Essays on optimal fiscal policy

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Abstract

This thesis examines the properties of optimal fiscal policy in the long-run and over the business cycle in general equilibrium models with agents that differ with respect to their skills and with production processes embodying capital-skill complementarity. To this end, the thesis is composed of four chapters which asses different aspects of optimal fiscal policy under various specifications incorporating labour skill and production differences as well as different assumptions regarding the policy instruments available to the government. The first two chapters focus on the long-run, while the last two concentrate on business cycle dynamics. The first and third chapters examine setups that allow households to differ with respect to their income and whose position in the labour market with respect to their skill is exogenously determined. In contrast, the second and fourth chapters consider setups where the labour force belongs to a single household, which guarantees consumption irrespective of skill level, and unskilled labour can endogenously acquire skills to become skilled.

Chapter 1 presents a detailed numerical analysis of the effects of optimal fiscal policy in an economy where the households are heterogeneous with respect to their labour and capital income. The production structure is characterised by a CES function allowing for capital, skilled and unskilled labour. In this setup, optimal fiscal policy in the long-run implies a non-zero and positive tax rate on capital income together with highly progressive labour income taxes. Moreover, the level of the optimal tax rate on capital income and the progressivity of labour income taxes are sensitive to the weight placed on the skilled agents in the objective function of the government.

Chapter 2 analyses optimal factor income taxation when there are different returns to skilled and unskilled workers, who belong to the same household, and to capital in structures and equipment, under capital-skill complementarity and endogenous skill acquisition. We find that when all factor inputs are taxed at separate rates, both capital income taxes are zero in the long-run, there is a subsidy to education and labour income taxes are progressive. The progressivity in labour income taxes is reduced if investment in

education cannot be subsidised, whereas if the government can only impose a single labour income tax, the tax on income from capital equipment will be non-zero. These results remain valid even if the government is restricted to satisfy a given level of debt to output ratio, although with welfare losses. Finally, we show that the transitional dynamics of the fiscal instruments from the exogenous to optimal taxation are not affected by the restrictions to the fiscal policy menu.

Chapter 3 examines how income taxes are optimally distributed over the business cycle in a model with high, middle and low income households when the government is restricted to balance its budget in each period. The findings of an empirically relevant model indicate that under optimal fiscal policy the income tax rate of the high income households has the lowest volatility and the income tax rate of the low income households exhibits the lowest counter-cyclicality. If the fiscal policy menu also includes a consumption tax, the progressivity of the income tax rates is even higher and the results regarding the volatilities of the income taxes are overturned. We further find that the progressivity of the income tax rates is optimally increased after an output-enhancing shock.

Chapter 4 undertakes a normative investigation of the quantitative properties of optimal tax smoothing in a business cycle model with state contingent debt, capital-skill complementarity, endogenous skill formation and stochastic shocks to public consumption, as well as total factor and capital equipment productivity. We also examine the properties of optimal taxation under a restriction on the debt to output ratio. Our main finding is that, an empirically relevant restriction which does not allow the relative supply of skilled labour to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. This result remains valid even in the presence of a budget rule that restricts the debt to output ratio. We show that the key to understanding this result is that the government finds it optimal to adjust labour income tax rates to alter the average net returns to skilled and unskilled labour hours.

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Dedicated to my wife.

"A person who never made mistakes never tried something new."
Albert Einstein

"A likely impossibility is always preferable to an unconvincing possibility."
Aristotle

"You have enemies? Good. That means you've stood up for something, sometime in your life."
Whiston Churchill

Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

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Preface

This thesis examines optimal fiscal policy in general equilibrium models with agents that differ regarding their skills and with capital-skill complementarity in the production process. The importance of the skill premium (wage inequality) and capital-skill complementarity is well documented in the literature. For example, Goldin and Katz (2008) have emphasized that wage inequality since 1980 has increased to levels not seen since 1910 and that capital-skill complementarity is an accurate way to characterize production over the 20th century (see also Hornstein *et al.* (2005) and Krusell *et al.* (2000)). Each chapter aims to contribute to a particular question in the optimal fiscal policy literature. Chapter 1 examines the optimal long-run value of capital income tax in a model with capital-skill complementarity and households that differ regarding their skill and their holdings of capital. Chapter 2 assesses optimal fiscal policy under restrictions to fiscal policy menu when there is endogenous skill acquisition. Chapter 3 examines optimal income taxation over the business cycle under an empirically relevant model with three types of households and two types of labour. Finally, Chapter 4 introduces state contingent debt and endogenous skill formation to assess optimal tax smoothing over the business cycle.

The first chapter extends Judd's (1985) model by assuming that households are heterogeneous regarding their labour skills, (i.e. there are skilled and unskilled workers). Following Krusell *et al.* (2000), the production function incorporates capital-skill complementarity. Although both types of households can save, following the literature on income inequality (see e.g. Aghion and Howitt, 1998) it is further assumed that capital market imperfections exist due to intermediation costs in capital transactions and that these differ for the two types of households. This introduces an additional source of heterogeneity between the two types of agents, in the form of differences in capital holdings.

We find that under optimal fiscal policy a government that wants to maximize the aggregate welfare of the economy should impose a positive optimal capital income tax rate together with progressive labour income tax rates. In this way, the government is able to redistribute income efficiently. We also find that imperfect capital markets are the main driving force for the positive optimal tax rate on capital income. However, when capital-skill complementarity is present and the government cannot impose two different labour income tax rates, the optimal capital income tax is positive even with perfect capital markets. Since skilled labour and capital are complements, a lower stock of capital, due to the positive capital income tax rate, reduces the demand for skilled labour and so lowers their pre-tax wage rate. Also, since unskilled labour are substitutes for capital and skilled labour, its wage rate increases causing a reduction in income inequality. These results are in line with the findings in Correia (1996) which state that under an incomplete set of tax instruments, the optimal capital income tax is non-zero in the long-run. Moreover, the results of this chapter are also consistent with the results of Judd (1997) and Guo and Lansing (1999) which show that when there is an imperfection in capital and/or labour markets then the zero capital income tax may not be obtained in the long-run.

The first chapter complements the literature of optimal fiscal policy by showing that when agents face different costs in accessing the capital market and when capital-skill complementarity is present, the optimal tax rate on capital income is positive and labour income taxation is progressive for a reasonable calibration. In addition, optimal fiscal policy is Pareto efficient, leading to higher welfare for each type of agent compared with the exogenous fiscal policy case. These results maintain when the government optimally chooses the level of consumption together with the tax rates, as in Judd (1985). Moreover, the optimal capital income tax rate remains non-zero even if the government is not Utilitarian, as in Chari and Kehoe (1999).

In the second chapter, we examine optimal factor income taxation in an environment with different skilled and unskilled labour services, endogenous skill creation, and production exhibiting capital-skill complementarity. We work with a representative agent framework, which allows us to focus on aggregate efficiency and abstract from potential equity considerations for optimal taxation. We assume that a representative household decides how to allocate its investment in the two types of capital stock and in creating skilled labour within the same period. Moreover, the representative household decides how to allocate its time endowment into leisure, labour supply in skilled and unskilled jobs, and in creating skill labour. Therefore, the model allows for endogenous skill acquisition. In this framework, we derive optimal tax policy under different scenarios regarding the policy menu available to the government and, in particular, which tax instruments are available as well as whether there are restrictions on issuing debt.

Our findings indicate that when the government can issue debt and can tax the different types of labour and capital income, as well as investment in education, at separate rates: (i) both capital income taxes are zero in the long-run; and (ii) there is a subsidy to education; and (iii) labour income taxation is progressive. This optimal policy results in a minor reduction in the skill premium compared with the data average. These results remain the same if the government can use a single tax for income from capital in structures and equipment.

When fiscal policy menu is restricted with respect to access to an education subsidy we find that: (i) the progressivity in labour income taxation falls relative to the benchmark case; and (ii) capital income taxation is still zero. However, when the government has access to education subsidy but cannot tax income from skilled and unskilled labour separately, we find that while the tax on income from structures remains zero in the long-run, there is a small positive tax on equipment capital. Finally, if the government can only implement a single labour income tax, without having access to education subsidies, the equipment tax becomes again positive but at a lower rate compared to the case with education subsidy and a single labour income tax.

The transition paths of the policy instruments from the exogenous fiscal policy to optimal fiscal policy regime are qualitatively similar in each case that we study. Our optimal policy findings are also similar if we restrict government debt by imposing a budget rule that requires that the debt to output ratio remains fixed at the data average. The restriction does imply, however, a reduction in the progressivity of optimal labour income taxes.

In the third chapter we develop a model with three types of households that are divided with respect to their income into low, middle and high. In addition, we have two labour markets, for skilled and unskilled labour, and we further assume that there are barriers that prevent agents from participating in both labour markets. In particular, we assume that high income households provide skilled labour, where skilled agents are those with a college degree or relevant professional qualification. The middle and low income households are assumed to provide unskilled labour, i.e. those without a college degree. Following Katz and Murphy (1992) and Krusell *et al.* (2000), we assume that the skill premium is driven by skill-biased technical change and capital-skill complementarities. Specifically, we assume that the production process follows the technology specified in Krusell *et al.* (2000).

The assumed capital market imperfections in our model imply that households differ with respect to their participation in the asset markets. Following the contributions of Campbell and Mankiw (1989), Mankiw (2000) and Galí *et al.* (2007), we assume that a subset of the households does not have any savings and thus earns only labour income, which it totally consumes. We further assume that these households offer unskilled labour services, so that the three types of households in the economy are defined as, high income skilled agents who own assets, middle income unskilled agents who also own assets and low income unskilled agents who do not have access to the capital market.

Using an empirically relevant model we assess the properties of optimal income tax rates over the business cycle. Moreover, we extend the set of fiscal instruments by allowing the government to optimally choose a consumption tax rate on top of the three income tax rates with the balanced budget restriction. We find that the cyclical properties of the income taxes differ significantly with each other and with those observed in the data. As expected, given the balanced budget restriction and the instruments available to the government, the tax rates are generally more volatile and more counter-cyclical than in the data. The overall counter-cyclicality of the taxes is driven by the balanced budget restriction because under a negative shock to the economy, output decreases and also capital and labour decrease, causing a reduction to labour and capital income and as a result the tax revenues decrease. Thus, the government needs to increase taxation to be able to finance its expenditures.

However, there are also important differences between the tax rates. These result from the trade-off that the government faces when deciding how to distribute the distortions reflected by the higher volatility and counter-cyclicality of the three tax rates over the business cycle. We find that optimal policy resolves this trade-off by keeping the lowest volatility for the tax rate for skilled and the lowest counter-cyclicality for the hand-to-mouth households. In contrast, the middle income group, made up by unskilled households with savings, receives very volatile and very counter-cyclical taxes. For the case where we also introduce the consumption tax we can see that most of the aforementioned results are preserved apart from the volatility of the income taxes. In this case we find that the unskilled agents that are able to save have the most volatile income tax, whereas the hand-to-mouth agents have the smoothest income tax.

We further analyse the optimal distribution of the tax burden in the short- and medium-run in response to temporary output-enhancing exogenous shocks. The government finds it optimal to respond to an increase in the productivity of capital equipment and to public spending cuts by increasing the progressivity of income taxes. The response to a positive total factor productivity (TFP) shock implies that the progressivity of the tax system increases after about two years. Finally, the aforementioned results and behaviour of the income taxes after a temporary shock remain unchanged with the introduction of a consumption tax that is optimally chosen by the government.

The fourth chapter contributes to the tax smoothing literature by focusing on an economy where the labour force is divided into skilled and unskilled workers. In particular, we examine the importance of differences in the complementarity between capital and skilled and unskilled labour as well as the endogenous determination of the relative skill supply for Ramsey tax policy over the business cycle. In contrast to Werning (2007), we focus on aggregate outcomes and abstract from redistribution incentives, by following the literature that examines a division of the labour force into two types of workers. To this end, we work with a representative household which guarantees its members the same level of consumption (see e.g. Arseneau and Chugh (2012)). We thus stay as close as possible to the representative agent Ramsey analysis of Chari *et al.* (1994) and extend their model to allow for capital-skill complementarity and endogenous skill formation.

The purpose of this chapter is to undertake a normative investigation of the quantitative properties of optimal taxation of capital and labour income, as well as skill-acquisition expenditure, in the presence of aggregate shocks to total factor productivity, capital equipment productivity and government spending. We further assume complete asset markets. However, to capture the importance of endogenous versus fixed relative skill supply, we also consider a labour market distortion that restricts the ratio of skilled to total workers to remain constant. Moreover, in our setup, the government can borrow by issuing state-contingent debt, tax skill acquisition expenditure, capital, skilled and unskilled labour income separately, to finance exogenous public spending.

Our main finding is that under capital-skill complementarity, a friction that does not allow the relative supply of skill to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. In particular, we first show that under endogenous relative skill supply, the optimal labour taxes for both skilled and unskilled labour income are very smooth, with the volatility of the unskilled income tax being marginally higher. We also find that the skilled tax moves pro-cyclically with output and the unskilled tax is mildly counter-cyclical.

However, when the relative skill supply is constrained to remain constant over the business cycle, the prescriptions for optimal policy markedly change. In particular, we find that the volatility of taxes increases significantly, so that the standard deviation of the effective average labour income tax is about twelve times higher than the perfect labour markets case, while the volatility of the skilled labour income tax is about two-and-a-half times higher than that of the unskilled labour income tax. Moreover, both taxes become strongly counter-cyclical. We show that these changes are driven from the fact that the government finds it optimal to minimise the effects of the relative labour supply distortion by keeping the marginal rates of substitution between leisure and consumption for the two types of labour at roughly the same levels as under a fully flexible labour market.

Our results further show that the skill heterogeneity considered, irrespective of the presence of the labour market friction, does not affect the results obtained in the literature regarding the cyclical behaviour of asset taxes. We also find that the skill-acquisition tax is the least smooth of the non-asset tax instruments when debt is state-contingent and fluctuates nearly as much as output. In addition, irrespective of the model variant examined, all of the policy instruments, except for the *ex post* capital tax and the private assets tax inherit the persistence properties of the shocks. Finally, we find that our main results are robust to the introduction of a budget rule, where the government must satisfy a given level of debt to output ratio over the business cycle.

Chapter 1: Optimal fiscal policy under skill heterogeneity and capital-skill complementarity

Abstract: This chapter presents a detailed discussion and empirical examination of the effects of optimal fiscal policy in an economy where the agents are heterogeneous with respect to their labour skills and capital holdings. It is further assumed that capital-skill complementarity is present. The findings indicate that, under these characteristics, the optimal fiscal policy suggests a non-zero and positive tax rate on capital income together with highly progressive labour income tax rates. By further analysing the model it is found that the driving force of the positive optimal tax rate on capital income is the heterogeneity in capital holdings. However, the effectiveness of the progressive labour income tax rates in reducing income inequality depends on the presence of capital-skill complementarity. In addition, we find that these results remain robust in the case where the government doesn't need to satisfy a given level of consumption. Finally, we show that, under a Partisan government, the level of the optimal tax rate on capital income is sensitive to the weight placed on the skilled agents in the objective function of the government and the progressivity of labour income tax rate is overturned. In particular, the latter becomes regressive when the weight placed on skilled agents exceeds a threshold value.

1.1 Introduction

The question of whether or not capital should be taxed in the long-run is of great interest and has been the focal point of numerous studies in the field of optimal fiscal policy. Using a neoclassical growth model Judd (1985), assuming two types of agents (capitalists and workers), and Chamley (1986), in a representative agent setup, are the first to show that under optimal fiscal policy a government should not tax capital income in the long-run. In particular, Judd (1985) shows that the zero capital income tax rate is independent of the weight attached to a certain group of agents from the government in its objective function. Moreover, Judd (1985) and Chamley (1986) state that their result does not depend on the government's ability to lend or borrow.

Following the seminal papers of Judd (1985) and Chamley (1986) there has been a growing literature concentrating on identifying the assumptions under which the result of zero optimal tax rate on capital income does not hold. For instance, Judd (1997) adds imperfectly competitive product markets and he shows that, under this setup, the optimal tax rate on capital income is negative. The government uses a subsidy on capital income to compensate for the loss of output and capital in the economy from the monopolistic competition.

Guo and Lansing (1999) extend Judd's work to include depreciation of physical capital and a tax allowance together with endogenous government expenditures. They show that the optimal capital income tax rate in this case can take any sign.[1] In another study, Conesa *et al.* (2009) find a positive optimal capital income tax rate using a model with endogenous labour supply together with life-cycle elements that can generate a labour supply that varies with age. They also show that the magnitude of the optimal tax rate on capital income is mainly affected by the elasticity of labour supply.[2]

Furthermore, optimal fiscal policy and its influence on income redistribution and welfare can depend on the presence of skill heterogeneity and whether the production function exhibits capital-skill complementarities.[3] In particular, Conesa *et al.* (2009) show that the presence of skill heterogeneity will lead to highly progressive labour income tax rates. Note that

[1] In particular, they show that the sign of the optimal capital income tax rate depends on the degree of monopoly power, the tax rate on monopoly profits, the magnitude of government expenditures and the magnitude of the depreciation allowance.

[2] It is also shown that under optimal fiscal policy, the capital income tax rate will be non-zero in the long-run if the government is not able to commit to its policies (see e.g. Klein *et al.*, 2008). Also, Lansing (1999), using a similar model to Judd (1985) but with logarithmic utility function, states that optimal capital income tax rate is non-zero. That happens because due to the logarithmic utility function, agents' savings decisions are not affected by future policies promised by the government.

[3] In particular, it is assumed that skilled agents are those with at least a college degree or a similar professional qualification.

when capital-skill complementarity is present it is assumed that unskilled agents are substitutes to both capital equipment and skilled agents, and that skilled agents and capital equipment are complements of each other.

The capital-skill complementarity hypothesis has been shown in the literature (see e.g. Katz and Murphy (1992), Krusell *et al.*, 2000 and Hornstein *et al.*, 2005) to explain most of the movements in the skill premium in the U.S. for the last three decades.[4] Moreover, the capital-skill complementarity assumption creates an additional channel through which the optimal fiscal policy can redistribute income and increase overall welfare.

For instance, in the case where returns to skill are exogenously determined, the central planner can only redistribute income through higher taxation of those agents in higher income brackets. When combined with the fact that agents with higher labour return hold more capital, this model shows that an increase in the tax rate of skilled agents will also result in a reduction of capital accumulation. This has two knock-on effects. Firstly, there is an increase in the returns of unskilled agents and secondly the skill premium declines. Under this setup, optimal fiscal policy is more effective in terms of income redistribution.

Taking the above into consideration, Judd's (1985) model will be extended in this chapter by assuming that agents are heterogeneous regarding their labour skills.[5] Moreover, building on Judd (1985), it is further assumed that both types of agents can save and work. Then, following Krusell *et al.* (2000), the production function will be extended to incorporate capital-skill complementarity. This way the calibrated model can replicate the wage premium and factor input elasticities suggested in the literature.

In addition, following the literature on income inequality (see e.g. Aghion and Howitt, 1998) it is further assumed that capital market imperfections are present due to different intermediation costs in capital transactions for the two types of agents. This will introduce an additional source of heterogeneity between the two types of agents, the capital holdings heterogeneity.[6]

[4] The skill premium is defined as the ratio of the wage rate of skilled relative to unskilled agents.

[5] Two types of agents will be assumed, skilled and unskilled agents.

[6] Note that it is also assumed that wealth and wage inequality always in the same

Through this feature the model is able to generate heterogeneity in savings as it is observed in the UK data. Thus, the model in this chapter is calibrated to the UK economy.

After calibrating the model to replicate the key great-ratios as well as the skill premium of the UK economy, the long-run solution is obtained by initially assuming an exogenous fiscal policy. Afterwards, the assumption of the exogenous fiscal policy is dropped and the steady-state results are obtained in the case of an endogenously determined fiscal policy (optimal fiscal policy), keeping the same calibration as in the exogenous case. The results show that under optimal fiscal policy a government that wants to maximize the aggregate welfare of the economy should impose a positive optimal capital income tax rate together with progressive labour income tax rates. In this way, the government is able to redistribute income efficiently.

Various versions of the model are examined to understand the main driving force(s) behind the positive optimal tax rate on capital income and the increase in the progressive nature of labour income tax under this setup. We find that imperfect capital markets are the main driving force of the positive optimal tax rate on capital income. However, when capital-skill complementarity is present and the government cannot impose two different labour income tax rates, the optimal tax rate on capital income will be positive even with perfect capital markets. This occurs because skilled agents and capital are complements meaning that a lower stock of capital, due to the positive capital income tax rate, will reduce the demand for skilled agents and so lower their pre-tax wage rate. Also, since unskilled agents are substitutes for capital and skilled agents, their wage rate will increase causing a reduction in income inequality.

These results are in line with the argument of Correia (1996) that under an incomplete set of tax instruments, or in other words when there is not a tax instrument for each input in the production process, the capital income tax may be non-zero in the long-run under optimal fiscal policy. Moreover, the results of this chapter verify the results of Judd (1997) and Guo and

direction. This is also suggested by the Panel Study of Income Dynamics data that Garcia-Mila *et al.* (2010) analyse.

Lansing (1999) which show that when there is an imperfection in capital and/or labour markets then the zero capital income tax may not be obtained.

A detailed analysis of the effect of optimal fiscal policy on income redistribution is provided as well as the interaction with various elements of the model. The results suggest that the key characteristic of the model that allows optimal fiscal policy to redistribute income efficiently is the capital-skill complementarity. In its absence, the optimal fiscal policy will increase skilled agents' share of total income and thus cause the income inequality to widen.

The effect of capital-skill complementarity together with capital market imperfection on optimal fiscal policy outcome has not been assessed before in the literature in a systematic way. Therefore, this chapter complements the literature of optimal fiscal policy in that when agents face different costs in accessing the capital market and when capital-skill complementarity is present the optimal tax rate on capital income will be positive, the labour income taxes will be progressive and the government can redistribute income efficiently. In addition, under the current setup, the optimal fiscal policy will be Pareto efficient, leading to higher welfare for each type of agent compared with the exogenous fiscal policy case.

The above results are robust even in the case where the government doesn't need to choose its tax rates so as to satisfy a given level of consumption/ expenditure, as in Judd (1985). Moreover, the optimal capital income tax rate remains non-zero even if the government places more weight to skilled agents, who are the majority of the population, as in Chari and Kehoe (1999).

The chapter is set out as follows. Section 2 provides a description of the benchmark model. Section 3 describes the calibration of the parameters. Section 4 shows the steady-state solution of the exogenous fiscal policy. Section 5 outlines the optimal fiscal problem and its solution. Section 6 compares the results of the exogenous and optimal fiscal policy. Section 7 provides a detailed assessment of optimal fiscal policy outcome. Section 8 provides a welfare and income inequality analysis. Section 9 discuss the main results. Sections 10 and 11 compare the models with and without capita-skill complementarity. Section 12 provides a discussion when government expenditures are endogenous. Section 13 contains a model comparison with respect to income inequality. Section 14 includes a case study when government places more weight to a certain group of agents. Section 15 provides the concluding remarks of the chapter.

1.2 The model

The model economy has a large number of two types of infinitely-lived identical households who own capital and rent it to firms. Each type of household has either skilled workers or unskilled workers that are able to save. The size of the overall population, N, is assumed to be constant. The population of the identical skilled workers is N_s and the population of the unskilled workers is N_u, such that $N = N_s + N_u$. For simplicity it is defined that the share of skilled agents is $n_s = N_s/N$, and the share of the unskilled agents is $n_u = N_u/N$, where $1 = n_s + n_u$. In addition, there is a large number of identical firms and a government. In each period, households are price takers and make decisions regarding how much to consume, work and save. Firms act competitively and use capital together with the two types of labour to produce a homogeneous consumption good. Government, on the other hand, runs a balanced budget and imposes capital and labour income tax rates. The government uses the revenue from these taxes to finance public consumption, which has a direct impact on households' utility.

1.2.1 Firms

All firms produce a homogeneous consumption good, Y_t, using labour and capital, and act in perfectly competitive markets, taking prices and policy variables as given:

$$\Pi_t = Y_t - w_{s,t}h_{fs,t} - w_{u,t}h_{fu,t} - r_{eq,t}K_{eq,t} - r_{st,t}K_{st,t} \tag{1}$$

subject to Krusell *et al.* (2000) type of production function:

$$Y_t = A_t K_{st,t}^{a^c} \times$$

$$\times \left[\lambda \left(\nu \left(A^{\rho}_{eq,t} K^{\rho}_{eq,t}\right) + (1-\nu)\, h^{\rho}_{fs,t}\right)^{\varphi/\rho} + (1-\lambda)\, h^{\varphi}_{fu,t}\right]^{\frac{1-a_c}{\varphi}} \tag{2}$$

where

$$a, \lambda, \nu \in (0,1)\,; \quad \varphi, \rho \in (-\infty, 1)\,.$$

Also, $h_{fs,t}$ and $h_{fu,t}$ denote the hours worked by skilled and unskilled labour respectively. $A_{eq,t}$ denotes the efficiency level of capital equipment and A_t is total factor productivity. $K_{st,t}$ and $K_{eq,t}$ denote the stock of capital structures and capital equipment respectively at the beginning of period t.

The elasticity of substitution between unskilled labour and skilled labour is equal to the elasticity of substitution between unskilled labour and capital equipment, $(1/(1-\varphi))$. Whereas, the elasticity of substitution between skilled labour and capital equipment is $(1/(1-\rho))$. In addition, the income share of capital structures is α_c, while the income share of capital equipment, skilled and unskilled labour is determined by λ and ν.

Under this setup the capital-skill complementarity hypothesis is present only when $\varphi > \rho$. If φ or ρ equals zero the CES production function will simplified to a Cobb-Douglas representation.[7]

Using the above production function and the fact that factors are being paid their marginal products (perfect competition), the skill premium can be written as the ratio of the two marginal products of skilled workers over unskilled workers as:

$$\frac{w_{st}}{w_{ut}} = \frac{\lambda(1-\nu)}{(1-\lambda)} \left[\nu \left(\frac{A_{eq,t} K_{eq,t}}{h_{fs,t}}\right)^{\rho} + (1-\nu)\right]^{(\varphi-\rho)/\rho} \left(\frac{h_{fu,t}}{h_{fs,t}}\right)^{1-\varphi}. \tag{3}$$

If capital-skill complementarity hypothesis is present $(\varphi > \rho)$ an increase in capital equipment, *ceteris paribus*, will increase the skill premium. This is called, following Krusell *et al.* (2000), "the capital-skill complementarity effect". In addition, if the ratio of unskilled to skilled labour increases the skill premium will increase as well, again assuming all the other factors remain constant. This is called by Krusell *et al.* (2000) "the relative supply effect".

Both of the productivity shocks, $A_{eq,t}$ and A_t, are assumed to follow

[7] If $\varphi, \rho = 1$ there is perfect substitutability and if $\varphi, \rho = -\infty$ there is perfect complementarity.

exogenous AR(1) processes with zero mean, constant variance and covariance equal to zero.

The law of motion for aggregate capital stock for the two types of agents, $j = s, u$ where s and u denote skilled and unskilled agents respectively, is:

$$K^j_{i,t+1} = (1 - \delta_i)K^j_{i,t} + I^j_{i,t} \tag{4}$$

note that $i = st, eq$, where st and eq denote capital structures and capital equipment respectively. The depreciation rate is $0 \leq \delta_i \leq 1$ and $I^j_{i,t}$ is the aggregate investment in new capital i for the agent of type j.

1.2.2 Households

The representative household $j \in \{s, u\}$ in each period maximizes its expected lifetime utility:

$$U_j = E_t \sum_{i=t}^{\infty} \beta^i u(C^*_{j,i}, l_{j,i}) \tag{5}$$

where $0 < \beta < 1$ is a constant discount factor and denotes the time preference of the individual; $C^*_{j,i}$ and $l_{j,i}$ are total effective consumption and leisure respectively at period i for the agent of type j; and $u(\cdot)$ is the utility function that is increasing, strictly concave and three times continuously differentiable with respect to its inputs.

Moreover, it is assumed that the effective consumption has the following constant elasticity of substitution (CES) representation:

$$C^*_{j,t} = \left[aC^{\mu}_{j,t} + (1 - a)\left(\bar{G}^c_t\right)^{\mu}\right]^{1/\mu} \tag{6}$$

where $C_{j,t}$ is private consumption for agent of type j and $\bar{G}^c_t$ is the average public consumption share of a representative agent ($\bar{G}^c_t = G^c_t/N$), which households take as given. Also, a and $1 - a$ are the share parameters on effective consumption ($0 < a < 1$) of private and public consumption respectively and $1/(1 - \mu)$ is the elasticity of substitution between public and private consumption.[8]

Utility function The utility function applied is the following non-separable utility function:

$$u(C_{j,t}, l_{j,t}) = \frac{\left(\left(C^{*}_{j,t}\right)^{\gamma} l_{j,t}^{1-\gamma}\right)^{(1-\sigma)}}{1-\sigma} \tag{7}$$

where $\sigma \geq 0$ represents the coefficient of relative risk aversion and γ, $1-\gamma$ ($\gamma \in (0,1)$) are the relative shares on utility of effective consumption and leisure respectively. Moreover, $1/\sigma$ is the elasticity of intertemporal substitution of effective consumption in any two periods. Therefore, the larger the elasticity, which means the smaller σ, the more willing is the individual to substitute consumption for leisure over time.

Budget constraint Each type of household faces the following time constraint:

$$1 = l_{j,t} + h_{j,t} \tag{8}$$

where $h_{j,t}$ is the amount worked in period t from agent of type j. The above equation states the fact that in each period households split their endowment of time between leisure and work. In this case it is normalized that the endowment in each period is equal to one.

In addition, the two types of household j have the following budget constraint:

$$\begin{aligned} C_{j,t} + I^{j}_{i,t} &= (1-\tau^{w}_{j,t}) w_{j,t} h_{j,t} - \psi_j \left[\left(K^{j}_{st,t}\right)^2 + \left(K^{j}_{eq,t}\right)^2\right] + \\ &\quad + (1-\tau^{r}_{t}) \left(r_{st,t} K^{j}_{st,t} + r_{eq,t} K^{j}_{eq,t}\right) \end{aligned} \tag{9}$$

where $\psi_j > 0$ captures the transaction costs of holding capital for each type of household. Therefore, ψ_j can be interpreted as a form of imperfection in capital markets and may be due to cost of information in legal issues or government regulations or even fees that need to be paid to intermediates. The transaction costs are being introduced to capture the heterogeneity among

[8] Note that CES representation can be transformed to a linear specification if $\mu = 1$. In the case where $\mu \to 0$ it will be transformed into a Cobb-Douglas specification.

the two types of agents in asset holdings, following Aghion and Howitt (1998). In addition, $\tau_{j,t}^{w}$ and τ_{t}^{r} are the tax rates on labour income for each type of agent and on income from capital in period t.[9] Therefore, the first term on the right-hand side of the above equation is the after tax return from labour and the last two terms represent the cost of holding capital and the after tax capital income.

1.2.3 The government budget constraint

The government runs a balanced budget in every period which is given by:

$$\begin{aligned} \bar{G}_t^c &= n_s\left(\tau_{s,t}^{w} w_{s,t} h_{st}\right) + n_u\left(\tau_{u,t}^{w} w_{u,t} h_{u,t}\right) + \\ &+\tau_t^r\left(r_{st,t} K_{st,t} + r_{eq,t} K_{eq,t}\right). \end{aligned} \tag{10}$$

The above equation assumes that the revenues from taxes are being used to finance public consumption/expenditures. Under the exogenous fiscal policy regime, the government's policy instruments (tax rates on capital and labour income) are calibrated and given for any period t. The long-run value of government consumption, $\bar{G}_t^c$, follows residually after the realization of the shocks.

1.2.4 Aggregate resource constraint and market clearing

It is further assumed that output can be used only for consumption (private and public) and for investment. That gives the following aggregate resource constraint:

$$\begin{aligned} NY_t &= N\bar{G}_t^c + N_s C_{s,t} + N_u C_{u,t} + N_s\left(I_{s,t}^{st} + I_{s,t}^{eq}\right) + N_u\left(I_{u,t}^{st} + I_{u,t}^{eq}\right) + \\ &+N_s\psi_s\left[\left(K_{st,t}^{s}\right)^2 + \left(K_{eq,t}^{s}\right)^2\right] + N_u\psi_u\left[\left(K_{st,t}^{u}\right)^2 + \left(K_{eq,t}^{u}\right)^2\right] \end{aligned}$$

[9] It is assumed that capital is owned by households and is rented out to firms. This assumption makes the firm's maximization problem static and as a result much easier to deal with.

or after replacing equation (4):

$$\begin{aligned} Y_t &= n_s\left(K^s_{st,t+1}-(1-\delta_{st})K^s_{st,t}+K^s_{eq,t+1}-(1-\delta_{eq})K^s_{eq,t}\right)+ \\ &\quad +n_sC_{s,t}+n_uC_{u,t}+\bar{G}^c_t+ \\ &\quad +n_u\left(K^u_{st,t+1}-(1-\delta_{st})K^u_{st,t}+K^u_{eq,t+1}-(1-\delta_{eq})K^u_{eq,t}\right)+ \\ &\quad +n_s\psi_s\left[\left(K^s_{st,t}\right)^2+\left(K^s_{eq,t}\right)^2\right]+n_u\psi_u\left[\left(K^u_{st,t}\right)^2+\left(K^u_{eq,t}\right)^2\right] \end{aligned} \tag{11}$$

Also, it should be noted that the following market clearing conditions exist in the labour market:

$$h_{fs}=n_sh_s \tag{12}$$

$$h_{fu}=n_uh_u \tag{13}$$

and in the capital market:

$$K_{i,t}=n_sK^s_{i,t}+n_uK^u_{i,t} \tag{14}$$

where $N_s/N=n_s$ and $N_u/N=n_u$ are the relevant shares of population of skilled and unskilled agents respectively.

The labour market clearing conditions show that the labour demanded by firms is equal to the labour supplied by households. Also, the capital market clearing condition shows that the stock of capital structures and capital equipment rented by firms is held entirely by skilled and unskilled agents.

1.3 Decentralized competitive equilibrium

Given an initial level of capital stock for structures ($K_{st,0}$) and equipment ($K_{eq,0}$), the three policy instruments ($\tau^w_{s,t}$, $\tau^w_{u,t}$, τ^r_t) and the exogenously set of stationary AR processes, the DCE system of equations is characterized by a sequence of allocations $\{C_{j,t},\ h_{j,t},\ K^j_{st,t+1},\ K^j_{eq,t+1},\ Y_t\}^\infty_{t=0}$, prices $\{w_{j,t},\ r_{st,t},\ r_{eq,t}\}^\infty_{t=0}$, and the remaining policy instrument $\{\bar{G}^c_t\}^\infty_{t=0}$ such that: (i) households maximize their welfare and firms their profits, taking policy and prices as given; (ii) government budget constraint is satisfied in each time period; (iii) all markets clear.[10]

1.4 Calibration

The model is calibrated according to the UK economy for the period 1960-2010. The sources that have been used to obtain the data for the UK economy are: the OECD, ECFIN, World Bank, Labour Force Survey and the Office for National Statistics.

Initially, following the data obtained from the Labour Force Survey (Quarter 4, 2010), we make the assumption that 60% of the population is skilled, which means that they have obtained at least a college degree or similar professional qualification. Therefore, the share of skilled agents, n_s, is set equal to 0.6 and the relevant share of unskilled agents, n_u, is set equal to 0.4. Moreover, the model assumes that skilled agents earn on average more income than the unskilled agents. As a result, following the Annual Survey of Hours and Earnings from the Office for National Statistics, the ratio of the income of the upper 60% of the population over the income of the lower 40% of the population for the UK economy is equal to 1.41. But since it is assumed that the agents with higher wealth are the skilled agents, it can be stated that the skill premium, which is determined as the ratio of the skilled agents' wage rate to the unskilled agents' wage rate, is also equal to 1.41. This is consistent with other studies that also report the skill premium of the UK (see e.g. Machin, 1996 and Angelopoulos *et al.*, 2012a).

The cost of capital holdings for the unskilled agents, ψ_u, is set two times higher than the relevant cost for the skilled agents, ψ_s, to replicate the wealth distribution observed in the data. This way the high-income agents (skilled agents) can hold two times more capital than the low-income agents (unskilled agents). This result is also consistent with the Panel Study of Income Dynamics (PSID) data, as Garcia-Milà *et al.* (2010) report. Note that the magnitude of the cost of capital holdings is calibrated so that the capital to output ratio observed in the data can be replicated, given the value of the depreciation rate and the rate of time preference.

[10] The DCE system of equations can be found in the Appendix A.

The parameters of the production function, a_c, ρ, φ, λ and ν affect the level of the skill premium through the returns to capital equipment, capital structures, skilled and unskilled labour, together with the complementarity between the four factors. For the calibration of those parameters we follow the works of Krusell *et al.* (2000) and Lindquist (2004) and as a result the elasticity of capital equipment to skilled labour, $1/(1-\rho)$, is set equal to 0.67 and the elasticity of capital equipment to unskilled labour, $1/(1-\varphi)$, is set equal to 1.67.[11] Meaning that $\sigma = 0.401$ and $\rho = -0.495$ and also that skilled agents are more complementary with capital equipment than the unskilled agents. Moreover, the income share of capital structures, a_c, is set equal to 0.118 and the relevant shares of composite input, λ, and capital equipment at the composite input, ν, are set equal to 0.7 and 0.595 respectively. These parameters are consistent and within the boundaries that the literature suggests (see Hornstein *et al.* (2005) for a review).

The total factor productivity parameter, A_t, is set equal to 1.5 with persistence parameter, ρ_A, that is calibrated using data from the Office for National Statistics (from 1998-2009) and taking into account the assumed AR(1) process. The persistence parameter is found to be $\rho_A = 0.92$ (significant at 1% level) with a standard deviation equal to 0.05. The efficiency level of capital equipment is normalised to unity and data are not available to calibrate accordingly its persistence and standard deviation.

Regarding the calibration of tax rates the OECD Statistics for the period 1970-2010 have been used and it is concluded that the average capital income tax rate, τ^r, is equal to 0.44 and that the average labour income tax rate is equal to 0.26. Since the effective labour income tax rate needs to be equal to the weighted average of the two labour income tax rates (skilled and unskilled labour income tax rates), the skilled labour income tax rate, τ_s^w, is set equal to 0.3, and the unskilled labour income tax rate, τ_u^w, equal to 0.2.[12]

The value of the private to public consumption elasticity, μ, is set equal to 0.6, which means that private and public consumption are substitutes.

[11] Note that Krusell *et al.* (2000) estimates the parameters of the production function using a two-step version of simulated pseudo-maximum likelihood.

[12] Thus, $0.26 = n_s\tau_s^w + n_u\tau_u^w$, where n_s and n_u are the relevant shares of the population for the skilled and unskilled agents respectively.

Furthermore, the relative weight of private consumption, α, in the composite consumption is set equal to 0.9, following the work of Baier and Glomm (2001). The depreciation rates are calibrated so as for the model to replicate the data average of the overall investment to output ratio for the UK. Therefore, the depreciation rate of capital structures, δ_s, is set equal to 0.035 and the depreciation rate of capital equipment, δ_e, is set equal to 0.07.

Regarding the weight of the composite consumption in utility the work of Kydland (1995) has been followed and γ is set equal to 0.34. This is equal to the ratio of total hours of work over the total number of hours available for either work or leisure, as obtained from the OECD database. Finally, the value of the intertemporal elasticity of consumption ($1/\sigma$) is set equal to 0.5 ($\sigma = 2$) which is common in the literature.

Table 1.1: Calibration

Param.	Definitions	Values
δ_s	depreciation rate of capital structures	0.035
δ_e	depreciation rate of capital equipment	0.070
β	time discount factor	0.976
γ	effective consumption weight in utility	0.340
a	private consumption weight on effective cons.	0.900
$\frac{1}{1-\mu}$	private to public consumption elasticity	2.500
σ	coefficient of relative risk aversion	2.000
ψ_s	cost of holding capital (skilled)	.0035
ψ_u	cost of holding capital (unskilled)	0.007
n_s	population share of skilled agents	0.600
n_u	population share of unskilled agents	0.400
α_c	income share of capital structures	0.118
$\frac{1}{1-\rho}$	capital equipment to skilled labour elasticity	0.670
$\frac{1}{1-\varphi}$	capital equipment to unskilled labour elasticity	1.670
λ	income share of composite input to output	0.700
ν	income share of capital eq. to composite input	0.595
$\bar{\tau}_s^w$	average wage tax rate for skilled agents	0.300
$\bar{\tau}_u^w$	average wage tax rate for unskilled agents	0.200
$\bar{\tau}^r$	average tax rate on capital income	0.440
$\bar{A}$	constant parameter productivity	1.500
$\bar{A}_{eq}$	efficiency level of capital equipment	1.000

1.5 Exogenous fiscal policy

Tables 1.2 and 1.3 present the steady-state results when the fiscal policy is exogenously determined.

Table 1.2: Exogenous steady-state results

Param.	Definitions	Exogen.
K_{st}^s	cap. structures skilled agents hold	0.54
K_{eq}^s	cap. equipment skilled agents hold	0.85
K_{st}^u	cap str. holdings of unskilled agents	0.27
K_{eq}^u	cap. eq. holdings of unskilled agents	0.43
h_s	working time by skilled workers	0.28
h_u	working time by unskilled workers	0.29
C^s	consumption of skilled agents	0.23
C^u	consumption of unskilled agents	0.18
Y	output	0.42
$\bar{G}^c$	government spending	0.14
τ^r	tax rate of returns on capital	0.44
τ_s^w	wage tax rate of skilled agents	0.30
τ_u^w	wage tax rate of unskilled agents	0.20
r_{st}	pre-tax returns on capital structures	0.11
r_{eq}	pre-tax returns on capital equipment	0.18
w_s	pre-tax wage rate of skilled agents	0.96
w_u	pre-tax wage rate of unskilled agents	0.68
w_s/w_u	pre-tax skill premium	1.41
$\frac{(1-\tau_s^w)w_s}{(1-\tau_u^w)w_u}$	after-tax skill premium	1.23

Table 1.4 includes the results from the data averages and the relevant results from the calibrated model for the great ratios. The calibrated model is able to replicate with accuracy the capital to output ratio and the skill premium. Also, at the same time the model is able to match the data for the UK for the investment to output ratio and government expenditures to output ratio.

1.6 Optimal fiscal policy

Under optimal fiscal policy the policy instruments are no longer exogenously determined. The government chooses optimally the level of labour and cap-

Table 1.3: Great ratios and welfare

Param.	Definitions	Exogen.
K^s_{st}/Y	K^s_{st} over output	0.78
K^s_{eq}/Y	K^s_{eq} over output	1.22
K^u_{st}/Y	K^u_{st} over output	0.26
K^u_{eq}/Y	K^u_{eq} over output	0.41
C^s/Y	cons. of skilled ag. over output	0.33
C^u/Y	cons. of unskilled ag. over output	0.17
$\bar{G}^c/Y$	government spending over output	0.34
I^s_{st}/Y	investment on K^s_{st} over Y	0.03
I^s_{eq}/Y	investment on K^s_{eq} over Y	0.09
I^u_{st}/Y	investment on K^u_{st} over Y	0.01
I^u_{eq}/Y	investment on K^u_{eq} over Y	0.03
U	average lifetime welfare[13]	-90.01
U^s	s.s. welfare of skilled agent	-86.88
U^u	s.s. welfare of unskilled agent	-94.71

Table 1.4: Great ratios and skill premium

Param.	Definitions	Data	Model
K/Y	capital to output	2.67	2.67
C/Y	consumption to output	0.62	0.51
I/Y	investment to output	0.15	0.15
G/Y	government cons. to output	0.38	0.34
w_s/w_u	pre-tax skill premium	1.41	1.41

ital income tax rates to achieve a maximum aggregate welfare of the two types of agents. Early studies, such as Judd (1985) and Chamley (1986), have shown that under optimal fiscal policy capital income tax rate should be zero in the long-run. To obtain these results government must commit to its policies and not re-optimise in the future, otherwise the equilibrium outcome from the optimal fiscal policy is not sustainable.[14]

Therefore, in this section it is assumed that the government choose and commits to a policy at $t = 0$ and it does not re-optimise in the future. Then the agents and firms form their decisions taking into account government's

[14]Note that the assumptions made in the previous section regarding the agents, firms, government's budget constraint and market clearing conditions continue to apply in this section.

policy. As a result, the government will take its decision of optimal taxation by taking into account the DCE system of equations, as it has been presented in the previous section and it is outlined in the Appendix A.

It is assumed that the government wants to maximize the aggregate welfare of the lifetime utility of the two types of agents choosing $\{\tau_t^k, \tau_{s,t}^w, \tau_{u,t}^w\}_{t=0}$ to finance its consumption, which is assumed to be constant and equal to the exogenously determined fiscal policy, having the following objective function:

$$\max \sum_{t=0}^{\infty} \beta^t \left(n_s U_s \left(C_{s,t}^*, h_{s,t} \right) + n_u U_u \left(C_{u,t}^*, h_{u,t} \right) \right) \tag{15}$$

subject to the DCE system of equations

Taking into consideration the setup of the model, as it has been outlined earlier, the government chooses optimally the following variables $C_{s,t}$, $C_{u,t}$, $h_{s,t}$, $h_{u,t}$, $K_{st,t+1}^s$, $K_{eq,t+1}^s$, $K_{st,t+1}^u$, $K_{eq,t+1}^u$, $w_{s,t}$, $w_{u,t}$, $r_{st,t}$, $r_{eq,t}$, Y_t, plus the three tax rates $\{\tau_t^r, \tau_{s,t}^w, \tau_{u,t}^w\}$.[15] Note that government consumption / expenditures is being targeted to be the same as in the exogenous case study. Therefore, it is assumed that the government needs to finance its expenditures through taxation without being able to borrow. This way the results of the model are comparable with the literature (see e.g. Garcia-Mila *et al.*, 2010).

The dual approach is applied for obtaining the steady-state results (see Tables 1.5 and 1.6 in the next section), taking into account the calibrated parameters as presented in the previous section.

In addition, following Lucas (1990) the welfare gains from adopting the optimal fiscal policy instead of the exogenous fiscal policy are being estimated. Thus, we calculate the percentage of consumption that the agent would be willing to give up while he/she is under the optimal fiscal policy in order to be indifferent between exogenous and optimal fiscal policy. Therefore, a positive value indicates that the agent is better off under the optimal

[15] This is the dual approach of optimal fiscal policy and it is presented in the Appendix A.

fiscal policy.

$$\xi_i = 1 - \left(\frac{LU^E_{i,ss}}{LU^R_{i,ss}}\right)^{\frac{1}{\gamma(1-\sigma)}} \quad (16)$$

where, $i = a, s, u$ for aggregate economy, skilled and unskilled agents respectively and ss indicates that the lifetime utility is calculated at the steady state. Also, LU represents the lifetime welfare at the steady state. The superscript on the lifetime welfare indicates the fiscal policy regime. So, it is either E for the exogenous fiscal policy or R for optimal fiscal policy.[16]

1.7 Exogenous vs. optimal fiscal policy

1.7.1 Steady-state

The results from the exogenous and Ramsey policy in the long-run are given in Tables 1.5 and 1.6. Comparing the two policies it can be seen that under optimal fiscal policy the capital income tax rate is reduced. Also, the skilled labour income tax rate has increased to balance the budget constraint and the unskilled labour income tax rate remained relatively stable.[17] Therefore, under the current setup of the model, with capital-skill complementarity and capital market imperfection, the optimal capital income tax rate will be positive and equal to 6%. This result contradicts the early studies of Judd (1985) and Chamley (1986) that suggested a zero optimal tax rate on capital income. But as it will become clear in the next session, the positive optimal tax rate on capital income is present under current setup due to the capital market imperfections, as in Judd (1997) and Guo and Lansing (1999).

The systematic assessment of how optimal fiscal policy is affected when the economy exhibits capital market imperfection, in the form of different cost of capital holdings, and capital-skill complementarity, is worth noting in this chapter and contributes the related literature. Therefore, in the next section we examine how the long-run optimal fiscal policy changes when there is

[16] Appendix A shows the derivation of equation (16).

[17] Note that this result is affected from the assumed calibration. For instance, if unskilled labour is less substitutable with capital then their wage tax rate will be higher than the exogenous case, but still very close to the exogenous case.

no capital market imperfection and/or no capital-skill complementarity and compare these results with the literature.

It is also important to note that the assumed skill heterogeneity leads to optimally progressive labour income tax system. As Conesa *et al.* (2009) suggest, skill heterogeneity leads to higher progressivity of the labour income tax rate. But also, under the current setup of the model, the government can affect optimally the after-tax skill premium and as a consequence labour income inequality. Thus, the government takes into account the effect of its policies on the returns to skill and income inequality and then forms the optimal progressive labour income tax rate.

The capital-skill complementarity assumption creates an additional channel through which the optimal fiscal policy can redistribute income and increase overall welfare. For instance, in the case that the returns to skill are endogenously determined and agents with higher labour return hold more capital, an increase in the tax rate of capital income will also result in a reduction of capital accumulation. The decrease of capital stock levels will result in an increase of the returns to unskilled agents and also skill premium will decline as a consequence. As a result, under this setup, optimal fiscal policy is more effective in terms of income redistribution.

In addition, skilled agents will keep working less than the unskilled agents but this gap will increase and both will work less in comparison to the exogenous case due to the increase in capital holdings for both agents.

Under optimal fiscal policy, skilled agents will choose similar amount of consumption as in the exogenous case and their capital holdings will increase. On the other hand, the unskilled agents will increase both their consumption and capital holdings. In particular, skilled agents hold the 66.7% of capital as it was expected from the imposed calibration. Thus the capital income distribution will remain the same as it was in the exogenous case.

Moreover, the after-tax skill premium will decrease under optimal fiscal policy. This result occurs because the effect from the higher progressivity of the labour income taxes is larger than the effect of the higher pre-tax wage rates. Since the capital income distribution remains constant, due to the exogenously determined cost of capital holdings, the change in the after-tax wage rates will be the main determinant of income inequality.

Therefore, now that the progressivity of labour income tax rates is endogenously determined the government will choose to impose a more progressive labour tax system to reduce income inequality.

Table 1.5: Steady-state results of exogenous and optimal fiscal policy

Param.	Definitions	Exogen.	Opt. Policy
K_{st}^{s}	cap. structures skilled agents hold	0.54	1.00
K_{eq}^{s}	cap. equipment skilled agents hold	0.85	1.55
K_{st}^{u}	cap str. holdings of unskilled agents	0.27	0.50
K_{eq}^{u}	cap. eq. holdings of unskilled agents	0.43	0.68
h_s	working time by skilled workers	0.28	0.25
h_u	working time by unskilled workers	0.29	0.28
C^s	consumption of skilled agents	0.23	0.24
C^u	consumption of unskilled agents	0.18	0.21
Y	output	0.42	0.48
$\bar{G}^c$	government spending	0.14	0.14
τ^r	tax rate of returns on capital	0.44	0.06
τ_s^w	wage tax rate of skilled agents	0.30	0.52
τ_u^w	wage tax rate of unskilled agents	0.20	0.19
r_{st}	pre-tax returns on capital structures	0.11	0.07
r_{eq}	pre-tax returns on capital equipment	0.18	0.11
w_s	pre-tax wage rate of skilled agents	0.96	1.41
w_u	pre-tax wage rate of unskilled agents	0.68	0.78
w_s/w_u	skill premium	1.41	1.81
$\frac{(1-\tau_s^w)w_s}{(1-\tau_u^w)w_u}$	after-tax skill premium	1.23	1.06

Therefore, the reduction of the capital income tax rate causes an increase in the total stock of capital through an increase in investment. The increase in the stock of capital will have a positive effect on output. Moreover, since it has been assumed that skilled agents are more complementary with capital equipment than the unskilled agents, the increase in the stock of capital equipment will cause an increase in the demand for skilled labour. This in turn will cause an increase in the pre-tax wage rate for skilled agents and as a result there will be greater disparity between the skilled and unskilled agents' pre-tax wage rates. This is the reason for obtaining a higher pre-tax skill premium under the optimal fiscal policy case study than the exogenous

Table 1.6: Great ratios and welfare under exogenous and optimal fiscal policy

Param.	Definitions	Exogen.	Opt. policy
K^s_{st}/Y	K^s_{st} over output	0.78	1.24
K^s_{eq}/Y	K^s_{eq} over output	1.22	1.68
K^u_{st}/Y	K^u_{st} over output	0.26	0.41
K^u_{eq}/Y	K^u_{eq} over output	0.41	0.56
C^s/Y	cons. of skilled ag. over output	0.33	0.30
C^u/Y	cons. of unskilled ag. over output	0.17	0.18
$\bar{G}^c/Y$	government spending over output	0.34	0.30
I^s_{st}/Y	investment on K^s_{st} over Y	0.03	0.04
I^s_{eq}/Y	investment on K^s_{eq} over Y	0.09	0.12
I^u_{st}/Y	investment on K^u_{st} over Y	0.01	0.02
I^u_{eq}/Y	investment on K^u_{eq} over Y	0.03	0.04
W	aggregate lifetime welfare	-90.01	-85.73
W^s	s.s. welfare of skilled agent	-86.88	-83.60
W^u	s.s. welfare of unskilled agent	-94.71	-88.93
ξ_a	-	-	13.34%
ξ_s	-	-	10.69%
ξ_u	-	-	16.90%

fiscal policy case study.

