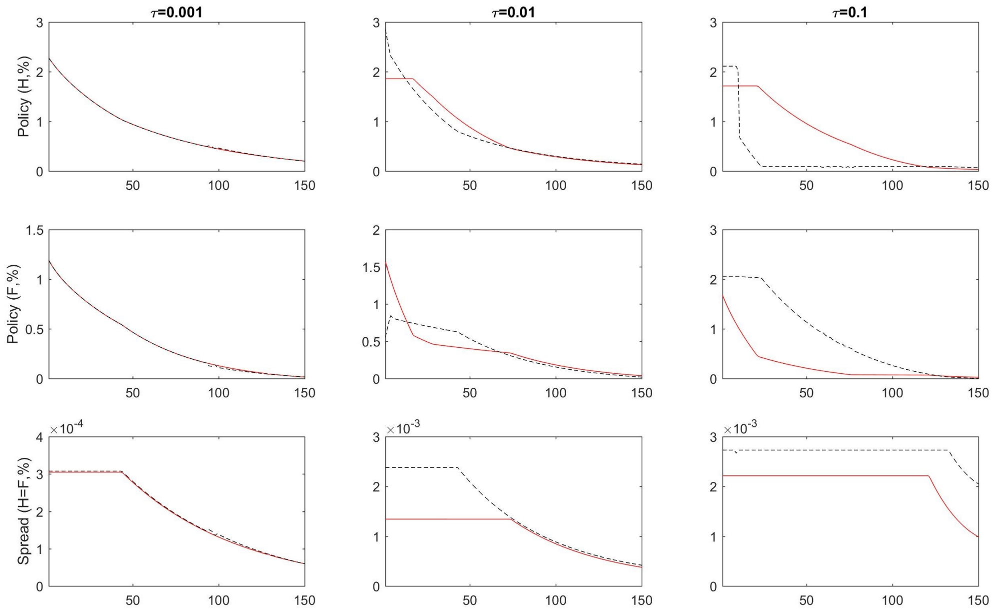
In models where there are true default risks and bank runs, such as those of Coimbra and Rey (2017) and Gertler et al. (2017), the precautionary effects are arguably stronger and hence the exit from policy is slower.

3.5 The Ramsey noncooperative policy

Without international cooperation, everything else in the cooperative policy problem applies, but each government now maximizes domestic welfare using domestic instruments, taking the entire path of foreign instruments as given. The equilibrium is an outcome of an open-loop dynamic Nash game. Following Coenen et al. (2007), taking the entire path of foreign instruments as given is an unrealistic assumption but a necessary simplification to the problem. Since policy cost plays an important role in noncooperative policy, it is important not to make assumption 1, which only provides reasonable approximation to the true Ramsey problem when the cost approaching zero. However, without assumption 1, I can only simulate the model reasonably fast with the EP alike solution. I confine my discussion to the most efficient policy, equity injections. Other policies generate similar results.

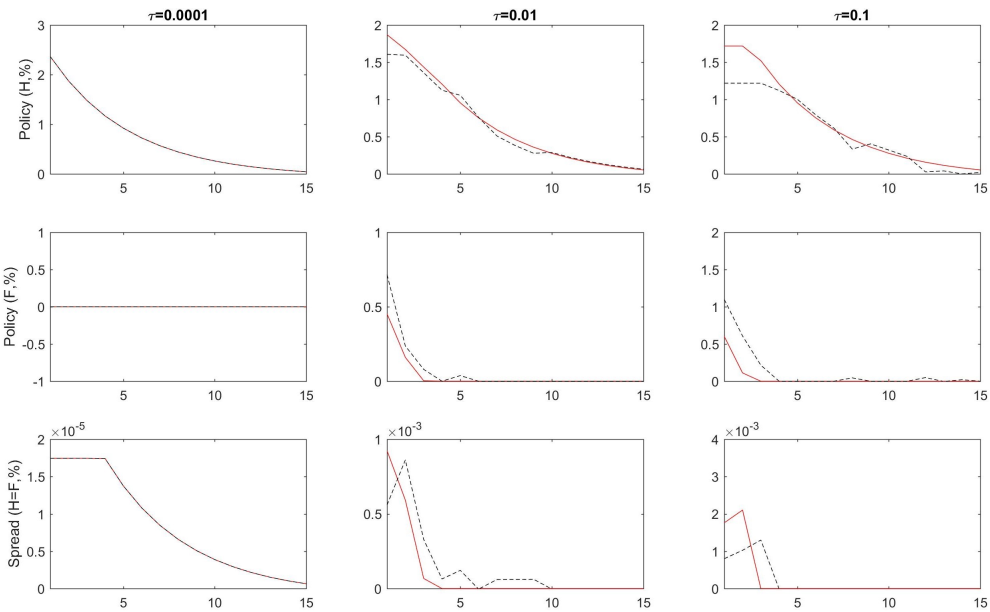
As shown in figures 3.4 and 3.5, the noncooperative equilibrium is identical to the cooperative equilibrium when the unconventional policy is very cheap to use ($\tau = 0.0001$). Increasing τ makes it favourable to share the intervention cost across countries. Given a Home shock, this means fewer interventions by the Home government and more by the Foreign government, which results in higher credit spreads globally. However, after a certain point, further increasing τ affects the noncooperative policy more than the cooperative policy, and the cooperation gain becomes a positive number. The tipping point is about $\tau = 0.01$ for the capital quality shock and about $\tau = 0.001$ for the financial shock. Under a Home shock, the noncooperative equilibrium features excessive interventions in Foreign and insufficient interventions in Home. The consequences of noncooperation for credit spreads also

Figure 3.4: Noncooperative policy responses to a Home capital quality shock



Note: The cooperative equilibrium is shown in red solid lines, and the noncooperative equilibrium is shown in black broken lines.

