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A Study of Learning Mathematics Related to some Cognitive Factors and to Attitudes

By

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A thesis submitted for the degree of Doctor of Philosophy (Ph.D.)
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بسم الله الرحمن الرحيم

{وَالرَّاسِخُونَ فِي الْعِلْمِ يَقُولُونَ آمَنَّا بِهِ كُلٌّ مِنْ عِنْدِ رَبِّنَا وَمَا يَذَّكَّرُ إِلَّا أُولُو الْأَلْبَابِ}
(آل عمران: من الآية7)

{And those who are firmly grounded in knowledge say:

"We believe in the Book; the whole of it is from our Lord:"

and none will grasp the Message except men of understanding.}
(Surah Al-i-'Imran / Verse 7)

Abstract

This study was conducted to look at some cognitive (working memory and field dependency) and attitudinal factors which relate to learning and teaching mathematics. The purpose was to suggest ways that might help to improve students' performance in mathematics. A multi-step strategy was used to examine the relationship between these variables and learning mathematics. The first and the second steps focussed on the students and the third step looked at the mathematics teachers and inspectors ideas about learning and teaching mathematics.

This research has investigated the influence of working memory capacity and field dependency on mathematics achievement. The working memory space and the degree of field dependency were measured for 1346 school students aged between 14-16 years from public schools in Kuwait. The Digit Backward Test was used to determine working memory space, and the Group Embedded Figure Test was used to measure the degree of field-dependency for the students, both these tests have been used widely and their validity is assured. However, absolute measurements were not important in this study, as rank order was all that was required.

In order to investigate the correlations between performance in different topics in mathematics and the working memory space and field dependency, mathematics tests were developed where some questions had high working memory demand and others had very low working memory demand. Furthermore, in order to investigate which versions of tasks will lead to improved mathematics performance, some questions were presented as symbolic tasks; others were presented as visual tasks; some of them presented as abstract tasks and others related to life.

This study also explored the attitudes of the students towards mathematics in the following areas: the importance of mathematics as a discipline; attitudes towards learning mathematics; confidence in mathematics classes; the relationship between attitudes and achievement; activities in mathematics classes, and opinions about mathematicians.

The perceptions of mathematics teachers and inspectors were investigated to see the extent to which their views related to the findings from work with students. A sample of 25 mathematics teachers and 4 mathematics inspectors was selected randomly and they were interviewed to compare their views. This step involved semi-structured interviews which offered an opportunity to focus on some key areas as well as giving freedom for the teachers to expand their views.

The results indicated that field dependent students with low working memory capacity perform badly in mathematics. This might be attributed to their inability to distinguish between relevant and irrelevant items, with consequent working memory overload. Evidence shows that the way the questions or the problems are given to the students is very important for the students to understand and to succeed in solving them. Complicated shapes or long involved text are both more likely to produce overloading of the working memory space. Therefore, the study recommends that teachers should organise their material with great care in order that students are not penalised for some personal characteristic over which they have no control.

This study also showed a clear evidence of a decline in attitudes with age and the excessively overloaded curriculum was a likely reason along with the perceptions that some topics were irrelevant. Furthermore, this study reflects the crucial role that the mathematics teacher plays in the formation of student attitudes towards mathematics. Thus, aiming to develop positive attitudes towards mathematics including confidence, enjoyment and an appreciation of it as a powerful tool should be parallel with the acquisition and the understanding of mathematics concepts and skills in mathematics education.

Finally, the interviews show that there is no agreement about the objectives of mathematics education in Kuwait between those who decide the syllabuses (mathematics inspectors) and those who are going to teach these syllabuses (mathematics teachers). When the issue of the purpose for mathematics education is agreed, then it may be possible to consider what topics might further these aims most fully. In fact, teachers are involved daily in the teaching processes and they know the population of their students very well. Thus, their views about the syllabuses should be taken into consideration and they should be involved in the process of deciding the syllabuses.

The study has major implications for the development of mathematics education in Kuwait but many of the findings will be widely applicable in other educational systems.

Dedication

This thesis is dedicated to my husband, ***Abdullah Aldelmany***, his love, his faith in my ability and his encouragements have opened the path for me to complete my PhD. His presence and unwavering support have been a continuous source of motivation and inspiration throughout the study.

My daughter ***Nawal***, the moon of my life, & my son ***Dhari***, the sun of my life, is a source of joy to my life, who kept asking "When will you come back mum?" they have been so patient and so loving through the journey, especially when they tried to cover their tears when I was leaving Kuwait.

To my three lovely moons, ***Hesah, Hasnaa, and Alymamah*** for their patient during my study

In the Name of Allah, the Beneficent, the Merciful

First and foremost, my deepest thanks and gratitude goes to God for providing me the opportunity, means and perseverance to complete my PhD journey. Without His will and generosity, none of this or any other accomplishment would have been possible.

I would like to express my sincere gratitude to my supervisors, Prof. ***Norman Reid*** for his guidance, encouragement, advice and invaluable help. I would like to thank him for pushing me and for his time in reading my drafts and for ceaselessly providing me with feedback and discussion.

I would like also to thank Prof ***Rex Whitehead*** for his guidance, sharing his deep thought in mathematics topics; his prove reading my thesis and correct the English mistakes.

I am greatly indebted to ***my mother*** who has show patience and support during my study. My sincerest appreciation goes to ***my father & mother- in-law*** for their tremendous support and encouragement during my study. Special thank to my best friend ***Siti*** (the baby sitter) for her patient and help during my study. I appreciate the assistance of my oldest brothers ***Mubark & Ahmad*** who help me by providing letters from Education ministry to facilitate this study. I wish to express my thanks for my brother ***Ebrahim*** and my nephews ***Yousef*** and ***Abdullah*** who help me applying the study tests in male schools in Kuwait and for their support. I am very grateful to my sister ***Maryam*** and her husband ***Salem*** for all the favours and endless support they provide for me during my study. I would like also to thank my brothers, sisters and my brothers and sisters- in- law for their support and encouragement.

Grateful acknowledgement is made to my colleagues in Science Education at Glasgow University for their direct and indirect support and help. I will never forget to express my deep thanks for all my friends in Kuwait (***Abeer Alenezi, Doaa Al-Batini & Huda Al-Hindal***) and in Glasgow (***Fathya Al-Ahamdi, Abeer felmban, Maryam Al-Awadi, Huda Ala-Rishi, Ebtihaj and Aminah Hormez***) for their support, encouragement and their prayers for me during my study.

I am grateful to the Public Authority for applied Education and Training in Kuwait for giving me the opportunity to complete my post-graduate studies. I also would to thank Kuwait cultural office in UK for their support.

Finally, I should not forget to thank all the students who accepted to involve in this study and all the schools staffs and teachers for their helps and interesting in applying the research tests.

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Chapter 1

Mathematics Education

1.1 Introduction

We live in a world where change is accelerating and where the need for mathematics as a way of representing, communicating and predicting events is ever increasing. It can be argued that the need to understand and be able to utilise mathematics in daily life and in the workplace become important requirements in the 21st century. The principles of mathematics appear in many ways in daily life: the world of finance, insurance issues, social decisions based in statistics and probability, as well as the routine use of number and shape.

For many, mathematics is an essential underpinning for careers and occupations: the world of the sciences, modern technologies, engineering, economics, medicine all have a heavy dependence on mathematical ideas. Mathematics is a universal language and the language of mathematics is not based fundamentally on languages like English, Arabic or Russian. It is a language and a way of thinking which all will need, in small or large measure, to make sense of the world around.

Mathematics is the study of the relations between objects or quantities. It is defined in the Cambridge dictionary (2003) as "*the study of numbers, shapes and space using reason and usually a special system of symbols and rules for organizing them.*" It can be argued that this definition is just an external description of mathematics and that mathematics is more inclusive and universal than this description. Steen (1990) defined mathematics as, "*an exploratory science that seeks to understand every kind of pattern –patterns that occur in nature, patterns invented by human mind, and even patterns created by other patterns.*" Thus, mathematics can be defined as a general way of thinking about the environment surrounding us, the relations between its elements and our interactions with it.

Booker (1993) stated that mathematics education is more than just a sum of appropriate learning subject matter content and provision of suitable pedagogy for teachers, but an understanding of the mathematical process or a coming to know what the "doing of mathematics" is all about. Thus, mathematics education can be defined as the study of practices and methods of teaching and learning mathematics, and the development of mathematics teaching tools that facilitate that exercise and practice.

Charles and Lester offer an overview of mathematics education:

- *"The study of the subject should provide students with certain basic life skills and processes that will prepare them to be productive members of society.*
- *The study of the subject should give students the necessary background knowledge and skills to enable them to make career decisions consistent with their interests and abilities.*
- *The study of the subject should have potential for enriching the students' lives in some way."*

Charles & Lester (1982: p. 3)

The first point stresses the importance of mathematics in terms of life skills and ability to function effectively in society, while the second emphasises the key role which mathematics holds in relation to many careers. The third is less tangible and may be suggesting that the methods of mathematics have some value in terms of the way we think although there is little evidence to support any idea of learning transfer.

Despite the accepted importance of mathematics, there is huge controversy and debate about the quantity and the quality of mathematics which should be included in any curriculum to guide what is taught to school students. Hiebert (1999) has noted that the presentation of many school mathematics topics has remained virtually unchanged for over 100 years despite the numerous attempts to improve mathematics curricula.

1.2 Mathematical Education in the State of Kuwait

The Kuwaiti educational system consists of three levels: primary, junior secondary school and high secondary levels. Primary level consists of five grades from age six until age eleven (6-11), junior secondary level consists of four grades from age twelve until the age fifteen (12-15), and the high secondary consists of three grades from age sixteen until the age eighteen (16-18).

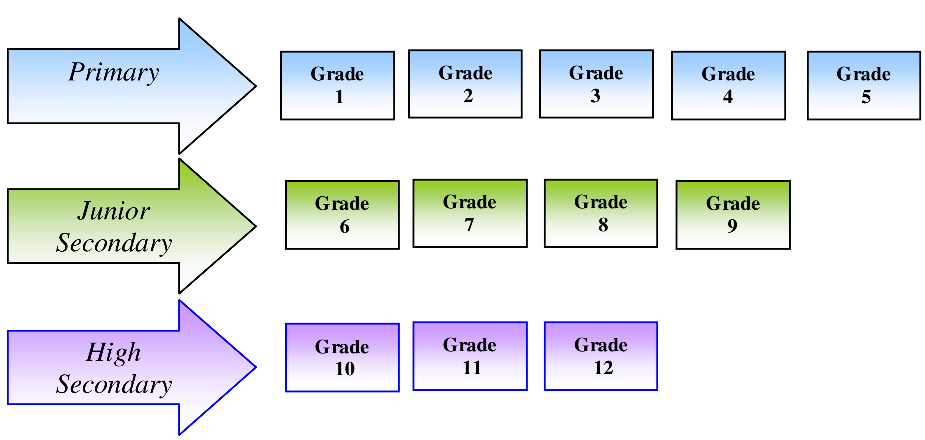


Figure 1-1: Kuwait educational system

School attendance in Kuwait is obligatory for all children between the ages of six and fifteen. The Ministry of Education controls the curriculum and teaching materials. Thus, the students receive the same education in any school in Kuwait. Mathematics occupies a prominent place in the Kuwaiti Educational system. Students receive considerable time in mathematics classes: 4-5 hours every week at all levels in the compulsory education system. As in many countries around the world, people in Kuwait seem to believe that mathematics results, along with English results, are a good measure for a prospective employee.

Aims form a fundamental part of any curriculum and come first in the definition of it. Thus, in many countries around the world, statements of aims or objectives preface the curricula in any subject. Mathematics is an immense field and, without clear aims, there is little way to define what is to be taught (Orton & Wain, 1994). Thus, there are general aims of mathematics education at every level in Kuwait. The general aims of teaching mathematics in junior secondary education (age 12-15) are:

- Acquiring mathematical knowledge relevant to daily life that help them to understand the world in which they live.
- Understand and use the vocabulary of mathematics language including symbols, formulae, pictures and graphics.
- Understanding the nature of mathematics which helps to explain natural phenomena.
- Understanding the number systems, algebra and geometry.
- Realizing the importance of mathematics in science and technology development and in other school subjects.
- Acquiring mathematical skills, which help in developing mathematical sense.
- Acquiring the ability on compilation and classification numerical and quantitative data, to tabulate and explain it.
- Using the language of mathematics and number effectively.
- The ability to question and argue rationally and to apply themselves to tasks, and physical skills.
- Developing algebra skills in handling mathematical processes.
- The ability to handle mathematical models and geometrical representations.
- Acquiring skills to use mathematical evidence and its logical approach.
- Using the scientific method in thinking.
- Acquiring the ability to solve the mathematical problems (numerical, geometrical, algebraic),
- Using different thinking methods (inferential, reflective, relational, synthetic, analytical), developing insights into truth and rationality.
- Inventing new methods for solving problems in mathematics.

www.moe.edu.kw

This list is translated from the Arabic. However, it is clear that these aims derive from the view of mathematics as an everyday, dynamic, socially determined, problem solving activity, that aims to empower the individual, and looks to be accessible to all people. Nonetheless, some of the aims need challenging.

Some may be highly desirable but there is little evidence to support that they are achievable by students of these ages (12-15), simply on grounds of their cognitive development. For example, skills to use mathematical evidence and developing insights into truth and rationality may be difficult to achieve at this age. The reference to problem solving implies that there is some agreement on a definition of this skill. The review by Reid and Yang (2002) in the sciences challenges this notion while the idea that school students of this age are likely to invent new methods of problem solving is exciting if unrealistic. The suggestion of thinking scientifically is curious. Very recent work (Al-Ahmadi, 2008) has offered some clear descriptions and it is very clear from these that mathematics has very little to do with scientific thinking. The logical thinking of mathematics is somewhat different.

However, aims and objectives do not always translate into practice in neat ways. Mathematics teachers may well not pay much attention to these aims but may seek, with commitment and enthusiasm, to impart the mathematical skills which they enjoyed themselves at school, often teaching in ways which reflect what they found most helpful when they were learning.

1.3 Diagnostic View of Mathematical Education in the State of Kuwait

There are some unfortunate trends in mathematics education in the state of Kuwait. These trends concentrate on acquiring mathematical skills and techniques to solve mathematical problems, ignoring their application in the real world and in other subjects. In fact, much of what is taught in school classrooms, as Burton (1996) described "immutable mathematics", are in fact skills and procedures, which were developed in a specific mathematics context. The socio-cultural view of mathematics does not receive general recognition, especially among the larger non-mathematical community (ibid).

Students face problems in recalling facts in mathematics and it is difficult to learn algorithms meaningfully. Thus, most of the concepts and procedures of mathematics are obscure to many students because rules and algorithms dominate them. It is quite possible to pass examinations by seeking to master the procedures with little understanding of their meaning. Thus, after several investigations, Feynman (1985) concluded that students in mathematics classes had memorized everything without any understanding:

> *"…so you see they could pass the examinations, and 'learn' all this stuff, and not know anything at all, except what they had memorised … Finally, I said that I couldn't see how anyone could be educated by this self-propagating system in which people pass exams, and teach others to pass exams, but nobody knows anything.*

Feynman (1985, P: 212-213)

This way of teaching mathematics as an isolated subject leads to the lack of any coherent understanding of mathematics. As a consequence of this, students tend to lose the sense of the importance of mathematics. In spite of the numerous attempts to improve mathematics curricula in Kuwait, the following can be noted:

- *The continued low-level achievement in mathematics compared with the rates of other subjects.*
- *There is a common belief that mathematics is complicated and difficult subject to learn. To some extent, some people think that mathematics is a meaningless and unpleasant subject due to its abstract nature.*
- *There are negative attitudes towards learning mathematics and this can be demonstrated by the numbers studying mathematics in higher education (see the table).*

Kuwait University	***Total Entries***	***Mathematics***
2000-2001	**4456**	**3**
2001-2002	**3745**	**13**
2002-2003	**3968**	**15**
2003-2004	**3831**	**12**
2004-2005	**4540**	**9**
2005-2006	**4843**	**16**
2006-2007	**5544**	**12**

Table 1-1: Kuwait University entries (Kuwait University)

Table 1-1 shows that students are turning away from learning mathematics in higher education and the highest percentage of students who choose mathematics does not reach 0.5% of the total entries of Kuwait University (Kuwait University is the only government university and the vast majority of the students in Kuwait study their).

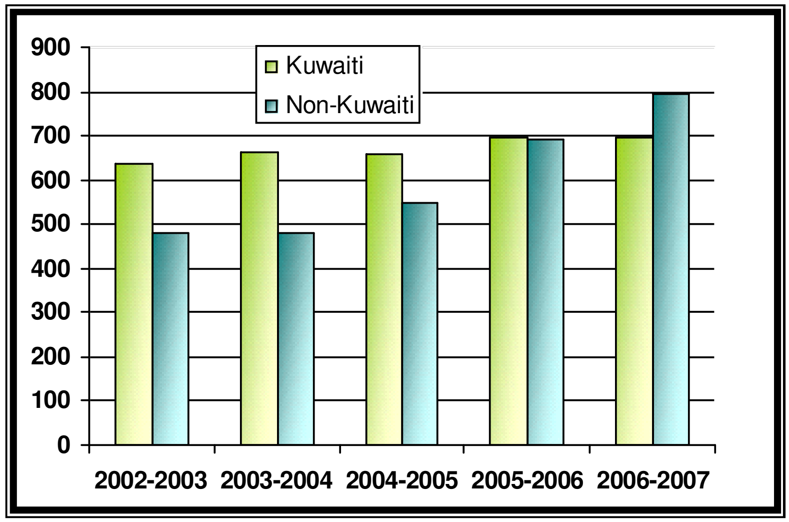


Figure 1-2: The numbers of mathematics teachers in junior secondary school

It can be seen from the chart above that, while the number of Kuwaiti mathematics teachers remains roughly the same for the last five years, the number of non-Kuwaiti mathematics teachers extremely increased. This also reflects the turning away from choosing mathematics for study in higher education.

1.4 Research Questions

This study attempts to explore three questions:

- *What cognitive demands does learning mathematics place on the learner?*
- *What are Kuwaiti students' attitudes towards mathematics?*
- *How do the view of teachers and inspectors relate to student experiences?*

The first question will be tackled by exploring the effects of two cognitive factors that may influence learning mathematics: working memory capacity and field dependency. Both of these are known strong correlates of learning in conceptual areas and this will also be related to formats of assessment. The second question will be addressed by exploring students' attitudes towards mathematics in three areas: students' attitudes towards mathematics subject; students' attitudes towards the learning of mathematics; and students' attitudes towards the topics and themes covered during the course. Interviews will be used to explore the third question.

1.5 The Outline of the Thesis

This thesis consists of two parts: the literature review is presented in the first part and this seeks to bring together some of the main findings relevant to the aims of this study. The second part describes how the study was conducted with the students in Kuwait, the data obtained and the conclusions which can be drawn.

Part one deals with literature surrounding the cognitive and the attitudinal factors affecting learning mathematics, and it is divided into four chapters. Firstly, the learning of mathematics is set in the context of the currently accepted models of learning. This is followed by a focus on the role of working memory, followed by an outline of the importance and measurement of field dependency. Finally, the area of attitudes relating to mathematics is reviewed, with a small selection from the immense literature of attitude development and measurement.

Although this study looks at learning models in supporting the process of learning mathematics (such as behaviourism, Piaget's insights and constructivism) as well as some theories of learning mathematics (such as Dienes theory of learning mathematics and the van Hiele theory of learning mathematics), the main focus of this study is on information processing. This model describes learning well and it is powerfully predicative in indicating how learning can be improved. The implications from this model are considered in detail.

Many researchers have claimed that working memory plays a crucial role in learning mathematics, and they have supported their claims by studies demonstrating close links between working memory capacity and measures of learning and academic achievement (e.g. Mclean & Hitch, 1999; Bull & Scerif, 2001; Christou, 2001; Alenezi, 2004; Holmes & Adams, 2006). Students' ability to hold and manipulate information has been found to be a crucial factor in mathematics performance for all ages. However, in mathematics classes much more is needed than having a working memory space to hold the information. The ability to distinguish between important items from unimportant ones is vitally important to solve many mathematics problems. This depends on the extent of field dependency.

The chapter of attitudes provides an overview of what attitudes are, why they are important, how attitudes can be measured and the literature surrounding attitudes development. Furthermore, the most important attitudes towards mathematics include a general perception and attitudes about mathematics, the perceived usefulness of mathematics, confidence in learning mathematics, attitudes towards different topics within mathematics; and the attitudes of mathematics teachers to their students. These are approached within this chapter.

After outlining the general way by which data will be obtained and handled, the remainder of the thesis describes a series of three major experiments. In the first two, the work is entirely with the students, seeking to find the impact of working memory capacity and extent of field dependency on performance and looking at some major issues relating to assessment in the light of these. Students' attitudes are also described. The third experiment involves teachers and school inspectors to see how they see the situation. The final chapter draws the findings together, making some suggestions for further work as well as identifying some implications from the study.

Chapter 2

Mathematics and Learning Theories

2.1 Introduction

In looking at the area of mathematics education and learning theories, two general kinds of theories are found. There are those which focus particularly on mathematics learning and there are general learning theories which can be applied to the learning of mathematics. It is assumed that general theories of learning have much to offer to the processes of teaching and learning mathematics (Orton, 2004). Orton wonders if it is possible to enhance learning mathematics through optimum sequencing, or is it a question of waiting until students are ready. He also asks whether students discover mathematics and if they can construct mathematical knowledge for themselves. It appears there are a variety of different learning theories and it is difficult to know which the appropriate one is. This chapter will examine several such ideas.

The research in this thesis focuses on information processing as a model which describes learning well and it is powerfully predicative in indicating how learning can be improved. The major theme of this chapter is to look at theories in supporting the process of learning mathematics. The implications from information processing model are considered in detail in the following chapter but, here, general views about other learning theories that link to learning mathematics are presented as follows:

- *Behaviourist Approach*
- *Piaget and cognitive developmental psychology*
- *Constructivism*
- *Theories of learning mathematics*
- *Dienes theory of learning mathematics*
- *The van Hiele theory of learning mathematics*
- *Ausubel's theory of meaningful-learning*
- *Information processing and cognitive theories of learning*

2.2 Behaviourist Approach

Behaviourism concentrates on behaviour observation and the behaviourists' belief that learning takes place through stimuli (events in the environment) and subsequent responses made by an individual. Human learning was first seen as response acquisition (Smith *et.al*, 1998). Early behaviourist psychologists, Watson, Pavlov and Skinner, started to study the human learning process based on the training of animals to associate a stimulus and a response. Then animals exhibit required patterns of behaviour to prove that conditioning worked (Atkinson *et.al*, 1993).

The Russian physiologist, Ivan Pavlov (1849-1936) in his salivation responses study in dogs, observed that dogs salivate not only when food is presented but also when food is about to be presented. He realised and described what is currently known as classical conditioning theory. He rang a bell as he fed some dogs several meals. Each time the dogs heard the bell they knew that a meal was coming, and they would begin to salivate. Pavlov then rang the bell without bringing food, but the dogs still salivated. They had been 'conditioned' to salivate at the sound of a bell. The principles outlined by Pavlov can be applied to learned emotional reactions, which are central to the educational process because an individual motivation to learn and their belief in their ability to learn will affect how they learn. As an example of learned emotional reactions, Bentham (2002) described to a very bright student 'Connie' who achieved ten grades for her GCSEs. Connie had developed a learned emotional reaction to maths (she becomes negative towards maths) because her math teacher punished her for her failure to answer the question 'what was five times five', and this could be explained in terms of classical conditioning theory.

Pavlov's study inspired psychologists in the United States such as E.L Thorndike (Hilgard & Bower, 1966). In his early work, Thorndike linked behaviour to physical reflexes and he went beyond Pavlov by showing that stimuli that occurred after behaviour had an influence on future behaviours. Thorndike (1922) proposed a number of laws, which have contributed to discussion. *The law of exercise: The response to a situation becomes associated with that situation, and the more it is used in a given situation the more strongly it becomes associated with it. On the other hand, disuse of the response weakens the association.* There is no assertion that practice guarantees mastery, but the majority still believe that practice is the best way to master knowledge (Orton, 2004). Mathematicians are still seeking to establish a strong bond between the stimulus (the question-type) and the response (the application of the method of solution leading to the correct answer), which seems to be direct application of the law of exercise. *The law of effect: responses that are accompanied or closely followed by satisfaction are more likely to happen again when the situation recurs, while responses accompanied or closely followed by discomfort will be less likely to recur.* When students behaviour is reinforced, the behaviour is sustained, and this is an example of the law of effect.

Orton (2004) remarks that, although these laws were suggested many years ago, it is interesting to consider how acceptable they are today in the teaching of mathematics, and he wonders if we could enhance learning mathematics through optimum sequencing. Given a proper task (stimulus) from the teacher, or from a book or programme, the correct answer (response) is obtained, and then slowly but surely, learning proceeds through a sequence or chain of stimulus-response links. Furthermore, feedback, reinforcement and reward have crucial places in the application of the theory; thus, a cycle of learning is generated. He argues that without other methods that involve repetition, learning may not retained effectively. He went on to observe that the purpose of learning multiplication tables could be considered to fit exactly through chanting, and then the student can repeat them in investigation of number patterns and relationships. Although, we are concerned about the student's understanding of why $7 \times 9 = 63$, *"we also hope that the stimulus* 7×9 *will produce the instant response 63*" (*ibid*, p: 27).

Skinner suggested another class of behaviour that he labelled operant conditioning. Operant conditioning states environmental contingencies or the environment's 'reaction' to that individual's behaviour controls that individual's behaviour. The operant conditioning principle has been supported in hundreds of experimental studies involving humans as well as animals. Like Thorndike, Skinner's operant conditioning study concentrated on the relation between behaviour and its consequences. Skinner (1938) states, if reinforcing consequences immediately follow individual behaviour, this behaviour is more likely to re-occur, and behaviours that are followed by unpleasant or punishing consequences are less likely to re-occur (Slavin, 2006). This means that pleasurable or reinforcing consequences strengthen behaviour; unpleasant or punishing consequences weaken it. For example, if students enjoy reading books, they will probably read more often, and if they find the stories boring, they may read less often.

Behavioural learning theories are useful for clarifying and explaining much of human behaviour; they are even useful in changing behaviour. However, it is important to recognize that behaviourism focuses almost exclusively on observable behaviour. Hence, behaviourism had limits. This explains Skinner's failure to provide any explanation of less visible learning processes, such as concept formation, learning from text, problem solving and thinking. He believed that the learner's mind was a 'black box' and that it was impossible to see what happens inside, and he preferred to keep explanatory concepts to a minimum and simply report data; relationships were unnecessary and unscientific (Asher, 2003). Later, a new era began with cognitive psychologists who attempt to look inside the human mind 'the black box'.

2.3 Piaget and Cognitive Developmental Psychology

Jean Piaget (1896-1980) is considered to be the most influential developmental psychologist in the twentieth century (see Flavell, 1996). After finishing his doctorate degree in biology, he devoted his life to study psychology, searching the mechanism of biological adaptation and analysing logical thought. His approach was based on an evolutionary epistemology. Piaget realised that any decent learning theory involves epistemological considerations and he called his own research programme 'Genetic Epistemology' (Piaget, 1972). Adaptation is the term that describes an individual's changes in response to the environment. Adaptation plays an essential role in Piaget's theory. Glasersfeld (1989) noticed that the most basic of all Piaget's ideas is that knowledge does not attempt to produce a copy of reality but, instead, assists adaptation purposes.

Piaget explored two questions: How do children manage to adapt to their environment?; and, How can we classify and order child development over time? Piaget's method in his experiments depends on asking children for their ideas about natural events and recording their answers with great attention. He believed that the highest form of human adaptation is cognition. In order to explain children's adaptation to the environment, he used features of biological adaptation and created his own distinctive terminology as explained below:

Schemas, according to Piaget, are the basic ways of organising patterns or units of action or thought that we construct to make sense of our interactions with the environment. Schemas can be thought of as files in which we store information, so each schema treats all objects and events in the same way. Piaget believed that thinking is internalised activity. Individuals interact with and make sense of the environment around them, and it is this physical interaction that becomes internalised to create thinking.

Assimilation and **Accommodation**: according to Piaget, the term adaptation is used to describe the process of adjusting schemas in response to the environment by means assimilation and accommodation. Assimilation, put simply, is taking in new information and trying to fit this information into existing schemas, or responding to the environment in terms of previously learned patterns of behaviour or schemas. Accommodation is the effort of organisms to change or modify an activity or ability to fit the new information, or responding to the environment in a new manner, if previously learned patterns of behaviour or schemas are not sufficient.

Equilibrium is when the individual's perception of the world fits into existing schemas. It is a state of continual activity in which an individual compensates for disturbances to the system. When existing schemas cannot deal with new experience there is dis-equilibrium.

In order to answer the question of how to classify and order child development, Piaget postulated four stages for cognitive development through which individuals' progress between birth and young adulthood, these stages being qualitatively different from each other. He claimed that children pass through a series of stages of thinking in this order and that no child can jump a stage, although some children would advance earlier or later to the next stage. Slavin (2006) listed these cognitive stages as shown below in table 2-1.

Piaget's Stages of Cognitive Development

People progress through four stages of cognitive development between birth and adulthood, according to Jean Piaget. Each stage is marked by the emergence of new intellectual abilities that allow people to understand the world in increasingly complex ways.

STAGE	APROXIMATE AGES	MAJOR ACCOMPLISHMENT
Sensor- motor	Birth to 2 years	Formation of concept of "object permanence" and gradual progression from reflexive behaviour to goal-directed behaviour.
Preoperational	2 to 7 years	Development of the ability to use symbols to represent objects in the world. Thinking remains egocentric and self centred.
Concrete operational	7 to 11 years	Improvement in ability to think logically. New abilities include the use of operations that are reversible. Thinking is decentred, and problem solving is less restricted by egocentrism. Abstract thinking is not possible.
Formal operational	11 years to adulthood	Abstract and purely symbolic thinking possible. Problems can be solved through the use of systematic experimentation.

Table 2-1: Piaget's stages of cognitive development (Slavin, 2006)

According to Piaget, all children move through all these levels and in the defined order, and they will not be able to reach one developmental stage until they master the previous one. Therefore, in the case of learning mathematics the consequence will be as Orton (2004, p: 52) said "*if a child is known to be operating at a particular Piagetian level, if it is known at what stage they are functioning, there is no possibility that they will be able to cope with any mathematics which depends on capabilities associated with a subsequent stage*". Piaget's theory would propose that the capability to cope with abstraction levels in learning mathematics depends on the development of formal operational thinking. Many mathematical topics and ideas confuse the students in mathematics classes due to the high level of abstraction demanded. For example, the early introduction of algebra, and its abstract nature (as a generalization and extension of arithmetic where the symbols and letters were used to represent number and quantity) requires high level of operational thinking. Students need to reach a certain developmental level in order to handle the ideas.

This tends to colour whole attitudes of students and their views about mathematics as abstract subject. Orton (2004, p: 53) stated:

> *"Many mathematical ideas require the kind of thinking skills which Piaget has claimed are not beginning to be available until the onset of the formal operational stage. It does not matter how carefully and systematically the teacher might try to build up a pupil's capabilities and knowledge – it is impossible to introduce concepts dependent on formal operational thought before the pupils has moved into that stage. The pupil is not yet ready for such abstract ideas. Pupils might, of course, be able to grasp the beginnings of an abstract idea in an intuitive or concrete way, but they cannot appreciate the idea as the teacher does. Explanations by the teacher will fail to make any impact unless such explanations are dependent only on skills available to pupils at the concrete operational stage."*

Doubtless, Piaget's work has been more influential than any other theorists' works in term of mathematics curricula development, especially in primary level. Evaluation and criticisms of Piaget work are considered in the following section.

2.4 Evaluation of Piagetian Theory

Piaget's theory of development revolutionised, and still dominates the study of human development. However, some of his principles have been questioned and criticised in more recent research. He has been criticised for the rigid and largely fixed developmental stages of his theory. Donaldson (1978) was strongly critical of the way in which Piaget asked children questions in experimental situations. The criticism of Piaget's device was summarised by Orton as follows:

- *"Many questions are not meaningful to the children – either they do not relate to the world in which the child lives or they do not motivate;*
- *Some questions might be regarded as ridiculous or frivolous for the above reason or because they contain questionable statements;*
- *The complexity of instructions in some questions, that is the language demands, are too much for some pupils;*
- *Some questions are not sufficiently free from context variables to produce results, from different backgrounds, which are comparable;*
- *Some questions, particularly those devised to test formal operational thinking skills, are too difficult even for most adults."*

Orton (2004, p: 63)

Researchers have created some tasks like the Piagetian tasks that require the same skills and found that they can be taught to children at earlier development stages (Black, 1981; Case, 1998; Siegler, 1998). Gelman (1979) found that, when the task was presented in a simpler way with simpler language, children could solve a conservation problem involving the number of blocks in a row. Sutherland (1992) noted criticisms of aspects of Piaget's stage theory on sensor-motor period, on concrete operation period and Piaget's clinical interview for the basis of the lack of scientific rigour.

Piaget has also been criticised because he did not use sufficiently large samples and he did not pay enough attention to statistical significance (Ausubel *et.al*, 1978). However, Orton (2004) argued that inadequate sampling is not a substantive issue because the conclusions which are drawn on the basis of work with small samples provides genuinely valuable information if the researcher is cautious. He also added that many experiments based on Piagetian tasks have been replicated by others in many countries around the world using large samples.

Piaget's work has also been criticised due to the underestimation of the young children's abilities of learning language. Piaget believed that the growth changes in the cognitive structure of the child produced linguistic development. Vygotsky (1978) took the opposite view of Piaget. On speech development, Piaget argued that the egocentric speech of children goes away with maturity, when it is transformed into social speech. On the contrary, for Vygotsky the child's mind and language are inherently social in nature.

A much more substantial criticism of Piaget theory is his statement that all children go through the stages in the same order though the age at which they progress to the next stage will differ. Thus, one of Piaget's greatest weaknesses was his failure to take individual differences into account: individual differences in personality, gender, intelligence and other factors that affect the ability to progress cognitively (Sutherland, 1992). These differences may be important and affect the rate of cognitive development.

In summary, Piaget's theory has been criticised for relying exclusively on broad, fixed, sequential stages through which all children progress and this gives an understanding of children's abilities. Nonetheless, Piaget established the basis that has led to much modern educational thought and he had a profound impact on the theory and practice of education (Donaldson, 1978; Miller, 1993; Orton, 2004). Overall, Piaget's observations were highly perceptive and offer a broadly correct picture of the cognitive development of children. He never attempted to explain what he observed. That was attempted by his followers and, especially, by those who developed the ideas of information processing. Piaget presented a picture of cognitive development which was, perhaps, slightly too rigid and which did not take enough account of language and social support.

2.4.1 Educational Implication of Piaget's Theory

Piaget's theories have led to all sorts of application in the real world, and have had impact on educational practice and research (Bentham, 2002). Berk (2001) summarises the educational applications drawn from Piaget's theories as follows:

> ***"A focus on the process of children's thinking, not just its products****. In addition to checking the correctness of children's answers, teachers must understand the processes children use to get to the answer. Appropriate learning experiences build on children's level of cognitive functioning, and only when teachers appreciate children's methods of arriving at particular conclusions are they in a position to provide such experiences.*
>
> ***Recognition of the crucial role of children's self-initiated, active involvement in learning activities.*** *In a Piagetian classroom the presentation of ready-made knowledge is deemphasized, and children are encouraged to discover for themselves through spontaneous interaction with the environment. Therefore, instead of teaching didactically, teachers provide a rich variety of activities that permit children to act directly on the physical world.*
>
> ***A deemphasis on practices aimed at making children adult like in their thinking.*** *Piaget referred to the question "How can we speed up development?" as "the American question." Among the many countries he visited, psychologists and educators in the United State seemed most interested in what techniques could be used to accelerate children's progress through the stages. Piagetian-based educational programs accept his firm belief that premature teaching could be worse than no teaching at all, because it leads to superficial acceptance of adult formulas rather than true cognitive understanding (May & Kundert, 1997).*
>
> ***Acceptance of individual differences in development progress.*** *Piaget's theory assumes that all children go through the same developmental sequence but that they do so at different rates. Therefore, teachers must make a special effort to arrange classroom activities for individuals and small groups of children rather than for the total class group. In addition, because individual differences are expected, assessment of children's educational progress should be made in terms of each child's own previous course of development, not in terms of normative standards provided by the performances of same – age peers."*

Piaget's work was welcomed as being helpful in relation to curricular design and to learning activities planning. Another major development rising out of Piaget's work and extending it in various ways is Constructivism.

2.5 Constructivism

Constructivism is one of the most important ideas in current educational psychology, and it draws heavily on Piaget's and Vygotsky's work (Slavin, 2006). It had considerable influence in science education research through the 1980s and 1990s. For example, instruction in mathematics (National Council of Teachers of Mathematics, 1989), and science (American Association for the advancement of Science, 1993) are increasingly grounded in constructivist theories of learning. The essence of constructivist theories of learning (Slavin, 2006) is the idea that learners must individually discover and transform complex information if they are to make it their own, and the learners are seen as constantly checking new information against old rules and then revising rules when they no longer work. It is a view of learning and development that emphasizes the active role of the learner in building understanding and making sense of the world (Eggen & Kauchak, 2007).

> *"Constructivists believe that making that knowledge results from individual constructions of reality. From their perspective, learning occurs through the continual creation of rules and hypotheses to explain what is observed. The need to create new rules and formulate new hypotheses occurs when students' present conceptions and new observations."*

(Brooks, 1990, p: 68)

According to constructivism, knowledge cannot be transmitted and teachers cannot simply give students knowledge. Instead, students' knowledge must be constructed in their own minds. The role of teachers is facilitating the learning process by teaching in ways that make information meaningful and relevant to students, by providing students with opportunities to discover or apply ideas themselves (Slavin, 2006). The works of Piaget and Vygotsky emphasized that cognitive change takes place only when previous conceptions go through a process of disequilibration in light of new information, and emphasized the social nature of learning (Slavin, 2006). There are different views of constructivism each with different implications for educational practice (Biggs, 1996). Constructivists disagree on the nature of knowledge and the importance of social interaction. These two varieties of constructivism will be discussed.

The first, originating largely in the work of Piaget, is called *the theory of personal constructivism*, which focuses on individual, internal constructions of knowledge (Greeno *et.al*, 1996). This view of construction emphasises learning activities that are learner-centred and discovery oriented. Children's everyday knowledge of natural phenomena is viewed as a coherent framework of ideas based on a common-sense interpretation of their experience of living in the world. For example, it is arguable that learning mathematics facts through discovery learning based on what the children already know, is more effective than having them presented by a teacher (Pressley *et.al*, 1992).

The second view, strongly inspired by Vygotsky's theories, is called *the social construction of knowledge,* which proposes that knowledge exists in a social context and is initially shared with others instead of being represented solely in the mind of an individual (Bruning *et.al*, 1999). Vygotsky developed a fully cultural psychology stressing on the primary role of communication and social life in meaning formation and cognition, and four key principles of his ideas have played an important role. First is his stress on the social nature of learning, where learning is viewed as more a cognitive structure used to interpret nature rather than physical events and phenomena themselves. In this approach the social context in which learning takes places is crucial. Social interaction plays a fundamental role of social construction in the development of cognition. Vygotsky (1978) states: "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). He asserted the significance of dialogue as a tool through which individuals collectively, or individually, could negotiate conceptual change (Boudourides, 1998).

The second key concept is the idea that the potential for cognitive development relies on the "zone of proximal development" (ZPD): a level of development attained when children engage in social behaviour. Vygotsky believed that there was a difference between what learners could achieve by themselves and what they could do with assistance from a more skilled individual. He developed a concept of a learning environment consisting not only of children and learning material and processes, but children, learning material and interactive communication. Vygotsky's findings suggest learning environments should involve guided interaction, permitting children to reflect on inconsistency and to change their conceptions not only through Piaget's intelligent action but also through speech and communication. The child's verbal and conceptual maturation can be achieved by exposure to increasingly more expert vocabularies through social interaction (Boudourides, 1998). Another aspect of Vygotsky's theory is cognitive apprenticeship, that the process by which a learner gradually acquires expertise is through interaction with an expert, either an adult or an older or more advanced peer (Salvin, 2006). Discussions between teacher and students and between students themselves enhance the quality of students' mathematical thinking as well as their ability to express themselves considerably (Cockcroft, 1982; Department of Education and Science, 1985). Finally, Vygotsky stressed scaffolding, or mediated learning. Scaffolding is a mechanism whereby, through the language of shared communication, a more skilled individual tries to impart knowledge to a less skilled individual, and it refers to the various types of support given by teachers (Bentham, 2002). The following table illustrates a comparison of Piaget's and Vygotsk's views of knowledge construction.

	Piaget	Vygotsky
Basic question	How is new knowledge created in all cultures?	How are the tools of knowledge transmitted in a specific culture?
Role of Language	Aids in developing symbolic thought; it does not qualitatively raise the level of intellectual functioning. (The level of functioning is raised by action.)	Is an essential mechanism for thinking, cultural transmission, and self-regulation. Qualitatively raises the level of intellectual functioning.
Social interaction	Provides a way to test and validate schemes.	Provides an avenue for acquiring language and the cultural exchange of ideas.
View of learners	Active in manipulating objects and ideas.	Active in social contexts and interactions.
Instructional implications	Design experiences to disrupt equilibrium.	Provide scaffolding. Guide interaction.

Table 2-2: A comparison of Piaget's and Vygotsk's views of knowledge construction (Eggen & Kauuchak, 2007)

Constructivist-learning theory emphasizes *top-down* instruction, which means that the learner begins with complex task to solve and then works out or discovers the basic skills required with the guidance of the teacher (Slavin, 2006). Lampert's method (1986) in introducing the multi-digit multiplication by top-down teaching is an instance of constructivist approach to mathematics teaching. The traditional, bottom-up approach to teaching the multiplication of two-digit numbers by one-digit numbers is to teach students a step-by-step procedure to get the right answer, and after basic skills have been mastered, the simple application tasks will be presented. The constructivist approach works in exactly the opposite order, starting by presenting problems and then helping students figure out how to do operations.

On the other hand, constructivist approaches to teaching emphasize *cooperative learning* and *discovery learning*. In cooperative learning, there are opportunities for discussion that lead to difficult concepts being more easily discovered and comprehended. In discovery learning, students are encouraged to learn largely on their own through active involvement with concepts and principles.

However, as Biggs (1996) indicates whatever particular constructivist theories may variously emphasize, a result would be that learners arrive at meaning by actively selecting, and cumulatively constructing, their knowledge, through both individual and social activity. Most constructivists, despite their differences, agree on four characteristics that influence learning as following:

- *"Learners construct their own understanding.*
- *New learning depends on current understanding.*
- *Learning is facilitated by social interaction.*
- *Meaningful learning occurs within authentic learning tasks."*

Bruning et.al, (1999)

However, it was argued by Millar (1989) and Jenkins (2000) that constructivism-learning theory requires a particular model of instruction or demands a progressive pedagogy. Of course, several writers have proposed instructional strategies based on constructivism ideas: greater emphasis on discourse relating to students concepts; discussion in the classroom; exchange of ideas; demonstration or experience with conflict situations; increasing the active involvement of students. Some suggested the use of modern audiovisual technologies and computer graphics can overcome difficulties with abstract, unobservable concepts (Garnett and Hackling, 1995). However, none of those strategies and techniques is 'unique' to constructivism as Jenkins (2000) stated:

> *"Selecting a strategy that is more, rather than less, likely to interest students and promote their learning is central to a teacher's professional competence".*

For that reason*, it can be wondered: "is the evidence which arises from the constructivist framework of pupils' ideas influential enough to affect straight away the teaching process?"* Of course, it should be clear that constructivism does not prescribe particular teaching techniques; it is nonetheless appropriate to attempt to discuss features of classroom practice and the problems which arise in trying to implement constructivist beliefs in the classroom (Orton & Wain, 1994).

Overall, constructivism does not provide us with a learning theory and does not prescribe to us what our teaching approach should be, and it has only a marginal impact on the theory and practice of scientific education. Undeniably, constructivism has given a challenge to consider on a relativist approach to the teaching and learning processes. Some of these considerations were rather critical against it (Suchting, 1992; Matthews, 1993; Phillips, 1995; Osborne, 1996) and some have urged caution in its adoption (Millar, 1989; Solomon, 1994). While many would disagree with the constructivist approach, few would silence the psychological influence on education brought about by the constructivist view of learning. In fact it is as *"a psychological theory about how beliefs are developed"*

(Matthews, 1998), where the original core of constructivism might be found. Kirschner *et.al* (2006) have brought together many modern approaches to teaching and learning and demonstrated that any model which does not take into account the limiting capacity of working memory does not lead to better ways of learning. They include constructivism in this describing as an excellent description of learning which, nonetheless, is of very poor predictive value.

2.6 Theories of Mathematics-Learning

The nature of mathematics as an objective, logical and abstract subject has consolidated resistance to educational shifts similar to the other subjects. Mathematics education researchers have sought to introduce psychosocial theories, to describe the process of learning in mathematics. Two theories of learning mathematics will be presented; one is *the Dienes theory of learning mathematics* (Dienes, 1960), and the other one is *the van Hiele theory of learning geometry* (from Orton, 2004 and Fuys *et.al,* 1988).

2.6.1 Dienes Theory of Learning Mathematics

Dienes started from the position that mathematics could not be learned in a stimulus-response manner since it did not address the problem that mathematics-learning was so dependent on understanding the structure (Orton, 2004). He derived his original inspiration from Piaget, Bruner (1966) and Bartlett (1958), but his theory was also found by research of his own. *Dienes theory* presents an early-learning environment intended to improve the construction of an understanding of place value, and it gives us a wealth of teaching ideas. Dienes's perspective of learning mathematics comprised the following four principles (Dienes, 1960):

1. *The constructive principle*: Dienes claimed that teacher must construct mathematical ideas. He considered the structure of a given mathematical idea cannot be abstracted from concrete objects, but alternatively must be abstracted from relational/ operational/organizational systems that humans require on sets of objects. For instance, when Dienes's arithmetic blocks (see Figure 2-1) are used to teach the "regrouping structure" of our base-ten numeration system, children must first organize the blocks using an appropriate system of relations and operations. Only after these organizational systems have been constructed, can children use the materials as a model that embodies the underlying structure.

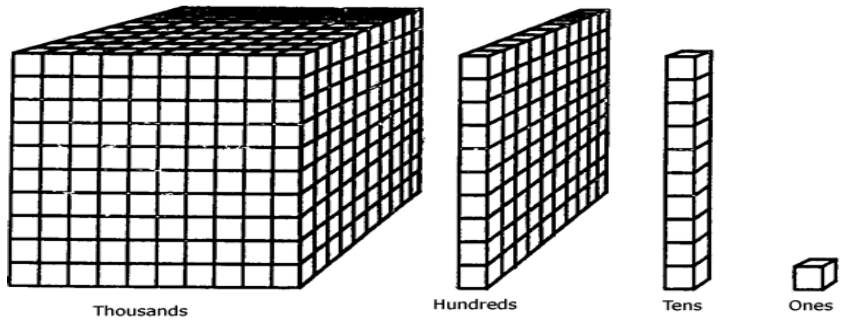


Figure 2-1: Dienes's Arithmetic Blocks

2. *The mathematical variability principle*: According to Dienes, mathematical ideas cannot be abstracted from single isolated patterns (or models which "embody" these patterns) any more than simpler abstractions can take place from single instances. Alternatively, mathematical abstractions occur when students recognize structural similarities shared by several related models.

3. *The dynamic principle*: According to Dienes, the schemes that must be abstracted from structurally related models are not simply "static patterns"; they are dynamic and correspond with other models.

4. *The perceptual variability principle*: Regardless of whether the object that used to represent a given model is a set of concrete objects, graphics, written symbols, spoken language, or some other representational system, models always have some features that the modelled system does not have – or, on the contrary, they fail to have some features that the modelled system does have. Therefore, to select a small number of particularly proper models to represent a given system, the following features should be taken into consideration:

- Irrelevant features should vary from one model to another so that these features will be 'washed out' of the resulting abstraction.
- Collectively, models should point out all of the most important structural features of the modelled system.

Dienes took the work of Piaget to propose that learning is an active process. He also drew the dynamic principle directly from the assumption that concept formation is promoted by providing suitable learning materials with which children can interact. The limitation of the

Dienes mathematics-learning theory is the neglect of the relationship between the learning of a new concept and the existing knowledge structure already held in the learner's mind. However, Dienes theory of mathematics-learning is much supported in number ways. It clearly builds on a cognitive approach, such as the work of Piaget, Bruner (1966), Bartlett (1958) and Wertheimer (1961), and takes into consideration how to accelerate learning and how to cope with individual differences and as Orton (2004, p: 181) stated:

> *"Certainly, it is clear that the community of mathematics teachers and educators has not accepted the theory as the ultimate answer to anything. Dienes proposed it as a feasible skeleton theory of learning mathematics, and not necessarily as an ultimate answer…What must also be gratefully acknowledged, however, is that Dienes has given us a wealth of teaching ideas."*

2.6.2 The van Hiele Theory of learning geometry

The van Hiele theory of learning geometry also drew heavily from the work of Piaget (1972) and Gestalt theory (Wertheimer, 1912). After studying Piaget's work, Pierre van Hiele and Dina van Hiele-Geldof thought that students' geometrical competence might well improve by progressing over a period of time through successive stages of thinking. There are five levels in van Hiele theory of learning geometry as follows (Orton, 2004):

Level (1) Visualization:

Students can recognize figures as whole entities (triangles, squares), but do not recognize properties of these shapes (right angles in a square); visual impression and appearance exert a storing influence, thus a square cannot also be a rectangle; drawings of shapes are based on holistic impressions and not on component parts; names may be invented for shapes according to their appearance, for example, 'Slanty rectangle' for parallelogram.

Level (2) Analysis:

Students can analyse figures' components such as sides and angles but cannot relate between figures and properties logically; properties and rules of a class of shapes may be discovered empirically (for example, by folding, measuring, or by using a grid or diagram); a figure can be identified from its properties; generalizations become possible, for example, all squares have four sides, the angles of triangles total 180º.

Level (3) Informal Deduction:

Students can establish interrelationship of properties within shapes and can make simple deductions, though the intrinsic meaning of deduction is not understood; a shape may be used to establish that a square is a rectangle; a statement cannot be separated from its converse.

***Level (4)* Deduction:**

At this level, students can appreciate the need for definitions and assumptions, and can present proofs within a postulational system; The interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof can be understood; proof as the final authority is accepted; inter-relationships among networks of theorems can be established.

***Level (5)* Rigor:**

Students at this level can work abstractly and can compare systems, can examine the consistency and independence of axioms and generalize a principle or theorem to find the broadest context. Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

All school geometry courses are taught at Level 3 (ibid). The van Hieles also recognized some features of their model, including the fact that a person must proceed through the levels in order. The most important features of van Hieles system as summarized by Fuys *et.al* (1988) are as follows: (a) the levels are consecutive; (b) each level has its own vocabulary, symbols and network of relations; (c) what is implicit at one level becomes explicit at the next level; (d) material taught to students above their level is dependent on reduction of level; (e) progress from one level to the next is more subject to instructional experience than on age or maturation; and (f) one goes through various 'phase' in arising from one level to the next.

2.7 Ausubel's Theory of Meaningful-Learning

Within cognitive psychology two significant approaches to the teaching process have proceeded: *Discovery learning* was advocated by Bruner (1966), and *Direct instruction learning* was advocated by Ausubel (1968).

2.7.1 Discovery Learning

Discovery learning is a method of inquiry based on the learner rather than a teacher-oriented view of the teaching and learning processes, and it is advocated in the work of Piaget (1972), Bruner (1966), and Papert (1980). It requires learners to proceed in the same way as scientists when investigating nature, using processes that are very similar to the processes of scientific discovery (Klahr & Dunbar, 1988; Jong & Njoo, 1993; Joolingen &

Jong, 1997). The central purpose of discovery learning is that the learner obtains or constructs knowledge by performing experiments. Discovery learning is considered to be a promising way of learning for a number of reasons, the main being that the active involvement of the learner with the field would result in a better knowledge base (Joolingen, 1999). Unfortunately, many students are still taught largely by exposition and are given little opportunity to learn by discovering (Orton, 2004).

Papert concentrated on the impact of new technology on learning, and he "*gained his enthusiasm for active, discovery-type learning environments directly from Piaget, with whom he worked for five years*" (Orton, 2004, p: 97). Papert expressed his belief that enriching the learning environment through the use of materials was more important than Piaget had suggested (ibid). He created Logo which is a computer programming language as a tool to improve the way that children may think and solve problems in mathematics.

A mobile electronic *'Logo turtle'* was developed and children were encouraged to solve problems and trace out shapes on a classroom floor. Papert (1980) argued that the usual mathematics curriculum was meaningless to most children, but the invention of Logo provided for them an opportunity to construct knowledge in meaningful way. However, Orton (2004) stated that there are some objections on the grounds that Logo is too difficult and it takes too much time, and that subsequently using Logo in ordinary classrooms has "*convinced many teachers that pupils cannot work entirely on their own in the way Papert seems to suggest is both possible and desirable, and that skilful teacher mediation between the children and the software is needed"* (ibid).

In teaching mathematics, words such as 'discovery', 'investigation', 'activity' and 'problem-solving' are famous and have became part of the mathematical language (Orton, 2004). At the present time, there is much awareness about the importance of the use of discovery learning in mathematics classes, especially at primary level. With older students, discovery learning might sometimes be a suitable method, but it is very rarely used. In practice, it is very hard to apply discovery learning in higher educational level and as Ausubel states what was created over the past four centuries cannot be rediscovered by our students in ten or fifteen years. Thus, meaningful verbal learning which provides expository teaching can be effective and in some ways better than other methods.

2.7.2 The Conditions of Meaningful-Learning- Ausubel's Terminology

One of the main researchers who has connection with the constructivist movement was David Ausubel. In 1968, he put forward the case that the most important thing for teachers to know at the outset of the teaching is what each student knows already. However, he held a different approach to how the teaching material should be presented in the classroom or by self-study than Bruner. He argued that students need guidance if they are to learn effectively and he advocated the direct instruction learning approach. Ausubel (1968) focussed on both the presentational methods of teaching and the acquisition of knowledge. He made a major contribution to learning by studying and describing the conditions that lead to 'meaningful learning'. He attempted to find 'the laws of meaningful classroom learning'.

According to Ausubel, meaningful learning presupposes:

- *"That the learning material itself can be nonarbitrarily (plausibly, sensibly, and nonrandomly) and substantively (nonverbatimly) related to any appropriate cognitive structure (possesses "logical meaning").*
- *That the particular learner's cognitive structure contains relevant anchoring idea(s) to which the new material can be related.*
- *The interaction between potentially new meanings and relevant ideas in the learner's cognitive structure gives rise to actual or psychological meanings. Because each learner's cognitive structure is unique, all acquired new meanings are perforce themselves unique."*

Ausubel et.al, (1978)

The meaningful learning processes exist when the new concept can be linked to the pre existing concept in the learners' cognitive structure (for example, already existing relevant aspect of knowledge of an image, an already meaningful symbol, a known concept or a proposition). The new concept interacts in a nonarbitrary (in the sense of plausibly, sensibly and nonrandomly), and substantive (nonverbatimly) basis with established ideas in cognitive structure. Thus, meaning derives directly from associations that exist among ideas, events, or objects. As the new knowledge is subsumed into the existing knowledge, it interacts and modifies it and the entire new matrix now becomes more elaborate and new linkages form between concepts.

Obviously, this theory can only become reality if the teacher finds out what the learner already knows. Orton (2004) argued that, if an attempt is made to force children to assimilate and accommodate new mathematical ideas that cannot link to knowledge which is already in an existing knowledge structure, then the ideas can only be learned by rote. In contrast with meaningful learning, rote learning results in arbitrary literal assimilation of new knowledge into cognitive structure. It occurs when no relevant concepts are accessible in the learner's cognitive structure. 'Rote-meaningful' learning is a continuum, which relies on the learner and differs from one learner to another. The feature of the cognitive structure of the learner interconnects in a diverse degree from topic to topic in the 'rote-meaningful' continuum. The learner's existing knowledge and the way that new knowledge is linked to existing knowledge involves subsumption. Ausubel postulates that cognitive structure is hierarchically organised which means the less inclusive sub-concepts and details of specific data are organised, under the more inclusive concepts. Therefore, good expository teaching should be given to the learner to ensure that a new concept is linked to relevant existing knowledge.

According to Ausubel (1968) the *advance organiser* is "an advanced introduction of relevant subsuming concepts (organisers) which can facilitate the learning and retention of unfamiliar but meaningful verbal material". The idea of *advance organizer* was introduced by Ausubel in the following two cases:

a) When the student does not process proper subsumers, e.g. when the material is totally new and the learner does not have relevant information to which they can relate the new material.
b) When the student obtains appropriate subsuming information, but the information is insufficiently developed and is not likely to be identified and linked to the new information.

The existing components of the knowledge structure to which new learning needed to be correlated; subsequently they become recognized also as 'anchoring' ideas or concepts. At this point Ausubel's idea of subsumers is similar to Bruner's view of readiness. So, if the subsumers are there the student is efficiently ready.

Orton (2004) stated that meaningful learning implies an understanding of constraints. He also added that any theory of learning mathematics should take into account the hierarchical nature of the subject, and there should not be many occasions when new knowledge cannot be linked to existing knowledge. For instance, it is not possible to learn about integers and about rational numbers unless the natural numbers are understood meaningfully (ibid).

2.8 Information Processing and Cognitive Theories of Learning

The three most basic assumptions of cognitive theories (mental processes exist, they can be studied scientifically, and humans are active information processors) have led to the information processing theories (Ashcraft, 2002). Information Processing is a cognitive theory that examines the way information enters our minds through our senses and is stored in and retrieved from memory. It has become the dominant theory of learning and memory since the mid-1970s and it is concerned with learning processes rather than with learner's nature. It does provide an explanation of why young children are poorer than adults at single-focus and complex multi-focus tasks. According to the information processing approach, the young child has a limited capacity for memorising, and this capacity is smaller than the average adult's capacity (Sutherland, 1992).

2.8.1 The Hypothetical Model of Human Memory

Cognitive psychology uses a metaphor borrowed from computer science. According to cognitive models, the brain functions somewhat like a computer and it has input and output devices (the sensori-motor systems), various classes of storage. The modal model of human memory, according to Ashcraft (1994), comprises three kinds of information storage:

1. The sensory memory (or sensory registrar or perception filter).
2. The short-term memory (or the working memory).
3. The long-term memory.

Ashcraft, (1994)

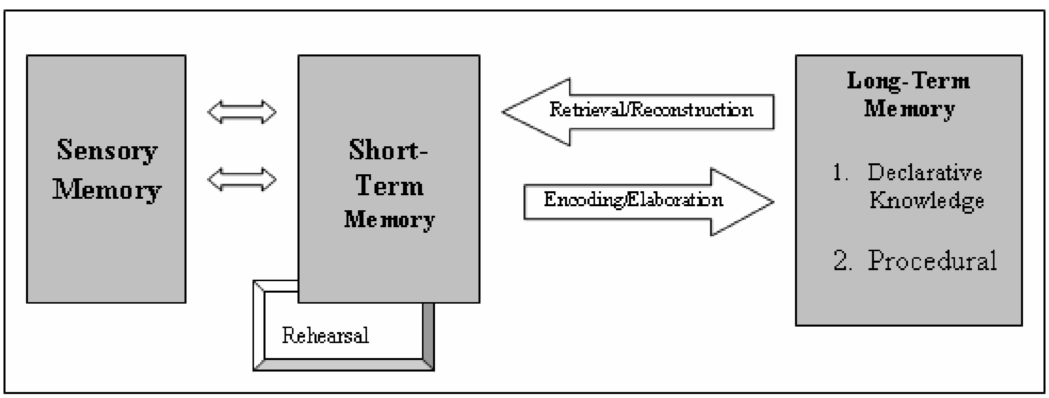


Figure 2-2: The Modal Model (Bruning *et.al*, 1999)

The differences between the three kinds of memory lie in the nature and extent of the processing that the information holds and in their capacity. Atkinson and Shiffrin's (1968) model of information processing outlined in figure 2-2 is not the only one accepted by cognitive psychologists. There are several models based on the basic assumptions of Atkinson and Shiffrin's models and elaborated aspects of it: for example, Ashcraft, 1994; Child, 1993; and Johnstone, 1993. However, differences lie in detail and extent of elaborate. The information processing model developed by Johnstone (1993) is the main focus of the following section.

2.8.2 The information Processing Model of Memory

The model (figure 2-3) suggested by Johnstone (1993) is based on a mechanism proposed by many psychologists. The key characteristics emphasised by Ashcraft (1994) are taken into consideration. The ideas of others theories such as Piaget's stage theory, Ausubel's importance of prior knowledge in meaningful learning, Pascual-Leone's idea of limited space related to age are observed. This model concentrates on learning processes and the nature of the learner. It proposes a simplified mechanism of the learning process and enables us to understand the limitations of learning.

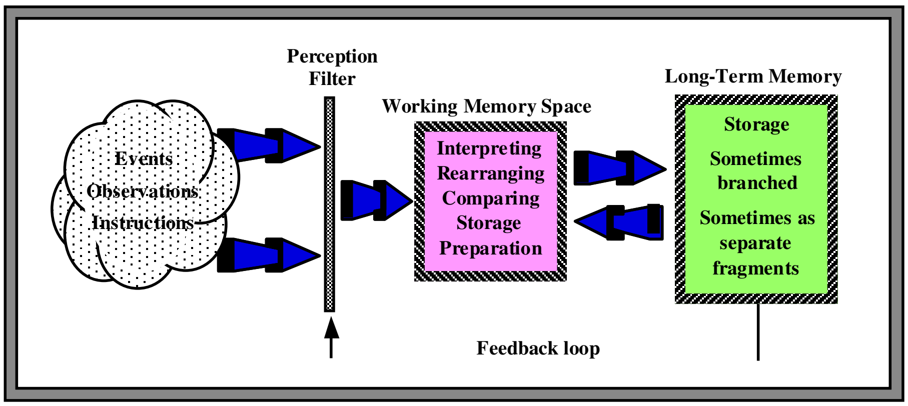


Figure 2-3: A Model of the Information Processing (Johnstone, 1993)

Sensory Memory-Perception Filter

The first component of the memory system is sensory memory, which is the information store that briefly holds stimuli from the environment until they can be processed and it consists of the sensory registers (Neisser, 1967). The sensory register is bombard with large amounts of information from the senses (sight, hearing, taste, touch and smell), and holds it for a very short time. There are two kinds of sensory memory, the auditory sensory memory and the visual sensory memory. The auditory sensory memory is a component of the sensory memory that responsible for receiving auditory information from the external environment (Ashcraft, 1994). The visual sensory memory refers to the part of the memory which holds visual sensations for a very brief duration (*ibid*).

The capacity of sensory memory is nearly unlimited, but if processing does not begin almost immediately, the memory trace quickly decays (Woolfolk, 2007). The estimated time that information can be held after the stimuli disappear varies from one second for vision and to four seconds for auditory (Driscoll, 2005; Leahey & Harris, 1997; Pashler & Carrier, 1996). In these moments, there is opportunity to select and organize information for further processing.

The process by which we select information is referred to as *perception*. In figure 2-3 the sensory memory is called the perception filter. The perception filter is controlled by information that is stored in long-term memory. Previously experiences, preferences, knowledge and prejudices control the perception filter and people respond and pay attention to certain stimuli (Johnstone, 1993). By paying *attention* for selecting stimuli and ignoring others, the possibilities for perceiving and processing will reduce (Woolfolk, 2007). Woolfolk argues that "what we pay attention to is guided to a certain extent by what we already know and what we need to know, so attention is involved in and influenced by all three memory processes" (p: 252). Paying attention is considered to be the first step in learning and without paying attention, students will not be able to process information that they do not recognize or perceive (Lachter et.al, 2004). Sensory memory holds information long enough to transfer it to the working memory, the next store.

Working Memory

Working memory is the store where new information is held for a relatively short period and combined with knowledge from long-term memory. Working memory is easily disrupted because of its limitations; it can hold only about seven plus or minus two (7 ± 2) items of information at a time (Miller, 1956) and holds information for limited period. Sweller, *et.al* (1998) argue "humans are probably only able to deal with two or three items of information simultaneously when required to process rather than merely hold information" (p: 252). Working memory has two functions:

- Hold information; and
- Process it into a form that can be used or stored in the long- term memory

Information processes such as selecting, comparing, and organizing also occupy working memory space, thus the number of items that can be dealt with is much less than the seven that can be simply held in working memory (Eggen & Kauchak, 2007). Working memory consists of three subcomponents, as following:

- The central executive system (CE): responsible for initiating and controlling processes, making decisions, and retrieving information from long-term memory.
- The phonological loop (PL): a subsidiary system for holding and handling sound and speech.
- The visuo-spatial working memory (VSWM): holding and manipulating nonverbal material.

Baddeley and Hitch (1974)

Working memory encodes information from sensory memory and long-term memory (Ashcraft, 1994). Thus, when a stimulus (sight, sound, smell, touch, or taste) is attended, by paying attention to this stimulus, the information transfers to the working memory. After, the information is processed in the working memory, it will be stored in the long-term memory. Because of its critical importance in learning, working memory is discussed in more detail in the following chapter.

Long-Term Memory

Long-term memory is the permanent information store. It is like a library with millions of entries and a network that allows them to be retrieved for reference and use (Eggan & Kauchak, 2007). The long-term memory is responsible for receiving the information from working memory and storing it on a relatively permanent basis for retrieving. Johnstone (1997) stated

> *"We store information which is potentially important, or interesting, or useful. We ignore or discard information which is more trivial or unimportant. This is a personal process and for that purpose memory uses a variety of functions such as: pattern recognition, rehearsal, elaborating, organizing. We seek for patterns as we try to connect the new information with existing information in order to make sense. We discard the new information when it does not make sense to us."*

The Long- term memory can be thought of in terms of three different kinds of storages:

- *Episodic memory,*
- *Semantic memory and*
- *Procedural memory*

(Eichenbaum, 2003; Squire el al., 1993; Tulving, 1993)

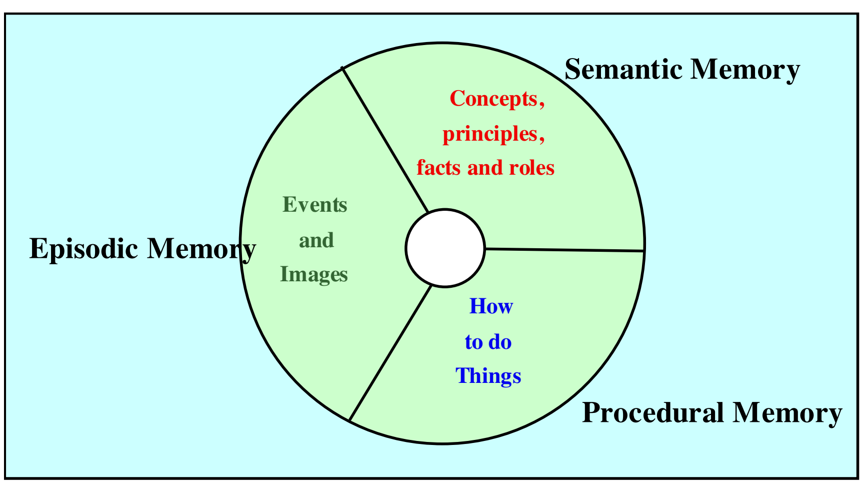


Figure 2-4: Long-Term Memory Subcomponents

Episodic memory comprises of events and images of experience organized by place and time (Tulving, 1993). It is “our memory of personal experiences, a mental movie of things we saw or heard” (Slavin, 2006). Long-term *semantic memory* comprises of the facts and generalized information that we know; concepts, principles, or roles and how to apply them, the skills of solving problems and learning strategies (Slavin, 2006). Semantic memory refers to a permanent memory that is responsible for storing general world information and knowledge such as the knowledge of language and other conceptual information (Ashcraft, 1994; Baddeley, 2004). *Procedural memory* refers to memory for procedural knowledge “knowing how” rather than “knowing what” (Solso, 2001). It stores the procedure for doing something, especially a physical tasks such as how to walk, how to drive, how to swim.

There are several differences between working and long-term memory in both capacity and duration:

- Working memory holds the information that is recently experienced and activated, whereas long-term memory holds the information that is well learned.
- Whereas working memory is limited to about seven plus or minus two, long term-memory's capacity is vast and durable.
- Working memory holds information for a matter of seconds. In long-term memory, once information is securely stored, it can remain there permanently.

Characteristics of Components of Cognitive Storage systems			
	Processes		
	Sensory Memory	**Working Memory**	**Long-term Memory**
Code	**Sensory feature**	**Acoustic, visual, semantic, sensory feature identified and named**	**Semantic, visual, knowledge; abstraction; meaningful images**
Capacity	**12-20 item …. to huge**	**7 ± 2 items**	**Enormous, virtually unlimited**
Duration	**250 msec to 4sec**	**About 12 sec. Longer with rehearsal**	**indefinite**
Retrieval	**Complete, given proper cueing**	**Complete, with each item being retrieved every 35 msec.**	**Specific and general information available, given proper cueing**
Failure to Recall Cause	**Masking or decay**	**Displacement, interference, decay**	**Interference organic dysfunctioning, inappropriate cues**

Figure 2-5: Characteristics of components of cognitive storage System (Slavin, 2006)

Long-term memory retention can be supported by several factors. The most important factor is the degree to which students have learned the material in the first place (Bahrick & Hall, 1991). Although, higher-ability students achieve more at the end of a course, they often lose the same proportion of what had to be learned as lower-ability students (Slavin, 2006). Another factor contributing to long-term memory is instructional strategies that involve students actively in the lessons (*ibid*). For example, Specht and Sandling's study (1991) compared undergraduate students who learned counting by traditional lessons with others who are taught through role-playing. They found after 6 weeks that students who are taught traditionally lost 54 % of their problem-solving performance whereas students who are taught through role-playing lost only 13 %. It is well known the role playing is a very powerful tool in attitudes development because it involves the bringing together of dissonant elements in long term memory (Reid, 1980). It is likely that role playing will also bring together cognitive elements and the presence of these links will aid problem solving (Reid & Yang, 2002b). This probably explains the finding of the Specht and Sandling's study (1991).

Processing in Long-term memory: Storage and Recall

The sensory memory transfers information to the working memory through the attention process. Through the rehearsal process the information remains in the working memory, and then transfers to the long-term memory. Encoding in long-term memory requires attention and rehearsal. Ashcraft (1994) defined rehearsal as "a deliberate mental process that can form a long-term memory trace, a record or representation of the information." (P: 58). Comprehension is another method that involves fundamentals of straightforward rehearsal (ibid). Atkinson and Shiffrin (1968) suggested that there are two effects of rehearsal: (1) to maintain information in the working memory; and (2) to also store the item in long-term memory. Johnstone (1997) referred to four ways for storing as following:

- *"The new knowledge finds a good fit to existing knowledge and is merged to enrich the existing knowledge and understanding (correctly filed).*
- *The new knowledge seems to find a good fit (or at least a reasonable fit) with existing knowledge and is attached and stored, but this may, in fact, be a misfit (a misfiling).*
- *Storage can often have a linear sequence built into it, and that may be the sequence in which things were taught.*
- *The last type of memorisation is that which occurs when the learner can find no connection on which to attach the new knowledge."*

Johnstone (1997)

Storing is one side of the processing, the other side is retrieving. Obviously, retrieving is as important as storing and as Ashcraft (1994) argued "What good does it do to store something in the memory if you can't retrieve it when you need it?" Retrieval depends on the way of encoding during learning. Baddeley (2004) stated that information can be encoded in long-term memory into the verbal coding system, which is linguistically modified information e.g. words, stories, discourse, or the imaginable coding system which is modified for non-verbal information such as pictures, sensations, sound. Tulving and Thompson's (1973) argued that information is encoded into long-term memory not as isolated items; instead each item is encoded into rich memory representation.

2.9 Conclusions

This chapter has attempted to review the general learning theories that link to learning mathematics, as well as the theories of learning mathematics. One example of general learning theories that offers a specific application to mathematics is the behaviourist approach, even though it is out of favour with most educationists today. Dienes (1973, p: 5) indicated that behaviourism was out of favour, in claming that "… no one today doubts any more the fact that the stimulus-response relation leads to a training which most of the time induces mental blockages…". Whereas, specialists on the brain functioning still emphasise the crucial role that recurrence plays in firming knowledge in the mind, and as Orton (2004, p:176) states "There is clearly an important distinction between thoughtful and necessary repetition and practice on the one hand, and mindless and potentially mind-numbing use of routine stimulus-response activities on the other."

The cognitive learning theories which were presented by Piaget are the alternatives to the behaviourist approach and an important insight into the growth of emphasis on cognition. *Information processing theories* and constructivism theories are the most significant ones among cognitive theories. Constructivism is seen as a view of cognitive development in which children actively build systems of meaning and understandings of reality through their experience and interactions arose from the earlier work of Piaget (DeVries, 1997). Meaningful learning which presented by David Ausubel (1968) incorporates results and concepts described by Piaget and also censures the enthusiastic belief in discovery learning efficiency. *Information processing theory* is concerned with the way information enters our minds through our senses, how it is stored in and retrieved from memory rather than with the learner's nature. It was appealing because it offers useful experimental methodology as well as an accessible language (Miller, 1993). In the information processing models, the structure of effective learning is seen in such a way that it can be stored usefully in the long-term memory. Knowledge is seen as something coherent and holistic, which provides sustenance for later learning (Atkins *et.al*, 1992). All mental learning processes take place in the working memory. Therefore, the characteristics of working memory and its' contributions to learning mathematics are highlighted in the next chapter.

Chapter 3

Working Memory and Learning Mathematics

3.1 Introduction

The dissatisfaction in the early 1970s with the idea of a single short-term storage and processing system, which is described in Atkinson and Shiffrin's (1968) model, led Baddeley and Hitch (1974) to suggest a multi-component working memory. The subcomponents of working memory are the executive control system and two slave systems: the visuo-spatial working memory, and the phonological loop.

Many cognitive psychologists have claimed that working memory plays a crucial role in learning processes, and they support their claims by studies demonstrating close links between working memory capacity and measures of learning and academic achievement (Adams & Hitch, 1997; Mclean & Hitch, 1999; Bull & Scerif, 2001; Christou, 2001; Towse & Houston, 2001; Jarvis & Gathercole, 2003; Alenezi, 2004; Holems & Adams, 2006). The aim of this chapter is to look at the literature of working memory and its implication in the mathematics field. This chapter is divided into six sections. The definition and subcomponents of working memory are explored in the first section. The second section discusses the function of working memory. The third section approaches the measurement of working memory. Working memory operation in mathematics education is approached in the fourth section in the following way:

- *Central executive and mathematics*
- *Phonological loop and mathematics*
- *Visuo-Spatial working memory and mathematics*
- *Working memory and achievement in mathematics*

Then, working memory limitations and overcoming working memory failure are discussed in the fifth and sixth sections.

3.2 Definition and Components of Working Memory

In recent years, the notion of short-term memory has been developed into the concept of working memory. Working memory is a system providing temporary storage and manipulation of the information that is supposed to be essential for creating important links between perception and controlled action. Reisberg (1997: 143) states: *"Working memory,* *it seems, deserves its name: it is indeed the workplace of cognition. But if working memory is to serve this function, then it must be capable of storing and working with all the diverse content that we can contemplate and attend – pictures and words and smells and abstract ideas, to name just a few."*

Johnstone (1984) distinguishes between short-term memory and working memory and introduces an explanation for the distinction between them. For remembering a set of numbers, like phone number, this process occurs in our short-term memory and no processing is taking place to recall it in the same order within a matter of seconds. However, to memorise them backwards, a working process must take place and the short-term memory is now called working memory. Essentially, short-term memory and working memory are the same space but the use defines the name.

