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ON THE RELATION BETWEEN THE STREAM AND THE ELLIPSOID CONSTANTS

D.G.Ewart
(Communicated by the Director, University Observatory, Glasgow.)

Summary.

The mean speeds, both in the positive and the negative directions, along and perpendicular to the axis of star-streaming are derived, both on the two-drifts and the ellipsoidal theories of the distribution of stellar linear velocities, the origin of velocity components being the centre of rest of all stars.

It is then found that, if the number of stars belonging to each drift are equal, it is possible to calculate the ratio of the axes of the velocity ellipsoid from the relative speeds of the two drifts.

The relation thus obtained is applied to the results of a number of analyses of proper motions and the calculated values of the axis-ratio are compared with the corresponding values derived from the analyses of the same material on the ellipsoidal hypothesis.

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1. An important feature of the results of analyses of the proper motions of the stars is the agreement in the direction of the vertex of star-streaming as found by the two mwthods of analysis - the Two-Drifts and the Ellipsoidal. As the two methods of analysis are very different - thw two-drifts analyses generally being performed by fitting theoretical curves to the observed distributiom in individual areas of the sky and the ellipsoidal analyses by a purely numerical method - the importance of the agreement is enhanced. It is therefore to be expected that there should exist some relationship, possibly only approximate, between certain of the parameters of the two distributions.

One such relation, which was derived by Smart (1) has been used with success in a number of practical applications. (2,3,4).

In theory, the use of Smart's relation is valid only when applied to restricted proper motions (those greater than a given value of μ) and when the following conditions are satisfied:-

(a) the space distribution of the stars is given by a density law of the form $N(r) = A\exp(-h^2k^2r^2)/r$, A, h, k being constants.,

(b) the ratio of k to μ is small

and (c) the stars are equally divided between the two-drifts.

It must also be noted that the relation only applies to a small region of the sky. To obtain the constants of the velocity ellipsoid for the whole sky, the results for each region must first be combined. The manner in which this may be accomplished has been described by Smart (2) and is basically one of succesive approximations to the true values of the unknowns from initially assumed values of the unknowns. In this paper an attempt is made to derive a more general relation.

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2. In the two-drifts theory, the stars are divided into two assemblies, for each of which the distribution of linear velocitie is random. The wto drifts are intermingled in space and their centres of rest are in relative motion along the axis of star-streaming, this latter axis defining the direction of the vertex.

From the centre of rest of all stars, the axis of star-streaming appears as an axis of greater mobility than in any other direction. In Schwarzschild's ellipsoidal theory, this feature is expressed analytically. The exponent of the distribution is of the form of the quadratic expression appearing in the equation of an ellipsoidax of revolution, and the mean velocity component in the direction of the vertex is greater than in any other direction.

Throughuot this paper, the following forms of the distribution functions will be used. For a single drift:-

$$\pi^{3/2} f(u,v,w) = nh^3 \exp(-h^2\{u^2+v^2+w^2\}),$$

the origin of velocity components being the centre of rest of the drift, u, v, and w being the velocity components, n the number of stars belonging ti thr drift and $h(= 1/s\sqrt{2})$ a constant, s being the mean velocity dispersion of stars in the drift. Thus for two drifts, we have:-

$$\pi^{3/2} f_\delta(u,v,w) = n_1 h_1^3 \exp(-h_1^2\{u_1^2+v_1^2+w_1^2\}) + n_2 h_2^3 \exp(-h_2^2\{u_2^2+v_2^2+w_2^2\})$$

where $n_1 u_1 v_1 w_1 h_1$ and $n_2 u_2 v_2 w_2$ and h_2 refer to Drift's I and II respectively. As the distribution of velocities in each drift is random, we may orient the axes in any convenient manner. We will take the u_1 and u_2 axes to be along the axis of star-streaming, found, in practice, to be in, or near, the galactic plane. The other axes are perpendicular to these, the w_1 and w_2 axes being approximately in the direction of the north gakactic pole.

We now take new axes (U,V,W) referred to the centre of rest of all stars and parallel to the u_1, v_1 and w_1 axes. Then, if the speeds of Drift I and Drift II, relative to the centre of rest of all stars, are Q_1 and Q_2 respectively, we have:-

$$n_1 Q_1 = n_2 Q_2 \tag{2,1}$$

and,

$$\left.\begin{aligned} u_1 &= U - Q_1, & v_1 &= V, & w_1 &= W \\ u_2 &= U + Q_2, & v_2 &= V, & w_2 &= W \end{aligned}\right\} \tag{2,2}$$

The distribution fmnction for the two-drifts theory then becomes:-

$$\pi^{3/2} F_1(U,V,W) = n_1 h_1^3 e^{-h_1^2(U-Q_1)^2 - h_1^2(V^2+W^2)} + n_2 h_2^3 e^{-h_2^2(U+Q_2)^2 - h_2^2(V^2+W^2)} \tag{2,3}$$

For the ellipsoidal theory the distribution function, referred to the same axes, is:-

$$\pi^{3/2} F_2(U,V,W) = nKH^2 e^{-K^2U^2 - H^2(V^2+W^2)} \tag{2,4}$$

where $n_1 + n_2 = n$ and $K < H$.

3. Let the number of stars on the two-drifts theory, with U 0, be $N' = N_1' + N_2'$, where N_1' is the number of stars belonging to Drift I and N_2' is the corresponding number for DriftII, for all values of V and W.

$$\therefore N_1' = \frac{n_1 h_1^3}{\pi^{3/2}} \int_0^{\infty} e^{-h_1^2(U-Q_1)^2} dU \iint_{-\infty}^{\infty} e^{-h_1^2(V^2+W^2)} dV dW.$$

Smart's values, except for the case of ALL stars, where the doubly erroneous 'old' values are not listed.

Zone	No.	1/K	1/H	K/H	G_o
0^o	167	37.78	32.24	0.853	$298^o.1$
		38.89	32.56	0.837	292 .5
	(619)	(33.70)	(20.90)	(0.619)	(340 .5)
20^o	277	39.63 ± 1.78	31.73 ± 2.81	0.801 ± 0.159	$327^o.9 ± 10^o.1$
		41.45	26.11	0.630	324 .4
	(981)	(30.10)	(26.40)	(0.879)	(332 .2)
45^o	296	53.63 5.89	19.17 7.48	0.357 0.165	$344^o.6$ $11^o.1$
		53.52	17.94	0.336	343 .7
	(1068)	(37.20)	(22.10)	(0.594)	(338 .8)
ALL	820	40.98 4.61	27.74 2.32	0.677 0.117	$331^o.4$ $6^o.4$
	(2668)	(33.70)	(23.10)	(0.697)	(339 .5 1.4)

3. As was noted in the previous paper, the only solution to which any weight can be attached is that for all stars. Again the value of the axis-ratio of the velocity ellipsoid agrees well with Smart's determination, but the velocity dispersions along the axes of the velocity ellipsoid are greater for the faint stars. This is to be expected both because of the lesser amuont of the material available for analysis and because of the greater probable errors of the solar motion. For the longitude of the major axis of the ellipsoid, the resultx is inconclusive as regards the vertex deviation because of the large probable error.

Summarising the revised results, it appears that although there are no definite indications of variation in the ellipsoidal constants between the brighter and the fainter stars - excepting the increased dispersions along the axes - there is a variation in the solar motion, in velocity and in the galactic longitude og the apex.

4. My thanks are due to Dr. T.R.Tannahill for caquainting me of the prescence of the error in the earlier analysis and to the Department of Scientific and Industrial Research for the award of a Maintainance Grant, during the tenure of which both analyses were performed.

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NOTE CONCERNING " THE CONSTANTS OF THE VELOCITY ELLIPSOID FROM THE RADIAL VELOCITIES OF 820 FAINT STARS."

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(Communicated by the Director, University Observatory, Glasgow.)

Summary.
Erroneous values of the solar motion and of the constants of the velocity ellipsoid published in a previous paper are corrected. The revised results indicate no definite variation in the constants of the velocity ellipsoid for the faint stars, but the variation in the solar motion is confirmed.

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1. In a previous paper (1), the derivation of the constants of the solar motion and of the velocity ellipsoid from the radial velocities of a sample group of faint stars - H.D. magnitudes 8.5 to 8.6, spectral types F to M - obtained at Lick Observatory by Moore and Paddock was described. It was recently brought to my notice by Dr. T.R.Tannahill that the solution for the ellipsoid constants for ALL stars was incorrect, the figures given not being the means of the values for the three zones. On examination, it was found that it had been assumed that the equations could be solved for the whole sky by taking the mean galactic latitude as 0°, which procedure is fallacious.

A new solution was consequently required, and it was decided to re-check the previous solutions simultaneously. In so doing, two errors of sign were discovered in the solution for the solar motion. A fresh analysis was therefore performed, the details and results of which are given below.

2. No changes were made in the methods of analysis. For the solar motion, the normal equations were fromed. For the calculation of the areal corrections, their approximate solutions were found to be:- $U = 21.05$ km./sec., $G = 41^{\circ}$, $g = +22^{\circ}$ and $K = +1.0$ km./sec. The complete solutions of the normal equations - including the corrections - were then found to be:-
$U = 22.31 \pm 1.20$ km./sec., $A = 280^{\circ}.0 \pm 5^{\circ}.8$, $D = +47^{\circ}.9 \pm 2^{\circ}.8$, $G = 44^{\circ}.3 \pm 3^{\circ}.5$, $g = +20^{\circ}.4 \pm 3^{\circ}.1$ with a K-term of $+1.17 \pm 0.79$ km./sec. The weighted mean results of Moore and Paddock are $U = 22.44$ km./sec. $A = 280^{\circ}$, $D = +44^{\circ}.5$.

The comstants of the velocity ellipsoid were again determined separately for the latitude zones 0°, 20° and 45°. For the whole sky solution, the iterative method of solution described by Smart and Chandrasekhar (2) was used, the initial values being the means of the zone results. The results, together with those of the previous solutic and those of Smart's (3) analysis of the radial velocities of Schkesinger's "Catalogue of Bright Stars, 1930" are given in the table below. It must be noted that the values for the whole sky from Smart's analysis are means of the three zomes - weighted for the longitude of the major axis. In the table, the first line gives the revised values of the constants, the second the 'old' values and the third ~~xxx~~ line

It can easily be seen that on putting $\rho = \lambda = 0$ in the above equations, then (8,19) reduces to (5,11), and (8,20) to H = J. Also $e^{-\lambda^2}\lambda\sqrt{\pi}\,\Theta(\lambda)$ is greater than one for λ greater than nothing. Thus if the direction of the vertex differs from one distribution to the oth then, even if H = J, the axis-ratio K/H is greater than if the direc ions were identical. If the differences in direction are - in galact co-ordinates - ΔG in longitude and Δg in latitude, then:-

$$\frac{\lambda}{(\tau^2 + \rho^2)^{1/2}} = \operatorname{Tan}\Delta g \quad \text{and} \quad \frac{\rho}{\tau} = \operatorname{Tan}\Delta G\,.$$

$$\therefore \quad \rho = \tau\operatorname{Tan}\Delta G \ , \ \lambda = \tau\operatorname{Sec}\Delta G.\operatorname{Tan}\Delta g \qquad (8,21\text{-}22)$$

The differences in either direction rarely exceed 5° in either co-ordinate, and are usually of the same order of magnitude as the probable errors in the co-ordinates. For $\Delta G = \Delta g = 5^{\circ}$ we have, $\rho = 0.09\tau$ and $\lambda = 0.09\tau$. The average value of τ for the first 15 values in Table II is 0.912, which gives K/H = 0.577. If the directions given by each method of analysis differed by 5° in each co-ordinate, then τ becomes 0.905 instead of 0.912, and K/H becomes 0,584 instead of 0.577. The effect is thus nagligible. Therefore, unless there exists a marked difference of direction between the results of the methods of analysis, the effect on the axis-ratio of the 'equivalent' ellipsoid to the two-drifts analysis is insignificant. This i especially the case when the probable errors are considered.

The extent to which the results depend on the assumed equality of the velocity dispersions of the two drifts is, however, unknown. Analyses of proper motions by themselves do not permit of t evaluation of these dispersions.

9 Acknowledgements. It is a pleasure to tender my thanks to Profes W.M.Smart for suggesting this investigation and for his advice whils it was in progress. My thanks are also due to the Department of Scientific and Industrial Research for the award of a Maintainance Grant, during the tenure of which the investigation was performed.

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where $\tau_1 = h_1 Q_1$ and $\tau_2 = h_2 Q_2$. Similarly, if N_1 is the number of stars with $U<0$, for all values of V and W, we have:-

$$2 N_1 = n_1\left(1 - \Theta(\tau_1)\right) + n_2\left(1 + \Theta(\tau_2)\right) \qquad (7,4)$$

In a similar amnner we can obtain, if N'' is the number of stars with $V>0$, for all values of U and W; $N_{''}$ the number of stars with $V<0$, for all values of U and W; N''' the number of stars with $W>0$, for all values of U and V amd $N_{'''}$ the number of stars with $W<0$ for all values of U and V:-

$$2N'' = n_1[1 + \Theta(\rho_1)] + n_2[1 - \Theta(\rho_2)] \qquad (7,5)$$
$$2N_{''} = n_1[1 - \Theta(\rho_1)] + n_2[1 + \Theta(\rho_2)] \qquad (7,6)$$
$$2N''' = n_1[1 + \Theta(\lambda_1)] + n_2[1 - \Theta(\lambda_2)] \qquad (7,7)$$
$$2N_{'''} = n_1[1 - \Theta(\lambda_1)] + n_2[1 + \Theta(\lambda_2)] \qquad (7,8)$$

where $\rho_1 = h_1 R_1$, $\rho_2 = h_2 R_2$, $\lambda_1 = h_1 S_1$ and $\lambda_2 = h_2 S_2$.

8. We now derive the mean speeds along the positive and negative directions of each axis. Letting these be U', U_1, V', V_1, W' and W_1, the subscrpts referring to the negative directions, we obtain;- in the notation of section 4,

$$U' = \phi/\psi, \quad U_1 = \frac{\phi}{\psi - 2G(\tau_1,\tau_2)}, \quad V' = \phi/\psi, \quad V_1 = \frac{\phi}{\psi - 2G(\rho_1,\rho_2)}, \quad W' = \frac{\phi}{\psi}, \quad W_1 = \frac{\phi}{\psi - 2G(\lambda_1,\lambda_2)} \qquad (8,1\text{-}6)$$

Since we have the relations

$$\frac{n_1^2(\tau_1^2+\rho_1^2+\lambda_1^2)}{h_1^2} = \frac{n_2^2(\tau_2^2+\rho_2^2+\lambda_2^2)}{h_2^2} \qquad (8,7)$$

and, as the drifts are moving in opposite directions,

$$\tau_1 = A\tau_2, \quad \rho_1 = A\rho_2, \quad \lambda_1 = A\lambda_2, \qquad (8,8)$$

so, from (8,7) and (8,8), we obtain,

$$\tau_1 = \alpha\beta\tau_2, \quad \rho_1 = \alpha\beta\rho_2, \quad \lambda_1 = \alpha\beta\lambda_2, \qquad (8,9)$$

where $\alpha = n_2/n_1$ and $\beta = h_2/h_1$.

The corresponding ~~e~~expressions on the ellipsoidal theory are:-

$$U' = U_1 = 1/K\sqrt{\pi}, \quad V' = V_1 = 1/H\sqrt{\pi}, \quad W' = W_1 = 1/J\sqrt{\pi} \qquad (8.10\text{-}12)$$

The condition for relaring the two theorues is once again,

$$G(\tau_1,\tau_2) = 0, \quad G(\rho_1,\rho_2) = 0, \quad G(\lambda_1,\lambda_2) = 0 \qquad (8.13\text{-}15)$$

As we ahve to take $h_1 = h_2 = h$(say), this being assumed in the two-drmfts analyses, the conditions (8,13) to (8,15) reduce to;-

$n_1 = n_2 = n/2$, $\tau_1 = \tau_2 = \tau$(say), $\rho_1 = \rho_2 = \rho$(say) and $\lambda_1 = \lambda_2 = \lambda$(say).

We thus have:-

$$\frac{1}{K} = \frac{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)}{h}, \quad \frac{1}{H} = \frac{e^{-\rho^2} + \rho\sqrt{\pi}\,\Theta(\rho)}{h}, \quad \frac{1}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{h} \qquad (8,16\text{-}18)$$

The ratios of the axes are thus, taking the polar axis to be the least:-

$$\frac{K}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)} = \frac{M(\tau)}{M(\lambda)} \qquad (8.19)$$

$$\frac{H}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{e^{-\rho^2} + \rho\sqrt{\pi}\,\Theta(\rho)} = \frac{M(\rho)}{M(\lambda)}. \qquad (8.20)$$

From the figures in the above table, it can thus be seen that the values of K H derived by the two methods agree well, especially the probable errors of the determinations are considered. A point of some interest is that quite good agreement is obtained even when the ratios of the numbers of stars belonging to the two drifts depart considerably from the assumed value of unity. This especially is the case for the values from the second volume of the Cape Astrographic Zone catalogues. In the paper describing the analyses of these proper motions, it is, however, stated that these abnormally hmgh values are probably spurious, being occasioned by the uncertainties in the determinations of the drift constants, especially those of Drift II. The above results would seem to confirm this - that is, the drift ratio for the stars of this catalogue is probably close to unity, as was the case for the first catalogue.

The belief that the stars are equally divided between the two drifts therefore, perhaps, receives some support from these results. Unfortunately, as oit is not possible to determine the probable errors of the values of the ratios of the numbers of stars in the two drifts, this question cannot be settled. The results of some recent investigations have suggested that the ratio is higher for the early type stars, of types B8 - F5, than for the later types, for which it is nearly unity. The higher values for the earlier type stars, however, may only be the effects of errors.

7. The analysis given in the preceding sections was based on the assumption that the direction of the major axis of the velocity ellipsoid was identical with the diretion of relative motion of the two drifts. The results of the application of the relation derived on this basis indicate that there is little difference between the observed and the 'equivalent' ellipsoids.

In this section, the analysis in the preceding sections is extended to consider the effects of a divergence between the two directions.

As in the previous analysis, the forms of the distribution functions that will be used are those referred to axes with origin at the centre of rest of all stars. We will take these axes to lie along the three axes of the velocity ellipsoid. Then for the velocity ellipsoid, we now have:-

$$\pi^{3/2}F_1(U,V,W) = nKHJ\exp(-K^2U^2-H^2V^2-J^2W^2) \qquad (7,1)$$

This is a generalised form of the function used in the previous sections. For the two-drifts theory, the distribution function will now be:-

$$\pi^{3/2}F_2(U,V,W) = n_1 h_1^3\exp[-h_1^2(U-Q_1)^2+(V-R_1)^2+(W-S_1)^2] + n_2h_2^3\exp[-h_2^2(U+Q_2)^2+(V+R_2)^2+(W+S_2)^2] \qquad (7,2)$$

Then, if N' is the number of stars with U O, for all values of V and W, on the two-drifts theory, we have:-

$$N' = \frac{n_1h_1^3}{\pi^{3/2}}\int_0^{\infty}e^{-h_1^2(U-Q_1)^2}dU\iint_{-\infty}^{\infty}e^{-h_1^2\{(V-R_1)^2+(W-S_1)^2\}}dVdW + \frac{n_2h_2^3}{\pi^{3/2}}\int_0^{\infty}e^{-h_2^2(U+Q_2)^2}dU\iint_{-\infty}^{\infty}e^{-h_2^2\{(V+R_2)^2+(W+S_2)^2\}}dVdW$$

$$\therefore 2N' = n_1(1+\Theta(\tau_1)) + n_2(1-\Theta(\tau_2)) \qquad (7,3)$$

Ko - M, F - M the group A5 - M and ALL normally is the group B8 -M. In the third column is given the value of the ratio of the numbers of stars belonging to each drift - in the sense N_1 to N_2. In the fourth, fifth and sixth columns are given, respectively, the values, together with their probable errors, where available, of the relative speeds of the drifts, the observed values of K/H, and the calculated values of K/H. For the two-drifts analyses, the values of 2τ are listed, this being the usually derived quantities.

The following sources have been used in compiling this table.

Source	Designation.	Investigators. Two-Drift		Investigators. Ellipsoidal	
Boss Preliminary General Catalogue	P.G.C.	Eddington Jones	(5) (6)	Raymond	(7)
Boss General Catalogue.	G.C.	Tannahill	(8)	Delhaye	(9)
Cape Astrographic Zone - Volume I	C.1.	Smart and Tannahill	(3,4)	Jackson	(10)
" Volume II	C.2.	Ewart	(11)	Ewart	(11)
Cape Photographic Zone Catalogues (Zones -30° to -48°)	C.3.	Ewart and Lourens	(12)	Ewart and Lourens	(12)

Also listed for the C.1. group are the values calculated by Smart and Tannahill from Smart's formula. It must be mentioned here that the groupings used by the various investigators of the P.G.C and the G.C. motions are not identical.

TABLE II

Source.	Sp. Group	N_1/N_2	2τ		K/H (obs)		K/H (calc)		Smart Form.
P.G.C.	ALL	1.5	1.868		0.52		0.567		
P.G.C.	ALL		1.924	±0.135	0.48		0.554	±0.031	
P.G.C.	ALL		1.962	0.053	0.52		0.546	0.011	
G.C.	F	1.22	2.075	0.039	0.574		0.519	0.008	
G.C.	K	1.02	1.638	0.038	0.67		0.623	0.010	
G.C.	F-M	1.19	1.816	0.030	0.62		0.579	0.007	
C.1.	F	1.17	1.873	0.05	0.552		0.565	0.012	0.62
C.1.	G	1.04	1.756	0.05	0.56		0.591	0.024	0.63
C.1.	K	1.08	1.606	0.06	0.65		0.632	0.015	0.67
C.1.	ALL	1.41	1.638	0.04	0.625		0.623	0.011	0.70
C.2.	ALL	1.71	1.648	0.04	0.602	±0.03	0.621	0.009	
C.2.	F	1.78	1.639	0.04	0.676	0.05	0.622	0.012	
C.2.	G	1.47	1.697	0.08	0.554	0.04	0.608	0.020	
C.2.	K	1.40	1.601	0.08	0.606	0.03	0.633	0.020	
C.2.	F-M	1.50	1.618	0.10	0.601	0.03	0.629	0.030	
C.3.	F	1.19	1.195	0.15	0.775	0.030	0.748	0.045	
C.3.	G	0.97	1.040	0.12	0.751	0.015	0.788	0.034	
C.3.	K	1.00	0.820	0.11	0.898	0.021	0.859	0.031	
C.3.	F-M	0.99	0.985	0.12	0.825	0.024	0.811	0.035	

two drifts. It must be noted that this relation does not imply that an analysis of proper motions on the ellipsoidal theory will yield an axis-ratio equal to that given by (5,11) from a two-drifts analysis of the same material. The value given by (5,11) is the axis-ratio of the velocity ellipsoid that will yield the same values for the mean speeds along the axis of star-streaming, that is, it is the axis-ratio of the ellipsoid 'equivalent' to the two-drifts solution.

As with Smart's formula, it is strictly only applicable when the stars are equally divided between the two drifts. The relation is not, however, limited to a region of the sky, as is Smart's formula, but applies to the whole sky. It is easily shown, however, by a similar analysis, that the relation also applies to a region of the sky. Here, if hV_1, θ_1, n_1; hV_2, θ_2, and n_2 are thedrift constants of a region, then the local value of τ, τ' (say), is given by:-

$$2\tau' = \sqrt{(hV_1 \operatorname{Sin}\theta_1 - hV_2 \operatorname{Sin}\theta_2)^2 + (hV_1 \operatorname{Cos}\theta_1 - hV_2 \operatorname{Cos}\theta_2)^2} \qquad (5,12)$$

By using the local value of τ' derived above in (5,11), the local value of the axis-ratio, usually written as k/h, can be found. The position angle of the vertex in the region can also be determined from the local drift constants, as, if it is θ_V, then:-

$$\operatorname{Tan}\theta_V = \frac{hV_1 \operatorname{Sin}\theta_1 - hV_2 \operatorname{Sin}\theta_2}{hV_1 \operatorname{Cos}\theta_1 - hV_2 \operatorname{Cos}\theta_2} \qquad (5,13)$$

Thus, from (5,12) and (5,13), we may calculate the constants of the velocity ellipsoid from the drift analysis of a region, it being assumed that there are equal numbers of stars in each drift. These can then be combined directly to give the vertex direction and the axis-ratio of the ellipsoid, without recourse to the metthod of successive approximations given by Smart.

It also may be noted that if the two-drifts and the ellipsoidal distributions are to have the same axes, then Schwarzschild's form of the ellipsoidal theory must be used. This follows from (5,7), (5,8) and (5,9).

6. In Table I, below, the values of the function $M(\tau)$ are given for a range of values of τ, the range covering the values obtaining in most investigations. In Table II, the values of K/H, calculated

TABLE I

τ	K/H	τ	K/H	τ	K/H	τ	K/H	τ	K/H
0.0	1.0000	0.3	0.9178	0.6	0.7463	0.9	0.5827	1.2	0.4602
0.1	0.9902	0.4	0.8652	0.7	0.6880	1.0	0.5372	1.3	0.4280
0.2	0.9618	0.5	0.8064	0.8	0.6331	1.1	0.4964	1.4	0.3994

from equation (5,11), for a number of two-drifts solutions, are compared with the values of K/H, derived from analyses of the same material, by Schwarzschild's automatic method. The frist column in the table gives the sources of the figures in each row. The second column gives the spectral group involved. In this column, the letter F represents the grouping A5 - F5, G the group F8 - G5, K the group from equating N8 to N'' and U' to U'' on the two-drifts theory. From (3,8) we have -

$$N'' = N' - G(\tau_1, \tau_2)$$

whence, for N' to equal N'', we must have -

$$G(\tau_1, \tau_2) = 0 \qquad (5,1)$$

Also from (4,4) and (4,5), for U' to equal U'', we must again have the above condition. This is thus the primary condition for relating the two theories. Writing it out fully, it is:-

$$n_1 \Theta(\tau_1) = n_2 \Theta(\tau_2) \qquad (5,2)$$

Substituting for in (5,2) from (2,1) and putting $\alpha = n_1/n_2$ and $\beta = h_2/h_1$ we obtain:-

$$\alpha\,\Theta(\tau_1) = \Theta(\alpha\beta\tau_1) \qquad (5,3)$$

Now in all practical applications of the two-drifts theory it is assumed that $h_1 = h_2 = h$(say), where $1/h$ is usually defined to be the theoretical unit of velocity. Hence in (5,3) we may write $\beta=1$. Then, on expanding the function $\Theta(\tau_1)$, (5,3) becomes:-

$$\alpha \int_0^{\tau_1} e^{-x^2}dx = \int_0^{\alpha\tau_1} e^{-y^2}dy$$

We now put $x = \tau_1 T$ and $y = \alpha\tau_1 t$. The condition now becomes:-

$$\alpha\tau_1 \int_0^1 e^{-T^2}dT = \alpha\tau_1 \int_0^1 e^{-\alpha^2\tau_1^2 t^2}dt$$

$$\therefore \alpha = 1 \qquad (5,4)$$

Thus the condition governing the relation of the rwo hteories is that, as h_1 is taken to be equal to h_2, we must have $n_1 = n_2$ and hence, from (2.1), $\tau_1 = \tau_2 = \tau$ (say). We then have

$$U' = U'' = \frac{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)}{h\sqrt{\pi}} \qquad (5,5)$$

and

$$N' = N'' = n/2 \qquad (5,6)$$

Now the number of stars which, for all values of U, have velocities between V and V+dV and W and W+dW, on the two-drifts theory is:-

$$\eta = \frac{n_1 h_1^2}{\pi} e^{-h_1^2(V^2+W^2)}dV\,dW + \frac{n_2 h_2^2}{\pi} e^{-h_2^2(V^2+W^2)}dV\,dW.$$

But we have just shown that in order to relate this result to the corresponding result given by the ellipsoidal theory, we must take $h_1 = h_2 = h$ and $n_1 = n_2 = n/2$.

$$\therefore \eta = \frac{nh^2}{\pi} e^{-h^2(V^2+W^2)}dV\,dW. \qquad (5,7)$$

On the ellipsoidal theory:-

$$\eta = \frac{nH^2}{\pi} e^{-H^2(V^2+W^2)}dV\,dW. \qquad (5,8)$$

It therefore follows that, ~~on the~~ to relate the two-drifts theory to the ellipsoidal, we must identify h with H, that is

$$h = H \qquad (5,9)$$

(5,5) now becomes

$$U' = U'' = \qquad (5,10)$$

Hence, as from (4,6) we have, on the ellipsoidal theory, $U'=U''=1\ K\ \pi$

$$\therefore \frac{K}{H} = \frac{1}{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)} = M(\tau) \qquad (5.11)$$

We have thus derived an expression for the ratio of the axes of the velocity ellipsoid in terms of the relative speeds of the

Let $x = h_1(U - Q_1)$ and $\tau_1 = h_1 Q_1$ (3,1)

Then we have:-

$$N_1' = \frac{n_1}{\sqrt{\pi}} \int_{-\tau_1}^{\infty} (x+\tau_1) e^{-x^2} dx$$

Define $\sqrt{\pi}\,\Theta(\alpha) = 2\int_0^{\alpha} e^{-x^2} dx$ (3,2)

Then, $2N_1' = n_1\left[(1+\Theta(\tau_1)\right)$ (3,3)

Similarly, $2N_2' = n_2(1-\Theta(\tau_2))$ (3,4)

where $= h\,Q$.

$$\therefore \quad 2N' = n + \left[n_1\Theta(\tau_1) - n_2\Theta(\tau_2)\right] \tag{3,5}$$

By the same methods, if $N'' = N_1'' + N_2''$ be the number of stars with $U < 0$ for all values of V and W, then:-

$$2N'' = n - \left[n_1\Theta(\tau_1) - n_2\Theta(\tau_2)\right] \tag{3,6}$$

Put $G(\tau_1,\tau_2) = n_1\Theta(\tau_1) - n_2\Theta(\tau_2)$ (3,7)

Then, $2N' = n + G(\tau_1,\tau_2)$

$2N'' = n - G(\tau_1,\tau_2)$ (3,8)

On the ellipsoidal theory we have,

$$N' = N'' = \frac{nk}{\sqrt{\pi}}\int_0^{\infty} e^{-k^2U^2} dU = n/2 \tag{3,9}$$

4. If we write U' for the mean speed in the positive direction of the U-axis, then on the two-drifts theory we have:-

$$N'U' = \frac{n_1 h_1}{\sqrt{\pi}}\int_0^{\infty} U e^{-h_1^2(U-Q_1)^2} dU + \frac{n_2 h_2}{\sqrt{\pi}}\int_0^{\infty} U e^{-h_2^2(U+Q_2)^2} dU \tag{4,1}$$

Writing τ_1 for $h_1 Q_1$ and τ_2 for $h_2 Q_2$ as above, and putting $x = h_1(U - Q_1)$ and $y = h_2(U + Q_2)$ we have, in (4,1):-

$$N'U' = \frac{n_1}{h_1\sqrt{\pi}}\int_{-\tau_1}^{\infty}(x+\tau_1) e^{-x^2} dx + \frac{n_2}{h_2\sqrt{\pi}}\int_{\tau_2}^{\infty}(y-\tau_2) e^{-y^2} dy$$

$$= \frac{n_1}{2h_1\sqrt{\pi}}\left[e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,(1+\Theta(\tau_1))\right] + \frac{n_2}{2h_2\sqrt{\pi}}\left[e^{-\tau_2^2} - \tau_2\sqrt{\pi}\,(1-\Theta(\tau_2))\right]$$

Rewriting (2,1) we obtain:- $n_1\tau_1/h_1 = n_2\tau_2/h_2$

$$\therefore \quad N'U' = \frac{n_1}{2h_1\sqrt{\pi}}\left(e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right) + \frac{n_2}{2h_2\sqrt{\pi}}\left(e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right]$$

$$\therefore \quad U' = \frac{{}^{n_1}/_{h_1\sqrt{\pi}}\left[e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right] + {}^{n_2}/_{h_2\sqrt{\pi}}\left[e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right]}{n + G(\tau_1,\tau_2)} \tag{4,2}$$

Similarly, if U" is the mean speed in the negative direction of the U-axis, then, by the above methods:-

$$U'' = \frac{\frac{n_1}{h_1\sqrt{\pi}}\left[e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right] + \frac{n_2}{h_2\sqrt{\pi}}\left[e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right]}{n - G(\tau_1,\tau_2)} \tag{4,3}$$

For convenience, we may write (4,2) as:-

$$U' = \frac{\phi\,(n_1, h_1, \tau_1; n_2, h_2, \tau_2)}{\psi\,(n, \tau_1, \tau_2)} = \phi/\psi \tag{4,4}$$

and hence, $U'' = \dfrac{\phi}{\psi - 2G(\tau_1,\tau_2)}$ (4,5)

The expressions for the mean speeds U' and U" may also be derived on the ellipsoidal theory. In this case we obtain:-

$$U' = U'' = 1/K\sqrt{\pi} \tag{4,6}$$

5. We have derived the facts that (i) on the two-drifts theory the number of stars with $U > 0$ can differ from the number with $U < 0$, and that (ii) the mean speed in the positive direction of the U-axis can differ from that in the negative direction. On the ellipsoidal theory these differences do not occur. Thus the first conditions for possible identity of the two theories are those which we shall obtain

INVESTIGATIONS

INTO

STELLAR MOTIONS.

BY

D.G.EWART.

PREFACE.

With the application of photographic methods to positional Astronomy as a supplement to, and in place of, visual meridian observations, the number of well determined stellar positions and proper motions has increased more than tenfold since the beginning of this century. Whereas in 1910 the number of accurate proper motions known was less than 10 000 - mostly in the "Preliminary General Catalogue" of L. Boss - almost all of which had been determined from visual observations, at the present date the number nears 200 000, of which only a sixth are visual determinations. The other five sixths have been determined either wholely or in part by photographic means.

There are two ways in which proper motions may be determined by photographic techniques. Firstly two photographs of a region of the sky, taken at different epochs separated by some thirty years or so, can be superimposed, and the motions of the stars on the plates measured relative to the faintest stars. These latter, having no conspicuous proper motion, are displaced only by the systematic effects of the parallactic motion and galactic rotation. The absolute proper motions of the stars measured can thus be determined. Alternatively proper motions can be determined from the combination of the photographic places of stars at one epoch, and the visual meridian positions at another.

By the first of these methods, the proper motions of some 41 000 stars between declinations -40° and -52° have been determined by the Royal Observatory, Cape of Good Hope, who are also using the second method to obtain modern positions and proper motions of stars south of declination -30°, excluding the above mentioned zone. This latter programme is an extension of the Yale programme of the re-observation of the A.G. zones between declinations $+60^{\circ}$ and -30°.

Although it has been estimated by Schlesinger that by photographic means, stellar positions can be determined to an accuracy approaching 0".2, the positions and motions found are liable to a number of errors, both of systematic and accidental nature. Instrumental errors due, for example, to flexure are less important, perhaps, as are presonal errors of observation, although both of these may enter through the visual observations of standard stars used in the reduction of the photographic places to catalogue places. Measurement errors are, however, of importance. The photographic image of a star is not a point, but is a circle of sensible diameter, the diameter depending on the magnitude of the star and the exposure given to the plate. The definition of the image also is liable to alter through the position of the image on the plate. Thus errors of measurement can enter, ~~due to~~ dependant on the magnitudes of the stars, their positions on the plates and , conceivably, on the graininess of the plate. Statistical corrections are usually applied in the reduction of the measurements, but there may remain accidental ~~xxxxx~~ residual errors together with possible systematic errors partly introduced from the visual observations of the standard stars. These errors may be largest for the brightest stars, their images having the greatest diameters.

The great advantage of the photographic methods of determining proper motions is that by these means, proper motions can be determined for stars too faint for visual observation, and they can be determined in bulk. Whereas visual meridian proper motions extend but to the eighth magnitude, photographic motions have been determined for tenth and eleventh magnitude stars, and, in some cases, even fainter. The Radcliffe motions include stars down to the fourteenth magnitude. However, for the faint stars, with their smaller proper motions, the accidental errors in their determination may equal the motions in size. A further trouble enters if the motions are to be reduced to a common fundamental system from the various catalogue systems. The corrections have usually been derived from the bright stars and not from the faint stars to which they are to be applied.

In Part I of this thesis, the proper motions - derived photographically - of the fainter stars in the declinationzones -40° to -52° and -30° to -40° are examined for the effects of preferential motion. These motions were determined by the Royal Observatory, Cape of Good Hope, the former zone by the purely photographic method and the latter by the combined meridian visual and photographic places. For this zone, the rqyisite material was kindly supplied by Mr. J.V.B.Lourens of the Cape Observatory in advance of the publication of the catalogues.

The aims of the analyses were to evaluate any differences in the constants of star-streaming dependant on spectral type of mean magnitude, to compare the behaviour of hte two types of proper motion as regards star-streaming and, for the -40° to -52° zone, to determine the effects of a change of the basic system of reference from the Cape system to that of FK3. The main interest is the determination of the effects of using only the more distant stars, it having been variously suggested by Blauuw and other investigators, that the character of the distribution of the peculiar linear velocities of the faint stars differs from that of the bright stars.

2.

Whereas proper motions can be determined visually, with perhaps greater accuracy at present than by photographic means, the visual observations covering a greater period of time than the photographic observations, the determination of the radial velocities of the stars have all been made photographically since Huggins applied photography to this problem. Previously only a very few radial velocities had been measured visually by Vogel. The number of known velocities is, however, much less than the number of known proper motions, since each star must be observed individually and must be observed a number of times. Also, for the fainter stars the exposure times necessary increase very much more than is the case for the determination of proper motions. Whereas for an eighth magnitude star an exposure of 3 to 6 minutes will give a strong image on a modern plate for the determination of position, to determine its radial velocity, the exposure will approach an hour and a half for the spectrograph plate - for a telescope of moderate aperture (36"). Thus radial velocity determinations have been made mostly for the brightest stars - thay are now complete to about the sixth magnitude - and for fainter stars of special astrophysical interest - the B stars, the giants, the late type stars for example. Analyses of radial velocities have been correspondingly limited. The motions of the bright stars have been fully examined by Nordstrom and Smart and Green and many analyses of the early type stars for the effects of galactic rotation have been made. The comparable data for the fainter main sequence stars is very limited.

In Part II of this thesis, the radial velocities of a sample group of faint stars of spectral types F to M have been analysed. Again the emphasis is on magnitude variations between these stars and the brighter stars examined by the same methods by Smart and Green.

Although variations with the magnitudes and spectral classes of the stars are sought, it is probable that both of these, if present, are mainly distance variations. The exact na ture of the distance variations cannot be decided unless the absolute magnitudes of the stars used and the amount of the interstellar absorption are both known. For the O and B type stars these latter can be estimated with a reasonable degree of accuracy. Also, these early type stars are at great enough distances to ~~xix~~ display detectable effects of differential galactic rotation. By usigg distance groupings of these stars, the variations with mean distance can be evaluated. An analysis of the radial velocities of about 1 600 such stars between galactic latitudes $+20^{\circ}$ and -20° contained in the recently published Mount Wilson "General Catalogue of Radial Velocities" compiled by R.E.Wilson,is outlined. Also to be used in the analyses are the interstellar velocities found from the spectra of a number of these stars.

Finally in Part III of the thesis a theoretical investigationof the relation between the two-drfits and the ellipsoidal theories of the distribution of stellar linear peculiar velocities is described. A relation between the relative velocities of the drifts and the axis-ratio of the velocity ellipsoid is derived,which agrees well with the observed values of these quantities.

3. The results of the analyses of the proper motions of the "Catalogue of the Proper Motions of 20 554 Faint Stars in the Cape Astrographic Zone", of the analysis of the radial velocities of 820 Faint Stars measured at Lick Observatory and of the investigation into the relationship between the two-drifts and the eelipsoidal theories dsecribed in Chapters IV, VIII and X of this thesis have been communicated to the Royal Astronomical Society and accetped for publication in the Monthly Notices thereof. Copies of the papers are included as an appendix to this thesis. The results of the analyses of the proper motions of the Cape Photographic Zone Catalogues, Zones -30° to -35° and -35° to -40°, described in Chapter V have not yet been published pending further discussion with Mr. J.v.B.Lourens.