Figure 3.5: Noncooperative policy responses to a Home financial shock



Note: The cooperative equilibrium is shown in red solid lines, and the noncooperative equilibrium is shown in black broken lines.

depend on the shock. Under a capital quality shock, the spreads are higher and more persistent in the noncooperative equilibrium than in the cooperative equilibrium. Under a financial shock, however, the noncooperative policy achieves smaller spreads in the short run. My results are similar to the results of the optimal noncooperative simple rule studied in Dedola et al. (2013) but different in the following ways. In their paper, increasing τ always reduces the policy responses in both countries. After a certain point, policy responses in the noncooperative equilibrium quickly approaching zero in both countries. Further increasing the cost also makes even policy responses in cooperative equilibrium equal to zero. By contrast, in my context, policy responses are re-balanced across countries to tackle the increase in the cost.

3.6 Conclusions

I study the Ramsey optimal unconventional monetary policy in a two-country version of Gertler and Kiyotaki (2010), with and without cross-country policy cooperation. The main findings are threefold. First, I suggest that unconventional policy should be designed to address the financial constraint that banks face. Second, after giving a strong initial response, the central bank should exit slowly from the policy even after a financial crisis has passed. Third, if cross-country policy cooperation is not imposed, the interventions are too strong in one country and too weak in the other. The cooperation gain is zero if the intervention cost is small. Increasing the intervention cost to a certain point lets the cooperation gain positive.

Naturally, this chapter is subject to several limitations. On one hand, this chapter focuses on the capital gain channel to keep the problems at hand relatively simple. Section 3.1 discuss many other possible channels. These channels may open for some but not all unconventional policies. For instance, forward guidance works mainly through the signalling channel. One important channel missing in this chapter is the bank's lending channel. In my model, there is no friction between banks and non-financial firms. Reducing credit spread automatically boosts lending and investment. This is not necessarily true in data according to Chakraborty et al. (2017) and Acharya et al. (2017). Thus we need better capture banks' behaviours and their heterogeneity. Nonetheless, I expect that the main conclusions of this chapter can be generalized to a more sophisticated model. On the other hand, following the literature, I assume a reduced form cost for the unconventional policy. The cost of the unconventional policy, while critical to the optimal policy analysis, is still little understood. A recent paper by Kandrac (2014) summarises several potential costs that have been discussed in the Federal Reserve. These costs introduce new policy trade-offs and may make new implications for optimal policy.

Appendix C

C.1 Equilibrium conditions

Competitive equilibrium conditions are given as follows. The cooperative and non-cooperative optimal policy equilibrium conditions are generated automatically by a Matlab routine using competitive equilibrium conditions as constraints.

External discount factors:

$$\beta_{t,t+1} = \bar{\beta} + \psi_\beta \log\left(\frac{C_t}{Y_t}\right),$$

$$\beta^*_{t,t+1} = \bar{\beta} + \psi_\beta \log\left(\frac{C^*_t}{Y^*_t}\right).$$

Euler equations:

$$\mathbb{E}_t\left[\Xi_{t,t+1} r_t\right] = 1,$$

$$\mathbb{E}_t\left[\Xi_{t,t+1} r^*_t\right] = 1,$$

$$\mathbb{E}_t\left[\Xi^*_{t,t+1} r^*_t\right] = 1.$$

Households' budget constraint:

$$C_t + D_{h,t} + B_t + D_{f,t} = (1-\tau_{w,t})\, w_t L_t + (D_{h,t-1} + B_{t-1})\, r_{t-1} + D_{f,t-1} r^*_{t-1} + \Pi_t - T_t.$$

Labour supply:

$$w_t\,(1-\tau_{w,t}) = \frac{\chi L_t^{\varphi}}{(C_t - hC_{t-1})^{-\sigma} - \beta_{t,t+1} h\,(C_{t+1} - hC_t)^{-\sigma}},$$

$$w^*_t\left(1-\tau^*_{w,t}\right) = \frac{\chi L_t^{*\varphi}}{\left(C^*_t - hC^*_{t-1}\right)^{-\sigma} - \beta^*_{t,t+1} h\left(C^*_{t+1} - hC^*_t\right)^{-\sigma}}.$$

Goods production functions:

$$Y_t = A_t\,(\xi_t K_{t-1})^{\alpha}\, L_t^{1-\alpha},$$

$$Y^*_t = A^*_t\left(\xi^*_t K^*_{t-1}\right)^{\alpha} L_t^{*1-\alpha}.$$

Labour demand equations:

$$w_t = (1-\alpha)\,\frac{Y_t\,(1-\tau_{y,t})}{L_t},$$

$$w_t^* = (1-\alpha)\frac{Y_t^*\left(1-\tau_{y,t}^*\right)}{L_t^*}.$$

Zero profit conditions of goods producers:

$$Y_t = z_t K_{t-1} + w_t L_t,$$

$$Y_t^* = z_t^* K_{t-1}^* + w_t^* L_t^*.$$

Capital accumulation:

$$K_t = I_t + (1-\delta)\,\xi_t K_{t-1},$$

$$K_t^* = I_t^* + (1-\delta)\,\xi_t^* K_{t-1}^*.$$

Long-term asset returns:

$$r_{k,t+1} = \frac{z_{t+1} + (1-\delta)\,\xi_{t+1} q_{t+1}}{q_t},$$

$$r_{k,t+1}^* = \frac{z_{t+1}^* + (1-\delta)\,\xi_{t+1}^* q_{t+1}^*}{q_t^*}.$$

Determination of asset prices:

$$q_t = 1 + \eta\left(\frac{I_t}{\delta K_{t-1}} - 1\right),$$

$$q_t^* = 1 + \eta\left(\frac{I_t^*}{\delta K_{t-1}^*} - 1\right).$$

Marginal value of banks' assets:

$$\begin{array}{rcl} \mathbb{E}_t \Xi_{t,t+1}\left(r_{n,t,t} + (1-r_{n,t,t})\,\nu_{n,t+1}\right)\left(r_{k,t+1} - r_t\right) & \equiv & \nu_{\omega,t} \\ & = & \frac{\lambda_t}{1+\lambda_t}\theta_t \end{array},$$

$$\begin{array}{rcl} \mathbb{E}_t \Xi_{t,t+1}^*\left(r_{n,t,t}^* + \left(1-r_{n,t,t}^*\right)\nu_{n,t+1}^*\right)\left(r_{k,t+1}^* - r_t^*\right) & \equiv & \nu_{\omega,t}^* \\ & = & \frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^* \end{array}.$$

Marginal value of discount window lending:

$$\begin{array}{rcl} \mathbb{E}_t \Xi_{t,t+1}\left(r_{n,t,t} + (1-r_{n,t,t})\,\nu_{n,t+1}\right)\left(r_{g,t} - r_t\right) & \equiv & \nu_{d_g,t} \\ & = & \frac{\lambda_t}{1+\lambda_t}\theta_t\theta_g \end{array},$$

$$\begin{array}{rcl} \mathbb{E}_t \Xi_{t,t+1}^*\left(r_{n,t,t}^* + \left(1-r_{n,t,t}^*\right)\nu_{n,t+1}^*\right)\left(r_{g,t}^* - r_t^*\right) & \equiv & \nu_{d_g,t}^* \\ & = & \frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^*\theta_g^* \end{array}.$$

Marginal value of deposits:

$$\nu_t \equiv \mathbb{E}_t \Xi_{t,t+1}\left(r_{n,t,t} + (1-r_{n,t,t})\,\nu_{n,t+1}\right) r_t,$$

$$\nu_t^* \equiv \mathbb{E}_t \Xi_{t,t+1}^*\left(r_{n,t,t}^* + \left(1-r_{n,t,t}^*\right)\nu_{n,t+1}^*\right) r_t^*.$$

Marginal value of net worth:

$$\nu_{n,t} = \nu_t \left(\frac{\nu_{\omega,t}}{\theta_t - \nu_{\omega,t}} + 1 \right),$$

$$\nu_{n,t}^* = \nu_t^* \left(\frac{\nu_{\omega,t}^*}{\theta_t^* - \nu_{\omega,t}^*} + 1 \right).$$

International arbitrage of long-term assets:

$$\begin{array}{rcl} \mathbb{E}_t \Xi_{t,t+1} \left(r_{n,t,t} + (1 - r_{n,t,t}) \nu_{n,t+1} \right) \left(r_{k,t+1} - r_{k,t+1}^* \right) & \equiv & \nu_{\alpha_p,t} \\ & = & 0 \end{array}.$$

Financial constraints and slackness conditions:

$$\nu_{\omega,t} = \max \left\{ \nu_{\omega,t} - \left[\nu_{n,t} N_t - \theta_t \left(\omega_t - \theta_g D_{g,t} \right) \right] \right\},$$

$$\nu_{\omega,t}^* = \max \left\{ \nu_{\omega,t}^* - \left[\nu_{n,t}^* N_t^* - \theta_t^* \left(\omega_t^* - \theta_g D_{g,t}^* \right) \right] \right\}.$$

Banks' balance sheets:

$$\omega_t = D_{h,t} + D_{h,t}^* + D_{g,t} + E_{h,t} + E_{g,t},$$

$$\omega_t^* = D_{f,t} + D_{f,t}^* + D_{g,t}^* + E_{h,t}^* + E_{g,t}^*.$$

Equity accumulation:

$$\begin{aligned} E_{h,t} &= \frac{E_{h,t-1}}{E_{h,t-1} + E_{g,t-1}} (1 - r_{n,t,t}) \left(q_{t-1} S_{h,t-1} r_{k,t} + q_{t-1}^* S_{f,t-1} r_{k,t}^* - D_{h,t-1} r_{t-1} - D_{h,t-1}^* r_{t-1}^* - D_{g,t-1} r_{g,t-1} \right) \\ &+ \varpi \left(\omega_{t-1} + AP_{1,t-1} \right), \end{aligned}$$

$$\begin{aligned} E_{h,t}^* &= \frac{E_{h,t-1}^*}{E_{h,t-1}^* + E_{g,t-1}^*} \left(1 - r_{n,t,t}^* \right) \left(q_{t-1} S_{h,t-1}^* r_{k,t} + q_{t-1}^* S_{f,t-1}^* r_{k,t}^* - D_{f,t-1} r_{t-1} - D_{f,t-1}^* r_{t-1}^* - D_{g,t-1}^* r_{g,t-1}^* \right) \\ &+ \varpi \left(\omega_{t-1}^* + AP_{1,t-1}^* \right). \end{aligned}$$

Government's budget constraints:

$$G_t + \Gamma_{i,t} + r_t B_{t-1} + AP_{i,t} = T_t + \tau_{w,t} w_t L_t + \tau_{y,t} Y_t + B_t + \mathcal{L}_{i,t},$$

$$G_t^* + \Gamma_{i,t}^* + r_t^* B_{t-1}^* + AP_{i,t}^* = T_t^* + \tau_{w,t}^* w_t^* L_t^* + \tau_{y,t}^* Y_t^* + B_t^* + \mathcal{L}_{i,t}^*.$$

All policy instruments are set to zero in competitive equilibrium.

