SEQUENCE FILTERS

A CRITICAL REVIEW OF SEQUENCE FILTERS , ANALYSIS OF DESIGN PRINCIPLES AND AN EXPERIMENTAL INVESTIGATION OF THE TRANSIENT RESPONSE OF SELECTED FILTERS

by

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SUMMARY

Although individual sequence filters, developed for specific purposes, appear from time to time in the relevant literature, there has been no comprehensive treatment of them to date. In existing published work there has been little indication of the importance of correct detail design.

Sequence filters were developed from the mathematical theory of symmetrical components. Filters are systematically reviewed on this basis. Initially the elements required for the construction of filters are considered. Generalised theories for the basic filters are developed and design principles established. A new design for an efficient negative sequence filter with a smaller frequency error than any previous filter is given.

The performance of -ve sequence filters during 3-ph symmetrical S.C. conditions in the power system have been analysed mathematically and investigated experimentally with good agreement in the results.

The development of complex filters from the basic types is considered and finally the roles performed by sequence filters in line protection is reviewed.

CONTENTS

SECTION 1

1.1 TRANSFORMING AND PHASE-SHIFTING ELEMENTS

The term sequence-filter is used to cover any eledtrical network designed to isolate one or more of the three sequence components from an unbalanced three phase supply

A sequence-filter essentially consists of some transforming and phase-shifting elements which contain combinations of resistance, inductance, capacitance, mutual inductance, current transformers and voltage transformers. The necessity for phase-shifting is described later (Art. 2.4)

A few methods of phase-shifting by resistance, inductance and capacitance are illustrated in Fig. 1.1 and will not be discussed further, but the use of transformers and mutual inductance for transforming and phase-shifting are worth examination.

1.2 MUTUAL INDUCTANCE

Mutual inductance is the essential basis of transformer action. A transformer whose magnetic core is of laminated iron without any air-gap has a saturation level lower than that of a transformer with an air-gapped iron core. Introduction of air-gaps maintains linearity over a wide range and gives negligible core-loss. Such transformers with air-gaps in the core are called by many names by different authors, such as

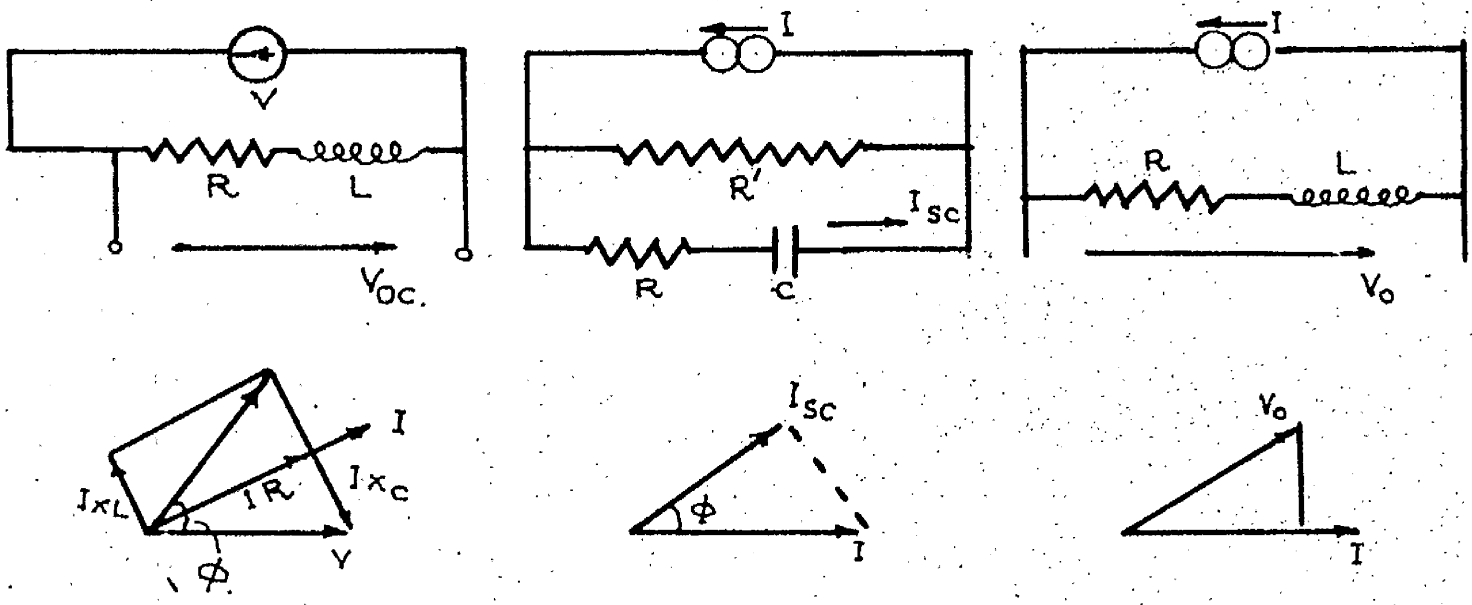
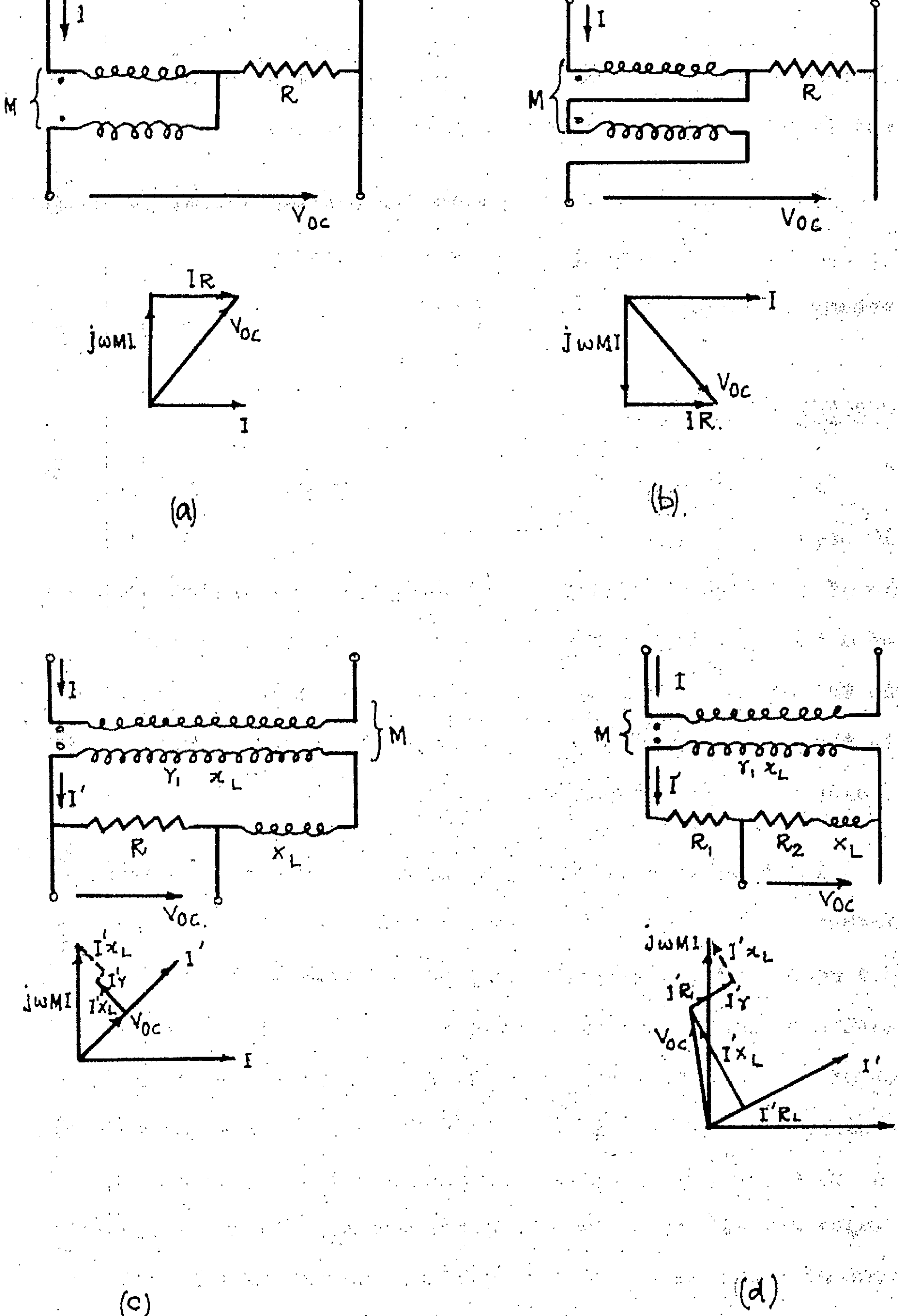


FIG 1.1



I
M
R
V_{oc}
IR
jωMI
V_{oc}
I
(a)
I
M
R
V_{oc}
I
jωMI
V_{oc}
IR.
(b)
I
M
I'
r_1 x_L
R
X_L
V_{oc}.
$I'x_L$
$I'r$
jωMI
$I'X_L$
V_{oc}
I'
I
(c)
I
M
I'
r_1 x_L
R_1
R_2
X_L
V_{oc}
jωMI
$I'x_L$
$I'R_1$
$I'r$
V_{oc}
$I'X_L$
I'
$I'R_L$
I
(d)

FIG 1.2

transformer-reactor or transactor. Here they will be called ''Mutual Inductors'' as distinct from transformers.

The use of a mutual inductor for phase-shifting is shown in fig. 1.2 with vector diagrams which are chiefly self-explanatory (Ref. 5).

1.3 CURRENT TRANSFORMERS

It is essential to know the polarity of transformers used in sequence filters. B.S.Specification no. 81 gives the details of marking polarities. The following convention (Fig.1.3) will be assumed in future discussions. If the current flows towards the dot on the dotted side of primary (as shown by arrow) then in the secondary side current will flow away from the dot of dotted side of secondary and vice-versa.

It is quite interesting to note that by using proper transformers connections only, it is possible to eliminate or obtain a zero-sequence output. The simple rule to follow for eliminating zero-sequence in a current in a circuit is that a current of the form ''$aI_a+bI_b+cI_c+nI_o$'' is so chosen to flow in the circuit that the quantity ''$a+b+c+n$'' becomes zero where a, b, c and n are scalar constants and I_a, I_b, I_c are the three input stimuli of the same nature, and I_o, the zero-sequence component of them, equals to one third of the neutral-current. Constants a, b, c and n can be positive, negative or zero scalar quantities, but they must add together to zero. As an example,

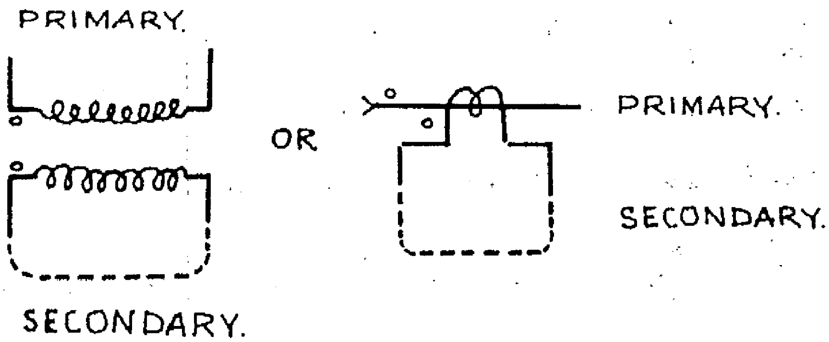


FIG 1.3

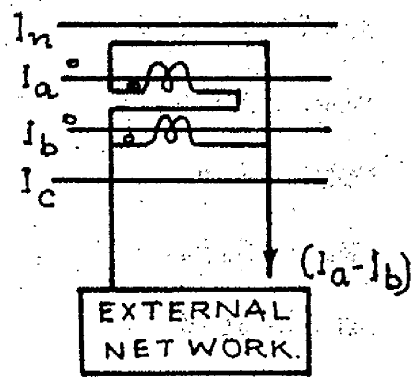


FIG 1.4

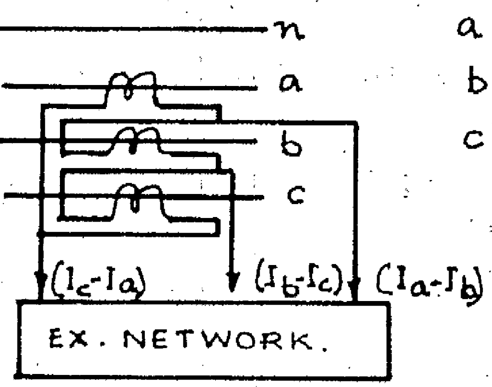


FIG 1.5

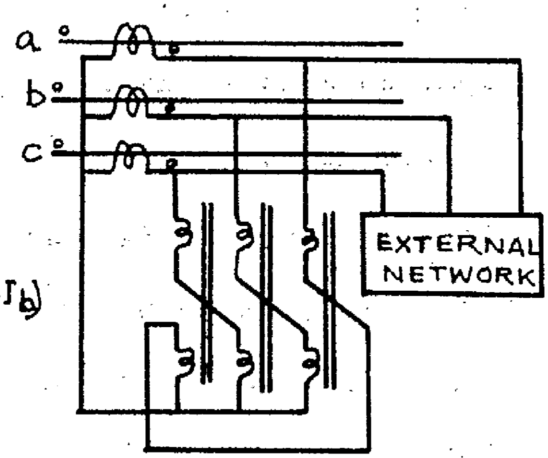


FIG 1.6.

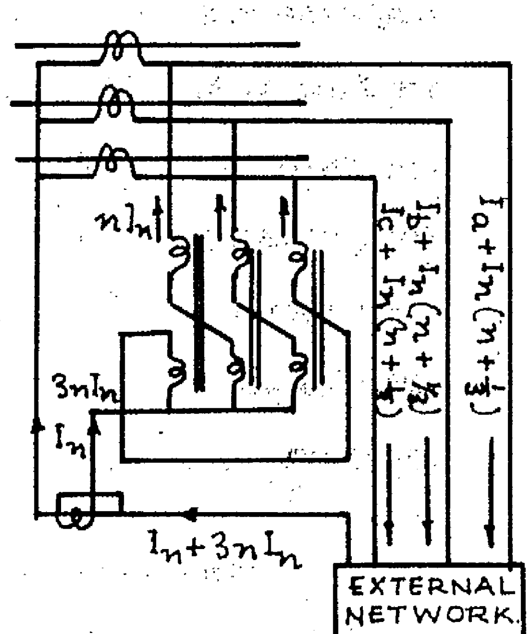


FIG 1.6 a.

let the current flowing in a circuit be ''$I_a - I_b$'', ---- here $a = 1$, $b = 1$, $c = 0$, $n = 0$ hence $a + b + c + n = 0$, thus it satisfies the aforesaid conditions for elimination of zero-sequence current. The desired network which is possible by two C.T.s only is shown in fig. 1.4.

Similarly, it is possible to derive currents $(I_b - I_c)$ or $(I_c - I_a)$ or $(I_a - I_o)$ or $(I_b - I_o)$ or $(I_c - I_o)$ to ahhieve the elimination of zero-sequence component. The familiar net-work of delta-connected C.T.s conforms to the above relationship. Three C.T.s are used to derive $(I_a - I_b)$, $(I_b - I_c)$ and $(I_c - I_a)$ as shown in fig. 1.5.

Another well-known connection for eliminating zero-sequence current by using zig-zag auxiliary transformers is shown in fig. 1.5. The usefulness of such a network can be fully appreciated, if we examine the network shown in fig. 1.6a. By this arrangement any required multiple of the zero-sequence component can be obtained and injected into the external circuit. This is one of the ways of obtaining current compensation in distance relaying for protection against earth faults.

The purpose of protective transformers, apart from providing insulation for protective gear against the high-voltage of the power circuit, is to supply the protective gear with quantities proportional to those of the power circuit but sufficiently reduced in magnitude so that the protective shheme can be made economic.

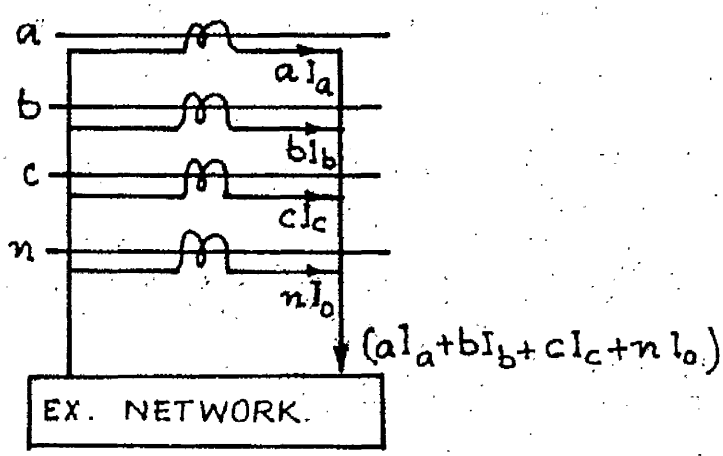

a
b
c
n
aIa
bIb
cIc
nIo
(aIa+bIb+cIc+n Io)
EX. NETWORK.

FIG 1.7.

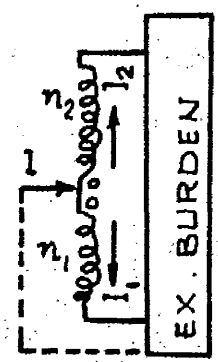

n2
I2
I
n1
I1
EX. BURDEN

FIG 1.8.

The connections of the C.T.s are such that the output is a single quantity of the generalised form ''$aI_a+bI_b+cI_c+nI_o$'', variants of which are widely used in sequence filter design. The generalised single quantity ''$aI_a+bI_b+cI_c+nI_o$'' is derived by using four C.T.s in the lines, where a, b, c and $\frac{n}{3}$ are the ratios of proportionality of the C.T.s in the respective phases and neutral and I_a, I_b, and I_c are the respective phase currents and $I_o = \frac{1}{3}$(neutral current), (fig. 1.7).

Re-examining the quantity ''$aI_a+bI_b+cI_c+nI_o$'' it can be rewritten as follows:-

$$aI_a + bI_b + cI_c + nI_o = aI_a + bI_b + cI_c + \frac{n}{3}(I_a+I_b+I_c)$$

$$= (a + \frac{n}{3})\, I_a + (b + \frac{n}{3})\, I_b + (c + \frac{n}{3})\, I_c = \underline{a'I_a + b'I_b + c'I_c}$$

where $a' = a+\frac{n}{3}$, $b' = b+\frac{n}{3}$ and $c' = c + \frac{n}{3}$

So instead of four C.T.s, it is possible to use three C.T.s (neutral C.T. is removed) to get the same result, but now the ratios of proportionity of C.T.s of the respective phases are a', b' and c' respectively. The constants a, b, c, n or a', b', c' are scalar quantities which can be either negative, positive or zero, the zero quantity can be obtained by removal of the C.T. from the desired phase.

Another useful connection of C.T.s is the ''auto-transformer''. The theory can be explained by reference to fig. 1.8. Here input current $I = I_1 + I_2$, but $n_1I_1 = n_2I_2$ giving $I_1 = \frac{n_2}{n_1 + n_2} I$, and $I_2 = \frac{n_1}{n_1 + n_2} I$ where n_1 and n_2 are the number of turns of the transformer and I_1 and I_2 are the current flowing

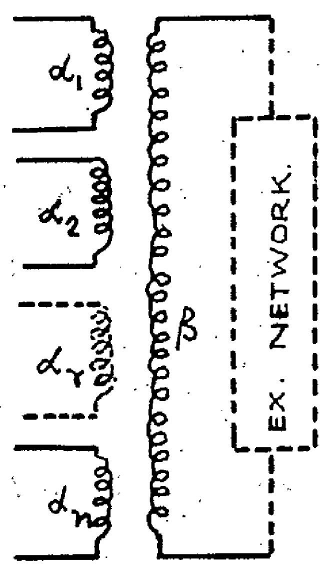


FIG 1.9.

in the two windings as shown.

The application of the above is shown later while discussing the design of sequence filter (Art. 3.10, Table 2 Filters 2e, 2f).

1.3.1 SUMMATION TRANSFORMERS:-

Discussion of C.T. connections used in conjunction with sequence filters would be incomplete without consideration of summation transformers.

Normally in the summation method multi-winding transformers are used. In the primary circuit different currents are fed in the different coils and from the secondary the sum of the currents depending on the turns ratio are taken out from two output terminals in the form of single phase. The theory behind the transformation is the well known relation of (Primary amp-turns = Secondary amp-turns). (Fig. 1.9)

Now let there be n number of coils and let the number of turns of the primary coils be $@_1$, $@_2$ - - $@_n$ and the number of turns of the secondary coil be λ. Following the lines of the previous discussion, let the currents in the primary coils be $(a_1I_a + b_1I_b + c_1I_c)$, $(a_2I_a + b_2I_b + c_2I_c)$, ---, $(a_nI_a + b_nI_b + c_nI_c)$, and let the current in the secondary coil be I.

$$I = (@_1(a_1I_a + b_1I_b + c_1I_c) + @_2(a_2I_a + b_2I_b + c_2I_c)$$

$$+ @_n(a_n I_a + b_n I_b + c_n I_c)$$

$$= \quad (a_1@_1 + a_2@_a + \text{---} + a_n@_n)\ I_a + (b_1@_1 + b_2@_2 + \text{---} + b_n@_n)\ I_b$$

$$+ (c_1@_1 + c_2@_2 + \text{---} + c_n@_n)\ I_c$$

or

$$1 = \frac{1}{\beta}(a_1@_1 + a_2@_2 + \text{---} + a_n@_n)\ I_a + (b_1@_1 + b_2@_2 + \text{---} + b_n@_n)\ I_b$$

$$+ (c_1@_1 + c_2@_2 + \text{---} + c_n@_n)\ I_c$$

$=$ $AI_a + BI_b + CI_c$. Where A, B and C are constants depending on the values of $\frac{1}{\beta}$, $@_1$, $@_2$ -- $@_n$, a_1, a_2, --- a_n. So it is again possible by the use of summation transformer to obtain the output current in the form ''$aI_a + bI_b + cI_c$''.

1.4 VOLTAGE TRANSFORMER

Knowledge of polarity of V.T. is as important as the knowledge of polarity of C.T. The following convention (fig.1.10) will be assumed:- if the voltage direction along the primary winding is towards the dot of the primary side, then the voltage direction in the secondary will be along the winding towards the dot of the secondary.

The fact to be considered to obtain zero-sequence component is that the ''zero-sequence component'' is proportional to the sum of the ''phase components''. Zero-sequence filters are explained later. Elimination of zero-sequence component is possible by similar way as used with C.T. connections. Thus like

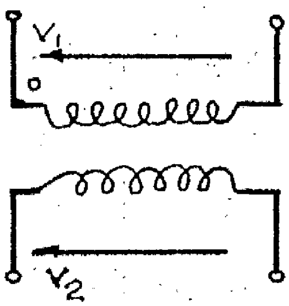


FIG 1.10

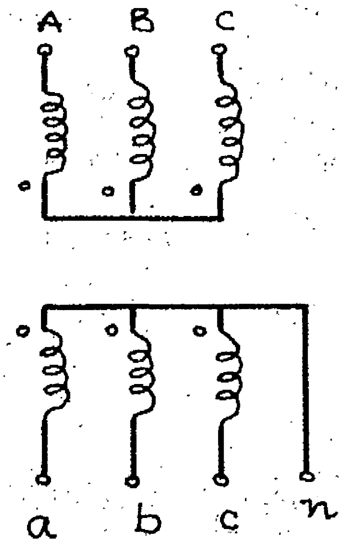


FIG 1.11

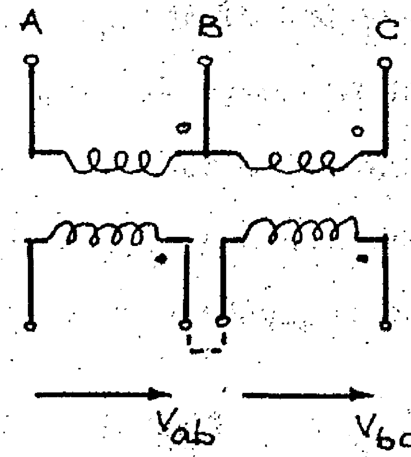


FIG 1.12

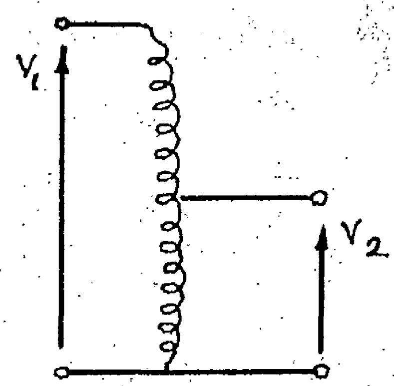


FIG 1.13.

$(I_a - I_b)$, $(V_a - V_b)$ does not contain any zero-sequence component. So avoiding re-discussion on zero-sequence elimination, only one useful but well known connection of V.T. is exemplified as shown in fig. 1.11 which has star-connected secondary V.T.s (dual to delta-connected C.T.s). Here any phase to phase voltage $(V_a - V_b)$, $(V_b - V_c)$ or $(V_c - V_a)$ does not contain zero-sequence components. Any two of the quantities $(V_a - V_b)$, $(V_b - V_c)$ and $(V_c - V_a)$ can be measured as shown in fig. 1.12 using only two pairs of windings and third quantity can be obtained by the series connection of the two secondaries. The variants of such a circuit are very useful in sequence filter designs.

Like C.T.s, voltage autotransformer connections (fig. 1.13) are used in filter designs. This is similar to a potential divider, the voltage relations of which can be formulated as

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$ where V_1 = input voltage

V_2 = output voltage

n_1 = windings on the primary side

n_2 = windings on the secondary side.

Transformers are further discussed while discussing errors in sequence filters.

The use of all those different circuit elements in the design of filters will be considered in the subsequent sections.

SECTION 2

2.1 GENERAL INTRODUCTION TO SEQUENCE FILTERS:-

A number of authors have described sequence-filters which are used in practice (Refs. 3, 5, 6, 11, 14, 15, 17, 20, 21, 22). However, this chapter attempts to generalise sequence-filter theories and the approach to the problems.

It has been shown in the previous section how to obtain the quantity ''$ax_a + bx_b + cx_c$'' by combination of transformers where x can be either voltage or current; a, b, c being scalar constants. And x_a, x_b and x_c can be expressed by well known sequence components (Refs. 2, 5, 6) as follows

$$x_a = x_+ + x_- + x_o$$

$$x_b = a^2x_+ + ax_- + x_o$$

$$x_c = ax_+ + a^2x_- + x_o$$

where x_+ = positive sequence component (vector) of three stimuli x_a, x_b and x_c

x_- = negative sequence component (vector) of three stimuli x_a, x_b and x_c

x_o = zero sequence component (vector) of three stimuli x_a, x_b and x_c

$a = e^{j\,120°}$, $a^2 = e^{-j\,120°}$ and in calculations ''a'' will be used as the refernce phase if not otherwise stated.

The above relations of the sequence components will be the basis for the design of filters.

It is already mentioned that the term sequence-filter is used to cover any electrical network designed to isolate one or more of the three sequence components from an unbalanced three phase supply. Any sequence- filter consists of an electrical circuit to which three input stimuli of the same nature x_a, x_b and x_c are applied and the output of the filter can be either single-phase or three-phase quantities. The term stimuli is used in a general sense to include voltage, current, mmf and flux.

A filter is named by the sequence component or components present in the output quantity or quantities, e.g. a filter is called a ''positive-sequence-voltage-filter'' when the output quantity of the filter contains only a positive-sequence component directly proportional to the positive-sequence component of the input quantities and when it is a voltage driven filter.

Filters with single-phase output will be, hereafter, called single-phase filters, the output of which may contain any one or any two of the sequence components. Filters with three-phase output will be called, hereafter, three-phase filters, the output of which may contain either positive or negative sequence components only, the reason being explained in a subsequent section on three-phase filters (Art. 5.4)

Before proceeding further, three more important points must be discussed. These are (1) Thevenin's or Norton's

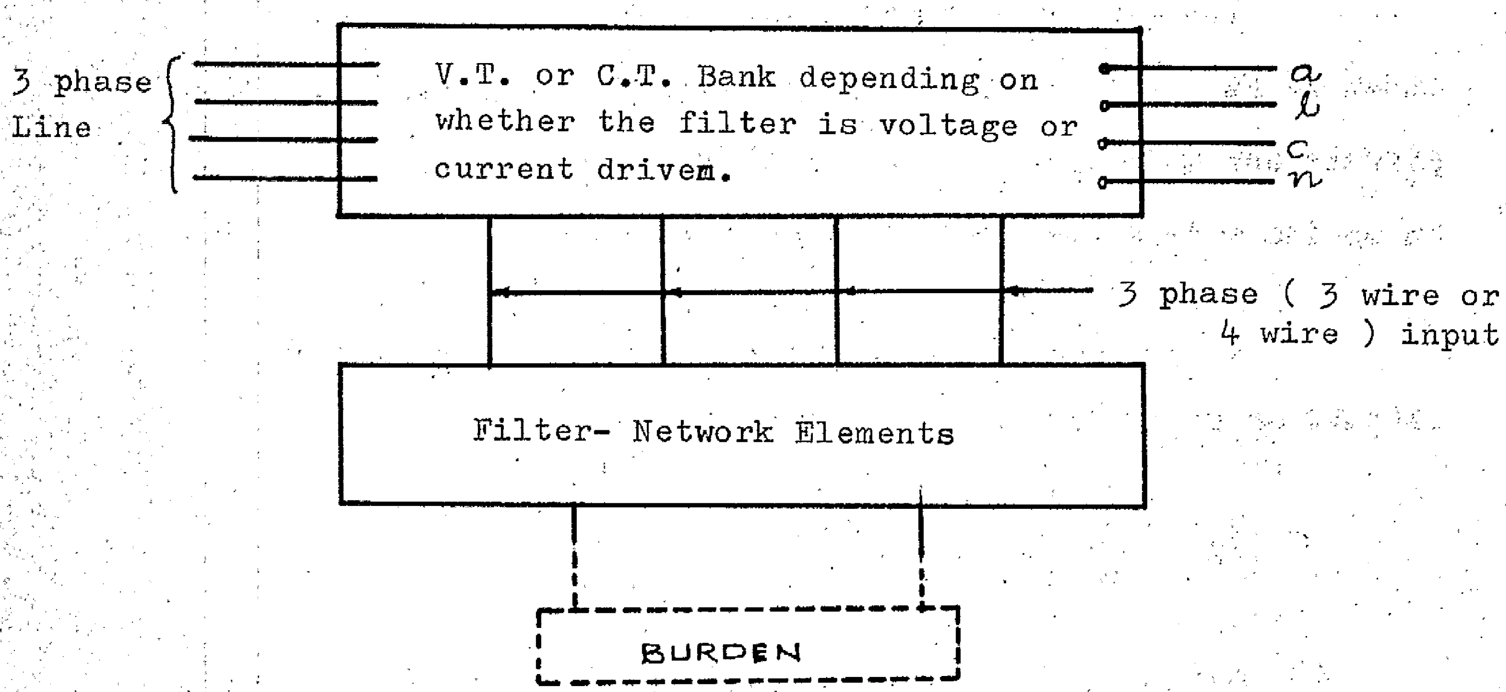


FIG 2.1

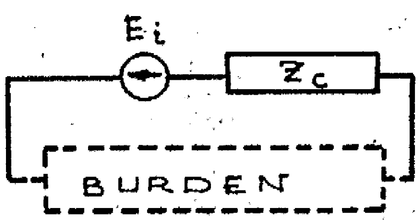


FIG 2.2 a.

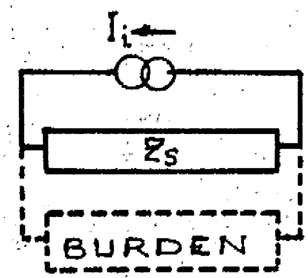


FIG 2.2 b.

equivalent of a sequence filter, (2) Dual circuit and (3) Necessity for phase shifting by circuit elements.

2.2 THEVENIN'S AND NORTON'S EQUIVALENT CIRCUITS

The arrangement for a single phase filter circuit is shown in Fig. 2.1. By applying Thevenin's Theorem, the filter circuit can be replaced by a constant voltage service E_i connected in series with an internal impedance Z_s as shown in Fig. 2.2(a).

It can also be represented by Norton's equivalent circuit as shown in Fig.2.2(b) where

$$I_i = \frac{E_i}{Z_s}$$

Assuming the filter circuit to be linear the following relations can be arrived at:-

$$E_i = a_v V_a + b_v V_b + c_v V_c \qquad \text{------------------(01a)}$$

or

$$E_i = a_i I_a + b_i I_b + c_i I_c \qquad \text{------------------(01b)}$$

where V_a, V_b of V_c and I_a, I_b and I_c are the input phase voltages or phase currents to the filters from the three phase system and a_v, b_v, c_v and a_i, b_i or c_i are constants (complex quantities) depending on the filter circuit elements. It is obvious that the output of a filter will contain the sequence component or components which are present in E_i or I_i.

2.3 DUAL CIRCUIT

A circuit designed as a voltage filter can be utilised to derive a corresponding circuit for use as a current filter and vice-versa, by satisfying the condition of duality (Ref. 5). To find the dual circuit corresponding to a given circuit, the following graphical method is followed as suggested in G.I. Atabekov's book (Ref. 5). ''A point is taken in each mesh of the given circuit and is examined as the future node of the dual circuit.

The nodes corresponding to each pair of adjacent meshes are inter-connected by parallel branches, the number of which is equal to the circuit common to the two meshes. The elements of the parallel branches are ''analogs'' of the elements of the given circuit, (a current source serves as the analog of a voltage source a capacity as that of an inductance, and conductance as that of a resistance)''. But this system is applicable only to co-planar networks: no method has yet been described for non-co-planar networks.

To avoid extra work for deriving generalised theories separately for voltage and current filters, deductions are made only for current filters; corresponding voltage filters can easily be derived by duality. Examples will follow later in the text. For non-co-planar circuits, current and voltage filters are designed separately.

2.4 NECESSITY OF PHASE SHIFTING

Let the quantity ''$ax_a + bx_b + cx_c$'' be examined, and this can be written as follows by putting the components x_+, x_-, and x_o :-

$$ax_a + bx_b + cx_c = a(x_+ + x_- + x_o) + b(@^2x_+ + @x_- + x_o)$$
$$+ c(@x_+ + @^2x_- + x_o)$$
$$= (a + @^2b + @c)\ x_+ + (a + @b + @^2c)x_- + a + b + c)x_o$$
$$= (a - \tfrac{1}{2}b - \tfrac{1}{2}c) - j\tfrac{\sqrt{3}}{2}\ (b - c)x_+ + (a - \tfrac{1}{2}b - \tfrac{1}{2}c) + j\tfrac{\sqrt{3}}{2}(b - c)x_-$$
$$+ (a + b + c)x_o \quad \text{------------------------------}(0.1c)$$

Now if it is required to eliminate the positive sequence components only then $(a - \tfrac{1}{2}b - \tfrac{1}{2}c) - j\tfrac{\sqrt{3}}{2}\ (b - c)$ must be equal to zero, i.e.

$$(a - \tfrac{1}{2}b - \tfrac{1}{2}c) = 0$$

and $\tfrac{\sqrt{3}}{2}(b - c) = 0$ which give $a = b = c$ --------(01d)

as a, b and c are scalar constants because the quantity $ax_a + bx_b + cx_c$ is in this case derived by transformer connections only.

So the expression (01c) becomes equal to $(a + b + c)x_o = 3ax_o$ which means instead of elimination of desired positive sequence component only, both the positive and negative sequence components are eliminated leaving the zero sequence component. In the previous chapter it has been shown that elimination of the zero sequence component is possible by transformer connections only and here it can be seen that it is also possible to obtain the zero sequence component by transformer connections only. But the positive and negative sequence components are inter-related, if one is eliminated, the other is also eliminated and if one is present, the other is also present holding a definite relation as shown below by re-orientation the expression (01c) which is equal to $h\,x_+ \angle{-\theta} + hx_- \angle{\theta} + h_o x_o$ where

$$h = \sqrt{\left[(a - \tfrac{1}{2} b - \tfrac{1}{2}c)^2 + (\tfrac{\sqrt{3}}{2}(b - c))^2\right]},\ h_o = a + b + c \text{ and}$$

$$\theta = \tan^{-1} \frac{\frac{\sqrt{3}}{2}(b - c)}{a - \frac{1}{2}b - \frac{1}{2}c}$$

here h and h_o are both scalar constants as a, b c are scalar constants too.

In the above expression, it can be seen that x_+ and x_- have both the same constant coefficient h and the same vector angle but negative in one case and positive in the other.

But if a, b and c, the constant coefficients of x_a, x_b and x_c respectively, are made vectors instead of scalar, then it is possible, by proper choice of these coefficients, to obtain or eliminate any of the sequence components. Phase shifting is necessary to obtain vector quantities a, b and c.

2.5 ZERO-SEQUENCE FILTERS:-

It has been shown by equation (01d) that when a = b = c then the quantity ''$ax_a + bx_b + cx_c$'' becomes equal to ''$3ax_o$'' which means that it contains only zero sequence components. Let x be replaced by I (current stimulus) and put a = b = c, then quantity ''$ax_a + bx_b + cx_c$'' becomes ''$a(I_a + I_b + I_c)$'' which

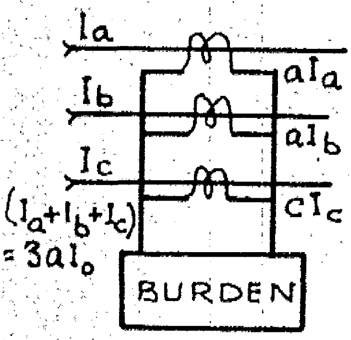


FIG 2.3.

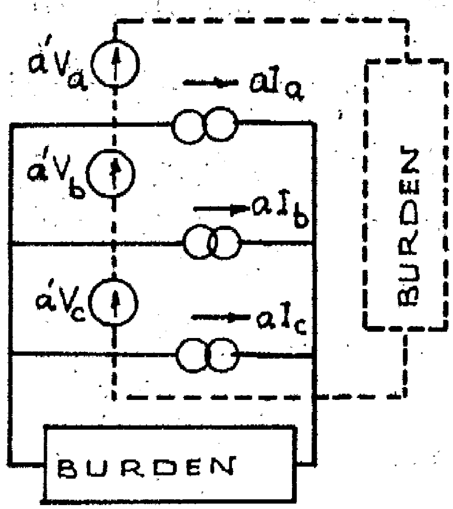


FIG 2.3a.

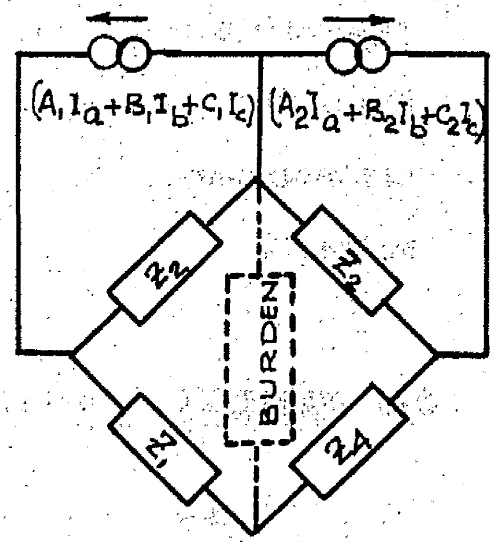


FIG 2.5

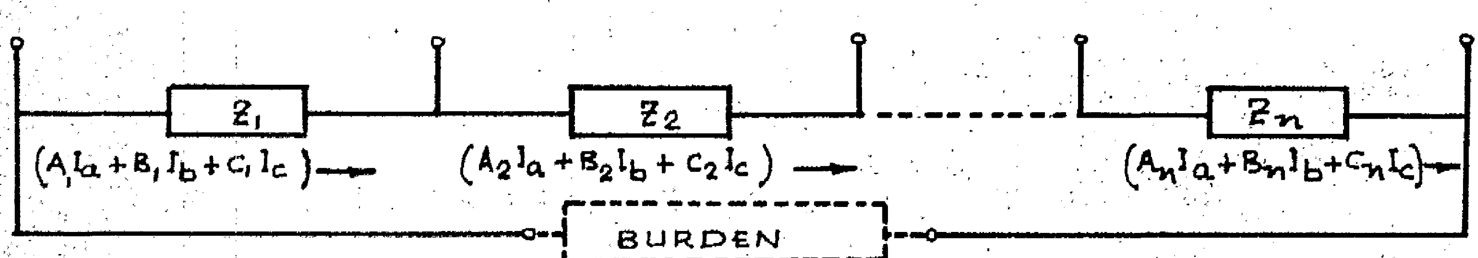


FIG 2.4

indicates how to construct a zero sequence filter; such a filter is shown in Fig. 2.3) where the output from the star-connected C.T.s contains only zero-sequence components. 'a' is the portion of current tapped out in the secondary side of the current transformers from the primary. So, as mentioned before, it is possible to obtain the zero-sequence filters by using transformers only. The Dual-voltage filter is shown in fig. 2.3a where the output from the open-delta connected V.T.s contains only zero-sequence components. A three-phase transformer of the shell type, with independent magnetic circuits, or three single-phase transformers, must be used to obtain accurate zero-sequence voltage.

2.6 NEGATIVE SEQUENCE FILTERS

These can be grouped into two major families (1) Series type and (2) Bridge type, which are discussed below.

2.6.1 SERIES TYPE

Let there be 'n' elements of impedance Z_1, Z_2, ... Z_n etc., connected in series, and let a current of the form $(AI_a + BI_b + CI_c)$ flow through each of these elements as shown in fig. 2.4. This current is derived from the appropriate C.T. bank or by the current summation method as discussed in the previous chapter. It can be shown (Appendix 1A) that to obtain a negative -sequence filter the following relation must hold good:

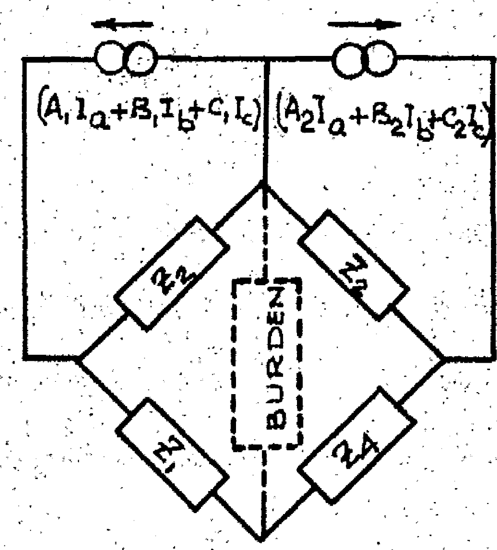


FIG 2.5

$$\frac{a}{@^2 - @} = \frac{b}{@ - 1} = \frac{c}{1 - @^2} \quad \text{------------------(1d)}$$

where $a = (A_1Z_1 + A_2Z_2 + \text{------} + A_nZ_n$

$b = (B_1Z_1 + B_2Z_2 + \text{------} + B_nZ_n)$

$c = (C_1Z_1 + C_2Z_2 + \text{------} + C_nZ_n)$

From the above equation (1d), it can be seen also that $a = b \neq c$.

2.6.2 BRIDGE TYPE

In this type of filter two stimuli $(A_1I_a + B_1I_b + C_1I_c)$ and $(A_2I_a + B_2I_b + C_2I_c)$ are fed in a bridge network consisting of four impedances Z_1, Z_2, Z_3 & Z_4 and output is taken from a diagonal of the bridge in the manner shown in Fig. 2.5.

From the conception of Norton's equivalent circuit, it can be said that if the ''short circuit'' current through the output terminals contains only negative sequence component then the network is a negative sequence filter. By ''short circuit'' is meant short circuiting of output terminals. So the basic idea of the network is to achieve a short circuit current which contains only negative-sequence component. To achieve this aim, $(A_1I_a + B_1I_b + C_1I_c)$ and $(A_2I_a + B_2I_b + C_2I_c)$ are given phase shifts by Z_1 in parallel with Z_2 and Z_4 in parallel with Z_3 respectively in the short circuited condition. It is to be noted that the short circuit current is the summation of currents through Z_1 and Z_4. It can be shown (Appendix 1B) that the following condition has to be satisfied to make it a desired negative-sequence filter:-

$$\frac{A_1k_1 + A_2k_2}{@ - @^2} = \frac{B_1k_1 + B_2k_2}{1 - @} = \frac{C_1k_1 + C_2k_2}{@^2 - 1} \qquad \text{-----(1g)}$$

where $k_1 = \frac{Z_2}{Z_1 + Z_2}$ and $k_2 = \frac{Z_3}{Z_3 + Z_4}$

2.7 POSITIVE-SEQUENCE FILTER

As the positive sequence is exactly the reverse of negative sequence totation, a positive-sequence filter can be obtained by interchanging the two input phase-terminals of a negative sequence filter while keeping the third fixed.

2.8 OTHER TYPES OF FILTER

So far it has been discussed the principles of basic types of filters - zero, negative and positive sequence filters, but there can be other types of filters also, e.g. combined filter with an output of positive and negative components together. Discussion of these can be dealt with more conveniently after developing design principles of the basic types of filter, which are considered in the next section.

SECTION 3

3.1 GENERAL

In the previous two sections the basic principles of transformation, phase-shifting and filter networks have been discussed, but more detailed analysis is required for design purposes. The first part of this section is devoted to an investigation of the properties of filter-elements and the latter part to actual design problems of basic filters.

3.2 CURRENT TRANSFORMERS (C.T.)

Two types of C.T.s are usually involved in filter networks. (B.S.2046)

(1) C.T.s which have low secondary leakage reactance are called low reactance C.T.s, e.g. symmetrical primary wound, ring-core with toroidal secondary windings, and

(2) C.T.s which have high secondary leakage reactance are called high reactance current transformers, e.g. unsymmetrical primary wound, built-up cores, with incomplete winding of the core.

Low reactance C.T.s are used in the majority of the cases, particularly while tapping the current from the system, but high reactance C.T.s find their applications as intermediate transformers in filter networks.

Descriptions of protective C.T.s can be found in the literature (Refs. 1,4), but for the present purpose the most

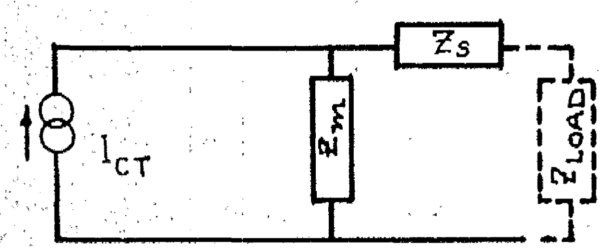


FIG 3.1.

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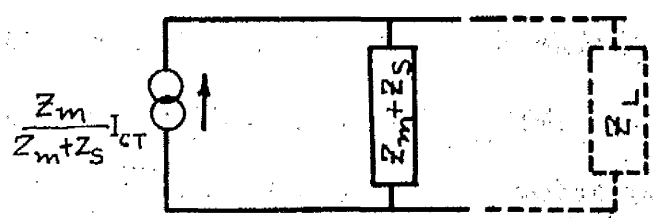


FIG 3.1a.

valuable thing to know is the equivalent circuit of a C.T. as shown in Fig. 3.1a, where Z_m is the impedance of the magnetising branch and Z_s is the total secondary impedance, including leakage impedance, lead, impedance, and any other series impedance (such as additional burden due to overcurrent relays) and Z_L is the load impedance.

In design the main consideration is that the series impedance, which consists of Z_s in series with the burden, must be very low in comparison with shunt impedance, Z_m, usually less than 10% of Z_m. Z_m is not a linear impedance, but it is quite justifiable to take it as a linear impedance within the working range, for approximate calculations, when accurate calculation or operation in the saturated region of the core is required, non-linearity of the shunt impedance must be considered.

It can be shown (Appendix 11) that instead of ''ideal (or theoretical) current output'' of the C.T., a different 'current output' obtains according to the following relations:-

''current output'' from a C.T. $= \left(I_{ct} - \frac{E_{ct}}{Z_{mag}}\right) n_{ct}$

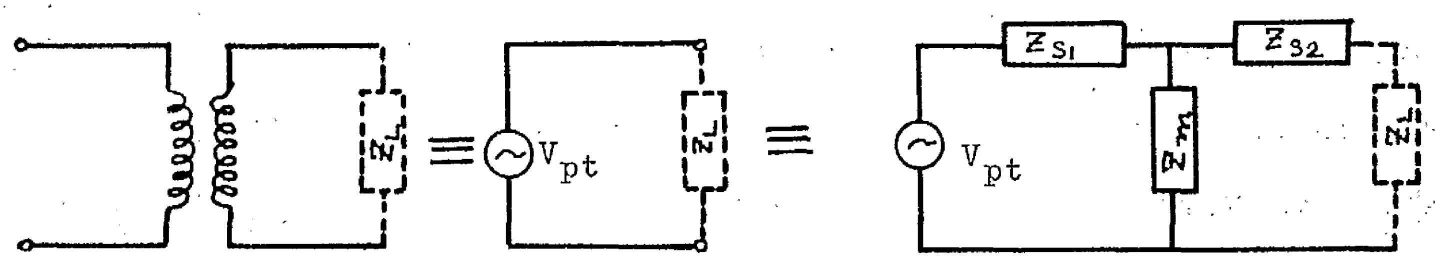
where Ict = '' ideal current output'' of the C.T.

E_{ct} = ''ideal voltage across'' the C.T.

$Z_{mag} = (Z_m + Z_s)$ and

n_{ct} = current output coefficient $= \frac{Z_m}{Z_m + Z_s}$

When Z_m is very large, as is usually in practice, then n_{ct} can be taken as unity and $Z_{mag} \doteqdot Z_m$.



ideal equivalent circuit

Actual equivalent circuit

FIG 3.1b.

From the above relation of current output, it can be appreciated that this diversion of ''output current'' from the ''ideal condition'' will cause an error in sequence filters.

3.3 INTERMEDIATE CURRENT TRANSFORMERS

In the above discussion on the equivalent circuit, no consideration has been given to the constants of the primary winding, due to the fact that the voltage required to drive the primary current through the primary winding is derived directly from the external system network. Intermediate Current Transformers are interposed between the main C.T.s and the sequence network and therefore to calculate the effect of these C.T.s on the main C.T. burden and overall transformation error, a full equivalent cirduit including both primary and secondary windings is required.

3.4 VOLTAGE TRANSFORMER

The principle of voltage transformer is similar to that of a power transformer and its equivalent circuit is exactly the same as that of a power transformer. Such an equivalent circuit is shown in Fig. 3.1b, all constants referred to secondary side. But in the case of V.T. the magnetising current is comparable to secondary current.

It can be shown from the equivalent circuit (Appendix III) that the output voltage is given by the expression:-

output voltage = $n_{pt}(V_{pt} - I_{pt}Z_{s2})$

where $n_{pt} = (1 - \frac{Z_{s1}}{Z_p})$, $Z_p = \frac{Z_m(Z_L + Z_{s2})}{Z_m + Z_L + Z_{s2}}$

V_{pt} = ideal output voltage

I_{pt} = Ideal output (secondary current)

Z_{s2} = secondary leakage impedance

Z_{s1} = primary '' ''

Z_m = magnetising branch ''

Z_L = load impedance

The above expression is on the basis, which is usually true, that Z_p or Z_L or $Z_m \gg Z_{s1}$ or Z_{s2}

Such a diversion of ''output quantity'' from ''ideal quantity'' is a source of error in a filter network.

3.5 MUTUAL-INDUCTANCE

The equivalent circuit of a ''mutual-inductor'' is exactly the same as that of a transformer. In a transformer, the magnetising impedance in the working region is usually very high in comparison to the burden, but that is not the case with ''Mutual -inductors''. Again due to the air-gap in the core, the mutual inductor has a very low iron-loss component and is less affected by saturation.