4. The investigations described in this thesis were performed while the author was a Research Student in the Department of Astronomy. The author wishes to acknowledge his indebtedness to Professor W.M.Smart and to Dr.T.R.Tannahill for their generous advice and encouragement during the performance of these investigations. He also wishes to thank the Department of Scientific and Industrial Research for the award of a Maintenance Grant during the tenure of which these investigations were performed.

CONTENTS

CHAPTER I

PROPER MOTION INVESTIGATIONS.

1. In this section of the thesis, investigations of the systematic motions of the stars, based on proper motions, are described. The proper motions used in these investigations are firstly, the proper motions of the Cape Astrographic Zone Catalogues and secondly those of the Cape Photographic Zone Catalogues for the zones -30° to -35° and -35° to -40°. The arrangement of this section is as follows:-

(i) in this chapter the methods of analysis are discussed and compared,

(ii) in chapter II, the Cape Astrographic Zone Catalogues and their proper motions are briefly described, the results of previous analyses of the proper motions in the first volume are discussed and the aims of the following investigations stated,

(iii) in Chapter III, the proper motions of the Cape Astrographic Zone Catalogues are reduced from the Cape system to that of the FK_3 and then analysed. It is concluded that the reductions are not satisfactory,

(iv) in Chapter IV, the proper motions of the faint star volume are re-analysed on the Cape system and the results of the analyses compared with those from the bright stars and from the G.C.

(v) In Chapter V , the proper motions of the Cape Photographic Zone Catalogues for the -30° to -35° and the -35° to -40° zones are analysed. It is found that to explain some of the results of the analyses further analyses would be required. These are stated but not performed, the necessary material not being yet available.

(vi) Finally, in Chapter VI, the results of the investigations of proper motions are summarised.

2.

That the distribution of stellar linear velocities is not random, as was originally postulated, was recognised by Kapteyn at the beginning of this century, and in 1904 he announced his discovery of the two star streams, thus originating the two-drift theory of the distribution of stellar linear velocities. This theory was developed by Eddington, who, in a paper published in 1906 (1) put the theory on a mathematical basis, and detailed a method of analysing proper motions on this theory. This method, known as Eddington's "Trial and Error" method is that used in such of the following investigations as are based on the two-drift theory.

An alternative view of the distribution of stellar linear velocities was suggested by Schwarzschild (2) in 1907 and gave rise to the ellipsoidal theory. In 1908 he published a convenient method, based on this thoery, of analysing the observed distribution of proper motions in position angle (3). This method is known as Schwarzschild's automatic method.

In the "Trial and Error" method, the distribution curve of the proper motions is drawn, with ordinate the number of stars moving in a given sector of position angle - usually 10° - and abscissa the position angle. Theoretical curves are then ~~drawn~~ calculated and compared with the observed curve until the best representation is obtained. There are five constants to be derived, these defining the theoretical curve. In practice these are estimated from the observed curve and the corresponding theoretical curve drawn. This is compared with the observed curve and revised values of the constants obtained. The new curve is now drawn and the process repeated until the best 'fit' is obtained.

In Schwarzschild's automatic method, no curves are drawn and the process is entirely a numerical method. The numbers of stars moving in quadrants are counted and the position angles at which certain combinations of the counts disappear are derived. These are then used to determine the other two constants of the observed distribution.

On each theory there is an alternative method of analysis of the position angle counts. The two-drift analysis can be performed by a numerical method based on a Fourier analysis of the observed distribution, but in practice this method has turned out to be indeterminate in some aspects. Similarly there is a graphical method of analysis based on the ellipsoidal theory which is rarely used and is laborious in application.

The question of whether one or other of these theoratical distributions gives a better representation of the observed distribution of stellar peculiar velocities than the other is not one that can yet be settled. Objections have at times been raised against the two-drift method of analysis by the "Trial and Error" method, on the grounds that it depends overmuch on the investigator's personal judgement of what constitutes a fit. This is especially the case when the observed curve is affected by irregularities. However, in this method of analysis the curve is drawn and the irregularities can be seen and their effects can be evaluated although it is a matter for the investigator to decide what weight he will attach to a given departure from smoothness. In the ellipsoidal analysis by the automatic method, the exact effects of irregularities are unknown at times, and they can have a great effect on the determinations of the position angles and, hence, on the other constants derived by the use of these position angles. The element of personal judgement also enters into this method of analysis when the irregularities are present. Two examples are shown in Figures I and II, the first referring to the detrmination of the position angle of the solar apex and the second to that

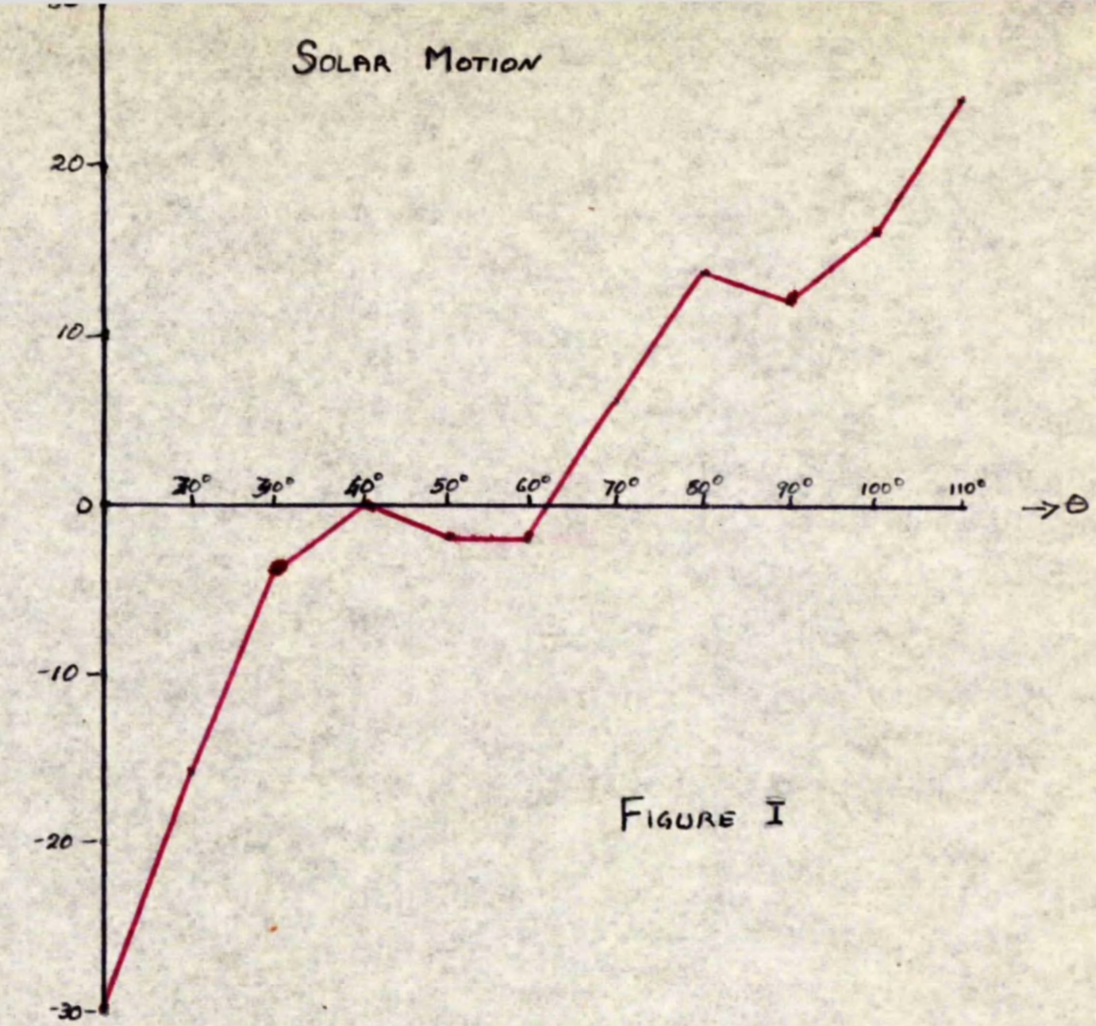

Figure I

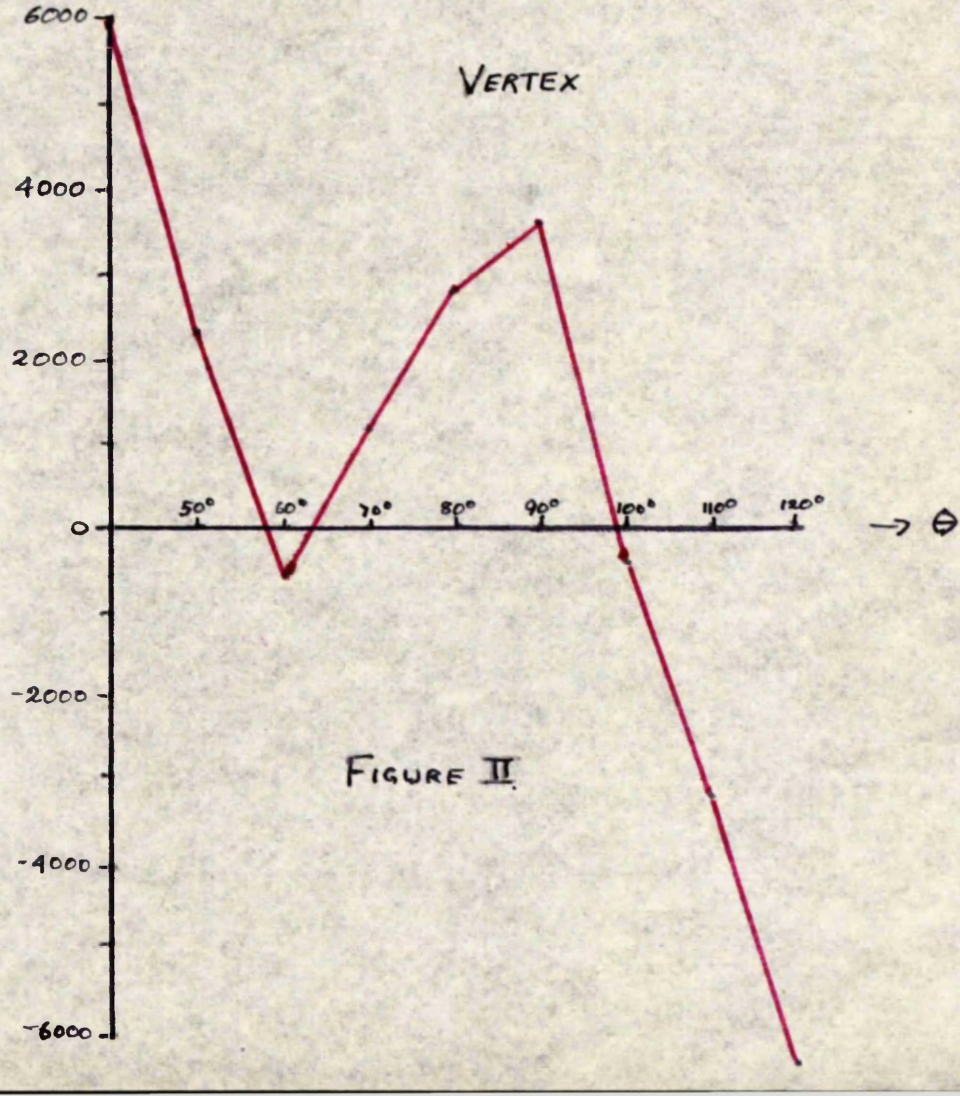

Figure II

of the major axis of the velocity ellipsoid. In each case it is required to determine the position angle at which the curves cross the angle axis, and in each case it is evident that there will be a considerable uncertainty.

It can also happen that the net effect of the irregularities ~~af~~ is zero. It is thus impossible to decide, at present, on the basis of the distribution of proper motions in position angle, that one theory gives a better representation than the other of the observed distribution.

In recent years a method of analysis, based on the magnitudes of the proper motions has been receiving more attention and has been used by a number of authors. The most striking feature of the use of this method is that it apparently gives no deviation of the vertex. - for the late type stars. The application of this method, however, requires ~~assuming~~ that the solar motion be known ~~and~~ as well as the mean parallaxes of the stars concerned. This method has not been used in the subsequent analyses.

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CHAPTER II

THE CAPE ASTROGRAPHIC ZONE CATALOGUES.

1. Following the successful introduction of photographic methods to Astronomy, the value of the new techniques as a method of determining stellar celestial positions was soon recognised, and, at a conference in Paris in 1891, the decision was taken that a photographic atlas of the sky should be constructed, the positions of all the brightest stars being thereby established to a high order of accuracy and each star being identified by a catalogue and a plate number. A number of observatories undertook the responsibility of the photography - and the subsequent reductions - of an area of the sky, each participating observatory being allotted a zone of declination as its contribution. The zone for the photography of which the Royal Observatory, Cape of Good Hope, undertook the responsibility - hereinafter referred to as the Cape Astrographic Zone (or the C.A.Z.) - extended from declinations -40° to -52°.

The photography of the zone, for the construction of the catalogue, was commenced in 1892 and was completed in 1896. In all the zone was divided into 1512 areas and each plate recorded an area of the sky 2° square, each plate being centred on an exact degree of declination. After this series, known as the 'old' series was completed, a second series, known as the 'catalogue' series was taken, starting in 1897 and finishing in 1910, although all but a few plates had been taken by 1905. The reason for this series being taken when the 'old' series had been intended for the construction of the catalogue was, to quote Jackson(1) in his introduction to the first catalogue of the proper motions in the zone:-

"....... in view of the relatively small amount of work involved in taking the plates compared with the measurement and reduction it was resolved to take a second series of plates for measurement. This second series was much better in quality and was taken at an epoch more closely corresponding to that of the meridian observations used in the reduction."

In addition to the photography, 8560 stars in the zone were observed with the 8-inch transit circle, these stars being intended as reference stars in the reduction of the plates, preference being given to the fainter, rather than the brightest stars. Each star was observed at least thrice. On the average there were 12 or 13 of these 'standard' stars on each plate.

After the compilation and publication of the C.A.Z. catalogue in 1923 - the catalogue listing 20 843 stars in the zone- it was decided to determine the proper motions of the stars in the zone. For this purpose, a third series of photographs was taken between 1923 and 1928. The proper motions were determined mainly from the 'old' series and the new - or 'P.M.' series. The measurements were not, however, confined to the stars in the published zone catalogue, butwere extended to a fainter magnitude limit.

The results of these determinations of positions and proper motions were published at various intervals, the principal publications being:-

(1) a catalogue of the positions for 1900.0 of the 8560 standard stars observed with the 8-inch transit circle and used for the reduction of the plates(1906) ,

(2) eleven volumes giving the rectangular co-ordinates and measured diameters of stars in the zone(1913 - 1926) ,

(3) the 'Zone Catalogue of 20 843 Stars, 1900 ' containing all the stars in (1) and extending to magnitude 9.0 inclusive on the C.P.D. scale. The positions were derived from a combination of meridian observations and measured photographic plages(1923) ,

(4) a volume giving for the above stars the Harvard spectral types, photographic and visual magnitudes, the photographic magnitudes derived from the measured diameters and the C.P.D. magnitudes reduced to a scale close to the international photographic scale(1927),

(5) a volume giving the photographic proper motions of these stars(1936) ,

and (6) a volume giving the positions, precessions, proper motions, spectral types and photographic magnitudes of a further 20 554 stars in the zone.(1939)

It is thus apparent that the material prepared from the astrographic plates affords a valuable collection suitable for many statistical investigations. The statistics of star counts, magnitudes and spectral types are discussed in the introductions to the four major volumes - those numbered above as (3), (4), (5) and (6). The proper motions of the 1936 volume are also discussed by Jackson(2) in the introduction, the details of analyses by Airy's method for the solar motion and by Schwarzschild's automatic method for the velocity ellipsoid being given.

In the investigations described below, attention has been given mainly to the determination of the variation of the constants of star streaming with spectral type for the 1939 volume.

2. As mentioned above, a series of plates was taken between 1923 and 1928 for the determination of proper motions. These plates were taken with the emulsion turned away from the object glass, exposure being made through the glass. Each plate was given four exposures - of 18^m, 6^m, 2^m and 40^s respectively - the telescope being moved in declination between each exposure. Although there was a loss of light through absorption by the plate, this was balanced by the increased sensitivity of the plates compared to those of the previous series. The images recorded were generally stronger than those of the earlier series, which had received exposures of 6^m, 3^m and 20^s.

For measuring, the old and the new plates were clipped together, film to film, a slight displacement being introduced of such magnitude that the 6^m exposure of the old plate was almost midway between the 18^m and the 6^m exposures on the new plate. The old and new images were thus close together. Measurements were then made on the 6^m exposures.

Although the zone catalogue had beeen compiled from the stars brighter than magnitude 9.0 on the C.P.D. scale, in determining the proper motions all stars were measured that were of magnitude down to 9.5 inclusive on the C.P.D. scale. There were also measured some fainter stars that were suspected of possessing considerable motions - greater than 5" per century. The only stars not measured were thus the very brightest stars - which could not be measured accurately - and those fainter than magnitude 9.5 with motions less than 5" per century.

Because of the overlap of the photographic plates there were generallytwo or three determinations for each star except for the stars in the extreme zones, namely -40 to -41 and -51 to -52 . The poorest determinations were for the stars at the edges of the plates, where the definition was falling off. The average period between the plates of the 'old' and the 'P.M.' series was 34 years, and between the 'catalogue' and the 'P.M.' series, 24 years. When a star was recorded on a number of plates, the interval might vary by up to 13 years for different pairs of plates. On each plate 10 fainter stars were also measured, in each of four areas, one area in each quadrant of the plate and not right at the centre. These stars were selected as being without conspicuous proper motion, and were to be used as reference stars in the reduction of the relative proper motions to absolute motions. From the average image diameter of these stars, their mean photo-graphic magnitudes were found to be $11^{m}.9$ on the Harvard scale.

As these stars were taken to be unaffected by proper motions, except for the effects of parallactic motion and galactic rotation, the measured displacements represented, for these stars, the displacements of one plate relative to the other. The dependance of this deliberately introduced displacement of the images of a star on one plate relative to the corresponding images on the other plate on the position of the images on the plate was then determined for each pair of plates. The measures Δx and Δy for the refence stars were equated to linear expressions in x and y, thus:-

$$\left.\begin{aligned} \Delta x &= ax + by + c \\ \Delta y &= dx + ey + f \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The scale and orientation constants a, b, d and e were determined from the measures in opposite areas and the comstants c and f were found from the means of the measures in all four areas.

The displacement corrections to be applied to the measures of the other stars on the plate were thus:-

$$\left.\begin{aligned} \Delta' x &= -ax - by - c \\ \Delta' y &= -dx - ey - f \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The relative proper motions in the interval between the epochs of the pairs of plates werd thus determined. The different measures for each star were them combined, being weighted according to the 'epoch-difference' of each determination.

3. As described above, the measures obtained are the proper motions in x and y relative to the eeference stars. To derive the absolute proper motions of the stars, three corrections have to be applied. These are:-

(i) a curvature correction to reduce the motions from x and y to right ascension and declination,

(ii) a correction for the motion of the refeerence stars due to parallactic motion and galactic rotation

and (iii) a correction due to the position of the star on the plate.

For curvature, the correction applied was:-

$$\left.\begin{aligned} \mu_\alpha \cos\delta &= \mu_x - f_x.\mu_y \\ \mu_\delta &= \mu_y + f_x.\mu_x \end{aligned}\right\} , \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{where} \qquad f_x = -\frac{\operatorname{Tan}\delta}{3437} . \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

According to Jackson(3), these corrections were negligible except for the few stars with large proper motions.

The corrections for the motion of thereference stars were derived from the tables given by van Rhijn and Bok(4), and were of the form:-

$$\left.\begin{aligned} \Delta\alpha &= \left(\frac{\bar{h}}{\rho}\right).P + Q \\ \Delta\delta &= \left(\frac{\bar{h}}{\rho}\right).P' + Q' \end{aligned}\right\} , \qquad \ldots\ldots\ldots\ldots(3)$$

where $\left(\frac{\bar{h}}{\rho}\right)$ is the secular parallax of the reference stars. ~~xxxxthex~~
The values of P,P',Q and Q' are taken from tables given by van Rhijn and Bok (Loc. cit. p23-24). In deriving these corrections ~~the solar apex id assumed to be at R.A. 270 and Dec. 34 . The~~

the solar apex was assumed to be at R.A. 270° and Dec. +31°. The values of $\left(\frac{\bar{h}}{\rho}\right)$ were also taken from tables in the same publication where it is tabulated for each of three galactic latitude zones for various Harvard visual magnitudes. Although the values are given for the latitude zones 0° to ± 20°, ± 20° to ± 40° and ± 40° to ± 90°, the nature of the variation with galactic latitude is not clear. Also the Cape zone does not lie in any one galactic latitude zone, but ranges from about galactic latitudes -75° to +18°. Accordingly the value used in the calculation of the corrections to the Cape relative motions was that independant of galactic latitude, and, as the photographic magnitude of the reference stars was, as stated above, 11.9 on the Harvard scale - with a mean deviation of $0^m\!.3$ - which equals $11^m\!.2$ on the Harvard visual scale, the value taken for $\left(\frac{\bar{h}}{\rho}\right)$ was 0".0092. The corrections thus applied for parallactic motion and galactic rotation of the reference stars were:-

$$\left.\begin{aligned} \Delta\alpha &= 0''.0092.P + Q \\ \Delta\delta &= 0''.0092.P' + Q' \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

The maximum amount in R.A. of this correction was 0".013.

The third correction, that due to the position of the star on the plate, was more complex, being possibly dependant on magnitude as well as position. As a result of a complete analysis of the material by Jackson(5), the corrections finally applied were: - on magnitude, no correction, and on position -

$$\left.\begin{aligned}\Delta\alpha &= \frac{34}{t}\left[3.7 - 0.25x^2 + 0.27xy - 0.04y^2\right]\\ \Delta\delta &= \frac{34}{t}\left[-3.5 - \quad -0.25xy - 0.27y^2\right]\end{aligned}\right\} \qquad (5)$$

with the unit being 0".001 and t being the 'epoch-difference' of the plates. Jackson concluded that the explanation of these latter corrections was a change of tilt of the telescope between the end of the 'catalogue' series and the start of the 'P.M.' series of amount of some 17' in each co-ordinate.

Thus by the application of these three corrections, the absolute proper motions were obtained. It is also necessary to know the accuracy of the proper motions thus derived. From a discussion of 6500 stars, it was found that the mean probable errors depended, as is to be expected, on the number of determinations made. The following figures are given by Jackson:-

No. of Determinations.	Probable Error.
1	± 0".008
2	± 0".0055
3	± 0".0044

The individual probable errors may, of course, vary considerably from the above, especially for the brightest stars.

4.

A number of investigations have been performed using the material of the 1936 volume as a basis. Those of most relevance to the investigations described hereinafter were:-

(i) a determination of the solar motion and apex by

Airy's method by Jackson(2),

(ii) an analysis of the proper motions by Schwarzschild's automatic method by Jackson(2),

and (iii) an analysis of the same proper motions by Eddington's 'Trial and Error' method by Smart and Tannahill (6,7).

The results of these investigations will only be summarised now, more detailed discussions being given where relevant later.

As was mentioned above, in removing the effects of the systematic motions of the reference stars to convert the relative proper motions to absolute motions, it was assumed that the apex of the solar motion relative to these stars lay at R.A. 270°, Dec. $+31^\circ$. No account was taken of the spectral types of these stars. If the apex of the solar motion derived from all stars of the catalogue was markedly different then, if this were not a magnitude effect, then the corrections applied might be in error. From an analysis of the motions of 17922 stars of all spectral types, and fainter than magnitude 6 on the catalogue scale, Jackson obtained the following elements of the solar motion for the apex:-

$$A = 273^\circ.6 \qquad D = +30^\circ.8 .$$

The apex thus derived from the motions of 86 per cent of the stars in the catalogue agrees well with the assumed apex. The probable errors of the determinations are not stated but are likely to be of the order of 1° to 2° in each co-ordinate. Thus there is no indication of the existence of any serious error introduced through the use of the assumed apex. Any difference that there may be could easily be ascribed to the difference in magnitude of the two groups of stars .

From the determinations of the constants of star-streaming made by Jackson and by Smart and Tannahill , very good agreement with the results of previous investigations of other proper motions is found. The interagreement of the two methods of analysis is also satisfactory. In all cases the determination of the declination of the solar apex is rather poor as was the determination of the Drift II apex declination by Smart and Tannahill. This can be attributed to the proximity of the zone to the apices, the declination differences being small. As is pointed out by Jackson, an error in the corrections applied for galactic rotation could also account for the variations.

5.

As stated above, in the measuring of the 'P.M.(plates to determine the proper motions of the stars of the Zone Catalogue, 1900, which referred to the stars brighter than C.P.D. magnitude 9.0 (inclusive), measurements were also made for all stars brighter than C.P.D. magnitude 9.5 inclusive and for such fainter stars as were suspected of possessing sensible motions. Of the latter category, those stars were retained that had absolute motions exceeding 5" per century. In all, these amounted to 2 432, 1 120 not being in the C.P.D. The additional stars in the C.P.D.

range $9^m.1$ to $9^m.5$ inclusive thus number 18 122. On the international scale these magnitudes are from $0^m.5$ tox $1^m.5$ fainter.

The catalogue of these stars and their motions was constructed at the same time and in the same way as the Zone catalogue, the same corrections and reductions being applied. The aatalogue thus contains for each star its position, precessions, magnitude on the international scale, epoch of observation for p position, proper motion and for about 90 per cent of hhe stars, the Harvard spectral types. Details are also given in the introduction of the magnitude and spectral distribution of the stars of both catalogues, together with other similar related statistics. The motions of the stars are not, however, discussed.

6.

In the present investigations, based mainly on the stars in the faint star volume, the aims have been:-

(i) to ascertain whether the low declination of the solar apex noted by Jackson and by Smart and Tannahill obtains also for the faint stars,

(ii) to confirm, or deny, the variations with spectral type of the constants of star-streaming found from the analyses of the bright star volume,

(iii) to determine whether there exist any intrinsic variations in the constants of star-streaming between the stars of the bright and faint star volumes,

and (iv) itx had originally been intended to determine whether , by reducing the Cape proper motions to the FK_3 system the low value of the declination of the solar apex might be removed. This was thought possible, as Jackson had remarked that an error in the rotation corrections might be the cause. This investigation did not achieve its aim, for reasons that will be discussed in the succeeding chapter.

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CHAPTER III

THE REDUCTION OF THE C.A.Z.MOTIONS TO THE SYSTEM OF FK$_3$ AND THE ANALYSES BASED THEREON.

1. As stated in section6 of the preceding chapter the original intention of this investigation had been to determine the extent to which the declination of the solar apex depended on the systematic corrections applied to the reference stars in the reduction of the plate measurements. It was also intended to make determinations of the constants of star-streaming, on the new system of reference, from the material of the two catalogues both separately and together for each spectral class.

In 1937 two catalogues of positions and proper motions of stars were published, both of a fundamental nature. The larger of these was the "General Catalogue of 33 342 Stars" compiled by B.Boss(1) - the G.C. - and the other the "Dritter Fundamentalkatalog des Berliner Astronomische Jahrbuchs" - the FK$_3$. Both catalogues recorded the positions and proper motions of the stars they contained to a very high degree of accuracy, but were not identical, there existing differences of a systematic nature between the two - largely owing to the methods of construction. The G.C. was compiled by combining almost all catalogues of ~~precision~~ positions and proper motions, and, as the number of stars for which such fundamental determinations have been made in the southern hemisphere is much less than the number in the northern hemisphere, the catalogue system of reference is perhaps weaker for the southern hemisphere. The FK_3 being of a somewhat different nature, and less extensive, is perhaps not quite affected to the same extent. This is not certain.

The Cape proper motion catalogues antedate these catalogues, the first volume - the bright star volume - being published in 1936. Its system of reference is thus not based on either of the above, nor are comparisons given, except with the "Preliminary General Catalogue" of L.Boss of 1910. Also, since the publication of the G.C. and the FK_3, almost all subsequent catalogues have been based on one or other of these, or else on a later version, Morgan's N30 catalogue (2). The same applies to investigations of proper motions.

Thus to facilitate comparison with recent analyses of proper motions - especially Tannahill's analysis of the G.C. proper motions (3) - and to see if the change of system affected the declination of the solar apex, it was decided to convert the Cape proper motions to either the G.C. or the FK_3 system.

The latter system was that finally adopted, the choice being based on a paper by Oort (4) in which, after considering both systems and their methods of construction, he concluded that the FK_3 system was preferable for the proper motions in the southern hemisphere at the declination of the Cape Astrographic Zone. This conclusion has been reached by other investigators - for example, the new Cape Photographic Zone-Catalogues, for the sky south of declination -30° are based on the FK_3 system, as are some of the Yale catalogues now being issued and revised.

To convert the Cape proper motions to the FK_3 system, corrections must be applied. A set of corrections for this purpose has been derived by Williams (5) as part of an investigation of the Leander- McCormick proper motions.(6). The Cape proper motions were to be included in the analyses in order to improve the solutions, the McCormick material all being confined to the sky north of declination -25°. As the McCormick material was reduced to FK_3, it was thus necessary that the Cape material be also reduced to that system.*

To obtain the corrections, the stars common to both the G.C. and the C.A.Z. catalogues were identified - G.C. stars with probable errors greater than 1" in eaither co-ordinate being omitted. In all some 2100 stars were thus found for comparison. These stars were then divided into groups by their Cape magnitudes and R.A. The material was now examined for two possible systematic effects depending respectively on R.A. and magnitude. In this work the differences between the components

* In the subsequent analyses , only the motions of the Cape reference stars were employed.

in R.A. and Dec. were treated separately.

The data thus derived consisted of the mean differences in R.A. and Dec. for each half-hour of R.A. for the magnitude groups 6.0 - 6.9, 7.0 - 7.9, and 8.0 on. The mean differences for the whole zone for each magnitude group are, in R.A. +0".57, -0".30 and -0".44 respectively, and, in Dec. -0".15, -0".15 and -0".37. In view of the large difference, 0".87, between the two groups 6.0-6.9 and 7.0 - 7.9 in R.A., Williams omitted the stars brighter than the seventh magnitude in the subsequent developments.

From the declination differences of the remaining two magnitude groups, Williams concluded that the differences between the 7.0 - 7.9 and the 8.0 on groups could be well represented as a constant magnitude effect of 0".22 ± 0".05 for all right ascensions. The differences in the proer motions in the eighth magnitude group were then derived by subtracting 0".22 from the seventh magnitude differences and obtaining the mean, weighted according to the numbers of stars, of these latter values and those found from the eighth magnitude stars. The values thus derived were then adopted as the corrections required to reduce the Cape proper motions in declination to the G.C. system. The seventh magnitude cor rections were then obtained by adding 0".22 to these eighth magnitude corrections. The reductions to FK_3 were then performed using the corrections published by Kopff (7). These values were finally smoothed to yield the adopted Cape - FK_3 corrections for the seventh and eighth magnitude stars.

The reductions in R.A. were found to be more complex. It appeared that the mean differences for the seventh and eighth magnitude stars, ($\Delta\mu_7$ and $\Delta\mu_8$), were notconnected by a constant magnitude effect as was the case in declination, but that there was an effect depending on R.A. The procedure adopted by Williams was:-- first the differences $\delta = \Delta\mu_8 - \Delta\mu_7$ were plotted against R.A. and a smooth curve drawn to represent the variation. Values were read from this curve and adopted as the corrections to reduce the observed values of $\Delta\mu_7$ to the system of $\Delta\mu_8$. Using these corrections, the mean values for $\overline{\Delta\mu_8}$ were then calculated, as were those in declination. These were then reduced to FK_3 by Kopff's tables, and then smoothed.

The corrections for the fainter stars were then derived from considering the mean values of the Cape proper motions of the stars earlier than type A_3 , which should be of roughly the same mean distance as the Cape reference stars, these latter being selected. Their variation with R.A. should then parallel the parallactic motion, this latter being their main component. From these, Williams found a well defined magnitude equation in the declination motions. That this was not a colour equation was found by considering the corresponding motions of the stars later than spectral types Ko between magnitudes 8.0 and 9.9 .

From these considerations, Williams arrived at the following values of the magnitude equation in declination, none being found in R.A.,

Magnitude	9.0 - 9.9	10.0 - 10.9	11.0 - 11.9	Ref. Stars
Correction	0	-0".10	-0".20	-0".25

The above values are all referred to the eighth magnitude stars.

Thus the corrections to reduce the proper motions of the stars of magnitudes between 8.0 and 9.9 from the Cape system to FK_3 are those listed in table 3.II in Williams' paper. cols. 7 and 8. For stars fainter than magnitude 9.9, the corrections in R.A. are unaltered and for stars of magnitudes 7.0 to 7.9 the values from the smooth values of $\Delta\mu_8 - \Delta\mu_7$ referred to above, are added. In declination, the corrections for each magnitude group to be added to the ~~above~~ values in column 8 of Williams' table are:-

Mag.	7.0 - 7.9	8.0 - 9.9	10.0 - 10.9	11.0 on
Corr^n.	+0".22	0	-0".10	-0".20

Finally, in the following investigations, a further constant correction for the whole zone of 0".08 in the G.C. - FK_3 reduction, recommended by Oort (4), was applied to the corrections for $\Delta\mu'$. The corrections thus applied are given in Table I, below, and are in units of 0".001 per annum - the same units as the listed motions in the Cape catalogues.

TABLE I

CAPE - FK_3 REDUCTIONS

R.A.	$\Delta\mu$ 7.0-7.9	8.0-	$\Delta\mu'$ 7.0-7.9	8.0-9.9	10.0-10.9	11.0-
0^h 15^m	+ 1.7	- 1.5	+ 8.0	+ 5.8	+ 4.8	+ 3.8
45	+ 3.0	- 0.8	+ 5.1	+ 2.9	+ 1.9	+00.9
1 15	+ 4.6	+ 0.3	+ 4.9	+ 2.7	+ 1.7	+ 0.7
45	+ 5.3	+ 0.6	+ 3.9	+ 1.7	+ 0.7	- 0.3
2 15	+ 2.5	- 2.6	+ 4.9	+ 2.7	+ 1.7	+ 0.7
45	- 0.6	- 6.1	+ 6.9	+ 4.7	+ 3.7	+ 2.7
3 15	- 3.0	- 8.7	+ 9.1	+ 6.9	+ 5.9	+ 4.9
45	- 2.2	- 8.1	+10.8	+ 8.6	+ 7.6	+ 6.6
4 15	- 2.7	- 8.8	+10.2	+ 8.0	+ 7.0	+ 6.0
45	- 0.5	- 6.6	+10.2	+ 8.0	+ 7.0	+ 6.0
5 15	- 2.8	- 8.9	+ 7.2	+ 5.0	+ 4.0	+ 3.0
45	- 2.6	- 8.5	+ 6.2	+ 4.0	+ 3.0	+ 2.0
6 15	- 4.4	-10.1	+ 5.6	+ 3.4	+ 2.4	+ 1.4
45	- 4.0	- 9.5	+ 6.8	+ 4.6	+ 3.6	+ 2.6
7 15	- 4.3	- 9.4	+ 6.7	+ 4.5	+ 3.5	+ 2.5
45	- 5.4	-10.1	+ 4.1	+ 1.9	+ 0.9	- 0.1
8 15	- 6.7	-11.0	+ 2.0	- 0.2	- 1.2	- 2.2
45	- 6.5	-10.3	+ 0.2	- 2.0	- 3.0	- 4.0
9 15	- 3.8	- 7.0	- 0.8	- 3.0	- 4.0	- 5.0
45	- 1.2	- 3.8	- 2.9	- 5.1	- 6.1	- 7.1
10 15	- 0.8	- 2.8	- 2.1	- 4.3	- 5.3	- 6.3
45	- 0.8	- 2.2	- 2.6	- 4.8	- 5.8	- 6.8
11 15	- 2.9	- 3.7	- 1.7	- 3.9	- 4.9	- 5.9
45	- 4.0	- 4.2	- 0.4	- 2.6	- 3.6	- 4.6
12 15	- 5.1	- 4.7	+ 0.2	- 2.0	- 3.0	- 4.0
45	- 5.5	- 4.5	+ 0.6	- 1.6	- 2.6	- 3.6
13 15	- 5.4	- 3.9	- 0.7	- 2.9	- 3.9	- 4.9
45	- 4.8	- 2.9	- 1.9	- 4.1	- 5.1	- 6.1
14 15	- 1.9	+ 0.4	- 1.0	- 3.2	- 4.2	- 5.2
45	- 1.1	+ 1.6	+ 1.7	- 0.5	- 1.5	- 2.5
15 15	- 0.4	+ 2.5	+ 4.4	+ 2.2	+ 1.2	+ 0.2
45	+ 0.5	+ 3.6	+ 5.9	+ 3.7	+ 2.7	+ 1.7
16 15	+ 1.0	+ 4.3	+ 6.7	+ 4.5	+ 3.5	+ 2.5
45	+ 3.3	+ 6.6	+ 9.7	+ 7.5	+ 6.5	+ 5.5
17 15	+ 3.6	+ 6.9	+10.9	+ 8.7	+ 7.7	+ 6.7
45	+ 3.3	+ 6.4	+13.5	+11.3	+10.3	+ 9.3
18 15	0	+ 2.9	+12.0	+ 9.8	+ 8.8	+ 7.8
45	- 2.9	- 0.2	+12.6	+10.4	+ 9.4	+ 8.4
19 15	- 3.8	- 1.5	+10.9	+ 8.7	+ 7.7	+ 6.7
45	- 3.2	- 1.3	+11.3	+ 9.1	+ 8.1	+ 7.1
20 15	- 3.6	- 2.1	+12.1	+ 9.9	+ 8.9	+ 7.9
45	- 2.1	- 1.1	+12.2	+10.0	+ 9.0	+ 8.0
21 15	+ 1.1	+ 1.5	+11.7	+ 9.5	+ 8.5	+ 7.5
45	+ 3.1	+ 2.9	+ 9.9	+ 7.7	+ 6.7	+ 5.7
22 15	+ 3.4	+ 2.6	+ 9.7	+ 7.5	+ 6.5	+ 5.5
45	+ 2.8	+ 1.4	+10.4	+ 8.2	+ 7.2	+ 6.2
23 15	+ 3.2	+ 1.2	+10.7	+ 8.5	+ 7.5	+ 6.5
45	+ 1.8	- 0.8	+10.0	+ 7.8	+ 6.8	+ 5.8

$\Delta = FK_3 - \text{Cape}$

Unit 0".001/annum

2. The corrections listed above in table I are given for four magnitude groups and for each half-hour of right ascension, $\Delta\mu$ being the correction in R.A. and $\Delta\mu'$ that in Dec. Accordingly, separate scatter diagrams were made for each half hour of R.A. for each of five spectral groups for each catalogue, the scatter diagrams for each catalogue being kept separate. In practice, as the faint star volume contained only one or two stars brighter than the 9th. magnitude on the interhational scale, scatter diagrams were made, for each half hour of R.A. of each spectral group, for the following magnitude groups for each volume of the catalogues:-

Bright Stars:- 7.0-7.9, 8.0-8.9, 9.0-9.9,10.0-10.9, 11.0-

Faint Stars :- 9.0-9.9, 10.0-10.9, 11.0-

This entailed preparing a maximum of 1 920 scatter diagrams. The actual total was somewhat less as not all the magnitude grpups were present for each spectral group in each region.

The spectral groups employed were of types B_8 - A_3, inclusive - called the A-type stars ; A_5 - F_5 inclusive - the F-type stars; F_8 - G_5 inclusive, the G-type stars; Ko - M inclusive - the K-type stars and finally the unclassified stars - the U- or Un. stars. Stars of spectral types earlier than B_8 were omitted because of their extreme galactic concentration and anomalous behaviour as regards star-streaming. Other omissions were the counting of binary stars as single stars and variable stars, since they could not be assigned to a magnitude group. After these omissions, 38 765 stars remained for analysis. The distribution of these stars by spectral group and R.A. is given in Table II.

