# THE QUASI-DEUTERON MODEL FOR THE PHOTO-EMISSION OF HIGH ENERGY NEUTRON-PROTON PAIRS FROM OXYGEN 16 AND CALCIUM 40

by

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# The quasi-deuteron model for the photo-emission of high energy neutron-proton pairs from oxygen 16 and calcium 40.


## Summary.

The benefit of using high energy $\gamma$-rays to study nuclear structure and the information which can be gained from their use is first mentioned, with particular reference to the reaction in the title. The early history of the quasi-deuteron model is then sketched which leads up to a treatment of it by K.Gottfried.

This is critically reviewed in detail and the main points of the theory, which are used as a basis of the latter part of the present work, are stressed.

In deriving the cross-section for the photo-emission of high energy neutron-proton pairs from closed shell nuclei, Gottfried uses the cross-section for the photo-disintegration of the deuteron to deal with the mechanism of the reaction and he then multiplies this by appropriate factors to account for the fact that the reaction is taking place inside a nucleus.

These factors include:

(A) a form factor $F(\underline{P})$ which is defined as the probability of finding a neutron-proton pair of combined momentum $\underline{P}$ at zero separation in the nucleus,

(B) a correction to allow for refraction of the photo-nucleons at the surface of the nucleus, and

(C) a constant $\frac{1}{\sqrt{2}}$ multiplying the cross-section to correct for absorption of the photo-pair on their way out of the nucleus.

After a brief mention of related work to Gottfried by S.Fujii,new work that has been performed is described after the reasons motivating it have been given.This work consists of extending Gottfrieds work by doing a more complete calculation of the factor (B) for oxygen 16 to check the validity of Gottfrieds simpler calculation, evaluating (A) for calcium 40 for the first time and treating (C) in a different way for both oxygen 16 and and calcium 40 following a suggestion by N.MacDonald. The formulae derived have been numerically evaluated, with programmes written by the author, on the KDF 9 computer of The University of Glasgow and the results are compared, where appropriate, with Gottfrieds results and the experimental results obtained from an experimental group at Glasgow University.

The agreement and shortcomings revealed by these comparisons are discussed and conclusions drawn.

# CONTENTS.

## PREFACE.

The work described here is a treatment of the photo-emission of neutron-proton pairs from oxygen 16 and calcium 40 using the quasi-deuteron model.

Chapter 1 suggests what information we might gain from studying such a phenomenon and then goes on to give a brief description of the growth of the model from the original idea by J.S.Levinger. This includes a critical, detailed review of a paper by K.Gottfried the basic theory of which underlies the work described in Chapter 2. The revelant points from Gottfried's theory are that, in deriving the cross-section, he uses the cross-section for the photo-disintegration of the deuteron to deal with the mechanism of the reaction and then multiplies this by appropriate factors to account for the fact that the reaction is taking place inside a nucleus.

These factors include (a) a function $F(|\underline{p}|)$ which is defined as the probability of finding a neutron-proton pair at zero separation in the nucleus and with total momentum $\underline{p}$ (b) a correction to allow for refraction of the photo-nucleons at the nuclear surface and (c) a constant $f_a$ multiplying the cross-section to correct for absorption of the photo-pair by the nucleus.

Chapter 2 describes new work performed by myself consisting of evaluating some of these factors for the cases of oxygen 16 and calcium 40. In detail, an extended calculation of (b) is performed to check the validity of Gottfried's simpler calculation, (a) is evaluated, using harmonic oscillator wave functions, for the case of calcium 40 for the first time, and the correction due to (c) is treated in anentirely different way for both nuclei following a suggestion by N.MacDonald.

The formulae derived have been numerically evaluated using the KDF 9 computer of The University of Glasgow, using programmes written by myself, and the results are compared, where appropriate, with Gottfried's and experimental results due to a group at Glasgow.

The agreement and shortcomings revealed by these comparisons and their implications are discussed and conclusions drawn.

# CHAPTER 1.

## DEVELOPMENT OF THE MODEL.

### 1.1 Introduction.

The main stimulus for interest in this subject is the use of high energy $\gamma$-rays to study the finer details of nuclear structure. At energies greater than 200 Mev the wavelength of the radiation is sufficiently small relative to the diameter of nuclei larger than carbon 12 to make it a reasonable assumption that the photons act on individual nucleons or clusters of nucleons and not on the nucleus as a whole. Thus information can be gained about ground state correlations between pairs of particles and also about nucleons which have abnormally high momenta in the nucleus.

The fact that such phenomena exist has been shown by high energy experiments other than those using $\gamma$-rays and the contradiction with the simple shell model picture of the nucleus has best been pointed out by K.A. Brueckner et al. in a well known series of papers (1). In particular these experiments, such as meson absorption by nuclei, proton-proton scattering in nuclei, deuteron pick-up etc., showed that some nucleons had higher momenta than might be expected. This suggested to them strong interactions between pairs of particles as distinct from the shell model picture of the nucleons moving in independent particle states of a uniform potential. To explain the apparent success of the shell model at low energies with the disagreement at higher energies, they drew the conclusion that the shell model wavefunctions were a description not of nucleon motion but of a collective particle motion which low energy experiments detect as ordinary nucleons due to the fact that they average over the time and spatial variations of the system.

By way of illustration they refer amongst other things to the photon induced emission of high energy (>50 Mev) neutron-proton pairs. Their description of this process is that the $\gamma$-ray interacts with a single nucleon which is momentarily closely bound to another with the result that both are emitted. Brueckners paper shows the type of information one might gain from a more detailed study of the photo-emission of neutron-proton pairs viz.

(1) the pair correlation function,

(2) the momentum distribution of nucleons in the ground state,

and hence the interest in this phenomenon.

## 1.2 Historical review.

The most fruitful method for describing the photo-emission of neutron-proton pairs has been the quasi-deuteron model of J.S.Levinger (2). He argues, in a paper on the ejection of fast protons, that the process is due to a neutron-proton pair absorbing dipole $\gamma$-rays due to their dipole moment which proton-proton and neutron-neutron pairs do not possess. Thus before emission the neutron and proton together are in a state resembling that of a deuteron and as a result he assumes the cross-section can be taken as essentially that for the photo-disintegration of the deuteron multiplied by modifying factors due to the presence of the other nucleons. Experiments by M.Q.Barton and J.H.Smith (3), J.W.Weil and B.C.McDaniel (4) and P.C.Stein et al.(5,11) have shown reasonable agreement with Levinger's theory considering the simplicity of his approach and, as he admits himself, its incompleteness.

The main deficiencies in his treatment apart from the fact that he is especially interested in the photo-emission of protons only, are that

(1) he takes no account of the interaction of the emitted proton with the rest of the nucleus,

(2) he uses an artificial momentum distribution for the quasi-deuteron,

(3) he neglects mesonic effects and photo-magnetic interactions.

It should also be mentioned that the photo-disintegration of the deuteron results he uses are also open to criticism and they have been improved on since.

The modest success of Levinger's pioneering work prompted K.A.Dedrick (6) to investigate the quasi-deuteron idea further. He calculated the photo-dissociation cross-section of a neutron and a proton which are scattering one another and confined to a volume $V$ which is later taken to be the volume of the nucleus. Electric dipole and quadrupole terms are taken into account but he neglects magnetic terms. To apply this calculation to the nuclear case the cross-section is averaged over all neutron-proton pairs. This is the most important part of the quasi-deuteron approach from a nuclear structure point of view since this averaging depends on the nuclear ground state neutron and proton distributions. It is at this point that important differences occur between one author and another. Levinger uses a triangular approximation to a fermi distribution with a temperature of 8 Mev. Weil and McDaniel (4), in interpretating their experimental results use a zero temperature fermi distribution. Dedrick approaches the problem by a random flight method using ground state nucleon momentum distributions that will submit to repeated integrations. Distributions of this type are zero temperature fermi distributions and gaussian distributions. Dedrick uses the latter.

An important development in Dedrick's work is that he attempts to take account of the interaction of the photo-nucleons with the residual nucleus. The distortion of the angular distribution due to refraction at the nuclear surface is neglected but he treats absorption of the particles by introducing what he calls a penetrability factor. This measures the probability of the nucleons getting through the coulomb and centrifugal barriers.

The importance of Dedrick s work is diminished by the fact that the tables he produces of the cross-section are for monoenergetic photon beams of energy 50,75,100,125 Mev while all the experiments use bremsstrahlung beams. As a result there are no experimental results with which his results may be compared directly.

## GOTTFRIED'S PAPER.

### 1.3 Basic assumptions.

The most influential theoretical paper after Levinger's and the one which is the basis of the latter part of the present work has been that of K.Gottfried (7). This sets out to determine the nuclear pair correlation function from the high energy photo-effect. The essential similarity with Levinger's work is that he retains the neat device of using the cross-section for the photo-disintegration of the deuteron to avoid dealing explicity with the mechanism of the interaction of the photon with the quasi-deuteron. In this respect he goes further than Levinger, using the experimentally determined cross-section for this process instead of a theoretically derived one.

We shall now give a critical, detailed review of Gottfried s paper, keeping as closely as possible to his notation and for ease of reference use his numbering of equations. In the process of this , reference will be made where appropriate, to a series of papers by A.Reitan and E.Ostgaard (8). They also consider the photon interacting with a pair of nucleons but differ from Gottfried by calculating explicity the interaction of the $\gamma$-ray with the photo-pair. As a result they cast an interesting light on some of Gottfried's assumptions.

Gottfried summarises these at the beginning of his paper. They are

(1) only two nucleons are involved in the absorption of the photon,

(2) the residual nucleus is never left in a highly excited state,

(3) the nuclear wave function is such that it leads to a two-nucleon density matrix or pair correlation function of the form

$$\rho(\underline{r}_1, \underline{r}_2) = \rho_0(r_1, r_2)\left(|g(|\underline{r}_1 - \underline{r}_2|)|\right)^2$$

where $\rho_0$ is the pair correlation function of a Slater Determinant wave function and $g$ is a modification thereof at small interparticle separations. We shall comment on these assumptions later when they arise in the analysis.

Having stated the main assumptions Gottfried then derives an expression for the cross-section which he does for two cases: the first neglecting final state interactions; the second including them by means of a complex optical potential calculation.

## 1.4 Derivation of the cross-section.

The hamiltonian for the reaction he writes as

$$H_{int} = \sum_{i<j=1}^{A} \mathcal{E}_{\lambda}^{ij} \quad \text{- - - -} \quad (2.)$$

using his first assumption, $\mathcal{E}_{\lambda}^{ij}$ being an electromagnetic operator of a photon of wavelength $\lambda$ which acts on two particles $i$ and $j$. This postulate seems reasonable and he gives some justification for it by pointing out that, as he assumes virtual pion emission and readsorption is the predominant disintegration mechanism which takes place at a distance less than one fermi, the probability of finding three nucleons within range of each other is negligible. He concludes that three nucleon effects can be disregarded.

The transition amplitude for a photon of wavelength $\lambda$ summed over all pairs is then

$$T_{f\rho}^{\lambda}(\omega) = \sqrt{\tfrac{1}{2}A(A-1)} \left\langle \Psi_{f\rho}^{(-)} \middle| \mathcal{E}_{\lambda} \middle| 0 \right\rangle \quad \text{- -} \quad (3.)$$

where

$|0\rangle$ is the ground state wave function of the nucleus

$$\Psi_{f\rho}^{(-)} = \phi_f \, \Xi_{\rho}$$

is an eigenstate of the nuclear hamiltonian

$\phi_f$ being a two-body antisymetric state

$\Xi_{\rho}$ being an (A-2)-body state of the residual nucleus.

Using well known methods Gottfried obtains from this the expression

$$T^{\lambda}_{f0}(\omega) = \sqrt{\tfrac{1}{2}A(A-1)}\left\{\langle \Xi_0 \chi_f^{(-)} | \mathcal{E}_\lambda | 0 \rangle + \langle \Xi_0 \chi_f^{(-)} | W G_E \mathcal{E}_\lambda | 0 \rangle\right\}$$

$$= \sqrt{\tfrac{1}{2}A(A-1)}\left\{\langle \Xi_0 \chi_f^{(-)} | (1 + W G_E) \mathcal{E}_\lambda | 0 \rangle\right\} \quad \cdots \quad (7.$$

$\chi_f^{(-)}$ is the 2-body scattering state corresponding to $\phi_f$ as the incident wave due to an interaction $V$. $W$ is the interaction between the photo-pair and the residual system.

$$G_E = \frac{1}{E - H + i\eta}$$

is the propagation function for the entire nuclear system of which $H$ is the hamiltonian and $E$ the total energy.

The transition amplitude has now been conveniently divided up into two terms, the first of which deals with the absorption of the photon giving rise to two emitted particles the second deals with the photo-particles interaction with the remaining nucleons. It should be noted at this point that if we wish to ignore the scattering of the photo-particles leaving the nucleus we just put $W = 0$.

To obtain the cross-section (7) is squared and summed over all states of the residual system.

Putting

$$\varphi_\lambda = (1 + W G_\varepsilon)\, \mathcal{E}_\lambda$$

this gives

$$d\sigma(\lambda) = \tfrac{1}{2} A(A-1)\, 2\pi\, \mho\, \frac{\mho\, d^3 k_1}{(2\pi)^3} \cdot \frac{\mho\, d^3 k_1}{(2\pi)^3} \cdot$$

$$\sum_{\rho} \int \frac{\mho\, d^3 p_r}{(2\pi)^3}\, \delta(\varepsilon - E_\rho - \mathcal{E}_r - B_\lambda + B_r) \cdot$$

$$\langle 0 | \varphi_\lambda^{+} | \chi_f^{-} \Xi_\rho \rangle \langle \Xi_\rho \chi_f | \varphi_\lambda | 0 \rangle \qquad \text{(9}$$

$\varepsilon$ = [photon energy]-[photo-particles energy]

$E_\rho$ = energy of excitation of state

$B_\lambda$, $B_r$ are the binding energies of the target and residual nuclei respectively.