3.6 APPRAOCH TO DESIGN PROBLEMS:-

The following two points are the basic and preliminary considerations for filter design problems:-

(a) Care in the design and selection of filter elements and

(b) Simplicity of the filter network.

(a) Concerning impedance elements, it has to be remembered that their value lies in the first and fourth quadrants of the rectangular co-ordinates.

To minimise error due to C.T.s, the first point is that the excitation current should be very low in comparison to burden current. Another quality to be maintained is low leakage impedance of the secondary side. It becomes sometimes uneconomical to maintain the above conditions. So as an alternative, turn compensation can be applied (Ref. 1) to compensate the magnitude of current. This can readily be done when using intermediate current transformers which usually have large number of secondary turns.

The errors due to V.T.s are usually negligible; however, if wanted, the turn compensation method can be suitably applied. Because of the large number of turns in the primary, high accuracy in compensation of the magnitude of voltage is possible. During fault conditions V.T.s have to work at much lower voltages than the rated condition which may cause a greater ratio of proportionity of exciting current to burden current than in the normal case, even then the difference between the magnitude of the derived voltage and the ideal voltage is negligible.

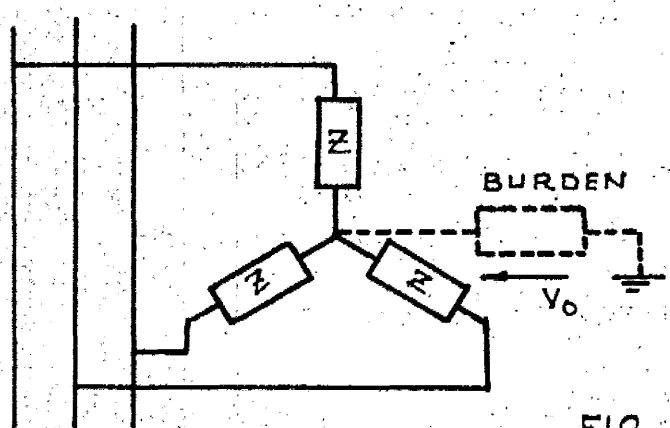


FIG 3.2.

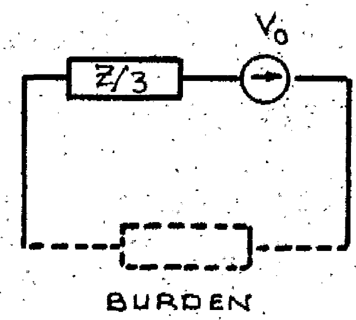


FIG 3.2a.

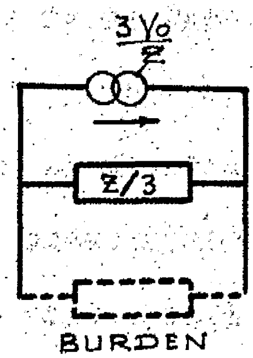


FIG 3.2b.

(b) Simplicity - the fewer circuit elements the better is a desirable feature in design from the economical and technical point of view.

It is not possible to further simplify the zero-sequence filters, discussed in the last section (Art. 2.5). However, if the neutral wire in the system is available then only one C.T. placed in the neutral wire will make a zero-sequence filter.

Another method (Ref. 6) of measuring zero-sequence voltage is shown in Fig. 3.2, where use of transformers is avoided. Thevenin's or Norton's equivalent circuits of this network is shown in Fig. 3.2a and Fig. 3.2b. As this filter constitutes of impedance elements, it is less efficient than the filter with open-delta connected secondaries (Fig. 2.3a).

For positive and negative sequence filters, generalised theories of series and bridge types of filters have been derived. In the series type, from the equation (Art. 2.6.1) it can be seen that a theoretically infinite number of filters can be designed by taking different values of Z_1, Z_2 ------ etc., and A_1, B_1, C_1, A_2, B_2, C_2 ------ etc. It has been shown in the Appendix 1VA that it is impossible to design positive or negative sequence filters with only one impedance element. Simplicity is of prime importance so the series type filters with two or three impedance elements and bridge type filters which may consist at the maximum of four impedance elements will be considered. As explained before, a negative sequence filter can work as a positive sequence filter by a mere inter-change of any two input quantities of the three-phase supply keeping the third fixed. The discussion on negative sequence filters, which have a wide field of application, will therefore provide the complete picture for both positive and negative sequence filters.

3.6.1a SERIES TYPE NEGATIVE SEQUENCE FILTER WITH TWO IMPEDANCE ELEMENTS:

From equation (1d) (Art. 2.6.1) it can be shown (Appendix 1VB) that the following conditions have to be satisfied to obtain a negative sequence filter:

$$\frac{Z_1}{Z_2} = \frac{A_2 + @^2 B_2 + @C_2}{A_1 + @^2 B_1 + @C_1} \angle 180° \qquad \text{------------(4c)}$$

$$A_1 + B_1 + C_1 = 0 \qquad \text{--------------------(4d)}$$

$$A_2 + B_2 + C_2 = 0 \qquad \text{--------------------(4e)}$$

where Z_1, Z_2, A_1, A_2 --- etc. represents the usual parameters as explained before (Art. 2.6) the number of impedance elements 'n' being equal to two.

Equations (4d) and (4e) show that the two input stimuli should be free from zero-sequence components. The following six stimuli $(I_a - I_b)$, $(I_b - I_c)$, $(I_c - I_a)$, $(I_a - I_o)$, $(I_b - I_o)$ and $(I_c - I_o)$ are of the required type which can be simply obtained. Combinations of two input stimuli can be chosen from any of the following categories (1) $l(I_a - I_b)$, $m(I_b - I_c)$

FIG 3.3a

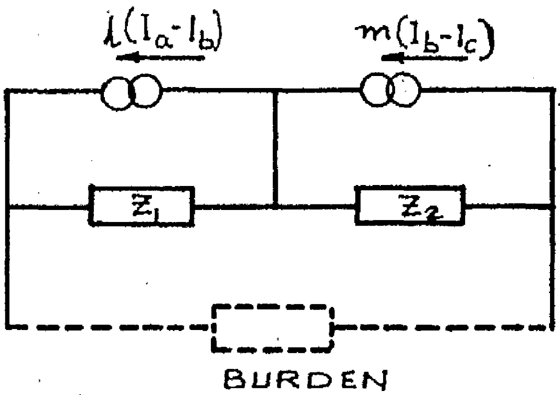


FIG 3.3b

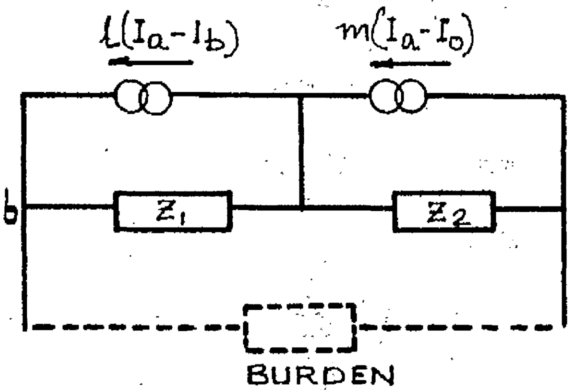


FIG 3.3c

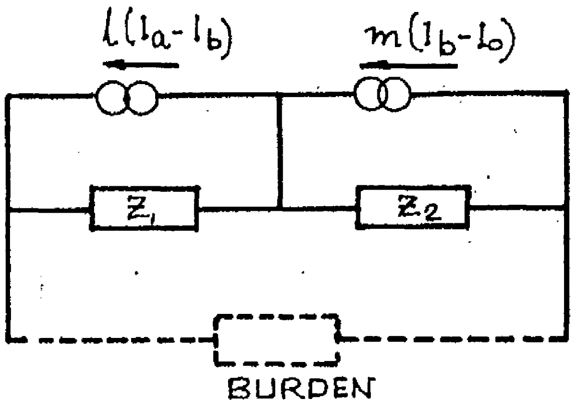
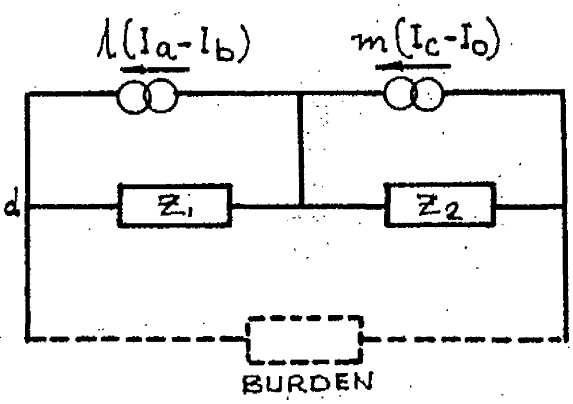


FIG 3.3d



$$(2)\ l(I_a - I_b),\ m(I_a - I_o)$$

$$(3)\ l(I_a - I_b),\ m(I_b - I_o)$$

$$(4)\ l(I_a - I_b),\ m(I_c - I_o)$$

where l and m are real positive or negative quantities. Because I_a, I_b and I_c maintains the cyclic order configuration it can be appreciated that any other combination of the above six stimuli falls under either one of the four categories, e.g $l(I_c - I_a)$, $m(I_a - I_b)$ falls under the first category.

From equation (4c), it can be shown (Appendix IVC) that the conditons to be satisfied to obtain negative sequence filters are:-

(1) $\frac{Z_1}{Z_2} = x\angle 60°$ for first category (Fig. 3.3a) ------(4f)

(2) $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}}\angle 150°$ '' second '' (Fig. 3.3b) ------(4g)

(3) $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}}\angle 30°$ '' third '' (Fig 3.3c) ------(4h)

and (4) $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}}\angle -90°$ '' fourth '' (Fig. 3.3d) ------(4i)

where $x = \frac{m}{l}$ in all cases.

A number of such filters are given in Table 1.

3.6.1b SERIES TYPE NEGATIVE SEQUENCE FILTERS WITH THREE IMPEDANCE ELEMENTS:

Again from equation (1d), (Art. 2.6.1), it can be shown (Appendix IVD) that to obtain a negative sequence filter, the following conditions have to be satisfied:-

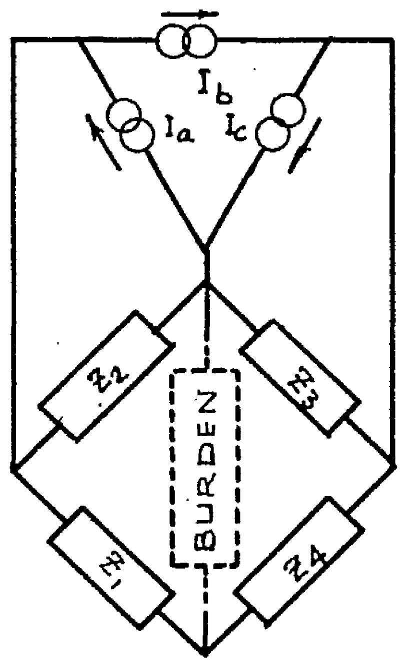
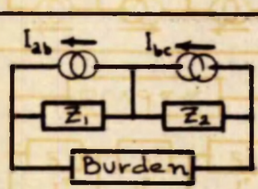
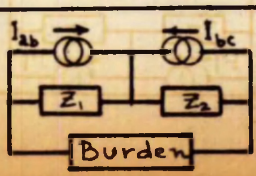
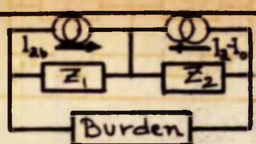
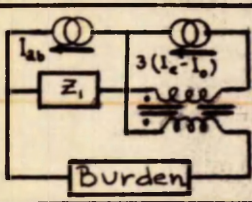
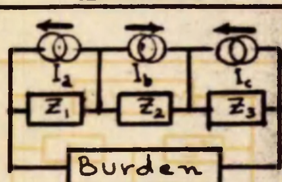
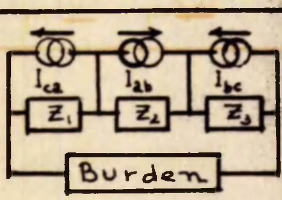
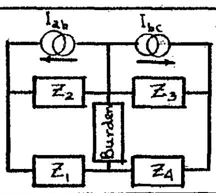
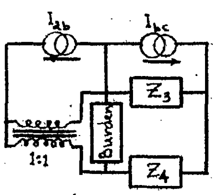
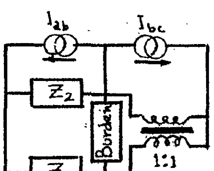
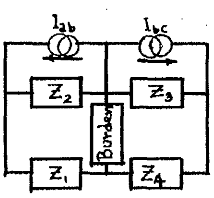


FIG 3.4.

TABLE I

Filter Network	Network Constants	Internal Impedance of the filter	Impedance quality	Output quantity		VA-quality		Frequency quality
				60° matched burden	90° matched burden •	60° matched burden	90° matched burden •	
 1a	$Z_1=R\angle 60°$ $Z_2=R$	$\sqrt{3}R\angle 30°$	30°	$0.91I\_\ \angle 15°$	$I\_$	0.27	0.333	0.5
Do 1b	$Z_1=R$ $Z_2=R\angle -60°$	$\sqrt{3}R\angle -30°$	-30°	$1.22I\_\ \angle -15°$	$1.73I\_\ \angle -30°$	0.5	1.0	0.5
Do 1c	$Z_1=R\angle -30°$ $Z_2=R\angle -90°$	$\sqrt{3}R\angle -60°$	-60°	$1.73I\_\ \angle -30°$	$3.35I\_\ \angle -15°$	1.0	3.76	0.5
 1d	$Z_1=R\angle -90°$ $Z_2=R\angle 30°$	$R\angle -30°$	-30°	$2.12I\_\ \angle -45°$	$3I\ \angle -60°$	1.5	3.0	0.765
Do 1e	$Z_1=R\angle -60°$ $Z_2=R\angle 60°$	R	0°	$1.73I\ \angle -30°$	$2.12I\_\ \angle -45°$	1.0	1.5	0.866
 1f	$Z_1=R\angle 30°$ $Z_2=\sqrt{3}R\angle 60°$	$2.64R\angle 49$	49°	$-0.33I\_\ \angle 6°$	$-0.35I\ \angle -93$	0.062	0.070	0.5
 1g	$Z_1=\sqrt{3}R$ Mutual Reactance = jR	$\pm 2R\angle 30°$	±30°	$\pm 1.56I\_\ \angle -165$	$\pm -1.73I\_$	± .462	± .577	0.5
 1h	$Z_1=R\angle -60°$ $Z_2=R$ $Z_3=R\angle 60°$	2R	0°	$0.865I\_\ \angle -90°$	$1.06I\_\ \angle -105$	0.75	1.125	0.29 0.58 For Zero Sequence
 1i	$Z_1=R\angle -60°$ $Z_2=R$ $Z_3=R\angle 60°$	2R	0°	$1.5I\_\ \angle -60°$	$1.84I\_\ \angle -75$	0.75	1.125	0.29

• NOTE : It is to be noted that a 90° matched burden is impossible to achieve, but it is chosen because it is the extreme case of pure inductive burden.

Filter Network	Network constants	Internal Impedance of the filter	Impedance quality	Output quantity		VA-quality		Frequency quality
				60° matched burden	90° matched burden ●	60° matched burden	90° matched burden ⊕	
 2a	Z_1=R Z_2=R/60° Z_3=R Z_4=R/60°	$\frac{\sqrt{3}}{2}$R/30°	30°	0.9I /15°	I /30°	0.2	0.25	0.5
Do 2b	Z_1=R/-60° Z_2=R Z_3=R/-60° Z_4=R	$\frac{\sqrt{3}}{2}$R/-30°	-30°	1.22I /-15°	1.73I /-30°	0.375	0.75	0.5
Do 2c	Z_1=R/-90° Z_2=$\sqrt{3}$R Z_3=$\sqrt{3}$R/-90 Z_4=R	$\frac{4R/-45°}{(1+\sqrt{3})\sqrt{2}}$	-45°	2.12I /-22.5	3.4I /-37.5	0.94	2.38	0.29
Do 2d	Z_1=R/-90° Z_2=2R/30° Z_3=2R/-90° Z_4=R/30°	R/-30°	-30°	2.45I /-15°	3.46I /-30°	1.0	2.0	0
 2e	Z_3=R/-90° Z_4=$\sqrt{3}$R	2R/-30°	-30°	1.06I /-45°	1.5I /-60°	0.866	1.732	0.5
 2f	Z_1=$\sqrt{3}$R/-90 Z_2=R	2R/-60°	-60°	1.5 I	2.9I /-15°	1.732	6.45	0.5
 2g	Z_1=$\sqrt{3}$R/-90 Z_2=R Z_3=$\sqrt{3}$R/-90 Z_4=$\sqrt{3}$R/-90	1.33R/-71	-71°	1.82I /-5.5°	4.6I /-20.6	0.96	6.25	0.5
Do 2h	Z_1=R Z_2=R/60° Z_3=R/-60° Z_4=R	R	0°	1.0 I	1.22I /-15°	0.333	0.5	0.58

⊕ NOTE : It is to be noted that a 90° matched burden is impossible to achieve, but it is chosen because it is the extreme case of pure inductive burden .

$$\frac{Z_1}{\begin{vmatrix} A_2 & A_3 & (@^2 - @) \\ B_2 & B_3 & (@ - 1) \\ C_2 & C_3 & (1 - @^2) \end{vmatrix}} = \frac{-Z_2}{\begin{vmatrix} A_3 & (@^2 - @) & A_1 \\ B_3 & (@ - 1) & B_1 \\ C_3 & (1 - @^2) & C_1 \end{vmatrix}} = \frac{Z_3}{\begin{vmatrix} (@^2 - @) & A_1 & A_2 \\ (@ - 1) & B_1 & B_2 \\ (1 - @^2) & C_1 & C_2 \end{vmatrix}}$$

$$= \frac{Q}{\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}} \quad \text{--} (4j)$$

where Q is an arbitrary constant and Z_1, Z_2, A_1, A_2 --- etc. represent the usual constant parameters as explained before, the number of impedance elements 'n' being equal to three.

A list of series type of filters is given in Table 1.

3.6.2 BRIDGE TYPE NEGATIVE SEQUENCE FILTERS:-

As with the series type of filters, and bearing in mind the requirement of simplicity, a group of filters, where the two stimuli I_{ab} and I_{bc} are derived from a delta-connected C.T. bank, are considered here (Fig. 3.4).

From equation (1g) (Art. 2.6.2) it can be shown (Appendix V) that the following relation has to be satisfied to obtain a negative sequence filter:

$$1 + \ddot{m}_1 \angle{-60°} + \ddot{m}_2 \angle{60°} = 0 \quad \text{---------------------} (5b)$$

where $\ddot{m}_1 = \dfrac{Z_1}{Z_2}$ and $\ddot{m}_2 = \dfrac{Z_4}{Z_3}$.

A number of such filters are given in Table 2, some of which have been in use for some time (Ref. 5).

In addition to the factors already described in the above articles, some further features must be considered in the design problems. These are:- Frequency quality, Impedance quality and VA quality.

3.7 FREQUENCY QUALITY:-

Since impedance elements other than reactance are frequency sensitive, an error is introduced in the filter output due to frequency variation. Errors caused by frequency deviation are mentioned by many authors (Ref. 5,6, 14) and in one publication, by Livingston (Ref. 14), a suitable explanation is given about its effect in relay applications. It has been shown how the error resulting from a variation in frequency of 10%, can be serious with only small values of unbalance factor (''unbalance factor'' is defined as negative sequence current, expressed as a fraction of positive sequence current) in a low-setting relay operated by a negative sequence filter.

All filters will behave very badly for large frequency deviation and some additional device is necessary to minimise this effect. But even for small frequency deviation the effect can be sometimes serious as discussed above. So in choosing a sequence filter proper care has to be given for its behaviour even for a small deviation.

Error due to any component is the ''difference between outputs of that component at nominal and deviated frequency respectively'' expressed as a fraction of desired output of the filter.

When a negative (as well as positive) sequence fiâter is freed from ''zero-sequence'' component by the use of ''cross-connected'' or'delta-connected' transformer secondary windings (e.g. $(I_a - I_b)$ or $(I_a - I_c)$) question of error with the presence of zero-sequence component due to frequency deviation does not arise because for small frequency deviation transformer-characteristics practically do not alter; but if the elimination of ''zero-sequence'' component is achieved by the combination of impedance elements then the ''error'' has to be considered.

Livingston has also shown that the greater portion of the resultant error in a negative sequence filter application, is contributed by the positive-sequence component, the contribution by negative - sequence component being relatively insignificant. So it is felt justifiable to consider ''positive-sequence error'' (i.e. error contributed by positive sequence component) as a criterion of performance of a negative-sequence filter while considering the effect of small frequency deviation.

Livingston, then, has shown how to compensate a filter against this error. The method, which involves more complex networks, is based on the mathematical logic that, while considering a small frequency deviation, if at nominal frequency the positive-sequence-error has zero rate of change with respect to frequency and is of zero magnitude, then the effect of error will be insignificant. The rate of change of positive-sequence-error with respect to frequency at nominal frequency is called the ''frequency quality''; frequency error and quality are usually expressed in terms of absolute values. Mathematically, frequency quality (denoted by μ) is expressed as follows

$$\mu = \frac{d}{dk}\left|(\text{positive sequence error})\right|_{k=1} \qquad \text{------------(6)}$$

where k = ratio of actual to nominal frequency (Appendix V1)

The following useful relations are derived from relation (6)

$$\mu = \frac{1}{V_{oc-}}\left|\frac{d}{dk}\, f_{oc}(k)\right|_{k=1} \qquad \text{------------------------(6a)}$$

$$\mu = \frac{1}{I_{sc-}}\left|\frac{d}{dk}\, f_{sc}(k)\right|_{k=1} \qquad \text{------------------------(6b)}$$

where V_{oc-} = o.c. voltage due to negative seq. component at nominal frequency

$f_{oc}(k)$ = o.c. voltage due to positive sequence component when frequency is k times nominal frequency

I_{sc-} = s.c. current due to negative sequence component at nominal frequency

$f_{sc}(k)$ = s.c. current due to positive sequence component when frequency is k times nominal frequency

The following formula derived from the same expression (Appendix V1, 6f) is very helpful for the calculation of frequency quality for a bridge filter:-

$$\mu = \frac{1}{\sqrt{3}}\left(1 + \frac{Z_4}{Z_3}\right)^{-1}\left|\frac{d}{dk}\,\frac{Z_1(k)}{Z_2(k)}\,\angle{-120°} + \frac{d}{dk}\,\frac{Z_4(k)}{Z_3(k)}\right|_{k=1} \qquad \text{(6f)}$$

where Z_1, Z_2 etc. are the impedance of the four bridge elements at nominal frequency and $Z_1(k)$, $Z_2(k)$ etc. are the corresponding impedances at K times nominal frequency.

The main drawback of Livingston's methods of compensation is that the filters become so inefficient and the networks so complex that they are not practically feasible.

It is shown later (Art. 3.10) that even without any compensation it is possible to achieve the same purpose in a filter which is also highly efficient, e.g. Filter 2d, Table 2, which is author's own design.

In conclusion, it can be mentioned again that the lower the ''frequency quality'' the less the frequency error. By term ''frequency quality'', we will mean the effect of positive-sequence component, if not otherwise stated, in a negative sequence filter.

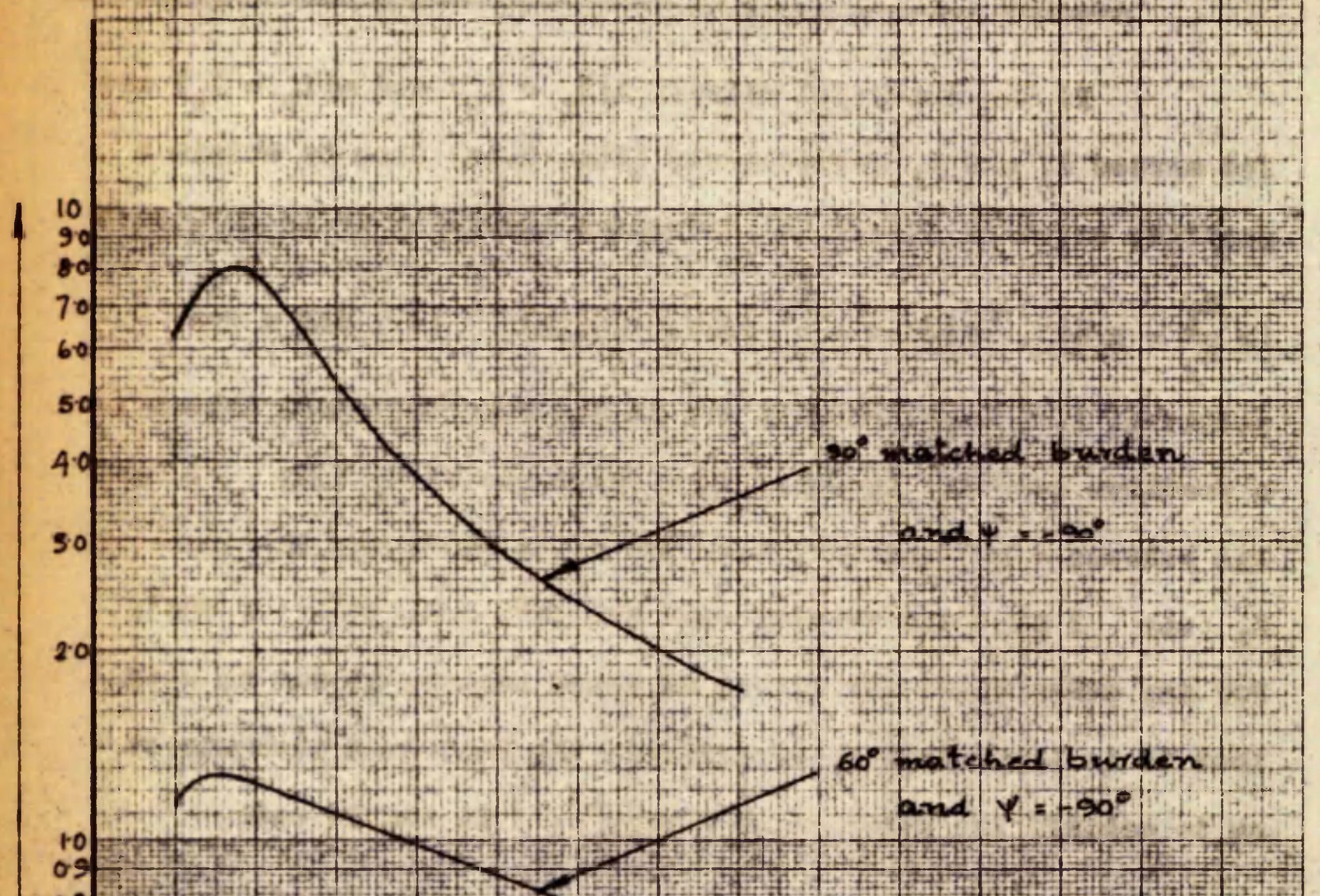
3.8 IMPEDANCE QUALITY AND VOLT-AMP (VA) QUALITY

The term ''matched impedance'' is well-known. The idea of matching of impedances is to obtain maximum volt-amp (VA) output from a known source of supply (e.g. a sequence filter). It can be shown (Appendix V11) that if $z_i \angle \phi_i$ and $z_L \angle \phi_L$ are source and burden impedances respectively then for matched condition $z_i = z_L$ and $(\phi_i - \phi_L) = \pm\pi$. In practice it is not possible to fulfil the second condition on account of the presence of active impedances (i.e. resistances) in the filters and in the load. So for matching purpose, apart from equal absolute magnitudes of impedances, the angles, when possible, should be close in magnitude to $\pi/2$ and opposite in sign. Usually burden (relay) impedances lie in the first quadrant (say 60° to 90°) so the source impedance angle should be near $-\pi/2$ for high VA output. This angle of the source impedance will be called ''impedance quality''.

And ''VA-quality'' will represent the (VA-output/VA-input) of the desired sequence. In a polyphase network, either balanced or unbalanced, the total vector volt-amperes is the square root of the sum of the squares of the total active power and the total reactive power.

It has been shown by one example (Appendix V11A, Art 3.10, Filter 2c, Table 2) that the positive-seuqance VA input is equal to the negative.sequence VA.input for the same supply quantities. So for easier calculations of ''VA-quality'' of a negative-sequence filter, ''VA-input'' is calculated with the positive-sequence component, and ''VA-output'' with negative sequence component. It is to be noted that it is not possible to calculate VA-input from the Thevenin's or Norton's equivalent circuit.

High ''VA-quality'' is a desired feature of a filter.



30° matched burden
and Ψ = -90°
60° matched burden
and Ψ = -90°
30° matched burden
and Ψ = 0°
60° matched burden
and Ψ = 0°
η (= VA - quality)
x (= m/l)
0·1
0·5
1
1·5
2
10
9·0
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2

FIG G1.

3.9 DESIGN OF SERIES TYPE FILTERS

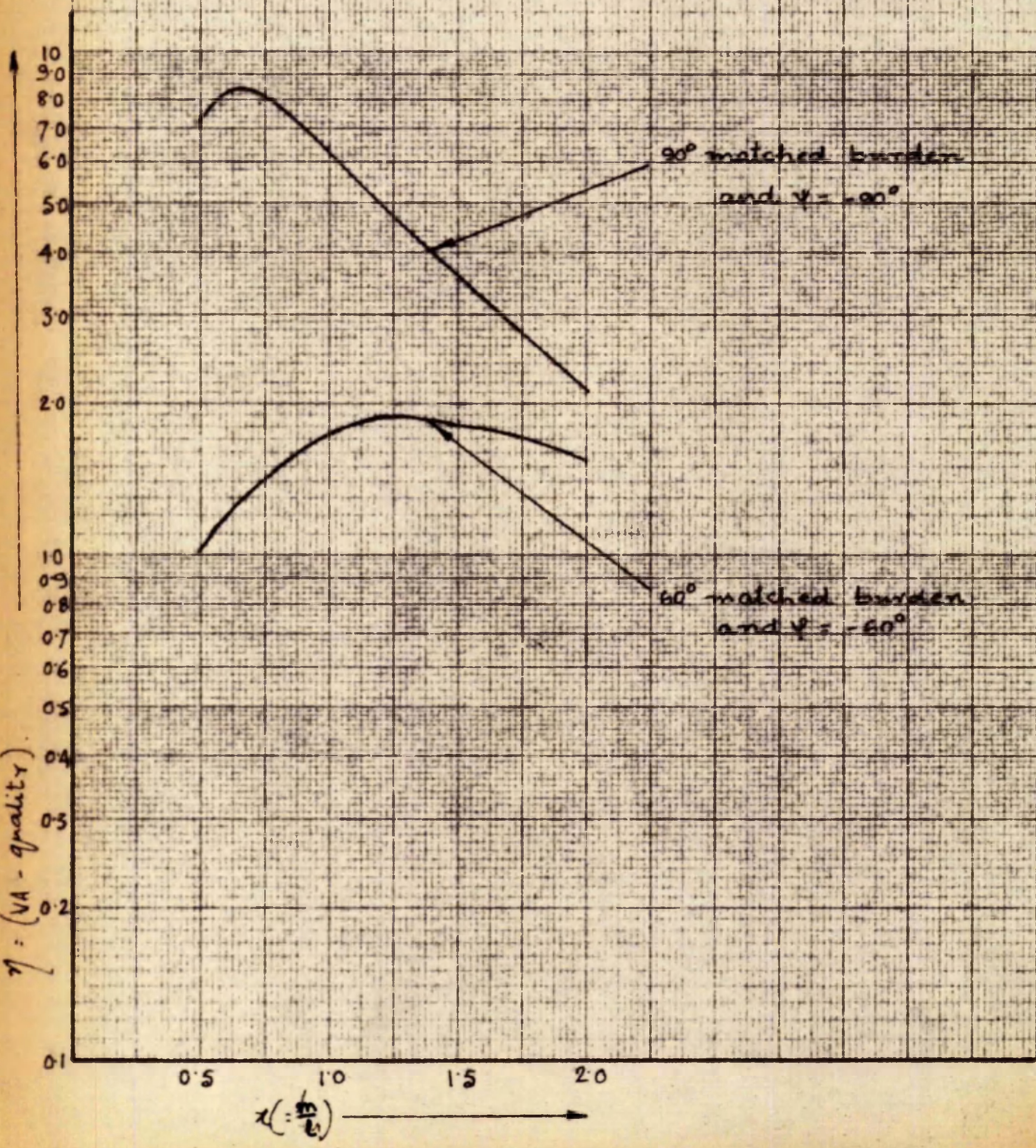
3.9.1a TWO-ELEMENT FILTERS WITH STIMULI $l(I_a - I_b)$ and $m(I_b - I_c)$

If the two input stimuli $l(I_a - I_b)$ and $m(I_b - I_c)$ are fed across Z_1 and Z_2 respectively (Fig. 3.3a) then to obtain a negative sequence filter the necessary condition to be satisfied is $\frac{Z_1}{Z_2} = x\underline{/60°}$ where $x = \frac{m}{l}$ (Art. 3.3.1a). If l and m are positive, i.e. input stimuli are in the same direction then x is a positive quantity in which case the angle of Z_1 (say ψ') must lie between $-30°$ and $90°$, i.e. $90° \geqslant \psi' \geqslant -30°$, and the angle of Z_2 (say ψ) must lie between $-90°$ and $30°$, i.e. $30° \geqslant \psi \geqslant -90°$. For any other value for either of these angles, it can be seen that it is impossible to co-ordinate the impedance elements so as to satisfy the relation $\frac{Z_1}{Z_2} = x\underline{/60°}$. It can be seen (Appendix VIIIa) that, everything else remaining fixed, the VA-quality (Fig. G1) varies as x varies. It starts with a minimum as $x \to 0$, then reaches maxiumum, then drops again as $x \to \infty$. Again it can be seen that the VA-quality improves as Z_2 is made capacitive, e.g. with 60° matched burden, choosing $\psi = -90°$ (i.e. $\psi' = -30°$) the VA-quality reaches about 1.24 when $x = {}^1/2$ whereas with $\psi = 0°$ (i.e. $\psi' = 60°$) the maxiumum VA-quality is .266. Impedance quality also improves as Z_2 is made capacitive. As for frequency quality it varies between .5 to .29 (Appendix VIII b). If $60° \geqslant \psi' \geqslant 0°$ and $0° \geqslant \psi \geqslant -60°$ then the frequency quality becomes less than .5, and it has a minimum value of .29 when $\psi' = \angle 30^\circ$ and $\psi = \angle -30^\circ$. ψ' and ψ lying in any other zone, the frequency quality is always .5. But even with these improvements it will be found later (Art. 3.10, Table 2) that in comparison with some bridge type of filters, these filters are inferior.

If any one of the input stimuli are reversed, then the relation $\frac{Z_1}{Z_2} = -\frac{m}{l}\angle 60^\circ$, i.e. $\frac{Z_1}{Z_2} = x\angle +120^\circ$ must hold good to to obtain a negative sequence filter. So ψ' must lie between -90° to -30°, i.e. $-30^\circ \geqslant \psi' \geqslant -90^\circ$ and ψ must lie between 30° and 90°, i.e. $90^\circ \geqslant \psi \geqslant 90^\circ$; thus Z_1 is always a capacitive element and Z_2 is an inductive element. Though the VA-quality and impedance quality are better with this arrangement than with previous arrangement, they suffer from poor frequency quality, the minimum value of which can be .765 (Appendix VIII c, Eqn. (8q), Table 1). Because of this drawback, they are never a good choice as a filter.

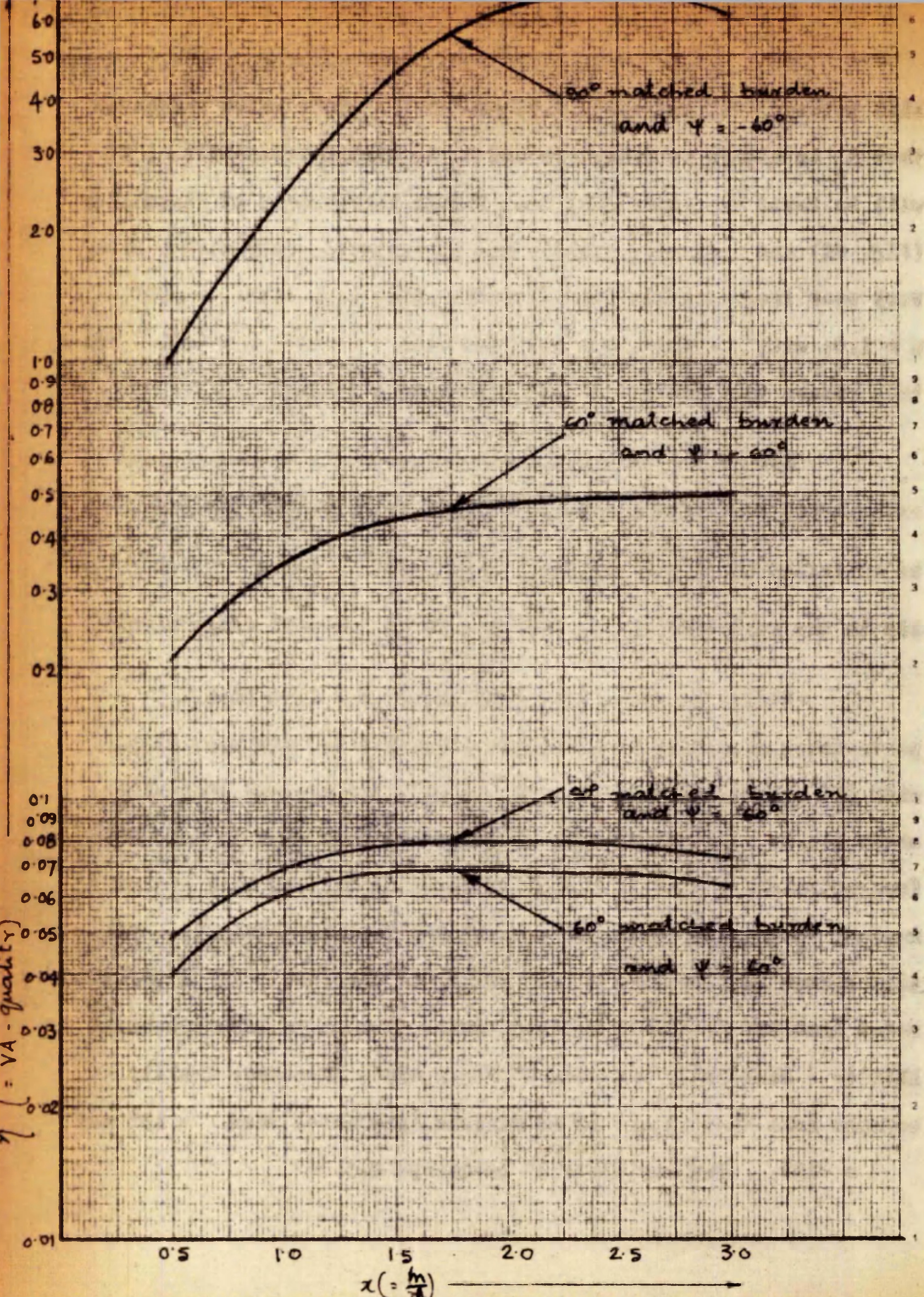
3.9.1b TWO-ELEMENT FILTERS WITH STIMULI $l(I_a - I_b)$ & $m(I_a - I_c)$:

It is already shown (Art. 3.6.1a, Appendix IVb, Fig. 3.3b) that for this type the condition to be satisfied to obtain a negative sequence filter is $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}}\angle 150^\circ$ where $x = \frac{m}{l}$. If l and m are positive quantities, i.e. stimuli flow in the same direction then x is a positive quantity. In such a case ψ' (angle of Z_1) lies between 60° and 90°, i.e. $90^\circ \geqslant \psi' \geqslant 60^\circ$ and ψ (angle of Z_2)



10
9·0
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
η : (VA - quality).
90° matched burden
and ψ = -90°
60° matched burden
and ψ = -60°
0·5
1·0
1·5
2·0

FIG G2.



6·0
5·0
4·0
3·0
2·0
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
0·5
1·0
1·5
2·0
2·5
3·0
η (= VA - quality)
x (= hm/rt)
90° matched burden and ψ = -60°
60° matched burden and ψ = -60°
90° matched burden and ψ = 60°
60° matched burden and ψ = 60°

FIG G3.

lies between -90° and -60°, i.e. $-60° \geqslant \psi \geqslant -90°$, which indicates that Z_1 is always on inductive and Z_2 is always capacitive. It will be found (Appendix 1Xa) that though improvement of VA-quality (Fig. G2) and impedance quality can be achieved, they suffer from very poor frequency quality (Appendix .Xb), e.g. with $\psi = -90°$, $\mu = 1.8$; with $\psi = -75°$, u = 1.85 and with $\psi = -60°$, $\mu = 1.8$. For this reason they are not a good choice as a filter.

Now if one of the stimuli is reversed, then with this arrangement to obtain a negative sequence filter the condition to be satisfied is $\frac{Z_1}{Z_2} = - \frac{m}{n\sqrt{3}} \angle 150° = \frac{x}{\sqrt{3}} \angle -30°$. Hence ' and lie in the region $60° \geqslant \psi' \geqslant -90°$ and $90° \geqslant \psi \geqslant -60°$ respectively.

It can be shown (Appendix 1X(c)),9m) that their performance regarding VA-qualities (Fig. G3) is very poor (e.g. with 60° matched burden and with $\psi = 60°$, the maximum VA-quality which can be achieved at this condition by varying x is only .068 (approximately); similarly with the same burden and with $\psi = -60°$ maximum $\overline{VA}$-quality (Fig. G3) is only .50 (approximately)). For this reason they are not a good choice as filters. It is to be noted that the frequency quality varies from .5 to .134 (Appendix 1X(d))- When $30° \geqslant \psi \geqslant 0°$ and $0° \geqslant \psi' \geqslant -30°$, frequency quality becomes less than .5 and it is minimum ($\mu = .134$) when $\psi = 15°$ and $\psi' = -15°$. For any other value of ψ it is always 0.5.

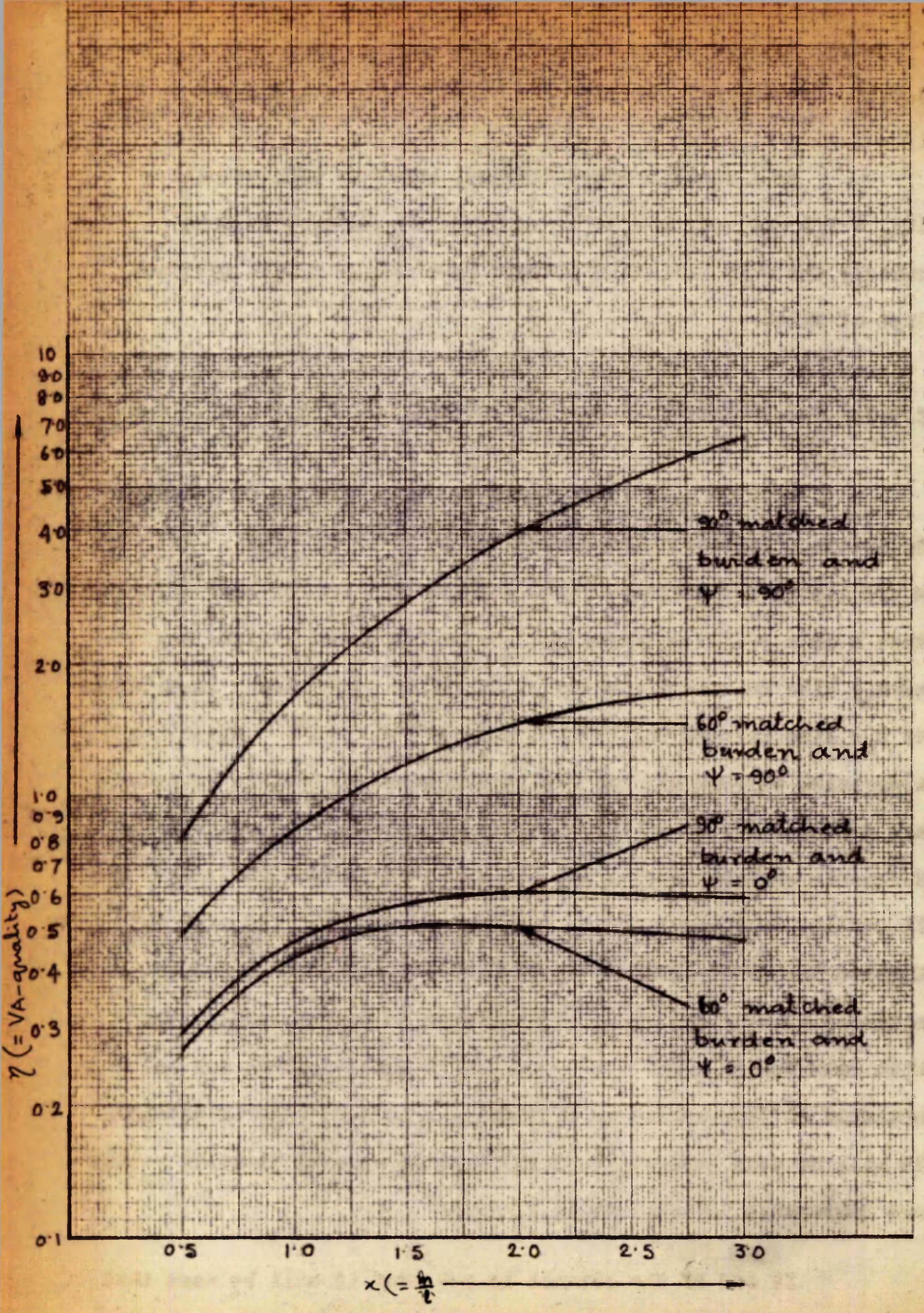

10
9·0
8·0
7·0
6·0
5·0
4·0
3·0
2·0
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
η (= VA-quality)
0·5
1·0
1·5
2·0
2·5
3·0
x (= h/l
90° matched burden and ψ = 90°
60° matched burden and ψ = 90°
90° matched burden and ψ = 0°
60° matched burden and ψ = 0°

FIG G4.

3.9.1c TWO-ELEMENT FILTERS WITH STIMULI $l(I_a - I_b)$ & $m(I_b - I_c)$

By a similar investigation, networks with stimuli $l(I_a - I_b)$ and $m(I_b - I_c)$ (Fig. 3.3c) can be seen to have exactly the same qualities as have been achieved with previous type of filters.

3.9.1d TWO-ELEMENT FILTERS WITH STIMULI $l(I_a - I_b)$ & $m(I_c - I_a)$

In this case (Fig. 3.3d) the condition to be satisfied to obtain a negative sequence filter is $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}} \angle -90°$. If l and m are positive quantities, i.e. stimuli flow in the same direction then x is a positive quantity. In such a case ψ'(i.e. angle of Z_1) lies between 0 and -90°, i.e. $0° \geq \psi' \geq -90°$ and ψ (i.e. angle of Z_2) lies between 90° and 0°, i.e. $90° \geq \psi \geq 0°$. Effecting a compromise between all the qualities, networks of this category are by far the best of two-element series type filters. Their frequency quality is always equal to .5 (AppendixX (10!)). The VA quality achieved is quite high, e.g. with -90° and with 60° matched burden the maximum VA quality (Fig. G4) which can be achieved at this condition by varying x is approximately .51. With same burden and $\psi = 0°$, maximum VA quality which can be achieved at this condition by varying x is 1.78 (approximately). It will be seen later (Art. 3.10) that as regards VA-quality they are comparable to bridge-type filters.

If one of the stimuli is reversed it will be seen that

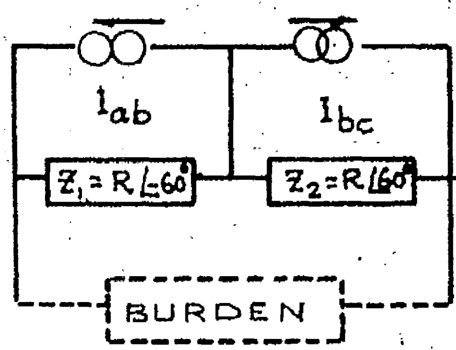


FIG 3.5

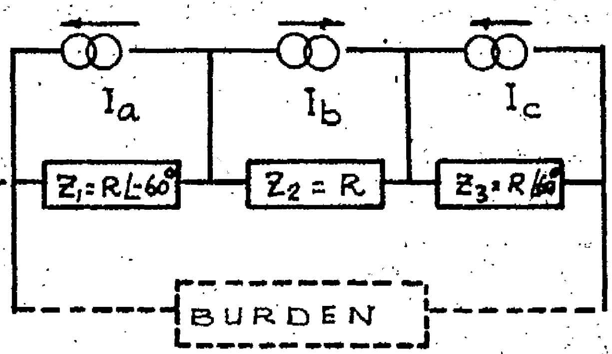
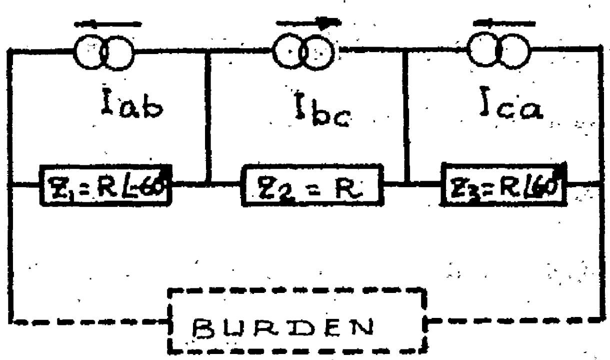


FIG 3.5a.

FIG 3.5b.



the similar result can be achieved.

3.9.2 THREE-ELEMENT SERIES FILTERS:

Here, as a result of the increased number of variables, a greater number of filters can be devised. As an example, let the Filter (1e) of Table 1 (Fig. 3.5), which has a poor frequency quality (.866), be considered. Because it is a negative sequence filter, the open circuit output voltage is independent of positive sequence components, and the general expression can be written as $E_{oc} = I_{ab}R\angle{-60°} - I_{bc}R\angle{60°}$. Re-orienting,

$$
\begin{aligned}
E_{oc} &= (I_a - I_b)R\angle{-60°} - (I_b - I_c)R\angle{60°} \\
&= I_aR\angle{-60°} - I_b(R\angle{-60°} + R\angle{60°}) + I_cR\angle{60°} \\
&= I_aR\angle{-60°} - I_bR + I_cR\angle{60°}
\end{aligned}
$$

This expression, with three items, immediately suggests that if I_a, $-I_b$ and I_c are passed through $R\angle{-60°}$, R and $R\angle{60°}$ respectively, and added together, an alternative to the above filter can be obtained and this leads to a three-element filter as shown in Fig. (3.5a).

It can be noted that many such two-element filters can be converted to three-element filters.