1.8 Comparison of welfare and income inequality

The welfare for both types of agents will increase under optimal fiscal policy due to the increase in consumption and the decrease in working hours for both agents. Therefore, the optimal fiscal policy under this setup is Pareto improving since both agents are better off than the exogenous case.

The last three rows of Table 1.6 report the percentage of consumption that the agent would be willing to give up under the optimal fiscal policy in order to be indifferent between the two policies. The results report that skilled agents are willing to give up 10.7% of their consumption before returning to the exogenous fiscal policy. Unskilled agents, on the other hand, are willing to give up 16.9% of their consumption. Both percentages are positive meaning that both agents are better off under optimal fiscal policy. Also, the optimal fiscal policy regime is more beneficial for the unskilled agents than the skilled agents because they are willing to give up almost 6% more from their consumption than the skilled agents so as to be indifferent between the two policy regimes ($\xi_u > \xi_s$).

Moreover, it can be examined how income inequality is affected between the two cases. For the income inequality analysis the two representative agents are being used and it is examined how the available income is split among them. In the exogenous case the representative skilled agent's share of total after-tax income is 58.5%.[18]

This can be split to labour income share and capital income share. Note that both agents have the same returns to capital and the same capital income tax rate. Since capital holdings for skilled agents are twice as much as the unskilled agents', due to the exogenous cost of capital holdings, the after-tax capital income ratio between skilled and unskilled agents will always be equal to 2. Meaning that skilled agents will always hold 2/3 of total capital income. This result will hold for both the exogenous fiscal policy and optimal fiscal policy.

Therefore, the after-tax labour income share (which is equivalent to the after-tax skill premium) will be the important factor for determining the overall income share of skilled and unskilled agents. The after-tax skill premium in the exogenous case study is 1.24 and under optimal fiscal policy it declines to 1.06.

Since the after-tax capital income share remained unaffected and the after-tax skill premium decreased, the overall income share of skilled agents has decreased as well. That means that under optimal fiscal policy the government manages to redistribute income more equally among the agents. In particular, under optimal fiscal policy the after-tax income share of the representative unskilled agent is 42.9%, which is 1.4% higher than the exogenous case.

The above analysis indicates that when both agents hold capital at constant rates and also have the same returns from capital, the only way that

[18]The income of the representative agent is calculated as the after tax income from labour plus the after tax income from capital holdings: $(1-\tau_j^w)w_j h_j + (1-\tau^r)(r^{st}K_{st}^j + r^{eq}K_{eq}^j)$ where, the subscript $j = s, u$ denotes the type of agent.

government can redistribute income efficiently is through the progressive tax rates of labour income.

As a result, under this setup, the optimal fiscal policy can be Pareto improving and also reduce income inequality in the economy.

1.9 Analysis of the results

The model presented in the previous section includes the following elements: (i) capital-skill complementarity, (ii) capital market imperfections through the cost of capital holdings $\psi_i > 0$, for $i = s, u$, (iii) two different labour tax rates for the two types of agents, (iv) the aggregate economy is being discussed instead of a simple representative agent model, and (v) the productivity of the two types of agents is being set to be equal and only the capital-skill complementarity drives the skill premium to increase.

To understand and isolate the main driving forces of the main findings: (i) the non-zero optimal capital income tax rate and its magnitude and (ii) the progressivity of the labour income tax rate; several versions of the benchmark model will be considered.[19] Table 1.7 summarizes the results of the optimal fiscal policy when several of the elements mentioned earlier are not present.[20]

The first column in the table takes into account the fact that the production function exhibits capital-skill complementarities. In the case that this element is not present the Cobb-Douglas production function, with constant returns to scale, is being used instead where the two types of agents are assumed to have the same share in the production process. The second column takes into account the presence of capital market imperfections. When this imperfection is not present it is assumed that only skilled agents have access to perfect capital market ($\psi_s = 0$).[21]

[19] The progressivity of labour income tax rate is determined as the difference between the skilled agents' labour income tax rate and the unskilled agents' labour income tax rate ($\tau_s^w - \tau_u^w$).

[20] Note that in every model the parameters that are re-calibrated to match the great ratios are in the production function and within the boundaries suggested in the literature.

[21] Note that when both agents have access to perfect capital markets, $\psi_s = \psi_u = 0$, the DCE system doesn't have a unique solution.

Table 1.7: Summary of quantitative results

Model	Cap.-Skill complem.	Capital markets imperfection	Aggregate economy	Same labour productivity	Diff. labour tax rates	τ^r	τ_s^w	τ_u^w	After-tax skill prem.	Progr. of lab. taxes
M1	No	No	No	Yes	No	0.00	0.20	0.20	1.27	0.00
M2	No	No	Yes	Yes	No	0.00	0.41	0.41	1.17	0.00
M3	No	Yes	Yes	Yes	No	0.04	0.37	0.37	0.99	0.00
M4	No	Yes	Yes	Yes	Yes	0.05	0.48	0.24	1.02	0.24
M5	No	No	Yes	Yes	Yes	0.00	0.50	0.36	0.76	0.14
M6	No	No	Yes	No	Yes	0.00	0.44	0.33	0.65	0.11
M7	No	Yes	Yes	No	No	0.04	0.37	0.37	0.83	0.00
M8	No	Yes	Yes	No	Yes	0.04	0.39	0.25	0.75	0.14
M9	Yes	Yes	No	Yes	No	0.09	0.39	0.39	1.67	0.00
M10	Yes	Yes	No	Yes	Yes	0.06	0.52	0.22	1.09	0.30
M11	Yes	Yes	Yes	Yes	No	0.09	0.40	0.40	1.67	0.00
M12	Yes	No	No	Yes	No	0.11	0.35	0.35	1.94	0.00
M13	Yes	No	No	Yes	Yes	0.00	0.80	0.17	0.65	0.63
M14	Yes	No	Yes	Yes	No	0.10	0.36	0.36	1.93	0.00
M15	Yes	No	Yes	Yes	Yes	0.00	0.67	-0.04	0.73	0.71
Bench	Yes	Yes	Yes	Yes	Yes	0.06	0.52	0.19	1.06	0.32
In the following models the government can also choose optimally the level of government consumption										
M16	Yes	No	Yes	Yes	Yes	0.00	0.28	-0.68	0.86	0.80
M17	Yes	Yes	Yes	Yes	Yes	0.01	0.19	-0.47	0.99	0.66
M18	Yes	No	Yes	Yes	No	0.13	-0.07	-0.07	1.83	0.00
M19	Yes	Yes	Yes	Yes	No	0.08	-0.04	-0.04	1.67	0.00

Moreover, when the representative agents are being used instead of the aggregate economy, the element presented in the third column is not present. When the elements in fourth column and fifth column are not present the agents have different productivity and they face the same labour income tax rate respectively.

The main findings from the analysis are: First, the capital-skill complementarity is an important element for the progressivity of the labour income tax rate and not for the non-zero optimal capital income tax rate. Second, the capital market imperfection is the main driving force of the positive optimal capital income tax rate. Third, the different labour income tax rate among the two types of agents, besides being the essential factor of the labour income tax rate progressivity, is also a key element for the optimal capital income tax rate when the production function exhibits capital-skill complementarity. Fourth, the productivity of the two types of agents and the aggregate economy affect only the level of the after-tax skill premium.

1.10 The model without capital-skill complementarity

In the case without capital-skill complementarity the Cobb-Douglas production function is being used instead. This case is helpful for reproducing the results of Judd (1985). The model in row M1 is a replication of Judd's model with a representative capitalist and worker, perfectly competitive markets and a single labour income tax rate. Therefore, the model in that row can replicate the result of zero optimal capital income tax rate.

Note that the model in row M1 makes the assumption that both agents can save and work. But in order for the model to have a unique solution is being assumed that only the capitalist (which is assumed to be the skilled agent) has access to perfect capital market ($\psi_s = 0$), whereas the worker (which is assumed to be the unskilled agent) needs to pay a cost of holding capital. However, in this case the optimal solution indicates that the worker will choose not to hold capital at all.

This result can be shown taking into account the two Euler equations for capital structures (A33, A37) of the two agents at the steady state:[22]

$$\frac{1}{\beta} = r_{st}(1-\tau^r) + (1-\delta_{st}) - 2\psi_s K^s_{st}$$

and

$$\frac{1}{\beta} = r_{st}(1-\tau^r) + (1-\delta_{st}) - 2\psi_u K^u_{st}$$

since the left hand side of the above equations is identical, the right hand side of the two equations should be equal:

$$r_{st}(1-\tau^r) + (1-\delta_{st}) - 2\psi_s K^s_{st} = r_{st}(1-\tau^r) + (1-\delta_{st}) - 2\psi_u K^u_{st}$$

taking into account the fact that $\psi_s = 0$:

$$r_{st}(1-\tau^r) + (1-\delta_{st}) = r_{st}(1-\tau^r) + (1-\delta_{st}) - 2\psi_u K^u$$

$$2\psi_u K^u_{st} = 0$$

and since, $\psi_u > 0$, the unskilled agents will not hold any capital at all no matter how large is the value of ψ_u:

$$K^u_{st} = 0$$

Therefore, setting $\psi_i = 0$, for $i = s$, or $i = u$, will always result to the capitalist and worker economy where only the capitalist holds capital.

Taking the above into consideration, Judd's (1985) theorem 4 is reinforced because even if only one of the two types of agents has access to perfect capital markets the optimal tax rate on capital income is zero. Specifically, when either of the two agents has no cost of holding capital the other agent will decide not to hold capital at all and the economy will be populated with capitalists that work and save and workers that only work.

When the production function doesn't exhibit capital-skill complementarities the zero optimal tax rate on capital income will remain valid even

[22] It makes no difference if the Euler equations for capital equipment are being chosen instead.

if the government can tax differently the labour income of capitalists and workers as the model in row M5. But in this case the government will choose to increase the progressivity of the labour income tax rate by taxing more the labour income of the agents that hold capital. As a result the after-tax skill premium is lower than the exogenous case which means that the government redistributes income to workers (the agents that do not hold capital) through labour income taxes.

As a result, with a single labour tax rate for both agents and no cost of capital holdings for the skilled agents (no capital market imperfection), the optimal policy suggests that capital income tax rate will be zero and the labour income tax rate will increase to balance the budget constraint. Thus the level of government consumption will remain constant, as in the exogenous fiscal policy case. But in the case where the government can impose two different wage tax rates, it will choose progressive labour income tax rates.

The difference in the productivity between skilled and unskilled agents affects only the progressivity of the labour income tax rate and as a consequence the after-tax skill premium (see the difference between the models in rows M5 and M6).

The progressivity that a model without capital-skill complementarity reports is at the range of 0.11-0.14 with an exemption of 0.24 for the case where all elements are present (model in row M4). The progressivity reported from these models is lower than what it is observed in the data, apart from the case where it is equal to 0.24.[23]

When the model assumes that both types of agents have access to imperfect capital markets (ψ_s and ψ_u are positive but $\psi_s < \psi_u$) the optimal tax rate on capital income becomes positive. This positive result on capital income tax rate is not affected from any of the other elements considered earlier (see models in rows M3, M4, M7 and M8).

Therefore, for the case without capital-skill complementarity the neces-

[23] The data from the UK government in 2012 report that the progressivity of income tax rate between the high income earners and the rest of the population is approximately equal to 30%.

sary condition of a non-zero optimal tax rate on capital income is that both agents have a positive cost of capital holdings. This result indicates that the main driving force of the non-zero optimal tax rate on capital income, under no capital-skill complementarity, is the imperfection in capital markets through the cost of holding capital.

1.11 The model with capital-skill complementarity

When the model exhibits capital-skill complementarity the lower optimal capital income tax rate will benefit more the skilled agents (all the models in rows M9-M19). That happens because the increase in the stock of capital, due to the lower capital income tax rate, will also increase the demand for skilled agents and their wage rate, driving the pre-tax skill premium to increase.

In the case with a single labour tax rate the government will not be able to redistribute efficiently the income among the agents (see model in rows M9, M11, M12 and M14) and as a result the after-tax skill premium will remain high.

In the models where only the skilled agents have access to perfect capital market ($\psi_s = 0$ and $\psi_u > 0$), the model collapses to the capitalist and worker situation, as it has been discussed in the previous subsection (see models in rows M12-M15). However, when capital-skill complementarity is present, the result of optimal tax rate on capital income is different than the Cobb-Douglas specification. In the case where the government cannot tax differently the labour income of the two types of agents, the optimal tax rate on capital income will be positive, as the models in rows M12 and M14. This result is valid even when a representative agent model is being solved or even when the productivity is different among the two types of agents.[24]

The intuition of this result is that in the case with capital-skill complementarity the government can redistribute income efficiently from skilled to unskilled agents through the tax rate on capital income when there is only a

[24] Also, Conesa *et al.* (2009) obtain a similar result. Using a life cycle model they show that when the government cannot impose a labour income tax rate that will depend on age, the optimal capital income tax will become positive.

single labour income tax rate. This result is different than the Cobb-Douglas case, presented in the previous section, because now skilled agents are complementary with capital. Thus, a lower stock of capital, due to the positive capital income tax rate, will also reduce the demand for skilled agents lowering their pre-tax wage rate. In addition, since unskilled agents are substitutes with capital and skilled agents, their wage rate will increase. As a result, in the case of capital-skill complementarity, if the government cannot impose two different labour income tax rates, it can still redistribute income through the taxation of capital income.

But when the government has access to two different labour income tax rates, as the models in rows M13 and M15, and when only skilled agents have access to perfect capital markets, the government will prefer to redistribute income efficiently by increasing the progressivity of the labour income tax rate. Therefore, in these models the optimal capital income tax rate is zero, as in the Cobb-Douglas cases. However, with capital-skill complementarity the progressivity of the labour income tax rate is much higher than the Cobb-Douglas case studies and it can replicate the progressivity observed in the UK economy. In addition, because of the higher labour income tax rate progressivity the after-tax skill premium is much lower than the previous case.

The result of high labour income progressivity is similar to Conesa *et al.* (2009) result. In a life cycle model with heterogeneous agents, with respect to labour productivity, they show that capital income tax rate will be positive and the government will choose to redistribute income through increasing the labour income tax rate progressivity.

In the case where both agents have access to imperfect capital markets (ψ_s and ψ_u are positive but with $\psi_s < \psi_u$), as in models in rows M9-M11, the government will always impose a positive capital income tax rate. This result is not affected from the other elements of the model.

To summarize, the above analysis indicates that the main driving force of the non-zero optimal tax rate on capital income is the imperfection in capital markets, as in Judd (1997) and Guo and Lansing (1999). In addition, there is a case study where the non-zero optimal tax rate on capital income can be obtained even with perfect capital markets. That happens when the government cannot impose two different labour income tax rates, which verifies the argument of Correia (1996). In this case, the government can still redistribute income through capital income taxation due to the presence of capital-skill complementarity.

1.12 Optimally chosen government consumption

The models in rows M16-M19 make the additional assumption that the government can also choose optimally the level of government consumption. In this case the qualitative results remain the same as before. However, progressivity of labour income tax rate is even higher when the government has access to two different labour income tax rates. Also, the unskilled labour income tax rate becomes negative (subsidy) and the optimal capital income tax rate remains positive but lower than the benchmark case (models in rows M16-M17).

Under this setup the level of government consumption that will be chosen depends on its weight in the effective consumption. This is denoted with the parameter α in Figure 1.1.[25] indicates how the government consumption, capital and labour income tax rates change as the share of government consumption to the effective consumption increases. The model presented and analysed in the figure is the model in row M17 from Table 1.7, where all the elements of the model are present.

[25] At the benchmark case the weight of the government consumption to the effective consumption is: $1 - \alpha = 1 - 0.9 = 0.1$. So as α decreases the weight of the government consumption on the effective consumption increases.

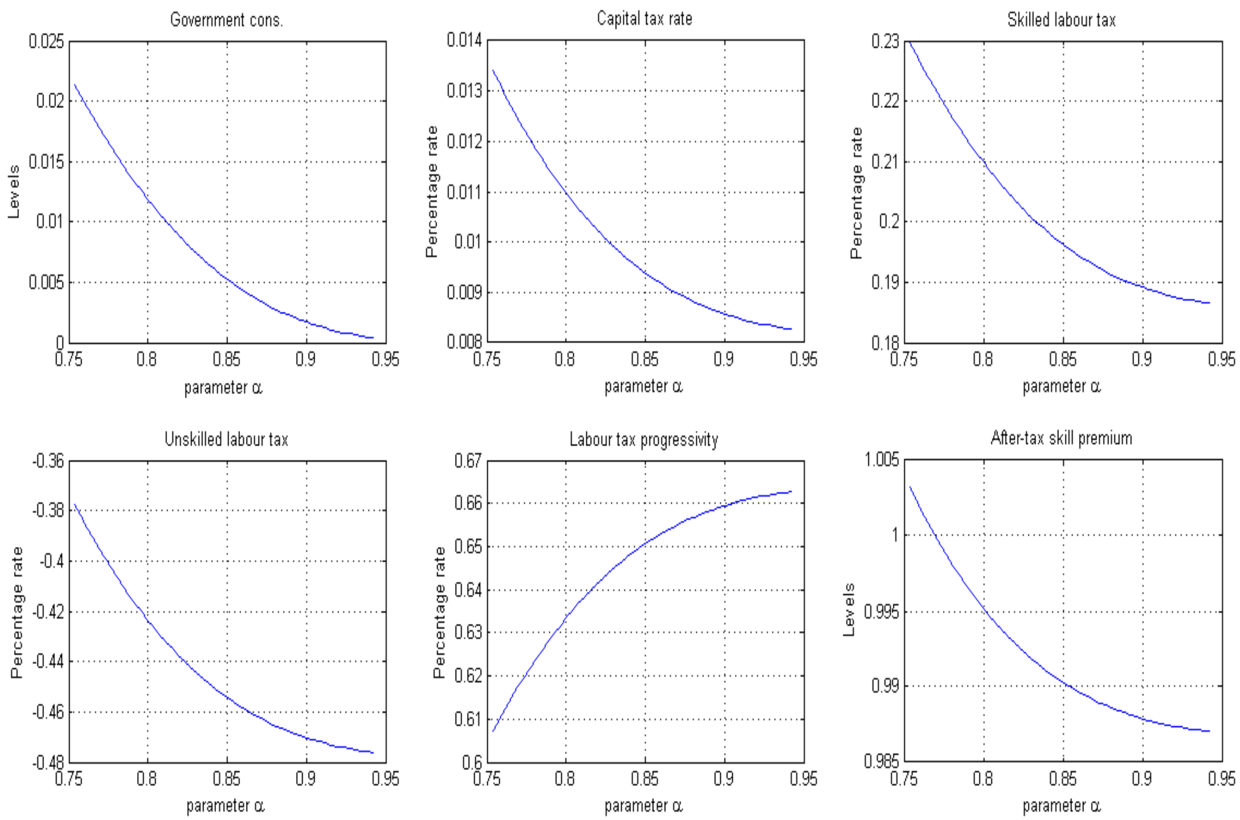

Figure 1.1: Sensitivity of the optimal taxation with respect to the weight on private consumption.

When the share of government consumption on the effective consumption, $(1-a)$, decreases so does the level of government consumption, all the tax rates and the after-tax skill premium. However, even though the labour income tax rates for both types of agents decrease the progressivity of the labour taxes increases. That occurs because when the tax rate on capital income decreases the government relies more on the labour income tax rates for income redistribution.

1.13 Assessing income inequality

Table 1.8 summarizes the results of income inequality under optimal fiscal policy for various versions of the model. The last two columns of the table compare the overall after-tax income share of the skilled agent. If this share declines, when the optimal fiscal policy is in place, it means that unskilled agents have higher share of the overall after-tax income and as a result income inequality is reduced.

Note that the table does not include a column for the change in the after-tax capital income because the capital income share of each type of agent is assumed to be fixed. In particular, skilled agents will always hold two times more capital stock than the unskilled agents due to the assumed cost of capital holdings. This will result in the skilled agents having two times higher capital income than the unskilled agents, given that the returns to capital and capital income tax rate is the same for both types of agents.

Table 1.8: Summary of income inequality results

Model	Capital-Skill complem.	Capital markets imperfection ($\psi_s, \psi_u > 0$)	Different labour tax rates ($\tau_s^w \neq \tau_u^w$)	after-tax skill premium (Exog.)	after-tax skill premium (Opt.)	skilled agents' share of total after-tax inc. (Exog.)	skilled agents' share of total after-tax inc. (Opt.)
L1	No	Yes	Yes	0.84	0.94	48.9%	53.7%
L2	No	No	Yes	0.90	0.76	55.4%	58.6%
L3	No	Yes	No	0.94	0.99	51.6%	54.8%
L4	No	No	No	1.02	1.17	58.2%	66.9%
L5	Yes	No	Yes	1.36	0.73	68.0%	61.7%
L6	Yes	Yes	No	1.40	1.67	60.7%	64.2%
L7	Yes	No	No	1.54	1.89	70.4%	75.8%
Bench	Yes	Yes	Yes	1.24	1.06	58.5%	57.1%

Taking the above into consideration the important factor that will reduce income inequality is labour income tax rates. The government will not be able to reduce income inequality unless it has access to two different labour income tax rates. In fact, optimal fiscal policy with a single wage tax rate will increase income inequality between the two types of agents (see models in rows L3, L4, L6 and L7).

The capital market imperfection element is affecting the level of income inequality and income distribution among the two types of agents. When there is no capital market imperfection (meaning that only skilled agents have access to perfect capital markets, $\psi_s = 0$ and $\psi_u > 0$, and as a result only skilled agents will hold capital) the optimal fiscal policy will result in even higher income inequality. Also, skilled agents will hold a larger proportion of total after-tax income when the same model is compared with the case where capital market imperfection is present (see models in rows L1-L2, L3-L4 and L6-L7).

When capital-skill complementarity is not present, the optimal fiscal policy cannot redistribute income efficiently. As a result skilled agents' share of income increases when the optimal fiscal policy is in place (see models in rows L1-L4). But when capital-skill complementarity is present and the government has access to two different labour income tax rates for each type of agent, the skilled agents' share of total after-tax income in the economy decreases but they still hold the majority of the available after-tax income.

1.14 Partisan policy

Throughout the analysis so far it has been assumed that under optimal fiscal policy the government wants to maximize the aggregate welfare in the economy. Under this assumption the central authority does not show any preference towards a certain type of agents. This means that in the objective function (17) the weight parameter for skilled agents will be equal to their share in the economy.[26]

[26] Therefore, under the calibration provided in a previous section: $\omega = n_s = 0.6$. As a result the relevant weight in the objective function for the unskilled agents is: $(1 - \omega) = n_u = 0.4$.

This section will re-assess the optimal fiscal policy outcome for the case where the central authority places more weight to one of the two types of agents. The only difference with the analysis provided in the optimal fiscal policy is that the objective function of the government will become:

$$\max \sum_{t=0}^{\infty} \beta^t \left(\omega \times U_s \left(C^*_{s,t}, h_{s,t} \right) + (1-\omega) \times U_u \left(C^*_{u,t}, h_{u,t} \right) \right) \tag{17}$$

subject to the same DCE system of equations as presented in the Appendix A for the optimal fiscal policy.

The procedure is identical to the optimal fiscal policy presented earlier with the only difference that here the variable that determines the weight the government places to a specific type of agents is not equal to the share of each type of agent. In this subsection the variable, ω, can take any value between 0 and 1. For example, when this value exceeds 0.6 the government places more weight to skilled agents, since at the calibration subsection earlier on we assumed that skilled agents are 60% of the population. As a result, the assumption of a Benthamite government, where $\omega = 0.6$, will be dropped and a Partisan government will be assumed instead.

Figures 1.2 and 1.3 present how the tax rates, income inequality and main macroeconomic variables react as the variable ω increases when the Partisan government sets the optimal fiscal policy.

In particular, Figure 1.2 shows that as the weight placed to skilled agents increases the optimal tax rate on capital income declines. But when $\omega \simeq 0.76$ the optimal tax rate on capital income reaches the lowest value of 5%. After that point optimal capital income tax rate increases again. Therefore, under the assumed setup and calibration the optimal tax rate on capital income will be positive irrespectively of the weight the government places to a certain group of agents.

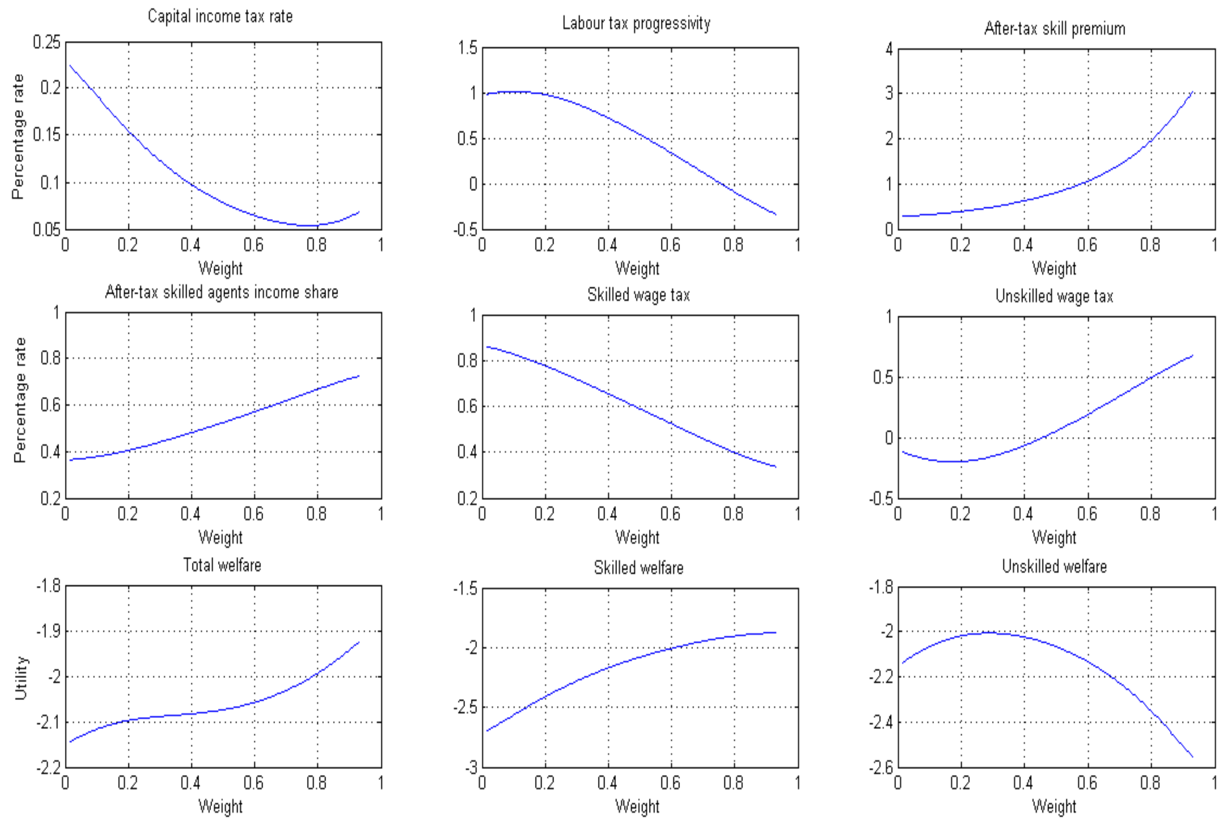

Figure 1.2: Optimal fiscal policy of a Partisan government.

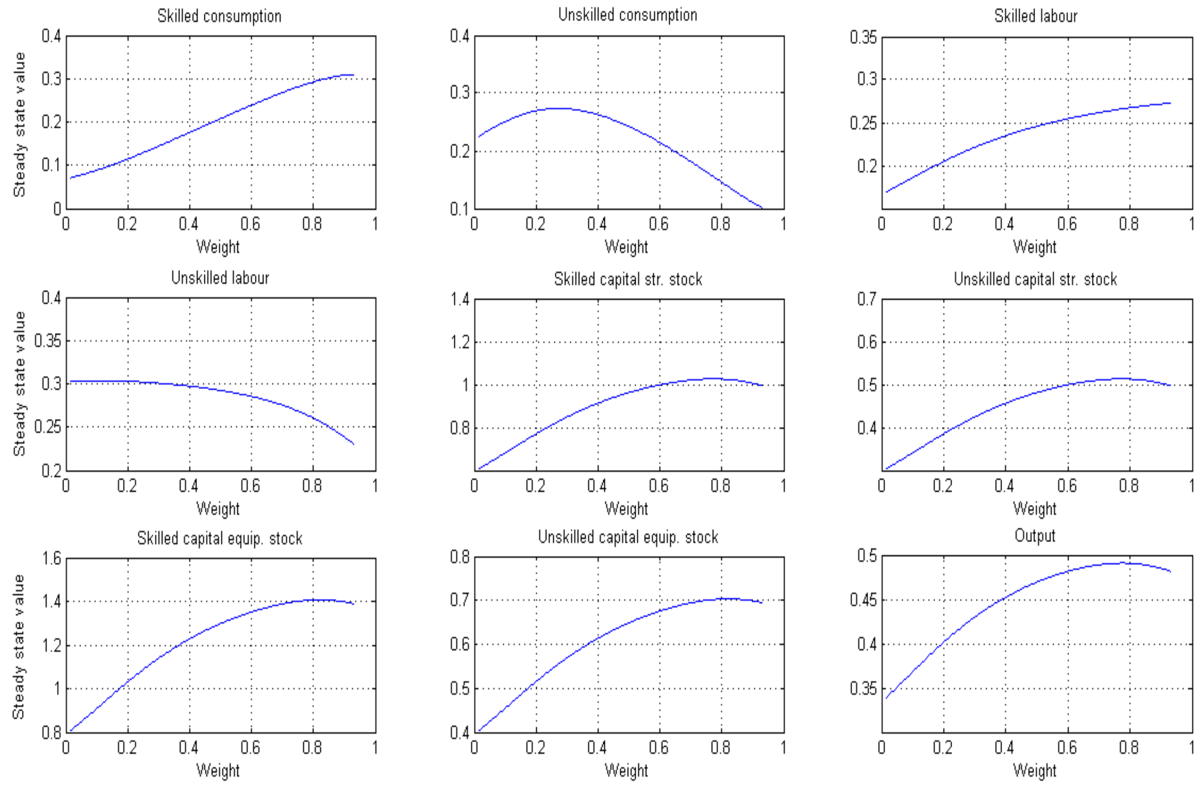

Figure 1.3: The effect of optimal fiscal policy on macroeconomic variables under a Partisan government.

The upper middle graph in Figure 1.2 shows the evolution of labour tax progressivity (defined as $\tau^w_{s,t} - \tau^w_{u,t}$). For very low values of the variable ω progressivity shows a slight increase but once it exceeds the value of 0.13 it starts to decline. The labour tax progressivity will become negative, which means that $\tau^w_s < \tau^w_u$, when the weight placed to skilled agents exceeds the value of 0.75.

The upper right hand side graph and the middle left hand side graph of Figure 1.2 show the evolution of after-tax skill premium and after-tax share of skilled agents on total income respectively. Both of these variables are positively correlated with the weight placed to skilled workers. In particular, the skilled agents will hold the majority of the after-tax income when ω becomes higher than 0.45.

The other two graphs in the middle row show the evolution of the wage tax rates for both agents. For skilled agents the wage tax rate decreases at a relative constant rate as variable ω increases. On the other hand, the unskilled wage tax rate decreases for low values of the variable ω and it starts to increase when the weight exceeds the value of 0.2.

The bottom row in Figure 1.2 shows the effects on the aggregate, skilled and unskilled agents' welfare. Regarding aggregate welfare the graph shows that it increases at an increasing rate as ω increases, with the exception of the range $0.2 - 0.4$ where aggregate welfare increases at a decreasing rate. Skilled agents' welfare has a positive relationship with ω. But the welfare of unskilled agents increases until the variable ω reaches the value of 0.3 and then it starts to decrease. The welfare of skilled and unskilled agents becomes equal when the government places a weight of 45% to skilled agents in the objective function.

Figure 1.3 includes graphs that show the evolution of the stock of capital held from each type of agent. The results are very similar between the two types of agents and the two types of capital (capital equipment and capital structures). Specifically, as the variable ω increases the stock of capital increases as well until the point where the optimal tax rate on capital income starts to increase again. That happens when ω exceeds the value of 0.75 for capital structures and 0.81 for capital equipment.

The graphs for the evolution of skilled consumption and skilled labour supply are positively correlated with ω. Whereas, the evolution of unskilled agents' consumption increases for small values of ω but when it exceeds 0.3 declines. The level of labour supply of the unskilled agents exhibits a negative correlation with ω. Finally, output follows the same pattern as the stock of capital, it increases as ω increase but it starts to decline when ω is higher than 0.78.

The main conclusions from the above analysis are: (i) the non-zero result of the optimal tax rate on capital income is not affected from the weight the government places to a certain group of agents, but the magnitude of the optimal capital income tax rate is affected; (ii) the progressivity of labour income tax rate turns negative, which means that $\tau_s^w < \tau_u^w$, when the weight placed to skilled agents exceeds the value of 0.75; (iii) aggregate welfare has a positive relationship with the weight the government places to skilled agents; (iv) as the optimal tax rate on capital income decreases the stock of capital increases and, due to the presence of capital-skill complementarity, skilled labour increases as well.

1.15 Concluding remarks

This chapter studied the outcome of optimal fiscal policy under skills and capital holdings heterogeneity. It is further assumed that the production function exhibits capital-skill complementarities. This way the model is able to reproduce the wage premium and factor input elasticities suggested in the literature.

The results suggested that under optimal fiscal policy a benevolent government will not choose to eliminate the tax rate on capital income, as Judd (1985) and Chamley (1986) suggest. The analysis showed that the main driving force of the non-zero optimal tax rate on capital income is the imperfection in capital markets, which is in line with the results of Judd (1997) and Guo and Lansing (1999). However, in the case where the government cannot impose two different labour income tax rates, the positive optimal tax rate on capital income will appear even if there is no capital market imperfection, verifying the argument of Correia (1996). In this case the government will redistribute income through capital income taxation due to the presence of capital-skill complementarity. But when capital-skill complementarity is not present the non-zero optimal tax rate on capital income will appear only if the capital market imperfection is present.

Another important result is the progressivity of labour income tax rates under optimal fiscal policy. In particular, when capital-skill complementarity is present and the government has access to two different labour income tax rates, income inequality will be reduced through the lower after-tax skill premium. But the government cannot reduce income inequality when capital-skill complementarity is not present. Therefore, under optimal fiscal policy the government is able to reduce income inequality and improve the welfare for both types of agents making the outcome Pareto efficient only when capital-skill complementarity is present.

These results remain valid even if the government doesn't need to satisfy a given level of consumption and as a result can optimally choose the tax rates and the level of government consumption. Finally, the conclusions remain valid even when the government places more weight to a certain group of agents, as in Chari and Kehoe (1999).

Finally, when we drop the assumption of the Benthamite planner and we allow for a partisan government, then we found that the non-zero result of the optimal tax rate on capital income is not affected from the weight the government places to skilled (or equivalently unskilled) agents. However, the level of the optimal tax rate on capital income is sensitive to the weight of the skilled agents in the objective function of the Partisan government. Moreover, the progressivity of labour income tax rate is overturned (it become regressive) when the weight placed to skilled agents exceeds the value of 75%. Also, aggregate welfare exhibits a positive relationship with the weight the government places to skilled agents.

Appendix A

A Chapter 1

A.1 First order conditions

A.1.1 Households

In period t households solve the following maximization problem:

$$\max_{C_{j,t},h_{j,t},K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(\left(C^*_{j,t}\right)^{\gamma} l_{j,t}^{1-\gamma}\right)^{(1-\sigma)}}{1-\sigma}$$

where, $l_{j,t} = 1 - h_{j,t}$ and $C^*_{j,t} = \left[aC^{\mu}_{j,t} + (1-a)\left(\bar{G}^c_t\right)^{\mu}\right]^{1/\mu}$, subject to their budget constraint depending on the type of the agent:

$$\begin{aligned} C_{j,t} + I^j_{st,t} + I^j_{eq,t} &= (1-\tau^w_{j,t})w_{j,t}h_{j,t} + (1-\tau^r_t)\left(r_{st,t}K^j_{st,t} + r_{eq,t}K^j_{eq,t}\right) - \\ &\quad -\psi_j\left[\left(K^j_{st,t}\right)^2 + \left(K^j_{eq,t}\right)^2\right] \end{aligned}$$

or if we replace the capital evolution equations:

$$\begin{aligned} &(1-\tau^w_{j,t})w_{j,t}h_{j,t} + (1-\tau^r_t)\left(r_{st,t}K^j_{st,t} + r_{eq,t}K^j_{eq,t}\right) - \\ &-\psi_j\left[\left(K^j_{st,t}\right)^2 + \left(K^j_{eq,t}\right)^2\right] \\ = \;& C_{j,t} + K^j_{st,t+1} - (1-\delta_{st})K^j_{st,t} + K^j_{eq,t+1} - (1-\delta_{eq})K^j_{eq,t} \end{aligned}$$

$$\begin{aligned} C_{j,t} + K^j_{st,t+1} + K^j_{eq,t+1} &= (1-\tau^w_{j,t})w_{j,t}h_{j,t} + \\ &\quad + \left[(1-\tau^r_t)r_{st,t} + (1-\delta_{st})\right]K^j_{st,t} + \\ &\quad + \left[(1-\tau^r_t)r_{eq,t} + (1-\delta_{eq})\right]K^j_{eq,t} - \\ &\quad -\psi_j\left[\left(K^j_{st,t}\right)^2 + \left(K^j_{eq,t}\right)^2\right]. \end{aligned} \quad \text{(A1)}$$

Also we should note that the stock of capital structures and capital equipment in period t are known and as a result households want to maximize the two types of capital in period $t+1$. In addition, E_0 is the conditional expectation in the initial period given the available information in that period.

If it assumed that Λ_t is the Lagrange multiplier associated with the budget constraint, equation (A1), the following Lagrange equation is obtained:

$$\begin{aligned} \mathcal{L} \;=\; & \max_{C_t,h_t,K_{t+1}} E_0\sum_{i=0}^{\infty}\beta^i\frac{(\left[aC^{\mu}_{j,t+i}+(1-a)\left(\bar{G}^{c}_{t+i}\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j,t+i})^{1-\gamma})^{1-\sigma}}{1-\sigma} \\ & +\Lambda_{t+i}((1-\tau^{w}_{j,t})w_{j,t+i}h_{j,t+i}+ \qquad\qquad (\text{A2}) \\ & +\left[(1-\tau^{r}_{t+i})r_{st,t+i}+(1-\delta_{st})\right]K^{j}_{st,t+i}+ \\ & +\left[(1-\tau^{r}_{t+i})r_{eq,t+i}+(1-\delta_{eq})\right]K^{j}_{eq,t+i}- \\ & -\psi_j\left[\left(K^{j}_{st,t+i}\right)^2+\left(K^{j}_{eq,t+i}\right)^2\right]-C_{j,t+i}-K^{j}_{st,t+i+1}-K^{j}_{eq,t+i+1}). \end{aligned}$$

Since the aim is for the maximization problem to be solved for a period $s \geq t$ it would be easier, for the derivations later on, to write equation (A2) as:

$$\begin{aligned} & E_0\{\sum_{i=0}^{s-t-1}\beta^i[\frac{(\left[aC^{\mu}_{j,t+i}+(1-a)\left(\bar{G}^{c}_{t+i}\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j,t+i})^{1-\gamma})^{1-\sigma}}{1-\sigma}+ \\ & +\Lambda_{t+i}(-(1-\tau^{w}_{j,t+i})w_{j,t+i}h_{j,t+i}+ \\ & +\left[(1-\tau^{r}_{t+i})r_{st,t+i}+(1-\delta_{st})\right]K^{j}_{st,t+i}- \\ & -\psi_j\left[\left(K^{j}_{st,t+i}\right)^2+\left(K^{j}_{eq,t+i}\right)^2\right]+ \\ & +\left[(1-\tau^{r}_{t+i})r_{eq,t+i}+(1-\delta_{eq})\right]K^{j}_{eq,t+i}- \\ & -C_{j,t+i}-K^{j}_{st,t+i+1}-K^{j}_{eq,t+i+1})]+ \\ & +\beta^{s-t}[\frac{(\left[aC^{\mu}_{j,s}+(1-a)\left(\bar{G}^{c}_{s}\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j.s})^{1-\gamma})^{1-\sigma}}{1-\sigma}+ \\ & +\Lambda_s((1-\tau^{w}_{j,s})w_{j,s}h_{j,s}-\psi_j\left[\left(K^{j}_{st,s}\right)^2+\left(K^{j}_{eq,s}\right)^2\right]+ \end{aligned}$$

$$
\begin{aligned}
&+\left[(1-\tau_s^r)r_{st,s}+(1-\delta_{st})\right]K_{st,s}^j-C_{j,s}-K_{st,s+1}^j-K_{eq,s+1}^j)+\\
&+\left[(1-\tau_s^r)r_{eq,s}+(1-\delta_{eq})\right]K_{eq,s}^j]+\\
&+\beta^{s-t+1}[\frac{(\left[aC_{j,s+1}^{\mu}+(1-a)\left(\bar{G}_{s+1}^c\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j,s+i})^{1-\gamma})^{1-\sigma}}{1-\sigma}+\\
&+\sum_{i=s-t+2}^{\infty}\beta^i[\frac{(\left[aC_{j,t+i}^{\mu}+(1-a)\left(\bar{G}_{t+i}^c\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j,t+i})^{1-\gamma})^{1-\sigma}}{1-\sigma}+\\
&+\Lambda_{t+i}((1-\tau_{j,t+i}^w)w_{j,t+i}h_{j,t+i}+\left[(1-\tau_{t+i}^r)r_{st,t+i}+(1-\delta_{st})\right]K_{st,t+i}^j+\\
&+\left[(1-\tau_{t+i}^r)r_{eq,t+i}+(1-\delta_{eq})\right]K_{eq,t+i}^j-\\
&-\psi_j\left[\left(K_{st,t+i}^j\right)^2+\left(K_{eq,t+i}^j\right)^2\right]-C_{j,t+i}-K_{st,t+i+1}^j-K_{eq,t+i+1}^j)]\}\ .
\end{aligned}
$$

Next the first-order conditions are presented at time $s \geq t$:[27]

Private consumption in period s:

$$
\frac{\partial\mathcal{L}}{\partial C_{j,s}}=\frac{\partial\{\beta^{s-t}[\frac{(\left[aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_s^c\right)^{\mu}\right]^{\gamma/\mu}(1-h_{j,s})^{1-\gamma})^{1-\sigma}}{1-\sigma}-\Lambda_s C_{j,s}]\}}{\partial C_{j,s}}=0
$$

$$
\begin{aligned}
0 &= \beta^{s-t}[\frac{\gamma(1-\sigma)\,aC_{j,s}^{\mu-1}\left(aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_s^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}}{1-\sigma}\times\\
&\times(1-h_{j,s})^{(1-\gamma)(1-\sigma)}-\Lambda_s]
\end{aligned}
$$

$$
\begin{aligned}
0 &= \beta^{s-t}[\gamma aC_{j,s}^{\mu-1}\left(aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_s^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}\times\\
&\times(1-h_{j,s})^{(1-\gamma)(1-\sigma)}-\Lambda_s].
\end{aligned}
$$

since $0<\beta<1$ and as a result it cannot be zero:

$$
\gamma aC_{j,s}^{\mu-1}\left(aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_s^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{j,s})^{(1-\gamma)(1-\sigma)}=\Lambda_s \quad \text{(A3)}
$$

[27] For simplicity all the factors that do not affect the derivations are removed from the first-order conditions.

Labour in period s:

$$\frac{\partial \mathcal{L}}{\partial h_{j,s}} = 0$$

$$\begin{aligned} 0 &= \frac{\partial}{\partial h_s}\{\beta^{s-t}[\frac{(\left[aC_{j,s}^{\mu} + (1-a)\left(\bar{G}_s^c\right)^{\mu}\right]^{\gamma/\mu}(1-h_s)^{1-\gamma})^{1-\sigma}}{1-\sigma} + \\ &+ E_s\Lambda_s(w_{j,s}(1-\tau_{j,s}^w)h_{j,s}]\} \end{aligned}$$

$$\begin{aligned} 0 &= \beta^{s-t}[\frac{(-1)(1-\gamma)(1-\sigma)\left(aC_{j,s}^{\mu} + (1-a)\left(\bar{G}_s^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)}}{1-\sigma} \times \\ &\times (1-h_{j,s})^{(1-\gamma)(1-\sigma)-1} + E_s\Lambda_s(w_{j,s}(1-\tau_{j,s}^w))] \end{aligned}$$

$$\begin{aligned} & (1-\gamma)\left(aC_{j,s}^{\mu} + (1-a)\left(\bar{G}_s^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{j,s})^{(1-\gamma)(1-\sigma)-1} \\ = \;& E_s\Lambda_s(w_{j,s}(1-\tau_{j,s}^w)) \end{aligned} \tag{A4}$$

Capital structures in period $s+1$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_{st,s+1}^j} &= \frac{\partial\{\beta^{s-t}\left(-\Lambda_s K_{st,s+1}^j\right)\}}{\partial K_{st,s+1}^j} + \\ &+ \frac{\partial\{\beta^{s-t+1}E_s\Lambda_{s+1}[((1-\tau_{s+1}^r)r_{st,s+1} + (1-\delta_{st}))K_{st,s+1}^j - \psi_j\left(K_{st,s+1}^j\right)^2]\}}{\partial K_{st,s+1}^j} = 0 \end{aligned}$$

$$-\beta^{s-t}\Lambda_s + \beta^{s-t+1}E_s[\Lambda_{s+1}\left(r_{st,s+1}(1-\tau_{s+1}^r) + (1-\delta_{st}) - 2\psi_j K_{st,s+1}^j\right)] = 0$$

$$-\Lambda_s + \beta E_s[\Lambda_{s+1}\left(r_{st,s+1}(1-\tau_{s+1}^r) + (1-\delta_{st}) - 2\psi_j K_{st,s+1}^j\right)] = 0$$

$$\Lambda_s = \beta E_s[\Lambda_{s+1}\left(r_{st,s+1}(1-\tau_{s+1}^r) + (1-\delta_{st}) - 2\psi_j K_{st,s+1}^j\right)] \tag{A5}$$

Capital equipment in period $s+1$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_{eq,s+1}^j} &= \frac{\partial\{\beta^{s-t}\left(-\Lambda_s K_{eq,s+1}^j\right)\}}{\partial K_{eq,s+1}^j} + \\ &+ \frac{\partial\{\beta^{s-t+1}E_s[\Lambda_{s+1}\left[\left[(1-\tau_{s+1}^r)r_{eq,s+1} + (1-\delta_{eq})\right]K_{eq,s+1}^j - \psi_j\left(K_{eq,s+1}^j\right)^2\right]\}}{\partial K_{eq,s+1}^j} = 0 \end{aligned}$$

$$-\beta^{s-t}\Lambda_s + \beta^{s-t+1}E_s[\Lambda_{s+1}\left(r_{eq,s+1}(1-\tau^r_{s+1}) + (1-\delta_{eq}) - 2\psi_j K^j_{eq,s+1}\right)] = 0$$

$$-\Lambda_s + \beta E_s[\Lambda_{s+1}\left(r_{eq,s+1}(1-\tau^r_{s+1}) + (1-\delta_{eq}) - 2\psi_j K^j_{eq,s+1}\right)] = 0$$

$$\Lambda_s = \beta E_s[\Lambda_{s+1}\left(r_{eq,s+1}(1-\tau^r_{s+1}) + (1-\delta_{eq}) - 2\psi_j K^j_{eq,s+1}\right)] \tag{A6}$$

Now, since we know that:

$$\frac{\partial u(\cdot)}{\partial C_{j,s}} = \gamma a C^{\mu-1}_{j,s}\left(aC^{\mu}_{j,s} + (1-a)\left(\bar{G}^c_s\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{j,s})^{(1-\gamma)(1-\sigma)}$$

equation (A3) can be written as:

$$\frac{\partial u(\cdot)}{\partial C_{j,s}} = \Lambda_s \tag{A7}$$

By using equation (A7) for period s and for one period ahead $(s+1)$ equations (A5) and (A6) will become:

$$\begin{aligned}
& \gamma a C^{\mu-1}_{j,s}\left(aC^{\mu}_{j,s} + (1-a)\left(\bar{G}^c_s\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{j,s})^{(1-\gamma)(1-\sigma)} \\
= \; & \beta E_s[(\gamma a C^{\mu-1}_{j,s+1}\left(aC^{\mu}_{j,s+1} + (1-a)\left(\bar{G}^c_{s+1}\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1} \times \\
& \times(1-h_{j,s+1})^{(1-\gamma)(1-\sigma)})\left(r_{st,s+1}(1-\tau^r_{s+1}) + (1-\delta_{st}) - 2\psi_j K^j_{st,s+1}\right)]
\end{aligned}$$

$$\frac{\partial u(\cdot)}{\partial C_{j,s}} = \beta E_s[\frac{\partial u(\cdot)}{\partial C_{j,s+1}}\left(r_{st,s+1}(1-\tau^r_{s+1}) + (1-\delta_{st}) - 2\psi_j K^j_{st,s+1}\right)] \tag{A8}$$

and

$$\begin{aligned}
& \gamma a C^{\mu-1}_{j,s}\left(aC^{\mu}_{j,s} + (1-a)\left(\bar{G}^c_s\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{j,s})^{(1-\gamma)(1-\sigma)} \\
= \; & \beta E_s[(\gamma a C^{\mu-1}_{j,s+1}\left(aC^{\mu}_{j,s+1} + (1-a)\left(\bar{G}^c_{s+1}\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1} \times \\
& \times(1-h_{j,s+1})^{(1-\gamma)(1-\sigma)})\left(r_{eq,s+1}(1-\tau^r_{s+1}) + (1-\delta_{eq}) - 2\psi_j K^j_{eq,s+1}\right)]
\end{aligned}$$

$$\frac{\partial u(\cdot)}{\partial C_{j,s}} = \beta E_s[\frac{\partial u(\cdot)}{\partial C_{j,s+1}}\left(r_{eq,s+1}(1-\tau^r_{s+1}) + (1-\delta_{eq}) - 2\psi_j K^j_{eq,s+1}\right)] \tag{A9}$$

The above equation is the well-known Euler equation and defines the consumption-savings optimality condition. Sets the marginal return of substitution between present and future consumption equal to the after tax returns on savings. Households equate the cost from saving one additional unit of today's consumption to the benefit of obtaining more consumption tomorrow. Therefore, the above Euler equation describes the optimal trade-off between current and future consumption.

Dividing equation (A4) by equation (A3) gives:

$$\frac{(1-\gamma)\left(aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_{s}^{c}\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{j,s})^{(1-\gamma)(1-\sigma)-1}}{\gamma aC_{j,s}^{\mu-1}\left(aC_{j,s}^{\mu}+(1-a)\left(\bar{G}_{s}^{c}\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{j,s})^{(1-\gamma)(1-\sigma)}}$$

$$= \frac{E_s\Lambda_s(w_{j,s}(1-\tau_{j,s}^{w}))}{\Lambda_s}$$

$$\frac{\partial u\left(\cdot\right)}{\partial h_{j,s}}=\frac{\partial u\left(\cdot\right)}{\partial C_{j,s}}E_s(w_{j,s}(1-\tau_{s}^{w})) \tag{A10}$$

Which is the consumption-leisure optimality condition. Under that condition the marginal rate of substitution between leisure and consumption equals the after tax wage rate. Shows that the agent (worker) equates the marginal utility from a marginal increase in leisure to the utility from working an equal amount and getting more consumption.

In addition to the above the transversality condition needs to be added:

$$\lim_{t\to\infty}\beta^{t}\frac{\partial u_t\left(\cdot\right)}{\partial C_{j,t}}K_{i,t+1}^{j}=0$$

The intuition of the above condition is that in an optimal path, the values of capital stock for a given period far enough into the future have zero weight to the maximization problem. In addition, the transversality condition states that the utility gains from accumulating capital eventually grow slower than $1/\beta$. This condition is crucial in order for the economy to reach a stationary state equilibrium, otherwise it would be optimal for the individual to postpone consumption forever.