$p_r$ is the recoil momentum of the residual nucleus.

In order to carry out the indicated summation Gottfried invokes his second assumption. By asumming that the residual nucleus is excited only to energies close to a well defined average energy, later put equal to zero, and inserting this in the delta-function the summation reduces to one term viz.

$$\delta(\varepsilon-\bar{\varepsilon})\langle 0|\varphi_\lambda^{\dagger}|\chi_f^{-}\rangle\langle\chi_f^{-}|\varphi_\lambda|0\rangle$$

Since the $\Xi_0$ are a complete orthonormal set. The only justification for this assumption is that it is a necessary simplification if progress is to be made. It has been found from cloud chamber studies that the excitation of the residual nucleus may be up to 30 Mev (9). This is a weak point of the theory.

Following Gottfried it is now convenient to define

$$u_\lambda = v^3 \tfrac{1}{2} A(A-1)\langle 0|\varepsilon_\lambda^{\dagger}|\chi_f^{-}\rangle\langle\chi_f^{-}|\varepsilon_\lambda|0\rangle \quad (12)$$

where we have made the impulse approximation by putting

$$W = 0$$

$u_\lambda$ is essentially the cross-section with kinematical factors suppressed.

This becomes written in a coordinate representation,

$$u_\lambda = v^3 \sum_{\zeta'\zeta''} \int d^3r \, \langle r_1''' r_2''' \zeta''|\rho|r_1'' r_2'' \zeta'\rangle \cdot$$

$$\langle r_1'' r_2'' \zeta'|\varepsilon_\lambda^{\dagger}|r_1 r_2 \zeta\rangle\langle r_1 r_2 \zeta|\chi_f^{-}\rangle \cdot$$

$$\langle\chi_f^{-}|r_1' r_2' \zeta\rangle\langle r_1' r_2' \zeta|\varepsilon_\lambda|r_1''' r_2''' \zeta''\rangle \cdot \quad (13)$$

where $\zeta$ represents $S, M_S, T, M_T$, the spin and isobaric quantum numbers for a pair of particles. The first factor is the two-particle density matrix which by Gottfried's third assumption is written as

$$\langle r_1 r_2 \zeta | \rho | r_1' r_2' \zeta' \rangle = g_\rho(x) \langle r_1 r_2 \zeta | \rho_0 | r_1' r_2' \zeta \rangle g_{\zeta\rho}^{*}(x')$$

This form follows from wave functions of the type

$$\Psi_0(1, \cdots, A) = \sum_{i>j}^{A} c_{ij}\, \phi_0(1, \cdots, A) \cdots \quad (19)$$

where the $\phi_0$ are Slater determinants. In (19) Gottfried has a product sign $\prod$ in place of $\sum$. This is wrong.

By a series of fourier transforms and assuming we are dealing with closed shell nuclei, (13) may be written as

$$\mu_\lambda = \frac{2\pi}{\omega} \sum_{\zeta'} \int d^3q\, d^3q'\, d^3x_1 \cdots d^3x_4 \langle \underline{q}, \underline{P}-\underline{q} | \rho_{ST} | \underline{q}', \underline{P}'-\underline{q}' \rangle \cdot$$
$$\left[\chi_f^*(\underline{x}_1 \zeta_f) \langle \underline{x}_1 \zeta_f | \varepsilon_\lambda | \underline{x}_2 \zeta' \rangle g_{\zeta'}(x_2)\, e^{i(\underline{q}-\frac{1}{2}\underline{P})\cdot\underline{x}_2}\right] \cdot$$
$$\left[\chi_f^*(\underline{x}_3 \zeta_f) \langle \underline{x}_3 \zeta_f | \varepsilon_\lambda | \underline{x}_4 \zeta' \rangle g_{\zeta'}(x_4)\, e^{i(\underline{q}'-\frac{1}{2}\underline{P}')\cdot\underline{x}_4}\right] \cdots \quad (20)$$

$\langle \underline{q}, \underline{P}-\underline{q} | \rho_{ST} | \underline{q}', \underline{P}'-\underline{q}' \rangle$ being the Slater determinant part of the density matrix in momentum space.

At this point Gottfried makes a crucial assumption. In effect it says that the photons only interact with pairs of nucleons which are so close together they may be considered to be at zero separation. This makes the exponentials in (26) disappear to give

$$w_\lambda = \frac{2\pi}{\omega} \sum_{S'T'} F_{S'T'}(P) \, | M_{\lambda f}(S' \to S_f) |^2 \quad \cdots \quad (27.)$$

with

$$M_{\lambda f}(S' \to S_f) = \int d^3x \, d^3x' \, \chi_f^*(x S_f) \langle x S_f | \epsilon_\lambda | x' S' \rangle \, g_{S'}(x') \quad - (29)$$

and

$$F_{ST}(P) = \int d^3q \, d^3q' \langle q, P - q | \mathcal{D}_{ST} | q', P - q' \rangle \quad \cdots (28).$$

Gottfried now shows by reference to earlier equations that $F_{ST}(P)$ is proportional to the probability for finding two particles of total momentum $P$ and at zero separation in the Slater determinant. Transforming back to coordinate space and carrying out the angular integrations, again using the fact that we are dealing with closed shell nuclei, gives

$$F(P) = \sum_{\substack{nn' \\ \ell\ell'}} \sum_{\varrho = |\ell - \ell'|}^{\ell + \ell'} (2\ell+1)(2\ell'+1) \, | \langle \ell \ell' 00 | \varrho 0 \rangle |^2 \cdot \left| \int_0^\infty R_{n\ell}(r) \, R_{n'\ell'}(r) \, j_\varrho(Pr) \, r^2 \, dr \right|^2 \quad - \quad (30)$$

The $R_{n\ell}(r)$ are the radial parts of the wave functions used and the dependence on S and T has been eliminated.

Part of the present work has been involved with evaluating $F(P)$ explicitly for the case of calcium 40 using harmonic oscillator wave functions. The result obtained from this will be given later.

Gottfried's final result for the cross-section neglecting final state interactions is thus

$$d\sigma = \frac{1}{(2\pi)^4} F(P) \, S_{fi} \, \delta(\varepsilon - \bar{\varepsilon}) \, d^3k_1 \, d^3k_2 \quad \text{---} \quad (32)$$

where

$$S_{fi} = \frac{1}{2\omega} \sum_{\lambda} \sum_{\substack{S_f S' \\ S' \neq T'}} \left| M_{\lambda f}(S' \rightarrow S_f) \right|^2 \quad \text{---} \quad (33)$$

From this it can be seen that the cross-section is made up of:

(1) the available phase space,

(2) the probability for finding two particles of total momentum $\underline{P}$ and at zero separation in the Slater determinant, and

(3) the probability that two particles in a state of relative motion given by the short range correlation function $g(x)$ perform a transition to the state $\chi_f^-$

The next important step is to relate this cross-section to that for the photo-disintegration of the deuteron.

## 1.5 Relationship with deuteron photo-disintegration.

Here Gottfried poses the question: "Can transitions from states other than the $^3S_1$ be neglected?". He answers this in the affirmative by an argument based on the idea that the photo-nuclear interaction is best described by the isobar model. This assumes that the photon produces a quasi-stable $T=\frac{3}{2}$, $J=\frac{3}{2}$ state from one nucleon which communicates some of its excitation energy to a nearby nucleon so that both move off with high kinetic energy. Since spin and isotopic spin are conserved in this reaction he then goes on to show from selection rule considerations and the fact that the process is observed to go approximately 80% by dipole absorption that transitions from the $^1S_0$ state can be neglected. At this juncture he draws attention to a point made earlier concerning the assumption of zero separation of the photo-pair, namely that it implies transitions from states of higher angular momentum than the zero state care neglected. Hence his "yes" to the question posed at the beginning of this paragraph.

With respect to the discussion Gottfried has given here, mention should be made of the conclusions reached by A.Reitan and E.Ostgaard (8). For the case of oxygen 16 Reitan calculates the cross-section for the photo-production of neutron-proton pairs by electric dipole

$\gamma$-rays of energy 100-200 Mev. Ostgaard has extended this calculation to include electric quadrupole and magnetic dipole $\gamma$-rays in the energy range from threshold to 200 Mev. An important point to notice is that no account is taken of mesonic effects, although these might be present at the higher energies. From their results Reitan points out that some of Gottfried's assumptions are suspect in the energy range they consider but adds that at 300 Mev, which is the energy Gottfried is mainly concerned with, they are acceptable due to pion effects. In particular, Reitan draws attention to the assumption that the main contribution comes from transitions from the $^3S_1$ state. They find that the s state contribution increases from 18 % of the total at E = 100 Mev to 90 % at 200 Mev, but even here the $^1S_0$ state is still more important than the $^3S_1$ state. He agrees with Gottfried, however, that according to the isobar model this is expected to change at higher energies when mesonic effects become important. A more disturbing discovery by Reitan, as far as Gottfried's theory is concerned, is that he finds that initial state correlations have but little influence on the cross-section for the ( $\gamma$, np ) reaction. A related conclusion to this is his statement that within the energies he is dealing with the interaction between $\gamma$-ray and the photo-pair takes place mainly at distances greater than, or of the order of, the mean inter particle spacing. Since Gottfried's theory depends on a close correlation between pairs of particles, before ejection this seems to undermine the basis of it . This difficulty can perhaps be surmounted like the rest by arguing that the range of mesonic forces is less than that of electromagnetic forces and hence, for Reitan's case, close spacing is not required, as it is in Gottfried's theory. As a result we would not expect close correlation to show up in Reitan's analysis. If this argument is not accepted then we can appeal to experiment to see if Gottfried's shows agreement with it. When compared with experiment Reitan and Ostgaard's work is too low particularly in the higher parts of the energy range they consider. Reitan suggests himself that this is probably due to their neglect of pion-connected additions to the cross-section.

We now return to the relationship of the nuclear photo-production of neutron-proton pairs cross-section to that for the photo-disintegration of the deuteron. Having shown that the spin and isobaric spin sums in equation (32) can be restricted to $S' = 1$, $T' = 0$ Gottfried considers the connection between $g_{10}(x)$ and $\phi_0(x)$, the deuteron ground state wave function. If these functions were proportional then $S_{fi}$ of equation (33) would be directly proportional to the corresponding quantity for the deuteron photo-disintegration. Unfortunately, this is not the case. $\phi_{10}(x)$ tends to zero and $g_{10}(x)$ goes to one for large $x$ so Gottfried is forced to assume proportionality for small $x \leq 10^{-13}$ cm. and hope that the results obtained justify this. He thus puts

$$|g_{10}(x)|^2 = \gamma^3 |\phi_0(x)|^2 \quad \text{for} \quad x \leq 10^{-13} \text{cm.}$$

where $\gamma$ is a constant having the dimension of length.

$S_{fi}$ can then be written as

$$S_{fi} = \frac{\gamma^3}{2\omega} \sum_{\lambda S_f S_p} \left| \int d^3x\, d^3x' \chi_f^*(x S_f) \langle x S_f | \varepsilon_\lambda \cdot p' S_0 \rangle \phi_0(x') \right|^2$$

$$= 3\gamma^3 D_{fi} \tag{34}$$

Noting that the cross-section for the photo-disintegration of the deuteron is

$$\frac{d\sigma_D}{d\Omega_{p0}} = \frac{k_{p0} E_{p0}}{4\pi} D_{fi}$$

where the subscript '0' indicates quantities measured in the centre of momentum frame, he thus obtains

$$\frac{\dfrac{d\sigma}{d\Omega_p}}{\left[\dfrac{d\sigma_D}{d\Omega_p}\right]_0} = \frac{3\gamma^3}{4\pi^3} F(P) \frac{k_p E_p}{[k_{p_D} E_{p_D}]_0} \delta(\varepsilon - \bar{\varepsilon}) d\varepsilon_p d^3k_n \tag{36}$$

Since $\omega_0$ and $k_{p0}$ are not independent quantities and the relationship between them is not the same in the two processes, he recommends that $\mathcal{B}_{G}$ should be evaluated for the appropriate value of $\omega_0$ since this would make the difference between the off- and on-shell $\mathcal{D}_{G}$ as small as possible. As a measure of this difference he later defines a quantity $\Delta$ given by

$$\Delta = 2(\varepsilon_{\rho_D} - \varepsilon_{\rho_0})$$

and this unfortunately becomes quite large $\sim 90$ Mev for large values $\sim 1{\cdot}5\,fm^{-1}$ of $|\rho|$ or equivalently for angles which differ considerably, from those of deuteron photo-disintegration kinematics.

At this point comparison can be made with experiment where agreement or disagreement would show whether final state interactions were unimportant or important resp.. Before doing this we shall indicate how Gottfried suggests these interactions should be at least partially accounted for.

## 1.6 Final state interactions.

In order to take final state interactions into account $W$ must be retained so that equation (13) becomes

$$u_\lambda = U^3 \tfrac{1}{2} A(A-1) \langle 0 | \left(1 + W \frac{1}{E - H + i\eta}\right)^* \mathcal{E}_\lambda^* | \chi_f^- \rangle \cdot$$

$$\langle \chi_f^- | \left(1 + W \frac{1}{E - H + i\eta}\right) \mathcal{E}_\lambda | 0 \rangle \quad \cdots \qquad (37)$$

To evaluate this explicitly further simplification is required. Gottfried argues, that, as the main processes which effect the photo-particles on their way out of the nucleus are absorption due to multiple scattering and refraction as they cross the edge of the nucleus, a good approximation would be to replace $W$ by a complex optical potential $\mathcal{W}$. $U_\lambda$ becomes

$$U_\lambda = U^3 \tfrac{1}{2} A(A-1) \langle 0 | \left(1 + \frac{1}{\varepsilon_f - K - V - \mathcal{W}^+ - i\eta}\right) \mathcal{W}^+ \mathcal{E}_\lambda^+ | \chi_f^- \rangle$$

$$\langle \chi_f^- | \left(1 + \frac{1}{\varepsilon_f - K - V - \mathcal{W} + i\eta}\right) \mathcal{W}\, \mathcal{E}_\lambda | 0 \rangle \quad \cdots \qquad (40)$$

where $\varepsilon_f$ = total energy of the photo-pair

$K$ = their kinetic energy operator.