Johnstone (1984) introduced a definition of working memory from the educational viewpoint as *"that part of the brain where we hold information, work upon it, organise it, and shape it, before storing it in long-term memory for further use."* The working memory space is very limited in terms of both its capacity (amount of information it can hold) and its duration (length of time it can hold information). Furthermore, working memory space depends on the age of the individual, and as Miller (1956) showed from memory experiments, the average capacity is about seven plus or minus two (7 ± 2) separate chunks. Chunking is the process of grouping into units which could be a single number, a letter, or many pieces of information, and the nature of the items plays a major role in the capability to recall.

Baddeley and Hitch (1974) suggested one of the most influential models of working memory. Based on their findings, they conceived that working memory as a multi-component system; the central executive is aided by two peripheral and independent systems – the phonological loop and the visuo-spatial working memory – that temporary store verbal and visuo-spatial information. This theoretical framework illustrates a development of earlier models of short-term storage (such as the model of Atkinson & Shiffrin, 1968).

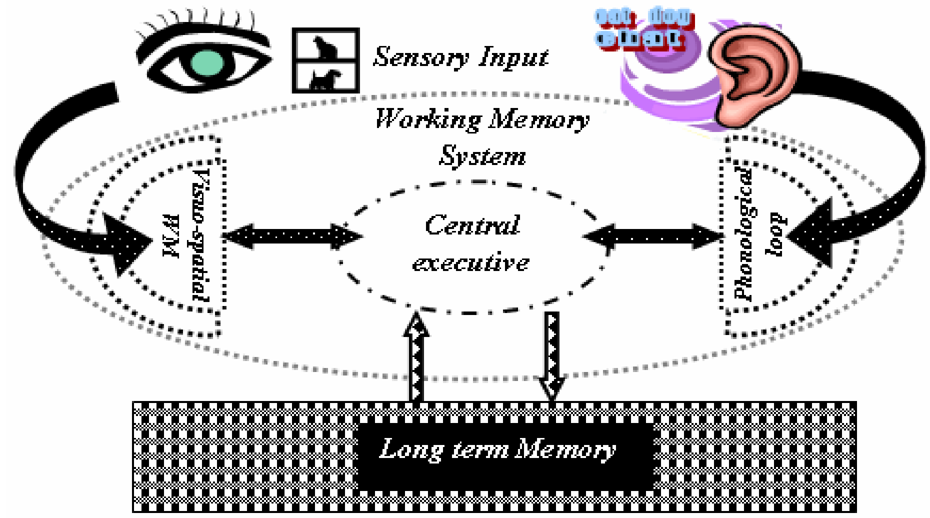


Figure 3-1: Model of Working Memory (derived from Baddeley's Model)

It is arguable that the *central executive* system is the most important of the three components of the working memory model because it controls the allocation of resources between the phonological loop and visuo-spatial working memory. The central executive is considered to be multi-functional and complex, and there is a considerable debate about the precise nature of its function (Bull & Espy, 2006). Miyake *et.al,* (2001) proposed that visuo-spatial working memory has a closer relationship with the central executive than the phonological loop, and visuo-spatial working memory can store more information as a whole. Although the central executive is the most important, it is the least understood component of the working memory (Baddeley, 2006).

The phonological loop is a slave system that specialises in processing the language-based information. Baddeley (1999) proposed that the phonological loop can be divided into two subcomponents: *Passive phonological store* is able to hold verbal information, for example before the memory trace vanishes or is refreshed by the *articulatory control process* (the second component), which is an active phonological rehearsal mechanism. Bull & Espy (2006) clarified the characteristic of the phonological loop as follows *"Information held in the phonological store is subject to decay, unless it can be refreshed by sub-vocal rehearsal, a process akin to repeating under one's breath the information one is trying to retain. Sub-vocal rehearsal, then, can be disrupted by secondary tasks that also use the verbal resources of the PL"*. Visually presented material (e.g. written material) can be also converted into an articulatory code by the articulatory control process and then it will be transferred to the phonological store (Baddeley, 1997).

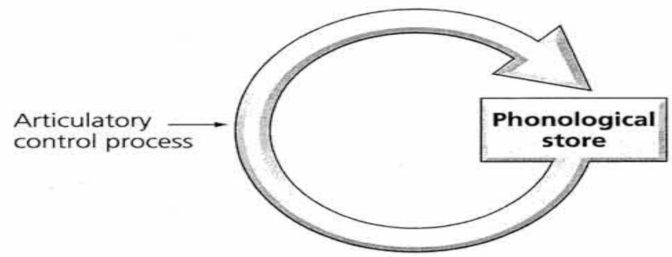


Figure 3-2: The phonological loop

(Source: www.simplypsychology.pwp.blueyonder.co.uk)

Many researchers have used this phonological loop characteristic to study its processes, using the secondary or dual-task methodology (Lee & Kang, 2002; Furst & Hitch, 2000; Logie *et.al,* 1994). There are basic phenomena associated with the phonological loop as follows (Logie, 1995; Baddeley, 2006):

The phonological similarity effect is the clearest phenomena associated with the phonological loop, and it refers to the fact that recall of a series of words or letters is more confusing when these words or the letters sound alike. For example, sequential recalling for a series such as *"Mat, Cat, Fat, Rat, Hat, Chat"* is more difficult than a sequence such as *"Bus, Clock, Spoon, Fish, Grate, Man"*. This phenomenon results from the confusion of the similarity of the verbal items that comprised in the phonological to one another (Baddeley, 1966a; Conrad, 1964).

Irrelevant Speech refers to the disruption that occurs from presenting irrelevant speech, while the recalling process is held. Salame & Baddeley (1982) discussed the disruptive effect of *irrelevant speech*. This kind of speech involves the conveying of ideas not relevant to the task in hand. The irrelevant speech accesses directly the phonological store, thereby disturbing its contents.

Word length effect is a phenomenon where recalling sequences of long words such as *"University, Aluminium, Hippopotamus, Mississippi, Refrigerator"* is less well done than short words sequences such as *"Pen, Book, Chair, Greece, Nail"* (Baddeley *et.al,*1975). The interpretation of the word length effect is that long words take a longer time to say or rehearse, allowing a greater degree of trace decay, thus will be less well retained (Baddeley *et.al,* 1975; Logie, 1995; Baddeley, 2006).

Articulatory Suppression: is a dramatical collapse in retaining a verbal sequence when subjects are concurrently required to repeat aloud an irrelevant speech sound such as *"the, the, the"* or *"hiya, hiya, hiya"* (Levy, 1971, 1975; Murray, 1965, 1968).

The Visuo-spatial working memory is a second slave system of working memory which parallels the phonological loop for processing visual and spatial information. Evidence from a number of sources now proposes that the visuo-spatial working memory may involve two subcomponents: one for maintaining visual information and the other for spatial information (Pickering *et.al*, 1998. Pickering *et.al*, 2001).

Hue and Ericsson (1988) found visual similarity effects in instant recall of unfamiliar Chinese characters. Frick (1988) argued that images in visual-spatial memory are unparsed and uncategorised. He reported when visual confusion errors occur in retention of letters there appears to be independent degradation of parts of the letter, so the letter 'P' might be recalled consequence of the similarity effect as an 'R'. However, the attention that has been paid to study this slave system is much less than the phonological loop. Baddeley (2006) justified this because of the absence of a rich and standardized set of stimuli such as those provided by the languages, visual information is much more difficult to quantify. He continued (p: 13), *"It seems likely that the visuo-spatial system will play a crucial role in the acquisition of our visual and spatial knowledge of the world: What color is a banana? How does a bicycle work? How do you play a DVD? How can I find my way around my hometown? Whereas we have many tests of language at the levels of phonology, individual word meaning, and text comprehension, we appear to lack well-developed measures of visuo-spatial world knowledge."* However, Baddeley draws attention to the compensating properties for studying the visuo-spatial working memory and the important of vision and visual attention.

3.3 The Function of Working Memory

According to Miller (1956), the function of short-term memory is to hold the information without manipulating. He found human beings can understand and remember no more than seven plus or minus two items of information at a time. Johnstone and Al-Naeme (1991) indicated that working memory is *"where a set of functions are dynamically taking place: selection of input, temporary memorization of sensory input, appeal to long-term memory for complementary input, searching and matching, 'sense making', sending of 'shaped' information to long-memory",* thus the emphasis should be placed more often on the 'working' and less on the 'memory'. Working memory has two important functions, and Johnstone (1997) refers to these functions of the working memory space (WMS);

i. It is the conscious part of the mind that is holding ideas and facts while it thinks about them. It is a shared holding and thinking space where new information coming through the perception filter consciously interacts with itself and with information drawn from the long-term memory store in order to make sense.

ii. It is a limited shared space in which there is trade-off between what has to be held in conscious memory and the processing activities required to handle it, transform it, manipulate it and get it ready for storage in long-term store. If there is too much to hold then there is not enough space for processing; if a lot of processing is required, it cannot hold much.

3.4 Measurement of Working Memory

Educators have been concerned with working memory measurement for more than century, and there are numerous techniques devised to measure the capacity of the working memory space. Baddeley (1997) indicated that Jacobs's attempt to the measure short-term memory in 1887 was the first recorded attempt for educational purposes. Jacobs invented a technique to assess their students' mental capacities, where the subject was presented with a sequence of items, such as numbers, and asked to repeat them back.

The Digit Span Task (DST) is one of the devised techniques which was used to measure working memory capacity for many years. In this task, the examiner reads a sequence of digits in a rate of time one item per second (e.g., "8,3,4") and the subject must immediately repeat them back. The sequential digits are read to subjects in an even monotone, in order to discourage any grouping likelihood of the items on the basis of intention and prosodic information (Pickering, 2006). If the subject succeeds in repeating the sequence without any error, he is given a slightly longer list, and so on. This procedure continues until the subject starts to make errors. One obvious limitation of the DST is that it measures the functioning of the phonological loop (Pickering, 2006), rather than the working memory. Others factors which may limit the individuals' performance on the DST include whether they pay attention when the items are presented, their hearing ability, and their capacity for speaking out (ibid). Nonetheless, the test gives consistent results when compared to the digit backwards test (see below) and the figural intersection test (see El-Banna, 1987)

The Block Recall Subtest (BRS) is a test based in visuo-spatial memory devised by Corsi (1972). Corsi's block task has been used in experimental and neuropsychological work to measure a visuo-spatial working memory. Recently, the task has been conceptualised as the measure of the spatial subcomponent of the visuo-spatial working memory (Logie, 1995). In this test the examiner uses a board on which nine identical cubes (blocks), where a number from (1 to 9) is printed on one side of each block and fixed in a random arrangement (Pickering, 2006). A sequence is tapped out on the blocks by the examiner, and then the subjects are required to repeat the sequence in the same order as they have been shown (ibid).

Visual Patterns Test (VPT) was devised by Della Sala and colleagues (1997) to measure the visuo-spatial working memory of adult neuropsychological patients. It is a measure of a different aspect of visuo-spatial working memory functioning from the Corsi blocks task, as experimental and neuropsychological research indicated (Pickering, 2006). Pickering (2006) explained the VPT as following: *"The task involves the recall of two-dimensional matrix patterns. Each pattern is formed by combination of equal numbers of black and white squares in a matrix. After having seen a pattern for 3 seconds, the participant is asked to recall the location of the black squares by marking onto an empty matrix of the same size. Patterns increase in complexity as the number of black and white squares increase. This allows the user to measure visual pattern span – the number of target (black) squares that can be held in immediate memory."* (P: 256).

The Digits Backwards Test (DBT) is a device that measures the working memory capacity using the auditory sensory memory. In this test, the examiner reads to the subjects a series of digits and asks them to write them in reverse order. For example, 76895 would return as 59867. Every digit is read to the subjects in a rate of one digit per second and the same time is given to recall after the reading of the whole series is over. After the subjects finish the task, they will receive a new task with a greater number of digits and so on. When the subject begins to make mistakes this infers that the working memory cannot hold that long series of digits and his upper limit is taken to be the capacity of his working memory. This test is more complex than the recall of digits in forward order and therefore it measures more than a simple function of the phonological loop. It is also known as The *Digits Span Backwards Test* (DSBT).

The standardised Figure Intersection Test (FIT) is another test to measure the working memory capacity, but it depends on the visual sensory memory in perception processes. This test was modified and used by Pascual-Leone and Smith in 1969. In every task, the subjects are asked to find the overlapping area of a set of simple shapes, which intersect to form a complex design. As the number of figures increases, the task becomes more complex. There are many complex designs and every one has from two to nine simple geometric shapes intersecting, and the subjects must determine the overlapping area between these shapes.

In this test, the separate shapes are located on the right hand side, the subject is required to look at these shapes, and then he must find and shade in the common area on the left hand side where the same shapes have intersected (see figure 3-3). Mathematically, the required area is called the intersection of the individual shapes. The highest number of shapes that a subject is able to find the intersection of these shapes is considered to be the size of his or her working memory space capacity. For example, if the working memory space capacity of a student equals five (X = 5), this means this student is able to find the common area of 2, 3, 4 and 5 overlapping shapes, but he fails to find the correct area for more than five overlapping shapes.

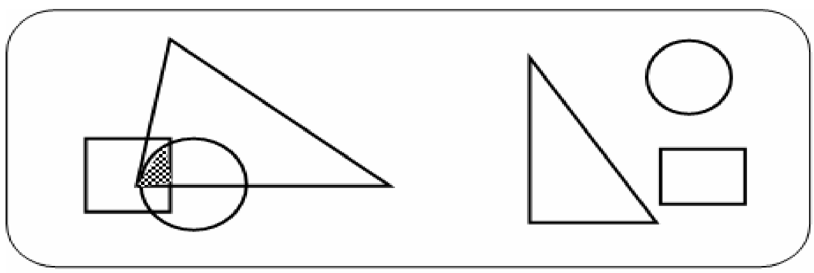


Figure 3-3: Example (1) of the Figure Intersection Test

In this case (figure 3-4) an extra shape has been added, (this irrelevant item may appear in the compound form of figures but not in the discrete form), to see if the subject is able to select only the relevant shapes.

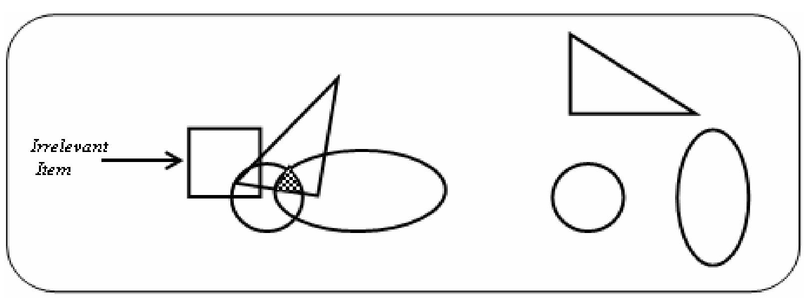


Figure 3-4: Example (2) of Figure Intersection Test

In order to test the consistency of these tests; El-Banna (1987) employed these two psychological tests in his project. He found that 529 out of 754 students (70% of the sample) obtained the same scores in both tests DBT and FIT, and this result reinforces the validity of these tests. It is worth noting that one test is based on number and other based on visual- spatial.

3.5 Working Memory in Mathematics Classes

Working memory is the mental workspace which can be used flexibly to hold cognitive activities that require both processing and storage. Mental arithmetic is a good example of activity that occupies working memory space (Alloway, 2006). Attempting to solve a multiplication task such 25×37 mentally for a beginner student, requires a space to hold these two numbers in it, and another two spaces for applying multiplication rules to calculate the products of sequential pairs of numbers. Then, adding the products to get the correct answer requires another space (experts may use another way to solve this task). Figure 3-5 illustrates the sequential mental processes required to solve an arithmetic task without using a pen and paper.

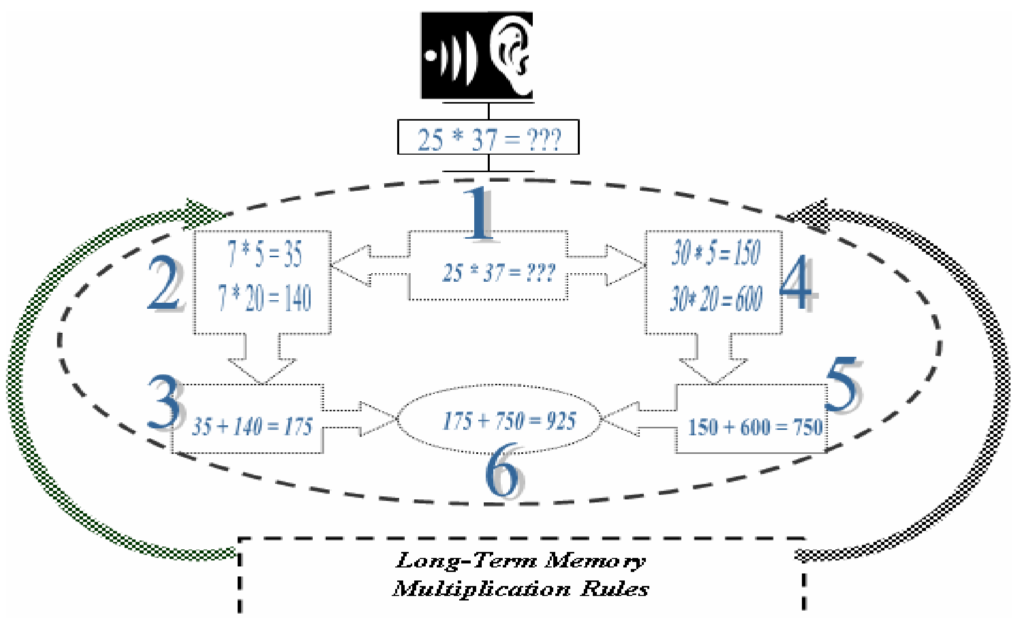


Figure 3-5: Mental processes of solving an arithmetic task

These processes occur in successive ways, and any interruption or distraction such as an unrelated thought springing to the mind, will resulting in losing the stored information completely (Alloway, 2006). He argues that the ability to perform such calculations is limited by the amount of information that has to be stored and processed; and we would not be able to keep in mind some information while processing other materials without working memory. He added multiplying larger numbers (e.g., 142 and 891) 'in our heads' is out of the question for the majority, even though it does not require more mathematical knowledge than the previous example. The interpretation of that is storage demands to solve such activities overloads the working memory.

Alloway's example was used by him to clarify the mental processes which are held in the working memory, so the restriction on using pen and paper or calculator was imposed. However, mathematics as a subject has many skills and much procedural knowledge that students must acquire by practice to support an array of problem solving strategies. Moreover, mathematics concepts are built in a hierarchical structure and lack of mastery of earlier content can lead to failure in understanding the higher-level concepts.

It is clear that working memory may be engaged several times in solving any mathematical problem. Solving mathematics' problems requires the student to hold information in working memory storage and retrieve other information from the long-term memory. Thus, student's ability to hold, handle and update information in the working memory is a crucial feature in mathematics competence for all ages. Furthermore, Bull & Espy (2006) indicated that many studies have shown that shifting ability, or mental flexibility, plays a crucial role in the mathematics competence of older students, where more complicated mathematics tasks such as multi-digit addition and multiplication may require the student to shift between procedures and interim solution, or even shift between multiplying and adding (Bull & Johnston, 1997; Bull *et.al*, 1999; Geary, 1990; Geary & Brown, 1991; Geary *et.al*, 1991; Geary *et.al,* 1992; Geary *et.al*, 2000; Jordan & Montani, 1997; Ostad, 1997). Furthermore, mathematics skills require not only basic storage functions engaging the working memory slave systems (the phonological loop and the visuo-spatial working memory), but also the intentional control functions of the central executive.

3.5.1 Central Executive and Mathematics

The functions of the *central executive* are involved in retrieval from long-term memory, planning, dual-task performance and switching of strategies (Baddeley, 1996b; Duncan, 1986; Baddeley, et. al 1991; Duff & Logie, 2000). In recent years, the function of the central executive and its relation to learning process has been considered carefully. Several studies correlated central executive functioning with learning disabilities (Russell et. al, 1996; Swanson, 1993, 1999), language and comprehension problems (Lorsbach *et .al*, 1996; Hughes, 1998) and mathematics skills (Bull & Scerif, 2001; Cirino, *et. al*, 2002; Gatherecole & Pickering, 2002a; Mclean & Hitch, 1999). The central executive may play an important role in mathematical calculation. This supported by various studies have been required to complete two tasks at the same time. Both tasks are thought to involve the central executive and there is evidence that one task interferes with the other (Hecht, 1999, 2002; Logie *et.al*, 1994). It is also related to subtraction (Geary *et.al,* 1993), multiplication (Seitz & Schumann-Hengsteler, 2000); and division (Lefevre & Morris, 1999).

Dual- task studies propose that the central executive may play an important role in calculation (Holmes and Adams, 2006). Concurrent executive functions have been found to disrupt single-digit addition and multiplication (Seitz & Schumman-Hengsteler, 2000, 2002); suggesting that the Concurrent functions may support the retrieval of number facts from long term memory (Holmes and Adams, 2006). Similar disruptions of Concurrent functions have been reported for multi-digit addition and multiplication problems (Lemaire *et al*, 1996; Seitz & Schumman-Hengsteler, 2000, 2002). Lemaire & Sigler (1995) thought that the central executive is crucial for the achievement of new solution strategies and for shifting between learned solution strategies – "*two key skills that are important for mathematics proficiency*" (Holmes and Adams, 2006).

To understand the functions of the central executive, it is possible to consider an example of mathematical activity that occupies the working memory space, even with the allowance of the use of pen and paper. In geometry, the task presented in figure 3-6 demands high working memory capacity to hold information that is retrieved from long-term memory to find the size of the angle.

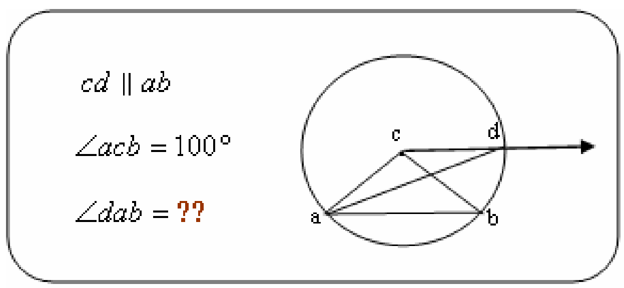


Figure 3-6: Geometry example from the syllabus of grade nine in Kuwait

Student should retrieve all the following geometrical knowledge to get the angle's size:

- All radii of the circle are equal (ac = cb).
- Isosceles triangle principles (angles opposite the congruent sides are equal).
- The three angles of a triangle add up to 180° ($180° - 100° = 80°$, $80° \div 2 = 40°$).
- Principle of Parallel Lines (two alternate angles are equal; $\angle cba = \angle bcd = 40°$).
- Central angle is twice the measure of the inscribed angle ($\angle dab = \frac{1}{2}\angle bcd = 20°$).

All this geometrical knowledge must be retrieved in sequential order to reach the correct answer, and lack of mastery of any earlier content can lead to failure to solve this question. Thus, the central executive works on controlling the process of recalling avoid working memory overload.

3.5.2 Phonological Loop and Mathematics

Many studies have investigated the relationship between the multi-components of working memory and learning mathematics. Researchers have found significant correlations between mental arithmetic in children and the phonologic loop (Adams & Hitch, 1997; Towse & Houston, 2001; Jarvis & Gathercole, 2003). The role of the phonological loop also has been explored in adult counting (Logie & Baddeley, 1987) and arithmetic (Ashcraft *et al.*, 1992; Logie *et al.*, 1994; Lemaire *et al.*, 1996). Holmes and Adams (2006) indicated the *phonological loop* is thought to be important for the attainment of number facts in early childhood. Learned number facts form complete networks in long term memory between mental arithmetic s and the phonological loop. However, the association was no longer significant in an adolescent population (Reuhkala, 2001).

The teaching and learning processes take place through the medium of language. Mathematics is considered to be the language of science, and as Baroody and Standifer (1993) indicate, "*For children, Mathematics is essentially a second or foreign language.*" The translation of ordinary language in mathematics into the symbolic language creates a conflict of exactitude which leads to overload of the working memory. The usage of common vocabularies in mathematics causes another language problem, because their meanings in the mathematics context differ from the normal English usages.

Gardner (1972) examined the accessibility of words to students at a number of levels in secondary school by testing commonly used words in science and the results showed:

- *"Pupils lacked precision in their use of words as they moved from context to context.*
- *Students were easily confused by words (which) 'sound alike' or 'look alike'.*
- *In a significant number of cases, students chose meaning exactly opposite to the accepted meaning.*
- *There was an improvement in performance with age."*

Durkin and Shire (1991) demonstrate that one of the feature in mathematics is that the meanings to convey them are often endowed with other meanings, which may be more familiar to children in every day language and the vocabulary of mathematics includes many words which have multiple meanings and there is evidence that students often fail to interpret the words as teachers intend them. Cassells and Johnstone (1983) have emphasised the great problems, which are caused when normal words are used with precise meanings. Macnab and Cummine (1986) specify words such *root, solution*, *product*, *matrix*, *differentiate*, *integrate, function*, *coordinate*, *prime*, *factor*, *multiply*, *power*, *index*, whose use in the mathematics context cause difficulties of the semantic confusion involved. Durkin and Shire (1991) also listed a table of individual common ambiguous words (multi-meaning), used in mathematics and they discuss the meaning of each word is likely to have for child before he or she encounters it in a mathematical context at school (Table p: 74). In this case, the mathematical meaning of the words and holding information occupy the student's working memory space. Therefore, there is no empty room or space for manipulating this information.

3.5.3 Visuo-Spatial Working Memory and Mathematics

Visuo-spatial ability is believed to play a crucial role in learning mathematics, particularly in geometry. Several studies have indicated the importance of visuo-spatial ability in mathematics performance in children (Reuhkala, 2001; Jarvis & Gathercole, 2003; Maybery & Do, 2003). Evidence from the adult population indicates that the *visuo-spatial working memory* proceeds as a "*mental blackboard*" to encode material, retain, and manipulate it during calculation (Heathcote, 1994; Trbovich & LeFevre, 2003).

Studies with children of specific mathematical difficulties have shown that they typically perform poorly on visuo-spatial span measures (Mclean & Hitch, 1999; White *et.al*, 1992), which suggests that they may have a deficit in visuo-spatial working memory (Holmes & Adams, 2006). Mazzocco *et.al,* (2006) argued that *"There are many possible routes by which visuospatial difficulties may interfere with mathematics performance. There may be deficits in basic skills, such as in the development of a mental number line; or in mathematical procedures, such as in the proper alignment of digits in arithmetical calculations".* It is noticed that there are several phenomena associated with visuo-spatial working memory from my experience as a mathematics teacher as follows:

Shapes Similarity effect: The confusion which occurs from attempting to distinguish between shapes that look alike, such as parallelograms, squares, rhombuses, and rectangles. The similarity of these shapes *A*, *B* and *C* in figure 3-7 may confuse the student's visuo-spatial working memory. The students have to recall the features of the diamond in order to answer a question like:

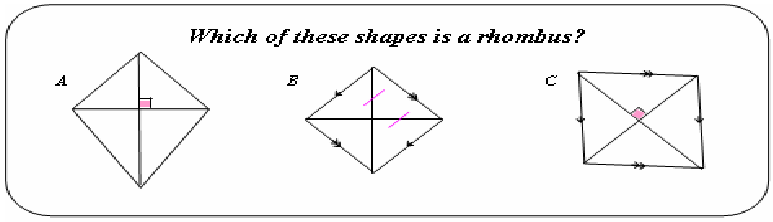


Figure 3-7: Example of shapes similarity effect

Irrelevant Pictures: The confusion occurs because irrelevant items are included in the task. For example, in figure 3-8 if the students were asked to get the intersection between A & B Sets, the set C will confuse their answer. Some of them will write {3, 4, 5} instead of {5}.

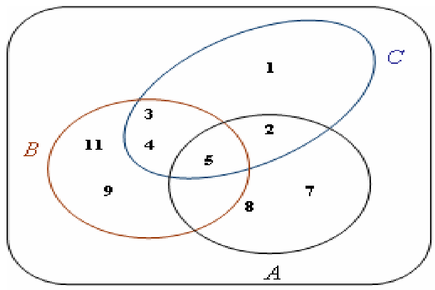


Figure 3-8: Example of Irrelevant item

Complicated shape effect: The confusion arises from very complicated figures containing a lot of information. Such figures as these impose on the visuo-spatial working memory a heavy load which may lead to failure (figure 3-9).

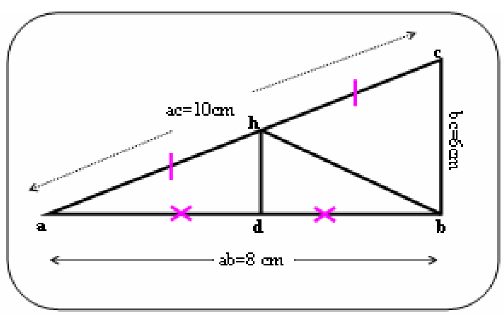


Figure 3-9: Example of Complicated Shape

Inaccurate spatial representation: The confusion arises from the inaccuracy in the drawing of geometrical figures. The inaccurate drawing of lengths in figure 3-10 may confuse the student. As it can be seen, $\overline{ad} = 3cm$ and $\overline{bc} = 7cm$ but, in figure 3-10, these two lengths are too close to each other.

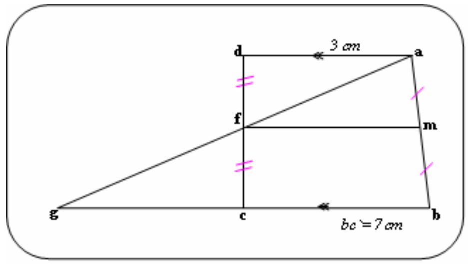


Figure 3-10: Example of inaccurate spatial diagram

3.5.4 Working Memory and Achievement in Mathematics

The main focus of this section is to investigate the correlation between working memory and achievement in mathematics. Mathematics is a complicated field and requires students to use cognitively demanding skills to solve many tasks. Many researchers have shown the close relationship between students' performance in mathematics and their working memory capacity (e.g. Mclean & Hitch, 1999; Bull & Scerif, 2001; Christou, 2001; Alenezi, 2004; Holems & Adams, 2006). Students' ability to hold and manipulate information has been found to be a crucial factor in mathematics performance for all ages.

It is assumed that the procedures used in maths problems are reliant on the working memory system. Thus, the competence of students with poor working memory capacity is affected. There is growing evidence that poor working memory function is a feature of children with learning disabilities in literacy or numeracy or in both areas (Bull & Scerif, 2001; de Jong, 1998; Mayringer & Wimmer. 2000; Siegel & Ryan. 1989; Swanson, 1994; Swanson *et.al,* 1996). Evidence shows the relationship between the task working memory demand and the sudden collapse of achievement. A study of Jonhstone and El-Banna (1989) showed a sudden collapse in performance in chemistry test items when the load of information exceeds the student's working memory capacity. They argued that if the student's working memory capacity is exceeded, his performance should fall unless he has some strategy which enables him to structure the question and to bring it within his capacity. The following chart (figure 3-11) shows a comparison of students' performance in chemistry examination for all groups of different X-space (X-space is the working memory capacity).

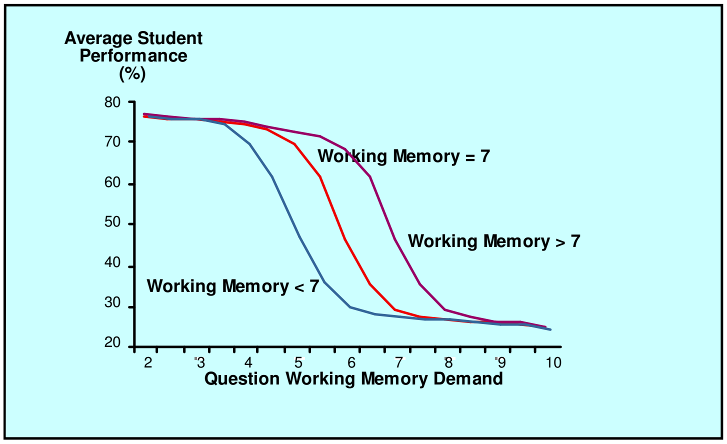


Figure 3-11: Comparison of the average student performance for all groups of different X-space (simplified from Johnstone & El-Banna study, 1989)

It can be seen from figure 3-11, for all groups of different X-space, a rapid collapse occurs in students' performance when the task demands working memory space to handle information more than the student working memory capacity.

Recent evidence shows that working memory is a reliable indicator of mathematical disabilities in the first year of schooling (Gersten *et.al,* 2005). Students with mathematics difficulties:

- Are less likely to use direct memory recovery to solve arithmetic tasks (Geary *et.al,* 1991; Bull & Johnston, 1997).
- Count more slowly and inaccurately than children with normal ability (Geary *et.al,* 1991; Geary *et.al,* 1992; Bull & Johnston, 1997).
- Have weak or incomplete networks of number facts in long term memory (Geary *et.al,* 1991; Hitch & McAuely, 1991)
- Typically perform poorly on measures of phonological loop function (Siegel & Linder 1984; Hitch & McAuely, 1991; Bull & Johnstone, 1997; Passolunghi & Siegel, 2001).

Holmes and Adams (2006)

Another study in mathematics showed a collapse in students' performance in solving algebra problems when the questions demanded more capacity that the working memory capacity of the student (Christou, 2001). Figure 3-12 shows the sudden collapse of students' performance in all groups of different X-space in solving algebra problem story when the questions demand higher working memory capacity.

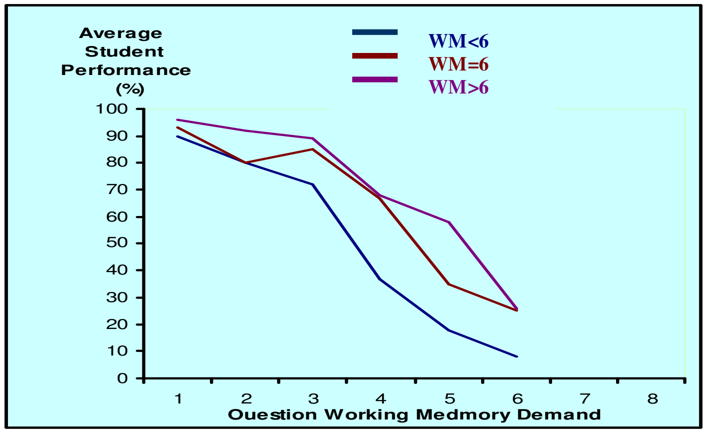


Figure 3-12: Comparison of the average student performance in mathematics for all groups of different X-space (Christou study, 2001)

Christou (2001) found high correlation between achievement in mathematics and working memory capacity ($r = 0.4$, $p < 0.001$). One of the most important results was reported by Alenezi Study (Alenezi, 2004). She found strong correlation between working memory capacity and mathematics' achievement in Kuwait ($r = 0.52$, $p < 0.001$). A study in England presented by Holmes & Adams (2006) examined the contribution of the different components of the working memory model to a range of mathematical skills in children (number and algebra, shapes space and measures, handling data, and mental arithmetic) , provides additional evidence for the involvement of working memory in children's mathematics. They found significant correlation between mathematics ability and the different components of the working memories follows:

- Central executive scores were significantly related to mathematical abilities (all *rs* > 0.30, $p < 0.01$).
- Visuo-spatial working memory scores were also related significantly to mathematics abilities (all *rs* > 0.30, $p < 0.01$)
- Phonological loop scores were only related to mental arithmetic ability ($r = 0.21$, $p < 0.01$).

Gathercole *et.al* (2006) presented two type of explanation for the association between working memory and learning:

Model (1): Apparent working memory limitations are consequences of difficulties in a particular processing field. By virtue of this, students with poor reading skills may gain low scores in translating the real-life problems in mathematics into algebra symbolic manner due to their deficit to meet the reading processing demands and, as a consequence of this, perform the task badly. In this situation, working memory capacity cannot be considered as the fundamental reason of the poor reading skills or the poor levels of academic achievement. They stated that *"The evidence in support [of] this model is not compelling"* (p: 221).

Model (2): The second model suggests that working memory capacity directly impairs the ability to learn complex skills and to acquire knowledge. This explanation has much support. For example, Adams & Hitch (1997) investigated the extent to which children's mental arithmetic is constrained by working memory, rather than their arithmetic performance. Mathematics calculation has been found more precise when the numbers to be added are visible during calculation, which reduces the load on working memory. They concluded that this is consistent with the notion that working memory is used to support storage and processing in the course of mental arithmetic.

3.6 Limitations of Working Memory

There is a general awareness that the capacity of working memory is limited and has to be shared for holding and operating processes. Overloading occurs when the learner tries to hold too many pieces of incoming data. Thus, if we attempt to do too much at once we simply overload. Barber (1988) argued if the information we are concerned with reaches the upper limits of our working space, an overloading in the capacity of working memory could occur, and a loss in productivity may arise. Studies (Johnstone and Wham, 1982; Johnstone and Letton, 1991) show that overloading of working memory appears when the learner is incapable of discriminating between the "noise" (irrelevant information or that which the teacher considers unimportant) and "signal" (relevant information the teacher thinks are important). As we can see in figure 3-13, it is necessary during an experiment to recall: theory, names of apparatus, and recognise material, recall skills, new written instructions, new skills, new verbal instructions.

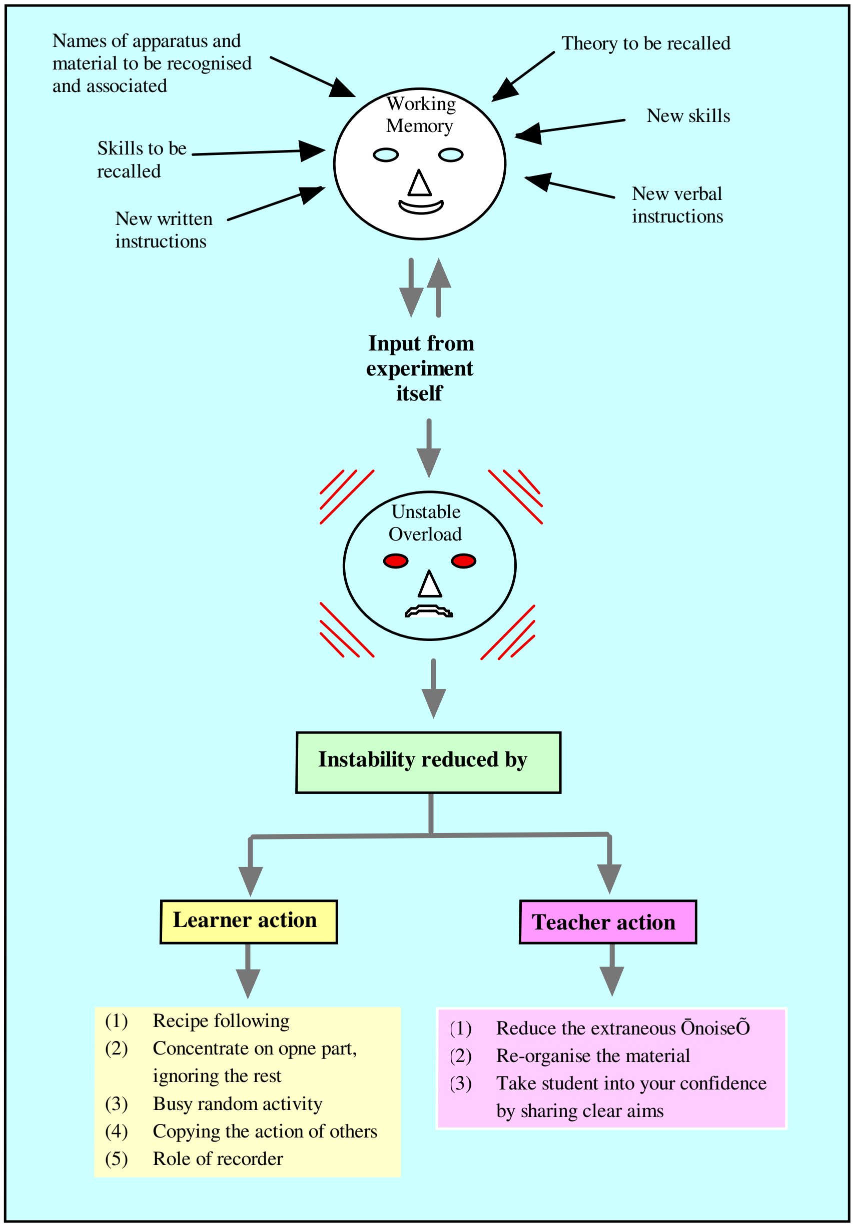


Figure 3-13: The effects of overloading working memory capacity in practical work (Jonhstone & Wham, 1982)

The study of Johnstone (1980) showed that a sudden drop in the learner's performance was apparent when any task load exceeded the upper limit of the learner's working memory capacity. Johnstone (1999) noted that overload happens very often during lectures. Because all students' working space is devoted to writing notes based on the lecturer-spoken words, little space is left for elaborating them and thus understanding them. This is very similar to what Jonhstone and Wham (1982) found with laboratory work (figure 3-13).

In the science field, Johnstone (1984) indicated there are three aspects which can hinder student learning: the nature of the science; the methods of teaching a science; and the methods by which students learn. Johnstone and El-Banna (1989) examined the relationship between working memory and problem solving success in chemistry. They showed that, if the number of the items of information the students are supposed to hold in the mind at the same time, in order to solve the problem exceeds their working memory capacity, their performance will drop dramatically.

In the mathematics field, Ashcraft (1994) declared that three aspects of mental calculation could overload working memory; the retention of temporary information, the length of time of that retention, and the number of operations involved in the calculation. The Christou study (2001), which investigated the difficulties of solving algebra story problems, showed that the phenomenon of overloading of working memory capacity could be responsible for students' difficulties in algebra story problems representation. He found a sharp drop in performance when a task demanded more working memory space than a student possessed.

Student ability to develop techniques to cope with information overload depends heavily on the conceptual framework already established in his long-term memory. It is known that working memory is not expandable but it can be used more efficiently. Miller (1965) put forward the idea of *"chunking"* which is the process of organising information and uses strategies to bring several items together into meaningful units. Gathercole *et.al*, (2006) indicated the next step after detecting a student with poor working memory ability is to determine the learning activities that will place heavy memory demands. Some types of learning activities demand storing a considerable amount of material that may be arbitrary in structure (such as a series of numbers or the precise wording of a fairly lengthy sentence), and other activities involve the storing of material while being engaged in another activity (such as spelling or reading a new word or making an arithmetic calculation).

Overall, working memory capacity is limited, so overloading can easily occur. When working memory is overloaded, the processing of information cannot take place unless this information can be effectively chunked. However, as Johnstone and El-Naeme (1991) stated *"there is an added limitation which leads to inefficient use of this working/ holding/ thinking space. This can occur at the selection stage in which the sensory input is filtered, to separate out or processing, that which is deemed to be relevant, important or interesting."* Overloading of working memory can lead to memory failures. Gathercole *et.al*, (2006) illustrated that memory failures occur when forgetting the instructions, failing to cope with Concurrent processing and storage demands, losing track in complex tasks, and episodic forgetting.

3.7 Overcoming Working Memory limitations

Working memory problems are identified as a cause of learning difficulty. Thus, it is important to minimize the working memory demands in the classroom activities if we are to help students in their learning. There are several effective strategies to reduce working memory demands and achieve success in learning situations. Cognitive load theory recognizes three methods that can help students to accommodate the limitations of working memory (Eggan & Kauchak, 2007):

- *Chunking*
- *Automaticity*
- *Dual processing*

Chunking is the process of grouping into units: it could be a single number, a letter, or many pieces of information. Miller (1956) found that human beings can remember no more than seven plus or minus two items at a time, and the amount of the information in short-term memory could be increased by chunking. The nature of the items plays a major role in the capability to recall. It is much easier, for instance, to recall 7 letters that make a word than to recall 7 unrelated letters. Another example of chunks can be found when we want to recall 14 digits for telephone number (00441413306565). It is very difficult to recall this number at once. However, we can recall this number easily if we remember that 0044 the international access for the United Kingdom, 141 the local access for Glasgow, and 330 the access for Glasgow University. After the chunking process, only four digits 6565 are needed to be recalled.

Automaticity refers to the ability to perform a task with low level of awareness without occupying the mind (Healy *et.al,* 1993; Schneider & Shiffrin, 1977). It is usually the consequence of learning, repetition, and practice. Ordinary activities such as walking, speaking, typing at keyboard, and driving a car are examples of automaticity. Stanovich (1990) stated that automaticity is a fundamental requirement for developing higher-level cognitive skills. It is possible after adequately practicing an activity, to concentrate the memory on other activities while preceding an automaticised activity. For example, people can hold a call or speak while driving a car. This ability can be applied in mathematics learning, where basic operations such as addition and multiplication must be automatic in the learner’s mind, to permit the space of working memory to be occupied for solving a task. In case these basic operations are not mastered automatically, the learner will think about the product of 7×9, for example, instead of solving the problem and not enough working memory space will be left to solve it (Eggan & Kauchak, 2007).

Dual processing attributes to benefiting from the feature of multi-components of working memory suggested by Baddeley (1992). Working memory consists of visual and auditory working memory, and while each is limited in capacity, they can work individually. This feature can be capitalized-on by presenting information in both visual and verbal forms (Mayer, 1997, 1998; Sweller *et.al*, 1998).

Whereas Gathercole *et.al*, (2006) indicated three ways that the teacher should take into consideration for managing and reducing the working memory demands:

- *Ensure that the child can remember the task*: Memorize activity instructions is an important step to achieve success in learning. Thus, the instructions should be as brief as possible for making them easy to remember. Gathercole *et.al*, (2006) advise to break the instructions down into smaller constituents where possible, which will have also the benefit of abbreviating a complicated task. The most successful way to enhance memory for the task instructions is the frequent repetition of them.

- *Use external memory aids*: The utility of external memory aids will facilitate the complex activities, which decree considerable processing as well as storage loads. Children, at the basic level, fall back on their fingers to aid them to get the answers for addition processes. Older children and adult do more complicated tasks and the use of the calculators in maths classes helps to reduce the processing loads on the working memory especially for students with low working memory capacity. For example, the volume of conic formula might look like:

  $$\frac{1}{3} \times 3.1416 \times 4.5^2 \times 12.7 .$$

  This complicated formula requires excessive processing and storage demands for retrieving the decimal multiplying operation if it is solved by paper and pencil.

- *Reduce processing loads*: Complex learning situations may cause a combination of excessive storage and processing demands, which generate a disruption of the student's performance. To avoid this disruption, the processing load of the task should be cut down (Gathercole *et.al*, 2006). Bull & Espy (2006) declared that cognitive limitations do lead to difficulties in learning basic arithmetic and mathematic skills, and to help students in their learning, these cognitive limitations need to be determined.

Furthermore, teachers should *avoid any question that may confuse the learner's mind*. Some teachers desire to introduce a task in the tests to confuse the learner rather than to assess his understanding of any topic. The square root of 16 ($\sqrt{16}$) is a good example of such task that cause confusion for the learner. When the students are asked 'what is the square root of 16, they answer 4. However, in multiple choice questions, the majority choose 8 as a root of 16 even when they understand the meaning of the square root. This incorrect choice is a consequence of the confusion that occurs in learner's mind from the similarity of adding two 8s. Adding is confused with squaring.

3.8 Conclusions

This chapter has highlighted the definition of working memory, its subcomponents and assessment. The working memory function in learning processes has also been considered. The most important finding from this chapter can be summarised as following:

- *Working memory is a system responsible for providing the temporary storage and manipulation required for any mental process.*
- *Working memory comprises three subcomponents: the central executive system, the phonological loop, and the visuo-spatial working memory, and every part has its own function and features.*
- *There are several techniques to assess the capacity of the working memory space, Digit Span Task (DST) measures the phonological loop; Block Recall Subtest (BRS) and Visual Patterns Test (VPT) measure the visuo-spatial working memory; Digit Backwards test (DBT) and Figure Intersection Test (FIT) measure the working memory capacity.*
- *Considerable evidence shows that working memory plays an important role in mathematics proficiency.*
- *Poor working memory is blamed for mathematics disabilities.*
- *All mathematical knowledge is mainly transmitted through the medium of language, which itself create various difficulties in the learner's mind.*
- *Working memory capacity is limited and the information can be held in it for a very limited duration.*

- *Working memory is not expandable but it can be used more efficiently by chunking, automaticity and dual task techniques.*
- *To overcome the limitations of working memory mathematics teacher should:*
- *Ensure that the child can remember the task;*
- *Attempt to reduce processing loads in any task;*
- *Allow the learner to use external aids if the task demands high working memory capacity, and*
- *Avoid questions that may confuse the learner's mind.*

The role of working memory cannot be neglected in learning mathematics; however, much more is needed to counter the abstract, conceptual and hierarchical nature of mathematics than providing storage for holding and manipulating information. Let assume that the learner masters all mathematical knowledge that is required to solve any task, and has high working memory capacity, is that enough to tackle mathematical tasks. To do this successfully, the learner needs not only working memory capacity but also a contractor or organizer to organize the perception and the retrieval processes. The idea of this *organizer,* the ability to select important information from flow information and to control the retrieval information from long term memory is called 'Field Dependency'. Field dependency which may be more important than the working memory space in solving any problem, will be the main focus of the next chapter.

Chapter 4

The Field Dependency Characteristic

4.1 Introduction

The previous chapter discussed working memory function in detail, and it can be concluded from the discussion that working memory plays a crucial role in the learning process. However, in mathematics classes, much more is needed than simply using working memory space to hold the information. The ability to distinguish between important or relevant items from unimportant or irrelevant ones, and the ability to choose between the various techniques that are stored in the long-term memory are required to solve any mathematics task. This ability to select what is important or relevant for a task in hand is called field dependency (Witkin *et.al*, 1977).

Green (1985) describes cognitive styles as consistencies in the ways in which individuals perceive, think, respond to others, and react to their environment. The idea of cognitive styles attempts to illustrate how different individuals tend to show patterns of approaches in learning and undertaking tasks, perhaps caused by personal preference, the way they have been taught or by the way their brain works best. Thus, the existence of such styles offers an explanation as to why some students achieve a high performance in some tasks while others do not (Kirton, 1989).

Field dependency has received the most attention by researchers of all cognitive styles (Daniels, 1996; Chinien & Boutin, 1993; Entwistle, 1981; Witkin & Goodenough, 1981). The exploration of the polar construct of field dependency began in the 1940s with Witkin's work on human perception of the upright position (Witkin *et.al*, 1977; Witkin & Goodenough, 1981; Goodenough, 1976). Witkin (1978) confirmed that field dependency is related to the theory of differentiation. The differentiation theory, on the other hand, refers to the complexity of structure of a psychological system. Students' learning ways differ according to their particular personality, their learning style, their ability and their preferences. Some students learn quickly with little practice, while others take a long time and may need constant repetition and revision to understand ideas and skills. Obviously in the field of mathematics, some students achieve better than others; hence, there are differences in receiving and dealing with the information. Orton (2004, p: 139) indicates that "*some pupils clearly do achieve more in their studies of mathematics than others, hence, there are differences. Abilities, preferences, attitudes and motivation all contribute* *to making some pupils more successful than others*". This chapter seeks to throw some light on the definition of cognitive style and discusses the more familiar convergent and divergent cognitive style before looking at field dependency under the following headings:

- *The construct of the field dependency characteristic*
- *The measurement of field dependency*
- *Field dependency and personality*
- *Field dependency and academic achievement*
- *Field dependency and information processing approaches*
- *The effects of field dependency and working memory capacity on achievement.*
- *Field dependency and mathematics ability*

4.2 Cognitive Style

Psychologists often see cognitive styles in terms of characteristics that the learners show when learning. In fact, the nature of cognitive styles is not clear: are they inbuilt or genetically determined characteristics; are they learned; or are they preferred ways for working. Of course they could be some combination of these.

The term 'cognitive style' is defined by Riding & Rayner as "*an individual's preferred and habitual approach to organizing and representing information*" (1998: p: 8). this implies some element of choice. Cognitive style is a reflection of the essential make-up of a person, and affects the individual's view about events and ideas (Riding, 2002). This implies a genetic basis. Thus, it affects his/her responses to these events and his /her decisions accordingly. It also influences individual attitudes towards other people, and the ways they relate to them (ibid).

The individual's style is an automatic approach dealing with information and situations, and the individual will not be conscious of their style since s/he has probably not experienced another. However, when the individual becomes aware of his/her style, s/he can develop strategies that help in using his/her strengths and avoid the effect of his/her weakness (ibid). Some researchers (for example, Riding & Cheema, 1991; Riding & Rayner, 1998) describe cognitive style as '*a fairly unchanging feature*' and '*tends to be relatively fixed and in-build characteristic of an individual*'. In this case, the cognitive styles are seen, essentially, as genetic features.

There is confusion between the concepts of cognitive style and ability. While Carroll (1993: 554) examined several measures of style and concluded that many of them are in fact ability measures, other researchers (McKenna, 1984; Riding & Pearson, 1994; Riding, 2002) referred to considerable distinction between cognitive styles and ability. McKenna (1984) highlighted four distinguishing features which differentiate between cognitive style and ability as following:

- *Ability is more focused on performance level, while style concerned with performance manner.*
- *Ability is a unipolar measure (less ability vs. more ability), while style is bipolar (visual vs. verbal).*
- *Ability has values associated with it such that one end of an ability dimension is valued and the other is not (performance on all tasks will develop as ability increase), while for a style dimension neither end is considered better overall.*
- *Ability has a narrower extent of application than style.*

McKenna (1984: p: 593-4)

The key insight here is on ‘performance manner’ and ‘performance level’. Thus, cognitive style can be defined as different characteristics relating to the way in which an individual tends to perceive, remember, think, solve problems, organize and represent information in his/her mind (Usama, 2002). After reviewing various investigators’ descriptions of style dimensions, Riding and Cheema (1991) brought many aspects of cognitive style together and gathered them into two basic cognitive style families as following:

- *“Wholist-analytic: affects cognitive style in terms of thinking, thinking about, and viewing and how, in responding to information and situations, an individual tends to process information as a whole or in parts.*
- *Verbal-imagery: affects the characteristic mode in which people represent information, either by thinking verbally or in images.”*

Wholist-analytic Style influences the individual’s ways of thinking, viewing and responding to information and situations. *Wholists* have a tendency to observe a situation as a whole, and are competent to overtake its total context. The ability to see the whole ‘picture’ assists individual to have a balanced view and attitudes. Whereas, *analytics* have a tendency to realise a situation as a set of components, and they often focus on one or two aspects of the situation (Riding, 2002; Riding & Rayner, 1997, 1998, Riding & Cheema, 1991). The positive strength of the analytics is their ability to analyse the parts of the situation which helps them to come quickly to the heart of the problem (Riding, 2002).

Between these two extreme views, *intermediates* will be able to benefit from the strengths of *wholist and analytic* styles (Riding, 2002).

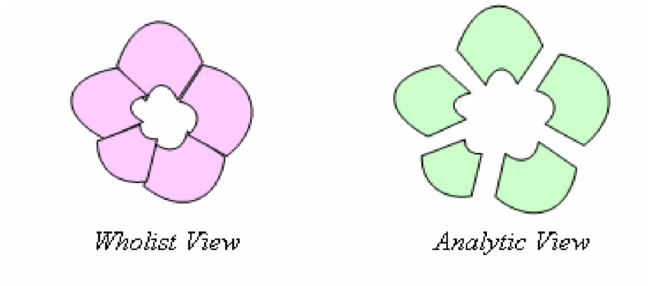


Figure 4-1: Wholist & Analytic views

The second dimension is *Verbal-imagery Style* which refers to individual's tendency to represent information during thinking verbally or in mental pictures (Rayner &, Riding 1997). This style has two essential effects that influence behaviour, teaching and relationships, (Riding, 2002):

- The way information is represented: this dimension categorises people in three types: verbalisers, bimodals or imagers.
- The external/ internal focus: it influences the focus and kind of person's activity – externally and stimulating in the case of verbalisers, and internally and more passive in terms of imagers.

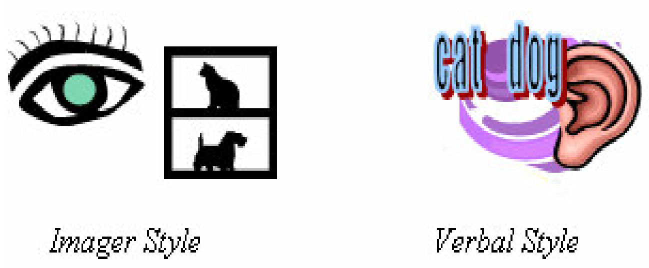


Figure 4-2: Verbal-Imager Style

Riding and Cheema (1991) split these two cognitive style families into dimensions (wholist-analytic and verbal-imagery), and every dimension has two ends (wholist, imagery, analytic and verbal). Riding (2002: p, 24) stated "*These two styles are independent of one another. A person's position on one dimension of cognitive style does not affect their position on the other. However, the way they behave will be the result of the joint influence of both dimensions*". This means one end of these dimensions may integrate with one end on the second dimension thus giving four styles and each of them has its own distinguishing characteristic (see Figure: 4-3).

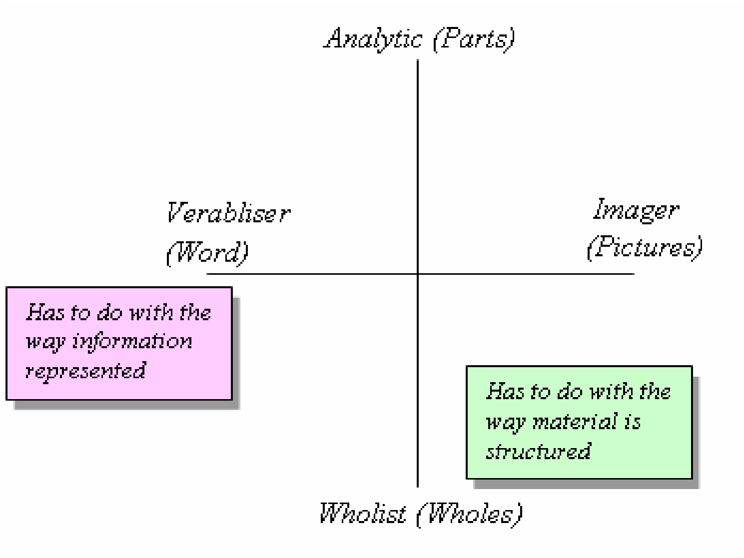


Figure 4-3: The cognitive style dimensions (Riding, 2002)

However, the Riding model (Figure 4-3) makes several major assumptions. While the assumption that the two dimensions are orthogonal is reasonable, the assumption that the two ends of each dimension are, in fact, opposites is more difficult to sustain. It is highly likely that there will be those who can function extremely well as, analysers as well as being able to think wholistically. Equally, it is perfectly possible that there are those who are poor at both skills. Similarly, there is no reason why some learners might not be extremely good at what he calls being a verbaliser while being able, also, to see things as pictures. Just to confuse things further, it is also possible that those who are good at seeing things as pictures might well tend to see things wholistically, simply because a picture is seeing a situation as a whole.

Rayner & Riding (1997) referred to models of style featuring the *wholist-analytic* dimension as following:

The wholist-analytic dimension		
Field dependency	***Individual dependency on a perceptual field when analysing a structure or form which is part of the field.***	**Witkin & Asch (1948a,1948b) Witkin (1964); Witkin *et.al* (1971, 1977).**
Levelling-sharpening	***A tendency to assimilate detail rapidly and lose detail or emphasise detail and changes in new information.***	**Klein (1954); Gardner *et.al* (1954).**
Impulsivity-reflectiveness	***Tendency for quick as against a deliberate response.***	**Kagan *et.al* (1964); Kagan (1966).**
Converging-diverging thinking	***Narrow, focused, logical, deductive thinking rather than broad, open-ended, associational thinking to solve problems.***	**Guiford *et.al* (1964); Hudson (1966, 1968)**
Holist-serialist thinking	***The tendency to work through learning tasks or problem solving incrementally or globally and assimilate detail.***	**Pask & Scott (1972); Pask (1976).**
Concrete Sequential/ concrete random/ abstract sequential/ abstract random	***The learner learns through experience concrete and abstraction either randomly or sequentially.***	**Gregorc (1982)**
Assimilator - explorer	***Individual preferences for seeking familiarity or novelty in the process of problem-solving and creativity.***	**Kaufmann (1989)**
Adaptors-innovators	***Adaptors prefer conventional, established procedures and innovators restructuring or new perspectives in problem solving.***	**Kirton (1976, 1987)**
Reasoning-Intuitive active-contemplative	***Preference for developing understanding through reasoning and or by spontaneity or insight and learning activity which allows active participation or passive reflection.***	**Allinson and Hayes (1996)**
The verbal-imager dimension		
Abstract versus concrete thinker	***Preferred level and capacity of abstraction.***	**Harvey et.al (1961)**
Verbaliser-visualiser	***The extent to which verbal or visual strategies are used to represent knowledge and in thinking.***	**Pavio (1971); Riding and Taylor (1976); Richardson (1977); Riding and Calvey (1981)**
An integration of the wholist-analytic and verbal-imagery dimensions		
Wholist-analytic Verbal-imagery	***Tendency for the individual to process information in parts or as a whole and think in words or pictures.***	**Riding (1991b, 1994, 1996); Riding and Cheema (1991); Riding and Rayner (1995)**

Table 4-1: Descriptions of style dimensions (Rayner & Riding 1997)

Ehrman and Leaver (2003) emphasised that various styles have been suggested several times under a variety of names. This problem has been a major source of confusion over the years. Thus, it is noticed from looking at the family of cognitive styles that every style has its reverse in the different dimension of the same family (for example see Table 4-2).

Analytics or Field independents	*Wholists or Field dependents*
Trend to organise information into clear-cut conceptual grouping	Tend to organise information into loosely clustered wholes.
See information as collection of parts	See information as whole
Focus on one or two of these a time	Able to have an overall perspective and appreciate total context
Possibility of getting the one aspect out of proportion to the total situation	Very difficult to distinguish the issues that make up the whole of a piece of information
The positive strength they can analyse information into the parts this allows them to come quickly to the heart of the problem	The positive strength can have a balanced view, extreme view or attitudes.

Table 4-2: Characteristics of the cognitive style (Hindal, 2007)

Furthermore, even with some agreement on what constitutes a cognitive style and some agreement on what some of the styles actually are, it is not easy to separate the various styles efficiently from each other. One style may have impact on another: for example, a study presented by Worley and Moore (2001) explored the influences of colours on learners of different cognitive style. Evidence from Worley and Moore's study indicates that performance scores are not affected for students classified by cognitive style when the image are offered using colour or black and white. However, it was predicted that use of highlight colour would assist the field-dependent students by attracting their attention to the important information in the image.

There is no doubt that learners tend to adopt certain styles. These may be genetic in origin, or be learned or simply be preferred ways of learning. Indeed, many styles may involve all three: some way of learning is preferred because it is genetically influenced and has been confirmed through previous experience as helpful, enabling the leaner to use this as a preferred approach because it works and is congenial. Much of the past work assumes that such styles have a bipolar nature but this may not always be true. With this background in mind, the next section looks at divergency and convergency to illustrate some of the key issues before addressing the whole area of field dependency which seems so important in mathematics.

4.3 Convergent and Divergent

The term *convergent thinking* refers to the ability to bring material from a variety of sources and to focus down, or converge on, the one correct answer in order to find the solution to a problem. The other term *divergent thinking* refers to the ability creatively to elaborate ideas to invent new ones. Convergent thinkers gain high scores in tasks demanding one conventionally correct solution obviously obtainable from the information available (as in intelligence and mathematics tests), while at the same time gaining low scores in tasks demanding the generation of several equally acceptable solutions. On the other hand, divergent thinking is the opposite approach, and is often regarded as more suited to arts specialists and study in the humanities (Hudson, 1966). Thus, convergent thinking requires close reasoning; divergent thinking requires fluency and flexibility (Child & Smithers, 1973).

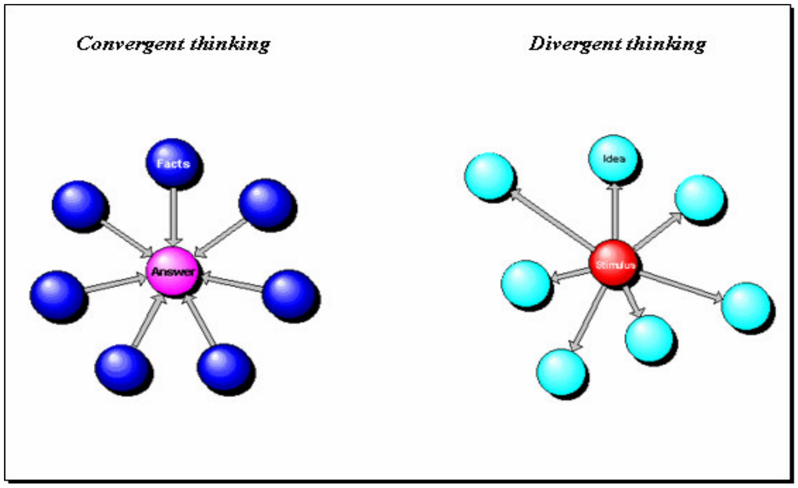


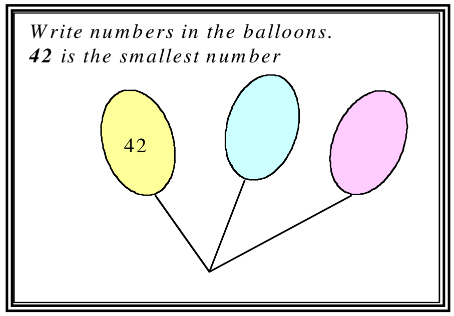
Figure 4-4: Convergent & divergent thinking. (Atherton, 2005)

Bahar (1999) outlined the general characteristics of convergent and divergent thinkers in Table 4-3.

Converger Characteristics	Divergers Characteristics
• Higher performance in intelligence tests • Good at the practical application of ideas • Specialised in physical science and classics • Prefer formal materials and logical arguments • Ability to focus hypothetical-deductive reasoning on specific problems • Better in abstract conceptualisation • Hold conventional attitudes • Like unambiguity • Emotionally inhibited	• Higher performance in open-ended tests • Fine at generating ideas and seeing things from different perspectives • Specialised in the arts • Better in concrete experience • Interested in people • Hold unconventional attitudes • Strong in imaginative ability • More likely to be witty

Table 4-3: General characteristics of convergent and divergent thinkers (Bahar: 1999)

After looking at the descriptions of convergent and divergent thinking, it seems that mathematical thinking as it is presented in mathematics classes is always convergent, and there may be no evidence that divergent thinking skills is required at all in mathematics classes (Orton, 2004). Looking at many test papers, tests questions which are divergent are not much used. The following examples may be the only questions that are divergent questions in mathematics classes:



Any number > 42 are acceptable

A + B = 1

There are infinite possible answers

Create an odd number containing three digits.

In these three examples, there are no fixed 'right' answers.

Currently, there are trends to encourage the use of mathematics investigations which are 'open-ended' tasks as a reaction against the highly convergent nature of the majority of the usual mathematics curricula (Ibid).

4.4 Field Dependency

From numerous types of cognitive styles, field dependency has received the most attention from researchers (e.g. Chinien & Boutin, 1993; Entwistle, 1981; Kent-Davis & Cochran, 1989; Witkin & Goodenough, 1981). Witkin (1948) found that some individuals show remarkable consistency in attending to different types of cues. Some subjects who tended to use the cues of the visual field were designated 'field-dependent', while others who appeared to rely on internal gravitational references (such as tactile, vestibular and kinaesthetic cues) were designated 'field-independent'. Jonassen and Grabowski (1993) stated that "*Field dependency describes the extent to which*

- *The surrounding framework dominates the perception of items within it,*
- *The surrounding organized field influences a person's perception of items within it,*
- *A person perceives part of the field as a discrete form,*
- *The organization of the prevailing field determines the perception of its components, or*
- *A person perceives analytically"*

Jonassen & Grabowski, (1993, p: 87)

According to Witkin *et.a*l (1962), field dependency is regarded as one expression of a more general individual-difference dimension. Frank and Keane (1993) claimed that the construct of field dependency points to a stable and pervasive preference of individuals for analytical or global information processing (*wholist-analytic style*), and these preference differences are reflected in the cognitive restructuring skills displayed by field-independent (FI) and field-dependent (FD) individuals. Witkin (1978) distilled the essences of field-dependent (FD) and field-independent (FI) as follows:

- *"The field-dependent (FD) and field-independent (FI) cognitive styles are process variables, so they represent techniques for moving toward a goal, rather than ability in achieving goals.*
- *Cognitive styles are pervasive dimensions of individual functioning. They express themselves across domains traditionally considered in isolation from each other. This pervasiveness need not be surprising in the case of the field-dependent and field-independent cognitive styles, since the tendencies to rely primarily on internal or external references, as a function of the extent of self-nonself segregation, represent rather deep cuts of the psyche."*

Goodenough (1976) defined field independence as,

> *"The ability to overcome embedding contexts in perceptual functioning, and it is considered to be the analytical aspect of an articulated mode of field approach as expressed in perception. Thus, the analytical cognitive style allows their experiences to be analysed and developed. In contrast, field dependence refers to people who take the organization of the field in perceptual and problem solving tasks as given, and they have difficulty in separating an item from its context."*

He emphasises that field-dependent people have a relatively global cognitive style, which governs their experience by the organisation of the field. The main features of the field-dependent and field-independent cognitive styles are defined by Witkin and Goodenough (1981) as:

- *"Field-Dependent (FD) individual who can insufficiently separate an item from its context and who readily accepts the dominating field or context.*
- *Field-Independent (FID) individual who can easily 'break up' an organised perceptual field and separate readily an item from its context."*

There are several factors affecting the field dependence-independence tendencies such childhood background and age stage. Witkin thought that field dependence-independence tendencies are a consequence of child-rearing practices that emphasize acquiring independence from parental controls (Korchin, 1986). Witkin showed in his early studies of child-rearing that, when there is strong emphasis on conformity to the authority of the parent and external control of impulses, the child will likely become relatively field dependent. If the family encourages the child to develop separate, autonomous functioning, the child will become relatively field independent. Generally, children are field dependent, but their field independence increases as they become adults and with the support of their family. Adults are more field independent, especially adult learners (Gurley, 1984). Field independence gradually declines throughout the rest of life; elderly people tend to be more field dependent (Witkin *et.al,* 1971).

4.5 The Measurement of Field Dependency

The *body adjustment test* (BAT) was originally used by Witkin (1948), and then (BAT) was replaced with the *rod and frame test* (RFT) to uncover field type. RFT estimates the individual perception of the location in relation to the upright of an item within a limited visual field (Witkin *et.al,* 1974). After experimenting with rod and frame tests, Witkin developed the *embedded figures Test* (EFT) in order to classify individual as field-dependent or field-independent cognitive style. This test is designed to measure disembedding skills where the subjects are required to separate a simple figure from a larger complex figure, and the figures were adopted from Gottschaldt (1926) figures which developed for his study (Witkin, *et.al*, 1974).

The *group embedded figures test* (GEFT) is a group version of the EFT and it was used by Witkin *et.al* (1977) to measure the field-dependence/ field-independence of an individual. It is paper-and-pencil test that requires a minimum level of language skills for performing; where the learners are asked to recognize a simple geometrical shape within a complex and confusing background. The simple shape has to be found in the same size, same properties, and the same orientation within the complex figure. The field-independent subject is the individual who is able to separate these shapes from the complex pattern. The more correct answers he identifies, the better at the separation process he is, and vice versa for field-dependent. The GEFT and EFT instruments are not absolute measures of field dependency but they have been used to categorize individuals into their abilities.

All these tests are reliable and valid (Witkin, *et.al*, 1974; Witkin, 1976). Reasonably high validity was reported in Witkin (1976) by correlating between the GEFT and EFT (0.63 for female undergraduates and 0.82 for male undergraduates). A reliability of 0.92 for EFT has been obtained after one week interval by test-retest correlation by Dana and Goocher

(1959). The test-retest correlations have been obtained for all these tests (see Witkin *et.al*, 1974). Figure 4-5 illustrates an example similar to those used in the EFT.

Figure 4-5: Sample of simple and complex figures similar to those used in the EFT (Witkin *et.al*, 1977)

Messick (1993) argues against sole reliance on either EFT or GEFT on the grounds that analytical ability will always be confused with test scores, and the cognitive style itself will remain unmeasured. Tinajero and Paramo (1997) confirm the argument of Messick (1993) and add to the debate, after using RFT and EFT to measure field articulation, that the RFT measures perception of the upright while embedded figures performance is a measure of cognitive analytical ability. In spite of the Messick's (1993) argument and Tinajero and Paramo's (1997) evidence, the utilization of the GEFT and EFT will likely remain the most common measure of field dependency because of the procedural requirements for administering the contrasting instrument (Hall, 2000).

4.6 Field Dependency and Personality

Investigations of Witkin and his colleagues (1974) ascertained that the way that a person accommodates himself in space is an expression of a more general favoured mode of perceiving which is linked to personal characteristics. Therefore, they considered the relationships between field dependency and some behavioural features of individuals.

Sense of separate identity: is the result of an individual's development of awareness of his own needs, feelings and characteristics as distinct from those of others. Witkin *et.al* (1974) considered three categories of behaviour aspects from which the extent of the development of a sense of separate identity may be deduced. The three aspects are:

1. Seeking for guidance,
2. Their own attitudes and values,
3. And their views about their selves.

Evidence from the Witkin *et.al,'s* study (1974) shows that subjects with a relatively analytical field approach (field independent) are able to function with the minimum guidance and support from others; they define their role; they are usually less influenced by authority; tending to be guided by values standards; they proceed with greater confidence and tend to show less tension and anxiety. Contrasting with subjects with a relatively global field approach (field dependent) who look for guidance from the examiner in many situations; less able to define their role; they lack confidence in their ability to perform the task; and as a consequence of that react with tension and anxiety (ibid).

Oltman (1980) argued that, when information seeking is not an issue or when the information that is available is unambiguous, no differences exist between field dependent and field independent people. In a study of differences in reaction with other people (Oltman *et.al*, 1975), students were paired in the laboratory according to their field dependency cognitive style: both subjects field dependent, both subjects field independent, and one field dependent and the other is field independent. The pairs discussed issues about which they initially disagree, and the researchers asked them to resolve their disagreement. The end results showed that: the most agreements were reached when the both members were field dependent; an intermediate number when one subject was field dependent and the other is field independent; and the least agreements when both members were field independent. The existence of field independent members together reduces the occurrence of conflict resolutions (ibid).

*Nature of control and Defences***:** is the relation's nature between field approach and defensive structures (the ability to control impulsive behaviour). Evidence concerning the nature of controls and defences confirm the view that individuals with a global field approach had less ability for the management of impulsive behaviour than individuals with analytical field approach (Witkin *et.al*, 1974). Individuals with analytical field approach (children or adult) tended to develop defensive structure and use relatively specialised complex defences. Field independent individuals prefer isolation and intellectualization, rather than primitive denial and massive regression. Witkin *et.al,* (1974) found that children with analytical field approach were shown to have better ability in modulating and mediating the ideas and feelings of aggression.