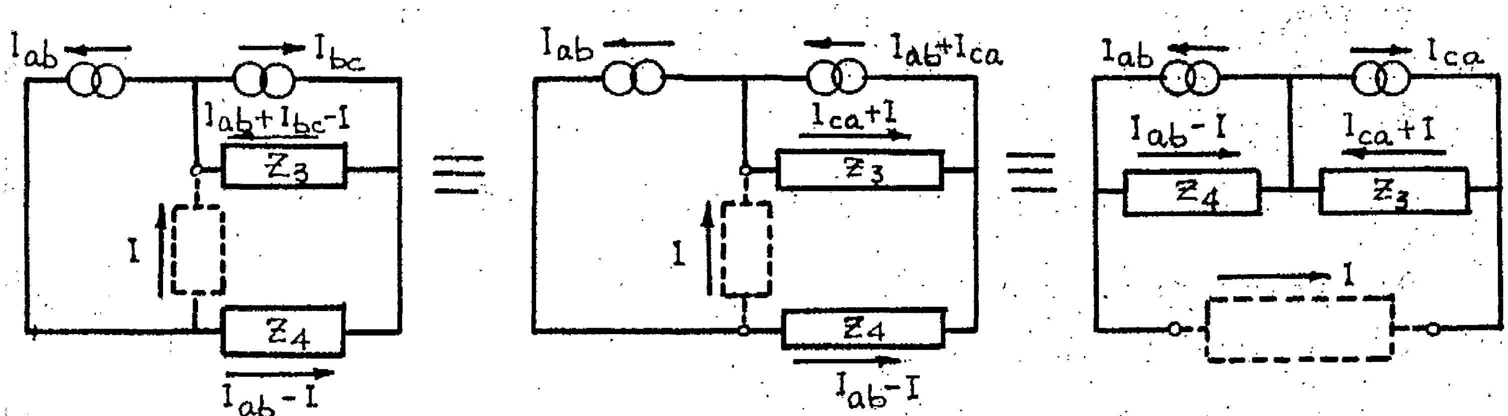
In the above arrangement it can be seen that ''frequency quality'' improves to .29 (Table 1) and the other qualities remain satisfactory. However, zero-sequence error is introduced. It can be verified from eqn. (4j) (Art. 3.6.1b) that to obtain a negative sequence filter with such input currents, the following relations must hold good:

$Z_1 : Z_2 : Z_3 = \angle{-60^\circ} : 1 : \angle{60^\circ}$. Zero-sequence frequency error can be eliminated by replacing I_a, I_b and I_c by I_{ab}, I_{bc} and I_{ca} respectively as shown in Fig. 3.5b. Even with this improvement it will be found later (Art. 3.10) that this is not as good as some of the bridge filters.

When the input currents are I_{ab}, $-I_{bc}$, I_{ca}, the following relation (Appendix XI) must hold good to obtain negative-sequence filter:

$$1 + m_1\angle{-60^\circ} + m_2\angle{60^\circ} = 0 \quad \text{where } m_1 = \frac{Z_1}{Z_2} \text{ and } m_2 = \frac{Z_2}{Z_3}.$$

This equation is similar to that of eqn. (5b) (Art. 3.6.2) of bridge-type filter. In the bridge type there are four impedance elements (Z_1, Z_2, Z_3 & Z_4) which are related to a similar equation where $m_1 = \frac{Z_1}{Z_2}$ and $m_2 = \frac{Z_4}{Z_3}$ (equ. 5b). In the series type there is a common term Z_2 in both m_1 and m_2, whereas in the bridge type there is no such common term in m_1 and m_2. So the flexibility of choice of m_1 and m_2 in bridge type is greater than that in series type. A treatment with such wider range of choice in bridge-type is given in the next article (Art. 3.10). As a matter of fact, fulfilment of overall good qualities can be achieved in bridge type filters quite successfully. Again as regards simplicity, it can be appreciated later (Art. 3.11) that in actual practice three-element filters cannot be simpler than two-element series type. So further rigorous treatment to three-element series filters is not given but two filters as described



I = BURDEN CURRENT.

FIG 3.6

above are given in Table 1 for comparison. Of course, in their design the emphasis is on ''simplicity'' as in other filters.

3.10 DESIGN OF BRIDGE TYPE FILTERS

Since there are four impedance variables in bridge type filters, it is possible to design innumerable filter networks with various combinations of these four variables. In filters with I_{ab} and I_{bc} as the two input stimuli, the condition to be satisfied is (Art. 3.6.2) $1 + \dot{m}_1\angle{-60^\circ} + \dot{m}_2\angle{60^\circ} = 0$ where $\dot{m}_1 = \frac{Z_1}{Z_2}$ and $\dot{m}_2 = \frac{Z_4}{Z_3}$ are complex variables. $\dot{m}_1$ and $\dot{m}_2$ can lie anywhere in the four quadrants of the impedance plane depending on the values of Z_1, Z_2, Z_3 and Z_4; thus the possibility of designing innumerable filter networks is clear from above. The main aim is to derive networks which possess good qualities while maintaining simplicity of circuitry. As a first step, assume $\dot{m}_1$ as zero which immediately gives Z_1 as zero or Z_2 as infinity; thus to obtain a negative sequence filter the relation $1 + \dot{m}_2\angle{60^\circ} = 0$, i.e. $\frac{Z_4}{Z_3} = \angle{120^\circ}$ must be satisfied. Such a filter is equivalent to a series type filter as shown in Fig. (3.6). Filters of such nature have already been discussed previously (Art. 3.9.1a). As the next step, assume $\dot{m}_1 = m\angle{\phi_1}$ and $\dot{m}_2 = m\angle{\phi_2}$, $\dot{m}_1$ and $\dot{m}_2$ have the same magnitude m. With such an arrangement, to obtain a negative sequence filter, the relations (1) $-\phi_1 = \phi_2$ (=ϕ say) and (2) $1 + 2m\cos(60^\circ + \phi) = 0$ must be satisfied (Appendix X11(a)12c). Relation $1 + 2m\cos(60^\circ + \phi) = 0$ traces a locus of a straight line

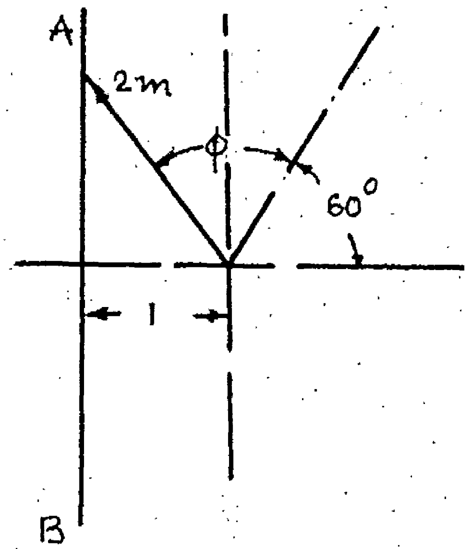
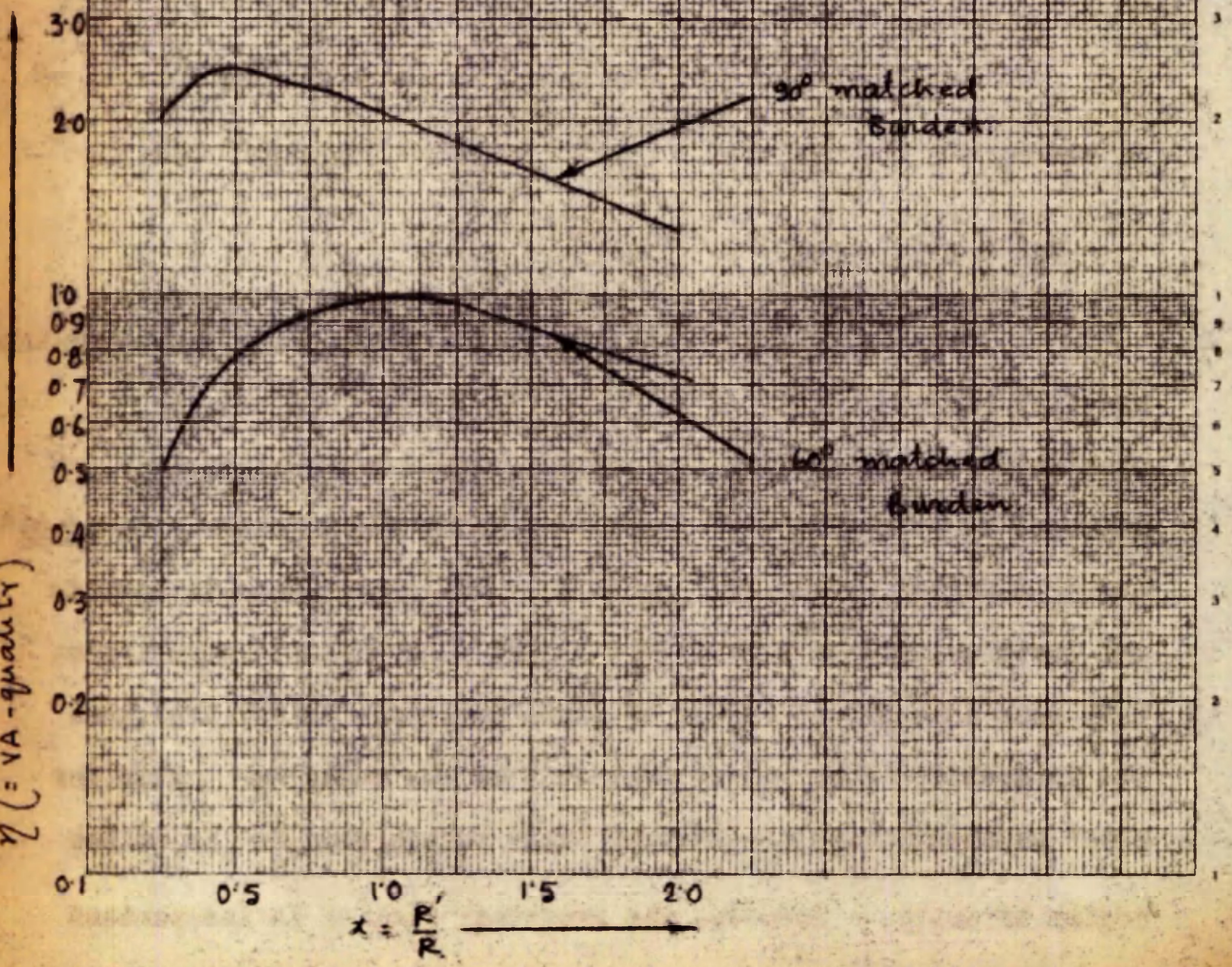


FIG 3.6a.


3·0
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1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·5
1·0
1·5
2·0
90° matched Burden.
60° matched Burden.
η (= VA-quality)
x = R'/R

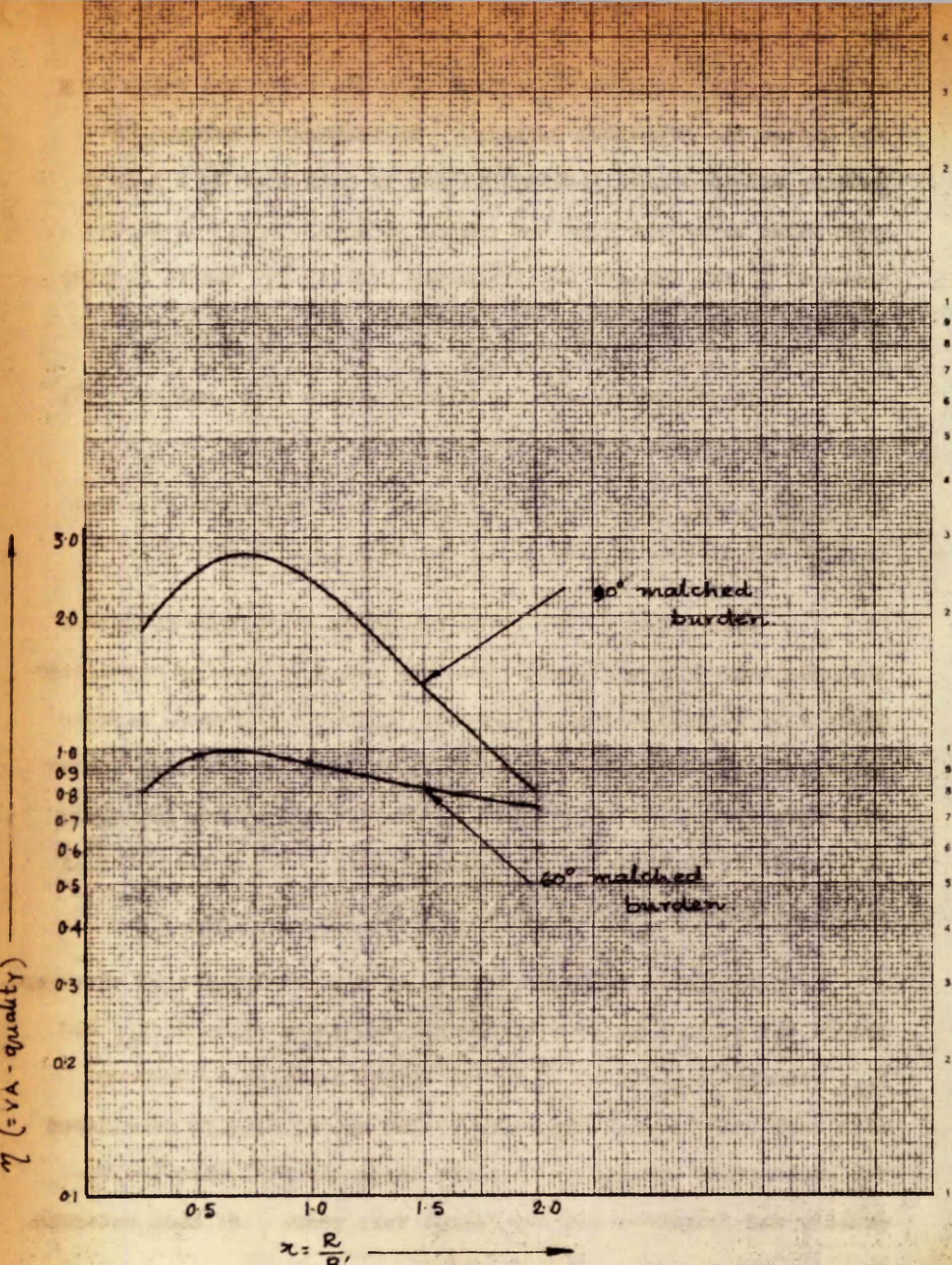
FIG G 5.

AB as shown in fig. (3.6a) from where it can be seen that 2m can possess a minimum value of unity and the maximum can extend to infinity on either side (positive or negative). The greater the value of m, the more difficult it will be to co-ordinate the values $Z_1 : Z_2$ and $Z_4 : Z_3$; so with limiting value of m (up to unity) a few suitable filter networks with (1) $m = {}^1/2$, $\phi = 120°$, (2) $m = \frac{1}{\sqrt{3}}$, $\phi = 90°$ and (3) $m = 1$, $\phi = 60°$ are considered as given in Table 2. It can be seen from the table that filters with $m = 1$, $\phi = 60°$ have a poor performance in VA-quality and frequency quality compared with the first two kinds of filters. Consequently they will not be discussed further.

In the first kind with $m = {}^1/2$, $\phi = 120°$, the values of m_1 and m_2 are given by $\dot{m}_1 (= \frac{Z_1}{Z_2}) = m\underline{/-120°} = {}^1/2\underline{/-120°}$ and

$$\dot{m}_2 (= \frac{Z_4}{Z_3}) = m\underline{/120°} = {}^1/2\underline{/120°}.$$

Capacitors are available practically without any impurities and they can be used as single elements. Thus not only is simplicity maintained but the internal impedance quality is improved by making it of a capacitive nature. Consider $Z_1 = R\underline{/-90°}$ and $Z_2 = 2R\underline{/30°}$, and $Z_4 = R'\underline{/30°}$ & $Z_3 = 2R\underline{/-90°}$ chosen to satisfy the above conditions where R and R' are real positive quantities. With such arrangement it can be shown (Appendix X11(b) 12q) that the VA-quality (Fig. G5) is dependent on the ratio $\frac{R'}{R}$. Thus for a 60° matched burden the maximum value occurs when $\frac{R'}{R}$ is in the region of unity. However, the frequency quality is independent



90° matched burden.
60° matched burden.
3·0
2·0
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0·5
1·0
1·5
2·0
η (= VA - quality)
x = R/R'

FIG G6.

of $\frac{R'}{R}$. Again it is to be noticed that the lower the value of x the better the impedance quality. By compromise between VA-quality and impedance quality, it can be said that $\frac{R'}{R} = 1$ is a good value to choose from the design point of view. A lower value of $\frac{R'}{R}$ may improve the impedance quality but the VA quality suffers.

In the second kind of filter with $m = \frac{1}{\sqrt{3}}$ and $\phi = 90°$, and with impedance values of $Z_1 = R\angle{-90°}$, $Z_2 = \sqrt{3}R$ and $Z_3 = \sqrt{3}R'\angle{-90°}$ and $Z_4 = R'$, it can be shown (Appendix X111, Fig. G6) that $\frac{R'}{R} = 1$ is also a good value for this type of network.

Comparison of the above two kinds of filters indicates that the second is preferable from the point of view of simplicity since only capacitance and resistance elements are used, whereas the first requires capacitance and inductive elements, the latter including resistance also. The first kind of network is better as regards VA-quality and frequency-quality. From the point of impedance quality / second kind is preferable.

To give a complete picture of the above types of networks, one of the stimuli is reversed, i.e. instead of stimulii I_{ab} and I_{bc}, stimulii I_{ab} and $-I_{bc}$ are fed in the network. A network, with such stimuli, corresponding to the second kind is considered for explanation and it can be shown (Appendix X1V) that the VA-quality and frequency quality become very poor. So such networks do not warrant further consideration.

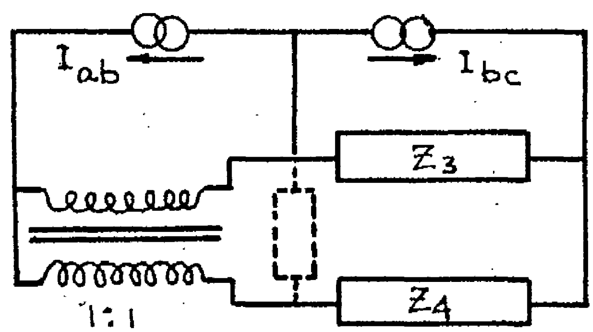


FIG 3.6b.

If, however, $m_1 = 1$, i.e. $Z_1 = Z_2$, then from equ.(5b) (Art. 3.6.2) $\frac{Z_4}{Z_3} = \sqrt{3}\angle 90°$. For the condition where the output is short-circuited (to obtain Norton's equivalent supply source), the currents passing through Z_1-branch and Z_2-branch are equal and each of them is equal to $^1/2I_{ab}$. The same result can be obtained by using auto-transformer with two windings of equal number of turns as shown in Fig. (3.6b). The use of an auto-transformer instead of impedance element improves the VA-quality as mentioned below. The values of Z_3 & Z_4 are chosen as $R\angle -90°$ & $\sqrt{3}R$; the obvious reason for choosing a capacitance element being for the improvement of impedance quality. The qualities of such a circuit are given in Table 2.

Now instead of taking $m_1 = 1$, let us take $m_2 = 1$, and use an auto-transformer with equal winding, as in the previous case. In this case, m_1 ($= \frac{Z_1}{Z_2}$) $= \sqrt{3}\angle -90°$ to satisfy the equ. (5b). The qualities of such a circuit (with $Z_1 = \sqrt{3}R\angle -90°$ and $Z_2 = R$) are given in Table . It will be seen that the VA- quality and impedance quality of this filter is better than the previous one, however frequency quality is the same for both.

If two equal capacitances are used for Z_1 and Z_2 instead of an auto-transformer then, as shown in Table 2, Filter 2g, though the impedance quality is improved, the VA-quality is lowered. Moreover when the use of intermediate transformer is necessary it will be seen that the circuitry of the filter using an auto-transformer becomes simple (Art. 3.11, Fig. 3.11b).

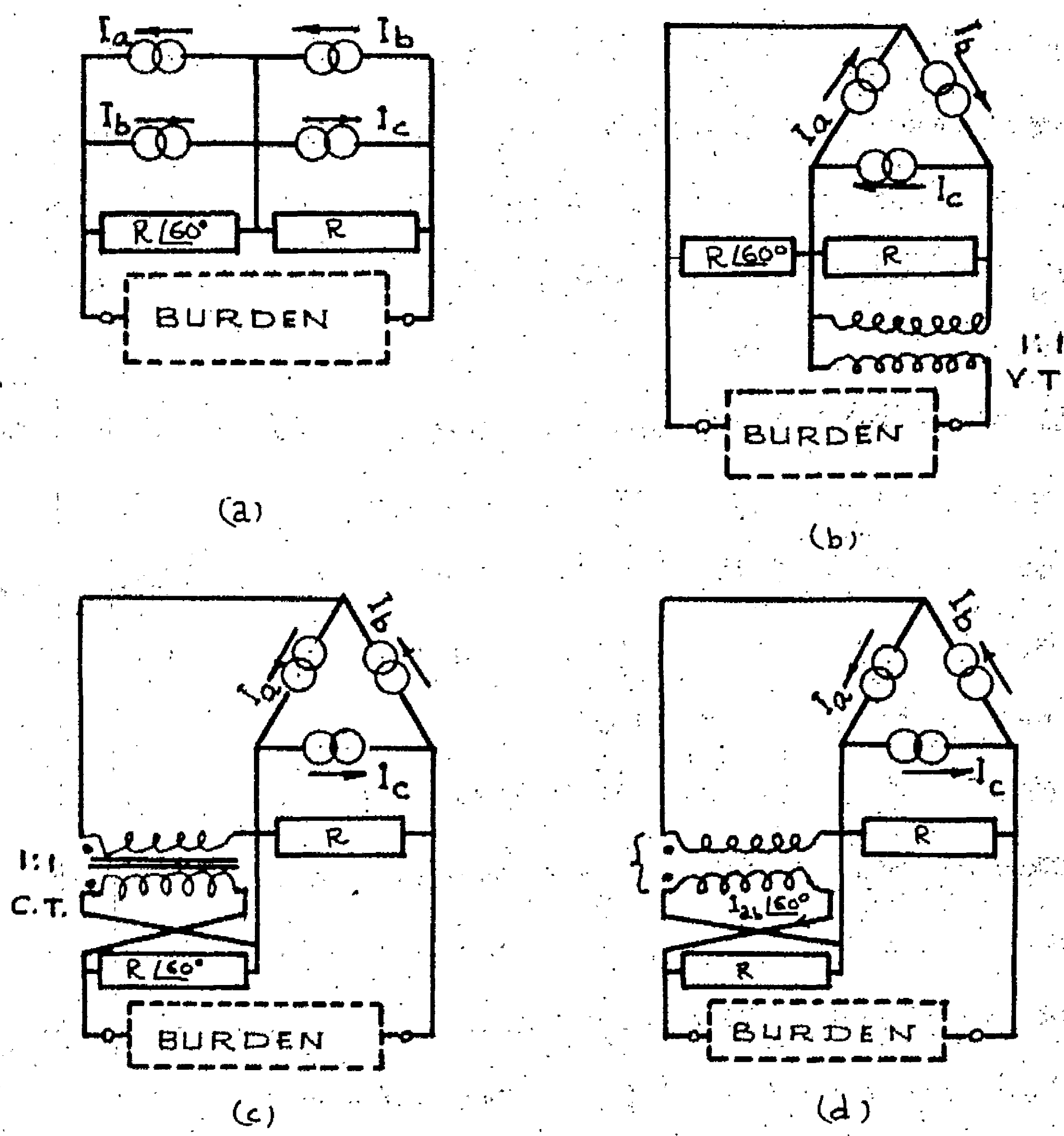

Ia
Ib
Ib
Ic
R∠60°
R
BURDEN
(a)
Ia
Ib
Ic
R∠60°
R
1:1
V.T
BURDEN
(b)
Ia
Ib
Ic
R
1:1
C.T.
R∠60°
BURDEN
(c)
Ia
Ib
Ic
R
R
BURDEN
(d)

FIG 3.7.

3.11 CHOICE OF FILTERS AND THEIR CIRCUITRY AS USED IN PRACTICE

It is to be noticed from Table 1 & 2, Art. 3.9 & 3.10, that it has not been possible to achieve all the best of the qualities simultaneously in any filter. So the following choice of filters is based on a compromise on qualities with 'simplicity' as the basic factor.

It has been explained before that among the two-element series filters, those with inputs $l(I_a-I_b)$ and $m(I_c-I_o)$ are best (Art.3.9.1d). Even the very popular type of filters, which have inputs as $l(I_a-I_b)$ and $m(I_b-I_c)$ (Art.3.9.1a) do not possess such overall good qualities as can be found in the above mentioned filters, e.g. Allcutt's filter (Filter 1a, Table 1). Allcutt's filter (named after the inventor) is no doubt a simple one, (different arrangements of Allcutt's filter-circuitry are shown in Fig. 3.7), but the previous mentioned filter is even simpler. Comparison of such filters can be found in Table 1.

There can be many choices, with different combinations of impedance elements, from filters with inputs $l(I_a-I_b)$ and $m(I_c-I_o)$, but taking 'simplicity' into consideration, a filter with a mutual inductance and resistance as the impedance elements is chosen as shown in Fig.3.8 corresponding to filter 1g , Table 1. For convinience of explanations in next sections, the stimuli I_a, I_b, I_c in filter 1g (Tabke 1) are replaced by I_b, I_c, I_a (derived from delta-connected C.T.s) respectively in Fig 3.8 ; by this change, the characteristics of the filter are not effected . In Article 3.9.1d it has been shown that the maximum VA-quality with this arrangement is .51 (for 60° burden) when $x = \sqrt{3}$. Impedance quality at this condition is 45°. Now this impedance quality can

FILTER NO 1.

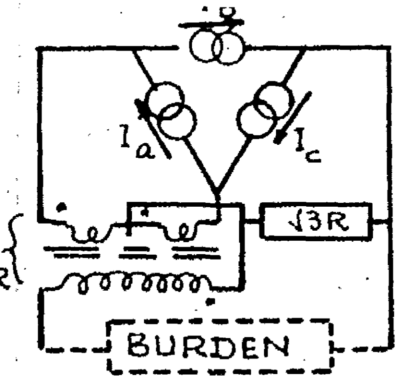


FIG 3.8

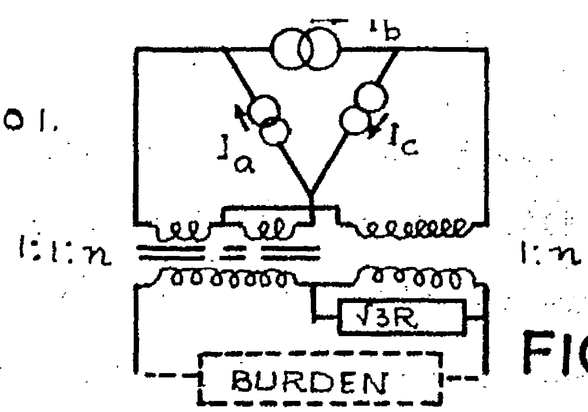


FIG 3.8 a

FILTER NO 2

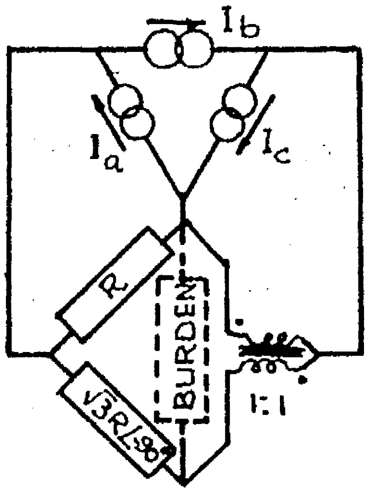


FIG 3.9

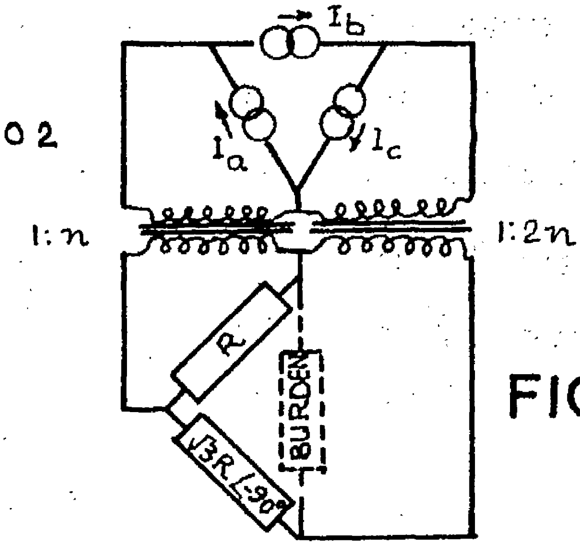


FIG 3.9 a

FILTER NO 3

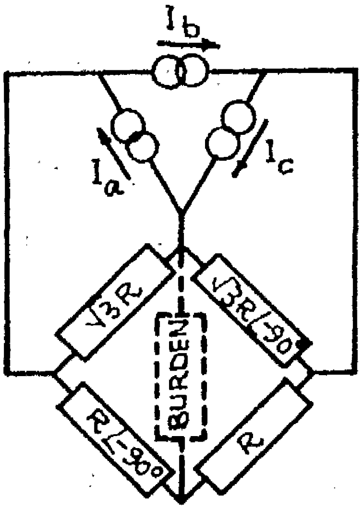


FIG 3.10

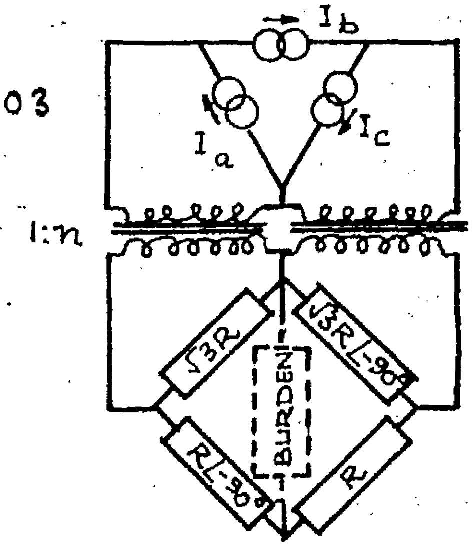


FIG 3.10 a

FILTER NO 4.

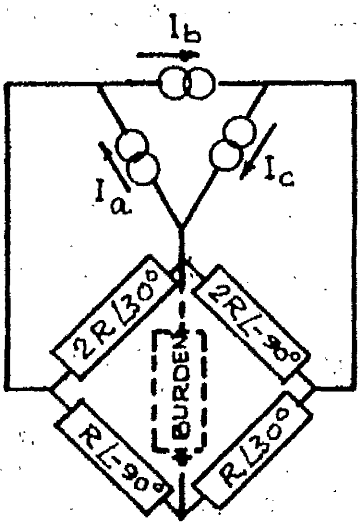


FIG 3.11

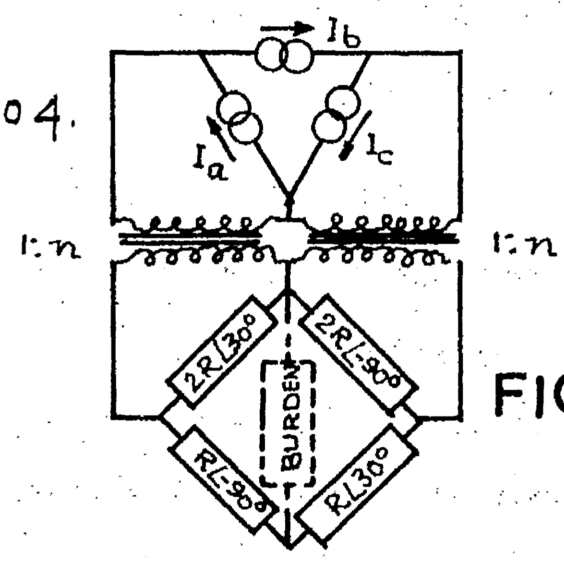


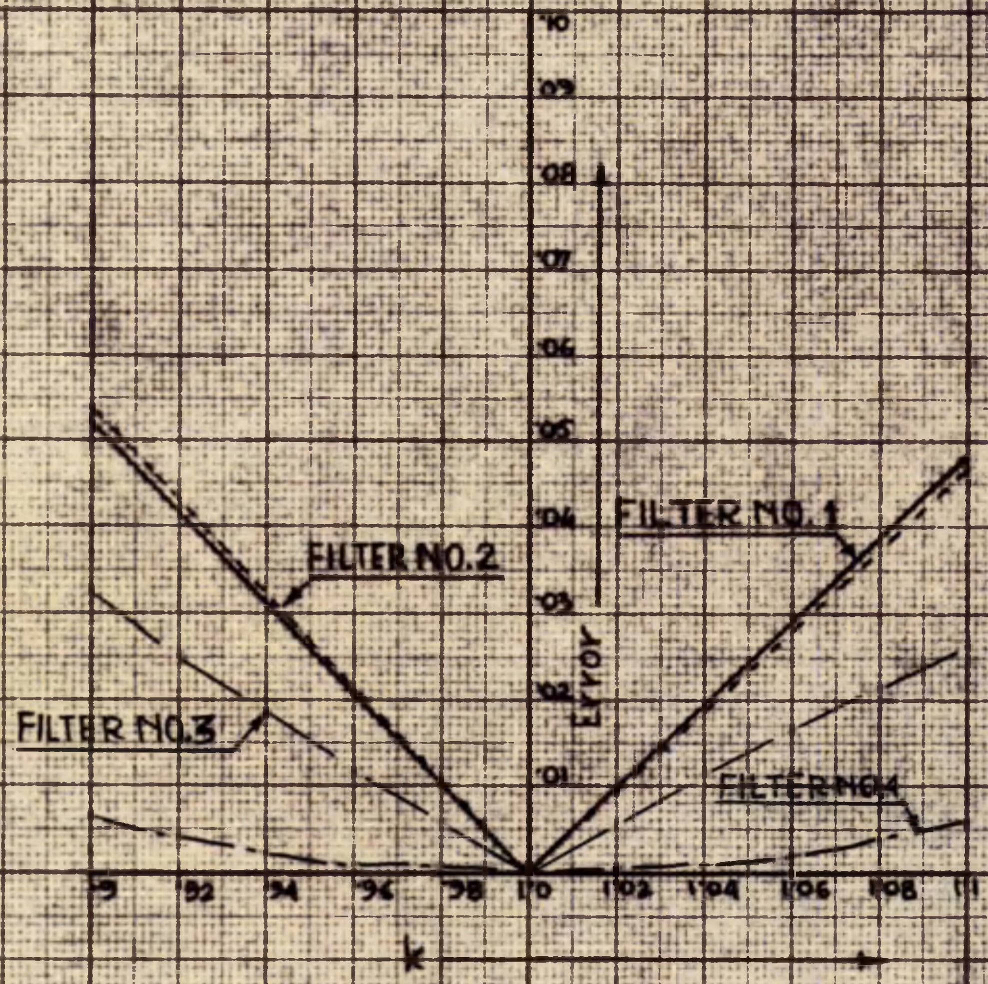
FIG 3.11 a

be bettered to 30° if x is increased to 3, but with a little sacrifice of VA-quality whose value is .463 at this new condition. So the selected filter has impedances $Z_1 = \sqrt{3}R$ and $Z_2 = jR$ (mutual reactance). It is to be noted here that the values of impedance quality and VA-quality are approximate in that the small resistance and leakage reactance of the mutual inductor-coil are neglected. This is to simplifyycalculations. For future reference this filter will be called as ''Filter No. 1''. Variants of this type are widely used (e.g. three phase or combined filters, Section five) and are unique in their simplicity.

Some of the bridge filters are chosen for their high qualities. Firstly, Filter 2f, Table 2, has been chosen for its high impedance and VA-qualities, though its frequency quality is .5, which is not better than the Filter No. 1. It has been shown in Fig. (3.9) and for future reference it will be called as ''Filter No. 2''.

Secondly, Filter 2c, Table 2, has been chosen for its reasonably good qualities in every item. Frequency quality is .29. The filter is shown in fig. (3.10) and for future reference it will be called as ''Filter No. 3''.

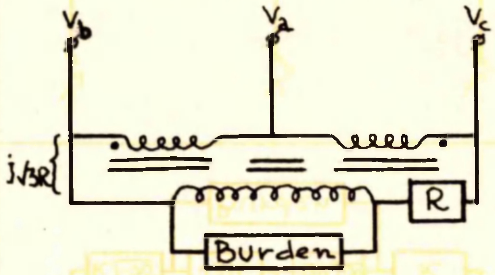
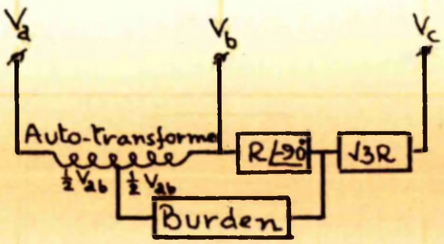
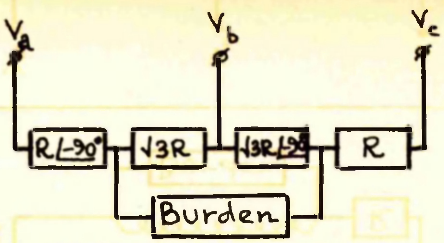
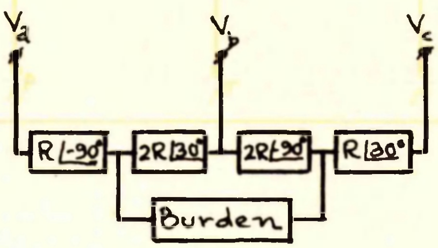
Thirdly, Filter no. 2d, Table 2, is chosen primarily because the ''frequency quality'' is zero with other qualities also very satisfactory. However, as far as circuitry is concerned, it is certainly inferior to the previous three filters. From a comparison with Filter No. 3 it can be seen, as explained



·10
·09
·08
·07
·06
·05
·04
·03
·02
·01
Error
FILTER NO. 1
FILTER NO. 2
FILTER NO. 3
FILTER NO. 4
·9
·92
·94
·96
·98
1·0
1·02
1·04
1·06
1·08
1·1
k

FIG. G.7

TABLE 3

Selected Negative Sequence Current Filters	Corresponding Negative Sequence Voltage Filters	Impedance quality	VA-quality		Frequency quality
			60° matched burden	90° matched burden	
Filter No.1		÷ 30°	÷ 0.61	÷ 0.77	0.5
Filter No.2		-60°	1.732	6.45	0.5
Filter No.3		-45°	0.94	2.38	0.29
Filter No.4		-30°	1.0	2.0	0

NOTE : The above filters are designed from the principle of duality, but they are not exactly dual circuits of the selected four current filters. Circuit constants are so chosen (maintaining the simplicity) that the qualities of these filters become as comparable as those of corresponding current filters.

before (Art. 3.10), that it has two resistors, two capacitors and two reactors as impedance elements whereas Filter No. 3 has two resistors and two capacitors. It is emphasised again that this filter can be chosen when high frequency quality is wanted with other satisfactory qualities. This filter for future reference will be called as ''Filter No. 4'' and is shown in Fig. (3.1).

Frequency error curves of the above four filters are shown in Fig. Q7.

Suitable arrangements of the above filter circuits as used in practice are shown in Figs. 3.8a, 3.9a, 3.10a & 3.11a. It has to be noted that intermediate transformers are necessary when the input currents are to be further reduced for reduction of relay size. In Filter No. 1, one three-winding mutual reactor can serve both the purpose of intermediate transform and mutual inductor by proper adjustment of the windings and the air-gap in the iron core.

3.12 NEGATIVE SEQUENCE VOLTAGE FILTERS

3.12 NEGATIVE SEQUENCE VOLTAGE FILTERS

Corresponding to above four-selected current filters, voltage filters are obtained from the principle of duality (Table 3). This has already been explained in Art. 2.3

The coils of the mutual inductor in voltage ''Filter No. 1'' should be wound on different cores of the magnetic circuit (Ref. 5) to avoid feedback between the two primary coils.

SECTION 4

4.1 TRANSIENT PROBLEMS

A filter is designed essentially to give a specific output with a sinusoidal input at one specific frequency. Transient waves are far from sinusoidal in nature, so it is likely that a different response than the desired one will be obtained from the filter. Thus during a three-phase symmetrical short circuit, a transient output will be obtained from a negative sequence filter even though under such condition no negative sequence component is present in the system. Similarly, during any other switching condition, a transient response is always present in the filter. It will be confined here to investigating the behaviour of negative sequence filters during a three-phase simultaneous symmetrical short circuit, because it has been found in practice that the presence of a transient wave due to such short circuits is sometimes important, e.g. unblocking of the main protection by a power-swing relay which is operated by a negative sequence filter, during a three-phase symmetrical short circuit with or without swing depends to some extent on the transient response (Ref. 5, Art 6.5). The transient response of the four selected filters (Art 3.11, Figs. 3.8a, 3.9a, 3.10a, 3.11a) has been investigated experimentally (Arts. 4.3, 4.4, 4.5). Theoretical treatment of the transient response of three of the four filters is mathematically very difficult and would probably require the use of a computer. The first filter however has been treated mathematically (Appendix XVI) . The response of this

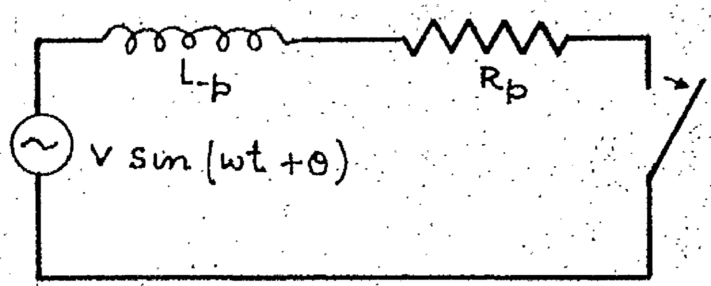


FIG4.1

circuit obtained experimentally was in close agreement with that predicted.

4.2 TRANSIENT RESPONSE TO THREE PHASE SYMMETRICAL SHORT CIRCUITS:-

4.2.1 TRANSIENT PRIMARY CURRENTS

The effect of transients is marked in high speed protective-gear operation. ''High speed'' means of the order of 3 cycles, and it is in this period that a transient predominates in power systems. For this period it is quite sufficient to make the simplifying assumption that the system comprises a constant impedance $(R_p + j\omega L_p)$ where ωL_p is a pessimistic reactance value and R_p is a resistance value of the system (Ref. 4) (Fig. 4.1). By ''pissimistic'' is meant that if stability of such gear is sought then the primary fault current has to be specified from the appropriate value of subtransient reactance of the generators of the system and if operation of such gear is sought then the primary fault current has to be specified from the appropriate value of synchronous reactance of the generators of the system. For further simplification other things assumed are:- (a) all the capacitance effect of the system are neglected and (b) the intial current flowing at the instant of switching is zero. Now if $\hat{V}$ Sin $(\omega t + \theta)$ represents the voltage inphase A of the balanced three-phase system then the currents flowing in three different lines under symmetrical three-phase short circuit condition are given by the following relations (Appendix XV1(a)):-

$$i_a = \frac{\hat{V}}{z_p} \quad \mathrm{Sin}(\omega t + \theta - \phi) - e^{-\gamma t}\mathrm{Sin}(\theta - \phi) \qquad \text{---(16c)}$$

$$i_b = \frac{\hat{V}}{z_p} \quad \mathrm{Sin}\ (\omega t + \theta - 120°) - e^{-\gamma t}\mathrm{Sin}(\theta - \phi - 120°) \qquad (16d)$$

$$i_c = \frac{\hat{V}}{z_p} \quad \mathrm{Sin}(\omega t + \theta - \phi + 120°) - e^{-\gamma t}\mathrm{Sin}(\theta - \phi + 120°) (16e)$$

where $z_p = \sqrt{\left[R_p^{\,2} + (\omega L_p)^2\right]}$ and $\phi = \tan^{-1} \frac{wL_p}{R_p}$ and $\gamma = \frac{R_p}{L_p}$

4.2.2 TRANSIENT OUTPUT OF THE FILTER NO. 1

Taking an ideal condition the output current of such a filter due to above transient currents (eqn. 16c, 16d, 16e) is given by the following expression(Appendix XV1(b))

$$i = \frac{3\hat{V}R}{\omega L_p} \cdot \frac{1}{L_s(\ \gamma' - \gamma\ )} \quad \left[e^{-\gamma t} - e^{-\gamma' t}\right] \mathrm{Sin}\ \theta \qquad (16n)$$

where $\gamma' = \frac{R_s}{L_s}$

L_s = Inductance of the Thevenin's equivalent circuit

R_s = Resistance of the Thevenin's equivalent circuit

R = Ohmic reactance value of the mutual inductor of the filter.

It will be found from the above expression that the transient output depends on the switching point on the normal sinusoidal wave, e.g. it is zero when $\theta = 0$ and it becomes maximum when $\theta = \pm \pi/2$. So the maximum transient output is given by

$$i = \frac{3\hat{V}R}{\omega L_p} \cdot \frac{1}{L_s(\ \gamma' - \gamma)} \quad \left[e^{-\gamma t} - e^{-\gamma' t}\right]$$

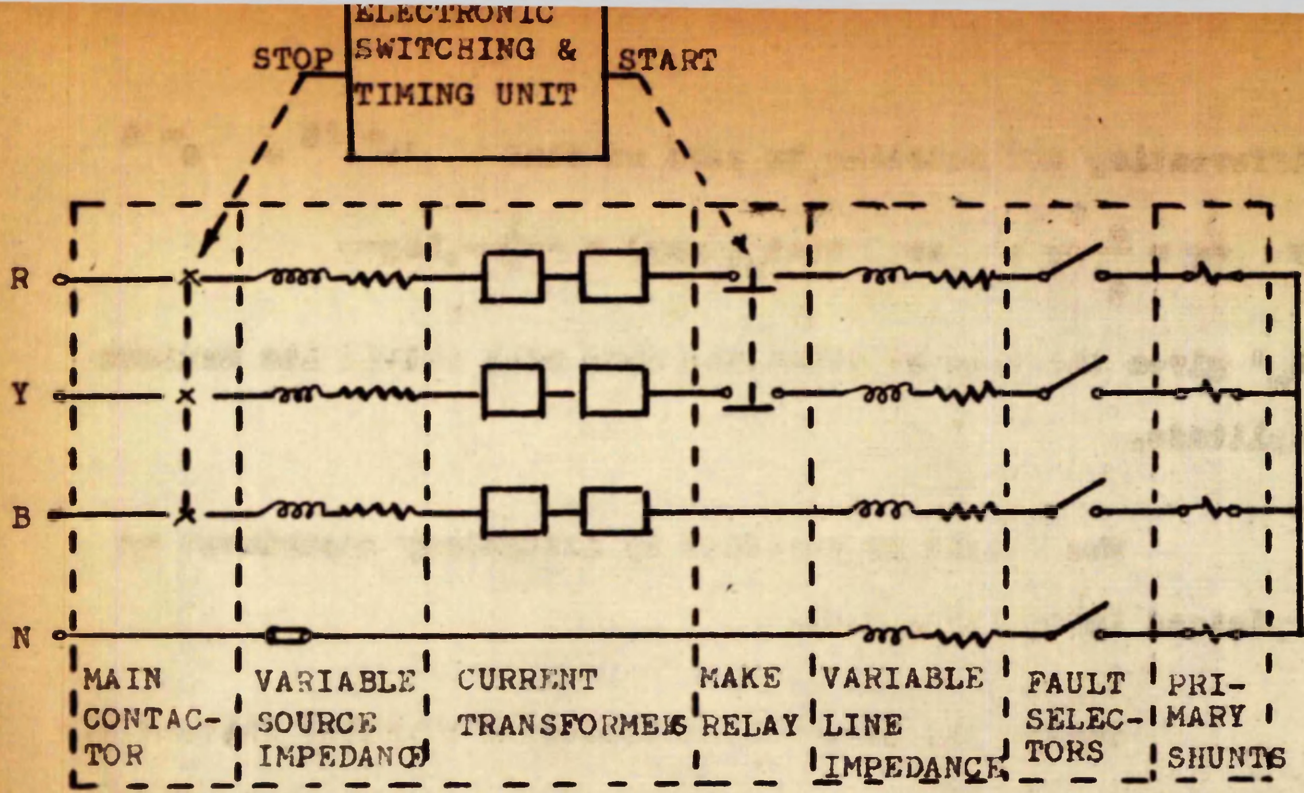
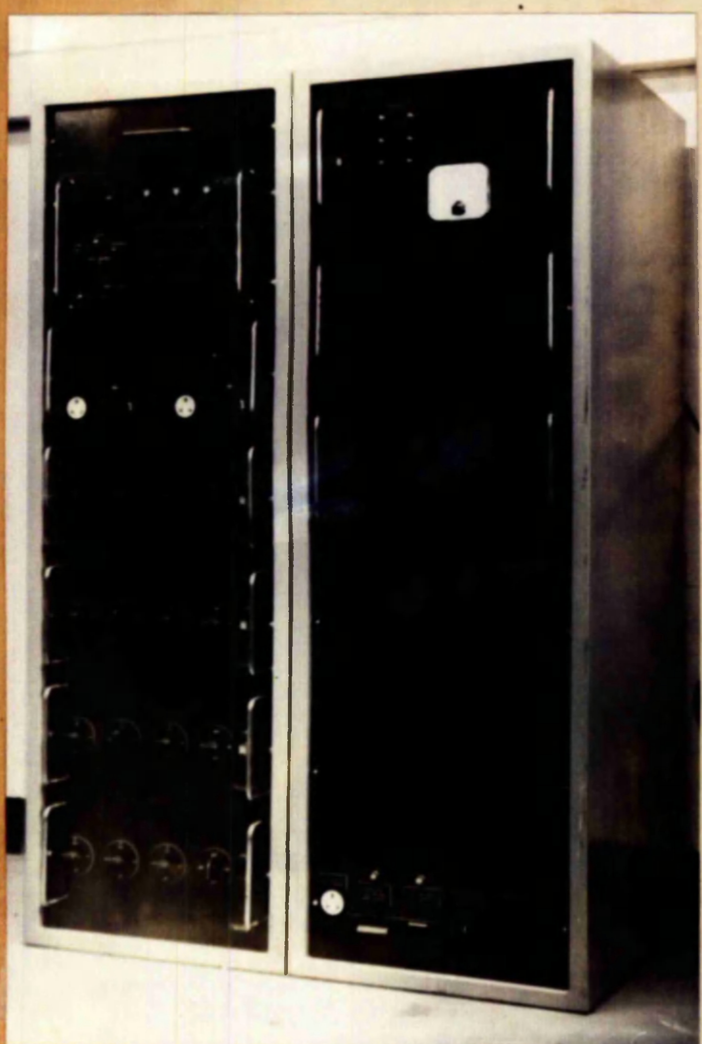


FIG 4.2a.



NOTE : Fault-selector switches are selected for three-phase fault (Neutral is left open). Three phase simultaneous short-circuit is applied by the simultaneous operation of make-relays in phases R & Y.

FIG 4.2.

Differenting and equating to zero we find $\gamma' e^{-\gamma' t} = \gamma e^{-\gamma t}$

or $\frac{\gamma}{\gamma'} = \frac{e^{\gamma t}}{e^{\gamma' t}}$; or $t (= t_m, \text{ say}) = \frac{1}{\gamma - \gamma'} \cdot \text{Log} \frac{\gamma}{\gamma'}$

't_m' gives the time at which the wave will obtain its maximum amplitude.

The result is verified by laboratoyy experiment as explained later (Art. 4.5).

Though the theoretical treatment of other filters was not possible, yet the results obtained by similar laboratory experiments are given in Table 4.

4.3 TEST EQUIPMENT

It should be emphasised that the equipment consists of components which were either already existing or could be quickly constructed. No attempt has been made to design a circuit to any specific limits of performance.