Market clearing conditions:

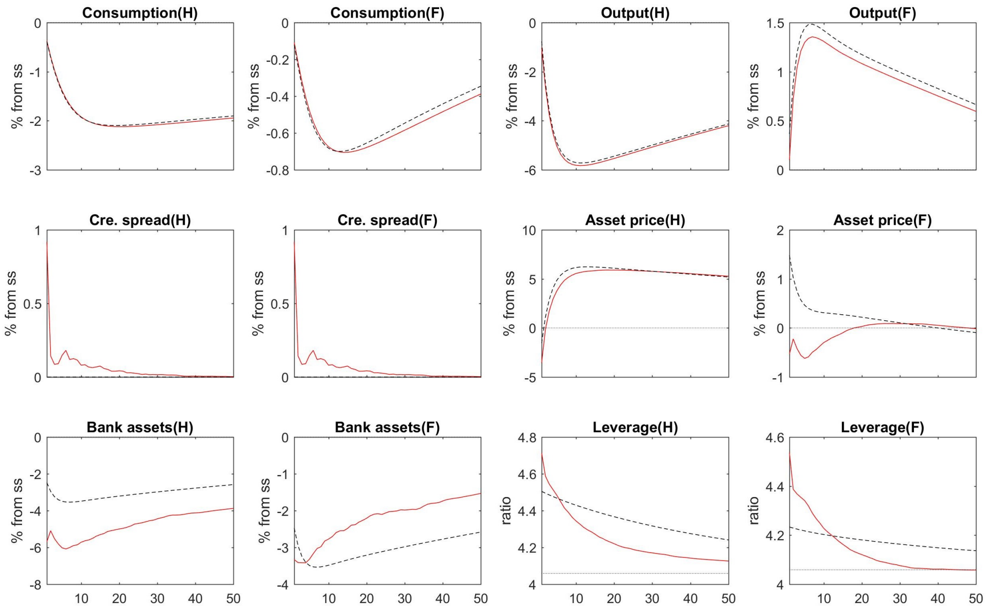
$$Y_t + Y_t^* = C_t + C_t^* + G_t + G_t^* + \Gamma_t + \Gamma_t^* + f\left(K_{t-1}, I_t \right) + f\left(K_{t-1}^*, I_t^* \right),$$

$$q_t S_t = q_t \left(S_{h,t} + S_{h,t}^* \right) + AP_{1,t},$$

$$q_t^* S_t^* = q_t^* \left(S_{f,t} + S_{f,t}^* \right) + AP_{1,t}^*.$$

Given that financial constraints do not bind and no policy in the steady state, and symmetry of the two countries, the static version of this model reduce to the model of Backus et al. (1992). To be specific,

Figure C.1: Response to Home capital quality shock



Note: Black broken lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and the dotted line represents the steady state.

steady-state capital, labour, and consumption are solved analytically using the following equations:

$$K^{\alpha}L^{(1-\alpha)} = \delta K + C,$$

$$\frac{\chi L^{\varphi}}{\left(\left(1-h\right)C\right)^{-\sigma}\left(1-\beta h\right)} = \left(1-\alpha\right)\left(\frac{K}{L}\right)^{\alpha},$$

$$\frac{1}{\beta} = \alpha\left(\frac{K}{L}\right)^{\alpha-1} + 1 - \delta.$$

The rest steady-state variables can be solved straightforwardly.

C.2 Competitive Equilibrium

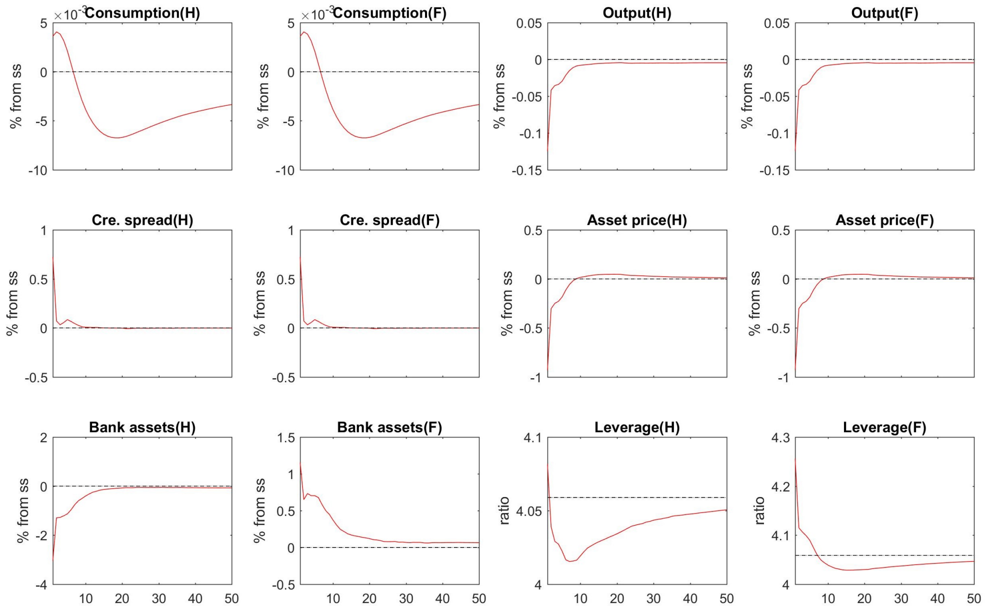
Here, I examine the quantitative behaviours of the model without policy interventions, and the size of precautionary effects that originate from the OBCs.