4.7 Field Dependency and Academic Achievement

The field dependency cognitive style has been considered to be the most critical variable that may affect achievement in various subject domains (Dubois & Cohen, 1970; Tinajero & Paramo, 1997; Vaidya & Chansky, 1980; Christou, 2001; Alenezi, 2004). Dubois and Cohen (1970) found significant correlations between the overall mark of university admission examination and scores in field dependency test. Dubois and Cohen's (1970) findings support Cohen's (1969) hypothesis that the greater restructuring ability of field-independent students contributed to achievement in the school environment, particularly in those tasks requiring analytical skills and the use of processing strategies based on the organisation and restructuring of information. Several studies have followed Dubois and Cohen's (1970) by considering the correlation between field dependency and academic achievement in several subjects such mathematics, language, natural science, social science, art and music.

Tinajero and Paramo's study (1997) investigated the association between the field dependency, and achievement in several disciplines such as English, mathematics, natural social science, Spanish and Galician. They concluded that field-independent subjects are superior to field-dependent subjects, whether assessment is of specific subjects or across the board.

Two studies (McLeod *et.al*, 1978; McLeod & Adams, 1979) considered the interaction between field independence with discovery learning in mathematics classes. They found field independent students learned the most in mathematics lessons with the minimum guidance and maximum opportunity for discovery, while field dependent students received the maximum guidance. In another study, Vaidya and Chansky (1980) investigated the relationship between achievement in mathematics and field dependency across grades. In all grades, field dependency was highly correlated with mathematics achievement: especially for concepts and applications, those who were field independent were best.

Studies of Christou (2001) and Alenezi (2004) investigated the relationship between field dependency and achievement in mathematics. Christou (2001) found that field independent students perform better than field dependent students in algebra story problems (see table 4-4).

	Number	***Mean Score in mathematics test***
Field dependent	**32**	**5.5**
Field intermediate	**33**	**6.6**
Field independent	**25**	**8.2**

Table 4-4: Field dependency related to performance in the mathematics test (Christou, 2001).

Alenezi (2004) found similar result with a very highly significant correlation and the differences between field dependent and field independent performance in mathematics is 30% of the means of mathematics marks (see table 4-5).

	Number	Mean Score in mathematics test
Field dependent	***67***	***56***
Field intermediate	***71***	***68***
Field independent	***45***	***86***

Table 4-5: Field dependency related to mathematics performance (Alenezi, 2004)

Indeed, there are no studies which have shown that field-dependent students are better in any cognitive task (Danili, 2004). This raises questions about the supposed neutrality of cognitive styles when applied to field dependency. Those who are field independent always seem to have the advantage in academic tasks although it has to be recognised that this may simply be a reflection of the tasks which are set in academic testing.

4.8 Field Dependency and Information Processing Approaches

The description of cognitive style as "*information processing habits*" by Messick (1970: p: 190), proposes that the features of field dependency are correlated with the three general stages of information processing. According to the information processing approach, there are considerable individual differences in learning arising from differences in a number of factors as follows:

- *"The component processes.*
- *The strategies into which these processes combine.*
- *The mental representations on which the processes and strategies act.*
- *The ways in which individuals allocate their attentional resources."*

Sutherland (1992)

Many researchers considered the differences in certain information processing stages such as attention, organization, and retrieval between field dependent and field independent individuals (Berger & Golberger, 1979; Goodenough, 1976, Davis & Frank, 1979; Annis, 1979; Pierce, 1980; Frank, 1984; Fehrenbach, 1994; Daniels, 1996). They believed these differences may influence the ways in which students perform in the classroom. The general tendencies of field dependent and independent individuals are summarized by Daniels (1996: p: 38) as follows:

"Field dependents:

- *Rely on the surrounding perceptual field.*
- *Have difficulty attending to, extracting, and using non salient cues.*
- *Have difficulty providing structure to ambiguous information.*
- *Have difficulty restructuring new information and forging links with prior knowledge.*

- *Have difficulty retrieving information from long-term memory.*

Conversely, field-independents:

- *Perceive objects as separate from the field.*
- *Can disembed relevant items from non-relevant items within the field.*
- *Provide structure when it is not inherent in the presented information.*
- *Recognizing information to provide a context for prior knowledge.*
- *Tend to be more efficient at retrieving items from memory."*

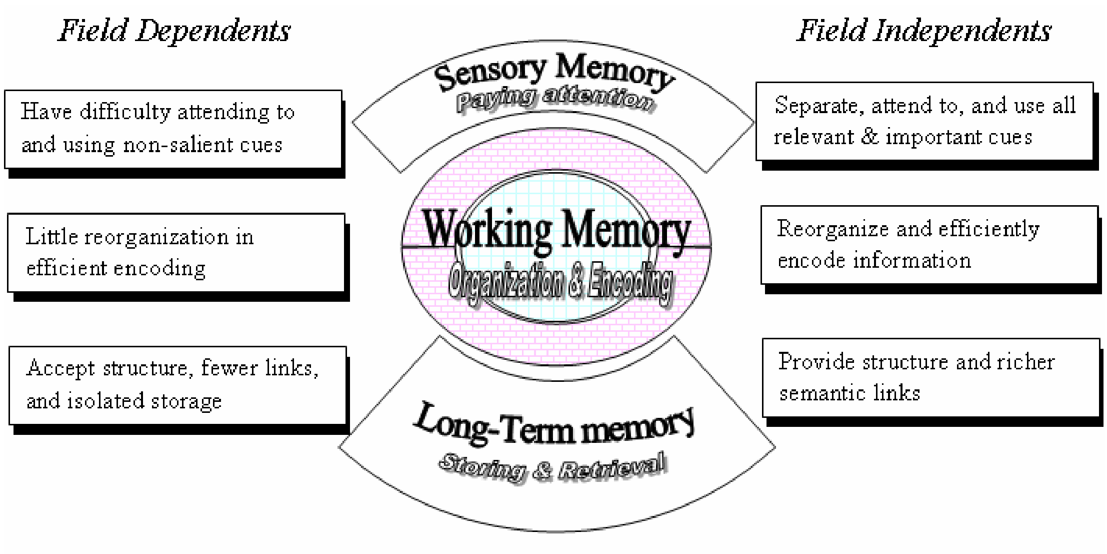


Figure 4-6: Information processing characteristics (the idea derived from Daniels, 1996)

Goodenough (1976) argued the idea that field dependency may be related to individual differences in learning and memory and this has been popular because it represents the confluence of two important streams of thought in the history of cognitive psychology. One stream takes place within the area of learning theory, where there has been a turn toward greater emphasis on the active role of the person in the processes of acquisition, storage, and retrieval of information (e.g. Neisser, 1967). The second stream occurs within the area of perception, where the theory of cognitive styles emerged during the 1950s, defining the boundaries between traditional areas of psychological study (Goodenough, 1976).

According to Davis and Frank (1979), field-independent students perform more efficiently than field-dependent students do and both sets of students employ different encoding strategies, or when they employ the same strategy, the effectiveness of this will vary. They indicated that the poor performance of field-dependent students is due to their less efficient memory as well as having difficulty in remembering the task, which has been set. They disagreed with the Goodenough finding (1976) which differentiates between field-dependent and field-independent people in the process they employ and, instead of that, they argued that field-independence people are more efficient than field-dependent people are.

Annis (1979) examined the effect of cognitive style on study technique effectiveness by having field-independent and field-dependent students read only or take notes on a logically organized or scrambled reading passage. She found that field-independent learners were better than field-dependent at recalling information of high structural importance, irrespective of whether the passage was organized or not.

Frank (1984) investigated the effect of field-dependence/independence and study technique on learning from a lecture. The results of his research presents evidence that field-independent and field-dependent individuals differ in the cognitive processes they use as well as in the effectiveness of their performances in certain situations. Frank found that the significant interaction between cognitive style and study techniques indicated that this difference was largely attributable to the condition of student notes. Because of more efficient note taking, field-independent students out-perform the field-dependent students. He suggested that field dependent students could be helped to improve their performance through a combination of training in note taking and the provision of the external organisational aids, such as lecture outlines. Fehrenbach (1994) confirm this by recording students' comments from different age groups (8th, 10th and 12th grades) about comprehension strategies used while reading a text. He found efficient use of summarising strategies by field independent students.

Pierce (1980) investigated the effect of imagery strategy in memorizing; the sample was collected from different age groups (5 to 6 and 8 to 10 years of age) with different cognitive style (GEFT was used). There were two conditions in the test of memorizing a story; the first condition is listening and the second condition is listening with a demand for generating images. She found field independent children achieved higher scores than field dependent children in the second memorizing conditions.

4.9 Field Dependency, Working Memory and Achievement

Baddeley and Hitch (1974) described working memory as a multipurpose central-processing system with limited processing capacity. Working memory capacity can be used to process various cognitive operations such as organizing and restructuring, or it can be used to recall from long term memory. Many researchers have investigated the effect of working memory capacity and field-dependence/field independence on the learning process (Pascual-Leone, 1970; El-Banna, 1987; Al-Naeme, 1988; Christou, 2001; Alenezi, 2004). Pascual-Leone (1970) assumed that the field dependency could restrict the learner from employing his full mental space in solving tasks. Therefore, field-dependent students may operate in a way that is below their actual X-space (X- means the working memory capacity that the student has).

The El-Banna study (1987) investigated the influence of working space and the field dependency in chemistry performance. He found among students with the same working memory capacity, the achievement in chemistry increases when the student is more field independent. Several studies (Al-Naeme, 1988; Danili, 2001) show little difference in performance in a chemistry examination between low working memory capacity field-independent students and high working memory capacity field-dependent students

In the mathematics domain, studies by Christou (2001) and Alenezi (2004) investigated the influence of working space, the field dependency learning style and mathematics achievement. The analysis of the data indicated that there is significant relationship between the two psychological factors measured in the research and the achievement in the mathematics test. They found that field-dependent students (FD) with high working memory capacity had the same mean score with field-intermediate students (FINT) with medium working memory capacity and almost the same with field-independent students (FI) with low working memory capacity (see table 4-6 and 4-7).

Group	*Field Dependent*	*Field intermediate*	*Field Independent*
Low (N=51)	*5.0*	*6.1*	*7.8*
Medium (N=20)	*5.9*	*7.3*	*8.3*
High (N=19)	*7.3*	*7.3*	*8.4*

Table 4-6: The field dependency and working memory classification versus the mean scores in mathematics test (Christou, 2001) N=90

Group	Field Dependent	Field intermediate	Field Independent
Low (N=92)	*49*	*64*	*67*
Medium (N=57)	*64*	*68*	*89*
High (N=34)	*77*	*87*	*95*

Table 4-7: The field dependency and working memory classification versus the mean scores in mathematics performance (Alenezi, 2004) N=183

Many years before, Johnstone (1993) offered a picture of what might be happening. Learners with a high working memory capacity who are field-dependent are occupied with 'noise' as well as with 'signal' because of the field dependence characteristic. Students of low capacity who are field-independent, on the other hand, will receive only signal and ignore the noise and they can use all their limited low working memory space for useful processing. Therefore, students of high capacity field-dependent cannot benefit from their larger working memory because it is reduced by the existence of "noise" (irrelevant information).

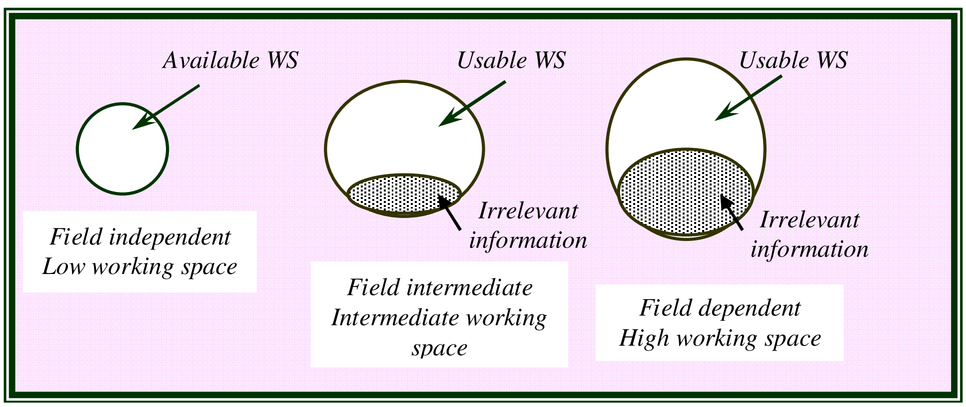


Figure 4-7: Usable working space capacity

4.10 Field Dependency and Mathematical Ability

Mathematics as a subject involves many skills as well as procedural knowledge that students must master first. They then must be able to select from the range of skills and procedures which they hold in their long-term memory to find the appropriate technique which can support any attempt to solve any mathematical task. As mentioned in the previous chapter, selecting or retrieving processes is heavily controlled by the perception processes. Thus, student's ability to recognize the important and useful items ('get the message from the noise') plays a critical role in solving any mathematical problem. Consider the following examples of mathematical tasks:

$4\frac{3}{5}+7\frac{2}{3}=$	$10-3\frac{2}{7}=$
$3\frac{3}{5}\times 5\frac{5}{6}=$	$3\frac{3}{7}\div 2\frac{2}{3}=$

These four mathematical tasks look like the basic operations; however, they require more mathematical knowledge than the basic operation demand. They are more complicated than they appear and every task has its own technique. If the students are not able to recognise the differences between them and distinguish between these techniques, they will not be able to solve these problems. To be able to solve these tasks, the student has to be a good problem-solver which requires:

- *"Ability to estimate and analyze,*
- *Ability to visualize and interpret quantitative facts and relationships,*
- *Ability to understand mathematical terms and concepts,*
- *Ability to note likeness, differences and analogies,*
- *Ability to select correct procedures and data,*
- *Ability to note irrelevant detail,*
- *Ability to generalize on the basis of a few examples,*
- *Ability to switch methods readily,*
- *Higher scores for self-esteem and lower scores for text anxiety."*

Suydam & Weaver (1977)

Or to have mathematical ability which comprises of:

- *"An ability to extract the formal structure from the content of a mathematical problem and to operate with that formal structure,*
- *An ability to generalize from mathematical results,*
- *An ability to operate with symbols, including numbers,*
- *An ability for spatial concepts, required in certain branches of mathematics,*
- *A logical reasoning ability,*
- *An ability to shorten the process of reasoning,*
- *An ability to be flexible in switching from one approach to another, including both the avoidance of fixations and the ability to reverse trains of thought,*
- *An ability to achieve clarity, simplicity, economy and rationality in mathematical argument and proof,*
- *A good memory for mathematical knowledge and ideas.*

Krutetskii (1976)

It is relevant to compare these two analytic views of mathematical ability and the good problem-solver with field independent characteristics that are described by Daniels (1996).

Independent individuals can recognize relevant items from non-relevant items within the field; provide structure when it is not inherent in the presented information; recognizing information to provide a context for prior knowledge; and tend to be more efficient at recalling items from memory.

After extensive studies into spatial ability, Smith (1964) concluded that spatial ability is a key component of mathematics ability. Sherman (1967) argued that the relationship between spatial ability and field independence is very strong by saying "*key measures of … [the field-independence] construct do not appear differentiable from the spatial factors… [and] the term analytical consequently implies unwarranted generality, especially since the construct appears unrelated to the verbal area*" (pp: 297-298). It can be argued after this comparison that field independent individuals have mathematical ability or they are good problem-solvers.

After consideration of the affects of field dependency in the learning process, two questions arise: can we teach field dependent individuals in some way to push them to be more field independent? Or is it better to consider the instructions of individual's cognitive style in preparing the teaching material? Based on the extensive research conducted on field dependency, Bertini (1986) concluded that field dependent learners are more likely to perform extremely well at learning functions such as:

- *Group-oriented and collaborative work situations where individuals need to be sensitive to social cues from others*
- *Situations where participants must follow a standardized pattern of performance*
- *Tests requiring learners to recall information in the form or structure that it was presented*
- *Knowledge domains that focus on social issues*

On the other hand, field dependents should be able to use the following learning strategies effectively:

- *Concentration on information*
- *Repetition or rehearsal of information to be recalled*

Conclusions

After the above discussion, it can be concluded that the,-

- *Field dependency influences individual's personality and affects his perception, interaction with the learning environment.*
- *Field dependency has impact on students' performance and achievement.*
- *Field dependency influences information processing (paying attention, encoding and retrieval).*
- *Field independent students with high working memory capacity achieve much better than field dependence with low working memory capacity in mathematics and in other subjects.*

Chapter 5

Attitudes towards Mathematics

5.1 Introduction

Mathematics attainment in secondary school could be attributed to a complex and dynamic interaction between cognitive and attitudinal factors (Volet, 1997). The previous chapters attempted to look at two cognitive factors that may affect mathematics achievement, working memory capacity and field dependency. The following chapter will deal with the attitudinal factors that affect mathematics achievement in junior secondary school (age 12-15 in Kuwait Education System). These attitudinal factors have emerged recently as salient variables affecting success and persistence in mathematics (Singh *et.al*, 2002). Unfortunately, many students are seeing mathematics as an abstract, complicated and difficult subject (Sharples, 1969; APU, 1980, 1981, 1982; Carpenter *et.al*, 1981; Dossey *et.al*, 1988). These negative attitudes towards mathematics may inhibit the learning process. Thus, a wider perspective and serious investigation of variables affecting achievement in mathematics in middle grades is needed because in these years students are thinking and negotiating tracks for their future.

Before looking at students' attitudes towards mathematics, we need to look at 'attitude' in general. Thus, this chapter provides an overview of what attitudes are, why they are important, how attitudes can be measured. The area of attitude development in science education is also discussed in that there has been considerable research effort in this field. Also, the literature surrounding attitudes towards mathematics are approached within this chapter.

5.2 The definition of Attitudes

What are attitudes? It is not easy to answer this question as it appears. The various definitions of the term 'attitude' and its interpenetration in terms of other psychological concepts create difficulties in determining a precise definition. Allport (1935) gave a variety of definitions of attitude that combine many ideas. They are listed below:

> *"Attitudes are individual mental processes, which determine both the actual and potential responses of each person in a social world. Since an attitude is always directed toward some object it may be defined as "a state of mind of the individual toward a value" (p: 6).*

> *"Attitude is a mental and neural state of readiness organized through experience exerting directive or dynamic influence upon the individual's response to all objects and situations with which it is related" (p: 8).*
>
> *"Attitude is a "degree of affect" for or against an object or a value" (p: 10)*

Halloran (1967) considered several features of Allport's descriptions: first, an attitude is a state of predisposition leading the individual to conceive things and people around him in certain ways. The second aspect is that attitudes are not inbred – they are learned; they develop and they are organized through experience. A third feature of Allport's definition follows from this, attitudes are dynamic, they are not merely latent states of preparedness awaiting the presentation of an appropriate object for their activation. Campbell (1950) drew attention to the weakness of Allport's definitions despite its obvious usefulness. He attempted to present a behavioural definition "*An individual's social attitude is a syndrome of response consistency with regard to social objects"* (P: 31).

Later, Katz (1960) defined an attitude as "the predisposition of an individual to evaluate some symbol or object or aspect of his world in a favourable or unfavourable manner." Krech (1960) introduced one of the clearest accounts of the nature and components of attitudes. He defined attitude as *"an enduring system of positive or negative evaluation, emotional feeling and pro or con action tendencies, with respect to a social object"* (P: 177). He specified three essential components of attitudes as follows:

- *The cognitive component: this has to do with beliefs about an object, including evaluative beliefs that are good or bad, appropriate or inappropriate. The cognitive components consist of thoughts or ideas about the attitude object. These thoughts are often conceptualized as beliefs, linkages that people establish between the attitude object and various attributes. They include the covert responses that occur when these associations are inferred or perceived as well as the overt responses of verbally stating one's beliefs. The features that are correlated with the attitude object express positive or negative evaluations and therefore can be located by psychologists on an evaluative continuum at any position from extremely positive to extremely negative, including the neutral point (Chaiken & Eagly, 1993).*
- *The affective or feeling component: this has to do with likes and dislikes. The affective component comprises of feeling, moods, emotions, and sympathetic nervous system activity that people experience in relation to attitude objects. (Ibid).*
- *Action or behavioural tendency. The behavioural component includes the overt actions that people exhibit in relation to the attitude object. These responses also*

> *range from extremely positive to extremely negative, so they can be located on an evaluative dimension of meaning (ibid).*

Many psychologists (e.g. Bagozzi and Burnkrant, 1979 and McGuire, 1985) have observed that attitudes comprise three components (cognitive, affective and behavioural), and these three components are not necessarily separable from each other and do not necessarily represent three independent factors. Halloran (1970: p: 22) described the place of attitude within such a theoretical framework in this way: *"in any given situation an individual may be shown to select some of available stimuli and neglect others. He processes or interprets the selected stimuli in certain ways, and reacts to the interpreted stimuli affectively and by behaviour tendencies which will emerge as behaviour under appropriate environmental conditions."* Attitudes are learned and they can develop as we develop with new input of a cognitive, affective or behavioural tendency (in interaction and relationships with other people). It is important to realize that attitudes will develop in learners whether it is the overt purpose of the teacher or not, and it is useless to ignore their importance.

5.3 The Important of Attitudes

Attitudes are considered to be the most important element to success in any endeavour. Bohner and Wanke (2002) put forward two main attitude functions. They state these attitude functions can be seen as the essence of different theoretical approaches: serving knowledge organisation and guiding approach and avoidance, and serving higher psychological needs. They also illustrate that the importance of attitudes becomes apparent at various levels of analysis that are all subjects of social psychological and social research:

- *At the individual level, attitudes influence perception, thinking, other attitudes and behaviour. Accordingly, attitudes contribute heavily to a person's psychological make-up.*
- *At the interpersonal level, information about attitudes is routinely requested and communicated. If we know others' attitudes, the world becomes a more predictable place.*
- *At the social level, attitude toward one's own groups and other groups are at the core of intergroup cooperation and conflict.*

Generations of psychologists have examined the question *"do attitudes play a major role in determining behaviour?"* (Fishbein & Ajzan, 1976; Fazio, 1990, Chaiken & Eagly, 1993), and they referred to this question as the relation between person attitudes (his knowledge and feeling toward some person, object, or event) and what he actually does (his reactions). For example, a student's attitude toward mathematics demand knowledge of what mathematics is, and what the student's feelings toward mathematics are, and this information may help to predict whether the student will choose to study mathematics in the future or not. Fazio (1990) stated, *"There can be no doubt that attitudes do sometimes relate to subsequent behaviour and that the field has achieved some understanding of just when that sometimes is"*. According to Fishbein and Ajzan's (1976) approach, it is possible to predict behaviour if the person's intentions to perform a particular behaviour is known. Chaiken & Eagly, (1993) noted that *"response to an inquiry about an attitude toward a specific behaviour directed toward a given target in a given context at a given time should predict the specific behaviour quite well because this attitude exactly corresponds to the specific behaviour"*.

Individual behaviour, perception, thinking and reaction toward any topic or person are controlled by his attitudes, which may affect his evaluations and decisions. Therefore, if a learner faces a difficulty in any subject, this may lead the person to block the process of learning. For instance, the student may have studied mathematics. During this process, the student acquires some knowledge of mathematics and, at the same time, may gain (negative or positive) attitudes toward mathematics. If negative attitudes towards mathematics are developed, these may lead the student to seek to avoid any further study in mathematics.

Overall, attitudes are important and central in all aspect of education because they enable students to make sense of an evaluation in terms of knowledge, feeling and behaviour.

5.4 Theory of Planned Behaviour

The *Theory of Planned Behaviour* was suggested by Ajzen (1985) as an extension of the *Theory of Reasoned Action*, which was proposed by Fishbein and Ajzen (1976), to account for behaviours that are completely under the subject's control. According to the *Theory of Reasoned Action,* a person's intention to behave can be predicted by knowing two things:

4. The person's *attitude towards the behaviour*;
5. The person's *subjective norm*.

The *attitude towards behaviour* refers to the individual's positive or negative feelings about engaging in the behaviour, and these feelings are a result of the information that an individual has about the attitude object and about engaging in the behaviour regarding this object. The second predictor is the *subjective norm*, which is the person's perception of the social pressures and norms to perform or not perform the behaviour. The theory of *Planned Behaviour* adds a third component which is the so-called *perceived behavioural control. Perceived behavioural control* refers to a person's belief as to how easy or difficult performance of the behaviour is likely to be and represents the extent to which the individual believes that behavioural performance is complicated by internal factors such as skills, abilities, and knowledge; and external factors such as time, lack of resources, opportunity, cooperation and behaviour of other people. Figure 5-1 summarizes the way the *Theory of Planned Behaviour* works.

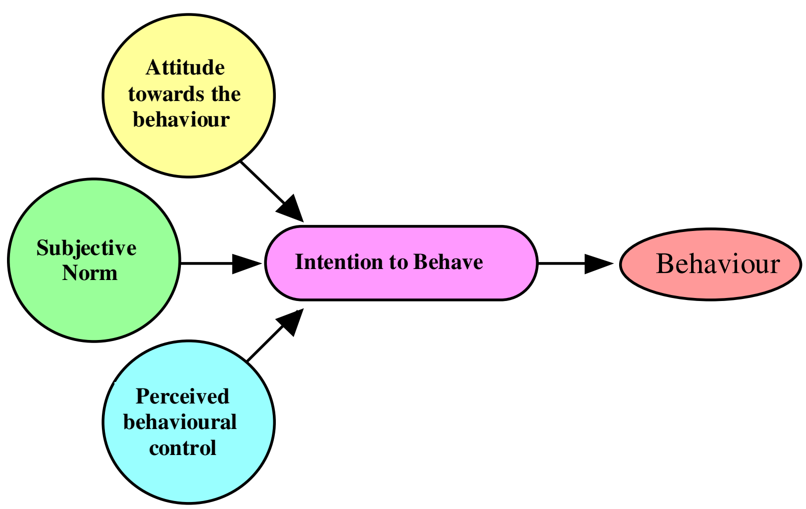


Figure 5-1: The Theory of Planned Behaviour

Attitudes often drive behaviour, and if we can develop attitudes then we are in position to influence other people's behaviour. Hence, measurement of attitude (intention to perform the behaviour) is the best single predictor of a person's behaviour. How attitudes are measured is the focus of the next section.

5.5 The Measurement of Attitudes

Ckaiken & Eagly (1993) stated *"The aim of measurement is to assign numbers to objects so that the properties of the numbers that are assigned reflect the relations of the objects to each other on the attribute being measured"* (P: 23). The importance of attitudes in the education process reflects the need for attitude measurement. However, an attitude is not something that can be examined and measured in the same way as the cells of a person can be examined with a microscope or the rate of heartbeat can be measured by a machine or a watch. Such measurements involve direct observation. Attitudes can only be measured indirectly and the only way is by observation of words and actions (Henerson *et.al*, 1987). It is important to recognize that attitudes cannot be measured in any absolute sense. In addition, it is not possible to measure the attitude of an individual with any degree of certainty. All that can be done is to compare the pattern of attitudes of one group with another (Reid, 2006).

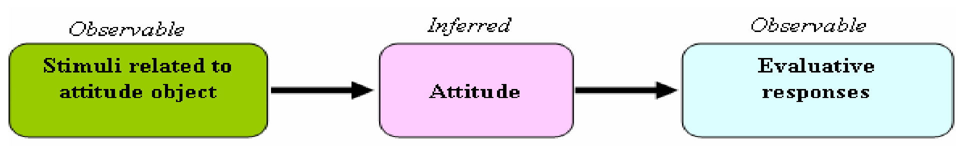


Figure 5-2: Attitude as a latent construct (after Chaiken & Eagly, 1993)

Observation of words or behaviour needs considered further. There is no certainty at all that behaviour (and writing and speaking can be seen as behaviour) will reflect the attitude a person holds with a degree of exactness. Indeed, the Theory of Planned Behaviour (Ajzen & Fishbein, 1980) would suggest otherwise although it has to be noted that attitudes are the best predictor of the intention to behave (which is perhaps closest to written attitude assessments). Thus, responses to interviews or questionnaires can best be regarded as indicators of inner attitudes but there is never any certainty that they reflect the held attitudes exactly. Nonetheless, as behaviour is often the outcome of attitudes held and the behaviour is what is so often important, such measures have immense value. For example, responses to a questionnaire may reflect attitudes to aspects of the learning of mathematics and such expressed attitudes may, indeed, be powerful indicators of the way students will choose to study more mathematics or the way they will approach such future studies.

Overall, this leads to an important principle: attitudes cannot be measured in any absolute sense nor can they be measured with any accuracy for individual students. All that can be done is to measure the attitudes of large groups and then use the data to compare between groups or to compare the attitude of a group at different times and under different circumstances.

5.6 General Way of Attitude Investigations

Attitudes cannot be observed directly. Thus, we have to find other ways of assessing them and we must rely on inference when we want to measure. Attitudes can be measured by asking questions about feeling, thought and likely behaviour toward the attitude object, or by techniques. Some of the more famous techniques for attitude measurement include:

- *Questionnaires*
- *Observation of apparent behaviour*
- *Physiological tests*
- *Partially formed stimuli (like projective tests).*
- *Performance of tasks*

Cook & Selltiz (1964)

Questionnaires and interviews are the most widely used approaches in an educational context. Questionnaires provide a large amount of information in a short time, while interviews provide rich and revealing insights although these are based on a small selected number of interviews. According to Oppenheim (1992, P: 100-102), a questionnaire is

> *"… an important instrument of research, a tool for data collection… it can be considered as a set of questions arranged in a certain order and constructed according to special rules. The questionnaire has a job to do: its function is measurement."*

There are two kinds of questions that may be included in any questionnaires; the *open-ended kind* and the *closed kind.* In the open-ended questions, the respondent enjoys full discretion in writing down what s/he thinks where, in the closed kind, the designer writes the anticipated answers. The closed kind may be harder for designer to form but they may be simpler to analyse.

5.7 Methods for Designing the Questions for Questionnaires

There are various methods that provide insights on how students' attitudes toward learning can be monitored. Five of them are approached in detail within this chapter; Thurstone method; The Likert method; The Semantic Differential; Rating methods; and interviews. Although the models that relate internal beliefs to outcomes are important because they provide the theoretical basis for studies, these theories cannot take the place of carefully constructed instruments for measuring the components of internal belief systems. Any device designed to measure attitude should be reliable and a valid indicator for this attitude. Reliably and validity definitions and techniques are discussed in the research methodology (chapter 6).

5.7.1 Thurstone Method

Thurstone's study was published in 1928, entitled *"Attitudes can be measured"*, and his scaling was constructed in the following steps:

- *Specification of the attitude variable to be measured.*
- *Collection of a wide variety of opinions relating to the specified attitude variable.*
- *Editing this material for a list of a bout 100 statements of opinion.*
- *Sorting the statements into an imaginary scale representing the attitude variable. This should be done by about 300 readers (judges).*
- *Calculation of the scale value of each statement.*
- *Elimination of some statements by the criterion of ambiguity.*
- *Elimination of some statements by the criterion irrelevance.*
- *Selection of a shorter list of about 20 statements evenly graduated along the scale.*

Thurstone (1928)

Psychologists argued that Thurstone's method is laborious, time-consuming and the statements were independent of the attitudes distribution of the readers who sort the statements (Likert 1932). Generally, this method is awkward and is rarely used in current research. Nonetheless, Thurstone opened the door of attitude measurement study and this stated to break down the views of the behaviourist psychologists who had long argued against the possibility of measuring latent constructs like attitudes.

5.7.2 The Likert Method

The Likert technique (1932) is one of the most popular measuring tools. Likert's method is used to measure attitudes, beliefs, preferences, and behaviours or affective reactions (e.g. Fishbein & Ajzen, 1976; Kothandapani, 1971; Ostrom, 1969). It comprises a series of statements, and it measures the extent to which a person agrees or disagrees with each statement. Participants are asked to indicate whether they strongly agree, agree, undecided, disagree or strongly disagree and, often, each point of the five-point scale is given a numerical value from one to five. Hence, a total numerical value can be calculated from all the responses. However, Reid (2006) argues that it is completely wrong to add up numbers which are ordinal in nature. A total can be obtained but this total may be meaningless. He illustrates his point by considering the responses of two fictitious students responding to ten questions in a Likert questionnaire (see Table 5-1 overleaf).

Questions	Responses				
	Strongly agree	Agree	Neutral	Disagree	Strongly disagree
	5	4	3	2	1
Q 1	X	Y			
Q 2		X	Y		
Q 3			X	Y	
Q 4				X	Y
Q 5	Y				X
Q 6		Y		X	
Q 7			X	Y	
Q 8		X	Y		
Q 9	X				Y
Q10	Y				X
Student 1 X		Student 2 Y			

Table 5-1: Imaginary example for two students' responses (Reid, 2006)

An ‘X’ shows the responses of the first student while ‘Y’ shows those for the second. It is clear that both students have an overall score of 30 but their attitudes are *totally* different (Ramsden, 1998, and Reid, 2006). This reveals another fundamental flaw in this summated rating approach. It hides important detail and, by reducing an attitude to a number, obscures the differences which actually exist.

For many years, the Likert method has been used to measure attitudes without using the Likert summated rating method. Thus, it is possible to use the Likert methods without adding up the scores on items. Each item can be analysed separately (Reid, 2006). This offers a more complex analysis but the details can be seen and these may be critical.

5.7.3 The Semantic Differential

The semantic differential was devised from the work of Osgood in the 1950s as a method for measuring the meanings of the words (Osgood *et.al*, 1957; Osgood, 1969a). Osgood wanted to create a technique that precisely mapped identification and localisation of the meanings of words by responding to several pairs of bipolar adjectives which are scored on a continuum running from +X to –X and when participants respond to a set of pairs, they are differentiating the meaning of that concept (Osgood *et.al*, 1957). In an extensive use of factor analysis of the meanings of the words, he and his colleagues found that the semantic space can be determined precisely by three factors labelled ‘Evaluation’, ‘Potency’ and

'Activity'. These three factors are loosely thought of as 'good-bad', 'powerful-powerless' and 'fast-slow' dimensions. The evaluation dimension was identified by Osgood and his colleagues (1957) as synonymous with attitudes. They said *"Our work in semantic measurement appears to suggest such an identification: if attitude is, indeed, some portion of the internal meditational activity, it is, by inference from our theoretical model, part of the semantic structure of an individual, and may be correspondingly indexed"*. Consequently, the semantic differential is used to measure the direction and intensity of an individual's attitudes (Osgood *et.al*, 1969).

Although the semantic differential was not developed for attitude measurement, it is a most popular technique of measuring attitude in current research. This technique comprises of bipolar adjective pairs (like good-bad), a series of unlabelled boxes (from 3 to 7) is deposited between the pair adjective, as shown in figure 5-3.

What are your opinions about your ***laboratory experiences in chemistry*** ?
Tick ONE box on each line.

Useful	☐	☐	☐	☐	☐	☐	Useless
Not helpful	☐	☐	☐	☐	☐	☐	Helpful
Understandable	☐	☐	☐	☐	☐	☐	Not understandable
Satisfying	☐	☐	☐	☐	☐	☐	Not satisfying
Boring	☐	☐	☐	☐	☐	☐	Interesting
Well organised	☐	☐	☐	☐	☐	☐	Not well organised
The best part of chemistry	☐	☐	☐	☐	☐	☐	The worst part of chemistry
Not enjoyable	☐	☐	☐	☐	☐	☐	Enjoyable

Figure 5-3: Several semantic different bipolar scales

The semantic differential technique has been found to be reliable (Osgood *et.al*, 1969, Hadden, 1981). Also, Brinton (1961) stated that the semantic differential validity *"appears to be high, based on its high correlation with scores obtained by traditional Thurstone, Likert and Guttman type of scales*". After using the semantic differential technique with school students, Brown and Brown (1972) listed six advantages in its use and one great drawback compared to Likert methods: it is has less applicability (Reid, 1978). Heise (1969) argued that "*Osgood's method is eminently suitable in terms of type of sample, administration, easy design, high reliability and validity when compared to other methods."* For these advantages of Osgood's technique, it is adopted in this research, along with other approaches.

This technique has the same problems as the Likert methods when used as a scaling technique. Nevertheless, it can be used in such a way that each bipolar line is treated separately like the Likert method (Reid, 2006).

5.7.4 Rating

Attitude rating questionnaires are developed by gathering statements, objects, situations or views, and then the participants are asked to rate these statements by using some kind of number system (sometimes they are asked to place statements in order, to compare statements, or to divide statements up in some way). This method can be very sensitive in determining attitude development. The weaknesses of this method is that it gives no final "score", it is difficult to collect evidence to draw final conclusion, and it is limited in the range of applications possible that are open to rating. Figure 5-4 illustrates an example of a rating question, derived from Reid and Serumola (2006). The aim of this question (which was set in Botswana) was to explore the school student attitudes towards ways of gaining evidence, the example being that of global warming.

(8) Tebogo has been studying global warming and wonders how scientists know what is actually the truth about global warming. Her friends suggest several ways to find the answers. These are listed in the shaded box.

A	Read Scientific books
B	Talk to experts like University professors
C	Carry out experiments to test the idea of global warming
D	Collect as much information as possible about global warming
E	Assume global warming is true and act accordingly
F	Use intelligent guesswork
G	Look at information which has already been gathered through research
H	Accept what majority of people believe is true about global warming

Arrange these suggested answers in order of their importance by placing the letters A, B, C...etc. in the boxes below. The letter which comes first is the <u>most</u> important and the letter which comes last is the least important for you.

Most important Least important

Figure 5-4: Example of rating (Reid & Serumola, 2006)

They analysed the question by comparing the response patterns of different groups of learners.

5.7.5 Interviews

An interview is a face-to-face meeting between two or more people in which the respondent answers questions posed by the interviewer (Henerson, *et.al,* 1987). There are several possible approaches: open ended interviews, highly structured interviews, fixed question interviews, interviews for validating questionnaires.

The advantages of interview are numerous. They can be used to obtain information from people who cannot read or for non-native speakers. They are very rich in the data obtained and interviews can clarify the questions and ensure that the subject understands them to avoid the possibility of ambiguities of language. The disadvantages of interviews is that they are very time consuming, often difficult to plan, give no final "score", and often difficult when trying to draw clear-cut final conclusions.

5.8 Attitude Development in Science Education

Attitude development refers to the formation or change of attitude. The literature tends to use the phrase 'change of attitude'. However, although not intended, this can carry the idea that people are being manipulated in some way and this is completely unacceptable in education. When the phrase 'attitude development' is used, this does not carry these unfortunate overtones. In addition, this phrase allows for changes which can be seen as positive or negative (Johnstone & Reid, 1981). Therefore, it is preferred to use the term 'development' in this thesis instead of 'change' to avoid any confusion. The learning environment should allow students to develop attitudes on a sound cognitive basis. Nonetheless, in spite of fact that psychological models use the term 'change' (with its possible overtones of manipulation), these models are still helpful to understand attitude development in science.

Although attitudes tend to be stable with time, change and development is possible in appropriate conditions. Attitudes can be developed by a number of sources including other people, family, media, classroom, worship places (mosque or church), or the object itself Attitude development assists people to understand themselves, the world around them and the relationships (Reid, 2003), and learners' attitudes will develop within the learning process whether it is the overt aim of the teacher or not. Attitudes development involves more than just the affective. The input may be a cognitive, effective or behavioural or any combination of the three (Reid, 1978).

In learning and teaching processes, students receive knowledge, information and acquire skills. At the same time they gain attitudes toward the subject, teachers or towards some topics. A crucial component of the educational processes is the attitudes that students bring into classroom regarding a specific subject area. Reid (1978) argued that a student's attitude towards science may well be more important than his understanding of science since his attitudes determine how he will use his knowledge. It is not the mission of teachers to make as many practising scientists as possible. Human society needs various professionals such as politicians, businessmen, artists etc, as well as scientists. Thus, the role of a science teacher is to make a contribution in developing educated students. Such students will have a balanced view of themselves as well as being able to relate their studies to culture, lifestyle and matters of social importance. Overall, the contribution of science education at the school level is to prepare students to take their place as citizens, to be informed in terms of the knowledge of science and its impact on modern society and to have developed attitudes based on sound knowledge and experience towards the sciences, their contributions to society and their potential for future impact.

The awareness of attitude 'target' is one of the essential features of attitudes: attitudes are evaluations of something or someone. Reid (2006: p: 7) identified broad targets of attitudes in science education:

- *"The science subject itself as a discipline;*
- *The learning of the science subject (and perhaps learning more generally);*
- *Topics and themes covered in a particular course (e.g. themes of social awareness).*
- *The methods of science (the so-called scientific attitude)."*

Figure 5-5 illustrates the four broad areas where attitude development is important in science education field.

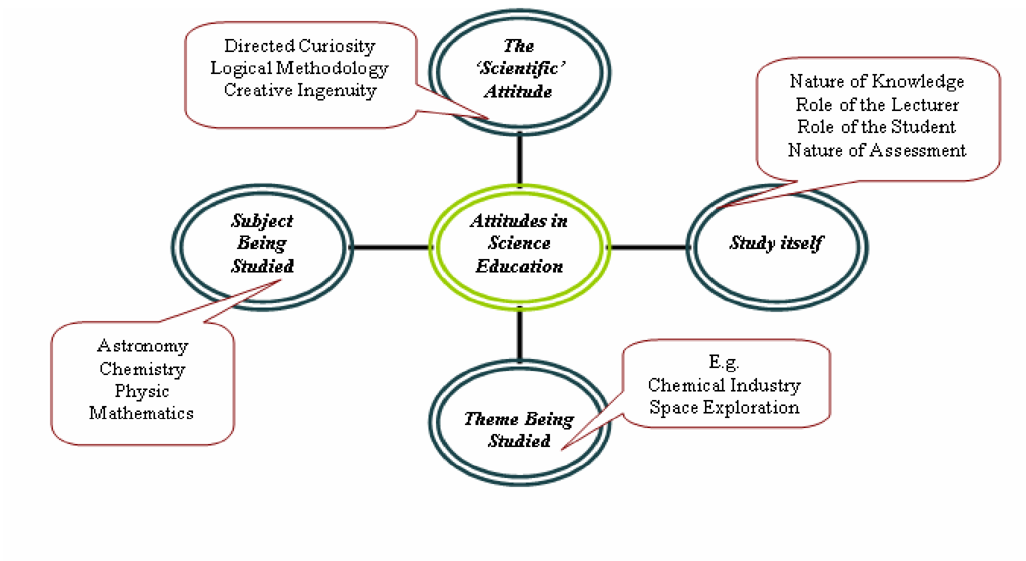


Figure 5-5: Four broad areas of attitudes in science education (Reid, 2006)

Singh *et.al* (2002) indicated that researchers have recommended that student's motivation to learn mathematics and science can be increased and improved when teachers create a curriculum that focuses on conceptualizing and creating meaning and relevance. Therefore, learners' attitudes must be taken into consideration while thinking of teaching and learning. Having looked briefly at some of the attitudinal work related to the sciences, attitudes towards mathematics are now considered.

5.9 Attitudes towards Mathematics

Orton and Wain (1994: 17) said,

> *"Many mathematics teachers believe that mathematics can provide pleasure and enjoyment. Thus, an aim in teaching mathematics could be that the learner comes to enjoy*

> *mathematics, gaining pleasure from the scope and intricacies of the subject, from its patterns, and from what it can reveal about learners and their worlds".*

However, most of the concepts and procedures of mathematics to many people are obscure because rules and algorithms dominate them. Haze and mystery are commonly regarded to be features of mathematics concepts. Russell (1921) defined pure mathematics, as "*The subject in which we never know what we are talking about, nor whether what we are saying is true."* Skemp (1989) suggests that mathematics is much more abstract than any of the other subjects which children are taught at the same age, and this leads to special difficulties of communication. Mathematics differs from other subjects where students can learn a wide range of topics at different levels of complexity and understanding. MacNab and Cummine (1986) argue: "*If this hierarchy of content is allowed to dominate the teaching sequence, not only will substantial learning difficulties be likely to arise, but also boredom and apathy*"

These negative attitudes towards mathematics may inhibit the learning process, and it seems clear that the problems of mathematical education cannot be solved from within mathematics itself. A wider perspective is needed to help students persist in mathematics. Thus, educational researchers have focused on the measurement of students' attitudes towards mathematics. Their attitudes merit concern because they affect achievement and participation in mathematics and other subjects in general. Moreover, Costello (1991) notes that attitudes may form the roots of personal qualities which persist into adult life and may be considered either beneficial or undesirable. He argues that positive attitudes can be considered as valid objectives of mathematics education in their own right, and affective learning outcomes – such as enjoyment, enthusiasm, fascination, appreciation – may be taken into account alongside the more cognitive aspects of learning mathematics which are measured in terms of achievement.

There are several possible approaches to determine students' attitudes and the choice depends rather on what it is intended to measure. The Fennema and Sherman (1976) Mathematics Attitude Scales (FS-MAS) are some of the most frequently used for measuring affective variables in mathematics. There are nine different scales in the Fennema-Sherman Mathematics Attitude Scales and they include the following mathematics attitudes: (a) mathematics confidence; (b) extrinsic mathematics motivation, which is described as the interest to achieve mathematics awards and recognition; (c) mathematics as a male domain – described as 'mathematics is a gender neutral subject'; (d) mathematics usefulness; and (e) intrinsic motivation to study mathematics – described as 'personal enjoyment' and 'pleasure in the study of mathematics'. Each of the scales contains 12 Likert-type items with five possible responses ranging from strongly disagree to strongly agree. Six of the items are positive statements, and six are negative. When the scales are administered, items from several of the scales are randomly mixed to form a single instrument. Scores on each scale can range from 12 to 60, with higher scores being indicative of positive attitudes. For example, high scores on the confidence and usefulness scales indicate more confidence and a greater appreciation for the usefulness of mathematics. Nonetheless, this approach uses the flowed method of scaling (see page 88-89).

The approach adopted in the Mathematics Attitude Scales relies on correlation and factor analysis defining each scale. This means that ordinal numbers are being used in an interval sense and the outcomes from studies using such scales will be likely to obscure important detail. However, it is helpful to use some of areas which have been used to describe the subscales as a means to group together research findings. Five areas will now be discussed: general perception and attitudes about mathematics; the usefulness of mathematics, confidence in learning mathematics, attitudes towards different topics within mathematics; and the attitudes of mathematics teachers to their students.

5.9.1 The Importance of Mathematics as Discipline

Student's perceptions of the usefulness of mathematics, both immediately and in their future, is a variable that has been shown to be strongly associated with mathematics participation and achievement. Mayer and Koehler (1990) stated that usefulness may affect participation on a short-term basis by increasing persistence when the material becomes harder. In the United States, Callahan (1971) discovered the general belief that mathematics is useful and 66% of students felt that mathematics is as important as (or more important than) any other subject. Hammouri (2004) studied self-perception of maths importance and found it significantly correlated with maths achievement ($r = 0.24$, $p < 0.05$). Students see mathematics as an important subject for the following reasons:

- Mathematics is useful in daily life
- Mathematics is important for some other subjects
- Mathematics can help to solve world problems
- Mathematics helps them to get careers
- Mathematics is important for many courses at university
- Mathematics is thought to teach logical thinking

Although a large proportion of students believe that mathematics is a useful subject, mathematics usually occupies a low position in term of liking, when it is compared with others subjects. This dislike is might be attributable to the anxiety**,** fear of failure and negative attitudes associated with mathematics

5.9.2 Students' Attitudes towards Learning Mathematics

The majority of school students, including some of the most able students in mathematics, do not like mathematics. Sharples (1969) has compared students' attitudes to various subjects in four junior schools by considering their reactions to the following statements of attitudes towards mathematics, reading, writing, stories, art and physical education in turn:

- *"I hate it;*
- *It is the worst thing we do in school;*
- *I can't stand it;*
- *It is all right sometimes;*
- *I think it is good;*
- *It is most enjoyable;*
- *It is good fun and I like it very much;*
- *I love it".*

Students indicated agreement with each statement by a tick.

Buxton (1981), in his book, 'Do you panic about Maths?' refers to the following beliefs about the nature of mathematics as typical of a general view of the subject.

"Mathematics is:

- *Fixed, immutable, external, intractable, unrealistic;*
- *Abstract and unrelated to reality;*
- *A mystique accessible to few;*
- *A collection of rules and facts to be remembered;*
- *An affront to common sense in some of the things it asserts;*
- *An area in which judgments, not only on one's intellect but on one's personal worth, will be made;*
- *Concerned largely with computation."*

Orton (1992) argues that mathematics does not involve the learner in indicating emotions or opinions. Thus, it is not surprising that anxiety and fear arise with such views of mathematics – perhaps emotions are being suppressed? The causes for mathematics anxiety are not that easily discernable as Martinez *et al* (1996: p: 6) observed: *"Mathematics anxiety is complex. It rarely follows a straightforward, single-cause/single-effect, linear progression. It has multiple causes and multiple effects"*. They state that identifying that someone is mathematics anxious only defines the symptom not the causes of it. Furner and Duffy (2002) state there are various components surrounding and influencing a student's mathematics anxiety such as: the school system, gender, socioeconomic status, and parental history and prejudices.

5.9.3 Confidence in Learning Mathematics

A student's confidence about his/her ability is often seen as an important variable in learning and teaching processes. There is awareness in an education context that lack of confidence may lead to the learner being prevented from making the required effort to reach the goals of education processes. Confidence has been identified by Reyes (1984) as one of the most crucial affective variables. He stated that, "*confidence in one's ability to learn mathematics appears consistently as a strong predictor of mathematics course election.*" (P: 562). Meyer and Koehler (1990) define confidence as one part of self-concept which has to do with how sure a student is of his or her ability to learn new mathematics and to do well on mathematics tasks. They argue that confidence affects a student's willingness to approach new topics and to persist when the material become more difficult.

It is argued that confidence is an attitude towards oneself and it depends heavily on experience (Oraif, 2007). Reid and Yang (2002b) noted that confidence was lacking when secondary school students faced a new and open-ended task but, with the completion of the first such task, confidence was observed to grow markedly when facing subsequent tasks even when the students were finding the tasks difficult. Yang (2000) also found that the growth of confidence did not necessarily seem to lead to better performance in the open-ended tasks, but it did mean that the school students approached these tasks more enthusiastically with more self-belief and assurance.

The effects of confidence on mathematics achievement and participation have been explored in many studies (see Fennema & Sherman, 1976, 1978; Sherman & Fennema, 1977; Hart, 1989; Pongboriboon, 1993; Hammouri, 2004; Engelbrecht *et al*, 2005), and there were significant correlations between confidence in mathematics' ability and mathematics' achievement. In the late 1970s, Fennema and Sherman carried out extensive research into the effects of confidence on mathematics achievement, and they reported that confidence was more strongly correlated with mathematics achievement ($r = 0.40$) than was any other affective variable (see Fennema & Sherman, 1976, 1978; Sherman & Fennema, 1977). Sherman (1982), in an analysis of longitudinal data for students at the Year 8 and at Year 11 levels, reported that the confidence in learning mathematics subscale emerged as a powerful predictor of how many years of college preparatory mathematics students would elect to do. Recently, a study presented by Hammouri (2004) examines the grade eight attitudinal and motivational variables related to mathematics achievement in Jordan. One of the most important results reported in this study is that confidence was more strongly correlated with mathematics achievement ($r = 0.38$, $p < 0.05$) than was any other affective variable employed in the study.

The question is how confidence might be developed in learners so that they can improve and apply these skills. The key factor is that success seems to lead to confidence among those students who have been more successful in school examinations (Oraif, 2007). The essential question is how to offer success to those who are not so good at formal examinations, particularly when based on recall of information or procedures. If success depends largely on confidence, there is a real danger that the examination system will generate many students who are unsuccessful, thus reducing their confidence. This may well lead to further poor performance in examinations. Thus, the system may lead to the destruction of confidence. It does not seem to be the style of examination but the fact of success in examination which is a crucial factor for the confidence. In that examinations are seen as a key part of most learning, the difficult question is how to generate success for all (thus enhancing confidence) without losing all sense of rigour.

5.9.4 Attitudes towards Different Topics within Mathematics

As a result of their experience in school mathematics, students learn knowledge, ideas and acquire skills. At the same time and through the work they do, they develop attitudes toward mathematics. Students' attitudes towards different topics within mathematics vary from topic to topic and depend on student's confidence about the topic and the easiness of it.

Students' attitudes to specific topics are examined in the APU Primary Survey (1980). This survey reveals a strong tendency for students to find mathematics useful but children appear more qualified in their views on other aspects of attitudes and mathematics. When asked whether they enjoyed mathematics, the answer was: liking and difficulty are not easily attributable to the whole subject – they are associated with specific topics and forms of presentation. This makes the discussion of particular topics relevant and important.

The work of Cresswell and Gubb (1987) is concerned with general attitudes to mathematics, but also the details of students' reactions to different topics and activities within the subject which were investigated through a set of 15 items. The items were designed to measure attitudes on three different dimensions and students were asked to respond to each activity on a five-point scale with respect to how important it was, how easy it was, and how much they enjoyed doing it. Their study confirms that mathematics is considered as important subject by both boys and girls. They also find there is a higher association between easiness and liking rating than between either of these and importance. They wondered whether the relationship between easiness and liking indicates that students feel more confident with the material, so that they like it, or conversely that they like working on topics they feel confident about.

As far as attitudes to more specific topics within mathematics are concerned, it can be recognised that certain topics are generally liked and others disliked; some topics are generally regarded as 'easy', others as 'hard'. In general, a student's willingness to approach new material and to continue when the material becomes difficult are influenced by how much s/he is confident about his or her mathematics ability. Despite the instant difficulty of the task, the student persists when s/he is confident that a solution will be found or that the material will be understood and the good teacher will be one who is keen to encourage his/her students and promote their confidence about their ability in order to obtain the desired result from them. In the light of the key role of the teacher, we now turn to discuss the interactions of mathematics teachers to their students.

5.10 The Attitudes of Mathematics Teachers to Their Students

Fennema and Sherman (1978) assert that,

> *"Teachers are the most educational influence on students' learning of mathematics"*

The students make the greatest development when positively encouraged by supportive and concerned relationships with knowledgeable and reasonable teachers. It was clear that the personality and the teaching style played an important role in students' experience of mathematics and the classroom, because all of the teacher's actions and words have a bearing, either directly or indirectly. It is argued by Cresswell and Gubb (1987) that some of these teachers opened up the beautiful world of mathematics for their students, motivating their curiosity, encouraging them, and exciting their interest to pursue learning mathematics, and some of their teachers' negative and boring portrayals of mathematics.

The role of the teacher in the classroom is the area that has been agreed upon by many researchers as an element key to mathematics anxiety (Fennema and Sherman, 1978; Grouws and Cramer, 1989; Martinez *et al*, 1996). Grouws and Cramer (1989) looked at the teachers' role within the framework of problem solving. They found that the classrooms of effective teachers of problem solving skills were very supportive environments. Costello (1991) found that, "*it appears usual for pupils to attribute good experience in mathematics to their own prowess but bad experience is less easily handled. Often it is blamed on the inadequacy of the teacher – this is hardly surprising, and is a good defence mechanism*" (p: 128). However, in the study of Christou (2001), he found that 38% of the sample answered the question why they liked mathematics in terms of the good teacher they had (in their previous study) and 21% because they understand the logic of mathematics. This shows that over one third of the students attributed their liking to their good teacher and reflects the crucial role that teachers play within the educational process in general and particularly in teaching mathematics. Therefore, teachers have to attach considerable importance to the promotion of favourable attitudes in their mathematics classes. Macnab and Cummine (1986) formulated four necessary qualities of good mathematics teachers, which enable teachers to build up positive attitudes to the subject. The qualities are:

> *"They cultivate with pupils relationships of encouragement and emotional warmth. Encouragement can rarely be overdone.*
>
> *They maintain, and are seen to maintain, a liking for an interest and involvement in mathematics.*
>
> *They seek to develop self-achievement in pupils through a pattern of activities in which such self-achievement is possible.*
>
> *They discuss mathematics with their pupils rather than simply transmit it, so that pupils can come to distinguish between mathematical fact and national convenience and practice, and, more generally, achieve a greater awareness of the process of mathematical development".*

5.11 The Relationship between Attitudes and Attainment

Attainment in mathematics in secondary school is a function of many interrelated factors: students' capability, attitudes and perceptions, socioeconomic factors, parent and peer influences, school-related factors, and so forth (Singh, *et.al*, 2002). Some of these variables are home and family related and are difficult to change being largely outside of the control of teachers and schools. However, other variables such as students' academic engagement, perception and attitudes are school-related variables and subject to change by educational interventions. Khan and Weiss (1973) discussed the variables in some detail in relation to science education and Reid (1978) summarised this as shown in figure 5-6.

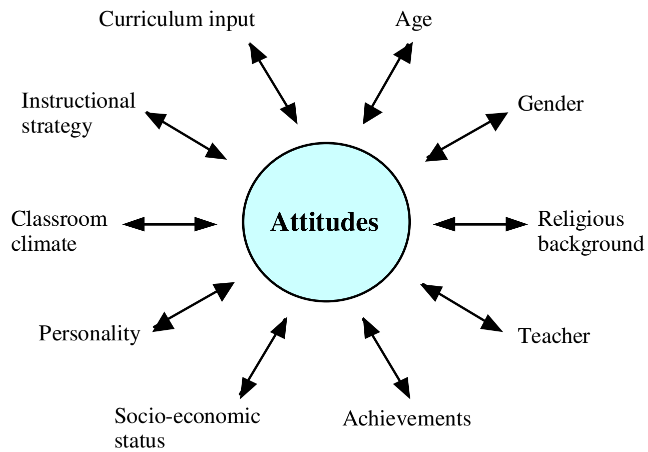


Figure 5-6: The variables in relation to science education (Reid, 1978)

The important point to note is how few of these variables are open to influence within the normal teaching situation: curriculum input, instructional strategy and classroom climate are obvious and, perhaps achievement can be influenced.

Recently, educational research has devoted a great deal of effort to the relationship between attitudes and mathematics attainment (Volet, 1997: Middleton, 1999; Christou, 2001; Singh *et.al,* 2002; Hammouri, 2004). There is evidence to support the seemingly fairly reasonable belief that favourable attitudes towards mathematics lead to higher achievement in the subject. Costello (1991) indicates that there is a common and reasonable belief that positive attitudes, particularly liking for, and interest in, mathematics, leads to greater effort and in turn to higher achievement. Aiken (1976) in his survey of work on attitudes towards mathematics does refer to some large-scale investigations, which show significant correlation between attitudes and achievement, but the correlations are still low. Another study is by Neale, Gill and Tismer (1970), who indeed find significant correlation between attitude and achievement among lower-secondary-age students. Christou (2001), in summarising his results, concluded that:

> *"Students' self confidence in mathematics was highly correlated with their performance in the mathematics test;*
>
> *The students of the sample who replied positively to the question whether they like mathematics, whether they enjoy solving mathematics problems and whether they believe they are good in mathematics performed better in the mathematics test.*
>
> *Students' motivation for further studying in university was highly correlated to achievement in mathematics test."*

He comments, "*In this case, it is possible to assume that a positive attitude towards mathematics helps achievement in mathematics. Equally, experience shows that achievement in mathematics helps to develop attitudes towards mathematics."* This two way effect is very important. Singh *et.al,* (2002) in a study examined mathematics and science achievement: effects of motivation, interest and academic engagement, found strong support for the hypothesized relationships and mediated effects of attitude on academic achievement. Cresswell and Gubb (1987) said "*None of this work is able to resolve the chicken and egg problem of whether positive attitudes towards a particular area of study enable students to succeed in that area or whether success in a subject breeds positive attitudes.*"

Despite the evidence supporting this, the link between attitudes and achievement is not so close as may be expected, and there are other affective variables that influence attitudes and achievement, such as home variables, general school variables, teaching variables , gender and demographic variables.

5.12 Gender and Attitudes towards Mathematics

This section attempts to specify the interaction of variables on the basis of gender and differentiates between males and females beliefs. Gardner (1975, p: 1) indicated that the influence of gender on attitudes towards science is large, remarking that, "*Sex is probably the single most important variable related to pupils' attitudes to science*". The most striking difference between the responses of the girls and boys appeared within liking or disliking parts. It is a common finding that girls are more likely to attribute their success to hard work or luck rather than to their ability. Thus, Reyes (1984) reported gender differences in patterns of attribution of success and failures in that girls are "*more likely to see success as caused by effort and less likely to see success by ability*" (p: 568). When we look at mathematics attribution studies using the Mathematics Attribution Scale (MAS), which was developed by Fennema, Wolleat, and Pedro, some similar findings were revealed. Pedro *et.al* (1981), in a study of 647 high school girls and 577 high school boys, found that the boys, more than the girls, attributed their success in mathematics to ability, and the girls, more often than the boys, attributed their success to effort. However, any experienced teacher might have come up with the same observation!

Taylor's study (1990) found that confidence was important to the male and female in three areas: teaching, learning, and research. He found that females were as confident as males in the areas to which they directed their energies, and there did not seem to be any gender-related differences in the levels of confidence expressed.

Fear of success was first defined by Horner (1968) as a variable useful in explaining gender differences in the research on achievement motivation. Mayer and Koehler (1990) state that fear of success describes the conflict, resulting fear, and decreased performance that many women experience because of the clash they perceive between attaining success and fulfilling the female role in our society. Fear of success is fear of the negative consequences that accompany success. Horner identified two sources for these negative consequences: (1) the individual's loss of her sense of femininity and self-esteem and (2) social rejection because of the success. Leder (1982) attempted in his study to investigate the relationship between fear of success, mathematics performance, and course-taking intentions for males and females. She found that, for high-achieving males, high fear of success was associated with the intention of leaving school or taking no further mathematics. Conversely, high-achieving females who were high in fear of success expressed their intentions of taking two additional mathematics courses. Mayer and Koehler (1990) indicate, "*Fear of success does not seem to provide clear explanations for gender differences in mathematics*".

These differentiations between female and male attitudes and beliefs might be attributable to the differential treatment of teachers. Researchers have examined the teacher's role in portraying mathematics as a male domain, and the basic question under consideration was, 'Are females and males treated differently within the classroom, and, if so, how?'

Fennema and Reyes (1981) found out that, "*teachers initiate more interactions with boys, ask boys more questions for discipline purposes, and ask boys more higher, lower, and non-mathematics questions*" (p: 21). The researchers concluded that, "*overall, girls are receiving less attention from teachers than are boys*" (p: 34). At the secondary level, Becker (1981) found that "*teachers called on males who volunteered to respond more often than females who volunteered to respond, teachers asked males more process or higher-order questions, and teachers acknowledged males' call-outs more often than females*".

Overall, some elements that could be contributing to the gap in mathematics performance between males and females have been illuminated. Koehler (1990) suggests that teachers promote equity by considering not only the quantity, but also the quality of their interactions with both female and male students, and teachers need to address higher-cognitive-level questions to females as often as to males.

5.13 Conclusions

This chapter has offered a brief summary of the nature of attitudes, their importance and how they can be measured. The importance of attitude development in science education is discussed before looking at the key role of attitudes in relation to learning in mathematics. The overall pattern is:

- Attitudes are an evaluation of something or someone, and they are based on the individual's knowledge, feelings and experiences. Attitudes have been widely measured using a variety of techniques but it is commonly assumed that it is difficult to infer behaviour accurately from these measurements.
- There are several methods for measuring attitudes such as: Likert's method, Osgood's method, ratings and interviews.
- Student's attitudes towards the science discipline may influence whether s/he will choose to study this discipline as an elective subject.
- Attitudes are important in an educational context because they cannot be neatly separated from study. A bad experience can lead to perceptions of mathematics which may be important in developing negative evaluations. Such evaluations generate negative attitudes in such a way that further learning is effectively blocked (Reid, 2003).
- Attitudes have been generally measured using a variety of methods but it is commonly supposed that it is difficult to infer behaviour precisely from these measurements. Many attitudes and beliefs have been studied in mathematics achievement and participation, and the ones which have shown the most consistent links, have been discussed.

Chapter 6

Research Methodology

6.1 Introduction

The key focus of this thesis is to explore the cognitive and attitudinal factors that affect learning and teaching mathematics. This is a huge area of investigation, so a multi-step strategy is used to examine the relationship between these variables and learning mathematics. The first and the second steps focus on the students and the third step looks at the mathematics teachers' and inspectors' ideas about learning and teaching mathematics. In this chapter, the main research questions and the data sources chosen to address the research question were identified. Moreover, all the procedures and methods which were used to conduct the data collection and data analysis of the study are presented within this chapter.

6.2 Study Aims

There were three main aims of this study:

- *To explore some cognitive factors affecting achievement in mathematics:*
- *(a) Working memory capacity;*
- *(b) Field-dependency.*
-
- *To find out junior secondary students' attitudes towards mathematics in Kuwait.*
- *To find out mathematics teachers' and inspectors' ideas about learning and teaching mathematics in Kuwait.*

6.3 Study Questions

This study attempted to identify the four main questions as follows:

- *Are there any relationships between students' working memory space and their achievement in mathematics?*
- *Are there any relationships between students' field dependency learning characteristic and their achievement in mathematics?*
- *What are students' attitudes towards mathematics?*
- *What is the perception of mathematics teachers and inspectors about mathematics education in the State of Kuwait?*

6.4 A Complementarity of Quantitative and Qualitative strategies: Triangulation

In studies on learning, it is important to check outcomes from one set of observations with the outcomes from another. This is sometimes described as *triangulation*, a word borrowed from navigation. It involves using different (or independent) methods to research the same issue. There is a controversy in the educational research literature when looking at qualitative and quantitative methods. Some researchers support only one of these approaches while others advocate the usage of a combination (or triangulation) of approaches which can offer different complementary strengths (Muijs, 2004; Cohen et.al, 2007 and Yin, 1984). The combining of several approaches helps to overcome the weakness, biases and limitation of using just a single approach and as Yin (1984, p:92) stated "*…any finding or conclusion in a case study is likely to be much more convincing and accurate if it is based on several different sources of information…"*. Furthermore, the usage of a mixture of approaches helps in collecting more comprehensive and robust data, and helps to make the researcher to be more confident that his findings are valid (Cohen & Manion 1994, p: 233-234).

This study employed both quantitative and qualitative approaches and used a variety of methods to collect data: cognitive tests, questionnaire and interviews. The reason for choosing to use several methods within this study was the distinctive contribution that each particular method could offer to the investigation of the research questions.

First step – Pilot study

In orders to reach the study aims, a multi-step strategy was used. The first step was a pilot study which helped the researcher to identify the key issues. Two psychological tests and a questionnaire was applied to identify the following questions:

- *Are there significant relationships (correlation) between students' performance in mathematics and their working memory space?*
- *Are there significant relationships (correlation) between students' performance in mathematics and their field-dependency characteristic?*
- *What are the attitudes of junior secondary school students (ages 12-15) toward mathematics?*
- *How do attitudes to mathematics change with age?*
- *How do students' attitudes toward mathematics relate to their gender, and does gender relate to attitude change?*

- *Are there any significant correlations between students' attitudes, their performance in mathematics, their working memory space and their field dependency?*

Second step – Main study

The second step is the main study and attempted to explore the same questions as in the first step. Furthermore, the correlations between performance in different topics in mathematics and the working memory space and field dependency are considered. For this purpose, mathematics tests were developed where some questions have high working memory demand and others have very low working memory demand. In order to investigate which versions of tasks will lead to improve mathematics performance, some questions are presented as symbolic tasks; others are presented as visual tasks; some of them presented as abstract tasks and others tasks related to life. The same psychological tests which were used in the first step were used again in the second step. In order to explore some aspects of students' attitudes towards mathematics, another questionnaire was developed.

Third step – Perception of Teachers &Inspectors

The third step focuses on the perceptions of mathematics teachers and inspectors to see the extent to which their views relate to the findings from work with students. It looks for mathematics teachers' and inspectors' views about the purpose of mathematics education at school level in Kuwait, as well as how they see various topics in the curriculum, and the focus is very much on topics which were found difficult for the students and the possible reasons why these difficulties arise. This step involved semi-structured interviews which offer an opportunity to focus on some key areas as well as giving freedom for the teachers to expand their views.

6.5 Statistics Methods Used

Several statistical techniques were used within this research. This section provides a brief explanation for every method used. The correlation, multiple regression and factor analysis methods were used to explore the relationships between variables while the t-test and the chi-square methods were use to explore differences between groups.

6.5.1 Correlation

In its simplest form, correlation measures the linear association between two scale variables. Correlation establishes if there is any relationship and whether that relationship is likely not to have been caused by chance. It does not establish why the relationship exists nor does it imply cause and effect between the variables. Thus, for example, if the height and weight of a large sample of women are measured, then these two measurements are highly correlated: those who are tall tend to weigh more.

There are three different types of correlation in statistics. The Pearson correlation coefficient is the most common one and it is used when the data comes from measurements from a scale (like heights, weights or exam scores). The Pearson correlation coefficient assumes an interval scale and that the measurements are approximately normally distributed. It is not appropriate for ordinal data or when there is a gross deviation from a normal distribution such as the presence of outliers. If the relationship between the variables is not linear (or there is an outlying point) then the Pearson statistic is not the appropriate method for the association.

Spearman correlation is used when one or both variables are not measured on an interval scale. It is based on ranking the two variables, and so makes no assumption about the distribution of the values. The Spearman correlation can cope with a few ties in the data (where responses are different but still show as one point on the scale); however, when there are a lot of tied values, the Kendall's tau-b correlation is more appropriate. This method is another nonparametric correlation coefficient and is an alternative to the Spearman correlation. Thus, Spearman may be used for relating marks when there is marked deviation from normality or the scale is very limited (e.g. 1 to 10). Kendall's Tau-b finds its place where data are ordinal as in questionnaire questions and there are few points on each scale (typically five or 6). For more details, see Hinton *et.al* (2004) and Pallant (2005). For all three correlation methods, the coefficients range from -1 to +1.

6.5.2 Multiple Regression

Multiple regression is a set of techniques that can be used to explore the relationship between one continuous dependent variable and a number of independent variables or predictors (Pallant, 2005). This technique is generally based on the correlation, but it is a more sophisticated exploration of the interrelationship among a group of variables (ibid). It can be used to address a variety of research questions (Pallant, 2005, P: 140), and some of the main types of research questions are:

- *"How well a set of variables is able to predict a particular outcome;*
- *Which variable in a set of variables is the best predictor of an outcome; and*

- *Whether a particular predictor variable is still able to predict an outcome when the effects of another variable are controlled for (e.g. socially desirable responding)."*

6.5.3 Factor Analysis

Factor analysis is a *"data reduction"* technique which is used to reduce or summarise a large set of variables to a smaller set of factors (Pallant, 2005).There are two main applications of the factor analytic technique: to reduce the variables number; or to detect the relationship between variables and classify them. Factor analysis was used with this study in order to detect the relationship between the cognitive factors and mathematics achievement. More details will be given later with the specific examples of its use.

6.5.4 Chi-square

Chi-square is a non-parametric test used to compare patterns of responses or frequencies. For example, it can be used to compare student response to a questionnaire item which was used before and after some teaching and learning experience to see if views have changed. It is used most frequently to test the statistical significance of results reported in bivariate tables.

There are two applications of chi-square: goodness of fit tests, and contingency tests. The former is used when it is appropriate to compare a pattern of responses to those of a control group. The latter is used to compare two patterns of response when neither can be considered as a control group (like comparing boys and girls). A contingency test was used in the present study to:

- *Compare grade eight and grade nine responses.*
- *Compare male and female responses.*

6.5.5 t- Test

The t-test compares the means of two sets of measurements to see if they are significantly different. The test assumes data are interval and approximately normally distributed. There are various types of t-test available, e.g. the independent-sample t-test and paired-samples t-test. If the comparison is between the mean scores of two different groups of people then the independent-sample t-test will be applied. However, if the comparison of mean scores of the same group of people on two different occasions, then the paired-samples t-test will be applied. The appropriate one for this study is the independent-sample t-test in order to compare the results of various test questions presented in various formats.

6.6 Measurement of Working Memory Space

In order to measure an individual's working memory space, the digits backwards test (DBT) was used. In this test, the examiner reads to the subjects a series of digits and asks them to write the digits in reverse order. For example, 76895 would return as 59867. Every digit is read to the subjects in a rate of one digit per second and the same time is given to recall after the reading of the whole series is over. After the subjects finish the task, they will receive a new task with more number of digits and so on. Two tasks are given for each number of digits. When the subject begins to make mistakes this indicates that the working memory has reached its capacity. It cannot hold any longer series of digits and this upper limit is taken to be the capacity of his working memory.

This test was marked in the following way. When the students fails to recall both sets of numbers containing the same number of digits, then the previous level was taken as the mark that represents the capacity of his/her working memory. Table 6-1 is an example of a subject who was classified to have a working memory capacity equal to 5 because s/he was able to recall the digits until level 5 but s/he failed in both attempt at level 6.

SET	NUMBERS						
2	4	2					√
	8	5					√
3	9	2	6				√
	5	1	4				√
4	9	7	2	3			√
	8	6	9	4			√
5	6	8	2	5	1		--
	3	4	8	1	6		√
6	8	1	4	3	1	5	--
	6	5	8	4	2	7	--

Table 6-1: The correction of Digit Backwards Test

The sample of students was divided into groups namely: *low*, *intermediate* and *high working memory space* capacity. Students who succeeded to remember in reverse way up to 4 digits (labelled as X = 4) were categorized as *low working memory space*. Students who able to recall 5 invert digits (labelled as X = 5) were classified as *intermediate working memory space* and the rest who memorize 6 or more overturned numbers, were classified as *high working memory space* (labelled as X = 6). While the test does give an accurate measurement, there are some important observations to make.

The digits *backwards* test will give the capacity of the working memory as one less than the actual capacity. This is because one space is used to reverse the number order. Thus, if the digits *forwards* test is used as well, the values for this are one more than those obtained for the digits span *backwards* test. All this can be confirmed by using the figural intersection test (a quite separate measurement tool). The result from this test compares accurately with those obtained from the digits *forwards* test. This was explored by El-Banna, 1987.

In practice, the digits backwards test is not always completely straightforward to mark in that the patterns obtained for a minority of students do not always neatly conform to an unambiguous cut-off point between success and failure. Thus, some students may make trivial mistakes (due to mishearing or occasional lack of concentration). However, the problem of marking is not too serious in that, in most studies, including this one, absolute measurements are not as important as the need to place students in rank order of working memory capacity. The whole problem of marking was considered by Mancy (2007).

6.7 Measurement of Field-Dependency

In order to measure the individual's degree of field-dependency, the Group Embedded Figure Test (GEFT) was used. This is based tightly on the work of Witkin (Witkin *et al*. 1977). The GEFT is a test which contains 20 items and the test was used here to place the students in order according to their ability to be able to discriminate a required item from its context. The test involves finding a simple geometrical shape which is embedded in a matrix of geometrical shapes. The more figures found the better the individual is at the process of separating an item from its context. The test was not used as an absolute measure of field dependency. It merely sought to place the students in order of their ability at this skill.

In each of 20 complex figures, the subject was asked to identify and recognise a specific simple geometric shape. The simple geometric shapes are on a separate sheet on the last page of the booklet and the subjects were asked to remove it to be always near them. The principle scoring scheme for the test is for every correct simple shape embedded in a complex figure one point is given, so the student can obtain up to 20 points. The whole test of the GEFT test, along with the correct answers can be seen in appendix (B). Figure 6-1 shows an example of the GEFT.