In general, apart from the filter-elements, the equipment used is assembled in an industrial type Test Bench:

(a) Test Bench:

An illustration of the test bench (excluding source reactors and main contactors) is shown in Fig. 4.2. The main primary circuit is supplied from special low-impedance 415-volt three-phase mains, and arrangement is shown in Fig.(4.2a). It can

FILTER NO 1

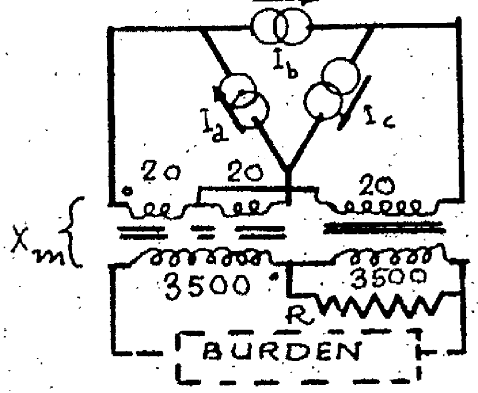


FIG 4.3 a.

R = 4540 Ω

X_m = 2620 Ω
Reffered to secondary

FILTER NO 2.

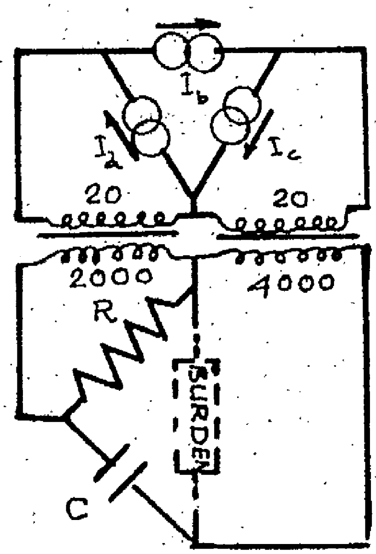


FIG 4.3 b.

R = 2770 Ω

C = 0·665 μF

X_C = 4800 Ω

FILTER NO 3.

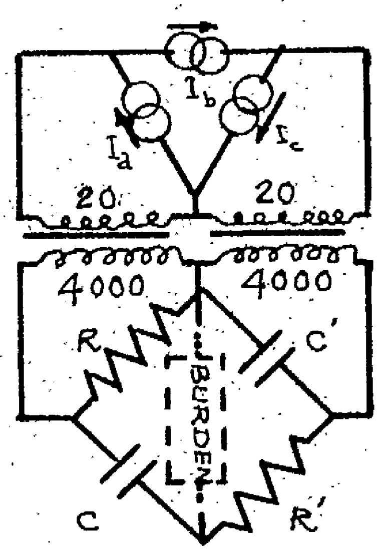


FIG 4.3 c.

R = 5400 Ω

R' = 9400 Ω

C = 0·59 μF

X_c = 5400 Ω

C' = 0·34 μF

X_c' = 9400 Ω

FILTER NO 4.

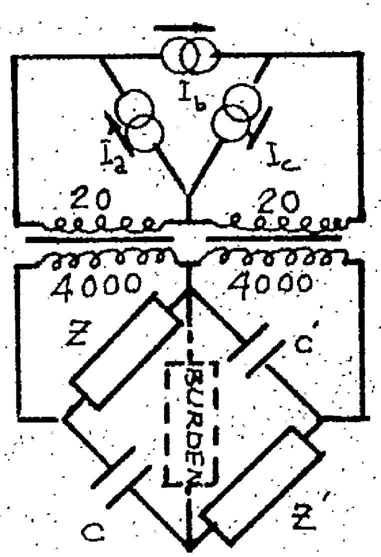


FIG 4.3 d

Z = 9600 + j 5540

Z' = 4800 + j 2770

C = 0·57 μF

X_c = 5540 Ω

C' = 0·286 μF

X_c' = 11080 Ω

be seen that the three-phase primary supply is applied through a ''fault-making'' switch, current transformer and two sets of variable impedances per phase, one of which represents a generating source impedance and the other the impedance of a transmission line. The impedances can be connected in various ways according to the particular fault distribution which it is required to reproduce.

The arrangement provides for the simulation of a wide range of fault conditions, with control of overall time-constant, the point-on-wave at which fault is applied, the type and magnitude of fault, and its duration.

(b) Filter elements:

Respective values of the impedance elements are given on the filter diagrams (Figs. (4.3a)(4.3b)(4.3c)(4.3d)). Impedance elements are so designed that they do not saturate in the working region. Their impedance values are very small in comparison to the magnetising impedance of the intermediate current transformers. The impedance burden imposed on the primary current transformers are reduced to a suitably low level by high turns-ratios of intermediate current transformers.

4.4 EXPERIMENTAL PROCEDURE

The requiremts is test procedure are, firstly, to obtain balance in supply currents and secondly to minimise inaccuracies

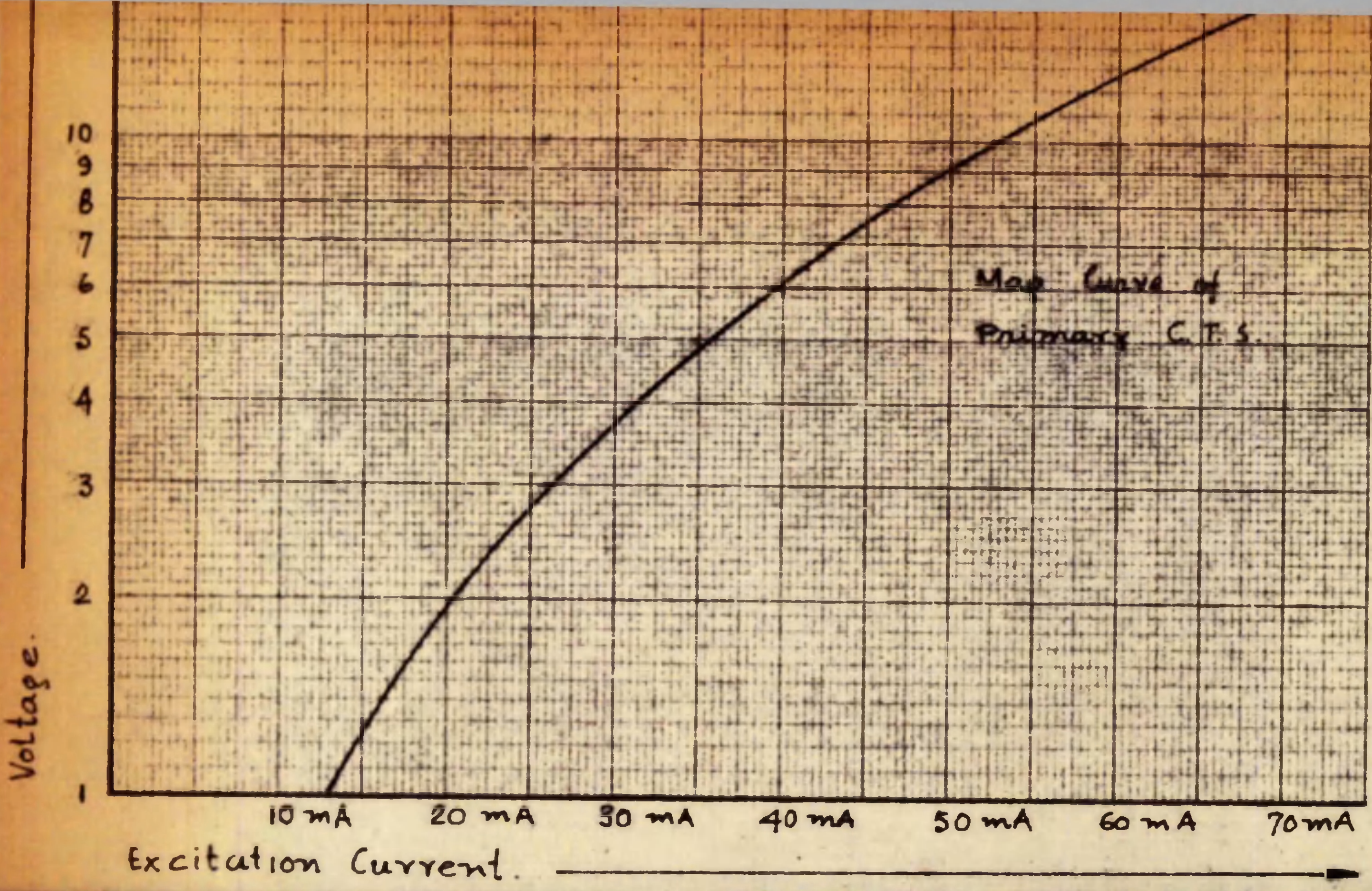

10
9
8
7
6
5
4
3
2
1
Voltage.
Mag Curve of
Primary C.T.S.
10 mA
20 mA
30 mA
40 mA
50 mA
60 mA
70 mA
Excitation Current

FIG. 4.4

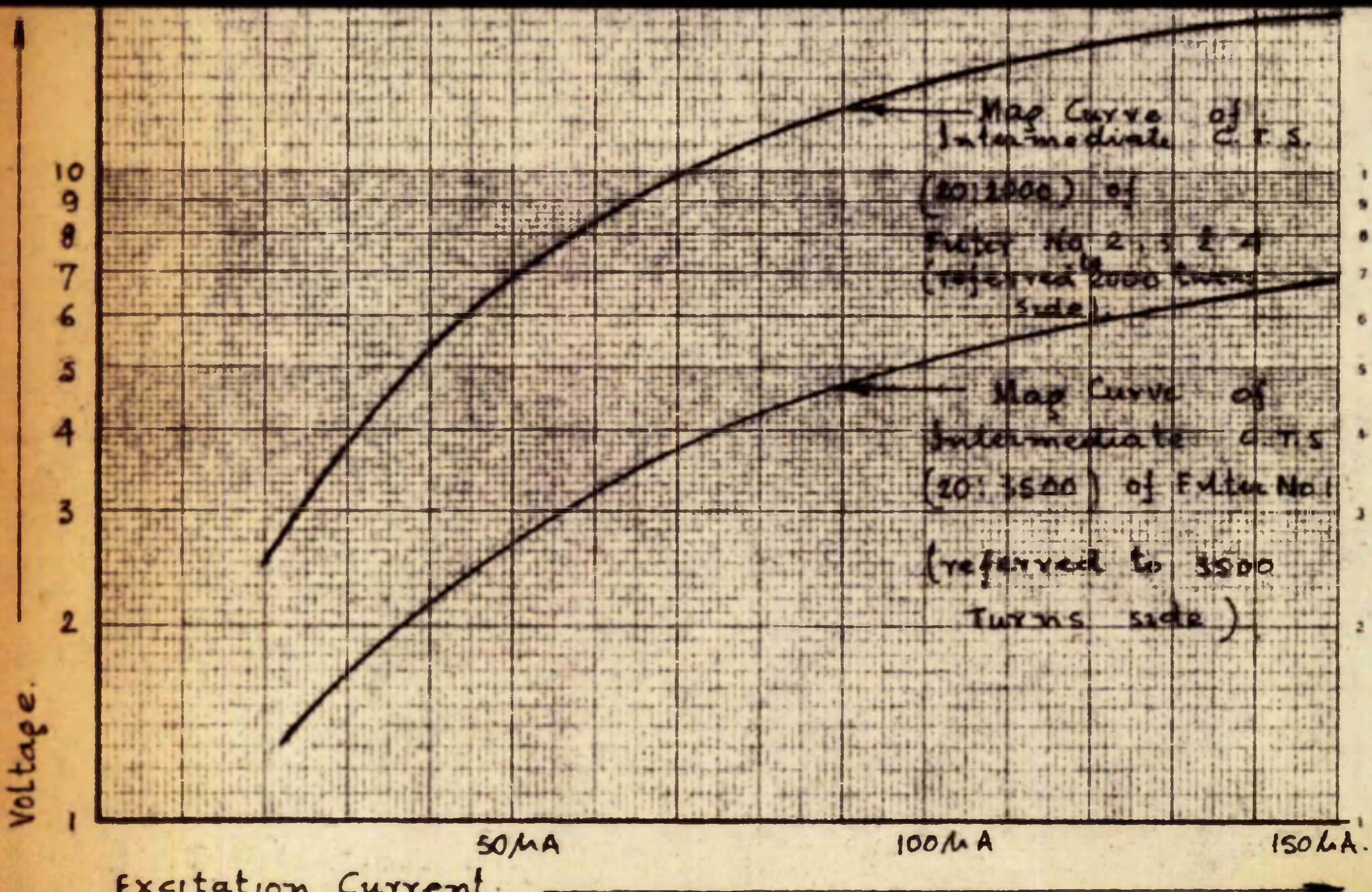
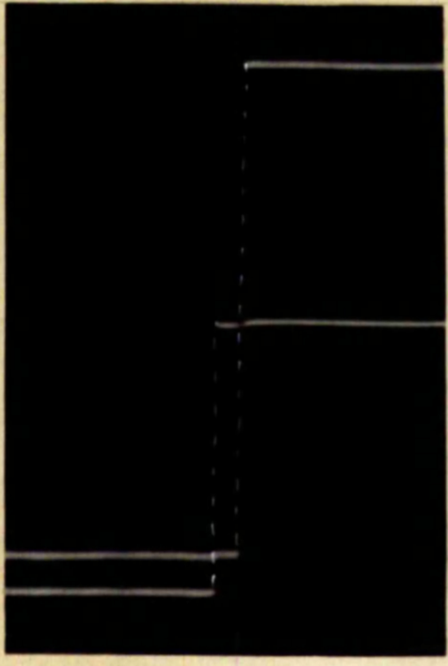
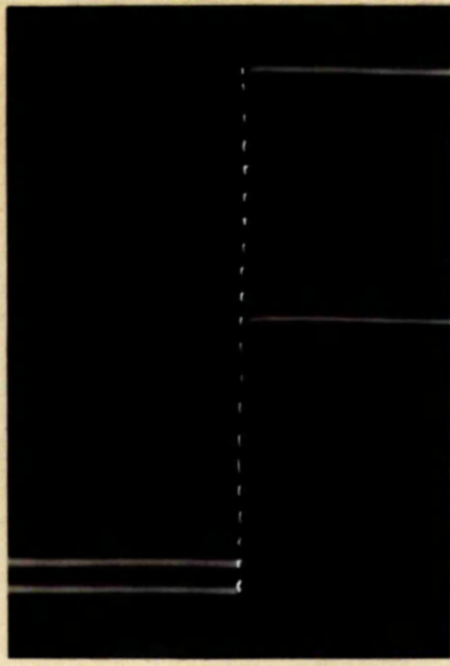

10
9
8
7
6
5
4
3
2
1
Voltage.
Mag Curve of
Intermediate C.T.S.
(20:2000) of
Filter No 2, 3 & 4
(referred to 2000 turns
side).
Mag Curve of
Intermediate C.T.S
(20: 3500) of Filter No.1
(referred to 3500
Turns side)
50 mA
100 mA
150 mA.
Excitation Current

FIG 4.5.



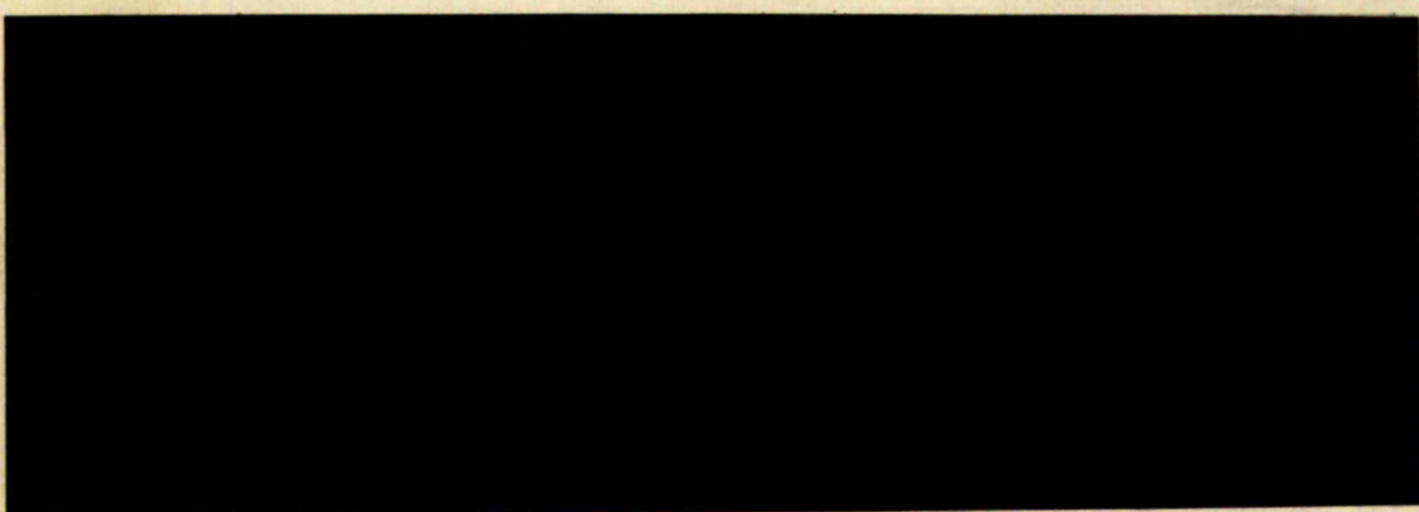
18° Before
(a)



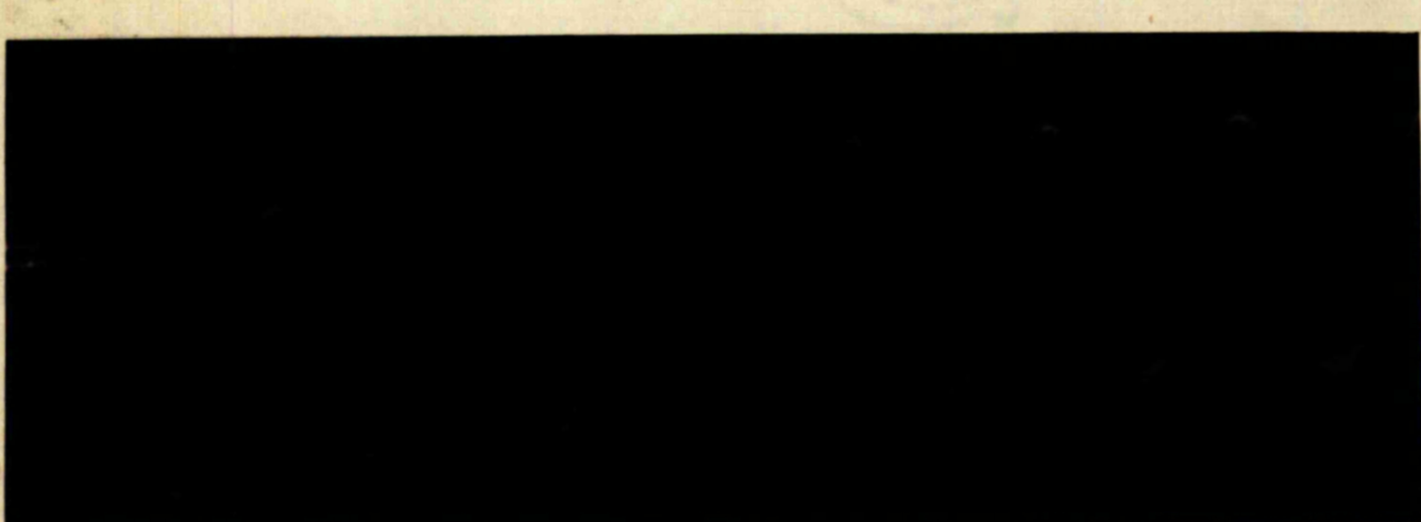
Instant of Coincidence
(b)

18° After
(c)

FIG 4.6



(a)



(b)

FIG 4.7

resulting from transformer magnetising impedances and saturation effects in network elements. The procedure, as adopted, is outlined below.

The magnetising characteristics of primary current transsformers and intermediate current transformers were examined (mag. characteristics Fig. 4.4 and Fig. 4.5).

The selected values of burdens to be imposed on the transformers were then checked to ensure their suitability. Linearity of the impedance elements was examined.

With the object of obtaining balance in the primary currents, preliminary estimates of individual phase resistance and reactance was made. First, the impedance of each phase was measured. Then the value of $\frac{R}{L}$ of the line was obtained from short circuit transient oscillograms. Suitable adjustments were made to external series resistors and line reactors until the reading of the line currents were equal and the reading on an ammeter connected in the neutral was very low (near zero).

Coincidence in switching of phases R and Y by ''make relays'' was checked by oscillogram (Fig. 4.6).

Transient records were taken simultaneously on the primary and secondary sides of the test-bench current transformers (Fig. 4.7). Their near agreement in wave form shows that the transformation from primary to secondary is quite suitable for the experiment. Any difference is due to magnetising branch of the current transformers.

Filter networks were checked by comparing the steady state output recorded with the calculated value. With positive-sequence input to the filter, the output should be zero. When it was found to be unsatisfactory an improvement was made by adjustment of the secondary turns of the current transformers. The oscillograms show a small 'error' in every case. This is of the order of 1 or 2 percent, and for the purpose of this investigation can be ignored.

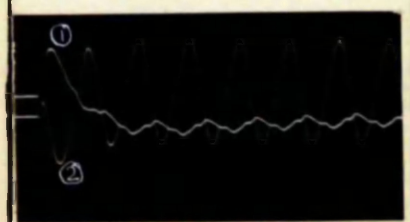
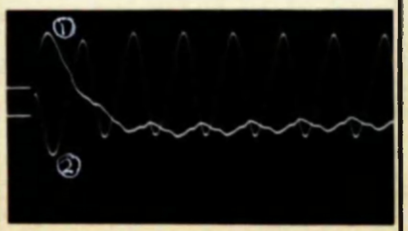
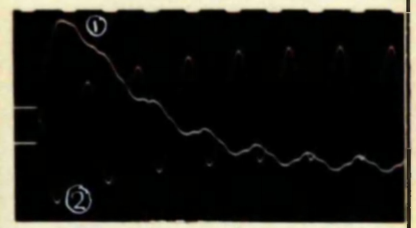
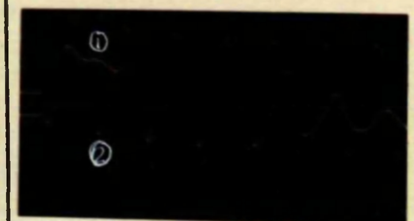
Transient outputs from all four -ve sequence filters were recorded for three phase simultaneous short circuits. Input transients were recorded from the R-Y phase. Two sets of transient records were taken: (1) one with $\tau \doteq 28$ (i.e. $\tan^{-1} \frac{\omega L_p}{R_p} = 85°$) and (2) another with $\tau \doteq 102$ (i.e. $\tan^{-1} \frac{\omega L_p}{R_p} = 72°$)

Again in each set, records were taken with 60° and approximately 90° matched burden. Records were taken at 15° interval on the wave over a period of 180° starting at 0°.

4.5 <u>TEST RESULTS</u>:-

It will be found from the oscillogram that, as expected, the wave depends on the instant of switching and the variation is cyclic.

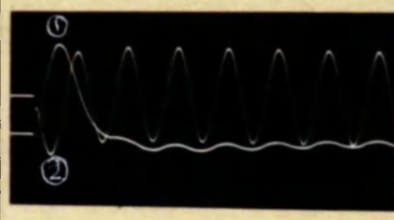
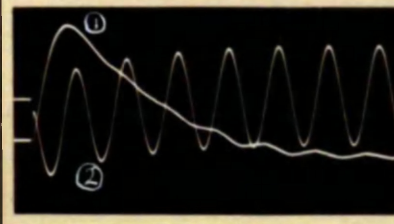
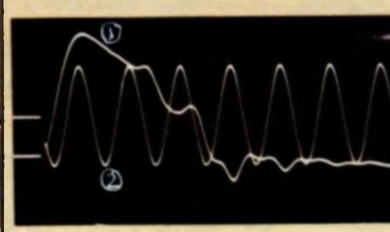
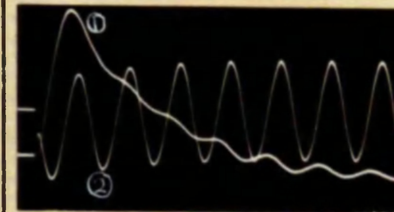
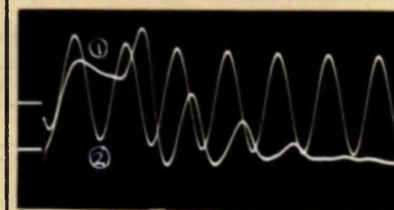
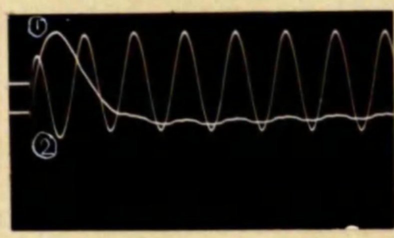
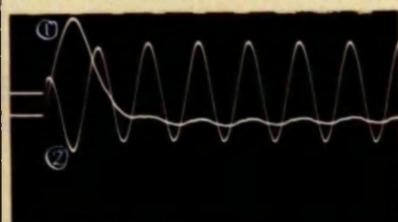
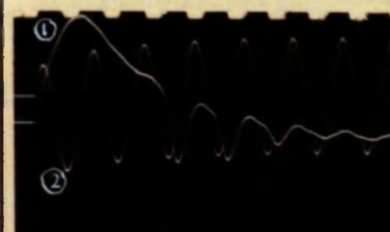
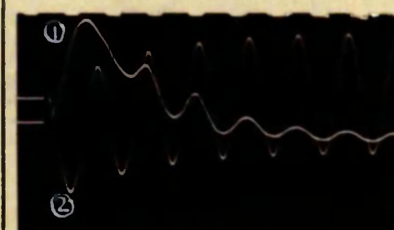
PLATE 1

	$\nu = 102$		$\nu = 28.3$		REMARKS
	60° Matched Burden	90° Matched Burden	60° Matched Burden	90° Matched Burden	
FILTER NO. 1					
FILTER NO. 2			 	 	Noticeable change in wav shape at diff rent instant o switching on the cycle wher $\nu = 28.3$

1. Output of the filter.
2. Input to the filter, (I_{ab})

Continued .....

PLATE 1 (Concluded)

	$\nu = 102$		$\nu = 28.3$		REMARKS
	60° Matched Burden	90° Matched Burden	60° Matched Burden	90° Matched Burden	
FILTER NO. 3			 	 	Noticeable change in wa shape at diffe- rent instant o switching on the cycle when $\nu = 28.3$
FILTER NO. 4					

1. Output of the filter
2. Input to the filter (I_{ab})

TABLE 4

FILTER	STEADY-STATE INPUT LINE CURRENT IN DELTA SIDE (Amp)	γ	MATCHED BURDEN	STEADY-STATE NEGATIVE SEQUENCE OUTPUT		TRANSIENT OUTPUT DUE TO POSITIVE SEQUENCE SWITCHING (EXPRESSED AS A FRACTION OF STEADY-STATE NEGATIVE SEQUENCE OUTPUT)		
				THEORITICAL (mA)	EXPERIMENTAL (mA)	OVER ONE CYCLE	OVER TWO CYCLES	OVER THREE CYCLES
FILTER NO.1	0.70	28.3	90°	3.7	3.4	0.915	0.77	0.182
			60°	3.38	3.1	0.74	0.595	0.65
	0.65	102	90°	3.4	3.2	0.51	0.375	0.304
			60°	3.15	2.9	0.41	0.297	0.248
FILTER NO.2	0.70	28.3	90°	9.8	9.1	0.20	0.225	0.212
			60°	6.0	5.6	0.375	0.385	0.35
	0.65	102	90°	9.1	8.3	0.20	0.143	0.116
			60°	5.55	5.1	0.27	0.192	0.164
FILTER NO.3	0.70	28.3	90°	6.1	5.8	0.274	0.485	0.415
			60°	4.3	4.1	0.66	0.68	0.62
	0.65	102	90°	5.7	5.4	0.353	0.25	0.207
			60°	3.98	3.8	0.417	0.30	0.25
FILTER NO.4	0.70	28.3	90°	6.45	6.0	0.445	0.40	0.34
			60°	4.95	4.25	0.525	0.495	0.42
	0.65	102	90°	6.0	5.5	0.383	0.275	0.226
			60°	4.6	4.25	0.435	0.316	0.263

** NOTE : It is to be noted that a 90° matched burden is impossible to achieve, but it is chosen because it is the extreme case of pure inductive burden. For the experiment 84.7° matched burden was taken.

• RMS values of Transient waves of the filters are calculated from Fig.4.8, Fig.4.8.1., Fig.4.8.2 and Fig.4.8.3

Typical transients for each filter are given in Plate 1. For three of the filters there is a noticeable change in wave form at different instants of switching.

If relay operation is assumed to take place between 2-3 cycles, r.m.s. values of the maximum transients of each filter over two cycles and three cycles are given in Table 4. They are expressed as a fraction of negative sequence output. For Filter No. 1 the predetermined wave is superimposed on the oscillogram for comparison. Sample calculation is given in Appendix (XV11).

4.6 DISCUSSIONS:

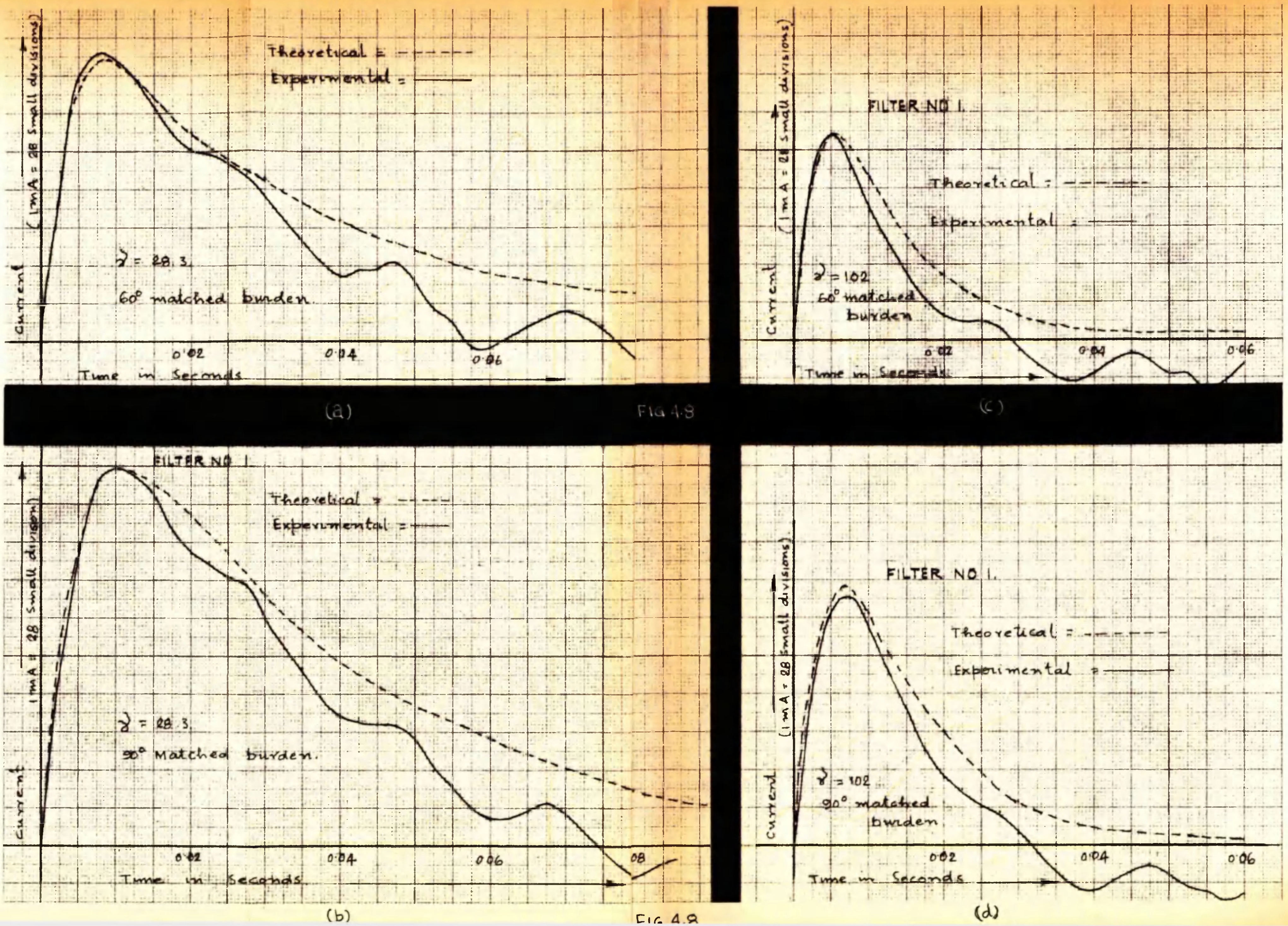
Though every attempt has been made to perform accurate experiments, erros may arise from any combination of the following causes:

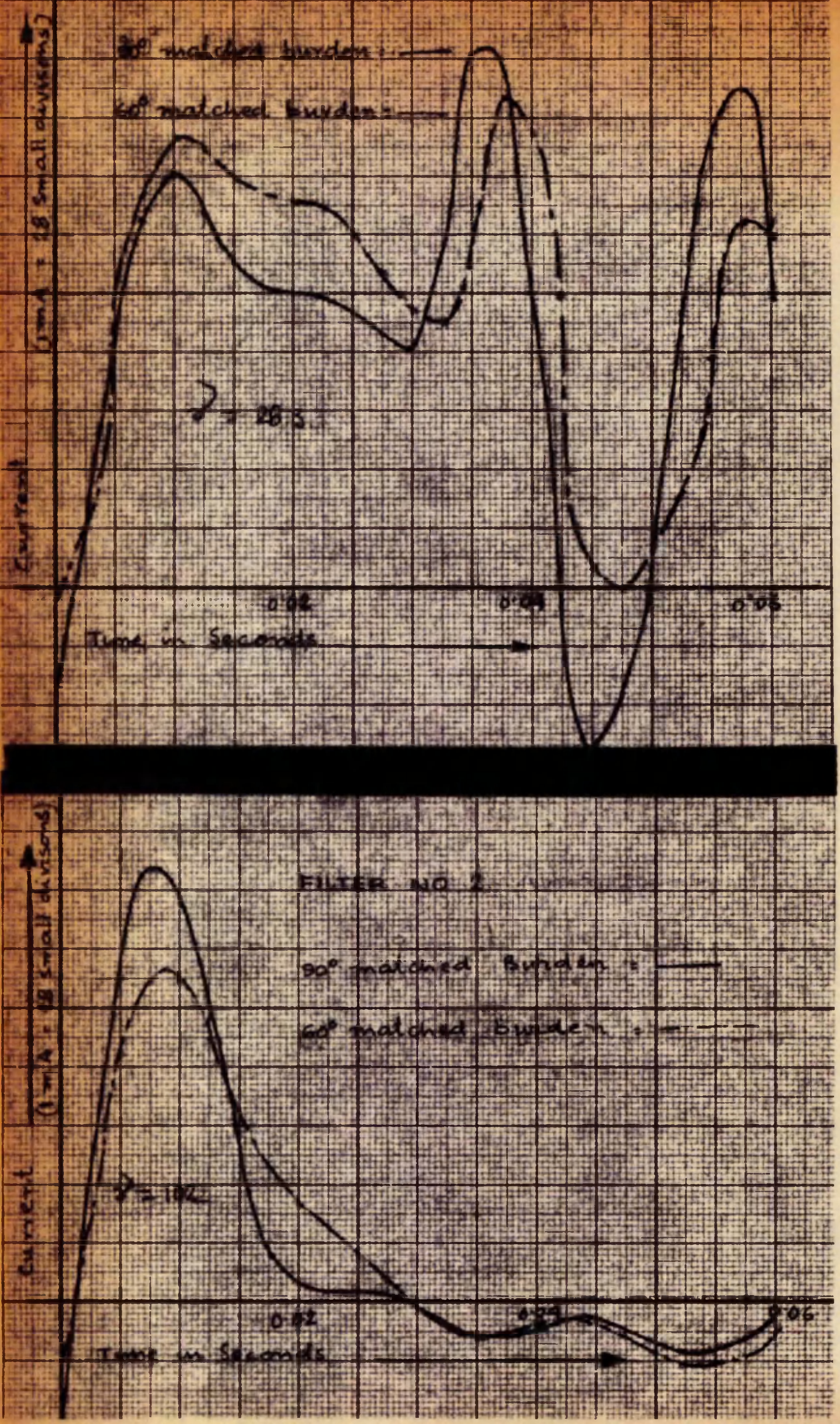
(1) Unavoidable small departure from synchronous in the operation of ''make relays'' in the primary.

(2) Unavoidable small out-of-balance in the input currents to filter.

(3) Slight non-sinusoidal character of the supply voltage and slight deviation from the normal frequency.

(4) ''Erross'' in the current transformers and distortions due to magnetising branches; and errors caused by series impedances in the current transformer circuits, e.g. lead resistances and shunt resistances used for oscillo-


Current
(1 mA = 28 small divisions)
Theoretical = - - - - -
Experimental = ———
γ = 28.3
60° matched burden
0·02
0·04
0·06
Time in Seconds
(a)
Fig 4.8
FILTER NO 1.
γ = 102
60° matched burden
(c)
γ = 28.3
90° Matched burden.
08
(b)
γ = 102
90° matched burden
(d)



90° matched burden : ——
60° matched burden : ——
Current (1 mA = 18 small divisions)
0·02
0·04
0·06
Time in Seconds
FILTER NO 2
90° matched Burden : ——
60° matched burden : — — —
Current (1 mA = 18 small divisions)
0·02
0·04
0·06
Time in Seconds

FIG. 4.8.1

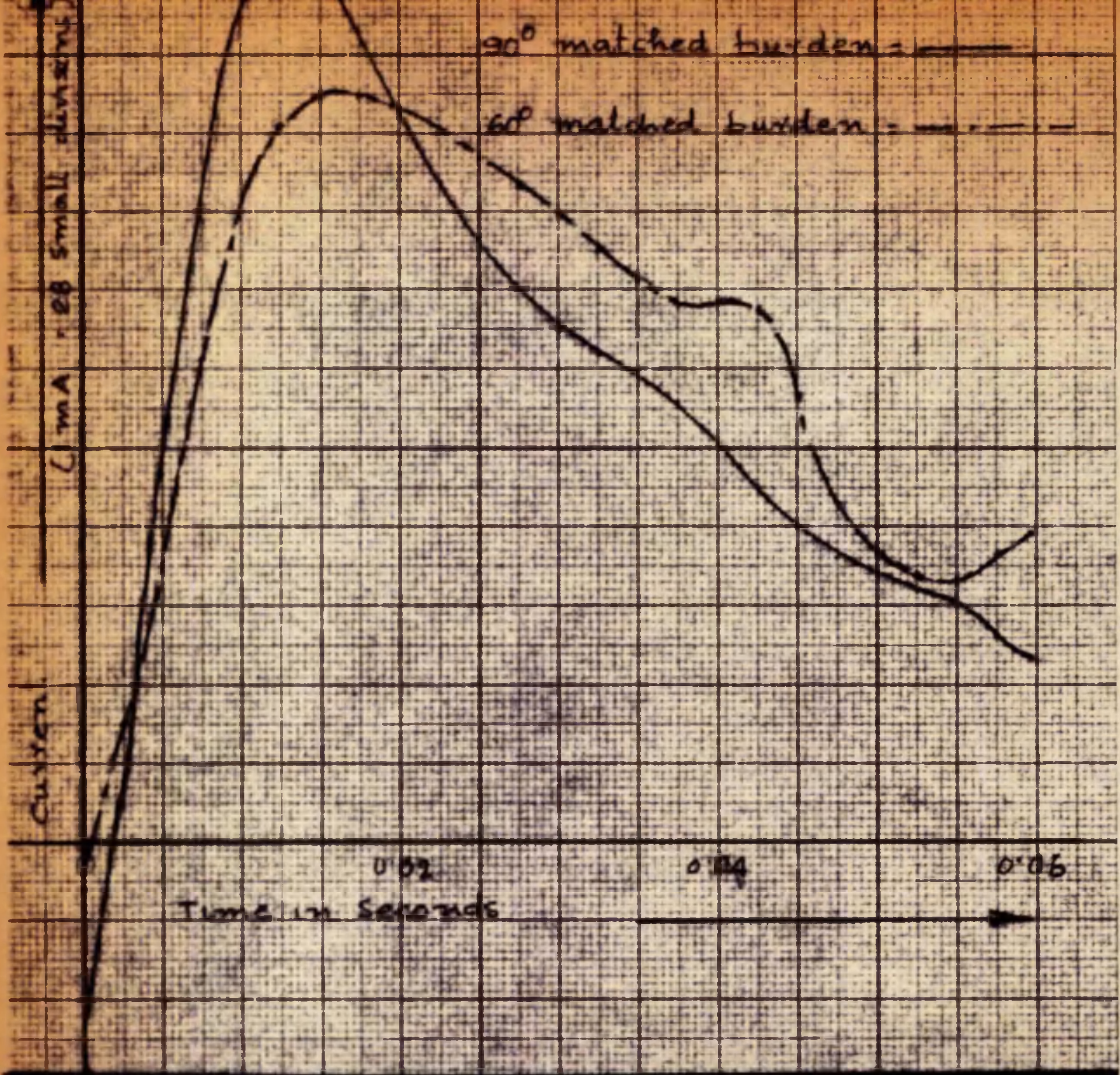
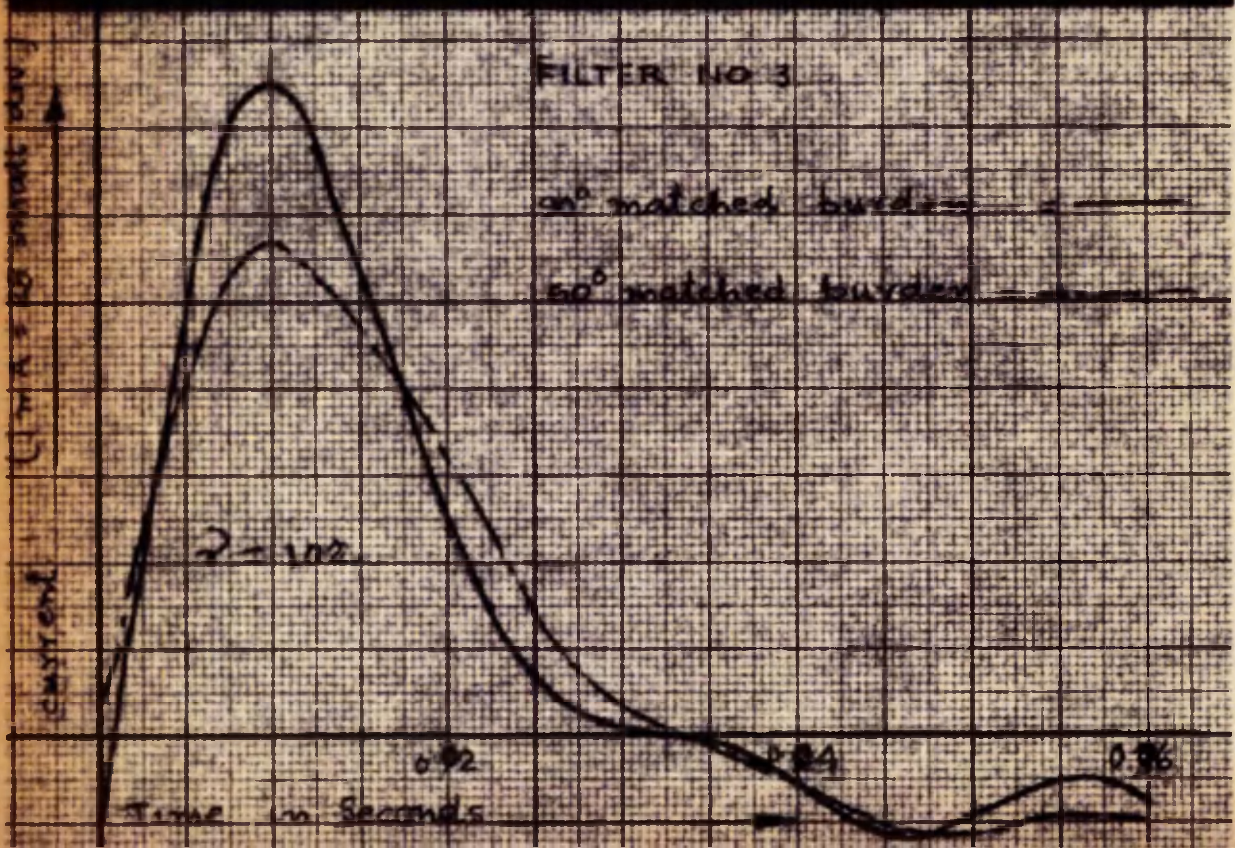

90° matched burden
60° matched burden
Current
0·02
0·04
0·06
Time in Seconds

FIG 4.8.2


FILTER NO 3
90° matched burden
60° matched burden
Current
0·02
0·04
0·06
Time in Seconds

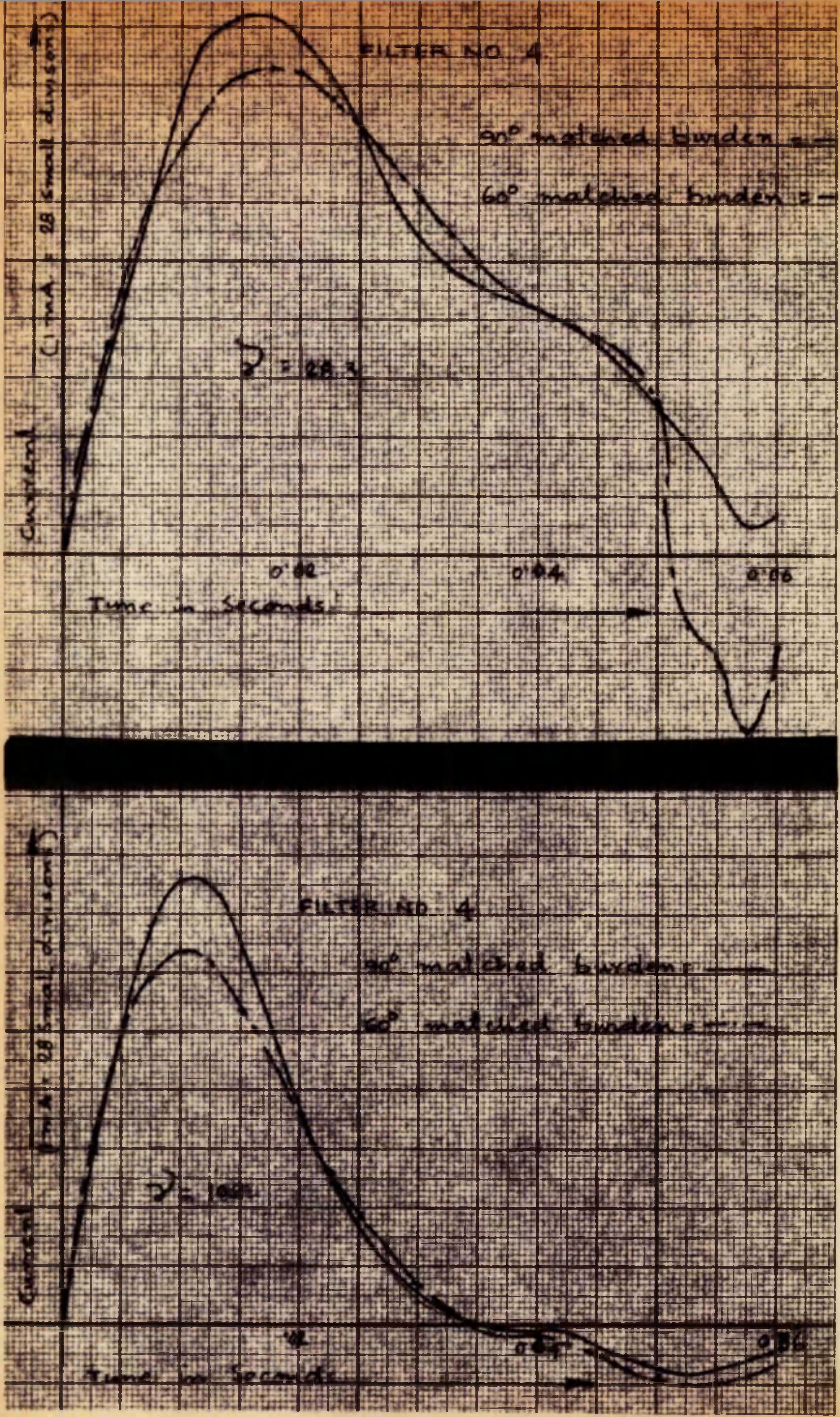
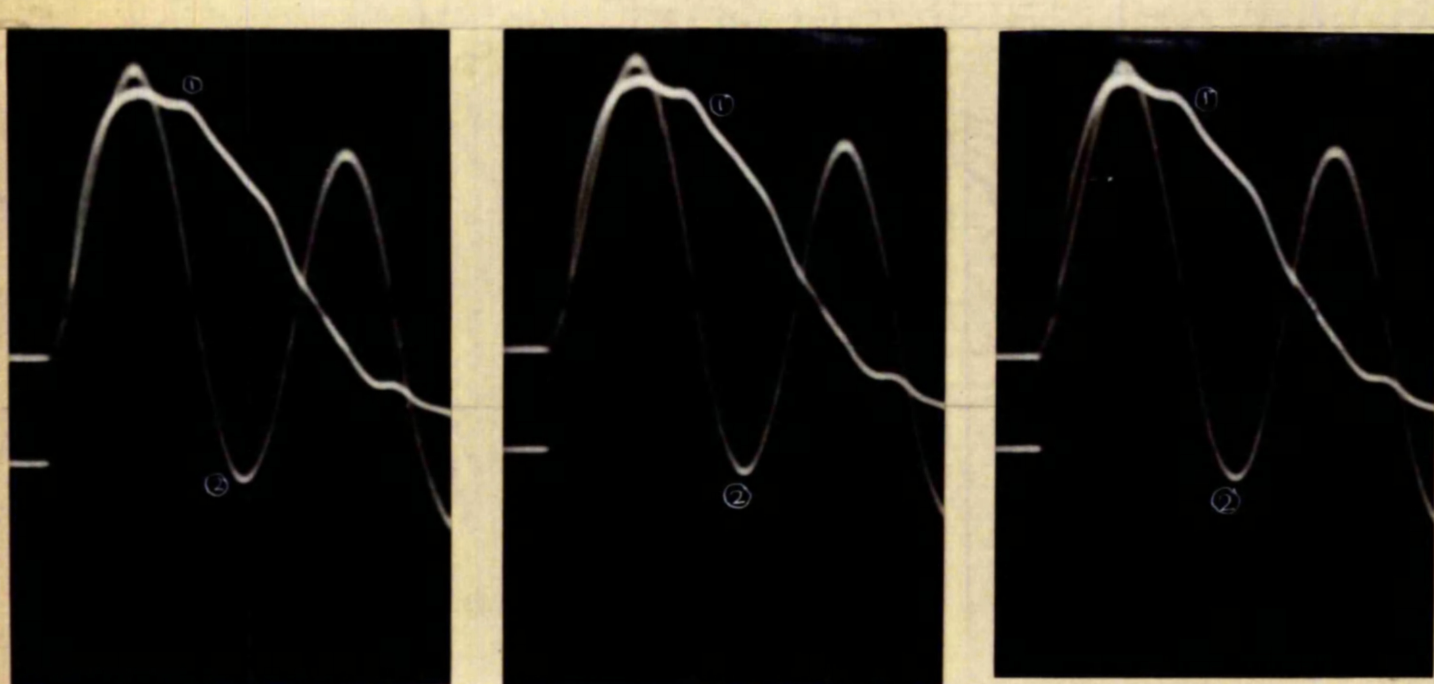

FILTER NO. 4
90° matched burden
60° matched burden
Current
0·02
0·04
0·06
Time in Seconds
FILTER NO 4
90° matched burden
60° matched burden
Current
Time in Seconds

FIG.4.8.3

(a) (b) (c)

FIG 4.9.



(a) (b) (c)

1. Output
2. Input

FIG 4.10

grams records of the input currents.

(5) Lack of perfection in the impedance elements of the filters.

