On Aspects of Inflation in the Context of Commodity and Futures Market

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I would like to dedicate this thesis to my loving parents.

Declaration

"I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution."

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Abstract

This thesis has developed alternative approaches for inflation forecasting and analysed the inflation risk premium in the context of commodity futures and options markets.

Chapter 1 proposes an approach to tackle the non-availability of exchange-traded inflation futures price data. The composition of the consumer price index enables us to recognise the commodities which correspond to the consumption goods in the CPI. By averaging the commodity futures prices in the same way as the CPI is composed, we construct a synthetic futures contract written on the consumer price index, i.e. a futures on the CPI proxy, based on which we derive a 'point' forecast of inflation rate.

Chapter 2 analyses the term structures of futures on the CPI proxy using the Schwartz (1997) method. Inspired by the Schwartz (1997)'s framework, we develop a two-factor valuation model filtering the spot consumer price index and the instantaneous real interest rate. The Kalman filter is applied to estimate the two-factor valuation model parameters. The filtered spot consumer price index may help alleviate the publication lag in the U.S. CPI-U index. What's more, the two-factor valuation model is capable of forecasting the downward trend in the U.S. CPI inflation rate during May 2014 to December 2014.

Chapter 3 forecasts the inflation rate from the perspective of commodity futures option market. We construct a synthetic option contract written on the futures on the CPI proxy. Based on a synthetic option implied volatility surface, we derive an interval estimate for the one-year ahead expected inflation rate. Moreover, the fact that commodity futures option market data is high-frequency enables our method of inflation forecasting to theoretically capture the market expectation of price level evolution in the real time.

Chapter 4 estimates the inflation risk premium using commodity market data. We derive a link between the inflation risk premium and the risk premium associated with the futures on the CPI proxy. The negative inflation risk premium estimates in our result are consistent with the recent inflation risk premium estimates in the macroeconomic inflation risk premium literature.

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Introduction

An inflation futures is a futures contract written on the inflation rate, which is used by market participants to hedge the risk associated with a volatile inflation rate. The attempts of issuing inflation futures remain unsuccessful due to a lack of a liquid underlying inflation-indexed bond market. Given that market participants are exposed to inflation risk, we propose an approach to tackle the non-availability of exchange-traded inflation futures price data by introducing a synthetic futures contract written on the consumer price index. The composition of the consumer price index indicates that most of the consumption goods in the consumer price index are actually commodities on which the futures contracts are actively traded. Collecting these commodity futures contracts and averaging them in the same way as the CPI is composed, will create a synthetic futures contract written on the consumer price index, i.e. a futures on the CPI proxy. The term structure of futures on the CPI proxy constructed on a certain date yields the contemporaneous expectations of spot price levels in corresponding maturity months of futures on the CPI proxy chain. To obtain a longer sample period of price level forecasts, we analyse the time series of the futures on the CPI proxy chain.

In the commodity pricing literature, the classical approach for analysing the term structures of futures prices is the Schwartz (1997)'s framework. Thus, in Chapter 2, we construct a two-factor valuation model and predict one-year ahead expected inflation rate using the filtered spot consumer price index and fitted forward curves. Although the two-factor valuation model is capable of forecasting the trend in the inflation rate, the prediction derived from the two-factor valuation model is only a 'point' forecast. No prediction of future evolution of price level can be made with absolute certainty. It is therefore more realistic to consider uncertainty when any prediction is made. Thus, there is considerable uncertainty surrounding the 'point' forecast of future price level. This implication motivates us to develop an approach to derive the uncertainty surrounding the 'point' forecast of the future price level.

One important feature of commodity markets is that commodity futures and commodity futures options are usually traded side by side in the same exchange. Besides, options contain information on their underlying assets' risk. This implies that we can use commodity futures options to derive the risk of underlying commodity futures and therefore figure out the uncertainties surrounding the 'point' forecast of future price level. Following this logic, in Chapter 3, we construct a synthetic option written on the futures on the CPI proxy. Using the synthetic option implied volatility surface, we derive the interval estimate for future price level, which yields one-year ahead inflation forecast.

The price level forecasts in Chapter 1 to Chapter 3 are all made under the risk-neutral measure. Under the risk-neutral measure, the commodity futures price is equal to the expected future commodity spot price. In this case, the commodity futures price is the accurate measure of the market expectation of the future price level. However, under the real-world measure, the existence of the risk premium in the commodity futures market renders the commodity futures price an inaccurate measure of the expected future commodity spot price. Thus, in Chapter 4, we extend our study from the risk-neutral measure to the real-world measure and estimate the risk premium associated with the futures on the CPI proxy. By deriving a link between the risk premium associated with the futures on the CPI proxy and the inflation risk premium, we estimate the inflation risk premium using commodity market data.

Chapter 1

Constructing a Futures on the CPI Proxy

Coauthored with CHRISTIAN-OLIVER EWALD

Abstract

The growth of modern inflation-linked bond markets has been strong since the early 2000s. Analogous to the nominal bond markets, the existence of an established underlying inflation-linked bond market gave rise to the emergence of a liquid inflation-linked derivative market, including the inflation futures. Compared to the over-the-counter traded inflation swaps, inflation futures are exchange-traded and accessible. However, due to low liquidity, inflation futures have been delisted from Eurex Exchange. There are no exchange-traded inflation-linked derivatives available on the U.S. and European exchanges in 2014. Given the fact that market participants such as pension funds and insurance companies are still exposed to inflation risk, we propose an approach to tackle the non-availability of inflation futures data. In this paper, we construct a CPI proxy by extracting information from commodity futures price data. The consumer price index is a composition of many components, on which active futures contracts are traded. Collecting these futures contracts and averaging them in the same way as the CPI is composed, will create a synthetic futures contract written on the CPI, i.e. a futures on the CPI proxy.

1.1 Introduction

Inflation-linked derivatives are financial derivatives linked to the inflation rate, which are used by market participants such as pension funds and insurance companies to hedge risks associated with volatile inflation rate. Inflation-linked derivatives provide market participants with additional choices to hedge inflation risk other than the existing inflation-indexed bonds. Furthermore, the entry requirement of the inflation-linked derivative market is lower than that of the inflation-indexed bond market and therefore small businesses can also benefit from inflation markets. In addition, compared with inflation-indexed bonds, inflation-linked derivatives are more flexible in terms of hedging inflation risk due to the fact that derivatives can be customized to meet individual investor's specific needs. Thus, inflation-linked derivative market is a powerful complement to the existing inflation-indexed bond market.

Since the early 2000s, the demand from market participants for increasingly complex inflation-linked derivatives kept growing. As a result, new inflation-linked derivatives other than the traditional inflation-linked swaps began to emerge, including the inflation futures. Inflation futures bridges the gap for hedging inflation risk at the front end of the inflation curve by giving expectation about the short-term inflation rate. However, due to low liquidity, all three attempts to issue inflation futures remain unsuccessful and nowadays data on inflation futures prices can hardly be collected from exchanges. In fact, our investigation suggests that there are no exchange-traded inflation-linked derivatives on inflation markets in 2014.

The existing inflation market literature mainly focus on the analysis of the development of both the inflation-indexed bond markets and the inflation-linked derivative markets. Deacon et al. (2004) is the most comprehensive guide for both the global inflation-indexed bond market and inflation-linked derivative market. Campbell et al. (2009a) explore the history of inflation-indexed bond markets in the U.S. and U.K.. Canty and Heider (2012) present the recent development in inflation markets and analyse the mechanism determining the value of inflation-linked products. The illiquidity of inflation futures is discussed in detail by Canty and Heider (2012), but no solution has been given to tackle this issue. By analysing the history and prospect of inflation futures, Ashton (2014) confirms the benefit that inflation futures brought to the inflation market and expresses optimism for the prospect of inflation futures.

In this paper, we contribute to the literature by proposing an approach to tackle the non-availability of exchange-traded inflation futures data. Based on the composition of U.S. CPI-U index, we construct a synthetic futures written on the consumer price index by extracting information from commodity futures contracts, i.e. a futures on the CPI proxy.

We organise this paper as follows. The next section describes the global inflation-linked derivative markets and our investigation for the availability of exchange-traded inflation-linked derivatives data. Section 1.3 introduces the approach to tackle the non-availability of inflation futures data. Section 1.4 describes the dataset. Section 1.5 discusses the empirical results. Finally, section 1.6 concludes the paper.

1.2 Global Market of Inflation-linked Derivatives

The global inflation market emerged three decades ago since the U.K. started to issue inflation-linked gilts in 1981. Although the modern global market of inflation-linked bonds is still relatively young, the growth has been strong since the early 2000s. Analogous to the nominal bond market, the existence of an established underlying inflation-linked bond market is necessary for the emergence of a liquid inflation-linked derivatives market. According to Canty and Heider (2012), the trading of inflation-linked derivatives has picked up since the early 2000s, particularly in Europe and later also in the U.S.

Typically, the most liquid inflation-linked derivatives are zero-coupon inflation swaps, year-on-year inflation swaps, inflation-linked asset swaps, inflation options and exchange-traded inflation futures contracts. Since most zero-coupon inflation swaps, year-on-year inflation swaps, inflation-linked asset swaps and inflation options are over-the-counter traded between two individual counterparties, these inflation-linked derivatives generally cannot be accessed from online financial data service platform such as Bloomberg.

Given the fact that there is no market for short-term government issued inflation-linked bills and that the inflation-linked swaps only hedge long-term risks beyond one year, the short-end of the inflation term structure remains undefined. According to Canty and Heider (2012), individual investor indeed faces short-term inflation risk as inflation-linked bonds and inflation-linked swaps roll down the inflation curve, and these risks will increase as inflation market matures. As a result, inflation futures contracts are constructed by the exchange to help investors to hedge short-term inflation risk. Next, we summarize briefly Canty and Heider (2012)'s study about the three inflation futures contracts issued by the Chicago Mercantile Exchange (CME) and Eurex Exchange and accordingly investigate the availability of exchange-traded inflation-linked derivatives.

The CME constructed a U.S. CPI futures contract in February 2004. The design of this contract was similar to the Eurodollar futures, with inflation defined as annualized quarterly inflation. The price of the contract for month was defined as 100 minus the annualized percentage change in the CPI over the three-month period preceding the contract month.

The CME U.S. CPI futures contract failed to gain traction, with the contract design in terms of quarterly inflation considered as the main culprit for the lack of interest. Because of CPI seasonality, the values of the contracts would vary visibly from one month to the other, making it more difficult to read the implied expectations of trend inflation. The CME has since closed the U.S. CPI contract.

In September 2005, aiming at picking up product's liquidity, the CME launched a euro area inflation futures contract, which-contrary to the previously issued U.S. CPI contract-was based on annual as opposed to quarterly inflation. Specifically, the price of the contract is defined as 100 minus the year-on-year change in HICPx, with each monthly contract referring to the HICPx released during the contract month. Contract for 12 consecutive calendar months are tradable. Given this design, contract prices go up when inflation expectations decline.

However, liquidity has never really picked up, i.e. the product has never managed to break out the circle of low liquidity/no investors/low liquidity. In late 2011, there was no open interest in this contract.

Eurex Exchange launched a new euro area inflation futures contract on 21^{st} of January 2008. The contract design is identical to the CME contract, but 20 consecutive contracts are available (instead of 12 for the CME contract) and trading stops only on the day of the HICP release (instead of one day earlier).

In order to overcome the problem of building up liquidity, Eurex Exchange proposed specified trading rules. In particular, two daily auctions of 15 minutes-opening and closing-were to be held where designated market makers would provide liquidity. During the auction, the market makers were required to quote for 12 out of 20 calendar months at a maximum spread of 20 ticks and be present at least 80% of the auction period. During continuous trading, market makers would quote upon request and respond to at least 70% of all incoming quote requests.

There was also some hope that the longer maturity contracts would attract additional interest, with the 20^{th} contract corresponding to the HICPx index just two months away from the liquid 2Y zero-coupon inflation swap point. Despite these improvements over the CME futures the Eurex HICP contract has not been a success. After some trading activity at the start, liquidity has dried up over time. At the end of 2011, there were no open positions in the Eurex HICP futures.

Accordingly, based on Canty and Heider (2012), until the end of 2011, the liquidity of Eurex HICP futures had dried up. However, given the fact that the Europe had been gradually recovering from the 2008 financial crisis, we hope that there were chances that their liquidity would be picked up since 2012. Therefore, we investigate the availability of data on Eurex HICP futures by contacting Eurex Exchange. It turned out that the Euro-inflation-futures, with product ID HICP, had only 4 closing prices during 2012 to 2013, i.e. this product was not liquid at all on the Eurex market since 2012. Finally, the management board of Eurex Exchange decided to de-list Eurex HICP futures on 11^{th} of June 2014.

The three attempts of issuing inflation futures contract indicate that there were hardly any interests for these products and consequently their liquidity has barely been picked up. Indeed, as Canty and Heider (2012) suggested, the main reason could be that, unlike the comparable nominal products such as the Eurodollar or Euribor futures contracts which are based on a very liquid underlying money market and regular issuance of short-term government debt, inflation futures contracts lack a liquid underlying inflation-linked bond market.

The existing literature suggests that the market of inflation futures contracts is illiquid. In order to investigate the availability of any other exchange-traded inflation-linked derivatives, we contacted the major exchange markets in U.S. and Europe, i.e. the CME group, Eurex Exchange, London Stock Exchange, GXG Markets, Spanish Exchanges, Boerse-Frankfurt Group, SuperDerivatives and European Stock Exchange and inquired about the availability of any exchange-traded inflation-linked derivatives, the feedbacks indicate that there are no exchange-traded inflation-linked derivatives on these major exchange markets in 2014.

However, an illiquid inflation futures market does not mean that market participants are free from inflation risk. The U.S. CPI-U index kept decreasing (from 238.031 to 233.707) for four consecutive months since September 2014, indicating potentially volatile inflation rate in the near future. Thus, in order to tackle the non-availability of inflation futures data, we introduce the futures on the CPI proxy. Next, we show step by step how we construct the futures on the CPI proxy.

1.3 An Approach to Tackle the Non-availability of Inflation Futures

We propose an approach to tackle the non-availability of exchange-traded inflation futures data. The idea of our approach is as follows: the consumer price index is a composition of many items, e.g. food, housing, transportation and services, etc. An inspection at the commodity futures exchange indicates that most of the items used to construct the CPI are actually commodities on which active futures contracts are traded. Collecting these futures contracts according to the composition of CPI and averaging them in the same way as the CPI is composed, will create a synthetic futures contract written on the CPI, i.e. a futures on the CPI proxy. Next, we present in detail how to construct the futures on the CPI proxy.

We focus on the U.S. exchanges, specifically the CME Group, to collect the commodity futures contracts, since the CME Group is the world's most diverse derivatives marketplace and the largest futures exchange. We do not include U.K. exchanges in our study, given that the majority of active commodity futures contracts are based in the U.S., focusing on the U.K. exchanges will give rise to the issue that most of the included futures contracts are being traded overseas, which leads to the exchange rate risks. Linking these futures contracts to foreign exchange products can only hedge part of the risks. Therefore, to avoid that potential risk, we focus on the U.S. exchanges.

Since we decide to focus on the U.S. exchanges, we then need to choose the reference U.S. consumer price index. The U.S. Bureau of Labour Statistics (BLS) publishes CPI index on a pre-specified date every calendar month. The three main U.S. CPI series are: CPI for All Urban Consumers (CPI-U); Chained CPI for All Urban Consumers (C-CPI-U); CPI for Urban Wage Earners and Clerical Workers (CPI-W). Specifically, CPI-U represents the buying habits of the residents of urban areas in the United States, while CPI-W covers only a subset of the urban population. C-CPI-U also represents the urban population as a whole, but it uses a distinct calculation formula which accounts for consumers' ability to achieve the same standard of living from alternative sets of consumer goods and services. This formula requires consumer spending data that are not immediately available and therefore C-CPI-U is published first in preliminary form and is subject to two subsequent scheduled revisions. Compared to CPI-W and C-CPI-U, CPI-U covers the largest urban population of the U.S. and is the least lagged price index. Therefore, we choose CPI-U as our reference consumer price index.

Given CPI-U as the reference consumer price index, we next reverse the procedure of U.S. Bureau of Labour Statistics constructing the CPI-U and investigate the composition of CPI-U. The official document 'Relative importance of components in the Consumer Price Indexes: U.S. city average' published by the Bureau of Labour Statistics gives us indication of the composition of CPI-U. Specifically, CPI-U consists of groups of items such as food and beverages, housing, apparel, transportation, medical care, recreation, education and communication and other goods and services, and each group is composed of individual items.

Given these items, we then try to find out their corresponding commodities on which futures contracts are actively traded in exchanges. Most of the items have the exact corresponding commodities. For those items which do not have exactly matching commodities, however, we find close substitute commodities based on the main ingredients of these items, e.g. bread can be treated as wheat; jewellery can be substituted with gold and silver, and so on.

Next, based on the underlying commodities, we search for the available data on corresponding commodity futures contracts in CME by using Bloomberg. Since the purpose of constructing the CPI proxy is to track the CPI-U as closely as possible, for each commodity futures contract table, we only include those futures contracts whose maturities follow closely the release date of CPI-U, e.g. the release date of January 2015 CPI-U is 26^{th} of February 2015, we only include the earliest-matured commodity futures contract among all futures contracts that matured after 26^{th} of February 2015. Once we have included all of the eligible futures contracts, we then collect data on the last price of each futures contracts observed on the CPI-U releasing date. If on the CPI-U release date, the last price is not available, we then use the settlement price of that futures contract observed on the same date instead since it is the best guess of the last price provided by the exchanges.

In order to ensure that the CPI proxy tracks the historical CPI-U index as closely as possible, we also need to figure out the optimum weight for each commodity. When determining the commodity's optimum weight, the key is to ensure that the optimum weight of each commodity reflects precisely the relative importance of the corresponding item in the CPI. One solution would be to take directly the original weight of item in the CPI and assign it to the corresponding commodity. However, this solution is viable only when every item in the CPI has an exactly matching commodity futures contract available in Bloomberg, which, in our study, is not the case. In more precise terms, for 79% of all items in the CPI, we can find either an exactly matching commodity futures contract or close substitute in Bloomberg; by contrast, the remaining 21% of components in the CPI have neither corresponding commodity futures nor any available substitute. Therefore, we need to renormalize the weights of the 79% of CPI components which have exactly matching commodity futures contract or close substitute in Bloomberg.

Given that we have two 'Relative importance of components in the Consumer Price Indexes: U.S. city average' documents in the dataset, we need to determine from which document of relative importance we derive the commodity's optimum weight. For each 'Relative importance of components in the Consumer Price Indexes: U.S. city average' document, we take the weights of CPI components that have matching or substitute commodities, then we renormalize these weights and assign them to each commodity. This gives us two groups of renormalized weights for commodities. Next, for each group of renormalized weights, we calculate the corresponding time series of CPI proxy. The CPI proxy corresponding to a specific reference CPI-U release date is defined as:

$$\sum_{i=1}^{n} W_i F_i(0,t),$$

where W_i is the renormalized weight for the i^{th} commodity; $F_i(0,t)$ is the CPI-U-release-date-observed last price of the i^{th} commodity's futures contract whose maturity follows closely the CPI-U release date; n is the number of commodities included in the construction of CPI proxy. For each one of the 26 CPI-U release dates in the dataset, we calculate one corresponding CPI proxy, which forms the time series of CPI proxy. We then compare each time series of CPI proxy with the time series of historical CPI-U indexes and see which time series of CPI proxy tracks the time series of historical CPI-U indexes more closely. Lastly, the group of renormalized weights that yields a time series of CPI proxy tracking the time series of historical CPI-U indexes more closely will be the optimum weights for commodities.

Given the commodity optimum weight, we then look into the future and construct the futures on the CPI proxy. The time 0 futures on the CPI proxy corresponding to a specific future time T is defined as:

$$\sum_{i=1}^{n} a_i F_i(0,T),$$

where a_i is the optimum weight for the i^{th} commodity; $F_i(0,T)$ is the time-0 last price of the i^{th} commodity's futures contract maturing in the future time T; n is the number of commodities in the dataset. The weighted average of commodity futures prices, i.e. $\sum_{i=1}^{n} a_i F_i(0,T)$, is therefore the futures on the CPI proxy.

1.4 Data Description

This section describes the data used in the empirical study. We have three datasets: U.S. CPI-U data, relative importance of components in the Consumer Price indexes: U.S. city average, and commodity futures price data.

1.4.1 U.S. CPI-U Data

We use the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers (CPI-U) as our reference consumer price index. We obtained this index from the U.S. Bureau of Labour Statistics CPI Databases. CPI-U is a measure of the average change over time in the prices paid by all U.S. urban consumers for a market basket of consumer goods and services. We choose the non-seasonally adjusted CPI-U because it is consistent with the seasonality observed in the commodity futures market.

We use monthly CPI-U data observed from December 2012 to January 2015. Table 1.1 presents the monthly CPI-U values from December 2012 to January 2015:

Table 1.1. CPI-U Index Values (December 2012 to January 2015)

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2012												229.601
2013	230.280	232.166	232.773	232.531	232.945	233.504	233.596	233.877	234.149	233.546	233.069	233.049
2014	233.916	234.781	236.293	237.072	237.900	238.343	238.250	237.852	238.031	237.433	236.151	234.812
2015	233.707											

1.4.2 Relative Importance of Components in the Consumer Price Indexes: U.S. City Average

The data on the 'Relative importance of components in the Consumer Price Indexes: U.S. city average' is available for downloading from 'CPI Tables' of U.S. Bureau of Labour Statistics website. The Relative importance of components in the Consumer Price Indexes is the component's expenditure or value weight expressed as a percentage of all items in an area. This percentage represents the component's average annual expenditures and shows how the indexed population distributes expenditures among all of the components. Every update of these percentages shows how consumers would distribute their expenditures on the components as prices change over time. The relative importance of components in the CPI gives us an indication of what commodity futures we need to include in the construction of the CPI proxy. Furthermore, we use the relative importance of components in the CPI to determine the optimum weight for each commodity.

Although the relative importance of components in the CPI data changes on monthly basis, BLS publishes the relative importance of each component in the CPI only once a year in every December as the monthly changes of the relative importance are deemed too small to perceive. During our analysis period, two sets of data on 'Relative importance of components in the Consumer Price Indexes' are available, namely '2011-2012 Weights, December 2013' and '2011-2012 Weights, December 2014', respectively. We present these two sets of data in the Appendix A.

1.4.3 Commodity Futures Price Data

Commodity futures price in our analysis refers to the daily last price or the daily settlement price for any commodity futures included in the construction of the futures on the CPI proxy. We choose the commodity futures contracts that are actively traded on CME Group exchanges. We collect the data on commodity futures price from Bloomberg.

As stated in the methodology, we start with the investigation of composition of the U.S. CPI-U index. The 'Relative importance of components in the Consumer Price Index' shows how the CPI-U is composed of different components and therefore is a precise indication of what commodity we need to include in the construction of CPI proxy. On one hand, we examine each item listed in the 'Relative importance of components in the Consumer Price Index' to see what commodity we need to include; on the other hand, we check what commodity futures contracts are available from CME Group exchanges. This 'examine and check' process yields two results: for items that have exactly matching commodity futures, we include these commodities directly; for other items which do not have exact matches, we find close substitutes. Additionally, for any included commodity that has more than one futures contracts written on it, we only choose the futures contract with the highest volume and open interest. Following this method, we include 34 commodity futures contract tables, with each contract table containing the chains of futures contracts with different maturities written on one commodity. In Appendix A, we present in detail the generic tickers of the 34 commodity futures contract tables and how we find close substitute commodities for items listed in the U.S. CPI-U index.

Given these 34 commodity futures contract tables, the next step is to collect the price data for these futures contracts. We classify the price data into two categories by their usage: price data for commodity futures with the shortest maturity; and price data for commodity futures maturing in the future.

Price Data for Commodity Futures with the Shortest Maturity

Our purpose of constructing the CPI proxy is to track the historical CPI-U as closely as possible. Two factors determine how closely the CPI proxy tracks the historical CPI-U: the weight of each commodity; and the prices of included commodity futures. In this section, we present how we choose the commodity futures price data in order to ensure that the CPI proxy tracks closely the historical CPI-U.

We use the time series of historical CPI-U index containing 26 consecutive observations from December 2012 to January 2015. Each historical CPI-U index has a release date, we present the 26 CPI-U release dates in Table 1.2.

Table 1.2. Schedule of Releases for CPI (December 2012-January 2015)

Reference Month	U.S. CPI Release Date	Unseasonally Adjusted U.S. CPI-U
January 2015	February 26, 2015	233.707
December 2014	January 16, 2015	234.812
November 2014	December 17, 2014	236.151
October 2014	November 20, 2014	237.433
September 2014	October 22, 2014	238.031
August 2014	September 17, 2014	237.852
July 2014	August 19, 2014	238.25
June 2014	July 22, 2014	238.343
May 2014	June 17, 2014	237.9
April 2014	May 15, 2014	237.072
March 2014	April 15, 2014	236.293
February 2014	March 18, 2014	234.781
January 2014	February 20, 2014	233.916
December 2013	January 16, 2014	233.049
November 2013	December 17, 2013	233.069
October 2013	November 20, 2013	233.546
September 2013	October 30, 2013	234.149
August 2013	September 17, 2013	233.877
July 2013	August 15, 2013	233.596
June 2013	July 16, 2013	233.504
May 2013	June 18, 2013	232.945
April 2013	May 16, 2013	232.531
March 2013	April 16, 2013	232.773
February 2013	March 15, 2013	232.166
January 2013	February 21, 2013	230.28
December 2012	January 16, 2013	229.601

source: U.S. Bureau of Labour Statistics

We set each CPI release date as the reference date for determining which commodity futures contract to include. Each commodity futures contract table contains a series of futures contracts with different maturities written on the underlying commodity, for each reference

CPI release date, we only choose the futures contract with the shortest maturity, e.g. for December 2014 CPI-U released on 16^{th} of January 2015, we only include the earliest-matured commodity futures contract among all futures contracts which matured after 16^{th} of January 2015. It should be noted that although only the futures contract with the shortest maturity are included, the CPI release date and included futures contract maturity are not necessarily in the same month. Due to the fact that different commodities have different number and timing of maturity months within a calendar year, it is common that certain commodity only has futures contract maturity in odd (or even) months. For such commodity futures, the 'shortest maturity' can be two or even three months away from the reference CPI release date.

For each commodity futures with the shortest maturity, we collect the historical daily last price of the futures contract observed on the reference CPI release date, e.g. for reference CPI release date 16^{th} of January 2015, the crude oil futures contract 'CLG5 Comdty' maturing on 20^{th} of January 2015 is the futures contract with the shortest maturity, then we collect the last price of 'CLG5 Comdty' observed on 16^{th} of January 2015. In the case that the last price is unobservable on the reference CPI release date, we use the settlement price instead since the settlement price is the 'best guess' made by exchanges of the average price of all trades happened on that day. In fact, even if there is no trade on a specific day, the settlement price is still observable. Therefore, the settlement price data is available on all trading days during our analysis period. Following this method, for each reference CPI release date, we collect 34 commodity futures price observations, producing a total of 884 price observations.

Price Data for Commodity Futures Maturing in the Future

We use price data for commodity futures with the shortest maturity to determine the optimum weight for each commodity. To construct the futures on the CPI proxy, we remove the restriction on the maturity of commodity futures contracts and collect the price data on commodity futures maturing in the future.

Specifically, for each one of the 34 commodities, we check the corresponding commodity futures contract table in Bloomberg and collect the historical daily last price data for all of the available futures contracts in contract table. The daily last price data for each futures contract observed from 1^{st} of January 2012 to 9^{th} of April 2015 (the most recent date in the analysis period) are collected. In the case that the daily last price of a futures contract is not available on a specific date, we use settlement price on that date instead.

It should be noted that the longest available futures contract maturity differs from one commodity to another. For example, for crude oil, we can observe available futures contract maturing in December 2021, which means that we can 'see far into the future'. Similarly, for gold, copper, electricity and natural gas, futures contracts maturing in 2020 can be observed. The longest available futures contract maturities of these five commodities are the longest among all the commodities in the dataset. For the remaining commodities in the dataset, a majority (28) of these commodities have futures contract maturities observable until May 2016; we also have commodity futures contract that has maturity month only available until September 2015. Therefore, we select May 2015 to May 2016 as our analysis period because during this period the majority of commodities have available futures contract maturity observable until May 2016.

It should also be noted that the number and timing of futures maturity months within a calendar year can vary from one commodity to another. As for our dataset, although there exists 'always-available' commodity futures such as crude oil futures which have maturity months available in every month of a year, most commodity futures only have maturity available in even or odd months of a year, or even every other quarter. As a matter of fact, during our analysis period (May 2015-May 2016), there are only 12 commodities which have futures contract maturity available in every month of a year. In other words, for the remaining commodities, the number of available maturity months within a calendar year is less than 12. Table 1.3 presents the number and timing of maturity months of commodity futures contracts.

In Table 1.3, the first column presents the ticker of the commodities; each number '1' in the table indicates that the commodity futures contract has observable maturity in that month. It can be seen from the table that different number and timing of commodity futures maturity months within a calendar year and distinct longest available futures contract maturity observed in the dataset give rise to the issue of missing commodity futures price observations in certain months. To tackle the issue of missing futures price observations, we apply the linear interpolation and extrapolation method to replace the missing futures prices. We present the result in section 1.5.

Our purpose is to extract information from the commodity futures maturing in the future and construct the futures on the CPI proxy. The consumer price index only describes the price level in current and historical periods, which indicates that the term structure of consumer price index forecasting the future price level does not exist. However, the commodity futures

Table 1.3. Number and Timing of Maturity Months of Commodity Futures Contracts

	2015												2016												2017												2018					
Ticker	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6
YP					1		1		1			1			1		1		1		1			1			1		1		1		1			1			1			
FC				1	1			1	1	1	1		1		1																										
LH				1	1	1	1	1		1		1		1		1	1	1	1																							
LC				1		1		1		1		1		1		1		1		1																						
DA				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1															
CHE				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1															
FSB					1		1			1					1		1		1			1					1															
LE				1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1																
JO					1		1		1		1		1		1		1				1		1		1		1		1				1		1		1		1				
FCC					1		1		1			1			1		1		1		1			1			1															
V6				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1															
DRW				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1															
FCO					1		1		1			1			1		1		1		1			1			1															
BO					1		1	1	1	1		1	1		1		1			1	1	1		1	1		1		1			1	1	1		1						
KV				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1		1															
CPI				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1															
DJE						1			1			1			1																										
NG					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LB					1		1		1		1		1		1		1																								
PH				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1
HRC				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1						
FC					1		1			1		1			1		1		1			1		1			1															
DFL				1	1	1	1	1	1	1	1	1	1	1	1																										
HG				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1		1		1			1			1		1	
OP					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1		1		1			1			1		1	
GC				1	1	1		1		1		1		1		1		1		1		1		1		1				1						1						1
CL					1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
IXC						1			1																																
SI				1	1	1	1		1			1	1		1		1		1		1			1	1						1					1						
MKC					1		1		1			1			1		1		1		1			1			1		1		1		1			1			1			
O					1		1		1			1			1		1		1		1			1			1		1		1		1									
RR					1		1		1		1		1		1		1																								
KW					1		1		1			1			1		1		1		1			1			1		1		1		1			1			1			
W					1		1		1			1			1		1		1		1			1			1		1		1											

Notes: The first column presents the ticker of the commodities; each number '1' in the table indicates that the commodity futures contract has observable maturity in that month.

Table 1.3 - continued. Number and Timing of Maturity Months of Commodity Futures Contracts

	2018						2019												2020												2021											
Ticker	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
YP																																										
FC																																										
LH																																										
LC																																										
DA																																										
CHE																																										
FSB																																										
LE																																										
JO																																										
FCC																																										
V6																																										
DRW																																										
FCO																																										
BO				1	1																																					
KV																																										
CPI																																										
DJE																																										
NG	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1																				
LB																																										
PH	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		1	1		1	1	1	1	1	1												
HRC																																										
FC																																										
DFL																																										
HG	1		1			1			1		1		1		1			1			1																				
OP	1		1			1			1																																
GC						1						1						1						1						1												
CL	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1						1						1
IXC																																										
SI	1					1												1																								
MKC																																										
O																																										
RR																																										
KW																																										
W																																										

Notes: The first column presents the ticker of the commodities; each number '1' in the table indicates that the commodity futures contract has observable maturity in that month.

market makes expectation for the future price level by pricing the commodity futures contracts and updating their prices continuously. The commodity futures prices observed on any date reveal the prices of that commodity to be delivered in a certain future month. The weighted sum of the prices of the 34 commodity futures contracts maturing in the same month creates the futures on the CPI proxy for that month; and the collection of futures on the CPI proxy corresponding to every month during May 2015 to May 2016 constructs the term structure of futures on the CPI proxy. It should be noted that, the term structure of futures on the CPI proxy may vary from one date to another because the underlying commodity futures price data updates continuously. Therefore, we pre-specify one reference date and construct the term structure of futures on the CPI proxy for that date, then the term structure of futures on the CPI proxy for other historical dates can be constructed by following the same method.

In fact, we cannot arbitrarily appoint any date as the reference date. Given that the start trading day of commodity futures maturing in the far future (e.g. 2 years in the future) can be close to the current date, the price quotes for such a commodity futures contract are only available during the most recent dates. For instance, cocoa futures contract ‘FCOH7’ maturing in March 2017 only has price quotes available from 2^{nd} of April 2015 to 8^{th} of April 2015. Therefore, in order to extract information from futures contracts maturing in the far future, we need to select the appropriate date as our reference date. As a result, we choose 7^{th} of April 2015 as the reference date for our analysis. We collect the reference date’s daily last price data for all available included commodity futures contracts. This dataset can be found in section 1.5.

Given the collected datasets, we now move on to determine the optimum weight for each commodity and construct the futures on the CPI proxy. In the next section, we present our empirical results.

1.5 Empirical Results

In this section, we present in detail the construction of the futures on the CPI proxy. Using the available datasets, we first determine the optimum weight for each commodity. Then we replace the missing futures prices. Next, based on the optimum weights and complete futures price dataset, we construct the term structure of futures on the CPI proxy.

1.5.1 Determining the Optimum Weight for Commodity

The purpose of constructing the CPI proxy is to track the time series of historical CPI-U index as closely as possible. To this end, we have selected the proper commodity futures price data by choosing the futures contract whose maturity follows closely the reference CPI release date. In order to fulfil the purpose, we also need to determine the optimum weight for each commodity.

When determining the commodity's optimum weight, the key is to ensure that the optimum weight of each commodity reflects precisely the relative importance of the corresponding item in the CPI. One solution would be to directly take the original weight of item in the CPI and assign it to the corresponding commodity. This solution is viable only when every item in the CPI has an exactly matching commodity futures contract available in Bloomberg, which is not the case in our study. There is no futures contract written on item such as tobacco and smoking products. Furthermore, services account for over 61% of total weight of components in the CPI. For the majority of the services such as housing and medical, we can find corresponding futures contracts; but for other services such as haircut, dental and legal services, neither corresponding futures contracts nor close substitutes are available in Bloomberg. Precisely, for 79% of all items in the CPI, we can find either an exactly matching commodity futures contract or close substitute in Bloomberg; by contrast, the remaining 21% of components in the CPI have neither corresponding commodity futures nor any available substitute. Therefore, we need to renormalize the weights of the 79% of CPI components which have exactly matching commodity futures contract or close substitute in Bloomberg.

As stated in the data description section, we have two 'Relative importance of components in the Consumer Price Indexes: U.S. city average' datasets available during the analysis period, i.e. 2011-2012 weights published in December 2013 and 2011-2012 weights published in December 2014. These two datasets are the benchmark to determine the optimum weights for commodities. The method for determining the renormalized weights is as follows: for items that have corresponding commodity futures contract or close substitute in Bloomberg, we take their weights from 'Relative importance of components in the Consumer Price Indexes: U.S. city average' and assign these weights to the corresponding commodities or close substitutes. Next, we renormalize these assigned weights to ensure the sum of these renormalized weights is equal to one while keeping the relative importance unchanged. We present in Table 1.4 the original assigned weights and renormalized weights for each commodity.

Table 1.4. Original Assigned Weights and Renormalized Weights for Commodities

commodity	Weights for December 2013		Weights for December 2014	
	original weights	renormalized weights	original weights	renormalized weights
YP	0.23%	0.30%	0.23%	0.30%
FC	5.70%	7.21%	5.84%	7.40%
LH	0.35%	0.44%	0.37%	0.47%
LC	0.83%	1.04%	0.91%	1.15%
DA	0.27%	0.34%	0.28%	0.36%
CHE	0.27%	0.34%	0.29%	0.36%
FSB	0.42%	0.53%	0.43%	0.54%
LE	0.20%	0.25%	0.20%	0.26%
JO	0.70%	0.89%	0.70%	0.89%
FCC	0.15%	0.19%	0.16%	0.20%
V6	0.25%	0.31%	0.25%	0.31%
DRW	0.28%	0.36%	0.29%	0.36%
FCO	0.33%	0.41%	0.33%	0.42%
BO	0.29%	0.36%	0.29%	0.37%
KV	0.05%	0.07%	0.06%	0.07%
CPI	0.44%	0.56%	0.44%	0.56%
DJE	32.03%	40.48%	32.71%	41.42%
NG	1.01%	1.27%	1.01%	1.28%
LB	2.67%	3.37%	2.60%	3.30%
PH	4.32%	5.46%	4.41%	5.58%
HRC	1.00%	1.27%	0.98%	1.24%
FCT	2.72%	3.44%	2.62%	3.32%
DFL	0.11%	0.13%	0.11%	0.14%
HG	0.56%	0.71%	0.56%	0.70%
OP	0.56%	0.71%	0.56%	0.70%
GC	0.11%	0.14%	0.11%	0.13%
CL	16.42%	20.75%	15.29%	19.36%
IXC	5.85%	7.39%	5.94%	7.53%
SI	0.11%	0.14%	0.11%	0.13%
MKC	0.12%	0.15%	0.12%	0.15%
O	0.25%	0.31%	0.25%	0.31%
RR	0.13%	0.16%	0.13%	0.16%
KW	0.19%	0.24%	0.19%	0.24%
W	0.23%	0.29%	0.23%	0.29%
SUM	**79.13%**	**100%**	**78.97%**	**100%**

Notes: Table 1.4 presents the original assigned weights and renormalized weights for each commodity futures contract table; ‘Weights for December 2013’ represents that the original weights are taken from ‘2011-2012 Weights, December 2013’ document; ‘Weights for December 2014’ represents that the original weights are taken from ‘2011-2012 Weights, December 2014’ document.