C.2.1 Impulse response analysis

I consider responses to two shocks, namely a standard deviation negative Home capital quality shock ξ_t and a standard deviation positive Home financial shock θ_t. I use black broken lines to represent variables that would be realized in a financially frictionless world (potential variables) and red solid lines to represent actual variables.

As shown in figure C.1, the capital quality shock creates a deep and persistent global recession. Thanks to the OBC setting, I can decompose the effects of this shock into two "stages". In the first stage, the shock has a real impact shown by the black broken lines. When the financial constraints are binding, there are second-stage "financial accelerator" effects. In this case, banks must firesale their assets, which

Figure C.2: Response to Home financial shock



Note: Black broken lines represent variables that would be realized in a financially frictionless world (potential variables), red solid lines represent actual variables, and the dotted line represents the steady state.

depresses asset prices and further impairs their net worth. As a result, the financial constraints bind even tighter. The positive spreads suggest that banks are inefficient financial intermediaries. Overall, the second-stage effects amplify the first-stage effects. The shock propagates to Foreign via the equalization of asset returns across countries, as suggested by equation 3.11, and a diversified portfolio. With a portfolio featuring home bias ($\alpha_p = 0.4$), Foreign banks suffer a smaller loss on their net worth than Home banks. Figure C.2 shows that the financial shock has only the second-stage impacts on the economy. The shock tightens the domestic financial constraint so domestic banks firesale their assets. Foreign banks would like to pick up those assets when the prices are low. However, Foreign banks have limited ability to do so due to their own financial constraint. Overall, the global investment drops and affects consumption and output in both countries symmetrically.

No matter whether the financial constraints bind forever or occasionally, the second-stage effects are much smaller than the first-stage effects. The literature notes at least two reasons why this is the case. Jakab and Kumhof (2015) suggest that banks in the real world provide financing through money creation but banks in most models accept pre-existing real resources from savers and then lend them to borrowers. They find that adding money creation to the model allows the same shocks to have much greater effects on the non-financial sector. Negro et al. (2017) highlight a role of the nominal rigidity and the zero lower bound without which the financial friction accounts for a drop in investment, but not in output, thanks to a rise in consumption.

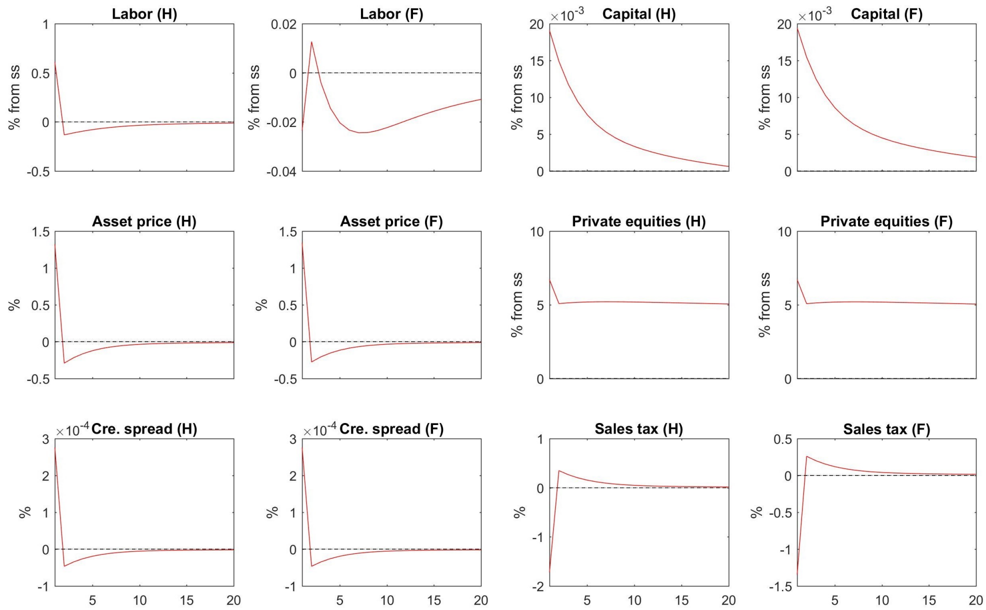
C.2.2 The precautionary effects of risk

If the financial constraints are always binding, banks always hold the maximum level of assets permitted by their net worth. However, in the OBCs setting, the amount of assets held by banks is also affected by the likelihood that financial constraints are binding in the future. This is known as the precautionary effects

Table C.1: Unconditional Mean (StD) of Home variables

Variables	$p\,(Crisis)$	Bank's assets	Annualized Spread	Consumption
EP alike	5.48%	4.72	2.89%	0.316
		(0.90)	(5.34)	(0.22367)
SEP alike	5.28%	4.47	2.33%	0.313
		(0.76)	(4.62)	(0.22375)

Figure C.3: Cooperative fiscal policy under a Home financial shock



of risk. To visualize the effects, I simulate the model with and without integrating over future uncertainty. The difference between the SEP alike solution and the EP alike solution shows the precautionary effects.