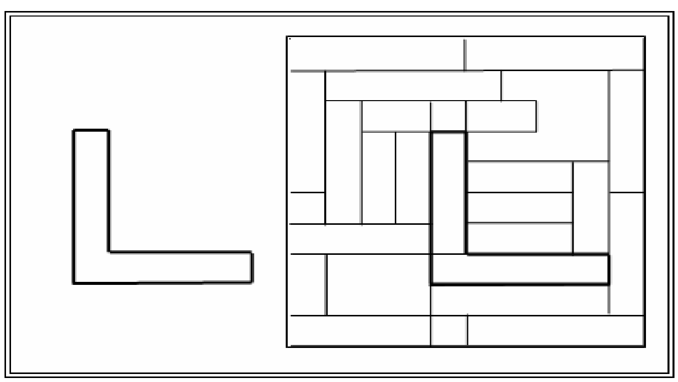


Figure 6-1: Example of Group Embedded Figure Test (GEFT)

The instructions given to the subjects were as follows:

- *The simple shape has to be found in the same size, same properties, and the same orientation within the complex figure.*
- *The subject is not allowed to use a ruler or any other means to measure the size of the simple shape in the complex figures.*

- *There is more than one simple shape embedded in some complex figures but the subject is required to locate only the simple shape which is in the same proportion, size, and orientation as the specimen.*
- *The test is timed (20 minutes).*

Different studies have used different approaches to classify individuals as field-dependent or field-independent. In Luk's study (1998), the median was used to classify the whole group: students gaining a score above the median of the overall scores were considered as field-independent; those were who gained below the median were considered as field-dependent. The field-intermediate category was omitted in this study in order to compare between contrasting groups (dependent versus independent).

In many research studies, the following method (see figure 6-2) was used to classify the students of the sample into categories: field-dependent students (FD) are regarded as those who scored at least a half-standard deviation below the mean. On the other hand, the students who scored more than a half-standard deviation above the mean are labelled as field-independent (FI) students. Moreover, the other students who scored between the above-mentioned classifications are labelled as field-intermediate (FIT) (El-Banna, 1987; Al-Naeme, 1988; Gray, 1997; Bahar, 1999; Danili 2000; Christou, 2001). This method has the advantage that it generates three approximately equal groups.

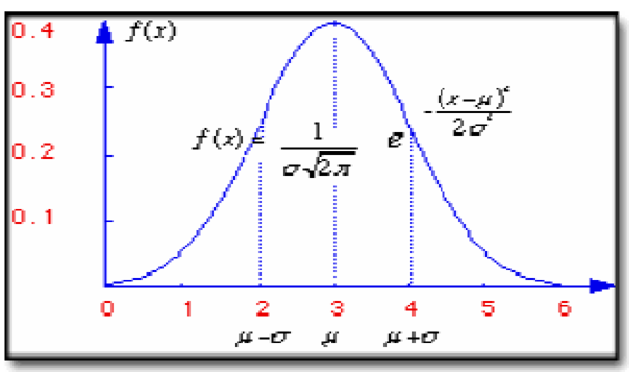


Figure 6-2: Method used to classify the students into field dependency

6.8 Mathematics Performance

Students' performance in mathematics was gained by taking the average of the student's marks in three tests added together. These tests ran for three hours of examination during the first term of the academic year. These were the tests used by the schools; all schools used the same tests. Thus, the mark in mathematics is a composite, covering many mathematical skills.

6.9 Mathematics Tests

Two mathematics tests were designed for eight and nine grades according to the Kuwaiti curricula for these grades. There are two versions for every test; some of the questions are common in both versions. Some of them differ in the presentation but not in the content. Thus, some of the questions are presented in a visual manner and some of them in symbolic manner; and some of them are abstract and the other are applied.

The aims of designing these tests are to explore the correlation between mathematics topics and the students' working memory space and their field dependency; and to explore which version of presentation may help the students to reach higher achievement in mathematics examinations. Does visual presentation or the use of symbolism aid performance? Do applied questions help students to achieve better compared to more abstract presentations?

The tests were corrected and the marks distributed according to the system that operates in the Kuwait education system. For the whole tests and the correction processes and marks distribution, see Appendix (C).

6.10 The Questionnaire

In order to measure students' attitudes toward mathematics, a questionnaire was constructed. The guidelines offered by Reid (2003) were used to guide development and scoring:

- *"Jot down as accurately as possible what you are trying to find out.*
- *Settle on what types of questions would be helpful.*
- *Be creative and write down as many ideas for questions as you can.*
- *Select what seem the most appropriate from your list – keep more than you need.*
- *Keep the English (the questionnaire language) simple and straightforward, avoid double negatives, keep negatives to a reasonable number, look for ambiguities, and watch for double questions.*
- *Find a critical friend to comment on your suggested questions.*

- *Pick the best, most appropriate and relevant questions, thinking of time available.*
- *Layout is everything!*
- *Try your questionnaire out on a small sample of students (e.g. a tutorial group) - ask for comments, criticisms. Check time required.*
- *Make modifications and only then apply to larger group.*
- *Analyze each question on its own."*

The questionnaire involved many different question formats, each having its own strengths and weaknesses:

(a) Likert format on a five point scale, ranging from *strongly agree* to *strongly disagree* (or from *always* to *never*), aimed to gain information of students' opinion about mathematics, mathematics classes and examinations (Likert, 1932).

(b) The semantic differential (Osgood *et.al,* 1969) was employed to investigate students' views about their confidence in mathematics performance, mathematics as a subject, their preferred topics within mathematics and their preference of mathematics compared with other subjects. This technique comprises bipolar adjectival pairs (boring, interesting), with a series of unlabelled 6 boxes deposited between them.

(c) The questionnaire also contained other type of questions aimed to gain insight about various aspects of learning mathematics:

- *Multiple tick questions, where students could choose as many options as they want. This type aimed to explore the reasons for liking mathematics.*
- *Yes or No questions and the reasons for their answers were used to explore the students' opinions about the importance of mathematics.*
- *Rating scales, where students are asked to place statements in order. This question asked students to order the reasons for why students should study mathematics at school according to the importance.*
- *Preference ranking questions, where students choose three things they feel most appropriate. Students were asked to choose three methods most helpful for them in mathematics studies.*

For more details about attitude measurement, see attitude chapter 5. See Appendix (D) for the questionnaires.

The questionnaires were first written in English, and then translated into Arabic ensuring the same sense as far as possible from English to Arabic. Arabic versions of the questionnaire were trialled using other Arabic postgraduate students at Glasgow University to ensure the sense of the items had not been lost in translation.

Attitudes and beliefs tend to be highly complex and multi-faceted. It is not easy to deduce an attitude precisely from one single question in a questionnaire. The aim here was more to 'paint a picture' of the way the junior secondary school learners saw mathematics. To enrich the picture, a group of individually analysed four or five questions (essentially used to measure different aspects of an attitude concept) will be *"qualitatively added"* (Reid, 2006, p: 18) to reach a final judgement about the attitude in question. This approach of treating the data has been used in several studies (e.g. Reid, 1980; Hadden & Johnstone, 1982; 1983a, 1983b, Reid & Skryabina, 2002a, 2002b) and it *"would allow for unmasking important and rich group variances"* (El-Sawaf, 2007).

It is well worth stressing that, in the statistical treatment, individuals' attitudes were not by any means measured in any absolute sense. Reid (2006) argues that, *"This simply cannot be done with attitudes. In the present state of knowledge, attitudes cannot be measured in any sense, with any degree of certainty. However, responses to attitude measures can be compared: before and after some experience; between two different groups such as male and female."* (P: 11). In this study, a picture was gained of student attitudes while comparison were made between subgroups (year groups, boys and girls), this comparison being analysed using chi-square as a contingency test.

6.11 The Interview

The main purpose of the interview was to explore the perceptions of mathematics teachers to see the extent to which their views relate to the findings from work with students. Questionnaire was not used with teachers because it was felt that they would be unwilling to complete these for fear of them been used to effect their careers. Having decided on the interview as a method to collect these data, the next step to be addressed is the format of the interview itself in particular, whether the interviews would be highly structured or totally open. Between these extremes lies the semi-structured interview (Reid, 2003, p: 29).

In a semi-structured interview, the interviewer sets up a general structure by deciding what ground was to be covered and what main questions were to be asked (Drever 1995, p: 1). Thus, this work involved semi-structured interviews which offer an opportunity to focus on some key areas as well as giving freedom for the teachers to expand their views (Reid, 2003). The interview is also flexible in the way that the interviewer can clarify the questions and ensure that the interviewees understand them (Henerson et.al, 1988, p: 25). While a semi-structured interview technique was used, open-ended questions were used to follow leads and introduce new questions. Open-ended questions permit flexibility, deeper probing of answers, clarification of misunderstanding and the testing of what the respondents truly believes in the interview situation.

Interviews were carried out with mathematics teachers and inspectors in their work places (schools for teachers and the mathematics department in the education ministry for the inspectors). The interviews were conducted in a fairly relaxed and comfortable atmosphere where the teachers were informed that this study will help to improve mathematics education and all the information in these interviews will be held securely. The interviewees were reassured that their names would not be associated with the notes taken in any way nor would the interviews affect them or their jobs in any way.

One key problem with all interviews is how to record the outcomes. The researcher can choose between note taking, either during or after the interview, or tape-recording and transcription (Reid, 2006, p: 30-31). Notes were taken in shorthand by the interviewer, which gives the interviewees a sense of ease and encourages them to talk freely. The duration of every interview varies from 25-35 minutes and final notes were developed immediately after the interview finished, this process taking approximately half an hour. The researcher translated all of the developed notes into English, so the interviews are written in the sense of the interviewees' words, not exactly what they said.

6.12 Reliability and Validity of Attitude Measures

Reliability refers to the degree to which a technique yields consistent scores or values when the attitude is measured a number of times (Shaw & Wright, 1967, and Chaiken & Eagly, 1993). However, this understanding of reliability is often confused with internal consistency. Thus the extent to which the various items are consistent with each other is what they are measuring. While internal consistency may be very important in measuring a specific latent construct, attitudes are so multi-faceted that such consistency is not appropriate at all. Nonetheless, internal consistency is still confused with reliability in various measures suggested. Reid (2006) argues that most reliability techniques are simply measures of internal consistency and offer more or less no evidence on test-retest reliability (see, for example, Gardner, 1995, who discusses this clearly).

Three main experimental methods which, supposedly, seek to assess the reliability of an attitude technique exist:

The Test-retest Method: The test-retest approach uses the same measure on two roughly equivalent occasions to see if the outcomes are similar. Thus, for example, Reid (2006) notes the use of an attitude survey with very large groups of students in two successive year groups where the two groups were seen as more or less equivalent populations. Equally, it is possible to use a questionnaire with one group on two occasions, separated by a short space of time although it is possible that, with a short questionnaire, students may recall answers from one occasions to the next. Both of these approaches consider the same test used on more than one occasion.

The Equivalent-forms Method: Here there are two equivalent forms of the test. These two forms are then administered to a group of participants, the reliability being assessed by comparing the two sets of outcomes. One major drawback of this approach is the practical difficulty in designing test items that are consistent in the measurement and the degree to which the two forms do measure the same attitude, and this is often not feasible.

The Split-half Method: Spilt-half methods of reliability measure the internal consistency of a test. The test items are divided in 2 (randomly or in a predetermines way such as odd-even questions). The outcomes from the two halves are compared and then, often, a statistical correction is applied to estimate the reliability of the whole test, known as the Spearman-Brown prophecy formula (Spearman, 1910; Brown, 1910). However, this technique is essentially a measure of internal consistency and is not helpful in typical educational settings (Reid, 2005, Shaw and Wright 1967).

Reid (2006) argued that most of these statistical techniques measure consistency instead of reliability and they "*offer more or less no evidence of test re-test reliability*" (see, for example, Gardner, 1995). Indeed, internal consistency is not a helpful idea in educational measurement. For example, if a mathematics test contains 10 questions, the last thing the setter wants is for the ten questions to measure the same thing or even be totally consistent with each other in the way the students perform. If there was high internal consistency then it would be much simpler to set only one question and save considerable time for everyone. The setter wants to explore ten different areas of the curriculum, ten different skills, or ten different aspects of mathematical ability (Reid, 1978, 2006). The evidence that internal consistency offers is that, *"if a student knows one area, he might well perform well in another, but that does not say anything about the reliability of the test"* (Reid, 2006, p:10). Split-half measures say nothing about reliability (Reid, 2006) while it becomes almost impossible to measure reliability by asking the same question in another form because it will no longer be the same question any more (Oppenheim, 1992). Therefore, test and re-test are recommended in checking the reliability of the used questionnaire. However, this is often not practical or even possible.

Therefore, Reid (2003) argues that, if tests or questionnaires are designed carefully to avoid ambiguity, the items are moderately difficult and the length of the tests or questionnaire are reasonable, using large samples, then reliability will not be a serious issue. He stated that evidence suggests that authentic reliability can be gained by:

- *"Using large samples;*
- *Careful pre-testing;*
- *Checking that test conditions are socially acceptable;*
- *Using enough questions, with cross checks*
  (e.g. repeated questions, similar questions)."

(Reid, 2006)

Validity is much more important than reliability, and it can be defined as the extent to which the instrument measures what it is supposed to measure (Chaiken & Eagly, 1993). There are four facets of validity which have been discussed (Apa Committee, 1954; Cronbach & Meehl, 1955): predictive validity, concurrent validity, content validity, and construct validity. Predictive and concurrent validity may be considered together as criterion-oriented validity.

Criterion-oriented Validity: A test can be said to be valid if it predicts some appropriate future performance accurately. If the performance is in the future it is known as predictive; if the performance is observed about the same time, it is called concurrent (Cronbach & Meehl, 1955; Shaw& Wright 1967). The problem with attitude measures is that attitudes do not necessarily predict future behaviour precisely anyway as the Theory of Planned Behaviour makes clear (Ajzen, 1985).

Content Validity: Content validity is the extent to which the questions being asked reflect the area under study with accuracy and relevance. It is of little value to assess abilities in simultaneous equations by setting questions on quadric equations! In typical mathematics testing, the questions must reflect the syllabus, the aims being explored and the level of difficulty appropriate to the group (Cronbach & Meehl, 1955; Shaw & Wright 1967).

Construct validity: The construct validity concept is more complicated than other categories of validity. The construct validity of an instrument refers to the degree to which you can be sure it represents the construct whose name appears in its title (Henerson et. al, 1988). This sounds straightforward but attitudes are highly complex and multi-faceted. Attitude measures do not consider one construct. Even a phrase like 'attitudes to mathematics' is complex in that this involves a wide range of quite disparate aspects such as interest, relevance, understanding, usefulness, teacher approaches, workloads and so on. Although it is argued that construct validity can be estimated by considering opinions of others, various correlation techniques, criterion-group studies and a even appeal to logic (see Cronbach & Meehl, 1955; Shaw& Wright 1967; Henerson et. al, 1988), in attitude measures for educational use, the key way forward is by discussing a proposed measure with others who are knowledgeable of the themes being explored and the population under consideration. Indeed, this is no different to the way mathematics examinations are often constructed by responding to the comments of colleagues. Overall, Reid (2006) has argued that, "*There is no absolute way to establish validity but sensible checks can be made which can offer some encouragement*". He points out that validity can be checked by:

> *"Seeking opinions of a group of those who know the population, the attitudes being considered and the social context.*
> *Developing questions based on the population (for example, by means of discussion or previous questionnaires).*
> *Sample interviewing.*
> *By comparing any conclusions drawn from the attitude measurements with other independent observation."*

(Reid, 2006)

In establishing instrument validity and reliability, several steps were taken in this project. All measurement instruments were carefully scrutinised by colleagues and their comments were incorporated into the improvement of the instruments. Second, the Arabic versions of the instruments were judged by other Arabic postgraduate students at Glasgow University to ensure the sense of the items had not been lost in translation. Third, a group of the students were interviewed by the researcher. The reliability of the study was achieved through using various methods in data collection. For example, large samples were used in this study, the questionnaires used both closed statements and open ended questions with cross checking questions to ensure the reliability of the instruments. There were high significant correlations between question such ‘I find my mathematics knowledge useful in daily life’ and ‘I think mathematics is useful subject’ ($r = 0.5$, $p < 0.001$), ‘I am getting better at mathematics’ and ‘I feel I am coping well’ ($r = 0.6$, $p <0.001$), and ‘Boring’ and ‘Mathematics classes are boring’ ($r = 0.7$, $p < 0.001$).

6.13 Conclusions

The main purpose of this research is to explore the cognitive and attitudinal factors affecting learning and teaching mathematics. This chapter has attempted to throw some light on the approaches adopted in this study. The study used many different techniques which involved both quantitative and qualitative approaches.

Chapter 7

Cognitive Factors and Mathematics Achievement

Phase One

7.1 Introduction

The main focus of this phase is to explore the relationship between achievement in mathematics and cognitive factors (working memory capacity and field dependency) as well as attitudinal factors. A sample of 472 Kuwaiti students was involved in this phase. The sample was collected from two different age group (grade 8 and grade 9), and the sample contains a roughly equal number of boys and girls. The findings of this phase are discussed in two chapters. The first chapter focuses in the cognitive factors affecting achievement in mathematics and the second one focuses on attitudinal factors. In this chapter the following questions are explored:

- *The relationship between working memory capacity and mathematics achievement.*
- *The relationship between field dependency and learning mathematics*
- *Are their any age differences in mathematics achievement?*

The steps can be summarized in the following chart (Figure 7-1).

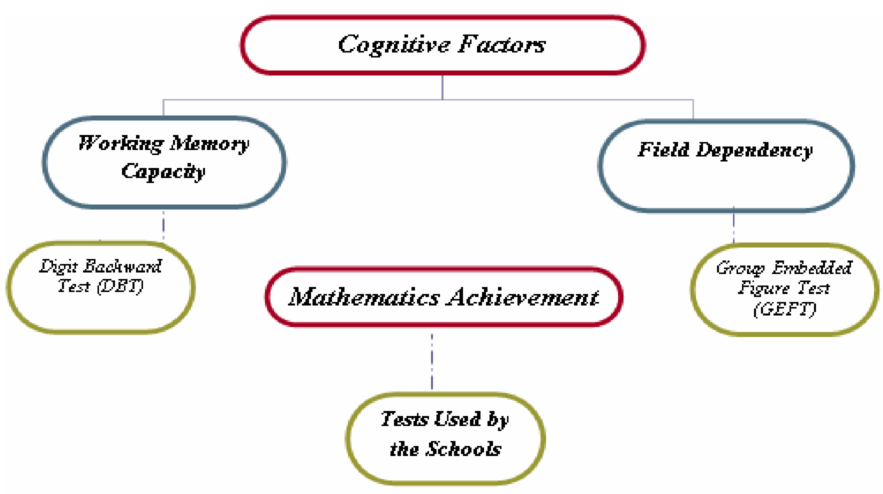


Figure 7-1: First phase procedures

7.2 Students' Sample Characteristics

The sample for the research was selected from five junior secondary schools in the state of Kuwait. Three of these schools are girls' schools and two are boys' schools. These schools reflect the makeup of Kuwaiti communities.

Junior level involves four grades: grade six, grade seven, grade eight and grade nine. The grade eight and grade nine were chosen to reflect students' attitudes towards mathematics after they spent at least two years in the junior secondary level. 233 students involved in this study were from grade eight and 239 students from grade nine. The sample description is shown in table 7-1 and the bar chart (Figure 7-2).

GROUP	GRADE 8 (14 years)	GRADE 9 (15 years)	TOTAL
BOYS	105	112	217
Girls	128	127	255
TOTAL	233	239	472

Table 7-1: Sample characteristics (First phase)

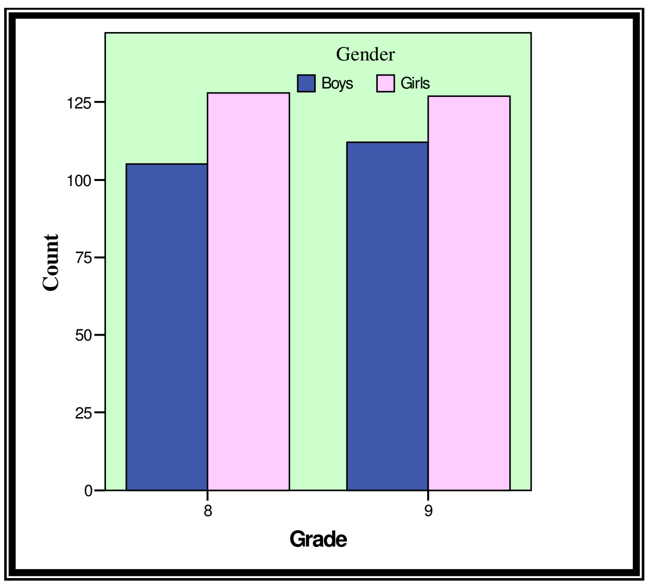


Figure 7-2: Sample characteristics (First phase)

Three instruments were used to collect data for this study. Two psychological factors were measured for every student of the sample in this project: working memory space and field-dependency. The third instrument measured students' attitudes towards mathematics (see chapter 8).

Students' performance in mathematics was gained by taking their marks in three tests added together. These were the tests used by all the schools. Thus, the mark in mathematics is a composite, covering many mathematical skills. These instruments were administrated in December 2005, the end of the first term.

7.3 Attainment in Mathematics

Students' attainments in mathematics were gained from using the average of three tests. These tests ran for three hours of examination during the first term of the academic year 2005/2006. The mean performance was 66%. In order to explore any relationships between their mathematics performance and their working memory capacity and extent of field dependency, Pearson correlation was employed.

7.4 Working Memory Measurement

The distribution of the students' Digit Backwards Test (DBT) total scores is shown in Figure 7-3.

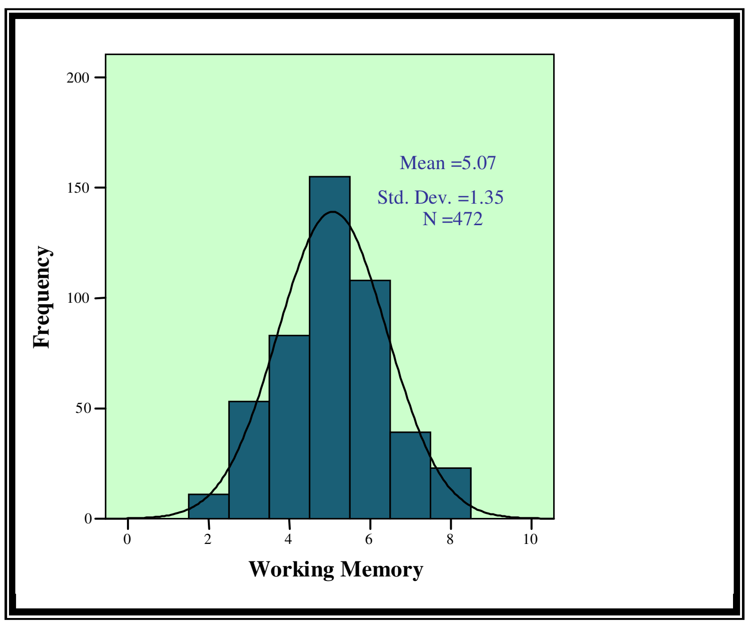


Figure 7-3: The distribution of the Digit Backwards Test scores

Descriptive statistics demonstrates that the mean of the scores is 5.1 and this is completely consistent with the established norms for this age (average age a little over 14) gives a mean of a little over 6. The digit span backwards test gives results about 1 less and that is consistent with the mean value of 5.1.

It is possible to divide the sample into three groups in order to *illustrate* the correlation (Danili, 2001). The sample of 472 students was categorised into groups namely: *low*, *intermediate* and *high working memory space* capacity. Students who succeeded to remember in reverse way up to 4 digits (shown as X = 4) were categorized as *low working memory space*. Students who able to recall 5 invert digits (shown as X = 5) were classified as *intermediate working memory space* and the rest who recall 6 or more reversed numbers, were classified as *high working memory space* (shown as X = 6). Table 7-2 shows the number of students in each category.

GROUP (X-SPACE)	NUMBERS OF STUDENTS	PERCENT
X=4	147	31%
X=5	155	33%
X=6	170	36%
TOTAL	472	100%

Table 7-2: The Classification of students into working memory space capacity groups

Students' working memory capacity (X-space) classification according to their grades is illustrated in the following bar chart (Figure 7-4).

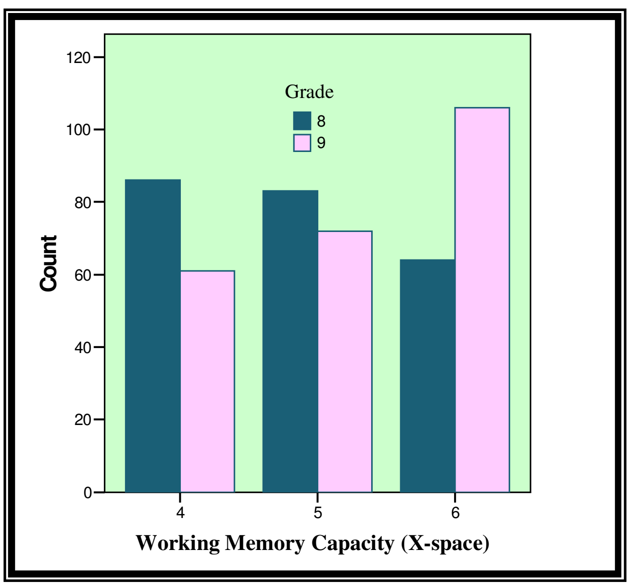


Figure 7-4: Students' working memory capacity (X-space) classification according to their grades

Figure 7-4 shows that the average working memory capacity grows with age. This supports the finding of Miller (1956) who showed that the average capacity is about seven plus or minus two (7 ± 2) separate chunks for adults, an adult being defined as 16 years or older. Miller found that the working memory space grows by about half a chunk each year on average. Table 7-3 shows the classification of the whole sample into X-space according to their grades.

Group	NUMBER OF STUDENTS			
	GRADE 8		GRADE 9	
X=4	86	37%	61	26%
X=5	83	36%	72	30%
X=6	64	27%	106	44%
TOTAL	233	100%	239	100%
Mean	4.85		5.28	

Table 7-3: Students' working memory capacity (X-space) classification according to their grade

7.5 Mathematics Attainment and Working Memory

Correlation between working memory space capacity and performance in mathematics in the Christou study (2001) gave a correlation value of 0.40 and Alenezi (2004) gave a correlation value of 0.52, these being significant at less than the 0.001 level. The data here gave a correlation value of 0.24, this being significant at less than the 0.01 level (mathematics performance has been standardised for both groups). The difference in the correlation might be attributable to the different mathematics tests that were used in these experiments. To illustrate this, table 7-4 provides a comparison between students' working memory space and their mean scores in mathematics. It can be seen from table 7-4, high working space capacity students (X=6) performed better in mathematics than these with lower working memory space capacity (X = 4).

GROUP (X-Space)	MEAN SCORE IN MATHEMATICS
X=4	60
X=5	69
X=6	70

Table 7-4: The relationship between working memory and mathematics performance

A scatter diagram was drawn for the two variables: Working memory capacity (X-capacity) and performance in mathematics (Figure 7-5), in addition to calculating the Pearson correlation coefficient (r).

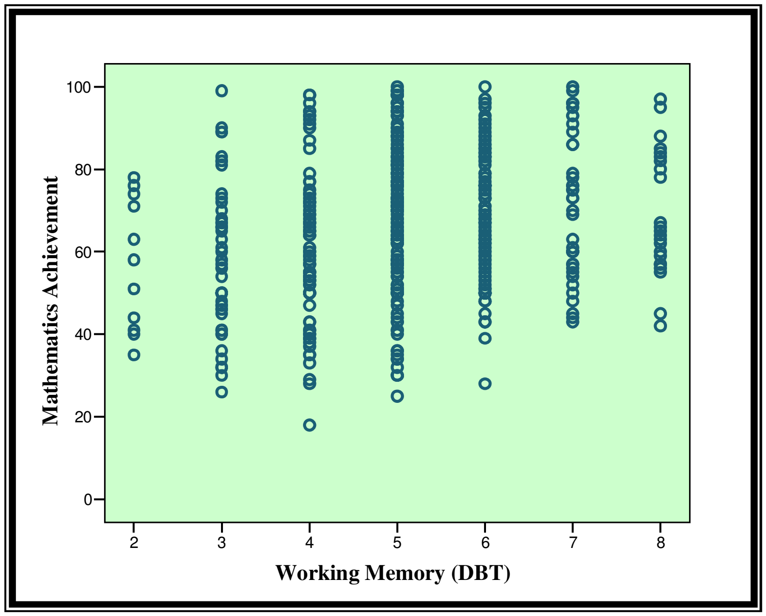


Figure 7-5: Scatter diagram of scores in DBT related to mathematics performance

Table 7-5 illustrates the relationship between students' working memory space and their performance in mathematics according to their grades.

Group	MEAN SCORE IN MATHEMATICS	
	GRADE 8	GRADE 9
X=4	61	58
X=5	71	66
X=6	71	70

Table 7-5: The relationship between working memory and mathematics performance according to grades

7.6 Field-Dependency Measurement

The sample of 472 students was divided into three learning style categories according to their scores in the Group Embedded Figure Test (GEFT). The distribution of students' scores in the GEFT test is shown in Figure 7-6.

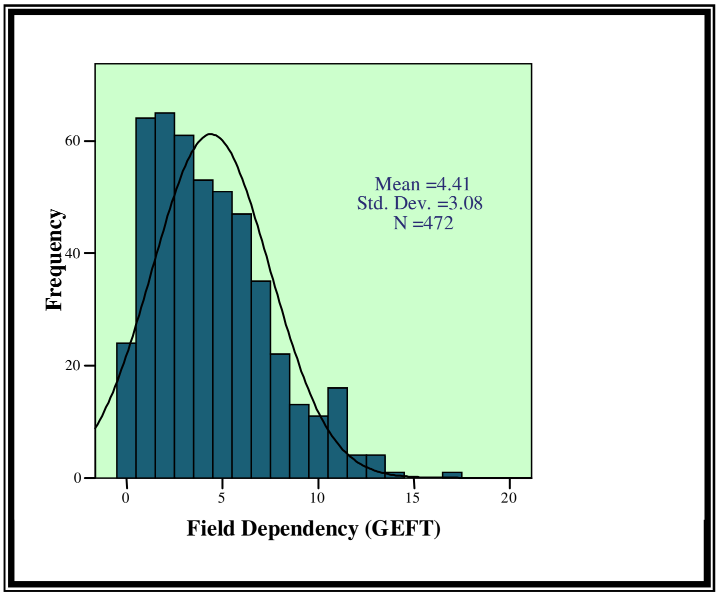


Figure 7-6: The distribution of Group Embedded Figure Test scores

- *Students' who scored less than half the standard deviation less than the mean in the GEFT were classified as Field Dependent (FD), and they form 32% of the sample. (Field Dependent <4.41- 3.078/2)*
- *Those who scored more than half standard deviation more than the mean were considered Field Independent (FI), 33% of the sample. (Field Independent > 4.41+3.078/2)*
- *The rest who scored between these values were labelled Field Intermediate (FIT), and they form the largest proportion of (35%). (4.41- 3.078/2 < Field Intermediate > 4.41+3.078/2)*

Table 7-6 shows the number of students in each learning style category.

GROUP	NUMBERS OF STUDENTS	PERCENT
Field Dependent	153	32%
Field Intermediate	165	35%
Field Independent	154	33%
TOTAL	472	100%

Table 7-6: The classification of the student into field dependency

The classification of the students' field depenedency is divided into groups according to their grade (Figure 7-7).

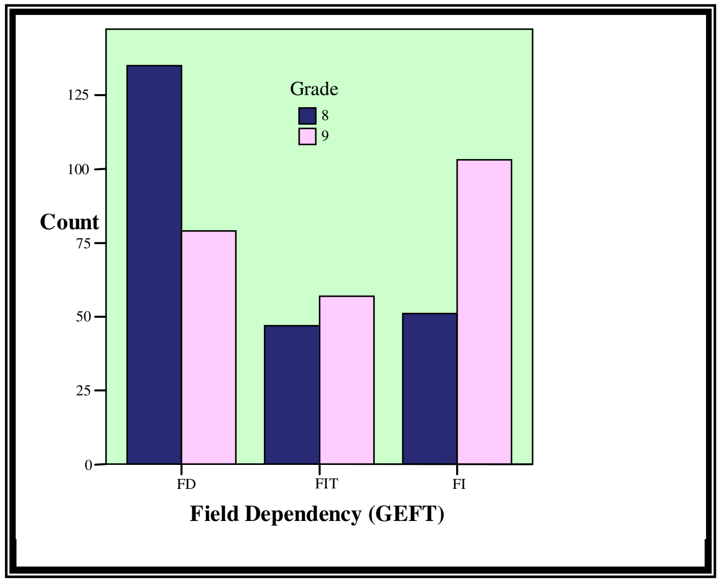


Figure 7-7: Students' field dependency classification according to their grades

42% of grade eight sample categorises as field dependent, whereas slightly more than this proportion (43%) of grade nine sample categorises as Field Independent. A t-test gave a value of $t = 6.0$, $p < 0.001$ and this supports the views of Witkin *et al.* (1971) and Gurley (1984) who thought that field dependency is affected by the subjects ages (see chapter 4). Field intermediate obtains similar proportion in grades eight and nine samples, (36%, 34% respectively). Table 7-7 shows the classification of the whole sample into their field dependency.

Group	NUMBER OF STUDENTS			
	GRADE 8		GRADE 9	
FD	99	42%	54	23%
FIT	83	36%	82	34%
FI	51	22%	103	43%
TOTAL	233	100%	239	100%

Table 7-7: Students' field dependency classification according to their grades

7.7 Mathematics Attainment and Field-Dependency

The same approach was adopted with field dependency. The Pearson correlation value was 0.43, significant at less than the 0.001 level, while in the Christou study (2001) gave a correlation value of 0.50 and the Alenezi (2004) study gave 0.60. The difference in the correlation might be attributable to the different mathematics tests that were used in these experiments. Table 7-8 illustrates that field-independent students achieved better than other groups of students. A scatter diagram for these variables is presented in figure 7-8.

GROUP	MEAN SCORE IN MATHEMATICS
FD	59
FIT	65
FI	75

Table 7-8: The relationship between students' field dependency characteristic and their performance in mathematics

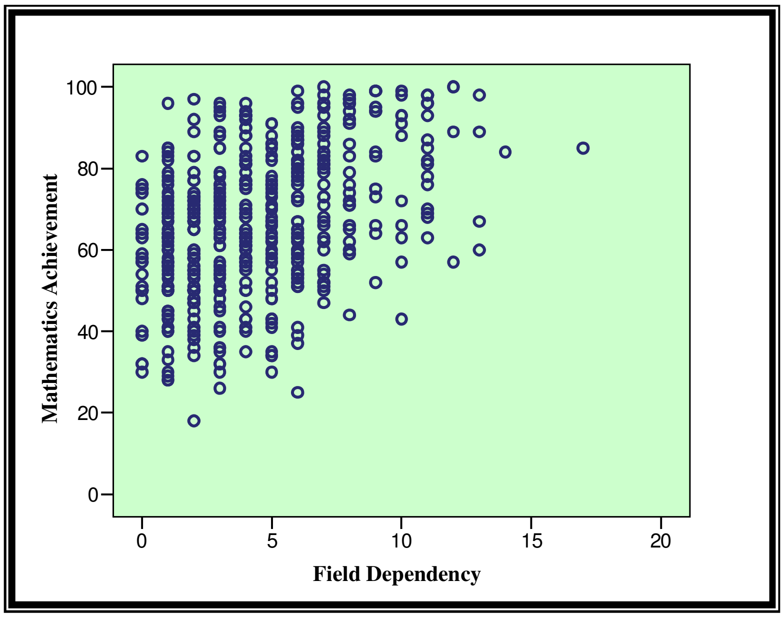


Figure 7-8: Scatter diagram of scores in GEFT related to performance in mathematics

Table 7-9 shows the relationship between students' field dependency characteristics and their performance in mathematics according to their grades.

Group	MEAN SCORE IN MATHEMATICS	
	GRADE 8	GRADE 9
FD	62	56
FIT	70	64
FI	79	74

Table 7-9: The relationship between field dependency and mathematics performance according to grades

7.8 Working Memory, Field-Dependency and Performance

Separate analyses of the performance in mathematics with working memory space and with field dependency indicate that there is a relationship between the two psychological factors and achievement in mathematics. It is worth looking at the influence of these two factors together on the performance in mathematics. When these independent variables, the students' X-space and their degree of field-dependency are put together in the regression model, they explain 21% of the total distribution of the students' performance in mathematics. This indicates a significant correlation between these factors and achievement in mathematics. Table 7-10 below shows the subgroups of students' X-space and their degree of field-dependency with the means of performance in mathematics for each subgroup. The effect of working memory capacity and extent of field dependency is quite considerable, table 7-10 showing a gain of 22% in performance between the least favoured group and the most favoured group.

GROUP	FD	FIT	FI
	Mean score	Mean score	Mean score
X=4	56	60	70
X=5	61	67	76
X=6	62	70	78

Table 7-10: The relationship between field dependency and working memory with mathematics performance

There is a direct relationship between students' achievement in mathematics and their scores in both psychological tests, working memory space and field-dependency. When the scores in these tests are increasing, the achievement in mathematics is also increasing.

Table 7-10 shows that field-dependent students with working memory capacity X = 5 had the same mean scores with field-intermediate students with working memory capacity X = 4. These results might be attributed to the ability of field-independent students with low or intermediate working memory capacity (X = 4, X = 5) to distinguish the important and relevant information from irrelevant information, allowing them to use their working memory space efficiently. Field-dependent students with high working memory capacity, on the other hand, do not have this ability; therefore unimportant and irrelevant items occupy their working memory space. This explanation was offered first by Johnstone (1993).

A Summary: This chapter has examined two cognitive factors that affect achievement in mathematics. The most important findings can be summarised as follows:

- Students with high working space capacity (X = 6) performed better in mathematics than those with lower working memory space capacity (X = 4).

- Field-independent students achieved better than other groups of students.

- There is significant relationship between the two psychological factors (working memory space and field-dependency) and the achievement in the mathematics test.

- Field-independent students with low or intermediate working memory capacity (X = 4, X = 5) perform better in mathematics because of their abilities to distinguish the important and relevant information from irrelevant one, allowing them to use their working memory space efficiently. Field-dependent students with high working memory capacity do not have this ability; therefore, unimportant and irrelevant items occupy their working memory space.

Having looked at two cognitive factors, the next stage is to look at the attitudinal data to see if this points to other factors influencing success in mathematics.

Chapter 8

Attitudes towards Mathematics

Phase One

8.1 Introduction

Several studies have attempted to look at student attitudes in relation to ways of teaching and learning (Thompson & Soyibo, 2002; Reid & Skryabina, 2002a; Berg *et al*, 2003). One study in Scotland (Johnstone & Reid, 1981) offered some guidelines about the fundamental principles that might underpin attitude development in the context of school science but this study was never followed up (Reid, 2006). This chapter explores students' attitudes towards mathematics as follows:

- *The importance of mathematics as discipline;*
- *Students' attitudes towards learning mathematics;*
- *Confidence in learning mathematics ;*
- *And compares students' attitudes towards different topics within mathematics;*
- *The relationship between attitudes and achievement;*
- *The gender difference of attitudes towards mathematics.*

The survey looked at grade eight (age 14) and grade nine (age 15) in typical schools in the State of Kuwait. A sample of 472 Kuwaiti students was involved in this questionnaire, the same sample as in the previous chapter.

GROUP	GRADE 8 (14 years)	GRADE 9 (15 years)	TOTAL
BOY	105	112	217
GIRL	128	127	255
TOTAL	233	239	472

Table 8-1: First phase sample characteristics

Each question was analysed separately. The tables show the response patterns for grade eight and grade nine groups together and then compare their responses. Later in the chapter, the patterns for boys and girls are compared. The data are shown as percentages for clarity. Chi-square was used as a contingency test to compare between groups and was calculated using the actual frequencies.

8.2 General Attitudes towards Mathematics

Tick the box which best represents your opinions	*SA*	*A*	*N*	*D*	*SD*
I usually understand mathematics idea easily	15	26	45	6	8
I do not enjoy mathematics lessons	18	11	25	17	28
I think every one should learn mathematics at secondary school	20	11	16	19	33
I think I am good in mathematics	23	25	23	13	16
You have to born with the right kind of brain, to be good in mathematic	14	11	21	21	33
To be good in mathematics, you have to spend more time studying it	36	23	21	10	10
I think mathematics is useful subject	42	22	20	5	10
I find my mathematics knowledge useful in daily life	32	23	25	10	10

Table 8-2: students' attitudes towards mathematics in general

The purpose of this question is to find out the students' attitudes and feeling towards mathematics in general. The majority indicates that their understanding of mathematics ideas is neutral. Perhaps their understanding varies from topic to topic according to the easiness and the importance of the topic. Despite two thirds of the sample seeing mathematics as a useful subject, half of them disagree that mathematics should be a compulsory subject in secondary school. One quarter of the sample believe that mathematics mastery is a genetic ability; on the other hand, a majority think mathematics mastery depends in the time and the effort that student puts in studying mathematics. The majority think mathematics is useful subject, and find their mathematics knowledge useful in daily life.

Grade 8 **Grade 9**	*SA*	*A*	*N*	*D*	*SD*	χ^2	**df**	**p**
I usually understand mathematics idea easily	16	28	44	6	7	2.7	4	n.s.
	14	24	48	6	9			
I do not enjoy mathematics lessons	14	10	25	17	34	11.4	4	<0.05
	23	12	26	18	22			
I think every one should learn mathematics at secondary school	20	12	20	17	31	5.9	4	n.s.
	19	11	13	21	36			
I think I am good in mathematics	20	12	20	17	31	3.3	4	n.s.
	19	11	13	21	36			
You have to born with the right kind of brain, to be good in mathematic	26	26	23	11	14	0.8	4	n.s.
	21	24	24	14	18			
To be good in mathematics, you have to spend more time studying it	14	10	21	22	32	3.7	4	n.s.
	13	12	20	20	34			
I think mathematics is useful subject	34	26	21	8	11	1.2	4	n.s.
	37	21	22	12	8			
I find my mathematics knowledge useful in daily life	45	23	19	5	9	14.1	4	<0.01
	40	22	22	5	12			

Table 8-3: A comparison between grades 8 & 9 attitudes towards mathematics

There are few differences between year groups. However, there is a decline with age in enjoyment of mathematics lessons and view of the useful of mathematics decline with age.

These changes perhaps can be attributed to increasing difficulties and more abstract concepts.

8.3 Like/Dislike Mathematics

I like mathematics because…	
16%	Hate Mathematics
10%	I am good in it
3%	I do not need to study before the exam
35%	It will help me in my career
11%	I understand its logic
12%	I have always liked it
9%	I always have high mark in it
20%	I have a good teacher
31%	I think it helps me in my life

Table 8-4: The like/dislike of mathematics

In this question students can select as much answers as they want. In spite of the importance of mathematics and the potential enjoyment that students can experience from it, there is a growth of dislike and hate mathematics among students: 16% of the sample state that they hate mathematics, (14% grade eight, 18% grade nine). 'I hate mathematics' is not a category offered in this questionnaire, but the students wrote it in, suggesting very strong views.

Two choices 'it will help me in my career' and 'I think it help me in my life' are rated highly. One fifth like mathematics because they have good teacher and this reflects the important role that teachers play in the learning process. It is interesting to note that the three main reasons for being attracted to mathematics relate to its usefulness (in terms of life and career potential) and the impact of the teacher. This is very similar to the findings in the sciences where, for example, Reid and Skryabina (2002) found that, for physics, the quality of the curriculum, the teacher and the career potential were all factors. By the quality of the curriculum, students were describing learning experiences where they could see what they were doing was related to their lifestyle. Perhaps, the responses of students in relation to mathematics when they suggested that they liked mathematics because it helped in their lives is saying something similar.

8.4 Importance of Mathematics

Do you think mathematics is important?		
Yes, Because 77%	46	Useful in daily life
	24	It will help me in my career
	1	I love mathematics
	1	I do not need to study for the exam
No, Because 22%	4	No, Because… very abstract
	2	I can't understand it
	12	Not related to the real life

Table 8-5: Importance of mathematics

The aim of this question was to find out students' opinions about mathematics importance. The belief that mathematics is important is fairly strong and does not change during the school life, where 83% of grade eight and 71% of grade nine think that mathematics is an important subject. As shown in this table, the vast majority of the sample sees mathematics as an important subject. Mathematics derives its importance from students' beliefs that mathematics is useful in daily life and will help them in their careers. One fifth of the sample believe that mathematics is unimportant and they explained this view mainly because mathematics is *not* seen as related to the real life. The importance of physics is being perceived as related to life was established by Reid and Skryabina (2002) and this seems a more general pattern in many subjects (e.g. Hussein, 2006). This is a real problem in mathematics. While it is of enormous importance in many areas of life and in numerous careers, this is not always obvious to young learners. It is not easy for teachers to bring in applications. The introduction of another level of thought while studying mathematics would almost certainly cause excessive working memory load. The procedures of mathematics have to be taught first before the applications are added.

8.5 Attitudes towards Learning Mathematics

What are your opinions?							
I am confident in mathematics classes	28	17	18	13	8	16	I am not confident
Mathematics is too abstract for me	19	7	16	16	18	25	Mathematics is too easy
I am getting worse at mathematics	14	5	12	18	17	35	I am getting better
I feel I am coping well	33	17	16	13	9	12	I feel I am not coping well
Mathematics classes are boring	28	5	13	18	11	24	interesting

Table 8-6: Attitudes towards learning mathematics

The aim of the question was to find out students' opinions about learning mathematics. In general, students tend to be positive although many express fairly neutral views. One exception relates to mathematics being seen as boring where a very considerable proportion hold a negative view. To clarify this trend a comparison between grades eight and nine is considered.

Grade 8 / Grade 9							χ^2	df	p
I am confident in mathematics classes	31	20	16	13	7	13	8.5	5	n.s.
	26	14	20	13	8	20			
Mathematics is too abstract for me	15	8	15	15	17	29	5.7	5	n.s.
	23	6	16	16	18	21			
I am getting worse at mathematics	10	5	12	16	20	37	8.5	5	n.s.
	18	4	12	19	14	34			
I feel I am coping well	33	21	17	13	7	10	8.6	5	n.s.
	32	14	16	13	11	15			
Mathematics classes are boring	21	4	14	19	12	31	18.4	5	<0.05
	35	7	12	18	10	18			

Table 8-7: A comparison between grades 8 & 9 attitudes towards learning mathematics

The above table shows the distribution of grade eight and grade nine student's responses and chi-square values. Again, there are few differences with age although it is a matter of concern that the perception of mathematics as boring grows with age. Several other comparisons are close to significance at the 5% level. Of considerable interest is the area of confidence. Views are generally polarised, suggesting that a significant minority lack confidence. Student's confidence about his/her ability is a crucial variable in the learning processes, and it is argued that confidence is an attitude towards oneself and it depends heavily on experience (Oraif, 2007).

8.6 Mathematics Preference among Other Subjects

Tick your class preferences:							
Arabic Language	38	6	14	9	6	27	**Mathematics**
Mathematics	24	5	11	10	6	45	**English**
Mathematics	18	6	10	9	6	50	**Geography**
Science	38	9	17	12	3	21	**Mathematics**

Table 8-8: The preference of mathematics among other subjects

In this question, students were asked to present their preferences between mathematics and Arabic language, English, Geography and Science. Students' views are highly polarised: they like mathematics or hate it relative to other subjects. Perhaps, the majority of the students prefer geography because of its usefulness and easiness. Furthermore, geography provides the students with information about their environment. In all cases, mathematics occupies a low position compared with other subjects. This negative attitude might be attributable to the fact that "*mathematics does not involve the learner in revealing emotions or opinions and hardly involves, of absolute necessity, any interaction with others*" (Orton, 2004: p: 154).

8.7 Attitudes towards Different Topics

Tick your class preferences:							
Fractions	18	3	14	16	7	41	**Geometry**
Sets	68	9	11	5	2	6	**Fractions**
Algebra	12	2	6	11	10	58	**Sets**
Geometry	38	5	10	12	3	31	**Linear equation**

Table 8-9: Attitudes towards different topics in mathematics

This question aimed to find out students' attitudes towards different topics in mathematics. Again, students' views are highly polarised. The above table shows that students' attitudes towards different topics within mathematics differ from topic to topic. This variation may depend on student's confidence about the topic and the easiness of it. Topics such geometry and sets are highly preferred. On the other hand, topics such as fractions and algebra occupy low positions because these topics are very difficult and demand a highly structured mind to work with them. However, overall, the differences may simply reflect the way various topics are treated and the level of demand set in the school syllabus

8.8 Mathematics as a Subject

Think about Mathematics as a subject:							
Abstract	19	9	16	18	12	26	**Not abstract**
Difficult	31	9	16	19	12	13	**Easy**
Unrelated to life	19	5	9	17	13	37	**Related to life**
Boring	30	6	12	19	10	23	**Interesting**
Not useful for careers	13	3	9	12	11	52	**Useful for careers**
Complicated	35	8	11	17	13	14	**Straightforward**

Table 8-10: Attitudes towards mathematics as a subject

The aim of this question was to explore students' opinions towards mathematics as a subject. The strong observation is that, despite students' beliefs that mathematics is important and related to life, the high proportions of students see mathematics as a difficult and complicated subject. The tendencies become more neutral in their views about the abstract nature of mathematics and their interest in mathematics as a subject. It is noteworthy that the view that the problem with mathematics is its abstractness is not really supported by the data here. Abstraction, on its own, is not a fundamental problem. When abstraction leads to working memory overload, it will be a problem and when abstraction leads to a loss of reality for the learner, then attitudes may deteriorate. The work of Hussein (2006) in chemistry illustrates these principles well.

It is possible to correlate the students' views of their confidence in mathematics with their attitudes towards mathematics as a subject. This is done using Kendall's Tau-b correlation, the data being ordinal. The correlation values between mathematics confidence and coping well in mathematics r =0.52; interest in mathematics classes r = 0.48; seeing mathematics as easy r = 0.47; and straightforward subject, r = 0.46, all highly significant at the 0.1% level. Students who lack the confidence in doing mathematics are seeing mathematics as abstract, difficult, boring and a complicated subject. It is difficult to know what causes what here. Do the abstraction, difficulty, complications and feeling of boredom cause lack of confidence or is it simply that these various views happen to tend to occur in the same students?

	Grade 8			Grade 9			χ^2	df	p
Abstract	14	10	17	15	14	29	15.7	5	<0.01
	23	7	16	21	10	23			
Difficult	27	11	15	20	13	14	5.8	5	n.s.
	35	8	16	18	12	11			
Unrelated to life	16	4	10	17	13	41	7.0	5	n.s.
	22	7	8	18	13	33			
Boring	21	5	11	23	12	27	20.8	5	<0.001
	39	7	12	15	9	19			
Not useful for careers	7	3	7	14	10	59	12.9	5	<0.05
	18	4	11	11	12	45			
Complicated	29	9	10	20	17	17	18.8	5	<0.01
	42	8	13	16	9	13			

Table 8-11: A comparison between grades 8 & 9 attitudes towards mathematics as a subject

In thinking about mathematics as a subject, there are considerable variations between the year groups. The proportions seeing mathematics as abstract, boring, not useful for careers and complicated grow with age while the proportions seeing mathematics as not abstract, interesting, useful for career and straight forward decline with age. These negative attitudes may be attributed again to the increasing demand levels as the student progresses through the mathematics curriculum.

8.9 Mathematics Classes

Think about your Mathematics classes	*SA*	*A*	*N*	*D*	*SD*
I do not understand what is taught	9	17	47	25	5
I find doing mathematics problems repetitive	11	20	38	20	11
The explanations are not clear	12	16	29	20	23
I am not sure what t should be doing	12	17	30	21	20
I find I make many mistakes	13	18	35	27	8
There is too much homework	20	16	25	25	14

Table 8-12: Attitudes towards mathematics classes

Students' reactions in mathematics classes are remarkably neutral with very positive views sometimes being expressed. The majority have a neutral tendency towards mathematics classes, they feel their understanding depends on the difficulty degree of any topic, the explanations are clear and they do not make many mistakes in mathematics classes. This tendency suggests that mathematics classes themselves are not the main cause of mathematics problem.

Grade 8 / Grade 9	*SA*	*A*	*N*	*D*	*SD*	χ^2	df	p
I do not understand what is taught	6	14	52	24	4	5.3	4	n.s.
	8	19	42	26	5			
I find doing mathematics problems repetitive	13	16	40	21	11	4.7	4	n.s.
	10	23	36	19	12			
The explanations are not clear	11	12	30	20	28	11.1	4	<0.05
	14	20	29	19	18			
I am not sure what should be doing	10	17	28	23	22	2.8	4	n.s.
	14	17	31	19	19			
I find I make many mistakes	11	17	34	29	9	3.5	4	n.s.
	15	19	35	24	7			
There is too much homework	17	14	28	25	17	7.8	4	n.s.
	23	18	23	25	11			

Table 8-13: A comparison between grades 8 & 9 attitudes towards mathematics classes

Few significant differences are found between grades eight and nine students with the older students tending to agree with statement ‘the explanations are not clear’.

8.10 Mathematics Tests

Think about Mathematics Tests	*SA*	*A*	*N*	*D*	*SD*
I tend to panic with difficult problems	37	20	27	9	7
They involve a lot of revision the day before	40	18	22	12	8
I find I am short of time	25	17	29	17	11
I often make mistakes	16	17	34	24	10
I cannot remember how to do things	18	18	33	21	11
There is little opportunity to explain things	27	19	27	17	10

Table 8-14: Attitudes towards mathematics examinations

The aim of this question was to explore students' opinions about their performance in mathematics tests and examinations. The vast majority of the students indicated that they tend to panic if they face difficult problems, they spend a long time in revision the day before, and they find themselves short of time during mathematics’ examinations. This negative tendency reflects the difficulty of mathematics examinations.

Students tend to see their problems in mathematics as not related to their abilities in mathematics, and this is clear from the responses to the statements ‘I often make mistakes’ and ‘I cannot remember how to do things’. Instead, they think difficult problems, short time, long curriculum and the little opportunity to explain things during the examination cause these problems.

Grade 8 Grade 9	SA	A	N	D	SD	χ^2	df	p
I tend to panic with difficult problems	31	20	33	10	7	11.0	5	<0.05
	43	20	21	8	7			
They involve a lot of revision the day before	41	19	20	13	7	1.9	4	n.s.
	40	16	24	11	9			
I find I am short of time	20	19	25	20	16	20.3	4	<0.001
	30	16	34	14	6			
I often make mistakes	13	17	31	25	13	8.3	4	n.s.
	18	17	36	23	6			
I cannot remember how to do things	16	19	33	22	10	1.0	4	n.s.
	19	17	33	21	11			
There is little opportunity to explain things	22	19	29	18	12	8.1	4	n.s.
	32	19	26	17	7			

Table 8-15: A comparison between grades 8 & 9 attitudes towards mathematics examinations

Few significant differences are found between grades eight and nine students but the older students tend to panic more with difficult problems and see a shortage of time during the examinations.

A Summary: tables (8-13), (8-14), and (8-15) offer a fascinating picture about problems in mathematics education. During mathematics classes, the majority indicate that they tend to understand what is taught, the explanations are clear for them, they are sure about what they should be doing, and do not make a lot of mistakes. On the other hand, in mathematics examinations, students tend to panic with difficult problems, find themselves short of time, they spend a long time in revision the day before, and there is a little opportunity to explain things during the examination. Overloaded curricula and examinations cause mathematics problems. It is clear that mathematics problems are seen to occur during the examinations and not in mathematics classes and, as a consequence of that, students lose their confidence and this may lead to prevent them from making the required effort for success in mathematics. However, this is what the students are saying. It might not reflect the reality. Examinations determine students' abilities and tend to make students conscious of what they see as their successes and failures.

8.11 Reasons for Studying Mathematics

Here are some reasons why students should study Mathematics at school Place these reasons in order, showing which is the most important for you	
1	A. It is useful in daily life
2	C. It is important for some other subjects
3	G. It is a useful way to make sense of the world
4	B. There are many jobs for mathematicians
5	D. It teaches me to think logically
6	F. It is important for many courses at university
7	E. Mathematics can help solve world problems
8	H. It is very satisfying

Table 8-16: Students' views about why they should study mathematics

The top three choices are A, C and G. It is not easy to see how mathematics, as taught in school, is useful in daily life or helps them to make sense of the world. Do students need to prove theories or solve equations in daily life? Indeed, these responses are surprising. On the other hand, it is easy to see that mathematics is important in other subjects.

8.12 Most Helpful Ways in Studying Mathematics

Here are some things, which can help me in my mathematics studies *Tick the three which are most helpful for you*	
64%	Practicing many mathematics exercises and problems until I get them right.
8%	Reading my textbook carefully.
32%	Working with my friends until I understand the ideas.
21%	Seeking help from my parents.
18%	Try to see things as pictures or diagrams.
64%	Following the methods taught by my teacher carefully.
31%	Making sure I understand what I am doing.
60%	Trying to find a method which always gives the right answer.

Table 8-17: Some ways help students in learning mathematics

Students were asked to choose three methods that they think are most helpful in mathematics studies. 'Practicing many mathematics exercises and problems until I get them right' and 'following the methods taught by my teacher carefully' were chosen as the most helpful ways for success in mathematics. 60% chose the last method which is 'trying to find a method which always gives the right answer' as the third most helpful way. It is interesting to note that all three are aspects of the very nature of mathematics: success comes when routine procedures can be identified and carried out with reliability. In the eyes of the students, understanding is seen as much less important, with about half the rating of the first three. However, this is not too surprising in Kuwait where, like many other countries, the rewards in mathematics come from the correct completion of procedures in order to get 'right' answers.

8.13 Sex-Related Differences in Attitudes towards Mathematics

This section discusses comparisons between boy's and girl's attitudes toward mathematics. In order to test the significance of the comparisons, the chi-square test was applied as a contingency test.

Tick the box which best represents your opinions	Boy			Girl		χ^2	df	p
I usually understand mathematics idea easily	18	27	40	5	11	10.2	4	<0.05
	13	24	51	7	6			
I do not enjoy mathematics lessons	15	8	26	18	33	11.0	4	<0.05
	21	14	25	17	23			
I think every one should learn mathematics at secondary school	29	9	16	13	34	28.1	4	<0.001
	12	14	17	25	33			
I think I am good in mathematics	23	23	25	11	18	2.3	4	n.s.
	23	26	22	14	14			
You have to born with the right kind of brain, to be good in mathematic	14	13	25	15	34	11.8	4	<0.05
	13	10	17	27	33			
To be good in mathematics, you have to spend more time studying it	37	22	20	10	12	2.7	4	n.s.
	35	24	23	10	8			
I think mathematics is useful subject	51	21	16	3	9	15.2	4	<0.01
	35	24	23	7	11			
I find my mathematics knowledge useful in daily life	44	25	19	4	8	33.7	4	<0.001
	23	22	29	14	11			

Table 8-18: A comparison between boy's & girl's attitudes towards mathematics in general

It can be seen from the table above that more boys than girls stated that

- *They understand mathematics ideas easily,*
- *They enjoy mathematics lessons,*
- *Think everyone should learn mathematics at secondary school,*
- *Think mathematics is useful subject*
- *Find their mathematics knowledge useful in daily life.*

Overall, the boys are more positive. This could reflect a genuine more positive attitude or it might simply be a function of the greater confidence of boys at this age in the Kuwaiti culture, with the girls being more hesitant and unsure.

What are your opinions? Boy Girl								χ^2	df	p
I am confident in mathematics classes	32	20	14	13	5	16	**I am not confident in mathematics classes**	10.3	5	n.s.
	26	14	21	13	10	17				
Mathematics is too abstract for me	20	7	18	17	17	21	**Mathematics is too easy for me.**	4.5	5	n.s.
	18	7	14	14	19	28				
I am getting worse at mathematics	13	5	11	18	18	37	**I am getting better at mathematics**	1.5	5	ns
	15	5	13	18	16	34				
I feel I am coping well	33	19	17	15	6	10	**I feel I am not coping well**	6.1	5	n.s.
	33	16	15	12	11	14				
Mathematics classes are boring	24	5	12	19	13	27	**Mathematics classes are interesting**	7.1	5	n.s.
	33	5	13	18	9	22				

Table 8-19: A comparison between boy's & girl's beliefs about their abilities in learning mathematics

The table above shows that there are no significant differences in the perceptions of the boys and girls.

Think about Mathematics as a subject: Boy Girl								χ^2	df	p
Abstract	20	5	20	18	12	26	**Not abstract**	9.3	5	n.s.
	18	11	13	19	13	26				
Difficult	29	9	15	19	10	18	**Easy**	10.1	5	n.s
	32	10	17	19	14	8				
Unrelated to life	17	6	8	11	13	46	**Related to life**	18.3	5	<0.01
	20	5	9	23	13	29				
Boring	25	4	12	21	12	27	**Interesting**	8.8	5	n.s.
	34	8	11	17	9	21				
Not useful for careers	14	1	10	13	9	53	**Useful for careers**	7.2	5	n.s.
	12	5	8	12	13	51				
Complicated	33	7	12	20	12	16	**Straight forward**	4.5	5	n.s.
	38	9	11	15	13	13				

Table 8-20: A comparison between boy's & girl's attitudes towards mathematics as a subject.

The table above shows more girls are seeing mathematics as unrelated to life.

Think about your Mathematics classes	Boy				Girl	χ^2	df	p
I do not understand what is taught	8	17	46	23	6	4.3	4	n.s.
	6	17	48	27	4			
I find doing mathematics problems repetitive	13	20	38	16	13	4.0	4	n.s.
	10	20	37	23	11			
The explanations are not clear	17	14	28	18	23	7.4	4	n.s.
	9	17	30	21	23			
I am not sure what I should be doing	17	13	28	24	20	15.2	4	<0.01
	8	21	32	18	21			
I find I make many mistakes	17	18	30	24	11	12.3	4	n.s.
	9	18	38	28	6			
There is too much homework	24	12	24	22	17	11.8	4	<0.05
	17	18	26	29	11			

Table 8-21: A comparison between boy's & girl's attitudes towards mathematics classes

The above table illustrates more boys than girls stated that they are not sure what should doing and that there is too much homework. It is not obvious why they should say the former but the latter probably reflects the lower levels of diligence of boys compared to girls.

Think about Mathematics Tests	Boy				Girl	χ^2	df	p
I tend to panic with difficult problems	32	20	31	8	9	9.7	4	<0.05
	42	21	23	10	5			
They involve a lot of revision the day before	40	24	21	9	7	15.3	4	<0.01
	40	12	23	15	9			
I find I am short of time	27	18	29	17	10	1.7	4	n.s.
	23	17	30	18	12			
I often make mistakes	18	14	36	20	11	6.9	4	n.s.
	13	19	32	27	9			
I cannot remember how to do things	20	17	34	20	11	2.3	4	n.s.
	16	19	32	23	11			
There is little opportunity to explain things	28	19	27	17	9	0.5	4	n.s.
	26	19	28	18	10			

Table 8-22: A comparison between boy's & girl's attitudes towards mathematics examinations

More girls tend to panic with difficult problems while boys are more concerned about revision. Boys tend to be confident, and girls tend to feel worried.

A Summary: Overall, there is considerable polarisation of views with some students holding very positive views while other hold very negative views. The most important findings from the data can be summarised as follows:

- Students' believe that mathematics is a useful subject in daily life, useful for their careers; and it is useful for other subjects.
- Students see mathematics as abstract, difficult and complicatedt but the abstraction itself is not the source of the difficulty.
- In spite of students' beliefs about the importance of mathematics and the potential enjoyment that students can experience from it, there is a growth of dislike for mathematics among students with age (from grade 8 to grade 9).
- The mathematics teacher plays a very important role in learning mathematics and in forming students' attitudes towards learning mathematics.
- Boys' attitudes towards mathematics are more positive than girls' attitudes, and this might be attributable to the masculine and feminine natures that boys tend to be confident, and girls tend to feel worried.

The next section considers how their attitudes relate to their performance in mathematics examinations.

8.14 Mathematics Performance and Attitudes towards Mathematics

Table 8-23 presents the correlations (using Kendall's tau-b) between mathematics performance and attitudes towards mathematics in general. In this question, responses ranged from *strongly agree* to *strongly disagree*. A negative correlation coefficient appears sometimes because the question is in negative form. Obviously, there are high correlations between mathematics performance and three questions 'understanding mathematics ideas easily', 'I do not enjoy mathematics lessons' and 'I think I am good in mathematics'.

Correlation between attitudes towards mathematics and performance in mathematics *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	**Correlation Coefficient**	
	Grade (8) ***N=(233)***	**Grade (9) N=(239)**
I usually understand mathematics idea easily	**0.24**	**0.36**
I do not enjoy mathematics lessons	**-0.22**	**-0.19**
I think every one should learn mathematics at secondary school	**0.09**	**0.20**
I think I am good in mathematics	**0.31**	**0.40**
You have to born with the right kind of brain, to be good in mathematic	**-0.06**	**-0.04**
To be good in mathematics, you have to spend more time studying it	**-0.09**	**-0.04**
I think mathematics is useful subject	**0.20**	**0.14**
I find my mathematics knowledge useful in daily life	**0.10**	**0.11**

Table 8-23: Correlations between mathematics performance and attitudes towards mathematics

It can be seen from the table above that students who obtained high marks in mathematics tend to feel that

- *They understand mathematics ideas easily*
- *They are enjoying mathematics lessons*
- *Their mathematics abilities are good.*
- *They believe mathematics is a useful subject*
- *They find their mathematics knowledge useful in daily life.*

This tendency is higher for grade nine than grade eight.

The correlations between students' performance in mathematics and their beliefs about their abilities in mathematics classes are presented in table 8-24.

Correlation between attitudes towards mathematics and performance in mathematics $P < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade (8) *N=(233)*	**Grade (9)** *N=(239)*
I am confident in mathematics classes	**0.22**	**0.29**
Mathematics is too abstract for me	**-0.12**	**-0.27**
I am getting worse at mathematics	**-0.20**	**-0.33**
I feel I am coping well	**0.30**	**0.38**
Mathematics classes are boring	**-0.18**	**-0.28**

Table 8-24: Correlations between mathematics performance & beliefs about abilities in mathematics

The above table shows students who achieved high marks in mathematics tend to feel

- *Confident in mathematics classes.*
- *Mathematics is somewhat abstract.*
- *They are getting better at mathematics.*
- *They are coping well*
- *Mathematics classes are interesting for them.*

Here also, these patterns are higher in grade nine than grade eight.

The correlations between students' opinions about mathematics as a subject and their performance in it can be seen in table 8-25. Students' views about mathematics as a subject are correlated significantly with their attainment in it. Students who have gained low marks in mathematics are seeing mathematics as abstract, difficult, boring and complicated subject, although they believe mathematics is a useful subject for daily life and for careers. The tendency is again stronger for grade nine.

Correlation between attitudes towards mathematics and performance in mathematics $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade (8) *N=(233)*	**Grade (9)** *N=(239)*
Abstract	**-0.17**	**-0.24**
Difficult	**-0.21**	**-0.31**
Unrelated to life	**-0.05**	**-0.11**
Boring	**-0.19**	**-0.29**
Not useful for careers	**-0.09**	**-0.13**
Complicated	**-0.13**	**-0.27**

Table 8-25: Correlations between attitudes about mathematics as a subject & performance

Students' attitudes towards mathematics classes and examinations are correlated with their performance in table 8-26 and table 8-27. In these questions, five (always, often, sometimes, rarely, never) responses were invited, and a negative correlation appears because all the questions are in the negative form.

Correlation between attitudes towards mathematics and performance in mathematics $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade (8) *N=(233)*	**Grade (9)** *N=(239)*
I do not understand what is taught	**-0.18**	**-0.22**
I find doing mathematics problems repetitive	**-0.13**	**-0.07**
The explanations are not clear	**-0.26**	**-0.25**
I am not sure what t should be doing	**-0.20**	**-0.33**
I find I make many mistakes	**-0.30**	**-0.32**
There is too much homework	**-0.32**	**-0.21**

Table 8-26: Correlations between attitudes towards mathematics classes & performance

Low performance students tend to feel

- *They do not understand what is taught*
- *They find doing mathematics problems repetitive.*
- *The explanations are not clear for them*
- *They are not sure what should be doing*
- *They are making many mistakes.*

Correlation between attitudes towards mathematics and performance in mathematics $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade (8) *N=(233)*	**Grade (9)** *N=(239)*
I tend to panic with difficult problems	**-0.19**	**-0.16**
They involve a lot of revision the day before	**-0.21**	**-0.26**
I find I am short of time	**-0.16**	**-0.17**
I often make mistakes	**-0.28**	**-0.32**
I cannot remember how to do things	**-0.25**	**-0.30**
There is little opportunity to explain things	**-0.14**	**-0.11**

Table 8-27: Correlations between attitudes towards mathematics examinations & performance

Students with low performances in mathematics tend to

- *Panic with difficult problems*
- *Spend long time in revision the day before the test.*
- *Find themselves short of time*
- *They often make mistakes*
- *They cannot remember how to do things.*

A summary: looking at the questions responses at:

- The question relating to performance correlate positively with the students performance ('I think I am good at mathematics' correlate with student performance in mathematics $r = 0.35$, $p < 0.001$, 'I feel I am coping well' correlate with student performance in mathematics, $r = 0.34$, $p < 0.001$). These data support the validity of the survey.

- The tendency in grade 9 is higher than grade 8

- Students' confidence in mathematics is highly correlated with students' performance in mathematics. Students who state that they are confident are those who perform better in mathematics classes and those who state they are unconfident are those who do not achieve in mathematics. Thus, in this case loss of confidence may lead the students to failure and failure in mathematics classes or examinations may lead the students to lose their confidence further. This is a vicious cycle.

- The students who responded positively to the questions about whether they like mathematics, whether they enjoy mathematics lessons, whether they are coping well and whether they believe they are good at mathematics performed better at mathematics. There is the question of what causes what: do the poor attitudes generate poor maths performance or, as is more likely, the poor performance generates poor attitudes. Each 'feeds off' the other.

This chapter has looked at attitude patterns in relation to mathematics for two age groups showing a slight deterioration with age. There are few differences in attitudes that are gender related. The relationship of attitudes to performance is also to be considered and this has identified some areas of concern. The next section considers cognitive factors.

8.15 Working Memory and Attitudes towards Mathematics

Working memory capacity is known to be a controlling factor in understanding (Johnstone, 1997). If understanding leads to positive attitudes, then it is possible that working memory capacity determines attitudes relating to mathematics. This was explored and this section discusses the outcomes.

The correlation between working memory space and attitudes towards mathematics is shown in the following table. There are very low but significant correlations with the understanding of mathematics; the compulsion of learning mathematics in secondary school; and the usefulness of mathematics (but only in grade 9 although the patterns of correlations are similar in grade 8).

Correlation between attitudes towards mathematics and working memory capacity space ($p < 0.05$, $p < 0.01$, $p < 0.001$)	**Correlation Coefficient**	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
I usually understand mathematics ideas easily	**0.03**	**0.19**
I do not enjoy mathematics lessons	**-0.09**	**-0.07**
I think every one should learn mathematics at secondary school	**0.10**	**0.12**
I think I am good in mathematics	**0.01**	**0.12**
You have to born with the right kind of brain, to be good in mathematic	**0.09**	**0.02**
To be good in mathematics, you have to spend more time studying it	**0.10**	**0.02**
I think mathematics is useful subject	**0.03**	**0.11**
I find my mathematics knowledge useful in daily life	**0.02**	**0.08**

Table 8-28: Correlations between working memory & attitudes towards mathematics

Grade 9 students with high working memory capacity tend slightly to:

- *Understand mathematics ideas easily;*
- *Think they are good in mathematics;*
- *Think that every one should study mathematics in secondary school; and*
- *Think mathematics is useful subject.*

In essence, higher working memory capacity offers an advantage in achieving success in mathematics and success is related to positive attitudes.

The correlations between students' beliefs about their abilities in mathematics classes and their working memory capacity are presented in the table 8-29. The same pattern of low but significant correlations is apparent for grade 9 with grade 8 showing a similar, if insignificant, pattern. Students with high working memory tend to feel confident and coping well in mathematics classes; where students with low working memory tend to feel that mathematics is too abstract for them; getting worse in mathematics classes and they weary from mathematics classes. The higher working memory is a small advantage.

Correlation between attitudes towards mathematics and working memory capacity space *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
I am confident in mathematics classes	**0.01**	**0.14**
Mathematics is too abstract for me	**-0.05**	**-0.09**
I am getting worse at mathematics	**-0.07**	**-0.11**
I feel I am coping well	**0.07**	**0.11**
Mathematics classes are boring	**-0.01**	**-0.18**

Table 8-29: Correlations between beliefs about abilities in mathematics classes & working memory

The correlation between students' beliefs about mathematics as a subject and their working memory capacities are shown in the following table. It can be seen from table 8-30 that low working memory grade 9 students to a slight extent believe that mathematics is abstract, difficult, unrelated to life, boring, not useful to careers and complicated compared to high working memory students. This tendency is higher in grade 9 because of the higher demand of the course at this level.

Correlation between attitudes towards mathematics and working memory capacity space *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
Abstract	**-0.09**	**-0.15**
Difficult	**0.03**	**-0.16**
Unrelated to life	**-0.04**	**-0.05**
Boring	**-0.01**	**-0.15**
Not useful for careers	**-0.02**	**-0.06**
Complicated	**0.01**	**-0.17**

Table 8-30: Correlations between beliefs about mathematics as a subject & working memory

Students' attitudes towards mathematics classes and their working memory capacity are correlated in the table 8-31. Low working memory grade 9 students to a slight extent believe they do not understand what is taught in mathematics classes; the explanations are not clear, make many mistakes and there is too much homework. It is likely that these negatives tendencies are being influenced in part by the limiting capacity of working memory which makes understanding difficult.

Correlation between attitudes towards mathematics and working memory capacity space $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
I do not understand what is taught	**-0.08**	**-0.12**
I find doing mathematics problems repetitive	**0.04**	**0.07**
The explanations are not clear	**0.01**	**-0.15**
I am not sure what t should be doing	**0.02**	**-0.10**
I find I make many mistakes	**-0.09**	**-0.13**
There is too much homework	**0.03**	**-0.16**

Table 8-31: Correlations between attitudes towards mathematics classes & working memory

The relationships between students' working memory capacities and their attitudes toward mathematics examinations are illustrated in table 8-32. Low working memory students tend slightly to feel they are short of time during the mathematics examinations; they make many mistakes and cannot remember how to do things. The same pattern for grades 8 and 9 can be seen.

Correlation between attitudes towards mathematics and working memory capacity space $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
I tend to panic with difficult problems	**0.01**	**-0.10**
They involve a lot of revision the day before	**-0.03**	**-0.10**
I find I am short of time	**-0.12**	**-0.17**
I often make mistakes	**-0.04**	**-0.22**
I cannot remember how to do things	**-0.06**	**-0.15**
There is little opportunity to explain things	**0.03**	**-0.05**

Table 8-32: Correlations between working memory & attitudes toward mathematics examinations

A Summary: It might be expected that working memory capacity would have weak correlations with various attitudes relating to the learning of mathematics. This has been shown to be the case for grade 9 although the grade 8 data are very similar but not as marked. Although significant, the correlations tend not to be strong. This could be explained because the effect is second order. Working memory capacity influences understanding in conceptual areas. Understanding is the natural process of learning which brings innate satisfaction. Where understanding is difficult, then attitudes will be less positive. However, the difference between grades 8 and 9 is interesting. Is it possible that grade 10 will show even further deterioration? Is this then the basis for the observed steady fall in positive attitudes relating to mathematics with age with significant samples of school students as they progress? Is working memory capacity, indirectly at least, a major factor in the generation of so many who do not like mathematics (indeed often strongly do not like it)?

It has been observed that there is a stronger polarisation of view about most aspects of mathematics when compared to other subject areas like the sciences and languages (Alhmali, 2007). In mathematics tests and examinations, answers tend to be 'right' or 'wrong' in very clear cut terms. Therefore, failure to understand is very obvious. There is a weakness in this argument in that it assumes that tests and examinations in mathematics do in fact test understanding. They may simply be testing the correct use of memorised procedures. However, this carries with it negative perceptions for it does appear that the natural (and satisfying) way to learn is to gain understanding.