In these scatter diagrams the proper motions were used as given in the Cape catalogues and the distribution counts on the FK_3 system prepared by introducing the corrections given in Table I as a change of origin.

The distribution counts were nade for every ten degrees of position angle for each scatter diagram. The counts for all magnitude groups of a given spectral group were then combined and, from these, a smoothed distribution obtained by taking running means of three successive sectors. These counts were then drawn on graph paper with ordinate the number of stars moving in a given sector of position angle and abscissa the position angles. These final counts were the subject of the subsequent analyses.

3.

The first analysis was for ALL stars - i.e. stars of known spectral types and the unclassified stars. This analysis was to be performed by Eddington's "Trial and Error" method. Before the analysis was commenced, however, it was necessary to know whether the omission of stars brighter than the seventh magnitude would affect the results, and, if so, to what extent. This is now investigated.

TABLE II

DISTRIBUTION OF STARS BY SPECTRAL TYPES AND BY REGIONS.

Region No.	A	F	G	K	ALL
1	12	58	171	91	374
2	15	62	175	73	372
3	16	61	142	102	366
4	16	54	131	78	333
5	14	61	136	76	331
6	17	73	147	107	396
7	22	77	150	128	445
8	24	83	178	119	447
9	57	89	160	105	455
10	41	127	184	134	515
11	57	114	150	151	490
12	74	118	198	121	542
13	99	151	149	154	630
14	167	186	212	202	823
15	251	229	185	210	925
16	482	176	169	196	1061
17	821	222	164	219	1455
18	804	298	172	226	1568
19	579	265	220	207	1349
20	509	260	233	238	1385
21	411	265	221	296	1301
22	305	208	226	273	1060
23	227	193	262	213	933
24	203	165	224	255	886
25	187	165	260	236	907
26	189	195	267	250	941
27	264	163	254	270	981
28	255	197	280	284	1053
29	295	170	240	198	923
30	305	153	239	204	908
31	555	223	296	278	1368
32	717	235	281	263	1527
33	665	166	198	218	1268
34	743	186	151	216	1328
35	721	196	265	278	1486
36	582	216	248	258	1319
37	276	143	209	186	837
38	141	142	183	181	674
39	101	107	194	167	579
40	59	135	198	165	566
41	47	98	168	148	477
42	33	90	169	130	446
43	32	90	178	121	439
44	32	78	168	171	466
45	26	63	218	151	473
46	20	80	209	136	468
47	22	58	183	151	449
48	22	81	157	124	440
TOTAL	11,512	7,025	9,572	8,758	38,765

On the two-drift theory, the distribution of stellar peculiar linear velocities is governed by a law of the form:-

$$n(u,v,w) = A e^{-h^2(u^2+v^2+w^2)} du\,dv\,dw, \qquad \ldots\ldots\ldots\ldots(1)$$

for each drift, where $n(u,v,w)$ is the number of stars belonging to the drift with velocities between u and $u+du$, v and $v+dv$ and w and $w+dw$ and A is a constant. If the total number of stars belonging to the drift is N, then we have:-

$$A = N h^5 \pi^{-3/2}, \qquad \ldots\ldots\ldots\ldots(2)$$

and N covers all magnitudes and distances.

Now let $D(r)$ be the density function such that the proportion of stars with distances between r and $r+dr$ and contained within a solid angle S is $S r^2 D(r)\,dr$.

Similarly let the proportional distribution functions of Apparent magnitudes and stellar linear velocities be $\phi(m)\,dm$ and $\psi(u,v)\,du\,dv$ respectively.

Then, if $N(m,r,u,v)$ is the number of stars with apparent magnitudes between m and $m+dm$, distances between r and $r+dr$ and transverse linear velocities between u and $u+du$ and v and $v+dv$, we have:-

$$N(m,r,u,v) = N S r^2 D(r)\,\phi(m)\,\psi(u,v)\,dm\,dr\,du\,dv. \qquad \ldots\ldots\ldots\ldots(3)$$

In the later stages we will take the drift form for $\psi(u,v)$ i.e.

$$\psi(u,v) = e^{-h^2(u^2+v^2)}. \qquad \ldots\ldots\ldots\ldots(4)$$

It is not necessary to specify the form of $D(r)$ except to state that at present

$$S' \int_0^\infty r^2 D(r)\, dr = 1 \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

The form to be taken for $\phi(m)$ must first be decided. That usually taken in statistical work is the Gaussian form ($\phi(m) = e^{-a^2(m-m_0)^2}$ where a and m_0 are constants) It must thus be determined whether this form applies to the Cape astrographic zone.

In the introduction to the Faint Stars volume, Jackson(8) finds that the catalogues are complete to about magnitude 10.2 . From tables in the introduction we obtain the following figures for the numbers of stars of given magnitudes.

TABLE III

NUMBERS OF STARS OF GIVEN MAGNITUDES - BOTH CATALOGUES.

m	n(m)	n'(m)	m	n(m)	n'(m)	m	n(m)	n'(m)
8.0	272	278	8.8	562	574	9.6	1085	1136
8.1	302	305	8.9	619	626	9.7	1265	1234
8.2	335	335	9.0	704	683	9.8	1388	1338
8.3	390	367	9.1	803	745	9.9	1457	1452
8.4	408	402	9.2	776	812	10.0	1588	1574
8.5	440	439	9.3	865	883	10.1	1734	1705
8.6	477	481	9.4	954	961	10.2	1866	1845
8.7	561	525	9.5	1057	1045	10.3	1983	1996
						10.4	2093	2157
						Total	23984	23898

In the above table m is the magnitude , $n(m)$ the observed ~~number of stars of that~~ number of stars of that magnitude and $n'(m)$ the number calculated from equation (12) of this section.

If we now take $n(m) = A e^{-a^2(m-m_0)^2}$ then we have:-

$$\log_e n(m) = \log_e A - a^2(m-m_0)^2$$

$$= P + Qm + Rm^2 \quad ? \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where,

$$\left.\begin{aligned} P &= \log_e A - a^2 m_0^2 \\ Q &= 2a^2 m_0 \\ R &= -a^2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Alternatively,

$$\log_{10} n(m) = P' + Q'm + R'm^2 \quad \ldots\ldots\ldots (8)$$

where, $P' = \dfrac{P}{2 \cdot 3026}$, $Q' = \dfrac{Q}{2 \cdot 3026}$, $R' = \dfrac{R}{2 \cdot 3026}$, ... (9)

and $a = \sqrt{-2 \cdot 3026 R'}$, $m_0 = -\dfrac{1}{2}\dfrac{Q'}{R'}$, $\log_e A = 2 \cdot 3026 \left\{ P' - \dfrac{Q'^2}{4R'} \right\}$ ... (10)

Applying (8) to the figures in table III we obtain 20 equations in the three unknowns.* These were solved by the method of least squares, the solutions being:-

$$\left.\begin{aligned} a &= 0 \cdot 18012 \\ m_0 &= 22 \cdot 36 \\ A &= 222,770 \end{aligned}\right\} \quad \ldots\ldots\ldots (11)$$

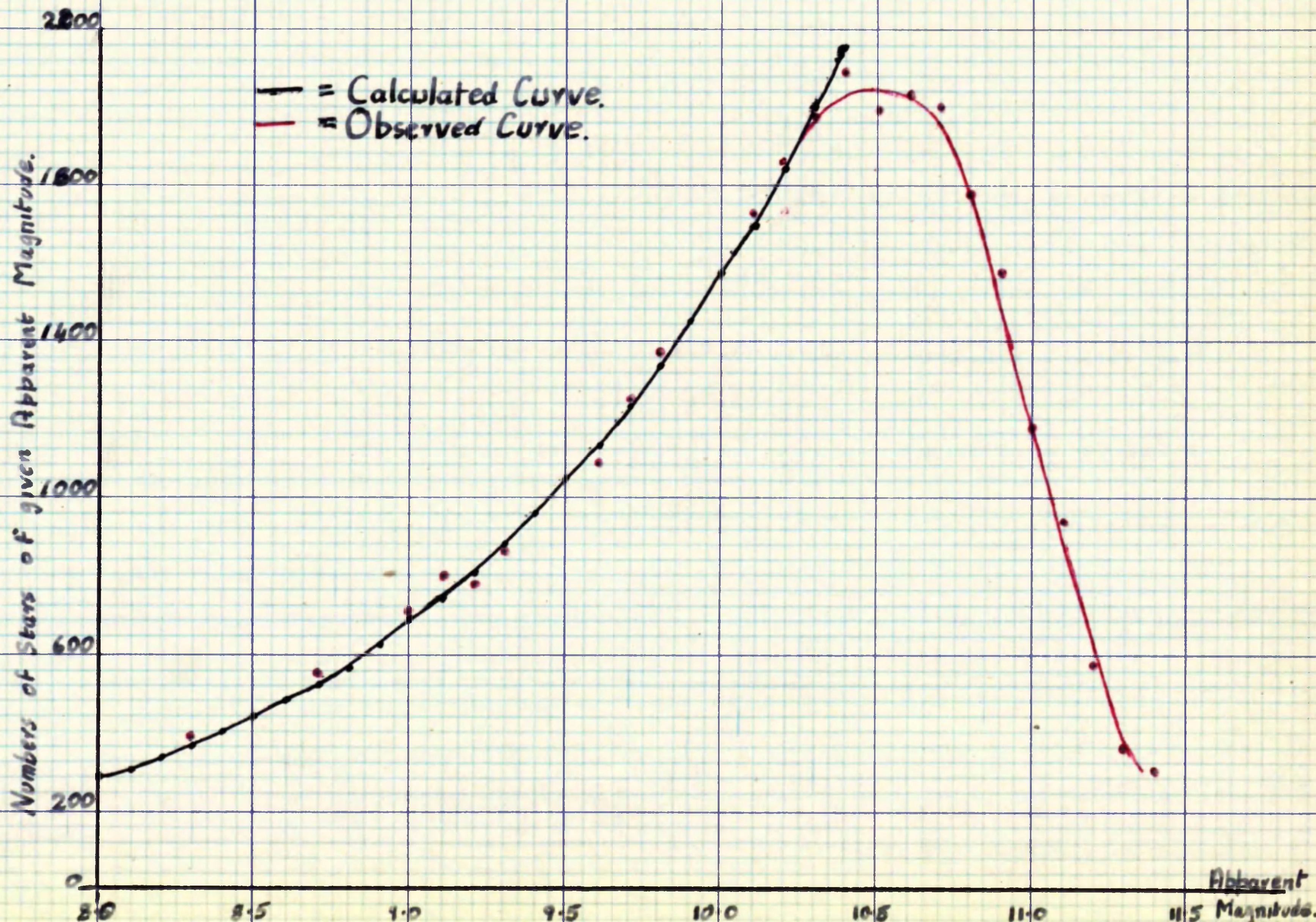
We thus have,

$$\log_{10} n(m) = -1 \cdot 69392 + 0 \cdot 62998m - 0 \cdot 01409m^2 \quad \ldots\ldots. (12)$$

and

$$n(m) = 222,770\, e^{-0 \cdot 03244(m - 22 \cdot 36)^2} \quad \ldots\ldots. (13)$$

* The points m = 8·7, 9·0, 9·1, 9·6, and 10·2 were not included.



= Calculated Curve.
= Observed Curve.
Numbers of Stars of given Apparent Magnitude.
2800
1800
1400
1000
600
200
0
8·0
8·5
9·0
9·5
10·0
10·5
11·0
11·5
Apparent Magnitude.

FIGURE I

The values of $n'(m)$ calculated from (12) for each value of m are given in the columns headed $n'(m)$ in Table III. It can be seen that there is a good measure of agreement between the observed and calculated values of the two. The values of $n(m)$ and $n'(m)$ are also plotted against m in Figure I, the observed values being plotted up to magnitude 11.4. The results fully confirm Jackson's estimate of $m = 10.2$ as the limit of completeness of the catalogue.

Three other points worthy of note arise from the above:- (i) the total number of stars in the zone as expected from (13) is $\frac{A}{a}\sqrt{\pi} = 2\ 192\ 300$. The expected number between magnitudes 7 and 10.4 is, from (13), 25 607. The observed number is 25 743. The observed numbers of stars up to the limit of completeness are thus well represented by (13).,

(ii) the effect of omitting stars brighter than the seventh magnitude would be to omit 0.0045 per cent.,

and (iii) from counts of stars between apparent magnitudes 2 and 17 on the Franklin-Adams charts, covering the whole sky, Chapman and Melotte[9] found the distribution function to be:-

$$n(m) = A e^{-0.0369\,(m-22.5)^2} \qquad \text{..........(14)}$$

Thus, it is apparent that only some 60 per cent. of the stars in the catalogues are distributed according to (13). From figure I it can be seen that the actual distribution is a skew-gaussian curve, and the distribution function is probably of the form:-

$$n(m) = A\Theta(m) + A_1\Theta'(m) + A_2\Theta''(m) + \text{———} \qquad \ldots\ldots(15)$$

where $\Theta(m) = e^{-b^2(m-m_1)^2}$, $\Theta'(m) = \frac{d}{dm}\left[\Theta(m)\right]$, ...(16)

and $A, A_1, A_2, \ldots A_n$ are constants.

Now from (3) we have:-

$$N(m,r,u,v) = N_1 S' r^2 D(r)\,\phi(m)\,\psi(u,v)\,dr\,dm\,du\,dv \qquad \ldots\ldots(17)$$

where N_1 is the total number of stars in the ~~zone~~ region of all magnitudes and distances. Now as

$$m = M + 5 - 5\log_{10} r \qquad \ldots\ldots(18)$$

where M = absolute magnitude, we have, on putting

$$M_0 = M + 5 \qquad \ldots\ldots(19)$$

$$\therefore\ N(m,r,u,v) = N_1 S' r^2 D(r)\,\phi(M_0 - 5\log r)\,\psi(u,v)\,dr\,dM_0\,du\,dv \quad \ldots\ldots(20)$$

Then, by omitting stars brighter than the seventh apparent magnitude, we have the number of stars with transverse linear velocities between u and $u+du$ and v amd $v+dv$ to be:-

$$N(u,v) = N_1\psi(u,v)\,du\,dv \int_0^\infty S' r^2 D(r)\,dr \int_{7+5\log r}^\infty \phi(M_0 - 5\log r)\,dM_0 \qquad \ldots(21)$$

The last two terms amount to a constant, so that, in effect, N_1 becomes N_1', and we have:-

$$N(u,v) = N_1'\psi(u,v)\,du\,dv \qquad \ldots\ldots\ldots(22)$$

and N_1' is the number of stars in the region of, or fainter than, the seventh apparent magnitude. In this case there is thus no effect on the determination of the drift constants the substitution of N_1' for N_1 acting proportionally over the whole of the region.

In the analyses that are to be performed now, two modifications are necessary to the above results. Firstly the function $\phi(m)$ must be replaced by that appropriate to the material - for example, (15) - we will write the function as $\rho(m)$. It will not be necessary to specify it as it will be a function of m alone. Secondly, however, there will also be a distance limit of some kind for the stars used. This will not be a definite limit, but there will probably be some mean maximum distance of the stars used - possibly corresponding to the mean minimum apparent magnitude of these stars (circa mag. 11.4) . Let this upper distance limit be R . The number of stars with transverse linear velocities between u and $u+du$ and v and $v+dv$ from the number of stars in the region will therefore be:-

$$N_1(u,v) = N_1\,\psi(u,v)\,du\,dv \int_0^R \omega r^2 D(r)\,dr \int_{7+5\log r}^{11.4} \rho(M_0 - 5\log r)\,dM_0 \quad \ldots(23)$$

The last two factors will again become a constant and we will have:-

$$N_1(u,v) = N'\,\psi(u,v)\,du\,dv \quad \ldots\ldots(24)$$

where N' is the number of stars in the region with $m \leqslant 7$.

Thus, there will be no change in the determination of the drift constants occasioned by omitting the stars brighter than the seventh apparent magnitude. This also applies to the velocity ellipsoid. It has, of course, been assumed that the function $\Psi(u,v)$ does not depend on the magnitudes of the stars or their distances.

4.

As was stated at the beginning of section 3 of this chapter the first analysis was for <u>all</u> stars and was performed on the two-drift theory by Eddington's " Trial and Error" method. 48 regions were used and a curve analysed for each region. The preliminary estimation of the drift constants for each region was made from the slope of the Drift I part of the curve, its amplitude and the position of its maximum. The Drift II constants were then estimated from the apparent position of its maximum and the number of stars available after estimating Drift I. Subsequently these original values were altered as was necessitated by the fit of the original calculated curve. The number of trials per region varied between one and seven, the average number being four. In all but eleven cases a very close fit was obtained. For the first eleven regions, however, the observed curves were very irregular. One feature of the analysis was the general prominence of Drift I , Drift II at times barely making any contribution to the shape of the curve and only fixing the 'background' numbers of stars. Indeed in two cases a local Drift II velocity of 0 was obtained, which results in the Drift II contribution in the region being equal in every direction. The details of the analysis for each region are given in Table IV, below.

The solutions for each region for each drift were then combined to determine the positions of the drift apices and the space velocities of the drifts - in terms of the theoretical unit of velocity $\frac{1}{h}$. Each region contributed an equation of the form:-

$$\left.\begin{aligned} -X\,\mathrm{Sin}\alpha + Y\,\mathrm{Cos}\alpha &= hV\,\mathrm{Sin}\theta \\ -X\,\mathrm{Cos}\alpha\mathrm{Sin}\delta - Y\,\mathrm{Sin}\alpha\mathrm{Sin}\delta + Z\,\mathrm{Cos}\delta &= hV\,\mathrm{Cos}\theta \end{aligned}\right\} \quad \ldots\ldots\ldots(1)$$

where (α,δ) are the equatorial co-ordinates of the centre of the region and (X,Y,Z) are the equatorial linear components of the space velocity of the drift relative to the sun. Then, if the drift-velocity is hW and the apex has equatorial co-ordinates (A,D),

$$\left.\begin{aligned} X &= hW\,\mathrm{CosACosD} \\ Y &= hW\,\mathrm{SinACosD} \\ Z &= hW\,\mathrm{SinD} \end{aligned}\right\} \quad \ldots\ldots\ldots(2)$$

For the Cape astrographic zone, δ is constant and is $-45^{\circ}47'$. The equations (1) for each drift and region were then solved by the method of least squares. The positions and velocities found, together with their probable errors, are given in Table V.

TABLE IV

DRIFT ANALYSIS FOR ALL STARS - ON THE FK_3 SYSTEM.

R	hV_1	N_1	θ_1	hV_2	N_2	θ_2	R	hV_1	N_1	θ_1	hV_2	N_2	θ_2
1	1.0	240	87°5	0.8	134	240°	25	1.9	552	260°	0.55	355	205°
2	0.9	250	80	0.7	122	240	26	1.9	601	255	0.65	340	200
3	1.2	180	80	0.5	186	220	27	1.9	695	247.5	0.5	286	200
4	1.1	167	70	0.4	166	230	28	1.9	714	237.5	0.65	339	170
5	0.8	171	55	0.6	160	215	29	1.3	753	230	0.9	170	147.5
6	1.1	183	50	0.3	213	230	30	1.3	713	230	0.8	195	145
7	1.2	156	30	0.3	289	230	31	1.3	1020	225	0.5	348	135
8	0.9	220	10	0.2	227	220	32	1.1	837	220	0.35	690	160
9	0.8	300	10	0.3	155	280	33	1.0	901	202.5	0.6	367	95
10	1.0	255	0	0.2	260	270	34	0.8	907	185	0.7	421	90
11	1.5	170	0	0.5	320	270	35	0.8	700	175	0.45	786	75
12	1.0	251	340	0.5	291	260	36	0.9	714	167.5	0.7	605	70
13	1.0	370	325	0.5	260	230	37	1.2	313	182.5	0.3	524	85
14	1.1	549	320	0.5	274	250	38	1.5	187	182.5	0.2	487	150
15	1.5	[illegible]	310	0.5	308	265	39	1.5	158	170	0	421	
16	1.5	782	296.25	0.55	279	245	40	1.1	220	160	0	346	
17	1.7	1042	287.5	0.7	413	245	41	0.8	169	140	0.1	308	150
18	1.7	1130	280	0.7	438	240	42	1.1	183	135	0.2	263	290
19	1.7	964	275	0.7	385	215	43	1.1	183	125	0.1	256	250
20	1.65	815	275	0.9	570	230	44	1.0	240	100	0.2	226	240
21	1.9	662	270	0.8	639	220	45	1.3	225	95	0.2	248	200
22	1.6	611	265	0.7	449	200	46	1.5	178	97.5	0.2	290	160
23	1.7	609	265	0.8	324	185	47	1.0	225	90	0.3	224	240
24	1.9	575	262.5	0.7	311	195	48	0.8	314	80	0.8	126	250

TABLE V

DRIFT CONSTANTS

	DRIFT I		DRIFT II	
X	0.30795 ±	0.03291	0.35166 ±	0.02782
	(0.052	0.035)	(0.093	0.029)
Y	1.41601	0.03291	-0.09269	0.02782
	(1.446	0.032)	(-0.140	0.026)
Z	-0.14984	0.04102	-0.38469	0.03468
	(-0.217	0.044)	(-0.628	0.036)
A	77°7	1°0	345°3	12°2
	(88.0	1.4)	(303.6	9.5)
D	- 5°9	1°7	- 46°6	12°2
	(- 8.5	0.6)	(- 75.0	2.4)
hW	1.457	0.035	0.529	0.034
	(1.463	0.035)	(0.650	0.035)

Smart and Tannahill's results are given in brackets.(10)

Having calculated the drift elements, the direction ~~of the vertex of star-streaming~~ of the vertex of star-streaming was derived, and the relative velocity of the drifts. If (ξ, η and ζ) be the latter's linear equatorial components, Ω the relative velocity of the drifts and (A_V, D_V) the equatorial co-ordinates of the vertex, then:-

$$\left.\begin{aligned} \xi &= X_1 - X_2 \\ \eta &= Y_1 - Y_2 \\ \zeta &= Z_1 - Z_2 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(3)$$

and

$$\left.\begin{aligned} \operatorname{Tan} A_V &= \frac{\eta}{\xi} \\ \operatorname{Tan} D_V &= \frac{\zeta}{\sqrt{\xi^2 + \eta^2}} \\ \Omega &= \sqrt{\xi^2 + \eta^2 + \zeta^2} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots(4)$$

Finally the elements of the solar motion were calculated. Writing these as X_0, Y_0 and Z_0 then: -

$$\left.\begin{aligned} (N_1 + N_2)\, X_0 &= N_1 X_1 + N_2 X_2 \\ (N_1 + N_2)\, Y_0 &= N_1 Y_1 + N_2 Y_2 \\ (N_1 + N_2)\, Z_0 &= N_1 Z_1 + N_2 Z_2 \end{aligned}\right\}, \qquad \ldots\ldots\ldots(5)$$

N_1 and N_2 being the numbers of stars in Drift's I and II respectively, and the position of the solar apex (A_0, D_0) and the speed of the sun's motion relative to the centre of rest of the stars concerned (hU_0) are given by:-

$$\operatorname{Tan} A_0 = \frac{Y_0}{X_0}, \quad \operatorname{Tan} D_0 = \frac{Z_0}{\sqrt{X_0^2 + Y_0^2}} \qquad \ldots\ldots\ldots(6)$$

$$\text{and } hU_0 = \sqrt{X_0^2 + Y_0^2 + Z_0^2}$$

The positions of the vertex and the solar motion are given in Table VI. Also given are the galactic co-ordinates of the vertex. (G_V, g_V). Again Smart and Tannahill's results are given in brackets for comparison.

TABLE VI

VERTEX AND SOLAR MOTION FROM DRIFT ANALYSIS-FK_3 SYSTEM

ξ	-0.0437 ±	0.0442	A_v	$91^\circ.6$ ±	$1^\circ.7$	G_v	$348^\circ.4$ ±	$2^\circ.8$
	(-0.041	0.046)		(91.5	1.6)		(343.2	1.0)
η	1.5087	0.0442	D_v	$8^\circ.7$	$2^\circ.7$	g_v	$3^\circ.2$	$1^\circ.2$
	(1.586	0.041)		(14.5	2.0)		(0.5	1.7)
ζ	0.2349	0.0537		1 527	0.057			
	(0.411	0.057)		(1.638	0.043)			

A_o	$247^\circ.9$	hU_o	0.8992
	(265.1)		(0.879)
D_o	$15^\circ.9$	N_1/N_2	1.454
	(26.1)		(1.41)

A number of points are immediately outstanding in ~~Taxkx~~ Table V. They are:-

(i) the low values of the right ascension of Drift I compared to Smart and Tannahill's determination,

(ii) the high value of the right ascension of the Drift II apex,

and (iii) the low southerly declination of the Drift II apex.

The latter two variations are not of as great concern as the first, since it is to be expected that the Drift II determination will be uncertain.- although in this case the uncertainty is rather more than usual. The Drift I position, however is well determined, as is partly shown by the small probable errors. Even stretching these probable errors to the limit, and similarly treating Smart and Tannahill's, there remains a discrepancy of some 8°, which cannot be disregarded.

There is also a large variation evident in Table VI. The vertex position agrees well with that of Smart and Tannahill although there is a difference in the declination of the vertex. The position of the solar apex on the FK_3 system, however, shows a considerable departure from that on the Cape system. This is especially evident in the declination of the apex and the declination found is even lower than the original value which was an object of this investigation.

The question arises as to which of these variations are significant - if any. The discrepancy of 8° in the right ascension of the Drift I apex must be regarded as definite, it being well determined. The variations in the Drift II apex are not, however, so important, since the position is bound to be somewhat uncertain - as was remarked by Smart and Tannahill. Further, as the apex is found to lie within the zone it cannot be said to be determined - as is borne out by the large probable errors of the determination. The uncertainty is further evidenced by the fact that in eleven of the regional analyses the local Drift II velocity was found to be 0.2 or less.

The position of the solar apex is not conclusive either, it being largely determined in right ascension by the Y component of the Drift I velocity and in declination by the declination of the Drift II apex, which being rather low will therefore yield a low declination for the solar apex. Very approximately, the position of the solar apex is given by

$\tan A_0 = \frac{Y_1}{X_1}$ and $\tan D_0 = \frac{Z_2}{Y_1}$. Thus the evidence from the solar motion can but be regarded as confirmatory and not conclusive. Further evidence can, however, be obtained in another way.

One of the features of the analysis of proper motions is that analyses ~~of~~ by both the two-drift and the ellipsoidal methods yield almost identical values of the solar motion . Hence, if an analysis on the ellipsoidal theory again yields the apex derived by the two-drift analysis, then it can be assumed that the divergence of the solar apex here obtained from that normally derived is real, and not an effect of the uncertainties of the two-drift analysis, the two methods of analysis being entirely different.

This test was made as follows. First a solution for the drift apices and the solar motion and the vertex of s star-streaming on the two-drift theory was made for a set of regions round the zone. Twelve regions were selected lying between R.A's. 0^h and $0^h\ 30^m$, 2^h and $2^h\ 30^m$ and every two hours of right ascension. The normal equations for the drift apices were immediately available from the solution for the whole zone. For these twelve regions, the solutions were:-

TABLE VII

	DRIFT I	DRIFT II	SOLAR MOTION	VERTEX
A	75°.9	342°.7	245°.8	91°.1
	(77.7±1.0)	(345.4±12.2)	(247.9)	(91.6±1.7)
D	−6°.8	−48°.1	17°.8	10°.3
	(−5.9±1.7)	(−46.6±12.2)	(15.9)	(8.7±2.7)
hW	1.410	0.582	0.894	1.498
	(1.457±0.035)	(0.529±0.034)	(0.899)	(1.527±0.057)

From Table VII it is evident that the twelve regions are typical of the behaviour of the zone for the purposes of analysis, the results being practically identical with those from the zone as a whole.

The distribution counts for these twelve regions were now analysed by Schwarzschild's automatic method. This method was chosen (a) for its simplicity in operation and (b) because it is diametrically opposite to the "Trial and Error" method of the two-drift analysis, being an entirely 'numerical' method, the operations on the observed numbers being completely automatic thus reducing the need for personal judgement to a minimum. The details of the analyses for each region are given in Table VIII, below.

TABLE VIII

R	θ_0	k/h	ζ_0	θ_1	Sh.	R.A.
1	73°2	0.544	1.541	277°1	0.424	$0^h - 0^h\ 30^m$
2	49.5	0.671	1.104	257.0	0.145	2 2 30
3	31.6	0.765	0.842	179.2	0.458	4 4 30
4	346.7	0.718	0.971	122.6	0.670	6 6 30
5	309.1	0.661	1.136	101.1	1.331	8 8 30
6	296.2	0.614	1.287	75.0	1.359	10 10 30
7	282.5	0.569	1.446	71.2	1.363	12 12 30
8	270.7	0.739	0.911	43.6	1.051	14 14 30
9	225.5	0.600	1.332	7.7	0.739	16 16 30
10	195.8	0.800	0.750	335.5	0.456	18 18 30
11	142.5	0.863	0.586	318.7	0.106	20 20 30
12	88.3	0.685	1.063	285.0	0.609	22 22 30

In the above table, θ_0 is the position angle of the vertex, k/h the ratio of the axes of the local velocity ellipse, $\zeta_0 = \left\{\frac{h^2}{k^2} - 1\right\}^{1/2}$, θ_1 the position angle of the solar motion and Sh. the local solar velocity. The equations of condition corresponding to (1) were then formed and solved by the method of least squares. The results were, for the vertex:-

$$\frac{K}{H} = 0.627\ ,\quad A_V = \underset{(91.1)}{92^{\circ}.2}\ ,\quad D_V = \underset{(10.5)}{13^{\circ}.0}\ ,\quad G_V = \underset{(346.7)}{344^{\circ}.9}\ ,\quad g_V = \underset{(-2.9)}{-0^{\circ}.6}\ ,$$

and for the solar motion:-

$$A_O = \underset{(245.8)}{245^{\circ}.6}\ ,\quad D_O = \underset{(17.8)}{15^{\circ}.5}\ ,\quad hU_O = \underset{(0.894)}{0.850}\ .$$

The figures in brackets are the corresponding results on the two-drift analysis. Thus, to all intents and purposes, the Two-drift and the ellipsoidal analyses yield identical results for the solar motion and the vertex. The conclusion which must thus be drawn is that the deviations in the positions of the drift apices and the solar motinn are significant and caused in some way, yet to be determined, by the motions and not by the method of analysis.

5.

There are four possibilities each of which might account for the anomalies found in the results of the analysis for ALL stars. They are:-

(i) they might be the result of including the stars of the faint star volume in the analysis - that is, of the nature of a magnitude effect - though this seems unlikely at first sight, the stars having been measured at the same timeand in the same way as those in the first volume,

(ii) they might be peculiar to the small proper motions,

(iii) they might be the effect of the Cape - FK_3 reductions, on the above or (iv) they might be the effect of the Cape - FK_3 reductions on all stars irrespective of the magnitudes of their proper motions.

To check the first possible explanation an analysis was made, confined to the bright stars of the first volume, the proper motions being reduced to the FK_3 system. This analysis was performed by Schwarzschild's automatic method. For the analysis, the zone was divided into 24 regions, each of an hour's width in right ascension, thus paralleling Jackson's (11) analysis of the stars of the first volume on the Cape system. Again, stars brighter than the seventh magnitude were omitted, no corrections having been derived for them. The spectral range of the group analysed was B8 - M, 62 stars of unclassified spectra being included. The total number of stars available for the analysis was thus 18 970 as against 18 350 treated by Jackson. The details of the analyses for each region are given below in Table IX, region 1 being centred at R.A. $0^h\ 30^m$.

One feature of note in table IX is that the irregularities in the distributions which caused poor determinations in the drift analysis also affect the determinations of the ellipsoid and solar motion constants.

Combining the results of the analyses for each

TABLE IX

R	θ_o	ζ_o	θ_1	Sh	R	θ_o	ζ_o	θ_1	Sh
1	73°2	1.164	264°5	0.357	13	277°5	1.343	67°9	1.363
2	58.4	1.044	266.5	0.329	14	268.9	1.372	66.2	1.902
3	50.5	1.068	218.2	0.037	15	261.0	0.883	37.8	0.929
4	34.8	1.117	177.2	0.428	16	244.1	0.775	26.7	0.734
5	20.1	0.822	166.3	0.464	17	223.0	0.935	357.7	0.670
6	7.1	1.194	129.9	0.583	18	203.0	0.984	324.6	0.607
7	347.6	1.045	136.7	0.933	19	177.1	0.558	342.1	0.479
8	331.7	1.061	116.1	1.134	20	159.7	0.622	350.3	0.027
9	312.9	0.810	99.2	1.226	21	130.5	0.688	307.3	0.322
10	307.9	1.252	85.1	1.393	22	116.9	0.653	295.0	0.316
11	291.5	1.327	71.0	1.122	23	94.7	0.850	278.5	1.026
12	298.3	1.094	73.7	1.206	24	74.2	0.943	269.7	0.296

region, the positions of the vertex and the elements of the solar motion are found to be:-

VERTEX

$$\frac{K}{H} = \underset{(0.625)}{0.656}\ ,\ A_v = \underset{(90.2)}{92^\circ.6}\ ,\ D_v = \underset{(13.4)}{15^\circ.4},\ G_v = \underset{(343.6)}{343^\circ.0}\ ,\ g_v = \underset{(2.1)}{0^\circ.9.},$$

SOLAR MOTION

$$A_o = \underset{(266.9)}{245^\circ.4}\ ,\ D_o = \underset{(28.3)}{11^\circ.6}\ ,\ hU_o = \underset{(0.94\ )}{0.862.}$$

The results given in brackets are Jackson's results from the first catalogue on the Cape system.

It is thus evident that the deviation in the solar apex position is still present for the first catalogue on the FK3 system. It can thus be concluded that the variations are not the result of including the stars of the faint star volume in the analysis. This result leaves only the possibility that the variations might be the effects of the small proper motions ~~of the fa~~ and/or the corrections employed to reduce the proper motions from the Cape system to FK3. To test these points, further analyses were made.

If the cause of the deviations from the normally derived positions was the effect of the corrections on the small proper motions, as now seemed probable, then the larger proper motions should be relatively unaffected. It was therefore decided to perform an analysis restricted to the stars with total proper motions greater than, or equal to, 4" per century, these stars being expected to be , to a great extent, independant of the system of reference. Then, if the analysis gave normal apices it would appear that it was the reductions to FK_3 affecting the small proper motions which were causing the anomalies. If, however, the analysis still gave deviations then it would appear that the corrections applied were affecting all the proper motions. A further possibility was also eliminated in these analyses by excluding the A stars - their distribution over the zone being uneven, which would mean that any effect that they had would vary from region to region. The analysis was thus made for the 8 482 stars of spectral types later than A_5 (inclusive) of magnitudes 7.0 and fainter, with proper motions exceeding 3".99 per century on the FK_3 system. Both catalogues were included, as were stars of unclassified spectra, these 1 latter forming a large proportion of the stars in some regions - as mentioned in the preceding chapter, 2 432 stars were included in the faint star volume because they possessed motiions exceeding 5" per century, (Cape), and 1 200 of these were of unclassified spectra.

To provide a standard of comparison, two further analyses were performed. The first of these was for all stars in the two catalogues of spectral types later than A_5, (inclusive), and fainter than magnitude 7 - their motions being on the FK_3 system. This analysis totalled 27 243 stars. The third analysis was designed to give the comparable results on the Cape system. The counts made by Smart and Tannahill wree used for the stars of this spectral group in the first volume and new counts were made for the faint star volume from the scatter diagrams. This gave a group comparable with, but not identical with, the above, totalling 26 272 stars. The main cause of the difference is that in the bright star volume, spectral types are not given for 2 399 stars and these were omitted by Smart and Tannahill. In the introduction to the faint star volume spectral types are given for 1 902 of these stars and they were thus included in the scatter diagrams prepared for this investigation. Smart and Tannahill's counts also include stars brighter than the seventh magnitude but their effect is likely to be small, since there are but 795 such stars in the catalogue, according to a table in the introduction to the faint star volume, and the exclusion of the stars earlier than spectral type A_5 will have removed many of these. Thus the difference in numbers is almost solely due to the inclusion of the additional spectral types in their respective groups in the FK_3 counts. Although the two groups are not identical, the effect of the 'Cape' group being smaller than it could have been will probably be too small to measure. It thus appears permissible to use these two groups as controls ar standards.

For these analyses the zone was divided into 16 regions, each of an hour and a half's width in R.A. - thereby ensuring that there would be sufficient numbers of large proper motion stars in each group for analysis. The analyses were performed by the two-drift theory, as the distribution curves demonstrate the effects of the corrections very clearly. As a preliminary test, six regions were selected and the numbers of stars in each of the three groups reduced - or increased to 1 000. Their distribution curves were then drawn and superimposed. These latter curves are shown in Figures II - VII, below. Unfortunately, as the magnitude of the corrections applied to reduce the motions from the Cape system to that of FK_3 vary with the magnitudes of the stars and their right ascensions, it is impossible to associate a given curve in these figures with a given value of the correction. Roughly, however, the magnitude of the corrections applied increase from region 1 to region 3 and for region 10 are less than those for regions 9 and 11.

From the figures it can be seen that the differences

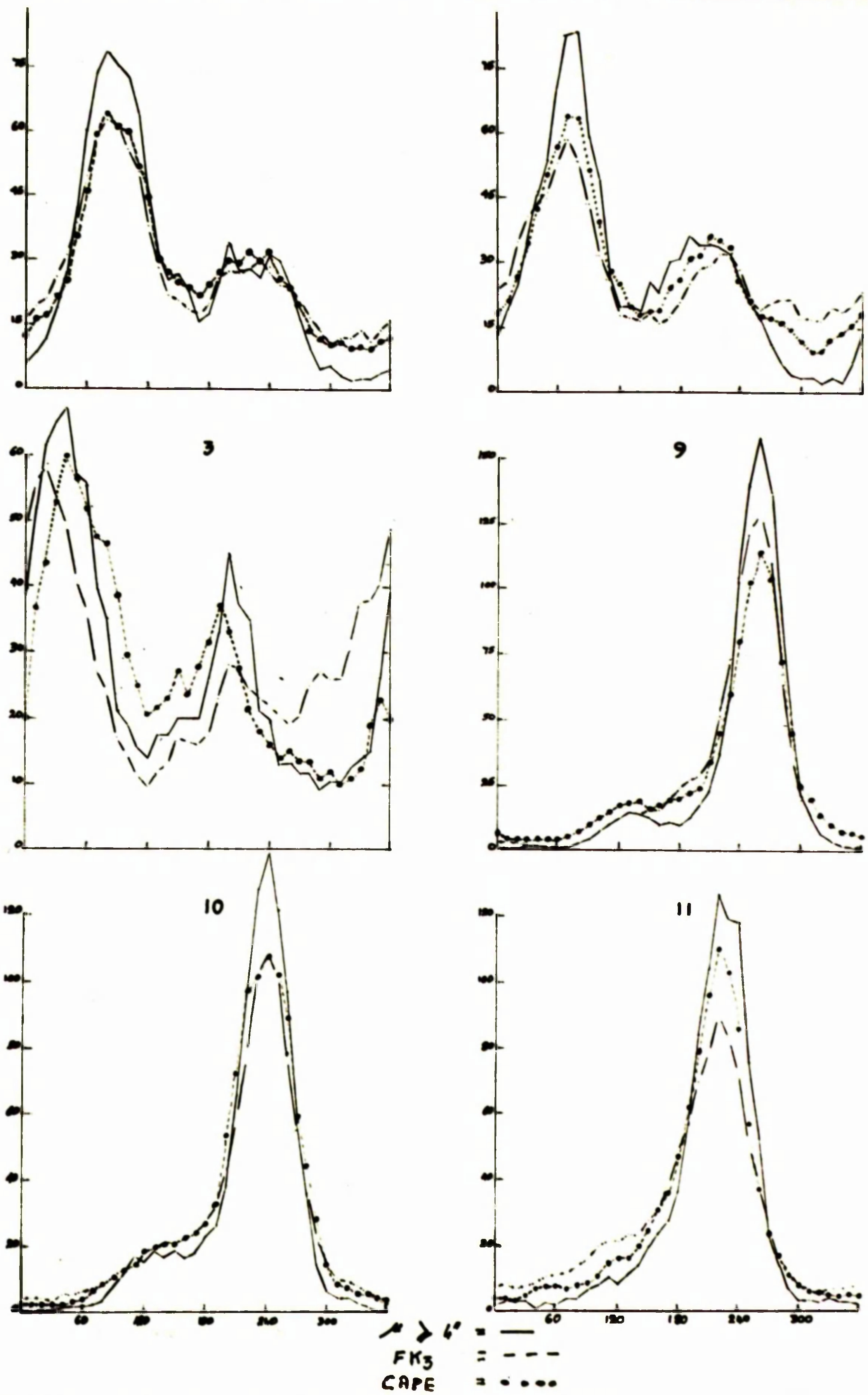


FIGURE II

between the Cape and the FK3 curves follow a similar trend. The Cape curves also seem to have lower minima which improves the determination of the drift constants. The curves for the restricted proper motions show high maxima and low minima as is to be expected, and the drifts are better separated and more evident.