A.1.2 The firm

Firms select the quantity of each type of labour that they are going to hire and the amount of capital (both structures and equipment) that they are going to rent in order to maximize their profits:

$$\begin{aligned}\Pi_t &= \max_{h_{s,t},h_{u,t},K_{st,t}K_{eq,t}} A_t K_{st,t}^{a_c}[\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho}+ \\ &+(1-\lambda)\,h_{fu,t}^{\varphi}]^{\frac{1-a^c}{\varphi}}-w_{s,t}h_{fs,t}- \\ &-w_{u,t}h_{fu,t}-r_{st,t}K_{st,t}-r_{eq,t}K_{eq,t}.\end{aligned}$$

The relevant first-order conditions for period t are:

For the skilled agent:

$$\frac{\partial \Pi_s}{\partial h_{fs,t}}=0$$

$$\begin{aligned}0 &= \lambda(1-\nu)\left(\frac{1-a^c}{\varphi}\right)\frac{\varphi}{\rho}\rho h_{fs,t}^{\rho-1}A_t K_{st,t}^{a_c}\times \\ &\times[\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho}+ \\ &+(1-\lambda)\,h_{fu,t}^{\varphi}]^{\frac{1-a^c}{\varphi}-1}-w_{s,t}\ .\end{aligned}$$

$$\begin{aligned}w_{s,t} &= \lambda(1-\nu)(1-a^c)\,h_{fs,t}^{\rho-1}A_t K_{st,t}^{a_c}\times \\ &\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\,h_{fu,t}^{\varphi}\end{array}\right]^{\frac{1-a^c}{\varphi}-1}.\end{aligned}$$

$$\begin{aligned}w_{s,t} &= \lambda(1-\nu)(1-a^c)\,h_{fs,t}^{\rho-1}A_t K_{st,t}^{a_c}\times \\ &\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\,h_{fu,t}^{\varphi}\end{array}\right]^{\frac{1-a^c}{\varphi}-1}.\end{aligned} \quad \text{(A12)}$$

For the unskilled agent:

$$\frac{\partial \Pi_s}{\partial h_{fu,t}}=0$$

$$
\begin{aligned}
0 &= (1-\lambda)\left(\frac{1-a^c}{\varphi}\right)\varphi h_{fu,t}^{\varphi-1} A_t K_{st,t}^{a_c} \times \\
&\times[\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho}+ \\
&+(1-\lambda)\,h_{fu,t}^{\varphi}]^{\frac{1-a^c}{\varphi}-1} - w_{u,t} \ .
\end{aligned}
$$

$$
\begin{aligned}
w_{u,t} &= (1-\lambda)\left(1-a^c\right) h_{fu,t}^{\varphi-1} A_t K_{st,t}^{a_c} \times \\
&\times \left[\begin{array}{c} \lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\,h_{fu,t}^{\varphi} \end{array}\right]^{\frac{1-a^c}{\varphi}-1} .
\end{aligned}
$$

$$
\begin{aligned}
w_{u,t} &= (1-\lambda)\left(1-a^c\right) h_{fu,t}^{\varphi-1} A_t K_{st,t}^{a_c} \times \\
&\times \left[\begin{array}{c} \lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\,h_{fu,t}^{\varphi} \end{array}\right]^{\frac{1-a^c}{\varphi}-1} .
\end{aligned}
\tag{A13}
$$

For capital structures:

$$
\frac{\partial \Pi_s}{\partial K_{st,s}} = 0
$$

$$
\begin{aligned}
0 &= \left[\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho}+(1-\lambda)\,h_{fu,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}} \times \\
&\times a^c A_t K_{st,t}^{a^c-1} - r_{st,t} \ .
\end{aligned}
$$

$$
\begin{aligned}
r_{st,t} &= a^c A_t K_{st,t}^{a^c-1} \times \\
&\times \left[\begin{array}{c} \lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{fs,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\,h_{fu,t}^{\varphi} \end{array}\right]^{\frac{1-a^c}{\varphi}} .
\end{aligned}
\tag{A14}
$$

For capital equipment:

$$
\frac{\partial \Pi_s}{\partial K_{eq,s}} = 0
$$

$$0 \;=\; \lambda\nu\left(\frac{1-a^{c}}{\varphi}\right)\frac{\varphi}{\rho}\rho A_{eq,t}^{\rho}K_{eq,t}^{\rho-1}A_{t}K_{st,t}^{a_{c}}\times$$
$$\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+\left(1-\nu\right)h_{fs,t}^{\rho}\right)^{\varphi/\rho}\\ +\left(1-\lambda\right)h_{fu,t}^{\varphi}\end{array}\right]^{\frac{1-a^{c}}{\varphi}-1}-r_{eq,t}\;.$$

$$r_{eq,t} \;=\; \lambda\nu\left(1-a^{c}\right)A_{eq,t}^{\rho}K_{eq,t}^{\rho-1}A_{t}K_{st,t}^{a_{c}}\times \tag{A15}$$
$$\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+\left(1-\nu\right)h_{fs,t}^{\rho}\right)^{\varphi/\rho}\\ +\left(1-\lambda\right)h_{fu,t}^{\varphi}\end{array}\right]^{\frac{1-a^{c}}{\varphi}-1}.$$

Where under the assumptions of homogeneous of degree one production function, perfect competition and free entry of new firms into the market, firms that already exist in the market make zero profits and as a result we have:

$$Y_{t}=w_{s,t}h_{fs,t}+w_{u,t}h_{fu,t}+r_{st,t}K_{st,t}+r_{eq,t}K_{eq,t} \tag{A16}$$

A.2 Decentralized competitive equilibrium

This section presents the solution of the decentralized competitive equilibrium (DCE) where households maximize their welfare and firms their profits, also all constraints mentioned above should be satisfied, including the market clearing conditions.

Therefore, given an initial level of capital stock for structures and equipment $(K_{s,0})$ and $(K_{e,0})$, and given the exogenous processes A_t, $A_{eq,t}$, a competitive equilibrium is characterized by the sequence: $\{C_{s,t},\ C_{u,t},\ h_{s,t},\ h_{u,t},\ K_{st,t+1}^{s},\ K_{eq,t+1}^{s},\ K_{st,t+1}^{u},\ K_{eq,t+1}^{u},\ w_{s,t},\ w_{u,t},\ r_{st,t},\ r_{eq,t},\ \bar{G}_{t}^{c},\ Y_{t}\}_{t=0}^{\infty}$satisfying the following conditions:

A.2.1 The first-order conditions of skilled households

$$\frac{\partial u\left(\cdot\right)}{\partial C_{s,t}}=\beta E_{s}[\frac{\partial u\left(\cdot\right)}{\partial C_{s,t+1}}\left(r_{st,t+1}(1-\tau_{t+1}^{r})+(1-\delta_{st})-2\psi_{s}K_{st,t+1}^{s}\right)] \tag{A17}$$

$$\frac{\partial u\left(\cdot\right)}{\partial C_{s,t}}=\beta E_{s}[\frac{\partial u\left(\cdot\right)}{\partial C_{s,t+1}}\left(r_{eq,t+1}(1-\tau_{t+1}^{r})+(1-\delta_{eq})-2\psi_{s}K_{eq,t+1}^{s}\right)] \tag{A18}$$

$$
\begin{aligned}
&\frac{(1-\gamma)\left(aC_{s,t}^{\mu}+(1-a)\left(\bar{G}_t^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{s,t})^{(1-\gamma)(1-\sigma)-1}}{\gamma aC_{s,t}^{\mu-1}\left(aC_{s,t}^{\mu}+(1-a)\left(\bar{G}_t^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{s,t})^{(1-\gamma)(1-\sigma)}} \\
= \;& E_t(w_{s,t}(1-\tau_{s,t}^w)) \qquad \text{(A19)}
\end{aligned}
$$

A.2.2 Budget constraint of skilled agents

$$
\begin{aligned}
C_{s,t}+K_{st,t+1}^s+K_{eq,t+1}^s \;=\; & (1-\tau_{s,t}^w)w_{s,t}h_{s,t}+ \qquad \text{(A20)}\\
&+\left[(1-\tau_t^r)r_{st,t}+(1-\delta_{st})\right]K_{st,t}^s+\\
&+\left[(1-\tau_t^r)r_{eq,t}+(1-\delta_{eq})\right]K_{eq,t}^s-\\
&-\psi_s\left[\left(K_{st,t}^s\right)^2+\left(K_{eq,t}^s\right)^2\right].
\end{aligned}
$$

A.2.3 The first-order conditions of unskilled households

$$
\frac{\partial u(\cdot)}{\partial C_{u,t}}=\beta E_s[\frac{\partial u(\cdot)}{\partial C_{u,t+1}}\left(r_{st,t+1}(1-\tau_{t+1}^r)+(1-\delta_{st})-2\psi_u K_{st,t+1}^u\right)] \qquad \text{(A21)}
$$

$$
\frac{\partial u(\cdot)}{\partial C_{u,t}}=\beta E_s[\frac{\partial u(\cdot)}{\partial C_{u,t+1}}\left(r_{eq,t+1}(1-\tau_{t+1}^r)+(1-\delta_{eq})-2\psi_u K_{eq,t+1}^u\right)] \qquad \text{(A22)}
$$

$$
\begin{aligned}
&\frac{(1-\gamma)\left(aC_{u,t}^{\mu}+(1-a)\left(\bar{G}_t^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{u,t})^{(1-\gamma)(1-\sigma)-1}}{\gamma aC_{u,t}^{\mu-1}\left(aC_{u,t}^{\mu}+(1-a)\left(\bar{G}_t^c\right)^{\mu}\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{u,t})^{(1-\gamma)(1-\sigma)}} \\
= \;& E_t(w_{u,t}(1-\tau_{u,t}^w)) \;. \qquad \text{(A23)}
\end{aligned}
$$

A.2.4 Firms' first-order conditions

$$
\begin{aligned}
w_{s,t} \;=\; & \lambda(1-\nu)(1-a^c)\,n_s^{\rho}h_{s,t}^{\rho-1}A_t\left(K_{st,t}^f\right)^{a^c}\times \qquad \text{(A24)}\\
&\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}^{\rho}\left(K_{eq,t}^f\right)^{\rho}\right)+(1-\nu)\,n_s^{\rho}h_{s,t}^{\rho}\right)^{\varphi/\rho}+\\ +(1-\lambda)\,n_u^{\varphi}h_{u,t}^{\varphi}\end{array}\right]^{\frac{1-a^c}{\varphi}-1}.
\end{aligned}
$$

$$
\begin{aligned}
w_{u,t} &= (1-\lambda)(1-a^c)\, n_u^{\varphi} h_{u,t}^{\varphi-1} A_t \left(K_{st,t}^f\right)^{a^c} \times \\
&\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}^{\rho}\left(K_{eq,t}^f\right)^{\rho}\right) + (1-\nu)\, n_s^{\rho} h_{s,t}^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\, n_u^{\varphi} h_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1} .
\end{aligned} \tag{A25}
$$

$$
\begin{aligned}
r_{st,t} &= a^c A_t A_{st,t}^{a^c} \left(K_{st,t}^f\right)^{a^c-1} \times \\
&\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}^{\rho}\left(K_{eq,t}^f\right)^{\rho}\right) + (1-\nu)\, A_{h_s,t}^{\rho} n_s^{\rho} h_{s,t}^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\, A_{h_u,t}^{\varphi} n_u^{\varphi} h_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}} .
\end{aligned} \tag{A26}
$$

$$
\begin{aligned}
r_{eq,t} &= \lambda\nu(1-a^c)\, A_{eq,t}^{\rho} \left(K_{eq,t}^f\right)^{\rho-1} A_t \left(K_{st,t}^f\right)^{a^c} \times \\
&\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}^{\rho}\left(K_{eq,t}^f\right)^{\rho}\right) + (1-\nu)\, n_s^{\rho} h_{s,t}^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\, n_u^{\varphi} h_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1} .
\end{aligned} \tag{A27}
$$

where $K_{i,t}^f = n_s K_{i,t}^s + n_u K_{i,t}^u$ for $i = st, eq$.

A.2.5 The behaviour of the exogenous processes

$$
\log(A_t) = (1-\rho_A)\log(A_0) + \rho_A \log(A_{t-1}) + \varepsilon_{At} \tag{A28}
$$

$$
\log(A_{eq,t}) = \left(1-\rho_{Aeq}\right)\log(A_{eq,0}) + \rho_{Aeq}\log(A_{eq,t-1}) + \varepsilon_{Aeq,t} \tag{A29}
$$

A.2.6 Government's budget constraint

$$
\begin{aligned}
\bar{G}_t^c &= n_s\left(\tau_{s,t}^w w_{s,t} h_{st}\right) + n_u\left(\tau_{u,t}^w w_{u,t} h_{u,t}\right) + \\
&+\tau_t^r\left(r_{st,t}(n_s K_{st,t}^s + n_u K_{st,t}^u) + r_{eq,t}(n_s K_{eq,t}^s + n_u K_{eq,t}^u)\right)
\end{aligned} \tag{A30}
$$

A.2.7 The aggregate resource constraint

$$\begin{aligned} Y_t &= n_s\left(K^s_{st,t+1}-(1-\delta_{st})K^s_{st,t}+K^s_{eq,t+1}-(1-\delta_{eq})K^s_{eq,t}\right)+ \\ &+n_sC_{s,t}+n_uC_{u,t}+\bar{G}^c_t+ \\ &+n_u\left(K^u_{st,t+1}-(1-\delta_{st})K^u_{st,t}+K^u_{eq,t+1}-(1-\delta_{eq})K^u_{eq,t}\right)+ \\ &+n_s\psi_s\left[\left(K^s_{st,t}\right)^2+\left(K^s_{eq,t}\right)^2\right]+n_u\psi_u\left[\left(K^u_{st,t}\right)^2+\left(K^u_{eq,t}\right)^2\right]. \end{aligned} \tag{A31}$$

A.2.8 The production function

$$\begin{aligned} Y_t &= A_t(n_sK^s_{st,t}+n_uK^u_{st,t})^{a^c}\times \\ &\times\{\lambda\left(\nu\left(A^\rho_{e,t}(n_sK^s_{eq,t}+n_uK^u_{eq,t})^\rho\right)+(1-\nu)\,n^\rho_s h^\rho_{s,t}\right)^{\varphi/\rho}+ \\ &+(1-\lambda)\,n^\varphi_s h^\varphi_{s,t}\}^{\frac{1-a^c}{\varphi}}. \end{aligned} \tag{A32}$$

Thus, there is a system of fourteen equations with fourteen unknowns which describes the DCE: $C_{s,t}$, $C_{u,t}$, $h_{s,t}$, $h_{u,t}$, $K^s_{st,t+1}$, $K^s_{eq,t+1}$, $K^u_{st,t+1}$, $K^u_{eq,t+1}$, $w_{s,t}$, $w_{u,t}$, $r_{st,t}$, $r_{eq,t}$, $\bar{G}^c_t$, Y_t, together with 2 exogenous processes $\{A_t, A_{eq,t}\}$.[28]

A.2.9 The steady-state

The equations that describe the equilibrium of the economy are nonlinear and as a result it may not permit analytical solutions. Such systems are usually solved using a log-linear approximation around the steady state and then solving the resulting log-linear system of equations. Therefore, the behaviour of the system will be determined in the steady-state. The steady-state value of any variable will be presented with a bar above the variable and by dropping the time index (i.e. $K_t = K_{t+1} = \bar{K}$). The deterministic steady-state is defined when all the errors of the AR(1) processes are equal to zero and all of the variables in equations (A17-A32) are constant.

Under these assumptions the steady-state conditions include the following.

[28] Where: $\frac{\partial u(\cdot)}{\partial C_{j,s}} = \gamma a C^{\mu-1}_{j,s}\left(aC^\mu_{j,s}+(1-a)\left(\bar{G}^c_s\right)^\mu\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_s)^{(1-\gamma)(1-\sigma)}$

The first-order conditions of the skilled household:

$$\frac{1}{\beta} = \left(\bar{r}_{st}(1-\bar{\tau}^{r}) + (1-\delta_{st}) - 2\psi_{s}K_{st}^{s}\right) \tag{A33}$$

$$\frac{1}{\beta} = \left(\bar{r}_{eq}(1-\bar{\tau}^{r}) + (1-\delta_{eq}) - 2\psi_{s}K_{eq}^{s}\right) \tag{A34}$$

$$\frac{(1-\gamma)\left(a\bar{C}_{s}^{\mu} + (1-a)\left(\bar{G}^{c}\right)^{\mu}\right)}{\gamma a\bar{C}_{s}^{\mu-1}(1-\bar{h}_{s})} = \bar{w}_{s}(1-\bar{\tau}_{s}^{w}) \tag{A35}$$

The budget constraint of the skilled agents:

$$\begin{aligned} \bar{C}_{s} &= \left[(1-\bar{\tau}_{t}^{r})\bar{r}_{st} - \delta_{s}\right]\bar{K}_{s}^{s} + \left[(1-\bar{\tau}_{t}^{r})\bar{r}_{eq} - \delta_{eq}\right]\bar{K}_{eq}^{s} + \\ &\quad +(1-\bar{\tau}_{s}^{w})\bar{w}_{s}\bar{h}_{s} - \psi_{s}\left[\left(\bar{K}_{st}^{s}\right)^{2} + \left(\bar{K}_{eq}^{s}\right)^{2}\right] \end{aligned} \tag{A36}$$

The first-order conditions of the unskilled household:

$$\frac{1}{\beta} = \left(\bar{r}_{st}(1-\bar{\tau}^{r}) + (1-\delta_{s}) - 2\psi_{u}\bar{K}_{st}^{u}\right) \tag{A37}$$

$$\frac{1}{\beta} = \left(\bar{r}_{eq}(1-\bar{\tau}^{r}) + (1-\delta_{e}) - 2\psi_{u}\bar{K}_{eq}^{u}\right) \tag{A38}$$

$$\frac{(1-\gamma)\left(a\bar{C}_{u}^{\mu} + (1-a)\left(\bar{G}^{c}\right)^{\mu}\right)}{\gamma a\bar{C}_{u}^{\mu-1}(1-\bar{h}_{u})} = \bar{w}_{u}(1-\bar{\tau}_{u}^{w}) \tag{A39}$$

The firms' first-order conditions:

$$\begin{aligned} \bar{w}_{s} &= \lambda\left(1-\nu\right)\left(1-a^{c}\right)n_{s}^{\rho}\bar{h}_{s}^{\rho-1}\bar{A}\bar{K}_{st}^{a^{c}} \times \\ &\quad \times \left[\begin{array}{c} \lambda\left(\nu\left(\bar{A}_{eq}^{\rho}\bar{K}_{eq}^{\rho}\right) + (1-\nu)\left(n_{s}\bar{h}_{s}\right)^{\rho}\right)^{\varphi/\rho} + \\ +\left(1-\lambda\right)\left(n_{u}\bar{h}_{u}\right)^{\varphi} \end{array}\right]^{\frac{1-a^{c}}{\varphi}-1} \end{aligned} \tag{A40}$$

$$\bar{w}_u = (1-\lambda)(1-a^c)\, n_u^{\varphi}\bar{h}_u^{\varphi-1}\bar{A}\bar{K}_{st}^{a^c} \times \tag{A41}$$
$$\times \left[\begin{array}{c} \lambda\left(\nu\left(\bar{A}_{et}^{\rho}\bar{K}_{eq}^{\rho}\right) + (1-\nu)\left(n_s\bar{h}_s\right)^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\left(n_u\bar{h}_u\right)^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1}$$

$$\bar{r}_{st} = a^c\bar{A}\bar{K}_{st}^{a^c-1} \times \tag{A42}$$
$$\times \left[\begin{array}{c} \lambda\left(\nu\left(\bar{A}_{eq}^{\rho}\bar{K}_{eq}^{\rho}\right) + (1-\nu)\left(n_s\bar{h}_s\right)^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\left(n_u\bar{h}_u\right)^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}}$$

$$\bar{r}_{eq} = \lambda\nu(1-a^c)\,\bar{A}_{eq}^{\rho}\bar{K}_{eq}^{\rho-1}\bar{A}\bar{K}_{st}^{a^c} \times \tag{A43}$$
$$\times \left[\begin{array}{c} \lambda\left(\nu\left(\bar{A}_{eq}^{\rho}\bar{K}_{eq}^{\rho}\right) + (1-\nu)\left(n_s\bar{h}_s\right)^{\rho}\right)^{\varphi/\rho} + \\ +(1-\lambda)\left(n_u\bar{h}_u\right)^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1}$$

where $K_i = n_s K_i^s + n_u K_i^u$ for $i = st, eq$.

The behaviour of the exogenous processes:

$$\log(\bar{A}) = (1-\rho_A)\log(A_0) + \rho_A\log(\bar{A}) \tag{A44}$$

$$\log(\bar{A}_{eq}) = \left(1-\rho_{Aeq}\right)\log(A_{eq,0}) + \rho_{Aeq}\log(\bar{A}_{eq}) \tag{A45}$$

Government budget constraint:

$$\bar{G}^c = n_s\left(\bar{\tau}_s^w\bar{w}_s\bar{h}_s\right) + n_u\left(\bar{\tau}_u^w\bar{w}_u\bar{h}_u\right) + \bar{\tau}^r\left(\bar{r}_{st}\bar{K}_{st} + \bar{r}_{eq}\bar{K}_{eq}\right) \tag{A46}$$

The market clearing condition:

$$\bar{Y}_t = n_s\bar{C}_s + n_u\bar{C}_u + n_s\left(\delta_{st}\bar{K}_{st}^s + \delta_{eq}\bar{K}_{eq}^s\right) + n_u\left(\delta_{st}\bar{K}_{st}^u + \delta_{eq}\bar{K}_{eq}^u\right) +$$
$$+n_s\psi_s\left[\left(\bar{K}_{st}^s\right)^2 + \left(\bar{K}_{eq}^s\right)^2\right] + n_u\psi_u\left[\left(\bar{K}_{st}^u\right)^2 + \left(\bar{K}_{eq}^u\right)^2\right] \tag{A47}$$

The production function:

$$\bar{Y}_t = \bar{A}\left(\left(n_s\bar{K}^s_{st} + n_u\bar{K}^u_{st}\right)^{a^c}\right)\times$$

$$\times\left[\begin{array}{c}\lambda\left(\nu\bar{A}^{\rho}_{eq}\left(n_s\bar{K}^s_{eq} + n_u\bar{K}^u_{eq}\right)^{\rho} + (1-\nu)\,n_s^{\rho}\bar{h}^{\rho}_s\right)^{\varphi/\rho}\\ +(1-\lambda)\,n_u^{\varphi}\bar{h}^{\varphi}_u\end{array}\right]^{\frac{1-a^c}{\varphi}} \tag{A48}$$

All the exogenous processes can be simplified even further. For example consider equation (A44):

$$\log(\bar{A}) = (1-\rho_A)\log(A_0) + \rho_A\log(\bar{A}) \Leftrightarrow$$

$$\log(\bar{A}) = \log(A_0)$$

which means that the steady-state of neutral technology shock ($\bar{A}$) depends on its long-run value(A_0). The long-run value of neutral technology affects only the scale of the economy (see King and Rebelo (1999)) and hence is normalize to be equal to:

$$\bar{A} = A_0 = 1.5$$

Moreover, the exogenous process of the capital equipment efficiency becomes:

$$\log(\bar{A}_{eq}) = \log(A_{eq,0})$$

which also means that its steady state depends on the long-run value.[29]

A.3 Optimal fiscal policy

The competitive equilibrium using the dual approach of the optimal fiscal policy is presented from the Euler equations of the representative skilled agent, the skilled agent budget constraint, the Euler equations of the representative unskilled agent, firms' first-order conditions, government's budget constraint, aggregate resource constraint and the production function, as presented in

[29] The long-run values of the exogenous processes are presented at the calibration subsection.

the decentralized competitive equilibrium section previously.

Government will choose: $C_{s,t}$, $C_{u,t}$, $h_{s,t}$, $h_{u,t}$, $K^s_{st,t+1}$, $K^s_{eq,t+1}$, $K^u_{st,t+1}$, $K^u_{eq,t+1}$, $w_{s,t}$, $w_{u,t}$, $r_{st,t}$, $r_{eq,t}$, Y_t, τ^r_t, $\tau^w_{s,t}$, $\tau^w_{u,t}$ to maximize equation (15) subject to the DCE system of equations. The associated Lagrange equation is:

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t\left(n_sU_s\left(C^*_{s,t},h_{s,t}\right)+n_uU_u\left(C^*_{u,t},h_{u,t}\right)\right)+\\
&+\Lambda^1_t(\beta[\frac{\partial u(\cdot)}{\partial C_{s,t+1}}\left(r_{s,t+1}(1-\tau^r_{t+1})+(1-\delta_s)-2\psi_sK^s_{s,t+1}\right)]-\frac{\partial u(\cdot)}{\partial C_{s,t}})+\\
&+\Lambda^2_t(\beta[\frac{\partial u(\cdot)}{\partial C_{s,t+1}}\left(r_{e,t+1}(1-\tau^r_{t+1})+(1-\delta_e)-2\psi_sK^s_{e,t+1}\right)]-\frac{\partial u(\cdot)}{\partial C_{s,t}})+\\
&+\Lambda^3_t(w_{s,t}(1-\tau^w_{s,t})-\\
&-\frac{(1-\gamma)\left(aC^\mu_{s,t}+(1-a)\left(\bar{G}^c_t\right)^\mu\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{s,t})^{(1-\gamma)(1-\sigma)-1}}{\gamma aC^{\mu-1}_{s,t}\left(aC^\mu_{s,t}+(1-a)\left(\bar{G}^c_t\right)^\mu\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{s,t})^{(1-\gamma)(1-\sigma)}})+
\end{aligned}
$$

$$
\begin{aligned}
&+\Lambda^4_t((1-\tau^w_{s,t})w_{s,t}h_{s,t}+[(1-\tau^r_t)r_{s,t}+(1-\delta_s)]\,K^s_{s,t}+\\
&+[(1-\tau^r_t)r_{e,t}+(1-\delta_e)]\,K^s_{e,t}-\\
&-\psi_s\left[\left(K^s_{s,t}\right)^2+\left(K^s_{e,t}\right)^2\right]-C_{s,t}-K^s_{s,t+1}-K^s_{e,t+1})+\\
&+\Lambda^5_t(\beta[\frac{\partial u(\cdot)}{\partial C_{u,t+1}}\left(r_{s,t+1}(1-\tau^r_{t+1})+(1-\delta_s)-2\psi_uK^u_{s,t+1}\right)]-\frac{\partial u(\cdot)}{\partial C_{u,t}})+\\
&+\Lambda^6_t(\beta[\frac{\partial u(\cdot)}{\partial C_{u,t+1}}\left(r_{e,t+1}(1-\tau^r_{t+1})+(1-\delta_e)-2\psi_uK^u_{e,t+1}\right)]-\frac{\partial u(\cdot)}{\partial C_{u,t}})+\\
&+\Lambda^7_t(w_{u,t}(1-\tau^w_{u,t})-\\
&-\frac{(1-\gamma)\left(aC^\mu_{u,t}+(1-a)\left(\bar{G}^c_t\right)^\mu\right)^{(\gamma/\mu)(1-\sigma)}(1-h_{u,t})^{(1-\gamma)(1-\sigma)-1}}{\gamma aC^{\mu-1}_{u,t}\left(aC^\mu_{u,t}+(1-a)\left(\bar{G}^c_t\right)^\mu\right)^{(\gamma/\mu)(1-\sigma)-1}(1-h_{u,t})^{(1-\gamma)(1-\sigma)}})+\\
&+\Lambda^8_t(w_{s,t}-\lambda(1-\nu)(1-a^c)\,h^{\rho-1}_{fs,t}A_tK^{a^c}_{s,t}\times\\
&\times\left[\lambda\left(\nu\left(A^\rho_{e,t}K^\rho_{e,t}\right)+(1-\nu)\,h^\rho_{fs,t}\right)^{\varphi/\rho}+(1-\lambda)\,h^\varphi_{fu,t}\right]^{\frac{1-a^c}{\varphi}-1})+\\
&+\Lambda^9_t(w_{u,t}-(1-\lambda)(1-a^c)\,h^{\varphi-1}_{fu,t}A_tK^{a^c}_{s,t}\times
\end{aligned}
$$

$$\times \left[\lambda \left(\nu \left(A_{e,t}^{\rho} K_{e,t}^{\rho}\right) + (1-\nu) h_{fs,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda) h_{fu,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}-1}) +$$
$$+\Lambda_t^{10}(r_{s,t} - (a^c Y_t / K_{s,t})) +$$
$$+\Lambda_t^{11}(r_{e,t} - \lambda\nu\left(1-a^c\right) A_{e,t}^{\rho} K_{e,t}^{\rho-1} A_t K_{s,t}^{a^c} \times$$
$$\times \left[\lambda \left(\nu \left(A_{e,t}^{\rho} K_{e,t}^{\rho}\right) + (1-\nu) h_{fs,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda) h_{fu,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}-1}) +$$
$$+\Lambda_t^{12}(\bar{G}_t^c - n_s \left(\tau_{s,t}^w w_{s,t} h_{st}\right) - n_u \left(\tau_{u,t}^w w_{u,t} h_{u,t}\right) -$$
$$\times \left[\lambda \left(\nu \left(A_{e,t}^{\rho} K_{e,t}^{\rho}\right) + (1-\nu) h_{fs,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda) h_{fu,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}-1}) +$$
$$+\Lambda_t^{12}(\bar{G}_t^c - n_s \left(\tau_{s,t}^w w_{s,t} h_{st}\right) - n_u \left(\tau_{u,t}^w w_{u,t} h_{u,t}\right) -$$
$$-\tau_t^r \left(r_{s,t} K_{s,t} + r_{e,t} K_{e,t}\right)) +$$
$$+\Lambda_t^{13}(Y_t - \bar{G}_t^c - n_s \left(K_{s,t+1}^s - (1-\delta_s) K_{s,t}^s + K_{e,t+1}^s - (1-\delta_e) K_{e,t}^s\right) -$$
$$-n_u \left(K_{s,t+1}^u - (1-\delta_s) K_{s,t}^u + K_{e,t+1}^u - (1-\delta_e) K_{e,t}^u\right) - n_s C_{s,t} - n_u C_{u,t} -$$
$$-n_s \psi_s \left[\left(K_{s,t}^s\right)^2 + \left(K_{e,t}^s\right)^2\right] - n_u \psi_u \left[\left(K_{s,t}^u\right)^2 + \left(K_{e,t}^u\right)^2\right]) +$$
$$+\Lambda_t^{14}(Y_t - A_t K_{s,t}^{a^c} \times$$
$$\times \left[\lambda \left(\nu \left(A_{e,t}^{\rho} K_{e,t}^{\rho}\right) + (1-\nu) h_{fs,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda) h_{fu,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}}).$$

The above procedure is known as the dual approach. One important step that is necessary to be taken into account using that approach is that the above set up needs to include besides periods t and $t+1$ for all of the constraints, as in the exogenous case in the previous section, also period $t-1$ in order to include all the appropriate variables when the first-order conditions are being calculated.

Then the system of equations is numerically solved using Matlab and the unique steady-state or long-run solution of the optimal fiscal policy is presented in Tables 1.5 and 1.6 in the main text.

A.4 Welfare gains between policy regimes

In order for the welfare gains to be estimated the following equation is used, as in Lucas (1990):[30]

$$\xi_i = 1 - \left(\frac{LU^E_{i,ss}}{LU^R_{i,ss}} \right)^{\frac{1}{\gamma(1-\sigma)}}$$

The value of ξ_i gives the compensating consumption supplement moving from the optimal fiscal policy regime to the exogenous fiscal policy regime. Therefore, a positive value of ξ_i indicates that the agent i is better off under the optimal taxation regime and the welfare improvement is measured in terms of consumption.

To get the above equation we have initially assumed that we want to calculate what percentage of their consumption the agents are willing to give up to be indifferent between the two policies, exogenous and optimal fiscal policy. Therefore, we use the following equation:

$$\sum_{t=0}^{\infty} \beta^t \left(U_i^R \left((1-\xi_i) C^*_{i,t}, h_{i,t} \right) \right) = \sum_{t=0}^{\infty} \beta^t \left(U_i^E \left(C^*_{i,t}, h_{i,t} \right) \right)$$

where $i = s, u$ determines if the agent is skilled or unskilled.

It can be shown that the above equation can be written as:

$$(1-\xi_i)_i^{\gamma(1-\sigma)} LU_i^R = LU_i^E$$

where LU_i is the lifetime welfare of the agent of type i.

$$(1-\xi_i)^{\gamma(1-\sigma)} = \frac{LU_i^E}{LU_i^R}$$

$$\xi_i = 1 - \left(\frac{LU_i^E}{LU_i^R} \right)^{\frac{1}{\gamma(1-\sigma)}} \tag{A49}$$

which is the equation that we have used to calculate the consumption that the agent is willing to give up so as to be indifferent between the two policies.

[30] Note that the superscript E stands for the exogenous fiscal policy case, while the superscript R denotes the optimal fiscal policy.

Chapter 2: Optimal factor income taxation with endogenous skill supply

Abstract: In this chapter we analyse optimal factor income taxation when there are different returns to skilled and unskilled labour as well as to capital in structures and equipment. In a setup employing capital-skill complementarity and endogenous skill acquisition, we consider restrictions on the tax policy menu, and on the debt to output ratio under exogenous spending policy. We find that when all factor inputs, as well as investment in skill acquisition, can be taxed at separate rates, both capital income taxes are zero in the long run. Moreover, there is a subsidy to education and the tax rate on skilled labour is higher than the tax rate on unskilled labour. The progressivity in labour income taxes is reduced if investment in education cannot be subsidised. If the government cannot tax income from skilled and unskilled labour separately, the tax on income from structures remains zero in the long-run, but there is a small positive tax on equipment capital. These results are not qualitatively affected by a budget rule that restricts debt to output, although the latter implies welfare losses. Finally, the transitional dynamics of the fiscal instruments from the exogenous to optimal taxation are not affected by the restrictions to the fiscal policy menu.

2.1 Introduction

The literature on optimal taxation has extensively examined the question of which factor of production should be taxed, assuming that the policy-maker wishes to maximise aggregate welfare under commitment and has access to distortionary taxation only. The prescriptions for optimal taxation are generally shown to depend on the underlying market structure and policy imperfections (see e.g. Mankiw *et al.* (2009) and Ljungqvist and Sargent (2012) for an evaluation of this literature).

A particular focus of research in this area has been the taxation of capital, following the seminal contributions of Judd (1985) and Chamley (1986), which showed that in the benchmark neoclassical model, although initially the tax rate on capital income can be very high, in the long-run it should be zero. Extensions to the benchmark neoclassical model have established the importance of market failures and restrictions on the policy menu. For instance, Judd (1997) and Guo and Lansing (1999) have shown that when markets are not fully competitive the optimal tax rate on capital income in the long-run will be non-zero. Moreover, Correia (1996) has shown that when there is an incomplete set of tax instruments, the optimal taxation on capital income will be non-zero to compensate for the missing tax instrument.

A second focus of research is the taxation of labour income when labour supply decisions have intertemporal implications. For instance, when labour income taxation has implications for the accumulation of labour augmenting technology in the form of human capital, there is an incentive for the government to use taxes to encourage human capital accumulation (see e.g. Jones *et al.* (1997)).

In this chapter, we revisit optimal factor income taxation in an environment where there are different skilled and unskilled labour services, endogenous skill creation, and production exhibits capital-skill complementarity. In particular, we consider the production technology in Krusell *et al.* (2000), also used in e.g. Lindqvist (2004), He and Liu (2008) and Pourpourides (2011), since this has been shown to match the data on the skill premium very well. In this framework, there are two types of labour services, capturing skilled and unskilled labour supply, and two types of capital stock, on structures and equipment, the latter of which complements skilled labour more than unskilled so that changes in its accumulation are skill biased. As is well documented in the literature (see e.g. Katz and Murphy (1992), Hornstein *et al.* (2005) and Acemoglu and Autor (2011)), this economic environment will lead to a wage premium accruing to skilled labour.

In this economic structure, new questions naturally develop within the two focal points in the literature, namely the optimal tax on capital and labour income. In particular, regarding capital income taxes, are both capital taxes optimally zero in the long-run? And, if there is an incomplete tax instrument set, which tax rate differs from zero? How does wage inequality due to capital-skill complementarity affect relative labour income taxation?

And, what implications does endogenous skill creation have for labour income taxation (perhaps under incomplete taxation)? We aim to answer these questions.

We work with a representative agent framework, which allows us to focus on aggregate efficiency and abstract from potential equity considerations for optimal taxation. Hence, our results regarding the optimal factor income taxation are directly comparable to the representative agent literature discussed above. We build on and extend the model in He and Liu (2008) to capture capital-skill complementarity and endogenous skill acquisition in the same economic environment.[31] In particular, we assume that a representative household decides how to allocate its investment in the two types of capital stock and in creating skilled labour within the same period. Moreover, it decides how to allocate its time endowment into leisure, labour supply in skilled and unskilled jobs, and in creating skill labour. Therefore, the model allows for endogenous skill acquisition. The production side is modelled as in Krusell *et al.* (2000). In this framework, we derive optimal tax policy under different scenarios regarding the policy menu available to the government and, in particular, which tax instruments are available as well as whether there are restrictions on issuing debt. We assume commitment and an exogenous spending policy.

In other recent work, Angelopoulos *et al.* (2014), we analyse optimal tax smoothing under skill heterogeneity and capital-skill complementarity, when the government has access to state-contingent debt and a complete set of state-contingent tax instruments, in a stochastic environment with endogenous and exogenous skill supply by different workers. In contrast, our interest here is in the long-run and transitional implications of optimal factor return taxation in a deterministic environment with a representative worker, allowing for capital-skill complementarity and endogenous skill accumulation.

We calibrate the model under exogenous policy to data averages for the US and calculate optimal tax policy for different options regarding the tax

[31] The model in He and Liu (2008) provides a convenient framework. However, He and Liu (2008) do not examine optimal policy. Since our aim here is to analyse optimal policy, we modify the model in He and Liu (2008) to allow for an endogenous labour-leisure choice, which is important when examining optimal labour taxes.

menu available to the government. We find that when the government can issue debt and can tax the different types of labour and capital income, as well as investment in education, at separate rates: (i) both capital income taxes are zero in the long-run; and (ii) there is a subsidy to education and progressive labour income taxation. This optimal policy results in a minor reduction in the skill premium compared with the data average. These results remain the same if the government can use a single tax for income from capital in structures and equipment.

However, when the policy menu is restricted with respect to access to an education subsidy and/or differential labour income taxation, the above results change. In particular, considering first the case where investment in education cannot be taxed/subsidised: (i) the progressivity in labour income taxation decreases relative to the benchmark case; (ii) capital income taxation is still zero. When the government cannot encourage skill acquisition by providing education subsidies, taxation of skilled labour income needs to be lowered relative to unskilled labour income to provide increased benefits to skill supply.

Next, considering the case where the government has access to the education subsidy but cannot tax income from skilled and unskilled labour separately, we find that while the tax on income from structures remains zero in the long-run, there is a small positive tax on equipment capital. Since equipment capital complements skilled labour, this tax on equipment capital allows the government to implement indirectly a higher tax burden on skilled labour supply, relative to unskilled, which is optimal as discussed above given the education subsidy. The economic structure considered here therefore provides an example where the two intertemporal wedges differ qualitatively in the long-run.

Finally, if the government can only implement a single labour income tax, without having access to education subsidies, the equipment tax becomes again positive but at a lower rate compared to the case with education subsidy and a single labour income tax. A relatively lower tax on equipment capital, which is complementing skilled labour, allows the government to implement indirectly a lower tax burden on skilled labour supply, relative to unskilled.

The transition paths of the policy instruments from the exogenous fiscal policy to optimal fiscal policy regime are qualitatively similar in each case we study and reflect the broad patterns obtained in the literature. In particular, there is a very high initial capital taxation, which facilitates the accumulation of assets, followed by quick reductions towards their steady-state values. Our optimal policy findings are similar if we restrict government debt by imposing a budget rule that requires that the debt to output ratio remains fixed at the data average. This restriction is motivated by the experience of the last decades in advanced economics and results in welfare losses of about half a percentage point over the lifetime relative to the absence of the budget rule. However, the results regarding optimal taxation in the long-run and in the transition are not affected qualitatively. The restriction does imply, however, a reduction in the progressivity of optimal labour income taxes.

The rest of the chapter is set out as follows. Section 2 provides a description of the model. Section 3 describes the calibration of the parameters under exogenous fiscal policy. In Section 4 we present and discuss the main results for optimal policy, together with the transition paths of the policy instruments. Finally, section 5 concludes the chapter.

2.2 The model

The economy is populated by a representative household which supplies skilled and unskilled labour services. Following He and Liu (2008) skilled labour supply requires the creation of skill, which is determined by time and goods. There is also a representative firm that uses two types of capital, structures and equipment, along with skilled and unskilled labour for the production of a homogeneous product. Following Krusell *et al.* (2000) and Hornstein *et al.* (2005), skilled labour is assumed to be more complementary to capital equipment than unskilled labour. Thus, capital equipment accumulation leads to higher skill premium, defined as the ratio of skilled wage rate over the unskilled wage rate. In contrast, increases in the relative supply of skilled labour tend to reduce the skill premium. Finally, the government finances exogenous public spending by issuing debt and taxing investment in skill creation and all sources of income.

2.2.1 The representative firm

The representative firm produces a homogeneous consumption good, Y_t, using labour and capital, acting in a perfectly competitive market, taking prices and policy variables as given:

$$\Pi_t = Y_t - w_{s,t}\widetilde{h}_{s,t} - w_{u,t}\widetilde{h}_{u,t} - r_{eq,t}\widetilde{K}_{eq,t} - r_{st,t}\widetilde{K}_{st,t} \tag{18}$$

subject to a Krusell *et al.* (2000) type production function:

$$\begin{aligned} Y_t = A_t \left(\widetilde{K}_{st,t}^{a^c}\right) \times \\ \times \left[\lambda \left(\nu \left(A_{eq,t}^{\rho}\widetilde{K}_{eq,t}^{\rho}\right) + (1-\nu)\,\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda)\,\widetilde{h}_{u,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}} \end{aligned} \tag{19}$$

where

$$a, \lambda, \nu \in (0,1)\,; \quad \varphi, \rho \in (-\infty, 1)$$

and $\widetilde{h}_{s,t}$ and $\widetilde{h}_{u,t}$ denote skilled and unskilled hours worked respectively; $A_{eq,t}$ is the efficiency level of capital equipment and A_t is the total factor productivity; $\widetilde{K}_{st,t}$ and $\widetilde{K}_{eq,t}$ denote the stock of capital structures and capital equipment respectively at the beginning of period t;[32] $w_{s,t}$ and $w_{u,t}$ are the returns to skilled and unskilled labour; and $r_{eq,t}$ and $r_{st,t}$ are the returns to capital holdings in equipment and structures respectively.

First-order conditions Firms select the quantity of each type of labour that they hire and the amount of capital (both structures and equipment) that they rent in order to maximize their profits:

$$\begin{aligned} \Pi_t = \max A_t \left(\widetilde{K}_{st,t}\right)^{a_c} [\lambda \left(\nu \left(A_{eq,t}\widetilde{K}_{eq,t}\right)^{\rho} + (1-\nu)\,\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} + \\ + (1-\lambda)\,\widetilde{h}_{u,t}^{\varphi}]^{\frac{1-a^c}{\varphi}} - w_{s,t}\widetilde{h}_{s,t} - w_{u,t}\widetilde{h}_{u,t} - r_{st,t}\widetilde{K}_{st,t} - r_{eq,t}\widetilde{K}_{eq,t} \end{aligned} \tag{20}$$

[32] For example, capital structures comprise buildings; and capital equipment scientific and professional equipment and machinery.

yielding the following first-order conditions:

$\widetilde{h}_{s,t}$:

$$w_{s,t} = \lambda (1-\nu)(1-a^c)\widetilde{h}_{s,t}^{\rho-1} A_t \left(\widetilde{K}_{st,t}\right)^{a_c} \times \tag{21}$$
$$\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}\widetilde{K}_{eq,t}\right)^{\rho} + (1-\nu)\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\widetilde{h}_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1}$$

$\widetilde{h}_{u,t}$:

$$w_{u,t} = (1-\lambda)(1-a^c)\widetilde{h}_{u,t}^{\varphi-1} A_t \left(\widetilde{K}_{st,t}\right)^{a_c} \times \tag{22}$$
$$\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}\widetilde{K}_{eq,t}\right)^{\rho} + (1-\nu)\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\widetilde{h}_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1}$$

$\widetilde{K}_{st,t}$:

$$r_{st,t} = a^c A_t \widetilde{K}_{st,t}^{a^c-1} \times \tag{23}$$
$$\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}\widetilde{K}_{eq,t}\right)^{\rho} + (1-\nu)\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\widetilde{h}_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}}$$

$\widetilde{K}_{ut,t}$:

$$r_{eq,t} = \lambda\nu (1-a^c) A_{eq,t}^{\rho}\widetilde{K}_{eq,t}^{\rho-1} A_t \left(\widetilde{K}_{st,t}\right)^{a_c} \times \tag{24}$$
$$\times \left[\begin{array}{c} \lambda \left(\nu \left(A_{eq,t}\widetilde{K}_{eq,t}\right)^{\rho} + (1-\nu)\widetilde{h}_{s,t}^{\rho}\right)^{\varphi/\rho} \\ +(1-\lambda)\widetilde{h}_{u,t}^{\varphi} \end{array} \right]^{\frac{1-a^c}{\varphi}-1}.$$

Where under the assumptions of: homogeneity of degree one production function, perfectly competitive markets and free entry of new firms into the market, firms that already exist into the market make zero profits giving:

$$Y_t = w_{s,t}\widetilde{h}_{s,t} + w_{u,t}\widetilde{h}_{u,t} + r_{st,t}\widetilde{K}_{st,t} + r_{eq,t}\widetilde{K}_{eq,t} \tag{25}$$

2.2.2 The representative household

The lifetime utility of the representative household is given by:

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, l_t) \tag{26}$$

where $0 < \beta < 1$ is a constant discount factor and denotes the time preference of the individual; C_t and l_t are total consumption and leisure for the representative household respectively at period t; and $u(\cdot)$ is the neoclassical utility function that is increasing, strictly concave and continuously differentiable.

The assumed utility function is the following:

$$u(C_t, l_t) = \frac{\left(C_t^{\gamma} l_t^{1-\gamma}\right)^{1-\sigma_1}}{1-\sigma_1} \tag{27}$$

where $(\sigma_1, \gamma) > 0$ represent the preference parameters of the representative household. Specifically, γ determines the weight given to consumption, and σ_1 is the risk aversion coefficient.

The representative household has the following time constraint:

$$1 = l_t + h_{s,t} + h_{u,t} + e_t \tag{28}$$

where $h_{s,t}$ and $h_{u,t}$ denote skilled and unskilled labour work time respectively in period t and e_t is time invested in education or other training for skills acquisition in period t. The above equation states that in each period the representative household splits its time endowment between leisure, education and work. The endowment of the representative household in each period is normalised to one.

The skill creation equation is similar to He and Liu (2008) and is given by:

$$h_{s,t} = f(I_{h,t}, e_t) = B\left[(I_{h,t})^{\alpha}(e_t)^{1-\alpha}\right]^{\xi} \tag{29}$$

where $I_{h,t} > 0$ determines the investment of goods in creating skills. The shares of goods and time in the creation of skills are given by α and $1-\alpha$ respectively, with $\alpha \in (0,1)$. The parameter $B > 0$ determines the efficiency of the skill-creation process. Finally, $\xi > 0$ is a measure of the returns to scale and should be less than one to ensure that the model has a unique solution (see also e.g. He and Liu (2008)).

The law of motion for the two types of capital stock , $i = st, eq$, where st and eq denote capital structures and capital equipment respectively, is given by:

$$\widetilde{K}_{i,t+1} = (1 - \delta_i)\widetilde{K}_{i,t} + I_{i,t} \tag{30}$$

where, the depreciation rate is $0 \leq \delta_i \leq 1$ and $I_{i,t}$ is the investment in new capital i at period t.

In addition, the household has the following budget constraint:

$$\begin{aligned} & C_t + I_{st,t} + I_{eq,t} + \left(1 - \tau_t^h\right) I_{h,t} + \frac{b_{t+1}}{r_t^b} \\ & = (1 - \tau_t^s) w_{s,t} h_{s,t} + (1 - \tau_t^u) w_{u,t} h_{u,t} + \\ & + (1 - \tau_{st,t}^r) r_{st,t} K_{st,t} + (1 - \tau_{eq,t}^r) r_{eq,t} K_{eq,t} + b_t \end{aligned} \tag{31}$$

where, b_{t+1} is the payout value of bonds bought by the household at period t; r_t^b is the return to bonds; τ_t^s, τ_t^u, $\tau_{st,t}^r$ and $\tau_{eq,t}^r$ are the tax rates on skilled and unskilled labour income and on income from capital structures and capital equipment in period t respectively. Moreover, τ_t^h is a subsidy for investment in goods for skills acquisition. Therefore, the first and second terms on the right-hand side of equation (31) give the after tax return from labour, the following two terms represent the after tax capital income and the last term denotes the revenues from bonds bought in the previous period.

The representative household chooses $\{C_t, h_{s,t}, h_{u,t}, e_t, I_{h,t}, K_{st,t+1}, K_{eq,t+1}, b_{t+1}\}_{t=0}^{\infty}$ given prices and taxes so as to maximize equation (26) subject to equations (27) - (31).

First order conditions Assuming that Λ_t and M_t are Lagrange multipliers associated with the budget constraint and the skill creation equation respectively, the first-order conditions for the problem of the household are given by the following Lagrange equation:

$$\mathcal{L} = \max \sum_{i=0}^{\infty} \beta^i \left(\frac{\left(C_{t+i}{}^{\gamma} l_{t+i}^{1-\gamma}\right)^{1-\sigma_1}}{1-\sigma_1} \right) + \tag{32}$$
$$+ \Lambda_{t+i}(C_{t+i} + \left(1-\tau_t^h\right) I_{h,t} - (1-\tau_{t+i}^s) w_{s,t+i} h_{s,t+i} -$$
$$- (1-\tau_{t+i}^u) w_{u,t+i} h_{u,t+i} + \frac{b_{t+i+1}}{r_{t+i}^b} - b_{t+i} -$$
$$- \left[(1-\tau_{st,t+i}^r) r_{st,t+i} + (1-\delta_{st})\right] K_{st,t+i} + K_{st,t+i+1} -$$
$$- \left[(1-\tau_{eq,t+i}^r) r_{eq,t+i} + (1-\delta_{eq})\right] K_{eq,t+i} + K_{eq,t+i+1}) +$$
$$+ M_{t+i} \left(h_{s,t+i} - B \left[(I_{h,t+i})^{\alpha} (e_{t+i})^{1-\alpha} \right]^{\xi_h} \right).$$

Maximisation implies:

C_t:

$$U_{C_t} = -\Lambda_t \tag{33}$$

$h_{s,t}$:

$$U_{h_{s,t}} + M_t - \Lambda_t (1-\tau_t^s) w_{s,t} = 0 \tag{34}$$

$h_{u,t}$:

$$U_{h_{u,t}} - \Lambda_t (1-\tau_t^u) w_{u,t} = 0 \tag{35}$$

$K_{st,t+1}$:

$$\Lambda_t = \beta[\Lambda_{t+1} \left(r_{st,t+1} (1-\tau_{st,t+1}^r) + (1-\delta_{st}) \right)] \tag{36}$$

$K_{eq,t+1}$:

$$\Lambda_t = \beta[\Lambda_{t+1} (r_{eq,t+1} (1-\tau_{eq,t+1}^r) + (1-\delta_{eq})] \tag{37}$$

e_t:

$$U_{e_t} - M_t f_{e_t} = 0 \tag{38}$$

$I_{h,t}$:

$$\Lambda_t \left(1-\tau_t^h\right) - M_t f_{I_{h,t}} = 0 \tag{39}$$

b_{t+1}:

$$\Lambda_t \frac{1}{r_t^b} = \beta \Lambda_{t+1} \tag{40}$$

where U_x is the derivative of the utility function with respect to the variable

x.

Equation (38) implies that: $M_t = U_{e_t}/f_{e_t}$. By substituting equations (33) and (38) to (34-40) we obtain:

$$0 = U_{h_{s,t}} + \frac{U_{e_t}}{f_{e_t}} + U_{C_t}(1 - \tau_t^s)w_{s,t} \tag{41}$$

$$0 = U_{h_{s,t}} + U_{C_t}(1 - \tau_t^u)w_{u,t} \tag{42}$$

$$U_{C_t} = \beta[U_{C_{t+1}}\left(r_{st,t+1}(1 - \tau_{st,t+1}^r) + (1 - \delta_{st})\right)] \tag{43}$$

$$U_{C_t} = \beta[U_{C_{t+1}}\left(r_{eq,t+1}(1 - \tau_{eq,t+1}^r) + (1 - \delta_{eq})\right)] \tag{44}$$

$$U_{C_t}(1 - \tau_t^h) + \frac{U_{e_t}(\cdot)}{f_{e_t}} f_{I_{h,t}} = 0 \tag{45}$$

$$r_t^b = \frac{U_{C_t}}{\beta U_{C_{t+1}}} \tag{46}$$

where 43 and 44 are the two Euler equations for capital structures and capital equipment respectively and demonstrate the effect of the two intertemporal wedges, $\tau_{st,t+1}^r$, and $\tau_{eq,t+1}^r$.