It should be noted that, although the absolute value of renormalized weight for each commodity is greater than the original weight of corresponding component in the CPI, the relative weight of commodity (after renormalization) remains the same to the relative importance of corresponding component in the CPI, as it is the relative weight of each commodity rather than the absolute value of weight that determines the importance of a commodity in the construction of futures on the CPI proxy.

Given the two sets of renormalized weights, we now move on to construct the time series of CPI proxy. Specifically, the CPI proxy corresponding to a specific reference CPI-U release date is defined as:

$$\sum_{i=1}^{34} W_i F_i(0,t),$$

where W_i is the renormalized weight for the i^{th} commodity; $F_i(0,t)$ is the reference-CPI-release-date-observed last price of i^{th} commodity's futures contract whose maturity follows closely the reference CPI release date. The weighted sum of futures prices, i.e. $\sum_{i=1}^{34} W_i F_i(0,t)$, is the CPI proxy corresponding to a reference CPI release date. Since we have 26 CPI release dates in our analysis, we calculate the corresponding 26 CPI proxies; and these 26 indexes construct the time series of CPI proxy which track closely the time series of 26 historical CPI-U index values. In Table 1.5, we present the time series of CPI proxy derived from both groups of renormalized weights.

Table 1.5. Time Series of CPI Proxies

U.S. CPI Release Dates	CPI-U	CPI Proxy 2013	CPI Proxy 2014
January 16, 2013	229.601	222.656	206.314
February 21, 2013	230.28	223.113	207.153
March 15, 2013	232.166	227.483	211.449
April 16, 2013	232.773	231.876	216.997
May 16, 2013	232.531	236.094	220.046
June 18, 2013	232.945	226.630	209.686
July 16, 2013	233.504	230.270	211.839
August 15, 2013	233.596	223.832	204.890
September 17, 2013	233.877	228.172	209.691
October 30, 2013	234.149	232.660	216.129
November 20, 2013	233.546	228.196	212.253
December 17, 2013	233.069	228.255	211.464
January 16, 2014	233.049	234.282	218.294
February 20, 2014	233.916	243.832	226.144
March 18, 2014	234.781	245.584	228.753
April 15, 2014	236.293	243.118	225.356
May 15, 2014	237.072	247.404	230.207
June 17, 2014	237.9	249.982	231.828
July 22, 2014	238.343	254.951	237.510
August 19, 2014	238.25	255.991	240.516
September 17, 2014	237.852	252.177	236.791
October 22, 2014	238.031	251.440	238.958
November 20, 2014	237.433	255.619	244.256
December 17, 2014	236.151	253.443	246.167
January 16, 2015	234.812	259.230	253.697
February 26, 2015	233.707	253.070	247.359

Notes: 'CPI Proxy 2013' is the abbreviation for 'the time series of CPI proxy derived from the renormalized 2013 weights'; 'CPI Proxy 2014' is the abbreviation for 'the time series of CPI proxy derived from the renormalized 2014 weights'. The shaded areas denote the periods when the price level is decreasing.

In Table 1.5, 'CPI Proxy 2013' is the abbreviation for 'the time series of CPI proxy derived from the renormalized 2013 weights'; analogously, 'CPI Proxy 2014' is the abbreviation for 'the time series of CPI proxy derived from the renormalized 2014 weights'. The shaded areas in Table 1.5 denote the periods when the price level is decreasing. The comparison between the shaded areas in the third and the fourth column indicates that the CPI proxy evolution in CPI proxy 2013 tracks the time series of historical CPI-U index more closely than CPI proxy 2014 does. We can also see from Table 1.5 that both time series of CPI proxy start with values that are lower than the CPI-U on 16^{th} of January 2013 (229.601) and end up with values that are higher than the CPI-U on 26^{th} of February 2015 (233.707). In order to see clearly the trend and fluctuations of both time series of CPI proxy and therefore compare them with the time series of historical CPI-U, we plot these time series in Figure 1.1 and Figure 1.2.

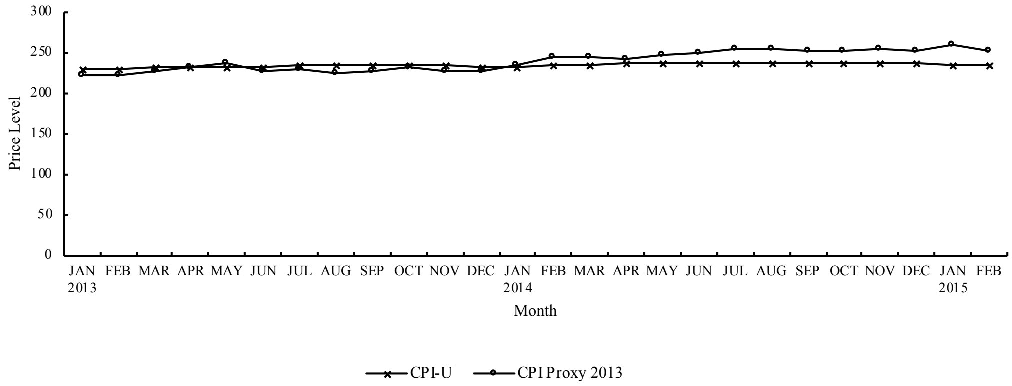


Figure 1.1. The line chart compares the time series of historical CPI-U (solid line with cross symbol) with the time series of CPI proxy constructed from the renormalized 2013 weights (solid line with circle symbol).

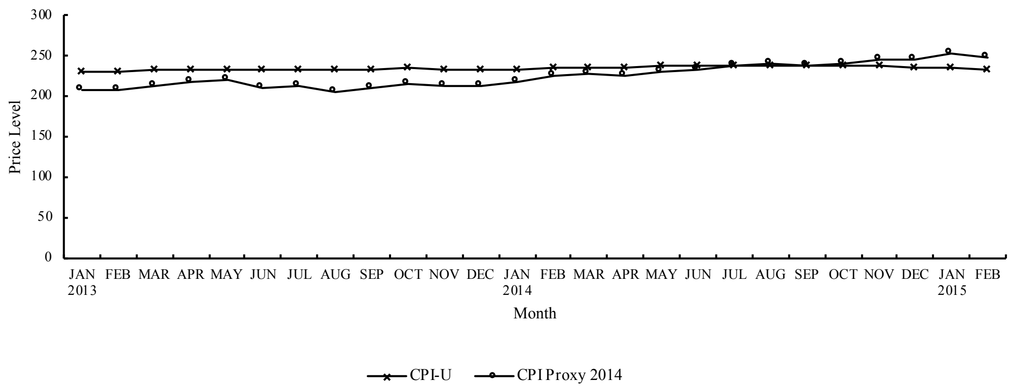


Figure 1.2. The line chart compares the time series of historical CPI-U (solid line with cross symbol) with the time series of CPI proxy constructed from the renormalized 2014 weights (solid line with circle symbol).

In Figure 1.1 and Figure 1.2, CPI Proxy 2013 and CPI Proxy 2014 are the abbreviations for CPI proxy constructed from renormalized 2013 weights and renormalized 2014 weights, respectively. We can see from these charts that both time series of CPI proxy seem to track closely the time series of historical CPI-U index.

In order to clearly observe which time series of CPI proxy tracks the time series of historical CPI-U index more closely, we measure the similarity between the time series of historical CPI-U index and each time series of CPI proxy. Following Wang et al. (2013), the similarity between two time series T_1 and T_2 is measured by the similarity function $Dist(T_1, T_2)$ that calculates the distance between time series T_1 and T_2. The smaller the value of similarity function $Dist(T_1, T_2)$, the more similar the two time series will be. In our case, the similarity between the time series of historical CPI-U index and each time series of CPI proxy is the lock-step-measured distance, i.e. comparing the time-t historical CPI-U to the contemporaneous time-t CPI proxy. Wang et al. (2013) suggest that, in the case of lock-step measures, the Euclidean distance is an accurate, simple and efficient method for measuring the similarity between two time series. Formally, for any two time series $T_1 = \{x_1, x_2, \ldots, x_n\}$ and $T_2 = \{y_1, y_2, \ldots, y_n\}$ of equal length n, the similarity measured by the Euclidean distance is defined as follows:

$$Dist\,(T_1, T_2) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{\frac{1}{2}}.$$

The Euclidean distance method follows the similar rationale of least square method for data fitting. The time series of historical CPI-U index is the observed time series; and the time series of CPI proxy can be treated as 'model-generated values' derived from either the 2013 weights or 2014 weights. The difference between the time-t historical CPI-U index and contemporaneous time-t CPI proxy can then be viewed as the residual. Thus, the time series of CPI proxy with a smaller sum of squared residuals fits the time series of historical CPI-U index better. More importantly, the similarity measured using the Euclidean distance fits our purpose for recognizing which time series of CPI proxy tracks the time series of historical CPI-U index more closely. Specifically, in Table 1.6, we generate four artificial time series $S1$, $S2$, $S3$ and $S4$. Time series $S2$ and $S3$ are generated by vertically shifting time series $S1$ by 1 unit and 3 units, respectively; while time series $S4$ is randomly generated. Obviously, all similarity measure methods including the Euclidean distance measure are capable of identifying time series $S4$ as being dissimilar to time series $S1$, which is demonstrated in panel C of Figure 1.3. The distinction between the Euclidean distance measure and other similarity measure methods can be illustrated by comparing panel A and panel B in Figure 1.3. Some similarity measure methods (e.g. elastic measure) may identify time series $S2$ and time series $S3$ as being equally similar to time series $S1$ due to the fact that both time series $S2$ and time series $S3$ are the vertical shifts of time series $S1$. This conclusion is not robust when similarity is measured by the Euclidean distance. That is, $Dist(S_1, S_2) = 3.16 < Dist(S_1, S_3) = 9.49$, i.e. compared with time series $S3$, time series $S2$ is more similar to time series $S1$. In other words, when measuring similarity, the Euclidean distance not only requires the measured time series to resemble the original time series, but also requires the time-t observation of measured time series to track closely the contemporaneous time-t observation of original time series. Therefore, the similarity measured by the Euclidean distance fits our purpose for identifying which time series of CPI proxy tracks the time series of historical CPI-U index more closely. Consequently, the smaller the value of Euclidean distance, the more closely the measured time series tracks the original time series.

Table 1.6. Artificial Time Series

S1	S2	S3	S4
10	9	7	12
10.2	9.2	7.2	8
10.1	9.1	7.1	6.5
10.5	9.5	7.5	12
10.1	9.1	7.1	11.9
10	9	7	7
10.4	9.4	7.4	10
10.2	9.2	7.2	11.8
10	9	7	6.3
10.1	9.1	7.1	8

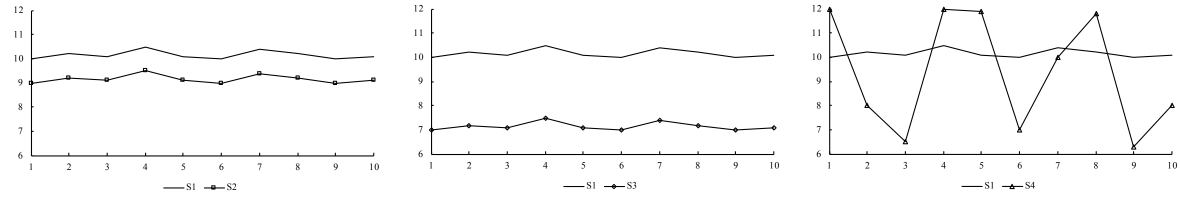


Figure 1.3. This figure plots the comparison between artificial time series. Line chart A compares time series S1 (solid line) with time series S2 (solid line with square symbol); line chart B compares time series S1 (solid line) with time series S3 (solid line with diamond symbol); line chart C compares time series S1 (solid line) with time series S4 (solid line with triangle symbol).

If we define the time series of historical CPI-U index as $T_{CPI} = \{CPI_1, CPI_2, \ldots, CPI_{26}\}$, the time series of CPI proxy derived from 2013 relative importance as

$$T_{CP}^{2013} = \{CP_1^{2013}, CP_2^{2013}, \ldots, CP_{26}^{2013}\},$$

and the time series of CPI proxy derived from 2014 relative importance as

$$T_{CP}^{2014} = \{CP_1^{2014}, CP_2^{2014}, \ldots, CP_{26}^{2014}\},$$

then the Euclidean distance between T_{CPI} and T_{CP}^{2013} is defined as:

$$Dist\left(T_{CPI}, T_{CP}^{2013}\right) = \left(\sum_{i=1}^{26} \left(CPI_i - CP_i^{2013}\right)^2\right)^{\frac{1}{2}}.$$

Similarly, the Euclidean distance between T_{CPI} and T_{CP}^{2014} can be defined as:

$$Dist\left(T_{CPI}, T_{CP}^{2014}\right) = \left(\sum_{i=1}^{26} \left(CPI_i - CP_i^{2014}\right)^2\right)^{\frac{1}{2}}.$$

Given the data in Table 1.5, $Dist\left(T_{CPI}, T_{CP}^{2013}\right)$ can be calculated as 58.82; and $Dist\left(T_{CPI}, T_{CP}^{2014}\right)$ is 82.41, which indicates that, compared with the time series of CPI proxy derived from 2014 relative importance, the time series of CPI proxy derived from 2013 relative importance tracks the time series of historical CPI-U index more closely.

Consequently, based on the analysis of Table 1.5, Figure 1.1, Figure 1.2 and similarity measure, the renormalized 2013 weights yield a time series of CPI proxy that tracks the time series of historical CPI-U indexes more closely. Therefore, the renormalized 2013 weights are the optimum weights for the commodities.

Given the optimum weights for commodities, we now look into the future and construct the term structure of futures on the CPI proxy. In the next section, we replace the missing commodity futures price observations.

1.5.2 Replacing the Missing Commodity Futures Prices

As stated in the data description section, due to different number and timing of commodity futures contract maturity months within a calendar year and distinct longest observable maturities of commodity futures contracts, some commodity futures prices are unobservable. In order to construct the complete term structure of futures on the CPI proxy, we need to replace the missing commodity futures prices. We apply the linear interpolation and extrapolation method to replace the missing commodity futures prices. The replaced commodity futures prices are presented in Table 1.7.

Table 1.7. Replacing the Unobservable Commodity Futures Prices

Ticker	May-15	Jun-15	Jul-15	Aug-15	Sep-15	Oct-15	Nov-15	Dec-15	Jan-16	Feb-16	Mar-16	Apr-16	May-16
YP	527.25	526.0625	524.875	529.125	533.375	537.75	542.125	546.5	550.9167	555.3333	559.75	564.125	568.5
FC	214.6	214.9917	215.3833	215.775	214.775	213.725	212.7	209.5625	206.425	204.8375	203.25	201.6625	200.075
LH	68.75	76.525	77.65	78.275	74.225	70.175	68.7625	67.35	68.675	70	70.9875	71.975	75.825
LC	157.1375	151.8	150.1375	148.475	149.425	150.375	150.7375	151.1	150.8125	150.525	150.0625	149.6	145.9125
DA	15.92	15.85	16.36	16.85	17.18	17.25	17.26	17.23	16.87	16.75	16.7	16.55	16.56
CHE	1.633	1.636	1.697	1.752	1.789	1.8	1.805	1.795	1.749	1.755	1.737	1.743	1.725
FSB	12.77	12.77	12.77	12.9467	13.1233	13.3	13.52	13.74	13.96	14.18	14.4	14.43	14.46
LE	101.975	105.75	108.1	111	115	117.5	118.925	119.05	124.5	127.5	129.75	132	135
JO	119	119.9	120.8	121.775	122.75	122.25	121.75	122.325	122.9	123.925	124.95	125.9	126.85
FCC	141.8	143.325	144.85	146.25	147.65	148.9333	150.2167	151.5	152.7667	154.0333	155.3	156.375	157.45
V6	178.525	180.05	181.25	183.75	184.25	185	186	186.5	181.9	173.5	173.5	173.5	173.5
DRW	43.5	41.75	40.55	39.525	40.025	40	39.5	39.975	40.475	40.975	42	41	40.525
FCO	2787	2759.5	2732	2748	2764	2759	2754	2749	2745	2741	2737	2734	2731
BO	31	31.11	31.22	31.28	31.32	31.3	31.345	31.39	31.59	31.72	31.85	31.955	32.06
KV	14.2	14.72	14.96	15.27	15.73	15.84	16.08	16	16.54	16.88	16.9	16.78	16.88
CPI	600.25	595.5	599.25	598.75	598	600.75	603.25	606	608.75	610.5	612	611.5	610.75
DJE	308.7	309	309.3	309.6	309.9	309.6	309.3	309	309	309	309	309	309
NG	2.661	2.708	2.767	2.794	2.799	2.825	2.933	3.105	3.214	3.207	3.154	3.01	3.011
LB	270	267.7	265.4	268	270.6	271.55	272.5	278.75	285	286	287	288	289
PH	22	21.95	25.45	25.45	21.95	21.85	22.75	25.2	34.65	34.65	25	25	21.75
HRC	505	510	510	510	515	520	520	520	540	550	554	555	555
FCT	66.33	66.12	65.91	65.9467	65.9833	66.02	65.83	65.64	65.53	65.42	65.31	65.49	65.67
DFL	393.5	393.5	392	394	394	405	405	405	422.5	422.5	422.5	422.5	422.5
HG	275.75	275.6	275.85	276.9	275.7	276.75	276.85	275.5	276.65	276.8	275.85	276.4	276.45
OP	2.756	2.764	2.76	2.768	2.766	2.768	2.768	2.764	2.766	2.768	2.764	2.764	2.762
GC	1208.3	1208.2	1209.75	1211.3	1211.15	1211	1211	1211	1212.8	1214.6	1215.25	1215.9	1216.4
CL	53.25	54.55	55.42	55.87	56.31	56.78	57.39	57.89	58.23	58.56	58.87	59.2	59.43
IXC	724.5	723.9	723.3	722.7	722.1	721.5	720.9	720.3	719.7	719.1	718.5	717.9	717.3
SI	16.825	16.86	16.87	16.8975	16.925	16.9433	16.9617	16.98	16.975	16.9725	16.97	17.003	17.036
MKC	567.375	569.375	571.375	577.375	583.375	588.0417	592.7083	597.375	600.1667	602.9583	605.75	609.25	612.75
O	273	273.125	273.25	275	276.75	277	277.25	277.5	279.8333	282.1667	284.5	286.875	289.25
RR	10.68	10.8225	10.965	11.0575	11.15	11.25	11.35	11.475	11.6	11.6025	11.605	11.605	11.605
KW	566.5	568.75	571	576	581	586	591	596	600.0833	604.1667	608.25	608.625	609
W	526	525.875	525.75	530.25	534.75	539.1667	543.5833	548	552.0833	556.1667	560.25	565.25	570.25

Notes: The shaded numbers represent the interpolated and extrapolated commodity futures prices.

In Table 1.7, the shaded numbers represent the interpolated and extrapolated commodity futures prices. It can be seen from Table 1.7 that, for the majority (22 out of 34) of commodities, the number of observable futures contract maturity months within a calendar year is less than 12; and the timing of futures contract maturity months varies from one commodity to another. Given that the commodity futures price dataset is complete, we now use the commodity's optimum weight and complete commodity futures price dataset to construct the term structure of futures on the CPI proxy.

1.5.3 Term Structure of Futures on the CPI Proxy

Given the optimum weights for commodities and complete commodity futures price dataset, we now look into the future and construct the term structure of futures on the CPI proxy. The consumer price index measures the present and historical price level, which suggests that the term structure of consumer price index predicting the future price level does not exist. However, the commodity futures market forecasts the future commodity price level by pricing the commodity futures contracts and updating their pricing continuously. The commodity futures prices reveal the prices of the underlying commodity to be delivered in a certain future month. Collecting today's last price data for all 34 available commodity futures contracts maturing in the same month and averaging these commodity futures prices using their optimum weights, will create the futures on the CPI proxy for that month. Collecting all of the futures on the CPI proxy corresponding to different months in the future will construct the term structure of futures on the CPI proxy for the analysis period. Specifically, the time 0 futures on the CPI proxy corresponding to a specific future time T is defined as:

$$\sum_{i=1}^{34} a_i F_i(0,T),$$

where a_i is the optimum weight for the i^{th} commodity; $F_i(0,T)$ is the time 0 last price for the futures contract written on the i^{th} commodity maturing in month T; we have 34 commodity futures prices for each maturity month T. The weighted average of commodity futures prices, i.e. $\sum_{i=1}^{34} a_i F_i(0,T)$, is the futures on the CPI proxy for maturity month T. The analysis period in our study consists of 13 consecutive months (from May 2015 to May 2016), collecting the futures on the CPI proxy corresponding to the 13 months will construct the term structure of futures on the CPI proxy. Table 1.8 presents the term structure of futures on the CPI proxy.

Table 1.8. Term Structure of Futures on the CPI Proxy

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2015					253.678	253.891	254.205	254.632	254.814	254.823	254.828	254.920
2016	255.790	255.898	255.410	255.413	255.185							

It should be noted that this term structure of futures on the CPI proxy is derived from the commodity futures price data observed on the reference date, i.e. 7^{th} of April 2015. For commodity futures price data observed on different historical date, the term structure of futures on the CPI proxy can be different because the commodity futures market keeps updating the commodity futures prices continuously, which suggests that the commodity futures market keeps updating its expectation about future commodity price level continuously.

In order to see clearly the trend of the term structure of futures on the CPI proxy, we plot the term structure in Figure 1.4.

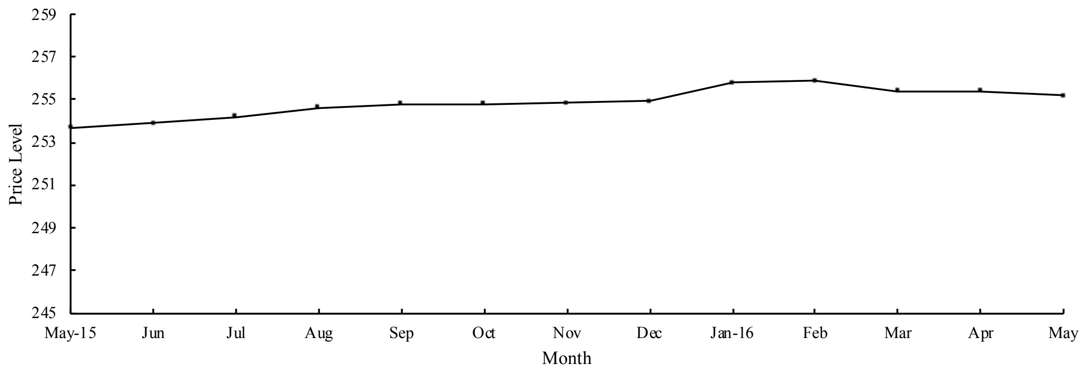


Figure 1.4. This line chart plots the term structure of futures on the CPI proxy constructed on 7^{th} of April 2015.

From Table 1.8 and Figure 1.4, we can see that, the term structure of futures on the CPI proxy starts with 253.678 at May 2015 and keeps increasing towards 255.898 at February 2016, then decreases slightly before finishing at 255.185 at May 2016. It also suggests that, the commodity futures market expects the price level during the coming 13 months to be higher than the latest U.S. CPI-U 234.722. Furthermore, based on the term structure of futures on the CPI proxy, the expected inflation rate between May 2015 and May 2016 is 0.59%, which indicates that the commodity futures market expects that the general level of commodity prices is going to increase by 0.59% from May 2015 to May 2016.

1.6 Conclusion and Future Research

In this paper, we describe an approach to tackle the non-availability of inflation futures data. Based on the 'Relative importance of components in the Consumer Price Indexes: U.S. city average' and historical price data of expired commodity futures contracts, we determine the optimum weight for each commodity. Based on the optimum weights and available active commodity futures contracts, we construct the term structure of futures on the CPI proxy, which will be useful in future research as the dataset for constructing a two-factor valuation model by following the Schwartz (1997) method.

Chapter 2

An Analysis of the Futures on the CPI Proxy Using the Schwartz' (1997) Method

Coauthored with CHRISTIAN-OLIVER EWALD

Abstract

In this paper, we construct a two-factor valuation model filtering the spot consumer price index and the instantaneous real interest rate. Based on the futures on the CPI proxy dataset, the parameters of the two-factor valuation model are estimated by applying the Kalman filter. The main results of our study are threefold. First, the parameter estimation results confirm the relevance of using the two-factor valuation model to analyse the futures on the CPI proxy dataset. Second, we introduce the spot consumer price index which may help alleviate the issue of U.S. CPI-U publication lag. Third, the two-factor valuation model is capable of forecasting the trend of inflation rate in the near future.

2.1 Introduction

Due to data collection and computation issues, the current U.S. CPI-U index is always reported with an unavoidable one-month lag. This one-month lag has an adverse impact on the effectiveness of using the inflation-indexed securities to hedge inflation risk. With this lag, inflation-indexed securities do not provide the exact real return but only approximate real return (Jarrow and Yildirim (2003)). This lag also indicates that the spot CPI-U index without lag is unobservable.

This paper aims to alleviate the issue of CPI-U publication lag by introducing the spot consumer price index (spot CPI). Inspired by the Fisher equation, given that the consumer price index is implicit in the expected inflation rate, the connection between the consumer price index, nominal interest rate and real interest rate is the foundation for introducing the spot CPI. However, the instantaneous real interest rate cannot be observed directly.

The non-observability of both spot CPI and instantaneous real interest rate is analogous to the case of unobservable spot commodity price and convenience yield. This inspires us to construct a valuation model analogous to the Schwartz (1997)'s framework to estimate both latent variables. Gibson and Schwartz (1990) introduce a two-factor reduced form commodity pricing model, in which spot commodity price and convenience yield follow a joint stochastic process. Schwartz (1997) develops a variation of the two-factor Gibson and Schwartz (1990) model and tests empirically the validity of the model using commodity futures dataset, which forms the classical commodity pricing framework that is widely applied in energy product evaluation literature.

Apart from intuitive analogy to the case of spot commodity price and convenience yield, evidence can be found to support applying Schwartz (1997)'s framework to estimate both spot CPI and instantaneous real interest rate. Schwartz (1997)'s model assumes that the two latent variables follow a joint stochastic process and the convenience yield is mean-reverting. In our case, the futures on the CPI proxy is constructed as the weighted sum of commodity futures prices, the spot CPI may be viewed as the spot price of a commodity basket. It is therefore reasonable to assume that the spot CPI has similar behaviour with the spot commodity price in the Schwartz (1997)'s two-factor model. Furthermore, Lai (2015)'s result indicates that the U.S. real interest rate is mean-reverting when a structural shift is taken into consideration, which supports our model in assuming that the instantaneous real interest rate follows an Ornstein-Uhlenbeck stochastic process.

The key to the success of Schwartz (1997)'s framework is that the closed form solution for the prices of futures contracts can be derived from the joint stochastic process. Once the model is cast into the state space form, values of latent variables can be filtered from the time series of the observable futures contracts, which in our case corresponds to the futures contracts written on the consumer price index. Inflation futures contracts written on the consumer price index have been launched since 2004. However, success remains very limited. Due to illiquidity, the Eurex HICP futures contract was delisted in 2014. In order to tackle the non-availability of exchange-traded inflation futures data, we introduce the futures on the CPI proxy in Chapter 1. The futures on the CPI proxy is the futures contracts written on the CPI. By applying the Kalman filter, we estimate the time series of both the spot CPI and instantaneous real interest rate from the futures on the CPI proxy dataset. Based on the parameter estimation results from the Kalman filter, the relevance of using the two-factor valuation model to analyse the term structures of futures on the CPI proxy is confirmed. Furthermore, we investigate the behaviour of spot CPI and instantaneous real interest rate and use the two-factor valuation model to forecast the expected inflation rate.

Our contribution is twofold. First, we explore to apply the Schwartz (1997)'s framework to estimate the spot CPI and instantaneous real interest rate. The spot CPI helps alleviate the issue of U.S. CPI-U publication lag. The publication lag gives rise to the indexation lag of inflation-indexed securities, so the inflation-indexed securities do not provide the exact real return. As Deacon et al. (2004) suggest, the indexation lag can be reduced by indexing to a price index that is published with a short lag. So our contribution is that the spot CPI helps further reduce the indexation lag by minimising the price index publication lag. Second, we use the forward curves generated from the two-factor valuation model to forecast the expected inflation rate. The two-factor valuation model is capable to predict the downward trend in the realised U.S. CPI inflation rate during May 2014 to December 2014.

We organise this paper as follows. In section 2.2, we construct the two-factor valuation model. Section 2.3 describes the construction of the futures on the CPI proxy dataset. Section 2.4 presents the state space form and analyses the empirical results. Finally, section 2.5 discusses the results and concludes the paper.

2.2 Valuation Model

In this section, we develop the two-factor valuation model and derive the corresponding analytical formula for pricing futures contract (the futures on the CPI proxy). Our model is based on the two-factor commodity pricing model from Schwartz (1997) which describes the joint stochastic process of commodity spot price and convenience yield. The two stochastic factors in our model are the spot consumer price index and instantaneous real interest rate. The spot consumer price index is assumed to be lognormal and the instantaneous real interest rate follows an Ornstein-Uhlenbeck stochastic process:

$$dCPI(t) = CPI(t)(\mu - r_R(t))dt + CPI(t)\sigma_{CPI}dW_1(t) \tag{2.1}$$

$$dr_R(t) = \kappa(\alpha - r_R(t))\,dt + \sigma_{r_R}dW_2(t), \tag{2.2}$$

where

• $CPI(t)$ is the spot consumer price index at time t (with no lag in publication), which cannot be observed directly;

• $r_R(t)$ is the instantaneous real interest rate at time t, which is also unobservable;

• μ is the drift;

• σ_{CPI} denotes the volatility of spot consumer price index, while σ_{r_R} is the volatility of instantaneous real interest rate;

• κ is the instantaneous real interest rate's speed of mean reversion;

• α is the long-run mean of instantaneous real interest rate;

• W_1 and W_2 are standard Brownian motions which are assumed to have correlation of $dW_1dW_2 = \rho dt$, ρ being constant. Specifically, equation (2.1) can be rewritten as follows,

$$\frac{dCPI(t)}{CPI(t)} = (\mu - r_R(t))dt + \sigma_{CPI}dW_1(t). \tag{2.3}$$

Taking the expectation of equation (2.3), we obtain

$$E\left[\frac{dCPI(t)}{CPI(t)}\right] = (\mu - r_R(t))dt, \tag{2.4}$$

where $E[\cdot]$ denotes the expectation under the real-world measure. The drift μ in the real-world measure dynamic of the spot CPI is the difference between the nominal interest rate and the inflation risk premium. Therefore, equation (2.4) can be linked to the inflation risk premium defined in the macroeconomic inflation risk premium literature, i.e.

$$E\left[\frac{dCPI(t)}{CPI(t)}\right] = ((r-\phi) - r_R(t))dt, \tag{2.5}$$

where r is the nominal interest rate; ϕ denotes the (annualised) inflation risk premium. Rearranging equation (2.5), we have

$$(r - r_R(t))dt = E\left[\frac{dCPI(t)}{CPI(t)}\right] + \phi dt. \tag{2.6}$$

The left-hand side of equation (2.6) is the difference between the nominal interest rate and the real interest rate; the right-hand side of equation (2.6) denotes the inflation compensation comprising the expected inflation rate $E\left[\frac{dCPI(t)}{CPI(t)}\right]$ and the inflation risk premium ϕdt. Thus, equation (2.6) replicates the definition of inflation risk premium used in the macroeconomic inflation risk premium literature (Bekaert and Wang (2010), Garcia and Werner (2010)). The definition of μ as described in equations (2.3) to (2.6) will prove useful in estimating the parameters of the two-factor valuation model.

Under the equivalent martingale measure, the joint stochastic process is as follows,

$$dCPI\,(t) = CPI\,(t)\,(r - r_R(t))\,dt + CPI(t)\sigma_{CPI} dW_1^Q(t) \tag{2.7}$$

$$dr_R\,(t) = \kappa\,(\overline{\alpha} - r_R(t))\,dt + \sigma_{r_R} dW_2^Q(t), \tag{2.8}$$

where

$$\overline{\alpha} = \alpha - \lambda/\kappa \tag{2.9}$$

λ being the scaled market price of instantaneous real interest rate risk; and W_1^Q, W_2^Q denote standard Brownian motions under equivalent martingale measure.

Next, based on the two-factor valuation model, we derive the analytical pricing formula for the futures on the CPI proxy, which is the futures contract written on the consumer price index. Applying Ito's formula, the solution to the spot CPI lognormal process is as follows,

$$CPI\,(T) = CPI\,(0)\,exp\left(\left(\mu - \frac{1}{2}\sigma_{CPI}^2\right)T - \int_0^T r_R(s)ds + \sigma_{CPI}\int_0^T dW_1(s)\right). \tag{2.10}$$

For analysis, we need the discounted spot CPI process,

$$\begin{aligned} e^{-rT}CPI(T) =& CPI(0)\,exp\left(\left(\mu-\frac{1}{2}\sigma_{CPI}^2-r\right)T-\int_0^T r_R(s)ds\right.\\ &\left.+\sigma_{CPI}\int_0^T dW_1(s)\right). \end{aligned} \tag{2.11}$$

Following Mastro (2013), the integral of instantaneous real interest rate in (2.11) is found by using the solution of integral of convenience yield in Bjerksund (1991),

$$\begin{aligned} \int_0^T r_R(s)ds =& \frac{1}{\kappa}\left(1-e^{-\kappa T}\right)\left(r_R(0)-\alpha\right)+\alpha T-\frac{1}{\kappa}\sigma_{r_R}e^{-\kappa T}\int_0^T e^{\kappa s}dW_2(s)\\ &+\frac{1}{\kappa}\sigma_{r_R}\int_0^T dW_2(s). \end{aligned} \tag{2.12}$$

The discounted spot CPI process can now be solved by inserting (2.12) into (2.11) and changing from the real-world measure to the equivalent martingale measure,

$$\begin{aligned} e^{-rT}CPI(T) =& CPI(0)\,exp\left(\left(-\frac{1}{2}\sigma_{CPI}^2-\overline{\alpha}\right)T+\frac{1}{\kappa}\left(\overline{\alpha}-r_R(0)\right)\left(1-e^{-\kappa T}\right)\right.\\ &-\frac{\sigma_{r_R}}{\kappa}\int_0^T dW_2^Q(s)+\frac{\sigma_{r_R}}{\kappa}e^{-\kappa T}\int_0^T e^{\kappa s}dW_2^Q(s)\\ &\left.+\sigma_{CPI}\int_0^T dW_1^Q(s)\right). \end{aligned} \tag{2.13}$$

For convenience, we rewrite equation (2.13) as follows,

$$e^{-rT}CPI(T) = CPI(0)\,e^{Z}. \tag{2.14}$$

After calculation and rearranging, the mean ζ and variance σ_Z^2 of the normally distributed factor Z under the risk-neutral measure are as follows,

$$\zeta = E^Q[Z] = \left(-\frac{1}{2}\sigma_{CPI}^2-\overline{\alpha}\right)T+\frac{1}{\kappa}(\overline{\alpha}-r_R(0))(1-e^{-\kappa T}) \tag{2.15}$$

$$\begin{aligned} \sigma_Z^2 =& E^Q\left[\left(Z-E[Z]\right)^2\right]\\ =& \frac{\sigma_{r_R}^2}{\kappa^2}T+\frac{1}{2}\frac{\sigma_{r_R}^2}{\kappa^3}\left(1-e^{-2\kappa T}\right)+\sigma_{CPI}^2T-2\frac{\sigma_{r_R}^2}{\kappa^3}\left(1-e^{-\kappa T}\right)\\ &-2\frac{\sigma_{r_R}\sigma_{CPI}\rho}{\kappa}T+2\frac{\sigma_{r_R}\sigma_{CPI}\rho}{\kappa^2}\left(1-e^{-\kappa T}\right). \end{aligned} \tag{2.16}$$

The price of a time-zero-observed futures on the CPI proxy maturing at time T is,

$$\begin{aligned} G(CPI(0), r_R(0), 0, T) &= E^Q[CPI(T)] = E^Q\left[e^{rT} CPI(0) e^Z\right] \\ &= e^{rT} CPI(0) E^Q\left[e^Z\right] = e^{rT} CPI(0) e^{\zeta + \frac{1}{2}\sigma_Z^2}. \end{aligned} \tag{2.17}$$

Combining expressions (2.15) to (2.17), we obtain the pricing formula of the futures on the CPI proxy for the joint stochastic process:

$$G\left(CPI\left(0\right), r_R\left(0\right), 0, T\right) = CPI\left(0\right) e^{\zeta + \frac{1}{2}\sigma_Z^2 + rT} = CPI(0) e^{A(T) + B(T) r_R(0)} \tag{2.18}$$

with

$$\begin{aligned} A(T) &= \left(r - \overline{\alpha} + \frac{1}{2}\frac{\sigma_{r_R}^2}{\kappa^2} - \frac{\sigma_{CPI}\sigma_{r_R}\rho}{\kappa}\right) T + \frac{1}{4}\sigma_{r_R}^2 \frac{1 - e^{-2\kappa T}}{\kappa^3} \\ &\quad + \left(\kappa\overline{\alpha} + \sigma_{CPI}\sigma_{r_R}\rho - \frac{\sigma_{r_R}^2}{\kappa}\right)\frac{1 - e^{-\kappa T}}{\kappa^2}, \\ B(T) &= -\frac{1 - e^{-\kappa T}}{\kappa}. \end{aligned}$$

The logarithm of the futures on the CPI proxy price can then be derived,

$$lnG\left(CPI\left(0\right), r_R\left(0\right), 0, T\right) = lnCPI\left(0\right) + A\left(T\right) + B(T) r_R(0). \tag{2.19}$$

The linear (affine) relationship between the (logarithm) futures on the CPI proxy price and underlying two factors is important for applying the Kalman filter to estimate the two-factor valuation model parameters in the following context.

2.3 Futures on the CPI Proxy Dataset

The futures on the CPI proxy dataset is constructed from weekly observations of last price data (or settlement price if last price is unavailable) for commodity futures written on the 34 commodities constructing the futures on the CPI proxy. The original commodity futures price data collected from Chicago Mercantile Exchange using Bloomberg terminal consists of daily observations covering 2^{nd} of January 2013 to 18^{th} of December 2013. Every Wednesday observation (to avoid weekend effects (Ramos Ribeiro and Hodges (2004))) is selected out of the daily observations to obtain the weekly observations. At every observation date we collect last (settlement) price data for all available commodity futures observed on that historical date for each commodity. Based on these weekly observations of commodity futures, we follow the method introduced in Chapter 1 to construct the futures on the CPI proxy observed on every Wednesday during 2^{nd} of January 2013 to 18^{th} of December 2013. In Figure 2.1, we plot the term structures of futures on the CPI proxy. The solid line denotes the time series of the price of the closest to maturity futures on the CPI proxy; the dotted lines represent the prices of futures on the CPI proxy chain constructed on each observation date. For each observation date, we construct a futures on the CPI proxy chain consisting of twelve futures on the CPI proxy observations whose time to maturity range from one month to twelve months in the future. The futures on the CPI proxy dataset will be used to estimate the two-factor valuation model parameters as well as filtering the state variables.