8.16 Field Dependency and Attitudes towards Mathematics

The correlations between the extent of field dependency and their attitudes towards mathematics are discussed in this section. Field dependency can be seen as one way to use limited working memory space more efficiently (Johnstone, 1997). It has also been shown repeatedly that being field independent is a considerable advantage in most learning. It never appears to be a disadvantage (Danili & Reid, 2006).

The extent of field dependency does correlate significantly with their understanding mathematics ideas; their enjoyment in mathematics lessons; the compulsion of learning mathematics in secondary school; the time that is spend in learning mathematics; and the usefulness of mathematics. The same pattern seen with working memory capacity shows again here: grades 8 and 9 are similar but the grade 9 effect is more marked.

Correlation between attitudes towards mathematics and field dependency $p < 0.05$ $p < 0.01$ $p < 0.001$	**Correlation Coefficient**	
	Grade 8 *(N=233)*	**Grade 9** *(N=239)*
I usually understand mathematics idea easily	**0.05**	**0.23**
I do not enjoy mathematics lessons	**-0.10**	**-0.14**
I think every one should learn mathematics at secondary school	**0.05**	**0.11**
I think I am good in mathematics	**0.08**	**0.32**
You have to born with the right kind of brain, to be good in mathematic	**-0.04**	**-0.04**
To be good in mathematics, you have to spend more time studying it	**0.11**	**0.09**
I think mathematics is useful subject	**0.03**	**0.11**
I find my mathematics knowledge useful in daily life	**0.02**	**0.07**

Table 8-33: Correlations between field dependency & attitudes towards mathematics

Field independent students tend to:

- *Understand mathematics idea easily;*
- *Enjoy mathematics lessons;*
- *Think every one should learn mathematics in secondary school;*
- *Think they are good in mathematics;*
- *Think spending more time in studying mathematics helps to improve their ability;*
- *Think mathematics is useful subject.*

The effects are more marked when considering their self-perceptions in relation learning in mathematics (table 8-34). Those who are field independent tend to be confident, coping well and getting better with those who are field dependent tending to see mathematics as abstract and boring.

Correlation between attitudes towards mathematics and field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 *(N=239)*	**Grade 9** *(N=239)*
I am confident in mathematics classes	**0.11**	**0.26**
Mathematics is too abstract for me	**-0.12**	**-0.27**
I am getting worse at mathematics	**-0.02**	**-0.20**
I feel I am coping well	**0.14**	**0.24**
Mathematics classes are boring	**-0.05**	**-0.22**

Table 8-34: Correlations between field dependency & beliefs about mathematics ability

Their views of mathematics also show significant correlations (table 8-35) although lower. Field dependent students tend to see mathematics as abstract; difficult; unrelated to life; boring; not useful to careers; and a complicated subject. Perhaps, those who are field dependent do not cope as well and this generates a range of negative perceptions.

Correlation between attitudes towards mathematics field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 *(N=239)*	**Grade 9** *(N=239)*
Abstract	**-0.06**	**-0.20**
Difficult	**-0.09**	**-0.29**
Unrelated to life	**-0.03**	**-0.11**
Boring	**-0.05**	**-0.21**
Not useful for careers	**-0.01**	**-0.07**
Complicated	**-0.12**	**-0.24**

Table 8-35: Correlations between field dependency & attitudes towards mathematics as a subject

The correlations between field dependency and their attitudes towards mathematics classes and examinations are shown in tables 8-36 and table 8-37.

Correlation between attitudes towards mathematics and field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 (N=239)	Grade 9 (N=239)
I do not understand what is taught	-0.09	-0.23
I find doing mathematics problems repetitive	-0.08	-0.04
The explanations are not clear	-0.16	-0.21
I am not sure what t should be doing	-0.13	-0.25
I find I make many mistakes	-0.16	-0.23
There is too much homework	-0.04	-0.12

Table 8-36: Correlations between field dependency & attitudes towards mathematics classes

Correlation between attitudes towards mathematics and field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade 8 (N=239)	Grade 8 (N=239)
I tend to panic with difficult problems	-0.10	-0.11
They involve a lot of revision the day before	-0.19	-0.18
I find I am short of time	-0.002	-0.11
I often make mistakes	-0.19	-0.24
I cannot remember how to do things	-0.16	-0.22
There is little opportunity to explain things	-0.13	-0.13

Table 8-37: Correlations between field dependency & attitudes towards mathematics examinations

Field dependent students feel

- *They do not understand what is taught;*
- *The explanations not clear for them;*
- *They are not sure about what they should do;*
- *They do make many mistakes in mathematics classes and examinations;*
- *They tend to panic if they face a difficult problem in the examinations;*
- *They are involved in a lot of revision on the day before the test; and*
- *There is little opportunity to explain things during the exam time.*

These negative tendencies are higher in grade nine than grade eight.

A Summary: Looking at the patterns of correlations obtained by relating the extent of field dependency to the responses to the attitude survey, there is a very similar pattern to those obtained when considering working memory capacity. In other words, while grade 9 shows the patterns more strongly, the two year groups are similar, and, in general, being field independent is related to holding more positive attitudes.

It is possible that those who are field independent are able to use their limited working memory capacity more efficiently and this means that they are more successful in term of understanding. Such understanding generates greater overall satisfaction and sense of achievement.

Looking at the two effects together, working memory capacity and extent of field dependency are clearly powerful correlates of success and they also correlate, although less powerfully, with attitudes relating to mathematics. It could be argued that those students who are better in mathematics are those who have these advantages: not all are equal. However, working memory capacity is genetically fixed while ways to develop field independency are not known. Does that mean that the student who happens to have a lower working memory and is field dependent is, therefore, unable to make much of mathematics? Is it legitimate to teach a subject where a sizeable proportion of the population are so disadvantaged?

There is another way to look at the problem. Can the teaching and learning of mathematics be re-thought so that those with these disadvantages are less hindered? This means two things: the actual teaching presentation is re-designed to lower working memory demand. This would make mathematics more accessible to those who happen to have a lower working memory and who are field dependent. The other important aspect is that the assessment of mathematics must not disadvantage those who happen to have a lower working memory and are field dependent. The way questions are designed will be explored in the next chapter.

Chapter 9

Cognitive Factors and Mathematics Achievement

Phase Two

9.1 Introduction

In the first phase, it was shown that both working memory capacity and extent of field dependency were related to performance in mathematics and, to a smaller extent, to attitudes related to mathematics. It was argued that, if mathematics was taught and assessed in such a way that these two factors were reduced in importance, then performance would rise and attitudes might be more positive. The area of testing is now explored and related to performance, attitudes and the two cognitive factors.

In this experiment four instruments were used to collect data. These instruments were administrated in Kuwait from October to December 2006, the end of the first term. These instruments are:

- *Two versions of mathematics tests were applied for each grade.*
- *Digits backwards test was again used to measure working memory space for every student of the sample.*
- *Group embedded figures test was again applied to classify the subjects by their field-dependency abilities.*
- *The fourth instrument measured students' attitudes towards mathematics and this will be discussed in the following chapter.*

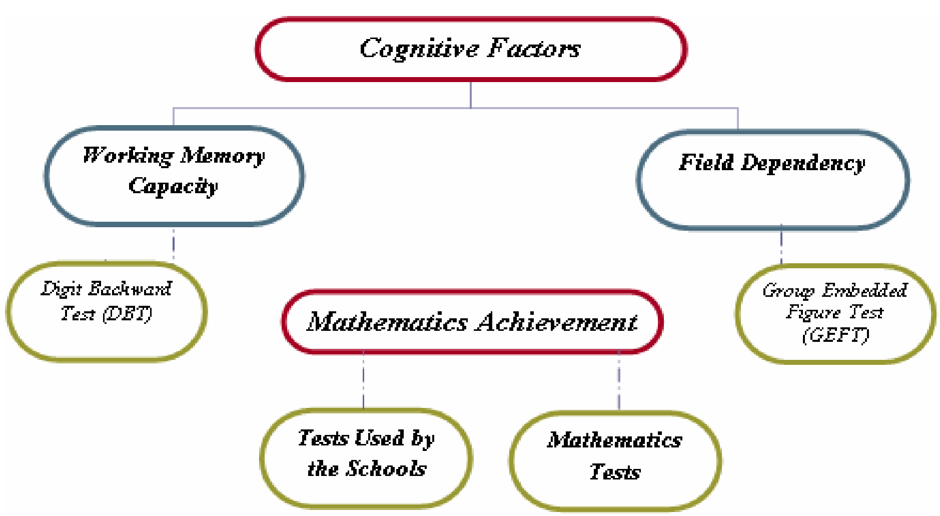


Figure 9-1: Second phase procedures

9.2 Students' Sample Characteristics

Three junior secondary schools in the state of Kuwait were involved in the second experiment of the research, two schools for girls, one for boys. The sample contains roughly equal populations of grade eight and nine (see table 9-1). Access to boys' schools is difficult for a female researcher, giving a gender imbalance.

GROUP	GRADE 8 (14YEARS)	GRADE 9 (15YEARS)	TOTAL
Boys	146	143	289
Girls	269	316	585
TOTAL	415	459	874

Table 9-1: Sample characteristics (second phase)

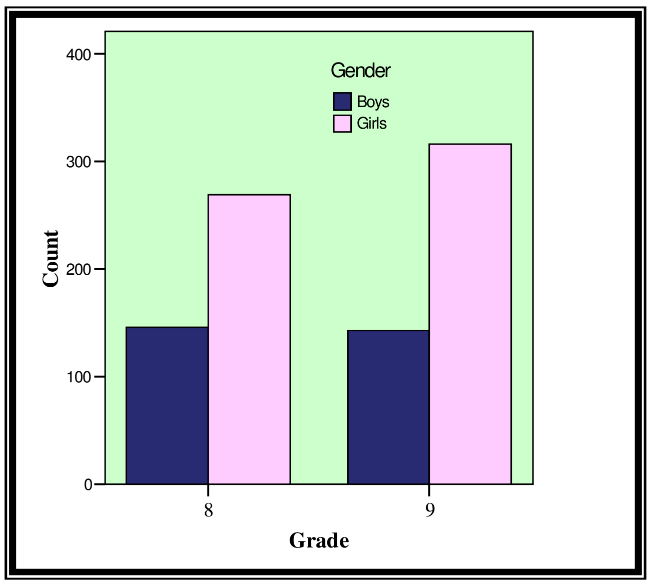


Figure 9-2: Sample characteristics (second phase)

9.3 Attainment in Mathematics

Students' achievements in mathematics were gained from two sources. Scores from three standard school tests during the first term of the academic year 2006/2007, with a total time of three hours, were combined. The mean performance in mathematics was 67%. In order to explore any relationships between their mathematics performance and their working memory capacity and extent of field dependency, Pearson correlation was employed

The second way involved using tests designed by the researcher. Four versions of mathematics tests were constructed, two tests for each grade, based on the curricula in Kuwait. The tests were constructed in the following way:

(a) Questions were carefully designed with different working memory demands. This is discussed further later.

(b) For each grade, some questions were common in both versions.

(c) For each grade, some questions were presented in a symbolic form while the equivalent question in the other paper was presented in a more visual form.

(d) For each grade, some questions were presented as abstract tasks while the equivalent question in the other paper was presented in a more applied form.

(e) For each grade, there was a mixture of questions of different working memory demand; symbolic and visual questions; abstract and more applied questions.

The aim was to make the two versions of the test for each grade of approximately the same standard. The grade eight tests consisted of twelve questions and the grade nine tests consisted of nine questions. The questions used in mathematics tests were based on questions typically used in Kuwait and questions generated for the use of this research (the full lay out as it was given to the students is presented in Appendix (C)).

The aims of applying these tests are to consider the relationship between mathematics topics and the students' working memory space and their field dependency; and to look at the best way to present the question in order to help the learners to reach higher achievement in mathematics examinations. Is the visual presentation the most appropriate manner or the symbolic? Or, do the applied or abstract questions assist students to achieve better? The instructions given to the students were:

- *Read the question carefully.*
- *If you are stuck, carry on with the next question.*
- *You have 45 minutes for the whole test.*

The tests were corrected (See figure 9-3) and the aim in correcting the tests was to explore the procedures that the students applied to show their understanding of the topic and not just the final answer.

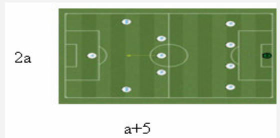
Version 1	***Version 2***
Calculate the area of the football patch 	**If the length of rectangle area is (a+5) cm, and its height is 2a calculate the area of this rectangle**
Rectangle area = Length X Width *(1/2)* **= (a+5) (2a)** *(1/2)* **=2a^2 + 10a** *(1/2) (1/2)* *Total : 2*	**Rectangle area = Length X Width** *(1/2)* **= (a+5) (2a)** *(1/2)* **=2a^2 + 10a** *(1/2) (1/2)* *Total : 2*

Figure 9-3: Example of correction process of the tests

The top score was 21 for both tests. For the whole tests and the correction processes and marks distribution, see Appendix (C). The samples that completed the various versions of the tests are shown in table 9-2, with a few students not included due to absence on the day of the test.

Test Version	Frequency	Percent	Total
Grade 8 (1)	**223**	**56%**	**396**
Grade 8 (2)	**173**	**44%**	
Grade 9 (1)	**249**	**56%**	**433**
Grade 9 (2)	**193**	**44%**	

Table 9-2: The number of grade 8 & 9 students who fulfilled Mathematics tests

The following histograms show the distribution of the marks of grade eight and grade nine mathematics tests.

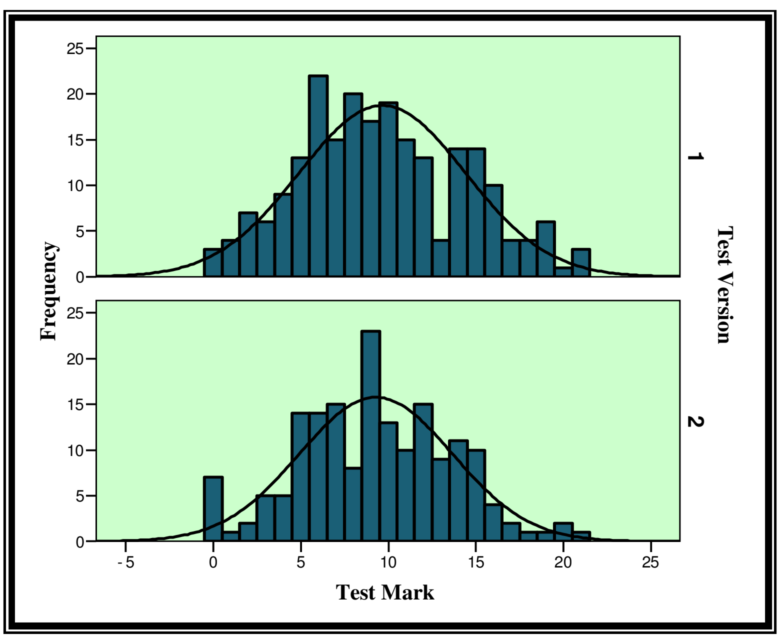


Figure 9-4: Histogram of Grade eight tests

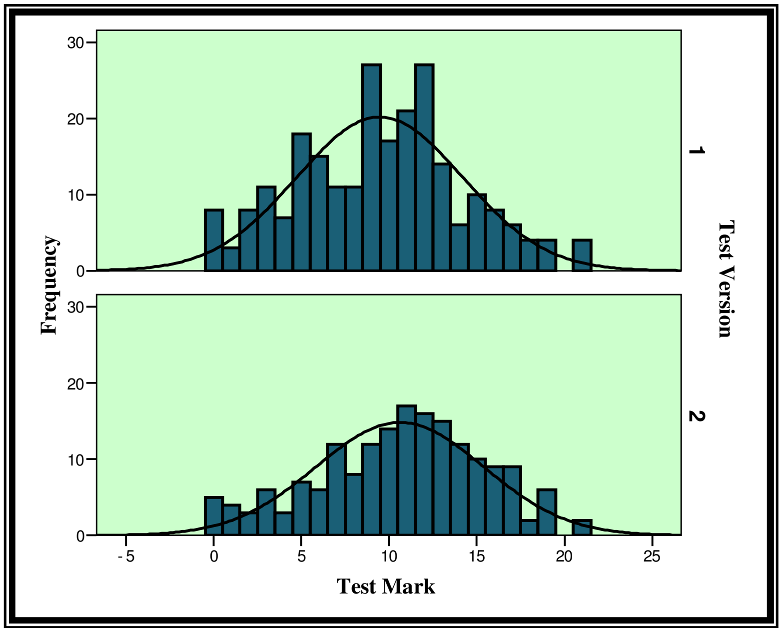


Figure 9-5: Histogram of Grade nine tests

All the distributions of marks show approximately normality and the use of Pearson correlation and the t-test is appropriate. The following table shows the descriptive statistics and the t-test results comparing the two versions of the mathematics tests. The two versions of grade eight tests are similar and there are no significant differences between the two versions. Grade nine tests slightly different from each other as it is shown from the t-test. Overall, the aim of making the two test of similar standard has been achieved.

Mathematics test	Mean	Std Deviation	t-test	p
Grade 8 (1)	**9.65**	**4.75**	**0.79**	**n.s.**
Grade 8 (2)	**9.29**	**4.37**		
Grade 9 (1)	**9.46**	**4.74**	**2.43**	**$p < 0.05$**
Grade 9 (2)	**10.61**	**4.78**		

Table 9-3: Descriptive statistics of mathematics tests

9.4 The Classification of X-space

The digit backwards test (DBT) was used to measure working memory space and the distribution of the results is shown in figure 9-6.

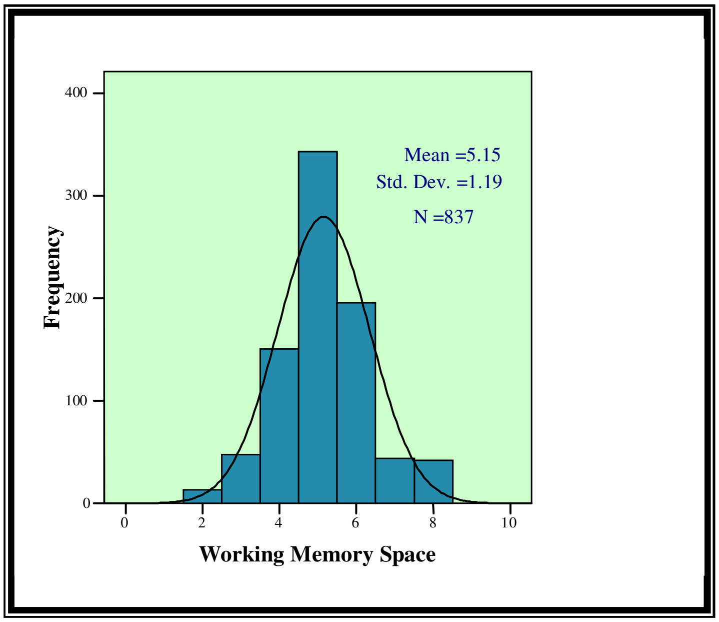


Figure 9-6: The distribution of the students Scores in Digits Backwards Test (DBT)

Descriptive statistics demonstrates that the mean of the scores is 5.2. For a sample aged 14 and 15, the mean working memory would be expected to be between 6.0 and 6.5. However, the digit span backwards test gives a result approximately one less (as one working memory chunk is used for the process of reversal. The mean obtained is roughly what would be expected. However, it has to be noted that the data from the digit span backwards test is not used here in any absolute sense. The important thing for correlation is the order.

The sample can be divided into three groups in order to *illustrate* the correlation (Danili, 2001). The sample of 837 students was categorised into groups namely: *low*, *intermediate* and *high working memory space* capacity. Students who succeeded to record 4 or fewer digits were classified as *low working memory space*. Students who able to recall 5 digits (shown as X = 5) were classified as *intermediate working memory space* and the rest who memorize 6 or more overturned numbers, were classified as *high working memory space* (shown as X = 6). Table 9-4 shows the number of students in each category.

GROUP (X-SPACE)	NUMBERS OF STUDENTS	PERCENT
X=4	212	25%
X=5	343	41%
X=6	282	33%
TOTAL	837	100%

Table 9-4: The classification of the student into working memory space capacity groups

Students' working memory capacity (X-space) classification according to their grades is illustrated in the following bar chart.

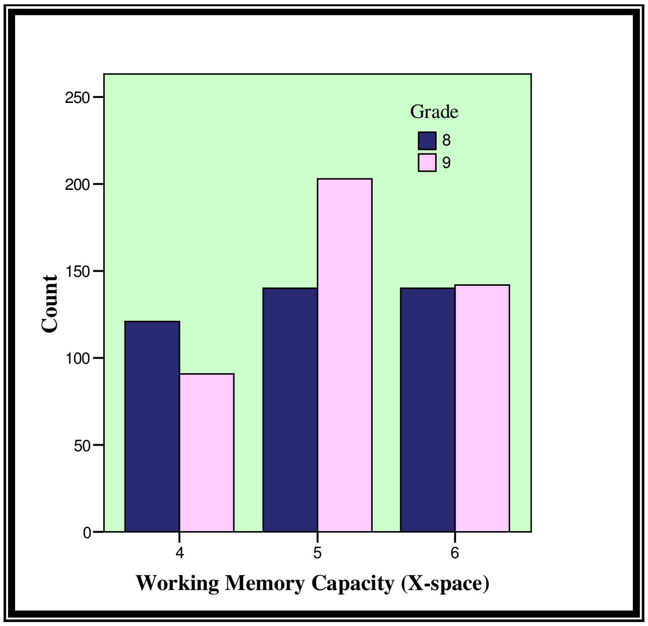


Figure 9-7: Students' working memory capacity (X-space) classification according to their grades

63% of grade eighth (age 14) students, their working memory capacity (X-space) ranges from 5 and less, whereas 80% of grade ninth (age 15) their working memory capacity (X-space) ranges from 5 and above. This supports the fact that working memory space depends on the age of the individual, and as Miller (1956) showed, after memory experiments, the average capacity for an adult is about seven plus or minus two (7 ± 2) separate chunks, growing by about 1 unit for every two years of age up to age 16. The table below shows the classification of the whole sample into X-space according to their classes.

Group	NUMBER OF STUDENTS			
	GRADE 8		GRADE 9	
X=4	121	30%	91	21%
X=5	140	35%	203	47%
X=6	140	35%	142	33%
TOTAL	401	100%	436	100%

Table 9-5: Students' working memory capacity classification according to their grades

9.5 Classification of Tests' Questions According to their Z-Demand

The questions were classified by means of their Z-demand. Johnstone & Wham (1982) defined the demand (Z) of a question as "*the maximum number of thought steps required by the least able pupil to reach the solution*". Johnstone and El-Banna (1986) found a sudden collapse occurred in chemistry tests when the demand of the questions exceeded the students' working memory space capacity (X-space). In this case, the working memory had reached what Johnstone & Wham (1982) described as a "*state of unstable overload*". Another study in mathematics showed a collapse in students' performance in solving algebra problem when the questions demand more capacity that the working memory capacity of the student (Christou, 2001).

In order to put the problem into a scale of difficulty according to their Z-demand in this way, the opinion of three 'experts' in this field was asked. Every expert classified the questions in Z-demand categories separately and then they sat together to look for agreement and to discuss the disagreement to give the test its final form. A lot of different mental actions are taking place in representing mathematical problems, like text comprehension translation from real language to mathematics notation, use of symbols, problem schemata, complexity of geometrical shapes and more many. Every student is more likely to have his/her own way to solve the question. Thus, it is not easy to determine the necessary thought steps which lead to the correct answer. Nonetheless, reasonable agreement was obtained.

Demand level is not the same as question difficulty. It is possible to create very difficult questions where the demand is very low (Reid, 2002). According to Z-demand, the gradual difficulty was developed by the following ways:

- *More information was given: the task to be produced was more complex;*
- *Different calculation had to be used (according to research and experience, some calculations are more complex in terms of thought steps to understand and to use than others: for example multiplication is more difficult than addition and division than subtraction; also students find fractions very difficult to manipulate etc.);*
- *Geometrical shapes often come with a lot of information: the task to be produced was more complex.*
- *An applied task may generate more complexity than an abstract task, simply because there is more information to consider.*

The questions were classified in five Z-demand categories. The Z-demand occurred in the amount and the type of information that was given to the students and the whole mental procedures that are needed for solving the task. Therefore, the Z-demand of the task is indicating the 'maximum of thought steps' to solve the task. The following question from grade eight tests was defined as a 4-demand task.

Question 10

Your bank account holds 20 KD. You enter your credits and debits with + and - signs, respectively. What do you own after writing down the entries +2.7 KD, -7.3 KD, - 7 KD, + 1.3 KD?

Thought steps

(1) + credit and
(2) - debits
(3) 20 + 2.7 + 1.3
(4) 24 – 7.3 – 7

An example of question from grade nine test with 6-demand is the following:

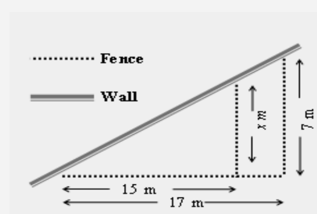
Question 9

Ali needs to replace fencing in his garden. He has taken measurements (shown) but has forgotten to measure the part of the fence mark x meters. The garden centre has only 28 metres of fencing stock. Is this enough to completely replace the existing fence?

Thought steps

(1) Two triangles in the same diagram is a sign of similarity
(2)........ fence
(3) $\frac{15}{17} = \frac{x}{7}$
(4) $x = \frac{15 \times 7}{17}$
(5) 15m part of the 17m, so it will not take when account the fence long
(6) Count the fence long and compare it to the 28 m available.

It is possible the stages 3 & 4 involve more than 1 step



Tables 9-6 and 9-7 show the performance of the grade 8, test 1 and 2 respectively. For each demand level (Z-demand) the performance is shown by taking all the questions at that Z-demand and showing the numbers who solve the questions correctly as a ratio of the whole sample.

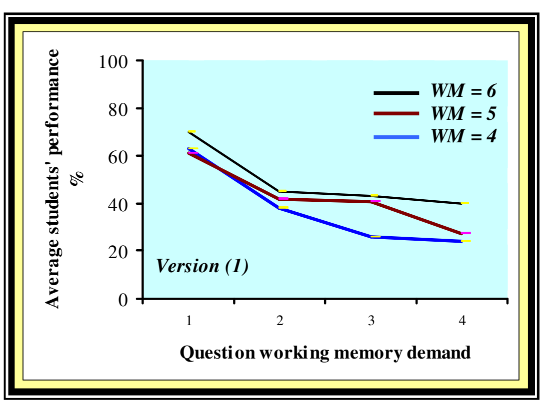
	Z=2	Z=3	Z=4	Z=6
X=4	**0.63**	**0.38**	**0.26**	**0.24**
X=5	**0.61**	**0.42**	**0.41**	**0.27**
X=6	**0.70**	**0.45**	**0.43**	**0.40**
Question	6,7,8	1, 3, 11	2,4, 9a, 9b, 10	5a, 5b, 12

Table 9-6: The performance of the students with X-space working memory capacity to questions of different Z-demand (Grade eight Test 1: there is no question Z = 5)

	Z=2	Z=3	Z=4	Z=5	Z=6
X=4	**0.54**	**0.33**	**0.29**	**0.27**	**0.10**
X=5	**0.70**	**0.53**	**0.37**	**0.32**	**0.10**
X=6	**0.73**	**0.57**	**0.52**	**0.43**	**0.14**
Question	*4,6,7,8*	*2,9a,9b*	*1,3,10,11*	*5a,5b*	12

Table 9-7: The performance of the students with X-space working memory capacity to questions of different Z-demand (Grade eight Test 2)

Looking at tables 9-6 and 9-7, the drop off in performance can be see when the working memory capacity of the students comes close to the demand level (the maximum number of thought steps required by the least able student to reach the solution) of the questions. In establishing the maximum number of thought steps required by the least able student to reach the solution, the assumption is that the student has to hold all these thought steps in the working memory at the same time (figure 9-8).



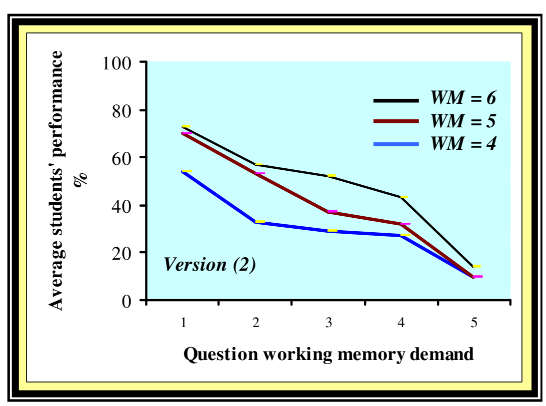


Figure 9-8: Comparison of the average student performance in Mathematics for all groups of different X-space (Grade 8)

The same analysis is now shown for grade 9 (tables 9-8 and 9-9).

	Z=2	Z=3	Z=4	Z=5	Z=6
X=4	**0.47**	**0.47**	**0.30**	**0.18**	**0. 01**
X=5	**0.65**	**0.66**	**0.48**	**0.32**	**0.10**
X=6	**0.66**	**0.76**	**0.55**	**0.49**	**0.30**
Question	*5, 8*	*2, 3*	*4, 7*	*6*	*9*

Table 9-8: The performance of the students with X-space working memory capacity to questions of different Z-demand (Grade nine Test 1)

	Z=2	Z=3	Z=4	Z=5	Z=6
X=4	**0.47**	**0.37**	**0.25**	**0.21**	**0.01**
X=5	**0.65**	**0.63**	**0.57**	**0.45**	**0.10**
X=6	**0.66**	**0.68**	**0.64**	**0.57**	**0.30**
Question	*8*	*2, 6*	*4, 7*	*3, 5*	*9*

Table 9-9: The performance of the students with X-working memory capacity to the questions of different Z-demand (Grade nine Test 2)

Figure 9-9 illustrates the drop off in performance as the demand level of the question approaches the average working memory capacity of the group.

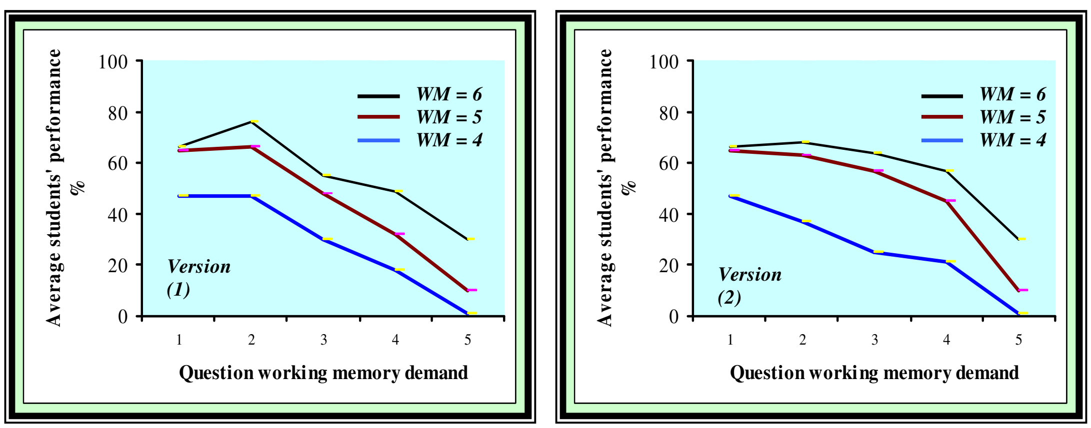


Figure 9-9: Comparison of the average student performance in Mathematics for all groups of different X-space (Grade 9)

These results illustrate that the overload of working memory is likely to be at least partly responsible for students' difficulties in solving mathematics tasks. This is discussed further later on in terms of correlation. However, other features of the questions may also be important. These include whether the question presentation is more applied or more abstract or whether there is a visual presentation or more symbolic presentation. Of course, the presentation may also influence possible working memory overload.

9.6 Presentation of Questions

According to information processing models, the way the questions or the problems are given to the students is very important for the students to understand and to succeed in solving them. Language is known to be important and can cause working memory overload (Johnstone and Selepeng, 2001) and it is likely that complicated shapes and the actual way the question is posed will also be factors that influence success. Noss and Hoyles (1996) indicated that the changing in setting produce quite dramatically different facility levels, and they refer to the Assessment Performance Unit (APU) in England as an example. The APU (1986, p: 836) showed that a question such $4.5+0.5=?$ was answered correctly by 63% of students (age 11) while the question: John saved £3.70 and then his mother gave him £1.50, how much did he have in all? was answered correctly by 82%. They justified this result by saying "*a possible explanation being that money problems are well-grounded in children's experience that such difficulties are 'overcome'*" (p: 32).

The two versions of the mathematics tests allowed for an exploration of some of these factors and this is now discussed. A t-test compared performance while the chi-square test (contingency) was used to compare the frequencies of right answers. The two approaches were adopted because answers being marked right and wrong do not give a clear cut distribution. In fact, in almost every case, the statistical findings were identical.

Applied versus Abstract Questions

Charles and Lester (1984: p: 10) indicated that applied problems provide an opportunity for students to use a variety of mathematical skills, processes, concepts, and facts to solve realistic problems, arguing that they may help the students to be aware of the value and usefulness of mathematics. However, applied questions may require high demands of working memory space to solve them. Comparisons of performance between students' performance in applied questions and abstract ones are now summarised. The comparisons are carried in the following way. A t-test is carried out to compare students' performance in the two versions. Because there is doubt that the scores from normal distributions (answers were marked right or wrong), a chi-square was calculated comparing the proportions getting each question right or wrong. The first example is a fraction division. Version (1) is abstract question and version (2) is applied question.

Version 1	*Version 2*
Find the solution 3.6 ÷ 1.2 =	**The length of a line is 3.6 m, and we want to divide it into several parts the length of each is 1.2m. How many pieces will we get?**
Correct 39%	**Correct 46%**
χ^2 = 2.0 (1), n.s.	**t=1.4, n.s.**

Example 1: Grade 8-Q1

Most of the students in the second version manipulate the question by drawing a line and they divided it into pieces without translating the applied question to the symbolic manner which may help them to tackle this task without overloading their working memory.

The second example is the addition of two fractions. One version is more applied question while the other is a straightforward abstract question.

Version 1	*Version 2*
Sara buys a jacket and skirt. The prices are $29\frac{3}{4}$ KD, $17\frac{1}{2}$ KD respectively. How much will she pay for them?	$29\frac{3}{4}+17\frac{1}{2}=$
Correct 58%	**Correct 75%**
$\chi^2 = 11.4\ (1),\ p < 0.001.$	$t=3.5,\ p = 0.001$

Example 2: Grade 8-Q2

Very clearly, the students performed better with the abstract version, almost certainly because it makes much less demand on working memory, layout being a factor.

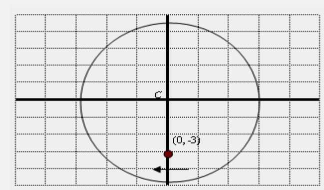
Example 3 is subtraction involving polynomials where version 1 is a statement and version 2 is in a symbolic form. .

Version 1	*Version 2*
Find the solution of subtraction $3x^2-5x+1$ **from** $x-2x^2+4$	$-\begin{array}{r} 3x^2-5x+1 \\ -2x^2+x+4 \\ \hline \end{array}$
Correct 53%	**Correct 82%**
$\chi^2 = 35.8\ (1),\ p < 0.001.$	$t = 6.5,\ p < 0.001$

Example 3: Grade 8-Q4

It is very clear that the use of a statement reduces performance very markedly. This might be attributable to the overload of the working memory space in 'translating' the statement into a symbolic manner, as well as arranging the polynomials in similar order to get the right answer. Furthermore, the semantic aspect of the symbolic language can cause confusion and the translation of ordinary language in mathematics into the symbolic language creates a 'conflict of exactitude' (Macnab & Cummine, 1986).

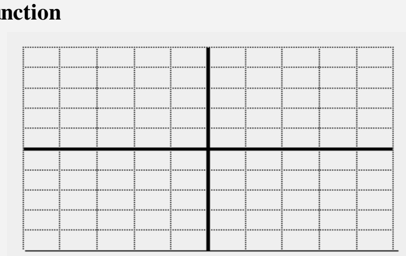
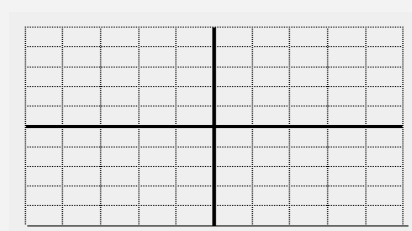
The following three examples (4, 5 and 6) are from the grade nine tests. Example 4 is a rotation question where version 1 is the abstract one and version 2 is much more applied.

Version 1	***Version 2***
Find the image of the point (0, -3) under rotation $90°$ clockwise	**The diagram shows the monitor of the control unit in Kuwait airport; the location of an air plane in the monitor is in the point (0, -3) (shown in the diagram as ● and the arrow shows the direction of the air plane). The controller asks the fight captain to make a rotation $90°$ clockwise around the centre point (shown in the diagram as C). Calculate the location of the air plane in the monitor after the rotation** 
Correct 65%	**Correct 43%**
$\chi^2 = 20.1$ (1), $p < 0.001$.	**$t = 4.6$, $p < 0.001$**

Example 4: Grade 9-Q3

This example also shows that the applied version is much more demanding and this again is probably because of considerable overload of the student's working memory space.

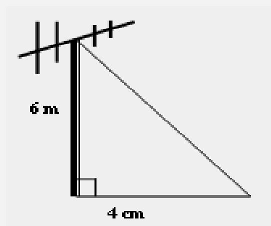
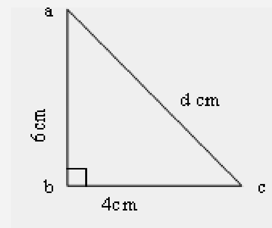
Example 5 asks the students to sketch the graph of a function where version 1 is the more applied question and version 2 is the more abstract one.

Version 1	***Version 2***
The function $y = x + 2$ describes the global warming, when average temperatures rise by two degrees. Sketch the graph of the function 	**Sketch the graph of the function** $y = x + 2$ 
Correct 28%	**Correct 43%**
$\chi^2 = 10.7$ (1), $p < 0.001$.	**$t = 7.3$, $p < 0.001$**

Example 5: Grade 9-Q6

It is clear that the more applied question hinders the students to solve the task.

Example 6 is a right-angle triangle question, where version 1 is applied question and version 2 is abstract one. .

<table>
<tr><th>Version 1</th><th>Version 2</th></tr>
<tr><td>What is the length of the wire that is used to fix the antenna?

</td><td>abc is a single right angle triangle
ab= 6 cm
bc=4cm
Find
ac=

</td></tr>
<tr><td>Correct 44%</td><td>Correct 53%</td></tr>
<tr><td>χ^2 = 3.5 (1), n.s.</td><td>t = 1.9, n.s.</td></tr>
</table>

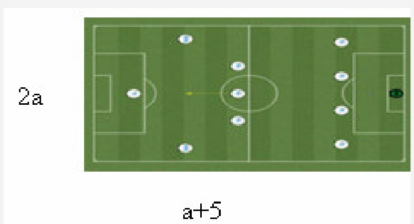
Example 6: Grade 9-Q7

Versions 1 is more applied but less wordy. Perhaps the two effects cancel each other out.

The overall evidence here shows that applied questions hinders good performance in four questions and makes no significant difference in the other two. This is almost certainly because the more applied question format increases working memory overload. In fact, taking all six questions together, performance in the more abstract format doubles the average performance when compared to the more applied question format.

Visual versus Symbolic Presentation

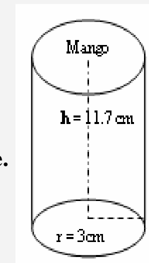
Several studies have indicated the importance of visuo-spatial ability in mathematics performance (Heathcote, 1994; Reuhkala, 2001; Trbovich & LeFevre, 2003; Jarvis & Gathercole, 2003; Maybery & Do, 2003) and it is widely believed that the visual presentation of any task plays a crucial role in facilitating this task, particularly in geometry. In order to examine the assistance of visual presentation in solving any task, similar questions were presented in the two different versions, one with a visual presentation and other without. Furthermore, a complicated geometrical shape with much information on it was presented to compare its assistance to another shape with no marking, giving the students opportunity to put the marks in by themselves. Example 1 compares the visual and the symbolic presentations.

Version 1	***Version 2***
Calculate the area of the football patch 	**If the length of rectangle area is (a+5) cm, and its height is 2a calculate the area of this rectangle**
Correct 59%	**Correct 58%**
χ^2 = 0.0 (1), n.s.	**t = 0.07, n.s.**

Example 1: Grade 8-Q3

There is no different in achievement in the two versions.

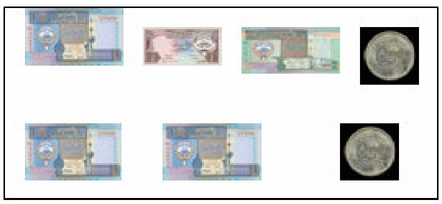
Example 2 looks at the assistance of the visual presentation for the student to solve the problem. Version 1 is applied question without any visual presentation and version 2 provides a visual picture to the students.

Version 1	***Version 2***
The shape of water tank is right circular cylinder. **The radius of its base r= 7cm and its height h = 10cm.** **a) Calculate the lateral surface area.** **b) The water volume if we are going to fill this tank.** $\pi = \frac{22}{7}$	**A company making various kinds of fruit juice decides to sell its product in 330 ml quantities. After considering possible containers they decide on metal in the shape right circular cylinder.**  **a) Lateral surface area of the container =** **b) Check that the container can in fact hold 330 ml of juice.**
Part (a) Correct 68%	**Part (a) Correct 37%**
$\chi^2 = 37.0$ **(1), p < 0.001**	**t = 6.4, p < 0.001**
Part (b) Correct 46%	**Part (b) Correct 56%**
$\chi^2 = 3.4$ **(1), n.s.**	**t = 1.8, n.s.**

Example 2: Grade 8-Q5

The visual presentation for the question hinders the correct calculation of surface area. It is possible that the students knew the formula and the first version offered an easier way to apply it while the second brought in more (unnecessary) information for them. Calculating volume shows no statistical difference.

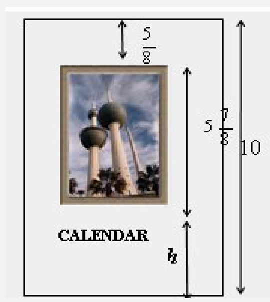
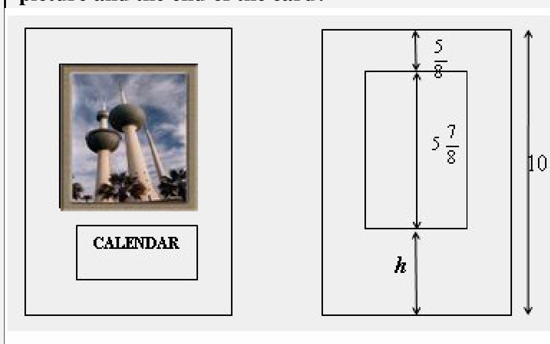
In the example 3, version 1 presents the pictures of Kuwait currency and gives the students opportunity to imagine the real situation, where version 2 gives them the money amount to Ali without any picture.

Version 1	***Version 2***
The fare charged for travelling by taxi is shown here. **a) How much does it cost to travel 2 miles by taxi?** **Fare** 1.500 KD for the first mile. 0,600 KD for every ½ mile **b) Ali has to travel 3 miles from the cinema to his home. Has he money enough to pay his taxi fare from the cinema to his home?** 	**The fare charged for travelling by taxi is shown here.** **a) How much does it cost to travel 2 miles by taxi?** **Fare** 1.500 KD for the first mile. 0,600 KD for every ½ mile **b) Ali has to travel 3 miles from the cinema to his home, he has 3,950 KD. Are his many enough to pay his taxi fare from the cinema to his hoe?**
Part (a) Correct 47%	**Part (a) Correct 41%**
χ^2 = 2.0 (1), n.s.	**t = 1.5, n.s.**
Part (b) Correct 31%	**Part (b) Correct 27%**
χ^2 = 0.9 (1), n.s.	**t = 1.0, n.s.**

Example 3: Grade 8-Q9

It makes no difference in either part of the question whether a diagram is used. This is an occasion where the diagram does not add to the question in any way.

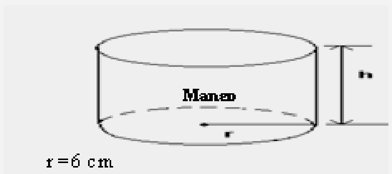
In the following example 4, there are visual presentations in both versions. However, in version 1, the measurements are presented in the same picture while, in version 2, the measurements are presented in separate shapes.

<table>
<tr><th>Version 1</th><th>Version 2</th></tr>
<tr><td>Huda has decided to make a calendar. She is going to stick a photograph onto a piece of card and leave space underneath for a calendar tab. The piece of card is 10 inches high. The photograph is $5\frac{7}{8}$ inches high and the space at the top is $\frac{5}{8}$ of an inch, as shown on the right. What is the height h of the space between the bottom of picture and the end of the card?
</td><td>Huda has decided to make a calendar. She is going to stick a photograph onto a piece of card and leave space underneath for a calendar tab. The piece of card is 10 inches high. The photograph is $5\frac{7}{8}$ inches high and the space at the top is $\frac{5}{8}$ of an inches, as shown on the right. What is the height h of the space between the bottom of picture and the end of the card?
</td></tr>
<tr><td>Correct 12%</td><td>Correct 8%</td></tr>
<tr><td>$\chi^2 = 1.7$ (1), n.s</td><td>t = 1.3, n.s.</td></tr>
</table>

Example 4: Grade 8-Q11

The majority of the sample fails to solve this task because the idea of the task is a new idea and they have not faced a similar task before. There is no statistical difference between the success rates for the two versions.

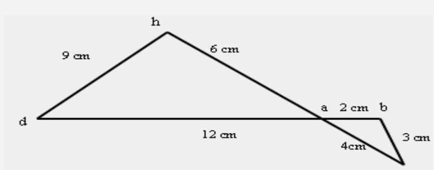
Example 5 presents the same question in both versions but version 1 with the visual presentation and version 2 without.

Version 1	*Version 2*
What would be the smallest possible height, to the nearest millimetre of this container so that it can hold 330ml of juice?  	**What would be the smallest possible height, to the nearest millimetre of cylinder the radius of its base r = 6 cm, so that it can hold 330ml of juice?**
Correct 12%	**Correct 10%**
$\chi^2 = 3.8\ (1)$, **n.s**	$t = 2.0,\ p < 0.05$

Example 5: Grade 8-Q12

There is no statistical difference between the success rates in the two versions when chi-square was applied. However, t-test shows very small differences in students' success rate between the two versions.

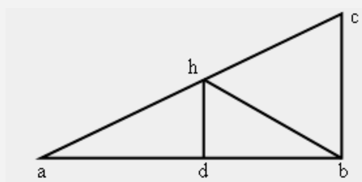
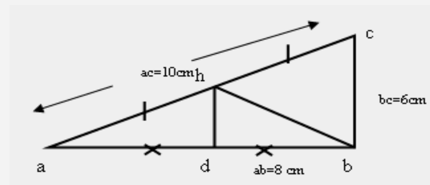
Examples 6 and 7 are from grade nine tests. Example 6 is about triangle similarity where there is a visual presentation in version 1 and version 2 without.

Version 1	*Version 2*
Prove triangle abc **is similar to triangle** ahd  	**abc is a triangle where ab= 2cm, ac= 4cm, bc= 3 cm. ahd is another triangle where ah = 6cm, ad =12cm, hd= 9 cm.** **Prove triangle** abc **is similar to triangle** ahd
Correct 65%	**Correct 61%**
$\chi^2 = 0.8\ (1)$, **n.s**	**t = 0.9, n.s.**

Example 6: Grade 9-Q2

The use of a diagram makes no difference.

Example 7 compares two different visual presentations where version 1 provides a simple shape and gives the students opportunity to mark dimensions on it and version 2 has the dimensions already added.

Version 1	*Version 2*
Δ abc is right angle triangle in b. **ac= 10 cm , bc= 6cm, ab = 8 cm** **ah=hc** **ad=db** **Find** **hd=** **hb=** 	**Find** **hd=** **hb=** 
Part (a) Correct 65%	**Part (a) Correct 58%**
χ^2 = 1.6 (1), n.s	**t = 1.3, n.s.**
Part (b) Correct 64%	**Part (b) Correct 46%**
χ^2= 13.6 (1), p < 0.001	**t = 3.7, p < 0.001**

Example 7: Grade 9-Q5

It seems that allowing the students to add the measurements on to the diagram is more helpful although only part (b) shows statistical significance.

The visual presentation of a task can provide much information as one chunk. This may help to minimize the load on the working memory and, therefore, the visual presentation assists the students to solve the task properly. However, a complicated picture with a lot of information may simply overload the working memory space. The visual presentation then hinders success.

Sometimes, it may be better to present a diagram without the detail marked on it. The student then has the task of marking each piece of individual information on the diagram. This may help them understand the question gradually and, indeed, enable them to see the information as a whole. Overload of working memory may be minimised.

9.7 Mathematics Attainment and Working Memory

The mathematics attainment, as measured by the mark gained by combining the three school tests, was related to the measured working memory capacity using Pearson correlation. The value obtained was 0.20, significant at $p < 0.01$. This correlation can be illustrated as in table 9-10. It can be seen that high working memory space capacity students ($X = 6$) performed better in mathematics than those with lower working memory space capacity ($X=4$).

GROUP (X-Space)	MEAN SCORE IN MATHEMATICS
X=4	63
X=5	67
X=6	73

Table 9-10: The relationship between working memory & mathematics performance

The correlation between working memory capacity and students' marks in the mathematics tests designed for this study (test data standardised) gave a correlation value of 0.36, $p < 0.001$.

GROUP (X-Space)	MEAN SCORE IN MATHEMATICS TEST
X=4	7
X=5	10
X=6	12

Table 9-11: The relationship between working memory & mathematics tests performance

The relationships can be illustrated using a scatter plot (figures 9-10 and 9-11)

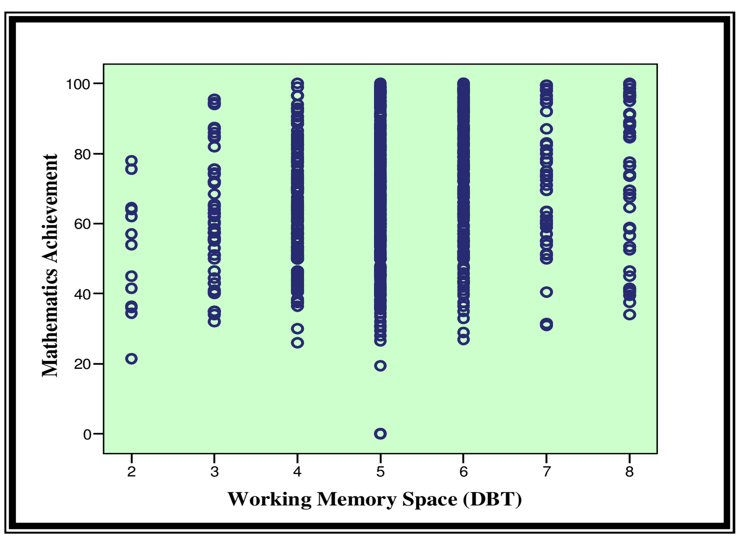


Figure 9-10: Scatter diagram of scores in DBT related to mathematics performance

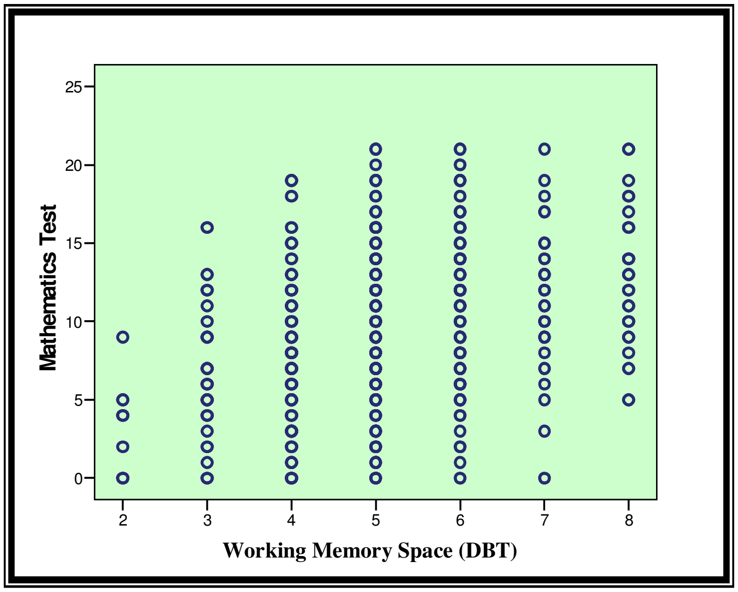


Figure 9-11: Scatter diagram of scores in DBT related to mathematics tests performance.

Tables 9-12 and 9-13 illustrate the relationship between students' working memory space and their performance in mathematics according to grade.

Group	MEAN SCORE IN MATHEMATICS	
	GRADE 8	GRADE 9
X=4	63	63
X=5	65	68
X=6	72	73

Table 9-12: The relationship between working memory & mathematics performance according to grades

Group	MEAN SCORE IN MATHEMATICS TEST	
	GRADE 8	GRADE 9
X=4	8	6
X=5	9	10
X=6	11	13

Table 9-13: The relationship between working memory & mathematics tests performance according to grades

The correlation values (0.20 and 0.36) are typical of the kinds of values found in other subjects at various ages and stages. Typical values range from 0.2 to 0.6 (Reid, 2008). However, they might appear not to be very large. However, the difference between the performance of above average and below average students is considerable (around 10% in the school tests rising to a doubling of the marks in grade 9 in the test set for this study).

9.8 Field Dependency Measurement

The sample of 874 students was classified into three learning style categories according to their scores in the Group Embedded Figures Test (GEFT). The distribution of students' scores in the GEFT Test is shown in Figure 9-12.

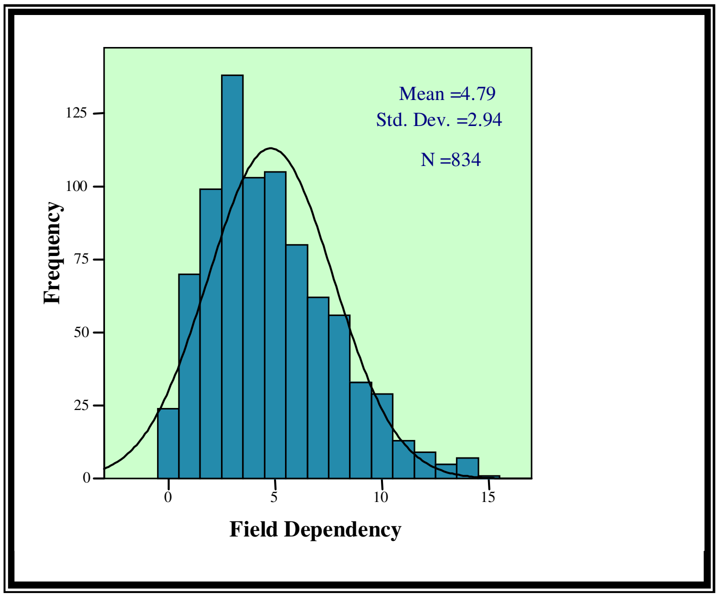


Figure 9-12: The distribution of students' scores in Group Embedded Figure Test (GEFT)

- Students who scored less than half the standard deviation less than the mean in the GEFT were classified as field dependent, and they form 40% of the sample. (FD <4.79- 2.942/2)
- Those who scored more than half standard deviation more than the mean were considered field independent, 35% of the sample.(FI> 4.79+2.942/2)
- The rest who scored between these values were labelled field intermediate, and they form the largest proportion of (25%).(4.79- 2.942/2 <FIT> 4.79+2.942/2)

Table 9-14 shows the number of students in each learning style category.

GROUP	NUMBERS OF STUDENTS	PERCENT
FD	331	40%
FIT	208	25%
FI	295	35%
TOTAL	834	100%

Table 9-14: The Classification of the student by field dependency

The classification of the students' field dependency is divided into two groups according to their grades.

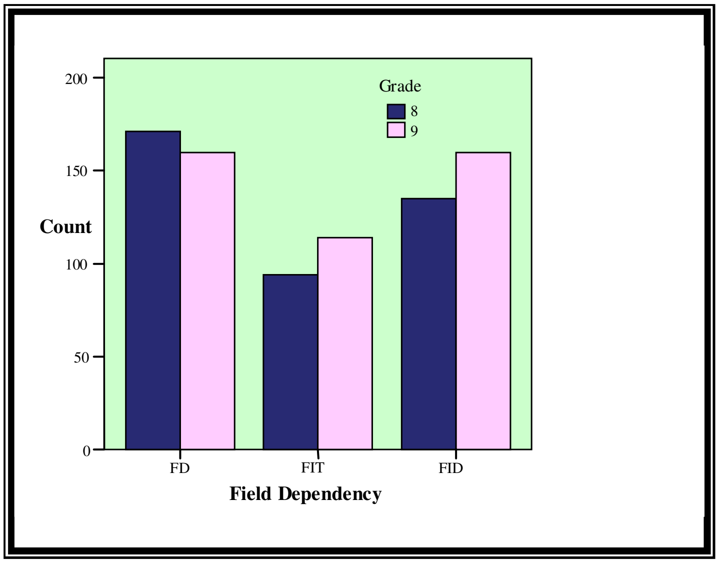


Figure 9-13: Students' field dependency classification according to their grades

43% of the grade eight sample categorises as field dependent while 37% of the grade nine sample categorises as field independent. The field intermediates give a similar proportion in grades eight and nine (24%, 26% respectively). The table below shows the classification of the whole sample into their field dependency characteristic.

Group	NUMBER OF STUDENTS			
	GRADE 8		GRADE 9	
FD	171	43%	160	37%
FIT	94	24%	114	26%
FI	135	34%	160	37%
TOTAL	400	100%	434	100%

Table 9-15: Students' field dependency classification according to their grades

9.9 Mathematics Attainment and Field Dependency

The scores on the field dependency test were correlated with the mathematics performance scores. Firstly, the school mathematics examinations were considered. The Pearson correlation value was 0.36, $p < 0.001$. The classification of the students according to the GEFT illustrates significant correlation between the students' scores in GEFT and their mean scores in mathematics. Table 9-16 illustrates that field-independent students achieved better than other groups of students. A scatter plot for these variables is presented in Figure 9.14.

GROUP	MEAN SCORE IN MATHEMATICS
Field Dependent	61
Field Intermediate	69
Field Independent	75

Table 9-16: The relationship between field dependency & mathematics performance

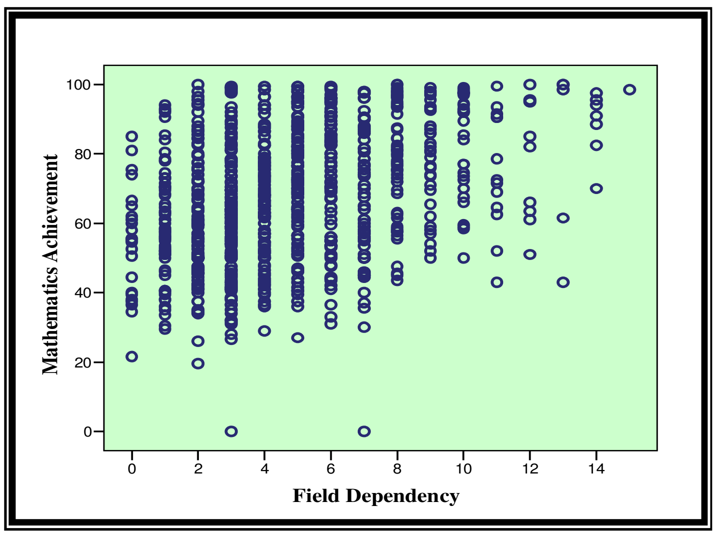


Figure 9-14: Scatter diagram of scores in GEFT related to performance

Secondly, The Pearson correlation between students' extent of field dependence and their marks in the mathematics tests developed for this study (versions standardised) gave a value of 0.57, $p < 0.001$ level. The scatter plot is shown in figure 9-15, illustrating the strength of the correlation.

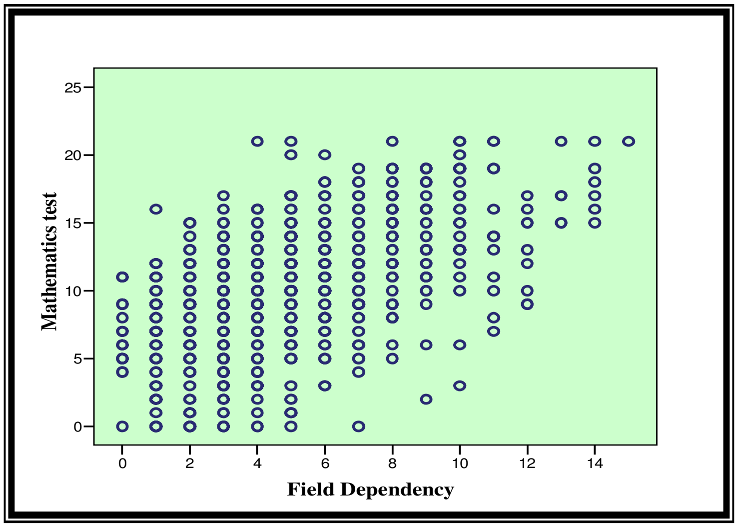


Figure 9-15: Scatter diagram of scores in GEFT related to mathematics tests performance.

Table 9-17 shows the relationship by looking at the three groups.

GROUP	MEAN SCORE IN MATHEMATICS TEST
Field Dependent	7
Field Intermediate	10
Field Independent	13

Table 9-17: The relationship between field dependency & mathematics tests performance

Table 9-18 shows the relationship between students' extent of field dependence and their performance in school mathematics examinations according to their grade.

Group	MEAN SCORE IN MATHEMATICS	
	GRADE 8	GRADE 9
Field Dependent	62	56
Field Intermediate	70	64
Field Independent	79	74

Table 9-18: The relationship between field dependency & mathematics performance according to grades

Table 9-19 shows the relationship between students' extent of field dependence and their performance in mathematics tests according to their grade.

Group	MEAN SCORE IN MATHEMATICS Test	
	GRADE 8	GRADE 9
Field Dependent	7	7
Field Intermediate	9	10
Field Independent	12	13

Table 9-19: The relationship between field dependency & mathematics tests performance according to their grades

The tests' questions were divided into two groups

(1) Questions which are essentially algorithmic in presentation;
(2) Questions which involve considerable 'noise' in terms of language or graphics

The first group are labelled 'algorithmic' and the second group as 'noisy'.

	Number	Mean	SD
Algorithmic	**32**	**0.54**	**19.9**
Noisy	**32**	**0.39**	**17.9**
T = 3.2, $p < 0.05$			

Table 9-20: The classification of tests' questions

It clear from the table above that there is significant difference between the performance means. Students' performance drop dramatically when the questions are 'noisy'.

9.10 Working Memory, Field Dependency and Performance

After the separate analyses of the performance in mathematics with working memory space and with field dependency, the two factors are considered together. When these independent variables, the students' X-space and their degree of field dependency are put together in the regression model, they account for 13% of the total distribution of the students' performance in mathematics. This indicates a significant correlation between these factors and achievement in mathematics. Table 9-21 shows the subgroups of students' X-space and their degree of field dependence, and there are the means of performance in mathematics for each subgroup.

GROUP	FD Mean score	FIT Mean score	FI Mean score
X=4	**59**	**64**	**70**
X=5	**60**	**68**	**75**
X=6	**65**	**73**	**77**

Table 9-21: The relationship between field dependency and working memory with mathematics performance

As we can see, there is a direct relationship between students' achievement in mathematics and their scores in both psychological tests, working memory space and field-dependency. When the scores in these tests are increasing, the achievement in mathematics is also increasing. The effect is quite marked: the mean performance in the school mathematics examinations of students who are field dependent and with a low working memory is 59 compared to 77 for those who are field independent and with a high working memory.

A similar pattern is obtained when looking at the two cognitive variables in relation to performance in the mathematics tests designed for this study (table 9-22). This explanation was offered first by Johnstone (1993).

GROUP	FD Mean score	FIT Mean score	FI Mean score
X=4	5	7	11
X=5	7	10	13
X=6	9	11	14

Table 9-22: The relationship between field dependency and working memory with mathematics tests performance

The multiple regression models were applied as a final stage in the analysis of the effect of the working memory space and field dependency on mathematics tests. The dependent variable is the students' scores in mathematics and there are two independent variables, the working memory capacity and field dependency. Both variables explain the total distribution of mathematics tests scores by 33% (significance level of $p < 0.001$). Of course, correlation does not of itself imply causality. However, the work of El-Banna and Al-Naeme show clearly that there is a large measure of causality involved here (El-Banna, 1987; Al-Naeme, 1988).

9.11 Factor Analysis

Factor analysis is a method of looking at correlations between several variables to explore whether there are any underlying reasons to account for the observed relationships. Principal Components Analysis using varimax rotation was employed here. The procedure does not identify what any factors (or components) are. It merely indicates how many factors exist and how the variables relate to these. The relationship is expressed as 'loadings'. These loading can be seen as correlations between the variables and the extracted factors (or components).

Four variables were considered: working memory capacity, extent of field dependency, test mark (versions standardised) and the school test marks. The scree plot indicated 2 factors and these accounted for 75% of the variance (see appendix F). This is satisfactory. The loadings are shown in table 9.23, with high loadings coloured in pink. Loadings are correlation of measurements with the factors.

Variables	Factors	
	1	2
Working memory capacity	0.12	0.98
Field dependency	0.83	-0.02
Test mark	0.81	0.34
Mathematics performance	0.74	0.11

Table 9-23: Loadings from Factor Analysis

In some ways, the outcomes from the analysis are a little surprising. Previous work (Hindal, 2007) has shown that working memory tends to load as a completely separate factor. This is because this is measuring the size of part of the brain and this is very different from performance – as in the other three measurements. The other three variables load onto the first factor. This almost implies that extent of field dependency is a performance factor like a mathematics test. Indeed, there is it considerable controversy about the nature of field dependency (Danili, 2004). Is ability to select information from noise the same as skill in mathematics?

Indeed, there is the possibility that the key to success in mathematics to seek to develop this skill in learners. The skill certainly grows with age (see table 7-7). It is not known if this is simply developmental, like working memory capacity, or is a function of learning and experience. If it is learning and experience, which is more likely, then an aim must be to explore what teaching strategies enhance the skill.

A Summary: This chapter attempts to explore the cognitive factors which affect achievement in mathematics. The most important findings can be summarised as follows:

- Overload of working memory is likely to be at least partly responsible for students' difficulties in solving mathematics tasks. High working memory students (X = 6) performed better in mathematics than those with lower working memory space capacity (X = 4).

- Field-independent students achieved better than other groups of students because their abilities to distinguish the important and relevant information from irrelevant ones, allowing them to use their working memory space efficiently. Field-dependent students do not have this ability; therefore unimportant and irrelevant items occupy their working memory space.

- The visual presentation of a task can provide much information as *one* chunk which helps to reduce the load on the working memory and, thus, the visual presentation assists the students to solve the task properly. However, a complicated picture with a lot of information may overload the working memory space in any attempt to understand.

- Applied questions hinder good performance because the more applied question format increases working memory overload.

The next chapter explores attitudinal areas further.

Chapter 10

Attitudes towards Mathematics

Phase Two

10.1 Introduction

In order to explore further aspects of students' attitudes towards mathematics, a second questionnaire was used. The aspects that the second questionnaire looked at are:

- Methods to help them understand mathematics;
- The importance of mathematics as discipline
- Attitudes towards different topics within the mathematics syllabus.
- Activities in mathematics classes.
- Opinion about mathematicians
- Attitudes towards learning mathematics; and
- Confidence in learning mathematics.

The survey involved the same sample as in chapter 9 (table 10-1)

GROUP	GRADE 8 (14YEARS)	GRADE 9 (15YEARS)	TOTAL
Boys	146	143	289
Girls	269	316	585
TOTAL	415	459	874

Table 10-1: Sample characteristics (second phase)

The questions were analysed separately. The tables show the response patterns for grade eight and grade nine groups, followed by the patterns for boys and girls. The data are shown as percentages for clarity. Chi-square was used as a contingency test to compare between groups and was calculated using the actual frequencies. Colour shading has been used to highlight key outcomes in the tables.

10.2 Methods to Help Understanding of Mathematics

G8 %	G9 %	*I think the following methods will help me to understand mathematics….. Tick THREE boxes which you think are the most important.*
64	58	Using a calculator.
41	31	Using a computer.
30	29	Have more mathematics lessons.
19	29	Using teaching aids such as models, pictures or diagrams.
31	28	Using games based in mathematics classes.
21	23	Use mathematics to solve real-life problem.
47	51	Teach mathematics more slowly.

Table 10-2: Question 1

The purpose of this question is to look at the methods that the students think will help them to understand mathematics. Students think using a calculator, teach mathematics more slowly and using a computer will help them to understand mathematics. The percentage differs from grade eight to grade nine according to the syllabuses for each grade. A high proportion of grade eight choose using a calculator because the syllabus of grade eight in the first term covers topics such as fractions and solving equations which depend considerably on calculation. On the other hand, in grade nine the syllabus covers triangle theories in the first term. Grade nine students think teaching mathematics more slowly will provide the opportunity to understand mathematics topics.

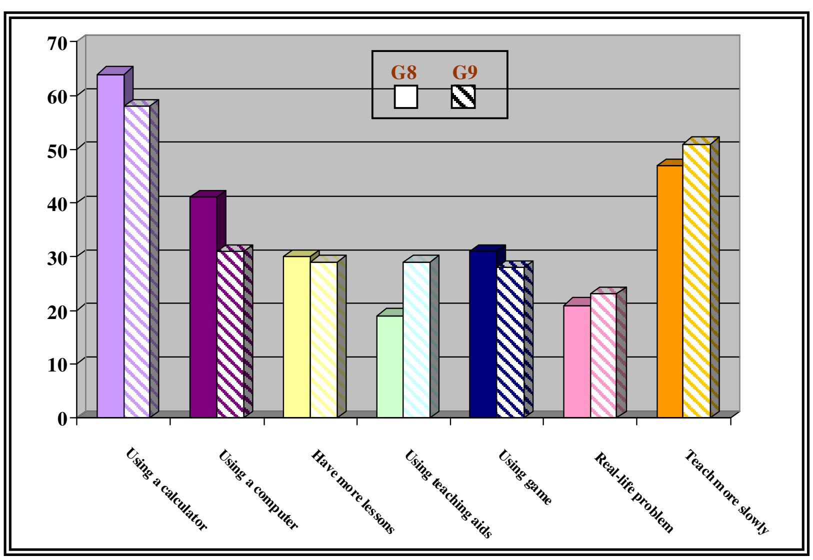


Figure 10-1: Question 1

10.3 Mathematics Importance

G8 %	G9 %	*I think mathematics is important* *Tick THREE boxes which you think are the most important.*
52	49	It is useful in daily life.
29	35	It is important for some other subjects.
22	22	Mathematics can help to solve world problems.
24	17	It is a useful way to make sense of the world.
32	33	There are many jobs for mathematicians.
31	31	It teaches me to think logically.
55	49	It is important for many courses at university

Table 10-3: Question 2

The aim of this question is to explore students' opinions about the reason for the importance of mathematics. Grade eight students think mathematics is important because it is required for many courses at university, it is useful in daily life and there are many jobs for mathematicians. Grade nine students agree with the first two choices but the third choice is different. They chose 'it is important for some other subjects', because in grade nine they start to use their mathematics knowledge in the science field.

While mathematics is perceived as a service subject for other tasks, the response that it is useful in daily life is interesting. It is not obvious why this should be so. The mathematics used in daily life rarely involves any more than very simple arithmetic skills.

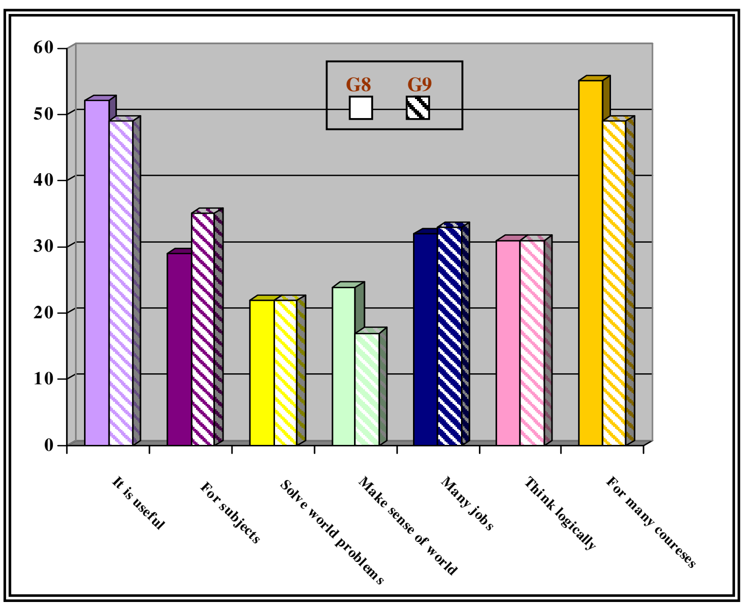


Figure 10-2: Question 2

10.4 Interesting Topics

The purpose of this question is to explore which topics interest students. The students are allowed to choose as many as they like.

G8 %	Which of the following topics interest you? *Tick as many as you wish*
50	Solving equations
18	Elementary sets theory
22	Quadratic equations
23	Fractions
9	Transformation geometry
23	Volume
12	Analytic geometry

Table 10-4: Question 3 (Grade 8)

The majority of grade eight students say that have an interest in solving equations. Students' interest in solving equations might be attributable to the challenge in such tasks. Only a small proportion of the grade eight sample chose Elementary sets and this have arisen because many had not reached the topic when the questionnaire was held.

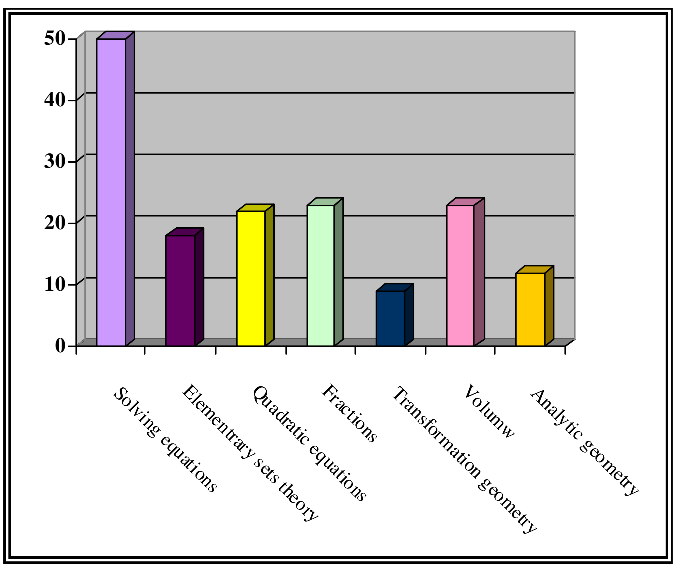


Figure 10-3: Question 3 (grade 8)

In looking at topics (table 10-5), sets and their operation gain a high percentage of grade nine preferences. Then, transformation geometry draws the second highest proportion. Student choices may be depend on their confidence about the topic and the easiness of it.