Combining equations 43, 44 and 46 we get the following equations:

$$r_t^b = r_{st,t+1}(1 - \tau_{st,t+1}^r) + (1 - \delta_{st}) \tag{47}$$

$$r_t^b = r_{eq,t+1}(1 - \tau_{eq,t+1}^r) + (1 - \delta_{eq}) \tag{48}$$

which define the no-arbitrage conditions for capital and bonds ensuring that the three assets have the same rate of return in equilibrium.

The following transversality conditions are also added for $i = st, eq$:

$$\lim_{t\to\infty} \beta^t U_{C_t} \frac{b_{t+1}}{r_t^b} = 0 \tag{49}$$

$$\lim_{t\to\infty} \beta^t U_{C_t} K_{i,t+1} = 0 \tag{50}$$

The intuition of the above conditions is that on an optimal path, the values of capital stock and bond holdings for a period far enough into the future have zero weight to the maximization problem. Therefore, the utility gains from accumulating capital and bonds eventually grow slower than $1/\beta$. This condition is crucial for the economy to reach a stationary equilibrium, otherwise it would be optimal for the household to postpone consumption forever.

2.2.3 The government budget constraint

The government needs to satisfy the following budget constraint in each period:

$$G_t^c + \tau_t^h I_{h,t} + b_t = \tau_t^s w_{s,t} h_{st} + \tau_t^u w_{u,t} h_{u,t} + \frac{b_{t+1}}{r_t^b} + \\ + \tau_{st,t}^r r_{st,t} K_{st,t} + \tau_{eq,t}^r r_{eq,t} K_{eq,t}. \tag{51}$$

The revenues from taxes and debt are being used to finance public consumption expenditures, a subsidy to investment in education and repayments on existing debt. The level of G_t^c is calibrated to obtain a debt to output ratio equal to 53%. Under optimal fiscal policy we keep the level of G_t^c fixed over time to the value obtained under the exogenous fiscal policy and we let the government choose optimally the tax rates and the level of debt.

2.2.4 Aggregate resource constraint and market clearing conditions

Output can be used for consumption (private and public) and for investment. That gives the following aggregate resource constraint:

$$Y_t = G^c + C_t + I_{st,t} + I_{eq,t} + I_{h,t}. \tag{52}$$

In addition, the following market clearing conditions for the input markets are always satisfied:

$$\widetilde{K}_{st,t} = K_{st,t} \tag{53}$$

$$\widetilde{K}_{eq,t} = K_{eq,t} \tag{54}$$

$$\widetilde{h}_{s,t} = h_{s,t} \tag{55}$$

$$\widetilde{h}_{u,t} = h_{u,t}. \tag{56}$$

2.2.5 Competitive equilibrium with exogenous fiscal policy

The decentralised competitive equilibrium (DCE) with exogenous policy is summarized by a sequence of allocations $\{C_t,\ K_{st,t+1},\ K_{eq,t+1},\ h_{s,t},\ h_{u,t},\ e_t,\ I_{h,t},\ b_{t+1},\ \widetilde{K}_{st,t},\ \widetilde{K}_{eq,t},\ \widetilde{h}_{s,t},\ \widetilde{h}_{u,t}\}_{t=0}^{\infty}$, prices $\{w_{s,t},\ w_{u,t},\ r_{st,t},\ r_{eq,t}\}_{t=0}^{\infty}$ and one residual policy instrument $\{G_t^c\}_{t=0}^{\infty}$, such that the representative household solves its optimisation problem and the firm maximizes profits, taking prices and tax rates as given; the government budget constraint is satisfied and all markets clear. The DCE system of equations is presented in the Appendix B.

2.3 Calibration

We calibrate the model under exogenous fiscal policy to match the key great-ratios of the annual data of the US economy for the period 1970-2011. Table 2.1 below reports the model's quantitative parameters. Starting with the share of leisure in utility, $(1 - \gamma)$, we calibrate it to 0.65 so that, in the steady-state, the household devotes about one third of its time to labour and education. The relative risk aversion parameter, $\sigma = 2$ is commonly employed in the literature.

The elasticities of substitution between skilled labour and capital and between unskilled labour and capital (or skilled labour) have been estimated by Krusell *et al.* (2000). Following the literature (see e.g. Lindquist (2004), and Pourpourides (2011)), we also use these estimates, to set $\varphi = 0.401$ and $\rho = -0.495$. Moreover, the income share of capital structures, a^c, is set equal to 0.12, as in Lindquist (2004). The remaining parameters in the production function are calibrated to ensure that the steady-state predictions of the model in asset and labour markets are consistent with the data. In more detail, the unskilled labour weight in composite input share, $(1 - \lambda) = 0.3$, is calibrated to obtain a skilled to unskilled labour of about 79% and the capital equipment weight share in composite input, $\nu = 0.47$, is calibrated to obtain a skill premium of about 1.64.[33] We also normalize the steady-state values of TFP and capital equipment efficiency to unity (i.e. $A = A_{eq} = 1$).

The depreciation rates of capital structures and capital equipment, $\delta_{st} = 0.08$ and $\delta_{eq} = 0.1$, are calibrated to obtain an annual capital to output ratio of about 1.94, which is consistent with the annual data reported by the BEA on capital stocks.[34] In addition, those values are in line with the works of Greenwood *et al.* (1997) and Krusell *et al.* (2000). The time discount factor, $\beta = 0.96$, is set to obtain a post-tax post-depreciation annual real rate of return on capital of roughly 4.17%, which coheres with the 4.19% obtained in the data from the World Bank.[35]

Regarding the endogenous skill acquisition equation, the returns to scale, ξ, is calibrated to be equal to 0.425, so as to obtain an investment in education to output ratio of about 1.8% which is similar to the average private expenditure on education in the US.[36] The weight on time investment, $1-\alpha$, is set equal to 0.45 to get an average time in education as a share of total non-leisure time of about 5%.[37] Finally, the efficiency of skills transformation, B, is normalised to unity.[38]

[33] The target value for the skill premium is obtained from U.S. Census data and the skilled to unskilled labour is obtained from the Acemoglu and Autor (2011) dataset for the past 20 years.

[34] Specifically, the BEA Table 1.1 on fixed-assets has been used to obtain the time series for capital stock for 1970-2011.

[35] The data refers to the annual real interest rate from World Bank Indicators database for the period 1970-2011 (i.e. FR.INR.RINR).

[36] Using annual data from U.S. National Center for Education Statistics,Digest of Education Statistics, for the period 1970-2011 the relevant share to output is about 2%.

[37] To obtain this value we assume that the total time spent in higher education is on average 4 years. Note that the average working period is 35 years. Therefore, the percentage of time spent in education is $\frac{4}{35} = 0.1143$. Taking into account that on average in the U.S. 40-45% of the overall population are college educated (see Table 4 of the Census Bureau, Survey of Income and Program Participation), we get that: $\frac{4}{35} \times 0.45 = 0.0514$. Therefore, in the U.S. the average time spent in higher education is about 5.1% of the labour force.

[38] These parameters are within the boundaries suggested in the related literature a (i.e. Heckman, 1976 and Stokey,1996) although in a different setup.

Table 2.1: Calibration

Param.	Definitions	Values
δ_s	depreciation rate of capital structures	0.080
δ_e	depreciation rate of capital equipment	0.100
β	time discount factor	0.960
γ	weight attached to consumption in utility	0.350
σ_1	coefficient of relative risk aversion	2.000
α	weight on goods inv. for skills creation	0.550
B	efficiency of skills transformation procedure	1.000
ξ	returns to scale in skills creation	0.425
α^c	income share of capital structures	0.120
$\frac{1}{1-\rho}$	capital equipment to skilled labour elasticity	0.670
$\frac{1}{1-\varphi}$	capital equipment to unskilled labour elasticity	1.670
λ	income share of composite input to output	0.700
ν	income share of capital eq. to composite input	0.470
τ^s	skilled labour income tax rate	0.250
τ^u	unskilled labour income tax rate	0.200
τ^r_{st}	tax rate on capital structures income	0.310
τ^r_{eq}	tax rate on capital equipment income	0.310
τ^h	subsidy for goods invest. in skills creation	0.000
$\bar{A}$	constant parameter productivity	1.000
$\bar{A}_{eq}$	efficiency level of capital equipment	1.000

Finally, we use the ECFIN effective capital and labour tax rates from Martinez-Mongay (2000) to obtain an average tax rate for capital and labour.[39] Therefore, we set the tax rate for both capital income $\tau^r_{st} = \tau^r_{eq} = 0.31$ and the two labour income tax rates $\tau^u = 0.20$ and $\tau^s = 0.25$.[40] Given that it

[39] In particular, we use the LITR and KITN rates for effective average labour and capital taxes respectively from 1970-2011, as they treat self-employed income as capital income in the calculations.

[40] Note that the calculation of the effective labour income tax rate is equal to 0.22. But since we assume that the skilled and unskilled labour income is taxed differently we decompose the labour income tax into skilled and unskilled tax so as the weighted average of the two tax rates equals 0.22.

is difficult to obtain data for the education investment subsidy, τ^h, we set it to zero under the exogenous fiscal policy. We finally set the value of government expenditures, $G^c = 0.0320$, to obtain a steady-state debt to output ratio, $b/Y = 53\%$, which is equal to the average debt to GDP ratio obtained in the data.[41]

2.4 Exogenous steady-state

Under exogenous fiscal policy we solve the decentralized competitive equilibrium system of equations, as it is outlined in the Appendix B, keeping the tax rates at their calibrated values. Table.2.2 presents the steady-state results of the exogenous fiscal policy model together with the US data averages for 1970-2011. We also present a more detailed set of results regarding the exogenous fiscal policy in Table.2.3.

Table 2.2: Steady state of exogenous fiscal policy

	model	data
C/Y	0.5631	0.6397
K/Y	1.9461	1.8951
I/Y	0.1810	0.1462
I_h/Y	0.0177	0.0212
b/Y	0.5281	0.5300
h_s/h_u	0.7953	0.7633
G^c/Y	0.2382	0.2031
w^s/w^u	1.6431	1.6590
r^{net}	0.0417	0.0419
$\frac{e}{h_s+h_u+e}$	0.0530	0.0514

2.5 Optimal fiscal policy

In this section we discuss the Ramsey solution of optimal taxation, where it is assumed that the government chooses the series of taxes and debt to finance

[41] The source of that time series is: FRED Economic Data on Gross Federal Debt as a percentage of GDP, 1970-2011.

exogenously determined public spending, with the objective to maximise the welfare of the household. The government, in other words, wishes to minimise the welfare costs of taxation. The optimisation is subject to the set of equations that characterize the DCE. To obtain the second best allocations, it is assumed that the government has access to a commitment technology in order not to re-optimize in the future.

To solve the optimal fiscal policy problem we follow the primal approach. Therefore, initially we derive the present discount value of the household's lifetime budget constraint using the transversality conditions for bonds and the two types of capital, presented earlier, and the Arrow-Debreu price of the bond. The next step of the primal approach is to substitute out of the present discount value of the budget constraint prices and taxes using the first order conditions as they have been presented in equations (41) - (46) and, in the Appendix B, in equations (21) - (24). These substitutions lead to the implementability constraint. The implementability constraint and the aggregate resource constraint are the restrictions to the maximisation problem of the government.

2.5.1 Implementability constraint

The implementability constraint is derived from the present value of the household's lifetime budget constraint. Starting from $t = 0$ and by adding future budget constraints for the household, taking the transversality conditions into account, we get:

$$\begin{aligned}
&\sum_{t=0}^{\infty}\left[\prod_{i=0}^{t-1}\left(r_i^b\right)^{-1}\right]\left(C_t+\left(1-\tau_t^h\right)I_{h,t}\right)= \\
&=\sum_{t=0}^{\infty}\left[\prod_{i=0}^{t-1}\left(r_i^b\right)^{-1}\right]\{(1-\tau_t^s)\,w_{s,t}h_{s,t}+[(1-\tau_t^u]\,w_{u,t})h_{u,t}\}+ \\
&+b_0+\left\{(1-\tau_{st,0}^r)r_{st,0}+(1-\delta_{st})\right\}K_{st,0}+ \\
&+\left\{(1-\tau_{eq,0}^r)r_{eq,0}+(1-\delta_{eq})\right\}K_{eq,0}.
\end{aligned} \tag{57}$$

Following Ljungqvist and Sargent (2012) we set the Arrow-Debreu price as: $q_t^0=\prod_{i=0}^{t-1}\left(r_i^b\right)^{-1}$, $\forall t\geq 1$, with $q_0^0=1$, and we re-write the above equation as:

$$
\begin{aligned}
&\sum_{t=0}^{\infty} q_t^0 \left(C_t + \left(1-\tau_t^h\right) I_{h,t}\right) = \\
&= \sum_{t=0}^{\infty} q_t^0 \{(1-\tau_t^s)\, w_{s,t} h_{s,t} + [(1-\tau_t^u]\, w_{u,t}) h_{u,t}\} + \\
&+ b_0 + \left\{(1-\tau_{st,0}^r) r_{st,0} + (1-\delta_{st})\right\} K_{st,0} + \\
&+ \left\{(1-\tau_{eq,0}^r) r_{eq,0} + (1-\delta_{eq})\right\} K_{eq,0}
\end{aligned}
\tag{58}
$$

Then if we use equations (41) - (46) to substitute out prices and taxes, we can write the household's lifetime budget constraint as:

$$
\sum_{t=0}^{\infty} \beta^t \left[U_{C,t} C_t - \left(\frac{U_{e_t}}{f_{e_t}} f_{I_{h,t}}\right) I_{h,t} + \left(U_{h_s,t} + \frac{U_{e_t}}{f_{e_t}}\right) h_{s,t} + U_{h_u,t} h_{u,t} \right] = A_0 \tag{59}
$$

where,

$$
A_0 = U_{C,0} \left\{ \begin{array}{c} b_0 + \left[(1-\tau_{st,0}^r) r_{st,0} + 1 - \delta_{st}\right] K_{st,0} + \\ + \left[(1-\tau_{eq,0}^r) r_{eq,0} + 1 - \delta_{eq}\right] K_{eq,0} \end{array} \right\}
$$

Equation (59) is the implementability constraint.

2.5.2 The primal approach

Under the primal approach the government maximises the following objective function:

$$
\max \sum_{t=0}^{\infty} \beta^t U\left(C_t, h_{s,t}, h_{u,t}, e_t\right) \tag{60}
$$

subject to the implementability constraint (59), the skill creation equation (29) and the aggregate resource constraint (52), by choosing: $\{C_t,\ h_{s,t},\ h_{u,t},\ K_{st,t+1},\ K_{eq,t+1}, e_t,\ I_{h_s,t}\}_{t=0}^{\infty}$, given $\{b_0,\ \tau_{eq,0}^r,\ \tau_{st,0}^r,\ K_{st,0},\ K_{eq,0}\}$.[42]

Following Ljungqvist and Sargent (2012) we initially determine a pseudo-value function and assuming that Φ is the Lagrange multiplier with respect

[42] We do not examine the problem of initial taxation and thus not allow the government to choose optimally the capital income taxes at $t = 0$.

to the implementability constraint we define:

$$V\left(C_t, h_{s,t}, h_{u,t}, e_t; \Phi\right) = U\left(C_t, h_{s,t}, h_{u,t}, e_t\right) + \Phi\{U_{C,t}C_t - \left(\frac{U_{e_t}}{f_{e_t}} f_{I_{h,t}}\right) I_{h,t} + $$
$$+ \left(U_{h_s,t} + \frac{U_{e_t}}{f_{e_t}}\right) h_{s,t} + U_{h_u,t} h_{u,t}\}. \tag{61}$$

Now we can write the Lagrangian equation for the optimal taxation under the primal approach:

$$\begin{aligned} J = \sum_{t=0}^{\infty} \beta^t V\left(C_t, h_{s,t}, h_{u,t}, e_t; \Phi\right) + \\ + \theta_t[Y_t - G_t^c - C_t - I_{h,t} - K_{st,t+1} + \\ + (1-\delta_{st})K_{st,t} - K_{eq,t+1} + (1-\delta_{eq})K_{eq,t}] + \\ + \zeta_t \left[h_{s,t} - f(I_{h,t}, e_t)\right] - \Phi A_0 \end{aligned} \tag{62}$$

where

$$Y_t = A_t \left(K_{st,t}^{a^c}\right) \left[\lambda \left(\nu \left(A_{eq,t}^{\rho} K_{eq,t}^{\rho}\right) + (1-\nu) h_{s,t}^{\rho}\right)^{\varphi/\rho} + (1-\lambda) h_{u,t}^{\varphi}\right]^{\frac{1-a^c}{\varphi}}$$

and $\theta_t, \zeta_t \geq 0 \ \forall t$, is a sequence of Lagrange multipliers with respect to the aggregate resource constraint and the skill creation constraint respectively.

Given the initial values of capital taxes, debt and the two stocks of capital, equation J is maximised with respect to $\{C_t,\ h_{s,t},\ h_{u,t},\ K_{st,t+1},\ K_{eq,t+1},\ e_t,\ I_{h_s,t}\}_{t=1}^{\infty}$ and for $t = 0$ equation J is maximised with respect to $\{C_0,\ h_{s,0},\ h_{u,0},\ e_0,\ I_{h_s,0}\}$. This means that at $t = 0$ the first order conditions are different from $t \geq 1$:

$$V_{C,t} = \theta_t \ , \ t \geq 1 \tag{63}$$

$$V_{h_s,t} = -\theta_t Y_{h_s,t} - \zeta_t \ , \ t \geq 1 \tag{64}$$

$$V_{h_u,t} = -\theta_t Y_{h_u,t} \ , \ t \geq 1 \tag{65}$$

$$V_{e,t} = \zeta_t \ f_{e,t}, \ t \geq 1 \tag{66}$$

$$V_{I_h,t} = \theta_t + \zeta_t \ f_{I_h,t} \ , \ t \geq 0 \tag{67}$$

$$\theta_t = \beta\theta_{t+1}\left[Y_{K_{st},t+1} + 1 - \delta_{st}\right], \ t \geq 0 \tag{68}$$

$$\theta_t = \beta\theta_{t+1}\left[Y_{K_{eq},t+1} + 1 - \delta_{eq}\right], \ t \geq 0 \tag{69}$$

$$V_{C,0} = \theta_0 + \Phi A_C \tag{70}$$

$$V_{h_s,0} = -\theta_0 Y_{h_s,0} - \zeta_0 + \Phi A_{h_s} \tag{71}$$

$$V_{h_u,0} = -\theta_0 Y_{h_u,0} + \Phi A_{h_u} \tag{72}$$

$$V_{e,0} = \zeta_0 \ f_{e,0} + \Phi A_e \tag{73}$$

where $Y_{x,t}$ is the derivative of Y_t, presented above, with respect to variable x at time t.

Therefore, the first order conditions for the government's problem are:

- for $t = 0$:

$$V_{h_s,0} = -(V_{C,0} - \Phi A_C)Y_{h_s,0} - \zeta_0 + \Phi A_{h_s} \tag{74}$$

$$V_{h_u,0} = -(V_{C,0} - \Phi A_C)Y_{h_u,0} + \Phi A_{h_u} \tag{75}$$

$$V_{e,0} = \zeta_0 \ f_{e,0} + \Phi A_e \tag{76}$$

$$V_{I_h,0} = \zeta_0 \ f_{I_h,0} + V_{C,0} - \Phi A_C \tag{77}$$

$$V_{C,0} - \Phi A_C = \beta V_{C,1}\left[Y_{K_{st},1} + 1 - \delta_{st}\right] \tag{78}$$

$$V_{C,0} - \Phi A_C = \beta V_{C,1}\left[Y_{K_{eq},1} + 1 - \delta_{eq}\right] \tag{79}$$

$$\begin{aligned} Y_0 = G^c + C_0 + I_{h,0} + K_{st,1} - (1 - \delta_{st})K_{st,0} + \\ + K_{eq,1} + (1 - \delta_{eq})K_{eq,0} \end{aligned} \tag{80}$$

$$h_{s,0} = f(I_{h,0}, e_0) \tag{81}$$

- for $t = 1, 2, 3...T - 1$:

$$V_{h_s,t} = -V_{C,t}Y_{h_s,t} - \zeta_t \tag{82}$$

$$V_{h_u,t} = -V_{C,t}Y_{h_u,t} \tag{83}$$

$$V_{e,t} = \zeta_t \; f_{e,t} \tag{84}$$

$$V_{I_h,t} = V_{C,t} + \zeta_t \; f_{I_h,t} \tag{85}$$

$$V_{C,t} = \beta V_{C,t+1} \left[Y_{K_{st},t+1} + 1 - \delta_{st} \right] \tag{86}$$

$$V_{C,t} = \beta V_{C,t+1} \left[Y_{K_{eq},t+1} + 1 - \delta_{eq} \right] \tag{87}$$

$$\begin{aligned} Y_t = G^c + C_t + I_{h,t} + K_{st,t+1} - (1-\delta_{st})K_{st,t} + \\ + K_{eq,t+1} + (1-\delta_{eq})K_{eq,t} \end{aligned} \tag{88}$$

$$h_{s,t} = f(I_{h,t}, e_t) \tag{89}$$

- for $t = T$:

$$V_{h_s,T} = -V_{C,T} Y_{h_s,T} - \zeta_T \tag{90}$$

$$V_{h_u,T} = -V_{C,T} Y_{h_u,T} \tag{91}$$

$$V_{e,T} = \zeta_T \; f_{e,T} \tag{92}$$

$$V_{I_h,T} = V_{C,T} + \zeta_T \; f_{I_h,T} \tag{93}$$

$$1 = \beta \left[Y_{K_{st},T} + 1 - \delta_{st} \right] \tag{94}$$

$$1 = \beta \left[Y_{K_{eq},T} + 1 - \delta_{eq} \right] \tag{95}$$

$$Y_T = G^c + C_T + I_{h,T} + \delta_{st} K_{st,T} + \delta_{eq} K_{eq,T} \tag{96}$$

$$h_{s,T} = f(I_{h,T}, e_T) \tag{97}$$

To solve this system, we initially guess a value for Φ and solve equations (74)-(97) for an allocation $\{C_t,\, h_{s,t},\, h_{u,t},\, K_{st,t+1},\, K_{eq,t+1},\, e_t,\, I_{h_s,t},\, \zeta_t\}_{t=0}^{\infty}$. The system has $[(8 \times T) + 1]$ equations and it is solved using standard non-linear numerical methods (see, e.g. Garcia-Milà *et al.* (2010), Adjemian *et al.* (2011)). Then we test if the implementability constraint (59) is binding and we increase or decrease accordingly the value of Φ until the implementability constraint is satisfied. We set the initial conditions for debt, the two stocks of capital and the two capital income taxes equal to their exogenous steady-state, to calculate the dynamic transition path from the exogenous to optimal fiscal policy steady-state. To ensure that the variables converge to the optimal fiscal policy steady-state, we set the value of $T = 250$. The results indicate that in model convergence has been achieved after the initial 150 periods.

2.5.3 Flexible debt: steady-state and lifetime welfare

We first present results from the case where no additional restrictions are placed on government debt, other than (59). Table 2.3 presents the results for the steady-state and for lifetime welfare for optimal fiscal policy together with the relevant steady-state of the exogenous fiscal policy for different assumptions regarding the policy menu set. In the penultimate row of the table we present the lifetime welfare in each case study by taking into account the transitional dynamics from the exogenous fiscal policy to optimal taxation (these will be examined below in more detail). The last row presents the compensating consumption supplement, ψ,for each model compared to the benchmark specification under a full instrument set in the second column. As a result, a negative value of ψ indicates that the lifetime welfare, taking into account the transitional dynamics, is lower under that restriction in the policy instrument set compared to the benchmark specification.[43]

The optimal policy results in the steady-state are consistent with the related literature (see e.g. Judd (1985), Chamley (1986) and Ljungqvist and Sargent (2012) ch. 16 for a review of the literature). Therefore, as it is expected from the literature, under a complete set of fiscal instruments and shown in column 2 in Table 2.3, the optimal tax rate on capital income is zero in the long-run. In our case study we find that this is true for both capital taxes and remains true even if we do not tax differently the returns from capital structures and capital equipment.[44] Compared to the exogenous fiscal policy at the steady-state of optimal fiscal policy capital accumulation is higher due to the elimination of the intertemporal wedges.

The optimal labour income taxation is progressive in our setup, with a

[43]The calculation of the compensating consumption supplement is presented in the Appendix B.

[44]These results are not shown here to save on space but are available on request.

larger difference between the two labour income taxes compared to exogenous fiscal policy. The increase in the returns to skilled labour, brought about by the skill biased increase in the capital equipment stock (in turn achieved by the elimination of the capital taxes) allows the government to tax skilled labour income more than unskilled, without reducing skilled hours. Indeed, under optimal fiscal policy, skilled hours are slightly higher than the unskilled hours, compared to the exogenous fiscal policy. This is further encouraged by a subsidy to spending in education. Therefore, skilled hours rise more relative to unskilled, and the representative household decides to invest more in skill creation, mainly through goods investment because of the subsidy in the investment of goods for skill acquisition. The increase in skilled hours reduces the skill premium compared with the exogenous fiscal policy case study, despite the rise in skill-biased equipment capital.

Next we restrict the choice of fiscal policy instruments for the government. First, we do not allow the government to use two different labour income tax rates, i.e. we impose that $\tau_t^s = \tau_t^u$; this is Model 1.[45] This allows us to examine the long-run optimal capital income tax and specifically we can assess if one of the two capital income tax rates will become non-zero in order to compensate for the missing labour income tax. In other words, under this specification we can examine optimal capital income taxation under incomplete set of fiscal instruments.

Then, as an alternative specification, we impose a different restriction to the set of fiscal policy instruments by not allowing the government to use a subsidy to goods investment for skill creation (Model 2). This is an interesting case study because we can examine under this setup how the government will incentivise the skill acquisition and what happens to the progressivity of the labour taxes.

Finally, we restrict even further the set of fiscal instruments by combining the previous restrictions (Model 4). In this extension of Model 2 we can investigate the effects on the optimal capital income tax rates.

[45] Note that we have also examined the case where the government is not able to impose two different capital income taxes. In this case the results are identical to our benchmark model and the government still optimally sets a zero capital income tax.

Table 2.3: Comparison of steady state optimal tax results

Param.	Exogenous policy	Opt. policy endog. b/Y (Benchmark)	Opt. Policy endog. b/Y with $\tau^s = \tau^u$ (Model 1)	Opt. policy endog. b/Y and $\tau^h = 0$ (Model 2)	Opt. policy endog. b/Y $\tau^s = \tau^u$ and $\tau^h = 0$ (Model 3)
Y	0.1342	0.1692	0.1678	0.1644	0.1640
C	0.0756	0.0899	0.0889	0.0882	0.0877
K_{st}	0.0913	0.1669	0.1655	0.1622	0.1618
K_{eq}	0.1698	0.2891	0.2866	0.2802	0.2794
h_s	0.1177	0.1351	0.1397	0.1305	0.1329
h_u	0.1480	0.1337	0.1236	0.1316	0.1268
e	0.0149	0.0145	0.0167	0.0177	0.0185
I_h/Y	0.0177	0.0297	0.0301	0.0200	0.0209
K_{st}/Y	0.6805	0.9863	0.9863	0.9863	0.9863
K_{eq}/Y	1.2656	1.7088	1.7076	1.7040	1.7036
$\bar{G}^c$	0.0320	0.0320	0.0320	0.0320	0.0320
C/Y	0.5631	0.5315	0..5298	0.5363	0.5350
$\bar{G}^c/Y$	0.2382	0.1890	0.1905	0.1944	0.1949
b/Y	0.5281	-0.4958	-0.5333	-0.5686	-0.5691
I/Y	0.1987	0.2498	0.2497	0.2493	0.2493
τ^r_{st}	0.3100	0.0000	0.0000	0.0000	0.0000
τ^r_{eq}	0.3100	0.0000	0.0240	0.0000	0.0116
τ^s	0.2500	0.3182	0.2762	0.2827	0.2663
τ^u	0.2000	0.2331	0.2762	0.2460	0.2663
τ^h	0.0000	0.4509	0.3783	-	-
w_s/w_u	1.6431	1.6408	1.5117	1.6609	1.5961
LU	-75.4549	-75.0366	-75.0744	-75.1884	-75.2002
ψ	-	-	-0.1440%	-0.5790%	-0.6241%

We start with the case where we reduce the available set of fiscal policy instruments by assuming that the government can only impose a single labour income tax rate for both types of labour, keeping the subsidy in the investment of goods for skills creation (Model 1). In this case we observe that the tax rate on capital equipment is not zero in the long-run. This result cohere with the argument of Correia (1996) that if there is a factor of production that cannot be taxed then the long-run value of the optimal capital income tax is not zero. However, in our case we have two different capital income tax rates, one for capital equipment and one for capital structures. Therefore, it is not obvious which of those fiscal instruments the central planner will use in order to compensate for the missing labour income tax. Under the current setup we can see that the optimal taxation in the long-run suggests that the tax rate on the returns to capital equipment will be non-zero and positive, while the tax on structures remains zero. Equipment capital complements skilled labour, thus the tax on equipment capital allows the government to implement indirectly a higher tax burden on skilled labour supply, relative to unskilled, which is optimal as discussed above given the increase in skill-biased capital.

Moreover, in Model 1 the unskilled labour income tax rate is higher relative to the benchmark case working as an incentive for the unskilled agents to create skills and as a result there is no need for the skill premium to be high to enhance that incentive. Thus, in this case we observe a lower skill premium compared to the benchmark model.

Since in our current setup we assume that there is a representative household, we cannot examine the effects of optimal fiscal policy on income inequality. However, we can evaluate the aggregate welfare losses associated with policy restrictions. In particular, we can see in Table 2.3 that when the government has a more restricted set of policy instruments, i.e. comparing Model 1 to the Ramsey case, the lifetime welfare of the representative household is lower compared to the benchmark case with a full set of fiscal policy instruments. This is expected since the government is restricted from using two different labour income taxes. Therefore, it is expected that as the restrictions to the set of fiscal policy menu increase, the compensating consumption supplement should get larger in absolute magnitude, but with a negative sign.

We then move to the case where we restrict the set of policy instruments by not allowing the government to use a subsidy to goods investment for skills acquisition (Model 2). We can see that we still obtain the optimal zero capital income tax in long-run. Moreover, the progressivity of the labour income taxes remains, although the unskilled labour income tax is higher and the skilled labour income tax lower compared to the benchmark case. Thus, when the government cannot encourage skill creation by providing education subsidies, taxation of skilled labour income needs to be decreased relative to unskilled labour income, to provide higher benefits to skill supply. Specifically, the skill labour income tax is almost three percentage points lower then the previous case, whereas the unskilled labour income tax has increased by one percentage point. In addition, in this case the lifetime welfare of the household has decreased compared to the benchmark case and compared to Model 1. This means that the use of education subsidy is an important tool for the government.

Finally, we reduce even further the set of fiscal policy instruments. In this case (Model 3), the central planner can only impose two different capital income tax rates and a single labour income tax. Therefore, now the planner doesn't have the subsidy to skill acquisition and the two different labour income taxes at its disposal. In this case we find again the result of non-zero optimal capital equipment income tax rate at the steady-state but this time the tax rate is lower than in Model 1. This implies that the tax on the returns to capital equipment will be the fiscal instrument that will compensate for the loss of the two different labour income taxes from the fiscal policy menu. Interestingly, even in this case the optimal tax rate on the income from capital structures remains zero. Thus, when there is a restricted fiscal policy menu the government finds it optimal to always use the tax rate on the returns from capital equipment accordingly so as to compensate for the missing fiscal policy instrument. Regarding the skill premium we can see that in this case it is lower compared to the benchmark case but higher compared to Model 1. This is observed because in this case the skilled labour income tax is the lowest compared to the benchmark and the other models and it is used as an incentive mechanism for skills acquisition instead of a high skill premium.

2.5.4 Transition path

Figure 2.1 presents the transition path associated with the optimal taxation for our benchmark model.[46] We present the dynamic paths implied by optimal policy for the capital structures tax, capital equipment tax, debt to output ratio, the two labour income taxes and the subsidy to investment in education as the economy transitions from the exogenous fiscal policy steady-state to the optimal taxation steady-state. The first row of Figure 2.1 shows that in the first period skilled and unskilled labour income are subsidised at rates 10.4% and 24.9% respectively. The subsidy to the investment in skill acquisition is equal to 5.3% in the first period. In the second period the skilled tax labour income tax increases to almost 35.7% and then it gradually decreases to its long-run steady-state reported in Table 2.3 (benchmark model). The unskilled labour income tax in the second period is equal to 25.5% and then it gradually converges to its long-run value under optimal taxation. The subsidy to the investment in skill acquisition becomes 45% in the second period and then follows the same pattern as the labour income taxes.

The second row of Figure 2.1 shows that in the first period, since capital is already in place (predetermined), the two capital income tax rates reach a confiscatory rate of about 268% and 245% for capital structures and capital equipment respectively. In the second period, the two tax rates become 1.3% and 0.48% for capital structures and equipment respectively and then slowly converge to zero. The high capital taxation in the first period allows the government to create a first period stock of assets via lending to households. Then in the second period the government assets decrease due to the sharp decrease in capital income taxes and then they slowly increase again mainly due to the decrease in the subsidy for skill acquisition. The income from assets is used to subsidise investment in goods for skill creation and to compensate for the zero capital income taxes, without the need to resort to significantly high labour income taxes.

[46] Note that we do not present the transition paths for every model presented in Table 2.3 since they are qualitatively and quantitatively similar.

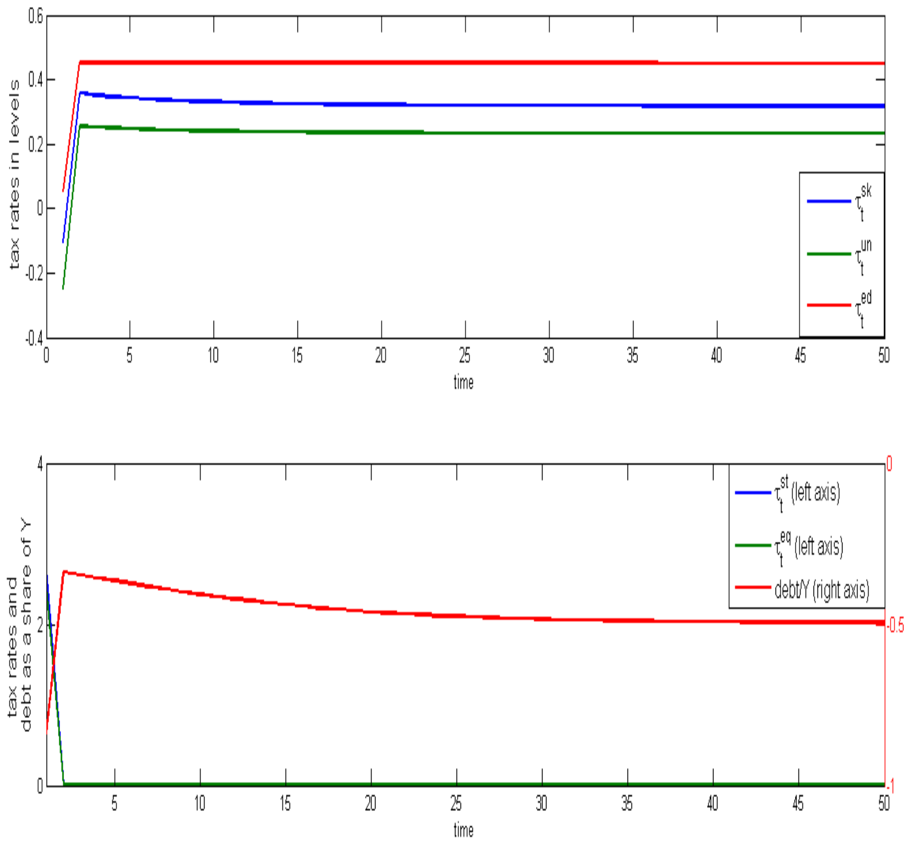

Figure 2.1: Transition path of the policy instruments

2.5.5 Budget rule

Table 2.4 presents the steady-state results under optimal fiscal policy under a budget rule. In particular, we assume that the government is required to keep a fixed debt to output ratio when choosing optimal taxation. This is an empirically relevant restriction since in recent years most advanced economies had debt targets to respect. Table 2.4 includes in the first column the results from the Ramsey case and this is followed by the results under the budget rule under the full set of tax instruments. The remaining columns present results obtained by imposing the same restrictions on the instrument set as in Table 2.3, for the case of budget rules.

Under a compete tax instrument set and a restriction that the government needs to keep a fixed debt to output ratio equal to the average debt to output ratio observed in the annual US data for the period 1970-2011 we have the results in the second column, under the label of Model 4. In this case we observe that both labour income taxes are higher and the subsidy is lower compared to the benchmark case in order for the government to be able to satisfy a given level of debt to output ratio. However, we still observe that the progressivity of the labour income taxes is present although the difference between the two labour income taxes has decreased compared to the benchmark case. Specifically, the skilled labour income tax is almost 1.5 percentage points higher and the unskilled labour income tax almost 3 percentage points higher than the benchmark case study. The optimal capital income taxes are again zero in this case. Finally, the compensating consumption supplement is negative 0.28% which means that the representative household is worse off under this setup compared to the benchmark case. This is expected since the government is restricted from issuing debt. Therefore, again we expect to see that as the restrictions to the set of fiscal policy menu increase, the compensating consumption supplement should get larger in absolute terms but with a negative sign. In addition, it is expected that the lifetime welfare under a fixed debt to output ratio should be lower compared to the cases where the government was able to issue debt.

In Model 5 we restrict the government from issuing two different labour income taxes, under the budget rule. In this case we observe that the optimal tax rate on capital equipment is non-zero, similarly to Model 1. Specifically, in this case the optimal tax on capital equipment is equal to 1.86%, which is lower compared to the 2.4% under Model 1. Therefore, the result that under an incomplete set of fiscal instruments the government is using the tax rate on capital equipment, and not that on capital structures, to compensate for the missing fiscal instrument remains even with a fixed debt to output ratio. Similarly to Model 1, the skill premium is being reduced compared to the benchmark and the exogenous fiscal policy model. That happens because in Model 5 the unskilled labour income tax rate is higher relative to Model 4 case study, working as an incentive for the unskilled agents to create skills and there is no need for the skill premium to be high so as to enhance that incentive. In particular, in this case the unskilled labour income tax is almost 3.7 percentage points higher than in Model 4, and the skilled labour income tax is almost 3.5 percentage points lower than in Model 4 with a full set of fiscal instruments but without flexible debt. In addition, we can see that in Model 5 the lifetime welfare of the representative household is lower compared to Model 4 and compared to the benchmark case with a full set of fiscal policy instruments.

In the case where we restrict the government from issuing a subsidy to goods investment for skills acquisition (Model 6) we observe that the optimal tax rate on both capitals is zero. In addition, the progressivity of the labour income taxes is still present although it has decreased compared to Model 4. Specifically, the labour income tax has increased by 1.5 percentage points and the skilled labour income tax has decreased by almost 3 percentage points compared to Model 4. Thus, similarly to Model 2, when the government cannot encourage skill creation by providing a subsidy to education, taxation of skilled labour income needs to be decreased more relative to unskilled labour income tax, so as to provide incentives for skills acquisition. Finally, the lifetime welfare of the household has decreased even further compared to the benchmark case and compared to Model 4.

In Model 7 we restrict even further the set of fiscal policy instruments. In this case the government cannot issue two different labour income taxes and also cannot impose a subsidy to the investment in education. Under this specification we get a non-zero tax on capital equipment which is lower compared to Model 5, at 0.88%. Regarding the level of the labour income taxes we observe that it is marginally lower than Model 5, similarly with the case studies where the government was able to issue debt (Models 1 and 3).

Table 2.4: Comparison of steady state optimal taxation results with fixed debt to output ratio

Param.	Opt. policy endog. b/Y (Ramsey)	Opt. policy fixed b/Y (Model 4)	Opt. policy fixed b/Y with τ^s=τ^u (Model 5)	Opt. policy fixed b/Y and τ^h= 0 (Model 6)	Opt. policy fixed b/Y τ^s= τ^u and τ^h= 0 (Model 7)
Y	0.1692	0.1653	0.1642	0.1613	0.1610
C	0.0899	0.0876	0.0867	0.0859	0.0856
K_{st}	0.1669	0.1630	0.1619	0.1590	0.1588
K_{eq}	0.2891	0.2828	0.2812	0.2754	0.2751
h_s	0.1351	0.1323	0.1362	0.1285	0.1306
h_u	0.1337	0.1300	0.1214	0.1278	0.1238
e	0.0145	0.0147	0.0166	0.0173	0.0180
I_h/Y	0.0297	0.0269	0.0270	0.0193	0.0200
K_{st}/Y	0.9863	0.9863	0.9863	0.9863	0.9863
K_{eq}/Y	1.7088	1.7108	1.7125	1.7077	1.7088
$\bar{G}^c$	0.0320	0.0320	0.0320	0.0320	0.0320
C/Y	0.5315	0.5297	0.5281	0.5327	0.5316
$\bar{G}^c/Y$	0.1890	0.1934	0.1947	0.1983	0.1986
b/Y	-0.4958	0.5281	0.5281	0.5281	0.5281
I/Y	0.2498	0.2500	0.2502	0.2497	0.2498
τ^r_{st}	0.0000	0.0000	0.0000	0.0000	0.0000
τ^r_{eq}	0.0000	0.0000	0.0186	0.0000	0.0088
τ^s	0.3182	0.3338	0.2983	0.3058	0.2924
τ^u	0.2331	0.2619	0.2983	0.2757	0.2924
τ^h	0.4509	0.3934	0.3199	-	-
w_s/w_u	1.6408	1.6329	1.5222	1.6454	1.5909
LU	-75.0366	-75.1101	-75.1550	-75.2752	-75.2888
ψ	-	-0.2801%	-0.4513%	-0.9112%	-0.9632%

2.6 Concluding remarks

In this chapter, we analysed optimal factor income taxation when there are different returns to skilled and unskilled labour and capital in structures and equipment, under capital-skill complementarity and endogenous skill acquisition. We worked with a representative agent framework, which allowed us to focus on aggregate efficiency and abstract from potential equity considerations for optimal taxation. In the model considered, a representative household decides how to allocate its investment in the two types of capital stock and in creating skill labour. Moreover, it decides how to allocate its time endowment into leisure, labour supply in skilled and unskilled jobs, and in creating skill labour through education time. Therefore, the model allows for endogenous skill acquisition, which is modelled following He and Liu (2008). The production side is modelled as in Krusell *et al.* (2000). In this framework, we derived optimal tax policy under commitment and exogenous spending policy, by considering various assumptions regarding the availability of tax instruments and debt to the policy-maker.

We found that when all factor inputs, as well as investment in skill acquisition, can be taxed at separate rates, both capital income taxes are zero in the long-run, there is a subsidy to education and the labour income taxes are progressive. However, if the government has access to the education subsidy but cannot tax income from skilled and unskilled labour separately, while the tax on income from structures remains zero in the long run, there is a small positive tax on equipment capital. If the government can only implement a single income tax, without having access to education subsidies, the equipment tax is again positive but with lower magnitude. Moreover, we found that the restrictions to fiscal policy menu do not change the result of progressive labour income taxes qualitatively, however, more restricted policy, in the form of unavailability of education subsidies and budget rules, is associated with decreases in progressivity.

Appendix B

B Chapter 2

B.1 DCE system of equations

Here we present the equations of the exogenous decentralized competitive equilibrium:

$$0 = U_{h_{s,t}} + \frac{U_{e_t}}{f_{e_t}} + U_{C_t}(1 - \tau_t^s) w_{s,t} \tag{B1}$$

$$0 = U_{h_{s,t}} + U_{C_t}(1 - \tau_t^u) w_{u,t} \tag{B2}$$

$$U_{C_t} = \beta[U_{C_{t+1}} \left(r_{st,t+1}(1 - \tau_{st,t+1}^r) + (1 - \delta_{st}) \right)] \tag{B3}$$

$$U_{C_t} = \beta[U_{C_{t+1}} \left(r_{eq,t+1}(1 - \tau_{eq,t+1}^r) + (1 - \delta_{eq}) \right)] \tag{B4}$$

$$U_{C_t}(1 - \tau_t^h) + \frac{U_{e_t}}{f_{e_t}} f_{I_{h,t}} = 0 \tag{B5}$$

$$r_t^b = \frac{U_{C_t}}{\beta U_{C_{t+1}}} \tag{B6}$$

$$w_{s,t} = \lambda (1 - \nu) (1 - a^c) h_{s,t}^{\rho - 1} A_t (K_{st,t})^{a_c} \times \\ \times \left[\begin{array}{c} \lambda \left(\nu (A_{eq,t} K_{eq,t})^{\rho} + (1 - \nu) h_{s,t}^{\rho} \right)^{\varphi/\rho} \\ + (1 - \lambda) h_{u,t}^{\varphi} \end{array} \right]^{\frac{1 - a^c}{\varphi} - 1} \tag{B7}$$

$$w_{u,t} = (1 - \lambda) (1 - a^c) h_{u,t}^{\varphi - 1} A_t (K_{st,t})^{a_c} \times \\ \times \left[\begin{array}{c} \lambda \left(\nu (A_{eq,t} K_{eq,t})^{\rho} + (1 - \nu) h_{s,t}^{\rho} \right)^{\varphi/\rho} \\ + (1 - \lambda) h_{u,t}^{\varphi} \end{array} \right]^{\frac{1 - a^c}{\varphi} - 1} \tag{B8}$$

$$r_{st,t}=a^c A_t K_{st,t}^{a^c-1}\times \tag{B9}$$
$$\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{s,t}^{\rho}\right)^{\varphi/\rho}\\+(1-\lambda)\,h_{u,t}^{\varphi}\end{array}\right]^{\frac{1-a^c}{\varphi}}$$

$$r_{eq,t}=\lambda\nu\left(1-a^c\right)A_{eq,t}^{\rho}K_{eq,t}^{\rho-1}A_t\left(K_{st,t}\right)^{a_c}\times \tag{B10}$$
$$\times\left[\begin{array}{c}\lambda\left(\nu\left(A_{eq,t}K_{eq,t}\right)^{\rho}+(1-\nu)\,h_{s,t}^{\rho}\right)^{\varphi/\rho}\\+(1-\lambda)\,h_{u,t}^{\varphi}\end{array}\right]^{\frac{1-a^c}{\varphi}-1}$$

$$\begin{aligned}G_t^c+\tau_t^h I_{h_{u,t}}+b_t=\tau_t^s w_{s,t}h_{st}+\tau_t^u w_{u,t}h_{u,t}+\frac{b_{t+1}}{r_t^b}\\+\tau_{st,t}^r r_{st,t}K_{st,t}+\tau_{eq,t}^r r_{eq,t}K_{eq,t}\end{aligned} \tag{B11}$$

$$\begin{aligned}Y_t=&\left(K_{st,t+1}-(1-\delta_{st})K_{st,t}+K_{eq,t+1}-(1-\delta_{eq})K_{eq,t}\right)+\\&+C_t+G_t^c+I_{h_{u,t}}\end{aligned} \tag{B12}$$

$$Y_t=w_{s,t}h_{s,t}+w_{u,t}h_{u,t}+r_{st,t}K_{st,t}+r_{eq,t}K_{eq,t} \tag{B13}$$

where,

$$Y_t=A_t\left(\left(K_{st,t}\right)^{a^c}\right)\times \tag{B14}$$
$$\times\left[\lambda\left(\nu\left(A_{e,t}^{\rho}(K_{eq,t})^{\rho}\right)+(1-\nu)\,h_{s,t}^{\rho}\right)^{\varphi/\rho}+(1-\lambda)\,h_{u,t}^{\varphi})\right]^{\frac{1-a^c}{\varphi}}.$$

B.2 Compensating consumption supplement

In order for the welfare gains to be estimated the compensating consumption supplement will be calculated, following Lucas (1990):

$$V^B(C,l)=V^A((1-\psi)C,l) \tag{B15}$$

where V^B is the lifetime welfare of the representative household under the Benchmark model and under optimal fiscal policy and V^A is the lifetime welfare of the representative household under optimal fiscal policy and under several restrictions to fiscal policy menu. Note that in both regimes we calculate the lifetime welfare by taking into account the transition period from the exogenous to optimal fiscal policy. The lifetime welfare is calculated as:

$$V^B(C,l) = \sum_{t=0}^{\infty} \beta^t \left(U^A\left(C_t, l_t\right)\right)$$

and

$$V^A((1-\psi)C,l) = \sum_{t=0}^{\infty} \beta^t \left(U^A\left((1-\psi)C_t, l_t\right)\right)$$

where U^B and U^A represent the within period welfare.

Therefore, equation (B15) will become:

$$\sum_{t=0}^{\infty} \beta^t \left(U^B\left(C_t, l_t\right)\right) = \sum_{t=0}^{\infty} \beta^t \left(U^A\left((1-\psi)C_t, l_t\right)\right)$$

$$V^B\left(C_t, l_t\right) = \sum_{t=0}^{\infty} \beta^t \frac{\left((1-\psi)^{\gamma} C_t{}^{\gamma} l_t^{1-\gamma}\right)^{1-\sigma_1}}{1-\sigma_1}$$

$$V^B\left(C_t, l_t\right) = \sum_{t=0}^{\infty} \beta^t (1-\psi)^{\gamma(1-\sigma_1)} \frac{\left(C_t{}^{\gamma} l_t^{1-\gamma}\right)^{1-\sigma_1}}{1-\sigma_1}$$

$$V^B\left(C_t, l_t\right) = (1-\psi)^{\gamma(1-\sigma_1)} \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t{}^{\gamma} l_t^{1-\gamma}\right)^{1-\sigma_1}}{1-\sigma_1}$$

$$V^B\left(C_t, l_t\right) = (1-\psi)^{\gamma(1-\sigma_1)} V^A$$

$$(1-\psi)^{\gamma(1-\sigma_1)} = \frac{\left(V^B\right)}{\left(V^A\right)}$$

$$\psi = 1 - \left(\frac{V^B}{V^A}\right)^{\frac{1}{\gamma(1-\sigma_1)}}$$

where we multiply by 100 to obtain a percentage rate:

$$\psi_{B \to A} = \left[1 - \left(\frac{V^B}{V^A}\right)^{\frac{1}{\gamma(1-\sigma_1)}}\right] \times 100 \qquad \text{(B16)}$$

Chapter 3: Optimal income taxation over the business cycle

Abstract: This chapter examines how income taxes are optimally distributed over the business cycle in a model with high, middle and low income agents when the government is restricted to balance its budget in each period. The model incorporates capital-skill complementarity and the agents are assumed to have different access to labour and capital markets. This allows us to develop an empirically relevant model that is able to match the cyclical characteristics of the US economy. The optimal fiscal policy results indicate that the income tax rate of the high income agents has the lowest volatility and the income tax rate of the low income agents exhibits the lowest counter-cyclicality. Moreover, the optimal income tax rate of the middle income agents is found to be very volatile and counter-cyclical. We also examine the effects on optimal income taxation under a different fiscal policy menu that also includes a consumption tax. In this case we find that the progressivity of the income tax rates is even higher and that the results regarding the volatilities of the income taxes are overturned. We further find that, in both of the case studies, the progressivity of the income tax rates is optimally increased immediately after a positive shock to capital equipment efficiency and a reduction to government expenditures, whereas it optimally increases after two years with a positive shock to total factor productivity.[47]

3.1 Introduction

There is a growing literature that examines the characteristics of optimal taxation over the business cycle (i.e. Chari *et al.* (1994) and Arseneau and Chugh (2012)). The optimal taxation literature acknowledges the fact that the set of fiscal policy instruments and the imperfections in capital and labour markets are essential for the behaviour of optimal fiscal policy. For instance, while in a frictionless labour market the labour income tax should optimally

[47] The CESifo 2013 working paper (4468) by Angelopoulos, Asimakopoulos and Malley entitled "The Optimal Distribution of the Tax Burden over the Business Cycle" is based on this Chapter.

not vary much over the business cycle and remain a-cyclical, Arseneau and Chugh (2012) show that under search frictions in the labour market, the optimal labour income tax becomes very volatile and counter-cyclical. Moreover, Stockman (2001) shows that a balanced-budget restriction leads to an increase of the optimal volatility of the labour relative to capital taxes.