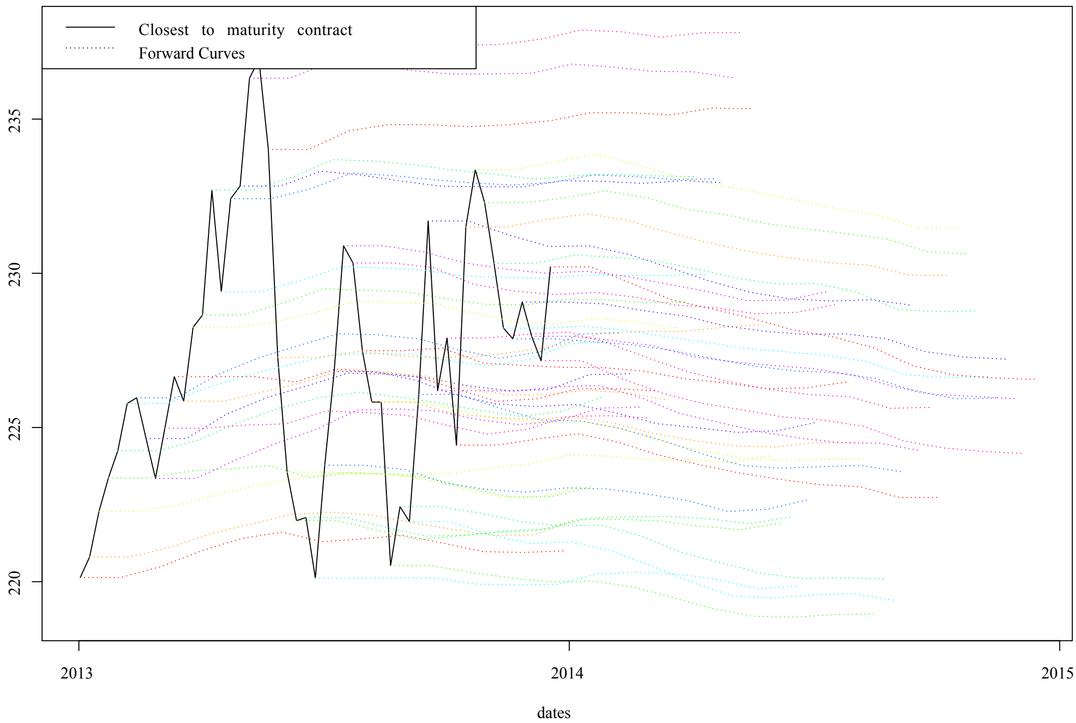


Figure 2.1. This figure plots the term structures of futures on the CPI proxy (or time series of futures on the CPI proxy chain, the two terminologies are used interchangeably in the thesis). The solid line denotes the time series of the price of the closest to maturity futures on the CPI proxy; each dotted line represents the prices of futures on the CPI proxy chain observed on a observation date.

2.4 Empirical Results

The available time series of U.S. CPI-U is the consumer price index with an inevitable one-month lag. The spot CPI with no lag, however, cannot be observed directly. Furthermore, the instantaneous real interest rate is also unobservable. By definition, the futures on the CPI proxy is the weighted sum of exchange-traded futures contracts on the commodities corresponding to the consumption goods comprising the U.S. CPI-U. Following Schwartz (1997), the non-observability of spot CPI and instantaneous real interest rate, plus the linear relationship between the (logarithm) futures on the CPI proxy and state variables indicates that the Kalman filter is the appropriate technique to estimate the two-factor valuation model parameters.

2.4.1 The State Space Form

In this section, we follow Erb et al. (2014) and put the two-factor valuation model into the state space form. The Kalman filter may only be applied if the two-factor valuation model is put in state space form. The state space representation applies to the time series of observable variables, which in our study is a time series of the futures on the CPI proxy maturing at different maturities. The observable variables are related to the unobservable state variables (i.e. the spot CPI and instantaneous real interest rate) via the measurement equation. The measurement equation is obtained from equation (2.19) by adding serially and cross-sectionally independent disturbances:

$$y_t = c_t + Z_t [x_t, r_{R_t}]' + Q_t \eta_t, \tag{2.20}$$

where

- $y_t = \begin{pmatrix} lnG_t(1) \\ \vdots \\ lnG_t(n) \end{pmatrix}$ is the (logarithm) futures on the CPI proxy chain observed at time t;
- $c_t = \begin{pmatrix} A(m_t(1)) \\ \vdots \\ A(m_t(n)) \end{pmatrix}$, and $m_t(i)$ denotes the remaining time to maturity of the ith closest to maturity futures on the CPI proxy $G_t(i)$;

• $Z_t = \begin{pmatrix} 1 & B(m_t(1)) \\ \vdots & \vdots \\ 1 & B(m_t(n)) \end{pmatrix}$;

• $x_t = ln(CPI(t))$ is the logarithm of spot CPI at time t;

• $Q_t Q_t^{'} = \begin{pmatrix} q_{11}^2 & & \\ & \ddots & \\ & & q_{nn}^2 \end{pmatrix}$ is the innovations in the measurement equation, which is assumed independent to each other;

• $\eta_t \sim \mathcal{N}(0, I_n)$ is the disturbance in the measurement equation.

The evolution of the unobservable state variables is described by the transition equation, which is given by the discretized form of the joint stochastic dynamics in (2.1) and (2.2):

$$[x_{t+\Delta t}, r_{R_{t+\Delta t}}]^{'} = d_t + T_t [x_t, r_{R_t}]^{'} + H_t \varepsilon_t, \tag{2.21}$$

where

• $d_t = \begin{pmatrix} \left(\mu - \frac{1}{2}\sigma_{CPI}^2 - \alpha\right)\Delta t + \frac{\alpha}{\kappa}(1 - e^{-\kappa \Delta t}) \\ \alpha(1 - e^{-\kappa \Delta t}) \end{pmatrix}$;

• $T_t = \begin{pmatrix} 1 & \frac{1}{\kappa}(e^{-\kappa \Delta t} - 1) \\ 0 & e^{-\kappa \Delta t} \end{pmatrix}$;

• $H_t H_t^{'} = \begin{pmatrix} \sigma_x^2(\Delta t) & \sigma_{xr_R}(\Delta t) \\ \sigma_{xr_R}(\Delta t) & \sigma_{r_R}^2(\Delta t) \end{pmatrix}$;

• $\varepsilon_t \sim \mathcal{N}(0, I_2)$.

We can see from expressions (2.20) and (2.21) that, the two-factor valuation model parameters are included in the matrices of measurement equation and transition equation. These parameters can be estimated by applying the Kalman filter via maximizing the log-likelihood function:

$$-\frac{T ln(2\pi)}{2} - \frac{1}{2}\sum_{t=1}^{T} ln\left(Var\left(y_{tP}\right)\right) - \frac{1}{2}\sum_{t=1}^{T} \frac{(y_t - E[y_{tP}])^2}{Var(y_{tP})}, \tag{2.22}$$

where y_{tP} denotes the predicted value of (logarithm) futures on the CPI proxy generated from the measurement equation at each time step.

The two-factor valuation model parameters estimation as well as the state variables filtration are realised by applying the R package ‘Schwartz97’. The parameter estimation results and corresponding interpretations are presented in the next section.

2.4.2 Empirical Results

Parameter estimation of Schwartz (1997) two-factor valuation model is statistically fragile. Multiple local maxima of the likelihood may exist which can result in absurd parameter estimates (Lüthi et al. (2014)). To obtain reasonable and meaningful parameter estimates, it is necessary to hold different subsets of parameters constant. For this reason, in our study, the initial spot CPI and initial instantaneous real interest rate are held constant; the risk-free interest rate r is equal to 0.032, which is reasonable given the U.S. 30-Year Treasury Constant Maturity Rate in 2013. Ideally, all the other parameters of two-factor valuation model can be estimated from within the Kalman filter. However, our calculation suggests that with leaving μ free as a parameter in the estimation, the Kalman filter does not converge very well. Therefore, we run the Kalman filter with different set values of μ. The candidate set values of μ are chosen based on the definition of μ as described in equations (2.3) to (2.6). In the inflation risk premium literature, the magnitude of inflation risk premium estimate is usually small, e.g. Buraschi and Jiltsov (2005) argue that the average 10-year inflation risk premium can be as small as 0.7%. In addition, recent studies such as Grishchenko and Huang (2013), d'Amico et al. (2016) and Chen et al. (2016) indicate that inflation risk premium estimates switch sign from positive to negative in 2008 and have been trending downward. Thus, given that $\mu = r - \phi$, the candidate set values of μ should be slightly larger than the nominal interest rate r, e.g. 0.0326. We run the Kalman filter with different set values of μ in the proximity of 0.032 and choose the estimate of μ that yields large log-likelihood and meaningful estimates for the remaining parameters. Using this method, the Kalman filter yields reasonable estimates for the parameters of two-factor valuation model. Table 2.1 presents the parameter estimation results based on the historical data for weekly observations of futures on the CPI proxy from 2^{nd} of January 2013 to 18^{th} of December 2013.

It can be seen from the parameter estimation results that the correlation coefficient ρ is large; the speed of mean reversion of instantaneous real interest rate κ and the scaled market price of instantaneous real interest rate risk λ are all positive and reasonable. The estimate of the market price of instantaneous real interest rate risk is $\bar{\lambda} = \frac{\lambda}{\sigma_{r_R}} = 0.0113$. Chernyakov (2013)'s study of the magnitude of market price of real interest rate risk suggests that, empirically, there is little evidence that real interest rate risk is priced at all. Our estimate for $\bar{\lambda}$ supports their observation. Furthermore, the long-run mean level of instantaneous real interest rate α is 0.0164. According to the World Bank Dataset, the U.S. 2013 real interest rate is 1.61%. Our estimate of long-term mean of instantaneous real interest rate indicates

Table 2.1. Parameter Estimation Results

Parameters	Estimates
mu	0.0328
sigma(CPI)	0.2397
kappa	2.2520
alpha	0.0164
sigma(r_R)	0.3968
rho	0.9201
lambda	0.0045
alpha-bar	0.0144
logLik	3039.8
Number of Iterations	1686

that the corresponding annual real interest rate is 1.65%, which is close to the actual U.S. 2013 real interest rate. This suggests that the parameter estimates are indeed economically meaningful. Therefore, the parameter estimation results indicate that the relevance of applying the two-factor valuation model to analyse the term structures of futures on the CPI proxy is confirmed.

Figure 2.2 depicts the parameter evolution of the two-factor valuation model parameter estimation. The relative tolerance decreases below 10^{-8} after 1500 iterations and the parameter values become more and more stationary, suggesting a good convergence.

The term structures of futures on the CPI proxy data from January 2013 to December 2013 are shown in Figure 2.3, where the left plot describes the actual term structures of futures on the CPI proxy data and the right plot presents the forward curves generated by the two-factor valuation model. By comparing these two plots, we can see that the two-factor valuation model's prediction looks reasonable. In general, the model captures the shapes of both short-term panels and long-term panels.

Based on the fitted forward curves in Figure 2.3, we extract the fitted futures on the CPI proxy values from the forward curves and calculate the inflation rate forecast using the filtered spot CPI and the value of fitted futures on the CPI proxy maturing twelve months from each observation date. Specifically, the one-year ahead expected inflation rate $i(t,T)$ is calculated as follows,

$$i(t,T) = \frac{F(t,T) - CPI(t)}{CPI(t)}, \tag{2.23}$$

where $CPI(t)$ is the filtered spot CPI on the observation date t; $F(t,T)$ is the time t price of the fitted futures on the CPI proxy maturing twelve months from the current month which the

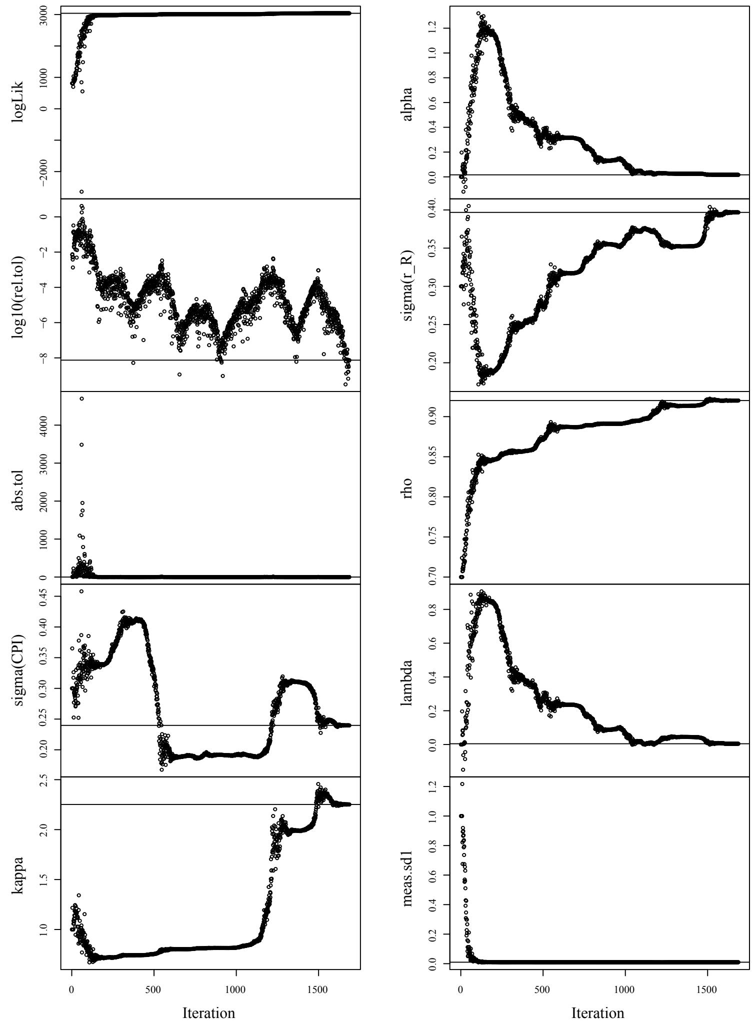


Figure 2.2. This figure depicts the parameter evolution of the two-factor valuation model parameter estimation.

observation date falls within. In Figure 2.4, we compare the time series of one-year ahead expected inflation rate (solid line) with the contemporaneous realised U.S. CPI inflation rate (dotted line), where X-axis denotes the months in 2014 and Y-axis represents the inflation rate in percentage. During the first half of 2014, the two-factor valuation model expects one-year ahead inflation. The inflation rate forecast switches sign from positive to negative in July 2014. Over the second half of 2014, the two-factor valuation model predicts one-year ahead deflation. Furthermore, the two-factor valuation model has been under-predicted the inflation rate during 2014. More importantly, the two-factor valuation model is capable to forecast the downward trend in the realised U.S. CPI inflation rate during May 2014 to December 2014.

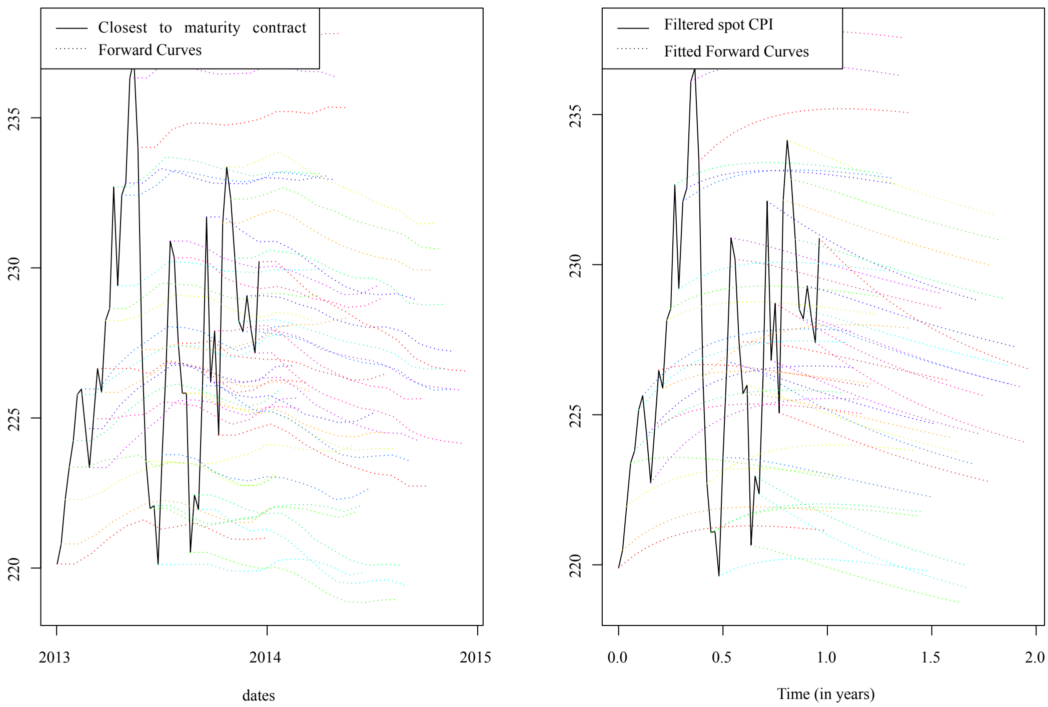


Figure 2.3. In the right plot, the unit of X-axis, 'Time', refers to time to maturity in years. For example, the right ending point of the fitted forward curve observed on the first observation date (i.e. 2^{nd} of January 2013) is the value of the fitted futures on the CPI proxy maturing in 1 year from 2^{nd} of January 2013; the right ending point of the corresponding forward curve in the left plot represents the value of the futures on the CPI proxy maturing in January 2014.

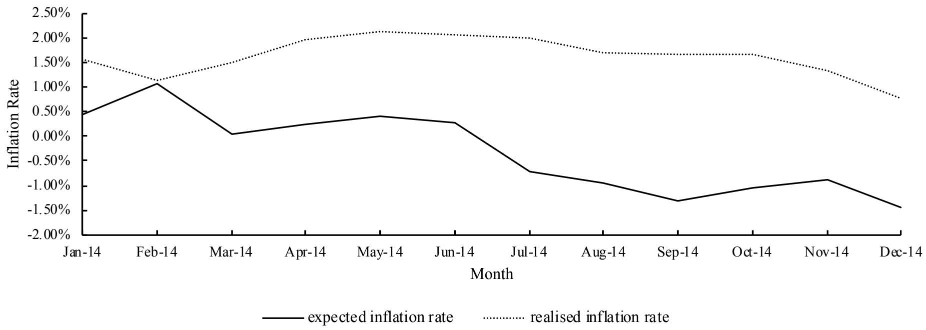


Figure 2.4. The line chart compares the time series of one-year ahead expected inflation rate with the contemporaneous realised U.S. CPI inflation rate. The original weekly forecasts are transformed into monthly forecasts which can be compared with the realised U.S. CPI inflation rate.

Figure 2.5 presents the filtered state variables, i.e. spot CPI and instantaneous real interest rate. It can be observed that the comovement between the two state variables is not obvious.

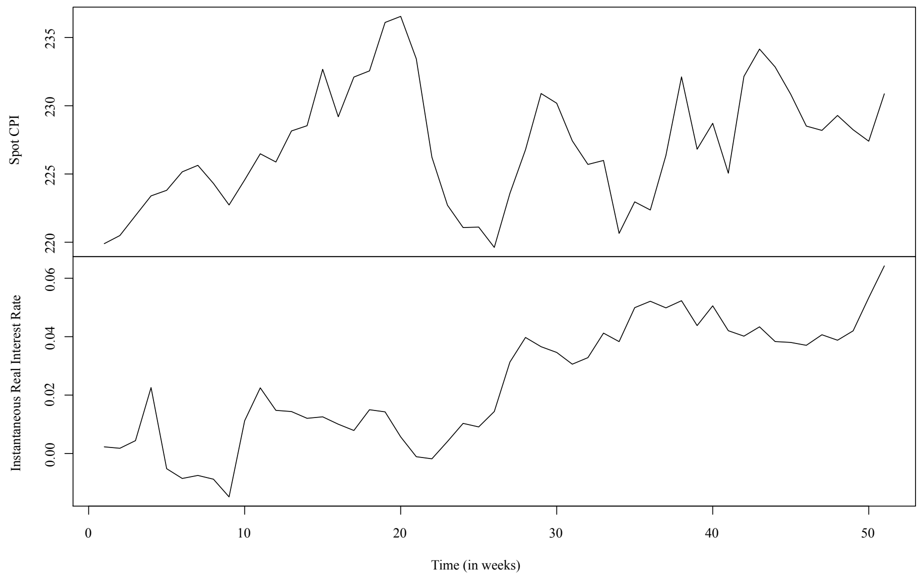


Figure 2.5. This figure plots the evolution of the filtered spot CPI and filtered instantaneous real interest rate.

In Figure 2.6, we compare the time series of filtered spot CPI (dotted line) with the time series of actual 2013 U.S. CPI (solid line). The original spot CPI is weekly-filtered, to obtain monthly observations we take the average of all weekly observations of the spot CPI falling within the same month. It can be observed that in each month the spot CPI is slightly smaller than the actual U.S. CPI, except in May where spot CPI is marginally larger than the U.S. CPI. In addition, the spot CPI is more volatile than the actual U.S. CPI, which is reasonable given the fluctuation observed in the closest to maturity futures on the CPI proxy time series.

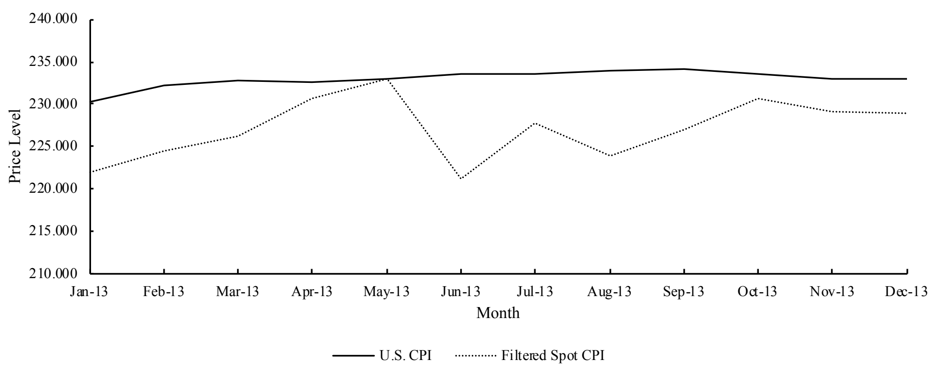


Figure 2.6. The line chart compares the time series of U.S. CPI (solid line) with the time series of filtered spot CPI (dotted line).

2.5 Discussion and Conclusions

We have analysed the behaviour of the term structures of futures on the CPI proxy using the Schwartz (1997)'s method and investigate their implications. Based on Schwartz (1997) two-factor commodity pricing model, we have constructed a two-factor valuation model filtering the spot CPI and instantaneous real interest rate. From the joint stochastic process, a closed-form pricing formula for the futures on the CPI proxy is derived. The two-factor valuation model is then cast into the state space form, which enables us to apply the Kalman filter to analyse the time series of the futures on the CPI proxy chain during 2^{nd} of January 2013 to 18^{th} of December 2013 and estimate the two-factor valuation model parameters.

The most significant two-factor valuation model parameter α, i.e. the long-run mean level of the instantaneous real interest rate is estimated as 1.64%. This implies that the corresponding annual real interest rate is estimated to be 1.65%, which is fairly close to the actual U.S. 2013 real interest rate 1.61%. The parameter estimation results are indeed economically meaningful. In addition, by comparing the term structures of futures on the CPI proxy with the fitted forward curves, it can be seen that the two-factor valuation model fits the dataset well. Therefore, the relevance of applying the two-factor valuation model to analyse the futures on the CPI proxy dataset is confirmed.

In addition, based on the futures on the CPI proxy dataset, we forecast the one-year ahead expected inflation rate. This method uses the filtered spot CPI as bases, and extracts the model-generated futures on the CPI proxy values from the fitted forward curves to forecast the inflation rate. The two-factor valuation model expects one-year ahead inflation during the first half of 2014, whereas it predicts one-year ahead deflation over the second half of 2014. More importantly, the two-factor valuation model is capable to predict the downward trend in the realised U.S. CPI inflation rate during May 2014 to December 2014.

What's more, we have introduced the consumer price index without any lag, i.e. the spot CPI. Our method of using the futures on the CPI proxy dataset to filter the spot CPI may help alleviate the unavoidable one-month lag in the publication of U.S. CPI-U index. Most importantly, the most significant advantage of our method is that, the underlying data (i.e. the commodity futures prices) is available at the exchanges immediately, therefore both the spot CPI and inflation rate forecast can be calculated immediately without any lag.

Chapter 3

Inflation Forecasting through Commodity Futures and Options

Coauthored with CHRISTIAN-OLIVER EWALD

Abstract

In this paper, we construct a synthetic option written on the futures on the CPI proxy and forecast inflation rate by analysing the option implied volatility surface. The futures on the CPI proxy derived from commodity futures price data predicts the CPI in the future. However, there are considerable uncertainties surrounding these 'point' forecasts of future CPI. Inspired by the Bank of England inflation fan chart, we estimate the volatility of the futures on the CPI proxy by analysing a synthetic option. Using Monte Carlo simulation, we construct a synthetic option written on the futures on the CPI proxy based on exchange-traded commodity futures options data. Our assumption of independent lognormal underlying commodity futures leads to a closed-form pricing formula for the European call option written on the futures on the CPI proxy. Using this Black's Model-type option pricing formula, we derive the implied volatility surface for the futures on the CPI proxy. This implied volatility surface provides us with information about the uncertainties surrounding the 'point' forecast of future price level, based on which we derive the inflation forecast.

3.1 Introduction

No prediction of future evolution of price level can be made with absolute certainty. It is more realistic to consider uncertainty when any prediction is made. The Bank of England publishes quarterly in its 'Inflation Report' the inflation fan chart depicting the Monetary Policy Committee (MPC)'s judgement of the probability of various outcomes for inflation in the future. Given the fact that each inflation fan chart is based on complex conditional assumptions, MPC key judgements and indicative projections, it is therefore difficult for any independent researcher to replicate the process of producing an inflation fan chart to predict future price level evolution.

However, the 'precisely engineered' Bank of England inflation fan chart is not the only way to figure out the uncertainty of future price level evolution. In this paper, we propose a method to estimate the uncertainty surrounding the consumer price index forecast from the perspective of commodity futures options.

In Chapter 1, we have constructed the futures on the CPI proxy based on the commodity futures price data. The futures on the CPI proxy is constructed as a synthetic futures contract written on the CPI. Several inflation futures contracts have been launched since 2004. However, success remains very limited. Due to illiquidity, the Eurex HICP futures contract was delisted in 2014. In order to tackle the non-availability of exchange-traded inflation futures data, we introduce the futures on the CPI proxy. The consumer price index is a composition of many items. An inspection at the futures exchange markets indicates that most of the items in the CPI are actually commodities on which active futures contracts are traded. Collecting these futures contracts and averaging them in the same way as the CPI is composed, will create a synthetic futures written on the consumer price index, that is, the futures on the CPI proxy. Specifically, based on the 'Relative importance of components in the Consumer Price Indexes: U.S. city average' published by the U.S. Bureau of Labor Statistics, we investigate the composition of U.S. CPI-U and locate the commodities on which commodity futures are traded; next, we determine optimum weight for each commodity by matching the time series of CPI proxy with the historical U.S. CPI-U time series; and lastly, we define the weighted sum of commodity futures prices to construct the futures on the CPI proxy.

The term structure of futures on the CPI proxy provide us with a term structure of single point CPI forecasts up to one year in the future. Inspired by the inflation fan chart, we seek for a method to estimate the uncertainty surrounding the 'point' forecast of future price level. One important feature of commodity markets is that commodity futures and commodity futures options are usually traded side by side in the same exchange. Besides, options contain information on their underlying assets' risk. This implies that we can use commodity futures options to derive the risk of underlying commodity futures and therefore figure out the uncertainties surrounding the 'point' forecast of future price level.

Following this implication, we construct a synthetic European call option written on the futures on the CPI proxy. The synthetic option implied volatility surface is derived using a Black's model-type option pricing formula. The implication of the implied volatility surface is analysed and consequent uncertainties of the price level forecast is discussed. Lastly, we derive the inflation forecast from the uncertainties surrounding the 'point' forecast of future price level.

The existing inflation forecasting literature use four main methods to forecast inflation. First, forecasts based on past inflation. For example, Atkeson et al. (2001) random walk model. Second, Phillips curve forecasts, whose performance has been shown by Stock and Watson (2008) as being episodic and instable. Third, survey-based forecasts. Fourth, forecasting inflation using information embedded in asset prices. An important study is Fleckenstein et al. (2017), which use inflation swaps and options price data to solve for the instantaneous expected inflation rate for horizon up to 30 years. Our approaches for inflation forecasting in Chapter 2 and 3 join this branch of literature. Our study makes two main contributions to the existing literature. First, compared with Fleckenstein et al. (2017) that use inflation swaps, our inflation forecast is derived from commodity futures and option prices that are exchange-traded and accessible. Second, compared with the existing literature which rely on term structure models, we use different approaches to forecast inflation. That is, in Chapter 2, we use a Schwartz (1997) two-factor valuation model to forecast inflation; in Chapter 3, we derive the uncertainty in inflation forecast by constructing a synthetic option written on the futures on the CPI proxy.

We organise the structure of this paper as follows. Section 3.2 describes the dataset. Section 3.3 presents the method to construct a synthetic option on futures on the CPI proxy. Section 3.4 then derives the synthetic option implied volatility surface and analyses its implications and consequent inflation forecast. Finally, section 3.5 discusses and concludes the paper.

3.2 Data Description

We have three datasets in our analysis: daily settlement price data for commodity futures, implied volatility surface data and European commodity futures options price data. The three datasets are used to construct the synthetic option and study the option implied volatility surface.

3.2.1 Commodity Futures Price

We use the daily settlement price data for included commodity futures contracts observed on 22^{nd} of January 2016. Data is imported from Bloomberg terminal using the Bloomberg Excel Add-in. Since not all commodities used for constructing the futures on the CPI proxy have available futures options, we only include the commodities which have corresponding futures options written on them. This gives us 21 commodities. Due to the commodity futures maturity month arrangement, the futures price for a specific maturity month may be unobservable, we use linear interpolation and extrapolation to calculate the unobservable commodity futures price data. Furthermore, for most of the included commodities, the longest time to maturity of corresponding futures options is within 11 months. Taking this into consideration, we only include commodity futures maturing within 12 months.

Table 3.1 presents the commodity futures price data, where texts in boldface refer to the price data calculated using linear interpolation or extrapolation. It should be noted that the commodity futures serve as the underlying assets of options. Therefore we index the commodity futures by their corresponding options' expiry month. The maturity month of each commodity future in Table 3.1 is the commodity futures option's expiry month, not the maturity month of futures. In the case of CME Exchange, commodity futures options usually expire several weeks earlier than their underlying futures do.

3.2.2 Commodity Futures Options Implied Volatility Surface

Each observed commodity futures in Table 3.1 (regular texts) has a corresponding option chain. Every option chain consists of options with different strikes written on the same underlying commodity futures. For each such option, Bloomberg calculates the Black-Scholes implied volatility so that the American option model prices match the option market prices. The model for the American option is a standard Black-Scholes American Option PDE Pricer, which is solved numerically using a trinomial method.

Table 3.1. Commodity Futures Daily Settlement Price

Tickers	Feb-16	Mar-16	Apr-16	May-16	Jun-16	Jul-16	Aug-16	Sep-16	Oct-16	Nov-16	Dec-16
wheat w	475.50	480.25	480.25	**482.75**	485.25	**489.50**	493.75	**498.33**	**502.92**	507.50	**510.83**
wheat kw	470.75	480.75	480.75	**485.75**	490.75	**497.38**	504.00	**510.33**	**516.67**	523.00	**529.33**
rough rice rra	10.96	**11.10**	11.24	**11.38**	11.52	**11.57**	11.62	**11.73**	11.84	**11.84**	11.84
oats o a	205.50	211.00	211.00	**213.63**	216.25	**219.00**	221.75	**221.42**	**221.08**	220.75	**220.75**
silver si	14.06	14.08	14.08	14.10	14.10	**14.12**	14.13	**14.14**	**14.15**	14.16	**14.18**
crude oil cla	32.19	33.51	34.67	35.65	36.44	37.06	37.56	37.99	38.41	38.84	39.18
gold gc	1096.30	1096.30	1096.50	1096.50	1097.00	1097.00	1097.60	1097.60	1098.40	1098.40	1099.30
copper hg	200.25	200.70	200.70	201.25	201.25	201.80	201.80	202.15	202.15	202.15	202.40
lumber lba	234.40	**234.20**	234.00	**235.55**	237.10	**239.80**	242.50	**243.95**	245.40	**246.85**	**248.30**
natural gas nga	2.14	2.22	2.29	2.35	2.41	2.44	2.44	2.47	2.55	2.72	2.84
milk classic IV kva	13.28	13.87	14.05	14.05	14.29	**14.75**	14.89	**15.19**	15.35	15.33	**15.38**
soybean oil boa	30.50	30.69	30.69	**30.80**	30.91	30.96	30.99	30.94	**30.97**	30.99	31.20
dry whey drwa	23.65	23.48	23.63	23.93	23.58	**24.26**	24.25	**24.90**	25.05	26.48	**26.75**
butter v6	206.93	224.03	225.24	225.03	223.45	**224.23**	225.00	**221.24**	221.03	219.45	**217.23**
lean hog lha	63.00	66.00	69.00	75.38	78.95	78.78	78.13	**72.66**	67.20	**65.14**	63.08
live cattle lca	132.08	133.08	133.08	**128.46**	123.85	**122.00**	120.15	**120.53**	120.90	**120.99**	121.08
cattle feeder fca	159.43	157.90	157.83	157.50	**157.97**	**158.43**	158.90	156.68	154.10	149.23	**144.35**
juice jo	121.65	122.10	122.10	122.90	122.90	**123.25**	123.60	**123.78**	123.95	**125.35**	126.75
non-fat milk lea	78.23	76.18	77.88	78.50	80.00	**84.70**	86.03	**90.01**	92.00	94.00	**95.88**
cheese che	1.52	1.52	1.52	1.53	1.56	**1.62**	1.64	**1.70**	1.71	1.73	**1.72**
milk daa	13.72	13.78	13.84	13.93	14.27	**14.86**	15.07	**15.70**	15.85	15.95	**15.91**

For instance, the implied volatility (observed from Bloomberg's OVML option pricing function) of wheat futures maturing in March 2016 (with ticker 'W H6') is presented in Table 3.2.

Table 3.2. OVDV Implied Volatility Surface Example

Contract	Futures	75%	90%	95%	97.50%	100%	102.50%	105%	110%	125%
W H6	475.5	19.72	18.69	19.17	19.97	21.06	22.22	23.4	26.14	33.72
		356.625	427.95	451.725	463.613	475.5	487.387	499.275	523.05	594.375

As shown in Table 3.2, moneyness for different options is indexed as %Moneyness (i.e. (strike price/underlying asset price)*100%). For each option, Bloomberg computes the Black-Scholes implied volatility (the 2^{nd} row of Table 3.2). The 3^{rd} row of Table 3.2 presents the corresponding strike prices.

For each option chain, we observe five implied volatility values: 90% moneyness option, 95% moneyness option, 100% moneyness option, 105% moneyness option and 110% moneyness option, corresponding to the implied volatilities for two in-the-money call options, one at-the-money call option and two out-of-the-money call options, respectively. All implied volatility surfaces are observed on 22^{nd} of January 2016 from Bloomberg's OVML option pricing function. The implied volatility data is used by Bloomberg's OVML option pricing function to price the commodity futures European options, which will be useful in constructing the synthetic option.

3.2.3 Commodity Futures Options Price

Commodity futures and futures options are traded side by side in CME Exchange. For the majority of commodities in CME Exchange, options written on their futures are traded. As for our analysis, among the 34 commodities used in the construction of futures on the CPI proxy, 21 commodities have available options written on their futures.

Traded futures options are usually American. Among the included 21 commodities, only crude oil and natural gas have both American and European futures options traded.

In order to construct the synthetic option, we need to find the commodity futures European options price data. Our research shows that we can derive implicitly the European options prices from the American options market prices by using the implied volatility as a bridge. Specifically, Bloomberg calculates the OVDV Black-Scholes implied volatility surface by matching Black-Scholes model prices with the market American option prices. These implied volatility surface data for the included 21 commodities are used by Bloomberg's OVML option pricing function as inputs of standard Black's option pricing model to calculate the European option prices. Bloomberg's OVML option pricing function is capable of matching automatically the Black's model's underlying commodity futures price, discount curve, time to maturity and implied volatility with the corresponding American option price data, therefore the calculated European option prices are accurate and reliable. As a result, we use OVML function to calculate the European futures options prices.

For each implied volatility value as described in section 3.2.2, we use OVML function to calculate the corresponding European option price. Due to the fact that the futures price may be unobservable for certain maturity months, some European option prices lack the underlying asset price and implied volatility as Black's model's inputs and therefore cannot be computed from OVML function. In such occasions, we use linear interpolation and extrapolation to calculate the missing European option prices. This gives us a total of 1155 European option price values and these price data will be useful in constructing the synthetic option.

3.3 A Synthetic Option on the Futures on the CPI Proxy

The purpose of the research is to find out the volatility of the futures on the CPI proxy from a forward-looking perspective. Inspired by the market convention that traders often quote the implied volatility of an option rather than the option price, we need to find an option contract written on the futures on the CPI proxy so that the implied volatility of futures on the CPI proxy can be derived. But in practice there is no such an option traded because the futures on the CPI proxy is itself synthetic.

Our solution is to use the available commodity futures European option price data to construct a synthetic European option written on the futures on the CPI proxy. Monte Carlo simulation is applied to construct the synthetic option. This section describes the construction of synthetic European option.

3.3.1 From Exchange-traded Futures Options to a Synthetic Option

The idea is to use weighted sum of observable commodity futures options prices as an index for the synthetic European option on futures on the CPI proxy. First, we define the 'data option' and the 'synthetic option'.