After introducing a complete set of two-body states $\chi_\alpha^{(-)}$ equation (40) becomes

$$U_\lambda = U^3 \tfrac{1}{2} A(A-1) \sum_{\alpha\beta} O_{f\alpha} O_{f\beta}^+ \langle 0 | \mathcal{E}_\lambda^+ | \chi_\beta^- \rangle \langle \chi_\alpha^- | \mathcal{E}_\lambda | 0 \rangle \qquad (41)$$

with

$$O_{f\alpha} = \langle \chi_f^{(-)} | \left(1 + \frac{1}{\varepsilon_\alpha - K - V - W - i\eta} W^\dagger\right) | \chi_\alpha^{(-)} \rangle$$

and the $\chi_\alpha^{(-)}$ satisfy

$$(K + V - \varepsilon_\alpha) \chi_\alpha^{(-)} = 0 \qquad (43)$$

Gottfried now ignores the photo-nucleons mutual interaction $V$ on their way out of the nucleus and this allows him to replace $\chi_f^{(-)}$ by a new two-body state given by

$$\psi_f^{(-)} = \phi_f + \frac{1}{\varepsilon_f - K - i\eta} W^\dagger \psi_f^{(-)} \qquad (44)$$

$\phi_f$ being the plane wave part of $\chi_f^{(-)}$.
Thus (43) becomes

$$O_{f\alpha} \xrightarrow[V \to 0]{} \delta_{f\alpha} + \frac{\langle \psi_f^{(-)} | W | \phi_\alpha \rangle}{\varepsilon_f - \varepsilon_\alpha + i\eta} \qquad (45)$$

Using the same techniques and assumptions that were used on equation (12), equation (41) may be reformulated as follows

$$U_\lambda = \frac{2\pi}{\omega} \sum_{S'} F_S(P_\alpha | P_\beta) M_{\lambda\alpha}(S' \to S_\alpha) M^*_{\lambda\beta}(S' \to S_\beta) \qquad (46)$$

where $M_\lambda(S' \to S_\alpha)$ takes the same form as before and $\underline{P_\alpha} + \underline{\omega}$, $\underline{P_\beta} + \underline{\omega}$ are the total momenta of $\chi_\alpha^-$, $\chi_\beta^-$ resp.

The form factor is a generalised version of that obtained previously. It is defined as

$$F_S(P_\alpha|P_\beta) = O_S \int d^3r\, d^3r'\, e^{-i(\underline{P}_\alpha . \underline{r} - \underline{P}_\beta . \underline{r}')} [\langle \underline{r}|\rho|\underline{r}'\rangle]^2$$

$$= O_S \sum_{\rho\mu\gamma} f_{\rho\gamma}(P_\alpha) f_{\rho\gamma}(P_\beta) Y_{\rho\mu}(\underline{P}_\alpha) Y_{\rho\mu}(\underline{P}_\beta) \qquad (48)$$

where $O_S$ is 0 or 1 according as (S + T) is even or odd and

$$f_{\rho\gamma}(P) = \left[\frac{4\pi(2l+1)(2l'+1)}{(2\rho+1)}\right]^{\frac{1}{2}} \langle ll'00|\rho 0\rangle \cdot$$
$$\int_0^\infty R_{nl}(r) R_{n'l'}(r) j_\rho(Pr) r^2 dr. \qquad (49)$$

noting that $\sum_\gamma$ implies summation over $(n, l; n', l')$

He finally obtains $U_\lambda$ in the form

$$U_\lambda = \frac{2\pi}{\omega} \sum_{\rho\mu\gamma} \sum_{\zeta'} O_{\zeta'} \left| \mathcal{J}_{\lambda\zeta'}^{\rho\mu\gamma} \right|^2$$

with

$$\mathcal{J}_{\lambda\zeta'}^{\rho\mu\gamma} = f_{\rho\gamma}(P) Y_{\rho\mu}(\underline{P})\left(M_{\lambda f}(\zeta' \to \zeta_f) + \sum_\alpha \frac{\langle \psi_f^{(-)}|W|\phi_\alpha\rangle}{\varepsilon_f - \varepsilon_\alpha + i\eta} M_{\lambda\alpha}(\zeta' \to \zeta_\alpha)\right) \qquad (51)$$

Neglecting the second part of this gives the result obtained previously , all the final state interactions being contained in this part.

In order to retain proportionality of the cross-section to that of the deuteron , $M_{\lambda\alpha}(\zeta' \to \zeta_\alpha)$ is put equal to $M_{\lambda f}(\zeta' \to \zeta_f)$. This is equivalent to neglecting transitions which are not allowed by free deuteron kinematics.

Even with this approximation (51) cannot be evaluated exactly since an analytic expression for the matrix element

$$\langle \psi_f^{(-)} | W | \varphi_\alpha \rangle$$

is required before the summation over $\alpha$ can be performed . To surmount these difficulties Gottfried makes what he calls 'drastic simplifications'. It is at this point he splits the final state interactions into the two distinct phenomena of

(1) absorption of the neutron-proton pair on their way out of the nucleus, and

(2) refraction at the nuclear surface.

He calculates the refraction correction in the Born approximation using $\mathrm{Re}\, W$ and the absorption he deals with by a mean free path calculation which involves $\mathrm{Im}\, W$.

## 1.7 Refraction.

To deal with the refraction effect he writes (51) as

$$\mathcal{T}^{\rho\mu\nu}_{\lambda\zeta'} = \mathcal{M}_{\lambda\alpha}(\zeta' \to \zeta_f)\Big\{ f_{\rho\nu}(P)\, Y_{\rho\mu}(\underline{P}) \; +$$

$$\frac{2M v}{(2\pi)^3}\int d^3q_n \frac{\langle k_n | W | q_n \rangle}{k_n^2 - q_n^2 + i\eta}\, f_{\rho\mu}(|\underline{P}+\underline{q}_n-\underline{k}_n|)\, Y_{\rho\mu}(\underline{P}+\underline{q}_n-\underline{k}_n)$$

$$+ \quad [n \leftrightarrow p] \Big\} \quad - \quad \cdot \quad \cdot \quad \cdot \quad (52)$$

where $[n \leftrightarrow p]$ signifies a similar term with the appropriate value of a quantity for the proton replacing that of the corresponding quantity for the neutron and vice-versa

Defining the coordinate representation of

$$\langle k_n | W | q_n \rangle$$

by

$$\langle k | W | q \rangle = v^{-1} \int d^3r\, e^{i(\underline{q}-\underline{k})\cdot \underline{r}}\, w(r) \quad - \quad - \quad (53)$$

and putting

$$f_{\rho\nu}(P)\, Y_{\rho\mu}(\underline{P}) = (2\pi)^{-\frac{3}{2}} \int d^3R\, e^{-i\,\underline{P}\cdot\underline{R}}\, h_{\rho\mu\nu}(\underline{R}) \quad - \quad \cdot \quad (54)$$

we obtain for (52)

$$\mathcal{J}_{\lambda\zeta'}^{\rho\mu\nu} = M_{\lambda f}\Big\{ f_{\rho\nu}(P)\, Y_{\rho\mu}(\underline{P}) \; - $$

$$\frac{M}{(2\pi)^{\frac{5}{2}}} \int \left[ \frac{e^{ik_n|\underline{r}-\underline{R}|}}{|\underline{r}-\underline{R}|}\, e^{-i\underline{k}_n\cdot\underline{r}}\, e^{-i(\underline{P}-\underline{k}_n)\cdot\underline{R}} + {}_{[n\leftrightarrow p]} \right] \cdot$$

$$W(r)\, h_{\rho\mu\nu}(\underline{R})\, d^3r\, d^3R \Big\} \quad - - - - - \quad (55).$$

where

$$h_{\rho\mu\nu}(\underline{R}) = \sum_{\mu}{}' \pi \left[ \frac{2(2\ell+1)(2\ell'+1)}{2\rho+1} \right]^{\frac{1}{2}} \langle \ell\ell'00|\rho 0\rangle \cdot$$

$$i^{\rho}\, R_{n\ell}(r)\, R_{n'\ell'}(r)\, Y_{\rho\mu}(\underline{R})$$

In this form $\mathcal{J}_{\lambda\zeta'}^{\rho\mu\nu}$ can be evaluated explicitly once a choice of the wave functions $R_{n\ell}(r)$ and potential $W(r)$ have been made. In his paper Gottfried goes on to do this but we shall defer discussion of it for the moment.

1.8 Absorption.

Gottfried asumes that absorption is an isotropic effect and hence its sole influence is the depletion of the cross-section by a numerical factor $f_a$ given by

$$f_a = \frac{1}{V_0}\int dW_0\, e^{-x/\lambda_a} \qquad (62)$$

where $V_0$ is the volume of the nucleus,

$x$ is the distance traversed by the photo-nucleons from their point of origin to the nuclear surface , and

$\lambda_a$ is the mean free path of a nucleon in nuclear matter.

Equation (62) was derived from a paper by R.Serber et al. (10)

Assuming that the nucleus is a sphere of constant density of radius $R_0$ and that the photo-particles are emitted in opposite directions enables the integration to be performed algebraically (11), yielding the result

$$f_a = 6\xi^{-3}\left[1 - e^{-\xi}\left(1+\xi+\tfrac{1}{2}\xi^2\right)\right] \qquad (63)$$

with

$$\xi = \frac{2R_0}{\lambda_a}.$$

The values used by Gottfried for oxygen 16 were $R_0$ =1.33 (16) $\times 10^{-13}$cm. and $\lambda_a = 4.0 \times 10^{-13}$cm. yield the result $f_a = 0.30$

Whether this result is of any real value is open to question. The nucleons in general do not obligingly come out back to back and this treatment deals with the s and p shell nucleons in the same way. T.Berggren and G.Jacob (12) indicate that absorption has a greater effect on the s shell nucleons than on the p shell ones because the latter extend further out and have therefore less chance of being scattered on their way out. At best Gottfried's result indicates that we might expect absorption to reduce the cross-section considerably. On a suggestion by N.MacDonald (13) the absorption effect has been treated by a completely different approach later in this dissertation with interesting results.

This completes the review of the general features of Gottfried s approach. The rest of his paper is mainly concerned with deriving numerical results and comparson with experiment with particular reference to oxygen 16.

## 1.9 Related work by S.Fujii.

We conclude this review with a brief discussion of other work, which draws much of its inspiration from Gottfried's theory, by S.Fujii (14). His work contains two significant differences. The first of these is that he does not use the cross-section for the photon induced break-up of the deuteron but instead calculates the neutron-proton pair photo-interaction using the conventional electromagnetic dipole interaction. The second difference is that instead of assuming proportionality between the short range correlation function $g_{10}(x)$ and the deuteron wave function he takes a definite form for $g_{10}(x)$ which contains two parameters. He then studies the variation of the cross-section with respect to these parameters.

This work also suggests a lack of correlation between neutron and proton in the initial state in support of Reitan's findings. A similar explanation might be offered for this as before, although the extent to which this conclusion is dependent on the author's choice of correlation function, which he gives no justification for, is unknown. The correlation function he uses is

$$g_{10}(x) = 1 - e^{-\beta^2 x^2} \cos \mu x$$

with typical values of $\beta$ and $\mu$ being $0.75 \times 10^{13}\ \text{cm}^{-1}$ and

$2.0\times 10^{13}\ cm^{-1}$ respectively.

Like Reitan and Ostgaard, Fujii's results are again too small (by a factor of 4) when compared with experiment and again the author suggests neglect of mesonic effects as the reason. A more thorough treatment of this approach perhaps bringing in some of the features of Reitan's work and including if possible the contributions to the cross-section due to mesonic effects would be more convincing.

# CHAPTER 2.

## EXTENSION OF GOTTFRIED'S WORK.

### 2.1 Summary and motivation.

The programme of work described here is

(a) production of refraction and absorption corrected form factors for neutron-proton pairs from oxygen 16.

(b) evaluating $F(\rho)$ for the case of calcium 40 using harmonic oscillator wave functions and then producing an $F(\rho)$ corrected for absorption in a similar way to the oxygen 16 case.