In a different literature, Hornstein *et al.* (2005) and Acemoglu and Autor (2011) provide empirical evidence that the capital-skill complementarity in the production process drives the growing wage inequality observed in the data. Moreover, Campbell and Mankiw (1989), Mankiw (2000) and Galí *et al.* (2007) show that the existence of a subset of the population that doesn't have access to capital markets, "hand-to-mouth" agents, is essential for shaping the economic policy under uncertainty. Despite the aforementioned research, the optimal taxation literature has not yet examined optimal income taxes over the business cycle under imperfections that limit the participation of households in markets for skilled labour and capital.

Income taxation has naturally been a focal point for the research on economic policy under income inequality (see e.g. the work reviewed and analysed in Kocherlakota (2010)). This is because, on one hand, progressive income taxation can be used to reduce income inequality and promote a fairer distribution of income. On the other hand, the disincentives associated with taxation and, in particular, with progressive taxation, need to be taken into account. In light of this, the normative properties relating to the progressivity of the tax system have been extensively analysed (see e.g. Mankiw et al. (2009) for an assessment of this literature). However, the response of optimal income taxes in business cycle frequencies to exogenous productivity and government spending shocks, under a balanced budget and both wage and asset inequalities, has not been examined. This is particularly relevant given the presence of these inequalities and the current economic reality that severely limits the use of debt to respond to economic fluctuations in most advanced economies. In such an environment, the revenue requirements for governments that are faced with exogenous aggregate shocks need to be financed by unpleasant taxes, so that a pertinent question for policymaking becomes how to distribute the tax burden over the business cycle to minimise the negative effect of distorting taxes. In addition, it is important to understand how optimal income taxation behaves when there is a richer set of fiscal instruments. For that reason we extend the fiscal policy menu of the government by introducing a consumption tax and we re-assess the reaction of the optimal taxation over the business cycle.

In a related literature, Aiyagari (1994) examines the effects of idiosyncratic shocks on income inequality. Specifically, he assumes that agents are *ex-ante* homogeneous but *ex-post* heterogeneous due to the idiosyncratic shocks. As a result agents tend to accumulate capital via a precautionary motive. This capital acts as self-insurance against potential future negative shocks. Moreover, Aiyagari (1994) examines only the steady state equilibrium and there are no aggregate shocks to the economy.

The New Dynamic Public Finance (NDPF) literature extends Aiyagari-Bewley model by assuming that agents are privately informed about their skills and productivity. This way optimal taxation depends on income, risk aversion and distribution of skills, and provides a trade-off for the government between incentives to labour and equality among the agents. Kocherlakota (2006) challenges the argument of Chamley (1986), that government can impose only linear taxes, because in practice governments are able to use nonlinear taxes. Thus, Kocherlakota (2006), following Mirrlees (1971), introduced nonlinear taxation in an economy populated with heterogeneous agents in terms of labour productivity. As in Mirrlees (1971), NDPF makes the assumption that people differ in their skills, which are private information, and also skills change stochastically over time.

Therefore, under the NDPF theory the government pre-commits to a tax schedule that can only depend on income and not skills. This way the government has an unrealistically large set of policy instruments at its disposal which is very difficult to monitor at each point in time.[48] Moreover, due to the large complexity of the model it is not feasible to be calibrated to capture some key characteristics of the economy and a static analysis is usually performed.

Taking the above into consideration, in this chapter we analyse the op-

[48] Note that in our model the government can observe the skills of the agents.

timal income taxation over the business cycle when the government runs a balanced budget. To this end, we develop a model that is able to replicate the cyclical properties of the economy and the empirical observations on wealth and wage inequality (skill premium).[49] Therefore, we build a model where the production process exhibits capital-skill complementarities, the labour market is fragmented regarding the skill supply and the agents have different access to asset markets with a subset of the population being excluded from holding assets.

Thus, the model has three types of households that are divided with respect to their income into low, middle and high. In addition, we have two labour markets, for skilled and unskilled labour and we further assume that there are barriers that prevent agents from participating in both labour markets. In particular, we assume that the high income households provide skilled labour, where skilled agents are those with a college degree or relevant professional qualification. The middle and low income households are assumed to provide unskilled labour, which means that are those without a college degree. The production structure implies that there are two wage rates in the model, skilled and unskilled wage rate, leading to a skill premium.[50] Following the contributions of Katz and Murphy (1992) and Krusell *et al.* (2000), we assume that the skill premium is driven by skill-biased technical change and capital-skill complementarities. In particular, we assume that the production process follows the technology specified in Krusell *et al.* (2000) which has received empirical support and has been shown to match the behaviour of the skill premium in the data.

Capital market imperfections imply that households in our model differ with respect to their participation in the asset markets. In particular, following the contributions of Campbell and Mankiw (1989), Mankiw (2000) and Galí *et al.* (2007), we assume that a subset of the households does not have any savings and is thus earning only labour income, which it totally consumes. We further assume that these households offer unskilled labour

[49] The skill premium is defined as the ratio of the wage rate of the skilled agent over the wage rate of the unskilled agent.

[50] In our setup we assume that skills are endowed, so taxes do not affect skill acquisition.

services, so that the three types of households in the economy are defined as, high income skilled agents who own assets, middle income unskilled agents who also own assets and low income unskilled agents who do not have access to the capital market.

Our work on this chapter deviates from the representative agent literature on optimal taxation, such as Chari *et al.* (1994) and Stockman (2001), by emphasizing on the importance of skill and asset market imperfections that generate wealth and wage inequality, as well as the balanced budget restriction on the behaviour of optimal taxation. Compared with the literature of optimal taxation with heterogeneous agents (i.e. Kocherlakota (2010) and references therein), our modeling emphasizes on wage inequalities that are generated by capital-skill complementarities together with the inequality from the asset market participation and focuses on the business cycle properties of income taxes.[51]

We calibrate a version of the model with exogenous tax policy to the U.S. quarterly data and find that the model fits the data very well with respect to key long-run stylized facts as well as the cyclical properties of the data, including the empirical findings that the skill premium is effectively a-cyclical and not volatile (see e.g. Lindquist (2004) and Pourpourides (2011)). Having established the empirical relevance of the model, we then characterize optimal policy, by letting the government choose the income tax rates optimally over the business cycle to maximise aggregate welfare given its revenue requirements. Moreover, we extend the set of fiscal instruments by allowing the government to choose optimally a consumption tax rate on top of the three income tax rates with the balanced budget restriction.

We find that the cyclical properties of the income tax rates differ significantly with each other and with those observed in the data. As expected, given the balanced budget restriction and the instruments available to the government, the tax rates are generally more volatile and more counter-cyclical than in the data, as in Stockman (2001). The counter-cyclicality of

[51] Specifically, this literature emphasizes on the unobservability of idiosyncratic labour productivity, which then generates wage inequality. In this chapter we make the assumption that skills are observable, since they depend on the level of education.

the taxes is also driven from the balanced budget restriction because under a negative shock to the economy, output decreases and also capital and labour decrease, causing a reduction to labour and capital income and as a result the tax revenues decrease. Thus, the government needs to increase taxation to be able to finance its expenditures.

However, there are also important differences between the tax rates. These result from the trade-off that the government faces when deciding how to distribute the distortions reflected by the higher volatility and counter-cyclicality of the three tax rates over the business cycle. On one hand, such distortions have a large impact on hand-to-mouth households, since they are less able to smooth shocks. There is thus an incentive to minimise the impact of policy for this type of household. On the other hand, tax-induced distortions to skilled households have the strongest propagation effects in the economy, given the complementarity of skilled hours with equipment capital. Therefore, there is also an incentive to minimise distortions to the choices of skilled households, since this acts to amplify external shocks. Optimal policy resolves this trade-off by keeping the lowest volatility for the tax rate for skilled and the lowest counter-cyclicality for the hand-to-mouth households. In contrast, the middle income group, made up by unskilled households with savings, receives very volatile and very counter-cyclical taxes. For the case where we also introduce the consumption tax we can see that most of the aforementioned results are preserved apart from the volatility of the income taxes. In this case we find that the unskilled agents that are able to save have the most volatile income tax, whereas the hand-to-mouth agents have the smoothest income tax.

We further analyse the optimal distribution of the tax burden in the short- and medium-run in response to temporary output-enhancing exogenous shocks. The government finds it optimal to respond to an increase in the productivity of capital equipment and to public spending cuts by in creasing the progressivity of income taxes. In the case of capital equipment technology shocks, in particular, the government finds it optimal to redistribute some of the gains to skilled workers, who are the main beneficiaries of such changes, to the more constrained households in the labour market.

This is achieved by increasing the high-income tax and reducing the other two taxes. Public spending cuts allow the government to reduce all income taxes, but the reduction is higher the lower the income level of the household. Finally, the response to positive total factor productivity (TFP) shocks implies that the progressivity of the tax system increases after about two years. The sensitivity of the income of hand-to-mouth households with respect to TFP shocks implies that the government needs to use a pro-cyclical tax on impact in this case to help smooth consumption. As a result, income taxes to low income agents increase immediately after positive TFP shocks so as to smooth consumption. The aforementioned results and behaviour of the income taxes after a temporary shock remain unchanged with the introduction of the consumption tax.

The rest of this chapter is organised as follows. Section 2 provides a description of the structure of the model. Sections 3 presents the cyclical properties of the data. Section 4 outlines the calibration of the model. Section 5 analyse the results of the exogenous fiscal policy case study. Section 6 provides a description of the cyclical properties of the model under optimal fiscal policy. Section 7 presents the optimal distribution of the tax burden over the business cycle. Section 8 presents a robustness analysis and section 9 concludes the chapter.

3.2 Model

Our model is developed to capture the key business cycle features of an economy characterised by imperfections that limit participation in labour and capital markets. We first consider a fragmented labour market, so that there exist separate markets for "skilled" and "unskilled" labour, defined as workers with and without college education, and assume that there exist socio-economic barriers that do not allow mobility between the two types of labour.[52] This is motivated by empirical evidence which suggests that in

[52] Under a setup with longer horizons it is logical to let the agents move between the two types of labour through human capital investment and/or university education (i.e. He (2012) and Angelopoulos *et al.* (2013d)). In that case, the social mobility is essential for assessing the long-run equilibrium together with the transition path (see e.g. Matsuyama

business cycle frequencies the share of college educated population in the data has low volatility and is effectively uncorrelated with output. In particular, using the data in Acemoglu and Autor (2011), we find that the standard deviation of the cyclical component of the skilled population share, relative to that of output, is 0.27, while its correlation with output is -0.18.[53] These findings suggest that the imperfections (in the form of, e.g. socio-economic barriers relating to access to education) which determine participation in labour markets are indeed more restrictive in shorter, business cycle horizons.

Then we follow Hornstein *et al.* (2005) and Acemoglu and Autor (2011), who examine the relation between skill premium and wage inequality, and we introduce two labour inputs in the production process, skilled and unskilled labour. In addition, we also introduce two types of capital, structures and equipment. Specifically, we make the assumption that skilled agents are complements with capital equipment and that the unskilled agents are substitutes with both capital equipment and skilled labour. This way the skill premium is sensitive to changes in the productivity of capital equipment, i.e. it will increase with positive shocks to capital equipment productivity.

Furthermore, following Schmitt-Grohé and Uribe (2003), Aghion and Howitt (2009, ch. 6) and Benigno (2009) we introduce a transaction cost for participating in capital markets. Given inequalities in asset ownership, and in particular, evidence that suggests higher wealth for skilled relative to unskilled workers[54], we distinguish these costs between skilled and unskilled households. This leads to different asset holdings across workers, and in particular, implies that a subset of the population is excluded from the asset markets (see e.g. Aghion and Howitt (2009, ch. 6) for capital market imperfections and agent heterogeneity). Excluded agents are thus not per-

(2006) and Aghion and Howitt (2009, ch. 6)). However, in this chapter we focus on business cycle frequencies and we take the barriers that lead to the split in the labour force to skilled and unskilled workers as given.

[53] We obtained those results using annual data for the period 1962-2008 from Acemoglu and Autor (2011) for the share of college educated population and GDP data for the same period from the U.S. National Income and Product Accounts (NIPA). Then we calculated the cyclical component of those series using the HP-filter with a smoothing parameter of 100.

[54] Data from the 2010 U.S. Census show that agents with at least a bachelor degree are two and half times wealthier than those without a bachelor degree.

mitted to accumulate capital stock to smooth consumption, as they consume all their (labour) income (see e.g. Campbell and Mankiw (1989), Mankiw (2000) and Galí et al. (2007) for hand-to-mouth consumers). We also assume that hand-to-mouth households offer unskilled labour services.

Taking the above into consideration we have an economy with agents that are heterogeneous with respect to capital holdings and skills. In particular, we have three types of households: (i) skilled households, s, that supply skilled labour and have access to asset markets; (ii) unskilled households, u, that supply unskilled labour and have access to asset markets; and (iii) hand-to-mouth households, h, that supply unskilled labour and do not have access to asset markets. Moreover, we assume that the shares of the population for each type of household are constant and exogenous. For simplicity, we also assume that the total size of the population, N, is constant. Setting as N_s the number of the skilled households, as N_u the number of the unskilled households and as N_h the number of the hand-to-mouth households we have that: $N = N_s + N_u + N_h$. We further define the relevant populations shares as $n_s = N_s/N$, $n_u = N_u/N$, and $n_h = 1 - n_s - n_u$. We further assume that the economy is populated with N identical firms and a government.

In each period, households act as price takers and make decisions regarding how much to consume, work and save. Firms act competitively and employ two types of capital stock together with the two types of labour to produce a homogeneous product. The government runs a balanced budget and imposes different tax rates on each income level. It also uses the revenue from these taxes to finance public spending.[55]

3.2.1 Households

We denote with the subscript $j = s, u, h$, the three types of households. Each household maximizes the expected lifetime utility:

$$U_j = E_t \sum_{t=0}^{\infty} \beta^t u(C_{j,t}, \overline{C}_{j,t-1}, l_{j,t}) \tag{98}$$

[55] Note that we also provide an experiment where the government can issue a single consumption tax on every household.

where E_t is the conditional expectations operator at period t; $0 < \beta < 1$ is a constant discount factor; $C_{j,t}$ and $l_{j,t}$ denote private consumption and leisure respectively at period t. In addition, we assume that each type of household faces external habits in consumption, as in Campbell and Cochrane (1999) and Ljungqvist and Uhlig (2000), that helps the model replicate the cyclical moments in the data.[56] This is captured by $\overline{C}_{j,t-1}$ that denotes the average consumption of the j-type household in period $t-1$. Finally, $u(\cdot)$ is the utility function that is increasing, concave and continuously differentiable.

The utility function is given by:

$$u(C_{j,t}, \overline{C}_{j,t-1}, l_{j,t}) = \frac{\left[\left(C_{j,t} - \omega \overline{C}_{j,t-1}\right)^{\gamma} l_{j,t}^{1-\gamma}\right]^{(1-\sigma)}}{1-\sigma} \tag{99}$$

where ω measures the weight attached to external consumption habits within each type of household; $\sigma > 1$ is the coefficient of relative risk aversion; and $0 < \gamma < 1$ is the weight of effective consumption in utility.

A household of type j faces the following time constraint:

$$1 = l_{j,t} + h_{j,t} \tag{100}$$

where $h_{j,t}$ is hours worked in period t.

Additionally, skilled and unskilled households face the following budget constraint:

$$\begin{aligned} C_{j,t} + I^q_{j,t} + I^e_{j,t} = (1-\tau_{j,t}) w_{j,t} h_{j,t} + (1-\tau_{j,t}) \left(r^q_t K^q_{j,t} + r^e_t K^e_{j,t}\right) - T_t - \\ - \psi_j \left(\left(K^q_{j,t}\right)^2 + \left(K^e_{j,t}\right)^2\right) \end{aligned} \tag{101}$$

while hand-to-mouth households face the following budget constraint:

$$C_{h,t} = (1-\tau_{h,t}) w_{u,t} h_{h,t} - T_t. \tag{102}$$

[56] Thus, we assume that there is "catching-up with the Joneses in the neighborhood", since each household compares its consumption level to that of its own socioeconomic class. Ljungqvist and Uhlig (2000) provide a detailed discussion on several versions of catching-up and keeping-up with the Joneses and internal versus external habits.

Note that the superscript $i = q, e$ in capital refers to structures, q, and equipment, e. In addition, $I^i_{j,t}$ is investment for i-type of capital from j-type of household at period t; $K^i_{j,t}$ is the capital stock for i-type of capital from j-type of household at period t; $w_{j,t}$ is the wage rate at period t for $j = s, u$ since we only have two types of labour (skilled and unskilled); ψ_j is the capital transaction cost for $j = s, u$; and T_t is a lump-sum tax.

Equations (101) and (102) capture several key features of the model. First, the households differ in their labour income, as there are different wage rates for skilled and unskilled households. Second, the households also differ in their capital income, since they face different transaction costs. In particular, the hand-to-mouth households implicitly face transaction costs that are infinite, so that they are excluded from the capital markets. The remaining households face finite transaction costs, modelled here as quadratic functions of the capital stock, following e.g. Persson and Tabellini (1992) and Benigno (2009). These may differ so that the households can be differentiated with respect to their steady state holdings of wealth. Third, there are two types of capital holdings, in structures and equipment, which pay different rates of return. The importance of allowing for the two types of capital is explained below in the discussion of skill-biased technology in production. Fourth, for each level of income, as reflected by the household type, there is a different income tax rate.

Finally the motion of the capital stock for i-type of capital and for $j = s, u$ type of household is:

$$K^i_{j,t+1} = (1 - \delta^i) K^i_{j,t} + I^i_{j,t} \tag{103}$$

where, $0 \leq \delta^i \leq 1$ is the depreciation rate for the i-type of capital.

The optimality conditions Each household $j = s, u$ chooses $\{C_{j,t},\ h_{j,t},\ K^i_{j,t+1},\ I^i_{j,t}\}^{\infty}_{t=0}$, to maximise (98) subject to (99), (100), (101) and (103), by taking tax rates, prices, and external consumption habits, $\overline{C}_{j,t-1}$, as given. Similarly, hand-to-mouth households, $j = h$, choose $\{C_{h,t}, h_{h,t}\}^{\infty}_{t=0}$, to maximise (98) subject to (99), (100) and (102), by taking tax rates, prices, and external consumption habits, $\overline{C}_{h,t-1}$, as given.

Optimality conditions skilled households

Consumption at time t:

$$0 = \frac{\left[(C_{s,t} - \omega C_{s,t-1})^{\gamma} (1 - h_{s,t})^{1-\gamma}\right]^{(1-\sigma)\gamma}}{C_{s,t} - \omega C_{s,t-1}} + \lambda_t^1 \tag{104}$$

Labour at time t:

$$0 = -\frac{\left[(C_{s,t} - \omega C_{s,t-1})^{\gamma} (1 - h_{s,t})^{1-\gamma}\right]^{(1-\sigma)(1-\gamma)}}{C_{s,t} - \omega C_{s,t-1}} - \lambda_t^1 (1 - \tau_{s,t}) w_{s,t} \tag{105}$$

Capital structures at time $t+1$:

$$0 = \lambda_t^2 + \beta \left\{ E_t \lambda_{t+1}^1 \left[(\tau_{s,t+1} - 1) r_{t+1}^q + 2\psi_s K_{s,t+1}^q \right] + E_t \lambda_{t+1}^2 (\delta^q - 1) \right\} \tag{106}$$

Capital equipment at time $t+1$:

$$0 = \lambda_t^3 + \beta \left\{ E_t \lambda_{t+1}^1 \left[(\tau_{s,t+1} - 1) r_{t+1}^e + 2\psi_s K_{s,t+1}^e \right] + E_t \lambda_{t+1}^3 (\delta^e - 1) \right\} \tag{107}$$

Investment in capital structures at time t:

$$0 = \lambda_t^1 - \lambda_t^2 \tag{108}$$

Investment in capital equipment at time t:

$$0 = \lambda_t^1 - \lambda_t^3 \tag{109}$$

Capital structures evolution equation:

$$0 = K_{s,t+1}^q - (1 - \delta^q) K_{s,t}^q - I_{s,t}^q \tag{110}$$

Capital equipment evolution equation:

$$0 = K_{s,t+1}^e - (1 - \delta^e) K_{s,t}^e - I_{s,t}^e \tag{111}$$

where λ_t^1, λ_t^2 and λ_t^3 represent the Lagrange multipliers associated to the budget constraint, capital structures and capital equipment evolution equations of the skilled agent.

Optimality conditions unskilled households

Consumption at time t:

$$0 = \frac{\left[(C_{u,t} - \omega C_{u,t-1})^{\gamma} (1 - h_{u,t})^{1-\gamma}\right]^{(1-\sigma)\gamma}}{C_{u,t} - \omega C_{u,t-1}} + \lambda_t^4 \tag{112}$$

Labour at time t:

$$0 = -\frac{\left[(C_{u,t} - \omega C_{u,t-1})^{\gamma} (1 - h_{u,t})^{1-\gamma}\right]^{(1-\sigma)(1-\gamma)}}{C_{u,t} - \omega C_{u,t-1}} - \lambda_t^4 (1 - \tau_{u,t}) w_{u,t} \tag{113}$$

Capital structures at time $t+1$:

$$0 = \lambda_t^5 + \beta \left\{E_t \lambda_{t+1}^4 \left[(\tau_{u,t+1} - 1) r_{t+1}^q + 2\psi_u K_{u,t+1}^q\right] + E_t \lambda_{t+1}^5 (\delta^q - 1)\right\} \tag{114}$$

Capital equipment at time $t+1$:

$$0 = \lambda_t^6 + \beta \left\{E_t \lambda_{t+1}^1 \left[(\tau_{u,t+1} - 1) r_{t+1}^e + 2\psi_u K_{u,t+1}^e\right] + E_t \lambda_{t+1}^6 (\delta^e - 1)\right\} \tag{115}$$

Investment in capital structures at time t:

$$0 = \lambda_t^4 - \lambda_t^5 \tag{116}$$

Investment in capital equipment at time t:

$$0 = \lambda_t^4 - \lambda_t^6 \tag{117}$$

Budget constraint:

$$\begin{aligned} 0 = {} & C_{u,t} + I_{u,t}^q + I_{u,t}^e - (1 - \tau_{u,t}) r_t^q K_{u,t}^q - (1 - \tau_{u,t}) r_t^e K_{u,t}^e - \\ & - (1 - \tau_{u,t}) w_{u,t} h_{u,t} + T_t + \psi_u \left[\left(K_{u,t}^q\right)^2 + \left(K_{u,t}^e\right)^2\right] \end{aligned} \tag{118}$$

Capital structures evolution equation:

$$0 = K^q_{u,t+1} - (1 - \delta^q)\, K^q_{u,t} - I^q_{u,t} \tag{119}$$

Capital equipment evolution equation:

$$0 = K^e_{u,t+1} - (1 - \delta^e)\, K^e_{u,t} - I^e_{u,t} \tag{120}$$

where λ^4_t, λ^5_t and λ^6_t denote the Lagrange multipliers associated with the budget constraint, the capital structures and capital equipment evolution equations of the unskilled agent.

Optimality conditions hand-to-mouth households

Consumption at time t:

$$0 = \frac{\left[(C_{h,t} - \omega C_{h,t-1})^{\gamma} (1 - h_{h,t})^{1-\gamma}\right]^{(1-\sigma)\gamma}}{C_{h,t} - \omega C_{h,t-1}} + \lambda^7_t \tag{121}$$

Labour at time t:

$$0 = -\frac{\left[(C_{h,t} - \omega C_{h,t-1})^{\gamma} (1 - h_{h,t})^{1-\gamma}\right]^{(1-\sigma)(1-\gamma)}}{C_{h,t} - \omega C_{h,t-1}} - \lambda^7_t (1 - \tau_{h,t})\, w_{u,t} \tag{122}$$

Budget constraint:

$$0 = C_{h,t} - (1 - \tau_{h,t})\, w_{u,t} h_{h,t} + T_t \tag{123}$$

where λ^7_t is the Lagrange multiplier with respect to the budget constraint of the hand-to-mouth agent.[57]

3.2.2 Production and firms

Each firm maximises its profits in perfectly competitive markets, by using labour and capital inputs to produce output, Y_t. The production function follows the specification in Krusell *et al.* (2000) which has been shown to

[57] Note that based on Walras's law only $N-1$ constraints are required in the decentralized competitive equilibrium. Thus, here we drop the budget constraint of the skilled agent.

match the behaviour of the skill premium in the data.[58] Specifically, there are two types of capital used in production, capital structures and capital equipment, denoted respectively as $K_t^{f,q}$ and $K_t^{f,e}$, and two types of labour, skilled and unskilled, denoted respectively as $h_{s,t}^f$ and $h_{u,t}^f$. The production function is given by a constant returns to scale technology assumed to take a constant elasticity of substitution (CES) specification, where it is further assumed that skilled labour, $h_{s,t}^f$, is relatively more complementary to capital equipment, $K_t^{f,e}$, than unskilled labour, $h_{u,t}^f$. This is captured by the following production function:

$$Y_t = A_t \left(K_t^{f,q}\right)^{\alpha} \times$$

$$\times \left[\lambda \left(\nu \left\{(A_t^e)^{\rho} \left(K_t^{f,e}\right)^{\rho}\right\} + (1-\nu)\left(h_{s,t}^f\right)^{\rho}\right)^{\varphi/\rho} + (1-\lambda)\left(h_{u,t}^f\right)^{\varphi}\right]^{\frac{1-a}{\varphi}} \tag{124}$$

where,

$$0 < a, \lambda, \nu < 1; \quad -\infty < \varphi, \rho < 1;$$

A_t denotes the total factor productivity (TFP) and A_t^e denotes the efficiency level of capital equipment. In addition, φ and ρ are the parameters determining the factor elasticities, i.e. $1/(1-\varphi)$ is the elasticity of substitution between capital equipment and unskilled labour and between skilled and unskilled labour, whereas $1/(1-\rho)$ is the elasticity of substitution between equipment capital and skilled labour. Moreover, the parameters a, λ, ν denote the factor shares.

Under this specification, capital-skill complementarity is obtained if $1/(1-\rho) < 1/(1-\varphi)$. Appendix C analytically confirms that the skill premium, defined as the skilled wage rate over the unskilled wage rate, is increasing in capital equipment, $K_t^{f,e}$, and decreasing in the relative supply of skilled labour, $\frac{h_{s,t}^f}{h_{u,t}^f}$, for the parameter restrictions considered.

Moreover, we assume that TFP, A_t, and the efficiency level of capital

[58] Similar studies that also incorporate the same specification of the production function are: Lindquist (2004), Pourpourides (2011) and He (2012).

equipment, A^e_t, follow a stochastic exogenous AR(1) process:

$$A_{t+1} = (1 - \rho_A) A + \rho_A A_t + \varepsilon^A_t \tag{125}$$

$$A^e_{t+1} = (1 - \rho_{A^e}) A^e + \rho_{A^e} A^e_t + \varepsilon^{A_e}_t \tag{126}$$

where ε^A_t and $\varepsilon^{A_e}_t$ are independently and identically distributed Gaussian random variables with zero means and standard deviations given respectively by σ_A and σ_{A^e}.[59]

Under this specification, when capital equipment productivity, A^e_t, increases the productivity of skilled workers will be enhanced more compared to the productivity of unskilled workers. This effect is called the skill-biased technology. Hence, the model is consistent with the empirical evidence that points to rising productivity of capital equipment and a rising skill premium over the recent decades (e.g. Katz and Murphy (1992), Krusell *et al.* (2000), Hornstein *et al.* (2005) and Acemoglu and Autor (2011)).

Taking prices and taxation as given, firms maximise their profits:

$$\Pi_t = Y_t - w_{s,t} h^f_{s,t} - w_{u,t} h^f_{u,t} - r^e_t K^{f,e}_t - r^q_t K^{f,q}_t \tag{127}$$

subject to the technology constraint in (124). Note that in equilibrium, profits are zero.

Optimality conditions Here we present the optimality conditions of the firms that choose $\{h^f_{s,t},\ h^f_{u,t},\ K^{f,e}_t,\ K^{f,q}_t\}^\infty_{t=0}$ to maximise (127) subject to (124), taking tax rates and prices as given.
Capital structures at period t:

$$0 = \frac{\alpha Y_t}{K^{f,q}_t} - r^q_t \tag{128}$$

[59] We assume that TFP and the efficiency level of capital equipment follow exogenous processes due to the significant role attached to them in the literature on the skill premium and economic fluctuations (see e.g. Lindquist (2004) and Pourpourides (2011)).

Capital equipment at period t:

$$0 = -\frac{K_t^{f,e}\lambda\nu Y_t}{K_t^{f,e}\Omega_t^1\left(\Omega_t^1\right)^{\frac{\alpha-1}{\varphi}}\left[\left(A_t^e\right)^\rho\left(K_t^{f,e}\right)^\rho + (1-\nu)\left(h_{s,t}^f\right)^\rho\right]} - r_t^e \tag{129}$$

where $\Omega_t^1 \equiv \lambda\left\{\nu\left[\left(A_t^e\right)^\rho\left(K_t^{f,e}\right)^\rho\right] + (1-\nu)\left(h_{s,t}^f\right)^\rho\right\}^{\varphi/\rho} + (1-\lambda)\left(h_{u,t}^f\right)^\varphi$.

Skilled labour at period t:

$$0 = \frac{A_t\left(K_t^{f,q}\right)^\alpha\lambda(1-\nu)(1-\alpha)\left(h_{s,t}^f\right)^\rho\Omega_t^2}{h_{s,t}^f\Omega_t^1\left(\Omega_t^1\right)^{\frac{\alpha-1}{\varphi}}\left[\nu\left(A_t^e\right)^\rho\left(K_t^{f,e}\right)^\rho + (1-\nu)\left(h_{s,t}^f\right)^\rho\right]} - w_{s,t} \tag{130}$$

where $\Omega_t^2 \equiv \left\{\nu\left[\left(A_t^e\right)^\rho\left(K_t^{f,e}\right)^\rho\right] + (1-\nu)\left(h_{s,t}^f\right)^\rho\right\}^{\varphi/\rho}$.

Unskilled labour at period t:

$$0 = \frac{A_t\left(K_t^{f,q}\right)^\alpha(1-\alpha)(1-\lambda)\left(n_u h_{u,t} + n_h h_{h,t}\right)^\varphi}{\Omega_t^1\left(\Omega_t^1\right)^{\frac{\alpha-1}{\varphi}}\left(n_u h_{u,t} + n_h h_{h,t}\right)} - w_{u,t}. \tag{131}$$

3.2.3 The government budget constraint

The government runs a balanced budget in every period given by:

$$\begin{aligned} G_t^c &= n_s\tau_{s,t}w_{s,t}h_{st} + n_u\tau_{u,t}w_{u,t}h_{u,t} + n_h\tau_{h,t}w_{u,t}h_{h,t} + \\ &+ \tau_{s,t}n_s\left(r_t^q K_{s,t}^q + r_t^e K_{s,t}^e\right) + \tau_{u,t}n_u\left(r_t^q K_{u,t}^q + r_t^e K_{u,t}^e\right) + T_t \end{aligned} \tag{132}$$

where G_t^c is the average government consumption per agent.

In this chapter we pay attention only to the revenue side of the budget constraint. Therefore, we assume that government consumption is wasteful and follows an exogenous AR(1) process. Thus its fluctuations act as exogenous spending shocks which require a change in the tax revenue collected (for a similar approach regarding G_t^c, see e.g. Chari *et al.* (1994), Stockman (2001) and Arseneau and Chugh (2012)):

$$G_{t+1}^c = \left(1-\rho_{G^c}\right)G^c + \rho_{G^c}G_t^c + \varepsilon_t^{G^c} \tag{133}$$

where ε_t^{Gc} is independently and identically distributed Gaussian random variable with zero means and standard deviations given by σ_{G^c}.

Regarding the tax rates, we consider below policy regimes where they are exogenously set or they are optimally chosen by the government. Following Arseneau and Chugh (2012), when we consider how the model economy behaves in response to exogenous fiscal policy, we use lump-sum taxes as the residual variable in the government budget constraint, since for this experiment we are not studying government financing issues. However, for the optimal policy analysis, again as in Arseneau and Chugh (2012), lump-sum taxes are fixed to zero.

In the literature that examines the optimality or not of tax smoothing (see e.g. Chari et al. (1994) and Arseneau and Chugh (2012)), the government budget constraint includes debt. In contrast, here we focus on the optimal allocation of the tax burden over the business cycle given the revenue requirements of the government. Hence we do not allow the government to issue debt to balance the budget (see also Stockman (2001), who considers optimal capital and labour taxes with and without access to debt, albeit in a different setup).

3.2.4 Resource constraint and market clearing conditions

The labour and capital market clearing conditions are given by:

$$h_s^f = n_s h_s \tag{134}$$

$$h_u^f = n_u h_u + n_h h_h \tag{135}$$

$$K_t^i = n_s K_{s,t}^i + n_u K_{u,t}^i . \tag{136}$$

which means that skilled and unskilled labour demanded by firms is equal to skilled and unskilled labour supplied by households in each period. Also, the supply of capital by households is equal to the capital demanded by firms.

The aggregate resource constraint is given by:

$$Y_t = G_t^c + n_s C_{s,t} + n_u C_{u,t} + n_h C_{h,t} + n_s \left(I_{s,t}^q + I_{s,t}^e\right) + n_u \left(I_{u,t}^q + I_{u,t}^e\right) + \\ + n_s \psi_s \left[\left(K_{s,t}^q\right)^2 + \left(K_{s,t}^e\right)^2\right] + n_u \psi_u \left[\left(K_{u,t}^q\right)^2 + \left(K_{u,t}^e\right)^2\right] \tag{137}$$

where $Y_t = A_t \left(K_t^{f,q}\right)^{\alpha} \times$

$$\times \left[\lambda \left\{\nu \left[(A_t^e)^{\rho} \left(K_t^{f,e}\right)^{\rho}\right] + (1-\nu)\left(h_{s,t}^f\right)^{\rho}\right\}^{\varphi/\rho} + (1-\lambda)\left(h_{u,t}^f\right)^{\varphi}\right]^{\frac{1-a}{\varphi}}.$$

3.3 Exogenous policy

In this section we examine the cyclical properties of our model when the tax rates are not optimally chosen by the government but instead they follow an exogenous stochastic process. Therefore, we calibrate our model so that it generates empirically relevant business cycle fluctuations. We concentrate on the key labour market dimension that determines inequality, i.e. the skill premium, when driven by the empirically relevant government spending and income tax rate processes. Thus, we further assume that the three income tax rates follow an exogenous AR(1) processes given by:

$$\tau_{j,t+1} = \left(1-\rho_j\right)\tau_j + \rho_j \tau_{j,t} + \varepsilon_t^{\tau_j} \tag{138}$$

where $j = s, u, h$ and $\varepsilon_t^{\tau_j}$ are independently and identically distributed Gaussian random variables with zero means and standard deviations given by σ_{τ_j}.

3.3.1 Decentralized competitive equilibrium

Taking into account the starting values for the stock of capital structures, K_0^q, and capital equipment, K_0^e, the three policy instruments $(\tau_{s,t}, \tau_{u,t}, \tau_{h,t})$ and the stationary stochastic processes $\{A_t, A_t^e, G_t^c\}_{t=0}^{\infty}$, the decentralized competitive equilibrium system of equations is characterized by a sequence of allocations $\{C_{s,t},\, C_{u,t},\, C_{h,t},\, h_{s,t},\, h_{u,t},\, h_{h,t},\, K_{s,t+1}^q,\, K_{s,t+1}^e,\, K_{u,t+1}^q,\, K_{u,t+1}^e,\, I_{s,t}^q,\, I_{s,t}^e,\, I_{u,t}^q,\, I_{u,t}^e\}_{t=0}^{\infty}$, prices $\{w_{s,t},\, w_{u,t},\, r_t^q,\, r_t^e\}_{t=0}^{\infty}$, and the residual policy instrument $\{T_t\}_{t=0}^{\infty}$ such that: (i) households maximise their welfare taking policy, prices and aggregate variables as given; (ii) firms maximise their profits taking policy, prices and aggregate variables as given; (iii) the government budget constraint is always balanced; (iv) market clearing conditions are satisfied and (v) the external consumption habit satisfies the following: $\overline{C}_{j,t-1} = C_{j,t-1}$.

Therefore, the full decentralized competitive equilibrium (DCE) system of equations is given by equations (104)-(138).

3.3.2 Data analysis and targets

We aim for the exogenous-policy model to replicate the long-run great ratios and key labour market averages as well as explaining the cyclical volatilities and correlations with output of key variables in the economy. We use quarterly data for U.S. economy, which are obtained from datasets constructed by Lindquist (2004), Piketty and Saez (2007), Castro and Coen-Pirani (2008), Pourpourides (2011), Arseneau and Chugh (2012) as well as data series from the Bureau of Economic Analysis (BEA).[60]

Table 3.1: Business cycle statistics of main endogenous variables

Variable	Correlation with output	Standard deviation
Y	1	0.013-0.014
C	0.83	0.011-0.012
I	0.76-0.91	0.037-0.063
$\frac{w_s}{w_u}$	0.09-0.19	0.006-0.013
h_s	0.42-0.69	0.008-0.010
h_u	0.59-0.73	0.006-0.012

Sources: The data ranges are constructed using the results reported in Lindquist (2004), Castro and Coen-Pirani (2008) and Pourpourides (2011).

Table 3.1 reports the data volatilities and correlations with output from existing studies for variables which correspond with key endogenous variables in our model. These are quarterly data for the period 1979-2002 (Lindquist (2004)) and 1979-2003 (Castro and Coen-Pirani (2008) and Pourpourides

[60] At this point I would like to mention my gratitude to Matthew Lindquist and Daniele Coen-Pirani for sharing their datasets.

(2011)). Their cyclical component has been obtained by taking the logarithms of the series and then using an HP-filter with a smoothing parameter of 1600.[61] As can be seen in Table 3.1, these studies document some interesting results regarding the labour market statistics. In particular, they point out that the skill premium is effectively uncorrelated with output and smoother than output in business cycle frequencies. Moreover, the cyclical properties of the labour supply of skilled and unskilled workers do not differ qualitatively, both having a positive correlation with output, while being less volatile than output. The statistics regarding consumption, investment and capital are similar to those commonly obtained in macroeconomic research.

Table 3.2: Data averages and business cycle statistics of policy variables

	G^c	τ_{60-100}	τ_{20-60}	τ_{0-20}
Averages	-	0.247	0.180	0.144
Autocorrelations	0.770	0.950	0.920	0.890
Correlations with Y	-0.066	0.587	0.654	0.198
Standard deviation	0.014	0.006	0.005	0.004

Sources: The statistics are obtained using the data from Piketty and Saez (2007), Arseneau and Chugh (2012) and BEA.

Moreover, Table 3.2 presents the averages, autocorrelations, volatilities and correlations with output of the government consumption and the three income tax series. The government spending series is obtained using quarterly data from the BEA for the period 1979 to 2002.[62] The income tax data

[61] The skill labour supply in those studies is obtained by dividing the labour force into two groups according to their education. For instance, the agents with 14 or more years of education form the skilled workers. This criterion makes an underlying assumption that workers with a college degree will be employed in high returns or high skilled occupations, Acemoglu and Autor (2011). However, it should be noted that Acemoglu and Autor (2011) have used annual data for the supply of skilled and unskilled labour and the associated skill premium, whereas in our analysis we use quarterly data. Even show the results of Acemoglu and Autor (2011) about the average skill premium and its volatility and correlation with output does not differ significantly from our results. Specifically, using their data we find that the mean value of the skill premium is 1.60, while its cyclical standard deviation and correlation with output are 0.49 and -0.13 respectively.

[62] The data we have used refer to government consumption expenditures and gross in-

are obtained using the Piketty and Saez (2007) dataset, which reports annual data on income tax rates per income group (in quantiles) for the period 1966-2001.[63] As we explain below, we calibrate the share of hand-to-mouth agents to be 20%, the share of unskilled workers who also have savings to be 40% and the share of skilled workers to 40%. Since our model predicts that the income levels of these three groups increase in the order mentioned above, we use the Piketty and Saez (2007) dataset to obtain three income tax rates, the first for the lowest quantile, the second as the average for the two middle quantiles, and the third as the average for the two top quantiles. We use these series of tax rates as proxies for $\tau_{j,t}$, for $j = h, u, s$, in our model.

In Table 3.2 we can see that the income tax of the hand-to-mouth agent, τ_{0-20} (bottom quantile), is 14.4%, the income tax rate of the unskilled agent, τ_{20-60}, is 18.0% and the income tax of the skilled agent, τ_{60-100}, is 24.7%. This also suggests that the income taxation is progressive in our setup.[64]

Regarding the business cycle statistics of the tax series, the results suggest that, as expected, these are highly persistent and have low volatility. The spending process is less persistent and more volatile. The correlations with output suggest that all the tax rates are pro-cyclical and the government spending is essentially uncorrelated with output. Finally, using data on the productivity of capital equipment from the BEA for the period 1988-2011, we estimate the autocorrelation of HP-filtered series to be 0.975 and its standard deviation to be 0.007.[65]

vestment as they are reported in NIPA Table 1.1.5. In order to obtain the statistical properties of the cyclical component of the series, we transform the series into logarithm and then we apply the HP-filter with a smoothing parameter of 1600.

[63] These tax rates refer to average tax rates by income groups. To obtain quarterly series from the annual data, so that the cyclical statistics from the tax series are comparable to the remaining data used in the analysis, we follow the interpolation method in Litterman (1983). We use as an indicator variable the quarterly time-series of labour income tax rates from Arseneau and Chugh (2012). To obtain the cyclical component of the series we again use the HP-filter with a smoothing parameter of 1600.

[64] Piketty and Saez (2007) pay particular attention on several aspects of the progressivity of income taxation. However, in our chapter we focus mainly on income taxation for the three income groups that best correspond to the household disaggregation in our model.

[65] We also used annual series for the three income taxes and we found that the main statistical results are qualitatively similar.

3.4 Calibration

The calibration of the parameters of the model is either based on data or by targeting the long-run values and cyclical properties (correlation and standard deviation) of the key endogenous variables to be consistent with the data. Table 3.3 summarises the calibrated parameters.

3.4.1 Population shares

Regarding the population shares we assume that $n_s = 0.4$, $n_u = 0.4$, $n_h = 0.2$. The share of skilled households is calibrated according to the data in Acemoglu and Autor (2011), where the share of the population with a college degree is about 45%. Then we split the unskilled households into those who can access the asset market and those that cannot (hand-to-mouth agents). The share of the hand-to-mouth population has been shown by the empirical evidence of Traum and Yang (2010) to be equal to 18%. In addition, Cogan *et al.* (2010) estimate the share of the hand-to-mouth population to be equal to 26.5%. Our calibration of 20% for the share of the hand-to-mouth agents falls within that range. The split in those population shares is also consistent with data from the 2010 U.S. Census, where the share of the population with a college degree is equal to 43%, whereas the share of the population without asset holdings is 18.7%.[66]

3.4.2 Tax-spending policy

As we mentioned earlier, the population shares we have assumed are helpful for approximating the effective income tax rate for each group using the income tax data per income quantile from Piketty and Saez (2007). As a result, under exogenous fiscal policy, where we assume that the three income tax rates follow an exogenous AR(1) process, we set their constant term equal to the average value of τ_{0-20}, τ_{20-60} and τ_{60-100}, for hand-to-mouth, unskilled and skilled agents respectively, as they are presented in Table 3.2. Moreover, we set accordingly, following the results in Table 3.2, the autocorrelation

[66] These data are obtained from Table 4 of the Census Bureau, Survey of Income and Program Participation.

parameters of the AR(1) processes, ρ_j, and the standard deviation of the cyclical components, σ_j for each type of income tax. Similarly we set the autocorrelation and standard deviation parameters of the AR(1) process for government consumption. We also calibrate the long-run value of government spending targeting a public spending to output ratio of 19%, consistent with the data discussed above.

3.4.3 Production and capital and labour markets

For the calibration of the elasticities of substitution between skilled labour and capital equipment, $(1/1-\varphi)$, and between unskilled labour and capital equipment (or skilled labour), $(1/1-\rho)$, we follow Krusell *et al.* (2000) and we set: $\varphi = 0.401$ and $\rho = -0.495$. For the rest of the parameters in the production function we follow Lindquist (2004), He and Liu (2008), Pourpourides (2011) and He (2012) so as our calibration is within the range of values they suggest and we also target the long-run values of our model in asset and labour markets to be consistent with the data. Therefore, we calibrate the income shares λ and ν aiming a skill premium of about 1.65 and a labour share of income of 69%. Both of these values are consistent with the U.S. data. Specifically, the value of the skill premium is obtained from U.S. Census data and is within the range of estimates in Table 3.1.[67] Also, the data for the labour income share as a percentage of GDP are obtained from BEA data on personal income for the period 1970-2011. The productivity of capital structures, α, is set as in Lindquist (2004). The calibrated parameters in the production function are generally very similar to those estimated or calibrated in the literature.

[67] The source is the Current Population Survey, 2011 Annual Social and Economic Supplement from the U.S. Census Bureau.

Table 3.3: Model parameters

Parameter	Value	Definition	Source
$0 \leq \delta^q \leq 1$	0.016	depreciation rate of capital structures	calibration
$0 \leq \delta^e \leq 1$	0.028	depreciation rate of capital equipment	calibration
$0 < \beta < 1$	0.990	time discount factor	calibration
$0 \leq \omega < 1$	0.580	habit persistence parameter	calibration
$0 < \gamma < 1$	0.225	weight attached to consumption in utility	calibration
$\sigma > 1$	2.000	coefficient of relative risk aversion	assumption
$0 \leq \alpha \leq 1$	0.130	income share of capital structures	assumption
$\frac{1}{1-\rho}$	0.669	capital equipment to skilled labour elasticity	assumption
$\frac{1}{1-\varphi}$	1.669	capital equipment to unskilled labour elasticity	assumption
$0 < \lambda < 1$	0.560	share of composite input to output	calibration
$0 < \nu < 1$	0.580	share of capital equipment to composite input	calibration
$0 < \frac{G^c}{Y} < 1$	0.190	government spending	calibration
$\psi_s > 0$	0.0002	transaction cost for skilled agents	calibration
$\psi_u > 0$	0.0018	transaction cost for unskilled agents	calibration
τ_s	0.247	average income tax rate, skilled	data
τ_u	0.180	average income tax rate, unskilled	data
τ_h	0.144	average income tax rate, hand-to-mouth	data
		Stochastic processes	
σ_A	0.004	standard deviation of TFP	calibration
ρ_A	0.950	AR(1) coefficient of TFP	data
σ_{A^e}	0.007	standard deviation of cap. equipment	data
ρ_{A^e}	0.975	AR(1) coefficient of cap. equipment	data
σ_{τ_s}	0.006	standard deviation of income tax, skilled	data
ρ_{τ_s}	0.950	AR(1) coefficient of income tax, skilled	data
σ_{τ_u}	0.005	standard deviation of income tax, unskilled	data
ρ_{τ_u}	0.920	AR(1) coefficient of income tax, unskilled	data
σ_{τ_h}	0.004	standard deviation of income tax, hand-to-mouth	data
ρ_{τ_h}	0.890	AR(1) coefficient of income tax, hand-to-mouth	data
σ_{G^c}	0.014	standard deviation of public spending	data
ρ_{G^c}	0.770	AR(1) coefficient of public spending	data

The depreciation rates of capital structures and capital equipment are calibrated to obtain a quarterly capital to output ratio equal to 6.95 in the steady state. This is within the range presented in Table 3.1 and is consistent with an annual capital to output ratio of 1.74, obtained using BEA annual data on capital stocks from 1970 to 2011. In particular, we set the depreciation rate of capital equipment as $\delta^e = 0.028$, which is within the range of the values that Lindquist (2004) and Pourpourides (2011) have used and then we calibrate the depreciation rate of capital structures residually as $\delta^q = 0.016$.[68]

We set the transaction cost parameters as $\psi_s = 0.0002$ and $\psi_u = 0.0018$. There are two targets for these parameters. The first is that the total capital holdings for skilled households in the deterministic steady state is 2.5 times higher than for unskilled households. This ensures that the model's steady state matches data from the 2010 Census,[69] which indicate that the wealth of the population with at least a bachelor degree is two and half times more than those without a bachelor degree. The second target is that in the steady state the transaction costs cohere with a real return to capital (that excludes depreciation and taxes) of about 1% per quarter.[70]

3.4.4 Utility function

As far as the utility function is concerned, we set the coefficient of relative risk aversion, $\sigma = 2$, following previous studies (e.g. Schmitt-Grohé and Uribe, 2007). The time discount factor, $\beta = 0.99$, is calibrated so as to replicate the investment to output ratio observed in the data. The weight of consumption to utility, $\gamma = 0.225$, is calibrated targeting a steady state value of labour time for each household equal to one third of its time. For the consumption habit parameter, ω, we follow Christiano *et al.* (2005) that suggest a value

[68] For example, Krusell *et al.* (2000) use the following depreciation rates $\delta^q = 0.0125$ and $\delta^e = 0.031$; whereas Pourpourides is using: $\delta^q = 0.014$ and $\delta^e = 0.027$; and Lindquist sets $\delta^q = 0.014$ and $\delta^e = 0.031$.

[69] The specific information is obtained using Table 1 from the 2010 U.S. Census Bureau, Survey of Income and Program Participation.

[70] The real return to capital at an annual frequency is 4%, using data from the World Bank.

within the range of $(0.52 - 0.71)$ and we also target the model's predicted volatility of consumption to be similar to the data presented in Table 3.1.

3.4.5 Technology

The two technological processes, TFP and productivity of capital equipment, are assumed to follow an exogenous AR(1) process. We normalize the constant terms of those processes to be equal to unity, $A = 1$ and $A^e = 1$. We also used a time series on the productivity of capital equipment as it is reported by BLS for the period 1988-2011, and we estimated its autocorrelation and standard deviation, after we HP-filtered the series, to be $\rho_{A^e} = 0.975$ and $\sigma_{A^e} = 0.007$ respectively, and these are the values we use for the respective exogenous process.[71] In addition, following Lindquist (2004) and Pourpourides (2011), we set the autocorrelation parameter of TFP equal to 0.95, while σ^A is calibrated to match the volatility of output observed in the data (see Table 3.1).

3.5 Solution and results

Table 3.4 compares the steady state solution of the decentralised competitive equilibrium (DCE) system of equations presented earlier for the key variables with the respective data averages. To study dynamics, we compute a first-order approximation of the equilibrium conditions around the deterministic steady state, by implementing the perturbation methods in Schmitt-Grohé and Uribe (2003).[72] We use the first-order accurate decision rules to simulate time paths of the equilibrium under shocks to total factor productivity, capital equipment augmenting technology, government spending, and income tax realizations, that are obtained from the distributions specified above. We conduct 1000 simulations, each 296 periods long. We drop the initial 200

[71] The time-series on the productivity of equipment capital is obtained from the Bureau of Labor Statistics and refers to annual data. The respective quarterly series is obtained using the methodology in Litterman (1983), where the indicator variable is the quarterly time-series of total investment for the same period. The series is then logged and HP-filtered with a smoothing parameter of 1600.

[72] Note that the results we present in this chapter are first-order approximations. Using a second-order approximation doesn't change our conclusions.

periods so that the remaining series length of 96 periods corresponds with the number of observations in the data, i.e. 1979:1-2002:4. For each simulation, we then compute the required moments and report the means of these moments across the simulations in Table 3.5. This table also reports for comparison the predicted business cycle statistics from the studies of Lindquist (2004) and Pourpourides (2011).

Table 3.4: Steady state of the exogenous policy model

Variable	Model	Data	Variable	Model	Data
$\frac{K}{Y}$	6.946	6.550	h_s	0.362	0.317
$\frac{I}{Y}$	0.151	0.159	h_u	0.364	0.348
$\frac{C}{Y}$	0.653	0.659	h_h	0.398	-
$\frac{G^c}{Y}$	0.189	0.195	$\frac{w_s}{w_u}$	1.649	1.659
r_{net}	0.010	0.010	$\frac{wh}{Y}$	0.699	0.686

In Table 3.4 we can see that the steady state values of the model are quantitatively similar to those observed in the data. At this point we need also to mention that in our model the ratio of average hours worked by unskilled workers over that of skilled workers is equal to 1.027, and is similar to the 1.099 suggested in the data (using U.S. data from the Bureau of Labor Statistics (BLS)).[73] The implied Frisch labour supply elasticities from our model are: 1.08 for skilled, 1.07 for unskilled and 0.93 for hand-to-mouth workers. These values are also consistent with the related literature (i.e. Browning *et al.* (1999), Chetty *et al.* (2011), and Keane and Rogerson (2012)). Finally, Table 3.4 indicates that the income share from labour is, $\frac{w_s n_s h_s + w_u n_u h_u + w_u n_h h_h}{Y} = 0.699$, which is similar to the value of 0.686 obtained in the data.[74]

[73] Castro and Coen-Pirani (2008) using quarterly U.S. data from 1979-2003 gets an unskilled to skilled labour supply ratio equal to 1.030. Our model prediction of 1.027 for the weighted ratio of unskilled to skilled labour coheres well with this figure.