With the commodity futures European options price data in hand, the 'data option' can be defined. As the name suggests, 'data option' is an option contract derived from the market data. Suppose we work in a risk-neutral world and risk-free interest rate r is a constant. Based on risk-neutral theorem, we define the time 0 price of a European call 'data option' as follows.

$$Doption = \sum_{i=1}^{21} \lambda_i E^Q \left[e^{-rT} \left(F_i - K_i \right)^+ \right], \tag{3.1}$$

where the upper limit of summation 21 is the number of included commodities; λ_i is the rescaled optimal weight for the ith commodity (see Table 3.8), the original optimal weight can be found in the construction of CPI proxy (see Chapter 1). The expected value $E^Q[\cdot]$ denotes the expectation under the risk-neutral measure. F_i is the futures price on the ith commodity observed on option's maturity date. Each F_i is carefully chosen to make sure that different commodity futures options expire in the same month. K_i is each option's strike price with identical moneyness level. For instance, when we consider constructing a deep-in-the-money European call 'data option', every K_i's moneyness is 90%moneyness.

By definition, $E^Q[e^{-rT}(F_i - K_i)^+]$ is the commodity futures European option price observed on 22nd of January 2016, which is ready to use. Multiplying each option price with corresponding optimal weight and sum them together yields the weighted sum of observed futures option prices, which is Doption. The key in defining Doption is twofold. First, all futures options in equation (3.1) must expire in the same month, which is realised by carefully choosing the identical time to maturity T and the underlying commodity futures contract. Second, each strike price K_i has identical %moneyness. As a result, each weighted sum of observed futures option prices defined this way gives a corresponding Doption maturing at time T with K_i's moneyness.

Next, we define the synthetic option written on the futures on the CPI proxy. In the risk-neutral world, the time 0 arbitrage-free price of the synthetic European call option is defined as follows.

$$Soption = E^Q\left[e^{-rT}\left(\sum_{i=1}^{21}\lambda_i F_i - k\right)^+\right] \tag{3.2}$$

with $k = \sum_{i=1}^{21}\lambda_i K_i$. $E^Q[\cdot]$ denotes the expectation under the risk-neutral measure. The optimal weight λ_i, underlying commodity futures F_i, strike price K_i and time to maturity T are identical to those in the definition of Doption. In contrast to Doption, the strike price k is now the weighted sum of K_i from Doption and therefore Soption and Doption share the same moneyness level. What's more, the synthetic option Soption's underlying asset $\sum_{i=1}^{21}\lambda_i F_i$ is now a weighted sum of the commodity futures prices. By definition, this is exactly the futures on the CPI proxy, yet observed at option's expiry date instead of current date. We will show later in the derivation of implied volatility that we only need the price of futures on the CPI proxy observed at current date. Consequently, we have now defined a synthetic European call option written on the futures on the CPI proxy. Using the same inputs, it is possible to compare the values of Doption and Soption.

In fact, the comparison between equation (3.1) and equation (3.2) leads to the Jensen's inequality. The Jensen's inequality is actually what initially inspired us to construct the Doption and Soption in this way.

Simply put, if $p_1, \ldots, p_n$ are positive numbers which sum to 1 and f is a real continuous function that is convex, then

$$f\left(\sum_{i=1}^{n} p_i x_i\right) \leq \sum_{i=1}^{n} p_i f(x_i). \tag{3.3}$$

As for our case, since function $f(x) = (x)^+$ is convex, comparison between Doption and Soption fits the Jensen's inequality in (3.3).

Since now we are aware that, with the same inputs, the price of Doption is supposed to be greater than or equal to that of Soption, we proceed to figure out the exact difference level between the prices of the two options.

By definition, the synthetic option Soption is actually a basket option, whose underlying asset is a portfolio of commodity futures contracts. Figuring out a closed-form pricing formula for basket option is challenging, because in most cases the density of the portfolio of underlying assets is unknown. For example, in the case of arithmetic Asian option, even if the underlying assets are assumed to follow geometric Brownian motion, an analytical pricing formula can hardly be found because the sum of lognormal variables is not lognormal anymore. Also, no such basket option is traded on the market. Therefore, numerical method is necessary to compute the price of basket option such as the synthetic option Soption. Next, we present step by step how we find out the general difference level between Doption and Soption and then evaluate Soption.

3.3.2 Evaluating the Synthetic Option Using Monte Carlo Method

In the risk-neutral world, each underlying commodity future in Doption follows a driftless lognormal process. However, the density of the weighted sum of these commodity futures in Soption is unknown. In such occasion, numerical method is needed to calculate option prices. This section describes how we set up the Monte Carlo simulation to compute the difference between (the prices of) Doption and Soption, and evaluate the synthetic option accordingly.

The Model As we work in the risk-neutral world, the futures price $F(t,T)$ observed at time t is assumed to follow the driftless lognormal process, i.e.

$$dF(t,T) = \sigma F(t,T)dW(t), \tag{3.4}$$

where σ is the volatility of futures price; $W(t)$ is the standard Brownian motion.

The solution to the stochastic differential equation in (3.4) is therefore

$$F(t,T) = F(0,T)exp\left(-\frac{1}{2}\sigma^2 t + \sigma W(t)\right), \tag{3.5}$$

where $F(0,T)$ is the futures price observed at time 0.

Since $W(t) \sim N(0,t)$ is a Brownian motion, we have $W(t) = \sqrt{t}N(0,1)$. This yields

$$F(t,T) = F(0,T)exp\left(-\frac{1}{2}\sigma^2 t + \sigma\sqrt{t}N(0,1)\right), \tag{3.6}$$

where $N(0,1)$ is the standard normal random variable. Equation (3.6) is the primary process for generating the underlying futures price evolution in Monte Carlo simulation.

Evaluating the Synthetic Option Inspired by the Jensen's inequality for convex function, we apply Monte Carlo simulation method to calculate the general difference level between Doption and Soption which contain the same inputs. The evaluation of the synthetic option consists of the following four steps.

1. Collecting option price simulation inputs for 21 different commodities with the same option maturity month and moneyness level. Simulating 21 commodity futures price paths to obtain the value of Doption and Soption. This gives a general difference level for this specific pair of maturity and moneyness level.

First, we collect inputs for the option price simulation, i.e. the underlying futures price of the ith commodity observed at initial time, the rescaled optimal weight for the ith commodity and the implied volatility for individual commodity futures option contract from OVML function. Here we take the difference level calculated for maturity month October 2016 at 90% moneyness as an example. The Matlab code for option parameters is presented as follows.

```
%%%%%%option parameters%%%%%%%%%%%%%%%%

f1=502.9167;q1=0.007421699;v1=0.225975;
f2=516.6667;q2=0.006125333;v2=0.2198;
f3=11.835;q3=0.004180783;v3=0.1868;
f4=221.0833;q4=0.007940246;v4=0.26905;
f5=14.15233;q5=0.003597418;v5=0.26975;
f6=38.41;q6=0.532093719;v6=0.4705;
f7=1098.4;q7=0.003597418;v7=0.1838;
f8=202.15;q8=0.018262566;v8=0.2631;
f9=245.4;q9=0.086498989;v9=0.1674;
f10=2.553;q10=0.032636032;v10=0.312;
f11=15.35;q11=0.001750095;v11=0.2148;
f12=30.965;q12=0.00933384;v12=0.1814;
f13=25.05;q13=0.009106976;v13=0.2402;
f14=221.025;q14=0.007940246;v14=0.2889;
f15=67.2;q15=0.011213572;v15=0.2039;
f16=120.9;q16=0.026737564;v16=0.2002;
f17=154.1;q17=0.184861894;v17=0.2403;
f18=123.95;q18=0.022783645;v18=0.2309;
f19=92;q19=0.006417015;v19=0.2948;
f20=1.714;q20=0.008653248;v20=0.1648;
f21=15.85;q21=0.008847703;v21=0.162;
T=9/12;
rep=5000000;
```

In the above code, f_i is the underlying futures price of the ith commodity observed at initial time, i.e. 22nd of January 2016; q_i is the rescaled optimal weight for the ith commodity; v_i is the implied volatility, which is used by Bloomberg as the inputs of OVML function to compute the commodity futures European option prices; T is the option's time to maturity; rep is the replication time of Monte Carlo simulation, here we replicate the simulation for 5 million times, which is practically large enough to obtain an accurate approximation of option price.

Second, we generate the underlying futures price evolution based on the solution to the dynamics of futures price in the risk-neutral world (equation (3.6)), i.e.

$$F(t,T) = F(0,T)exp\left(-\frac{1}{2}\sigma^2 t + \sigma\sqrt{t}N(0,1)\right).$$

The corresponding Matlab code example is as follows.

```
%%%%%%generate underlying futures price evolution%%%

ai=fi*exp(-0.5*((vi)∧2)*T+vi*sqrt(T)*randn(rep,1));
```

In the above code, a_i is the futures price for the ith commodity at option's maturity, it corresponds to the time t futures price $F(t,T)$ in equation (3.6); randn(rep,1) is a 5000000-by-1 column vector consisting of standard normal random numbers.

A key assumption for underlying futures price generation process is that we assume the futures price is independent to each other,[1] i.e. each standard Brownian motion driving the futures price process is independent to each other. This assumption enables us to derive a closed-form pricing formula for the European call option written on the futures on the CPI proxy.

Third, we evaluate the futures option price for each of the 21 commodities. The relevant Matlab code is:

```
%%%%%%define option's moneyness level%%%%%%%%

ki=0.9*fi;

%%%%%%option evaluation%%%%%%%%%%%%%%%%%

doptionpayoffi=max(ai-ki,0);
doptiondiscounti=doptionpayoffi*exp(-r*T);
doptionmeani=mean(doptiondiscounti);
```

In the above code, k_i is option strike price for the ith commodity, in this example, we let every k_i equal to 90% of corresponding underlying futures price, this setting enables us

[1]We assume the futures price is independent to each other. The case where the futures prices are correlated in the Monte Carlo simulation is discussed in Appendix B.

to investigate the general difference between Doption and Soption when both options are deep-in-the-money (90%moneyness).

For each commodity, we simulate the option payoff $max(a_i - k_i, 0)$. Then, discounting this payoff back to the initial date and taking the average gives us the estimated futures option price for each commodity.

Fourth, we multiply each estimated futures option price with corresponding optimal weight and sum them up. This yields the price of Doption for the predetermined option maturity month and moneyness level, i.e.

```
%%%%%%evaluate Doption%%%%%%%%%%%%%%%%%

doption=q1*doptionmean1+q2*doptionmean2
+q3*doptionmean3+q4*doptionmean4+q5*doptionmean5
+q6*doptionmean6+q7*doptionmean7+q8*doptionmean8
+q9*doptionmean9+q10*doptionmean10
+q11*doptionmean11+q12*doptionmean12
+q13*doptionmean13+q14*doptionmean14
+q15*doptionmean15+q16*doptionmean16
+q17*doptionmean17+q18*doptionmean18
+q19*doptionmean19+q20*doptionmean20
+q21*doptionmean21;
```

Next, we evaluate the synthetic option based on the above inputs and underlying futures price evolutions:

```
%%%%%%evaluate the synthetic option%%%%%%%%%%


j=q1*k1+q2*k2+q3*k3+q4*k4+q5*k5+q6*k6+q7*k7+q8*k8
+q9*k9+q10*k10+q11*k11+q12*k12+q13*k13+q14*k14
+q15*k15+q16*k16+q17*k17+q18*k18+q19*k19+q20*k20
+q21*k21;
Soptionpayoff=max((q1*a1+q2*a2+q3*a3+q4*a4+q5*a5
+q6*a6+q7*a7+q8*a8+q9*a9+q10*a10+q11*a11+q12*a12
+q13*a13+q14*a14+q15*a15+q16*a16+q17*a17+q18*a18
+q19*a19+q20*a20+q21*a21)-j,0);
Soptiondiscount=Soptionpayoff*exp(-r*T);
Soption=mean(Soptiondiscount);
```

In the above code, we define j as the weighted sum of k_i, and each k_i is a replicate of the option strike price in the construction of Doption. The payoff of Soption has identical form with the payoff in the Soption's definition (equation (3.2)). Discounting this payoff back to the initial date and taking the average, we find the estimated value of Soption.

Finally, we calculate the difference level between Doption and Soption.

```
%%%%%%calculate the difference level%%%%%%%%%%%


difference=Soption-doption;
```

We run the code for at least ten times. Since the replication time is large enough, the difference level between Doption and Soption is always stable at two decimal place accuracy.

2. With the option's time to maturity held fixed, we estimate the general difference levels for the remaining four different moneyness levels. That is, we repeat the process in step 1 for another four times, but each time we run the code with different implied volatility inputs for corresponding %moneyness level (95%, 100%, 105%, 110%). This yields a difference level chain describing respectively the general difference levels for two in-the-money, one at-the-money and two out-of-the-money options maturing in the same month.

3. We now repeat the first and second steps to calculate the difference level chains for all other option's expiry months. This gives a difference level surface as shown in Table 3.3.

Table 3.3. Difference Level Surface

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Feb 2016	-0.74	-1.24	-1.48	-1.24	-0.83
Mar 2016	-1.49	-2.1	-2.36	-2.04	-1.49
Apr 2016	-2.01	-2.62	-2.86	-2.58	-2.03
May 2016	-2.37	-2.93	-3.12	-2.86	-2.33
Jun 2016	-2.67	-3.18	-3.33	-3.08	-2.58
Jul 2016	-2.94	-3.43	-3.58	-3.33	-2.84
Aug 2016	-3.16	-3.65	-3.78	-3.55	-3.07
Sep 2016	-3.36	-3.83	-3.96	-3.73	-3.26
Oct 2016	-3.52	-3.98	-4.1	-3.88	-3.42
Nov 2016	-3.7	-4.26	-4.26	-4.04	-3.59
Dec 2016	-3.81	-4.27	-4.38	-4.16	-3.72

Notes: 'Expiry' in the first column denotes the synthetic option expiry month; each number represents a difference level for a specific pair of expiry month and %moneyness level, the difference level is defined as '$Soption - Doption$'.

All difference level values in Table 3.3 are negative, i.e. for all expiry months and moneyness levels, Soption price is always smaller than Doption price. Not surprisingly, our result fits the Jensen's inequality for convex function.

Figure 3.1 plots the 3D bar of difference level surface. Interesting patterns can be observed. First, as the month to maturity increases, the magnitude of the difference level between Doption and Soption increases. Second, for Doption and Soption maturing in the same month, as moneyness level increases from 90% to 110%, the magnitude of difference level keeps rising until 100% moneyness and then falls to a level roughly equal to difference level at 90% moneyness. That is, difference levels at 90% and 110% moneyness, difference levels at 95% and 105% moneyness, are roughly symmetrical with respect to difference level at 100% moneyness, respectively.

4. Given the general difference level, we now proceed to evaluate the synthetic option. Given the commodity futures European option price data, the price of synthetic option Soption is the sum of corresponding Doption price and difference level. Table 3.4 presents the price data for the synthetic options.

Table 3.4. Synthetic Option Price

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Feb 2016	9.09	4.58	1.11	0.03	0.00
Mar 2016	9.36	5.20	2.15	0.60	0.10
Apr 2016	9.62	5.65	2.72	1.06	0.33
May 2016	9.81	5.92	3.25	1.32	0.50
Jun 2016	10.01	6.21	3.37	1.60	0.70
Jul 2016	10.23	6.49	3.67	1.87	0.89
Aug 2016	10.48	6.78	3.99	2.14	1.09
Sep 2016	10.62	6.99	4.22	2.37	1.27
Oct 2016	10.73	7.16	4.43	2.56	1.42
Nov 2016	10.79	7.17	4.59	2.72	1.56
Dec 2016	10.86	7.40	4.74	2.88	1.69

Notes: 'Expiry' in the first column denotes the synthetic option expiry month. In Table 3.4, due to the fact that the absolute difference level is larger than the 110% moneyness Doption price maturing in February 2016, the calculated Soption price is actually negative. But option price must be greater than or equal to zero, therefore, to make the price economically meaningful, we equate this price to zero.

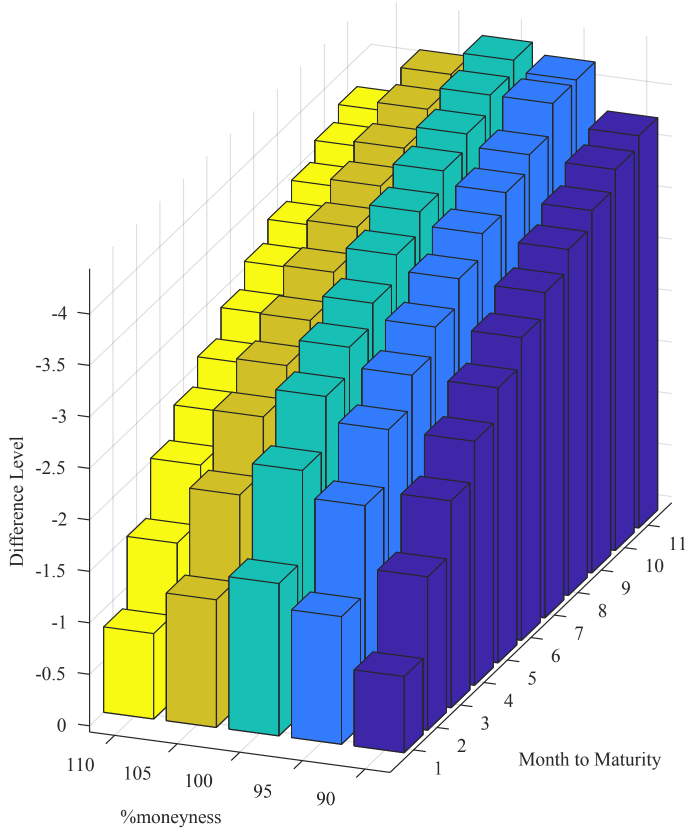


Figure 3.1. This figure plots the difference level surface data of Table 3.3 in the form of 3D bar.

In Table 3.4, due to the fact that the absolute difference level is larger than the 110% moneyness Doption price maturing in February 2016, the calculated Soption is actually negative. But option price must be greater than or equal to zero, therefore to make the price economically meaningful, we equate this price to zero.

Figure 3.2 plots the 3D bar of synthetic option price. We can see from Figure 3.2 that, for fixed option expiry month, Soption price decreases dramatically as the %moneyness increases. Furthermore, as the month to maturity increases, Soption price rises gradually.

Analogous to any exchange-traded option contract, the synthetic option price contains implicitly the volatility of the underlying asset. In our case, this is the implied volatility of the futures on the CPI proxy. In the next section, we derive this implied volatility surface.

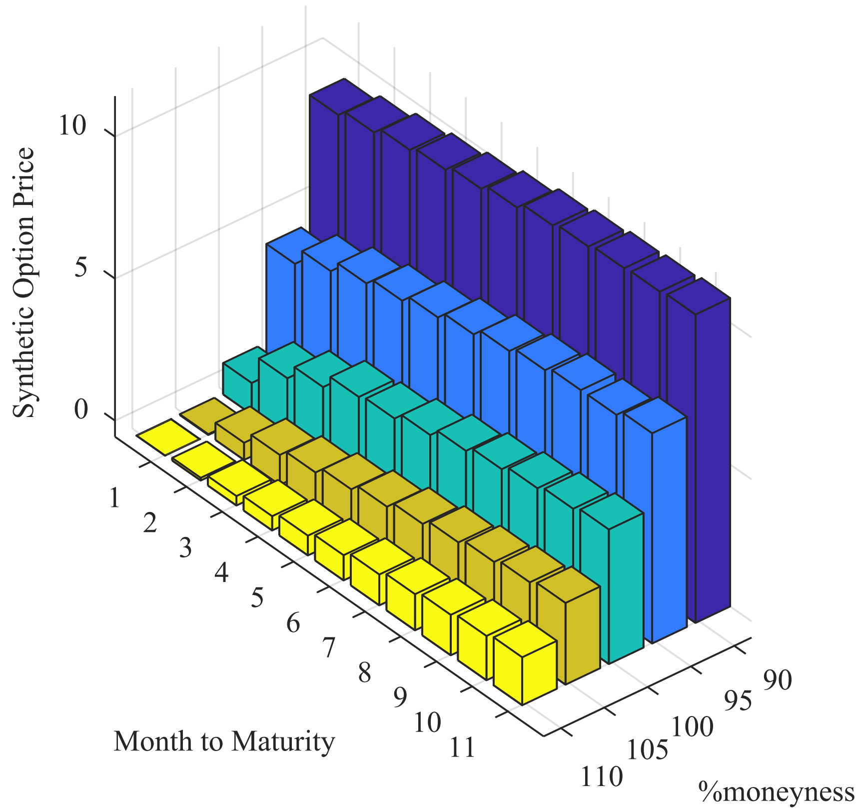


Figure 3.2. This figure plots the synthetic option price data of Table 3.4 in the form of 3D bar.

3.4 The Synthetic Option Implied Volatility Surface

This section describes how we derive the synthetic option implied volatility surface. Given the fact that the synthetic option prices are derived from exchange-traded commodity futures options price data, the synthetic option prices can therefore be treated as the observed market prices for the synthetic options. The synthetic option implied volatility is the volatility value such that, when inserted into the option pricing formula, equates the theoretical option model price with the observed market price of synthetic option. Before we reverse this process to find the synthetic option implied volatility, we need a proper pricing formula for the synthetic option written on the futures on the CPI proxy. Now we derive this pricing formula.

3.4.1 Pricing the Synthetic Option in a Black's Model Type Framework

In the risk-neutral world, the futures price is assumed to follow a driftless lognormal process, i.e.

$$dF(t,T) = \sigma F(t,T)dW(t). \tag{3.7}$$

Therefore in the risk-neutral world, the futures price is a martingale.

Since we assume that the futures price is independent to each other, the linearity of conditional expectation implies that the sum of futures prices is also a martingale. More specifically, we consider a continuous trading economy. The uncertainty in the economy is characterized by a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ where Ω is the state space, $\mathscr{F}$ is the set of all possible events, $\mathbb{P}$ is the statistical probability measure and $\widetilde{\mathbb{P}}$ is the equivalent martingale measure. Let $\{\mathscr{F}_t : t \in [0,T]\}$ be the standard filtration generated by the standard Brownian motion $W(t)$. By the linearity of conditional expectation,

$$\begin{aligned} E^Q[FCP(t,T)|\mathscr{F}_0] &= E^Q\left[\sum_{i=1}^{21}\lambda_i F_i(t,T)|\mathscr{F}_0\right] \\ &= \sum_{i=1}^{21}\lambda_i E^Q[F_i(t,T)|\mathscr{F}_0] \\ &= \sum_{i=1}^{21}\lambda_i F_i(0,T) \\ &= FCP(0,T), \end{aligned} \tag{3.8}$$

where $FCP(t,T)$ is by our definition the maturity T futures on the CPI proxy observed at time t; $F_i(t,T)$ is the ith maturity T commodity futures observed at time t; λ_i is the optimal weight for the ith commodity. Equation (3.8) proves that the futures on the CPI proxy $FCP(t,T)$ is indeed a martingale in the risk-neutral world.

Therefore, we assume that the futures on the CPI proxy $FCP(t,T')$ follows the driftless lognormal process.

$$dFCP(t,T') = \sigma_{FCP}FCP(t,T')dW_{FCP}(t), \tag{3.9}$$

where σ_{FCP} is the volatility of futures on the CPI proxy $FCP(t,T')$.

Under the assumption that the futures on the CPI proxy follows the dynamics given in (3.9), the price of a European call option on the futures on the CPI proxy with strike price K, maturity T and with maturity payoff

$$C\left(T, FCP\left(T, T'\right)\right) = \left(FCP(T, T') - K\right)^{+} \tag{3.10}$$

is at time 0 given by

$$C\left(0, FCP\left(0, T'\right)\right) = e^{-rT}\left[FCP\left(0, T'\right)N\left(d_1\right) - KN(d_2)\right] \tag{3.11}$$

with $N(\cdot)$ denotes the cumulative density function and

$$\begin{aligned} d_1 &= \frac{ln\left(\frac{FCP(0,T')}{K}\right) + \frac{1}{2}\sigma_{FCP}^2 T}{\sigma_{FCP}\sqrt{T}} \\ d_2 &= \frac{ln\left(\frac{FCP(0,T')}{K}\right) - \frac{1}{2}\sigma_{FCP}^2 T}{\sigma_{FCP}\sqrt{T}} = d_1 - \sigma_{FCP}\sqrt{T}, \end{aligned}$$

where $FCP\left(0, T'\right)$ denotes the futures on the CPI proxy observed at current time 0; $FCP\left(T, T'\right)$ is the futures on the CPI proxy observed at a future time T with maturity $T' > T$. The derivation of equation (3.11) is presented in Appendix C. Using the closed-form option pricing formula given by (3.11), we can now move on to derive the synthetic option implied volatility.

3.4.2 Derive the Synthetic Option Implied Volatility

With the closed-form option pricing formula in hand, we now take as inputs the synthetic option prices, risk-free interest rate, strike prices, initial price of the futures on the CPI proxy and time to maturity for this formula and calculate the implied volatility surface. Newton-Raphson method is applied to compute the implied volatility. This process can be realised by using the 'blkimpv' function from the Matlab Financial Toolbox. The result is presented in Table 3.5.

It can be seen from Table 3.5 that the synthetic option implied volatility is time and strike dependent. To see the time and strike-dependency of implied volatility more clearly, we plot the implied volatility surface in Figure 3.3.

From Figure 3.3 we can see that, for every option expiry month, the volatility skew is observed, i.e. the implied volatility value decreases as strike price increases. Although not pronounced, a slight volatility smile can also be observed for April 2016 to November 2016 as the implied volatility for these months rise slightly from 105%moneyness option to 110%moneyness option. Next, we analyse the implication of the implied volatility surface.

Table 3.5. Implied Volatility Surface

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Mar 2016	0.1743	0.1499	0.1440	0.1417	0.1364
Apr 2016	0.1799	0.1560	0.1489	0.1476	0.1480
May 2016	0.1740	0.1516	0.1544	0.1426	0.1444
Jun 2016	0.1722	0.1509	0.1435	0.1416	0.1433
Jul 2016	0.1710	0.1507	0.1426	0.1409	0.1424
Aug 2016	0.1726	0.1519	0.1438	0.1413	0.1423
Sep 2016	0.1723	0.1522	0.1437	0.1412	0.1422
Oct 2016	0.1719	0.1522	0.1434	0.1404	0.1410
Nov 2016	0.1711	0.1478	0.1427	0.1396	0.1399
Dec 2016	0.1719	0.1517	0.1425	0.1391	0.1390

Notes: 'Expiry' in the first column denotes the synthetic option expiry month. Due to the fact that the price of 110% moneyness Soption maturing in February 2016 is zero, we only calculate the implied volatility for synthetic options maturing from March 2016 to December 2016.

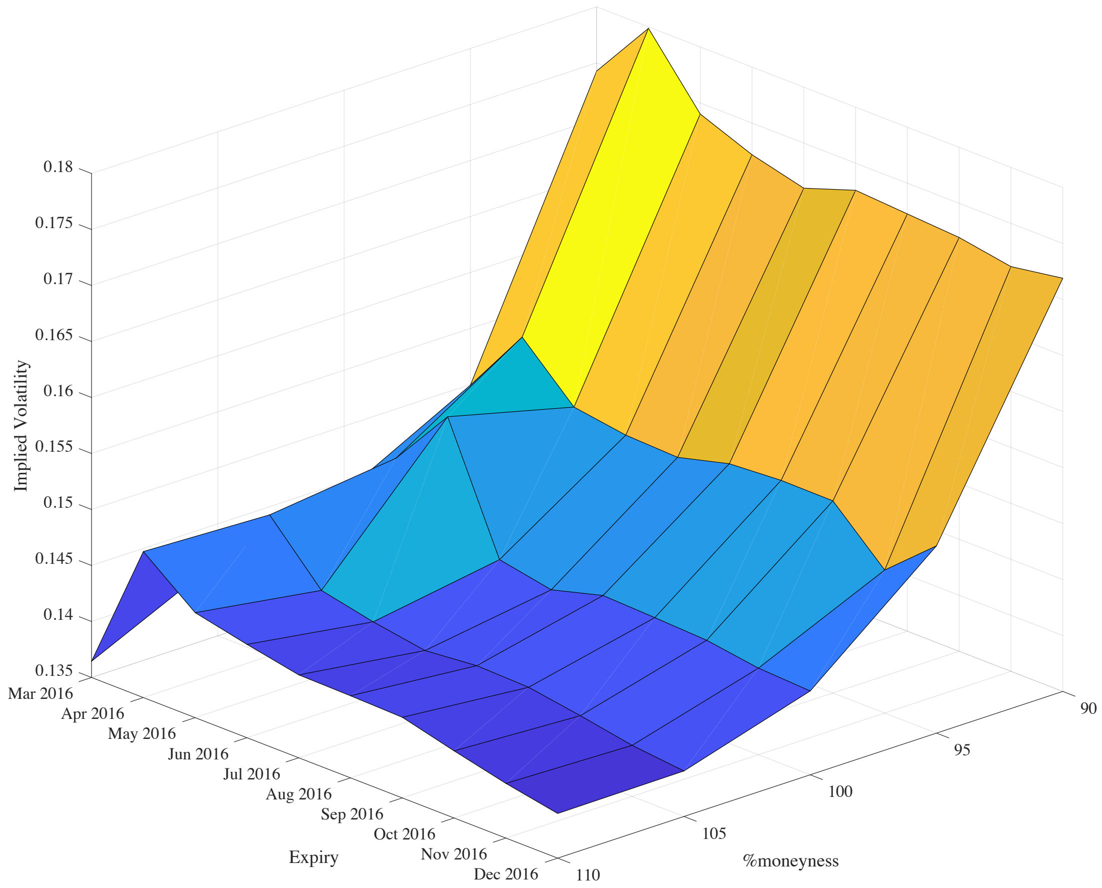


Figure 3.3. This figure plots the implied volatility surface corresponding to the data in Table 3.5.

3.4.3 Implication of Implied Volatility Surface

Based on the current value of the futures on the CPI proxy, the implied volatility indicates an interval estimate of the futures on the CPI proxy at some future time T, i.e. the range of swing of the futures on the CPI proxy at a future time T.

Formally, assuming the futures on the CPI proxy follows the lognormal dynamics in (3.9),

$$dFCP(t,T') = \sigma_{FCP} FCP(t,T') dW_{FCP}(t).$$

The probability distribution of the natural logarithm of the futures on the CPI proxy $lnFCP(T,T')$ is

$$lnFCP(T,T') \sim \phi \left[lnFCP(0,T') - \frac{1}{2}\sigma_{FCP}^2 T, \;\; \sigma_{FCP}^2 T \right], \tag{3.12}$$

where $\phi[a,b]$ denotes the normal distribution with mean a and variance b. Note that the volatility of futures on the CPI proxy σ_{FCP} is calculated by matching the theoretical option price derived from option pricing formula (3.11) with the observed synthetic option price, therefore the volatility of futures on the CPI proxy σ_{FCP} is equal to the implied volatility, i.e. $\sigma_{FCP} = \sigma_{imp}$. Thus, we have

$$lnFCP(T,T') \sim \phi \left[lnFCP(0,T') - \frac{1}{2}\sigma_{imp}^2 T, \;\; \sigma_{imp}^2 T \right]. \tag{3.13}$$

Using the probability distribution of $lnFCP(T,T')$ in (3.13), we calculate the 90%, 95% and 99% level confidence intervals of $lnFCP(T,T')$ for each futures on the CPI proxy of the futures on the CPI proxy chain $FCP(T,T')$ with maturity T' ranging from April 2016 to January 2017. We use Figure 3.4 and an example to illustrate the timing of the futures on the CPI proxy and corresponding synthetic option; and how we calculate the confidence interval of $lnFCP(T,T')$ and the corresponding interval estimate of the futures on the CPI proxy.

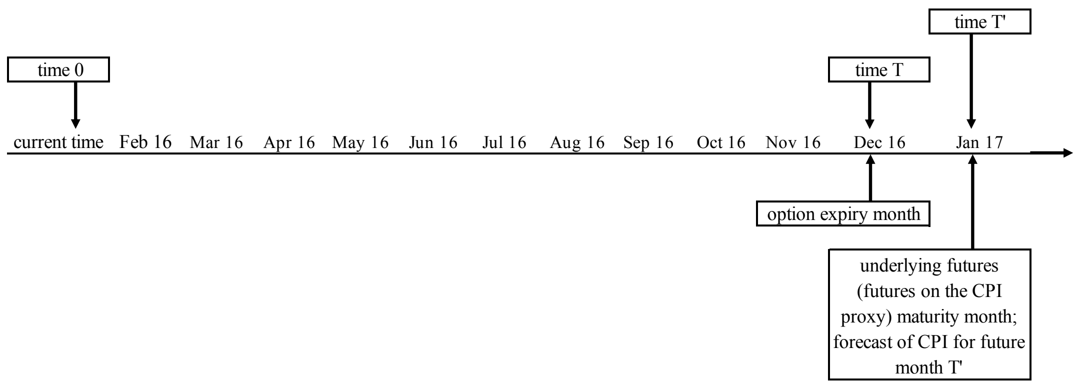


Figure 3.4. This timeline chart plots the timing of futures on the CPI proxy and corresponding synthetic option.

In Figure 3.4, the current month (time 0) is January 2016. We take the synthetic option maturing in eleven months (December 2016) as an example. Usually, for commodity futures options traded at the CME, there is a gap of one to three months between the commodity futures option expiry and the underlying commodity futures maturity. In our case, most commodity futures options expire one month earlier than the underlying commodity futures contracts. Therefore, we assume that the synthetic option expires one month earlier than the underlying futures on the CPI proxy. This means that, in Figure 3.4, the synthetic option maturing in December 2016 has an underlying futures on the CPI proxy maturing in January 2017 (time T').

By definition, the futures on the CPI proxy $FCP(0,T')$ in Figure 3.4 is the current-time forecast of consumer price index at a future time T' (January 2017). The way we constructed the futures on the CPI proxy indicates that the futures on the CPI proxy $FCP(0,T')$ is a 'point' forecast, i.e. a single value which is the weighted average of commodity futures last prices (or settlement prices) observed on current date. No prediction of future evolution of price level can be made with absolute certainty. It is therefore more realistic to consider uncertainty when the prediction is made. Thus, we calculate the 90%, 95% and 99% level confidence intervals of $lnFCP(T,T')$ where time T is the synthetic option expiry. Using the confidence intervals of $lnFCP(T,T')$, we then calculate the corresponding contemporaneous interval estimates of $FCP(T,T')$. Given that the futures on the CPI proxy $FCP(T,T')$ is essentially a futures contract, the time T interval estimates of futures on the CPI proxy $FCP(T,T')$ are actually the interval estimates for the consumer price index at the futures on the CPI proxy maturity T' (January 2017). Consequently, we have derived the uncertainty surrounding the 'point' forecast of consumer price index at time T' (January 2017).

We now use a numerical example to demonstrate the above-described procedure of deriving the uncertainty surrounding the 'point' forecast of consumer price index at some future time T'. The current value of futures on the CPI proxy $FCP(0,T')$ maturing in January 2017 is 95.53044, which is the time 0 'point' forecast for January 2017 price level. The implied volatility of this futures on the CPI proxy (implied from the 105% moneyness level synthetic option chain) is 13.91% per annum. From equation (3.13) the probability distribution of the futures on the CPI proxy $FCP(T,T')$ in eleven months' time is given by

$$lnFCP(T,T') \sim \phi[4.55, 0.02].$$

There is a 95% probability that a normally distributed variable has a value within 1.96 standard deviations of its mean. In our case, the standard deviation is 0.13. Therefore, with 95% confidence,

$$4.55 - 1.96 \times 0.13 < lnFCP(T,T') < 4.55 + 1.96 \times 0.13,$$

based on which we can derive the corresponding contemporaneous interval estimates of $FCP(T,T')$

$$e^{4.55-1.96\times 0.13} < FCP(T,T') < e^{4.55+1.96\times 0.13},$$

this is

$$72.94 < FCP(T,T') < 122.92.$$

Since the futures on the CPI proxy is essentially a futures contract, the interval estimate of time T value of $FCP(T,T')$ is actually an interval estimate of price level at time T', i.e. there is 95% probability that the price level in January 2017 will lie between 72.94 and 122.92. Using the same method, we can calculate the 90% and 99% level confidence intervals of $lnFCP(T,T')$ and derive the corresponding interval estimates of price level in January 2017. The three interval estimates constitute the uncertainty surrounding the 'point' forecast of price level in January 2017.

Following this logic, we calculate the time T (synthetic option expiry month) 90%, 95% and 99% level confidence intervals of $lnFCP(T,T')$ for each futures on the CPI proxy in the futures on the CPI proxy chain whose maturities range from April 2016 to January 2017. For expository purpose, the calculated confidence intervals and corresponding contemporaneous interval estimates of $FCP(T,T')$ are presented in Appendix D. By our definition of the futures on the CPI proxy, these interval estimates of $FCP(T,T')$ can be translated into a time series of monthly interval estimates for future price level from April 2016 to January 2017. We calculate the time series of interval estimates for price level with respect to each %moneyness level synthetic option chain in our study. We plot these interval estimates for price level in Figure 3.5.