Before dealing with (a) and (b) in detail mention should be made for the motivation behind it . An experiment carried out by J.Garvey et al. (9)in Glasgow to investigate correlated neutron-proton pairs from the photo-disintegration of oxygen 16 at energies 300 MeV was analysed using Gottfried's theory.Using an $F(\rho)$ derived using harmonic oscillator wave functions they were able to obtain a best fit for their experimental points as shown in Fig.1. $F(\rho)$ having the form

$$e^{-\frac{\rho^2}{2\alpha_0^2}} + \left(\frac{\alpha_1^5\alpha_0^3}{k^8}\right)\frac{\rho^2}{k^2}\,e^{-\frac{\rho^2}{2k^2}} + \left(3 - \frac{\rho^2}{\alpha_1^2} + \frac{\rho^4}{4\alpha_1^4}\right)e^{-\frac{\rho^2}{2\alpha_1^2}}$$

where $k^2 = \frac{1}{2}\left(\alpha_0^2 + \alpha_1^2\right)$ and the terms are due to

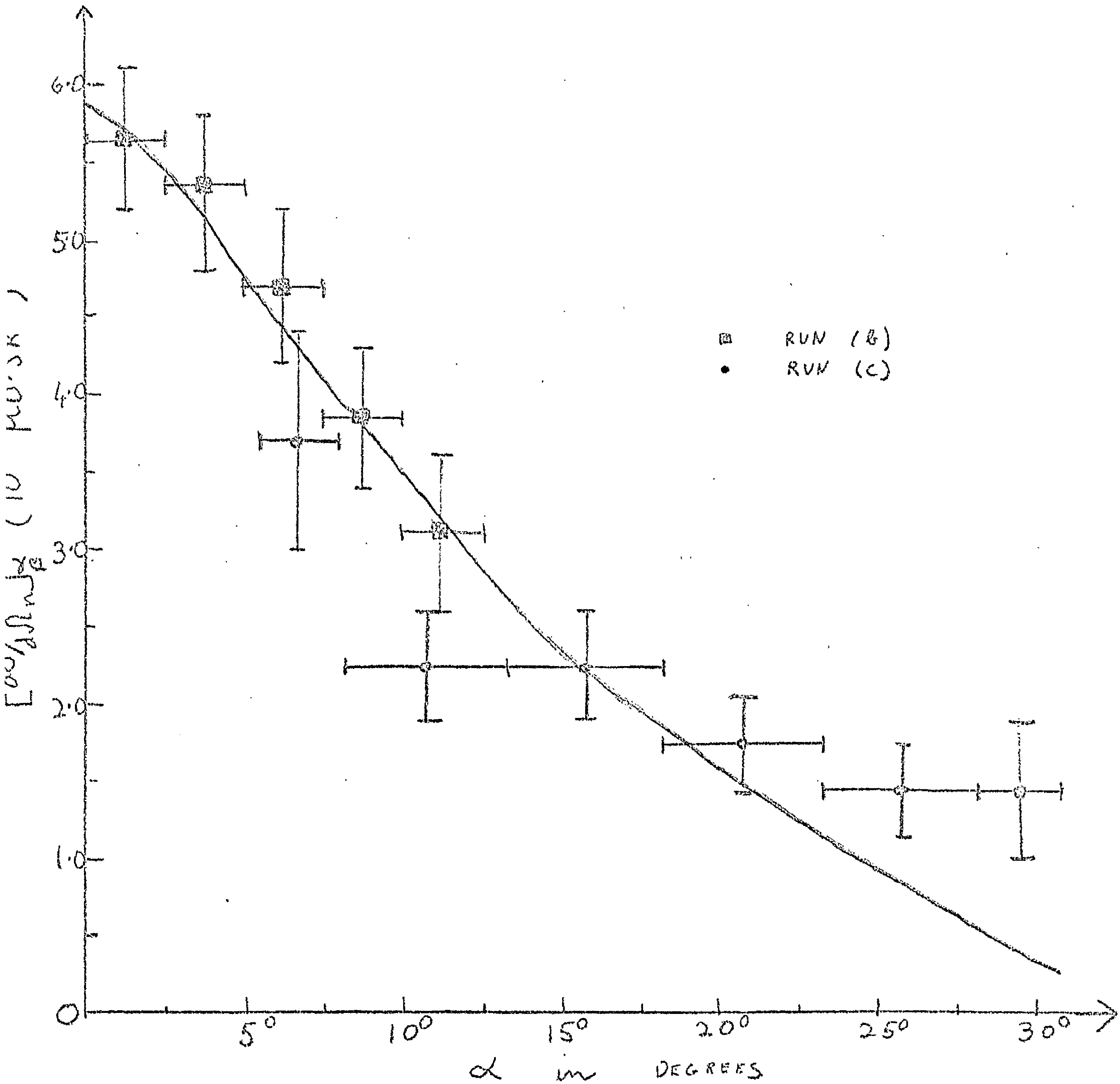



FIG.1. This is Fig.10 of ref.(9).It gives the distribution of 88.8 to 142.5 Mev protons detected at $\beta$=65 in coincidence with 76.5± 17 Mev neutrons at 90°in the lab. $\alpha_C$=0.54 fm$^{-1}$; $\alpha_1$=0.32 fm$^{-1}$with rms radius 4.45 fm.

two 1s shell, a 1s and 1p shell, and two 1p shell nucleons respectively. As can be seen from the graph the agreement is satisfactory up to $\alpha = 15^{o}$, where angle $\alpha$ is defined in Fig. 2., but the theoretical curve fails to match the experimental tail. Another serious defect is that the parameters chosen for the theoretical curve imply a root mean square radius of 4.45fm. which is considerably larger than the most recent value of 2.25 fm (15) obtained from other measurements. The authors of this paper therefore suggested that a first step to resolving these anomalies might be a detailed calculation of the effects of the absorption and refraction of s and p shell nucleons. This I have attempted to do.

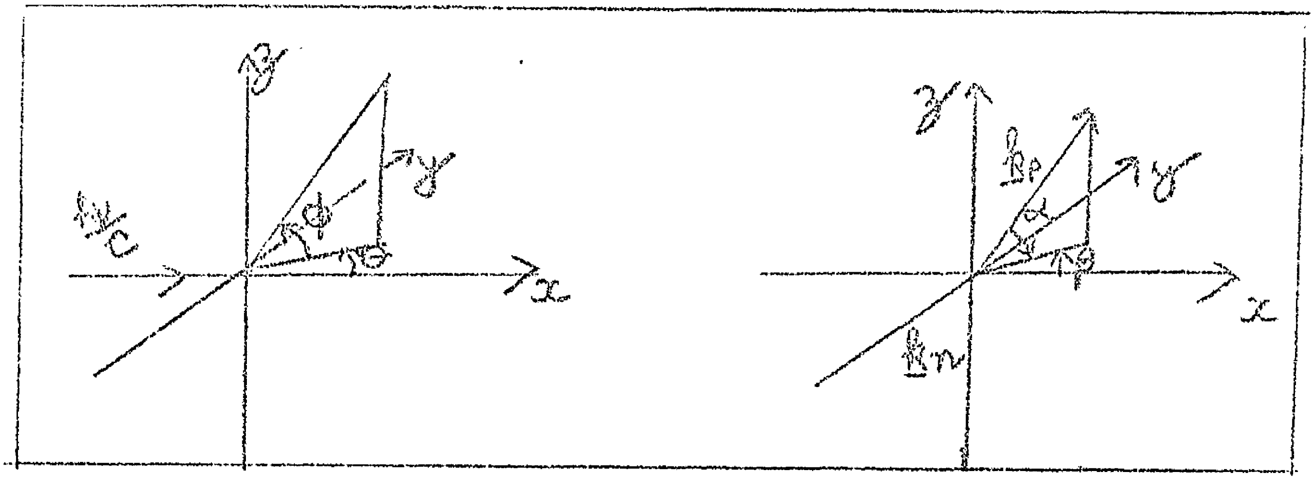



FIG.2. Definition of angles used. $k_p, k_n$ represent photo-proton and photoneutron momenta resp..

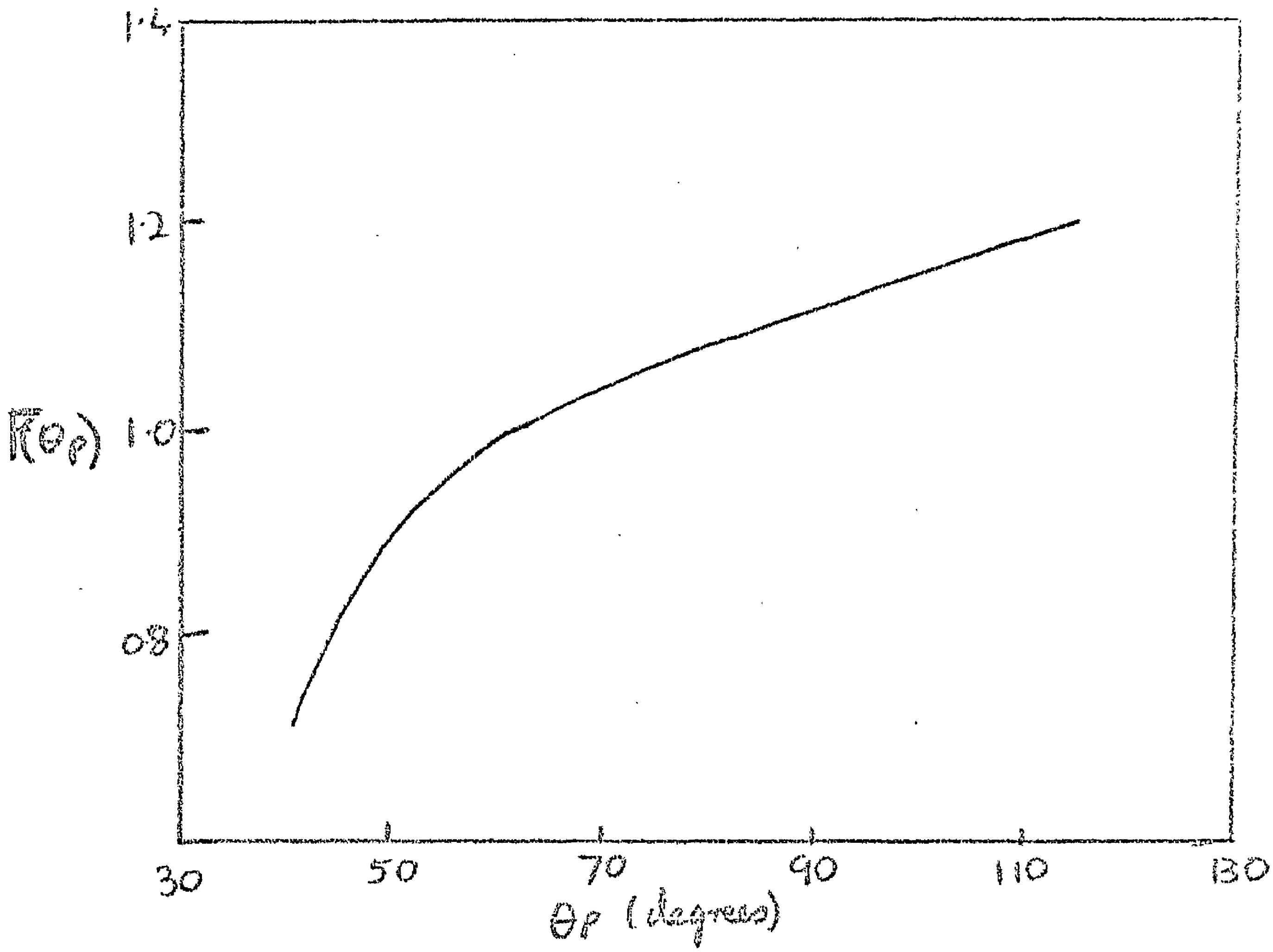



FIG.3. Fig.2. of ref.(7). This is Gottfrieds refraction correction for $\mathcal{E}_n = \mathcal{E}_p =$ 130 Mev, $\theta_n = 75^o$, $\phi_p = \phi_n$, $W_0 = -12$ Mev.

WORK ON OXYGEN 16

## 2.2 Refraction corrected form factor.

In evaluating equation (55) Gottfried assumed that all the nucleons in oxygen 16 are in an s state of a harmonic oscillator potential and he used for the real part of his optical potential a gaussian of the form

$$W(r) = W_0 e^{-(r/D)^2} \tag{56}$$

He also chose the same parameter $D$ for the harmonic oscillator wave functions taking them to be of the form $e^{-\frac{1}{2}(r/D)^2}$. Since it is generally accepted that the optical potential should extend beyond the nucleons producing it an obvious improvement is to take different parameters here. His only justification also for taking all the nucleons in s-states is given in a footnote where he states "In view of all our other approximations, it is doubtful whether the inclusion of states with both angular momenta would lead to a significant improvement." It seemed worthwhile to check this so a calculation was performed using both s and p shell harmonic oscillator wave functions.

After inserting his form of wave functions and optical potential in equation (55) Gottfried obtained

$$\sigma = (\text{const.}) \cdot \left[ e^{-\frac{\rho^2 D^2}{4}} - \frac{1}{\sqrt{8}} M D^2 e^{-\frac{\rho^2 D^2}{8}} W_0 \mathcal{F} \right] \tag{57}$$

where

$$\mathcal{F} = \frac{2}{D^2}\int_0^\infty \left\{ e^{ik_n x} j_0(\chi_n x)\, e^{-\frac{1}{2}\left(\frac{x}{D}\right)^2} + [n \leftrightarrow p] \right\} x\, dx \quad (60)$$

$$\chi_n = \left| \tfrac{1}{2}\underline{P} - \underline{k}_n \right|$$

He derives the refraction corrected cross-section from this by squaring (57) and taking only the leading and interference terms. Since the cross-section is real he neglects the imaginary parts of the result and finds that the cross-section should be proportional to

$$|\mathcal{T}|^2 = |\mathcal{T}_0|^2 \left( 1 - \frac{1}{\sqrt{2}} W_0 M D^2 e^{\frac{P^2 D^2}{8}} \operatorname{Re} \mathcal{F} \right) \quad (61)$$

$$= |\mathcal{T}_0|^2\, \Gamma(\varepsilon_p, \theta_p, \varphi_p; \varepsilon_n, \theta_n, \varphi_n)$$

where $\mathcal{T}_0$ is the amplitude in the absence of final state interactions.

The multiplying factor $\Gamma$ is the modification due to refraction, the angles $\theta, \varphi$ being the usual spherical polar angles measured with respect to the photon beam direction (Fig.2.) Gottfried graphs $\Gamma$ as a function of $\theta_p$ for $\varphi_p = \varphi_n = 75^\circ$, $\varepsilon_p = \varepsilon_n = 130$ Mev and $W_0 = -12$ Mev with $D$ chosen so that the wave-function gave the measured root mean square radius of the charge distribution. His result is shown in Fig. 3.

This graph appears to suggest that the refraction correction may be important for certain angles. This is misleading as except for $60^0 < \theta_\rho < 80^0$ when $\Gamma$ is very near the value 1.0 the cross-section is very small due to the fact that the value of $\underline{\rho}$ required to satisfy the kinematics is so large that the function $\Gamma(|\rho|)$ is less than 1 % of its value for $|\underline{\rho}| = 0.0$.