[74] See, BEA Table 2.1 for the period 1979-2002.

Table 3.5: Business cycle statistics of the exogenous policy model

	Correlation with Output			Standard deviation		
Variable	Model	Lindquist (2004)	Pourpourides (2011)	Model	Lindquist (2004)	Pourpourides (2011)
Y	1	1	1	0.014	0.013	0.014
C	0.89	0.81	0.93	0.012	0.006	0.012
I	0.80	0.96	0.76	0.048	0.053	0.040
$\frac{w_s}{w_u}$	0.15	0.06	-0.09	0.005	0.003	0.005
h_s	0.68	0.81	0.95	0.005	0.006	0.007
h_u	0.28	0.95	0.54	0.004	0.006	0.012
h_h	-0.42	N/A	N/A	0.007	N/A	N/A

Turning to the business cycle statistics in Table 3.5, the overall fit is comparable to existing research on business cycle models with the skill premium (see e.g. Lindquist (2004) and Pourpourides (2011)). In particular, the model matches the key stylized facts of the skill premium in the data, i.e. that it is effectively not correlated with output and that its volatility is less than that of output (refer to Table 3.1). In addition, the model predictions regarding the second moments of the hours worked are generally consistent with the data both qualitatively and quantitatively. However, the model quantitatively under-predicts the correlation of unskilled hours with output. The model also matches the second moments of consumption and investment. Overall, the model's predictions regarding the key endogenous variables are empirically relevant.[75]

[75] Note that the data sources for the series in Table 4 include: (i) BEA, NIPA Table 1.1.5 for output, investment and consumption; (ii) BEA, NIPA Table 1.1 (line 3 plus line 21 minus line 7) and Tables 7.1A (line 30) plus Table 7.2B (line 32) for the capital stock; (iii) BLS, Current Employment Statistics survey for hours worked; (iv) World Bank for the real rate of return; (v) BEA, NIPA Table 2.1 for labour's share in income; and (vi) U.S. Census Bureau, Survey of Income and Program Participation for the skill premium. Comparable averages are obtained using the dataset in Lindquist (2004), for those variables that are similar in both studies.

3.6 Optimal taxation over the business cycle

Having established the empirical relevance of the calibration, we now discard the exogenous processes for the income tax rates in (138) and instead assume that the paths of these tax rates are optimally chosen by a government that seeks to maximise a utilitarian objective function under commitment, taking the revenue requirements as given.

3.6.1 The problem of the government

The government chooses optimally the paths of the income taxes so as to maximise the aggregate welfare taking into account the decentralised system of equations, presented earlier, under the following assumptions:[76] (i) the government takes spending as given. Specifically, government expenditures will still follow an exogenous stochastic process as presented in equation (133); (ii) the government is able to tax only the total income of each agent and as a result the government is restricted to implement an incomplete tax system;[77] and (iii) the government is not able to issue debt. Treating the spending side of the budget as given is a common assumption in the analysis of optimal taxation and allows us to focus on the revenue side of the budget. The assumption of having an incomplete set of taxes, via allowing only for income taxation to be present, is motivated from the current fiscal policy framework where governments are not able to tax each source of income differently. Finally, the assumption of no debt is again driven from the current economic framework where most of the developed economies have high levels of debt and as a result they are not able to use it as a smoothing instrument against economic fluctuations.

Given the imperfections in labour and capital markets and the aforemen-

[76] We also keep lump-sum taxes fixed to zero, as is common for optimal taxation analysis. We note, however, that our results do not change qualitatively if we keep the lump-sum instrument fixed to its steady-state value obtained from the model under exogenous fiscal policy.

[77] We do not allow the government to has a complete tax system at its disposal (see e.g. Chari and Kehoe (1999), for the definition of a complete tax system) motivated from the practical policy implementation and the fact that the governments are restricted to impose a single tax rate on capital and labour income.

tioned restrictions on fiscal policy, the key question is: what is the optimal income taxation and how the government distributes the distortions over the business cycle? The requirements imposed on the tax system certainly dictate some properties that optimal income taxation must satisfy over the business cycle. In particular, we would expect income taxes on average to be more volatile compared to the data since tax smoothing via public debt or expenditure management is not an option. We would also expect them to be generally counter-cyclical, given that negative shocks that reduce the tax bases necessitate a rise in the tax rates to make up for the loss in the tax revenue (see Stockman (2001) for a similar result). However, allowing the government to choose different tax rates for each income group, implies that the government still has to decide on whether these tax rates should have the same volatility and co-movement with output over the business cycle and, if not, how to set these cyclical properties for each tax rate.

We examine the problem of a government that has Utilitarian preferences, so that its objective function is given by the expected lifetime utility of the weighted average of the welfare of the three types of households, where the weights attached to each type are equal to the population share of that type, n_j. Thus, in our setup, the government chooses the three income tax rates $\{\tau_{s,t}, \tau_{u,t}, \tau_{h,t}\}_{t=0}^{\infty}$ and the following variables$\{C_{j,t}, h_{j,t}, K_{j,t+1}^{i}, I_{j,t}^{i}, w_{j,t}, r_{t}^{i}\}_{t=0}^{\infty}$, for $i = s, u$ and $j = s, u, h$, to maximise:

$$U^g = E_t \sum_{t=0}^{\infty} \beta^t \left[\sum\nolimits_{j=s,u,h} n_j u(C_{j,t}, C_{j,t-1}, l_{j,t}) \right] \tag{139}$$

subject to the decentralised competitive equilibrium system of equations as they have been presented earlier in equations (104) − (137). Note that following Arseneau and Chugh (2012) we set $\{T_t \equiv 0\}_{t=0}^{\infty}$. In addition, the government takes into account the externalities that appear through the consumption habits, when implementing its optimal policy. Finally, we assume that the government commits to its policy.

As in the exogenous-policy baseline, we first compute the deterministic steady state equilibrium under optimal policy and next approximate the dynamic equilibrium paths using the first-order approximation of the equilibrium conditions under optimal policy for time $t > 0$ around the deterministic steady state of these conditions. As is common in the literature (see e.g. Arseneau and Chugh (2012)), when characterizing asymptotic policy dynamics, we also make the auxiliary assumption that the initial state of the economy at $t = 0$ is the steady state under optimal policy. We use the first-order accurate decision rules to simulate the optimal policy equilibrium under shocks to TFP, equipment capital efficiency and government consumption spending that are obtained as in the exogenous policy experiments in the previous section.[78]

3.6.2 Properties of optimal taxes over the business cycle

Table 3.6 presents the optimal properties of the tax system under shocks to all stochastic processes. As it can be seen by the steady state income taxes, optimal tax policy is progressive and, in fact, relatively more progressive compared with the data averages. In particular, $\tau_s > \tau_{60-100}$, $\tau_u < \tau_{20-60}$, $\tau_h < \tau_{0-20}$.[79] This is noteworthy since the progressivity of the tax system as captured by the three tax rates considered here has indeed increased since the mid-1960s (see e.g. the data in Piketty and Saez (2007)).[80] Therefore, the assumed imperfections and inequalities in our model justify progressive income taxation.

[78] The business cycle statistics are calculated in this case as in the exogenous policy case. Thus, we perform 1000 simulations under shocks to the exogenous processes, each simulation is 296 periods long and then we drop the initial 200 periods. Afterwards, we get the business cycle statistics for each simulation and we calculate the averages across the simulations.

[79] Note that we observe the largest difference between the optimal income tax and the data in the case of the hand-to-mouth households. In practice, though, subsidies that are targeted to a specific group of agents alleviate part of the tax burden for that group, leading to a more realistic distribution of the tax burden in our model that is closer to reality. However, we do not discuss further about the targeted subsidies since for the optimal taxation case we do not include any lump-sum transfers.

[80] Using the dataset of Piketty and Saez (2007) we find that for the period 1966-2001 the income tax of the high income earners has increase 5 percentage points, whereas the income tax of the middle income group agents has decreased 4 percentage points and for the low income groups agents has decreased 5 percentage points, similar to our results.

Table 3.6: Optimal tax policy

	τ_s	τ_u	τ_h
Steady state	0.285	0.143	-0.041
Autocorrelations	0.754	0.832	0.891
Correlations with Y	-0.461	-0.732	-0.110
Standard deviation	0.012	0.070	0.173

These results in Table 3.6 further suggest that the volatility and co-movement of optimal taxes with output differ significantly with each other and with the data reported in Table 3.2. As discussed above, the policy problem we consider implies that the tax rates need to be generally more volatile and more counter-cyclical, compared with the data. However, Table 3.6 also reveals important differences between the tax rates. These result from the trade-off that the government faces when deciding how to distribute the distortions reflected by the higher volatility and counter-cyclicality of the three tax rates over the business cycle. On one hand, such distortions have a larger impact on hand-to-mouth households, since they are less able to smooth shocks. There is thus an incentive to minimise the impact of policy for this type of household. On the other hand, tax-induced distortions to skilled households have the strongest propagation effects in the economy, given the complementarity of skilled hours with equipment capital. Therefore, there is also an incentive to minimise distortions to the choices of skilled households, since this acts to amplify external shocks

Delving deeper into the cyclical properties of each income tax rate under optimal fiscal policy we can see that the volatilities are higher than those observed in the data. This is expected given the balanced budget restriction we imposed together with the restricted set of fiscal instruments. Note that the volatilities observed in the data are similar across the three income tax rates with the income tax for the high income earners having the highest volatility and the income tax for the lowest income earners the lowest volatility. However, under the optimal taxation the volatilities of the three income taxes have significant differences. In this case the income tax of the lowest income group is the most volatile and the income tax of the highest income group is the smoothest.[81]

A smoother income tax creates fewer distortions in the household's optimisation problem. Since income taxes are the only choices for the policy-maker in this framework, implying that smoothing is not possible for all tax rates, the government finds it optimal to keep the tax rate which distorts incentives the most, the smoothest. Given the higher complementarities of skilled hours with equipment capital, implying that tax-induced fluctuations in skill supply propagate more in the economy via the equipment capital channel, the government finds it optimal to keep the tax to skilled households the least volatile. In contrast, since hand-to-mouth households do not own capital stock, their choices affect the endogenous propagation mechanism in the economy the least, so that their taxes are optimally the most volatile.

Regarding the cyclicality of the tax rates over the business cycle, we found in the data (Table 3.2) that taxes are positively correlated with output. Specifically, the data indicate that the correlation of the income taxes with output are: 0.587 for skilled income tax; 0.654 for unskilled income tax; and 0.198 for hand-to-mouth income tax. As expected, the results in Table 3.6 confirm that the correlations become negative when the tax instruments are the only optimally chosen instruments in the government's budget constraint. However, the results also indicate that the optimal correlations of the tax rates are symmetrically opposite to the data correlations. For example, the strongest pro-cyclical tax in the data becomes optimally the strongest counter-cyclical, while the least cyclical in the data becomes the least counter-cyclical. Hence, the requirement of the government to make the tax system generally counter-cyclical, is not translated into a proportional reduction of the correlation coefficients of all tax rates.[82]

[81] The results we obtain regarding the magnitude and range in volatilities of the three income tax rates are within the range of the optimal volatilities for tax instruments considered in the model with search frictions and government debt in Arseneau and Chugh (2012) and the neoclassical model with a balanced budget restriction in Stockman (2001).

[82] The literature has not examined the behaviour of different income taxes under optimal fiscal policy, like in our chapter. However, it has been shown that market frictions are important for defining the optimal cyclicality of the tax rates. For example, under

Counter-cyclical taxes intensify fluctuations in income, as they amplify the effects of exogenous productivity shocks. Therefore, a government that needs to make use of counter-cyclical taxes over the business cycle, does so with a view to minimise the distortions that they cause. In this setup, it is optimal to minimise such policy distortions to the income of the hand-to-mouth households, by making their tax rate to be the least counter-cyclical. This is because these agents are the most exposed to economic fluctuations. Comparing skilled and unskilled households, it is optimal to least distort the choices of skilled, given the higher complementarities of skilled hours with equipment capital. As a result, the unskilled workers face the most counter-cyclical income tax.

Finally, the persistence of the income taxes (autocorrelation) is optimally chosen to be similar across the three taxes. Compared with the data, the persistence of the income taxes is slightly lower under optimal fiscal policy, especially the skilled income tax.

3.6.3 Changing the set of fiscal instruments

In this section we extend the set of fiscal instruments and we allow for the government to optimally choose a consumption tax that is the same for each type of household, together with the three income tax rates. Through that experiment we assess the properties of optimal income taxes over the business cycle and especially the differences compared to the previous case where we had a more restricted fiscal policy menu. Note that in this case we keep the same calibration presented earlier.[83]

Table 3.7 presents the results of the optimal taxation under the different set of fiscal instruments. The optimal long run value of consumption tax is 27.8%, leading to a reduction of the optimal income tax rates compared to the previous case, with the unskilled income tax rate becoming also a

a neoclassical model the labour income taxes are generally pro-cyclical, while in model with labour marker frictions, like Arseneau and Chugh (2012), the labour income tax is optimally counter-cyclical.

[83] In other words, we make the assumption that under the exogenous fiscal policy the consumption tax was zero.

subsidy.[84] Moreover, the progressivity of the income taxation has increased even further. Now the difference between skilled and unskilled income tax is 20.7% from 14.2%, also the difference between skilled and hand-to-mouth income taxes is approximately 50% from 32%.

The persistence of the three income taxes is similar with the previous case study. In addition, the autocorrelation of the consumption tax is on the same range with the income taxes and it is also similar with the data (0.813).

As in the case with a more restricted set of fiscal instruments, we still observe that the correlation with output and volatility of the income tax rates is different compared to the data reported in Table 3.2 and they are also different compared to each other. Specifically, the correlation of the income tax rates with output remains on the same level as in the case without the consumption tax. The correlation of the consumption tax with output is optimally chosen to be procyclical.

However, we observe significant differences in the volatility of the tax rates compared to the previous case. In this case the income tax rate for the hand-to-mouth agents becomes the smoothest (it was the most volatile in the previous case) because now those agents face an additional distortion in their income coming from the volatility of the consumption tax. Therefore, the government finds it optimal to smooth out their income tax rate since those agents are more exposed to shocks, due to their exclusion from the asset markets.

Regarding the skilled and unskilled income tax, we can see that in this case again the skilled labour income tax is the least volatile because the government has an incentive to minimise tax-distortions to their income. That happens because under the capital-skill complementarity channel the volatility of skilled labour supply, driven from tax fluctuations, can propagate more in the economy than the unskilled labour supply, and thus it can amplify the effect of the shocks to the economy. Finally, the optimal consumption

[84] At this point we should mention that we used quarterly data from 1970-2012 on sales taxes, so as to create a time series for the consumption tax. The data have been downloaded from FRED. Then we have HP-filtered the data and we found that the mean is 4.7%, the standard deviation is 0.018, autocorrelation is 0.813 and the correlation with output is equal to 0.76.

tax is set to be the least volatile of all fiscal instruments because of its direct effect to households' utility.

Table 3.7: Optimal tax policy with consumption tax

	τ_s	τ_u	τ_h	τ^c
Steady state	0.142	-0.065	-0.361	0.278
Autocorrelations	0.794	0.797	0.869	0.827
Correlations with Y	-0.375	-0.712	-0.112	0.385
Standard deviation	0.019	0.150	0.017	0.011

3.7 The optimal distribution of the tax burden over the business cycle

3.7.1 The case with three income taxes

In this section we analyse how optimal income taxes and the key macroeconomic variables behave following a temporary standard deviation shock to total factor productivity, productivity of capital equipment and government spending. Figure 3.1 presents the impulse responses (IRs) from an output-enhancing shock.[85] This allows us to evaluate how the government optimally distributes the tax burden in the short- and medium-run in response to output-enhancing exogenous shocks.

[85]Note that Figure 3.1 plots the percent deviations from the steady state for every variable.

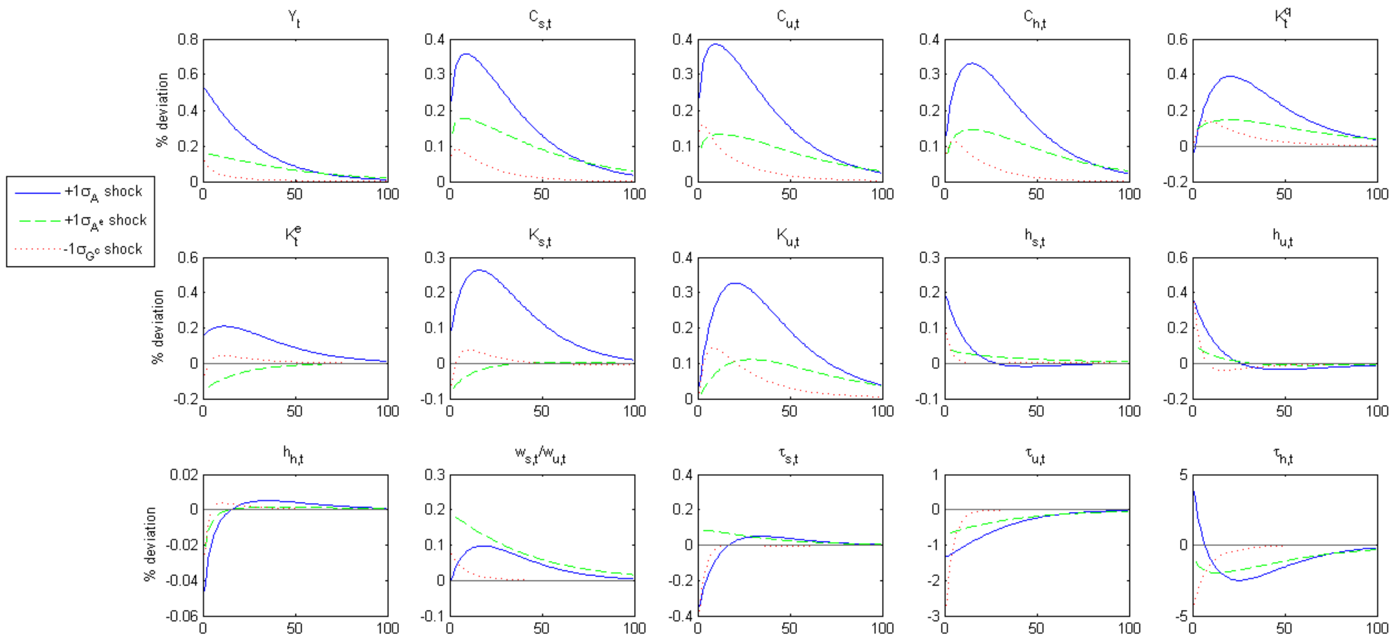

Figure 3.1: Impulse responses of optimal policy (Benchmark case)

A positive shock to total factor productivity will lead to higher immediate income tax rate for the hand-to-mouth agents (τ_h). The increase of the income tax of the hand-to-mouth agents is only present in the short-run because the government wants to smooth out their consumption. In addition, in the short-run the income taxes of the other two types of agents decrease, as the government can reduce these taxes and maintain the same level of tax revenue, given the rise in tax bases. The income tax of the unskilled agents exhibits the highest reduction compared to the skilled income tax. As a result, the progressivity of the income taxes is mixed after a TFP shock, since the income tax of the low income agents increases, whereas the income tax of the middle income agents decreases more compared to that of the high income agents.

In the medium-run we can see that the income tax of skilled agents increases above its steady state. As the economy returns to the steady state, it is optimal for the government to generate the tax revenue required by increasing faster the tax rate for the largest income source, so that it can maintain a low tax rate for unskilled and hand-to-mouth for a longer time. Figure 3.1 also shows that the increase in equipment capital and decrease in relative skill supply induced by the changes in TFP and income taxation increase the skill premium. As a result, the skill premium is procyclical.

The impulse responses of the optimal income taxes are different under a positive shock to the productivity of capital equipment. In particular, this shock creates an increase in wage inequality, reflecting the rise in the productivity of skilled labour via capital-skill complementarity, to which the government responds, by increasing the tax for skilled workers and decreasing the tax for unskilled workers.[86] Hence, in this case, the government finds it optimal, given the exogenous productivity gains for skilled households, to redistribute the tax burden in favour of the unskilled households. As a result,

[86] However, the stock of capital equipment decreases following the positive shock to its productivity. That happens because of the important income gains of skilled workers as a result of the increase in the returns to capital equipment. This effect causes a reduction of the investment in capital equipment, leading to an increase in consumption. Lindquist (2004) gets a similar reaction of equipment capital although in a model without optimal taxation.

income taxation becomes more progressive.

After an output-enhancing temporary reduction to government spending all taxes can fall, since the government needs to generate less tax revenue. However, driven by the incentive to support the households who are most constrained in the asset market, the reduction is bigger for the income tax of the hand-to-mouth agents, followed by that of the unskilled agents. Therefore, a temporary spending cut is followed by an increase in the progressivity of income taxes. The reductions in the income tax rates lead to a proportionately larger increase for the supply of unskilled labour, relative to skilled, which in turn increases the skill premium.

3.7.2 The case with different fiscal policy menu

In this section we present the reaction of the optimal income taxes and the key macroeconomic variables following a temporary standard deviation shock to total factor productivity, productivity of capital equipment and government spending. Note that, as we presented in the previous section, the shock to government spending is an output-enhancing shock (negative shock to government spending). Figure 3.2 presents the impulse responses under each shock.[87]

Comparing the responses of the model with the consumption tax and the model without, we can see that there are no qualitative differences in the reaction of the optimal income taxes after a temporary shock to the economy. Regarding the reaction of the optimal consumption tax, we can see that under a TFP shock it increases in the short-run so as for the government to smooth out shocks to consumption. In the medium-run the optimal consumption tax decreases and falls below its steady state and then it converges slowly to its long run value. Under a capital equipment productivity shock the consumption tax has a similar reaction with the TFP shock but with a smaller magnitude. Finally, under an output-enhancing government spending shock the optimal consumption tax decreases in the short-run below its long-run value, as the rest of the income taxes.

[87] Note that again Figure 3.2 plots the percent deviations from the steady state for every variable.

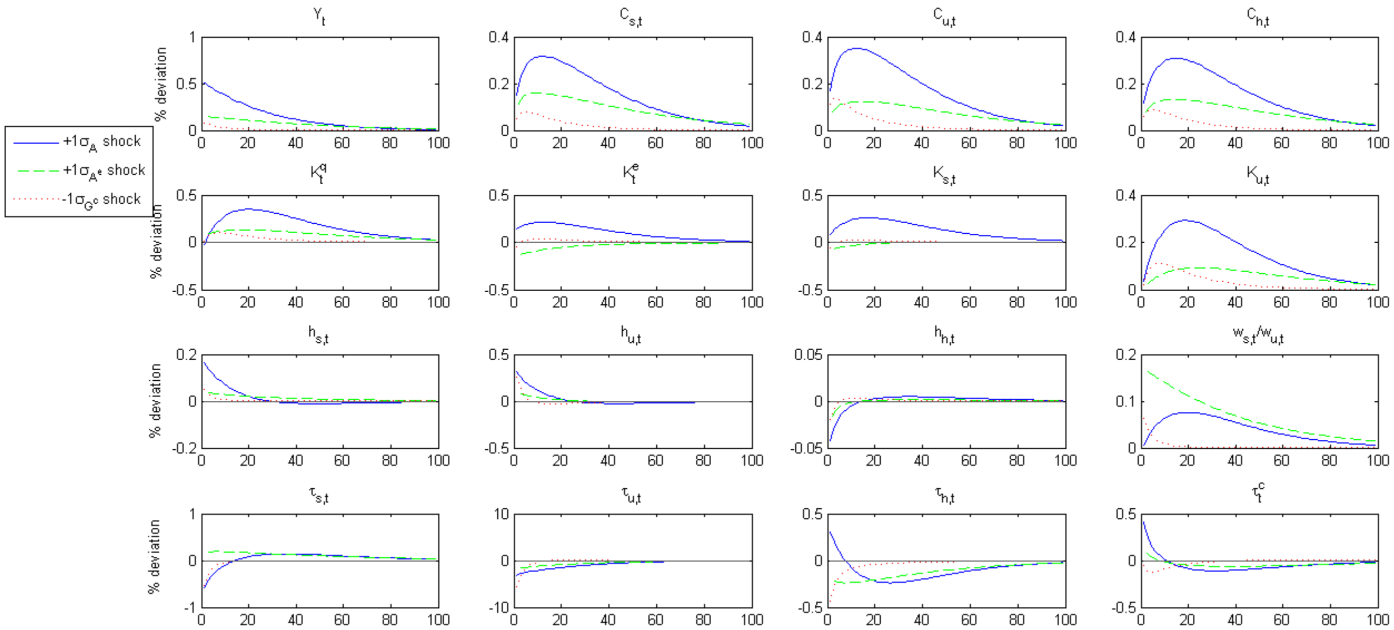

Figure 3.2: Impulse responses of optimal policy with the inclusion of the consumption tax

3.8 Robustness of results

In this section we analyse the robustness of the optimal reaction of the income taxes under the exogenous shocks we assumed in the previous sections and for our benchmark case study (without consumption tax). In Figure 3.3, we show, for each exogenous shock, the optimal path of the three income tax rates starting from the non-stochastic steady state. For comparison, we also present the optimal paths under the benchmark case analysed above.

Specifically, we consider three alternative calibrations and present the optimal taxes in each case, in response to exogenous output-enhancing shocks. In Model 1, we set the transaction costs to be five times higher, for both skilled and unskilled workers, compared to their benchmark calibration (see Table 3.3 for the benchmark calibration). In Model 2 we impose a higher capital-skill substitutability compared to the benchmark case, by setting the parameter ρ equal to -0.40.[88] Finally, in Model 3 we impose a lower capital-skill substitutability compared to the benchmark case, by setting the parameter φ equal to 0.30.[89]

The general message from Figure 3.3 is that the responses of optimal policy are quantitatively very similar across the different model variants. Hence, our results regarding the optimal reaction to exogenous shocks are robust to changes in the magnitude of transaction costs and production function elasticities.

[88] Following Cantore and Levine (2012), in Model 2 we re-calibrate the following parameters in the production function, $\lambda = 0.5676$ and $\nu = 0.5243$, so as the factor shares remain the same and the production function is uniquely identified.

[89] In Model 3 we re-calibrate $\lambda = 0.5727$ and $\nu = 0.5808$.

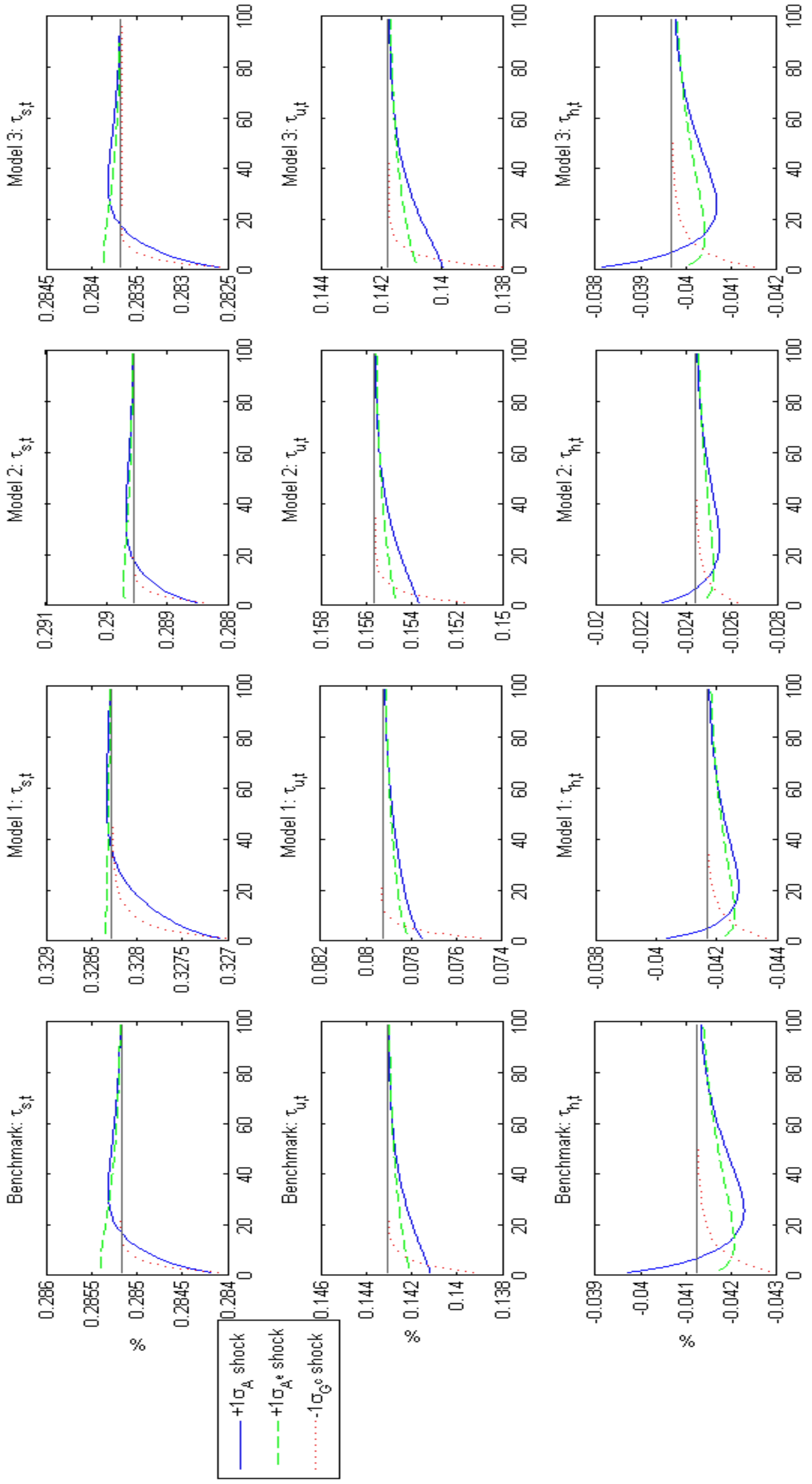

Figure 3.3: Optimal tax rates for benchmark model and alternative calibrations

3.9 Conclusions

In this chapter, we studied the optimal progressivity and cyclical properties of the income tax system over the business cycle in a model with capital-skill complementarity, a wage premium to skilled workers and labour and asset market imperfections. In particular, we assumed that a share of the population provides skilled labour services, whereas the rest of the population provided unskilled labour services. In addition, we assumed that a subset of the population did not have access to asset markets due to participation premia. It was shown that the model was able to replicate the empirical characteristics of the key macroeconomic variables over the business cycle as well as the key features of wealth inequality and labour markets.

Our analysis considered the problem of a government that chose the paths of income tax rates to maximise aggregate welfare. For that reason we constrained the set of fiscal instruments of the government and we focused only on income tax rates. In a different experiment, we also introduced a single consumption tax to the fiscal policy menu. Afterwards, we assessed the cyclical properties of the optimal income taxes under this setup and their optimal short-run and medium-run behaviour under several temporary output-enhancing exogenous shocks.

With respect to business cycle properties, we found that each optimal income tax had a different correlation with output and a volatility that was also different from the data. In particular, the optimal fiscal policy results indicated that the income tax rate of the high income agents had the lowest volatility and the income tax rate of the low income agents exhibited the lowest counter-cyclicality. Moreover, the optimal income tax rate of the middle income agents was the most countercyclical and volatile. In addition, we examined the effects on optimal income taxation under a different fiscal policy menu that also included a consumption tax. In this case we found that the progressivity of the income tax rates was even higher and that the results regarding the volatilities of the income taxes were overturned.

Finally, we found that, in both of the case studies, the progressivity of the income tax rates was optimally increased immediately after a positive shock to capital equipment efficiency and a reduction to government expenditures, whereas it optimally increased after two years with a positive shock to total factor productivity.

Appendix C

C Chapter 3

C.1 The skill premium

Using equations (130) and (131) implies the following expression for the skill premium:

$$\frac{w_{s,t}}{w_{u,t}} = \frac{\lambda(1-\nu)}{(1-\lambda)} \frac{\left(h_{s,t}^{f}\right)^{\rho-1}}{\left(h_{u,t}^{f}\right)^{\varphi-1}} \left(\Xi_t^1\right)^{\varphi/\rho-1} \tag{C1}$$

where $\Xi_t^1 \equiv \nu\left[(A_t^e)^\rho \left(K_t^{f,e}\right)^\rho\right] + (1-\nu)\left(h_{s,t}^{f}\right)^\rho$.

The skill premium is increasing with respect to capital equipment as long as the equipment-skill complementarity is present, i.e. $\rho < 0$, and the unskilled agents are substitutes to both of them, i.e. $0 < \varphi < 1$. Also, for the $\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(K_t^{f,e}\right)} > 0$ condition to hold it is necessary that $0 < \lambda, \nu < 1$ which is satisfied through our calibration as in Krusell *et al.* (2000):

$$\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(K_t^{f,e}\right)} = \frac{\lambda(1-\nu)}{(1-\lambda)} \frac{\left(h_{s,t}^{f}\right)^{\rho-1}}{\left(h_{u,t}^{f}\right)^{\varphi-1}} (\varphi-\rho)\,\nu\,(A_t^e)^\rho \left(K_t^{f,e}\right)^{\rho-1} \left(\Xi_t^1\right)^{\varphi/\rho-2}. \tag{C2}$$

Moreover the skill premium is decreasing to skilled labour supply, $\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(h_{s,t}^{f}\right)} < 0$:

$$\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(h_{s,t}^{f}\right)} = \frac{\lambda(1-\nu)}{(1-\lambda)} \frac{\left(h_{s,t}^{f}\right)^{\rho-1}}{\left(h_{u,t}^{f}\right)^{\varphi-1}} \left(\Xi_t^1\right)^{\varphi/\rho-1} \times \tag{C3}$$

$$\times\left[\rho + (\rho-\varphi)(1-\nu)\left(h_{s,t}^{f}\right)^{\rho-1}\right]$$

where the terms that define the sign are: $\frac{\lambda(1-\nu)}{(1-\lambda)} > 0$ and the term inside the squared brackets $\rho + (\rho-\varphi)(1-\nu)\left(h_{s,t}^{f}\right)^{\rho-1} < 0$ (due to the fact that $\rho < 0$

and $0 < \varphi < 1$).

Also, the skill premium is increasing to unskilled labour supply, $\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(h^f_{u,t}\right)} > 0$:

$$\frac{\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)}{\partial\left(h^f_{u,t}\right)} = \frac{\lambda\left(1-\nu\right)}{\left(1-\lambda\right)}\left(1-\varphi\right)\frac{\left(h^f_{s,t}\right)^{\rho-1}}{\left(h^f_{u,t}\right)^{\varphi}}\times \\ \times\left\{\nu\left[\left(A^e_t\right)^{\rho}\left(K^{f,e}_t\right)^{\rho}\right]+\left(1-\nu\right)\left(h^f_{s,t}\right)^{\rho}\right\}^{\varphi/\rho-1} \tag{C4}$$

now the crucial terms are: $\frac{\lambda(1-\nu)}{(1-\lambda)}\left(1-\varphi\right)$, where $\frac{\lambda(1-\nu)}{(1-\lambda)} > 0$ and also $(1-\varphi)$ since $0 < \varphi < 1$.

The last two derivatives, $(C3 - C4)$, imply that the skill premium is decreasing with respect to the relative labour supply of skilled over unskilled agents, i.e. $\partial\left(\frac{w_{s,t}}{w_{u,t}}\right)/\partial\left(\frac{h^f_{s,t}}{h^f_{u,t}}\right) < 0$.

Chapter 4: Tax smoothing in a business cycle model with capital-skill complementarity

Abstract: This chapter undertakes a normative investigation of the quantitative properties of optimal tax smoothing in a business cycle model with state contingent debt, capital-skill complementarity, endogenous skill formation and stochastic shocks to public consumption as well as total factor and capital equipment productivity. We also examine the properties of optimal taxation under a restriction on the debt to output ratio. Our main finding is that, an empirically relevant restriction which does not allow the relative supply of skilled labour to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. This result remains valid even in the presence of a budget rule that restricts the debt to output ratio. We show that the key to understanding this result is that the government finds it optimal to adjust labour income tax rates to alter the average net returns to skilled and unskilled labour hours.[90]

4.1 Introduction

The celebrated labour tax smoothing result of Barro (1979) in a partial equilibrium setting has lead to a number of important studies on optimal fiscal policy over the business cycle in representative agent general equilibrium models. For example, Lucas and Stokey (1983) formalised labour tax smoothing within a complete markets neoclassical setup without capital when the government has access to state-contingent debt. Chari *et al.* (1994) generalised this result in a model with capital taxation and showed that Ramsey policy dictates that the labour income tax fluctuates very little in response to aggregate shocks and the *ex ante* capital income tax is approximately zero in each period.

The literature has also examined the implications of policy frictions and incomplete asset markets for optimal tax and debt policy, through a variety

[90]The CESifo 2014 working paper (4744) by Angelopoulos, Asimakopoulos and Malley entitled "Tax smoothing in a business cycle model with capital-skill complementarity" is based on this Chapter.

of restrictions to the policy instrument set, government debt and capital income taxation (see e.g. Stockman (2001), Aiyagari *et al.* (2002), Angeletos (2002), Buera and Nicolini (2004) and Farhi (2010)). In contrast, assuming complete asset markets, Arseneau and Chugh (2012) consider labour market frictions which imply a division of the labour force into employed and unemployed workers. Their model, with state-contingent debt but no capital, suggests that optimal labour tax volatility depends on whether wages are set efficiently.

Another important division of the labour force is with respect to the type of labour services workers provide and, in particular, how these complement capital in the production process. This is especially pertinent given the empirical relevance of the wage premium accruing to skilled labour and the roles attributed to capital-skill complementarity, the relative supply of skilled labour and capital augmenting technical progress (see e.g. Katz and Murphy (1992), Krusell *et al.* (2000) and Hornstein *et al.* (2005)). In an important related contribution, Werning (2007) establishes the conditions under which optimal labour tax smoothing holds in a model with redistribution under complete asset markets when workers differ with respect to their productivity. However, since the distinct types of labour are treated as perfect substitutes in production, this setup does not capture how labour may exhibit different degrees of complementarity with capital as in e.g. Katz and Murphy (1992) and Krusell *et al.* (2000). Moreover, since the distribution of productivity differentials is taken as exogenous, this approach also does not explain how differences in employment type can be endogenously determined (see e.g. Matsuyama (2006), who also reviews this literature).

In this chapter we aim to contribute to the tax smoothing literature by focusing on the above two features of an economy where the labour force is divided into skilled and unskilled workers. In particular, we examine the importance of differences in the complementarity between capital and skilled and unskilled labour as well as the endogenous determination of the relative skill supply for Ramsey tax policy over the business cycle. In contrast to Werning (2007), we focus on aggregate outcomes and abstract from redistribution incentives, by following the literature that examines a division of the labour force into two types of workers. To this end, we work with a representative household which guarantees its members' the same level of consumption (see e.g. Arseneau and Chugh (2012)). We thus stay as close as possible to the representative agent Ramsey analysis of Chari *et al.* (1994) and extend their model to allow for capital-skill complementarity and endogenous skill formation.[91]

Our goal is thus to undertake a normative investigation of the quantitative properties of optimal taxation of capital and labour income, as well as skill-acquisition expenditure, in the presence of aggregate shocks to total factor productivity (TFP), capital equipment productivity and government spending. We assume complete asset markets, however, to capture the importance of endogenous versus fixed relative skill supply, we also consider a labour market distortion that restricts the ratio of skilled to total workers to remain constant. This assumption is supported by empirical evidence suggesting that the share of college educated or skilled workers in the data has low relative volatility and is effectively uncorrelated with output over the business cycle. For example, the standard deviation of the cyclical component of this share relative to the standard deviation of output is 0.27 and its correlation with output is -0.18.[92]

In our setup, the government can borrow, tax skill acquisition expenditure, capital, skilled and unskilled labour income separately, to finance exogenous public spending. All policy instruments are allowed to be state-contingent. In this environment, the optimal taxes on labour income and skill acquisition expenditure are uniquely determined. However, as is well known, when the government has access to both state contingent debt and state contingent capital taxation, the second-best Ramsey allocations do not uniquely pin down optimal debt and capital taxes (see Chari *et al.* (1994)). Hence, following the literature, in this instance we discuss the properties of

[91] Given that employment in skilled jobs is observable, we also abstract from issues related to Mirrleesian taxation.

[92] These calculations are based on annual data for the share of college educated to total working population measured in efficiency units (1963-2008) from Acemoglu and Autor (2011) and GDP data from the US NIPA accounts (1963-2008). The cyclical component of the series is obtained using the HP-filter with a smoothing parameter of 100.

the *ex ante* capital tax rate. Moreover, we also examine the case where debt is restricted to be state uncontingent, which allows us to calculate the *ex post* capital tax or, if we also allow for state-contingent taxation of income from bonds, the private assets tax.[93]

Our main finding is that under capital-skill complementarity, a friction that does not allow the relative supply of skill to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. In particular, we first show that under endogenous relative skill supply, the optimal labour taxes for both skilled and unskilled labour income are very smooth, with the volatility of the unskilled income tax being marginally higher. We also find that the skilled tax moves pro-cyclically with output and the unskilled tax is mildly counter-cyclical. These results are largely consistent with the literature and extend previous findings to a setup with capital-skill complementarity and endogenous skill supply.

However, when the relative skill supply is constrained to remain constant over the business cycle, the prescriptions for optimal policy markedly change. In particular, we find that the volatility of taxes increases significantly, so that the standard deviation of the effective average labour income tax is about twelve times higher than the perfect labour markets case, while the volatility of the skilled labour income tax is about two-and-a-half times higher than that of the unskilled labour income tax. Moreover, both taxes become strongly counter-cyclical. We show that the key to understanding these changes is that the government finds it optimal to minimise the effects of the relative labour supply distortion by keeping the marginal rates of substitution between leisure and consumption for the two types of labour at roughly the same levels as under a fully flexible labour market. In other words, the government adjusts labour income tax rates to alter the average net returns to skilled and unskilled labour hours.

Compared with the extension of Chari *et al.* (1994) undertaken by Werning (2007), our extension does not allow for redistribution. However, our

[93] As shown by Zhu (1992) and Chari *et al.* (1994), state-contingent capital income taxes allow the government to implement the complete asset markets outcome, despite the lack of access to state-contingent debt.

results add to the findings in Werning (2007) in the following way. Werning (2007) shows that exogenous skill heterogeneity does not alter the basic optimal tax smoothing results for a large class of utility functions, when the assumption regarding the neoclassical production function is maintained and the different skill-adjusted labour inputs are perfect substitutes in the production function. In contrast, we analyse a case where skill-adjusted labour inputs have different degrees of complementarity with capital and find that whether this skill heterogeneity is endogenous or exogenous does indeed matter for the cyclical properties of optimal labour taxes.

Our results further show that the skill heterogeneity considered, irrespective of the presence of the labour market friction, does not affect the results obtained in the literature regarding the cyclical behaviour of asset taxes. In particular, the *ex ante* tax rate on capital is around zero for every period, the state contingent private assets and *ex post* capital taxes are near zero and are the most volatile of the non-asset tax instruments. We also find that the skill-acquisition tax is the least smooth of the non-asset tax instruments when debt is state-contingent and fluctuates nearly as much as output. Finally, irrespective of the model variant examined, all of the policy instruments, except for the *ex post* capital tax and the private assets tax inherit the persistence properties of the shocks.

When we introduce a budget rule, where the government is not able to issue debt but needs to satisfy a given level of debt to output ratio over the business cycle instead, we find that the main results remain valid. In particular, we find that again under an empirically relevant restriction to relative skill supply, the volatility of the labour income tax rates increases significantly and the effective labour income tax volatility becomes almost nine times higher compared to the flexible relative skill supply case study. Moreover, under the budget rule all the fiscal instruments become countercyclical and they inherit the persistence properties of the shocks.

The remainder of the chapter is organised as follow. Sections 2 and 3 present the theoretical model and the Ramsey problem respectively. Section 4 contains the quantitative results. Section 5 presents the results under a fixed debt to output ratio and Section 6 draws the conclusions.

4.2 Model

We develop a model that extends the complete markets neoclassical setup in Zhu (1992) and Chari *et al.* (1994) by allowing for a division of the labour force into skilled and unskilled workers, an endogenous skill supply on the household side and capital-skill complementarity on the production side. This setup implies a wage premium for skilled labour, the relative supply of which can be increased by a cost to the household in the form of earmarked training expenditure.[94] As in Chari *et al.* (1994) households save in the form of physical capital and state-contingent government bonds.

The household is modelled as an infinitely-lived representative dynasty. The head of the household makes all choices on behalf of its members by maximising the aggregate welfare of the family, ensuring that each household member experiences the same level of consumption irrespective of individual labour market status. This is a commonly employed assumption since Merz (1995), given that it allows for tractability when studying aggregate fluctuations under heterogeneities in the labour market (see e.g. Arseneau and Chugh (2012) for an example with optimal tax policy).

Firms use capital, skilled and unskilled labour to produce a homogeneous product. Following Katz and Murphy (1992), Krusell *et al.* (2000) and Hornstein *et al.* (2005), skilled labour is assumed to be more complementary to capital than unskilled labour. Hence, capital accumulation as well as technological developments and government policies that are capital augmenting, increase the skilled wage premium. In contrast, increases in the relative supply of skilled labour reduce the skill premium. Finally, the government can borrow, tax skill acquisition expenditure, capital, skilled and unskilled labour income separately, to finance exogenous public spending.

4.2.1 Notation

The notation employed throughout follows Ljungqvist and Sargent (2012). In particular, we assume that in every period $t \geq 0$, there is a realization

[94] This is consistent with the literature on upward professional mobility, where there is a cost associated with achieving the higher professional status (see e.g. Matsuyama (2006) for a review of several models).

of shocks (stochastic events) $s_t \in S$. Therefore, at each period t there is a history of events $s^t = [s_0, s_1, s_2, ..., s_t]$ which is known. The unconditional probability of observing a specific history of events s^t is defined as $\pi_t(s^t)$. For $t > \tau$, the conditional probability of having s^t sequence of events given the realization of s^τ is defined as: $\pi_t(s^t \mid s^\tau)$.

4.2.2 Households

A representative household is comprised of two types of members who provide skilled and unskilled labour services.[95] The household can invest in capital and in state-contingent sequentially traded government bonds that mature fully within a period. The objective function of the representative household is given by:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t\left(s^t\right)u\left\{c_t\left(s^t\right), \psi_t(s^t)l_t^s\left(s^t\right), \left[1-\psi_t(s^t)\right]l_t^u\left(s^t\right)\right\} \tag{140}$$

where $u(.)$ is increasing, strictly concave and three times continuously differentiable with respect to its inputs; $c_t(s^t)$ is average consumption of all household members at time t given the history of events s^t;[96] $l_t^s(s^t)$ and $l_t^u(s^t)$, denote, respectively, per skilled and unskilled member leisure time; and $\psi_t(s^t)$ is the share of skilled to total household members or the relative skill supply. Thus $\psi_t(s^t)l_t^s(s^t)$ and $[1-\psi_t(s^t)]\,l_t^u(s^t)$ represent average skilled and unskilled leisure time respectively. The time constraints facing each type of member are given by:

$$h_t^s\left(s^t\right) + l_t^s\left(s^t\right) = 1 \tag{141}$$

$$h_t^u\left(s^t\right) + l_t^u\left(s^t\right) = 1 \tag{142}$$

where, $h_t^s(s^t)$ and $h_t^u(s^t)$ denote, respectively, skilled and unskilled labour hours per member. The household can determine its relative skill supply

[95] Note that the unit mass of household members is equal to the sum of its skilled and unskilled members.

[96] Since consumption is the same for all members of the household, average and per member consumption are the same.

by incurring an average (over all its members) skill-acquisition cost, $e_t\left(s^t\right)$, according to the following relation:

$$\psi_t(s^t) = \widetilde{g}\left[e_t\left(s^t\right)\right] \tag{143}$$

where $\widetilde{g}(.)$ is increasing, strictly concave and three times continuously differentiable with respect to $e_t\left(s^t\right)$.

The household also faces a sequence of budget constraints given by:

$$\begin{array}{l} c_t\left(s^t\right) + k_{t+1}\left(s^t\right) + \sum\limits_{s^{t+1}} p_t\left(s_{t+1} \mid s^t\right) b_{t+1}\left(s_{t+1} \mid s^t\right) + \\ + \left[1 + \tau_t^a\left(s^t\right)\right] g\left[\psi_t(s^t)\right] = \left[1 - \tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right) \times \\ \times \psi_t(s^t) h_t^s\left(s^t\right) + \left[1 - \tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right)\left[1 - \psi_t(s^t)\right] h_t^u\left(s^t\right) + \\ + (1-\delta)\, k_t\left(s^{t-1}\right) + \left[1 - \tau_t^k\left(s^t\right)\right] r_t\left(s^t\right) k_t\left(s^{t-1}\right) + b_t\left(s_t \mid s^{t-1}\right)\ \forall t \end{array} \tag{144}$$

where $p_t\left(s_{t+1} \mid s^t\right)$ is the pricing kernel for government bonds in terms of t goods and $b_{t+1}\left(s_{t+1} \mid s^t\right)$ is the state s_{t+1} contingent payout value of bonds bought per member at period t;[97] $e_t\left(s^t\right)$ has been substituted out of equation (143) using the inverse function of $\widetilde{g}$ defined as $g\left[\psi_t(s^t)\right] = e_t\left(s^t\right)$; $\tau_t^s\left(s^t\right)$, $\tau_t^u\left(s^t\right)$, $\tau_t^k\left(s^t\right)$, $\tau_t^a\left(s^t\right)$ are the tax rates on skilled and unskilled labour, capital income and skill-acquisition expenditure respectively; $w_t^s\left(s^t\right)$ and $w_t^u\left(s^t\right)$ are the wage rates of skilled and unskilled labour respectively; $r_t\left(s^t\right)$ is the return to capital; $k_t\left(s^{t-1}\right)$ is the per member stock of capital at time t given the history of events s^{t-1}; and $0 < \delta < 1$ is the capital depreciation rate.

4.2.3 First order conditions for households

Substituting the constraints (141)-(142) into the utility function $u(.)$, the household maximises the resulting objective function subject to the sequence of constraints in (144), by choosing $\{c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t), k_{t+1}\left(s^t\right)$ $\forall s^t\}_{t=0}^{\infty}$ and $\{b_{t+1}\left(s_{t+1}, s^t\right);\ \forall s^t\}_{t=0}^{\infty}$, given initial values for b_0, k_0. In each time period t and given history s^t, $\{b_{t+1}\left(s_{t+1}, s^t\right)\}_{t=0}^{\infty}$ is a vector of government bonds with one element of the vector for each possible realisation of

[97] Given the period t state $s_t \mid s^{t-1}$ (or else the history s^t), the income side of the household budget includes revenue from bonds dated $b_t\left(s_t \mid s^{t-1}\right)$.

s_{t+1}. This yields six first-order conditions which are reported in the Appendix.

Combining the first-order conditions for consumption, skilled and unskilled labour supply as well as the relative skill supply gives the following *a-temporal* equilibrium conditions:

$$-\frac{u_{h^s}(s^t)}{u_c(s^t)} = \psi_t(s^t) w_t^s\left(s^t\right)\left[1-\tau_t^s(s^t)\right] \tag{145}$$

$$-\frac{u_{h^u}(s^t)}{u_c(s^t)} = \left[1-\psi_t(s^t)\right] w_t^u\left(s^t\right)\left[1-\tau_t^u(s^t)\right] \tag{146}$$

$$\begin{aligned} -\frac{u_{\psi}(s^t)}{u_c(s^t)} &= h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right) - \\ &- h_t^u\left(s^t\right)\left[1-\tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right) - \left[1+\tau_t^a\left(s^t\right)\right]\left[g_{\psi}\left(s^t\right)\right]. \end{aligned} \tag{147}$$

Conditions (145)-(146) equate the marginal rates of substitution between consumption and each type of labour with the wage rates net of taxes. The final relation given by (147) states that the marginal rate of substitution between consumption and the relative skill supply is equal to the net marginal benefit of increasing the household's share of skilled workers. The latter includes the post-tax labour income from an additional skilled member, $h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right)$, less the post-tax labour income from one less unskilled member, $h_t^u\left(s^t\right)\left[1-\tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right)$, less the post-tax cost for an additional skilled member, $\left[1+\tau_t^a\left(s^t\right)\right]\left[g_{\psi}\left(s^t\right)\right]$.