G9 %	Which of the following topics interest you? *Tick as many as you wish*
48	Sets and their operation
15	Inequalities
30	Solving equations
27	Triangle geometry
12	Circle geometry
38	Transformation geometry
7	Polynomials

Table 10-5: Question 3 (Grade 9)

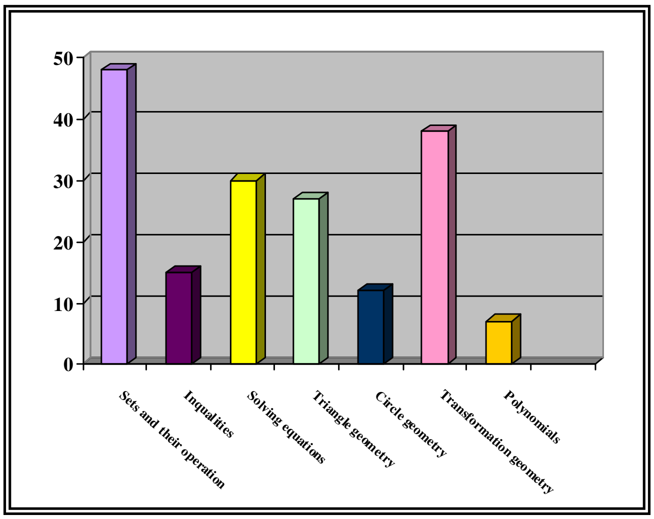


Figure 10-4: Question 3 (Grade 9)

10.5 Mathematics Difficulty

The aim of this question is to explore where students place their reliance when facing difficulties. Students are allowed to choose three boxes which they think are the most important. Table 10-6 shows the percentage of students' responses in grade 8 and 9.

G8 %	G9 %	*When I have difficulty in studying mathematics, I rely on …..* *Tick THREE boxes which you think are the most important*
43	41	School textbook
39	38	Family member
43	47	School teacher
28	32	Out-of-school teacher
11	11	General mathematics book
15	6	Internet
35	37	Self-teaching manual
23	32	Friends

Table 10-6: Question 4

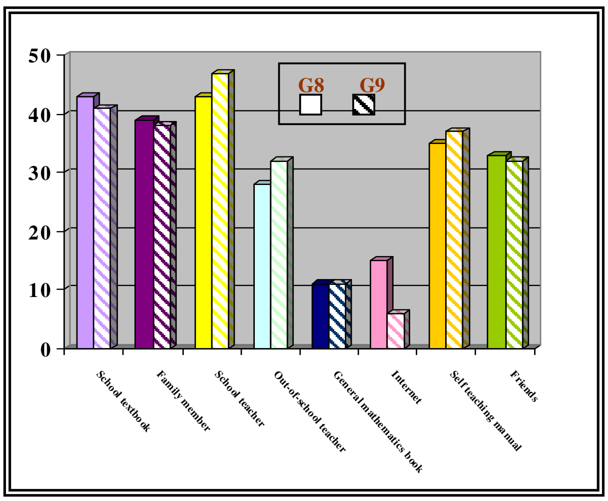


Figure 10-5: Question 4

A high proportion of the sample (almost half) indicated that they rely on the school teacher if they face difficulty in mathematics. Students need someone to show them what is important, to demonstrate the techniques and to respond to their questions. A computer or a book will rarely be able to interact with them in this fashion (Krantz, 1993). A high proportion of the students also indicated that they rely on the school textbook if they face difficulty in studying mathematics. This reveals the importance of having quality school textbooks. The high reliance on the out-of-school teacher (private tutor) is a matter of concern, with the least successful relying most (see table 10-19).

10.6 Activities in Mathematics Classes

The aim of this question is to explore which activity the students prefer in mathematics classes. Table 10-7 illustrates the percentage of students' responses in grade 8 and 9.

G8 %	G9 %	What type of activity do you like in mathematics classes? *Tick ONE box*
24	20	Solving exercises and problem
4	3	Discovering
6	7	Working on my own
1	4	Theory proving
18	11	Using a computer
2	4	Reasoning and proving
10	8	Working as a group
13	12	Listening to the teacher
12	21	Discussion

Table 10-7: Question 5

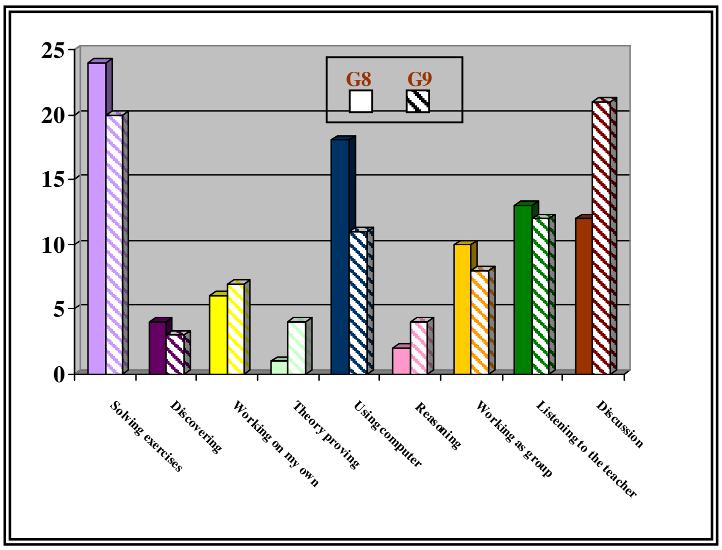


Figure 10-6: Question 5

Solving problems and discussion show a high proportion of students' preferences in grade eight and grade nine. Solving exercises and problems, for the students, means the completion of exercises, usually from the textbook. Success in mathematics tests and examinations depends so heavily on being able to carry out procedures correctly; the importance of practice is obvious. However, the question relates to students saying that they like the activity. There is, perhaps, a satisfaction in mastering a technique so that it can be carried out automatically and correctly. There is also a satisfaction in getting a right answer in that the students then know that they can cope with the test and examinations when they come. The perceptions of students are perhaps rather practical and sensible here!

Discussion can help the students to share where they have problems and where they do not understand but, perhaps, are uncertain about revealing such difficulties to their teacher. There can be cognitive conflict in such discussion and ideas can be corrected, modified and challenged in useful ways. Students *think* that discussion will help them but their experience may be too limited to make such a judgement. The main reason may well be that such discussion gets them away from a teacher dominated lesson.

Using a computer also shows a high proportion of the students' preference. As Garnett and Hackling (1995) suggested, the use of modern audiovisual technologies and computer graphics can overcome difficulties with abstract, unobservable concepts (Garnett and Hackling, 1995). However, using modern technologies may create another difficulty which is unfamiliarity with usage of such technologies. Also, the problem of finding quality software is very real and there may be difficulties in gaining access to appropriate hardware when the need arises. Of course, new technology has novelty value and this sometimes influences its use when evidence for its effectiveness is not apparent.

A greater problem relates to working memory. In a very recent study with first year university students, the key problem of information overload in much typical software was identified (Chandi, 2008). It is absolutely essential that software design takes this limiting factor into account. One study in chemistry looked at the use of computer software and found that it brought about no advantage at all in learning (AlJumailly, 2006). The author considered working memory overload and regarded this as the main problem.

Proving theories and the skills of reasoning and proving are rated poorly. The former is a very boring task with little apparent benefit while the lack of support for the latter may reflect the developmental stage of the learners here. Such skills will not yet be fully accessible to them.

10.7 Secondary Mathematics versus Primary Mathematics

The question aims to explore students' ideas about why secondary mathematics is often seen as more difficult than primary mathematics. The vast majority see secondary mathematics as very complicated and involves difficult explanations. Unfortunately, only a very small minority of the whole sample see secondary mathematics as no more difficult than primary and it is less in grade nine than grade eight.

G8 %	G9 %	**Secondary mathematics is often seen as more difficult than primary mathematics.** *Tick ONE box which best describes* **Secondary mathematics**
7	8	Not related to the real-life
2	3	Very abstract
27	26	Secondary mathematics involves difficult explanation
38	41	Very complicated
17	11	Secondary mathematics is no more difficult than primary

Table 10-8: Question 6

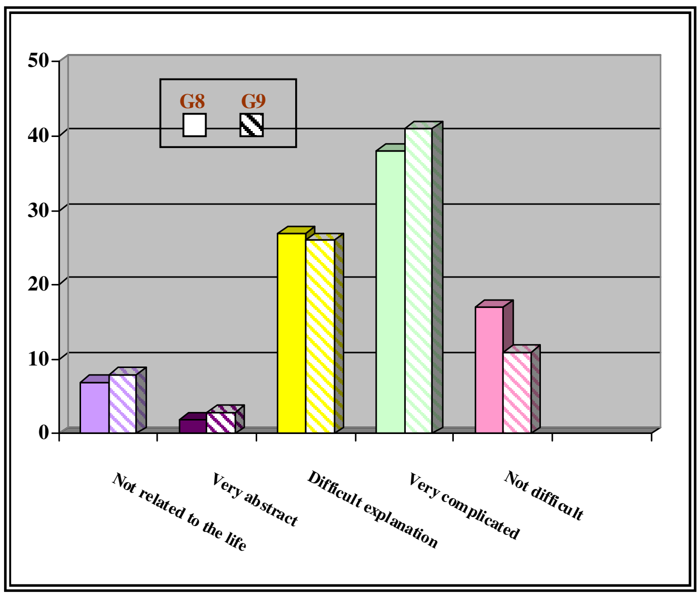


Figure 10-7: Question 6

10.8 Image of Mathematicians

The aim of question 7 is to explore the image of mathematicians among grade eight and grade nine students. The data are shown as percentages for clarity. Chi-square was used as a contingency test to compare between groups and was calculated using the actual frequencies. The vast majority of the students see mathematicians as clever and valuable in the society. On the other hand, they see the mathematician as a hard worker but unlikely to be wealthy. A very small minority see mathematicians as dull and they justify their view by writing "because they invent such complicated things".

What is your opinion about mathematicians? *Tick ONE box in each line* Grade 8 Grade 9								χ^2	df	p
Clever	52	14	11	4	1	10	**Dull**	5.9	5	ns
	57	9	11	3	1	9				
Valuable to the society	46	16	15	4	2	8	**Worthless to the society**	18.5	5	<0.01
	52	9	10	5	2	12				
Popular	17	4	17	13	6	34	**Not Popular**	18.0	5	<0.01
	44	5	12	11	6	11				
Hard worker	45	10	14	6	2	13	**Not a hard worker**	5.3	5	ns
	45	11	9	5	2	16				
Rich	15	10	31	18	3	13	**Poor**	8.5	5	<0.05
	12	7	28	25	3	13				
Doing a dangerous job	13	1	16	15	8	37	**Doing a safe job**	36.3	5	<0.001
	6	2	8	13	8	53				

Table 10-9: The distribution of grades 8 & 9 student's responses and chi-square values (Q7)

Table 10-9 shows a growth of negative attitudes toward the valuation of the mathematician from grade eight to nine. Grade 9 students hold slightly more polarised views in thinking about the value of mathematicians to society. In looking at the data, it is important to note that their main (maybe only) evidence about mathematicians is their teachers. They may well be describing their own teachers.

10.9 Presentation of Questions

The aim of this question is to explore students' ideas about the easier presentation of mathematics tasks. As we can see from the table below that students' views polarise between the two dimensions.

The mathematics tasks are easier for me, if they are presented… *Tick ONE box in each line.* **Grade 8**							**Grade 9**	χ^2	df	p
Pictures, like diagrams	37	5	7	6	5	29	**symbol, like algebra**	9.8	5	<0.05
	41	5	10	5	3	24				
abstract tasks	35	5	5	4	6	34	**real-world tasks**	2.2	5	ns
	34	4	5	5	4	36				

Table 10-10: The distribution of grades 8 & 9 student's responses and chi-square values (Q8)

Table 10-10 shows the distribution of grade eight and grade nine student's responses and chi-square values. Little significant difference is found between eight and grade nine but slightly more grade nine students think the presentation of mathematics tasks in term of picture like diagrams is easier. This difference might be attributable to their recent mathematics curricula. The syllabus in grade nine is highly focus in proving theories and geometry, so the presentation of mathematics tasks in term of pictures may be easier. By contrast, in grade eight the syllabus is highly focus in equations and algebra and the presentation of the tasks in the symbol manner is easier for them. However, in both questions, views are highly polarised suggesting strong views. The polarisation poses some very real problems for the mathematics teacher. It is very difficult to meet the needs of the students with such diverse preferences. The only way forward is to seek to provide a variety of experiences.

10.10 Studying Mathematics

This question aims to look at the students views about their learning style. The following table shows the distribution of grade eight and grade nine student's responses and chi-square values.

When I study mathematics… *Tick ONE box in each line*	Grade 8 / Grade 9							χ^2	df	P
I rely on memorizing	19	4	8	8	4	47	understanding	10.6	3	<0.05
	15	2	5	6	4	56				
I enjoy challenging activities	31	8	11	8	2	29	I do not enjoy	3.7	2	ns
	26	7	10	7	3	34				
I enjoy repetitive tasks	42	8	12	8	3	16	I do not enjoy	3.5	5	ns
	42	8	11	5	2	19				
I like to master one way of achieving a task	41	8	8	7	3	23	many ways	5.3	4	ns
	43	5	7	5	4	24				
I find exercises boring	33	5	11	10	6	24	interesting	5.8	4	ns
	34	3	8	11	7	25				
I depend on the teacher most	46	9	12	9	1	11	on the text book most	4.2	3	ns
	51	7	12	5	2	10				
I can hold all the ideas in my head easily	21	10	12	10	5	31	I cannot hold	8.7	3	<0.05
	16	8	10	10	5	38				
I am not quite sure what is important	28	8	16	11	6	21	I am quite sure	12.2	4	<0.05
	36	5	11	9	5	19				

Table 10-11: The distribution of grades 8 & 9 student's responses and chi-square values (Q9)

The vast majority of the students

- *rely on understanding in studying mathematics,*
- *prefer the repetitive tasks,*
- *like to master one way of a achieving a task, and*
- *depend on the teacher most.*

These views might be attributable to the nature of mathematics which means that understanding is heavily dependent on the quality of the explanation from the teacher. Thus, they highly depend on the teacher in mathematics classes, prefer the repetitive tasks and like to master one way of achieving a task to avoid failure in mathematics.

By contrast, the students' views polarise in the

- *enjoyment of challenging activities,*
- *enjoyment in doing the exercises,*
- *their ability to hold all the ideas in their head easily, and*
- *their assurance about what is important.*

More students in grade 9 state that they rely on understanding; they face difficulties when trying to hold all the ideas in their head, and they are not quite sure what is important. This growth of negative attitudes among grade nine students might be attributable to the higher demand levels of the syllabus. The syllabus in grade nine is highly focussed on proving theories and there are 20 theories (10 triangle theories; 10 circle theories) that students are required to understand, prove them and hold them in mind in order to apply them in other tasks. Problems are to be expected, caused largely by an inappropriate syllabus emphasis.

10.11 Confidence and Mathematics

The aim of this question is to explore students' confidence in learning mathematics, and how sure a student is of his/her ability in learning mathematics. Mayer and Koehler (1990) referred to the importance of confidence in learning process by saying *"Confidence influences a student's willingness to approach new material and to persist when the material becomes difficult"* (P: 61). The following table shows the distribution of grade 8 and 9 student's responses. The data are shown as percentages for clarity. Chi-square was used as a contingency test to compare between groups and was calculated using the actual frequencies.

How do you describe yourself in mathematics classes? **Grade 8** **Grade 9**	*SA*	*A*	*N*	*D*	*SD*	χ^2	df	p
I am generally a confident person in mathematics classes	**21**	**24**	**28**	**6**	**10**	**38.4**	**4**	**<0.001**
	16	**24**	**30**	**10**	**9**			
I feel more confident when I succeed in solving a task	**55**	**22**	**9**	**3**	**3**	**2.3**	**4**	**ns**
	53	**22**	**9**	**2**	**4**			
I feel confident when I study mathematics	**28**	**29**	**17**	**8**	**9**	**12.3**	**4**	**<0.05**
	21	**26**	**25**	**9**	**9**			
I feel confident when I really understand what is being taught in mathematics classes	**51**	**24**	**8**	**4**	**3**	**2.3**	**4**	**ns**
	48	**24**	**10**	**3**	**4**			
I feel confident taking part in a discussion group in mathematics classes	**47**	**22**	**14**	**3**	**4**	**6.0**	**4**	**ns**
	44	**26**	**14**	**1**	**5**			
I feel confident in mathematics examinations	**16**	**21**	**18**	**12**	**21**	**7.4**	**4**	**ns**
	12	**18**	**24**	**14**	**21**			
I am confident even when facing difficult material to understand in mathematics classes	**10**	**14**	**18**	**17**	**32**	**3.9**	**4**	**ns**
	10	**12**	**22**	**16**	**29**			

Table 10-12: The distribution of grades 8 & 9 student's responses and chi-square values (Q10)

Looking at the questions overall, it is obvious that students feel confident in mathematics classes if they succeed in solving a task, when they really understand what is being taught in mathematics classes and if they take part in a discussion group. The majority of the students tend to lose their confidence in mathematics examinations and when they face difficult material to understand in class. Unfortunately, three quarters of the sample see secondary mathematics as abstract, complicated and difficult (see question 5). Thus, in this case losing of confidence may lead the students to failure and failure to achieve any tasks in mathematics classes or examinations may lead the students to lose their confidence. The following chart shows the path to confidence in learning mathematics (or in learning in general) derived from students' beliefs.

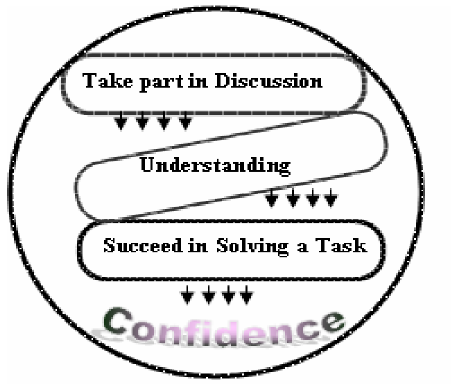


Figure 10-8: Confidence path

There are few differences between the grades. However, there is significant loss of confidence in mathematics classes among grade nine students. This may simply reflect the syllabus.

10.12 Like/Dislike Mathematics

Question 11 is an open one, asking the students to write three sentences to explain why they like or dislike mathematics. The response categories shown in table 10-13 were created from reading their actual sentences in an attempt to summarise the data.

Write *Three* sentences to explain why you like or dislike mathematics.					
45% Like			55% Dislike		
$\chi2 = 14.3$ (1), $p < 0.001$					
G8 %	G9 %	G8 (44%) G9 (47%)	G8 %	G9 %	G8 (56%) G9 (53%)
37	36	I have good teacher $\chi2 = 0.0$ (1), n.s.	18	22	I have bad teacher $\chi2 = 0.9$ (1), n.s.
42	46	useful in daily life $\chi2 = 0.5$ (1), n.s.	17	19	Not related to the real-life $\chi2 = 0.2$ (1), n.s.
24	24	I always love math $\chi2 = 0.0$ (1), n.s.	10	17	I always hate math $\chi2 = 4.6$ (1), $p < 0.05$
38	23	easy subject $\chi2 = 9.9$ (1), $p < 0.005$	81	89	Difficult subject $\chi2 = 4.7$ (1), $p < 0.05$
16	16	I do not need to study $\chi2 = 0.0$ (1), n.s.	82	83	complicated subject $\chi2 = 3.7$ (1), n.s.
4	6	useful to solve problems $\chi2 = 0.7$ (1), n.s.	36	30	the explanation not clear $\chi2 = 1.8$ (1), n.s.
38	31	useful to my career in the future $\chi2 = 1.8$ (1), n.s.			
17	36	challenging subject $\chi2 = 15.9$ (1), $p < 0.001$			
18	39	interesting subject $\chi2 = 19.7$ (1), $p < 0.001$			

Table 10-13: Question 11

The first thing to be notice is that most of the students state that they dislike mathematics. They attributed their dislike to the difficulties that they face in mathematics classes and to the complicated nature of the subject. Students who indicated that they like mathematics attributed their view to the usefulness of mathematics in daily life and for their career in the future.

The second thing to be notice is that mathematics teacher play a crucial role in the formation student attitudes towards mathematics. Student answers in question 11 draw a very clear picture about the characteristic of their teachers whether they have ‘good’ teacher (so they like mathematics because they love him/her), or they have ‘bad’ teacher (so they dislike mathematics because they hate him/her). Indeed, it was possible to see which classes have a ‘good’ teacher and which ones have a ‘bad’ teacher.

The third thing to be noticed is that there are significant differences in some of the student responses. More students in grade 8 indicated that they like mathematics because it is an easy subject while more students in grade 9 indicated that they dislike mathematics because it is a difficult subject. The two responses patterns indicated that grade 9 syllabus is more difficult than grade 8 syllabus, and this is emphasised by the teachers' interviews in the following chapter.

10.13 Sex-related Differences in Attitudes towards Mathematics

This section discusses boys and girls attitudes toward mathematics. The data are shown as percentages for clarity. Chi-square was used as a contingency test to compare between groups and was calculated using the actual frequencies. In questions 1-6, the boys and girls responses are almost identical and they are not discussed further here.

	BOY					GIRL		$\chi 2$	df	p
Clever	**69**	**11**	**11**	**2**	**1**	**7**	**Dull**	**12.2**	**5**	**<0.05**
	57	**13**	**13**	**4**	**1**	**12**				
Valuable to the society	**69**	**13**	**8**	**3**	**2**	**6**	**Worthless to the society**	**36.7**	**5**	**<0.001**
	48	**14**	**16**	**6**	**2**	**14**				
Popular	**21**	**6**	**17**	**13**	**6**	**38**	**Not Popular**	**12.5**	**5**	**<0.05**
	13	**5**	**15**	**15**	**5**	**47**				
Hard worker	**59**	**10**	**11**	**6**	**4**	**10**	**Not a hard worker**	**20.5**	**5**	**<0.001**
	47	**13**	**14**	**6**	**2**	**20**				
Rich	**21**	**8**	**33**	**25**	**3**	**10**	**Poor**	**16.8**	**5**	**<0.005**
	12	**10**	**34**	**24**	**3**	**17**				
Doing a dangerous job	**13**	**1**	**8**	**16**	**9**	**53**	**Doing a safe job**	**13.7**	**5**	**<0.05**
	9	**2**	**16**	**15**	**8**	**50**				

Table 10-14: The distribution of boys & girls responses and chi-square values (Q7)

Table 10-14 shows that more boys than girls think the mathematician is

- *Clever*
- *Valuable to society*
- *Hard worker*
- *Rich*
- *Doing a safe job*

Some girl students justify their view about doing a dangerous job by writing "because mathematics is a dangerous subject".

The mathematics tasks are easier for me, if they are presented… BOY GIRL								χ2	df	p
In term of pictures, like diagrams	50	7	12	6	3	23	In term of symbol, like algebra	16.2	5	<0.01
	42	5	8	6	5	34				
As abstract tasks	48	6	6	6	6	29	As real-world tasks	18.9	5	<0.001
	35	4	6	6	5	45				

Table 10-15: The distribution of boys & girls responses and chi-square values (Q8)

The above table shows that more boys indicated that mathematics tasks are easier for me, if they are presented in term of pictures and as abstract tasks. Whereas, more girls state that mathematics tasks are easier for them, if they are presented in term symbol and as real-world tasks. Fruchter (1954, p: 392) indicated that "*it has been found with some consistency that boys on the average excel over girls on spatial tasks [and] that the spatial functions mature between the ages of eleven and fifteen*". Researchers have constantly observed that boys tend to score higher than girls on measures of spatial skills (Fennema and Leder, 1990: p:44), and Hilton and Berglund (1974) indicated that gender differences in mathematics achievement might be attributable at least in part to 'gender-typed interests'. Thus, this difference of preferences between boys and girls may be attributed as Schonberger (1976: p: 43) stated "*According to Mitchelmore's (1975) survey of cross-cultural research, groups who hunt and wander have more highly developed spatial skills than those who farm. If boys in our [any] culture wander more in their play than girls, this could be a cause of sex differences in spatial ability*".

	BOY					GIRL		χ2	df	p
I rely on memorizing	23	3	7	5	5	57	I rely on understanding	8.4	5	ns
	17	3	8	9	5	58				
I enjoy challenging activities	39	10	12	8	2	29	I do not enjoy challenging	13.3	5	<0.01
	29	8	12	8	4	39				
I enjoy repetitive tasks	50	11	10	6	2	21	I do not enjoy repetitive	8.1	5	ns
	47	8	15	8	3	19				
I like to master one way of achieving a task	47	5	8	6	3	32	I like to think of many ways of achieving a task	7.3	5	ns
	48	8	9	7	4	24				
I find exercises boring	35	4	11	9	9	32	I find exercises interesting	8.2	5	ns
	40	4	11	14	6	26				
I depend on the teacher most	57	6	15	8	2	13	I depend on the text book	7.8	5	ns
	54	11	13	9	2	11				
I can hold all the ideas in my head easily	23	10	14	11	6	36	I cannot hold all the ideas	2.7	5	ns
	20	10	12	12	5	41				
I am not quite sure what is important	32	7	16	11	7	28	I am quite sure	5.9	5	ns
	39	8	15	11	6	21				

Table 10-16: The distribution of boys & girls responses and chi-square values (Q9)

The above table shows a very few significant differences between boys and girls in studying mathematics. More boys than girls state that they enjoy challenging tasks: perhaps boys tend to take risks and like challenging tasks more.

BOY GIRL	SA	A	N	D	SD	χ2	df	p
I am generally a confident person in mathematics classes	27	29	27	7	11	11.6	4	<0.01
	18	26	35	10	11			
I feel more confident when I succeed in solving a task	62	21	9	2	5	4.0	4	Ns
	58	26	10	3	4			
I feel confident when I study mathematics	33	32	19	7	10	11.1	4	<0.05
	24	30	26	11	10			
I feel confident when I really understand what is being taught in mathematics classes	57	24	11	3	4	2.3	4	Ns
	54	28	10	4	4			
I feel confident taking part in a discussion group in mathematics classes	52	22	18	2	5	5.5	4	Ns
	50	28	14	3	5			
I feel confident in mathematics examinations	21	23	24	13	19	9.9	4	<0.05
	14	21	24	15	26			
I am confident even when facing difficult material to understand in mathematics classes	15	14	22	15	34	12.6	4	<0.05
	9	15	23	20	34			

Table 10-17: The distribution of boys & girls responses and chi-square values (Q10)

More boys than girls state that:

- *I am generally a confident person in mathematics*
- *I feel confident when I study mathematics*
- *I feel confident in mathematics examinations*
- *I am confident even when facing difficult material to understand in mathematics.*

Generally, boys feel more confident in mathematics classes and this difference may be related to the masculine and feminine natures. Boys tend to be confident, and girls tend to feel worried more easily. Meyer and Koehler (1990) indicated that "*gender differences in confidence were also found even when there were no differences in achievement. At both the middle and high school levels, females reported lower levels of confidence in their ability to learn mathematics than did males* " (p: 61).

Having looked at the student responses to the questionnaire and considered any gender differences, the next questions are whether the students' attitudes are related in any way to their achievement, to their measured working memory and their measured extent of field dependency. The next section looks at achievement.

10.14 Mathematics Performance and Attitudes towards Mathematics

The correlations between students' achievement in mathematics and their attitudes towards mathematics are discussed in this section.

Correlation between attitudes and performance in mathematics *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	**Correlation Coefficient**	
	Grade (8) *N=(415)*	**Grade (9) N=(459)**
Using a calculator.	**-0.11**	**-0.17**
Using a computer.	**-0.04**	**-0.11**
Have more mathematics lessons.	**0.02**	**-0.06**
Using teaching aids such as models, pictures or diagrams.	**0.08**	**0.05**
Using game based in mathematics classes.	**0.08**	**0.04**
Use mathematics to solve real-life problem.	**0.05**	**0.10**
Teach mathematics more slowly.	**-0.04**	**0.05**

Table 10-18: Correlations between students' responses in Q1 and performance in mathematics

There is a significant negative correlation between students' responses that the calculator and computer help them in studying mathematics and their achievement. Low achievement students tend to believe that the usage of the calculator and computer will help them to in studying mathematics. In fact, using a calculator in mathematics classes for low achievement may help them in the high working memory demanding tasks. Many mathematicians support students' view and they wonder '*what is the purpose of spending years teaching these procedures which will be rarely used in adult life, especially for less able pupils, with the availability of cheap powerful electronic calculators'* (Macnab and Cummine; 1986). However, the students may well *think* that calculators would help rather than have any assurance that they do help.

Correlation between attitudes and performance in mathematics *$p <0.05$* *$p <0.01$* *$p <0.001$*	**Correlation Coefficient**	
	Grade (8) *N=(415)*	**Grade (9)** **N=(459)**
School textbook	**0.10**	**0.01**
Family member	**0.06**	**0.10**
School teacher	**0.17**	**0.12**
Out-of-school teacher	**-0.21**	**-0.25**
General mathematics book	**-0.11**	**-0.10**
Internet	**-0.16**	**0.10**
Self-teaching manual	**0.03**	**-0.03**
Friends	**0.01**	**0.07**

Table 10-19: Correlations between students' responses in Q4 and performance in mathematics

There are low but highly significant correlations between students' achievement and some things that they rely on when facing difficulty in studying mathematics. High achieving students tend to rely more on the school teacher while low achievers rely more on the out-of-school teacher, perhaps because low achievers need the extra personal support in answering their questions (Krantz, 1993). It is argued that high achieving students tend to teach themselves, with instructors to point out, guide or clarify some points (Krantz, 1993), the school teacher providing this very adequately.

Correlation attitudes and performance in mathematics *$p <0.05$* *$p <0.01$* *$p <0.001$*	**Correlation Coefficient**	
	Grade (8) *N=(415)*	**Grade (9)** **N=(459)**
I rely on memorizing	**-0.09**	**-0.03**
I enjoy challenging activities	**0.14**	**0.12**
I enjoy repetitive tasks	**-0.10**	**0.01**
I like to master one way of achieving a task	**-0.06**	**0.06**
I find exercises boring	**-0.17**	**-0.04**
I depend on the teacher most	**-0.05**	**0.08**
I can hold all the ideas in my head easily	**0.12**	**0.09**
I am not quite sure what is important	**-0.10**	**-0.03**

Table 10-20: Correlations between students' responses in Q9 and performance in mathematics

Table 10-20 shows that students who obtained higher marks in mathematics tend to:

- *rely on understanding*
- *enjoy challenging activities*
- *find exercises interesting*
- *feel they can hold all the ideas in their head easily*

- *they are quite sure what is important.*

It is interesting to note that being able to hold all ideas in your head easily (high working memory space) and being sure what is important (being field independent) will lead to performing better in mathematics (Christou, 2001; Alenezi, 2004).

Correlation attitudes and performance in mathematics *$p < 0.05$ $p < 0.01$ $p < 0.001$*	**Correlation Coefficient**	
	Grade (8) ***N=(415)***	**Grade (9)** **N=(459)**
I am generally a confident person in mathematics classes	**0.20**	**0.20**
I feel more confident when I succeed in solving a task	**0.19**	**0.18**
I feel confident when I study mathematics	**0.19**	**0.12**
I feel confident when I really understand what is being taught in mathematics classes	**0.17**	**0.12**
I feel confident taking part in a discussion group in mathematics classes	**0.14**	**0.16**
I feel confident in mathematics examinations	**0.20**	**0.12**
I am confident even when facing difficult material to understand in mathematics classes	**0.12**	**0.04**

Table 10-21: Correlations between students' responses in Q10 and performance in mathematics

There are significant correlations between high achievement and confidence in mathematics classes with the higher achieving students tending to feel more confident. The fundamental question is how to develop confidence in learners so that they can improve and apply these skills? In her study, Oraif (2007) found that the only thing that seems to lead to confidence among students is past success, that success being limited to success in academic areas. The essential question is how to offer success to those who are not so good at formal examinations, particularly when these are based on recall. If success is the key to confidence, there is a real danger that the examination system will generate many of the population who are unsuccessful, thus reducing confidence. The system may lead to the destruction of confidence. It does not seem to be the style of examination but the fact of success in examination, which is a crucial factor for the confidence. This is important in that confidence may lead to further success or even a willingness to try**.**

10.15 Working Memory and Attitudes towards Mathematics

The correlation between students' working memory space and their beliefs about their learning style is shown in the following table (using Kendall's Tau-b).

Correlation attitudes and working memory space *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	**Correlation Coefficient**	
	Grade (8) ***N=(415)***	**Grade (9)** **N=(459)**
I rely on memorizing	**0.03**	**0.05**
I enjoy challenging activities	**0.05**	**0.04**
I enjoy repetitive tasks	**-0.05**	**-0.07**
I like to master one way of achieving a task	**-0.01**	**0.02**
I find exercises boring	**-0.08**	**-0.04**
I depend on the teacher most	**-0.02**	**0.04**
I can hold all the ideas in my head easily	**0.05**	**0.03**
I am not quite sure what is important	**-0.08**	**-0.03**

Table 10-22: Correlations between students' responses in Q9 and their working memory space

The first thing to notice is that, in most questions, there are almost no significant correlations between students' working memory capacity and their beliefs about their learning preferences. Even the question of 'I can hold all the ideas in my head easily' (coloured in red see table 10-22) does not correlate significantly with their working memory capacity. It is possible that they cannot see themselves as they are. In a recent study Hindal (2007) found that school students of about the same age saw themselves in ways which bore no relation to the measurement made on learning characteristics. She concluded that students were unable to see themselves as they really were (perhaps seeing themselves as they would like to be) and, therefore, it was not a useful way forward to try to measure any kind of learner characteristic by means of questionnaires.

Correlation attitudes and working memory space *p <0.05* *p <0.01* *p <0.001*	Correlation Coefficient	
	Grade (8) *N=(415)*	Grade (9) N=(459)
I am generally a confident person in mathematics classes	**0.03**	**0.13**
I feel more confident when I succeed in solving a task	**0.09**	**0.06**
I feel confident when I study mathematics	**0.11**	**0.06**
I feel confident when I really understand what is being taught in mathematics classes	**0.03**	**0.01**
I feel confident taking part in a discussion group in mathematics classes	**0.01**	**0.08**
I feel confident in mathematics examinations	**-0.01**	**0.06**
I am confident even when facing difficult material to understand in mathematics classes	**0.01**	**0.03**

Table 10-23: Correlations between students' responses in Q10 and their working memory space

In most cases, no significant correlation was obtained between students' working memory space and their confidence in learning mathematics. Only the data for two questions were found significant. In grade 9, students who have high working memory stated that they are generally confident persons in mathematics classes. Where in grade 8, the students who have high working memory space indicated that they feel confident when they study mathematics.

10.16 Field Dependency and Attitudes towards Mathematics

The correlations between students' field dependency and their attitudes towards mathematics are represented in this section (using Kendall's Tau-b).

Correlation between attitudes and field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade (8) *N=(415)*	**Grade (9)** *N=(459)*
I rely on memorizing	**-0.07**	**-0.07**
I enjoy challenging activities	**0.11**	**0.19**
I enjoy repetitive tasks	**-0.18**	**0.02**
I like to master one way of achieving a task	**-0.12**	**-0.07**
I find exercises boring	**-0.10**	**-0.10**
I depend on the teacher most	**-0.05**	**0.04**
I can hold all the ideas in my head easily	**0.02**	**0.06**
I am not quite sure what is important	**-0.02**	**-0.03**

Table 10-24: Correlations between responses in Q9 and field dependency

Correlations are low and only a few are significant. The result shows that field independent students tend to prefer the challenging activities and those who are field dependent tend to prefer repetitive tasks and find exercises boring. Here again, the question 'I am not quite sure what is important' (coloured in red see table 10-24) does not correlate with the students' field dependency, reflecting their inability to see themselves as they are.

Correlation between attitudes and field dependency *$p < 0.05$* *$p < 0.01$* *$p < 0.001$*	Correlation Coefficient	
	Grade (8) *N=(415)*	**Grade (9)** N=(459)
I am generally a confident person in mathematics classes	**0.10**	**0.19**
I feel more confident when I succeed in solving a task	**0.07**	**0.13**
I feel confident when I study mathematics	**0.03**	**0.10**
I feel confident when I really understand what is being taught in mathematics classes	**0.06**	**0.11**
I feel confident taking part in a discussion group in mathematics classes	**-0.03**	**0.11**
I feel confident in mathematics examinations	**0.10**	**0.12**
I am confident even when facing difficult material to understand in mathematics classes	**0.11**	**0.10**

Table 10-25: Correlations between students' responses in Q10 and field dependency

The first thing to be notice is, in every question, there is a significant correlation between grade 9 responses and their field dependency and grade 8 responses correlate only with three questions. However, the pattern of responses is similar. The result shows that field independent students' tend to describe themselves generally as confident persons in mathematics classes and field dependent students tend to be less confident in mathematics classes. The field independent students are more successful and this will lead to confidence.

A Summary: This chapter has attempted to offer a general view about students' attitudes towards mathematics learning and the relationship between their attitudes and their achievement in mathematics, their working memory capacity and their field dependency. The most important findings from this questionnaire can be summarised as follows:

- Less able students' believe that the calculator is the most helpful method in learning mathematics.
- Mathematics possesses its importance from the students' views that mathematics is important for many courses in the university, important in daily life and there are many jobs for mathematicians.
- There is high proportion of the students who depend on out-of-school teacher if they face difficulty in mathematics.
- Solving problems and discussion attracts the highest proportion of the students' preferences in mathematics classes.
- Students believe that secondary mathematics is more difficult, complicated and abstract than primary mathematics.
- Mathematicians are seen by almost all as clever and valuable in the society.
- Most students say that they rely on understanding when they study mathematics.
- Students believe that their confidence can be improved by taking part in discussion, understanding and then succeed in solving tasks.
- Boy students have more positive attitudes and more confidence in mathematics classes than girl students.
- Field independent students are more confident than field dependent students.

10.17 Review of Findings from Students

From working with students, the importance of working memory and extent of field dependency has been established in relation to success in mathematics. It was also established that the use of a more visual approach had to be applied with considerable care while the introduction of a more applications orientated approach raised the major difficulty of working memory overload.

Indeed, there is the possibility that the key to success in mathematics is to seek to develop ability to select information from noise in learners. There was clear evidence of a decline in positive attitudes with age and the excessively overloaded curriculum was a likely reason along with the perceptions that some topics were irrelevant. Can we reduce the 'noise' amount in mathematics curriculum, which will help the students to be able to select from the techniques variety that they have studied in mathematics classes. The perceptions of mathematics teachers and inspectors will be explored in the next experiment. The next experiment will seek teachers' views about some topics in mathematics which are believed to confuse the students in mathematics such as fractions and triangle theories.

Chapter 11

Mathematics Teachers Interviews

Phase Three

11.1 Introduction

In the two previous phases, the students' working memory capacity, field-dependency and their attitudes have been related to their achievement in mathematics. This was followed by a detailed analysis of two mathematics tests to explore the ways by which students gained success. From this work, the importance of working memory and extent of field dependency has been established in relation to success in mathematics. It was also established that the use of a more visual approach had to be applied with considerable care while the introduction of a more applications orientated approach raised the major difficulty of working memory overload. There was clear evidence of a decline in positive attitudes with age and the excessively overloaded curriculum was a likely reason along with the perceptions that some topics were perceived as irrelevant.

The first two phases had focused on the learners and the curriculum experiences they had. In this phase, the aim is to focus on the perceptions of mathematics teachers and school inspectors to see the extent to which their views relate to the findings from work with students.

This work involved semi-structured interviews which offer an opportunity to focus on some key areas as well as giving freedom for the teachers to expand their views. Interviews were carried out with mathematics teachers and inspectors in their workplaces. This was thought to make them more comfortable with the experience. The interviews were conducted in a relaxed atmosphere. Each teacher or inspector was informed that the aim of the interview was to explore their view of mathematics education and to seek to establish ways by which this could be improved. They were told that all the information in the interview would be held securely and that their names would not be associated with records of the interview in any way. The outcomes of the interview would not affect their job at all. The aim was to encourage a relaxed freedom so that teachers and inspectors would feel free to respond honestly. Notes were taken in shorthand by the interviewer and expanded immediately after each interview, this taking a considerable time.

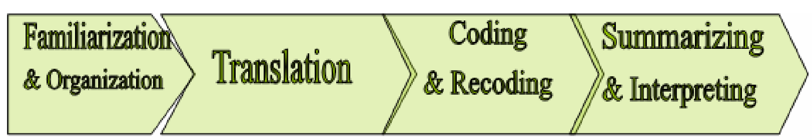
11.2 Main Interview Themes

After a few very simple questions were first asked to establish some factual details about each interviewee and to enable them to relax, the first major area under discussion was how each saw the aims of mathematics education. At that stage, seven topics were considered to establish how these were seen in terms of the stage when they should be taught and aspects of how they should be taught. The idea of ‘readiness for learning’ a topic was introduced and the interviewees were asked to consider which topics they considered caused most of the problems for the students.

At this point, the interviewer gave a very quick summary of the main features of information processing and there was a discussion as to the extent to which this model offered insights to explain why certain topics in mathematics were found difficult. Finally, opportunity was given for the interviewees to suggest any ideas or thoughts about ways to improve mathematics education in Kuwait. The interview schedule used is shown in full overleaf. However, this was a guideline and, where interviewees wished to expand issues, time and freedom was offered for this purpose.

11.3 Stages of the Quantitative Data Analysis

The following diagrams summarises the data analysis stages of the interviews.



Analysing qualitative data involves three main stages: familiarization and organisation, coding and recoding and summarizing and interpreting (Ary et.al, 2006, p: 490). However in this study in addition to the three main steps, an additional step of translation from Arabic to English is involved. The first stage was familiarization and organization. All interviews’ notes were read and reread by the researcher to be familiar with the data, and to be able to organise the open-ended questions. The second stage was translation from Arabic to English. The researcher attempted to translate all the interviews without losing or changing the meaning of interviewees’ words (for more detail see chapter 6). Coding and recoding was the third stage where the researcher identified the main categories and themes. The last stage was the summarizing and interpreting data. In this stage, the researcher attempted to present the massive data in a well-organized way.

<u>Mathematics Education</u>
<u>Mathematics Teachers Interview</u>

Q1: How many years have you been a teacher of mathematics?

Q2: Which age group do you teach now?

Q3: Do you think the ***objectives of mathematics education*** in Kuwait aim to help the students to ***create a mathematical sense*** about the world around them, or aim to ***create mathematicians***?

Q4: Think about the following topics

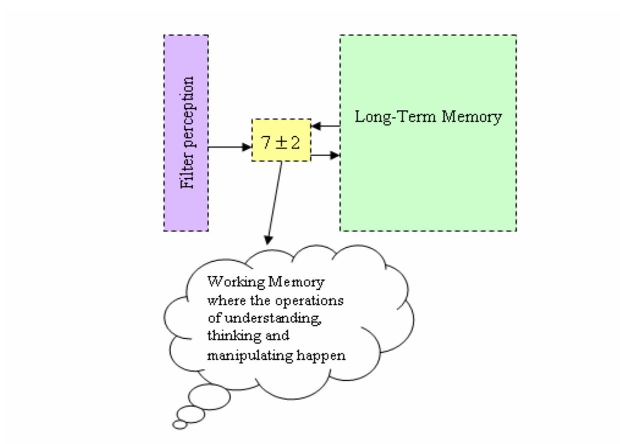
This table will be filled with the cooperation of the teachers

Topic	Essential by age 16	Not essential by age 16	Better as Now	Better Later	Comments
Fractions					
Negative numbers					
Sets and their operation					
Triangle Theories					
Circle Theories					
Quadratic equations					
Percentage					

Q4: According to the idea of ***readiness***, do you think that the early introduction of ***Fractions*** cause a real problem in learning mathematics?

Q5: What is the greatest problems area in mathematics?

This diagram will be used to explain the function of the working memory to the teachers



Q6: Does working memory space make sense for the difficulty of this area?

Q7: Any ***ideas*** or ***thoughts*** that may improve mathematics education in Kuwait?

11.4 Sample Characteristics

25 teachers were selected randomly from different schools in Kuwait. All taught in junior secondary schools (ages 11-15 years old). 4 mathematics inspectors were also interviewed to compare their views. The interviews were held in the autumn of 2007 and each interview lasted from 30 to 35 minutes.

The semi-structured nature of the interviews offered an opportunity to focus on some key areas as well as giving freedom for the teachers to expand their views. The opening questions gave details of the nature of the sample and this is summarised in table 11.1.

Sample	
Male	7
Female	22
Mathematic Teachers	25
Mathematics Inspector	4
Experience	
5-9 Years	10
10-19 Years	16
20 Years & more	3

Table 11-1: Sample Descriptions (Third Phase)

The outcomes from the interviews are now summarised briefly. Each starter question is shown and the pattern of responses obtained brought together. Typical statements made by teachers are offered, trying as far as possible to catch the meaning from the Arabic original.

1.5 Data Analysis

Q3: Do you think the objectives of mathematics education in Kuwait aim to help the students to create a mathematical sense about the world around them, or aim to create mathematicians?

The majority of mathematics teachers think that the specified objectives of mathematics education in Kuwait aim is to create mathematicians rather than help the students to create a mathematical sense about the world around them. In addition, they think the curricula concern about the high achievement students and ignore the low achieving and their attitudes towards mathematics. They seemed to appreciate that what they were being asked to teach was not what was required by the majority. They argued by saying…

> *"Most of the topics are wasting of time, and only the person who will be mathematician will benefit of them."*
>
> *"Mathematics objectives aim to create mathematicians instead of creating the mathematical sense and cover topics more than the students need."*
>
> *"The majority of mathematics syllabus not useful and do not serve the students because it very abstract"*
>
> *"The objectives aim to create a student without any ambition to continue learning mathematics. They will forget all maths that they have been taught at school because of the large amount of mathematical skills which will not help them, they don't need them."*

In complete contrast, the mathematics inspectors and a few mathematics teachers thought the mathematics education objectives aim to help the students to create mathematical sense about the world around them. Typical statements were:

> *"The objectives take on consideration all different levels high and low achievements of the students and all the knowledge that they study are useful in daily life".*
>
> *"The syllabuses are very easy, but the students nowadays differ from those in the past, they don't make an all-out of effort as they did in the past."*
>
> *"The objectives aim to help the students to create mathematical sense about the world around them, and help them to improve their scientific thinking."*
>
> *"The objectives aim to help the students to create mathematics sense and to create positive attitudes towards mathematics."*

From the point of view of the interviewees, it was clear that, there is no agreement about the aims and the objectives of mathematics educations in Kuwait. There is a big gap between the views of those who decide how the syllabuses will be and which topics will be included in them, and those who will teach these syllabuses. In fact, teachers are involved daily in the teaching processes and they know the population of their students very well. Thus, their views about the syllabuses should be taken into consideration and they should be involved in the process of deciding the syllabuses. Overall, the teachers seem to see that what they are being asked to teach is not meeting the needs of the majority of the students. This seems consistent with student views.

Q4: This asked the interviewees to think about the following topics

This question aimed to explore the views of teachers and the inspectors of mathematics about some topics in the mathematics syllabuses which were found to be difficult for the students. Teachers' views will be discussed in detail and then will be compared to inspectors' views which were often very different.

Fractions

The vast majority of mathematics teachers see fractions as an essential topic in mathematics education and that all the students should study this topic in junior secondary school (before the age 16) but they preferred to delay the introduction of fractions from grade five (10 years old) to grade six or seven (11-12 years old) or divide teaching fraction into two years. A typical remark was:

> *"It is better to teach fraction in stages, start with addition and subtraction of fraction and in the following year teach them the multiplication and division of the fraction, to avoid the ambiguity of fraction."*

A small minority of mathematics teachers thought fractions is not an essential topic for all the students in junior secondary school at all. They thought that decimal fractions can be taught to them instead. They also think fractions can be taught to the students who will be specialising in mathematics. However, they offered no insights into how this could be achieved.

Most interviewees thought the topic was an essential topic for the majority of students and that it should be studied in low secondary schools, but it was recognised that it caused real difficulties for the students. Thus, it is better to delay teaching fractions from grade five to grade six or teach it in stages as the teachers suggested.

Mathematics inspectors agreed with the vast majority of the teachers about the importance of the topic but, by contrast, they thought the stage when the topic was to be taught was an appropriate time.

Negative Numbers

The views of the mathematics teachers about teaching negative numbers in junior secondary school were divided into two separate groups, with strongly opposing views. The first group thought that negative numbers is an essential topic in mathematics education and should be taught in junior secondary school because they think the negative numbers help the students to describe the weather temperature and profit and loss ideas. This view corresponds with the inspectors views.

However, the second group think negative numbers is not an essential topic in junior secondary school and they think this topic is very complicated for the students. A typical quotation from this group was:

> *"The difficulties which are caused by this topic make the benefits of this topic of limited value".*

They added in that the teaching of multiplications in primary school starts by seeing multiplication as repeated adding (See figure 11-1). So they wondered, "What is the meaning of (-3) x (-2)?"

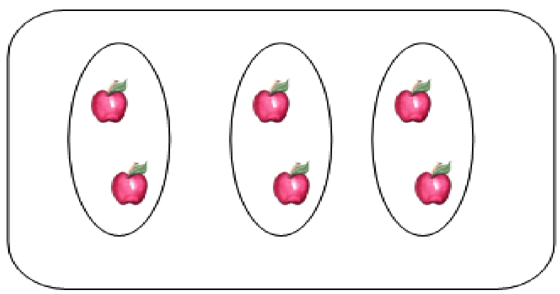


Figure 11-1: Multiplication example (the picture was drawn by the teacher as example)

One teacher stated:

> *"In grade eight, the negative fractions will be introduced to the students and in this case the students will be asked to solve such task: $(-4^{1}/_{2} \div 2.5)$? So, they will face double difficulties occurring from the difficulties of the negative number and fractions."*

This view is supported by Martinez (2006, p: 1) in his book '*Negative Math: How mathematical rules can be positively bent*'. He wondered,

> *"Is it really true that -4 x -4 =16? And what does this mean physically?*

and he continued by wondering,

> *"Can we construct a system in which, say, -4 x -4 = -16"*

The idea of negative numbers arising from loss or from the under zero measurements like temperatures, will be familiar from daily life for students. However, the difficulties in these topics might be attributable to the complicated rules and procedures that the students have to apply to solve any problems involving negative numbers. For example, here are the procedures which the students have to follow in solving any tasks with negative number:

> *"When adding numbers of the same sign, we add their absolute values, and give the result the same sign.*
>
> *When adding numbers of the opposite signs, we take their absolute values, subtract the smaller from the larger, and give the result the sign of the number with the larger absolute value.*
>
> *Subtracting a number is the same as adding it's opposite.*
>
> *To multiply a pair of numbers if both numbers have the same sign, their product is the product of their absolute values (their product is positive). If the numbers have opposite signs, their product is the opposite of the product of their absolute values (their product is negative). If one or both of the numbers is 0, the product is 0.*
>
> *To divide a pair of numbers if both numbers have the same sign, divide the absolute value of the first number by the absolute value of the second number.*
>
> *To divide a pair of numbers if both numbers have different signs, divide the absolute value of the first number by the absolute value of the second number, and give this result a negative sign."*
>
> (www.mathleague.com/help/posandneg/posandneg.htm)

Because of the complexity of these rules, it is better to avoid such complicated rules and ideas which do not have any physical meaning in learning mathematics.

Sets and their operations

Here also the mathematics teachers were divided into two groups. Group one thought that sets and their operations was an essential topics and it should be taught to the students in junior secondary school because it is easy to understand and helps them in categorization and classification. The other group think it is not an essential topic for the students in junior secondary school, a typical remark being:

> *"Teaching this topic just wasting time without any real benefit from it".*

They also thought that it occupies a large area in mathematics education in junior secondary school. This time could be used much more profitably for more useful and important topics such percentages etc. Some mathematicians such Errett Bishop (1928-1983) support the teachers view about sets theory. Furthermore, they have objected to using set theory as foundation for mathematics by claiming that it is just a game which includes elements of imagination (http://en.wikipedia.org/wiki/Set_theory).

While there was no agreement about the value of sets, teachers saw no problem in the area in that the topic was seen as an easy topic by the majority of the students. Similarly, a typical inspector view was expressed:

> *"This topic can provide the enjoyment in mathematics classes and one of the topics which shows that the syllabuses take the low achievement in consideration"*

The discussion illustrates an important issue. On what basis is a topic or theme selected for inclusion in a syllabus? Does level of easiness or difficulty determine its inclusion? Is perceived usefulness a major factor? Is a topic included because it underpins other future areas of learning or because it is seen as an important feature of a discipline?

Triangle Theories

Almost all mathematics teachers (20 out of 25) thought that triangle theories are not essential topics and they thought that it would be better to delay them to high secondary school when the students are able to specialise. They wondered:

> *"What is the benefit of teaching the low achievement students such a difficult topic?"*

There was a consistent view that the inclusion of such topics may lead the low achieving students to become less interested in learning mathematics. A typical view was:

> *"There are ten triangle theories, students need to know all these theories and know how to prove these theories and know how to apply these theories to solving many different tasks. It is too much. We just push them to failure"*

It is clear that there was agreement among mathematics teachers about the difficulties of triangle theories. Thus, some of them suggest to delay teaching triangle theories to secondary school where the students are able to decide to study mathematics or not while others suggested to delete the proofs of the theories and just teach the statement of the theory and its application. In complete contrast, the mathematics inspectors thought that triangle theories were essential topics in junior secondary school and they thought it is introduced at the appropriate time.

Circle Theories

The vast majority of mathematics teachers think that circle theories are not essential topics that students should study in junior secondary school. They prefer to delay these topics to high secondary school.

It was clear, here again, that mathematics teachers agreed to delay this topic to high secondary school and think it is very complicated topic. In the other hand, the mathematics inspectors think these topics are essential topics and the students should learn them in junior secondary school.

Quadratic Equations

Mathematics teachers think that equations are an essential topic in low secondary school but that the quadratic equation is not an essential topic to be taught in low secondary school. Some of them said:

> *"It is better to delay the quadratic equations to high secondary school",*

While others said:

> *"It is better to delete the quadratic equations completely"*

Teachers' view about solving equations corresponds to students' views about the most interesting topic, where half the sample prefers solving equations but just a fifth of them express interest in quadratic equations. Again, the views of the inspectors stand in stark contrast: they think that a quadratic equation is an essential topic in junior secondary school.

Percentage

All mathematics teachers and inspectors thought that a percentage is an essential topic in low secondary school because this topic is useful topic in banking, sales and in statistics. A small minority preferred to delay the introduction of percentage from grade six (about age 12) to grade seven or eight, and they thought that this delay would help the students to understand this topic very well. Some of them comment on this question by saying:

> *"If the percentage is deleted or delayed, what we are going to teach them in mathematics classes!"*

There was a general agreement among teachers and inspectors that percentages is an essential topic to have studied by age 16 because of the importance of this topic and the benefits that students gain from this topic in their daily life situations. Although a small minority prefer to delay the introduction of the percentage, the vast majority agreed about the current situation.

Q5: According to the idea of readiness, do you think that the early introduction of fractions cause a real problem in learning mathematics?

The vast majority of mathematics teachers thought that the early introduction of fractions in grade four and five can cause a real problem in learning mathematics. They thought fractions and their operations are very complicated and the students are required to remember many techniques which confuse their minds. A typical view was:

> *"The tasks which involve fractions often become confused with each other. This may be because some of the tasks appear very similar and the students may be wondering "what is the proper technique to solve this task?"*

While it was noted that:

> *"Fractions are depending heavily on the multiplication table and unless the students master it they will not be able to solve any fractions task".*

Thus, they preferred to delay the introduction of fractions to junior secondary school and focus on the basics of addition, subtraction, multiplication and division in primary school until the students masters the basics which can be built on. One of the teachers told us a story about one student called 'Gumanh'. Gumanh cannot cope with the fractions although she performs very well in mathematics. She always describes herself by saying:

> *"I'm stupid in fractions, I can't understand them".*

The teacher thinks Gumanh problem arises from the early introduction to fractions in grade four and five and continued by saying:

> *"If we delay the introduction of fractions to grade six, Gumanh will understand fractions because she is really good at mathematics".*

According to the teacher's view, the early introduction of fractions for the students without the feeling of readiness or the real need of it in daily life situations can cause a real problem in learning mathematics and the students fail to cope in fraction topics.

Q6: What is the greatest problems area in mathematics?

The teachers' views did vary some what according to the grade they taught but, overall, they thought the following topics were the greatest problem areas (not in any order) in mathematics:

- Geometry
- Triangle theories
- Long division
- Fractions
- Percentage

One of teacher who thought that geometry was the greatest problematic area in mathematics said:

> *"Geometry is very difficult and demands high levels of thinking and imagination. Students have to retrieve all the geometrical knowledge which have been studied in the previous years to be able to understand the new topic which is built on it. So, no wonder that students will lose the enjoyment in geometry"*

All mathematics teachers who teach grade nine thought that triangle theories and their proofs are the most problematic area in mathematics and the mathematics inspectors supported this by saying "*proving triangle theories is the most problematic area*". Although they thought that they were being taught at the appropriate time. Fractions are also seen by the majority of mathematics teachers and inspectors as a most problematic topics in learning mathematics.

Q7: Does working memory space make sense for the difficulty of this area?

At this point, the interviewer gave a very brief summary of what is known about the way people process information. The following diagram was used to explain the function of the working memory to the teachers. The approach was built upon:

Cognitive psychology uses a metaphor borrowed from computer science. According to cognitive models, the brain functions somewhat like a computer and it has input and output devices and various classes of storage. The modal model of human memory comprises of three kinds of information storage:

(1) The sensory memory (or sensory registrar or perception filter).
(2) The working memory.
(3) The long-term memory.

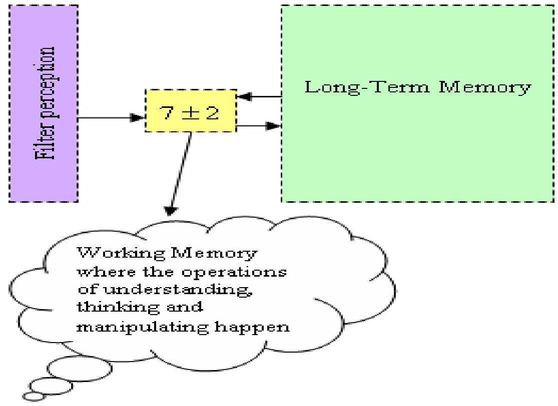


Figure 11-2: The Modal model of human mind (simplified).

The sensory memory involves the five senses: sight, hearing, taste, touch and smell but, in the classroom, we depend heavily on sight and hearing. The long term memory is the permanent information store and stores information for a long time. The working memory is the store where new information is held for relatively short periods and combined with knowledge from long-term memory and holds information for a limited period. Adult (age 16+) can hold in their working memory only about seven plus or minus two (7 ± 2) items of information at a time (Miller, 1956). For students age 14-16, the researcher after experiments on Kuwaiti students found the average capacity of their working memory is about six.

After this quick outline of the way the brain works in learning, the interviewees were asked if they thought the limited working memory space made sense of the difficulties in this area? The majority of mathematics teachers and inspectors thought that this model offered a good explanation for the reasons for difficulties in the previous topics. They said:

"Ah…We ask the students to retrieve a huge amount of information not just 5 or 6 items, no wonder why they fail in mathematics."

"We don't leave any space for thinking. We fill their working space… ha…ha (laughing)."

"Am… proving theories need more than this space, for that it is difficult"

However, some of them think the working memory space was just a *part* of the problems and typical comments were:

"Am…I don't think this is the only reason of the difficulty. I think the nature of mathematics and students' attitudes towards it play a major rule"

"It may explain some part of the real problem but not the whole of it. The problems of geometry arise from its hierarchical nature. Students need to retrieve all the information that they have studied before"

However, even in these comments, the potential for working memory is implicit. For example, the conceptual nature of mathematics more or less demands that much information has to be held in the working memory at the same time in order to achieve understanding while the hierarchical nature of mathematics also implies working memory demand.

Indeed, more negative attitudes may also be related to working memory overload (Jung, 2005) perhaps because overload leads to a failure to understand which, in turn, may lead to loss of positive attitudes towards the learning process in mathematics. Building up ideas hierarchically does place a great emphasis on previous ideas being mastered thoroughly and then being able to be applied by drawing them back into working memory or inter-relate with new ideas. The original understanding and storing is highly dependent on the perception filter and the working memory.

Furthermore, attitudes influence the filter perception, controlling whether we pay attention or not and then they control our working memory. Thus, if a student has negative attitudes towards mathematics, he may not pay attention to understand any new ideas in mathematics classes because he already blocks his mind from learning any mathematics.

Q8: Any ideas or thoughts that may improve mathematics education in Kuwait?

The teachers and inspectors were invited to suggest ways by which mathematics education in Kuwait might be improved. Their views were different and are summarised here. Teachers made the following suggestions:

- Revise the syllabuses and reduce the contents, especially in grade nine syllabus the number of theories needs to be minimised. They described grade nine syllabus as "full cream syllabus" which means that the syllabus just was built for the high achieving students.
- Teach mathematics in an attractive manner by using problem solving or using the computer.
- Reduce the classes that mathematics teacher should take, to give him the opportunity to look after the low achieving students.
- Teach the students the theories without asking them to prove them.
- Primary school should focus on the basics (addition, subtraction, multiplication and division) to guarantee that all the students master them.
- Teach the students in separate classes or group according to their levels in mathematics (setting).

- Provide training courses for mathematics teacher to give them opportunity to know about the latest learning models and the best methods in teaching mathematics.
- Avoid complicated topics in order to create positive attitudes towards mathematics.

A very strong and consistent view was that mathematics syllabuses need to be revised and the contents reduced, especially in grade nine. Thus, their views correspond with the views of students about the most helpful methods to understand mathematics: half of the student sample in the second questionnaire thought that *'teach mathematics more slowly'* is the best way to help them to understand mathematics. In other words, give them the time to reflect and practise what they have learnt.

Teaching mathematics in an attractive manner by using problem solving also corresponds with students' views about the type of activity that they like in mathematics classes where, 'solving exercises and problems' obtains the highest proportion of the students' preferences among other activities. However, this needs clarified. What students seemed to be suggesting was that they gained confidence by completing sets of exercises (which they call problems). There is often confusion between exercises and problems. Teachers were suggesting the same view as their students. However, the use of computers is uncertain as there was little evidence that teachers saw clearly how a computer might help.

The inspectors of mathematics held very different views in many areas and these are summarised:

- Provide an adult to help the mathematics teachers in the classes.
- Relate mathematics to real life situations.
- Create a motivation to learn mathematics by using attractive methods in teaching mathematics.
- Use the touch teaching aids especially in primary level.
- Using the computer in teaching mathematics.

Their views seem to be dominated by an emphasis on seeing the teacher as the source of the problem. Essentially, they want better teaching and different teaching methods. They give no clear indication how motivation can be created and the use of computers is not thought out at all. There is a definite impression of ‘blame the teacher’, without offering any evidence that this is the case.

A Summary: This chapter discussed the analyses of the interviews and the data were gathered from two groups: mathematics teachers and mathematics inspectors. The aim was to explore the perceptions of mathematics teachers and inspectors to see the extent to which their views relate to the findings from work with students. The most important finding from the interviews can be summarised as follows:

- There is little agreement about the objectives of mathematics education in Kuwait between those who decide the syllabuses (mathematics inspectors) and those who are going to teach these syllabuses (mathematics teachers).
- Fractions is seen as essential topic in mathematics education. However, the early introduction of it in primary four and five and then build on previous knowledge before the mastery of them can cause a real problem for many in mathematics classes.
- There is a growing voice from mathematicians against teaching sets and their operation and teachers think this topic takes too large a part in syllabuses, more than it deserves.
- Topics such triangle theories, circle theories and quadratic equations are seen a difficult topics and it would be preferable to delay them to high secondary school where the students will be able to choose to study mathematics or not.
- There is agreement among mathematics teachers and inspectors about the importance of percentages and it is an essential topic in junior secondary school.
- Triangle theories and fractions are seen as the most difficult topics to teach and to learn.

Looking at the interviews overall, taking into account the voice of the students as seen in the questionnaires, the strong impression is gained that there was considerable consistency of views between the teachers and the students. By contrast, the inspectors seemed to be out of step. Mathematics cannot justify its place in a school on the basis of the production of mathematicians as this tends to ignore the needs of the vast majority. The place of mathematics must be justified in terms of what its study can do for the majority, including those who perform less well. If this issue was to be addressed fully, then decisions about topics can then be discussed on an informed basis.

Teachers and students are acutely aware of the difficulties in some topics and certain emphases (like memorisation of theorem proofs) are difficult to justify. Nonetheless, it is not appropriate simply to devise a curriculum by removing difficult topics. However, if topics are found to be excessively difficult, then the reasons for the difficulties need to be explored and there needs to be a clear justification for their inclusion at the specified age. The views of the inspectors seemed woefully lacking in such insights.

Chapter 12

Conclusions and Recommendations

12.1 Overview of the Project

The present study has looked at two cognitive factors as well as attitudinal factors which relate to learning and teaching mathematics. The overall aim was to suggest ways that might help to improve students' performance in mathematics.

This study looked at learning theories in supporting the process of learning mathematics such as behaviourist theory, Piaget's theory and constructivism; as well as the theories of learning mathematics such as Dienes theory of learning mathematics and the van Hiele theory of learning mathematics. However, the main focus of this study is on information processing as a model. This describes learning well and the model absorbs most of the findings from other models. In addition, the model is powerful predicative in indicating how learning can be improved (see Johnstone, 1997; Reid, 2008).

The *information processing model* is based on a metaphor that emerges from the development of information processing technology. In fact, the findings which brought information processing into prominence happened to occur as information technology made great progress and this technology offered a language which was found useful.

The model is concerned with the way information enters our minds through our senses and how it is stored in and retrieved from memory. The model has been derived from strong evidence which suggests that all learning takes place in the same way. At the moment, individual differences and personal factors have received little attention in the model although there are signs that this is rapidly changing (see Hindal, 2007). The model assumes that meaningful learning is related to the way knowledge is stored in long term memory: ideas are linked together correctly to form a complex matrix. Thus, knowledge is seen as something coherent and holistic, which provides the basis for later learning (Atkins *et.al,* 1992).

This research has investigated the influence of working memory capacity and field dependency on mathematics achievement. The working memory space and the degree of field dependency were measured for 1346 school students aged between 14-16 years from public schools in Kuwait. The Digit Backward Test (**DBT**) was used to determine working memory space, and the Group Embedded Figure Test (**GEFT**) was used to measure the degree of field-dependency for the students, both these test being tests which have been used widely and their validity is assured. However, absolute measurements were not important in this study, as rank order was all that was required.

Table 12-1 summarises the student samples involved.

	Phase 1		Phase 2		Total
	G 8	G 9	G 8	G 9	
Male	105	112	146	143	506
Female	128	127	269	316	840
Total	233	239	415	459	1346

Table 12-1: Research samples

Mathematics is usually seen as holding an important place in the school curriculum. Because of the importance of mathematics as a discipline and because it forms an important part of the school curriculum, students' attitudes towards mathematics will be important so they can gain as much as possible from their studies. In order to explore some aspects of students' attitudes towards mathematics, two questionnaires were developed and used. This study explored the attitudes of the students towards mathematics in the following areas: the importance of mathematics as discipline; attitudes towards learning mathematics; confidence in mathematics classes; the relationship between attitudes and achievement; activities in mathematics classes, and opinions about mathematicians.

This study also looked at the views of mathematics teachers and inspectors about the purpose of mathematics education at school level in Kuwait, as well as how they see various topics in the curriculum. The focus was very much on topics which were found to be difficult for the students and the possible reasons why these difficulties arose. 25 mathematics teachers and 4 mathematics inspectors were interviewed. The aim was to explore the perceptions of mathematics teachers and inspectors to see the extent to which their views relate to the finding from work with students.

12.2 The Main Findings

The most important findings of this study can be summarised under three headings: cognitive factors influencing learning in mathematics; attitudes of the learners; the views of teachers and inspectors:

(a) Cognitive Factors:

- Overload of working memory is likely to be at least partly responsible for students' difficulties in solving mathematics tasks. High working memory students (X = 6) performed better in mathematics than these with lower working memory space capacity (X=4).
- Field-independent students achieved better than other groups of students because their abilities enable them to distinguish the important and relevant information from the irrelevant, allowing them to use their working memory space efficiently. Field-dependent students do not have this ability; therefore, unimportant and irrelevant items occupy their working memory space.
- There was a drop in performance with those questions which placed a high demand on working memory space.
- The way the questions or the tasks are given to the students is very important for the students to understand and to succeed in solving them. The evidence shows that questions which are more applied tend to hinder good performance. This is almost certainly because the more applied question format increases working memory demand. On the other hand, usual visual presentation tends to assist the students to solve the task properly. This is because visual presentation of a task can provide much information as one chunk and this may help to minimize the load on the working memory. However, some complicated pictures with a lot of information may hinder the students understanding and may overload the working memory space.

There are key issues here. If working memory capacity is genetically determined, then it raises ethical issues if a low working memory capacity places a hindrance on learning or on assessment. There is no clear evidence that working memory capacity is neatly related to intellectual ability. It is, however, almost always related to performance in the kinds of tests used at school and university. The issue of field dependency is more complex in that there is no certainty about whether these skills can be taught. The ability of being able to focus on what is important for a task in hand is clearly a most valuable skill.

(b) Attitudinal Aspects:

- Mathematics is believed to be an important subject and it possesses its importance from the belief of students that mathematics is a useful subject in daily life, useful for their careers, and it is useful for other subjects.
- In spite of the belief that mathematics is an important subject, mathematics is seen as an abstract, difficult and complicated subject although abstraction, in itself, is not seen as the source of its difficulty.
- The attitudes of boys towards mathematics are more positive than girls' attitudes, and this might be attributable to the different masculine and feminine natures at this age: males tend to be more confident while females tend to feel worried.
- There are significant correlations between students' attitudes towards mathematics and their mathematics achievement, their working memory capacities and their field dependency characteristic.
- Mathematics teachers play a central role in learning mathematics and in forming students' attitudes towards learning mathematics.
- Confidence in mathematics classes can be improved when the emphasis is on the mastery of procedures leading to correct solutions to exercises. Students also considered that providing opportunities for discussion would help.

One huge problem is knowing what causes what. Do negative attitudes cause poor performance or does poor performance lead to negative attitudes? It is almost certainly a two-way effect (Christou, 2000). However, one marked feature is the way attitudes related to mathematics are often so polarised, with sizeable minorities holding *very* negative views. The power of the teacher is very evident although there is little indication that the teachers in Kuwait are the source of any negative attitudes. The subject is perceived as difficult, irrelevant and unnecessarily complicated.

(c) Teachers and Inspectors:

- ❖ There is no agreement about the objectives of mathematics education in Kuwait between those who decide the syllabuses (mathematics inspectors) and those who are going to teach these syllabuses (mathematics teachers).
- ❖ Fractions is seen as essential topic in mathematics education. However, the early introduction of it in primary four and five, the aim being to build on this previous knowledge before the mastery at a later stage, can cause a real problem for many in mathematics classes.
- ❖ There is a growing voice from mathematicians against teaching some topics: this includes negative numbers (at too early a stage), triangle theories, and 'useless' topic such as sets.

The strong overall impression given from the interviews is the marked discrepancies between the views of the teachers and the views of the inspectors. Indeed, the views of the teachers seem much closer to the views of the students, leaving the inspectors holding very isolated perceptions. In general, the inspectors seem to think that things are generally fine and that all the problems can be laid at the feet of the teachers. Naturally, there is no evidence that the teachers agree! Probably, the most fundamental issues relates to why mathematics is taught, the view of the inspectors being unsustainable.

12.3 Overall Conclusions

The study of mathematics has held a central role in school education systems from early times. Originally, the work focussed mainly on what today would be known as arithmetical skills, these being seen as important for life and, specifically, for many occupations. Algebra and geometry started to take their places as secondary education developed while, later, calculus was added. With the advent of the ubiquitous calculator, the need for basic arithmetical skills was reduced but the topics were retained in school education simply because it was felt that a clear grasp of such skills was essential for all students. While a calculator could complete the procedures very accurately and quickly, there was a need for the learner to understand what was being done.

There are some parallels with the use of statistics in research. Few researchers have a good understanding of statistics but many put data into a statistical computer package to reach conclusions. The dangers are considerable and there are endless papers which have discussed the problems (Reid, 2006). It might be argued that the correct use of a statistical package needs some understanding of the processes involved in statistical calculation and interpretation.

This leads to a fundamental issue which has arisen in this study. What are the arguments for all students studying mathematics? Mathematics cannot justify its place in a school on the basis of the production of mathematicians as this tends to ignore the needs of the vast majority. The place of mathematics must be justified in terms of what its study can do for the majority, including those who perform less well. If this issue was to be addressed fully, then decisions about topics can then be discussed on an informed basis. Teachers and students realise the difficulties in some topics but it is not appropriate simply to devise a curriculum by removing difficult topics. However, if topics are found to be excessively difficult, then the reasons for the difficulties need to be investigated and there needs to be a clear justification for their inclusion at the specified age. The views of the inspectors by contrast seemed lacking in such insights.

When the issue of the purpose for mathematics education is agreed, then it will be possible to consider what topics might further these aims most fully. In fact, teachers are involved daily in the teaching processes and they know the population of their students very well. Thus, their views about the syllabuses should be taken into consideration and they should be involved in the process of deciding the syllabuses. At that stage, the whole question of topic order and pedagogy can be addressed.

Working memory is a system responsible for providing temporary storage and manipulation required for any mental process, and its role in learning mathematics cannot be neglected. This is where thinking, understanding and problem solving (in its genuine sense) take place. Extent of field dependency relates, at least in part, to the efficiency by which working memory can operate.

Working memory capacity is genetically fixed. However, it is perfectly possible to reduce its importance without sacrificing the rigour of the instruction or testing (see Reid, 2002; Danili and Reid, 2004; Hussein, 2006; Chu, 2008). In all these studies, the demands of the curriculum were maintained but the way the teaching or assessment carried out was aimed at minimising demands on working memory so that those with lower capacities were not too greatly disadvantaged.

In mathematics, this can be done by:

- Attempt to reduce processing loads in any task given, especially in early stages of understanding;
- Avoid the use of applications which are unnecessary: those which add nothing to the question being considered but simply generate noise;
- Allow learners to use external aids if the task demands high working memory capacity: paper notes, discussion.
- Provide a clear visual presentation, where appropriate, where the 'noise level is not increased but many ideas are 'chunked' together.

It is often argued that mathematics involves the abstract and this, on its own, explains the difficulties. This study has shown that the difficulties are very much related to working memory overload and the lack of field independence when solving exercises and problems. Nonetheless, mathematics is abstract, conceptual and hierarchical. These features place considerable stress on the working memory.

Perhaps, the key lies in considering field dependency. This offers a way to control the flow of information, releasing more capacity for holding and manipulating information. The skill certainly grows with age. It is not known if this is simply developmental, like working memory capacity, or is a function of learning and experience. If it is learning and experience, which is more likely, then an aim must be to explore what teaching strategies enhance the skill.

This can be brought together in figure 12-1.