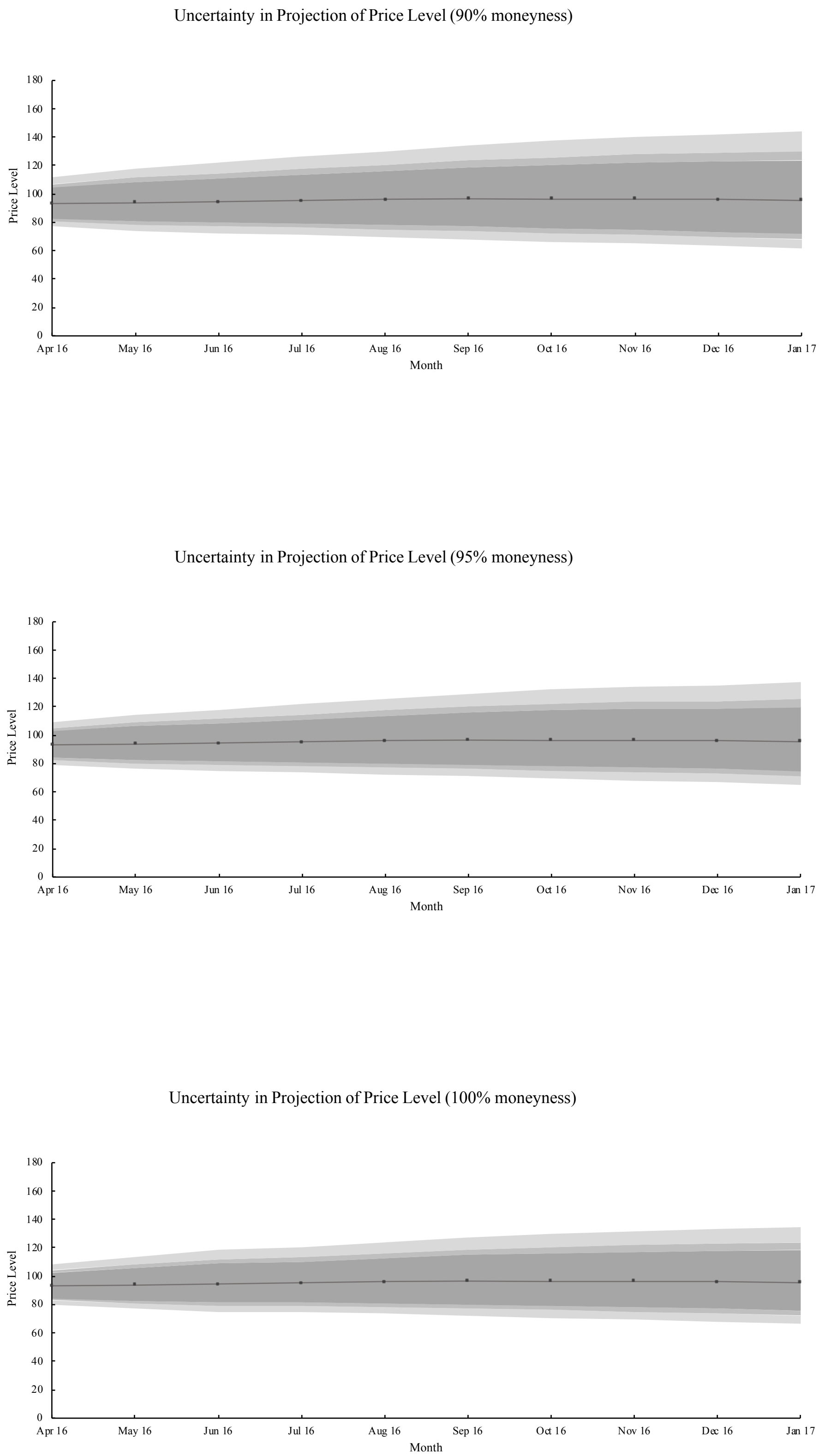
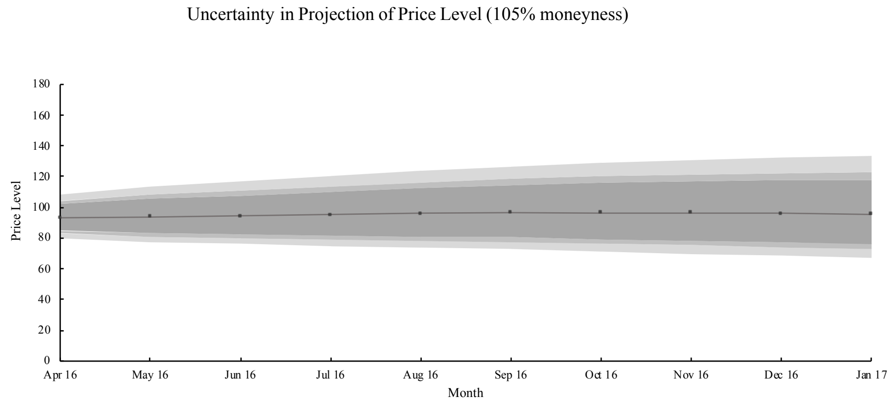


Figure 3.5. The three fan charts (from top to bottom) plot the uncertainty in projection of future price level derived from deep-in-the-money call option chain (90%moneyness), in-the-money call option chain (95%moneyness) and at-the-money call option chain (100%moneyness), respectively.



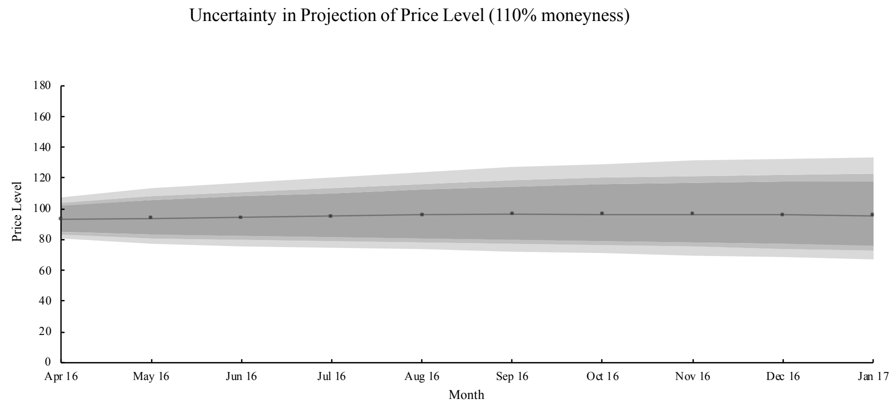


Figure 3.5 - continued. The two fan charts (from top to bottom) plot the uncertainty in projection of future price level derived from out-of-the-money call option chain (105%moneyness) and deep-out-of-the-money call option chain (110%moneyness), respectively.

There are five fan charts in Figure 3.5, each plots the uncertainty in projection of future price level derived from deep-in-the-money call option chain (90%moneyness), in-the-money call option chain (95%moneyness), at-the-money call option chain (100%moneyness), out-of-the-money call option chain (105%moneyness) and deep-out-of-the-money call option chain (110%moneyness), respectively.

In each fan chart, X-axis denotes the calendar months from April 2016 to January 2017, corresponding to the expiry months of futures on the CPI proxy chain; Y-axis is the forecast of future price level. Every dot on the solid line represents a current 'point' forecast for future consumer price index, forming a time series of 'point' forecasts for price level from April 2016 to January 2017, which correspond to the values of time 0 futures on the CPI proxy chain $FCP(0,T')$ with maturity T' ranging from April 2016 to January 2017.

To illustrate the uncertainty surrounding the 'point' forecast, coloured bands are drawn alongside the solid line. The coloured bands become lighter as the distance from the solid line increases, forming a fan chart. For every 'point' forecast of future price level , we derive three interval estimates of price level forecast surrounding the 'point' forecast. Different interval estimates are characterized by different shades of grey, reflecting the relative probability of future price level lying in a particular interval estimate. The central interval estimate in the deepest shade of grey represents the interval estimate of future price level derived from the 90% level confidence interval of $lnFCP(T,T')$; the pair of interval estimates in the second deepest shade of grey on either side of the central interval estimate denote the range of interval estimate of future price level derived from the 95% level confidence interval of $lnFCP(T,T')$ that exceeds the interval estimate of future price level derived from the 90% level confidence interval of $lnFCP(T,T')$; the pair of interval estimates in the lightest shade of grey on both edges of the fan chart stand for the portion of interval estimate of future price level derived from the 99% level confidence interval of $lnFCP(T,T')$ that exceeds the interval estimate of future price level derived from the 95% level confidence interval of $lnFCP(T,T')$. The time series of interval estimates derived from the 90% level confidence interval of $lnFCP(T,T')$ are chronologically connected to form the band in the deepest shade of grey at the centre of the fan chart; each pair of interval estimates in the second deepest shade of grey on either side of the central interval estimate are chronologically connected to form the bands in the second deepest shade of grey; and analogously, the time series of each pair of interval estimates in the lightest shade of grey form the bands in the lightest shade of grey. Altogether, the coloured bands construct the fan chart.

We can see from Figure 3.5 that, in each fan chart, the bands widen as forecast horizon extends, i.e. the magnitude of uncertainty surrounding the 'point' forecast of price level increases as month to maturity becomes longer. This result makes economic sense. It is natural when forecasting to assume that the future degree of uncertainty is greater than that at present; and the increasing degree of uncertainty in price level forecast as horizon extends can also be observed from the Bank of England inflation fan chart. What's more, the trend in the evolution of the degree of price level forecast uncertainty can be understood from the financial derivatives point of view. Given the observed persistency in the time series of U.S. CPI for a short period, it is reasonable to assume when forecasting that the CPI to remain persistent for three months in the future. However, as forecast horizon extends from three months to one year ahead, the uncertainty is expected to increase. Canty and Heider (2012) indicate that, the very front end of the inflation curve remains relatively undefined, because there is no government inflation-linked bills market; and the inflation-linked swaps trading is mostly beyond one year and often two-year maturities. Additionally, the inflation futures hedging inflation risk one year ahead have been delisted due to illiquidity. Therefore, market participants lack the proper financial derivatives to speculate inflation expectation for one year in the future. As a result, the uncertainty of price level forecast increases as forecast horizon extends from three month to one year ahead.

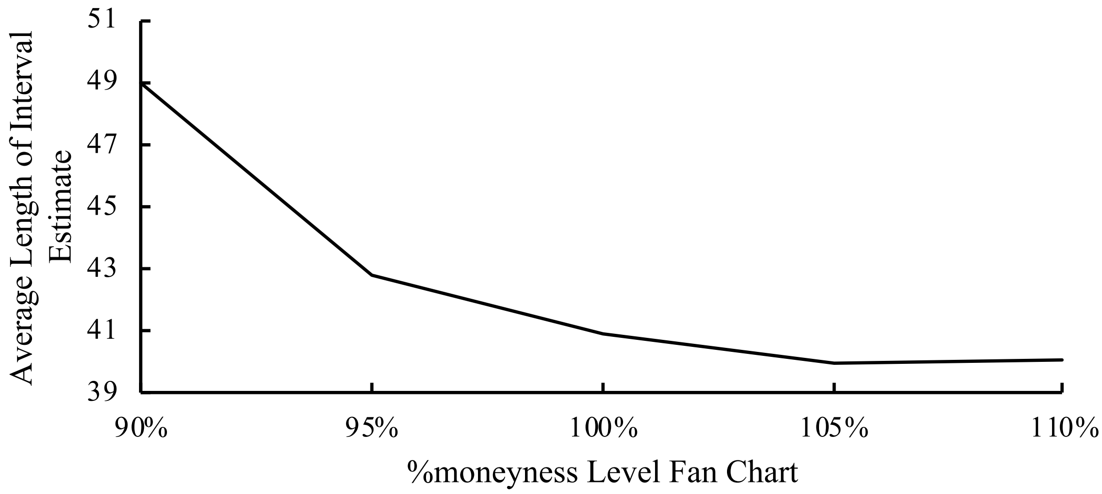


Figure 3.6. The line chart plots the average length of interval estimate of future price level of the five fan charts.

Furthermore, in Figure 3.6, we compare the average length of interval estimates of future price level across the five fan charts. We define the length of interval estimates of future price level as the difference between the maximum value and minimum value of each interval estimate of future price level, which reflects the degree of uncertainty surrounding the 'point' forecast of future price level. We can see from Figure 3.6 that, as the %moneyness level increases from deep-in-the-money call option chain (90%moneyness) to deep-out-of-the-money call option chain (110%moneyness), the average length of interval estimate of future price level keeps decreasing towards its bottom level at out-of-the-money call option chain (105%moneyness) and then rises slightly at deep-out-of-the-money call option chain (110%moneyness), suggesting that the lowest average uncertainty surrounding the 'point' forecast of future price level appears at out-of-the-money call option chain. In other words, we are overall more certain about the price level forecast when we use out-of-the-money call option chain. One explicit reason is the volatility skew (slight smile) of synthetic options.

Another implicit reason is that, the implied volatility surface used by Bloomberg to compute the European commodity futures options prices is derived from out-of-the-money exchange-traded American commodity futures options. Therefore, compared with in-the-money and at-the-money option chains, implied volatility derived from out-of-the-money options has superior capability of capturing the market expectation of future price level.

3.4.4 Inflation Forecast

In this section, we discuss the inflation forecast implied from the fan charts in Figure 3.5 and compare our result with the Bank of England inflation fan chart's inflation prediction.

Our fan charts depict the uncertainty in projection of future price level, and the U.S. inflation forecast can be implied implicitly from the interval estimate of future price level. By contrast, the Bank of England inflation fan chart predicts the probability density of U.K. CPI inflation and therefore inflation prediction can be read explicitly from the fan chart. Our fan charts are technically distinct in many ways from the Bank of England inflation fan chart, a comparison between the two types of fan charts yields a more comprehensive understanding of our results.

Figure 3.7 presents an inflation fan chart taken from the Bank of England February 2016 Inflation Report.

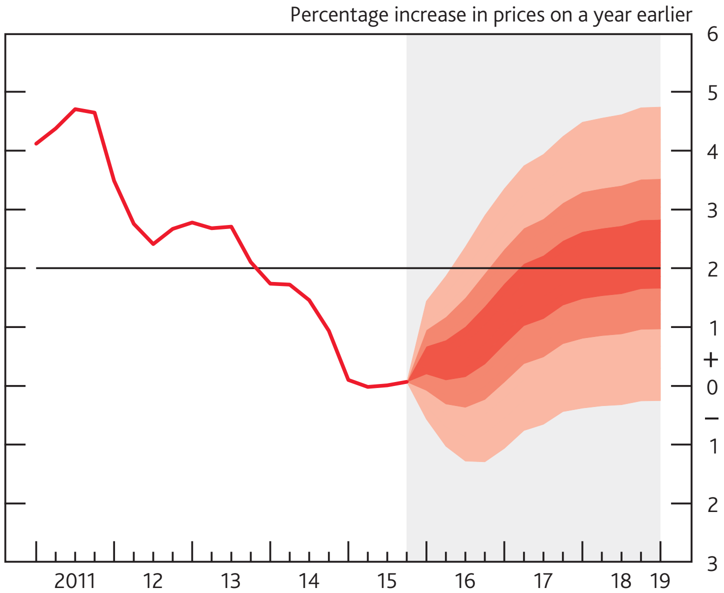


Figure 3.7. This figure presents an inflation fan chart taken from the Bank of England February 2016 Inflation Report. The Inflation Report is produced quarterly (i.e. in February, May, August and November) by Bank staff under the guidance of the members of the Monetary Policy Committee.

The Bank of England Monetary Policy Committee (MPC)'s view of the likely outcome for inflation in any future quarter is represented by a 'two-piece' normal probability density (see Britton et al. (1998) and pages 48-49 of the May 2002 Inflation Report). Such a probability density is produced for each of all future quarters in the forecast. The inflation fan charts correspond to the bird's-eye view of the interpolated time series of the probability densities of CPI inflation. For any time point in the forecast period, the shade of red reflects the relative probability of inflation lying in a particular band. For example, at the end of forecast period, i.e. the 4^{th} quarter of 2018, the darkest red band in the centre of the chart represents a 30% probability, so there is a 30% probability that inflation will lie approximately between 1.7% and 2.8%. Moving away from the central band, the area covered by each pair of successive identically shaded bands-one on each side of the central band-also represents a 30% probability. Therefore, at the 4^{th} quarter of 2018 there is judged to be a 90% probability that the outturn of CPI inflation will be approximately between -0.3% and 4.7%.

Because our fan charts are forward-looking forecast with no historical time series of 'point' forecast of future price level, we compare our fan charts with the forward-looking part of the Bank of England inflation fan charts, i.e. the red fan charts with grey background.

First, we notice that for both our fan charts and the Bank of England inflation fan charts, the coloured bands widen as the forecast horizon extends. In other words, the magnitude of forecast uncertainty increases as the prediction looks further into the future. This implies that, the further ahead in time, the less certain both the commodity futures options market and the MPC tend to be about the future price level.

The horizon of our interval estimate of future price level indicates that we are able to predict an interval estimate for inflation rate 12 months in the future. By contrast, the Bank of England inflation forecast covers 13 quarters ahead in time. The reason that our forecast horizon is limited to 12 months is that, for the majority of commodities included in the construction of the futures on the CPI proxy, the longest time to maturity of the available futures options is 11 months. Therefore, the constructed synthetic options can only imply volatilities for the underlying futures on the CPI proxy with time to maturity up to 12 months (i.e. corresponding to the one-year ahead price level forecast). In other words, for the futures on the CPI proxy with time to maturity longer than 12 months, we lack the corresponding synthetic option to derive the implied volatility for the underlying futures on the CPI proxy, which limits the forecast horizon to 12 months in the future.

Furthermore, the uncertainty of our one-year ahead inflation forecast is larger than that of inflation prediction from the Bank of England inflation fan chart. We compare the Bank of England inflation prediction with our one-year ahead inflation forecast derived from the out-of-the-money call option chain (105% moneyness), as the 105% moneyness level synthetic option chain yields the lowest average uncertainty surrounding the 'point' forecast of future price level among all 5 call option chains (see Figure 3.6). Furthermore, we compute the inflation forecast using only the interval estimate of future price level derived from the 90% level confidence intervals of $lnFCP(T,T')$, in order that the confidence level of our forecast matches the coverage of the probability of the Bank of England inflation fan chart (90% probability). Specifically, based on the interval estimate of price level in January 2017 (76.11 to 117.79) and the CPI proxy one year earlier at January 2016 (93.662), we expect that there is a 90% probability that January 2017's inflation rate will lie between -18.74% and 25.76%. By contrast, the Bank of England inflation fan chart predicts that January 2017's inflation rate will be between -1% and 3.5% with a 90% chance. Although we predict U.S. inflation rate while the Bank of England forecasts U.K. CPI inflation, given that both U.S. (the Federal Open Market Committee) and U.K. target 2% inflation rate, the magnitude of these inflation rate forecasts are comparable. Clearly, the length of the interval estimate of our inflation rate forecast is larger than the uncertainty of inflation prediction from the Bank of England inflation fan chart. Next, we show that it is the source of uncertainty that gives rise to the distinct magnitudes of the two inflation forecasts.

To evaluate the complete inflation forecast distribution, the Bank of England forecast team assesses a number of economic shocks affecting the inflation forecast. The variance of inflation is derived from the underlying variances of economic shocks. Simulation is then used to identify the contribution of relevant economic shock variances to the variance of the inflation forecast. The inflation forecast variance can then be obtained by a weighted sum of underlying individual economic shocks. But rather than simply adding up all the variances, the past inflation forecast error variance is taken as a starting point and then adjusted upwards or downwards. Above all, since 1994, U.K. inflation has been much less volatile than it was in the past, helping to lower forecast error. Thus, a lower starting forecast error variance helps curb the future forecast uncertainty within a narrow range. What's more, the Bank of England has concentrated on systematically building up a forecast distribution of inflation in a manner that reflects MPC's subjective judgment, rather than simply the mapping from the objective economic shocks. Therefore, by adjusting the underlying economic shock variances, the inflation forecast variance is thus changed to match the degree of uncertainty as viewed by the MPC, which limits the forecast uncertainty within a satisfying range. Thus, the uncertainty of Bank of England inflation forecast is limited subjectively to a low level to match the MPC's view as well as the small historical forecast error.

By contrast, the uncertainty of our inflation forecast comes directly from the commodity futures options market, with no subjective adjustments. We now analyze step by step how the inflation forecast uncertainty is generated from the commodity futures options market. Specifically, every observed CME commodity futures in our study has a corresponding option chain; each option chain consists of options with different strikes written on the same underlying commodity futures. For each one such option, Bloomberg OVDV function calculates the Black-Scholes implied volatility so that the American option model prices match the exchange-traded option prices. The Bloomberg OVDV Black-Scholes implied volatility surface is then used by Bloomberg OVML option pricing function to calculate the corresponding European commodity futures options prices. These European commodity futures options price data are then used to construct the synthetic European call options, which imply the implied volatility for the underlying futures on the CPI proxy. The implied volatility determines directly the magnitude of the confidence interval of $lnFCP(T,T')$, which yields the length of the interval estimate of future price level. Finally, based on these interval estimates of future price level, the uncertainty of one-year ahead inflation forecast can be derived.

To figure out the origin of large inflation forecast uncertainty, we calculate in Table 3.6 for each option expiry month and %moneyness level the weighted sum of Bloomberg OVDV implied volatility for individual commodity futures option observed in Bloomberg OVML option pricing function.

Table 3.6. Weighted Sum of Bloomberg OVDV Individual Option Implied Volatility (proxy)

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Feb 2016	0.4585	0.4320	0.4585	0.4214	0.4585
Mar 2016	0.4496	0.4356	0.4252	0.4188	0.4177
Apr 2016	0.4316	0.4209	0.4127	0.4065	0.4029
May 2016	0.4118	0.4018	0.3941	0.3882	0.3845
Jun 2016	0.3968	0.3874	0.3798	0.3741	0.3704
Jul 2016	0.3849	0.3765	0.3696	0.3642	0.3603
Aug 2016	0.3741	0.3665	0.3602	0.3551	0.3511
Sep 2016	0.3651	0.3579	0.3520	0.3471	0.3433
Oct 2016	0.3560	0.3493	0.3434	0.3385	0.3345
Nov 2016	0.3492	0.3492	0.3377	0.3330	0.3293
Dec 2016	0.3419	0.3363	0.3311	0.3265	0.3227

Notes: 'Expiry' in the first column denotes the individual option expiry month. For each option expiry month and %moneyness level, the number represents the weighted sum of Bloomberg OVDV implied volatility for individual commodity futures option observed in Bloomberg OVML option pricing function.

Each weighted sum in Table 3.6 is a proxy indicating the overall view of how big the individual implied volatility is on average. It can be seen from Table 3.6 that, overall, for each expiry month and %moneyness level, the volatility proxy is large. To understand the magnitude of these proxies more intuitively, we compare in Table 3.7 in percentage the ratio of synthetic option implied volatility to the corresponding weighted sum of individual option implied volatility with the same expiry month and %moneyness level.

Table 3.7. The Ratio of Synthetic Option Implied Volatility to the Corresponding Proxy (in percentage)

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Mar 2016	39%	34%	34%	34%	33%
Apr 2016	42%	37%	36%	36%	37%
May 2016	42%	38%	39%	37%	38%
Jun 2016	43%	39%	38%	38%	39%
Jul 2016	44%	40%	39%	39%	40%
Aug 2016	46%	41%	40%	40%	41%
Sep 2016	47%	43%	41%	41%	41%
Oct 2016	48%	44%	42%	41%	42%
Nov 2016	49%	42%	42%	42%	42%
Dec 2016	50%	45%	43%	43%	43%

Notes: 'Expiry' in the first column denotes the synthetic option expiry month. For each option expiry month and %moneyness level, the percentage denotes the ratio of synthetic option implied volatility to the corresponding weighted sum of individual option implied volatility (i.e. corresponding proxy in Table 3.6).

We can see from Table 3.7 that, on average, the synthetic option implied volatility accounts for only 41% of the corresponding weighted sum of individual option implied volatility, indicating that the proxy volatility is indeed much larger than the corresponding synthetic option implied volatility.

What's more, we figure out which commodity contributes the most to the large proxy volatility. In Table 3.8, we present the weight of each commodity as well as corresponding commodity's average implied volatility over all expiry months and %moneyness levels. We can see clearly from Table 3.8 that 'crude oil' stands out among all commodities, i.e. not only its weight accounts for over half of total weight (53%), but also its average commodity futures option implied volatility is significantly higher than that of the remaining commodities. The result is reasonable, as the crude oil futures price is known to be volatile. As a result, the volatile crude oil futures prices drive up the overall proxy volatility.

Accordingly, results from Table 3.6, 3.7 and 3.8 indicate that the Bloomberg OVDV Black-Scholes implied volatilities are overall large, especially the implied volatility of crude oil futures. As the input data used by Bloomberg OVML option pricing function, large implied volatilities yield high European commodity futures call option prices. The higher the European commodity futures call option prices, the higher the prices of the constructed synthetic option will be, which imply larger implied volatility for the underlying futures on

Table 3.8. Commodity Weight and Average Implied Volatilities

Ticker	Weight	Average Implied Volatilities
wheat w	0.74%	0.2358
wheat kw	0.61%	0.2276
rough rice rra	0.42%	0.2051
oats o a	0.79%	0.2964
silver si	0.36%	0.2608
crude oil cla	53.21%	0.5098
gold gc	0.36%	0.1704
copper hg	1.83%	0.2690
lumber lba	8.65%	0.1968
natural gas nga	3.26%	0.3533
milk class IV kva	0.18%	0.2064
soybean oil boa	0.93%	0.1885
dry whey drwa	0.91%	0.2402
butter v6	0.79%	0.2817
lean hog lha	1.12%	0.2086
live cattle lca	2.67%	0.2004
cattle feeder fca	18.49%	0.2299
juice jo	2.28%	0.2612
non-fat milk lea	0.64%	0.2680
cheese che	0.87%	0.1564
milk daa	0.88%	0.1565

Notes: 'Average Implied Volatilities' represent the commodity's average implied volatility for individual commodity futures option observed in Bloomberg OVML option pricing function over all expiry months and %moneyness levels.

the CPI proxy. These implied volatilities then translate into greater length of interval estimate of future price level, leading to higher uncertainty of corresponding inflation forecast.

Consequently, the Chicago Mercantile Exchange prices the commodity futures options in the way that the options' implied volatilities are large, which then translate into the large uncertainty of inflation forecast in our study. In other words, by analysing the option contracts, we conclude that the commodity futures options market expects high uncertainty for one-year ahead inflation forecast; and this uncertainty implied from the commodity futures options market is higher than the magnitude of inflation forecast uncertainty assumed by the Bank of England.

3.4.5 Rescaling the Synthetic Option Implied Volatility

The previous section analyses the inflation forecast purely from the perspective of the commodity futures options market. In this section, we consider the fact that the consumption good price is generally less volatile than the corresponding commodity futures price, based on which we rescale the synthetic option implied volatility and therefore reduce the uncertainty of inflation forecast.

Specifically, we collect from U.S. Bureau of Labor Statistics the 2011 to 2015's time series of data on U.S. city average prices of consumption goods in the CPI, with each consumption good corresponding to one commodity in the commodity basket of the futures on the CPI proxy, e.g. the time series of U.S. bread prices corresponds to wheat commodity. Next, we use the time series to calculate the historical volatility v_i of the price of the ith consumption good. Then, we compare, over the same period, the consumption good price historical volatility v_i with the corresponding commodity futures option implied volatility V_i. We observe the commodity futures option implied volatility surface from Bloomberg OVML option pricing function for each commodity on 21st of January 2011, 20th of January 2012, 22nd of January 2013, 22nd of January 2014 and 22nd of January 2015, respectively, corresponding to the 2011 to 2015's time series of consumption goods average price data. The ratio $u_i = \frac{v_i}{V_i}$ is calculated for each period (see Table 3.9) and each %moneyness level to estimate the proportion of variation in the price of the ith consumption good to the option implied volatility of the corresponding ith commodity. The average value of u_i calculated across five years' data gives us, in general, the proportion of the ith consumption good price historical volatility to the ith commodity futures option implied volatility for a specific pair of option maturity (period) and %moneyness level. Lastly, we define

$$U = q_1 u_1 + q_2 u_2 + ... + q_{21} u_{21}$$

as the weighted sum of ratio u_i for the same pair of option maturity (period) and %moneyness level across 21 commodities, where q_i denotes the optimal weight for the ith commodity. Since our basket of commodities includes all the commodities with options written on them, based on rich data sample covering the past five years (2011 to 2015), the ratio U can be defined as the general proportion of historical volatility for all consumption goods to the corresponding commodity basket's option implied volatility. The results for ratio U are presented in Table 3.9.

The general proportion of historical volatility for all consumption goods to the corresponding commodity basket's option implied volatility ranges from 23.52% to 56.91%, with an average value of 42.93%, i.e. across all option maturities (period) and %moneyness levels,

Table 3.9. General Proportion of Historical Volatility to Implied Volatility

Period	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Jan-Mar 2016	23.52%	25.60%	27.46%	28.58%	28.39%
Jan-Apr 2016	28.65%	30.60%	32.45%	33.76%	34.22%
Jan-May 2016	33.24%	35.36%	37.46%	39.22%	40.17%
Jan-Jun 2016	39.34%	41.67%	44.14%	46.44%	48.03%
Jan-Jul 2016	39.73%	41.90%	44.21%	46.50%	48.50%
Jan-Aug 2016	42.02%	44.22%	46.60%	48.95%	50.94%
Jan-Sep 2016	42.52%	44.70%	47.05%	49.33%	51.15%
Jan-Oct 2016	43.98%	46.17%	48.50%	50.75%	52.53%
Jan-Nov 2016	46.48%	48.81%	51.28%	53.68%	55.67%
Jan-Dec 2016	47.83%	50.10%	52.50%	54.87%	56.91%

Notes: 'Period' in the first column denotes the period over which we compare the consumption good price historical volatility v_i with the corresponding commodity futures option implied volatility V_i. Each percentage represents the general proportion of historical volatility for all consumption goods to the corresponding commodity basket's option implied volatility.

averagely, only 42.93% of the volatility implied by the commodity futures option can be reflected in the variation of the consumption goods prices.

Next, since the ratio U is generated using rich data sample covering the past five years, we now treat U as an 'attribute' of commodity basket indicating a 'fixed' proportion of historical volatility for all consumption goods to the corresponding commodity basket's option implied volatility. Using this 'attribute', we predict the volatility of consumption goods prices in 2016 by rescaling the synthetic option implied volatility, i.e. by multiplying each synthetic option implied volatility with corresponding ratio U at the same option maturity (period) and %moneyness level. The result is shown in Table 3.10.

Table 3.10. Rescaled Synthetic Option Implied Volatility

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Mar 2016	0.0410	0.0384	0.0396	0.0405	0.0387
Apr 2016	0.0515	0.0477	0.0483	0.0498	0.0507
May 2016	0.0578	0.0536	0.0579	0.0559	0.0580
Jun 2016	0.0677	0.0629	0.0633	0.0658	0.0688
Jul 2016	0.0679	0.0631	0.0630	0.0655	0.0691
Aug 2016	0.0725	0.0672	0.0670	0.0692	0.0725
Sep 2016	0.0732	0.0680	0.0676	0.0696	0.0727
Oct 2016	0.0756	0.0702	0.0696	0.0713	0.0741
Nov 2016	0.0796	0.0721	0.0732	0.0749	0.0779
Dec 2016	0.0822	0.0760	0.0748	0.0763	0.0791

Notes: 'Expiry' in the first column denotes the synthetic option expiry month.

Following the same procedure of forecasting future price level in section 3.4.3, we use the rescaled synthetic option implied volatility data to recalculate the interval estimate of the future price level and therefore reproduce the fan chart in Figure 3.8.

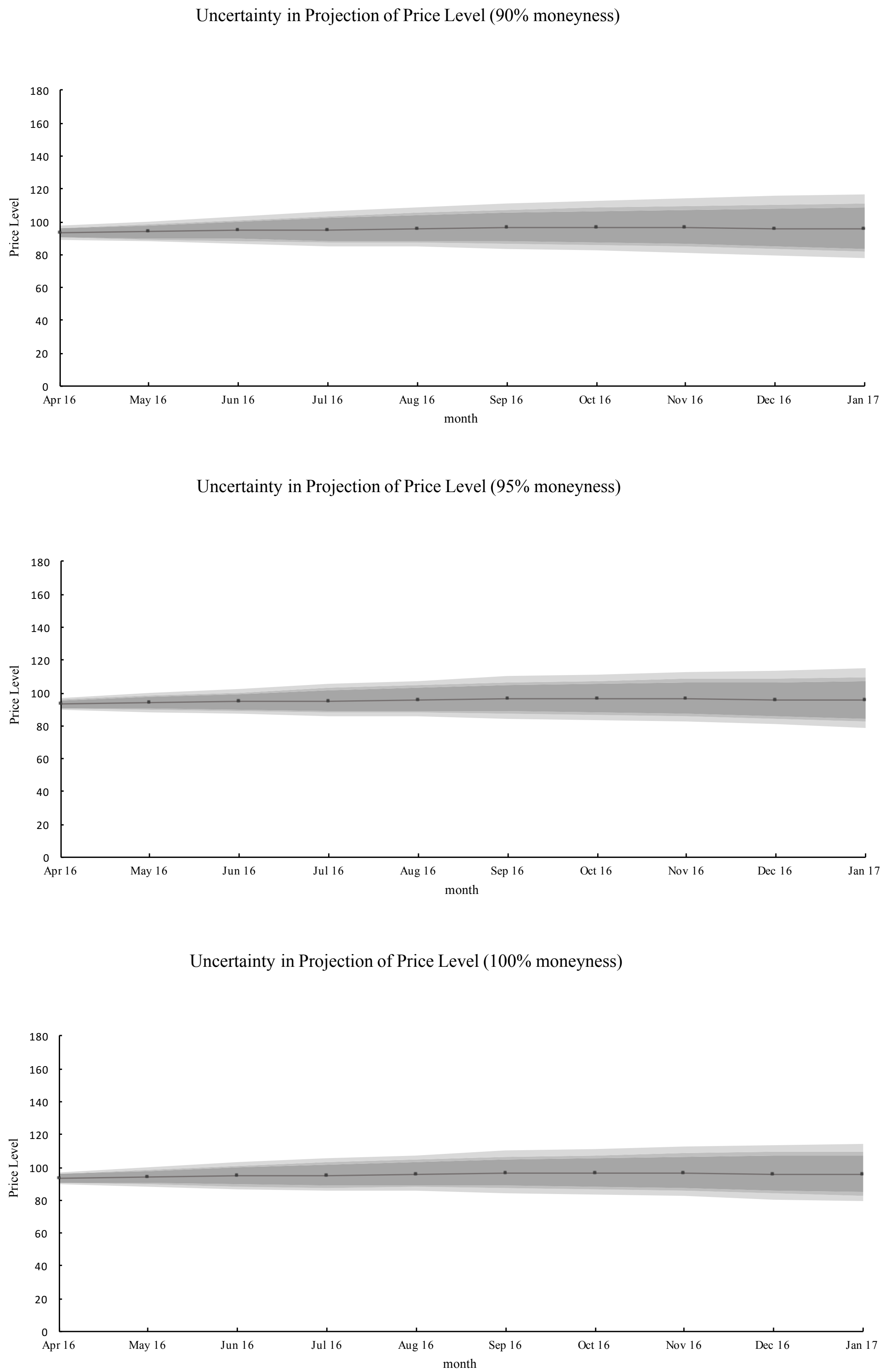
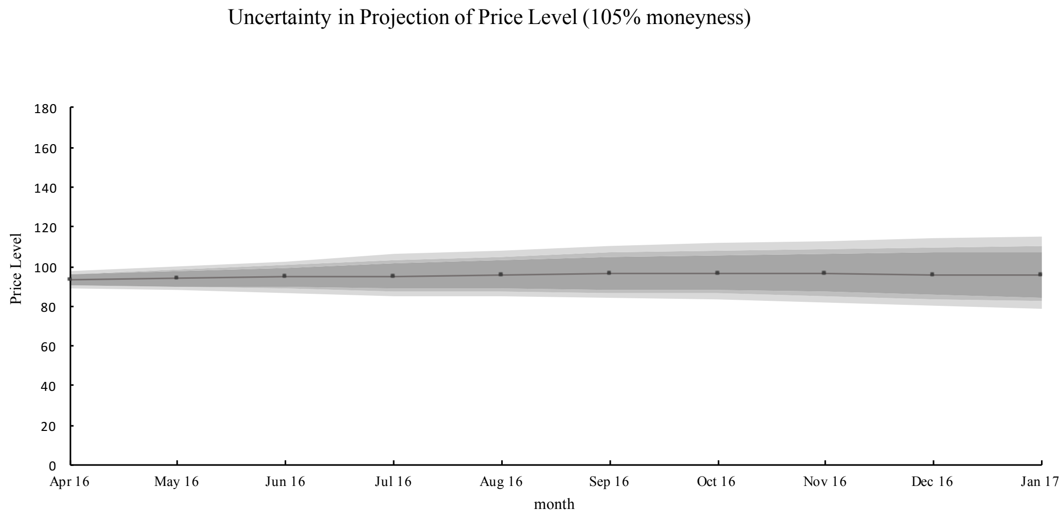


Figure 3.8. The three reproduced fan charts (from top to bottom) plot the uncertainty in projection of future price level derived from deep-in-the-money call option chain (90%moneyness), in-the-money call option chain (95%moneyness) and at-the-money call option chain (100%moneyness), respectively.



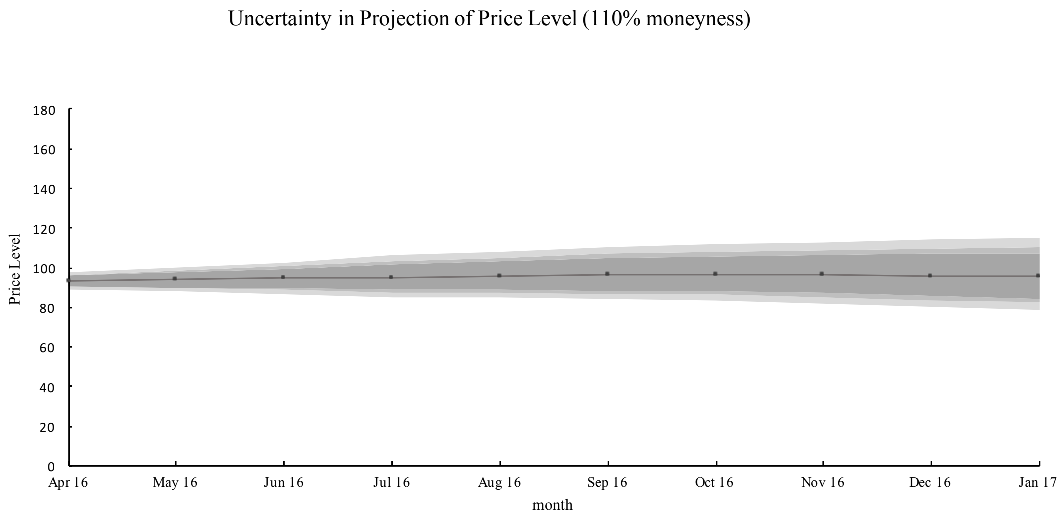


Figure 3.8 - continued. The two reproduced fan charts (from top to bottom) plot the uncertainty in projection of future price level derived from out-of-the-money call option chain (105%moneyness) and deep-out-of-the-money call option chain (110%moneyness), respectively.

We can see from Figure 3.8 that, in each reproduced fan chart, the uncertainty surrounding the 'point' forecast of future price level has been decreased significantly due to the rescaled synthetic option implied volatility.

What's more, based on the interval estimate of price level in January 2017 recalculated using the 105% moneyness level out-of-the-money call option chain (84.52 to 107.4) and the CPI proxy one year earlier at January 2016 (93.662), we now predict that there is a 90% chance that January 2017's inflation rate will lie between -9.76% and 14.67%. Therefore, the uncertainty of inflation forecast has almost been halved.