Comparison of theoretical and test results of Filter No. 1 shows that the curves are similar but not in full agreement (Fig.4.8). It will be noticed from the above curves that the theoretical curve does not pass through except at the start and the steady state, but the test curve reverses (i.e. crosses the zero line) in between starting and steady state position. This is due to any combination of the above-mentioned reasons.

Two interesting phenomena were observed during the experiments in which high input currents to the filters were used. Figs.(4.9a),(4.9b),(4.9c) show the filter output for three consecutive identical short circuits. It is seen that the wave form differs each time and that there is a discontinuity in each wave.

The explanation for the change in wave shape is probably due to C.T. remanence build up which occurs with large transients currents but not with low currents,(Figs. 4.10a, 4.10b, 4.10c). No explanation can be offered for the discontinuity which also did not appear with lower input currents to the filters.

SECTION 5

5.1 INTRODUCTION

We have so far discussed the basic types of filters, i.e. zero, negative and positive sequence. In this section other types of filters will be considered. They are variations of the basic types.

5.2 DOUBLE FILTERS

In general, positive and negative sequence components can be obtained by positive and negative sequence filters respectively. But the idea of ''double filters'' is to obtain both positive and negative sequence components individually from the same filter.

It has been shown before that for every negative-sequence filter there is a corresponding positive-sequence filter.. In ''double filter'' they are combined together to form a single network which will have two outputs, positive and negative sequence, from two different terminal-pairs. To show how this can be achieved let an example (say Allcutt's filter, Art. 3.11) be taken. The corresponding negative and positive filters can be shown in Fig. (5.1a) and Fig. (5.1b). Now if these two filters are examined it will be found that the difference of positive sequence filter from the negative sequence filter is that terminals C-F

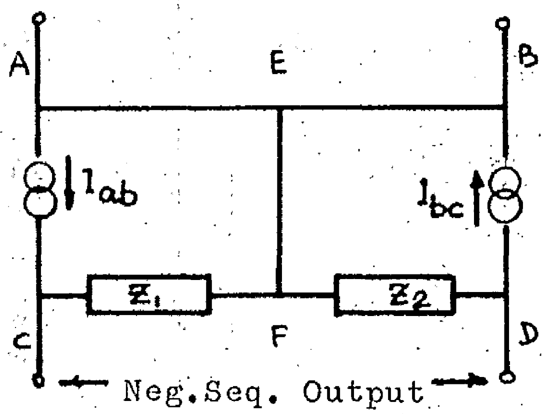


FIG 5.1a

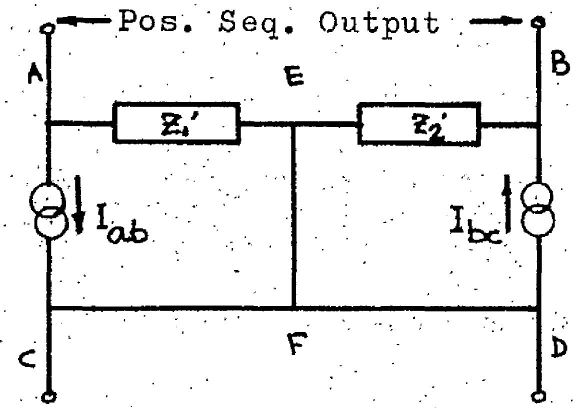
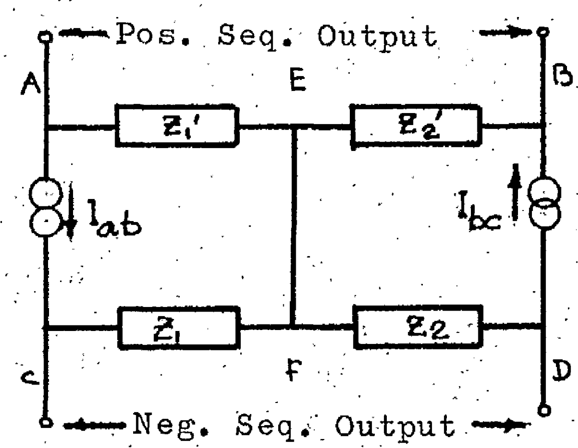
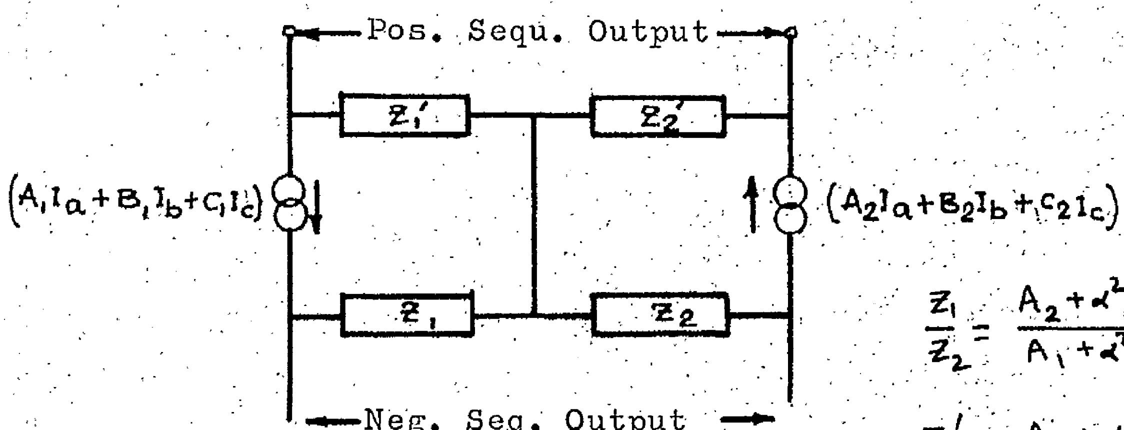


FIG 5.1b



$$\frac{Z_1}{Z_2} = \frac{Z_2'}{Z_1'} = \angle 60^\circ$$

FIG 5.1c



$$\frac{Z_1}{Z_2} = \frac{A_2 + \alpha^2 B_2 + \alpha C_2}{A_1 + \alpha^2 B_1 + \alpha C_1}$$

$$\frac{Z_1'}{Z_2'} = \frac{A_2 + \alpha B_2 + \alpha^2 C_2}{A_1 + \alpha B_1 + \alpha^2 C_1}$$

and $A_1 + B_1 + C_1 = 0$

$A_2 + B_2 + C_2 = 0$

FIG 5.2

and D-F are short circuited, and Z_1' and Z_2' are put across A-E and B-E respectively (which were short circuited in the case of negative-sequence filter); $Z_2':Z_1'$ has to be $\angle 60°$ for positive-sequence output whereas $Z_1:Z_2$ was $\angle 60°$ for negative-sequence output. It will be noticed that I_{ab} flows through Z_1 as well as Z_1' and I_{bc} through Z_2 as well as Z_2'. Positive-sequence output is the sum of the series voltage across Z_1' and Z_2' respectively whereas negative-sequence output is the sum of the series voltages across Z_1 and Z_2 respectively.

Now for double-filter short-circuited terminals are filled by respective branch-impedances of the two filters as shown in Fig. 5.1c), and it will be seen that I_{ab} still flows through Z_1 and Z_1' and I_{bc} flows through Z_2 and Z_2'. So after fulfillment of such conditions, it is quite obvious that the negative-sequence output and positive-sequence output are exactly the same as before, i.e. negative-sequence output is the sum of the series voltage across Z_1 and Z_2 respectively and positive-sequence output is the sum of the series voltages Z_1' and Z_2' respectively.

Economy of the double-filter can be followed from the fact that to segregate positive and negative sequence components by separate filters the sources of supply have to be duplicated, whereas for double-filter such duplication of sources of supply does not arise.

Following the line of above example it will be found that similar double-filters can be designed corresponding to all

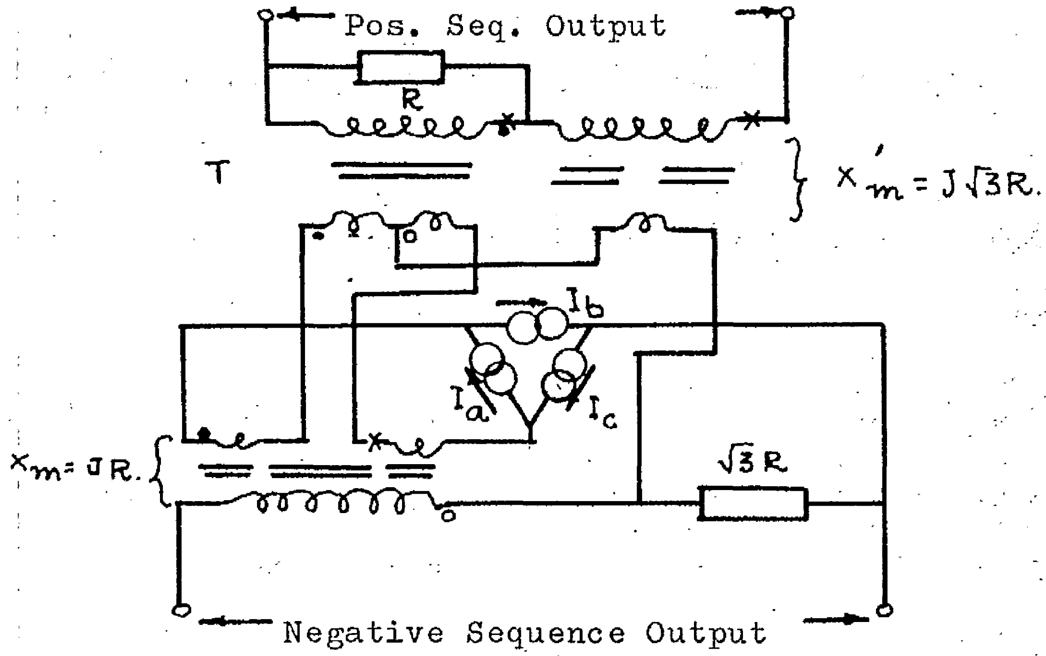


FIG 5.3a

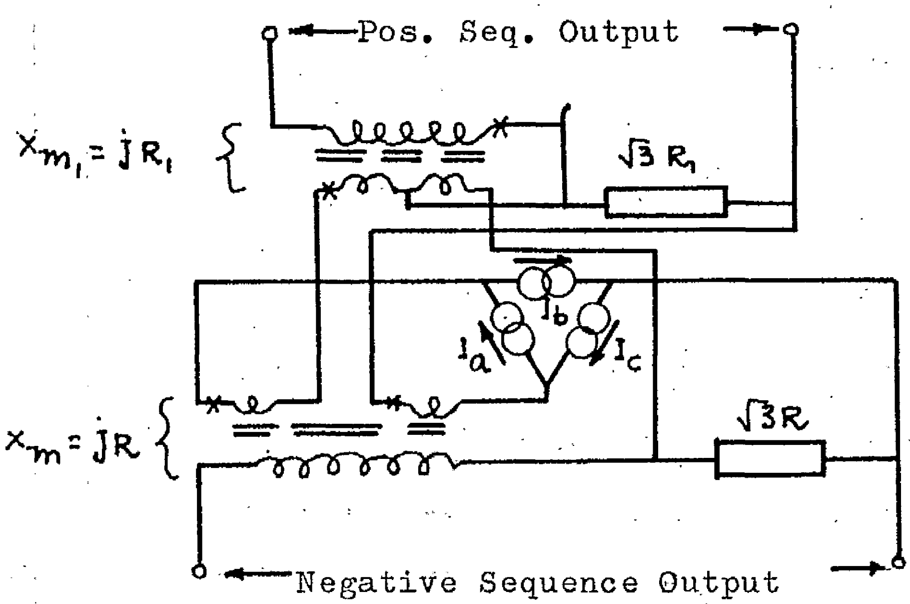


FIG 5.3b

two-element series filters discussed in the previous section. However, from the above discussion, and from eqn. (4c), (4d) & (4e) (Art. 3.6.1a) a generalised ''double filter'' can be designed shown by Fig. 5.2, corresponding to a two-element series filter.

It is sometimes possible to simplify further a filter derived from the generalised filter (Fig. 5.2), depending on the circuitry of the filter itself. One of the examples of such cases is the double filter corresponding to Filter No. 1. From the generalised network, a possible double-filter-network designed is as shown in Fig. (5.3a). Here the positive-sequence output voltage (open circuit) is given by

$$(I_{ab} + I_{ca})R' + (I_b - I_c)j\sqrt{3}R' = 6I_+R'$$

But a positive-sequence output will still be obtained if the equation is

$$(I_{ab} - I_{bc})jR_1 + (I_c - I_a)\sqrt{3}R_1 \; (= 6\, I_+R_1 \underline{/150^\circ}\;)$$

From this equation and from Fig. (5.3a) a simplified filter can be designed as shown in Fig. 5.3.b. By such an arrangement it will be noticed that the intermediate transformer T of Fig. (5.3a) is not required. Simplification, in this case, lies in the removal of one intermediate transformer. By such change, circuit parameters have to be changed from that of Fig. (5.3.a), but the purpose of double filter is fulfilled and its filter-circuitry is simplified.

Double-filters corresponding to three-element-series

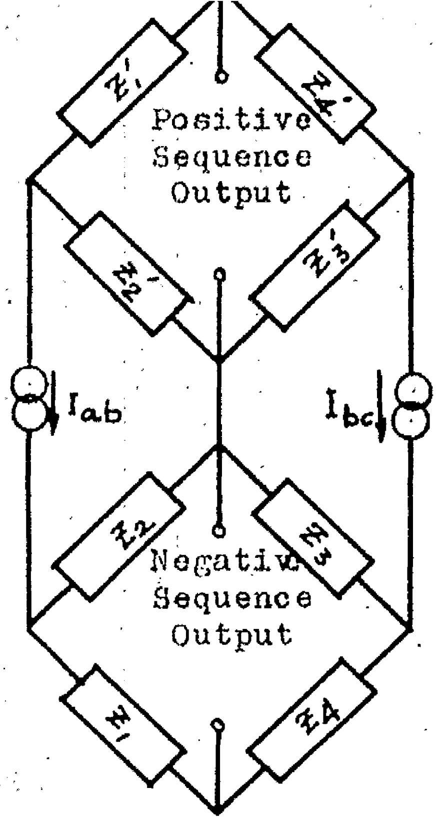


FIG 5.4

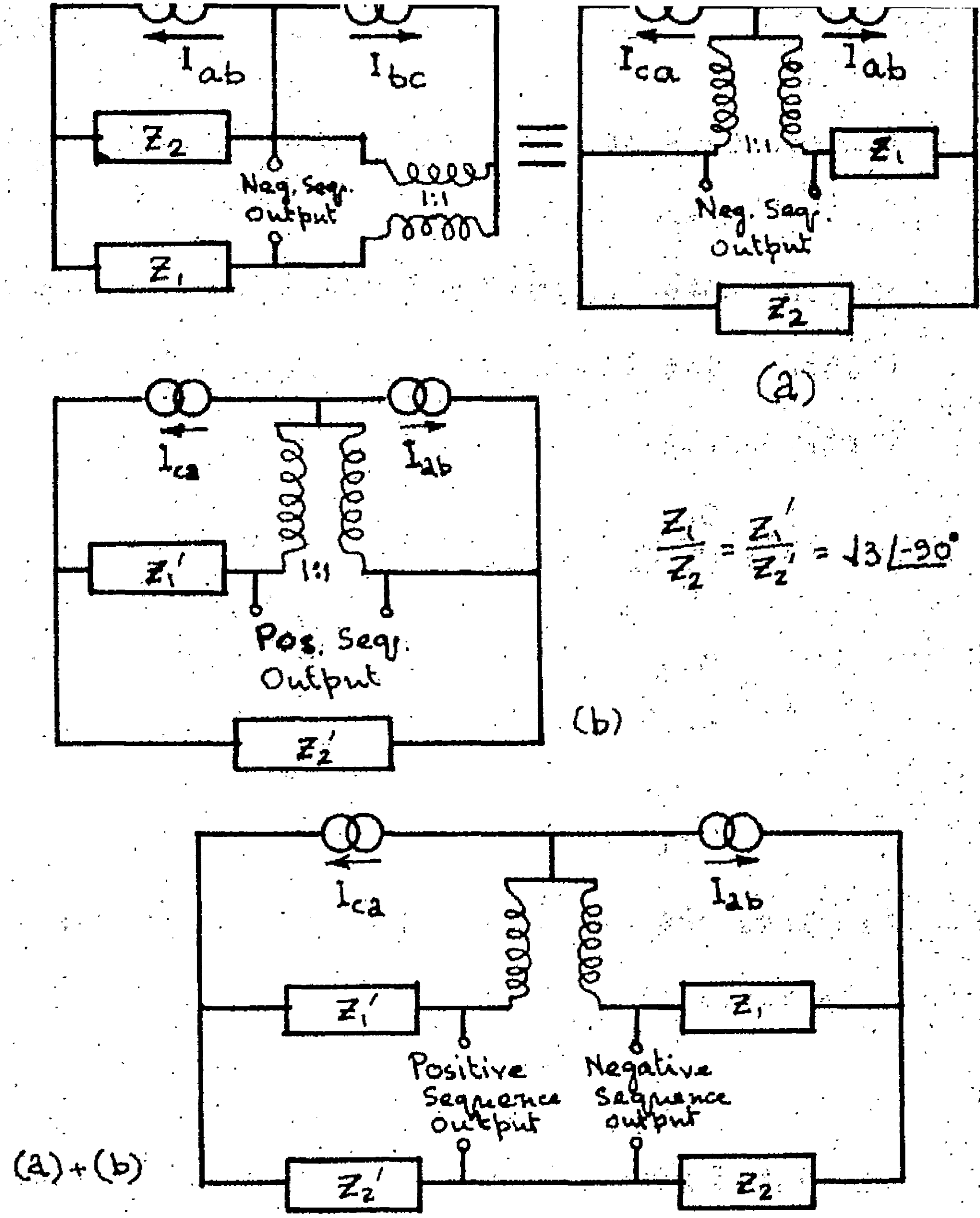


FIG 5.5

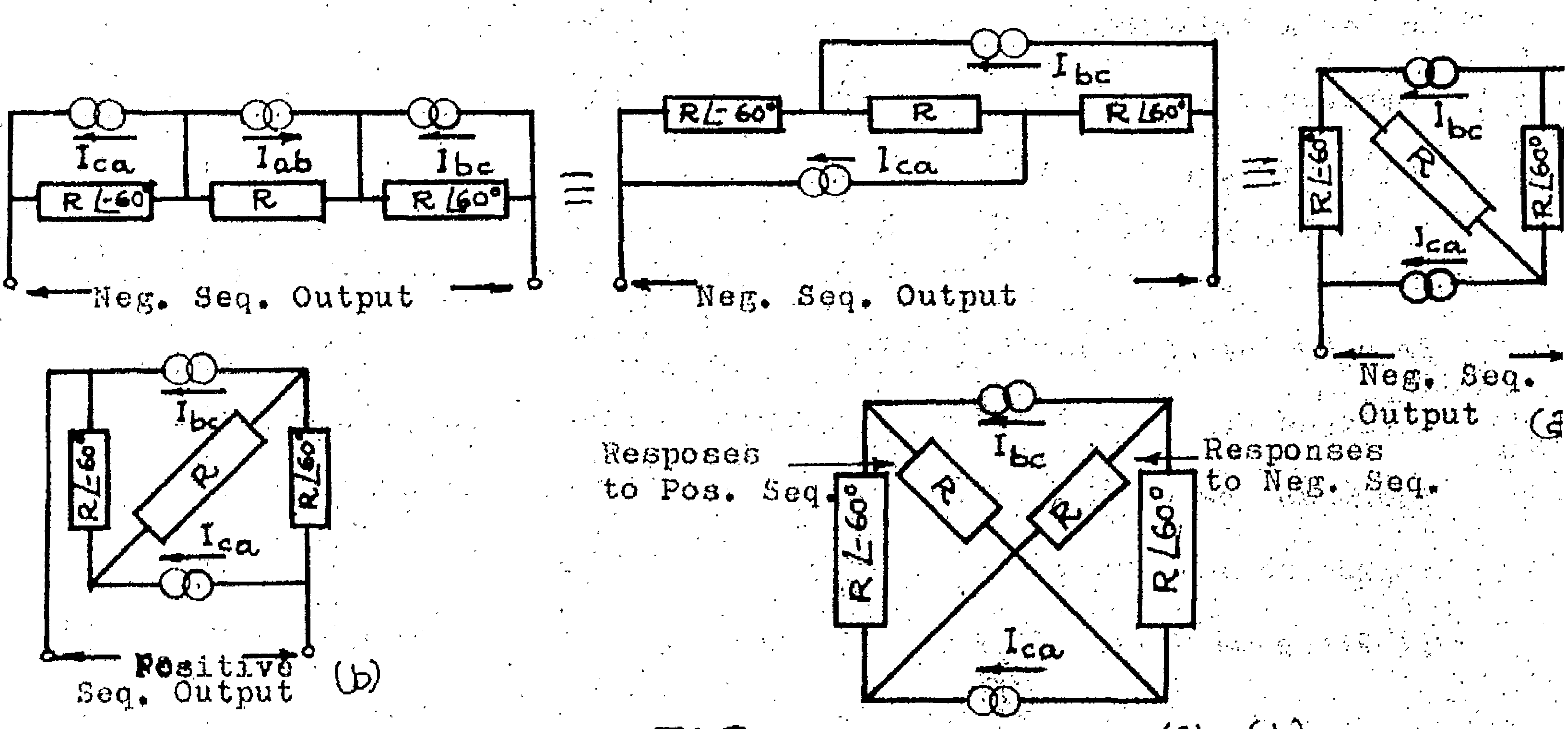


FIG 5.6

filters have been discussed later in the article for convenience.

In the same way, a generalised ''double-filter'' network corresponding to bridge filter can be designed as shown in Fig. (5.4). The following relations must hold good for this filter (Art. 3.6.2).

$$1 + \frac{Z_1}{Z_2} \angle{-60^\circ} + \frac{Z_4}{Z_3} \angle{60^\circ} = 0$$ (to obtain a -ve seq. output)

and $$1 + \frac{Z_1}{Z_2} \angle{60^\circ} + \frac{Z_4}{Z_3} \angle{-60^\circ} = 0$$ (to obtain +ve seq. output).

It can be easily seen that their circuitry is somewhat complex, but one filter of this group corresponding to Filter No. 2 (Art. 3.11) has been simplified as shown in Fig. 5.5 (Ref. 5).

It has been mentioned before (Art. 3.9.2) that a three-element series filter has no advantage over a two-element series filter or bridge filter. So only one double-filter of this group is mentioned here (Fig. 5.6). This filter is suggested in reference 5. It will be noticed here that the burdens themselves act as filter elements.

It should be emphasised that double-filters are designed from the economy point of view and as a result the overall performance may suffer, e.g. their VA-qualities are likely to decrease due to increased number of elements. However, their impedance qualities remain the same as has been explained in a previous section. Voltage double-filters can be designed in a

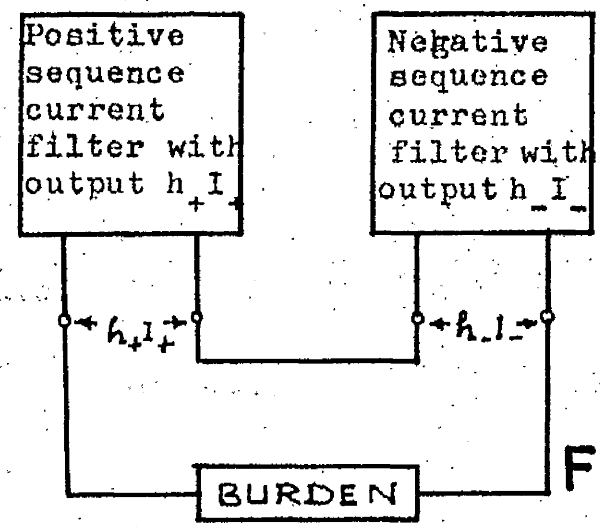

Positive sequence current filter with output h+I+
Negative sequence current filter with output h_I_
h+I+
h-I-
BURDEN

FIG 5,7

similar manner as explained above or from the principle of duality.

5.3 COMBINED FILTERS

From this type of filter, a single-phase output can be obtained which will contain more than one component in predetermined proportions. In general, it can be said that the different components are segregated by separate filters (e.g. negative-sequence component by negative sequence filter and positive sequence component by positive sequence filter) and these output quantities are connected in series (in case of current filters) or in parallel (in case of voltage filters) across the burden.

For example, the general arrangement of a current combined filter of the nature $(h_+I_+ + h_-I_-)$ where h_+ and h_- are predetermined constants, is shown in the block diagram (Fig. 5.7). But it can be seen that because of the use of two separate filters it loses its simplicity. But from the knowledge of Filter No. 1 it has been possible to derive a few simplified combined filters which were originally invented in 1937 (Ref. 21). Their principles are explained below:-

From Filter No. 1 (Fig. 3.8), the open circuit output voltage =

$$(I_{ab} - I_{ca})jR + I_{bc}\sqrt{3}R = (3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R = h_-I_- \quad \text{-------------(05.1)}$$

where $h_- = j6R$

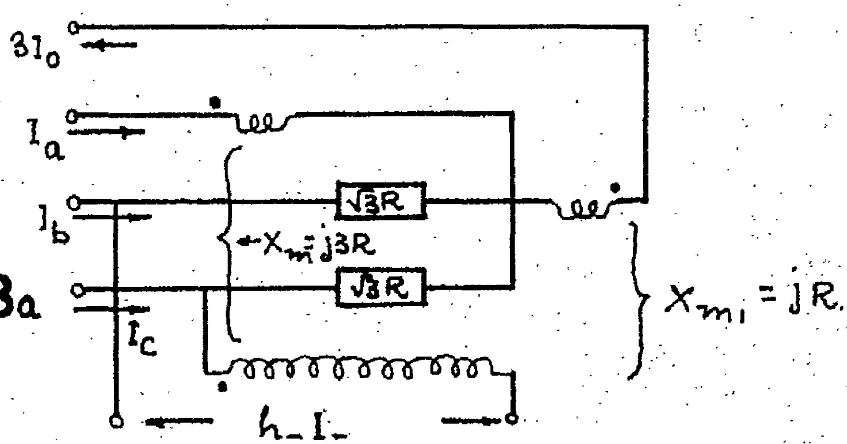


FIG 5.8a

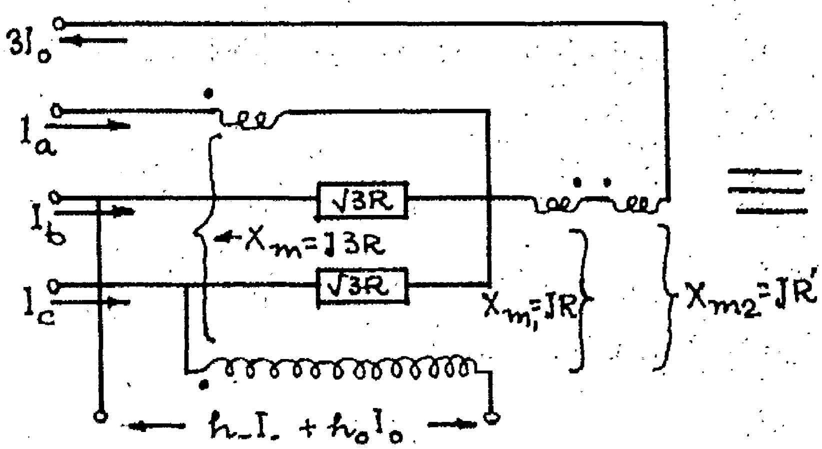


FIG 5.8b

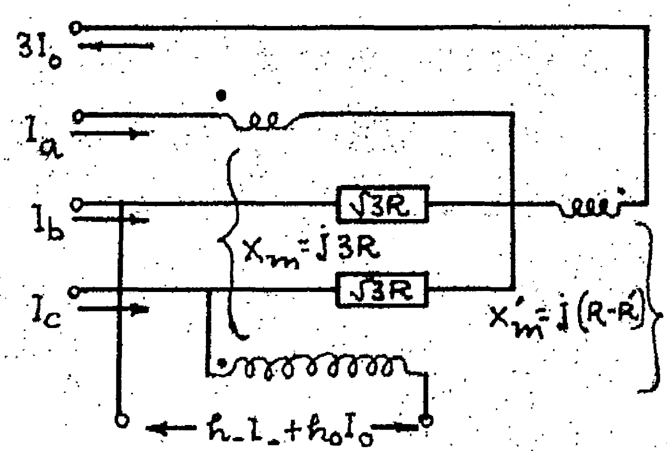


FIG 5.8c

(1) Combined filter of nature ''$h_-I_- \pm h_oI_o$'':-

(a) From the expression $(3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R$ of eqn (05.1) it can be seen that the desired simple negative sequence filter will be as shown in Fig. (5.8a).

Now if in the neutral circuit another coil which has a mutual inductance of value R' with the secondary coil is connected to the output terminal as shown in the Fig. (5.8b) then the open circuit output of the filter will be '$h_-I_- + h_oI_o$' where h_o = $j3R'$. Here same neutral current $3I_o$ is flowing through both coils in the neutral, so easily they can be combined into one coil as shown in Fig. 5.8c.

Polarity shown on the coil in the neutral is on the assumption that R > R'; if R' > R, the polarity has to be changed to other side, and in this case the filter is'$h_-I_- - h_oI_o$' type. It has to be notcied that R and R' both have same dimensions. To change 'h_o' by replacing corresponding mutual inductor by any other type of element will make the filter network complex.

(b) Referring back to equation (05.1), if the expression $(3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R$ is multiplied by (-j) the expression will still be proportional to 'I_-', i.e.

$$-j\ (3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R =$$

$$(3I_a - 3I_o)R + (I_c - I_b)j\sqrt{3}R = h_-I_- \qquad \text{---(05.2)}$$

but here $h_- = 6R$

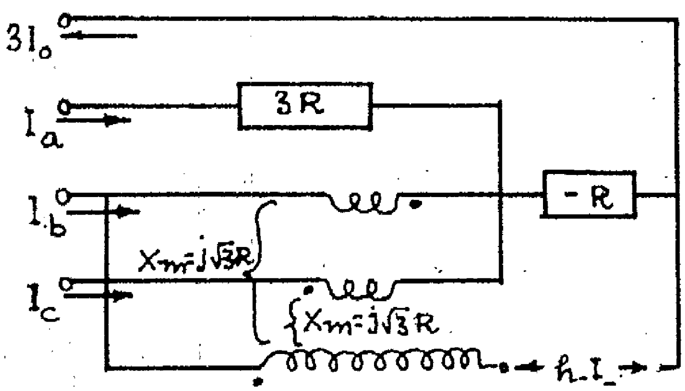


FIG 5.9a

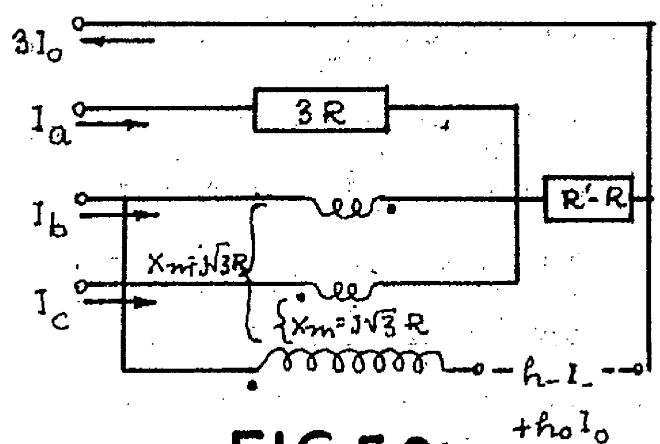


FIG 5.9b

The reason for multiplying by '-j' is that from the expression $(3I_a - 3I_o)R + (I_c - I_b)j\sqrt{3}R$ a new variation of the filter similar to Fig. (5.8a) can be visualized as shown in Fig. 5.9a. But it has to be noticed that such a filter is impracticable due to the fact that it is impossible to get a negative resistance (as shown in the network) for the neutral circuit. Now if (R' - R) is inserted instead of -R and if R' > R then not only will the resistance be positive but also the output will be of the nature ''$h_- I_- + h_o I_o$'' where $h_o = 3R'$ as shown in Fig. (5.9b). From this filter it is not possible to obtain an output of the nature ''$h_- I_- - h_o I_o$'' because R' is always greater than R. Again its impedance quality is likely to be inferior because its nature is less resistive than the previous one, if R' is not suitably adjusted. The previous filter suffers from zero-sequence frequency error, whereas the last filter is independent of zero-sequence frequency error. This is because in the last filter the mutual inductor is so arranged that the currents taken from phases B and C get the effect of cross-connection. In both the cases, however, the ''frequency quality'' (due to positive sequence) is .5 (as described in Art.3.9.4). In choosing either of these filters it is suggested that they have to be compared in relation to the particular application.

(2) Combined filter of nature ''$h_+ I_+ \mp h_o I_o$''

Their design principle is exactly the same as dicussed above and they can be obtained by mere inter-changing any two input currents of I_a, I_b and I_c and keeping the third fixed.

(3) Combined filter of nature "$h_+I_+ \pm h_-I_-$"

Referring back to expression $(3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R$ of eqn. (05.1), it can be seen that the zero-sequence component is eliminated in the stimulii $(3I_a - 3I_o)$ and $(I_b - I_c)$ and thus zero-sequence component is eliminated from the whole expression. Let $jR = A$ and $\sqrt{3}R = B$. It will be noticed that if either of these constants is varied with respect to the other, still the expression will remain free from zero-sequence component. Before such variation, the expression was free from both positive and zero sequence components (eqn. 05.) but after variation it is quite obvious that some other components will tend to appear: it is already seen zero-sequence component still does not appear, so it is quite likely that a positive sequence component will appear. Mathematically it is explained as follows:

Let $A(= jR)$ be varied from jR to $j(R + R')$, so that the expression becomes

$$(3I_a - 3I_o)j(R + R') + (I_b - I_c)\sqrt{3}R \qquad \text{----------(05.3)}$$

$$= (3I_a - 3I_o)jR + (I_b - I_c)\sqrt{3}R + (3I_a - 3I_o)jR$$

$$= j6R\,I_- + j3R'I_+ + j3R'I_- = h_+I_+ + h_-I_-$$

where $h_+ = j3R'$ and $h_- = j(6R + 3R')$.

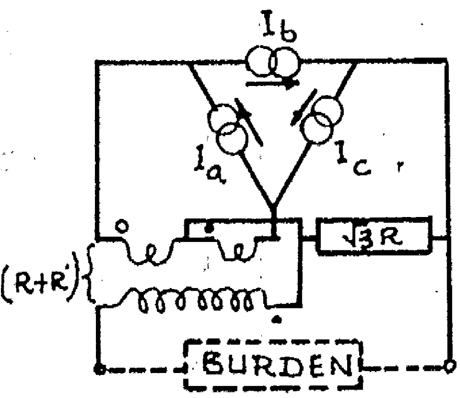


FIG 5.10

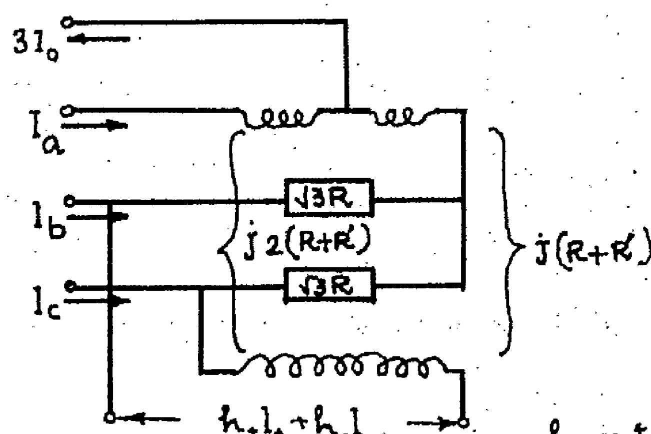


FIG 5.11

$h_+ = j3R'$

$h_- = j(6R+3$

From above it can be seen how ''$h_+I_+ + h_-I_-$'' is obtained. From eqn. (05.3) and from Filter No. 1, the derived filter network is as shown in Fig. (5.10).

So it can be noticed here, a filter which was originally a negative-sequence filter can be converted to a mixed filter of type ''$h_+I_+ + h_-I_-$'' by detuning the elements of the original filter. The terms ''detuning'' or ''unbalancing'' are commonly used to express such predetermined variations of networks elements.

A variant of the above filter is suggested in Ref. (5) where, instead of taking input currents from delta-connected C.T.s they are taken from star-connected C.T.s (Fig. 5.11). The principle which is derived from expression (05.1) can be explained as follows:

The expression $(3I_a - 3I_o)j(R + R') + (I_b - I_c\sqrt{3}R$

can be re-oriented to

$$(3I_a - I_a - I_b - I_c)j(R + R') + (I_b - I_c)\sqrt{3}R \qquad (5.11)$$

$$= \quad 2I_a j(R + R') - (I_b + I_c)j(R + R') + (I_b - I_c)\sqrt{3}R$$

which gives the clue to design the new filter (Fig. 5.11). The open-circuit output voltage of such a filter is exactly the same as from previous filter (Fig. 5.10); here the impedance quality is more resistive than the previous one but the VA-quality likely to be reduced due to increase of resistive elements.

Proceeding exactly in the similar manner as described above the following filters (Ref. 5) of nature ''$h_+I_+ + h_-I_-$''

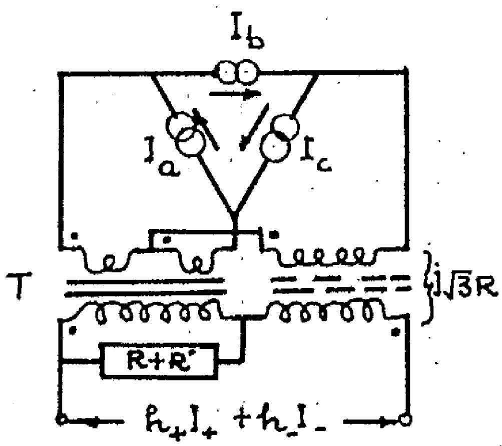


FIG 5.12a

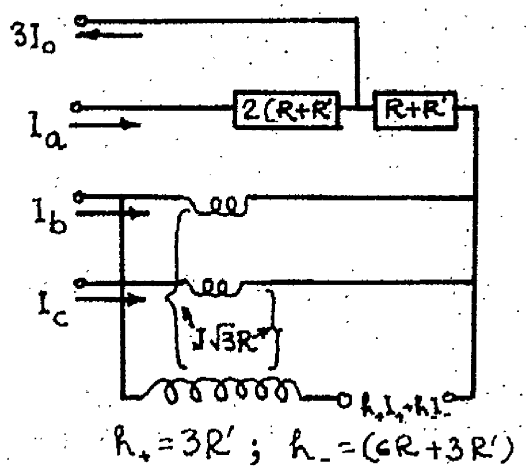


FIG 5.12b

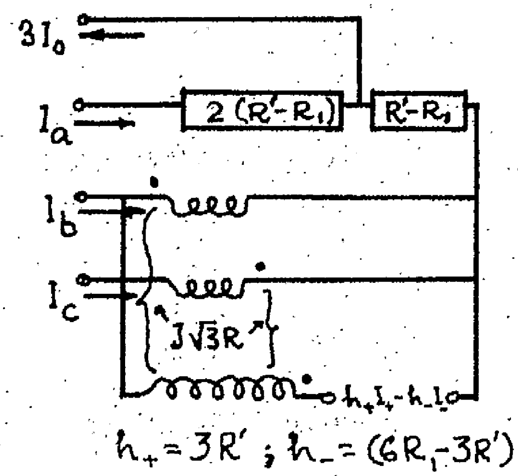


FIG 5.12c

can be obtained from the expression $(3I_a - 3I_o)R + (I_c - I_b)j\sqrt{3}R$ of eqn. (05.2) by multiplication of $(3I_a - 3I_o)$ by $(R + R')$ instead of by R. (Fig 5.12a)

The second variant of the filter (Fig. 5.12.b) is derived from the re-orientation of expression $(3I_a - 3I_o) + (I_c - I_b)j\sqrt{3}R$ to $2I_aR - (I_b + I_c)R + (I_c - I_b)j\sqrt{3}R$. It will be noticed that the first variant uses an extra transformer T. In the above filters, $h_+ = 3R'$ & $h_- = (6R + 3R')$. It is possible to obtain ''$h_+I_+ - h_-I_-$'' by choosing R as a negative value (say $R = -R_1$) such that $R' - R_1 > 0$ and $2R_1 - R' > 0$ as shown in Fig. 5.12c. Here $h_+ = 3R'$ and $h_- = -(6R - 3R')$.

An extreme generalisation of combined filters, where all the three components are present in the output in predetermined proportions, can be done from the expression (01b), (Art. 2.2). If $E_i = aI_a + bI_b + cI_c = h_+I_+ + h_-I_- + h_oI_o$, where a, b, c and h_+, h_- and h_o are complex constants, then

$$a + @^2b + @c = h_+$$

$$a + @b + @c = h_-$$

$$a + b + c = h_o$$

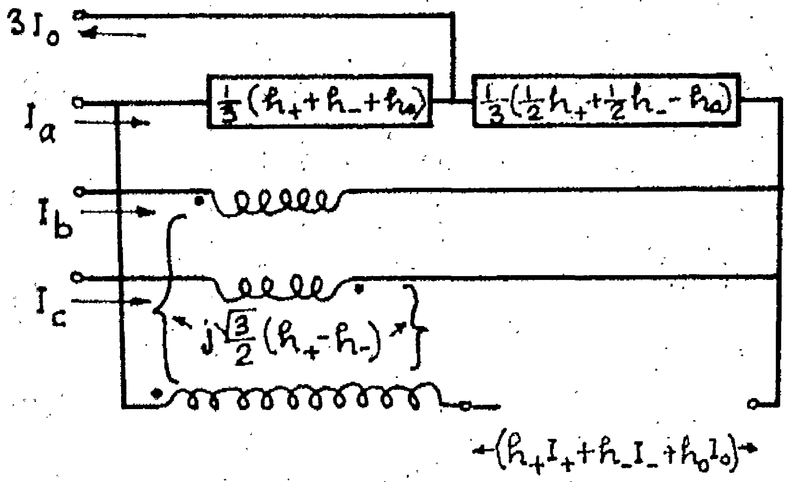
or

$$a = {}^1/_3(h_+ + h_- + h_o)$$

$$b = {}^1/_3(@h_+ + @^2h_- + h_o) \qquad \text{------------(05.4)}$$

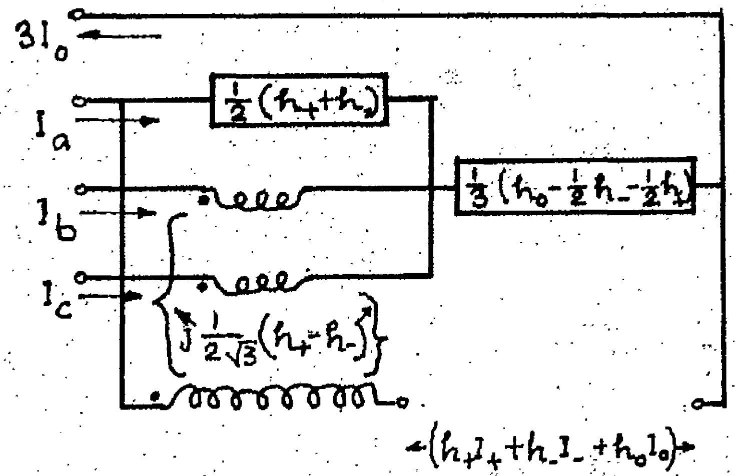
$$c = {}^1/_3(@^2h_+ + @h_- + h_o)$$

Now if h_+, h_- and h_o are real quantities, i.e. they have the dimension of resistance, then from (05.4)



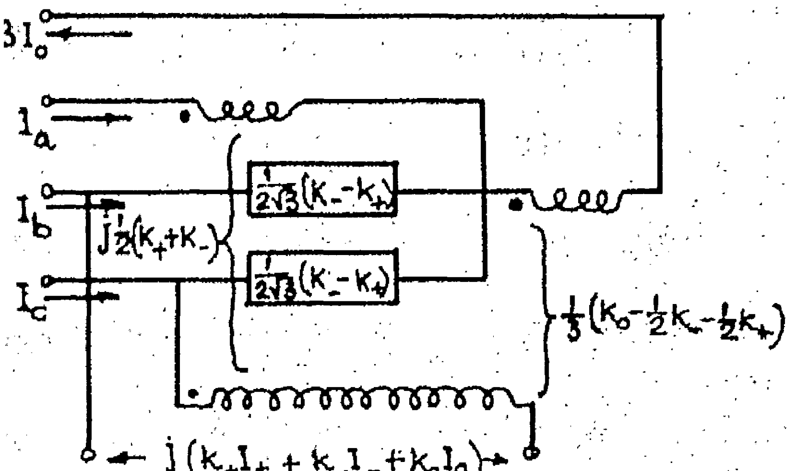
$\left(\frac{1}{2}h_+ + \frac{1}{2}h_-\right) \geqslant h_o$

FIG 5.13a



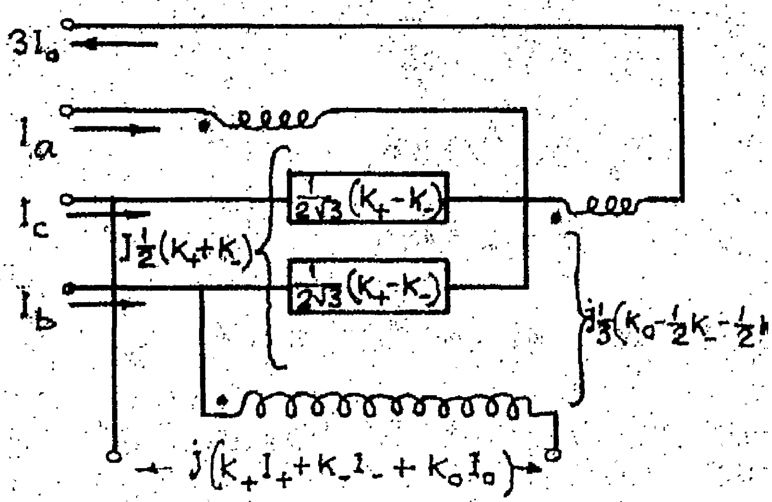
$h_o \geqslant \frac{1}{2}(h_+ + h_-)$

FIG 5.13b



$K_+ > K_-$

FIG 5.14a



$K_+ > K_-$

FIG 5.14b

$$a = {}^1/3(h_+ + h_- + h_o)$$

$$b = {}^1/3 \quad (-{}^1/2h_+ - {}^1/3h_- + h_o) + j\frac{\sqrt{3}}{2}(h_+ - h_-)$$

$$c = {}^1/3 \quad (-{}^1/2h_+ - {}^1/2h_- + h_o) - j\frac{\sqrt{3}}{2}(h_+ - h_-)$$

$$\therefore \quad E_i = {}^1/3(h_+ + h_- + h_o)I_a + {}^1/3 \quad (-{}^1/2h_+ - {}^1/2h_- + h_o) + j\frac{\sqrt{3}}{2}(h_+ - h_-) \quad I_b + {}^1/3 \quad (-{}^1/2h_+ - {}^1/2h_- + h_o) - j\frac{\sqrt{3}}{2}(h_+ - h_-) \quad I_c \quad \text{------(05.5a)}$$

$$\text{or} \quad E_i = {}^1/2(h_+ + h_-)I_a + j\frac{1}{2\sqrt{3}}(h_+ - h_-)I_b - j\frac{1}{2\sqrt{3}}(h_+ - h_-)I_c + {}^1/3(h_o - {}^1/2h_- - {}^1/2h_+)(I_a + I_b + I_c) \text{------(05.5b)}$$

The two networks which follow from (05.5a) and (05.5b) are as shown in Fig. (5.13a) & Fig. (5.13b) (Ref. 5) respectively.

Now if h_+, h_-, h_o are imaginary quantities, say $h_+ = jk_+$, $h_- = jk_-$ and $h_o = jk_o$ then proceeding as before,

$$E_i = {}^1/2j(k_+ + k_-)I_a + \frac{1}{2\sqrt{3}}(k_- - k_+)I_b - \frac{1}{2\sqrt{3}}(k_- - k_+)I_c + {}^1/3j(k_o - {}^1/2k_- - {}^1/2k_+)(I_a + I_b + I_c) \quad \text{----(05.6a)}$$

$$\text{or} \quad E_i = {}^1/2j(k_+ + k_-)I_a - \frac{1}{2\sqrt{3}}(k_+ - k_-) + \frac{1}{2\sqrt{3}}(k_+ - k_-)I_c + {}^1/3j(k_o - {}^1/2k_- - {}^1/2k_+)(I_a + I_b + I_c) \quad \text{----(05.6b)}$$

The two networks corresponding to (05.6a) and (05.6b) are as shown in Fig. (5.14a) & Fig. (&.14b) respectively.

Virtually all the combined networks designed previously are special cases of either of the four networks Fig. (5.13a), Fig. (5.13b), Fig. (5.14a) and Fig. (5.14b); e.g. the filter network of Fig. (5.8c) is a special case of Fig. (5.14a) when $h_+ = 0$

It is to be noticed that Figs. (5.13a), (5.13b), (5.14a) and (5.14b) are designed on the basis that h_+, h_-, h_o are either real or imaginary. This is done only to maintain the simplicity of the filters.

5.4 THREE PHASE FILTERS:-

If

$$aI_a + bI_b + cI_c = h_+I_+ + h_-I_- + h_oI_o \quad \text{----------(05.7a)}$$

then $bI_a + cI_b + aI_c = h_+(@^2I_+) + h_-(@I_-) + h_oI_o$ ---(05.7b)

and $cI_a + aI_b + bI_c = h_+(@I_+) + h_-(@^2I_-) + h_oI_o$ ---(05.7c)

where a, b, c, h_+, h_- and h_o are complex quantities holding the following relations

$$h_+ = a + @^2b + @c$$

$$h_- = a + @b + @^2c$$

$$h_o = a + b + c$$

Now if h_+ and h_o are zero, as for instance in the case of negative-sequence filter whose output quantity does not contain positive and zero sequence components, then the above three expressions will become three phase balanced quantities (h_-I_-, $@h_-I_-$, $@^2h_-I_-$). This is the basis of three phase filters. It will be noticed here that a, b, c are so chosen that h_+ and h_o become zero and I_a, I_b and I_c are replaced by each other in rotation in the successive expressions.