Substituting the first-order condition for consumption and its one-period lead into the first-order conditions for the two assets gives the following *inter-temporal* conditions equating the current cost of investing in bonds and capital to the future state-contingent and expected benefits respectively:

$$u_c(s^t) p_t\left(s_{t+1} \mid s^t\right) = \beta \pi_{t+1}\left(s^{t+1} \mid s^t\right) u_c(s^{t+1}) \tag{148}$$

$$u_c(s^t) = \beta E_t\left\{u_c(s^{t+1})\left[\left(1-\tau_{t+1}^k(s^{t+1})\right) r_{t+1}\left(s^{t+1}\right)+1-\delta\right]\right\} \tag{149}$$

where $\frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)} = \pi_{t+1}\left(s^{t+1} \mid s^t\right)$ and E_t is the expectation conditional on information available at time t (i.e. history s^t), $E_t x_{t+1}(s^{t+1}) = \sum_{s^{t+1}|s^t} \frac{\pi_{t+1}(s^{t+1})}{\pi_t(s^t)} \times$ $\times x_{t+1}(s^{t+1})$, and the summation over s^{t+1} denotes the sum over all possible histories $\widetilde{s}^{t+1}$ such that $\widetilde{s}^t = s^t$.

By combining the *inter-temporal* conditions we obtain:

$$1 = \sum_{s_{t+1}} p_t \left(s_{t+1} \mid s^t\right) \left\{ \left[1 - \tau^k_{t+1}\left(s^{t+1}\right)\right] r_{t+1}\left(s^{t+1}\right) + (1-\delta) \right\} \tag{150}$$

which ensures no-arbitrage between the investment opportunities in bonds and capital.

4.2.4 Firms

Firms rent capital as well as skilled and unskilled labour from households to maximize their profits using a production function $F(\cdot)$ that exhibits constant returns to scale in its three inputs:

$$\begin{aligned} \Pi_t = F &\left[(h^{s,f}_t(s^t), h^{u,f}_t(s^t), k^f_t(s^{t-1}) \right] - \\ &- w^s_t(s^t) h^{s,f}_t(s^t) - w^u_t(s^t) h^{u,f}_t(s^t) - r_t(s^t) k^f_t(s^{t-1}). \end{aligned} \tag{151}$$

This yields the standard first-order conditions:

$$w^s_t(s^t) = F_{h^{s,f}}(s^t) \tag{152}$$

$$w^u_t(s^t) = F_{h^{u,f}}(s^t) \tag{153}$$

$$r_t(s^t) = F_{k^f}(s^t). \tag{154}$$

4.2.5 Government budget and market clearing

Given a history s^t, the government finances an exogenous stream of expenses $g^e_t(s^t)$ and its debt obligation $b_t\left(s_t \mid s^{t-1}\right)$, by taxing capital and labour income and skill acquisition expenditure, and by issuing state-contingent debt.

Hence, the within-period government budget constraint is given by:

$$
\begin{aligned}
&g_t^e(s^t) = \tau^s(s^t)w_t^s(s^t)\psi_t(s^t)h_t^s(s^t) + \tau^u(s^t)w_t^u(s^t)\left[1-\psi_t(s^t)\right]\times \\
&\times h_t^u(s^t) + \tau_t^k(s^t)r_t(s^t)k_t(s^{t-1}) + \tau_t^a(s^t)\, g\left[\psi_t(s^t)\right] + \\
&+\sum_{s_{t+1}} p_t\left(s_{t+1}\mid s^t\right) b_{t+1}\left(s_{t+1}\mid s^t\right) - b_t\left(s_t\mid s^{t-1}\right).
\end{aligned}
\tag{155}
$$

Finally, the aggregate consistency condition and market clearing conditions for skilled labour, unskilled labour and capital are given respectively by:

$$
F(\cdot) = c_t(s^t) + g_t^e(s^t) + g\left[\psi_t(s^t)\right] + k_{t+1}(s^t) - (1-\delta)\, k_t(s^{t-1}) \tag{156}
$$

$$
\psi_t(s^t)h_t^s\left(s^t\right) = h_t^{s,f}(s^t) \tag{157}
$$

$$
\left[1-\psi_t(s^t)\right] h_t^u\left(s^t\right) = h_t^{u,f}(s^t) \tag{158}
$$

$$
k_t(s^{t-1}) = k_t^f(s^{t-1}). \tag{159}
$$

4.3 The Ramsey problem

To solve the Ramsey problem we follow the primal approach and first derive the present discounted value (PDV) of the household's lifetime budget constraint using the Arrow-Debreu price of the bond and the transversality conditions for bonds and capital. Second, we derive the implementability constraint by substituting out prices and tax rates from the household's present value budget constraint using the first-order conditions for the household and firm. Finally, we derive the optimal Ramsey allocations by maximising the planner's objective function subject to the implementability constraint and the aggregate resource constraint.

4.3.1 Present value of budget constraint

Starting from period 0 and by repeatedly substituting forward one-period budget constraints for the household, we obtain the PDV of the household's lifetime budget constraint:

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)c_t(s^t)=\sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)\times\\
&\times\{[(1-\tau_t^s\left(s^t\right)]\,w_t^s\left(s^t\right)\psi_t(s^t)h_t^s\left(s^t\right)+[(1-\tau_t^u\left(s^t\right)]\,w_t^u\left(s^t\right)\times\\
&\times[1-\psi_t(s^t)]h_t^u\left(s^t\right)-[1+\tau_t^a\left(s^t\right)]\,g\left[\psi_t(s^t)\right]\}+b_0+\\
&+\left\{\left[(1-\tau_0^k\left(s_0\right)\right]r_0\left(s_0\right)+(1-\delta)\right\}k_0\ \text{(delete dot)}
\end{aligned}
\tag{160}
$$

where we have imposed the series of no-arbitrage conditions (150) $\forall t$ and the following transversality conditions for any s^∞:

$$
\lim_{t\to\infty}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)k_{t+1}\left(s^t\right)=0 \tag{161}
$$

$$
\lim_{t\to\infty}\sum_{s_{t+1}}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)p_t\left(s_{t+1}\mid s^t\right)b_{t+1}\left(s_{t+1}|s^t\right)=0 \tag{162}
$$

which specify that for any possible future history the household does not hold positive or negative valued wealth at infinity. Defining $\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)\equiv q_t^0(s^t)$, $\forall t\geq 1$, with $q_0^0(s^0)\equiv 1$, where $q_t^0(s^t)$ is the Arrow-Debreu price, we can re-write (160) as:

$$
\begin{aligned}
&\sum_{t=0}^{\infty}\sum_{s^t}q_t^0(s^t)c_t(s^t)=\sum_{t=0}^{\infty}\sum_{s^t}q_t^0(s^t)\{\,[(1-\tau_t^s\left(s^t\right)]\,w_t^s\left(s^t\right)\psi_t(s^t)h_t^s\left(s^t\right)+\\
&+[(1-\tau_t^u\left(s^t\right)]\,w_t^u\left(s^t\right)[1-\psi_t(s^t)]h_t^u\left(s^t\right)-[1+\tau_t^a\left(s^t\right)]\,g\left[\psi_t(s^t)\right]\,\}+\\
&+b_0+\left\{\left[(1-\tau_0^k\left(s_0\right)\right]r_0\left(s_0\right)+(1-\delta)\right\}k_0.
\end{aligned}
\tag{163}
$$

Notice that the Arrow-Debreu price satisfies the recursion:

$$
q_{t+1}^0(s^{t+1})=p_t\left(s_{t+1}\mid s^t\right)q_t^0(s^t). \tag{164}
$$

Using the first-order condition from the sequential equilibrium for pricing contingent claims (148) and noting that $\pi_0\left(s^0\right)=1$, since, at period 0 the state s^0 is known, the above recursion can be written as:

$$q_{t+1}^0(s^{t+1}) = \beta^{t+1}\pi_{t+1}\left(s^{t+1}\right)\frac{u_c(s^{t+1})}{u_c(s^0)}. \tag{165}$$

4.3.2 Implementability constraint

First, notice that (165) implies:

$$q_t^0(s^t) = \beta^t\pi_t\left(s^t\right)\frac{u_c(s^t)}{u_c(s^0)}. \tag{166}$$

Substituting (166) for $q_t^0(s^t)$; the first-order conditions of the firm, (152), (153) and (154) for $w_t^s(s^t)$, $w_t^u(s^t)$ and r_0, respectively; and the first-order conditions of the household, (145), (146), and (147) for $\tau_t^s(s^t)$, $\tau_t^u(s^t)$ and $\tau_t^a(s^t)$, respectively; into the present value budget constraint (163), we obtain the implementability constraint:[98]

$$\begin{aligned}&\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t\left(s^t\right)[u_c\left(s^t\right)c_t\left(s^t\right)+u_{h^s}\left(s^t\right)h_t^s\left(s^t\right)+\\&+u_{h^u}\left(s^t\right)h_t^u\left(s^t\right)+\Omega_t\left(s^t\right)]-A=0\end{aligned} \tag{167}$$

where $\Omega_t\left(s^t\right) \equiv \left[u_\psi(s^t) - h_t^s\left(s^t\right)\frac{u_{h^s}(s^t)}{\psi_t(s^t)} + h_t^u\left(s^t\right)\frac{u_{h^u}(s^t)}{1-\psi_t(s^t)}\right] g\left[\psi_t(s^t)\right]\left[g_\psi\left(s^t\right)\right]^{-1}$; $A \equiv A(c_0\left(s^0\right), h_0^s\left(s^0\right), h_0^u\left(s^0\right), \psi_0(s^0); b_0, k_0, \tau_0^k) = u_c\left(s^0\right)\{b_0 + [(1-\tau_0^k)\widetilde{F}_k(s^0) + (1-\delta)]k_0\}$ and $\widetilde{F}_k(s^0)$ is obtained by substituting the market clearing condition (159) into $F_{k^f}(s^0)$.

4.3.3 Pseudo value function

Substituting the constraints (141)-(142) into the utility function $u(.)$, the government maximises the resulting objective function subject to the implementability constraint (167) and the aggregate resource constraint (156) by

[98] Note that the *inter-temporal* first-order condition (150) has been used already in deriving (163), while the government budget constraint is redundant, since it is a linear combination of the household's budget constraint and the aggregate resource constraint. Therefore, (167) and (156) summarise all the constraints that the government needs to respect.

choosing $\{c_t(s^t), h_t^s(s^t), h_t^u(s^t), \psi_t(s^t), k_{t+1}(s^t) \forall \mathbf{s^t}\}_{t=0}^{\infty}$, given $\{b_0, k_0, \tau_0^k\}$.[99] To achieve this, we follow Ljungqvist and Sargent (2012) and first specify the following within-period pseudo value function:

$$V[c_t(s^t), h_t^s(s^t), h_t^u(s^t), \psi_t(s^t); \Phi] = u[c_t(s^t), 1 - h_t^s(s^t), \\ 1 - h_t^u(s^t), \psi_t(s^t)] + \Phi[u_c(s^t) c_t(s^t) + u_{h^s}(s^t) h_t^s(s^t) + \\ + u_{h^u}(s^t) h_t^u(s^t) + \Omega_t(s^t)] \tag{168}$$

where Φ is the Lagrange multiplier with respect to the implementability constraint.[100] The Lagrangian of the Ramsey planner is defined as:

$$J = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \pi_t(s^t)\{V(c_t(s^t), h_t^s(s^t), h_t^u(s^t), \psi_t(s^t); \Phi) + \\ + \theta_t(s^t)[\widetilde{F}(\cdot) - c_t(s^t) - g_t^e(s^t) - g[\psi_t(s^t)] - k_{t+1}(s^t) + \\ + (1-\delta)k_t(s^{t-1})]\} - \Phi A \tag{169}$$

where $\widetilde{F}(\cdot)$ is obtained by substituting market clearing conditions (157)-(159) into $F(\cdot)$; $\{\theta_t(s^t); \forall s^t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers attached to the aggregate resource constraint. For a given level of $\{b_0, k_0, \tau_0^k\}$, J is maximized with respect to $\{c_t(s^t), h_t^s(s^t), h_t^u(s^t), \psi_t(s^t), k_{t+1}(s^t); \forall s^t\}_{t=1}^{\infty}$ and $c_0(s^0), h_0^s(s^0), h_0^u(s^0), \psi_0(s^0)$ yielding the following first-order conditions respectively:

$$V_c(s^t) = \theta_t(s^t), \ t \geq 1 \tag{170}$$

$$V_{h^s}(s^t) = -\theta_t(s^t)\widetilde{F}_{h^s}(s^t), \ t \geq 1 \tag{171}$$

$$V_{h^u}(s^t) = -\theta_t(s^t)\widetilde{F}_{h^u}(s^t), \ t \geq 1 \tag{172}$$

$$V_{\psi}(s^t) = \theta_t(s^t)[g_{\psi}(s^t)], \ t \geq 1 \tag{173}$$

$$\theta_t(s^t) = \beta E_t \theta_{t+1}(s^{t+1})\left[\widetilde{F}_k(s^{t+1}) + 1 - \delta\right], \ t \geq 0 \tag{174}$$

$$V_c(s^0) = \theta_0(s^0) + \Phi A_c \tag{175}$$

[99] Note that following the literature we do not examine the problem of initial capital taxation and thus do not allow the government to choose τ_0^k.

[100] Note that the multiplier Φ is non-negative and measures the disutility of future tax distortions.

$$V_{h^s}\left(s^0\right)=-\theta_0\left(s^0\right)\widetilde{F}_{h^s}\left(s^0\right)+\Phi A_{h^s} \tag{176}$$

$$V_{h^u}\left(s^0\right)=-\theta_0\left(s^0\right)\widetilde{F}_{h^u}\left(s^0\right)+\Phi A_{h^u} \tag{177}$$

$$V_{\psi}\left(s^0\right)=\theta_0\left(s^0\right)\left[g_{\psi}\left(s^0\right)\right]+\Phi A_{\psi}. \tag{178}$$

where $\{\widetilde{F}_{h^s}(t), \widetilde{F}_{h^u}(t), \widetilde{F}_{k}(t);\ \forall s^t\}_{t=0}^{\infty}$ are obtained by substituting market clearing conditions (157)-(159) into $\{F_{h^s}(t), F_{h^u}(t), F_{k}(t);\ \forall s^t\}_{t=0}^{\infty}$ respectively. The first-order conditions derived in (170)-(178) imply that the system of equations to be solved will be different for $t=0$ and for $t>0$. These conditions in a non-stochastic environment are presented in the Appendix.

4.4 Quantitative implementation

In this section we quantitatively solve both the non-stochastic and stochastic optimal policy models. Our solution approach follows Arseneau and Chugh (2012). In particular, we first calibrate the non-stochastic model with exogenous policy. Next, we solve the deterministic Ramsey problem, starting from the exogenous policy steady state, using non-linear methods. Since we are interested in tax smoothing over the business cycle, we then approximate around the steady state of the deterministic Ramsey problem to solve the stochastic problem and obtain near steady state dynamics.

4.4.1 Functional forms

Following Chari *et al.* (1994) and Stockman (2001), we use a CRRA utility function:

$$u(\cdot)=\frac{\left\{\left[c_t\left(s^t\right)\right]^{1-\sigma_1-\sigma_2}\left[\psi_t(s^t)l_t^s\left(s^t\right)\right]^{\sigma_1}\left[\left[1-\psi_t(s^t)\right]l_t^u\left(s^t\right)\right]^{\sigma_2}\right\}^{\sigma_3}}{\sigma_3} \tag{179}$$

where, σ_1 and σ_2 are the weights to leisure in the utility function and σ_3 is the relative risk aversion parameter.

The production side is given by a CES production function that allows for capital-skill complementarity, since the latter has been shown to match the dynamics of the skill premium in the data (see e.g. Krusell *et al.* (2000), Lindquist (2004), and Pourpourides (2011)):

$$F(\cdot) = A_t \left\{ \mu \left(h_t^{u,f} \right)^{\alpha} + (1-\mu) \left[\rho \left(A_t^k k_t^f \right)^{\nu} + (1-\rho) \left(h_t^{s,f} \right)^{\nu} \right]^{\frac{\alpha}{\nu}} \right\}^{\frac{1}{\alpha}} \quad (180)$$

where, A_t is total factor productivity; A_t^k is the efficiency level of capital equipment; $\alpha < 1$, and $\nu < 1$ are the parameters determining the factor elasticities, i.e. $1/(1-\alpha)$ is the elasticity of substitution between capital and unskilled labour and between skilled and unskilled labour, whereas $1/(1-\nu)$ is the elasticity of substitution between equipment capital and skilled labour; and $0 < \mu, \rho < 1$ are the factor share parameters. In this specification, capital-skill complementarity is obtained if $1/(1-\alpha) > 1/(1-\nu)$.

The above functional form implies that the skill premium, defined as $\frac{w^s(s^t)}{w^u(s^t)}$, can be obtained as:

$$\frac{w^s(s^t)}{w^u(s^t)} = \frac{\widetilde{F}_{h^s}(s^t)}{\widetilde{F}_{h^u}(s^t)} = \frac{(1-\mu)(1-\rho)}{\mu} \frac{\left[\psi(s^t) h_t^s(s^t)\right]^{\nu-1}}{\left\{\left[1-\psi(s^t)\right] h_t^u(s^t)\right\}^{\alpha-1}} \left(\Xi_t\right)^{\frac{\alpha}{\nu}-1} \quad (181)$$

where $\Xi_t \equiv \rho \left[\left(A_t^k(s^t) \right)^{\nu} \left(k_t(s^{t-1}) \right)^{\nu} \right] + (1-\rho) \left(\psi(s^t) h_t^s(s^t) \right)^{\nu}$. The restrictions placed above on the parameters of the production function imply that the skill premium is decreasing in $\psi(s^t)$ and increasing in $k_t(s^{t-1})$, see Appendix.

The functional form for the relative skill supply is:

$$\widetilde{g}\left[\cdot\right] = \Psi \left[e_t \left(s^t \right) \right]^{\gamma} \quad (182)$$

where $\Psi > 0$ is the productivity of skill-acquisition; and $0 \leq \gamma < 1$ is the elasticity of the relative skill supply with respect to skill-acquisition expenditure.

Finally, we calculate the effective labour tax rate as the ratio of total tax revenues from both skilled and unskilled sources as a share of total labour income:

$$\tau_t^n\left(s^t\right) = \frac{\tau_t^s\left(s^t\right) w_t^s\left(s^t\right) \psi_t\left(s^t\right) h_t^s\left(s^t\right) + \tau_t^u\left(s^t\right) w_t^u\left(s^t\right) \left(1-\psi_t\left(s^t\right)\right) h_t^u\left(s^t\right)}{w_t^s(s^t) \psi_t(s^t) h_t^s(s^t) + w_t^u(s^t)(1-\psi_t(s^t)) h_t^u(s^t)}. \quad (183)$$

4.4.2 Exogenous policy and calibration

We next present the calibration and steady state for the exogenous policy model under both endogenous and exogenous relative skill supply. In particular, we obtain the steady state of the following decentralised competitive equilibrium (DCE):

Definition 1. Non-stochastic DCE with exogenous policy

Given initial levels of k_0 and b_0, and the five policy instruments $\{\tau_t^s, \tau_t^u, \tau_t^k, \tau_t^a, g_t^e\}$, the non-stochastic DCE system is characterized by a sequence of allocations $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^{\infty}$, prices $\{w_t^s, w_t^u, r_t, p_t\}_{t=0}^{\infty}$, and the residual policy instrument $\{b_{t+1}\}_{t=0}^{\infty}$ such that: (i) households maximise their welfare and firms maximise their profits, taking policy and prices as given; (ii) the government budget constraint is satisfied in each time period and (iii) all markets clear. Thus, imposing the market-clearing conditions (157)-(159), the non-stochastic DCE is comprised of the non-stochastic version of the first-order conditions of the household (145)-(149), the three first-order conditions of the firm (152)-(154), the government budget constraint (155) and the aggregate resource constraint (156).

4.4.3 Calibration

The non-stochastic model with exogenous policy is calibrated so that its steady state is consistent with the annual US data for 1970-2011.

Utility Table 4.1 below reports the model's quantitative parameters along with an indication of their source. Starting with the share of leisure for each skill type in utility, σ_1 and σ_2, we calibrate these to 0.35 each so that, in the steady state, the household devotes about one third of its time to labour. The relative risk aversion parameter, $\sigma_3 = -2$ is commonly employed in business cycle models.

Production The elasticities of substitution between skilled labour and capital and between unskilled labour and capital (or skilled labour) have been estimated by Krusell *et al.* (2000). Following the literature (see e.g.

Lindquist (2004), and Pourpourides (2011)), we also use these estimates, to set $a = 0.401$ and $\nu = -0.495$. The remaining parameters in the production function are calibrated to ensure the steady state predictions of the model in asset and labour markets are consistent with the data. More specifically, the unskilled labour weight in composite input share $\mu = 0.272$ is calibrated to obtain a labour share of income of approximately equal to 70% and the capital weight in composite input share, $\rho = 0.518$, is calibrated to obtain a skill premium of about 1.64. Both of these targets are consistent with the U.S. data for the period 1970-2011. The target value for the skill premium is obtained from U.S. Census data and the share of labour income in GDP is from the BEA data on personal income.[101] We also normalize the steady state values of TFP and capital equipment to unity (i.e. $A = A^k = 1$).

Depreciation and time preference The depreciation rate of capital $\delta = 0.07$ is calibrated to obtain an annual capital to output ratio of about 1.94, which is consistent with the annual data reported by the BEA on capital stocks.[102] The time discount factor, $\beta = 0.96$, is set to obtain a post-tax post-depreciation annual real rate of return on capital of roughly 4.17%, which coheres with the 4.19% obtained in the data from the World Bank.[103]

Relative skill supply To match ψ, the share of skilled workers in total population of roughly 44% in the data, we set the elasticity of relative skill supply with respect to skill-acquisition, γ, equal to 0.2334. This share is consistent with the data from the 2010 U.S. Census which indicates that 43% of the population has a college degree.[104] It also coheres with a related data set by Acemoglu and Autor (2011) which implies that the average share of the labour force with a college degree is approximately 45%. We normalise

[101] The data source is the Current Population Survey, 2011 Annual Social and Economic Supplement from the U.S. Census Bureau.

[102] Specifically, the BEA Table 1.1 on fixed-assets has been used to obtain the time series for capital stock for 1970-2011.

[103] The data refers to the annual real interest rate from World Bank Indicators database for the period 1970-2011 (i.e. FR.INR.RINR).

[104] This information is obtained from Table 4 of the Census Bureau, Survey of Income and Program Participation.

skill-acquisition productivity, Ψ to unity.

Table 4.1: Model parameters

Parameter	Value	Definition	Source
$0 < \sigma_1 < 1$	0.350	weight to skilled leisure in utility	calibration
$0 < \sigma_2 < 1$	0.350	weight to unskilled leisure in utility	calibration
$\sigma_3 < 0$	-2.000	coefficient of relative risk aversion	assumption
$\frac{1}{1-\alpha} > 0$	1.669	cap. equip. to unskilled labour elasticity	assumption
$0 < \frac{1}{1-\nu} < \frac{1}{1-\alpha}$	0.669	cap. equip. to skilled labour elasticity	assumption
$0 < 1-\mu < 1$	0.728	share of composite input to output	calibration
$0 < \rho < 1$	0.518	share of cap. equip. to composite input	calibration
$A > 0$	1.000	TFP	assumption
$A^k > 0$	1.000	capital equipment productivity	assumption
$0 \leq \delta \leq 1$	0.070	depreciation rate of capital	calibration
$0 < \beta < 1$	0.960	time discount factor	calibration
$0 \leq \gamma < 1$	0.189	relative skill supply elasticity	calibration
$\Psi > 0$	1.000	productivity of skill-acquisition	assumption
τ^k	0.310	capital income tax rate	data
τ^u	0.200	unskilled labour tax rate	data
τ^s	0.250	skilled labour tax rate	data
τ^n	0.220	effective labour tax rate	data
τ^a	0.000	skill-acquisition expenditure tax rate	assumption
$g^e > 0$	0.047	government spending	calibration

Tax rates and government spending Finally, we use the ECFIN effective capital and labour tax rates from Martinez-Mongay (2000) to obtain an average tax rate for capital and labour.[105] Therefore, we set the tax rate for capital income $\tau^k = 0.31$ and the two labour income tax rates $\tau^u = 0.20$ and $\tau^s = 0.25$.[106] Given that it is difficult to obtain data which cohere well

[105] In particular, we use the LITR and KITN rates for effective average labour and capital taxes respectively for 1970-2011, as they treat self-employed income as capital income in the calculations.

[106] Note that the calculation of the effective labour income tax rate is equal to 0.22. But since we assume that the skilled and unskilled labour income is taxed differently we

with the skill-acquisition expenditure tax rate, τ^a, we set it to zero for the exogenous policy model. We finally set the steady state value $g^e = 0.0469$, to obtain a steady state debt to output ratio, $b/Y = 53\%$, which is equal to the average debt to GDP ratio obtained in the data.[107]

Steady state The steady state of the DCE defined and calibrated above is presented in Table 4.2. The results indicate that the model's predictions for the great ratios match those implied by the data quite well. For example, in the data for 1970-2011: $\frac{k}{y} = 1.895$, $\frac{c}{y} = 0.640$, $\frac{i}{y} = 0.146$, $\frac{g^e}{y} = 0.203$ and $\frac{b}{y} = 0.530$.[108] Moreover, the share of skill acquisition expenditure in GDP, $\frac{e}{y}$, roughly coheres with US total expenditures for colleges and universities as a share of output equal to 6% for 1970-2010. This data is obtained from the U.S. National Center for Education Statistics, Digest of Education Statistics. As pointed out above, the remaining steady state variables in the exogenous model, have been calibrated to match their values in the data.

Table 4.2: Steady state of exogenous policy

$\frac{c}{y}$	$\frac{k}{y}$	$\frac{i}{y}$	$\frac{e}{y}$	$\frac{b}{y}$	$\frac{g^e}{y}$	$\frac{w^s}{w^u}$	r^{net}	ψ
0.5613	1.9444	0.1361	0.0659	0.5272	0.2367	1.6344	0.0417	0.4400

4.5 Deterministic Ramsey

The deterministic version of the Ramsey problem in (170)-(178) is summarised in Appendix D, (D7-D22) and is solved iteratively, conditional on the calibration described in the previous section. In particular, we first guess a value for Φ and solve equations (D7-D21) for an allocation $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^T$. Then we test whether equation (D22) is binding and we increase or decrease the value of Φ if the budget is in deficit or surplus respectively.

decompose the labour income tax into skilled and unskilled tax so as the weighted average of the two tax rates equals 0.22.

[107]The source of that time series is: FRED Economic Data on Gross Federal Debt as a percentage of GDP, 1970-2011.

[108]Note that if model prediction for the cost of becoming skilled, $\frac{e}{y} = 0.0659$, is added to the $\frac{c}{y}$ ratio from the model, the sum is very close to the $\frac{c}{y}$ ratio in the data.

The initial conditions for the model's state variables are given by the non-stochastic exogenous steady state (see Table 4.2). For the terminal values of the forward looking variables, we assume that after T years the dynamic system has converged to its Ramsey steady state. This implies that the appropriate terminal conditions are obtained by setting the values for these variables equal to those of the preceding period.

The final system is given by $[(5 \times T) + 1]$ equations, which is solved non-linearly using standard numeric methods (see, e.g. Garcia-Milà *et al.* (2010), Adjemian *et al.* (2011), and Angelopoulos *et al.* (2013b)). This gives the dynamic transition path from the exogenous to the optimal steady state. We set $T = 250$ to ensure that convergence is achieved. Our results show that this occurs for all endogenous variables within 150 years.[109] After we find the optimal allocation for $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^{T}$ we obtain $w_t^s = \widetilde{F}_{h^s}(t)$, $w_t^u = \widetilde{F}_{h^u}(t)$ and $r_t = \widetilde{F}_k(t)$. Additionally, we solve for τ_t^s, τ_t^u, τ_t^a, τ_t^k and τ_t^n using the non-stochastic versions of (145), (146), (147), (149) and (183) respectively.

The Ramsey steady state is reported in Table 4.3. The results are consistent with the messages from the literature initiated by Chamley (1986) on dynamic Ramsey taxation in a deterministic environment (see e.g. Ljungqvist and Sargent (2012), ch. 16 for a review of this literature). As expected, allowing the government a complete instrument set results in a zero capital tax rate in the long-run. Compared with the steady state of exogenous policy, a Ramsey government would increase capital accumulation in the steady state, by eliminating the *inter-temporal* wedge. Moreover, since skilled labour is complementing capital more than unskilled, the Ramsey government would find it optimal to encourage an increase in the relative skill supply, since a higher relative quantity of skilled labour increases the returns to, and thus the accumulation of, physical capital. This is achieved by a small subsidy to skill acquisition expenditure. The fall in the skill premium under Ramsey policy suggests that the increase in the relative skill supply has a relatively stronger quantitative impact than the increase in the capital stock. The Ramsey equilibrium also implies a mild regressivity regarding the long-run

[109] See Figure 1 below for an illustration of convergence using the tax policy instruments.

labour income taxes, revealing an incentive to encourage the labour supply of skilled hours, consistent with the discussion above. Finally, the government is able to reduce the overall burden of taxation, since it can finance part of the required public spending from accumulated assets.

Table 4.3: Steady state of optimal policy

$\frac{c}{y}$	$\frac{k}{y}$	$\frac{i}{y}$	$\frac{e}{y}$	$\frac{b}{y}$	$\frac{g^e}{y}$	$\frac{w^s}{w^u}$
0.5444	2.6413	0.1849	0.0719	-0.7212	0.1987	1.5002

τ^s	τ^u	τ^n	τ^k	τ^a	r^{net}	ψ
0.2432	0.2591	0.2475	0.0000	-0.0644	0.0417	0.4624

We further study the transition dynamics associated with Ramsey policy. Figure 4.1 illustrates the dynamic paths implied by optimal policy for the capital tax, the two labour taxes, the skill-acquisition expenditure tax and debt to output as the economy evolves from the exogenous steady state to the Ramsey steady state. The first row of Figure 4.1 shows that in period 1 skilled and unskilled labour are subsidised at rates of 30.1% and 23.7% respectively; and skill-acquisition expenditure is taxed at a rate of 47.3%. In period 2, skilled and unskilled labour taxes are 26.9% and 27.6% respectively and eventually converge to their steady state values reported in Table 4.3. Also in period 2, skill-acquisition is subsidised at a rate of 4.1% and converges to 6.4% in the steady state.

The second row of Figure 4.1 shows that in period 1, since capital already in place, capital income is taxed at a confiscatory rate (approximately 318%). In period 2, the capital tax is 0.85% and then converges slowly to zero. The high capital taxation in the first period allows the government to create a first period stock of assets of approximately the size of GDP, by lending to the household. Government assets increase in future periods, after an immediate drop in the second period due to the decrease in capital income tax, and their income is used to subsidise skill-acquisition expenditure and to compensate for the losses from foregone capital income taxation, without the need to resort to high labour income taxes. These transition paths are

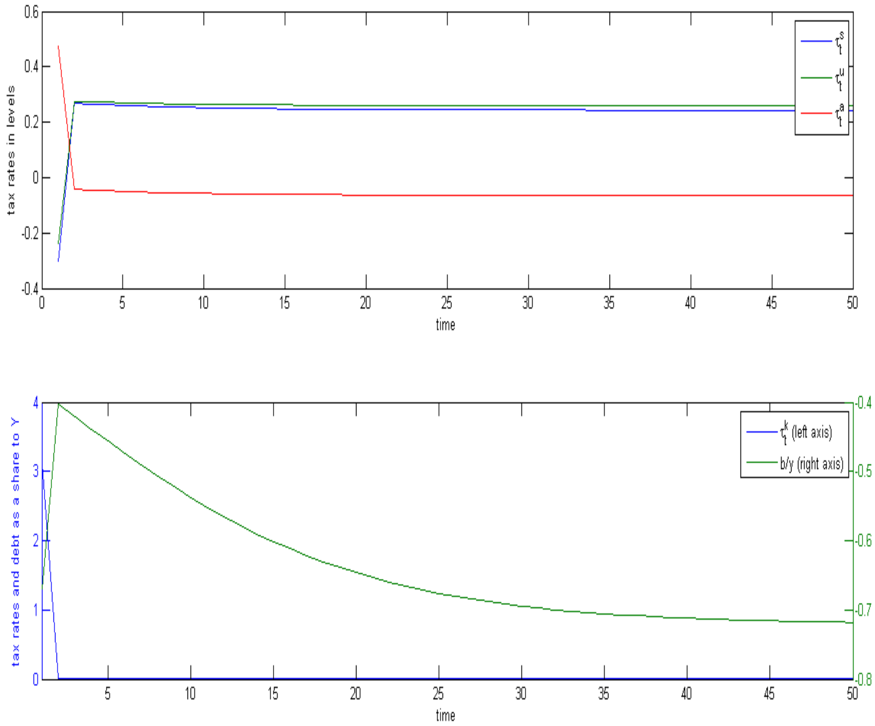

Figure 4.1: Transition paths of the policy instruments

consistent with previous research.

4.6 Stochastic processes

To move to the analysis of the stochastic Ramsey problem, we need to define the stochastic processes that drive economic fluctuations. In what follows we designate a stochastic state s^t at time t that determines exogenous shocks to both the firm's production technologies, $(A_t,\, A_t^k)$, and to government expenditures (g_t^e). Therefore, the optimal allocation of households will depend on the history of events s^t at time t. Following the literature, A_t, A_t^k and g_t^e are assumed to follow stochastic $AR(1)$ processes:

$$A_{t+1} = (1 - \rho_A) A + \rho_A A_t + \varepsilon_{t+1}^{A} \tag{184}$$

$$A_{t+1}^k = (1 - \rho_{A^k}) A^k + \rho_{A^k} A_t^k + \varepsilon_{t+1}^{A_k} \tag{185}$$

$$g_{t+1}^e = \left(1 - \rho_{g^e}\right) g^e + \rho_{g^e} g_t^e + \varepsilon_{t+1}^{g^e} \tag{186}$$

where ε_t^A, $\varepsilon_t^{A_k}$ and $\varepsilon_t^{g^e}$ are independently and identically distributed Gaussian random variables with zero means and standard deviations given respectively by σ_A, σ_{A^k} and σ_{g^e}.

The values for the $AR(1)$ coefficients and the standard deviations for the government expenditures and capital productivity exogenous processes are data based and are estimated to be: $\rho_{A^k} = 0.90$, $\rho_{g^e} = 0.70$, $\sigma_{A^k} = 0.007$ and $\sigma_g = 0.012$.[110] The autocorrelation parameter of TFP is set equal to 0.95, following Lindquist (2004) and Pourpourides (2011), while σ_A is calibrated to match the volatility of output observed in the BEA data.[111] More specifically, the standard deviation for TFP is set $\sigma_A = 0.8\%$ to obtain a volatility for output from 1970-2011 equal to 1.2%.

Table 4.4: Parameters for stochastic processes

Parameter	Value	Definition	Source
σ_A	0.008	standard deviation of TFP	calibration
ρ_A	0.950	AR(1) coefficient of TFP	data
σ_{A^k}	0.007	standard deviation of capital equipment	data
ρ_{A^k}	0.900	AR(1) coefficient of capital equipment	data
σ_{g^e}	0.012	standard deviation of public spending	data
ρ_{g^e}	0.700	AR(1) coefficient of public spending	data

[110] The government spending series refers to government consumption expenditures and gross investment from NIPA Table 1.1.5 (1970-2011). The capital series refers to productive capital stock and is from the Bureau of Labour Statistics Table 4.1 (1988-2011). Note that there is no data available prior to 1988 for the productivity of capital. To calculate the statistical properties of the cyclical component of the series, we take logs and apply the HP-filter with smoothing parameter equal to 6.25.

[111] The time series for GDP from 1970-2011 is obtained from NIPA Table 1.1.5. Cyclical output is again calculated using the HP-filter as above.

4.7 Stochastic Ramsey

We next approximate the dynamic equilibrium paths due to three exogenous shocks using first-order accurate decision rules of the equilibrium conditions under optimal policy in (170)-(174), around the optimal deterministic steady-state of these conditions described above.[112] As is common in the literature when characterizing policy dynamics, we also make the auxiliary assumption that the initial state of the economy at $t = 0$ is the steady state under optimal policy.

As is well known (see e.g. Zhu (1992), Chari *et al.* (1994) and Ljungqvist and Sargent (2012)), the Ramsey problem with state-contingent debt cannot uniquely pin down the capital tax rate. Hence, we follow the literature and calculate the optimal *ex-ante* capital income tax rate (see Appendix D for details):

$$\overline{\tau}^k_{t+1}(s^t) = \frac{\beta E_t u_c(s^{t+1})\left[\widetilde{F}_k(s^{t+1}) + 1 - \delta\right] - u_c(s^t)}{\beta E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1})}. \tag{187}$$

Alternatively, by assuming that government debt is not state-contingent, we can calculate the *ex post* state contingent capital tax (see Appendix D for the derivation):

$$\begin{aligned}\widetilde{\tau}^k_t(s^t) = \left(\tfrac{1}{r_t(s^t)k_t(s^{t-1})}\right)\{g_t(s^t) - \tau^a_t\left(s^t\right) g\left[\psi_t(s^t)\right] - \tfrac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} + b_t\left(s^{t-1}\right) - \\ - \tau^s(s^t)w^s_t(s^t)\psi_t\left(s^t\right) h^s_t(s^t) - \tau^u(s^t)w^u_t(s^t)\left[1 - \psi_t\left(s^t\right)\right] h^u_t(s^t)\}\end{aligned} \tag{188}$$

where $\overline{R}_t\left(s_t\right)$ is the state uncontingent return to holding government debt. Alternatively, assuming the government employs a state-contingent tax on income from government bonds, we can calculate the private assets tax, $\xi\left(s^{t+1}|s^t\right)$ that applies to taxing jointly the income from assets as (see Ap-

[112] We use the perturbation methods in Schmitt-Grohé and Uribe (2003) to solve the dynamic model.

pendix D for the derivation):

$$\xi_t\left(s^{t+1}|s^t\right)=\left(\frac{1}{F_k(s^{t+1})k_{t+1}(s^t)+b_{t+1}(s^t)}\right)\times\{g_{t+1}(s^{t+1})+b_{t+1}\left(s^t\right)-\frac{b_{t+2}\left(s^{t+1}\right)}{\overline{R}_{t+1}(s^{t+1})}-$$
$$-\tau^s(s^{t+1})w^s_{t+1}(s^{t+1})\psi_{t+1}\left(s^{t+1}\right)h^s_{t+1}(s^{t+1})-\tau^u(s^{t+1})w^u_{t+1}(s^{t+1})\times$$
$$\times\left[1-\psi_{t+1}\left(s^{t+1}\right)\right]h^u_{t+1}(s^{t+1})-\tau^a_{t+1}\left(s^{t+1}\right)g\left[\psi_{t+1}(s^{t+1})\right]\}. \tag{189}$$

To calculate the business cycle statistics of the relevant quantities of the model under optimal policy, we conduct simulations by shocking all of the exogenous processes, obtain the required moments for each simulation and then calculate their mean value across the simulations. We undertake 1000 simulations, each 242 periods long and drop the first 200 periods to ensure that the initial conditions do not affect the results. We retain 42 periods in our analysis to match the number of years between 1970 and 2011 used in the calibration.

4.8 Cyclical properties

We first present the results regarding the key second moments of the stochastic optimal policy problems, under different scenaria regarding the policy set available to the government and an endogenous or exogenous relative skill supply. This is followed by an impulse response analysis, which allows to investigate the channels through which tax policy works over the business cycle.

4.8.1 Second moments

We start with the cyclical properties of Ramsey taxation under endogenous relative skill supply. The results on standard deviations and correlations with output, for the endogenous variables of the model as well as the various tax rates that were explained above are summarised in Table 4.5. The results regarding optimal taxation are largely consistent with the literature and thus extend previous findings to a setup with capital-skill complementarity and endogenous skill supply.

Table 4.5: Stochastic results

	endogenous ψ			exogenous ψ		
x_i	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$
y	0.2358	0.0226	1	0.2358	0.0213	1
c	0.1283	0.0254	0.9777	0.1283	0.0262	0.9778
k	0.6226	0.0201	0.6021	0.6227	0.0185	0.5941
h^s	0.3797	0.0021	0.3464	0.3797	0.0018	0.3933
h^u	0.1825	0.0099	-0.5724	0.1825	0.0092	-0.5563
ψ	0.4623	0.0034	0.9575	0.4623	0.0000	0.0000
$\frac{w^s}{w^u}$	1.5004	0.0041	-0.9740	1.5000	0.0020	0.2942
τ^s	0.2432	0.0007	0.4642	0.2433	0.0071	-0.9309
τ^u	0.2591	0.0009	-0.3662	0.2591	0.0033	-0.9278
$\psi(1-\tau^s)w^s$	0.2414	0.0257	0.9831	0.2414	0.0266	0.9819
$(1-\psi)(1-\tau^u)w^u$	0.1831	0.0237	0.9878	0.1831	0.0247	0.9874
τ^n	0.2475	0.0005	0.1022	0.2476	0.0061	-0.9350
τ^a	-0.0645	0.0169	0.0951	-0.0645	0.0000	0.0000
$\overline{\tau}^k$	-1.8e-5	0.0010	0.5765	-3.6e-5	0.0008	0.6178
$\widetilde{\tau}^k$	0.0142	0.1330	-0.2085	0.0152	0.1393	-0.2238
ξ	-0.0125	0.0855	0.2015	-0.0149	0.0943	0.2150

In particular, the *ex ante* tax rate on capital is effectively zero and is around zero for every period. Moreover, when debt is not allowed to be state-contingent, the state contingent private assets and *ex post* capital taxes are near zero, have low correlations with output and are the most volatile of the tax instruments. These results are similar to findings in the literature to date. Also consistent with the labour tax-smoothing results in the literature, both labour taxes have very low standard deviations relative to output, as the government finds it optimal to minimise the distortions introduced by labour taxes over the business cycle by keeping them relatively smooth and letting the remaining state-contingent policy instruments respond to exogenous shocks. However, they exhibit different correlations with output. The tax rate on skilled labour income is pro-cyclical, whereas the tax rate on unskilled labour income is mildly counter-cyclical. The skill-acquisition tax is the least smooth of the tax instruments when debt is state-contingent and fluctuates nearly as much as output. Moreover, it is mildly pro-cyclical.

Finally, the labour income taxes and the *ex ante* capital income tax in this model inherit the properties of the exogenous processes. As can be seen in Table 4.6, the autocorrelations of these instruments follow the autocorrelations of the exogenous processes, so that when shocks are autocorrelated as in Table 4.4, so are the tax rates. However, if we assume that the shocks follow *iid* processes, the autocorrelation of the tax rates generally becomes very small.[113] On the contrary, the autocorrelations of the *ex post* capital tax and of the private assets tax do not follow the autocorrelations of the exogenous processes. This is again similar to previous findings.

Table 4.6: Autocorrelations

	autocorrelated shocks		*iid* shocks	
	endogenous ψ	exogenous ψ	endogenous ψ	exogenous ψ
τ^s	0.7848	0.9054	-0.0290	0.0778
τ^u	0.9231	0.9213	-0.0068	0.7635
τ^n	0.7738	0.9114	-0.0630	0.1658
τ^a	0.8048	1.0000	0.0462	1.0000
$\overline{\tau}^k$	0.7353	0.7441	-0.0436	-0.0388
$\widetilde{\tau}^k$	-0.1621	-0.1604	-0.5029	-0.4999
ξ	-0.1744	-0.1602	-0.5062	-0.5013

We next examine how the prescriptions for optimal policy are affected by a friction in the labour market that does not permit changes in the relative skill supply over the business cycle. As discussed in the introduction, this restriction is empirically relevant.[114] This is also consistent with the literature on optimal policy under heterogeneity in labour productivity (see e.g.

[113]Note that under exogenous ψ, τ^a is constant since skill acquisition expenditure is constant. Thus τ^a has a unit AR(1) parameter for both the autocorrelated and *iid* cases.

[114]Note that the benchmark model developed earlier does not capture this empirical feature when relative skill supply is endogenously chosen. In particular, when the model is simulated under the exogenous processes in Section 4.4, it produces an HP filtered series for $\psi_t(s^t)$, which has a correlation with similarly detrended output of about 60% and a relative-to-output standard deviation of 50%.

Werning (2007) for a review of this literature), which typically assumes that the distribution of labour productivity is exogenously determined.

To analyse the effects of a fixed relative skill supply over the business cycle, we obtain the first-order conditions for optimal policy incorporating this rigidity and then approximate these conditions around the Ramsey deterministic steady state with endogenous $\psi_t(s^t)$ in Table 4.3. The latter avoids approximating around the steady state in which the relative skill supply is restricted over both the short- and long-run. Thus, we set $\psi_t(s^t)$, for each possible history s^t, to be equal to the steady state value from the deterministic Ramsey problem with endogenous $\psi_t(s^t)$ in Table 4.3. This also means that skill-acquisition expenditure $e_t(s^t)$ and the respective tax rate $\tau_t^a(s^t)$ are also set to their respective values in Table 4.3.[115] The results pertaining to the business cycle properties of the economy under optimal policy in this case are presented in the last three columns of Table 4.5.

These results first suggest that the properties of asset taxation do not change. However, there are important differences regarding labour income taxation. In particular, the two labour income taxes become quantitatively more volatile, so that the effective labour income tax rate, $\tau_t^n(s^t)$, is about twelve times more volatile. Also note that the labour tax volatility increases asymmetrically, so that $\tau_t^s(s^t)$ is about two-and-a-half times more volatile than $\tau_t^u(s^t)$. Finally, both labour taxes become strongly counter-cyclical. Thus, under capital-skill complementarity, imposing the restriction that the relative skill supply does not change over the business cycle has important implications for the business cycle properties of labour income taxation.

The key to understanding these changes is to note that the rigidity of $\psi_t(s^t)$ over the business cycle creates a distortion in the labour markets that is reflected in the difference between the *a-temporal* first-order conditions of the household given by (145) and (146) and the corresponding conditions when the relative skill supply is fixed, $\overline{\psi}$. This distortion drives a wedge

[115] Note that $\tau_t^a(s^t)$ needs to remain constant if skill-acquisition expenditure remains constant. Otherwise it would turn to a lump-sum tax, which is ruled out in Ramsey second-best analysis. Moreover, if skill-acquisition expenditure remains constant, there is no margin in the household decision for $\tau_t^a(s^t)$ to affect. Thus there is no reason why it should change.

between the average net returns to labour supply in the perfect and imperfect labour markets, or, alternatively, a wedge between the marginal rates of substitution between leisure and consumption. Thus these wedges for skilled and unskilled workers respectively can be defined as follow:[116]

$$lw_t^s\left(s^t\right)=\left[1-\tau_t^s\left(s^t\right)\right]\psi_t\left(s^t\right)w_t^s\left(s^t\right)-\left[1-\widehat{\tau}_t^s\left(s^t\right)\right]\overline{\psi}\widehat{w}_t^s\left(s^t\right) \tag{190}$$
$$lw_t^u\left(s^t\right)=\left[1-\tau_t^u\left(s^t\right)\right]\left[1-\psi_t\left(s^t\right)\right]w_t^u\left(s^t\right)-\left[1-\widehat{\tau}_t^u\left(s^t\right)\right]\left(1-\overline{\psi}\right)\widehat{w}_t^u\left(s^t\right)$$

where hatted variables denote the distorted case.

Our results make clear that the government wishes to minimise these wedges over the business cycle and this is achieved by setting $\widehat{\tau}_t^s\left(s^t\right)$ and $\widehat{\tau}_t^u\left(s^t\right)$ so that paths for the skilled and unskilled average net return under the market distortion are as close as possible to the paths of the corresponding quantities without the market distortion. Table 4.5 clearly shows that second moments of these returns are very similar and this is further confirmed when we examine the impulse responses below. On the contrary, the *inter-temporal* margins are not directly affected by the rigidity in the relative skill supply. Hence, the optimal policies regarding asset taxation are not qualitatively different between the two models.

4.8.2 Impulse responses

To further explain the previous results and examine the optimal response of taxation to changes in exogenous productivity and government spending, we plot the impulse responses of key endogenous variables after a temporary 1% shock to the exogenous distributions in ε_t^A, $\varepsilon_t^{A_k}$ and $\varepsilon_t^{g^e}$. These plots are shown in Figures 4.2-4.4 below.

[116]Given that in both models considered here (i.e. with flexible and rigid relative skill supply) it is assumed that the government needs to resort to distortionary taxation, the first-best cannot be achieved in either case. Hence, the best that the government can do in the presence of the market friction in the form of rigid relative skill supply is to achieve the second-best allocations in the labour markets, represented by Ramsey taxation under flexible relative skill supply in (145) and (146). This is reflected in the definition of the labour wedge created by the rigidity in the relative skill supply in (190).