However, the uncertainty of our inflation forecast is still relatively large compared to that of the Bank of England's inflation forecast. The reason for our inflation forecast uncertainty being large even after rescaling the synthetic option implied volatility is that the implied volatility of crude oil futures in 2016 is especially high compared to those observed during the past five years. To see this clearly, we compare the annual average implied volatility of crude oil futures observed during 2011 to 2016 in Figure 3.9.

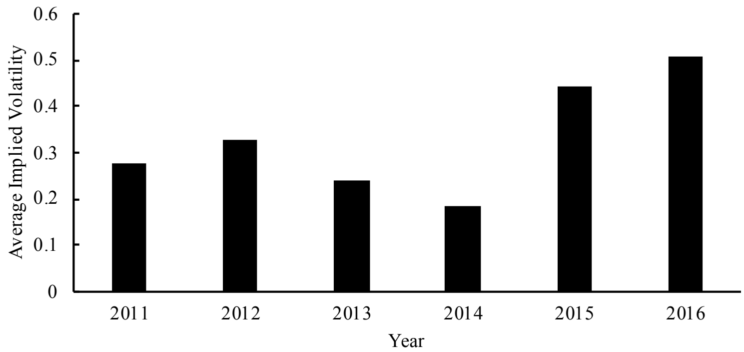


Figure 3.9. Each bar in Figure 3.9 denotes the average implied volatility of crude oil futures price observed on the observation date of the indexed year. For example, the bar indexed with '2011' represents the average value of the implied volatility surface of crude oil futures observed on 21^{st} of January 2011 from Bloomberg OVML option pricing function.

It can be seen from Figure 3.9 that the annual average implied volatility of crude oil futures in 2016 is significantly higher than those observed during the past five years; it increases considerably from the lowest annual average implied volatility 0.1861 in 2014 to 0.5098 in 2016. Given that the crude oil accounts for over half of the weight in commodity basket (53%), it is not surprising that the high implied volatility of crude oil futures has been driving the uncertainty of our inflation forecast to a relatively high level. In addition, the implied volatility surface updates continuously, therefore, if we apply our method to the period when the crude oil futures option implied volatility is low, then the uncertainty of our inflation forecast can be narrowed down to similar magnitude of the inflation prediction uncertainty of the Bank of England inflation fan chart.

3.5 Discussion and Conclusions

The futures on the CPI proxy derived from commodity futures price data predicts the price level up to one year in the future. However, there are considerable uncertainties surrounding these 'point' forecasts. In this paper, we have investigated the uncertainty of consumer price index prediction by studying the implication of a synthetic option's implied volatility surface.

The major difficulty we encountered in the analysis is that most exchanged-traded commodity futures options are American options. In order to construct the synthetic option, we need the corresponding European options price data. This is realized by using Bloomberg OVML option pricing function which uses Bloomberg OVDV function implied volatility surface to price consistently the corresponding commodity futures European options.

Based on the commodity futures European option price data, we apply Monte Carlo method and construct a synthetic European option written on the futures on the CPI proxy. Our assumption of independent lognormal commodity futures leads to a closed-form Black's model type option pricing formula. We then use this option pricing formula to derive the implied volatility surface.

A volatility skew (slight smile) is observed in the synthetic option's implied volatility surface. We analyse the implication of implied volatility surface by calculating the interval estimate of the future price level at the maturity of the futures on the CPI proxy. Our result has two major implications. First, for one-year horizon, the further we look into the future, the less certain we are about the price level prediction. This is due to a lack of predicting tool such as inflation futures which covers the price level prediction over one year in the future. Second, compared to in-the-money and at-the-money option chains, we are more certain about the price level forecast when the prediction is derived from the out-of-the-money option chain (105%moneyness). This can be explained by the implied volatility skew (slight smile) and the fact that out-of-the-money option chain has superior capability of capturing the market expectation of future price level compared to in-the-money and at-the-money option chains.

Furthermore, based on the uncertainty surrounding the 'point' forecast of future price level, we derive the one-year ahead inflation forecast, which reflects the commodity futures options market's expectation for future inflation rate. Our analysis indicates that the commodity futures options market expects high uncertainty for one-year ahead inflation forecast; and this uncertainty implied from the commodity futures options market is higher than the uncertainty of inflation prediction of the Bank of England inflation fan chart. The main factor driving the uncertainty of our inflation forecast to a high level proves to be the especially high crude oil futures option implied volatility in 2016. By comparing the consumption goods price historical volatility with corresponding commodity futures option implied volatility, we significantly reduce the uncertainty of inflation forecast.

Most importantly, one of the most important features of the commodity futures option market is that tradings are happening continuously, leading to high frequency in the update of market data, including the commodity futures option implied volatility surface. This feature indicates that, the continuously-updated market data yields continuously-updated inflation forecast. We can narrow down the uncertainty of inflation forecast to the same range as the Bank of England's inflation forecast by applying our method to the period when the crude oil futures option implied volatility is low, but what is important in our study is that the inflation forecast derived from our method tracks closely the commodity futures option market's expectation about future inflation rate in the real time, i.e. inflation forecasting through commodity futures and options.

Chapter 4

A Study of the Inflation Risk Premium in the Context of Commodity Futures Market

Coauthored with CHRISTIAN-OLIVER EWALD

Abstract

In this paper, we estimate and analyse the inflation risk premium based on a link between the inflation risk premium and the risk premium associated with the futures on the CPI proxy. We estimate the risk premium associated with the futures on the CPI proxy as the predictable component in the prediction error. We then use the most accurate estimate of the risk premium associated with the futures on the CPI proxy to estimate the inflation risk premium. The negative inflation risk premium estimated from our method is consistent with the post-Lehman evidence for negative inflation risk premium in the macroeconomic inflation risk premium literature. Given that the price of the futures on the CPI proxy is more volatile than the consumer price index, the magnitude of our inflation risk premium estimate is larger than that of the inflation risk premium estimate in the macroeconomic inflation risk premium literature.

4.1 Introduction

Traditionally, the standard practice among central bankers and policy makers has been to interpret the price of a commodity futures contract as the market expectation of the spot price of the commodity, e.g. many researchers have treated the crude oil futures price as the measure of the market expectation of the spot crude oil price at the expiry of the futures contract. Indeed, under the risk-neutral measure, the commodity futures price is equal to the expected spot commodity price at the expiry of the futures contract, therefore the error from predicting the spot commodity price based on the commodity futures price should be zero on average. However, as the central bankers and policy makers usually make forecast under the real-world measure, the existence of a time-varying risk premium in the commodity futures market will render the commodity futures price an inaccurate measure of the expected future spot price.

In the commodity futures market risk premium literature, there has been emerging consensus on the existence of a time-varying risk premium in the commodity futures market. Sadorsky (2002), Pagano and Pisani (2009), Hamilton and Wu (2014) and Baumeister and Kilian (2017) all document the existence of a time-varying risk premium in the oil futures market. More importantly, the time-varying risk premium exists in a wide range of commodity futures markets. Evidence includes Fama and French (1987), Bessembinder and Chan (1992), Bjornson and Carter (1997), Lucia and Torró (2008) and Hambur et al. (2016). These studies all provide evidence supporting that the time-varying risk premium exists in the energy, metal, agricultural and electricity futures market. In Chapter 2, we construct the term structures of futures on the CPI proxy dataset. Given that the futures on the CPI proxy is constructed as the weighted sum of commodity futures prices, the evidence of the existence of time-varying risk premium in a wide range of commodity futures markets inspires us to investigate and estimate the time-varying risk premium associated with the futures on the CPI proxy.

The purpose of constructing the futures on the CPI proxy dataset is to offer an approach to predict the future U.S. price level. The forecast of future price level implies explicitly the expected inflation rate. Our result in Chapter 3 shows that, a volatile futures on the CPI proxy yields large degree of uncertainty in the inflation rate prediction. Thus, the risk premium associated with the futures on the CPI proxy is connected to the inflation risk premium. In this paper, we derive a link between the risk premium associated with the futures on the CPI proxy and the inflation risk premium, based on which we estimate the inflation risk premium.

In the existing inflation risk premium literature, the mainstream approach to estimate the unobservable inflation risk premium is to use the no-arbitrage term structure model, where data on the nominal yield and inflation rate are a must. For example, Ang et al. (2008) specify a no-arbitrage term structure model with both nominal yield and inflation data to identify the term structure of real rates and inflation risk premium. Apart from data on nominal yield and inflation, more recent study such as Grishchenko and Huang (2013) use TIPS data and Survey of Professional Forecast to estimate the inflation risk premium. An important recent study, Fleckenstein et al. (2017), differentiates itself from the previous literature by using the inflation swaps and options prices to infer the inflation risk premium. They construct a continuous-time model describing the dynamics of instantaneous expected inflation rate, and price the inflation swaps and options accordingly. Our study makes two contributions to the literature. First, we do not need TIPS or inflation swaps data to infer the real yield and expected inflation rate, our approach is based on the widely available commodity futures and spot prices data. Second, we derive a link between the inflation risk premium and the risk premium associated with the futures on the CPI proxy and use this link to estimate the inflation risk premium.

The reminder of the paper is organised as follows. We start by describing the method for estimating the time-varying risk premium associated with the futures on the CPI proxy in section 4.2; we then derive the relationship between the risk premium associated with the futures on the CPI proxy and the inflation risk premium in section 4.3; section 4.4 describes the construction of datasets; next, sections 4.5 and 4.6 estimate and select the accurate estimates for the risk premium associated with the futures on the CPI proxy; section 4.7 estimates the inflation risk premium and assesses our estimate against the inflation risk premium estimates in the macroeconomic inflation risk premium literature; finally, section 4.8 concludes the paper.

4.2 Estimating the Risk Premium Associated with the Futures on the CPI Proxy

We follow the approach proposed by Baumeister and Kilian (2017) to estimate the time-varying risk premium associated with the futures on the CPI proxy. Let FCP_t^h denote the current price of the futures on the CPI proxy maturing h periods from now; and CP_{t+h} the corresponding spot CPI proxy at the expiry of the futures on the CPI proxy. As shown in

Shreve (2004), the marking to margin mechanism of the futures market implies that, if an agent holds a long futures position between time t and $t+1$, then at time $t+1$ the agent receives a payment

$$FCP_{t+1}^{h-1} - FCP_t^h.$$

Furthermore, the futures price FCP_t^h satisfies

$$FCP_{t+h}^0 = CP_{t+h}.$$

Therefore, the sum of the payments received by the agent holding a long position from the current time until the expiry of the futures on the CPI proxy is

$$CP_{t+h} - FCP_t^h.$$

As stated in Hansen and Renault (2010), the principle of no arbitrage implies that

$$E_t[Q_{t+h}(CP_{t+h} - FCP_t^h)] = 0, \quad (4.1)$$

where Q_{t+h} is the stochastic discount factor and $E[\cdot]$ denotes the expectation under the real-world measure. Using the definition of covariance, we rearrange equation (4.1) and obtain

$$FCP_t^h = E_t[CP_{t+h}] + \frac{cov(CP_{t+h}, Q_{t+h})}{E_t[Q_{t+h}]}, \quad (4.2)$$

where $\frac{cov(CP_{t+h}, Q_{t+h})}{E_t[Q_{t+h}]}$ refers to the risk premium. The commodity futures pricing theory that explains equation (4.2) is the risk premium theory, which originates from the theory of normal backwardation introduced by Keynes (1934). As shown in equation (4.2), the risk premium theory states that the futures on the CPI proxy is equal to the sum of the expected spot CPI proxy at the expiry of the futures on the CPI proxy, and the risk premium associated with the futures on the CPI proxy. While the theory of normal backwardation from Keynes (1934) argues that the risk premium is negative, more recent evidence in commodity risk premium literature such as Cootner (1960), Bessembinder (1992), Brooks et al. (2013), Gorton et al. (2013) and Hambur et al. (2016) show that the sign of the risk premium can vary cross-sectionally and inter-temporally, depending on the net hedging pressure, i.e. the net supply of the futures contracts. For example, if the commodity producers' hedging activity is greater than that of the commodity consumers, there will be net short hedging position. As Cootner

(1960) argues, to induce the speculator to buy, the futures price must be depressed below the price expected by the speculator to offer them an adequate return, leading to a negative risk premium. If, on the other hand, the commodity consumers' hedging activity is greater than that of the commodity producers, then there will be net long hedging position. In this case, the futures price will be set above the expected future spot price, corresponding to a positive risk premium.

Furthermore, it follows from equation (4.2) that, when the risk premium $\frac{cov(CP_{t+h},Q_{t+h})}{E_t[Q_{t+h}]}$ is zero, the futures on the CPI proxy is equal to the expected spot CPI proxy at the expiry of the futures on the CPI proxy, i.e.

$$FCP_t^h = E_t[CP_{t+h}], \tag{4.3}$$

rearranging, we have

$$E_t[CP_{t+h} - FCP_t^h] = 0,$$

where $CP_{t+h} - FCP_t^h$ denotes the prediction error from predicting the spot CPI proxy at the expiry of the futures on the CPI proxy based on the current price of the futures on the CPI proxy. According to Baumeister and Kilian (2017), evidence of a predictable component in the prediction error $CP_{t+h} - FCP_t^h$ such that

$$E_t[CP_{t+h} - FCP_t^h] \neq 0$$

would be consistent with the existence of a time-varying risk premium.

As stated in Baumeister and Kilian (2017), the prediction error $CP_{t+h} - FCP_t^h$ is not stationary, in order to estimate the predictable component in the prediction error by regression method, we need to normalize the prediction error $CP_{t+h} - FCP_t^h$ as follows,

$$\frac{CP_{t+h} - FCP_t^h}{FCP_t^h}.$$

The risk premium associated with the futures on the CPI proxy may be estimated from the regression

$$\frac{CP_{t+h} - FCP_t^h}{FCP_t^h} = \alpha_h + \beta_h x_t + v_{t+h}, \tag{4.4}$$

where α_h and β_h are the horizon-specific regression coefficients; x_t denotes the predictor; and v_{t+h} is the mean zero error term. Compared with the traditional return regression method based on the holding-period return, estimating the risk premium as the predictable component in the prediction error as in equation (4.4) yields estimates of risk premium at the horizon towards the expiry of the futures on the CPI proxy, which enables us to measure the implied spot CPI proxy expectations. Based on the implied spot CPI proxy expectations, we may estimate the risk premium associated with the futures on the CPI proxy. Solving equation (4.4) for the spot CPI proxy at the expiry of the futures on the CPI proxy CP_{t+h}, we obtain

$$CP_{t+h} = FCP_t^h(1+\alpha_h+\beta_h x_t+v_{t+h}). \tag{4.5}$$

Taking the expectation of equation (4.5), we obtain the expectation of the spot CPI proxy at the expiry of the futures on the CPI proxy

$$E_t[CP_{t+h}] = FCP_t^h(1+\alpha_h+\beta_h x_t). \tag{4.6}$$

The risk premium associated with the futures on the CPI proxy RP_t^h then may be estimated as

$$RP_t^h = FCP_t^h - E_t[CP_{t+h}]. \tag{4.7}$$

4.3 A Link between the Inflation Risk Premium and the Risk Premium Associated with the Futures on the CPI Proxy

Based on the method of estimating the risk premium associated with the futures on the CPI proxy, we now show that the inflation risk premium found in the macroeconomic inflation risk premium literature can be derived from the risk premium associated with the futures on the CPI proxy.

As shown in equation (4.7), the futures on the CPI proxy is equal to the sum of the expectation of the spot CPI proxy at the expiry of the futures on the CPI proxy and the risk premium, i.e.

$$FCP_t^h = E_t[CP_{t+h}] + RP_t^h. \tag{4.8}$$

In the macroeconomic inflation risk premium literature (Bekaert and Wang (2010), Garcia and Werner (2010)), the difference between the yield of a nominal bond and the yield of a real bond is the break-even inflation rate, and the break-even inflation rate comprises the expected inflation rate and the inflation risk premium:

$$y_t^h - r_t^h = E_t[\pi_{t+h,h}] + \phi_t^h, \tag{4.9}$$

where y_t^h denotes the yield of a nominal bond with maturity of h periods; r_t^h is the yield of a real bond with maturity of h periods; $E_t[\pi_{t+h,h}]$ is the expected inflation rate; and ϕ_t^h denotes the inflation risk premium.

The inflation rate between time t and time $t+h$ is:

$$\pi_{t+h,h} = \frac{CP_{t+h} - CP_t}{CP_t}. \tag{4.10}$$

If we take the expectation of equation (4.10), we have:

$$E_t[\pi_{t+h,h}] = \frac{E_t[CP_{t+h}] - CP_t}{CP_t}. \tag{4.11}$$

Furthermore, the real bond pays off:

$$1 + \frac{CP_{t+h} - CP_t}{CP_t}. \tag{4.12}$$

The price of the real bond is equal to the expected discounted payoff in (4.12), where $E^Q[\cdot]$ denotes the expectation under the risk-neutral measure:

$$\begin{aligned} & E_t^Q\left[\frac{1}{1+y_t^h}\left(1+\frac{CP_{t+h}-CP_t}{CP_t}\right)\right] \\ & = \frac{1}{1+y_t^h}\left(1+\frac{FCP_t^h - CP_t}{CP_t}\right) \\ & \approx (1-y_t^h)\left(1+\frac{FCP_t^h - CP_t}{CP_t}\right) \\ & \approx 1 - y_t^h + \frac{FCP_t^h - CP_t}{CP_t}. \end{aligned} \tag{4.13}$$

The real yield is computed as the difference between the real payoff (which is equal to 1) and the real price, divided by the real price:

$$r_t^h = \frac{1 - \left(1 - y_t^h + \frac{FCP_t^h - CP_t}{CP_t}\right)}{1 - y_t^h + \frac{FCP_t^h - CP_t}{CP_t}} \approx y_t^h - \frac{FCP_t^h - CP_t}{CP_t}. \tag{4.14}$$

Therefore,

$$y_t^h - r_t^h = \frac{FCP_t^h - CP_t}{CP_t}. \tag{4.15}$$

We now substitute equation (4.15) and (4.11) into (4.9):

$$\frac{FCP_t^h - CP_t}{CP_t} = \frac{E_t[CP_{t+h}] - CP_t}{CP_t} + \phi_t^h,$$

which is:

$$FCP_t^h - E_t[CP_{t+h}] = CP_t \times \phi_t^h. \tag{4.16}$$

Therefore, substituting equation (4.8) into (4.16), we have derived the relationship between the risk premium associated with the futures on the CPI proxy and the inflation risk premium in the macroeconomic inflation risk premium literature:

$$\phi_t^h = \frac{RP_t^h}{CP_t}, \tag{4.17}$$

i.e. the inflation risk premium is equal to the ratio of the risk premium at time t associated with the futures on the CPI proxy maturing h periods from time t to the spot CPI proxy at time t. Equation (4.17) enables us to estimate the inflation risk premium from the estimates of the risk premium associated with the futures on the CPI proxy.

4.4 Data Description

Based on the approach for estimating the inflation risk premium as described above, we now describe the construction of the datasets. Our analysis uses three datasets, namely, the futures on the CPI proxy, the spot CPI proxy, and the predictors.

4.4.1 Futures on the CPI Proxy

We apply the futures on the CPI proxy dataset constructed in Chapter 2 to estimate the inflation risk premium. This application indicates that the futures on the CPI proxy dataset may potentially be fitted into various frameworks in both the commodity market literature and macroeconomic inflation risk premium literature to gain new findings. The original commodity futures price data consists of daily observations from 2^{nd} of January 2013 to 18^{th} of December 2013. Every Wednesday observation is selected out of the daily observations to obtain the weekly observations. At every observation date we collect last price (or settlement price if last price is unavailable) for the available commodity futures chain with time to maturity up to twelve months in the future. Based on these weekly observations of commodity futures prices we then construct the futures on the CPI proxy chain observed on every Wednesday from 2^{nd} of January 2013 to 18^{th} of December 2013. Specifically, we construct the time t price of the futures on the CPI proxy maturing in the future month $T > t$ as the weighted sum of individual commodity futures price, i.e.

$$FCP(t,T) = \sum_{i=1}^{34} a_i F_i(t,T), \tag{4.18}$$

where $i = 1,2,3,...,34$ represents the i^{th} commodity of the 34 commodities in the construction of the futures on the CPI proxy; a_i denotes the weight for the i^{th} commodity; $F_i(t,T)$ is the time t price of the futures contract maturing in month T written on the i^{th} commodity. Given that all 34 commodity futures contracts mature in the same month T, the way we construct the futures on the CPI proxy as shown in expression (4.18) suggests that $FCP(t,T)$ is the time t expectation of the average spot price level in month T. More importantly, although all 34 commodity futures contracts expire in the same month T, the fact that each commodity has its unique futures maturity month arrangement indicates that the last trading date in month T differs from one commodity to another. As a result, the maturity T of the futures on the CPI proxy can be viewed as a maturity month and therefore the futures on the CPI proxy does not have a specific maturity date. Defining the maturity of the futures on the CPI proxy as a maturity month is our desired result and this definition is consistent with our interpretation of the futures on the CPI proxy, i.e. the time t expectation of consumer price index for month T.

However, for the purpose of using the futures on the CPI proxy dataset to estimate the inflation risk premium empirically in this study, the futures on the CPI proxy needs to have a specific maturity date. Thus, we follow the approach suggested by Fama and French (1987)

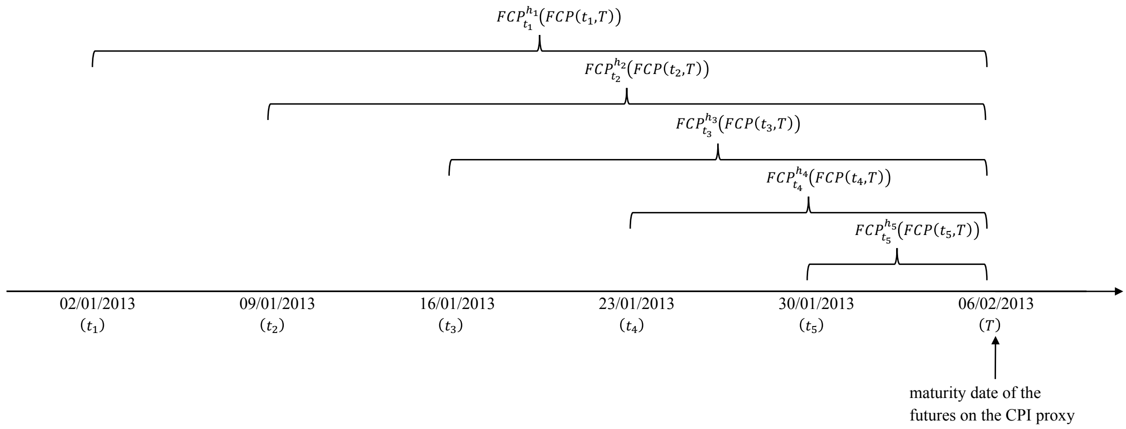


Figure 4.1. This figure illustrates the timing of the futures on the CPI proxy, which explains the connection between $FCP(t,T)$ and FCP_t^h.

and assume that the futures on the CPI proxy matures on the first Wednesday of the expiry month. This assumption facilitates us to apply consistently the construction of the futures on the CPI proxy $FCP(t,T)$ as described in (4.18) to the definition of the futures on the CPI proxy FCP_t^h in this study. Given that the futures on the CPI proxy is constructed to forecast the price level in the future, we define the nearest futures on the CPI proxy on each observation date as the futures on the CPI proxy maturing in the month following the month which time t falls within, e.g. on observation date 2^{nd} of January 2013, the nearest futures on the CPI proxy is the futures on the CPI proxy maturing in February 2013. For each observation date, the futures on the CPI proxy chain consists of twelve consecutive futures on the CPI proxy, i.e. from the nearest futures on the CPI proxy to the futures on the CPI proxy maturing twelve months in the future. Accordingly, the way we construct the futures on the CPI proxy chain indicates that all observation dates within the same month correspond to the same futures on the CPI proxy chain. In other words, the futures on the CPI proxy chain observed on the dates falling within the same month are actually different observations of the same futures on the CPI proxy chain.

Specifically, we use an example in Figure 4.1 to illustrate the timing of the futures on the CPI proxy, which explains the connection between $FCP(t,T)$ and FCP_t^h. In Figure 4.1, 2^{nd} of January 2013 (t_1), 9^{th} of January 2013 (t_2), 16^{th} of January 2013 (t_3), 23^{rd} of January 2013 (t_4) and 30^{th} of January 2013 (t_5) are five observation dates in January 2013; 6^{th} of February 2013 (T) is the maturity date of the five observation dates' nearest futures on the CPI proxy. $FCP(t_i,T)$ denotes the time t_i price of the futures on the CPI proxy maturing on time T; $FCP_{t_i}^{h_i}$ represents the corresponding contemporaneous price of the futures on the CPI proxy maturing h_i weeks from time t_i. Following this logic, we define the horizon h_i as follows,

$$h_i = \frac{T - t_i}{7}, \tag{4.19}$$

i.e. the number of weeks between the observation date t_i and the maturity date T of the futures on the CPI proxy. Under this definition, the 2^{nd} of January 2013 price of the futures on the CPI proxy maturing on 6^{th} of February 2013 ($FCP(t_1,T)$) is the 2^{nd} of January 2013 price of the futures on the CPI proxy maturing in five weeks ($FCP_{t_1}^{h_1}$). Accordingly, we define the connection between the definition of FCP_t^h and the corresponding dataset $FCP(t,T)$ as follows,

$$FCP_t^h = FCP(t,T) \quad \Leftrightarrow \quad h = \frac{T - t}{7}, \tag{4.20}$$

i.e. FCP_t^h and $FCP(t,T)$ refer to the same futures on the CPI proxy if and only if the horizon h is equal to the number of weeks between the observation date t and the futures on the CPI proxy maturity T.

4.4.2 Spot CPI Proxy

The spot CPI proxy is constructed as the weighted sum of the commodity spot prices, i.e.

$$CP_t = \sum_{i=1}^{34} a_i S_{it}, \tag{4.21}$$

where CP_t is the time t spot CPI proxy; $i = 1,2,3,...,34$ represents the ith commodity of the 34 commodities in the construction of the futures on the CPI proxy as shown in (4.18); a_i denotes the same weight for the ith commodity as described in (4.18); S_{it} is the time t spot price for the ith commodity. We construct the spot CPI proxy with two category of timings. The first category of timing denotes the spot CPI proxy on each observation date of the futures on the CPI proxy, which is the weighted sum of commodity spot prices on the observation date as shown in definition (4.21). This category of spot CPI proxy will be applied in equation (4.17) to estimate the inflation risk premium. The second category of timing refers to the spot CPI proxy at the expiry of the corresponding futures on the CPI proxy, i.e.

$$CP_{t+h} = \sum_{i=1}^{34} a_i S_{i(t+h)}, \tag{4.22}$$

where CP_{t+h} is the spot CPI proxy at the expiry of the corresponding futures on the CPI proxy; $S_{i(t+h)}$ denotes the spot price of the ith commodity observed on the last trading date of the corresponding futures contract written on the ith commodity. Analogous to the relationship between the timing of individual commodity futures contract maturity and the timing of the constructed futures on the CPI proxy maturity, due to the fact that the last trading date arrangement of futures contract differs from one commodity to another, the subscript $t+h$ of CP_{t+h} actually denotes a period of time which is the same expiry month T of $FCP(t,T)$ as described in definition (4.18). In order to keep the timing of the spot CPI proxy and corresponding futures on the CPI proxy consistent, we assume that the subscript $t+h$ of CP_{t+h} to be a specific date that is the same date as the assumed maturity date of corresponding futures on the CPI proxy.

We use Bloomberg to collect commodity spot price data. Energy commodity and metal commodity spot price data are widely available. For agricultural commodities, we read the commodity futures contract's description and note about the underlying commodity and obtain relevant information regarding the grade and quality of the commodity, then we use Bloomberg 'AGGP' function to search for the spot price data of commodity that matches the grade and quality of the commodity underlying the corresponding commodity futures contract. For those futures contracts whose underlying assets' spot price data is not available, we follow the approach in Fama and French (1987) and use the relevant futures price instead. Specifically, for the spot CPI proxy on the observation date as constructed in definition (4.21), we use the futures price of the observation date's nearest futures contract as the measurement of the unavailable spot price; for the spot CPI proxy at the expiry of the futures on the CPI proxy as described in definition (4.22), we use the futures price observed on the last trading day of the futures contract as the substitution for the unobservable spot price.

4.4.3 Predictors

As suggested by Pagano and Pisani (2009) and Baumeister and Kilian (2017), in the absence of a time-varying risk premium, the prediction error from predicting the spot CPI proxy at the expiry of the futures on the CPI proxy based on the current price of the futures on the CPI proxy should be zero on average, and the prediction error is expected to be uncorrelated with any variable in the information set at the time the prediction is made. In the presence of the time-varying risk premium, however, the prediction error may be predictable based on the information available at the time the forecast is made. Therefore, we include twelve candidate predictors which are conjectured to be correlated with the prediction error.

The candidate predictors are divided into three categories based on the way they are connected with the prediction error. First, given that both the futures on the CPI proxy and spot CPI proxy are based on price data from the commodity market, we include indices that are broad measures of the U.S. commodity spot and futures market, i.e. Bloomberg Commodity Index, Bloomberg Commodity Index Total Return, Commodity Research Bureau BLS/U.S. Spot All Commodities, Commodity Research Bureau BLS/U.S. Spot Raw Industrials, Thomson Reuters/Core Commodity CRB Commodity Index, Dow Jones Commodity Index and Rogers International Commodity Index Total Return. Second, the purpose of constructing the futures on the CPI proxy is to forecast future U.S. consumer price index level. Given that the ratio between S&P 500 Index and consumer price index is closely monitored by the Federal Reserve Bank due to the connection between the two indices, we include S&P 500 Index, S&P 500 Total Return Index, Dow Jones Industrial Average and Dow Jones Industrial Average Total Return Index. Third, the market price of the non-diversifiable risk is known as the risk premium. We include the Chicago Board Options Exchange SPX Volatility Index (VIX Index) as a proxy of market risk estimate. The daily observations of the twelve candidate predictors are collected from Bloomberg. The description of the twelve candidate predictors are presented in Table 4.1.

4.5 Estimation Results of the Risk Premium Associated with the Futures on the CPI Proxy

Using the three datasets as described in the previous section, we apply the OLS regression as displayed in (4.4)

$$\frac{CP_{t+h} - FCP_t^h}{FCP_t^h} = \alpha_h + \beta_h x_t + v_{t+h}$$

to estimate the time-varying risk premium associated with the futures on the CPI proxy based on the information available at time t. For each horizon $h \in \{1,2,3,...,52\}$, the time series of the dependent variable is paired with the contemporaneous time series of each candidate predictor to estimate the horizon-specific regression coefficients α_h and β_h, and therefore for every horizon the twelve candidate predictors yield twelve estimates of regression coefficients α_h and β_h. Any regression result with insignificant p-value of β_h is excluded from further

Table 4.1. Description of Predictors

Predictor	Description
S&P 500 Total Return Index (SPXT Index)	Calculated intraday by S&P based on the price changes and reinvested dividends of SPX Index with a starting date of 4th January 1988.
Bloomberg Commodity Index (BCOM Index)	Bloomberg Commodity Index (BCOM) is calculated on an excess return basis and reflects commodity futures price movements. The index rebalances annually weighted 2/3 by trading volume and 1/3 by world production and weight-caps are applied at the commodity, sector and group level for diversification. Roll period typically occurs from 6th -10th business day based on the roll schedule.
Bloomberg Commodity Index Total Return (BCOMTR Index)	The Bloomberg Commodity Total Return index is composed of futures contracts and reflects the returns on a fully collateralized investment in the BCOM. This combines the returns of the BCOM with the returns on cash collateral invested in 13 weeks (3 Month) U.S. Treasury Bills.
Commodity Research Bureau BLS/U.S. Spot All Commodities (CRB CMDT Index)	This data represents Commodity Research Bureau BLS Spot Indices. The prices used in the index are obtained from trade publications or from other government agencies. Prices for cocoa beans, corn, steers, sugar, wheat, burlap, copper scrap, cotton, lead scrap, print cloth (spot), rubber, steel scrap, wool tops, and zinc, are of the same specification and market source as those used in the comprehensive monthly Wholesale Price Index. Prices for butter, hides, hogs, lard, rosin, tallow, and tin are either differently specified spot prices or from different markets.
Commodity Research Bureau BLS/U.S. Spot Raw Industrials (CRB RIND Index)	This data represents Commodity Research Bureau BLS Spot Indices. CRB BLS Raw Industrials: Hides, tallow, copper scrap, lead scrap, steel scrap, zinc, tin, burlap, cotton, print cloth, wool tops, rosin, and rubber.
Thomson Reuters/Core Commodity CRB Commodity Index (CRY Index)	The TR/CC CRB Excess Return Index is an arithmetic average of commodity futures prices with monthly rebalancing.
Dow Jones Commodity Index (DJCI Index)	Dow Jones Commodity Index.
Dow Jones Industrial Average Total Return Index (DJITR Index)	The Dow Jones Industrial Average Total Return Index tracks the total return of the member stocks of the DJI Index. Dividends are reinvested. This index prices once per day at market close by Dow Jones. The index is quoted in USD.
Dow Jones Industrial Average (INDU Index)	The Dow Jones Industrial Average is a price-weighted average of 30 blue-chip stocks that are generally the leaders in their industry. It has been a widely followed indicator of the stock market since 1st October 1928.
Rogers International Commodity Index Total Return (RICIGLTR Index)	This data represents Rogers International commodity index data for Agricultural, Energy and Metals markets.
S&P 500 Index (SPX Index)	Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries. The index was developed with a base level of 10 for the 1941-43 base period.
Chicago Board Options Exchange SPX Volatility Index (VIX Index)	The Chicago Board Options Exchange Volatility Index reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of strikes. 1st and 2nd month expirations are used until 8 days from expiration, then the 2nd and 3rd are used.

consideration, so that we only use the statistically significant OLS regression results to estimate the risk premium associated with the futures on the CPI proxy.

Based on the datasets and estimates of the regression coefficients, we follow the procedure as described in equation (4.5), (4.6) and (4.7) to estimate the risk premium associated with the futures on the CPI proxy. The estimation results are presented in Figure 4.2.

For expository purpose, Figure 4.2 focuses on 13-week, 26-week, 39-week and 52-week horizons. Each line chart in Figure 4.2 illustrates estimates of the risk premium associated with the futures on the CPI proxy based on statistically significant regression results of alternative candidate predictors. The X-axis is the observation date; the Y-axis denotes the estimates of the risk premium associated with the futures on the CPI proxy; different candidate predictors

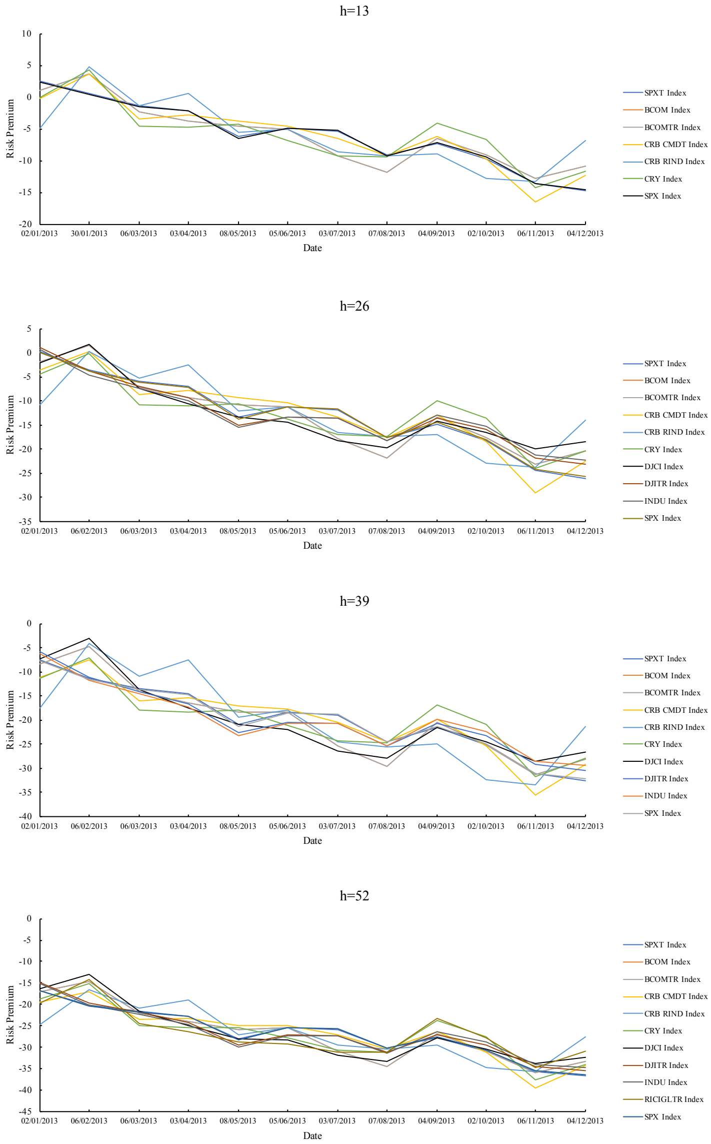


Figure 4.2. The line charts in Figure 4.2 presents the alternative estimates of the risk premium associated with the futures on the CPI proxy.

are characterised by different colours. In our study, a risk premium of 5, for instance, means that the futures on the CPI proxy exceeds the expected spot CPI proxy at the expiry of futures on the CPI proxy by 5, whereas a risk premium of -10 denotes that the expectation of spot CPI proxy exceeds the futures on the CPI proxy by 10. The estimates for all four horizons agree that there is a downward trend in the risk premium associated with the futures on the CPI proxy from January 2013 to December 2013. The sign and magnitude of risk premium, however, differ across four horizons. Specifically, for the 13-week horizon, the risk premium starts with an estimate slightly above zero and ends up with an average value of -11.72. The 26-week horizon saw the risk premium decreases from an average value slight below zero to

-21.29 averagely. As for the 39-week horizon, the risk premium starts further below zero and ends up with an average of -28.58. Finally, the 52-week horizon yields the lowest values, i.e. the risk premium estimation declines averagely from -17.89 to -33.51. Accordingly, in Figure 4.2, as horizon increases from 13 weeks to 52 weeks, the risk premium estimates decrease from a slightly positive average value to an average value of -33.51.