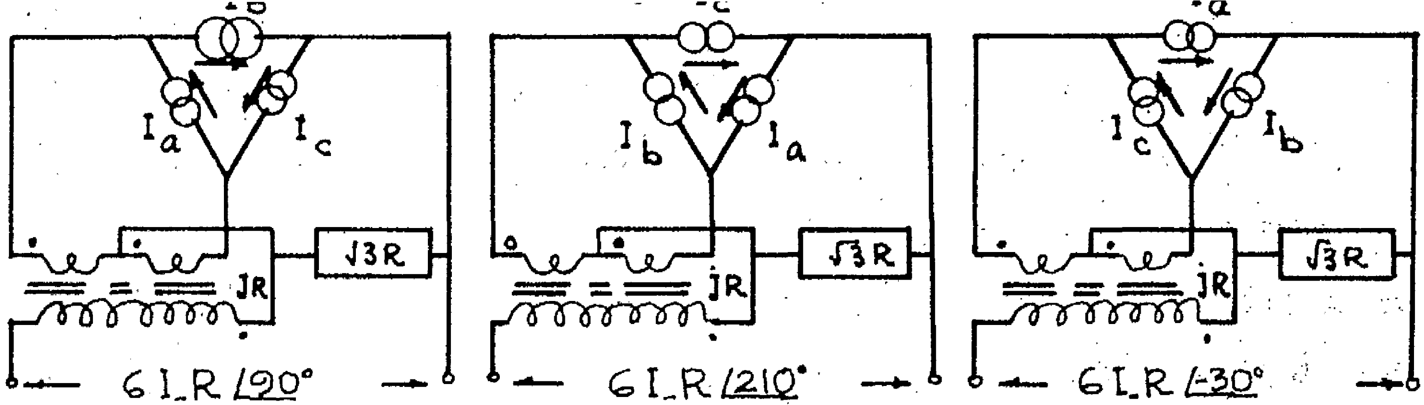


FIG 5.15

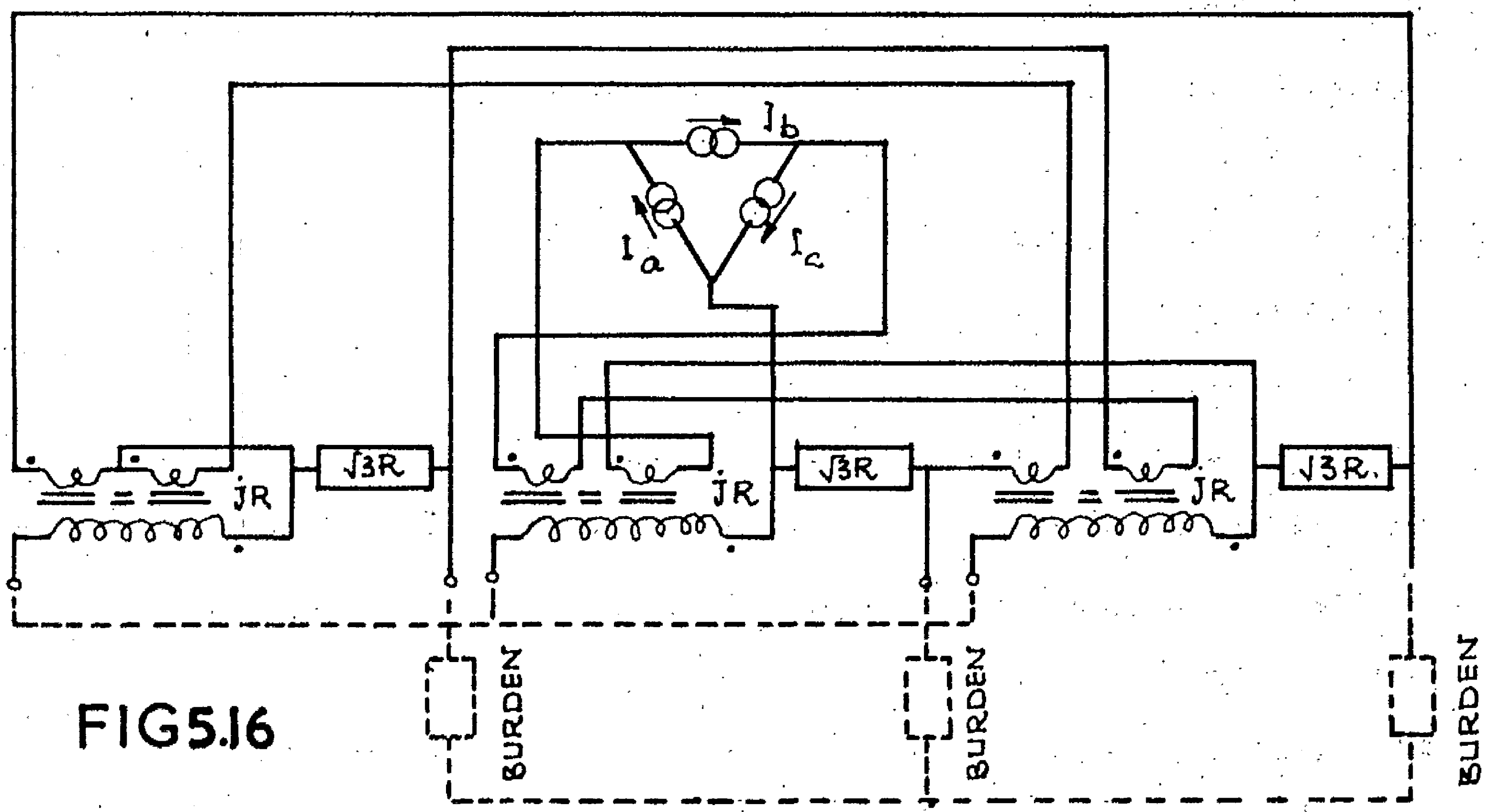
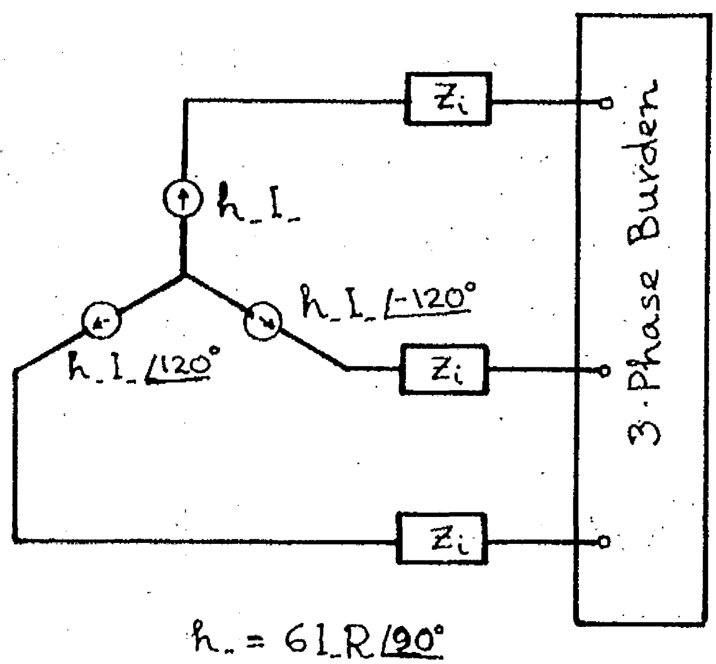
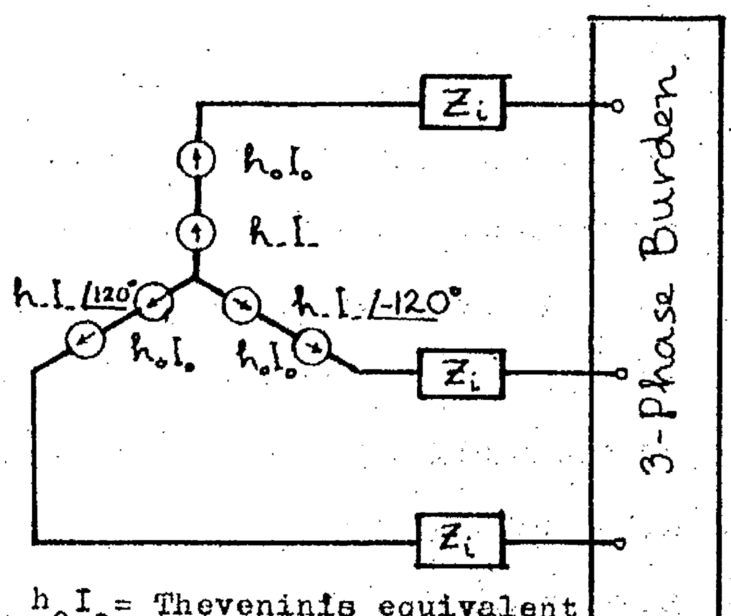


FIG 5.16



$h_- = 6I_-R\angle 90°$

FIG 5·17.



h_oI_o = Theveninis equivalent voltage source due to zero-sequence component.

FIG 5.18.

For every single-phase negative sequence filter there is a corresponding three-phase filter. For example, the three-phase filter corresponding to Filter No. 1 (Art. 3.11) can be obtained as follows:-
three single-phase outputs, mutually 120° apart, are shown in Fig. 5.15. These three networks can be used to form a three-phase system by connecting them in the usual manner. This can be easily simplified as shown in Fig. 5.16. Here the source of primary supply is used once instead of three times as in Fig. 5.15. An important point is that the circuitry should be such that there is no unwanted feedback in any phase from other phases. The use of intermediate Current Transformers may sometimes be necessary to avoid such feedback. The output connection can be either star or delta. A star-connection is shown by dotted lines in Fig. 5.16. Thevenin's three-phase equivalent circuit of this filter is shown in Fig. 5.17, where ''Z_1'' is the internal impedance of each phase.

In the expressions (05.7a), (05.7b), (05.7c) a zero sequence component is also present along with a negative-sequence component (i.e. only positive sequence component is eliminated) then the expected three-phase equivalent circuit will be as shown in Fig. 5.18. It is to be noticed here that the flow of currents in the burden will not be affected at all due to zero-sequence components (h_oI_o) i.e. the network will still behave as a three-phase negative sequence current filter.

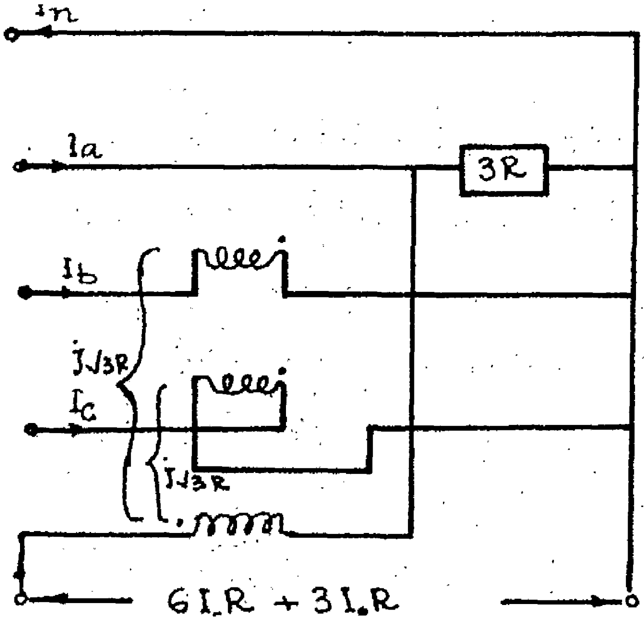


FIG 5.19.

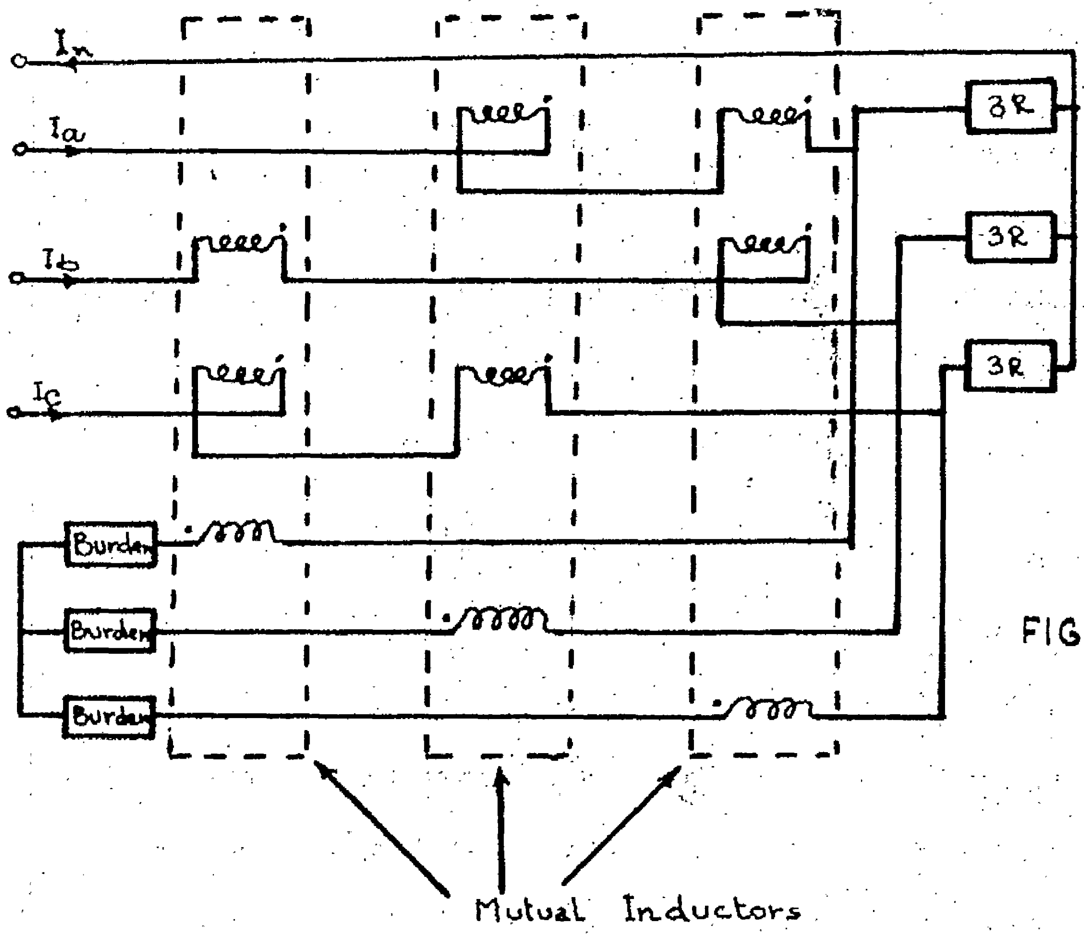


FIG 5.19a.

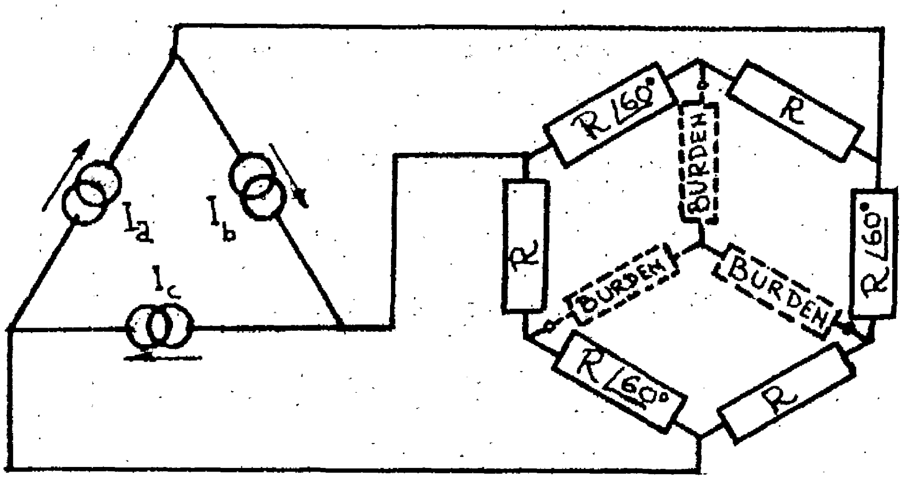


FIG 5.20.

So It is not always necessary to use additional measures such as cross-connected C.T.s to eliminate zero-sequence component while obtaining a three-phase negative sequence filter. Based on the above knowledge, a simple three-phase filter corresponding to the single-phase filter of Fig. 5.19 is designed (Ref. 21) as shown in Fig. 5.19a. It is to be noticed that the above-mentioned single phase filter is a special case of the filter shown in Fig. 5.9b (''$h_- I_- + h_o I_o$'' type) where R' = R.

In this three-phase filter, if there is a neutral return path between burden and filter, zero-sequence current will flow thus upsetting the three-phase system. So a neutral path through the burden should be avoided in such filters; zero-sequence current should only flow through the filter-elements.

Thus, for each single phase ''negative sequence filter'' or ''combined filter of negative and zero-sequence components'' there can be a corresponding three-phase negative sequence filter. But their circuitry, which may need intermediate current transformers to isolate different phases, is usually complex. One simple filter, however, corresponding to a bridge filter (Filter 2a, Table 2) is shown in Fig. 5.20 as suggested in Refs. 3, 6).

Similarly, for each single phase ''positive sequence filter'' or ''combined filter of positive or zero-sequence components'' there can be a corresponding three-phase positive-sequence filter.

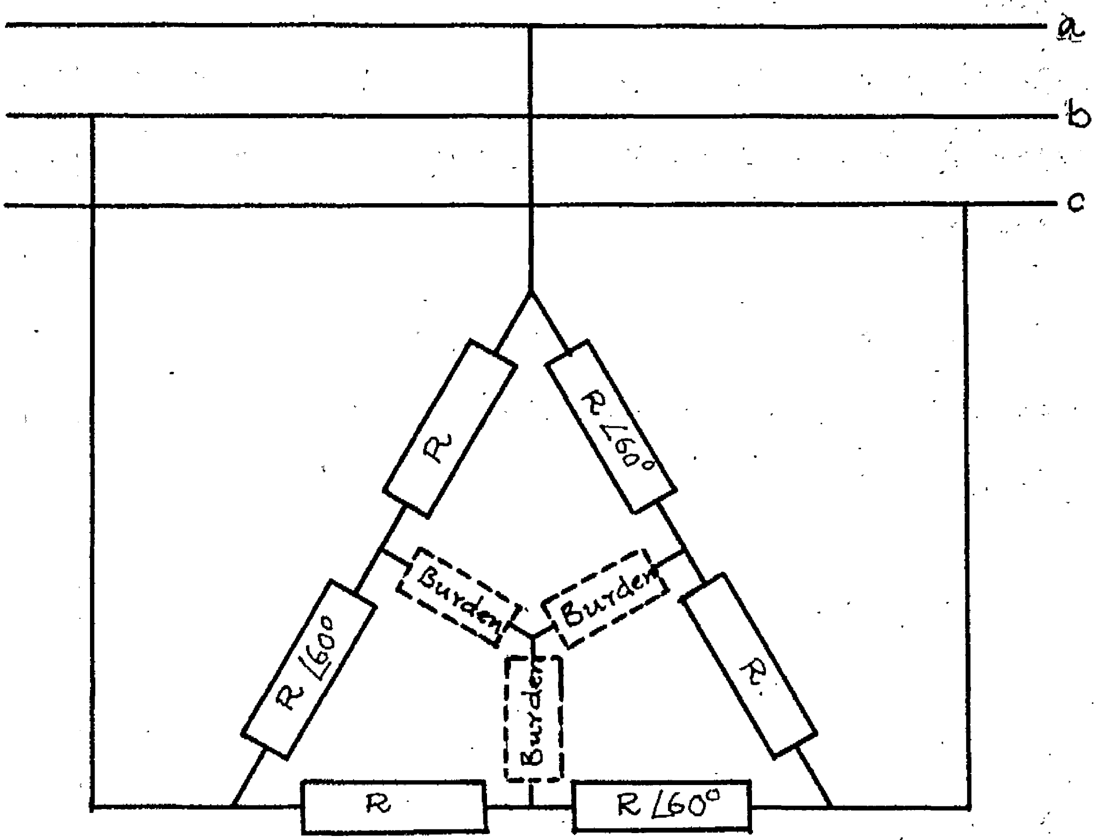


FIG 5. 21.

It is to be noticed that since the zero-sequence components in the three expressions are in phase with one another, it is not possible to obtain a three-phase zero-sequence filter. Again it is to be noticed that the positive and negative sequence components have 120° phase shifts in successive expressions (05.7a), (05.7b) (05.7c) in two opposite directions of rotation. Thus any three-phase filter of the combined type ''$h_+I_+ \pm h_-I_-$'' cannot be achieved.

Three-phase voltage filters can be designed in a similar manner. One such filter corresponding to current filter of Fig. 5.20 is shown in Fig. 5.21 (Refs. 3, 6).

5.5 SEGREGATION OF α, β, 0 COMPONENTS

The three line currents can be related to α, β, 0 components by the following equations:-

$$I_a = I_\alpha + I_o$$

$$I_b = -{}^1/2 I_\alpha + \frac{\sqrt{3}}{2} I_\beta + I_o$$

$$I_c = -{}^1/2 I_\alpha - \frac{\sqrt{3}}{2} I_\beta + I_o$$

Now, say, it is required to achieve a filter of the general type ''$h_\alpha I_\alpha + h_\beta I_\beta + h_o I_o$'' where h_α, h_β, h_o are the arbitrary chosen constants.

In general, let the input current of the general form ''$AI_a + BI_b + CI_c$'' be examined. Resolving I_a, I_b and I_c by ''α, β and 0'' components and equating the respective co-efficients of I_α, I_β and I_o with that of h_α, h_β and h_o the following sets of equations can be obtained

$$A - {}^1/_2B - {}^1/_2C = h_\alpha$$

$$ {}^1/_2B - {}^1/_2C = h_\beta$$

$$A + B + C = h_o$$ which gives

$$A = {}^2/_3h_\alpha + {}^1/_3h_o$$

$$B = -{}^1/_3h_\alpha + \frac{1}{\sqrt{3}}h_\beta + {}^1/_3h_o$$

$$C = -{}^1/_3h_\alpha - \frac{1}{\sqrt{3}}h_\beta + {}^1/_3h_o$$

From above it can be seen that if h_α, h_β and h_o are scalar quantities then A, B, C are also scalar quantities. So it is possible to achieve such filters by proper transformer connections or summation transformers as explained in Section 1.

It is to be noted, as mentioned before in Art , that it is not possible to isolate positive or negative sequence components by mere transformer connections.

No application of α, β filters has been mentioned in relaying schemes. But if the summators (or summation-transformers) (Ref. 5) are taken as the combined α, β filters then their application can be found in practice as auxiliary mechanisms in differential line protection.

The advantage of summators or α, β filters is that they are very simple in that they work on the transformer-principles and such being the case, they are very efficient also.

SECTION 6

6.1. INTRODUCTION :

To provide a complete picture on sequence filters, it is considered valuable to give a brief review of the usage of these filters in general, in connection with the protection of transmission and distribution lines. This section outlines briefly the roles performed by the sequence filters in protective schemes, 'Single-phase-auto-reclosing', 'Pilot-channel relaying', 'Distance relaying' and 'Power-swing blocking'.

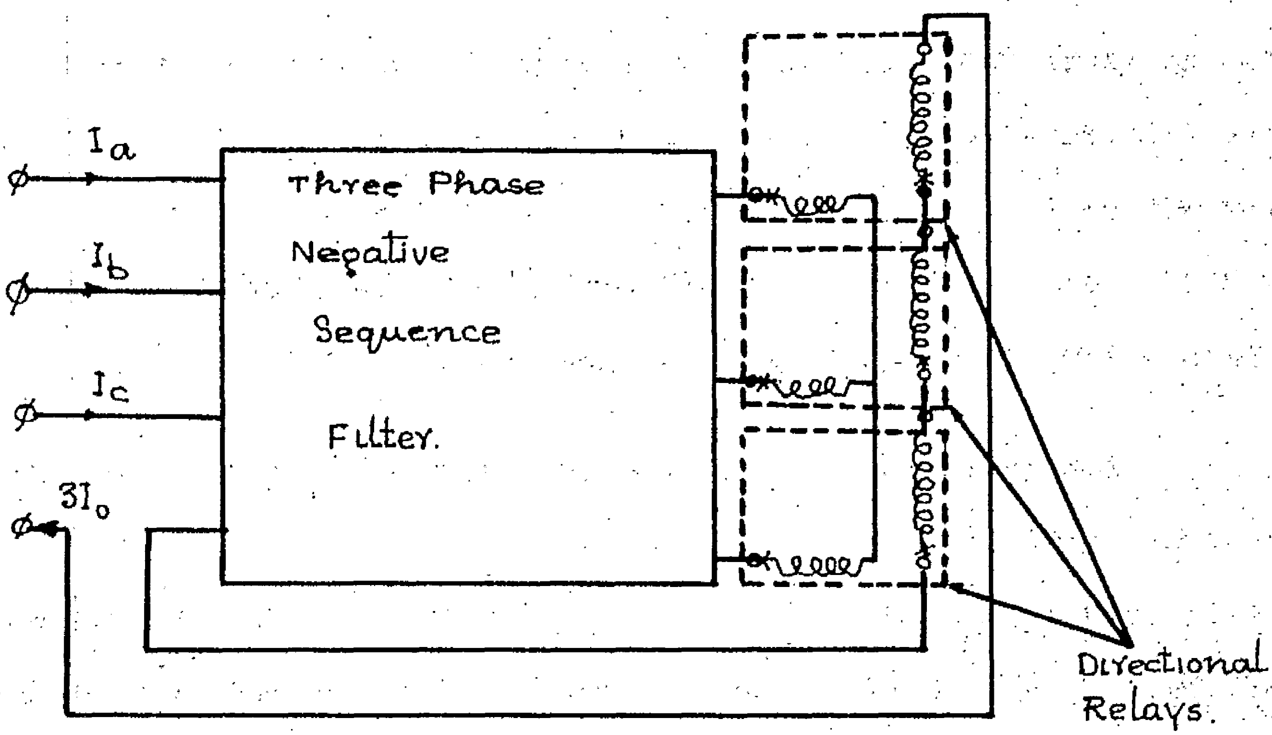
The following facts are basic clues for designing relaying-schemes :-

(a) All faults contain positive-sequence currents

(b) During power-swing only positive sequence current is present

and (c) Any fault involving earth, except symmetrical three-phase to earth fault, contains zero-sequence component.

The generalised equivalent circuits for determining the fault currents for series and shunt faults are given in reference 5 using symmetrical components.



I_a
I_b
I_c
$3I_0$
Three Phase
Negative
Sequence
Filter.
Directional
Relays.

FIG 6.1.

6.2 Single-Phase-Auto-Reclosing

Previously three-pole tripping and reclosing was widely practised but, later, by the introduction of single-pole tripping and auto-reclosing it was made possible to isolate the faulty line leaving the healthy lines to transmit power, ensuring increased and improved stability and reliability of the system. On a phase-to-earth fault (it is assumed that the system neutrals are grounded) the scheme operates only to open and reclose the faulty phase and on all other types of fault all three phases are opened. All the phases are tripped if the fault persists after the first reclosure and further reclosing is prevented.

The establishment of sequence-filter theory resulted in the development of a sensitive directional relay in 1942 which is capable of distinguishing the faulty phase from the healthy phases (Ref. 20). It depends solely on fault conditions and is independent of all normal conditions.

The scheme utilises a three-phase negative-sequence-filter whose output contains three symmetrically displaced balanced negative-sequence currents. Three similar directional relays, each of which is polarised by zero-sequence current, are connected to the output terminals of the sequence-filter one in each phase as shown in Fig. 6.1. The directional relays consist of power-elements which depend on the cosine of the angle of the input quantities and they are arranged such that if one of them reacts to I_-(negative-sequence current) and I_0(zero-sequence-current), the second will react to $@I_-$ and I_o and the third to $@^2I_-$ and I_o where @ is the conventional 120° operator.

In the grounded-system, during the single-phase to earth fault, the phase angle relations between I_- and I_o are as follows:-

(1) A-phase to ground fault : I_o is in phase with I_-
(2) B-phase to ground fault : I_o leads I_- by 120°
(3) C-phase to ground fault : I_o lags I_- by 120°

All vectors are referred to phase current vector of phase A

So the directional relay which reacts to I_- and I_o will act in one direction for A-E fault, but will act in opposite direction for B-E and C-E faults. Thus this directional relay is capable of detecting the ground fault on phase A. Similarly the other two relays will behave exactly similarly where one of them will detect the ground fault on phase B and other on phase C respectively.

When one phase is tripped, the currents passing in the other two phases forms the following phase-angle relations between I_- and I_o :-

(1) Phase-A tripped : I_o is in phase with I_-
(2) Phase-B tripped : I_o leads I_- by 120°
(3) Phase-C tripped : I_o lags I_- by 120°

All vectors are referred to current vector of phase A.

The above relations further ensure the selection of the damaged phase.

During two phase-to-earth faults the phase-angle relations which depend on zero-sequence and negative-sequence impedance-angles of the protected system, between I_- and I_o may be as follows:-

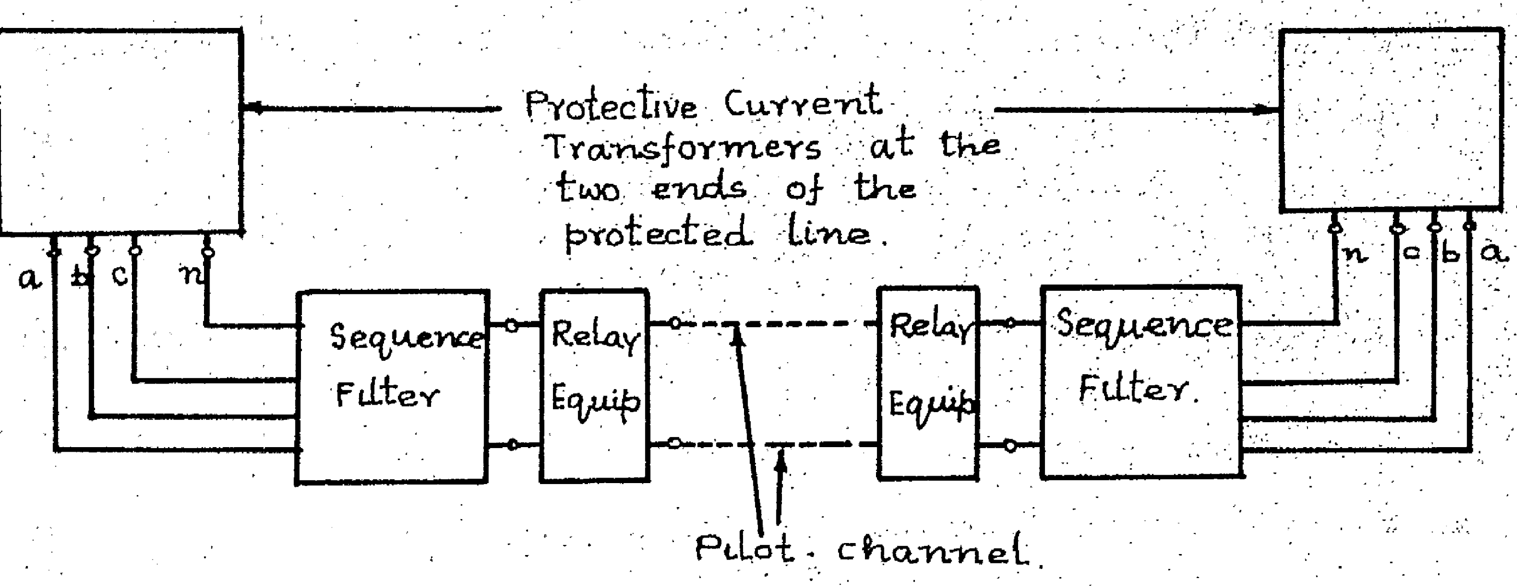

Protective Current Transformers at the two ends of the protected line.
a
b
c
n
Sequence Filter
Relay Equip
Relay Equip
Sequence Filter.
n
c
b
a
Pilot channel.

FIG 6.2.

(1) BC-ground fault : I_o is nearly in phase with I_-
(2) CA-ground fault : I_o leads I_- nearly by 120°
(3) AB-ground fault : I_o lags I_- nearly by 120°

} All vectors are referred to current vector of ph.A

The above relations show that the undamaged line will be falsely selected for tripping which is not wanted and this is a disadvantage of the scheme. To overcome this disadvantage an additional device is required. This is explained elsewhere (Ref. 20).

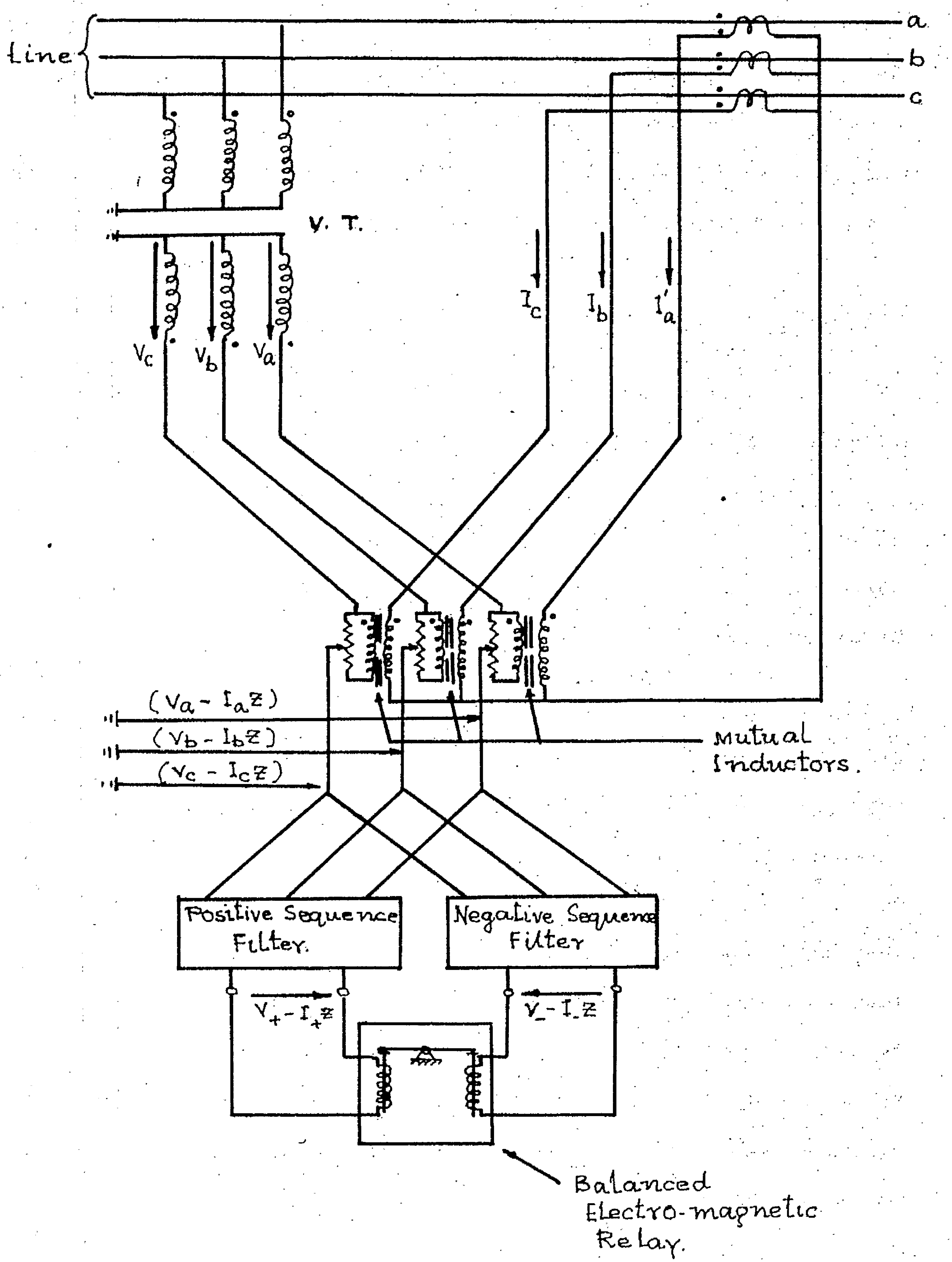
6.3 Pilot-Channel-Relaying

Full details of different types of Pilot-relaying schemes have been described elsewhere. (Refs. 11, 16, 17, 18, 21, 22). These systems are basically adaptations of the principles of differential relaying. Modification and economic feasability of these protective schemes were largely due to the application of sequence-filters by which a representative single-phase quantity can be derived from the three-phase and ground currents at the terminals of a protected line and these single-phase quantities can be compared over a pilot-channel. Because of these derived single-phase quantities by sequence-filters it has been possible to limit the pilot-channel-paths to a minimum of two only, which has not been feasible otherwise before by any other efficient way.

The block-diagram of the pilot-channel-relaying devices by using sequence-filters is shown in Fig. 6.2.

The typical phenomenon of all types of faults is that they all contain positive-sequence current (I_+). A sequence-filter with single-phase positive-sequence output is therefore a good choice for the required single-phase quantity in the pilot-relaying schemes. But in the case of ground fault, the magnitude of I_+ may be very small if the zero-sequence impedance of the protected section is high. To overcome this difficulty it is suggested in one of the schemes (using current comparison principle) to use a sequence-filter whose output is a single-phase quantity of amplified zero-sequence current (I_o) in addition to positive-sequence current (I_+), i.e. the quantity is of the form '$I_+ + k_oI_o$' where k_o is an amplifying factor for I_o (Ref. 21).

But in the above scheme the quantity '$I_+ + k_oI_o$' varies over a considerable range for different fault-conditions at the same location where I_+^+ is maximum for a three-phase fault and minimum for a ground fault. Again, the positive-sequence component I_+ of different types of faults is likely to lose some sensitivity due to the influence of through-current of the system loads which, as loads being usually balanced, contain only positive sequence currents. This may cause, sometimes, faulty operation in some schemes, e.g. incorrect blocking ina protective scheme using carrier channel. To overcome these difficulties, instead of using '$I_+ + k_oI_o$', '$I_- + k_oI_o$' is used as single phase quantity in a scheme using carrier-channel. But the scheme will fail to work during internal three-phase fault, because fault current will consist only of positive sequence and no negative and zero components.



a
b
c
Line
V. T.
V_c
V_b
V_a
I_c
I_b
I'_a
$(V_a - I_a Z)$
$(V_b - I_b Z)$
$(V_c - I_c Z)$
Mutual Inductors.
Positive Sequence Filter.
Negative Sequence Filter
$V_+ - I_+ Z$
$V_- - I_- Z$
Balanced Electro-magnetic Relay.

FIG 6.3.

So for three-phase fault, it is necessary to introduce an extra device in the scheme so that the single-phase quantity produced consists of positive-sequence current, the magnitude of which should, however, be greater than normal load current (Ref. 17).

6.4 Distance Relaying

In 1948, a scheme was produced requiring one relay only (Fig. 6.3) which uses two sequence filters (one positive and another negative) with single-phase output quantities, by which protection against all types of interphase short circuits is feasible. The principle of this scheme is briefly discussed below (Ref. 5, 19).

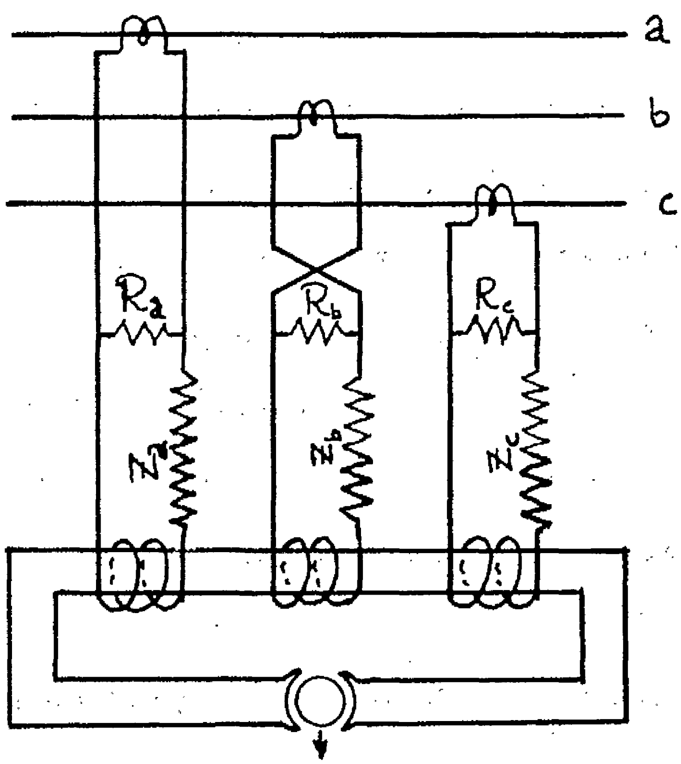
The input to the sequence filters are three voltages $(V_a - I_a Z)$, $(V_b - I_b Z)$ and $(V_c - I_c Z)$ where V_a, V_b and V_c are the derived voltages from the potential transformers of phases a, b and c respectively with respect to ground and I_a, I_b and I_c are the phase currents of the corresponding phases derived from current transformers and Z is an impedance the value of which depends on the impedance of the line protected. The output quantities of sequence filters are of the form $(V_+ - I_+ Z)$ and $(V_- - I_- Z)$ respectively where $(V_+ - I_+ Z)$ and $(V_- - I_- Z)$ are the equivalent positive and negative sequence somponents respectively of the input quantities to the common terminal of the sequence filters.

The restraining coil and the operating coil of the balanced electromagnetic relay are operated by the single-phase quantities $(V_+ - I_+Z)$ and $V_- - I_-Z)$ respectively. The impedance quantity Z can be chosen in such a way that for any type of metallic-interphase short circuits at the end of the protected zone the quantity $(V_+ - I_+Z)$ is equal to the quantity $(V_- - I_-Z)$, thus preventing the operation of the relay, i.e. the relay is in the balanced condition. When the interphase fault is outside the protected zone the relay operates in the opening direction and for faults within the protected zone it operates in the closing direction of the relay (Appendix XIX).Working on the same principle, a new device has been developed which the three input voltages $(V_a - I_aZ)$, $(V_b - I_bZ)$ and $(V_c - I_cZ)$ are directly applied to an induction-cup relay which works on induction-motor principle, i.e. when the negative sequence torque equals to the positive sequence torque the relay remains at standstill. When the negative sequence torque is greater than the positive sequence torque the relay moves in one direction and when the negative sequence torque is less than the positive sequence torque the relay moves in the other direction. These are the cases which arise depending on whether the inerphase fault is at the end, outside or inside the protected zone respectively (Ref. 13).

In the same scheme a separate relay working on similar principle protects against the three-phase fault.

6.5 Power-Swing Blocking

The only sequence component which is present during the swing (without fault) is positive sequence component. So the sequence filters which have an output containing only negative sequence component or only zero sequence component or both will have no output during swings. With this idea in mind, and using a negative-sequence (or combined negative and zero sequence) filter, a relaying device can be developed by which the main-protection can be unblocked during faults with or without swings and can be blocked during swing only (Refs. 5, 12). The disadvantage of such a scheme is that it may fail to work during three-phase fault which contains only positive sequence components. But practical experience has shown that there is momentary appearance of negative sequence component across the filter due to non-simultaneous short-circuiting of the three phases. It is to be noted also that there is some momentary voltage appearance across the filters due to transients during the three-phase or any fault.



$$\frac{R_a}{Z_a+R_a} = K\underline{/-60°}$$

$$\frac{R_b}{Z_b+R_b} = K\underline{/0°}$$

$$\frac{R_c}{Z_c+R_c} = K\underline{/60°}$$

Fig. 7

CONCLUSION

In protective schemes which are designed to react to the sequence components which arise during abnormal conditions, it is usual for a relay to act in conjunction with a seperate sequence filter. Such filters have been discussed in this work. This combination however is not necessarily the best arrangement. It may be sometime more convinient in practice, and more efficient, to combine the two units. An example of this procedure is the current filter 1c of Table 1(Art.3.6.1). Using the same network constants, an electromagnetic relay can be constructed (Ref.6) as shown in Fig.7, where the resulting magnetic flux is proportional to the sum of the currents in the three special symmetrical windings. Although design principles of such combined units have not been treated here, the filter theories which have been considered still apply.

Apart from the obvious possibility of developing new relaying schemes using sequence filters, further investigation of the transient behaviour of filters for different fault conditions is required. There is also considerable scope for thorough theoretical treatment of the performance of sequence filters in different relaying schemes.

This work is not presented with any degree of finality, but rather as an initial systematic analysis of a subject which has hitherto been treated in fragments. It is hoped that it may be of assistance in enabling filters for specific purposes to be synthesised from the general principles discussed.

CONDITIONS FOR OBTAINING NEGATIVE SEQUENCE FILTERS

Appendix 1A

By applying Thevenin's principle, the open circuit voltage of the filter (Fig. 2.4) is given by

$$E_{oc} = (A_1I_a + B_1I_b + C_1I_c)Z_1 + (A_2I_a + B_2I_b + C_2I_c)Z_2$$

$$+ \text{-----------} + (A_nI_a + B_nI_b + C_nI_c)Z_n$$

$$= (A_1Z_1 + A_2Z_2 + \text{-------} + A_nZ_n)I_a +$$

$$(B_1Z_1 + B_2Z_2 + \text{-------} + B_nZ_n)I_b +$$

$$(C_1Z_1 + C_2Z_2 + \text{-------} + C_nZ_n)I_c$$

or $E_{oc} = aI_a + bI_b + cI_c$ ---------------------------(1a)

where $a = A_1Z_1 + A_2Z_2 + \text{-------} + A_nZ_n$

$b = B_1Z_1 + B_2Z_2 + \text{-------} + B_nZ_n$

$c = C_1Z_1 + C_2Z_2 + \text{-------} + C_nZ_n$

Replacing I_a, I_b and I_c by sequence components in eqn. (1a),

$$E_{oc} = a(I_+ + I_- + I_o) + b(@^2I_+ + @I_- + I_o) + c(@I_+ + @^2I_- + I_o)$$

$$= (a + @^2b + @c)I_+ + (a + @b + @^2c)I_- + (a + b + c)I_o$$

In the above equation if

$a + @^2b + @c = 0$ ---------------------------(1b)

and $a + b + c = 0$ ---------------------------(1c)

then E_{oc} is a function of I_- only.

The necessary condition to obtain the desired negative sequence filter, found by solving equns. (1b) and(1c) is

$$\frac{a}{@^2 - @} = \frac{b}{@ - 1} = \frac{c}{1 - @^2} \qquad \text{------------------------(1d)}$$

Appendix 1B

From Fig. (2.5),

$$\text{Short circuit current} = I_{sc} = (A_1I_a + B_1I_b + C_1I_c)\frac{Z_2}{Z_1 + Z_2} + (A_2I_a + B_2I_b + C_2I_c)\frac{Z_3}{Z_3 + Z_4}$$

or $$I_{sc} = (A_1k_1 + A_2k_2)I_a + (B_1k_1 + B_2k_2)I_b + (C_1k_1 + C_2k_2)I_c \quad \text{.....(1d)}$$

where $k_1 = \dfrac{Z_2}{Z_1 + Z_2}$ and $k_2 = \dfrac{Z_3}{Z_3 + Z_4}$

To obtain a negative sequence filter, the co-efficients of I_+ and I_o must be zero,

hence from eqn. (1d)

$$(A_1k_1 + A_2k_2) + (B_1k_1 + B_2k_2) + (C_1k_1 + C_2k_2) = 0 \quad \text{..(1e)}$$

and $$(A_1k_1 + A_2k_2) + @^2(B_1k_1 + B_2k_2) + @(C_1k_1 + C_2k_2) = 0 \quad \text{...(1f)}$$

∴ from eqns. (1e) and(1f) the following relation can be obtained:

$$\frac{A_1k_1 + A_2k_2}{@ - @^2} = \frac{B_1k_1 + B_2k_2}{1 - @} = \frac{C_1k_1 + C_2k_2}{@^2 - 1} \qquad \text{-----(1g)}$$

APPENDIX 11 ERROR IN OUTPUT CURRENT OF C.T.

Norton's variant of the C.T. equivalent circuit is shown in Fig. (3.1a). If Z_L is the burden across the C.T. then the voltage across the load = E_L (say) = $n_{ct} I_{ct} \cdot \frac{Z_L Z_{mag}}{Z_L + Z_{mag}}$

$$= n_{ct}I_{ct}Z_L(1 + \frac{Z_L}{Z_{mag}})^{-1} = n_{ct}I_{ct}Z_L(1 - \frac{Z_L}{Z_{mag}}) \text{ as } Z_{mag} \gg Z_L$$

$$= n_{ct}Z_L(I_{ct} - \frac{I_{ct}Z_L}{Z_{mag}}) \quad \text{-----------------------------}(2.1)$$

In the ideal case voltage across the C.T. (or load) = $I_{ct}Z_L$

$$= E_{ct} \text{ (say)} \ldots (2.2)$$

∴ from (2.2) and (2.1)

$$E_L = n_{ct}Z_L(I_{ct} - \frac{E_{ct}}{Z_{mag}})$$

Output current of C.T. = current in Z_L = $\frac{E_L}{Z_L} = n_{ct}(I_{ct} - \frac{E_{ct}}{Z_{mag}})$

when $Z_m \gg Z_s$, then $Z_{mag} = Z_m + Z_s \doteqdot Z_m$ and

$$n_{ct} = \frac{Z_m}{Z_m + Z_s} \doteqdot 1$$

Output current of C.T. $\doteqdot (I_{ct} - \frac{E_{ct}}{Z_m})$

It can be argued that a current sequence filter consists of several current sources instead of a single source and hence the output current may be different. But it can be seen that the same result will be arrived at as shown by the following calculations:-

Let a black box containing a network with several current sources be put across the C.T. concerned as shown in Fig. (App. 2.1).

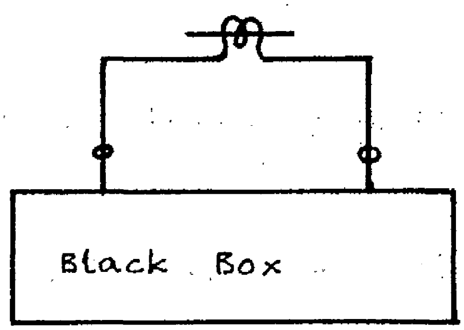


FIG App 2.1.

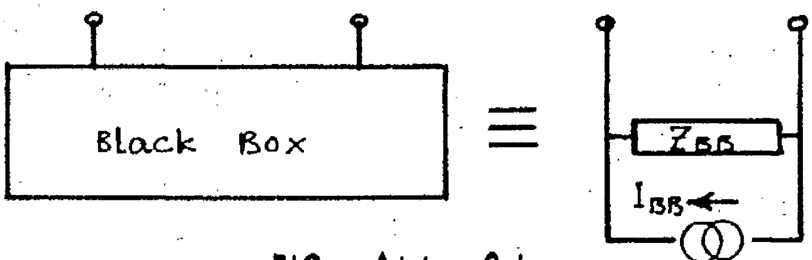


FIG App 2.1a.

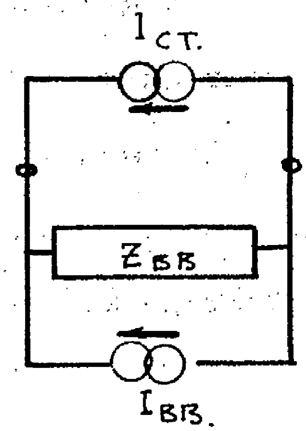


FIG App 2.1b.

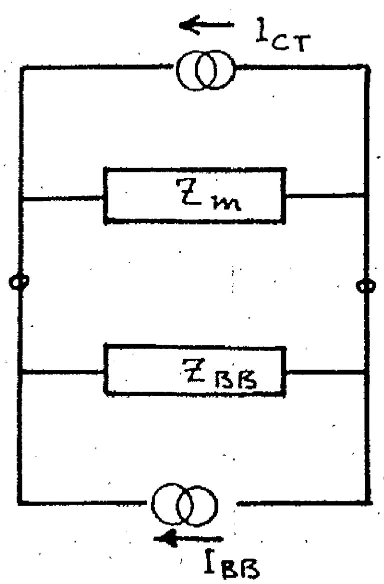


FIG App 2.1.C.

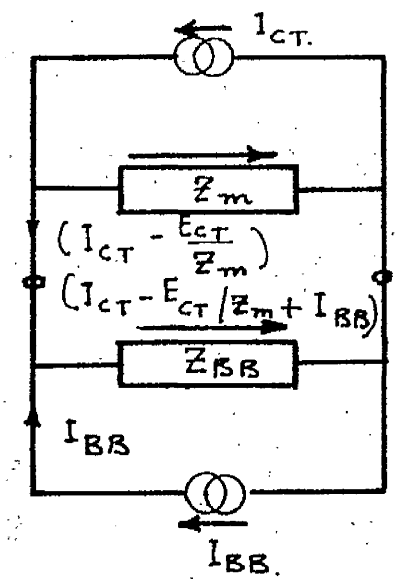


FIG App 2.1d.