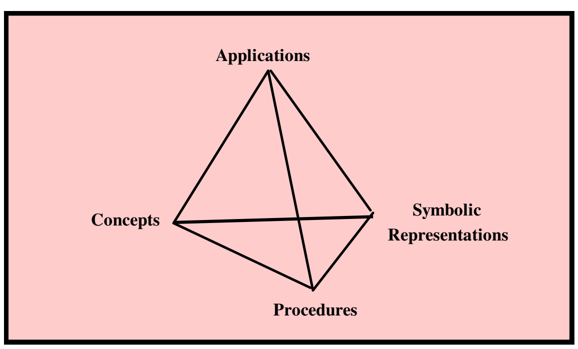


Figure 12-1: Tetrahedral relationships between performance types

This figure was derived from earlier work by Johnstone in Chemistry (Johnstone 1997) where he developed a triangle. The figure illustrates the problem with mathematics. Clearly, the ultimate aim of mathematics teaching must be to enable the learner to apply procedures correctly in new situations. To reach that point, the learner must have a good conceptual grasp and confidence with the symbolisms used. Thus, this involves all four corners of the tetrahedron. The 'expert' can do this but the 'novice' will fail simply because the working memory cannot handle the ideas from all four vertices *at the same time*. The highly field independent person can select more efficiently exactly what is needed for a task and thus reduce the working memory overload.

This offers a key to a way forward. It is essential that learners are *not* required to work at all four corners of the tetrahedron. Thus, the teacher should train them on the procedures of solving any task until they master them and these procedures are automated. This means that, like driving a car, the procedures require little thought. Indeed, in terms of working memory, the procedures are 'chunked' into one and are thus occupying only one space. However, this still involves mastering the procedures and essential symbolic representations in the learning stage. These need to be made as straightforward as possible in terms of working memory load.

After procedures and symbolic representations are mastered and automated, then the teacher can move on to develop conceptual understandings. At this stage, the students are able to carry out the procedures with high levels of success but may not know why they are doing or what it means. Learners are seeking to make sense of their world around (Piaget, 1962) and this final stage of developing some kind of conceptual understanding is critical to generate some satisfaction in their learning experience. Nonetheless, because of working memory limitations the development of conceptual understanding needs to *follow* procedural mastery.

At that stage, applications can be introduced. The students now know and understand the procedures and they just focus in interpreting the applied question into a symbolic form and then apply the procedures automatically without the need for thinking about these procedures.

This study offered some clear evidence of a decline in positive attitudes with age and the excessively overloaded curriculum was a likely reason along with the perceptions that some topics were perceived as irrelevant. Furthermore, this study reflects the crucial role that the mathematics teacher plays in the formation of student attitudes towards mathematics. Thus, aiming to develop positive attitudes towards mathematics including confidence, enjoyment and an appreciation of it as a powerful tool should be parallel with the acquisition and the understanding of mathematics concepts and skills in mathematics education

12.4 Strengths and Weaknesses of this Study

Any research study has its own strengths and weakness.

- This study involved very large sample with two age groups, making it likely there is high reliability in the measurements.
- The working memory capacity and the field dependency tests are well established.
- In attitudes work, there is no certainty that students responded to reflect the reality of their views but their responses may have reflected their aspirations. However, there was considerable consistency of views between the teachers and the students and questions relating to performance correlate positively with the students' performance. All this supports the validity of the survey.
- It would be interesting to involve other age groups (grade six or grade ten) and to interview more students to explore their attitudes towards mathematics. However, time did not permit this to take place.
- One major limitation of this study is that is was carried out only in Kuwait. Nevertheless, it is possible to generalise the outcomes because mathematics education seems to be similar in most countries around the world.
- Another limitation regarding this study is that it did not attempt to solve the problems of mathematics education. The aim was to provide some explanation of the causes of the problem. Given these explanations, the next stages are to apply them in modifying the way mathematics is structured, taught and assessed in schools. However, this raises a very real issue. There is a pressing need to enable those who take the decisions in Kuwait and elsewhere relating to the aims of mathematics education, the curriculum and the assessment procedures so that changes, based on sound empirical evidence can be introduced. This will not be easy.

12.5 Suggestion for Further Work

This study has raised many issues and the following areas are suggested for further work:

(1) Explore the effects of visual-spatial working memory on achievement in geometry and mathematics in general.

(2) Explore whether field dependency is simply developmental, like working memory capacity, or is a function of learning and experience. If it is learning and experience which is more likely, then the question is what are the learning and teaching strategies that enhance the field dependency skill in mathematics classes?

(3) Is teaching the students in separate classes or groups according to their levels of ability in mathematics classes (setting) helpful in enhancing learning mathematics?

(3) Explore the rationale behind mathematics syllabuses and evaluate mathematics topics. The inclusion of many topics needs more reasoned justification and this applies particularly to some complicated topics.

(5) Construct attractive mathematics syllabuses, based on a sound rationale that also takes into account the nature of the learner and the place of mathematics education as an integral part of the preparation of young people as effective members of their society.

(6) The widespread introduction of computers into schools has its impact on teaching mathematics. Computers bring a new way of thinking about mathematics and introduce new areas of mathematics. How can we use this technology effectively? Can we design programs that help students to learn individually?

(7) In the light of the observation that mathematics is a highly unpopular subject in Kuwait at upper school and university level, it will be a useful survey of attitudes related to mathematics from about age11 to age 16 to see if there are clear pointers to the origins of the problem.

12.6 Endpiece

Taking into account previous research findings, this study has aimed to offer some insights into the problems associated with the learning of mathematics in Kuwait. It is very clear that mathematics has major problems in Kuwait in terms of its popularity with students. It is hoped that it will contribute to future improvements there and, perhaps, elsewhere as the exciting world revealed by studying that most demanding discipline are opened up to increasing numbers of students to give satisfaction and benefit as well as meaning and enjoyment.

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Appendices

Appendices A: The Digit Backwards Test

Appendices B: The Group Embedded Figure Test

Appendices C: Mathematics Tests

Appendices D: Questionnaire

Appendices E: Interviews Matrix

Appendices F: Statistical Analysis

Appendix A

Digit Backwards Test

(4) Here are the numbers used in this work:

5	8	2						
6	9	4						
6	4	3	9					
7	2	8	6					
4	2	7	3	1				
7	5	8	3	6				
6	1	9	4	7	3			
3	9	2	4	8	7			
5	9	1	7	4	2	8		
4	1	7	9	3	8	6		
5	9	1	9	2	6	4	7	
3	8	2	9	5	1	7	4	
2	7	5	8	6	2	5	8	4
7	1	3	9	4	2	5	6	8

(5) When this is finished, allow a short break and then....

You now give a second set of instructions.
"Now I am going to give you another set of numbers. However, there is an added complication!
When I have finished saying the numbers, I want you to write them down in ***reverse*** order.
For example, if I say "7, 1, 9", you write it down as "9, 1, 7".
Now, no cheating!! You must not write the numbers down backwards.
You listen carefully, turn the numbers round in your head and then write them down normally.
Have you got this? Let's begin."

(6) Here are the numbers:

2	4						
5	8						
6	2	9					
4	1	5					
3	2	7	9				
4	9	6	8				
1	5	2	8	6			
6	1	8	4	3			
5	3	9	4	1	8		
7	2	4	8	5	6		
8	1	2	9	3	6	5	
4	7	3	9	1	2	8	
9	4	3	7	6	2	5	8
7	2	8	1	9	6	5	3

Student Answer Sheet

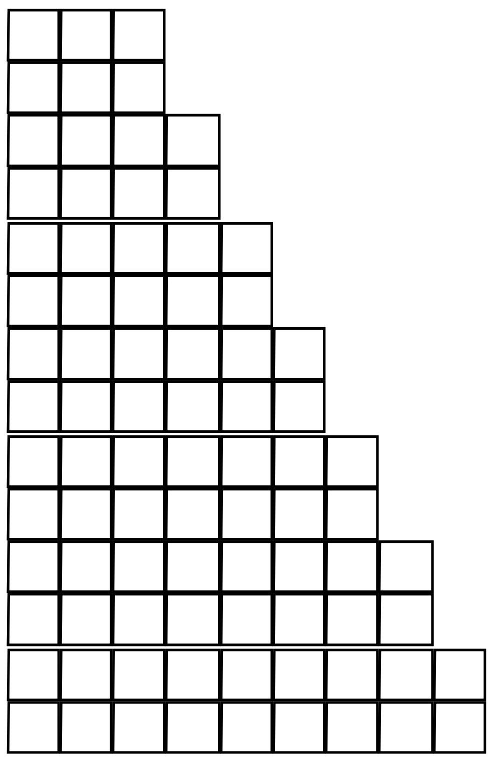
Name: ……………………………………………………………………………………………………

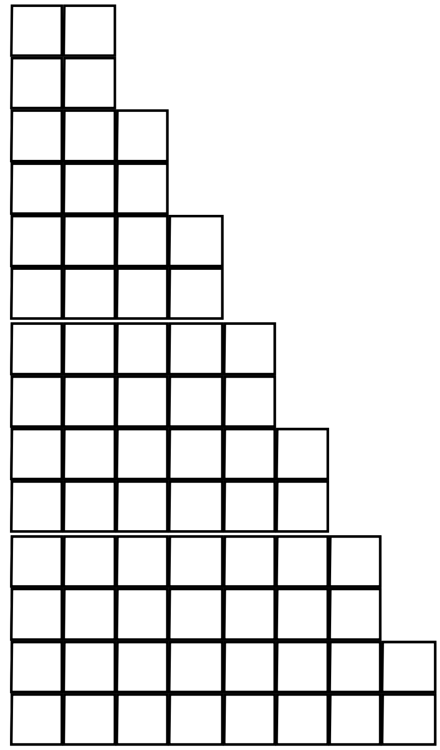
School: ....……………………………………………………………….

Class: …………………………………………………………………………………………………

Write the numbers in the boxes below

Digit Backwards Tests





Appendix B

The Group Embedded Figure Test

Notes

I. The field dependency tests were presented to pupils as a booklet.
II. The answers to the Shapes are included, beginning on page appendix. B-17.

Name: ....

School: ....

Class: ....

SHAPES

Shape recognition within complex patterns

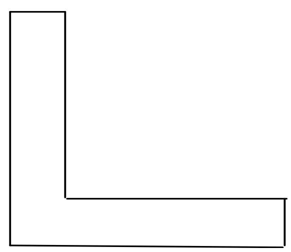
This is a test of your ability to recognize simple SHAPES, and to pick out and trace HIDDEN SHAPES within complex patterns. The results will not affect your course assessment in any way.

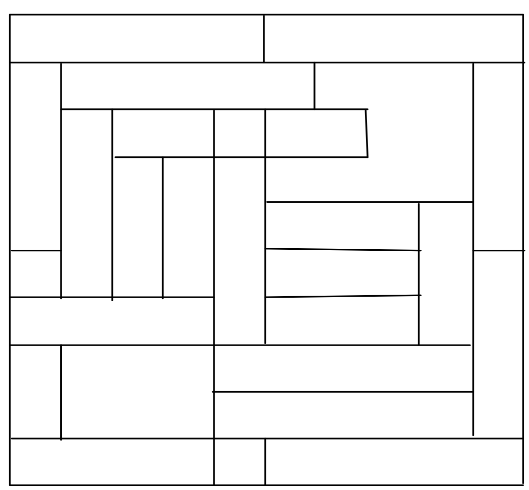
YOU ARE ALLOWED ONLY 20 MINUTES TO ANSWER ALL THE ITEMS.
TRY TO ANSWER EVERY ITEM, BUT DON'T WORRY IF YOU CAN'T.
DO AS MUCH AS YOU CAN IN THE TIME ALLOWED.
DON'T SPEND TOO MUCH TIME ON ANY ONE ITEM

DO NOT START UNTIL YOU ARE TOLD TO DO SO

LOOKING FOR HIDDEN SHAPES

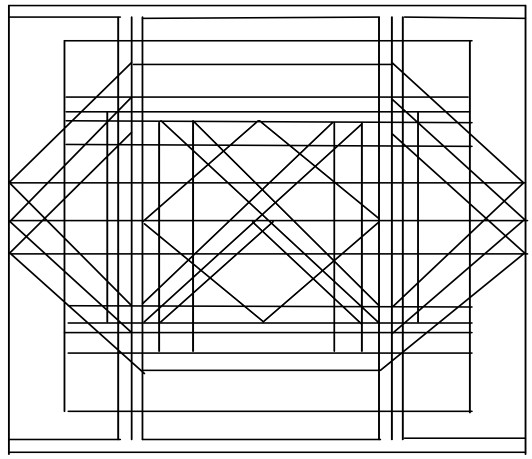
A simple geometric figure can be 'hidden' by embedding it in a complex pattern of lines. For example, the simple L-shaped figure on the left has been hidden in the pattern of lines on the right. Can you pick it out?





Using a pen, trace round the outline of the L- shaped figure to mark the position.

The same L-shaped figure is also hidden within the more complex pattern below. It is the same size, the same shape and faces in the same direction as when it appears alone. Mark its position by tracing round its outline using a pen.



(To check your answers, see page 14)

More problems of this type appear on the following pages. In each case, you are required to find a simple shape ‘hidden’ within a complex pattern of lines, and then, using a pen, to record the shape’s position by tracing its outline.

There are TWO patterns on each page. Below each pattern there is a code letter (A, or B, or C etc.) to identify which shape is hidden in that pattern.

In the last page of this booklet, you will see all the shapes you have to find, along with their corresponding code letters. Keep this page opened out until you have finished all the problems.

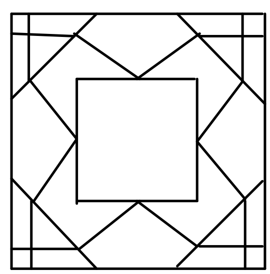
Note these points:

1. **You can refer to the page of simple shapes as often as necessary.**
2. **When it appears within a complex pattern, the required shape is always:**
   - **the same size,**
   - **has the same proportion,**
   - **and faces in the same direction as when it appears alone**
3. **Within each pattern, the shape you have to find appears only once.**

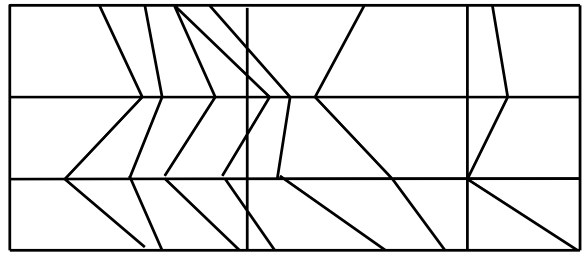
4. **Trace the required shape and only that shape for each problem.**

5. **Do the problems in order – don't skip one unless you are absolutely stuck.**

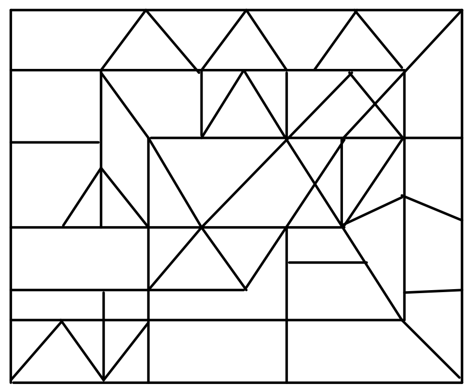
START NOW



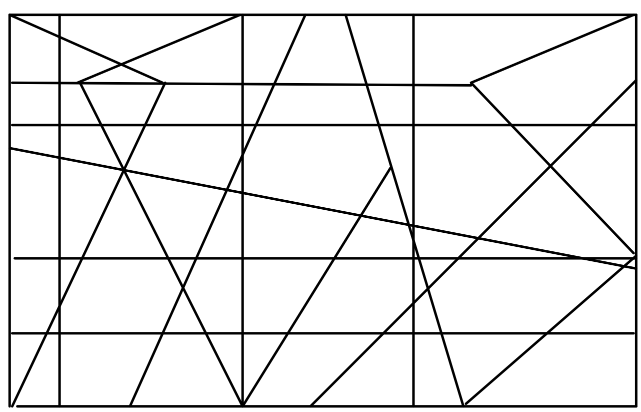
Find shape B



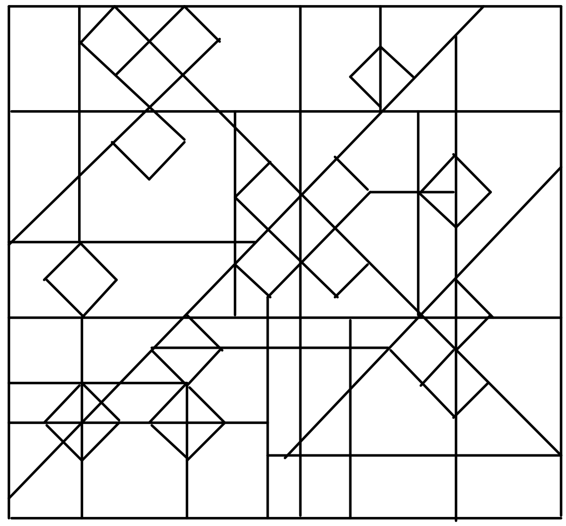
Find shape D



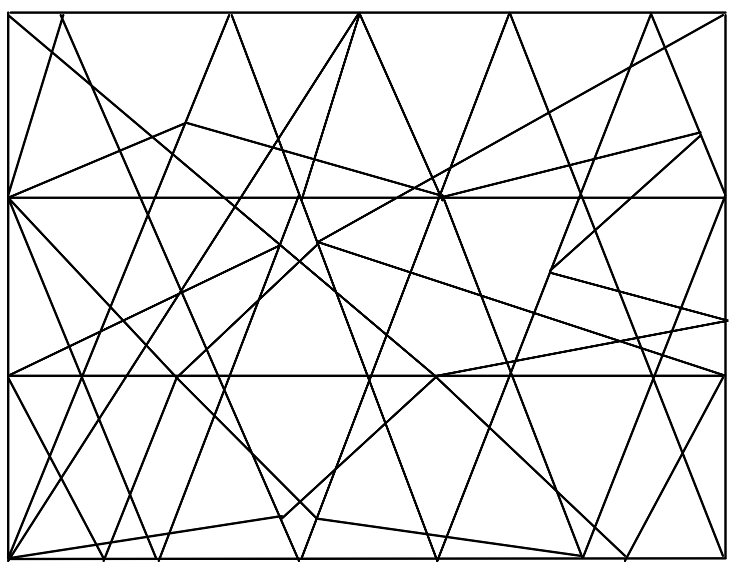
Find shape H



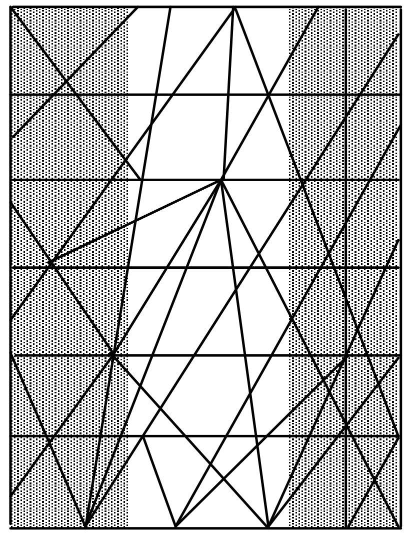
Find shape E



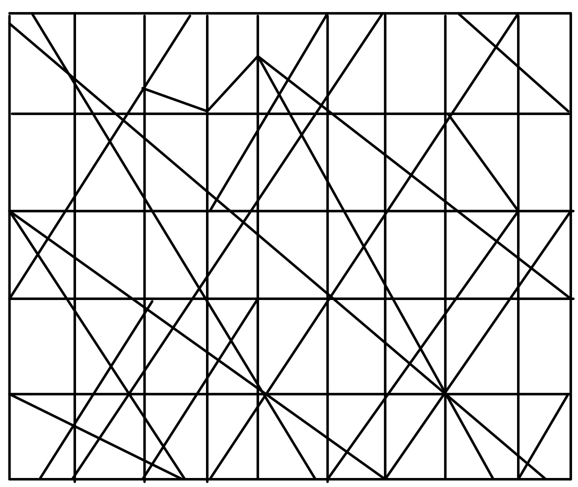
Find shape F



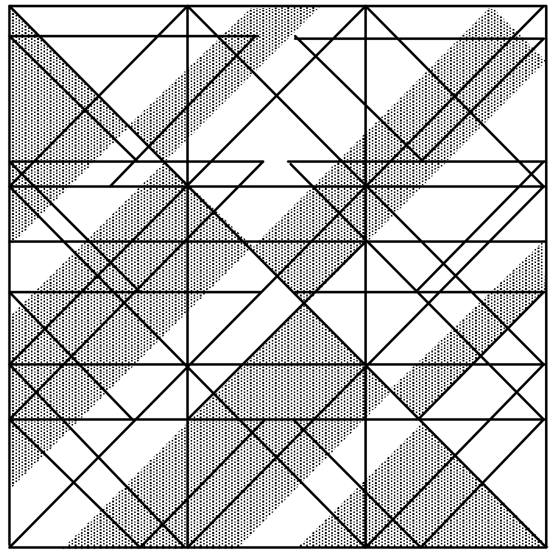
Find shape A



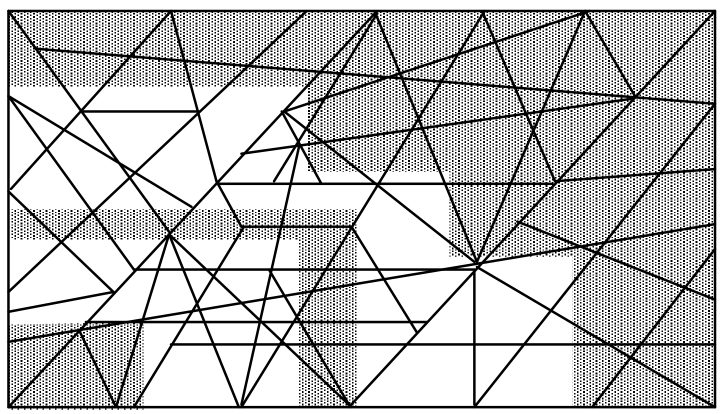
Find shape E



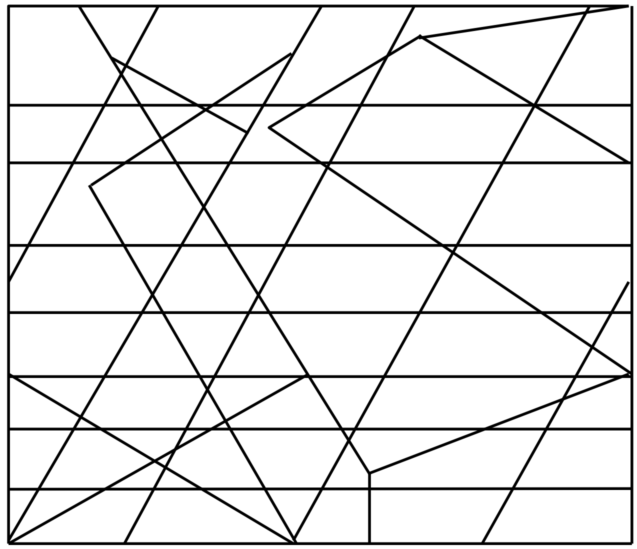
Find shape H



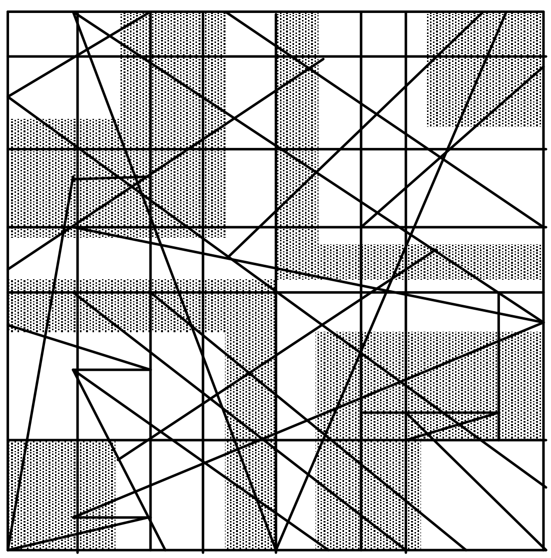
Find shape D



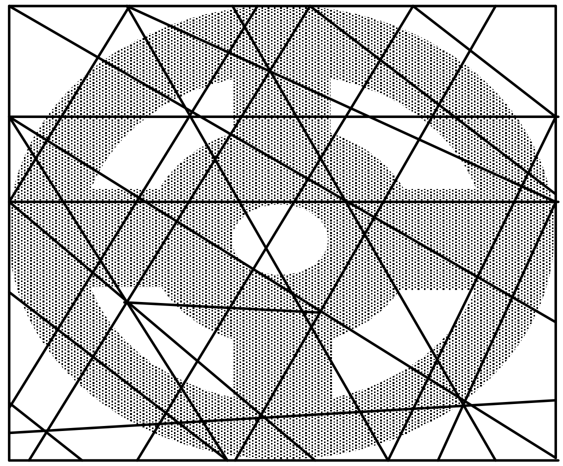
Find shape G



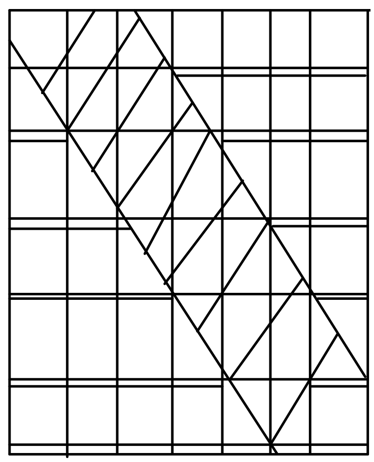
Find shape C



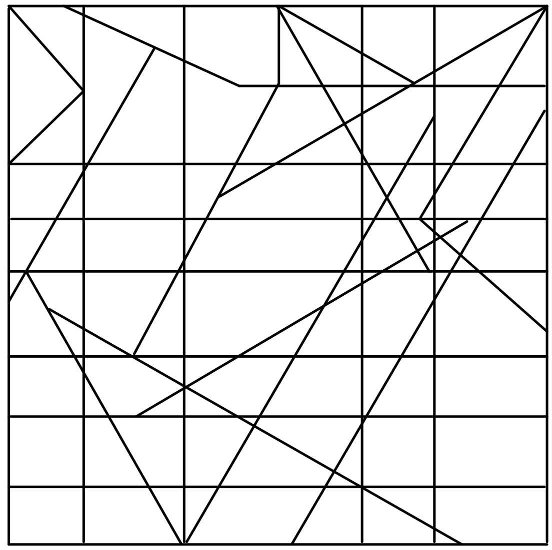
Find shape B



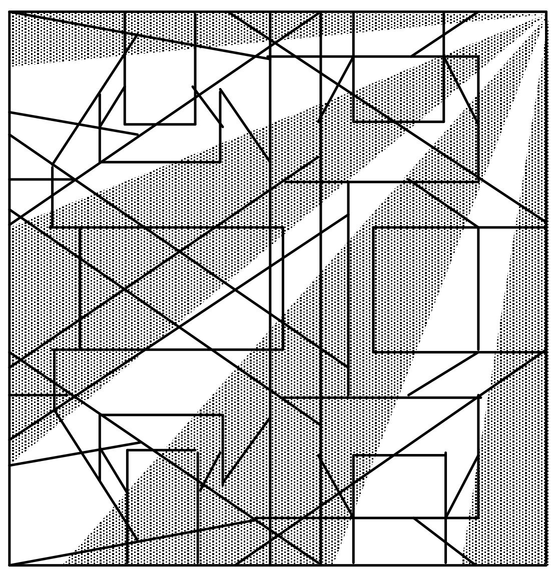
Find shape G



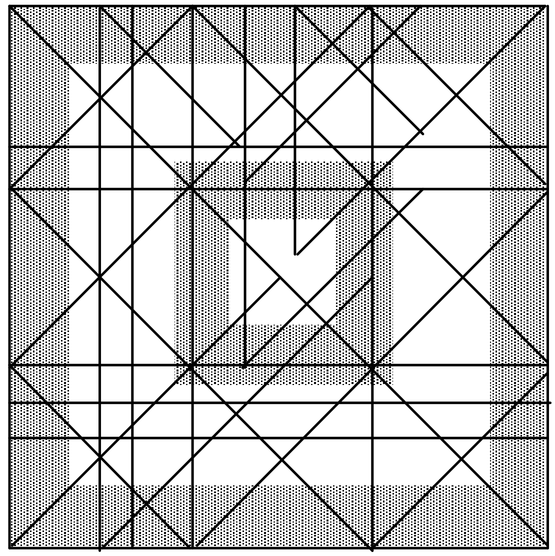
Find shape H



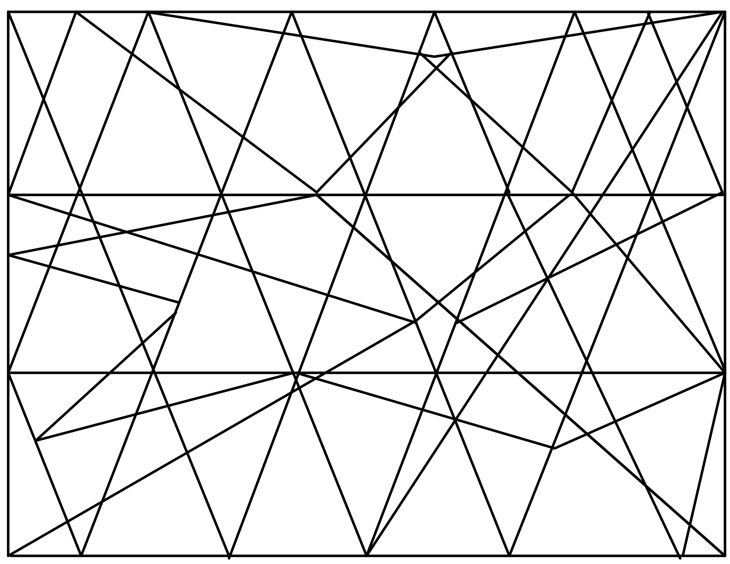
Find shape C



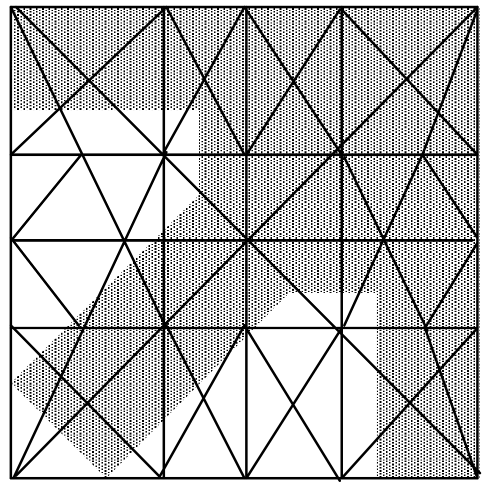
Find shape B



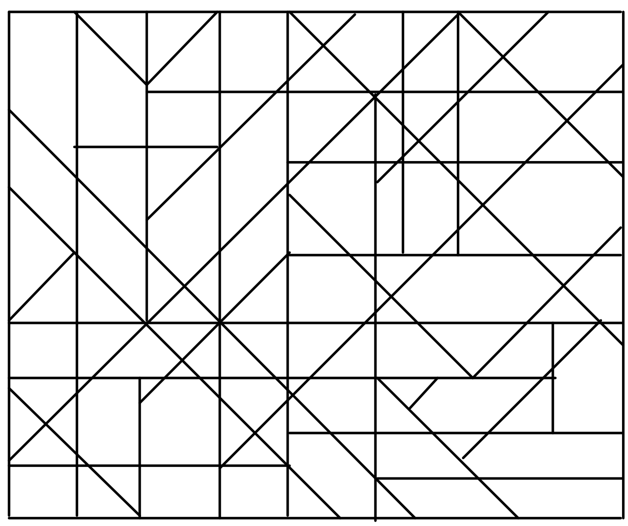
Find shape D



Find shape A

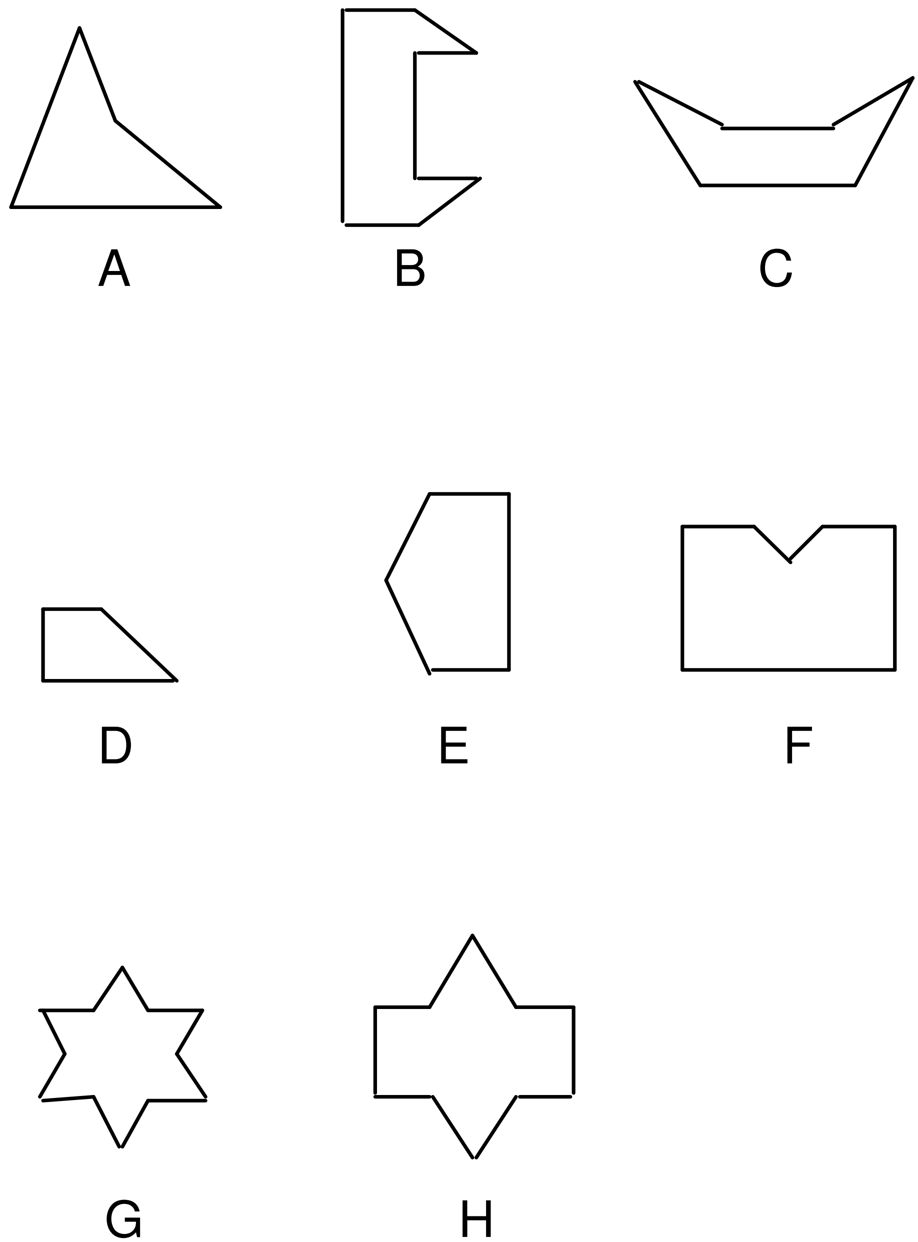


Find shape E

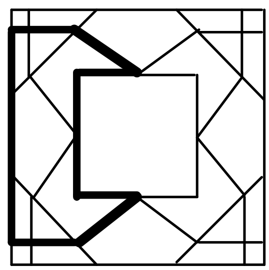


Find shape F

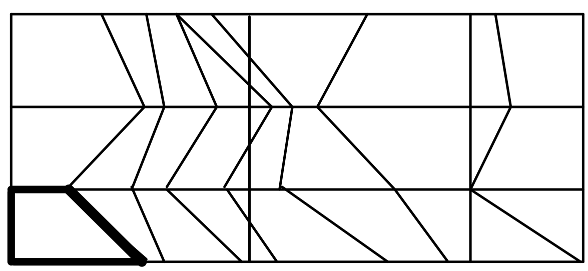
The shapes you have to find



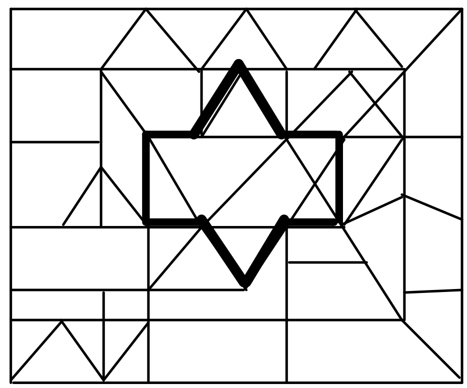
ANSWERS TO SHAPES



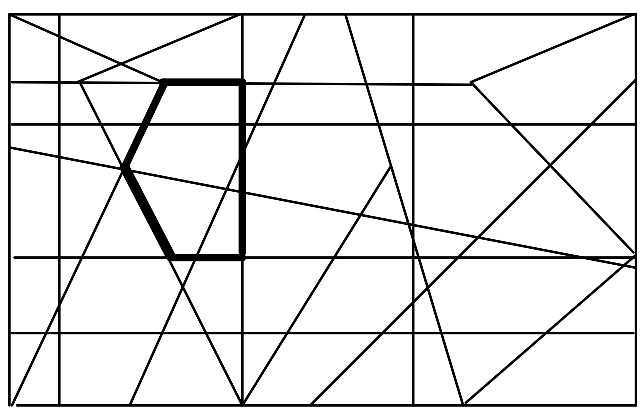
Find SHAPE B



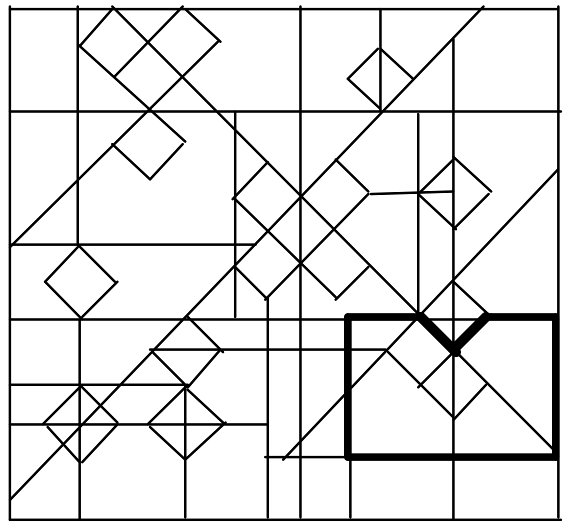
Find SHAPE D



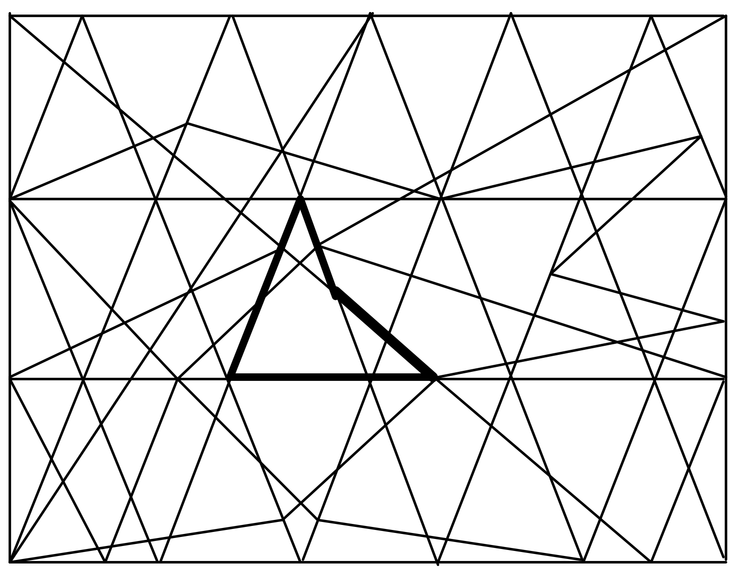
Find SHAPE H



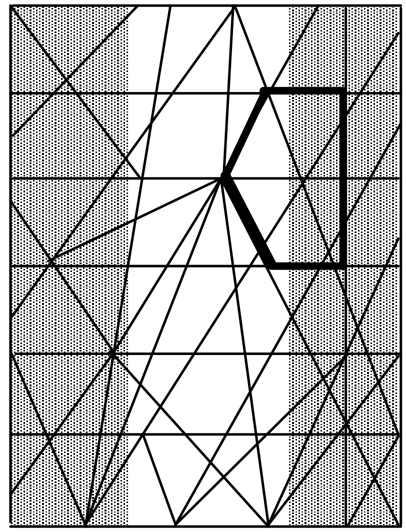
Find SHAPE E



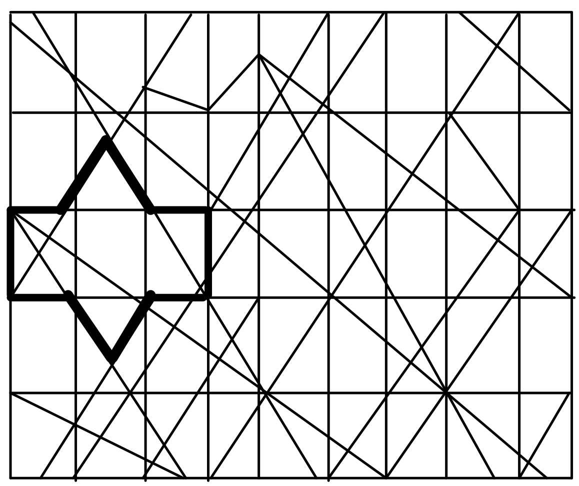
Find SHAPE F



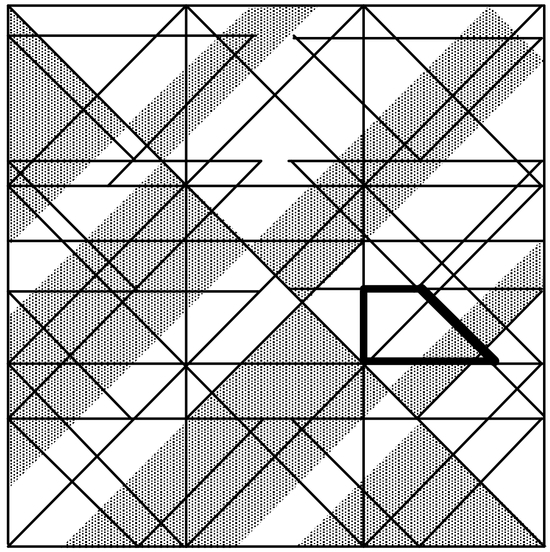
Find SHAPE A



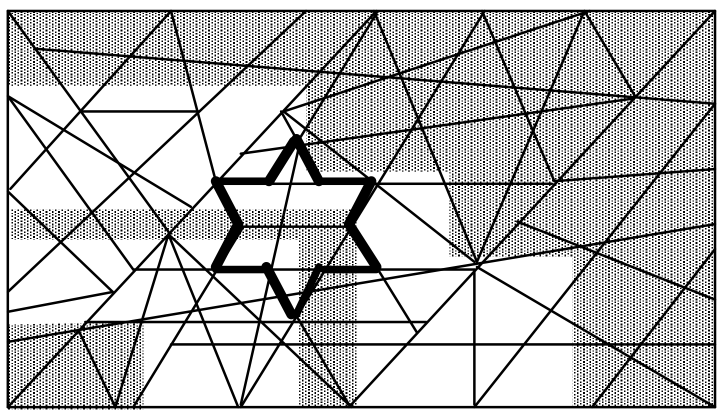
Find SHAPE E



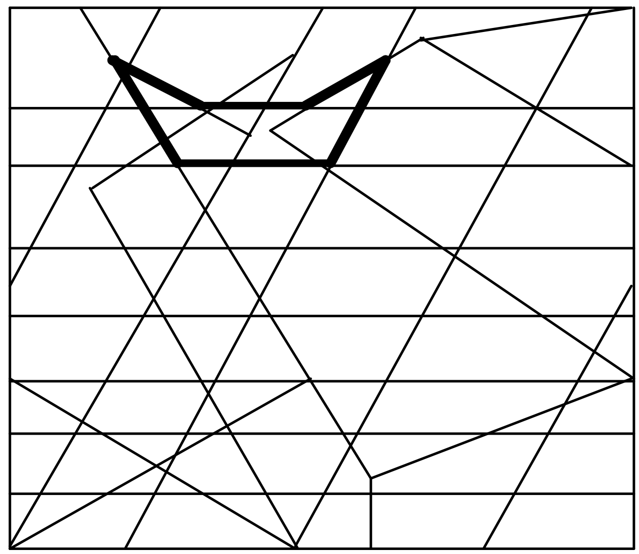
Find SHAPE H



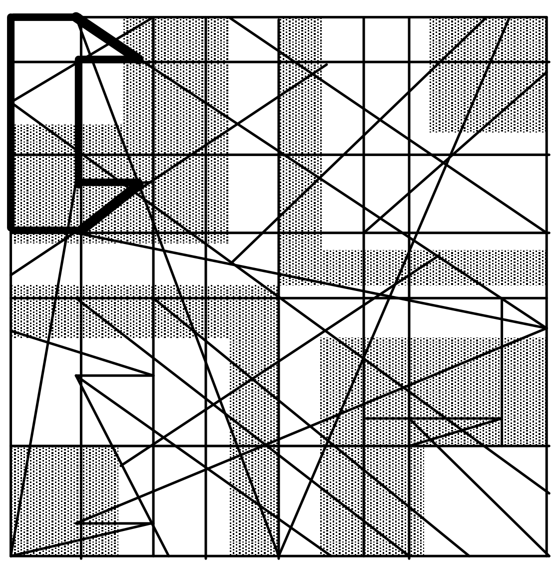
Find SHAPE D



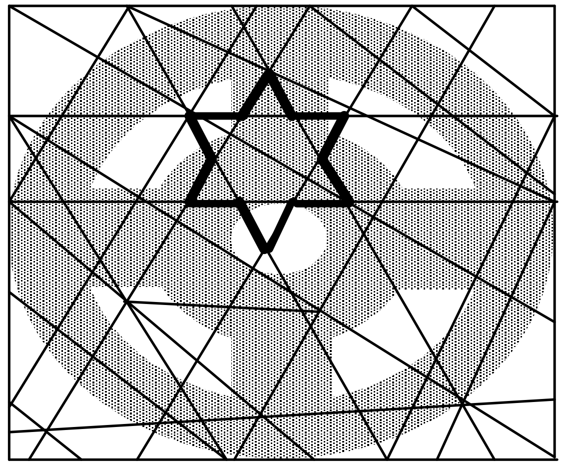
Find SHAPE G



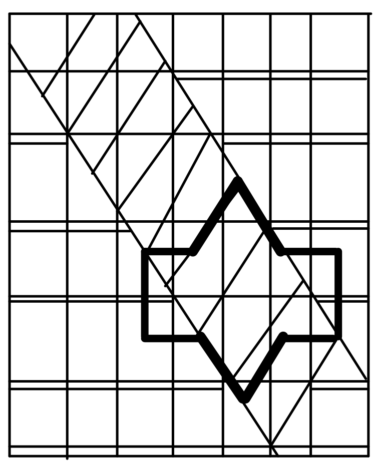
Find SHAPE C



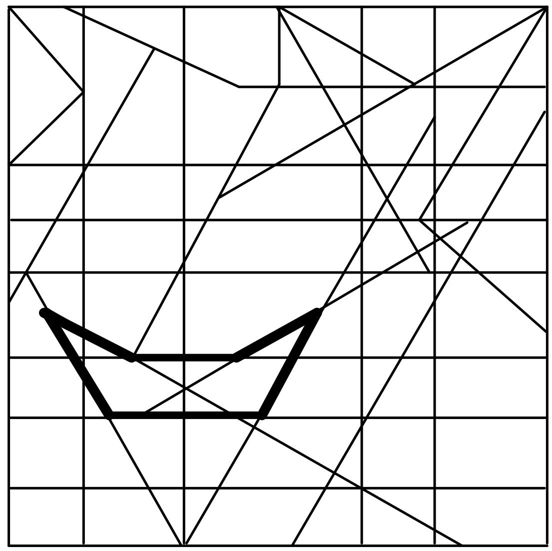
Find SHAPE B



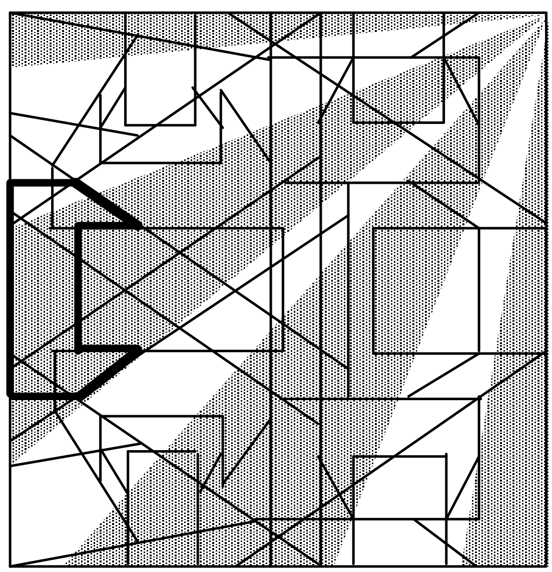
Find SHAPE G



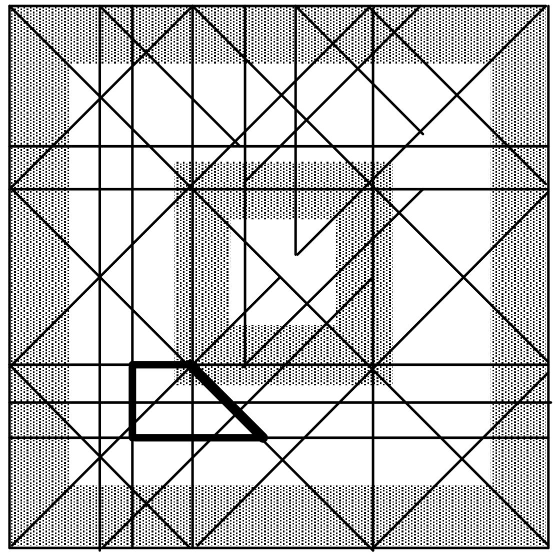
Find SHAPE H



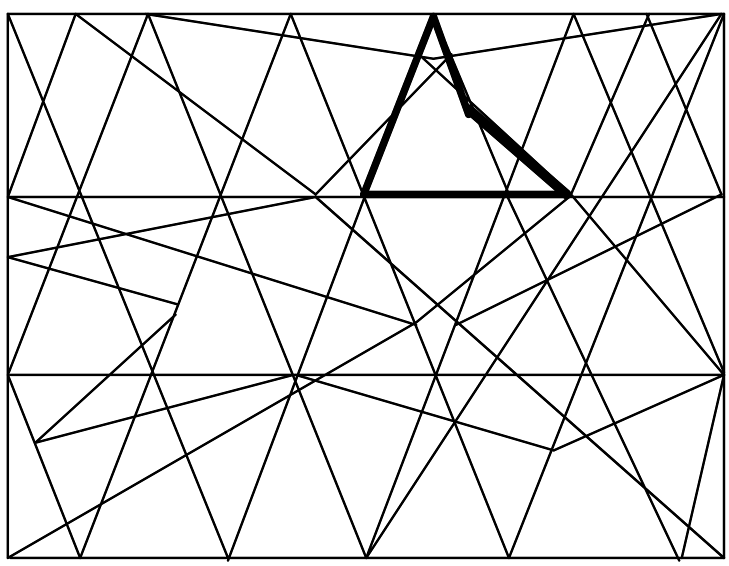
Find SHAPE C



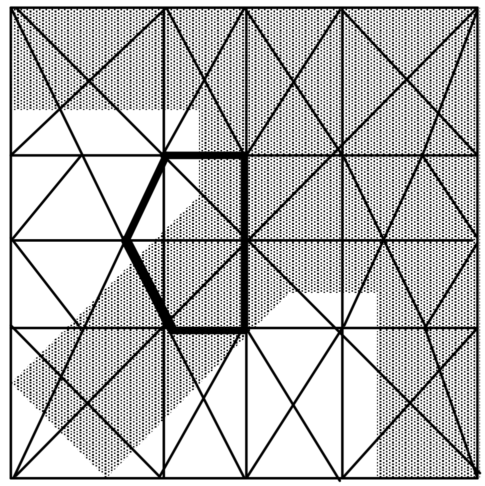
Find SHAPE B



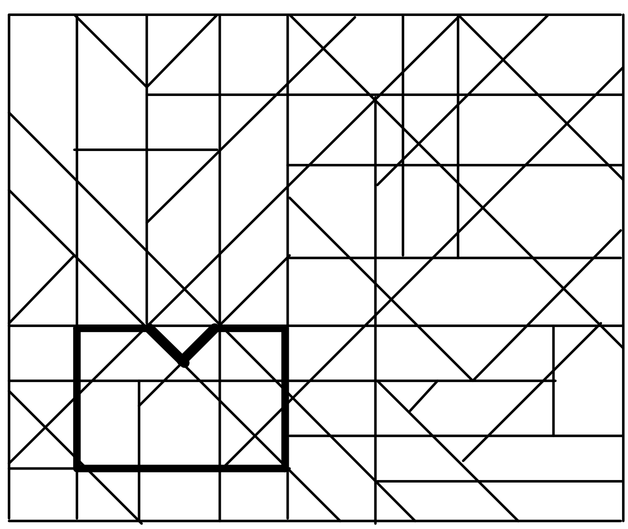
Find SHAPE D



Find SHAPE A



Find SHAPE E



Find SHAPE F

Appendix C

Mathematics Tests

University of Glasgow

Science Education Center

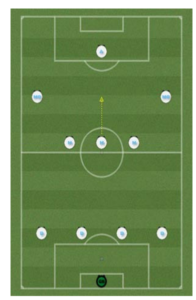
Name:……………….

Class:………

12.7 Grade Eight Test(1)

<table>
<tr><td>Find the solution

$3.6 \div 1.2 =$</td><td>Sara buys a jacket and skirt. the prices are $29\frac{3}{4}$ KD $17\frac{1}{2}$ KD respectively. How much she will pay for them?</td></tr>
<tr><td colspan="2">Calculate the area of the football patch

2a
a+5
</td></tr>
<tr><td colspan="2">Find the solution of subtraction $3x^2 - 5x + 1$ from $x - 2x^2 + 4$</td></tr>
<tr><td colspan="2">The shape of water tank is right circular cylinder.
The radius of its base r= 7cm and its height h = 10cm.
Calculate the lateral surface area and the water volume if we going to fill this tank. $\pi = \frac{22}{7}$</td></tr>
<tr><td>a) Lateral surface area=</td><td>b) The volume=</td></tr>
</table>

Solve for x: $x^2 - 5 = 11$	$4x - 3 = 29$

Find the solution:

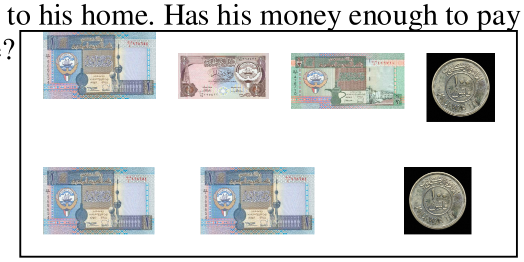
$$\frac{-3}{7} \times \frac{2}{5} =$$

The fare charged for traveling by taxi is shown here

1) How much does it cost to travel **2** miles by taxi?

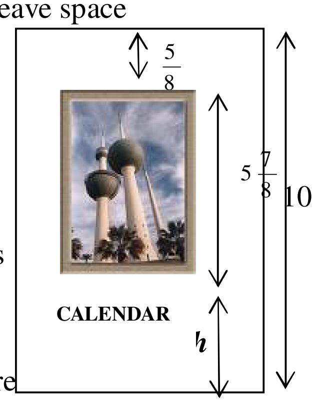
1.1.1 Fare

2) Ali has to travel **3** miles from the cinema to his home. Has his money enough to pay his taxi fare from the cinema to his home?



Your bank account holds 20 KD. You enter your credits and debits with + and - signs, respectively. What do you own after writing down the entries +2,7 KD, -7,3 KD, - 7 KD, + 1,3 KD?

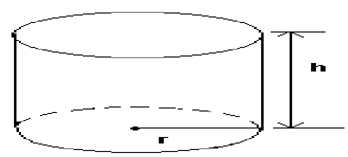
Huda has decided to make a calendar.
She is going to stick a photograph onto a piece of card and leave space underneath for a calendar tab.



The piece of card is 10 inches high
The photograph is $5\frac{7}{8}$ inches high and the space at the top is of an inches, as shown on the right.

What is the height of the space between the bottom of picture and the end of the card?

What would be the smallest possible height, to the nearest millimeter of this container so that it can hold 330ml of juice?



r =6 cm

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12.8 Grade Eight Test (2)

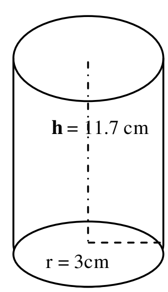
Name:…………………

Class:………………..

<table>
<tr><td>The length of a line is 3.6 m, and we want to divide it to several parts the length of each is 1.2m. How many pieces we will get?</td><td>$29\frac{3}{4}+17\frac{1}{2}=$</td></tr>
<tr><td colspan="2">If the length of rectangle area is (a+5) cm, and its height is 2a calculate the area of this rectangle.</td></tr>
<tr><td colspan="2">$$\begin{array}{r} 3x^2-5x+1 \\ -\quad -2x^2+x+4 \\ \hline \end{array}$$</td></tr>
<tr><td colspan="2">A company making various kinds of fruit juice decides to sell its product in 330 ml quantities. After considering possible containers they decide on metal in the shape right circular cylinder.

a) Lateral surface area of the container =

b) Check that the container can in fact hold 330 ml of juice.


</td></tr>
<tr><td>$x^2-5=11$</td><td>$4x-3=29$</td></tr>
</table>

Find the following solution:

$$\frac{-3}{7} \times \frac{2}{5} =$$

1.1.2 Fare

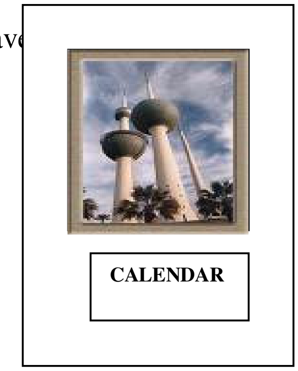
The fare charged for traveling by taxi is shown here

3) How much does it cost to travel **2** miles by taxi?

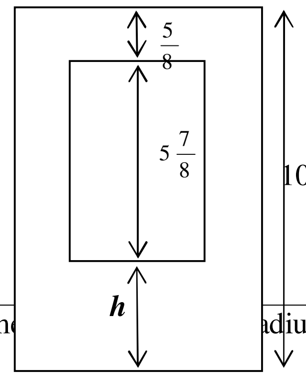
4) Ali has to travel **3** miles from the cinema to his home, he has 3,950 KD. Are his many enough to pay his taxi fare from the cinema to his hoe?

Your bank account holds 20 KD. You enter your credits and debits with + and - signs, respectively. What do you own after writing down the entries +2,7 KD, -7,3 KD, - 7 KD, + 1,3 KD?

Huda has decided to make a calendar.
She is going to stick a photograph onto a piece of card and leav
underneath for a calendar tab.



The piece of card is 10 inches high
The photograph is $5\frac{7}{8}$ inches high and the space at the top is $\frac{5}{8}$ of an inches, as shown on the right.
What is the height of the space at the bottom, shown as ***h*** in the diagram?



What would be the smallest possible height, to the nearest millim adius of its base r = 6 cm, so that it can hold 330ml of juice?

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Name:……………….
Class:………………..

Science Education Centre

Grade Nine Test (1)

If U = { $a : a \in N,\ \ a < 8$ } is a universal set, and C = {1, 2, 6}, D = {2, 3, 4, 1}

Find the elements of

U ={

$C \cap D = \{$

$C \cup D = \{$

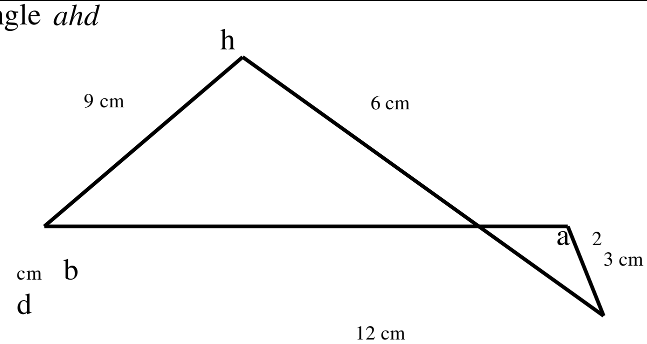
$C' = \{$

$D' = \{$

$D - C =$

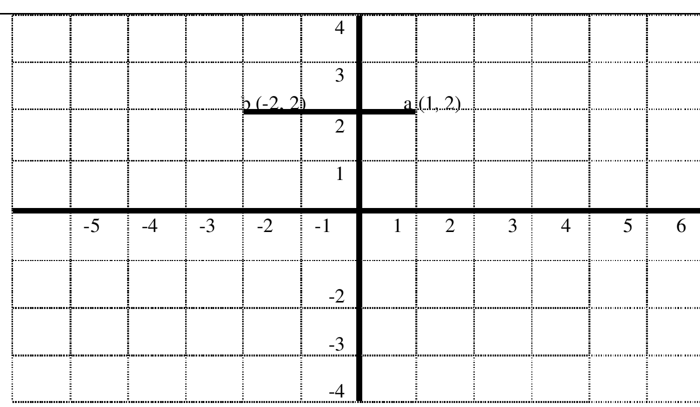
$C' \cap C =$

Prove triangle abc is similar to triangle ahd

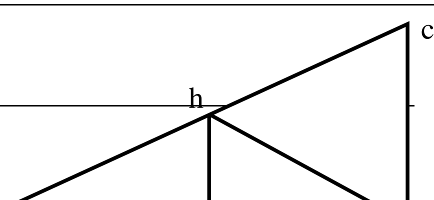


Find the image of the point (0, -3) under rotation 90° clockwise

Draw the image of ab translation 3 units in the negative side of x-axis



Δ abc is right angle triangle in b.



ac= 10 cm , bc= 6cm, ab = 8 cm

ah=hc

ad=db

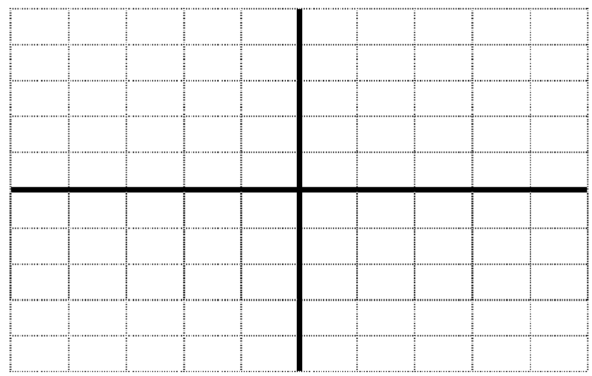
Find

hd=

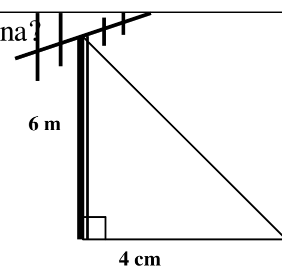
hb=

The function $y = x + 2$ describes the global warming, when average temperatures rise by two degrees

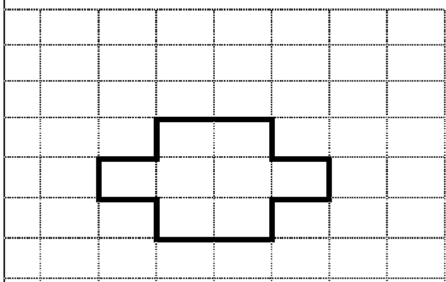
Sketch the graph of the function

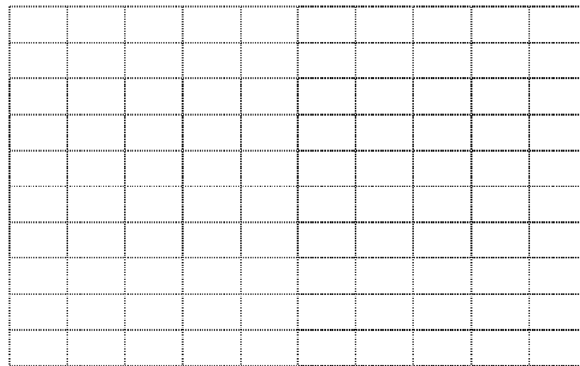


What is the length of the wire that is used to fix the antenna?

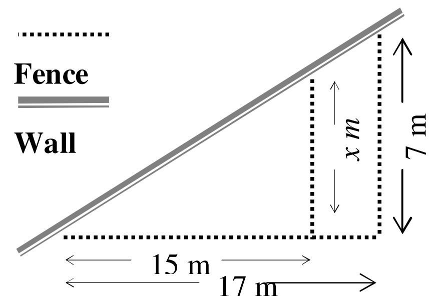


Draw a shape the same as this but make all the sides twice as long





Ali needs to replace fencing in his garden . he has taken measurments (shown) But has forgotten to measure the part of the fence marke *x* meters. The garden centre has only 28 metres of fencing stock. Is this enough to completely replace the exsiting fence?



Hint: seeing two tringle in adigram is often a sign of similar trinagle.

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Science Education Centre

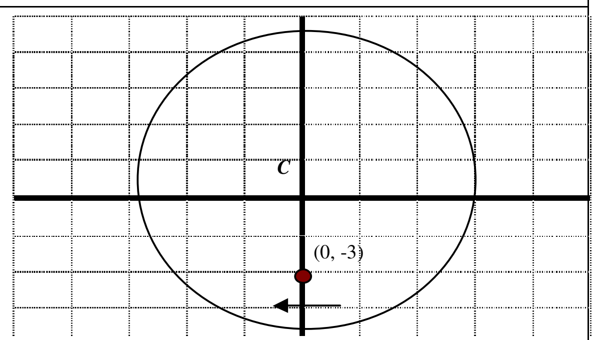
Name:……………….
Class:………………..

Grade Nine Test(2)

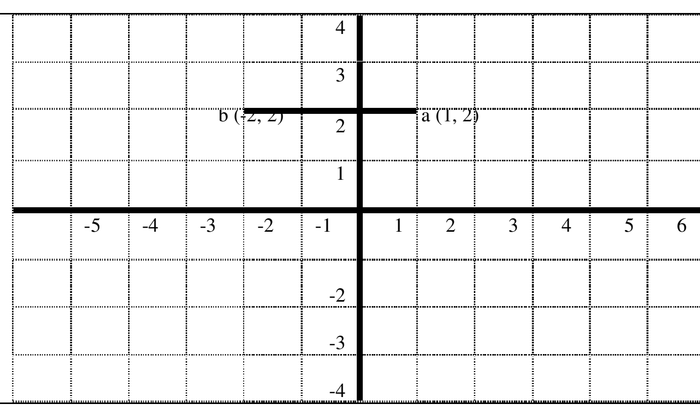
If U = { $a : a \in N,\ \ a < 8$ } is a universal set, and C = {1, 2, 6}, D = {2, 3, 4, 1}
Find the elements of
U ={
$C \cap D = \{$ $D' = \{$
$C \cup D = \{$ $D - C =$
$C' = \{$ $C' \cap C =$

abc is a triangle where ab= 2cm, ac= 4cm, bc= 3 cm.
ahd is another triangle where ah = 6cm, ad =12cm, hd= 9 cm.
Prove triangle *abc* is similar to triangle *ahd*

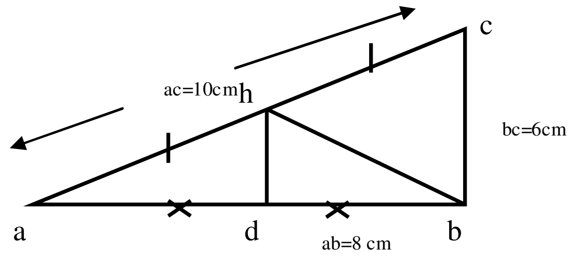
The diagram shows the monitor of the control unit in Kuwait airport, the location of an air plane in the monitor is in the point (0, -3) (shown in the diagram as , and the arrow shows the direction of the air plane)
The controller asks the fight captain to make a rotation $90°$ clockwise around the centre point (shown in the diagram as ***C***).
Calculate the location of the air plane in the monitor after the rotation.



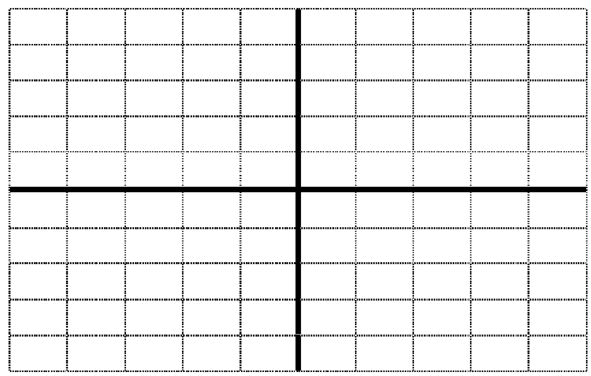
Draw the image of ab translation 3 units in the negative side of x-axis



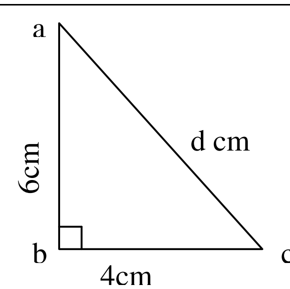
Find
hd=
hb=



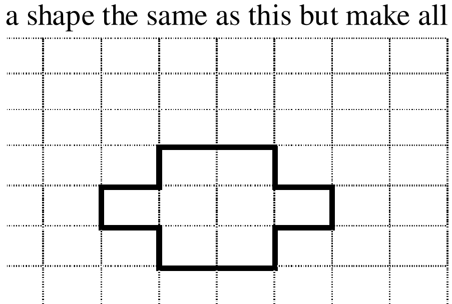
Sketch the graph of the function
$y = x + 2$

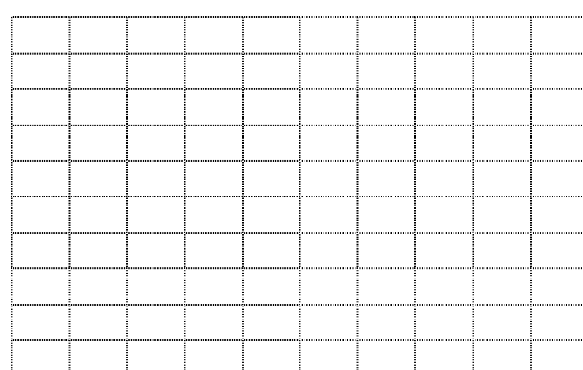


abc is a single right angle triangle
ab= 6 cm
bc=4cm
Find
ac=

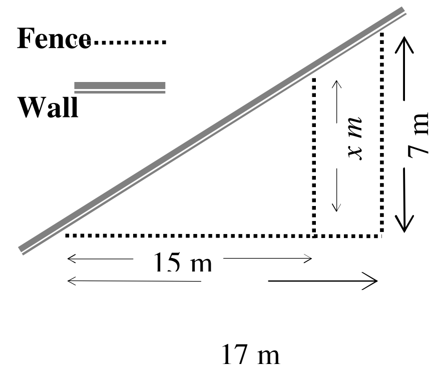


Draw a shape the same as this but make all the sides twice as long





Ali needs to replace fencing in his garden . he has taken measurments (shown) But has forgotten to measure the part of the fence marke *x* meters. The garden centre has only 28 metres of fencing stock. Is this enough to completely replace the exsiting fence?



Hint: seeing two tringle in adigram is often a sign of similar trinagle.

Mathematics Tests

Marks Distribution

Name:……………….
Class:………………..

University of Glasgow
Science Education Centre

14.1 Grade Eight Test (1)

<table>
<tr>
<td>Find the solution

3.6 ÷ 1.2 =
→ →
× 10 × 10 ½
36 ÷ 12 = 3 ½
Total mark = 1</td>
<td>Sara buys a jacket and skirt. the prices are $29\frac{3}{4}$ KD $17\frac{1}{2}$ KD respectively. How much she will pay for them?
$29\frac{3}{4}+17\frac{1}{2}=29\frac{3}{4}+17\frac{2}{4}=46\frac{5}{4}=47\frac{1}{4}$
¼ ¼ ¼ ¼
Total mark = 1</td>
</tr>
<tr>
<td colspan="2">Calculate the area of the football patch

2a
a+5


Rectangle area = Length X Width ½
= (a+5) (2a) ½
=$2a^2$ + 10a
½ ½
Total mark : 2</td>
</tr>
<tr>
<td colspan="2">Find the solution of subtraction $3x^2-5x+1$ from $x-2x^2+4$

$3x^2-5x+1$</td>
</tr>
</table>

½ $\quad -$

$$-2x^2 + x + 4$$

$$5x^2 - 6x - 3$$

½ ½ ½

Total mark = 2

The shape of water tank is right circular cylinder.
The radius of its base r= 7cm and its height h = 10cm.

Calculate the lateral surface area and the water volume if we going to fill this tank. $\pi = \dfrac{22}{7}$

Lateral surface $2\pi rh$ area=
½

$= 2 \times 22/7 \times 7 \times 10$ *1*
$= 440\ \text{cm}^2$
½
Total mark = 2

$V = \pi r^2 h$ ½
$= 22/7 \times (7)^2 \times 10$ *1*
$= 1540\ cm^3$ ½
Total mark = 2

Solve for x:
$x^2 - 5 = 11$
$x^2 = 16$ ½
$x = 4$ ½
Total Mark = 1

$4x - 3 = 29$
$4\ x = 32$ ½
$x = 8$ ½
Total Mark = 1

Find the solution:

$\frac{-3}{7} \times \frac{2}{5} = \frac{-6}{35}$ ½

½

Total mark = 1

The fare charged for traveling by taxi is shown here

5) How much does it cost to travel **2** miles by taxi?

1.1.3 Fare

1,500 + 600 + 600 = 2,700 KD
¼ ¼ ¼ ¼ Total mark = 1

6) Ali has to travel **3** miles from the cinema to his home. Has his money enough to pay his taxi fare from the cinema to his home?

1.500 + 1.200 + 1.200 = 3.900 KD
¼ ¼ ¼ ¼ Total mark= 1

Your bank account holds 20 KD. You enter your credits and debits with + and - signs, respectively. What do you own after writing down the entries +2.7 KD, -7.3 KD, - 7 KD, +

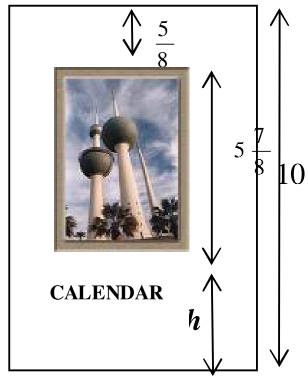
1.3 KD?

20 + 2.7 + 1.3 = 24 KD 1

24 – 7.3 – 7 = 9.7 KD 1

Total mark = 2

Huda has decided to make a calendar.
She is going to stick a photograph onto a piece of card and leave space underneath for a calendar tab. The piece of card is 10 inches high
The photograph is $5\frac{7}{8}$ inches high and the space at the top is $\frac{5}{8}$ of an inches, as shown on the right.



What is the height of the space between the bottom of picture and the end of the card?

$10-(5\frac{7}{8}+\frac{5}{8}) = 10-5\frac{12}{8} = 10-6\frac{4}{8} = 9\frac{8}{8}-6\frac{4}{8} = 3\frac{4}{8}$

½ ½ ¼ ¼ ¼ ¼

Total mark = 2

What would be the smallest possible height, to the nearest millimeter of this container so that it can hold 330ml of juice?