More importantly, it can be seen from each line chart that there is disagreement on the magnitude and even the sign of the estimates of risk premium on the same date. To be more specific, for 13-week horizon, on the 2^{nd} of January 2013, SPXT Index estimates a risk premium of 2.5, whereas CRB RIND Index yields a risk premium of -4.85. In terms of disagreement on the magnitude, alternative estimates of risk premium on the same date can differ by as much as 12.12. Given that the purpose of this empirical study is to estimate the inflation risk premium by using the estimates of risk premium associated with the futures on the CPI proxy, dispersion in the estimates of risk premium associated with the futures on the CPI proxy on the same date will yield mutually inconsistent estimates of inflation risk premium. Therefore, not all of the estimates of risk premium associated with the futures on the CPI proxy are equally valid. As a result, we need to select the most accurate and reliable estimate of risk premium, so that we can derive the accurate estimate of inflation risk premium.

4.6 Selecting the Most Credible Estimate of the Risk Premium Associated with the Futures on the CPI Proxy

We follow closely the approach proposed by Baumeister and Kilian (2017) to select the most accurate estimate of risk premium associated with the futures on the CPI proxy for a specific horizon.

As shown in equation (4.7), the risk premium theory argues that the futures on the CPI proxy is the sum of the risk premium associated with the futures on the CPI proxy and the expected spot CPI proxy at the expiry of the futures on the CPI proxy, i.e.

$$FCP_t^h = RP_t^h + E_t[CP_{t+h}]. \tag{4.23}$$

A credible estimate of the risk premium associated with the futures on the CPI proxy RP_t^h, when inserted into equation (4.23), should yield an implied spot CPI proxy expectation

$E_t[CP_{t+h}]$ that is as close as possible to the realised spot CPI proxy CP_{t+h}. Thus, the credibility of the estimate of risk premium associated with the futures on the CPI proxy may be assessed by measuring the accuracy of the implied spot CPI proxy expectation. The accuracy of the implied spot CPI proxy expectation can be assessed by the mean squared prediction error of $E_t[CP_{t+h}]$, that is,

$$E[CP_{t+h} - E_t[CP_{t+h}]]^2. \tag{4.24}$$

According to Baumeister and Kilian (2017), the conditional expectation, i.e. $E_t[CP_{t+h}]$ in this case, minimises the mean squared prediction error.

The credibility of the estimate of risk premium associated with the futures on the CPI proxy is assessed by comparing the mean squared prediction error derived from alternative specifications of the implied spot CPI proxy expectation $E_t[CP_{t+h}]$. We first assume that the time-varying risk premium associated with the futures on the CPI proxy does not exist. In this case, as described by equation (4.3), the futures on the CPI proxy is equal to the expected spot CPI proxy at the expiry of the futures on the CPI proxy, i.e.

$$FCP_t^h = E_t[CP_{t+h}].$$

Therefore, in the absence of time-varying risk premium, FCP_t^h minimises the mean squared prediction error, and the corresponding specification of the mean squared prediction error is

$$E[CP_{t+h} - FCP_t^h]^2. \tag{4.25}$$

Now, in the presence of the time-varying risk premium, the risk premium adjusted futures on the CPI proxy instead minimises the mean squared prediction error. On this occasion, the corresponding specification of the mean squared prediction error is

$$E[CP_{t+h} - (FCP_t^h - RP_t^h)]^2. \tag{4.26}$$

Given that our result shows clearly that the time-varying risk premium associated with the futures on the CPI proxy indeed exists in reality, a credible estimate of the risk premium associated with the futures on the CPI proxy is supposed to yield an implied spot CPI proxy expectation $FCP_t^h - RP_t^h$ with mean squared prediction error (4.26) being smaller than the mean squared prediction error (4.25). Thus, any estimate of risk premium yielding an implied spot CPI proxy expectation $FCP_t^h - RP_t^h$ with mean squared prediction error (4.26) being higher than mean squared prediction error (4.25) is inadmissible and should therefore be discarded.

An immediate implication of the mean squared prediction error (4.26) is that, the most credible estimate of risk premium associated with the futures on the CPI proxy, among all candidate risk premium estimates for a specific horizon, is the estimate yielding the smallest mean squared prediction error (4.26). This implication enables us to rank the risk premium estimated from alternative candidate predictors and therefore recognise the most credible risk premium estimate. In Table 4.2, we use the mean squared prediction error derived from 13-week horizon risk premium estimates as an example to illustrate the above selection procedure. The first column denotes the mean squared prediction error (4.25) of the 13-week horizon

Table 4.2. Selecting the Most Credible Estimate of the 13-week Horizon Risk Premium

$E\left[CP_{t+h} - FCP_t^h\right]^2$	SPXT Index	BCOM Index	BCOMTR Index	CRB CMDT Index	CRB RIND Index	CRY Index	SPX Index
101.13	40.58	42.44	42.46	**39.50**	41.01	43.73	41.72

Notes: $E\left[CP_{t+h} - FCP_t^h\right]^2$ denotes the mean squared prediction error (4.25); 'SPXT Index' represents the mean squared prediction error (4.26) derived from the risk premium estimated using the candidate predictor 'SPXT Index', this explanation applies to all remaining candidate predictors in the first row; boldface indicates that 'CRB CMDT Index' yields the most credible risk premium estimate.

futures on the CPI proxy; the second column to the eighth column represent the mean squared prediction error (4.26) of the risk premium adjusted futures on the CPI proxy derived from alternative candidate predictors. According to the criterion defined by the comparison between mean squared prediction error (4.25) and (4.26), all candidate predictors and corresponding risk premium estimates are admissible. Furthermore, since the mean squared prediction error (4.26) of the risk premium adjusted futures on the CPI proxy derived from CRB CMDT Index is the smallest among all candidate predictors, we select the risk premium estimated from CRB CMDT Index as the most credible risk premium estimate for 13-week horizon. Following this criterion and selection procedure, we select the most credible risk premium estimate out of candidate risk premium estimates for all other horizons, the result is presented in Table 4.3. We can see from Table 4.3 that, for each horizon, all predictors and the corresponding risk premium estimates are admissible. For each horizon, the predictor yielding the most accurate risk premium estimate is highlighted in boldface. Next, based on the selected risk premium estimates, we estimate the contemporaneous inflation risk premium.

Table 4.3. Selecting the Most Accurate Risk Premium Estimates (all horizons)

Horizon	$E[CP_{t+h}-FCP_t^h]^2$	SPXT Index	BCOM Index	BCOMTR Index	CRB CMDT Index	CRB RIND Index	CRY Index	DJCI Index	DJITR Index	INDU Index	RICIGLTR Index	SPX Index	VIX Index
11	91.48	44.94			**42.96**							45.85	
12	120.77	61.27	**53.08**	53.09	56.94		59.57						
13	101.13	40.58	42.44	42.46	**39.50**	41.01	43.73					41.72	
14	98.60	25.92	**23.44**	23.47	30.99	35.14	32.60	31.88	27.41	29.61		26.82	
15	116.00	**31.89**	35.07	35.11	38.00		42.47	47.33	38.63	41.63	84.19	33.04	
16	132.23	**39.85**	41.81	41.87	41.63		51.35	59.76	49.78	53.78		41.56	
17	144.94	39.24	42.54	42.60	**32.36**	42.91	45.08	48.91	55.19		59.15	41.91	
18	127.56	**14.07**	30.57	30.65	25.22	34.89		40.56	21.04	24.89		15.29	
19	137.89	33.52	**30.34**	30.38	30.83	42.29	42.04	46.38	43.94	48.82		35.65	
20	149.12	26.11	35.15	35.22	**25.90**		47.17	52.91	34.56	37.94		27.51	
21	192.81	36.36	41.54	41.64	**34.18**	63.37	49.54	49.67	54.44	61.71	63.02	39.45	
22	179.94	**15.45**	30.70	30.77	24.15	40.75	49.40	41.77	25.01	28.93		16.58	
23	189.18	**26.35**	37.72	37.80	39.74				34.94	39.27		28.10	
24	224.42	32.87	37.05	37.12	**31.99**		61.33	59.74	43.52	47.83		34.65	
25	271.90	53.41	55.07	55.18	**47.61**		83.53	82.15	72.09	79.93		57.11	
26	264.42	47.69	50.06	50.13	**45.72**	55.05	65.61	60.48	60.21	65.78		50.05	
27	284.97	**51.70**	59.98	60.07	61.37				60.42			54.11	
28	319.79	**66.52**	74.74	74.81	74.81				78.48			68.81	
29	335.81	**64.26**	83.28	83.39	77.58				78.37	84.32		66.81	
30	367.72	79.95	67.99	68.05	**67.86**	78.36	88.27	86.26				83.76	
31	337.50	**47.49**	62.40	62.48	59.97				58.87	63.47		49.27	
32	376.06	75.62	**72.39**	72.43	81.39							78.53	
33	361.21	**43.14**	58.66	58.73	46.83				54.86	58.63		44.43	
34	437.02	69.49	**56.14**	56.19	63.00	77.09	77.07	72.66	87.15			72.88	
35	436.20	**45.16**	55.80	55.87	56.02	55.41		70.20	54.96	59.43		46.63	
36	447.88	61.90	56.71	56.78		**52.72**		68.35	68.78			64.14	
37	454.85	51.89	**50.66**	50.72	54.47	59.77		67.80	61.23	65.28		53.55	
38	513.22	79.31	**50.45**	50.47	68.65	76.07	90.19	77.00	92.68			83.06	82.15
39	508.00	65.04	54.17	54.22	62.99	**41.66**	74.48	55.66	72.14	77.41		67.46	
40	522.81	55.71	48.65	48.74	63.95	**46.65**		55.16	54.09	58.20		57.71	
41	514.57	**40.05**	44.95	45.02	56.69	64.27		50.15	41.28	44.34		41.27	
42	530.54	**36.40**	39.65	39.75	51.79			52.80	37.38	40.82		37.88	
43	568.99	57.39	41.82	41.90	56.07	43.78	67.25	**39.77**	65.59	72.17	73.13	60.27	
44	548.73	40.49	**34.62**	34.69	46.55	41.23		38.89	41.33	44.64		41.98	
45	588.88	57.75	**33.71**	33.73	55.90	55.61	56.72	45.84	57.62	61.93	62.49	59.99	
46	578.26	29.37	**26.95**	27.00	31.04		48.00	38.72	27.24	28.64	53.59	30.13	
47	652.58	28.51	**23.15**	23.22	37.58	55.04	39.01	23.63	31.60	36.24	45.66	30.28	
48	660.72	24.93	20.86	20.91	33.12	43.96	40.63	24.40	**20.11**	21.72	48.16	25.71	
49	695.28	29.13	**27.06**	27.11		32.83		28.90	28.09	30.07		30.13	
50	710.07	29.18	**27.83**	27.87	29.28			35.48	31.22	32.99		29.95	
51	760.51	38.53	**28.33**	28.38	37.66		47.45	41.32	43.43	47.47	53.55	40.54	
52	773.60	32.00	**22.95**	22.98	28.76	31.95	28.89	23.65	33.26	35.93	35.73	33.30	

Notes: Only the statistically significant OLS regression results are included to estimate the risk premium; for 1-week horizon to 10-week horizon, no candidate predictor has statistically significant linear relationship with the corresponding dependent variables; blank indicates that the candidate predictor has no statistically significant linear relationship with the corresponding dependent variable; $E[CP_{t+h}-FCP_t^h]^2$ denotes the mean squared prediction error (4.25); 'SPXT Index' represents the mean squared prediction error (4.26) derived from the risk premium estimated using the candidate predictor 'SPXT Index', this explanation applies to all remaining candidate predictors in the first row; boldface denotes the smallest mean squared prediction error (4.26) for a specific horizon.

4.7 The Inflation Risk Premium

4.7.1 Estimating the Inflation Risk Premium

In this section, we apply the selected estimates of the risk premium associated with the futures on the CPI proxy and the spot CPI proxy dataset to equation (4.17)

$$\phi_t^h = \frac{RP_t^h}{CP_t}$$

to estimate the inflation risk premium. The inflation risk premium ϕ_t^h estimated using equation (4.17) is the h-week horizon inflation risk premium at time t, which is equal to the ratio of the contemporaneous h-week horizon risk premium associated with the futures on the CPI proxy to the spot CPI proxy at time t. The estimates of inflation risk premium are presented in Figure 4.3. For expository purpose, Figure 4.3 focuses on 13-week, 26-week, 39-week and

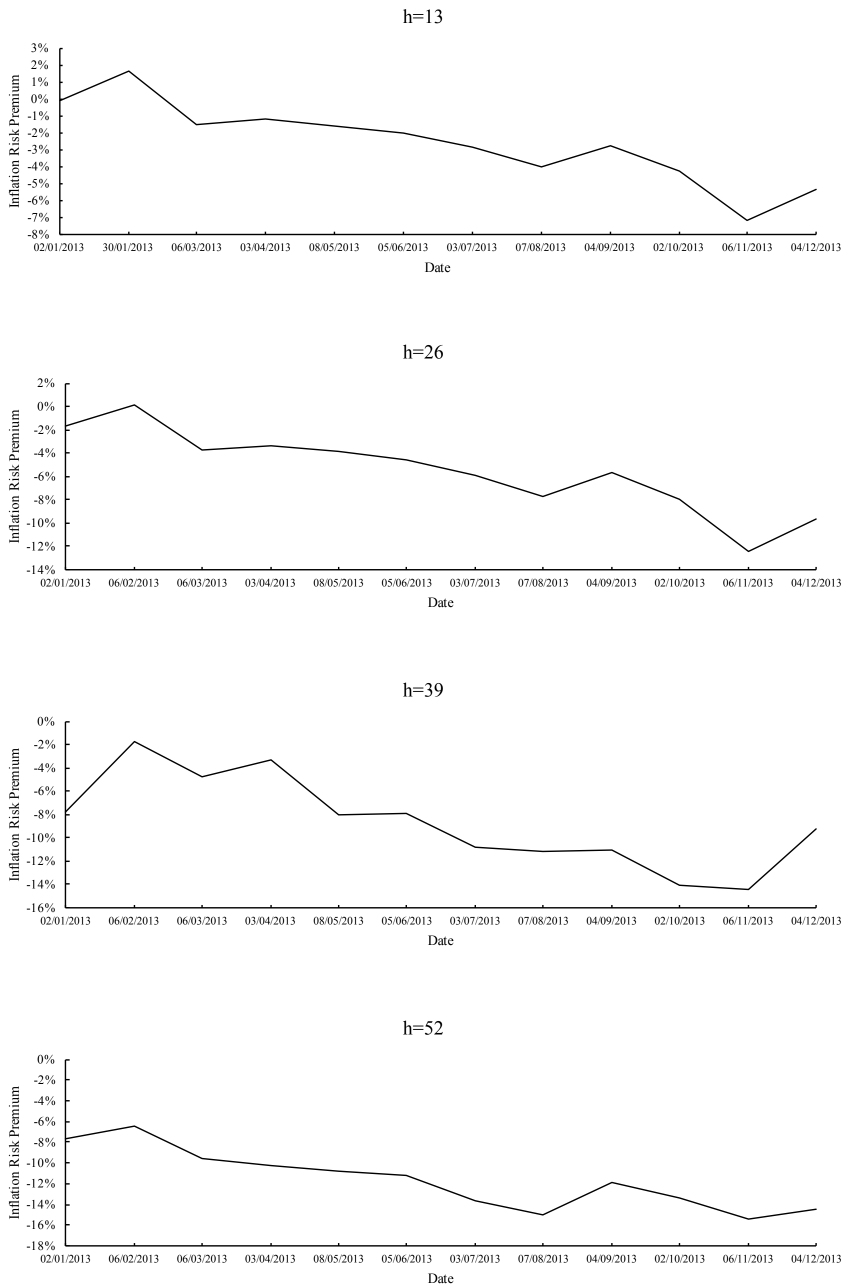


Figure 4.3. The line charts in Figure 4.3 presents the 13-week, 26-week, 39-week and 52-week horizon inflation risk premium estimates.

52-week horizon inflation risk premium estimates. In each line chart, X-axis is the observation date; Y-axis denotes the inflation risk premium estimates in percentage. As illustrated in Figure 4.3, across all four horizons, the inflation risk premium estimates have been trending down during 2013. For all four horizons, the trough in inflation risk premium estimates are in November 2013. The peak of 26-week, 39-week and 52-week horizon inflation risk premium estimates are in February 2013, whereas the 13-week horizon inflation risk premium estimates have peak value in January 2013. Furthermore, all inflation risk premium estimates are negative except the peak values of 13-week and 26-week inflation risk premium estimates. In addition, as horizon extends from 13 weeks to 52 weeks, the magnitude of the average inflation risk premium estimates keeps increasing from 2.59% to 11.64%. Figure 4.3 clearly

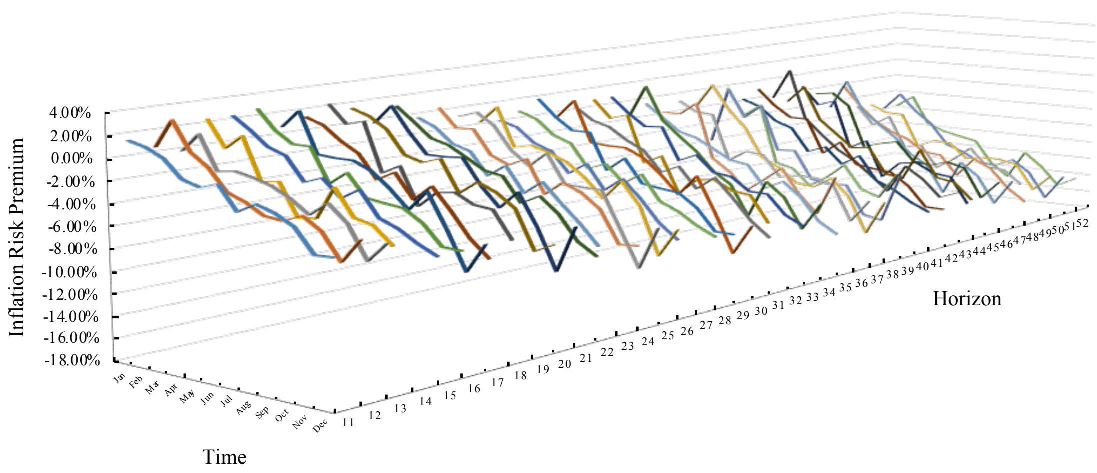


Figure 4.4. This figure presents the inflation risk premium estimates for all horizons (i.e. 11-week to 52-week horizons).

visualises the trend in the inflation risk premium estimates for four specific horizons. In order to see the trend of inflation risk premium for all horizons, we plot Figure 4.4. Figure 4.4 suggests that, for every horizon, the inflation risk premium estimate shows downward trend. In terms of the sign of the inflation risk premium estimates, for 11-week to 22-week horizons, 24-week horizon and 26-week horizon, positive inflation risk premium estimates can be observed during the first quarter of 2013; for all remaining horizons, the inflation risk premium estimates are negative. Moreover, in Figure 4.5, we plot the average inflation risk premium estimates for each horizon. As horizon extends from 11 weeks to 52 weeks, the magnitude of the average inflation risk premium increases steadily.

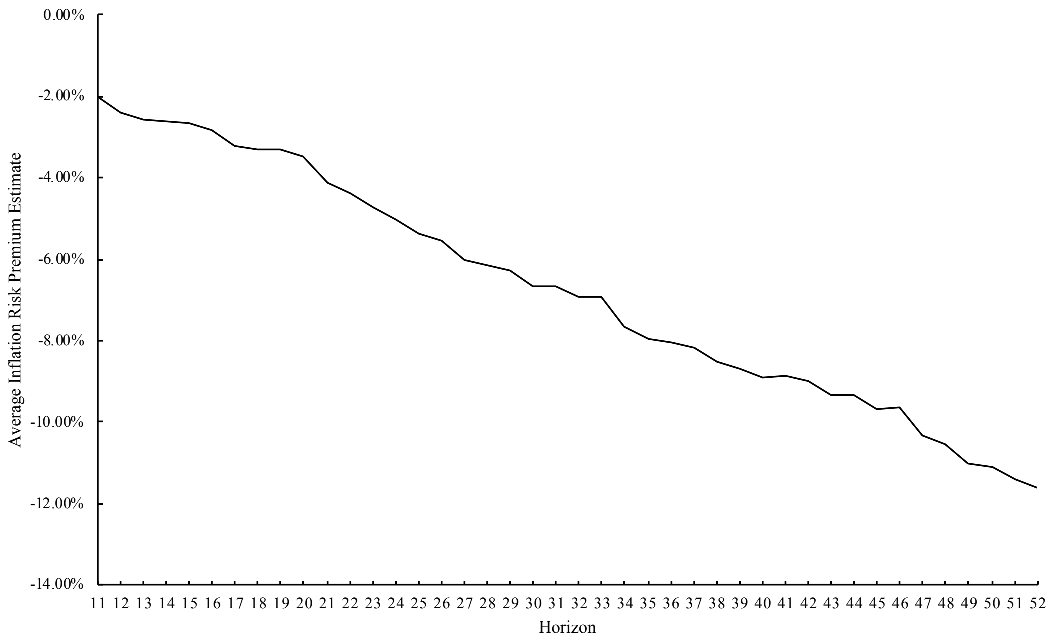


Figure 4.5. This line chart plots the average inflation risk premium estimates for each horizon.

4.7.2 Recent Estimates of the Inflation Risk Premium

Campbell and Shiller (1996) discuss two methods of estimating the magnitude of the inflation risk premium. First, the direct estimates from the average excess return on a nominal 5-year bond. Second, the indirect estimates from the covariances of the return on the nominal 5-year bond with relevant state variables. Their results suggest that there is a 50 to 100 basis points inflation risk premium in the returns on the 5-year zero-coupon nominal bond. Campbell and Viceira (2001) use a two-factor term structure model to estimate the inflation risk premium, their findings indicate that the inflation risk premium is 35 bps for three-months nominal zero-coupon bond and 1.1% for ten-year zero-coupon bond. Buraschi and Jiltsov (2005) estimate the inflation risk premium based on a structural monetary version of a real business cycle model. Their estimate for 10-year inflation risk premium is on average 70 basis points.

Based on three alternative proxies for inflation expectations, Grishchenko and Huang (2013) extract the inflation risk premium from monthly yields on zero-coupon TIPS and nominal Treasury bonds of 5, 7 and 10-year maturities. They document negative inflation risk premium estimates: -37 basis points for 5-year horizon, -26 basis points for 7-year horizon and -10 basis points for 10-year horizon. Camba-Mendez and Werner (2017) construct model-free and model-based indicators for the inflation risk premium. They show that the inflation risk premium turned negative during the post-Lehman period. Their estimate for 1-year ahead inflation risk premium is on average -40 basis points. Fleckenstein et al. (2017) measure deflation risk by extracting the objective distribution of inflation from the market prices of inflation swaps and options. Their estimates for 1-year, 5-year and 10-year can be as small as -1.57 basis points, -23.89 basis points and -42.53 basis points, respectively. The recent estimates of inflation risk premium are summarised in Table 4.4.

Table 4.4. Recent Estimates of the Inflation Risk Premium

	sample period prior to recent financial crisis			sample period covers recent financial crisis		
	Campbell and Shiller (1996)	Campbell and Viceira (2001)	Buraschi and Jiltsov (2005)	Grishchenko and Huang (2013)	Fleckenstein et al. (2017)	Camba-Mendez and Werner (2017)
Sample Period	1953-1994	1952-1996	1960-2000	2000-2008	2009-2015	2008-2016
Data	5-year zero-coupon nominal bond	nominal zero-coupon yields at maturities three months, one year, three years, and ten years; equity return; inflation	zero-coupon yields and forward curves from one month to ten years; inflation; money supply (M2 as measure of the money stock)	monthly yields on zero-coupon TIPS and nominal Treasury bonds of 5, 7, and 10-year maturities	prices of inflation swaps and options	the quoted prices for the ILS contracts and the survey data from Consensus
Method	information from 5-year bond	two-factor term structure model	a structural monetary version of a real business cycle model	'model free' approach based on three proxies of inflation expectations	measure deflation risk by extracting the objective distribution of inflation from the market prices of inflation swaps and options	first approach compares inflation forecasts from Consensus Economics with ILS rates; second approach applies a standard affine term structure model to the term structure of ILS rates
3-month		35 bps				
1-year					as small as -1.57 bps	-40 bps
5-year	between 50 bps and 100 bps			-37 bps	as small as -23.89 bps	
7-year				-26 bps		
10-year		110 bps	on average 70 bps (time-varying between 20 to 140 bps)	-10 bps	as small as -42.53 bps	

Table 4.4 summarises the sample period, data, method and inflation risk premium estimates of alternative studies. We divide the studies on inflation risk premium into two groups based on their sample period. The first group of studies (i.e. Campbell and Shiller (1996), Campbell and Viceira (2001) and Buraschi and Jiltsov (2005)) use data sample which is prior to the recent financial crisis; whereas the second group of studies (i.e. Grishchenko and Huang (2013), Camba-Mendez and Werner (2017) and Fleckenstein et al. (2017)) include the recent financial crisis in their sample period. Table 4.4 shows that, studies using data sample period

prior to the recent financial crisis usually yield positive inflation risk premium estimates. By contrast, studies that cover the recent financial crisis, especially the post-Lehman period, obtain negative inflation risk premium estimates. In addition to the above-mentioned studies, Campbell et al. (2009a) review the inflation-indexed bond markets and argue that, when inflation is positively correlated with stock prices, the inflation risk premium in nominal Treasury bonds is likely negative. Chen et al. (2016) also indicate that the consumption-based asset pricing model and the capital asset pricing model suggest that the long-run inflation risk premium has trended down over time, and is likely to be negative in the current macroeconomic environment. What's more, d'Amico et al. (2016)'s inflation risk premium estimates trended down over time and turned negative at the 5-year maturity during the recent financial crisis. They argue that the downward trend in inflation risk premium reflects the substantial risk of deflation at the time.

The fact that negative inflation risk premium estimates can be observed for all horizons in our result is consistent with the evidence from the post-Lehman studies which argue that the inflation risk premium is negative. From equation (4.17), given that the spot CPI proxy CP_t is strictly positive, the sign of the inflation risk premium ϕ_t^h is determined by the sign of the risk premium associated with the futures on the CPI proxy RP_t^h. From the definition of the risk premium associated with the futures on the CPI proxy as described in equation (4.7), a negative RP_t^h suggests that the implied spot CPI proxy expectation $E_t[CP_{t+h}]$ exceeds the futures on the CPI proxy FCP_t^h. Given the estimates of negative risk premium associated with the futures on the CPI proxy, the corresponding inflation risk premium are therefore negative. Furthermore, as suggested by d'Amico et al. (2016) and Camba-Mendez and Werner (2017), inflation risk premium is likely to be negative during times of deflation fears and higher and positive when hyperinflation is a concern. In Figure 4.6, we plot the realised U.S. CPI inflation rate during 2014 to 2015. We can see that, from May 2014 to April 2015, the inflation rate has been trending down; deflation indeed occurs during January 2015 to May 2015, which suggests that the deflation fears are likely to exist during 2013 to 2014. Thus, the negative inflation risk premium estimates in our result may be explained by the deflation fears prevailing during 2013 to 2014.

An inspection of the inflation risk premium estimates in Table 4.4 indicates that, there is disagreement on the magnitude of the inflation risk premium for the same horizon estimated from alternative studies. For instance, Campbell and Viceira (2001) estimate the 10-year inflation risk premium to be 110 basis points on average; whereas Buraschi and Jiltsov

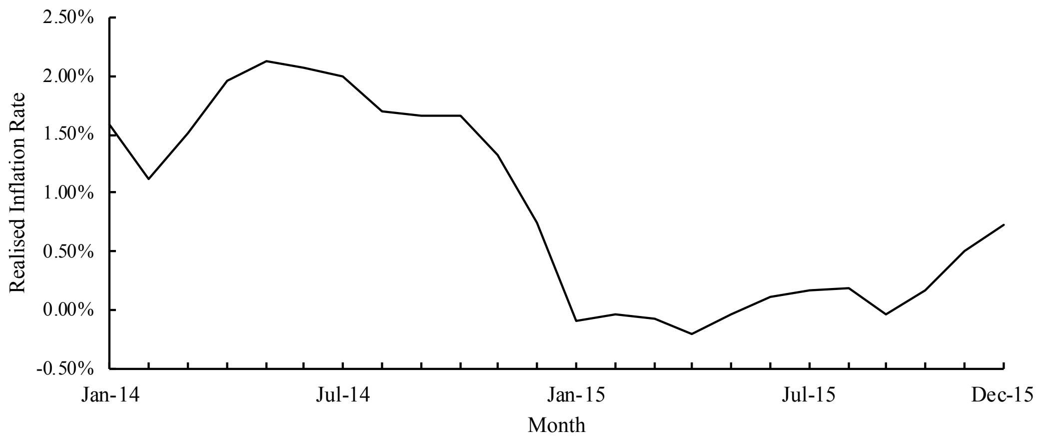


Figure 4.6. This line chart plots the realised U.S. CPI inflation rate during 2014 to 2015.

(2005)'s result argues that the 10-year inflation risk premium is on average 70 basis points and can be as small as 20 basis points. The different magnitude of inflation risk premium estimates may be attributed to the distinct sample period, data and method used in these studies. Indeed, as Bekaert and Wang (2010) suggest, the variation in the estimates across the different studies reflects not only different methodologies, but also simply the use of different data and sample periods. Grishchenko and Huang (2013) also point out that there appears to be no consensus so far in the literature as to the magnitude of the inflation risk premium estimates.

The magnitude of our estimates of the average 1-year horizon inflation risk premium (11.64%) is larger than that from Camba-Mendez and Werner (2017) (0.4%), as the method and data used in our study are different from those used by the inflation risk premium literature. The inflation risk premium literature usually depend on the TIPS and nominal Treasury bond, inflation expectation survey, inflation-linked derivatives and alternative specifications of term structure models to estimate the inflation risk premium. By contrast, our study uses commodity market data and estimates the inflation risk premium by deriving the relationship between the inflation risk premium and the risk premium associated with the futures on the CPI proxy. The meaning of the negative inflation risk premium in our study is different from that in the inflation risk premium literature. In our study, the sign of the inflation risk premium is determined by the sign of the risk premium associated with the futures on the CPI proxy. As the futures on the CPI proxy is essentially a futures contract, a negative inflation risk premium actually reflects net short hedging position in the commodity futures market. By contrast, in the inflation risk premium literature, as discussed by Campbell et al. (2009b) and Imakubo et al. (2015), a negative inflation risk premium corresponds to the negative covariance between the representative agents' intertemporal marginal rate of substitution and their inflation expectation. If inflation falls unexpectedly, when the marginal utility is high, the representative agents benefit from holding the short-term nominal bond, as the nominal bond hedges real risk in this occasion. Therefore the representative agents command a negative risk premium. Since the meaning of the negative inflation risk premium in our study is different from that in the inflation risk premium literature, we focus on the magnitude of the inflation risk premium estimates. In Chapter 3, section 3.4.5, we compare the variation in the price of the consumption good in the CPI with the corresponding commodity futures option implied volatility. Our result shows that the commodity futures market is more volatile than the market for the actual consumption good. For example, the price of bread in the supermarket changes much slower than the futures price of wheat at the CME. Thus, the volatility of the price of the futures on the CPI proxy is higher than the volatility of the consumer price index. As a result, the magnitude of the inflation risk premium estimates in our study is larger than that of the inflation risk premium estimate in the macroeconomic inflation risk premium literature.

4.8 Conclusions

In this paper, we study the inflation risk premium in the context of commodity futures market. Following the approach proposed by Baumeister and Kilian (2017), we estimate the risk premium as the predictable component in the prediction error. We confirm the existence of the time-varying risk premium associated with the futures on the CPI proxy by documenting the time series of downward-trending risk premium estimates. Given that not all risk premium estimates are equally valid, we select the most accurate risk premium estimate by comparing alternative specifications of the mean squared prediction error. Based on the link between the risk premium associated with the futures on the CPI proxy and the inflation risk premium, we use the selected risk premium estimates to estimate the inflation risk premium. The fact that negative inflation risk premium can be observed for all horizons in our result is consistent with the post-Lehman estimates of inflation risk premium estimate in the macroeconomic inflation risk premium literature. Due to the fact that the price of the futures on the CPI proxy is more volatile than the consumer price index, the magnitude of our inflation risk premium estimates is larger than that of the inflation risk premium estimates in the macroeconomic inflation risk premium literature.

Conclusions

In Chapter 1, we have constructed the futures on the CPI proxy. The term structure of futures on the CPI proxy yields one-year ahead 'point' forecast of inflation rate. The data underlying the construction of the futures on the CPI proxy (i.e. the commodity futures price) is exchange-traded and therefore more accessible compared to the over-the-counter traded inflation-linked derivatives. This point is important because the futures on the CPI proxy offers an accessible approach for inflation forecasting given that exchange-traded inflation futures contract had been delisted. Thus, the construction of the futures on the CPI proxy tackles the non-availability of exchange-traded inflation futures price data.

Chapter 2 has analysed the term structures of futures on the CPI proxy using a two-factor valuation model inspired by Schwartz (1997)'s framework. The most significant two-factor valuation model parameter α, i.e. the long-run mean level of the instantaneous real interest rate, yields corresponding annual real interest rate which is fairly close to the actual U.S. 2013 real interest rate. This result confirms the relevance of applying the two-factor valuation model to analyse the term structures of futures on the CPI proxy. Based on the filtered spot consumer price index and the fitted forward curves, we predict the one-year ahead expected inflation rate. Our result suggests that the two-factor valuation model is capable of forecasting the downward trend of U.S. CPI inflation rate during May 2014 to December 2014. Furthermore, the spot consumer price index may help alleviate the lag in the publication of U.S. CPI-U index. Indeed, since the data used by the two-factor valuation model (i.e. the commodity futures price) is available from the exchange immediately, the spot consumer price index can be calculated immediately without any lag.

In Chapter 3, we have used the synthetic option implied volatility surface to derive the interval estimate for future price level, which is illustrated by the fan chart. The fan chart suggests that, for one-year horizon, the further we look into the future, the less certain we are about the price level prediction. In addition, by comparing the average length of interval estimate of future price level across the fan charts, we find that, compared with in-the-money and at-the-money option chain, we are more certain about the price level forecast when the prediction is derived from the out-of-the-money option chain. Furthermore, the uncertainty of one-year ahead inflation forecast derived from the fan chart is larger than that of inflation prediction of the Bank of England inflation fan chart due to the high implied volatility of commodity futures prices. More importantly, given that the commodity futures option market data is high-frequency, our method of inflation forecasting is theoretically capable of capturing the commodity futures option market's expectation of inflation rate in the real time.

In Chapter 4, we have documented the time-varying risk premium associated with the futures on the CPI proxy. By deriving the link between the inflation risk premium and the risk premium associated with the futures on the CPI proxy, we estimate the inflation risk premium. The negative inflation risk premium estimated from our method is consistent with the post-Lehman inflation risk premium estimates in the macroeconomic inflation risk premium literature. Due to the fact that the volatility of the price of futures on the CPI proxy is higher than the variation in the consumer price index, the magnitude of our inflation risk premium estimates is larger than that of the inflation risk premium estimates in the macroeconomic inflation risk premium literature.

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Appendix A

Relative Importance of Components in the Consumer Price Indexes: U.S. City Average

The first column contains all of the items in the U.S. CPI, this is the precise composition of U.S. CPI. The second and third columns present the original weight for each item in the CPI taken from the 'Relative importance of components in the Consumer Price Index: U.S. city average' published in December 2013 and December 2014, respectively. The weights for some of the items changed slightly as time evolves from December 2013 to December 2014.

We look at every item in the CPI, this gives us indication of which item we need to include. At the same time, we examine the CME commodity futures contract table to see which commodity futures contracts are available. Comparing the items in the CPI and available commodity futures contracts in Bloomberg enables us to locate the commodities that we need to include. If the item in the CPI has an exactly matching commodity, then that commodity is included directly. For those items which do not have exactly matching commodities, we find close substitutes. For example, for transportation, we use crude oil as crude oil is the primary energy used in transportation. For apparels, we use lumber and cotton because lumber and cotton are the primary resources used to make apparels. By analysing the main ingredients of items in the CPI, we find close substitute commodities for these items. The fifth column shows the exact matches and close substitute commodities for all included items in the construction of CPI proxy.

Given included commodities, we then look for the futures contracts written on these commodities in Bloomberg. For commodities which have more than one futures contract written on them, we only include the futures contract with the highest volume and open interest. Following this method, we include 34 commodity futures contract tables, which are listed in the fourth column.

We can see from the fourth column that some commodity futures appear more than once. Then the weights for these commodity futures are just the summation of individual weight of the same commodity futures appearing for different items. For clarification, the ‘weight for commodity futures’ in this appendix actually denotes the weight for each futures contract table written on the underlying commodity, which holds the same meaning to the expression ‘optimum weight for each commodity’ in the thesis because one commodity corresponds to one futures contract table containing all futures contracts written on this commodity.