As stated the curves were analysed by the "Trial and Error" method. The details of the analyses for each region and group are given below in Table X.

The positions of the Drift apices were then calculated. In the case of the restricted proper motions, however, the normal method is not strictly applicable. The distribution in position angle has been investigated for such proper motions by Smart (12) for the case of a stellar density law of the form $\frac{A}{r} e^{-(hkr)^2}$. It was found that the position angle of the drift apex would be unaffected but the drift velocity would be exaggerated. Thus apices calculated from the analyses in the usual way for restricted proper motions will be 'pseudo-drift apices'. Hence the drift apices for the proper motions greater than 4" per century were calculated both by the normal method and also from the position angles alone - the latter by a method of successive approximations from assumed apices. For the latter, Smart and Tannahill's apices at $88^\circ\!.0$, $-8^\circ\!.5$ and $303^\circ\!.6$, $-75^\circ\!.0$ were used. Each region then contributes an equation of the form:-

TABLE X

$\mu \gg 4''$ per century.

R	hV_1	N_1	θ_1	hV_2	N_2	θ_2
1	1.3	281	85°	0.8	208	215°
2	1.4	290	70	0.8	198	205
3	1.0	366	40	1.1	171	205
4	1.5	224	10	0.5	258	200
5	1.6	313	332.5	0.55	189	180
6	1.5	412	302.5	1.05	109	175
7	1.9	442	285	0.85	207	165
8	2.0	488	272.5	0.7	195	155
9	1.9	570	260	0.8	118	145
10	1.5	561	237.5	1.2	73	130
11	1.65	332	225	0.8	137	180
12	1.5	270	195	0.6	109	170
13	1.4	192	185	0.7	176	170
14	0.9	320	150	0.6	148	175
15	1.4	350	107.5	0.8	196	215
16	1.3	373	97.5	0.7	206	205
			FK3			
1	0.9	685	85°	0.65	383	225°
2	0.9	654	65	0.7	359	220
3	0.8	722	20	0.4	523	230
4	1.0	742	350	0.5	633	235
5	1.1	1296	315	0.4	564	220
6	1.3	1569	290	0.7	408	210
7	1.9	1391	277.5	0.7	1146	210
8	1.7	1358	265	0.6	786	185
9	1.9	1453	257.5	0.7	726	190
10	1.4	1453	232.5	0.5	576	135
11	1.5	1146	220	0.3	1080	165
12	1.05	1083	185	0.4	1004	95
13	1.1	778	177.5	0.2	794	30
14	0.8	725	145	0.05	625	280
15	1.1	714	105	0.4	574	225
16	1.1	714	90	0.4	579	250
			Cape			
1	0.9	649	85°	0.7	364	225°
2	1.4	390	70	0.5	515	210
3	0.9	630	50	0.5	500	190
4	1.05	610	20	0.45	658	190
5	1.1	946	340	0.4	806	200
6	1.3	961	310	0.3	950	230
7	1.6	1598	290	0.5	870	185
8	1.5	1345	275	0.5	699	165
9	1.9	1202	260	0.5	933	180
10	1.3	1537	240	0.7	475	140
11	1.6	1349	225	0.6	801	170
12	1.5	1447	195	0.3	612	180
13	1.5	707	172.5	0.6	877	205
14	1.5	487	145	0.5	864	185
15	1.3	590	120	0.5	671	210
16	1.5	515	100	0.4	718	205

TABLE X

R						
1	1.3	281	35°	0.8	208	215°

TABLE XI

	CAPE	$\mu \geqslant 4''$ cent. (a)	(b)	FK_3
A_1	90°.6 ± 1°.3	87°.9	88°.3 ± 1°.4	79°.6 ± 3°.4
D_1	-8°.4 ± 2°.8	-2°.4	-7°.3 ± 2°.4	-7°.3 ± 6°.1
hW_1	1.571 ± 0.03	1.69		1.400 ± 0.05
A_2	296°.3 ± 19°.7	290°.3	280°.0 ± 9°.0	321°.0 ± 9°.6
D_2	-79°.3 ± 2°.2	-73°.7	-71°.0 ± 1°.9	-55°.3 ± 10°.8
hW_2	0.655 ± 0.04	1.15		0.571 ± 0.05
A_V	92°.5	91°.4		90°.1
D_V	14°.0	26°.4		10°.5
	1.72	2.22		1.59
A_0	269°.0	265°.8		251°.4
D_0	26°.0	20°.8		20°.5
hU_0	0.93	1.14		0.84

Note.

In the results for $\mu \geqslant 4''$ per century, the column headed (a) gives the 'pseudo-drift apices' calculated in the usual way and the column headed (b) the drift apices as calculated from the directions off the apices only.

TABLE XI

CASE		µ	Per cent.		PK3
A 90.6	1.3	296.3	19.7		

$$a.dA + b.dD = c \qquad \text{..........(1)}$$

the values of dA and dD being found to be $0^{\circ}.3 \pm 1^{\circ}.4$ and $1^{\circ}.2 \pm 2^{\circ}.4$ for Drift I and $-23^{\circ}.9 \pm 9^{\circ}.0$ and $4^{\circ}.0 \pm 1^{\circ}.9$ for Drift II respectively.

The main features of the results of these analyses is that the apices of the drifts as given by the restricted proper motions on the FK_3 system do not agree with those from the unrestricted proper motions on the FK_3 system, but are practically identical with those on the Cape system. This also applies, though not quite so well to the 'pseudo- drift apices'. It is thus apparent that the anomalous positions found for the drift apices on the FK_3 system in the original analysis are to be attributed to the effects of the corrections applied to reduce the proper motions from the Cape system to that of the FK_3 and that their effect is to cause a radical alteration in the character of the distribution of the small proper motions in position angle.

It is not, however, possible to say whether the large proper motions are affected by the corrections. From the above results, the only signs of variation are those between thepositions of the Drift II apex for the'Cape'motions and the large proper motions. These, however, are affected by their large probable errors so nothing can be said except that the effect, if any, of the corrections is small.

If the results from the unrestricted proper motions on the FK3 system are compared with those from the original analysis, an improvement is evident - assuming that the drift apices usually obtained are correct and that positions departing from them are suspicious. From table V we have, for the drift apices for ALL stars:-

A_1	$77^{\circ}.7 \pm 1^{\circ}.0$	A_2	$345^{\circ}.4 \pm 13^{\circ}.2$
D_1	$-5^{\circ}.9 \pm 2^{\circ}.6$	D_2	$-46^{\circ}.6 \pm 10^{\circ}.2$
hW_1	1.457 ± 0.04	hW_2	0.529 ± 0.04

and from table XI, for the same stars, after omitting the A stars:-

A_1	$79^{\circ}.6 \pm 3^{\circ}.4$	A_2	$321^{\circ}.0 \pm 9^{\circ}.6$
D_1	$-7^{\circ}.3 \pm 6^{\circ}.1$	D_2	$-55^{\circ}.3 \pm 10^{\circ}.8$
hW_1	1.400 ± 0.05	hW_2	0.571 ± 0.04

Although there is little change in the Drift I apex, there is an improvement in the Drift II apex - taking as normal $308^{\circ} > A_2 > 270^{\circ}$ and $D_2 \sim -75^{\circ}$. The extent of the improvement is not certain because of the considerable probable errors of each co-ordinate. Nevertheless, if the belief that the anomalies are occasioned by the effects of the corrections and that they alter the character of the distribution of the small proper motions in a radical manner, is correct, then the improvement is of the right order of magnitude since the new positions would still be bound to be anomalous, yet they should be less affected than was the case in the first analysis, since by the omission of the A stars in the latter a considerable number of small proper motions are removed together with their uneven distribution over the zone, although the remaining small proper motions will still yield an anomalous determination of the drift apex.

A limitted example is shown in figure VIII. Here two curves are drawn. The first of the curves - the full line curve - gives the distribution of 771 stars of spectral types B to A_3 from the bright star volume and between R.A's. 16^h and 17^h. The second curve - the dotted curve - gives the distribution, in position angle, of 681 of these stars on the FK_3 system. Even the addition of the 90 stars of spectral types B to B_7 to the FK_3 curve would not bring the two into even approximate agreement, and it is evident that the Cape - FK_3 corrections have caused a radical alteration in the character of the distribution in position angle of these stars.

The conclusion may thus be drawn from the analyses described in this section that the effect of the corrections derived by Williams and applied to the Cape proper motions to reduce them to the FK_3 system has been to occasion anomalous determinations of the constants of star-streaming. Further, the corrections have defeated their purpose to some extent, in that one of the reasons for applying the co rrections was to ascertain whether the low declination of the solar apex found from the bright star volume would be removed thereby whereas it is found that instead it is further lowered. Since there is no reason to

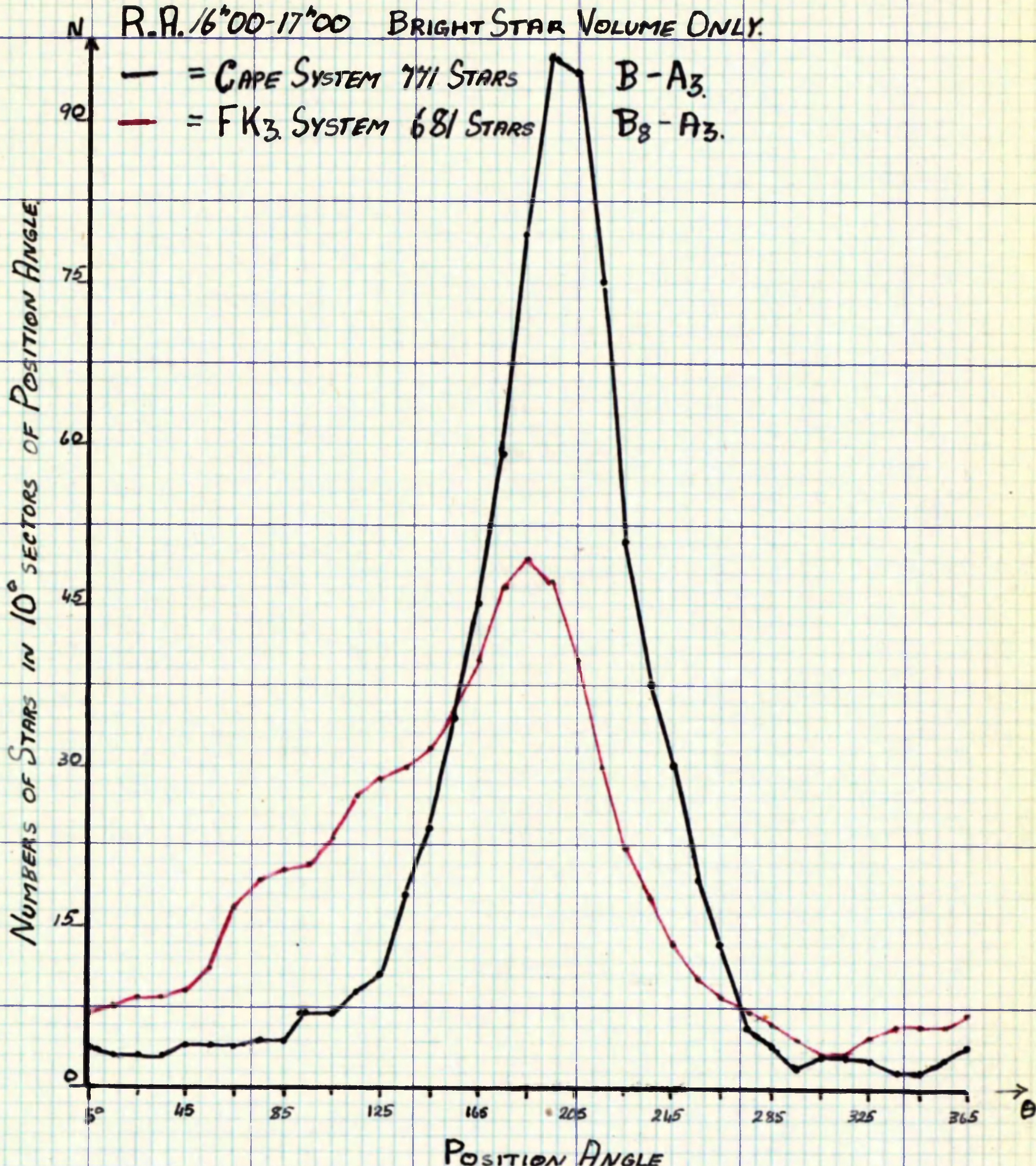

N
R.A. 16h00-17h00 BRIGHT STAR VOLUME ONLY.
= CAPE SYSTEM 771 STARS B-A3.
= FK3. SYSTEM 681 STARS B8-A3.
NUMBERS OF STARS IN 10° SECTORS OF POSITION ANGLE.
90
75
60
45
30
15
0
5°
45
85
125
166
206
245
285
325
365
θ
POSITION ANGLE.

FIGURE. III

believe that the systems of the Cape catalogues, the G.C. or FK3 are seriously in error and as the departures of the stream constants from the accepted values - which there is no reason to believe to be erroneous - are so marked, the simplest explanation perhaps is that the Cape - G.C. corrections are at fault, these being perhaps the least well determined. One reason for this is that they are to a great extent based on the brighter stars of the Cape catalogue and these are those which are most likely to be in error. It must not, however, be neglected that the G.C. - FK3 corrections are also based on bright stars - even brighter than those of the Cape - G.C. reductions, the FK3 being a very much smaller catalogue than the G.C.

There are one or two points in the derivation of the Cape - G.C. corrections by Williams which may require more detailed examination. One step in the derivation, about which the present author has had increasing doubts in the course of this investigation, is the reduction of the observed seventh magnitude differences in R.A. to the system of the eighth magnitude differences; as performed by Williams, namely the fitting of a smooth curve to the observed values of $\Delta\mu_8 - \Delta\mu_7$. A second doubt arose after discussions with Dr. T.R.Tannahill. He offered to commence an investigation into the nature and derivation of the corrections after he had completed the investigation in the performance of which he was then engaged.

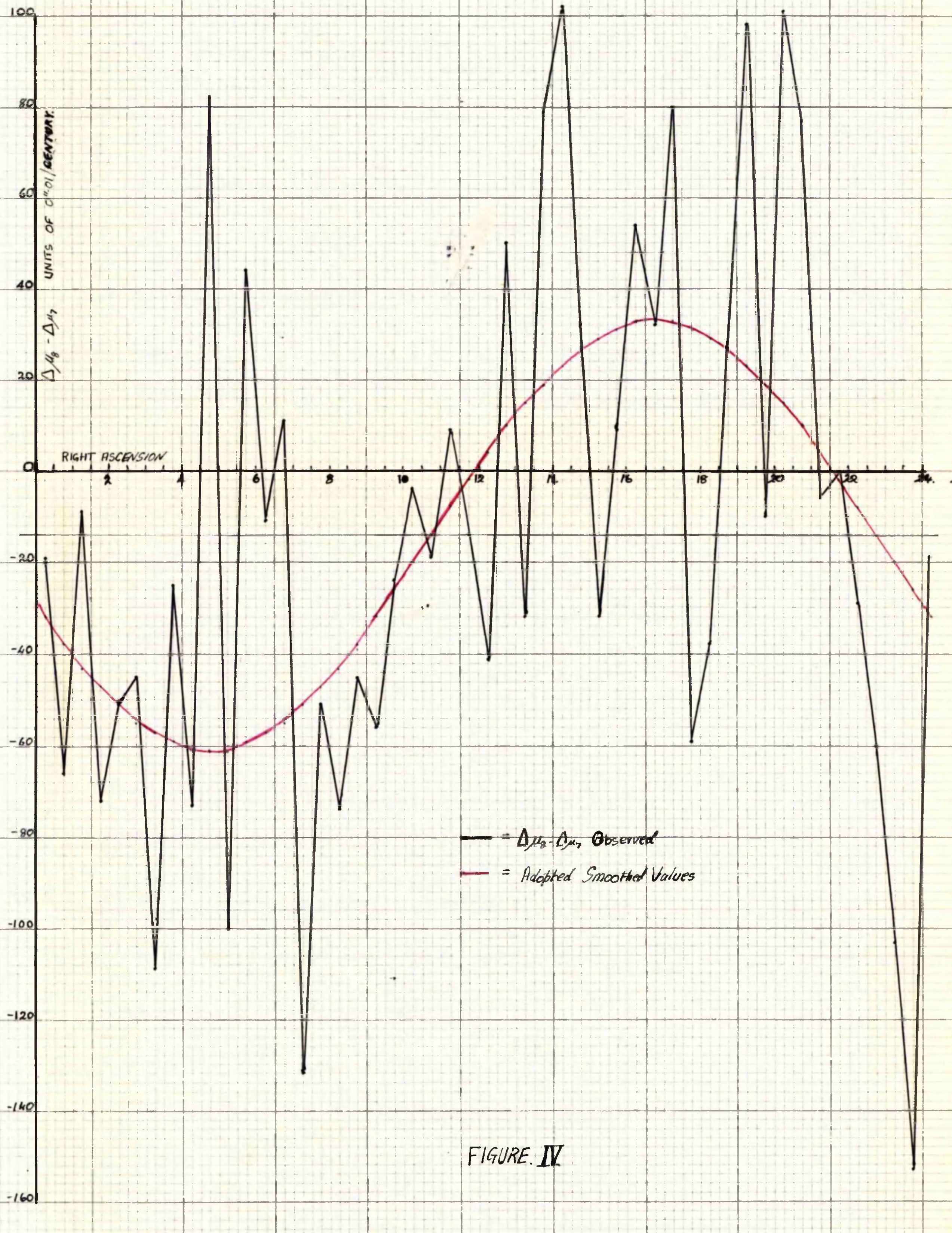
As a preliminary, in June 1954, the present author and Dr. Tannahill identified rather more than 3 500 stars common to the G.C. and the Cape Astrographic Zone catalogues as against 2 100 found by Williams - it now appears that these 2 100 refer to those stars with probable errors less than one second in either co-ordinate and that if a star had a probable error of less than one second in one co-ordinate and more than that in the other, then it was not used. The stars were identified by finding these stars between declinations $-39^{\circ} 20'$ and $-52^{\circ}40'$ in the G.C., noting their H.D. number, obtaining their 1900 co-ordinates from the H.D. catalogues and then identifying them in the Cape catalogues by their co-ordinates, spectral types and magnitudes. The increased number of stars so found has necessarily increased the scope of the investigation and the results are not yet to hand. The general aim decided upon is to first reproduce Williams' figures and find their standard deviations and to determine the effect of the omissions. One feature of Williams' corrections is that at times but four stars or so are used to give the value of a mean difference. It is not possible to say to what extent a change in these corrections will affect the results of the subsequent analysesperformed by Vyssotsky and Williams, as,although the Cape material was given twice the weight of their own material, it but contributes 96 regions to their general solution as against 530 from the McCormick material, which thus determines the solutions.

In figure IV the observed values of $\Delta\mu_8 - \Delta\mu_7$ are plotted against R.A. as are the smooth values used by Williams and taken from column 2 of table 3.II of her paper. It can be seen that only a dozen of the observed points lie close to the smooth curve. From the figure, it appears that the smooth curve is given by:-

$$\Delta\mu_8 - \Delta\mu_7 \quad =) \quad -0''.14 \; + 0''.47 \,\mathrm{Sin}(\alpha - 10^h\, 45^m) \quad .....(2)$$

In figures V and VI, the observed and the adopted values of $\Delta\mu_8$ and $\Delta\mu_7$ respectively are plotted against R.A.. It appears that $\Delta\mu_8$ method by which the adopted values of $\Delta\mu_8$ and $\Delta\mu_7$ were derived gives values agreeing fairly well with the observed values but there are quite large discrepancies at times. There is some indication that these values could be well represented by a smooth variation with R.A. and that departures from the curve are accidental. To derive any such curve, it would, however, be advantageous, if not essential to know the accuracy ofthe values of the Cape - G.C. differences.

In figure VII are drawn the final'smoothed' values of the corrections for the eighth magnitude stars on the FK3 system and the adopted values of the eighth magnitude Cape-G.C. differences. The method by which the FK3 values were smoothed was not stated by Williams. It is evident from these curves that the adopted Cape-FK3 corrections consist almost entirely of the Cape- G.C. corrections and that there is only a small contribution from the G.C. - FK_3 reduction. The mean value of



$\Delta\mu_8 - \Delta\mu_7$ UNITS OF $0''.01$/CENTURY.
RIGHT ASCENSION
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
2
4
6
8
10
12
14
16
18
20
22
24
= $\Delta\mu_8 - \Delta\mu_7$ Observed
= Adopted Smoothed Values

FIGURE. IV

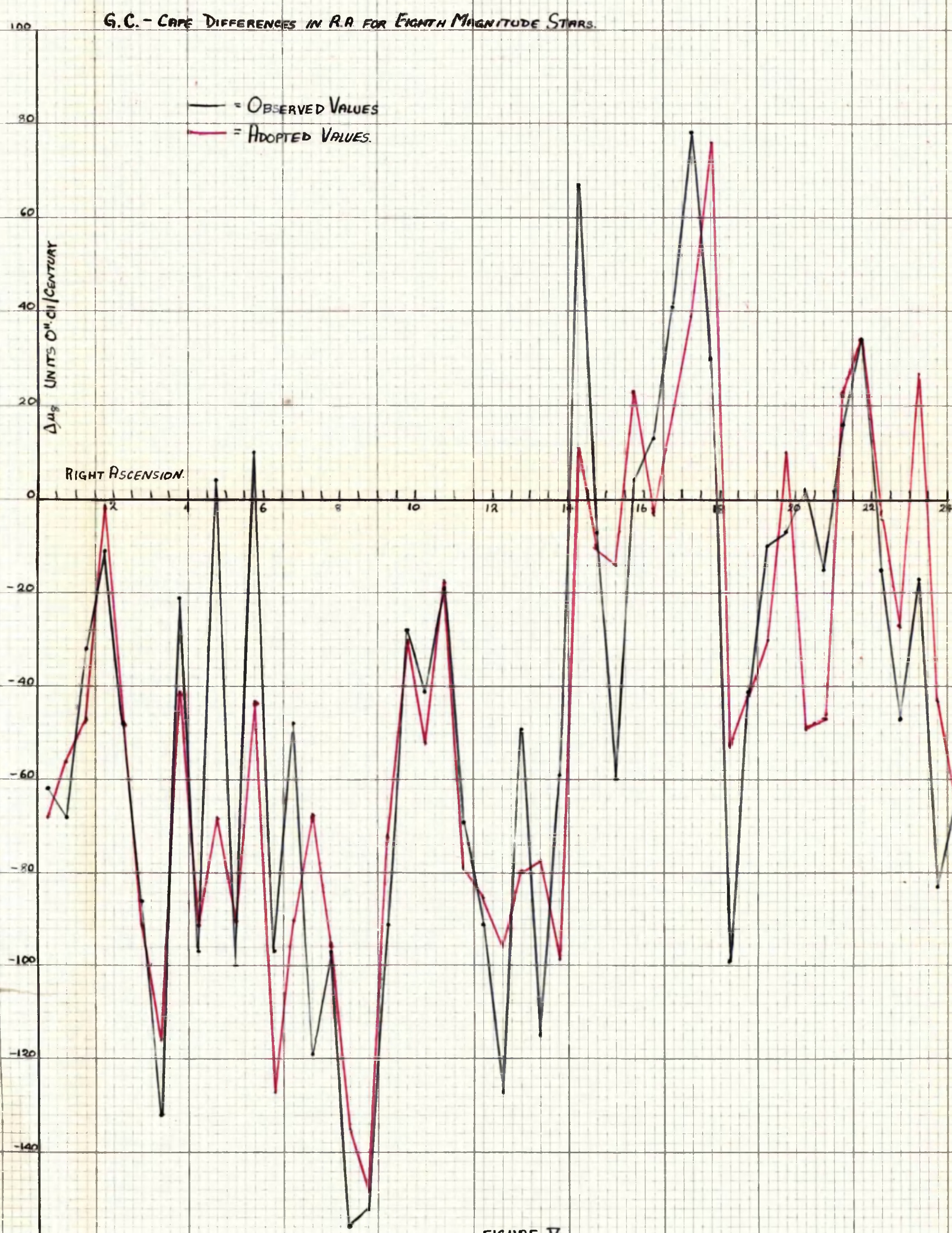

G.C. - Cape Differences in R.A for Eighth Magnitude Stars.
= Observed Values
= Adopted Values.
$\Delta\mu_8$ Units $0''.01$/Century
Right Ascension.
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
2
4
6
8
10
12
14
16
18
20
22
24

FIGURE V.

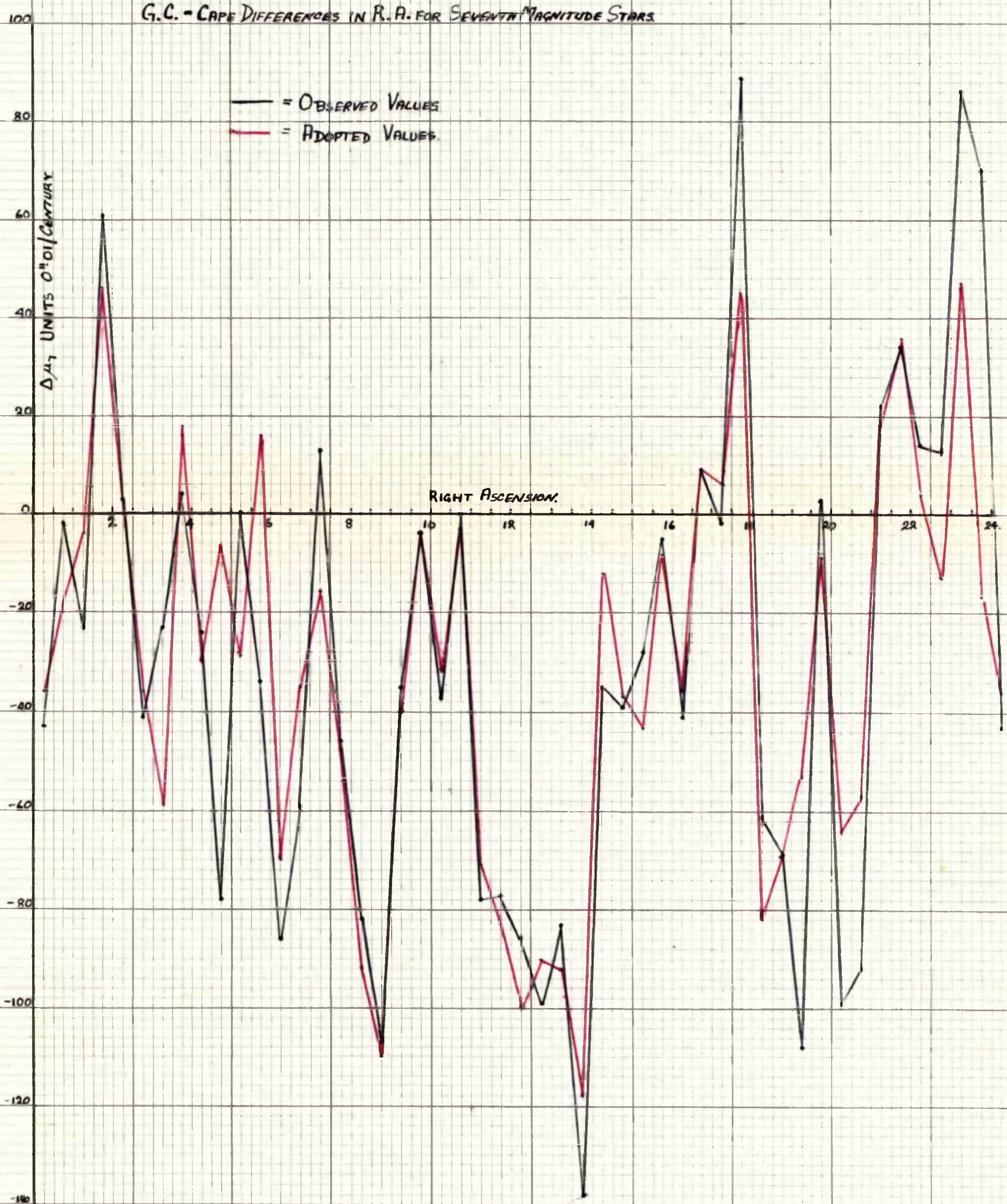
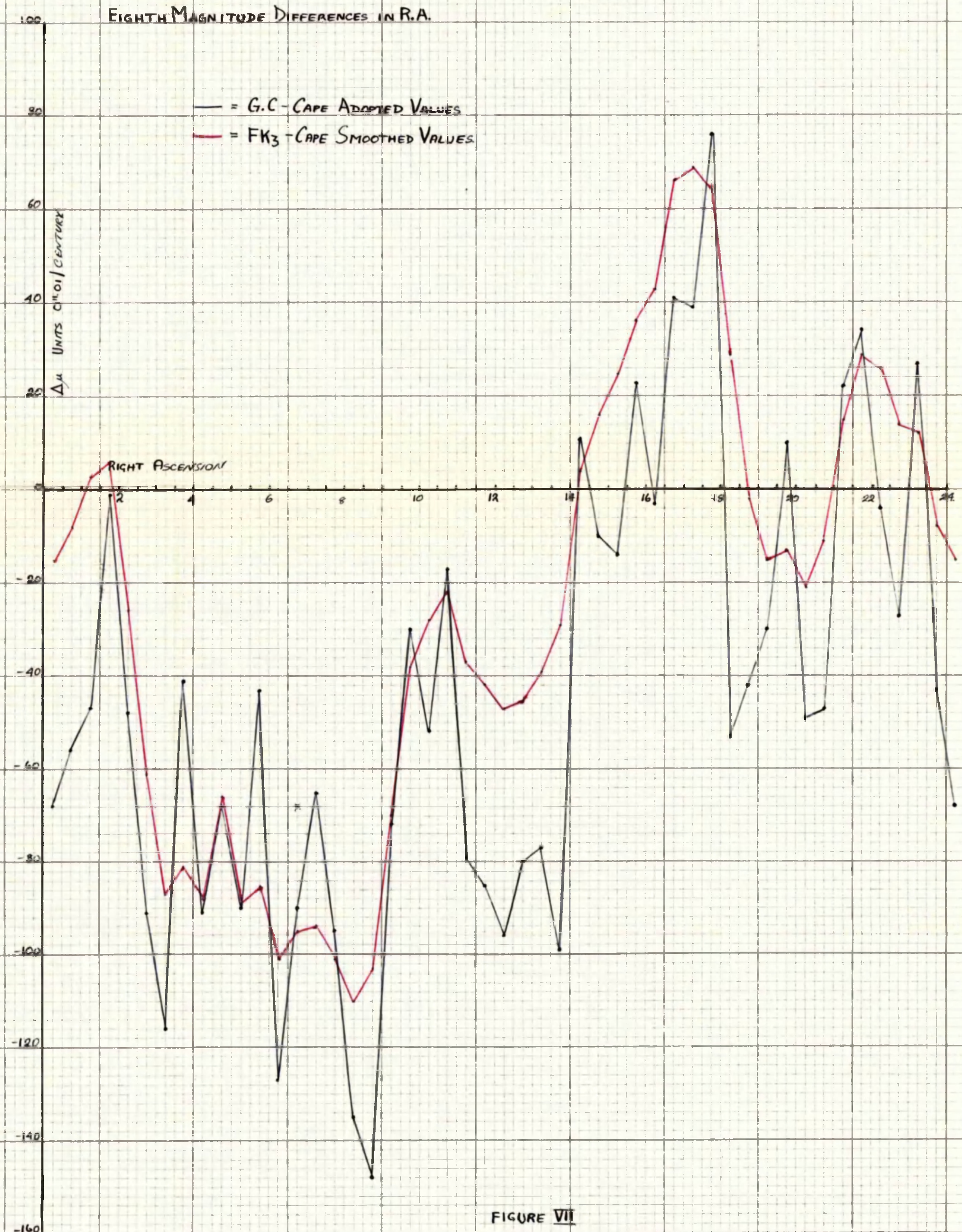
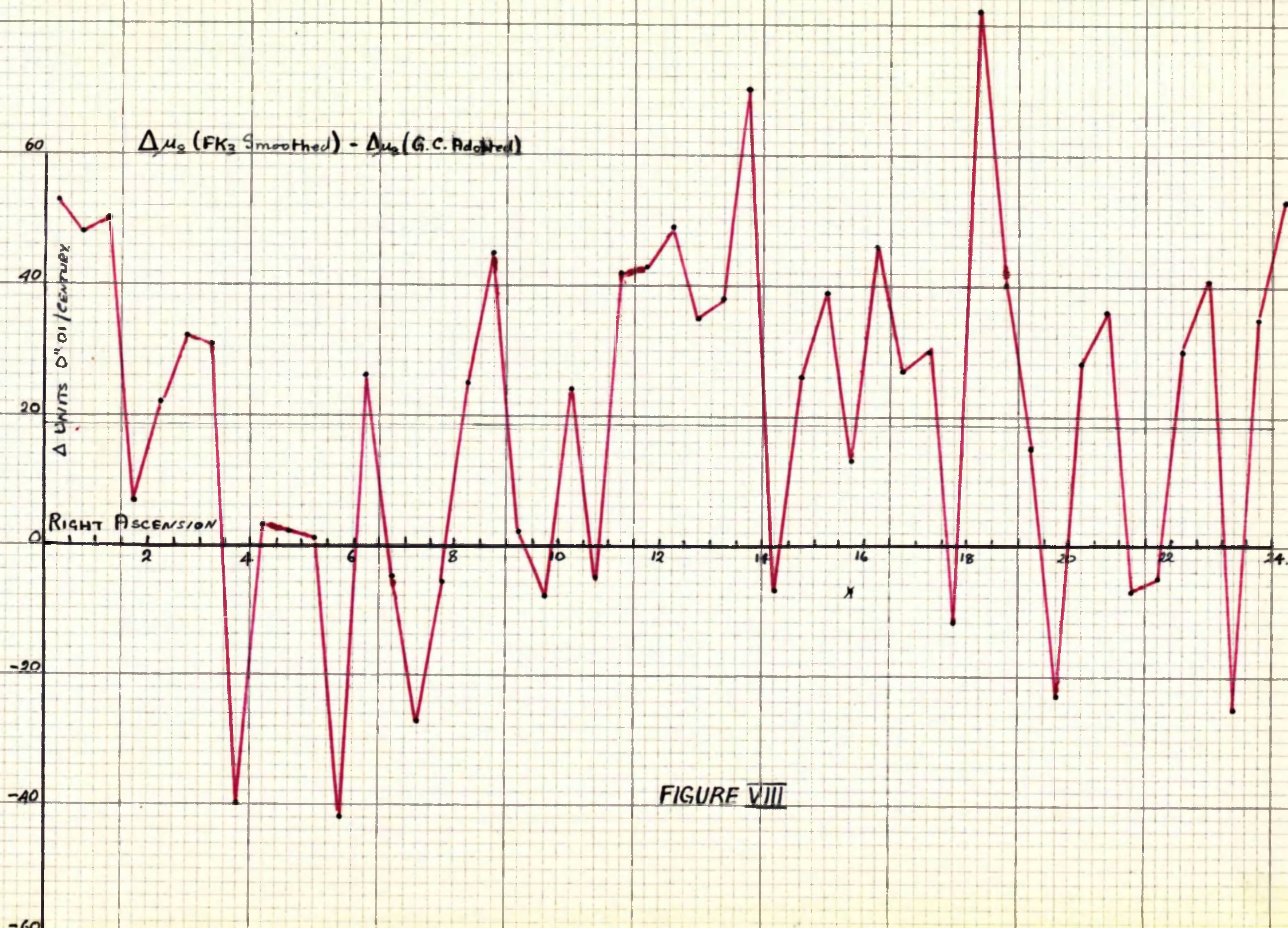

G.C. - CAPE DIFFERENCES IN R.A. FOR SEVENTH MAGNITUDE STARS
— = OBSERVED VALUES
— = ADOPTED VALUES
Δμ7 UNITS 0".01/CENTURY
RIGHT ASCENSION
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
2
4
6
8
10
12
14
16
18
20
22
24

FIGURE VI



Eighth Magnitude Differences in R.A.
= G.C - Cape Adopted Values
= FK3 - Cape Smoothed Values
$\Delta\mu$ Units 0s.01/Century
Right Ascension
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
2
4
6
8
10
12
14
16
18
20
22
24

FIGURE VII



$\Delta\mu_\delta$ (FK3 Smoothed) - $\Delta\mu_\delta$ (G.C. Adopted)
Δ UNITS 0".01/CENTURY
RIGHT ASCENSION
60
40
20
0
-20
-40
-60
2
4
6
8
10
12
14
16
18
20
22
24.

FIGURE VIII

the Cape- G.C. correction is $-0''.44$ and of the Cape - FK3 correction is $-0''.25$, so that the G.C. - FK3 reduction thus amounts to $0''.19$, which is less than half the Cape-G.C. correction. In figure XIII, the differences between the last two curves are plotted against the R.A. - these are in effect the adopted G.C. - FK3 corrections. There is a great degree of scatter evident, but whether this scatter is about a mean value of $0''.19$ or about a smooth curve it is difficult to say. It would be possible to represent these corrections by a curve of the form $0''.19 + 0''.30 \sin(\alpha - 11^h\ 00^m)$ with reasonable accuracy and possibly without increasing the mean errors and similarly the adopted Cape- G.C. corrections could be represented to a similar degree of accuracy by an expression of the approximate form

$$-0''.44 + 0''.66 \sin(\alpha - 13^h\ 00^m) .$$

We would then obtain the final Cape - FK3 corrections to be:- $-0''.25 + A.\sin\alpha + B.\cos\alpha$, A and B being constants. These might possibly be as accurate as the corrections given by Williams. A full investigation would, however, be necessary, such as Dr. Tannahill's. It does appear from the above that, if the order of magnitude of the corrections and the form of their variation with R.A. is as derived by Williams, then the periodic form given above may be equally applicable, with a similar form for the declination corrections. The major factor will be the accuracy of the mean differences derived from the Cape - G.C. comparisons and the effects of the inclusion of the stars omitted by Williams.

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CHAPTER IV

THE ANALYSIS OF THE PROPER MOTIONS OF THE FAINT STARS IN THE CAPE ASTROGRAPHIC ZONE ON THE CAPE SYSTEM.

1\. As it was found from the analyses described in the preceding chapter that the corrections applied to reduce the Cape proper motions to the FK3 system were to be regarded with suspicion as regards the determination of the constants of star-streaming, it was decided that in the investigations of the variations in the constants of star-streaming with spectral type - and possibly with magnitude - the proper motions would be used on the Cape system of reference. The aims of the investigation as stated in section 6 of chapter II, were not changed. They were:-

(i) to ascertain whether the low declination of the solar apex still obtains for the faint stars,

(ii) to comfirm, or deny, the variations of the constants of star-streaming with spectral type noted from the analyses of the stars of the 1936 volume, and

(iii) to determine whether there is any variation in the constants of star-streaming between the stars of the bright star volume and those of the faint star volume.

In the following investigations only the proper motions of the faint star volume were used.