Table C.1 compares the sample mean and the standard deviation of a few variables computed from the two simulations. First, the precautionary effects reduce the probability of a financial crisis (defined in the main text) by 0.2%. To avoid being constrained, banks would like to hold fewer assets on average. If banks do not do this, they suffer from a larger volatility of asset holding. The precautionary effects also reduce both the mean and the standard deviation of the spread. However, the precautionary effects are small on non-financial variables. In a smaller open economy model with an occasionally binding collateral constraint, Mendoza (2010) also finds that long-run business cycle moments are largely unaffected by precautionary savings. Gertler et al. (2017) show that the nonlinearity induced by the OBCs are quantitatively small relative to the nonlinearity induced by the bank run mechanism they add to their early model.

C.3 Fiscal distortion

While policy cost plays an important role in shaping the optimal unconventional policy, there is no hard evidence to quantify it in the reduced form. Another possible form of policy cost is a distorting effect of a tax by which the unconventional policy must be financed. To simulate the model in this case, again, I do not make assumption 3.4.

Suppose that the most efficient policy, i.e. equity injections, is solely financed by a sales tax, figure C.3 plots the cooperative responses to a Home financial shock. The unconventional instruments are not shown because they are only used passively. The true instrument here is the sales subsidy, which is financed by a negative unconventional policy. This fiscal stimulation boosts investment and hence asset prices. Banks earn a fortune from their investment and escape from the financial constraints. In contrast to the unconventional policy, the fiscal policy addresses a financial crisis from the demand side of capital. However, the credit spreads are very large, suggesting inefficient financial markets. Since these results are obtained from maximizing welfare, varying the unconventional instrument actively must worsen welfare because the necessary sales tax to finance the active unconventional policy would have a strong distortionary effect on the economy. In other words, the distorting tax is an expensive source of fund to finance the unconventional policy. This result is robust to the nature of the shock, the chosen unconventional instrument, the chosen fiscal instrument (such as a labour tax), and an alternative reasonable parameterization of the model such as a smaller Frisch elasticity of labour supply.[1] Nonetheless, this result is not very surprising because the benefits of the unconventional policy are bounded from above by the welfare losses caused by the second-stage effect of a shock. In this and similar models, such losses are relatively small (see, Dedola et al. 2013, and appendix C.2) so the benefits of the unconventional policy are dominated by the distorting effect of the tax. If a smaller proportion of unconventional policy is financed by the tax, the active unconventional policy becomes cheaper but the active sales subsidy becomes more expensive. The threshold below which the unconventional policy is active is about 15% of the unconventional policy financed by the sales tax.

[1] Brendon et al. (2011) study a similar question where the tax follows a simple rule and the government budget is balanced by government debts. It is not surprising that the distorting tax plays a minor role in their paper.

Chapter 4

Optimal Unconventional Monetary Policy rules

4.1 Introduction

As is well known, Ramsey policy is silent regarding implementation. In extreme cases, for instance the capital control policy analysed in Schmitt-Grohé and Uribe (2016), there could be no policy intervention in Ramsey equilibrium. Whether optimal outcomes can be supported by such Ramsey policy, however, depends on policy implementation.

This chapter follows closely the previous chapter and study implementation of optimal unconventional monetary policy via simple rules. Based on the intuition of optimal policy obtained in the previous chapter, I find that the Ramsey policy can be characterised by a rule responding to gaps in asset prices. Unfortunately, this rule requires knowledge of asset prices that would be realized in a world free of financial friction so cannot be used to guide unconventional monetary policy in practice. In searching of appropriate practical rules, I find the best rule being the one proposed by Foerster (2015), i.e., policy responding to its own lag and credit spreads. I argue that the superiority of this rule comes from its effects on the expectation of asset prices, not the slow unwinding suggested by Foerster (2015).

The contribution of this chapter is to design optimal simple rules based on intuitions of Ramsey policy and formally evaluate different simple rules. I also show that insights for policy design apply to similar unconventional policies, i.e., public asset purchases, and equity injections and lending to banks. By contrast, related papers in the literature often consider public asset purchases only. And they have different focuses. Dedola et al. (2013) focuses on the difference between cooperative and noncooperative policy using a relatively inefficient credit spread rule. They find that the noncooperative policy may feature too large or too small policy response parameters, depending on the policy costs. This chapter does not examine the noncooperative policy because searching policy parameters in the model with occasionally binding constraints is computationally expensive. Furthermore, rules considered exclusively in this chapter are unlikely to feature noncooperative equilibrium that is significantly different from that of Dedola et al. (2013). Ellison and Tischbirek (2014) and Quint and Rabanal (2017) focus on asset purchase programs in normal times. The former paper jointly optimises the parameters in the interest rate and the asset purchase rules while the latter optimises the asset purchase rules conditional on a estimated Taylor rule.

In the next section, I examine desirability of three rules against their unconditional welfare loss. Then, To build intuitions of the welfare implications, I compare impulse responses of the economy under each of these rules. I conclude this chapter in the last section.