But the black box also can be represented by a Norton's variant as shown in Fig. (App. 2.1a).

So the above circuit (Fig. App. 2.1) can be represented as shown in Fig. (App. 2.1b).

$\therefore$ Ideal voltage across the C.T. concerned = $(I_{ct} + I_{BB})\, Z_{BB}$.

Due to impurities in the C.T. concerned the circuit will be different from the ideal circuit as shown in Fig. (Appp 2.1c). Here Z_m is assumed to be vary large compared to Z_s and hence Z_s is ignored to simplify the calculation:-

$$\text{Voltage across the output terminal} = E_o = (I_{ct} + I_{BB})\, Z_{BB} \left(1 + \frac{Z_{BB}}{Z_m}\right)^{-1}$$

It has been mentioned before in the main text that the impedance seen at the C.T. terminal must be very low compared to Z_m; i.e.

$$Z_{BB} \ll Z_m \;\therefore\; E_o = (I_{ct} + I_{BB})\, Z_{BB}\left(1 - \frac{Z_{BB}}{Z_m}\right) \text{ as } Z_{BB} \ll Z_m$$

$$= Z_{BB}\left(I_{ct} + I_{BB} - \frac{E_{CT}}{Z_m}\right)$$

$$\therefore \quad \text{Current through } Z_{BB} = \frac{E_o}{Z_{BB}} = \left(I_{ct} - \frac{E_{ct}}{Z_m}\right) + I_{BB}$$

$\therefore$ Distribution of currents in the circuit should be as shown in Fig. (App. 2.1d) which shows that the current output from the C.T.

$$= \left(I_{ct} - \frac{E_{ct}}{Z_m}\right)$$

When Z_s cannot be ignored in comparison to Z_m, the above expression should be multiplied by n_{ct} and Z_m is replaced by $Z_{mag} (= Z_m + Z_s)$

APPENDIX III : ERROR IN VOLTAGE TRANSFORMER OUTPUT

From Fig.3.1b(actual equivalent circuit),

the impedance of Z_m in parallel with $(Z_{s2} + Z_L)$

$$= \frac{Z_m(Z_{s2} + Z_L)}{Z_m+Z_{s2}+Z_L} = Z_p \text{ (say)}$$

$$\therefore \text{ Voltage across } Z_p = V_{pt}\, \frac{Z_p}{Z_p+Z_{s1}} = (1- \frac{Z_{s1}}{Z_p})\, V_{pt} = n_{pt}\, V_{pt}$$

$$(\text{as } Z_p \gg Z_{s1})$$

$$\therefore \text{ Voltage across } Z_L = (\text{Voltage across } Z_p) \times \frac{Z_{s2}}{Z_L+Z_{s2}}$$

$$= n_{pt}\, V_{pt}\, (1- \frac{Z_{s2}}{Z_L}) \qquad (\text{as } Z_L \gg Z_{s2})$$

$$= n_{pt}\, (V_{pt} - \frac{V_{pt}}{Z_L} Z_{s2}) = n_{pt}(V_{pt}-I_{pt}Z_{s2})$$

$$\text{where } I_{pt} = \frac{V_{pt}}{Z_L} = \text{Ideal output current.}$$

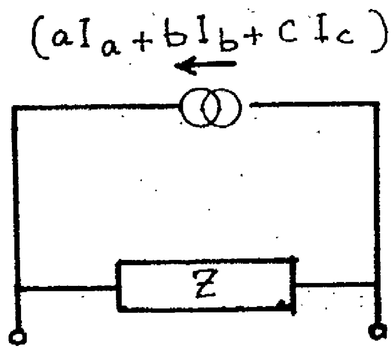


FIG. APP 4.1.

BASIC CONDITIONS FOR DESIGNING DIFFERENT KINDS OF NEGATIVE SEQUENCE FILTERS

Appendix IV A

Let the current in the impedance element Z (Fig. App. 4.1) be $(aI_a + bI_b + cI_c)$. So the output quantity (current) in the burden is proportional to $(aI_a + bI_b + cI_c)$

i.e. output current

$$\propto (aI_a + bI_b + cI_c)$$

$$\propto \left[(a - {}^1/2b - {}^1/2c) - j\frac{\sqrt{3}}{2}(b - c)\right] I_+$$

$$+ \left[(a - {}^1/2b - {}^1/2c) + j\frac{\sqrt{3}}{2}(b - c)\right] I_-$$

$$+ \left[a + b + c\right] I_o$$

which is similar to expression (1) (Art. 2.4). It has been explained in Art. 2.4 that it is not possible to isolate I_+ or I_-, one at a time, under the above condition.

Appendix IV B

From eqn. 1d (Appendix 1A)

$$\frac{a}{@^2 - @} = \frac{b}{@ - 1} = \frac{c}{1 - @^2} = Q \text{ (say)}$$

For a two-element filter n = 2

$\therefore$ putting
$$\left.\begin{aligned} a &= A_1Z_1 + A_2Z_2 \\ b &= B_1Z_1 + B_2Z_2 \\ c &= C_1Z_1 + C_2Z_2 \end{aligned}\right\}$$
in the above equation, the following sets of equations can be obtained.

A_1

$$A_1Z_1 + A_2Z_2 = Q(@^2 - @)$$

$$B_1Z_1 + B_2Z_2 = Q(@ - 1)$$

$$C_1Z_1 + C_2Z_2 = Q(1 - @^2)$$

giving $(A_1 + @^2B_1 + @C_1)Z_1 + (A_2 + @^2B_2 + @C_2)Z_2 = 0$ (4a)

and $(A_1 + B_1 + C_1)Z_1 + (A_2 + B_2 + C_2)Z_2 = 0$ -------(4b)

from (4a) $\frac{Z_1}{Z_2} = \frac{A_2 + @^2B_2 + @C_2}{A_1 + @^2B_1 + @C_1} \angle 180°$ -----------(4c)

$\therefore$ from (4b) and (4c)

$$(A_1 + B_1 + C_1)\frac{A_2 + @^2B_2 + @C_2}{A_1 + @^2B_1 + @C_1} \angle 180° + (A_2 + B_2 + C_2) = 0$$

or $(A_1 + B_1 + C_1)(A_2 + @^2B_2 + @C_2) - (A_2 + B_2 + C_2)(A_1 + @^2B_1 + @C_1) = 0$ ---(

To satisfy the above condition the real and the imaginary parts of the above equation should be zero, i.e.

$$\left[(A_1 + B_1 + C_1)(A_2 - {}^1/2B_2 - {}^1/2C_2) - (A_2 + B_2 + C_2)\times (A_1 - {}^1/2B_1 - {}^1/2C_1)\right] = 0$$

and $\left[(A_1 + B_1 + C_1)(-\frac{\sqrt{3}}{2}B_2 + \frac{\sqrt{3}}{2}C_2) - (A_2 + B_2 + C_2)\times (-\frac{\sqrt{3}}{2}B_1 + \frac{\sqrt{3}}{2}C_1)\right] = 0$

Solving the above pair of equations, the following relations can be obtained

$A_1 + B_1 + C_1 = 0$ ----------------------------(4d)

and $A_2 + B_2 + C_2 = 0$ ----------------------------(4e)

Appendix lV C

(a) If the input stimuli be $l(I_a - I_b)$ and $m(I_b - I_c)$ as shown in Fig. 3.3a,

then $A_1 = l$, $B_1 = -l$, $C_1 = 0$

and $A_2 = 0$, $B_2 = m$, $C_2 = -m$

$\therefore$ from eqn. (4c)

$$\frac{Z_1}{Z_2} = \frac{@^2 m - @m}{l = @^2 l} = x\angle 60^\circ \qquad \text{------------------------(4f)}$$

where $x = \frac{m}{l}$

(b) If the input stimuli be $l(I_a - I_b)$ and $m(I_a - I_o)$, Fig. 3.3b,

then $A_1 = l$, $B_1 = -l$, $C_1 = 0$

and $A_2 = {}^2/3m$, $B_2 = -{}^1/3m$, $C_2 = -{}^1/3m$

as $m(I_a - I_o) = m({}^2/3I_a - {}^1/3I_b - {}^1/3I_c)$

$\therefore$ from eqn. (4c)

$$\frac{Z_1}{Z_2} = \frac{m}{l}\,\frac{{}^2/3 - {}^1/3(@^2 + @)}{(1 - @^2)}\angle 180^\circ = \frac{x}{\sqrt{3}}\ \angle 150^\circ \qquad \text{--(4g)}$$

where $x = \frac{m}{l}$

(c) If the two input stimuli be $l(I_a - I_b)$ and $m(I_b - I_o)$, Fig. 3.3c,

then $A_1 = l$, $B_1 = -l$, $C_1 = 0$

and $A_2 = -{}^1/3m$, $B_2 = {}^2/3m$, $C_2 = -{}^1/3m$

as $m(I_b - I_o) = m({}^1/3I_a + {}^2/3I_b - {}^1/3I_c)$

$\therefore$ from eqn. (4c)

$$\frac{Z_1}{Z_2} = \frac{m}{l}\frac{(-{}^1/3 + @^2\,{}^2/3 - @\,{}^1/3)}{(1 - @^2)}\angle 180^\circ = \frac{x}{\sqrt{3}}\angle 30^\circ \quad (4h)$$

where $x = \frac{m}{l}$.

(d) If the two input stimulii be $l(I_a - I_b)$ and $m(I_c - I_o)$, Fig. 3.3d

then $A_1 = l$, $B_1 = -l$, $C_1 = 0$

and $A_2 = -{}^1/3m$, $B_2 = -{}^1/3m$, $C_2 = {}^2/3m$

as $m(I_c - I_o) = m(-{}^1/3I_a - {}^1/3I_b + {}^2/3I_c)$

$\therefore$ from eqn. (4c)

$$\frac{Z_1}{Z_2} = \frac{m}{l}\;\frac{-{}^1/3 - @^2\,{}^1/3 + @\,{}^2/3}{1 - @^2}\angle 180^\circ = \frac{x}{\sqrt{3}}\angle -90^\circ \ldots (4i)$$

where $x = \frac{m}{l}$

Appendix lV D

From eqn. (1d) (Appendix lA)

$$\frac{a}{@^2 - @} = \frac{b}{@ - 1} = \frac{c}{1 - @^2} = Q \text{ (say)}$$

Because it is a three element network n = 3,

$\therefore$ putting
$$\left.\begin{aligned} a &= A_1Z_1 + A_2Z_2 + A_3Z_3 \\ b &= B_1Z_1 + B_2Z_2 + B_3Z_3 \\ c &= C_1Z_1 + C_2Z_2 + C_3Z_3 \end{aligned}\right\}$$

in the above equation the following relations can be obtained

$$A_1Z_1 + A_2Z_2 + A_3Z_3 = Q(@^2 - @)$$

$$B_1Z_1 + B_2Z_2 + B_3Z_3 = Q(@ - 1)$$

$$C_1Z_1 + C_2Z_2 + C_3Z_3 = Q(1 - @^2)$$

which give

$$\frac{Z_1}{\begin{vmatrix} A_2 & A_3 & (@^2 - 1) \\ B_2 & B_3 & (@ - 1) \\ C_2 & C_3 & (1 - @^2) \end{vmatrix}} = \frac{-Z_2}{\begin{vmatrix} A_3 & (@^2 - @) & A_1 \\ B_3 & (@ - 1) & B_1 \\ C_3 & (1 - @^2) & C_1 \end{vmatrix}}$$

$$= \frac{Z_3}{\begin{vmatrix} (@^2 - @) & A_1 & A_2 \\ (@ - 1) & B_1 & B_2 \\ (1 - @^2) & C_1 & C_2 \end{vmatrix}} = \frac{Q}{\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}} \qquad \text{---------(4j)}$$

APPENDIX V BASIC CONDITION FOR DESIGNING BRIDGE FILTERS

Here, $A_1 I_a + B_1 I_b + C_1 I_c = I_a - I_b$

and $A_2 I_a + B_2 I_b + C_2 I_c = I_b - I_c$

$\therefore \quad A_1 = 1, \quad B_1 = -1, \quad C_1 = 0$

$A_2 = 0, \quad B_2 = 1, \quad C_2 = -1$

Putting these values in eqn. (1g) the following relations can be obtained

$$\frac{k_1}{@ - @^2} = \frac{k_2 - k_1}{1 - @} = \frac{-k_2}{@^2 - 1}, \quad \text{giving}$$

$$(1 - @^2)k_1 = k_2(@ - @^2)$$

or $k_1 \angle 30° = k_2 \angle 90°$

or $k_1 = k_2 \angle 60°$

or $$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_3}{Z_4 + Z_3} \angle 60° \qquad \text{------------(5a)}$$

or $$1 + \ddot{m}_1 \angle -60° + \ddot{m}_2 \angle 60° = 0 \qquad \text{------------(5b)}$$

where $\ddot{m}_1 = \frac{Z_1}{Z_2}$ & $\ddot{m}_2 = \frac{Z_4}{Z_3}$

APPENDIX VI   FREQUENCY QUALITY

(a) Let $f_{oc}(k)$ = Open circuit voltage due to positive sequence component when frequency is k times nominal frequency.

V_{oc-} = Open circuit voltage due to negative sequence component at nominal frequency.

$Z(k)$ = (Internal impedance of the filter + Burden impedance) when frequency is k times nominal frequency.

Z_i = Internal impedance of the filter at nominal frequency.

Z_L = Burden impedance at nominal frequency.

$f_{sc}(k)$ = Short circuit current due to positive sequence component when frequency is k times nominal frequency.

I_{sc-} = Short-circuit due to negative sequence component at nominal frequency.

Let a current filter be considered. Then from the knowledge of Thevenin's equivalent circuit and from Art. 3.7 :

Error due to positive sequence component

$$= E_{rror}\ (\text{say}) = \left| \left\{ f_{oc}(k)/Z(k) \right\} \left\{ (Z_i + Z_L)/V_{oc-} \right\} \right|$$

(as burden current = $V_{oc-} / (Z_i + Z_L)$)

$$\therefore \mu = \left| \frac{d}{dk}(E_{rror}) \right|_{k=1} = \frac{Z_i + Z_L}{V_{oc-}} \left| \frac{d}{dk} \frac{f_{oc}(k)}{Z(k)} \right|_{k=1}$$

$$= \frac{Z_i + Z_L}{V_{oc-}} \left| f_{oc}(k) \frac{d}{dk} \frac{1}{Z(k)} + \frac{1}{Z(k)} \frac{d}{dk} f_{oc}(k) \right|_{k=1}$$

$$= \frac{Z_i + Z_L}{V_{oc-}} \left[0 + \frac{1}{Z_i + Z_L} \left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} \right]$$

$$= \frac{1}{V_{oc-}} \left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6a)$$

Similarly it can be shown also that

$$\mu = \frac{1}{I_{sc-}} \left| \frac{d}{dk} f_{sc}(k) \right|_{k=1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6b)$$

6b) In a bridge filter, (Art. 3.6.2)

$$f_{sc}(k) = I_{ab} \frac{Z_2(k)}{Z_1(k) + Z_2(k)} + I_{bc} \frac{Z_3(k)}{Z_3(k) + Z_4(k)}$$

where $Z_1(k)$, $Z_2(k)$ etc. are the impedances of Z_1, Z_2 etc. at k times nominal frequency.

$$\therefore \frac{d}{dk} f_{sc}(k) \Big|_{k=1} = \left| -I_{ab} \left\{ 1 + \frac{Z_1(k)}{Z_2(k)} \right\}^{-2} \frac{d}{dk} \left\{ \frac{Z_1(k)}{Z_2(k)} \right\} - I_{bc} \left\{ 1 + \frac{Z_4(k)}{Z_3(k)} \right\}^{-2} \frac{d}{dk} \left\{ \frac{Z_4(k)}{Z_3(k)} \right\} \right|_{k=1} \ldots\ldots (6c)$$

From (5a) & (6c),

$$\frac{d}{dk} f_{sc}(k) \Big|_{k=1} = \left(1 + \frac{Z_4}{Z_3}\right)^{-2} \left| I_{ab} \angle 120° \frac{d}{dk} \left\{ \frac{Z_1(k)}{Z_2(k)} \right\} + I_{bc} \frac{d}{dk} \left\{ \frac{Z_4(k)}{Z_3(k)} \right\} \right|_{k=1}$$

$$= \sqrt{3}\ I_+ \left(1 + \frac{Z_4}{Z_3}\right)^{-2} \left| \frac{d}{dk} \frac{Z_1(k)}{Z_2(k)} \angle -120^\circ + \frac{d}{dk} \frac{Z_4(k)}{Z_3(k)} \right|_{k=1} \quad \ldots(6d)$$

$$I_{sc-} = I_{ab} \frac{Z_2}{Z_1+Z_2} + I_{bc} \frac{Z_3}{Z_3+Z_4} = 3I_- \frac{Z_3}{Z_3+Z_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6e)$$

∴ From (6b), (6d) and (6e)

$$\mu = \frac{1}{\sqrt{3}} \left\{1 + \frac{Z_4}{Z_3}\right\}^{-1} \left| \frac{d}{dk} \frac{Z_1(k)}{Z_2(k)} \angle -120^\circ + \frac{d}{dk} \frac{Z_4(k)}{Z_3(k)} \right|_{k=1} \quad \ldots\ldots(6f)$$

'Frequency error' and 'Frequency quality' are usually expressed in absolute values.

APPENDIX VII : IMPEDANCE QUALITY

Let E_{oc} = Open circuit voltage of the filter

$z_i \angle \phi_i$ = Internal impedance of the filter

$z_L \angle \phi_L$ = Burden impedance.

$$\therefore \text{ Volt-ampere output } = \left| \frac{E_{oc}}{z_i \angle \phi_i + z_L \angle \phi_L} \right|^2 z_L$$

$$= \left| \frac{E_{oc}^2 \; z_L}{z_i^2 + z_L^2 + 2\, z_i\, z_L \cos(\phi_i - \phi_L)} \right|$$

Differentiating the above expression with respect to z_L (taking z_L as variable) and equating to zero, gives $z_i = z_L$. So for maximum VA-output z_i should be equal to z_L.

$$\therefore \; \left| VA \right|_{\text{maximum}} = \left| \frac{E_{oc}^2}{2z_i \left\{ 1 + \cos(\phi_i - \phi_L) \right\}} \right|$$

Now if $(\phi_i - \phi_L)$ is made variable, then the maximum VA-output can be seen to be further increased if $(\phi_i - \phi_L) = \pm \pi$ which is not possible to be fulfilled in practice as explained in Art.3.8.

APPENDIX VII A : VA-INPUT TO THE FILTER

Filter 2c of Table 2 is chosen to show that the 'VA-input to the filter' calculated on the negative-sequence basis is equal to that calculated on the positive-sequence basis. Again it will be seen here that, in comparison, the calculation on the negative-sequence basis is more complex than that on the positive sequence basis.

Let I be the circulating current through the impedance elements on the open circuit condition of the filter, as shown in Fig.APP.7a.

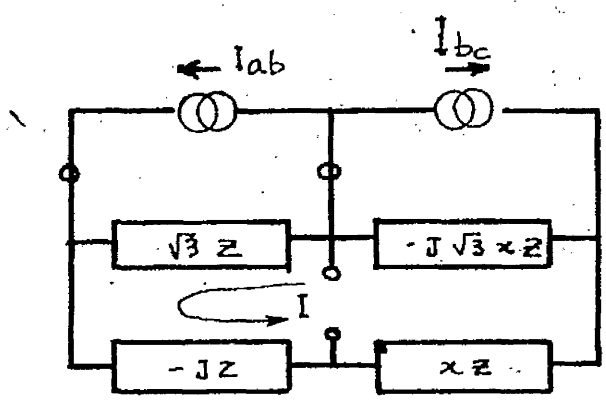


FIG App 7a.

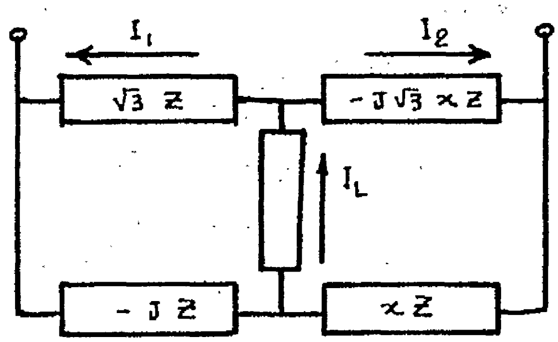


FIG App 7b.

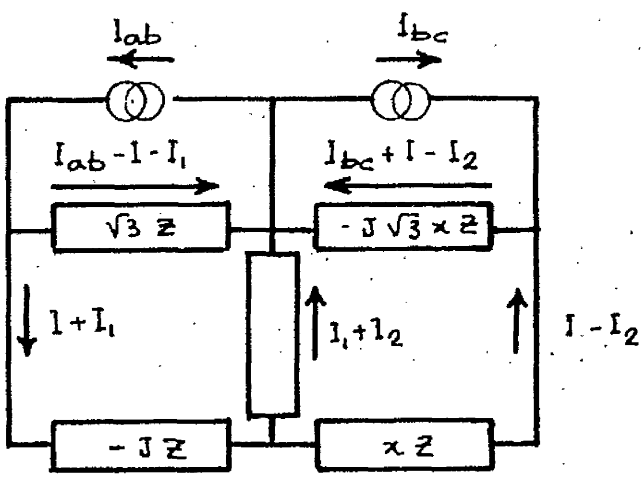


FIG App 7.C.

∴ From the loop,

$$I(xz + \sqrt{3}z - jz - jx\sqrt{3}z) - jx\sqrt{3}zI_{bc} - \sqrt{3}zI_{ab} = 0$$

or, $I(x+\sqrt{3}) - jI(1+x\sqrt{3}) = -3I_{-}x + 3I_{-}\angle{-30^\circ}$

(considering negative sequence component)

or, $$I = 3I_{-}\frac{\sqrt{(x^4-x^2+1)}}{2(x^2+\sqrt{3}x+1)}\angle{-\theta} \quad \text{.........................(7a)}$$

$$\text{where } \cos\theta = \frac{2-x^2}{2\sqrt{(x^4-x^2+1)}} \quad \& \quad \sin\theta = \frac{\sqrt{3}x^2}{2\sqrt{(x^4-x^2+1)}}$$

Now let I_L be the burden current,

∴ The distribution of currents I_1 and I_2 as shown in Fig.APP.7b is given by

$$I_1 = I_L\frac{xz-jx\sqrt{3}z}{\sqrt{3}z-jz+xz-j\sqrt{3}xz} = I_L'\,x(1-j\sqrt{3})$$

$$= I_L'\,2x\angle{-60^\circ} \quad \text{.....(7b)}$$

$$\text{where } I_L' = \frac{I_L}{(x+\sqrt{3}) - j(1+\sqrt{3}x)}$$

Similarly $$I_2 = I_L'(\sqrt{3} - j) = I_L'2\angle{-30^\circ} \quad \text{..............(7c)}$$

∴ By superposition of (7a), (7b) and (7c) the actual distribution of currents are shown in Fig.APP.7c.

∴ Volt-amp Input = $(VA)_{input}$ = $\Big[$(VA-input from source I_{ab}) + (VA-input from source I_{bc})$\Big]$

$$= \Big[\Big\{|I_{ab}^2|\sqrt{3}z - |I_{ab}.I|\sqrt{3}z\angle{-(\theta-30^\circ)} - |I_{ab}.I_1|\sqrt{3}z\angle{-(60^\circ-30^\circ)}$$

$$+ |I_{bc}^2|x\sqrt{3}z + |I_{bc}.I|x\sqrt{3}z\angle{(-90^\circ-\theta)-90^\circ} - |I_{bc}.I_2|x\sqrt{3}z\angle{-(90^\circ+30^\circ)-90^\circ}\Big\}\Big]$$

......................(7d)

On the negative sequence basis $|I_{ab}| = |I_{bc}| = |\sqrt{3}I_{-}|$ ....(7e)

Putting the values of I, I_1, I_2, $|I_{ab}|$ and $|I_{bc}|$ from (7a), (7b), (7c) and (7e) respectively in equation (7d), $(VA)_{input}$, after simplification, will be obtained as,

$$(VA)_{input} = \frac{3\sqrt{3}}{2} I_-^2 R \sqrt{(x^2+\sqrt{3}x+1)} \quad \text{.........................(7f)}$$

It is to be noted in expression (7d) that,

$$\left|I_{ab}\cdot I_1\right| \sqrt{3}z\angle{-30°} + \left|I_{bc}\cdot I_2\right| x\sqrt{3}z\angle{-210°} = 0$$

When the calculation is done on the positive-sequence basis, there is no current in the burden, i.e. the burden is in between equipotential points, and so this condition will remain uneffected even if the two equipotential points (or burden) are short-circuited(or open circuited). With this previlige, it can be appreciated that the calculation of $(VA)_{input}$ on the positive-sequence basis is simplified. It will be seen in Appendix XIII, equ.(13h) that the same result as (7f) is arrived by calculating on the positive sequence basis for the same filter.

APPENDIX VIII DESIGN FORMULAE OF 1st CATEGORY OF SERIES FILTERS

(a) The open circuit voltage = E_{oc}

$$= 1\ (I_a - I_b)\ Z_1 + m\ (I_b - I_c)\ Z_2$$

$$= 1Z_2 \left[(I_a - I_b) \frac{Z_1}{Z_2} + x(I_b - I_c) \right]$$

$$= 1 \left[\sqrt{3} I_- Z_2 \left\{ x\angle 30° + x\angle 90° \right\} \right] \left(\text{as } \frac{Z_1}{Z_2} = x\angle 60° \text{ , equ. 4f} \right)$$

$$= 1 \cdot (3 I_- x Z_2 \angle 60°) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

Internal impedance of the filter = Z_i

$$= Z_1 + Z_2 = Z_2 (1 + \frac{Z_1}{Z_2}) = Z_2 (1 + x\angle 60°) \quad \ldots\ldots\ldots\ldots\ldots (8b)$$

Let $Z_2 = z_2 \angle \psi$

$\therefore$ from (8b), $Z_i = z_2 (\angle \psi + x\angle 60° + \psi)$

or, $Z_i = z_2 \left\{ \cos\psi + x \cos(60° + \psi) + j \sin\psi + j\ x \sin(60° + \psi) \right\}$

$$= z_2 \sqrt{(x^2 + x + 1)} \angle \delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8c)$$

$$\text{where } \cos\delta = \frac{\cos\psi + x \cos(60° + \psi)}{\sqrt{(x^2 + x + 1)}}$$

$$\text{and } \sin\delta = \frac{\sin\psi + x \sin(60° + \psi)}{\sqrt{(x^2 + x + 1)}}$$

$\therefore$ Burden current = $I_L = \dfrac{E_{oc}}{Z_i + z_i \angle \theta}$ where $z_i \angle \theta$ is the matched burden

$$= 1 \cdot \frac{3 I_- x Z_2 \angle 60°}{z_2 \sqrt{(x^2 + x + 1)} \cdot (\angle \delta + \angle \theta)} \quad \text{(from (8a))}$$

$$\therefore \left| I_L \right| = 1\ \frac{3x}{\sqrt{(x^2 + x + 1)}} \left| \frac{I_-}{\angle \delta + \angle \theta} \right|$$

∴ Volt-amp output = $(VA)_{output}$

$$= \left|I_L^2 z_i\right| = 1^2 \frac{9x^2 z_2}{\sqrt{(x^2+x+1)}} \left|\frac{I_-}{\angle\delta + \angle\theta}\right|^2 \quad \text{.....................(8d)}$$

Volt-amp input (calculated on the positive sequence basis)

$$= (VA)_{input} = 1^2 \cdot 3\left|I_+\right|^2 \left|xZ_1 + \angle x^2 Z_2\right|$$

$$= 1^2 \cdot 3\left|I_+^2 x z_2 (\angle 60^\circ + x)\right|$$

$$= 1^2 \cdot 3\left|I_+\right|^2 x z_2 \sqrt{(x^2+x+1)} \quad \text{..................(8e)}$$

∴ from (8d) & (8e)

$$\text{VA-quality} = \eta = \frac{3x}{x^2+x+1} \left|\frac{1}{\angle\delta + \angle\theta}\right|^2$$

$$= \frac{1.5x}{x^2+x+1} \cdot \frac{1}{1 + \cos\theta \cdot \cos\delta + \sin\theta \cdot \sin\delta} \quad \text{......(8f)}$$

Two sets of VA-quality curves from (8f) with two suitable values of ψ ($\psi = 0^\circ$ and $\psi = -90^\circ$) are shown in Figs.G1 for 60° (i.e., θ=60°) and 90° (i.e., θ=90°) matched burdens for comparisons. It is to be noted here that a 90° matched burden is impossible to achieve, but it is chosen because it is the extreme case of pure inductive burden.

(b) It can be seen from the relation (4f) that the impedance elements can be chosen such that :

Z_1 and Z_2 can both be inductive

or, Z_1 can be inductive and Z_2 can be capacitive

or, Z_1 and Z_2 can both be capacitive .

If Z_1 and Z_2 are both be inductive, i.e. $Z_2 = z_2\angle\psi$, say,

and $Z_1 = xz_2\angle 60^\circ + \psi$

at nominal frequency,

then

then $Z_1(k) = xz_2 \cos(60° + \psi) + j\, xkz_2 \sin(60° + \psi)$ ....(8g)

and $Z_2(k) = z_2 \cos\psi + j\, k\, z_2 \sin\psi$ ....(8h)

$\therefore$ from (8g) and (8h)

$$f_{oc}(k) = 1\Big\{\big[x\, z_2 \cos(60° + \psi) + j\, k\, x\, z_2 \sin(60° + \psi)\big] \cdot I_{ab} + \big[z_2 \cos\psi + j\, k\, z_2 \sin\psi\big] \cdot x\, I_{bc}\Big\}$$

$$\therefore \left|\frac{d}{dk} f_{oc}(k)\right|_{k=1} = \sqrt{3}\, I_+\, z_2 \cdot 1 \cdot \left|x \sin(60° + \psi)\angle 30° + x \sin\psi \angle 90°\right|$$

$$= 1\left\{\frac{3}{2} I_+\, x\, z\right\} \text{ ....(8i)}$$

from (8a), $V_{oc-} = |E_{oc}| = 1.\ 3\ I_-\ x\ z_2$ ....(8j)

$\therefore$ from (6a), (8i) and (8j),

$$\mu = 0.5 \text{ ....(8k)}$$

Similarly it can be shown that the frequency quality is 0.5 if Z_1 and Z_2 are both capacitive.

Now let Z_1 be inductive and Z_2 be capacitive, e.g.

$$\left.\begin{aligned} Z_2 &= z_2 \angle -\psi \\ \text{and}\quad Z_1 &= x\, z_2 \angle 60° - \psi \end{aligned}\right\} \text{(at nominal frequency)}$$

$$\therefore\ Z_1(k) = x\, z_2 \cos(60° - \psi) + j\, k\, x\, z_2 \sin(60° - \psi)$$

$$\&\ \ Z_2(k) = z_2 \cos\psi - j \frac{z_2}{k} \sin\psi$$

$$\therefore\ f_{oc}(k) = 1\left\{I_{ab}\left[x\, z_2 \cos(60° - \psi) + j\, k\, x\, z_2 \sin(60° - \psi)\right] + x\, I_{bc}\left[z_2 \cos\psi - j \frac{z_2}{k} \sin\psi\right]\right\}$$

$$\therefore \left|\frac{d}{dk} f_{oc}(k)\right|_{k=1} = \sqrt{3}\, I_+\, x\, z_2\, 1 \left|\sin(60° - \psi) + \sin \angle -120°\right|$$

$$= \sqrt{3}\, x\, z_2\, I_+\, 1 \sqrt{(.75 - \frac{\sqrt{3}}{2} \sin 2\psi + \sin^2\psi)} \text{ ...(8l)}$$

$\therefore$ from (6a), (8j) and (8l)

$$\mu = \frac{1}{\sqrt{3}} \sqrt{(.75 - \frac{\sqrt{3}}{2} \sin 2\psi + \sin^2\psi)} \text{ ....(8m)}$$

the minimum value of which is .29 at $\psi = 30°$.

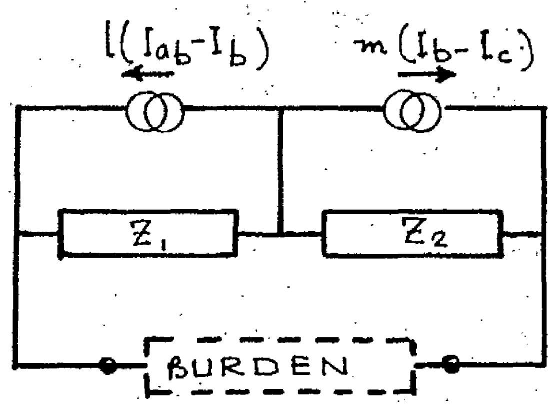


FIG APP B.1.

(c) If one of the stimulli is reversed (Fig. APP 8.1) then to obtain the negative sequence filter, the condition to be satisfied is

$$\frac{Z_1}{Z_2} = x\angle{-120^\circ} \; .$$

Let $Z_2 = z_2\angle{\psi}$ } (at nominal frequency)
and $Z_1 = x\, Z_2\angle{-(120^\circ-\psi)}$ }

$$\therefore Z_1(k) = x\, z_2 \cos(120^\circ-\psi) - j\,\frac{x\, z_2}{k} \sin(120^\circ-\psi)$$

and $Z_2(k) = z_2 \cos\psi + j\, k\, z_2 \sin\psi$

$$\therefore f_{oc}(k) = \left[I_{ab}\left\{ x\, z_2 \cos(120^\circ-\psi) - j\,\frac{x\, z_2}{k} \sin(120^\circ-\psi) \right\} - I_{bc}\left\{ x\, z_2 \cos\psi + j\, x\, k\, z_2 \sin\psi \right\} \right] . 1$$

$$\therefore \left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} = \sqrt{3}\, I_+\, x\, z_2 \left| \sin(120^\circ-\psi) - \sin\psi\angle{-120^\circ} \right| .1 \qquad \text{..........(8n)}$$

$$V_{oc-} = 3\, I_-\, x\, z_2 \, . \, 1 \qquad \text{..............................(8p)}$$

$\therefore$ from (6a), (8n) and (8p),

$$\mu = \frac{1}{\sqrt{3}} \left| \sin(120^\circ-\psi) - \sin\psi\angle{-120^\circ} \right| \qquad \text{...........(8q)}$$

From above $\mu = .765$ when $\psi = 30^\circ$
$\mu = .866$ when $\psi = 60^\circ$
and $\mu = .765$ when $\psi = 90^\circ$

APPENDIX IX DESIGN FORMULAE OF 2nd CATEGORY OF SERIES FILTERS

(a) Open circuit voltage = E_{oc}

$$= \left[\sqrt{3} I_- \; Z_1 \angle -30^\circ + x \; I_- \; Z_2\right] . 1 = \left[I_- \; x \; Z_2 \angle 60^\circ\right] . 1 \quad \text{......(9a)}$$

$$\left(\text{as } \frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}} \angle 150^\circ \text{ ; from equ. 4g}\right)$$

Now let $Z_2 = z_2 \angle \psi$

then $Z_i = Z_1 + Z_2 = \frac{Z_2}{\sqrt{3}} (\sqrt{3} + x \angle 150^\circ)$

$$= \frac{Z_2}{\sqrt{3}} \left| \sqrt{3} \cos \psi - \cos(\psi - 30^\circ) + j \sqrt{3} \sin \psi - j \sin(\psi - 30^\circ) \right|$$

$$= \frac{Z_2}{\sqrt{3}} \sqrt{(3 - 3x + x^2)} \angle \delta \quad \text{..............................(9b)}$$

where $\cos \delta = \frac{\sqrt{3} \cos \psi - \cos(\psi - 30^\circ)}{\sqrt{(3 - 3x + x^2)}}$

$\sin \delta = \frac{\sqrt{3} \sin \psi - \sin(\psi - 30^\circ)}{\sqrt{(3 - 3x + x^2)}}$

$\therefore$ From (9a) and (9b), Burden current = $|I_L|$

$$= 1. \frac{\sqrt{3} x}{\sqrt{(3 - 3x + x^2)}} \left| \frac{I_-}{\angle \delta + \angle \theta} \right|$$

, where $\angle \theta$ = angle of the matched burden.

$\therefore$ $(VA)_{output} = \left| I_L^2 \; z_i \right|$

$$= 1^2 . \frac{\sqrt{3} x^2 \; Z_2}{\sqrt{(3 - 3x + x^2)}} \left| \frac{I_-}{\angle \delta + \angle \theta} \right|^2 \quad \text{.................(9c)}$$

$$(VA)_{input} = \left| I_+^2 (3 Z_1 + x^2 Z_2) \right| . 1^2 = = x \; z_2 \left| I_+^2 (\sqrt{3} \angle 150^\circ + x) \right| . 1^2$$

$$= x \; z_2 \; I_+^2 \sqrt{(3 - 3x + x^2)} \; . \; 1^2 \quad \text{....................(9d)}$$

$\therefore$ From (9c) and (9d), VA-quality = $= \frac{\sqrt{3} x}{(3 - 3x + x^2)} \left| \frac{1}{\angle \delta + \angle \theta} \right|^2$

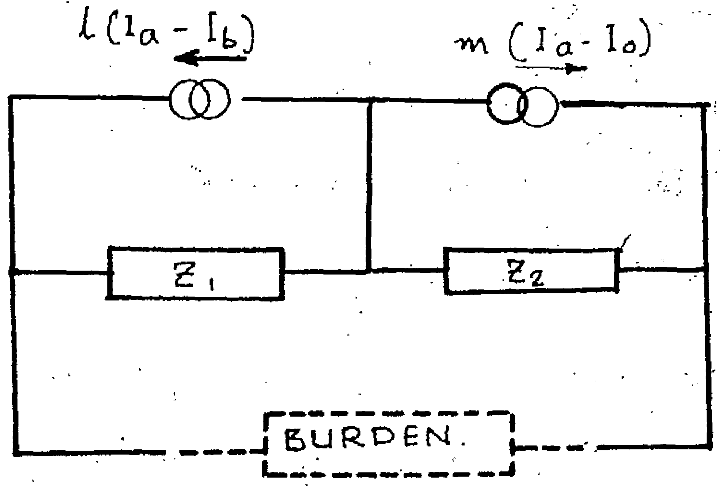


FIG App 9.1.

$$= \frac{\sqrt{3}x}{(3-3x+x^2)} \cdot \frac{1}{2(1 + \cos\theta \cos\delta + \sin\theta \sin\delta)}$$

A set of curves with 60° and 90° matched burdens is shown in Fig.G2 corresponding to the above equation when $\Psi = -90°$.

(b) Let $Z_1 = \frac{x}{\sqrt{3}} z_2 \angle 150° - \Psi$ and $Z_2 = z_2 \angle -\Psi$ } (at nominal frequency)

$$\therefore Z_1(k) = \frac{x\ z_2}{\sqrt{3}} \cos(150°-\Psi) + j\ k \frac{x\ z_2}{\sqrt{3}} \sin(150°-\Psi)$$

$$\text{and } Z_2(k) = z_2 \cos\Psi - j \frac{z_2}{k} \sin\Psi$$

$$f_{oc}(k) = 1.\left[I_{ab}\ Z_1(k) + x\ (I_a - I_o)\ Z_2(k)\right]$$

$$\therefore \quad \frac{d}{dk} f_{oc}(k) \Big|_{k=1} = 1.\left|\sqrt{3}\ I_+\ k \frac{x\ z_2}{\sqrt{3}} \sin(150°-\Psi)\angle 30° + x\ I_+ \frac{z_2}{k^2} \sin\Psi\right|_{k=1}$$

$$V_{oc-} = E_{oc} \quad = 1.I_+\ x\ z_2 \left|\sin(150°-\Psi)\angle 30° + \sin\Psi\right| \quad \text{...(9e)}$$

$$V_{oc-} = \left|E_{oc}\right| = I_-\ x\ z_2.\ 1 \quad \text{........................(9f)}$$

$\therefore$ from (6a), (9e) and (9f)

$$\mu = \sin(150°-\Psi)\angle 30° + \sin\Psi \quad \text{..........(9g)}$$

which gives $\mu = 1.8$ when $\Psi = 90°$
$\mu = 1.75$ when $\Psi = 75°$
$\mu = 1.8$ when $\Psi = 60°$

(c) If one of the stimuli is reversed (Fig.APP 9.1) then to obtain a negative sequence filter the condition to be satisfied is

$$\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}} \angle -30° \quad \text{..................................(9h)}$$

where $x = \frac{m}{1}$

$$E_{oc} = 1\ I_-\ x\ Z_2 \angle{-120^\circ} \quad \cdots\cdots (9i)$$

Let $Z_2 = z_2 \angle\psi$

$$\therefore Z_i = z_2 \left(\angle\psi + \frac{x}{\sqrt{3}} \angle{-30^\circ + \psi} \right)$$

$$= \frac{1}{\sqrt{3}} z_2 \sqrt{(x^2+3x+3)} \angle\delta \quad \cdots\cdots (9j)$$

where $\cos = \dfrac{\sqrt{3}\cos\psi + x\cos(30^\circ - \psi)}{\sqrt{(x^2+3x+3)}}$

and $\sin = \dfrac{\sqrt{3}\sin\psi + x\sin(30^\circ - \psi)}{\sqrt{(x^2+3x+3)}}$

$\therefore$ From (9i) and (9j)

$$\text{Burden current} = |I_L| = \left| \frac{E_{oc}}{z_i(\angle\delta + \angle\theta)} \right| = \frac{1.\ \sqrt{3}\ x}{\sqrt{(x^2+3x+3)}} \left| \frac{I_-}{\angle\delta + \angle\theta} \right|$$

$$\therefore (VA)_{output} = I_L{}^2 Z_i = \frac{\sqrt{3}\ x^2\ z_2}{\sqrt{(x^2+3x+3)}} \left| \frac{I_-}{\angle\delta + \angle\theta} \right|^2 .\ 1^2 \quad \cdots\cdots (9k)$$

$$(VA)_{input} = \left| \, |I_{ab}|^2\ Z_1 + x^2 |I_a - I_o|^2\ Z_2 \, \right| .\ 1^2$$

$$= x\ z_2\ I_+{}^2 . 1^2 . \sqrt{(x^2+3x+3)} \quad \cdots\cdots (9l)$$

$\therefore$ From (9k) and (9l)

$$\eta = \frac{\sqrt{3}\ x}{(x^2+3x+3)} \left| \frac{1}{\angle\delta + \angle\theta} \right|^2$$

$$= \frac{\sqrt{3}\ x}{(x^2+3x+3)} \ \frac{1}{2\ (1 + \cos\theta\ \cos\delta + \sin\theta\ \sin\delta)} \quad \cdots (9m)$$

Two sets of VA-quality curves from equ.(9m) with two suitable values of ($\psi = 60^\circ$ and $\psi = -60^\circ$) are shown in Fig. G3 for 60° and 90° matched burdens for comparisons.

(d) Let, at nominal frequency,

$$Z_1 = \frac{x\ z_2}{\sqrt{3}} \underline{/-30° + \psi}$$
$$\& \quad Z_2 = z_2 \underline{/\psi}$$

Both are inductive i.e., $90° > \psi > 30°$

$$\therefore Z_1(k) = \frac{x\ z_2}{\sqrt{3}} \cos(-30° + \psi) + j \frac{k\ x\ z_2}{\sqrt{3}} \sin(-30° + \psi) \quad \ldots\ldots\ldots\ldots (9n)$$

$$Z_2(k) = z_2 \cos\psi + j\, k\, z_2 \sin\psi \quad \ldots\ldots\ldots\ldots (9p)$$

$$f_{oc}(k) = \left[I_{ab}\ Z_1(k) - x\ (I_a - I_o)\ Z_2(k) \right] .1 \quad \ldots\ldots\ldots\ldots (9q)$$

$\therefore$ From (9n), (9p) and (9q)

$$\left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} = x\ z_2\ I_+ \left| \sin(\psi - 30°)\underline{/30°} - \sin\psi \right| .1$$
$$= \frac{1}{2}\ x\ z_2\ I_+ .1 \quad \ldots\ldots\ldots\ldots (9r)$$

From (9i), $V_{oc-} = \left| E_{oc} \right| = 1.\ I_-\ x\ z_2 \quad \ldots\ldots\ldots\ldots (9s)$

$\therefore$ From (6a), (9r) and (9s) , $\mu = 0.5$

Similarly for $0° \geqslant \psi \geqslant -60°$, $\mu = 0.5$.

Now if $30° \geqslant \psi \geqslant 0°$

then $Z_2 = z_2 \underline{/\psi}$ (inductive)

and $Z_1 = (x\ z_2 / \sqrt{3})\ \underline{/-(30° - \psi)}$ (capacitive)

} at nominal frequency

$$\therefore Z_1(k) = \frac{x\ z_2}{\sqrt{3}} \cos(30° - \psi) - j \frac{x\ z_2}{k\sqrt{3}} \sin(30° - \psi) \quad \ldots\ldots\ldots\ldots (9t)$$

$$\&\ Z_2(k) = z_2 \cos\psi + j\, k\, z_2 \sin\psi \quad \ldots\ldots\ldots\ldots (9u)$$

$\therefore$ from (9q), (9t) and (9u),

$$\left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} = z_2\ x\ I_+ \left| \sin(30° - \psi)\underline{/30°} - \sin\psi . \right| 1$$
$$= z_2\ x\ I_+ \sqrt{\left(\frac{1}{4} - \sqrt{3} \cos\psi . \sin\psi + 3 \sin^2\psi\right)} \quad \ldots\ldots (9v)$$

$\therefore$ from (6a), (9s) and (9v)

$$\mu = \sqrt{\left(\frac{1}{4} - \sqrt{3} \cos\psi . \sin\psi + 3 \sin^2\psi \right)} \quad \ldots\ldots\ldots\ldots (9w)$$

minimum of which is at $= 15°$ giving $\mu = 0.134$

APPENDIX X DESIGN FORMULAE OF 4th CATEGORY OF SERIES FILTERS

(a) Open circuit voltage = $E_{oc} = \left[(I_a - I_b)\,Z_1 + x(I_c - I_o)Z_2\right]\,1$

$$= \left[2\,x\,Z_2\,I_-\,\angle{-120^\circ}\right]\,1 \quad \text{........(10a)}$$

(as $\frac{Z_1}{Z_2} = \frac{x}{\sqrt{3}}\angle{-90^\circ}$, from equ.(4i))

$$Z_i = Z_1 + Z_2 = z_2\left(\angle\psi - j\,\frac{x}{\sqrt{3}}\,\angle\psi\right), \quad \text{where} \quad Z_2 = z_2\angle\psi$$

$$= z_2\,\frac{1}{\sqrt{3}}\,\sqrt{(3+x^2)}\,\angle\delta \quad \text{.......................(10b)}$$

$$\text{where} \quad \cos\delta = \frac{\sqrt{3}\cos\psi + x\sin\psi}{\sqrt{(3+x^2)}}$$

$$\text{and} \quad \sin = \frac{\sqrt{3}\sin\psi - x\cos\psi}{\sqrt{(3+x^2)}}$$

∴ From (10a) and (10b)

$$|I_L| = \left|\frac{E_{oc}}{z_i(\angle\delta + \angle\theta)}\right|$$

, where θ is the angle of the matched burden

$$= \frac{2\sqrt{3}\,x}{\sqrt{(3+x^2)}}\left|\frac{I_-}{\angle\delta + \angle\theta}\right|\,1$$

$$\therefore (VA)_{output} = \left|I_L^{\,2} z_i\right| = \frac{4\sqrt{3}\,x^2\,1^2}{\sqrt{(3+x^2)}}\,z_2\left|\frac{I_-}{\angle\delta + \angle\theta}\right|^2 \quad \text{..........(10c)}$$

$$(VA)_{input} = \left[\left|I_a - I_b\right|^2 Z_1 + x^2\left|I_c - I_o\right|^2 Z_2\right]1^2$$

$$= x\,z_2\,I_+^{\,2}\,\sqrt{(3+x^2)}.\ 1^2 \quad \text{.....................(10d)}$$

∴ From (10c) and (10d),

$$\text{VA-quality} = \eta = \frac{4\sqrt{3}\,x}{(3+x^2)}\left|\frac{1}{\angle\delta + \angle\theta}\right|^2$$

$$= \frac{2\sqrt{3}\,x}{(3+x^2)}\cdot\frac{1}{1 + \cos\delta.\cos\theta + \sin\theta.\sin\delta} \quad \text{......(10e)}$$

Two sets of VA-quality curves from equ.(10e) with two suitable values of ($\psi = 90^\circ$ and $\psi = 0^\circ$) are shown in Fig.G4 for 60° and 90° matched burdens for comparisons.

(b) At nominal frequency let

$$Z_2 = z_2 \angle \psi \qquad \text{(Inductive)}$$

$$Z_1 = (x\, z_2/\sqrt{3}) \angle -(90° - \psi) \qquad \text{(Capacitive)}$$

$$\therefore \quad Z_1(k) = \frac{x\, z_2}{\sqrt{3}} \sin\psi - j\,\frac{x\, z_2}{k} \cos\psi \quad \ldots\ldots (10f)$$

$$Z_2(k) = z_2 \cos\psi + j\, k\, z_2 \sin\psi \quad \ldots\ldots (10g)$$

$$f_{oc}(k) = \left[I_{ab}\, Z_1(k) + x(I_c - I_o)\, Z_2(k) \right] .1 \quad \ldots\ldots (10h)$$

$\therefore$ from (10f), (10g) and (10h),

$$\left| \frac{d}{dk} f_{oc}(k) \right|_{k=1} = \left| I_+ \left\{ x\, z_2 \cos\psi \angle 30° + x\, z_2 \sin\psi \angle 120° \right\} \right| 1$$

$$= \left| I_+\, x\, z_2 \right| 1 \quad \ldots\ldots (10j)$$

From (10a), $V_{oc-} = E_{oc} = 2\, x\, z_2\, I_-$ ......(10k)

$\therefore$ From (6a), (10j) and (10k),

$$\mu = 0.5 \quad \ldots\ldots (10l)$$

APPENDIX XI CONDITION FOR A THREE ELEMENT NEGATIVE SEQUENCE SERIES FILTER

$$E_{oc} = I_{ab} Z_1 - I_{bc} Z_2 + I_{ca} Z_3$$

$$= \sqrt{3}\, I_+ \left(Z_1\underline{/30^\circ} + Z_2\underline{/90^\circ} - Z_3\underline{/30^\circ} \right)$$

$$+ \sqrt{3}\, I_- \left(Z_1\underline{/-30^\circ} - Z_2\underline{/90^\circ} - Z_3\underline{/30^\circ} \right)$$

∴ To obtain a negative sequence filter,

$$Z_1\underline{/30^\circ} + Z_2\underline{/90^\circ} - Z_3\underline{/-30^\circ} = 0$$

or, $$1 + \frac{Z_1}{Z_2}\underline{/-60^\circ} + \frac{Z_3}{Z_2}\underline{/60^\circ} = 0$$

APPENDIX XII DESIGN FORMULAE OF BRIDGE FILTERS

(a) If $\ddot{m}_1 = m\underline{/\,\phi_1}$ and $\ddot{m}_2 = m\underline{/\,\phi_2}$

then the equation (5b) becomes

$$1 + m\,\underline{/-60°+\phi_1} + m\underline{/60°+\phi_2} = 0$$

or, $1 + m\cos(60°-\phi_1) + m\cos(60°+\phi_2)$

$$- j\,m\sin(60°-\phi_1) + j\,m\sin(60°+\phi_2) = 0$$

$\therefore\ 1 + m\cos(60°-\phi_1) + m\cos(60°+\phi_2) = 0$ ....(12a)

and $m\sin(60°+\phi_2) - m\sin(60°-\phi_1) = 0$ ....(12b)

from (12b), $\phi_2 = \phi_1$ (= ϕ, say)

Putting this value in (12a)

$$1 + m\cos(60°+\phi) + m\cos(60°+\phi) = 0$$

or, $1 + 2m\cos(60°+\phi) = 0$ ....(12c)

(b) If $m = \frac{1}{2}$ and $\phi = 120°$ which satisfy the equation (12c),

then $\frac{Z_1}{Z_2} = \frac{1}{2}\,\underline{/-120°}$ and $\frac{Z_4}{Z_3} = \frac{1}{2}\underline{/120°}$.