2. From the scatter diagrams prepared for the analyses described in the previous chapter, the distribution counts were prepared. For these analyses the half-hour wide regions wree combined to give regions of an hour and a half's width in R.A. In combining the regions, a correction for the convergency of the meridians, of magnitude 5° , was included. These regions were choosen to give sufficient numbers of stars in the regions for analysis. Even so, it was necessary, in the case of the F stars, to omit four regions, because they contained toofew stars to yield curves amenable to accurate ahalysis. The same spectral groupings were used, as also were the unclassified stars.

The analyses were made for five groups - ALL stars, (B_8 - M, including Un. stars) ; A_5 - F_5 , F_8 - G5 , Ko - M and A_5 - M. No analysis was made for the A stars by themselves since but half of the regions would have yieled curves to the analysis of which any weight could have been attached. The distribution of the stars by spectral types and regions is given in Table I.

The analyses were performed both by the "Trial and Error" method and by Schwarzschild's automatic method. Before the analysis, the observed distribution counts were smoothed bky taking running means of three successive sectors. The details of the analyses for each region and group are given in Tables II to XI. It will be noted that in tablkes II to VI, fractions of a star occur. This is because it was found more convenient to use as the number of stars in a sector of position angle the actual number of stars plus the numbers in the sectors on either side of it rather than a third of this total.

From the analyses for each region, the elements of the drift motions, the relative velocity of the drifts and the solar motion were derived in the usual way as were the corresponding quantities for the velocity ellipsoid. These are given in Tables XII to XIV. In Table XV the results from the drift analyses and those from the ellipsoidal analyses are compared for the quantities in common.

Finally, from the elements found above for the position angles of the drifts in each region and their velocities, the position angles of the major axis of the velocity ellipsoid and of the solar motion were calculated and compared with the observed values. These are given in Tables XVI to XXI.

The value of these latter tables is that from them the agreement between the observed and the calculated values of the constants of a region can be evaluated. For Drift I the agreement is quite satisfactory, the position angles having normally been determined to the nearest 5° and the velocities to the nearest 0.1 unit. For Drift II, there is a wider departure of the observed from the calculated values. Generally, the determniations are not quite as good as Smart and Tannahill's (1,2) analyses of the bright stars. This is possibly because

TABLE I

Region.	R.A.	ALL	F	G	K	F-K
1	$0^h 45^m$	422	39	173	79	291
2	2 15	394	38	122	84	244
3	3 45	525	73	160	128	361
4	5 15	630	122	237	164	523
5	6 45	1175	251	297	293	841
6	8 15	2137	435	302	345	1082
7	9 45	2437	494	455	431	1380
8	11 15	1604	324	469	381	1174
9	12 45	1526	271	488	407	1166
10	14 15	1519	275	466	375	1116
11	15 45	2308	379	484	428	1291
12	17 15	2419	353	441	464	1258
13	18 45	955	151	315	265	731
14	20 15	627	89	264	196	549
15	21 45	600	61	264	207	532
16	23 15	561	50	231	158	439
TOTAL		19 839	3405	5168	4405	12 978

F = Spectral Group A_5 - F5

G = " " F8 - G5

K = " " Ko - M

F-K= " " A_5 - M

ALL= " " B8 - M

TABLE II
DRIFT ANALYSES - ALL STARS

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	1 1.35	85°	206	0.65	210°	216
2	1.0	70	264.7	0.90	215	129.3
3	1.0	40	253.7	0.60	175	271.3
4	1.0	20	253.7	0.25	180	376.3
5	1.0	340	600.3	0.3	230	574.7
6	1.2	305	1354.3	0.2	210	782.7
7	1.5	290	1601.3	0.45	210	835.7
8	1.6	275	1075	0.5	170	529
9	1.5	260	1006.7	0.35	160	519.3
10	1.4	240	1124	0.6	150	395
11	1.5	222.5	1555.3	0.5	170	752.7
12	1.5	197.5	1715.7	0.5	155	703.3
13	1.3	175	598	0.3	200	357
14	1.0	157.5	350.3	0.3	180	276.7
15	1.4	115	307	0.6	215	293
16	1.5	100	247	0.5	200	314

TABLE III
A_5 - F_5

4	0.7	0°	87.7	0.7	215°	34.3
5	1.1	335	122.7	0.5	240	128.3
6	1.1	305	317	0.2	180	118
7	1.6	287.5	343	0.5	200	151
8	1.7	270	201.3	0.5	180	122.7
9	1.4	255	190	0.5	165	81
10	1.5	245	171.7	0.7	200	103.3
11	1.5	220	233.3	0.4	175	145.7
12	1.9	195	209	0.55	150	144
13	1.4	170	85.3	0.4	175	65.7
14	1.0	150	51.7	0.4	240	37.3
15	1.2	120	39.7	0.8	240	21.3

TABLE IV

F8 - G5

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	1.1	90°	89.7	0.5	200°	83.3
2	0.9	60	73.3	0.8	230	48.7
3	0.8	30	94.7	0.9	190	65.3
4	0.9	20	113.3	0.5	160	123.7
5	1.1	335	153.3	0.4	210	143.7
6	1.2	305	164.7	0.4	190	137.3
7	1.7	295	222.3	0.45	195	232.7
8	1.6	276.25	298.7	0.55	160	170.3
9	1.5	257.5	305.3	0.4	185	182.7
10	1.6	237.5	308.7	0.6	140	157.3
11	1.5	225	336.3	0.7	160	147.7
12	1.5	195	286	0.5	160	155
13	1.4	175	175.3	0.4	200	139.7
14	1.1	160	175.3	0.1	190	88.7
15	1.3	115	152.3	0.7	230	111.7
16	1.2	110	129	0.6	220	102

TABLE V

Ko - M

1	0.9	90°	44.3	0.9	220°	34.7
2	0.9	60	52.3	0.8	220	31.7
3	0.9	40	65	0.5	165	63
4	1.1	20	66.7	0.3	170	97.3
5	0.8	335	166.3	0.3	255	126.7
6	1.1	305	209	0.5	190	136
7	1.5	290	257.3	0.3	190	173.7
8	1.3	275	285	0.6	160	96
9	1.6	262.5	258.7	0.5	170	148.3
10	1.5	242.5	242.3	0.4	180	132.7
11	1.5	222.5	256.3	0.4	180	171.7
12	1.5	195	254	0.4	230	210
13	1.5	185	108.7	0.3	180	156.3
14	0.9	150	113.3	0.5	230	82.7
15	1.2	120	91.3	0.5	220	115.7
16	1.1	100	97.7	0.8	235	60.3

TABLE VI

A_5 - M

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	1.2	85°	118.3	0.4	200°	172.7
2	0.9	57.5	157.3	0.8	220	86.7
3	0.7	40	224.3	0.7	190	136.7
4	0.9	20	263.7	0.4	185	259.3
5	1.0	335	467	0.4	220	374
6	1.2	310	649	0.5	210	433
7	1.5	290	846.3	0.4	190	533.7
8	1.5	272.5	772	0.5	160	402
9	1.5	257.5	777.7	0.5	180	388.3
10	1.5	242.5	709	0.5	175	407
11	1.6	221.25	796	0.3	180	495
12	1.5	195	773	0.5	185	485
13	1.5	175	377.3	0.3	205	353.7
14	1.1	155	338.7	0.3	240	210.3
15	1.3	115	280	0.6	220	252
16	1.2	102.5	231.3	0.5	200	207.7

TABLE VII

ELLIPSOIDAL ANALYSIS - ALL STARS

R	k/h	θ_0	hS	θ_1
1	0.570	67°.1	0.536	292°.7
2	581	48 .6	523	263 .6
3	654	22 .1	382	257.4
4	708	12 .0	314	208 .1
5	683	344 .6	449	140 .4
6	703	324 .4	0.805	118 .4
7	599	306 .3	1.022	100.8
8	551	290 .1	1.087	86 .0
9	656	276.0	1.076	71 .9
10	667	265 .0	1.116	54 .4
11	668	237 .1	1.090	35 .0
12	807	204 .4	1.154	12..8
13	816	170 .8	0.957	357 .7
14	799	132 .2	660	343 .0
15	649	92 .2	699	321 .3
16	0.598	83 .9	680	304 .7

TABLE VIII
$A_5 - F_5$

R	k/h	θ_0	hS	θ_1
4	0.745	19°.1	0.385	178°.9
5	700	0 .5	0.490	132 .0
6	751	318 .9	0.740	119 .2
7	532	306 .2	1.098	98 .8
8	631	291 .3	1.115	82 .8
9	624	275 .0	0.956	69 .7
10	667	281..3	1.015	55 .0
11	555	240 .6	1.094	29 .8
12	882	197 .2	1.465	10 .3
13	909	177 .4	0.909	349 .9
14	816	135 .3	0.595	329 .3
15	0.944	105 .5	0.843	306 .7

TABLE IX
$F_8 - G_5$

1	0.707	65°.0	0.529	290°.6
2	459	50 .4	0.442	256 .2
3	588	19 .7	0.292	244 .5
4	725	8 .9	0.250	220.0
5	646	346 .1	0.388	140 .6
6	587	325 .0	0.518	105 .0
7	526	312 .1	0.852	95 .0
8	497	294 .8	1.000	80 .9
9	575	277 .1	1.084	69 .4
10	631	264 .1	0.874	50 .4
11	637	234 .1	1.307	36 .0
12	719	204 .1	1.317	11 .7
13	617	161 .3	1.046	0 .8
14	726	150 .0	0.745	341 .3
15	575	88 .8	0.731	319 .7
16	0.732	88 .2	0.671	310 .8

TABLE X

Ko - M

R	k/h	θ_0	hS	θ_1
1	0.534	67°.5	0.431	320°.0
2	619	53.9	0.444	248.2
3	649	13.5	0.4444	261.0
4	657	17.7	0.322	213.2
5	536	334.2	0.483	150.0
6	638	327.5	0.949	122.9
7	599	302.5	0.891	103.3
8	593	289.2	0.932	88.4
9	650	276.7	1.086	74.8
10	677	261.8	0.962	61.4
11	785	238.3	1.064	39.0
12	848	222.5	1.116	17.7
13	798	176.8	0.736	3.9
14	783	124.0	0.641	348.3
15	713	104.0	0.605	326.4
16	0.538	86.7	0.493	308.1

TABLE XI

A5 - M

1	0.670	69°.5	0.443	294°.3
2	608	49.0	0.511	242.0
3	685	22.4	0.317	250.8
4	721	13.6	0.324	205.0
5	688	347.4	0.504	137.5
6	655	323.6	0.727	115.0
7	575	307.6	0.968	99.3
8	574	292.8	1.036	85.0
9	510	276.8	1.062	71.0
10	662	266.5	1.041	51.1
11	688	237.7	1.073	35.0
12	810	204.0	1.165	13.4
13	798	167.7	0.888	359.2
14	798	138.0	0.693	341.2
15	659	93.8	0.689	320.4
16	0.587	86.7	0.591	308.1

TABLE XII

DRIFT CONSTANTS

DRIFT I

	ALL STARS		A5 – F5		F8 – G5	
X_1	-0.013	± 0.053	-0.000	± 0.072	0.001	± 0.074
	(0.052	0.035)	(-0.028	0.054)	(-0.019	0.042)
Y_1	1.486	0.053	1.548	0.074	1.449	0.074
	(1.446	0.032)	(1.677	0.041)	(1.533	0.038)
Z_1	-0.190	0.054	-0.296	0.092	-0.226	0.076
	(-0.217	0.044)	(-0.220	0.067)	(-0.136	0.053)
A_1	90°.5	2°.0	90°.0	2°.6	90°.0	3°.0
	(88°.0	1°.4)	(91°.0	1°.8)	(90°.7	1°.6)
D_1	- 7°.3	2°.1	-10°.8	4°.5	- 8°.9	2°.4
	(- 8°.5	0°.6)	(- 7°.5	2°.3)	(- 5°.1	2°.0)
hW_1	1.498	0.053	1.576	0.074	1.615	0.068
	(1.463	0.035)	(1.692	0.042)	(1.539	0.038)
N_1	12513		2052.3		3078.3	
	(10745)		(1760.0)		(2015.0)	

DRIFT II

	ALL STARS		A5 – F5		F8 – G5	
X_2	0.037	± 0.067	0.081	± 0.083	0.056	± 0.070
	(0.093	0.029)	(0.182	0.033)	(0.046	0.028)
Y_2	-0.108	0.067	-0.064	0.086	-0.189	0.070
	(-0.140	0.026)	(-0.141	0.030)	(-0.139	0.025)
Z_2	-0.603	0.068	-0.583	0.107	-0.666	0.071
	(-0.628	0.036)	(-0.619	0.040)	(-0.670	0.034)
A_2	289°.0	33°.3	321°.7	40°.0	286°.5	20°.5
	(303°.6	9°.5)	(322°.2	7°.7)	(288°.4	10°.9)
D_2	-79°.0	7°.1	-80°.0	19°.2	-74°.1	5°.9
	(-75°.0	2°.4)	(-69°.4	2°.9)	(-77°.7	2°.2)
hW_2	0.614	0.068	0.592	0.106	0.694	0.023
	(0.650	0.035)	(0.661	0.039)	(0.6868	0.034)
N_2	7326		1152.7		2089.7	
	(7578)		(1503.0)		(1941.0)	

TABLE XII (Cont.)

DRIFT CONSTANTS

DRIFT I

	Ko	± M	A5	± M
X_1	-0.036	0.078	0.013	0.068
	(-0.029	0.052)	(0.022	0.045)
Y_1	1.369	0.078	1.433	0.068
	(1.442	0.047)	(1.538	0.041)
Z_1	-0.212	0.080	-0.246	0.069
	(-0.099	0.064)	(-0.144	0.056)
A_1	91°.5	3°.3	89°.5	2°.7
	(91°.1	2°.1)	(90°.8	1°.7)
D_1	- 8°.8	3°.3	- 9°.7	2°.7
	(- 3°.9	2°.5)	(- 5°.3	2°.1)
hW_1	1.385	0.078	1.454	0.067
	(1.446	0.046)	(1.545	0.041)
N_1	2568.3		7781	
	(2191.0)		(6002)	

DRIFT II

	Ko	± M	A5	± M
X_2	-0.047	0.079	0.024	0.058
	(0.040	0.037)	(0.092	0.024)
Y_2	-0.188	0.079	-0.145	0.058
	(-0.086	0.033)	(-0.131	0.022)
Z_2	-0.585	0.081	-0.604	0.058
	(-0.587	0.046)	(-0.649	0.030)
A_2	256°.0	23°.4	279°.5	22°.4
	(295°.2	21°.2)	(304°.9	8°.4)
D_2	-71°.6	7°.4	-76°.3	5°.3
	(-80°.4	3°.1)	(-76°.2	2°.0)
hW_2	0.616	0.084	0.622	0.058
	(0.595	0.046)	(0.668	0.029)
N_2	1836.7		5197	
	(2030.0)		(5446)	

TABLE XIII

THE VERTEX OF STAR-STREAMING AND THE SOLAR MOTION

	ALL STARS	A5 - F5	F8 - G5	Ko - M	A5 - M
A_v	271°.8 ± 2°.3	272°.9 ± 3°.9	271°.9 ± 3°.6	269°.6 ± 4°.1	270°.4 ± 3°.2
	(271°.5 1°.6)	(270°.6 2°.0)	(272°.2 1°.7)	(272°.6 2°.4)	(273°.8 1°.8)
D_v	-14°.5 1°.4	-10°.1 3°.1	-15°.0 3°.5	-13°.5 4°.0	-12°.8 3°.4
	(-14°.5 2°.0)	(-12°.3 2°.5)	(-18°.2 2°.2)	(-17°.7 3°.0)	(-16°.8 2°.2)
G_v	343°.4	347°.8	343°.0	343°.2	344°.2
g_v	+0°.1	+1°.5	-0°.1	+2°.6	+2°.0
(G_v)	(343°.2)	(344°.5)	(340°.2)	(340°.6)	(342°.0)
(g_v)	(+0°.5)	(+2°.8)	(-1°.2)	(-1°.4)	(-1°.8)
Ω	1.648±0.041	1.639±0.044	1.697±0.075	1.601±0.084	1.618±0.100
	(1.638 0.043)	(1.873 0.052)	(1.756 0.049)	(1.606 0.060)	(1.747 0.048)
A_o	269°.6	268°.3	268°.6	273°.2	268°.8
	(265°.1)	(265°.3)	(268°.9)	(269°.5)	(267°.5)
D_o	25°.4	22°.4	25°.5	27°.8	25°.9
	(26°.1)	(25°.6)	(29°.2)	(25°.3)	(27°.3)
hU_o	0.800	1.047	0.972	0.808	0.891
	(0.879)	(0.934)	(0.817)	(0.780)	(0.838)
$\frac{N_1}{N_2}$	1.708	1.781	1.473	1.398	1.497
	(1.410)	(1.173)	(1.038)	(1.080)	(1.102)

TABLE XIV

ELLIPSOIDAL CONSTANTS AND THE SOLAR MOTION.

	ALL STARS	A5 - F5	F8 - G5	Ko - M	A5 - M
A_v	271°.6 ± 2°.0 (270°.2)	278°.7 ± 4°.3 (276°.6)	270°.7 ± 3°.6 (272°.6)	269°.6 ± 2°.6 (268°.5)	271°.6 ± 2°.6
D_v	-17°.2 2°.0 (-13°.4)	-15°.4 5°.2 (-11°.7)	-15°.5 3°.5 (-16°.4)	-19°.7 2°.6 (-16°.2)	-16°.2 2°.5
G_v	340°.0 (343°.6)	345°.8 (348°.1)	342°.0 (342°.1)	337°.8 (340°.3)	341°.8
g_v	+0°.8 (+2.1)	-6°.0	+0°.6	-0°.4	-0°.3
$\frac{K}{\bar{H}}$	0.602 ± 0.036 (0.625)	0.676 ± 0.072 (0.552)	0.554 ± 0.075 (0.56)	0.606 ± 0.046 (0.65)	0.601±0.0
A_o	271°.5 ± 4°.2 (266°.9)	266°.9 ± 3°.5 (266°.0)	271°.2 ± 4°.4 (269°.9)	275°.4 ± 5°.5 (272°.3)	269°.5 ± 4°.6
D_o	25°.0 3°.9 (28°.3)	19°.8 4°.1 (23°.2)	30°.9 3°.8 (31°.6)	24°.2 5°.3 (31°.8)	29°.5 4°.2
hU_o	0.948 ± 0.06 (0.94)	1.056±0.064 (1.08)	0.950±0.063 (0.89)	0.831±0.074 (0.82)	0.819±0.065

TABLE XV

COMPARISON OF TWO-DRIFTS AND ELLIPSOIDAL VERTICES AND SOLAR MOTION.

	G_v		g_v		A_o		D_o		hU_o	
	T.D.	Ell.	T.D.	Ell.	T.D.	Ell.	T.D.	Ell.	T.D.	Ell.
ALL	343°.4	340°.0	+0°.1	+0°.8	269°.6	271°.5	+25°.4	+25°.0	0°.80	0°.95
F	347°.8	345°.8	1°.5	-6°.0	268°.3	266°.9	22°.4	19°.8	1.05	1.06
G	343°.0	342°.0	-0°.1	0°.6	268°.6	271°.2	25°.5	30°.9	0.97	0.95
K	343°.2	337°.8	2°.6	-0°.4	273°.2	275°.4	27°.8	24°.2	0.81	0.83
F-K	344°.2	341°.8	2°.0	-8°.3	268°.8	269°.5	25°.9	29°.5	0.89	0.82

TABLE XVI

ALL STARS

R	λ_1	$\Delta\theta_1$	$\Delta(hV_1)$	λ_2	$\Delta\theta_2$	$\Delta(hV_2)$
1	77°.3	+2°.3	+0.11	43°.9	-14°.6	-0.22
2	61.9	+0.1	+0.32	47.9	-21.0	-0.44
3	48.5	+8.2	+0.12	51.3	+15.8	-0.12
4	39.8	-1.6	-0.04	53.7	+ 6.6	+0.25
5	39.6	+3.2	-0.04	54.8	-48.5	+0.20
6	48.0	+8.0	-0.09	54.6	-33.4	+0.30
7	61.3	+0.8	-0.19	53.0	-38.2	+0.04
8	76.6	-1.8	-0.14	50.2	- 2.1	-0.03
9	92.2	-2.8	0	46.5	+ 5.3	+0.10
10	106.7	0	0.04	42.4	+14.6	-0.19
11	118.7	-3.0	-0.19	38.4	- 3.7	-0.12
12	125.9	-3.1	-0.29	35.2	+17.1	-0.15
13	126.0	-8.2	-0.09	33.6	-22.4	+0.04
14	119.2	-16.0	+0.31	34.0	+ 5.0	+0.04
15	107.3	+5.8	+0.03	36.2	-24.0	-0.24
16	92.9	+3.5	0	39.8	- 5.4	-0.11

TABLE XVII

A5 - F5

4	36°.3	+17°.9	+0.23	49°.4	-23°.2	-0.25
5	36.3	+ 7.1	-0.17	52.1	-51.8	-0.03
6	45.3	+ 4.8	+0.02	53.8	+ 3.8	+0.28
7	59.0	+ 0.5	-0.25	54.2	-21.0	-0.02
8	74.5	+ 0.1	-0.18	53.3	- 5.6	-0.03
9	90.0	- 0.9	+0.18	51.2	+ 5.2	-0.04
10	104.3	- 3.3	+0.03	48.1	-32.8	-0.26
11	115.8	- 2.7	-0.08	44.3	- 9.4	+0.01
12	122.5	- 2.4	-0.57	40.5	+16.2	-0.17
13	122.5	- 2.6	+0.07	37.1	- 5.8	+0.04
14	115.8	- 7.3	+0.42	34.8	-65.4	-0.06
15	104.3	- 1.7	+0.33	34.3	-58.6	-0.47

$$\Delta\theta_1 = (\theta_1)_{calc.} - (\theta_1)_{obs.}$$

$$\Delta(hV_1) = (hV_1)_{calc.} - (hV_1)_{obs.}$$

TABLE XVIII

F8 - G5

R	λ_1	$\Delta\theta_1$	ΔhV_1	λ_2	$\Delta\theta_2$	ΔhV_2
1	75°.8	+2°.2	+0.47	45°.0	+2°.7	-0.01
2	66.8	+10.9	+0.58	50.8	-30.2	-0.26
3	46.9	+18.8	+0.30	55.5	+ 4.8	-0.33
4	38.1	- 2.7	+0.10	58.7	+28.6	+0.09
5	38.1	+ 7.7	-0.10	60.1	-28.3	+0.20
6	46.9	+ 6.2	-0.02	59.5	-15.4	+0.20
7	66.8	- 5.9	+0.22	57.2	-27.0	+0.13
8	75.8	- 4.0	-0.03	53.1	+ 2.4	+0.01
9	90.1	- 2.1	+0.11	47.8	-26.6	+0.11
10	105.8	+ 1.1	-0.03	41.8	+17.0	-0.14
11	117.5	- 6.8	-0.07	35.8	- 1.1	-0.29
12	124.4	- 2.0	-0.17	30.9	+ 5.6	-0.14
13	124.4	- 8.0	-0.07	28.4	-23.0	-0.07
14	117.5	-18.2	+0.33	29.3	- 0.5	+0.24
15	105.8	+ 6.4	+0.27	33.3	-31.4	-0.32
16	90.1	-17.8	+0.41	38.9	-17.3	-0.16

TABLE XIX

K5 - M

1	76°.9	-00°.8	+0.45	54°.1	-19°.4	-0.40
2	61.5	-12.0	+0.32	58.9	-25.6	-0.27
3	47.7	+10.4	+0.12	61.8	+22.0	-0.04
4	38.6	- 0.5	+0.24	62.6	+ 9.3	+0.25
5	37.9	+ 9.2	-0.05	61.2	-83.2	+0.24
6	46.2	+ 8.0	+0.10	57.9	-26.0	+0.02
7	59.5	+ 0.5	-0.31	52.7	-22.0	+0.19
8	74.8	- 4.7	+0.04	46.3	- 5.8	-0.15
9	90.4	- 5.3	-0.21	39.3	-16.8	-0.11
10	105.0	- 2.6	-0.16	32.5	-23.2	-0.07
11	117.0	- 2.7	-0.27	27.5	-13.4	-0.12
12	124.2	+ 0.3	-0.35	25.8	-48.9	-0.13
13	124.8	-16.7	-0.36	28.5	+16.4	-0.01
14	118.9	- 6.8	+0.41	34.1	-25.4	-0.15
15	106.8	+ 2.5	+0.13	41.0	-13.2	-0.10
16	92.4	+ 5.2	+0.28	48.0	-29.9	-0.34

TABLE XX

A5 - M

R	λ_1	$\Delta\theta_1$	ΔhV_1	λ_2	$\Delta\theta_2$	ΔhV_2
1	74.°9	+3.°4	+0.20	46.°3	- 0.°9	+0.05
2	59.5	+13.5	+0.35	51.1	-23.9	-0.32
3	45.9	+ 8.8	+0.34	54.8	+ 1.5	-0.19
4	37.2	- 2.3	-0.02	57.2	- 0.7	+0.12
5	37.5	+ 5.6	-0.12	57.8	-40.4	+0.13
6	46.4	+ 0.1	+0.15	56.9	-36.7	+0.02
7	60.2	- 1.9	-0.24	54.3	-22.2	+0.11
8	75.6	- 1.7	-0.09	50.4	+ 3.2	-0.02
9	90.1	- 2.7	-0.05	45.5	-19.1	-0.06
10	105.4	- 4.8	-0.10	40.9	-11.1	-0.09
11	117.0	- 4.0	-0.30	35.2	-11.5	+0.06
12	123.7	- 1.4	-0.29	31.7	-10.7	-0.17
13	123.5	- 9.0	-0.30	30.6	-24.2	+0.02
14	116.6	-13.3	+0.20	32.1	-49.5	+0.03
15	104.8	+ 6.5	+0.11	36.0	-22.9	-0.23
16	90.0	+ 1.9	+0.25	41.0	-20.3	-0.09

TABLE XXI

	ALL		F		G		K		F-K	
R	$\Delta\theta_o$	$\Delta\theta_i$	$\Delta\theta_o$	$\Delta\theta_i$	$\Delta\theta_o$	$\Delta\theta_i$	$\Delta\theta_o$	$\Delta\theta_i$	$\Delta\theta_o$	$\Delta\theta_i$
	°	°	°	°	°	°	°	°	°	°
1	+ 2.7	-11.0	+ 2		+ 6.8	- 4.0	+ 0.5	-37.9	+2.3	-10.1
2	+ 4.6	+ 2.5			+ 4.6	+16.0	- 2.2	+18.3	+6.3	+27.4
3	+11.9	-11.4			+16.1	+ 9.0	+19.4	-14.1	+13.7	+11.3
4	- 0.4	+ 2.5	+2.5	+18.4	+ 4.2	- 2.5	- 6.6	+ 2.4	+0.2	+ 6.0
5	+ 0.7	+15.6	-3.3	+ 8.9	+ 2.4	+ 8.3	+13.7	+16.6	+2.1	+ 9.0
6	+ 0.8	- 0.8	+14.4	- 0.9	+0 .6	+ 4.2	- 1.5	+ 3.1	+3.0	- 5.6
7	- 0.2	- 4.7	+ 6.3	- 1.2	- 6.1	- 5.4	+ 6.0	- 1.2	-0.5	- 7.8
8	- 0.4	- 5.8	+ 3.7	- 1.4	- 5.5	- 5.9	+ 2.1	- 3.4	-2.5	- 9.8
9	- 3.0	- 6.3	+ 4.1	- 3.5	- 3.4	- 8.2	- 0.7	- 4.7	-2.1	- 9.9
10	- 7.7	- 4.3	-18.3	- 5.4	- 7.3	- 4.0	- 2.4	- 5.8	-855	- 5.1
11	- 1.9	- 2.6	+ 2.9	+ 2.0	+ 0.9	- 6.3	- 0.8	- 2.1	-1.2	- 6.1
12	- 3.3	- 0.6	+18.2	- 1.8	- 1.8	- 0.7	-20.2	- 1.3	0	- 3.7
13	-13.4	- 7.0	- 2.2	- 4.6	- 0.9	- 9.6	-21.0	- 9.7	-6.3	-9.7
14	- 8.3	-12.9	+ 2.0	- 5.2	-23.2	- 9.0	- 3.0	-15.8	-10.9	-10.9
15	+ 9.8	- 9.2	+ 6.1	- 1.0	+16.7	- 4.4	- 2.7	-12.8	-10.7	- 6.3
16	+ 1.4	- 8.4			- 0.8	- 6.4	- 3.4	-10.9	-0.7	- 9.9

the smaller motions of the faint stars are more susceptible to error.

3\. As was the case with the bright stars, it was found possible in nearly every case to obtain a good fit between the observed and the calculated curves, although the fits were not, perhaps, as good as those for the bright stars. As mentioned earlier it was necessary to omit four regions for the F stars, as no weight could be attached to their analyses.

In table XXII, below, the positions of the Drift apices found from this investigation are compared with those from the bright star volume and with Tannahill's results (3) from his analysis of the motions of the G.C.

On comparing the positions of the Drift I apex from each group, it is evident that the pairs of values are so close together that the differences are accidental, and that it can be taken that the two volumes of the Cape proper motions give identical Drift I positions. The differences for the pairs of values of hW, are greater in comparison, but as the mean probable error of the bright star determinations is about 0.04 and that for the faint stars about 0.07, it seems that the differences are unlikely to be significant, especially as they do not appear to be systematic.

A similar conclusion appears probable for the Drift II determinations. Here the differences lie mainly in R.A. For these, however, the probable errors are large - of the order of 10° for the bright stars and 20° for the faint stars. - Thus again it can be taken that the apices are identical.

On comparing the Cape results with those from the G.C. a number of differences appear. For the right ascension of the Drift I apex, the G.C. positions are all greater than those from the Cape, whilst for Drift II the Cape values are higher than those for the G.C. For the declinations, the Drift I values are much the same, but for Drift II they appear to be greater, numerically, for the Cape. Nothing can be deduced about the Drift I velocities in this respect, but the G.C. gives higher Drift II velocities. This latter is to be expected, since the G.C. covers the whole sky and not solely a zone near the Drift II apex, and has better determined motions.

The question of variations with spectral type can now be considered. Taking the two sets of Cape results separately, the only variation fully evident from the bright stars is that of hW_1, which decreased from the F type stars to the K types. This cannot be definitely confirmed from the faint stars, but taking the two sets of results to be the same, it then can be confirmed. For the bright stars, the declination of the Drift I apex increases from types F to K, but for the faint stars all that can be said is that they do not deny this variation.

For Drift II, the bright stars show a decrease of the declination of the apex from types F to K. For the faint stars the trend is reversed. The faint stars also show a decrease

TABLE XXII

COMPARISON OF DRIFT APICES

	F			G			K		
A_1	91°.0	90°.0	91°.1	90°.7	90°.0	92°.9	91°.1	91°.5	96°.0
D_1	-7°.5	-10°.8	-8°.9	-5°.1	-8°.9	-10°.3	-3°.9	-8°.8	-10°.8
hW_1	1.692	1.576	1.770	1.539	1.615	1.564	1.446	1.385	1.385
A_2	322°.2	321°.7	301°.0	288°.4	286°.5	273°.5	295°.2	256°.0	265°.8
D_2	-69°.4	-80°.0	-63°.4	-77°.7	-74°.1	-71°.0	-80°.4	-71°.6	-74°.4
hW_2	0.661	0.592	0.735	0.686	0.694	0.889	0.595	0.676	0.770
	(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)

TABLE XXIV

COMPARISON OF VERTICES AND SOLAR MOTION -FROM DRIFTS

G_v	344°.5	347°.8	348°.6	340°.2	343°.0	341°.7	340°.6	343°.0	342°.4
g_v	+ 2°.8	+ 1°.5	- 1°.2	- 1°.2	- 0°.1	- 2°.0	- 1°.4	+ 2°.6	- 3°.4
Ω	1.673	1.639	2.075	1.756	1.697	1.912	1.606	1.601	1.638
A_o	265°.3	268°.3	266°.0	268°.9	268°.6	272°.8	269°.5	273°.2	277°.8
D_o	+25°.6	+22°.4	+28°.1	+29°.2	+25°.5	+37°.4	+25°.3	+27°.8	+40°.3
hU_o	0.934	1.047	0.948	0.817	0.972	0.883	0.780	0.808	0.770
N_1/N_2	1.173	1.781	1.22	1.038	1.473	1.18	1.080	1.398	1.02

TABLE XXV

COMPARISON OF ELLIPSOIDAL CONSTANTS AND SOLAR MOTION

G_v	348°.1	345°.8	342°.6	342°.1	342°.0		340°.3	337°.8	335°.6
g_v		- 6°.0	- 6°.7		+ 0°.6			- 0°.4	- 5°.1
K/H	0.55	0.676	0.574	0.56	0.554		0.65	0.606	0.67
A_o	266°.0	266°.9	263°.5	269°.9	271°.2		272°.3	275°.4	277°.6
D_o	+23°.2	+19°.8	+28°.3	+31°.6	+30°.9		+31°.8	+24°.2	+31°.2
hU_o	1.05	1.056	0.92	0.89	0.950		0.82	0.831	0.66

(a) CAPE - BRIGHT STARS

(b) CAPE - FAINT STARS.

(c) G. C.

in the right ascension of the Drift II apex from types F to K that is only slightly evident for the bright stars. Thus, taking the bright and the faint stars separately, variations with spectral type in the positions of the drift apices and their velocities cannot be fully established because of the magnitudes of the various errors. The above refer to regular variations.

For the G.C. - for the same spectral groups- four regular variations can be traced and one irregular variation. they are that, as spectral type changes from F to K:-

(i) the R.A. of Drift I increases,

(ii) the Drift I velocity decreases,

(iii) the R.A. of Drift II decreases,

(iv) the declination of Drift II decreases, and

(v) the Drift II velocity increases and then decreases.

As remarked above the Cape results are apparently identical for each volume. The definitive apices were therefore calculated for each of the spectral groups F, G and K by combining the pairs of values weighted according to their probable errors. The values so obtained are given in Table XXIII

TABLE XXIII

	F	G	K		F	G	K
A_1	$90^\circ.6$	$90^\circ.5$	$91^\circ.3$	A_2	$322^\circ.1$	$287^\circ.8$	$276^\circ.6$
D_1	$-8^\circ.6$	$-6^\circ.8$	$-6^\circ.0$	D_2	$-70^\circ.8$	$-76^\circ.8$	$-77^\circ.8$
hW_1	1.650	1.565	1.423	hW_2	0.642	0.691	0.602

From these values it is evident that most of the variations found for the G.C. are repeated for the Cape motions. The exception is that there is no evidence of any definite variation in the R.A. of the Drift I apex. The variation in the declination of the Drift I apex is too indefinite to be regarded as significant. The variation in hW_2 follows that of the G.C. exactly. No information can be gathered about the variations of these elements with galactic latitude found by Tannahill.

In Table XXIV the positions of the vertex and the solar apex derived from the drift analysis are compared with those from the bright stars and from the G.C. Again it is evident that the bright stars and the faint stars give identical results for the vertex , although for the F stars there is a large difference between the values of Ω. The positions also agree well with those from the G.C. and a deviation of the vertex of some 20° magnitude is present. Again the velocities vary with spectral class, being greater for the F stars than for the K stars. It is also apparent that the longitude of the vertex is greater for the F type stars than for the later type stars, thus confirming previous results

The greatest variation evident in Table XXIV is in the values of the drift ratios - N_1 to N_2. In every case the values for the faint stars are much greater than for the bright stars or for the G.C. Otherwise their variation with spectral class parallels those for the bright stars and for the

G.C. The high values foor the faint stars here found are probably spurious, being a result of the uncertainties in the analyses, especially in the determinations of Drift II. It was at first feared that these high values might have a vitiating effect on the determniations of the elements of the solar motion. That such is not the case is evident from the excellent agreement between the determinations of the latter from the drift analyses and those from the ellipsoidal analyses.

In the case of the solar motion the Cape results are again in agreement. It will be seen that the low declination of the solar apex still obtains for the faint stars, but the results of the analyses by the ellipsoidal theory show that the probable errors of these declinations are of the order of 5°. In all cases the values of the declination are much lower than those from the G.C. The three sets of results for the R.A. of the apex are all in close accord and the variations with spectral class are well marked, the R.A. increasing steadily from type F to type K. This variation is extended by Tannahill's value of $264^{\circ}.8$ for the A stars. Asteady variation is also seen in the speed of the sun relative to the centres of rest of the various groups, it being much higher for the Ftype stars than for the K types. In this latter respect it is interesting to note that the velocities found from the faint stars are in every case greater than those from the bright stars and from the G.C.. This is confirmed by the results of the ellipsoidal analysis. This may well be a result of the greater distances of the faint stars.

Finally the results of the analyses by the ellipsoidal thory are compared, in Table XXV, with those from Jackson's analysis of the bright stars (4) and Delhaye's analysis of the G.C. (5). It must be noted that in his analysis of the G.C. Delhaye used somewhat different spectral groups - his being B8 - A_5 , Fo - F_9 and Ko - K_2 - and further divided his groups into regions of high and low galactic latitude. The figures given here are thus approximate means of his results.

As was the case with the drift analyses, the two volumes of the Cape proper motions give almost identical results both for the direction of the major axis of the velocity ellipsoid and for the solar motion. There is a slight suggestion that the galactic longitude of the major axis is systematically less for the fainter stars but the evidence is most uncertain. The decrease in the longitude of the major axis with spectral class is well marked much more so than was the case for the drift analysis. For the solar motion the results reproduce almost exactly those for the drift analysis. Again the faint stars yield higher velocities.

4.

It cannow be determined to what extent the aims of the investigation nave been accomplished.

First, the low declination of the solar apex found for the bright stars. From the above it is seen that although the declinations found are still low, they are accompanied by quite large probable errors - of the order of 5°. The significance of the low declination is thus diminished. Although a revision of the corrections applied in the preparation of the catalogues for the systematic motions of the reference stars might improve the determination, as the drift apices agree well with those from the G.C., it appears that any such corrections will necessarily be small.

The second aim was the determination of any magnitude effects between the two volumes of the Cape catalogue. From the analyses only three differnces are found. Firstly there were the very high values found for the ratio of N_1 to N_2. As the errors of these determinations are unknown, their significance cannot be established, but it seems likely that these extreme values are mostly spurious. Secondly there were the higher values found in both analyses for the solar speed for the faint stars. The third difference was that for the major axis of the velocity ellipsoid, the longitude was less for the faint stars in all cases. Of these two differences, the first appears to be the only one of significance and may well be a result of the greater distances of the faint stars. The vertex difference is too nebulous to be evaluated and is not supported by the results of the drift analysis. When the probable errors are considered it is even more unlikely that this difference is significant. There is certainly no evidence at all that the deviation of the vertex disappears for the faint stars, as has been suggested by some authors.

The third aim of the investigation has, however, been fully achieved, the variations with spectral class noted by Smart and Tannahill having been confirmed and extended to confirm those found by Tannahill from his analysis of the G.C.

The results of these investigations have been communicated to the Royal Astronomical Society and have been published in the Monthly Notices thereof. (6). A copy of the paper is included in the appendix. It is to be mentioned here that the full discussion of the variations with spectral class given above were not all given in the paper referred to above, that by Tannahill which has been referred to in this chapter not having then been available.

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CHAPTER V.

THE CAPE PHOTOGRAPHIC CATALOGUES, ZONES -30° to -35° AND -35° to -40° AND THE ANALYSES OF THE PROPER MOTIONS THEREOF.

1.

The Cape Photographic Zone-Catalogues, when completed, will cover the southern sky south of declination -30°, and are intended to give accurate places of standard stars for use in the reduction of the Astrographic Catalogues as well as the proper motions of the stars. To date, the catalogue for the first zone, -30° to -35°, has been published and distributed and that for the second zone, -35° to -40°, is to be published shortly. The -40° to -52° zone is, of course, covered by the Cape Astrographic Zone Catalogues.

In this chapter the proper motions of the stars in the first two zones are analysed, both on the two-drift and on the ellipsoidal theories. The distribution counts of the proper motions in position angle were prepared for me by Mr. J.v.B.Lourens of the Royal Observatory, Cape of Good Hope, from the manuscripts of the catalogues.