In performing the evaluation of the refraction correction using both s and p wavefunctions it was found expedient to neglect the $(1p, 1p)\, \ell=2$ term when it came to compute the numerical value of the correction. This term turned out to be difficult to handle due to the occurence of quadrupole spherical harmonics which when real and imaginary parts were taken made the term on its own approximately five times the complexity of the rest of the correction. As the $(1p, 1p)\, \ell=2$ term is itself small we would be very surprised if any correction to it were large. We were, however, prepared to evaluate it if our result differed substantially from Gottfried's. As luck would have it this was not the case.

Neglecting the $(1p, 1p)\, \ell=2$ term the oxygen 16 refraction corrected form factor can be written in four parts as, with $W(r) = W_0\, e^{-\beta^2 r^2}$

$$R_{10} = 2\left(\frac{\alpha^3}{\pi}\right) e^{-\frac{1}{2}\alpha^2 r^2} \qquad R_{11} = 2\left(\frac{2\alpha^5}{3\sqrt{\pi}}\right) e^{-\frac{1}{2}\alpha^2 r^2}\, \alpha r$$

$$4\left(1 + \frac{\rho^4}{16\alpha^4}\right) e^{-\frac{\rho^2}{2\alpha^2}} \quad - - - - \quad (2)$$

$$- \frac{4MW_0}{(1+a)^3} \frac{\alpha^3}{D^3} e^{-\left(\frac{\rho^2}{4\alpha^2}+\frac{\rho'^2}{4D^2}\right)} \left( I_1(n,c,x) + (n \Leftrightarrow \rho) \right) \quad \cdots \quad (b)$$

$$- \frac{8\sqrt{3}\, PMW_0}{(1+a)^4} e^{-\left(\frac{\rho^2}{4\alpha^2}+\frac{\rho'^2}{4D^2}\right)} \frac{\alpha^3}{D^3} \left[ \frac{\rho'}{2D^2} I_1(n,c,x) \right.$$

$$\left. - \frac{\chi_n' \cdot \rho'}{|\chi_n'|\,|\rho'|} I_2(n,c,x) + (n \Leftrightarrow \rho) \right] \quad \cdots \quad (c)$$

$$- \frac{8MW_0}{3(1+a)^5} \frac{\alpha^5}{D^5} \left( 3 - \frac{\rho^2}{2\alpha^2} \right) e^{-\left(\frac{\rho^2}{4\alpha^2}+\frac{\rho'^2}{4D^2}\right)} \cdot$$

$$\left[ D^2 I_3(n,c,x) + \frac{1}{2}\left(3 - \frac{\rho'^2}{2D^2}\right) I_1(n,c,x) + \right.$$

$$\left. \frac{2\chi_n' \cdot \rho'}{|\chi_n'|} I_2(n,c,x) + (n \Leftrightarrow \rho) \right] \quad \cdots \quad (d)$$

The definitions of symbols not used previously are

$$a = \frac{\alpha^2}{\beta^2}$$

$$c^2 = \frac{\alpha^2 + a^2\beta^2}{(1+a)^2} \qquad D^2 = \frac{\alpha^2 + \beta^2}{(1+a)^2}$$

$$\rho' = \frac{1}{1+a}\, \rho$$

$$I_1(n,c,x) = \frac{1}{2\chi_n'} \int_0^\infty \left[ \sin(\chi_n' + k_n)x + \sin(\chi_n' - k_n)x \right] e^{-c^2x^2} dx$$

$$\chi_n' = |k_n - \rho'|$$

$$I_2(n,c,x) = \frac{1}{2x_n'}\left[2I_1(n,c,x) - \int_0^\infty \{\cos(x_n'-k_n)x + \cos(x_n'+k_n)x\}e^{-c^2x^2}\,dx\right]$$

$$I_3(n,c,x) = \frac{1}{2x_n'}\int_0^\infty \{\sin(x_n'+k_n)x + \sin(x_n'-k_n)x\}e^{-c^2x^2}\,dx$$

Part (a) of the form factor is the simple uncorrected form factor with all the $\alpha$'s equal. (b) is the correction to the $(1S, 1S)$ term, (c) and (d) the corrections to the $(1S, 1P)$ and $(1P, 1P)$ $\rho = 0$ terms respectively.

Part (b) reduces to Gottfried's correction when his assumptions are made.

In Fig.4 the refraction corrected form factor is plotted with the uncorrected form factor for comparison. The corrected curve has been multiplied by 0.826 to make comparison of shape easy. The values of the parameters used were obtained from T.Berrgren and G.Jacob (12).The same optical potential well of -32 MeV was used for both s and p shells which is the mean value of those author's separate values and the value of $\beta$ was obtained from the formula

$$\frac{1}{\beta^2} = \frac{2}{3}\left(a^2 + c^2\right).$$

which is slightly modified form of that quoted by them,where $a^2$ is the mean square radius and $c^2 = 3.5$ Fm$^2$.

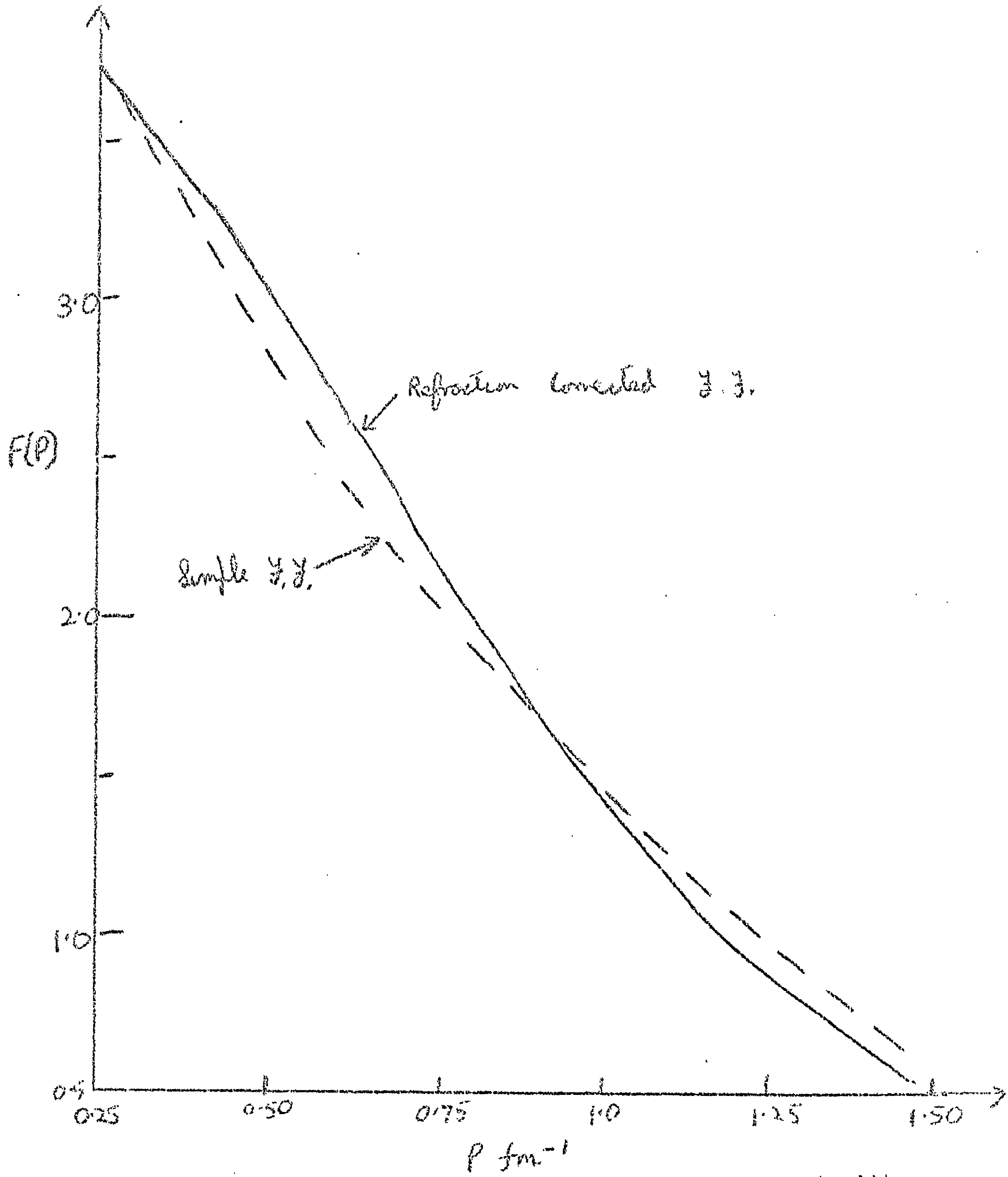
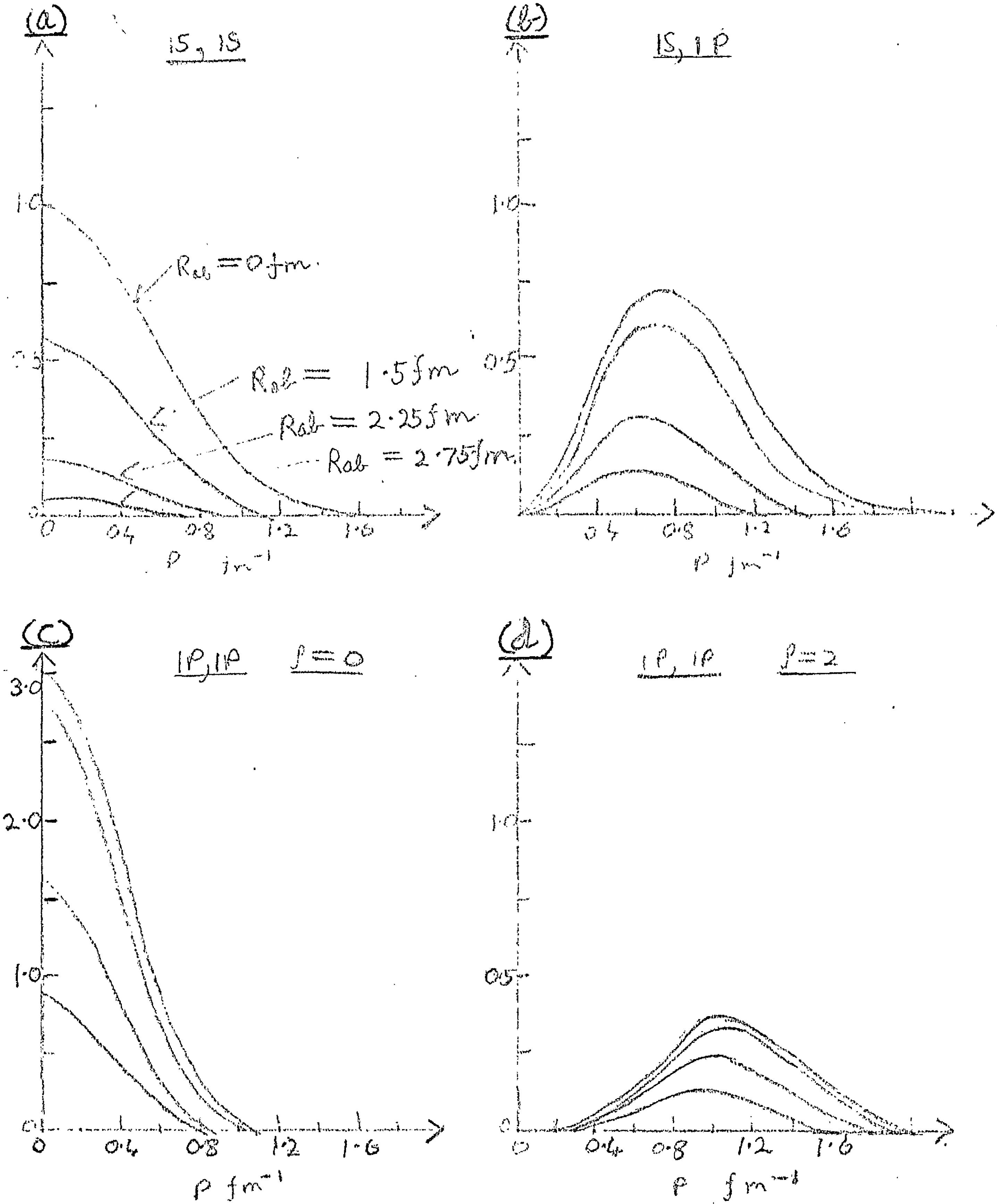



FIG.4. Refraction corrected form factor compared with simple F(P). $\alpha$ =0.525 $fm^{-1}$ giving rms radius 2.86 fm.W=-32Mev.Corrected curve is x 0.826.

Gottfried's correction when normalised in a similar way by multipling by 0.88 produces a curve indistinguishable from the uncorrected one for the scale used. In view of the slight difference between the two curves of Fig.4. we conclude that Gottfried is justified in not including p shell nucleons in his calculation.

## 2.3. Absorption correction.

Criticism of Gottfried's treatment of absorption has already been given. The way absorption has been treated here essentially consists in saying that all photonucleons produced in the centre of the nucleus out to a radius Rab do not get out. This is taken account of by calculating the contribution to F(p) from Rab to in equation (31). We now define $F_{ab}(p, R_{ab})$ as the probability of finding two nucleons of total momentum $\underline{P}$ at zero separation in the region of the nucleus outside a sphere of radius $R_{ab}$ centred on the nucleus centre. An immediate advantage of this treatment is that it reduces the contribution of the s shell nucleons to the form factor considerably more than the contribution of the p shell nucleons thus reflecting the expected stronger absorption of the former. This is illustrated in the four diagrams of Fig.5. Diagram (a) labelled showing that both nucleons come from the s shell indicates that for Rab = 2.75 fm the contribution from this term is down to less than 10% of its original value while (c) and (d) show that if both nucleons come from the p shell the contribution is only down by a third. In diagram (b) in which one nucleon comes from each shell the decrease in magnitude is intermediate between these two cases as might be expected.