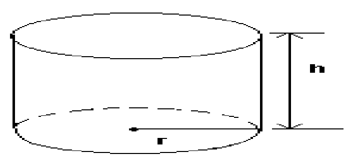
$V = \pi r^2 h$

$330 = 3.14 \times 6^2 \times h$ 1

h = 330 / 3.14 × 36 ½

h = 0.08 cm ½

Totla mark = 2



r =6 cm

Name:……………….
Class:………………..

University of Glasgow

Science Education Centre

14.2 Grade Eight Test (2)

The length of a line is 3.6 m, and we want to divide it to several parts the length of each is 1.2m. How many pieces we will get? $3.6 \div 1.2 =$ $\rightarrow \quad \rightarrow$ $\times 10 \quad \times 10$ ½ $36 \div 12 = 3$ ½ *Total mark = 1*	$29\frac{3}{4}+17\frac{1}{2} = 29\frac{3}{4}+17\frac{2}{4} = 46\frac{5}{4} = 47\frac{1}{4}$ ½ ¼ ¼ Total mark = 1
If the length of rectangle area is (a+5) cm, and its height is 2a calculate the area of this	

rectangle.

Rectangle area = Length X Width ½

= (a+5) (2a) ½

$=2a^2 + 10a$

½ ½

Total mark : 2

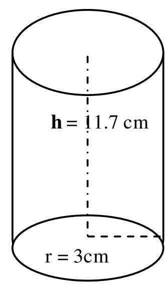
$$\begin{array}{r} 3x^2 - 5x + 1 \\ - \\ -2x^2 + x + 4 \\ \hline 5x^2 - 6x - 3 \end{array}$$

½

½ ½ ½

Total mark = 2

A company making various kinds of fruit juice decides to sell its product in 330 ml quantities. After considering possible containers they decide on metal in the shape **right circular cylinder.**



a) Lateral surface area of the container =

Lateral surface $2\pi rh$ area= ½

$= 2 \times 3.14 \times 3 \times 11.7$ *1*

$= 220.428 \text{ cm}^2$ ½

Total mark = 2

b) Check that the container can in fact hold 330 ml of juice.

$V = \pi r^2 h$ ½

$= 3.14 \times (3)^2 \times 10$ *1*

$= 282.6\ cm^3$ ½

Total mark = 2

$x^2 - 5 = 11$

$x = 16$ ½

$x = 4$ ½

Total Mark = 1

$4x - 3 = 29$

$4x = 32$ ½

$x = 8$ ½

Total Mark = 1

Find the following solution:

$\frac{-3}{7} \times \frac{2}{5} = \frac{-6}{35}$ ½

½

Total mark = 1

1.1.4 Fare

The fare charged for traveling by taxi is shown here

1) How much does it cost to travel **2** miles by taxi?

1,500 + 600 + 600 = 2,700 KD
¼ ¼ ¼ ¼ Total mark = 1

2) Ali has to travel **3** miles from the cinema to his home, he has 3,950 KD. Are his many enough to pay his taxi fare from the cinema to his hoe?

1.500 + 1.200 + 1.200 = 3.900 KD
¼ ¼ ¼ ¼ Total mark= 1

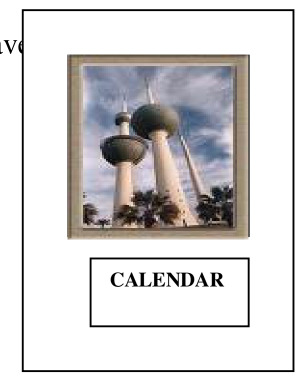
Your bank account holds 20 KD. You enter your credits and debits with + and - signs, respectively. What do you own after writing down the entries +2,7 KD, -7,3 KD, - 7 KD, + 1,3 KD?

20 + 2.7 + 1.3 = 24 KD 1

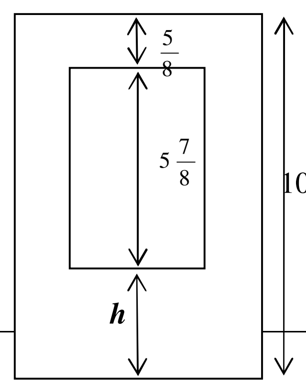
24 – 7.3 – 7 = 9.7 KD 1

Total mark = 2

Huda has decided to make a calendar.
She is going to stick a photograph onto a piece of card and leave underneath for a calendar tab.



The piece of card is 10 inches high
The photograph is $5\frac{7}{8}$ inches high and the space at the top is $\frac{5}{8}$ of an inches, as shown on the right.
What is the height of the space at the bottom, shown as ***h*** in the diagram?



$$10-(5\frac{7}{8}+\frac{5}{8}) = 10-5\frac{12}{8} = 10-6\frac{4}{8} = 9\frac{8}{8}-6\frac{4}{8} = 3\frac{4}{8}$$

½ ½ ¼ ¼ ¼ ¼
Total mark = 2

What would be the smallest possible height, to the nearest millimetre of cylinder the radius of its base r = 6 cm, so that it can hold 330ml of juice?

$V = \pi r^2 h$

$330 = 3.14 \times 6^2 \times h$ 1

$h = 330 / 3.14 \times 36$ ½

$h = 0.08$ cm ½

Totla mark = 2

Name:……………….
Class:………………..

University of Glasgow

Science Education Centre

Grade Nine Test (1)

If U = $\{ a : a \in N,\ \ a < 8 \}$ is a universal set, and C = {1, 2, 6}, D = {2, 3, 4, 1}

Find the elements of

U = { 0,1,2,3,4,5,6,7 } 1

$C \cap D = \{1,2\ \}$ ¼

$D' = \{0,5,6,7\ \}$ ½

$C \cup D = \{1,2,3,4,6\ \}$ ¼

$D - C = \{3,4\ \}$ ¼

$C' = \{0,3,4,5,7\ \}$ ½

$C' \cap C = \phi$ ¼

Total mark= 3

Prove triangle *abc* is similar to triangle *ahd*

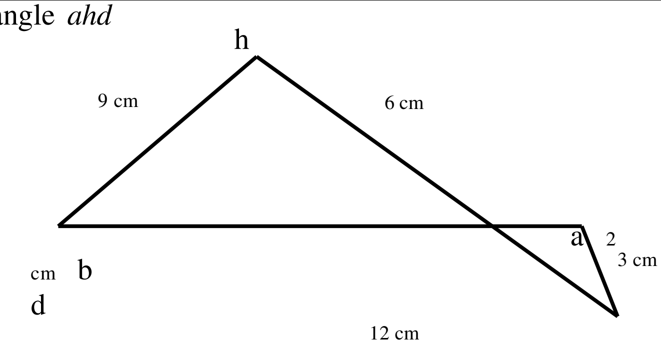
$\frac{hd}{bc} = \frac{ha}{ab} = \frac{da}{ac} = \frac{9}{3} = \frac{6}{2} = \frac{12}{4} = \frac{3}{1}$

¼ ¼ ¼ ¼ ¼ ¼

$abc \cong ahd$

¼

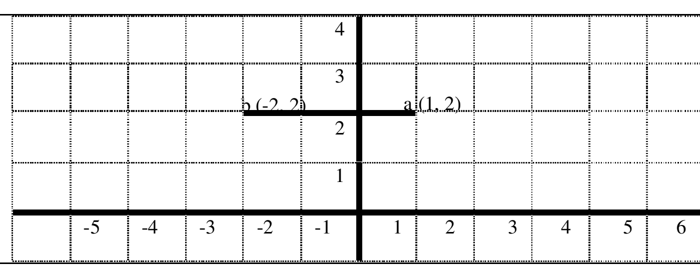
* $\cong$ *similar*

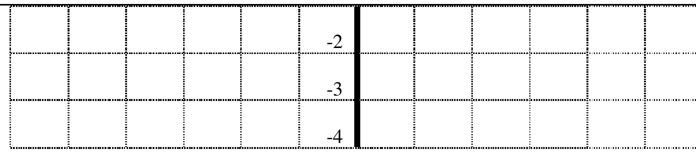


Total mak =2

Find the image of the point (0, -3) under rotation 90° clockwise

Total mark = 1

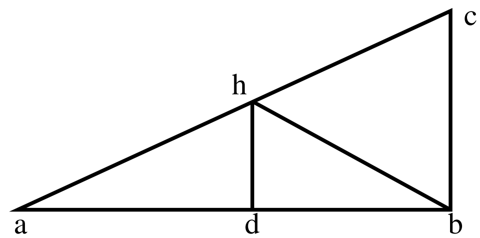




Draw the image of ab translation 3 units in the negative side of x-axis

1 for every point
Total mark =2

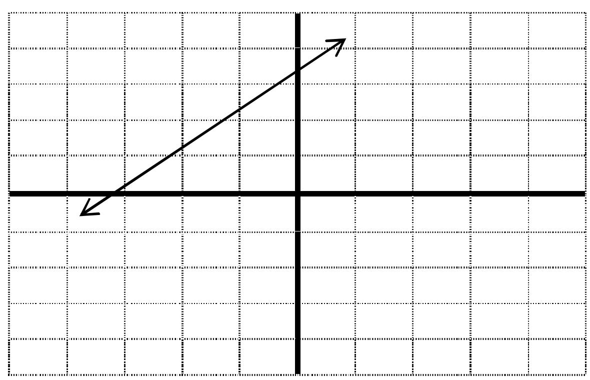
Δ abc is right angle triangle in b.
ac= 10 cm , bc= 6cm, ab = 8 cm
ah=hc
ad=db
Find
hd= 3 cm (Theory) 1
hb= 5 cm (Theory) 1



Total mark = 2

The function $y = x + 2$ describes the global warming, when average temperatures rise by two degrees
Sketch the graph of the function

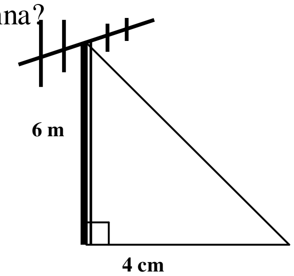
X	Y	(x, y)
-1	1	½ $(-1,1)$
0	2	½ $(0,2)$
1	3	½ $(1,3)$



(½) for every point sketched in the graph. A student who sketched the graph without the table, he had the total mark.
Total mark =3

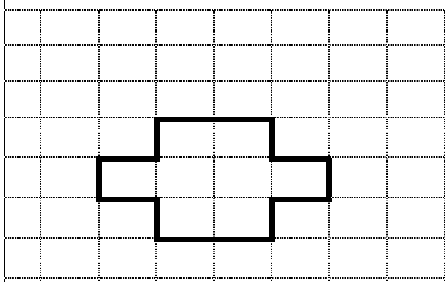
What is the length of the wire that is used to fix the antenna?

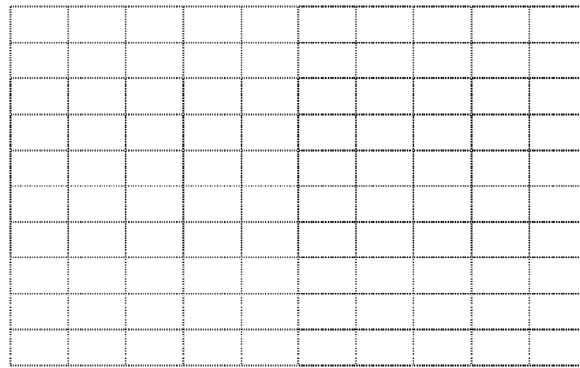
The length of the wire = $\sqrt{6^2 + 4^2}$ 1
= $\sqrt{36+16}$ 1
= $\sqrt{52}$ 1



Total mark = 3

Draw a shape the same as this but make all the sides twice as long





Total mark =2

Ali needs to replace fencing in his garden . he has taken measurments (shown) But has forgotten to measure the part of the fence marke x meters. The garden centre has only 28 metres of fencing stock. Is this enough to completely replace the exsiting fence?

$\frac{x}{7} = \frac{15}{17}$ 1

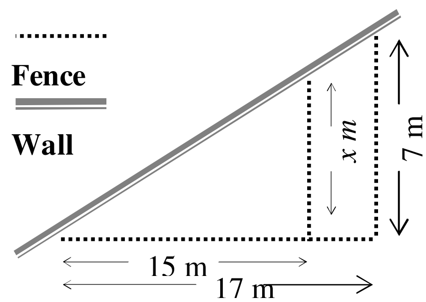
$x = \frac{15 \times 7}{17} = 6.176$ m 1

The fencing of the garden= 17 + 7 + 6.176 = 30.176 m

½ ½ ½ ½

28 metters not enough

Total mark = 4



Hint: seeing two tringle in adigram is often a sign of similar trinagle.

Name:……………….
Class:………………..

University of Glasgow

Science Education Centre

Grade Nine Test (2)

If U = { $a : a \in N,\ a < 8$ } is a universal set, and C = {1, 2, 6}, D = {2, 3, 4, 1}

Find the elements of

U = { 0,1,2,3,4,5,6,7 } 1

$C \cap D = \{1,2\}$ ¼ | $D' = \{0,5,6,7\}$ ½

$C \cup D = \{1,2,3,4,6\}$ ¼ | $D - C = \{3,4\}$ ¼

$C' = \{0,3,4,5,7\}$ ½ | $C' \cap C = \phi$ ¼

Total mark= 3

---

abc is a triangle where ab= 2cm, ac= 4cm, bc= 3 cm.
ahd is another triangle where ah = 6cm, ad =12cm, hd= 9 cm.
Prove triangle abc is similar to triangle ahd

$$\frac{hd}{bc} = \frac{ha}{ab} = \frac{da}{ac} = \frac{9}{3} = \frac{6}{2} = \frac{12}{4} = \frac{3}{1}$$

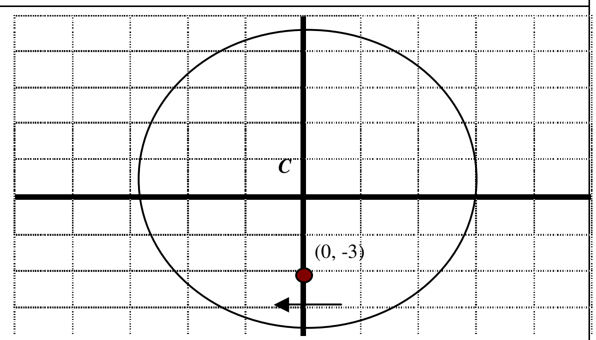
¼ ¼ ¼ ¼ ¼ ¼

$abc \cong ahd$ * $\cong$ *similar*

¼

Total mark =2

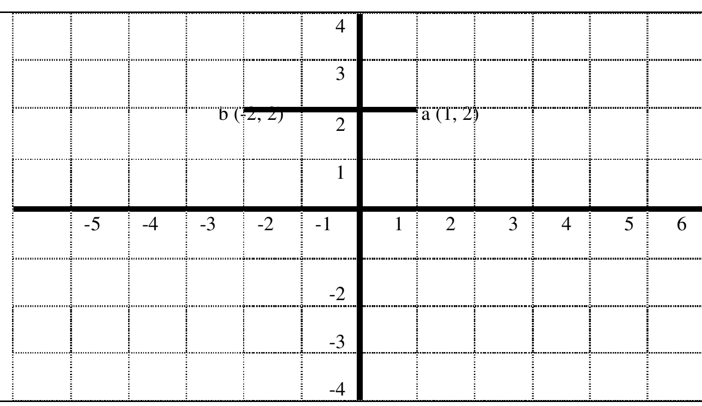
The diagram shows the monitor of the control unit in Kuwait airport, the location of an air plane in the monitor is in the point (0, -3) (shown in the diagram as , and the arrow shows the direction of the air plane)
The controller asks the fight captain to make a rotation 90° clockwise around the centre point (shown in the diagram as ***C***).
Calculate the location of the air plane in the monitor after the rotation.



Total mark = 1

Draw the image of ab translation 3 units in the negative side of x-axis

1 for every point
Total mark =2

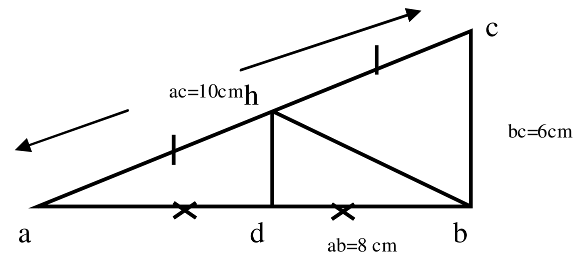


Find

hd= 3 cm (Theory) 1

hb= 5 cm (Theory) 1

Total mark = 2



Sketch the graph of the function

$y = x + 2$

X	Y	(x, y)
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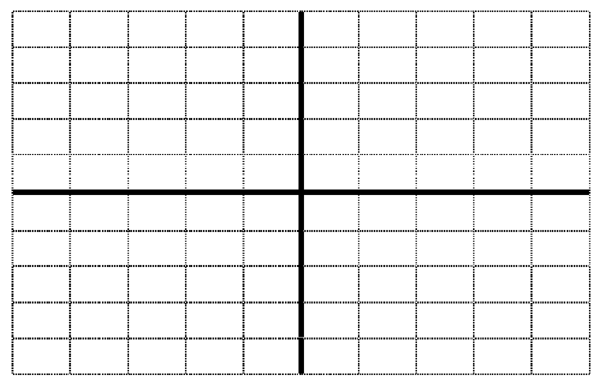
1	3	½ $(1,3)$



(½) for every point sketched in the graph.

A student who sketched the graph without the table, he had the total mark.

Total mark =3

abc is a single right angle triangle

ab= 6 cm

bc=4cm

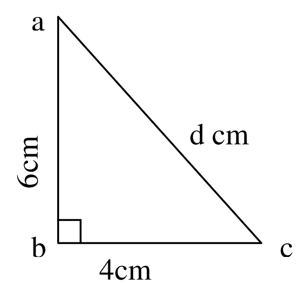
Find

ac= = $\sqrt{6^2 + 4^2}$ 1

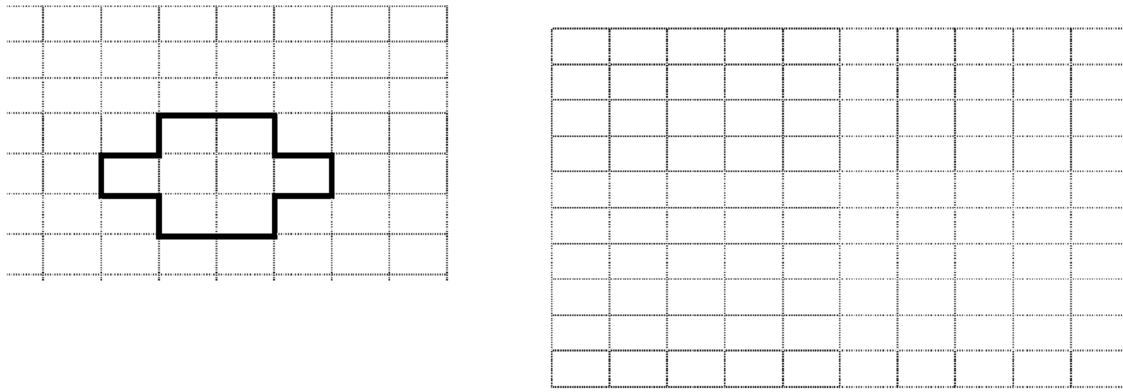
$= \sqrt{36+16}$ 1

$= \sqrt{52}$ 1

Total mark = 3



Draw a shape the same as this but make all the sides twice as long



Total mark = 2

Ali needs to replace fencing in his garden . he has taken measurments (shown) But has forgotten to measure the part of the fence marke x meters. The garden centre has only 28 metres of fencing stock. Is this enough to completely replace the exsiting fence?

$\frac{x}{7} = \frac{15}{17}$ 1

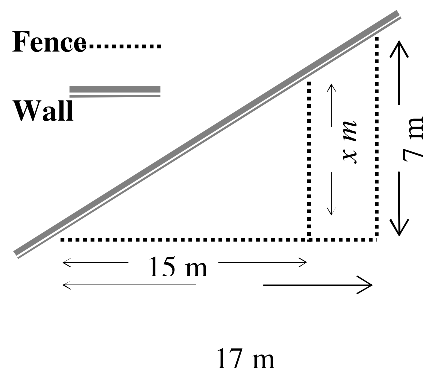
$x = \frac{15 \times 7}{17} = 6.176$ m 1

The fencing of the garden= 17 + 7 + 6.176 = 30.176 m

½ ½ ½ ½

28 metters not enough

Total mark = 4



Hint: seeing two tringle in adigram is often a sign of similar trinagle.

Appendix D

Questionnaires

Phase (1)

University of Glasgow

Centre of Science Education

Name………………………….. Are you Male Female?

> Most questions can be answered simply by putting (**X**) in the relevant **box** (es) or by writing your answer.

(1) Tick the box which best represents your opinions
(Tick one box in each line)

	Strongly Agree	Agree	Neutral	Disagree	Strongly
(a) I usually understand mathematics idea easily. ………. …………….………………………………	☐	☐	☐	☐	☐
(b) I do not enjoy mathematics lessons……………………………………………………………	☐	☐	☐	☐	☐
(c) I think every one should learn mathematics at secondary school………………………………..	☐	☐	☐	☐	☐
(d) I think I am good in mathematics………………………………………………………...............	☐	☐	☐	☐	☐
(e) You have to born with the right kind of brain, to be good in mathematic………………………..	☐	☐	☐	☐	☐
(f) To be good in mathematics, you have to spend more time studying it……………………………	☐	☐	☐	☐	☐
(g) I think mathematics is useful subject……………………………………………………………….	☐	☐	☐	☐	☐
(i) I find my mathematics knowledge useful in daily life……………………………………………	☐	☐	☐	☐	☐

(2) I like mathematics because………….

☐ I am good in it ☐ I understand its logic ☐ I have a good teacher

☐ I do not need to study before the exam ☐ I have always liked it ☐ I think it helps me in my life

☐ It will help me in my career ☐ I always have high mark in it

(3) Do you think mathematics is important?

☐ Yes, Because………………………………

☐ No, Because……………………………….

Here is a way to describe a racing car.

quick	☑	☐	☐	☐	☐	☐	slow
important	☐	☐	☑	☐	☐	☐	unimportant
safe	☐	☐	☐	☐	☑	☐	dangerous

> The positions of the ticks between the word pairs show that you consider it as very quick, slightly more important than unimportant and quite dangerous.

(4) Use the same method to show your opinions below.
<u>*Tick **one** box on each line.*</u>

I am confident in mathematics classes	☐	☐	☐	☐	☐	☐	I am not confident in mathematics classes
Mathematics is too abstract for me	☐	☐	☐	☐	☐	☐	Mathematics is too easy for me.
I am getting worse at mathematics	☐	☐	☐	☐	☐	☐	I am getting better at mathematics
I feel I am coping well	☐	☐	☐	☐	☐	☐	I feel I am not coping well
Mathematics classes are boring	☐	☐	☐	☐	☐	☐	Mathematics classes are interesting

(5) Tick your class preferences:
(The closest tick to the answer, the strongest preferences)

Arabic Language	☐	☐	☐	☐	☐	☐	Mathematics
Mathematics	☐	☐	☐	☐	☐	☐	English
Mathematics	☐	☐	☐	☐	☐	☐	Geography
Science	☐	☐	☐	☐	☐	☐	Mathematics

Fractions	☐	☐	☐	☐	☐	☐	Geometry
Sets	☐	☐	☐	☐	☐	☐	Fractions
Algebra	☐	☐	☐	☐	☐	☐	Linear Equation
Geometry	☐	☐	☐	☐	☐	☐	Sets

(6) Think about ***Mathematics as a subject***

Abstract	☐	☐	☐	☐	☐	☐	Not abstract
Difficult	☐	☐	☐	☐	☐	☐	Easy
Unrelated to life	☐		☐	☐	☐	☐	Related to life
Boring	☐	☐	☐	☐	☐	☐	Interesting
Not useful for careers	☐	☐	☐	☐	☐	☐	Useful for careers
Complicated	☐	☐	☐	☐	☐	☐	Straight forward

(7) Think about your ***Mathematics classes***

	Always	Often	Sometimes	Rarely	Never
I do not understand what is taught………………………………………….	☐	☐	☐	☐	☐
I find doing mathematics problems repetitive………………………………	☐	☐	☐	☐	☐
The explanations are not clear……………………………………………….	☐	☐	☐	☐	☐
I am not sure what t should be doing………………………………………..	☐	☐	☐	☐	☐
I find I make many mistakes………………………………………………….	☐	☐	☐	☐	☐
There is too much homework………………………………………………….	☐	☐	☐	☐	☐

(8) Think about mathematics ***tests and examinations***

	Always	Often	Sometimes	Rarelv	Never
I tend to panic with difficult problems………………………………………	☐	☐	☐	☐	☐
They involve a lot of revision the day before……………………………….	☐	☐	☐	☐	☐
I find I am short of time…………………………………. ………………	☐	☐	☐	☐	☐
I often make mistakes………………………………………………………..	☐	☐	☐	☐	☐
I cannot remember how to do things………………………………………..	☐	☐	☐	☐	☐
There is little opportunity to explain things…………………………………	☐	☐	☐	☐	☐

(9) Here are some reasons ***why pupils should study Mathematics at school***

A. It is useful in daily life

B. There are many jobs for mathematicians

C. It is important for some other subject

D. It teaches me to think logically

E. Mathematics can help solve world problems

F. It is important for many courses at university

G. It is a useful way to make sense of the world

H. It is very satisfying

Place these reasons **<u>in order</u>**, showing which is the most important for you

☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Most important ***Least important***

(10) Here are some things which can help me in my mathematics studies

17.1 Tick the **three** which are most helpful for you

☐ Practising many mathematics exercises and problems until I get them right;

☐ Reading my textbook carefully;

☐ Working with my friends until I understand the ideas;

☐ Seeking help from my parents;

☐ Try to see things as pictures or diagrams;

☐ Following the methods taught by my teacher carefully;

☐ Making sure I understand what I am doing.
☐ Trying to find a method which always gives the right answer.

Thank you very much for your cooperation

How You See Mathematics
Grade Eight

Your Name:…………………………………………. Your Class: …………..

This survey wants to find out what you think of your studies in mathematics.

Please be completely honest!!

(1) I think the following methods will help me to understand mathematics…..
*Tick **THREE** boxes which you think are the most important.*

☐ Using a calculator.
☐ Using a computer.
☐ Have more mathematics lessons.
☐ Using teaching aids such as models, pictures or diagrams.
☐ Using game based in mathematics classes.
☐ Use mathematics to solve real-life problem.
☐ Teach mathematics more slowly.

(2) I think mathematics is important.
*Tick **THREE** boxes which you think are the most important.*

☐ It is useful in daily life.
☐ It is important for some other subjects.
☐ Mathematics can help to solve world problems.
☐ It is a useful way to make sense of the world .
☐ There are many jobs for mathematicians.
☐ It teaches me to think logically.
☐ It is important for many courses at university.

(3) Which of the following topics interest you?
*Tick **as many** as you wish*

☐ Solving equations ☐ Quadratic equations ☐ Volume
☐ Elementary sets theory ☐ Fractions ☐ Analytic geometry
☐ Transformation geometry

(4) When I have difficulty in studying mathematics, I rely on …..
*Tick **THREE** boxes which you think are the most important*

☐ School textbook ☐ Out-of-school teacher ☐ Self-teaching manual
☐ Family member ☐ General mathematics book ☐ Friends
☐ School teacher ☐ Internet

(5) What type of activity do you like in mathematics classes?
*Tick **ONE** box.*

☐ Solving exercises and problem proving ☐ Theory
☐ Working as a group
☐ Discovering ☐ Using computer ☐ Listening to the teacher
☐ Working on my own ☐ Reasoning and proving ☐ Discussion

(6) Secondary mathematics is often seen as more difficult than primary mathematics.
*Tick **ONE** box who best describes you.*

Secondary mathematics is….

☐ Not related to the real-life ☐ Secondary mathematics involves difficult explanation.
☐ Very abstract ☐ Very complicated
☐ Secondary mathematics is no more difficult than primary

(7) What is your opinion about mathematicians?
*Tick **ONE** box in each line*

Clever	☐	☐	☐	☐	☐	☐	Dull
Valuable to the society	☐	☐	☐	☐	☐	☐	Worthless to the society
Popular	☐	☐	☐	☐	☐	☐	Not Popular
Hard worker	☐	☐	☐	☐	☐	☐	Not a hard worker
Rich	☐	☐	☐	☐	☐	☐	Poor
Doing a dangerous job	☐	☐	☐	☐	☐	☐	Doing a safe job

(8) The mathematics tasks are easier for me, if they are presented…
*Tick **ONE** box in each line*

In term of pictures, like diagrams	☐	☐	☐	☐	☐	☐	In term of symbol, like algebra
As abstract tasks	☐	☐	☐	☐	☐	☐	As real-world tasks

(9) When I study mathematics…
*Tick **ONE** box in each line*

I rely on memorizing	☐	☐	☐	☐	☐	☐	I rely on understanding
I enjoy challenging activities	☐	☐	☐	☐	☐	☐	I do not enjoy challenging activities
I enjoy repetitive tasks	☐	☐	☐	☐	☐	☐	I do not enjoy repetitive tasks
I like to master one way of achieving a task	☐	☐	☐	☐	☐	☐	I like to think of many ways of achieving a task
I find exercises boring	☐	☐	☐	☐	☐	☐	I find exercises interesting
I depend on the teacher most	☐	☐	☐	☐	☐	☐	I depend on the text book most
I can hold all the ideas in my head easily	☐	☐	☐	☐	☐	☐	I cannot hold all the ideas in my head easily
I am not quite sure what is important	☐	☐	☐	☐	☐	☐	*I am quite sure what is importan*

(10) How do you describe yourself in mathematics classes?
Tick **ONE** box in each line.

	Strongly Agree	*Agree*	*Neutral*	*Disagree*	*Strongly Disagree*
I am generally a confident person in mathematics classes	☐	☐	☐	☐	☐
I feel more confident when I succeed in solving a task	☐	☐	☐	☐	☐
I feel confident when I study mathematics	☐	☐	☐	☐	☐
I feel confident when I really understand what is being taught in mathematics classes	☐	☐	☐	☐	☐
I feel confident taking part in a discussion group in mathematics classes	☐	☐	☐	☐	☐
I feel confident in mathematics examinations	☐	☐	☐	☐	☐
I am confident even when facing difficult material to understand in mathematics classes	☐	☐	☐	☐	☐

(11) Write THREE sentences to explain why you like or dislike mathematics.

……

……

……

17.2 Thank You

Centre for Science Education
University of Glasgow, Scotland

How You See Mathematics
Grade Nine

Your Name:…………………………………………. Your Class: …………..

This survey wants to find out what you think of your studies in mathematics.

Please be completely honest!!

(1) I think the following methods will help me to understand mathematics…..
*Tick **THREE** boxes which you think are the most important.*

- ☐ Using a calculator.
- ☐ Using a computer.
- ☐ Have more mathematics lessons.
- ☐ Using teaching aids such as models, pictures or diagrams.
- ☐ Using game based in mathematics classes.
- ☐ Use mathematics to solve real-life problem.
- ☐ Teach mathematics more slowly.

(2) I think mathematics is important.
*Tick **THREE** boxes which you think are the most important.*

- ☐ It is useful in daily life.
- ☐ It is important for some other subjects.
- ☐ Mathematics can help to solve world problems.
- ☐ It is a useful way to make sense of the world .
- ☐ There are many jobs for mathematicians.
- ☐ It teaches me to think logically.
- ☐ It is important for many courses at university.

(3) Which of the following topics interest you?
*Tick **as many** as you wish*

☐ Sets and their operation ☐ Solving equations ☐ Transformation geometry
☐ Inequalities ☐ Triangle geometry ☐ Polynomials
☐ Circle geometry

(4) When I have difficulty in studying mathematics, I rely on …..
*Tick **THREE** boxes which you think are the most important*

☐ School textbook ☐ Out-of-school teacher ☐ Self-teaching manual
☐ Family member ☐ General mathematics book ☐ Friends
☐ School teacher ☐ Internet

(5) What type of activity do you like in mathematics classes?
*Tick **ONE** box.*

☐ Solving exercises and problem proving ☐ Theory
☐ Working as a group
☐ Discovering ☐ Using computer ☐ Listening to the teacher
☐ Working on my own ☐ Reasoning and proving Discussion

(6) Secondary mathematics is often seen as more difficult than primary mathematics.
*Tick **ONE** box who best describes you.*
Secondary mathematics is…..

☐ Not related to the real-life ☐ Secondary mathematics involves difficult explanation.
☐ Very abstract ☐ Very complicated
☐ Secondary mathematics is no more difficult than primary

(7) What is your opinion about mathematicians?
*Tick **ONE** box in each line*

Clever	☐	☐	☐	☐	☐	☐	Dull
Valuable to the society	☐	☐	☐	☐	☐	☐	Worthless to the society
Popular	☐	☐	☐	☐	☐	☐	Not Popular
Hard worker	☐	☐	☐	☐	☐	☐	Not a hard worker
Rich	☐	☐	☐	☐	☐	☐	Poor
Doing a dangerous job	☐	☐	☐	☐	☐	☐	Doing a safe job

(8) The mathematics tasks are easier for me, if they are presented…
*Tick **ONE** box in each line*

In term of pictures, like diagrams	☐	☐	☐	☐	☐	☐	In term of symbol, like algebra
As abstract tasks	☐	☐	☐	☐	☐	☐	As real-world tasks

(9) When I study mathematics…
*Tick **ONE** box in each line*

I rely on memorizing	☐	☐	☐	☐	☐	☐	I rely on understanding
I enjoy challenging activities	☐	☐	☐	☐	☐	☐	I do not enjoy challenging activities
I enjoy repetitive tasks	☐	☐	☐	☐	☐	☐	I do not enjoy repetitive tasks
I like to master one way of achieving a task	☐	☐	☐	☐	☐	☐	I like to think of many ways of achieving a task

I find exercises boring ☐ ☐ ☐ ☐ ☐ ☐ I find exercises interesting

I depend on the teacher most ☐ ☐ ☐ ☐ ☐ ☐ I depend on the text book most

I can hold all the ideas in my head easily ☐ ☐ ☐ ☐ ☐ ☐ I cannot hold all the ideas in my head easily

I am not quite sure what is important ☐☐☐☐☐☐ *I am quite sure what is importan*

(10) How do you describe yourself in mathematics classes?
Tick **ONE** box in each line.

	Strongly Agree	*Agree*	*Neutral*	*Disagree*	*Strongly Disagree*
I am generally a confident person in mathematics classes	☐	☐	☐	☐	☐
I feel more confident when I succeed in solving a task	☐	☐	☐	☐	☐
I feel confident when I study mathematics	☐	☐	☐	☐	☐
I feel confident when I really understand what is being taught in mathematics classes	☐	☐	☐	☐	☐
I feel confident taking part in a discussion group in mathematics classes	☐	☐	☐	☐	☐
I feel confident in mathematics examinations	☐	☐	☐	☐	☐
I am confident even when facing difficult material to understand in mathematics classes	☐	☐	☐	☐	☐

(11) Write THREE sentences to explain why you like or dislike mathematics.

……

……

……

17.3 Thank You

Centre for Science Education
University of Glasgow, Scotland

Appendix E

Interview Matrix

E- 2

	Q3	*Q4a*	*Q4b*	*Q4c*	*Q4d*	*Q4e*	*Q4f*	*Q4g*	*Q5*	*Q6*	*Q7*	*Q8*
1 F 9 6/9	*Help to create mathematical sense. The objectives take on consideration all different levels high and low achievements of the students and all the knowledge that they study are useful in daily life.*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Not essential, better later*	*Not essential, better later*	*Essential, better as now*	*Essential, better as now*	*No, just in time*	*Geometry*	*Yes, sure. Notice that in geometry*	*Reduce the curriculum*
2 F 12 6/8	*Create mathematician, Most of the topics are wasting of time, and only the person who will be mathematician will benefit of them.*	*Essential, better latter G7*	*Not Essential, better later (-4x-2)*	*Not Essential, better later*	*Not Essential, better later*	*Not Essential, better later*	*Not Essential, better later*	*Essential, better later G8*	*Yes, better to delay to G7. focus in the basic (+, -, x,)*	*Geometry and its theories*	*Explain part of the problem. The hierarchical nature play role as well*	*Reduce the curriculum. Give more time for the same topic until the pupils handle it.*

E- 3

3 F 19 6/8	*Help to create mathematical sense. The syllabuses are very easy, but the students nowadays differ from those in the past, they don't make all-out of their effort as suppose.*	*Essential, better latter*	*Not Essential, better later (-2x-3) Diagram 11-2*	*Not Essential, better later*	*Not Essential, better later*	*Not Essential, better later*	*Not Essential, better later*	*Essential, better as now*	*No, but they need to handle the basics first to achieve*	*Geometry theories*	*Ah…We ask the students to retrieve a huge amount of information not just 5or 6 items, no wonder why they fail in mathematics*	*Reduce the curriculum "full cream"*

	Q3	*Q4a*	*Q4b*	*Q4c*	*Q4d*	*Q4e*	*Q4f*	*Q4g*	*Q5*	*Q6*	*Q7*	*Q8*
7 F 11 7	*Create mathematician*	*Not Essential, better latter*	*Not Essential, better latter The difficulties which are caused by this topic make the benefits of this topic of*	*Essential, better as now*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*Yes, it is very difficult*	*Fractions*	*Yes, it clarifies what happen*	*Attractive learning, reduce curriculum.*

E- 4

			limited value									
8 F 12 7/9	*Create mathematician, focus on high achievement.*	*Essential, better as now*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*No, it is introduced in the proper time*	*Fractions*	*Maybe why not? It explains*	*Reduce the curriculum, G9 "full cream" Teach them the skills until they handle it and then explain to them how to apply them.*
9 M 5 7/8	*"Mathematics objectives aim to create mathematicians instead of creating the mathematical sense and cover topics more than the students need."*	*Essential, better latter*	*Not Essential, better latter (high Sec level)*	*Essential, better as now*	*Not Essential, better latter (high Sec level)*	*Not Essential, better latter (Sec level)*	*Not Essential, better latter (Sec level)*	*Essential, better as now(sales, banking, get the zakat and Almearath*	*Yes, it does not take in consideration when they design the syllabus*	*Fractions and their operations*	*It may explain the cause of the problem. There are many ideas and techniques which need high capacity of memory*	*Train the students until they handle it, there is need to prove theories,*
10 F 17 9	*Create mathematician*	*Essential, better as now*	*Not Essential, better later*	*Not Essential, better later*	*Essential, better as now*	*Essential, better as now*	*Not Essential, better later It is better to delete the quadratic equations at all*	*Essential, better as now (attendance percentage,*	*Fractions are depending heavily on the multiplication table and unless the students master it they will not be able to*	*Long Division*	*It could be their memory capacities*	*Focus in the basic (+, -,x, …). Improve learning system.*

Appendix E

E- 5

								banking)	*solve any fractions task*			*teaching ways and methods*

	Q3	Q4a	Q4b	Q4c	Q4d	Q4e	Q4f	Q4g	Q5	Q6	Q7	Q8
11 M 5 7/9	*Create mathematician*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better later (take more than it deserve)*	*Not Essential, better latter (many theories)*	*Not Essential, better later*	*Not Essential, better latter*	*Essential, better as now*	*Yes, very difficult and complicate the students' minds*	*Geometry, and long division*	*Long division is highly depend in previous knowledge (Times table) and geometry as well (No wonder)*	*Reduce the G9 curriculum. And reduce the classes for the teacher*
12 M 17 9/7	*Create mathematical sense*	*Essential, better as now*	*Essential, better now*	*Not Essential at all no need to teach it Teaching this topic just wasting time without any real benefit from it*	*Essential, better latter*	*Essential, better latter*	*Not Essential, better latter*	*Essential, better as now (useful in daily life) If the percentage is deleted or delayed, what we are going to teach them in mathematics classes*	*No, there is no problem in the introduction of the Fractions*	*Multiplication and division (Times tables)*	*Maybe, but they need to focus in their study*	*Attractive teaching by using teaching aid, reduce curriculum in G9.*

Appendix E

E- 6

13 M 8 6/8	*Create mathematician*	*Essential, better latter*	*Not Essential, better later (- x -)*	*Essential, better as now*	*Not Essential, better later*	*Essential, better as now*	*Not Essential, better later*	*Essential, better as now(very useful in daily life Zakat and al mearath)*	*We have to focus in basic and just introduce the fraction without its operations*	*Geometry & Long Division*	*It clarifies, because the syllabus design for those with high working memory (high achievement)*	*Reduce curriculum. Individual differences. Setting classes (Groups)*
14 M 5 6/7	*Create mathematician*	*Essential, better latter*	*Essential, better as now*	*Not Essential at all (wasting time)*	*Essential, better latter*	*Essential, better later*	*Essential, better latter*	*Essential, better as now*	*Yes, very difficult and demand a lot from students*	*Long Division*	*Maybe*	*Focus in teaching with suit the student's age.*

E- 7

	Q3	Q4a	Q4b	Q4c	Q4d	Q4e	Q4f	Q4g	Q5	Q6	Q7	Q8
15 F 10 8/9	*Create mathematician (focus in high achievement)*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*Not Essential, better latter*	*Essential, better now*	*Not Essential, better latter*	*Essential, better as now*	*Yes, it is very difficult and it is the reason for difficulty in maths*	*Geometry is very difficult and demands high levels of thinking and imagination. Students have to retrieve all the geometrical knowledge which have been studied in the previous years to be able to understand the new topic which is built on the. So, no wonder that students will lose the enjoyment in geometry*	*Of course, it clarifies what happen and I totally agree that the syllabuses don't suit the students' age.*	*Reduce curriculum.*
16 F 10 9	*Help to create mathematical sense and take students level in consideration.*	*Essential, better Latter (G6/7)*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*Yes, it is introduced very early and better to delay the operations until G6 or G7 and teach decimal instead.*	*Percentage*	*No, It explains a small part of the problem but the problem from mathematics itself. (hierarchical nature)*	*Reduce the curriculum, Teach them the skills until they handle it and then explain to them how to apply them. Train teachers*
17 F 11 7/8	*Help to create mathematical sense and take students level in*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Not Essential, better later*	*Not Essential, better later*	*Essential, better as now*	*Essential, better as now*	*No, it introduce in the proper time*	*Geometry & Proving theory*	*May be, it explain part of the problem but the problem*	*Give the teacher freedom to choose the proper way of teaching*

Appendix E

E- 8

	consideration.										*because the lesson time, the teacher doesn't have enough time to clarify the idea*	
18 F 13 69	*Help to create mathematical sense and take students level in consideration*	*Essential, better latter*	*Essential, better as now*	*Essential, better as now*	*Not Essential, better latter*	*Essential, better later*	*Not Essential, better latter*	*Essential, better as now*	*Yes, very difficult and demand a lot from students. The student don't understand it*	*Geometry theories (Triangle and Circle Theories)*	*Yes of course, it does explain because proving theory need space for recall all the previous information.*	*Serious learning and teaching? Don't teach for the exams teach for learning*

	Q3	*Q4a*	*Q4b*	*Q4c*	*Q4d*	*Q4e*	*Q4f*	*Q4g*	*Q5*	*Q6*	*Q7*	*Q8*
19 F 8 7/8	*Create mathematician*	*Essential, better latter (G6)*	*Essential, better latter*	*Essential, better as now*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*No, this not a problem is the hierarchical nature of mathematics and unless the student masters the previous knowledge he will not be able to understand the new information which depend heavily in the previous knowledge*	*Geometry and Triangle theories*	*Yes of course, it does explain because there is a huge amount of information in mathematics classes*	*Reduce curriculum.*

E- 9

20 F 6 6/9	*Create mathematician.*	*Essential, better latter (G6)*	*Not Essential, better later*	*Essential, better as now*	*Not Essential, better later*	*Not Essential, better later It is better to delay the quadratic equations to high secondary school*	*Not Essential, better later*	*Essential, better as now*	*Actually, grade 5 student where start teaching fraction doesn't realize the importance of fractions and does not feel he need to learn these difficult operations that introduce for him which cause a real problem for the students and the teacher at the same time*	*Geometry*	*Of course, he will not understand for this reason, he doesn't have enough space to hold the information. We don't leave any space for thinking. We fill their working space… ha…ha (laughing)."*	*Setting (teach them in group according to their level of achievement)*
21 F 9 6/9	*Create mathematician, do not take all levels into consideration*	*Essential, better latter*	*Essential, better as now*	*Not Essential, better latter*	*Not Essential, better latter. There are ten triangle theories, students need to know all these theories and know how to prove these theories and know how to apply these theories to solving many different tasks. It is too much. We just push them to failure*	*Not Essential, better later*	*Not Essential, better latter*	*Essential, better as now*	*Yes, very difficult*	*Triangle theories*	*Maybe, why not students don't have enough space!!*	*Reduce the curriculum. Setting (teach them in group according to their level of achievement*
22 F 25	*The aims just focus on those who love maths*	*Not Essential, better*	*Essential, better as now*	*Essential, better as now*	*Essential, better latter*	*Essential, better as now*	*Essential, better as now*	*Essential, better latter*	*Yes, it is very difficult it should be delay to G6 or 7*	*Percentag e& Geometry*	*Yes, it clarifies what happen, they need space to hold*	*Give the student the freedom to choose whether*

6	*and neglect other who hate it.*	*latter (require a lot from the students)*						*(G8)*			*information (maths topic require more space than this)*	*study maths or no (before the age of 17). Concern about students attitude towards maths*

	Q3	Q4a	Q4b	Q4c	Q4d	Q4e	Q4f	Q4g	Q5	Q6	Q7	Q8
23 F 12 7/9	*Create mathematician, focus on high achievement. Many of the topics just wasting time (useless) even for the high achievement*	*Essential, better latter It is better to teach fraction in stages, start with addition and subtraction of fraction and in the following year teach them the multiplication ad division of the fraction, to avoid the ambiguity of fraction.*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Not Essential, better latter*	*Essential, better as now*	*Of course, we introduce the topic without the student feeling of readiness*	*Proving theories*	*"Am… proving theories need more than this space, for that it is difficult. Maybe why not? It explains, with all these information, the student can't cope*	*Reduce the curriculum, use calculator in maths classes because in realty they use it in their life and in other subject where they need maths*
24 F 7 7/8	*Help to create mathematical sense and take students level in consideration*	*Essential, better as now*	*Essential, better as now*	*Not Essential, better latter*	*Essential, better as now*	*Essential, better as now*	*Not Essential, better latter*	*Essential, better as now*	*Actually the problem of maths because the huge amount of ideas (full cream*	*Geometry*	*I think so*	*Focus on basic in primary level until they handle them (+, -, x…)*

E- 11

									curriculum)			
25 F 9 6/8	Create mathematician	Essential, better latter	Not Essential, better latter	Essential, better as now	Not Essential, better latter. What is the benefit of teaching the low achievement students such a difficult topic	Not Essential, better latter	Essential, better as now	Essential, better as now	It real problem and the students don't realize the importance of fractions at that age	Fractions	I feel this theory is right and explain why we suffer with them.	Focus on basic in primary level until they handle them (+, -, x…) Reduce the curriculum

	Q3	Q4a	Q4b	Q4c	Q4d	Q4e	Q4f	Q4g	Q5	Q6	Q7	Q8
26 M 20 I	The objectives aim to help the students to create mathematical sense about the world around them, and help them to improve their scientific thinking	Essential, better as now	Essential, better as now	Essential, better as now, This topic can provide the enjoyment in mathematics classes and one of the topics which shows that the syllabuses take the low	Essential, better as now	Essential, better as now	Essential, better as now	Essential, better as now	No, it introduces in the proper time	Geometry & proving theories	Am…I don't think this is the only reason of the difficulty. I think the nature of mathematics and students' attitudes towards it play a major rule	Use the computer in teaching maths

				achievement in consideration								
27 M 19 I	*The objectives aim to help the students to create mathematics sense*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now (classification)*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*No, I don't think so. I just introduce in the proper time*	*proving theories in Geometry*	*It is play a role and there is another reasons (student attitude)*	*Change the teaching ways in some topics Provide assistant teacher (adult to help the math teacher)*
28 F 14 I	*The objectives aim to help the students to create mathematics sense and to create positive attitudes towards mathematics.*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now. (for low achievement)*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*it introduces in the proper time*	*Fractions*	*may be, because some tasks*	*Relate maths to the real life. Create a motivation to learn mathematics by attractive teaching,*
29 F 27 I	*help the students to create mathematical sense about the world around them, and help them to improve their scientific thinking*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*Essential, better as now*	*it doesn't cause any problem=, it introduces in the proper time*	*proving theories in Geometry*	*It may explain some part of the real problem but not the whole of it. The problems of geometry arise from its hierarchical nature of it. Students need to retrieve all the information that they have studied before*	*Improve teaching ways , use concrete aids in teaching*

Appendix F

Statistical Analysis

The Chi-square Test (χ^2)

The chi-square test is said to to be one of the most widely used tests for statistical data generated by non-parametric analysis. There are two different of applications of chi-square test.

(1) Goodness of Fit Test

This tests how well the experimental (sampling) distribution fits the control (hypothesised) distribution. An example of this could be a comparison between a group of experimentally observed responses to a group of control responses. For example,

Positive	**Neutral**	**Negative**	**Negative**	
Experimental	55	95	23	N(experimental) = 173
Control	34	100	43	N(control) = 177
				(using raw numbers)

A calculation of observed and expected frequencies lead to

	Positive	**Neutral**	**Negative**
fo = observed frequency	*55*	95	23
fe = expected frequency	33	97	42

Where *fe* = [N(experimental)/N(control)] X (control data) or (173/177) X (control data)

The degree of freedom (df) for this comparison is 2. This comparison is significant at two degrees of freedom at greater than 1%. (χ^2 critical at 1% level = 9.21)

(2) Contingency Test

This chi-square test is commonly used in analysing data where two groups or variables are compared. Each of the variable may have two or more categories which are independent from each other. The data for this comparison is generated from the frequencies in the categories. In this study, the chi-square as a contingency test was used, for example, to compare two or more independent samples like, year groups, gender, or ages. The data is generated from one population group. For example,

	Positive	Neutral	Negative
Male (experimental)	55	95	23
Female (experimental)	34	100	43

(Actual data above)

	Positive	Neutral	Negative	N
Male (experimental)	55 (44)	95 (96)	23 (33)	173
Female (experimental)	34 (45)	100 (97)	43 (33)	177
Totals	89	195	66	350

(Expected frequencies above in brackets)

The expected frequencies are shown in brackets ( ), and are calculated as follows:

e.g. 44 = (173/350) x 89

$$\chi^2 = 2.75 + 0.01 + 3.03 + 2.69 + 0.09 + 3.03$$
$$= 11.60$$

At two degrees of freedom, this is significant at 1%. (χ^2 critical at 1% level = 9.21)
The degree of freedom (df) must be stated for any calculated chi-square value. The value of the degree of freedom for any analysis is obtained from the following calculations:

df = (r-1) x (c-1)

Where r is the number of rows and c is the number of columns in the contingency table.

Limitations on the Use of χ^2

It is known that when values within a category are small, there is a chance that the calculation of χ^2 may occasionally produce inflated results which may lead to wrong interpretations. It is safe to impose a 10 or 5% limit on all categories. When the category falls below either of these, then categories are grouped and the df falls accordingly.

Correlation

It frequently happens that two measurements relate to each other: a high value in one is associated with a high value in the other. The extent to which any two measurements are related in this way is shown by calculating the correlation coefficient. There are three ways of calculating a correlation coefficient, depending on the type of measurement:

(a) With integer data (like examination marks), Pearson correlation is used. This assumes an approximately normal distribution.
(b) With ordered data (like examination grades), Spearman correlation is used. This does not assume a normal distribution.
(c) With ordered data where there are only a small number of categories, Kendall's Tau-b correlation used. This does not assume a normal distribution.

Sometimes, the two variables to be related use different types of measurement. In this case, none of the methods is perfect and it is better to use more than one and compare outcomes. It is possible to use a Pearson correlation when one variable is integer and other is dichotomous. The coefficient is now called a point biserial coefficient.

Phase (1) Kendall's tau_b

	Wm	FD	MATH	Q1A	Q1B	Q1C	Q1D	Q1E	Q1F	Q1G	Q1H	Q2A	Q2B	Q2C	Q3A	Q3B	Q4A	Q4B	Q4C	Q4D	Q4E	Q5A	Q5B	Q5C	Q5D
Q1A	.095	.125	.299																						
Q1B	-.064	-.084	-.203	-.222																					
Q1C	.005	.067	.146	.315	-.181																				
Q1D	.045	.180	.355	.477	-.235	.346																			
Q1E	-.035	-.033	-.050	-.011	.029	-.028	-.033																		
Q1F	-.040	-.093	-.063	.029	.058	.072	.006	.129																	
Q1G	.059	.058	.169	.339	-.232	.323	.330	-.072	.124																
Q1H	.005	.007	.106	.323	-.162	.248	.291	-.009	.169	.484															
Q2A	-.042	-.023	.051	.123	-.101	.132	.057	.022	-.002	.062	.097														
Q2B	.017	.066	.075	.129	-.062	.109	.139	-.138	.019	.184	.091	-.034													
Q2C	-.024	.063	.089	.167	-.101	.109	.157	-.095	.033	.136	.127	-.135	.580												
Q3A	-.054	-.053	-.150	-.223	.190	-.267	-.230	.109	-.089	-.428	-.381	-.046	-.133	-.131											
Q3B	.001	-.035	-.005	-.089	.091	-.114	-.094	.023	.066	-.146	-.143	.081	-.014	.001	.448										
Q4A	.056	.165	.258	.380	-.311	.262	.412	-.047	.058	.357	.294	.067	.067	.165	-.298	-.096									
Q4B	-.058	-.167	-.197	-.289	.201	-.192	-.305	.055	-.008	-.214	-.172	-.066	-.038	-.108	.147	.044	-.321								
Q4C	-.075	-.092	-.266	-.379	.266	-.271	-.401	.092	-.021	-.368	-.248	-.094	-.140	-.169	.307	.086	-.389	.401							
Q4D	.079	.176	.342	.420	-.275	.290	.444	-.060	.019	.343	.256	.056	.111	.163	-.276	-.096	.520	-.371	-.557						
Q4E	-.059	-.091	-.229	-.375	.342	-.355	-.392	.019	-.072	-.403	-.346	-.105	-.137	-.191	.371	.150	-.464	.377	.487	-.406					
Q5A	-.092	-.148	-.276	-.286	.184	-.293	-.295	.046	-.014	-.220	-.217	-.088	-.073	-.092	.296	.139	-.285	.264	.296	-.314	.411				
Q5B	.019	-.009	.098	.206	-.107	.162	.133	-.033	-.040	.206	.181	.017	.104	.123	-.159	-.100	.168	-.114	-.144	.164	-.240	-.245			
Q5C	.048	.128	.253	.260	-.202	.223	.274	-.031	-.001	.187	.146	.013	.070	.102	-.200	-.119	.255	-.147	-.175	.254	-.274	-.380	.305		
Q5D	-.049	-.032	-.117	-.146	.120	-.187	-.190	.094	-.048	-.186	-.138	-.037	-.173	-.149	.193	.073	-.164	.164	.211	-.153	.268	.287	-.303	-.298	
Q5E	.017	-.071	-.020	-.050	.036	-.057	-.068	.056	.040	-.074	-.018	-.034	-.021	-.023	.045	.013	-.027	.038	.030	-.046	.115	.107	-.035	-.104	.052
Q5F	.054	.017	.035	-.081	.025	-.032	-.016	.003	-.086	-.033	-.137	.013	-.064	-.098	.156	.113	-.018	.001	-.029	.041	.081	.096	-.070	-.082	.079
Q5G	-.074	-.010	-.026	.091	-.056	.054	.079	.045	.050	.085	.079	-.014	-.006	.056	-.130	-.072	.084	.017	.007	.053	-.076	-.110	.106	.124	-.049
Q5H	.040	.090	-.001	-.021	.035	-.018	-.058	-.111	-.085	-.049	-.035	.007	.003	-.016	.049	.040	-.004	.039	.057	.008	.070	.046	-.053	-.022	.100
Q6A	-.105	-.102	-.212	-.307	.249	-.231	-.343	.062	-.058	-.325	-.263	-.015	-.093	-.159	.234	.085	-.356	.470	.402	-.373	.427	.308	-.133	-.209	.170
Q6B	-.060	-.165	-.262	-.406	.268	-.304	-.433	.042	-.006	-.366	-.335	-.071	-.106	-.157	.324	.126	-.473	.349	.435	-.446	.547	.445	-.196	-.287	.235
Q6C	-.024	-.026	-.088	-.203	.174	-.263	-.233	.060	-.102	-.363	-.405	-.075	-.085	-.117	.425	.168	-.274	.188	.298	-.268	.337	.175	-.130	-.174	.178
Q6D	-.045	-.088	-.240	-.333	.351	-.313	-.406	.013	-.042	-.400	-.370	-.102	-.078	-.163	.398	.150	-.481	.355	.443	-.429	.694	.426	-.240	-.318	.249
Q6E	-.016	.006	-.117	-.174	.180	-.247	-.254	.051	-.092	-.335	-.366	-.033	-.069	-.126	.392	.181	-.266	.152	.312	-.275	.324	.218	-.122	-.136	.158
Q6F	-.055	-.136	-.205	-.370	.287	-.338	-.397	.009	-.010	-.355	-.304	-.098	-.095	-.147	.357	.117	-.459	.335	.370	-.386	.588	.378	-.167	-.264	.224
Q7A	-.106	-.151	-.203	-.301	.254	-.141	-.259	.059	-.005	-.273	-.167	-.004	-.124	-.135	.199	.114	-.307	.225	.287	-.286	.289	.135	-.146	-.191	.116
Q7B	.064	-.048	-.104	-.045	.100	-.064	-.071	.029	.031	-.124	-.090	.026	-.041	-.021	.194	.090	-.099	.067	.112	-.095	.133	.102	-.060	-.122	.077
Q7C	-.050	-.156	-.254	-.274	.310	-.232	-.247	.061	.030	-.258	-.169	-.111	-.020	-.064	.210	.097	-.346	.286	.341	-.367	.374	.217	-.173	-.140	.177
Q7D	-.031	-.168	-.314	-.357	.280	-.215	-.343	.110	-.006	-.309	-.229	-.007	-.150	-.219	.268	.103	-.360	.298	.374	-.345	.404	.247	-.146	-.206	.181
Q7E	-.093	-.174	-.322	-.386	.204	-.241	-.361	.102	.058	-.275	-.194	-.048	-.113	-.131	.187	.035	-.380	.261	.307	-.393	.296	.225	-.177	-.200	.168
Q7F	-.048	-.034	-.125	-.148	.138	-.061	-.108	.108	-.026	-.185	-.147	.012	-.085	-.081	.161	.092	-.139	.071	.160	-.127	.208	.205	-.121	-.184	.086

Q8A	-.028	-.075	-.173	-.280	.224	-.177	-.293	.056	.040	-.233	-.173	-.046	-.081	-.133	.164	.082	-.339	.200	.257	-.280	.319	.149	-.122	-.130	.092
Q8B	-.063	-.181	-.233	-.251	.146	-.096	-.279	.062	.157	-.137	-.069	-.005	-.120	-.106	.133	.093	-.218	.167	.195	-.245	.204	.195	-.107	-.185	.084
Q8C	-.114	-.024	-.171	-.207	.146	-.140	-.175	.090	-.002	-.190	-.188	.007	-.094	-.078	.142	.083	-.204	.134	.213	-.190	.244	.168	-.097	-.117	.096
Q8D	-.113	-.187	-.301	-.329	.186	-.209	-.358	.094	.028	-.196	-.187	-.031	-.117	-.138	.155	.039	-.377	.221	.318	-.405	.313	.196	-.179	-.191	.125
Q8E	-.108	-.188	-.277	-.350	.212	-.241	-.347	.062	.012	-.259	-.158	-.070	-.136	-.188	.187	.061	-.332	.258	.387	-.396	.335	.241	-.167	-.215	.146
Q8F	.006	-.096	-.125	-.137	.154	-.089	-.126	.055	-.016	-.113	-.111	-.044	-.118	-.082	.102	.053	-.138	.125	.140	-.127	.190	.110	-.037	-.145	.092

Phase (1) Kendall's tau_b

	Q5G	Q5H	Q6A	Q6B	Q6C	Q6D	Q6E	Q6F	Q7A	Q7B	Q7C	Q7D	Q7E	Q7F	Q8A	Q8B	Q8C	Q8D	Q8E
Q5F																			
Q5G																			
Q5H	-.115																		
Q6A	-.001	.032																	
Q6B	-.069	.040	.464																
Q6C	-.029	.022	.249	.301															
Q6D	-.084	.041	.468	.609	.381														
Q6E	-.059	.043	.269	.280	.526	.352													
Q6F	-.096	.063	.464	.593	.306	.623	.316												
Q7A	-.075	-.018	.293	.306	.141	.328	.122	.289											
Q7B	.018	.063	.060	.098	.184	.165	.123	.124	.162										
Q7C	.022	.027	.263	.320	.189	.341	.219	.303	.308	.179									
Q7D	-.025	-.027	.365	.400	.252	.429	.228	.347	.353	.149	.380								
Q7E	-.070	-.029	.288	.361	.184	.307	.172	.280	.311	.149	.360	.423							
Q7F	-.051	.069	.181	.224	.124	.198	.115	.134	.153	.060	.185	.185	.150						
Q8A	-.050	.000	.300	.359	.134	.348	.135	.345	.269	.128	.263	.334	.341	.145					
Q8B	-.072	.015	.120	.256	.063	.180	.081	.235	.195	.131	.232	.277	.330	.179	.350				
Q8C	-.053	.034	.212	.214	.167	.233	.203	.216	.205	.073	.235	.281	.241	.224	.299	.318			
Q8D	-.049	-.029	.305	.366	.148	.351	.166	.345	.340	.118	.348	.375	.460	.195	.374	.347	.290		
Q8E	-.042	-.001	.306	.362	.194	.354	.177	.328	.339	.098	.305	.421	.431	.182	.389	.342	.274	.503	
Q8F	-.010	.054	.096	.129	.115	.159	.098	.153	.140	.135	.281	.256	.173	.190	.216	.255	.235	.218	.193

Correlation is significant at the 0.01 level (2-tailed).

Phase (2): Correlations Kendall's tau_b

Grade Eight

Correlation is significant at the 0.01 level (2-tailed).

	Wm	FD	Q7A	Q7B	Q7C	Q7D	Q7E	Q7F	Q8A	Q8B	Q9A	Q9B	Q9C	Q9D	Q9E	Q9F	Q9G	Q9H	Q10A	Q10B	Q10C	Q10D	Q10E	Q10F
FD	.091																							
Q7A	.035	.009																						
Q7B	-.008	.010	.586																					
Q7C	.031	.036	.107	.140																				
Q7D	.032	.029	.279	.325	.043																			
Q7E	.007	.034	.248	.249	.248	.289																		
Q7F	-.023	.011	-.049	-.045	.137	.135	.166																	
Q8A	-.041	-.026	.083	.023	.075	.110	.003	.054																
Q8B	-.043	.006	.010	.058	-.019	.024	.009	-.025	.244															
Q9A	-.002	-.075	-.074	-.058	.069	-.106	.054	.106	.085	-.006														
Q9B	.022	.110	.115	.158	.145	.079	.085	-.097	.028	.064	-.008													
Q9C	-.044	-.180	.128	.131	-.004	.105	.096	.017	.063	.019	.104	.008												
Q9D	-.001	-.121	.059	.059	-.051	.097	.024	.112	.058	-.013	.107	-.164	.269											
Q9E	-.043	-.088	-.186	-.142	-.051	-.049	-.091	.120	.058	-.013	.186	-.185	.161	.336										
Q9F	-.019	-.046	.218	.242	.015	.131	.105	-.035	.059	-.068	.001	.042	.222	.206	.105									
Q9G	.030	.021	.187	.153	.165	.092	.150	-.007	.032	.020	.028	.304	-.039	-.088	-.250	.015								
Q9H	-.055	-.019	-.119	-.058	.007	-.041	-.087	.113	.082	.050	.165	-.140	.159	.292	.524	.167	-.363							
Q10A	.021	.084	.219	.173	.084	.064	.123	-.082	-.068	-.021	-.038	.193	-.005	-.102	-.219	.093	.337	-.224						
Q10B	.046	.076	.228	.214	.097	.107	.001	-.021	.029	-.002	.026	.185	.044	-.059	-.212	.062	.143	-.170	.325					
Q10C	.059	.031	.327	.224	.107	.049	.111	-.050	.005	-.034	.001	.173	.067	-.074	-.231	.175	.243	-.197	.388	.338				
Q10D	.013	.061	.261	.240	.080	.134	.092	-.064	-.046	-.094	-.065	.130	.052	-.008	-.141	.131	.168	-.140	.342	.500	.358			
Q10E	-.007	-.029	.217	.209	.050	.057	.073	-.032	.015	-.033	.021	.170	.076	.009	-.136	.170	.202	-.125	.337	.364	.355	.524		
Q10F	-.012	.101	.129	.100	.178	.061	.126	-.061	.007	.037	-.016	.220	-.081	-.190	-.224	.008	.350	-.264	.387	.279	.333	.258	.248	
Q10G	.013	.108	.076	.068	.116	.004	.094	-.075	-.027	.059	-.025	.213	-.046	-.212	-.226	-.026	.311	-.213	.395	.161	.245	.160	.212	.515

Grade Nine

	Wm	FD	Q7A	Q7B	Q7C	Q7D	Q7E	Q7F	Q8A	Q8B	Q9A	Q9B	Q9C	Q9D	Q9E	Q9F	Q9G	Q9H	Q10A	Q10B	Q10C	Q10D	Q10E	Q10F
FD	.206																							
Q7A	.117	.046																						
Q7B	.077	.081	.490																					
Q7C	-.051	.068	-.065	.044																				
Q7D	.035	.021	.142	.320	.104																			
Q7E	-.006	.036	.061	.235	.329	.254																		
Q7F	-.003	.013	-.131	-.057	.123	.101	.141																	
Q8A	.107	.023	.101	.124	.067	.113	.111	.065																
Q8B	.011	.037	.022	.071	.026	.179	.051	.057	.282															
Q9A	-.005	-.071	-.091	-.085	.070	.052	.087	.116	.053	.046														
Q9B	.038	.185	.083	.148	.268	.121	.198	-.007	.075	.070	-.024													
Q9C	-.072	.015	-.037	.028	-.027	.147	.029	-.014	-.007	.022	.007	-.035												
Q9D	.016	-.069	-.063	-.087	-.189	-.027	-.127	.140	.038	.067	.097	-.171	.255											
Q9E	-.041	-.082	-.159	-.166	-.153	-.037	-.093	.198	-.026	-.022	.181	-.262	.188	.455										
Q9F	.040	.038	.044	.050	-.124	.080	-.021	-.017	.089	.112	-.059	.011	.134	.197	.191									
Q9G	.030	.060	.092	.142	.231	.117	.190	-.006	.040	.040	.011	.408	-.079	-.179	-.234	.024								
Q9H	-.026	-.033	-.086	-.075	-.144	.025	-.057	.104	.051	.074	.074	-.183	.185	.383	.430	.250	-.346							
Q10A	.133	.190	.199	.206	.223	.104	.169	-.040	.048	.040	-.013	.298	-.081	-.202	-.270	-.049	.345	-.325						
Q10B	.064	.130	.158	.136	.118	.114	.071	-.046	.014	.000	-.035	.172	.001	-.117	-.133	.049	.187	-.177	.367					
Q10C	.063	.091	.219	.284	.132	.094	.177	-.040	.050	.085	.017	.222	.017	-.105	-.218	.007	.269	-.200	.442	.321				
Q10D	.011	.106	.120	.129	.140	.092	.133	-.006	.095	.047	.005	.146	.052	-.061	-.069	.057	.217	-.093	.343	.478	.365			
Q10E	.080	.111	.119	.155	.132	.084	.127	.044	.039	.007	-.020	.190	.008	-.059	-.120	-.014	.166	-.123	.347	.422	.357	.489		
Q10F	.062	.123	.119	.186	.262	.064	.210	-.058	.065	.037	-.064	.279	-.039	-.207	-.287	-.044	.322	-.326	.485	.268	.419	.276	.235	
Q10G	.029	.091	.170	.209	.195	.078	.217	-.052	.068	.029	.010	.302	-.046	-.185	-.247	-.007	.329	-.303	.462	.248	.385	.205	.216	.596

Regression Statistics (Phase 1)

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' working memory space*

R	R Square	Adjusted R Square	Std. Error of the Estimate
.231(a)	.054	.052	17.219

	Sum of Squares	df	Mean Square	F	Sig.
Regression	7878.743	1	7878.743	26.574	.000(a)
Residual	139347.697	470	296.484		
Total	147226.441	471			

Model	Coefficients		Standardized Coefficients	T Stat
	B	Std. Error	Beta	
Mathematics Performance	51.009	3.076		16.583
X-space	3.022	.586	.231	5.155

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' Field dependence/independence*

R	R Square	Adjusted R Square	Std. Error of the Estimate
.431(a)	.185	.184	15.975

	Sum of Squares	df	Mean Square	F	Sig.
Regression	27286.324	1	27286.324	106.925	.000(a)
Residual	119940.117	470	255.192		
Total	147226.441	471			

Model	Coefficients		Standardized Coefficients	T Stat
	B	Std. Error	Beta	
Mathematics Performance	55.428	1.285		43.120
FDI	2.473	.239	.431	10.340

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' working memory space*
: *Students' Field dependence/independence*

R	R Square	Adjusted R Square	Std. Error of the Estimate
.456(a)	.208	.205	15.767

	Sum of Squares	df	Mean Square	F	Sig.
Regression	30639.208	2	15319.604	61.627	.000
Residual	116587.232	469	248.587		
Total	147226.441	471			

Model	Coefficients		Standardized Coefficients	T Stat
	B	Std. Error	Beta	
Mathematics Performance	45.994	2.865		16.054
X-space	2.009	.547	.154	3.673
FDI	2.302	.241	.401	9.569

Regression Statistics (Phase 2)

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' working memory space*

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.200(a)	.040	.039	18.6155

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	12101.231	1	12101.231	34.920	.000
	Residual	289359.242	835	346.538		
	Total	301460.473	836			

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta	B	Std. Error
Mathematics Performance	42.325	4.331		9.772	.000
X-space	4.979	.843	.200	5.909	.000

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' Field dependence/independence*

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.331	.110	.109	17.7984

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	32442.688	1	32442.688	102.413	.000
	Residual	263563.829	832	316.783		
	Total	296006.517	833			

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta	B	Std. Error
Mathematics Performance	46.248	2.213		20.894	.000
FDI	4.718	.466	.331	10.120	.000

Dependent Variable: *Students' performance in mathematics*

Independent Variable: *Students' working memory space*

: *Students' Field dependence/independence*

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.360(a)	.130	.128	17.6062

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	38415.126	2	19207.563	61.964	.000
Residual	257591.391	831	309.978		
Total	296006.517	833			

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta	B	Std. Error
Mathematics Performance	29.867	4.327		6.903	.000
X-space	3.564	.812	.145	4.389	.000
FDI	4.336	.469	.304	9.239	.000

Factor Analysis

Communalities

	Initial	Extraction
Working memory	1.000	.975
FD	1.000	.690
Mathematics performance	1.000	.560
Mathematics test	1.000	.765

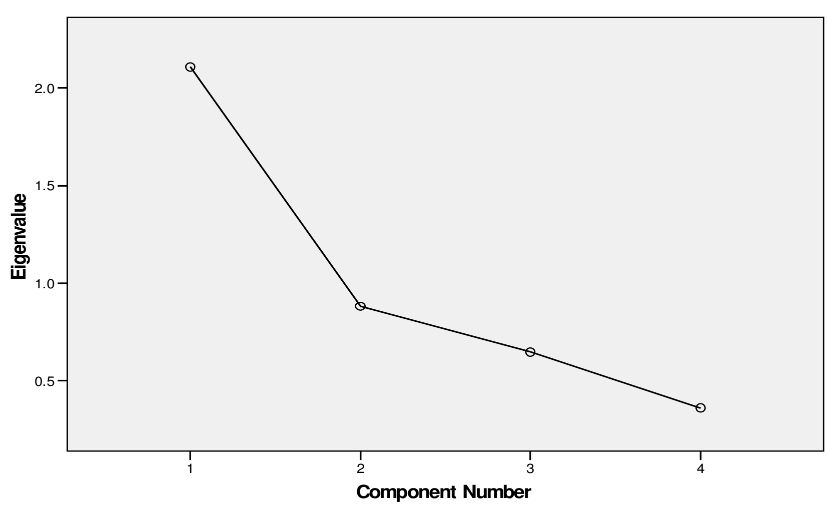
Extraction Method: Principal Component Analysis.

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.108	52.702	52.702	2.108	52.702	52.702	1.901	47.518	47.518
2	.882	22.048	74.750	.882	22.048	74.750	1.089	27.232	74.750
3	.648	16.211	90.961						
4	.362	9.039	100.000						

Extraction Method: Principal Component Analysis.

Scree Plot



Component Matrix(a)

	Component

	1	2
Working memory	.511	.845
FD	.750	-.356
Mathematics performance	.721	-.203
Mathematics test	.874	-.021

Extraction Method: Principal Component Analysis.
a 2 components extracted.

Rotated Component Matrix(a)

	Component	
	1	2
Working memory	.119	.980
FD	.830	-.016
Mathematics performance	.740	.111
Mathematics test	.806	.341

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a Rotation converged in 3 iterations.

Component Transformation Matrix

Component	1	2
1	.912	.411
2	-.411	.912

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Kuwait University Entries

KUWAIT UNIVERSITY
Admissions & Registration

جامعة الكويت
عمادة القبول والتسجيل

Date: 14 FEB 2008

الرقم : ٠٣٨٦

الأستاذ الدكتور / جمال فاخر النكاس المحترم

القائم بأعمال العميد

كلية الحقوق

تحية طيبة،

بالإشارة إلى كتابكم بتاريخ 2008/2/10 بشأن تزويدكم بأعداد الطلبة المقبولين بالجامعة منذ العام الجامعي 2001/2000 إلى العام الجامعي 2008/2007 والطلبة المقبولين بكلية العلوم قسم الرياضيات، تجدون طيه البيانات المطلوبة.

شاكرين لكم حسن تعاونكم، وتفضلوا بقبول فائق الاحترام ،،،

القائم بأعمال عميد القبول والتسجيل

د.شفيقة عبدالحميد العوضي

ت : (٠٠٩٦٥) ٤٨٤٢٠٤٠/ ٤٨٤٢٠٧٩/ ٤٨٤٢٦١٧ / ٤٨٤٢٧٠٥ – ص. ب ١٩٨٣ الصفاة، الرمز البريدي : 13020 الكويت
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أعداد الطلبة المقبولين بالجامعة للعام الجامعي 2000/2001 - 2007/2008

العام الجامعي	عدد المقبولين
2001/2000	4456
2002/2001	3745
2003/2002	3968
2004/2003	3831
2005/2004	4540
2006/2005	4843
2007/2006	5544
2008/2007	5229

أعداد الطلبة المقبولين في كلية العلوم -قسم الرياضيات من العام الجامعي 2000/2001 - 2007/2008

[illegible]	[illegible]
الاول 2001/2000	1
الثانى 2001/2000	1
الصيفى 2001/2000	1
الأول 2002/2001	5
الثاني 2002/2001	8
الأول 2003/2002	12
الثاني 2003/2002	3
الأول 2004/2003	3
الثاني 2004/2003	9
الأول 2005/2004	4
الثاني 2005/2004	5
الأول 2006/2005	8
الثاني 2006/2005	6
الصيفي 2006/2005	2
الأول 2007/2006	6
الثاني 2007/2006	6
الأول 2008/2007	10