It had originally been hoped that it would be possible to combine the analyses of these zones with those of the Astrographic Zone, thereby improving the determinations of hte constants of star-streaming, but this plan had to be abandonned, since (a) the C.P.Z. catalogues are already reduced to the system of FK_3 and (b) the reduction of the C.A.Z. motions to the FK_3

system had proved unsatisfactory. In comsequence, the C.P.Z. proper motions were treated independantly to determine the constants of star-streaming for the purpose of comparing them with those from the C.A.Z. proper motions. This comparison is of some interest as, whereas the C.A.Z. motions were determined by purely photographic methods, plates taken at different epochs being superimposed and the proper motions derived from the measured displacements, for the C.P.Z. motions they were found by other methods, and indeed, might be called 'Photographic Meridian Proper Motions' , being determined as follows.

Each photographic plate used in the compilation of the C.P.Z. catalogues was exposed when the region recorded thereon was near the meridian. The stars to be measured were then selected, in the case of the -30° to -35° zone, from the Cordoba "B" and "C" catalogues, these being visual meridian ~~xat~~ catalogues. Their Cordoba positions were then reduced to the epoch 1950.0,using the catalogue precessions corrected to Newcombe's system. From these positions, their standard co-ordinates were calculated and the difference between these and their measured ~~dxx~~ co-ordinates derived. These differences were then analysed and corrections derived to be applied to the measured positions. Next the residuals Cape - Cordoba were formed and analysed for errors depending on position. From the results of this analysis corrections were evaluated and applied to the residuals giving the proper motions in the interval -

almost 35 years - and when added to the Cordoba position, the observed position. The proper motions are thus half derived from visual observations and half from photographic measures.

The proper motions were then reduced to the FK3 system by applying the correction:-

$$(\text{G.C.} - \text{Cape 1925}) - (\text{G.C.} - \text{Cordoba}) + \tfrac{1}{3}.\ 100\ [\Delta\mu(\text{FK}_3 - \text{G.C.})]$$

the reference stars having been observed on the system of the 'First Cape Catalogue for 1925.0'.

As, at the time of writing, the catalogue of the -35° to -40° zone has not been published its exact construction is not known, except from information received in private letters from Mr. Lourens, from which the following quotations are taken.

"..........About 20 per cent. of the stars (in the -35° to -40° zone) are without proper motions since for the zone -37° to -40° earlier meridian catalogues of precision contain only a limited number of stars and we had to supply "extra stars" mainly from the C.P.D. where positions are not precise enough for P.M. determinations,........"

(13 April 1954)

" Accidental errors of P.M. are expected to be higher for the -30° to -35° zone where only one source was used for the first epoch of the proper motions. In the -35° to -40° zone where more than one initial source, where available, was used for the determination or the proper motions ...."

(9 December 1954)

The second catalogue thus appears to have been constructed in basically the same manner as the first catalogue, except that the larger number of sources has increased the accuracy. For the first catalogue the probable error of a P.M. in R.A. or Dec. is given as having a maximum value of 0".014.

2.

The first material received from the Cape consisted of the distribution counts for the -30° to -35° zone. These were prepared for each of five spectral groups $B8 - A3$, $A_5 - F_5$, $F_8 - G_5$, Ko - M and unclassified, for five magnitude groups as with the astrographic zone for each half hour of R.A. This grouping was selected to be parallel to the astrographic zone, being choosen before it was known that it would not be possible to combine the photographic and the astrographic zones. Stars omitted in forming the counts were (a) stars of spectral types earlier than B_7 (b) stars brighter than magnitude 7.0 as explained above, (c) variable stars with a large range of variation and (d) a few close doubles of indefinite category. After these omissions, totalling 531, 12 315 stars remained for analysis. These counts were prepared by Mr. Lourens. Scatter diagrams were not prepared, but for every star, the position angle was entered beside it in the catalogue and then the counts were formed. The distribution of these stars by spectral group and right ascension is given in Table I.

It was decided to perform the first analysis on the two-drift theory and to use all the material, including

TABLE I.

No. and Centre of Region.		$B_8 - A_3$	$A_5 - F_5$	$F_8 - G_5$	Ko - M	ALL (inc. Un)
1	$0^h - 1^h30$	20	64	244	140	528
2	$1^h30 - 3^h$	27	67	252	101	501
3	$3^h - 4^h30$	41	97	267	170	614
4	$4^h30 - 6^h$	57	152	311	225	787
5	$6^h - 7^h30$	183	147	183	257	842
6	$7^h30 - 9^h$	267	99	131	243	806
7	$9^h - 10^h30$	166	127	181	281	824
8	$10^h30 - 12^h$	79	156	217	298	841
9	$12^h - 13^h30$	85	117	250	337	852
10	$13^h30 - 15^h$	104	148	252	339	902
11	$15^h - 16^h30$	132	135	253	243	807
12	$16^h30 - 18^h$	382	105	160	242	918
13	$18^h - 19^h30$	259	114	216	308	940
14	$19^h30 - 21^h$	67	112	273	301	831
15	$21^h - 22^h30$	36	86	279	220	709
16	$22^h30 - 24^h$	26	75	287	166	613
	TOTAL	1 931	1 801	3 756	3 871	12 315

956 stars of unclassified spectra. From the colour indices of the latter Lourens found that their probable distribution was

$B_8 - A_3$	1.9 per cent.
$A_5 - F_5$	16.5 " "
$F_8 - G_5$	22.6 " "
Ko - M	59.0 " "

For the analysis, the zone was divided into the 16 regions given in table I, each region being of an hour and a half's width in R.A. and 5^{o} in declination. The distribution curves were then drawn and analysed by Eddington's "Trial and Error" mathod. Immediately noticeable was the irregularity or lack of smoothness of the curves. For regions 6 - 11, it was possible to obtain quite a good fit of the calculated curves to the observed curves. This part of the zone, from right ascension $7^h\ 30^m$ to $16^h\ 30^m$ corresponds roughly to that part of the zone north of the galactic equator. For the other regions the fit was less perfect and there was a trend for the fit to decrease in exactness the further the region was from the middle of the above mentioned area - i.e. the quality of the analyses diminished with increasing south galactic latitude. The distribution curves are shown in Figures I and II.

The principal features of these curves are the breadth of the maxima - that is, the breadth of Drift I in practice - and the high minima. In other words, there is a considerable 'background' of stars - a more descriptive term being 'noise level'. The breadth of the maxima had the effect

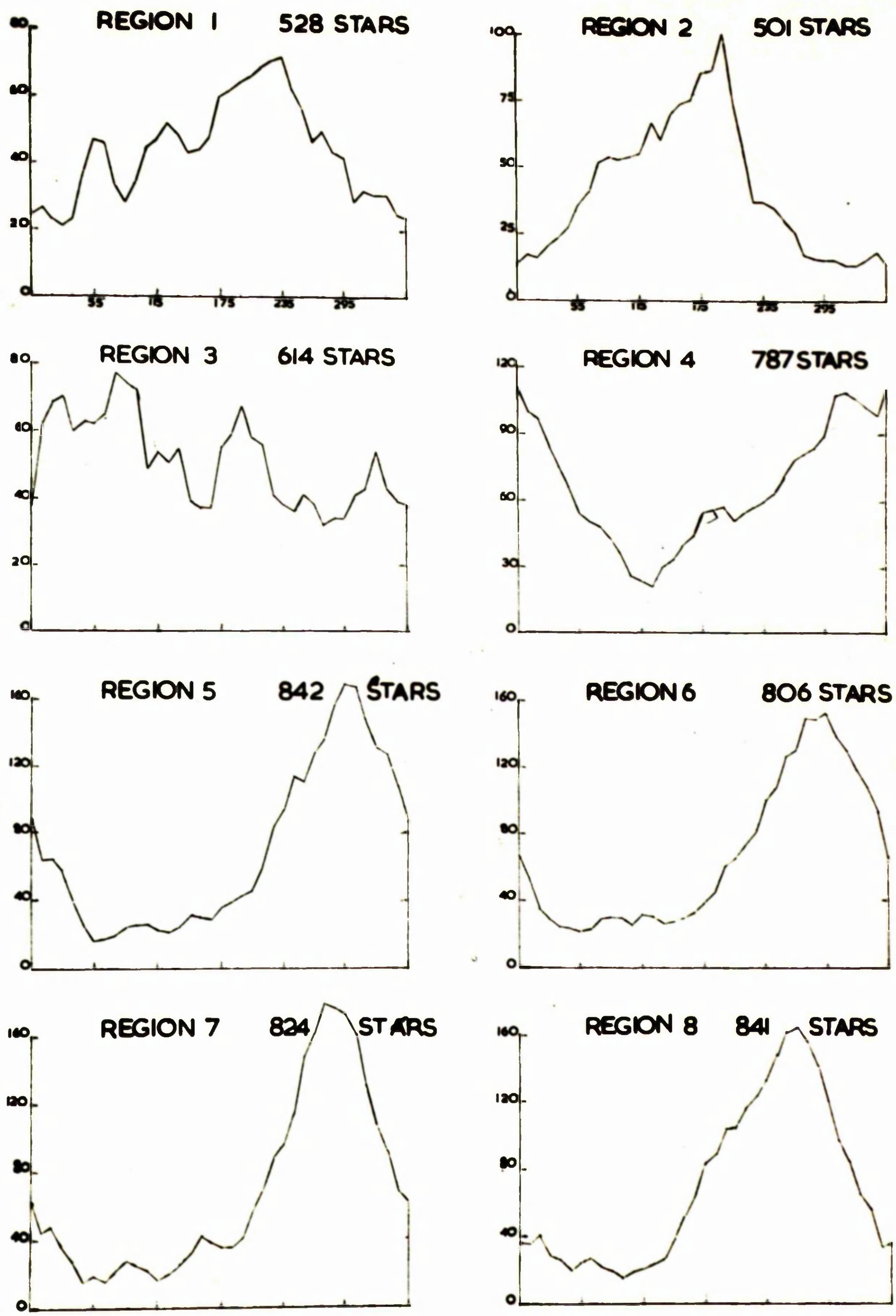
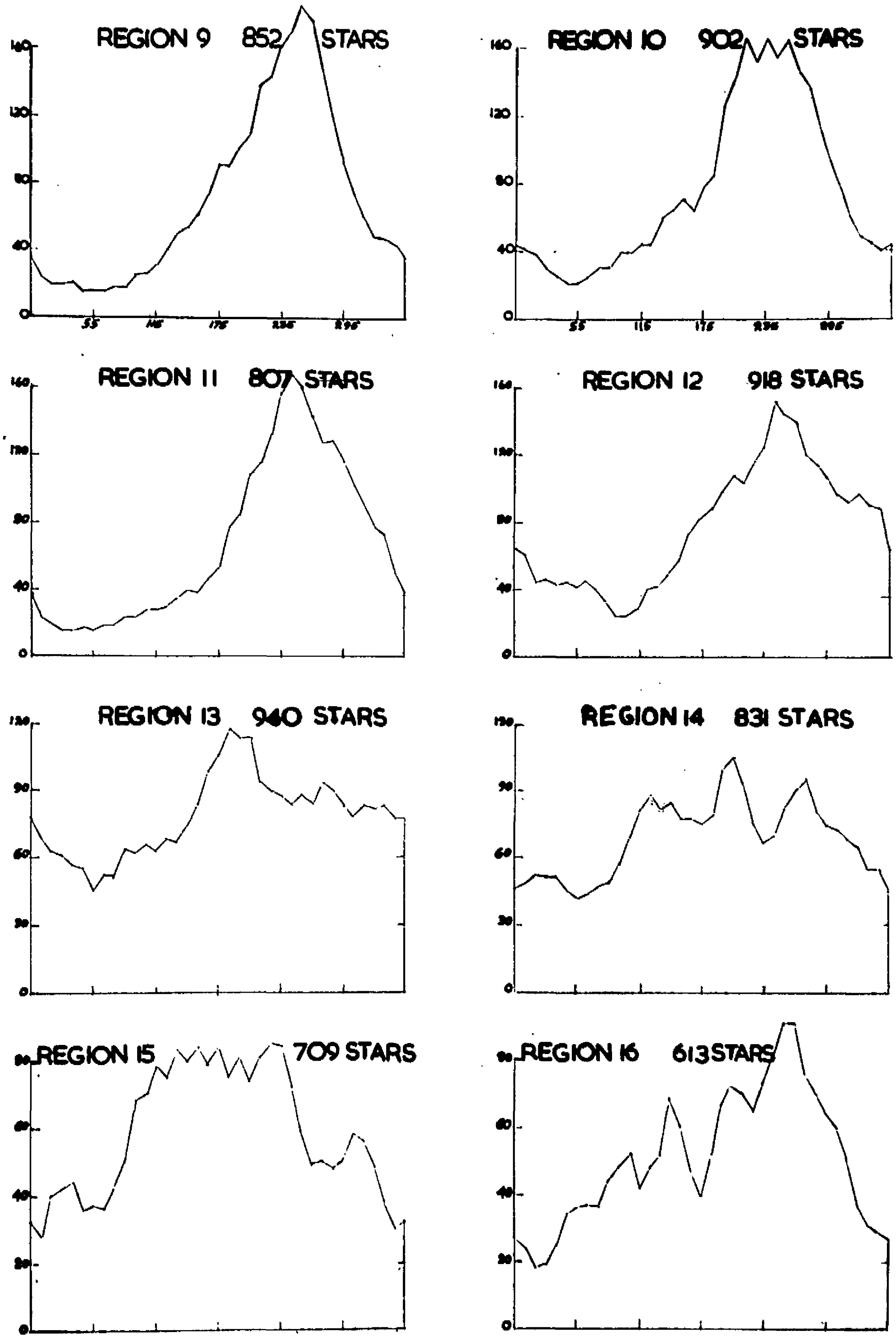

REGION 1 528 STARS
REGION 2 501 STARS
REGION 3 614 STARS
REGION 4 787 STARS
REGION 5 842 STARS
REGION 6 806 STARS
REGION 7 824 STARS
REGION 8 841 STARS
55
115
175
235
295

FIGURE 1



REGION 9 852 STARS
REGION 10 902 STARS
REGION 11 807 STARS
REGION 12 918 STARS
REGION 13 940 STARS
REGION 14 831 STARS
REGION 15 709 STARS
REGION 16 613 STARS

FIGURE II

of lowering the Drift I velocity and the high 'noise level' lowers the Drift II velocity and also renders the position of the Drift II maximum indeterminate. In all these figures the ordinate is thrice the number of stars and the abscissae the position angles. Contrary to the case of Drift II, the Drift I position angle is usually well defined although for some regions the curves are barely recognisable as drift curves.- especially regions 3 and 15. In all the analyses, the sole aim was to obtain the best possible fit, even though the constants thus obtained were completely anomalous in comparison to their normal run. No guide to the probable values, derived from adjacent regions. was used. The results of the complete analysis for each region are given in Table II and the derived positions of the Drift apices in Table III.

Three solutions for the drift apices were made. The first was for regions 4 to 11, inclusive., the second for regions 12 to 3 inclusive and the third for the whole zone.

As was to be expected from the regional analyses, the solutions for the two halves of the zone are completely discordant, the first group consisting of the best regions - from the point of view of the fits - and the second group of the worst. The result for the whole zone is in fair agreement with the usual values for positions, but the drift velocities are abnormally low.-This latter feature is to be expected, the velocities found in the analyses being low. Further, as the

TABLE II

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	0.3	$115^\circ.0$	243	0.5	$225^\circ.0$	285
2	0.4	115	364	1.5	185	137
3	0.5	55	223	0.05	135	391
4	0.6	345	391	0.2	245	396
5	0.8	320	467	0.8	260	375
6	1.0	305	384	0.5	245	422
7	0.9	290	534	0.4	230	290
8	0.9	270	476	0.6	200	365
9	1.1	255	433	0.45	190	419
10	0.9	245	476	0.3	190	426
11	0.9	260	494	0.3	215	313
12	0.6	275	404	0.3	225	514
13	1.5	195	91	0.1	235	849
14	0.7	145	242	0.3	260	589
15	0.6	135	276	0.4	225	433
16	0.4	130	287	0.6	260	326

TABLE III

Solution.	DRIFT I			DRIFTII		
	A_1	D_1	hW_1	A	D	hW
4 - 11	$99^\circ.8$	$21^\circ.5$	1.265	$332^\circ.5$	$-19^\circ.6$	0.451
12 - 3	$113^\circ.3$	$-30^\circ.4$	0.609	$244^\circ.5$	$-41^\circ.7$	0.564
ALL	$96^\circ.4$	$-8^\circ.6$	0.831	$317^\circ.7$	$-79^\circ.0$	0.365

TABLE IV

R	ΔhV_1	$\Delta\theta_1$	ΔN_1	ΔhV_2	$\Delta\theta_2$	ΔN_2
2	+0.4	-25°	-206	-0.8	$+5^\circ$	+206
4		+10		+0.05		
5	+0.2	+5	- 92	-0.2		+ 92
11	-0.2	-35	-181	+0.25	+60	+181
12	-0.1	-60	+ 55	+0.2	+80	- 55
15		-10				

zone lies quite near to the Drift I apex and the solar antapex lies within the zone, the determinations will be uncertain.

This analysis having been completed, a second analysis was now performed. The aim of this analysis was to obtain better fits, if possible, for the doubtful curves and to attempt to obtain better agreement between the two halves of the zone. As a guide in this analysis, the position angles for Drift I in each region were calculated for an assumed apex at R.A. 90°, Dec. −7°.5. In all it was found that for six regions a better fit between the observed and the calculated curves could be obtained, these regions being numbers 2, 4, 5, 11, 12 and 15. These regions are mostly in the southern galactic hemisphere, only regions 5 and 11 having any extensions into the nothern hemisphere.

That such changes should be possible demonstrates what is, perhaps, a weakness in the analysis of the distribution of proper motions in position angle by the "Trial and Error" method, namely, that when the distributuion curves are affected by irregularities it is sometimes possible to obtain two sets of curves, with sometimes widely differing constants, which give equally good fits to the observed curve, one taking account of a hump (or hollow) that the other does not, and vice-versa. The selection of the curve that is to be accepted is thus a matter for the investigators personal judgement of the particular case and of the significance of any particular irregularity in the curve and also on any other criterions that he may apply. One such criterion used in these investigations has been that, in all possible cases, the theoretical curve should intersect the observed curve at <u>least</u> twelve times.

A possible criterion for accepting a curve might be that its constants should follow the general run of those in the adjacent regions, but this does not seem to be a valid criterion - there might be, for example, some other phenomenon affecting the region, a natural consequence of which being a departure from the 'expected' constants. It also assumes that there is no error in the 'expected' values.

The changes made in the present case are shown in Table IV, ΔhV_1 indicating the change made in hV_1, etc.

After this re-analysis, there still remained two regions where no satisfactory fit could be obtained and a number for which the fit was very poor. It was therefore decided that, in deriving the Drift constants, the regions should be weighted. These weights were assigned on the basis of the fit between the observed and the calculated curves. In Table V the details of the analyses for each region and the weights attached to each region are given. In Table VI, the derived positions and velocities of the drifts are listed together with the elements of the vertex and the solar motion. It is apparent that there is little improvement as a result of the re-analysis of the zone. The mean position from the two solutions are $A_1 = 91^\circ.9$, $D_1 = -4^\circ.2$ and $A_2 = 275^\circ.3$, $D_2 = -73^\circ.2$.

TABLE V

R	Wt.	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	0	0.3	115°	243	0.5	225°	285
2	1	0.8	90	158	0.7	180	343
3	1	0.3	55	223	0.05	135	391
4	3	0.6	355	391	0.25	245	396
5	3	1.0	325	375	0.6	260	467
6	2	1.0	305	384	0.5	245	422
7	3	0.9	290	534	0.4	230	290
8	3	0.9	270	476	0.6	200	365
9	3	1.1	255	433	0.45	190	419
10	3	0.9	245	476	0.3	190	426
11	3	0.7	225	313	0.55	275	494
12	3	0.5	215	459	0.5	305	459
13	0	1.5	195	91	0.1	235	849
14	2	0.7	145	242	0.3	260	589
15	1	0.6	125	276	0.4	225	433
16	1	0.4	130	287	0.6	260	326

TABLE VI

	Drift I	Drift II		Vertex		Solar Motion
A	87°.4	232°.8	G_V	343°.0	A_0	271°.1
D	0°.3	-67°.4	g_V	- 3°.0	D_0	15°.2
hW	0.98	0.23			hU_0	0.42

It is clearly apparent from the above results that the proper motions of the stars in the -30° to -35° zone catalogue exhibit marked departures from the normal drift solutions. Perhaps the main features are the very low velocities found for the drifts, these being but half the usual values. This depression of the drift velocities explains most of the other differences, since the low velocities found in the regions makes the other determinations uncertain. Although these results with their variation of the drift apices may be indicative of a real effect peculiar to these stars, this seems unlikely, as these stars are little, if at all, fainter than those of the Faint Star volume of the Cape Astrographic Zone Catalogues, which gave normal drifts and apices. Rather it seems that the explanation may have its origin in the effects of accidental errors on the small proper motions or else in the manner of reduction of the observations to the final values. The high 'noise level' does suggest that either of these two possibilities might be the cause. The effect of the A type stars which, besides being unevenly distributed over the zone, are possibly more prone to the effects of small errors must, however, not be neglected. It is of interest to mention here that, in referring to this zone Dr. J.Jackson , under whose direction most of the reduction was performed, says (1) , " .... The proper motions (of the -30° to -35° zone) will not be of the highest quality."

3. As the results of the analysis for ALL stars in the -30° to -35° zone had proved unsatisfactory, it was decided that, before an analysis for each spectral group was performed, the above zone would be combined with the -35° to -40° zone, the distribution counts for which having just been received from Mr. Lourens. This was decided because it would yield greater numbers of stars per region, especially in those regions which had given the weakest analyses for ALL stars. Furthermore, the larger regions - now of an hour and a half's width in R.A. and 10° in declination - would possibly smooth out the irregularities to some extent. It was further decided to omit the A type stars. In all, 16 538 stars remained for analysis, divided into four groups - $A_5 - F_5$, $F_8 - G_5$, Ko - M and A_5 - M. The distribution of these stars by regions and spectral groups is given in Table VII. A noticeable feature is the more even distribution over the zone of the stars of the various spectral groups compared to that in the Astrographic zone. Only the K type stars show much variation with galactic latitude. This is partly because the zone has a lesser range of galactic latitude and partly because it is not complete to the same extent as the Astrographic Zone Catalogues.

The distribution curves were then drawn for each region and group. In all cases they were smoother than was the case for the -30° to -35° zone, especially for those regions which were weak for ALL stars.

TABLE VII

Centre and No. of Region.			$A_5 - F_5$	$F_8 - G_5$	Ko - M	$A_5 - M$
1	0^h	45^m	124	412	241	777
2	2	15	128	411	231	770
3	3	45	187	475	291	953
4	5	15	256	479	391	1126
5	6	45	280	308	467	1055
6	8	15	212	217	450	879
7	9	45	241	315	538	1094
8	11	15	280	404	534	1218
9	12	45	223	421	590	1234
10	14	15	256	447	574	1277
11	15	45	248	441	405	1094
12	17	15	202	269	398	869
13	18	45	216	405	506	1127
14	20	15	202	417	471	1090
15	21	45	191	465	388	1044
16	23	15	143	469	319	931
TOTAL			3 389	6 355	6 794	16 538
Avge. No. per Region.			212	397	424	1 033

TABLE VIII
DRIFT ANALYSES
A_5 - F_5

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	0.6	95°	63	0.6	255°	61
2	0.6	95	60	0.5	200	68
3	0.7	70	110	0.5	255	77
4	0.7	325	112	0.2	225	144
5	1.2	300	113	0.6	230	167
6	1.0	305	110	0.6	225	102
7	1.0	285	151	0.5	195	90
8	1.3	260	131	0.5	235	149
9	1.1	255	140	0.5	185	83
10	0.8	255	181	0.4	195	75
11	1.1	225	152	0.5	255	96
12	1.4	205	68	0.3	275	134
13	0.8	175	141	0.8	305	75
14	0.5	155	125	0.6	270	77
15	0.6	135	89	0.3	225	102
16	0.5	115	94	1.2	255	49

TABLE IX

F_8 - G_5

1	0.7	95°	166	0.6	215°	246
2	0.6	105	206	0.6	185	205
3	0.5	65	250	0.4	215	225
4	0.6	355	226	0.4	225	253
5	1.0	315	123	0.4	245	185
6	1.0	295	100	0.4	235	117
7	1.1	280	185	0.4	215	130
8	1.0	265	250	0.6	195	154
9	1.1	245	229	0.4	205	192
10	1.0	245	255	0.5	195	192
11	1.1	230	268	0.5	255	173
12	1.5	205	76	0.5	235	193
13	0.5	185	203	0.3	205	202
14	0.6	165	211	0.4	245	206
15	0.6	135	201	0.5	235	264
16	0.6	115	188	0.6	250	281

TABLE X

Ko - M

R	hV_1	θ_1	N_1	hV_2	θ_2	N_2
1	0.3	145°	107	0.3	225°	134
2	0.5	125	142	0.4	195	89
3	0.8	55	121	0.2	175	170
4	0.6	350	228	0.4	200	163
5	0.7	285	247	0.3	220	220
6	0.9	295	175	0.4	230	275
7	0.9	285	274	0.4	225	264
8	1.0	265	250	0.5	195	284
9	1.0	250	300	0.6	185	290
10	1.0	247.5	255	0.4	195	319
11	0.7	235	202	0.3	275	203
12	0.6	215	263	0.3	345	135
13	0.6	200	284	0.2	335	222
14	0.2	165	236	0.2	205	234
15	0.6	145	201	0.35	235	187
16	0.5	135	119	0.3	235	200

TABLE XI

A_5 - M

1	0.6	105°	335	0.5	235°	442
2	0.7	105	408	0.5	195	362
3	0.6	60	482	0.4	195	471
4	0.6	345	583	0.3	215	543
5	0.9	290	433	0.3	235	622
6	1.0	295	333	0.4	245	546
7	1.0	285	610	0.4	215	484
8	1.1	265	631	0.6	205	587
9	1.1	250	616	0.6	195	618
10	1.0	245	677	0.4	195	600
11	1.0	230	622	0.4	260	472
12	1.1	215	407	0.3	285	462
13	0.7	180	609	0.4	295	518
14	0.4	165	572	0.3	240	518
15	0.6	135	490	0.5	235	554
16	0.6	125	433	0.6	255	498

TABLE XII

ELLIPSOIDAL ANALYSIS

$A_5 - F_5$

R	k/h	ζ_o	θ_o	Sh	θ'
1	0.707	1.000	79°.3	0.144	53°.0
2	730	0.936	53.2	389	320.0
3	887	521	72.0	129	360.8
4	801	747	11.3	298	122.3
5	811	722	343.0	581	88.9
6	798	756	337.3	715	94.7
7	820	697	307.5	645	91.0
8	936	0.377	306.6	879	70.9
9	674	1.096	284.1	855	54.7
10	793	0.767	295.8	678	60.0
11	874	0.556	314.9	794	53.5
12	899	0.486	192.9	442	41.5
13	681	1.074	157.9	384	22.8
14	847	0.627	124.4	290	9.5
15	969	0.256	109°.8	168	311°.8
16	0.786	0.788	91°.7	0.174	1°.7

TABLE XIII

$F_8 - G_5$

1	0.752	0.877	67°.8	0.328	350°.0
2	768	834	51.1	430	329.1
3	749	883	33.6	163	294.8
4	838	650	17.0	162	122.3
5	849	622	344.0	480	106.3
6	822	0.692	297.9	613	94.4
7	700	1.020	313.8	690	87.5
8	823	0.691	292.3	764	66.9
9	798	755	285.1	755	54.2
10	801	749	277.0	735	51.5
11	927	405	222.9	755	56.3
12	867	574	186.8	655	40.9
13	933	386	172.6	357	18.2
14	873	560	133.1	374	12.9
15	759	867	103.4	332	8.3
16	0.735	0.924	94°.1	0.298	32°.9

TABLE XIV

ELLIPSOIDAL ANALYSIS

K_6 - M

R	k/h	ζ_o	θ_o	Sh	θ_s
1	0.912	0.432	80°.0	0.218	19°.2
2	929	398	87 .0	422	330 .2
3	842	641	58 .4	297	264 .3
4	827	679	351 .8	249	143 .5
5	957	302	325 .5	415	82 .9
6	918	434	294 .1	559	86 .9
7	902	479	327 .9	539	91 .9
8	941	361	327 .0	586	64 .5
9	931	393	324 .2	590	52 .8
10	956	308	296 .3	545	50 .0
11	797	758	306 .5	659	61 .5
12	942	356	220 .4	367	42 .7
13	902	480	175 .5	267	31 .8
14	941	361	166 .7	201	358 .9
15	957	300	95 .7	386	345 .7
16	0.800	0.750	106°.5	0.237	14°.3

TABLE XV

A_5 - M

R	k/h	ζ_o	θ_o	Sh	θ_s
1	0.789	0.779	73°.5	0.243	4°.9
2	787	785	53 .2	423	328 .2
3	816	710	40 .6	190	272 .4
4	857	601	5 .1	203	130 .7
5	886	524	336 .1	460	92 .0
6	865	581	307 .6	586	91 .9
7	795	763	316 .8	614	90 .0
8	913	448	300 .5	732	67 .3
9	847	627	292 .6	707	53 .7
10	874	556	289 .2	637	52 .9
11	936	377	307 .9	618	57 .1
12	924	413	197 .1	461	41 .8
13	845	633	165 .4	340	24 .2
14	932	388	139 .1	284	8 .0
15	875	553	103 .9	350	356 .0
16	0.803	0.742	94°.2	0.251	24°.8

Analyses were performed for each group by both the "Trial and Error" method and Schwarzschild's automatic method, the agreement between the observed and the calculated curves in the two-drift analysis being very much better than was the case with the -30° to -35° zone by itself. The details of the analyses for each group and region are given in Tables VIII to XV. From these it can be seen that the drift - and the ellipsoidal - constants do not run smoothly, but , at times, vary immensely from region to region. Of the two analyses, it is impossible that the two-drift is to be preferred, as better fitting the observed distribution. As the zone lies so near to the solar antapex , the determination of the position angle of the latter, in the ellipsoidal analysis, is accompanied by a considerable uncertainty in some regions, with consequent results on the determinations of the solar speed and the axis ratio. However, as the drift curves still show a considerable 'noise level', it is probable that the two methods of analysis are equally accurate - in the sense that the determined constants are those that would be given by perfect proper motions in that zone. The elements of the Drift motions, etcetera, are given in Tables XVI and XVII.

The first feature of these is that the celestial co-ordinates of the drift apices are essentially normal - that is, in agreement with those normally obtained - especially when the large probable errors are considered. The

TABLE XVI

	$A_5 - F_5$		$F_8 - G_5$		Ko - M		A_5 - M	
A_1	87°.1	± 5°.7	96°.0	± 4°.9	92°.6	± 6°.4	90°.6	± 5°.3
D_1	-11°.1	5°.6	-12°.9	4°.8	-11°.7	6°.4	-13°.5	5°.2
hW_1	0.992	0.097	0.989	0.083	0.755	0.086	0.901	0.081
A_2	268°.5	±30°.6	300°.2	66°.4	305°.8	43°.3	258°.7	43°.9
D_2	-50°°8	19°.3	-80°.1	11°.4	-72°.0	13°.4	-70°.8	14°.5
hW_2	0.348	0.117	0.403	0.080	0.279	0.065	0.323	0.082
A_v	87°.4	7°.3	92°.5	6°.4	95°.9	7°.6	89°.4	6°.7
D_v	3°.7	7°.3	9°°8	6°.4	7°.9	7°.5	5°.5	6°.7
G_v	350°.7		347°.8		351°.1		350°.1	
g_v	9°.3		2°.0		-0°.1		6°.7	
Ω	1.195	0.152	1.040	0.115	0.820	0.108	0.985	0.115
A_o	266°.8		268°.4		268°.6		272°.3	
D_o	28°.0		34°.8		31°.9		33°.9	
hU_o	0.485		0.543		0.394		0.463	
$\frac{N_1}{N_2}$	1 .19		0 .97		1 .00		0 .99	

TABLE XVII

	$A_5 - F_5$		$F_8 - G_5$		Ko - M		A_5 - M	
A_v	97°.1	± 5°.8	89°.5	± 2°.7	106°.0	± 7°.2	92°.7	± 4°.6
D_v	14°.9	5°.6	12°.5	3°.1	16°.3	6°.9	16°.4	3°.7
G_v	345°.4		344°.6		348°.1		342°.1	
g_v	-4°.5		-3°.2		-12°.7		-1°.4	
$\frac{K}{H}$	0.775	0.030	0.751	0.015	0.898	0.021	0.825	0.024
A_o	262°.0	11°.0	276°.0	11°.0	271°.4	11°.7	270°.3	11°.1
D_o	27°.9	9°.7	36°.2	8°.8	35°.3	9°.6	35°.0	9°.1
hU_o	0.489	0.083	0.506	0.078	0.394	0.066	0.449	0.071

latter for Drift II are particularly large so that there may at first be some doubt whether the positions are convergents. However the probable errors in R.A. are given in equatorial values. At the declination of the apex these become but 10° or so, so that the position is a convergent. The drift velocities remain at about half their usual values.

In the same way, it appears that a vertex deviation of some 20° remains. The positions found for the solar apex agree very well with those from other analyses, especially when it is considered that the positions given mostly lie within the zone. In both cases, the velocities are low and of about half their usually determined anounts. Perhaps the most interesting features of the results of the drift analyses are the values of N_1 to N_2, which are unity for the last three groups and slightly greater than unity for the F stars. These values are in such excellent agreement with the values to be expected that one wonders whether they are real at all. Unfortunately, their probable errors are not known. They bear out fully the results found by Smart and Tannahill (2) from the bright stars in the C.A.Z. and by Tannahill (3) from the G.C.

When the results of the analysis by Schwarzschild's automatic method are similarly examined it is again found that the vertex deviation and direction is in good agreement with that from the drift analysis and with the normal values and the elements of the solar motion are to all intents identical with those from the drift analysis. One valuable consequence of this latter agreement is that it indicates that the drift analyses were not in error - that is, in fitting the curves those taken as being the bestrepresentations of the observed curves were also those appropriate to the ellipsoidal analysis - in other words, in accepting a set of constants as being the drift analysis of the region, even though they might appear anomalous, no serious error was introduced. This is confirmed by the high values obtained for the axis-ratio $\frac{K}{H}$. The low drift velocities indicate that the distribution is more nearly random than usual, and hence the axis-ratio should more nearly approach unity.

Further support can be gained by the use of the relation between $\frac{K}{H}$ and Ω derived in a later chapter. From this relation, the values of $\frac{K}{H}$ corresponding to the observed values of Ω - which we will write as $\left(\frac{K}{H}\right)_{calc.}$ - are those in the tpp line of the underneath table, those in the lower being the observed values of $\frac{K}{H}$.

	$A_5 - F_5$	$F_8 - G_5$	Ko - M	A_5 - M
$\left(\frac{K}{H}\right)$ calc.	0.748 $\pm$ 0.045	0.788 $\pm$ 0.034	0.859 $\pm$ 0.031	0.811 $\pm$ 0.035
$\left(\frac{K}{H}\right)$ obs.	0.775 $\pm$ 0.030	0.751 $\pm$ 0.015	0.898 $\pm$ 0.021	0.825 $\pm$ 0.024

Thus, on considering the probable errors the pairs of values of $\frac{K}{H}$ are in full agreement. Unfortunately this evidence can be used both ways - as support for the for the agreement of the analyses and as evidence that the theoretical relation agrees with observation.

There are thus two conclusions to be drawn from the above. They are:-

(i) that the proper motions of the Cape Photographic Zone-Catalogues, zones -30° to -35° and -35° to -40°, display the effects of preferential motion and yield normal positions for the apices of the drifts, the vertex of star-streaming and the solar motion, and

(ii) that because of the high 'noise level' - that is probably indicative that the distribution of the small proper motions is almost random - the velocities determined are abnormally low. This is confirmed by the high values of the axis-ratios of the velocity ellipsoid.

It also appears that there has been no great change in these determinations by the inclusion of the -35° to -40° zone in the analysis or by the exclusion of the A type stars. Further it is impossible to say whether there are any variations with spectral class, as the uncertainties of the various determinations are so great. There is some slight sign of a trend for Ω to decrease from types F to types K but it is not definite. The only other variation is the drop of the ratio N_1 to N_2 for the later spectral classes.

The question of whether there is any great difference between the behaviour of the two component zones cannot be fully settled from the above results. If the curves for each zone for each region are drawn separately, it appears that the Drift I determination for the combined zones depends mainly on the -35° to -40° zone and that for Drift II on the -30° to -35° zone, the latter having the greater (noise level' occasioning the low Drift II velocities. The indications are that the -35° to -40° zone, if examined separately, would give only slightly higher Drift I velocities, but somewhat greater Drift II velocities and more accurate positions for the apices. However, it appears that the velocities would still depart from the 'normal' values. Thus it can be concluded that the cause of the low velocities operates on both zones but perhaps to a greater extent on the -30° to -35° zone, in which the proper motions are expexted to be less accurate than in the other zone.

One way in which more information might be gained would be by an analysis confined to the larger proper motions and thus deriving the 'pseudo -drift velocities'. These would normally be very much greater than those from analyses of all proper motions, and it might thus be possible to draw further conclusions from their values. There are two possible causes for the low velocities found in this investigation that appear more probable than others. They are, firstly, the effect of accidental errors on the small proper motions, and secondly the effect of the corrections applied in the preparation of the catalogues. Then,if analyses were made for motions greater than, say, 1 ,2,3,4,5 and 10 seconds of arc per century,then , if the cause be accidental errors affecting the smallest proper motions, there should be a sudden change in the drift velocities found between,perhaps,the first two limits. If, alternatively, the results from the largest motions displayed no abnormallity and the other motions showed an increasing departure as the limit decreased, then this might indicate that the systematic corrections were occasioning the differences. It would be necessary to perform a parallel analysis for the C.A.Z. or some other catalogue,which had previously yielded normal apices and velocities,to provide a control and to indicate what variations, if any, were the results of selection in using only the large proper motions.

There is one other point which must be considered - namely that the reductions Cape to FK3 are based mainly on the brightest stars, the G.C, extending but to magnitude eight and the FK3 ,which contains many fewer stars, to a brighter limit. The reductions are thus, as stated, based on the brightest stars, and thesee are theye whose photographic places are usually weakest. A magnitude correction, however, was derived in the preparation of the catalogues and applied. Thus if the large proper motions did not yield any futther information,it is still possible that an analysis by magnitude groups might.

However, there would also enter a selection factor, in the first place, and secondly, any analysis by magnitude groups, to perform which it would be necessary to combine spectral groups to obtain sufficient numbers of stars, which yielded variations with magnitude, would also contain an effect depending on spectral type, since the mean Colour Index varies with the magnitude group and the mean magnitude with spectral group in these catalogues.

As the necessary material for such analyses of the restricted proper motions of the two zones is not available at the time of writing, these investigations, which might have provided further insight unto the explanation of the anomalies, cannot be preformed. The present author hopes, however, that, when the material becomes available, he will be able to complete the investigation on the lines detailed above.

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CHAPTER VI

SUMMARY OF THE INVESTIGATIONS OF PROPER MOTIONS.

1.

The investigations of the proper motions of the Cape Astrographic Zone Catalogues and of the Cape Photographic Zone Catalogues, Zones -30° to -35° and -35° to -40°, which have been described in the preceding chapters, were generally concerned with three objectives. They were, in the order described, firstly, the effect of a change of the system of reference on the constants of star-streaming, with special reference to the declination of the solar apex; secondly, the variations in the constants of star-streaming with spectral class and with magnitude, and thirdly, the comparison of the two methods of determining proper motions. The resultsof these investigations will now be summarised.

The first investigation, in which the proper motions of the Cape Astrographic Zone were reduced to the system of the FK_3, did not accomplish its purpose and the effects of a change to another system could not be evaluated, it being found that the corrections applied to accomplish the reduction were possibly not satisfactory, there being some question whether their method of derivation was correct, the magnitudes of the changes in the constants of star-streaming being so great as to indicate that the proper motions were possibly not on the true FK_3 system after the reduction.