FIGS.5. These show the effect of increasing R on the contributions of the different shells to the form factor. $\alpha$ = 0.525 fm.

In comparing the magnitude of the contribution of each term to the total form factor it should be noted that the vertical scale of diagram (c) is half that of the others.

Another feature of this treatment is that it produces an anisotropic absorption effect since the reduction of the proportion of nucleons with momentum $\underline{\rho}$ for different values of $\underline{\rho}$ is not uniform. This in turn affects the angular distribution since this depends on $\underline{\rho}$ due to kinematical considerations.

This variation in the shape of the form factor is shown in Fig. 6. where the curves have been normalised to the value of the uncorrected form factor at $|\underline{\rho}| = 0.2\,fm^{-1}$ The normalisation factor, which is given for each curve, is a measure of the reduction of the cross-section produced by the choice of the corresponding value of Rab. This anisotropic effect has the useful result that it enables a fit to be made to the experimental points as good as, if not better than, that due to the simple form factor while uusing only one wave function parameter which gives the accepted root mean square radius. The fit obtained choosing $\alpha = 0.525\ fm^{-1}$, Rab = 2.75 fm. is shown in Fig. 7.. Better fits can be obtained if different wave function parameters are used for the s and p shell and the normalisation is altered slightly. This variety

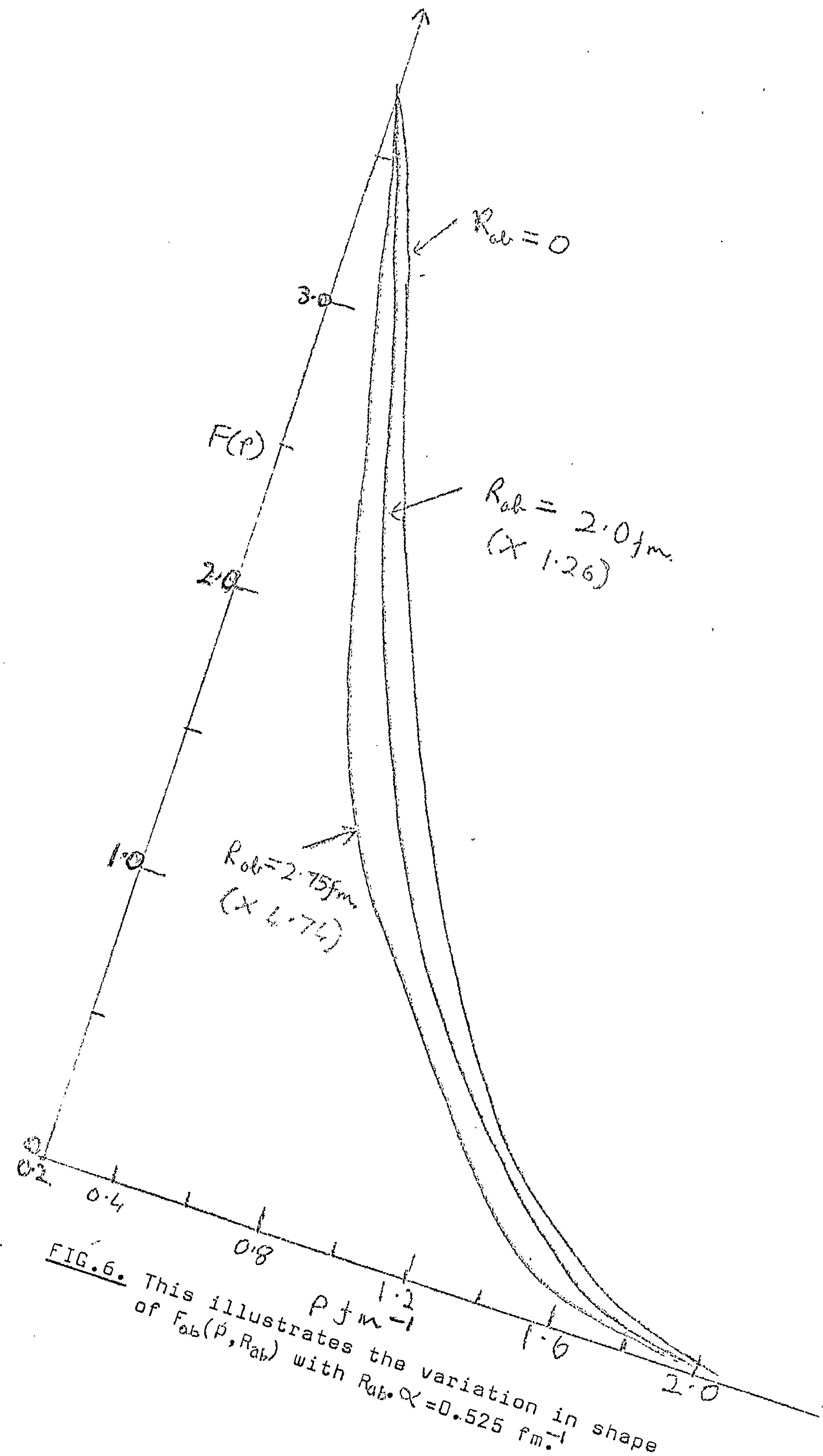



FIG.6. This illustrates the variation in shape of $F_{ab}(\rho, R_{ab})$ with $R_{ab}$. $\alpha$ = 0.525 fm$^{-1}$

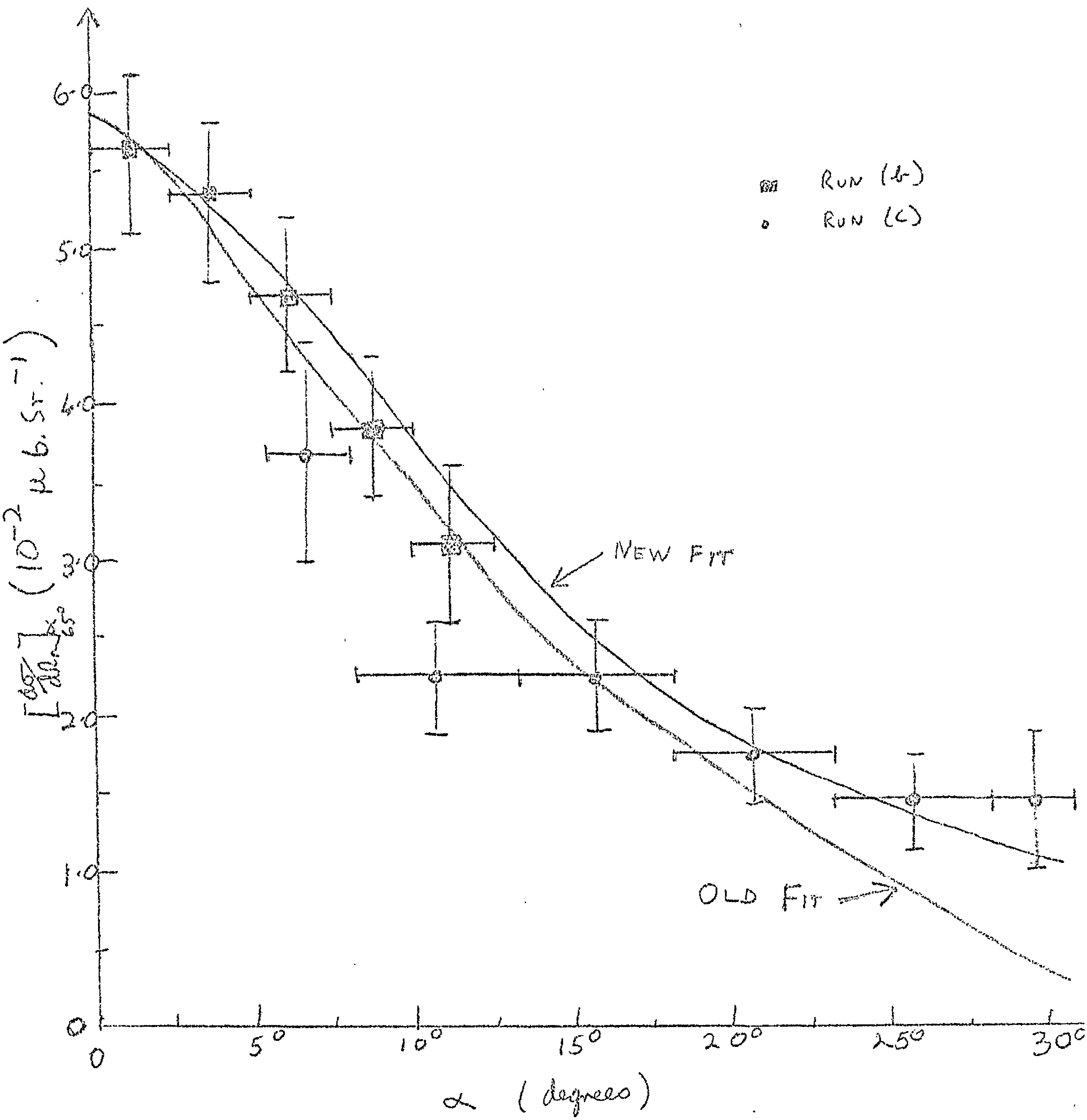



Fig. 7. A fit to the points of fig.1 using $F_{ab}(P,2.75)$, $\alpha = 0.525 fm.^{-1}$

of fits is only possible due to the large errors in the experimental points and it is doubtful whether any useful information can be deduced from them. The errors in the points and the slow variation in shape of $F_{ab}(\rho, R_{ab})$ with Rab allow us also to have a free hand in the choice of the value of Rab. This can not be fixed from the size of the curve either due to normalisation difficulities.One way round this would be a reliable independent estimate of the magnitude of the reduction of the cross-section due to absorption. Gottfried's method indicates a reduction of 70 % which in turn implies an Rab $\sim$ 2.5 fm..

Assuming absorption is of this order of magnitude, dealing with absorption by the present method suggests that the photo-emission of neutron-proton pairs only takes place fairly close to the surface of the nucleus and for oxygen 16 mainly involves p shell nucleons.

## 2.4 Absorption using exponential wave functions.

Gottfried found that the shape of the form factor was very similar for harmonic oscillator and infinite square well wave functions and he suggested that realistic Hartree-Fock wave functions would lead to an $F(P)$ lying between the form factors obtained from them. To investigate the dependence of $F_{ab}(\rho R_{ab})$ on the wave function used it was suggested by N.MacDonald that the form factor should be calculated using wave functions of the form

$$\psi_{\ell}^{m}(r) = \alpha_{\ell}^{\frac{3}{2}} \sqrt{\frac{2^{2\ell+3}}{(2\ell+2)!}} \, (\alpha_{\ell} r)^{\ell} e^{-\alpha_{\ell} r} \, Y_{\ell}^{m}(\theta,\varphi)$$

with $\alpha_{\ell} = \frac{1}{\hbar} \sqrt{2M\beta_{\ell}}$

which are given by T.Berggren and G.Jacob(12) and that the variation of shape as Rab was altered ascertained. These wave functions arise from a potential which varies as $r^{-1}$ and they extend further into space than the harmonic oscillator functions. The $\beta_{\ell}$ occurring them is the separation energy of the different shells as determined in ( p ,2p ) experiments.

In Fig. 8 the form factor for exponential wave functions is compared with that for harmonic oscillator wave functions, the values of $\beta_0$ , $\beta_1$ used being those quoted by Berggren and Jacob giving a root mean square radius of 2.86 fm. . Two features which are immediately obvious are , first, the exponential function form

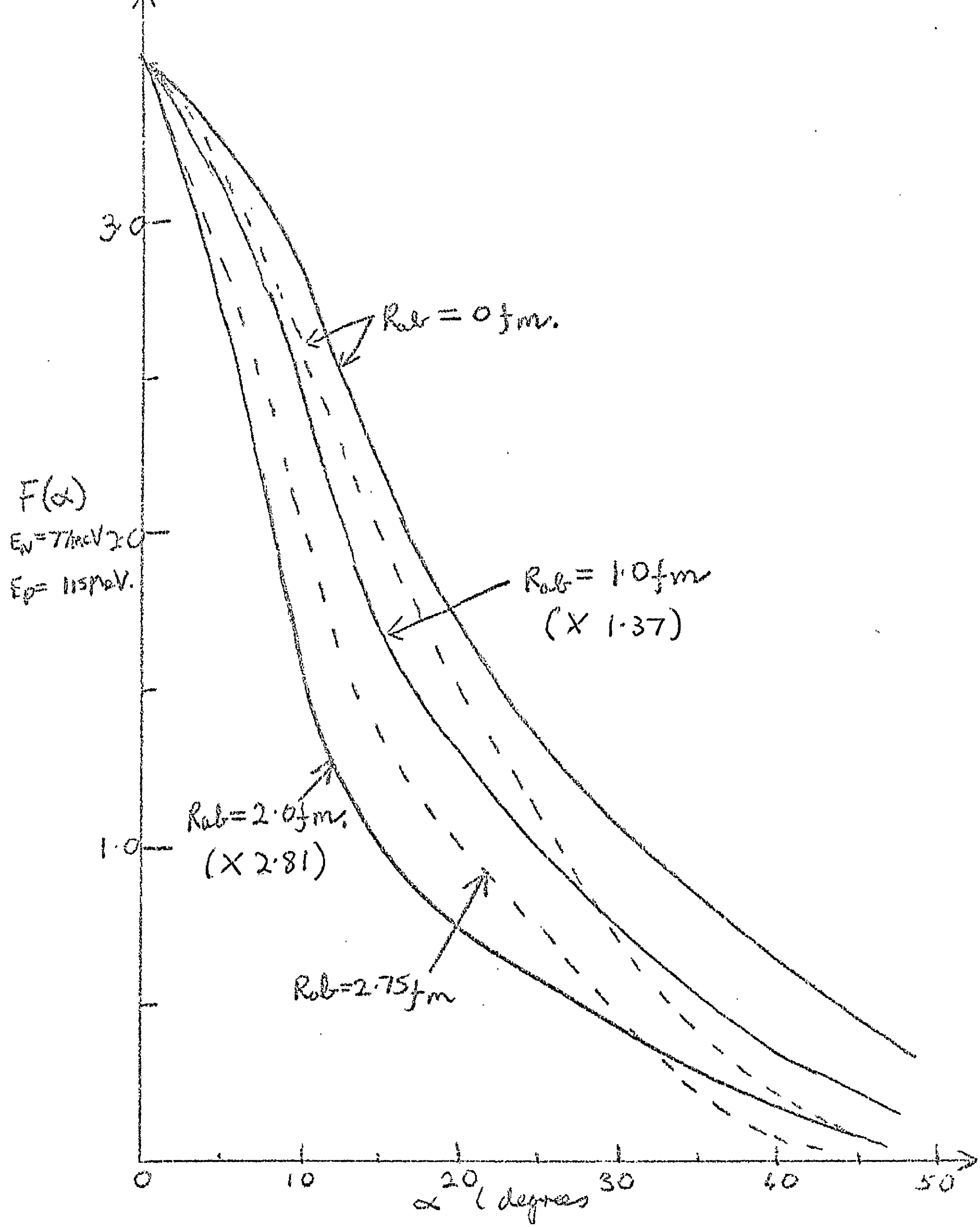



FIG.8. Absorption corrected exp. wave function and harmonic oscillator wave function form factors compared.