Table 4.1: Unconditional welfare loss under optimized simple rules

	Asset pur.	Dis. win.	Equity inj.
Spread rule	6.84 (150)	7.43 (150)	4.30 (115)
AR spread rule	5.80 (150, 0.9)	6.20 (150, 0.9)	4.18 (115, 0.9)
Price rule	3.83 (450)	4.66 (450)	2.49 (450)

Note: The numbers outside the brackets are base points of the unconditional welfare loss under an optimized rule relative to that under the optimal allocation. The numbers inside the brackets are the parameters of the optimized rules.

4.2 Simple rule design

Based on the model studied in chapter 3, I consider three simple rules. The benchmark rule that has been most popular in the literature is

$$\mathcal{P}_{i,t} = \kappa \mathbb{E}_t \left(r_{k,t+1} - r_t \right) \tag{4.1}$$

in Home and similarly in Foreign where $\mathcal{P}_{i,t}$ is the policy spending proportional to domestic asset value, i indexes public asset purchases, equity injections and lending to banks, and parameter κ determines the aggressiveness of the interventions. I refer to this rule as the spread rule. Foerster (2015) proposes an improvement on equation 4.1 by adding an autoregressive term (AR spread rule):

$$\mathcal{P}_{i,t} = \kappa \left(1 - \rho_{\mathcal{P}}\right) \mathbb{E}_t \left(r_{k,t+1} - r_t \right) + \rho_{\mathcal{P}} \mathcal{P}_{i,t-1}. \tag{4.2}$$

As discussed in chapter 3, unconventional monetary policy in this model works via a capital gain channel. It's therefore natural to consider the rule that responds to asset price gaps (price rule)

$$\mathcal{P}_{i,t} = -\kappa \left(\ln q_t - \ln q_{t,potential} \right), \tag{4.3}$$

where $q_{t,potential}$ is the asset price that would occur in a world free of financial constraints, and $\ln q_t - \ln q_{t,potential}$ is the asset price gaps (in percentage). The negative sign before κ reflects the fact that the asset price is low during financial stresses.

To evaluate each of these rules for each of the policies, I calculate a second order approximation of the model, including the global welfare loss defined as

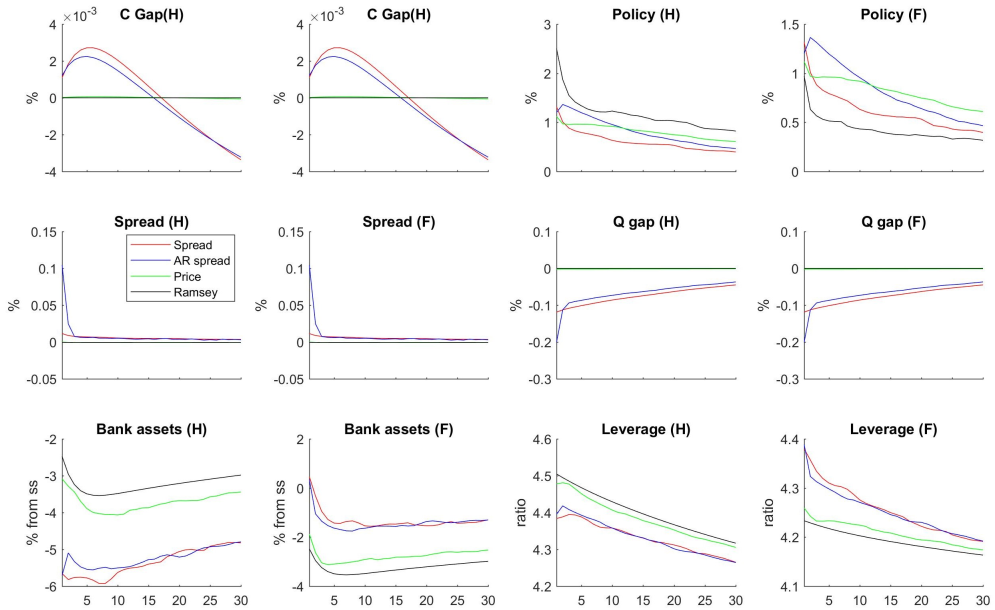
$$\min WEL_g \equiv$$
$$-\mathbb{E}_t \sum_{j=t}^{\infty} \beta^g_{t,t+j} \frac{1}{2} \left[\frac{\left(c_{t+j} - hc_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{1+\varphi}}{1+\varphi} + \frac{\left(c^*_{t+j} - hc^*_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l_{t+j}^{*1+\varphi}}{1+\varphi} \right].$$

The unconditional welfare loss is calculated by simulating the model for 10000 periods and taking the average. Again, I employ the method of Holden (2016a,b) to allow occasionally binding constraint.

The unconditional welfare loss and the optimized policy rule parameters are reported in table 4.1. Once more, the government needs to pay a reduced form cost with τ=0.01. First note that, regardless of the rule, the most efficient policy is again equity injections, followed by public asset purchases, and discount window lending. Second, regardless of the policy, the price rule always generate smallest welfare loss among all three. The AR spread rule is an improvement over the naive spread rule but only at a margin.