The first investigation having been unsatisfactory, the proper motions of the faint star volume of the Cape A Astrographic Zone Catalogue were then analysed to determine whether there were any variations in the constants of star-streaming with the differing magnitudes of the two volumes of the Astrographic Zone catalogue and also to ascertain the nature of the variations of these constants with spectral class. But one variation, which appeared significant, of the constants with magnitude was found - namely that the speed of the solar motion was higher for each spectral group for the faint stars. It was also found that each element of the drift motions relative to the sun varied with spectral class. For the Drift I apex the variations were too small to be regarded as established but the other variations were all of such magnitude as to be considered as having been established, especially when taken along with those found by Tannahill from his analyses of the G.C. Not all of the variations were regular, and when the A type stars are included, as was done by Tannahill, some of these irregular variations are emphasised. Two suggestions of the causes of these irregular variations have been made by Tannahill. Firstly it may be that an analysis in which the later spectral groupswere separated into giant and dwarf stars might resolve the discrepancies. Alternatively, it may be that these discrepancies are an indication of the inadequacy of the present theories of the distribution of stellar peculiar velocities.

The purposes of the third investigation, in which the proper motions of the Cape Photographic Zone Catalogues were analysed, were twofold. The first was to attempt to obtain further information about the variations with spectral class, with especial reference to the discrepancies previously noticed. Secondly, the purpose was to compare the two photographic methods of determining proper motions. This investigation was inconclusive, the results of the analyses being anomalous as regards the velocities and axis-ratios determined and the positions found having sudh large probable errors that no variations with spectral class could be established, except for the drift ratios, which confirmed the results from the Astrographic zone and from the G.C. Neither could the origin of the anomalies be established, the requisite material not being yet available. The results of the analyses do, however, indicate that of the two methods of deriving proper motions, the purely photographic method, in which two plates of the same region are superimposed and the changes in position measured, is preferable. In this method the positions do not need to be accurately known, nor the precessions, whereas in the other method, in which photographic plates are used only at the second epoch, both the positions and the precessions need to be known at one epoch.

Although in both cases where the proer motions analysed were on the system of FK3 anomalous results were obtained, it is probable that the reason is that the reductions to FK3 are based on the brightest stars in the catalogues, whereas the stars used in these analyses were all fainter than the seventh magnitude.

CHAPTER V11

ANALYSES BASED ON RADIAL VELOCITIES

In this section of the thesis two investigations based on radial velocities are described. The second of these investigations is still in progress.

Generally speaking, our knowledge of the nature of the systematic motions of the stars derived from radial velocities is scant in comparison to that from proper motions. With regard to star-streaming the total of radial velocity evidence to date can be regarded as being but confirmatory of proper motion results and only slightly as having brought new knowledge. This does not, however imply that radial velocity data is secondary to proper motion data. For certain types of stars and stellar associations the radial velocity data and results are superior to the corresponding proper motion material.

Nevertheless, on comparing the amount of radial velocity material available for analysis to that for proper motions, the paucity is evident. The explanation is neither that the measurement of proper motion antedates that of radial velocity by more than a century nor that radial velocities, to be accurate, must be measured photographically, but that each star must have its velocity measured individually, and, to obtain accurate results, a number of independent determinations are required, whereas proper motions can be determined in mass, merely by taking two plates of the same region with a suitable period of time between them. Another factor is that to photograph stellar spectra, for radial velocity determinations, longer exposures are required than for, say, Astrographic work. It is thus not surprising that, whilst proper motions have now been measured for almost all stars brighter than the tenth apparent magnitude, radial velocity determinations are barely complete to the sixth.

A further consideration is that radial velocity determinations are frequently a by-product of some other investigation and the known velocities are thus weighted towards stars of other astrophysical interest, such as the O and B type stars, binary stars (especially spectroscopic binaries), the late type stars, etcetcetc.

The limited amount of material available for analysis is balanced to some extent by the fact that the actual velocity along the line of sight is measured, in kilometres per second, whereas to convert proper motions to velocities the parallaxes need to be known. This can be of particular value in such matters as the determination of the solar motion, the constants of galactic rotation and the dispersions of velocity along the axes of the velocity ellipsoid. Of great importance is the fact that radial velocities are largely independent of the distances of the stars. The radial velocities of the distant O and B stars can thus be measured as accurately as those of the near stars of equal apparent magnitude, whereas their proper motions become too small for accurate measurement.

The main feature of analyses of radial velocities is the presence of a constant term - the 'K' term. This was discovered by Campbell (7) in 1911 , and has been confirmed by subsequent analyses. Its origin is uncertain, but, as it only becomes appreciable for the O and B stars, it has been associated with the gravitational red shift.

Whether or not the gravitational red shift accounts for the observed K term can not yet be said to be decided, especially in view of the uncertainties in the masses of the O and B stars, some of which may be in need of revision. For the late type stars, however, it semms probable that the K terms are of accidental origin.

Generally speaking, radial velicity analyses have mostly been concerned with the determination of the solar motion and, for the distant early type stars, galactic rotation. Only restricted use has been made of radial velocities in determining the constants of star-streaming, as they are not as amenable to analyses as proper motions. The results of such analyses as have been made, have been, however, confirmation of proper motion results. There is also some slight evidence that the observed distribution can be better represented on the ellipsoidal theory than on the two-drift's theory. A defect of most methods of analyses has been that it was generally necessary first to remove from the observed velocities the parellactic component and then to analyse the residual velocities.

A convenient method by which the observed radial velocities could be analysed on the ellipsoidal hypothesis was was derived in 1937 by Smart and Chandrasekhar (1) and does not require the removal of the parallactic component before analysis. It does, however, still require a knowledge of the solar motion appropriate to the stars treated. The method has been applied by Smart (2) to the stars of Schlesinger's "Catalogue of Bright Stars, 1930" (3), and the results agreed well with Nordström's (4) results from his analyses of radial velocities.

In Chapter V111, the radial velocities of a sample group of fainter stars, measured at Lick Observatory, are analysed both for the determination of the solar motion and of the constants of the velocity ellipsoid. The aim of this investigation was to ascertain any points worthy of more detailed analysis when more extensive data became available for faint stars, with the then forthcoming Mount Wilson catalogue in mind.

In Chapter 1X, an analysis of the radial velocities of some 1,600 O and B type stars between galactic latitudes ± 20° is outlined, it being still in progress. Attention has been directed to the stars, since:

(a) their great distances improved the determination of effects depending on distance, and

(b) their absolute magnitudes are known with reasonable accuracy, and their distances can be determined fairly well, although the effects of interstellar absorption cannot be fully taken into account. However, the amount of the absorption can be estimated for their stars.

Other advantages are that (a) the question of giant and dwarf classification only enter slightly, (b) the

O and B stars in the catalogue are more likely to be near completion to some limit than the later stars, much attention having been paid to them observationally, because of their importance in numerous astrophysical and cosmological problems, and (c) interstellar velocities are frequently available from their specra and can then be included in the analysis. A drawback, however is that they often form associations and moving clusters, which cannot always be recognised as such from the catalogue data.

The extent of Chapter V111 was communicated to the Royal Astronomical Society and published in the Monthly Notices thereof (5) A copy of the paper is included in the appendix.

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CHAPTER VIII

ANALYSES BASED ON THE LICK RADIAL VELOCITIES OF 820 FAINT STARS.

1\.

Because of the restricted nature of the material available, investigations based on radial velocity analyses have necessarily been confined to the brighter stars - usually those brighter than the sixth magnitude. It was with this in mind that Moore and Paddock decided, at the end of a programme of determinations of the radial velocities of all stars brighter than visual magnitude 5.51, to extend the observations to include a sample group of fainter stars.

In order not to duplicate determinations of radial velocities being made at other observatories, it was decided that, for this sample group, the members would be restricted to stars of photographic magnitudes between 8.5 and 8.6 in the H.D. catalogues and of spectral types F to M. The observations were commenced in July 1928 and completed in December 1937. The stars used were selected on the basis of a random distribution of the line of sight, thus giving a fairly uniform distribution over the sky north of declination -21°. For the F, G and Ko - K_4 stars a further restriction was required to give a total number that might be observed in a reasonable time. For these stars, only those lying in the even zones of declination were selected. In all, 820 faint stars were observed.

For the determinations of the velocities single prism spectrographs were used, the dispersion being 75 A per mm. Afurther 260 bright stars whose radial velocities had previously been determined with the three prism Mills spectrograph were also included in the single prism observations. These atars were used to reduce the single prism determinations to the Lick three prism system.

It was found to be generally sufficient to take two plates for each star, although originally three per star were taken. The plates were measured by Paddock and Miller, the average probable error of one observation being found to be, for unit weight , 3.6 km. per sec. The catalogue of their positions, magnitudes, radial velocities, spectral classes and luminosity classifications - from the " Atlas of Stellar Spectra " by Morgan, Keenan and Kellman (1) - was prepared for publication (2) by N.U.Mayall after the retirement of Dr. Paddock and the death of Dr. Moore.

3.

Besides publishing the catalogue, its compilers also gave details of analyses for the solar motion and for the galactic rotation of the Class III giants.

In the analysis for the solar motion the material was divided into three groups - luminosity classes II to III-IV, IV and V. Five stars with velocities greater than 80 km. per second, 63 of variable or possibly variable velocity, 74 stars of luminosity class V in the spectral ranges Fo - F_4 and G_5 - K_4, 3 stars of luminosity class Ib and 9 of unclassified luminosities were excluded, leaving 666 stars for the analyses. The group of 74 stars of luminosity class V of which 62 were in the spectral group Fo - F_4 and 12 in the G_5 - K_4 group appear to have been omitted because their luminosity classes were uncertain.

The results of the analyses - the method used was not specified - are given in Table I.

TABLE I

Class	Vo (km. sec.)	A_o	D_o	K	No. of Stars
II to III-IV.	24.2 ± 1.8	282°.2 ± 5°.2	46°.7 ± 2°.3	1.5 ± 1.1	381
IV	23.0 2.3	275°.7 6°.4	43°.6 3°.0	3.0 1.5	139
V (F_5-G_4)	18.7 3.1	285°.0 9°.5	43°.0 5°.0	-2.0 1.7	146

Because of the high probable errors, no variations between the luminosity groups can be established, and the determinations all lie within their probable errors. A mean apex was therefore taken at A_o = 280°, D_o = 44°.5 with, for each group, the velocity found above. This was used in finding the residual velocities. The main feature of the above is the rather high value found for the declination of the solar apex.

After this analysis, an attempt was nade to see whether there was any evidence of galactic rotation effects for the giants - their mean distance being found to be 300 parsecs. For this analysis, 348 stars of luminosity classes between II-III and III-IV were used. The first solution was for the solar motion and for galactic rotation, a zero K term being assumed and the centre of rotation taken at galactic longitude 325°. The results were:-

$V_0 = 24.7 \pm 3.3$ km. per sec. , $A_0 = 281^{\circ}.9 \pm 1^{\circ}.5$
$(24.2 \quad 1.8)$ $(282^{\circ}.2 \quad 5^{\circ}.2)$
$D_0 = +41^{\circ}.3 \pm 0^{\circ}.9$ and $\bar{r}A = -4.2 \pm 1.9$ km. per sec.
$(+46^{\circ}.7 \quad 2^{\circ}.3)$

The solar motion is thus almost exactly the same as for the first group in the previous analysis - its results being given in brackets above. A second solution was now made using the ~~radial~~ Residual velocities sfter removing a solar motion of 24.2 km. per sec. towards the mean apex derived from the first analysis. In this analysis only atars between galactic latitudes -20° and $+20^{\circ}$ were used - in all 147 stars. The rotational constant was derived, as was the longitude of the centre of rotation. The results were — $\bar{r}A = 6.7 \pm 1.5$ km.per sec. and $L_0 = 309^{\circ}.2 \pm 7^{\circ}.0$ which value for $\bar{r}A$ agrees with the usual value of A(17 to 20km.sec.) the distance being 300 parsecs.

3. There are two main features of interest in the above results. They are that the values for the solar motion show significant increases over Smart and Green's (3) results for the stars of Schlesinger's "Catalogue of Bright Stars"(1930). The limiting magnitude of this latter catalogue was about $6^{m}.5$. For the solar speed Smart and Green found, for their A to M group, a value of 18.20 km. per sec. whereas the Lick result is about 23 km. per sec. The Lick results also indicate an increase of some 12° in the declination of the apex and about 10° in its R.A. This may well be a magnitudeeffect, or else it may arise solely because so few stars were used in the Lick determination. It was therefore decided that an analysis of the Lick material would be made, with, as its main object, the determination of the constants of the velocity ellipsoid - to be able to compare them with those from the faint star volume of the Cape Astrographic Zone Catalogue. Also, it was decided to make a new solution for the solar motion by the method used by Smart and Green. The two sets of results for the solar motion cannot really be compared unless they have both been made by the same method, and the method used by the Lick investigators is unspecified.

Because there were so few stars available for analysis, it was decided that the material would not be divided into groups as was the case in the Lick analyses, since, in the statistical methods of analysis to be used, it is necessary that the distribution of the stars ov er the regions be uniform. This is the case for all the stars which were selected to give a uniform distribution over the area of the sky covered. A division of the material into groups would destroy this uniformity. Further, the division of the material into the groups used in the Lick analyses would mean that, when the sky was divided into regions for analysis, there would be too few stars in many of the regions to give accurate means for the velocities.

For these reasons all 820 stars were used in the analyses. The primary intention was the determination of the constants of the velocity ellipsoid and the analysis for the solar motion was repeated by the method used by Smart and Green, firstly, because the Lick results did not apply to all 820 stars, and secondly, so that the results of the analysis might be exactly comparable with those of Smart (4) for the stars of Schlesinger's catalogue. Although, because of the limited number of stars available, no great significance could be attached to the results, it was hoped that the results derived might indicate features worthy of more detailed analysis, when the Mount Wilson catalogue of radial velocities was published, to which analyses, this investigation was intended to serve as a pilot investigation.

4.

The first stage of the investigation* was the determination of the elements of the solar motion for the 820 stars. No stars were omitted.

The method used was that given by Smart (5). This is a modification of the method used by Campbell, and takes into consideration the systematic corrections required by the use of large regions in the analyses and areas of the sky as large as 1000 square degrees can be used, thus reducing the number of equations of condition from several hundred to two score or so.

First of all, the sky was divided into 34 regions, according to galactic latitude, as follows.

Region	Limits of Gal. Latl.	Limits of Gal. Long.
A	+60° to +90°	90° to 270°
s	+60 to +90	270 to 90
B	+30 to +60	90 to 150
C	+30 to +60	150 to 210
D	+30 to +60	210 to 270
p	+30 to +60	270 to 330
q	+30 to +60	330 to 30
r	+30 to +60	30 to 90
E	+10 to +30	90 to 150
F	+10 to +30	150 to 210
G	+10 to +30	210 to 270
l	+10 to +30	270 to 330
m	+10 to +30	330 to 30
n	+10 to +30	30 to 90

* The original published results were found to be erroneous. A new solution was made. Details of both are given.

H	-10° to $+10^\circ$	90° to 150°
J	-10 to +10	150 to 210
K	-10 to +10	210 to 270
h	-10 to +10	270 to 330
j	-10 to +10	330 to 30
k	-10 to +10	30 to 90
		90
L	-30 to -10	90 to 150
M	-30 to -10	150 to 210
N	-30 to -10	210 to 270
e	-30 to -10	270 to 330
f	-30 to -10	330 to 30
g	-30 to -10	30 to 90
P	-60 to -30	90 to 150
Q	-60 to -30	150 to 210
R	-60 to -30	210 to 270
b	-60 to -30	270 to 330
c	-60 to -30	330 to 30
d	-60 to -30	30 to 90
S	-90 to -60	90 to 270
a	-90 to -60	270 to 90

For each region the algebraic sum of the catalogue radial velocities was formed and then antipodal areas were combined. The values of $\sum R$ and the numbers of stars for each region are given in Table II. From theses the equations of condition for each region were formed and solved by the method of least squares for the elements of the solar motion and for the K term. In this solution, the corrections C_z, for the sizes of the regions, were omitted. The basic equation of condition used was:-

$$l.X + m.Y + n.Z + K = \frac{1}{N}.\sum R \qquad \text{................(1)}$$

TABLE 11

Region	$\sum R$	N
a	3	11
A	41	26
b	0	0
B	-585	40
c	-198	15
C	+121	39
d	-190	35
D	+155	29
e	0	0
E	-168	30
f	-223	22
F	+624	36
g	-457	37
G	+227	11

Region	$\sum R$	N
s	-286	32
S	+ 90	11
p	+ 55	32
P	+62	29
q	-612	27
Q	+491	22
r	-345	28
R	0	0
l	+ 57	8
L	-257	38
m	-682	34
M	+276	26
n	-495	35
N	0	0

Region	$\sum R$	N
h	0	0
H	+ 58	21
j	-302	44
J	+1086	56
k	-974	46
K	0	0

The approximate solutions of the normal equations were:-

$V_0 = \cancel{19.38}$ 21.05 km.per sec., $G_0 = \cancel{36^\circ.1}$ 41°, $g_0 = \cancel{21^\circ.5}$ 22° with a

K term of ~~-0.18~~ +1.0 km.per sec.

From the values thus derived, the values of the correction C_z were now calculated from its literal expression:-

$$C_z = V_0 . \sin g_0 . \sin g \, (1 - \cos\theta)$$
$$+ V_0 . \cos g_0 . \cos g . \cos(G_0 - G) \left\{ 1 - \frac{\sin\phi}{\phi} \left(\frac{2\theta + \sin 2\theta \cos 2g}{4 \sin\theta \cos^2 g} \right) \right\}$$

, ...(2)

where:- ϕ is half the width of the region in galactic longitude and θ the half-width in galactic latitude. The values calculated for C_z are given in Table III.

TABLE III

a	+1.1844	g	-0.7968	n	-0.8788
b	+0.2794	h	+0.1847	p	-0.1006
c	-0.1635	j	-0.7306	q	-0.5435
d	-0.2529	k	-0.9153	r	-0.6329
e	+0.2101	l	+0.1280	s	+0.6656
f	-0.6277	m	-0.7097		

These corrections were now inserted in the normal equations, the equation of condition for a single region now being:-

$$l.X + m.Y + n.Z = \frac{1}{N}.\sum R + C_z \qquad ....(3)$$

The revised equations were now solved, the solutions were:-

$V_0 = 22.31 \pm 1.20$ km./sec. ; $A = 280^{\circ}.0 \pm 5^{\circ}.8$; $D = +47^{\circ}.9 \pm 2^{\circ}.8$
$K = +1.17 \quad 0.79$ km./sec. ; $G = 44^{\circ}.3 \quad 3^{\circ}.5$; $g = +20^{\circ}.4 \quad 3^{\circ}.1$

For comparison the mean Lick results were V_0 = 22.44km.per sec., $A_0 = 280^{\circ}$ and $D_0 = 44^{\circ}.5$. The two solutions thus agree well and the possible magnitude variations are still apparent.

5.

For the determination of the constants of the velocity ellipsoid, the same division of the sky was used as in the solar motion analysis. The method used was that developed by Smart and Chandrasekhar (6) and applied by Smart (4) to the radial velocities of Schlesinger's catatogue.

First, the observed radial velocities were corrected by an amount C_z - K from the results of the above analysis. The mean radial speeds for each region were then formed - which we denote by $\bar{R}_o$. The values of $\bar{R}_o$ are given in Table IV along with the numbers of stars per region.

TABLE IV

Region	$\bar{R}_o$	N	Region	$\bar{R}_o$	N
a	15.84	37	k	25.50	46
b	21.50	40	l	18.89	46
c	19.23	54	m	24.32	60
d	14.29	64	n	20.29	35
e	23.91	30	p	16.10	61
f	23.62	58	q	29.19	49
g	22.44	48	r	19.21	28
h	21.71	21	s	13.36	43
j	22.59	100			

Next, from the elements of the solar motion derived above, the quantities $V_o \cos\lambda$ were calculated, V_o being the solar velocity and λ the angular distance from the centre of the region to the solar apex or antapex, whichever is less than 90°. These values were then used in conjunction with those of $\bar{R}_o$ to find the values of a quantity t, where $t = \dfrac{\pi^{\frac{1}{2}}\bar{R}_o}{V_o \cos\lambda}$.

The method now is to solve the equation

$$t = \frac{1}{\xi} . F(\xi) \qquad \text{.................(1)}$$

where
$$F(\xi) = e^{-\xi^2} + 2\xi \int_0^{\xi} e^{-x^2} dx \qquad \text{..............(2)}$$

for ξ. The values of t corresponding to various values of ξ have been tabulated by Smart(4). The method of solution of this equation depends on the value of t. For large values of t, that is t greater than 5, the equation can be written to a sufficient degree of accuracy:-

$$t = \frac{1}{\xi} + \xi \qquad \text{.........(3)}$$

Hence, by obtaining the approximate value of ξ for the value of t, from Smart's table, which will be denoted by ξ_o, we have -

$$\xi = \frac{1}{t - \xi_o} \qquad \text{........(4)}$$

For small values of t the equation must be solved graphically.

Having thus obtained the value of ξ for each region, the quantity C can then be calculated, C being given by

$$C = \left(\frac{V_o \cos\lambda}{\xi}\right)^2 \qquad \text{.........(5)}$$

We thus obtain for each region an equation of condition of the form -

$$P + Q \cos 2G + R \sin 2G = C \qquad \text{......(6)}$$

where
$$P = \frac{1}{H^2} + \frac{1}{2}\left(\frac{1}{K^2} - \frac{1}{H^2}\right)\cos^2 g \qquad \text{......(7)}$$

The method now is to solve an equation

$$t = \xi F($$

$$Q = \frac{1}{2}\left(\frac{1}{K^2} - \frac{1}{H^2}\right)\cos^2 g \cos 2G_0 \qquad \ldots\ldots\ldots\ldots(8)$$

$$R = \frac{1}{2}\left(\frac{1}{K^2} - \frac{1}{H^2}\right)\cos^2 g \sin 2G_0 \qquad \ldots\ldots\ldots\ldots(9)$$

It is assumed in this work that the major axis of the velocity ellipsoid lies in the galactic plane., at longitude G_0. The values of t, ξ and C for each region are given in Table V.

TABLE V

Region.	t	ξ	C	Region.	t	ξ	C
a	14.80	0.0679	780.130	k	2.25	0.594	1148.003
b	4.20	0.253	1287.762	l	15.27	0.0658	1110.976
c	6.04	0.1702	1098.134	m	2.58	0.472	1254.704
d	2.69	0.443	451.835	n	1.67	-----	--------
e	5.64	0.1832	1681.180	p	18.70	0.0536	810.552
f	3.67	0.297	1473.500	q	3.19	0.352	2129.906
g	2.45	0.512	1007.688	r	1.71	-----	--------
h	7.45	0.1367	1426.928	s	1.88	0.9625	171.930
j	2.68	0.446	1125.435				

g	2.84	0.4118	1113	r	1.84	1.0630	278
h	20.05	0.0500	1505	s	1.88	0.9500	171
j	2.63	0.4580	1126				

A number of solutions by galactic zones were made as well as for the whole sky. The results are given in Table VI. For comparison the results obtained by Smart(4) in his original application of this method are given in brackets. The results from Smart's analysis here used are those for stars of spectral types A to M. *The solution for ALL stars was performed by iteration from the means of 1/K, 1/H and Go (weighted mean) for the three zones.*

TABLE VI

ZONE	No. of Satrs	$\frac{1}{K}$	$\frac{1}{hH}$	$\frac{K}{H}$	G_o
0^o	167 (619)	37.78 (33.70)	32.24 (20.90)	0.853 (0.619)	$298^o.1$ ($340^o.5$)
20^o	277 (981)	39.63±1.78 (30.10)	31.73±2.81 (26.40)	0.801±0.159 (0.879)	$327^o.9$±$17^o.1$ ($332^o.2$)
45^o	296 (1068)	53.63 5.89 (37.20)	19.17 7.48 (22.10)	0.357 0.165 (0.594)	$344^o.6$ $11^o.1$ ($338^o.8$)
ALL	820 (2668)	40.98 4.61 (33.70)	27.74 2.32 (23.10)	0.677 0.117 (0.697)	$331^o.4$ $6^o.4$ ($339^o.5$ $1^o.4$)

6\.

The results of the analyses can now be considered. Firstly the solar motion. It is seen that the Lick results, derived in some other way than that here employed, are fully confirmed as regards the greater values of the solar velocity and the declination of the solar apex. There is, however, no definite indication of any great change in the R.A. of the solar apex. Similarly, although the change in the declination of hte solar apex appears well, its probable error is shown by the Lick results to be of the order of $7^o.5$, or so. The change in the solar velocity is, however, supported to some extent by the results for the solar motion derived from the faint stars in the Cape Astrographic Zone.

For the velocity ellipsoid there appears to be evidence of a magnitude effect, but as the amount of the original data was so small, it cannot be taken as definite. There are, however, a number of features which suggest the need of further examination when the Mount Wilson material is analysed.

Compared to the brighter stars analysed by Smart, the main feature is the greater values obtained for $1/K$ and $1/H$ - that is, the velocity dispersions along the axes of the velocity ellipsoid are greater for the faint stars than for the bright stars. This is to be expected from the lesser amount of material available and from the greater uncertainties in the solar motion. Further, the solar motion from the whole sky was used and not the value for each latitude zone. For the 45° zone, a marked change in the axis ratio is found, but the probable errors are so great that it becomes of little, if any, significance. Similarly, the variations in the longitude of the major axis cannot be taken as significant.

The only result to which any weight can be atteached is that for ALL stars - the iterative solution. Here, the axis-ratio agrees well with Smart's value - which, however, is the mean of his zone results - and again the dispersions are greater. For the longitude of the major axis, especially as regards the deviation from the centre of the galaxy, the faint star result is quite inconclusive, being $6^{\circ}.4 \pm 6^{\circ}.4$, the centre being taken at 325°.

Thus as with the proper motion results, the only definite evidence of a magnitude effect is in the solar velocity. This result for the radial velocities is perhaps preferable to that from the proper motions, since the solar motion elements for the latter depend, to some extent, on the position assumed for the apex in reducing them from relative to absolute motions.

The original results of these analyses were communicated to the Royal Astronomical Society and published in the Monthly Notices thereof, (7). The revised results given in this chapter have also beem communicated.

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CHAPTER IX

ANALYSES OF THE MOUNT WILSON "GENERAL CATALOGUE OF RADIAL VELOCITIES"

1.

As stated in the preceding chapter, the analyses of the Lick radial velocities were of the nature of a pilot investigation, for the purpose of indicating features requiring more detailed investigation when the necessary material became available.

In the autumn of 1954, a new catalogue of radial velocities was published. This catalogue, the "General Catalogue of Radial Velocities", was compiled by R.E.Wilson (1) and takes the place of Moore's catalogue (2). In all it refers to 15 106 stars - and nebulae and also lists the emission line and interstellar velocities. North of declination -30°, the catalogue affords a good statistical sample, but south of that declination the material is limited by the small number of observations until the new reflectors at Cordoba, Pretoria and Mount Stromlo provide the necessary observations.

Besides listing the magnitudes, Boss proper motions when available, and spectra the luminosity classifications are given, when possible, for the stars of spectral type later than A5 . There is no limiting magnitude and the catalogue cannot be regarded as complete atall . What may be the major defect is that for the fainter stars, radial velocities programmes at different observatories are governed frequently by other considerations and the stars examined are often members of a group being studied for some other feature of astrophysical interest. In a paper describing the catalogue, (3), Wilson gives details of the distribution of the stars in the catalogue by spectral class. It appears that the numbers of O and B type stars, of F type, of G type and of type M are about equal, the F types having the fewest members, whilst the Atypes are represented by an extra 5 per cent. The greatest number of stars is for the K types, there being about twice as many of these as of the F types. There also seemes to be a deficiency of dwarf K and M type stars. Thus when the fainter stars are analysed, there will probably enter an unknown selection factor.

A valuable feature of the catalogue is that each radial velocity is assigned a quality classification based on (a) the number of observations, (b) the dispesion of the spectrograph used and (c) on the interagreement of the various determinations. Five classes are used, the fifth referring to stars whose radial velocities are of very low weight and which have been includede only because they have been published. However, about 87 per cent are of value for statistical investigations although lower weights must be attached to 31 per cent of these.

2. The first investigation based on the radial velocities of this catalogue, and which is still in progress, is an analysis of the velocities of the O and the Bo - B_7 type stars. Tt is intended to derive (a) the solar motion with reference to these stars (b) the constants of galactic rotation and (c) the constants of the velocity ellipsoid. In these analyses the interstellar velocities will also be included and solutions will be made (a) for the O and B stars alone, (b) for the interstellar velocities alone and (c) for thw combined material. In these analyses it is intended to use the method outlined by Milne (4) where terms are included for the possible expansion of the galaxy,with a K term depending on the distance. Solutions ~~will~~ may also be made, probably, on the normal 'planetary type' rotation theory. For the analysis on Milne's theory,it will be necessary to conduct a preliminary analysis to determine the 'local solar motion' as this must be removed from the velocities before the analysis. This will have to be determnied from the nearest stars. The equation of condition will then be of the form:-

$$\rho = K_1 + K_2\bar{r}\cos^2 g + \bar{r}.\cos^2 g\sin 2(G - G_o)\,.\,P.\cos 2\lambda_o$$
$$- \bar{r}.\cos^2 g.\cos 2(G - G_o).\,P.\sin 2\lambda_o \quad .....(1)$$

Here λ_o is a possible deviation of the centre of rotation from the centre of the galaxy G_o which is derived by other methods, K_2 is the expansion factor and K_1 a possible systematic effect of accidental origin in the catalogue radial velocities and will include the gravitational red shift for the early type s stars.

In these equations there enters the factor $\bar{r}$ - the mean distance of the group of stars used in the solution. To derive the value of the rotational constant, it is necessary that the mean distance be known. To derive this, two factors must be known - the absolute magnitudes of the stars concerned and also the amount of the interstellar absorption. For a single star neither of these factors are known, but for a group of stars they can be derived statistically with a fair degree of accuracy. There are two ways in which the effects of interstellar absorption can be taken into account. From a study of the radial velocities of 156 Cepheid variables, Joy (5) concludes that a mean absorption of 0.85^{mag}. per parsec is present. The other method is based on the use of colour excesses. These have been measured by Stebbins, Huffer and Whitford (6) for 1332 B stars and they find that the absrption is represented by seven times their colour excess. This latter method of deriving the effects of absorbtion has been used by Ali (7) and by Pismis and Prieto (8) in their analyses of the radial velocities of the B stars. This method is that used in the present analyses., adjustments being made for the revised spectral types given in the Mount Wilson catalogue.

The absolute amgnitudes of the B stars have been derived by a number of authors in a variety of ways. In their investigations Ali and Pismis and Prieto used those given by Stebbins, Huffer and Whitford(6) but Pismis and Prieto conclude from their results that these mean absolute amgnitudes are too great. In this investigation the mean absolute magnitudes derived by Wilson(9) have been used. We then have:-

$$\log r = 0.2(m - 7E_1 - M) + 1 \qquad \text{......(2)}$$

where r is the mean distance of the star, E1 is its colour excess and M its mean absolute magnitude.

This method applies to the stars common to the Mount Wilson catalogue and the list of Stebbins, Huffer and Whitford. For the stars not on the list the distances are found by the method used by Ali - the mean colour excess can be plotted against apparent magnitude for a given spectral type and the mean colour excess for each star not on the list thus derived and hence use(2)as above to obtain the mean distance.

For the interstellar lines it will be necessary, after deriving the value of $\bar{r}$A to assume that their distances are half those of the star in whose spectrum they appear. The stars and the interstellar lines will be grouped by distances for these analyses. In all there will be some 1620 stars available for analysis all between galactic latitudes +20° and -20°. The number of interstellar velocities is not yet ascertained.

3 As stated above, Ali's results, and those of Pismis and Prieto, suggest that the absolute magnitudes used by Stebbins, Huffer and Whitford are systematically too great, and, on comparing Wilson's absolute magnitudes with those of Stebbins, Huffer and Whitford, the difference found is of about the amount which would bring Ali's results into full agreement with a value of the rotational constant of 17 to 20 km/sec/kiloparsec. The changes in the spectral types assigned to some stars between Stebbin's list and Wilson's catalogue will, however, have little effect, but the distances derived will probably be affected by a further factor, namely, that it has been suggested by Oort (10), that the C_1 colours used by Stebbins, Huffer and Whitford require a correction of - 0·04 and this has also been found by Gascoigne (11) from his determinations of the relative gradients of 166 southern stars. This would have the effect of decreasing the calculated distances by about 10%. As the distances, however, are not perfectly determined, the change will not have a great effect when it is recalled that for the O- type stars and the emission stars the assigned absolute magnitudes are of low weight and that this is even more the case for the 'c' characteristic stars.

There is one other factor which may have a great effect on the results - that, in using Milne's theory, the motions must be connected for the effects of the local solar motion - with reference to the nearest, and brightest stars. The question thus arises as to the values to be taken for this motion on the value derived from the nearest O and B stars or that from the nearest stars of ALL spectral types - and this latter group is much nearer than the former. This correction may possible be found by forming for each distance group and galactic region it's equation of condition of the form.

$$V_1(G,g) + V_2(r,G,g) = P. \qquad \text{-------- (1)}$$

where $V_1(G,g)$ represents the term(s) due to the solar motion and $V_2(r,G,g)$ those raised by galactic rotation and expansion. The normal procedure would be to solve the equations of conclusion for each distance group separately, the appropriate solar motion for each distance group being found. As on Milne's theory the solar motion component $V_1(G,g)$ must be taken as constant in a given region for all possible distance groupings, the best procedure may thus be to form the equation (1) for each distance group and then to make the solution for all the distance groups and not for each individual distance group. Then the solar motion thus derived can be removed from each distance group and the rotation and expansion effects derived for each group separately.

A normal solution for the solar motion for each distance group is also of interest, as this may yield more in formation on the variations found from the proper motion analyses detached in Chapter IV.

Of special interest also will be the determination of the constants of the velocity ellipsoid since Tannahill (12) has found that the B stars do demonstrate the effects of star-streaming contrary to previous beliefs. It will also be of interest to find whether these constants are effected by correcting each star individually for the effects of galactic rotation and expansion instead of treating each group en masse.

4 INTERSTELLAR LINES.

Finally with regard to the use of the interstellar velocities, one point arises. Although they have the advantage that they are not affected by the stars, from the spectra of which they are determined, being possible cluster members, their distances can not be directly determined. Generally they behave as though their origin was at half the distance of the "parent" star, but this depends on the distribution of the interstellar material, and the light from the star may have penetrated anything from one uniform spread of the material to a number of individual clouds of material. Although, in using these velocities, it is usually assumed that yheir origin is at half the distance of the star concerned, which is probably true for the whole number of stars, it will be best to analyse these velocities as the stellar velocities, using the undiminished stellar distances. The from a companion of the "stellar" and the 'interstellar' values of the rotational and expansion coefficients, the average distances of their origin can be determined and used in the combined solution. The, if the 'stellar' value of a constant is, for example, about twice the 'interstellar' value, then the origin of the interstellar lines can be taken as half that of the mean distance of the group of stars. If, however, the 'stellar' values is markedly different from twice the 'interstellar' value, it would not be correct to assume the interstellar origin to be at half the stellar distance.

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CHAPTER X

THE RELATION BETWEEN THE STREAM AND THE ELLIPSOID CONSTANTS

1\. One of the features of the analyses of the proper motions of the C.A.Z. catalogues and the C.P.Z. catalogues described in the earlier chapters, is the agreement between the two-drift and the ellipsoid methods of analysis for the direction of the vertex of star-streaming and the solar motion. As the two methods of analysis are entirely different, the agreement is of greater significance, the two-drift analyses having been performed by fitting theoretical curves to the observed distribution curves of the proper motions in position angle whereas the ellipsoidal analyses were performed by an entirely numerical method. This agreement has been found in almost all analyses of proper motions that have been performed by both methods. It thus is to be expected that it should be possible to derive the constants of one distribution from the observed constants of the other. Complete agreement between calculated and observed values of such constants is not to be expected, as the two distributions are not identical, but, as the difference between the two is too small to be detected from analyses of proper motions - as is evident from the fact that no investigations have yet given any indication of any such difference - it is to be anticipated that it should be possible to calculate the values of one set of constants from the other to a good degree of accuracy.

Before the derivation of such a relation is attempted, it is possible to obtain a certain amount of information from the results of analyses of proper motions which will indicate the approximate nature of the relationship. In analyses on the ellipsoidal theory, the object of the investigations is the determination of the axis-ratio $\frac{K}{H}$, K and H being related to the dispersions of the peculiar linear velocity components along and perpendicular to the axis of star-streaming. In two-drift analyses, the first determined quantities are the drift velocities relative to the sun, and then, from these, the relative velocity of the drifts, Ω., this latter being independant of the sun's motion. The relationship will thus be between the axis-ratio and the relative velocity of the drifts, i.e. we will have

$$\frac{K}{H} = F(\Omega) \qquad \text{......(1)}$$

This will apply both to the distribution over the whole sky and to the distribution of velocities in a small region of the sky, with certain minor modifications. Further, for a random distribution of the peculiar velocities the axis-ratio becomes unity and the greater the degree of preferential motion the more the axis-ratio tends to zero. For the two-drift distribution, random distribution of the stellar peculiar linear velocities implies zero relative velocity of the drifts, whilst large relative velocities show a high degree of preferential motion. Thus the expected relationwill be of the form

$$\frac{K}{H} = \frac{1}{f(\Omega)} \qquad \text{.........(2)}$$

where $f(\Omega)$ increases with Ω.

A further point can be obtained from the observation that the value of the solar velocity in theoretical units, as derived by the two methods of analysis from a given proper motion analysis, is the same for both methods of analysis. - within the limits of probable error. Since in the two-drift analysis the solar motion is expressed in terms of $\frac{1}{h}$ and in the ellipsoidal analysis in terms of $\frac{1}{H}$, it appears that, in practice, we have $h = H$.

2.

One relation between the two-drift and the ellipsoidal theories has been derived by Smart (1) and has been applied with reasonable success to the results of a number of two-drift analyses of proper motions (2,3,4). This relation was derived as a result of an investigation of the behaviourof restricted proper motions on the two theories. Smart found that, when the space distribution of the stars is governed by a density law of the form:-

$$N(r) = \frac{A}{r} e^{-h^2k^2r^2} \quad \text{........(1)}$$

where A,h and k are ~~small~~ constants, then on both theories the number of stars in a small region of the sky with proper motions greater than a given amount are identical, provided the ratio of k to μ, where μ is the limiting proper motion, is small. On the basis of this agreement, a relation between the axis-ratio of the local velocity ellipse and the drift velocities for the region and the projected transverse solar velocity is derived. It is also necessary that the stars be equally divided between the two-drifts.

This relation has two disadvantages - namely, that it can only be used to derive the axis-ratio of the velocity ellipse from the drift constants for the region and not vice-versa, and secondly, that to derive the axis-ratio for the whole sky the results for each region must be combined. A method by which this may be accomplished has been described by Smart (2) - basically one of successive approximations to the true values of the unknowns from initially assumed values - but the use of the relation is hampered by the fact that it is, in theory, only applicable when the three conditions apply and the method is cumbersome in application. Further, since the main purpose, in practice, of such a relation would be to compare the results of an analysis by one method with those of an analysis by the other method - perhaps by another investigator - a relation is required that is immediately applicable to the results for the whole sky. It should, also, be of such a form that it can be used either way.

For these reasons , the approach to deriving a relation should be from the distribution functions for the whole sky and not from the distribution for a single region. Further, if a relation is derived from the whole sky, it can be easily modified to apply to a region of the sky, whereas the reverse is not necessarily true.

There are two forms that the ellipsoidal theory can take. The first, due to Schwarzschild, assumes that the distribution is symmetrical about the axis of star-streaming. The second form is a generalisation of this , and permits all axes to be unequal. As in practice it is found that, when the general form is applied, the axes perpendicular to that of star-streaming are practically equal, Schwarzschild's form is treated first. The relation is then extended to cover the case of unequal axes.

3.

In the two-drifts theory, the stars are divided into two assemblies, for each of which the distribution of linear velocities is random. The two drifts are intermingled in space and their centres of rest are in relative motion along the axis of star-streaming, this latter axis defining the direction of the vertex of star-streaming.