Item and Group	Weights of Items in CPI-U (December 2013)	Weights of items in CPI-U (December 2014)	Corresponding Commodity Futures Contract Ticker	Corresponding Commodity
All items	100	100		
Food and beverages	**14.901**	**15.272**		
Food	**13.891**	**14.257**		
Food at home	**8.187**	**8.427**		
Cereals and bakery products	**1.141**	**1.138**		
Cereals and cereal products	**.374**	**.370**		
Flour and prepared flour mixes	.049	.048	O A Comdty (O H5 COMB Comdty)	oats
Breakfast cereals	.196	.197	O A Comdty (O H5 COMB Comdty)	oats
Rice, pasta, cornmeal	.129	.126	RRA Comdty (RRH5 COMB Comdty)	rough rice
Bakery products	**.766**	**.767**		
Bread	.229	.230	W A Comdty (W H5 COMB Comdty)	wheat
Fresh biscuits, rolls, muffins	.115	.116	MKCA Comdty)(MKCH5 COMB Comdty)	mini kc hrw wheat
Cakes, cupcakes, and cookies	.189	.189	KWA Comdty (KWK5 COMB Comdty)	wheat
Other bakery products	.234	.233	YPA Comdty (YPH5 COMB Comdty)	mini-sized wheat
Meats, poultry, fish, and eggs	**1.859**	**2.014**		
Meats, poultry, and fish	**1.737**	**1.880**		
Meats	**1.099**	**1.229**		
Beef and veal	.494	.582	LCA Comdty (LCG5 COMB Comdty)	live cattle
Pork	.346	.372	LHA Comdty (LHJ5 COMB Comdty)	lean hog
Other meats	.258	.275	FCA Comdty (FCH5 COMB Comdty)	cattle feeder
Poultry	.357	.360		
Fish and seafood	.281	.291		
Eggs	.122	.134		

Dairy and related products	**.860**	**.898**		
Milk	.273	.283	DAA Comdty (DAG5 COMB Comdty)	milk
Cheese and related products	.267	.286	CHEA Comdty (CHEG5 Comdty)	cheese
Ice cream and related products (sugar)	.122	.126	FSBA Comdty (FSBH5 Comdty)	sugar
Other dairy and related products	.198	.204	LEA Comdty (LEG5 COMB Comdty)	non-fat milk
Fruits and vegetables	**1.346**	**1.379**		
Fresh fruits and vegetables	**1.042**	**1.076**		
Fresh fruits	**.560**	**.575**		
Apples	.085	.083		
Bananas	.088	.087		
Citrus fruits	.140	.146		
Other fresh fruits	.246	.259		
Fresh vegetables	**.482**	**.500**		
Potatoes	.077	.075		
Lettuce	.069	.072		
Tomatoes	.089	.102		
Other fresh vegetables	.247	.251		
Processed fruits and vegetables		**.303**		
Non-alcoholic beverages and beverage materials	**.955**	**.955**		
Juices and non-alcoholic drinks	**.703**	**.699**	**FCOJ Comdty (JOH5 Comdty)**	**juice**
Carbonated drinks	.283	.285		
Frozen non-carbonated juices and drinks	.014	.014		
Non-frozen non-carbonated juices and drinks	.406	.400		
Beverage materials and including coffee and tea	**.252**	**.256**		
Coffee	.153	.158	FCCA Comdty (FCCK5 Comdty)	coffee
Other beverage materials including tea	.099	.099		

Other food at home	**2.027**	**2.043**		
Sugar and sweets	**.298**	**.299**	**FSBA Comdty (FSBH5 Comdty) 2**	**sugar**
Sugar and artificial sweeteners	.054	.054		
Candy and chewing gum	.183	.185		
Other sweets	.060	.060		
Fats and oils	**.245**	**.245**	**V6A Comdty (V6G5 Comdty)**	**butter**
Butter and margarine	.069	.077		
Salad dressing	.065	.062		
Other fats and oil including peanut butter	.111	.107		
Other foods	**1.485**	**1.499**		
Soups	.094	.093		
Frozen and freeze dried prepared food (dry whey)	.281	.285	DRWA Comdty (DRWG5 Comdty)	dry whey
Snacks (cocoa)	.326	.330	FCOA Comdty (FCOK5 Comdty)	cocoa
Spices, seasonings, condiments, sauces (soybean oil)	.288	.292	BOA Comdty (BOH5 COMB Comdty)	soybean oil
Baby food	.054	.055	KVA Comdty (KVG5 Comdty)	classic IV milk
Other miscellaneous foods (palm oil)	.440	.444	CPIA Comdty (CPIG5 Comdty)	crude palm oil
Food away from home	**5.704**	**5.830**		
Full service meals and snacks	2.759	2.823	FCA Comdty (FCH5 COMB Comdty) 2	cattle feeder
Limited service meals and snacks	2.356	2.413		
Food at employee sites and schools	.210	.212		
Food from vending machines and mobile vendors	.064	.064		
Other food away from home	.315	.319		
Alcoholic beverages	**1.010**	**1.015**		
Alcoholic beverages at home	.597	.597		
Beer, ale, and other malt beverages at home	.274	.274		
Distilled spirits at home	.073	.073		
Wine at home	.250	.250		

Alcoholic beverages away from home	.412	.418		
Housing	**41.448**	**42.173**		
Shelter	**32.029**	**32.711**	**DJEA Index (DJEH5 Index), (Z5)**	**shelter**
Rent of primary residence	**6.977**	**7.159**		
Lodging away from home	**.795**	**.839**		
Housing at school, excluding board	.169	.172		
Other lodging away from home including hotels and motels	.626	.666		
Owners' equivalent rent of residence	**23.900**	**24.339**		
Owners' equivalent rent of primary residence	22.505	22.918		
Unsampled owners' equivalent rent of secondary residences	1.395	1.421		
Tenants' and household insurance	**.358**	**.375**		
Fuels and utilities	**5.158**	**5.273**		
Household energy	**3.980**	**4.051**		
Fuel oil and other fuels	**.275**	**.236**		
Fuel oil	.173	.139	NGA Comdty (NGH5 COMB Comdty	natural gas
Propane, kerosene, and firewood	.102	.097	LBA Comdty (H6) (LBH5 COMB Comdty)	lumber
Energy services	**3.705**	**3.815**		
Electricity	2.872	2.940	PHA Comdty (Z20) (PHG5 COMB Comdty)	electricity
Utility (piped) gas service	.834	.875	NGA Comdty (NGJ5 COMB Comdty) 2	natural gas
Water and sewer and trash collection services	**1.177**	**1.222**		
Water and sewer maintenance	.902	.945		
Garbage and trash collection	.275	.277		
Household furnishings and operations	**4.262**	**4.189**		

Window and floor coverings and other linens	**.278**	**.266**	**FCTA Comdty (FCTK5 Comdty)**	**cotton**
Floor covering	.047	.047		
Window coverings	.055	.053		
Other linens	.176	.166		
Furniture and bedding	**.787**	**.769**	**FCTA Comdty (FCTK5 Comdty) 2, LBA Comdty (LBH5 COMB Comdty) 3**	**cotton, lumber**
Bedroom furniture	.276	.268		
Living room, kitchen, and dining room furniture	.373	.363		
Other furniture	.128	.128		
Unsampled furniture	.010	.009		
Appliances	**.288**	**.271**	**LBA Comdty (LBH5 COMB Comdty) 2, HRCA Comdty (HRCH5 Comdty)**	**lumber, hot rolled steel**
Major appliances	.159	.147		
Other appliances	.124	.120		
Unsampled appliances	.004	.004		
Other household equipment and furnishings	**.503**	**.479**		
Clocks, lamps, and decorator items	.275	.257	HRCA Comdty (HRCH5 Comdty) 2, FCTA Comdty (FCTK5 Comdty) 3	hot rolled steel, cotton
Indoor plants and flowers	.106	.107	DFLA Comdty (DFLH5 Comdty	fertilizer
Dishes and flatware	.045	.041		
Non-electric cookware and tableware	.077	.074	HGA, Comdty (HGH5 COMB Comdty), OPA Comdty (OPJ5 Comdty)	copper
Tools, hardware, outdoor equipment and supplies	**.715**	**.710**	**HGA Comdty (HGH5 COMB Comdty) 2 , OPA Comdty (OPJ5 Comdty), HRCA Comdty**	**copper, hot rolled steel**
Tools, hardware and suppliers	.189	.189		
Outdoor equipment and supplies	.371	.367		

Un-sampled tools, hardware, outdoor equipment and supplies	.155	.154	**(HRCH5 Comdty)**	
Housekeeping supplies	**.860**	**.847**	**HGA Comdty (HGH5 Comdty) 3, OPA Comdty (H20) (OPJ5 Comdty), HRCA Comdty (HRCH5 Comdty)**	**copper, hot rolled steel**
Household cleaning products	.343	.337		
Household paper products	.251	.247		
Miscellaneous household products	.267	.263		
Household operations	**.831**	**.848**		
Domestic services	.277	.279		
Gardening and lawncare services	.269	.279		
Moving, storage, freight expense	.115	.116		
Repair of household items	.064	.066		
Un-sampled household operations	.104	.107		
Appeal	**3.437**	**3.343**		
Men's and boys' apparel	**.866**	**.834**	**LBA Comdty (LBH5 Comdty), FCTA Comdty (FCTK5 Comdty)**	**lumber, cotton**
Men's apparel	**.678**	**.653**		
Men's suits, sport coats, and outerwear	.113	.104		
Men's furnishings	.191	.185		
Men's shirts and sweaters	.207	.196		
Men's pants and shorts	.160	.160		
Un-sampled men's apparel	.007	.007		
Boys' apparel	**.188**	**.181**		
Women's and girls' apparel	**1.504**	**1.439**		
Women's apparel	**1.263**	**1.210**		
Women's outerwear	.114	.118		
Women's dresses	.154	.155		
Women's suits and separates	.604	.550		

Women's underwear, nightwear, sportswear and accessories	.382	.378		
Un-sampled women's apparel	.010	.010		
Girls' apparel	**.240**	**.229**		
Footwear	**.710**	**.725**		
Men's footwear	.216	.218		
Boys' and girls' footwear	.169	.178		
Women's footwear	.326	.329		
Infants' and toddlers' apparel	**.136**	**.135**		
Jewellery and watches	**.222**	**.211**	**GCA Comdty (GCH5 COMB Comdty), SI1 Z19**	**gold, silver**
Watches	.047	.046		
Jewellery	.175	.164		
Transportation	**16.418**	**15.289**	**CLA Comdty (CLJ5 COMB Comdty)**	**crude oil**
Private transportation	**15.254**	**14.167**		
New and used motor vehicles	**5.815**	**5.720**		
New vehicles	3.559	3.551		
Used cars and trucks	1.673	1.591		
Leased cars and trucks	.401	.397		
Car and truck rental	.073	.073		
Un-sampled new and used motor vehicles	.109	.109		
Motor fuel	**5.065**	**3.979**		
Gasoline (all types)	4.979	3.904		
Other motor fuels	.086	.075		
Motor vehicle parts and equipment	**.441**	**.435**		
Tires	.292	.285		
Vehicle accessories other than tires	.148	.150		
Motor vehicle maintenance and repair	**1.153**	**1.168**		

Motor vehicle body work	.056	.057		
Motor vehicle maintenance and servicing	.485	.492		
Motor vehicle repair	.580	.587		
Un-sampled service policies	.032	.032		
Motor vehicle insurance	**2.213**	**2.300**		
Motor vehicle fees	**.567**	**.565**		
State motor vehicle registration and license fees	.318	.312		
Parking and other fees	.231	.235		
Un-sampled motor vehicle fees	.018	.018		
Public transportation	**1.164**	**1.122**		
Airline fare	.742	.702		
Other intercity transportation	.159	.157		
Intra-city transportation	.259	.260		
Un-sampled public transportation	.004	.004		
Medical care	**7.551**	**7.716**		
Medical care commodities	**1.704**	**1.772**		
Medicinal drugs	**1.628**	**1.696**		
Prescription drugs	1.274	1.345		
Non-prescription drugs	.354	.351		
Medical equipment and supplies	**.076**	**.076**		
Medical care services	**5.847**	**5.944**	**IXCA Index (Spot) (IXCH5 Index)**	**e-mini health care**
Professional services	**3.003**	**3.032**		
Physician’s services	1.579	1.590		
Dental services	.795	.804		
Eyeglasses and eye care	.279	.284		
Services by other medical professionals	.350	.354		
Hospital and related services	**2.081**	**2.159**		
Hospital services	1.780	1.853		

Nursing homes and adult day services	.170	.174		
Care of invalids and elderly at home	.131	.132		
Health insurance	**.763**	**.753**		
Recreation	**5.793**	**5.750**		
Video and audio	**1.867**	**1.847**		
Televisions	.161	.133		
Cable and satellite television and radio service	1.448	1.468	PHA Comdty (PHG5 COMB Comdty) 2	electricity
Other video equipment	.030	.029		
Video discs and other media, including rental of video and audio	.093	.090		
Audio equipment	.072	.066		
Audio discs, tapes and other media	.046	.044		
Un-sampled video and audio	.017	.016		
Pets, pet products and services	**1.053**	**1.058**		
Pets and pet products	.662	.659	LCA Comdty (LCG5 COMB Comdty) 2, FCA Comdty (FCH5 COMB Comdty) 2	live cattle, cattle feeder
Pet services including veterinary	.391	.399		
Sporting goods	**.412**	**.400**	**HRCA Comdty (HRCH5 Comdty), LBA Comdty (LBH5 COMB Comdty)**	**hot rolled steel, lumber**
Sports vehicles including bicycles	.185	.181		
Sports equipment	.222	.214		
Un-sampled sporting goods	.005	.005		
Photography	**.121**	.120		
Photographic equipment and supplies	.059	.058		
Photographers and film processing	.061	.062		
Un-sampled photography	.001	.001		

Other recreational goods	**.399**	**.381**		
toys	.295	.277	FCTA Comdty (FCTK5 Comdty)	cotton
Sewing machines, fabric and supplies	.051	.050		
Music instruments and accessories	.041	.042		
Un-sampled recreation commodities	.012	.011		
Other recreational services	**1.723**	**1.724**		
Club dues and fees for participant sports and group exercises	.604	.602		
Admissions	.641	.640		
Fees for lessons or instructions	.208	.211		
Un-sampled recreation services	.270	.271		
Recreational reading materials	**.216**	**.220**	**LBA Comdty (LBH5 COMB Comdty)**	**lumber**
Newspapers and magazines	.119	.123		
Recreational books	.096	.094		
Un-sampled recreational reading materials	.002	.002		
Educational and communication	**7.087**	**7.062**		
Education	**3.244**	**3.325**		
Educational books and supplies	**.195**	**.203**		
Tuition, other school fees, and childcare	**3.049**	**3.122**		
College tuition and fees	1.806	1.853		
Elementary and high school tuition and fees	.365	.377		
Child care and nursery school	.714	.725		
Technical and business school tuition and fees	.039	.039		
Un-sampled tuition, other school fees, and childcare	.125	.128		
Communication	**3.843**	**3.737**		
Postage and delivery services	**.140**	**.144**		
Postage	.126	.130		

Delivery services	.014	.014		
Information and information processing	**3.703**	**3.593**		
Telephone services	**2.534**	**2.462**		
Wireless telephone services	1.706	1.624		
Land-line telephone services	.828	.837		
Information technology, hardware and services	**1.169**	**1.132**		
Personal computers and peripheral equipment	.306	.272		
Computer software and accessories	.069	.068		
Internet services and electronic information providers	.705	.711		
Telephone hardware, calculators, and other consumer information items	.076	.068		
Un-sampled information and information processing	.013	.012		
Other goods and services	**3.365**	**3.394**		
Tobacco and smoking products	**.703**	.718		
Cigarettes	.647	.661		
Tobacco products other than cigarettes	.050	.050		
Un-sampled tobacco and smoking products	.006	.006		
Personal care	**2.662**	**2.676**		
Personal care products	**.727**	**.724**		
Hair, dental, shaving, and miscellaneous personal care products	.373	.369		
Cosmetics. Perfume, bath, nail preparations and implements	.347	.348		
Un-sampled personal care products	.007	.007		
Personal care services	**.633**	.638		
Haircuts and other personal care services	.633	.638		

Miscellaneous personal services	**1.107**	**1.122**		
Legal services	.314	.316		
Funeral expenses	.172	.173		
Laundry and dry cleaning services	.273	.276		
Apparel services other than laundry and dry cleaning	.033	.034		
Financial services	.222	.228		
Un-sampled items	.093	.095		
Miscellaneous personal goods	**.195**	.192		

Appendix B

Monte Carlo Simulation with Correlated Commodity Futures Prices

In section 3.3.2, we assume the futures price is independent to each other in the Monte Carlo simulation. We now consider the case where the commodity futures prices are correlated in the Monte Carlo simulation, and accordingly evaluate the synthetic option based on the simulated correlated commodity futures prices.

Following Sieczka and Hołyst (2009), the correlation between futures price of commodity i and futures price of commodity j is defined as follows:

$$Corr_{ij} = \frac{cov\left(r_i, r_j\right)}{\sigma_{r_i}\sigma_{r_j}},$$

where $r_i = ln\left(\frac{F_i(t+1)}{F_i(t)}\right)$ and $r_j = ln\left(\frac{F_j(t+1)}{F_j(t)}\right)$ are the logarithmic returns of the commodity futures prices; $F_i\left(t\right)$ and $F_j\left(t\right)$ denote the time t daily settlement price of the closest to maturity commodity futures contract written on commodity i and commodity j, respectively.

For each one of the 21 commodities listed in Table 3.1, we collect the daily settlement price data for the closest to maturity commodity futures contract on every Wednesday from 8^{th} of January 2014 to 8^{th} of June 2016 (i.e. 127 observations for each commodity).

We calculate the correlation coefficient for all pairs of commodities listed in Table 3.1. The lower triangular matrix of the correlation coefficient matrix is presented in Table B.1.

For every iteration in the Monte Carlo simulation, simulating 21 correlated commodity futures price paths requires simulating 21 correlated random numbers. Following the method described by Hull (2012), let x denote independent random numbers; θ denote correlated random numbers; ρ_{ij} denote the correlation between θ_i and θ_j. For every iteration in the Monte

Table B.1. Correlations of Commodity Futures Prices

Ticker	w	kw	rr	o	si	cl	gc	hg	lb	ng	kv	bo	drw	v6	lh	lc	fc	jo	le	che	da
w	1																				
kw	0.927	1																			
rr	0.271	0.266	1																		
o	0.177	0.160	0.074	1																	
si	0.048	0.074	0.143	0.081	1																
cl	0.000	0.022	0.121	0.021	0.221	1															
gc	0.090	0.094	0.031	0.080	0.767	0.055	1														
hg	0.044	0.103	0.064	0.072	0.319	0.493	0.157	1													
lb	0.189	0.196	0.057	0.234	0.123	0.133	0.154	0.114	1												
ng	0.043	0.015	0.126	0.047	-0.035	0.051	-0.123	-0.052	-0.004	1											
kv	0.048	0.115	0.110	0.027	0.236	0.203	0.066	0.174	-0.054	-0.127	1										
bo	0.244	0.255	0.123	0.224	0.281	0.349	0.114	0.256	0.093	0.087	0.175	1									
drw	-0.037	-0.033	-0.109	-0.011	0.101	0.059	0.112	-0.035	0.144	0.051	-0.032	0.045	1								
v6	0.005	0.060	0.015	0.093	0.212	0.123	0.073	0.150	-0.078	-0.148	0.822	0.135	-0.064	1							
lh	0.016	0.016	0.065	0.014	0.168	0.200	0.121	0.106	-0.022	0.123	-0.022	0.191	0.071	-0.014	1						
lc	-0.028	0.027	0.131	0.026	0.143	0.248	0.084	0.200	0.133	0.001	0.079	0.157	-0.046	-0.021	0.058	1					
fc	-0.079	-0.074	0.071	0.074	0.137	0.182	0.090	0.146	0.022	0.122	-0.016	0.007	-0.030	-0.079	0.074	0.671	1				
jo	0.030	0.008	-0.091	0.108	0.072	0.103	0.108	0.089	0.320	0.010	0.031	0.071	0.227	0.050	-0.130	0.015	0.017	1			
le	0.080	0.111	0.137	0.010	0.188	0.242	0.111	0.140	0.019	-0.100	0.613	0.206	0.101	0.205	0.027	0.188	0.158	0.003	1		
che	-0.048	0.009	0.099	0.050	0.126	0.120	-0.009	0.067	-0.094	0.091	0.353	0.133	0.134	0.413	0.099	0.002	-0.045	0.022	0.116	1	
da	-0.038	0.018	0.085	0.060	0.154	0.130	0.012	0.067	-0.076	0.087	0.408	0.150	0.261	0.469	0.102	-0.010	-0.052	0.054	0.149	0.984	1

Carlo simulation, we first sample 21 independent random numbers $x_i\,(1 \le i \le 21)$ from the standard normal distribution. The corresponding correlated random numbers $\theta_i\,(1 \le i \le 21)$ can be generated as follows:

$$\begin{aligned}
\theta_1 &= \beta_{11}x_1 \\
\theta_2 &= \beta_{21}x_1 + \beta_{22}x_2 \\
\theta_3 &= \beta_{31}x_1 + \beta_{32}x_2 + \beta_{33}x_3 \\
&\vdots
\end{aligned} \tag{B.1}$$

The coefficients β_{ij} are chosen as follows: set $\beta_{11} = 1$; choose β_{21} so that $\beta_{21}\beta_{11} = \rho_{21}$; choose β_{22} so that $\beta_{21}^2 + \beta_{22}^2 = 1$; choose β_{31} so that $\beta_{31}\beta_{11} = \rho_{31}$; choose β_{32} so that $\beta_{31}\beta_{21} + \beta_{32}\beta_{22} = \rho_{32}$; choose β_{33} so that $\beta_{31}^2 + \beta_{32}^2 + \beta_{33}^2 = 1$; and so on. This procedure is known as the Cholesky decomposition.

Let *Corr* denote the correlation coefficient matrix of commodity futures prices. The Cholesky decomposition of correlation coefficient matrix *Corr* means that *Corr* can be factorized as $Corr = DD^*$, where D is the lower triangular matrix whose elements are the coefficients β_{ij} in (B.1):

$$\begin{bmatrix} \beta_{11} & 0 & 0 & \cdots \\ \beta_{21} & \beta_{22} & 0 & \cdots \\ \beta_{31} & \beta_{32} & \beta_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix};$$

and D^* is the conjugate transpose of D. The lower triangular matrix of Cholesky decomposition of the correlation coefficient matrix of commodity futures prices is calculated using the Matlab function 'chol(Corr,'lower')'. Now the correlated random numbers $\theta_i\,(1 \le i \le 21)$ can be generated from the independent random numbers $x_i\,(1 \le i \le 21)$ as follows:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_{21} \end{bmatrix} = D \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{21} \end{bmatrix}. \tag{B.2}$$

For every iteration in the Monte Carlo simulation, we first generate 21 independent random numbers $x_i\,(1 \le i \le 21)$, then the corresponding correlated random numbers $\theta_i\,(1 \le i \le 21)$ are generated using equation (B.2). The correlated random numbers are used to generate the corresponding correlated commodity futures price paths. Based on large number of samples (5000000 replications), we follow the same simulation procedure as described in substep 3 to substep 6 of step 1 as described in section 3.3.2 to calculate the general difference level between Soption and Doption. The difference level surface is presented in Table B.2.

Table B.2. Difference Level Surface (Correlated Commodity Futures Prices)

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Feb 2016	-0.70	-1.08	-1.20	-1.08	-0.78
Mar 2016	-1.35	-1.77	-1.92	-1.72	-1.34
Apr 2016	-1.78	-2.18	-2.33	-2.16	-1.78
May 2016	-2.07	-2.43	-2.55	-2.38	-2.03
Jun 2016	-2.31	-2.63	-2.72	-2.56	-2.22
Jul 2016	-2.52	-2.84	-2.92	-2.77	-2.44
Aug 2016	-2.71	-3.02	-3.10	-2.94	-2.62
Sep 2016	-2.86	-3.16	-3.24	-3.09	-2.78
Oct 2016	-2.99	-3.28	-3.35	-3.21	-2.90
Nov 2016	-3.13	-3.50	-3.48	-3.34	-3.04
Dec 2016	-3.22	-3.51	-3.58	-3.43	-3.14

Notes: 'Expiry' in the first column denotes the synthetic option expiry month.

We now proceed to evaluate the synthetic option. Using the commodity futures European option price data as described in section 3.2.3, the price of synthetic option Soption is the sum of corresponding Doption price and difference level. The prices of the synthetic options are presented in Table B.3.

Table B.3. Synthetic Option Price (Correlated Commodity Futures Prices)

Expiry	90% moneyness level	95% moneyness level	100% moneyness level	105% moneyness level	110% moneyness level
Feb 2016	9.13	4.74	1.39	0.19	0.00
Mar 2016	9.50	5.54	2.59	0.93	0.25
Apr 2016	9.86	6.08	3.25	1.48	0.58
May 2016	10.11	6.41	3.82	1.80	0.80
Jun 2016	10.38	6.75	3.98	2.13	1.05
Jul 2016	10.65	7.08	4.32	2.43	1.29
Aug 2016	10.93	7.42	4.67	2.75	1.53
Sep 2016	11.11	7.66	4.95	3.02	1.76
Oct 2016	11.26	7.86	5.18	3.24	1.94
Nov 2016	11.35	7.93	5.37	3.43	2.11
Dec 2016	11.45	8.16	5.54	3.60	2.26

Notes: 'Expiry' in the first column denotes the synthetic option expiry month. Due to the fact that the absolute difference level is larger than the 110% moneyness Doption price maturing in February 2016, the calculated Soption price is actually negative. But option price must be greater than or equal to zero, therefore, to make the price economically meaningful, we equate this price to zero.

The comparison between the synthetic option prices in Table B.3 and the synthetic option prices in Table 3.4 indicates that, when the correlation between commodity futures prices is taken into consideration, the synthetic option prices increase for all maturity and %moneyness level synthetic options. Since the synthetic option is constructed as an European option, higher synthetic option prices implies that the synthetic option implied volatilities will be higher, which indicates that the corresponding uncertainty of inflation rate forecast will be higher than that derived from the Monte Carlo simulation based on independent commodity futures prices. Therefore, to keep the uncertainty of inflation rate forecast at a reasonably low level, we assume the commodity futures price is independent to each other in the Monte Carlo simulation.

Appendix C

Derivation of Black's Model Type Option Pricing Formula

The risk-neutral pricing theorem implies that the European call option written on the futures on the CPI proxy with maturity T and strike price K has time 0 price as follows,

$$C\left(0,FCP\left(0,T'\right)\right)=E^{Q}[e^{-rT}\left(FCP\left(T,T'\right)-K\right)^{+}].$$

Since the risk-free interest rate r is assumed constant, we have

$$C\left(0,FCP\left(0,T'\right)\right)=e^{-rT}E^{Q}[\left(FCP\left(T,T'\right)-K\right)^{+}].$$

Given the assumption that the underlying futures on the CPI proxy $FCP(t,T')$ follows the driftless lognormal process in (3.9), we can use the solution for stochastic differential equation in (3.9) to write the value of futures on the CPI proxy $FCP(T,T')$ at time T as

$$FCP\left(T,T'\right)=FCP\left(0,T'\right)exp\left(-\frac{1}{2}\sigma_{FCP}^{2}T+\sigma_{FCP}\sqrt{T}\varepsilon\right),$$

where $\varepsilon\sim\mathcal{N}(0,1)$ is the standard normal random variable. Taken the value of $FCP(T,T')$ into consideration, the time 0 option price can be written as

$$C\left(0,FCP\left(0,T'\right)\right)=e^{-rT}E^{Q}\left[\left(FCP\left(0,T'\right)exp\left(-\frac{1}{2}\sigma_{FCP}^{2}T+\sigma_{FCP}\sqrt{T}\varepsilon\right)-K\right)^{+}\right]$$

$$=e^{-rT}\int_{-\infty}^{\infty}\left(FCP\left(0,T'\right)exp\left(-\frac{1}{2}\sigma_{FCP}^{2}T+\sigma_{FCP}\sqrt{T}\varepsilon\right)-K\right)^{+}f(\varepsilon)d\varepsilon,$$

where $f(\varepsilon) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{\varepsilon^2}{2}\right)}$ is the standard normal density function.

The European call option will be exercised at maturity only when futures on the CPI proxy observed at maturity $FCP(T,T')$ is greater than the strike price K, i.e.

$$\varepsilon > \frac{ln\left(\frac{K}{FCP(0,T')}\right) + \frac{1}{2}\sigma_{FCP}^2 T}{\sigma_{FCP}\sqrt{T}} = \varepsilon_0,$$

which yields

$$\begin{aligned} C\left(0, FCP\left(0,T'\right)\right) =& e^{-rT}\left(\int_{\varepsilon_0}^{\infty}\left(FCP\left(0,T'\right) exp\left(-\frac{1}{2}\sigma_{FCP}^2 T + \sigma_{FCP}\sqrt{T}\varepsilon\right)\right) f(\varepsilon)\, d\varepsilon \right.\\ &\left. + \int_{\varepsilon_0}^{\infty} (-K) f(\varepsilon)\, d\varepsilon\right) \\ =& e^{-rT}(A+B). \end{aligned}$$

We now calculate the value of A and B respectively.

$$\begin{aligned} A &= \int_{\varepsilon_0}^{\infty}\left(FCP\left(0,T'\right) exp\left(-\frac{1}{2}\sigma_{FCP}^2 T + \sigma_{FCP}\sqrt{T}\varepsilon\right)\right) f(\varepsilon)\, d\varepsilon \\ &= \frac{FCP(0,T')}{\sqrt{2\pi}} \int_{\varepsilon_0}^{\infty} exp\left(-\frac{1}{2}\sigma_{FCP}^2 T + \sigma_{FCP}\sqrt{T}\varepsilon - \frac{1}{2}\varepsilon^2\right) d\varepsilon \\ &= \frac{FCP(0,T')}{\sqrt{2\pi}} \int_{\varepsilon_0}^{\infty} exp\left(-\frac{1}{2}(\varepsilon - \sigma_{FCP}\sqrt{T})^2\right) d\varepsilon \\ &= FCP(0,T') \int_{\varepsilon_0 - \sigma_{FCP}\sqrt{T}}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{(\varepsilon')^2}{2}\right) d\varepsilon' \\ &= FCP(0,T')(1 - N(\varepsilon_0 - \sigma_{FCP}\sqrt{T})) \\ &= FCP\left(0,T'\right) N(-\varepsilon_0 + \sigma_{FCP}\sqrt{T}), \end{aligned}$$

and

$$\begin{aligned} B &= \int_{\varepsilon_0}^{\infty} (-K) f(\varepsilon) d\varepsilon \\ &= -K \int_{\varepsilon_0}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\varepsilon^2}{2}\right) d\varepsilon \\ &= -K(1 - N(\varepsilon_0)) \\ &= -KN(-\varepsilon_0). \end{aligned}$$

Therefore, the time 0 European call option price is

$$
\begin{aligned}
C\left(0, FCP\left(0, T'\right)\right) &= e^{-rT}(A+B) \\
&= e^{-rT}\left(FCP\left(0, T'\right) N\left(-\varepsilon_0 + \sigma_{FCP}\sqrt{T}\right) - KN\left(-\varepsilon_0\right)\right) \\
&= e^{-rT}\left(FCP\left(0, T'\right) N\left(d_1\right) - KN\left(d_2\right)\right),
\end{aligned}
$$

where $N(\cdot)$ denotes the cumulative density function and

$$
\begin{aligned}
d_1 &= \frac{ln\left(\frac{FCP(0,T')}{K}\right) + \frac{1}{2}\sigma_{FCP}^2 T}{\sigma_{FCP}\sqrt{T}} \\
d_2 &= \frac{ln\left(\frac{FCP(0,T')}{K}\right) - \frac{1}{2}\sigma_{FCP}^2 T}{\sigma_{FCP}\sqrt{T}} = d_1 - \sigma_{FCP}\sqrt{T}.
\end{aligned}
$$

This completes the derivation of option pricing formula.

Appendix D

Confidence Interval and Interval Estimates of Futures on the CPI Proxy

	90% Confidence Level				95% Confidence Level				99% Confidence Level			
	Confidence Interval		Interval Estimate		Confidence Interval		Interval Estimate		Confidence Interval		Interval Estimate	
	90% Moneyness Level											
Synthetic Option Expiry	lnlc90	lnrc90	lc90	rc90	lnlc95	lnrc95	lc95	rc95	lnlc99	lnrc99	lc99	rc99
Mar-16	4.42	4.65	82.77	104.52	4.39	4.67	80.91	106.93	4.35	4.72	77.42	111.75
Apr-16	4.39	4.69	80.69	108.38	4.36	4.71	78.40	111.54	4.31	4.77	74.15	117.94
May-16	4.38	4.71	79.78	110.92	4.35	4.74	77.26	114.54	4.28	4.80	72.59	121.91
Jun-16	4.37	4.73	78.83	113.50	4.33	4.77	76.08	117.61	4.26	4.84	71.01	126.00
Jul-16	4.36	4.75	78.11	116.12	4.32	4.79	75.15	120.70	4.24	4.87	69.72	130.10
Aug-16	4.35	4.78	77.14	118.86	4.30	4.82	73.95	123.98	4.22	4.90	68.15	134.54
Sep-16	4.33	4.79	75.92	120.43	4.28	4.84	72.58	125.97	4.20	4.92	66.52	137.45
Oct-16	4.31	4.80	74.75	121.82	4.27	4.85	71.27	127.76	4.17	4.94	64.98	140.12
Nov-16	4.30	4.81	73.43	122.58	4.25	4.86	69.85	128.87	4.15	4.96	63.40	141.97
Dec-16	4.28	4.82	71.95	123.45	4.22	4.87	68.26	130.12	4.12	4.97	61.64	144.10
	95% Moneyness Level											
Synthetic Option Expiry	lnlc90	lnrc90	lc90	rc90	lnlc95	lnrc95	lc95	rc95	lnlc99	lnrc99	lc99	rc99
Mar-16	4.43	4.63	84.18	102.90	4.41	4.65	82.55	104.94	4.38	4.69	79.48	109.00
Apr-16	4.41	4.67	82.37	106.39	4.39	4.69	80.34	109.07	4.34	4.74	76.54	114.48
May-16	4.40	4.69	81.59	108.72	4.37	4.72	79.34	111.81	4.32	4.77	75.15	118.04
Jun-16	4.39	4.71	80.74	111.13	4.36	4.74	78.27	114.65	4.30	4.80	73.68	121.78
Jul-16	4.38	4.73	80.10	113.61	4.35	4.77	77.42	117.55	4.28	4.83	72.47	125.57
Aug-16	4.37	4.75	79.32	116.04	4.34	4.79	76.43	120.43	4.26	4.86	71.12	129.41
Sep-16	4.36	4.77	78.16	117.49	4.32	4.81	75.11	122.25	4.24	4.88	69.54	132.05
Oct-16	4.34	4.78	77.06	118.73	4.30	4.82	73.88	123.84	4.22	4.90	68.08	134.38
Nov-16	4.33	4.78	76.28	118.74	4.29	4.82	73.06	123.97	4.21	4.90	67.20	134.79
Dec-16	4.31	4.79	74.49	119.96	4.26	4.83	71.11	125.66	4.17	4.92	64.98	137.51
	100% Moneyness Level											
Synthetic Option Expiry	lnlc90	lnrc90	lc90	rc90	lnlc95	lnrc95	lc95	rc95	lnlc99	lnrc99	lc99	rc99
Mar-16	4.44	4.63	84.53	102.51	4.42	4.65	82.95	104.46	4.38	4.69	79.98	108.34
Apr-16	4.42	4.66	82.87	105.80	4.39	4.69	80.92	108.35	4.35	4.73	77.27	113.47
May-16	4.40	4.69	81.36	109.00	4.37	4.72	79.07	112.15	4.32	4.78	74.82	118.53
Jun-16	4.40	4.70	81.42	110.31	4.37	4.73	79.04	113.63	4.31	4.79	74.63	120.34
Jul-16	4.39	4.72	80.91	112.61	4.36	4.76	78.34	116.30	4.30	4.82	73.59	123.80
Aug-16	4.38	4.74	80.18	114.96	4.35	4.78	77.41	119.07	4.28	4.85	72.31	127.47
Sep-16	4.37	4.76	79.12	116.25	4.33	4.79	76.21	120.70	4.26	4.87	70.86	129.80
Oct-16	4.36	4.77	78.10	117.38	4.32	4.81	75.06	122.14	4.24	4.88	69.49	131.91
Nov-16	4.34	4.77	76.91	117.92	4.30	4.81	73.77	122.94	4.22	4.89	68.04	133.28
Dec-16	4.33	4.77	75.67	118.38	4.28	4.82	72.44	123.66	4.20	4.90	66.57	134.57

	90% Confidence Level				95% Confidence Level				99% Confidence Level			
	Confidence Interval		Interval Estimate		Confidence Interval		Interval Estimate		Confidence Interval		Interval Estimate	
	105% Moneyness Level											
Synthetic Option Expiry	lnlc90	lnrc90	lc90	rc90	lnlc95	lnrc95	lc95	rc95	lnlc99	lnrc99	lc99	rc99
Mar-16	4.44	4.63	84.67	102.36	4.42	4.65	83.11	104.27	4.38	4.68	80.18	108.08
Apr-16	4.42	4.66	82.97	105.69	4.39	4.68	81.03	108.21	4.35	4.73	77.41	113.28
May-16	4.41	4.68	82.32	107.85	4.38	4.71	80.18	110.73	4.33	4.76	76.19	116.53
Jun-16	4.40	4.70	81.58	110.11	4.37	4.73	79.23	113.38	4.32	4.79	74.87	119.99
Jul-16	4.40	4.72	81.08	112.40	4.36	4.75	78.53	116.04	4.30	4.82	73.83	123.43
Aug-16	4.39	4.74	80.45	114.62	4.35	4.78	77.72	118.65	4.29	4.84	72.69	126.86
Sep-16	4.37	4.75	79.41	115.89	4.34	4.79	76.53	120.24	4.27	4.86	71.25	129.15
Oct-16	4.36	4.76	78.46	116.91	4.32	4.80	75.46	121.55	4.25	4.88	69.98	131.07
Nov-16	4.35	4.77	77.30	117.40	4.31	4.81	74.21	122.29	4.23	4.89	68.58	132.34
Dec-16	4.33	4.77	76.11	117.79	4.29	4.81	72.94	122.92	4.21	4.89	67.16	133.50
	110% Moneyness Level											
Synthetic Option Expiry	lnlc90	lnrc90	lc90	rc90	lnlc95	lnrc95	lc95	rc95	lnlc99	lnrc99	lc99	rc99
Mar-16	4.44	4.63	84.98	102.01	4.42	4.64	83.47	103.85	4.39	4.68	80.64	107.49
Apr-16	4.42	4.66	82.93	105.72	4.39	4.68	80.99	108.26	4.35	4.73	77.36	113.34
May-16	4.41	4.68	82.18	108.02	4.38	4.71	80.02	110.94	4.33	4.76	75.99	116.82
Jun-16	4.40	4.70	81.43	110.30	4.37	4.73	79.05	113.61	4.31	4.79	74.65	120.32
Jul-16	4.39	4.72	80.92	112.59	4.36	4.76	78.36	116.28	4.30	4.82	73.61	123.77
Aug-16	4.39	4.74	80.34	114.76	4.35	4.78	77.59	118.82	4.28	4.85	72.54	127.10
Sep-16	4.37	4.75	79.29	116.04	4.34	4.79	76.40	120.43	4.26	4.86	71.09	129.42
Oct-16	4.36	4.76	78.39	117.00	4.32	4.80	75.39	121.66	4.25	4.88	69.89	131.23
Nov-16	4.35	4.77	77.26	117.45	4.31	4.81	74.17	122.35	4.23	4.89	68.52	132.43
Dec-16	4.33	4.77	76.12	117.78	4.29	4.81	72.95	122.90	4.21	4.89	67.17	133.47

Notes: 'lnlc' denotes the lower end of the confidence interval; 'lnrc' denotes the upper end of the confidence interval; 'lc' denotes the minimum value of interval estimate of futures on the CPI proxy; 'rc' represents the maximum value of interval estimate of futures on the CPI proxy.