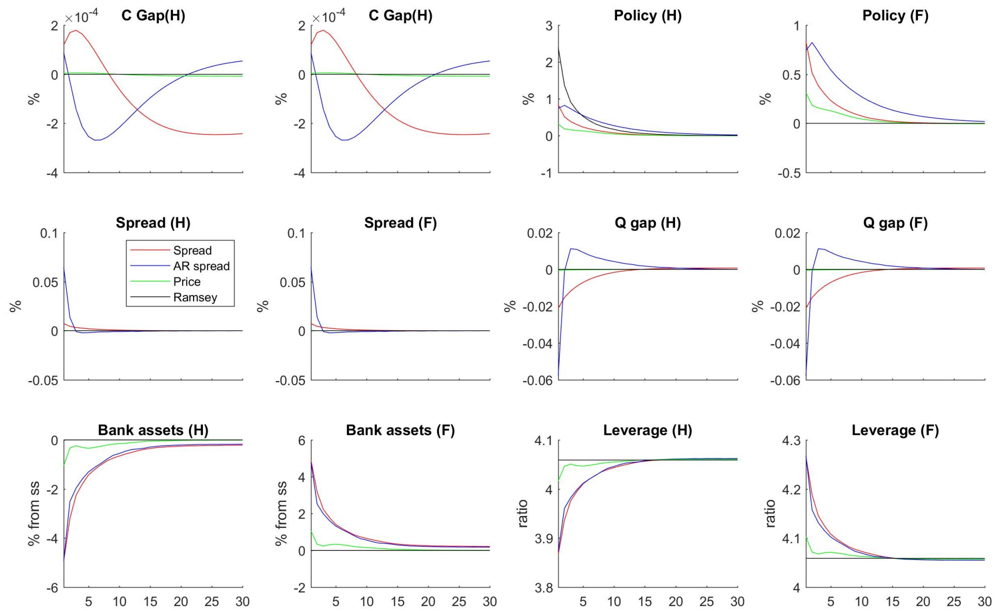
To better understand how these rules differ, it is useful to consider the impulse response of the economy under each rule. For the purpose of illustration, I only show the responses of equity injections in figures 4.1 and 4.2. Consider first the spread rule shown in red. Subject to either shock, the spread rule is not aggressive enough to stabilize the financial sector, leaving a significant gap in asset prices and

Figure 4.1: Response to Home capital quality shock, with different policy rules



Note: The gap variable is the difference between the actual variable that is realized in the model and its counterpart that would be realized in a world without the financial constraints. C denotes consumption and Q denotes the asset price.

Figure 4.2: Response to Home financial shock, with different policy rules



Note: The gap variable is the difference between the actual variable that is realized in the model and its counterpart that would be realized in a world without the financial constraints. C denotes consumption, Q denotes the asset price.

spreads. As a result, there is also a substantial fluctuation in the consumption gap. However, it exits from interventions at roughly the same speed as the Ramsey results. Adding an AR term (shown in blue) does not necessary make the interventions more persistent as we expect. For example, AR spread rule exits faster than the spread rule upon a capital quality shock. Generally, the AR spread rule features hump-shape responses that are stronger than the naive spread rule. The hump-shape responses seems capturing the observation that central banks tend to strengthen unconventional policy at the early stage of the crisis. By employing the AR spread rule, a central bank allows relatively large spreads and price gaps in the first few periods, but makes them smaller thereafter by better anchoring the expectation of asset prices. Therefore a central bank faces additional trade-offs between the short run and the long run financial efficiency, which pins down the optimal AR parameter. With optimized parameters, the AR spread rule reduces fluctuations in consumption and improve the life-time utility. This improvement is particularly clear for the capital quality shock.

The price rule (shown in green) characterises the Ramsey outcomes. There is barely any fluctuation in the consumption gaps, the asset price gaps, and the spreads. This is expected because the unconventional policy works through the capital gain channel. The price rule still generates a substantial welfare loss because the government cannot customize rule parameters to shocks with different nature and sources. Consequently, facing a Home shock, interventions are relatively weak in Home but relatively strong in Foreign, and visa versa when facing a Foreign shock. In addition, the price rule is not practicable facing a capital quality shock, or any other real shock, because the potential asset price is not observable.

4.3 Conclusion

This chapter completes the discussion of optimal unconventional monetary policy by studying how optimal policy can be implemented in practice. I find that a central bank should respond to asset price gaps facing a financial shock but respond to the credit spread with a certain degree of inertia facing other shocks. In choosing the optimal degree of inertia, a central bank face additional trade-offs between short run and long run financial efficiency.

It may be natural to consider a policy shock in this chapter. For example, there have been concerns that excessive unconventional interventions may create asset price inflation. I left this exercise to further research, however, because policy shocks bring no interesting dynamics in this model. A positive shock on the policy perfectly crowds out private funds and hence cannot push asset prices beyond their optimal level. Because of this unexpected crowding out effect, though, the policy must be longer lasting.

Conclusion

In this thesis, I address two issues in the field of international macroeconomics. The first is the PPP puzzle. I propose an empirical model called integral correction mechanism (ICM) to capture real exchange rate dynamics. It implies that the real exchange rate responds to an unidentified shock non-monotonically. The real exchange rate reverts to its long-run equilibrium quickly in the short-run but persistently move around the equilibrium. Hence the PPP puzzle can be understood by distinguishing between degrees of persistence in the short-run and the long-run. The micro-foundation underlying this type of dynamics is partially a time-varying price elasticity of tradable goods such that international trade is less responsive to terms of trade in the short-run. The second issue is the optimal cooperative and non-cooperative unconventional monetary policy. I suggest that unconventional policy should be designed to address the financial constraint that banks face. The central bank should exit slowly from the policy slowly even after a financial crisis has passed. In practice, the central bank can respond to asset price gaps if observable, or credit spreads with inertia. Cross-country policy cooperation is welfare improving. The welfare gain depends on policy cost.

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