From the centre of rest of all stars, the axis of star-streaming appears as an axis of greater mobility than in any other direction. In Schwarzschild's theory this appearance is expressed analytically. The exponent of the distribution function is of the form of the quadratic expression appearing in the equation of an ellipsoid of revolution, and the mean velocity component in the direction of the vertex is greater than in any other direction.

For a single drift the distribution function of the stellar linear peculiar velocity components will be taken as:-

$$f(u,v,w) = \frac{nh^3}{\pi^{3/2}}.e^{-h^2(u^2+v^2+w^2)} \qquad \ldots\ldots\ldots(1)$$

the origin of velocity components u, v and w being the centre of rest of the drift, n the number of stars belonging to the drift and $h\left(=\frac{1}{s\sqrt{2}}\right)$ a constant, s being the velocity dispersion of the stars in the drift. Thus for two drifts, we have

$$f'(u,v,w) = \frac{n_1h_1^3}{\pi^{3/2}}.\ e^{-h_1^2(u_1^2+v_1^2+w_1^2)} + \frac{n_2h_2^3}{\pi^{3/2}}.e^{-h_2^2(u_2^2+v_2^2+w_2^2)}$$

$$\ldots\ldots\ldots\ldots(2)$$

where n_1, u_1, v_1, w_1 and h_1 and n_2, u_2, v_2, w_2 and h_2 refer to Drifts I and II respectively. As the distribution of velocities in each drift is random, the axes may be oriented in any convenient manner. We will take the u_1 and u_2 axes to be along the axis of star-streaming, found, in practice, to be in, or near, the galactic plane. The other axes are perpendicular to these, the w_1 and w_2 axes being approximately in the direction of the galactic pole.

New axes are now taken, (U,V,W), referred to the centre of rest of all stars and parallel to the u_1, v_1 and w_1 axes. Then, if the speeds of Drift I and Drift II, relative to the centre of rest of all stars, are Q_1 and Q_2 respectively, we have:-

$$n_1 Q_1 = n_2 Q_2 \quad \text{........(3)}$$

and

$$\left.\begin{array}{lll} u_1 = U - Q_1 , & v_1 = V , & w_1 = W \\ u_2 = U + Q_2 , & v_2 = V , & w_2 = W \end{array}\right\} \text{....(4)}$$

The distribution function of the two-drift theory then becomes:-

$$\pi^{3/2} F_1(U,V,W) = n_1 h_1^3 e^{-h_1^2(U-Q_1)^2 - h_1^2(V^2+W^2)}$$
$$+ n_2 h_2^3 e^{-h_2^2(U+Q_2)^2 - h_2^2(V^2+W^2)} \quad \text{......(5)}$$

For the ellipsoidal theory the distribution function, referred to the same axes, is:-

$$\pi^{3/2} F_2(U,V,W) = nKH^2 e^{-K^2U^2 - H^2(V^2+W^2)} \quad \text{......(6)}$$

where $n = n_1 + n_2$ and $K < H$.

4.

From the forms of (5) and (6) of the preceding section, it is apparent that the two are not identical. For them to be identical would require that, when the stars are equally divided between the drifts, we have

$$e^{(H^2-K^2)U^2} = \operatorname{Cosh}(2h^2 Q.U) \quad \text{.......(1)}$$

There is, however, one way in which a relation may be derived. Let the number of stars on the two-drifts theory, with $U>0$, be N' ($\equiv N_1' + N_2'$), where N_1' is the number of such stars belonging to Drift I and N_2' is the corresponding number for Drift II, for all values of V and W.

$$\therefore\ N_1' = \frac{n_1 h_1^3}{\pi^{3/2}}\int_0^{\infty} e^{-h_1^2(U-Q_1)^2}\,dU \iint_{-\infty}^{\infty} e^{-h_1^2(V^2+W^2)}\,dV\,dW$$

$$= \frac{n_1}{\sqrt{\pi}}\int_0^{\infty} e^{-h_1^2(U-Q_1)^2}\,dU \quad \text{...........(2)}$$

Let $x = h_1(U-Q_1)$ and $\tau_1 = h_1 Q_1$. ....(3)

Then we have $N_1' = \frac{n_1}{\sqrt{\pi}}\int_{-\tau_1}^{\infty} e^{-x^2}\,dx$ ....(4)

Define $\Theta(\alpha) = \frac{2}{\sqrt{\pi}}\int_0^{\alpha} e^{-x^2}\,dx$ ....(5)

Then, $N_1' = \frac{n_1}{2}\left\{1 + \Theta(\tau_1)\right\}$ ....(6)

and similarly, $N_2' = \frac{n_2}{2}\left\{1 - \Theta(\tau_2)\right\}$ ....(7)

where $\tau_2 = h_2 Q_2$.

$$\therefore \quad N' = \frac{n}{2} + \frac{1}{2}\left\{ n_1\Theta(\tau_1) - n_2\Theta(\tau_2)\right\} \qquad \ldots\ldots(8)$$

By the same methods, if N'' ($\equiv N_1'' + N_2''$) be the number of stars with $U < 0$ for all values of V and W., then

$$N'' = \frac{n}{2} - \frac{1}{2}\left\{ n_1\Theta(\tau_1) - n_2\Theta(\tau_2)\right\} \qquad \ldots\ldots\ldots(9)$$

Put $\quad G(\tau_1, \tau_2) = n_1\Theta(\tau_1) - n_2\Theta(\tau_2).$ $\qquad \ldots\ldots\ldots(10)$

Then,

$$\left.\begin{aligned} N' &= \frac{n}{2} + \frac{1}{2}G(\tau_1, \tau_2) \\ N'' &= \frac{n}{2} - \frac{1}{2}G(\tau_1, \tau_2) \end{aligned}\right\} \qquad \ldots\ldots\ldots(11)$$

On the ellipsoidal theory, we have,

$$N' = N'' = \frac{nK}{\sqrt{\pi}}\int_0^{\infty} e^{-K^2U^2}dU = \frac{n}{2}. \qquad \ldots\ldots\ldots(12)$$

5. If we write U' for the mean speed in the positive direction of the U-axis, then on the two-drifts theory

$$N'U' = \frac{n_1 h_1}{\sqrt{\pi}}\int_0^{\infty} U e^{-h_1^2(U-Q_1)^2}dU + \frac{n_2 h_2}{\sqrt{\pi}}\int_0^{\infty} U e^{-h_2^2(U+Q_2)^2}dU. \qquad \ldots\ldots\ldots(1)$$

Writing τ_1 for h_1Q_1 , τ_2 for h_2Q_2 , x for $h_1(U-Q_1)$ and y for $h_2(U+Q_2)$ ~~for~~ then (1) becomes:-

$$N'U' = \frac{n_1}{h_1\sqrt{\pi}}\int_{-\tau_1}(x+\tau_1)e^{-x^2}dx + \frac{n_2}{h_2\sqrt{\pi}}\int_{\tau_2}(y-\tau_2)e^{-y^2}dy.$$

$$= \frac{n_1}{2h_1\sqrt{\pi}}\left\{e^{-\tau_1^2} + \tau_1\sqrt{\pi}\left(1+\Theta(\tau_1)\right)\right\} + \frac{n_2}{2h_2\sqrt{\pi}}\left\{e^{-\tau_2^2} - \tau_2\sqrt{\pi}\left(1-\Theta(\tau_2)\right)\right\} \ldots\ldots\ldots(2)$$

Rewriting (3) of section 3 in terms of τ_1 and τ_2 we obtain

$$\frac{n_1 \tau_1}{h_1} = \frac{n_2 \tau_2}{h_2} \qquad \text{......(3)}$$

$$\therefore \quad N'U' = \frac{n_1}{2h_1\sqrt{\pi}}\left\{e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right\} + \frac{n_2}{2h_2\sqrt{\pi}}\left\{e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right\}$$

$$\therefore \quad U' = \frac{\frac{n_1}{h_1\sqrt{\pi}}\left\{e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right\} + \frac{n_2}{h_2\sqrt{\pi}}\left\{e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right\}}{n + G(\tau_1, \tau_2)} \qquad \text{....(4)}$$

Similarly, if U'' is the mean speed in the negative direction of the U-axis, then, by the above methods:-

$$U'' = \frac{\frac{n_1}{h_1\sqrt{\pi}}\left\{e^{-\tau_1^2} + \tau_1\sqrt{\pi}\,\Theta(\tau_1)\right\} + \frac{n_2}{h_2\sqrt{\pi}}\left\{e^{-\tau_2^2} + \tau_2\sqrt{\pi}\,\Theta(\tau_2)\right\}}{n - G(\tau_1, \tau_2)} \qquad \text{...(5)}$$

For convenience, we may write (4) as:-

$$U' = \frac{\phi\{n_1, h_1, \tau_1 \,;\, n_2, h_2, \tau_2\}}{\Psi\{n_1, \tau_1, \tau_2\}} = \frac{\phi}{\Psi} \qquad \text{....(6)}$$

(5) then becomes,

$$U'' = \frac{\phi}{\Psi - 2G(\tau_1, \tau_2)} \qquad \text{.....(7)}$$

The expressions for U' and U'' can also be derived on the ellipsoidal thoery. In this case we obtain:-

$$U' = U'' = \frac{1}{K\sqrt{\pi}} \qquad \text{......(8)}$$

6.

Thus from sections 4 and 5 the following facts have been derived - (i) on the two-drifts theory, the number of stars with $U > 0$ can differ from the number with $U < 0$, and (ii) on the two-drifts theory, the mean speed in the positive direction of the U-axis can differ from that in the negative direction. On the ellipsoidal theory these differences do not occur. Thus the conditions for a relationship to exist between the constants of the two distribution are those under which, on the two-drifts theory, N' equals N'' and U' equals U''.

From (11) of section 4 and from (7) and (8) of section 5, it is evident that the condition in each case is that:-

$$G(\tau_1, \tau_2) = 0 \qquad \text{......(1)}$$

or, from the definition of the function G,

$$n_1 \Theta(\tau_1) = n_2 \Theta(\tau_2) \qquad \text{......(2)}$$

If we now write $\alpha = \frac{n_1}{n_2}$ amd $\beta = \frac{h_2}{h_1}$, then, from (3) of section 5, ~~xx~~ (2) becomes:-

$$\alpha \Theta(\tau_1) = \Theta(\alpha\beta\tau_1). \qquad \text{........(3)}$$

Now in all practical applications of the two-drifts theory, it is assumed that $h_1 = h_2 = h$ say, where $1/h$ is usually defined to be the theoretical unit of velocity. Hence we take $\beta = 1$, and putting this in (3) it is apparent that the only solution is that $\alpha = 1$, that is that the numbers of stars belonging to each drift be equal.

Since the numbers of stars belonging to each drift are equal, as well as the velocity dispersions of each drift, it then follows that $\tau_1 = \tau_2 = \tau$, say. We then have,

$$U' = U'' = \frac{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)}{h\sqrt{\pi}} \qquad \text{.....(4)}$$

$$N' = N'' = \frac{n}{2} \qquad \text{.....(5)}$$

Now the numbers of stars which, for all values of U have velocities between V and $V+dV$ and W and $W+dW$ on the two-drifts theory is:-

$$n = \frac{n_1 h_1^2}{\pi} e^{-h_1^2(V^2+W^2)} dV dW + \frac{n_2 h_2^2}{\pi} e^{-h_2^2(V^2+W^2)} dV dW \text{....(6)}$$

But, in order to relate the two-drifts theory to the ellipsoidal theory, it has been found that it must be taken that $n_1 = n_2 = \frac{n}{2}$ and $h_1 = h_2 = h$.

$$\therefore \quad n = \frac{nh^2}{\pi} e^{-h^2(V^2+W^2)} dV dW \qquad \text{......(7)}$$

The corresponding result for the ellipsoidal theory is that

$$n = \frac{nH^2}{\pi} e^{-H^2(V^2+W^2)} dV dW \qquad \text{.......(8)}$$

Hence, for the two theories to give the same distribution of velocities about the U-axis, we must have

$$h = H \qquad \text{......(9)}$$

(4) now becomes:-

$$U' = U'' = \frac{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)}{H\sqrt{\pi}} \quad \text{..........(10)}$$

Hence, as from (8) of section 5, we have, on the ellipsoidal theory, $U' = U'' = \frac{1}{K\sqrt{\pi}}$, then:-

$$\frac{K}{H} = \frac{1}{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)} = M(\tau) \quad \text{...........(11)}$$

There has thus been derived an expression for the ratio of the axes of the velocity ellipsoid in terms of the relative speeds of the drifts. It must be noted that this relation does not imply that an analysis of proper motions om the ellipsoidal theory will yield an axis ratio equal to that given by (11) from a two-drfits analsyis of the same material. The value given by (11) is the axis-ratio of the velocity ellipsoid that will yield the same values for the mean speeds along both directions of the axis of star-streaming and will give the same distribution of velocities perpendicular to this axis. It is instead the axis-ratio of the velocity ellipsoid 'equivalent' to the two-drifts solution.

It is to be noted that the relation (11) follows the expected lines discussed in section 1, and is of the form given by (2) of that section and, also, it is found that, if the two drifts and the ellipsoidal theories are to give the same distribution of the velocities perpendicular to the axis of star-streaming, then the equality of h and H, anticipated from section 1 must hold.

As with Smart's relation, the relation derived above is strictly only applicable when the stars are equally ~~ax~~ divided between the two drifts. The relation is not, however, limited in application, to a small region of the sky, as is Smart's form, but applies to the whole sky. It is easily shown that the relation, in a suitably mofified form, also applies to a region of the sky. In that case, if $hV_1, \Theta_1, n_1; hV_2, \Theta_2, n_2$ are thedrift constants of a region, then the local value of τ, (τ' say), is given by:-

$$2\tau' = \sqrt{(hV_1 \sin\Theta_1 - hV_2 \sin\Theta_2)^2 + (hV_1 \cos\Theta_1 - hV_2 \cos\Theta_2)^2} \quad \text{.........(12)}$$

By using the value derived above in (11), the local value of the axis-ratio - usually written as k/h - can be found. The position angle of the vertex in the region can also be determined from the local drift constants, as if it is Θ_v, then

$$\tan\Theta_v = \frac{hV_1 \sin\Theta_1 - hV_2 \sin\Theta_2}{hV_1 \cos\Theta_1 - hV_2 \cos\Theta_2} \quad \text{......(13)}$$

Thus from (12) and (13) the constants of the velocity ellipse for a region can be calculated from the stream constants of that region, it being assumed that there are equal numbers of stars in each drift. These can then be combined directly to give the vertex direction, without recourse to the method of successive approximations given by Smart.

It might also be noted that if the two-drifts and the ellipsoidal distributions are to have the same axes, then Schwarzschild's form of the ellipsoidal theory must be used. This follows from (7),(8) and (9).

7.

In Table I, below, the values of the function

are given for a range of values of τ, the range covering the values obtaining in most investigations. In Table II the

TABLE I

τ	K/H	τ	K/H	τ	K/H
0.0	1.0000	0.5	0.8064	1.0	0.5372
0.1	0.9902	0.6	0.7463	1.1	0.4964
0.2	0.9618	0.7	0.6880	1.2	0.4602
0.3	0.9178	0.8	0.6331	1.3	0.4280
0.4	0.8652	0.9	0.5827	1.4	0.3994

values of K/H calculated from (11), for a number of two-drifts analyses of proper motions, are compared with the values derived from the analyses of the same material by Schwarzschild's automatic method. The first column in the table gives the source of the figures in each row, the second the spectral group involved. In this column, the letter F represents the group $A_5 - F_5$, G the group $F_8 - G_5$, K the group Ko - M, F-M the group A_5 - M and ALL normally is the group B_8 - M. In the third column is given the value of the ratio of the numbers of stars belonging to each drift - in the sense N_1 to N_2. In the fourth, fifth and sixth columns are given, respectively, the values, together with their probable errors, when available, of the relative speeds of the drifts, the observed values of K/H and the calculated values. For the two-drifts analyses, the values of 2τ are listed, this being the quantity usually derived.

The following sources have been used in compiling this table.

Source	Designation	Investigators: Two-Drift	Investigators: Ellipsoidal
Boss Preliminary Gemeral Catalogue.	P.G.C.	Eddington (5) Jones (6)	Raymond (7)
Boss General Catalogue.	G.C.	Tannahill (8)	Delhaye (9)
Cape Astrographic Zone - Volume I	C.1.	Smart and Tannahill. (3,4)	Jackson (10)
Cape Astrographic Zone - Volume II.	C.2.	Ewart - Chapter IV.	
Cape Photographic Zone Catalogues. (Zomes -30° to -40°)	C.3.	Ewart - Chapter V	

Also listed in the table are the values of K/H for the C.1. group calculated from Smart's relation. It must also be noted that the groupings used in the P.G.C and the G.C. analyses were not the same for the two methods of analysis.

From the figures in the table, it can thus be seen that the values of K/H derived by the two methods agree well, especially when their probable errors are considered. Most noticeable is the agreement even when the values given for N_1 to N_2 depart considerably from the value of unity required by the theory. This is the case for the C.2. group and for the ALL groups. The agreement for the C.2. group thus can be taken as confirming

TABLE II

(1) Source.	(2) S.G	(3) N_1/N_2	(4) 2τ	(5) K/H (obs)	(6) K/H (calc)	Smart's Formula
P.G.C	ALL	1.5	1.868	0.52	0.567	
P.G.C.	ALL	—	1.924 ±0.135	0.48	0.554±0.031	
P.G.C.	ALL		1.962 0.053	0.52	0.546 0.011	
G.C.	F	1.2	2.075 0.039	0.574	0.519 0.008	
G.C.	K	1.0	1.638 0.038	0.67	0.623 0.010	
G.C.	F-M	1.2	1.816 0.030	0.62	0.579 0.007	
C.1	F	1.2	1.873 0.05	0.552	0.565 0.012	0.62
C.1	G	1.0	1.756 0.05	0.56	0.591 0.024	0.63
C.1	K	1.1	1.606 0.06	0.65	0.632 0.015	0.67
C.1	ALL	1.4	1.638 0.04	0.625	0.623 0.011	0.70
C.2	ALL	1.7	1.648 0.04	0.602±0.030	0.621 0.009	
C.2	F	1.8	1.639 0.04	0.676 0.05	0.622 0.012	
C.2	G	1.5	1.697 0.08	0.554 0.04	0.608 0.020	
C.2	K	1.4	1.601 0.08	0.606 0.03	0.633 0.020	
C.2	F-M	1.5	1.618 0.10	0.601 0.03	0.629 0.030	
C.3	F	1.2	1.195 0.15	0.775 0.030	0.748 0.045	
C.3	G	1.0	1.040 0.12	0.751 0.015	0.788 0.034	
C.3	K	1.0	0.820 0.11	0.898 0.021	0.859 0.031	
C.3	F-M	1.0	0.985 0.12	0.825 0.024	0.811 0.035	

the view expressed in Cahpter IV that these high ~~xxx~~ values are probably spurious and are caused by the uncertainties of the analysis - especially for Drift II.

The belief that the stars are really equally divided between the two drifts thus receives some support from the above, as does the assumption that the velocity dispersions of the two drifts are equal. Since fair agreement is obtained in Table II even when the ratio is high, as for the ALL groups, these high values may be affected by considerable errors. This cannot be decided as the probable errors of the ratio are unknown from the "Trial and Error" method of the two-drifts analysis. For the A type stars, Tannahill found a value of 1.54 (11) but for the B type stars the value was 1.0 -(12) - but this value is uncertain.

As stated above, two-drifts solutions for the vertex assume that the velocity dispersions of the drifts are equal, and $\frac{1}{h}$ is defined as the theoretical unit of velocity. This is essential for analyses by the "Trial and Error" method, which does not give solutions for the actual value of h. The elements of the relative motion of the drifts, ξ, η and ζ, are given by $\xi = X_1 - X_2$, $\eta = Y_1 - Y_2$ and $\zeta = Z_1 - Z_2$. No information being derivable from the analysis as to the true value of the ratio β, in order to derive the values of ξ, η and ζ β must be taken as unity. This is also the case for the solar motion. A more general method of analysis, which would probably rquire the use of the magnitudes of the motions, and not the directions, might permit the evaluation of the true value of β, which might be such that the condition (3) of section 6 still holds. The two distributions would still then give the two main features to be identical, i.e. $N' = N''$ and $U' = U''$, but the two-drifts distribution would no longer be symmetrical about the U-axis. However, as the two-drifts determinations of the solar motion - derived on the assumption that $\beta=1$ - and the ellipsoidal values agree so well, it appears that the true value of β, if not unity, departs only slightly from unity, and hence, if a solution was made that gave the actual value of β, then the solar motion- which woild then be found in terms of h_1 - most probably, Drift I being the main velocity component - would probably still agree with the ellipsoidal value and it could be taken that $h_1 = H$, and the axes-ratio of the velocity ellipsoid could then be derived from a suitable modification of (11) of section 6.

8. The analyses given in the precedeing sections was based on the assumption that the direction of the major axis of the velocity ellipsoid was identical with the direction of relative motion of the two drifts. The results of the application of the relation derived on this basis indicate that the 'equivalent' ellipsoid differs little from the observed ellipsoid.

In this section the analysis in the precedeing sections is extended to cover th case when the two axes are not coincident. As before, the forms of the distribution functions that will be used are those referred to axes with origin at the centre of rest of all stars. The axes will again be taken to be those of the velocity ellipsoid.

The ellipsoidal distribution now becomes:-

$$\pi^{3/2} F_1(U,V,W) = nKHJ\, e^{-K^2U^2 - H^2V^2 - J^2W^2}, \quad \ldots\ldots\ldots(1)$$

and the two-drifts:-

$$\pi^{3/2} F_2(U,V,W) = n_1 h_1^3 e^{-h_1^2\{(U-Q_1)^2+(V-R_1)^2+(W-S_1)^2\}} + n_2 h_2^3 e^{-h_2^2\{(U+Q_2)^2+(V+R_2)^2+(W+S_2^2)\}} \quad \ldots\ldots\ldots(2)$$

Then if N' is the number of stars with $U>0$, for all values of V and W, on the two-drifts theory, and N_1 the corresponding number with $U<0$, we have, as before,

$$N' = \frac{n}{2} + \frac{1}{2} G(\tau_1, \tau_2) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

and $$N_1 = \frac{n}{2} - \frac{1}{2} G(\tau_1, \tau_2) \qquad \ldots\ldots\ldots\ldots(4)$$

where, $\tau_1 = h_1 Q_1$, $\tau_2 = h_2 Q_2$ and $G(\tau_1, \tau_2) = n_1 \Theta(\tau_1) - n_2 \Theta(\tau_2)$ ....(5)

Similarly, if N'' be the number of stars with $V>0$, for all values of U and W, N_{11} the corresponding number for $V<0$, N''' the number of stars with $W>0$, for all U and V, and N_{111} the corresponding number with $W<0$, then:-

$$N'' = \frac{n}{2} + \frac{1}{2} G(\rho_1, \rho_2) \qquad \ldots\ldots\ldots\ldots(6)$$

$$N_{11} = \frac{n}{2} - \frac{1}{2} G(\rho_1, \rho_2) \qquad \ldots\ldots\ldots\ldots(7)$$

$$N''' = \frac{n}{2} + \frac{1}{2} G(\lambda_1, \lambda_2) \qquad \ldots\ldots\ldots\ldots(8)$$

$$N_{111} = \frac{n}{2} - \frac{1}{2} G(\lambda_1, \lambda_2) \qquad \ldots\ldots\ldots\ldots(9)$$

where, $\rho_1 = h_1 R_1$, $\rho_2 = h_2 R_2$, $\lambda_1 = h_1 S_1$ and $\lambda_2 = h_2 S_2$ ....(10)

9.

The mean speeds along the directions of the axes are now derived as before. If U' be the mean speed along the positive direction of the U axis, U'' the mean speed along the negative direction of the U axis, and V', V'', W' and W'' be the corresponding quantities for the V and the W axes, respectively, then, in the notation of section 5, we have:-

$$U' = \frac{\phi\{n_1, h_1, \tau_1;\, n_2, h_2, \tau_2\}}{\psi\{n_1, \tau_1, \tau_2\}} \qquad \ldots\ldots\ldots\ldots(1)$$

$$U'' = \frac{\phi\{n_1, h_1, \tau_1;\, n_2, h_2, \tau_2\}}{\psi\{n_1, \tau_1, \tau_2) - 2G(\tau_1, \tau_2)} \qquad \ldots\ldots\ldots\ldots(2)$$

$$V' = \frac{\phi\{n_1, h_1, \rho_1 \,;\, n_2, h_2, \rho_2\}}{\psi(n, \rho_1, \rho_2)} \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

$$V'' = \frac{\phi\{n_1, h_1, \rho_1 \,;\, n_2, h_2, \rho_2\}}{\psi(n_1, \rho_1, \rho_2) - 2G(\rho_1, \rho_2)} \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

$$W' = \frac{\phi\{n_1, h_1, \lambda_1 \,;\, n_2, h_2, \lambda_2\}}{\psi(n_1, \lambda_1, \lambda_2)} \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

$$W'' = \frac{\phi\{n_1, h_1, \lambda_1 \,;\, n_2, h_2, \lambda_2\}}{\psi\{n_1, \lambda_1, \lambda_2\} - 2G(\lambda_1, \lambda_2)} \qquad \ldots\ldots\ldots\ldots\ldots(6)$$

Now, $$\frac{n_1^2\{\tau_1^2 + \lambda_1^2 + \rho_1^2\}}{h_1^2} = \frac{n_2^2\{\tau_2^2 + \lambda_2^2 + \rho_2^2\}}{h_2^2}, \qquad \ldots\ldots\ldots\ldots(7)$$

and, as the drifts are moving in opposite directions,

$$\tau_2 = A\tau_1 \;,\quad \rho_2 = A\rho_1 \;,\quad \lambda_2 = A\lambda_1. \qquad \ldots\ldots\ldots\ldots\ldots(8)$$

Thus, from (7) and (8), puttıng $\alpha = \frac{n_1}{n_2}$ and $\beta = \frac{h_2}{h_1}$,

$$\tau_2 = \alpha\beta\,\tau_1 \;,\quad \rho_2 = \alpha\beta\,\rho_1 \;,\quad \lambda_2 = \alpha\beta\,\lambda_1 \qquad \ldots\ldots\ldots\ldots\ldots(9)$$

Now, on the ellipsoidal theory, we have,

$$U' = U'' = \frac{1}{K\sqrt{\pi}} \qquad \ldots\ldots\ldots\ldots\ldots(10)$$

$$V' = V'' = \frac{1}{H\sqrt{\pi}} \qquad \ldots\ldots\ldots\ldots\ldots(11)$$

$$W' = W'' = \frac{1}{J\sqrt{\pi}} \qquad \ldots\ldots\ldots\ldots\ldots(12)$$

Thus, as was the case for coincident axes, we again have, if the two-drifts and the ellipsoidal theories are to give equal values for the numbers of stars moving along the opposite directions of the axes and are to give the same mean sppeeds along the respective axes,

$$G(\tau_1, \tau_2) = 0 \qquad \text{.............}(13)$$

$$G(\rho_1, \rho_2) = 0 \qquad \text{.............}(14)$$

$$G(\lambda_1, \lambda_2) = 0 \qquad \text{.............}(15)$$

As β has to be taken to be equal to one, the conditions (13) to (15) again reduce to $n_1 = n_2 = \frac{n}{2}$ and $\tau_1 = \tau_2 = \tau$ (say), $\rho_1 = \rho_2 = \rho$ (say) and $\lambda_1 = \lambda_2 = \lambda$ (say). Then:-

$$\frac{1}{K} = \frac{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)}{h} \qquad \text{..............}(16)$$

$$\frac{1}{H} = \frac{e^{-\rho^2} + \rho\sqrt{\pi}\,\Theta(\rho)}{h} \qquad \text{..............}(17)$$

$$\frac{1}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{h} \qquad \text{..............}(18)$$

The ratios of the axes are then, taking the polar axis to be the least ,

$$\frac{K}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{e^{-\tau^2} + \tau\sqrt{\pi}\,\Theta(\tau)} = \frac{M(\tau)}{M(\lambda)} \qquad \text{.............}(19)$$

$$\frac{H}{J} = \frac{e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)}{e^{-\rho^2} + \rho\sqrt{\pi}\,\Theta(\rho)} = \frac{M(\rho)}{M(\lambda)} \qquad \text{.............}(20)$$

It is evident that, by putting $\rho = \lambda = 0$ in the above relations, then (19) reduces to (11) of section 6, and (20-) reduces to $H = J$. Also, $e^{-\lambda^2} + \lambda\sqrt{\pi}\,\Theta(\lambda)$ is greater than one for λ greater than nothing. Thus, if the direction of the vertex differs from one distribution to the other, then, even if $H = J$, the axis-ratio $\frac{K}{H}$ is greater than if the directions were identical. If the differences in direction are - in galactic coordinates - ΔG in longitude and Δg in latitude, then:-

$$\frac{\lambda}{\sqrt{(\tau^2 + \rho^2)}} = \operatorname{Tan} \Delta g \quad \text{and} \quad \frac{\rho}{\tau} = \operatorname{Tan} \Delta G. \qquad \ldots\ldots\ldots(21)$$

i.e. $$\rho = \tau \operatorname{Tan} \Delta G. \qquad \ldots\ldots\ldots(22)$$

and $$\lambda = \tau \operatorname{Sec} \Delta G \operatorname{Tan} \Delta g \qquad \ldots\ldots\ldots(23)$$

The differences in direction rarely exceed 5° in either co-ordinate and are usually of the same order of magnitude as their probable errors. For $\Delta G = 5^{\circ} = \Delta g$, we have, $\rho = 0.09$, and $\lambda = 0.09$, The average value of $\frac{K}{H}$ for the first 15 values in Table II is 0.577 - calculated from the average value of τ which is 0.912. If both co-ordinates differed by 5°, then τ becomes 0.905 - a drop of 0.007 - and $\frac{K}{H}$ becomes 0.584 - an increase of 0,007. The effect is thus negligible. Accordingly, unless there exists a marked difference of direction between the results of the two methods of analysis, the effect on the axis-ratio of the 'equivalent' ellipsoid to the two-drifts solution is insignificant. This is especially so when the probable errors are considered.

10. From the relation here derived, it is evident that it does not supply a criterion which might decide in favour of one method of analysis as being a better representation of the observed distribution of stellar linear velocities as regards the results of the many analyses made so far. If, however, an analysis of proper motions yielded values of α and β significantly different from unity and also gave a good fit

to the observed distribution, then the ellipsoidal theory would probably not give an adequate representation. It can also be noted that, if the peculiar velocity components with reference to the centre of rest of all stars were known, and their frequency distribution analysed as a power series in powers of U, V and W, then, if $\beta \neq 1$, the two-drifts theory would permit the prescence of odd powers of U, at least, and also of V and W, if the vertex directions of the two distributions are not coincident, whereas the ellipsoid would not yield such terms. Their abscence would not , however, imply that the two-drifts theory was inapplicable.

Afurther point is that, whereas Smart's relation does not permit the calculation of the stream constants from those of the velocity ellipsoid, the relation (11) of section 6 does. Suppose (G,g) are the co-ordinates of the vertex in galactic co-ordinates, and let ω be the derived value of 2τ from the relation. Then if ξ, η and ζ are the elements of the relative motion of the drifts, we have:-

$$\xi = \omega \cos G \cos g \qquad \text{........(1)}$$

$$\eta = \omega \sin G \cos g \qquad \text{........(2)}$$

$$\zeta = \omega \sin g \qquad \text{........(3)}$$

Thus ξ, η and ζ are known. Now we have,

$$\xi = X_1 - X_2 \qquad \text{........(4)}$$

$$\eta = Y_1 - Y_2 \qquad \text{........(5)}$$

and $$\zeta = Z_1 - Z_2 \qquad \text{........(6)}$$

where X_1, Y_1, Z_1 and X_2, Y_2 and Z_2 are the galactic elements of the drift motions relative to the sun. Now we also have, in practice, the same solar motion from the two theories, say hU_o towards an apex at (G_o , g_o). Then, as the stars are taken, by the relation, to be equally divided between the drifts, we have:-

$$2\ hU_o \cos G_o \cos g_o = X_1 + X_2 \qquad \text{.........(7)}$$

$$2\ hU_o \sin G_o \cos g_o = Y_1 + Y_2 \qquad \text{.........(8)}$$

$$2\ hU_o \sin g_o = Z_1 + Z_2 \qquad \text{.........(9)}$$

Hence from (4) to (6) and (7) to (9) the values of $X_1 \ldots Z_2$ can be calculated and from these the positions of the drift apices and their velocities relative to the sun.

Most of the content of sections 1 to 9 of this chapter have been prepared for publication in a paper which has been accepted by the Royal Astronomical Society for publication in the Monthly Notices thereof.(13). A script of this paper is included in the appendix.

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CHAPTER XI

SUMMARY OF RESULTS

1. PROPER MOTIONS.

In the analyses of proper motions, the main aim of the investigations was the determination of the nature of the variations with spectral type and magnitude, and the second the comparison of the two methods of determining proper motions through the use of photography.

From the C.A.Z. motions a number of variations with spectral type were found. Regular variations - that is, progressive variations - were found, for the Drift Apices and velocities in the following elements:-

(1) The R.A. of Drift II - decrease from F to K types
(2) The Dec. of Drift II - decrease " "
(3) The Drift I velocity - decrease " "

An irregular variation was found for the Drift II velocity, relative to the sun. In the position of Drift I, no definite variations could be established, but the suggestions are that the R.A. and the Dec. both increase from types F to K.

From the C.P.Z. motions, the results possess too great probable errors to permit any variations to be regarded as significant, but there are indications that the variations in the R.A. and velocity of Drift I and the velocity of Drift II follow the above pattern.

When the results of Tannahill's analyses of the proper motions of the G.C. are cosidered, it is seen that all the variations noted above are reproduced,with the sole exception of the declination variation for Drift I. Some of the variations also lose their progressive nature when the A stars are included, but for the F,G and K types the three sets of results are self confirmative.

For the vertex of star-streaming and the relative velocities of the drifts, variations with spectral class are only evident for the latter element, the relative velocities of the drifts in all three sets of results decreasing from the F type stars to the K stars. The only variation in thr longitude ovf the vertex is that it is higher for the F types than for the later types, which latter yield the same direction. ~~For the solar motion~~

For the solar motion, the variations with spectral class are well marked. From spectral type F to K, the C.A.Z. results demonstrate an increase in the R.A. of the apex and a decrease in the solar velocity relative to the centres of rest of the groups No significant variation in the declination is found, however. For the C.P.Z., the results are too uncertain to admit of any conclusions, but for the G.C. the above variations are fully comfirmed, with the addition of an increase in the declination of the apex with spectral class. Finally , for the stream ratio N_1 to N_2, all the analyses show that it is higher for the F type stars than for the later stars, for which it may well be unity. Since the errors of these values are not known, its variation with spectral class cannot be assessed.

Taking the ellipsoidal results, a number of these variations are repeated. For the C.A.Z. motions a decrease in the longitude of the vertex is apparent with spectral class but no regular variation in the axis-ratio can be established. The C.P.Z. results do, however, indicate the possibility of a variation in the axis-ratio concordant with the two-drifts variation in the relative velocity of the drifts. This is borne out by the G.C. results, as is the variation in the longitude of the vertex, but the spectral groups are not fully comparable.

The elements of the solar motion as derived from the ellipsoidal analyses show, for the C.A.Z. motions, the same variations as did the two-drifts results, as do the C.P.Z. and the G.C. values. The declination variations cannot, however, be established from the results.

For magnitude variations, the results are few. But one variation can be regarded as being established, namely, that the solar velocity is systematically greater for the faint stars in the C.A.Z. catalogues than for the bright stars and for the G.C. stars, which also are brighter than the C.A.Z. faint stars. A slight indication that the longitude of the vertex for the faint stars is systematically less than for the bright stars is apparent from the ellipsoidal results, but it is not confirmed by the two-drifts results.

Although it is clear that the variation in the solar motion for the faint stars compared to the bright stars is a distance effect - the faint stars of the C.A.Z. being possibly as much as twice as distant as the bright stars - it is not possible to say what part of the variations with spectral class consists of a distance effect and what part is a true spectral variation. To determine this, it would be necessary to make an analysis for each group, using only stars in each group of the same mean distance. Then the variations, if any, would be of spectral origin.

Further , before the variations can be interpreted, more detailed analyses would be required, in which the giant and dwarf members of each spectral group were treated separately.

2. RADIAL VELOCITIES.

From the analysis of the radial velocites of 820 stars of H.D. magnitudes 8.5 to 8.6, it is again seen that the solar velocity increases for the fainter stars and that there is possibly an increase in the R.A. and Dec. of the apex. For the velocity ellipsoid, no definite information can be obtained, but there is a moderate probability that the vertex longitude is less than for the brighter stars.

3. TWO-DRIFT AND ELLIPSOID RELATION.

The main conclusion that can be drawn from this investigation is that, despite the high values of the stream ratio, the true value must differ little from unity, or, otherwise, the agreement between the observed and the calculated values of the axis-ratios would not be so good.

4

Although half a century has elapsed since the two theories of the distribution of stellar linear velocities were announced by their originators, they are still in use, unaltered. The theories are both observational - that is, they were chosen in their forms because they would represent the observed distribution to a satisfactory extent. Whether they can be related to the dynamical theory of the galaxy has been attempted, a number of explanations of the two-drifts theory having been advanced shortly after its ennunciation. An example was Turner's.

A possible basis of both theories would be from the fact that the peculiar velocities, to the distribution of which the theories refer, are of the form:-

$$V_p = (V_{ot} - V_{oc}) + (V_{gt} - V_{gc}) \quad ....(1)$$

where, V_p is the peculiar velocity of a star, V_{ot} the velocity of the sun relative to the star, V_{oc} the velocity of the sun relative to the centre of rest of the group of stars of which the star under consideration is a member, V_{gt} the velocity of the star due to differential galactic rotation and V_{gc} the velocity of the centre of rest of that group of stars due to differential galactic rotation. Consider a group of stars. For each star, the peculiar velocity will be given by ah expression of the form of (1). It is possible that for the whole group of stars each of the components $V_{ot} - V_{oc}$ and $V_{gt} - V_{gc}$ will have a mean value differing from zero. Put $V' = V_{ot} - V_{oc}$ and $V'' = V_{gt} - V_{gc}$ and let their mean values be $\bar{V}'$ and $\bar{V}''$. Then from the centre of rest of the V' 's , the V'' 's will be moving in some direction with a velocity $V_{\odot}$. It is also possible that the distributions of the V' 's and of the V'' 's are random about the values of $\bar{V}'$ and $\bar{V}''$ reepectively. If we could then take the number of stars with peculiar velocities between V_p and $V_p + dV_p$ to be given by a function $F(V_p)$ where,

$$F(V_p) = Ae^{-a^2(V' - \bar{V}')^2} + Be^{-b^2(V'' - \bar{V}'')^2} \quad ....(2)$$

then we could identify the two component functions with the two drifts, and by taking axes so that the U-axis was along the direction of relative motion of the component distributions the two-drifts distribution function used in Chapter IX would be obtained. Similarly, the ellipsoidal distribution might be obtainable by taking the distribution of the V_p's to be random and then putting the $V_{\prime}$ and $V_{\prime\prime}$'s in terms of the components U,V,W and so forth. Also, in general, each star would have a value for each of its peculiar velocity components $V_{\prime}$ and $V_{\prime\prime}$ and there would thus be equal numbers of stars in each drift. The variations with spectral type and distance might then be capable of interpretation. This view would also hold if the factor V_{gc} is not allowed for, that is , if it is taken to be zero. Further, for a particular group of stars being examined, either $V_{\prime}$ or $V_{\prime\prime}$ might be always small, or might be zero for a large number of the stars and thus an unequal number of stars might be found for each drift. A single drift, as was thought to obtain for the B stars, could also be obtained.

It is certainly possible that a formal, instead of a tentative postulative, approach on these lines might result in the derivation of a general distribution function of the peculiar velocities which, on making one approximation or another, would yield the two-drift and the ellipsoidal theories, and which would also predict both qualitatively and quantitatively the variations that would occur with the distances of the stars concerned.