$\alpha_0 = 1.35\ \mathrm{fm}^{-1}$, $\alpha_1 = 0.92\ \mathrm{fm}^{-1}$, $\alpha = 0.525\ \mathrm{fm}^{-1}$

$\beta_0 = 38\ \mathrm{MeV}$, $\beta_1 = 16\ \mathrm{MeV}$.

factors have a longer tail, and second, the variation with Rab is more pronounced. Comparing these curves with those of Fig. 7. suggests that the exponential wave function form factor would also provide a reasonable fit with experiment.

In case it might be required for this purpose the uncorrected version of this form factor is given below.

$$F(\rho) = \left[1 + \left(\frac{\rho}{2\alpha_0}\right)^2\right]^{-4} \quad \text{-- (a)}$$

$$+ 32\frac{\alpha_0^3\alpha_1^5\rho^2}{(\alpha_0+\alpha_1)^{10}}\left[1 + \left(\frac{\rho}{\alpha_0+\alpha_1}\right)^2\right]^{-6} \quad \text{-- (b)}$$

$$+ 3\left[1 - \left(\frac{\rho}{2\alpha_1}\right)^2\right]^2\left[1 + \left(\frac{\rho}{2\alpha_1}\right)^2\right]^{-8} \quad \text{-- (c)}$$

$$+ 24\left(\frac{\rho}{2\alpha_1}\right)^4\left[1 + \left(\frac{\rho}{2\alpha_1}\right)\right]^{-8} \quad \text{-- (d)}$$

Term (a) is due to two 1s shell nucleons, term (b) to one 1s and one 1p shell nucleons, and (c) and (d) to two 1p shell nucleons with $\mathcal{J} = 0$ and $\mathcal{J} = 2$ respectively.

# WORK ON CALCIUM 40.

## 2.5 Simple form factor.

The simple uncorrected $F(\rho)$ was calculated for calcium 40 using the harmonic oscillator wave functions given below.

$$R_{10} = 2\left(\frac{\alpha_0^3}{\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\alpha_0^2 r^2}$$

$$R_{11} = 2\left(\frac{2\alpha_1^3}{3\sqrt{\pi}}\right) e^{-\frac{1}{2}\alpha_1^2 r^2}\alpha_1 r$$

$$R_{12} = 4\left(\frac{\alpha_2^3}{15\sqrt{\pi}}\right) e^{-\frac{1}{2}\alpha_2^2 r^2}\alpha_2^2 r^2$$

$$R_{20} = \left(\frac{2\alpha_2^3}{3\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{1}{2}\alpha_2^2 r^2}\left(2\alpha_2^2 r^2 - 3\right)$$

Different $\alpha$'s were used for the 1s, 1p and 1d,2s shells to allow them to be altered independently when fitting a theoretically derived curve to the experimental points. This also helps to show up the dependence of $F(\rho)$ on the various shells.

The result obtained,showing the shells from which the various terms came,is shown in Fig. 9.. The terms are arranged in rows of powers of $\rho^2$ and the columns give the contributions from the various shells. Each term in a column should be multiplied by the exponential factor at the bottom and all the terms are added together to give the form factor. The (1s,1s), (1s,1p) and (1p,1p)

| | 1S, 1S | 1S, 1P | 1S, 1D | 1S, 2S | 1P, 1P | 1P, 1D | 1P, 2S | 1D, 2S | 1D, 1D | 2S, 2S |
|---|---|---|---|---|---|---|---|---|---|---|
| $\rho^0$ | $1$ | | | $3\left(\frac{\alpha_0^3\alpha_2^3}{k^6}\right)\left(1-\frac{2\alpha_2^2}{k^2}+\frac{\alpha_2^4}{k^4}\right)$ | $3$ | | | | $5$ | $1$ |
| $\rho^2$ | | $\frac{\alpha_1^5\alpha_0^3\rho^2}{k^{10}}$ | | $\frac{\alpha_0^3\alpha_2^5}{k^8}\left(1-\frac{\alpha_2^2}{k^2}\right)\frac{\rho^2}{k^2}$ | $-\frac{\rho^2}{\alpha_1^2}$ | $\frac{10}{3}\left(\frac{\alpha_1^5\alpha_2^7}{k^{12}}\right)\frac{\rho^2}{k^2}$ | $\frac{1}{6}\left(\frac{\alpha_1^5\alpha_2^3}{k^8}\right)\frac{\rho^2}{k^2}\left(9-\frac{30\alpha_2^2}{k^2}+\frac{25\alpha_2^4}{k^4}\right)$ | | $-\frac{10}{3}\frac{\rho^2}{\alpha_2^2}$ | $-\frac{2}{3}\frac{\rho^2}{\alpha_2^2}$ |
| $\rho^4$ | | | $\frac{\alpha_2^7\alpha_0^3\rho^4}{6\,k^{14}}$ | $\frac{\alpha_0^3\alpha_2^7\rho^4}{12\,k^{14}}$ | $\frac{\rho^4}{4\alpha_1^4}$ | $-\frac{3}{3}\left(\frac{\alpha_1^5\alpha_2^7}{k^{12}}\right)\frac{\rho^4}{k^4}$ | $\frac{1}{6}\left(\frac{\alpha_1^5\alpha_2^5}{k^{10}}\right)\cdot\left(3-\frac{5\alpha_2^2}{k^2}\right)\frac{\rho^4}{k^4}$ | $\frac{4}{9}\frac{\rho^4}{\alpha_2^4}$ | $\frac{10}{9}\frac{\rho^4}{\alpha_2^4}$ | $\frac{7\rho^4}{36\alpha_2^4}$ |
| $\rho^6$ | | | | | | $\frac{1}{12}\left(\frac{\alpha_1^5\alpha_2^7}{k^{12}}\right)\frac{\rho^6}{k^6}$ | $\frac{1}{24}\left(\frac{\alpha_1^5\alpha_2^7}{k^{12}}\right)\frac{\rho^6}{k^6}$ | $-\frac{\rho^6}{9\alpha_2^6}$ | $-\frac{1}{9}\frac{\rho^6}{\alpha_2^6}$ | $-\frac{\rho^6}{36\alpha_2^6}$ |
| $\rho^8$ | | $k^2=\frac{1}{2}(\alpha_0^2+\alpha_1^2)$ | $k^2=\frac{1}{2}(\alpha_0^2+\alpha_2^2)$ | $k^2=\frac{1}{2}(\alpha_0^2+\alpha_2^2)$ | | $k^2=\frac{1}{2}(\alpha_1^2+\alpha_2^2)$ | $k^2=\frac{1}{2}(\alpha_1^2+\alpha_2^2)$ | $\frac{\rho^8}{144\alpha_2^8}$ | $\frac{\rho^8}{864\alpha_2^8}$ | $\frac{\rho^8}{576\alpha_2^8}$ |
| EXP | $\frac{-\rho^2}{2\alpha_0^2}$ | $\frac{-\rho^2}{\alpha_0^2+\alpha_1^2}$ | $\frac{-\rho^2}{\alpha_0^2+\alpha_2^2}$ | $\frac{-\rho^2}{\alpha_0^2+\alpha_2^2}$ | $\frac{-\rho^2}{2\alpha_1^2}$ | $\frac{-\rho^2}{\alpha_1^2+\alpha_2^2}$ | $\frac{-\rho^2}{\alpha_1^2+\alpha_2^2}$ | $\frac{-\rho^2}{2\alpha_2^2}$ | $\frac{-\rho^2}{2\alpha_2^2}$ | $\frac{-\rho^2}{2\alpha_2^2}$ |

Fig. 9. Ca 40 Form Factor. For explanation see text.

terms by themselves give the oxygen 16 form factor.

Putting the $\alpha$'s equal the form factor reduces to

$$F(\rho) = \left(10 + \frac{5}{4}\frac{\rho^4}{\alpha^4} - \frac{1}{6}\frac{\rho^6}{\alpha^6} + \frac{1}{64}\frac{\rho^8}{\alpha^8}\right)e^{-\frac{\rho^2}{2\alpha^2}}$$

I.L.Smith (15) has used the form factor to provide a comparison curve for his data from an experiment investigating the photo-production of neutron-proton pairs from calcium 40 using $\gamma$-rays in the energy range 200-300 Mev. Fig. 10. shows some of the results he obtained along with a theoretical fit in a graph which corresponds to a similar experimental situation to that of oxygen 16 in Fig.1.. At first glance the agreement seems reasonable but on closer inspection several flaws become apparent. The primary defect is the number of variable parameters we have at our disposal. The first is that normalisation is by eye. Secondly,a different parameter is used for each shell,the 1s and 1p shell parameters being those used for oxygen 16 in Fig.1. by J.Garvey et al. (9). Thirdly,these parameters yield a root mean square radius of 5.51 fm. which is considerably larger than the value of 3.47 fm. (17) obtained by more direct methods. The fact that the experimental errors are large is another unfortunate feature.

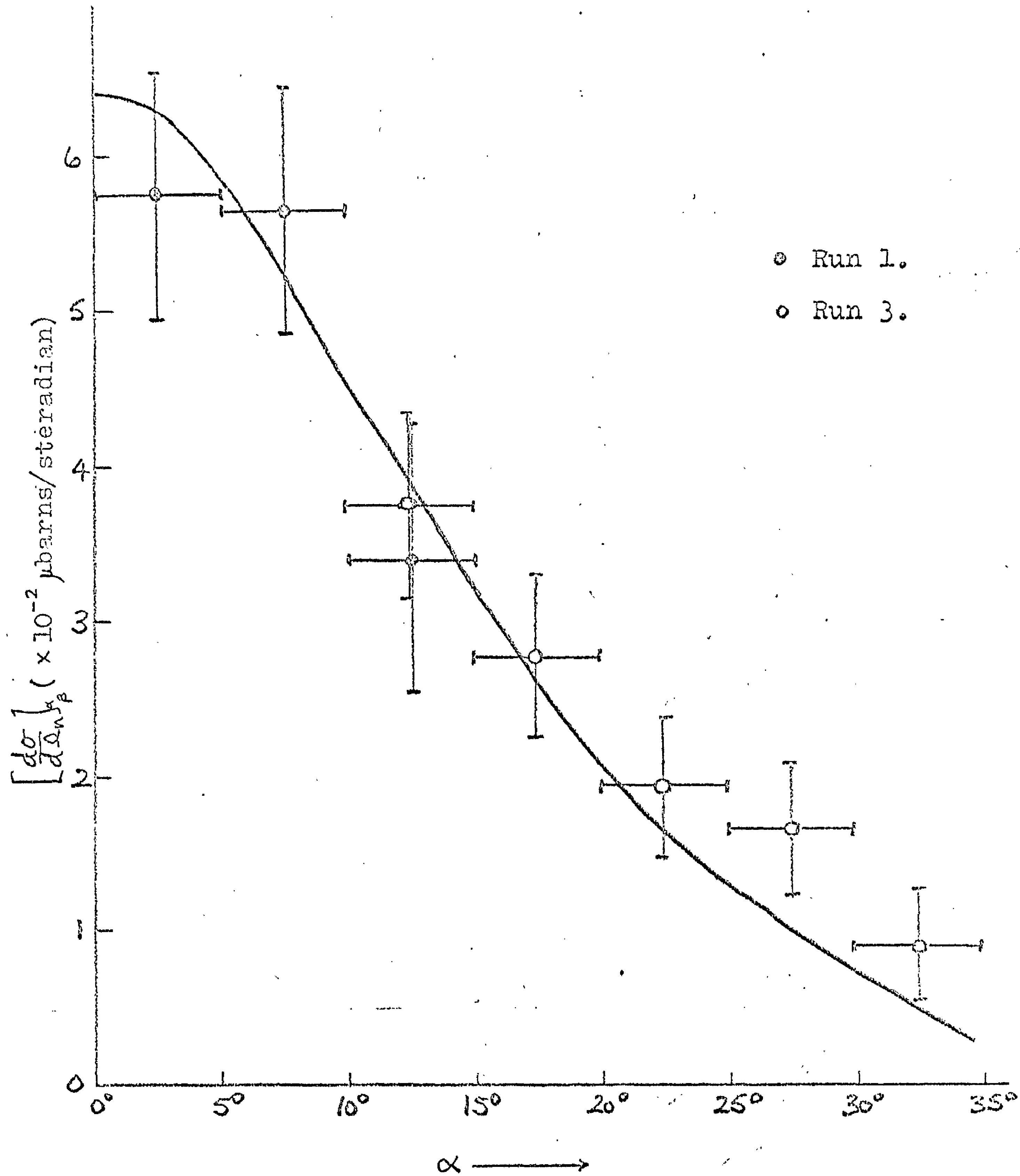



Fig. 10. Calcium: Comparison between experiment and theory. $\alpha_0 = 0.54 fm^{-1}$ $\alpha_1 = 0.32 fm^{-1}$ $\alpha_2 = 0.30 fm^{-1}$

## 2.6 Absorption correction.

To see if this situation could be improved upon the form factor was corrected for absorption by the same method that was used for the oxygen 16 case. The variation of shape of $F_{ab}(\rho, R_{ab})$ with Rab is shown in Fig. 11 with the curves normalised at $\alpha = 0^o$ with the normalising factor used indicated. It should be noted that even with Rab as large as 3.5fm. which is just greater than the root mean square radius the form factor is only down by a factor of 2.1 whereas the corresponding factor for a similar situation for oxygen was 3.2 . The method therefore indicates that we require a proportionally larger Rab for calcium 40 than oxygen 16 to reduce the cross-section by a similar amount.This in turn implies that the photo-pair come from a region even closer to the surface of the nucleus and that absorption is playing an even bigger role than before. All of which we would expect.