Let $Z_1 = z\underline{/-90°}$ and $Z_2 = 2z\underline{/30°}$

and $Z_3 = 2z'\,\underline{/-90°}$ and $Z_4 = 2z'\,\underline{/30°}$

and let $\frac{z'}{z} = x$, where z and z' are arbitary constants.

$\therefore\ Z_1+Z_2 = z\,(\,\underline{/-90°} + 2\underline{/30°}\,) = \sqrt{3}\,z$ ....(12d)

and $Z_3+Z_4 = x\,z\,(\,2\underline{/-90°} + \underline{/30°}\,) = \sqrt{3}\,x\,z\,\underline{/-60°}$ ....(12e)

$$Z_i = \frac{(Z_1+Z_2)(Z_3+Z_4)}{(Z_1+Z_2) + (Z_3+Z_4)}$$ ....(12f)

$\therefore$ From (12d), (12e) and (12f),

$$Z_i = \frac{\sqrt{3}z.\sqrt{3}\ x\ z\ \angle{-60^\circ}}{\sqrt{3}\ z + \sqrt{3}\ x\ z\ \angle{-60^\circ}} = \frac{\sqrt{3}\ x}{\sqrt{(x^2+x+1)}}\ z\ \angle{-\delta} \quad \text{.............(12g)}$$

$$\text{where} \quad \cos\delta = \frac{1 + 2x}{2\sqrt{(x^2+x+1)}}$$

$$\text{and} \quad \sin\delta = \frac{\sqrt{3}}{2\sqrt{(x^2+x+1)}}$$

$$\text{Short circuit current} = I_{sc} = I_{ab}\cdot\frac{Z_2}{Z_1+Z_2} + I_{bc}\cdot\frac{Z_3}{Z_3+Z_4}$$

$$= 3\ I_-\ \frac{Z_3}{Z_3+Z_4}\angle{60^\circ} \quad \text{.................................(12h)}$$

$$\left(\text{as, from (5a)}, \frac{Z_2}{Z_1+Z_2} = \frac{Z_3}{Z_3+Z_4}\angle{60^\circ}\right)$$

$$\therefore \text{From (12h)}, \quad I_{sc} = 3\ I_-\ \frac{1}{\{1 + (Z_4/Z_3)\}}\angle{60^\circ}$$

$$= 3\ I_-\ \frac{\angle{60^\circ}}{\{1 + (1/2)\angle{120^\circ}\}} = 2\sqrt{3}\ I_-\ \angle{30^\circ} \quad \text{.......(12i)}$$

$$\text{Burden current} = |I_L| = |I_{sc}|\left|\frac{Z_i}{Z_i + z_i\angle{\theta}}\right| = \left|\frac{I_{sc}}{\angle{-\delta} + \angle{\theta}}\right| \quad \text{....(12j)}$$

where θ is the angle of the matched burden.

$$\therefore \text{Volt-amp output} = (VA)_{output} = \left|I_L^2\ z_i\right| = \left|\frac{I_{sc}}{\angle{-\delta} + \angle{\theta}}\right|^2 z_i \quad \text{...(12k)}$$

$\therefore$ From (12i), (12k) and (12f),

$$(VA)_{output} = \frac{12\ I_-^2\ z_i}{\left|\angle{-\delta} + \angle{\theta}\right|^2} = 12\left|\frac{I_-}{\angle{-\delta} + \angle{\theta}}\right|^2 \frac{\sqrt{3}\ x\ z}{\sqrt{(x^2+x+1)}}$$

$$= \frac{6\sqrt{3}\ x\ z\ I_-^2}{\sqrt{(x^2+x+1)}.\ (1+\cos\delta\cos\theta - \sin\theta\sin\delta)} \quad \text{..(12l)}$$

$$\text{Impedance of } Z_1 \text{ in parallel with } Z_2 = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{2}{\sqrt{3}}\ z\ \angle{-60^\circ} \quad \text{..(12m)}$$

Similarly impedance of Z_3 in parallel with $Z_4 = \frac{2}{\sqrt{3}} x z$ ....(12n)

∴ From (12m) and (12n),

$$(VA)_{input} = \left| \left|I_{ab}\right|^2 \frac{2}{\sqrt{3}} z\angle{-60°} + \left|I_{bc}\right|^2 \frac{2}{\sqrt{3}} x z \right|$$

$$= 2\sqrt{3}\sqrt{(x^2+x+1)}\, z\, I_+^2 \quad \text{..........................(12p)}$$

∴ From (12l) and (12p),

$$= \frac{3x}{(x^2+x+1)} \cdot \frac{1}{(1+\cos\delta\cos\theta - \sin\theta\sin\delta)} \quad \text{........(12q)}$$

VA-quality curves from equ.(12q) are shown in Fig.G5 for 60° and 90° matched burden.

(c) $Z_1(k) = -j\frac{z}{k}$, $Z_2(k) = 2z(\frac{\sqrt{3}}{2} + j\frac{1}{2}k)$

$Z_3(k) = -2j\frac{z'}{k}$, $Z_4(k) = z'(\frac{\sqrt{3}}{2} + j\frac{1}{2}k)$

$$\therefore \frac{d}{dk}\frac{Z_1(k)}{Z_2(k)} = \frac{d}{dk}\frac{-j\frac{z}{k}}{2z(\frac{\sqrt{3}}{2}+j\frac{1}{2}k)} = j\frac{\sqrt{3}+j2k}{(\sqrt{3}k+jk)^2} \quad \text{.......(12r)}$$

$$\text{Similarly } \frac{d}{dk}\frac{Z_4(k)}{Z_3(k)} = j\frac{1}{4}(\sqrt{3}+j2k) \quad \text{...................(12s)}$$

∴ From (6f), (12r) and (12s),

$$\mu = \frac{1}{\sqrt{3}}\left(1+\frac{Z_4}{Z_3}\right)^{-1}\left| j\frac{\sqrt{3}+j2k}{(\sqrt{3}k+jk)^2}\angle{-120°} + j\frac{1}{4}(\sqrt{3}+j2k)\right|_{k=1}$$

$$= 0 \quad \text{...................................(12t)}$$

APPENDIX XIII DESIGN FORMULAE OF A BRIDGE FILTER (SECOND KIND)

(a) If $m = \frac{1}{\sqrt{3}}$ and $\phi = 90°$ which satisfy the equation (12c),

then $\frac{Z_1}{Z_2} = \frac{1}{\sqrt{3}} \angle -90°$ and $\frac{Z_4}{Z_3} = \frac{1}{\sqrt{3}} \angle 90°$.

Let $Z_1 = z\angle -90°$, $Z_2 = \sqrt{3}\, z$

$Z_3 = \sqrt{3}\, z' \angle -90°$, $Z_4 = z'$ and let $\frac{z'}{z} = x$

where z and z' are arbitary constants.

$\therefore$ $Z_1 + Z_2 = z(\sqrt{3} - j) = 2z \angle -30°$ ....(13a)

$Z_3 + Z_4 = x\, z\, (1 - j\sqrt{3}) = 2\, x\, z \angle -60°$ ....(13b)

$\therefore$ From (13a), (13b) and (12f),

$$Z_i = \frac{2\, x\, z}{\sqrt{(x^2 + \sqrt{3}x + 1)}} \angle -\delta \qquad \text{.................................(13c)}$$

where $\cos\delta = \frac{1 + \sqrt{3}x}{2\sqrt{(x^2 + \sqrt{3}x + 1)}}$

and $\sin\delta = \frac{\sqrt{3} + x}{2\sqrt{(x^2 + \sqrt{3}x + 1)}}$

From (12h), $I_{sc} = 3I_- \dfrac{1}{1 + \frac{Z_4}{Z_3}} \angle 60° = 1.5\sqrt{3}\, I_- \angle 30°$ ..(13d)

From (12k) and (13d),

$$(VA)_{output} = 3.\ 2.25\, I_-^{\,2} \left| \frac{1}{\angle -\delta + \angle \theta} \right|^2 z_i \qquad \text{.................(13e)}$$

Impedance of Z_1 in parallel with Z_2

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{\sqrt{3}}{2}\, z \angle -60° \qquad \text{.....................................(13f)}$$

Similarly impedance of Z_3 in parallel with $Z_4 = \frac{\sqrt{3}}{2}\, x\, z\angle -30°$

.....(13g)

∴ From (13f) and (13 g), $(VA)_{input}$ (calculated on the positive sequence basis) $= \left| |I_{ab}|^2 \frac{\sqrt{3}}{2} z \angle -60° + |I_{bc}|^2 \frac{\sqrt{3}}{2} x z \angle -30° \right|$

$$= \frac{3\sqrt{3}}{2} I_-^2 z \sqrt{(x^2+\sqrt{3}x+1)} \quad \text{.................(13h)}$$

∴ From (13c), (13e) and (13h)

$$(VA)\text{- quality} = \eta = \frac{3\sqrt{3}\ x}{|\angle -\delta + \angle \theta|^2 \sqrt{(x^2+\sqrt{3}x+1)}} \quad \text{........(13i)}$$

VA-quality curves from equ.(13i) are shown in Fig.G6 for 60° and 90° matched burdens.

(b)

$$Z_1(k) = -j \frac{z}{k}, \quad Z_2(k) = \sqrt{3}\ z, \quad Z_3(k) = -j \frac{\sqrt{3}}{k} z' \quad \& \quad Z_4(k) = z'$$

$$\therefore \frac{d}{dk} \frac{Z_1(k)}{Z_2(k)} = j \frac{1}{\sqrt{3}\ k^2} \quad \text{..............................(13j)}$$

$$\text{and} \quad \frac{d}{dk} \frac{Z_4(k)}{Z_3(k)} = j \frac{1}{\sqrt{3}} \quad \text{......................................(13k)}$$

$$\left(1 + \frac{Z_4}{Z_3}\right)^{-1} = \frac{\sqrt{3}}{2} \angle -30° \quad \text{..............................(13l)}$$

∴ From (6f), (13j), (13k) and (13l),

$$\mu = \left| \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} \left[j \frac{\angle -120°}{\sqrt{3}\ k^2} + j \frac{1}{\sqrt{3}} \right] \right|_{k=1} = 0.29 \quad \text{..(13m)}$$

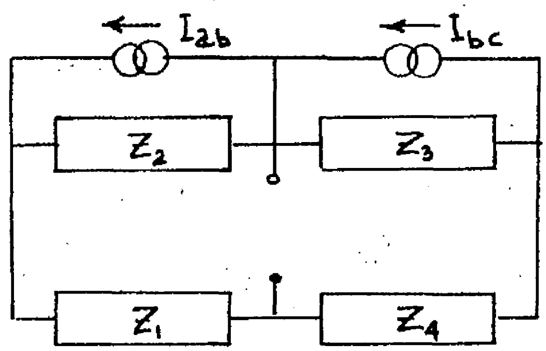


FIG. APP. 14.1

APPENDIX XIV DESIGN OF A BRIDGE FILTER WITH I_{ab} AND I_{bc} IN THE SAME DIRECTION

(a) From Fig. APP.14.1, short circuit current = I_{sc}

$$= I_{ab} \frac{Z_2}{Z_1+Z_2} + I_{bc} \frac{Z_3}{Z_3+Z_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

For a negative sequence filter the co-efficient of I_+ must be zero,

hence, from the above equation, $\frac{Z_2}{Z_1+Z_2}\angle 30° - \frac{Z_3}{Z_3+Z_4}\angle -90° = 0$

$$\text{or,} \quad 1 + \frac{Z_1}{\sqrt{3}\ Z_2}\angle 30° + \frac{Z_4}{\sqrt{3}\ Z_3}\angle -30° = 0° \quad \ldots\ldots\ldots\ldots\ldots (14b)$$

To keep the same input as that in filter 2c, Table 2, but still to satisfy the above equation (14b),

let $Z_1 = \sqrt{3}\ z$, $Z_2 = z\angle -90°$

$Z_3 = z$, $Z_4 = \sqrt{3}\ z\angle -90°$

Putting these values in (14a),

$$I_{sc} = \sqrt{3}\ I_- \left[\frac{Z_2}{Z_1+Z_2}\angle -30° - \frac{Z_3}{Z_3+Z_4}\angle 90°\right] = 1.5 I_- \angle -60° \quad ..(14c)$$

$$Z_1+Z_2 = z(\sqrt{3}-j) = 2z\angle -30° \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14d)$$

$$\text{and} \quad Z_3+Z_4 = z(1-j\sqrt{3}) = 2z\angle -60° \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14e)$$

∴ From (12f), (14d) and (14e),

$$Z_i = \frac{4\ z}{\sqrt{2}\ \ (1+\sqrt{3})}\angle -45° \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14f)$$

From (12j) and (14 f),

$$\text{Burden current} = \left|I_L\right| = \left|\frac{1.5\ I_-}{\angle -45° + \angle \theta}\right| ,$$

for $\theta = 60°$ (i.e. 60° matched burden)

$$\left|I_L\right| = \left|\frac{1.5\ I_-}{\angle -45° + \angle 60°}\right| = 1.232\ I_- \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14g)$$

∴ From (14f) and (14g),

$$(VA)_{output} = \left|I_L^2\ z_i\right| = \left|1.56\ I_-^2\ z\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots (14h)$$

Impedance of Z_1 in parallel with $Z_2 = \dfrac{Z_1 Z_2}{Z_1+Z_2} = \dfrac{\sqrt{3}}{2}\, z\underline{/-60°}$ ...(14i)

Similarly impedance of Z_3 in parallel with $Z_4 = \dfrac{\sqrt{3}}{2} z\underline{/-30°}$ ..(14j)

∴ From (14i) and (14j), $(VA)_{input} = \left| 3\, I_+^2 \left[\dfrac{\sqrt{3}}{2}\, z\underline{/-60°} + \dfrac{\sqrt{3}}{2} z\underline{/-30°}\right]\right|$

$$= 1.5\sqrt{3}\sqrt{(2+\sqrt{3})}\, I_+^2 z \quad \text{.................(14k)}$$

∴ From (14h) and (14k),

VA-quality $= \eta = \dfrac{1.56}{1.5\sqrt{3}\sqrt{(2+\sqrt{3})}} = 0.313$ (for 60° matched burden)

(b)

$$f_{sc}(k) = \left| I_{ab}\left\{1+\frac{Z_1(k)}{Z_2(k)}\right\}^{-1} - I_{bc}\left\{1+\frac{Z_4(k)}{Z_3(k)}\right\}^{-1}\right|$$

$$\left|\frac{d}{dk} f_{sc}(k)\right|_{k=1} = \left|\left\{1+\frac{Z_1}{Z_2}\right\}^{-2}\right| \left| I_{ab}\frac{d}{dk}\frac{Z_1(k)}{Z_2(k)} - I_{bc}\frac{d}{dk}\frac{Z_4(k)}{Z_3(k)}\underline{/240°}\right|_{k=1}$$

$$= \sqrt{3} I_+ \left|\left\{1+\frac{Z_1}{Z_2}\right\}^{-2}\right| \left|\frac{d}{dk}\frac{Z_1(k)}{Z_2(k)}\underline{/60°} + \frac{d}{dk}\frac{Z_4(k)}{Z_3(k)}\right|_{k=1}$$

...(14l)

$Z_1(k) = \sqrt{3}z$, $Z_2(k) = -j\dfrac{z}{k}$, $Z_3(k) = z$, and $Z_4(k) = -j\dfrac{\sqrt{3}z}{k}$

∴ $\dfrac{d}{dk}\dfrac{Z_1(k)}{Z_2(k)} = \underline{/90°}$ ....(14m)

and $\dfrac{d}{dk}\dfrac{Z_4(k)}{Z_3(k)} = \dfrac{1}{k^2}\underline{/90°}$ ....(14n)

∴ From (14l), (14m) and (14n), $\left|\dfrac{d}{dk} f_{sc}(k)\right|_{k=1} = \dfrac{3\sqrt{3}}{4} I_+$ .....(14p)

∵ From (6b), (14c) and (14p), $\mu = \dfrac{3\sqrt{3}}{4.(1.5)} = 0.866$

APPENDIX XV FREQUENCY ERRORS IN SELECTED FILTERS

(a) To find out ''Frequency Error'' in filter No. 1 (Fig. 3.8) :

$$f_{oc}(k) = -3I_+ k R\angle 90^\circ + \sqrt{3} I_+ \sqrt{3} R \angle 90^\circ$$
$$= j\, 3\, I_+ R (1-k) \quad \text{.......(15a)}$$

$$Z_i = \sqrt{3}R + jR = 2R \angle 30^\circ$$

$\therefore$ 60° matched burden = $Z_L = 2R\angle 60^\circ = j\sqrt{3} R + R$

(Here a typical impedance angle of 60° is chosen for matched burden angle to find out frequency error.)

Now, $Z_i(k) = \sqrt{3}R + jkR$

and $Z_L(k) = R + j\sqrt{3}kR$

$$\therefore Z_i(k) + Z_L(k) = (1+\sqrt{3})(1+jk)R \quad \text{.......(15b)}$$

$\therefore$ From (15a) and (15b) , Burden current due to positive sequence component = $\left|I_{L+}\right| = \left|\dfrac{j\, 3I_+ R (1-k)}{(1+\sqrt{3})(1+jk) R}\right|$

$$= \left|\frac{3}{(1+\sqrt{3})} \frac{1-k}{\sqrt{(1+k^2)}} I_+\right| \quad \text{.....(15c)}$$

Open circuit voltage due to negative sequence component

$$= -3I_- R\angle 90^\circ - 3I_- R\angle 90^\circ = -6I_- R\angle 90^\circ \quad \text{.......(15d)}$$

From (15b) $Z_i + Z_L = (1+\sqrt{3})(1+j) R = \sqrt{2}(1+\sqrt{3})R\angle 45^\circ$ ......(15e)

$\therefore$ From (15d) and (15e), Burden current due to negative sequence component = $\left|I_{L-}\right| = \left|\dfrac{6I_-}{(1+\sqrt{3})\sqrt{2}}\right|$ ....(15f)

$\therefore$ From (15c) and (15f),

$$\text{Error} = \left|\frac{I_{L+}}{I_{L-}}\right| = \sqrt{2}\frac{1-k}{\sqrt{(1+k^2)}} \quad \text{.......(15g)}$$

Curve of the above Error equation is shown in Fig.G7.

(b) To find out 'Frequency Error' in Filter No. 2 (Fig.3.9):

$$Z_1(k) = -j\frac{\sqrt{3}R}{k} \quad ; \quad Z_2(k) = R \; .$$

Short circuit current due to positive sequence component

$$= I_{sc+} = \left[\sqrt{3}\, I_+ \angle 30^\circ \; \frac{R}{R - j\frac{\sqrt{3}R}{k}} + \frac{1}{2}\sqrt{3}\, I_+ \angle -90^\circ\right]$$

$$\text{or,} \quad \left|I_{sc+}\right| = \left|1.5\,\frac{k^2 - 1}{k^2 + 3}\, I_+\right| \quad \ldots\ldots(15h)$$

$$Z_i = R - j\sqrt{3}\,R = 2\,R\angle -60^\circ$$

Let a 60° matched burden be considered,

$$\therefore Z_L = R + j\sqrt{3}R$$

$$\therefore Z_i(k) = R - j\frac{\sqrt{3}R}{k} \quad \ldots\ldots(15i)$$

$$Z_L(k) = R + j\sqrt{3}\,k\,R \quad \ldots\ldots(15j)$$

From (15h), (15i) and (15j)

$$\left|I_{L+}\right| = \left|I_{sc+}\;\frac{Z_i(k)}{Z_i(k) + Z_L(k)}\right|$$

$$= \left|1.5\,\frac{k^2-1}{\sqrt{(k^2+3)}}\;\frac{1}{\sqrt{(3k^4 - 2k^2 + 3)}}\, I_+\right| \quad \ldots\ldots(15k)$$

From Table 2, filter 2f ,

$$\left|I_{L-}\right| = \left|1.5\, I_-\right| \quad \ldots\ldots(15l)$$

∴ From (15k) and (15l),

$$\text{Error} = \left|\frac{k^2 - 1}{\sqrt{(k^2+3)}}\;\frac{1}{\sqrt{(3k^4 - 2k^2 + 3)}}\right| \quad \ldots\ldots(15m)$$

Curve of above error equation is shown in Fig.G7.

(c) To find out ''Frequency Error'' in Filter No.3 (Fig.3.10) :

$$Z_1(k) = -j\frac{R}{k}\ ; \quad Z_2(k) = \sqrt{3}R\ ; \quad Z_3(k) = -j\frac{\sqrt{3}R}{k}\ ; \quad Z_4(k) = R$$

$$\left|I_{sc+}\right| = \left|\sqrt{3}I_+\angle 30^\circ \frac{Z_2(k)}{Z_1(k) + Z_2(k)} + \sqrt{3}I_+\angle -90^\circ \frac{Z_3(k)}{Z_3(k) + Z_4(k)}\right|$$

From above, by simplification,

$$\left|I_{sc+}\right| = \left|3\,\frac{\sqrt{(k^2-k+1)}}{\sqrt{(3k^2+1)}\,\sqrt{(k^2+3)}}\,(k-1)\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15n)$$

$$Z_i(k) = \frac{\{Z_1(k) + Z_2(k)\}\ \{Z_3(k) + Z_4(k)\}}{Z_1(k) + Z_2(k) + Z_3(k) + Z_4(k)}$$

$$= \frac{R}{k(k^2+1)(1+\sqrt{3})}\left[k(\sqrt{3}k^2-\sqrt{3}+4) - j(4k^2-\sqrt{3}k^2+\sqrt{3})\right] \quad \ldots\ldots(15o)$$

From above, $Z_i = \frac{R}{2(1+\sqrt{3})}(4-j4) = \frac{4R}{\sqrt{2}\ (1+\sqrt{3})}\angle -45^\circ$

$\therefore$ 60° matched burden = $Z_L = \frac{4R}{\sqrt{2}\ (1+\sqrt{3})}\angle 60^\circ$

$$\therefore \quad Z_L(k) = \frac{4R}{\sqrt{2}\ (1+\sqrt{3})}\left[\frac{1}{2} + j\ k\ \frac{\sqrt{3}}{2}\right] = \frac{R}{1+\sqrt{3}}(\sqrt{2} + j\sqrt{6}k) \quad \ldots\ldots(15p)$$

Now $\left|I_{L+}\right| = \left|I_{sc+}\ \frac{Z_i(k)}{Z_i(k) + Z_L(k)}\right|$ ....(15q)

where the simplified expression of $\frac{Z_i(k)}{Z_i(k) + Z_L(k)}$ is given by (15o) and (15p) as follows :

$$\frac{Z_i(k)}{Z_i(k) + Z_L(k)} = \left[\frac{k(\sqrt{3}k^2-\sqrt{3}+4) - j(4k^2-\sqrt{3}k^2+\sqrt{3})}{\left\{(\sqrt{3}+\sqrt{2})k^3 + (4+\sqrt{2}+\sqrt{3})k\right\} + j\left\{\sqrt{6}k^4+(\sqrt{6}+\sqrt{3}-4)k^2-\sqrt{3}\right\}}\right] \quad \ldots\ldots\ldots\ldots(15q1)$$

From Table 2, Filter 2c, $\left|I_{L-}\right| = \left|2.12\ I_-\right|$ ....(15r)

$\therefore$ From (15n), (15q) and (15r),

$$\text{Error} = \left| 3 \frac{\sqrt{(k^2-k+1)}}{\sqrt{(3k^2+1)}.\sqrt{(k^2+3)}} (k-1). \frac{Z_i(k)}{Z_i(k)+Z_L(k)} \right|_{\times\frac{1}{2.12}} \quad \ldots\ldots (15s)$$

where $\dfrac{Z_i(k)}{Z_i(k)+Z_L(k)}$ is given by (15q1).

Curve of above error equation is shown in Fig.G7. Computer has been used for Calculation.

(d) Frequency Error of Filter No.4 :

From Fig.3.11, $Z_1(k) = -j\frac{R}{k}$; $Z_2(k) = \sqrt{3}R + jkR$

$Z_3(k) = -j\frac{2R}{k}$; and $Z_4(k) = \frac{\sqrt{3}R}{2} + j\,\frac{R}{2k}$

$$\left|(I_{sc})_+\right| = \left| \frac{\sqrt{3}R+jkR}{\sqrt{3}R-j\frac{R}{k}+jkR}\sqrt{3}I_+\underline{/30^\circ} + \frac{-j\frac{2R}{k}}{R(\frac{\sqrt{3}}{2}+j\frac{1}{2}k-j\frac{2}{k})}\sqrt{3}I_+\underline{/-90^\circ} \right|$$

$$= \left| \frac{-(k-1)^2(k^2+2k-2) + j2\sqrt{3}(k-1)^2(k+1)}{\sqrt{3}k+j(k^2-1) \quad \sqrt{3}k+j(k^2-4)}\sqrt{3}I_+ \right| \quad \ldots\ldots (15t)$$

$$Z_i(k) = \frac{\left[Z_1(k)+Z_2(k)\right]\left[Z_3(k)+Z_4(k)\right]}{Z_1(k)+Z_2(k)+Z_3(k)+Z_4(k)}$$

$$= \frac{\left[\sqrt{3}+j(k-\frac{1}{k})\right]\left[\frac{\sqrt{3}}{2}+j(\frac{k}{2}-\frac{2}{k})\right]}{\sqrt{3}+\frac{\sqrt{3}}{2}+j\,(\frac{3}{2}k-\frac{3}{k})}R \quad \ldots\ldots (15u)$$

From above $Z_i = R\underline{/-30^\circ}$

∴ 60° matched burden = $R\underline{/60^\circ}$

$$\therefore Z_L(k) = R(\frac{1}{2}+j\,\frac{\sqrt{3}}{2}k) \quad \ldots\ldots (15v)$$

From Table 2, Filter 2d ,

$$\left|I_{L-}\right| = \left|2.45\ I_-\right| \quad \ldots\ldots (15w)$$

$\therefore$ From (15q), (15t), (15u), (15v) and (15w),

$$\text{Error} = \frac{1}{2.45} \left| \frac{-(k-1)^2(k^2+2k-2) + j2\sqrt{3}(k-1)^2(k+1)}{\left[\sqrt{3}k+j(k^2-1)\right]\left[\sqrt{3}k + j(k^2-4)\right]} \quad \frac{Z_i(k)}{Z_i(k)+Z_L(k)} \right| \qquad \text{..............(15x)}$$

where $\frac{Z_i(k)}{Z_i(k)+Z_L(k)}$ can be obtained from (15u) and (15v).

A curve of the above Error expression is shown in Fig.G7. A computer was used for the calculation.

APPENDIX : XVI : TRANSIENT PRIMARY CURRENTS AND OUTPUT OF FILTER NO.1 DUE TO POSITIVE SEQUENCE SWITCHING.

(a) Referring to Fig.5.1 , let the sinusoidal voltage $\hat{V}\sin(\omega t+\theta)$ [= imaginary part of $\hat{V}\, e^{j(\omega t+\theta)}$] be switched in the circuit.

Let i_p be the current due to switching of $Ve^{j(\omega t+\theta)}$ whose Laplace Transform is $\hat{V}e^{j\theta}\,\frac{s}{s-j\omega}$:

then applying Laplace-mathematics in the network (Fig.5.1),

$$\bar{i}_p(sL_p + R_p) = \hat{V}e^{j\theta}\,\frac{s}{s-j\omega}$$

or, $\bar{i}_p L_p(s+\gamma) = \hat{V}e^{j\theta}\,\frac{s}{s-j\omega}$ where $\gamma = R_p / L_p$

$$\text{or, } \bar{i}_p = \frac{\hat{V}e^{j\theta}}{L_p}\,\frac{s}{(s+\gamma)(s-j\omega)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

∴ If $\bar{i}_p$ is the current in one phase of the three phase balanced system due to simultaneous symmetrical three phase switching then the currents in other phases are $\bar{i}_p e^{j120°}$ and $\bar{i}_p e^{-j120°}$ respectively because of 120° displacements of phases.

From equ.(16a), transform becomes

$$i_p = \frac{\hat{V}}{L_p}\,e^{j\theta}\,\frac{e^{-\gamma t} - e^{j\omega t}}{-j\omega-\gamma} = \frac{\hat{V}}{L_p\sqrt{(\omega^2+\gamma^2)}}\;\frac{e^{j(\omega t+\theta)} - e^{-\gamma t}\,.\,e^{j\theta}}{e^{j\phi}}$$

where $\phi = \tan^{-1}\frac{\omega}{\gamma} = \tan^{-1}\frac{\omega L_p}{R_p}$

$$\text{or, } i_p = \frac{\hat{V}}{Z_p}\left[e^{j(\omega t+\theta-\phi)} - e^{-\gamma t}\,.\,e^{j(\theta-\phi)}\right] \ldots\ldots\ldots\ldots\ldots\ldots (16b)$$

where $Z_p = \sqrt{(R_p^2 + \omega^2 L_p^2)}$.

But the above current is due to the switching of voltage $Ve^{j(\omega t+\theta)}$, the imaginary part of which is $V\sin(\omega t+\theta)$. So the current due to switching of voltage $V\sin(\omega t+\theta)$ is the imaginary part of the expression in equation (16b). Let this current be called i_a.

$$\therefore i_a = \frac{V}{Z_p}\left[\sin(\omega t+\theta-\phi) - e^{-\gamma t}\sin(\theta-\phi)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(16c)$$

Similarly the currents in other phases, in a balanced three phase system due to simultaneous symmetrical three phase switching, are

$$i_b = \frac{V}{Z_p}\left[\sin(\omega t+\theta-\phi-120°) - e^{-\gamma t}\sin(\theta-\phi-120°)\right] \quad \ldots\ldots\ldots\ldots(16d)$$

$$i_c = \frac{V}{Z_p}\left[\sin(\omega t+\theta-\phi+120°) - e^{-\gamma t}\sin(\theta-\phi+120°)\right] \quad \ldots\ldots\ldots\ldots(16e)$$

(b) Referring to Fig. 3.8,

Let L_m is the inductive value of the mutual inductor,

$\therefore L_m = \frac{R}{\omega}$ (as $X_m = jR$).

From (16c), (16d), and (16e),

$$(i_a-i_b) = \frac{\sqrt{3}V}{Z_p}\left[\sin(\omega t+\theta-\phi+30°) - e^{-\gamma t}\sin(\theta-\phi+30°)\right] \quad \ldots\ldots\ldots(16f)$$

$$(i_b-i_c) = \frac{\sqrt{3}V}{Z_p}\left[\sin(\omega t+\theta-\phi-90°) - e^{-\gamma t}\sin(\theta-\phi-90°)\right] \quad \ldots\ldots\ldots(16g)$$

$$(i_c-i_a) = \frac{\sqrt{3}V}{Z_p}\left[\sin(\omega t+\theta-\phi-30°) - e^{-\gamma t}\sin(\theta-\phi-30°)\right] \quad \ldots\ldots\ldots(16h)$$

Now the open circuit voltage of the filter is

$$e_{oc} = \sqrt{3}R(i_b-i_c) + L_m\frac{d}{dt}\left[(i_a-i_b)-(i_c-i_a)\right]$$

$$= \sqrt{3}R(i_b - i_c) + L_m \frac{d}{dt}(2i_a - i_b - i_c) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16i)$$

But $i_a + i_b + i_c = 0$; $\therefore i_b + i_c = -i_a$ ....(16j)

$\therefore$ From (16c),(16g),(16i) and (16j)

$$e_{oc} = \sqrt{3}R(i_b - i_c) + L_m \frac{d}{dt}(3i_a) = 3R\left[(i_b - i_c) + \frac{1}{\omega}\frac{d}{dt}(i_a)\right]$$

$$= \frac{3\hat{V}R}{Z_p}\left[\left\{-\cos(\omega t+\theta-\phi) + e^{-\gamma t}\cos(\theta-\phi)\right\} + \frac{1}{\omega}\left\{\omega\cos(\omega t+\theta-\phi) + \gamma e^{-\gamma t}\sin(\theta-\phi)\right\}\right]$$

$$= \frac{3\hat{V}R}{Z_p}\left[e^{-\gamma t}\ \cos(\theta-\phi) + \frac{\gamma}{\omega}\sin(\theta-\phi)\right]$$

$$= \frac{3\hat{V}R}{Z_p}\cdot\frac{1}{\omega}\cdot\left[\ \omega\cos(\theta-\phi) + \gamma\sin(\theta-\phi)\ \right] e^{-\gamma t}$$

$$= \frac{3\hat{V}R}{\omega Z_p}\sqrt{(\omega^2+\gamma^2)}\ \left[\sin\phi\ \cos(\theta-\phi) + \cos\phi\ \sin(\theta-\phi)\right] e^{-\gamma t}$$

(as, stated before, $\phi = \tan^{-1}\frac{\omega}{\gamma}$)

$$\therefore e_{oc} = \frac{3\hat{V}R}{\omega Z_p}\sqrt{(\omega^2+\gamma^2)}\ \sin\theta . e^{-\gamma t}$$

$$= \frac{3\hat{V}R}{\omega Z_p}\sqrt{(\omega^2+\frac{R_p^2}{L_p^2})}\ e^{-\gamma t}\sin\theta$$

$$= \frac{3\hat{V}R}{\omega L_p}e^{-\gamma t}\sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16k)$$

Let the impedance of the mutual inductive coil be $Z = r + j\omega L$ seen from the secondary side.

$\therefore$ Open circuit impedance of the filter = $\sqrt{3}R + r + j\omega L$ ....(16l)

Again let the ipedance of the burden be $Z_b = r_b + j\omega L_b$ ....(16m)

$\therefore$ From (16k), (16l) and (16m), the thevenin's equivalent circuit for Laplace operation is as shown in Fig.APP16a.

Now from (16k), Laplace Transform of e_{oc} is

$$\bar{e}_{oc} = \frac{3\hat{V}R}{\omega L_p} \sin\theta \frac{s}{s+\gamma}$$

∴ Output current of the filter (in Laplace Transform)

$$= \frac{3\hat{V}R}{\omega L_p} \sin\theta \frac{s}{s+\gamma} \frac{1}{(r+r_b+\sqrt{3}R) + s(L+L_b)}$$

$$= \frac{3\hat{V}R}{\omega L_p} \sin\theta \frac{s}{(s+\gamma)(R_s + sL_s)} \quad ;\text{(where } R_s = r+r_b+\sqrt{3}R$$

$$\text{and } L_s = L+L_b)$$

$$= \frac{3\hat{V}R}{\omega L_p} \sin\theta \frac{s}{(s+\gamma)L_s(s+\gamma')} \quad , \quad (\text{ where } \gamma' = \frac{R_s}{L_s})$$

$$= \frac{3\hat{V}R}{\omega L_p L_s} \sin\theta \frac{s}{(s+\gamma)(s+\gamma')}$$

∴ from above, output current

$$= i = \frac{3\hat{V}R}{\omega L_p L_s} \frac{1}{(\gamma' - \gamma)} (e^{-\gamma t} - e^{-\gamma' t}) \sin\theta \quad \ldots\ldots\ldots\ldots\ldots(16n)$$

APPENDIX XVII : SAMPLE CALCULATIONS FOR TRANSIENT WAVES

Equation (16n), Fig.4.3a (Art.4.3) will be referred for calculations of filter no.1.

$V = \sqrt{2}V_{rms}$; $\therefore \frac{3V}{\omega L_p} = \frac{3\sqrt{2}\ V_{rms}}{X_p}$, where X_p = reactance of the system.

$$\therefore \frac{3V}{\omega L_p} = \frac{3\sqrt{2}\ V_{rms}}{Z_p} \frac{Z_p}{X_p} = 3\sqrt{2}.(\text{Phase current}).\frac{Z_p}{X_p}$$

But,

Phase current = (Line current in delta-side)/$\sqrt{3}$

$$\therefore \frac{3V}{\omega L_p} = \frac{\text{Line current in delta-side}}{\sqrt{3}}.\ 3\sqrt{2}.\ \frac{Z_p}{X_p}$$

$$= \sqrt{6}.(\text{Line current in delta-side}).\ \frac{Z_p}{X_p}\ .$$

If n is the turns ratio of the intermediate current transformer,

then, $\frac{3V}{\omega L_p} = \sqrt{6}.(\text{Line current in delta-side}).n.\ \frac{Z_p}{X_p}$ ....(17a)

$\therefore$ from (16n) and (17a), Maximum Transient Output

$$= \sqrt{6}.(\text{Line current in delta-side}).n.\frac{Z_p}{X_p}.\frac{R}{L_s}.\frac{1}{\gamma'-\gamma}(e^{-\gamma t}-e^{-\gamma' t})$$

$$= \sqrt{6}.(\text{Line current in delta-side}).n.\frac{\sqrt{(\omega^2+\gamma^2)}}{\omega}.\frac{R}{L_s}.\frac{(e^{-\gamma t}-e^{-\gamma' t})}{\gamma'-\gamma}$$

$$= \sqrt{6}.(\text{Line current in delta-side}).n.\frac{\sqrt{(\omega^2+\gamma^2)}}{\gamma'-\gamma}.\frac{R}{X_s}.(e^{-\gamma t}-e^{-\gamma' t})$$

..............(17b)

From Fig.4.3a, $\sqrt{3}R=4540$, $R=2620\ \Omega$ ....(17c)

Internal Impedance of the network = 4540+288+j2680 (where r=288)Ω

= 4828+j2680 = 5540$\angle 29.1°$ ohm

90° matched burden is chosen for sample calculations :

Impedance of 90° matched burden =(522 + j5540) ohm

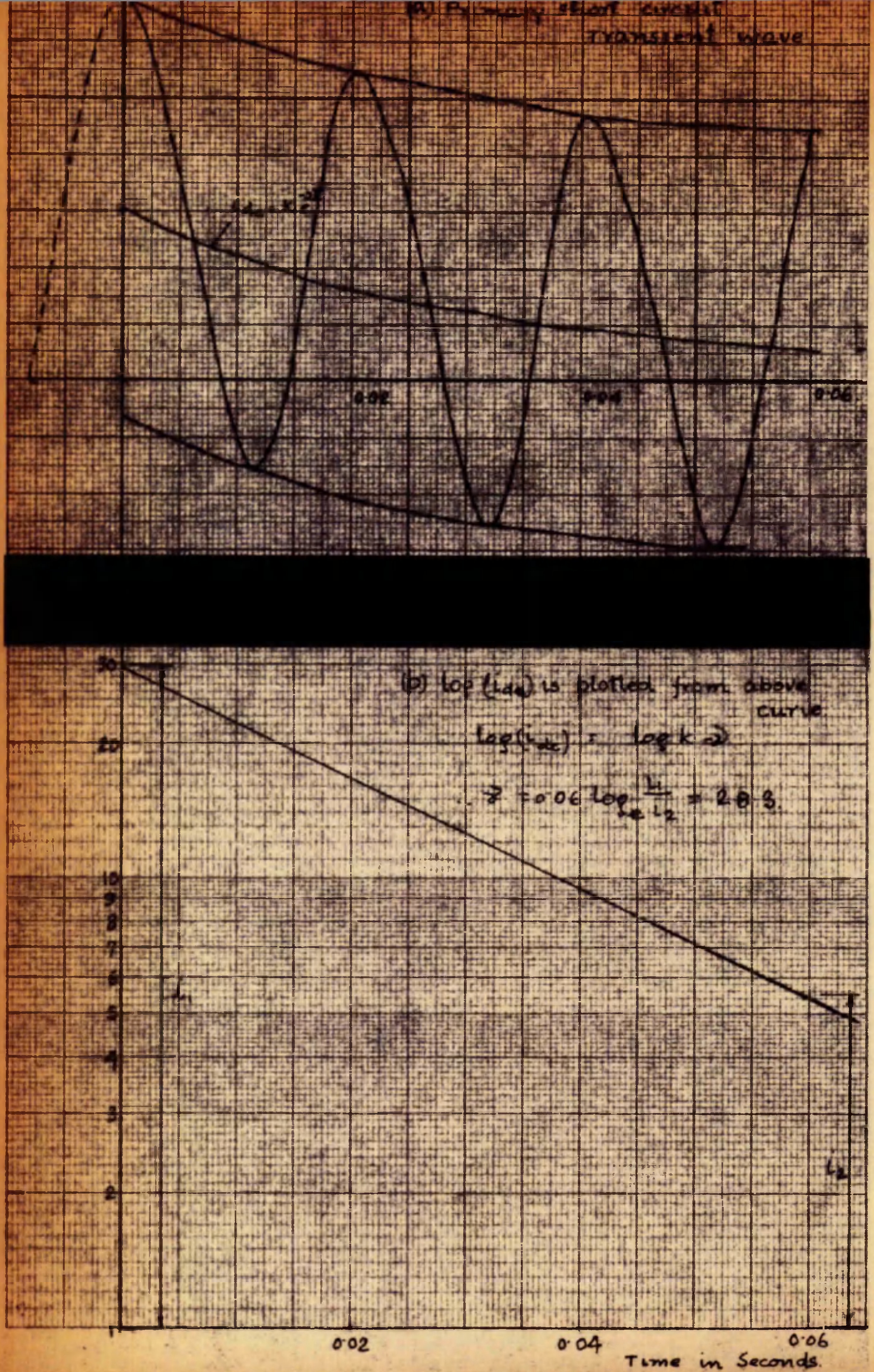

Transient wave
0·02
0·04
0·06
(b) log (idc) is plotted from above curve
30
20
10
9
8
7
6
5
4
3
2
1
0·02
0·04
0·06
Time in Seconds

FIG. G.17a.

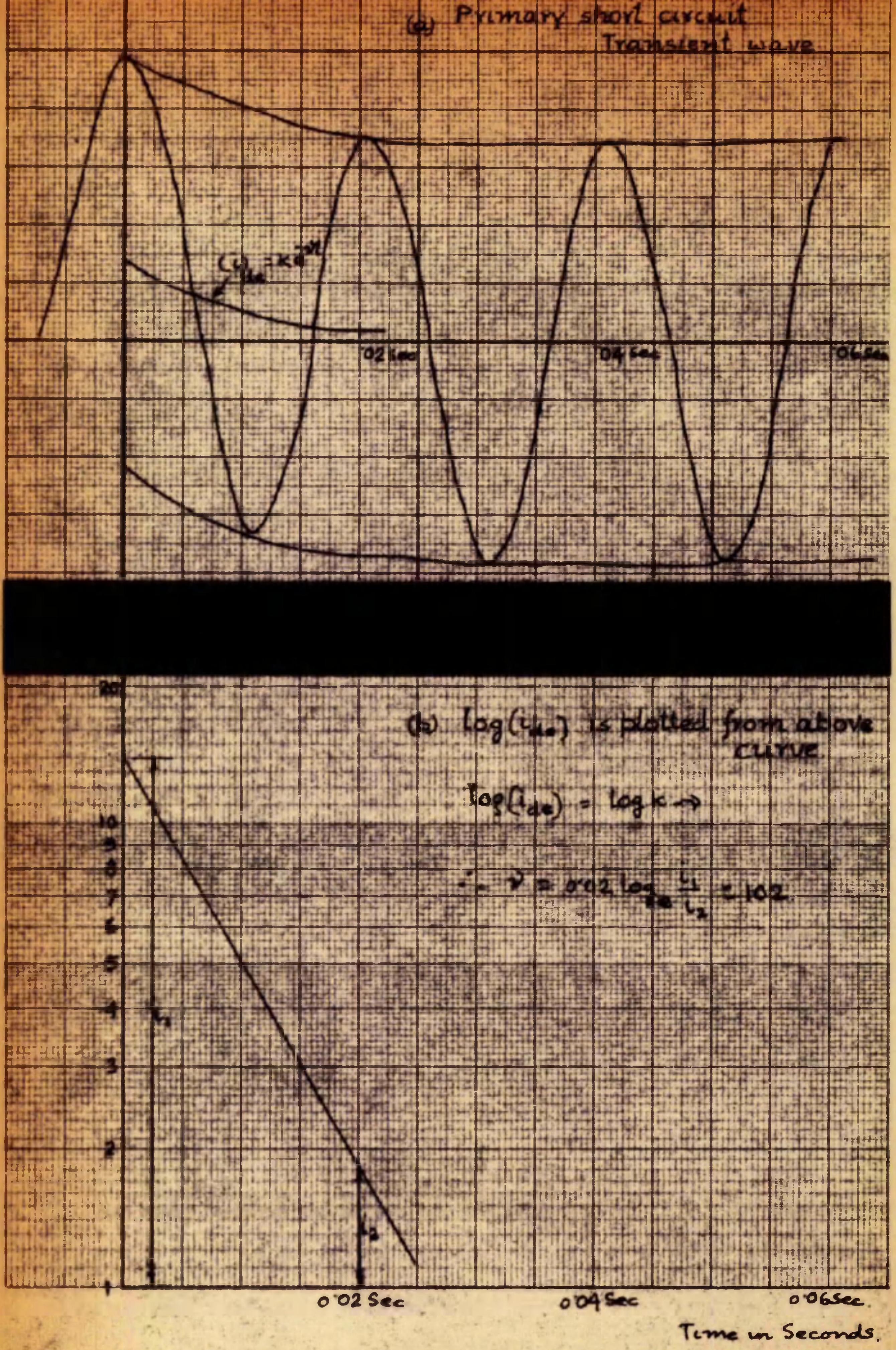

(a) Primary short circuit
Transient wave
$(i)_{dc} = Ke^{-\gamma t}$
·02 Sec
·04 Sec
·06 Sec
20
10
9
8
7
6
5
4
3
2
1
i_1
i_2
(b) Log(i_{dc}) is plotted from above curve
Log(i_{dc}) = log K →
∴ γ = 0·02 log$_e$ $\frac{i_1}{i_2}$ = 102
0·02 Sec
0·04 Sec
0·06 Sec
Time in Seconds.

FIG. G.17b

$\therefore R_s = 4828+522 = 5350$ ohm

and $X_s = \omega L_s = 2680+5540 = 8220$ .... (17d)

$\therefore \gamma' = \frac{R_s}{L_s} = \frac{5350}{8220} \cdot 314 = 204$ ....(17e)

The values of can be found from primary transient waves as shown in Figs.APP.G.17a and APP.G.17b. Let the chosen value of γ for sample calculation be $\gamma = 28.3$ ....(17f)

From Fig.4.3a, $n = \frac{20}{3500}$ ....(17g)

When $\gamma = 28.3$, from Table 4., column 1, Line current in delta-side = 0.70 amp ....(17h)

From Art.4.2 and from (17e) and (17f),

t_m (= time at which the wave reaches its maximum amplitude)

$$= \frac{1}{\gamma' - \gamma} \log \frac{\gamma'}{\gamma} = \frac{1}{204-28.3} \log \frac{204}{28.3} = 0.0112 \text{ sec.} \quad \ldots\ldots(17i)$$

$\therefore$ From (17b), (17c), (17d), (17e), (17f), (17g), (17h) and (17i), the maximum amplitude of the wave

$$= \sqrt{6} \times .70 \times \frac{\sqrt{(314^2+28.3^2)}}{204 - 28.3} \times \frac{2620}{8220} \left(e^{-28.3 \times .0112} - e^{-204 \times .0112} \right)$$

$= 3.52$ mA.

Similarly amplitudes at different instants are calculated as tabulated below :

Time in seconds	Amplitude of current (mA)
0	0
0.0112	3.52
0.0200	3.10
0.0300	2.35
0.0400	1.74
0.0600	1.01
0.1000	0.33

From above, the wave is plotted as shown by dotted line in Fig.4.8(b) (Art.4.6) .

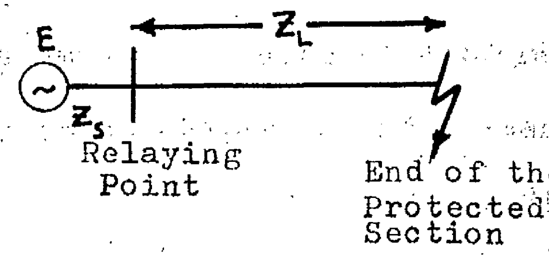
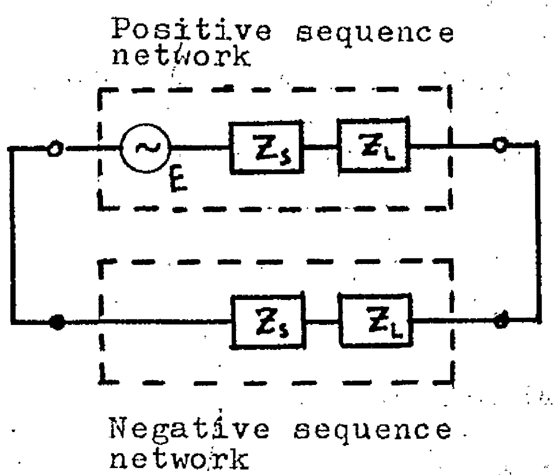


FIG. APP.18a



(The diagram is drawn for b-c fault at the end of the protected section)

Fig. APP.18b

APPENDIX : XVIII : IMPEDANCE MEASUREMENT IN DISTANCE RELAYING

It is assumed that the positive and negative sequence impedances of the source are equal, i.e., $Z_{s+} = Z_{s-} = Z_s$ (say). Similar assumption is made for protected line also, i.e., $Z_{L+} = Z_{L-} = Z_L$ (say) where Z_L is the impedance of the protected section as shown in Fig.APP.18a.

At the balance condition, $V_- - I_- Z = V_+ - I_+ Z$ ....(18.a)

$$\left.\begin{aligned} \text{But} \quad V_+ &= E - I_+ Z_s \\ \text{and} \quad V_- &= 0 - I_- Z_s \end{aligned}\right\} \quad \text{.....................................(18b)}$$

The equivalent circuit for the calculation of sequence components in a phase to phase fault at the end of the protected zone is shown in Fig.APP.18b.

From Fig.APP.18b. $I_+ = -I_- = \dfrac{E}{2(Z_s+Z_L)}$ ....(18c)

∴From (18a), (18b) and (18c),

$$-I_-(Z_s+Z) = E - I_+(Z+Z_s) \quad ; \text{ or, } 2I_+(Z+Z_s) = E \; ;$$

or, $\dfrac{(Z + Z_s)E}{Z_L+Z_s} = E$; or, $Z= Z_L$ ....(18d)

which shows that the impedance setting of the relay should be such that it is equal to the impedance of the protected section.

Now let the fault be beyond the protected section and the impedance of the protected section be Z_L' which is obviously greater than Z_L (=Z).

∴ the voltage across the operating coil,

$$= V_- - I_- Z = -I_- (Z + Z_s) \quad \text{(from 18b)}$$

$$= \frac{E(Z+Z_s)}{2(Z_s+Z_L')} \quad \text{..(18e)}$$

and the voltage across the restraining coil,

$$= V_+ - I_+ Z = E - I_+ (Z_s + Z)$$

$$= E - \frac{E(Z+Z_s)}{2(Z_s+Z_L')} = \frac{E(Z_s+2Z_L'-Z)}{2(Z_s + Z_L')} \quad \text{.......................(18f)}$$

∴ From (18e) and (18f) it can be seen that

(Voltage across operating coil) < (Voltage across restraining coil)

(as $Z_L' > Z$ $(=Z_L)$).

Hence the operation of the relay is prevented.

Similarly it can be shown that the opposite action will take place (i.e. operation of the relay will take place) if the fault is within the protected section.

INDEX TO APPENDIX

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