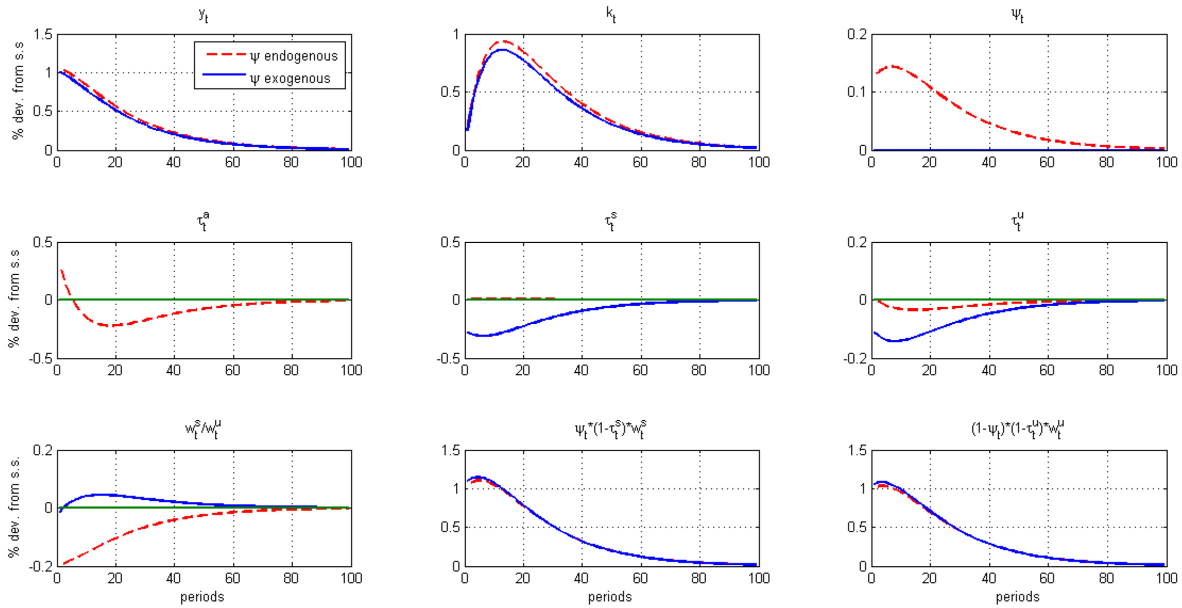

Figure 4.2: Impulse responses to 1% temporary shock to TFP

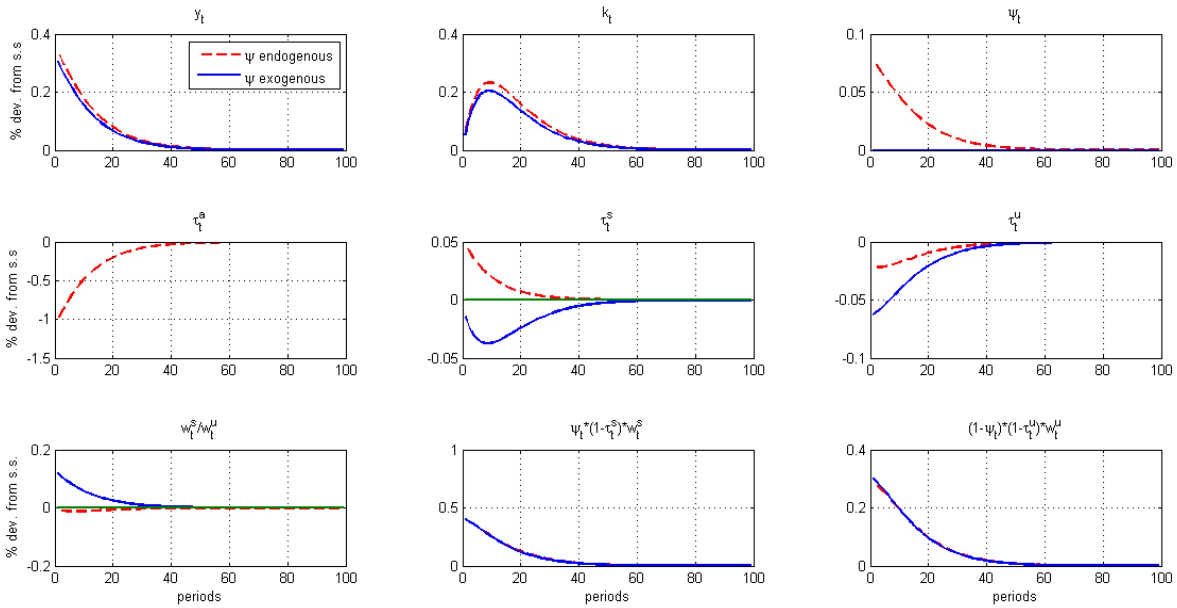

Figure 4.3: Impulse responses to 1% temporary shock to capital equipment productivity

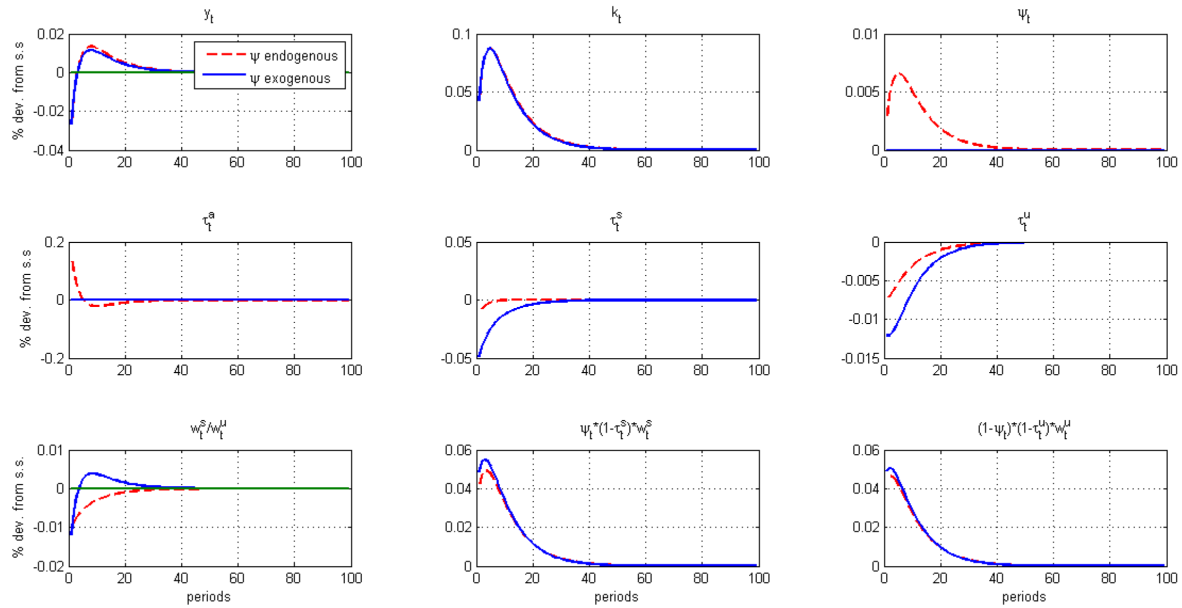

Figure 4.4: Impulse responses to 1% temporary shock to government spending

After a positive TFP or capital equipment shock (see Figures 4.2 and 4.3), the capital stock, k_t, increases, since the productivity of capital increases. As shown earlier (see Appendix), this tends to increase the returns to skilled hours more than the return to unskilled, given capital-skill complementarity. In the flexible labour markets model, the increase in the returns to skilled labour also leads to an increase in the relative supply of skill, ψ_t, which, other things equal, tends to decrease the skill premium (see Appendix). These two forces, on balance, lead to a decrease in the skill premium, $\frac{w_t^s}{w_t^u}$ shown in the Figures. The government finds it optimal to respond to these shocks by keeping the labour income taxes (τ_t^s and τ_t^u) relatively smooth , consistent with the tax smoothing literature. Optimal policy also encourages the accumulation of skill by decreasing τ_t^a.[117]

Under the relative skill supply restriction, the increase in the capital stock cannot be followed by an increase in ψ_t (see again Figures 4.2 and 4.3). Therefore, the returns to skilled and unskilled labour, w_t^s and w_t^u, respectively, now follow different paths, summarised by the increase in the skill premium. *Ceteris paribus*, this drives a wedge between the average net returns **to** skilled and unskilled labour hours under the restricted model, relative to those from the flexible labour markets model. To minimise the effects of the relative labour supply distortion, by keeping the marginal rates of substitution between leisure and consumption for the two types of labour at roughly the same levels as under a fully flexible labour market, the government adjusts the optimal response of the labour income taxes, as can be seen in the plots for these returns. Indeed, the response becomes more counter-cyclical, to smooth the response of average net returns to skilled and unskilled labour, so that these last two quantities exhibit, post shock, effectively identical responses with their corresponding quantities in the flexible labour market. Note also that the change in τ_t^s is larger than τ_t^u, since, given capital-skill complementarity, w_t^s is affected more by the increase in the capital stock than w_t^u. Thus a larger adjustment in policy is required.

[117]However, note that the smoothness of the labour income taxes is not due to the skill-acquisition subsidy, since a version of the model where $\tau_t^a(s^t)$ is fixed over the business cycle provides very similar second moments and impulse responses. These results are not presented here to save on space but are available on request.

A temporary reduction in government spending in Figure 4.4, does not have direct productivity effects in these models. However, it allows the government to temporarily reduce the tax burden on labour income and thus encourage labour supply. In the model with endogenous relative skill supply, a small reduction in τ_t^s increases the average net return to skilled labour both directly and indirectly, via the induced increase in ψ_t. The latter happens because the increase in skilled labour raises the return to capital as well and thus the returns to investing into skill-acquisition. On the contrary, under the restricted relative skill supply assumption, the indirect effect is missing and thus τ_t^s needs to be increased by more, to maintain the same average net return to skilled labour hours. The unskilled labour supply does not affect capital accumulation as much (given capital-skill complementarity). Hence it does not need to be changed by as much under endogenous relative skill supply. In turn, this implies that no big changes are required in the optimal response to τ_t^u when relative skill supply is fixed, to maintain the same average net return to unskilled labour hours.

4.9 Imposing a budget rule

Table 4.7 presents the steady state results of optimal fiscal policy under a budget rule. In particular, we assume that the government is required to keep a fixed debt to output ratio when choosing optimal taxation. This is an empirically relevant restriction since in recent years most advanced economies have had debt targets to respect. The Ramsey steady state with a fixed share of debt to output is reported in Table 4.7 along with the steady state results from our benchmark model with flexible debt.

Table 4.7: Steady state of optimal policy with fixed debt to output ratio

	$\frac{c}{y}$	$\frac{k}{y}$	$\frac{i}{y}$	$\frac{e}{y}$	$\frac{b}{y}$	$\frac{g^e}{y}$	$\frac{w^s}{w^u}$
Benchmark	0.5444	2.6413	0.1849	0.0719	-0.7212	0.1987	1.5002
Fixed b/Y	0.5418	2.6356	0.1845	0.0702	0.5272	0.2036	1.5190

	τ^s	τ^u	τ^n	τ^k	τ^a	r^{net}	ψ
Benchmark	0.2432	0.2591	0.2475	0.0000	-0.0644	0.0417	0.4624
Fixed b/Y	0.2688	0.2820	0.2724	0.0000	-0.0394	0.0417	0.4581

The results are similar with the benchmark model under flexible debt. The main difference is that under a budget rule the labour taxes are higher and the subsidy to skill acquisition expenditure is lower, in order for the government to finance a given level of government expenditures without the use of debt. These results are consistent with Stockman (2001), although he imposes a fixed level of debt instead of a fixed debt to output ratio.

We further examine the transition dynamics associated with Ramsey policy under a budget rule. Figure 4.2 presents the dynamic paths of the fiscal policy instruments implied by optimal policy as the economy evolves from the exogenous steady state to the Ramsey steady state with a fixed debt to output ratio. Figure 4.2 shows that under a budget rule, in period 1 skilled and unskilled labour are subsidised at rates of 14.1% and 10.4% respectively; and skill-acquisition expenditure is taxed at a rate of 36.4%. Therefore, the labour income subsidy in the initial period is almost half under a fixed debt to output ratio compared with the flexible debt case study presented earlier. In the second period, skilled and unskilled labour taxes are 29.3% and 29.7% respectively and eventually converge to their long-run values. Moreover, in period 2, skill-acquisition is subsidised at a rate of 2.6% and converges slowly to its steady state. As a result, the subsidy to skill acquisition is lower under the budget rule and it exhibits smaller fluctuations in the first two periods. The capital income tax in the first period reaches again a confiscatory rate (approximately 278%). However, in period 2, the capital income tax is subsidised in this case at 0.28% and then converges slowly to zero.

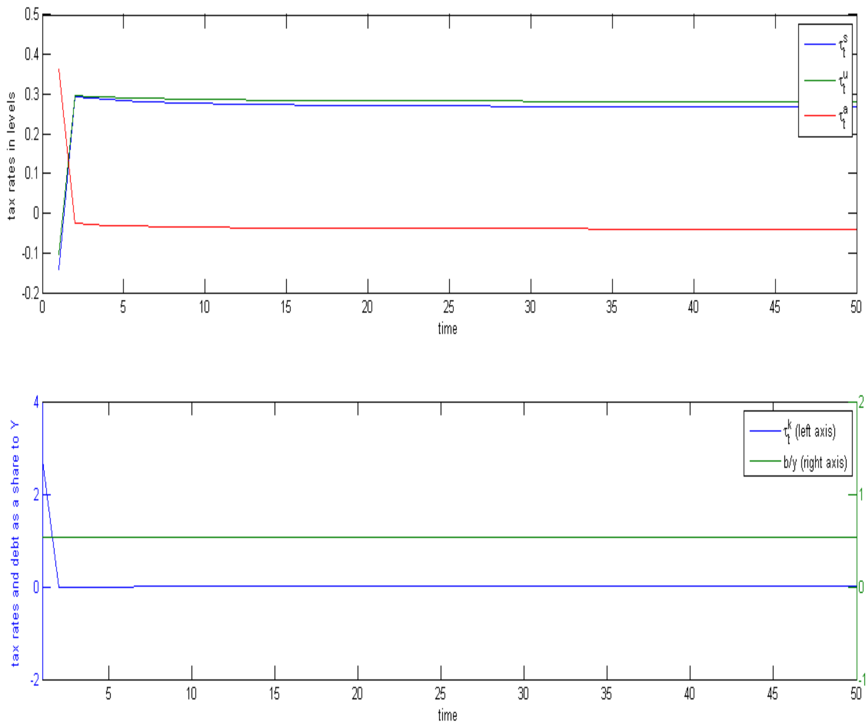

Figure 4.5: Transition paths of policy instruments under a fixed debt to output ratio

4.9.1 Cyclical properties under the budget rule

Following the same approach as before, we initially present the results regarding the key second moments of the stochastic optimal policy with a fixed debt to output ratio, under two different cases: endogenous and exogenous relative skill supply. Table 4.8 presents the cyclical properties of Ramsey taxation under endogenous and under exogenous relative skill supply and Table 4.9 presents the autocorrelation of the policy instruments.

In particular, we find that the labour income taxes are higher and the skill-acquisition subsidy is lower under the budget rule compared to the flexible debt regime on average over the business cycle. This is expected because under the budget rule and the zero optimal capital income tax, the government is not able to borrow to balance the budget and it needs to satisfy its expenditures through labour income tax revenues. The capital income and private asset taxes in this case are again close to zero but larger in absolute terms compared to the flexible debt regime. Moreover, both labour taxes are again smooth over the business cycle and counter-cyclical to output. The skill-acquisition and capital income tax are the least smooth of the tax instruments. These results cohere well with the related literature (see e.g. Chari *et al.* (1994) and Stockman (2001)).

Moreover, apart from the labour income taxes, we find that under the budget rule the capital income tax and the private asset tax also inherit the properties of the exogenous processes. Thus, the autocorrelations of these instruments follow the autocorrelations of the exogenous processes and under *iid* processes of the shocks the autocorrelation of the policy instruments generally become very small.

From Table 4.8 we can see again that even under the budget rule the government wants to minimise the labour wedges over the business cycle by setting the paths for the skilled and unskilled average net return under the market distortion as close as possible to the paths of the corresponding quantities without the market distortion. Therefore, the second moments of these returns are very similar under flexible and fixed relative skill supply.

Table 4.8: Stochastic results under fixed debt to output ratio

	endogenous ψ			exogenous ψ		
x_i	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$
y	0.2302	0.0220	1	0.2302	0.0206	1
c	0.1247	0.0255	0.9718	0.1247	0.0262	0.9725
k	0.6067	0.0177	0.6112	0.6067	0.0162	0.6044
h^s	0.3725	0.0023	0.1536	0.3725	0.0020	0.1381
h^u	0.1795	0.0106	-0.5581	0.1795	0.0098	-0.5456
ψ	0.4581	0.0033	0.9601	0.4581	0.0000	0.0000
$\frac{w^s}{w^u}$	1.5192	0.0042	-0.9726	1.5188	0.0018	0.2620
τ^s	0.2688	0.0005	-0.6211	0.2689	0.0061	-0.9569
τ^u	0.2820	0.0009	-0.7595	0.2820	0.0031	-0.9654
$\psi(1-\tau^s)w^s$	0.2318	0.0255	0.9803	0.2318	0.0262	0.9792
$(1-\psi)(1-\tau^u)w^u$	0.1773	0.0238	0.9841	0.1773	0.0246	0.9839
τ^n	0.2724	0.0006	-0.8611	0.2725	0.0053	-0.9627
τ^a	-0.0395	0.0193	-0.2436	-0.0395	0.0000	0.0000
τ^k	0.0003	0.0137	-0.8388	0.0004	0.0101	-0.8396
ξ	0.0386	0.0110	-0.6631	0.0307	0.0102	-0.8488

Focusing on the effects of a fixed relative skill supply over the business cycle with given skill-acquisition expenditure $e_t(s^t)$ and respective tax rate $\tau_t^a(s^t)$ on optimal taxation under the budget rule, we find that the properties of optimal asset taxes do not change. However, as in the case with flexible debt, there are important differences regarding labour income taxation. Specifically, the two labour income taxes become quantitatively more volatile, so that the effective labour income tax rate, $\tau_t^n(s^t)$, is now about nine times more volatile. In addition, under the budget rule we again obtain the asymmetric increase in labour tax volatility. The $\tau_t^s(s^t)$ is about two times more volatile than $\tau_t^u(s^t)$. Therefore, the fixed relative skill supply has significant implications of the business cycle properties of labour income taxation even under a budget rule.

Table 4.9: Autocorrelations under fixed debt to output ratio

	autocorrelated shocks		*iid* shocks	
	endogenous ψ	exogenous ψ	endogenous ψ	exogenous ψ
τ^s	0.3814	0.9114	-0.0388	0.0166
τ^u	0.6866	0.8519	-0.0274	-0.0139
τ^n	0.5083	0.9066	-0.0340	0.0092
τ^a	0.7987	1.0000	-0.0581	1.0000
τ^k	0.8897	0.8743	-0.0522	-0.0439
ξ	0.5475	0.5810	-0.2513	-0.3934

4.10 Conclusions

Motivated by the empirical relevance of the wage-skill premium and the roles played by capital-skill complementarity, the relative supply of skilled labour and capital augmenting technical change, this chapter contributed to the tax smoothing literature by undertaking a normative investigation of the quantitative properties of optimal taxation of capital and labour income, as well as skill-acquisition expenditure, under flexible or fixed debt to output ration, in the presence of aggregate shocks to total factor productivity (TFP), capital equipment productivity and government spending.

Our main finding was that under capital-skill complementarity, a friction that did not allow the relative supply of skill to adjust in response to aggregate shocks, significantly changed the cyclical properties of optimal labour taxes. In particular, we first showed that under endogenous relative skill supply, the optimal labour taxes for both skilled and unskilled labour income were very smooth, with the volatility of the skilled income tax being marginally higher. We also found that the skilled tax moves pro-cyclically with output and the unskilled tax was mildly counter-cyclical. These results were largely consistent with the literature and extended previous findings to a setup with capital-skill complementarity and endogenous skill supply.

We also found that, when the relative skill supply was constrained to remain constant over the business cycle, the prescriptions for optimal policy markedly changed. In particular, we found that the volatility of taxes increased significantly, so that the standard deviation of the effective average labour income tax was about twelve times higher than the perfect labour markets case, while the volatility of the skilled labour income tax was about two-and-a-half times higher than that of the unskilled labour income tax. Moreover, both taxes became strongly counter-cyclical. We further showed that the key to explaining these changes was that the government found it optimal to adjust labour income tax rates to alter the average net returns to skilled and unskilled labour hours.

Our results further showed that the skill heterogeneity considered, irrespective of the presence of the labour market friction, did not affect the results obtained in the literature regarding the cyclical behaviour of asset taxes. In particular, the *ex ante* tax rate on capital was around zero for every period, the state contingent private assets and *ex post* capital taxes were near zero and are the most volatile of the non-asset tax instruments. We also found that the skill-acquisition tax was the least smooth of the non-asset tax instruments when debt was state-contingent and fluctuated nearly as much as output. Irrespective of the model variant examined, all of the policy instruments, except the *ex post* capital tax and the private assets tax inherited the persistence properties of the shocks.

Finally, we found that the introduction of the budget rule doesn't affect our key results. However, under a fixed debt to output ratio all fiscal instruments are counter-cyclical and inherit the persistence properties of the shocks.

Appendix D

D Chapter 4

D.1 Household's first-order conditions

The household's first-order conditions for consumption, skilled labour supply, unskilled labour supply, debt, capital and the relative skill supply are given respectively by the following relations:

$$u_c(s^t) = \lambda_t\left(s^t\right) \tag{D1}$$

$$u_{h^s}(s^t) = -\lambda_t(s^t)\psi_t(s^t)\left\{\left[1-\tau_t^s(s^t)\right]w_t^s\left(s^t\right)\right\} \tag{D2}$$

$$u_{h^u}(s^t) = -\lambda_t(s^t)\left[1-\psi_t(s^t)\right]\left[1-\tau_t^u(s^t)\right]w_t^u\left(s^t\right) \tag{D3}$$

$$\pi_t\left(s^t\right)\lambda_t(s^t)p_t\left(s_{t+1}\mid s^t\right) = \beta\pi_{t+1}\left(s^{t+1}\right)\lambda_{t+1}(s^{t+1}) \tag{D4}$$

$$\begin{aligned}\pi_t\left(s^t\right)\lambda_t(s^t) = \beta\sum_{s_{t+1}}\{&\pi_{t+1}\left(s^{t+1}\right)\lambda_{t+1}(s^{t+1})\times\\ \times&\left[r_{t+1}\left(s^{t+1}\right)\left[1-\tau_{t+1}^k\left(s^{t+1}\right)\right]+(1-\delta)\right]\}\end{aligned} \tag{D5}$$

$$\begin{aligned}u_\psi(s^t) = -\lambda_t(s^t)\{&h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right]w_t^s\left(s^t\right)-h_t^u\left(s^t\right)\times\\ \times&\left[1-\tau_t^u\left(s^t\right)\right]w_t^u\left(s^t\right)-\left[1+\tau_t^a\left(s^t\right)\right]\left[g_\psi\left(s^t\right)\right]\}.\end{aligned} \tag{D6}$$

D.2 Deterministic Ramsey system

In a non-stochastic environment, the first-order conditions derived in (170)-(178) of the main text become:

- for $t=0$:

$$V_{h^s}(0) = -\left[V_c(0)-\Phi A_c\right]\widetilde{F}_{h^s}(0)+\Phi A_{h^s} \tag{D7}$$

$$V_{h^u}(0) = -\left[V_c(0)-\Phi A_c\right]\widetilde{F}_{h^u}(0)+\Phi A_{h^u} \tag{D8}$$

$$V_\psi(0) = \left[V_c(0)-\Phi A_c\right]\left[g_\psi(0)\right]+\Phi A_\psi \tag{D9}$$

$$V_c(0) = \beta V_c(1)\left[\widetilde{F}_k(1) + 1 - \delta\right] + \Phi A_c \tag{D10}$$

$$\widetilde{F}[\cdot(0)] = c_0 + g^e + g(\psi_0) + k_1 - (1-\delta)k_0 \tag{D11}$$

- for $t = 1, 2, 3 ... T-1$:

$$V_{h^s}(t) = -V_c(t)\,\widetilde{F}_{h^s}(t) \tag{D12}$$

$$V_{h^u}(t) = -V_c(t)\,\widetilde{F}_{h^u}(t) \tag{D13}$$

$$V_\psi(t) = V_c(t)\,[g_\psi(t)] \tag{D14}$$

$$V_c(t) = \beta V_c(t+1)\left[\widetilde{F}_k(t+1) + 1 - \delta\right] \tag{D15}$$

$$\widetilde{F}[\cdot(t)] = c_t + g^e + g(\psi_t) + k_{t+1} - (1-\delta)k_t \tag{D16}$$

- for $t = T$:

$$V_{h^s}(T) = -V_c(T)\,\widetilde{F}_{h^s}(T) \tag{D17}$$

$$V_{h^u}(T) = -V_c(T)\,\widetilde{F}_{h^u}(T) \tag{D18}$$

$$V_\psi(T) = V_c(T)\,[g_\psi(T)] \tag{D19}$$

$$1 = \beta\left[\widetilde{F}_k(T) + 1 - \delta\right] \tag{D20}$$

$$\widetilde{F}[\cdot(T)] = c_T + g^e + g(\psi_T) + \delta k_T \tag{D21}$$

- lifetime implementability constraint:

$$\sum_{t=0}^{T}\beta^t[u_c(t)\,c_t + u_{h^s}h_t^s + u_{h^u}h_t^u + \Omega_t] - A = 0 \tag{D22}$$

where $A = u_c(0)\left\{b_0 + \left[(1-\tau_0^k)\widetilde{F}_k(0) + (1-\delta)\right]k_0\right\}$, the Lagrange multiplier θ_t has been replaced with $V_c(t)$ using (170) and (175) in the main text and the notation $X(t)$ denotes the time period t quantity of X.

D.3 The effects of k_t and ψ_t on the skill premium

Differentiating the skill premium, given by (181) in the main text,with respect to k_t we have (note that we do not use the s^t notation to keep the presentation simpler):

$$\frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial k_t} = \{A_t^k \nu\rho\left(A_t^k k_t\right)^{\nu-1}\left(\psi_t h_t^s\right)^{\nu-1}\left[(1-\rho)\left(\psi_t h_t^s\right)^{\nu}+\rho\left(A_t^k k_t\right)^{\nu}\right]^{\frac{a}{\nu}-2}\times$$
$$\times\left(\left(1-\psi_t\right)h_t^u\right)^{1-\alpha}(1-\mu)(1-\rho)\left(\frac{a}{\nu}-1\right)\}\div\mu$$

This is positive if $a > \nu, a, \nu < 1, 0 < \rho, \mu < 1$.

Differentiating (181) with respect to ψ_t gives:

$$\frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial \psi_t} = -h_t^u\left(\psi_t h_t^s\right)^{\nu}(1-\mu)(1-\rho)\left[(1-\rho)\left(\psi_t h_t^s\right)^{\nu}+\rho\left(A_t^k k_t\right)^{\nu}\right]^{\frac{a}{\nu}-2}\times$$
$$\times\left[\begin{array}{c}(1-\rho)\left(\psi_t h_t^s\right)^{\nu}+\rho\left[A_t^k k_t\right]^{\nu}-a(\psi_t h_t^s)^{\nu}-\\ -\nu\rho\left(A_t^k k_t\right)^{\nu}+a\rho(\psi_t h_t^s)^{\nu}+\\ +\nu\psi_t\rho\left(A_t^k k_t\right)^{\nu}-a\psi_t\rho\left(A_t^k k_t\right)^{\nu}\end{array}\right]\div$$
$$\div\left[\mu h_t^s\psi_t^2\left(\left(1-\psi_t\right)h_t^u\right)^{\alpha}\right]$$

Focusing on the term in the squared brackets we need to test if that term is positive. If this is true then the above derivative is negative due to the negative sign at the beginning of the right hand side.

$$\left[\begin{array}{c}(1-\rho)\left(\psi_t h_t^s\right)^{\nu}+\rho\left[A_t^k k_t\right]^{\nu}-a(\psi_t h_t^s)^{\nu}-\\ -\nu\rho\left(A_t^k k_t\right)^{\nu}+a\rho(\psi_t h_t^s)^{\nu}+\\ +\nu\psi_t\rho\left(A_t^k k_t\right)^{\nu}-a\psi_t\rho\left(A_t^k k_t\right)^{\nu}\end{array}\right]$$

having in the first row the terms with labour and in the rest the terms with capital:

$$\left[\begin{array}{c}(1-\rho)\left(\psi_t h_t^s\right)^{\nu}+a\rho(\psi_t h_t^s)^{\nu}-a(\psi_t h_t^s)^{\nu}-\\ -\nu\rho\left(A_t^k k_t\right)^{\nu}+\rho\left[A_t^k k_t\right]^{\nu}+\\ +\nu\psi_t\rho\left(A_t^k k_t\right)^{\nu}-a\psi_t\rho\left(A_t^k k_t\right)^{\nu}\end{array}\right]$$

$$\left[\begin{array}{c} (\psi_t h_t^s)^\nu [1-\rho+a\rho-a]+ \\ +\left(A_t^k k_t\right)^\nu [\rho-\nu\rho]+ \\ +\psi_t \left(A_t^k k_t\right)^\nu [\rho\nu-a\rho] \end{array}\right]$$

$$\left[\begin{array}{c} (\psi_t h_t^s)^\nu [(1-\rho)(1-a)]+ \\ +\rho\left(A_t^k k_t\right)^\nu [(1-\nu)]+ \\ +\psi_t\rho \left(A_t^k k_t\right)^\nu [(\nu-a)] \end{array}\right]$$

$$\left[\begin{array}{c} (\psi_t h_t^s)^\nu [(1-\rho)(1-a)]+ \\ +\rho\left(A_t^k k_t\right)^\nu [(1-\nu)+\psi_t(\nu-a)] \end{array}\right]$$

The first row is positive because $(1-\rho)>0$ and $(1-a)>0$. Therefore, if $(1-\nu)+\psi_t(\nu-a)>0$ the term in the squared brackets is positive.

$$(1-\nu)+\psi_t(\nu-a)>0$$

$$(1-\nu)>-\psi_t(\nu-a)$$

$$(1-\nu)>\psi_t(\alpha-\nu)$$

$$\frac{(1-\nu)}{(\alpha-\nu)}>\psi_t$$

Note that: $\frac{(1-\nu)}{(\alpha-\nu)}>1$, since $1>\alpha \Rightarrow 1-\nu>\alpha-\nu$. Since $0<\psi_t<1$ we finally have that $\frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial\psi_t}<0$.

To summarize:

$$\begin{aligned} \frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial\psi_t} &= -h_t^u (\psi_t h_t^s)^\nu (1-\mu)(1-\rho)\left[(1-\rho)(\psi_t h_t^s)^\nu+\rho\left(A_t^k k_t\right)^\nu\right]^{\frac{a}{\nu}-2}\times \\ &\times\left[\begin{array}{c} (\psi_t h_t^s)^\nu [(1-\rho)(1-a)]+ \\ +\rho\left(A_t^k k_t\right)^\nu [(1-\nu)+\psi_t(\nu-a)] \end{array}\right]\div \\ &\div\left[\mu h_t^s \psi_t^2 \left((1-\psi_t) h_t^u\right)^\alpha\right] \end{aligned}$$

is negative if $[(1-\nu)+\psi_t(\nu-a)]>0$ or $\frac{1-\nu}{\alpha-\nu}>\psi_t$, which is true because $\frac{1-\nu}{\alpha-\nu}>1$, since $1>\alpha\Rightarrow 1-\nu>\alpha-\nu$ and $0<\psi_t<1$.

D.4 Ex ante capital tax

Assume that the government uses a capital tax that is not state-contingent, so that its value for period $t+1$ is decided using the history s^t. Define this uncontingent tax as $\overline{\tau}^k_{t+1}(s^t)$ and note that it needs to satisfy the Euler-equation from (149) in the main text, so that the Ramsey allocations are preserved:

$$u_c(s^t) = \beta E_t \left\{ u_c(s^{t+1}) \left[\widetilde{F}_k(s^{t+1}) \left[1 - \tau^k_{t+1}(s^{t+1}) \right] + 1 - \delta \right] \right\} \tag{D23}$$

where we have used $\widetilde{F}_k(s^{t+1}) = r_{t+1}(s^{t+1})$. Hence, $\overline{\tau}^k_{t+1}(s^t)$ needs to satisfy:

$$u_c(s^t) = \beta E_t \left\{ u_c(s^{t+1}) \left[\widetilde{F}_k(s^{t+1}) \left[1 - \overline{\tau}^k_{t+1}(s^t) \right] + 1 - \delta \right] \right\}. \tag{D24}$$

By comparing (D24) with (D23), we see that $\overline{\tau}^k_{t+1}(s^t)$ needs to satisfy:

$$E_t \left\{ u_c(s^{t+1}) \left[\widetilde{F}_k(s^{t+1}) \left[1 - \overline{\tau}^k_{t+1}(s^t) \right] + 1 - \delta \right] \right\} = \tag{D25}$$

$$= E_t \left\{ u_c(s^{t+1}) \left[\widetilde{F}_k(s^{t+1}) \left[1 - \tau^k_{t+1}(s^{t+1}) \right] + 1 - \delta \right] \right\}$$

implying that:

$$\overline{\tau}^k_{t+1}(s^t) = \frac{E_t u_c(s^{t+1}) \left[\tau^k_{t+1}(s^{t+1}) \widetilde{F}_k(s^{t+1}) \right]}{E_t u_c(s^{t+1}) \widetilde{F}_k(s^{t+1})}. \tag{D26}$$

This gives $\overline{\tau}^k_{t+1}(s^t)$ the *ex ante* capital tax interpretation, since, by multiplying both numerator and denominator in (D26) by $k_{t+1}(s^t)$, this expression provides the expected tax revenue from capital income as share of the expected capital income, where the expectation is calculated using information at period t.

To obtain the *ex ante* rate stated in equation (187) of the main text, we first expand the Euler-equation (D23):

$$u_c(s^t) = \beta E_t u_c(s^{t+1}) \widetilde{F}_k(s^{t+1}) - \beta E_t u_c(s^{t+1}) \tau^k_{t+1}(s^{t+1}) \widetilde{F}_k(s^{t+1}) + \tag{D27}$$
$$+ \beta E_t u_c(s^{t+1}) (1 - \delta)$$

and note that $E_t u_c(s^{t+1})\tau^k_{t+1}(s^{t+1})\widetilde{F}_k(s^{t+1})$ in (D27) equals $\overline{\tau}^k_{t+1}(s^t)E_t u_c(s^{t+1})\times$ $\times\widetilde{F}_k(s^{t+1})$, using (D26). Substituting this expression back into (D27) we obtain:

$$u_c(s^t) = \beta E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1}) - \beta\overline{\tau}^k_{t+1}(s^t)E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1}) + \\ + \beta E_t u_c(s^{t+1})(1-\delta). \tag{D28}$$

Finally solving (D28) for $\overline{\tau}^k_{t+1}(s^t)$ gives the *ex ante* capital tax rate reported in equation (187) of the main text.

D.5 Uncontingent debt

D.5.1 Ex-post capital tax

The treatment of state-uncontingent debt and presentation follows Chari *et al.* (1994) and Ljungqvist and Sargent (2012, ch. 16). Assume that the government issues uncontingent debt, $b_{t+1}(s^t)$ which has a risk-free return $\overline{R}_t(s^t)$. The budget constraint of the government in period t is written as:

$$g_t(s^t) = \tau^s(s^t)w^s_t(s^t)\psi_t\left(s^t\right)h^s_t(s^t) + \tau^u(s^t)w^u_t(s^t)\left[1-\psi_t\left(s^t\right)\right]h^u_t(s^t) + \\ +\tau^a_t\left(s^t\right)g\left[\psi_t(s^t)\right] + \tau^k_t(s^t)r_t(s^t)k_t(s^{t-1}) + \frac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} - b_t\left(s^{t-1}\right) \tag{D29}$$

The budget constraint of the household in period t is given by:

$$c_t\left(s^t\right) + k_{t+1}\left(s^t\right) + \frac{b_{t+1}\left(s^t\right)}{\bar{R}_t(s^t)} + \left[1+\tau^a_t\left(s^t\right)\right]g\left[\psi_t(s^t)\right] = \left(1-\tau^s_t\left(s^t\right)\right)\times \\ \times w^s_t\left(s^t\right)\psi_t\left(s^t\right)h^s_t\left(s^t\right) + \left[1-\tau^u_t\left(s^t\right)\right]w^u_t\left(s^t\right)\left[1-\psi_t\left(s^t\right)\right]h^u_t\left(s^t\right) + \\ +(1-\delta)k_t\left(s^{t-1}\right) + \left[1-\tau^k_t\left(s^t\right)\right]r_t\left(s^t\right)k_t\left(s^{t-1}\right) + b_t\left(s^{t-1}\right) \tag{D30}$$

which implies that the first-order condition with respect to holding bonds is given by:

$$\frac{1}{\overline{R}_t\left(s^t\right)} = \beta E_t\frac{u_c(s^{t+1})}{u_c(s^t)}. \tag{D31}$$

Note that the right-hand side of (D4) needs to be the same as the right-hand side of the first-order condition with respect to bonds in the case of state-contingent debt, so that the implied allocations from the two problems (i.e. with and without state-contingent debt) are the same. In turn, this implies that the risk-free (or uncontingent) return needs to satisfy:

$$\frac{1}{\overline{R}_t\left(s^t\right)}=\sum_{s^{t+1}|s^t} p_t\left(s_{t+1} \mid s^t\right). \tag{D32}$$

To obtain an expression for $b_{r+1}\left(s^r\right)$ for a given period r, we work as follows. We multiply the budget constraint of the household in (D30) for periods r and $r+1$ by $\pi_r\left(s^r\right)$ and $\pi_{r+1}\left(s^{r+1}\right)$ respectively, sum the resulting budget constraint in $r+1$ over all possible realisations s_{r+1} and add it to the budget constraint in period r. We then use the first-order conditions of the household to simplify the expression and continue this forward process until time period $T \rightarrow \infty$. By imposing the appropriate transversality conditions we obtain an expression for $b_{r+1}\left(s^r\right)$ as a function of identified equilibrium paths given in:

$$\begin{aligned} b_{r+1}\left(s^r\right)= & \bar{R}_r\left(s^r\right) \sum_{t=r+1}^{\infty} \sum_{s^t} \frac{\beta^{t-r} \pi_t\left(s^t\right)\left[u_c\left(s^t\right) c_t\left(s^t\right)+u_{h^s}\left(s^t\right) h_t^s\left(s^t\right)+u_{h^u}\left(s^t\right) h_t^u\left(s^t\right)+\Omega_t\left(s^t\right)\right]}{\pi_r\left(s^r\right) u_c\left(s^r\right)}- \\ & -\bar{R}_r\left(s^r\right) k_{r+1}\left(s^r\right) \end{aligned} \tag{D33}$$

where, $\Omega_t\left(s^t\right)$ is defined in the main text under equation (167). Hence we can use (D31) to obtain $\overline{R}_t\left(s^t\right)$, (D33) to find $b_{t+1}\left(s^t\right)$ and finally (D29) to calculate the *ex-post* capital tax reported in equation (188) of the main text.

D.5.2 Private assets tax

Assume that the government issues uncontingent debt, $b_{t+1}\left(s^t\right)$, which has a risk-free return $\overline{R}_t\left(s^t\right)$, satisfying (D32), but which is taxed using a state-contingent tax $\upsilon_{t+1}\left(s^{t+1}\right)$. The budget constraint of the government is now written as:

$$\begin{aligned}g_t(s^t) &= \tau^s(s^t)w_t^s(s^t)\psi_t\left(s^t\right)h_t^s(s^t) + \tau^u(s^t)w_t^u(s^t)\left[1-\psi_t\left(s^t\right)\right]h_t^u(s^t)+ \\ &+ \tau_t^a\left(s^t\right)g\left[\psi_t(s^t)\right] + \tau_t^k(s^t)r_t(s^t)k_t(s^{t-1})+ \\ &+ \frac{b_{t+1}\left(s^t\right)}{\overline{R}_t\left(s^t\right)} - \left[1-\upsilon_t\left(s^t\right)\right]b_t\left(s^{t-1}\right)\end{aligned} \tag{D34}$$

while the budget constraint of the household becomes:

$$\begin{aligned}&c_t\left(s^t\right) + k_{t+1}\left(s^t\right) + \tfrac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} + \left[1+\tau_t^a\left(s^t\right)\right]g\left[\psi_t(s^t)\right] = \\ &= \left[1-\tau_t^s\left(s^t\right)\right]w_t^s\left(s^t\right)\psi_t\left(s^t\right)h_t^s\left(s^t\right) + \left[1-\tau_t^u\left(s^t\right)\right]w_t^u\left(s^t\right)\times \\ &\times\left[1-\psi_t\left(s^t\right)\right]h_t^u\left(s^t\right) + (1-\delta)\,k_t\left(s^{t-1}\right) + \left[1-\tau_t^k\left(s^t\right)\right]r_t\left(s^t\right)\times \\ &\times k_t\left(s^{t-1}\right) + \left[1-\upsilon_t\left(s^t\right)\right]b_t\left(s^{t-1}\right)\end{aligned} \tag{D35}$$

which implies that the first-order condition with respect to holding bonds becomes:

$$\frac{1}{\overline{R}_t\left(s^t\right)} = \sum_{s^{t+1}\mid s^t}\beta\pi_{t+1}\left(s^{t+1}\mid s^t\right)\frac{u_c(s^{t+1})}{u_c(s^t)}\left[1-\upsilon_{t+1}\left(s^{t+1}\right)\right]. \tag{D36}$$

The introduction of the new assets tax has to be such that the equilibrium allocations obtained without it are respected. Hence the asset tax must be such that makes the right-hand side of (D4) and (D36) equal. Hence, the asset tax must satisfy:

$$E_t u_c(s^{t+1})\upsilon_{t+1}\left(s^{t+1}\right) = 0 \tag{D37}$$

which implies that at time period t, the expected value of the asset tax in period $t+1$, valued in terms of utility, has to be equal to zero. Therefore, (D37) implies that $\overline{R}_t\left(s^t\right)$ in this case is given by (D31) as well. Moreover, by working as above, we substitute household budget constraints in (D35) forward, using the household first-order conditions, the transversality conditions, the restriction in (D37) and the restriction that the asset tax in the initial period under consideration is zero (equivalent to the zero capital tax in the initial period), we can show that debt can be obtained by the same expression as (D33).

The private assets tax is defined as the tax revenue from assets over income from assets. In particular:

$$\xi_t\left(s^{t+1}|s^t\right) = \frac{\tau^k_{t+1}(s^{t+1})F_k(s^{t+1})k_{t+1}(s^t) + \upsilon_{t+1}\left(s^{t+1}\right)b_{t+1}\left(s^t\right)}{F_k(s^{t+1})k_{t+1}(s^t) + b_{t+1}\left(s^t\right)}. \tag{D38}$$

Solving (D34) for $\upsilon_t\left(s^t\right)b_t\left(s^{t-1}\right)$ and substituting this into (D38) we have the expression for $\xi_t\left(s^{t+1}|s^t\right)$ reported in equation (189) of the main text.

References

[1] Acemoglu, D. and D. Autor (2011). 'Skills, tasks and technologies: implications for employment and earnings', in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, 4b, 1043-1171, Amsterdam: Elsevier.

[2] Adjemian, S., Bastani, H., Karamé, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. and S. Villemot (2011). 'Dynare: Reference Manual Version 4', Dynare Working Papers 1, CEPREMAP, revised Jan 2014.

[3] Aghion, P. and P. Howitt (1998). 'Endogenous growth theory', Cambridge, MA: MIT Press.

[4] Aiyagari, S. (1994). 'Uninsured idiosyncratic risk and aggregate saving', *The Quarterly Journal of Economics*, 109 (3). 659–684.

[5] Aiyagari, S., Marcet, A., Sargent, T. and J. Seppala (2002). 'Optimal taxation without state-contingent debt', *Journal of Political Economy*, 110 (6). 1220–1254.

[6] Angeletos, G.M. (2002). 'Fiscal policy with non-contingent debt and the optimal maturity structure', *Quarterly Journal of Economics*, 117 (3), 1105-1131.

[7] Angelopoulos, K., Malley, J. and A. Philippopoulos (2012a). 'Tax structure, growth, and welfare in the UK', *Oxford Economic Papers*, 64 (2), 237-258.

[8] Angelopoulos, K., Malley, J. and A. Philippopoulos (2012b). 'Optimal taxation and the skill premium', University of Glasgow, Discussion Paper.

[9] Angelopoulos, K., Asimakopoulos, S. and J. Malley (2013a). 'The optimal distribution of the tax burden over the business cycle', *CESIfo Working paper series*, No. 4468.

[10] Angelopoulos, K., Fernandez, B. and J. Malley (2013b). 'The Distributional Consequences of Tax Reforms Under Capital-Skill Complementarity', *Economica*, (in press).

[11] Angelopoulos, K., Jiang, W. and J. Malley (2013c). 'Tax reforms under market distortions in product and labour markets', *European Economic Review*, 61, 28–42.

[12] Angelopoulos, K., Malley, J. and A. Philippopoulos (2013d). 'Human capital, social mobility and the skill premium', *CESifo Working Paper 4388*.

[13] Angelopoulos, K., Asimakopoulos, S. and J. Malley (2014). 'Tax smoothing in a business cycle model with capital-skill complementarity', *CESIfo Working paper series,* No. 4744.

[14] Arseneau, D. and S. Chugh (2012). 'Tax smoothing in frictional labor markets', *Journal of Political Economy*, 120, 926-985.

[15] Baier, S. and G. Glomm (2001). 'Long-run growth and welfare effects of public policies with distortionary taxation', *Journal of Economic Dynamics and Control*, 25, 2007-2042.

[16] Barro, R.J. (1979). 'On the Determination of the Public Debt', *Journal of Political Economy*, 87 (6), 940-971.

[17] Benigno, P. (2009). 'Price stability with imperfect financial integration', *Journal of Money, Credit and Banking*, 41, 121-49.

[18] Bewley, T. (1986). 'Stationary monetary equilibrium with a continuum of independently fluctuating consumers', In *Cntributions to Mathematical Economics in Hinir of Gerard Debreu*, ed. by W. Hildenbrand and A. Mas-Colell. North Holland, Amsterdam.

[19] Browning, M., Hansen, L. and J. Heckman (1999). 'Micro data and general equilibrium models', in J. Taylor and M. Woodford (eds.), Handbook of Macroeconomics, 1, 543-633, Amsterdam: Elsevier.

[20] Bueraa, F. and Nicolini, J.P. (2004). 'Optimal maturity of government debt without state contingent bonds', *Journal of Monetary Economics*, 51 (3). 531–554.

[21] Campbell, J. and G. Mankiw (1989). 'Consumption, income, and interest rates: Reinterpreting the time-series evidence', in O.J. Blanchard and S. Fischer (eds.), *NBER Macroeconomics Annual*, 185-216, MIT Press.

[22] Campbell, J. and J. Cochrane (1999). 'By force of habit: A consumption-based explanation of aggregate stock market behavior', *Journal of Political Economy*, 107, 205–251.

[23] Cantore, C. and P. Levine (2012). 'Getting normalization right: Dealing with 'dimensional constants' in macroeconomics', *Journal of Economic Dynamics and Control*, 36, 1931-1949.

[24] Castro ,R. and D. Coen-Oirani (2008). 'Why have aggregate skilled hours become so cyclical since the mid-1980s?', *International Economic Review*, 49(1), 135-184.

[25] Chamley, C. (1986). 'Optimal taxation of capital income in general equilibrium with infinite lives', *Econometrica*, 54, 607-622.

[26] Chari, V., Christiano, L.J. and P. Kehoe (1994). 'Optimal fiscal policy in a business cycle model', *Journal of Political Economy*, 102 (4), 617-52.

[27] Chari, V. and P. Kehoe (1999). 'Optimal fiscal and monetary policy', in Handbook of Macroeconomics, Vol. 1A, edited by J.B. Taylor and M. Woodford, North-Holland.

[28] Chetty, R., Guren, A., Manoli, D. and A. Weber (2011). 'Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins', *American Economic Review*, 101, 471-75.

[29] Christiano, C., Eichenbaum, M. and C. Evans (2005). 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Economy*, 113 (1), 1-45.

[30] Cogan, J., Cwik, T., Taylor, J. and V. Wieland (2010). 'New Keynesian versus old Keynesian government spending multipliers', *Journal of Economic Dynamics and Control*, 34, 281-295.

[31] Conesa, J.C., Kitao, S. and D. Krueger (2009). 'Taxing capital? Not a bad idea after all!', *American Economic Review*, 99 (1), 25-48.

[32] Correia, I. (1996). 'Consumption taxes and redistribution', *American Economic Review*, 100(4): 1673-94.

[33] Domeij, D. and J. Heathcote (2004). 'On The Distributional Effects Of Reducing Capital Taxes', *International Economic Review*, 45 (2), 523-554.

[34] Farhi, E. (2010). 'Capital taxation and ownership when markets are incomplete', *Journal of Political Economy*, 118 (5), 908-948.

[35] Floden, M. (2009). 'Why Are Capital Income Taxes So High?', *Macroeconomic Dynamics*, 13 (3), 279-304.

[36] Galí, J., Lopez-Salido, D. and J. Valles (2007). 'Understanding the effects of government spending on consumption', *Journal of the European Economic Association*, 5 (1), 227-270.

[37] Garcia-Milà, T., Marcet, A. and E. Ventura (2010). 'Supply Side Interventions and Redistribution', *Economic Journal*, 120(543), 105-130.

[38] Goldin C. and L. Katz (2008). 'The race between education and technology', The Belknap Press of Harvard University Press.

[39] Greenwood, J., Hercowitz, Z. and P. Krusell (1997). 'Long-run implications of investment-specific technological change', *American Economic Review*, 87(3), 342–362.

[40] Guo, J.T and K.J. Lansing (1999). 'Optimal taxation of capital income with imperfectly competitive product markets', *Journal of Economics Dynamics and Control*, 23, 967-995.

[41] He, H. (2012). 'What Drives the Skill Premium: Technological change or demographic variation?', *European Economic Review*, 56, 1546-1572.

[42] He, H and Z. Liu (2008). 'Investment-specific technological change, skill accumulation, and wage inequality', *Review of Economic Dynamics*, 11, 314-334.

[43] Heckman, J.J. (1976). 'A life-cycle model of earnings, learning and consumption', *Journal of Political Economy*, 84(4), S11-S44.

[44] Hornstein, A., P. Krusell, and G.I. Violante (2005). 'The effects of technical change on labour market inequalities', *International Macroeconomics and Labour Economics*, Discussion Paper Series, No. 5025.

[45] Jones, L., R. Manuelli and P. Rossi (1997). 'On the optimal taxation of capital income', *Journal of Economic Theory*, 73(1), 93–117.

[46] Judd, K. (1985). 'Redistributive taxation in a simple perfect foresight model', *Journal of Public Economics*, 23, 967-995.

[47] Judd, K. (1997). 'The optimal tax on capital income is negative', National Bureau of Economic Research, Working Paper 6004.

[48] Katz, L. and K. Murphy (1992). 'Changes in relative wages, 1963-1987: supple and demand factors', *The Quarterly Journal of Economics*, 107, 35-78.

[49] Keane, M. and E., Psarad (1993). 'Skill levels and the cyclical variability of employment, hours and wages', IMF Staff Papers, 40(4), 711-743.

[50] Mirrlees, J.A. (1971). 'An exploration in the theory of optimum income taxation'. *The Review of Economic Studies*, 38(2), 175-208.

[51] Klein, P., Krusell, P. and V. Rios-Rull (2008). 'Time consistent fiscal policy', *Review of Economic Studies*, 75(3), 789-908.

[52] Kocherlakota, N. (2006). 'Advances in dynamic optimal taxation', In *Advances in Economics and Econometrics*: Theory and applications, Ninth World Congress, Volume I (ed. R. Blundell, W.K. Newey, and T. Persson). Cambridge University Press.

[53] Kocherlakota, N. (2010). 'The new dynamic public finance', Princeton University Press, Princeton, New Jersey.

[54] Krusell, P., Ohanian, L.E., Rios-Rull, J.V. and G.L. Violante (2000). 'Capital-skill complementarity and inequality', *Econometrica,* 68(5), 1029-1053.

[55] Kydland, F.E. (1984). 'Labour force heterogeneity and the business cycle', Carnegie-Rochester Conference Series on Public Policy, 21, 173-208.

[56] Kydland, F. (1995). 'Business cycles and aggregate labour market fluctuations', in Frontiers of Business Cycle Research, edited by T. Cooley, Princenton University Press.

[57] Lansing, K. J. (1999). 'Optimal redistributive capital taxation in a neoclassical growth model', *Journal of Public Economics*, 73 (3), 423-453.

[58] Lindquist, M. J. (2004). 'Capital-skill complementarity and inequality over the business cycle', *Review of Economic Dynamics*, 7, 519-540.

[59] Litterman, R. (1983). 'A random walk, Markov model for the distribution of time-series', *Journal of Business & Economic Statistics*, 1 (2), 169-173.

[60] Ljungqvist L. and T. Sargent (2012). 'Recursive macroeconomic theory', 3rd edition, The MIT Press, Cambridge, Massachusetts.

[61] Ljungvist, L. and H. Uhlig (2000). 'Tax policy and aggregate demand management under catching up with the Joneses', *American Economic Review*, 90 (3), 356-366.

[62] Lucas, R. (1990). 'Supply-side economics: an analytical review', *Oxford Economic Papers*, 42, 293-316.

[63] Lucas, R. and N. Stokey (1983). 'Optimal fiscal and monetary policy in an economy without capital', *Journal of Monetary Economics*, 12 (1). 55-93.

[64] Machin, S. (1996). 'Wage inequality in the UK', *Oxford Review of Economic Policy*, 42, 293-316.

[65] Mankiw, G. (2000). 'The savers-spenders theory of fiscal policy', *American Economic Review*, 90 (2), 120-125.

[66] Mankiw, G., Weinzierl, M. and D. Yagan (2009). 'Optimal taxation in theory and practice', *Journal of Economic Perspectives*, 23, 147-174.

[67] Martinez-Mongay, C. (2000). 'ECFIN's effective tax rates. Properties and comparisons with other tax indicators', Economic Paper No. 146, (October), Brussels, European Commission.

[68] Matsuyama K. (2006). 'The 2005 Lawrence R. Klein Lecture: emergent class structure', *International Economic Review*, 47, 327-360.

[69] Merz, M. (1995). 'Search in the labor market and the real business cycle', *Journal of Monetary Economics*, 36 (2), 269-300.

[70] Mukoyama, T. and A., Sahim (2006). 'Costs of business cycles for unskilled workers', *Journal of Monetary Economics*, 53, 2179-2193.

[71] Persson, T. and G. Tabellini (1992). 'The politics of 1992: fiscal policy and European integration', *Review of Economic Studies*, 59, 689-701.

[72] Piketty, T. and E. Saez (2007). 'How progressive is the U.S. federal tax system? A historical and international perspective', *Journal of Economic Perspectives*, 21 (1), 3-24.

[73] Pourpourides, P. (2011). 'Implicit contracts and the cyclicality of the skill premium', *Journal of Economic Dynamics and Control*, 35, 963-979.

[74] Prasad, E. (1996). 'Skill heterogeneity and the business cycle', *The Canadian Journal of Economics*, 29(4), 910-929.

[75] Schmitt-Grohé, S. and M. Uribe (2003). 'Closing small open economy models', *Journal of International Economics*, 61, 163-185.

[76] Schmitt-Grohé, S. and M. Uribe (2007). 'Optimal simple and implementable monetary and fiscal rules', *Journal of Monetary Economics*, 54, 1702–1725.

[77] Sims, C.A (2001). 'Solving linear rational expectations models', *Computational Economics*, 20, 1-20.

[78] Stockman, D. (2001). 'Balanced-budget rules: Welfare loss and optimal policies', *Review of Economic Dynamics*, 4, 438-459.

[79] Stokey, N.L. (1996). 'Free trade, factor returns, and factor accumulation', *Journal of Economic Growth*, 1(4), 421-447.

[80] Traum, N. and S. Yang (2010). ' Does government debt crowd out investment? A Bayesian DSGE approach', Congressional Budget Office Working Paper 2010-02.

[81] Werning, I. (2007). 'Optimal fiscal policy with redistribution', *The Quarterly Journal of Economics*, 122 (3), 925-967

[82] Zhu, X. (1992). 'Optimal fiscal policy in a stochastic growth model', *Journal of Economic Theory*, 58 (2), 259-289.