Using the corrected form factor we get a comparison with experiment as shown in Fig. 12. At the very least the agreement is as good as before but there are two latent improvements: only one parameter is used for the oscillator wave functions and this has been fixed by requiring that it produces the root mean square radius of 3.45 fm. of other measurements. Unfortunately,like the oxygen case, lack of knowledge about the magnitude

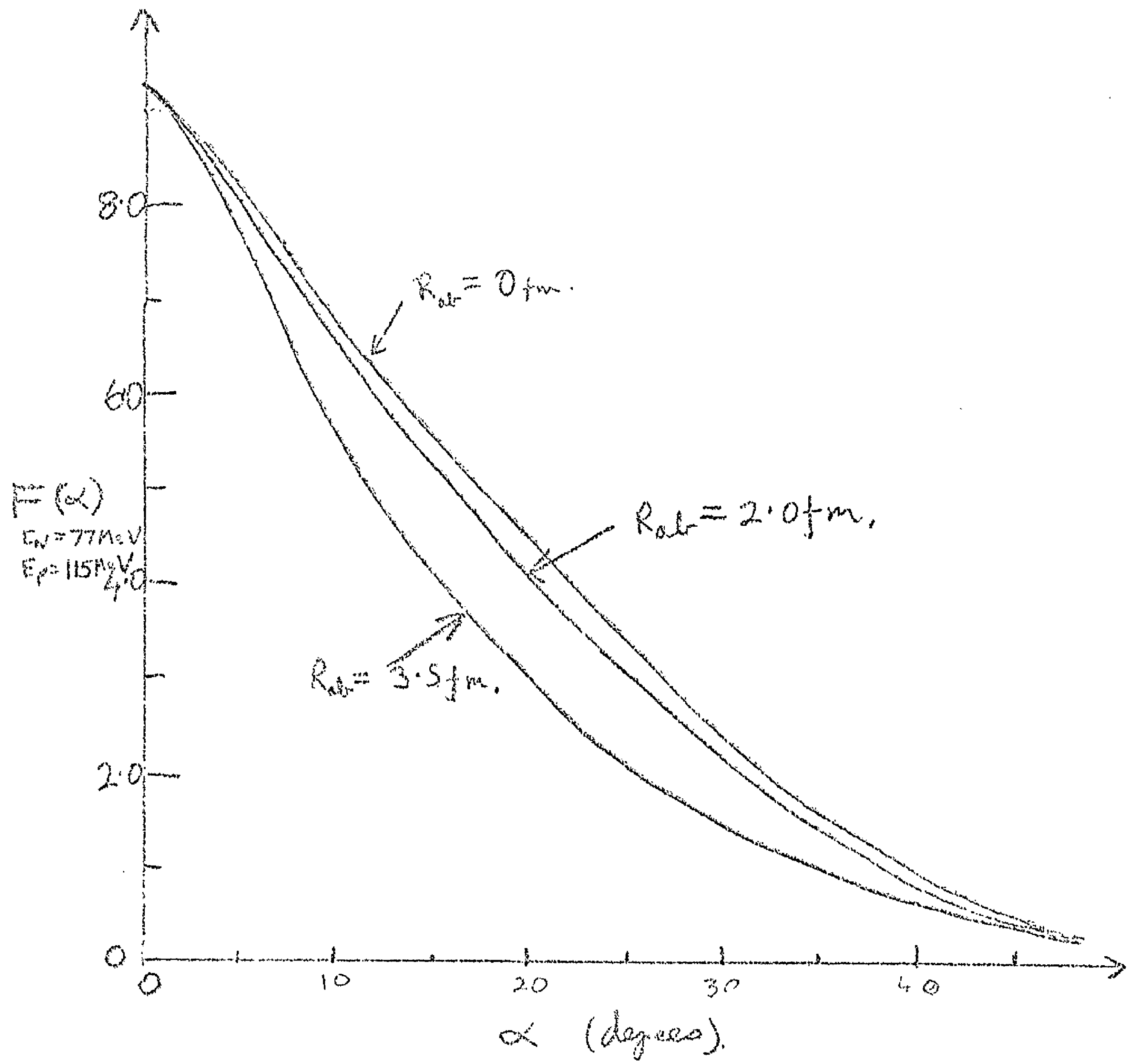
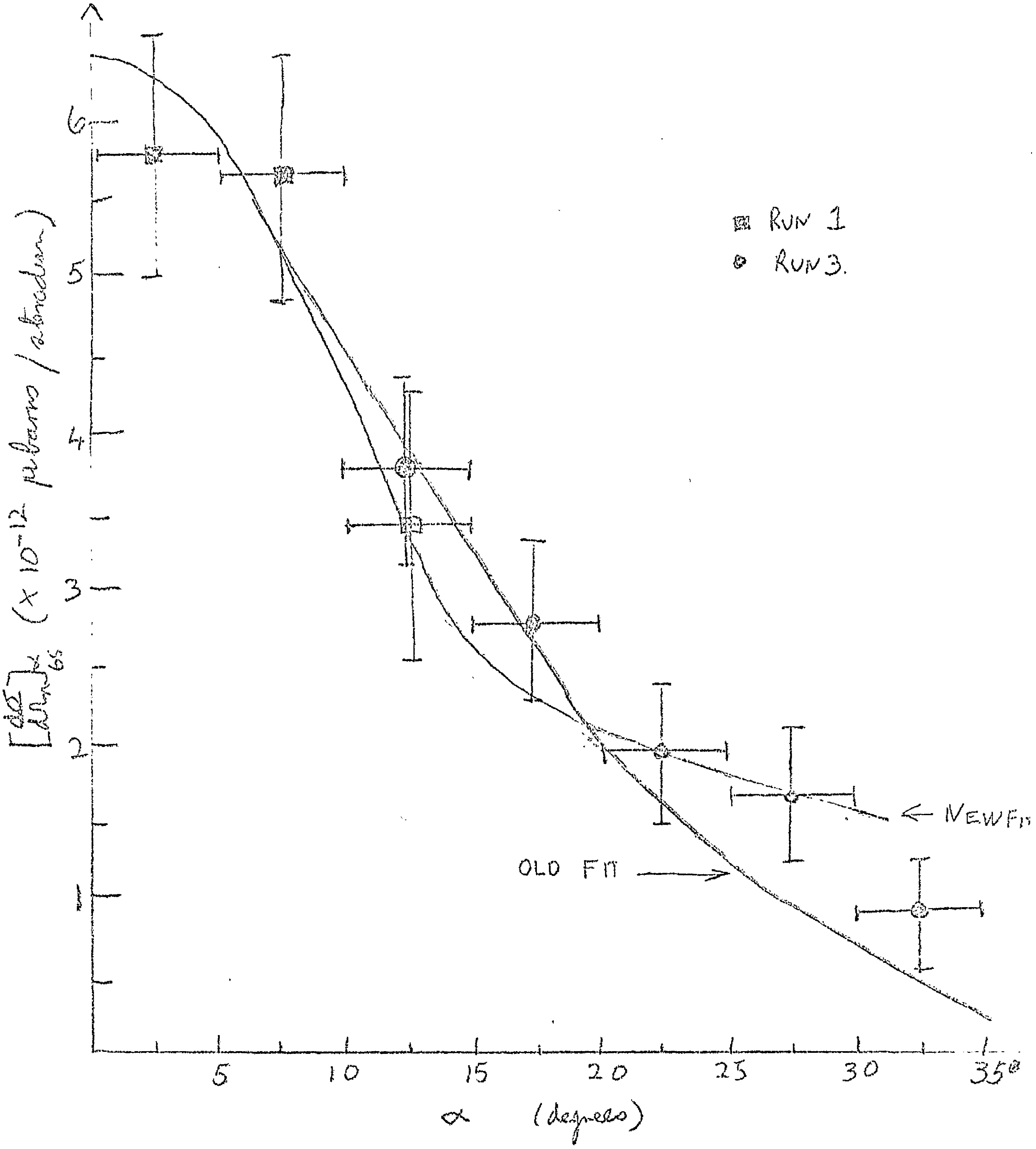



FIG.11. This illustrates the variation in shape of F (P,R ) for Ca 40 with $R_{ab}.\alpha$ =0.497 $fm^{-1}$ Curves (b) and (c) have been multiplied by 1.19 and 2.08 resp.




<u>FIG.12.</u> A theoretical fit to the points of Fig.10 using the absorption corrected form factor with $R_{ab}$=3.5 fm Normalisation is by eye. $\alpha$ =0.497 $fm^{-1}$

of the absorption and the large experimental errors means that the normalisation problem is present here and Rab is undefined.

Gottfried's absorption correction factor $f_a$ when evaluated for calcium, turns out to be 0.20 which would mean an Rab $\sim$ 4.0 fm. which is 0.55 fm. larger than the root mean square radius.

The other feature of the method reflecting the stronger absorption, of the inner, shells is also present and more noticeable. For Rab = 3.5 fm. the 1s shell contribution to $F(\rho, R_{ab})$ being down to one or two per cent of its original value while the 2s and 1d shell contributions are still approximately three-quarters of what they were. In this case also therefore, correcting for absorption in this way seems worthwhile .

## 2.7 Discussion and conclusions.

The work, which has been described here, tends to confirm the view that the quasi-deuteron model and in particular Gottfried's version of it is basically sound. As a result of this, the assumption that the neutron and proton are very close together before emission takes place becomes more acceptable although a more detailed and satisfactory explanation of the emission process is desirable. The extension of the analysis to include non-closed shell nuclei would also be worth investigating in order that a more rigorous test of the theory could be carried out by increasing its applicability.

The comparison with experiment in the present work has many shortcomings. As has been mentioned already, there is too much freedom in the choice of parameters and normalisation. A side effect of this is that by arrangement the curves tend to fit the experimental points for low $\alpha$ but show disagreement for large $\alpha$. This situation tends to obscure the effect at these angles of the large value of $\Delta$ which implies that we might not be justified in assuming proportionality of the transition amplitude for deuteron photo-disintegration and the one for the photo-emission of neutron-proton pairs in the nuclear case.

Another unknown factor is how good an approximation the form factor derived from harmonic oscillator wave functions is to one derived from more realistic nuclear wave functions. Gottfried suggests that they should be good enough for most applications. Similarly, cutting a hole out of the centre of the nucleus to account for absorption although yielding interesting results, is also an artificial procedure.

Shortcomings on the experimental side are that the energy of the photons used was in the range 200-300 Mev so that for some of them Gottfried's assumptions might not be valid, and the more obvious defect, that there are large errors in the points.

In conclusion, it seems that more accurate experiments are required so that flaws in the theory can not hide behind large experimental errors.

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## REFERENCES


(1) K.A.Brueckner,R.J.Eden and N.C.Francis Phys. Rev. 98 (1955) 1445.

See this paper for other references.

(2) J.S.Levinger Phys. Rev. 84 (1951) 43.

(3) M.Q.Barton and J.H.Smith Phys. Rev. 95 (1954) 573.

(4) J.W.Weil and B.C.McDaniel Phys. Rev. 92 (1953) 391.

(5) A.C.Odian,P.C.Stein,A.Wattenberg and B.T.Feld Phys. Rev. 102 (1956) 837.

A.Wattenberg,A.C.Odian,P.C.Stein and H.Wilson Phys. Rev. 104 (1956) 1710.

(6) K.A.Dedrick Phys. Rev. 100 (1955) 58.

(7) K.Gottfried Nuc. Phys. 5 (1958) 557.

(8) A.Reitan Nuc. Phys. 64 (1965) 113.

E.Ostgaard Nuc. Phys. 64 (1965) 289.

See these papers for other references.

(9) J.Garvey,B.H.Patrick,J.G.Rutherglen and I.L.Smith Nuc. Phys. 70 (1965) 241.

(10) S.Fernbach,R.Serber and T.B.Taylor Phys. Rev. 75 (1949) 1352.

(11) P.C.Stein,A.C.Odian,A.Wattenberg and R.Weinstein Phys. Rev. 119 (1960) 348.

(12) T.Berggren and G.Jacob Nuc. Phys. 47 (1963) 481.

(13) N.MacDonald Private communication.

(14) S.Fujii Nuv. Cim. 25 (1962) 995.

(15) D.H.Wilkinson and M.E.Mafethe Nuc. Phys. 85 (1966) 97.

(16) I.L.Smith Thesis, University of Glasgow, 1966.

(17) R.R.Shaw Proc. Roy. Soc. 86 (1965) 53.